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A Method to Quantify the Radiation Characteristics of an Unknown Interference Source

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A Method to Quantify the Radiation Characteristics of an Unknown Interference Source

Mark T. Ma
Galen H. Koepke

Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

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A Method to Quantify the Radiation Characteristics of an Unknown Interference Source

Mark T. Ma and Galen H. Koepke

Electromagnetic Fields Division
National Bureau of Standards
Boulder, Colorado 80303

A new method for determining the radiation characteristics of leakage from electronic equipment for interference studies is described in this report. Basically, an unintentional leakage source is considered to be electrically small, and may be characterized by three equivalent orthogonal electric dipole moments and three equivalent orthogonal magnetic dipole moments. When an unknown source object is placed at the center of a transverse electromagnetic (TEM) cell, its radiated energy couples into the fundamental transmission mode and propagates toward the two output ports of the TEM cell. With a hybrid junction inserted into a loop connecting the cell output ports, one is able to measure the sum and difference powers and the relative phase between the sum and difference outputs. Systematic measurements of these powers and phases at six different source object positions, based on a well-developed theory, are sufficient to determine the amplitudes and phases of the unknown component dipole moments, from which the detailed free-space radiation pattern of the unknown source and the total radiated power can be determined. Results of simulated theoretical examples and an experiment using a spherical dipole radiator are given to illustrate the theory and measurement procedure.

Key words: dipole moments; electrically small; interference source; leakage; phase measurements; power measurements; radiation pattern; TEM cell; total radiated power.

1. Introduction

As part of a continuing effort to devise measurement methods for quantifying radio frequency leakage from electronic equipment, a practical method has been developed. The measurement system shown in figure 1 uses a transverse electromagnetic (TEM) cell to isolate the equipment under test (EUT) from the environment while providing the coupling mechanism for the necessary power and phase measurements.

The premise for this approach is twofold. The first is that the cell is well constructed and operation is limited to the dominant TEM mode only. The second is that leakage currents on the exterior surface of the EUT may be modeled with equivalent electric and magnetic short dipole sources [1,2]. These dipoles may then be vectorially combined to yield a composite equivalent source consisting of three orthogonal electric and three orthogonal magnetic dipole moments as represented in figure 2.

It has been shown that the total power radiated in free space by the unknown leakage source under study can be determined with an experimental setup similar to that given in figure 1 by measuring the sum and difference powers [1,2]. The detailed radiation pattern, however, can be obtained only when the unknown source is characterized by equivalent three orthogonal electric dipole moments <u>or</u> three orthogonal magnetic dipole moments, <u>but not both</u>. The reason for this is clearly shown in the next section. For a general unknown radiator characterized by a composite source consisting of both electric and magnetic dipole moments, such as that represented in figure 2, the amplitudes and phases of the component dipole moments will be equally important for determining the radiation pattern for such a composite source. Hence, it is the objective of this report to describe additional required measurements and their theoretical justifications to determine uniquely the equivalent six unknown amplitudes and six unknown phases displayed in figure 2.

To avoid the complication of having to establish a phase reference physically connected to the EUT, the phase difference from sum to difference ports in figure 1, namely $\phi_\Sigma - \phi_\Delta$, is measured for each of the six EUT orientations required previously for power measurements. Thus, the measurement setup described in this report remains essentially the same as before [1,2] except that an additional instrument capable of measuring the relative sum-difference phase is inserted into the system.

For easy reference and clarity, the previous theoretical work is very briefly outlined in Section 2. It is there that the notation and definition of terms are also established. Section 3 describes the power and relative phase measurements and shows how the individual phase information associated with each component dipole moment (both types) can be extracted from these measurements. Specific results for two theoretically simulated examples and an experiment using a spherical dipole are presented in Section 4. A computer algorithm giving instructions of measurement sequences and numerical results is included as Appendix.

2. A Short Summary of Fundamental Theory for Determining the Radiation Characteristics of an Electrically Small Source

The electric and magnetic fields appearing at the output ports of a waveguide of arbitrary cross section, such as one half of a TEM cell shown in figure 3, generated by a current source located inside the waveguide may be expressed as [1,2]:

$$\bar{E}^{(+)} = \text{the vector electric field appearing at the right-hand port} \quad = \sum_n a_n \bar{E}_n^{(+)} \tag{1a}$$

$$\bar{E}^{(-)} = \text{the vector electric field appearing at the left-hand port} \quad = \sum_n b_n \bar{E}_n^{(-)} \tag{1b}$$

$$\bar{H}^{(+)} = \text{the vector magnetic field appearing at the right-hand port} \quad = \sum_n a_n \bar{H}_n^{(+)} \tag{1c}$$

$$\bar{H}^{(-)} = \text{the vector magnetic field appearing at the left-hand port} \quad = \sum_n b_n \bar{H}_n^{(-)} \tag{1d}$$

where $\bar{E}_n^{(\pm)}$ and $\bar{H}_n^{(\pm)}$ are respectively the vector orthonormal electric and magnetic basis functions describing the field structure for each of the n modes that can exist in the waveguide, and a_n and b_n are the expansion coefficients.

When the size of the waveguide cross section and operating frequency are such that only the dominant TEM mode (n = 0) exists, and the current source is placed at z = 0, we obtain, with an application of the Lorentz reciprocity theorem for perfectly conducting waveguide walls [1],

$$a_0 = b_0 = -\frac{1}{2}\bar{m}_e \cdot \bar{e}_0 \tag{2a}$$

when the source is a short infinitesimally thin current filament with an electric dipole moment $\bar{m}_e = J\,\overline{d\ell}$, where J is a normalized filament current and $\overline{d\ell}$ is the directed length vector of the short dipole; or

$$a_0 = -\,b_0 = -\,\frac{1}{2}jk\,(\bar{m}_m \times \bar{z}) \cdot \bar{e}_0 \tag{2b}$$

when the source is a small current loop with a magnetic dipole moment $\bar{m}_m = J'\overline{ds}$, where J' is the normalized loop current, $\overline{ds}$ is the vector loop area, k is the free space wave number, and $\bar{z}$ is the unit vector along the direction of propagation.

In (2a) and (2b), $\bar{e}_0$ is the normalized transverse vector electric field inside the TEM cell. Physically, it is the fundamental mode field generated at z = 0 when a power of one watt is supplied to the TEM cell. It has mostly the vertical component (y-directed) at the cell center. When the TEM cell size and frequency are specified, $\bar{e}_0$ in v/m can be computed theoretically [3].

When a general EUT is represented by a combination of both small electric and magnetic dipoles, the principle of superposition yields,

$$a_0 = -\,\frac{1}{2}\,(\bar{m}_e + jk\,\bar{M}) \cdot \bar{e}_0 \tag{3a}$$

and

$$b_0 = -\,\frac{1}{2}\,(\bar{m}_e - jk\,\bar{M}) \cdot \bar{e}_0 \tag{3b}$$

where

$$\bar{M} = \bar{m}_m \times \bar{z}\ . \tag{3c}$$

The unit for $\bar{m}_e$ is in meters (or amp-m normalized with respect to a unit current), and that for $\bar{m}_m$ is in meter-squares (or amp-m^2 normalized with respect to a unit current). The units for a_0 and b_0 are in volts.

Taking the sum and difference of (3a) and (3b), we obtain, for the sum power,

$$P_s = |a_0 + b_0|^2 = |\bar{m}_e \cdot \bar{e}_0|^2\ , \tag{4a}$$

and for the difference power,

$$P_d = |a_0 - b_0|^2 = k^2|\bar{M} \cdot \bar{e}_0|^2\ . \tag{4b}$$

Note that the sum power depends only on the electric dipole moment while the difference power depends only on the magnetic dipole moment. Note further that only the components of $\bar{m}_e$ and $\bar{M}$ that lie along the direction of the transverse vector field $\bar{e}_0$ contribute to the output. From the analysis point of view, when the current sources ($\bar{m}_e$ and $\bar{M}$) are specified, a_0 and b_0 can be calculated to give the field amplitudes at the cell output ports. Or equivalently, the sum and difference powers can be calculated from (4). From the synthesis point of view, P_s and P_d are obtained from measurements, and $\bar{m}_e$ and $\bar{M}$ can be determined from (4) to represent the unknown EUT. After $\bar{m}_e$ and $\bar{M}$ (and hence $\bar{m}_m$) are determined, the corresponding far-field power pattern radiated in free space may be written as [1]

$$
\begin{aligned}
P(\theta,\phi) = \frac{15\pi}{r^2\lambda^2} [& (m_{ex}^2 + k^2\, m_{mx}^2)\,(\cos^2\theta\,\cos^2\phi + \sin^2\phi) \\
& + (m_{ey}^2 + k^2\, m_{my}^2)\,(\cos^2\theta\,\sin^2\phi + \cos^2\phi) + (m_{ez}^2 + k^2\, m_{mz}^2)\,\sin^2\theta \\
& - 2\{m_{ex}m_{ey}\cos(\psi_{ex} - \psi_{ey}) + k^2 m_{mx}m_{my}\cos(\psi_{mx} - \psi_{my})\}\,\sin^2\theta\,\sin\phi\,\cos\phi \\
& - 2\{m_{ey}m_{ez}\cos(\psi_{ey} - \psi_{ez}) + k^2 m_{my}m_{mz}\cos(\psi_{my} - \psi_{mz})\}\,\sin\theta\,\cos\theta\,\sin\phi \\
& - 2\{m_{ez}m_{ex}\cos(\psi_{ez} - \psi_{ex}) + k^2 m_{mz}\, m_{mx}\cos(\psi_{mz} - \psi_{mx})\}\,\sin\theta\,\cos\theta\,\cos\phi \\
& + 2k\{m_{ex}m_{my}\,\sin(\psi_{ex} - \psi_{my}) - m_{ey}m_{mx}\,\sin(\psi_{ey} - \psi_{mx})\}\,\cos\theta \\
& + 2k\{m_{ey}m_{mz}\,\sin(\psi_{ey} - \psi_{mz}) - m_{ez}m_{my}\,\sin(\psi_{ez} - \psi_{my})\}\,\sin\theta\,\cos\phi \\
& + 2k\{m_{ez}m_{mx}\,\sin(\psi_{ez} - \psi_{mx}) - m_{ex}m_{mz}\,\sin(\psi_{ex} - \psi_{mz})\}\,\sin\theta\,\sin\phi] \;, \qquad (5)
\end{aligned}
$$

where m_{ex}, m_{ey}, and m_{ez} are the amplitudes of the three orthogonal components of the electric dipole moment $\bar{m}_e$, ψ_{ex}, ψ_{ey}, and ψ_{ez} are the phases associated with the components of $\bar{m}_e$; m_{mx}, m_{my}, m_{mz}, ψ_{mx}, ψ_{my}, and ψ_{mz} have similar meanings for the magnetic dipole moment $\bar{m}_m$; λ is the operating wavelength; r is the distance measured from the radiator; and θ and ϕ are the spherical coordinates relative to a chosen origin at the radiator.

