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U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

Size of Letters Required for Visibility as a Function of Viewing Distance and Observer Visual Acuity

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ABSTRACT

A formula is derived giving the letter stroke-width needed for legibility of words on a sign at any given distance by an observer with any given visual acuity. The stroke width, in turn, determines the letter size, depending upon the characteristics of the type face used. The derivation is strictly mathematical and is based on the assumption that beyond a distance of a few meters, a person's visual acuity is specifiable by a fixed visual angle, independent of the distance. The information implicit in the formula is also presented graphically, in four plots that apply to four different combinations of length units for measuring stroke width and viewing distance. Also presented are formulas and graphs for correcting the critical stroke width for nonstandard contrast or background luminance. These correction formulas are based on a body of data on visual acuity as a function of contrast and background luminance, and a formula fitting the mid-ranges of the data, both published recently by other researchers.

Keywords: Acuity, visual; angle, visual; contrast; distance, viewing; letters; luminance; resolution, eye; signs; Snellen chart; stroke width; visual acuity; visual angle.

FOREWORD

This report is one of a series documenting the results of NBS research in support of the Occupational Safety and Health Administration (OSHA), in fulfillment of OSHA Interagency Agreement (IAG) contract No. J-9-F-7-0146 entitled "Criteria for Signage in Workplaces." Work on this contract was conducted in the period December 1977 through May 1983.

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PREFACE

The purpose of this report is to provide practical guidance for all those who need to know how large the lettering on signs ought to be. The basic message underlying the entire analysis is that the size of the lettering required is not an absolute for all signs, but depends, for each particular sign, on the maximum distance from which the sign needs to be read, and on the keenness of eyesight (visual acuity) of the people who need to read the sign. The letter size required will also depend on the contrast with which the sign is printed, and the light level with which it is illuminated.

Nontechnical readers who are concerned only with using the formulas, and not with knowing how they are derived, may prefer to confine their attention to section 6, which is a list of step-by-step rules for finding the answer needed. Readers who are interested in checking the validity of the formulas, and who are comfortable with ordinary algebra and a bit of trigonometry, may find the full report of interest.

ALPHABETICAL LIST OF SYMBOLS

Roman Letters

C = contrast (as fraction or decimal)

$\tilde{C}$ = 100 C: contrast in percent

d = viewing distance

g = height-to-strokewidth ratio of a letter

h = height of a letter

k = multiplicative constant in Kaneko's formula, Eq. (22)

L_b = luminance of background

L_t = luminance of "target" (lettering)

p = power to which L_b is raised in Kaneko's formula, Eq. (22)

p_r = $\pi r/216{,}000$: the proportionality constant in the relationship $w_s = p_r\, Sd$, depending on the choice of units for d and w_s (Appendix A)

p_1 = 1.45×10^{-5} [Eq. (15)]: the value of the constant in Eq. (13)

q = power to which $\tilde{C}$ is raised in Kaneko's formula, Eq. (22)

r = U_d/U_w: ratio of unit used for viewing distance to unit used for stroke width (Appendix A)

R = L_b/L_t: luminance ratio, background to target

S = denominator of Snellen index, 20/S

S_s = Snellen denominator corresponding to a standard visual acuity, V_s

U_d = unit of length in which viewing distance is measured (Appendix A)

U_w = unit of length in which stroke width is measured (Appendix A)

V = visual acuity (reciprocal of threshold visual angle in minutes of arc)

V_s = visual acuity of a group or individual measured under standard conditions of contrast (=90%) and luminance (=85 cd/m^2)

w = linear size (length or width) of an object

w_S = stroke width, in length units, needed by someone with 20/S vision in order to recognize letters

Greek Letters

θ = visual angle subtended by an object at the eye

θ_S = stroke width, in angular terms, needed by someone with 20/S vision in order to recognize letters

π = 3.14159...: the ratio of the circumference of a circle to its diameter

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SIZE OF LETTERS REQUIRED FOR VISIBILITY AS A FUNCTION OF VIEWING DISTANCE AND OBSERVER VISUAL ACUITY

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1. INTRODUCTION

In designing a sign containing a word message, a primary objective is to have the sign legible to the people who will see it, at the maximum distance at which it would be useful or necessary for the message information to be conveyed to the viewers. Prescribing a specific size for the lettering on a type of sign to be used in a variety of locations may lead to legibility in typical cases, but it is more universally effective to let the prescribed letter size vary with the maximum distance from which the sign might be viewed in the specific location in which it is installed. Thus, exit-sign letters 6 inches* high, with a stroke width of 1 inch, may be large enough to attract attention and to be legible in a typical office building hallway, but for people working near the center of a huge warehouse floor, such letters may be too small to be read if the signs are located on the outer walls, more than 300 feet from the viewers.

At a distance of about 286 feet, letters with a stroke width of 1 inch will be about as legible as are the letters on the 20/20 ("normal") line on an eye chart at the standard testing distance of 20 feet. Thus, if allowance is to be made for people with worse than 20/20 vision -- and there are many of them -- a stroke width greater than 1 inch would be necessary for legibility at

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* The following exact equations relate customary units used in this report to SI (metric) units: 1 inch = 25.4 millimeters, 1 foot = 12 inches = 0.3048 meter, and 1 yard = 3 feet = 36 inches = 0.9144 meter.

the same 286-foot viewing distance. It is worth noting, by reference to Table 1, that more than a quarter of adults, wearing whatever glasses they usually wear, have worse than 20/20 vision (Roberts, 1964). The purpose of this paper is to permit a sign designer or specifier to select letters large enough to provide legibility for any given viewing distance, for viewers characterized by any given level of visual acuity. In section 5, information will also be provided on how to allow for variations in the two additional factors of contrast between the letters and the sign background, and the luminance of the background.

1.1 VISIBILITY OF SYMBOLS

The significance of the stroke width of a letter -- at least for block capitals -- is that the thickness of the strokes making up a letter represents the finest detail within the letter that it is necessary to see in order to recognize the letter. When the strokes making up a set of capital letters are all of uniform thickness, it is easy to specify the "critical detail": it is simply the width of the strokes. When the typeface used does not have all its strokes of constant width, but contains tapering strokes of varying width, it may no longer be obvious by simple inspection what the critical detail is for each letter.

When symbols, as opposed to words, are considered, the situation is considerably more complex. An occasional symbol is composed of "stick figures" and resembles letters in the sense that all the component strokes are of the same, uniform width. In cases where such a symbol is easily interpreted when visible, the distance of bare visibility would be about the same as for a letter of the same stroke width. On the other hand, if it takes most people a little thought to figure out what the symbol means even when they can see

Table 1. Cumulative Distribution of Visual Acuity Levels in the Adult Population

Proportion of Population 18-79 Years Old Reaching or Exceeding the Test Levels for Binocular Distance Vision: United States, 1960-62 (From Roberts, 1964)

Test Level	Proportion for Distance Vision	
	Un-Corrected	"Corrected"[a]
20/10 or better --	1.1	1.5
20/15 or better --	30.3	40.0
20/20 or better --	53.9	72.9
20/30 or better --	69.3	90.6
20/40 or better --	75.8	95.1
20/50 or better --	80.4	96.8
20/70 or better --	83.9	97.7
20/100 or better --	93.5	99.2
20/200 or better --	97.6	99.6[b]

[a] Uncorrected testing was without glasses. "Corrected" testing was with glasses, if worn to the examination; otherwise, without them.

[b] Not regarded by Roberts as a reliable figure. She believes it is safe to say that no more than 1 percent of adults have "corrected" acuities worse than 20/200. Not all of that group are legally blind; some are correctable to a better acuity but simply didn't have glasses, or did not wear them to the test.

it fairly well, it might be expected that the average recognition distance for such a symbol would be somewhat less than the stroke width would suggest (i.e., they would seem to be harder to see).

Most symbols do not contain strokes of only a single width, but contain a range of details from fine to coarse. In this common situation, it is impossible, in most cases, to predict by inspection what the critical detail of the symbol is. Here, the element of meaning becomes deeply confounded with the visibility aspect. It may be obvious what the finest details of the symbol are, but is it necessary to see those particular details in order to recognize the symbol? Sometimes the finest details are not essential, and the critical detail -- which must be made out for recognition to take place -- may be of somewhat larger size. Attempts to guess the size of the critical detail of a complex symbol are not likely to be reliably correct.

The only recourse is to test each such symbol with a representative group of observers, who might be instructed to approach the symbol slowly from well beyond the visibility range and state the meaning of the symbol as soon as recognition occurs. For each symbol, there is then an associated mean recognition distance. It is possible to calculate, by means to be described in this report, what stroke width would be needed in order for recognition of a letter to occur at this same distance, for observers of any specified visual acuity. Thus, an "equivalent stroke width" -- the effective size of the unknown critical detail -- can be determined from the symbol recognition-distance data. The visual acuity level used in the equivalence would be that characterizing the observers used in the symbol study.

Once this equivalent stroke width is known for a given symbol, it is reasonable to assume -- pending contrary evidence -- that the methods of this report will allow for a prediction of how big the symbol needs to be made in order to be just recognizable at any specified distance by people with any specified visual acuity. In summary, each symbol should be "calibrated" in a recognition-distance study (which automatically takes account of both the meaning factor and the simple visibility factor), and thereafter, the symbol should be treatable as if it were a letter (specifically, a letter having a stroke width equal to the equivalent stroke width determined for the symbol).

The above methodology is subject to experimental verification and is presented as a possible approach for linking the visibility of symbols to the visibility of words. The remainder of this report will be confined to a discussion of letters and words.

2. VISUAL ANGLE

2.1 SIGNIFICANCE OF THE VISUAL ANGLE

The eye, like a camera, forms an image of external objects on its internal back surface. Instead of the flat film that permanently captures the image in a camera, the eye has a network -- called the retina -- of light-sensitive nerve cells that generate a continuously changing electrical pattern. This pattern is interpreted as a visual image by the higher centers of the brain. The retina, following the contours of the eyeball, is spread over a spherical surface. Our ability to make out fine details in an object depends upon the size and sharpness of the image of the object on the retina.

The matter of sharpness relates to the visual acuity of the eye in question. This subject will be covered in section 3 of this report. As for the size of the retinal image, it is a function of the angle subtended by the object at the eye; i.e., the angle formed by the light rays joining the ends of the object to the point inside the eye known as the nodal point or optical center (Duke-Elder and Abrams, 1970, p. 121; Brown, 1965, p. 56). This "visual angle," in turn, is dependent both on the physical size of the object (positive dependence) and the distance of the object from the eye (negative dependence).

2.2 CALCULATING VISUAL ANGLE

The angle subtended by an object at the eye of an observer is easy to calculate by elementary trigonometry. It can be seen in Fig. 1, in the right triangle ACO, for example, that

$$\tan \frac{\theta}{2} = \frac{w/2}{d} .$$

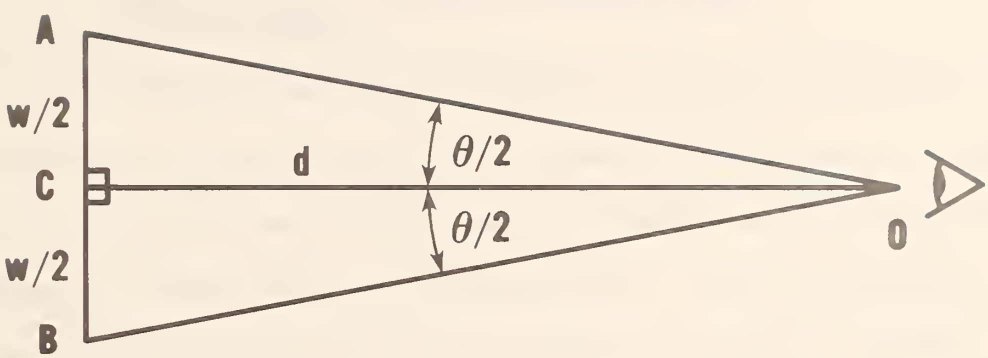

Figure 1. Determination of visual angle. The nodal point (optical center) of the observer's eye is at O. The object, AB, of length or width _w_, has its center at C. The viewing distance OC is _d_, and _θ_ is the visual angle subtended by AB at O.

Accordingly,

$$\theta = 2 \tan^{-1} \frac{w}{2d} . \tag{1}$$

Fortunately, for small angles, the trigonometric expression in Eq. (1) can be closely approximated by a simple algebraic quantity. For small angles, the sine of the angle, the tangent of the angle, and the angle itself (expressed in radians), are all nearly equal. Therefore, when the arctangent of an argument corresponds to a small angle, it is nearly equal to the argument (the tangent), and we can simply eliminate the $\tan^{-1}$ symbol in Eq. (1), leaving

$$\theta = \frac{w}{d} . \tag{2}$$

In words,

$$\text{visual angle (radians)} = \frac{\text{size of object}}{\text{viewing distance}} , \tag{3}$$

where the object's size and viewing distance are measured in the same units of length. The approximate formula given by Eq. (2) or Eq. (3) is correct to four decimal places, relative to the exact value given by Eq. (1), for angles up to 0.0843 rad (4°50'), and is correct to three places for angles up to 0.182 rad (10°26'). Since we are concerned in this report with the limit of resolution of the eye, the angles of interest are of the order of a few minutes of arc. For such tiny angles, Eq. (3) is more than sufficiently accurate for any practical purpose.

