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Cooling of Bitumen During Construction of Built-Up Roofing Systems— A Mathematical Model

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Cooling of Bitumen During Construction of Built-Up Roofing Systems— A Mathematical Model

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COOLING OF BITUMEN DURING CONSTRUCTION OF BUILT-UP ROOFING SYSTEMS - A MATHEMATICAL MODEL

by

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ABSTRACT

Construction of bituminous built-up roofing systems in the United States generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Adequate adhesion of the bitumen to the roofing component materials may be obtained only when the hot bitumen is applied at a viscosity sufficient to flow uniformly, to cover the component surfaces or substrate completely and to provide the proper thickness. During construction, rapid cooling of hot bitumen increases its viscosity significantly. If the viscosity becomes too high, poor adhesion between components, voids within the bitumen, and an excessive and non-uniform thickness of the bitumen may result.

This report describes a mathematical model based on finite-difference equations for calculating transient heat flow to estimate the cooling time of hot roofing bitumen. Estimates of the time required for hot bitumen to cool from its application temperature to 300°F (149°C) were computed as a function of material and environmental factors, including: quantity of applied bitumen, bitumen application or contact temperature, air temperature, wind speed, and thermal properties of the bitumen and of the roofing components. The model was used to predict cooling times expected for hot asphalt applied to typical substrates with thermal properties representative of those of polyurethane foam and glass fiber insulation boards, insulating concrete, plywood, concrete and steel decks, and roofing felt on decks or insulations. In addition, the model was used to predict cooling times for hot coal tar pitch applied to concrete and to felt adhered to glass fiber insulation. The results of the calculations demonstrate the widely varying bitumen cooling times which depend upon the component material to which the bitumen is applied and the environmental conditions during application. Under certain environmental conditions, hot bitumen applied to some substrates cools extremely rapidly. In these cases, sufficient time for proper application may not be available.

Key words: Bitumen application temperatures; bituminous roofing; built-up roofing; cooling of roofing bitumens; mathematical model; roofing.

1. INTRODUCTION

The most widely accepted waterproofing system for low-sloped roofs in the United States is bituminous built-up roofing. Construction of bituminous built-up roofing systems generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Bitumens commonly used as waterproofing materials and adhesives in built-up membranes are asphalt and coal tar pitch.

Roofing bitumens must be applied at elevated temperatures so that their viscosities will be sufficiently low to enable uniform flow, complete coverage of component surfaces, and proper thickness. After hot bitumen is applied to the substrate, it cools rapidly. Little time is available to place roofing felts or insulation on the hot bitumen while its temperature is sufficiently high and its viscosity is sufficiently low to provide for adequate adhesion and proper thickness. Laboratory data are available on the relationship between temperature and viscosity of roofing asphalts [1,2].* However, little data were found in the literature on the cooling time of hot bitumen during built-up roof construction. A field study has been reported wherein the cooling time of asphalt during cold weather construction was determined [3].

Since the rate at which roofing bitumens cool is important to the quality of built-up roofing construction, a mathematical model was developed to enable quantitative assessment of the effects of various factors on the cooling time of roofing bitumens. The mathematical model is based on well-known heat transfer principles, uses finite-difference equations [4-6], and makes use of representative roofing materials and their properties. It also incorporates the principal environmental variables associated with roof construction. The mathematical model to predict the cooling time of bitumen assumes one dimensional heat transfer; that is, the roof is taken to be a flat plate with its components at uniform temperature across the horizontal planes of the plate. During built-up roofing construction, hot bitumen is applied over large areas of the roof. Because large areas of the roof are considered to be at uniform temperature under these conditions, the heat transfer was assumed to occur in the direction perpendicular to the plane of the roof and not laterally within its plane.

Cooling time of bitumen can in principle be determined experimentally by means of field studies. However, measurement of cooling times applicable to many different types of roofing systems under a variety of environmental conditions are precluded in practice because of the excessive time and expense required to conduct investigations involving many variables. Nevertheless, selective field studies to obtain data on bitumen cooling times are needed to assess the validity of the model presented.

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* Figures in brackets indicate references listed in Section 6.

1.1 BACKGROUND

Rossiter and Mathey [1] measured the viscosities of ASTM Type I and Type III roofing asphalts from different sources over a range of application temperatures. They recommended that roofing asphalts be applied at temperatures based on viscosity rather than on empirically determined temperature limits. From a similar investigation, Ducy recommended that roofing asphalts be applied based on an equiviscous temperature (EVT) [2]. It was suggested by Rossiter and Mathey [1] that in order to maintain the recommended viscosity range during application, consideration should be given to factors affecting the rate of cooling such as air temperature, wind speed, substrate temperature, and the specific heat of the substrate. A viscosity criterion for the application of asphalt has not generally been used in roofing construction nor included in specifications. It is noted, for example, that the National Roofing Contractors Association (NRCA) [7] recommends a minimum application temperature of 350°F (177°C) and a maximum heating temperature of 475°F (246°C) for asphalt. Likewise, the Tri-Service Roofing Manual [8] states that asphalt should not be heated above 450°F (232°C) and should not normally be lower than 350°F (177°C) when applied to the roof. Current trends in the roofing industry indicate an acceptance of the selection of application temperatures based upon the viscosity of the bitumen.

During application, if bitumen cools too rapidly with an accompanying increase in its viscosity, the bitumen layers may be too thick, uneven, contain voids, and result in inadequate adhesion between roof components. These effects result in unsatisfactory built-up bituminous membrane performance. A thick or uneven application of interply bitumen can lead to poor adhesion between plies [1]. Voids between plies or between the membrane and insulation are sources of blistering [9]. Excessive thickness of between-ply bitumen under certain conditions may contribute to membrane slippage [10]. Since the bitumen has a larger coefficient of thermal expansion than other membrane materials, excessive thickness of bitumen between plies may result in higher thermally induced forces in the membrane. Lee, Dupuis, and Johnson [11] showed that temperature-induced forces measured for 2-ply membranes increased as the quantity of interply asphalt was increased. Temperature-induced forces due to a temperature change from 70 to -20°F (21 to -29°C) at an unspecified rate, measured for "thin, normal, and thick" interply thicknesses of asphalt, were about 54, 55, and 80 lbf/in (9.4, 9.6, and 14.0 kN/m), respectively [11].

In a field investigation of cooling times of asphalt applied during cold weather, Dupuis, Lee, and Johnson [3] measured temperatures of hot asphalt in kettles and the contact temperatures (temperatures at the time the asphalts came in contact with the substrates). They reported that during asphalt application, peak contact temperatures* were usually 50 - 75°F (28 - 42°C) lower than the temperature in the asphalt kettle. Asphalt cooling times

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* Peak contact temperature was defined as the highest temperature recorded during asphalt application [3].

were recorded at various locations within insulated roofing systems under construction in four cities in Wisconsin with air temperatures ranging from about 20 to 40°F (-7 to 4°C) and wind speeds from about 3 to 14 mph (1.4 to 6.3 m/s). Under those weather conditions, they found that the asphalt cooled rapidly. For example, in the application of asphalt to insulation, the asphalt temperatures dropped from the contact temperature to less than 300°F (149°C) in about 2 to 7 seconds.

The rapid cooling of asphalt observed by Dupuis, Lee, and Johnson [3] raised a question about adhesion of roofing components. They reported that rapid temperature decay of hot asphalt makes good adhesion extremely difficult to attain. They also stated that hot asphalt applied at subfreezing temperatures to light gauge steel decks congeals so rapidly that mechanical fastening is probably the only effective technique for securely anchoring the insulation. With regard to membrane application, they indicated that raising peak asphalt contact temperatures during cold weather may increase the time the asphalt remains sufficiently hot to adhere a ply of felt adequately.

In presenting the rate of heat loss (time dependency) of roofing asphalts, Dupuis, Lee, and Johnson [3] expressed the temperature of the asphalt with time as:

$$T = T_s + (T_o - T_s)\, e^{-\kappa\tau}$$

where T = the asphalt temperature at time τ
T_s = the temperature of the surroundings
T_o = the initial temperature of the asphalt at application
κ = an empirical constant

It is noted that this model was empirical and did not include terms for all the factors which affect the cooling time of asphalt. The cooling data obtained by Dupuis, Lee, and Johnson have not been compared with data generated by the model presented in this paper.

The roofing industry in the United States has not employed mathematical models to predict the cooling time of hot bitumen during construction of built-up roofing systems. A mathematical model would benefit the roofing industry since it would provide guidelines for the proper application of hot bitumen without excessive cooling on various components under a range of environmental conditions. Mathematical models were developed by Dickson and Corlew [12,13] for predicting temperatures of hot asphaltic concrete pavement layers placed in cold weather. These layers must be compacted while the viscosity of the asphalt is low enough to enable compaction of a pavement of proper density. Dickson and Corlew [12,13] used established finite-difference equations as a basis for evaluating temperature profiles in the hot asphaltic concrete pavement layers. Several references [14-18] identify the rationale for preheating the base to permit cold weather highway and airfield paving. The techniques used in estimating temperatures of paving mixtures are adaptable, with some modification, for use in estimating the cooling times of bitumen during the application of built-up roofing systems.

1.2 OBJECTIVE

The objective of this study was to develop a mathematical model for predicting the cooling time or time interval for hot bitumen to cool from its application temperature to a specified lower temperature during construction of built-up roofing systems. This paper presents examples of temperature decay charts generated from the model to enable the prediction of bitumen cooling times for various environmental conditions and for some roofing systems typical of those encountered in built-up roofing construction. The computer program based on the model is given to make it available to those individuals who wish to calculate bitumen cooling times for built-up roofing systems other than those presented here. Busching [19] has presented some early results from this study in a paper concerning the effects of moisture and temperature on roofing membranes.

Data developed through the use of the model may be useful in applying viscosity criteria and specifications effectively. In addition, information on the time of cooling for a wide range of variables may be used to help assure proper application of hot bitumen and other roofing components to improve construction practice. Data may be obtained from a computerized model at much lower cost than would be encountered if the same data were obtained through field experiment. However, before full reliance is placed on the results of the model calculations, data should be obtained as recommended in section 1 through selective field studies to assess the validity of the model.

2. COOLING OF BITUMEN

Hot bitumen is normally applied to roofs by mopping or by mechanical application. The method for applying hot bitumen was not a consideration in the development of the model to predict cooling times of bitumen. The model assumes that the temperature of the hot bitumen at the time of contact with the surface to which it is applied is known. The hot bitumen begins to cool immediately upon contact with the surface. The cooling time is dependent on factors associated with environmental conditions and materials properties.

2.1 FACTORS AFFECTING COOLING

Many factors affect the cooling of hot bitumen during roof construction. These factors include application temperature and thermal properties of the bitumen, quantity of bitumen applied per unit area (spread quantity), ambient air temperature, wind speed on the roof, and temperature, thickness and thermal properties of the component materials below the hot bitumen.

Thermal properties of the bitumen, substrate, and other component materials which affect the cooling are thermal conductivity, specific heat, and density. The application temperature of the hot bitumen is defined as the temperature at which the bitumen makes contact with the surface to which it is applied. The factors which affect cooling were incorporated in the mathematical model for predicting the cooling time of hot bitumen occurring in the construction of built-up roofing systems.

Most individuals experienced with roofing construction would attest that many of the factors listed previously affect the cooling of hot bitumen. However, the effect of these factors on bitumen cooling is only understood qualitatively. For example, a decrease in either the ambient air or substrate temperature will result in a decrease in the cooling time. The time of cooling will also decrease with an increase in wind speed. In addition, an increase in either the application temperature or quantity of hot bitumen will increase the time available to apply component roofing materials before the viscosity of the bitumen becomes too high to enable good adhesion. A mathematical model to predict cooling time of bitumen would quantify the effect of these factors.

The effect of the thermal properties (thermal conductivity, specific heat, and density) of the substrate and component materials on the cooling of hot bitumen may not be as evident to those experienced with roofing construction as for the environmental factors. Calculations using the mathematical model also allow quantitative determination of the effect of these thermal properties on bitumen cooling. Definitions of these thermal properties and related parameters are as follows:

- Thermal conductivity (k) - Time rate of heat flow through a homogeneous material under steady-state conditions through unit area, per unit temperature gradient. $Btu \cdot in/h \cdot ft^2 \cdot {}^\circ F$ ($W/m \cdot K$)

- Specific heat (C_p) - The quantity of heat required for a one degree temperature change in a unit mass of material. $Btu/lbm \cdot {}^\circ F$ ($J/kg \cdot K$)

- Density (ρ) - The mass of a material per unit volume. lbm/ft^3 (kg/m^3)

- Heat capacity (ρC_p) - The quantity of heat required for a one degree temperative change in a unit volume of material. $Btu/ft^3 \cdot {}^\circ F$ ($J/m^3 \cdot K$)

- Thermal diffusivity (α) - The ratio of a material's capacity to conduct thermal energy to its capacity to store thermal energy. ft^2/h (m^2/s) Thermal diffusivity is defined by the relationship:

$$\alpha = \frac{k}{\rho C_p}$$

2.2 MODEL FOR PREDICTING COOLING TIME

2.2.1 Assumptions in the Development of the Model

Assumptions made in the development of the mathematical model using finite-difference equations for predicting cooling time of hot bitumen are as follows:

- o heat transfer from the hot bitumen is one-dimensional perpendicular to to the plane of the roof;
- o the values of the properties of the bitumen and other roofing system components remain constant with time as the bitumen cools;
- o the thickness of the bitumen does not change as it cools;
- o absorbed solar radiation during bitumen application is negligible and neglected;
- o heat is lost from the bottom of the substrate by radiation and convection unless the substrate contains more than 100 elements;
- o the elements for each of the roofing system components have the same thickness except for the half elements at the top and bottom surfaces of the system;
- o the temperature of the bitumen was taken as the average of the temperature of all elements in the bitumen component;
- o the temperature of the air and sky are taken to be the same; and
- o the thermal resistance at the interfaces of the roofing system components is taken to be zero.

2.2.2 Finite-Difference Heat Flow Equations

2.2.2.1 Description of the System

During the construction of built-up roofing systems, hot bitumen is applied to cooler roofing materials and components such as decks, insulations and felts. Figure 1 shows schematically the modes of heat transfer that are considered in the analysis. Heat is lost from the hot bitumen to the cooler surroundings by convection and radiation at the top surface, and by conduction at the bottom surface which interfaces with the roofing component to which the bitumen is applied. In addition, as heat is lost from the surfaces of the bitumen layer, heat is transfered by conduction from the interior of the bitumen layer towards the cooler surfaces.

As shown in figure 1, heat flow from the hot bitumen is directed up from its top surface and down into the substrate. The heat flow from the upper surface of the bitumen is dependent on environmental conditions at the surface and on the temperature of the bitumen. Heat flow from the lower surface of the hot bitumen is dependent upon the temperature and thermal properties of the substrate as well as the temperature of the bitumen.

Radiant energy from the sun may also be incident on the roof and partially absorbed by the surface of the bitumen during built-up roofing construction. The quantity of radiant energy is negligible during bitumen application because of the relatively short time (usually a few seconds) during which the hot bitumen is exposed to solar radiation prior to being covered with insulation board or ply of felt.

The mathematical model to predict the cooling time of hot bitumen uses well-known finite-difference equations to calculate heat transfer. The treatment presented is based in general on that described by Dusinberre [5,6]. The calculations incorporated one-dimensional balanced heat flow between discrete nodes at specific locations within the hot bitumen. One-dimensional heat flow equations were developed that included terms which account for heat transfer by conduction, convection, and radiation depending upon the location of the discrete nodes selected. Figure 2 presents a diagram of the hot bitumen layer divided into elements for development of the finite-difference equations. The diagram in figure 2 shows the location of the nodes, the distance between nodes, the thickness of the elements, and the notation used to describe the temperature of the elements. Finite-difference equations were developed for three types of elements: the elements within the bitumen and the two boundary elements of the top and bottom surfaces. The development of these finite-difference equations is given in the sections which follow.

2.2.2.2 Heat Flow Within the Bitumen

Heat transfer by conduction within a material can be described by the partial differential equation for one-dimensional heat flow:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where α is the thermal diffusivity of the material.

As applied to the cooling of hot bitumen, this equation describes the relationship between bitumen temperature, T_b, time, t, thermal diffusivity of the bitumen, α_b, and a position coordinate, y. The finite-difference form of equation (1) for the cooling of bitumen can be expressed as:

$$\frac{T'_{bi} - T_{bi}}{\Delta t} = \alpha_b \frac{(T_{b(i-1)} - T_{bi}) + (T_{b(i+1)} - T_{bi})}{\Delta y^2} \tag{2}$$

where $T_{b(i-1)}$, T_{bi}, and $T_{b(i+1)}$ are temperatures in adjacent bitumen elements having a thickness of Δy (figure 2). The terms T'_{bi} and T_{bi} are the temperatures in bitumen element i at times t + Δt, and t, respectively. Substituting $\alpha_b = k_b/C_{pb}\rho_b$ into equation (2) and rearranging the equation gives:

$$\frac{k_b(T_{b(i-1)} - T_{bi})}{\Delta y} + \frac{k_b(T_{b(i+1)} - T_{bi})}{\Delta y} = \frac{C_{pb}\rho_b(\Delta y)(T'_{bi} - T_{bi})}{\Delta t} \tag{3}$$

where subscript b refers to bitumen.

Equation (3) can also be interpreted as the one-dimensional heat flow balance between three adjacent nodes whereby heat flows from the two outer nodes into the center node, as shown in figure 3a. The sum of the heat flow into node i from node i+1 and node i-1 must equal the energy stored in node i during the time, Δt, of heat flow. The heat flow per unit area into node i from either outer node is the temperature difference between nodes (ΔT_b) divided by the thermal resistance between nodes ($R_b = \Delta y/k_b$). The energy stored in node i during the time of heat flow (Δt) equals the heat capacity ($C_{pb}\rho_b$) times the volume of the element of node i ($\Delta y \cdot 1$) times the increase in temperature of node i ($T'_{bi} - T_{bi}$).

Equation (3) may be rearranged to yield equation (4) and then equation (5):

$$(T_{b(i-1)} - T_{bi}) + (T_{b(i+1)} - T_{bi}) = \frac{C_{pb}\rho_b(\Delta y)^2}{k_b\Delta t} (T'_{bi} - T_{bi}) \tag{4}$$

$$(T_{b(i-1)} - T_{bi}) + (T_{b(i+1)} - T_{bi}) = M_b(T'_{bi} - T_{bi}) \tag{5}$$

The term $M_b = C_{pb}\rho_b(\Delta y)^2/k_b\Delta t$ contains thermal, geometric and time parameters. In order to provide a stable solution using the finite-difference method for conduction, values of the modulus, M_b, must be chosen to be equal or greater than 2 [5,6]. The requirement, $M_b \geq 2$, assures that in the heat flow calculation, the second law of thermodynamics is satisfied whereby heat flow is from a warm to a cold node. Because of this restriction on M_b, values of Δy and Δt can not be selected independently. Solving for T'_{bi} in equation (5) yields the equation to calculate the future temperature of the bitumen interior element after heat flow during the time, Δt:

$$T'_{bi} = T_{bi} + \frac{1}{M_b} (T_{b(i-1)} - 2T_{bi} + T_{b(i+1)}) \tag{6}$$

2.2.2.3 Heat Flow at the Surface Boundary of the Bitumen

The finite-difference equation describing heat flow per unit area at the surface boundary of the bitumen is obtained by performing a heat balance on the surface node 1 (figure 3b). It is noted that the thickness of the first element is only one half that of the other bitumen elements. Consequently the volume of this element is equal to $(\Delta y/2).1$. For a heat balance at node 1 (figure 3b), the heat flow from node a to node 1 and the heat flow from node 2 to node 1 must equal the thermal energy stored at node 1. The heat flow from node a to node 1 is expressed as the product of the bitumen surface coefficient of heat transfer (h_b) and the temperature difference between the air and the surface element ($T_a - T_{bi}$). Heat flow per unit area from node 2 to node 1 is the temperature difference of the nodes ($T_{b2} - T_{b1}$) divided by the thermal resistance ($R_b = \Delta y/k_b$). The expression for stored thermal energy at node 1 in Figure 3b considers that the volume of the element containing node 1 is only half that of the element containing node i (figure 3a). The equation for heat balance at the surface boundary is expressed as follows:

$$h_b (T_a - T_{b1}) + \frac{k_b(T_{b2} - T_{bi})}{\Delta y} = \frac{C_{pb}\rho_b(\Delta y/2)(T'_{b1} - T_{b1})}{\Delta t} \tag{7}$$

Equation (7) may be rearranged:

$$\frac{h_b\Delta y}{k_b}(T_a - T_{b1}) + (T_{b2} - T_{b1}) = \frac{C_{pb}\rho_b(\Delta y)^2(T'_{b1} - T_{b1})}{2k_b\Delta t} \tag{8}$$

$$N_b(T_a - T_{b1}) + (T_{b2} - T_{b1}) = \frac{M_b}{2}(T'_{b1} - T_{b1}) \tag{9}$$

The term, $N_b = h_b\Delta y/k_b$, defines a Biot or Nusselt number. The term, M_b, has been previously defined and must be chosen such that $M_b \geq (2N_b + 2)$ in order for equation (9) to provide a stable solution and to satisfy the thermodynamic requirements concerning the direction of heat flow [5,6]. Rearrangement of equation (9) results in the expression to determine the future temperature (T'_{b1}) of node 1 after heat has flowed during the time, Δt:

$$T'_{b1} = T_{b1}\left[1.0 - \frac{2}{M_b}(N_b + 1.0)\right] + \frac{2}{M_b}(T_{b2} + N_bT_a) \tag{10}$$

For this study, the surface coefficient of heat transfer, h, is taken as:

$$h = h_c + h_r$$

where h_c = convection coefficient $Btu/h\cdot ft^2\cdot °F$ ($W/m^2\cdot K$)
h_r = radiation coefficient = $\sigma\varepsilon(T_s^2 + T_{sky}^2)(T_s + T_{sky})$

where σ = Stefan-Boltzman constant, $Btu/h\cdot ft^2\cdot °R^4$ ($W/m^2\cdot K^4$)
ε = emittance of the surface, dimensionless
T_s = temperature of the surface, °R (K)
T_{sky} = sky temperature, °R (K); for purposes of this model, the sky temperature was taken to be the same as the ambient air temperature.