The total radiated power is given by

$$P_T = \int_{4\pi} P(\theta,\phi)\,d\Omega$$

$$= \frac{40\pi^2}{\lambda^2} \{m_{ex}^2 + m_{ey}^2 + m_{ez}^2 + k^2 (m_{mx}^2 + m_{my}^2 + m_{mz}^2)\}$$

$$= \frac{40\pi^2}{\lambda^2} \left(|\bar{m}_e|^2 + k^2 |\bar{m}_m|^2 \right) . \tag{6}$$

It is clear that the total radiated power in (6) depends only on the amplitudes of dipole moments, and that the power pattern in (5) is much more involved.

Based on the above observation and various properties noted earlier for P_s and P_d, an experimental procedure was developed to measure the sum and difference powers for six different EUT positions, from which the total radiated power and the radiation pattern for an EUT made of one kind of dipole moment can be determined [1].

Outlining this method, we establish a coordinate system (x,y,z) with respect to the TEM cell with the origin at the geometric center of the cell, place the EUT at $(0, y_0, 0)$, assign another coordinate system (x', y', z') with respect to the center of the EUT. Initially, we align x-x', y-y', and z-z' as shown in figure 4a. We then rotate the EUT counterclockwise by an angle of $\pi/4$ about the z'-axis so that its position relative to the TEM cell can be seen in figure 4b, measure the sum and difference powers, and designate them respectively as P_{s1} and P_{d1}. Remembering the transverse electric field $\bar{e}_0 = p\bar{x} + q\bar{y}$, we see from (4) that only the x- and y-components of the "rotated" transverse electric field $\bar{e}_0' = \frac{1}{\sqrt{2}} (p+q)\bar{x} + \frac{1}{\sqrt{2}} (q-p)\bar{y}$ and the x- and y-components of $\bar{m}_e$ and $\bar{M}$ contribute to P_{s1} and P_{d1}.

We next rotate the EUT by an additional $\pi/2$, also counterclockwise about the z'-axis as displayed in figure 4c, and measure the sum and difference powers P_{s2} and P_{d2}. Clearly, this time $\bar{e}_0'' = \frac{1}{\sqrt{2}} (-p + q)\bar{x} - \frac{1}{\sqrt{2}} (p + q)\bar{y}$ and the x- and y-components of $\bar{m}_e$ and $\bar{M}$ make contribution to P_{s2} and P_{d2}.

We then align the coordinate frames such that x = y', y = z' and z = x' as shown in figure 5a. Now, we rotate the EUT counterclockwise by an angle of $\pi/4$ about the x'-axis to have a geometric condition given in figure 5b, proceed to make the sum and difference power measurements, and call them respectively P_{s3} and P_{d3}. Note the $\bar{e}_0'$ and the y- and z-components of $\bar{m}_e$ and $\bar{M}$ contribute to P_{s3} and P_{d3}. The EUT is then rotated counterclockwise by another $\pi/2$ about the x'-axis with its position displayed in figure 5c yielding measurements of P_{s4} and P_{d4} due to contributions by $\bar{e}_0''$ and the y- and z-components of the dipole moments.

Finally, we align the coordinate frames in accordance with x = z', y = x', and z = y', rotate in a similar manner as indicated in figures 6a, b, c, and measure P_{s5}, P_{d5}, P_{s6} and P_{d6}. Now, $\bar{e}_0'$ and the x- and z-components of $\bar{m}_e$ and $\bar{M}$ contribute to P_{s5} and P_{d5} while $\bar{e}_0''$ and the x- and z-components of $\bar{m}_e$ and $\bar{M}$ contribute to P_{s6} and P_{d6}.

After collecting the measured sum and difference powers, we obtain the following [1]:

$$\begin{pmatrix} m_{ex}^2 \\ m_{ey}^2 \\ m_{ez}^2 \end{pmatrix} = [C]\ [P_s]/2(p^2 + q^2) \tag{7a}$$

$$\begin{pmatrix} m_{mx}^2 \\ m_{my}^2 \\ m_{mz}^2 \end{pmatrix} = [C]\ [P_d]/2k^2(p^2 + q^2) \tag{7b}$$

$$\begin{pmatrix} m_{ex}m_{ey}\cos(\psi_{ex} - \psi_{ey}) \\ m_{ey}m_{ez}\cos(\psi_{ey} - \psi_{ez}) \\ m_{ez}m_{ex}\cos(\psi_{ez} - \psi_{ex}) \end{pmatrix} = [D]\ [P_s]/2(q^2 - p^2) \tag{7c}$$

and

$$\begin{pmatrix} m_{mx}m_{my}\cos(\psi_{mx} - \psi_{my}) \\ m_{my}m_{mz}\cos(\psi_{my} - \psi_{mz}) \\ m_{mz}m_{mx}\operatorname{cox}(\psi_{mz} - \psi_{mx}) \end{pmatrix} = [D]\ [P_d]/2k^2(q^2 - p^2)\ , \tag{7d}$$

where

$$[P_s] = \begin{pmatrix} P_{s1} \\ P_{s2} \\ P_{s3} \\ P_{s4} \\ P_{s5} \\ P_{s6} \end{pmatrix} \tag{7e}$$

$$[P_d] = \begin{pmatrix} P_{d2} \\ P_{d1} \\ P_{d4} \\ P_{d3} \\ P_{d6} \\ P_{d5} \end{pmatrix} \tag{7f}$$

$$[C] = \begin{pmatrix} 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{7g}$$

$$[D] = \begin{pmatrix} 1 & -1 & f & f & -f & -f \\ -f & -f & 1 & -1 & f & f \\ f & f & -f & -f & 1 & -1 \end{pmatrix} \tag{7h}$$

and

$$f = 2pq/(p^2 + q^2) \ . \tag{7i}$$

From (6) and (7a,b) we see why the sum and difference power measurements alone are sufficient to determine the total radiated power, in free space, of the unknown EUT. From (5) and (7c,d) we see why the radiation pattern can also be determined by the power measurements under the condition that the EUT may be characterized by either $\bar{m}_e$ or $\bar{m}_m$ (but not both) even though the phases associated with the component dipole moments have not been treated yet. For a general EUT consisting of a combination of both types of dipole moments, the phases as well as the amplitudes of the component dipole moments will be important for determining the radiation pattern in view of the last three lines in (5).

Before considering the phase measurements to be presented in the next section, we note from (7a) that the electric dipole moment components are related only to the sum powers, and from (7b) that the magnetic dipole moment components are related only to the difference powers. Thus, an observation of the relative values of the measured sum and difference powers may reveal the basic characteristics of the unknown EUT as to whether it is of electric type, magnetic type, or both.

3. Phase Considerations for Determining an Unknown General Source Consisting of Both Electric and Magnetic Dipole Moments

The results in (7c,d) contain limited phase information already. Once the amplitudes of the component electric dipole moments are obtained from (7a), application of (7c) yields

$$\psi_{ex} - \psi_{ey} = \cos^{-1}\left\{\frac{P_{s1} - P_{s2} + f(P_{s3} + P_{s4} - P_{s5} - P_{s6})}{2(q^2 - p^2)\, m_{ex}\, m_{ey}}\right\} \equiv \theta_{e1} \tag{8a}$$

$$\psi_{ey} - \psi_{ez} = \cos^{-1}\left\{\frac{P_{s3} - P_{s4} + f(-P_{s1} - P_{s2} + P_{s5} + P_{s6})}{2(q^2 - p^2)\, m_{ey}\, m_{ez}}\right\} \equiv \theta_{e2} \tag{8b}$$

and

$$\psi_{ez} - \psi_{ex} = \cos^{-1}\left[\frac{P_{s5} - P_{s6} + f(P_{s1} + P_{s2} - P_{s3} - P_{s4})}{2(q^2 - p^2)\, m_{ez}\, m_{ex}}\right] \equiv \theta_{e3} , \tag{8c}$$

where the results have been designated as θ_{ei}, i= 1,2, and 3.

