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Eigenmodes and the Composite Quality Factor of a Reverberating Chamber

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Eigenmodes and the Composite Quality Factor of a Reverberating Chamber

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FOREWORD

This report describes theoretical and experimental analyses developed by staff of the University of Colorado at Boulder in collaboration with the Electromagnetic Fields Division of the National Bureau of Standards (NBS), under a contract sponsored by NBS. Professor David C. Chang heads the University team. Dr. Mark T. Ma of NBS serves as the technical contract monitor. The period covered by this report extends from January 1982 to July 1983.

The work presented in this report represents an initial effort to establish a theoretical basis for the design of a reverberating chamber, and for the analysis and interpretation of the measurement results made inside the chamber.

The particular topics addressed herein are to determine (1) the total number of possible electromagnetic eigenmodes, for a given chamber size and operating frequency, which may exist inside the chamber for the stirring and tuning purpose, (2) the mode density within a frequency range, (3) the dependence of mode degeneracy on the dimensions of the chamber, (4) how easy a uniformly homogenous and isotropic field may be generated within a test zone inside the chamber for performing electromagnetic interference/compatibility tests, and (5) a composite quality factor to represent the chamber as a whole. The theoretical results obtained for each of these topics are considered very useful for the design purpose.

Previous publications under the same effort include:

Tippet, J. C.; Chang, D. C. Radiation characteristics of dipole sources located inside a rectangular coaxial transmission line. Nat. Bur. Stand. (U.S.) NBSIR 75-829; 1976 January.

Tippet, J. C.; Chang, D. C.; Crawford, M. L. An analytical and experimental determination of the cut-off frequencies of higher-order TE modes in a TEM cell. Nat. Bur. Stand. (U.S.) NBSIR 76-841; 1976 June.

Tippet, J. C.; Chang, D. C. Higher-order modes in rectangular coaxial line with infinitely thin inner conductor. Nat. Bur. Stand. (U.S.) NBSIR 78-873; 1978 March.

Sreenivasiah, I.; Chang, D. C. A variational expression for the scattering matrix of a coaxial line step discontinuity and its application to an over moded coaxial TEM cell. Nat. Bur. Stand. (U.S.) NBSIR 79-1606; 1979 May.

Tippet, J. C.; Chang, D. C. Dispersion and attenuation characteristics of modes in a TEM cell with a lossy dielectric slab. Nat. Bur. Stand. (U.S.) NBSIR 79-1615; 1979 August.

Sreenivasiah, I.; Chang, D. C.; Ma, M. T. Characterization of electrically small radiating sources by tests inside a transmission line cell. Nat. Bur. Stand. (U.S.) Tech. Note 1017; 1980 February.

Wilson, P. F.; Chang, D. C.; Ma, M. T. Excitation of a TEM cell by a vertical electric Hertzian dipole. Nat. Bur. Stand. (U.S.) Tech. Note 1037; 1981 March.

Sreenivasiah, I.; Chang, D. C.; Ma, M. T. A method of determining the emission and susceptibility levels of electrically small objects using a TEM cell. Nat. Bur. Stand. (U.S.) Tech. Note 1040; 1981 April.

Wilson, P. F.; Chang, D. C.; Ma, M. T. Input impedance of a probe antenna exciting a TEM cell. Nat. Bur. Stand. (U.S.) Tech. Note 1054; 1982 April.

CONTENTS

Eigenmodes and the Composite Quality Factor of a Reverberating Chamber

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The total number N of electromagnetic eigenmodes, with eigenfrequencies not greater than some given value, which can exist inside a rectangular mode-stirred or mode-tuned reverberating chamber is important in that it reveals how many modes can be available at an operating frequency for the "stirring" or tuning purpose. This is calculated analytically via a lattice-point counting technique in the k-space (k = wave number), leading to an exact expression for N, which can be split into a smooth component and a fluctuating part. The former contains, in addition to Weyl's volume term, an edge term as a second-order correction. The latter is sensitive to the dimensions of the chamber. Simple design criteria are then derived in view of the number of available modes and the uniformity of their distribution. To take into account the ohmic loss in metal walls of the chamber, a composite Q-factor is also proposed for design purposes. This is achieved by taking a suitable average of 1/Q-values of all possible modes within a small frequency interval. Comparison with numerical Q-values for individual modes shows that the composite Q can be used as a practical design parameter.

Key words: cavity; composite quality factor; eigenfrequency; eigenmode; electromagnetic field; mode density; mode number; reverberating chamber.

1. Introduction

In performing electromagnetic interference (EMI) or compatibility (EMC) measurements, a uniform plane wave in open space can be used as a reference environment. Such waves can be obtained, for example, from the far field of a radiating source or by means of a large equiphase radiator; however, it is not easy to produce high level fields necessary for measurement purposes. Also, the poor isolation from external interferences and the suffering from the

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effects of ground reflections at lower operating frequencies (below a few hundred MHz) are inconvenient for performing measurements. Anechoic chambers may be designed to simulate free-space environments without interferences from external sources and to produce an approximate plane wave, but the cost is usually prohibitively high.

Transverse electromagnetic (TEM) cells have been developed recently to provide a shielded environment for test purposes [1, 13 - 16, 27 - 29]. In the lower frequency range (which is upper bounded by the cutoff frequency of the first higher-order mode), and for electrically small equipment under test (EUT), a plane wave condition can be achieved and successful methods for EMI measurements have been proposed [24, 27 - 29]. For medium-sized TEM cells, such as the one at the National Bureau of Standards (NBS) which measures 1.2 m x 1.2 m x 2.4 m, the limitation on operating frequencies (less than 125 MHz) could be a drawback. Since the polarization of the field so generated is fixed, the EUT placed inside a TEM cell has to be physically rotated in order to make a compatibility assessment. This requirement of EUT rotation could also be an inconvenient aspect.

There is increasing interest in developing newer EMI/EMC measurement techniques by utilizing another kind of test field, namely that of a uniformly homogenous and isotropic field within a local region inside a metal enclosure. The mode-stirred or mode-tuned reverberating chambers have been introduced [3-12, 17, 21, 22, 26] to meet this demand. The isolation from external interferences being equally good, respectful bandwidths of these reverberating chambers can also be achieved. As a rule, these chambers are large enough compared to the wavelengths of the operating frequencies--which are usually in the microwave range. For example, the NBS reverberating chamber is a steel-welded, rf-tight rectangular room 3.05 m wide, 4.57 m long, and 2.74 m high with a mode-stirrer or mode-tuner mounted near the middle of the ceiling.

Conceptually, by rotating the irregularly shaped mode-stirrer or paddle-wheel tuner, the associated boundary conditions are changing either continuously or in steps so that the eigenmodes, which exist simultaneously inside the shielded metallic chamber, are perturbed accordingly. In this way, a uniformly random field or an average homogenous field can be created in a local region inside the chamber, and this region can then be used as a test zone. Theoretically and experimentally, there are a number of questions and open research topics such as general properties, test regions, measurement methods, designs, interpretations of the measured results, etc., pertaining to a mode-stirred or mode-tuned reverberating chamber.

To treat electromagnetic field problems associated with reverberating chambers and to provide basic knowledge for design purposes, two analytical approaches are possible: the direct approach and the indirect one. The former involves the direct solution of field problems containing boundaries with time-varying configurations. A formal solution using this approach is very difficult to obtain. In the latter approach, suitable linear combinations of eigenmodes of the unperturbed cavity (i.e., without mode stirrer or tuner) are used a priori to approximately satisfy the boundary conditions on the surface of the rotating mode stirrer. The coefficients are generally time dependent. The main advantage of this approach is the fact that the unperturbed eigenfrequencies and eigenmodes are usually easier to calculate; therefore, the problem can be reduced to a more familiar one. A necessary condition for the validity of this method is that the total number N of eigenmodes, with eigenfrequencies less than or equal to the operating frequency f, be large enough. Moreover, by knowing N as a function of f, we can also judge whether there are enough eigenmodes for "stirring" at any given frequency. Thus, an analytical solution of N(f) is indispensable to the study of reverberating chambers. This is indeed the approach presented in this report.

In section 2, it will be shown that, starting from a lattice-point counting technique in the k-space (k = wave number), the problem of total number N of possible TE and TM eigenmodes for an unperturbed lossless rectangular cavity can be treated via Poisson's summation formulas. The degeneracy of eigenmodes will be suitably taken into account. Besides Weyl's volume term [30, 32], the smooth component of N also contains an edge term as a second-order correction, while the first-order, or surface correction, term will be seen to vanish. The comparison with exact computer counting reveals that this result can be used for design purposes in frequency ranges with $ka_0 \gtrsim \pi$, where a_0 represents the smallest side of the chamber. The total number of eigenmodes related to an operating microwave frequency can thus be readily determined and, simultaneously, a criterion for the applicability of the indirect approach is also provided.

The ohmic loss of metal walls or, equivalently, the finite Q of a reverberating chamber will be investigated in section 3. In microwave frequency ranges, a convenient expression for the composite quality factor $\tilde{Q}$, as a function of frequency, of the chamber will be introduced. This is achieved by considering a suitable average of 1/Q-values of all possible eigenmodes in the k-space. The comparison with exact numerical Q-values of individual modes will be made with the help of normalized, cumulative distribution curves of the latter. The new expression for composite Q is useful for the practical design of reverberating chambers with slightly lossy walls.

Section 4 summarizes our results from a design point of view. For a better understanding of reverberating chambers, further investigations based on this report are necessary.

2. Number of electromagnetic eigenmodes in a rectangular chamber

The problem of the number of electromagnetic resonant modes existing inside a cavity was treated by Jeans [19] and Rayleigh [23] as early as 1905 in their studies of thermal radiation of large, rectangular cavities (isothermal black bodies). Weyl [30 - 33] investigated the same problem for more general cavities by utilizing integral equation techniques. The related problem of mode density was treated by Brownell [4] via a logarithmic Gaussian smoothing procedure and more recently by Baltes and Kneubühl [2] via computational methods. Due to strong and irregular fluctuations of the total number N of eigenmodes, most results so far available seem to have been reported either in an asymptotic sense or in terms of smoothed or averaged expressions.

In this section, a geometric approach will be adopted which, essentially, consists of a lattice-point counting technique in the k-space (k = wave number). Complete expressions for both the number N and the mode density dN/df can be achieved for the case of a rectangular chamber.

2.1 Field solution: eigenmodes and eigenfrequencies

Consider first a closed cavity with lossless metallic walls. The $\overline{E}$, $\overline{H}$ fields of the free electromagnetic oscillations obey the Maxwell equations

$$\nabla \times \overline{E} = -j\omega\mu\,\overline{H} \tag{1}$$

$$\nabla \times \overline{H} = j\omega\varepsilon\,\overline{E} \tag{2}$$

throughout the interior region of the cavity and the boundary conditions

$$\left.\begin{aligned} \overline{n} \times \overline{E} &= 0 \\ \overline{n} \cdot \overline{H} &= 0 \end{aligned}\right. \tag{3}$$

on the enclosing walls, where $\overline{n}$ is the outward unit normal vector. Equations (1) and (2) are equivalent to the vector Helmholtz equation

$$(\nabla^2 + k^2)\left\{\begin{matrix}\overline{E}\\ \overline{H}\end{matrix}\right. = 0 \tag{4}$$

plus the divergence condition

$$\nabla \cdot \left\{\begin{matrix}\overline{E}\\ \overline{H}\end{matrix}\right. = 0\ , \tag{5}$$

if the interior region of the cavity is filled with an isotropic, homogenous material of permittivity ε and permeability μ. Here we have assumed (and suppressed) the time harmonic $e^{j\omega t}$ for all the field quantities and the wave number k in (4) is given by $k^2 = \omega^2\mu\varepsilon$.

In order to treat the problem of rectangular cavities with sides a, b, c specifically, we will choose, in an arbitrary way, the z-direction to be that parallel to side c as shown in figure 1. Following standard procedures [18, p. 129-132], we can formally construct a $TE^{(z)}$ set and a $TM^{(z)}$ set of eigenmode solutions:

$TE^{(z)}_{mnp}$ modes:

$$\begin{cases} E_z = 0 \\ E_x = \frac{n\pi}{b} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sin\frac{p\pi z}{c} \\ E_y = -\frac{m\pi}{a} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sin\frac{p\pi z}{c}\ ; \end{cases} \tag{6a}$$

$$\begin{cases} H_z = \frac{1}{j\omega\mu}\left(k^2 - \frac{p^2\pi^2}{c^2}\right) \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sin\frac{p\pi z}{c} \\ H_x = \frac{1}{j\omega\mu} \frac{-m\pi}{a} \frac{p\pi}{c} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \cos\frac{p\pi z}{c} \\ H_y = \frac{1}{j\omega\mu} \frac{-n\pi}{b} \frac{p\pi}{c} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \cos\frac{p\pi z}{c}\ ; \end{cases} \tag{6b}$$

$TM^{(z)}_{mnp}$ modes:

$$\begin{cases} E_z = \frac{1}{j\omega\varepsilon}\left(k^2 - \frac{p^2\pi^2}{c^2}\right) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \cos\frac{p\pi z}{c} \\ E_x = \frac{1}{j\omega\varepsilon} \frac{m\pi}{a} \frac{-p\pi}{c} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sin\frac{p\pi z}{c} \\ E_y = \frac{1}{j\omega\varepsilon} \frac{n\pi}{b} \frac{-p\pi}{c} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sin\frac{p\pi z}{c}\ ; \end{cases} \tag{7a}$$

$$\begin{cases} H_z = 0 \\ H_x = \frac{n\pi}{b} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \cos\frac{p\pi z}{c} \\ H_y = -\frac{m\pi}{a} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \cos\frac{p\pi z}{c} \end{cases} \tag{7b}$$

where m, n, p are nonnegative integers that determine the number of nodes and antinodes in the standing wave pattern of the corresponding eigenmode. In both sets of eigenmode solutions, the k and ω are determined by the relation

$$\omega_{mnp}^2 \mu\varepsilon = k_{mnp}^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + (\frac{p\pi}{c})^2 , \quad m, n, p \geq 0 . \tag{8}$$

A closer investigation into (6) and (7) reveals that there are only two categories of <u>true</u> eigenmode solutions to the boundary value problem defined by (3) - (5) as discussed below.

