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# A Radio-Frequency Power Delivery System: Procedures for Error Analysis and Self-Calibration

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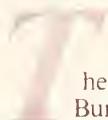
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# A RADIO-FREQUENCY POWER DELIVERY SYSTEM: PROCEDURES FOR ERROR ANALYSIS AND SELF-CALIBRATION

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An expression is developed for net power delivered to a load in terms of the indicated forward and reflected power and the system S-parameters and reflection coefficients. The dual directional coupler is treated as nonideal with power reflections assumed between all ports. The system itself is used to evaluate the major S-parameter terms in net power computation, and uncertainty in the computed power is derived from origins in the power meter readings and incompletely known S-parameters.

Key words: errors; error analysis; radio-frequency power; scattering coefficients; S-parameters.

## 1. Introduction

Standard-gain horn antennas are employed to establish known electromagnetic (EM) fields in the National Bureau of Standards anechoic chamber. Part of the uncertainty in our knowledge of these fields arises from the uncertainty in the net power delivered to the horn. In turn, this uncertainty reflects our lack of knowledge of the amplitudes and phases of the various reflection and transmission coefficients in the power delivery system, as well as the uncertainty in measurements of the power incident upon and reflected from the horn. To determine how the net power to an antenna depends on system components, we now derive an expression for net power in terms of power meter readings and the system S (scattering)-parameters.

## 2. System S-Parameters

We derive our net power expression from a set of simultaneous S-parameter equations which describe voltage waves at an n-port microwave junction. In figure 1, the voltage wave $a_j$ is incident on port j when all other ports have matched terminations. The voltage wave $b_i$ emerging into the termination on port i is obtained from the incident wave $a_j$ at port j by the relation defining the S-parameter $S_{ij}$ [1]:

$$b_i = S_{ij} \, a_j. \tag{1}$$

The symbols $b_i$ and $a_j$ are complex and represent the magnitude and phase of the waves at ports i and j; $S_{ij}$, therefore, is also complex. (For convenience in writing the power equations that appear later, we let a and b represent voltage, i.e. E field, waves normalized to $\sqrt{Z_0}$, the square root of the characteristic impedance of the system.) When i = j, the S-parameter is a reflection coefficient; when i ≠ j, the S-parameter is a transmission coefficient.

If all the ports are terminated in unmatched loads, there will be an incident $a_j$ at every $j^{th}$ port contributing to the voltage wave $b_i$ emerging from the $i^{th}$ port:

$$b_i = S_{i1}a_1 + S_{i2}a_2 + \ldots S_{ij}a_j + \ldots S_{in}a_n. \tag{2}$$

Because a similar equation can be written for the voltage wave emerging from every $i^{th}$ port, we have a set of n simultaneous equations in which all emerging waves are related to all incident waves through the S-parameters $S_{ij}$ of the junction.

In our system for establishing standard radio-frequency fields, we compute net power delivered to the transmitting antenna from measurements of incident and reflected power obtained with a dual directional coupler. Our power delivery and measurement system can, therefore, be represented as shown in figure 2 where the port terminations and numbering are:

1. Power meter to monitor forward power
2. Power meter to monitor power reflected from antenna
3. Rf generator
4. Antenna (load).

We now confine our attention to that part of the system in figure 2 within the dashed rectangle and, except for their action as "reflectors" (imperfect matches), we exclude the four terminations. This four-port system, including such components as dual directional coupler, switches, and cables is described by four simultaneous S-parameter equations:

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4$$
$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4$$
$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \qquad (3)$$
$$b_4 = S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4$$

Again, each $b_i$ represents the magnitude and phase of a voltage wave emerging from the $i^{th}$ port, and $a_j$ the magnitude and phase of a voltage wave incident at the $j^{th}$ port. Because the system (everything within the dashed-line rectangle in figure 2) described by these equations is reciprocal [1], we use the reciprocity condition

$$S_{ij} = S_{ji} \qquad (4)$$

to simplify the simultaneous equations. (The reciprocity condition is $S_{ij}Z_j = S_{ji}Z_i$; but we assume $Z_j = Z_i$.) Solving these equations for $b_i/b_j$ is further simplified by relating each $b_i$ to $a_i$ through the voltage reflection coefficient $\Gamma_i$ of the termination on the $i^{th}$ port:

$$a_1 = b_1\Gamma_1; \quad a_2 = b_2\Gamma_2; \quad a_4 = b_4\Gamma_4. \qquad (5)$$

## 3. Expressions for Net Delivered Power

An expression for net power absorbed by the load can be obtained from inspection of figure 2 if we assume an ideal dual directional coupler. By that, we mean a coupler in which there is complete isolation between ports 1 and 4, 2 and 3, and 1 and 2; i.e., $S_{14} = S_{23} = S_{12} = 0$. Thus, the power emerging from port 1 is derived only from the generator power entering at port 3; and the power emerging from port 2 is derived only from power entering port 4 in the voltage wave reflected from the load on port 4. The power absorbed by a load on port 4 is the difference between incident and reflected power at the load: $P_{net} = P_{inc} - P_{refl}$. Then, where $b_4$ and $a_4$ are the emerging and incident waves at port 4 and $\Gamma_4$ is the voltage reflection coefficient of the port 4 load,

$$P_{net} = |b_4|^2 - |a_4|^2 = |b_4|^2 - |b_4\Gamma_4|^2 = |b_4|^2 (1 - |\Gamma_4|^2). \qquad (6)$$