In the application of the model, the convection coefficient, h_c, was maintained constant for each set of given conditions and changed only to account for the effect of different wind speeds on the bitumen cooling. The radiation coefficient, h_r, was changed at each time increment to account for heat radiation which is nonlinearly dependent upon surface temperature. It is noted that the value of the convection coefficient, h_c, may be difficult to approximate. The surface coefficient may vary according to the rate of air movement across the surface, the surface length, and roughness.

2.2.2.4 Heat Flow at the Bottom (Interface) Boundary of the Bitumen

The finite-difference equation for heat flow at the bottom boundary element of the bitumen is determined by considering the heat balance at this element. The bottom boundary element interfaces with the top element of the roofing component to which the hot bitumen is applied, and the thermal resistance across this interface is considered to be zero. Figure 3c indicates the heat flow at node n. The expressions for heat flow from node n-1 to node n and for stored energy at node n may be determined similarly to the expressions given in figure 3a. The heat flow per unit area from roofing component node x1 to node n is the temperature difference $(T_{x1} - T_n)$ divided by the thermal resistance between the nodes (R_{bx}) as shown in figure 3c. The resistance R_{bx} is the sum of the resistance of the bitumen $(\Delta y/2k_b)$ and that of the component element $(\Delta y/2k_x)$ and may be expressed as:

$$R_{bx} = \frac{\Delta y}{2k_b} + \frac{\Delta y}{2k_x} = \frac{\Delta y}{2}\left(\frac{k_b + k_x}{k_b k_x}\right) \tag{11}$$

From Figure 3c, the heat balance equation at node n is expressed as:

$$\frac{k_b(T_{b(n-1)} - T_{bn})}{\Delta y} + \frac{2}{\Delta y}\left(\frac{k_b k_x}{k_b + k_x}\right)(T_{x1} - T_{bn}) = \frac{C_{pb}\rho_b(\Delta y)(T'_{bn} - T_{bn})}{\Delta t} \tag{12}$$

Rearrangement of equation (12) gives:

$$(T_{b(n-1)} - T_{bn}) + \frac{2k_x}{k_b + k_x}\;(T_{x1} - T_{bn}) = \frac{C_{pb}\rho_b(\Delta y)^2(T'_{bn} - T_{bn})}{k_b\Delta t} \tag{13}$$

$$(T_{b(n-1)} - T_{bn}) + \frac{2k_x}{k_b + k_x}\;(T_{x1} - T_{bn}) = M_b(T'_{bn} - T_{bn}) \tag{14}$$

The term M_b has been previously defined. Solving for T'_{bn}, the future temperature at node n after heat has flowed for time, Δt, results in the finite-difference equation for heat flow at the bottom boundary element of bitumen:

$$T'_{bn} = T_{bn} + \frac{1}{M_b}\left[T_{b(n-1)} - 1 + \left(\frac{2k_x}{k_b + k_x}\right)T_{bn} + \left(\frac{2k_x}{k_b + k_x}\right)T_{x1}\right] \tag{15}$$

2.2.2.5 <u>Heat Flow within the Roofing Components Other than Bitumen</u>

The mathematical model, as developed here, is applicable to roof systems which may contain two, three, or four components. In the four component system hot bitumen is applied to a felt which is bonded with a bitumen (referred to as bitumen-cement for purpose of classification of the model) to a substrate. The three-component system contains hot bitumen, felt, and substrate; and the two-component system is applicable to hot bitumen applied to a substrate. The finite-difference equations used to describe heat flow in the typical elements of the felt, bitumen-cement, and substrate components may be derived from heat balance equations similar to those presented in sections 2.2.2.2 through 2.2.2.4. For the convenience of the reader, the typical finite-difference equations used to calculate cooling time of hot bitumen in the four-, three-, and two-component systems are given in appendix A.

2.2.2.6 Solution to the Model

To calculate the cooling time of hot bitumen during roofing construction, each roofing component was divided into a number of elements each with the same thickness, Δy, with the exception of the top and bottom surface elements which had thickness of $\Delta y/2$. The number of elements for the individual components was based on the thickness of the component. Each element was assigned an appropriate finite-difference equation for calculating the change in temperature with time within that element. Each element was also assigned an initial temperature. The temperature of each element as a function of an increment of time, Δt, was calculated sequentially from the top surface element of the bitumen to the lowest element of the substrate. This process started with the initial bitumen temperature and continued iteratively until the bitumen temperature reached a limiting value. The temperature of the bitumen was taken as the average of the temperature of each element in the bitumen component for each increment of time, Δt. In order to satisfy the condition that the limiting value of the modulus, M, be greater than or equal to 2, values of Δy and Δt cannot be chosen independently and must be chosen relatively small to provide a stable solution for the heat flow calculation (section 2.2.2.2).

To facilitate the calculations, the model was computerized. The listing of the computer program is given in appendix B. The program uses the finite-difference equations for two-, three-, and four-component roofing systems given in appendix A. It is noted that the program contains a control statement that directs the calculation for roofing systems depending on the number of components.

3. APPLICATION OF MODEL FOR PREDICTING BITUMEN COOLING

3.1 VALUES NEEDED TO SOLVE MODEL

Because the mathematical model was developed to predict cooling time of hot bitumen applied to specific roofing components and under specific environmental conditions, the program prepared for the solution of the model requires values for each of the factors discussed in section 2.1 which affect cooling. In this regard, values for the following factors are read into the program for each set of calculations: initial temperature, number of elements, thermal conductivity, and thermal diffusivity of the roofing components; air temperature; and convection coefficient for the top surface of the bitumen and the bottom surface of the substrate. Emissivities of the bitumen and substrate are constants included in the program and the values of emissivity must be changed as appropriate for different bitumens and substrates. The initial temperature of any component is taken to be uniform throughout its thickness. Other values for operation of the program are the minimum temperature to which the bitumen cools and the maximum time allowed for bitumen to cool during program execution. Values are also needed for the number of components, the thickness of the elements, Δy, and the increment of time, Δt.

Although not directly included as part of the program input needed for the cooling calculations, values of the following parameters are necessary since values of input are dependent on these parameters. The thermal diffusivity of a material is dependent on its density, thermal conductivity, and specific heat. The convection coefficient is related to the wind speed. The number of finite-elements of each component is determined by dividing the component thickness by the thickness of the element, Δy, with the exception that at a surface a half element is used in the calculation. It is noted that the computer program limits the number of substrate elements to 100.

In order to illustrate the applicability of the mathematical model, specific values of material, environmental, and physical properties for selected model roofing systems were needed for use in the cooling calculations. Values for materials properties and convection coefficients as a function of wind speed were obtained from literature sources. Environmental factors influencing cooling (air temperature, roof component temperature, and wind speed) were chosen to represent a range of conditions encountered in built-up roofing applications. Roofing component thicknesses were selected on the basis of current roofing practice.

The thermal properties of the roofing component materials considered in this study are given in table 1. Selected values for roofing component temperatures and environmental factors are given in table 2. Table 3 gives thicknesses of roofing component materials and the number of finite-elements for each material used in the calculations. The element thickness, Δy, for computer program input was selected as 0.003 in or 0.00025 ft (0.076 mm). Values of the time increment, Δt, ranged from 0.000000045 to 0.000001 h (0.000162 to 0.0036 s). These values of time were selected to satisfy the conditions for heat flow calculation by finite-difference techniques [5,6] and to minimize computation time.

Using input data in tables 1, 2 and 3 and selected values of Δy and Δt within the ranges given above, the average bitumen temperature was computed as a function of time for various model roofing systems (configurations) given in table 4. The model roofing systems contain two, three, or four components and included bitumens, felts, insulations, and deck materials. The solution of the mathematical model for a given set of conditions was a tabular listing of bitumen temperatures corresponding to half second time intervals. The temperatures of the applied bitumen during the cooling period were computed over a range beginning with the selected application temperature (table 2) to the selected lower limit of 300°F (149°C). Tabular data from the model were used to prepare the bitumen temperature versus cooling time plots given in figures 4-17.

3.2 RESULTS FOR SPECIFIC ROOFING SYSTEMS

This section presents the results (figures 4-17) of the calculations for various two-, three-, and four-component model roofing systems (table 4) conducted to illustrate the applicability of the mathematical model. The results in figures 4-15 give the temperature-time relationships for the cooling of hot bitumen applied at an application quantity of about 20 lbm/100 ft^2 (0.98 kg/m^2) for asphalt and about 25 lbm/100 ft^2 (1.2 kg/m^2) for coal tar pitch, at temperatures of 500, 450, 400, and 350°F (260, 232, 204, and 177°C) to a lower temperature of 300°F (149°C) under specific environmental conditions. These environmental conditions were air and initial component temperatures of 120, 70, and 20°F (49, 21, and -7°C) and wind speeds of 0, 10, 20, and 30 mph (0, 4.5, 9.0, and 13.5 m/s). The asphalt and coal tar pitch application quantities correspond to a thickness of 0.04 in (1.0 mm).

Figure 16 gives results for the three-component roofing system of asphalt applied to felt on polyurethane foam insulation and shows the effect of the quantity of applied asphalt (asphalt thickness) on cooling time for application temperatures of 500 and 400°F (260 and 204°C) and a wind speed of zero. The air and initial component temperatures in this figure were also 120, 70, and 20 °F (49, 21, and -7°C). Figure 17 compares the cooling time of asphalt applied at 500°F (260°C) to various substrates. In this figure the air and initial substrate temperatures were 70°F (21°C) and the wind speed was zero.

A comparison of the results given in figures 4-15 indicates the influence of the various factors that affect the cooling rate of hot bitumen. The time for the bitumen to cool from 500 to 300°F (260 to 149°C) ranged from less than 4 to more than 50 seconds depending upon the wind speed, air temperature, initial component material temperatures, and the type of roofing system. As expected, the cooling time decreased for a particular roofing system with a decrease in application temperature, an increase in wind speed, and a decrease in the air and initial component material temperatures.

The type of substrate to which the hot asphalt is applied may have a significant effect on the cooling time. Figures 4-9 illustrate this effect. In comparing these two-component systems, the cooling time was much longer for the cases of asphalt applied to low thermal conductivity and low heat capacity polyurethane foam and fiber glass insulations than the cooling time for asphalt applied to the more conductive and higher heat capacity concrete and steel. The cooling times for asphalt applied to insulating concrete and plywood, figures 6 and 7, were about the same, and fell between those for the low and high thermal conductivity and heat capacity materials. It is noted from table 1 that insulating concrete and plywood have comparable thermal properties.

Because steel has a higher thermal conductivity and heat capacity than concrete, it might be expected that asphalt applied on steel would cool faster than on concrete. From figures 8 and 9 it can be seen that the calculated asphalt cooling times were shorter for a concrete substrate than for a steel substrate. This can be explained in part by considering the thickness of the substrate. The thin steel substrate rapidly heats upon application of the asphalt until its temperature is nearly uniform through its thickness. At this point, the steel is less of a heat sink for the asphalt than the concrete. In the case of asphalt applied to the thick concrete substrate, the concrete has a considerably larger volume, heats more slowly, and continuously absorbs heat from the hotter asphalt.

A comparison is made from figures 8 and 10 of the cooling times of asphalt and coal tar pitch applied to concrete. The slightly longer cooling time for coal tar pitch is attributed to its lower thermal diffusivity and higher heat capacity. It is noted that identical values of thermal conductivity and specific heat were assigned for these two bitumens. The lower diffusivity of the coal tar pitch is accounted for by its higher density.

Figures 11 and 12 illustrate the use of the mathematical model for three-component systems, whereby hot bitumen is applied on a felt which is attached to a substrate. In these examples, the bitumen was asphalt, the felt was type 15 asphalt organic, and the substrates were polyurethane foam insulation (figure 11) and plywood (figure 12). The plots in figures 11 and 12 indicate that the felt and substrate combination influence the cooling of the applied bitumen. For these three-component systems, the asphalt cooled faster when applied to the felt on the substrate with the higher heat capacity and thermal conductivity (plywood). In these two cases, the thermal properties of the felt were identical.

The substrate in these cases influences the cooling of the asphalt because the felt is relatively thin. Heat flowing from the asphalt into the thin felt flows into the substrate before the asphalt cools to the lower temperature limit of 300°F (149°C). Since heat flows from the hot asphalt first into the felt and then into the substrate, the substrate below the felt has little effect on the cooling time in the initial seconds after application of the asphalt. In comparing figures 11 and 12, it may be seen that the temperature-time cooling curves for the asphalt applied to felt on polyurethane insulation or to felt on plywood are essentially identical during the first two to three seconds after application. Then the time of cooling increases for the case of asphalt applied to felt on polyurethane foam.

The effect of the substrate below the felt on the cooling time of the applied asphalt may be seen by examining the temperature distribution of the elements within the roofing system as a function of time. Figure 18 presents temperature profiles through the thickness of the three-component roofing systems, asphalt applied to felt on either polyurethane foam insulation or plywood substrate. In figure 18, asphalt at 500°F (260°C) was applied to the felt and substrate at 120°F (49°C) with a zero wind speed. The temperature profiles are obtained by plotting the temperature of the element versus the number of the element. The thickness of the substrate was limited to 100 elements. The temperature profiles given in figure 18 are for times of 2, 4, 6, 8, 10, and 20 seconds after application of the asphalt (figures 18a through 18f, respectively).

It can be seen from figure 18 that the temperature profiles for the two three-component systems are very similar at times corresponding to a few seconds after application of the asphalt. For example, 2 seconds after application (figure 18a), the distribution of temperature through the asphalt and felt components are almost identical for the two systems. The temperatures within the polyurethane substrate are seen to be only slightly higher than those of the plywood substrate for a few elements below the felt-substrate interface. Four seconds after application (figure 18b), the temperature distributions within the asphalt components remain similar for the two three-component systems. As the time after application increases, the temperature profiles of the two three-component systems exhibit greater disparity. The temperatures of the asphalt applied to felt on plywood are lower than the temperatures of asphalt applied to felt on polyurethane. In addition, the temperatures within the polyurethane foam insulation substrate rise higher than the temperatures within the plywood substrate which has greater heat capacity and thermal conductivity. The result is a slower cooling of asphalt applied to felt on polyurethane foam in comparison to asphalt applied to felt on plywood.

A comparison may be made between figures 4 and 11, or figures 7 and 12 to show the effect of the felt on the time of bitumen cooling. Figure 4 gives the calculated results for asphalt applied directly to polyurethane foam, while in figure 11, the asphalt is applied to felt on polyurethane foam. The asphalt applied to felt on foam cools more rapidly than that applied directly on foam, because of the effect of the relatively high heat capacity and thermal conductivity of the felt compared to the heat capacity and thermal conductivity of the foam. The cooling time for asphalt applied to plywood, figure 7, was essentially the same as that for asphalt applied to felt over plywood, figure 12. The thermal properties of the felt and plywood were considered to be comparable (table 1).

Examples of asphalt cooling time for two four-component roofing systems are presented in figures 13 and 14. The cooling time for hot asphalt applied to felt on asphalt on concrete was essentially the same as for hot asphalt applied to felt on asphalt on polyurethane foam, even though the thermal properties of the concrete and polyurethane foam differ significantly. The cooling time to reach 300°F (149°C) for these four-component systems was influenced only by the felt and asphalt and not the substrate.

Figure 15 allows a comparison to be made for a four-component system containing coal tar pitch instead of asphalt. The cooling times for coal tar pitch in figure 15 were slightly longer than for the asphalt cooling times given in figures 13 and 14. Assuming that the substrate has little influence on the cooling time for a four-component system, this observation is consistent with the previously noted comparison between figures 8 and 10 for the case of these two bitumens applied to the same substrate.

The effect of the quantity of applied asphalt on the cooling time is illustrated in figure 16. In figures 4 through 15, the bitumens were applied at a constant thickness of about 0.04 in (1.0 mm). This corresponds to an application quantity of about 20 lbm/100 ft^2 (0.98 kg/m^2) for asphalt and

25 lbm/100 ft^2 (1.2 kg/m^2) for coal tar pitch. As noted in figure 16, increasing the amount of applied bitumen significantly increased the cooling time to reach a given temperature. For example, in figure 16 the cooling time to reach 300°F (149°C) for asphalt at 500°F (260°C) applied to components at 20°F (-7°C) ranged from about 8 to 45 seconds for application quantities of about 15 and 30 lbm/100 ft^2 (0.73 and 1.5 kg/m^2), respectively. These quantities correspond to asphalt thicknesses of about 0.03 and 0.06 in (0.7 and 1.5 mm).

Figure 17 shows the effect of the substrate on asphalt cooling time for a specific set of environmental conditions, application temperature, and asphalt thickness. The air and substrate temperatures are 70°F (21°C) with a zero wind speed and with an asphalt application temperature of 500°F (260°C). The asphalt thickness corresponds to about 20 lbm/100 ft^2 (0.98 kg/m^2). In general, the curves in this figure are grouped according to the thermal properties of the substrate, and in particular, the heat capacities of the substrates. Table 5 presents the heat capacities of the substrates and the times for the asphalt to cool to 300°F (149°C) for the specific set of environmental conditions in figure 17. It can be seen from table 5 that the asphalt cooling times generally decrease as the heat capacities of the substrates increase. The thickness of the steel substrate is considerably less then that of the other substrates (table 3). This accounts for the cooling time being comparable to that for concrete, as shown in table 5. It is noted that the cooling time of asphalt applied to felt is influenced by the polyurethane foam insulation substrate. Cooling time of asphalt applied to felt alone was not calculated, since this is not applicable to roofing construction.

3.3 USES FOR MODEL

The model for predicting the cooling time of hot bitumen has practical application to the construction of bituminous built-up roofing systems. The model can provide quantitative information about bitumen cooling time with regard to the time available to apply insulations or felts properly. Bitumen cooling is a complex process which depends upon a combination of environmental factors and thermal properties of the materials in the roofing system.

The model has shown in section 3.2 that for certain roofing systems under specific conditions bitumen will cool extremely rapidly. In cases where cooling may be more rapid than expected, there may be insufficient time to place insulation or to broom felts to achieve adequate adhesion between roofing components. The model permits a quantitative determination of bitumen cooling time as opposed to empirical or intuitive approaches. Use of the model indicates that under some conditions, the application temperature may have to be raised above normally accepted limits to provide sufficient time to apply component materials, or hot bituminous roofing should not be applied under some conditions.

Although the use of the model was demonstrated in this paper for specific roofing systems and environmental conditions, it may be applied to other roofing systems and environmental conditions as well. This paper presented the results of the cooling calculations as plots of bitumen temperature versus time for specific bitumen application temperatures at various wind speeds and at specific air and substrate temperatures. Other types of plots may be used to express the results of the cooling calculations. One such plot relates the dimensionless temperature parameter, $(T_b - T_{ac})/(T_a - T_{ac})$, to time. The term T_b is the bitumen temperature at a given time, T_a is the application temperature of the bitumen, and T_{ac} is the temperature of the air and the initial temperature of components other than bitumen. Figure 19 is an example of a plot of dimensionless temperature versus time for the application of hot asphalt to steel, and was generated from the results of the cooling calculations from which figure 9 was prepared. Whereas figure 9 contains forty-eight curves, figure 19 contains only four curves. These four curves may be used to approximate the bitumen temperature (T_b) at any given time for any bitumen application temperature (T_a) and air and initial roof component temperature (T_{ac}) for each of the four wind speeds.