<u>Category I</u>: <u>None of m, n, p is zero</u> (i.e., $m \geq 1$, $n \geq 1$, $p \geq 1$). In this category, (6) and (7) represent two independent polarizations of field corresponding to each set of values m, n, p. Each eignenvalue k_{mnp} is of mutiplicity 2 or, in other words, two independent eigenmodes are associated with each eigenvalue k_{mnp}. Thus, the degeneracy of mode is 2.

For a given eigenvalue k_{mnp}, the $\overline{E}$, $\overline{H}$ fields of the associated (degenerated) eigenmodes can be represented by

$$(\overline{E}, \overline{H})^t = a_1 \underset{TE_{mnp}^{(z)}}{(0, E_x, E_y, H_z, H_x, H_y)^t} + b_1 \underset{TM_{mnp}^{(z)}}{(E_z, E_x, E_y, 0, H_x, H_y)^t} , \tag{9a}$$

where $(\cdot \cdot \cdot \quad \cdot \cdot \cdot)^t$ means the column matrix obtained by transposing the row matrix $(\cdot \cdot \cdot \quad \cdot \cdot \cdot)$ and where a_1, b_1 are two arbitrary coefficients corresponding to the degeneracy of 2. By $(\overline{E}, \overline{H})^t$, we mean the six components of the $\overline{E}$, $\overline{H}$ fields under consideration. The six components of $(0, E_x, E_y, H_z, H_x, H_y)^t$ of the $TE_{mnp}^{(z)}$ modes and the six components of $(E_z, E_x, E_y, 0, H_x, H_y)^t$ of the $TM_{mnp}^{(z)}$ modes are explicitly shown in (6) and (7). The superscript (z) indicates that the terminology TE or TM has its usual meaning with respect to z-direction. Equivalently, the $\overline{E}$, $\overline{H}$ fields of the left-hand side of (9a) can also be represented by

$$a_2(E_z, 0, E_y, H_z, H_x, H_y)^t + b_2(E_z, E_x, E_y, H_z, 0, H_y)^t \qquad (9b)$$
$$\qquad TE^{(x)}_{mnp} \qquad\qquad\qquad TM^{(x)}_{mnp}$$

or

$$a_3(E_z, E_x, 0, H_z, H_x, H_y)^t + b_3(E_z, E_x, E_y, H_z, H_x, 0)^t$$
$$\qquad TE^{(y)}_{mnp} \qquad\qquad\qquad TM^{(y)}_{mnp}$$

with a_2, b_2 or a_3, b_3 being arbitrary coefficients. The expressions for the field components of $TE^{(x)}_{mnp}$, $TM^{(x)}_{mnp}$, $TE^{(y)}_{mnp}$, $TM^{(y)}_{mnp}$ modes can be obtained by suitably permuting the parameters, a, b, c, m, n, p and x, y, z. In Table 1 we have arbitrarily chosen (9a) to represent this category.

Table 1

Case	Restrictions	Nonvanishing components	Designation	Number of modes
1	$m\geq1$, $n\geq1$, $p\geq1$	E_x, E_y, E_z, H_x, H_y	$TM^{(z)}_{mnp}$ modes	$N_1(k)$
2	$m\geq1$, $n\geq1$, $p\geq1$	E_x, E_y, H_x, H_y, H_z	$TE^{(z)}_{mnp}$ modes	$N_2(k)$

Category II: Only one of m, n, p is zero. There are three cases as shown in Table 2. The names of $TM^{(z)}_{mno}$, $TE^{(z)}_{onp}$, and $TE^{(z)}_{mop}$ modes are adopted from the literature [18, p. 190], although the designation of E_z, E_x, and E_y modes in the fourth column seems simpler. In each of these three cases, the eigenvalue k_{mnp} is simple and the degeneracy of mode is 1.

Table 2

Case	Restrictions	Nonvanishing components	Designation	Number of modes
3	$m\geq1$, $n\geq1$, $p=0$	E_z, H_x, H_y	$TM^{(z)}_{mno}$ or E_z modes	$N_3(k)$
4	$m=0$, $n\geq1$, $p\geq1$	E_x, H_y, H_z	$TE^{(z)}_{onp}$ or E_x modes	$N_4(k)$
5	$m\geq1$, $n=0$, $p\geq1$	E_y, H_z, H_x	$TE^{(z)}_{mop}$ or E_y modes	$N_5(k)$

In the last column of Table 1 and Table 2 we have, for the purpose of convenience, also included the items $N_1(k)$, $N_2(k)$, . . . , $N_5(k)$, where $N_1(k)$ means the number of all the $TM^{(z)}_{mnp}$ modes with eigenvalues k_{mnp} less than or equal to some value k. The quantities $N_2(k)$, . . . , $N_5(k)$ have similar meaning. Thus, the total number N(k) of electromagnetic resonant modes with resonant "frequencies" smaller than or equal to some "frequency" k,† with all degeneracies taken into account, is

$$N(k) = N_1(k) + N_2(k) + N_3(k) + N_4(k) + N_5(k)$$

$$= 2N_1(k) + N_3(k) + N_4(k) + N_5(k) \tag{10}$$

The last step of (10) is due to the fact that $N_1(k) = N_2(k)$. In the following we will calculate N(k) and dN/dk via geometrical considerations in the three-dimensional k space. Note that N(k) is invariant with respect to the choice of z-axis.

2.2 Calculation of number of modes and mode density

From (8) it is apparent that each value k_{mnp} can be represented geometrically by a lattice point P having the coordinates $(\frac{m\pi}{a}, \frac{n\pi}{b}, \frac{p\pi}{c})$ in the three-dimensional Euclidian k-space as shown in figure 2. The value k_{mnp} is then equal to the distance from the origin O to the point P. To find the number $N_1(k)$ of $TM^{(z)}_{mnp}$ eigenmodes with eigenfrequencies $k_{mnp} \leqslant k$ where $m \geqslant 1$, $n \geqslant 1$, $p \geqslant 1$ (see case 1 of Table 1), we need only count the number of lattice points with $k_{mnp} \leqslant k$ in the first open octant. Thus,

$$N_1(k) = \sum_{m,n,p\geqslant 1}^{\infty} H(k - k_{mnp}) \; , \tag{11}$$

where H(x) is the Heaviside unit-step function defined by

†In this report, we will occasionally use the same word "frequency" to denote the frequency f, the wave number $k = \omega\sqrt{\mu\varepsilon}$, or the normalized wave number ka. Its meaning should be clear from the context.

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} .$$

By geometrical observations, the expression (11) can be rewritten as follows:

$$N_1(k) = \frac{1}{8} \sum_{m,n,p=-\infty}^{\infty} H(k - k_{mnp})$$

$$- \frac{1}{2} \{ \sum_{m,n,\geq 1} H(k - k_{mno}) + \sum_{n,p\geq 1} H(k - k_{onp}) + \sum_{m,p\geq 1} H(k - k_{mop}) \}$$

$$- \frac{1}{4} \{ \sum_{m\geq 1} H(k - k_{moo}) + \sum_{n\geq 1} H(k - k_{ono}) + \sum_{p\geq 1} H(k - k_{oop}) \}$$

$$- \frac{1}{8} H(k - k_{ooo}) . \tag{12}$$

Since

$$\sum_{m\geq 1} H(k - k_{moo}) = \frac{1}{2} \sum_{m=-\infty}^{\infty} H(k - k_{moo}) - \frac{1}{2} H(k - k_{ooo})$$

$$\sum_{n\geq 1} H(k - k_{ono}) = \frac{1}{2} \sum_{n=-\infty}^{\infty} H(k - k_{ono}) - \frac{1}{2} H(k - k_{ooo}) \tag{13}$$

$$\sum_{p\geq 1} H(k - k_{oop}) = \frac{1}{2} \sum_{p=-\infty}^{\infty} H(k - k_{oop}) - \frac{1}{2} H(k - k_{ooo}) ,$$

we have

$$N_1(k) = \frac{1}{8} \sum_{m,n,p=-\infty}^{\infty} H(k - k_{mnp})$$

$$- \frac{1}{2} \{ \sum_{m,n\geq 1} H(k - k_{mno}) + \sum_{n,p\geq 1} H(k - k_{onp}) + \sum_{m,p\geq 1} H(k - k_{mop}) \}$$

$$- \frac{1}{8} \{ \sum_{m=-\infty}^{\infty} H(k - k_{moo}) + \sum_{n=-\infty}^{\infty} H(k - k_{ono}) + \sum_{p=-\infty}^{\infty} H(k - k_{oop}) \}$$

$$+ \frac{1}{4} H(k - k_{ooo}) \ . \tag{14}$$

The quantities $N_3(k)$, $N_4(k)$, $N_5(k)$ can be written down immediately. Thus,

$$N_3(k) = \sum_{m,n \geq 1} H(k - k_{mno}) \ ,$$

$$N_4(k) = \sum_{n,p \geq 1} H(k - k_{onp}) \ , \tag{15}$$

$$N_5(k) = \sum_{m,p \geq 1} H(k - k_{mop}) \ .$$

Substituting (14) and (15) into (10), we arrive at the result

$$N(k) = \frac{1}{4} \sum_{m,n,p=-\infty}^{\infty} H(k - k_{mnp})$$

$$- \frac{1}{4} \{ \sum_{m=-\infty}^{\infty} H(k - k_{moo}) + \sum_{n=-\infty}^{\infty} H(k - k_{ono}) + \sum_{p=-\infty}^{\infty} H(k - k_{oop}) \}$$

$$+ \frac{1}{2} H(k - k_{ooo}) \ , \tag{16}$$

expressing the total number of resonant modes which can exist inside an unperturbed lossless chamber when the operating frequency is k. By taking the derivative, we also have the mode density

$$\frac{dN}{dK} = \frac{1}{4} \sum_{m,n,p=-\infty}^{\infty} \delta(k - k_{mnp})$$

$$- \frac{1}{4} \{ \sum_{m=-\infty}^{\infty} \delta(k - k_{moo}) + \sum_{n=-\infty}^{\infty} \delta(k - k_{ono}) + \sum_{p=-\infty}^{\infty} \delta(k - k_{oop}) \}$$

$$+ \frac{1}{2} \delta(k - k_{ooo}) \tag{17}$$

where $\delta(x)$ is the Dirac's δ-function. For lossless chambers, the mode density consists of only δ functions as expected.

Expression (17) can be put into a more convenient form by utilizing the following three-dimensional and one-dimensional Poisson's summation formulas for Fourier transforms [20, 25]:

$$\sum_{m,n,p=-\infty}^{\infty} f(m\alpha, n\beta, p\gamma) = \frac{1}{\alpha\beta\gamma} \sum_{m,n,p=-\infty}^{\infty} F\left(m \frac{2\pi}{\alpha}, n \frac{2\pi}{\beta}, p \frac{2\pi}{\gamma}\right) \tag{18}$$

$$\sum_{m=-\infty}^{\infty} g(m\alpha) = \frac{1}{\alpha} \sum_{m=-\infty}^{\infty} G\left(m \frac{2\pi}{\alpha}\right) \tag{19}$$

where the Fourier transforms F and G are defined by

$$F(\xi, \eta, \zeta) = \iiint_{-\infty}^{\infty} f(x,y,z)\, e^{i(x\xi+y\eta+z\zeta)}\, dxdydz \tag{20a}$$

$$G(\xi) = \int_{-\infty}^{\infty} g(x)\, e^{ix\xi}\, dx . \tag{20b}$$

A straightforward, though somewhat tedious, manipulation can lead to the following Fourier transform pairs:

$$f(x,y,z) = \delta(q - \sqrt{x^2 + y^2 + z^2}) \leftrightarrow F(\xi,\eta,\zeta) = 4\pi q^2 \frac{\sin\sqrt{\xi^2 + \eta^2 + \zeta^2}\, q}{\sqrt{\xi^2 + \eta^2 + \zeta^2}\, q} \quad (21)$$

$$g(x) = \delta(q - |x|) \qquad \leftrightarrow \qquad G(\xi) = 2 \cos q \xi \, . \quad (22)$$

Thus, the triple sum in (17) can be rewritten, after (18) and (21) are used, as

$$\sum_{m,n,p=-\infty}^{\infty} \delta(k - k_{mnp}) = \sum_{m,n,p=-\infty}^{\infty} \delta\left(k - \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}\right)$$

$$= 4\, abc \frac{k^2}{\pi^2} \sum_{m,n,p=-\infty}^{\infty} \frac{\sin 2\sqrt{a^2m^2 + b^2n^2 + c^2p^2}\, k}{2\sqrt{a^2m^2 + b^2n^2 + c^2p^2}\, k} \quad (23)$$

while the single sums in (17) can be rewritten, with the help of (19) and (22), as

$$\sum_{m=-\infty}^{\infty} \delta(k - k_{moo}) = \sum_{m=-\infty}^{\infty} \delta\left(k - \left|\frac{m\pi}{a}\right|\right) = \frac{2a}{\pi} \sum_{m=-\infty}^{\infty} \cos 2amk \quad (24)$$

$$\sum_{n=-\infty}^{\infty} \delta(k - k_{ono}) = \sum_{n=-\infty}^{\infty} \delta\left(k - \left|\frac{n\pi}{b}\right|\right) = \frac{2b}{\pi} \sum_{n=-\infty}^{\infty} \cos 2bnk \quad (25)$$

$$\sum_{p=-\infty}^{\infty} \delta(k - k_{oop}) = \sum_{p=-\infty}^{\infty} \delta\left(k - \left|\frac{p\pi}{c}\right|\right) = \frac{2c}{\pi} \sum_{p=-\infty}^{\infty} \cos 2cpk \, . \quad (26)$$