In order to express Eq. (3) directly in minutes of arc, we need to know the number of minutes in a radian. A complete circle is 2π radians, and also 360°, so that

$$2\pi \text{ rad} = 360° ,$$

or

$$1 \text{ rad} = \frac{360°}{2\pi} . \tag{4}$$

Since there are 60 minutes in a degree, it follows from Eq. (4) that

$$1 \text{ rad} = \frac{360 \times 60'}{2\pi} = \frac{10{,}800'}{\pi} \approx 3438. \tag{5}$$

Using the 4-digit approximation in Eq. (5), we can rewrite Eq. (3) as

$$\text{visual angle (minutes)} = 3438 \times \frac{\text{size of object}}{\text{viewing distance}} . \tag{6}$$

The true value of the constant in Eq. (5) is a decimal that goes on forever (because π is an irrational number). However, the rounded quantity 3438 will serve quite well for most practical purposes. Most of the basic quantities in this paper will be given to three or four significant figures, a range consonant with the typical field-measurement accuracies of the empirical quantities entering into the formulas. To allow for any possible need for still greater accuracy, exact expressions for the key quantities are also given, as in Eq. (5), just before the final number. Later, in section 5, five significant figures will be used to agree with the practice of the quoted authors.

3. THE SNELLEN INDEX OF VISUAL ACUITY

The expression "visual acuity" refers in casual usage to sharpness of vision, or the limiting resolution of the eye. It is traditionally assumed that the finest detail that can just be made out by an eye with normal visual acuity, viewing black lines on a white background, with moderate levels of illumination, subtends a visual angle of 1 minute of arc. This assumption is apparently rather conservative in terms of population statistics; that is, the average observer can actually make out details subtending somewhat less than 1 minute (Le Grand, 1967; Roberts, 1964). Nevertheless, the most commonly used kind of eye chart, the Snellen-type chart, takes normal acuity as 1 minute, as embodied in the line of the chart with the Snellen notation 20/20. It is not the entire letter of this size that subtends a visual angle of 1 minute, but rather the finest detail of the letter -- the stroke width -- that subtends 1 minute at the standard viewing distance of 20 feet. Snellen letters are five times the stroke width in height, so the letters corresponding to 20/20 vision have a height subtending 5 minutes of arc.

A capital letter E has three horizontal strokes and two spaces between the strokes. In the Snellen E, the strokes and spaces are all equal in thickness, so that the total height of the letter is five times the stroke width. In addition, the width of the letter is taken to be equal to the height. Other Snellen letters, which are all capitals, are designed to have the same stroke width, overall height, and overall width as the E.

It is intuitively clear that if the horizontal spaces in an E were substantially thinner than the strokes, then the space would be the limiting factor in visually resolving the letter. That is, if the letter were viewed at such a distance that the strokes were just barely visible, the spaces would be invisible, and the letter would appear, ideally, as a solid black rectangle. On the other hand, if the spaces were substantially wider than the strokes, the strokes would provide the limiting detail. Consequently, at a distance at which the spaces could just be seen, the strokes could not be seen, and the letter would appear, ideally, as a white rectangle (i.e., it would disappear entirely against the white background of the chart).

In reality, vision does not work in such an all-or-nothing fashion. Acuity is not perfectly constant either over time or over space. What is actually seen in looking at a letter not far beyond the resolution limit is a grayish blob that has dark and light features that are elusive and changing, and do not form a pattern sufficiently familiar to recognize. Nevertheless, the principles of the preceding paragraph are correct, and, for a given letter height, the highest acuity scores are in fact obtained when the strokes and spaces of a capital E are of equal width, as on the Snellen chart. Snellen letters, with a 5:1 height-to-strokewidth ratio, appear "blocky" and unesthetic to most people, apparently including graphic designers. Letter heights are never ordinarily any less than 5 times the stroke width, and are usually a higher multiple. A ratio within the range of 6 to 10 is most commonly recommended in human engineering handbooks as best for good legibility (Morgan et al., 1963, p. 102; McCormick, 1970, p. 166; Huchingson, 1981, p. 128; Woodson, 1981, p. 491).

When the stroke width, rather than the overall height, of a capital E is kept fixed, then extra-high spaces between the strokes will not harm visibility and may even increase it, so height-to-strokewidth ratios in the neighborhood of 10 may then be optimum for visibility; but the price paid is bigger letter size. Because of the considerable variation in the ratio of height to stroke width in the type faces actually used in current systems of signs, it would be inaccurate to specify letter heights as being visible at various distances by normal or other observers. Instead, this report will state its recommendations in terms of stroke width. Letter heights for whatever type face is desired can then be obtained by multiplying the specified stroke width by the height-to-strokewidth ratio characterizing that face. Thus, for 20/20 vision at 20 feet, a stroke width of 1 minute of arc is required. If Snellen letters are used, the letter height will be 5 minutes of arc; if more typical letter styles are used, the height may be 6 to 10 minutes, or more.

3.1 MEANING OF THE SNELLEN NOTATION

People with 20/40 vision can just see at 20 feet the same letters that the average ("normal") person can just see at 40 feet. They need to see stroke widths subtending 2 minutes of arc, in order to recognize letters. They can accomplish this by looking at the 20/20 line of the chart from a distance of only 10 feet; or, if they stay at a distance of 20 feet, they need letters that are twice as large as the 20/20 letters that the person with normal acuity can see from 20 feet. The 20/40 letters are in fact exactly twice as large as the 20/20 letters. On the other hand, people with visual acuity twice as good as normal can read the 20/10 line on the chart, which has a stroke width subtending half a minute at 20 feet. They see at 20 feet what the normal observer can only make out at 10 feet. The 20/10 letters are half the size of the 20/20 letters.

In general, we can refer symbolically to a Snellen index of 20/$\underline{S}$. Here, $\underline{S}$ is the distance at which the normal observer must stand to just make out the same letters that people with 20/$\underline{S}$ vision can just make out at the standard viewing distance of 20 feet. (That is, $\underline{S}$ is the distance at which the width of the strokes comprising these letters subtends a visual angle of 1 minute of arc.) $\underline{S}$ values greater than 20 represent subnormal vision; $\underline{S}$ values less than 20 represent superior vision. Accordingly, the Snellen denominator $\underline{S}$ is an <u>inverse</u> measure of visual acuity.

Another inverse measure of visual acuity is the size of the threshold object -- the object that can just barely be seen -- measured in angular terms. If the threshold object, such as the stroke width of a Snellen-type letter, is small, then visual acuity is high; if the threshold stroke width is large, then visual acuity is low. Accordingly, the established <u>direct</u> measure of visual acuity is the reciprocal of the threshold visual angle, when the angle is measured in minutes of arc. This quantity is, in fact, the technical definition of the term "visual acuity." Thus, visual acuity is measured in units of reciprocal minutes of arc (min^{-1}). Further reference in this report to visual acuity, or simply acuity, in a quantitative context, will refer specifically to this measure.

Since the Snellen denominator $\underline{S}$ is also an inverse measure of visual acuity, we know that: $\underline{S}$ is proportional to the threshold stroke width (specifically, $\underline{S}$ is equal to 20 times the latter quantity, measured in minutes of arc); and the quantity $20/\underline{S}$ would be a direct acuity measure. This quantity is simply the Snellen notation, interpreted as an actual fraction. Not only is the Snellen fraction directly proportional to visual acuity, it is actually numerically equal to it. This follows from the observation above that $\underline{S}$ is equal to 20 times the stroke width measured in minutes of arc; the factors of 20 in the fraction cancel out. For a person with normal vision, the Snellen fraction is $20/20 = 1$, and the visual acuity is 1 min^{-1}, since the stroke width of the letters on the 20/20 line of the chart subtends an angle of 1 minute of arc at the standard viewing distance of 20 feet. A person whose acuity threshold is reached with the 20/40 line of the chart has a Snellen fraction equal to 0.5 and a visual acuity of 0.5 min^{-1}, because the stroke width of those letters subtends 2 minutes of arc at 20 feet. Finally, a person who can make out the 20/10 letters on the chart has a Snellen fraction equal to 2.0 and a visual acuity of 2.0 min^{-1}, since the stroke width of those letters subtends 1/2 minute at 20 feet.

In general, a person with 20/$\underline{S}$ vision needs a stroke width of $\underline{S}$/20 minutes of arc to just see letters and that person has a visual acuity of 20/$\underline{S}$ min^{-1}.

3.1.1 Metric Snellen Index

The numerator of the customary Snellen fraction is always 20, because the standard viewing distance for administering the Snellen eye test is 20 feet. Since feet are not a metric unit, a version of the Snellen index suitable for use with the metric International System of units (SI) has been developed, and is found in some of the clinical visual literature (i.e., optometric and ophthalmological journals). Conveniently, 20 feet is almost exactly 6 meters. (The actual figure is exactly 6.096 m.) Hence, the base distance in meters for administration of the metric Snellen test, and the fixed numerator of the metric Snellen fraction, have been taken as 6. (Six meters equals approximately 19.685 feet.) The metric Snellen notations equivalent to the seven customary Snellen indices used in the graphs of this report are shown in Table 2. As we have seen earlier, the Snellen notation, when interpreted as an actual numerical fraction, is equal to the reciprocal of the critical detail (stroke width) in minutes of arc. The same is true for the metric notation, since the numerator and denominator have the same ratio as in the equivalent customary notation.

Recently, the NAS-NRC Vision Committee (National Academy of Sciences, 1980) issued recommended standards for the design of visual acuity tests, the procedures to be used in administering them, and the method for specifying the results of the tests. They concluded, for both computational and visual reasons discussed in their paper, that the optimum metric distance for viewing acuity test displays is four, rather than six meters, even though this distance (a little over 13 feet) is well short of the traditional 20 feet. As a result, it can be anticipated that Snellen notations with a numerator of 4 will soon be showing up in clinical literature. For future reference, these notations are also included in Table 2.

Table 2. Equivalent[a] Metric and Customary Snellen Notations

| Snellen Notation | | | Visual Acuity, | Critical Visual Angle | Angular Height of Snellen |
Customary	Metric-6	Metric-4	Reciprocal Minutes[b]	(Stroke Width), Minutes	Letter, Minutes
20/10	6/3	4/2	2.00	0.5	2.5
20/20	6/6	4/4	1.00	1	5
20/40	6/12	4/8	0.50	2	10
20/60	6/18	4/12	0.33	3	15
20/100	6/30	4/20	0.20	5	25
20/200	6/60	4/40	0.10	10	50
20/400	6/120	4/80	0.05	20	100

[a] The equivalences are exact with respect to the visual acuities represented, even though the standard metric viewing distance of 6 meters is a bit less than the exact equivalent (6.096 m) of 20 feet. The metric denominators are of course off by the same factor, so that the ratios are exactly equal. The same is true of the 4-meter metric notations, even though this viewing distance (approximately 13.123 feet) is grossly different from 20 feet.

[b] Note that visual acuity is numerically equal to the Snellen fraction (customary or metric), if the latter is interpreted as an actual arithmetic quantity.

4. MATHEMATICAL ANALYSIS OF VISUAL ACUITY

We recall at this point, from the considerations of section 3, that a normal observer needs stroke widths of 1 minute to just recognize letters, and that the person with 20/*S* vision needs stroke widths of *S*/20 minutes. If we denote by Θ_S the stroke width needed by someone with 20/*S* vision, expressed in angular terms, then we can write

$$\Theta_{20} = 1 \text{ min},$$

and, in general,

$$\Theta_S = S/20 \text{ min}. \tag{7}$$

We can obtain an expression for Θ_S in radians by inverting Eq. (5):

$$1 \text{ min} = \frac{\pi}{10{,}800} \text{ rad} \approx 2.91 \times 10^{-4} \text{ rad}; \tag{8}$$

and then converting Eq. (7) with the use of Eq. (8). The exact result is

$$\Theta_S = \left(\frac{S}{20}\right)\left(\frac{\pi}{10{,}800}\right)\text{rad} = \frac{\pi S}{216{,}000} \text{ rad}, \tag{9}$$

or, numerically,

$$\Theta_S = 1.45 \times 10^{-5} \times S \text{ rad}. \tag{10}$$

If we now denote by w_S the stroke width needed by someone with 20/*S* vision, expressed in length units, we can make use of Eq. (2) to write

$$\Theta_S = w_S/d, \tag{11}$$

where *d* is the viewing distance. We can then substitute for Θ_S in Eqs. (9) and (10) the expression given by Eq. (11), the results being

$$w_S/d = \frac{\pi S}{216{,}000}$$

and

$$w_S/d = 1.45 \times 10^{-5} \times S,$$

respectively. The results are the following expressions for w_S:

$$w_S = \frac{\pi S d}{216{,}000} \tag{12}$$

or

$$w_S = 1.45 \times 10^{-5} \times S \times d, \tag{13}$$

where w_S and *d* are expressed <u>in the same units of length</u>.

Ordinarily, it is convenient to use smaller units for specifying the stroke width than are used for the viewing distance. If w_S and d in Eq. (13) are measured in meters, it is easy enough to then convert w_S to units of centimeters or millimeters. Similarly, if the two quantities are measured in feet, w_S can be converted to inches. It is possible to modify Eq. (13) to take explicit account of any possible units of measurement for w_S and d, and such a treatment has been included in Appendix A.

