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A Statistical Characterization of Electroexplosive Devices Relevant to Electromagnetic Compatibility Assessment

Dennis S. Friday
John W. Adams

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Electroexplosive devices (EEDs) are electrically fired explosive initiators used in a wide variety of applications. The nature of most of these applications requires that the devices function with near certainty when required and remain inactive otherwise. Recent concern with pulsed electromagnetic interference (EMI) and nuclear electromagnetic pulse (EMP) made apparent the lack of methodology for assessing EED vulnerability. A new and rigorous approach for characterizing EED firing levels is developed in the context of statistical linear models and is demonstrated in this paper. We combine statistical theory and methodology with thermodynamic modeling to determine the probability that an EED, of a particular type, fires when excited by a pulse of a given width and amplitude. The results can be applied to any type of EED for which the hot-wire explosive binder does not melt below the firing temperature. Included are methods for assessing model validity and for obtaining probability plots, called "Firing Likelihood Plots". A method of measuring the thermal time constant of an EED is given. This parameter is necessary to evaluate the effect of a train of pulses. These statistical methods are both more general and more efficient than previous methods for EED assessment. The results provide information which is crucial for evaluating the effects of currents induced by impulsive electromagnetic fields of short duration relative to the EEDs thermal time constant.

Key words: EED; EED response to pulsed currents; electroexplosive device; electromagnetic compatibility; EMC; firing likelihood plots; thermal time constant.

1. Background and Introduction

A hot-wire electroexplosive device (EED) is an initiator that sets off a small charge of primary explosive that surrounds a wire. Electrical current flowing through this wire causes joule heating. When the primary explosive reaches its critical temperature due to this heating, it explodes, detonating a secondary explosive which serves as an actuator. A typical EED is shown in figure 1. Commonly used parameters describe this process, but additional parameters are needed for electromagnetic compatibility (EMC) analysis.

It is necessary to quantify both electrical and heat flow characteristics of the EED. The heating power (p) is a function of current (i), and electrical resistance (R_e). Parameters that must be measured are the critical temperature (Θ_c) of the explosive, the thermal resistance (R) of the EED, the thermal capacity (C) of the EED and the firing energy (U). U is the energy in a single pulse that will fire the EED when the pulse width is so short that almost no heat energy has time to leak out during the duration of the pulse. The thermal time constant (τ) of the EED may be calculated from the RC product. For single pulses, a family of curves may be generated that relate pulse width and peak power of a rectangular pulse to the likelihood of firing.

There are three distinct measurement procedures needed to obtain these parameters and curves. Not all of these parameters can be measured on any one EED since each measurement destroys the EED. Subsequent measurements must be made on other EEDs. For example, the firing current and thermal resistance may be measured by using a slowly increasing current ramp. The thermal capacity and energy to fire may be measured with another measurement procedure which uses a very short pulse of current. These data may then be used to calculate other parameters and to design the third measurement procedure, the statistical experiment. All of the EEDs used in the three measurement procedures should be randomly chosen from the same batch of EEDs.

The electromagnetic environment which may induce stray currents in the wire of the EED is usually poorly known. The theory of how energy may be transferred from this environment by unintended antennas (any electrical conductor) is also poorly understood. These two very relevant topics are not within the scope of this paper, but motivate the work reported. They must be considered in any comprehensive EMC analysis.

There are several widely used standards or guidelines for evaluating and handling EEDs in the presence of EMI [1,2,3,4]. The Bruceton up-down procedure [5] has been used for years to measure the DC firing current of EEDs. It will be described in the next section but in summary it provides little information on extreme firing levels. In practice, since the extremes of the firing current distribution are of interest, e.g., the minimum all-fire current from an operational standpoint, and the maximum no-fire current from a safety standpoint, alternate procedures are used. In one such procedure, fifty EEDs randomly selected from a lot are tested for five minutes at an arbitrarily set no-fire current level. These same EEDs are then fired at an arbitrarily set all-fire level. If any of the samples fire at the no-fire level, or if any do not fire at the all-fire level, the lot fails.

The reason that better, more direct measurement methods are not more widely used is due to a practical problem with the binder used to hold the primary explosive around the wire of some of the older designs of EEDs. The binder softens at a temperature below the critical temperature of the primary explosive. This allows flow of the explosive-binder mixture, and changes the thermal characteristics of the EED. Later designs do not have this problem, and direct measurement of the DC firing current gives a better measure of the average and standard deviation for a given sample size than can be obtained with the Bruceton method.

An omission in existing analyses of the effect of EMI on EEDs is the effect of impulsive EM fields, either from periodic pulses such as may be generated by radar, or aperiodic pulses such as may be generated by lightning, nuclear EMP, or arcing of DC machinery. Whether these transient problems are of consequence is not clearly known. The methodology presented later in this paper may help to answer these questions.

We propose a new way of characterizing the response of EEDs to impulsive fields. The probability of firing is determined statistically as a function of width and power of a rectangular input pulse. We present general statistical procedures that can be used for characterizing EEDs and describe the proper experimental methodology. The results of our experiments are presented to illustrate the methodology. These results are not given for the purpose of characterizing the EED type used in this study. The EEDs used, while all of the same type, were not suitably controlled prior to testing. They were readily available, but the lot numbers were not recorded and the storage time and conditions were not fully known. They were used to develop and to demonstrate the method, and any variability from these unknown factors, if any, was confounded with our random errors.

2. Sensitivity Testing - Prior Methods

Sensitivity testing is the name that has been used for the general methodology associated with EED testing. The class of experiments is characterized by a binary response, fire or no-fire in this case, and a continuous stimulus. The stimulus is adjusted to a predetermined set of levels and the proportion of "fire" responses at each level is determined. Many test specifications specify such procedures [2].

A procedure called the Bruceton method or the up-down method has been used for such tests [5, 6, 7]. The stimulus in general may represent very different attributes such as input voltage, height of drop, temperature, etc. The experiment consists of selecting an equispaced lattice of stimulus levels:... s_{-2}, s_{-1}, s_0, s_1, s_2... centered at a nominal "50 percent" firing level. Begin by applying the stimulus to a randomly selected EED at level s_0. The remaining settings are determined by the previous outcomes. If the first EED fires the second one is tested at level s_{-1}. If the first EED does not fire the second is tested at level s_1. Each subsequent EED is tested according to this procedure; one level up if no response and one level down if a response. The advantage of this test is that it concentrates the test levels near the mean and improves accuracy of that estimate. Fewer EEDs are therefore required on the average for a given accuracy. The disadvantages of the up-down method are that it obtains relatively poor estimates of the dispersion, it requires one-at-a-time sequential testing, and it deals with only one-dimensional stimuli. Procedures for computing the estimates of the mean firing level and standard deviation are given in the references.

Another possible approach to characterizing such devices is to adapt methodology used in statistical bioassay known as quantal-response models. In these methods the probability of response is expressed as a linear function of the levels of the stimuli. Multiple stimuli are possible with this approach. Considerable methodology is available on this topic because of its many years of use and development in biological experiments. Two models have evolved as the most commonly used. The Probit model is based on a Gaussian probability distribution and the Logit model on a logistic distribution. They are very similar in their results and assumptions, but the Logit model is computationally simpler and appears to be the more commonly used procedure. See [8, 9] for details.

Both procedures require information on the proportion of (in this case) EEDs in independent trials that fired at preset levels of the stimuli. To our knowledge Probit and Logit methods have not been used in EED characterizations, but they are the natural choice for certain EED experiments. They differ from the Bruceton method in several ways. They are not sequential in nature and the statistical design is predetermined based on preliminary experimentation or a priori information. Stimulus levels and the number of EEDs tested at each level are fixed. The model is more specific than the Bruceton method and must be validated with data for the results to be defensible. Good diagnostic tools are available for these methods, however. All three methods are suited to experiments that are somewhat wasteful of information since exact firing levels are not observed. EEDs not fired are wasted unless it can be determined that they have not been affected by attempts at firing and can possibly be reused. We originally used the Logit method for the EED characterization problem addressed here but since it was possible with our measurement system and the particular EEDs tested to obtain more information than is necessary for the Logit model, we developed the more efficient procedure discussed herein. The statistical basis for the methodology we propose will be developed over the next three sections.

3. The General Linear Model

Some statistical concepts and notation are now introduced. The general linear model is given by eq (1).

$$\underline{Y} = \underline{X}\,\underline{\beta} + \underline{\varepsilon} \qquad (1)$$

Where:

- $\underline{Y}$ is an n x 1 vector of response variables.
- $\underline{X}$ is an n x r matrix of regressor variables. Its elements are fixed, not random variables. The dimension r (r<n) equals the number of unknown parameters in the model. The structure of this matrix fully specifies the form of the linear model.
- $\underline{\beta}$ is a r x 1 vector of unknown parameters.
- $\underline{\varepsilon}$ is an n x 1 vector of random errors. Statistical assumptions are zero mean ($E(\underline{\varepsilon}) = \underline{0}$) and, in the most general case, that its variances and covariances are specified by a positive definite symmetric n x n matrix $\underline{\Sigma}$. The exact form of $\underline{\Sigma}$ is often unknown.

The elements ε_i of $\underline{\varepsilon}$ are the errors in the i^{th} observation. When the ε_i are uncorrelated, the off-diagonal elements of $\underline{\Sigma}$ will be zero. If the ε_i also have constant variance σ^2, then $\underline{\Sigma} = \sigma^2\underline{I}$ where $\underline{I}$ is the n x n identity matrix. However, this assumption is not made in this paper. When it can also be assumed that the ε_i have a Gaussian distribution, then uncorrelated ε_i are also statistically independent. With Gaussian errors more can be said about the statistical properties of the model and its parameter estimates than with arbitrary distributions.

The best (minimum variance) linear unbiased estimate $\hat{\underline{\beta}}$ of the parameters $\underline{\beta}$ is then given by the expression

$$\hat{\underline{\beta}} = (\underline{X}' \underline{\Sigma}^{-1} \underline{X})^{-1} \underline{X}' \underline{\Sigma}^{-1} \underline{Y} \qquad (2)$$

(where exponents ' and -1 are the transpose and inverse respectively). This is the estimator for the general case with nondiagonal covariance matrix Σ. If ε_i are uncorrelated but variances are unequal, then Σ^{-1} is a diagonal matrix with components $(\sigma_i^2)^{-1}$. It is usually written in terms of a weight matrix when a common variance is factored out. That is;

$$\Sigma^{-1} = \begin{bmatrix} (\sigma_1^2)^{-1} & & \underline{0} \\ & \ddots & \\ \underline{0} & & (\sigma_n^2)^{-1} \end{bmatrix} = \frac{1}{\sigma^2} \underline{W} = \frac{1}{\sigma^2} \begin{bmatrix} w_1 & & \underline{0} \\ & \ddots & \\ \underline{0} & & w_n \end{bmatrix} \qquad (3)$$

with weights w_i; $i = 1,2,...,n$. Observations Y_i with small w_i have larger variance than observations with large w_i. The estimate, eq (2), then reduces to

$$\hat{\beta} = (\underline{X}' \underline{W} \underline{X})^{-1} \underline{X}' \underline{W} \underline{Y} . \qquad (4)$$

This case is often called weighted least squares. An excellent reference for linear models is the book by Graybill [10].

4. The General EED Model and Its Symmetry

In the classical linear model the response variable $\underline{Y}$ and the columns of $\underline{X}$, the regressor variables, represent distinct attributes. It is sometimes possible to interchange attributes in a different linear model but the parameter estimates and the error structure are different. The two arrangements are distinct linear models. This restriction does not apply to the type of EEDs considered here. We will demonstrate that a symmetry exists between the two variables chosen for this study which is not generally possible. Furthermore, it can be expressed within the linear model given by eq (1).

The notation must first be generalized. The random vector $\underline{Y}$ will always denote the response variable and the columns of $\underline{X}$ the regressor variables. The symmetry arises because the attributes which represent $\underline{X}$ and $\underline{Y}$ may be interchanged within the same linear model. We denote these two possible arrangements as experiment 1 and experiment 2. The arrays $\underline{X}$ and $\underline{Y}$ are partitioned according to which experiment (or attribute arrangement) is represented.

Let

$$\underline{Y} = \begin{bmatrix} \underline{Y}^{(1)} \\ \underline{Y}^{(2)} \end{bmatrix} \quad \text{and} \quad \underline{X} = \begin{bmatrix} \underline{X}^{(1)} \\ \underline{X}^{(2)} \end{bmatrix} \tag{5}$$

where superscripts (1) and (2) identify the experiment. Vectors $\underline{Y}^{(1)}$ and $\underline{Y}^{(2)}$ are of dimensions $n_1 \times 1$ and $n_2 \times 1$ respectively, where $n_1 + n_2 = n$. Elements of $\underline{Y}$ are written $y_i^{(k)}$; $i = 1,2,\ldots,n_k$; $k = 1,2$. Likewise the matrices $\underline{X}^{(k)}$ are $n_k \times r$ with elements $x_{i,j}^{(k)}$; $i = 1,2,\ldots,n_k$; $j = 1,2,\ldots,r$; $k = 1,2$.

A general design matrix for this problem which admits an intercept parameter and a parameter for experiment effects is the following (dimension n x 3):

$$\underline{X} = \begin{bmatrix} 1 & +1 & x_{1,3}^{(1)} \\ & \vdots & \vdots \\ \cdot & +1 & x_{n_1,3}^{(1)} \\ \cdot & & \\ \cdot & -1 & x_{1,3}^{(2)} \\ & \vdots & \vdots \\ 1 & -1 & x_{n_2,3}^{(2)} \end{bmatrix} . \tag{6}$$

A more restricted design might be to eliminate column 1 or column 2 of this matrix, or even both columns 1 and 2. The final design is not dictated by the analyst but by the data; the most parsimonious design for which a good fit of the data is manifested in the diagnostic analysis.

While the above design matrix will be shown to be more general than necessary for the particular EEDs which were tested for this study, we feel it is important when applying this methodology to a new type of EED to develop the model from first principles. Column 1 allows for a nonzero mean response when the value of the regressor $x_{i,j}^{(k)}$ is set to zero. While counterintuitive in a physical sense, one cannot rule out a priori that such a shift might be exhibited by the data in the region where the experiment is performed. The second column is perhaps more reasonable and is necessary to ascertain whether there is any significant difference in mean value of the two experiments. The question, "Does the phenomenon under study (the firing level of the EED) depend on which attribute is fixed and which is random?" is another way of expressing this. The symmetry can be illustrated by considering two attributes u and v. Let indicator variable I(k) denote the experiment. I(k) = +1 if k = 1 and I(k) = −1 if k = 2. I(k) changes with the arrangement of u and v. Let β_0, β_1, β_2 denote the elements of $\underline{\beta}$ and let $\underline{\varepsilon}$ have covariance matrix $\sigma^2\underline{I}$. If experiment 1 means u is the response and v is the regressor, and experiment 2 is the reverse, then (ignoring row subscripts) either of the following arrangements is permitted in our linear model.

$$u = \beta_0 + \beta_1 I(1) + \beta_2 v^{-1} + \varepsilon v^{-1} = \beta_0 + \beta_1 + \beta_2 v^{-1} + \varepsilon v^{-1} \tag{7a}$$

$$v = \beta_0 + \beta_1 I(2) + \beta_2 u^{-1} + \varepsilon u^{-1} = \beta_0 - \beta_1 + \beta_2 u^{-1} + \varepsilon u^{-1} . \tag{7b}$$

Variables u and v are present in both $\underline{Y}$ and $\underline{X}$ under the same model with the same parameters $\underline{\beta}$ and σ^2. This is a situation which does not normally arise in a linear model. The coefficients of ε in eq (7) change the variance of each row and correspond to the weights in eq (3) (i.e., $w = v^2$ or u^2).

In the case where the intercept is indeed the origin ($\beta_0 = 0$) and there are no effects due to the different experiments ($\beta_1 = 0$), the first two columns of $\underline{X}$ are unnecessary. In this case eqs (7) reduce to

$$u = \beta v^{-1} + \varepsilon v^{-1} \tag{8a}$$

or

$$v = \beta u^{-1} + \varepsilon u^{-1} . \tag{8b}$$

These eqs (8) admit an interpretation of bilinearity when written in the equivalent hyperbolic form

$$uv = \beta + \varepsilon . \tag{9}$$

While mathematically equivalent to eq (8), the form in eq (9) is not an adequate basis for a statistical analysis where u and v are being varied as separate factors, and the region of model validity must be determined. It will be shown later that eq (9) enables simple computation of $\hat{\beta}$ once the model has been proven valid for the given data from a particular type of EED.

5. Parameter Estimation in the EED Model

We now consider the statistical estimation of the parameter vector $\underline{\beta}$ in the linear model given by eq (1) where the design matrix is given by eq (6). First it is necessary to establish the statistical structure of the error vector $\underline{\varepsilon}$. Each component of $\underline{\varepsilon}$ corresponds to a different EED. It is assumed that the random perturbation in firing level for each EED, about their common firing level, is not affected by the errors of the other EEDs.

They were chosen randomly from the batch of EEDs and the time order of the design levels and experiments was randomized. Therefore the elements of $\underline{\varepsilon}$ are mutually uncorrelated. The variances of the ε_i are not assumed to be constant, but to vary with the regressor variable as suggested by eqs (7). The ε_i may be represented as an independent random error ε with variance σ^2 scaled by the value of the corresponding $x_{i,3}^{(k)}$.