Substituting (23), (24), (25), (26) into (17), we get

$$\frac{dN}{dk} = abc\,\frac{k^2}{\pi^2}\sum_{m,n,p=-\infty}^{\infty}\frac{\sin 2\sqrt{a^2m^2+b^2n^2+c^2p^2}\,k}{2\sqrt{a^2m^2+b^2n^2+c^2p^2}\,k}$$

$$- \left\{\frac{a}{2\pi}\sum_{m=-\infty}^{\infty}\cos 2amk + \frac{b}{2\pi}\sum_{n=-\infty}^{\infty}\cos 2bnk + \frac{c}{2\pi}\sum_{p=-\infty}^{\infty}\cos 2cpk\right\} + \frac{1}{2}\,\delta(k)\ . \tag{27}$$

The last term of the above expression has been formally changed to $\frac{1}{2}\,\delta(k)$, since $k_{000} = 0$. Both (17) and (27) are complete expressions for the mode density of a lossless rectangular chamber. By analogy to trigonometrical Fourier series, we can express (27) as follows:

$$\frac{dN}{dk} = abc\,\frac{k^2}{\pi^2} - \frac{a+b+c}{2\pi} + \frac{1}{2}\,\delta(k) + abc\,\frac{k^2}{\pi^2}\sum_{m,n,p=-\infty}^{\infty}{}' \frac{\sin 2\sqrt{a^2m^2+b^2n^2+c^2p^2}\,k}{2\sqrt{a^2m^2+b^2n^2+c^2p^2}\,k}$$

$$- \left\{\frac{a}{2\pi}\sum_{m=-\infty}^{\infty}{}'\cos 2amk + \frac{b}{2\pi}\sum_{n=-\infty}^{\infty}{}'\cos 2bnk + \frac{c}{2\pi}\sum_{p=-\infty}^{\infty}{}'\cos 2cpk\right\} \tag{28}$$

where the prime (') in the triple sum denotes that the term with $m = n = p = 0$ is to be excluded, while the prime in the single sums denotes that the term with $m = 0$, or $n = 0$, or $p = 0$ is not included. The right-hand side of (28) is thus separated into two constituents: the first three terms may be called the "smooth component," and the rest the "fluctuating part" in accordance with the terminology to be adopted for $N(k)$ below.

The total number $N(k)$ of modes can now be calculated by integration, namely,

$$N(k) = \int_0^k \frac{dN}{dk}\,dk\ , \tag{29}$$

with the integration limits so selected that N(0) = 0 as required by physical reasoning.† For $k > 0$, we have

$$N(k) = \frac{abc}{3\pi^2} k^3 - \frac{a + b + c}{2\pi} k + \frac{1}{2}$$

$$+ \frac{abc}{4\pi^2} k \sum_{m,n,p=-\infty}^{\infty}{}' \frac{1}{r_{mnp}^2} \left(\frac{\sin 2 r_{mnp}k}{2 r_{mnp}k} - \cos 2 r_{mnp}k\right)$$

$$- \frac{1}{4\pi} \left\{ \sum_{m=-\infty}^{\infty}{}' \frac{\sin 2 amk}{m} + \sum_{n=-\infty}^{\infty}{}' \frac{\sin 2 bnk}{n} + \sum_{p=-\infty}^{\infty}{}' \frac{\sin 2 cpk}{p} \right\} \tag{30}$$

with

$$r_{mnp} = \sqrt{a^2m^2 + b^2n^2 + c^2p^2} \ . \tag{31}$$

We can separate the right-hand side of (30) into two constituents--a smooth component and a fluctuating part. The first three terms are the <u>smooth component</u> N_s, i.e.,

$$N_s(k) = \frac{abc}{3\pi^2} k^3 - \frac{a + b + c}{2\pi} k + \frac{1}{2} \qquad (k > 0) \tag{32}$$

or, in terms of frequency f,

$$N_s(f) = \frac{8\pi}{3} abc \frac{f^3}{\upsilon^3} - (a + b + c) \frac{f}{\upsilon} + \frac{1}{2} \qquad (f > 0) \tag{33}$$

where υ stands for the speed of light in the medium (usually air) inside the chamber. Note that the first term of (33) coincides with Weyl's asymptotic expression [30 - 33], which is proportional to the volume abc of the chamber and the third power of frequency. The second term is the "edge term" which modifies Weyl's result, especially in lower frequency ranges. The comparison of (32) or (33) with exact number N by computer-counting (16) reveals that these formulas are applicable in the frequency range

†There are <u>no</u> static eigenmode solutions to the type of hollow cavity as shown in figure 1.

$$ka_0 \gtrsim \pi \tag{34}$$

where a_0 represents the smallest side of the rectangular chamber.

Some typical results are depicted in figures 3, 4, and 5 where the step-like solid curves with label 1 represent the exact values of N, the curves with label 2 represent our result (33) of the smooth component, and those with label 3 correspond to Weyl's asymptotic formula. It can be seen mathematically that our smooth-component expression (33) does, on the one hand, approach Weyl's classical formula as $f \to \infty$ and, on the other hand, gives better agreement with exact N values in lower frequency ranges (e.g., in microwave frequency ranges of interest) if the ratios $a^2 : b^2 : c^2$ of the rectangular chamber are not too rational (see figures 3 and 4). In this sense, our smooth component $N_s(f)$ (which is deduced via geometrical approach) is an improvement of Weyl's formula (which was derived by integral equation technique) for the case of rectangular chambers. Also, our $N_s(f)$ provides a clear insight into the meaning of Weyl's asymptotic formula.

From (28), we can also write down the "smooth component" D_s of the mode density:

$$D_s(k) = \frac{dN_s}{dk} = \frac{abc}{\pi^2} k^2 - \frac{a + b + c}{2\pi} + \frac{1}{2}\,\delta(k) \tag{35}$$

or, in terms of frequency f,

$$D_s(f) = \frac{dN_s}{df} = \frac{dN_s}{dk}\,\frac{dk}{df}$$

$$= 8\,\pi\,abc\,\frac{f^2}{\upsilon^3} - \frac{a + b + c}{\upsilon} + \frac{1}{2}\,\delta(f)\ . \tag{36}$$

The curves shown in figures 3, 4, and 5 with label 4 correspond to the mode density (36).

The triple sum and the three single sums in (30) represent the fluctuating part N_f of the number of modes N(k). The single sums are purely periodic oscillations with k-periods equal to π/a, π/b, or π/c respectively:

$$\sum_{m=-\infty}^{\infty}{}' \frac{\sin 2\, amk}{m} = \pi - 2\, ka \; ; \qquad 0 < ka < \pi \qquad (\mathrm{mod}\ \pi)$$

$$\sum_{n=-\infty}^{\infty}{}' \frac{\sin 2\, bnk}{n} = \pi - 2\, kb \; ; \qquad 0 < kb < \pi \qquad (\mathrm{mod}\ \pi) \tag{37}$$

$$\sum_{p=-\infty}^{\infty}{}' \frac{\sin 2\, cpk}{p} = \pi - 2\, kc \; ; \qquad 0 < kc < \pi \qquad (\mathrm{mod}\ \pi) \; .$$

The triple sum represents an irregular, nonperiodic function whose details vary depending upon the a, b, c values and the frequency k. This can also be seen qualitatively from figures 3, 4, and 5.

2.3 Design considerations of reverberating chambers

As already mentioned in the introduction, one of the purposes of investigating the total number N(f) of resonant modes which can exist inside an unperturbed chamber at a given microwave frequency f, is to establish analytical criteria for the validity of the indirect approach to solving the electromagnetic field problem associated with a reverberating chamber. One such criterion is now supplied by the expression (33), namely,

$$N_s(f) = \frac{8\pi}{3}\, abc\, \frac{f^3}{\upsilon^3} - (a + b + c)\, \frac{f}{\upsilon} + \frac{1}{2} \tag{38}$$

which is applicable for $k \gtrsim \max\left\{\frac{\pi}{a}, \frac{\pi}{b}, \frac{\pi}{c}\right\}$. For example, at frequencies higher than 200 MHz, the indirect approach can be reasonably applied to the NBS chamber (2.74 m x 3.05 m x 4.57 m) since there are more than 83 resonant modes available for eigenmode expansions as predicted by (38) (see curve 2 of figure 3). As another application, we know from (38) that, for frequencies lower than 110 MHz, the number of resonant modes available in the NBS chamber is less than 12, so that a uniformly homogenous and isotropic field inside any local region in the chamber can hardly be produced even with the help of the rotating mode-stirrer or tuner.

From a design point of view, we learn from (38) also that, in the microwave range, it is the volume abc of the chamber that essentially determines the number of modes available. There is no term appearing in (38) which is related to the inner surface area $S = 2(ab + bc + ca)$ of the chamber. The edge term, i.e., the second term of (38), has only limited influence on $N_s(f)$ at microwave frequencies because it is of f^2-order smaller when compared with the volume term. In order to have a better stirring or tuning effect and to achieve a more uniform field, a larger number of modes should be available. In other words, a chamber of bigger volume will be preferable as is clearly shown by (38).

Next let us consider another design question such as "what are the best dimensions for a rectangular chamber, provided that its volume is given and fixed?" If the volume abc is constant, the inner surface area $2(ab + bc + ca)$ and the total edge length $4(a + b + c)$ of the chamber, according to (38), have little or no effect with respect to the smooth component $N_s(f)$ of the total number of modes at microwave frequencies or above. However, they do affect the degeneracy of modes and, hence, the fluctuating part, significantly. Since the resonant frequencies of eigenmodes are determined by

$$\frac{k^2}{\pi^2} = \frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{c^2}, \tag{39}$$

we see that, if the ratios $a^2 : b^2 : c^2$ are rational, there may exist more than one set of (m, n, p) values giving the same value of k/π and the degeneracy of modes arises. Thus, we should design the dimensions of the chamber in such a way as to keep the ratios $a^2 : b^2 : c^2$ mutually not too rational in order to reduce the degeneracy of modes and, hence, to increase the uniformity in distribution of resonant modes. The cubic chamber ($a = b = c$) should especially be avoided.

To see this design principle, let us compare three rectangular chambers which have the same volume of 38.191 m^3 but different dimensions:

$$\text{Chamber 1 (NBS chamber)} \quad \begin{cases} a = 2.74 \text{ m} \\ b = 3.05 \text{ m} \\ c = 4.57 \text{ m} \end{cases} \quad \ldots \quad \text{Figure 3 and figure 6} \qquad (40a)$$

$$\text{Chamber 2 (square-base chamber)} \quad \begin{cases} a = 2.1755 \text{ m} \\ b = 4.1899 \text{ m} \\ c = b \end{cases} \quad \ldots \quad \text{Figure 4 and figure 7} \qquad (40b)$$

$$\text{Chamber 3 (cubic chamber)} \quad \begin{cases} a = 3.3676 \text{ m} \\ b = a \\ c = a \end{cases} \quad \ldots \quad \text{Figure 5 and figure 8} \qquad (40c)$$

For the frequency range $40 < f < 220$ MHz considered in figures 3, 4, and 5, the smooth components $N_s(f)$ are essentially the same for all three chambers (see curves with label 2). The fluctuation of exact N-values (solid curves with label 1) around the smooth components is mild for the NBS chamber (figure 3), relatively strong for the square-base chamber (figure 4), and violent for the cubic chamber (figure 5), in agreement with our criterion on $a^2 : b^2 : c^2$ as stated above.

Figures 6, 7, and 8 are further examples to illustrate the same fact, where the frequency range $140 \leqslant f \leqslant 230$ MHz has been chosen. The three chambers have 110, 116, or 113 modes, respectively, in this frequency range. They have about the same number of modes. The numbers ΔN of modes in each interval $\Delta f = 1$ MHz are computer-counted in order to show the distribution of resonant modes. We can also see that the uniformity in mode distribution is good for the NBS chamber (figure 6), fair for the square-base chamber (figure 7), and poor for the cubic chamber (figure 8) where no mode exists in a larger number of frequency sub-bands.

In summary, based on the separation of the total number of eigenmodes into a smooth component and a fluctuating part, we have established the following two criteria which can be used for the design of rectangular reverberating chambers for microwave interference measurements:

<u>Criterion A.</u> The volume abc of the chamber should be as large as possible in order to have large values of $N_s(f)$ for stirring or tuning purposes. The total edge length $4(a + b + c)$ can be set nearly equal to its minimal value if there is no possible contradiction to criterion B. (41)

<u>Criterion B.</u> The ratios $a^2 : b^2 : c^2$ of edge lengths squared should not be too rational in order to reduce the fluctuating part $N_f(f)$ and, hence, to increase the uniformity in mode distribution. (42)

2.4 A brief review of mode degeneracy

As already mentioned, there are two causes of eigenmode degeneracy. The first cause is due to the vector property of Maxwell's equations or, equivalently, the transverse property of electromagnetic wave motions. As seen in Table 1, there are <u>two</u> independent eigenmodes corresponding to each three-dimensional lattice point (m, n, p) with $m \geqslant 1$, $n \geqslant 1$, $p \geqslant 1$.