4. SUMMARY

This paper presented a mathematical model using finite-difference equations to predict the cooling times of hot bitumen applied during the construction of built-up roofing. The application or contact temperature of the hot bitumen was taken as the temperature at which the bitumen made contact with the surface to which it was applied. The quality of built-up roofing is dependent upon the application of bitumen which is hot enough (i.e., has the proper viscosity) to provide a uniform continuous layer of bitumen and adequate adhesion between roofing components. A model for the prediction of cooling times was needed, since it would not be practicable to measure these rates for the numerous roofing systems to which hot bitumen may be applied under many different environmental conditions. Up to now, mathematical modeling of the cooling of bitumen has not been employed in the roofing industry.

The effect of various factors on the cooling times of hot bitumen was investigated. These factors included materials properties and environmental conditions. The materials properties were the initial temperature, thermal conductivity, specific heat, density, and thickness of the roofing components. Environmental conditions affecting cooling were air temperature and wind speed. Values for each of these factors were incorporated in the model to determine quantitatively their effect on the cooling. The model was computerized for application to roofing systems having two, three, or four components.

Examples of the use of the model to predict the cooling times of hot bitumen applied to various roofing components were given. These components included felts, insulations such as fiber glass and polyurethane foam boards, and typical decks such as concrete, steel, and plywood. Times for bitumens to cool to 300°F (149°C) were computed for bitumen application temperatures ranging from 500 to 350°F (260 to 177°C). For the selected examples, cooling times ranged from less than 4 to more than 50 seconds depending upon the type of model roofing system in question, environmental conditions, and component material temperatures. Cooling times of the hot bitumen were graphically presented using computer-generated plots of bitumen temperature versus time.

The model indicated that for some systems under certain environmental conditions, hot bitumen cools extremely rapidly. In these cases, sufficient time may not be available to place insulation or felts properly. To provide adequate time for proper application, the temperature of the bitumen may need to be increased above normally accepted limits, or hot bituminous roofing should not be applied under some conditions.

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Table 1. Thermal Properties of the Roofing Component Materials for Use in the Model.

Roofing Component Material	Density ρ lbm/ft^3 (kg/m^3)	Thermal Conductivity k Btu·ft/h·ft^2·°F (W/m·K)	Specific Heat C_p Btu/lbm·°F (kJ/kg·K)	Thermal Diffusivity[1] α ft^2/h (mm^2/s)	Heat Capacity[2] ρC_p Btu/ft^3·°F (kJ/m^3·K)	Data Source Reference
o Bitumen (Hot)[3]						
Asphalt	56 (896)	0.067 (0.12)	0.53 (2.22)	0.0023 (0.059)	30 (1989)	[20]
Coal Tar Pitch[4]	72 (1153)	0.067 (0.12)	0.53 (2.22)	0.0018 (0.046)	38 (2560)	[20]
o Felt						
Type 15 - Asphalt Organic	63 (1008)	0.047 (0.08)	0.27 (1.13)	0.0028 (0.072)	17 (1139)	[21]
Type 15 - Coal Tar Pitch Organic	69 (1105)	0.047 (0.08)	0.35 (1.46)	0.0019 (0.049)	24 (1613)	[21]
o Bitumen - Cement						
Asphalt	64 (1025)	0.089 (0.15)	0.42 (1.76)	0.0033 (0.085)	27 (1804)	[20]
Coal Tar Pitch[4]	80 (1281)	0.089 (0.15)	0.42 (1.76)	0.0026 (0.067)	34 (2255)	[20]
o Substrate						
Steel	489 (7829)	26.2 (45.3)	0.12 (0.50)	0.45 (11.61)	59 (3915)	[22]
Concrete	140 (2241)	0.75 1.30	0.22 (0.92)	0.024 (0.62)	31 (2062)	[23]
Insulating Concrete	40 (640)	0.096 (0.17)	0.21 (0.88)	0.011 (0.62)	8 (563)	[23]
Plywood	34 (544)	0.067 (0.12)	0.29 (1.21)	0.0068 (0.18)	10 (658)	[23]
Glass Fiber Insulation Board	6.5 (104)	0.021 (0.036)	0.23 (0.96)	0.014 (0.36)	1.5 (100)	[23]
Polyurethane Foam Insulation Board	1.5 (24)	0.013 (0.022)	0.38 (1.59)	0.023 (0.59)	0.57 (38)	[23]

(1) Calculated from the relationship, $\alpha = k/\rho C_p$.

(2) Calculated from the values of density and specific heat.

(3) Density, thermal conductivity, and specific heat values have been adjusted for the elevated bitumen application temperatures. The adjustment was made for a temperature of 450°F (232°C).

(4) According to Mack [20], the thermal conductivity and specific heat of coal tar pitches are of the same order of magnitude as those of asphalts. Thus, for purposes of this paper, values of thermal conductivity and specific heat of coal tar pitch were chosen to be identical to those of asphalt.

Table 2. Values of Roofing Component Temperatures and Environmental Factors Used in the Calculations

Factor	Units	Numerical Values			
Bitumen Application Temperature	°F (°C)	500 (260)	450 (232)	400 (204)	350 (177)
Felt, Bitumen-Cement and Substrate Temperatures	°F (°C)	120 (49)	70 (21)	20 (-7)	
Air Temperature	°F (°C)	120 (49)	70 (21)	20 (-7)	
Wind Speed	mph (m/s)	0 (0)	10 (4.5)	20 (9.0)	30 (13.5)
and					
Corresponding Convection Coefficient(1)	$Btu/h\cdot ft^2\cdot °F$ ($W/m^2\cdot K$)	1.0 (5.7)	3.6 (20.4)	6.1 (34.6)	8.5 (48.2)

(1) Wind speeds are those blowing across the surface of the cooling bitumen. The values of the convection coefficient corresponding to the selected wind speeds were estimated from the curve for smooth surfaced materials in figure 1, chapter 22 of ASHRAE Handbook 1977 Fundamentals [24]. The radiation contribution to the overall surface coefficient has been subtracted to estimate the convection coefficient.

Table 3. Thickness of the Roofing Component Materials and Corresponding Number of Elements Used in the Calculations.

Roofing Component Material	Component Thickness		Number of Elements[1]
	in	mm	
o Bitumen			
Asphalt[2]	0.0285	0.72	10
	0.0405	1.03	14
	0.0495	1.26	17
	0.0585	1.49	20
Coal Tar Pitch[2]	0.0405	1.03	14
o Felt			
Type 15 Asphalt Organic	0.033	0.84	11
Type 15 Coal Tar Pitch Organic	0.033	0.84	11
o Bitumen-Cement			
Asphalt	0.042	1.1	14
Coal Tar Pitch	0.042	1.1	14
o Substrate[3]			
Steel[2]	0.0285	0.72	10
Concrete	6.0	152.4	100
Insulating Concrete	4.0	101.6	100
Plywood	0.75	19.1	100
Glass Fiber Insulation Board	1.0	25.4	100
Polyurethane Foam Insulation Board	1.0	25.4	100

(1) The number of finite elements is in general determined by dividing the component thickness by the thickness of a single element, Δy. In this report, Δy was chosen equal to be 0.003 in (0.076 mm).

(2) The component thickness considers the half-thickness element at the surface.

(3) The computer program limits the number of substrate elements to 100.

Table 4. Configurations of Roofing Components for Which the Cooling Time of the Applied Bitumen was Calculated.

Roofing Configuration			Figure Number[(2)]
Number of Components	Applied Bitumen	Other Roofing Components[(1)]	
2	asphalt	polyurethane foam insulation board	4
	asphalt	fiber glass insulation board	5
	asphalt	insulating concrete	6
	asphalt	plywood	7
	asphalt	concrete	8
	asphalt	steel	9
	coal tar pitch	concrete	10
3	asphalt	felt on polyurethane foam insulation board	11, 16
	asphalt	felt on plywood	12
4	asphalt	felt on asphalt on polyurethane foam insulation board	13
	asphalt	felt on asphalt on concrete	14
	coal tar pitch	felt on coal tar pitch on fiber glass insulation board	15

(1) For roofing configurations which contain three and four components, the components are listed in the sequence in which they are located in the system, the first listed being uppermost.

(2) The figure numbers refer to the figures of bitumen temperature versus cooling time given in section 3.2.

Table 5. Heat Capacities of Various Substrates and Cooling Times for Asphalt When Applied on Them[a]

Substrate	Heat Capacity ρC_p Btu/ft^3·°F (kJ/m^3·K)	Asphalt Cooling Time[a] s
Polyurethane Foam Insulation Board	0.57 (38)	59
Glass Fiber Insulation Board	1.5 (100)	49
Insulating Concrete	8 (563)	16
Plywood	10 (658)	17
Felt[b]	17 (1139)	_[b]
Felt on Polyurethane Foam Insulation	_[c]	28
Concrete	31 (2062)	4.5
Steel[d]	59 (3915)	6.5

[a] Time to cool to 300°F (149°C). Asphalt applied at 500°F (260°C) with air and initial substrate temperatures of 70°F (21°C) and zero wind speed. The asphalt application quantity corresponds to about 20 lbm/100 ft^2 (0.98 kg/m^2).

[b] Asphalt is not applied to felt alone, and felt is not considered to be a substrate.

[c] Heat capacities of the felt and polyurethane foam are listed separately.

[d] The thickness of the steel is considerably less than that of the other substrates (table 3).

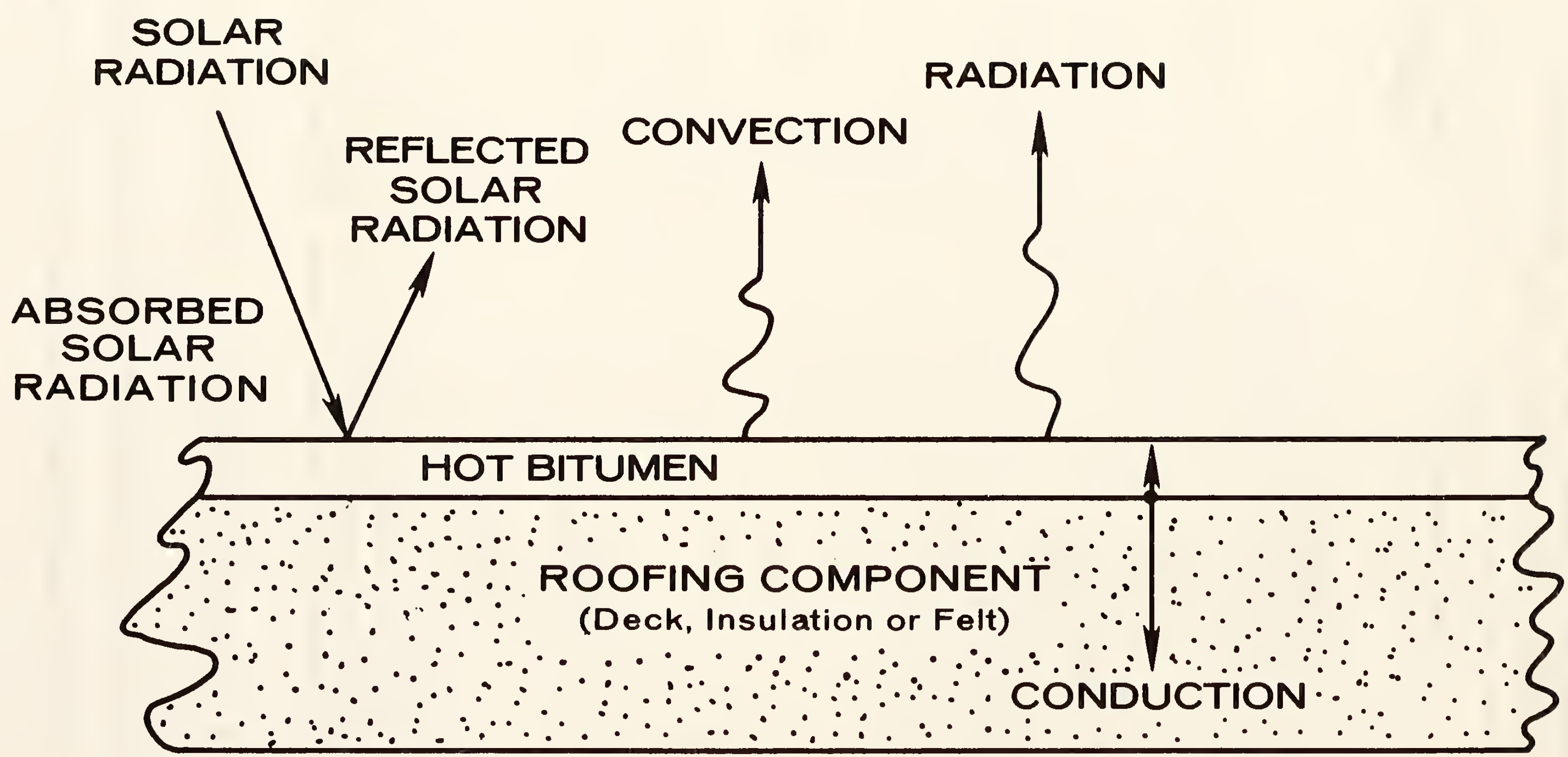
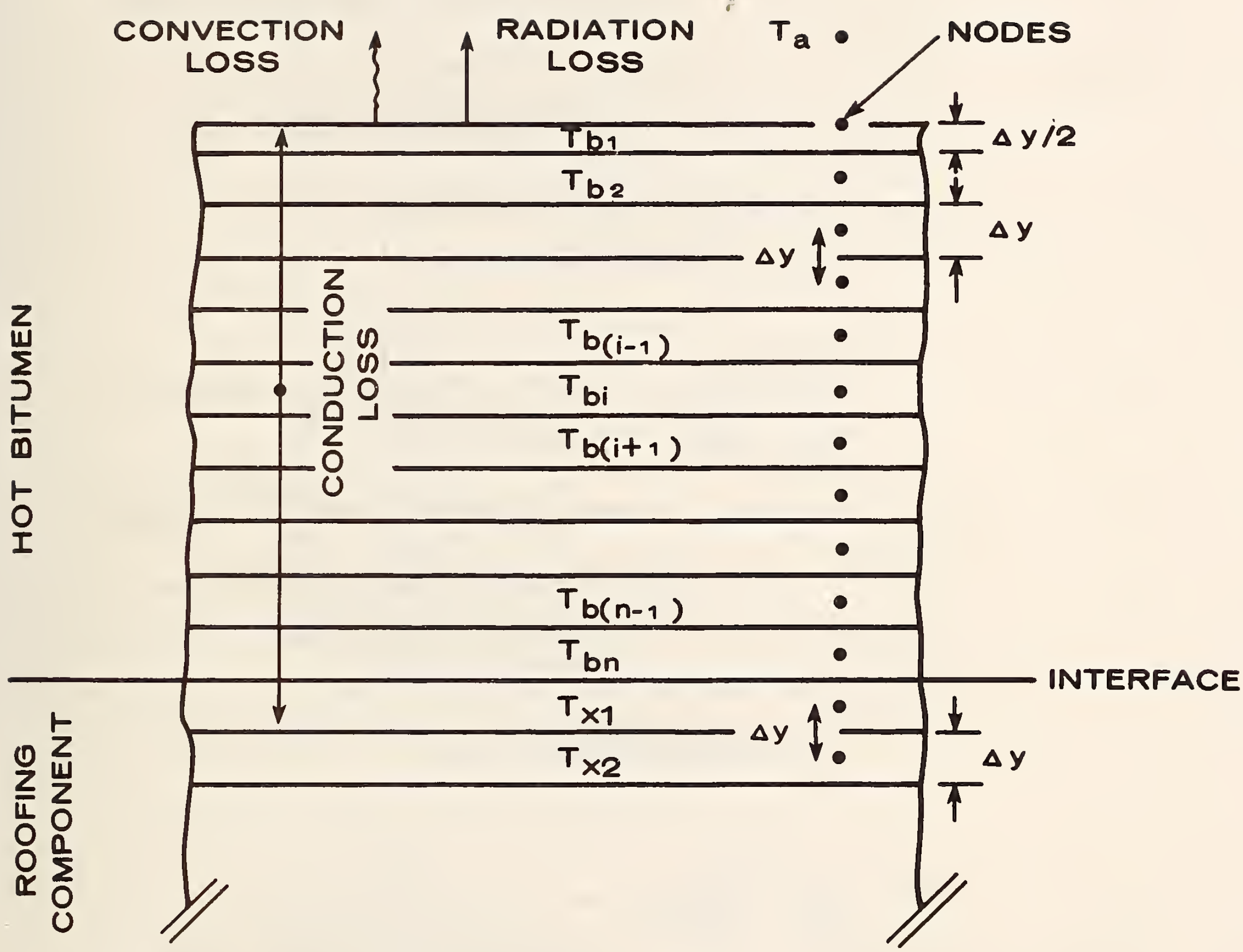
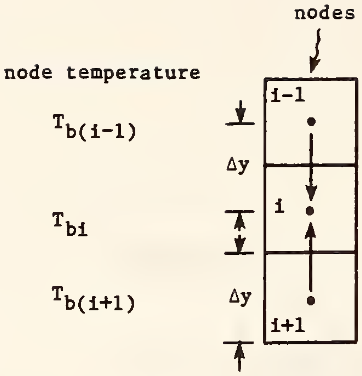


Figure 1. Modes of Heat Transfer During Construction of Bituminous Built-Up Roofing Systems. The Effect of Solar Radiation was not Included in the Mathematical Model. Heat Transfer due to Convection and Radiation from the Underside of Thin Decks Without Insulation was Included in the Model.



Key: T_a = temperature of the air
T_{b1} = temperature of the first bitumen element
T_{bi} = temperature of an interior bitumen element
T_{bn} = temperature of the last bitumen element
T_{x1} = temperature of the first element of the roofing component below the bitumen
Δy = the thickness of an element and the distance between nodes

Figure 2. Diagram of the Elements Within the Bitumen and Top Elements of the Roofing Component Below the Bitumen.



$Q_{i-1,i}$ = heat flow from node i-1 to node i

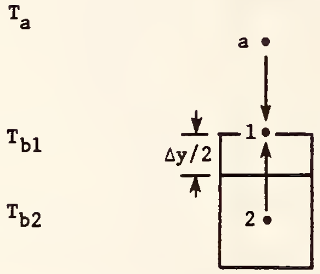
$$= \frac{(T_{b(i-1)} - T_{bi})}{R_b} = \frac{k_b(T_{b(i-1)} - T_{bi})}{\Delta y}$$

$$\text{Stored heat at node } i = \frac{C_{pb}\rho_b(\Delta y)(T'_{bi} - T_{bi})}{\Delta t}$$

$Q_{i+1,i}$ = heat flow from node i+1 to node i

$$= \frac{(T_{b(i+1)} - T_{bi})}{R_b} = \frac{k_b(T_{b(i+1)} - T_{bi})}{\Delta y}$$

3a. Heat flow within an element within the bitumen



$Q_{a,1}$ = heat flow from air node a to surface node 1

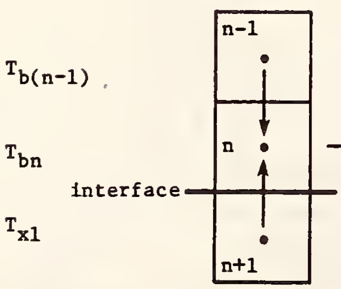
$$= h_b(T_a - T_{b1})$$

$$\text{Stored heat at node } 1 = \frac{C_{pb}\rho_b(\Delta y/2)(T'_{bi} - T_{b1})}{\Delta t}$$

$Q_{1,2}$ = heat flow from node 2 to surface node 1

$$= \frac{(T_{b2} - T_{b1})}{R_b} = \frac{k_b(T_{b2} - T_{b1})}{\Delta y}$$

3b. Heat flow at the surface boundary element of the bitumen



$Q_{n-1,n}$ = heat flow from node n-1 to node n

$$= \frac{(T_{b(n-1)} - T_{bn})}{R_b} = \frac{k_b(T_{b(n-1)} - T_{bn})}{\Delta y}$$

$$\text{Stored heat at node } n = \frac{C_{pb}\rho_b(\Delta y)(T'_{bn} - T_{bn})}{\Delta t}$$

$Q_{x1,n}$ = heat flow from node x1 to node n

$$= \frac{(T_{x1} - T_{bn})}{R_{bx}} = \frac{2}{\Delta y}\left(\frac{k_b k_x}{k_b + k_x}\right)(T_{x1} - T_{bn})$$

3c. Heat flow at the bottom boundary element of the bitumen

Figure 3 Diagram of the Elements and Nodes Used to Develop the Finite-Difference Equations for Heat Flow Within the Bitumen, at the Surface of the Bitumen, and at the Bottom Boundary of the Bitumen.