We may write

$$\varepsilon_i = \{ x_{i,3}^{(k)} \} \varepsilon \qquad (10)$$

Therefore $$v(\varepsilon_i) = \{ x_{i,3}^{(k)} \}^2 \sigma^2 .$$

The covariance matrix of $\underline{\varepsilon}$ is then:

$$\underline{\Sigma} = \sigma^2 \begin{bmatrix} \{x_{1,3}^{(1)}\}^2 & & & & & \\ & \ddots & & & \underline{0} & \\ & & \{x_{n_1,3}^{(1)}\}^2 & & & \\ & & & \{x_{1,3}^{(2)}\}^2 & & \\ & \underline{0} & & & \ddots & \\ & & & & & \{x_{n_2,3}^{(2)}\}^2 \end{bmatrix} . \qquad (11)$$

The least squares estimate of $\underline{\beta}$ is therefore given by

$$\hat{\underline{\beta}} = (\underline{X}' \, \underline{W} \, \underline{X})^{-1} \, \underline{X}' \, \underline{W} \, \underline{Y} , \qquad (12)$$

where

$$\underline{W} = \begin{bmatrix} \{x_{1,3}^{(1)}\}^{-2} & & & & & \\ & \ddots & & & \underline{0} & \\ & & \{x_{n_1,3}^{(1)}\}^{-2} & & & \\ & & & \{x_{1,3}^{(2)}\}^{-2} & & \\ & \underline{0} & & & \ddots & \\ & & & & & \{x_{n_2,3}^{(2)}\}^{-2} \end{bmatrix} \qquad (13)$$

is consistent with eqs (2) and (3), and our assumptions. No assumptions were made on the distribution of $\underline{\varepsilon}$ up to this point. We have simply derived the least squares estimates. If a further assumption that the ε_i have a the least squares estimates. If a further assumption that the ϵ_i have a Gaussian distribution is possible, then $\hat{\underline{\beta}}$ is the maximum likelihood estimator of $\underline{\beta}$. This estimator is unbiased and has the smallest variance of all unbiased estimators. In this sense it is an optimal procedure. A practical advantage of this approach is that most statistical packages contain software for computing these estimates and, equally important, provide diagnostic information for assessing the quality of the fit of the model to the data.

6. Heat Flow Equations

Some relevant thermodynamic concepts will now be introduced. These concepts are important for a deeper understanding of the statistical model and its limitations, for obtaining prior information on the region of model validity, and for identifying thermodynamic parameters relevant for EED characterizations. The methodology introduced in these sections is based on physical principles rather than statistical modeling. Only a small number of EEDs could be allocated to these measurements and extensive model validation was not possible. Parameter estimates are obtained directly from the equations, not from statistical principles. These procedures are adequate for the intended purposes. They follow basic heat flow equations similar to those used in determining temperature distribution around a barretter wire [11]. In situations of temperatures less than red glow of the wire, heat flow by radiation is minimal. Also, for small volumes, the Grashoff number is low, indicating minimal heat flow due to convection. Thus the simple models based on conductive heat flow should be adequate. They provide approximate "estimates" of the thermodynamic parameters and a priori information on design boundaries for the statistical model. The final statistical results demonstrate no inconsistencies with these initial thermodynamic measurements. This theory is now developed.

The general heat flow equation that applies during joule heating is

$$\Theta(t) = \Theta_a + p(t)R(1-e^{-t/\tau}) \; ; \; t>0 \tag{14}$$

where

$\Theta(t)$ is temperature as a function of time (degrees Celsius),

Θ_a is ambient temperature (degrees Celsius),

t is time after application of current (seconds),

p(t) is power due to joule heating, $p(t) = i(t)^2 R_e$ (watts), where i(t) is current as a function of time, R_e is the electrical resistance of the EED wire (ohms). The current is often constant over a given interval of time.

R is the thermal resistance of the heat-leak path out of the EED (degrees Celsius per watt),

τ is the thermal time constant (seconds). τ is the product of R and C, where C is the thermal capacity of the EED wire and explosive-binder mixture (joules per degree Celsius).

Differentiating eq (14) gives the rate of rise of temperature

$$d\Theta(t)/dt = (p(t)/C)e^{-t/\tau}. \qquad (15)$$

During cooling, the temperature decreases from initial temperature (Θ_o) according to

$$\Theta(t) = \Theta_a+(\Theta_o-\Theta_a)\ e^{-t/\tau}\ . \qquad (16)$$

These equations have some approximations in them due to individual variations of geometry and parameters among individual EEDs. There are also nonlinearities due to parameter dependence on temperature. These effects have been simulated for a similar structure, a barretter wire in air, with an iterative mathematical procedure [11], and found to cause relatively minor variations in absolute temperature.

The steady state temperature may be obtained from eq (14) by letting time be much greater than the thermal time constant. For most EEDs, this means time greater than 300 milliseconds. If R, p(t) and τ are known, this is a straight forward calculation. For most operational applications, the temperature will exceed the critical temperature of the explosive well before the steady state equilibrium conditions are reached. For some EED designs, the binder-explosive mixture may soften at a temperature below the critical temperature of the explosive, resulting in a change of characteristics, often indicated by dudding.

The temperature rise and fall due to a single impulse function of current may be calculated using eqs (14) and (16). For times much less than the thermal time constant of the EED, the t = 0 limit of eq (15) may be used as given by

$$d\Theta(t)/dt = p(t)/C. \qquad (17)$$

This is a useful form to determine power and hence current needed to fire an EED within a specified time.

For the case where there is a train of pulses, either periodic or aperiodic, where the cooling time between pulses is of the same order as the thermal time constant, a combination of eqs (14) and (16) may be used to determine cumulative temperature rise, commonly known as stacking. This is illustrated in figure 2.

If there is a single impulse with known amplitude and short duration compared to the thermal time constant of an EED, the curves shown later in this paper can be used to determine probability of firing.

6.1 Measurement Procedures

Three distinct measurement procedures are used to determine the parameters of each EED. Since each measurement procedure destroys the EED, only a subset of the relevant parameters can be measured on any one EED.

The first measurement procedure is as follows. Ambient temperature is recorded. The critical temperature of the primary explosive is either obtained from the manufacturer or measured with a special oven with five solid walls and a sixth weak wall made of some easily replaceable material that can serve as a pressure-release blast wall. The electrical resistance of an EED is measured with special instrumentation that applies much less current than the no-fire current. A slow current ramp is used to heat the EED to detonation. The value of current at which the EED fires is recorded as I_f. The average and standard deviation may be calculated for the electrical resistance, the firing current, the firing power, and the thermal resistance. The thermal resistance is obtained by

$$R = (\Theta_c - \Theta_a)/P_f \tag{18}$$

where

Θ_c is the critical temperature of the primary explosive (degrees Celsius)

Θ_a is the ambient temperature (degrees Celsius),

P_f is the power (watts) applied at the time of firing, (i.e., $P_f = I_f^2 R_e$).

If the wire in the EED has a significant temperature coefficient of resistance, a correction may be needed. This gives measured values of four independent parameters, R_e, I_f, Θ_a, and R. The fifth parameter, P_f, is calculated from the first two.

The second measurement procedure also requires that critical temperature of the primary explosive, ambient temperature and electrical resistance be measured. A signal generator and amplifier are used to generate a train of current pulses whose width is much less than the expected time constant of the EED, and whose period is much greater than this same time constant. The amplitude of the pulse can then be increased slowly until the EED under test fires. The amplitude at which the EED fires is recorded as I_p. Since the pulse width is much less than the thermal time constant of the EED, almost no heat energy has had time to leak out during the duration of the pulse. Also, since the time between pulses is much greater than the thermal time constant, the temperature of the wire nearly returns to ambient temperature between each pulse. Therefore, the energy to fire with a single rectangular pulse may be calculated as follows:

$$U_f = P\, w \tag{19}$$

where

U_f is energy (joules) to fire with a single impulse,

P is peak power (watts), $P = I_p^2 R_e$,

w is the pulse width (seconds).

The thermal capacity of the EED may be calculated as follows:

$$C = U_f/(\Theta_c - \Theta_a) \tag{20}$$

where

C is the thermal capacity of the EED in joules per degree Celsius,

U_f is defined in eq (19),

Θ_c and Θ_a are critical and ambient temperatures respectively.

The thermal time constant can be calculated as follows:

$$\tau = R\,C \tag{21}$$

where R is obtained from eq (18) and C is obtained from eq (20).

Four independent parameters are measured: R_e, dt, Θ_a, and I_p. U_f and C are calculated from the above four, plus knowledge of Θ_c.

The third measurement procedure, the statistical experiment, is discussed in section 7. Information obtained from the first two procedures is used to determine the appropriate region for the statistical experiment.

6.2 Measured Data

A set of measurements were made on some squibs. A squib is an EED of low explosive charge, usually without any secondary explosive. The particular squibs used simulate a common form of EED, an electrically fired commercial blasting cap. Due to the age and unsealed design of these squibs, the data obtained are not considered typical of data that would be obtained from normal EEDs. The purpose in making the measurements was to allow evaluation and improvement in the procedures, test equipment, and statistics needed to obtain the desired parameters and curves for EMC analysis.

Data from the first measurement procedure are summarized in the table below. The average and standard deviation are calculated from 10 EEDs.

Table 6.1 Average and standard deviation of four parameters obtained using first measurement procedure.

Parameter	Average	SD	Units
Electrical Resistance	1.42	0.123	ohms
DC firing Current	0.39	0.015	amperes
Thermal Resistance	1365.76	188.4	degrees Celsius per watt
Power to Fire	0.22	0.027	watts

Data from the second measurement procedure are summarized in Table 6.2, based on 11 EEDs.

Table 6.2 Average and standard deviation of three parameters obtained using the second measurement procedure.

Parameter	Average	SD	Units
Electrical resistance	1.45	0.09	ohms
Energy to Fire	11.8	2.2	millijoules
Thermal Capacity	39.1	7.4	microjoules per degree Celsius

The product of the average thermal resistance and the average thermal capacity gives a calculated average value of thermal time constant, τ, of 53.4 milliseconds.

The variation in the range of values for the thermal time constant of individual EEDs must be estimated since this parameter cannot be measured explicitly on any one EED. The destruction of each EED during either measurement procedure precludes use of the same EED for subsequent measurements.

The lower and upper confidence limits of thermal resistance may be calculated from the average and standard deviation of the measured values for specified confidence levels. Since the two measurement procedures are independent, a second set of confidence limits may be calculated for the thermal capacity. Lower and upper 95 percent confidence limits are given in the table below (10 degrees of freedom).

Table 6.3 Calculated values of confidence limits of thermal parameters from measured values in previous tables.

Parameter	Lower	Average	Upper	Units
Thermal resistance	1230.9	1365.7	1500.5	degrees Celsius per watt
Thermal capacitance	33.8	39.1	44.4	microjoules per degree Celsius

An estimate of the thermal time constant may be obtained by taking the product of the thermal resistance and the thermal capacitance. The thermal time constant calculated from the two average values is 53.4 milliseconds.

6.3 Instrumentation

The instrumentation to make these measurements consists of four pieces of equipment plus a restricted area for setting off the EEDs. See figure 3 for a block diagram. The pulse generator, storage oscilloscope and digital multimeter are commercially available items. The pulse amplifier is a special design; it should be able to reach peak pulse currents of 20 amperes or more with a rise time of about 10 microseconds. It will require some substantial ancillary power supplies. See figure 4 for one such design. The multimeter must be able to measure resistance in the one ohm range to three significant figures. Most good pulse generators and oscilloscopes meet the rise time requirements. A dummy 1 ohm load (in place of the EED) that can handle 10 amperes is needed to make adjustments on the range scales of the pulse generator and oscilloscope.

The requirements for the area used for setting off the EEDs are based mainly on safety and convenience. A 200 liter (55 gallon) drum with about 30 cm of sand in the bottom serves as an absorber and director for blast energy and shrapnel particles. Firing current is fed through a coaxial cable and a connector on the side of the drum. This allows final activation of the circuit by connecting the cable to the outside of the drum, and the person making the connection is never exposed to an armed EED. EED resistance measurements are made through this same cable, subtracting short circuit resistance of the cable from total resistance. This also assures reliable electrical connection which prevents apparent dudding.

7. The Statistical Experiment

In this section we describe the actual physical experiment and its statistical design. This is the direct application of the statistical methodology developed in sections 3, 4 and 5 combined with the thermodynamic concepts of section 6. The two attributes discussed in sections 4 and 5 are pulse width and pulse amplitude. The two experiments are defined according to which of these attributes is fixed and which is measured.

> Experiment 1. The pulse amplitude is fixed within a set of predetermined design levels and the pulse width at which each each EED fires is measured.
>
> Experiment 2. The pulse width is fixed within a set of predetermined design levels and the pulse amplitude at which each EED fires is measured.

The physical interpretation of the symmetry discussed in eq (7) should now be apparent. The pulse width, whether fixed or measured, is in units of time (milliseconds).

The choice of units for pulse amplitude, however, is not as simple. Two considerations arise here, namely model and error sources. Possible choices for units are voltage, current or power. While the firing current and voltage can be measured directly, the firing power must be computed as a function of either measured current (or voltage) and measured electrical resistance.

Within the region where the instrumentation supplies a rectangular pulse, the energy in a single pulse is simply the product of pulse width and pulse power. If the pulse is short enough that there is minimal heat loss during the pulse, then temperature rise is proportional to pulse energy. Within these limitations we expect a simple pulse energy dominant model for EED firing level. This reasoning suggests the symmetry discussed in section (4).

The second consideration is related to minimizing the random error in the experiment due to sources that can be eliminated by proper statistical design. In this case the electrical resistance of the EED is such a factor. If units of voltage or current were used for pulse amplitude, the firing levels would implicitly depend on the electrical resistance of each EED. This resistance, due to manufacturing variability, material inhomogeneity and other reasons, fluctuates randomly among EEDs of identical type and lot. Using power units for amplitude requires measuring this resistance, but once it is known, a source of error is removed. While there may be pragmatic reasons, depending on the applications of the results, to include or exclude electrical resistance, our intent is to explore the nature of the EED firing level in as error-free an experiment as possible.

The two considerations of model simplicity and error minimization therefore lead to a common approach, power-width based measurements. Each EED, after its electrical resistance is measured, will be excited by a train of rectangular pulses of known width and amplitude. The amplitude, in units of power, is determined for the particular EED under test, based on its known electrical resistance. The relatively high accuracy of this measurement justifies our assumption that electrical resistance is known. The interpulse spacing is chosen so that the temperature returns to ambient between pulses. We will return to this issue later. The pulse width or pulse amplitude, depending on the experiment, is increased by an infinitesimal amount for each succeeding pulse until the EED fires. The width and amplitude of the pulse that fired each EED are the basic data for the analysis.

The type of EED tested had the property that the binder material was not affected below the firing temperature. This is not true of all EEDs, as some binders soften at temperatures below the critical temperature of the primary explosive. Such EEDs must be tested by a different procedure, as was stated earlier.

7.1 Statistical and Thermodynamic Design of the Experiment

The design of the experiment is done in two stages. The first stage is based largely on physical considerations. Its objective is to determine a range of pulse amplitudes and widths within which it is possible to perform the experiment and be assured that crucial assumptions hold. The second stage is the statistical design of the experiment within this feasible region.

Limits of the instrumentation used in the experiment, thermodynamic properties of the EED, and mathematical-statistical assumptions underlying the analysis must be considered for the first stage of the design. A feasible region will then be established within which pulse width and pulse amplitude may be set. The minimum interpulse spacing is also determined at this time. This variable does not arise explicitly in the experiment but is crucial for model validity. The details for determining the feasible regions of pulse width, pulse amplitude and interpulse spacing are now given.

There is a limitation on both the maximum pulse width and the minimum pulse width. The minimum pulse width is limited by instrumentation capabilities. The pulse amplifier rise time will distort the leading edge of a pulse. For long pulse widths this rise time distortion is negligible and the pulses may be assumed to be rectangular. For very short pulses the imperfect leading edge will dominate its shape and the pulse can no longer be considered rectangular. Pulse amplifier rise time limitations can be determined from the amplifier design specifications and should also be measured in the test system with a suitably fast oscilloscope. It was determined that pulses of about 0.1 ms minimum duration could be considered rectangular. In the final statistical design the shortest pulse width assigned for experimental purposes was 0.2 ms or twice the minimal pulse width. The requirement of rectangular pulses enables use of the model described in eq (1) and its special case eq (8). This is true because the pulse can then be characterized by its width-power product, or equivalently, its energy. The EED temperature is related to pulse energy in eq (14), but in order to simplify this relationship, a limit must be placed on the maximum pulse width.

The maximum feasible width is not determined by instrument limitations but by thermodynamic properties of the EED and the desire to minimize the complexity of the model and the analysis. The rectangular pulse requirement of the previous paragraph enables simple computation of energy. Statistically this would enable the use of a simple model except that the binary response (fire or no-fire) is in general not linear in energy but in temperature. In order for the EED input energy and EED bridgewire temperature to be approximately linearly related, there must be minimal heat loss from the EED during the pulse. In this case (see eq 17) the temperature at firing will be linearly related to the product of input pulse width and pulse power, a highly desirable property. Using eq (17) we can determine the percent error between the linear and exponential models for various pulse widths. This error is 0.5 percent for $\tau/100$, 2.5 percent for $\tau/20$, 5.1 percent for $\tau/10$, 8.2 percent for $\tau/6$, and 13 percent for $\tau/4$. To enable as wide a range of pulse widths as possible $\tau/20$ was chosen as the maximum preset pulse width for the experiment. The value of $\tau = 53.4$ ms in table 6.3 therefore limits pulse width to at most 2.7 ms in the statistical design.

The maximum limits on pulse amplitude are due to the instrumentation. The pulse amplifier exhibits some distortion at high powers and begins to saturate above 50 watts. It was possible to generate pulses over 50 watts, but for levels much greater, some distortion in the rectangular shape was visible. Therefore the upper power limit is set at 50 watts. There is no lower limit on pulse amplitude per se, but it is limited implicitly by the pulse width maximum. As pulse power decreases a longer pulse width is required to fire an EED. Since, as discussed above, large pulse widths allow heat leakage and therefore require more complex modeling and experimentation. The pulse amplitude will, for this reason, have a lower limit to its range.

The minimal spacing between pulses must also be predetermined. While not explicitly required for the statistical design, it is necessary to space the pulses sufficiently far apart to allow all, or almost all, of the heat from previous pulses to dissipate. The response of each pulse can therefore be related to its dimensions independently of previous pulses. Equation (16), based on first order thermodynamic theory, describes how the hot wire temperature returns to ambient after a pulse is turned off. Approximately 95 percent of the heat energy from any pulse will be dissipated in three time constants, 98 percent in four time constants, and over 99 percent in five time constants. This is true for any ambient temperature and any pulse width or power -- below the firing level. A minimum interpulse spacing of at least five time constants (267 ms) was chosen. In the experiments described here an interpulse spacing of 500 ms was used throughout.

