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Uncertainties in Extracting Radiation Parameters for an Unknown Interference Source Based on Power and Phase Measurements

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Uncertainties in Extracting Radiation Parameters for an Unknown Interference Source Based on Power and Phase Measurements

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A method for determining the radiation characteristics of a leaking interference source has been reported in a previous publication [1], in which the unintentional, electrically small leakage source was modeled by two vectors representing a combination of equivalent electric and magnetic dipole moments. An experimental setup, measurement procedures, and the necessary theoretical basis were all described therein to explain how the relevant source parameters can be extracted from the measurement data of output powers and phases taken when the interference source is placed inside a transverse electromagnetic (TEM) cell. Simulated examples were also given to show that the equivalent source parameters of unknown vector dipole moments and, thus, the detailed radiation pattern and the total power radiated by the source in free space could be uniquely determined by the method if the measurement data were not contaminated by noise. This report presents the mathematical analysis of the uncertainties in the final, extracted results when the experimental data are degraded by the background noise and measurement inaccuracies.

Key words: dipole moments, electrically small source, error analysis, interference sources, phase measurements, power measurements, radiation pattern, TEM cell, total radiated power, uncertainties.

1. Introduction

It was shown in previous reports [1,2] that a general unknown leakage interference source may be characterized by a composite set of parameters representing both the equivalent vector electric and magnetic dipole moments as displayed in figure 1. Necessary theoretical justifications were also given therein to demonstrate that these parameters of twelve unknowns (six dipole moment amplitudes and six dipole moment phases) may be determined, with the experimental setup shown in figure 2 when the unknown source is placed inside a TEM cell, by taking the output sum and difference powers and the relative phases between the sum and difference ports for six different source orientations. The corresponding radiation pattern and the total power radiated by the source in free space can also be computed. Naturally, when the experimental power and phase readings are not contaminated by noise or other inaccuracies, the source parameters so extracted and the radiation characteristics so computed are accurate and unique. In the practical world, however, the experimental data are always degraded somewhat by the background noise, equipment limitations, and the reading accuracy. The question is then: how will these measurement imperfections affect the final results of source parameters? This report is intended to answer this question by giving necessary mathematical derivations for, and performing the analysis of, these uncertainties.

For the convenience of presenting the material, all the equations required for this report are reproduced as follows with the detailed derivations and related measurement procedures referred to the previous report [1]:

$$m_{ex}^2 = (P_{s1} + P_{s2} - P_{s3} - P_{s4} + P_{s5} + P_{s6})/(2q^2) \tag{1}$$

$$m_{ey}^2 = (P_{s1} + P_{s2} + P_{s3} + P_{s4} - P_{s5} - P_{s6})/(2q^2) \tag{2}$$

$$m_{ez}^2 = (-P_{s1} - P_{s2} + P_{s3} + P_{s4} + P_{s5} + P_{s6})/(2q^2) \tag{3}$$

$$m_{ex}m_{ey} \cos\theta_{e1} = (P_{s1} - P_{s2})/(2q^2) \tag{4}$$

$$m_{ey}m_{ez} \cos\theta_{e2} = (P_{s3} - P_{s4})/(2q^2) \tag{5}$$

$$m_{ez}m_{ex} \cos\theta_{e3} = (P_{s5} - P_{s6})/(2q^2) \tag{6}$$

$$\theta_{e1} \equiv \psi_{ex} - \psi_{ey} , \qquad \theta_{e2} \equiv \psi_{ey} - \psi_{ez} , \qquad \theta_{e3} \equiv \psi_{ez} - \psi_{ex} ; \tag{7}$$

$$m_{mx}^2 = (P_{d1} + P_{d2} - P_{d3} - P_{d4} + P_{d5} + P_{d6})/(2k^2q^2) \quad (8)$$

$$m_{my}^2 = (P_{d1} + P_{d2} + P_{d3} + P_{d4} - P_{d5} - P_{d6})/(2k^2q^2) \quad (9)$$

$$m_{mz}^2 = (-P_{d1} - P_{d2} + P_{d3} + P_{d4} + P_{d5} + P_{d6})/(2k^2q^2) \quad (10)$$

$$m_{mx}m_{my} \cos\theta_{m1} = (P_{d2} - P_{d1})/(2k^2q^2) \quad (11)$$

$$m_{my}m_{mz} \cos\theta_{m2} = (P_{d4} - P_{d3})/(2k^2q2) \quad (12)$$

$$m_{mz}m_{mx} \cos\theta_{m3} = (P_{d6} - P_{d5})/(2k^2q^2) \quad (13)$$

$$\theta_{m1} \equiv \psi_{mx} - \psi_{my} , \qquad \theta_{m2} \equiv \psi_{my} - \psi_{mz} , \qquad \theta_{m3} \equiv \psi_{mz} - \psi_{mx} . \quad (14)$$

Here m_{ex}, m_{ey}, and m_{ez} are the x-, y-, and z-components of the amplitude of the equivalent vector electric dipole moment; ψ_{ex}, ψ_{ey}, and ψ_{ez} are the associated phases of the electric dipole moment; m_{mx}, m_{my}, and m_{mz} are the x-, y-, and z-components of the amplitude of the equivalent vector magnetic dipole moment; ψ_{mx}, ψ_{my}, and ψ_{mz} are the associated phases of the magnetic dipole moment; P_{si}, i = 1, 2, . . ., 6 are the measured sum powers at the required six different source orientations; P_{di}, i = 1, 2, . . ., 6 are the measured difference powers at the same six source orientations; q is the normalized amplitude of the vertical electric field which would exist at the center of an empty TEM cell when it is operated in a receiving mode and is excited by an input power of one watt; and k is the wave number.

In obtaining (1) through (13), we have assumed that the TEM cell is made of perfectly conducting material, symmetric, and impedance-matched at 50 Ω so that there exists no horizontal component of the electric field at the center of a TEM cell. The frequency of the unknown interference source to be detected by the spectrum analyzer included in the measurement instrumentation as shown in figure 2 is assumed to allow only propagation of the dominant mode inside a given TEM cell chosen for this study [2]. The size of the interference source to be tested is also required to be small relative to the test volume of the cell in order to minimize the potential perturbation to the field distribution inside the cell [3].

Under the above conditions, the amplitudes of all the dipole moment components, and the relative phases among the dipole moment components of the same kind can be extracted based on the power (sum and difference) measurements alone [2]. The relative phases among the dipole moment components of the mixed kinds are still undetermined. This is the reason why measurements of relative phases between the sum and difference signals are needed.

Before summarizing the equations for phase information, it is instructive to examine the dependence of the power pattern, $P(\theta,\phi)$, and the total power, P_T, radiated by the source in free space on the source parameters. The expressions for computing $P(\theta,\phi)$ and P_T are [1, 2]:

$$
\begin{aligned}
P(\theta,\phi) = \frac{15\pi}{r^2\lambda^2} \Big[& (m_{ex}^2 + k^2\, m_{mx}^2)\, (\cos^2\theta \cos^2\phi + \sin^2\phi) \\
& + (m_{ey}^2 + k^2\, m_{my}^2)\, (\cos^2\theta \sin^2\phi + \cos^2\phi) + (m_{ez}^2 + k^2\, m_{mz}^2) \sin^2\theta \\
& - 2\{m_{ex}m_{ey} \cos(\psi_{ex} - \psi_{ey}) + k^2 m_{mx}m_{my} \cos(\psi_{mx} - \psi_{my})\} \sin^2\theta \sin\phi \cos\phi \\
& - 2\{m_{ey}m_{ez} \cos(\psi_{ey} - \psi_{ez}) + k^2 m_{my}m_{mz} \cos(\psi_{my} - \psi_{mz})\} \sin\theta \cos\theta \sin\phi \\
& - 2\{m_{ez}m_{ex} \cos(\psi_{ez} - \psi_{ex}) + k^2 m_{mz}m_{mx} \cos(\psi_{mz} - \psi_{mx})\} \sin\theta \cos\theta \cos\phi \\
& + 2k\{m_{ex}m_{my} \sin(\psi_{ex} - \psi_{my}) - m_{ey}m_{mx} \sin(\psi_{ey} - \psi_{mx})\} \cos\theta \\
& + 2k\{m_{ey}m_{mz} \sin(\psi_{ey} - \psi_{mz}) - m_{ez}m_{my} \sin(\psi_{ez} - \psi_{my})\} \sin\theta \cos\phi \\
& + 2k\{m_{ez}m_{mx} \sin(\psi_{ez} - \psi_{mx}) - m_{ex}m_{mz} \sin(\psi_{ex} - \psi_{mz})\} \sin\theta \sin\phi \Big] \,. \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
P_T &= \int_{4\pi} P(\theta,\phi)\, d\Omega \\
&= \frac{40\pi^2}{\lambda^2} \{m_{ex}^2 + m_{ey}^2 + m_{ez}^2 + k^2\, (m_{mx}^2 + m_{my}^2 + m_{mz}^2)\} \,, \qquad (16)
\end{aligned}
$$

where (θ,ϕ) are the spherical coordinates measured from the source center, and λ is the wavelength.

Note that if one is interested only in determining the total power radiated by the source, which may be the case in practice, all we need to extract are m_{ei}^2 and m_{mi}^2, i = 1, 2, 3 from (1), (2), (3), (8), (9), and (10) based on the measured powers. It is the radiation pattern for which the relative phases are required. In particular, for the last three lines in (15), we also need the relative phases among the mixed components, $\psi_{ex} - \psi_{my}$, $\psi_{ey} - \psi_{mx}$, $\psi_{ey} - \psi_{mz}$, $\psi_{ez} - \psi_{my}$, $\psi_{ez} - \psi_{mx}$, and $\psi_{ex} - \psi_{mz}$, which are related to some of the quantities given in (1) through (14) and some of the measured phases ϕ_i, i = 1, 2, . . ., 6 between the sum and difference ports as follows:

$$\psi_{ex} - \psi_{my} = \phi_1 + \alpha_1 - \beta_1 \tag{17}$$

$$\psi_{ey} - \psi_{mx} = \psi_{ex} - \psi_{my} - \theta_{e1} - \theta_{m1} \; ; \tag{18}$$

$$\psi_{ey} - \psi_{mz} = \phi_3 + \alpha_3 - \beta_3 \tag{19}$$

$$\psi_{ez} - \psi_{my} = \psi_{ey} - \psi_{mz} - \theta_{e2} - \theta_{m2} \; ; \tag{20}$$

$$\psi_{ez} - \psi_{mx} = \phi_5 + \alpha_5 - \beta_5 \tag{21}$$

$$\psi_{ex} - \psi_{mz} = \psi_{ez} - \psi_{mx} - \theta_{e3} - \theta_{m3} \; ; \tag{22}$$

where

$$\alpha_1 = \tan^{-1}\left(\frac{m_{ey} \sin\theta_{e1}}{m_{ex} + m_{ey} \cos\theta_{e1}}\right) \tag{23}$$

$$\beta_1 = \tan^{-1}\left(\frac{m_{mx} \cos\theta_{m1} - m_{my}}{m_{mx} \sin\theta_{m1}}\right) \; ; \tag{24}$$

$$\alpha_3 = \tan^{-1}\left(\frac{m_{ez} \sin\theta_{e2}}{m_{ey} + m_{ez} \cos\theta_{e2}}\right) \qquad (25)$$

$$\beta_3 = \tan^{-1}\left(\frac{m_{my} \cos\theta_{m2} - m_{mz}}{m_{my} \sin\theta_{m2}}\right) ; \qquad (26)$$

and

$$\alpha_5 = \tan^{-1}\left(\frac{m_{ex} \sin\theta_{e3}}{m_{ez} + m_{ex} \cos\theta_{e3}}\right) \qquad (27)$$

$$\beta_5 = \tan^{-1}\left(\frac{m_{mz} \cos\theta_{m3} - m_{mx}}{m_{mz} \sin\theta_{m3}}\right) . \qquad (28)$$

Alternatively, eqs (17), (19), and (21) may be replaced respectively by:

$$\psi_{ex} - \psi_{my} = \phi_2 + \alpha_2 - \beta_2 , \qquad (29)$$

$$\psi_{ey} - \psi_{mz} = \phi_4 + \alpha_4 - \beta_4 , \qquad (30)$$

and

$$\psi_{ez} - \psi_{mx} = \phi_6 + \alpha_6 - \beta_6 , \qquad (31)$$

where

$$\alpha_2 = \tan^{-1}\left(\frac{-m_{ey} \sin\theta_{e1}}{m_{ex} - m_{ey} \cos\theta_{e1}}\right) , \qquad (32)$$

$$\beta_2 = \tan^{-1}\left(\frac{-m_{mx} \cos\theta_{m1} - m_{my}}{-m_{mx} \sin\theta_{m1}}\right) , \qquad (33)$$

$$\alpha_4 = \tan^{-1}\left(\frac{-m_{ez}\sin\theta_{e2}}{m_{ey} - m_{ez}\cos\theta_{e2}}\right) , \tag{34}$$

$$\beta_4 = \tan^{-1}\left(\frac{-m_{my}\cos\theta_{m2} - m_{mz}}{-m_{my}\sin\theta_{m2}}\right) , \tag{35}$$

$$\alpha_6 = \tan^{-1}\left(\frac{-m_{ex}\sin\theta_{e3}}{m_{ez} - m_{ex}\cos\theta_{e3}}\right) , \tag{36}$$

and

$$\beta_6 = \tan^{-1}\left(\frac{-m_{mz}\cos\theta_{m3} - m_{mx}}{-m_{mz}\sin\theta_{m3}}\right) . \tag{37}$$

It should be noted that, in principle, either (17) or (29) is required for computing the mixed phase difference $\psi_{ex} - \psi_{my}$. They both should probably be computed for the benefit of checking data consistency. The same note applies for (19), (30), (21), and (31).

