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U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

Calculated Operating Temperatures of Thermally Insulated Electric Cables

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c_p - specific heat of air [W·s/g·°C]
D - diameter of outer surface of a cylinder [cm]
F - analytic function
g - acceleration of gravity [cm/s^2]
Gr - Grashof number [dimensionless]
k - thermal conductivity of insulation material [W/cm °C]
k_a - thermal conductivity of insulation A [W/cm °C]
k_b - thermal conductivity of insulation B [W/cm °C]
k_T - effective thermal conductivity [W/cm °C]
L - insulation thickness [cm]
Nu - Nusselt number [dimensionless]
Pr - Prandtl number [dimensionless]
$\dot{q}'$ - rate of heat production per unit length of conductor [W/cm]
$\dot{q}''_c$ - net rate of convective heat transfer per unit surface area [W/cm^2]
$\dot{q}''_r$ - net rate of radiative heat transfer per unit surface area [W/cm^2]
r - radial distance [cm]
$\hat{r}$ - dimensionless radial distance r/r_s
r_j - cable jacket radius [cm]
$\hat{r}_j$ - dimensionless cable jacket radius r_j/r_s
r_s - surface radius [cm]
$\hat{r}_s$ - dimensionless outer surface radius r_s/r_s
t - time [s]
T - temperature [°C or K]
T_j - cable jacket temperature [°C or K]
T_o - ambient temperature [°C or K]
T_s - surface temperature [°C or K]
u - variable substitution for T [°C or K]
w - image point of $\hat{z}$ under transformation given by equation 10
x - dimensional coordinate [cm]
$\hat{x}$ - dimensionless coordinate, x/r_s
y - dimensional coordinate [cm]
$\hat{y}$ - dimensionless coordinate, y/r_s
z - dimensional coordinate x + iy [cm]
$\hat{z}$ - dimensionless coordinate, z/r_s

α	-	parameter equal to $[(1-\hat{\xi})/(1+\hat{\xi})]$ [dimensionless]
β	-	coefficient of thermal expansion for air [1/K]
λ	-	thermal conductivity of air [W/cm °C]
μ	-	viscosity of air [g/cm·s]
ρ	-	density of air [g/cm^3]
ξ	-	dimensional distance of the conductor from center of cable [cm]
$\hat{\xi}$	-	dimensionless distance of the conductor from center of cable, ξ/r_s

CALCULATED OPERATING TEMPERATURES OF THERMALLY INSULATED ELECTRIC CABLES

David D. Evans

Steady-state operating temperatures of current-carrying electric cables buried axially in cylinders of thermal insulation were calculated. Combinations of six types of nonmetallic electric cable, two thermal insulation materials, five thermal insulation thicknesses and a range of currents both greater and less than the typical rated service currents for the cables were studied.

These calculations show that thermally insulated electric cables carrying the rated current may exceed the rated operating temperature limit for common cable jacket materials.

Key words: Electric cable; electrical fire; models; thermal insulation; wiring system.

1. INTRODUCTION

Accompanying the present increased use of thermal insulation have been questions about the potential harmful effects of thermally insulating building electrical wiring systems. Existing electric cables routed through the wall cavity and attic spaces in a building may be encapsulated by thermal insulation.

Electric current limits for cables are established by considering the temperature limits for the electrical insulation material used in construction of the cable. In establishing the operating temperatures of current-carrying electric cables, the heat generated by the current flow in the conductors is considered to be dissipated through the cable jacket directly into air at a temperature of 30°C [1][1]. The established principle for general wiring is that no conductor shall be used under such conditions that its temperature, even when carrying current, will exceed the specified maximum operating temperature for the type of (electrical) insulation involved.

[1] Numbers in brackets refer to the references listed in section 7.

All thermal insulation materials, such as glass fiber, cellulose fiber, mineral fiber and foam plastics when surrounding electric cables inhibit the dissipation of the heat generated by the electric current in the conductors. For thermally insulated cables, the heat that normally would be dissipated from a current-carrying cable directly from the cable surface to the surrounding air, must overcome the added resistance of the thermal insulation before dissipation to the air. This additional resistance causes an increase in the temperature of the cable above that for a cable exposed directly to air under an identical current load. The increase in cable temperature, when thermally insulated and carrying the rated current, should not be so large that the cable operates at temperatures greater than the maximum design operating temperature.

The calculations presented in this report predict the expected steady-state operating temperature of long electric cables wrapped circumferentially with thermal insulation forming a cylinder of insulation. All materials are treated as chemically inert so that the only heat generated is by the current flow through the electrical conductors. The adoption of cylindrical geometry for the insulation layers around the cable facilitates the calculation of cable jacket temperatures. Although a cylinder is not an exact representation of the way insulation may surround electric cables in attics and wall spaces, the calculated cable jacket temperatures demonstrate the sensitivity of cable operating temperatures to encapsulation in thermal insulation. These predicted steady operating temperatures can serve as a basis for discussion of conditions that may exist in practical building installations.

2. MODEL FOR CALCULATION

The current-carrying electric cable is modeled as a cylindrical heat source with uniform temperature. The temperatures of materials interior to the outer cable jacket are assumed to be equal to the cable jacket temperature. For all thermally insulated cases, cylindrical symmetry is maintained by considering the cable encapsulated in a cylinder of insulation. The thermal conductivity of the insulation material is temperature dependent. All cable jacket temperatures are calculated for the steady-state condition, in which heat generated by the current flow within the conductors per unit length is conducted radially through the insulation cylinder, and dissipated from the outer surface by natural convection of air and radiation to the surroundings, both at the ambient temperature of 30°C.

2.1 Representation of Cables

For ease of calculation cylindrical symmetry is essential, but none of the cables of interest were circular in cross section. For all calculations, the non-circular cable cross section was represented by a circular cross section of equal circumference. This approximation for the cable diameter is diagramed for a AWG 4 aluminum 3 CDRS SE cable in figure 1. Table 1 lists the effective diameters for all cables studied.

The numerical model is not intended to calculate the temperatures of components inside the cable jacket. The temperatures for the conductors in the cable are necessary only to determine the heat generated by the current flow. The conductor temperatures were assumed to be equal to the cable jacket temperature. For thermally insulated cables, the temperature differences between the conductors and the cable jacket are generally a small fraction of the temperature rise of the cable jacket above ambient temperature. In tests conducted by Beausoliel et al. [2], the temperature difference between the conductors and the cable jacket was typically only one or two degrees Celsius.

2.2 Heat Generated by Current Flow

With the temperatures of components internal to the cable jacket set equal to the jacket temperature, the temperature of the current-carrying conductors is also equal to the jacket temperature for this calculation. Knowing the temperature of the conductor, its resistance per unit length can be calculated. For a given electric current, the heat generated per unit length of conductor can be calculated as the product of the square of the current and the electrical resistance per unit length (I^2R). In the calculations, this heat generation is modeled as if it were generated uniformly along the axis of the cylinder.

Two conductors in each cable were assumed to be carrying equal current with no current flow in the grounding conductor. It is essential in these calculations to consider the increase in wire electrical resistance with increasing temperature. For copper the electrical resistance per unit length doubles over the range of 20°C to 275°C [3]. Aluminum doubles its electrical resistance over the range of 20°C to 268°C [4]. Table 2 lists the equations used to represent the electrical resistance of various size copper and aluminum conductors as a function of temperature. These equations are based on data collected over the temperature range of 0°C to 200°C for standard annealed copper [3] and 0°C to 100°C for aluminum ECH19 [4]. For the calculations performed, values of electrical resistance outside these ranges were generated by straight line extrapolation.

2.3 Thermal Insulation Layer

To maintain cylindrical symmetry, the solutions for the jacket temperatures of the cables were generated by considering a layer of thermal insulation of a given thickness wrapped circumferentially around the cable. The general case is represented pictorially in figure 2. Heat generated within the cable must be conducted through the insulation layer and dissipated from the outer surface by heat transfer to the surroundings.

The heat conduction calculations take into account the increase in thermal conductivity of fibrous insulation materials with increasing temperature. The thermal characteristics of two different fibrous insulations were used in these calculations. Type A (see figure 3 [5]) is a 11 Kg/m^3 bulk density glass fiber insulation which is representative of residential insulation material. Type B (see figure 3 [6]) is representative of a high density mineral fiber industrial pipe insulation and would not be expected to be used as a building insulation. Type B material was included in this study strictly to demonstrate the sensitivity of predicted cable jacket temperatures to changes in the thermal conductivity of the thermal insulation. The thermal conductivity of material A is greater than that of material B over the temperature range of interest for this study as shown in figure 3. It is expected that the thermal conductivities of most other common building insulation materials not included in this study, such as cellulosic fiber and foam materials, will be at or between the values for the glass fiber and high density mineral fiber insulations A and B respectively. Therefore the range of predicted cable jacket temperatures between those for material type A and B should be representative of the expected variations of results with many different possible building insulation materials.

The thermal conductivity of each insulation, k_a and k_b for Type A and Type B, respectively, can be calculated from the following equations representing the data fits shown in figure 3:

$$k_a = \exp\left[-7.9440 + 0.005194\ T\ (°C)\right]\ W/cm°C \quad [5] \qquad (1)$$

$$k_b = \exp\left[-8.094 + 0.003834\ T\ (°C)\right]\ W/cm°C \quad [6] \qquad (2)$$

Thermal conductivity data outside the range of experimental values were calculated by extrapolation of the data fits using equations 1 and 2.

2.4 Surface Heat Dissipation

Heat generated by the flow of electric current in the conductors is transferred through the insulation materials and dissipated to the surrounding air at the outer surface. Heat is dissipated at the outer surface by two mechanisms, convection to the surrounding air at 30°C and radiation to the surrounding medium at 30°C.

Radiative heat transfer was calculated from the familiar Stefan-Boltzmann law. The net rate of radiative heat transfer per unit area of the cylinder surface with unit emissivity, $\dot{q}_r''$, at absolute temperature T_s to the surroundings at absolute temperature T_o is:

$$\dot{q}_r'' = 5.672 \times 10^{-12} \, (T_s^4 - T_o^4) \ \mathrm{W/cm^2} \tag{3}$$

where the units of both temperatures are kelvin.

Convective heat transfer to quiescent air at one atmosphere pressure and 30°C was calculated from an empirical equation obtained by Bosworth [7] representing experimental data for natural convection from cylinders over a wide range of variable Gr · Pr. In conventional notation this relationship is:

$$(Nu)^{1/2} = 0.62 + 0.35 \, (Gr \cdot Pr)^{1/6} \tag{4}$$

Using this relationship, the rate of convective heat transfer per unit area of the cylinder surface $\dot{q}_c''$ at temperature T_s to the air at T_o is:

$$\dot{q}_c'' = \left(\frac{\lambda}{D}\right) \cdot \left[0.62 + 0.35 \left(\frac{D^3 \rho^2 g \beta (T_s - T_o)}{\mu^2} \cdot \frac{\mu c_p}{\lambda}\right)^{1/6}\right]^2 \cdot \left(T_s - T_o\right) \tag{5}$$

All air properties in this expression are evaluated at the film temperature, the average of surface and ambient temperatures. Values for λ and $(g\beta\rho^2/\mu^2)$ as a function of film temperature were obtained by interpolation of tabular values [8] given in tables 3 and 4. The value of $Pr = \mu c_p/\lambda$ was taken as 0.72 for all temperatures.

