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Automated Measurement of Frequency Response of Frequency-Modulated Generators Using the Bessel Null Method

J. R. Major
E. M. Livingston
R. T. Adair

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CONTENTS

Automated Measurement of Frequency Response of Frequency-Modulated Generators Using the Bessel Null Method

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This paper describes a Bessel null technique to measure the frequency response of a frequency-modulated rf carrier and a program to automate frequency response measurements of signal generators with output frequencies from 0.450 to 2000 MHz. The measurements obtained using this technique are more precise than those obtained by a highly trained technician using a manual system.

Automated measurement of this process is desirable since the manual method is subject to the following problems: (1) excessive time, (2) error in finding the null, and (3) lack of assurance that the null is the first Bessel null. Automated measurements can be performed using a system controller, a spectrum analyzer, a function generator, and a voltmeter (all of which must be compatible and controllable remotely).

The nonlinear relationship between the modulating signal amplitude and the center frequency amplitude of the carrier is a major obstacle to automated measurement. This problem was solved by obtaining an approximate formula for this nonlinear curve.

Assurance that the null found is the first Bessel null is provided by the analysis of the frequency response of the signal generator under test as displayed on the spectrum analyzer.

Key words: automated; Bessel null; curve fitting; frequency modulated generators; frequency response; linearization.

1. Introduction

One of the criteria of the performance of a signal generator is its ability to produce a specified flatness of output amplitude (e.g., ±2 dB) during frequency modulation over its intended frequency range. The actual response of a generator to a given range of frequencies of modulating signal can be determined with the aid of the Bessel null technique as described herein. When a carrier of radio frequency is frequency modulated by a signal of lower frequency, variations in the amplitude of the modulating signal are converted to corresponding variations in the carrier frequency [1]. The instantaneous radian frequency ω_i varies about the unmodulated carrier frequency ω_c at the frequency of the modulating signal ω_m with a maximum frequency deviation of $\Delta\omega$ radians. By definition the ratio of the frequency deviation to the modulating frequency, $\Delta\omega/\omega_m$, is the modulation index and is labeled β. An increase in the amplitude of the modulating signal corresponds to an increase in the bandwidth occupied by the frequency-modulated carrier, and hence increases the frequency deviation and consequently the modulation index β.

The average power associated with the frequency-modulated carrier is independent of the modulating signal and is the same as the average power of the unmodulated carrier.

As the modulation index, β, increases, the energy in the carrier begins to decrease, thereby transferring more and more energy to frequency sidebands, which increase in number. For a value of β of 2.4 the carrier amplitude reaches a minimum or null point, and maximum energy is transferred to the sidebands. The power series expansion of this power spectrum is expressed in terms of its coefficients. These are the amplitudes of the carrier frequency and sidebands. The amplitudes are defined as Bessel functions of the first kind (J_n).

An accurate measurement of the frequency deviation $\Delta\omega$ of the modulated carrier may be obtained by determining the Bessel null point.

An amplitude-versus-time display of a frequency-modulated signal is shown in figure 1 along with the unmodulated carrier signal and the modulating signal.

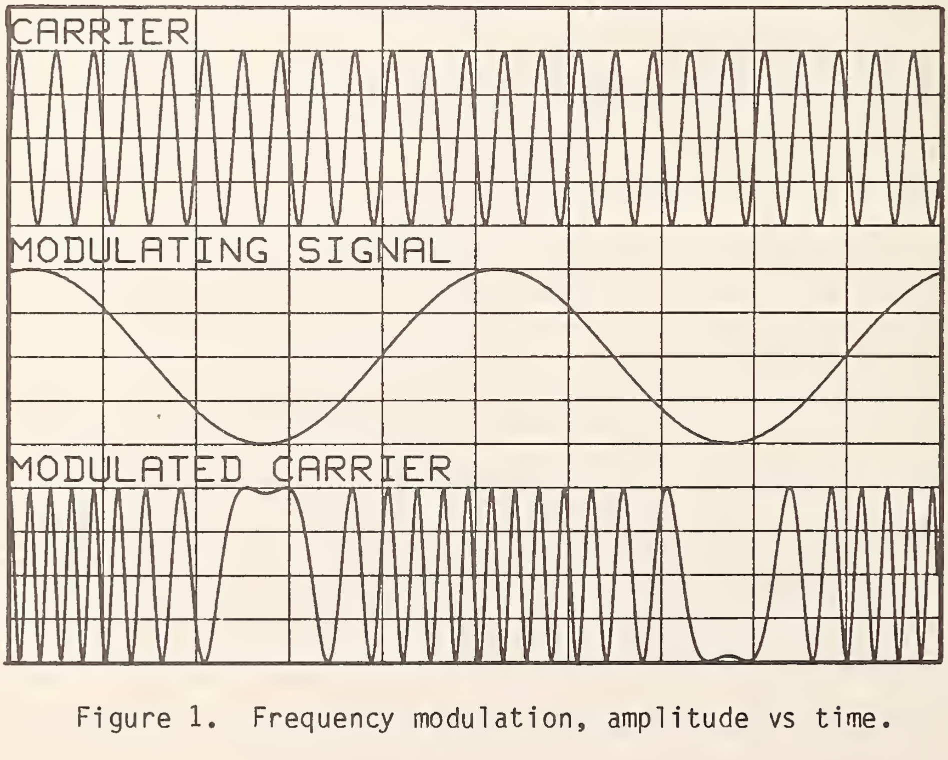

Figure 1. Frequency modulation, amplitude vs time.

Measurement of deviation from this type of display would be difficult at best, especially with a wide variation in the frequency of the modulating signal. Since the measurement of frequency response of the modulated carrier is dependent on an accurate means of measuring deviation, it is necessary to consider an alternate measurement method.

Figures 2 and 3 show a display of amplitude versus frequency of an unmodulated carrier and a frequency-modulated signal, respectively, as observed on a spectrum analyzer. These figures provide the observer with a clear concept of frequency deviation. However, actual measurement of deviation from measurements shown in figure 3 is difficult due to the wide variations in the frequency of the modulating signal.

Modulation analyzers can be used to make accurate measurements, providing of course that the frequency range of the analyzer is adequate. In this case, the 2.0 GHz upper limit of the frequency range of the signal generator to be tested for frequency response exceeded the range of available modulation analyzers.

The Bessel null method has a distinct advantage in obtaining a precise measurement of deviation. With this method the amplitude of the modulating signal is increased until the amplitude of the center frequency of the modulated carrier reaches a minimum (see fig. 4). The disadvantage of this method is the excessive time required to perform the test. This disadvantage can be overcome by automating this measurement procedure.

2. Bessel Null Background

With a fixed frequency of modulation the carrier amplitude can be reduced to a minimum by increasing the amplitude of the modulating signal. This null point is not dependent on the modulating frequency. The Bessel formulas and functions provide verification. Those shown below are taken from reference [2]. If a sinusoidal modulating signal of the form $A_m \cos \omega_m t$ is used and has a maximum frequency deviation $\Delta\omega$ then the instantaneous frequency is

$$\omega_i(t) = \omega_c + \Delta\omega\ (\cos\ \omega_m t) \qquad (1)$$

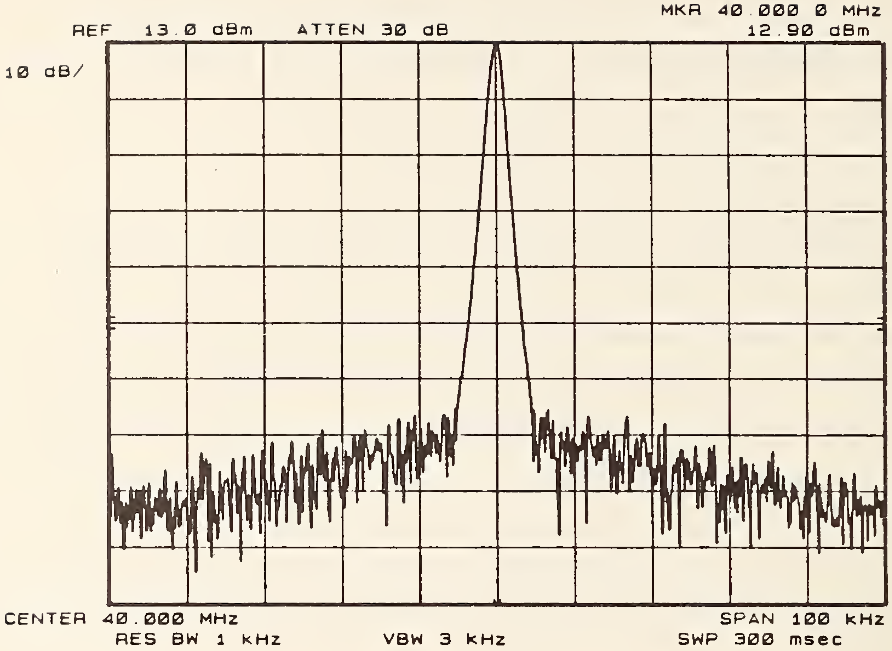

Figure 2. Frequency spectrum of unmodulated carrier.