In addition, the possible sign ambiguity for θ_{ei} and θ_{mi}, i = 1, 2, 3 obtained from (4), (5), (6), (11), (12), and (13) may be resolved by the obvious constraints,

$$\sum_{i=1}^{3} \theta_{ei} = 0 , \qquad \sum_{i=1}^{3} \theta_{mi} = 0 \tag{38}$$

and the following special relationships:

$$\sin\theta_{mi} = M_i/N_i , \qquad \sin\theta_{ei} = m_i/n_i \tag{39}$$

where

$$M_1 = 2(m_{mx}^2 - m_{my}^2)\, m_{ex} m_{ey} \sin\theta_{e1} + (m_{ex}^2 - m_{ey}^2)\,(m_{mx}^2 - m_{my}^2)\tan(\phi_1 - \phi_2)\ , \tag{40}$$

$$N_1 = -\,2(m_{ex}^2 - m_{ey}^2)\, m_{mx} m_{my} + 4 m_{ex} m_{ey} m_{mx} m_{my} \tan(\phi_1 - \phi_2) \sin\theta_{e1}\ , \tag{41}$$

$$m_1 = 2(m_{ex}^2 - m_{ey}^2)\, m_{mx} m_{my} \sin\theta_{m1} + (m_{ex}^2 - m_{ey}^2)\,(m_{mx}^2 - m_{my}^2)\tan(\phi_1 - \phi_2)\ , \tag{42}$$

$$n_1 = -\,2(m_{mx}^2 - m_{my}^2)\, m_{ex} m_{ey} + 4 m_{ex} m_{ey} m_{mx} m_{my} \tan(\phi_1 - \phi_2) \sin\theta_{m1}\ , \tag{43}$$

$$M_2 = 2(m_{my}^2 - m_{mz}^2)\, m_{ey} m_{ez} \sin\theta_{e2} + (m_{ey}^2 - m_{ez}^2)\,(m_{my}^2 - m_{mz}^2)\tan(\phi_3 - \phi_4)\ , \tag{44}$$

$$N_2 = -\,2(m_{ey}^2 - m_{ez}^2)\, m_{my} m_{mz} + 4 m_{ey} m_{ez} m_{my} m_{mz} \tan(\phi_3 - \phi_4) \sin\theta_{e2}\ , \tag{45}$$

$$m_2 = 2(m_{ey}^2 - m_{ez}^2)\, m_{my} m_{mz} \sin\theta_{m2} + (m_{ey}^2 - m_{ez}^2)\,(m_{my}^2 - m_{mz}^2)\tan(\phi_3 - \phi_4)\ , \tag{46}$$

$$n_2 = -\,2(m_{my}^2 - m_{mz}^2)\, m_{ey} m_{ez} + 4 m_{ey} m_{ez} m_{my} m_{mz} \tan(\phi_3 - \phi_4) \sin\theta_{m2}\ , \tag{47}$$

$$M_3 = 2(m_{mz}^2 - m_{mx}^2)\, m_{ez} m_{ex} \sin\theta_{e3} + (m_{ez}^2 - m_{ex}^2)\,(m_{mz}^2 - m_{mx}^2)\tan(\phi_5 - \phi_6)\ , \tag{48}$$

$$N_3 = -2(m_{ez}^2 - m_{ex}^2)\, m_{mz} m_{mx} + 4 m_{ez} m_{ex} m_{mz} m_{mx} \tan(\phi_5 - \phi_6) \sin\theta_{e3} , \tag{49}$$

$$M_3 = 2(m_{ez}^2 - m_{ex}^2)\, m_{mz} m_{mx} \sin\theta_{m3} + (m_{ez}^2 - m_{ex}^2)\,(m_{mz}^2 - m_{mx}^2) \tan(\phi_5 - \phi_6) , \tag{50}$$

and

$$n_3 = -2(m_{mz}^2 - m_{mx}^2)\, m_{ez} m_{ex} + 4\, m_{ez} m_{ex} m_{mz} m_{mx} \tan(\phi_5 - \phi_6) \sin\theta_{m3} . \tag{51}$$

In summary, eqs (17) through (28) constitute the complete set of expressions for extracting the necessary relative phases required for computing the detailed radiation pattern. Equations (29) through (37) serve as alternatives for checking or averaging purposes. Equations (38) through (51) are necessary for resolving the sign-ambiguity problem for θ_{ei} and θ_{mi}, i = 1, 2, and 3.

2. Derivation of Potential Errors

Examination of (1) reveals that when all the six sum-power readings are not contaminated by measurement imperfections, the quantity, m_{ex}, so computed contains no error. On the other hand, when the measurements are not perfect due to the background noise, equipment imperfections, measurement environment, or even human errors, the result of m_{ex} will also be in error. To perform a quantitative analysis of potential errors, we denote the measured values and the parameters computed therefrom with the primes such as

$$P'_{si} = P_{si} + dP_{si} , \quad i = 1, 2, \ldots, 6 \tag{52}$$

where P_{si} represents the true value, and dP_{si} is the small error contributed by the measurement system.

For the general case, we also assume that the actual vertical electric field, q, inside the TEM cell be changed to q' by a perturbation dq due to the presence of source equipment placed inside the cell, and possible construction defect and imperfect impedance matching of the cell,

$$q' = q + dq \ . \tag{53}$$

We then have, in accordance with (1),

$$\begin{aligned} m_{ex}'^2 &= (P_{s1}' + P_{s2}' - P_{s3}' - P_{s4}' + P_{s5}' + P_{s6}')/(2q'^2) \\ &\cong m_{ex}^2 + (dP_{s1} + dP_{s2} - dP_{s3} - dP_{s4} + dP_{s5} + dP_{s6})/(2q'^2) \\ &- (P_{s1} + P_{s2} - P_{s3} - P_{s4} + P_{s5} + P_{s6})\ dq/q^3 \ , \end{aligned} \tag{54}$$

or

$$\frac{dm_{ex}}{m_{ex}'} \cong \frac{dP_{s1} + dP_{s2} - dP_{s3} - dP_{s4} + dP_{s5} + dP_{s6}}{2(P_{s1}' + P_{s2}' - P_{s3}' - P_{s4}' + P_{s5}' + P_{s6}')} - \frac{dq}{q'} \ ,$$

$$m_{ex}' > 0 \ , \tag{55}$$

which gives the percentage error in the source parameter m_{ex}' computed directly from the measurement data of sum powers P_{si}' involving imperfections dP_{si}, $i = 1, 2, \ldots, 6$.

In (54) and (55), we have neglected the second-order terms such as $(dm_{ex})^2$, $(dq)(dm_{ex})$, etc., for the simplifying purpose. This practice is certainly justified if the first-order errors, dm_{ex}, dq, etc., are small. When the situation warrants, the second-order terms can be included to improve the error analysis.

Note from the first expression in (54) that $m_{ex}'^2$ may be negative even though the true value m_{ex}^2 is always nonnegative. This can occur only when $dP_{s3} + dP_{s4}$ is relatively large and m_{ex} almost vanishes. Under this condition, we just assume that $dm_{ex}/m_{ex}' \cong 1$, giving $m_{ex} = 0$.

The above analysis also applies for m_{ey}^2, m_{ez}^2, m_{mx}^2, m_{my}^2, and m_{mz}^2 respectively in (2), (3), (8), (9), and (10). For m_{mi}^2, $i = 1, 2, 3$, we may reasonably assume $dk = 0$ implying no perturbation in frequency. Therefore, we also have

$$\frac{dm_{ey}}{m'_{ey}} = \frac{dP_{s1} + dP_{s2} + dP_{s3} + dP_{s4} - dP_{s5} - dP_{s6}}{2(P'_{s1} + P'_{s2} + P'_{s3} + P'_{s4} - P'_{s5} - P'_{s6})} - \frac{dq}{q'} , \tag{56}$$

$$\frac{dm_{ez}}{m'_{ez}} = \frac{- dP_{s1} - dP_{s2} + dP_{s3} + dP_{s4} + dP_{s5} + dP_{s6}}{2(- P'_{s1} - P'_{s2} + P'_{s3} + P'_{s4} + P'_{s5} + P'_{s6})} - \frac{dq}{q'} , \tag{57}$$

$$\frac{dm_{mx}}{m'_{mx}} = \frac{dP_{d1} + dP_{d2} - dP_{d3} - dP_{d4} + dP_{d5} + dP_{d6}}{2(P'_{d1} + P'_{d2} - P'_{d3} - P'_{d4} + P'_{d5} + P'_{d6})} - \frac{dq}{q'} , \tag{58}$$

$$\frac{dm_{my}}{m'_{my}} = \frac{dP_{d1} + dP_{d2} + dP_{d3} + dP_{d4} - dP_{d5} - dP_{d6}}{2(p'_{d1} + p'_{d2} + P'_{d3} + P'_{d4} - P'_{d5} - P'_{d6})} - \frac{dq}{q'} , \tag{59}$$

and

$$\frac{dm_{mz}}{m'_{mz}} = \frac{- dP_{d1} - dP_{d2} + dP_{d3} + dP_{d4} + dP_{d5} + dP_{d6}}{2(- P'_{d1} - P'_{d2} + P'_{d3} + P'_{d4} + P'_{d5} + P'_{d6})} - \frac{dq}{q'} . \tag{60}$$

The predicted values for m_{ei} and m_{mi}, i = x, y, z, then become

$$m_{ei} = m'_{ei} - dm_{ei} = m'_{ei}(1 - dm_{ei}/m'_{ei})$$

and (61)

$$m_{mi} = m'_{mi} - dm_{mi} = m'_{mi}(1 - dm_{mi}/m'_{mi}) .$$

The above implies that if we have known a priori (or have some basis) to estimate dP_{si}, dP_{di}, and dq, the percentage errors (dm_{ei}/m'_{ei} and dm_{mi}/m'_{mi}) can be evaluated to predict approximately the true values. Or, at least, these percentage errors can be computed as a function of assumed values of dP_{si}, dP_{di}, and dq.

If, however, there is strong belief that dP_{si}, dP_{di}, and dq are all truly random, the problem is much simpler. Then, the average values of them, and thus the average values of dm_{ei} and dm_{mi} in accordance with (55) through (60) all vanish. That is,

$$\overline{dP}_{si} = \overline{dP}_{di} = \overline{dq} = 0 \ , \tag{62}$$

and

$$\overline{dm}_{ei} = \overline{dm}_{mi} = 0 \ . \tag{63}$$

We conclude that

$$\overline{m'_{ei}} = m_{ei} \quad \text{and} \quad \overline{m'_{mi}} = m_{mi} \ . \tag{64}$$

This implies that, if repeated sets of power measurements are taken, we can compute the corresponding sets for m'_{ei} and m'_{mi} . The average values should approach the true values when all errors involved in the measurement are random in nature.

Let us now consider an expression involving the relative phase such as (4). Using the same notation convention and neglecting the second-order terms, we obtain

$$\cos\theta_{e1} \cong \cos\theta'_{e1} \left(1 + \frac{dm_{ex}}{m'_{ex}} + \frac{dm_{ey}}{m'_{ey}} - \frac{dP_{s1} - dP_{s2}}{P'_{s1} - P'_{s2}} + \frac{2dq}{q'}\right) \ , \tag{65}$$

where

$$\cos\theta'_{e1} = (P'_{s1} - P'_{s2})/2q'^{2}\, m'_{ex}\, m'_{ey}) \ , \tag{66}$$

which can readily be computed based on the power measurements alone, and the remaining terms in (65) can also be computed if the values for dP_{si} and dq are either known or assumed. Taking the inverse cosines of (65) and (66) will yield θ_{e1} and θ'_{e1} with the appropriate sign determined by (38) and (39). The difference, $d\theta_{e1} = \theta'_{e1} - \theta_{e1}$, represents the error incurred in the relative phase deduced from the power measurements.