Similarly, we obtain the following with the aid of (7b) and (7d):

$$\psi_{mx} - \psi_{my} = \cos^{-1}\left[\frac{-P_{d1} + P_{d2} + f(P_{d3} + P_{d4} - P_{d5} - P_{d6})}{2k^2(q^2 - p^2)\, m_{mx}\, m_{my}}\right] \equiv \theta_{m1} \tag{9a}$$

$$\psi_{my} - \psi_{mz} = \cos^{-1}\left[\frac{-P_{d3} + P_{d4} + f(-P_{d1} - P_{d2} + P_{d5} + P_{d6})}{2k^2(q^2 - p^2)\, m_{my}\, m_{mz}}\right] \equiv \theta_{m2} \tag{9b}$$

$$\psi_{mz} - \psi_{mx} = \cos^{-1}\left[\frac{-P_{d5} + P_{d6} + f(P_{d1} + P_{d2} - P_{d3} - P_{d4})}{2k^2(q^2 - p^2)\, m_{mx}\, m_{mz}}\right] \equiv \theta_{m3} . \tag{9c}$$

Note that (8) and (9) give some phase relationship between the component dipole moments of the <u>same kind only</u>. The phase relationship between the components of mixed types, which is also necessary for determining the power pattern in (5), has to be obtained by other means.

To avoid the difficulty of having to establish a phase reference with respect to a physical point on the EUT, and to keep the measurement setup simple, we only insert an additional instrument into the previous system [1,2] that has the ability to measure the relative phase between the sum and difference ports, as shown in figure 1. We will now present the necessary derivations to demonstrate that the relative sum and difference phases measured at the same six EUT orientations as those for measuring the sum and difference powers are sufficient to extract the individual phase information for the component dipole moments.

Referring to (3a), (3b), and figure 4b where the first sum and difference power measurements are made, we have, for the sum port,

$$\begin{aligned} a_0 + b_0 &= -\bar{m}_e \cdot \bar{e}_0' = -(\bar{x}\, m_{ex} e^{j\psi_{ex}} + \bar{y}\, m_{ey} e^{j\psi_{ey}}) \cdot (\bar{x}p' + \bar{y}q') \\ &= -(A_1 + jB_1) \quad , \end{aligned} \tag{10a}$$

where

$$A_1 = p' m_{ex} \cos\psi_{ex} + q' m_{ey} \cos\psi_{ey} \tag{10b}$$

$$B_1 = p' m_{ex} \sin\psi_{ex} + q' m_{ey} \sin\psi_{ey} \tag{10c}$$

$$p' = p\cos(\pi/4) + q\,\sin(\pi/4) = (p + q)/\sqrt{2} \tag{10d}$$

$$q' = q\cos(\pi/4) - p\,\sin(\pi/4) = (-p + q)/\sqrt{2} \tag{10e}$$

and p and q are respectively the x- and y-components of the normalized transverse electric field vector $\bar{e}_0$.

For the difference port, we have

$$\begin{aligned} a_0 - b_0 &= -jk\,\bar{M} \cdot \bar{e}_0' = -jk(\bar{m}_m \times \bar{z}) \cdot \bar{e}_0' \\ &= -k(C_1 + jD_1) \end{aligned} \tag{11a}$$

where

$$C_1 = q'm_{mx}\,\sin\psi_{mx} - p'm_{my}\,\sin\psi_{my} \tag{11b}$$

and

$$D_1 = -q'm_{mx}\,\cos\psi_{mx} + p'm_{my}\,\cos\psi_{my} \;. \tag{11c}$$

The relative phase between the sum and difference ports for this first measurement orientation is then,

$$\phi_1 = \tan^{-1}(B_1/A_1) - \tan^{-1}(D_1/C_1) \quad . \tag{12a}$$

When the relation $\psi_{ey} = \psi_{ex} - \theta_{e1}$ from (8a) is substituted above for A_1 and B_1, and $\psi_{mx} = \psi_{my} + \theta_{m1}$ from (9a) is substituted for C_1 and D_1, we obtain

$$\tan^{-1}(B_1/A_1) = \psi_{ex} - \alpha_1 \tag{12b}$$

and

$$\tan^{-1}(D_1/C_1) = \psi_{my} - \beta_1 \tag{12c}$$

where

$$\alpha_1 = \tan^{-1}\left[\frac{q'm_{ey}\,\sin\theta_{e1}}{p'm_{ex} + q'm_{ey}\,\cos\theta_{e1}}\right] \tag*{12d)}$$

and

$$\beta_1 = \tan^{-1}\left[\frac{q'm_{mx}\,\cos\theta_{m1} - p'm_{my}}{q'm_{mx}\,\sin\theta_{m1}}\right] \tag{12e}$$

Thus, equation (12a) becomes

$$\phi_1 = \psi_{ex} - \alpha_1 - (\psi_{my} - \beta_1) , \tag{12f}$$

or

$$\psi_{ex} - \psi_{my} = \phi_1 + \alpha_1 - \beta_1 . \tag{12g}$$

Note that α_1 and β_1 are obtainable from the measured sum and difference powers in accordance with (7a,b), (8a) and (9a). Since ϕ_1 is also available through a direct measurement, the mixed phase $\psi_{ex} - \psi_{my}$ required in (5) can then be obtained from (12g).

Alternatively, when the relation $\psi_{ex} = \psi_{ey} + \theta_{e1}$ and $\psi_{my} = \psi_{mx} - \theta_{m1}$ are used in (12a), we have

$$\phi_1 = \psi_{ey} - \alpha_1' - (\psi_{mx} - \beta_1') , \tag{12h}$$

or

$$\psi_{ey} - \psi_{mx} = \phi_1 + \alpha_1' - \beta_1' , \tag{12i}$$

where

$$\alpha_1' = \tan^{-1}\left[\frac{-p'm_{ex}\sin\theta_{e1}}{p'm_{ex}\cos\theta_{e1} + q'm_{ey}}\right] \tag{12j}$$

and

$$\beta_1' = \tan^{-1}\left[\frac{q'm_{mx} - p'm_{my}\cos\theta_{m1}}{p'm_{my}\sin\theta_{m1}}\right] \tag{12k}$$

Taking the difference between (12g) and (12i), and using (8a) and (9a) yields

$$\theta_{e1} + \theta_{m1} = \alpha_1 - \beta_1 - (\alpha_1' - \beta_1') \tag{12ℓ}$$

which may serve as a useful check to determine data consistency.

The truth of (12ℓ) may also be evidenced by showing

$$\alpha_1 - \alpha_1' = \theta_{e1} \tag{12m}$$

from (12d,j), and

$$\beta_1' - \beta_1 = \theta_{m1} \tag{12n}$$

from (12e,k).

Clearly, either the set of (12d,e,g) or that of (12i,j,k) is necessary from the computation point of view, provided, of course, that a measured ϕ_1 is available. This means that we may either compute the relative mixed phase $\psi_{ex} - \psi_{my}$ from (12d,e,g) and then obtain its counterpart $\psi_{ey} - \psi_{mx}$ also required in (5) by

$$\psi_{ey} - \psi_{mx} = \psi_{ex} - \psi_{my} - \theta_{e1} - \theta_{m1} , \tag{12o}$$

or compute $\psi_{ey} - \psi_{mx}$ from (12i,j,k) and then obtain $\psi_{ex} - \psi_{my}$ by

$$\psi_{ex} - \psi_{my} = \psi_{ey} - \psi_{mx} + \theta_{e1} + \theta_{m1} . \tag{12p}$$

When the measured quantities (sum and difference powers, and the relative phase ϕ_1 between the sum and difference ports) are not contaminated by noise, there will be no inaccuracy contained in the computed values. The computation from either of the above two sets is then equally good and yields identical results. However, when the computed values are relatively inaccurate because of the noisy measured quantities, more reasonable results may be obtained from one set of computations than the other, as will be evident later in Example 3. For this reason only, the derivations for both sets of computations are presented above.

At the second measurement orientation shown in figure 4c, we have

$$a_0 + b_0 = -\bar{m}_e \cdot \bar{e}_0'' = -(A_2 + jB_2) \tag{13a}$$

$$a_0 - b_0 = -jk(\bar{m}_m \times \bar{z}) \cdot \bar{e}_0'' = -k(C_2 + jD_2) , \tag{13b}$$

where

$$A_2 = q'm_{ex}\cos\psi_{ex} - p'm_{ey} \cos\psi_{ey} \tag{13c}$$

$$B_2 = q'm_{ex} \sin\psi_{ex} - p'm_{ey} \sin\psi_{ey} \tag{13d}$$

$$C_2 = p'm_{mx} \sin\psi_{mx} - q'm_{my} \sin\psi_{my} \tag{13e}$$

and

$$D_2 = p'm_{mx} \cos\psi_{mx} + q'm_{my} \cos\psi_{my} . \tag{13f}$$

Note that A_2, B_2, C_2 and D_2 above can be obtained from (10b,c) and (11b,c) merely by replacing p' by q' and q' by -p' in the latter expressions. This is so because the measurement orientation for EUT in figure 4c differs by 90° from that in figure 4b. Thus, the relative sum-to-difference phase now becomes

$$\phi_2 = \tan^{-1}(B_2/A_2) - \tan^{-1}(D_2/C_2) , \tag{14a}$$

which is of the same form as that in (12a) except a change in subscript from 1 to 2.