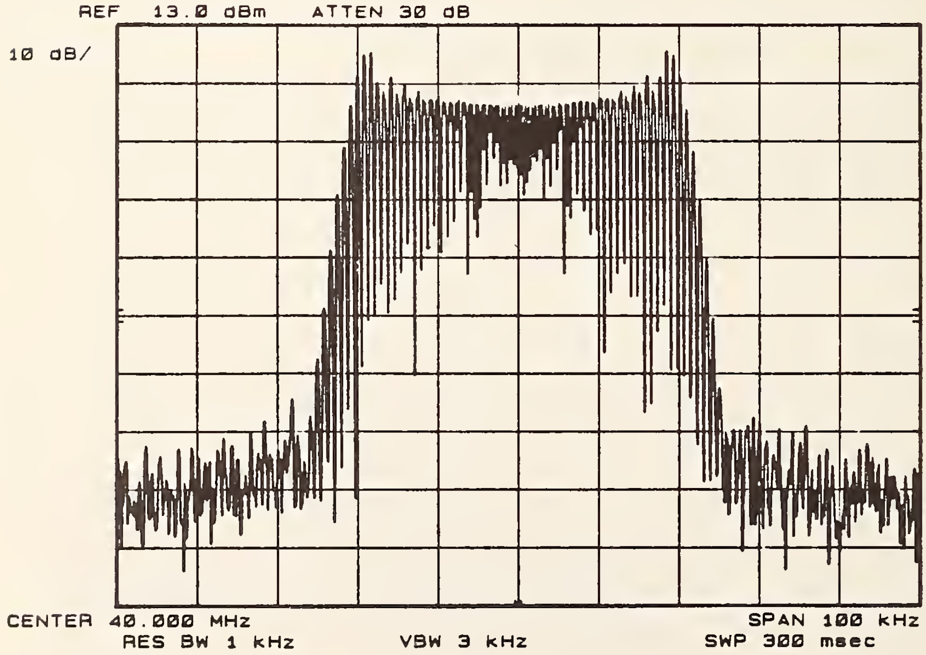

Figure 3. Frequency spectrum of frequency-modulated carrier.

where $\Delta\omega$ is independent of the modulating frequency, ω_m, and is proportional to the amplitude of the modulating signal. The Bessel function is derived from spectral analysis as shown by Carlson [3], from the power spectral series

$$X_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos (\omega_c + n\omega_m)t, \tag{2}$$

where $J_n(\beta)$ is the nth order Bessel function. Examination of this power spectral series shows that the Bessel function represents the relative amplitude of the carrier and is a function of the modulation index. When the zero-order Bessel function $J_0(\beta) = 0$, $\beta = 2.4, 5.5$, etc. $(\beta = \frac{\Delta\omega}{\omega_m})$.

3. System Description

A spectrum analyzer is used to monitor the amplitude of the carrier and is connected to the unit under test, the modulating signal generator, and the system controller as shown in figure 5. The system controller must be able to control the amplitude and frequency of the signal generator using the program discussed below (see appendix A). Also, the system controller must be able to control functions of the spectrum analyzer such as center frequency, resolution bandwidth, video bandwidth, attenuation, marker, delta marker, and marker amplitudes.

The program used with the system controller contains the solution to the problems of (1) working with a nonlinear relationship between the modulating signal amplitude and the carrier amplitude, and (2) the requirement that the system use the first null of the zero order Bessel function rather than any of the subsequent nulls.

The null as described above cannot always be obtained. Instead, a minimum is obtained in this test procedure.

A nonlinear relationship exists between the amplitude of the modulating frequency and the amplitude of the carrier frequency. The change required to bring the first tenth of reduction in carrier frequency amplitude is many times greater than the change required for the last tenth of reduction. This is a substantial problem for a manual test and a potentially difficult problem for an automatic test. A conventional approach to the automation of this test

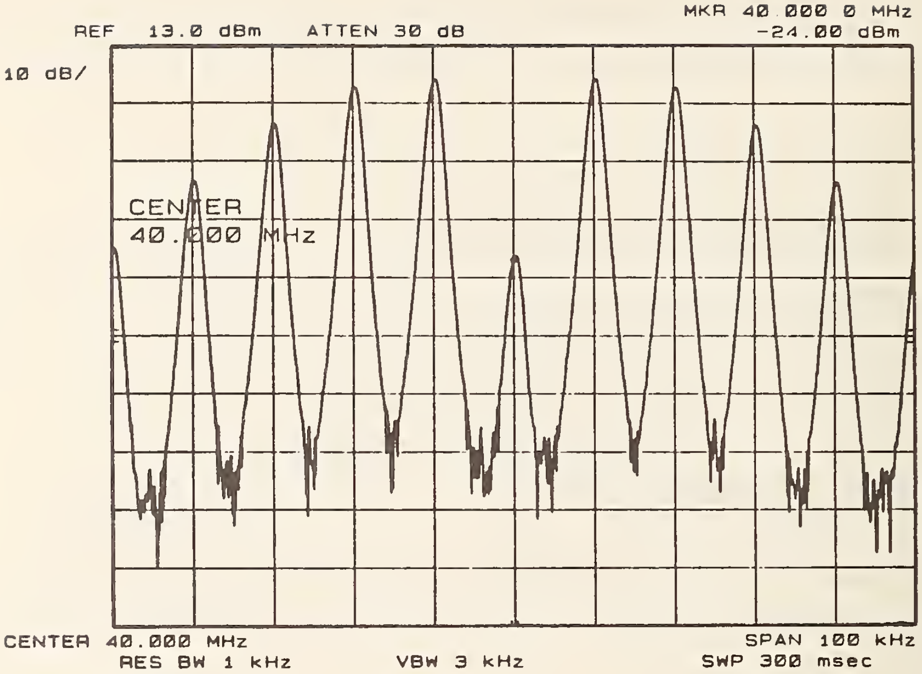

Figure 4. Frequency spectrum of frequency-modulated signal with amplitude of modulating signal adjusted to "null" carrier.