4.1 PLOTTING THE GRAPH

The relationship (13) is of a very simple form: it says that w_S is simply proportional to d, the constant of proportionality being a fixed constant times S. It is easy to see what the result would be if we were to plot w_S as a function of d, with S as a parameter. The graph would consist of a family of lines through the origin of the w_S against d plot, each line corresponding to a particular value of S. The problem with actually drawing such a graph is that we want to cover a range of values for d (and consequently for w_S) of several orders of magnitude; that is, we would be interested in viewing distances of 2 meters, 20 meters, and 200 meters -- occasionally even 2000 meters. Any graph that includes 2000 m would have 2 m located so close to 0 m that accurate readings would be impossible. Frequently, the solution to such a situation is to plot the logarithms of the variables against each other, rather than the variables themselves. Taking the logs of both sides of Eq. (13) gives

$$\log w_S = \log p_1 + \log S + \log d, \tag{14}$$

where p_1 stands for the numerical constant, i.e.,

$$p_1 = 1.45 \times 10^{-5}. \tag{15}$$

Inspection of Eq. (14) shows that a plot of log $\underline{w}_S$ against log $\underline{d}$ always yields a straight line of unit slope (at 45°), regardless of the observer's visual acuity (indexed by $\underline{S}$). Hence, the pencil ("fan") of lines through the origin obtained by plotting $\underline{w}_S$ against $\underline{d}$ is replaced, in the log-log plot, by a family of parallel lines of unit slope. The spacing between these lines, each line corresponding to a particular value of $\underline{S}$, is fixed, once a particular selection of $\underline{S}$ values is chosen.

We proceed, then, to choose the set of $\underline{S}$ values to be displayed. The considerations governing this choice are: (1) the highest and lowest $\underline{S}$ values (lowest and highest acuity, respectively) should embrace virtually the entire population of interest; (2) the values of $\underline{S}$ used should correspond to values appearing on actual Snellen charts; and (3) the values chosen should lead to lines approximately equally spaced on the log-log plot, i.e., the ratios of successive $\underline{S}$ values should be approximately constant. In accordance with these criteria, seven values of $\underline{S}$ have been selected: 10, 20, 40, 60, 100, 200, and 400. Few people have vision better than 20/10, and there is virtually never a need to design displays for people with such super-acute vision. At the other extreme, all degrees of visual defect exist, but it seems impractical even to consider allowing visually for Snellen acuities worse than 20/400. Auditory or tactual information transfer is more suitable for people with such poor acuity. (Legal blindness is usually defined as corrected vision not exceeding 20/200 in the better eye, and, as indicated in Table 1, the percentage of adults who are legally blind is less than 1 percent by an unknown amount.) It must be recalled that a person with 20/$\underline{S}$ vision requires letters $\underline{S}$/20 times as large as a person with normal 20/20 vision. Hence, those who are just barely legally blind (20/200) need letters 10 times as large as those required by people of normal acuity, at the same viewing distance. In a situation where 6-inch letters are appropriate for normal viewers, the person with 20/200 vision needs 5-foot letters, and 20/400 vision requires 10-foot letters.

In reaching a decision as to what limiting acuity should be chosen as determining the size of sign lettering, economics (the cost of the signs) and esthetics tend to hold down the size of the signs, while the importance of the sign's message (e.g., calling attention to a severe safety hazard) will tend to encourage use of a large enough sign so that a high proportion of the population will be alerted by the visual message. Decisions as to the optimum balance will have to be made either on a case-by-case basis, or by general agreement reached through a government board or a standardizing committee. Because of recent federal regulations (ATBCB, 1981), public buildings will be equipped with auditory and tactual signals for the benefit of those without sight. The decision on a cutoff acuity is therefore not a matter of abandoning all people with worse vision than the cutoff level, but rather a question of where to switch from reliance on reading signs to reliance on signals through the other senses (Collins, Danner, and Tibbott, 1981). A behavioral survey of substantial numbers of people with very low visual acuity might reveal a fairly sharply defined acuity level dividing those who rely principally on their visual sense for information about the outside world, from those more dependent on other senses. People with worse than that level of acuity would tend to consider themselves functionally blind and to operate in public more on the basis of sound and touch (including dependence on guide dogs), rather than on the basis of their very limited vision. If such a level exists, it would be interesting to see how close it is to the 20/200 visual acuity that has been chosen as the cutoff for legal blindness.

No recommendation on the cutoff acuity is made in this report. By presenting a graph that allows for any likely choice of the cutoff level, the report presents directly the information needed by sign designers who have made the choice for themselves. In any event, Eq. (13) would serve to provide the required information on letter height to a designer who chooses any limiting acuity whatever, even a level outside the range of 20/10 to 20/400 included in the graph.

Figure 2 shows the final graph, based on measuring both $\underline{w_S}$ and $\underline{d}$ in meters. Because the functions drawn in the figure are straight lines, they can be plotted by locating only two points on each line and simply connecting the points. For the line corresponding to an acuity of 20/$\underline{S}$, Eq. (13) tells us that the values of $\underline{w_S}$ for $\underline{d} = 10^{-1}$ and $\underline{d} = 10^{3}$ m are, respectively, $1.45 \times 10^{-5} \times S \times 10^{-1}$ and $1.45 \times 10^{-5} \times S \times 10^{-3}$ m, or $1.45 \times 10^{-6} \times S$ and $1.45 \times 10^{-2} \times S$ m.

Thus, for example, to plot the 20/400 line ($\underline{S} = 400$) on the graph, the $\underline{w_S}$ values indicated above become specifically $1.45 \times 10^{-6} \times 400$ and $1.45 \times 10^{-2} \times 400$ m, or 5.80×10^{-4} and 5.80 m, respectively. In short, we plot points at (0.1 m, 5.80×10^{-4} m) and (1000 m, 5.80 m) and draw the line connecting them. The other lines in the figure are constructed similarly.

In Appendix A, three additional figures are presented, analogous to Fig. 2. Those graphs are for situations in which $\underline{w_S}$ and $\underline{d}$ are measured in different units. The figures in Appendix A were selected as representing combinations of units that are likely to be of practical interest.

As was pointed out earlier, Eq. (13), on which Fig. 2 is based, is valid provided the same units are used for measuring distance and stroke width. Accordingly, the very same equation would apply if both variables were measured in, for example, inches, instead of meters. However, the graph corresponding

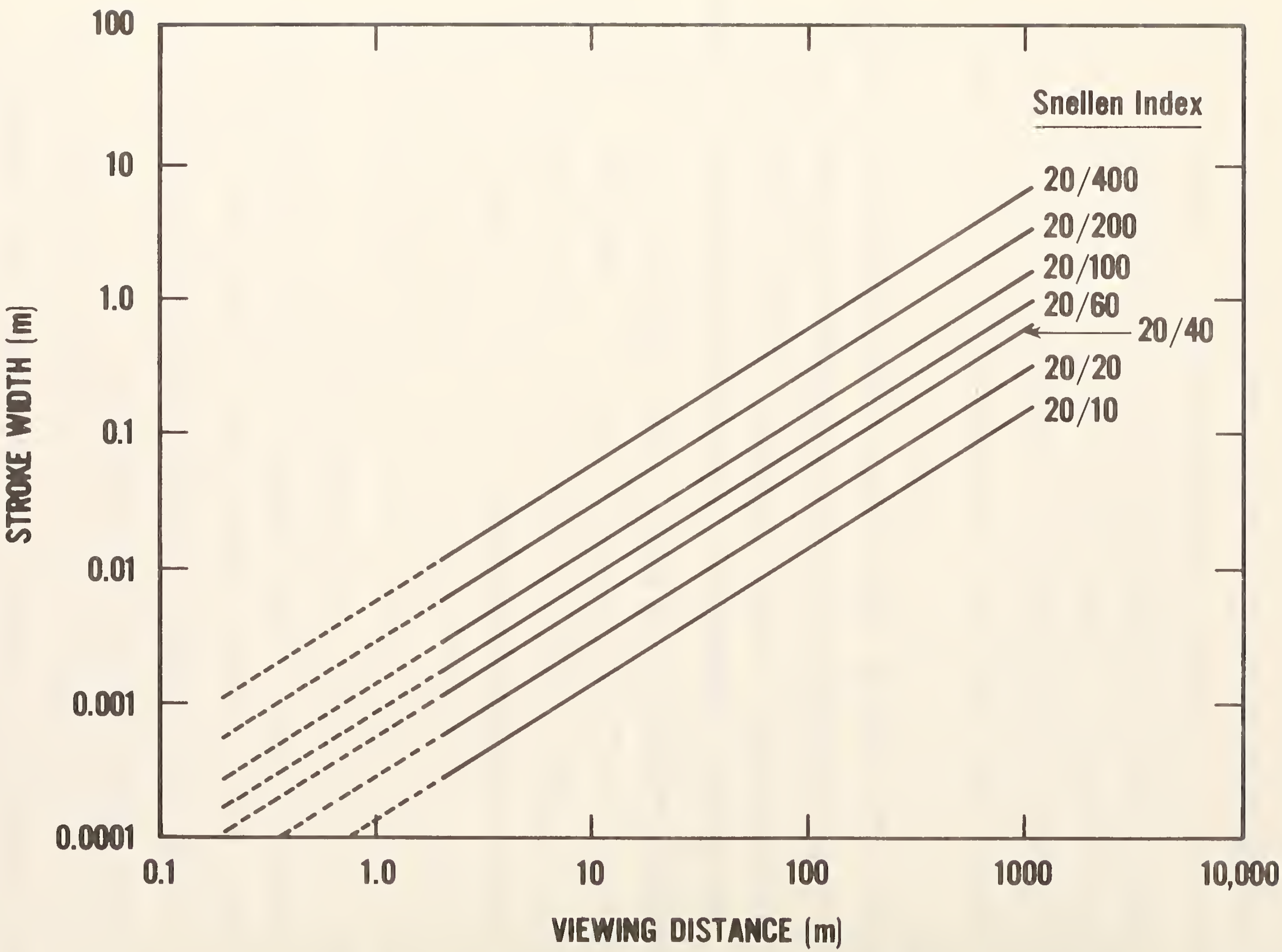

Figure 2. Log-log plot of minimum stroke width (meters) required for letter visibility as a function of viewing distance (meters), with observer visual acuity as a parameter. The lines correspond to Eq. (13), for seven values of the Snellen denominator, $\underline{S}$.

to this choice of units would not be identical to Fig. 2. The family of lines would indeed be the same, but the "inch-vs.-inch" graph would have numbers on both axes larger by a factor of approximately 39 than the numbers used in Fig. 2. This means that the powers of 10 labeling corresponding points on the graphs would be greater by about 1.6 on the "inch-vs.-inch" graph.

4.2 SIGNIFICANCE OF THE GRAPHS

The basis of the straight lines in the graphs -- including those in Appendix A -- is the same as the basis of the Snellen system: the assumption that only visual angle counts, and not absolute distance. If a person has 20/20 vision at a viewing distance of 6 meters, it is assumed that he still has 20/20 vision (that is, a threshold of 1 minute of arc) at 60 or 600 meters. Some researchers who have tested this assumption directly have found it to be true (Beebe-Center et al., 1945; Berger, 1941; Dimmick and Rudolph, 1948), but others have found deviations (Giese, 1946; Chapanis and Scarpa, 1967). Until the matter is settled definitively, it is simplest to assume that visual acuity does not vary with distance (beyond some minimum distance). We know that "near" acuity, operative at reading distance, can be very different from "far" acuity; that is why many people need bifocals. The lines in the graphs have been dashed below 2 meters, because the transition from far to near vision begins somewhere around that distance. When the distance gets down to 0.4 meter (16 inches), one is dealing with full-fledged near vision, and below 0.2 meter (8 inches), where the graph lines have been discontinued, many adults cannot focus their eyes fully and visual acuity deteriorates. Duke-Elder and Abrams (1970) give the eye's near point -- the distance of shortest focus -- as 0.1 meter (4 inches). Fortunately, most signs in buildings are meant to be viewed at distances of 2 meters or more.

The lines in the graphs actually apply at <u>any</u> distance because the information conveyed by the lines is a matter of definition and not of fact. (For example, 20/20 vision means an ability to just make out stroke widths subtending 1 minute of arc, and the 20/20 line in each graph indicates what stroke width subtends 1 minute at each distance.) However, two cautions are important in interpreting and applying the graphs: (1) the vision of a particular person may be characterized by one of the lines in the graph at long viewing distances, but by a different line at reading distances; and (2) it is widely accepted that the average vision of the population is 20/20 when testing is at the customary viewing distance of 20 feet, but is the population average also 20/20 at long viewing distances, such as 200 feet, or at short distances, such as 2 feet? No published data bearing on the variation of population acuity norms with viewing distance have been discovered by the author, but it seems to be widely assumed (perhaps as a convenience) that 20/20 is normal vision at all distances down to about a foot and a half. Although the conclusions of this report may apply fairly well to near as well as to distance vision, it is safest to regard the report as being concerned with the legibility of signs viewed from beyond "reading distance."

5. ALLOWING FOR LOW CONTRAST AND LUMINANCE

5.1 INTRODUCTION

Standard procedure in administering visual acuity tests by means of wall charts involves printing the chart with high contrast (i.e., very black print on a very white background) and lighting it to adequate levels that put the luminance of the white chart background well up into the photopic ("day-vision") range (the lower limit of which is at about 1 footlambert). It is obvious that strong reduction in the contrast of the letters with the background -- e.g., using light gray instead of black letters on the same white background -- would interfere with anyone's ability to resolve the letters, and would accordingly result in a poorer visual acuity score. Similarly, a marked reduction in luminance would lead to the same result.