The same application of $\psi_{ey} = \psi_{ex} - \theta_{e1}$ and $\psi_{mx} = \psi_{my} + \theta_{m1}$ in (14a) yields

$$\tan^{-1}(B_2/A_2) = \psi_{ex} - \alpha_2 \quad , \tag{14b}$$

$$\tan^{-1}(D_2/C_2) = \psi_{my} - \beta_2 \ , \tag{14c}$$

$$\phi_2 = \psi_{ex} - \alpha_2 - (\psi_{my} - \beta_2) \quad , \tag{14d}$$

or

$$\psi_{ex} - \psi_{my} = \phi_2 + \alpha_2 - \beta_2 \quad , \tag{14e}$$

where

$$\alpha_2 = \tan^{-1}\left[\frac{-p'm_{ey}\ \sin\theta_{e1}}{q'm_{ex} - p'm_{ey}\ \cos\theta_{e1}}\right] , \tag{14f}$$

and

$$\beta_2 = \tan^{-1}\left[\frac{-p'm_{mx}\ \cos\theta_{m1} - q'm_{my}}{-p'm_{mx}\ \sin\theta_{m1}}\right] . \tag{14g}$$

Note that (14e) and (12g) should give an identical result for $\psi_{ex} - \psi_{my}$ if no error is present in the measured quantities. When errors are present because of the noisy measurement background, inaccurate readings of the instruments, or other reasons, the results from (12g) and (14e) are no longer identical. The difference between them may be indicative of the quality of the measured quantities involved. In practise, we may be forced to take the average of them as the final result for $\psi_{ex} - \psi_{my}$.

Alternatively, we obtain the following, if the relations $\psi_{ex} = \psi_{ey} + \theta_{e1}$ and $\psi_{my} = \psi_{mx} - \theta_{m1}$ are used in (14a),

$$\phi_2 = \psi_{ey} - \alpha_2' - (\psi_{mx} - \beta_2') \quad , \tag{14h}$$

or

$$\psi_{ey} - \psi_{mx} = \phi_2 + \alpha_2' - \beta_2' \quad , \tag{14i}$$

where

$$\alpha_2' = \tan^{-1}\left[\frac{-q'm_{ex}\ \sin\theta_{e1}}{q'm_{ex}\ \cos\theta_{e1} - p'm_{ey}}\right] \quad , \tag{14j}$$

and

$$\beta_2' = \tan^{-1}\left[\frac{-p'm_{mx} - q'm_{my}\ \cos\theta_{m1}}{q'm_{my}\ \sin\theta_{m1}}\right] \quad . \tag{14k}$$

The identities,

$$\alpha_2 - \alpha_2' = \theta_{e1} \tag{14ℓ}$$

and

$$\beta_2' - \beta_2 = \theta_{m1} \tag{14m}$$

may also be useful for the purpose of checking data consistency.

Furthermore, the equality of (12g) and (14e) also helps to establish the following important phase relationships between θ_{e1} and θ_{m1}:

$$\sin\theta_{m1} = M_{12}/N_{12} \tag{15a}$$

or

$$\sin\theta_{e1} = M_{12}'/N_{12}' \quad , \tag{15b}$$

where

$$\begin{aligned} M_{12} = {} & 2(m_{mx}^2 - m_{my}^2)\, m_{ex} m_{ey} \sin\theta_{e1} \\ & + (m_{ex}^2 - m_{ey}^2)(m_{mx}^2 - m_{my}^2)\tan(\phi_1 - \phi_2) \end{aligned} \tag{15c}$$

$$\begin{aligned} N_{12} = {} & -2(m_{ex}^2 - m_{ey}^2)\, m_{mx} m_{my} \\ & + 4 m_{ex} m_{ey} m_{mx} m_{my} \tan(\phi_1 - \phi_2) \sin\theta_{e1} \end{aligned} \tag{15d}$$

$$\begin{aligned} M_{12}' = {} & 2(m_{ex}^2 - m_{ey}^2)\, m_{mx} m_{my} \sin\theta_{m1} \\ & + (m_{ex}^2 - m_{ey}^2)(m_{mx}^2 - m_{my}^2)\tan(\phi_1 - \phi_2) \end{aligned} \tag{15e}$$

and

$$\begin{aligned} N_{12}' = {} & -2(m_{mx}^2 - m_{my}^2)\, m_{ex} m_{ey} \\ & + 4\, m_{ex} m_{ey} m_{mx} m_{my} \tan(\phi_1 - \phi_2) \sin\theta_{m1} \quad . \end{aligned} \tag{15f}$$

The importance of (15a,b) can be seen when we refer to (8) and (9). In addition to the obvious constraints,

$$\theta_{e1} + \theta_{e2} + \theta_{e3} = 0 \tag{16a}$$

and

$$\theta_{m1} + \theta_{m2} + \theta_{m3} = 0 \quad , \tag{16b}$$

there is still a sign ambiguity for θ_{ei} and θ_{mi}, i = 1,2,3, based on the power measurement data and the operation of taking inverse cosines. Equation (15a) or (15b) will solve this problem of sign ambiguity in the sense that only one particular choice of sign set will satisfy (15a) or (15b).

Similarly, the other four EUT orientations discussed in Section 2 yield:

$$\phi_i = \tan^{-1}(B_i/A_i) - \tan^{-1}(D_i/C_i) \quad , \quad i = 3,4,5,6 \tag{17}$$

where A_3, B_3, C_3 and D_3 may be obtained from (10b,c) and (11b,c) when the subscript x in the latter expressions is replaced by y and y by z; A_4, B_4, C_4 and D_4 from (13c,d,e,f) by doing the same changes in suscripts; A_5, B_5, C_5 and D_5 from (10b,c), (11b,c) and A_6, B_6, C_6 and D_6 from (13c,d,e,f) by changing the subscript x to z and y to x.

More explicitly, we have

$$A_3 = p'm_{ey} \cos\psi_{ey} + q'm_{ez} \cos\psi_{ez} \ , \tag{18a}$$

$$B_3 = p'm_{ey} \sin\psi_{ey} + q'm_{ez} \sin\psi_{ez} \ , \tag{18b}$$

$$C_3 = q'm_{my} \sin\psi_{my} - p'm_{mz} \sin\psi_{mz} \ , \tag{18c}$$

$$D_3 = -q'm_{my} \cos\psi_{my} + p'm_{mz} \cos\psi_{mz} \ ; \tag{18d}$$

$$A_4 = q'm_{ey} \cos\psi_{ey} - p'm_{ez} \cos\psi_{ez} \ , \tag{19a}$$

$$B_4 = q'm_{ey} \sin\psi_{ey} - p'm_{ez} \sin\psi_{ez} \ , \tag{19b}$$

$$C_4 = -p'm_{my} \sin\psi_{my} - q'm_{mz} \sin\psi_{mz} \ , \tag{19c}$$

$$D_4 = p'm_{my} \cos\psi_{my} + q'm_{mz} \cos\psi_{mz} \ ; \tag{19d}$$

$$A_5 = p'm_{ez} \cos\psi_{ez} + q'm_{ex} \cos\psi_{ex} \ , \tag{20a}$$

$$B_5 = p'm_{ez} \sin\psi_{ez} + q'm_{ex} \sin\psi_{ex} \ , \tag{20b}$$

$$C_5 = q'm_{mz} \sin\psi_{mz} - p'm_{mx} \sin\psi_{mx} \ , \tag{20c}$$

$$D_5 = -q'm_{mz} \cos\psi_{mz} + p'm_{mx} \cos\psi_{mx} \; ; \tag{20d}$$

and

$$A_6 = q'm_{ez} \cos\psi_{ez} - p'm_{ex} \cos\psi_{ex} \; , \tag{21a}$$

$$B_6 = q'm_{ez} \sin\psi_{ez} - p'm_{ex} \sin\psi_{ex} \; , \tag{21b}$$

$$C_6 = -p'm_{mz} \sin\psi_{mz} - q'm_{mx} \sin\psi_{mx} \; , \tag{21c}$$

$$D_6 = p'm_{mz} \cos\psi_{mz} + q'm_{mx} \cos\psi_{mx} \; . \tag{21d}$$

With the substitution of $\psi_{ez} = \psi_{ey} - \theta_{e2}$ from (8b) and $\psi_{my} = \psi_{mz} + \theta_{m2}$ from (9b) in (17) for i = 3 and 4, we obtain:

$$\phi_3 = \psi_{ey} - \alpha_3 - (\psi_{mz} - \beta_3) \tag{22a}$$

and

$$\phi_4 = \psi_{ey} - \alpha_4 - (\psi_{mz} - \beta_4) \quad ; \tag{22b}$$

yielding

$$\psi_{ey} - \psi_{mz} = \phi_3 + \alpha_3 - \beta_3 = \phi_4 + \alpha_4 - \beta_4 \quad , \tag{22c}$$

where

$$\alpha_3 = \tan^{-1}\left(\frac{q'm_{ez} \sin\theta_{e2}}{p'm_{ey} + q'm_{ez} \cos\theta_{e2}}\right), \tag{22d}$$

$$\beta_3 = \tan^{-1}\left(\frac{q'm_{my} \cos\theta_{m2} - p'm_{mz}}{q'm_{my} \sin\theta_{m2}}\right), \tag{22e}$$

$$\alpha_4 = \tan^{-1}\left(\frac{-p'm_{ez} \sin\theta_{e2}}{q'm_{ey} - p'm_{ez} \cos\theta_{e2}}\right), \tag{22f}$$

and

$$\beta_4 = \tan^{-1}\left(\frac{-p'm_{my} \cos\theta_{m2} - q'm_{mz}}{-p'm_{my} \sin\theta_{m2}}\right), \tag{22g}$$