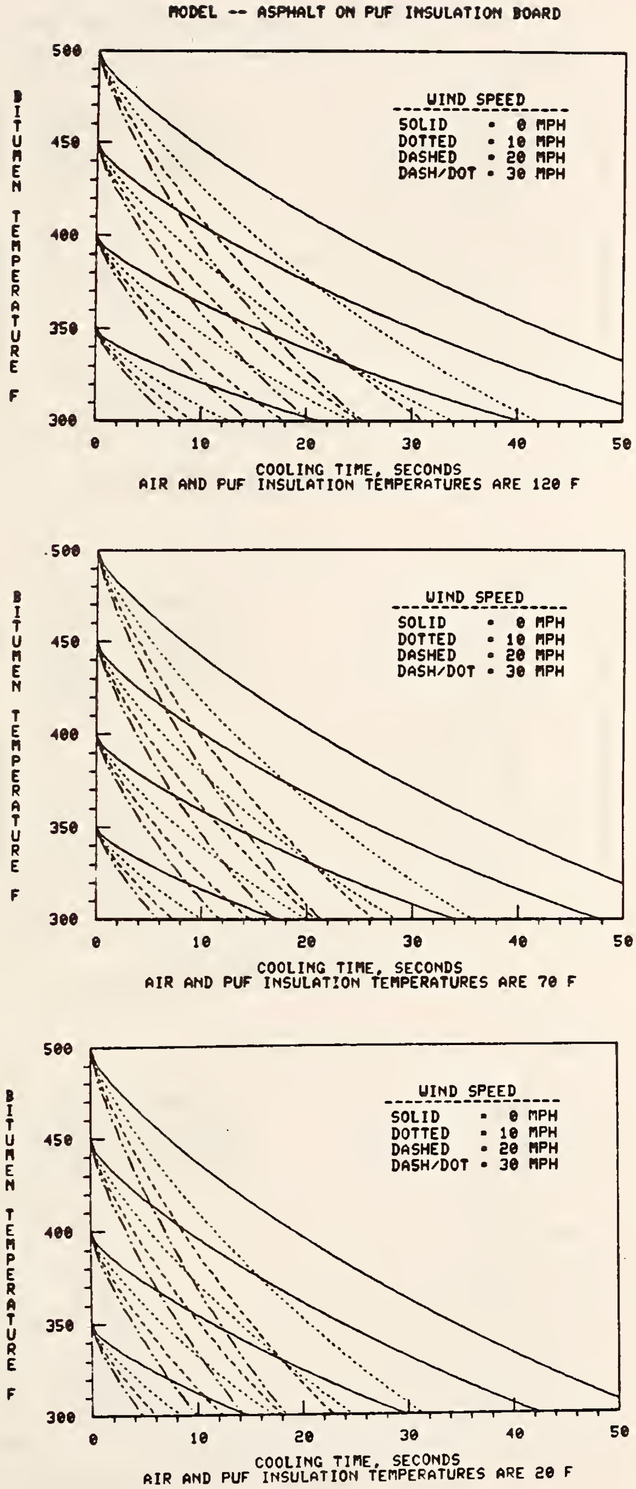


Figure 4. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

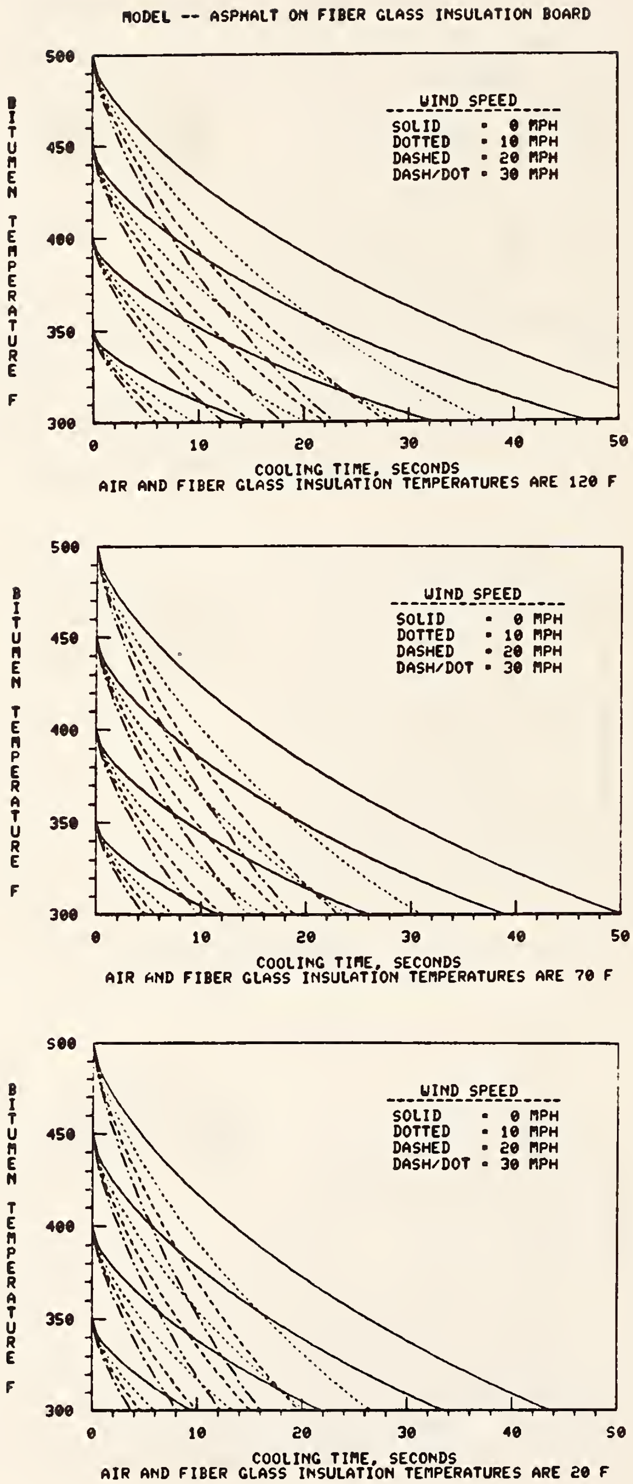


Figure 5. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

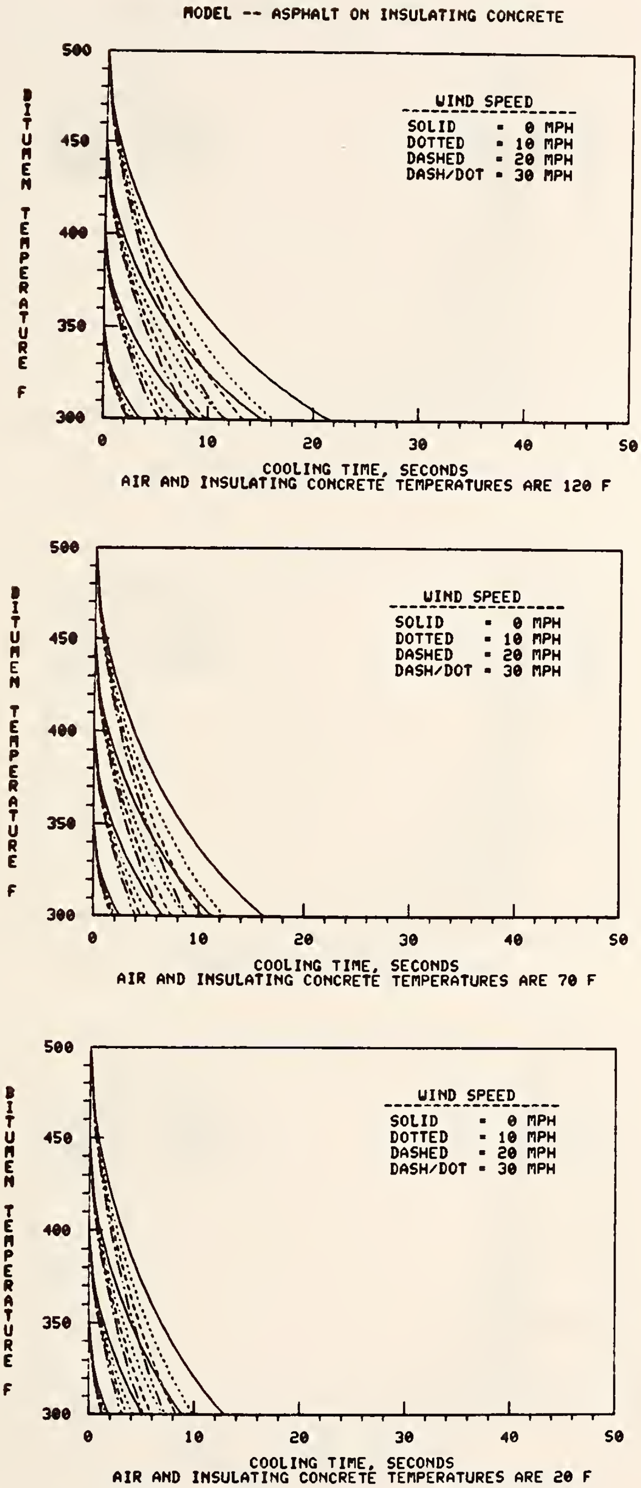


Figure 6. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Insulating Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

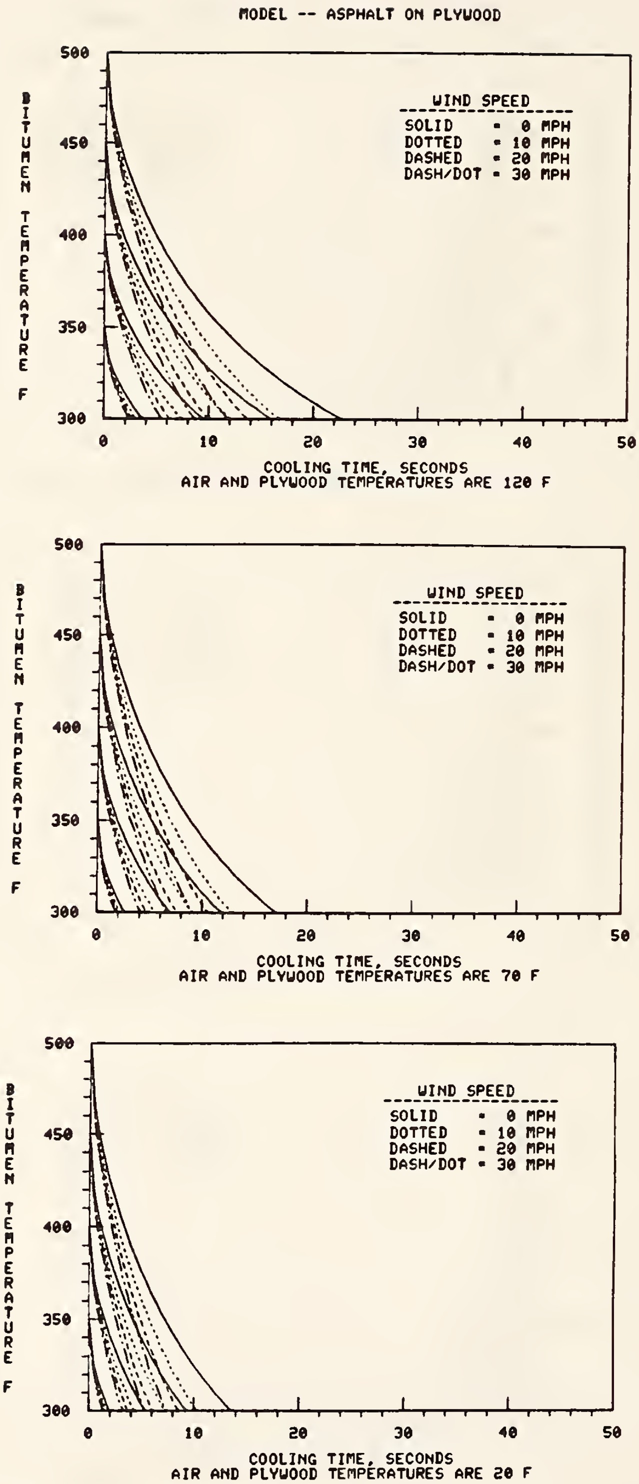


Figure 7. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

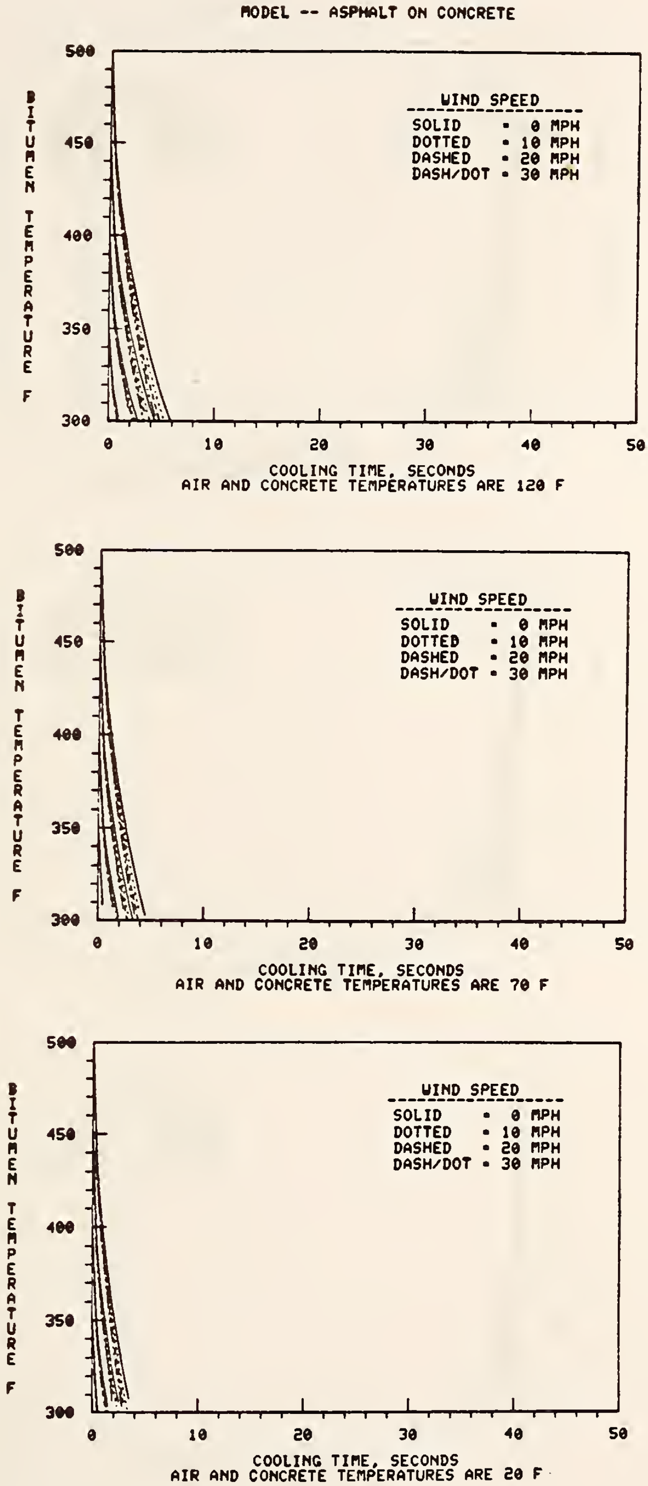


Figure 8. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

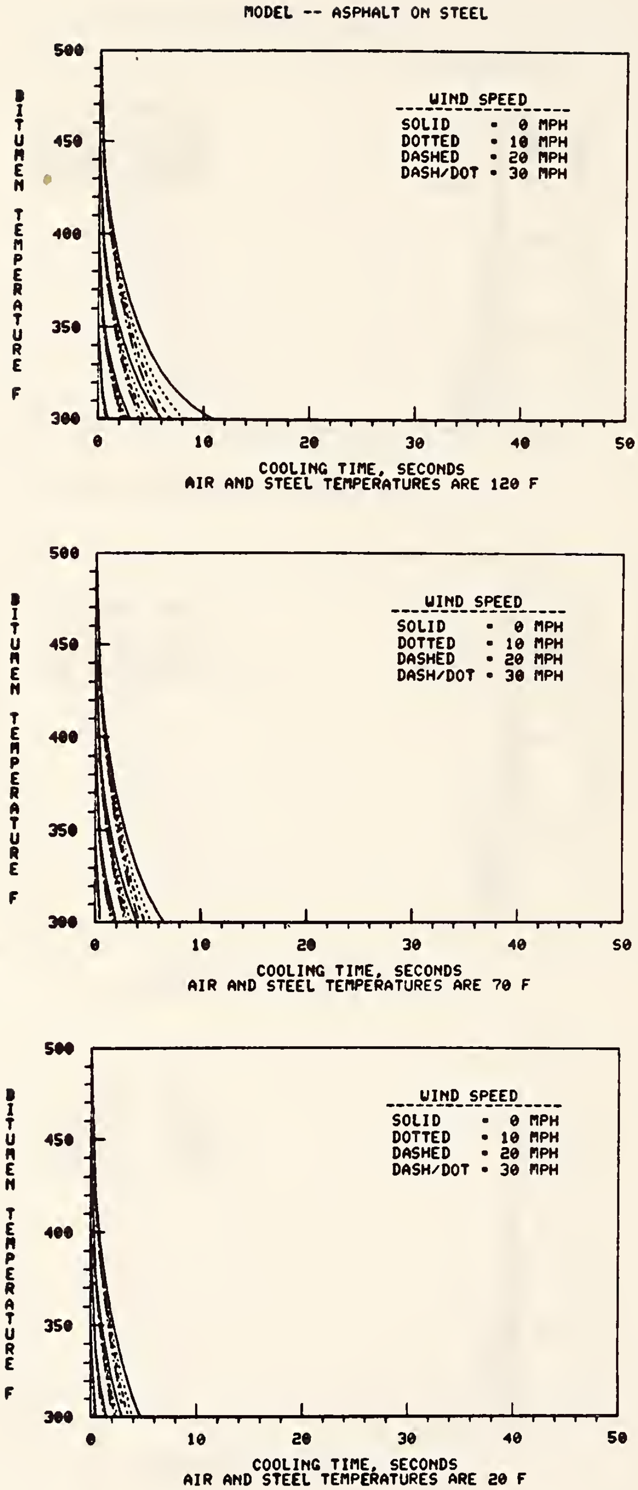


Figure 9. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Asphalt Applied on Steel. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

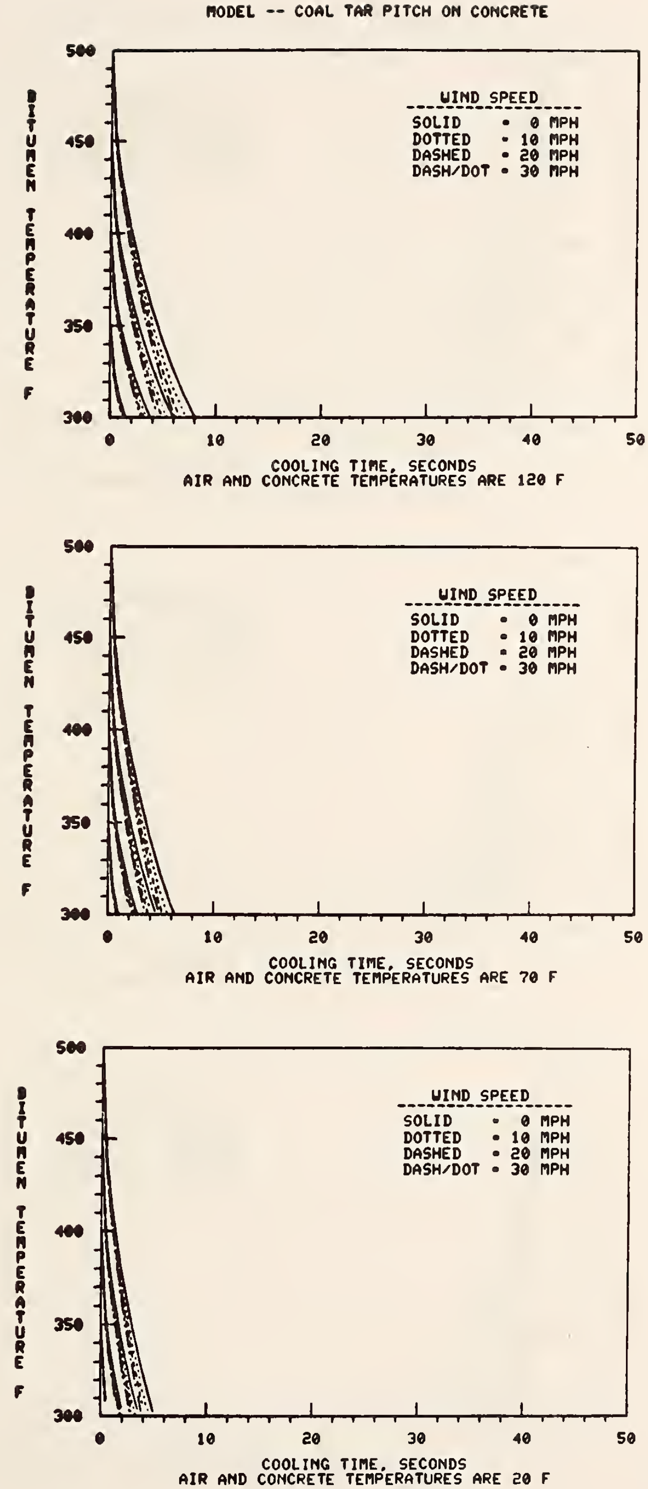


Figure 10. Relationship Between Bitumen Temperature and Cooling Time for the Two-Component Model, Coal Tar Pitch Applied on Concrete. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft^2 (1.2 kg/m^2).