The numerical values for these bounds are relevant only for the particular type of EED and instrumentation used. The methods described here may be used for obtaining the corresponding limits for any other EED for which the binder material does not melt before firing. The experimental limits could have been obtained by other methods, statistical for example. The methods given, however, are simple to implement, have useful physical interpretations, and appear to work reasonably well with minimal data (using few EEDs). Note that the bounds which have been determined are used for the preset values of pulse width or pulse amplitude only. If measured values exceed these bounds, the effects will have to be evaluated statistically. Given this preliminary information we chose to allocate the approximately 100 EEDs that remained in a 2x5 experiment. The two level factor is the experiment type. The two experiments, described previously (section 7) are characterized by which variable is fixed and which is observed. The second factor is the level at which the fixed variable is set. Five equally spaced levels were chosen within the predetermined experimental region for each experiment. The chosen levels are given in table 7.1.

Table 7.1 Factor levels chosen for pulse width and pulse power

Width:	0.2	0.8	1.4	2.0	2.6	ms
Power:	2.0	14.0	26.0	38.0	50.0	W

Ideally the available EEDs should be allocated equally among all 10 design points. This could not be done because a small proportion of the EEDs were expected to be defective, i.e., duds. Remember that we used unsealed squibs (EEDs with only bridge wire and primary explosive present) which are not as reliable as sealed EEDs. Since each EED is destroyed upon functioning, the defective ones could be identified only when selected and tested.

After the levels of the experiment have been determined, it is then desirable to randomize the order in which the experiment is performed. It was decided to randomize across the 10 levels of the two factors since to randomly assign each EED would have been impractical. Setting the fixed variable once and testing a set of EEDs insured better repeatability and required considerably less time. While there is no reason to believe that there is a time effect in this measurement system, the randomization is done to insure against inadvertent time trends. Table A-36 from the book by

Natrella [5] was used and the ordering in table 7.2 was obtained. The selection is done simply by assigning an integer to each level in table 7.1 and moving sequentially, in any direction, through Table A36 from a randomly chosen starting point. The levels are then chosen without replacement as their integer code arises in the sequence.

Table 7.2 Randomized design sequence for measurement of EED design levels.

Test sequence	Fixed variable	Level
1	Width	0.8 ms
2	Width	2.0 ms
3	Power	38.0 W
4	Width	0.2 ms
5	Power	2.0 W
6	Width	1.4 ms
7	Power	50.0 W
8	Power	26.0 W
9	Width	2.6 ms
10	Power	14.0 W

8. Experimental Results

The experiment was conducted according to the chosen experimental design. The resulting data are given in tables 8.1 and 8.2, along with some relevant transformations of the data. Table 8.1 contains the data from experiment 1 (pulse power fixed) and table 8.2, the data from experiment 2 (pulse width fixed). All levels of experiment 1 have 7 or 8 observations. This is reasonably balanced considering the random nature of the EED reliability discussed previously. In experiment 2 all levels have 7 or 8 observations except one, the 0.2 ms width level, which contains 16 observations. This is due to an equipment failure that occurred in the system which generates and measures the pulses. After the equipment was repaired and recalibrated, the level which was completed just prior to the failure was repeated. When the new measurements exhibited no significant change, the new data were combined with the old and the experiment continued. Partly as a result of this failure, there were 46 EEDs tested in experiment 2 as compared to 36 in experiment 1.

Lineprinter plots of the data are given in figures 5 through 10. Figure 5 shows the 36 data points for experiment 1 (power fixed) and figure 6 shows the 46 data points for experiment 2. In these plots the horizontal axis is the fixed variable and the vertical axis the measured variable. Figure 7 shows the combined data and figure 8 a log-log plot of the data. The conjectured bilinear relationship and hyperbolic form are clearly exhibited in the plots. We will demonstrate that the proposed model is also supported by the statistical analysis.

Some additional plots are discussed before proceeding. The firing energy (power-width product) for each EED tested is plotted in figure 9. They are plotted in the order in which they were tested. There are two outlying data, EEDs which required larger firing energies than was typical of the majority. The corresponding log energy for each EED is plotted in figure 10. The choice of power for specifying pulse amplitude was based on

Table 8.1 Listing of data and relevant transformations of the data from experiment 1

EXPERIMENT 1 (1-POWER FIXED, 2-WIDTH FIXED)

I	EX	NR	LV	WIDTH	POWER	CURRENT	RESISTANCE	ENERGY	LOGWIDTH	LOGPOWER	LOGENERGY	FIXEDVAR	MEAS VAR	FIXLOGVAR	MSDLOGVAR
1	1	1	4	.305E+00	.380E+02	.521E+01	.140E+01	.116E+02	-.119E+01	.364E+01	.245E+01	.380E+02	.305E+00	.364E+01	-.119E+01
2	1	2	4	.285E+00	.380E+02	.479E+01	.166E+01	.108E+02	-.126E+01	.364E+01	.238E+01	.380E+02	.285E+00	.364E+01	-.126E+01
3	1	3	4	.300E+00	.380E+02	.489E+01	.159E+01	.114E+02	-.120E+01	.364E+01	.243E+01	.380E+02	.300E+00	.364E+01	-.120E+01
4	1	4	4	.350E+00	.380E+02	.495E+01	.155E+01	.133E+02	-.105E+01	.364E+01	.259E+01	.380E+02	.350E+00	.364E+01	-.105E+01
5	1	5	4	.270E+00	.379E+02	.511E+01	.145E+01	.102E+02	-.131E+01	.363E+01	.232E+01	.379E+02	.270E+00	.363E+01	-.131E+01
6	1	6	4	.250E+00	.380E+02	.524E+01	.139E+01	.950E+01	-.139E+01	.364E+01	.225E+01	.380E+02	.250E+00	.364E+01	-.139E+01
7	1	7	4	.325E+00	.380E+02	.537E+01	.132E+01	.123E+02	-.112E+01	.364E+01	.251E+01	.380E+02	.325E+00	.364E+01	-.112E+01
8	1	8	4	.330E+00	.380E+02	.529E+01	.136E+01	.125E+02	-.111E+01	.364E+01	.253E+01	.380E+02	.330E+00	.364E+01	-.111E+01
9	1	9	1	.570E+01	.202E+01	.115E+01	.153E+01	.115E+02	.174E+01	.705E+00	.245E+01	.202E+01	.570E+01	.705E+00	.174E+01
10	1	10	1	.590E+01	.200E+01	.114E+01	.155E+01	.118E+02	.177E+01	.694E+00	.247E+01	.200E+01	.590E+01	.694E+00	.177E+01
11	1	11	1	.630E+01	.200E+01	.121E+01	.138E+01	.126E+02	.184E+01	.693E+00	.253E+01	.200E+01	.630E+01	.693E+00	.184E+01
12	1	12	1	.690E+01	.200E+01	.116E+01	.148E+01	.138E+02	.193E+01	.692E+00	.262E+01	.200E+01	.690E+01	.692E+00	.193E+01
13	1	13	1	.580E+01	.200E+01	.115E+01	.152E+01	.116E+02	.176E+01	.693E+00	.245E+01	.200E+01	.580E+01	.693E+00	.176E+01
14	1	14	1	.560E+01	.200E+01	.113E+01	.157E+01	.112E+02	.172E+01	.692E+00	.241E+01	.200E+01	.560E+01	.692E+00	.172E+01
15	1	15	1	.870E+01	.200E+01	.130E+01	.119E+01	.174E+02	.216E+01	.693E+00	.286E+01	.200E+01	.870E+01	.693E+00	.216E+01
16	1	16	5	.235E+00	.500E+02	.608E+01	.136E+01	.118E+02	-.145E+01	.391E+01	.246E+01	.500E+02	.235E+00	.391E+01	-.145E+01
17	1	17	5	.200E+00	.500E+02	.605E+01	.137E+01	.100E+02	-.161E+01	.391E+01	.230E+01	.500E+02	.200E+00	.391E+01	-.161E+01
18	1	18	5	.250E+00	.500E+02	.612E+01	.134E+01	.125E+02	-.139E+01	.391E+01	.253E+01	.500E+02	.250E+00	.391E+01	-.139E+01
19	1	19	5	.205E+00	.500E+02	.624E+01	.129E+01	.103E+02	-.158E+01	.391E+01	.233E+01	.500E+02	.205E+00	.391E+01	-.158E+01
20	1	20	5	.240E+00	.500E+02	.582E+01	.148E+01	.120E+02	-.143E+01	.391E+01	.248E+01	.500E+02	.240E+00	.391E+01	-.143E+01
21	1	21	5	.240E+00	.500E+02	.599E+01	.140E+01	.120E+02	-.143E+01	.391E+01	.248E+01	.500E+02	.240E+00	.391E+01	-.143E+01
22	1	22	5	.196E+00	.500E+02	.601E+01	.139E+01	.980E+01	-.163E+01	.391E+01	.228E+01	.500E+02	.196E+00	.391E+01	-.163E+01
23	1	23	3	.520E+00	.260E+02	.418E+01	.149E+01	.135E+02	-.654E+00	.326E+01	.260E+01	.260E+02	.520E+00	.326E+01	-.654E+00
24	1	24	3	.420E+00	.260E+02	.408E+01	.157E+01	.109E+02	-.868E+00	.326E+01	.239E+01	.260E+02	.420E+00	.326E+01	-.868E+00
25	1	25	3	.450E+00	.260E+02	.429E+01	.142E+01	.117E+02	-.799E+00	.326E+01	.246E+01	.260E+02	.450E+00	.326E+01	-.799E+00
26	1	26	3	.520E+00	.260E+02	.433E+01	.139E+01	.135E+02	-.654E+00	.326E+01	.260E+01	.260E+02	.520E+00	.326E+01	-.654E+00
27	1	27	3	.370E+00	.260E+02	.427E+01	.143E+01	.962E+01	-.994E+00	.326E+01	.226E+01	.260E+02	.370E+00	.326E+01	-.994E+00
28	1	28	3	.510E+00	.260E+02	.445E+01	.132E+01	.133E+02	-.673E+00	.326E+01	.259E+01	.260E+02	.510E+00	.326E+01	-.673E+00
29	1	29	3	.410E+00	.260E+02	.441E+01	.134E+01	.107E+02	-.892E+00	.326E+01	.237E+01	.260E+02	.410E+00	.326E+01	-.892E+00
30	1	30	2	.840E+00	.140E+02	.328E+01	.130E+01	.118E+02	-.174E+00	.264E+01	.246E+01	.140E+02	.840E+00	.264E+01	-.174E+00
31	1	31	2	.730E+00	.140E+02	.317E+01	.139E+01	.102E+02	-.315E+00	.264E+01	.232E+01	.140E+02	.730E+00	.264E+01	-.315E+00
32	1	32	2	.770E+00	.140E+02	.315E+01	.141E+01	.108E+02	-.261E+00	.264E+01	.238E+01	.140E+02	.770E+00	.264E+01	-.261E+00
33	1	33	2	.870E+00	.140E+02	.331E+01	.128E+01	.122E+02	-.139E+00	.264E+01	.250E+01	.140E+02	.870E+00	.264E+01	-.139E+00
34	1	34	2	.920E+00	.140E+02	.316E+01	.140E+01	.129E+02	-.834E-01	.264E+01	.256E+01	.140E+02	.920E+00	.264E+01	-.834E-01
35	1	35	2	.780E+00	.140E+02	.319E+01	.138E+01	.109E+02	-.248E+00	.264E+01	.239E+01	.140E+02	.780E+00	.264E+01	-.248E+00
36	1	36	2	.880E+00	.140E+02	.328E+01	.131E+01	.123E+02	-.128E+00	.264E+01	.251E+01	.140E+02	.880E+00	.264E+01	-.128E+00

Table 8.2 Listing of data and relevant transformations of the data from experiment 2

EXPERIMENT 2 (1-POWER FIXED, 2-WIDTH FIXED)

I	EX	NR	LV	WIDTH	POWER	CURRENT	RESISTANCE	ENERGY	LOGWIDTH	LOGPOWER	LOGENERGY	FIXEDVAR	MEAS VAR	FIXLOGVAR	MSOLOGVAR
37	2	1	2	.800E+00	.150E+02	.304E+01	.163E+01	.120E+02	-.223E+00	.271E+01	.249E+01	.800E+00	.150E+02	-.223E+00	.271E+01
38	2	2	2	.800E+00	.135E+02	.327E+01	.126E+01	.108E+02	-.223E+00	.260E+01	.238E+01	.800E+00	.135E+02	-.223E+00	.260E+01
39	2	3	2	.800E+00	.133E+02	.310E+01	.139E+01	.106E+02	-.223E+00	.259E+01	.237E+01	.800E+00	.133E+02	-.223E+00	.259E+01
40	2	4	2	.800E+00	.132E+02	.292E+01	.155E+01	.106E+02	-.223E+00	.258E+01	.236E+01	.800E+00	.132E+02	-.223E+00	.258E+01
41	2	5	2	.800E+00	.144E+02	.321E+01	.139E+01	.115E+02	-.223E+00	.266E+01	.244E+01	.800E+00	.144E+02	-.223E+00	.266E+01
42	2	6	2	.800E+00	.164E+02	.345E+01	.138E+01	.132E+02	-.223E+00	.280E+01	.258E+01	.800E+00	.164E+02	-.223E+00	.280E+01
43	2	7	2	.800E+00	.149E+02	.333E+01	.134E+01	.119E+02	-.223E+00	.270E+01	.248E+01	.800E+00	.149E+02	-.223E+00	.270E+01
44	2	8	2	.800E+00	.167E+02	.345E+01	.141E+01	.134E+02	-.223E+00	.282E+01	.259E+01	.800E+00	.167E+02	-.223E+00	.282E+01
45	2	9	4	.200E+01	.679E+01	.226E+01	.133E+01	.135E+02	.693E+00	.192E+01	.261E+01	.200E+01	.679E+01	.693E+00	.192E+01
46	2	10	4	.200E+01	.605E+01	.214E+01	.132E+01	.121E+02	.693E+00	.180E+01	.249E+01	.200E+01	.605E+01	.693E+00	.180E+01
47	2	11	4	.200E+01	.584E+01	.208E+01	.135E+01	.117E+02	.693E+00	.176E+01	.246E+01	.200E+01	.584E+01	.693E+00	.176E+01
48	2	12	4	.200E+01	.555E+01	.202E+01	.136E+01	.111E+02	.693E+00	.171E+01	.241E+01	.200E+01	.555E+01	.693E+00	.171E+01
49	2	13	4	.200E+01	.500E+01	.185E+01	.147E+01	.100E+02	.693E+00	.161E+01	.230E+01	.200E+01	.500E+01	.693E+00	.161E+01
50	2	14	4	.200E+01	.501E+01	.182E+01	.152E+01	.100E+02	.693E+00	.161E+01	.230E+01	.200E+01	.501E+01	.693E+00	.161E+01
51	2	15	4	.200E+01	.637E+01	.214E+01	.139E+01	.127E+02	.693E+00	.185E+01	.254E+01	.200E+01	.637E+01	.693E+00	.185E+01
52	2	16	4	.200E+01	.510E+01	.191E+01	.141E+01	.102E+02	.693E+00	.163E+01	.232E+01	.200E+01	.510E+01	.693E+00	.163E+01
53	2	17	1	.200E+00	.531E+02	.619E+01	.139E+01	.106E+02	-.161E+01	.397E+01	.236E+01	.200E+00	.531E+02	-.161E+01	.397E+01
54	2	18	1	.200E+00	.512E+02	.619E+01	.134E+01	.102E+02	-.161E+01	.393E+01	.233E+01	.200E+00	.512E+02	-.161E+01	.393E+01
55	2	19	1	.200E+00	.553E+02	.631E+01	.139E+01	.111E+02	-.161E+01	.401E+01	.240E+01	.200E+00	.553E+02	-.161E+01	.401E+01
56	2	20	1	.200E+00	.440E+02	.571E+01	.135E+01	.880E+01	-.161E+01	.378E+01	.218E+01	.200E+00	.440E+02	-.161E+01	.378E+01
57	2	21	1	.200E+00	.487E+02	.619E+01	.127E+01	.973E+01	-.161E+01	.388E+01	.228E+01	.200E+00	.487E+02	-.161E+01	.388E+01
58	2	22	1	.200E+00	.525E+02	.643E+01	.127E+01	.105E+02	-.161E+01	.396E+01	.235E+01	.200E+00	.525E+02	-.161E+01	.396E+01
59	2	23	1	.200E+00	.562E+02	.643E+01	.136E+01	.112E+02	-.161E+01	.403E+01	.242E+01	.200E+00	.562E+02	-.161E+01	.403E+01
60	2	24	1	.200E+00	.524E+02	.631E+01	.132E+01	.105E+02	-.161E+01	.396E+01	.235E+01	.200E+00	.524E+02	-.161E+01	.396E+01
61	2	25	1	.200E+00	.481E+02	.619E+01	.126E+01	.962E+01	-.161E+01	.387E+01	.226E+01	.200E+00	.481E+02	-.161E+01	.387E+01
62	2	26	1	.200E+00	.457E+02	.536E+01	.159E+01	.914E+01	-.161E+01	.382E+01	.221E+01	.200E+00	.457E+02	-.161E+01	.382E+01
63	2	27	1	.200E+00	.581E+02	.655E+01	.136E+01	.116E+02	-.161E+01	.406E+01	.245E+01	.200E+00	.581E+02	-.161E+01	.406E+01
64	2	28	1	.200E+00	.517E+02	.583E+01	.152E+01	.103E+02	-.161E+01	.394E+01	.234E+01	.200E+00	.517E+02	-.161E+01	.394E+01
65	2	29	1	.200E+00	.539E+02	.619E+01	.141E+01	.108E+02	-.161E+01	.399E+01	.238E+01	.200E+00	.539E+02	-.161E+01	.399E+01
66	2	30	1	.200E+00	.634E+02	.679E+01	.138E+01	.127E+02	-.161E+01	.415E+01	.254E+01	.200E+00	.634E+02	-.161E+01	.415E+01
67	2	31	1	.200E+00	.880E+02	.607E+01	.239E+01	.176E+02	-.161E+01	.448E+01	.287E+01	.200E+00	.880E+02	-.161E+01	.448E+01
68	2	32	1	.200E+00	.585E+02	.630E+01	.148E+01	.117E+02	-.161E+01	.407E+01	.246E+01	.200E+00	.585E+02	-.161E+01	.407E+01
69	2	33	3	.140E+01	.978E+01	.232E+01	.182E+01	.137E+02	.336E+00	.228E+01	.262E+01	.140E+01	.978E+01	.336E+00	.228E+01
70	2	34	3	.140E+01	.966E+01	.250E+01	.155E+01	.135E+02	.336E+00	.227E+01	.260E+01	.140E+01	.966E+01	.336E+00	.227E+01
71	2	35	3	.140E+01	.957E+01	.262E+01	.140E+01	.134E+02	.336E+00	.226E+01	.260E+01	.140E+01	.957E+01	.336E+00	.226E+01
72	2	36	3	.140E+01	.866E+01	.244E+01	.146E+01	.121E+02	.336E+00	.216E+01	.250E+01	.140E+01	.866E+01	.336E+00	.216E+01
73	2	37	3	.140E+01	.897E+01	.250E+01	.144E+01	.126E+02	.336E+00	.219E+01	.253E+01	.140E+01	.897E+01	.336E+00	.219E+01
74	2	38	3	.140E+01	.719E+01	.232E+01	.134E+01	.101E+02	.336E+00	.197E+01	.231E+01	.140E+01	.719E+01	.336E+00	.197E+01
75	2	39	3	.140E+01	.853E+01	.250E+01	.137E+01	.119E+02	.336E+00	.214E+01	.248E+01	.140E+01	.853E+01	.336E+00	.214E+01
76	2	40	5	.260E+01	.461E+01	.185E+01	.136E+01	.120E+02	.956E+00	.153E+01	.248E+01	.260E+01	.461E+01	.956E+00	.153E+01
77	2	41	5	.260E+01	.458E+01	.179E+01	.144E+01	.119E+02	.956E+00	.152E+01	.248E+01	.260E+01	.458E+01	.956E+00	.152E+01
78	2	42	5	.260E+01	.423E+01	.179E+01	.133E+01	.110E+02	.956E+00	.144E+01	.240E+01	.260E+01	.423E+01	.956E+00	.144E+01
79	2	43	5	.260E+01	.495E+01	.185E+01	.146E+01	.129E+02	.956E+00	.160E+01	.256E+01	.260E+01	.495E+01	.956E+00	.160E+01
80	2	44	5	.260E+01	.445E+01	.173E+01	.150E+01	.116E+02	.956E+00	.149E+01	.245E+01	.260E+01	.445E+01	.956E+00	.149E+01
81	2	45	5	.260E+01	.363E+01	.167E+01	.131E+01	.943E+01	.956E+00	.129E+01	.224E+01	.260E+01	.363E+01	.956E+00	.129E+01
82	2	46	5	.260E+01	.376E+01	.173E+01	.126E+01	.977E+01	.956E+00	.132E+01	.228E+01	.260E+01	.376E+01	.956E+00	.132E+01