3. METHOD FOR CALCULATION AND DISCUSSION

The calculated cable jacket temperatures as a function of electric current flow in the conductors, and thermal insulation type and thickness were generated by iteration. An assumed jacket temperature was used to calculate the outer surface temperature at the insulation-air interface. The heat transfer rate from the outer thermal insulation surface was compared to the heat generated by the conductors for the given electric current flow and resistance. The assumed jacket temperature was adjusted until the heat generated by the conductors equaled the heat loss from the outer surface. This value of the jacket temperature is the steady-state value for the conditions of interest. Steady-state jacket temperatures as a function of current loads for six common electric cables, under various thermal insulation conditions, were determined in this study.

A basic part of the above iteration method is the calculation of the outer thermal insulation surface temperature for a given rate of heat production and jacket temperature of the cable. The method for performing this calculation is discussed below.

3.1 Heat Flow Between Concentric Cylinders

For the calculations performed, the electric cable jacket and surrounding insulation are modeled as two concentric cylinders. Within the cable jacket there are two conductors that together produce a known rate of heat per unit length of cable, $2\,\dot{q}'$, when there is an electric current flow through the conductors. The heat flow between the cylindrical outer surface of the cable jacket and the cylindrical outer surface of the thermal insulation is assumed to be entirely axisymmetric. The cable is also assumed to be long so that conduction is radial. If the temperature of either the cable jacket or the outer surface is known, the other may be calculated.

The temperature distribution between the two concentric cylinders, representing the outer surfaces of the cable jacket and insulation layer under steady-state conditions, can be obtained by observing that the heat flow through any concentric cylindrical surface within the thermal insulation is constant.

This can be represented by the equation:

$$-\,2\pi kr\,\frac{dT}{dr} = \text{constant} = 2\,\dot{q}' \qquad (6)$$

The solution to equation 6 for the temperature distribution between the bounding cylindrical surfaces for a given cable jacket temperature and constant thermal conductivity is:

$$T = \frac{\dot{q}'}{\pi k}\,\ln\left(r_j / r\right) + T_j \qquad (7)$$

Similarly, for a given outer thermal insulation surface temperature the temperature distribution is:

$$T = \frac{\dot{q}'}{\pi k} \quad \ell n \left(r_s / r\right) + T_s \tag{8}$$

Temperature dependent thermal conductivity is discussed in section 3.3.

The above solutions assume that the electric cable may be represented as a circular cylinder in order to simplify the calculations, but actual cables are not circular in cross section. This approximation will introduce distortions in the temperature distribution within the thermal insulation. The greatest distortions will be in the immediate area of the cable jacket. The amount of variation in predicted temperature between the axisymmetic solution given by equation 8 and a more refined model of the thermally insulated electric cable will be examined in the next section.

3.2 Heat Flow in a Cylinder Containing One or Two Heat Sources

In the previous section, a solution for temperature distribution within the cylindrical layer of thermal insulation surrounding an electric cable was presented. This solution was based on representing the cable as a single uniform axisymmetric heat source. In an actual electric cable the centers of the two current-carrying conductors are separated by a distance of approximately one-half the diameter of the cable. This separation of the heat sources will produce a temperature distribution within the cylinder of thermal insulation that is not axisymmetric. Deviations from the axisymmetric temperature distribution will be greatest near the heat sources.

As an illustrative example of the possible differences between the solution for the single axisymmetric heat source and the solution for two heat sources representing the separate conductors within the cable a specific case of a AWG-14 two conductor cable was studied.

Specific parameters for the calculation were: a 30 ampere current load, a heat generation rate of 0.1265 W/cm per conductor, a 5.91 cm diameter for the outer surface of the insulation cylinder, a constant thermal conductivity for the thermal insulation of 0.00036 W/cm °C, and a fixed insulation outer surface temperature of 43.9°C.

Using equation 8 the axisymmetic temperature distribution within the insulation can be calculated with T_s = 43.9°C, r_s = 5.91/2 = 2.955 cm and $\dot{q}'$ = 0.1265 W/cm and k = 0.00036 W/cm °C. The resulting expression for the temperature distribution is:

$$T = 43.9 + \frac{0.1265}{\pi\ 0.00036}\ \ell n \left(\frac{2.955}{r}\right)\ [C°] \tag{9}$$

where r is measured in centimeters.

Figure 4 shows the constant temperature lines for the axisymmetic heat source calculated from equation 9 as the broken lines. Only one quarter of the insulation cylinder cross section is shown. The size of the cylindrical model cable for the AWG-14 cable, 0.91 cm diameter (see table 1) is also shown. Based on the constant temperature lines shown in figure 4 the predicted temperature for the cylindrical model AWG-14 cable jacket is slightly greater than 250°C.

The solution for the temperature distribution produced by two heat sources representing the two conductors within the cable was obtained by conformal transformations [9] and by the method of images. Figure 5 shows a dimensional sketch of two line heat sources representing the separation of conductors within a AWG-14 cable enclosed by a cylinder of insulation in the dimensional z-plane.

Figure 6 shows the same configuration in the dimensionless $\hat{z}$-plane, in which all dimensions are normalized by dividing them by the radius of the outer surface of the insulation cylinder, r_s = 2.955 cm.

Figure 7 shows the mapping of the cylindrical cross section to the w-plane by the transformation

$$w = \frac{i\ (1 - \hat{z})}{\hat{z} + 1} \tag{10}$$

The labels in figure 6 help to locate the image points in figure 7. The result of the transformation is to map the area inside the cylinder onto the upper half plane. The circumference of the cylinder becomes the real axis in the w-plane. To satisfy the boundary conditions for this problem, the real axis in the w-plane must be a constant potential line. By introducing two heat sources of equal strength along the imaginary axis above the real axis and as counterparts two identical heat sinks along the imaginary axis below the real axis, the two heat sources within the cylinder representing the cable conductors may be represented as two corresponding points on the w-plane above the real axis, see figure 7. The two points representing sinks below the real axis cause the real axis to become a constant potential line. The complex potential for heat flow between the sources and sinks is represented by the analytical function, F(w) as:

$$F(w) = \frac{\dot{q}'}{2\pi k}\Big[[\ln(w - i\alpha) - \ln(w + i\alpha)] + [\ln(w - i/\alpha) - \ln(w + i/\alpha)]\Big] \tag{11}$$

where a = $(1 - \hat{\xi})/(1 + \hat{\xi})$ in which $\hat{\xi}$ is dimensionless distance of the conductor from the center of the cable. The real part of F(w) is zero everywhere along the real axis.

Substituting for w in terms of $\hat{z}$, yields the form of the solution in the $\hat{z}$-plane.

$$F(\hat{z}) = \frac{\dot{q}'}{2\pi k}\left[\ln\left(\frac{i(1+\alpha)\hat{z} - i(1-\alpha)}{i(1-\alpha)\hat{z} - i(1+\alpha)}\right) + \ln\left(\frac{i(1+\frac{1}{\alpha})\hat{z} - i(1-\frac{1}{\alpha})}{i(1-\frac{1}{\alpha})\hat{z} - i(1+\frac{1}{\alpha})}\right)\right] \tag{12}$$

Substituting for $\hat{z} = \hat{x} + i\hat{y}$ and extracting the negative of the real part yields the temperature distribution in the $\hat{z}$-plane as a function of $T-T_s$:

$$-\mathrm{Re}(F(\hat{z})) = T-T_s = \frac{\dot{q}'}{2\pi k}\left[\ln\left(\sqrt{\frac{[(1-\alpha)\hat{y}]^2 + [(1+\alpha) - (1-\alpha)\hat{x}]^2}{[(1+\alpha)\hat{y}]^2 + [(1-\alpha) - (1+\alpha)\hat{x}]^2}}\right) + \right.$$
$$\left.\ln\left(\sqrt{\frac{[(1-\frac{1}{\alpha})\hat{y}]^2 + [(1+\frac{1}{\alpha}) - (1-\frac{1}{\alpha})\hat{x}]^2}{[(1+\frac{1}{\alpha})\hat{y}]^2 + [(1-\frac{1}{\alpha}) - (1+\frac{1}{\alpha})\hat{x}]^2}}\right)\right] \tag{13}$$

Equation 13 was used to generate the temperature distribution from two heat sources representing the separate conductors within the electric cable. This distribution is shown for comparison with the temperature distribution produced by the single heat source as the solid lines in figure 4. Only one quarter of the cylindrical cross section is shown. For scale, an outline representing the cable jacket of the 2 conductor AWG-14 cable is shown. It can be seen from figure 4 that the temperature distribution calculated from the single heat source and cylindrical cable approximation fall between the minimum and maximum temperatures of the distribution predicted from the two heat source solution. For this study temperatures at the cable jacket (250°C contour) are of interest. Even though deviations of the cylindrical approximation from the better two heat source solution are greatest there, these deviations are tolerable in this study.

All of the solutions for cable jacket temperatures generated in this report are calculated on the basis of the cylindrical cable approximation and a single axial heat source. Even though a solution for two heat sources was available, it is cumbersome and, as just demonstrated, does not provide much more information than the single source solution. Predicted cable jacket temperatures, using the above approximations, should be regarded as an average value for the circumference of the cable.

A special case of equation 13 results if both line sources are superimposed at the origin by setting $\hat{\xi} = 0$ thus making $\alpha = 1$. The resulting temperature distribution in normalized coordinates is

$$T-T_s = \frac{\dot{q}'}{\pi k} \ \ell n \ 1 \Big/ \sqrt{\hat{x}^2 + \hat{y}^2} \tag{14}$$

Noting that a radial distance from the origin satisfies $\hat{r}^2 = \hat{x}^2 + \hat{y}^2$, equation 14 becomes in dimensional coordinates:

$$T-T_s = \frac{\dot{q}'}{\pi k} \ \ell n \ r_s \big/ r \tag{15}$$

Equation 15 agrees with the result obtained by other means, equation 8.

The technique used to calculate the temperature distribution produced by two line sources representing a single cable, may be extended by adding the additional source-sink pairs to calculate the temperature distribution produced by several cables.

3.3 Heat Flow in a Cylinder With Temperature Dependent Thermal Conductivity

As shown in figure 3, the thermal conductivity of fibrous insulation materials is a strong function of the temperature of the material. For the two insulation materials, A and B, the thermal conductivity increases exponentially with increasing material temperature. This exponential temperature dependence for the thermal conductivity can be incorporated into a solution for the temperature distribution from one axial heat source within a cylinder of insulation. This modification of the calculation for the single heat source detailed in section 3.2 was used in the calculation of cable jacket temperature presented in this report.

The case of the constant thermal conductivity was useful in demonstrating the differences between temperature distribution produced by the single and dual heat sources. The increase in thermal insulation conductivity with increasing temperature is significant enough over the temperature range of interest in this study, that the overall thermal resistance of the insulation layer surrounding the cable is affected. Better predictions of the operating temperatures of thermally insulated electric cables are obtained by accounting for thermal conductivity variations.

To incorporate non-constant thermal conductivity in the solution for the temperature distribution between cylinders, a different method of solution from that given in section 3.2 for constant thermal conductivity must be used. For steady-state conduction between cylinders, the rate of heat conduction through cylindrical shells at all radii must be equal. Using Fourier's law of heat conduction applied at any radius between the cylinders representing an electric cable (2 conductors) and the outer surface of the thermal insulation, the rate of heat conduction per unit length is:

$$2\dot{q}' = -2\pi r k \frac{dT}{dr} \qquad (16)$$

Equation 16 is identical to equation 6, except that the integration of the equation is complicated by the fact that thermal conductivity, k, is not assumed constant and is a function of the local temperature, T.