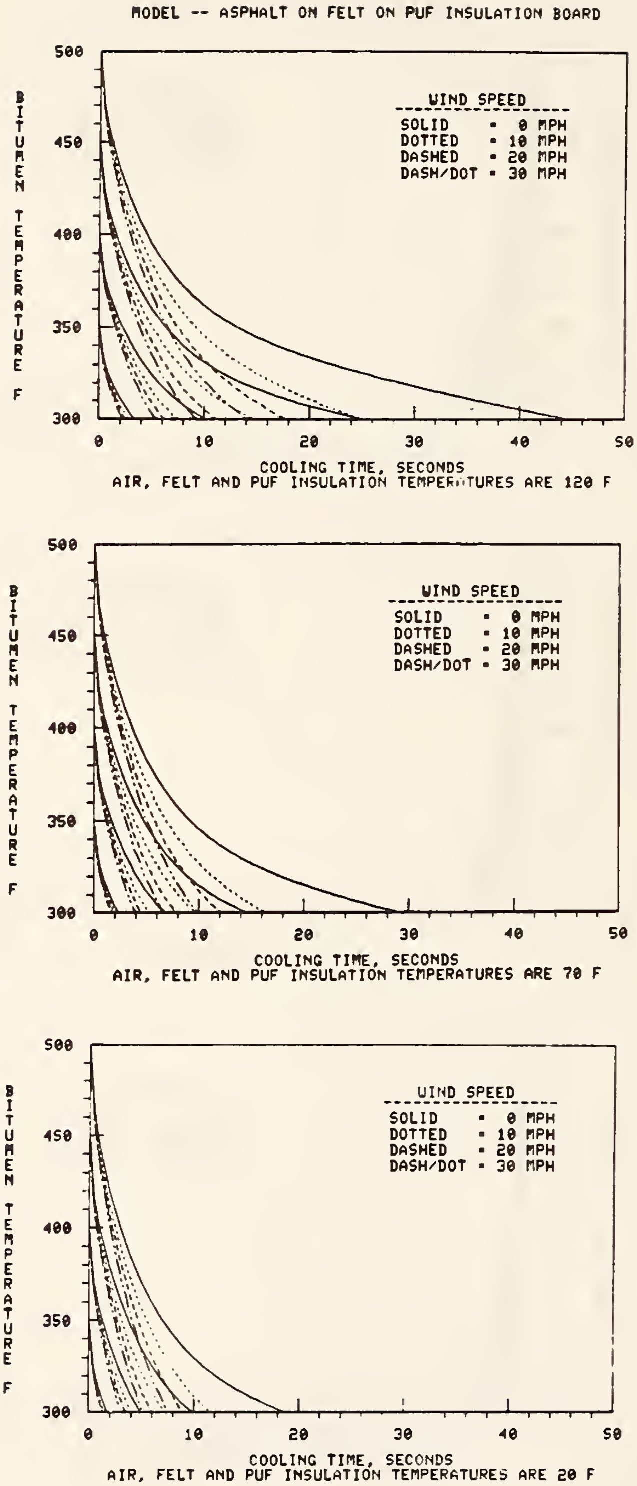


Figure 11. Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

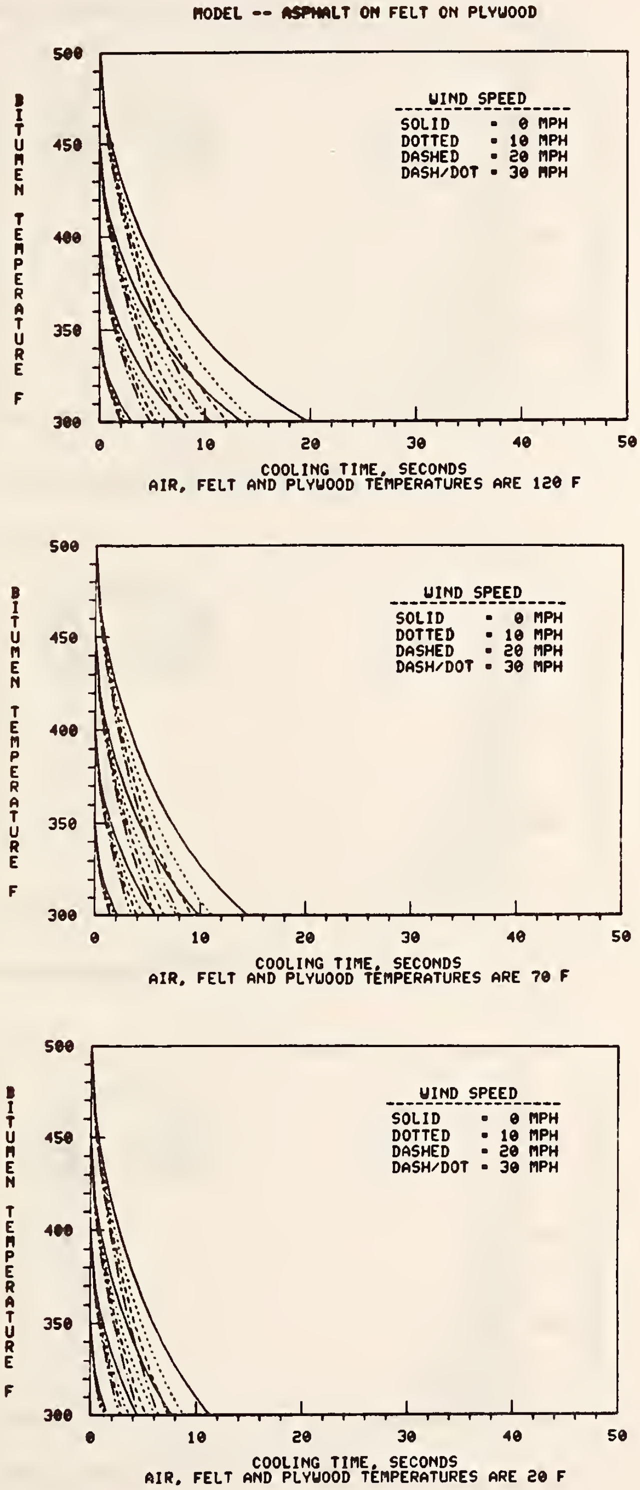


Figure 12. Relationship Between Bitumen Temperature and Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Plywood. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

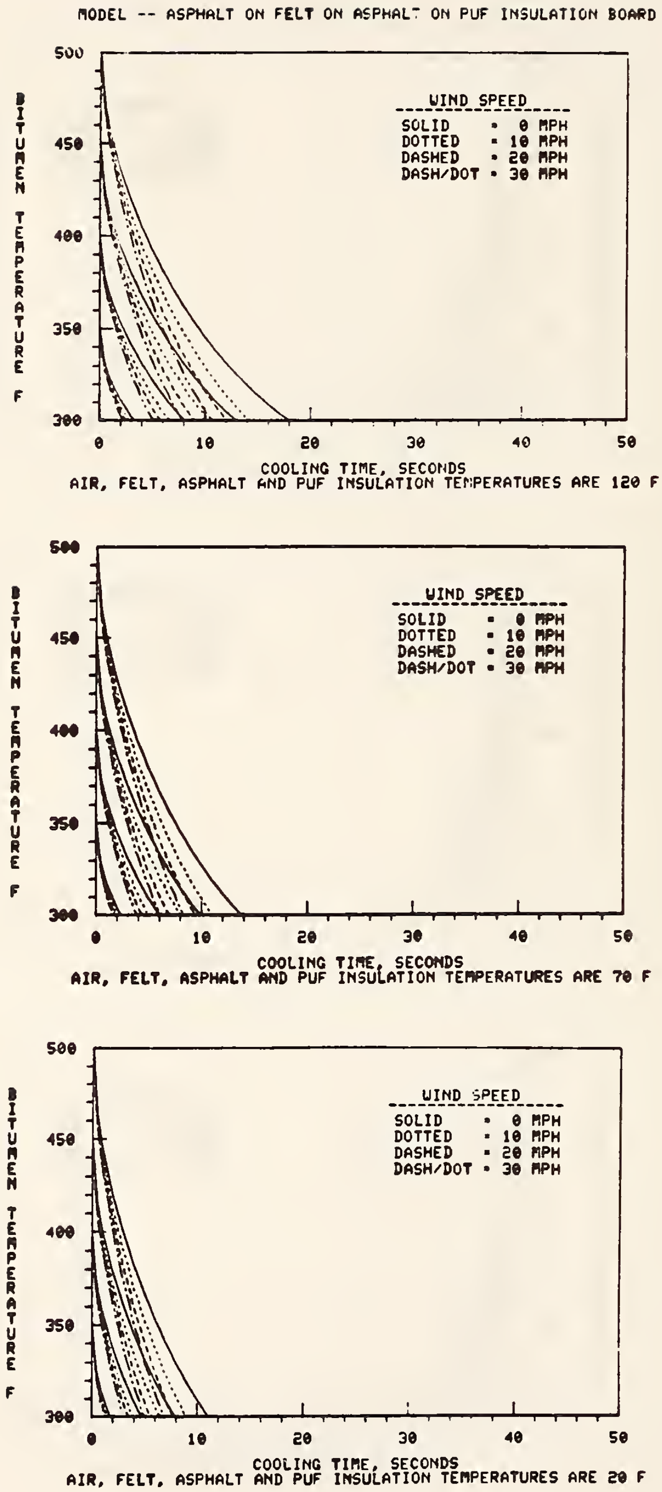


Figure 13. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Polyurethane Foam Insulation. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

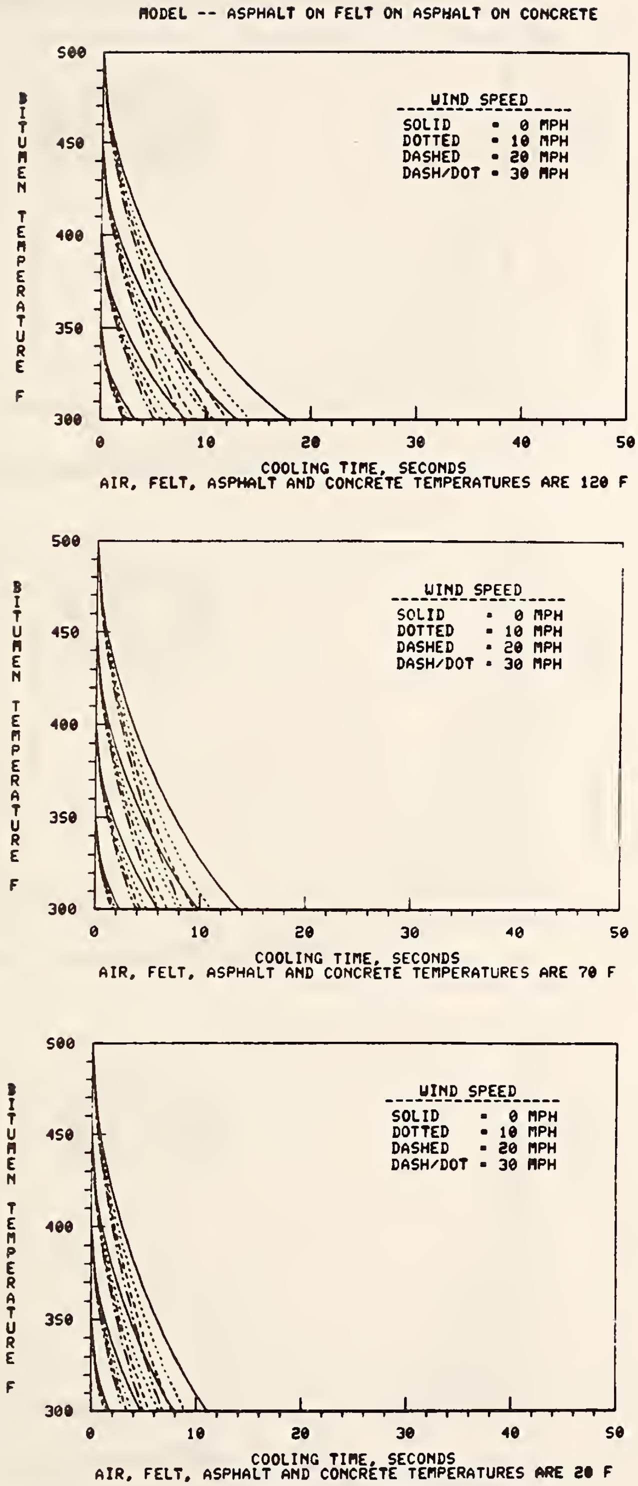


Figure 14. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Asphalt Applied on Felt over Asphalt on Concrete. The Quantity of Applied Bitumen is Approximately 20 lbm/100 ft^2 (0.98 kg/m^2).

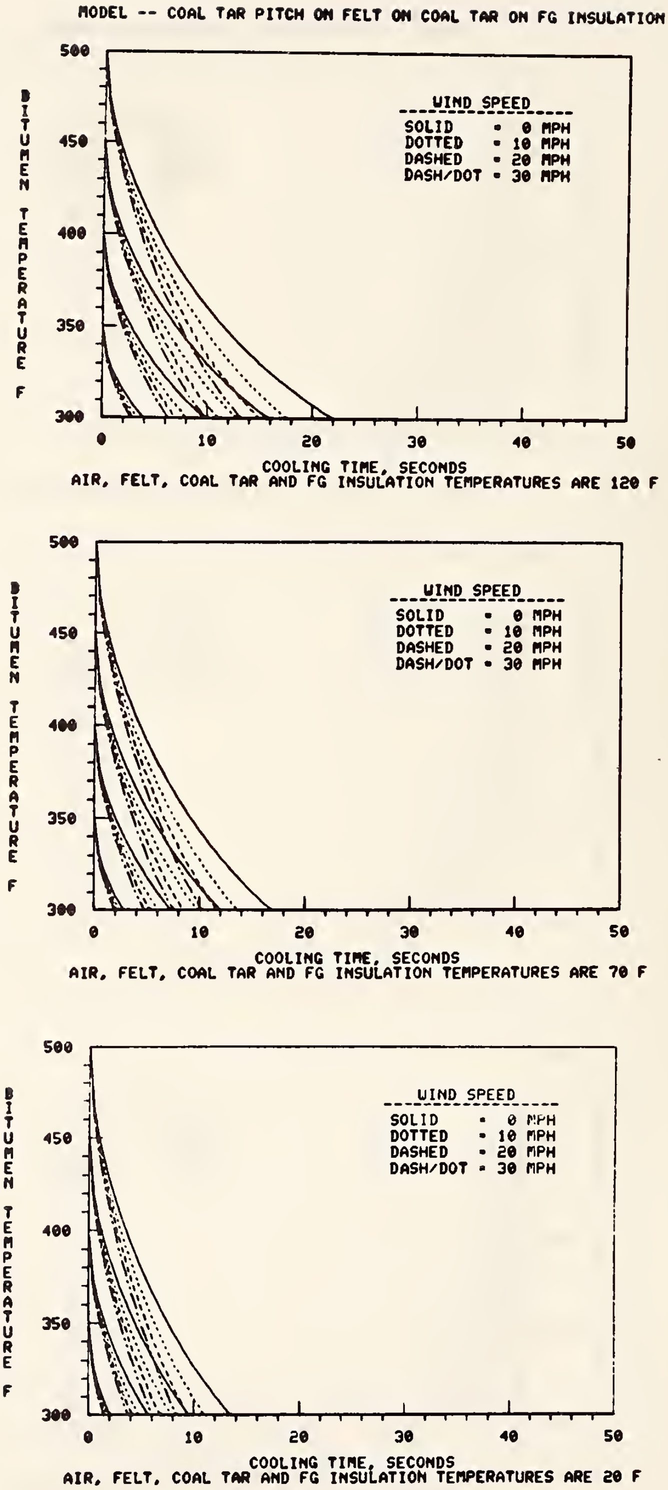


Figure 15. Relationship Between Bitumen Temperature and Cooling Time for the Four-Component Model, Coal Tar Pitch Applied on Felt over Coal Tar Pitch on Fiber Glass Insulation. The Quantity of Applied Bitumen is Approximately 25 lbm/100 ft^2 (1.2 kg/m^2).

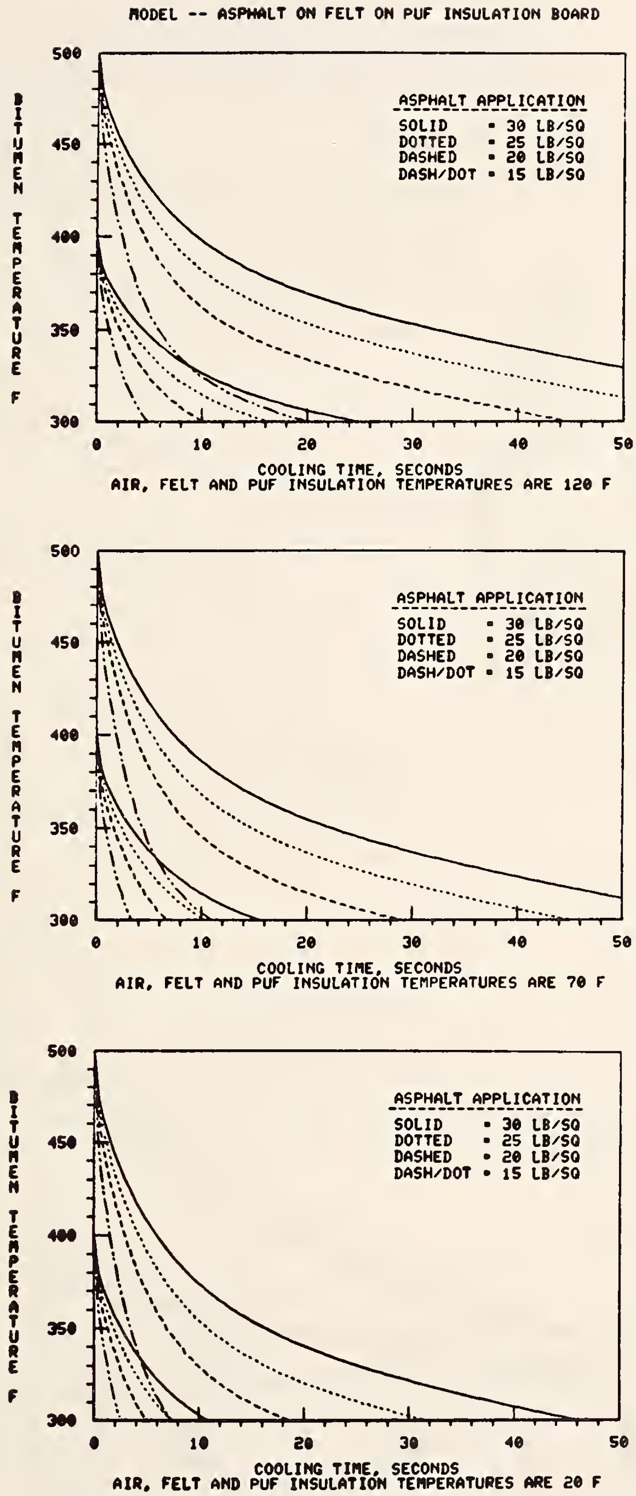


Figure 16. Effect of Asphalt Thickness on the Cooling Time for the Three-Component Model, Asphalt Applied on Felt over Polyurethane Foam Insulation.