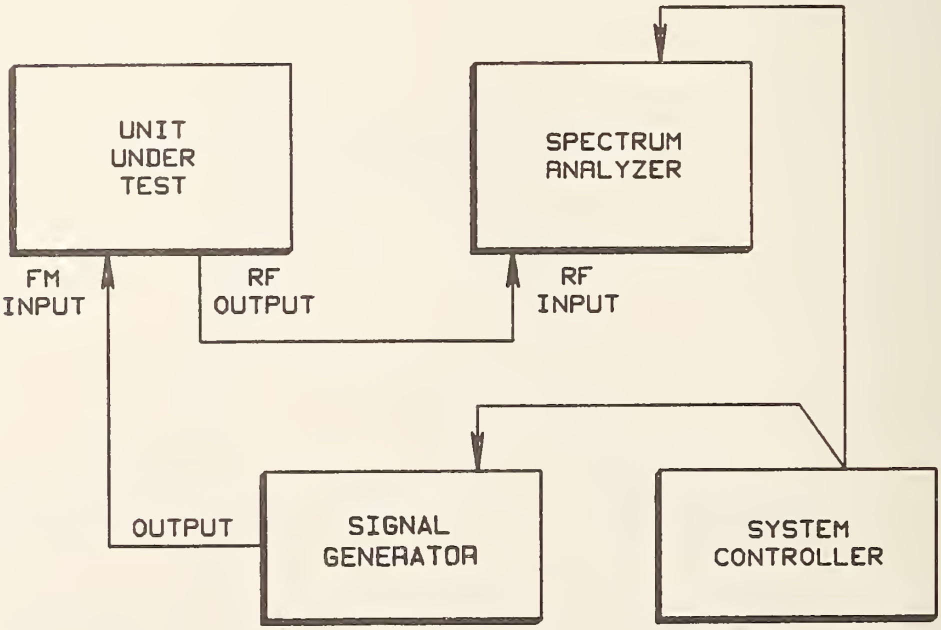

Figure 5. Equipment setup for automated measurements of frequency response of frequency modulated generators using the Bessel null method.

is to make the increments sufficiently small to provide the accuracy required by the specifications. For example, with a carrier frequency of 40 MHz the amplitude of a 100 kHz modulating signal at the null is approximately 500 mV. The increment in modulating signal at the null point that would produce changes in the carrier amplitude is less than 0.5 mV. Therefore, over 1000 of these increments would be required at each modulating frequency. A first-order approximation of the time required for this test is greater than the time required for the manual test.

To characterize the nonlinear relationship, one could use a look-up table of measured values or a mathematical formula derived from a curve fitted to the measured data. To provide for adequate resolution, a look-up table would require an excessive number of values. This would create a measurement problem in that a large amount of time would be required to acquire the data.

The solution to the above nonlinear problem is the determination of an equation that adequately describes the nonlinear relationship. Therefore, a "curve-fit" program was developed to determine an equation that would fit the data of the relationship between the modulating signal amplitude and the carrier amplitude.

Since the plot of the data of modulating signal amplitude and carrier amplitude appears to be an exponential function, a simple trial and error curve fit program was implemented. This program allows the user to select a formula and determine its fit to the data.

The following discussion provides an explanation of the curve-fit program using three different trial equations:

$$y = x^B \text{ where } B \text{ is a constant,} \tag{3}$$

$$y = C \log x \text{ where } C \text{ is a constant,} \tag{4}$$

$$y = \frac{A^{(x + C)}}{D} \text{ where } A, C \,\&\, D \text{ are constants.} \tag{5}$$

(The values of these constants will be determined during the curve-fitting process.) The data shown in table 1 were measured using the spectrum analyzer and are the basis of information from which an equation is to be determined.

Table 1. Amplitude of modulating signal versus carrier frequency amplitude.

Amplitude of 10 kHz modulating signal (V), x	Incremental amplitude of 10 kHz modulating signal (V), (0.498 - x)	40 MHz carrier amplitude (dBm), y
0	0.498	13
0.384	0.114	3
0.463	0.035	-7
0.491	0.007	-17
0.498	0	-22.8 (minimum)
0.515	-0.017	-17

The curve-fit program allows the user first to plot the data of the nonlinear relationship from table 1, as shown in figure 6.

The next step in the curve-fit program is to select a trial formula; for example, eq (3). The program then plots the curve and allows the user to make changes as needed to find an adequate curve fit. The curve fit of eq (3) is shown in figure 7.

Figure 8 is a plot of the next trial formula which is:

$$y = C \log x. \tag{4}$$

Figure 8 contains a set of data points which describe the increments in modulating signal amplitude to reach minimum carrier amplitude as shown in table 1 (points marked with a "y"). The "null" in table 1, which is a minimum of carrier amplitude, was achieved at a modulating signal amplitude of 0.498 V. This corresponds to an increment to achieve "null" or zero volts (see table 1). The incremental data were useful since the program to control the modulating signal amplitude uses increments of increasingly finer resolution to "zero in" on the "null" of the carrier.

The third trial formula is of the form

$$y = \frac{A^{(x + C)}}{D} \tag{5}$$

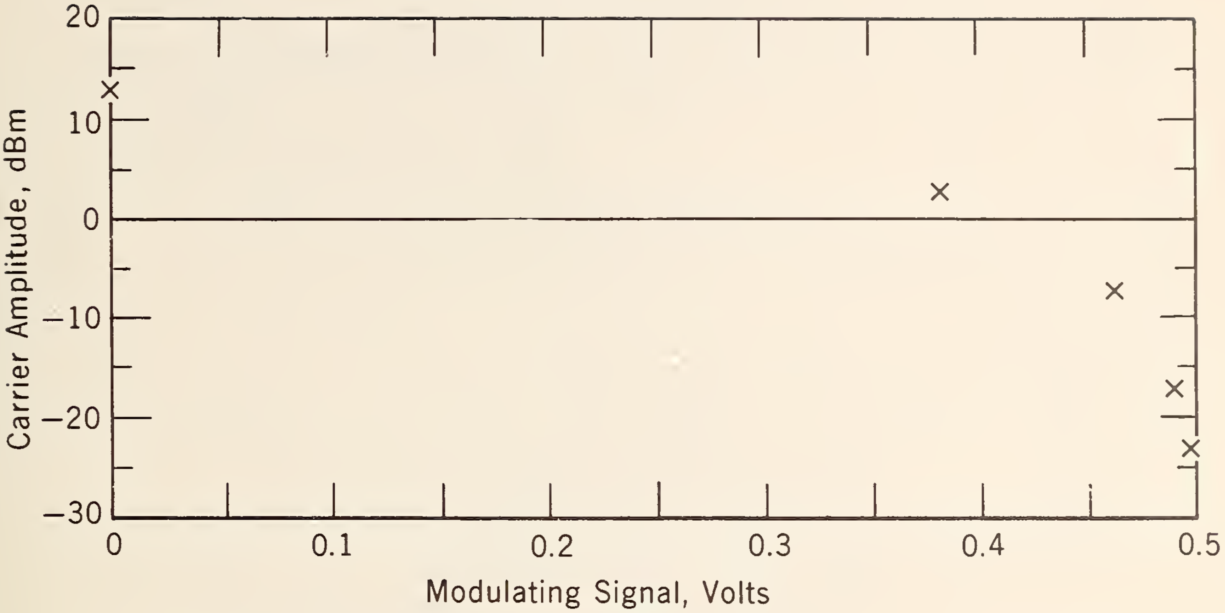

Figure 6. Plot of data from table 1 to be used in "curve-fit" program.

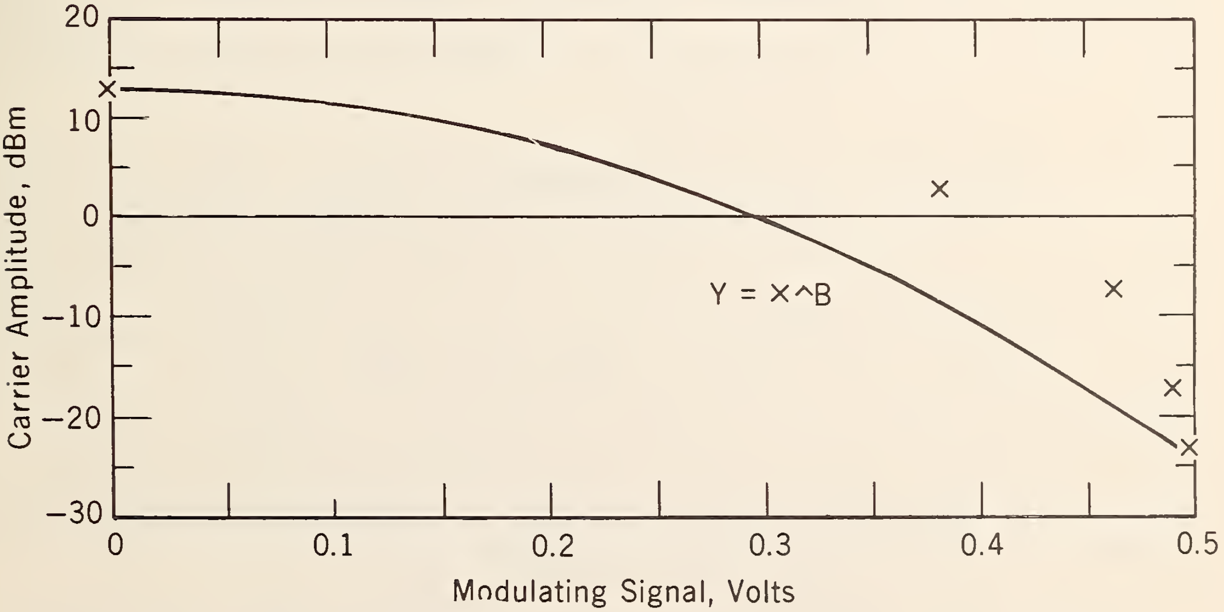

Figure 7. Curve-fit result of $y = x^B$.