knowledge of the electrical resistance of each EED. The values of the individual EED resistances, which were measured with relatively high accuracy, are plotted in figure 11. The ordering corresponds to the previously discussed energy plots. The two EEDs which were outliers in firing energy are also outliers in electrical resistance. In this case the 15th EED is a moderate outlier with lower resistance than all the others and the 67th EED is a high resistance outlier. Both, however, required a higher energy to fire than was typical of the other EEDs. Figure 12 is a plot of firing energy vs resistance for all of the EEDs tested. The same two EEDs, which fall in the upper left and right corners, are, once again, atypical.

Since the EEDs which required excessive firing energies also exhibited nontypical resistances the possibility of screening EEDs by measuring their electrical resistance is suggested. While this is possible, it is not done here. The present study was not directed toward studying outliers and there is insufficient information in these data to say anything definite about them. The history of the EEDs used is unknown. Therefore crucial knowledge about differences in age, manufacture, storage conditions, etc. is unavailable. All of the data are therefore included in the main analysis as it is possible that such behavior is typical. The effects on the main analysis of adjusting for the outliers will be discussed, however.

8.1 Statistical Analysis

The linear model given in eq (1) is a general model which enables a wide choice of specific models. The exact model is specified by the choice of the design matrix. It is good practice to fit several likely models to the data and choose the best one. "Best" can be interpreted as the model that provides a good fit determined by statistical methods, yet is parsimonious (contains only necessary parameters).

Models were fit to the data beginning with the general design matrix given by eq (6). Several iterations in the analysis determined that the intercept parameter (column 1) and the experimental effect parameter (column 2) were unnecessary. The independent variable in column 3 is sufficient to provide a good fit to the data. The firing level behavior of the EEDs can therefore be described by the relatively simple eqs (8) and (9).

The chosen design matrix has dimensions 82x1 (82 EEDs tested), and its components are the inverse values of the fixed variable (width or power). The raw residual variances were nonhomogeneous; therefore weighted least squares was necessary. The form of the nonhomogeneity is specified in eq (8), and was suggested by statistical theory. The weights are the inverse of the error variance under this model and proved effective at stabilizing the residuals on the given data. The parameter vector $\underline{\beta}$ reduces, in this case, to a scalar β.

The weighted least squares estimate of β is 11.60 and the standard deviation of the estimate is 0.17 based on the data from 82 EEDs. The units of β are watt-milliseconds. Therefore corresponding relationships

$$P = 11.60\ w^{-1} \qquad (22a)$$

$$w = 11.60\ P^{-1} \qquad (22b)$$

are the best predictors of P and w respectively within the range of experimentation. The values must be in the correct units, milliseconds for width (w) and watts for power (P). Equations (22) exhibit the symmetry of eq (8).

Simplifying eq (2) for this model shows that the estimate of β is simply the arithmetic mean of the firing energies. This fact suggests a simpler analysis. It is not recommended, however, except as an exploratory tool or perhaps for testing a batch of EEDs for which this model has already been proven correct and its valid experimental region determined. The full linear model approach that we describe makes possible a thorough assessment of the appropriateness of the model for the data. Foremost, it allows the effects of the two factors, width and power, to be separated, whereas they would be confounded in a simple energy model. Since the utility of the method suggested here, and of the resulting graphics, is to determine the likelihood of firing for a given pulse configuration (width and amplitude), these factors must remain explicit in the experimentation and analysis. Another advantage, since computation is done almost exclusively on computers, is that good statistical software for linear models is readily available. Such software usually provides excellent diagnostic tools for use in the model validation. Good graphics and exploratory capabilities are also common.

Some diagnostic statistical plots which are of use in assessing the fit of the model in eq (22) to the data (Tables 8.1 and 8.2) are displayed in figures 13 through 16. The standardized residuals are plotted in figure 13. They should exhibit a random fluctuation with constant mean and homogeneous variance. With the exception of the two outliers on the upper margin and a cluster of low points in the right half, they appear reasonable. Figure 14 is a Normal probability plot of the residuals which, except for two outliers, is linear. This indicates the residuals are Normally distributed, an important assumption for some of the diagnostic tests and confidence intervals. The plot in figure 15 shows the standardized residuals as a function of predicted values. This plot should show homogeneity with respect to both location and variance of each group. The plot exhibits this property with the exception of a slight drop in mean on the right most group of points. Figure 16 is a plot of the standardized residuals plotted against their adjacent (lag one) values. There should be no diagonal pattern, only a random cluster. Except for outliers this is exhibited in the plot. Overall the plots indicate a good fit of the model to the data. Except for the outliers there is only a slight suggestion that the model is beginning to change at the extremes of the experimental region. This will be discussed in more detail in the next section. We conclude with the decomposition of the sum of the squares of the data given in table 8.3, where df is degrees of freedom.

Table 8.3 Sum of squares decomposition for the fitted model in eq (22).

Total sum of squares:	11234.64	(82 df)
Sum of squares due to model:	11041.05	(1 df)
Residual sum of squares:	193.59	(81 df)

The model therefore accounts for more than 98.25 percent of the variation in the data.

8.2 Effects of Extremal Data

Various models and subsets of the data were considered in the exploratory phase of this study. We will first evaluate the effects of the two outlying data points which were discussed in the previous section. These are the 15th observation in experiment 1 and the 31st observation in experiment 2. Both points represented energies in excess of 17.4 millijoules while all the remaining data ranged between 8.8 and 13.7 millijoules with a mean of 11.45 and a standard deviation of 1.24. The outliers are at least 4.8 standard deviations from the mean.

If the random variation has a Gaussian distribution, as it appears to have, then this is an extremely unlikely event. If occasional outliers are typical of EED behavior then our model must be extended to account for the outliers. Further experimentation and analysis must be done to determine if this is necessary. These points are therefore included in the main analysis; it may be typical for this type of EED (squib). If these two points are deleted, the point estimate of β is then 11.45 and its standard deviation is 0.14. The classical estimates of mean and variance are known to be sensitive to outliers. In this case, 2 out of 82 EEDs required extreme firing energies and caused a shift in the mean of one standard deviation. Considering the consequences of EED failures, the issue of outliers requires a more complete investigation than was possible here. In this case 98.8 percent of the variation is accounted for by the model and the estimates are more typical of the majority of EEDs.

Another type of extreme is related to measurements falling on the boundaries of the design region. In section 7.1 bounds were determined using thermodynamic arguments for a region in which the model was likely to be valid. The fixed factors in the experiments were constrained to remain within these bounds. The measured factors, however, could not be constrained a priori; the values were determined by the EED characteristics. It can be seen from the data tables 8.1 and 8.2 or from plots of the data that in each experiment responses in one of the levels exceeded these limits. This occurred in level 1 of experiment 1 (EEDs 9 through 15) and level 1 of experiment 2 (EEDs 53 through 68). In the former, the power was fixed at nominal level of 2 watts and the measured pulse width ranges from 5.6 to 8.7 milliseconds which exceeded the chosen limit of 2.7 milliseconds. In the latter, the width was fixed at 0.2 milliseconds and the measured pulse power ranged from 44 to 87.9 watts whereas the chosen limit was 50 watts. Recall that heat loss during the pulse becomes significant for wide pulses and that amplifier limitations cause pulse shape distortion at high power levels.

All the data were retained in the main analysis but it was indicated that some slight, but not significant, effects were noticeable. The two levels discussed here, on the design boundary, contained the two outliers. This is believed to be coincidental since the external behavior of those EEDs is not believed to be related to the thermodynamic and instrumentation limits.

Models were fit to the 59 data values which remain after deleting these two levels. There was some improvement in the diagnostic plots due to removal of the marginal points. The analysis, however, showed little change from the one with all the data. The estimate of β is 11.61, almost identical to the original, and the standard deviation of the estimate is 0.159. The total sum of squares is 7963.67 and 98.9 percent of this is accounted for by the model.

Since the difference between the trimmed 59 point analysis and the original 82 point analysis was insignificant, we retain the full analysis. Note, however, that data in the extreme regions of the experiment are exhibiting some slight changes. These are due to the thermodynamic and instrumentation considerations discussed in section 7.1. In this sense there is reasonable agreement between the statistical results and the non-statistical thermodynamic analysis. Extrapolation to higher power levels and correspondingly shorter pulse durations can be done if a different pulse source that can generate these higher levels is available.

Physical theory suggests that these limits could be extended in all but one direction. Pulse power may grow arbitrarily high and width arbitrarily small and the relationship given by the model should still hold. Statistically this is only a conjecture since we could not generate pulses outside the given limits. The upper limit on pulse width, however, is a restriction of the model. Heat loss during the pulse would cause nonlinearities which were not accounted for in the present model. Extrapolation in this direction is not possible except as a bound. Clearly more pulse power will be required for a given width due to heat energy loss. Approximate firing likelihoods might also be obtained for non rectangular pulses as long as the width is approximately correct (and less than $\tau/6$) and total energy the same. The firing likelihood plots can therefore provide approximations outside the established bounds based on physical considerations.

9. Implementation of the Firing Probability Plots

A most useful tool for assessing EED behavior which is made possible by this research is the firing likelihood plot (FLP). It graphically summarizes relevant information from the analysis in a format that is useful and easily interpreted. In this section we describe how to implement these plots from the data analysis which was described in section 8. It is first necessary to discuss two distinct topics related to the FLPs. This is done in sections 9.1 and 9.2.

9.1 Tolerance Intervals

A commonly used statistical interval estimate is the confidence interval. It is an interval which, given validity of the assumptions, has a known probability of containing the unknown parameter of interest. In this case a 99 percent confidence interval could be derived for parameter β. It would provide information on where the mean firing energy is for this type of EED. An interval for the mean firing energy would be of little use however. Concern in EED applications is for the probability that a given type of EED fires (or does not fire) given a particular input pulse or train of pulses.

Tolerance intervals address this issue more directly. They are intervals within which, with a given probability γ, a chosen proportion α of the population will lie. In the case of EEDs, a γ percent tolerance interval estimates the range of firing energies, expressed in terms of pulse width and pulse amplitude, which would fire 99 percent of the EEDs. The coefficient γ is the probability that the resulting interval is correct. It is analogous to the confidence coefficient in a confidence interval. There are various types of tolerance intervals. One-sided intervals provide either lower or upper bounds on the percentile of the distribution. While these may also have applications to EED evaluation, we will develop only the two-sided intervals. The one-sided intervals are simple adaptations of the two-sided ones. Note that different tables are required depending on whether one-sided or two-sided intervals are being used. A good reference for both tables and instructions for implementation of tolerance intervals is Natrella [5]. They can also be implemented without distributional assumptions.

The firing energy, the estimate, $\hat{\beta}$, and the standard deviation, s, are central in adapting tolerance intervals for firing likelihood plots. A proportion P_r of the underlying population, Normal in this case, must be chosen. Then a confidence coefficient γ, which is a probability that the interval is correct, must be chosen. Table A6 of [5], a look-up table of values of K for calculating two-sided tolerance limits, is then used to obtain the specific value of K for the desired γ, P_r and df. Upper and lower bounds for the tolerance interval are then given by the limits:

$$\hat{\beta} + Ks \tag{23a}$$

$$\hat{\beta} - Ks \; . \tag{23b}$$

Each probability contour on the firing likelihood plot requires a pair of these bounds. We must digress to discuss plot implementation before we can use these intervals.

9.2 Plot Implementation

It is possible that a problem may arise in the algorithm for generating plots. This will apply to most graphics software. The issue is the hyperbolic nature of the function being plotted. Choosing equally spaced points along either axis will generate plots which have a dense grid on the chosen axis but are very sparse on the orthogonal axis. Even logarithmic scaling will not alleviate this problem.

The approach taken was to derive a parametric equation, develop an equispaced index for the parameter, and then generate the plotted points from these values. The procedure is as follows:

The basic contours are described by the equation $Pw = \Theta^2$ where P is the pulse power value and w is the pulse width value. We choose the following pair of parametric equations:

$$w = \Theta^{1-t}$$

$$P = \Theta^{1+t}.$$

To determine bounds on t observe that:

$$w_{max} = \Theta^{1-t_{min}}$$

$$P_{max} = \Theta^{1+t_{max}}.$$

These equations are equivalent to the following:

$$t_{min} = 1 - \frac{\log w_{max}}{\log \Theta}$$

$$t_{max} = \frac{\log P_{max}}{\log \Theta} - 1 \quad .$$

Given the maximum w and P chosen for the plots, find t_{min} and t_{max}. Then choose n, the number of points to be plotted.
Compute:

$$t_i = t_{min} + \frac{(t_{max} - t_{min})}{n-1} (i-1) \ ; \ i = 1,...,n$$

and the corresponding pairs of points

$$w_i = \Theta^{1-t_i} \qquad (24a)$$

$$;i = 1,...,n$$

$$p_i = \Theta^{1+t_i}. \qquad (24b)$$

The chosen parameterization corresponds to $pw = \Theta^2$, not Θ. Therefore, if $\hat{\beta}$ is the estimate in the linear model, the proper relationship for plotting the mean firing level would be $\Theta = (\hat{\beta})^{1/2}$. Details for relating this parameterization to the plotting of other contours will be given in the following section.

9.3 Generating the Graphs

A pair of contours for the firing likelihood plots are now simply implemented. For a given P_r and γ, obtain the limits given by eqs 23a and 23b. Then let $\Theta_1 = (\hat{\beta} + Ks)^{1/2}$ and $\Theta_2 = (\hat{\beta} - Ks)^{1/2}$, where Θ_1 and Θ_2 are each substituted for Θ in eqs (24a) and (24b). Generate a set of points to be plotted, as explained in section 9.2 for both Θ_1 and Θ_2. These families of points will generate the 100 P_r percent firing likelihood curves. This process is repeated for each P_r for which a curve is desired. Examples are given in figures 17 and 18 for the EEDs tested.

9.4 Interpreting the Graphs

Firing likelihood plots provide a convenient representation of an operating characteristic of a class of EEDs. One can graphically assess the probability of firing for an EED from this class when it is subjected to a rectangular input pulse of a given width and amplitude. Once the experiment described herein is performed on a representative sample of EED's and the resulting statistical analysis is completed, then any relevant probability level (P_r) may be chosen for the plot contours. The contours define a set of pulse dimensions which are almost certain (i.e., with probability γ; sect. 9.1) to cause a proportion P_r of the EED's to fire. In most cases the extreme values of P_r will be of interest, near zero and near one, corresponding to maximum no-fire and minimum all-fire levels, respectively. In the example given the contours may be thought of as joint no-fire/all-fire limits.

Alternative versions of firing likelihood plots are also possible. for example, the one discussed above is based on a two-sided tolerance interval and the two contours only have meaning when considered as a pair. An alternative might be to use one-sided tolerance intervals which have an individual interpretation. A P_r = 0.01 one-sided contour would provide a set of pulse widths and pulse amplitudes above which 99 percent of the EED's will fire. Since the Gaussian distribution is symmetric, the same constant is also used for a one-sided P_r = 0.99 contour. Natrella [5] also includes tables and methods for one-sided tolerance intervals. Other alternatives include plots based on distributional quantile estimates or confidence intervals.

It is possible to relate the curves directly to electromagnetic field intensity for a given physical configuration. Since pulse power is proportional to E^2, electric field strength squared, the power axis of the plots may also be expressed in terms of the peak field strength squared of an impulsive field. A value of E^2 that corresponds to a specific power level is required for determining the position of the E^2 scale. This may be obtained by measuring steady state values of E and power for a particular EED support structure. Usually the coupling of electromagnetic energy from a field to the structure and hence to the EED is poorly known.

