Cooling of Bitumen During Construction of Built-Up Roofing Systems—A Mathematical Model
NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards\(^1\) was established by an act of Congress on March 3, 1901. The Bureau’s overall goal is to strengthen and advance the Nation’s science and technology and facilitate their effective application for public benefit. To this end, the Bureau conducts research and provides: (1) a basis for the Nation’s physical measurement system, (2) scientific and technological services for industry and government, (3) a technical basis for equity in trade, and (4) technical services to promote public safety. The Bureau’s technical work is performed by the National Measurement Laboratory, the National Engineering Laboratory, and the Institute for Computer Sciences and Technology.

THE NATIONAL MEASUREMENT LABORATORY provides the national system of physical and chemical and materials measurement; coordinates the system with measurement systems of other nations and furnishes essential services leading to accurate and uniform physical and chemical measurement throughout the Nation’s scientific community, industry, and commerce; conducts materials research leading to improved methods of measurement, standards, and data on the properties of materials needed by industry, commerce, educational institutions, and Government; provides advisory and research services to other Government agencies; develops, produces, and distributes Standard Reference Materials; and provides calibration services. The Laboratory consists of the following centers:


THE NATIONAL ENGINEERING LABORATORY provides technology and technical services to the public and private sectors to address national needs and to solve national problems; conducts research in engineering and applied science in support of these efforts; builds and maintains competence in the necessary disciplines required to carry out this research and technical service; develops engineering data and measurement capabilities; provides engineering measurement traceability services; develops test methods and proposes engineering standards and code changes; develops and proposes new engineering practices; and develops and improves mechanisms to transfer results of its research to the ultimate user. The Laboratory consists of the following centers:


THE INSTITUTE FOR COMPUTER SCIENCES AND TECHNOLOGY conducts research and provides scientific and technical services to aid Federal agencies in the selection, acquisition, application, and use of computer technology to improve effectiveness and economy in Government operations in accordance with Public Law 89-306 (40 U.S.C. 759), relevant Executive Orders, and other directives; carries out this mission by managing the Federal Information Processing Standards Program, developing Federal ADP standards guidelines, and managing Federal participation in ADP voluntary standardization activities; provides scientific and technological advisory services and assistance to Federal agencies; and provides the technical foundation for computer-related policies of the Federal Government. The Institute consists of the following centers:

- Programming Science and Technology — Computer Systems Engineering.

\(^1\)Headquarters and Laboratories at Gaithersburg, MD, unless otherwise noted; mailing address Washington, DC 20234.

\(^2\)Some divisions within the center are located at Boulder, CO 80303.
Cooling of Bitumen During Construction of Built-Up Roofing Systems—A Mathematical Model

Walter J. Rossiter, Jr.¹
Robert G. Mathey¹
Herbert W. Busching²
William C. Cullen³

¹ Center for Building Technology
National Engineering Laboratory
National Bureau of Standards
Washington, DC 20234

² Department of Civil Engineering
Clemson University
Clemson, SC 29613

³ Office of Engineering Standards
National Engineering Laboratory
National Bureau of Standards
Washington, DC 20234

U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

Issued March 1981
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Objective</td>
<td>4</td>
</tr>
<tr>
<td>2. COOLING OF BITUMEN</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Factors Affecting Cooling</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Model for Predicting Cooling Time</td>
<td>5</td>
</tr>
<tr>
<td>2.2.1 Assumptions in the Development of the Model</td>
<td>5</td>
</tr>
<tr>
<td>2.2.2 Finite-Difference Heat Flow Equations</td>
<td>6</td>
</tr>
<tr>
<td>3. APPLICATION OF MODEL FOR PREDICTING BITUMEN COOLING</td>
<td>11</td>
</tr>
<tr>
<td>3.1 Values Needed to Solve Model</td>
<td>11</td>
</tr>
<tr>
<td>3.2 Results for Specific Roofing Systems</td>
<td>13</td>
</tr>
<tr>
<td>3.3 Uses for Model</td>
<td>16</td>
</tr>
<tr>
<td>4. SUMMARY</td>
<td>17</td>
</tr>
<tr>
<td>5. ACKNOWLEDGMENTS</td>
<td>18</td>
</tr>
<tr>
<td>6. REFERENCES</td>
<td>18</td>
</tr>
<tr>
<td>APPENDIX A. LISTING OF FINITE-DIFFERENCE EQUATIONS</td>
<td>A1</td>
</tr>
<tr>
<td>A.1 Finite-Difference Equations for the Four-Component System</td>
<td>A3</td>
</tr>
<tr>
<td>A.2 Finite-Difference Equations for the Three-Component System</td>
<td>A4</td>
</tr>
<tr>
<td>A.3 Finite-Difference Equations for the Two-Component System</td>
<td>A5</td>
</tr>
<tr>
<td>APPENDIX B. LISTING OF COMPUTER PROGRAM</td>
<td>B1</td>
</tr>
<tr>
<td>B.1 Check on Computer Program Execution</td>
<td>B13</td>
</tr>
<tr>
<td>APPENDIX C. NOTATION</td>
<td>C1</td>
</tr>
</tbody>
</table>
Figure 1. Modes of Heat Transfer During Construction of Bituminous Built-Up Roofing Systems. The Effect of Solar Radiation was not Included in the Mathematical Model. Heat Transfer due to Convection and Radiation from the Underside of Thin Decks Without Insulation was Included in the Model .......... 26

Figure 2. Diagram of the Elements Within the Bitumen and Top Elements of the Roofing Component Below the Bitumen ............. 27

Figure 3. Diagram of the Elements and Nodes Used to Develop the Finite-Difference Equations for Heat Flow Within the Bitumen, at the Surface of the Bitumen, and at the Bottom Boundary of the Bitumen .......................... 28

Figure 4. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 29

Figure 5. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 30

Figure 6. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Insulating Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 31

Figure 7. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 32

Figure 8. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 33

Figure 9. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Steel. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²) ....................... 34
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Coal Tar Pitch Applied on Concrete. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft² (1.2 kg/m²)</td>
<td>35</td>
</tr>
<tr>
<td>11.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²)</td>
<td>36</td>
</tr>
<tr>
<td>12.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²)</td>
<td>37</td>
</tr>
<tr>
<td>13.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²)</td>
<td>38</td>
</tr>
<tr>
<td>14.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²)</td>
<td>39</td>
</tr>
<tr>
<td>15.</td>
<td>Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Coal Tar Pitch Applied on Felt over Coal Tar Pitch on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft² (1.2 kg/m²)</td>
<td>40</td>
</tr>
<tr>
<td>16.</td>
<td>Effect of Asphalt Thickness on the Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation</td>
<td>41</td>
</tr>
<tr>
<td>17.</td>
<td>Effect of Substrate on the Asphalt Cooling Time</td>
<td>42</td>
</tr>
<tr>
<td>18.</td>
<td>Temperature Profiles of the Three-Component Roofing Systems, Asphalt Applied to Felt on Plywood and Asphalt Applied to Felt on Polyurethane Foam Insulation, for Various Times after Bitumen Application. Asphalt at 500°F (260°C) was Applied to the Felt and Substrate at 120°F (49°C) with Zero Wind Speed</td>
<td>43</td>
</tr>
<tr>
<td>19.</td>
<td>Plot of Dimensionless Temperature Versus Time for the Two-Component Roofing System, Asphalt Applied on Steel</td>
<td>44</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (Continued)

Figure A1. Diagram of the Finite-Elements Within the Components of the Roofing System Used for the Calculation of Bitumen Cooling Time by Finite-Difference Techniques .......... A2

LIST OF TABLES

Table 1. Thermal Properties of the Roofing Component Materials for Use in the Model .................................................. 21

Table 2. Values of Roofing Component Temperatures and Environmental Factors Used in the Calculations .............................. 22

Table 3. Thickness of the Roofing Component Materials and Corresponding Number of Elements Used in the Calculations ... 23

Table 4. Configurations of Roofing Components for Which the Cooling Time of the Applied Bitumen Was Calculated ..................... 24

Table 5. Heat Capacities of Various Substrates and Cooling Times for Asphalt When Applied on Them ...................................... 25

Table B1. Temperature Distribution in the Finite Elements of the Flat Plate as Determined from the Computer Program and the Temperature Charts ................................................. B14

Table C1. Notation and Description of Terms ........................................ C1
COOLING OF BITUMEN DURING CONSTRUCTION OF BUILT-UP ROOFING SYSTEMS - A MATHEMATICAL MODEL

by

Walter J. Rossiter, Jr.
Robert G. Mathey
Herbert W. Busching
William C. Cullen

ABSTRACT

Construction of bituminous built-up roofing systems in the United States generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Adequate adhesion of the bitumen to the roofing component materials may be obtained only when the hot bitumen is applied at a viscosity sufficient to flow uniformly, to cover the component surfaces or substrate completely and to provide the proper thickness. During construction, rapid cooling of hot bitumen increases its viscosity significantly. If the viscosity becomes too high, poor adhesion between components, voids within the bitumen, and an excessive and non-uniform thickness of the bitumen may result.

This report describes a mathematical model based on finite-difference equations for calculating transient heat flow to estimate the cooling time of hot roofing bitumen. Estimates of the time required for hot bitumen to cool from its application temperature to 300°F (149°C) were computed as a function of material and environmental factors, including: quantity of applied bitumen, bitumen application or contact temperature, air temperature, wind speed, and thermal properties of the bitumen and of the roofing components. The model was used to predict cooling times expected for hot asphalt applied to typical substrates with thermal properties representative of those of polyurethane foam and glass fiber insulation boards, insulating concrete, plywood, concrete and steel decks, and roofing felt on decks or insulations. In addition, the model was used to predict cooling times for hot coal tar pitch applied to concrete and to felt adhered to glass fiber insulation. The results of the calculations demonstrate the widely varying bitumen cooling times which depend upon the component material to which the bitumen is applied and the environmental conditions during application. Under certain environmental conditions, hot bitumen applied to some substrates cools extremely rapidly. In these cases, sufficient time for proper application may not be available.

Key words: Bitumen application temperatures; bituminous roofing; built-up roofing; cooling of roofing bitumens; mathematical model; roofing.

vii
1. INTRODUCTION

The most widely accepted waterproofing system for low-sloped roofs in the United States is bituminous built-up roofing. Construction of bituminous built-up roofing systems generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Bitumens commonly used as waterproofing materials and adhesives in built-up membranes are asphalt and coal tar pitch.

Roofing bitumens must be applied at elevated temperatures so that their viscosities will be sufficiently low to enable uniform flow, complete coverage of component surfaces, and proper thickness. After hot bitumen is applied to the substrate, it cools rapidly. Little time is available to place roofing felts or insulation on the hot bitumen while its temperature is sufficiently high and its viscosity is sufficiently low to provide for adequate adhesion and proper thickness. Laboratory data are available on the relationship between temperature and viscosity of roofing asphalts [1,2]. However, little data were found in the literature on the cooling time of hot bitumen during built-up roof construction. A field study has been reported wherein the cooling time of asphalt during cold weather construction was determined [3].

Since the rate at which roofing bitumens cool is important to the quality of built-up roofing construction, a mathematical model was developed to enable quantitative assessment of the effects of various factors on the cooling time of roofing bitumens. The mathematical model is based on well-known heat transfer principles, uses finite-difference equations [4-6], and makes use of representative roofing materials and their properties. It also incorporates the principal environmental variables associated with roof construction. The mathematical model to predict the cooling time of bitumen assumes one dimensional heat transfer; that is, the roof is taken to be a flat plate with its components at uniform temperature across the horizontal planes of the plate. During built-up roofing construction, hot bitumen is applied over large areas of the roof. Because large areas of the roof are considered to be at uniform temperature under these conditions, the heat transfer was assumed to occur in the direction perpendicular to the plane of the roof and not laterally within its plane.

Cooling time of bitumen can in principle be determined experimentally by means of field studies. However, measurement of cooling times applicable to many different types of roofing systems under a variety of environmental conditions are precluded in practice because of the excessive time and expense required to conduct investigations involving many variables. Nevertheless, selective field studies to obtain data on bitumen cooling times are needed to assess the validity of the model presented.

* Figures in brackets indicate references listed in Section 6.
1.1 BACKGROUND

Rossiter and Mathey [1] measured the viscosities of ASTM Type I and Type III roofing asphalts from different sources over a range of application temperatures. They recommended that roofing asphalts be applied at temperatures based on viscosity rather than on empirically determined temperature limits. From a similar investigation, Ducy recommended that roofing asphalts be applied based on an equiviscous temperature (EVT) [2]. It was suggested by Rossiter and Mathey [1] that in order to maintain the recommended viscosity range during application, consideration should be given to factors affecting the rate of cooling such as air temperature, wind speed, substrate temperature, and the specific heat of the substrate. A viscosity criterion for the application of asphalt has not generally been used in roofing construction nor included in specifications. It is noted, for example, that the National Roofing Contractors Association (NRCA) [7] recommends a minimum application temperature of 350°F (177°C) and a maximum heating temperature of 475°F (246°C) for asphalt. Likewise, the Tri-Service Roofing Manual [8] states that asphalt should not be heated above 450°F (232°C) and should not normally be lower than 350°F (177°C) when applied to the roof. Current trends in the roofing industry indicate an acceptance of the selection of application temperatures based upon the viscosity of the bitumen.

During application, if bitumen cools too rapidly with an accompanying increase in its viscosity, the bitumen layers may be too thick, uneven, contain voids, and result in inadequate adhesion between roof components. These effects result in unsatisfactory built-up bituminous membrane performance. A thick or uneven application of interply bitumen can lead to poor adhesion between plies [1]. Voids between plies or between the membrane and insulation are sources of blistering [9]. Excessive thickness of between-ply bitumen under certain conditions may contribute to membrane slippage [10]. Since the bitumen has a larger coefficient of thermal expansion than other membrane materials, excessive thickness of bitumen between plies may result in higher thermally induced forces in the membrane. Lee, Dupuis, and Johnson [11] showed that temperature-induced forces measured for 2-ply membranes increased as the quantity of interply asphalt was increased. Temperature-induced forces due to a temperature change from 70 to -20°F (21 to -29°C) at an unspecified rate, measured for "thin, normal, and thick" interply thicknesses of asphalt, were about 54, 55, and 80 lbf/in (9.4, 9.6, and 14.0 kN/m), respectively [11].

In a field investigation of cooling times of asphalt applied during cold weather, Dupuis, Lee, and Johnson [3] measured temperatures of hot asphalt in kettles and the contact temperatures (temperatures at the time the asphalts came in contact with the substrates). They reported that during asphalt application, peak contact temperatures* were usually 50 - 75°F (28 - 42°C) lower than the temperature in the asphalt kettle. Asphalt cooling times

------------------------
* Peak contact temperature was defined as the highest temperature recorded during asphalt application [3].