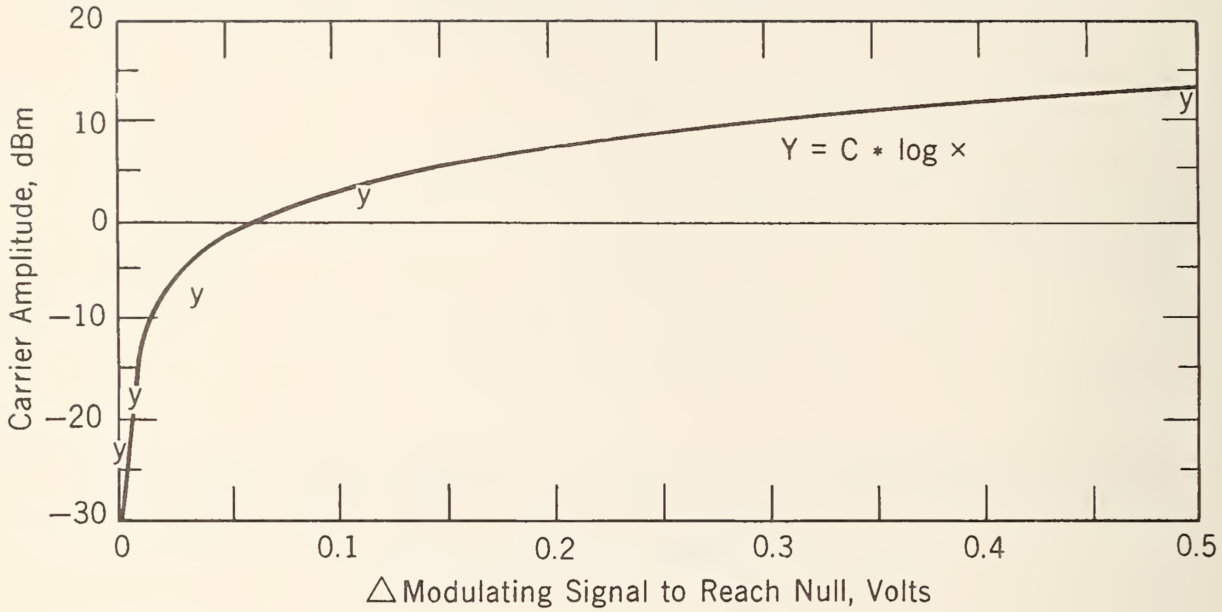

Figure 8. Curve-fit result of y = C log x.

(where A, C, and D are constants that may be changed to provide a curve fit). This formula was found to be adequate to describe the nonlinear relationship of the data in table 1 (see figure 9).

As the frequency of the modulating signal is varied, the curve (of modulation signal amplitude versus carrier amplitude) changes in amplitude. The data points Z in figure 9 are typical for a modulating signal whose frequency is less than 10 kHz. Therefore, this requires an equation that can be easily changed. Changes in the constant A of eq (5) as shown in figure 9 provide the necessary flexibility. In the computer program, A is changed to a smaller value in two cases. The first case occurs when a minimum is obtained and second and third resolutions are performed. The second case occurs when the frequency of the modulating signal is reduced.

The curve-fitting program (see appendix B for a complete listing) was sufficient and adequate for this test. Additional programs can be obtained to provide more information on curve fitting. For example, there are programs

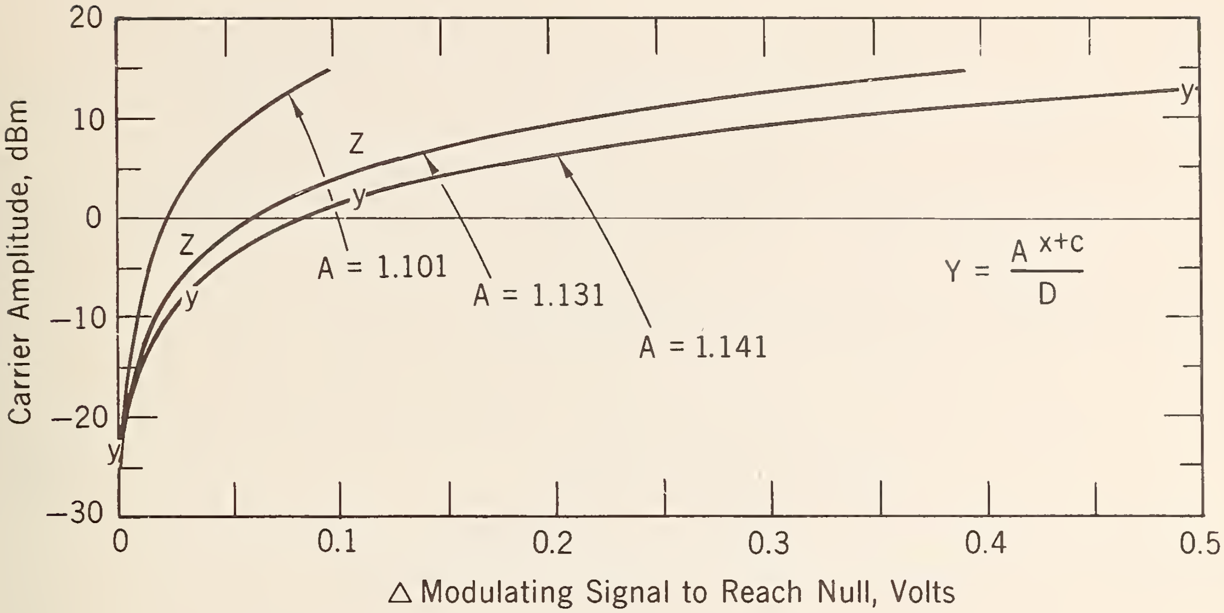

Figure 9. Curve-fit result of $y = \frac{A^{(x+C)}}{D}$.

which are sufficiently sophisticated to provide a formula that fits the data precisely. If this type of program is not available, the book "Fitting Equations to Data" [4] will be useful. The process of linearization is shown in chapter 3. An example follows which demonstrates the use of this linearization process. The data in table 2 (which was generated from the formula $a = x^{1.2}$) will be used in the example.

The first step is the selection of a formula type to investigate. A good choice for the data in table 2 is an exponential function in the form $y = x^B$ (even if the answer is not known ahead of time).

Table 2. Data to provide a case study for the linearization process (generated from $a = x^{1.2}$).

x	a	x	a
1	1	6	8.586
2	2.297	7	10.33
3	3.737	8	12.13
4	5.278	9	13.97
5	6.899	10	15.85

If

$$y = x^B, \tag{6}$$

then

$$\ln y = B \ln x; \tag{7}$$

Therefore, ln y is linear in ln x as shown in [4].

Also

$$\frac{\ln y}{B} = \ln x, \tag{8}$$

and solving for x gives

$$x = e^{\frac{\ln y}{B}}. \tag{9}$$

Since a linear plot is sought the data in table 2 are plotted in the form of $e^{(\ln a)/B}$ versus x, where the data in table 2 are values of a for corresponding x values. The result is the straight line shown in figure 10 where $B = 1.2$. Therefore, the correct formula for the data in table 2 is $y = x^{1.2}$. If the incorrect formula had been selected, the resulting curve would not have been a straight line and the obvious task would have been to select another formula. If, however, the incorrect value of a constant such as B in this example were selected, a few trials would show that the plot would quickly approach the data points.