Similarly, we have

$$\cos\theta_{e2} \cong \cos\theta'_{e2} \left(1 + \frac{dm_{ey}}{m'_{ey}} + \frac{dm_{ez}}{m'_{ez}} - \frac{dP_{s3} - dP_{s4}}{P'_{s3} - P'_{s4}} + \frac{2dq}{q'}\right) \ , \tag{67}$$

$$\cos\theta_{e3} \cong \cos\theta'_{e3} \left(1 + \frac{dm_{ez}}{m'_{ez}} + \frac{dm_{ex}}{m'_{ex}} - \frac{dP_{s5} - dP_{s6}}{P'_{s5} - P'_{s6}} + \frac{2dq}{q'}\right) , \tag{68}$$

$$\cos\theta_{m1} \cong \cos\theta'_{m1} \left(1 + \frac{dm_{mx}}{m'_{mx}} + \frac{dm_{my}}{m'_{my}} - \frac{dP_{d2} - dP_{d1}}{P'_{d2} - P'_{d1}} + \frac{2dq}{q'}\right) , \tag{69}$$

$$\cos\theta_{m2} \cong \cos\theta'_{m2} \left(1 + \frac{dm_{my}}{m'_{my}} + \frac{dm_{mz}}{m'_{mz}} - \frac{dP_{d4} - dP_{d3}}{P'_{d4} - P'_{d3}} + \frac{2dq}{q'}\right) , \tag{70}$$

and

$$\cos\theta_{m3} \cong \cos\theta'_{m3} \left(1 + \frac{dm_{mz}}{m'_{mz}} + \frac{dm_{mx}}{m'_{mx}} - \frac{dP_{d6} - dP_{d5}}{P'_{d6} - P'_{d5}} + \frac{2dq}{q'}\right) , \tag{71}$$

where

$$\cos\theta'_{e2} = (P'_{s3} - P'_{s4})/(2q'^2\, m'_{ey}\, m'_{ez}) , \tag{72}$$

$$\cos\theta'_{e3} = (P'_{s5} - P'_{s6})/(2q'^2\, m'_{ez}\, m'_{ex}) , \tag{73}$$

$$\cos\theta'_{m1} = (P'_{d2} - P'_{d1})/(2k^2 q'^2\, m'_{mx}\, m'_{my}) , \tag{74}$$

$$\cos\theta'_{m2} = (P'_{d4} - P'_{d3})/(2k^2 q'^2\, m'_{my}\, m'_{mz}) , \tag{75}$$

and

$$\cos\theta'_{m3} = (P'_{d6} - P'_{d5})/(2K^2 q'^2\, m'_{mz}\, m'_{mx}) . \tag{76}$$

Note from (66) that the situation $|\cos\theta'_{e1}| > 1$ may arise if $dP_{s1} - dP_{s2}$ is relatively large and θ_{e1} nearly vanishes. In this case, we just set $\cos\theta'_{e1} \cong 1$ (or $\theta'_{e1} \cong 0$). Then we have

$$d\theta_{e1} \cong -\theta_{e1} \cong \sqrt{2}\left(\frac{dP_{s1} - dP_{s2}}{P'_{s1} - P'_{s2}} - \frac{dm_{ex}}{m'_{ex}} - \frac{dm_{ey}}{m'_{ey}} - \frac{2dq}{q'}\right)^{1/2} . \tag{77}$$

Similarly, we have

$$d\theta_{e2} = -\theta_{e2} \cong \sqrt{2}\left(\frac{dP_{s3} - dP_{s4}}{P'_{s3} - P'_{s4}} - \frac{dm_{ey}}{m'_{ey}} - \frac{dm_{ez}}{m'_{ez}} - \frac{2dq}{q'}\right)^{1/2} , \tag{78}$$

$$d\theta_{e3} = -\theta_{e3} = \sqrt{2}\left(\frac{dP_{s5} - dP_{s6}}{P'_{s5} - P'_{s6}} - \frac{dm_{ez}}{m'_{ez}} - \frac{dm_{ex}}{m'_{ex}} - \frac{2dq}{q'}\right)^{1/2} , \tag{79}$$

$$d\theta_{m1} = -\theta_{m1} = \sqrt{2}\left(\frac{dP_{d2} - dP_{d1}}{P'_{d2} - P'_{d1}} - \frac{dm_{mx}}{m'_{mx}} - \frac{dm_{my}}{m'_{my}} - \frac{2dq}{q'}\right)^{1/2} , \tag{80}$$

$$d\theta_{m2} = -\theta_{m2} = \sqrt{2}\left(\frac{dP_{d4} - dP_{d3}}{P'_{d4} - P'_{d3}} - \frac{dm_{my}}{m'_{my}} - \frac{dm_{mz}}{m'_{mz}} - \frac{2dq}{q'}\right)^{1/2} , \tag{81}$$

and

$$d\theta_{m3} = -\theta_{m3} = \sqrt{2}\left(\frac{dP_{d6} - dP_{d5}}{P'_{d6} - P'_{d5}} - \frac{dm_{mz}}{m'_{mz}} - \frac{dm_{mx}}{m'_{mx}} - \frac{2dq}{q'}\right)^{1/2} . \tag{82}$$

Thus, eqs (65) through (76) provide the necessary means for estimating $d\theta_{ei}$ and $d\theta_{mi}$ when θ'_{ei} and θ'_{mi} computed from the power measurements are not near zero. Equations (77) through (82) serve the same purpose when $\theta_{ei} \cong 0$ and $\theta'_{mi} \cong 0$, with the values for dP_{si}, dP_{di}, and dq known or assumed. On the other hand, if dP_{si}, dP_{di} and dq are random, we see that $\overline{d\theta_{ei}} = \overline{d\theta_{mi}} = 0$, yielding $\overline{\theta'_{ei}} = \theta_{ei}$ and $\overline{\theta'_{mi}} = \theta_{mi}$.

Corresponding error expressions for the relative mixed phases, (17) through (22), can also be derived. Since they involve taking the derivatives of inverse tangent functions, the final forms are quite complicated. For this reason, they are not presented here. One alternative but simple method of achieving these is to (i) compute the primed quantities based on the measurement data, (ii) compute the unprimed quantities based on the predicted amplitudes of the component dipole moments and the predicted relative phases among the same kind, which have already been obtained, and then (iii) take the difference between the results derived above.

Consider (17) first. Initially, we have the computed value for $\psi'_{ex} - \psi'_{my}$ derived from the measurements,

$$\psi'_{ex} - \psi'_{my} = \phi'_1 + \alpha'_1 - \beta'_1 , \tag{83}$$

where ϕ'_1 represents the actual measured relative phase between the sum and difference signals for one of the measurement positions of the interference source, and α'_1 and β'_1 can be calculated respectively in accordance with (23) and (24),

$$\alpha'_1 = \tan^{-1}\left(\frac{m'_{ey} \sin\theta'_{e1}}{m'_{ex} + m'_{ey} \cos\theta'_{e1}}\right) , \tag{84}$$

$$\beta'_1 = \tan^{-1}\left(\frac{m'_{mx} \cos\theta'_{m1} - m'_{my}}{m'_{mx} \sin\theta'_{m1}}\right) . \tag{85}$$

Of course, all the primed quantities in (84) and (85) are based on the measured data, thus, containing some measurement errors.

The error involved in $\psi'_{ex} - \psi'_{my}$ may be expressed as

$$d(\psi_{ex} - \psi_{my}) = d\phi_1 + d\alpha_1 - d\beta_1 , \tag{86}$$

where $d\phi_1$ represents the actual measurement error in the relative phase for the first source position, $d\alpha_1 = \alpha'_1 - \alpha_1$, $d\beta_1 = \beta'_1 - \beta_1$ with α_1 and β_1 to be computed also from (23) and (24) but with the unprimed quantities deduced in (61), (65), and (69).

Note that the phase error $d\phi_1$ may be estimated (depending on the type of equipment involved in the measurement system) or assumed for computational purpose. The predicted true value for $\psi_{ex} - \psi_{my}$ then becomes

$$\psi_{ex} - \psi_{my} = \psi'_{ex} - \psi'_{my} - d(\psi_{ex} - \psi_{my}) \ . \tag{87}$$

Substituting (87) into (18), we also have $\psi_{ey} - \psi_{mx}$.

Similarly, from (19) and (21), we obtain

$$\psi'_{ey} - \psi'_{mz} = \phi'_3 + \alpha'_3 - \beta'_3 \tag{88}$$

and

$$\psi'_{ez} - \psi'_{mx} = \phi'_5 + \alpha'_5 - \beta'_5 \ , \tag{89}$$

where ϕ'_3 and ϕ'_5 are the actual measured phases between the sum and difference signals for two other measurement positions of the source,

$$\alpha'_3 = \tan^{-1}\left(\frac{m'_{ez} \sin\theta'_{e2}}{m'_{ey} + m'_{ez} \cos\theta'_{e2}}\right) \ , \tag{90}$$

$$\beta'_3 = \tan^{-1}\left(\frac{m'_{my} \cos\theta'_{m2} - m'_{mz}}{m'_{my} \sin\theta'_{m2}}\right) \ , \tag{91}$$

$$\alpha'_5 = \tan^{-1}\left(\frac{m'_{ex} \sin\theta'_{e3}}{m'_{ez} + m'_{ex} \cos\theta'_{e3}}\right) \ , \tag{92}$$

and

$$\beta_5' = \tan^{-1}\left(\frac{m_{mz}' \cos\theta_{m3}' - m_{mx}'}{m_{mz}' \sin\theta_{m3}'}\right) . \tag{93}$$

The errors involved in these phases are:

$$d(\psi_{ey} - \psi_{mz}) = d\phi_3 + d\alpha_3 - d\beta_3 , \tag{94}$$

$$d(\psi_{ez} - \psi_{mx}) = d\phi_5 + d\alpha_5 - d\beta_5 , \tag{95}$$

$$d\alpha_3 = \alpha_3' - \alpha_3 , \qquad d\beta_3 = \beta_3' - \beta_3 , \tag{96}$$

and

$$d\alpha_5 = \alpha_5' - \alpha_5 , \qquad d\beta_5 = \beta_5' - \beta_5 , \tag{97}$$

where $d\phi_3$ and $d\phi_5$ represent the phase errors in the measurements, and α_3, β_3, α_5, β_5 may be computed from (25) through (28) with the predicted values for m_{ei}, m_{mi}, θ_{e2}, and θ_{m2} (unprimed) deduced from (61), (67), (68), (70), and (71).

The predicted true values for the mixed phases being considered then become

$$\psi_{ey} - \psi_{mz} = \psi_{ey}' - \psi_{mz}' - d(\psi_{ey} - \psi_{mz}) , \tag{98}$$

and

$$\psi_{ez} - \psi_{mx} = \psi_{ez}' - \psi_{mx}' - d(\psi_{ez} - \psi_{mx}) . \tag{99}$$

The phases deduced above can then be used in (20) and (22) to obtain $\psi_{ez} - \psi_{my}$ and $\psi_{ex} - \psi_{mz}$.

Alternatively, eqs (87) (98), and (99) can also be derived by starting from (29), (30), and (31) based on the measured phases ϕ_2', ϕ_4', and ϕ_6'. Either the mixed phases obtained here or those obtained in (87), (98), and (99), or the average values between them may be inserted in (15) to compute the predicted power pattern, which may be compared with that computed based on the primed values to reveal the final error in power pattern.

Of course, we may also conclude that, when repeated measurements of P_{si}', P_{di}', and ϕ_i' are available and dP_{si}, dP_{di}, and $d\phi_i$ are believed to be random,

$$\overline{\psi_{ex}' - \psi_{my}'} = \psi_{ex} - \psi_{my}, \qquad \overline{\psi_{ey}' - \psi_{mx}'} = \psi_{ey} - \psi_{mx} \tag{100}$$

$$\overline{\psi_{ey}' - \psi_{mz}'} = \psi_{ey} - \psi_{mz}, \qquad \overline{\psi_{ez}' - \psi_{my}'} = \psi_{ez} - \psi_{my} \tag{101}$$

and

$$\overline{\psi_{ez}' - \psi_{mx}'} = \psi_{ez} - \psi_{mx}, \qquad \overline{\psi_{ex}' - \psi_{mz}'} = \psi_{ex} - \psi_{mz} . \tag{102}$$

Under this condition, the average power pattern will approach the true power pattern.

3. A Simulated Example with Assumed Unbiased Measurement Errors

For the purpose of checking the formulation and derivation, an ideal simulated example was given in the previous report [1], where an unknown arbitrary interference source was assumed to be characterized by the equivalent electric and magnetic dipole moments with the following values:

$$\begin{aligned} &m_{ex} = 1.4, \quad m_{ey} = 1.8, \quad m_{ez} = 1.6, \quad \psi_{ex} = 0, \quad \psi_{ey} = 80, \quad \psi_{ez} = 60, \\ &m_{mx} = 0.8, \quad m_{my} = 0.6, \quad m_{mz} = 0.4, \quad \psi_{mx} = -80, \quad \psi_{my} = -60, \quad \psi_{mz} = -45, \end{aligned} \tag{103}$$

where the normalized amplitude of the electric dipole moment is in meters, that of the magnetic dipole moment is in meters-squared, and the phase is in degrees. The value used for the vertical electric field existing inside the particular TEM cell considered therein was q = 11.83 V/m, and the frequency was 30 MHz. With the source parameters assumed in (103), we computed in [1] that the sum and difference powers in watts and the relative sum-to-difference phases in degrees should be:

$$
\begin{aligned}
&P_{s1} = 425.107856\,, && P_{s2} = 302.626424\,, && P_{s3} = 784.597583\,,\\
&P_{s4} = 27.106038\,, && P_{s5} = 473.027282\,, && P_{s6} = 159.541746\,;\\
\\
&P_{d1} = 2.704333\,, && P_{d2} = 52.545279\,, && P_{d3} = 1.556813\,,\\
&P_{d4} = 27.172985\,, && P_{d5} = 7.617338\,, && P_{d6} = 36.582351\,;\\
\\
&\phi_1 = -103.0261\,, && \phi_2 = -77.0300\,, && \phi_3 = -113.5502\,,\\
&\phi_4 = 105.5593\,, && \phi_5 = 48.116\,, && \phi_6 = 91.9132\,.
\end{aligned}
\qquad (104)
$$

In the previous report [1], we also demonstrated that if the values in (104) were the actual measurement results, we would recover uniquely the source parameters exactly as those given in (103). In this case, of course, there would be no error involved.