2
were recorded at various locations within insulated roofing systems under construction in four cities in Wisconsin with air temperatures ranging from about 20 to 40°F (-7 to 4°C) and wind speeds from about 3 to 14 mph (1.4 to 6.3 m/s). Under those weather conditions, they found that the asphalt cooled rapidly. For example, in the application of asphalt to insulation, the asphalt temperatures dropped from the contact temperature to less than 300°F (149°C) in about 2 to 7 seconds.

The rapid cooling of asphalt observed by Dupuis, Lee, and Johnson [3] raised a question about adhesion of roofing components. They reported that rapid temperature decay of hot asphalt makes good adhesion extremely difficult to attain. They also stated that hot asphalt applied at subfreezing temperatures to light gauge steel decks congeals so rapidly that mechanical fastening is probably the only effective technique for securely anchoring the insulation. With regard to membrane application, they indicated that raising peak asphalt contact temperatures during cold weather may increase the time the asphalt remains sufficiently hot to adhere a ply of felt adequately.

In presenting the rate of heat loss (time dependency) of roofing asphalts, Dupuis, Lee, and Johnson [3] expressed the temperature of the asphalt with time as:

\[ T = T_s + (T_o - T_s) e^{-\kappa t} \]

where \( T \) = the asphalt temperature at time \( t \)
\( T_s \) = the temperature of the surroundings
\( T_o \) = the initial temperature of the asphalt at application
\( \kappa \) = an empirical constant

It is noted that this model was empirical and did not include terms for all the factors which affect the cooling time of asphalt. The cooling data obtained by Dupuis, Lee, and Johnson have not been compared with data generated by the model presented in this paper.

The roofing industry in the United States has not employed mathematical models to predict the cooling time of hot bitumen during construction of built-up roofing systems. A mathematical model would benefit the roofing industry since it would provide guidelines for the proper application of hot bitumen without excessive cooling on various components under a range of environmental conditions. Mathematical models were developed by Dickson and Corlew [12,13] for predicting temperatures of hot asphaltic concrete pavement layers placed in cold weather. These layers must be compacted while the viscosity of the asphalt is low enough to enable compaction of a pavement of proper density. Dickson and Corlew [12,13] used established finite-difference equations as a basis for evaluating temperature profiles in the hot asphaltic concrete pavement layers. Several references [14-18] identify the rationale for preheating the base to permit cold weather highway and airfield paving. The techniques used in estimating temperatures of paving mixtures are adaptable, with some modification, for use in estimating the cooling times of bitumen during the application of built-up roofing systems.
1.2 OBJECTIVE

The objective of this study was to develop a mathematical model for predicting the cooling time or time interval for hot bitumen to cool from its application temperature to a specified lower temperature during construction of built-up roofing systems. This paper presents examples of temperature decay charts generated from the model to enable the prediction of bitumen cooling times for various environmental conditions and for some roofing systems typical of those encountered in built-up roofing construction. The computer program based on the model is given to make it available to those individuals who wish to calculate bitumen cooling times for built-up roofing systems other than those presented here. Busching [19] has presented some early results from this study in a paper concerning the effects of moisture and temperature on roofing membranes.

Data developed through the use of the model may be useful in applying viscosity criteria and specifications effectively. In addition, information on the time of cooling for a wide range of variables may be used to help assure proper application of hot bitumen and other roofing components to improve construction practice. Data may be obtained from a computerized model at much lower cost than would be encountered if the same data were obtained through field experiment. However, before full reliance is placed on the results of the model calculations, data should be obtained as recommended in section 1 through selective field studies to assess the validity of the model.

2. COOLING OF BITUMEN

Hot bitumen is normally applied to roofs by mopping or by mechanical application. The method for applying hot bitumen was not a consideration in the development of the model to predict cooling times of bitumen. The model assumes that the temperature of the hot bitumen at the time of contact with the surface to which it is applied is known. The hot bitumen begins to cool immediately upon contact with the surface. The cooling time is dependent on factors associated with environmental conditions and materials properties.

2.1 FACTORS AFFECTING COOLING

Many factors affect the cooling of hot bitumen during roof construction. These factors include application temperature and thermal properties of the bitumen, quantity of bitumen applied per unit area (spread quantity), ambient air temperature, wind speed on the roof, and temperature, thickness and thermal properties of the component materials below the hot bitumen.

Thermal properties of the bitumen, substrate, and other component materials which affect the cooling are thermal conductivity, specific heat, and density. The application temperature of the hot bitumen is defined as the temperature at which the bitumen makes contact with the surface to which it is applied. The factors which affect cooling were incorporated in the mathematical model
for predicting the cooling time of hot bitumen occurring in the construction of built-up roofing systems.

Most individuals experienced with roofing construction would attest that many of the factors listed previously affect the cooling of hot bitumen. However, the effect of these factors on bitumen cooling is only understood qualitatively. For example, a decrease in either the ambient air or substrate temperature will result in a decrease in the cooling time. The time of cooling will also decrease with an increase in wind speed. In addition, an increase in either the application temperature or quantity of hot bitumen will increase the time available to apply component roofing materials before the viscosity of the bitumen becomes too high to enable good adhesion. A mathematical model to predict cooling time of bitumen would quantify the effect of these factors.

The effect of the thermal properties (thermal conductivity, specific heat, and density) of the substrate and component materials on the cooling of hot bitumen may not be as evident to those experienced with roofing construction as for the environmental factors. Calculations using the mathematical model also allow quantitative determination of the effect of these thermal properties on bitumen cooling. Definitions of these thermal properties and related parameters are as follows:

- **Thermal conductivity (k)** - Time rate of heat flow through a homogeneous material under steady-state conditions through unit area, per unit temperature gradient. Btu·in/h·ft²·°F (W/m·K)

- **Specific heat (C_p)** - The quantity of heat required for a one degree temperature change in a unit mass of material. Btu/lbm·°F (J/kg·K)

- **Density** (ρ) - The mass of a material per unit volume. lbm/ft³ (kg/m³)

- **Heat capacity** (ρC_p) - The quantity of heat required for a one degree temperature change in a unit volume of material. Btu/ft³·°F (J/m³·K)

- **Thermal diffusivity (α)** - The ratio of a material's capacity to conduct thermal energy to its capacity to store thermal energy. ft²/h (m²/s) Thermal diffusivity is defined by the relationship:

\[
\alpha = \frac{k}{\rho C_p}
\]

### 2.2 MODEL FOR PREDICTING COOLING TIME

#### 2.2.1 Assumptions in the Development of the Model

Assumptions made in the development of the mathematical model using finite-difference equations for predicting cooling time of hot bitumen are as follows:
heat transfer from the hot bitumen is one-dimensional perpendicular to the plane of the roof;

the values of the properties of the bitumen and other roofing system components remain constant with time as the bitumen cools;

the thickness of the bitumen does not change as it cools;

absorbed solar radiation during bitumen application is negligible and neglected;

heat is lost from the bottom of the substrate by radiation and convection unless the substrate contains more than 100 elements;

the elements for each of the roofing system components have the same thickness except for the half elements at the top and bottom surfaces of the system;

the temperature of the bitumen was taken as the average of the temperature of all elements in the bitumen component;

the temperature of the air and sky are taken to be the same; and

the thermal resistance at the interfaces of the roofing system components is taken to be zero.

2.2.2 Finite-Difference Heat Flow Equations

2.2.2.1 Description of the System

During the construction of built-up roofing systems, hot bitumen is applied to cooler roofing materials and components such as decks, insulations and felts. Figure 1 shows schematically the modes of heat transfer that are considered in the analysis. Heat is lost from the hot bitumen to the cooler surroundings by convection and radiation at the top surface, and by conduction at the bottom surface which interfaces with the roofing component to which the bitumen is applied. In addition, as heat is lost from the surfaces of the bitumen layer, heat is transferred by conduction from the interior of the bitumen layer towards the cooler surfaces.

As shown in figure 1, heat flow from the hot bitumen is directed up from its top surface and down into the substrate. The heat flow from the upper surface of the bitumen is dependent on environmental conditions at the surface and on the temperature of the bitumen. Heat flow from the lower surface of the hot bitumen is dependent upon the temperature and thermal properties of the substrate as well as the temperature of the bitumen.

Radiant energy from the sun may also be incident on the roof and partially absorbed by the surface of the bitumen during built-up roofing construction. The quantity of radiant energy is negligible during bitumen application.
because of the relatively short time (usually a few seconds) during which the hot bitumen is exposed to solar radiation prior to being covered with insulation board or ply of felt.

The mathematical model to predict the cooling time of hot bitumen uses well-known finite-difference equations to calculate heat transfer. The treatment presented is based in general on that described by Dusinberre [5,6]. The calculations incorporated one-dimensional balanced heat flow between discrete nodes at specific locations within the hot bitumen. One-dimensional heat flow equations were developed that included terms which account for heat transfer by conduction, convection, and radiation depending upon the location of the discrete nodes selected. Figure 2 presents a diagram of the hot bitumen layer divided into elements for development of the finite-difference equations. The diagram in figure 2 shows the location of the nodes, the distance between nodes, the thickness of the elements, and the notation used to describe the temperature of the elements. Finite-difference equations were developed for three types of elements: the elements within the bitumen and the two boundary elements of the top and bottom surfaces. The development of these finite-difference equations is given in the sections which follow.

2.2.2.2 Heat Flow Within the Bitumen

Heat transfer by conduction within a material can be described by the partial differential equation for one-dimensional heat flow:

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{1}
\]

where \(\alpha\) is the thermal diffusivity of the material.

As applied to the cooling of hot bitumen, this equation describes the relationship between bitumen temperature, \(T_b\), time, \(t\), thermal diffusivity of the bitumen, \(\alpha_b\), and a position coordinate, \(y\). The finite-difference form of equation (1) for the cooling of bitumen can be expressed as:

\[
\frac{T_{bi} - T_{bi}}{\Delta t} = \alpha_b \left( \frac{T_{b(i-1)} - T_{bi}}{\Delta y^2} + \frac{T_{b(i+1)} - T_{bi}}{\Delta y^2} \right) \tag{2}
\]

where \(T_{b(i-1)}\), \(T_{bi}\), and \(T_{b(i+1)}\) are temperatures in adjacent bitumen elements having a thickness of \(\Delta y\) (figure 2). The terms \(T_{bi}^\prime\) and \(T_{bi}\) are the temperatures in bitumen element \(i\) at times \(t + \Delta t\), and \(t\), respectively. Substituting \(\alpha_b = \frac{k_b}{C_p \rho_b}\) into equation (2) and rearranging the equation gives:

\[
\frac{k_b(T_{b(i-1)} - T_{bi})}{\Delta y} + \frac{k_b(T_{b(i+1)} - T_{bi})}{\Delta y} = \frac{C_p \rho_b(\Delta y)(T_{bi}^\prime - T_{bi})}{\Delta t} \tag{3}
\]

where subscript \(b\) refers to bitumen.
Equation (3) can also be interpreted as the one-dimensional heat flow balance between three adjacent nodes whereby heat flows from the two outer nodes into the center node, as shown in figure 3a. The sum of the heat flow into node $i$ from node $i+1$ and node $i-1$ must equal the energy stored in node $i$ during the time, $\Delta t$, of heat flow. The heat flow per unit area into node $i$ from either outer node is the temperature difference between nodes ($\Delta T_b$) divided by the thermal resistance between nodes ($R_b = \Delta y/k_b$). The energy stored in node $i$ during the time of heat flow ($\Delta t$) equals the heat capacity ($C_pb \rho_b$) times the volume of the element of node $i$ ($\Delta y \cdot 1$) times the increase in temperature of node $i$ ($T_{bi}^2 - T_{bi}$).

Equation (3) may be rearranged to yield equation (4) and then equation (5):

\[
(T_b(i-1) - T_{bi}) + (T_b(i+1) - T_{bi}) = \frac{C_pb \rho_b (\Delta y)^2}{k_b \Delta t} (T_{bi}^2 - T_{bi}) \quad (4)
\]

\[
(T_b(i-1) - T_{bi}) + (T_b(i+1) - T_{bi}) = M_b (T_{bi}^2 - T_{bi}) \quad (5)
\]

The term $M_b = C_pb \rho_b (\Delta y)^2/k_b \Delta t$ contains thermal, geometric and time parameters. In order to provide a stable solution using the finite-difference method for conduction, values of the modulus, $M_b$, must be chosen to be equal or greater than $2$ [5,6]. The requirement, $M_b > 2$, assures that in the heat flow calculation, the second law of thermodynamics is satisfied whereby heat flow is from a warm to a cold node. Because of this restriction on $M_b$, values of $\Delta y$ and $\Delta t$ cannot be selected independently. Solving for $T_{bi}^2$ in equation (5) yields the equation to calculate the future temperature of the bitumen interior element after heat flow during the time, $\Delta t$:

\[
T_{bi} = T_{bi} + \frac{1}{M_b} (T_b(i-1) - 2T_{bi} + T_b(i+1)) \quad (6)
\]

2.2.2.3 Heat Flow at the Surface Boundary of the Bitumen

The finite-difference equation describing heat flow per unit area at the surface boundary of the bitumen is obtained by performing a heat balance on the surface node 1 (figure 3b). It is noted that the thickness of the first element is only one half that of the other bitumen elements. Consequently the volume of this element is equal to $(\Delta y/2) \cdot 1$. For a heat balance at node 1 (figure 3b), the heat flow from node a to node 1 and the heat flow from node 2 to node 1 must equal the thermal energy stored at node 1. The heat flow from node a to node 1 is expressed as the product of the bitumen surface coefficient of heat transfer ($h_b$) and the temperature difference between the air and the surface element ($T_a - T_{bi}$). Heat flow per unit area from node 2 to node 1 is the temperature difference of the nodes ($T_{b2} - T_{bi}$) divided by the thermal resistance ($R_b = \Delta y/k_b$). The expression for stored thermal energy at node 1 in figure 3b considers that the volume of the element containing node 1 is only half that of the element containing node i (figure 3a). The equation for heat balance at the surface boundary is expressed as follows:
h_b (T_a - T_{b1}) + \frac{k_b(T_{b2} - T_{b1})}{\Delta y} = \frac{C_p \rho_b (\Delta y/2) (T_{b1} - T_{b1})}{\Delta t} \tag{7}

Equation (7) may be rearranged:

\[ \frac{h_b \Delta y}{k_b} (T_a - T_{b1}) + (T_{b2} - T_{b1}) = \frac{C_p \rho_b (\Delta y)^2 (T_{b1} - T_{b1})}{2k_b \Delta t} \tag{8} \]

N_b(T_a - T_{b1}) + (T_{b2} - T_{b1}) = \frac{M_b}{2} (T_{b1} - T_{b1}) \tag{9}

The term, \(N_b = h_b \Delta y/k_b\), defines a Biot or Nusselt number. The term, \(M_b\), has been previously defined and must be chosen such that \(M_b > (2N_b + 2)\) in order for equation (9) to provide a stable solution and to satisfy the thermodynamic requirements concerning the direction of heat flow \([5,6]\). Rearrangement of equation (9) results in the expression to determine the future temperature \(T_{b1}'\) of node 1 after heat has flowed during the time, \(\Delta t\):

\[ T_{b1}' = T_{b1} \left[1.0 - \frac{2}{M_b} (N_b + 1.0)\right] + \frac{2}{M_b} (T_{b2} + N_b T_a) \tag{10} \]

For this study, the surface coefficient of heat transfer, \(h\), is taken as:

\[ h = h_c + h_r \]

where

\[ h_c = \text{convection coefficient} \ Btu/h \cdot \text{ft}^2 \cdot \degree F \ (W/m^2 \cdot K) \]

\[ h_r = \text{radiation coefficient} = \sigma \varepsilon (T_s^2 + T_{\text{sky}}^2) (T_s + T_{\text{sky}}) \]

where

\[ \sigma = \text{Stefan-Boltzman constant, } Btu/h \cdot \text{ft}^2 \cdot \degree F^4 (W/m^2 \cdot K^4) \]

\[ \varepsilon = \text{emittance of the surface, dimensionless} \]

\[ T_s = \text{temperature of the surface, } \degree R \ (K) \]

\[ T_{\text{sky}} = \text{sky temperature, } \degree R \ (K); \text{ for purposes of this model, the sky temperature was taken} \]

\[ \text{to be the same as the ambient air temperature.} \]

In the application of the model, the convection coefficient, \(h_c\), was maintained constant for each set of given conditions and changed only to account for the effect of different wind speeds on the bitumen cooling. The radiation coefficient, \(h_r\), was changed at each time increment to account for heat radiation which is nonlinearly dependent upon surface temperature. It is noted that the value of the convection coefficient, \(h_c\), may be difficult to approximate. The surface coefficient may vary according to the rate of air movement across the surface, the surface length, and roughness.