It is necessary to know that the null found is the first null of the zero order Bessel function in both the manual and the automatic tests using the Bessel null method. Figure 4 is a plot of frequency vs amplitude of the output of a frequency-modulated signal generator at the first null Bessel function as displayed on a spectrum analyzer. Figure 11 shows this display for a second Bessel null.

The basis for the solution of the problem of determining the desired Bessel null is the difference in the "filtered response" of the first null and that of the second null. Figures 12 and 13 are, respectively, plots of the first and second null except that the video bandwidth and resolution bandwidth of the spectrum analyzer have been reduced and produce a "filtered response."

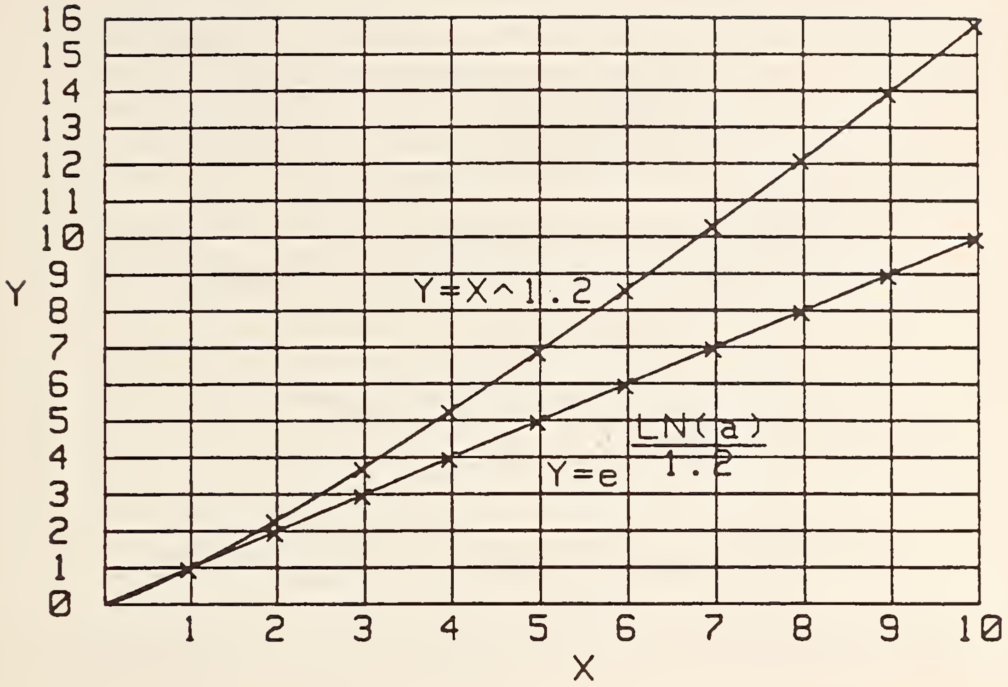

Figure 10. Curve-fit to data points using linearization.

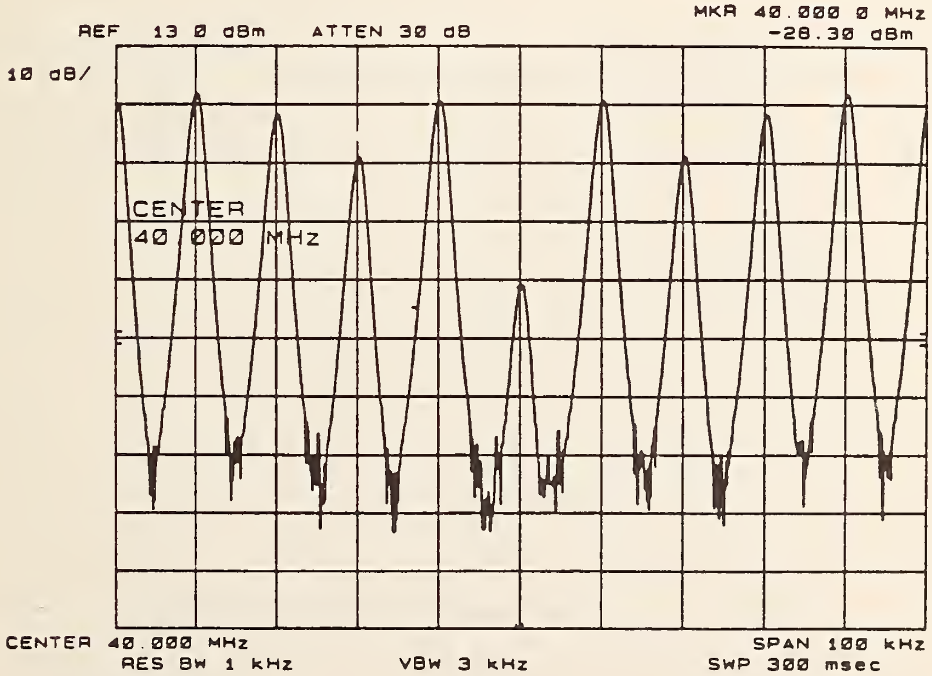

Figure 11. Frequency spectrum showing frequency-modulated signal at second null of zero order Bessel function.

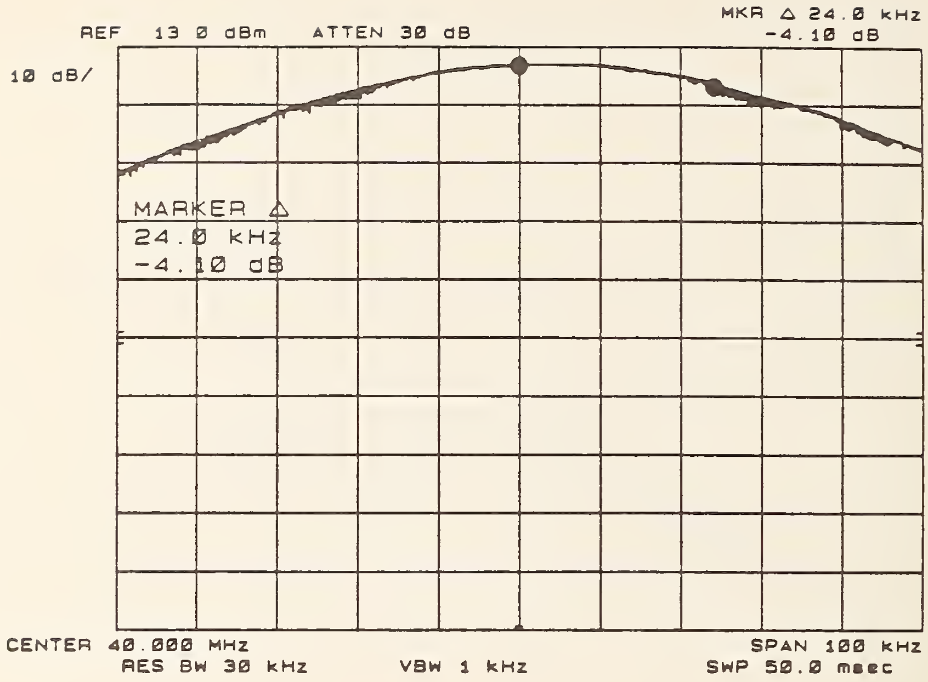

Figure 12. Frequency spectrum showing "filtered" first null of the zero order Bessel function.