Since this is an example aimed for analyzing the final errors in the extracted source parameters for given errors in the simulated "measurement data," we treat the problem here in a deterministic manner. The averaging technique, by taking repeated measurements, is useful only when we are sure that the measurement errors are indeed random for the equipment involved in the measurement system.

Now, let us consider the problem from the practical point of view. Suppose that the hardware in the measurement system is not capable of taking perfect readings of the outputs accurate to the fourth or sixth digit after the decimal point as demanded in (104). Instead, all the readings are only accurate to the second digit after the decimal point. In addition, suppose that the combined effect of background noise, equipment imperfections, and human factors also degrades the readings somewhat such that the actual measured powers in watts and phases in degrees are as follows:

$$
\begin{aligned}
P'_{s1} &= 446.36 \text{ (increasing approximately by 5\%)},\\
P'_{s2} &= 287.50 \text{ (decreasing approximately by 5\%)},\\
P'_{s3} &= 745.37 \text{ (decreasing approximately by 5\%)},\\
P'_{s4} &= 28.46 \text{ (increasing approximately by 5\%)},\\
P'_{s5} &= 496.68 \text{ (increasing approximately by 5\%)},\\
P'_{s6} &= 151.56 \text{ (decreasing approximately by 5\%)},\\
P'_{d1} &= 2.57 \text{ (decreasing approximately by 5\%)},\\
P'_{d2} &= 49.92 \text{ (decreasing approximately by 5\%)},\\
P'_{d3} &= 1.63 \text{ (increasing approximately by 5\%)},\\
P'_{d4} &= 28.53 \text{ (increasing approximately by 5\%)},\\
P'_{d5} &= 8.00 \text{ (increasing approximately by 5\%)},\\
P'_{d6} &= 34.75 \text{ (decreasing approximately by 5\%)},\\
\phi'_1 &= -\ 92.72 \text{ (increasing approximately by 10\%)},\\
\phi'_2 &= -\ 69.33 \text{ (increasing approximately by 10\%)},\\
\phi'_3 &= -124.91 \text{ (decreasing approximately by 10\%)},\\
\phi'_4 &= 95.00 \text{ (decreasing approximately by 10\%)},\\
\phi'_5 &= 43.30 \text{ (decreasing approximately by 10\%)},
\end{aligned}
\tag{105}
$$

and

$$\phi'_6 = 101.10 \text{ (increasing approximately by 10\%)}.$$

For simplifying the problem, we also assume that there is no perturbation in the vertical electric field inside the TEM cell. That is, $dq = 0$, $q' = q = 11.83$ V/m. We then obtain

$$
\begin{aligned}
m'^2_{ex} &= 2.1732 , & m'_{ex} &= 1.474 ,\\
m'^2_{ey} &= 3.0706 , & m'_{ey} &= 1.752 ,\\
m'^2_{ez} &= 2.4588 , & m'_{ez} &= 1.568 ,\\
m'^2_{mx} &= 0.5890 , & m'_{mx} &= 0.767 ,\\
m'^2_{my} &= 0.3611 , & m'_{my} &= 0.601 ,\\
m'^2_{mz} &= 0.1848 , & m'_{mz} &= 0.430 ;
\end{aligned}
$$

$dm_{ex}/m'_{ex} = \ \ 0.050$ (meaning an error of +5% in m'_{ex} deduced above),

$dm_{ey}/m'_{ey} = -0.026$ (meaning an error of -2.6% in m'_{ey} deduced above),

$dm_{ez}/m'_{ez} = -0.019$ (meaning an error of -1.9% in m'_{ez} deduced above),

$dm_{mx}/m'_{mx} = -0.042$ (meaning an error of -4.2% in m'_{mx} deduced above),

$dm_{my}/m'_{my} = \ \ 0.003$ (meaning an error of +0.3% in m'_{my} deduced above),

$dm_{mz}/m'_{mz} = \ \ 0.068$ (meaning an error of +6.8% in m'_{mz} deduced above);

m_{ex} (predicted true value) $= m'_{ex}(1 - dm_{ex}/m'_{ex})$

$= 1.400$ (practically no error compared with the true value of 1.4),

$m_{ey} = 1.798$ (small error compared with the true value of 1.8),

$m_{ez} = 1.598$ (small error compared with the true value of 1.6), (106)

$m_{mx} = 0.799$ (small error compared with the true value of 0.8),

$m_{my} = 0.599$ (small error compared with the true value of 0.6),

and

$m_{mz} = 0.401$ (small error compared with the true value of 0.4).

Thus, the method presented does predict very good true values for the amplitudes of the component dipole moments as long as the arbitrary errors in power measurements are within 5% of the actual readings. Results with other percent errors in power readings are presented in figure 3a,b, where the manner with which the errors occur is assumed to remain the same as that in (105), and the second-order correction terms are also excluded. From figure 3a, we see that, even with the power measurement error as large as 40% and with the second-order correction terms neglected, the predicted value for m_{ey} is still about 1.68 vs. 1.80 for the true value, a decrease of only 6.67%.

From (66), (72) through (76), and the constraints given in (38) and (39), we obtain approximately,

$$\begin{aligned} \cos\theta'_{e1} &= 0.219778 \ , & \theta'_{e1} &= -77.30° \ , \\ \cos\theta'_{e2} &= 0.932363 \ , & \theta'_{e2} &= 21.19° \ , \\ \cos\theta'_{e3} &= 0.533491 \ , & \theta'_{e3} &= 57.76° \ , \end{aligned} \tag{107}$$

$$\cos\theta'_{m1} = 0.929589 , \qquad \theta'_{m1} = -21.63° ,$$
$$\cos\theta'_{m2} = 0.941998 , \qquad \theta'_{m2} = -19.61° ,$$
$$\cos\theta'_{m3} = 0.734008 , \qquad \theta'_{m3} = 42.78° .$$

Since none of the above values is near zero, we use (65) and (67) through (71) to yield

$$\begin{aligned}
&\cos\theta_{e1} = 0.174289 , && \theta_{e1} = -79.96° \text{ (increased by 0.05\%)} , \\
&\cos\theta_{e2} = 0.940726 , && \theta_{e2} = 19.83° \text{ (decreased by 0.85\%)} , \\
&\cos\theta_{e3} = 0.499926 , && \theta_{e3} = 60.00° \text{ (completely recovered)} ; \\
&\cos\theta_{m1} = 0.939814 , && \theta_{m1} = -19.98° \text{ (increased by 0.1\%)} , \\
&\cos\theta_{m2} = 0.961780 , && \theta_{m2} = -15.89° \text{ (decreased by 5.93\%)} , \\
&\cos\theta_{m3} = 0.811744 , && \theta_{m3} = 35.73° \text{ (increased by 2.09\%)} .
\end{aligned} \tag{108}$$

Thus, the final errors in θ_{ei} and θ_{mi} in degrees are

$$\begin{aligned}
&d\theta_{e1} = \theta'_{e1} - \theta_{e1} = 2.66 , && d\theta_{m1} = \theta'_{m1} - \theta_{m1} = -1.65 \\
&d\theta_{e2} = \theta'_{e2} - \theta_{e2} = 1.36 , && d\theta_{m2} = \theta'_{m2} - \theta_{m2} = -3.72 \\
&d\theta_{e3} = \theta'_{e3} - \theta_{e3} = -2.24 , && d\theta_{m3} = \theta'_{m3} - \theta_{m3} = 7.05 .
\end{aligned} \tag{109}$$

We, next analyze the mixed phases. From (84), (85), and (83), we have the following results in degrees,

$$\alpha'_1 = -42.59 , \qquad \beta'_1 = 158.39 , \qquad \phi'_{ex} - \phi'_{my} = 66.30 . \tag{110}$$

Then, from (23), (24), and (86), we have (all results in degrees):

$$\alpha_1 = -45.94 , \qquad \beta_1 = 150.91$$

giving

$$d\alpha_1 = \alpha'_1 - \alpha_1 = 3.35 , \qquad d\beta_1 = \beta'_1 - \beta_1 = 7.48$$

$$d(\phi_{ex} - \phi_{my}) = d\phi_1 + d\alpha_1 - d\beta_1 = 6.18 ,$$

which, in turn, yields

$$\begin{aligned}\phi_{ex} - \phi_{my} &= \phi'_{ex} - \phi'_{my} - d(\phi_{ex} - \phi_{my}) \\ &= 60.12 \quad \text{(increased by 0.2\%)} \ ,\end{aligned} \tag{111}$$

and

$$\begin{aligned}\phi_{ey} - \phi_{mx} &= \phi_{ex} - \phi_{my} - \theta_{e1} - \theta_{m1} \\ &= 160.06 \quad \text{(increased by 0.04\%)} \ .\end{aligned}$$

Similarly, from (90, 91, 88), (92, 93, 89), (25, 26), and (27, 28), we obtain the following in degrees:

$$\alpha'_3 = 10.00 \ , \qquad \beta'_3 = 145.98 \ , \qquad \phi'_{ey} - \phi'_{mz} = 99.11 \ ;$$

$$\alpha'_5 = 27.90 \ , \qquad \beta'_5 = -57.10 \ , \qquad \phi'_{ez} - \phi'_{mx} = 128.30 \ ;$$

$$\alpha_3 = 9.33 \ , \qquad \beta_3 = 133.12 \ ;$$

and

$$\alpha_5 = 27.82 \ , \qquad \beta_5 = -63.68 \ . \tag{112}$$

Then,

$$d\alpha_3 = \alpha'_3 - \alpha_3 = 0.67, \qquad d\beta_3 = \beta'_3 - \beta_3 = 12.86$$

$$d\alpha_5 = \alpha'_5 - \alpha_5 = 0.08, \qquad d\beta_5 = \beta'_5 - \beta_5 = 6.58 \ ;$$

$$d(\phi_{ey} - \phi_{mz}) = d\phi_3 + d\alpha_3 - d\beta_3 = -23.55 \ ,$$

and

$$d(\phi_{ez} - \phi_{mx}) = d\phi_5 + d\alpha_5 - d\beta_5 = -11.32$$

yield

$$\begin{aligned}\phi_{ey} - \phi_{mz} &= \phi'_{ey} - \phi'_{mz} - d(\phi_{ey} - \phi_{mz}) \\ &= 122.66 \quad \text{(decreased by 1.87\%)} \ ,\end{aligned}$$

$$\psi_{ez} - \psi_{mx} = \psi'_{ez} - \psi'_{mx} - d(\psi_{ez} - \psi_{mx})$$
$$= 139.62 \quad \text{(decreased by 0.27\%)} \;, \tag{113}$$

$$\psi_{ez} - \psi_{my} = \psi_{ey} - \psi_{mz} - \theta_{e2} - \theta_{m2}$$
$$= 118.72 \quad \text{(decreased by 1.07\%)} \;,$$

and

$$\psi_{ex} - \psi_{mz} = \psi_{ez} - \psi_{mx} - \theta_{e3} - \theta_{m3}$$
$$= 43.89 \quad \text{(decreased by 2.47\%)} \;.$$