In a real world situation the pulses are not likely to be rectangular, and even if they are, most antennas will cause ringing due to phase distortion. Probabilities which are based on an ideal (rectangular) test situation are therefore likely to be conservative because of the decrease in actual energy coupled into the EED by an irregularly shaped pulse whose maximum values of amplitude and duration are equal to those of a rectangular pulse. Such a representation can extend the usefulness of the firing likelihood plots to cover most irregular pulse shapes.

There is no physical reason that the left end of the firing likelihood plots cannot be extended beyond the experimental region. The limitation was imposed due to the inability of the test system to generate rectangular pulses of higher power and lesser pulse width. These limitations are related to maximum pulse amplifier power and/or oscilloscope rise time specifications. Extension beyond this limit could not be experimentally verified and statistical prediction was not considered. Such an extension of the plots should therefore be used with discretion as it is possible that unanticipated factors may affect the model in this region. Extension to the right of the experimental region is not possible under the given model. Physical reasons previously stated limit extensions to the right.

The interpretation assumes adequate cooling time between pulses. If pulse repetition occurs in less than approximately 5 thermal time constants then eqs (14) and (16) will have to be used to estimate the cumulative heating effect. The consequent cumulative heating will cause the the EED to fire at a lower pulse energy level. Since temperature increases proportionally with power, it is possible to calculate a correction factor for closely spaced periodic pulses. It is also possible to relate the firing likelihood plots to aperiodic pulse trains but computations are more complex and require consideration of specific aperiodic sequences.

10. Conclusions

A new method which integrates both statistical and engineering concepts has been proposed for characterizing EEDs. The method provides a useful description of performance for a class of EEDs based on rigorous and efficient statistical procedures. The methodology has been proven and demonstrated in an actual experiment and is applicable to a wide class of EEDs. The resulting firing likelihood plots provide information which is relevant and not previously available.

While these contributions are unique, they suggest further extensions of this work. These include models where the electrical resistance is not measured, statistical methods for efficient estimation of the thermodynamic parameters, extension of the present model outside of the given limits, the link from this model to induced currents caused by electromagnetic fields, and the extension to EED types which are affected by previous heating (binder melts). We feel this work is only an important first step toward new and rigorous methods of characterizing EED performance. With the increasing use of EEDs, the increasingly complex electromagnetic environment and the nature of many EED applications it becomes a very important problem. The consequences of an inadvertent firing or a failure to fire could be extremely costly. Optimal statistical procedures for characterizing EEDs will help minimize the likelihood of such events.

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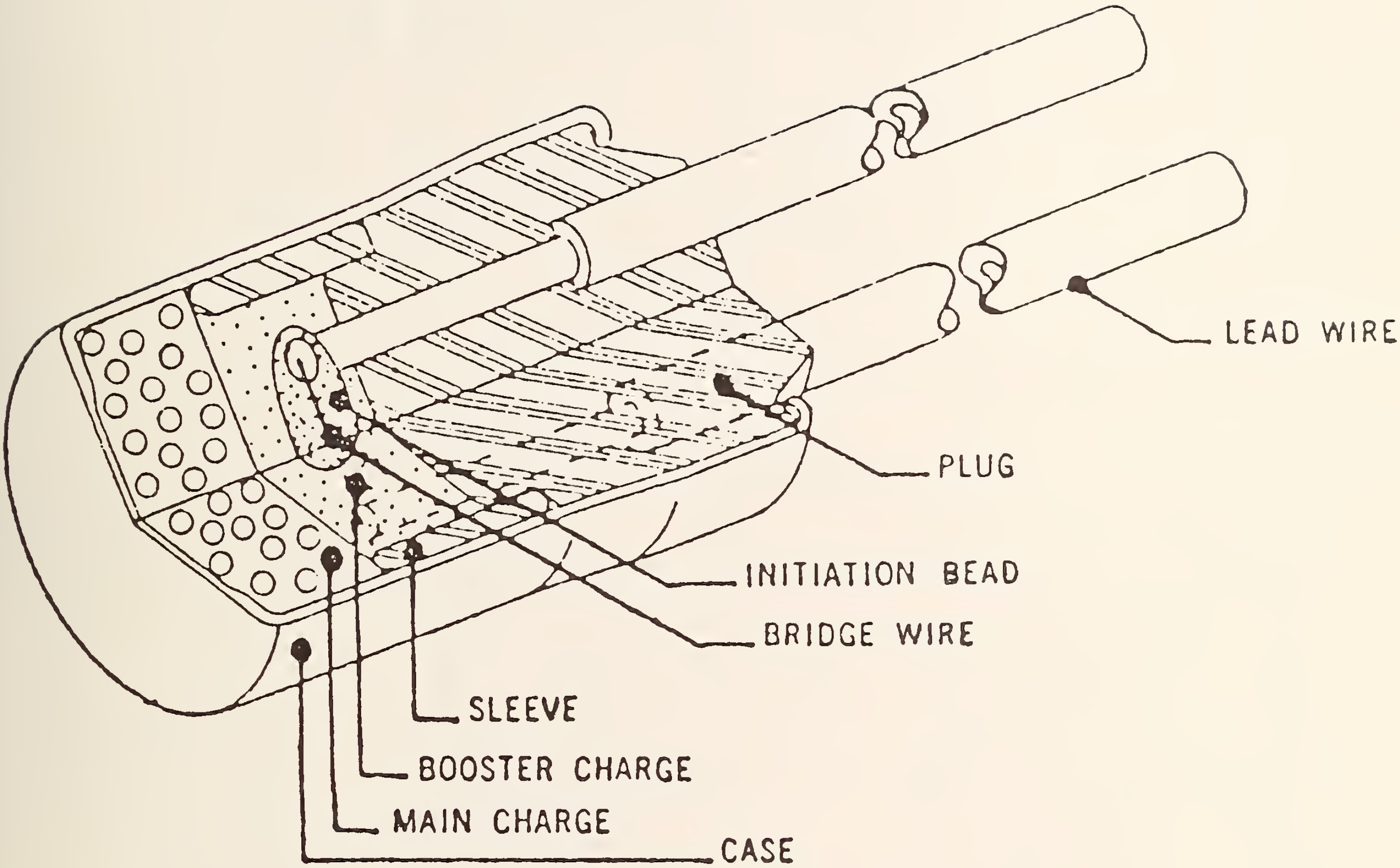


Figure 1. Structural diagram of a typical EED.

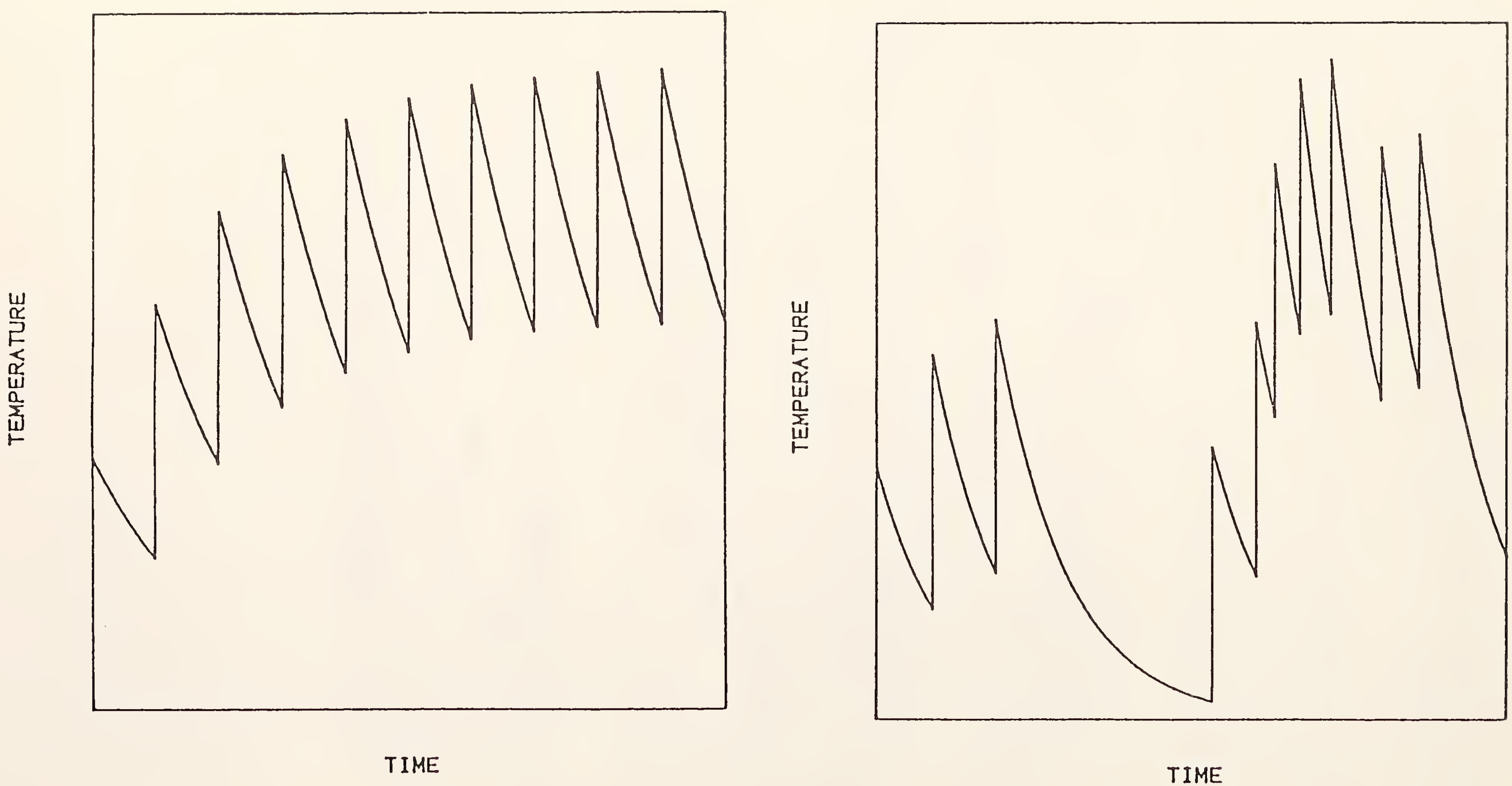


Figure 2. Illustration of the cumulative heating (stacking) of an EED due to a train of periodic (left) or aperiodic (right) pulses.

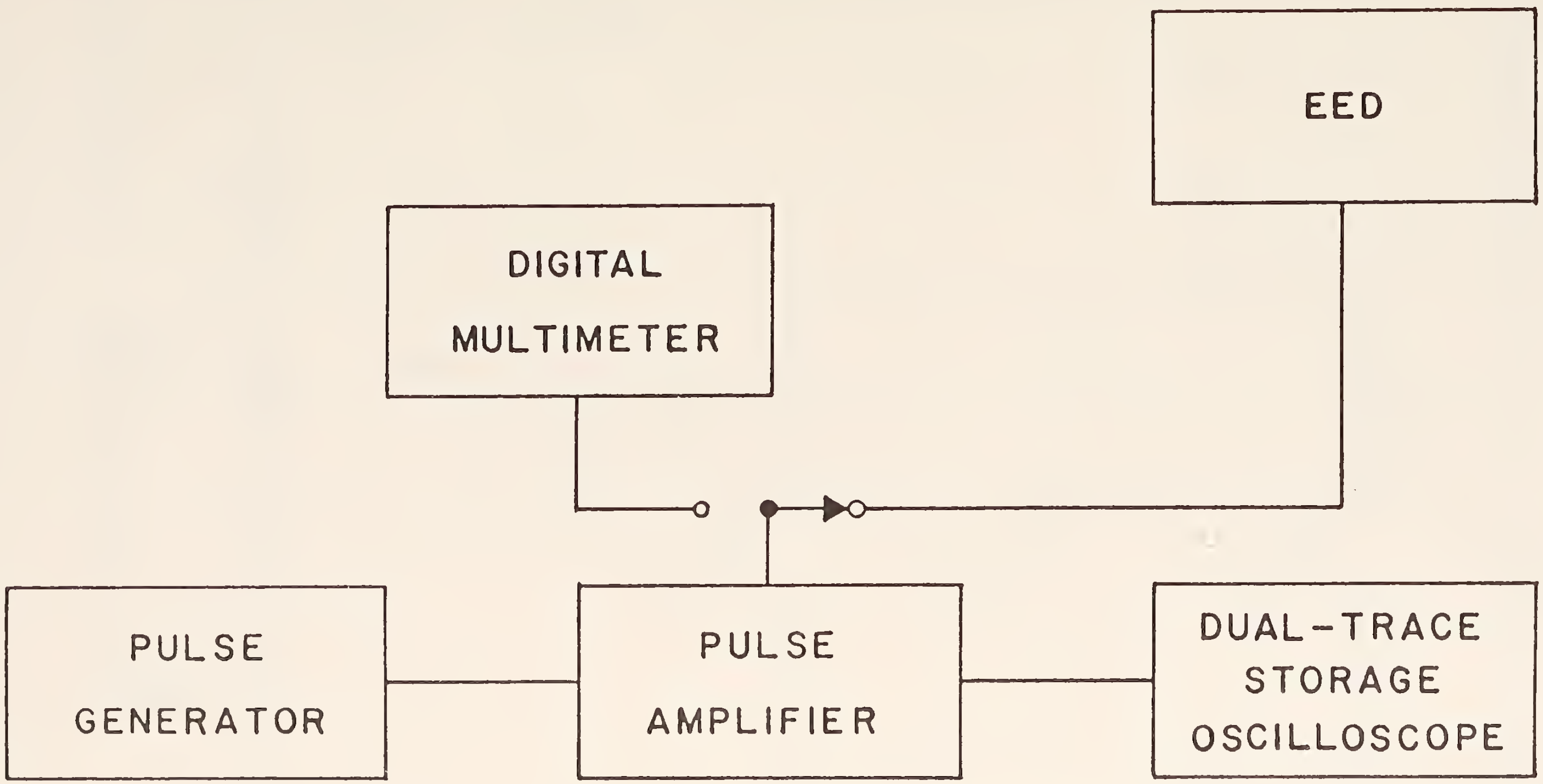


Figure 3. Block diagram of equipment setup.