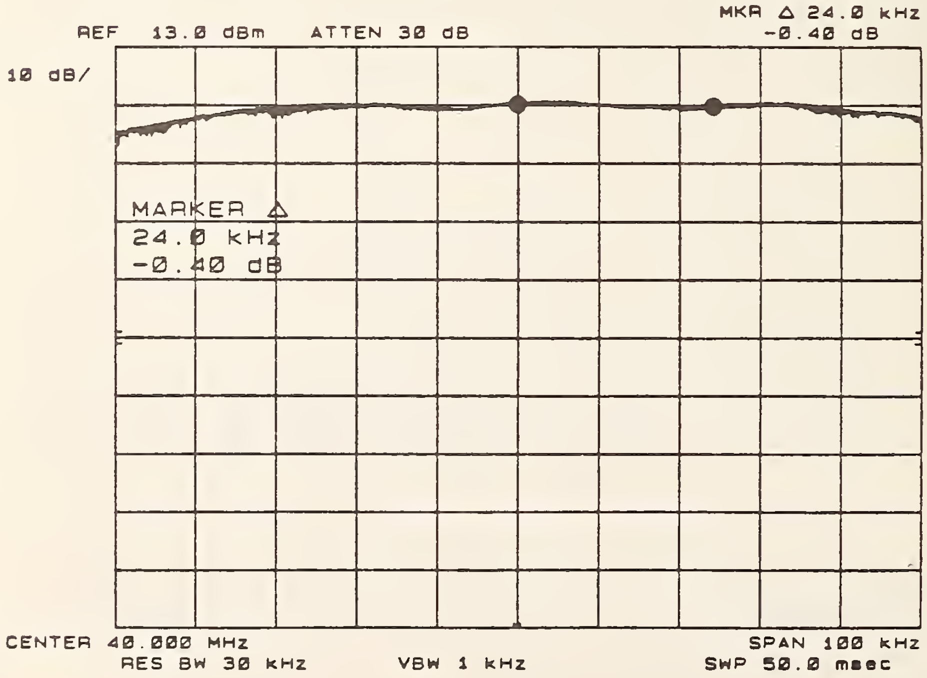

Figure 13. Frequency spectrum showing "filtered" second null of the zero order Bessel function.

A comparison of figure 12 and 13 shows that the "filtered response" of the second null is much flatter (also true of subsequent nulls) than that of the first null. Therefore, the measurement of the difference in amplitudes at the markers indicated in figures 12 and 13 is the information that can be used to determine if the null found is the first Bessel null. If the difference is greater than 1.5 dB (a number selected between the 4.1 dB difference in fig. 12 and the 0.4 dB difference in fig. 13) then the null found is the first Bessel null.

A block diagram of the computer program is shown in figure 14 and a complete listing is given in appendix A.

4. Frequency Response Calculations

Here, we define a response parameter R which for an ideal FM modulator would equal unity (0 dB). In the process of finding the first null at any modulating frequency, β is held constant.

Recall that since $\beta = \frac{\Delta\omega}{\omega_m}$ [1] or $\beta = \frac{\Delta f}{f_m}$ where Δf is the deviation from center frequency and f_m is the modulating frequency, then Δf must decrease as fm decreases. Therefore, to normalize Δf (and make relative comparisons of modulating signals and deviations from center frequency), we multiply the rms voltage of the modulating signal at null (V_{mod}) by the ratio of the reference frequency (f_{ref}) to the frequency of modulation (f_{mod}).

To normalize to a given chosen reference modulation frequency, e.g., 100 KHz, we divide by the rms voltage required to reach null at the reference frequency (V_{ref}). The normalized reading N is then:

$$N = V_{mod} \times \frac{f_{ref}}{f_{mod}} \times \frac{1}{V_{ref}} \qquad (10)$$

or

$$= \frac{\dfrac{V_{mod}}{V_{ref}}}{\dfrac{f_{mod}}{f_{ref}}} \qquad (11)$$

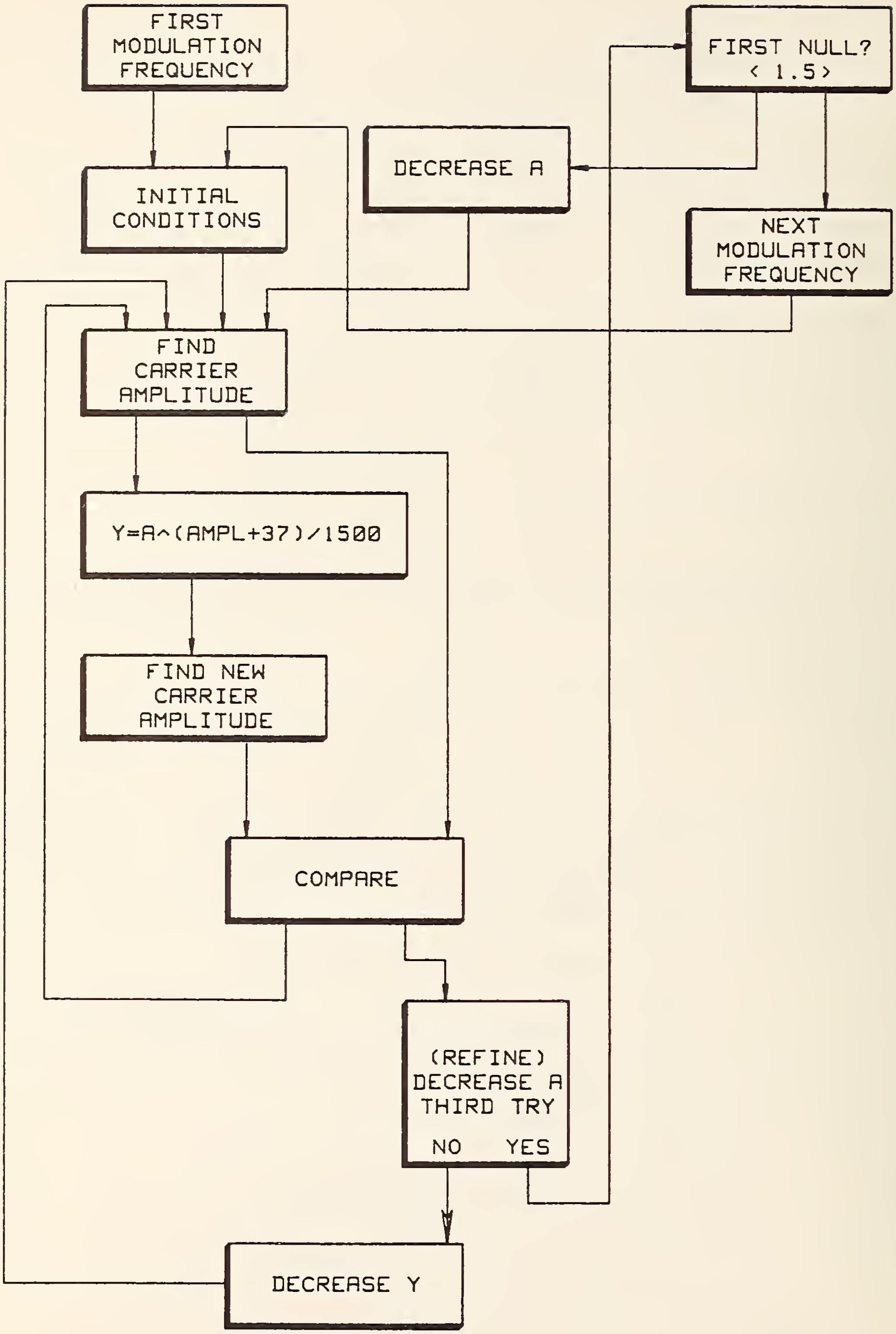

Figure 14. Flow diagram of program to find the first null of the zero order Bessel function.

Expressed in dB the response is

$$R = 20 \log N \tag{12}$$

or

$$= 20 \log \frac{V_{mod}}{V_{ref}} - 20 \log \frac{f_{mod}}{f_{ref}}. \tag{13}$$

Typical measurement results have been recorded in table 3 (using equipment setups as shown in fig. 5 and computer programs as described herein). The program output is a printout showing the response R using eq (10) for each modulating frequency.