The second cause of degeneracy is due to the number-theoretical property of the eigenfrequency expression (39), where, at most, one of the integers m, n, p is allowed to be zero. Evidently, there may be more than one set of integers (m, n, p) corresponding to a given eigenfrequency k, especially when the ratios $a^2 : b^2 : c^2$ are rational.

To sum up, let $z_3(k; m, n, p)$ be the number of sets of integers $(m\ n, p)$, $m \geqslant 1$, $n \geqslant 1$, $p \geqslant 1$, satisfying (39); and let $z_{21}(k; n, p)$ denote the number of sets of integers $(0, n, p)$, $m = 0$, $n \geqslant 1$, $p \geqslant 1$, satisfying (39) with similar denotation of $z_{22}(k; m, p)$ and $z_{23}(k; m, n)$. Then the <u>total</u> degeneracy $G(k)$ of modes associated with the eigenfrequency k is given by

$$G(k) = 2\ z_3(k;\ m,\ n,\ p) + z_{21}(k;\ n,\ p) + z_{22}(k;\ m,\ p) + z_{23}(k;\ m,\ n).$$

At higher frequencies where $k \gg 1$, this can be approximated by

$$G(k) \cong 2\ z_3(k;\ m,\ n,\ p)\ .$$

Since the total number of electromagnetic eigenmodes with eigenfrequencies less than or equal to some given value k' is

$$N(k') = \sum_{k \leqslant k'} G(k)\ ,$$

k satisfying (39), and since G(k) increases very rapidly with k, a rough approximation for large k' can be made as follows

$$N(k') \simeq 2 \sum_{k \leqslant k'} z_3(k;\ m,\ n,\ p)\ , \qquad k' \gg 1\ ,$$

which means that, at higher frequencies, the total number of electromagnetic eigenmodes is roughly twice that of scalar (such as acoustic) eigenmodes. Again, this can be attributed to the transverse property of electromagnetic wave motions in contrast to the longitudinal property of acoustic wave motions.†

Unfortunately, little knowledge concerning the "analytic" properties of the discrete, number-theoretical functions z_{2i} and z_3 is available, so we will not try to deal with mode degeneracy in more detail. Instead, Criterion B has been set up in section 2.3 to serve for practical design purposes.

3. A composite Q-factor for lossy rectangular chambers

So far, we have considered only rectangular chambers with perfectly conducting metal walls. In practice, however, the steel, aluminum, or other

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†In some technical papers, e.g., [6], the mode number and mode density are in error by a factor of, roughly, 2. The expression $N \simeq 4\pi\ abc\ f^3/(3\ \upsilon^3)$ is often used instead of the correct one given by the leading term of our result (38) for $N_s(f)$.

metal walls are not perfect so that the electromagnetic fields will penetrate into walls and cause ohmic losses. For general structures, this phenomenon may be described with the help of the skin depth and/or the surface resistance. For the case of cavity resonators, the quality factor Q is a more suitable quantity to account for dielectric and wall losses. In this section we will be concerned with the calculation of Q-factor of a rectangular chamber due to its wall losses.

Once the z-direction is chosen to be parallel to one of the edges, say edge c, of the rectangular chamber, the Q-factors of the $TE^{(z)}_{onp}$, $TE^{(z)}_{mop}$, $TE^{(z)}_{mnp}$, $TM^{(z)}_{mno}$, and $TM^{(z)}_{mnp}$ modes are already known and explicitly given [18, p. 190]. These, however, are not convenient for practical applications. Since there are a lot of resonant modes (hundreds, thousands, ten thousand, or more modes) which can exist and be stirred or tuned inside a reverberating chamber at microwave frequencies, it is surely more convenient to define a composite Q in order to describe the ohmic loss behavior of the chamber. In this section, a composite Q will be derived by considering a suitable average of 1/Q in the k-space.

3.1 Previous results of Q's for rectangular cavities

With the coordinate system chosen, as shown in figure 1, such that the z-direction is parallel to edge c, the Q due to conductor losses for the various modes in a rectangular cavity [18 p. 190] as outlined in Tables 1 and 2 are listed below.

$TE^{(z)}_{onp}$ modes:
$$Q^{TE}_{onp} = \frac{\eta\ abc\ k_r^3}{2R(bc\ k_r^2 + 2ac\ k_y^2 + 2ab\ k_z^2)} \tag{43}$$

$TE^{(z)}_{mop}$ modes:
$$Q^{TE}_{mop} = \frac{\eta\ abc\ k_r^3}{2R(ac\ k_r^2 + 2bc\ k_x^2 + 2ab\ k_z^2)} \tag{44}$$

$TE_{mnp}^{(z)}$ modes: $$Q_{mnp}^{TE} = \frac{\eta \; abc \; k_{xy}^{2} \; k_{r}^{3}}{4R[bc(k_{xy}^{4} + k_y^2 k_z^2) + ac(k_{xy}^{4} + k_x^2 k_z^2) + ab \; k_{xy}^2 k_z^2]} \tag{45}$$

$TM_{mno}^{(z)}$ modes: $$Q_{mno}^{TM} = \frac{\eta \; abc \; k_{r}^{3}}{2R(ab \; k_r^2 + 2bc \; k_x^2 + 2ac \; k_y^2)} \tag{46}$$

$TM_{mnp}^{(z)}$ modes: $$Q_{mnp}^{TM} = \frac{\eta \; abc \; k_{xy}^{2} \; k_{r}}{4R[b(a + c) \; k_x^2 + a(b + c) \; k_y^2]} \tag{47}$$

with

$$k_x = \frac{m\pi}{a}, \qquad k_y = \frac{n\pi}{b}, \qquad k_z = \frac{p\pi}{c}, \qquad k_{xy} = \sqrt{k_x^2 + k_y^2},$$

$$k_r = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2} = k_{mnp} \; . \tag{48}$$

Here η denotes the intrinisic impedance of the medium inside the chamber ($\eta = \sqrt{\mu_0/\varepsilon_0} = 120\pi$ ohms for air medium), and R is the surface resistance of the metal wall given by

$$R = \frac{1}{\sigma\delta_s} \qquad \text{(ohms)}$$

where σ (siemens/m) is the conductivity of the metal and δ_s the skin depth. Expressions (43) - (47) are the starting point of our consideration.

3.2 Construction of a composite Q

The central idea of constructing a composite Q is to form a lattice average of 1/Q-values of all resonant modes, taking into account the fact that each three-dimensional lattice point (m, n, p) with $m \geqslant 1$, $n \geqslant 1$, $p \geqslant 1$ corresponds to two modes, and that the number of these three-dimensional lattice points is much larger than that of two-dimensional lattice points when the frequency k_r under consideration is high enough. When k is large (see figure 9), the lattice points (m, n, p) corresponding to (48) inside the elliptical shell $k < k_r < k + \Delta k$ are densely distributed, so that lattice sums over this shell can be approximately replaced by suitable integrals.

Let us consider the three-dimensional lattice sum of $1/Q^{TM}_{mnp}$ over the shell $k < k_r < k + \Delta k$ in the first open octant, namely,

$$I_1 = \sum_{m,n,p} \frac{1}{Q^{TM}_{mnp}(m, n, p)} \cong \iiint_{k_r=k}^{k+\Delta k} \frac{1}{Q^{TM}_{mnp}(m, n, p)} \, dm \, dn \, dp \; . \tag{49}$$

After introducing the spherical coordinates k_r, θ, ψ according to

$$\begin{aligned} \frac{m\pi}{a} &= k_r \sin\theta \cos\psi & k_r &\in (k, k + \Delta k) \\ \frac{n\pi}{b} &= k_r \sin\theta \sin\psi & \theta &\in (0, \pi/2) \\ \frac{p\pi}{c} &= k_r \cos\theta & \psi &\in (0, \pi/2) \end{aligned} \tag{50}$$

and rewriting $1/Q^{TM}_{mnp}$ in terms of these new coordinates as

$$\frac{1}{Q^{TM}_{mnp}} = \frac{4R}{\eta \, abc} \frac{1}{k_r} [ab + c(b \cos^2\psi + a \sin^2\psi)] \; . \tag{51}$$

The integral I_1 can be performed, leading to the result

$$I_1 \cong \frac{2Rk}{\eta\pi^2} \; [ab + \frac{1}{2} c(a + b)] \, \Delta k \; . \tag{52}$$

The three-dimensional lattice sum of $1/Q^{TE}_{mnp}$ over the shell $k < k_r < k + \Delta k$ in the first open octant

$$I_2 = \sum_{m,n,p} \frac{1}{Q^{TE}_{mnp}(m, n, p)} \cong \iiint_{k_r=k}^{k+\Delta k} \frac{1}{Q^{TE}_{mnp}(m, n, p)} \, dm \, dn \, dp \tag{53}$$

can be treated analogously. Using the coordinates (50) and the corresponding expression

$$\frac{1}{Q_{mnp}^{TE}} = \frac{4R}{\eta\ abc}\frac{1}{k_r}\left[c(a + b)\sin^2\theta + ab\cos^2\theta\right.$$

$$\left. + \frac{1}{2}c(b + a)\cos^2\theta\left(1 + \frac{b - a}{b + a}(\sin^2\phi - \cos^2\phi)\right)\right] , \tag{54}$$

we have

$$I_2 \cong \frac{2Rk}{\eta\pi^2}\left[\frac{1}{3}(ab + bc + ca) + \frac{1}{2}c(a + b)\right]\Delta k \quad . \tag{55}$$

Next, let us turn to the two-dimensional lattice sum of $1/Q_{mno}^{TM}$ over the strip $k < k_r < k + \Delta k$ in the first open quadrant of the mn-plane, namely,

$$I_3 = \sum_{m,n}\frac{1}{Q_{mno}^{TM}(m, n)} \cong \iint_{k_r=k}^{k+\Delta k}\frac{1}{Q_{mno}^{TM}(m, n)}\,dm\,dn \ . \tag{56}$$

The introduction of the polar coordinates k_r, ϕ according to

$$\begin{cases} \dfrac{m\pi}{a} = k_r\cos\phi & k < k_r < k + \Delta k \\[2ex] \dfrac{n\pi}{b} = k_r\sin\phi & 0 < \phi < \dfrac{\pi}{2} \end{cases} \tag{57}$$

and the rewriting of $1/Q_{mno}^{TM}$ in the following polar coordinate form

$$\frac{1}{Q_{mno}^{TM}} = \frac{2R}{\eta\ abc}\frac{1}{k_r}\left[ab + 2c(b\cos^2\phi + a\sin^2\phi)\right] \tag{58}$$

lead to the result

$$I_3 \cong \frac{R}{\eta c\pi}(ab + bc + ca)\Delta k \ . \tag{59}$$

By applying the same technique, we can evaluate the other two-dimensional lattice sums

$$I_4 = \sum_{n,p} \frac{1}{Q^{TE}_{onp}(n, p)} \tag{60}$$

$$I_5 = \sum_{m,p} \frac{1}{Q^{TE}_{mop}(m, p)} \tag{61}$$

over one-fourth of the strip $k < k_r < k + \Delta k$ in the np-plane or mp-plane respectively. The results are

$$I_4 \cong \frac{R}{\eta a \pi} (ab + bc + ca)\Delta k \tag{62}$$

$$I_5 \cong \frac{R}{\eta b \pi} (ab + bc + ca)\Delta k \ . \tag{63}$$

In order to calculate the average of 1/Q-values, we now need the total number of modes ΔN corresponding to the shell $k < k_r < k + \Delta k$ in the first octant. This can be approximated by twice the number of three-dimensional lattice points (m, n, p) inside that shell. Using the terminology of Tables 1 and 2 of section 2.1, this is equal to†

$$\begin{aligned} \Delta N &= \Delta N_1 + \Delta N_2 + \Delta N_3 + \Delta N_4 + \Delta N_5 \\ &\cong 2\ \Delta N_1 \\ &\cong 2 \iiint_{k_r=k}^{k+\Delta k} dm\ dn\ dp = \frac{abck^2}{\pi^2} \Delta k \ . \end{aligned} \tag{64}$$

The average of all the 1/Q-values with z-axis parallel to edge c can be defined as

$$\langle \frac{1}{Q} \rangle = \frac{1}{\Delta N} (I_1 + I_2 + I_3 + I_4 + I_5) \ . \tag{65}$$

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†In fact, the approximation $\Delta N \cong (abc/\pi^2)\ k^2 \Delta k$ used here could have also been obtained by taking $\Delta N \cong (dN/dk)\ \Delta k$ and retaining only the first term of (28) for dN/dk.

The substitution of (52), (55), (59), (62) - (64) into (65) gives

$$\langle\frac{1}{Q}\rangle = \frac{2RS}{\eta\ abck} \cdot \frac{2}{3}\left\{1 + \frac{3\pi}{8k}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\right\} , \tag{66}$$

where

$$S = 2(ab + bc + ca) \tag{67}$$

is the total inner surface area of the chamber. The form of (66) is invariant with respect to the choice of z-direction, so that it can be taken as a generally valid result, although it is derived under the arbitrary choice of z-axis along the edge c. Taking the inverse of $\langle\frac{1}{Q}\rangle$, a composite quality factor $\tilde{Q}$ can be defined for the rectangular chamber:

$$\tilde{Q} = \frac{1}{\langle 1/Q\rangle} = \frac{\eta}{2}\frac{abc}{R\ S}k \cdot \frac{3}{2} \cdot \frac{1}{1 + \frac{3\pi}{8k}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)} . \tag{68}$$

Noting that

$$\frac{\eta}{2}\frac{abc}{R\ S}k = \frac{V}{S\ \delta_s} \tag{69}$$

we have the alternative form

$$\tilde{Q} = \frac{V}{S\ \delta_s} \cdot \frac{3}{2}\frac{1}{1 + \frac{3\pi}{8k}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)} , \tag{70}$$

where V denotes the volume of the chamber. At high frequencies with $ka_o \gg 1$, $a_o = \min\{a, b, c\}$, our result thus predicts for the reverberating chamber a composite Q-factor which is 1.5 times larger than that of the conventional $V/(S\ \delta_s)$ estimate.