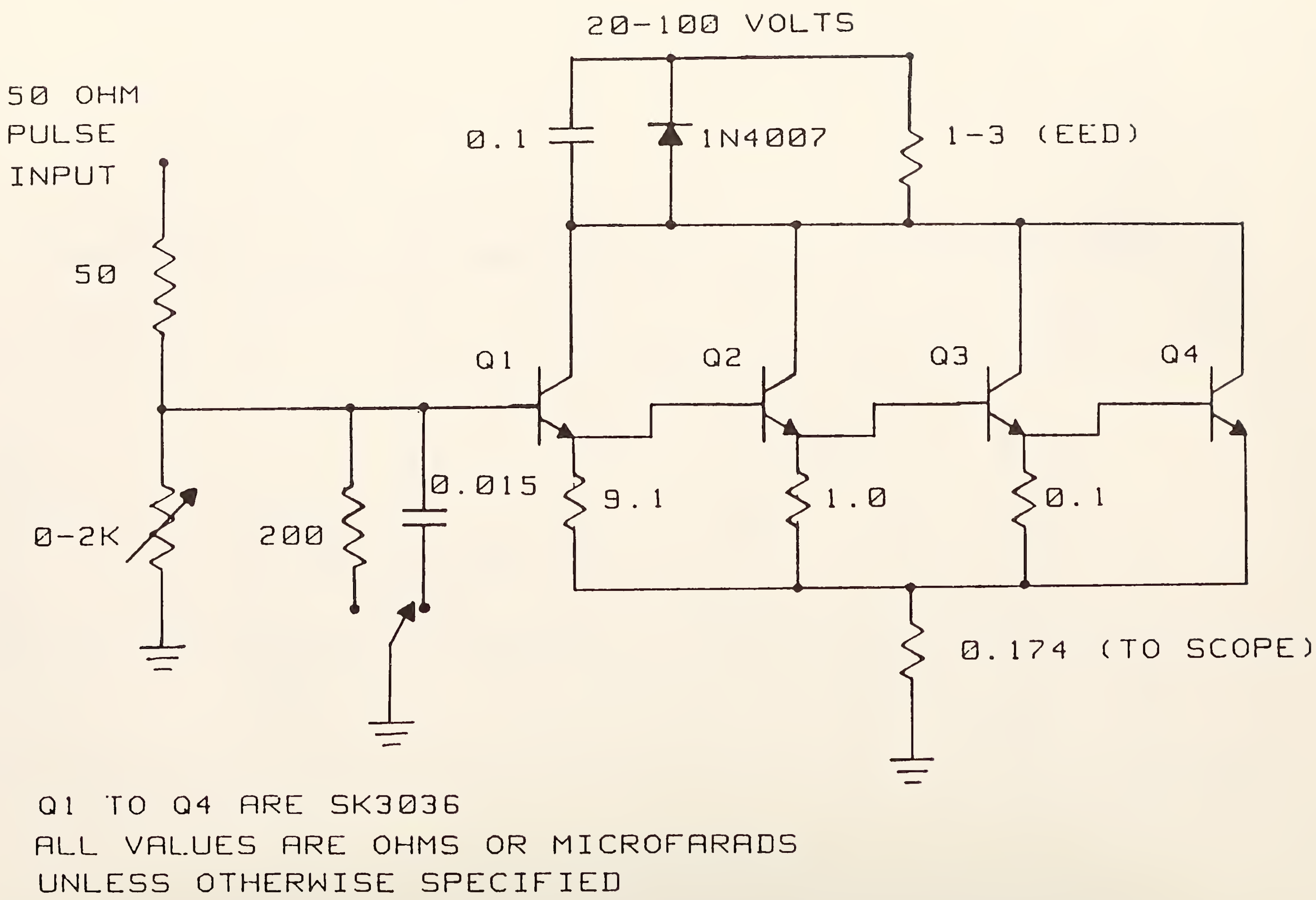
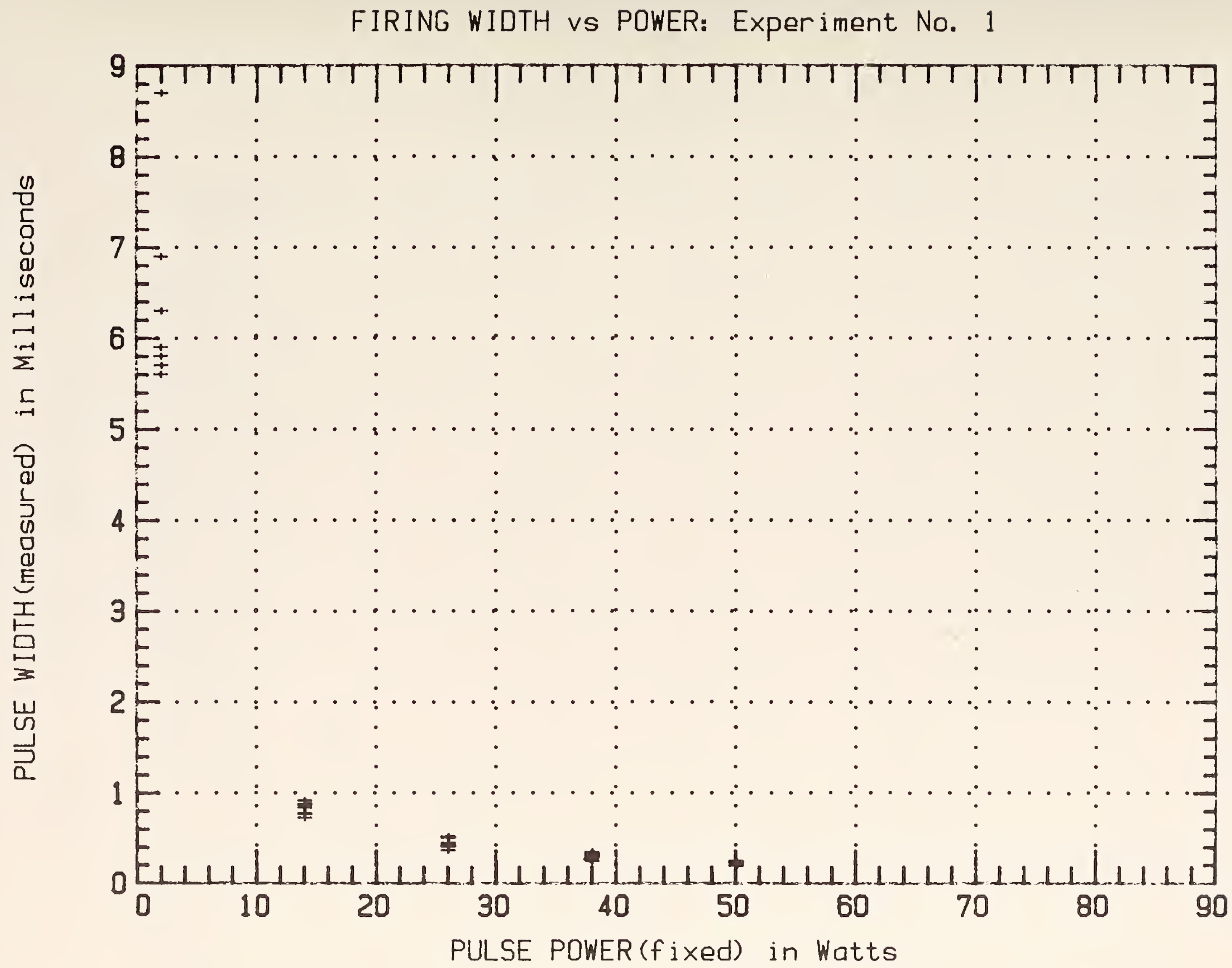
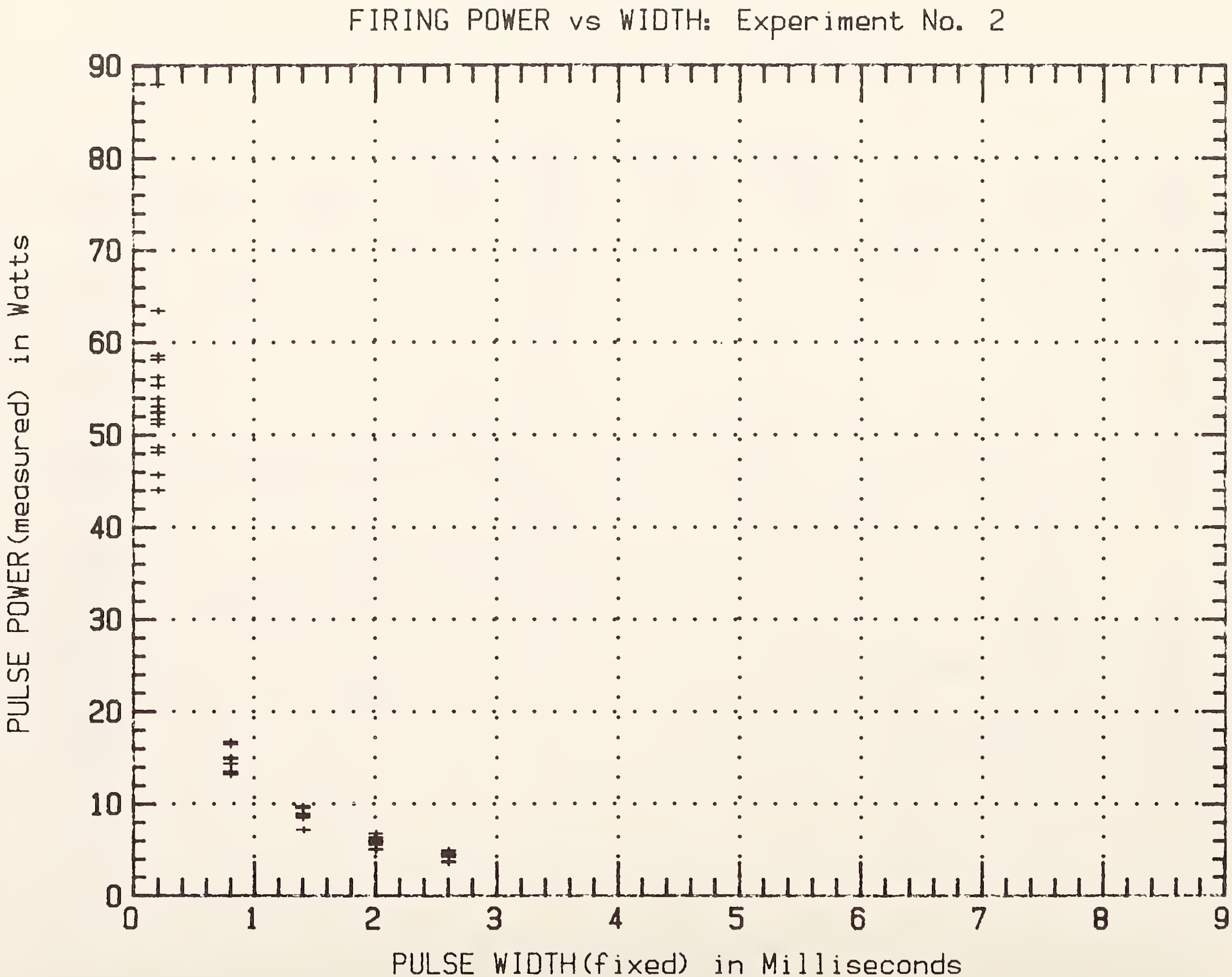


Figure 4. Pulse amplifier schematic.



Power (watts)

Figure 5. Plot of data for experiment 1 (fixed power), 36 points.



Width (milliseconds)

Figure 6 . Plot of data for experiment 2 (fixed width), 46 points.

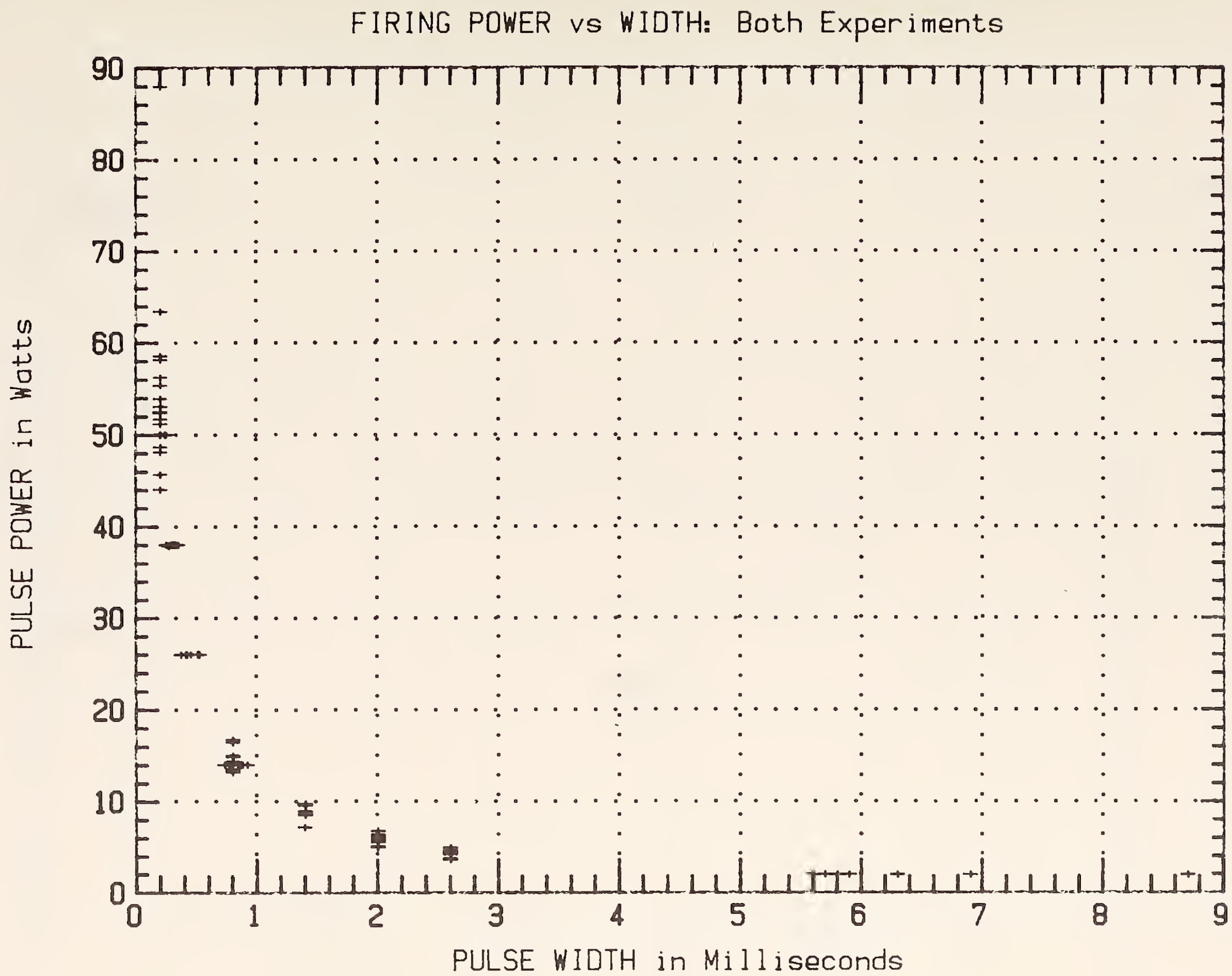
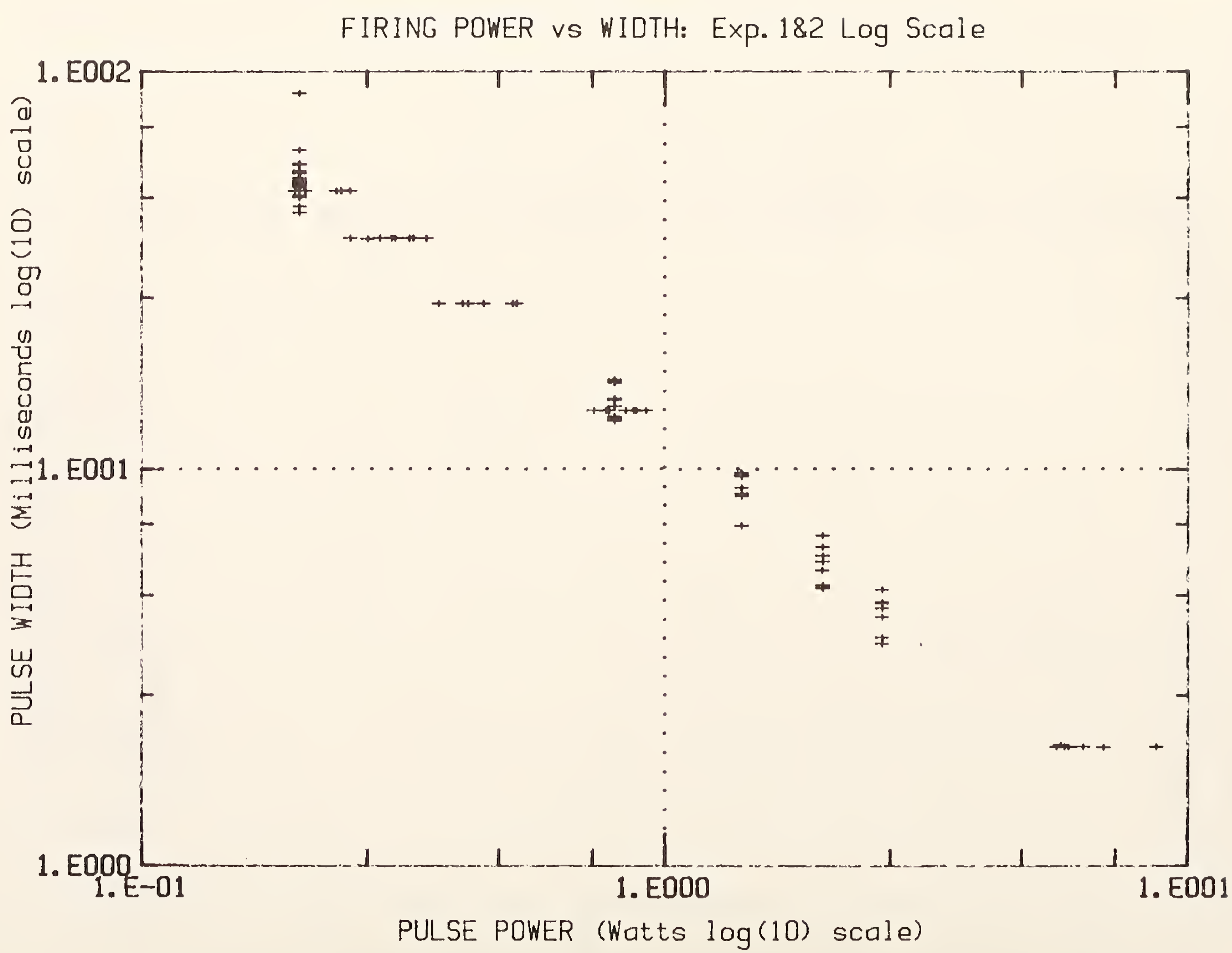
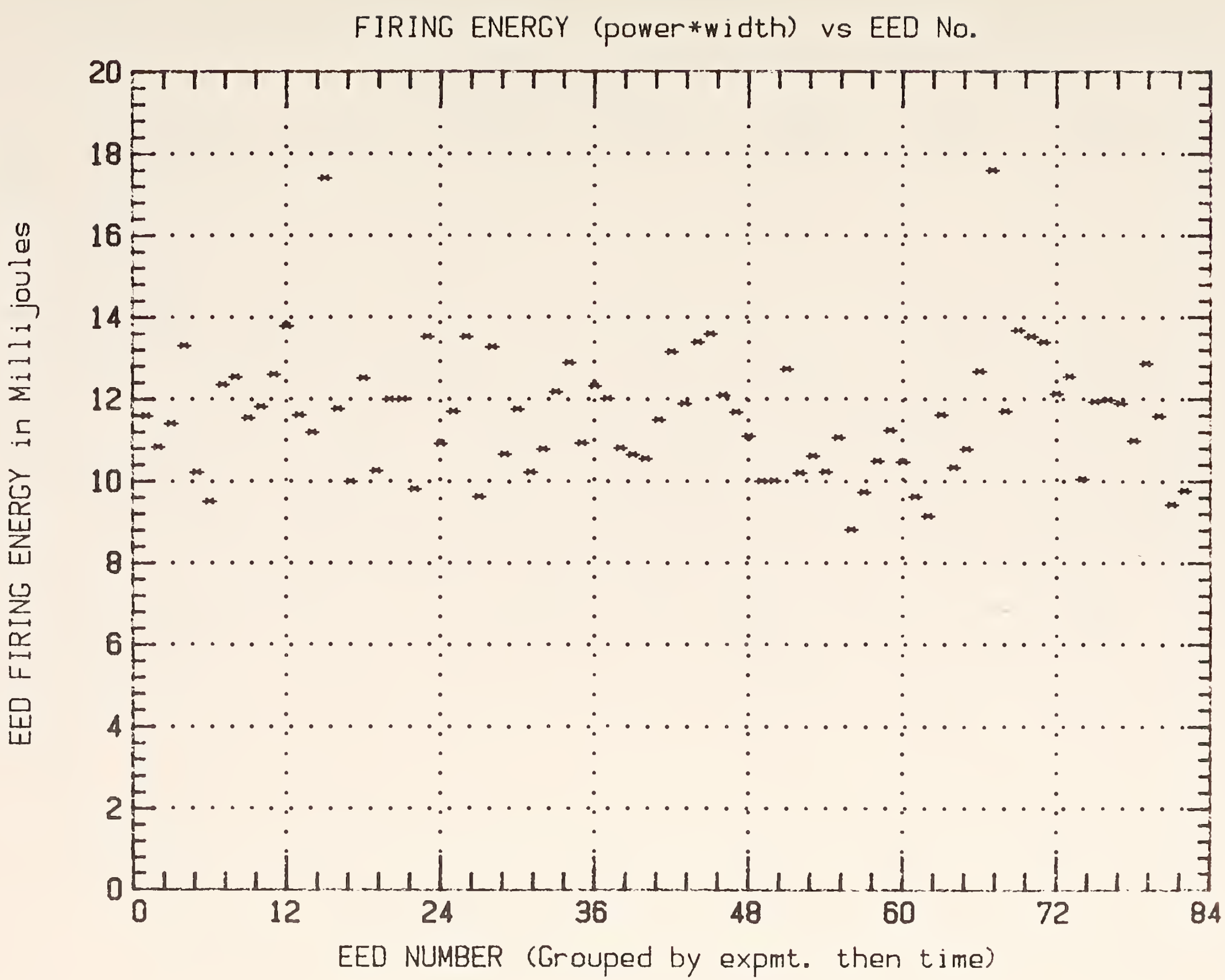


Figure 7. Plot of data from both experiments, 82 points.