The figure of 1 minute of arc as the limiting detail that can be discriminated by the normal observer (i.e., 20/20 vision) is based on testing with high-contrast charts lit to relatively high luminance levels (in the neighborhood of 25 footlamberts). The designer responsible for the specification of any sign -- particularly a sign relating to safety -- should make it a point to use very high contrast between the lettering and the background, and to have the sign brightly lit. However, if some compelling circumstance requires sacrificing some degree of contrast or luminance, it would be desirable to be able to estimate quantitatively how much visual acuity would be reduced; and, accordingly, how much larger the sign lettering should be made in order to compensate for the worsening of visual resolution under these "reduced" (visually degraded) conditions. That is the purpose of this section of the present report.

5.2 CONTRAST

5.2.1 Color Contrast

A visual "target" and its background may differ in luminance, chromaticity (colloquially, "color"), or both. The luminance of an object determines its lightness (if it is a reflecting surface) or its brightness (if it is a light emitter). Much is known about the effects of luminance contrast on target visibility, and some of the implications of this knowledge for the visibility of lettered messages will be discussed in section 5.2.2.

On the other hand, the effects of chromatic differences on visibility have been less fully explored, partly because they are intrinsically more complex. No attempt will be made in this report to provide any method of allowing for the contribution of chromatic differences to the visibility of lettering on signs. Some of the problems slowing the development of such a method are discussed below.

In the analysis of luminance or reflectance differences, the concept of contrast has proved very useful in many applications, and will be discussed at length in section 5.2.2. Although there are several different formal definitions of contrast, the one we will work with is essentially a percentage measure of luminance or reflectance difference between the target and background, relative to the value for the background. No analogous measure is widely accepted in the area of chromatic differences. The quantity often loosely labeled "color contrast" or "chromatic contrast" is not a contrast in the preceding sense of a percentage difference, but is actually an absolute measure of amount of color difference.

Unfortunately, there is not yet even an agreed upon formula for color difference. Many formulas have been proposed over the years, and a number of them have proved useful in limited industrial applications (Wyszecki and Stiles, 1967, p. 459 ff.). The whole matter is still so controversial that the CIE -- the International Commission on Illumination, which makes formal recommendations for practice in the area of light and color -- was unable to agree on any single formula as the best available, and instead recommended two different formulas for use and further study (CIE, 1978).

It is known that even when the target and background are viewed from a short enough distance to appear large, more visual difference can be contributed by a large lightness difference than by even the largest chromatic difference. Moreover, as one or both of the target and background become smaller in angular size (down in the range below half a degree), the contribution of any chromatic difference to the overall perceived difference drops off more rapidly than the contribution of the lightness difference (Judd, 1973). As a result, when the target is a sign message not far above the limits of legibility, the colors of the lettering and background make hardly any contribution to the perceived contrast, and only the luminance or reflectance difference is important in determining visibility. For purposes of reading continuous text, Tinker (1963, 1965) has pointed out that the colors of ink and paper are essentially irrelevant, and that maximum speed of reading will result as long as the luminous reflectance of the paper exceeds that of the ink by a factor of about 8.

For purposes of sign specification, the most useful advice that can be given here is to make the lettering as dark as possible and the background as light as possible, or vice versa. Most of us have seen a red on green or red on blue (or the reverse) sign or product label that was virtually unreadable, despite the fact that these color combinations provide a great deal of chromatic contrast. The problem arises when the lightnesses or reflectances of the two colors are too close together in value. There is no harm in introducing some coloration of the letters and/or background, but only to the extent that a reflectance ratio of 7 or 8 can be maintained.

The ANSI (1979) standard governing the use of colors on signs for safety purposes takes cognizance of these rules. In order to maximize lightness contrast with white lettering or symbols, which are required to have a luminous reflectance of at least 73.40 percent, the reflectances of contrasting colors are allowed to be rather low. For example, the minimum reflectance allowed for red or green is 9.00 percent, and for blue 6.56 percent. On the other hand, black must have a reflectance not exceeding 3.13 percent, or 4.61 percent if the black is matte. Thus, black contrasts well not only with white, but also with yellow, which must have a reflectance of at least 50.68 percent. Orange, too, which the standard restricts to a reflectance range between 24.58 and 36.20 percent, contrasts adequately with black if the orange is chosen toward the lighter end of the range.

Collins (1983) has made some contrast measurements of actual signs of various colors, both in the laboratory and in place in a coal mine and in a limestone mine. Her results show that in the real world, many safety signs, despite dirt accumulation, do have a reflectance ratio of 7 or 8 between their light and dark parts, but many others have a ratio closer to 4. Most of the latter group were red and white signs.

5.2.2 Luminance Contrast

Contrast can be defined and measured quantitatively. This section is devoted entirely to lightness or brightness contrast, which depends only on the luminous reflectances or luminances of the print and background, and is not at all concerned with their chromaticities. If we let L_t be the luminance of the "target" -- in this case, the lettering on the sign -- and L_b be the luminance of the background on which the lettering is printed, then the usual definition of contrast, C, is

$$C = \frac{L_b - L_t}{L_b} . \tag{16}$$

In various applications, the sign of the numerator of Eq. (16), and hence of C, is defined in different ways. Since vision is ordinarily tested with dark letters on a light background, the situation of most concern here involves L_b being much greater than L_t. Thus, contrast was defined in Eq. (16) so that it would be positive for dark print on a light background. By this choice, light print on a dark background involves negative contrast values. We can now state quantitatively what was meant in section 5.1 by "high" contrast: a C of at least 85 percent (0.85) is considered acceptable in an eye chart (National Academy of Sciences, 1980).

Note that C in Eq. (16) can be written in the form

$$C = 1 - \frac{L_t}{L_b} , \tag{17}$$

or

$$C = 1 - \frac{1}{R} , \tag{18}$$

where

$$R = \frac{L_b}{L_t} , \tag{19}$$

the luminance ratio of the bright background to the dark target.

Solving Eq. (18) for R results in the reverse equation

$$R = \frac{1}{1 - C} . \tag{20}$$

According to Eq. (20), when $\underline{C} = 0.85$, then $\underline{R} = 6.67$. In other words, a good eye chart has a background-to-ink luminance (or, equivalently, luminous reflectance) ratio of around 7 or more. In fact, the reflectance ratio, $\underline{R}$, on a chart of high quality, may be 10 ($\underline{C} = 0.90$) or even more than 25 ($\underline{C} = 0.96$).

Recently, a body of data on visual acuity as a function of both luminance and contrast was collected in Japan (Nakane and Ito, 1978). Kaneko (1982), in an American publication, has discussed these data and presented an equation of simple form that he has fitted to the middle ranges of the data. Figure 3 shows that Kaneko's formula, represented by the straight lines in the graph, fits the mean Nakane-Ito data quite well.

Kaneko regards his formula as accurate within the followings limits:

$$1 \leq L_b \leq 1000 \text{ cd/m}^2, \quad 10 \leq \tilde{C} \leq 90\%, \quad 0.2 \leq V \leq 2.0 \text{ min}^{-1}; \tag{21}$$

where $\underline{L_b}$ denotes the background luminance of the white screen on which the dark symbols were presented; $\underline{\tilde{C}}$ denotes contrast as defined in Eq. (16), but multiplied by 100 to make it a percentage; and $\underline{V}$ denotes visual acuity defined as the reciprocal of the finest resolvable detail measured in minutes of arc. Kaneko's formula is

$$V = k\, L_b^p\, \tilde{C}^q, \tag{22}$$

where the values of the constants $\underline{k}$, $\underline{p}$, and $\underline{q}$ are

$$k = 0.06298, \quad p = 0.21310, \quad q = 0.53158. \tag{23}$$

Figure 3 is a log-log plot of $\underline{V}$ against $\underline{L_b}$ with $\underline{\tilde{C}}$ as the parameter. By taking the logarithms of both sides of Eq. (22), we obtain

$$\log V = \log k + p \log L_b + q \log \tilde{C}. \tag{24}$$

With $\underline{\tilde{C}}$ regarded as a parametric constant, Eq. (24) shows that a plot of log $\underline{V}$ against log $\underline{L_b}$ yields a straight line with the fixed slope $\underline{p}$ and a y-intercept of log $\underline{k}$ + $\underline{q}$ log $\underline{\tilde{C}}$, which depends on the value of $\underline{\tilde{C}}$. Thus, as

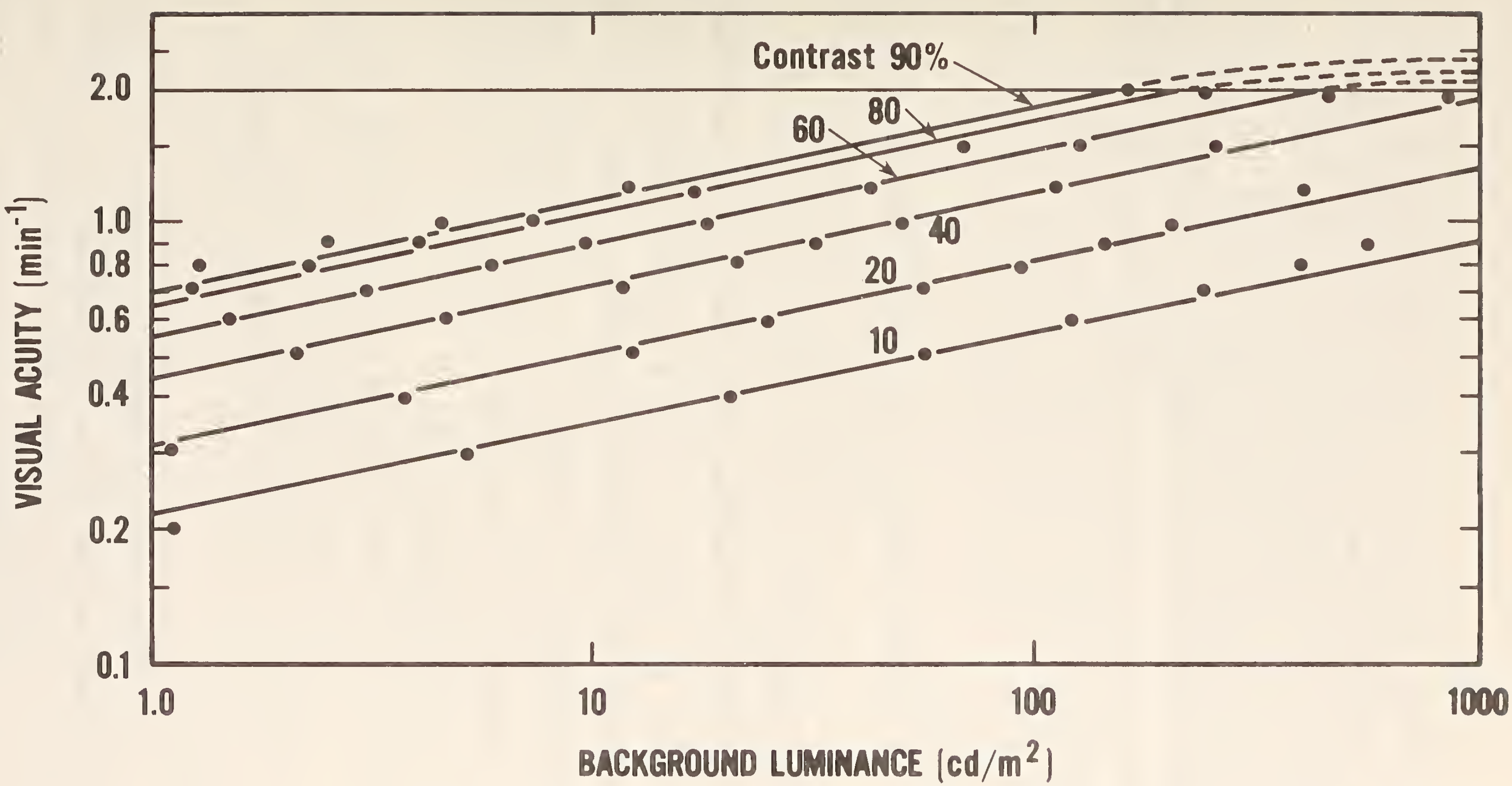

Figure 3. The good fit of Kaneko's (1982) formula [Eq. (22)] to the data of Nakane and Ito (1978). The graph is a log-log plot of visual acuity against background luminance, with contrast as the parameter. The solid diagonal lines correspond to the formula, the filled dots to the data. The curved, dashed extensions of the lines at the upper right lie outside the ranges of validity [Eq. (21)] claimed by Kaneko. Modified from Kaneko (1982).

confirmed in Fig. 3, the family of curves for various values of $\underline{\tilde{C}}$ is a set of parallel straight lines with common slope $\underline{p}$ = 0.21310. For any two values of $\underline{\tilde{C}}$ -- let us say $\underline{\tilde{C}_1}$ and $\underline{\tilde{C}_2}$ -- the vertical separation between the lines is $(\log \underline{k} + \underline{q} \log \underline{\tilde{C}_1}) - (\log \underline{k} + \underline{q} \log \underline{\tilde{C}_2}) = \underline{q} \log (\underline{\tilde{C}_1}/\underline{\tilde{C}_2})$. We see, then, that the spacing between lines is determined by the ratio of the associated contrasts. It follows that the lines for $\underline{\tilde{C}}$ = 80 and 40 are the same distance apart as the lines for $\underline{\tilde{C}}$ = 40 and 20, as well as the lines for $\underline{\tilde{C}}$ = 20 and 10, since all three line pairs correspond to a contrast ratio of 2 to 1. This equality of spacing is apparent in Fig. 3.