The above results in (111) and (113) are based on the measured phases ϕ'_1, ϕ'_3, ϕ'_5 and the assumed values for $d\phi_1$, $d\phi_3$, and $d\phi_5$. Alternatively, the results may also be derived from the measured ϕ'_2, ϕ'_4, ϕ'_6 and the assumed values for $d\phi_2$, $d\phi_4$, and $d\phi_6$ in accordance with (29) through (37):

$$\alpha'_2 = 57.50 \;, \qquad \beta'_2 = -77.86 \;, \qquad \psi'_{ex} - \psi'_{my} = 66.03$$
$$\alpha'_4 = -62.90 \;, \qquad \beta'_4 = -78.55 \;, \qquad \psi'_{ey} - \psi'_{mz} = 110.65$$
$$\alpha'_6 = -57.91 \;, \qquad \beta'_6 = -105.10 \;, \qquad \psi'_{ez} - \psi'_{mx} = 148.29 \;;$$

$$\alpha_2 = 58.46 \;, \qquad \beta_2 = -78.57$$
$$\alpha_4 = -61.47 \;, \qquad \beta_4 = -80.47$$
$$\alpha_6 = -53.47 \;, \qquad \beta_6 = -101.76 \;;$$

$$d\alpha_2 = \alpha'_2 - \alpha_2 = -0.96 \;, \qquad d\beta_2 = \beta'_2 - \beta_2 = 0.71$$
$$d\alpha_4 = \alpha'_4 - \alpha_4 = -1.43 \;, \qquad d\beta_4 = \beta'_4 - \beta_4 = 1.92$$
$$d\alpha_6 = \alpha'_6 - \alpha_6 = -4.44 \;, \qquad d\beta_6 = \beta'_6 - \beta_6 = -3.34 \;;$$

$$d(\psi_{ex} - \psi_{my}) = d\phi_2 + d\alpha_2 - d\beta_2 = 6.03$$
$$d(\psi_{ey} - \psi_{mz}) = d\phi_4 + d\alpha_4 - d\beta_4 = -13.91$$
$$d(\psi_{ez} - \psi_{mx}) = d\phi_6 + d\alpha_6 - d\beta_6 = 8.09 \;;$$

and

$$\psi_{ex} - \psi_{my} = \psi'_{ex} - \psi'_{my} - d(\psi_{ex} - \psi_{my})$$
$$= 60.00 \quad \text{(completely recovered)}$$

$$\psi_{ey} - \psi_{mz} = \psi'_{ey} - \psi'_{mz} - d(\psi_{ey} - \psi_{mz})$$
$$= 124.56 \quad \text{(decreased by 0.35\%)} \tag{114}$$

$$\psi_{ez} - \psi_{mx} = \psi'_{ez} - \psi'_{mx} - d(\psi_{ez} - \psi_{mx})$$
$$= 140.20 \quad \text{(increased by 0.14\%)} \ .$$

The results obtained above by the alternative method may be used either to compare with those obtained in (111) and (113) to check the consistency of the measured data, or to be averaged with those in (111) and (113) to minimize the overall errors in the deduced results, because, in reality, the actual measurement accuracy is unknown. If the averaged values are desirable, we have:

$$(\psi_{ex} - \psi_{my})_{av} = 60.06 \quad \text{(vs. the true value of 60.00)} \ ,$$

$$(\psi_{ey} - \psi_{mz})_{av} = 123.61 \quad \text{(vs. the true value of 125.00)} \ ,$$

$$(\psi_{ez} - \psi_{mx}) = 139.91 \quad \text{(vs. the true value of 140.00)} \ , \tag{115}$$

$$(\psi_{ey} - \psi_{mx})_{av} = (\psi_{ex} - \psi_{my})_{av} - \theta_{e1} - \theta_{m1} = 160.00 \quad \text{(exact)} \ ,$$

$$(\psi_{ez} - \psi_{my})_{av} = (\psi_{ey} - \psi_{mz})_{av} - \theta_{e2} - \theta_{m2} = 119.67$$
$$\text{(vs. the true value of 120.00)} \ ,$$

and

$$(\psi_{ex} - \psi_{mz})_{av} = (\psi_{ez} - \psi_{mx})_{av} - \theta_{e3} - \theta_{m3} = 44.18$$
$$\text{(vs. the true value of 45.00)} \ .$$

The above predicted results together with the predicted relative phases obtained in (108) and the amplitudes obtained in (106) may be used in (15) to compute the predicted radiation pattern, which may then be compared with that computed by the primed values (based on the measured results directly) for any θ and ϕ. This comparison should reveal the error caused by the measurement inaccuracy assumed in (105), and the contribution by the predicting process discussed here. Typical graphical results are presented in figure 4, where the original ideal case without involving measurement errors is also included.

The predicted amplitudes of the component dipole moments (given in (106)) alone may be used in (16) to predict the total power radiated in free space by the interference source being investigated. That is,

$$P_T = 0.4\ \pi^2\ [m_{ex}^2 + m_{ey}^2 + m_{ez}^2 + k^2(m_{mx}^2 + m_{my}^2 + m_{mx}^2)]$$
$$= 32.39\ \text{W} \quad \text{(vs. the true value of 32.44 W).}$$

Should the primed values based on direct measurements be used, we would have

$$P_T' = 32.17\ \text{W} \quad \text{(decreased by 0.83\% compared to the true value).}$$

Thus, from the anlaysis made so far for this particular simulated example, we may conclude that the final predicted results are not substantially affected by the measurement errors (e.g., ± 5% in power measurements and ± 10% in phase measurements) arbitrarily assumed in (105). Selected results with other combinations of percentage errors in power and phase measurements, with the same manner of error occurrence as assumed in (105), are presented in figure 5. From these results, one may make a quantitative assessment as to how much percentage errors in measurement can be tolerated in order to achieve a certain accuracy in the final predicted source characteristics.

4. Another Simulated Example with Assumed Biased Measurement Errors

Sometimes, we experience biased errors due to the measurement system. In this case, the measurement readings are consistently higher than the actual status. For simulating this situation, we assume that $dP_{si} = dP_{di} = 20$ W and $d\phi_i = 20°$ for the example considered in the previous section. Thus, the measured powers in watts and the measured phases in degrees become

$$P_{s1}' = 445.11\ , \qquad P_{s2}' = 322.63\ , \qquad P_{s3}' = 804.60\ ,$$
$$P_{s4}' = 47.11\ , \qquad P_{s5}' = 493.03\ , \qquad P_{s6}' = 179.54\ ;$$

$$P_{d1}' = 22.70\ , \qquad P_{d2}' = 72.55\ , \qquad P_{d3}' = 21.56\ ,$$
$$P_{d4}' = 47.17\ , \qquad P_{d5}' = 27.62\ , \qquad P_{d6}' = 56.58\ ; \tag{116}$$

$\phi_1' = -83.03$, $\phi_2' = -57.03$, $\phi_5' = -93.55$,

$\phi_4' = 125.56$, $\phi_5' = 68.11$, $\phi_6' = 111.91$.

Note that in the above, the errors in P_{d1}', P_{d3}', and P_{d5}' are quite enormous as compared to the true values given in (104). Consequently, we should expect large errors to appear in the final deduced source parameters related to these difference powers. Here, we once again assume dq = 0 for simplicity. Then, from (54) through (61), we obtain

$$
\begin{aligned}
m_{ex}'^2 &= 2.1029 , & m_{ex}' &= 1.450 , & dm_{ex}/m_{ex}' &= 0.034 ; \\
m_{ey}'^2 &= 3.3829 , & m_{ey}' &= 1.839 , & dm_{ey}/m_{ey}' &= 0.021 ; \\
m_{ez}'^2 &= 2.7029 , & m_{ez}' &= 1.644 , & dm_{ez}/m_{ez}' &= 0.026 ; \\
m_{mx}'^2 &= 1.0020 , & m_{mx}' &= 1.001 , & dm_{mx}/m_{mx}' &= 0.181 ; \\
m_{my}'^2 &= 0.7220 , & m_{my}' &= 0.850 , & dm_{my}/m_{my}' &= 0.251 ; \\
m_{mz}'^2 &= 0.5220 , & m_{mz}' &= 0.722 , & dm_{mz}/m_{mz}' &= 0.347 ;
\end{aligned}
$$

$$
\begin{aligned}
m_{ex} \text{ (the predicted value)} &= m_{ex}'(1 - dm_{ex}/m_{ex}') \\
&= 1.401 \quad \text{(vs. 1.40, +0.07\%)} , \\
m_{ey} &= 1.800 \quad \text{(vs. 1.80, no error)} , \\
m_{ez} &= 1.601 \quad \text{(vs. 1.60, +0.06\%)} , \\
m_{mx} &= 0.820 \quad \text{(vs. 0.80, +2.50\%)} , \\
m_{my} &= 0.637 \quad \text{(vs. 0.60, +6.17\%)} ,
\end{aligned}
\tag{117}
$$

and

$$m_{mz} = 0.471 \quad \text{(vs. 0.40, +17.75\%)} ,$$

where, for simplicity, the plus sign in parenthesis denotes increasing while the minus sign denotes decreasing.

From (66), (72) through (76), and the constraint requirements in (38) and (39), we also obtain the relative phases in degrees among the same kind of dipole components directly from the measured data,

$$\begin{aligned}
\cos\theta'_{e1} &= 0.164103\ , & \theta'_{e1} &= -80.55 \\
\cos\theta'_{e2} &= 0.895146\ , & \theta'_{e2} &= 26.47 \\
\cos\theta'_{e3} &= 0.469845\ , & \theta'_{e3} &= 61.98\ ; \\
\\
\cos\theta'_{m1} &= 0.530216\ , & \theta'_{m1} &= -57.98 \\
\cos\theta'_{m2} &= 0.377654\ , & \theta'_{m2} &= -67.81 \\
\cos\theta'_{m3} &= 0.362634\ , & \theta'_{m3} &= 68.74\ .
\end{aligned} \tag{118}$$

Equations (65) and (67) through (71) yield the predicted relative phases in degrees:

$$\begin{aligned}
\cos\theta_{e1} &= 0.173129\ , & \theta_{e1} &= -80.03 \quad (\text{vs. } -80.0,\ -0.04\%) \\
\cos\theta_{e2} &= 0.937218\ , & \theta_{e2} &= 20.41 \quad (\text{vs. } 20.0,\ +2.05\%) \\
\cos\theta_{e3} &= 0.498036\ , & \theta_{e3} &= 60.13 \quad (\text{vs. } 60.0,\ +0.22\%)\ ; \\
\\
\cos\theta_{m1} &= 0.759269\ , & \theta_{m1} &= -40.60 \quad (\text{vs. } -20.0,\ -103.00\%) \\
\cos\theta_{m2} &= 0.603491\ , & \theta_{m2} &= -52.88 \quad (\text{vs. } -15.0,\ -252.53\%) \\
\cos\theta_{m3} &= 0.554105\ , & \theta_{m3} &= 56.35 \quad (\text{vs. } 35.0,\ +61.00\%)\ .
\end{aligned} \tag{119}$$

Note that the θ'_{mi}s obtained above still deviate substantially from the constraint requirement in (38) while the θ'_{e1}s so deduced are very satisfactory. Note further that only the first-order correction terms are retained in (69) to (71). This will yield a reasonable predicted result only when the percentage errors involved are small. Since the actual percent errors for dm_{mi}/m'_{mi}, i = x, y, z are all relatively large in view of the measurement errors assumed for P'_{d1}, P'_{d3}, and P'_{d5}, it is not surprising to see that the results for θ_{mi} predicted in (119) are what they are. Should the second-order correction terms also be included in the formulation, the predicted results for θ_{mi} would be more reasonable.

Comparing the results in (118) and (119), we have the errors in degrees for the relative phases among the same kind of dipole components for the example considered herein,

$$d\theta_{e1} = \theta'_{e1} - \theta_{e1} = -0.52\,, \qquad d\theta_{m1} = \theta'_{m1} - \theta_{m1} = -17.38$$
$$d\theta_{e2} = \theta'_{e2} - \theta_{e2} = 6.06\,, \qquad d\theta_{m2} = \theta'_{m2} - \theta_{m2} = -14.93 \qquad (120)$$
$$d\theta_{e3} = \theta'_{e3} - \theta_{e3} = 1.85\,, \qquad d\theta_{m3} = \theta'_{m3} - \theta_{m3} = 12.39\,.$$

Now, we treat the mixed phases. In accordance with (83, 84, 85) and (17, 23, 24), we obtain the following in degrees:

$$\alpha'_1 = -46.00\,, \qquad \alpha_1 = -45.99\,, \qquad d\alpha_1 = -0.01$$
$$\beta'_1 = -159.39\,, \qquad \beta_1 = -178.45\,, \qquad d\beta_1 = 19.06$$

$$\begin{aligned} \psi'_{ex} - \psi'_{my} &= \phi'_1 + \alpha'_1 - \beta'_1 = 30.36 \\ d(\psi_{ex} - \psi_{my}) &= d\phi_1 + d\alpha_1 - d\beta_1 = 0.93 \\ \psi_{ex} - \psi_{my} &= \psi'_{ex} - \psi'_{my} - d(\psi_{ex} - \psi_{my}) = 29.43\,. \end{aligned} \qquad (121)$$

The above relative phase can also be deduced from ϕ'_2. That is,

$$\alpha'_2 = 57.67\,, \qquad \alpha_2 = 58.43\,, \qquad d\alpha_2 = -0.76$$
$$\beta'_2 = -58.42\,, \qquad \beta_2 = -67.04\,, \qquad d\beta_2 = 8.62$$

$$\begin{aligned} \psi'_{ex} - \psi'_{my} &= \phi'_2 + \alpha'_2 - \beta'_2 = 59.06 \\ d(\psi_{ex} - \psi_{my}) &= d\phi_2 + d\alpha_2 - d\beta_2 = 10.62 \\ \psi_{ex} - \psi_{my} &= \psi'_{ex} - \psi'_{my} - d(\psi_{ex} - \psi_{my}) = 48.44 \end{aligned} \qquad (122)$$

which may be used to average with that in (121), yielding

$$(\psi_{ex} - \psi_{my})_{av} = 38.94 \quad (\text{vs. } 60.0, -35.11\%)\,, \qquad (123)$$

$$\begin{aligned} (\psi_{ey} - \psi_{mx})_{av} &= (\psi_{ex} - \psi_{my})_{av} - \theta_{e1} - \theta_{m1} \\ &= 159.57 \quad (\text{vs. } 160.0, -0.27\%)\,. \end{aligned} \qquad (124)$$