Table 3. Calculated values of FM frequency response, R, for f_{ref} = 100 KHz and V_{ref} = 0.498 volts

EXTERNAL MODULATION FREQUENCY (KHz)	DEVIATION AS INDICATED BY UUT	MODULATING SIGNAL Vmod (VOLTS)	CALCULATED RESPONSE R (DB)
0.4	*	0.00149	-2.52
1	0.01	0.0389	-2.15
3	0.09	0.012	-1.90
10	0.22	0.052	+0.375
30	0.72	0.146	-0.200
100	2.4	.498	0.00

* NO METER MOVEMENT DETECTED

5. Measurement Accuracy

A preliminary estimate of the measurement uncertainty using the automatic technique described is ±5 percent. The signal generator used to provide the modulating signal has a flatness specification of ±4 percent. Therefore, the measurement accuracy could be improved markedly by using a signal generator for modulation with a better flatness specification. Thus, the modulating signal source is the major contributor to the measurement uncertainty.

The actual method of finding a null as described in this document appears to be highly precise. A comparison was made between the results of the automatic null measurement technique and the reading of the deviation meter on the signal generator under test. Readings were taken with a deviation of 2.4 as indicated on the unit under test deviation meter. Errors due to parallax in reading the meter were minimized by using the meter glass as a mirror and aligning the reflection of the operator's eye with the meter needle. Each reading was taken with the meter indicator at a specific mark (2.4) to maximize the accuracy and repeatability of reading the meter. Several readings were taken with each method. Variations observed with the automatic null measurement method were ±0.1 dB compared to ±0.8 dB using the deviation meter on the unit under test. Thus, the resolution of the measurements using the automatic null technique is better than that of the manual method using the built-in deviation meter. This verifies that the accuracy of frequency response measurements using the automatic null measurement technique is considerably better than manufacturers' stated accuracy based on conventional techniques.

6. Summary

A method to determine the frequency response of frequency-modulated signal generators is described. The Bessel null method is used to control deviation since frequency deviation is independent of the modulating frequency. Automated measurements are necessary to eliminate the tedious manual "nulling" and possibility of nulling on other than first nulls of the zero order Bessel function.

An equation for the nonlinear relationship of modulating-signal amplitude versus carrier amplitude is supplied by a computer program developed for curve fitting. This equation is a necessary part of the automated measurements.

This technique provides a unique, faster, and more accurate method of measuring signal-generator frequency response than those commonly used.

7. Conclusions

The Bessel null method of measuring frequency response is potentially highly precise due to the lack of dependence of modulating frequency on frequency deviation. The measurements must be made automatically or with a highly skilled technician. If many measurements are necessary the manual method is a very tedious process.

Automated measurements have problems such as nonlinear relationships and multiple Bessel nulls. This document describes how these problems can be resolved in a way that is tractable for automatic application.

Additional considerations for future work include the use of a programmable step attenuator placed between the output of the modulating signal generator and the external frequency modulation input of the signal generator under test. Then the sub-millivolt increments in modulation voltage will remain a very small part of the total modulation voltage required to obtain a first Bessel null.

Also, the "loading effects" of a variable input impedance of the external frequency modulation must be considered. For example, an ac-coupled input could possibly cause a significant phase shift at lower modulating signal frequencies.

Although this test provided results that were repeatable within the sub-millivolt increments of the modulating signal, a more thorough data analysis should be performed to establish more accurate uncertainty limits. Other areas of investigation include, but are not limited to, "sharpness" of null, frequency deviation other than the first Bessel null, and distortion of the modulating signal, as well as distortion of the modulated signal with respect to finding the Bessel null.

8. Acknowledgments

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[4] Daniel, Cuthbert; Wood, Fred S. Fitting equations to data. New York: Wiley-Interscience; 1971. pp. 19-21.

Appendix A Computer Program for Determining Frequency Response of Frequency-Modulated Generators Automatically

```
100   PRINTER IS 1
105   PRINT "FM_RESPONS"
110   PRINTER IS 701
115   PRINT "        FREQUENCY RESPONSE USING BESSEL NULL"
120   PRINT
125   !   THIS PROGRAM FINDS THE RESPONSE WITH RESPECT TO 100KHZ OF 30, 10,
130   !   3, 1, AND 0.4KHZ.  TO FIND THE RESPONSE AT LOWER FREQUENCIES USE
135   !   AN ATTENUATOR BETWEEN THE OUTPUT OF THE OSCILLATOR AND THE INPUT
140   !   TO THE FM INPUT OF THE UNIT UNDER TEST.
145   !
150   ! THE 2:1 FACTOR IN THE AUDIO ANA OUTPUT CAN BE AVOIDED BY CONNECTING
155   !   THE OSCILLATOR OUTPUT TO THE INPUT. THIS CONNECTION IS NECESSARY
160   !   ALSO TO OBTAIN VOLTAGE READINGS FOR CALCULATIONS IN RESPONSE.
165   ASSIGN @Aa TO 728
170   ASSIGN @Sa TO 718
175   ASSIGN @Hpib TO 7
180   CLEAR @Hpib
185   A=1.141            ! THIS IS A FACTOR IN THE NON LINEAR EXPRESSION
190                      ! OF AMPLITUDE VS. NULL (USED IN "FINDNULL")
195   Span=1000          ! OPERATING CONDITIONS
200   Mdfrq=100          ! FOR THE FIRST
205   Dltfrq=240         ! MODULATION FREQUENCY  (100KHZ)
210   Rb=300                 ! CHANGES IN RES BW AND VIDEO BW
215   Vb=30                  ! FOR SECOND BESSEL NULL CHECK
220   GOSUB Sub_gosub
225   V100khz=Rms
230   PRINT "MODULATION FREQUENCY=100KHZ";"   VMOD";Rms;"V";"   R=0DB"
235   PAUSE
240   Span=300           !
245   Mdfrq=30           !    (30KHZ)
250   Dltfrq=72          !
255   Rb=100                 !
260   Vb=3                   !
265   GOSUB Sub_gosub
270   GOSUB Calc
275   PAUSE
280 Ten_k:   !          !
285   Span=100          !
290   Mdfrq=10          !    (10KZ)
295   Dltfrq=24         !
300   Rb=30                 !
305   Vb=.3                 !
310   A=1.091
315   GOSUB Sub_gosub
320   GOSUB Calc
325   PAUSE
330 Three_k:   !        !
335   Span=30           !
340   Mdfrq=3           !     (3KHZ)
```

```
345   Dltfrq=7.2        !
350   Rb=10                 !
355   Vb=.1                 !
360   A=1.061
365   GOSUB Sub_gosub
370   GOSUB Calc
375   PAUSE
380   Span=10           !
385   Mdfrq=1           !
390   Dltfrq=2.4        !    (1KHZ)
395   Rb=3                  !
400   Vb=.03                !
405   A=1.041
410   GOSUB Sub_gosub
415   GOSUB Calc
420   PAUSE
425   Mdfrq=.4          !
430   Dltfrq=.96        !    (400HZ)
435   Rb=1                 !
440   Vb=.03               !
445   A=1.001
450   GOSUB Sub_gosub
455   GOSUB Calc
460   GOTO End
465 Outputs:   !  COMMANDS TO AUDIO ANALYZER AND SPECTRUM ANALYZER
470   OUTPUT @Aa;"AU FR";Mdfrq;"KZ AP0VL L0"
475   OUTPUT @Sa;"LF S2 CF40MZ SP1MZ RL20DM TS E1 MT1 TS"
480   OUTPUT @Sa;"SP";Span;"KZ"
485   OUTPUT @Sa;"TS M2 TS E1 TS E2 TS E4 TS MA"
490   RETURN    ! (FOR Outputs)
495 Sub_gosub:  !
500   GOSUB Findnull
505   GOSUB Secd_bes_ck  ! A CHECK TO MAKE SURE THAT THE NULL THAT WAS FOUND
510                      ! WAS THE FIRST NULL.  (IF THE VOLTAGE INCREMENTS
515                      ! TO THE OSCILLATOR WERE TOO LARGE, THE FIRST NULL
520                      ! COULD BE JUMPED OVER AND A NULL FOUND ON OTHER
525                      ! THAN THE FIRST NULL)
530   GOSUB Ntmdfq_rpt_fdnl
535              ! GOTO NEXT MODULATION FREQUENCY OR IF THE SECOND BESSELL
540              ! TEST INDICATES; REPEAT FINDNULL WITH A LOWER A (SMALLER
545              !  INCREMENTS TO AVOID JUMPING OVER FIRST NULL)
550   RETURN          ! (FOR Sub_gosub)
555 Ntmdfq_rpt_fdnl:!
560 Rpt_nqfl:    !
565   IF Delta<-1.5 THEN GOTO Rtn_nqfl
570   A=A-.02
575   GOSUB Findnull
580   GOSUB Secd_bes_ck
585   GOTO Rpt_nqfl
```

```
590 Rtn_nqfl:     !
595    RETURN             !    (FOR Ntmdfq_rpt_fdnl)
600 Findnull:!
605   B=1
610   GOSUB Outputs
615 Loop1:            !    SUBLOOP OF FINDNULL
620   OUTPUT @Sa;"TS MA"
625   ENTER @Sa;Ampl
630   OUTPUT @Aa;"M1"
635   ENTER @Aa;Rms
640   Y=A^(Ampl+37)/1500     ! APPROXIMATE FORMULA OF BESSEL NULL
645   OUTPUT @Aa;"AN";Y;"VL UP"
650   WAIT .3
655   OUTPUT @Sa;"TS MA"
660   ENTER @Sa;Newamp
665   IF Newamp>Ampl THEN GOTO Refine
670   GOTO Loop1
675 Refine:    ! SUBLOOP OF FINDNULL (2ND AND 3RD CHECK OF NULL WITH
680   B=B+1    ! SUCCESSIVELY SMALLER AMPLITUDE INCREMENTS)
685   IF B>3 THEN GOTO Endloop1
690   A=A-.01
695   Z=Y*2
700   OUTPUT @Aa;"AN";Z;"VL DN"
705   WAIT .3
710   GOTO Loop1
715 Endloop1:  !
720   RETURN              !  (FOR FINDNULL)
725 Secd_bes_ck:    !
730   OUTPUT @Sa;"RB";Rb;"KZ VB";Vb;"KZ TS M3";Dltfrq;"KZ TS MA"
735   ENTER @Sa;Delta
740   RETURN              !  (FOR Secd_bes_ck)
745 Calc:   !      CALCULATION OF FREQUENCY RESPONSE
750   R=20*LGT(Rms/V100khz)-20*LGT(Mdfrq/100)
755   PRINT "MODULATION FREQUENCY="
760   PRINT Mdfrq;"KHZ";"   VMOD";Rms;"V";"   R=";R;"DB"
765   RETURN              !   (FOR Calc)
770 End:   !
775   LOCAL @Hpib
780   ABORT @Hpib
785   END
```