In addition, equation (22c) also yields a relationship between θ_{e2} and θ_{m2},

$$\sin\theta_{m2} = M_{34}/N_{34} \tag{23a}$$

or

$$\sin\theta_{e2} = M'_{34}/N'_{34} \quad , \tag{23b}$$

where

$$\begin{aligned} M_{34} &= 2(m_{my}^2 - m_{mz}^2)\, m_{ey} m_{ez} \sin\theta_{e2} \\ &\quad + (m_{ey}^2 - m_{ez}^2)\,(m_{my}^2 - m_{mz}^2) \tan(\phi_3 - \phi_4) \quad , \end{aligned} \tag{23c}$$

$$\begin{aligned} N_{34} &= -2(m_{ey}^2 - m_{ez}^2)\, m_{my} m_{mz} \\ &\quad + 4 m_{ey} m_{ez} m_{my} m_{mz} \tan(\phi_3 - \phi_4) \sin\theta_{e2} \quad , \end{aligned} \tag{23d}$$

$$\begin{aligned} M'_{34} &= 2(m_{ey}^2 - m_{ez}^2)\, m_{my} m_{mz} \sin\theta_{m2} \\ &\quad + (m_{ey}^2 - m_{ez}^2)\,(m_{my}^2 - m_{mz}^2) \tan(\phi_3 - \phi_4) \;, \end{aligned} \tag{23e}$$

and

$$\begin{aligned} N'_{34} &= -2(m_{my}^2 - m_{mz}^2)\, m_{ey} m_{ez} \\ &\quad + 4\, m_{ey} m_{ez} m_{my} m_{mz} \tan(\phi_3 - \phi_4) \sin\theta_{m2} \quad . \end{aligned} \tag{23f}$$

Alternatively, with the substitution of $\psi_{ey} = \psi_{ez} + \theta_{e2}$ and $\psi_{mz} = \psi_{my} - \theta_{m2}$ in (17) for i = 3 and 4, we obtain:

$$\phi_3 = \psi_{ez} - \alpha'_3 - (\psi_{my} - \beta'_3) \tag{24a}$$

and

$$\phi_4 = \psi_{ez} - \alpha'_4 - (\psi_{my} - \beta'_4) \quad ; \tag{24b}$$

yielding

$$\psi_{ez} - \psi_{my} = \phi_3 + \alpha'_3 - \beta'_3 = \phi_4 + \alpha'_4 - \beta'_4 \quad , \tag{24c}$$

where

$$\alpha'_3 = \tan^{-1}\left[\frac{-p' m_{ey} \sin\theta_{e2}}{p' m_{ey} \cos\theta_{e2} + q' m_{ez}}\right] , \tag{24d}$$

$$\beta'_3 = \tan^{-1}\left[\frac{q' m_{my} - p' m_{mz} \cos\theta_{m2}}{p' m_{mz} \sin\theta_{m2}}\right] , \tag{24e}$$

$$\alpha_4' = \tan^{-1}\left(\frac{-q'm_{ey}\sin\theta_{e2}}{q'm_{ey}\cos\theta_{e2} - p'm_{ez}}\right), \tag{24f}$$

and

$$\beta_4' = \tan^{-1}\left(\frac{-p'm_{my} - q'm_{mz}\cos\theta_{m2}}{q'm_{mz}\sin\theta_{m2}}\right). \tag{24g}$$

Of course, the second relation in (24c) yields the same relationship between θ_{e2} and θ_{m2} as those in (23a,b). The following identities,

$$\alpha_3 - \alpha_3' = \alpha_4 - \alpha_4' = \theta_{e2} \tag{24h}$$

and

$$\beta_3' - \beta_3 = \beta_4' - \beta_4 = \theta_{m2} \tag{24i}$$

may be useful for checking the correctness of various quantities.

Applying the relationships of $\psi_{ex} = \psi_{ez} - \theta_{e3}$ from (8c) and $\psi_{mz} = \psi_{mx} + \theta_{m3}$ from (9c) to (17) with i = 5 and 6, we obtain:

$$\phi_5 = \psi_{ez} - \alpha_5 - (\psi_{mx} - \beta_5) , \tag{25a}$$

and

$$\phi_6 = \psi_{ez} - \alpha_6 - (\psi_{mx} - \beta_6) , \tag{25b}$$

yielding

$$\psi_{ez} - \psi_{mx} = \phi_5 + \alpha_5 - \beta_5 = \phi_6 + \alpha_6 - \beta_6 , \tag{25c}$$

where

$$\alpha_5 = \tan^{-1}\left(\frac{q'm_{ex}\sin\theta_{e3}}{p'm_{ez} + q'm_{ex}\cos\theta_{e3}}\right) , \tag{25d}$$

$$\beta_5 = \tan^{-1}\left(\frac{q'm_{mz}\cos\theta_{m3} - p'm_{mx}}{q'm_{mz}\sin\theta_{m3}}\right) , \tag{25e}$$

$$\alpha_6 = \tan^{-1}\left(\frac{-p'm_{ex}\sin\theta_{e3}}{q'm_{ez} - p'm_{ex}\cos\theta_{e3}}\right) , \tag{25f}$$

and

$$\beta_6 = \tan^{-1}\left(\frac{-p'm_{mz}\cos\theta_{m3} - q'm_{mx}}{-p'm_{mz}\sin\theta_{m3}}\right) . \tag{25g}$$

Also, equation (25c) yields a relationship between θ_{e3} and θ_{m3} ,

$$\sin\theta_{m3} = M_{56}/N_{56} \tag{26a}$$

or

$$\sin\theta_{e3} = M'_{56}/N'_{56} \quad , \tag{26b}$$

where

$$\begin{aligned} M_{56} &= 2(m_{mz}^2 - m_{mx}^2)m_{ez}m_{ex}\sin\theta_{e3} \\ &\quad + (m_{ez}^2 - m_{ex}^2)(m_{mz}^2 - m_{mx}^2)\tan(\phi_5 - \phi_6) \quad , \end{aligned} \tag{26c}$$

$$N_{56} = -2(m_{ez}^2 - m_{ex}^2)m_{mz}m_{mx} + 4m_{ez}m_{ex}m_{mz}m_{mx}\tan(\phi_5 - \phi_6)\sin\theta_{e3} \quad , \tag{26d}$$

$$\begin{aligned} M'_{56} &= 2(m_{ez}^2 - m_{ex}^2)m_{mz}m_{mx}\sin\theta_{m3} \\ &\quad + (m_{ez}^2 - m_{ex}^2)(m_{mz}^2 - m_{mx}^2)\tan(\phi_5 - \phi_6) \end{aligned} \tag{26e}$$

and

$$N'_{56} = -2(m_{mz}^2 - m_{mx}^2)m_{ez}m_{ex} + 4m_{ez}m_{ex}m_{mz}m_{mx}\tan(\phi_5 - \phi_6)\sin\theta_{m3} \quad . \tag{26f}$$

Alternatively, if we use $\psi_{ez} = \psi_{ex} + \theta_{e3}$ and $\psi_{mx} = \psi_{mz} - \theta_{m3}$ in (17) for i = 5 and 6, we obtain:

$$\phi_5 = \psi_{ex} - \alpha'_5 - (\psi_{mz} - \beta'_5) \quad , \tag{27a}$$

and

$$\phi_6 = \psi_{ex} - \alpha'_6 - (\psi_{mz} - \beta'_6) \quad ; \tag{27b}$$

yielding

$$\psi_{ex} - \psi_{mz} = \phi_5 + \alpha'_5 - \beta'_5 = \phi_6 + \alpha'_6 - \beta'_6 \quad , \tag{27c}$$

where

$$\alpha'_5 = \tan^{-1}\left(\frac{-p'm_{ez}\sin\theta_{e3}}{p'm_{ez}\cos\theta_{e3} + q'm_{ex}}\right), \tag{27d}$$

$$\beta'_5 = \tan^{-1}\left(\frac{q'm_{mz} - p'm_{mx}\cos\theta_{m3}}{p'm_{mx}\sin\theta_{m3}}\right), \tag{27e}$$

$$\alpha'_6 = \tan^{-1}\left(\frac{-q'm_{ez}\sin\theta_{e3}}{q'm_{ez}\cos\theta_{e3} - p'm_{ex}}\right), \tag{27f}$$

and

$$\beta_6' = \tan^{-1}\left(\frac{-p' m_{mz} - q' m_{mx} \cos\theta_{m3}}{q' m_{mx} \sin\theta_{m3}} \right) . \tag{27g}$$

Of course, equation (27c) yields the same relationship between θ_{e3} and θ_{m3} as given in (26a, b). The following identities for possible checking purpose are also obvious,

$$\alpha_5 - \alpha_5' = \alpha_6 - \alpha_6' = \theta_{e3} , \tag{27h}$$

and

$$\beta_5' - \beta_5 = \beta_6' - \beta_6 = \theta_{m3} . \tag{27i}$$

In addition, from (12f), (14d), (22a,b) and (25a,b), we have another useful checking relationship:

$$\begin{aligned} \phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6 = {} & \alpha_2 + \alpha_4 + \alpha_6 - (\alpha_1 + \alpha_3 + \alpha_5) \\ & + \beta_1 + \beta_3 + \beta_5 - (\beta_2 + \beta_4 + \beta_6) \quad . \end{aligned} \tag{27j}$$

Similarly, from (12h), (14h), (24a,b) and (27a,b), we also have

$$\begin{aligned} \phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6 = {} & \alpha_2' + \alpha_4' + \alpha_6' - (\alpha_1' + \alpha_3' + \alpha_5') \\ & + \beta_1' + \beta_3' + \beta_5' - (\beta_2' + \beta_4' + \beta_6') \quad . \end{aligned} \tag{27k}$$

Thus, the phases obtained in (12g) [or (14e)], (12i) [or (14i)], (22c), (24c), (25c) and (27c) together with (7a-d) derived previously constitute the complete set of information characterizing an unknown emitter being investigated, from which the detailed radiation pattern may be plotted in accordance with (5).