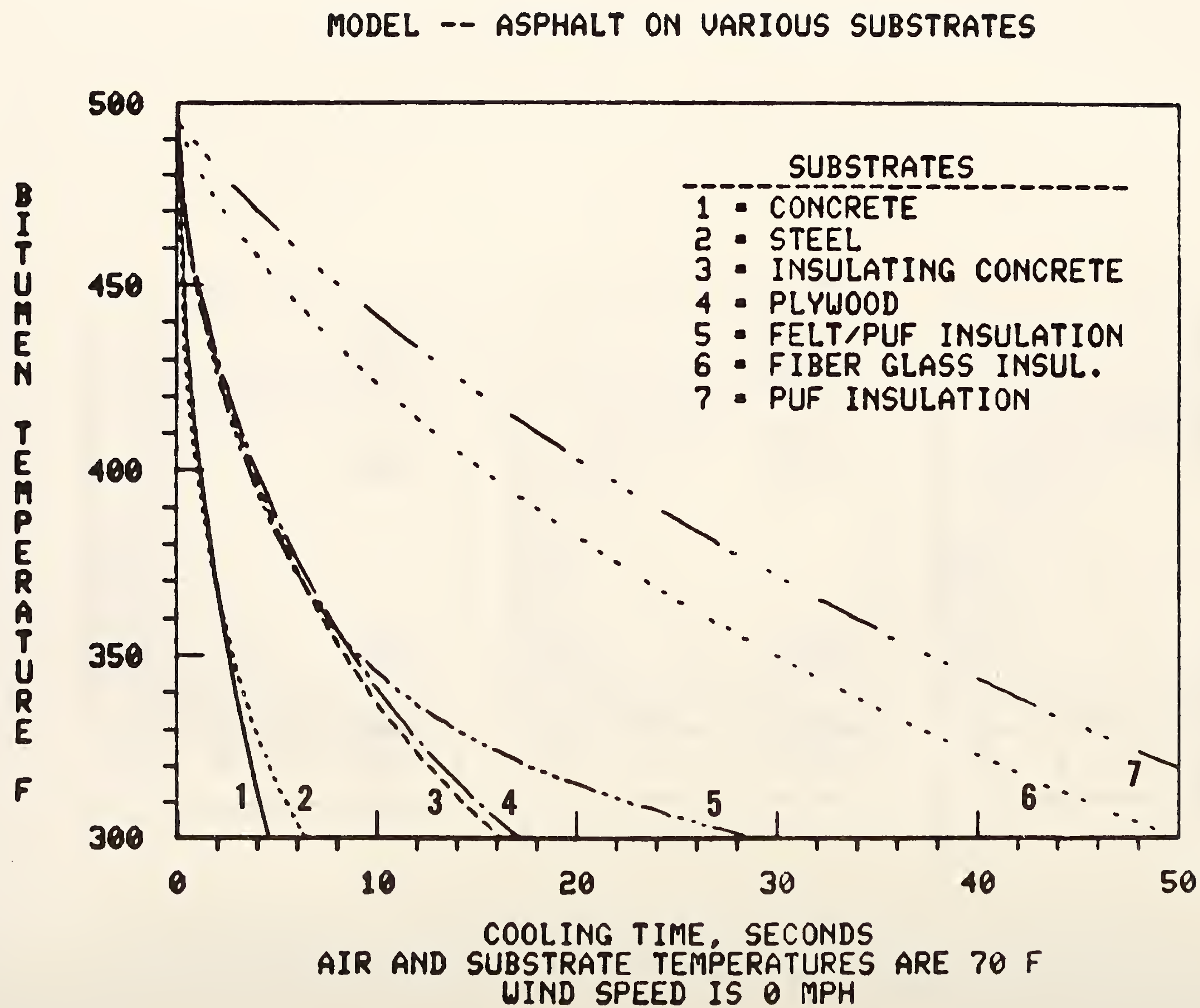


Figure 17. Effect of Substrate on the Asphalt Cooling Time.

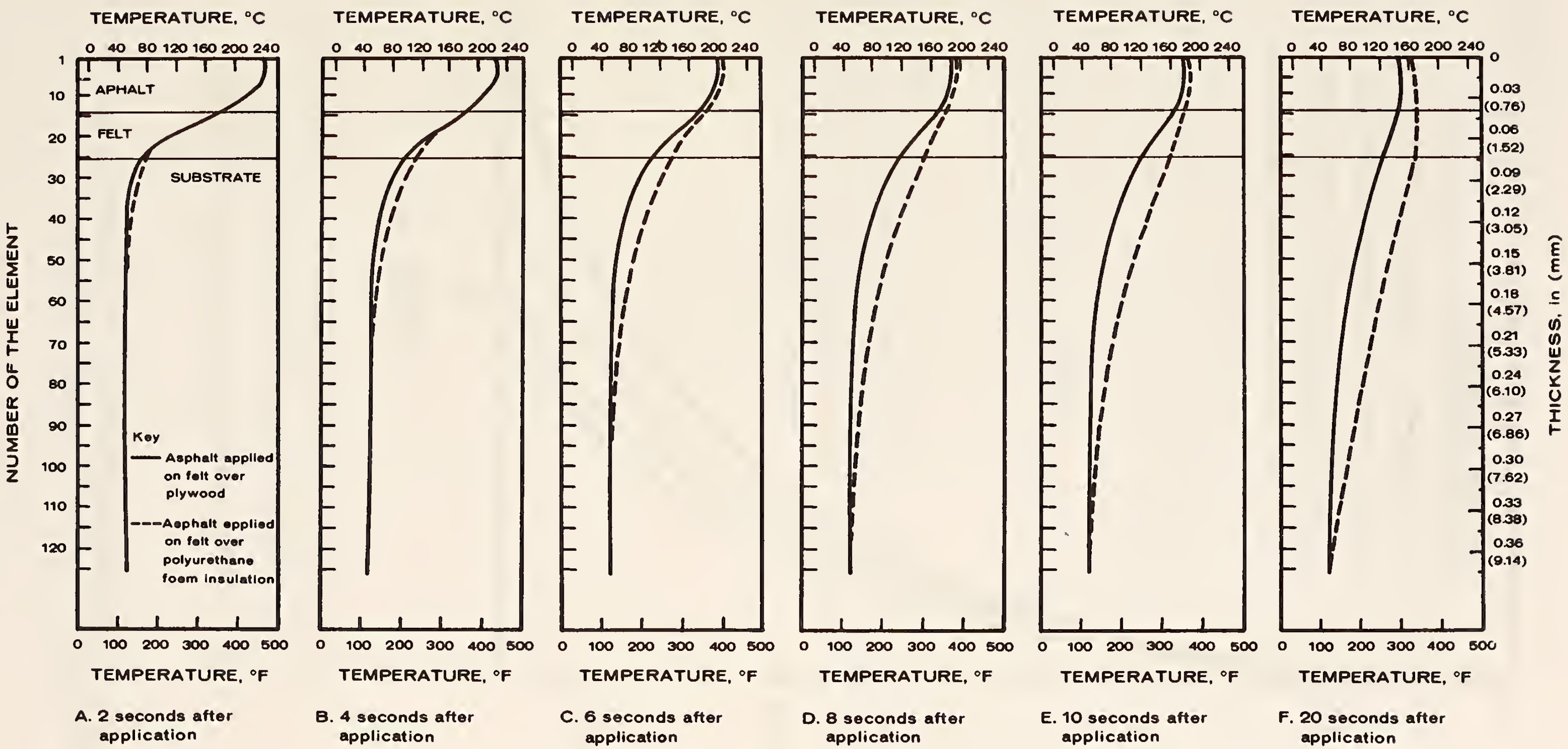
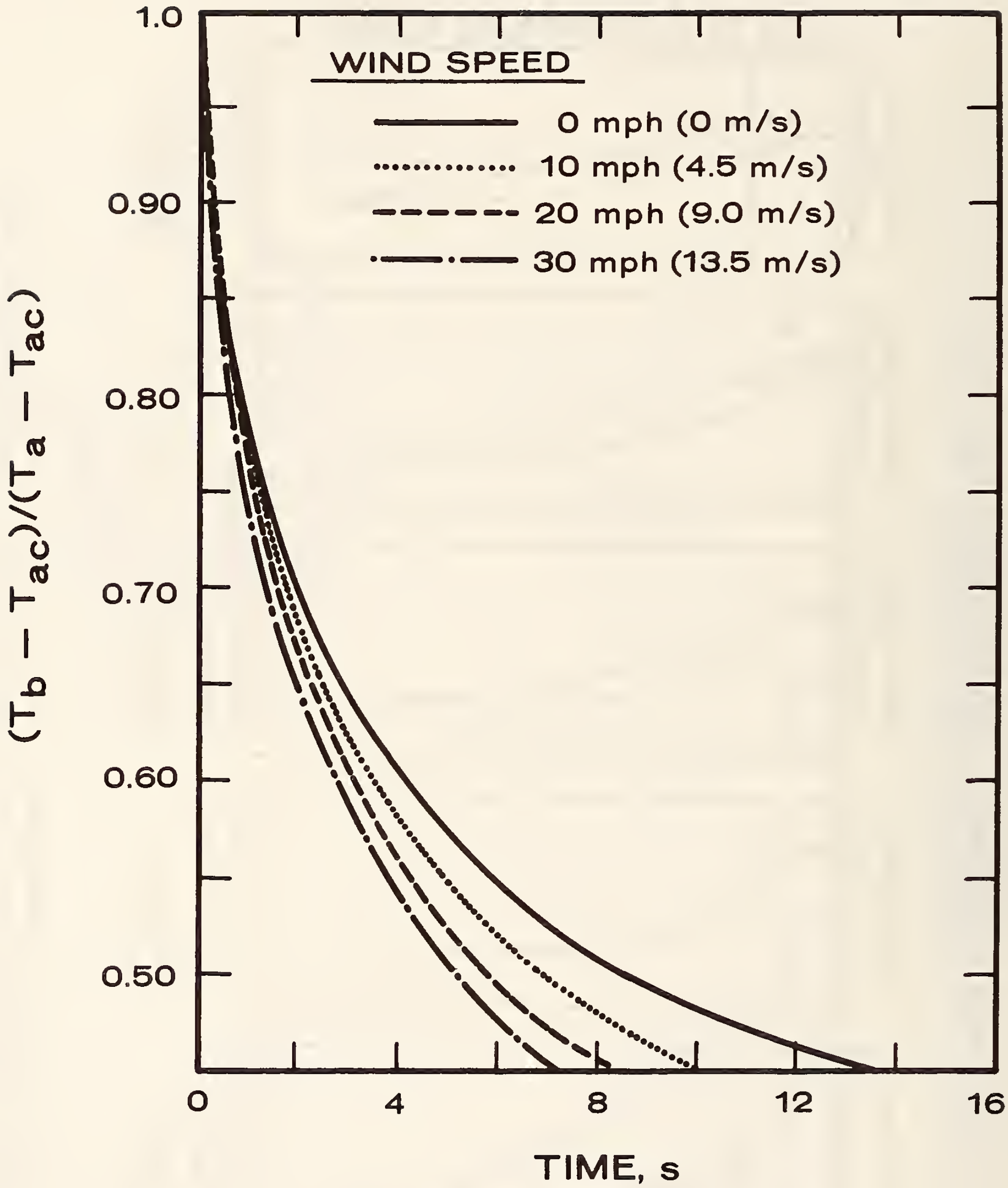


Figure 18. Temperature Profiles of the Three-Component Roofing Systems, Asphalt Applied to Felt on Plywood and Asphalt Applied to Felt on Polyurethane Foam Insulation, for Various Times after Bitumen Application. Asphalt at 500°F (260°C) was Applied to the Felt and Substrate at 120°F (49°C) with Zero Wind Speed.



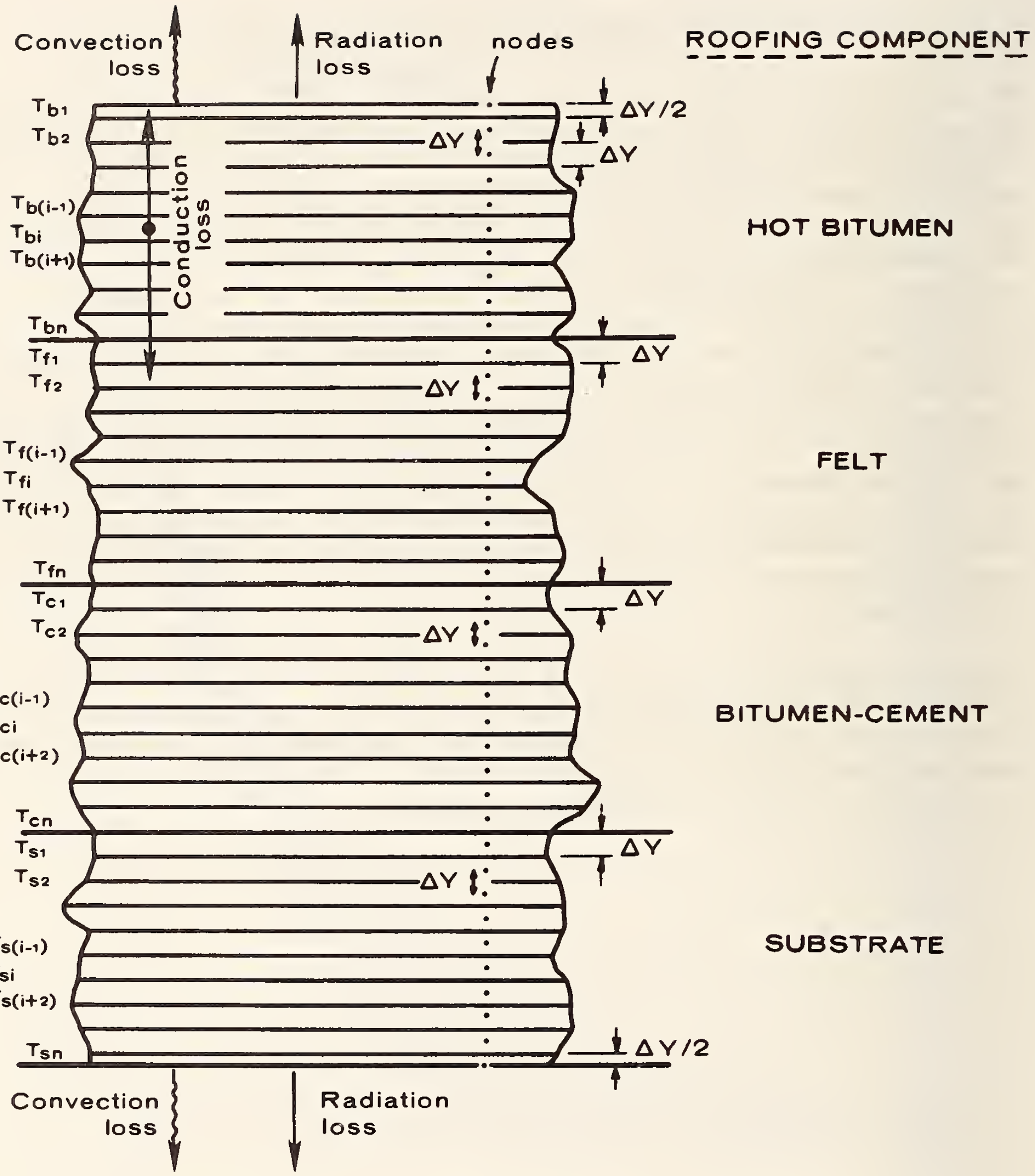
T_b = bitumen temperature at a given time
T_a = bitumen application temperature
T_{ac} = temperature of air and substrate

Figure 19. Plot of Dimensionless Temperature Versus Time for the Two-Component Roofing System, Asphalt Applied on Steel.

APPENDIX A. LISTING OF FINITE-DIFFERENCE EQUATIONS

The mathematical model for predicting the cooling times of hot bitumen during built-up roofing construction, as developed here, is applicable to roof systems which may contain two, three, or four components. The model contains three types of finite-difference equations for each component of the roofing system; that is, the model uses 6, 9 or 12 types of equations for calculating cooling times for systems containing two, three, or four components, respectively. For the convenience of the reader, this appendix presents the types of finite-difference equations used for the two-, three-, or four-component roofing systems. These equations may be developed by the treatment given in section 2.2. The subscripts b, f, c, and s in the equations which follow refer to bitumen, felt, bitumen-cement, and substrate, respectively. It is noted that the types of finite-difference equations for similar elements in different components (e.g., the equations within the bitumen-cement and within the substrate) are identical in form. The difference between the equations is the values of the constants for the thermal conductivity, specific heat, and density properties of the components in question.

Figure A1 presents a diagram of the elements and mathematical notation used to denote the temperature of the elements for the four-component roofing system. This figure was prepared for development of the finite-difference equations for the four-component system. Similar diagrams may be prepared for the three- and two-component systems.



KEY: T_{bi} = temperature of any bitumen element
T_{bn} = temperature of the last bitumen element
T_{fi} = temperature of any felt element
T_{fn} = temperature of the last felt element
T_{ci} = temperature of any bitumen-cement element
T_{cn} = temperature of the last bitumen-cement element
T_{si} = temperature of any substrate element
T_{sn} = temperature of the last substrate element

Figure A1. Diagram of the Finite-Elements Within the Components of the Roofing System Used for the Calculation of Bitumen Cooling Time by Finite-Difference Techniques.

A.1 FINITE-DIFFERENCE EQUATIONS FOR THE FOUR-COMPONENT SYSTEM

° At the bitumen surface, T_{b1}

$$T'_{b1} = T_{b1}\left[1.0 - \frac{2}{M_b}(N_b + 1.0) + \frac{2}{M_b}(T_{b2} + N_b T_a)\right] \quad \text{(A1)}$$

° Within the bitumen, T_{bi}

$$T'_{bi} = T_{bi} + \frac{1}{M_b}(T_{b(i-1)} - 2T_{bi} + T_{b(i+1)}) \quad \text{(A2)}$$

° In the bottom elemental layer of bitumen, T_{bn}

$$T'_{bn} = T_{bn} + \frac{1}{M_b}\left[T_{b(n-1)} - \left(1 + \frac{2k_f}{k_b + k_f}\right)T_{bn} + \left(\frac{2k_f}{k_b + k_f}\right)T_{f1}\right] \quad \text{(A3)}$$

° In the top elemental layer of the felt, T_{f1}

$$T'_{f1} = T_{f1} + \frac{1}{M_f}\left[\left(\frac{2k_b}{k_b + k_f}\right)T_{bn} - \left(1 + \frac{2k_b}{k_b + k_f}\right)T_{f1} + T_{f2}\right] \quad \text{(A4)}$$

° Within the felt, T_{fi}

$$T'_{fi} = T_{fi} + \frac{1}{M_f}(T_{f(i-1)} - 2T_{fi} + T_{f(i+1)}) \quad \text{(A5)}$$

° In the bottom elemental layer of felt, T_{fn}

$$T'_{fn} = T_{fn} + \frac{1}{M_f}\left[T_{f(n-1)} - \left(1 + \frac{2k_c}{k_c + k_f}\right)T_{fn} + \left(\frac{2k_c}{k_c + k_f}\right)T_{c1}\right] \quad \text{(A6)}$$

° In the top elemental layer of bitumen-cement, T_{c1}

$$T'_{c1} = T_{c1} + \frac{1}{M_c}\left[\left(\frac{2k_f}{k_c + k_f}\right)T_{fn} - \left(1 + \frac{2k_f}{k_c + k_f}\right)T_{c1} + T_{c2}\right] \quad \text{(A7)}$$

° Within the bitumen-cement, T_{ci}

$$T'_{c1} = T_{ci} + \frac{1}{M_c}(T_{c(i-1)} - 2T_{ci} + T_{c(i+1)}) \quad \text{(A8)}$$

° In the bottom elemental layer of bitumen-cement, T_{cn}

$$T'_{cn} = T_{cn} + \frac{1}{M_c}\left[T_{c(n-1)} - \left(1 + \frac{2k_s}{k_c + k_s}\right)T_{cn} + \left(\frac{2k_s}{k_c + k_s}\right)T_{s1}\right] \quad \text{(A9)}$$

° In the top elemental layer of the substrate, T_{s1}

$$T'_{s1} = T_{s1} + \frac{1}{M_s}\left[\left(\frac{2k_c}{k_c + k_s}\right)T_{cn} - \left(1 + \frac{2k_c}{k_c + k_s}\right)T_{s1} + T_{s2}\right] \tag{A10}$$

° Within the subtrate, T_{si}

$$T'_{si} = T_{si} + \frac{1}{M_s}(T_{s(i-1)} - 2T_{si} + T_{s(i+1)}) \tag{A11}$$

° In the bottom elemental layer of the substrate, T_{sn}

$$T'_{sn} = T_{sn}\left[1.0 - \left(\frac{2}{M_s}\right)(N_s + 1)\right] + \left(\frac{2}{M_s}\right)(T_{s(n-1)} + N_sT_a) \tag{A12}$$

A.2 FINITE-DIFFERENCE EQUATIONS FOR THE THREE-COMPONENT SYSTEM

The nine types of finite-difference equations for the three-component system are:

o At the bitumen surface, T_{b1}

$$T'_{b1} = T_{b1}\left[1.0 - \left(\frac{2}{M_b}\right)(N_b + 1.0)\right] + \left(\frac{2}{M_b}\right)(T_{b2} + N_bT_a) \tag{A13}$$

o Within the bitumen, T_{bi}

$$T'_{bi} = T_{bi} + \frac{1}{M_b}(T_{b(i-1)} - 2T_{bi} + T_{b(i+1)}) \tag{A14}$$

o In the bottom elemental layer of bitumen, T_{bn}

$$T'_{bn} = T_{bn} + \frac{1}{M_b}\left[T_{b(n-1)} - \left(1 + \frac{2k_f}{k_b + k_f}\right)T_{bn} + \left(\frac{2k_f}{k_b + k_f}\right)T_{f1}\right] \tag{A15}$$

o In the top elemental layer of felt, T_{f1}

$$T'_{f1} = T_{f1} + \frac{1}{M_f}\left[\left(\frac{2k_b}{k_b + k_f}\right)T_{bn} - \left(1 + \frac{2k_b}{k_b + k_f}\right)T_{f1} + T_{f2}\right] \tag{A16}$$

o Within the felt, T_{fi}

$$T'_{fi} = T_{fi} + \frac{1}{M_f}(T_{f(i-1)} - 2T_{fi} + T_{f(i+1)}) \tag{A17}$$