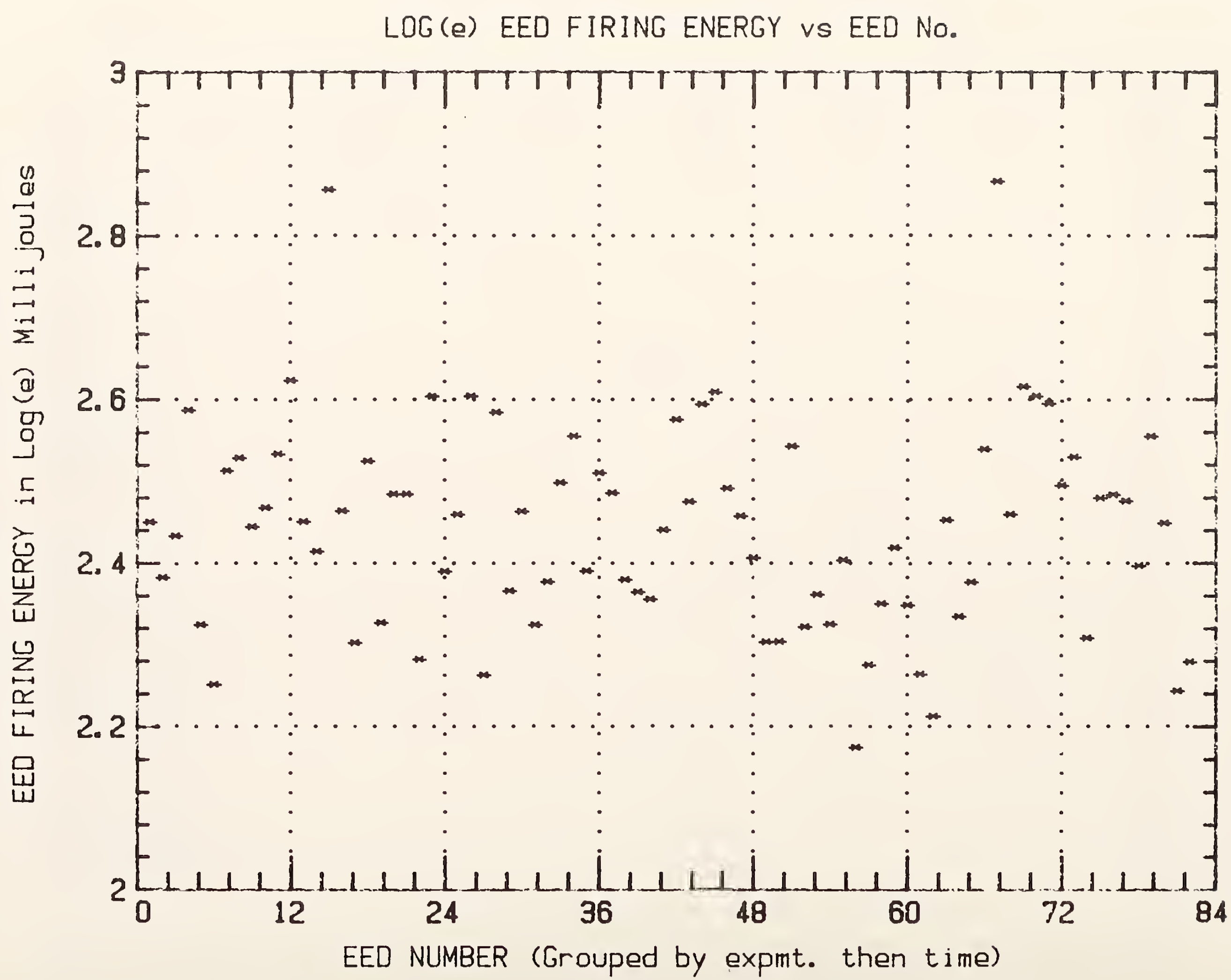
Log Power

Figure 8 . Plot of log data for both experiments, 82 points.



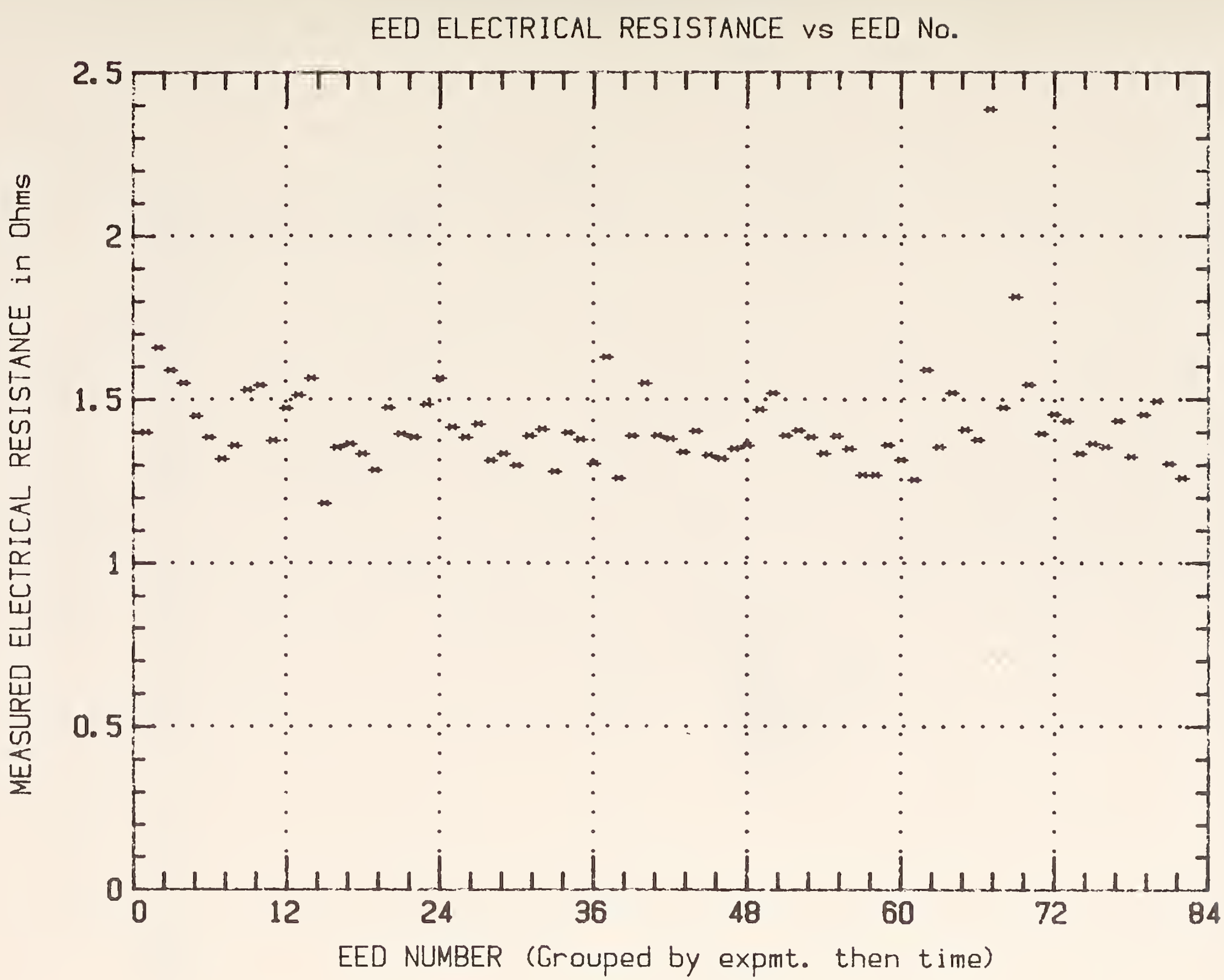
EED Number

Figure 9. Plot of firing energy (width-power product) for each EED, 82 points.



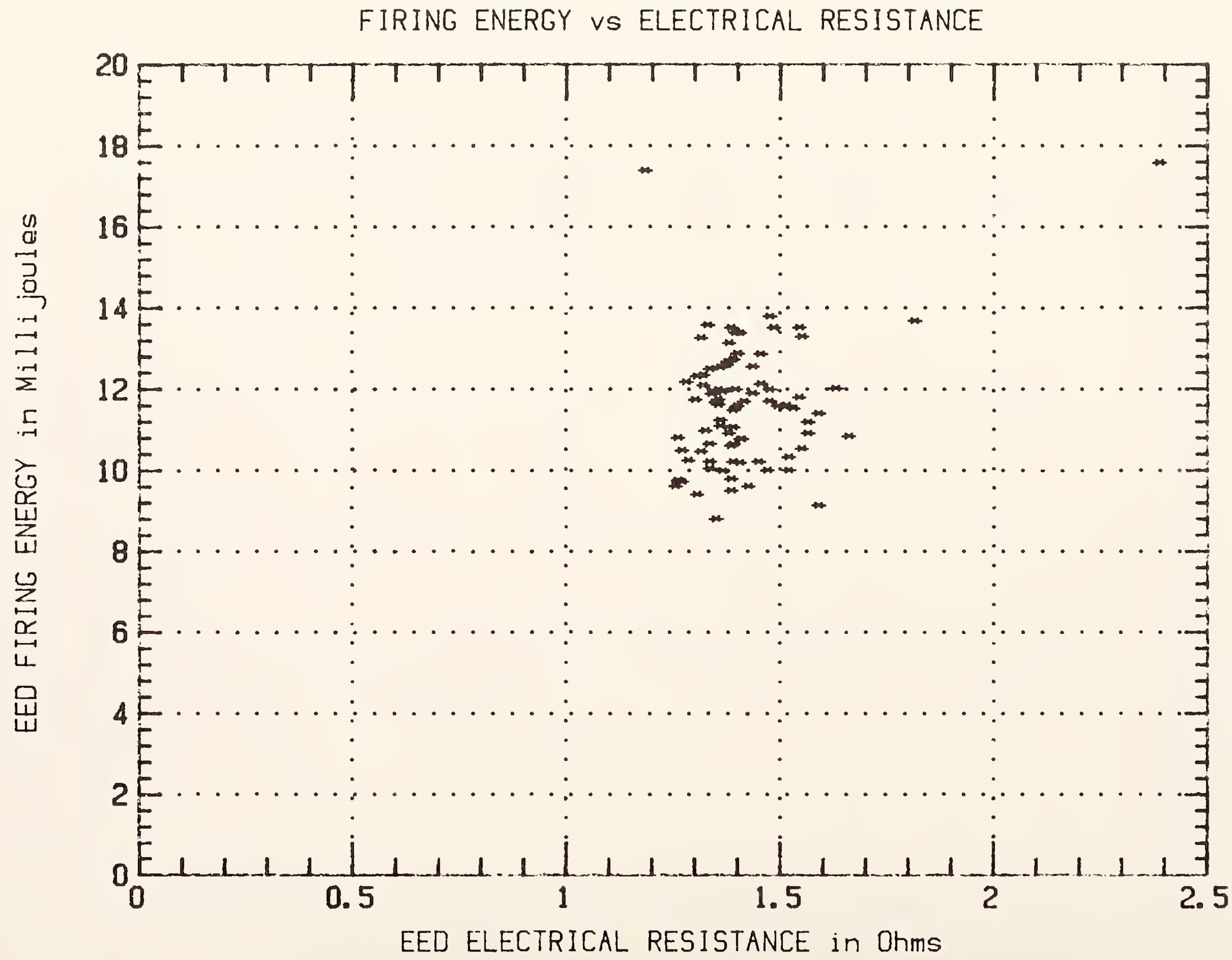
EED Number

Figure 10. Plot of log energy for each EED, 82 points.



EED Number

Figure 11. Plot of electrical resistance of each EED, 82 points.



Electrical Resistance (ohms)
Figure 12. Plot of firing energy of each EED vs. its electrical resistance, 82 points.

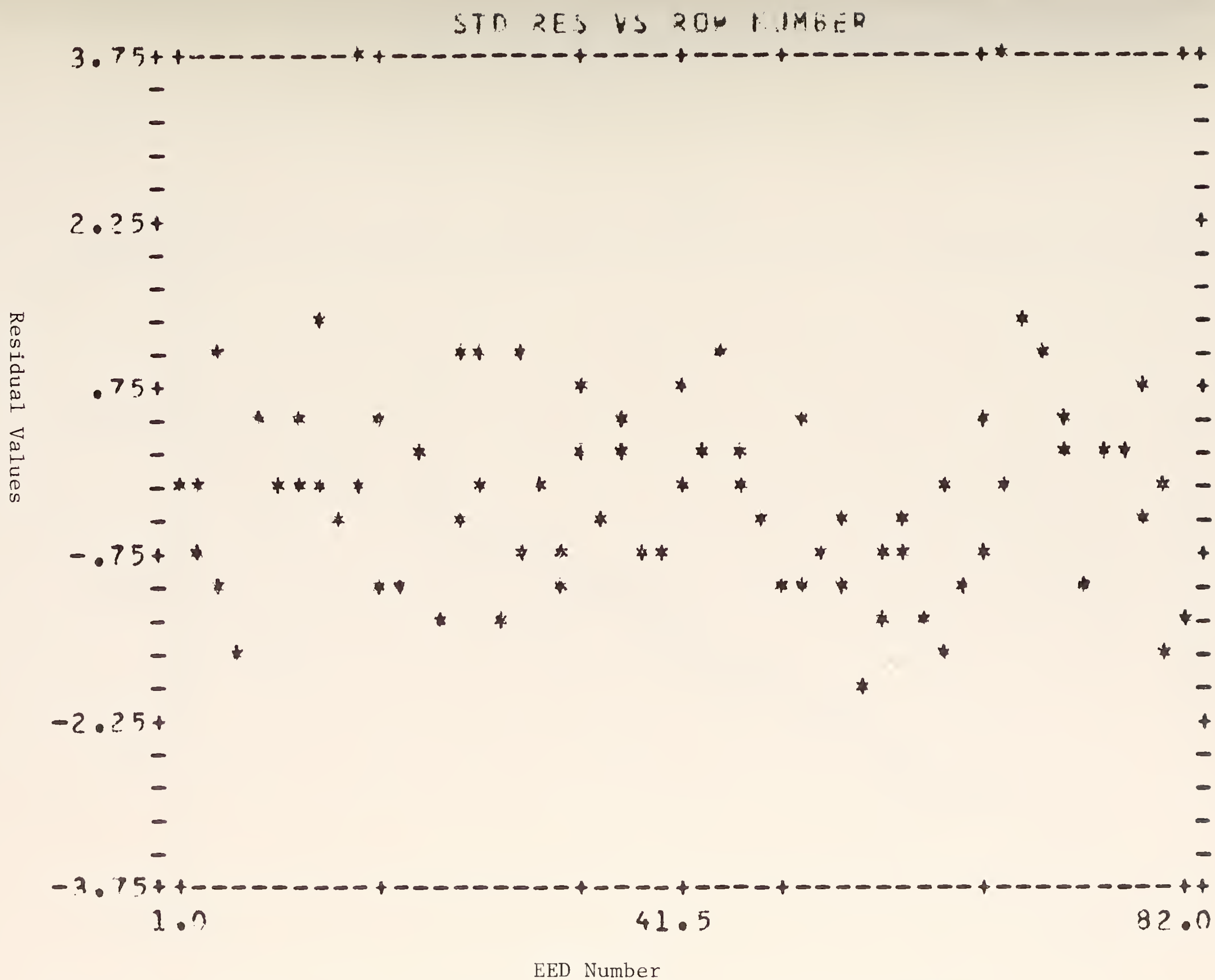


Figure 13. Diagnostic plot of standardized residuals from total experiment.

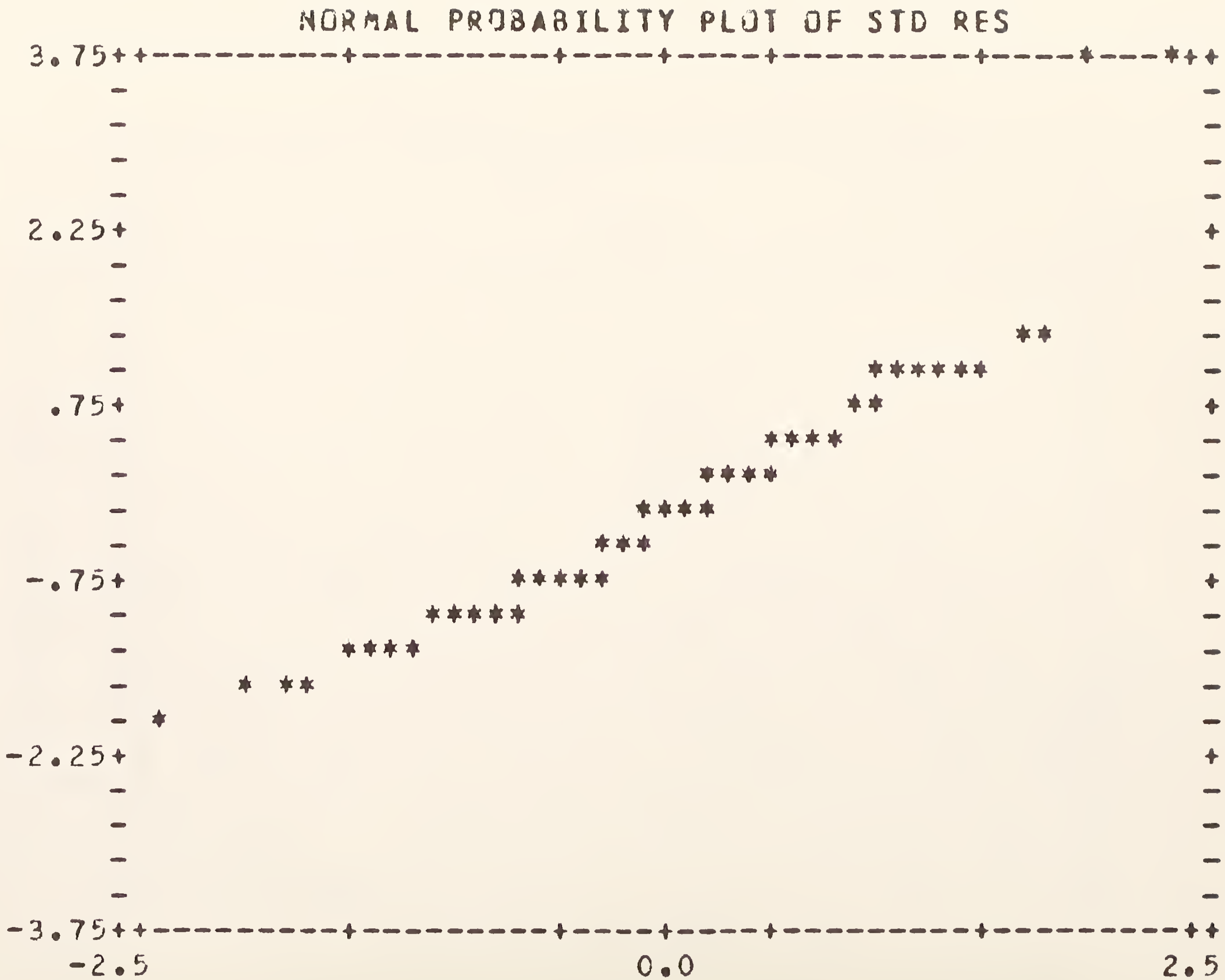


Figure 14. Diagnostic Normal Q - Q plot of standardized residuals from combined experiment.