2.2.2.4 Heat Flow at the Bottom (Interface) Boundary of the Bitumen

The finite-difference equation for heat flow at the bottom boundary element of the bitumen is determined by considering the heat balance at this element. The bottom boundary element interfaces with the top element of the roofing
component to which the hot bitumen is applied, and the thermal resistance across this interface is considered to be zero. Figure 3c indicates the heat flow at node n. The expressions for heat flow from node \( n-1 \) to node \( n \) and for stored energy at node \( n \) may be determined similarly to the expressions given in figure 3a. The heat flow per unit area from roofing component node \( x_1 \) to node \( n \) is the temperature difference \((T_{x1} - T_n)\) divided by the thermal resistance between the nodes \((R_{bx})\) as shown in figure 3c. The resistance \( R_{bx} \) is the sum of the resistance of the bitumen \((\Delta y/2k_b)\) and that of the component element \((\Delta y/2k_x)\) and may be expressed as:

\[
R_{bx} = \frac{\Delta y}{2k_b} + \frac{\Delta y}{2k_x} = \frac{\Delta y}{2} \left( \frac{k_b + k_x}{k_b k_x} \right) 
\]  

(11)

From Figure 3c, the heat balance equation at node \( n \) is expressed as:

\[
\frac{k_b(T_{b(n-1)} - T_{bn})}{\Delta y} + \frac{2}{\Delta y} \left( \frac{k_b k_x}{k_b + k_x} \right) (T_{x1} - T_{bn}) = \frac{C_p b \rho b (\Delta y)(T_{bn} - T_{bn})}{\Delta t} 
\]

(12)

Rearrangement of equation (12) gives:

\[
(T_{b(n-1)} - T_{bn}) + \frac{2k_x}{k_b + k_x} (T_{x1} - T_{bn}) = \frac{C_p b \rho b (\Delta y)^2(T_{bn} - T_{bn})}{k_b \Delta t} 
\]

(13)

\[
(T_{b(n-1)} - T_{bn}) + \frac{2k_x}{k_b + k_x} (T_{x1} - T_{bn}) = M_b(T_{bn} - T_{bn}) 
\]

(14)

The term \( M_b \) has been previously defined. Solving for \( T_{bn}' \), the future temperature at node \( n \) after heat has flowed for time, \( \Delta t \), results in the finite-difference equation for heat flow at the bottom boundary element of bitumen:

\[
T_{bn}' = T_{bn} + \frac{1}{M_b} \left[ T_{b(n-1)} - 1 + \left( \frac{2k_x}{k_b + k_x} \right) T_{bn} + \left( \frac{2k_x}{k_b + k_x} \right) T_{x1} \right] 
\]

(15)

2.2.2.5 Heat Flow within the Roofing Components Other than Bitumen

The mathematical model, as developed here, is applicable to roof systems which may contain two, three, or four components. In the four component system hot bitumen is applied to a felt which is bonded with a bitumen (referred to as bitumen-cement for purpose of classification of the model) to a substrate. The three-component system contains hot bitumen, felt, and substrate; and the two-component system is applicable to hot bitumen applied to a substrate. The finite-difference equations used to describe heat flow in the typical elements of the felt, bitumen-cement, and substrate components may be derived from heat balance equations similar to those presented in sections 2.2.2.2 through 2.2.2.4. For the convenience of the reader, the
typical finite-difference equations used to calculate cooling time of hot bitumen in the four-, three-, and two-component systems are given in appendix A.

2.2.2.6 Solution to the Model

To calculate the cooling time of hot bitumen during roofing construction, each roofing component was divided into a number of elements each with the same thickness, \( \Delta y \), with the exception of the top and bottom surface elements which had thickness of \( \Delta y/2 \). The number of elements for the individual components was based on the thickness of the component. Each element was assigned an appropriate finite-difference equation for calculating the change in temperature with time within that element. Each element was also assigned an initial temperature. The temperature of each element as a function of an increment of time, \( \Delta t \), was calculated sequentially from the top surface element of the bitumen to the lowest element of the substrate. This process started with the initial bitumen temperature and continued iteratively until the bitumen temperature reached a limiting value. The temperature of the bitumen was taken as the average of the temperature of each element in the bitumen component for each increment of time, \( \Delta t \).

In order to satisfy the condition that the limiting value of the modulus, \( M \), be greater than or equal to 2, values of \( \Delta y \) and \( \Delta t \) cannot be chosen independently and must be chosen relatively small to provide a stable solution for the heat flow calculation (section 2.2.2.2).

To facilitate the calculations, the model was computerized. The listing of the computer program is given in appendix B. The program uses the finite-difference equations for two-, three-, and four-component roofing systems given in appendix A. It is noted that the program contains a control statement that directs the calculation for roofing systems depending on the number of components.

3. APPLICATION OF MODEL FOR PREDICTING BITUMEN COOLING

3.1 VALUES NEEDED TO SOLVE MODEL

Because the mathematical model was developed to predict cooling time of hot bitumen applied to specific roofing components and under specific environmental conditions, the program prepared for the solution of the model requires values for each of the factors discussed in section 2.1 which affect cooling. In this regard, values for the following factors are read into the program for each set of calculations: initial temperature, number of elements, thermal conductivity, and thermal diffusivity of the roofing components; air temperature; and convection coefficient for the top surface of the bitumen and the bottom surface of the substrate. Emissivities of the bitumen and substrate are constants included in the program and the values of emissivity must be changed as appropriate for different bitumens and substrates. The initial temperature of any component is taken to be uniform throughout its thickness. Other values for operation of the program are the minimum temperature to which the bitumen cools and the maximum time allowed for bitumen to cool during program execution. Values are also needed for the
number of components, the thickness of the elements, \( \Delta y \), and the increment of time, \( \Delta t \).

Although not directly included as part of the program input needed for the cooling calculations, values of the following parameters are necessary since values of input are dependent on these parameters. The thermal diffusivity of a material is dependent on its density, thermal conductivity, and specific heat. The convection coefficient is related to the wind speed. The number of finite-elements of each component is determined by dividing the component thickness by the thickness of the element, \( \Delta y \), with the exception that at a surface a half element is used in the calculation. It is noted that the computer program limits the number of substrate elements to 100.

In order to illustrate the applicability of the mathematical model, specific values of material, environmental, and physical properties for selected model roofing systems were needed for use in the cooling calculations. Values for materials properties and convection coefficients as a function of wind speed were obtained from literature sources. Environmental factors influencing cooling (air temperature, roof component temperature, and wind speed) were chosen to represent a range of conditions encountered in built-up roofing applications. Roofing component thicknesses were selected on the basis of current roofing practice.

The thermal properties of the roofing component materials considered in this study are given in table 1. Selected values for roofing component temperatures and environmental factors are given in table 2. Table 3 gives thicknesses of roofing component materials and the number of finite-elements for each material used in the calculations. The element thickness, \( \Delta y \), for computer program input was selected as 0.003 in or 0.00025 ft (0.076 mm). Values of the time increment, \( \Delta t \), ranged from 0.0000000045 to 0.000001 h (0.000162 to 0.0036 s). These values of time were selected to satisfy the conditions for heat flow calculation by finite-difference techniques [5,6] and to minimize computation time.

Using input data in tables 1, 2 and 3 and selected values of \( \Delta y \) and \( \Delta t \) within the ranges given above, the average bitumen temperature was computed as a function of time for various model roofing systems (configurations) given in table 4. The model roofing systems contain two, three, or four components and included bitumens, felts, insulations, and deck materials. The solution of the mathematical model for a given set of conditions was a tabular listing of bitumen temperatures corresponding to half second time intervals. The temperatures of the applied bitumen during the cooling period were computed over a range beginning with the selected application temperature (table 2) to the selected lower limit of 300°F (149°C). Tabular data from the model were used to prepare the bitumen temperature versus cooling time plots given in figures 4-17.
3.2 RESULTS FOR SPECIFIC ROOFING SYSTEMS

This section presents the results (figures 4-17) of the calculations for various two-, three-, and four-component model roofing systems (table 4) conducted to illustrate the applicability of the mathematical model. The results in figures 4-15 give the temperature-time relationships for the cooling of hot bitumen applied at an application quantity of about 20 lbm/100 ft² (0.98 kg/m²) for asphalt and about 25 lbm/100 ft² (1.2 kg/m²) for coal tar pitch, at temperatures of 500, 450, 400, and 350°F (260, 232, 204, and 177°C) to a lower temperature of 300°F (149°C) under specific environmental conditions. These environmental conditions were air and initial component temperatures of 120, 70, and 20°F (49, 21, and -7°C) and wind speeds of 0, 10, 20, and 30 mph (0, 4.5, 9.0, and 13.5 m/s). The asphalt and coal tar pitch application quantities correspond to a thickness of 0.04 in (1.0 mm).

Figure 16 gives results for the three-component roofing system of asphalt applied to felt on polyurethane foam insulation and shows the effect of the quantity of applied asphalt (asphalt thickness) on cooling time for application temperatures of 500 and 400°F (260 and 204°C) and a wind speed of zero. The air and initial component temperatures in this figure were also 120, 70, and 20°F (49, 21, and -7°C). Figure 17 compares the cooling time of asphalt applied at 500°F (260°C) to various substrates. In this figure the air and initial substrate temperatures were 70°F (21°C) and the wind speed was zero.

A comparison of the results given in figures 4-15 indicates the influence of the various factors that affect the cooling rate of hot bitumen. The time for the bitumen to cool from 500 to 300°F (260 to 149°C) ranged from less than 4 to more than 50 seconds depending upon the wind speed, air temperature, initial component material temperatures, and the type of roofing system. As expected, the cooling time decreased for a particular roofing system with a decrease in application temperature, an increase in wind speed, and a decrease in the air and initial component material temperatures.

The type of substrate to which the hot asphalt is applied may have a significant effect on the cooling time. Figures 4-9 illustrate this effect. In comparing these two-component systems, the cooling time was much longer for the cases of asphalt applied to low thermal conductivity and low heat capacity polyurethane foam and fiber glass insulations than the cooling time for asphalt applied to the more conductive and higher heat capacity concrete and steel. The cooling times for asphalt applied to insulating concrete and plywood, figures 6 and 7, were about the same, and fell between those for the low and high thermal conductivity and heat capacity materials. It is noted from table 1 that insulating concrete and plywood have comparable thermal properties.

Because steel has a higher thermal conductivity and heat capacity than concrete, it might be expected that asphalt applied on steel would cool faster than on concrete. From figures 8 and 9 it can be seen that the calculated asphalt cooling times were shorter for a concrete substrate than for a steel substrate. This can be explained in part by considering the
thickness of the substrate. The thin steel substrate rapidly heats upon application of the asphalt until its temperature is nearly uniform through its thickness. At this point, the steel is less of a heat sink for the asphalt than the concrete. In the case of asphalt applied to the thick concrete substrate, the concrete has a considerably larger volume, heats more slowly, and continuously absorbs heat from the hotter asphalt.

A comparison is made from figures 8 and 10 of the cooling times of asphalt and coal tar pitch applied to concrete. The slightly longer cooling time for coal tar pitch is attributed to its lower thermal diffusivity and higher heat capacity. It is noted that identical values of thermal conductivity and specific heat were assigned for these two bitumens. The lower diffusivity of the coal tar pitch is accounted for by its higher density.

Figures 11 and 12 illustrate the use of the mathematical model for three-component systems, whereby hot bitumen is applied on a felt which is attached to a substrate. In these examples, the bitumen was asphalt, the felt was type 15 asphalt organic, and the substrates were polyurethane foam insulation (figure 11) and plywood (figure 12). The plots in figures 11 and 12 indicate that the felt and substrate combination influence the cooling of the applied bitumen. For these three-component systems, the asphalt cooled faster when applied to the felt on the substrate with the higher heat capacity and thermal conductivity (plywood). In these two cases, the thermal properties of the felt were identical.

The substrate in these cases influences the cooling of the asphalt because the felt is relatively thin. Heat flowing from the asphalt into the thin felt flows into the substrate before the asphalt cools to the lower temperature limit of 300°F (149°C). Since heat flows from the hot asphalt first into the felt and then into the substrate, the substrate below the felt has little effect on the cooling time in the initial seconds after application of the asphalt. In comparing figures 11 and 12, it may be seen that the temperature-time cooling curves for the asphalt applied to felt on polyurethane insulation or to felt on plywood are essentially identical during the first two to three seconds after application. Then the time of cooling increases for the case of asphalt applied to felt on polyurethane foam.