As numerical checks of the validity of expression (70), we considered the NBS chamber at three different frequency "shells" of bandwidth 20 MHz for each,

$180 \leqslant f \leqslant 200$ MHz	(23 modes)	-----Figure 10	
$330 \leqslant f \leqslant 350$ MHz	(87 modes)	-----Figure 11	(71)
$480 \leqslant f \leqslant 500$ MHz	(178 modes)	-----Figure 12	

and computed the (normalized) 1/Q-values of resonant modes of various types by using formulas (43) - (47). Each value of 1/Q was computed three times according to the three orthogonal directions. For example, 69 1/Q-values were obtained for the case of $180 \leqslant f \leqslant 200$ MHz. These values are plotted as a normalized cumulative distribution curve depicted in figure 10; their arithmetical mean and standard deviation are also shown. From figures 10, 11, and 12, it is seen that, as the frequency goes higher, the arithmetical mean of the normalized 1/Q-values moves closer and closer to the theoretical limit of 2/3 = 0.667 as predicted by (70), while the standard deviation decreases from 0.090 to 0.074, showing that more 1/Q-values become concentrated near the theoretical limit. Hence, the validity of the composite quality factor (70), which is a relatively simple expression, is checked and can be used for design purposes.

3.3 Discussion of results

From the derivation leading to (66) and (70), or equivalently, from the factor

$$1 + \frac{3\pi}{8k}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \tag{72}$$

appearing in (66) and (70), we see that the main contribution to the final result of $\langle\frac{1}{Q}\rangle$ or $\tilde{Q}$ comes from the TE_{mnp} and TM_{mnp} modes (with respect to x-, y-, or z-direction) that correspond to the three-dimensional lattice points (m, n, p) with $m \geqslant 1$, $n \geqslant 1$, $p \geqslant 1$. The contribution from the TE_{onp}, TE_{mop}, and TM_{mno} modes (with respect to x-, y-, or z-direction), that correspond to two-dimensional lattice points (see figures 2 and 9) is of k-order smaller, as the second term of (72) shows.

On the other hand, a clear insight into the individual Q-values of the various eigenmodes can be obtained by considering the special case of a cubic cavity with a = b = c. Starting from (43) - (47) and noting (69), we can readily arrive at the interesting, extremely simple result, namely,

TE_{onp} modes: $$Q^{TE}_{onp} = \frac{2V}{S\,\delta_s} \tag{73a}$$

TE_{mop} modes: $$Q^{TE}_{mop} = \frac{2V}{S\,\delta_s} \tag{73b}$$

TM_{mno} modes: $$Q^{TM}_{mno} = \frac{2V}{S\,\delta_s} \tag{73c}$$

TE_{mnp} modes: $$Q^{TE}_{mnp} = \frac{3}{2}\left(\frac{V}{S\,\delta_s}\right) \tag{74a}$$

TM_{mnp} modes: $$Q^{TM}_{mnp} = \frac{3}{2}\left(\frac{V}{S\,\delta_s}\right) \tag{74b}$$

where TE or TM means transversal electric or magnetic with respect to any of the x-, y-, and z-directions which are now equivalent because of the cubic structure of cavity. From (73) and (74) we see that, for the cubic cavity, the Q associated with <u>each</u> of the TE_{mnp} and TM_{mnp} modes (corresponding to three-dimensional lattice points) has the value $\frac{3V}{2S\,\delta_s}$, while the Q associated with <u>each</u> of the TE_{onp}, TE_{mop}, and TM_{mno} modes (corresponding to two-dimensional lattice points) has the value $\frac{2V}{S\,\delta_s}$. If the dimension of the cavity deviates from that of a cube, then the Q-values of the TE_{mnp} and TM_{mnp} modes will be expected to spread out around $\frac{3V}{2S\,\delta_s}$, while those of the TE_{onp}, TE_{mop}, and TM_{mno} modes will spread out into a vicinity of $\frac{2V}{S\,\delta_s}$.

Based on these observations, we can now give a plausible interpretation to the composite quality factor Q for the rectangular chamber with sides a, b, c. Since $\langle\frac{1}{Q}\rangle$, as given by (66), is <u>invariant</u> with respect to the choice of z-direction, that is,

$$\langle\frac{1}{Q}\rangle\Big|_{z\parallel c} = \langle\frac{1}{Q}\rangle\Big|_{z\parallel b} = \langle\frac{1}{Q}\rangle\Big|_{z\parallel a} = \langle\frac{1}{Q}\rangle\ , \tag{75}$$

we can consider it to be the average of three (identical) quantities obtained by choosing z-axis parallel to edges c, b, and a, respectively:

$$\langle\frac{1}{Q}\rangle = \frac{1}{3}\left\{\langle\frac{1}{Q}\rangle\Big|_{z\parallel c} + \langle\frac{1}{Q}\rangle\Big|_{z\parallel b} + \langle\frac{1}{Q}\rangle\Big|_{z\parallel a}\right\}\ . \tag{76}$$

The quantities $<\frac{1}{Q}>|_{z\parallel c}$, $<\frac{1}{Q}>|_{z\parallel b}$, $<\frac{1}{Q}>|_{z\parallel a}$ themselves are lattice averages of the individual 1/Q-values which are now distributed either in the vicinities of $\frac{2}{3}(S\ \delta_s/V)$ (corresponding to three-dimensional lattice points) or in the vicinities around $\frac{1}{2}(S\ \delta_s/V)$ (corresponding to two-dimensional lattice points), due to deviations from "cubicness." The three cases of deviation from cubicness, corresponding to three different choices of z-axis, namely, $z\parallel c$, $z\parallel b$, and $z\parallel a$ respectively, will cancel one another to some extent when the overall average (76) is taken. In other words, the final result of $<\frac{1}{Q}>$ for a rectangular chamber should not differ much from that for a cubic cavity of the same volume. Indeed, by comparing (73) and (74) with our result (66), and recalling the fact that the three-dimensional lattice points are dominant in number at higher frequencies, we see that the above interpretation of $<\frac{1}{Q}>$ and, hence, of $\tilde{Q}$ is satisfactory and can be used to shed some physical insight into the expressions (70) for $\tilde{Q}$.

It seems interesting to intuitively account for the factor 2/3 appearing in the expression (66) for $<\frac{1}{Q}>$ via arguments similar to those used in [5]. For a randomly varying field, there are 3 degrees of freedom with respect to the $\overline{E}$-polarization, while there are only 2 degrees of freedom concerning the $\overline{E}$ -polarization of the partial plane waves that add together to form the standing wave patterns in a rectangular chamber.

4. Conclusions

Among many problems pertaining to the operation and design of mode-stirred or mode-tuned reverberating chambers, we have solved here two problems from a design point of view, namely that of the distribution of resonant modes and that of the composite quality factor of the chamber.

A complete solution is obtained for the total number N(f) of resonant modes with resonant frequencies less than or equal to f. Due to its stepwise character, it is conveniently split into a smooth component N_s and a fluctuating part N_f. The former depends at microwave frequencies essentially only on the volume, while the latter is sensitive to the dimensions of the rectangular chamber. Simple criteria are then formulated concerning the design of the dimensions of a rectangular chamber. Exact expressions for the mode density dN/df are also obtained.

To take into account the ohmic loss in the metallic walls, a composite quality factor $\tilde{Q}$ is proposed; it is simple and closed in form. The limiting value of this unloaded $\tilde{Q}$ differs from the conventional $V/(S\ \delta_s)$ estimation by a factor 3/2, where V represents the volume of the chamber, S its inner surface area, and δ_s the skin depth of the lossy walls of the chamber at the operating frequency.

Since compromises between low conductor loss (high Q) and broad modal coverage (low Q) are almost always necessary in the practical design of reverberating chambers, our results on the distribution of resonant modes and on the composite quality factor presented in this report will be found convenient and helpful in design applications. We emphasize the fact, however, that these factors should be used for preliminary design considerations. Under practical conditions, when equipment is placed inside the chamber for test, the volume and surface of the chamber and the surface resistance will change, causing the loaded Q and distribution of resonant modes to be different depending on the size and shape of the equipment under test and the material of which the equipment is made.

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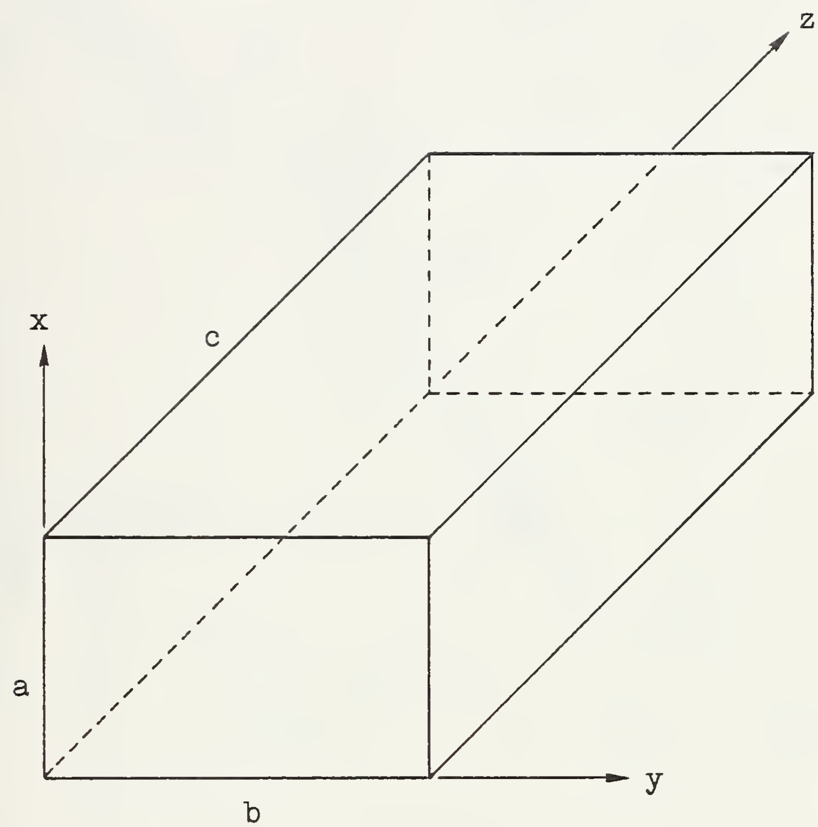

Figure 1. A rectangular chamber showing orientation conventions adopted in this report.

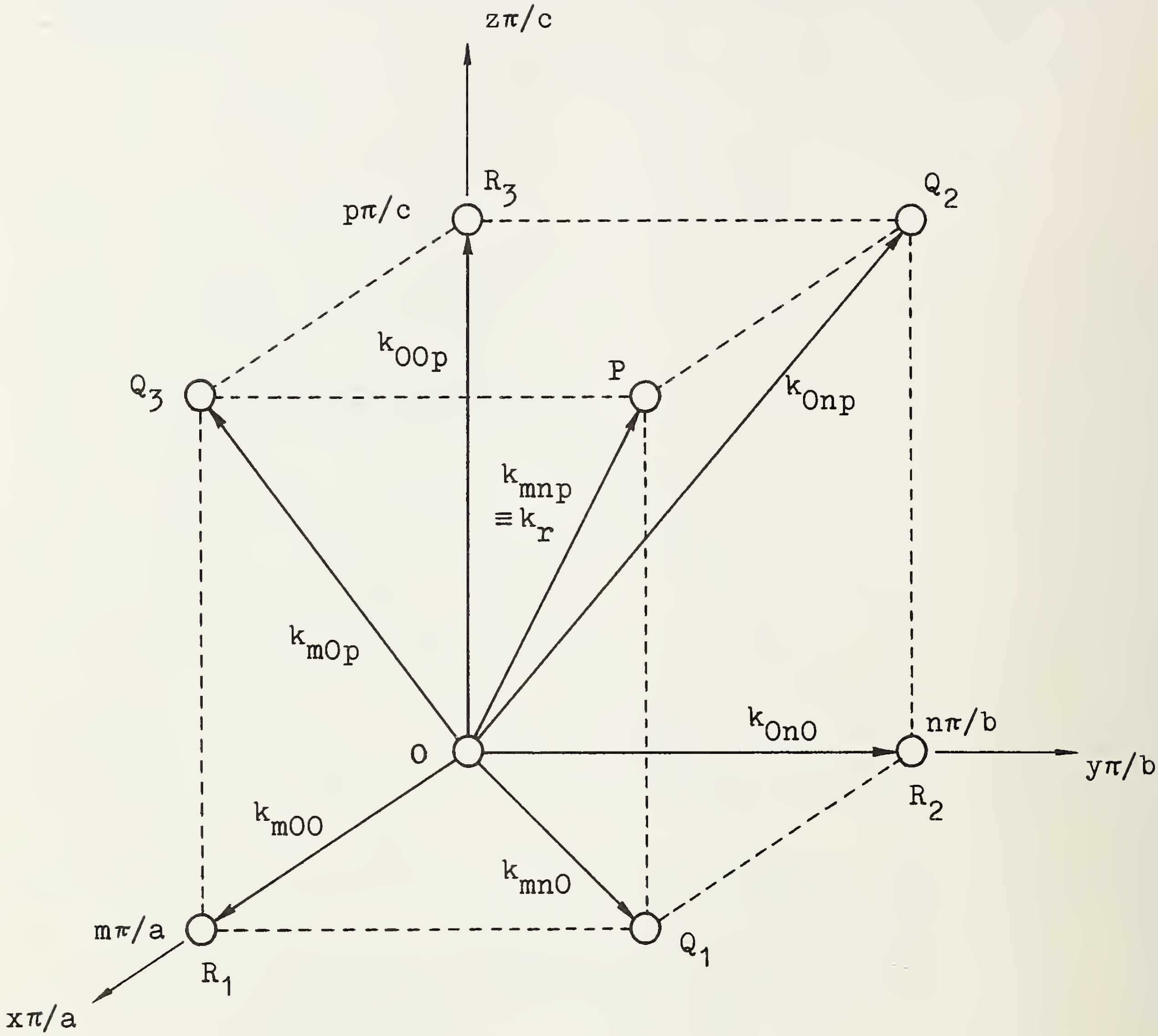

Figure 2. Lattice points in k-space. Point P represents a general 3-dimensional lattice point. Also shown are the 2-dimensional lattice points Q_1, Q_2, Q_3; the 1-dimensional lattice points R_1, R_2, R_3; and the origin.