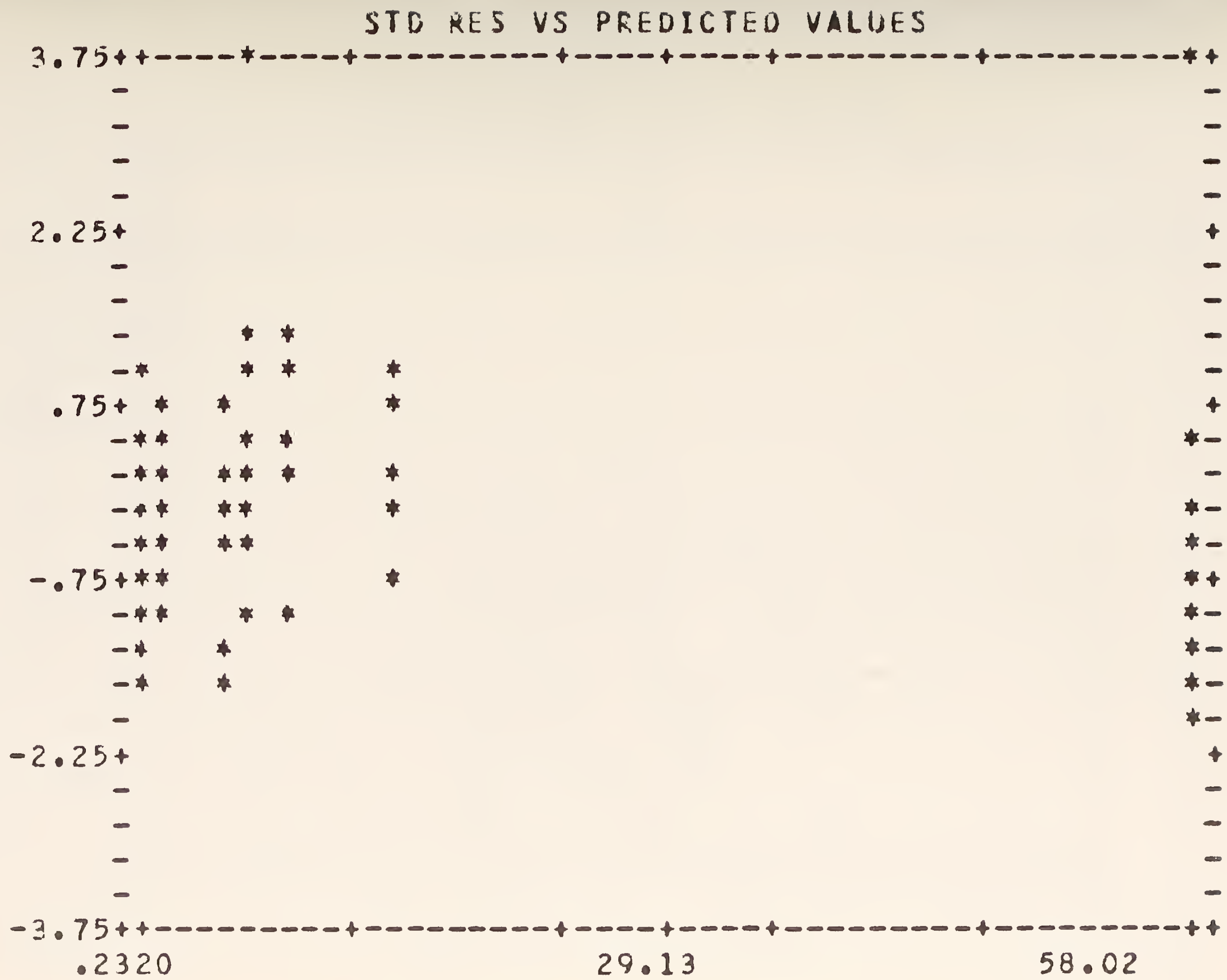


Figure 15. Diagnostic plot of standardized residuals vs. predicted values.

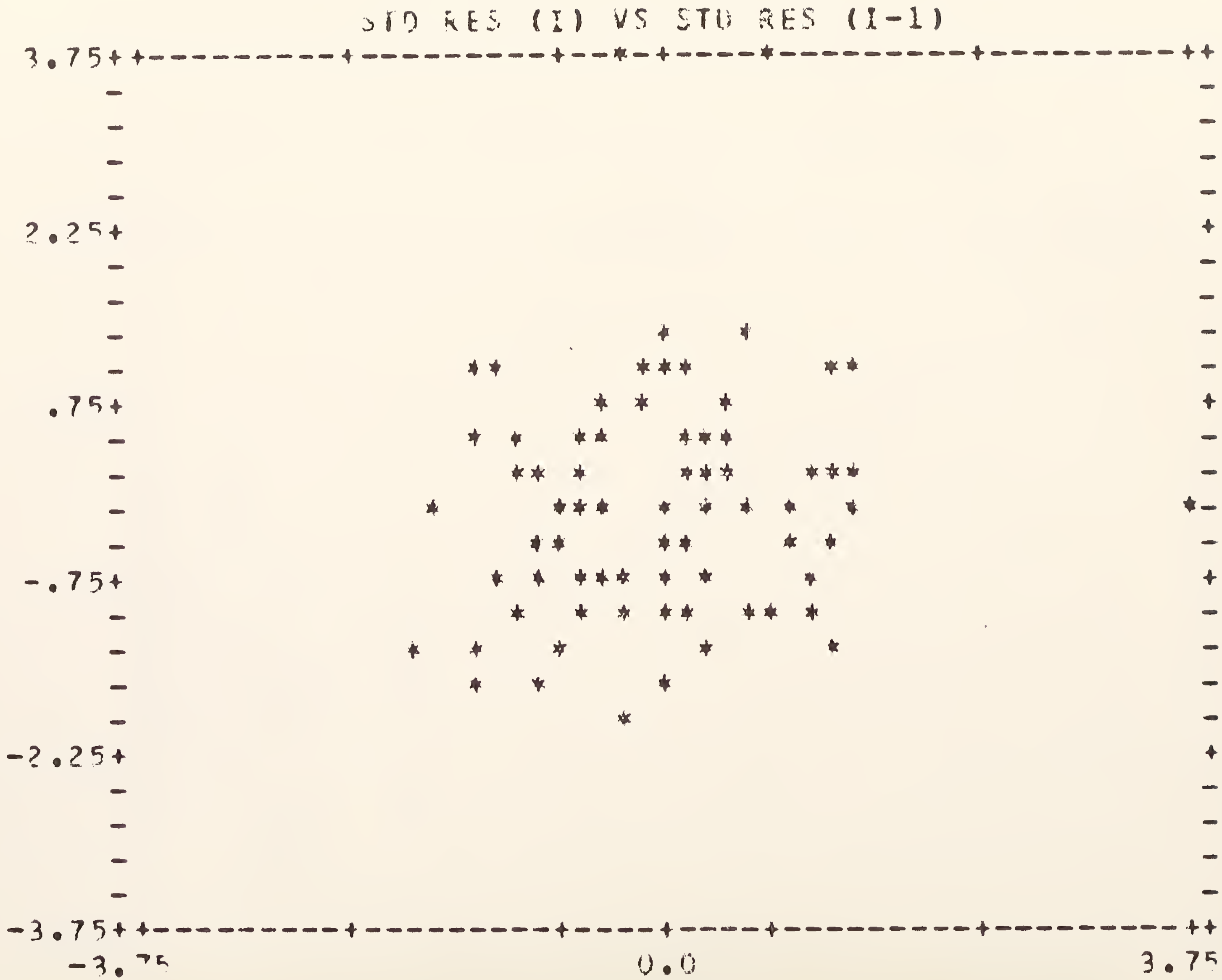


Figure 16. Diagnostic plot of standardized residuals vs. lagged standardized residuals.

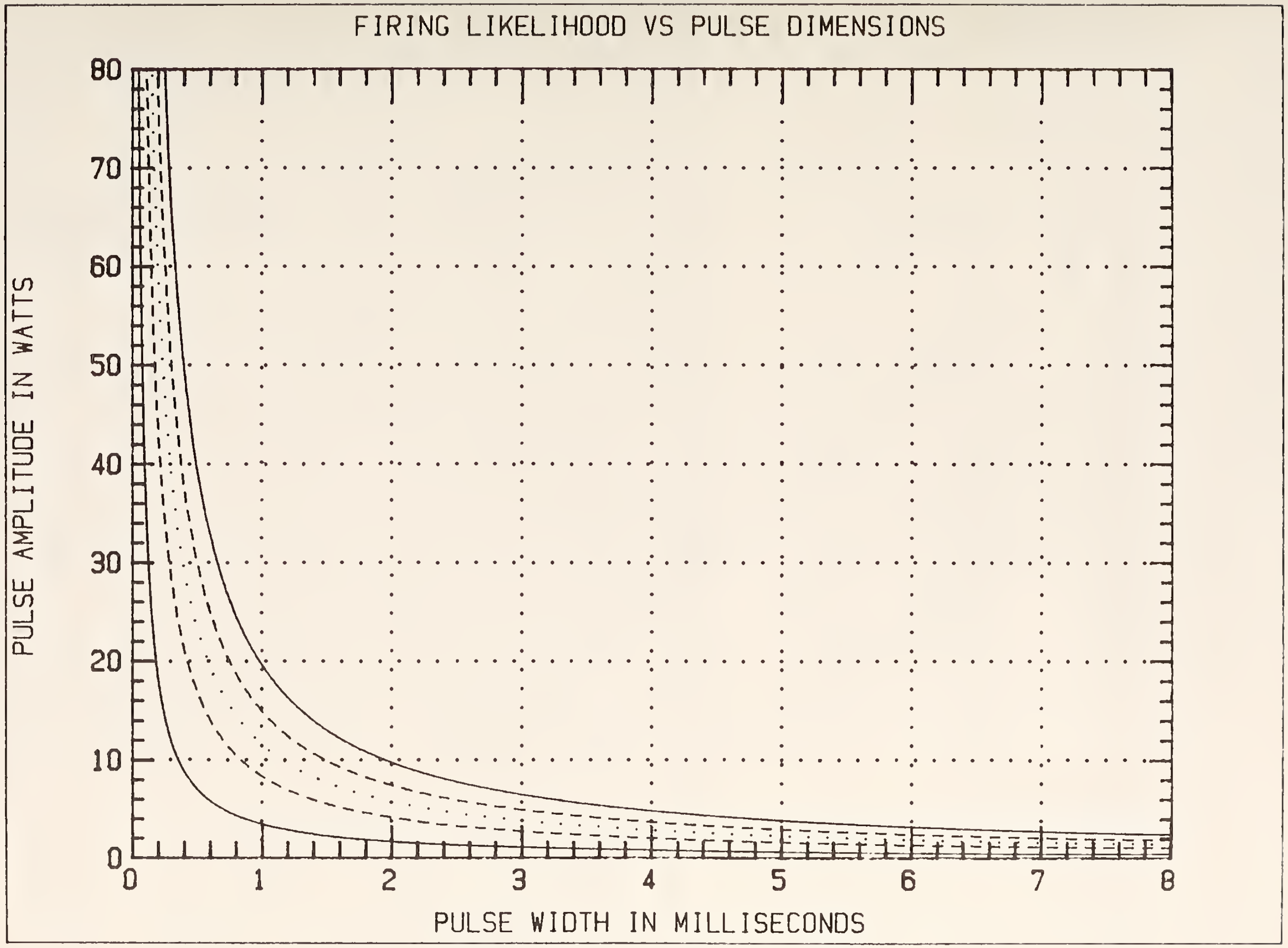


Figure 17. Firing Likelihood Plots for EED type tested.
Dotted line: mean firing level, Dashed lines: upper and lower one-sided 95% tolerance intervals, Solid lines: upper and lower one-sided 99.999% tolerance intervals.

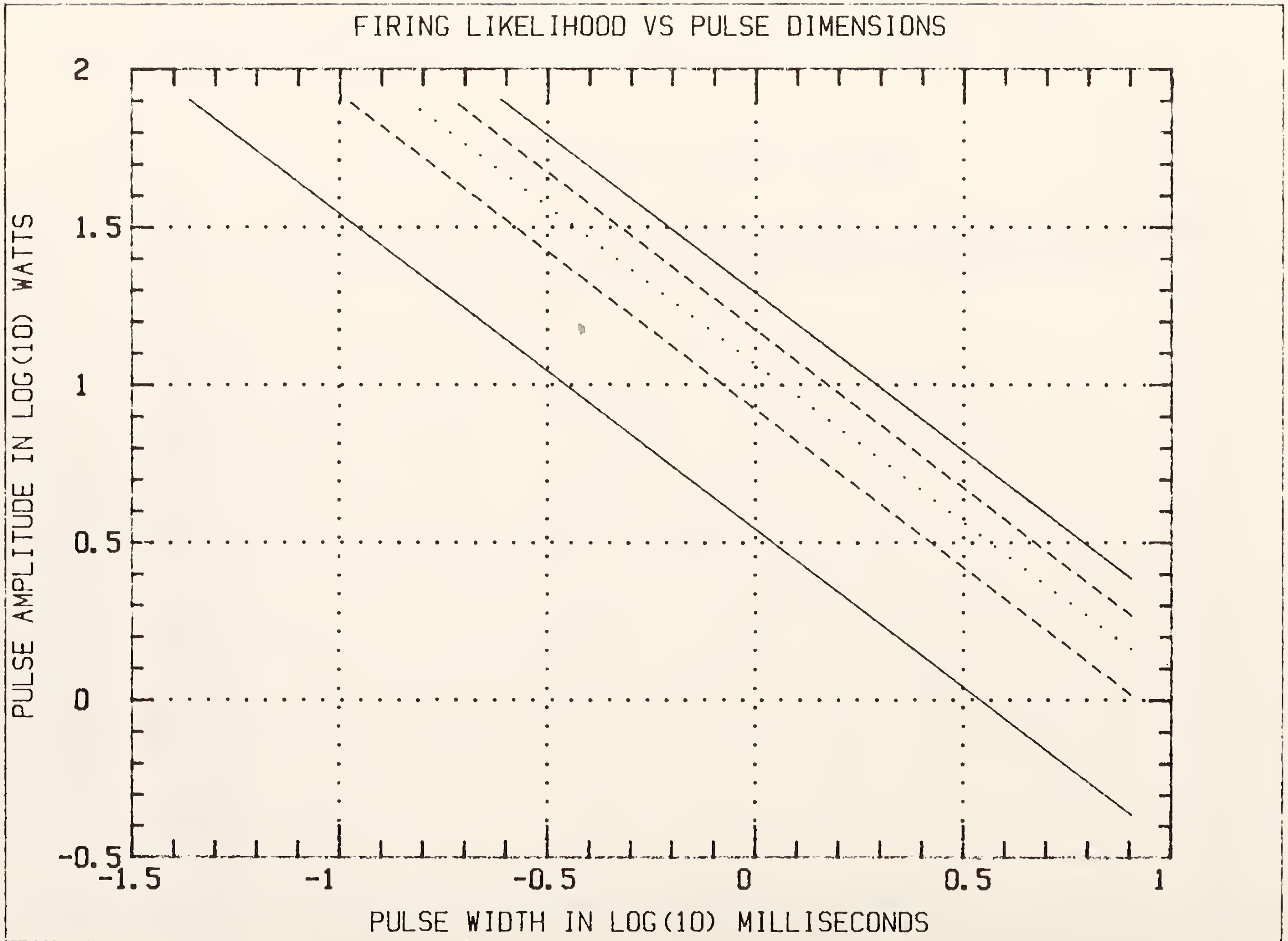


Figure 18. Logarithmic Firing Likelihood Plots for EED type tested. Dotted line: mean firing level, Dashed lines: upper and lower one-sided 95% tolerance intervals, Solid lines: upper and lower one-sided 99.999% tolerance intervals.

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

Electroexplosive devices (EEDs) are electrically fired explosive initiators used in a wide variety of applications. The nature of most of these applications requires that the devices function with near certainty when required and remain inactive otherwise. Recent concern with pulsed electromagnetic interference (EMI) and nuclear electromagnetic pulse (EMP) made apparent the lack of methodology for assessing EED vulnerability. A new and rigorous approach for characterizing EED firing levels is developed in the context of statistical linear models and is demonstrated in this paper. We combine statistical theory and methodology with thermodynamic modeling to determine the probability that an EED, of a particular type, fires when excited by a pulse of a given width and amplitude. The results can be applied to any type of EED for which the hot-wire explosive binder does not melt below the firing temperature. Included are methods for assessing model validity and for obtaining probability plots, called "Firing Likelihood Plots". A method of measuring the thermal time constant of an EED is given. This parameter is necessary to evaluate the effect of a train of pulses. These statistical methods are both more general and more efficient than previous methods for EED assessment. The results provide information which is crucial for evaluating the effects of currents induced by impulsive electromagnetic fields of short duration relative to the EEDs thermal time constant.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

EED; EED response to pulsed currents; electroexplosive device; electromagnetic compatibility; EMC; firing likelihood plots; thermal time constant

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