After the rather lengthy formal derivations presented above, it is instructive to summarize a few points. (i) The sum and difference power measurements taken at the proposed six EUT positions are sufficient to determine the equivalent six unknown dipole moments amplitude (m_{ex}, m_{ey}, m_{ez}; m_{mx}, m_{my}, m_{mz}), and the total power radiated by the unknown EUT in free space. (ii) The same power measurements are not enough to determine the detailed radiation pattern if the EUT is made of both types of dipole moments. To achieve this objective for the general case, the relative phase measurements between the sum and difference ports taken at the six prescribed EUT positions are required. (iii) An examination of the power pattern expression given by (5) shows that all phase terms appear as relative quantities (i.e., $\psi_{ex} - \psi_{my}$). This permits specifying one phase angle as a reference, assigning an arbitrary value to it, and resolving the five remaining angles relative to the chosen reference. Of course, the reference value assigned will have no effect on the final radiation power pattern. (iv) To extract the remaining five relative phase angles from the measured power and phase data (P_{si}, P_{di}, ϕ_i, $i=1,2,\ldots,6$), after choosing one of the six phases ψ_{ex}, ψ_{ey}, ψ_{ez}; ψ_{mx}, ψ_{my}, ψ_{mz} as a reference, it is necessary to use:

(A) One set of three equations from either (8) or (9) with the constraints given by (16a) or (16b),

(B) (12g) or (14e),

(C) one of the relations in (22c)

and

(D) one of the relations in (25c)

for a total of five independent equations. The sign ambiguity in (8) or (9) is solved by (15a), (23a), and (26a) [or (15b), (23b), and (26b)].

Three examples are given in the next section to illustrate the above summary.

4. Illustrative Examples

Since the equations required for extracting the unknown phases are quite involved, we wish first to verify their usefulness and validity by two simulated theoretical examples so that the problem is not futher complicated by practical considerations such as measurement inaccuracy caused by the background noise and imperfect readings of the instrument.

Example 1. Suppose the equivalent electric and magnetic dipole moments in meters and square-meters respectively and phases in degrees representing an unknown EUT are:

$$m_{ex} = 1.4, \quad m_{ey} = 1.8, \quad m_{ez} = 1.6, \quad \psi_{ex} = 0, \quad \psi_{ey} = 80, \quad \psi_{ez} = 60 \;;$$
$$m_{mx} = 0.8, \quad m_{my} = 0.6, \quad m_{mz} = 0.4, \quad \psi_{mx} = -80, \quad \psi_{my} = -60, \quad \psi_{mz} = -45 \;.$$

In addition, suppose the particular TEM cell used has a cross-sectional area of 1.2mX1.2m so that at the frequency of 30 MHz, only the dominant TEM mode can propagate and the vector transverse electric field is computed as $\bar{e}_0 = \bar{x}p + \bar{y}q$, where $p = 0$ and $q = 11.83$ v/m [3].

In accordance with the material presented in Section 2, we have the following sum and difference powers,

$$P_{s1} = \frac{1}{2}q^2[m_{ex}^2 + m_{ey}^2 + 2m_{ex}m_{ey}\cos(\psi_{ex} - \psi_{ey})] = 425.107856 \;, \tag{28a}$$

$$P_{s2} = \frac{1}{2}q^2[m_{ex}^2 + m_{ey}^2 - 2m_{ex}m_{ey}\cos(\psi_{ex} - \psi_{ey})] = 302.626424 \;, \tag{28b}$$

$$P_{s3} = \frac{1}{2}q^2[m_{ey}^2 + m_{ez}^2 + 2m_{ey}m_{ez}\cos(\psi_{ey} - \psi_{ez})] = 784.597583 \;, \tag{28c}$$

$$P_{s4} = \frac{1}{2}q^2[m_{ey}^2 + m_{ez}^2 - 2m_{ey}m_{ez}\cos(\psi_{ey} - \psi_{ez})] = 27.106038 , \quad (28d)$$

$$P_{s5} = \frac{1}{2}q^2[m_{ez}^2 + m_{ex}^2 + 2m_{ez}m_{ex}\cos(\psi_{ez} - \psi_{ex})] = 473.027282 , \quad (28e)$$

$$P_{s6} = \frac{1}{2}q^2[m_{ez}^2 + m_{ex}^2 - 2m_{ez}m_{ex}\cos(\psi_{ez} - \psi_{ex})] = 159.541746 , \quad (28f)$$

$$P_{d1} = \frac{1}{2}k^2q^2[m_{mx}^2 + m_{my}^2 - 2m_{mx}m_{my}\cos(\psi_{mx} - \psi_{my})] = 2.704333 , \quad (28g)$$

$$P_{d2} = \frac{1}{2}k^2q^2[m_{mx}^2 + m_{my}^2 + 2m_{mx}m_{my}\cos(\psi_{mx} - \psi_{my})] = 52.545279 , \quad (28h)$$

$$P_{d3} = \frac{1}{2}K^2q^2[m_{my}^2 + m_{mz}^2 - 2m_{my}m_{mz}\cos(\psi_{my} - \psi_{mz})] = 1.556813 , \quad (28i)$$

$$P_{d4} = \frac{1}{2}k^2q^2[m_{my}^2 + m_{mz}^2 + 2m_{my}m_{mz}\cos(\psi_{my} - \psi_{mz})] = 27.172985 , \quad (28j)$$

$$P_{d5} = \frac{1}{2}k^2q^2[m_{mz}^2 + m_{mx}^2 - 2m_{mz}m_{mx}\cos(\psi_{mz} - \psi_{mx})] = 7.617338 , \quad (28k)$$

and

$$P_{d6} = \frac{1}{2}k^2q^2[m_{mz}^2 + m_{mx}^2 + 2m_{mz}m_{mx}\cos(\psi_{mz} - \psi_{mx})] = 36.582351 . \quad (28\ell)$$

The relative sum-to-difference phases in degrees can be obtained from (12f), (14d) and (22a), (22b), (25a) and (25b) in Section 3 as:

$$\begin{array}{lll} \phi_1 = -103.0261 , & \phi_2 = -77.0300 , & \phi_3 = -113.5502, \\ \phi_4 = 105.5593 , & \phi_5 = 48.1116 , & \phi_6 = 91.9132. \end{array} \quad (29)$$

We, now, assume that the EUT is unknown and the values given in (28) and (29) are the "measured" sum and difference powers and phases. The problem at hand is then to verify the validity and usefulness of the development presented in Sections 2 and 3 to see whether the unknown dipole moments and phases can be recovered to compute the radiation pattern. Using (7) with p = 0 and q = 11.83, we obtain

$$m_{ex}^2 = (P_{s1} + P_{s2} - P_{s3} - P_{s4} + P_{s5} + P_{s6})/(2q^2) = 1.96 , \quad (30a)$$

$$m_{ey}^2 = (P_{s1} + P_{s2} + P_{s3} + P_{s4} - P_{s5} - P_{s6})/(2q^2) = 3.24 \ , \tag{30b}$$

$$m_{ez}^2 = (-P_{s1} - P_{s2} + P_{s3} + P_{s4} + P_{s5} + P_{s6})/(2q^2) = 2.56 \ , \tag{30c}$$

$$m_{mx}^2 = (P_{d1} + P_{d2} - P_{d3} - P_{d4} + P_{d5} + P_{d6})/(2k^2q^2) = 0.64 \ , \tag{30d}$$

$$m_{my}^2 = (P_{d1} + P_{d2} + P_{d3} + P_{d4} - P_{d5} - P_{d6})/(2k^2q^2) = 0.36 \ , \tag{30e}$$

$$m_{mz}^2 = (-P_{d1} - P_{d2} + P_{d3} + P_{d4} + P_{d5} + P_{d6})/(2k^2q^2) = 0.16 \ ; \tag{30f}$$

$$\cos(\psi_{ex} - \psi_{ey}) = (P_{s1} - P_{s2})/(2q^2 m_{ex} m_{ey}) = 0.173648 \tag{30g}$$
$$\text{or} \quad \psi_{ex} - \psi_{ey} = \theta_{e1} = \pm\ 80^0 \ ,$$

$$\cos(\psi_{ey} - \psi_{ez}) = (P_{s3} - P_{s4})/(2q^2 m_{ey} m_{ez}) = 0.939693 \tag{30h}$$
$$\text{or} \quad \psi_{ey} - \psi_{ez} = \theta_{e2} = \pm\ 20^0 \ ,$$

$$\cos(\psi_{ez} - \psi_{ex}) = (P_{s5} - P_{s6})/(2q^2 m_{ez} m_{ex}) = 0.500000 \tag{30i}$$
$$\text{or} \quad \psi_{ez} - \psi_{ex} = \theta_{e3} = \pm\ 60^0 \ ,$$

$$\cos(\psi_{mx} - \psi_{my}) = (P_{d2} - P_{d1})/(2k^2q^2 m_{mx} m_{my}) = 0.939693 \tag{30j}$$
$$\text{or} \quad \psi_{mx} - \psi_{my} = \theta_{m1} = \pm\ 20^0 \ ,$$

$$\cos(\psi_{my} - \psi_{mz}) = (P_{d4} - P_{d3})/(2k^2q^2 m_{my} m_{mz}) = 0.965926 \tag{30k}$$
$$\text{or} \quad \psi_{my} - \psi_{mz} = \theta_{m2} = \pm\ 15^0 \ ,$$

$$\cos(\psi_{mz} - \psi_{mx}) = (P_{d6} - P_{d5})/(2k^2q^2 m_{mz} m_{mx}) = 0.819152 \tag{30ℓ}$$
$$\text{or} \quad \psi_{mz} - \psi_{mx} = \theta_{m3} = \pm\ 35^0 \ .$$

Clearly, all the amplitudes of dipole moments have been recovered, (16a,b) can be satisfied with two possibilities for each. The sign ambiguity for θ_{ei} and θ_{mi}, $i = 1,2,3$ may be resolved by (15), (23) and (26).