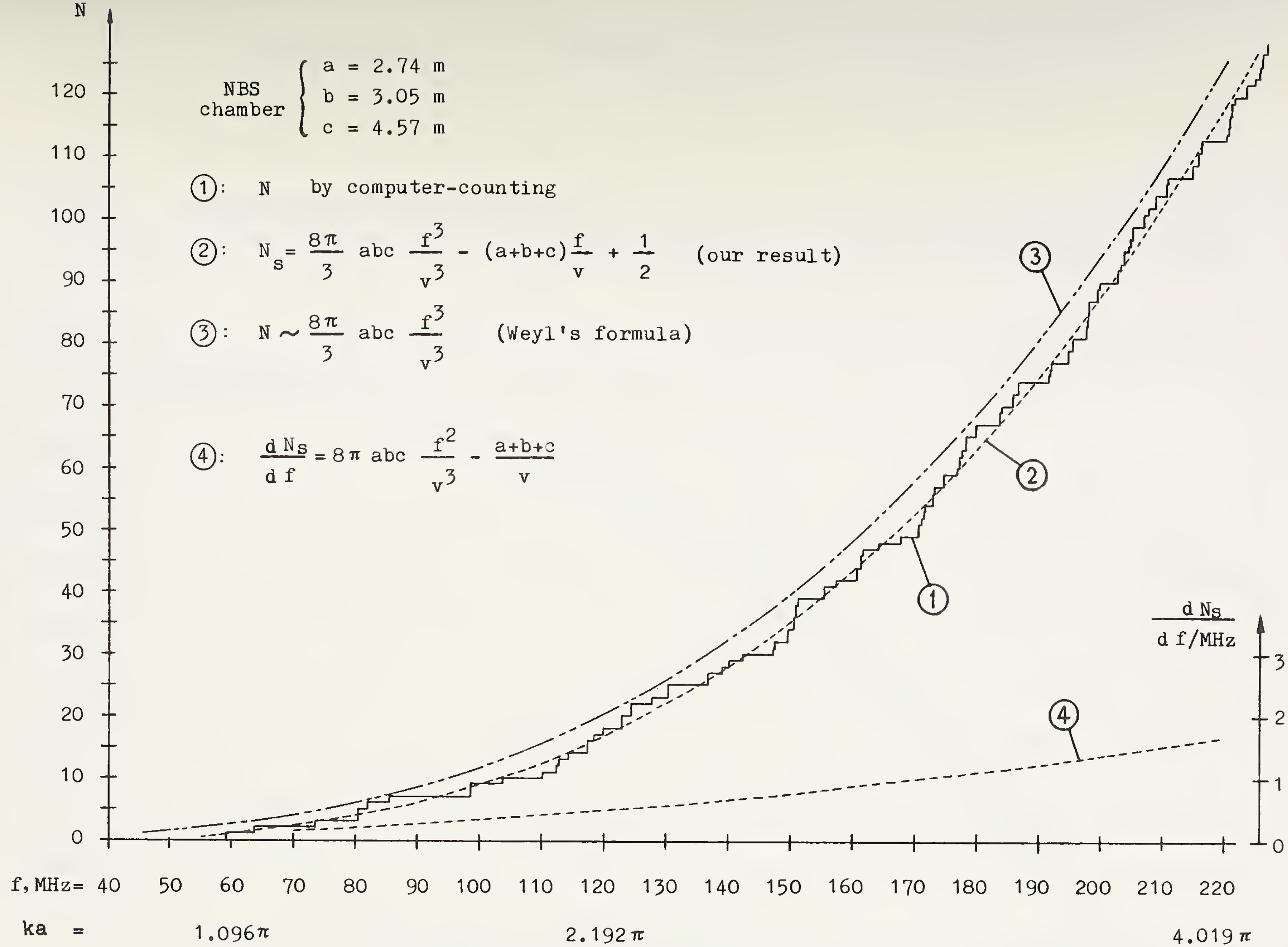

Figure 3. Total number of modes and mode density as a function of operating frequency for the NBS chamber.

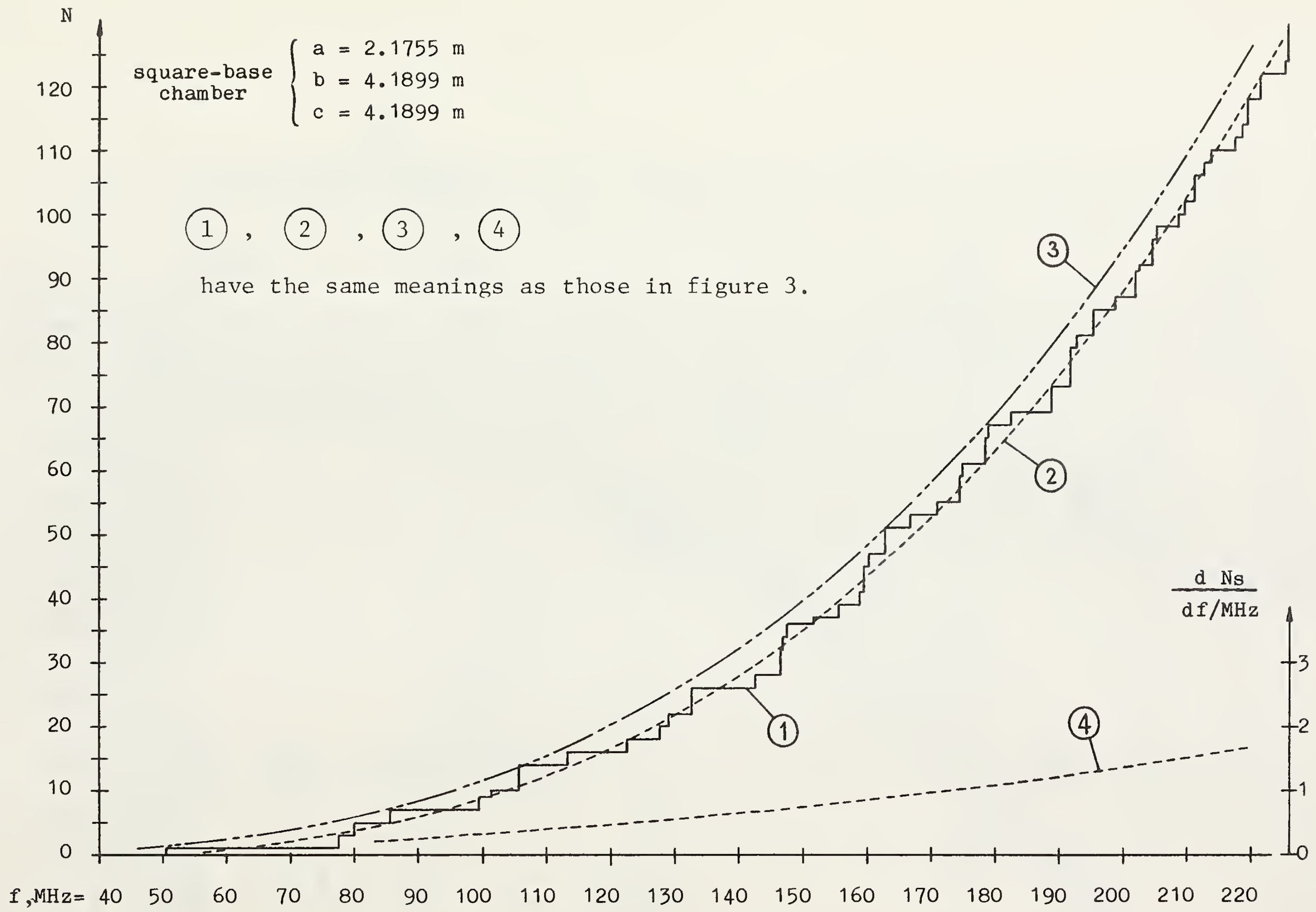

Figure 4. Total number of modes and mode density as a function of operating frequency for a square-base chamber whose volume is the same as that of NBS chamber.

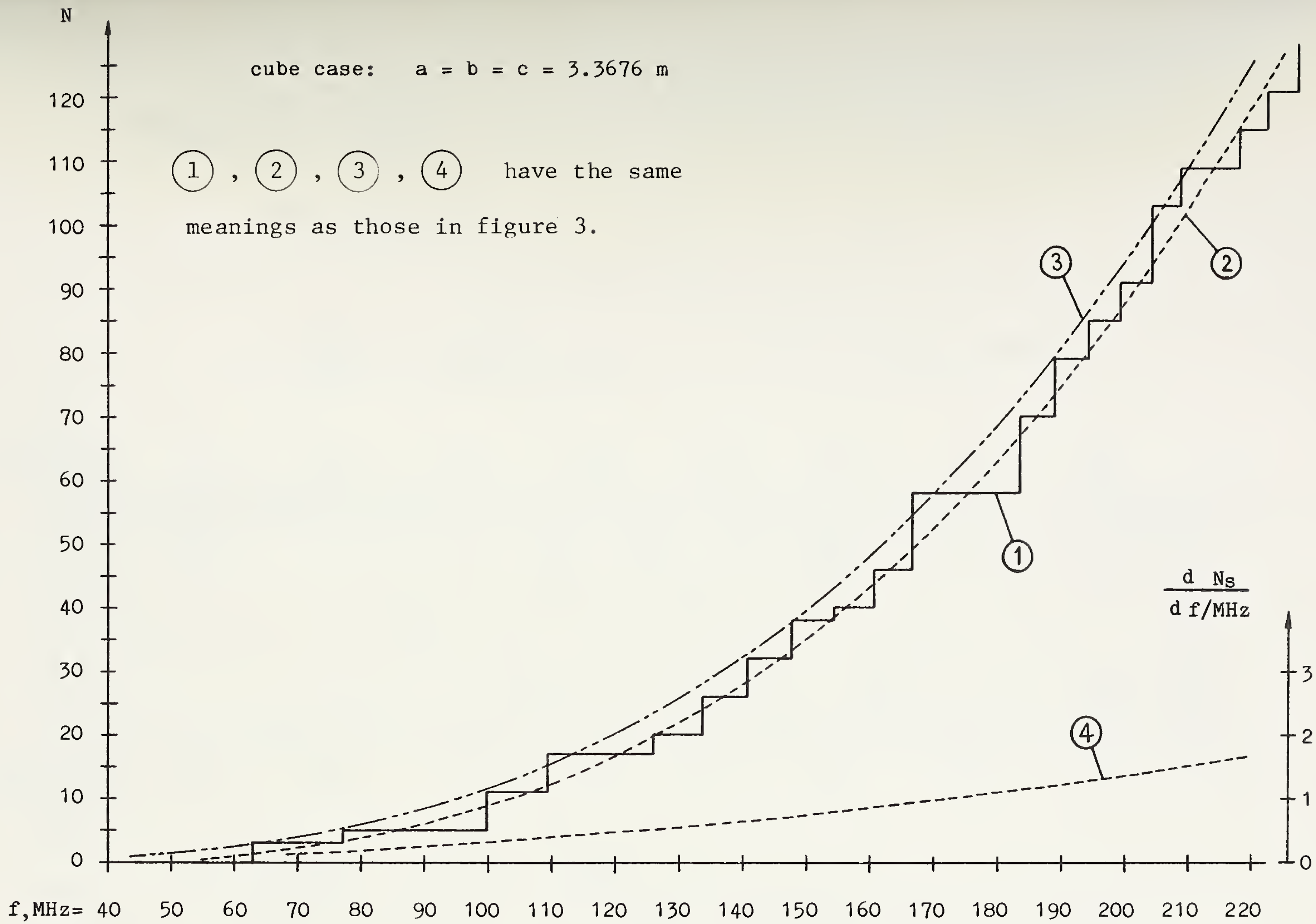

Figure 5. Total number of modes and mode density as a function of operating frequency for a cubic chamber whose volume is the same as that of NBS chamber.