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Keeping in mind that the power transmission coefficient $|S_{ij}|^2$ represents an attenuation and that $P_i$ is the reading of a power meter whose sensor has a reflection coefficient $\Gamma_i$, we inspect figure 2 and write an expression for the power incident on the port 4 load in terms of $P_1$. The power $|b_1|^2$ emerging from port 1 is $P_1$ divided by $1 - |\Gamma_1|^2$ (i.e., $P_1 = |b_1|^2 \times (1 - |\Gamma_1|^2)$); $|b_1|^2$ divided by $|S_{13}|^2$ is the power entering the coupler at port 3; and this power multiplied by $|S_{34}|^2$ is the power emerging from port 4. Using similar reasoning to relate $|b_2|^2$ to the power reflected from the load, we have

$$P_{inc} = \frac{P_1 |S_{34}|^2}{|S_{13}|^2 (1 - |\Gamma_1|^2)} \quad ; \quad P_{refl} = \frac{P_2}{|S_{24}|^2 (1 - |\Gamma_2|^2)}. \tag{7}$$

The power absorbed by the load antenna in the idealized system is now

$$P_{net} = \frac{P_1 |S_{34}|^2}{|S_{13}|^2 (1 - |\Gamma_1|^2)} - \frac{P_2}{|S_{24}|^2 (1 - |\Gamma_2|^2)}$$

$$= \left|\frac{S_{34}}{S_{13}}\right|^2 \frac{P_1}{1 - |\Gamma_1|^2} - \frac{1}{|S_{24}|^2} \frac{P_2}{1 - |\Gamma_2|^2}. \tag{8}$$

This equation tells us which parameters of this ideal system we must know in order to compute $P_{net}$ from power measurements $P_1$ and $P_2$.

We now derive an expression for $P_{net}$ for a real (i.e., nonideal) four-port system in which none of the $S_{ij} = 0$.

$$P_{net} = P_{inc} - P_{refl} = |b_4|^2 - |a_4|^2; \quad a_4 = b_4 \Gamma_4 \tag{9}$$

where $\Gamma_4$ is the reflection coefficient of the load. Because our power measurements are made at ports 1 and 2, we want to express $b_4$ in terms of $b_1$, and $a_4$ in terms of $b_2$. We do this by solving eq (3) for $b_4/b_1$ and $b_4/b_2$:

$$\frac{b_4}{b_1} = g(S,\Gamma); \quad \frac{b_4}{b_2} = \frac{a_4}{b_2 \Gamma_4} = h(S,\Gamma) \tag{10}$$

where $g(S,\Gamma)$ and $h(S,\Gamma)$ are functions of the system S-parameters and the reflection coefficients of terminations on ports 1, 2, and 4. Now

$$P_{net} = |b_1|^2 |g|^2 - |b_2|^2 |\Gamma_4|^2 |h|^2. \tag{11}$$

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The quantities $b_1$ and $b_2$ can be expressed in terms of the power meter readings at ports 1 and 2 by using the final form of eq (6):

$$P_1 = |b_1|^2(1 - |\Gamma_1|^2); \quad P_2 = |b_2|^2(1 - |\Gamma_2|^2) \tag{12}$$

and so

$$P_{net} = \frac{P_1}{1 - |\Gamma_1|^2} |g|^2 - \frac{P_2 |\Gamma_4|^2}{1 - |\Gamma_2|^2} |h|^2 . \tag{13}$$

To obtain g and h, we modify eq (3) by using the reciprocity relation (eq (4)) and eq (5) ($a_i = b_i \Gamma_i$). The voltage wave $b_3$ is not involved in our computations for $P_{net}$ and so we use only the first two and the fourth of eq (3). These are now:

$$b_1 = S_{11}\Gamma_1 b_1 + S_{12}\Gamma_2 b_2 + S_{13}a_3 + S_{14}\Gamma_4 b_4$$

$$b_2 = S_{12}\Gamma_1 b_1 + S_{22}\Gamma_2 b_2 + S_{23}a_3 + S_{24}\Gamma_4 b_4 \tag{14}$$

$$b_4 = S_{14}\Gamma_1 b_1 + S_{24}\Gamma_2 b_2 + S_{34}a_3 + S_{44}\Gamma_4 b_4.$$

A convenient form for these equations can be obtained by grouping the two common $b_i$ terms within each equation, then solving the first for $a_3$ and substituting this for $a_3$ in the second and third equations (this treatment is based on [2]). We then have

$$- Bb_1 = - Ab_2 + Cb_4$$

$$- Eb_1 = Fb_2 - Db_4 \tag{15}$$

where

$$A = S_{13} (1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2$$

$$B = S_{23} (1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1$$

$$C = (S_{13}S_{24} - S_{14}S_{23})\Gamma_4 \tag{16}$$

$$D = S_{13} (1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4$$

$$E = S_{34} (1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1$$

$$F = (S_{13}S_{24} - S_{12}S_{34})\Gamma_2.$$

Equation (15) can now be solved for the ratios $g(S,\Gamma) = b_4/b_1$ and $h(S,\Gamma) = b_4/b_2$, giving the following expressions for g and h in eq (13) for $P_{net}$:

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$$g(S,\Gamma) = \frac{S_{34}}{S_{13}} \times \frac{1 - S_{11}\Gamma_1}{1 - S_{44}\Gamma_4} \times \frac{1 + \dfrac{S_{13}S_{14}\Gamma_1}{S_{34}(1 - S_{11}\Gamma_1)}}{1 + \dfrac{S_{14}S_{34}\Gamma_4}{S_{13}(1 - S_{44}\Gamma_4)}} \tag{17}$$

$$\times \frac{1 + \dfrac{[S_{23}(1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1](S_{13}S_{24} - S_{12}S_{34})\Gamma_2}{[S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2][S_{34}(1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1]}}{1 - \dfrac{(S_{13}S_{24} - S_{14}S_{23})(S_{13}S_{24} - S_{12}S_{34})\Gamma_2\Gamma_4}{[S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2][S_{13}(1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4]}}$$

$$h(S,\Gamma) = \frac{1}{S_{24}} \times \frac{1 - S_{22}\Gamma_2}{\Gamma_4} \times \frac{1 + \dfrac{S_{12}S_{23}\Gamma_2}{S_{13}(1 - S_{22}\Gamma_2)}}{1 - \dfrac{S_{14}S_{23}}{S_{13}S_{24}}} \tag{18}$$

$$\times \frac{1 + \dfrac{[S_{23}(1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1](S_{13}S_{24} - S_{12}S_{34})\Gamma_2}{[S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2][S_{34}(1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1]}}{1 + \dfrac{[S_{23}(1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1][S_{13}(1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4]}{[S_{34}(1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1](S_{13}S_{24} - S_{14}S_{23})\Gamma_4}}$$

We see that the initial factors in g and h are $S_{34}/S_{13}$ and $1/S_{24}$, respectively. These factors are present in eq (8) for the $P_{net}$ in an idealized system. Thus, the rest of g and h corrects eq (8) for a system which is not ideal. (Note that the second term in h contains $1/\Gamma_4$ which removes the considerable dependence on $\Gamma_4$ that $P_{net}$ appears to have in eq (13)). Section 6 further discusses the role of g and h in determining the uncertainty in $P_{net}$ due to approximating eq (13) with eq (8).

If the S-parameters, reflection coefficients, and their uncertainites are known for our four-port system, we can use eq (13) to compute $P_{net}$ and its uncertainty from the power measurements $P_1$ and $P_2$ and their uncertainties. The amplitudes of some of the $S_{ij}$ (see page 11) are so much less than unity that $P_{net}$ should be a weak function of their phases. To confirm this we computed g and h using these $S_{ij}$ and $\Gamma_i$ magnitudes. We allowed for a complete lack of phase information for these parameters by replacing each plus and minus sign in eqs (17) and (18) with a ± sign. The uncertainty in $P_{net}$ introduced by this phase

ambiguity is only ±1.2 percent when $|\Gamma_4| = 0.05$. For this reason, an order-of-magnitude knowledge of some of the $S_{ij}$ amplitudes is sufficient for evaluating g and h. Uncertainty components in $P_{net}$ are discussed further in section 6.

## 4. A Self-Calibrating System

The functions $g(S,\Gamma)$ and $h(S,\Gamma)$ were written in the forms given in eqs (17) and (18) to show which S-parameters have the most direct influence on $P_{net}$. These parameters ($S_{13}$, $S_{24}$, and $S_{34}$) predominate in g and h in the forms $S_{34}/S_{13}$ and $1/S_{24}$, and in $P_{net}$ as squares of magnitudes $|S_{34}/S_{13}|^2$ and $|1/S_{24}|^2$ which do not involve the parameter phases. One of our intentions in developing an automated system for establishing standard electromagnetic fields is to make the system largely self-calibrating in the sense that the system itself is used to evaluate $|S_{34}/S_{13}|^2$ and $|1/S_{24}|^2$. This section presents the procedures for such a "self analysis".

In figure 3a, we have replaced the load on port 4 with a short ($\Gamma_4 = -1$). Using eq (12), the ratio of power measurements $P_2$ and $P_1$ is

$$\frac{P_2}{P_1} = \left|\frac{b_2}{b_1}\right|^2 \frac{1 - |\Gamma_2|^2}{1 - |\Gamma_1|^2} . \tag{19}$$

We solve eq (15) for $b_2/b_1$ and obtain

$$\left|\frac{b_2}{b_1}\right|^2 = |\Gamma_4|^2 \times \left|\frac{S_{24}S_{34}}{S_{13}}\right|^2 \times \left|\frac{1 - S_{11}\Gamma_1}{(1 - S_{44}\Gamma_4)(1 - S_{22}\Gamma_2)}\right|^2$$

$$\times \left|\frac{1 - \dfrac{S_{14}S_{23}}{S_{13}S_{24}}}{1 + \dfrac{S_{12}S_{23}\Gamma_2}{S_{13}(1 - S_{22}\Gamma_2)}}\right|^2 \times \left|\frac{1 + \dfrac{S_{13}S_{14}\Gamma_1}{S_{34}(1 - S_{11}\Gamma_1)}}{1 + \dfrac{S_{14}S_{34}\Gamma_4}{S_{13}(1 - S_{44}\Gamma_4)}}\right|^2 \tag{20}$$

$$\times \left|\frac{1 - \dfrac{[S_{23}(1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1][S_{13}(1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4]}{[S_{34}(1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1](S_{14}S_{23} - S_{13}S_{24})\Gamma_4}}{1 - \dfrac{(S_{14}S_{23} - S_{13}S_{24})(S_{12}S_{34} - S_{13}S_{24})\Gamma_2\Gamma_4}{[S_{13}(1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4][S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2]}}\right|^2$$