Equation 16 may be integrated by introducing a new variable u for T and a constant thermal conductivity k_T. The new variable, u is defined such that,

$$k_T \left(\frac{du}{dr}\right) = k\left(\frac{dT}{dr}\right) \qquad (17)$$

with the boundary conditions,

$$u = T_j \text{ at } r = r_j \qquad (18)$$

$$u = T_s \text{ at } r = r_s \qquad (19)$$

Integrating equation 17 using boundary conditions given in equations 18 and 19, the effective value of constant thermal conductivity k_T can be determined as:

$$k_T = \frac{1}{T_s - T_j} \int_{T_j}^{T_s} k dT \tag{20}$$

Thus k_T is the average thermal conductivity between the temperature extremes at the boundaries of the insulation layer. It is interesting to note that by introducing the temperature dependent thermal conductivity both boundary conditions at r_j and r_s must be used in the integrate process for equation 16. In integrating equation 6, only one was needed.

Substituting u for T and k_T for k, equation 16 may be integrated using either boundary condition equation 18 or 19. Using equation 18, the temperature distribution within the insulation layer is:

$$T = \frac{\dot{q}'}{2\pi k_T} \ln (r_j / r) + T_j \tag{21}$$

Equation 21 is identical to equation 7 except for the effective substitution of k_T for k.

The thermal conductivity of both the glass fiber insulation material A and mineral fiber material B may be expressed in the form k = exp (a + bT) as given in equations 1 and 2. Using equation 20 the effective or average thermal conductivity is

$$k_T = \frac{1}{(T_s - T_j) b} \left[\exp (a + bT_s) - \exp (a + bT_j) \right] \tag{22}$$

In this study, a surface temperature at the outer surface of the insulation layer, T_s, was found by iteration using equations 21 and 22 for each assumed value of the cable jacket temperature, T_j.

4. EXPLANATION OF CURVES FOR CABLE JACKET TEMPERATURES

The results of the calculations performed are presented in figures 8 through 19. In each of these figures, predicted cable jacket temperatures are plotted as a function of the current for a specific cable and insulation material. Results are shown for a range of currents, above and below the rated current. The typical rated service current for each cable is recorded in each figure. For ease of comparison, predicted temperatures for each cable buried in insulation B are immediately following those for insulation A. Predictions are presented for 14, 12, 10 AWG copper and 8, 6, 4 AWG aluminum cables. An ambient temperature of 30°C is assumed for all calculations.

To demonstrate the effect of thermal insulation on the cable jacket temperature, predicted values for the cable alone are given in each plot labeled, "no insulation". In each figure 8 through 19, curves for cable jacket temperature are shown for 2.5 cm, 5 cm, 10 cm, and 20 cm thick layers of insulation surrounding the cable as diagramed in figure 2.

5. DISCUSSION OF RESULTS

Comparing the results for insulations A and B for like conditions, the cable jacket temperatures for the cable buried in insulation B are in all cases substantially greater than for the cable in insulation A. The cable jacket temperature is quite sensitive to the thermal conductivity of the insulation material.

All cables can exceed the common temperature limits for the jacket materials when the cable is sufficiently insulated and carrying the typical rated current. In all cases, these calculations show that a 60°C temperature limit common for cable jacket materials is exceeded at typical rated current for all cables studied, when wrapped with a 5 cm thick layer of either insulation A or B. For many cases this temperature limit was exceeded with only a 2.5 cm thick layer. As wiring systems in buildings may be buried in insulation layers that are substantially thicker and provide more resistance to heat dissipation, cable jacket temperatures may routinely exceed recommended limits in normal service.

Cables that were loaded beyond the recommended current limits can become hot enough to ignite combustibles. Ignition of cellulosic insulation by buried electric cables has been reported by Gross [10] for a 14 gauge cable carrying 28.5 amperes, which was 190 percent of the typical rated 15 ampere current load.

It should be reemphasized that the predictions shown in figures 8 through 19 are for steady-state conditions and a 30°C ambient temperature. Normally, a few hours would be required for the cable and insulation material initially at ambient conditions to reach steady-state temperatures, but more than 60 percent of the final temperature rise can be expected to occur within the first hour. Ambient temperatures above 30°C will elevate the cable temperature above the predicted values. At low final cable temperatures, the change in predicted cable jacket temperature will be almost identical to the change in ambient temperature. At high final cable temperatures, the change in cable temperatures will be less than the change in ambient temperature.

6. CONCLUSIONS

Predictions of the steady-state operating temperatures of thermally insulated current-carrying electric cables have been presented for the special case of a cable buried axially in a cylinder of thermal insulation. These predictions show that electric cables carrying the typical rated current can exceed the common 60°C operating temperature limit for common jacket materials with a 5 cm thick layer and often just a 2.5 cm thick layer of thermal insulation wrapped circumferentially around the cable. This suggests that the jacket materials of electric cables that are frequently encapsulated in layers of building insulation could be exceeding recommended upper temperature limits for the materials, if the typical rated current load is maintained long enough to establish nearly steady conditions. For cables operated for an extended period of time above typical rated current, sufficiently high temperatures may be reached to induce ignition of combustible insulation materials.

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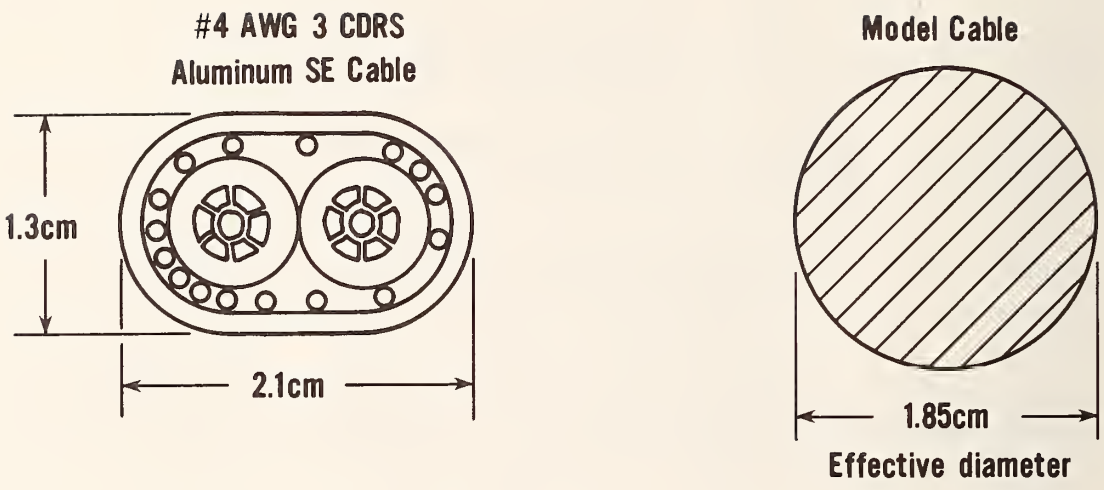