The effect of the substrate below the felt on the cooling time of the applied asphalt may be seen by examining the temperature distribution of the elements within the roofing system as a function of time. Figure 18 presents temperature profiles through the thickness of the three-component roofing systems, asphalt applied to felt on either polyurethane foam insulation or plywood substrate. In figure 18, asphalt at 500°F (260°C) was applied to the felt and substrate at 120°F (49°C) with a zero wind speed. The temperature profiles are obtained by plotting the temperature of the element versus the number of the element. The thickness of the substrate was limited to 100 elements. The temperature profiles given in figure 18 are for times of 2, 4, 6, 8, 10, and 20 seconds after application of the asphalt (figures 18a through 18f, respectively).
It can be seen from figure 18 that the temperature profiles for the two three-component systems are very similar at times corresponding to a few seconds after application of the asphalt. For example, 2 seconds after application (figure 18a), the distribution of temperature through the asphalt and felt components are almost identical for the two systems. The temperatures within the polyurethane substrate are seen to be only slightly higher than those of the plywood substrate for a few elements below the felt-substrate interface. Four seconds after application (figure 18b), the temperature distributions within the asphalt components remain similar for the two three-component systems. As the time after application increases, the temperature profiles of the two three-component systems exhibit greater disparity. The temperatures of the asphalt applied to felt on plywood are lower than the temperatures of asphalt applied to felt on polyurethane. In addition, the temperatures within the polyurethane foam insulation substrate rise higher than the temperatures within the plywood substrate which has greater heat capacity and thermal conductivity. The result is a slower cooling of asphalt applied to felt on polyurethane foam in comparison to asphalt applied to felt on plywood.

A comparison may be made between figures 4 and 11, or figures 7 and 12 to show the effect of the felt on the time of bitumen cooling. Figure 4 gives the calculated results for asphalt applied directly to polyurethane foam, while in figure 11, the asphalt is applied to felt on polyurethane foam. The asphalt applied to felt on foam cools more rapidly than that applied directly on foam, because of the effect of the relatively high heat capacity and thermal conductivity of the felt compared to the heat capacity and thermal conductivity of the foam. The cooling time for asphalt applied to plywood, figure 7, was essentially the same as that for asphalt applied to felt over plywood, figure 12. The thermal properties of the felt and plywood were considered to be comparable (table 1).

Examples of asphalt cooling time for two four-component roofing systems are presented in figures 13 and 14. The cooling time for hot asphalt applied to felt on asphalt on concrete was essentially the same as for hot asphalt applied to felt on asphalt on polyurethane foam, even though the thermal properties of the concrete and polyurethane foam differ significantly. The cooling time to reach 300°F (149°C) for these four-component systems was influenced only by the felt and asphalt and not the substrate.

Figure 15 allows a comparison to be made for a four-component system containing coal tar pitch instead of asphalt. The cooling times for coal tar pitch in figure 15 were slightly longer than for the asphalt cooling times given in figures 13 and 14. Assuming that the substrate has little influence on the cooling time for a four-component system, this observation is consistent with the previously noted comparison between figures 8 and 10 for the case of these two bitumens applied to the same substrate.

The effect of the quantity of applied asphalt on the cooling time is illustrated in figure 16. In figures 4 through 15, the bitumens were applied at a constant thickness of about 0.04 in (1.0 mm). This corresponds to an application quantity of about 20 lbm/100 ft² (0.98 kg/m²) for asphalt and
25 lbm/100 ft² (1.2 kg/m²) for coal tar pitch. As noted in figure 16, increasing the amount of applied bitumen significantly increased the cooling time to reach a given temperature. For example, in figure 16 the cooling time to reach 300°F (149°C) for asphalt at 500°F (260°C) applied to components at 20°F (-7°C) ranged from about 8 to 45 seconds for application quantities of about 15 and 30 lbm/100 ft² (0.73 and 1.5 kg/m²), respectively. These quantities correspond to asphalt thicknesses of about 0.03 and 0.06 in (0.7 and 1.5 mm).

Figure 17 shows the effect of the substrate on asphalt cooling time for a specific set of environmental conditions, application temperature, and asphalt thickness. The air and substrate temperatures are 70°F (21°C) with a zero wind speed and with an asphalt application temperature of 500°F (260°C). The asphalt thickness corresponds to about 20 lbm/100 ft² (0.98 kg/m²). In general, the curves in this figure are grouped according to the thermal properties of the substrate, and in particular, the heat capacities of the substrates. Table 5 presents the heat capacities of the substrates and the times for the asphalt to cool to 300°F (149°C) for the specific set of environmental conditions in figure 17. It can be seen from table 5 that the asphalt cooling times generally decrease as the heat capacities of the substrates increase. The thickness of the steel substrate is considerably less than that of the other substrates (table 3). This accounts for the cooling time being comparable to that for concrete, as shown in table 5. It is noted that the cooling time of asphalt applied to felt is influenced by the polyurethane foam insulation substrate. Cooling time of asphalt applied to felt alone was not calculated, since this is not applicable to roofing construction.

3.3 USES FOR MODEL

The model for predicting the cooling time of hot bitumen has practical application to the construction of bituminous built-up roofing systems. The model can provide quantitative information about bitumen cooling time with regard to the time available to apply insulations or felts properly. Bitumen cooling is a complex process which depends upon a combination of environmental factors and thermal properties of the materials in the roofing system.

The model has shown in section 3.2 that for certain roofing systems under specific conditions bitumen will cool extremely rapidly. In cases where cooling may be more rapid than expected, there may be insufficient time to place insulation or to broom felts to achieve adequate adhesion between roofing components. The model permits a quantitative determination of bitumen cooling time as opposed to empirical or intuitive approaches. Use of the model indicates that under some conditions, the application temperature may have to be raised above normally accepted limits to provide sufficient time to apply component materials, or hot bituminous roofing should not be applied under some conditions.

Although the use of the model was demonstrated in this paper for specific roofing systems and environmental conditions, it may be applied to other
roofing systems and environmental conditions as well. This paper presented the results of the cooling calculations as plots of bitumen temperature versus time for specific bitumen application temperatures at various wind speeds and at specific air and substrate temperatures. Other types of plots may be used to express the results of the cooling calculations. One such plot relates the dimensionless temperature parameter, \((T_b - T_{ac})/(T_a - T_{ac})\), to time. The term \(T_b\) is the bitumen temperature at a given time, \(T_a\) is the application temperature of the bitumen, and \(T_{ac}\) is the temperature of the air and the initial temperature of components other than bitumen.

Figure 19 is an example of a plot of dimensionless temperature versus time for the application of hot asphalt to steel, and was generated from the results of the cooling calculations from which figure 9 was prepared. Whereas figure 9 contains forty-eight curves, figure 19 contains only four curves. These four curves may be used to approximate the bitumen temperature \((T_b)\) at any given time for any bitumen application temperature \((T_a)\) and air and initial roof component temperature \((T_{ac})\) for each of the four wind speeds.

4. SUMMARY

This paper presented a mathematical model using finite-difference equations to predict the cooling times of hot bitumen applied during the construction of built-up roofing. The application or contact temperature of the hot bitumen was taken as the temperature at which the bitumen made contact with the surface to which it was applied. The quality of built-up roofing is dependent upon the application of bitumen which is hot enough (i.e., has the proper viscosity) to provide a uniform continuous layer of bitumen and adequate adhesion between roofing components. A model for the prediction of cooling times was needed, since it would not be practicable to measure these rates for the numerous roofing systems to which hot bitumen may be applied under many different environmental conditions. Up to now, mathematical modeling of the cooling of bitumen has not been employed in the roofing industry.

The effect of various factors on the cooling times of hot bitumen was investigated. These factors included materials properties and environmental conditions. The materials properties were the initial temperature, thermal conductivity, specific heat, density, and thickness of the roofing components. Environmental conditions affecting cooling were air temperature and wind speed. Values for each of these factors were incorporated in the model to determine quantitatively their effect on the cooling. The model was computerized for application to roofing systems having two, three, or four components.

Examples of the use of the model to predict the cooling times of hot bitumen applied to various roofing components were given. These components included felts, insulations such as fiber glass and polyurethane foam boards, and typical decks such as concrete, steel, and plywood. Times for bitumens to cool to 300°F (149°C) were computed for bitumen application temperatures ranging from 500 to 350°F (260 to 177°C). For the selected examples, cooling times ranged from less than 4 to more than 50 seconds depending upon the
type of model roofing system in question, environmental conditions, and component material temperatures. Cooling times of the hot bitumen were graphically presented using computer-generated plots of bitumen temperature versus time.

The model indicated that for some systems under certain environmental conditions, hot bitumen cools extremely rapidly. In these cases, sufficient time may not be available to place insulation or felts properly. To provide adequate time for proper application, the temperature of the bitumen may need to be increased above normally accepted limits, or hot bituminous roofing should not be applied under some conditions.

5. ACKNOWLEDGMENTS

The authors wish to acknowledge the assistance provided by discussion with Mr. Douglas M. Burch and Mr. Bradley A. Peavy, National Bureau of Standards, Building Thermal and Service Systems Division; Dr. Geoffrey J. C. Frohnsdorff, National Bureau of Standards, Structures and Materials Division; Dr. Philip F. Dickson, Colorado School of Mines, Department of Chemical and Petroleum Refining Engineering; and, Dr. Edwin Mertz, formerly of the National Roofing Contractors Association.

Special thanks are given to Dr. James J. Filliben, National Bureau of Standards, Statistical Engineering Division, for his contributions to this report. The authors appreciate his valuable assistance in the development of the computer programs to predict the cooling times of hot bitumen and to plot the temperature versus cooling time curves.

6. REFERENCES


Table 1. Thermal Properties of the Roofing Component Materials for Use in the Model.

<table>
<thead>
<tr>
<th>Roofing Component Material</th>
<th>Density $\rho$ (lbm/ft$^3$)</th>
<th>Thermal Conductivity $k$ (Btu<em>ft/h</em>ft$^2$*°F)</th>
<th>Specific Heat $C_p$ (Btu/1lbm*°F)</th>
<th>Thermal Diffusivity $\alpha$ (ft$^2$/h)</th>
<th>Heat Capacity $\rho C_p$ (Btu/ft$^2$*°F)</th>
<th>Data Source Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>o Bitumen (Hot)</strong> (3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt</td>
<td>56 (896)</td>
<td>0.067 (0.12)</td>
<td>0.53</td>
<td>0.0023</td>
<td>30 (1989)</td>
<td>[20]</td>
</tr>
<tr>
<td>Coal Tar Pitch (4)</td>
<td>72 (1153)</td>
<td>0.067 (0.12)</td>
<td>0.53</td>
<td>0.0018</td>
<td>38 (2560)</td>
<td>[20]</td>
</tr>
<tr>
<td><strong>o Felt</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 15 - Asphalt Organic</td>
<td>63 (1008)</td>
<td>0.047 (0.08)</td>
<td>0.27</td>
<td>0.0028</td>
<td>17 (1139)</td>
<td>[21]</td>
</tr>
<tr>
<td>Type 15 - Coal Tar Pitch Organic</td>
<td>69 (1105)</td>
<td>0.047 (0.08)</td>
<td>0.35</td>
<td>0.0019</td>
<td>24 (1613)</td>
<td>[21]</td>
</tr>
<tr>
<td><strong>o Bitumen - Cement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt</td>
<td>64 (1025)</td>
<td>0.089 (0.15)</td>
<td>0.42</td>
<td>0.0033</td>
<td>27 (1804)</td>
<td>[20]</td>
</tr>
<tr>
<td>Coal Tar Pitch (4)</td>
<td>80 (1281)</td>
<td>0.089 (0.15)</td>
<td>0.42</td>
<td>0.0026</td>
<td>34 (2255)</td>
<td>[20]</td>
</tr>
<tr>
<td><strong>o Substrate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>489 (7829)</td>
<td>26.2 (45.3)</td>
<td>0.12</td>
<td>0.45</td>
<td>59 (3915)</td>
<td>[22]</td>
</tr>
<tr>
<td>Concrete</td>
<td>140 (2241)</td>
<td>0.75 (1.30)</td>
<td>0.22</td>
<td>0.024</td>
<td>31 (2062)</td>
<td>[23]</td>
</tr>
<tr>
<td>Insulating Concrete</td>
<td>40 (640)</td>
<td>0.096 (0.17)</td>
<td>0.21</td>
<td>0.011</td>
<td>8 (563)</td>
<td>[23]</td>
</tr>
<tr>
<td>Plywood</td>
<td>34 (544)</td>
<td>0.067 (0.12)</td>
<td>0.29</td>
<td>0.0068</td>
<td>10 (658)</td>
<td>[23]</td>
</tr>
<tr>
<td>Glass Fiber Insulation Board</td>
<td>6.5 (104)</td>
<td>0.021 (0.036)</td>
<td>0.23</td>
<td>0.014</td>
<td>1.5 (100)</td>
<td>[23]</td>
</tr>
<tr>
<td>Polyurethane Foam Insulation Board</td>
<td>1.5 (24)</td>
<td>0.013 (0.022)</td>
<td>0.38</td>
<td>0.023</td>
<td>0.57 (38)</td>
<td>[23]</td>
</tr>
</tbody>
</table>

(1) Calculated from the relationship, $\alpha = \frac{k}{\rho C_p}$.

(2) Calculated from the values of density and specific heat.

(3) Density, thermal conductivity, and specific heat values have been adjusted for the elevated bitumen application temperatures. The adjustment was made for a temperature of 450°F (232°C).

(4) According to Mack [20], the thermal conductivity and specific heat of coal tar pitches are of the same order of magnitude as those of asphalts. Thus, for purposes of this paper, values of thermal conductivity and specific heat of coal tar pitch were chosen to be identical to those of asphalt.
Table 2. Values of Roofing Component Temperatures and Environmental Factors Used in the Calculations

<table>
<thead>
<tr>
<th>Factor</th>
<th>Units</th>
<th>Numerical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bitumen Application Temperature</td>
<td>°F</td>
<td>500 (260)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>450 (232)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400 (204)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>350 (177)</td>
</tr>
<tr>
<td>Felt, Bitumen-Cement and Substrate Temperatures</td>
<td>°F</td>
<td>120 (49)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70 (21)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 (-7)</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>°F</td>
<td>120 (49)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70 (21)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 (-7)</td>
</tr>
<tr>
<td>Wind Speed and Corresponding Convection Coefficient(1)</td>
<td>mph</td>
<td>0 (0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 (4.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 (9.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 (13.5)</td>
</tr>
<tr>
<td></td>
<td>m/s</td>
<td>(0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9.0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(13.5)</td>
</tr>
<tr>
<td></td>
<td>Btu/h•ft²•°F</td>
<td>1.0 (5.7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.6 (20.4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.1 (34.6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.5 (48.2)</td>
</tr>
<tr>
<td></td>
<td>(W/m²•K)</td>
<td></td>
</tr>
</tbody>
</table>

(1) Wind speeds are those blowing across the surface of the cooling bitumen. The values of the convection coefficient corresponding to the selected wind speeds were estimated from the curve for smooth surfaced materials in figure 1, chapter 22 of ASHRAE Handbook 1977 Fundamentals [24]. The radiation contribution to the overall surface coefficient has been subtracted to estimate the convection coefficient.
Table 3. Thickness of the Roofing Component Materials and Corresponding Number of Elements Used in the Calculations.