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in which $|\Gamma_4| = 1$. Writing this equation in this form (with $|\Gamma_4| = 1$) shows that $|b_2/b_1|^2$ depends primarily on the first term containing the squares of magnitudes of $S_{24}$, $S_{34}$, and $S_{13}$. For a typical dual directional coupler, the $S_{ij}$ magnitudes are such that the numerators and denominators of the last four terms can be written as unity with a small number added or subtracted. Because we do not know the phases of the S-parameters, the value of each $S_{ij}$ may be anywhere from $+ |S_{ij}|$ to $- |S_{ij}|$ (similarly for the $\Gamma$s). Therefore, the last four terms in eq (20) can each be put in the form $|1 \pm (a \text{ small number})|^2$ and then the expression for $|b_2/b_1|^2$ reduced to

$$\left|\frac{b_2}{b_1}\right|^2 = \left|\frac{S_{24}S_{34}}{S_{13}}\right|^2 (1 \pm \Delta) \tag{21}$$

where $\Delta$ is the uncertainty in $|b_2/b_1|^2$. The order of $\Delta$ can be obtained by substituting nominal values for the $S_{ij}$ and $\Gamma$s into eq (20). From eqs (19,21) we now have

$$\left|\frac{S_{24}S_{34}}{S_{13}}\right|^2 = \frac{P_2}{P_1(1 \pm \Delta)} \frac{1 - |\Gamma_1|^2}{1 - |\Gamma_2|^2} \tag{22}$$

where $\Delta$ and the uncertainty in $P_2/P_1$ make up the total uncertainty in $|S_{24}S_{34}/S_{13}|^2$. The reflection coefficients $\Gamma_1$ and $\Gamma_2$ are given in the manufacturer's specifications for the power meter sensors on ports 1 and 2. (Because our uncertainty estimates are derived from phase ambiguities, the $\Gamma_1$ and $\Gamma_2$ magnitudes do not contribute to the total uncertainty in eq (22)).

For the next step in evaluating the predominant S-parameter terms in $P_{net}$, the system is in the configuration shown in figure 3b where the port 4 short has been replaced with the port 2 power meter, and port 2 is terminated with a matched load, ($\Gamma_2 \cong 0.05$). The ratio of the two power measurements is (analogous to eq (19))

$$\frac{P_1}{P_4} = \left|\frac{b_1}{b_4}\right|^2 \frac{1 - |\Gamma_1|^2}{1 - |\Gamma_4|^2} \tag{23}$$

where we solve eq (15) (or take the reciprocal of eq (17)) to obtain

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$$\left|\frac{b_1}{b_4}\right|^2 = \left|\frac{S_{13}}{S_{34}}\right|^2 \times \left|\frac{1 - S_{44}\Gamma_4}{1 - S_{11}\Gamma_1}\right|^2 \times \left|\frac{1 + \dfrac{S_{14}S_{34}\Gamma_4}{S_{13}(1 - S_{44}\Gamma_4)}}{1 + \dfrac{S_{13}S_{14}\Gamma_1}{S_{34}(1 - S_{11}\Gamma_1)}}\right|^2 \tag{24}$$

$$\times \left|\frac{1 - \dfrac{(S_{13}S_{24} - S_{12}S_{34})(S_{13}S_{24} - S_{14}S_{23})\Gamma_2\Gamma_4}{[S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2][S_{13}(1 - S_{44}\Gamma_4) + S_{14}S_{34}\Gamma_4]}}{1 + \dfrac{(S_{13}S_{24} - S_{12}S_{34})[S_{23}(1 - S_{11}\Gamma_1) + S_{12}S_{13}\Gamma_1]\Gamma_2}{[S_{13}(1 - S_{22}\Gamma_2) + S_{12}S_{23}\Gamma_2][S_{34}(1 - S_{11}\Gamma_1) + S_{13}S_{14}\Gamma_1]}}\right|^2$$

in which every $\Gamma_2$ is then set equal to 0.05. Again, because many of the $S_{ij}$ magnitudes are small, and the phases not known, we reduce eq (24) to the form

$$\left|\frac{b_1}{b_4}\right|^2 = \left|\frac{S_{13}}{S_{34}}\right|^2 (1 \pm \Delta) \tag{25}$$

where $\Delta$ includes all that we do not know about the $S_{ij}$. From eqs (23,25) we now have

$$\left|\frac{S_{13}}{S_{34}}\right|^2 = \frac{P_1}{P_4(1 \pm \Delta)} \frac{1 - |\Gamma_4|^2}{1 - |\Gamma_1|^2} \tag{26}$$

which is the predominant S-parameter dependence in $g(S,\Gamma)$ and, therefore, one of the S-parameter multipliers we need for computing $P_{net}$ from eq (13). Again, $\Delta$ and the uncertainty in the ratio $P_1/P_4$ give the uncertainty in $|S_{13}/S_{34}|^2$. From eqs (22) and (26) we can now obtain a value for $|1/S_{24}|^2$ which is the predominant S-parameter dependence in $h(S,\Gamma)$.

## 5. Summary of Self-Calibration Procedure

In this section we summarize the computation of system S-parameters from readings of forward and reflected power at the side arms of a dual directional coupler. System self-calibration proceeds as follows.

1. We must know which parameter groups are to be evaluated. For an ideal system, eq (8) shows the S-parameters that must be known to compute

the net power to the load antenna in figure 2. For a nonideal system, eqs (13), (17), and (18) show how all the S-parameters of the system are involved in the net power measurement, and also that the S-parameters with major influence in this measurement are the same as those identified in eq (8).