What is the effect of reducing contrast? For a given luminance level, we move straight downward in Fig. 3 from one of the higher lines to one of the lower lines. What is more, regardless of what the luminance level is, the amount of downward shift in Fig. 3 is the same, since the lines corresponding to the two levels of contrast are parallel. Thus, a particular reduction of contrast results in a fixed amount of reduction in log visual acuity; which is to say, in reduction of visual acuity by a fixed ratio. By applying Kaneko's formula, Eq. (22), we can express the correspondence between contrast reduction and the drop in visual acuity in quantitative terms.

Suppose we initially have a contrast of $\underline{\tilde{C}_1}$ and are concerned with the effect of reducing the contrast to a level $\underline{\tilde{C}_2}$. Then the visual acuities, $\underline{V_1}$ and $\underline{V_2}$, corresponding to these two contrast levels at the fixed luminance level $\underline{L_b}$, are, by Eq. (22),

$$V_1 = kL_b^p\tilde{C}_1^q \quad \text{and} \quad V_2 = kL_b^p\tilde{C}_2^q \quad . \tag{25}$$

The ratio of the two visual acuities in Eqs. (25) is then

$$\frac{V_2}{V_1} = \frac{\tilde{C}_2^q}{\tilde{C}_1^q} = \left(\frac{\tilde{C}_2}{\tilde{C}_1}\right)^q \quad , \tag{26}$$

the $\underline{k}$ and $\underline{L}_b^p$ terms having divided out of the ratio.

Equation (26) is a relatively simple relationship which tells us that the ratio of the visual acuities is determined by the ratio of the contrasts; and, specifically, that the $\underline{V}_2/\underline{V}_1$ ratio is obtained by raising the $\underline{\tilde{C}}_2/\underline{\tilde{C}}_1$ ratio to the fixed power $\underline{q} = 0.53158$.

As an example, let us look at the result of reducing contrast from the high level of 90 percent to the medium level of 40 percent. The contrast ratio (of the second level to the first) is 40/90 = 0.44444. Therefore, Eq. (26) tells us that the ratio of the visual acuities corresponding to this contrast reduction is $0.44444^{0.53158} = 0.64981$.

As Eq. (26) shows, this result is independent of the initial level of contrast, and depends only on the contrast reduction factor (i.e., the ratio). Thus, if we start with a contrast level of 40 percent and reduce the contrast to 17.778 percent -- again 0.44444 times the initial value -- then the visual acuity again goes down to a fraction of its initial value equal to $0.44444^{0.53158} = 0.64981$. Note that the effect of the fractional exponent is to make the reduction factor for visual acuity less extreme (i.e., closer to unity) than the factor for the contrast reduction, the discrepancy being greater the more the contrast ratio departs from unity.

Note also that, although we have been concerned with contrast <u>reduction</u> up to this point, there is nothing in Eq. (26) that restricts $\underline{\tilde{C}}_2$ to being less than $\underline{\tilde{C}}_1$. The formula, in fact, does apply just as well to contrast <u>increases</u>. As a check of this, consider the reverse of the earlier problem: what is the effect on visual acuity of increasing contrast from $\underline{\tilde{C}}_1 = 40$ to $\underline{\tilde{C}}_2 = 90$? This time the contrast ratio (second to first) is 2.25, and $2.25^{0.53158} = 1.5389$. This visual-acuity ratio is again less extreme (closer to unity) than the contrast ratio, and, as expected, is the reverse (i.e., reciprocal) of the ratio found earlier for the decreasing direction between the same contrast levels (since $1.5389 \times 0.64981 = 0.99999$).

Equation (26), then, provides the simple method sought for allowing for changes in contrast in the calculation of letter stroke-widths needed for various levels of visual acuity. However, it is important to realize that a very strong assumption is involved in applying Eq. (26) for this purpose. What we have actually shown is that, <u>for the relatively high-acuity observers used by Nakane and Ito</u> (1978), Eq. (26) tells us how the visual acuity of those observers varies with changes in contrast, for the average of that group. What we have to assume is that the same rule applies to any group of observers, even if their visual acuity measured under standard test conditions is well below the normal 20/20. The data needed would come from a repeat of the Nakane-Ito study using several groups of observers, the group means spreading over a wide range of standardized visual acuity scores, with each group covering a relatively narrow range. Does Eq. (22) apply in general form to people with 20/200 vision under standard test conditions, as well as to people with 20/20 vision? If so, does the exponent $\underline{q}$ in Eq. (22) have the same numerical value for the low-vision group, so that Eq. (26) can be applied universally? We do not yet know the answers to these questions, so in order to be able to make a quantitative estimate of the effect of contrast changes, it appears reasonable to use Eq. (26) until such time as either practical experience in applying the equation to sign design, or formal vision studies, indicate that it is not always valid. Fortunately, a figure presented by Nakane and Ito (1978), showing data for individual observers covering the narrow range of acuity (2 to 1) represented within the group, suggests that the foregoing assumptions may well be true.

In actual practice, the "standard" conditions for administering acuity screening tests are not fully standardized. In particular, the white-background luminance level recently recommended as standard by the NAS-NRC Vision Committee (National Academy of Sciences, 1980) -- namely 85 $\pm$ 5 cd/m^2 (candelas per square meter) -- is far from universally observed. (The equivalent level in customary units is 25 $\pm$ 1.5 fL, the approximate conversion factor being 1 fL = 3.42626 cd/m^2.) It is not only lower luminance levels, but also higher levels, that cause a problem. Figure 3 shows that exceeding the minimum required contrast level of 85 percent (coincidentally the same number as the standard luminance) does not importantly increase acuity, but it also shows that substantially exceeding the 85-cd/m^2 standard luminance level _will_ result in significantly higher acuity ratings. We can use Kaneko's (1982) formula -- Eq. (22) -- to determine what the mean visual acuity of Nakane and Ito's observers would be under the NAS-recommended conditions. Since the NAS (1980) only requires a _minimum_ contrast of 85 percent, we have to choose a particular $\tilde{C} \geq 85$ as _the_ standard. $\tilde{C} = 90$ is convenient, since 90 is a round number, is not a difficult level of contrast to achieve in practice, and corresponds to the upper limit of application of Kaneko's formula [see Eq. (21)]. Accordingly, let us substitute the values $L_b = 85$ and $\tilde{C} = 90$ into Eq. (22). The result is a visual acuity value of $V = 1.7750$, corresponding to a Snellen notation of approximately 20/11 -- very sharp vision indeed, for a group.

Kaneko (1982) and Nakane and Ito (1980) report that the latter's 52 observers had a somewhat lower mean V of 1.4 (a Snellen score of 20/14), apparently according to a preliminary screening test administered at an unspecified luminance level, rather than from the experimental data themselves. It is understandable that this group of observers had superior acuity: Nakane and

Ito's (1980) list of the observers' ages shows that all but four of them were in the youthful age range of 19 to 28 years. [For some reason, the Kaneko (1982) paper reports this range of ages as "10 to 20."] The total range of visual acuity scores represented in the screeening test results -- including the four older observers -- was from a <u>low</u> of 20/20 (!), up to 20/10 ($\underline{V}$ = 1.0 to 2.0). Note that 20/14 is not significantly better average vision than is to be found in Americans of comparable age. Roberts (1964) reports that 58 percent of Americans aged 18 to 24 years have an acuity level of 20/15 or better for "corrected" distance vision.

The discrepancy between the screening-test mean ($\underline{V}$ = 1.4) and the mean results ($\underline{V}$ = 1.7750) of the experiment itself for the "standard" contrast and luminance is not particularly puzzling: viewing conditions and procedures in the experiment probably differed considerably from those used in the presumably Snellen-chart-type screening test (which was not actually described in either paper). In particular, the uniformly lit white background in the experiment filled a large part of the observer's visual field (160° by 90°, according to Kaneko), whereas the white background of a clinical testing chart is comparatively small, and the rest of the room is typically lit to lower levels than is the chart.

We are now in a position to hypothesize explicitly what the generalized form of Eq. (22) is, if indeed there is a universal formula applying to people with all levels of visual acuity. The simplest result would be for Fig. 3 to be regarded as applying specifically to people of superior standard acuity, and for the entire family of lines to be simply shifted downward by an amount depending on the standard visual acuity of the observer group of interest. The lower the standard acuity of the group, the further down the lines would be pushed. Such a fixed-distance downward shift of log visual acuity (the ordinate dimension in Fig. 3) corresponds, as we have noted earlier, to division by a fixed constant (i.e., to a fixed ratio of visual acuities). If we denote by $\underline{V}_s$ the mean standard visual acuity of a group of observers (or of a single individual), and if we define the standard conditions to be $\underline{L}_b = 85$, $\underline{\tilde{C}} = 90$, then we can correct the $\underline{V}$ value obtained from Eq. (22) by multiplying by the factor $\underline{V}_s/1.7750$. Explicit application of this correction will be postponed to section 5.5.

For situations in which high accuracy is not needed in the evaluation of the power function of Eq. (26), it is easy to construct a graph that permits $\underline{V}_2/\underline{V}_1$ to be read off for any value of $\underline{\tilde{C}}_2/\underline{\tilde{C}}_1$. Taking the logarithm of both sides of Eq. (26) yields the relationship

$$\log \frac{V_2}{V_1} = q \log \frac{\tilde{C}_2}{\tilde{C}_1} \quad . \tag{27}$$

A log-log plot of the $\underline{V}$-ratio against the $\underline{\tilde{C}}$-ratio is seen in Eq. (27) to consist of a straight line with slope q (= 0.53158). This plot is shown in Fig. 4, which can be used to determine the $\underline{V}$-ratio from the $\underline{\tilde{C}}$-ratio when accuracy is not important. If we take the greatest $\underline{V}$ of interest to be 2.0 (20/10), and the smallest to be 0.05 (20/400), then the greatest $\underline{V}$-ratio it is necessary to deal with is 2.0/0.05 = 40. Similarly, when the ratio is of the smaller to the larger $\underline{V}$ value, the minimum ratio of interest is 1/40. Thus, the log $\underline{V}$-ratio axis should cover at most 4 log cycles, from 0.01 up to 100. Because of the limitation of $\tilde{C}$ to the range from 10 to 90 percent [Eq. (21)] in order for Eq. (22) to be valid, the minimum and maximum $\tilde{C}$-ratios with which we need to deal are 1/9 and 9. Accordingly, only two log cycles are needed for

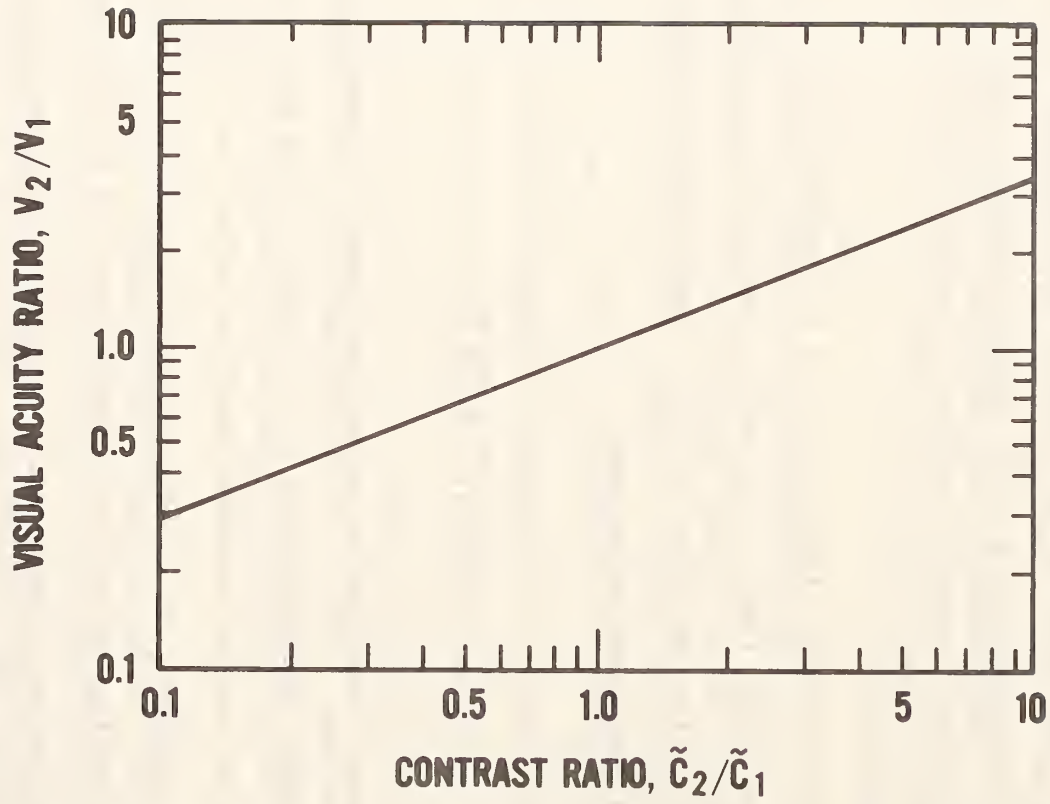

Figure 4. Log-log plot of visual-acuity ratio as a function of percent-contrast ratio [Eq. (27)]. This figure allows for approximate estimates of the change in V resulting from any change in $\tilde{C}$.

the $\tilde{C}$-ratio axis, from 0.1 up to 10. Since the V-ratios corresponding to these two extreme values on the $\tilde{C}$-ratio axis are about 0.29 and 3.4, respectively, we see that on this graph only two log cycles are needed for the log V-ratio axis as well, covering the same range of 0.1 to 10.