Figure 1. Comparison of actual and model cable

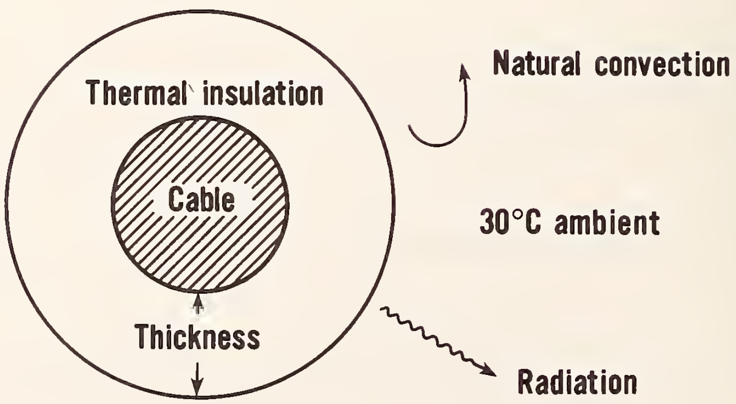

Figure 2. Geometry of the insulated cable

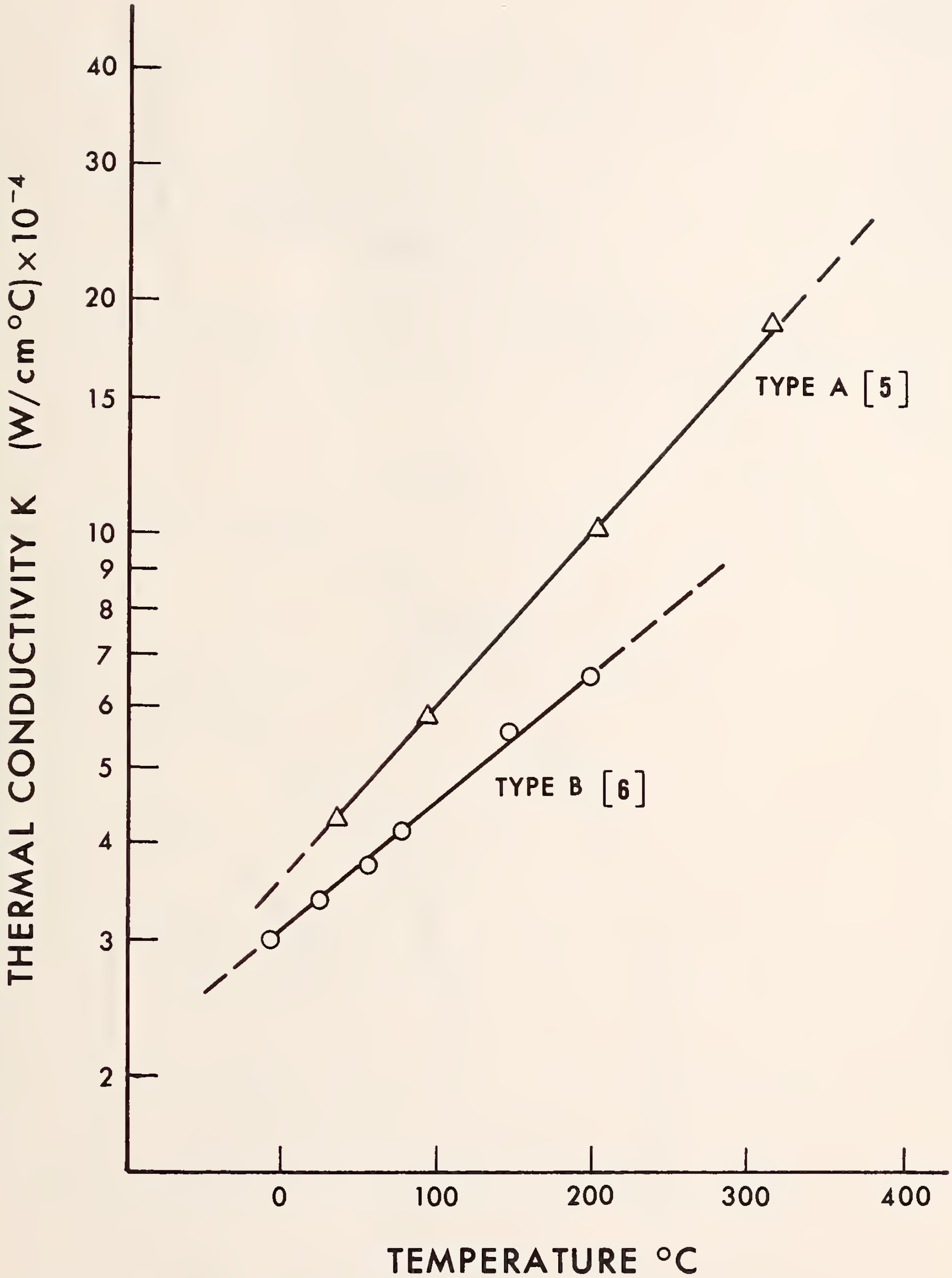

Figure 3. Thermal conductivity for fibrous insulation

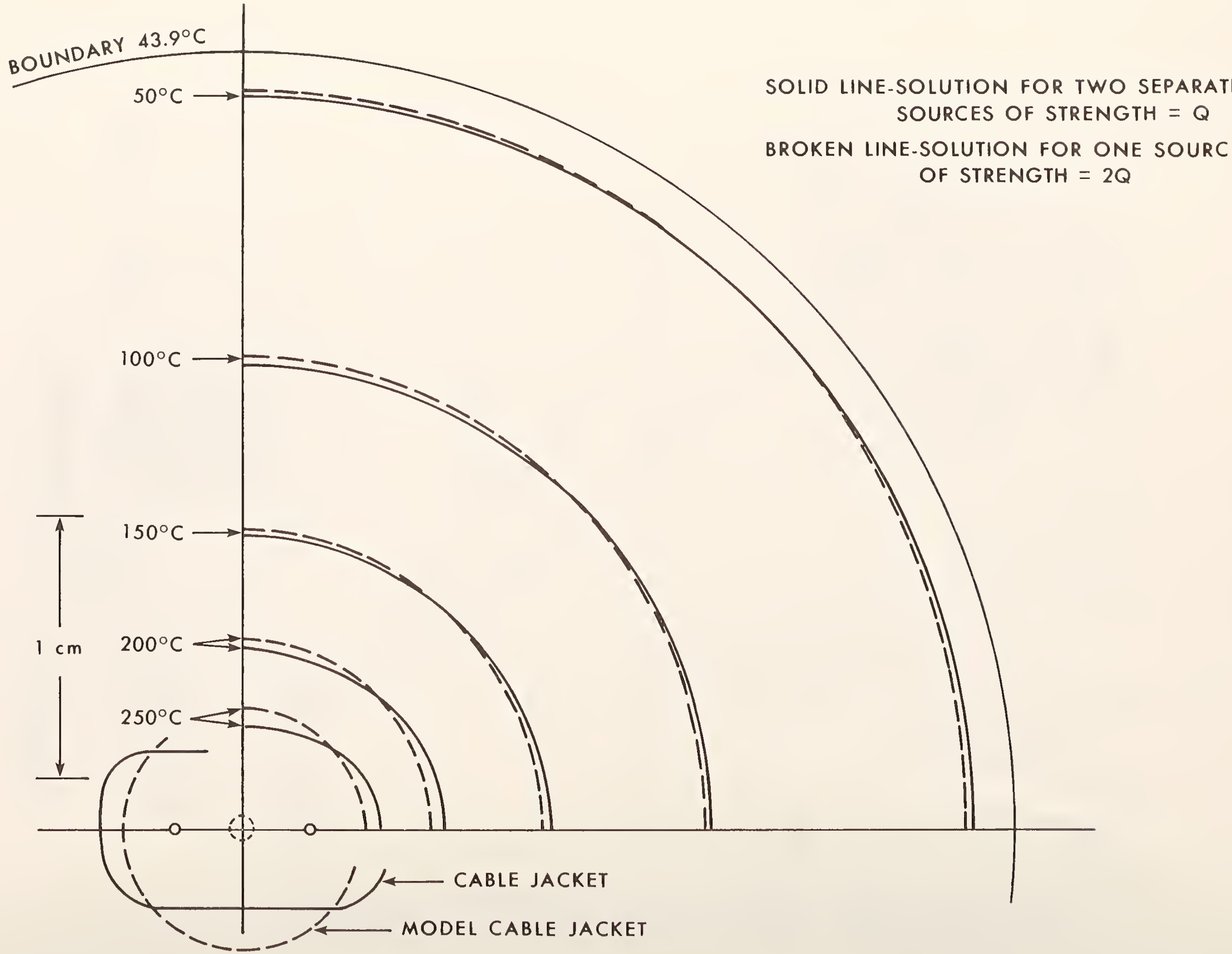

Figure 4. Temperature distribution within the insulation layer for a thermally insulated 14 gauge NM cable

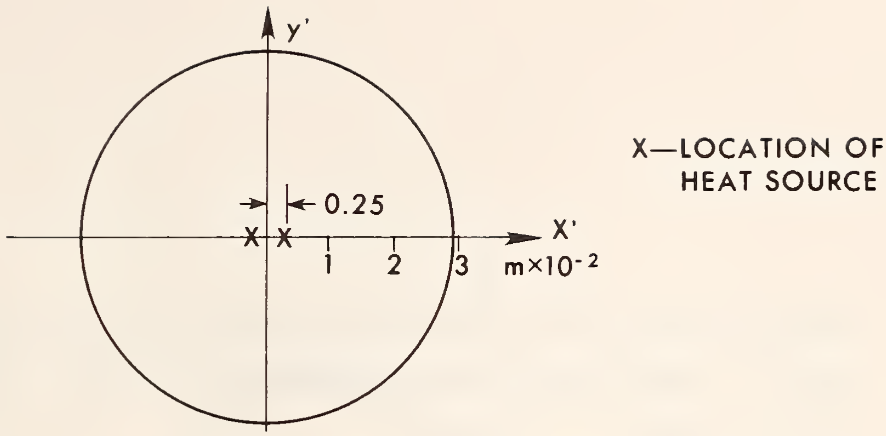

Figure 5. Dimensional sketch of the insulation cylinder with two heat sources representing the conductors in a 14 gauge cable

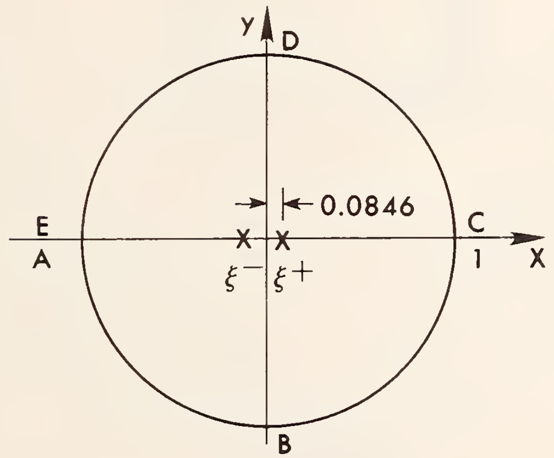

Figure 6. Normalized dimensional sketch of the insulation cylinder with two heat sources

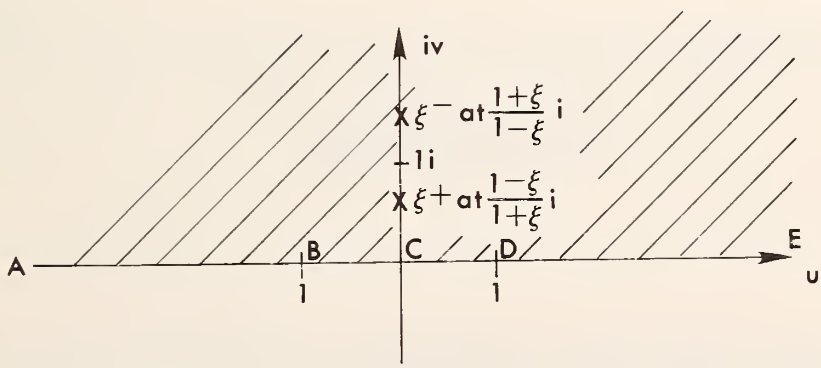

Figure 7. Transformation of cylinder onto the upper w-plane

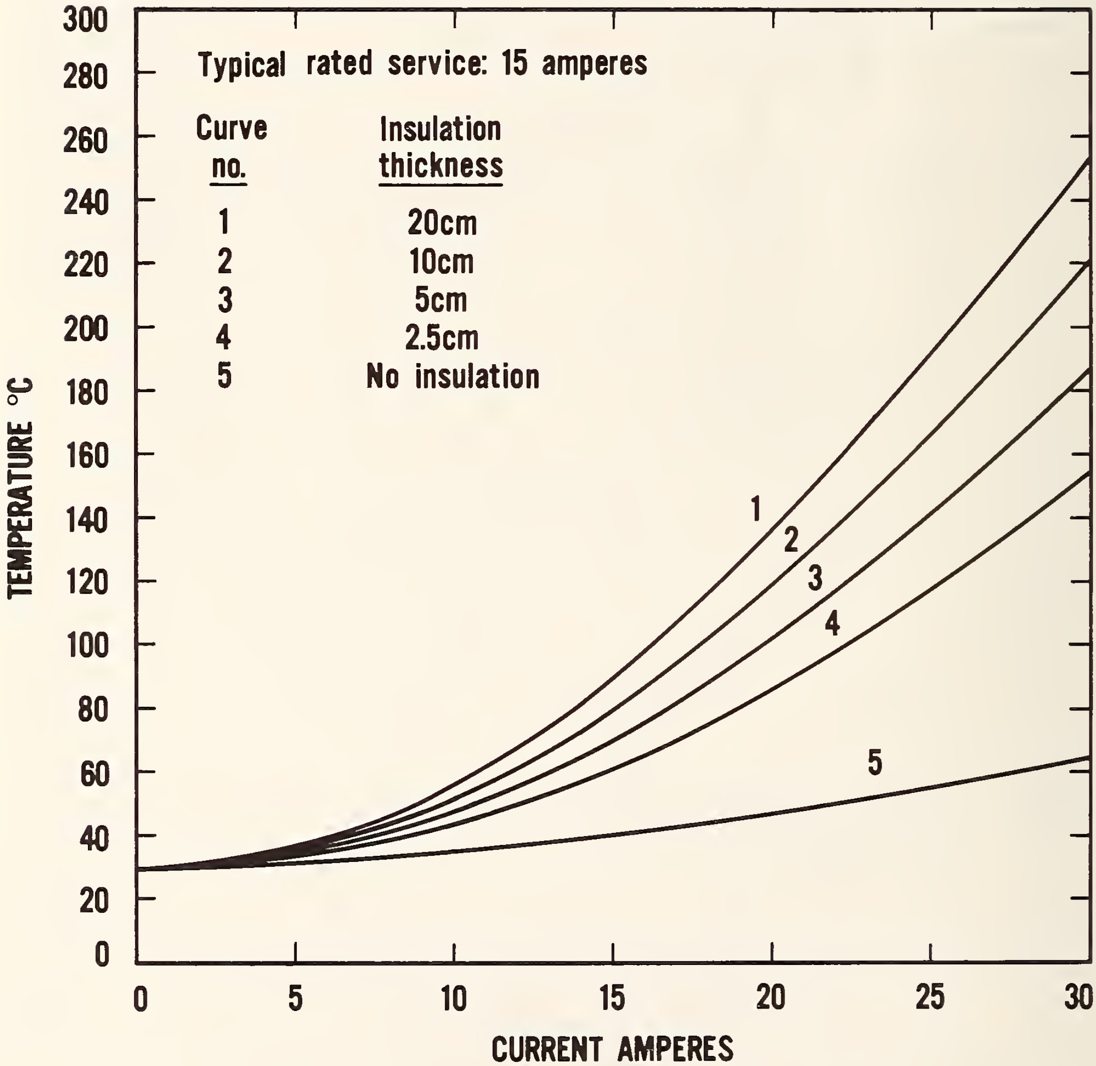

Figure 8. Steady-state jacket temperature for AWG #14 copper 2-G NM cable surrounded by insulation A