2. Values for the S-parameter multipliers in eq (8) are obtained from measurements of forward and reflected power with the system in the two configurations shown in figure 3. For the system as in figure 3a, values for power measurements $P_1$ and $P_2$ and their uncertainties are entered into eq (22) to compute the quantity $|S_{24}S_{34}/S_{13}|^2$. The uncertainty $\Delta$ is computed from the last four terms in eq (20). Then, for the system as in figure 3b, power measurements $P_1$ and $P_4$ and their uncertainties are entered into eq (26) to compute the quantity $|S_{13}/S_{34}|^2$, for which the uncertainty $\Delta$ is computed from the last three terms in eq (24). We can now compute $|1/S_{24}|^2$ and its uncertainty, and thus obtain values for the leading terms in $|g|^2$ and $|h|^2$ of eq (13), where g and h are given by eqs (17) and (18).

3. The computed values for $|S_{34}/S_{13}|^2$ and $|1/S_{24}|^2$ are substituted into $|g|^2$ and $|h|^2$, respectively, in eq (13) for the net power to the load antenna in figure 2. The remaining terms in $|g|^2$ and $|h|^2$ cannot be completely evaluated because the phases of the S-parameters and reflection coefficients are not known. Therefore, each S and $\Gamma$ is treated as a scalar quantity having the range of values $\pm|S|$ and $\pm|\Gamma|$, and the terms multiplying $|S_{34}/S_{13}|^2$ and $|1/S_{24}|^2$ in $|g|^2$ and $|h|^2$ are reduced to the form $1\pm\Delta$ in the same manner as in eqs (20,21) and (24,25).

4. The total uncertainty in the net power delivered to the port 4 load has three parts which are entered into eq (13): 1) the uncertainty in the S-parameter groups $|S_{34}/S_{13}|^2$ and $|1/S_{24}|^2$ in $|g|^2$ and $|h|^2$, respectively; 2) the uncertainty, $1\pm\Delta$, which represents our lack of knowledge of the S-parameters and reflection coefficients in the remaining terms in $|g|^2$ and $|h|^2$; and 3) the uncertainty in the power readings $P_1$ and $P_2$.

## 6. Estimated Uncertainty in Computed Net Power

The uncertainty terms that we have represented by the symbol $\Delta$ are computed using nominal values for the magnitudes of S-parameters and reflection coefficients. These values are obtained from the manufacturer's specifications for the dual directional coupler and for the power sensors. Conservative values for percent uncertainty in the power meter readings are also obtained from specifications for the meter and power sensors. We have computed the estimated uncertainties using these magnitudes

- $|S_{11}| = |S_{22}| = |S_{44}| = 0.05$
- $|S_{13}| = |S_{24}| = 0.1$
- $|S_{14}| = |S_{23}| = 10^{-3}$
- $|S_{12}| = 10^{-6}$
- $|S_{34}| = 0.95$
- $|\Gamma_1| = |\Gamma_2| = 0.05$
- $|\Gamma_4| = 0.05$ for a standard gain horn antenna (see page 14)

The power meter reading contains ±1 percent instrumentation uncertainty arising in the metering circuits, ±1.5 percent mismatch uncertainty in the sensor calibration factor, and ±2 percent uncertainty in the power linearity of the sensor. We therefore set a ±4.5 percent uncertainty on each power meter reading. Using eq (13) for net power delivered to a load antenna on port 4, the system self-calibration procedure described in this report then gives an estimated worst-case uncertainty of ±16 percent (+ 0.64 dB, - 0.75 dB) in the computed value of the delivered power $P_{net}$.

The quoted uncertainties in the power readings $P_1$ and $P_2$ are for a dual channel power meter with two sensors. It is possible to reduce this uncertainty by using the meter in the single channel mode (or using a single channel meter) with a single power sensor. The instrumentation uncertainty in the metering circuits is then quoted as ±0.5 percent (instead of ±1.0 percent). In addition, the power sensor mismatch uncertainty (a function of frequency) will cancel in the ratios $P_1/P_2$ and $P_1/P_4$ because both readings in each ratio are at the same frequency. However, the mismatch uncertainty remains in the power readings $P_1$ and $P_2$ in eq (13). A single channel power meter with one sensor reduces the estimated worst-case uncertainty in $P_{net}$ to ±11.4 percent (+ 0.47 dB, -0.53 dB), but at the expense of increased system complexity in

added circuits to switch the single power sensor between coupler ports 1, 2, and 4.

Although the preceding computations give an estimate of the maximum uncertainty inherent in the self-calibration method, we emphasize that the S-parameter magnitudes on page 11 describe only the dual directional coupler for which they are given. In practice, the procedures set forth in this report measure parameters for the entire system within the dashed rectangle in figure 2. For example, the S-parameter $S_{24}$ will be the transmission coefficient not just for the path between ports 2 and 4 of the coupler, but for the path between a measurement plane on the rectangle outside port 2 and a measurement plane on the rectangle outside port 4. Physically, these measurement planes are at the power sensor for meter $P_2$ and at the input connector on the antenna terminating port 4.