- In the bottom elemental layer of felt, T_{fn}

$$T'_{fn} = T_{fn} + \frac{1}{M_f}\left[T_{f(n-1)} - \left(1 + \frac{2k_s}{k_f + k_s}\right)T_{fn} + \left(\frac{2k_s}{k_f + k_s}\right)T_{s1}\right] \tag{A18}$$

- In the top elemental layer of the substrate, T_{s1}

$$T'_{s1} = T_{s1} + \frac{1}{M_s}\left[\left(\frac{2k_f}{k_f + k_s}\right)T_{fn} - \left(1 + \frac{2k_f}{k_f + k_s}\right)T_{s1} + T_{s2}\right] \tag{A19}$$

- Within the substrate, T_{si}

$$T'_{si} = T_{si} + \frac{1}{M_s}(T_{s(i-1)} - 2T_{si} + T_{s(i+1)}) \tag{A20}$$

- In the bottom elemental layer of the substrate, T_{sn}

$$T'_{sn} = T_{sn}\left[1.0 - \left(\frac{2}{M_s}\right)(N_s + 1)\right] + \left(\frac{2}{M_s}\right)(T_{s(n-1)} + N_sT_a) \tag{A21}$$

A.3 FINITE-DIFFERENCE EQUATIONS FOR THE TWO-COMPONENT SYSTEM

The six types of finite-difference equations for the two component system are:

- At the bitumen surface, T_{b1}

$$T'_{b1} = T_{b1}\left[1.0 - \left(\frac{2}{M_b}\right)(N_b + 1.0)\right] + \left(\frac{2}{M_b}\right)(T_{b2} + N_bT_a) \tag{A22}$$

- Within the bitumen, T_{bi}

$$T'_{bi} = T_{bi} + \frac{1}{M_b}(T_{b(i-1)} - 2T_{bi} + T_{b(i+1)}) \tag{A23}$$

- In the bottom elemental layer of bitumen, T_{bn}

$$T'_{bn} = T_{bn} + \frac{1}{M_b}\left[T_{b(n-1)} - \left(1 + \frac{2k_s}{k_b + k_s}\right)T_{bn} + \left(\frac{2k_s}{k_b + k_s}\right)T_{s1}\right] \tag{A24}$$

o In the top elemental layer of the substrate, T_{s1}

$$T'_{s1} = T_{s1} + \frac{1}{M_s}\left[\left(\frac{2k_b}{k_b + k_s}\right)T_{bn} - \left(1 + \frac{2k_b}{k_b + k_s}\right)T_{s1} + T_{s2}\right] \tag{A25}$$

o Within the substrate, T_{si}

$$T'_{si} = T_{si} + \frac{1}{M_s}(T_{s(i-1)} - 2T_{si} + T_{s(i+1)}) \tag{A26}$$

o In the bottom elemental layer of the substrate, T_{sn}

$$T'_{sn} = T_{sn}\left[1.0 - \left(\frac{2}{M_s}\right)(N_s + 1)\right] + \left(\frac{2}{M_s}\right)(T_{s(n-1)} + N_s T_a) \tag{A27}$$

APPENDIX B. LISTING OF COMPUTER PROGRAM

The computer program for the mathematical model for predicting the cooling time of hot bitumen applied to various substrates under different environmental conditions was written specifically for the preparation of the majority of the figures (4 to 16) presented in section 3.2. The Fortran program is given in this appendix. Only one set of input data was necessary to produce the 48 distinct curves given in the majority of the figures (4 to 16).

The initial set of input data described a model roofing system in which a given thickness of bitumen at 500°F (260°C) was applied to other components at 120°F (49°C) with zero wind speed. After completing the calculation for predicting the cooling time of bitumen under these initial conditions, the program repeated the cooling calculations using new input data which was generated from the initial input data. The bitumen application temperatures selected in the study were 500, 450, 400, and 350°F (260, 232, 204, and 177°C) and the air and initial component temperatures were 120, 70, and 20°F (49, 21, and -7°C). The wind speeds used in the calculations were 0, 10, 20, and 30 mph (0, 4.5, 9.0, and 13.5 m/s). The cooling calculation using new input data repeated in a stepwise fashion until the output to plot the 48 distinct curves in each of the figures 4 to 16 was obtained.

Although the program was written to produce data from which the sets of plots in figures 4 to 16 were generated, minor changes may readily be made to allow the prediction of bitumen cooling times under environmental and temperature conditions different than those presented here. Also, minor changes in the program may be made to alter in an incremental fashion the thickness of the hot bitumen or other roofing components. For example, the curves shown in figure 17 were plotted after the program was altered to allow an iteration of the quantity of applied bitumen of about 15, 20, 25, and 30 lbm/100 ft^2 (0.73, 0.98, 1.2, and 1.5 kg/m^2) in the absence of wind.

The Fortran program contains two subroutines which were useful to the NBS research staff in the development of the program and the interpretation of the preliminary output. Because these subroutines were useful, reference to them was kept within the program reproduced here, but the subroutines are not given. The first of these subroutines is called "TRACEH." Its purpose was to facilitate "debugging" the program and to verify whether the program was operating sequentially as expected. The second subroutine, called "PLOTT," allowed the generation of a single temperature versus time cooling curve for a distinct set of environmental conditions and component temperatures.

The Fortran program prepared to predict the cooling times of hot bitumen on various substrates under different environmental conditions follows.

ROSSITER*MAIN(1).ROSSITER3(29)

```
C     A PROGRAM TO ESTIMATE THE RATE OF COOLING OF HOT ROOFING BITUMENS
C     ON VARIOUS SUBSTRATES.
C
C     NOTE--THE ROOFING SYSTEM HEREIN MODELED MAY INCLUDE 2, 3, OR 4
C           COMPONENTS--BITUMEN, FELT, BITUMEN-CEMENT, AND SUBSTRATE.
C
C     ************************************
C     ************************************
C     **  STEP 1--                      **
C     **  DEFINITIONS OF                **
C     **  PROGRAM PAPAMETERS AND        **
C     **  DIMENSION STATEMENTS          **
C     ************************************
C     ************************************
C
C     CB      = THERMAL CONDUCTIVITY OF HOT BITUMEN    , BTU, H-1, FT-1.
C     CF      = THERMAL CONDUCTIVITY OF FELT           , BTU, H-1, FT-1.
C     CC      = THERMAL CONDUCTIVITY OF BITUMEN-CEMENT, BTU, H-1, FT-1.
C     CS      = THERMAL CONDUCTIVITY OF SUBSTRATE      , BTU, H-1, FT-1.
C
C     DB      = THERMAL DIFFUSIVITY  OF HOT BITUMEN    , FT 2, H-1.
C     DF      = THERMAL DIFFUSIVITY  OF FELT           , FT 2, H-1.
C     DC      = THERMAL DIFFUSIVITY  OF BITUMEN-CEMENT, FT 2, H-1.
C     DS      = THERMAL DIFFUSIVITY  OF SUBSTRATE      , FT 2, H-1.
C
C     DELY    = THICKNESS OF ANT FINITE ELEMENT IN THE SYSTEM,FT
C     DELT    = INCREMENTAL TIME,H
C     NUMCOM = NUMBER OF COMPONENTS IN THE MODELED SYSTEM.
C
C     DR      = CONSTANT TO CONVERT TEMPERATURES TO DEGREES RANKINE
C     EMB     = EMITTENCE OF BITUMEN,DIMENSIONALLESS
C     EMS     = EMITTENCE OF SUBSTRATE,DIMENSIONALLESS
C     SIGMA   = STEFAN-BOLTZMAN CONSTANT,BTU,FT-2,H-1,R-4
C     PFACT   = FACTOR TO ADJUST THE NO. OF DATA POINTS PRINTED AS OUTPUT
C
C     HBC     = CONVECTION COEFFICIENT FOR BITUMEN
C     HSC     = CONVECTION COEFFICIENT FOR SUBSTRATE
C
C     NB      = NUMBER OF FINITE ELEMENTS IN BITUMEN
C     NF      = NUMBER OF FINITE ELEMENTS IN FELT
C     NC      = NUMBER OF FINITE ELEMENTS IN BITUMEN-CEMENT
C     NS      = NUMBER OF FINITE ELEMENTS IN THE SUBSTRATE
C
C     TA      = TEMPERATURE OF AIR,F
C     TBO     = INITIAL TEMPERATURE OF HOT BITUMEN,F
C     TFO     = INITIAL TEMPERATURE OF FELT,F
C     TCO     = INITIAL TEMPERATURE OF BITUMEN-CEMENT,F
C     TSO     = INITIAL TEMPERATURE OF SUBSTRATE,F
C
C     TB(I)   = TEMPERATURE OF A BITUMEN ELEMENT IN TIME,F
C     TF(I)   = TEMPERATURE OF A FELT ELEMENT IN TIME,F
C     TC(I)   = TEMPERATURE OF A BITUMEN-CEMENT ELEMENT IN TIME,F
C     TS(I)   = TEMPERATURE OF A SUBSTRATE ELEMENT IN TIME,F
C
C     AVEBIT = AVERAGE TEMPERATURE OF THE BITUMEN IN TIME,F
C
C     TEMPMN = MINIMUM TEMPERATURE TO WHICH THE BITUMEN COOLS
```

```
C             DURING THE EXECUTION OF THE PROGRAM,F
C      TSECMX = MAXIMUM TIME OVER WHICH THE BITUMEN COOLS
C             DURING THE EXECUTION OF THE PROGRAM
C
C
C
C      ***************************************************************
C
       DIMENSION TB(1000)
       DIMENSION TF(1000)
       DIMENSION TC(1000)
       DIMENSION TS(1000)
       DIMENSION TEMP(500)
       DIMENSION TIME(500)
C
C      ***************************************************************
C
       IRD=5
       IPR=6
C
       DR=460.0
       EMB=0.95
       EMS=0.23
       SIGMA=.1714E-08
       PFACT=2.0
C
C      ***************************
C      ***************************
C      **  STEP 2--             **
C      **  READ IN PARAMETERS.  **
C      ***************************
C      ***************************
C
       READ(IRD,211)BUG
       IBUG=BUG+0.5
C
C
       IF(IBUG.EQ.1)CALL TRACEH('2','ROSSIT')
C
       WRITE(IPR,998)
  998  FORMAT(1H1)
       WRITE(IPR,999)
  999  FORMAT(1H )
       WRITE(IPR,999)
       WRITE(IPR,999)
C
  211  FORMAT(F15.0)
  212  FORMAT(A6)
C
       READ(IRD,212,END=9000)JUNK
       IF(JUNK.EQ.'END')GOTO9000
       READ(IRD,211)ANUMCO
       READ(IRD,211)DELT
       READ(IRD,211)DELY
       NUMCOM=ANUMCO+0.5
C
       IF(IBUG.EQ.1)WRITE(IPR,221)ANUMCO,DELT,DELY
  221  FORMAT(1H ,'ANUMCO,DELT, DELY = ',3E15.7)
```

```
      READ(IRD,212)JUNK
      READ(IRD,211)CB
      READ(IRD,211)CF
      READ(IRD,211)CC
      READ(IRD,211)CS
      IF(IBUG.EQ.1)WRITE(IPR,222)CB,CF,CC,CS
  222 FORMAT(1H ,'CB,CF,CC,CS = ',4F15.7)
C
      READ(IRD,212)JUNK
      READ(IRD,211)DB
      READ(IRD,211)DF
      READ(IRD,211)DC
      READ(IRD,211)DS
      IF(IBUG.EQ.1)WRITE(IPR,223)DB,DF,DC,DS
  223 FORMAT(1H ,'DB,DF,DC,DS = ',4F15.7)
C
      READ(IRD,212)JUNK
      READ(IRD,211)HBC
      READ(IRD,211)HSC
      IF(IBUG.EQ.1)WRITE(IPR,224)HBC,HSC
  224 FORMAT(1H ,'HBC,HSC = ',2F15.7)
C
      READ(IRD,212)JUNK
      READ(IRD,211)ANB
      READ(IRD,211)ANF
      READ(IRD,211)ANC
      READ(IRD,211)ANS
      NB=ANB+0.5
      NF=ANF+0.5
      NC=ANC+0.5
      NS=ANS+0.5
      IF(IBUG.EQ.1)WRITE(IPR,225)NB,NF,NC,NS
  225 FORMAT(1H ,'NB,NF,NC,NS = ',4I8)
C
      READ(IRD,212)JUNK
      READ(IRD,211)TA
      READ(IRD,211)TBO
      READ(IRD,211)TFO
      READ(IRD,211)TCO
      READ(IRD,211)TSO
      IF(IBUG.EQ.1)WRITE(IPR,226)TA,TBO,TFO,TCO,TSO
  226 FORMAT(1H ,'TA,TBO,TFO,TCO,TSO = ',5F15.7)
C
      READ(IRD,212)JUNK
      READ(IRD,211)TEMPMN
      READ(IRD,211)TSECMX
      IF(IBUG.EQ.1)WRITE(IPR,227)TEMPMN,TSECMX
  227 FORMAT(1H ,'TEMPMN,TSECMX = ',2F15.7)
C
C
C     ***********************************************************
C     ***********************************************************
C     **  STEP 3--
C     **  SET UP A LARGE LOOP INCREMENTING
C     **  THE TEMPERATURES OF THE AIR AND SUBSTRATE(S),
C     **  THE WIND SPEED, AND THE BITUMEN APPLICATION TEMP.
C     ***********************************************************
C     ***********************************************************
```

```
C
C
      TASAV=TA
      TBOSAV=TBO
      TFOSAV=TFO
      TCOSAV=TCO
      TSOSAV=TSO
C
      HBCSAV=HBC
C
      ICOUNT=0
      L2=0
C
      DO310ISTEMP=1,3
      TA=TASAV
      TFO=TFOSAV
      TCO=TCOSAV
      TSO=TSOSAV
      IF(ISTEMP.EQ.1)GOTO319
      IF(ISTEMP.EQ.2)GOTO312
      IF(ISTEMP.EQ.3)GOTO313
      GOTO319
C
  312 CONTINUE
      TA=TA-50
      TFO=TFO-50
      TCO=TCO-50
      TSO=TSO-50
      GOTO319
C
  313 CONTINUE
      TA=TA-100
      TFO=TFO-100
      TCO=TCO-100
      TSO=TSO-100
      GOTO319
C
  319 CONTINUE
      DO320MPH=1,4
      HBC=HBCSAV
      IF(MPH.EQ.1)GOTO329
      IF(MPH.EQ.2)GOTO322
      IF(MPH.EQ.3)GOTO323
      IF(MPH.EQ.4)GOTO324
      GOTO329
C
  322 CONTINUE
      HBC=3.6
      GOTO329
C
  323 CONTINUE
      HBC=6.1
      GOTO329
C
  324 CONTINUE
      HBC=8.5
      GOTO329
C
```

```
  329 CONTINUE
      DO330IBTEMP=1,4
      TBO=TBOSAV
      IF(IBTEMP.EQ.1)GOTO0339
      IF(IBTEMP.EQ.2)GOTO0332
      IF(IBTEMP.EQ.3)GOTO0333
      IF(IBTEMP.EQ.4)GOTO0334
      GOTO0339
C
  332 CONTINUE
      TBO=TBO-50
      GOTO0339
C
  333 CONTINUE
      TBO=TBO-100
      GOTO0339
C
  334 CONTINUE
      TBO=TBO-150
      GOTO0339
C
  339 CONTINUE
      ICOUNT=ICOUNT+1
      WRITE(6,999)
      WRITE(6,341)ICOUNT
  341 FORMAT(1H ,'BEGIN COMPUTATION WITH DATA SET = ',I8)
C
      WRITE(6,999)
      WRITE(6,342)TA,TBO,TFO,TCO,TSO,HBC
  342 FORMAT(1H ,'TA,TBO,TFO,TCO,TSO,HBC = ',6F12.6)
C
C     ***********************************
C     ***********************************
C     **  STEP 4--                     **
C     **  SET UP A LARGE LOOP          **
C     **  BASED ON INCREMENTING TIME.  **
C     ***********************************
C     ***********************************
C
      IF(IBUG.EQ.1)CALL TRACEH('4','ROSSIT')
      T=0.0
      L=0
      L=L+1
      L2=L2+1
      TEMP(L)=TBO
      TIME(L)=0
      ISTEP=-1
  400 CONTINUE
      ISTEP=ISTEP+1
      IF(ISTEP.EQ.0)WRITE(IPR,411)ISTEP,TBO
      IF(ISTEP.EQ.0)GOTO0500
      IREM=ISTEP-1000*(ISTEP/1000)
      IF(IREM.EQ.0)WRITE(IPR,411)ISTEP,AVEBIT
  411 FORMAT(1H ,'ENTERING STEP ',I8,' WITH AVE. BIT. TEMP. = ',E15.7)
C
C     ******************************************
C     ******************************************
C     **  STEP 5--                            **
```

```
C      **  INITIALIZE VALUES IN TB(.), TF(.),   **
C      **  TC(.), AND TS(.).                    **
C      *****************************************
C      *****************************************
C
   500 CONTINUE
       IF(IBUG.EQ.1)CALL TRACEH('5','ROSSIT')
C
       IF(ISTEP.NE.0)GOTO590
C
       DO510I=1,NB
       TB(I)=TB0
   510 CONTINUE
C
       DO520I=1,NF
       TF(I)=TF0
   520 CONTINUE
C
       DO530I=1,NC
       TC(I)=TC0
   530 CONTINUE
C
       DO540I=1,NS
       TS(I)=TS0
   540 CONTINUE
C
   590 CONTINUE
C
C      *****************************************
C      *****************************************
C      **  STEP 6--                             **
C      **  APPLY THE SET OF EQUATIONS           **
C      **  (SURFACE, MIDDLE STEPS, AND BOTTOM)  **
C      **  FOR COMPONENT 1 (BITUMEN).           **
C      *****************************************
C      *****************************************
C
       IF(IBUG.EQ.1)CALL TRACEH('6','ROSSIT')
C
       I=1
       IP1=I+1
       HBR=SIGMA*EMB*((TB(I)+DR)**2+(TA+DR)**2)*((TB(I)+DR)+(TA+DR))
       HB=HBC+HBR
       ABB=DELY*HB/CB
       AMB=(1.0/DB)*(DELY*DELY/DELT)
       TB(I)=TB(I)*(1.0-(2.0/AMB)*(ABB+1.0)) + (2.0/AMB)*(TB(IP1)+ABB*TA)
       IF(IBUG.EQ.1)WRITE(6,611)TA,DR,HBR,HB,ABB,AMB,TB(I)
   611 FORMAT(1H ,'TA,DR,HBR,HB,ABB,AMB,TB(I) = ',7E10.3)
C
       IMAX=NB-1
       DO600I=2,IMAX
       IM1=I-1
       IP1=I+1
       TB(I)=TB(I)+(1.0/AMB)*(TB(IM1)-2.0*TB(I)+TB(IP1))
   600 CONTINUE
C
       I=NB
       IM1=I-1
```

```
C
      IF(NUMCOM.LE.2)GOTO631
      GOTO632
  631 CONTINUE
      FACTOR=(2.0*CS)/(CB+CS)
      TB(I)=TB(I)+(1.0/AMB)*(TB(IM1)-(1.0+FACTOR)*TB(I)+FACTOR*TS(1))
      GOTO680
  632 CONTINUE
      FACTOR=(2.0*CF)/(CB+CF)
      TB(I)=TB(I)+(1.0/AMB)*(TB(IM1)-(1.0+FACTOR)*TB(I)+FACTOR*TF(1))
      GOTO680
  680 CONTINUE
      IF(NUMCOM.LE.2)GOTO910
C
C     ******************************************
C     ******************************************
C     **  STEP 7--                            **
C     **  APPLY THE SET OF EQUATIONS          **
C     **  (SURFACE, MIDDLE STEPS, AND BOTTOM) **
C     **  FOR COMPONENT 2 (FELT).             **
C     ******************************************
C     ******************************************
C
      IF(IBUG.EQ.1)CALL TRACEH('7','ROSSIT')
C
      I=1
      IP1=I+1
      AMF=(1.0/DF)*(DELY*DELY/DELT)
      FACTOR=(2.0*CB)/(CB+CF)
      TF(I)=TF(I)+(1.0/AMF)*(FACTOR*TB(NB)-(1.0+FACTOR)*TF(I)+TF(IP1))
C
      IMAX=NF-1
      DO700I=2,IMAX
      IM1=I-1
      IP1=I+1
      TF(I)=TF(I)+(1.0/AMF)*(TF(IM1)-2.0*TF(I)+TF(IP1))
  700 CONTINUE
C
      I=NF
      IM1=I-1
C
      IF(NUMCOM.LE.3)GOTO731
      GOTO732
  731 CONTINUE
      FACTOR=(2.0*CS)/(CF+CS)
      TF(I)=TF(I)+(1.0/AMF)*(TF(IM1)-(1.0+FACTOR)*TF(I)+FACTOR*TS(1))
      GOTO780
  732 CONTINUE
      FACTOR=(2.0*CC)/(CC+CF)
      TF(I)=TF(I)+(1.0/AMF)*(TF(IM1)-(1.0+FACTOR)*TF(I)+FACTOR*TC(1))
      GOTO780
  780 CONTINUE
      IF(NUMCOM.LE.3)GOTO910
C
C     ******************************************
C     ******************************************
C     **  STEP 8--                            **
C     **  APPLY THE SET OF EQUATIONS          **
```

```
C     **  (SURFACE, MIDDLE STEPS, AND BOTTOM)  **
C     **  FOR COMPONENT 3 (BITUMEN-CEMENT)     **
C     ******************************************
C     ******************************************
C
      IF(IBUG.EQ.1)CALL TRACEH('8','ROSSIT')
C
      I=1
      IP1=I+1
      AMC=(1.0/DC)*(DELY*DELY/DELT)
      FACTOR=(2.0*CF)/(CF+CC)
      TC(I)=TC(I)+(1.0/AMC)*(FACTOR*TF(NF)-(1.0+FACTOR)*TC(I)+TC(IP1))
C
      IMAX=NC-1
      DO800I=2,IMAX
      IM1=I-1
      IP1=I+1
      TC(I)=TC(I)+(1.0/AMC)*(TC(IM1)-2.0*TC(I)+TC(IP1))
  800 CONTINUE
C
      I=NC
      IM1=I-1
      FACTOR=(2.0*CS)/(CC+CS)
      TC(I)=TC(I)+(1.0/AMC)*(TC(IM1)-(1.0+FACTOR)*TC(I)+FACTOR*TS(1))
C
C     *****************************************
C     *****************************************
C     **  STEP 9--                           **
C     **  APPLY THE SET OF EQUATIONS         **
C     **  (SURFACE, MIDDLE STEPS, AND BOTTOM) **
C     **  FOR COMPONENT 4 (SUBSTRATE).       **
C     *****************************************
C     *****************************************
C
  910 CONTINUE
C
      IF(IBUG.EQ.1)CALL TRACEH('9','ROSSIT')
C
      I=1
      IP1=I+1
      AMS=(1.0/DS)*(DELY*DELY/DELT)
C
      IF(NUMCOM.LE.2)GOTO931
      IF(NUMCOM.EQ.3)GOTO932
      IF(NUMCOM.GE.4)GOTO933
  931 CONTINUE
      FACTOR=(2.0*CB)/(CB+CS)
      TS(I)=TS(I)+(1.0/AMS)*(FACTOR*TB(NB)-(1.0+FACTOR)*TS(I)+TS(IP1))
      GOTO980
  932 CONTINUE
      FACTOR=(2.0*CF)/(CF+CS)
      TS(I)=TS(I)+(1.0/AMS)*(FACTOR*TF(NF)-(1.0+FACTOR)*TS(I)+TS(IP1))
      GOTO980
  933 CONTINUE
      FACTOR=(2.0*CC)/(CC+CS)
      TS(I)=TS(I)+(1.0/AMS)*(FACTOR*TC(NC)-(1.0+FACTOR)*TS(I)+TS(IP1))
      GOTO980
  980 CONTINUE
```

```
C
      IF(NS.GT.100)IMAX=100
      IF(NS.LE.100)IMAX=NS-1
      DO900I=2,IMAX
      IM1=I-1
      IP1=I+1
      TS(I)=TS(I)+(1.0/AMS)*(TS(IM1)-2.0*TS(I)+TS(IP1))
  900 CONTINUE
C
      IF(NS.GT.100)GOTO990
      I=NS
      IM1=I-1
      IF(ISTEP.EQ.0)TS(I)=TS0
      HSR=SIGMA*EMS*((TS(I)+DR)**2+(TA+DR)**2)*((TS(I)+DR)+(TA+DR))
      HS=HSC+HSR
      ABS=DELY*HS/CS
      AMS=(1.0/DS)*(DELY*DELY/DELT)
      TS(I)=TS(I)*(1.0-(2.0/AMS)*(ABS+1.0)) + (2.0/AMS)*(TS(IM1)+ABS*TA)
  990 CONTINUE
C
C
C     ******************************************************
C     ******************************************************
C     **  STEP 10
C     **  COMPUTE SUM AND AVEBIT                            **
C     **  SUM IS THE SUMMATION OF TEMPERATURES              **
C     **  OF ELEMENTS IN THE BITUMEN COMPONENT              **
C     **  AVEBIT IS THE AVERAGE TEMPERATURE OF BITUMEN      **
C     ******************************************************
C     ******************************************************
C
      IF(IBUG.EQ.1)CALL TRACEH('10','ROSSIT')
C
      SUM=0.0
      DO1032I=1,NB
      SUM=SUM+TB(I)
      IF(IBUG.EQ.1)WRITE(IPR,1034)I,TB(I)
 1034 FORMAT(1H ,'I,TB(I) = ',I8,E15.7)
 1032 CONTINUE
      AVEBIT=SUM/ANB
C
C     *******************************************************
C     *******************************************************
C     **  STEP 11--                                          **
C     **  COMPARE AVERAGE TEMPERATURE TO TEMPMN.             **
C     **  COMPARE TIME OF COOLING TO TSECMX.                 **
C     *******************************************************
C     *******************************************************
C
      IF(IBUG.EQ.1)CALL TRACEH('11','ROSSIT')
C
      IF(IBUG.EQ.1)WRITE(IPR,1111)AVEBIT,TEMPMN
 1111 FORMAT(1H ,'AVEBIT,TEMPMN = ',2F20.10)
      IF(AVEBIT.LT.TEMPMN)GOTO1200
      TSOLD=T*3600.0
      T=T+DELT
      TSNEW=T*3600.0
      IF(IBUG.EQ.1)WRITE(IPR,1112)TSNEW,TSECMX
```

```
 1112 FORMAT(1H ,'TSNEW,TSECMX = ',2F20.10)
      IF(TSNEW.GT.TSECMX)GOTO1200
C
      TSOLD2=PFACT*TSOLD
      TSNEW2=PFACT*TSNEW
      ITSOL2=TSOLD2
      ITSNE2=TSNEW2
      IF(IBUG.EQ.1)WRITE(IPR,1113)ITSNE2,ITSOL2
 1113 FORMAT(1H ,'ITSNE2,ITSOL2 = ',2I8)
      IF(ITSNE2.EQ.ITSOL2)GOTO400
C
      L=L+1
      L2=L2+1
      TEMP(L)=AVEBIT
      TIME(L)=TSOLD
      GOTO400
C
C     ****************************************
C     ****************************************
C     **  STEP 12--                         **
C     **  WRITE DATA TO FILE                **
C     **  PRINT AND PLOT                    **
C     **  THE TEMPERATURE VERSUS TIME TRACE.  **
C     ****************************************
C     ****************************************
C
C
 1200 CONTINUE
      IF(IBUG.EQ.1)CALL TRACEM('12','ROSSIT')
C
      WRITE(IPR,999)
      WRITE(IPR,999)
      WRITE(IPR,999)
      WRITE(IPR,999)
      WRITE(IPR,1210)L2
 1210 FORMAT(1H ,'NUMBER OF CUMULATIVE POINTS COMPUTED = ',I8)
      WRITE(IPR,999)
      WRITE(IPR,999)
      WRITE(IPR,999)
      DO1220I=1,L
      WRITE(IPR,1221)I,TEMP(I),TIME(I),ICOUNT
      WRITE(7,1221)I,TEMP(I),TIME(I),ICOUNT
 1221 FORMAT(1H ,I8,2F15.7,I8)
 1220 CONTINUE
C
      CALL PLOTT(TEMP,TIME,L)
      WRITE(IPR,998)
      IMAX=NB
      IF(NUMCOM.GE.3.AND.NF.GT.IMAX)IMAX=NF
      IF(NUMCOM.GE.4.AND.NC.GT.IMAX)IMAX=NC
      IF(100.GT.IMAX)IMAX=100
      DO1230I=1,IMAX
      WRITE(IPR,1231)I,TB(I),TF(I),TC(I),TS(I)
 1231 FORMAT(1H ,'I,TB(I),TF(I),TC(I),TS(I) = ',I8,4F15.7)
 1230 CONTINUE
C
  330 CONTINUE
  320 CONTINUE
```

```
  310 CONTINUE
C
C
C
C     ****************
C     ****************
C     **  STEP 90--  **
C     **  EXIT.      **
C     ****************
C     ****************
C
C
 9000 CONTINUE
      IF(IBUG.EQ.1)CALL TRACEH('90','ROSSIT')
C
      WRITE(7,9001)
 9001 FORMAT('END')
C
      ENDFILE 7
      ENDFILE 7
C
 9002 CONTINUE
      STOP
      END
END PRT
@PRT,S DATA.4
```