<table>
<thead>
<tr>
<th>Roofing Component Material</th>
<th>Component Thickness</th>
<th>Number of Elements&lt;sup&gt;(1)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in</td>
<td>mm</td>
</tr>
<tr>
<td>o Bitumen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt&lt;sup&gt;(2)&lt;/sup&gt;</td>
<td>0.0285</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>0.0405</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>0.0495</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>0.0585</td>
<td>1.49</td>
</tr>
<tr>
<td>Coal Tar Pitch&lt;sup&gt;(2)&lt;/sup&gt;</td>
<td>0.0405</td>
<td>1.03</td>
</tr>
<tr>
<td>o Felt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 15 Asphalt Organic</td>
<td>0.033</td>
<td>0.84</td>
</tr>
<tr>
<td>Type 15 Coal Tar Pitch Organic</td>
<td>0.033</td>
<td>0.84</td>
</tr>
<tr>
<td>o Bitumen-Cement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.042</td>
<td>1.1</td>
</tr>
<tr>
<td>Coal Tar Pitch</td>
<td>0.042</td>
<td>1.1</td>
</tr>
<tr>
<td>o Substrate&lt;sup&gt;(3)&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel&lt;sup&gt;(2)&lt;/sup&gt;</td>
<td>0.0285</td>
<td>0.72</td>
</tr>
<tr>
<td>Concrete</td>
<td>6.0</td>
<td>152.4</td>
</tr>
<tr>
<td>Insulating Concrete</td>
<td>4.0</td>
<td>101.6</td>
</tr>
<tr>
<td>Plywood</td>
<td>0.75</td>
<td>19.1</td>
</tr>
<tr>
<td>Glass Fiber Insulation Board</td>
<td>1.0</td>
<td>25.4</td>
</tr>
<tr>
<td>Polyurethane Foam Insulation</td>
<td>1.0</td>
<td>25.4</td>
</tr>
</tbody>
</table>

(1) The number of finite elements is in general determined by dividing the component thickness by the thickness of a single element, \( \Delta y \). In this report, \( \Delta y \) was chosen equal to be 0.003 in (0.076 mm).

(2) The component thickness considers the half-thickness element at the surface.

(3) The computer program limits the number of substrate elements to 100.
Table 4. Configurations of Roofing Components for Which the Cooling Time of the Applied Bitumen was Calculated.

<table>
<thead>
<tr>
<th>Number of Components</th>
<th>Applied Bitumen</th>
<th>Other Roofing Components&lt;sup&gt;(1)&lt;/sup&gt;</th>
<th>Figure Number&lt;sup&gt;(2)&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>asphalt</td>
<td>polyurethane foam insulation board</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>fiber glass insulation board</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>insulating concrete</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>plywood</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>concrete</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>steel</td>
<td>9</td>
</tr>
<tr>
<td>coal tar pitch</td>
<td>coal tar pitch</td>
<td>concrete</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>felt on polyurethane foam insulation board</td>
<td>11, 16</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>asphalt</td>
<td>felt on plywood</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>asphalt</td>
<td>felt on asphalt on polyurethane foam insulation board</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>asphalt</td>
<td>felt on asphalt on concrete</td>
<td>14</td>
</tr>
<tr>
<td>coal tar pitch</td>
<td>coal tar pitch</td>
<td>felt on coal tar pitch on fiber glass insulation board</td>
<td>15</td>
</tr>
</tbody>
</table>

<sup>(1)</sup> For roofing configurations which contain three and four components, the components are listed in the sequence in which they are located in the system, the first listed being uppermost.

<sup>(2)</sup> The figure numbers refer to the figures of bitumen temperature versus cooling time given in section 3.2.
Table 5. Heat Capacities of Various Substrates and Cooling Times for Asphalt When Applied on Them<br>

<table>
<thead>
<tr>
<th>Substrate</th>
<th>Heat Capacity $\rho C_p$ Btu/ft$^3$$^\circ$F (kJ/m$^3$$^\circ$K)</th>
<th>Asphalt Cooling Time (a) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyurethane Foam Insulation Board</td>
<td>0.57 (38)</td>
<td>59</td>
</tr>
<tr>
<td>Glass Fiber Insulation Board</td>
<td>1.5 (100)</td>
<td>49</td>
</tr>
<tr>
<td>Insulating Concrete</td>
<td>8 (563)</td>
<td>16</td>
</tr>
<tr>
<td>Plywood</td>
<td>10 (658)</td>
<td>17</td>
</tr>
<tr>
<td>Felt (b)</td>
<td>17 (1139)</td>
<td>_ (b)</td>
</tr>
<tr>
<td>Felt on Polyurethane Foam Insulation</td>
<td>_ (c)</td>
<td>28</td>
</tr>
<tr>
<td>Concrete</td>
<td>31 (2062)</td>
<td>4.5</td>
</tr>
<tr>
<td>Steel (d)</td>
<td>59 (3915)</td>
<td>6.5</td>
</tr>
</tbody>
</table>

(a) Time to cool to 300°F (149°C). Asphalt applied at 500°F (260°C) with air and initial substrate temperatures of 70°F (21°C) and zero wind speed. The asphalt application quantity corresponds to about 20 lbm/100 ft$^2$ (0.98 kg/m$^2$).

(b) Asphalt is not applied to felt alone, and felt is not considered to be a substrate.

(c) Heat capacities of the felt and polyurethane foam are listed separately.

(d) The thickness of the steel is considerably less than that of the other substrates (table 3).
Figure 1. Modes of Heat Transfer During Construction of Bituminous Built-Up Roofing Systems. The Effect of Solar Radiation was not Included in the Mathematical Model. Heat Transfer due to Convection and Radiation from the Underside of Thin Decks Without Insulation was Included in the Model.
CONVECTION LOSS

RADIATION LOSS

T_a

NODES

Δy/2

Δy

HOT BITUMEN

CONDUCTION LOSS

T_{b1}

T_{b2}

Δy

Δy

INTERFACE

ROOFING COMPONENT

T_{b(i-1)}

T_{bi}

T_{b(i+1)}

T_{b(n-1)}

T_{bn}

T_{x1}

T_{x2}

Key:

T_a = temperature of the air
T_{b1} = temperature of the first bitumen element
T_{bi} = temperature of an interior bitumen element
T_{bn} = temperature of the last bitumen element
T_{x1} = temperature of the first element of the roofing component below the bitumen
Δy = the thickness of an element and the distance between nodes

Figure 2. Diagram of the Elements Within the Bitumen and Top Elements of the Roofing Component Below the Bitumen.
3a. Heat flow within an element within the bitumen

\[ Q_{i-1,i} = \text{heat flow from node } i-1 \text{ to node } i = \frac{(T_{b(i-1)} - T_{bi})}{R_b} = \frac{k_b(T_{b(i-1)} - T_{bi})}{\Delta y} \]

\[ \text{Stored heat at node } i = \frac{c_{pb} \rho_b (\Delta y)(T_{bi}^r - T_{bi})}{\Delta t} \]

\[ Q_{i+1,i} = \text{heat flow from node } i+1 \text{ to node } i = \frac{(T_{b(i+1)} - T_{bi})}{R_b} = \frac{k_b(T_{b(i+1)} - T_{bi})}{\Delta y} \]

3b. Heat flow at the surface boundary element of the bitumen

\[ Q_{a,1} = \text{heat flow from air node } a \text{ to surface node } 1 = h_b(T_a - T_{b1}) \]

\[ \text{Stored heat at node } 1 = \frac{c_{pb} \rho_b (\Delta y/2)(T_{bi}^r - T_{b1})}{\Delta t} \]

\[ Q_{1,2} = \text{heat flow from node } 2 \text{ to surface node } 1 = \frac{(T_{b2} - T_{b1})}{R_b} = \frac{k_b(T_{b2} - T_{b1})}{\Delta y} \]

3c. Heat flow at the bottom boundary element of the bitumen

\[ Q_{n-1,n} = \text{heat flow from node } n-1 \text{ to node } n = \frac{(T_{b(n-1)} - T_{bn})}{R_b} = \frac{k_b(T_{b(n-1)} - T_{bn})}{\Delta y} \]

\[ \text{Stored heat at node } n = \frac{c_{pb} \rho_b (\Delta y)(T_{bn}^r - T_{bn})}{\Delta t} \]

\[ Q_{x1,n} = \text{heat flow from node } xl \text{ to node } n = \frac{(T_{x1} - T_{bn})}{R_{bx}} = \frac{2}{\Delta y} \left( \frac{k_b k_x}{k_b + k_x} \right)(T_{x1} - T_{bn}) \]

Figure 3  Diagram of the Elements and Nodes Used to Develop the Finite-Difference Equations for Heat Flow Within the Bitumen, at the Surface of the Bitumen, and at the Bottom Boundary of the Bitumen.
Figure 4. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 5. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 6. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Insulating Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 7. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 8. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 9. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Steel. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft$^2$ (0.98 kg/m$^2$).
Figure 10. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Coal Tar Pitch Applied on Concrete. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft$^2$ (1.2 kg/m$^2$).
Figure 11. Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 12. Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft\(^2\) (0.98 kg/m\(^2\)).
Figure 13. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 14. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft² (0.98 kg/m²).
Figure 15. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Coal Tar Pitch Applied on Felt over Coal Tar Pitch on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft² (1.2 kg/m²).
Figure 16. Effect of Asphalt Thickness on the Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation.
MODEL -- ASPHALT ON VARIOUS SUBSTRATES

1 = CONCRETE
2 = STEEL
3 = INSULATING CONCRETE
4 = PLYWOOD
5 = FELT/PUF INSULATION
6 = FIBER GLASS INSUL.
7 = PUF INSULATION

COOLING TIME, SECONDS
AIR AND SUBSTRATE TEMPERATURES ARE 70 F
WIND SPEED IS 0 MPH

Figure 17. Effect of Substrate on the Asphalt Cooling Time.
Figure 18. Temperature Profiles of the Three-Component Roofing Systems, Asphalt Applied to Felt on Plywood and Asphalt Applied to Felt on Polyurethane Foam Insulation, for Various Times after Bitumen Application. Asphalt at 500°F (260°C) was Applied to the Felt and Substrate at 120°F (49°C) with Zero Wind Speed.
Figure 19. Plot of Dimensionless Temperature Versus Time for the Two-Component Roofing System, Asphalt Applied on Steel.

\[ \frac{T_b - T_{ac}}{T_a - T_{ac}} \]

- \( T_b = \) bitumen temperature at a given time
- \( T_a = \) bitumen application temperature
- \( T_{ac} = \) temperature of air and substrate
APPENDIX A. LISTING OF FINITE-DIFFERENCE EQUATIONS

The mathematical model for predicting the cooling times of hot bitumen during built-up roofing construction, as developed here, is applicable to roof systems which may contain two, three, or four components. The model contains three types of finite-difference equations for each component of the roofing system; that is, the model uses 6, 9 or 12 types of equations for calculating cooling times for systems containing two, three, or four components, respectively. For the convenience of the reader, this appendix presents the types of finite-difference equations used for the two-, three-, or four-component roofing systems. These equations may be developed by the treatment given in section 2.2. The subscripts b, f, c, and s in the equations which follow refer to bitumen, felt, bitumen-cement, and substrate, respectively. It is noted that the types of finite-difference equations for similar elements in different components (e.g., the equations within the bitumen-cement and within the substrate) are identical in form. The difference between the equations is the values of the constants for the thermal conductivity, specific heat, and density properties of the components in question.

Figure A1 presents a diagram of the elements and mathematical notation used to denote the temperature of the elements for the four-component roofing system. This figure was prepared for development of the finite-difference equations for the four-component system. Similar diagrams may be prepared for the three- and two-component systems.
KEY: $T_{bi}$ = temperature of any bitumen element  
$T_{bn}$ = temperature of the last bitumen element  
$T_{fi}$ = temperature of any felt element  
$T_{fn}$ = temperature of the last felt element  
$T_{ci}$ = temperature of any bitumen-cement element  
$T_{cn}$ = temperature of the last bitumen-cement element  
$T_{si}$ = temperature of any substrate element  
$T_{sn}$ = temperature of the last substrate element

Figure A1. Diagram of the Finite-Elements Within the Components of the Roofing System Used for the Calculation of Bitumen Cooling Time by Finite-Difference Techniques.
A.1 **FINITE-DIFFERENCE EQUATIONS FOR THE FOUR-COMPONENT SYSTEM**

° At the bitumen surface, $T_{b1}$

\[ T_{b1}^* = T_{b1} \left[ 1.0 - \frac{2}{M_b} (N_b + 1.0) + \frac{2}{M_b} (T_{b2} + N_b T_a) \right] \]  

(A1)

° Within the bitumen, $T_{b1}$

\[ T_{b1}^* = T_{b1} + \frac{1}{M_b} (T_{b(i-1)} - 2T_{b1} + T_{b(i+1)}) \]  

(A2)

° In the bottom elemental layer of bitumen, $T_{bn}$

\[ T_{bn}^* = T_{bn} + \frac{1}{M_b} \left[ T_{b(n-1)} - \left( 1 + \frac{2k_f}{k_b + k_f} \right) T_{bn} + \left( \frac{2k_f}{k_b + k_f} \right) T_{f1} \right] \]  

(A3)

° In the top elemental layer of the felt, $T_{f1}$

\[ T_{f1}^* = T_{f1} + \frac{1}{M_f} \left[ \left( \frac{2k_b}{k_b + k_f} \right) T_{bn} - \left( 1 + \frac{2k_b}{k_b + k_f} \right) T_{f1} + T_{f2} \right] \]  

(A4)

° Within the felt, $T_{f1}$

\[ T_{f1}^* = T_{f1} + \frac{1}{M_f} (T_{f(i-1)} - 2T_{f1} + T_{f(i+1)}) \]  

(A5)

° In the bottom elemental layer of felt, $T_{fn}$

\[ T_{fn}^* = T_{fn} + \frac{1}{M_f} \left[ T_{f(n-1)} - \left( 1 + \frac{2k_c}{k_c + k_f} \right) T_{fn} + \left( \frac{2k_c}{k_c + k_f} \right) T_{c1} \right] \]  

(A6)

° In the top elemental layer of bitumen-cement, $T_{c1}$

\[ T_{c1}^* = T_{c1} + \frac{1}{M_c} \left[ \left( \frac{2k_f}{k_c + k_f} \right) T_{fn} - \left( 1 + \frac{2k_f}{k_c + k_f} \right) T_{c1} + T_{c2} \right] \]  

(A7)

° Within the bitumen-cement, $T_{ci}$

\[ T_{ci}^* = T_{ci} + \frac{1}{M_c} (T_{c(i-1)} - 2T_{ci} + T_{c(i+1)}) \]  

(A8)