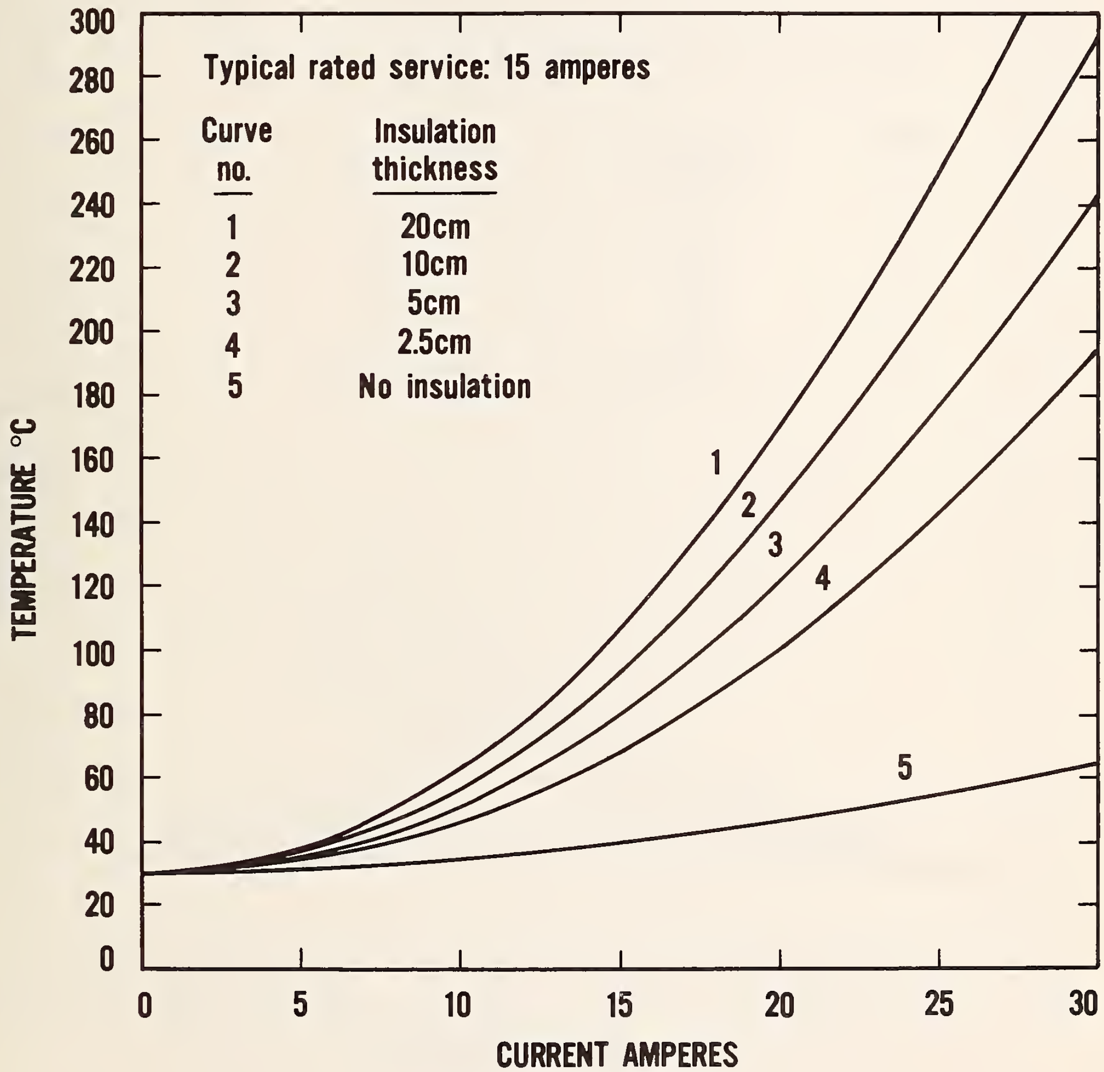

Figure 9. Steady-state jacket temperature for AWG #14 copper 2-G NM cable surrounded by insulation B

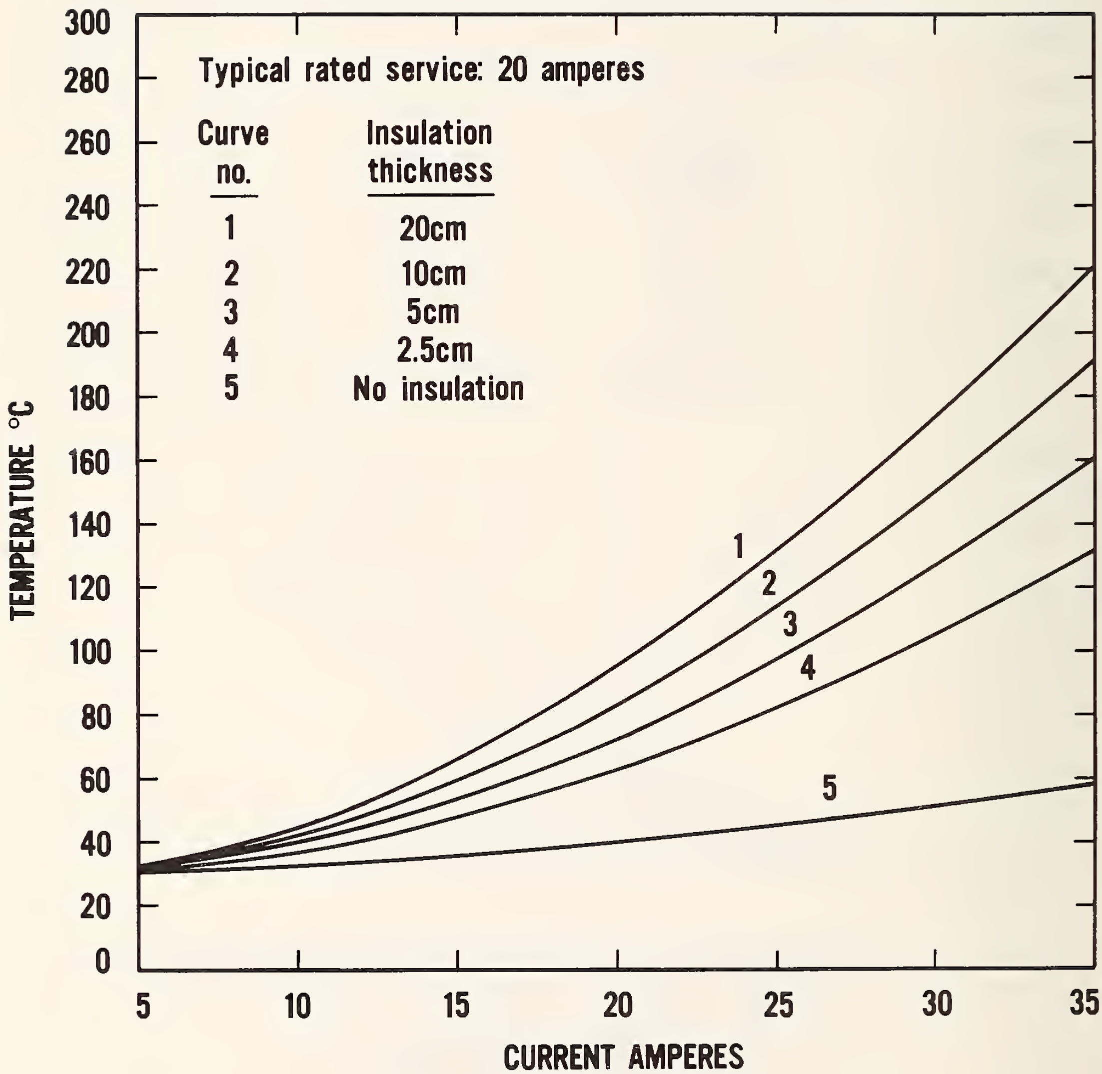

Figure 10. Steady-state jacket temperature for AWG #12 copper 2-G NM cable surrounded by insulation A

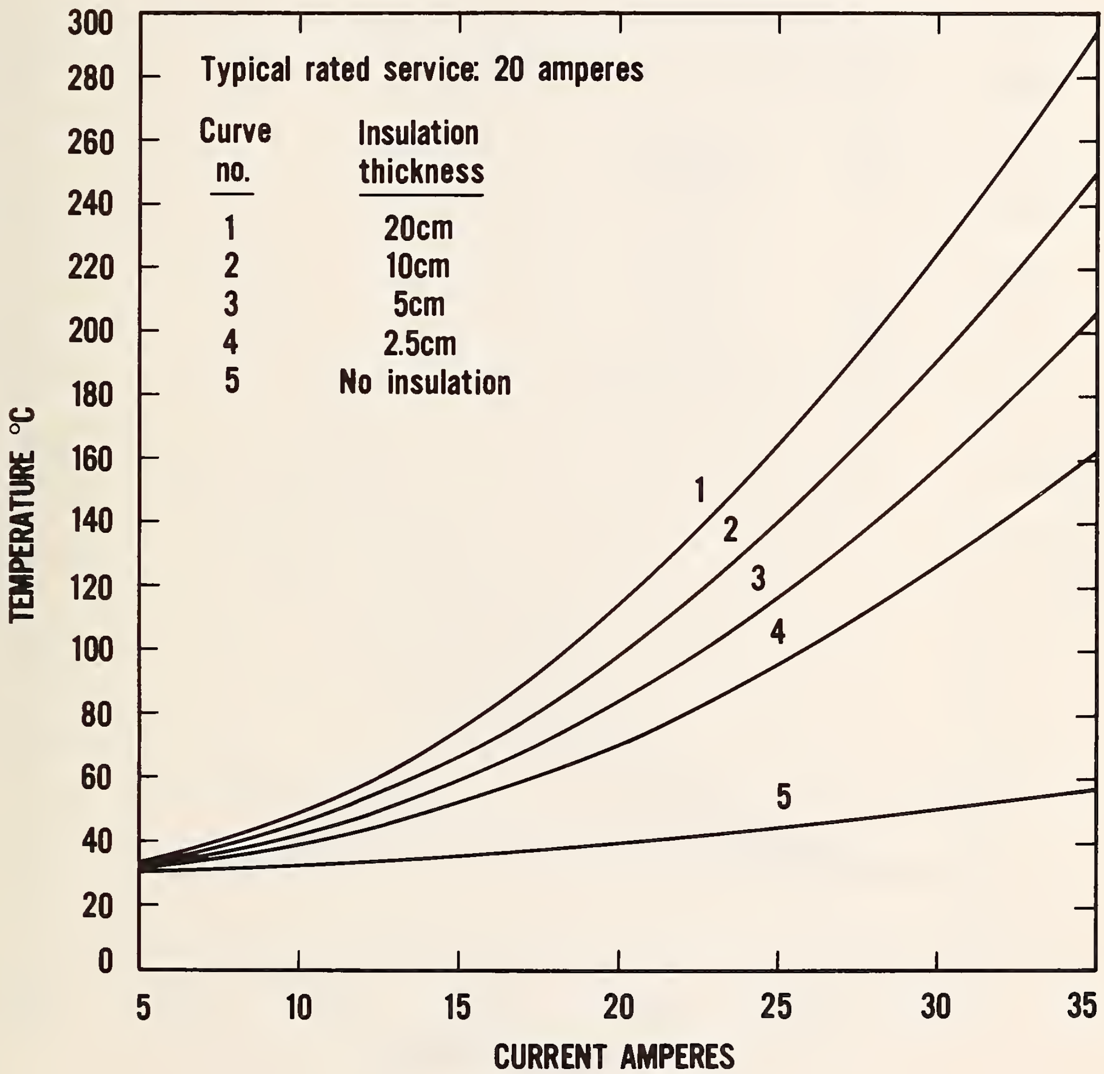

Figure 11. Steady-state jacket temperature for AWG #12 copper 2-G NM cable surrounded by insulation B