B.1 CHECK ON COMPUTER PROGRAM EXECUTION

The model presented in this paper has not been validated through field experimentation. To determine whether the model for predicting the cooling time of hot bitumen was functioning in a reasonable manner, the computer program was altered to simulate a condition in which no heat could flow from the bottom layer (infinitely insulated bottom surface) of the substrate to the surroundings. The computer program could thus approximate heat transfer in an infinite flat plate which is initially at a uniform temperature and then subjected to a hot environment such that heat flows from the surroundings into the flat plate. It is noted that the four-component model roofing system is equivalent to a flat plate if the physical properties (i.e., temperature, thermal conductivity, and density) of the four components are taken to be identical. Transient heat flow and temperature distribution in a flat plate have been calculated and the results have been given in charts. Kreith has presented these charts and examples of their use [B1]. The treatment here is based on the charts given by Kreith.

The following calculation was performed to compare the output of the computer program with the charts given by Kreith. A 0.025 ft (7.6 mm) thick plate containing 100 finite elements and having an initial temperature of 100°F (38°C) was subjected to elevated temperatures at the surface. Heat was allowed to flow into the plate for 100 seconds. The temperatures of the top, twentieth, fourtieth, sixtieth, eightieth, and bottom finite elements within the plate were calculated from the computer program. The thermal conductivity and diffusivity of the plate were taken as 0.54 Btu•in/h•ft^2•°F (0.078 W/m•K) and 0.0187 ft^2/h (0.48 mm^2/s), respectively. The convection coefficient was taken as 5.4 Btu/h•ft^2•°F (30.6 W/m^2•K). Ambient temperatures to which the surface of the plate was subjected were 200, 300, and 400°F (93, 149, and 204°C). The results from the computer program output and those obtained from the temperature charts are given in table B1.

A comparison of the temperature distribution results from the computer program output and the temperature charts shows differences between 2 and 6 percent, depending upon the temperature to which the plate was subjected and the position of the finite element within the slab. The percent differences were considered acceptable, since it was difficult to interpolate the temperature charts given by Kreith because of their small size. Interpolation was considered a major source of error. The results indicated that the computer program for determining the temperature distribution in the flat plate was functioning as expected. Moreover, since only a minor change was made in converting the bitumen cooling program into the flat plate program, the bitumen cooling program was also considered to be functioning in a reasonable manner.

B.1.1 Reference

B1. Kreith, Frank, Principles of Heat Transfer, International Textbook Company, Scranton, Pennsylvania, pp. 148-152, (July 1968).

Table B1. Temperature Distribution in the Finite Elements of the Flat Plate as Determined from the Computer Program and the Temperature Charts.

Temperature to Which the Plate Was Subjected	Finite Element	Finite Element Temperature			
		Program Output		Charts[a]	
		°F	°C	°F	°C
200°F (93°C)	top	114	46	120	49
	20th	112	44	118	48
	40th	111	44	116	47
	60th	110	43	115	46
	80th	109	43	115	46
	bottom	109	43	114	45
300°F (149°C)	top	132	56	140	60
	20th	128	53	136	58
	40th	125	52	132	56
	60th	122	50	130	54
	80th	121	49	129	54
	bottom	120	49	128	53
400°F (204°C)	top	157	69	160	71
	20th	150	66	154	68
	40th	144	62	148	64
	60th	139	59	145	63
	80th	137	58	144	62
	bottom	136	58	142	61

(a) from Kreith [B1].

APPENDIX C. NOTATION

This appendix lists the notation used in this report in the development of both the mathematical model and the computerized program for the prediction of the cooling time of bitumen during the construction of built-up roofing. Table C1 includes the algebraic notation, Fortran code, the description or definition, and the customary and S.I. units of measurement for each term used.

Table C1. Notation and Description of Terms

Algebraic Notation	Fortran Code	Description	Units Customary	Units SI
α	** (1)	thermal diffusivity; $\alpha = k/C_p \cdot \rho$	ft^2/h	mm^2/s
* (2)	DB	thermal diffusivity of applied bitumen	ft^2/h	mm^2/s
*	DC	thermal diffusivity of bitumen-cement	ft^2/h	mm^2/s
*	DF	thermal diffusivity of roofing felt	ft^2/h	mm^2/s
*	DS	thermal diffusivity of substrate	ft^2/h	mm^2/s
C_p	**	specific heat	Btu•lbm/°F	J/kg•K
C_{pb}	**	specific heat of applied bitumen	Btu•lbm/°F	J/kg•K
C_{pc}	**	specific heat of bitumen-cement	Btu•lbm/°F	J/kg•K
C_{pf}	**	specific heat of roofing felt	Btu•lbm/°F	J/kg•K
C_{ps}	**	specific heat of substrate	Btu•lbm/°F	J/kg•K
ϵ	**	surface emittance	- (3)	- (3)
*	EMB	surface emittance of bitumen	-	-
*	EMS	surface emittance of substrate	-	-
h	**	surface emittance of heat transmission	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
h_c	**	surface convection coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
h_r	**	surface radiation coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
h_b	HB	bitumen surface coefficient of heat transmission	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
*	HBC	bitumen surface convection coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
*	HBR	bitumen surface radiation coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$

(1) The double asterisk indicates that the term was not used in the paper in Fortran code.
(2) The asterisk indicates that the term was not used in algebraic notation.
(3) The dash indicates that the term is dimensionless.

Table C1. (Continued)

Algebraic Notation	Fortran Code	Description	Units	
			Customary	S.I.
h_s	HS	substrate surface coefficient of heat transmission	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
*	HSC	substrate surface convection coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
*	HSR	substrate surface radiation coefficient	$Btu/h \cdot ft^2 \cdot °F$	$W/m^2 \cdot K$
k	**	thermal conductivity	$Btu \cdot ft/h \cdot ft^2 \cdot °F$	$W/m \cdot K$
k_b	CB	thermal conductivity of applied bitumen	$Btu \cdot ft/h \cdot ft^2 \cdot °F$	$W/m \cdot K$
k_c	CC	thermal conductivity of bitumen-cement	$Btu \cdot ft/h \cdot ft^2 \cdot °F$	$W/m \cdot K$
k_f	CF	thermal conductivity of roofing felt	$Btu \cdot ft/h \cdot ft^2 \cdot °F$	$W/m \cdot K$
k_s	CS	thermal conductivity of substrate	$Btu \cdot ft/h \cdot ft^2 \cdot °F$	$W/m \cdot K$
M	**	modulus relating Δy and Δt	-	-
M_b	AMB	modulus for bitumen	-	-
M_c	AMC	modulus for bitumen-cement	-	-
M_f	AMF	modulus for roofing felt	-	-
M_s	AMS	modulus for substrate	-	-
N	**	Nusselt or Biot number; $N = h \cdot \Delta y/k$	-	-
N_b	ABB	Nusselt or Biot number for bitumen	-	-
N_s	ABS	Nusselt or Biot number for substrate	-	-
*	NB	number of finite-elements in the bitumen	-	-
*	NC	number of finite-elements in the bitumen-cement	-	-
*	NF	number of finite-elements in the felt	-	-
*	NS	number of finite-elements in the substrate	-	-
ρ	**	density	lbm/ft^3	kg/m^3
ρ_b	**	density of bitumen	lbm/ft^3	kg/m^3
ρ_c	**	density of bitumen-cement	lbm/ft^3	kg/m^3
ρ_f	**	density of roofing felt	lbm/ft^3	kg/m^3
ρ_s	**	density of substrate	lbm/ft^3	kg/m^3
σ	SIGMA	Stefan-Boltzman constant;	$Btu/h \cdot ft^2 \cdot °R^4$	$W/m^2 \cdot K^4$
T	**	temperature	°F	°C
T_a	TA	air temperature	°F	°C

Table C1. (Continued)

Algebraic Notation	Fortran Code	Description	Units	
			Customary	SI
T_s	**	temperature of a surface	°F	°C
T_{b1}	TB(1)	temperature of the top bitumen surface element	°F	°C
T'_{b1}	TB(1)	future temperature of the top bitumen surface element	°F	°C
T_{bi}	TB(I)	temperature of a bitumen element	°F	°C
T'_{bi}	TB(I)	future temperature of a bitumen element	°F	°C
T_{bn}	TB(NB)	temperature of the bottom bitumen element	°F	°C
T'_{bn}	TB(NB)	future temperature of the bottom bitumen element	°F	°C
T_{c1}	TC(1)	temperature of the top bitumen-cement element	°F	°C
T'_{c1}	TC(1)	future temperature of the top bitumen-cement element	°F	°C
T_{ci}	TC(I)	temperature of a bitumen-cement element	°F	°C
T'_{ci}	TC(I)	future temperature of a bitumen-cement element	°F	°C
T_{cn}	TC(NC)	temperature of the bottom bitumen-cement element	°F	°C
T'_{cn}	TC(NC)	future temperature of the bottom bitumen-cement element	°F	°C
T_{f1}	TF(1)	temperature of the top felt element	°F	°C
T'_{f1}	TF(1)	future temperature of the top felt element	°F	°C
T'_{fi}	TF(I)	temperature of a felt element	°F	°C
T'_{fi}	TF(I)	future temperature of a felt element	°F	°C
T_{fn}	TF(NF)	temperature of the bottom felt element	°F	°C
T'_{fn}	TF(NF)	future temperature of the bottom felt element	°F	°C
T_{s1}	TS(1)	temperature of the top substrate element	°F	°C
T'_{s1}	TS(1)	future temperature of the top substrate element	°F	°C
T_{si}	TS(I)	temperature of a substrate element	°F	°C
T'_{si}	TS(I)	future temperature of a substrate element	°F	°C
T_{sn}	TS(NS)	temperature of the bottom substrate element	°F	°C
T'_{sn}	TS(NS)	future temperature of the bottom substrate element	°F	°C
*	TBO	initial temperature of bitumen	°F	°C
*	TCO	initial temperature of bitumen-cement	°F	°C

Table Cl. (Continued)

Algebraic Notation	Fortran Code	Description	Units	
			Customary	SI
*	TFO	initial temperature of roofing felt	°F	°C
*	TSO	initial temperature of substrate	°F	°C
t	**	time	h	s
Δt	DELT	a small finite time increment or interval	h	s
y	**	a space coordinate for the heat flow	ft	m
Δy	DELY	a small finite distance	ft	m

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

Construction of bituminous built-up roofing systems in the United States generally involves the application of hot bitumen to the roofing components, including deck, insulation, and felts, to adhere them to each other and to form a waterproof membrane. Adequate adhesion of the bitumen to the roofing component materials may be obtained only when the hot bitumen is applied at a viscosity sufficient to flow uniformly, to cover the component surfaces or substrate completely, and to provide the proper thickness. During construction, rapid cooling of hot bitumen increases its viscosity significantly. If the viscosity becomes too high, poor adhesion between components, voids within the bitumen, and an excessive and non-uniform thickness of the bitumen may result.

This report describes a mathematical model based on finite-difference equations for calculating transient heat flow to estimate the cooling time of hot roofing bitumen. Estimates of the time required for hot bitumen to cool from its application temperature to 300°F (149°C) were computed as a function of material and environmental factors including: quantity of applied bitumen, bitumen application or contact temperature, air temperature, wind speed, and thermal properties of the bitumen and of the roofing components. The model was used to predict cooling times expected for hot asphalt applied to typical substrates with thermal properties representative of those of polyurethane foam and glass fiber insulation boards, insulating concrete, plywood, concrete and steel decks, and roofing felt on decks or insulations. In addition, the model was used to predict cooling times for hot coal tar pitch applied to concrete and to felt adhered to glass fiber insulation. The results of the calculations demonstrate the widely-varying bitumen cooling times which depend upon the component material to which the bitumen is applied and the environmental conditions during application. Under certain environmental conditions, hot bitumen applied to some substrates cools extremely rapidly. In these cases, sufficient time for proper application may not be available.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Bitumen application temperatures; bituminous roofing; built-up roofing; cooling of roofing bitumens; mathematical model; roofing.

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