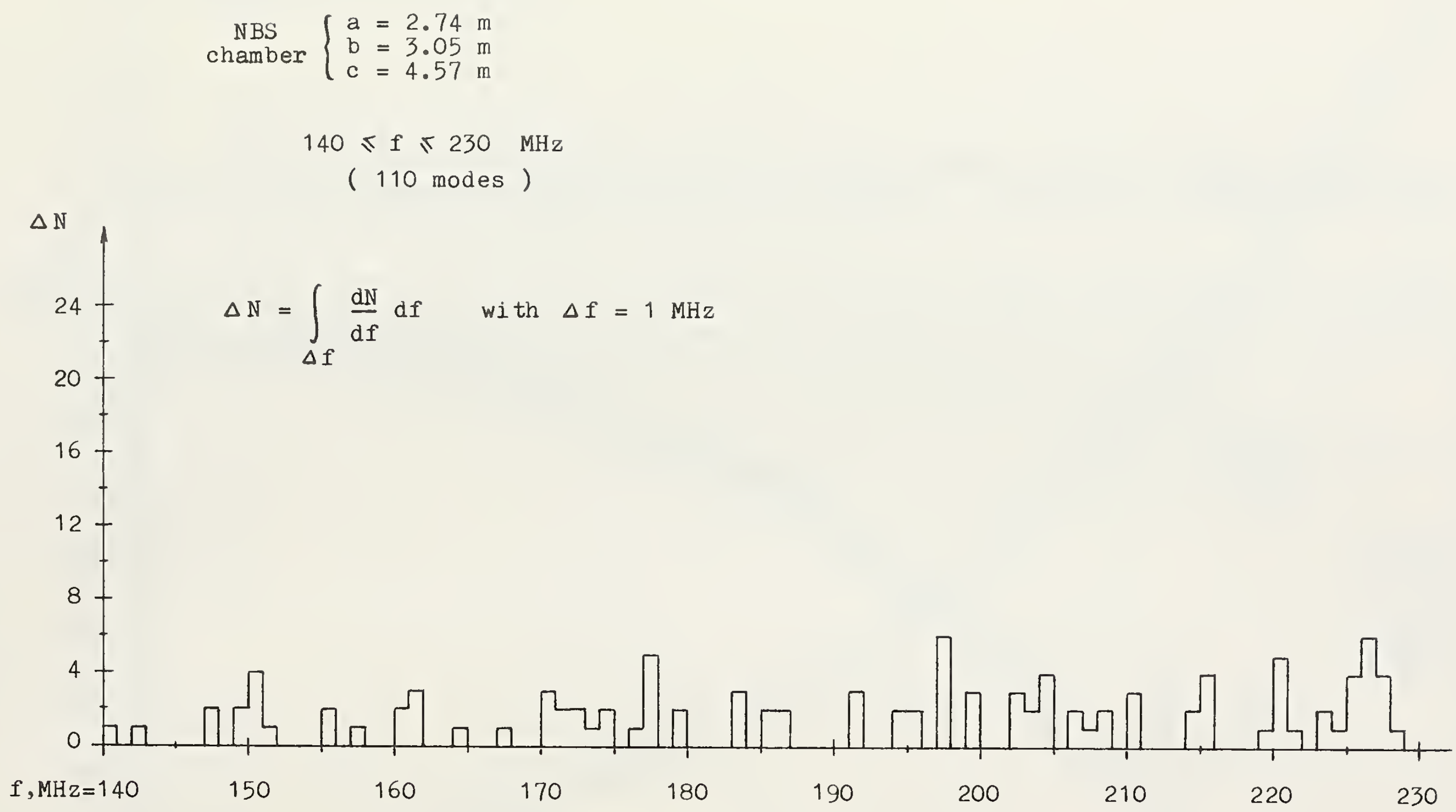

Figure 6. An illustration of mode degeneracy (NBS chamber).

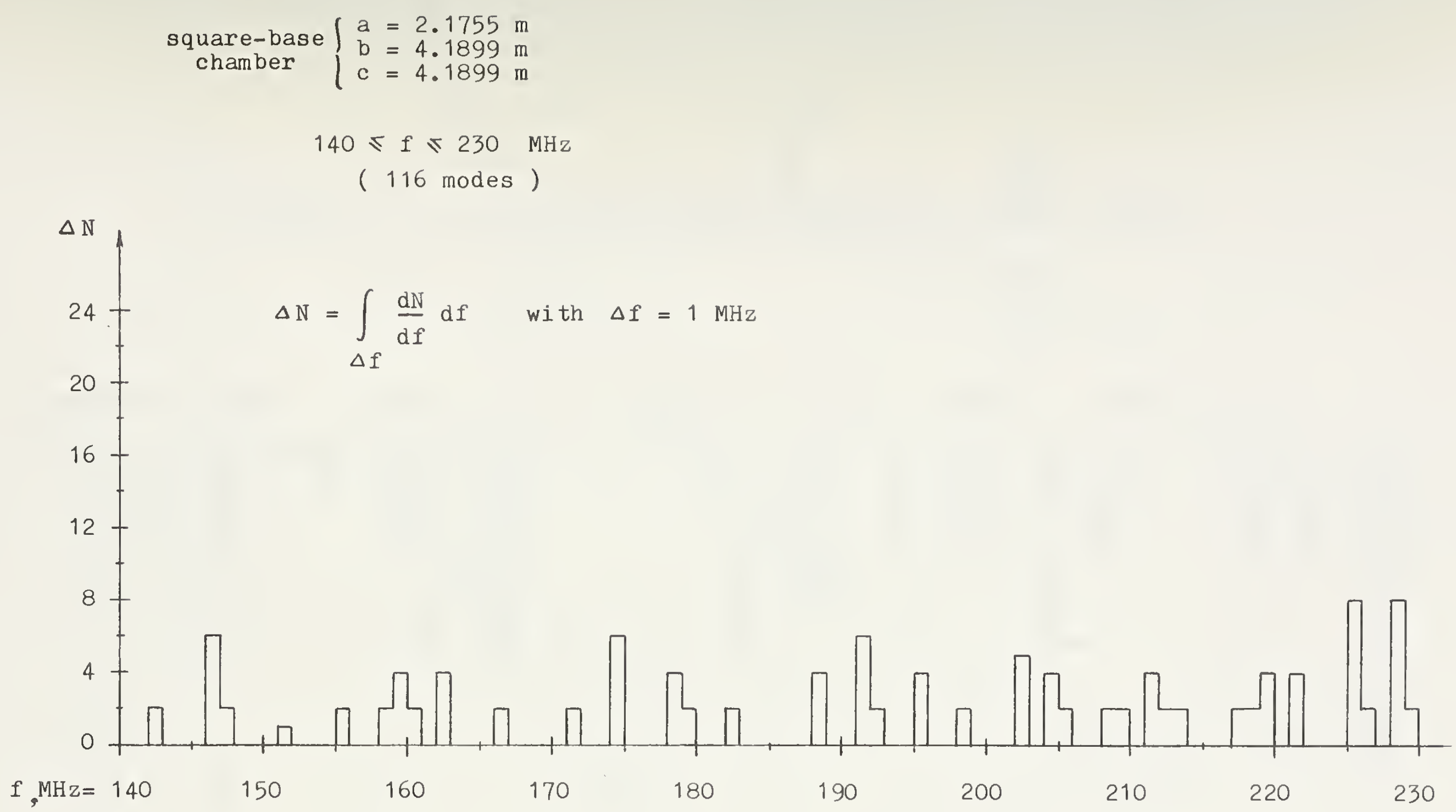

Figure 7. An illustration of mode degeneracy (square-base chamber).

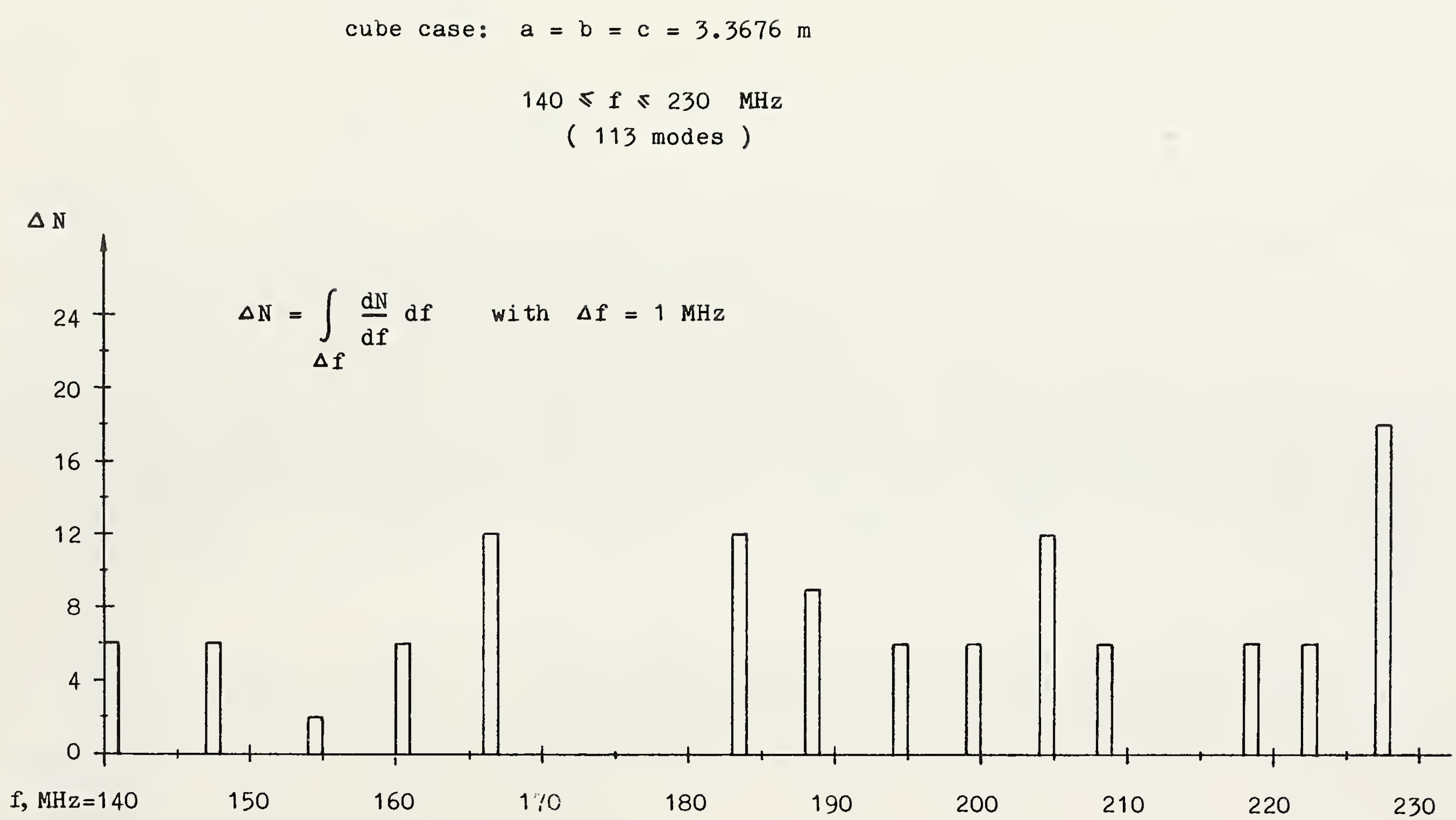

Figure 8. An illustration of mode degeneracy (cubic chamber).

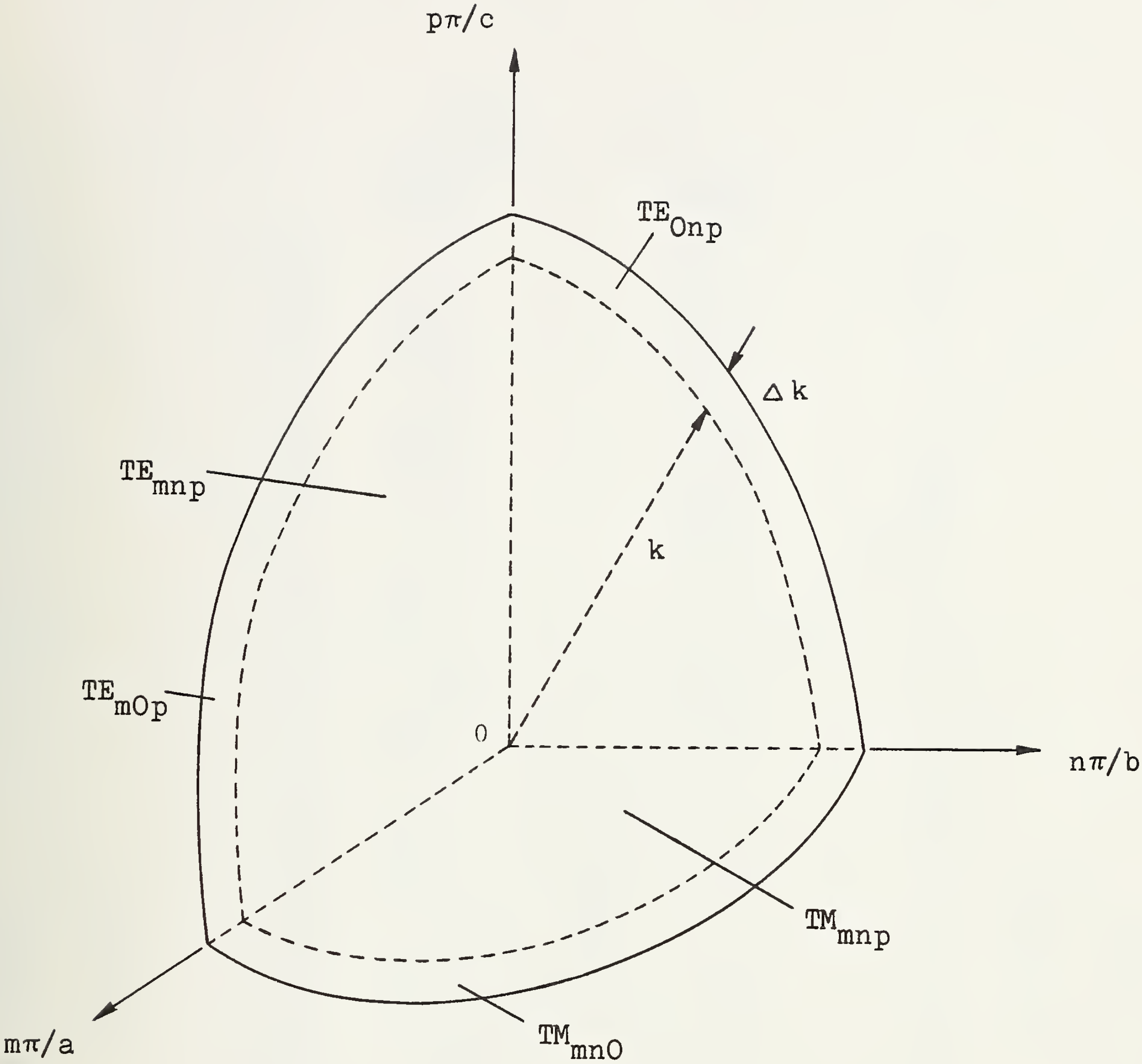

Figure 9. A k-shell for computing the composite quality factor.