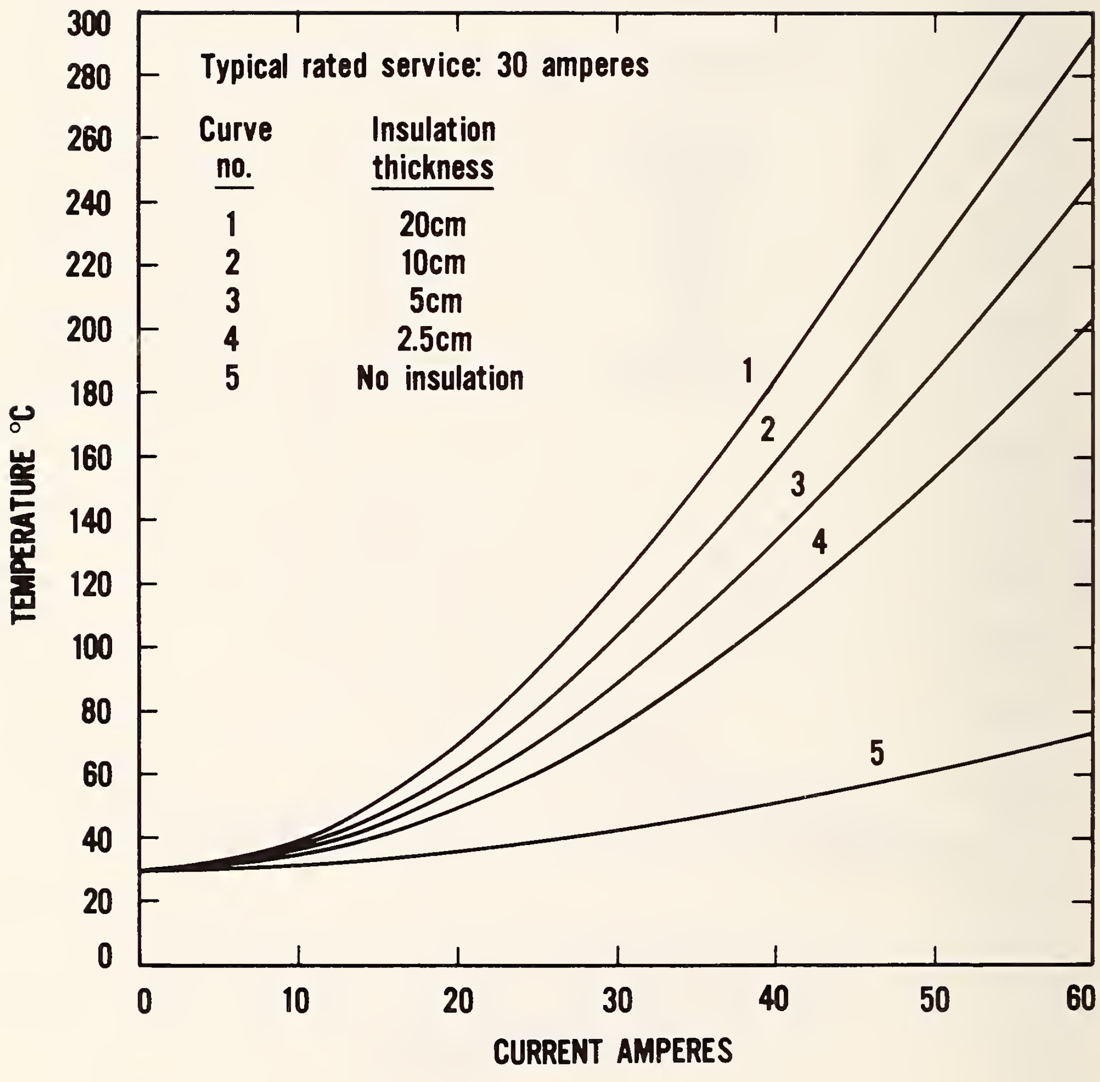

Figure 12. Steady-state jacket temperature for AWG #10 copper 2-G NM cable surrounded by insulation A

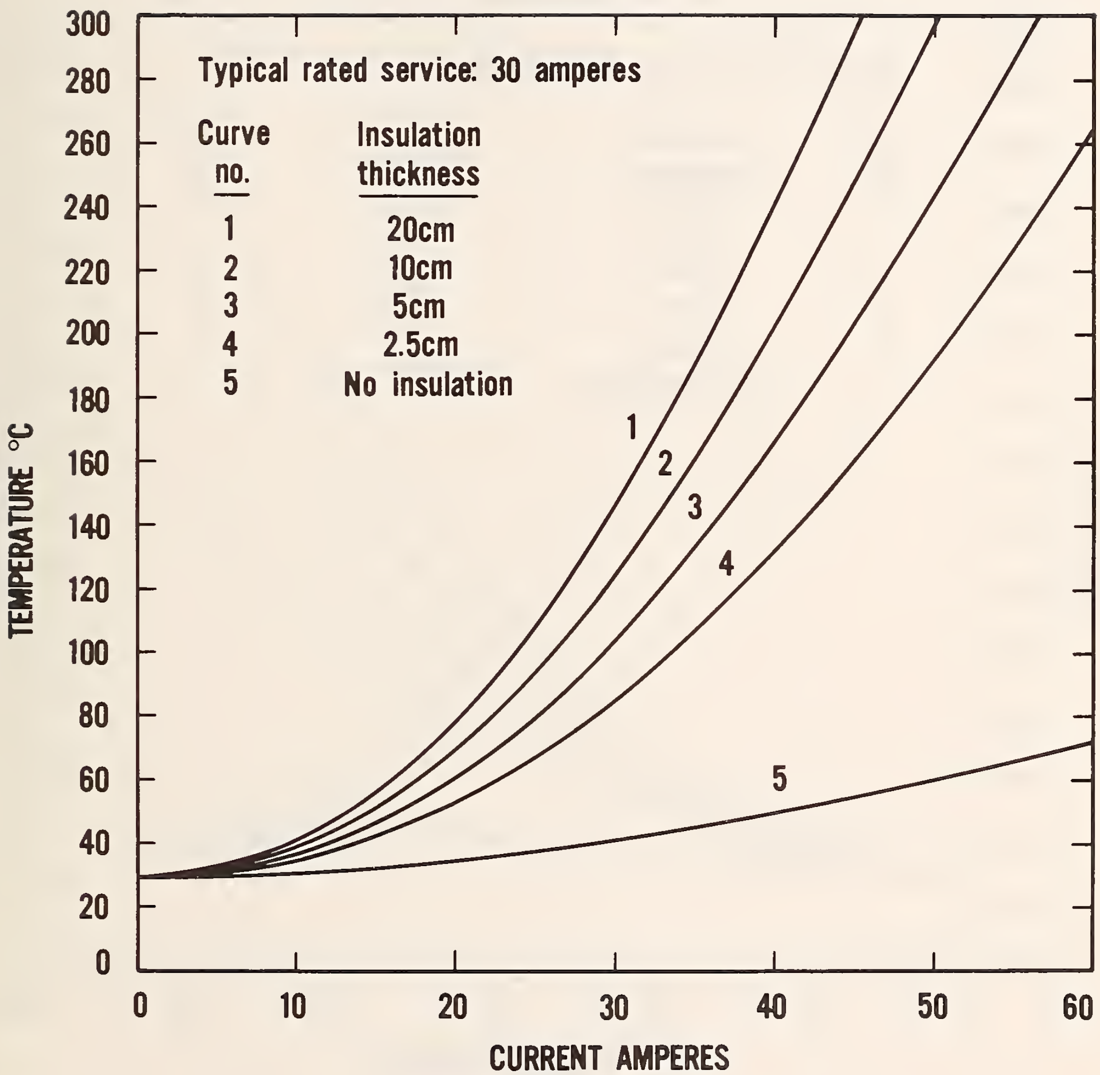

Figure 13. Steady-state jacket temperature for AWG #10 copper 2-G NM cable surrounded by insulation B

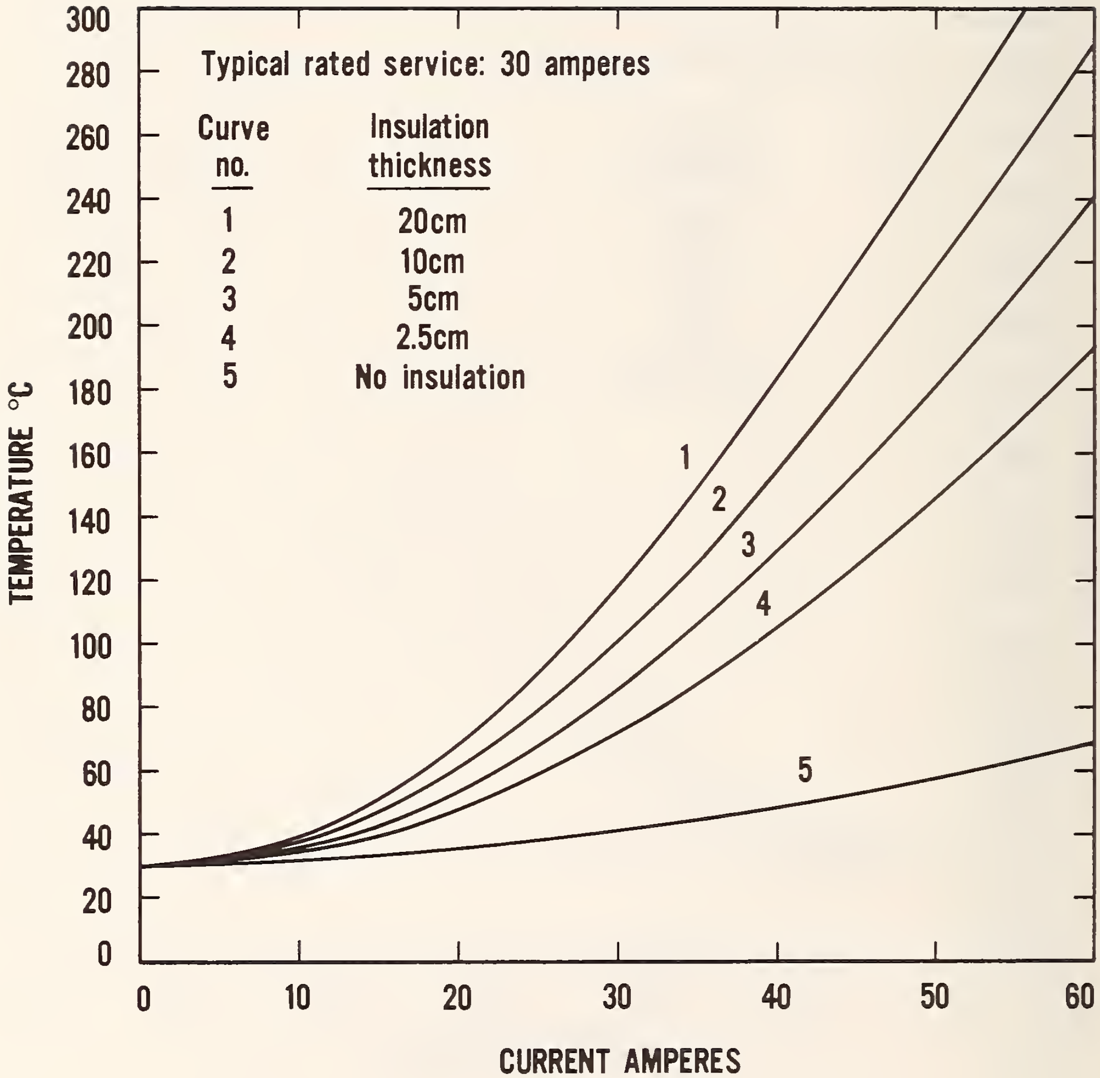

Figure 14. Steady-state jacket temperature for AWG #8 aluminum 3 CDRS SE cable surrounded by insulation A