° In the bottom elemental layer of bitumen-cement, $T_{cn}$

\[ T_{cn}^* = T_{cn} + \frac{1}{M_c} \left[ T_{c(n-1)} - \left( 1 + \frac{2k_s}{k_c + k_s} \right) T_{cn} + \left( \frac{2k_s}{k_c + k_s} \right) T_{s1} \right] \]  

(A9)
In the top elemental layer of the substrate, $T_{s1}$

$$T_{s1}' = T_{s1} + \frac{1}{M_s} \left[ \left( \frac{2k_c}{k_c + k_s} \right) T_{c1n} - \left( 1 + \frac{2k_c}{k_c + k_s} \right) T_{s1} + T_{s2} \right]$$  \hspace{1cm} (A10)

Within the substrate, $T_{si}$

$$T_{si}' = T_{si} + \frac{1}{M_s} \left( T_{s(i-1)} - 2T_{s1} + T_{s(i+1)} \right)$$  \hspace{1cm} (A11)

In the bottom elemental layer of the substrate, $T_{sn}$

$$T_{sn}' = T_{sn}\left[ 1.0 - \left( \frac{2}{M_s} \right) (N_s + 1) \right] + \left( \frac{2}{M_s} \right) (T_{s(n-1)} + N_s T_{a})$$  \hspace{1cm} (A12)

### A.2 FINITE-DIFFERENCE EQUATIONS FOR THE THREE-COMPONENT SYSTEM

The nine types of finite-difference equations for the three-component system are:

- At the bitumen surface, $T_{bl}$

$$T_{b1}' = T_{b1}\left[ 1.0 - \left( \frac{2}{M_b} \right) (N_b + 1.0) \right] + \left( \frac{2}{M_b} \right) (T_{b2} + N_b T_{a})$$  \hspace{1cm} (A13)

- Within the bitumen, $T_{bi}$

$$T_{bi}' = T_{bi} + \frac{1}{M_b} \left( T_{b(i-1)} - 2T_{bi} + T_{b(i+1)} \right)$$  \hspace{1cm} (A14)

- In the bottom elemental layer of bitumen, $T_{bn}$

$$T_{bn}' = T_{bn} + \frac{1}{M_b} \left[ T_{b(n-1)} - \left( 1 + \frac{2k_f}{k_b + k_f} \right) T_{bn} + \left( \frac{2k_f}{k_b + k_f} \right) T_{f1} \right]$$  \hspace{1cm} (A15)

- In the top elemental layer of felt, $T_{f1}$

$$T_{f1}' = T_{f1} + \frac{1}{M_f} \left[ \left( \frac{2k_b}{k_b + k_f} \right) T_{bn} - \left( 1 + \frac{2k_b}{k_b + k_f} \right) T_{f1} + T_{f2} \right]$$  \hspace{1cm} (A16)

- Within the felt, $T_{fi}$

$$T_{fi}' = T_{fi} + \frac{1}{M_f} \left( T_{f(i-1)} - 2T_{fi} + T_{f(i+1)} \right)$$  \hspace{1cm} (A17)
\[ T_{fn} = T_{fn} + \frac{1}{M_f} \left[ T_{f(n-1)} - \left( 1 + \frac{2k_s}{k_f + k_s} \right) T_{fn} + \left( \frac{2k_s}{k_f + k_s} \right) T_{s1} \right] \] (A18)

\[ T_{s1} = T_{s1} + \frac{1}{M_s} \left[ \left( \frac{2k_f}{k_f + k_s} \right) T_{fn} - \left( 1 + \frac{2k_f}{k_f + k_s} \right) T_{s1} + T_{s2} \right] \] (A19)

\[ T_{si} = T_{si} + \frac{1}{M_s} (T_{s(i-1)} - 2T_{si} + T_{s(i+1)}) \] (A20)

\[ T_{sn} = T_{sn} \left[ 1.0 - \left( \frac{2}{M_s} \right) (N_s + 1) \right] + \left( \frac{2}{M_s} \right) (T_{s(n-1)} + N_s T_a) \] (A21)

A.3 \textit{FINITE-DIFFERENCE EQUATIONS FOR THE TWO-COMPONENT SYSTEM}

The six types of finite-difference equations for the two component system are:

\[ T_{bl1} = T_{bl1} \left[ 1.0 - \left( \frac{2}{M_b} \right) (N_b + 1.0) \right] + \left( \frac{2}{M_b} \right) (T_{b2} + N_b T_a) \] (A22)

\[ T_{bi} = T_{bi} + \frac{1}{M_b} (T_{b(i-1)} - 2T_{bi} + T_{b(i+1)}) \] (A23)

\[ T_{bn} = T_{bn} + \frac{1}{M_b} \left[ T_{b(n-1)} - \left( 1 + \frac{2k_s}{k_b + k_s} \right) T_{bn} + \left( \frac{2k_s}{k_b + k_s} \right) T_{s1} \right] \] (A24)
In the top elemental layer of the substrate, $T_{s1}$

$$T_{s1}' = T_{s1} + \frac{1}{M_s} \left[ \left( \frac{2k_b}{k_b + k_s} \right) T_{bn} - \left( 1 + \frac{2k_b}{k_b + k_s} \right) T_{s1} + T_{s2} \right]$$  \hspace{1cm} (A25)

Within the substrate, $T_{s1}$

$$T_{s1}' = T_{s1} + \frac{1}{M_s} (T_{s(i-1)} - 2T_{s1} + T_{s(i+1)})$$  \hspace{1cm} (A26)

In the bottom elemental layer of the substrate, $T_{sn}$

$$T_{sn}' = T_{sn} \left[ 1.0 - \left( \frac{2}{M_s} \right) (N_s + 1) \right] + \left( \frac{2}{M_s} \right) (T_{s(n-1)} + N_sT_a)$$  \hspace{1cm} (A27)
APPENDIX B. LISTING OF COMPUTER PROGRAM

The computer program for the mathematical model for predicting the cooling time of hot bitumen applied to various substrates under different environmental conditions was written specifically for the preparation of the majority of the figures (4 to 16) presented in section 3.2. The Fortran program is given in this appendix. Only one set of input data was necessary to produce the 48 distinct curves given in the majority of the figures (4 to 16).

The initial set of input data described a model roofing system in which a given thickness of bitumen at 500°F (260°C) was applied to other components at 120°F (49°C) with zero wind speed. After completing the calculation for predicting the cooling time of bitumen under these initial conditions, the program repeated the cooling calculations using new input data which was generated from the initial input data. The bitumen application temperatures selected in the study were 500, 450, 400, and 350°F (260, 232, 204, and 177°C) and the air and initial component temperatures were 120, 70, and 20°F (49, 21, and -7°C). The wind speeds used in the calculations were 0, 10, 20, and 30 mph (0, 4.5, 9.0, and 13.5 m/s). The cooling calculation using new input data repeated in a stepwise fashion until the output to plot the 48 distinct curves in each of the figures 4 to 16 was obtained.

Although the program was written to produce data from which the sets of plots in figures 4 to 16 were generated, minor changes may readily be made to allow the prediction of bitumen cooling times under environmental and temperature conditions different than those presented here. Also, minor changes in the program may be made to alter in an incremental fashion the thickness of the hot bitumen or other roofing components. For example, the curves shown in figure 17 were plotted after the program was altered to allow an iteration of the quantity of applied bitumen of about 15, 20, 25, and 30 lbm/100 ft² (0.73, 0.98, 1.2, and 1.5 kg/m²) in the absence of wind.

The Fortran program contains two subroutines which were useful to the NBS research staff in the development of the program and the interpretation of the preliminary output. Because these subroutines were useful, reference to them was kept within the program reproduced here, but the subroutines are not given. The first of these subroutines is called "TRACEH." Its purpose was to facilitate "debugging" the program and to verify whether the program was operating sequentially as expected. The second subroutine, called "PLOTT," allowed the generation of a single temperature versus time cooling curve for a distinct set of environmental conditions and component temperatures.

The Fortran program prepared to predict the cooling times of hot bitumen on various substrates under different environmental conditions follows.
A program to estimate the rate of cooling of hot roofing bitumens on various substrates.

Note—the roofing system herein modeled may include 2, 3, or 4 components—bitumen, felt, bitumen-cement, and substrate.

** Step 1—
** Definitions of
** Program Parameters and
** Dimension Statements

**----------------------------------------------------------------------

CB = thermal conductivity of hot bitumen  BTU/ft·h·ft
CF = thermal conductivity of felt  BTU/ft·h·ft
CC = thermal conductivity of bitumen-cement  BTU/ft·h·ft
CS = thermal conductivity of substrate  BTU/ft·h·ft

DB = thermal diffusivity of hot bitumen  ft²/h
DF = thermal diffusivity of felt  ft²/h
DC = thermal diffusivity of bitumen-cement  ft²/h
DS = thermal diffusivity of substrate  ft²/h

DELY = thickness of any finite element in the system  ft
DELT = incremental time  h
NUMCOM = number of components in the modeled system

DR = constant to convert temperatures to degrees Rankine
EMB = emissivity of bitumen dimensionalless
EMS = emissivity of substrate dimensionalless
SIGMA = Stefan-Boltzman constant  BTU·ft²/h·°R·ft
PFACT = factor to adjust the no. of data points printed as output

HBC = convection coefficient for bitumen
HSC = convection coefficient for substrate

NB = number of finite elements in bitumen
NF = number of finite elements in felt
NC = number of finite elements in bitumen-cement
NS = number of finite elements in the substrate

TA = temperature of air  °F
TB0 = initial temperature of hot bitumen  °F
TF0 = initial temperature of felt  °F
TC0 = initial temperature of bitumen-cement  °F
TS0 = initial temperature of substrate  °F

TB(i) = temperature of a bitumen element in time  °F
TF(i) = temperature of a felt element in time  °F
TC(i) = temperature of a bitumen-cement element in time  °F
TS(i) = temperature of a substrate element in time  °F

AVEBIT = average temperature of the bitumen in time  °F

TEMPMN = minimum temperature to which the bitumen cools
C TSECXM = MAXIMUM TIME OVER WHICH THE ATUMEN COOLS
DURING THE EXECUTION OF THE PROGRAM

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

DIMENSION TB(1000)
DIMENSION TF(1000)
DIMENSION TC(1000)
DIMENSION TS(1000)
DIMENSION TEMP(500)
DIMENSION TIME(500)

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

C IRO=5
C IPR=6
C
C OR=46.0
C EMB=0.95
C EMS=0.23
C SIGMA=1714E-08
C PFACT=2.0
C
C ***
C ** STEP 2 -
C ** READ IN PARAMETERS. **
C ***
C
C READ(IRD,211)BUG
C IBUG=BUG+0.5
C
C IF(IBM.EQ.1)CALL TRACEN('2','ROSSIT')
C
C WRITE(IPR,998)
C 998 FORMAT(1H1)
C WRITE(IPR,999)
C 999 FORMAT(1H1)
C WRITE(IPR,999)
C WRITE(IPR,999)
C
C 211 FORMAT(F15.0)
C 212 FORMAT(A6)
C
C READ(IRD,212,END=9000)JUNK
C IF(JUNK.EQ.'END')G0T09000
C READ(IRD,211)ANUMCO
C READ(IRD,211)DELT
C READ(IRD,211)DELY
C NUMCOM=ANUMCO*DELT+DELY
C
C IF(IBM.EQ.1)WRITE(IPR,221)ANUMCO,DELT,DELY
C 221 FORMAT(1H,'ANUMCO,DELT,DELY = ',E15.7)
116 READ(IRD+212)JUNK
117 READ(IRD+211)CB
118 READ(IRD+211)CF
119 READ(IRD+211)CC
120 READ(IRD+211)CS
121 IF(IBUG.EQ.1)WRITE(IPR+222)CB,CF,CC,CS
122 222 FORMAT(IH, 'CB,CF,CC,CS = ',4F15.7)
123 C
124 READ(IRD+212)JUNK
125 READ(IRD+211)DB
126 READ(IRD+211)DF
127 READ(IRD+211)DC
128 READ(IRD+211)DS
129 IF(IBUG.EQ.1)WRITE(IPR+223)DB,DF,DC,DS
130 223 FORMAT(IH, 'DB,DF,DC,DS = ',4F15.7)
131 C
132 READ(IRD+212)JUNK
133 READ(IRD+211)DBC
134 READ(IRD+211)HSC
135 IF(IBUG.EQ.1)WRITE(IPR+224)DBC,HSC
136 224 FORMAT(IH, 'DBC,HSC = ',4F15.7)
137 C
138 READ(IRD+212)JUNK
139 READ(IRD+211)ANB
140 READ(IRD+211)ANF
141 READ(IRD+211)ANC
142 READ(IRD+211)TANS
143 NB=ANB+0.5
144 NF=ANF+0.5
145 NC=ANC+0.5
146 NS=ANS+0.5
147 IF(IBUG.EQ.1)WRITE(IPR+225)NB,NF,NC,NS
148 225 FORMAT(IH, 'NB,NF,NC,NS = ',4F15.7)
149 C
150 READ(IRD+212)JUNK
151 READ(IRD+211)TA
152 READ(IRD+211)TB0
153 READ(IRD+211)TF0
154 READ(IRD+211)TCC
155 READ(IRD+211)TS0
156 IF(IBUG.EQ.1)WRITE(IPR+226)TA,TB0,TF0,TCC,TS0
157 226 FORMAT(IH, 'TA,TB0,TF0,TCC,TS0 = ',5F15.7)
158 C
159 READ(IRD+212)JUNK
160 READ(IRD+211)TSMN
161 READ(IRD+211)TSCMNX
162 IF(IBUG.EQ.1)WRITE(IPR+227)TSMN,TSCMNX
163 227 FORMAT(IH, 'TSMN,TSCMNX = ',2F15.7)
164 C
165 C
166 C
167 C
168 C ** STEP 3 --
169 C ** SET UP A LARGE LOOP INCREMENTING
170 C ** THE TEMPERATURES OF THE AIR AND SUBSTRATE(S),
171 C ** THE WIND SPEED, AND THE BITUMEN APPLICATION TEMP.
172 C
173 C
329 CONTINUE  
330 DO330 ITEMP=1,4  
331 TBO=TBO5AV  
332 IF(ITEMP.EQ.1)GOTO339  
333 IF(ITEMP.EQ.2)GOTO332  
334 IF(ITEMP.EQ.3)GOTO333  
335 IF(ITEMP.EQ.4)GOTO334  
336 GOTO339  
337  
338 CONTINUE  
339 TBO=TBO-50  
340 GOTO339  
341  
342 CONTINUE  
343 TBO=TBO-100  
344 GOTO339  
345  
346 CONTINUE  
347 TBO=TBO-150  
348 GOTO339  
349  
350 CONTINUE  
351 ICOUNT=ICOUNT+1  
352 WRITE(6,999)  
353 WRITE(6,341)ICOUNT  
354  
355 FORMAT(1H"*BEGIN COMPUTATION WITH DATA SET = 't',l0)  
356  
357 WRITE(6,999)  
358 WRITE(6,342)TATB0+TFO+TC0+TS0+HBC  
359  
360 FORMAT(1H"*TA=TBO+TFO+TC0+TS0+HBC = ',6F12,6)  
361  
362 CONTINUE  
363 **********  
364 **********  
365 ** STEP 4== **  
366 ** SET UP A LARGE LOOP **  
367 ** BASED ON INCREMENTING TIME. **  
368 **********  
369 **********  
370 IF(ITUG.EQ.1)CALL TRACEH('t',t',ROSTIT')  
371 T=0.0  
372 L=0  
373 L2=L+1  
374 TEMP(L)=TBO  
375 TIME(L)=0  
376 ISTEP=1  
377  
378 CONTINUE  
379 ISTEP=ISTEP+1  
380 IF(ITEG.EQ.0)WRIT(411)ISTEP+TBO  
381 IF(ITEG.EQ.0)GOTO500  
382 IF(ITEG.EQ.1000)ISTEP=0  
383 RTEM=ISTEP-1000/ISTEP/1000  
384 IF(RTEM=ISTEP)ISTEP,AVEBIT  
385  
386 FORMAT(1H"*ENTERING STEP 't'18' WITH AVE. RIT. TEMP. = ',E15.7)  
387  
388 **********  
389 **********  
390 **********  
391 **********  
392 **********  
393 **********
** INITIALIZE VALUES IN TB(,), TF(,), **
** TC(,), AND TS(,). **