We have also used the S and $\Gamma$ magnitudes on page 11 to determine the uncertainty component which appears in $P_{net}$ as a consequence of considering the system to be nonideal; that is, the uncertainty incurred in $P_{net}$ through using eq (13) instead of eq (8) (the uncertainties in $P_1,P_2$, $|S_{34}/S_{13}|^2$, and $|1/S_{24}|^2$ being the same in each equation). To compare uncertainties in the computed $P_{net}$ for ideal and nonideal systems, the expressions $|g|^2$ and $|h|^2$ are inserted into eq (13) in the forms

$$|g|^2 = \left|\frac{S_{34}}{S_{13}}\right|^2 (1 \pm \Delta g); \quad |h|^2 = \left|\frac{1}{S_{24}}\right|^2 (1 \pm \Delta h) \tag{27}$$

where the $|1/\Gamma_4|^2$ in $|h|^2$ is omitted because it cancels the $|\Gamma_4|^2$ in eq (13). For a nonideal system, $P_{net}$ is then

$$P_{net} = \left|\frac{S_{34}}{S_{13}}\right|^2 \frac{P_1}{1 - |\Gamma_1|^2} (1 \pm \Delta g) - \left|\frac{1}{S_{24}}\right|^2 \frac{P_2}{1 - |\Gamma_2|^2} (1 \pm \Delta h). \tag{28}$$

Equation (8) for an ideal system is

$$P_{net} = \left|\frac{S_{34}}{S_{13}}\right|^2 \frac{P_1}{1 - |\Gamma_1|^2} - \left|\frac{1}{S_{24}}\right|^2 \frac{P_2}{1 - |\Gamma_2|^2}. \tag{8}$$

Equations (28) and (8) now closely resemble one another except for the terms $(1 \pm \Delta g)$ and $(1 \pm \Delta h)$ in eq (28). Therefore, neglecting the quantities $\Delta g$ and $\Delta h$ is tantamount to assuming an ideal system and identifies $\Delta g$ and $\Delta h$ as uncertainties in $P_{net}$ which are overlooked in this assumption.

To obtain the uncertainty in $P_{net}$ in eq (28) due to the uncertainty $\Delta g$ in the forward power term and the uncertainty $\Delta h$ in the reflected power term, we write $P_1$ and $P_2$ in terms of $P_0$ defined as the incident minus reflected power at port 3. Referring to figure 2, an approximation to the power $P_1/(1 - |\Gamma_1|^2)$ emerging from port 1 in a nonideal system is

$$\frac{P_1}{1 - |\Gamma_1|^2} \tilde{=} P_0 |S_{13}|^2 + P_0 |S_{34}|^2 |\Gamma_4|^2 |S_{14}|^2 \tag{29}$$

Even for $|\Gamma_4|^2 = 1$, the second term is ~ $10^4$ times smaller than the first term, so we let

$$\frac{P_1}{1 - |\Gamma_1|^2} = P_0 |S_{13}|^2 \tag{30}$$

The approximate power emerging from port 2 in a nonideal system is

$$\frac{P_2}{1 - |\Gamma_2|^2} \tilde{=} P_0 |S_{34}|^2 |\Gamma_4|^2 |S_{24}|^2 + P_0 |S_{23}|^2 \tag{31}$$

where, for $|\Gamma_4|^2 \cong 0.05$, the second term is about 25 times smaller than the first term. Because the expression in eq (30) is then about 400 times larger than that in eq (31), we let

$$\frac{P_2}{1 - |\Gamma_2|^2} = P_0 |S_{34}|^2 |\Gamma_4|^2 |S_{24}|^2 \tag{32}$$

with negligible effect on the computed uncertainty in $P_{net}$.

Equations (30) and (32) are good approximations for a nonideal system, and are exact for an ideal system. Substituting eqs (30) and (32) into eq (28), we have for a nonideal system

$$\begin{aligned} P_{net} &= |S_{34}|^2 P_0 (1 \pm \Delta g) - |S_{34}|^2 |\Gamma_4|^2 P_0 (1 \pm \Delta h) \\ &= |S_{34}|^2 P_0 [(1 \pm \Delta g) - |\Gamma_4|^2 (1 \pm \Delta h)] \\ &= |S_{34}|^2 P_0 [(1 - |\Gamma_4|^2) \pm (\Delta g + |\Gamma_4|^2 \Delta h)] \end{aligned} \tag{33}$$

where the absolute uncertainties $\Delta g$ and $\Delta h$ are grouped to form

$$\Delta P_{net} = \pm |S_{34}|^2 P_0 (\Delta g + |\Gamma_4|^2 \Delta h) \qquad (34)$$

which is the contribution to the total $P_{net}$ uncertainty neglected in the "ideal system" assumption. As a percent of $P_{net}$, $\Delta P_{net}$ becomes

$$\Delta P_{net}(\%) = \pm \frac{\Delta g + |\Gamma_4|^2 \Delta h}{1 - |\Gamma_4|^2} \times 100. \qquad (35)$$

Now putting eqs (30) and (32) into eq (8), we have for an ideal system

$$P_{net} = |S_{34}|^2 P_0 - |S_{34}|^2 |\Gamma_4|^2 P_0 = |S_{34}|^2 P_0 (1 - |\Gamma_4|^2). \qquad (36)$$

Comparing eq (36) with the first term of eq (33) shows that the mathematical descriptions of ideal and nonideal systems give the same expression for $P_{net}$. However, eq (33) also gives an additional uncertainty (beyond that common to $P_{net}$ in eqs (8) and (13)) involving the terms $\Delta g$ and $\Delta h$ which characterize the nonideal nature of the power delivery system.

Because $\Delta g$, $\Delta h$, and the relative size of $P_1$ and $P_2$ depend on $\Gamma_4$, we computed $\Delta P_{net}(\%)$ for three values of $|\Gamma_4|$. Numerical values for $\Delta g$ and $\Delta h$ were obtained by substituting the S and $\Gamma$ magnitudes into eqs (17) and (18) and consolidating the second, third, and fourth terms in each of these equations into one term of the form $(1 \pm \Delta)$. For the following values for $|\Gamma_4|$, eq (35) yields

- $|\Gamma_4| = 0.05$ ($|\Gamma_4|^2 = 0.0025$), $\Delta P_{net} = \pm 1.2\%$
- $|\Gamma_4| = 0.1$ ($|\Gamma_4|^2 = 0.01$), $\Delta P_{net} = \pm 2.1\%$ $\qquad (37)$
- $|\Gamma_4| = 0.224$ ($|\Gamma_4|^2 = 0.05$), $\Delta P_{net} = \pm 5.7\%$

(The maximum VSWR obtained by Slayton [3] for a pyramidal horn was 1.25, for which $|\Gamma| = 0.11$. Our own measurements of $|\Gamma|^2$ for a horn and an open-ended waveguide were about 0.001 ($|\Gamma| \cong 0.03$) and 0.01 ($|\Gamma| \cong 0.1$), respectively. These values suggested those we chose for $|\Gamma_4|$.)