5.3 LUMINANCE

Adjustments to visual acuity because of changes in luminance level can be treated in a manner analogous to the way we have dealt with contrast. Now we are concerned with a change from a luminance of L_{b1} to a luminance of L_{b2}. According to Eq. (22), the visual acuities V_1 and V_2 corresponding to these two luminance levels, with the fixed contrast $\tilde{C}$, are

$$V_1 = kL_{b1}^p\tilde{C}^q \quad \text{and} \quad V_2 = kL_{b2}^p\tilde{C}^q \quad . \tag{28}$$

The ratio of the two visual acuities in Eqs. (28) is then

$$\frac{V_2}{V_1} = \frac{L_{b2}^p}{L_{b1}^p} = \left(\frac{L_{b2}}{L_{b1}}\right)^p \quad , \tag{29}$$

with the fixed terms k and $\tilde{C}^q$ having divided out of the ratio. The basic forms of the relationships (26) and (29) are the same -- namely, they are power functions -- and only the exponents are different. Since p (= 0.21310), as well as q, is less than unity, the acuity ratio is less than the luminance ratio, just as we have seen it is less than the contrast ratio. However, since p is even smaller than q, the acuity ratio differs less with luminance changes than with contrast changes, on a ratio basis.

As a numerical example, let us consider the result of reducing luminance by the same ratio that we used in our first example for contrast reduction. This time we reduce an initial luminance of 90 cd/m^2 to 40 cd/m^2. The luminance ratio (of the second level to the first) is 0.44444. Equation (29) tells us that the ratio of the visual acuities corresponding to this luminance reduction is $0.44444^{0.21310} = 0.84130$. This compares to the acuity ratio of 0.64981 that corresponds to the same numerical reduction in contrast, a confirmation of the stronger role played by contrast, compared to luminance, in determining visual acuity. (The smaller fraction corresponds to the more drastic acuity reduction.)

The reverse shift, from $\underline{L}_b = 40$ cd/m^2 to $\underline{L}_b = 90$ cd/m^2, involves a luminance ratio of 90/40 = 2.25, and a corresponding visual acuity ratio of $2.25^{0.21310} = 1.1886$. This is, as it must be, the reciprocal of the acuity ratio of 0.84130 that corresponds to the reverse luminance shift ($1.1886 \times 0.84130 = 0.99997$).

The logarithmic version of Eq. (29) is

$$\log \frac{V_2}{V_1} = p \log \frac{L_{b2}}{L_{b1}} . \tag{30}$$

A log-log plot of the $\underline{V}$-ratio against the $\underline{L}_b$-ratio consists of a straight line of slope p = 0.21310. Figure 5 shows this graph, which may be used for low-accuracy conversion of luminance ratio to the corresponding acuity ratio. Because we may deal validly [Eq. (21)] with luminances from 1 to 1000 cd/m^2, the luminance ratios of interest cover a range from 0.001 to 1000, or six logarithmic cycles. However, the V-ratio values corresponding to these extreme L_b-ratios are approximately 0.23 and 4.4, respectively, a surprisingly narrow range resulting from the small value of the exponent $\underline{p}$. Consequently, the V-ratio range in Fig. 5 is the same as in Fig. 4: namely, the two log cycles from 0.1 to 10.

5.4 CONTRAST AND LUMINANCE COMBINED

It is easy to combine the formulas (26) and (29), to allow for changes of both contrast and luminance. The formulas can be applied one after the other,

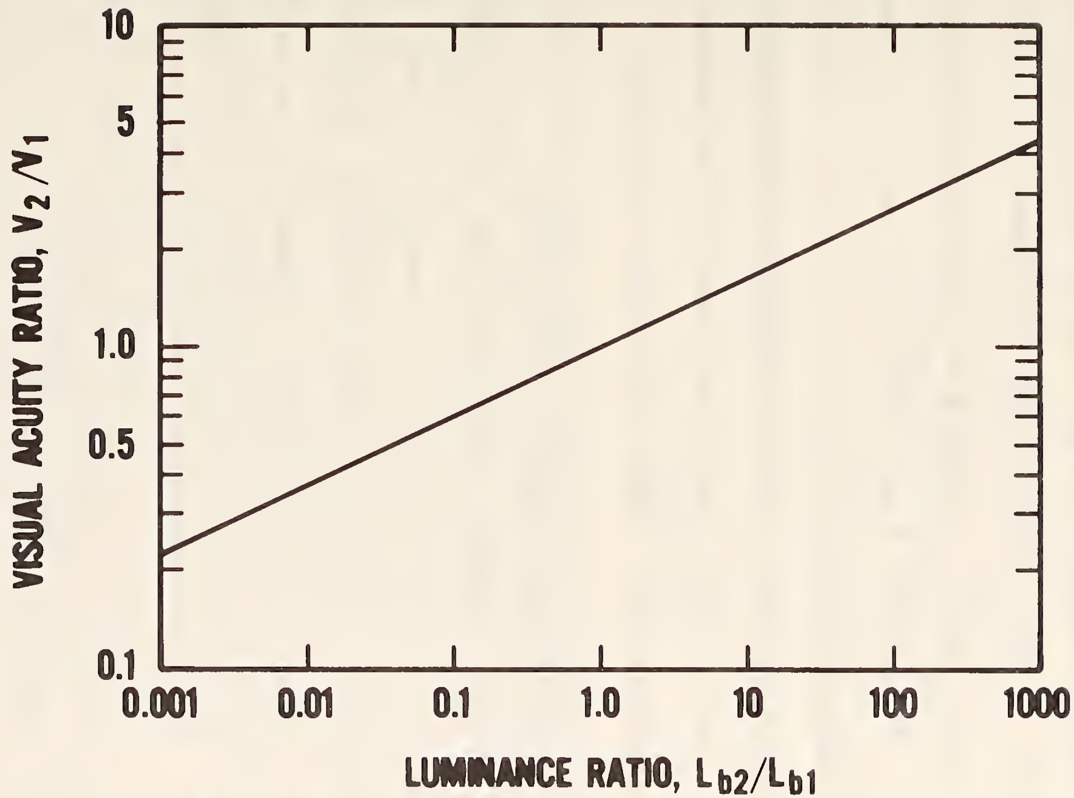

Figure 5. Log-log plot of visual-acuity ratio as a function of background-luminance ratio [Eq. (30)]. This figure allows for approximate estimates of the change in $\underline{V}$ resulting from any change in $\underline{L_b}$.

in either order. To derive the combined formula directly, we go back to Eq. (22) and allow for two sets of conditions: the first with contrast and luminance $\tilde{C}_1$ and $\underline{L}_{b1}$, and the second with corresponding quantities $\underline{\tilde{C}_2}$ and $\underline{L}_{b2}$. The visual acuities corresponding to these two sets of conditions, $\underline{V}_1$ and $\underline{V}_2$, respectively, are, according to Eq. (22),

$$V_1 = kL_{b1}^p \tilde{C}_1^q \quad \text{and} \quad V_2 = kL_{b2}^p \tilde{C}_2^q \quad . \tag{31}$$

The ratio of the two visual acuities in Eqs. (31) is

$$\frac{V_2}{V_1} = \frac{L_{b2}^p \tilde{C}_2^q}{L_{b1}^p \tilde{C}_1^q} = \left(\frac{L_{b2}}{L_{b1}}\right)^p \left(\frac{\tilde{C}_2}{\tilde{C}_1}\right)^q \quad , \tag{32}$$

the $\underline{k}$ factors having divided out of the ratio.

Thus, correcting visual acuity to allow for changes in both luminance and contrast [Eq. (32)] is done by simply taking the product of the separate correction factors for luminance [Eq. (29)] and contrast [Eq. (26)].

5.5 APPLICATION OF THE CONTRAST AND LUMINANCE ADJUSTMENTS

We have seen that for our standard viewing conditions of 90 percent contrast, and a luminance of 85 cd/m^2, formula (22) predicts a visual acuity of 1.7750. We are concerned with being able to make predictions of the effects of deviations from these standard viewing conditions. Therefore, we can choose in Eq. (32) to regard condition number 1 as referring always to the standard situation, i.e.,

$$L_{b1} = 85 \text{ cd/m}^2; \quad \tilde{C}_1 = 90\%; \quad V_1 = 1.7750 \ . \tag{33}$$

Substituting these values into Eq. (32), we obtain

$$\frac{V_2}{1.7750} = \left(\frac{L_{b2}}{85}\right)^p \left(\frac{\tilde{C}_2}{90}\right)^q , \tag{34}$$

with $\underline{p}$ and $\underline{q}$ still being the constants defined in Eqs. (23).

We can now drop the subscript 2 from Eq. (34) and write it in the form

$$V = 1.7750\left(\frac{L_b}{85}\right)^p \left(\frac{\tilde{C}}{90}\right)^q . \tag{35}$$

It is clear by inspection of Eq. (35) that if we choose the standard conditions $L_b = 85$ and $\tilde{C} = 90$, the two rightmost factors are both unity, and $V = 1.7750$, as it should.

As a final adjustment, we introduce at this point the factor $V_s/1.7750$ discussed in section 5.2.2. This factor corrects the visual acuity value predicted by Eq. (35) to the value V_s, the standard visual acuity of the specific group or individual for which we want to make predictions. The result is

$$V = V_s\left(\frac{L_b}{85}\right)^p \left(\frac{\tilde{C}}{90}\right)^q . \tag{36}$$

Since the standard visual acuity ($L_b = 85$ and $\tilde{C} = 90$) of the Nakane-Ito observers to whom Eq. (22) applies is 1.7750, for them $V_s = 1.7750$, and Eq. (36) reduces to Eq. (35), as it should. If we want to deal with a population with the classically assumed "normal" resolution of 1 minute of arc (20/20 vision), then $V_s = 1$ for them, and the constant in Eq. (36) disappears. In the latter case, all acuities predicted by Eq. (36) are lower, by a factor of 1/1.7750, than the acuities predicted for the Nakane-Ito group. This is the nature of the simple correction postulated in the discussion of V_s in section 5.2.2.

Finally, we want to be able to translate Eq. (36) into a form involving S, the Snellen denominator, rather than V, the visual acuity. Since the Snellen fraction, $20/S$, is directly equal to the visual acuity, V, in reciprocal minutes of arc -- i.e.,

$$V = 20/S; \tag{37}$$

then we can substitute for V and V_s in Eq. (36) the equivalent expressions given by Eq. (37), and obtain

$$\frac{20}{S} = \frac{20}{S_s} \left(\frac{L_b}{85}\right)^p \left(\frac{\tilde{C}}{90}\right)^q . \tag{38}$$

The factors of 20 in Eq. (38) divide out, and, taking the reciprocals of both sides, we obtain the final formula we are seeking:

$$S = S_s \left(\frac{85}{L_b}\right)^p \left(\frac{90}{\tilde{C}}\right)^q \tag{39}$$

where p and q are given numerically in Eqs. (23), and S_s is the Snellen denominator for the observer group for which we want to specify letter stroke widths on a sign. What Eq. (39) tells us is the proper S value to use in entering Eq. (13) to predict the just-visible stroke width, for an observer with $20/S_s$ vision under standard conditions. Of course, there is no need to bother with Eq. (39) unless L_b deviates from the standard value of 85 cd/m^2, or $\tilde{C}$ deviates from the standard value of 90 percent. When those two parameters have their standard values, then Eq. (39) reduces to simply

$$S = S_s , \tag{40}$$

which means that the S value to use in Eq. (13) is the S value applying to the observer in question under standard viewing conditions.

As an example, suppose we are concerned with allowing for observers with standard Snellen acuities of 20/60 (V = 1/3). Suppose that we want to allow for a heavy coating of dust all over the sign, which would reduce the contrast to 45 percent, but we are designing for a very brightly lit room which produces a luminance of 170 cd/m^2 on the dusty white sign background. Then, to determine the equivalent Snellen denominator for these non-standard conditions, we substitute into Eq. (39) the values $\underline{S}_s = 60$, $\underline{L}_b = 170$, and $\underline{\tilde{C}} = 45$. The result is

$$S = 60 \times (\frac{1}{2})^{0.21310} \times 2^{0.53158}; \qquad (41)$$

$$S = 60 \times 0.86268 \times 1.4455; \qquad (42)$$

$$S = 60 \times 1.2470; \qquad (43)$$

$$S = 74.82. \qquad (44)$$

Since the right-hand factor in Eq. (43) is greater than unity, we see that reducing the contrast to half its standard value, while at the same time doubling the standard luminance, results in dominance by the contrast loss; i.e., a net degradation of vision from 20/60 to [Eq. (44)] 20/75. (This confirms our previous observation about the greater potency of contrast changes, relative to luminance changes of the same ratio.) In this case, we would apply Eq. (13) to determine the needed stroke width with a value of $\underline{S} = 74.82$, rather than 60, and so the corresponding letters would be about 25 percent larger than they would have been if we had been designing for the same 20/60 observers under standard viewing conditions.