500 CONTINUE

IF(IBUG.EQ.1)CALL TRACEN('5','ROSSIT')

IF(ISTEP.NE.0)GOTo590

DO510I=1,NB
TB(I)=TB0
510 CONTINUE

DO520I=1,NF
TF(I)=TF0
520 CONTINUE

DO530I=1,NC
TC(I)=TC0
530 CONTINUE

DO540I=1,NS
TS(I)=TS0
540 CONTINUE

590 CONTINUE

** APPLY THE SET OF EQUATIONS **
** (SURFACE, MIDDLE STEPS, AND BOTTOM) **
** FOR COMPONENT 1 (BITUNEN). **

IF(IBUG.EQ.1)CALL TRACEN('6','ROSSIT')

I=1
IP1=I+1
HBR=SIGMA*EMB((TB(I)+DR)*0.2+(TA+DR)*0.2)+((TB(I)+DR)+(TA+DR))
HB=HBC+HBR
ABB=DELY/HB/CO
AMB=(1.0/DB)*DELY/DELY/DELT)
TB(I)=TB(I)+(1.0-2.0/AMB)*(AMB+1.0) + (2.0/AMB)*(TB(IP1)+AMB*TA)
611 IF(IBUG.EQ.1)WRITE(6.611)TA,DR,HBR,HB,ABB,AMB,TB(I)

IMAX=NB-1
DO600I=2,IMAX
IM1=I-1
IP1=I+1
TB(I)=TB(I)+1.0/AMB*(TB(IM1)-2.0*TB(I)+TB(IP1))
600 CONTINUE

I=NB
IM1=I-1
C IF(NUMCON.LE.2) GOTO 631
GOTO 632
631 CONTINUE
FACTOR=(2.0*CS)/(CS+CS)
TB(1)=TB(1)+(1.0/AMB)*TB(IM1)-(1.0*FACTOR)*TB(1)+FACTOR*TS(1))
GOTO 630
632 CONTINUE
FACTOR=(2.0*CF)/(CS+CF)
TB(1)=TB(1)+(1.0/AMB)*TB(IM1)-(1.0*FACTOR)*TB(1)+FACTOR*TF(1))
GOTO 630
680 CONTINUE
IF(NUMCON.LE.2) GOTO 60910
C
C =============================================================================
C ** STEP 7--**
C ** APPLY THE SET OF EQUATIONS**
C ** (SURFACE, MIDDLE STEPS, AND BOTTOM)**
C ** FOR COMPONENT 2 (FELT).**
C =============================================================================
C
C IF(IBUG. EQ. 1) CALL TRACE(17, 'ROSSIT')
C I=1
IPI=I+1
AMP=(1.0/DF)/(DF+DF/DELT)
FACTOR=(2.0*CB)/(CB+CF)
TF(1)=TF(1)+(1.0/AMF)*(FACTOR+TAO(IM1)+(1.0+FACTOR)*TF(1)+TF(IPI))
C IMAX=NF-1
DOTOPI=2*IMAX
IM1=I-1
IM1=I+1
TF(1)=TF(1)+(1.0/AMF)*TF(IM1)-2.0*TF(I)+TF(IPI))
700 CONTINUE
C
C I=NF
IM1=I-1
C IF(NUMCON.LE.3) GOTO 731
GOTO 732
731 CONTINUE
FACTOR=(2.0*CS)/(CS+CS)
TF(1)=TF(1)+(1.0/AMF)*TF(IM1)-(1.0+FACTOR)*TF(1)+FACTOR*TS(1))
GOTO 730
732 CONTINUE
FACTOR=(2.0*CF)/(CS+CF)
TF(1)=TF(1)+(1.0/AMF)*TF(IM1)-(1.0+FACTOR)*TF(1)+FACTOR*TC(1))
GOTO 730
780 CONTINUE
IF(NUMCON.LE.3) GOTO 60910
C
C =============================================================================
C ** STEP 8--**
C ** APPLY THE SET OF EQUATIONS**
C =============================================================================
C
** (SURFACE, MIDDLE STEPS, AND BOTTOM) **
** FOR COMPONENT 3 (BITUMEN-CEMENT) **
*******************************************************************************
IF(IBUG.EQ.1)CALL TRACEH('8', 'ROSSIT')

I=1
IPI=I+1
AMC=(1.0/QC)*(DELY*DELY/DELT)
FACTOR=(2.0*CF)/(CF*CC)
TC(I)=TC(I)+(1.0/AMC)*(FACTOR*TF(NF)-(1.0+FACTOR)*TC(I)+TC(I1))
IMAX=NC-1
DO8010=2,IMAX
I=1
I1=I+1
TC(I1)=TC(I)+(1.0/AMC)*(TC(I1)-2.0*TC(I)+TC(I1))
8010 CONTINUE

I=NC
I1=I+1
FACTOR=(2.0*CS)/(CC*CS)
TC(I1)=TC(I1)+(1.0/AMC)*(TC(I1)-1.0+FACTOR)*TC(I1)+FACTOR*TS(I1))
*******************************************************************************
** STEP 9 -- **
** APPLY THE SET OF EQUATIONS **
** (SURFACE, MIDDLE STEPS, AND BOTTOM) **
** FOR COMPONENT 4 (SUBSTRATE). **
*******************************************************************************
910 CONTINUE

IF(IBUG.EQ.1)CALL TRACEH('9', 'ROSSIT')

I=1
I1=I+1
AMR=(1.0/DS)*(DELY/DELY/DELT)
IF(NUMCOM.LE.2)GOTO931
IF(NUMCOM.EQ.3)GOTO932
IF(NUMCOM.GE.4)GOTO933
931 CONTINUE
FACTOR=(2.0*CB)/(CB*CS)
TS(I)=TS(I)+(1.0/AMR)*(FACTOR*TB(NR)-(1.0+FACTOR)*TS(I)+TS(I1))
GOTO980

GOTO980
920 CONTINUE
FACTOR=(2.0*CF)/(CF*CS)
TS(I)=TS(I)+(1.0/AMR)*(FACTOR*TF(NF)-(1.0+FACTOR)*TS(I)+TS(I1))
GOTO980

GOTO980
930 CONTINUE
FACTOR=(2.0*CC)/(CC*CS)
TS(I)=TS(I)+(1.0/AMR)*(FACTOR*TC(NR)-(1.0+FACTOR)*TS(I)+TS(I1))
GOTO980

980 CONTINUE
C
464 IF(NS.GT.100)IMAX=100
465 IF(NS.LE.100)IMAX=NS-1
466 DO900=1,IMAX
467 IM1=1
470 ITZ(K(I)+1)+TS(I)+(1.0/AMS)*((TS(IM1)-2.0*TS(I))*TS(IP1))
471 900 CONTINUE
472 C
473 IF(NS.GT.100)GOTO990
474 IM1=1
476 IF(ISTEP.EQ.0)TS(I)=TS0
477 HSR=SIGMA*ENETS*(TS(I)+DR)*2+(TA+DR)*2*(TS(I)+DR)+(TA+DR)
478 HS=HS/HSR
479 ABS=DELTY*HS/CS
480 AMS=(1.0/D5)*(DELTY*DELT/DELT)
481 TS(I)=TS(I)+(1.0-(2.0/AMS)*(ABS+1.0))+(2.0/AMS)*(TS(IM1)+ABS+TA)
482 990 CONTINUE
483 C
484 C
485 C
486 C
487 C ** COMPUTE SUM AND AVEBIT
488 C ** SUM IS THE SUMMATION OF TEMPERATURES
489 C ** OF ELEMENTS IN THE BITUMEN COMPONENT
490 C ** AVEBIT IS THE AVERAGE TEMPERATURE OF BITUMEN
491 C
492 C
494 C
495 C IF(IBUT.EQ.1)CALL TRACEH('10','ROSSIT')
496 C
497 C SUM=0.0
498 DO1032=1,NB
499 SUM=SUM+TS(I)
500 IF(IBUT.EQ.1)WRITE(IPR,1034)I,TS(I)
501 1034 FORMAT(1H,'1T',I,TS(I),' ',I8,E15.7)
502 1032 CONTINUE
503 AVEBIT=SUM/NB
504 C
505 C
506 C
507 C ** STEP 11--
508 C ** COMPARE AVERAGE TEMPERATURE TO TEMPMN.
509 C ** COMPARE TIME OF COOLING TO TSEC
510 C
511 C
512 C
513 C IF(IBUT.EQ.1)CALL TRACEH('11','ROSSIT')
514 C
515 C IF(IBUT.EQ.1)WRITE(IPR,1111)AVEBIT,TEMPMN
516 1111 FORMAT(1H,'AVERAGE , TEMP = ',2F20.10)
517 IF(AVEBIT.LT.TEMPMN)GOTO1200
518 TSOLD=T*3600.0
519 T=T*DELT
520 TSNEW=T*3600.0
521 IF(IBUT.EQ.1)WRITE(IPR,1112)TSNEW,TSEC
1112 FORMAT(1H,**TSNEW,TSECXM = **,2F20.10)
1113 FORMAT(1H,**TSNE2,ITSOL2 = **,2B16)

1120 CONTINUE
1130 IF(ITSNE2.EQ.ITSO2)GO TO 1220

1210 FORMAT(1H,**NUMBER OF CUMULATIVE POINTS COMPUTED = **,I8)
1220 CALL PLOT(TMP,TIME,*)
1230 CONTINUE
310 CONTINUE

** STEP 90 -- **
**  EXIT.  **

9000 CONTINUE

IF(IBUG.EQ.1)CALL TRACE('90','ROSSIT')

C WRITE(7,9001)

9001 FORMAT('END')

ENDFILE 7

END FILE 7

6002 CONTINUE

STOP

603 END

END PRT

0PRT,S DATA,B
B.1 CHECK ON COMPUTER PROGRAM EXECUTION

The model presented in this paper has not been validated through field experimentation. To determine whether the model for predicting the cooling time of hot bitumen was functioning in a reasonable manner, the computer program was altered to simulate a condition in which no heat could flow from the bottom layer (infinitely insulated bottom surface) of the substrate to the surroundings. The computer program could thus approximate heat transfer in an infinite flat plate which is initially at a uniform temperature and then subjected to a hot environment such that heat flows from the surroundings into the flat plate. It is noted that the four-component model roofing system is equivalent to a flat plate if the physical properties (i.e., temperature, thermal conductivity, and density) of the four components are taken to be identical. Transient heat flow and temperature distribution in a flat plate have been calculated and the results have been given in charts. Kreith has presented these charts and examples of their use [Bl]. The treatment here is based on the charts given by Kreith.

The following calculation was performed to compare the output of the computer program with the charts given by Kreith. A 0.025 ft (7.6 mm) thick plate containing 100 finite elements and having an initial temperature of 100°F (38°C) was subjected to elevated temperatures at the surface. Heat was allowed to flow into the plate for 100 seconds. The temperatures of the top, twentieth, fourtieth, sixtieth, eightieth, and bottom finite elements within the plate were calculated from the computer program. The thermal conductivity and diffusivity of the plate were taken as 0.54 Btu·in/h·ft²·°F (0.078 W/m·K) and 0.0187 ft²/h (0.48 mm²/s), respectively. The convection coefficient was taken as 5.4 Btu/h·ft²·°F (30.6 W/m²·K). Ambient temperatures to which the surface of the plate was subjected were 200, 300, and 400°F (93, 149, and 204°C). The results from the computer program output and those obtained from the temperature charts are given in table B1.

A comparison of the temperature distribution results from the computer program output and the temperature charts shows differences between 2 and 6 percent, depending upon the temperature to which the plate was subjected and the position of the finite element within the slab. The percent differences were considered acceptable, since it was difficult to interpolate the temperature charts given by Kreith because of their small size. Interpolation was considered a major source of error. The results indicated that the computer program for determining the temperature distribution in the flat plate was functioning as expected. Moreover, since only a minor change was made in converting the bitumen cooling program into the flat plate program, the bitumen cooling program was also considered to be functioning in a reasonable manner.
Table B1. Temperature Distribution in the Finite Elements of the Flat Plate as Determined from the Computer Program and the Temperature Charts.

<table>
<thead>
<tr>
<th>Temperature to Which the Plate Was Subjected</th>
<th>Finite Element</th>
<th>Finite Element Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Program Output</td>
</tr>
<tr>
<td></td>
<td></td>
<td>°F</td>
</tr>
<tr>
<td>200°F (93°C)</td>
<td>top</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>20th</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>40th</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>60th</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>80th</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>bottom</td>
<td>109</td>
</tr>
<tr>
<td>300°F (149°C)</td>
<td>top</td>
<td>132</td>
</tr>
<tr>
<td></td>
<td>20th</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>40th</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>60th</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>80th</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>bottom</td>
<td>120</td>
</tr>
<tr>
<td>400°F (204°C)</td>
<td>top</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>20th</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>40th</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>60th</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>80th</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>bottom</td>
<td>136</td>
</tr>
</tbody>
</table>

(a) from Kreith [Bl].
APPENDIX C. NOTATION

This appendix lists the notation used in this report in the development of both the mathematical model and the computerized program for the prediction of the cooling time of bitumen during the construction of built-up roofing. Table C1 includes the algebraic notation, Fortran code, the description or definition, and the customary and S.I. units of measurement for each term used.