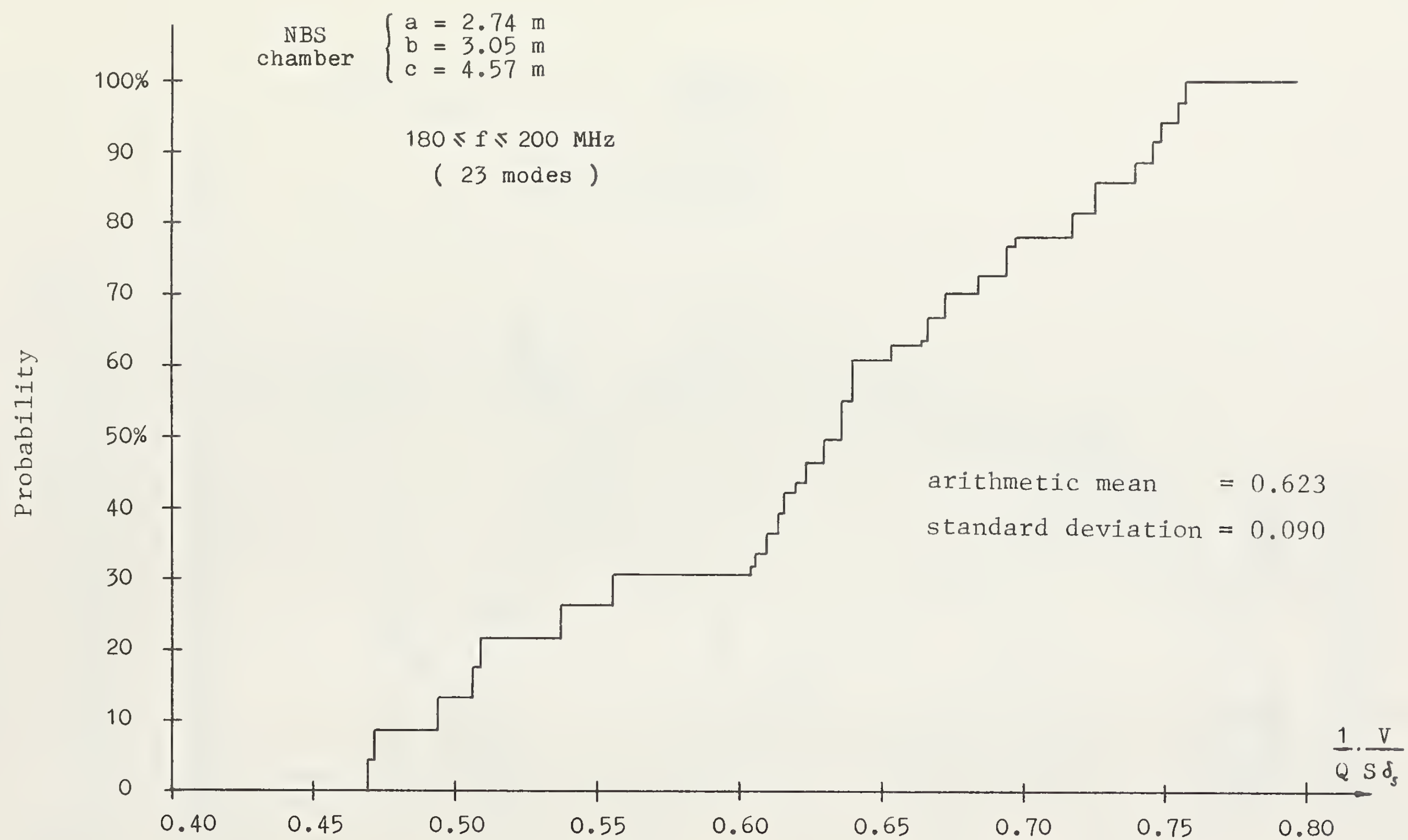

Figure 10. Cumulative distribution curve of the normalized 1/Q values in the 180 MHz to 200 MHz frequency band for the NBS chamber.

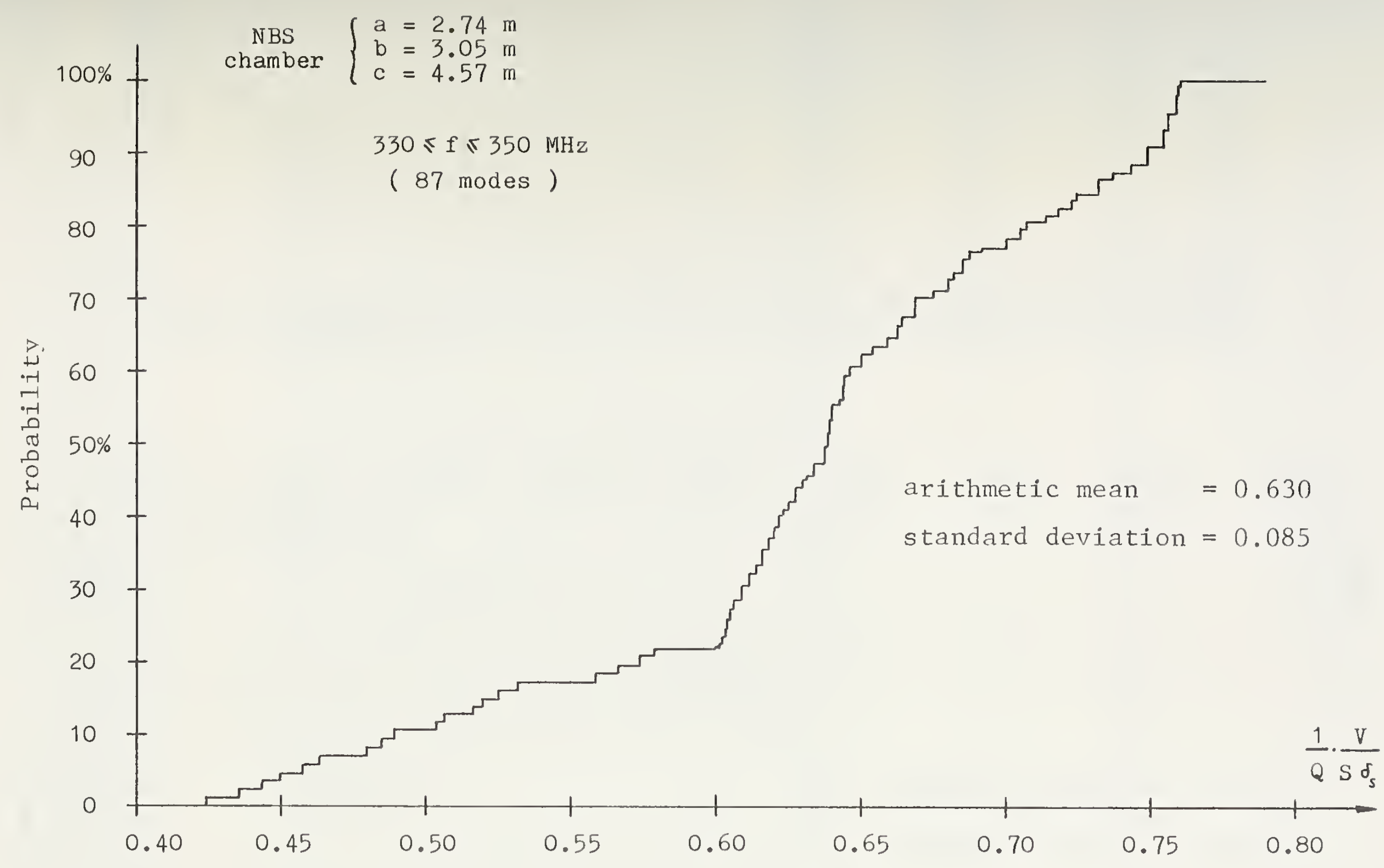

Figure 11. Cumulative distribution curve of the normalized 1/Q values in the 330 MHz to 350 MHz frequency band for the NBS chamber.

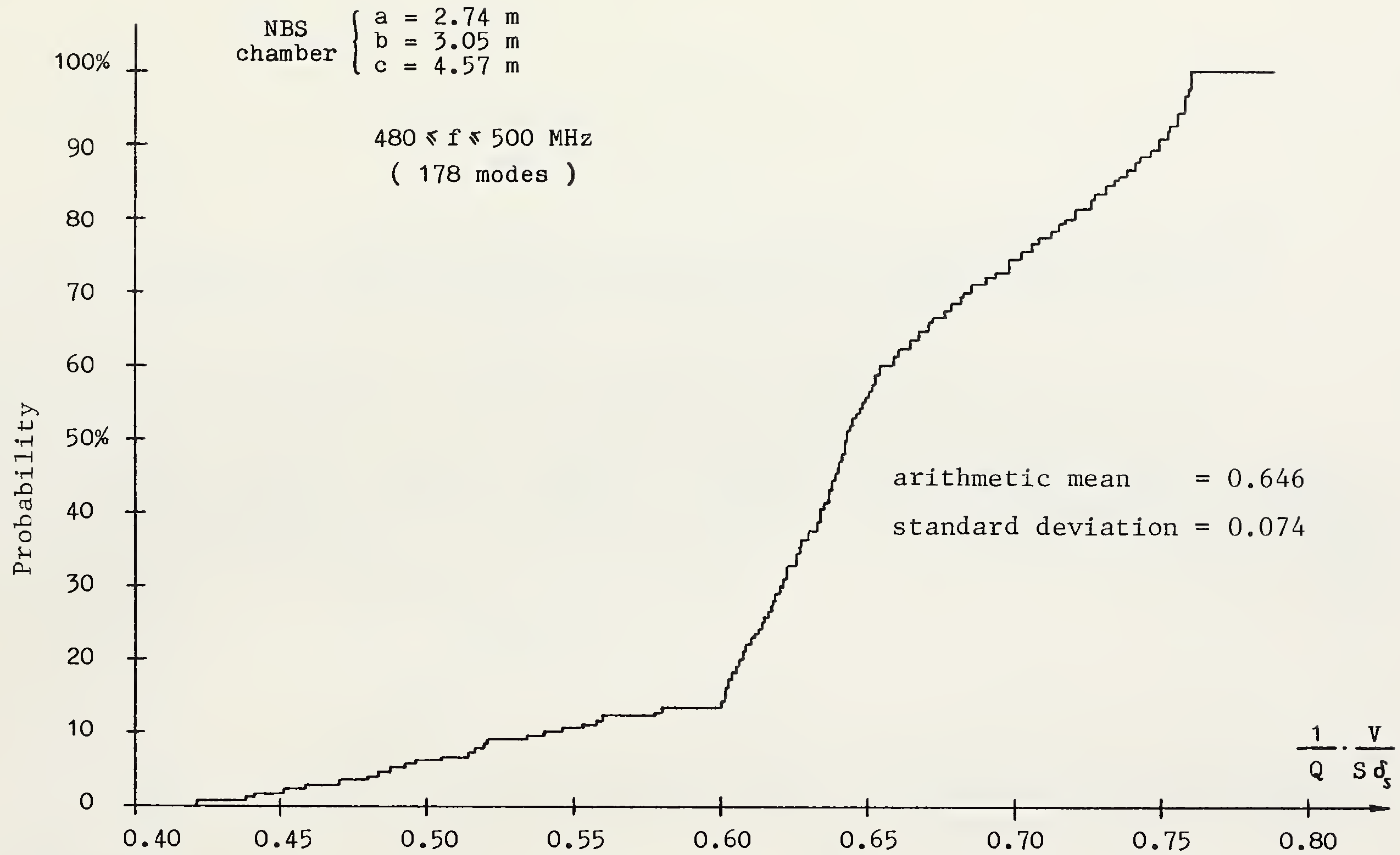

Figure 12. Cumulative distribution curve of the normalized 1/Q values in the 480 MHz to 500 MHz frequency band for the NBS chamber.

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The total number N of electromagnetic eigenmodes, with eigenfrequencies not greater than some given value, which can exist inside a rectangular mode-stirred or mode-tuned reverberating chamber is important in that it reveals how many modes can be available at an operating frequency for the "stirring" or tuning purpose. This is calculated analytically via a lattice-point counting technique in the k-space (k = wave number), leading to an exact expression for N, which can be split into a smooth component and a fluctuating part. The former contains, in addition to Weyl's volume term, an edge term as a second-order correction. The latter is sensitive to the dimensions of the chamber. Simple design criteria are then derived in view of the number of available modes and the uniformity of their distribution. To take into account the ohmic loss in metal walls of the chamber, a composite Q-factor is also proposed for design purposes. This is achieved by taking a suitable average of 1/Q-values of all possible modes within a small frequency interval. Comparison with numerical Q-values for individual modes shows that the composite Q can be used as a practical design parameter.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

cavity; composite quality factor; eigenfrequency; eigenmode, electromagnetic field; mode density; mode number; reverberating chamber.

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