Case 1. Choosing $\theta_{e1} = 80^0$, $\theta_{e2} = -20^0$, and $\theta_{e3} = -60^0$ for (15a), (23a) and (26a), we obtain

$$\sin\theta_{m1} = -1.428572, \text{ no solution ;}$$

$$\sin\theta_{m2} = -1.140644, \text{ no solution ;}$$

$$\sin\theta_{m3} = 1.070753, \text{ no solution .}$$

Case 2. Choosing $\theta_{e1} = -80^0$, $\theta_{e2} = 20^0$, and $\theta_{e3} = 60^0$ for (15a), (23a) and (26a) we obtain

$$\begin{aligned} \sin\theta_{m1} &= -0.342020 \; , \\ \sin\theta_{m2} &= -0.258819 \; , \\ \sin\theta_{m3} &= 0.573605 \; , \end{aligned} \tag{31a}$$

which yield

$$\theta_{m1} = -20^0, \quad \theta_{m2} = -15^0, \quad \theta_{m3} = 35^0 \tag{31b}$$

or

$$\theta_{m1} = -160^0, \quad \theta_{m2} = -165^0, \quad \theta_{m3} = 145^0 \quad . \tag{31c}$$

A quick check of (16b) and (30j,k,ℓ) confirms that the solution in (31b) is correct. The false solution in (31c) should be discarded. Thus,

$$\begin{aligned} &\psi_{ex} - \psi_{ey} = -80^0, \quad \psi_{ey} - \psi_{ez} = 20^0, \quad \psi_{ez} - \psi_{ex} = 60^0 \; , \\ &\psi_{mx} - \psi_{my} = -20^0, \quad \psi_{my} - \psi_{mz} = -15^0, \quad \psi_{mz} - \psi_{mx} = 35^0 \; . \end{aligned} \tag{32}$$

Should we start with θ_{m1}, θ_{m2} and θ_{m3} obtained in (30j,k,ℓ) and then use (15b), (23b) and (26b), we would reach the same conclusion.

The application of (12d,e,g), (14e,f,g), (22c,d,e,f,g) and (25c,d,e,f,g) yields

$$\alpha_1 = -45.9877^0 \quad , \quad \beta_1 = 150.9862^0 \quad , \tag{33a}$$

$$\alpha_2 = 58.4730^0 \quad , \quad \beta_2 = -78.5570^0 \quad , \tag{33b}$$

$$\alpha_3 = 9.4057^0 \quad , \quad \beta_3 = 130.8555^0 \quad , \tag{33c}$$

$$\alpha_4 = -61.5510^0 \quad , \quad \beta_4 = -80.9917^0 \quad , \tag{33d}$$

$$\alpha_5 = 27.7958^0 \quad , \quad \beta_5 = -64.0926^0 \quad , \tag{33e}$$

$$\alpha_6 = -53.4132^0 \quad , \quad \beta_6 = -101.5000^0 \quad , \tag{33f}$$

and

$$\psi_{ex} - \psi_{my} = 60^0 \ , \ \psi_{ey} - \psi_{mz} = 125^0 \ , \ \psi_{ez} - \psi_{mx} = 140^0 \quad . \tag{33g}$$

The application of (12i,j,k), (14i,j,k), (24c,d,e,f,g) and (27c,d,e,f,g) yields

$$\alpha_1' = 34.0123^0 \quad , \quad \beta_1' = 130.9862^0 \quad , \tag{34a}$$

$$\alpha_2' = 138.4730^0 \quad , \quad \beta_2' = -98.5570^0 \quad , \tag{34b}$$

$$\alpha_3' = -10.5943^0 \quad , \quad \beta_3' = 115.8554^0 \quad , \tag{34c}$$

$$\alpha_4' = -81.5511^0 \quad , \quad \beta_4' = -95.9917^0 \quad , \tag{34d}$$

$$\alpha_5' = -32.2042^0 \quad , \quad \beta_5' = -29.0926^0 \quad , \tag{34e}$$

$$\alpha_6' = -113.4132^0 \quad , \quad \beta_6' = -66.5000^0 \quad , \tag{34f}$$

and

$$\psi_{ey} - \psi_{mx} = 160^0 \ , \ \psi_{ez} - \psi_{my} = 120^0 \ , \ \psi_{ex} - \psi_{mz} = 45^0 \quad . \tag{34g}$$

Note that the quantities obtained in (32), (33g) and (34g) fulfill the relative phase requirement for the radiation pattern in (5). Note further that the relationships in (12ℓ,m,n), (14ℓ,m), (24h,i), and (27h,i,j,k) are all exactly satisfied.

If, in addition, $\psi_{ex} = 0$ is chosen as the phase reference, the other phases will be $\psi_{ey} = 80°$, $\psi_{ez} = 60^0$, $\psi_{mx} = -80^0$, $\psi_{my} = -60^0$, and $\psi_{mz} = -45^0$, which are exactly the ones assigned in this particular example.

At this point, one may doubt the validity of the formal presentation if some of the six amplitudes of the dipole moments vanish. To eliminate this doubt, another theoretical example is given below.

Example 2. Suppose the input data remain the same as those presented in Example 1, except that $m_{mz} = 0$. Naturally, under this condition, ψ_{mz} has no meaning. The sum powers and the first two difference powers should also remain as those obtained in (28) because they do not involve m_{mz}. The other difference powers will be different. They are:

$$P_{d3} = P_{d4} = k^2 q^2 m_{my}^2 / 2 = 9.944930 \tag{35a}$$

and

$$P_{d5} = P_{d6} = k^2q^2m_{mx}^2/2 = 17.679876 \quad . \tag{35b}$$

By the same reason, ϕ_1 and ϕ_2 should have the same values as those given in (29). The other sum-to-difference phases in degrees may be obtained from (17) as:

$$\phi_3 = -139.4057, \quad \phi_4 = 111.5511, \quad \phi_5 = 22.2042 \quad \text{and} \quad \phi_6 = 103.4132 \; . \tag{36}$$

Note that, for this special case, we not only have $P_{d3} = P_{d4}$ and $P_{d5} = P_{d6}$ as given in (35), but also have $2(P_{d3} + P_{d5}) - P_{d1} - P_{d2} = 0$. In fact, these relations insure $m_{mz} = 0$ as can be easily seen from (7). The sum-to-difference phases ϕ_i have, however, no simple relationship.

Again, from the testing point of view, we assume that the powers given in (28a through h) and (35a,b) and the sum-to-difference phases given in (36) together with $\phi_1 = -103.0261°$ and $\phi_2 = -77.0300°$ are the "measured" values. Obviously, all the dipole-moment amplitudes can be totally recovered by the same exercise presented in (30a through f). The relative phases associated with the electric dipole moments also remain the same as before, namely, $\theta_{e1} = \pm 80°$, $\theta_{e2} = \pm 20°$, and $\theta_{e3} = \pm 60°$. The only relative phase associated with the magnetic dipole moment is still $\theta_{m1} = \pm 20°$. The other relative phases associated with the magnetic dipole moments, θ_{m2} and θ_{m3}, are undefined because they both involve the meaningless quantity ψ_{mz}. Fortunately, they are not required, as can be seen later.

Case 1. Choosing $\theta_{e1} = 80^0$, $\theta_{e2} = -20^0$, and $\theta_{e3} = -60^0$ for (15a), we have

$$\sin\theta_{m1} = -1.428572, \quad \text{no solution.}$$

Case 2. Choosing $\theta_{e1} = -80^0$, $\theta_{e2} = 20^0$, and $\theta_{e3} = 60^0$ for (15a), we have

$$\sin\theta_{m1} = -0.342020, \text{ or } \theta_{m1} = -20^0,$$

which implies that the correct solution should be:

$$\psi_{ex} - \psi_{ey} = -80^0, \; \psi_{ey} - \psi_{ez} = 20^0, \; \psi_{ez} - \psi_{ex} = 60^0, \text{ and } \psi_{mx} - \psi_{my} = -20^0 \; . \tag{37a}$$

Note that the other possible solution of $\theta_{m1} = -160^0$ is discarded in view of (28g) and (28h). So long as $P_{d2} > P_{d1}$, we always have $|\theta_{m1}| < 90^0$.

The conditions (12g) and (12i) yield respectively

$$\psi_{ex} - \psi_{my} = 60^0 \quad , \tag{37b}$$

and

$$\psi_{ey} - \psi_{mx} = 160^0 \quad ; \tag{37c}$$

the condition (24c) yields

$$\psi_{ez} - \psi_{my} = 120^0 \quad . \tag{37d}$$

and the condition (25c) yields

$$\psi_{ez} - \psi_{mx} = 140^0 \quad . \tag{37e}$$

The relative phases so extracted above are all required by (5) for computing the power pattern for the emitter being considered. The other conditions, (22c) and (27c) are meaningless in this case. Note that the values for α_i, α_i' , (i = 1,2,3,4,5,6), β_1, β_2, β_1', β_2' remain the same as those given in (33) and (34) since they involve only θ_{ei} (i = 1,2,3) and θ_{m1}. The other β_i and β_i' become

$$\begin{aligned} &\beta_3, \beta_4, \beta_5', \beta_6' \ \text{undefined,} \\ &\beta_3' = 90^0 \ , \ \beta_4' = -90^0 \ , \ \beta_5 = -90^0 \ , \ \beta_6 = -90^0 \quad . \end{aligned} \tag{38}$$

Of course, when $\psi_{ex} = 0$ is chosen as the phase reference, we obtain from (37)

$$\psi_{ey} = 80^0, \quad \psi_{ez} = 60^0, \quad \psi_{mx} = -80^0 \text{ and } \psi_{my} = -60^0 \tag{39}$$

Thus, we demonstrated that the theoretical development presented in this report is also applicable to the special case when one of the component dipole moments vanishes. The same conclusion is valid when any one of the other five component dipole moments is zero. For example, when $m_{my} = 0$, we will have $P_{d1} = P_{d2}$, $P_{d3} = P_{d4}$, and $2(P_{d1} + P_{d3}) - P_{d5} - P_{d6} = 0$; and θ_{m1} and θ_{m2} will be meaningless. As another example, if $m_{ex} = 0$, we will then have $P_{s1} = P_{s2}$, $P_{s5} = P_{s6}$, and $2(P_{s1} + P_{s5}) - P_{s3} - P_{s4} = 0$; and θ_{e1} and θ_{e3} will have no meaning.