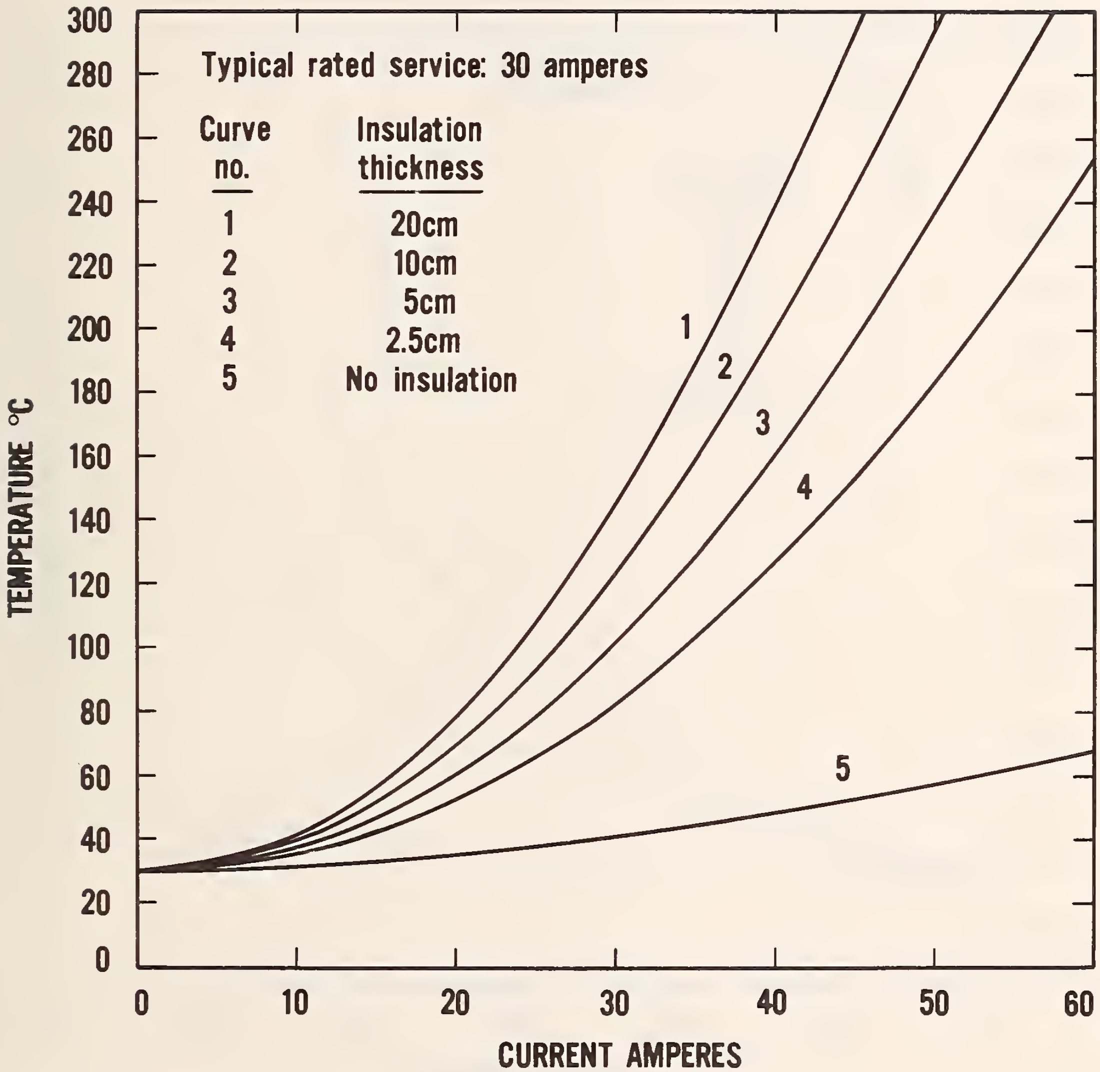

Figure 15. Steady-state jacket temperature for AWG #8 aluminum 3 CDRS SE cable surrounded by insulation B

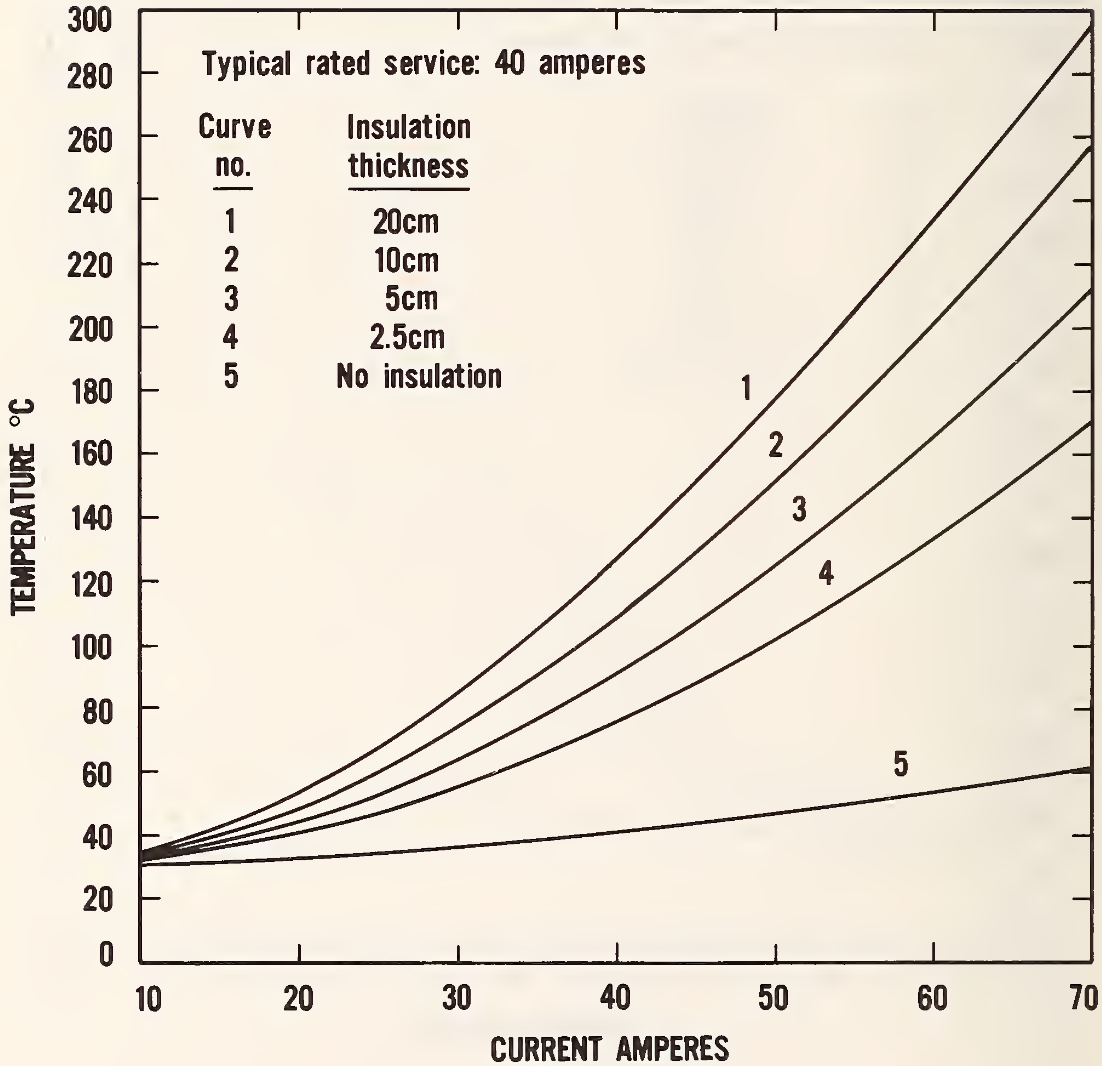

Figure 16. Steady-state jacket temperature for AWG #6 aluminum 3 CDRS SE cable surrounded by insulation A