Referring to eq (28), the total uncertainty in $P_{net}$ comprises $\Delta P_{net}$ and the uncertainties in the S-parameter multipliers and in $P_1$ and $P_2$. For

smaller values of $\Gamma_4$, $\Delta P_{net}$ is a smaller part of this uncertainty. In a power delivery system using a dual channel power meter and having $|\Gamma_4| = 0.05$, $\Delta P_{net}$ = ± 1.2% out of a total $P_{net}$ uncertainty of ± 16%. If $|\Gamma_4| = 0.224$, then $\Delta P_{net}$ = ± 5.7% out of a total uncertainty of ± 23%. Thus when $\Gamma_4$ is small, the assumption of an ideal system (i.e., the use of eq (8) instead of eq (13) to determine $P_{net}$) incurs only a small discrepancy in the estimated uncertainty in $P_{net}$.

We are currently testing the system self-calibration procedure by computing $P_{net}$ with eq (13) and comparing that value with a direct power measurement at port 4. Results of this work will be given in a later report.

## 7. Conclusion

We derived an equation for computing the net power to a load antenna using a power delivery system for which all of the S-parameters are assumed to be nonzero. An S-parameter description of the system yields equations by which the system can be self-calibrated using known terminations on port 4. When the system uses a dual channel power meter and is calibrated by the procedure described, the maximum uncertainty in $P_{net}$ obtained from eq (28) is ± 16% (for $|\Gamma_4| = 0.05$). This uncertainty can be reduced to 11.4 percent by using a single channel power meter. We have shown that a $P_{net}$ computation based on the "ideal system" assumption may be sufficient if the reflection coefficient of the load on port 4 is small.

## 8. Acknowledgment

It is our pleasure to acknowledge the cheerful and expert assistance of Wilbur Anson who wrote and ran a computer program which allowed us to observe the behavior of $g(S,\Gamma)$ and $h(S,\Gamma)$ versus many combinations of the $S_{ij}$ and $\Gamma_i$.

## 9. References

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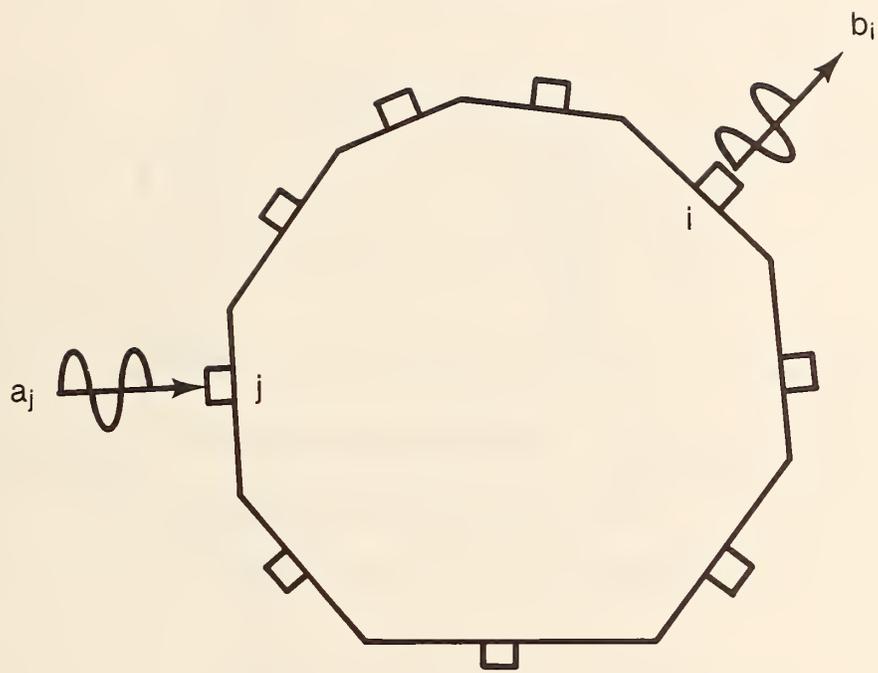
Figure 1. An n-port microwave junction