6. SUMMARY OF STEPS TO DETERMINE LETTER SIZE

To determine: the size of letters on a sign.

Given: the maximum distance, d, at which anyone will need to read the sign; and the Snellen index, $20/S$, corresponding to the most visually handicapped people for whom allowance is to be made.

Significance of solution: using the letter size (height), h, to be derived, should allow people with $20/S$ vision or better to read the sign at any distance up to d.

Steps:

1. Decide on the units (e.g., feet, meters) in which the distance d is to be measured. Decide on the units (e.g., inches, centimeters) in which the letter-height h is to be measured. The stroke width, w_S, will be measured in the same units.

2. Decide on the type face to be used. Determine the height-to-strokewidth ratio, g, characterizing letters in that face.

3. Substitute the given viewing distance, d, and the denominator, S, of the given Snellen index, into Eq. (13) to obtain the required stroke width, w_S. The equation is

$$w_S = 1.45 \times 10^{-5} \times S \times d,$$

and at this point w_S will be expressed in the same units as are used for d.

4. Convert w_S as determined in step 3 into the units selected for it in step 1. Examples: to convert w_S in feet to be in inches, multiply by 12; to convert w_S in meters to be in millimeters, multiply by 1000.

5. Multiply the stroke width, w_S, from step 4, by the ratio, g, from step 2, to determine the letter height, h.

6. If feasible, order the sign made with very black letters on a very white background (not feasible if the sign is to be color-coded); and light the sign, when it is installed, to a luminance level of at least 85 candelas per square meter (cd/m^2) [or 25 footlamberts (fL)] on the white background. If both conditions are met, the problem is solved. If either of these two conditions is not feasible or not desired, proceed to step 7 and/or step 8, respectively.

7. If the lettering or the background is noticeably colored, or if the black of the letters is not a really deep black, or if the white of the background is not a really brilliant white, then contrast measurements will have to be made. One alternative would be to use a luminance meter. Obtain a test sign with the exact colors of lettering and background to be used, and set it up under the same type of light source that is going to be used (e.g., high pressure sodium or cool white fluorescent). Use any size letters large enough to make luminance measurements easy. Since the geometry (positioning) of both the light distribution and the measuring instrument, with respect to the sample, can affect contrast, position the sign with respect to the light source in the same way it will be positioned when installed. Hold the luminance meter so that it "sees" the sign at the same angle that most people will view the sign when it is installed. In the absence of other information, let the meter's line of view be perpendicular to the sign surface. Measure the luminance of the letters, $\underline{L}_t$, and the luminance of the background, $\underline{L}_b$. A second alternative is to measure not luminance but luminous reflectance. Again, the same type of light source and geometry must be used as will be used in the area in which the sign will be located. Unfortunately, most reflectance-

measuring instruments contain their own fixed light source, so luminance measurement is more likely to be practical.

8. If it is expected that the luminance of the sign's light background (L_b) may be lower than 85 cd/m^2 when the sign is installed, then luminance measurements similar to those described in step 7 will have to be made. If the sign is to be installed in an already existing lighted space (as opposed to the situation where signs are being planned for a space not yet constructed or not yet lit), then the test sign should be installed in the exact spot where the final sign is to be placed. If only this step, and not step 7 is necessary, there is no need to measure L_t, the luminance of the letters.

9. If step 7 was not carried out, proceed to step 10. If step 7 was necessary, substitute L_b and L_t into Eq. (16) to determine the luminance contrast, C. The equation is

$$C = \frac{L_b - L_t}{L_b} .$$

Then multiply C by 100 to obtain percentage contrast, $\tilde{C}$.

10. Substitute L_b and $\tilde{C}$ into Eq. (39). The equation is

$$S = S_s \left(\frac{85}{L_b}\right)^p \left(\frac{90}{\tilde{C}}\right)^q ,$$

where [Eqs. (23)]

$$p = 0.21310 \text{ and } q = 0.53158.$$

In Eq. (39), the quantity S_s corresponds to the given Snellen denominator, S. The output of Eq. (39) is called S, and is intended to replace the original S. If step 7 was unnecessary (no contrast measured), simply omit the last (contrast) term in Eq. (39); then only L_b, p, and S_s are needed.

11. Using the adjusted $\underline{S}$ value obtained in step 10, go back and carry out steps 4 and 5. Step 5 completes the solution.

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APPENDIX A: TREATMENT OF CASE OF STROKE WIDTH AND VIEWING DISTANCE MEASURED IN DIFFERENT UNITS

It is frequently inconvenient to express the widths of letter strokes in the same units of length in which the viewing distance is measured. If, for example, we want to express $\underline{w_S}$ in millimeters and $\underline{d}$ in meters, we use Eq. (13) to obtain $\underline{w_S}$ in meters, and then simply multiply the answer by 1000 to obtain $\underline{w_S}$ in millimeters. This unit-conversion step can be combined with the basic Eq. (13) to yield, in this case,

$$w_S = 0.0145 \times S \times d. \tag{45}$$

To derive Eq. (45) from Eq. (13), we had to multiply the constant on the right side by 1000, the number of millimeters in a meter. In general, the absolute sizes of the units used for $\underline{w_S}$ and $\underline{d}$ are irrelevant; it is only the <u>ratio</u> of these unit sizes that affects the constant in the equation. Thus, Eq. (45) -- and its exact form, derived from Eq. (12) --

$$w_S = \frac{\pi S d}{216}, \tag{46}$$

apply to <u>any</u> combination of units such that the stroke width $\underline{w_S}$ is measured in units 1/1000 the size of the units in which the distance $\underline{d}$ is measured. For example, Eqs. (45) and (46) apply also when $\underline{d}$ is measured in inches and $\underline{w_S}$ in mils (thousandths of an inch).

We can treat the situation in general. Let $\underline{U_d}$ be the unit (e.g., the meter) in which we want to measure $\underline{d}$. Let $\underline{U_w}$ be the unit (e.g., the millimeter) in which we want to measure $\underline{w}$. Then let

$$r = U_d/U_w. \tag{47}$$

Finally, we can generalize Eq. (12) by inserting $\underline{r}$ on the right side. The result is

$$w_S = \frac{\pi r S d}{216{,}000} \quad . \tag{48}$$

A similar generalization of the numerical form, Eq. (13), yields

$$w_S = 1.45 \times 10^{-5} \times r \times S \times d. \tag{49}$$

When the units of measurement of $\underline{w_S}$ and $\underline{d}$ are the same, then $\underline{r} = 1$, and Eq. (48) reduces to Eq. (12), as it should. Similarly, when $\underline{d}$ is measured in meters and $\underline{w_S}$ in millimeters, then $\underline{r} = 1000$, and Eq. (48) reduces to Eq. (46), as required.

We have established that regardless of the choice of units for $\underline{d}$ and $\underline{w_S}$, the required stroke width, $\underline{w_S}$, for a person with 20/$\underline{S}$ vision looking at a display at a distance $\underline{d}$ from him is

$$w_S = p_r S d, \tag{50}$$

where $\underline{p_r}$ is a constant -- depending on the choice of units -- defined as

$$p_r = \frac{\pi r}{216{,}000} \quad . \tag{51}$$

The explicit (three-significant-figure) equivalent of Eq. (51) is

$$p_r = 1.45 \times 10^{-5} \times r. \tag{52}$$

Table 3 gives the values of the ratio $\underline{r}$ corresponding to a variety of possible choices of the units of measurement for $\underline{w_S}$ and $\underline{d}$. Table 4 gives expressions for the exact values of the constant $\underline{p_r}$ corresponding to the same combinations of measurement units. The numerical equivalents, to three significant figures, are given in Table 5. For most ordinary applications, it will be necessary to consult only Table 5.

The relationship (50) -- like Eq. (12), of which it is a generalization -- is of a very simple form: it says that $\underline{w_S}$ is simply proportional to $\underline{d}$, the constant of proportionality being $\underline{p_r S}$. If we were to make a particular choice of $\underline{r}$, thus fixing a numerical value for $\underline{p_r}$, we could then plot $\underline{w_S}$ as a function

Table 3. Ratios of Distance Units to Stroke-Width Units.[a] These Ratios Correspond to Values of the Constant $r=U_d/U_w$, in Eq. (47).

Stroke-Width Units	Distance Units: Feet (Exact)	Distance Units: Yards (Exact)	Distance Units: Meters: Exact	Distance Units: Meters: Approximate
Thousandths of an inch (mils)	12,000	36,000	$\frac{50,000}{1.27}$	39,370.
Hundredths of an inch	1,200	3,600	$\frac{5,000}{1.27}$	3,937.
Sixty-fourths of an inch	768	2,304	$\frac{3,200}{1.27}$	2,520.
Millimeters	304.8	914.4	1,000	1,000.[b]
Tenths of an inch	120	360	$\frac{500}{1.27}$	393.7
Centimeters	30.48	91.44	1.00	100.0[b]
Inches	12	36	$\frac{50}{1.27}$	39.37
Meters			1	1.000[b]

[a] The ratios for stroke width in meters and distance in feet and yards are omitted as combinations unlikely to be applied in practice.

[b] These three ratios are exact; all other numbers in this column are approximate.

Table 4. Exact Values of the Constant $p_r = \pi r/216{,}000$, From Eq. (51).[a] Values of r for the Same Combinations of Distance Units and Stroke-Width Units Are Given in Table 3.

Stroke-Width Units	Distance Units: Feet	Yards	Meters
Thousandths of an inch (mils)	$\frac{\pi}{18}$	$\frac{\pi}{6}$	$\frac{625\pi}{3{,}429}$
Hundredths of an inch	$\frac{\pi}{180}$	$\frac{\pi}{60}$	$\frac{125\pi}{6{,}858}$
Sixty-fourths of an inch	$\frac{4\pi}{1{,}125}$	$\frac{4\pi}{375}$	$\frac{40\pi}{3{,}429}$
Millimeters	$\frac{127\pi}{90{,}000}$	$\frac{127\pi}{30{,}000}$	$\frac{\pi}{216}$
Tenths of an inch	$\frac{\pi}{1{,}800}$	$\frac{\pi}{600}$	$\frac{25\pi}{13{,}716}$
Centimeters	$\frac{127\pi}{900{,}000}$	$\frac{127\pi}{300{,}000}$	$\frac{\pi}{2{,}160}$
Inches	$\frac{\pi}{18{,}000}$	$\frac{\pi}{6{,}000}$	$\frac{5\pi}{27{,}432}$
Meters			$\frac{\pi}{216{,}000}$

[a] The values for stroke width in meters and distance in feet or yards are omitted as combinations unlikely to be applied in practice.

Table 5. Three-Figure Numerical Values of the Constant p_r of Eq. (50).[a] Exact Expressions for These Values Are Given in Table 4.

Stroke-Width Units	Distance Units		
	Feet	Yards	Meters
Thousandths of an inch (mils)	1.75×10^{-1}	5.24×10^{-1}	5.73×10^{-1}
Hundredths of an inch	1.75×10^{-2}	5.24×10^{-2}	5.73×10^{-2}
Sixty-fourths of an inch	1.12×10^{-2}	3.35×10^{-2}	3.66×10^{-2}
Millimeters	4.43×10^{-3}	1.33×10^{-2}	1.45×10^{-2}
Tenths of an inch	1.75×10^{-3}	5.24×10^{-3}	5.73×10^{-3}
Centimeters	4.43×10^{-4}	1.33×10^{-3}	1.45×10^{-3}
Inches	1.75×10^{-4}	5.24×10^{-4}	5.73×10^{-4}
Meters			1.45×10^{-5}

[a] The values for stroke width in meters and distance in feet or yards are omitted as combinations unlikely to be applied in practice.

of d, with S as a parameter. The result would be a family of lines through the origin of the w_S against d plot, each line corresponding to a particular value of S.

Just as we derived Eq. (14) from Eq. (13), we take the logs of both sides of Eq. (50) and obtain

$$\log w_S = \log p_r + \log S + \log d. \tag{53}$$

Inspection of Eq. (53) shows that a plot of log w_S against log d always yields a straight line of unit slope (at 45°), regardless of the choice of measurement units (p_r) or of the observer's visual acuity (indexed by S). Hence, the pencil ("fan") of lines through the origin obtained by plotting w_S against d is replaced, in the log-log plot, by a family of parallel lines of unit slope. The spacing between these lines, each line corresponding to a particular value of S, is fixed, once a particular selection of S values is chosen. The choice of units of measurement, embodied in the constant p_r, affects the graph only by moving the entire parallel bundle of lines rigidly up or down along the w_S axis, in accordance with the value of the additive constant log p_r.

Just as we were able to plot Fig. 2 using Eq. (13), we can plot analogous graphs for any combination of units for w_S and d by using Eq. (50). If the combination of units of interest is listed in Table 5, the value of p_r can be read from the table and substituted directly into Eq. (50). Otherwise, the value of p_r for any combination of units can be calculated from Eq. (52), based on the r value defined in Eq. (47).

Three additional graphs of this sort are presented in Figs. 6 through 8. The unit combinations represented were chosen to be of likely practical interest.

Figure 6, like Fig. 2, is metric, applying for viewing distance measured in meters, and stroke width in millimeters. Figures 7 and 8 deal with customary (English) units, Fig. 7 applying for viewing distance in feet and stroke width in inches, and Fig. 8 applying when the viewing distance is given instead in yards.