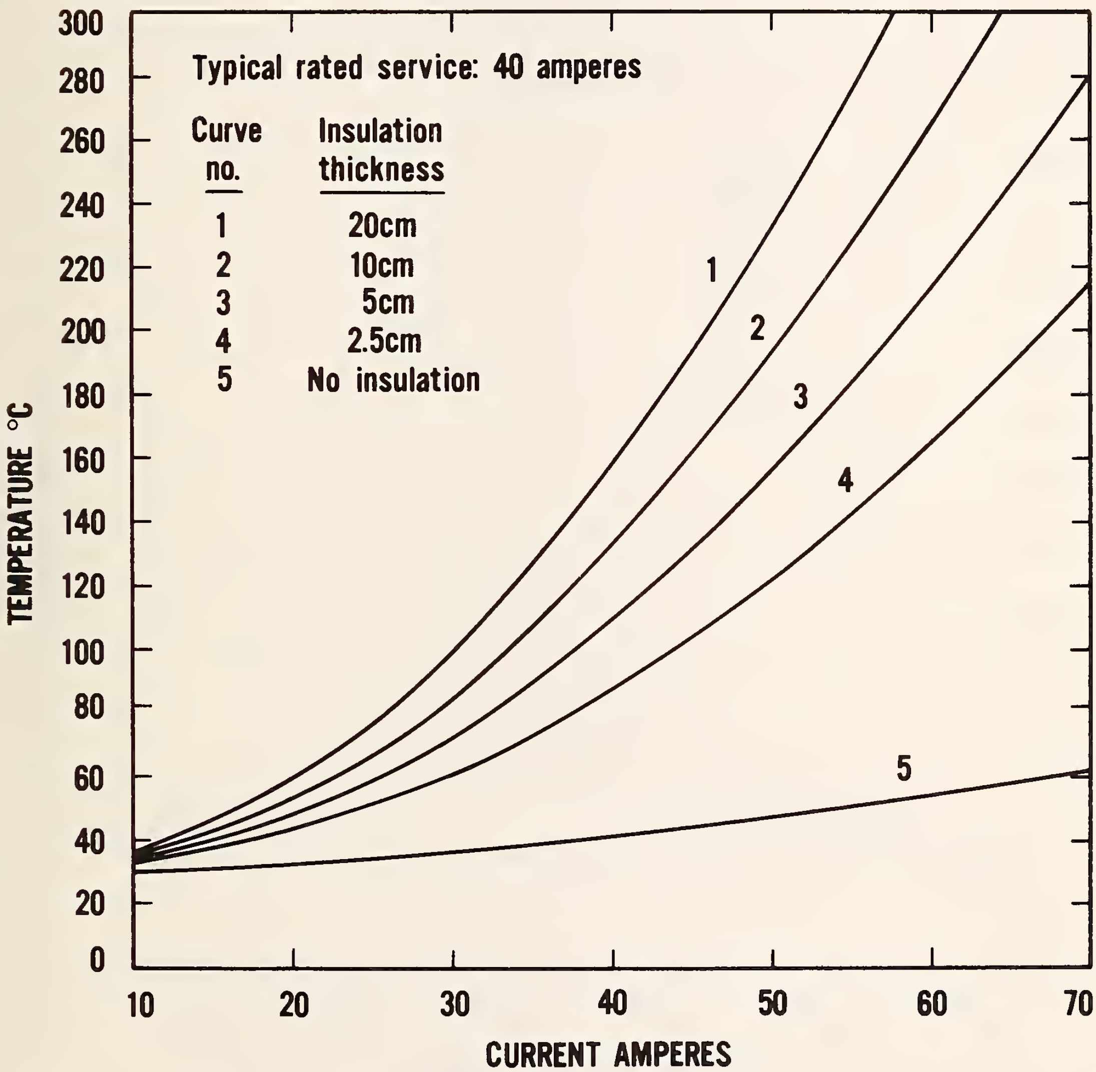

Figure 17. Steady-state jacket temperature for AWG #6 aluminum 3 CDRS SE cable surrounded by insulation B

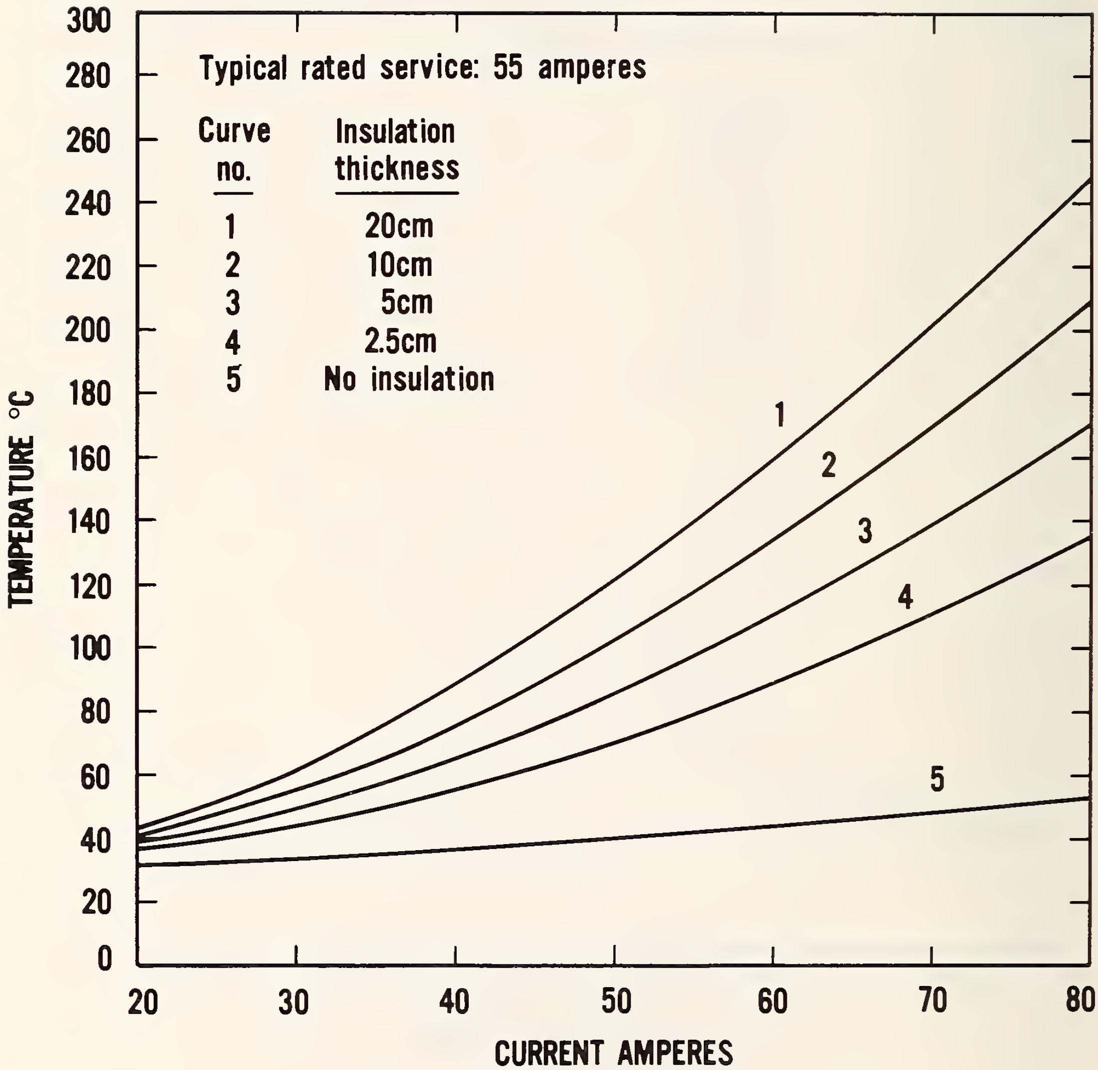

Figure 18. Steady-state jacket temperature for AWG #4 aluminum 3 CDRS SE cable surrounded by insulation A

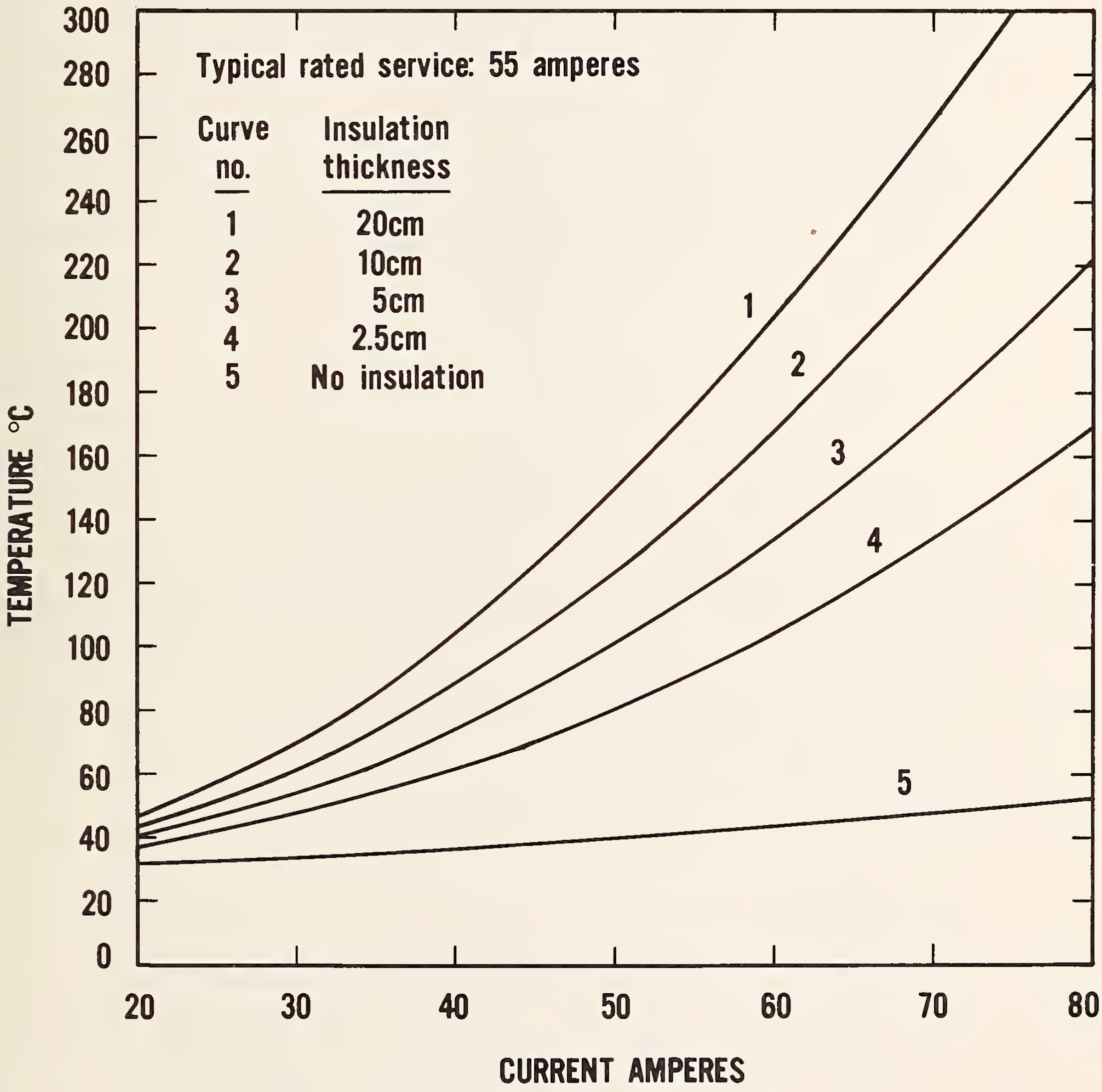

Figure 19. Steady-state jacket temperature for AWG #4 aluminum 3 CDRS SE cable surrounded by insulation B

Table 1. Effective diameter of cables

2-G AWG Gauge/Metal	Effective Diameter (cm)
14/Cu	0.91
12/Cu	1.02
10/Cu	1.15
4/Al	1.85
6/Al	1.54
8/Al	1.46

Table 2. Electrical resistance of metal conductors

General Form:

$$R = A\,[1 + BT(°C)] \times 10^{-5} \text{ ohms/cm}$$

Copper $B = 0.00427\ [1/°C]$

Aluminum $B = 0.00438\ [1/°C]$

AWG Gauge/Metal	A* [ohms/cm]
14/Cu	7.63
12/Cu	4.80
10/Cu	3.02
4/Al	1.229
6/Al	1.954
8/Al	3.106

*For stranded aluminum conductors, increase table value of A by 2%.

Table 3. Values of thermal conductivity for air at atmospheric pressure

Temperature °C	Thermal Conductivity W/cm°C
0	0.000242
38	0.000266
148	0.000334
260	0.000400
371	0.000464
482	0.000524
815	0.000692

Table 4. Values of $\frac{g\beta\rho^2}{\mu^2}$ for air at atmospheric pressure

Temperature °C	$\frac{g\beta\rho^2}{\mu^2}$ 1°/C cm^3
0	201
38	112
148	28
260	10.1
371	4.48
482	2.29
815	0.447

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

Steady-state operating temperatures of current-carrying electric cables buried axially in cylinders of thermal insulation were calculated. Combinations of six types of nonmetallic electric cable, two thermal insulation materials, five thermal insulation thicknesses and a range of currents both greater and less than the typical rated service currents for the cables were studied.

These calculations show that thermally insulated electric cables carrying the rated current may exceed the rated operating temperature limit for common cable jacket materials.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

Electric cable; electrical fire; models; thermal insulation; wiring system.

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