17
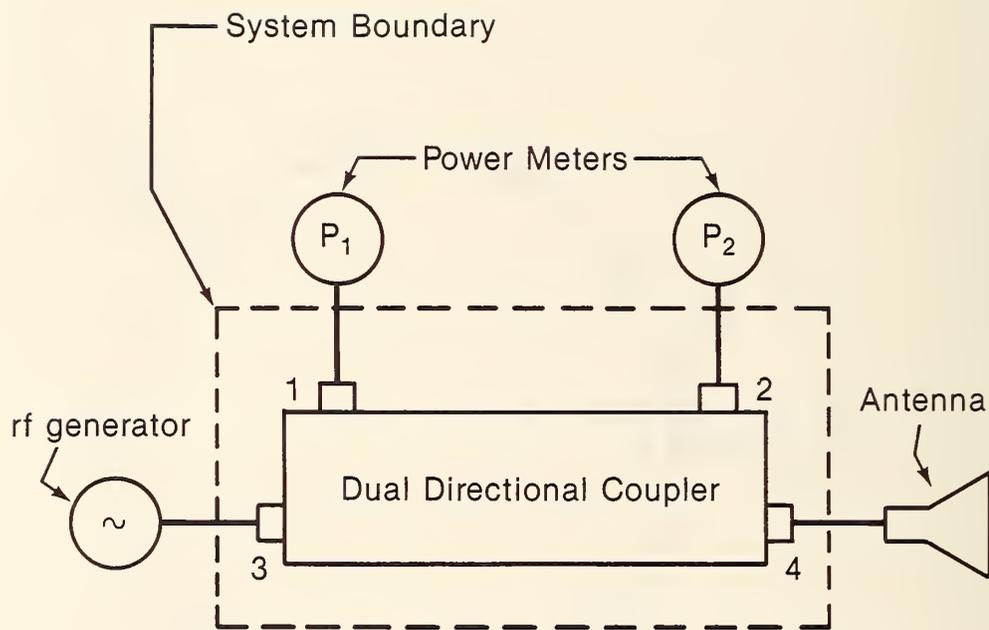
Figure 2. System for measuring rf power delivered to an antenna

a)

Power Meters

P₁    P₂

1    2

Short Circuit

Dual Directional Coupler

3    4

rf generator

b)

P₁

Matched Load

1    2

Dual Directional Coupler

3    4

P₂

rf generator
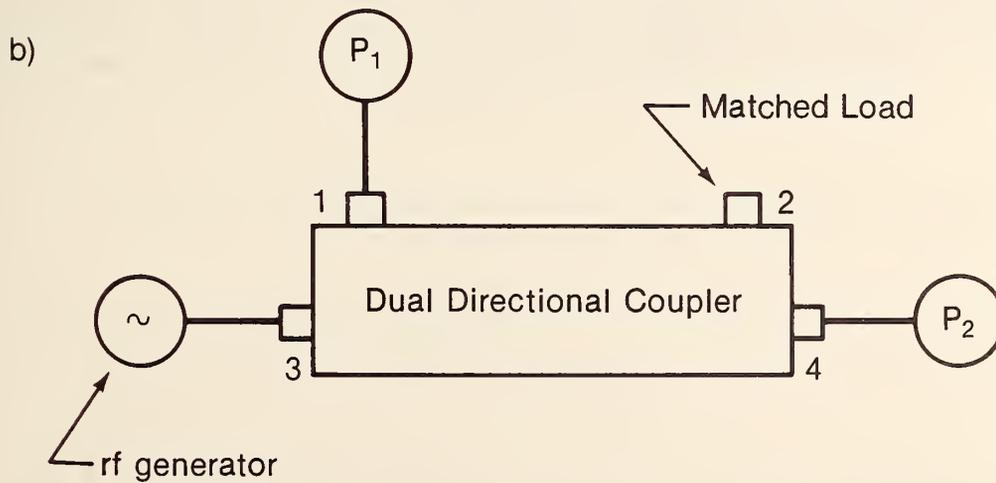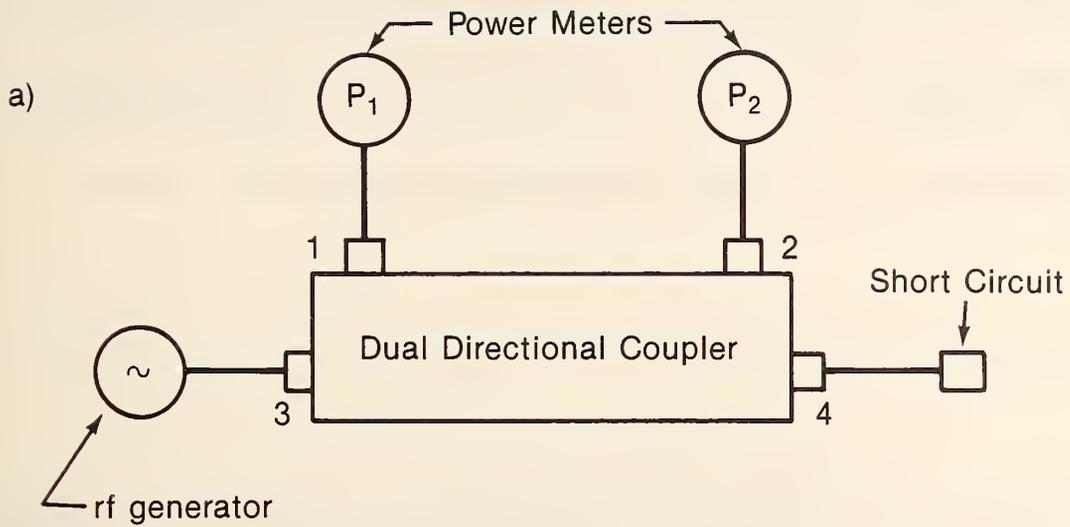
Figure 3. Configurations for measuring system s-parameters.

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An expression is developed for net power delivered to a load in terms of the indicated forward and reflected power and the system S-parameters and reflection coefficients. The dual directional coupler is treated as nonideal with power reflections assumed between all ports. The system itself is used to evaluate the major S-parameter terms in net power computation, and uncertainty in the computed power is derived from origins in the power meter readings and incompletely known S-parameters.

**12. KEY WORDS** *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

errors; error analysis; radio-frequency power; scattering coefficients, S-parameters

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