Figures 2 and 6 to 8 can be used for rapid, order-of-magnitude estimates, but Eq. (50) is so easy to use that the algebraic procedure outlined two paragraphs back should be the method of choice for serious applications in which repeated unit conversions are to be avoided.

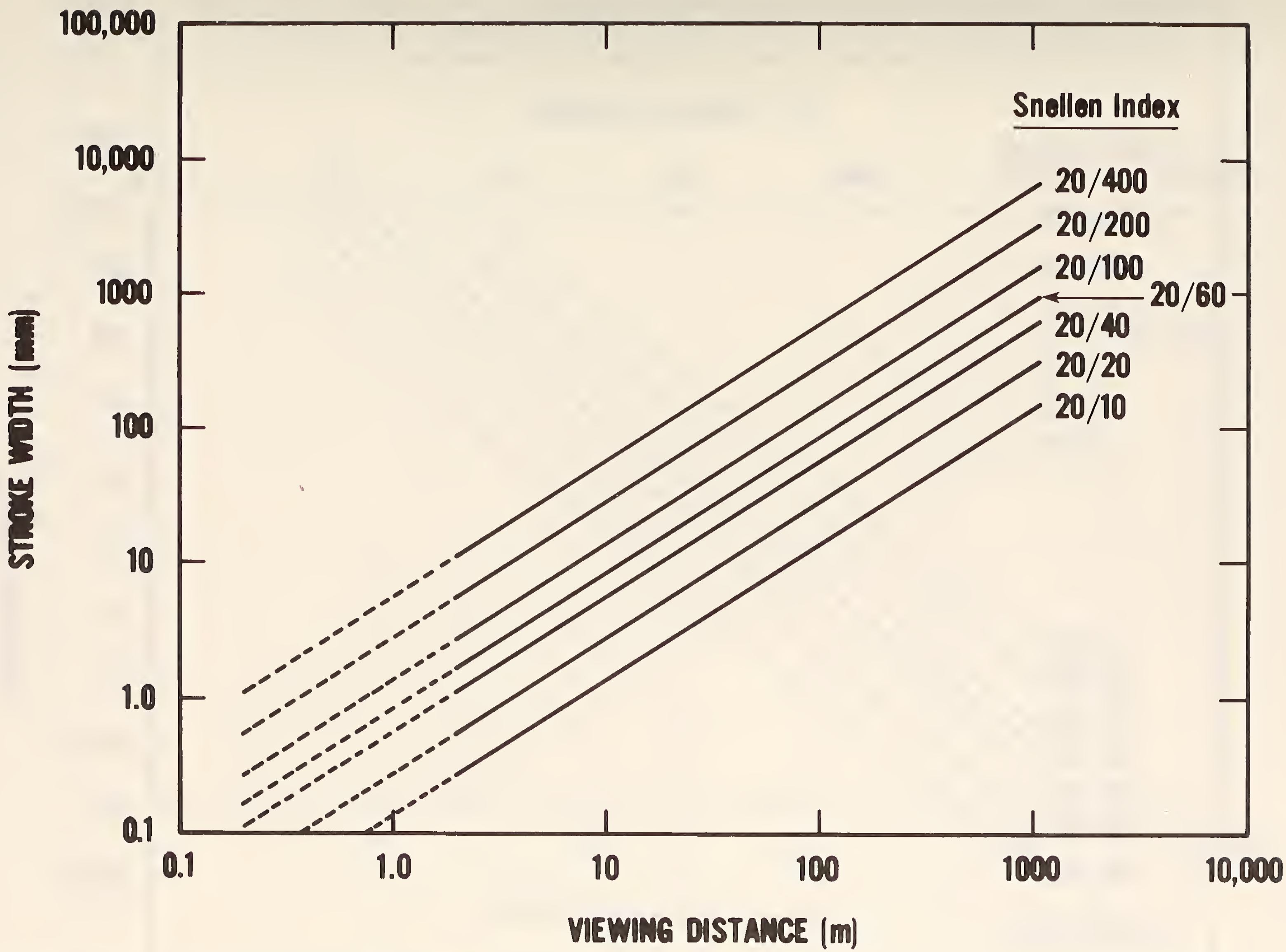

Figure 6. Log-log plot of minimum stroke width (millimeters) required for letter visibility as a function of viewing distance (meters), with observer visual acuity as a parameter. The lines correspond to Eq. (50), for seven values of the Snellen denominator, *S*.

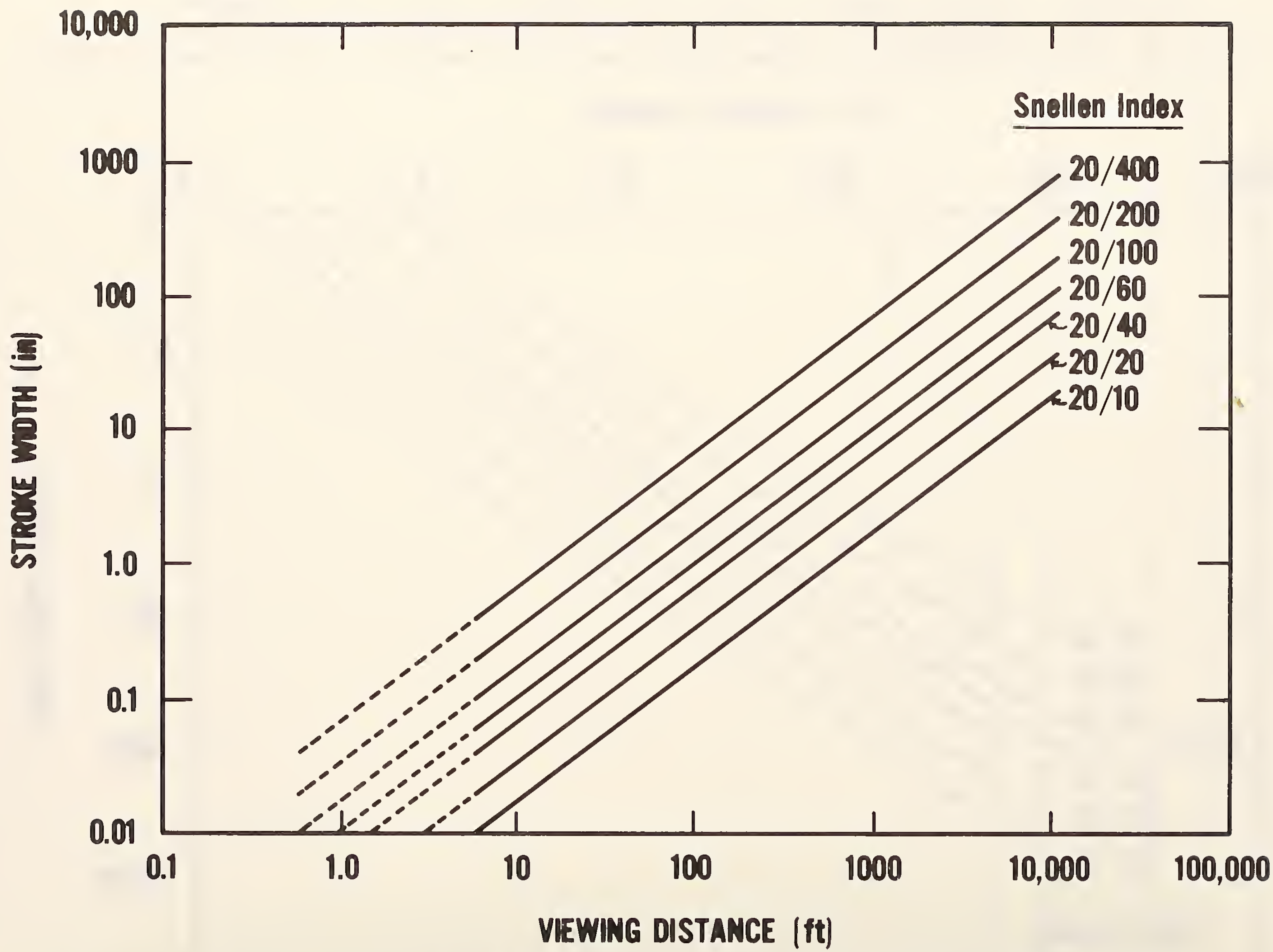

Figure 7. Log-log plot of minimum stroke width (inches) required for letter visibility as a function of viewing distance (feet), with observer visual acuity as a parameter. The lines correspond to Eq. (50), for seven values of the Snellen denominator, $\underline{S}$.

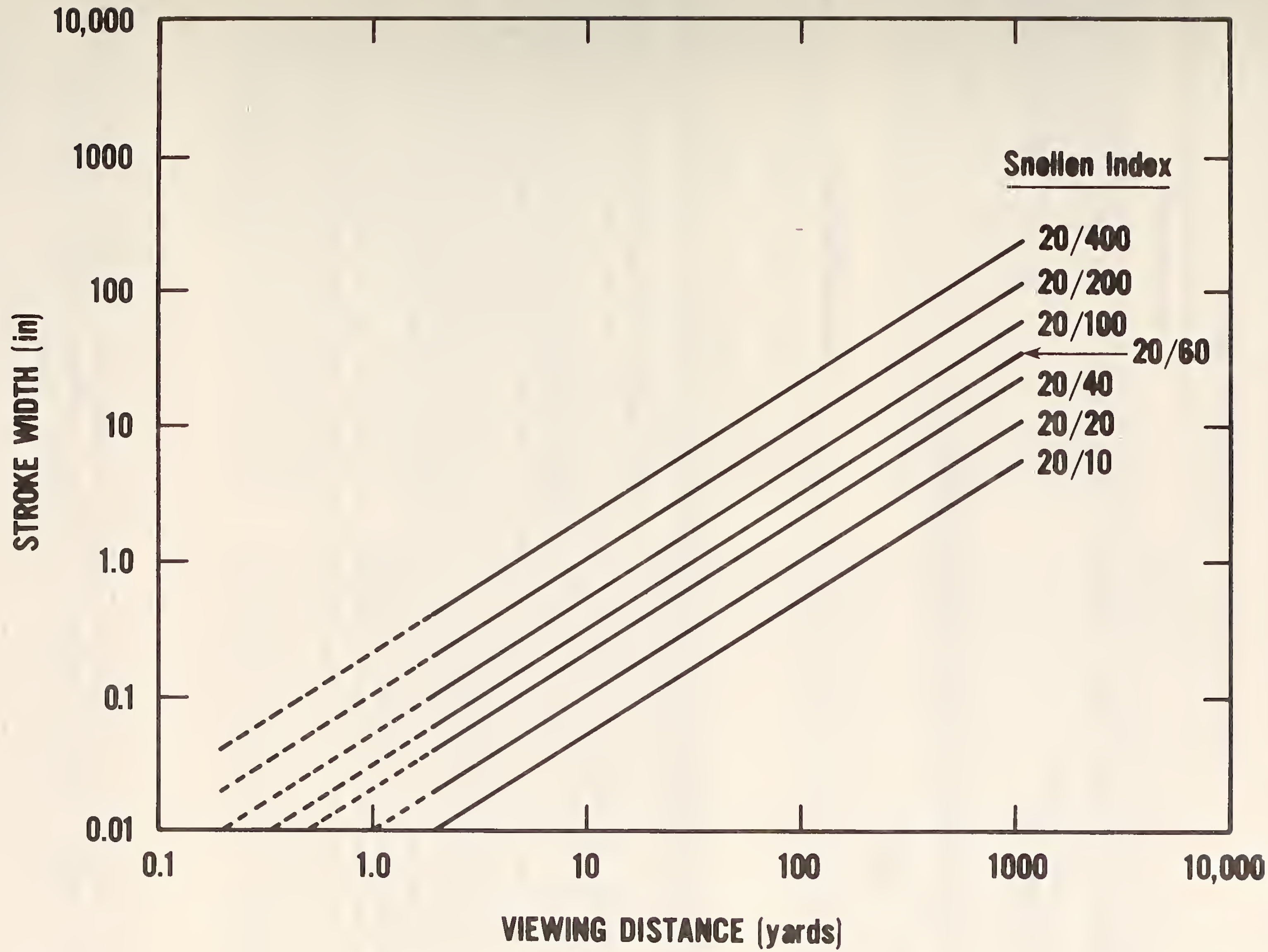

Figure 8. Log-log plot of minimum stroke width (inches) required for letter visibility as a function of viewing distance (yards), with observer visual acuity as a parameter. The lines correspond to Eq. (50), for seven values of the Snellen denominator, S.

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

A formula is derived giving the letter stroke-width needed for legibility of words on a sign at any given distance by an observer with any given visual acuity. The stroke width, in turn, determines the letter size, depending upon the characteristics of the type face used. The derivation is strictly mathematical and is based on the assumption that beyond a distance of a few meters, a person's visual acuity is specifiable by a fixed visual angle, independent of the distance. The information implicit in the formula is also presented graphically, in four plots that apply to four different combinations of length units for measuring stroke width and viewing distance. Also presented are formulas and graphs for correcting the critical stroke width for nonstandard contrast or background luminance. These correction formulas are based on a body of data on visual acuity as a function of contrast and background luminance, and a formula fitting the mid-ranges of the data, both published recently by other researchers.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Acuity, visual; angle, visual; contrast; distance, viewing; letters; luminance; resolution, eye; signs; Snellen chart; stroke width; visual acuity; visual angle.

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