Table C1. Notation and Description of Terms

<table>
<thead>
<tr>
<th>Algebraic Notation</th>
<th>Fortran Code</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td><strong>(1)</strong></td>
<td>thermal diffusivity; ( a = \frac{k}{C_p \cdot \rho} )</td>
<td>ft(^2)/h, mm(^2)/s</td>
</tr>
<tr>
<td>*(2)</td>
<td>DB</td>
<td>thermal diffusivity of applied bitumen</td>
<td>ft(^2)/h, mm(^2)/s</td>
</tr>
<tr>
<td>*</td>
<td>DC</td>
<td>thermal diffusivity of bitumen-cement</td>
<td>ft(^2)/h, mm(^2)/s</td>
</tr>
<tr>
<td>*</td>
<td>DF</td>
<td>thermal diffusivity of roofing felt</td>
<td>ft(^2)/h, mm(^2)/s</td>
</tr>
<tr>
<td>*</td>
<td>DS</td>
<td>thermal diffusivity of substrate</td>
<td>ft(^2)/h, mm(^2)/s</td>
</tr>
<tr>
<td>C</td>
<td>**</td>
<td>specific heat</td>
<td>Btu/\text{lbm/°F}</td>
</tr>
<tr>
<td>C(_{pb})</td>
<td>**</td>
<td>specific heat of applied bitumen</td>
<td>Btu/\text{lbm/°F}</td>
</tr>
<tr>
<td>C(_{pc})</td>
<td>**</td>
<td>specific heat of bitumen-cement</td>
<td>Btu/\text{lbm/°F}</td>
</tr>
<tr>
<td>C(_{pf})</td>
<td>**</td>
<td>specific heat of roofing felt</td>
<td>Btu/\text{lbm/°F}</td>
</tr>
<tr>
<td>C(_{ps})</td>
<td>**</td>
<td>specific heat of substrate</td>
<td>Btu/\text{lbm/°F}</td>
</tr>
<tr>
<td>e</td>
<td>**</td>
<td>surface emittance</td>
<td>-</td>
</tr>
<tr>
<td>*</td>
<td>EMB</td>
<td>surface emittance of bitumen</td>
<td>-</td>
</tr>
<tr>
<td>*</td>
<td>EMS</td>
<td>surface emittance of substrate</td>
<td>-</td>
</tr>
<tr>
<td>h</td>
<td>**</td>
<td>surface emittance of heat transmission</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
<tr>
<td>h(_c)</td>
<td>**</td>
<td>surface convection coefficient</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
<tr>
<td>h(_r)</td>
<td>**</td>
<td>surface radiation coefficient</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
<tr>
<td>h(_b)</td>
<td>HB</td>
<td>bitumen surface coefficient of heat transmission</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
<tr>
<td>*</td>
<td>HBC</td>
<td>bitumen surface convection coefficient</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
<tr>
<td>*</td>
<td>HBR</td>
<td>bitumen surface radiation coefficient</td>
<td>Btu/\text{h/\text{ft}^2/°F}</td>
</tr>
</tbody>
</table>

(1) The double asterisk indicates that the term was not used in the paper in Fortran code.
(2) The asterisk indicates that the term was not used in algebraic notation.
(3) The dash indicates that the term is dimensionless.
<table>
<thead>
<tr>
<th>Algebraic Notation</th>
<th>Fortran Code</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_s )</td>
<td>HS</td>
<td>substrate surface coefficient of heat transmission</td>
<td>Btu/ft^2°F</td>
</tr>
<tr>
<td>( * )</td>
<td>HSC</td>
<td>substrate surface convection coefficient</td>
<td>Btu/ft^2°F</td>
</tr>
<tr>
<td>( * )</td>
<td>HSR</td>
<td>substrate surface radiation coefficient</td>
<td>Btu/ft^2°F</td>
</tr>
<tr>
<td>( k )</td>
<td>**</td>
<td>thermal conductivity</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( k_b )</td>
<td>CB</td>
<td>thermal conductivity of applied bitumen</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( k_c )</td>
<td>CC</td>
<td>thermal conductivity of bitumen-cement</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( k_f )</td>
<td>CF</td>
<td>thermal conductivity of roofing felt</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( k_s )</td>
<td>CS</td>
<td>thermal conductivity of substrate</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( M )</td>
<td>**</td>
<td>modulus relating ( \Delta y ) and ( \Delta t )</td>
<td>-</td>
</tr>
<tr>
<td>( M_b )</td>
<td>AMB</td>
<td>modulus for bitumen</td>
<td>-</td>
</tr>
<tr>
<td>( M_c )</td>
<td>AMC</td>
<td>modulus for bitumen-cement</td>
<td>-</td>
</tr>
<tr>
<td>( M_f )</td>
<td>AMF</td>
<td>modulus for roofing felt</td>
<td>-</td>
</tr>
<tr>
<td>( M_s )</td>
<td>AMS</td>
<td>modulus for substrate</td>
<td>-</td>
</tr>
<tr>
<td>( N )</td>
<td>**</td>
<td>Nusselt or Biot number; ( N = h*\Delta y/k )</td>
<td>-</td>
</tr>
<tr>
<td>( N_b )</td>
<td>ABB</td>
<td>Nusselt or Biot number for bitumen</td>
<td>-</td>
</tr>
<tr>
<td>( N_s )</td>
<td>ABS</td>
<td>Nusselt or Biot number for substrate</td>
<td>-</td>
</tr>
<tr>
<td>( * )</td>
<td>NB</td>
<td>number of finite-elements in the bitumen</td>
<td>-</td>
</tr>
<tr>
<td>( * )</td>
<td>NC</td>
<td>number of finite-elements in the bitumen-cement</td>
<td>-</td>
</tr>
<tr>
<td>( * )</td>
<td>NF</td>
<td>number of finite-elements in the felt</td>
<td>-</td>
</tr>
<tr>
<td>( * )</td>
<td>NS</td>
<td>number of finite-elements in the substrate</td>
<td>-</td>
</tr>
<tr>
<td>( \rho )</td>
<td>**</td>
<td>density</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>( \rho_b )</td>
<td>**</td>
<td>density of bitumen</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>**</td>
<td>density of bitumen-cement</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>**</td>
<td>density of roofing felt</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>**</td>
<td>density of substrate</td>
<td>lbm/ft^3</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>SIGMA</td>
<td>Stefan-Boltzman constant;</td>
<td>Btu<em>ft/hr</em>ft^2°F</td>
</tr>
<tr>
<td>( T )</td>
<td>**</td>
<td>temperature</td>
<td>°F, °C</td>
</tr>
<tr>
<td>( T_a )</td>
<td>TA</td>
<td>air temperature</td>
<td>°F, °C</td>
</tr>
<tr>
<td>Algebraic Notation</td>
<td>Fortran Code</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$T_s$</td>
<td>**</td>
<td>temperature of a surface</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bl}$</td>
<td>TB(1)</td>
<td>temperature of the top bitumen surface element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bl}'$</td>
<td>TB(1)</td>
<td>future temperature of the top bitumen surface element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bi}$</td>
<td>TB(I)</td>
<td>temperature of a bitumen element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bi}'$</td>
<td>TB(I)</td>
<td>future temperature of a bitumen element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bn}$</td>
<td>TB(NB)</td>
<td>temperature of the bottom bitumen element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{bn}'$</td>
<td>TB(NB)</td>
<td>future temperature of the bottom bitumen element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{cl}$</td>
<td>TC(1)</td>
<td>temperature of the top bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{cl}'$</td>
<td>TC(1)</td>
<td>future temperature of the top bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{ci}$</td>
<td>TC(I)</td>
<td>temperature of a bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{ci}'$</td>
<td>TC(I)</td>
<td>future temperature of a bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{cn}$</td>
<td>TC(NC)</td>
<td>temperature of the bottom bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{cn}'$</td>
<td>TC(NC)</td>
<td>future temperature of the bottom bitumen-cement element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fl}$</td>
<td>TF(1)</td>
<td>temperature of the top felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fl}'$</td>
<td>TF(1)</td>
<td>future temperature of the top felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fi}$</td>
<td>TF(I)</td>
<td>temperature of a felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fi}'$</td>
<td>TF(I)</td>
<td>future temperature of a felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fn}$</td>
<td>TF(NF)</td>
<td>temperature of the bottom felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{fn}'$</td>
<td>TF(NF)</td>
<td>future temperature of the bottom felt element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{sl}$</td>
<td>TS(1)</td>
<td>temperature of the top substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{sl}'$</td>
<td>TS(1)</td>
<td>future temperature of the top substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{si}$</td>
<td>TS(I)</td>
<td>temperature of a substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{si}'$</td>
<td>TS(I)</td>
<td>future temperature of a substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{sn}$</td>
<td>TS(NS)</td>
<td>temperature of the bottom substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>$T_{sn}'$</td>
<td>TS(NS)</td>
<td>future temperature of the bottom substrate element</td>
<td>°F</td>
</tr>
<tr>
<td>*</td>
<td>TBO</td>
<td>initial temperature of bitumen</td>
<td>°F</td>
</tr>
<tr>
<td>*</td>
<td>TC0</td>
<td>initial temperature of bitumen-cement</td>
<td>°F</td>
</tr>
<tr>
<td>Algebraic Notation</td>
<td>Fortran Code</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-------------------</td>
<td>-------------</td>
<td>-------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>*</td>
<td>TFO</td>
<td>initial temperature of roofing felt</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td>TSO</td>
<td>initial temperature of substrate</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>**</td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>Δt</td>
<td>DELT</td>
<td>a small finite time increment or interval</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>**</td>
<td>a space coordinate for the heat flow</td>
<td></td>
</tr>
<tr>
<td>Δy</td>
<td>DELY</td>
<td>a small finite distance</td>
<td></td>
</tr>
</tbody>
</table>
Cooling of Bitumen During Construction of Built-Up Roofing Systems - A Mathematical Model

Walter J. Rossiter, Jr., Robert G. Mathey, Herbert W. Busching, and William C. Cullen

Construction of bituminous built-up roofing systems in the United States generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Adequate adhesion of the bitumen to the roofing component materials may be obtained only when the hot bitumen is applied at a viscosity sufficient to flow uniformly, to cover the component surfaces or substrate completely, and to provide the proper thickness. During construction, rapid cooling of hot bitumen increases its viscosity significantly. If the viscosity becomes too high, poor adhesion between components, voids within the bitumen, and an excessive and non-uniform thickness of the bitumen may result.

This report describes a mathematical model based on finite-difference equations for calculating transient heat flow to estimate the cooling time of hot roofing bitumen. Estimates of the time required for hot bitumen to cool from its application temperature to 300°F (149°C) were computed as a function of material and environmental factors including: quantity of applied bitumen, bitumen application or contact temperature, air temperature, wind speed, and thermal properties of the bitumen and of the roofing components. The model was used to predict cooling times expected for hot asphalt applied to typical substrates with thermal properties representative of those of polyurethane foam and glass fiber insulation boards, insulating concrete, plywood, concrete and steel decks, and roofing felt on decks or insulations. In addition, the model was used to predict cooling times for hot coal tar pitch applied to concrete and to felt adhered to glass fiber insulation. The results of the calculations demonstrate the widely-varying bitumen cooling times which depend upon the component material to which the bitumen is applied and the environmental conditions during application. Under certain environmental conditions, hot bitumen applied to some substrates cools extremely rapidly. In these cases, sufficient time for proper application may not be available.

Bitumen application temperatures; bituminous roofing; built-up roofing; cooling of roofing bitumens; mathematical model; roofing.
NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH—The Journal of Research of the National Bureau of Standards reports NBS research and development in those disciplines of the physical and engineering sciences in which the Bureau is active. These include physics, chemistry, engineering, mathematics, and computer sciences. Papers cover a broad range of subjects, with major emphasis on measurement methodology and the basic technology underlying standardization. Also included from time to time are survey articles on topics closely related to the Bureau’s technical and scientific programs. As a special service to subscribers each issue contains complete citations to all recent Bureau publications in both NBS and non-NBS media. Issued six times a year. Annual subscription: domestic $13; foreign $16.25. Single copy, $3 domestic; $3.75 foreign.

NOTE: The Journal was formerly published in two sections: Section A “Physics and Chemistry” and Section B “Mathematical Sciences.”

DIMENSIONS/NBS—This monthly magazine is published to inform scientists, engineers, business and industry leaders, teachers, students, and consumers of the latest advances in science and technology, with primary emphasis on work at NBS. The magazine highlights and reviews such issues as energy research, fire protection, building technology, metric conversion, pollution abatement, health and safety, and consumer product performance. In addition, it reports the results of Bureau programs in measurement standards and techniques, properties of matter and materials, engineering standards and services, instrumentation, and automatic data processing. Annual subscription: domestic $11; foreign $13.75.

NONPERIODICALS

Monographs—Major contributions to the technical literature on various subjects related to the Bureau’s scientific and technical activities.

Handbooks—Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Special Publications—Include proceedings of conferences sponsored by NBS, NBS annual reports, and other special publications appropriate to this grouping such as wall charts, pocket cards, and bibliographies.

Applied Mathematics Series—Mathematical tables, manuals, and studies of special interest to physicists, engineers, chemists, biologists, mathematicians, computer programmers, and others engaged in scientific and technical work.

National Standard Reference Data Series—Provides quantitative data on the physical and chemical properties of materials, compiled from the world’s literature and critically evaluated. Developed under a worldwide program coordinated by NBS under the authority of the National Standard Data Act (Public Law 90-396).

NOTE: The principal publication outlet for the foregoing data is the Journal of Physical and Chemical Reference Data (JPCR) published quarterly for NBS by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements available from ACS, 1155 Sixteenth St., NW, Washington, DC 20036.

Building Science Series—Disseminates technical information developed at the Bureau on building materials, components, systems, and whole structures. The series presents research results, test methods, and performance criteria related to the structural and environmental functions and the durability and safety characteristics of building elements and systems.

Technical Notes—Studies or reports which are complete in themselves but restrictive in their treatment of a subject. Analogous to monographs but not so comprehensive in scope or definitive in treatment of the subject area. Often serve as a vehicle for final reports of work performed at NBS under the sponsorship of other government agencies.

Voluntary Product Standards—Developed under procedures published by the Department of Commerce in Part 10, Title 15, of the Code of Federal Regulations. The standards establish nationally recognized requirements for products, and provide all concerned interests with a basis for common understanding of the characteristics of the products. NBS administers this program as a supplement to the activities of the private sector standardizing organizations.

Consumer Information Series—Practical information, based on NBS research and experience, covering areas of interest to the consumer. Easily understandable language and illustrations provide useful background knowledge for shopping in today’s technological marketplace.


Order the following NBS publications—FIPS and NBSIR’s—from the National Technical Information Services, Springfield, VA 22161.


NBS Interagency Reports (NBSIR)—A special series of interim or final reports on work performed by NBS for outside sponsors (both government and non-government). In general, initial distribution is handled by the sponsor; public distribution is by the National Technical Information Services, Springfield, VA 22161, in paper copy or microfiche form.