Appendix B Computer Program for Curve Fitting

```
100   !  ANALIT
105   PRINTER IS 1
110   DIM Title$[39]
115   PRINT CHR$(12)
120   PRINT TABXY(1,10),"ANALIT"
125   PRINT TABXY(1,18),"<    TITLE ?  MAXIMUM 38 CHARACTERS    >"
130   LINPUT "123456789 123456789 123456789 12345678",Title$
135   IF Title$="" THEN
140   Title$="ANALIT"
145   ELSE
150   END IF
155   GOSUB Gph_setup  !commands to get into graphics mode and set up grid
160   GOSUB Incremtl_curve
165   GOSUB Curve_to_fit
170   GOSUB Plot_a_tothe_x
175   GOSUB Plot_nexta_x
180   GOSUB Plot_nex_a_x
185 ! GOSUB Plot_log
190 ! GOSUB Plot_x_tothe_b
195   GOSUB Y_scale
200   GOSUB X_scale
205   GOSUB Title
210   GOTO End
215 Gph_setup:  !
220   GINIT
225   GRAPHICS ON
230   ALPHA OFF
235   WINDOW -.10,.55,-70,30
240   CLIP 0,.5,-60,15
245   GRID .05,5,0,0
250   CLIP OFF
255   RETURN
260 Incremtl_curve:!
265   DATA .498,13,.114,3,.035,-7,.007,-17,0,-22.8
270   LORG 5
275   FOR I=1 TO 5
280   READ X
285   READ Y
290   MOVE X,Y
295   LABEL "y"
300   NEXT I
305   RETURN
310 Curve_to_fit:!
315   DATA 0,13,.384,3,.463,-7,.491,-17,.498,-22.8
320   LORG 5
325   FOR I=1 TO 5
330   READ X
335   READ Y
340   MOVE X,Y
```

```
345   LABEL "x"
350   NEXT I
355   RETURN
360 Plot_a_tothe_x:  !
365   MOVE 0,0
370   FOR J=-25 TO 15 STEP .2
375   A=1.141
380   X=A^(J+37)/1500
385   PLOT X,J
390   IF X>.5 THEN GOTO Ret
395   NEXT J
400 Ret:  !
405   RETURN
410 Plot_nexta_x:  !
415   MOVE 0,0
420   FOR J=-25 TO 15
425   A=1.131
430   X=A^(J+37)/1500
435   PLOT X,J
440   NEXT J
445   RETURN
450 Plot_nex_a_x:  !
455   MOVE 0,0
460   FOR J=-25 TO 15
465   A=1.101
470   X=A^(J+37)/1500
475   PLOT X,J
480   NEXT J
485   RETURN
490 Plot_log:!
495   MOVE 0,0
500   FOR J=.001 TO .5 STEP .01
505   Y=(LOG(J)*(20)+55)/3
510   PLOT J,Y
515   NEXT J
520   RETURN
525 Plot_x_tothe_b:  !
530   MOVE 0,0
535   FOR J=.001 TO .5 STEP .01
540   Y=-((J*10)^2-(13/1.5))*(1.5)
545   PLOT J,Y
550   NEXT J
555   RETURN
560 Y_scale:  !
565   FOR I=-60 TO 10 STEP 10
570   MOVE -.03,I
575   LABEL I
580   NEXT I
585   RETURN
590 X_scale:!
```

```
595   FOR I=0 TO .7 STEP .1
600   MOVE I,19
605   LABEL I
610   NEXT I
615   RETURN
620 Title:  !
625   MOVE .25,25
630   LABEL Title$
635   RETURN
640 Remove:  !
645   OFF KEY
650   ON KBD GOTO End
655   GOTO Spin
660 End:    !
665   ON KEY 0 LABEL "DUMP GRAPHICS",3 GOTO G_dump
670   ON KEY 1 LABEL "END",3 GOTO Endit
675   ON KEY 2 GOTO Spin
680   ON KEY 3 GOTO Spin
685   ON KEY 4 GOTO Spin
690   ON KEY 5 LABEL "   REMOVE THIS",3 GOTO Remove
695   ON KEY 6 LABEL "SOFTKEY DISP",3 GOTO Remove
700   ON KEY 7 LABEL "(PRESS ANY NUM",3 GOTO Remove
705   ON KEY 8 LABEL " OR LETTER KEY",3 GOTO Remove
710   ON KEY 9 LABEL " TO REGAIN)",3 GOTO Remove
715 Spin:    GOTO Spin
720 G_dump:!
725   PRINTER IS 701
730   OUTPUT 2;" N"
735   GOTO Spin
740 Endit:!
745   GRAPHICS OFF
750   PRINT CHR$(12)
755   END
```

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

This paper describes a Bessel null technique to measure the frequency response of a frequency-modulated rf carrier and a program to automate frequency response measurements of signal generators with output frequencies from 0.450 to 2000 MHz. The measurements obtained using this technique are more precise than those obtained by a highly trained technician using a manual system.

Automated measurement of this process is desirable since the manual method is subject to the following problems: (1) excessive time, (2) error in finding the null, and (3) lack of assurance that the null is the first Bessel null. Automated measurements can be performed using a system controller, a spectrum analyzer, a function generator, and a voltmeter (all of which must be compatible and controllable remotely).

The nonlinear relationship between the modulating signal amplitude and the center frequency amplitude of the carrier is a major obstacle to automated measurement. This problem was solved by obtaining an approximate formula for this nonlinear curve.

Assurance that the null found is the first Bessel null is provided by the analysis of the frequency response of the signal generator under test as displayed on the spectrum analyzer.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

automated; Bessel null; curve fitting; frequency-modulated generators; frequency response; linearization

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