Similarly, we also have the following in degrees:

$$\alpha'_3 = 12.48\,, \qquad \alpha_3 = 9.60\,, \qquad d\alpha_3 = 2.88$$
$$\beta'_3 = -153.00\,, \qquad \beta_3 = -170.33\,, \qquad d\beta_3 = 17.33$$

$$\psi'_{ey} - \psi'_{mz} = \phi'_3 + \alpha'_3 - d\beta_3 = 71.93$$
$$d(\psi_{ey} - \psi_{mz}) = d\phi_3 + d\alpha_3 - d\beta_3 = 5.55$$
$$\psi_{ey} - \psi_{mz} = \psi'_{ey} - \psi'_{mz} - d(\psi_{ey} - \psi_{mz}) = 66.38 \; ; \qquad (125)$$

$$\alpha'_4 = -63.38 \, , \qquad \alpha_4 = -61.79 \, , \qquad d\alpha_4 = -1.59$$
$$\beta'_4 = -52.96 \, , \qquad \beta_4 = -59.30 \, , \qquad d\beta_4 = 6.34$$

$$\psi'_{ey} - \psi'_{mz} = \phi'_4 + \alpha'_4 - \beta'_4 = 115.14$$
$$d(\psi_{ey} - \psi_{mz}) = d\phi_4 + d\alpha_4 - d\beta_4 = 12.07$$
$$\psi_{ey} - \psi_{mz} = \psi'_{ey} - \psi'_{mz} - d(\psi_{ey} - \psi_{mz}) = 103.07 \; ; \qquad (126)$$

$$(\psi_{ey} - \psi_{mz})_{av} = 84.73 \quad (\text{vs. } 125.0, -32.22\%) \, , \qquad (127)$$

$$(\psi_{ez} - \psi_{my})_{av} = (\psi_{ey} - \psi_{mz})_{av} - \theta_{e2} - \theta_{m2}$$
$$= 117.20 \quad (\text{vs. } 120.0, -2.33\%) \; ; \qquad (128)$$

$$\alpha'_5 = 28.83 \, , \qquad \alpha_5 = 27.86 \, , \qquad d\alpha_5 = 0.97$$
$$\beta'_5 = -47.69 \, , \qquad \beta_5 = -54.95 \, , \qquad d\beta_5 = 7.26$$

$$\psi'_{ez} - \psi'_{mx} = \phi'_5 + \alpha'_5 - \beta'_5 = 144.63$$
$$d(\psi_{ez} - \psi_{mx}) = d\phi_5 + d\alpha_5 - d\beta_5 = 13.71$$
$$\psi_{ez} - \psi_{mx} = \psi'_{ez} - \psi'_{mx} - d(\psi_{ez} - \psi_{mx}) = 130.92 \; ; \qquad (129)$$

$$\alpha'_6 = -53.05 \, , \qquad \alpha_6 = -53.37 \, , \qquad d\alpha_6 = 0.32$$
$$\beta'_6 = -118.05 \, , \qquad \beta_6 = -109.94 \, , \qquad d\beta_6 = -8.11$$

$$\psi'_{ez} - \psi'_{mx} = \phi'_6 + \alpha'_6 - \beta'_6 = 176.91$$
$$d(\psi_{ez} - \psi_{mx}) = d\phi_6 + d\alpha_6 - d\beta_6 = 28.43$$
$$\psi_{ez} - \psi_{mx} = \psi'_{ez} - \psi'_{mx} - d(\psi_{ez} - \psi_{mx}) = 148.48 \; ; \qquad (130)$$

$$(\psi_{ez} - \psi_{mx})_{av} = 139.70 \quad (\text{vs. } 140.0, -0.21\%), \qquad (131)$$

$$(\psi_{ex} - \psi_{mz})_{av} = (\psi_{ez} - \psi_{mx}) - \theta_{e3} - \theta_{m3}$$
$$= 23.22 \quad (\text{vs. } 45.0, -48.40\%) \, . \qquad (132)$$

The predicted amplitudes of the dipole moment obtained in (117) may be used in (16) to compute the predicted total power radiated by the source emitter under study. That is,

$$P_T = 32.69 \text{ W} \quad (\text{vs. the true value of } 32.44 \text{ W}, +0.76\%) \ , \tag{133}$$

which is still very reasonable in view of the large errors incurred in the magnetic dipole moments. If the primed source parameters are used to compute the total radiated power, we would get

$$P_T' = 35.82 \text{ W} \quad (\text{vs. } 32.44 \text{ W}, +10.42\%) \ . \tag{134}$$

The above result implies that the measured errors, $dP_{si} = dP_{di} = 20$ W and $d\phi_i = 20°$ assumed in this example, will contribute more than 10% of error toward the derived radiated power if no application of the predicting process as demonstrated is made. Once the predicting process is used, the overall error in the deduced total radiated power amounts only to less than 1%.

In addition, the predicted phases among the same kind of dipole components obtained in (119) and the predicted average mixed phases obtained in (123, 124, 127, 128, 131, 132), together with the predicted amplitudes just considered can be used in (15) to evaluate the detailed normalized free-space radiation pattern. This pattern may then be compared with that computed by the primed source parameters to reveal the error caused by the measurement inaccuracy and the correction contributed by the process so presented.

Thus, both the example considered in this section and that given in the previous section show that the predicting process outlined in this report (i.e., obtaining the necessary unprimed source parameters from the primed ones computed directly from the experimental data containing measurement errors) is indeed useful to extract the source parameters and, therefore, to predict the radiation characteristics of an unknown inteference source, provided that the approximate measurement error is known or can be estimated.

5. Sensitivity of Extracted Parameters with Respect to Measurement Accuracy

Another useful concept for error analysis is via the sensitivity function. Mathematically, the sensitivity of a parameter A with repect to a quantity B may be defined as

$$S_B^A = \frac{dA/A}{dB/B} \cong \frac{B}{A}\frac{\partial A}{\partial B} \tag{135}$$

giving the percentage change in A for a given percentage change in B, or indicating the sensitivity of dependence of A on B.

Refering to (1), if we let $A = m_{ex}$, and $B = P_{si}$, we can derive an expression for the percentage error incurred in the deduced source parameter m_{ex} due to a given percentage error in the measured quantity P_{si}:

$$S_{P_{si}}^{m_{ex}} = \frac{P_{si}}{m_{ex}}\frac{\partial m_{ex}}{\partial P_{si}} = \pm \frac{P_{si}}{4m_{ex}^2 q^2} \begin{cases} + \text{ for } i = 1,2,5,6 \\ - \text{ for } i = 3,4 \end{cases} . \tag{136}$$

Similarly, we obtain

$$S_{P_{si}}^{m_{ey}} = \pm \frac{P_{si}}{4m_{ey}^2 q^2} \begin{cases} + \text{ for } i = 1,2,3,4 \\ - \text{ for } i = 5,6 \end{cases} \tag{137}$$

$$S_{P_{si}}^{m_{ez}} = \pm \frac{P_{si}}{4m_{ez}^2 q2} \begin{cases} + \text{ for } i = 3,4,5,6 \\ - \text{ for } i = 1,2 \end{cases} \tag{138}$$

$$S_{P_{di}}^{m_{mx}} = \pm \frac{P_{di}}{4m_{mx}^2 k^2 q^2} \begin{cases} + \text{ for } i = 1,2,5,6 \\ - \text{ for } i = 3,4 \end{cases} \tag{139}$$

$$S_{P_{di}}^{m_{my}} = \pm \frac{P_{di}}{4m_{my}^2 k^2 q^2} \begin{cases} + \text{ for } i = 1,2,3,4 \\ - \text{ for } i = 5,6 \end{cases} \tag{140}$$

$$S_{P_{di}}^{m_{mz}} = \pm \frac{P_{di}}{4m_{mz}^2 k^2 q^2} \begin{cases} + \text{ for } i = 3,4,5,6 \\ - \text{ for } i = 1,2 \end{cases} \tag{141}$$

$$S_{P_{si}}^{P_T} = \frac{P_{si}}{P_T} \frac{\partial P_T}{\partial P_{si}} = (20\pi^2/\lambda^2 q^2) \frac{P_{si}}{P_T} \tag{142}$$

and

$$S_{P_{di}}^{P_T} = (20\pi^2/\lambda^2 q^2) \frac{P_{di}}{P_T} . \tag{143}$$

The corresponding expressions for $A = \theta_{ei}$, θ_{mi}, or $\psi_{ei} - \psi_{mj}$ (i, j = x,y,z, $i \neq j$) and $B = P_{si}$, P_{di}, or ϕ_1 are much more involved, and thus not presented here.

For the simulated example (103) and (104) considered in section 3, we have

$S_{P_{s1}}^{m_{ex}} = 0.387$, $S_{P_{s2}}^{m_{ex}} = 0.276$, $S_{P_{s3}}^{m_{ex}} = -0.715$,

$S_{P_{s4}}^{m_{ex}} = -0.025$, $S_{P_{s5}}^{m_{ex}} = 0.431$, $S_{P_{s6}}^{m_{ex}} = 0.145$;

$S_{P_{s1}}^{m_{ey}} = 0.234$, $S_{P_{s2}}^{m_{ey}} = 0.167$, $S_{P_{s3}}^{m_{ey}} = 0.433$,

$S_{P_{s4}}^{m_{ey}} = 0.015$, $S_{P_{s5}}^{m_{ey}} = -0.261$, $S_{P_{s6}}^{m_{ey}} = -0.088$;

$$S_{P_{s1}}^{m_{ez}} = -0.297 , \quad S_{P_{s2}}^{m_{ez}} = -0.211 , \quad S_{P_{s3}}^{m_{ez}} = 0.547 ,$$

$$S_{P_{s4}}^{m_{ez}} = 0.019 , \quad S_{P_{s5}}^{m_{ez}} = 0.330 , \quad S_{P_{s6}}^{m_{ez}} = 0.111 ;$$

$$S_{P_{d1}}^{m_{mx}} = 0.019 , \quad S_{P_{d2}}^{m_{mx}} = 0.372 , \quad S_{P_{d3}}^{m_{mx}} = -0.011 ,$$

$$S_{P_{d4}}^{m_{mx}} = -0.192 , \quad S_{P_{d5}}^{m_{mx}} = 0.054 , \quad S_{P_{d6}}^{m_{mx}} = 0.259 ;$$

$$S_{P_{d1}}^{m_{my}} = 0.034 , \quad S_{P_{d2}}^{m_{my}} = 0.660 , \quad S_{P_{d3}}^{m_{my}} = 0.020 ,$$

$$S_{P_{d4}}^{m_{my}} = 0.342 , \quad S_{P_{d5}}^{m_{my}} = -0.096 , \quad S_{P_{d6}}^{m_{my}} = -0.460 ; \qquad (144)$$

$$S_{P_{d1}}^{m_{mz}} = -0.076 , \quad S_{P_{d2}}^{m_{mz}} = -1.486 , \quad S_{P_{d3}}^{m_{mz}} = 0.044 ,$$

$$S_{P_{d4}}^{m_{mz}} = 0.768 , \quad S_{P_{d5}}^{m_{mz}} = 0.215 , \quad S_{P_{d6}}^{m_{mz}} = 1.035 ,$$

$$S_{P_{s1}}^{P_T} = 0.185 , \quad S_{P_{s2}}^{P_T} = 0.132 , \quad S_{P_{s3}}^{P_T} = 0.341 ,$$

$$S_{P_{s4}}^{P_T} = 0.012 , \quad S_{P_{s5}}^{P_T} = 0.206 , \quad S_{P_{s6}}^{P_T} = 0.069 ;$$

$$S_{P_{d1}}^{P_T} = 0.001 , \quad S_{P_{d2}}^{P_T} = 0.023 , \quad S_{P_{d3}}^{P_T} = 0.001 ,$$

$$S_{P_{d4}}^{P_T} = 0.012 , \quad S_{P_{d5}}^{P_T} = 0.003 , \quad S_{P_{d6}}^{P_T} = 0.016 .$$

From the above numbers for this particular example, we may interpret the results as follows:

1) The accuracy in measuring P_{s3} has the most effect on m_{ex}. In particular, the deduced source parameter m_{ex} <u>decreases</u> from the true value by approximately 0.7% whenever the measured P_{s3} is caused to <u>increase</u> from its true value by 1% due to the measurement error. The accuracy

of the measured P_{s3} has also relatively large effect on m_{ey} and m_{ez}. This observaton is, of course, obvious in view of the fact that P_{s3} is the largest reading in (104). The implication is that more care should be exercised for taking the stronger data.

2) The accuracy in measuring P_{d2} and P_{d6} has more effect on m_{mi}, i = x,y,z than that in measuring P_{d1}, P_{d3}, P_{d4}, and P_{d5}.

3) The total radiated power, P_T, by the interference source is not very sensitive to the accuracy in measuring the sum and difference powers, because $S_{\alpha}^{P_T} < 0.4$, $\alpha = P_{si}$ and P_{di}.

6. Concluding Remarks

Based on the previous formulation for quantifying the radiation characteristics of an unknown interference source from power and phase measurements with the aid of a TEM cell, we have derived in this report the deviation expressions, due to possible errors in measuring these powers and phases, for all those source parameters required for computing the radiation pattern and the total power radiated by the subject source in free space. From these deviation expressions, we can make corrections to those source parameters deduced directly from the measured data in order to obtain results closer to the true values, when the measurement errors are provided either by a manufacturer's estimation or by assumption. If, on the other hand, the measurement errors are believed to be random in nature, the derivations contained herein indicate that repeated measurements are required, from which a set of source parameters may be directly computed. The averages of these respective computed source parameters should approach the true values.

Two numerical examples based on two sets of measurement errors--one is arbitrarily assumed and the other is based on a systematic biased error--have been given to illustrate the general procedure of making error analysis. Utilization of the sensitivity function to yield a percentage change in one of the deduced source parameters for a given percentage error in one of the measurement data has also been applied.

In practice, the accuracy in power and phase measurements may be estimated in terms of worst-case bounds such as ± x% in power measurements and ± y% in phase measurements (or in an absolute term such as ± x watts and ± y degrees respectively). Under this situation, the formulation outlined in this report is still applicable by considering all the possible combinations of power and phase readings to derive an upper and lower bounds for the deduced source parameters and, consequently, for the radiation characteristics of the unknown source. When one of the source parameters so computed based on the measurement data is dominant, more attention can then be paid to the bounds of this particular parameter, because its accuracy should have more impact on the accuracy in the final result. For example, if $m_{ex}'^2$ is the largest among the source parameters based on the direct sum power measurements P_{si}', i = 1, 2, . . . , 6, then, in accordance with the first expression given in (54), we easily obtain the upper and lower bounds for m_{ex}^2 as follows:

$$
\begin{aligned}
m_{ex}^2\big|_{upper} &= [P_{s1}'(1 + x\%) + P_{s2}'(1 + x\%) - P_{s3}'(1 - x\%) \\
&\quad - P_{s4}'(1 - x\%) + P_{s5}'(1 + x\%) + P_{s6}'(1 + x\%)] /(2q^2) \\
m_{ex}^2\big|_{lower} &= [P_{s1}'(1 - x\%) + P_{s2}'(1 - x\%) - P_{s3}'(1 + x\%) \\
&\quad - P_{s4}'(1 + x\%) + P_{s5}'(1 - x\%) + P_{s6}'(1 - x\%)] /(2q^2)
\end{aligned}
\tag{145}
$$

where dq = 0 is also assumed.

Based on the same sequence of assumed errors as above, m_{ey}^2 and m_{ez}^2 can then be computed. After applying a similar consideration for m_{mi}^2, the upper and lower bounds for the total radiated power P_T may be estimated by (16).

Strictly speaking, the formulation outlined in this report applies only to the symmetric TEM cell, where the center septum is exactly midway between the top and bottom outer conductors. For an asymmetric cell designed to provide more test zone inside either top or bottom half of the cell [4], the electric fields at the centers of the two halves are no longer identical. When the vertical component of the electric field in either half, q_a, is known (by theoretical computation or by actual measurement) and the corresponding horizontal component is relatively insignificant--as it should be for a well-designed cell--the formulation and analysis presented herein can still be applicable by merely replacing the parameter q in (1) through (13) by q_a.

7. References

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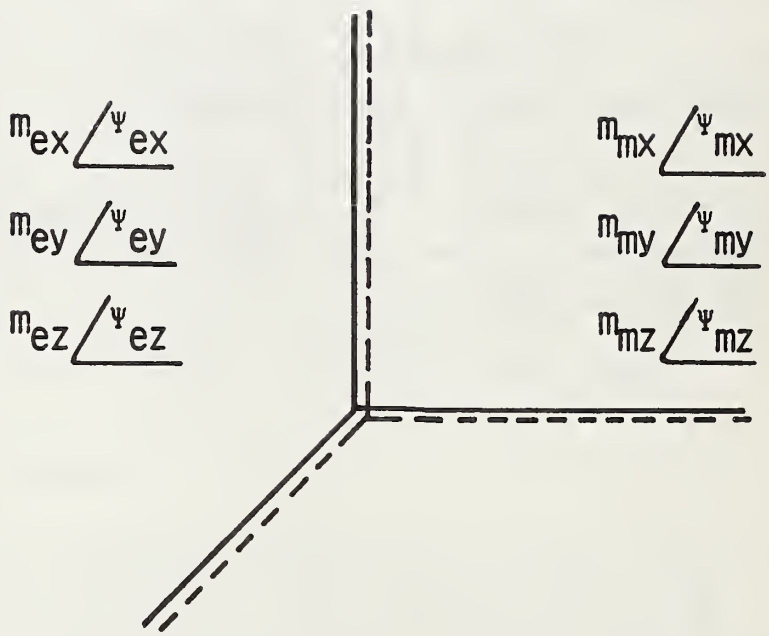

Figure 1. Unknown equipment under test (EUT) is made of equivalent three orthogonal electric and three orthogonal magnetic dipoles.

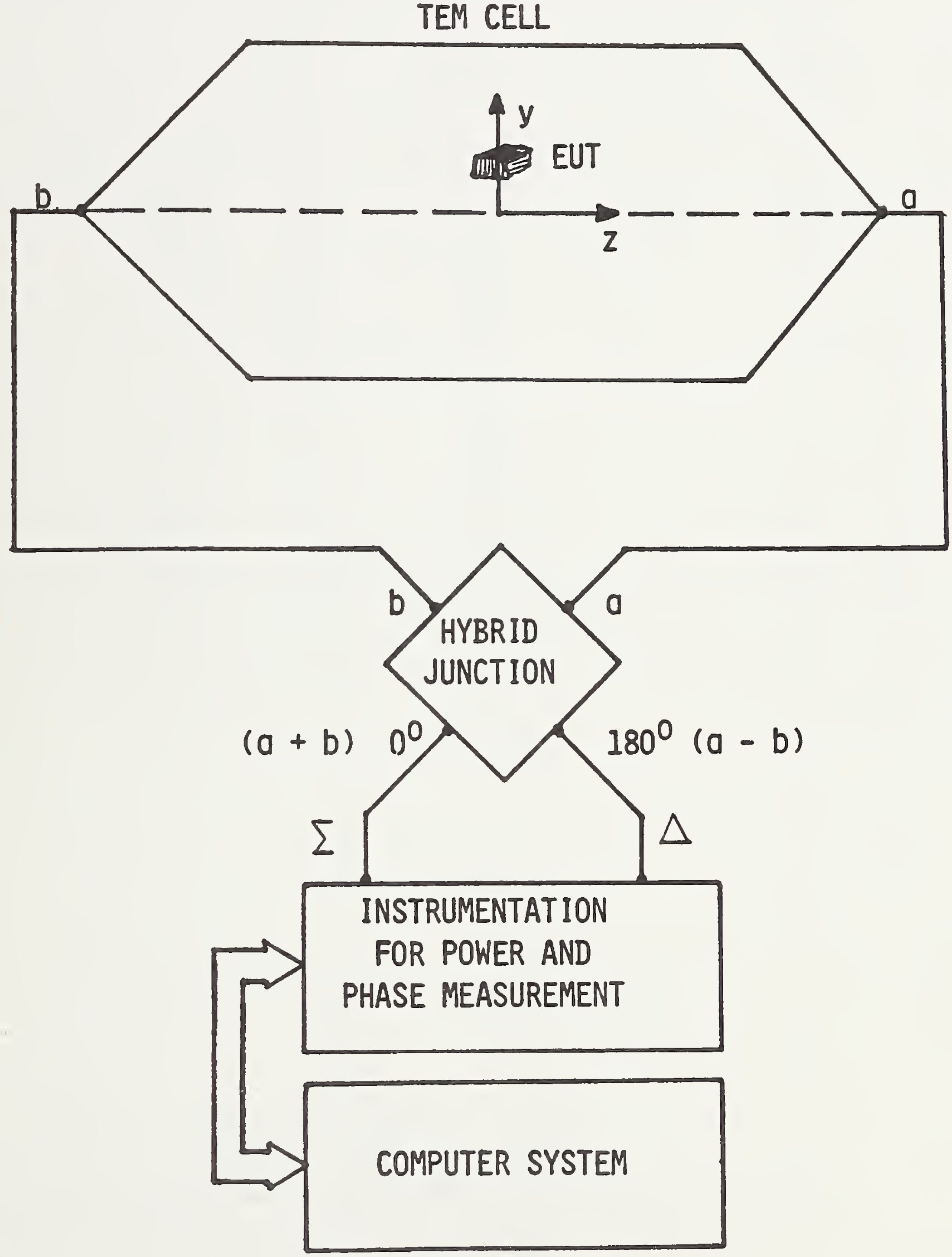

Figure 2. Emissions testing measurement system.

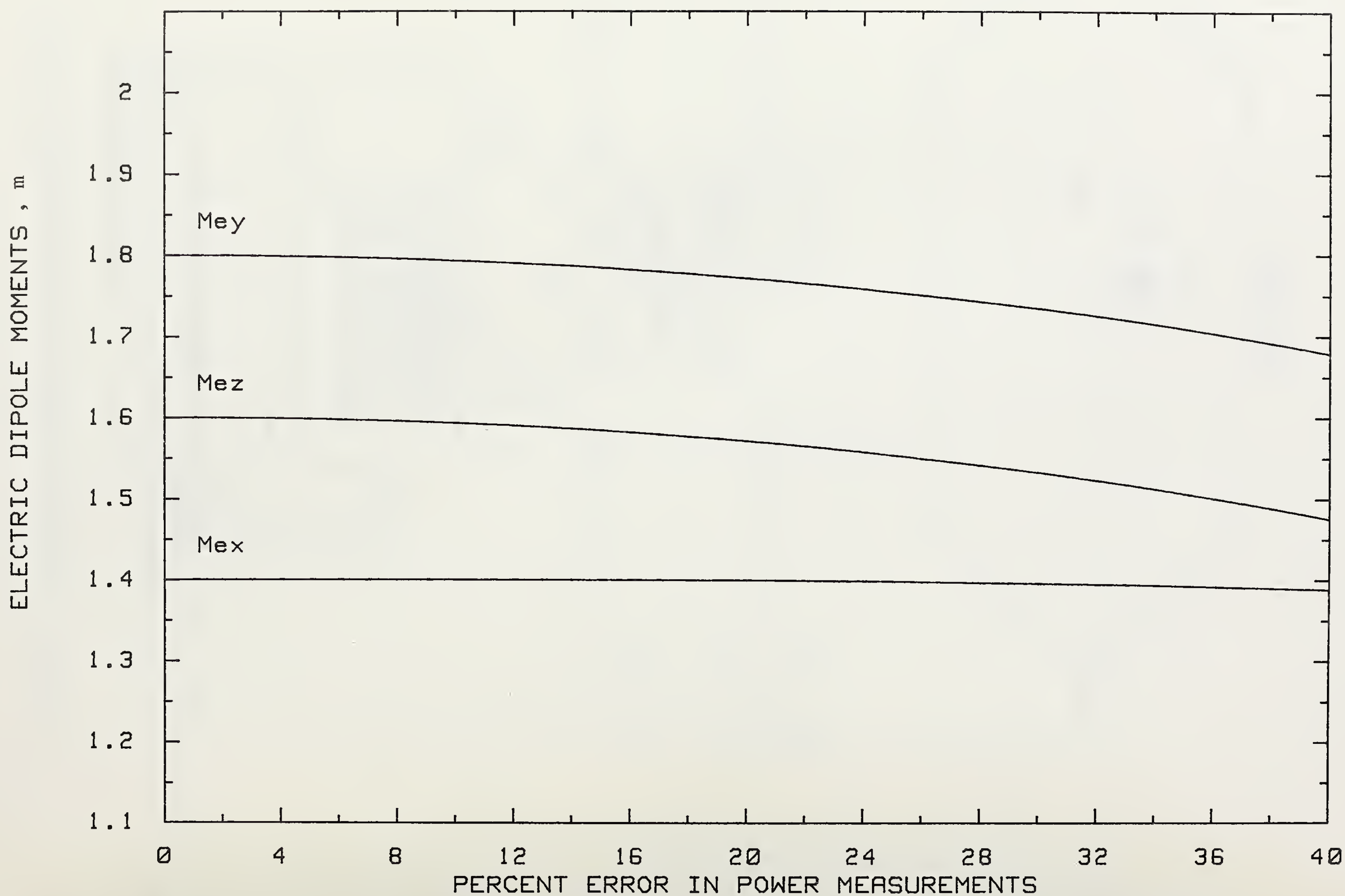

Figure 3a. Predicted electric dipole moments versus percent error in sum power measurements as assumed in (105), p.19.

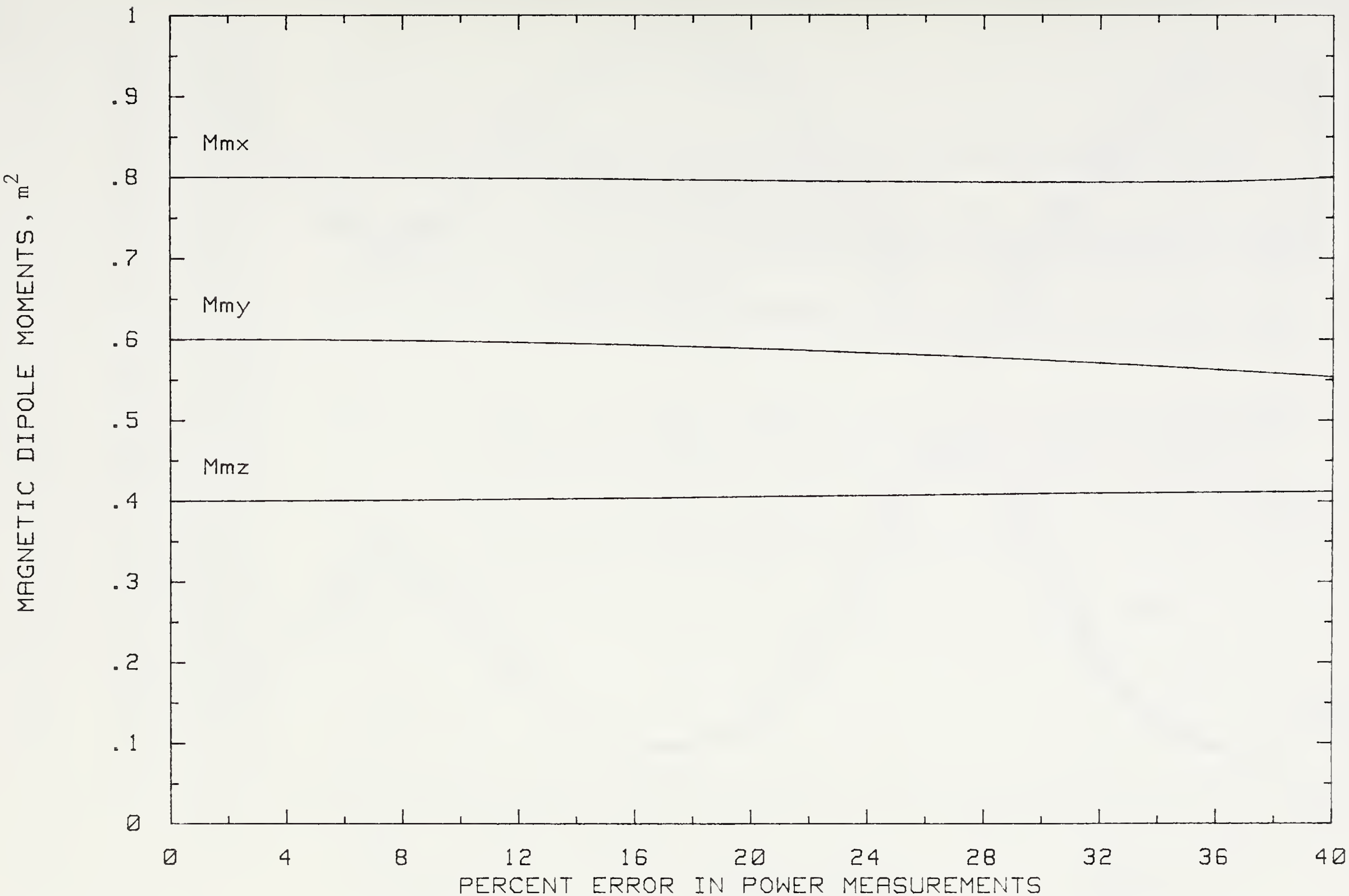

Figure 3b. Predicted magnetic dipole moments versus percent error in difference power measurements, as assumed in (105), p.19.

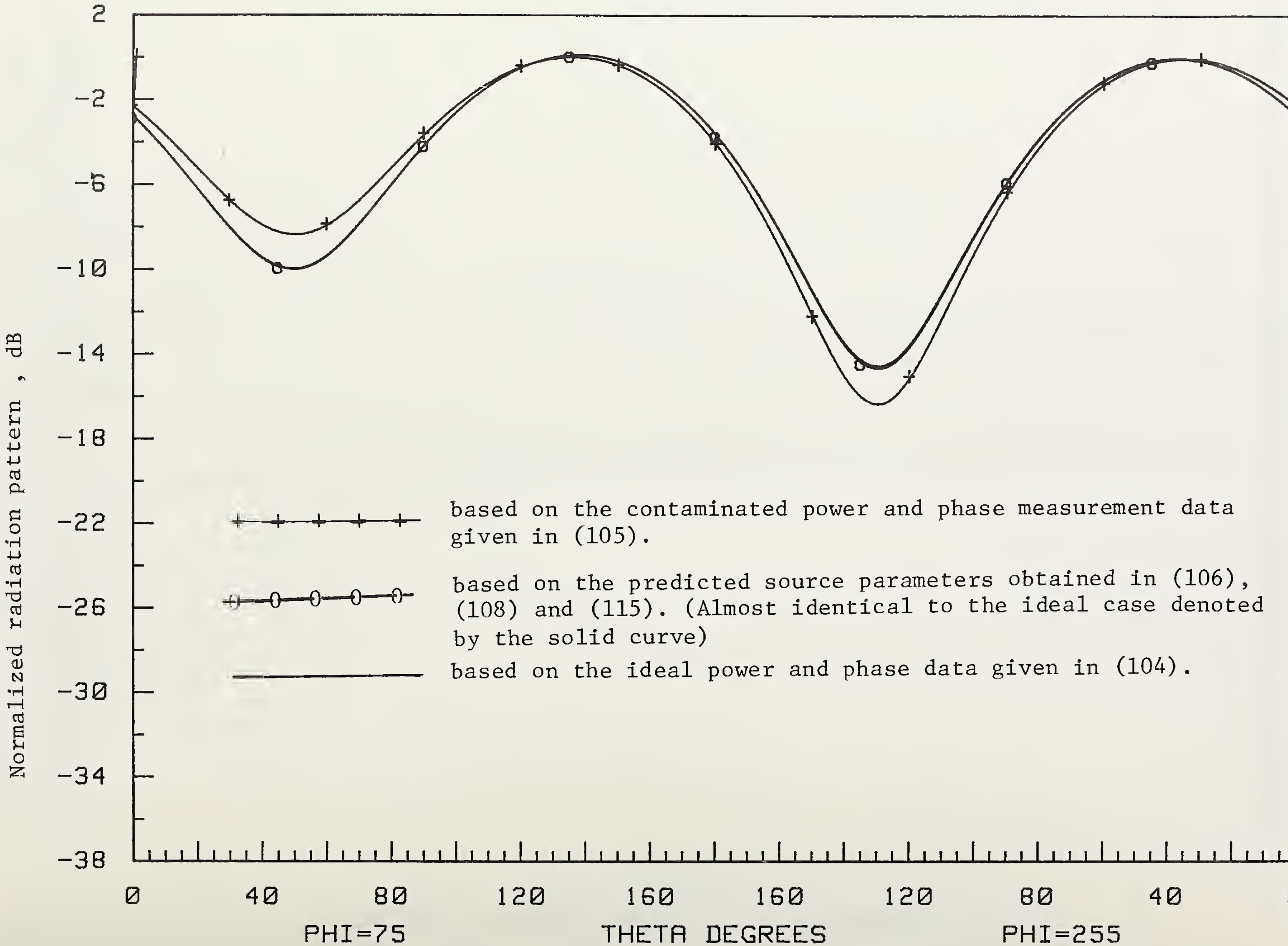

Figure 4. Normalized free-space radiation patterns of the interference source.

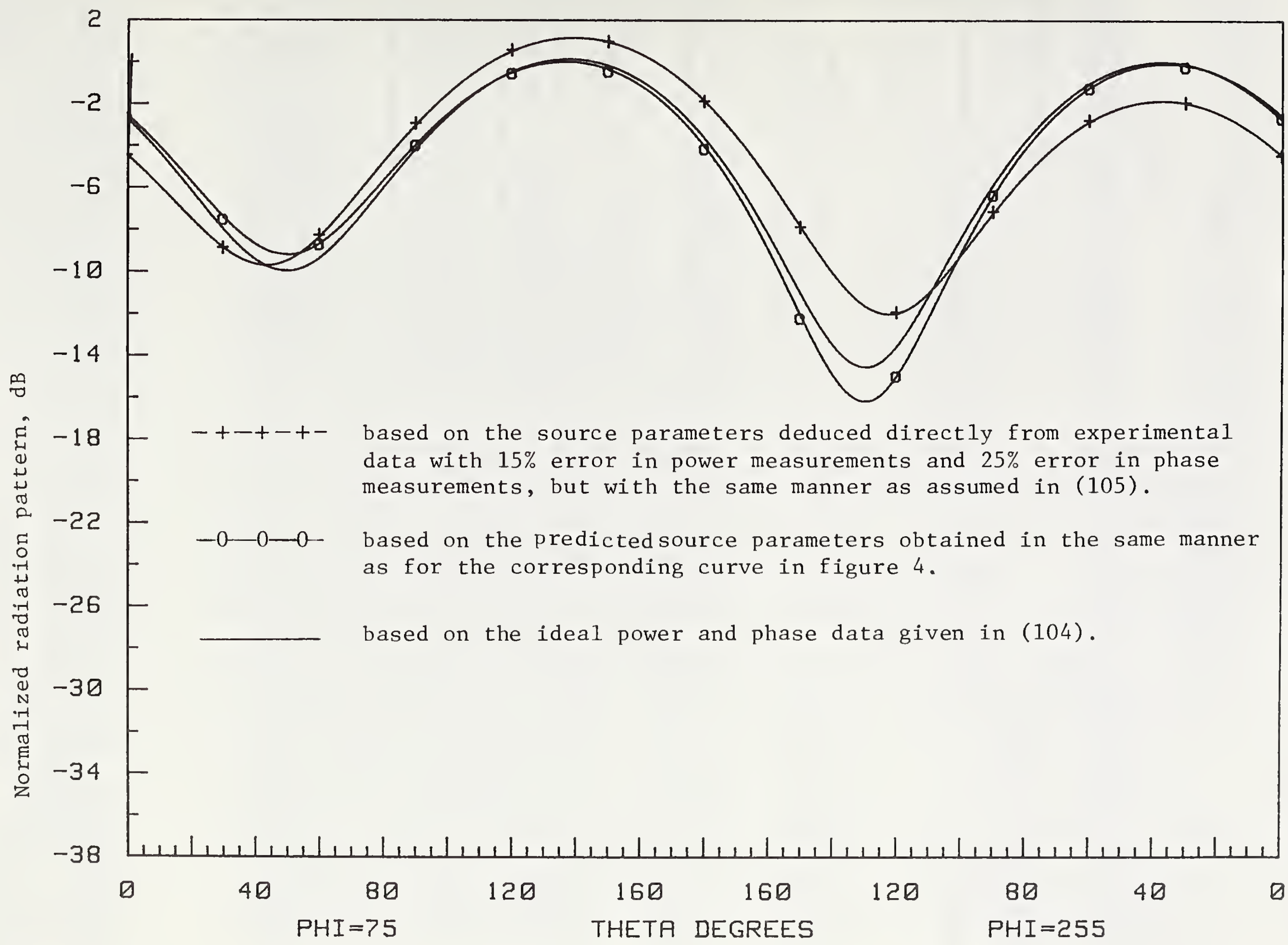

Figure 5. Normalized free-space radiation patterns of the interference source.

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5. AUTHOR(S)

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

A method for determining the radiation characteristics of a leaking interference source has been reported in a previous publication [1], in which the unintentional electrically small leakage source was modeled by two vectors representing a combination of equivalent electric and magnetic dipole moments. An experimental setup, measurement procedures, and the necessary theoretical basis were all described therein to explain how the relevant source parameters can be extracted from the measurement data of output powers and phases taken when the interference source is placed inside a transverse electromagnetic (TEM) cell. Simulated examples were also given to show that the equivalent source parameters of unknown vector dipole moments and thus the detailed radiation pattern and the total power radiated by the source in free space could be uniquely determined by the method if the measurement data were not contaminated by noise. This report presents the mathematical analysis of the uncertainties in the final, extracted results when the experimental data are degraded by the background noise and measurement inaccuracies.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

dipole moments; electrically small source; error analysis; interference sources; phase measurements; power measurements; radiation pattern; TEM cell; total radiated power; uncertainties.

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