The formulation is also good when any two or more of the six component dipole moments vanish. For example, when $m_{my} = m_{mz} = 0$, we will have $P_{d1} = P_{d2} = P_{d5} = P_{d6}$, and $P_{d3} = P_{d4} = 0$; and all θ_{mi} (i=1,2,3) will have no meaning. Then θ_{e1}, θ_{e2}, θ_{e3}, and $\psi_{ez} - \psi_{mx}$ from (25c) will be sufficient to give the correct solution. As another example, if $m_{ez} = m_{mz} = 0$, we then have $P_{s3} = P_{s4}$, $P_{s5} = P_{s6}$, $2(P_{s3} + P_{s5}) - P_{s1} - P_{s2} = 0$; $P_{d3} = P_{d4}$, $P_{d5} = P_{d6}$, $2(P_{d3} + P_{d5}) - P_{d1} - P_{d2} = 0$; and

θ_{e2}, θ_{e3}, θ_{m2}, and θ_{m3} will all have no meaning. Then, θ_{e1}, (12g), (14i), and (15a) will give an unambiguous solution for the relative phases.

After demonstrating the usefulness and validity of the theory and computation procedures outlined in Sections 2 and 3 by two theoretical examples, we now give a practical example based on actual experimental data. It should be anticipated that the procedures involved will not be as straightforward as those for the theoretical examples because the measured inaccuracy and noise will mask the consistency required by various equations.

Example 3. In this experiment, the unknown EUT is represented by a spherical dipole radiator with a radius of 5 cm. The radiator consists of two hemispherical shells which are fed at the poles and held together by threading onto a dielectric disc such that there is a gap of 3 mm between them. The electronic circuitry feeding the dipole is enclosed within the shells and consists of a battery-operated power supply and a 30-MHz crystal oscillator followed by an amplifier. Thus, the radiator is self-contained and no external connections are needed.

The radiator, with an arbitrary orientation, was placed at the center of the upper chamber of a TEM cell of 1.20 m X 1.20 m X 2.40 m. The measured sum powers in watts, difference powers in watts, and relative phases in degrees between the sum and difference outputs at the six positions depicted in figures 4, 5, and 6, as recorded by the computer in the experimental system, are as follows:

$$
\begin{array}{lll}
P_{s1} = 9.935735(10^{-6}), & P_{s2} = 8.855233(10^{-10}), & P_{s3} = 2.224334(10^{-6}), \\
P_{s4} = 2.674238(10^{-6}), & P_{s5} = 2.329164(10^{-6}), & P_{s6} = 2.800271(10^{-6}), \\
P_{d1} = 2.640584(10^{-8}), & P_{d2} = 5.391381(10^{-12}), & P_{d3} = 5.031529(10^{-9}), \\
P_{d4} = 5.911531(10^{-9}), & P_{d5} = 5.268658(10^{-9}), & P_{d6} = 1.179506(10^{-8}); \\
\phi_1 = -32.94, & \phi_2 = \text{unstable because of weak levels of } P_{s2} \text{ and } P_{d2}, & \\
\phi_3 = 166.5, & \phi_4 = -13.5 & \\
\phi_5 = 168.66, & \phi_6 = -48.6. &
\end{array}
$$

An examination of the relative values of the sum and difference powers confirms that this radiator is essentially an electric-type source, as it should be. We, nevertheless, treat it as a combination of both types even though the magnetic dipole moments are not as important as the electric counterparts. The frequency of 30 MHz assures that only the dominant mode may exist inside this cell. The size of the radiator is, indeed, small compared to the cross section of the cell and to the wavelength of 10 m. The normalized transverse electric field at the radiator center for this particular example is estimated at $\bar{e}_0 = 11.825\bar{y}$ v/m [3]. That is, p = 0, q = 11.825, and the field is purely y-directed.

The amplitudes of the dipole moments extracted from these measurement data in accordance with (7a,b) are:

$$
\begin{array}{lll}
m_{ex} = 1.906736(10^{-4}), & m_{ey} = 1.862939(10^{-4}), & m_{ez} = 0.180769(10^{-4}), \\
m_{mx} = 1.716559(10^{-5}), & m_{my} = 1.355661(10^{-5}), & m_{mz} = 0.380153(10^{-5}).
\end{array} \quad (40)
$$

The relative phases between the dipole components of the same kind may be extracted in accordance with (7c,d) [or (8) and (9)] as:

$$
\begin{aligned}
&\cos\theta_{e1} \cong 1.0, && \theta_{e1} = \psi_{ex} - \psi_{ey} = 0°, \\
&\cos\theta_{e2} = -0.477711, && \theta_{e2} = \psi_{ey} - \psi_{ez} = \pm\, 118.5360°, \\
&\cos\theta_{e3} = -0.488734, && \theta_{e3} = \psi_{ez} - \psi_{ex} = \pm 119.2574°, \\
\\
&\cos\theta_{m1} \cong -1.0, && \theta_{m1} = \psi_{mx} - \psi_{my} = \pm 180°, \\
&\cos\theta_{m2} = 0.154661, && \theta_{m2} = \psi_{my} - \psi_{mz} - \pm 81.1028°, \\
&\cos\theta_{m3} = 0.905867, && \theta_{m3} = \psi_{mz} - \psi_{mx} - \pm 25.0596°.
\end{aligned}
\qquad (41)
$$

Since the measurement data involve inaccuracies due to background noise, instrument errors, etc., obviously we should not expect a perfect satisfaction of various restraining equations as for the theoretical examples considered previously. For example, to satisfy (16a), we may choose (a) $\theta_{e1} = 0$, $\theta_{e2} = 118.5360°$, and $\theta_{e3} = -119.2574°$, or (b) $\theta_{e1} = 0$, $\theta_{e2} = -118.5360°$, and $\theta_{e3} = 119.2574°$. Either of these two cases yields a small error of less than 1°, which implies that the accuracy on the measured sum powers and the values of m_{ei}, $i = 1,2,3$ deduced from them is quite acceptable. However, the values of θ_{mi} do not nearly meet the requirement in (16b). In fact, the smallest deviation among various combinations of θ_{mi} from (16b) is of the order of 74°. This is not surprising in view of the relatively weak levels of m_{mi}. Since $\cos\theta_{m1}$ involves $P_{d2} - P_{d1}$ and the measured P_{d2} is a few orders smaller than the measured P_{d1}, we are certain that $|\theta_{m1}|>90°$. Whether or not the absolute value of θ_{m1} is closer to 180° as obtained in (41) depends naturally on the actual accuracy. Similarly, since the measured P_{d6} is quite stronger than the measured P_{d5}, we may be certain that $\cos\theta_{m3}$ determined by $P_{d6} - P_{d5}$ is positive, giving an absolute value for θ_{m3} smaller than 90°. Of course, the true value for θ_{m3} may be much different from 25° as approximately shown in (41). As for θ_{m2}, the situation is very different from θ_{m1} and θ_{m3}. Since the measured values for P_{d3} and P_{d4} are quite close, the true value for $\cos\theta_{m2}$ determined by $P_{d4} - P_{d3}$ may be small-positive or small-negative. Thus, the absolute value for θ_{m2} may be greater or smaller than 90°. The above analysis explains that the condition (16b) is still likely to be met if the accuracy involved in the measurement of difference powers is more reasonable. In any case, we still have other conditions such as (15), (23), and (26) to be examined.

Case 1. Choosing $\theta_{e1} = 0°$, $\theta_{e2} = 118.5360°$, and $\theta_{e3} = -119.2574°$, we obtain respectively from (23a) and (26a):

$$\sin\theta_{m2} = -0.282754 \text{ yielding } \theta_{m2} = -16.4246° \text{ or } -163.5754°$$

and

$$\sin\theta_{m3} = 9.364986/4.105186 = 2.281247, \text{ no solution.}$$

In examining (26a) more closely for this case, we find that the result of an unrealizable θ_{m3} is mainly due to the chosen negative sign for θ_{e3} even though the values for m_{ex}, m_{ez}, m_{mx}, m_{mz} and the measured ϕ_5 and ϕ_6 are all somewhat responsible. Even a substantial deviation in all of these values in view of the implicit measurement errors is not likely to yield a realizable value for θ_{m3} so long as a negative sign is chosen for θ_{e3}.

The value for θ_{m1} cannot even be determined from (15a) because it requires ϕ_1 and ϕ_2, and yet the weak levels of P_{s2} and P_{d2} fail to produce a stable measured value for ϕ_2. Thus, we are forced to use $\theta_{m1} = 180°$ as obtained in (41).

Case 2. Choosing $\theta_{e1} = 0°$, $\theta_{e2} = -118.5360°$, and $\theta_{e3} = 119.2574°$, we obtain from (23a)

$$\sin\theta_{m2} = 0.282754 \text{ giving } \theta_{m2} = 16.4246° \text{ or } 163.5754°,$$

and from (26a)

$$\sin\theta_{m3} = 5.994492/5.299348 = 1.131175.$$

Although θ_{m3} also appears unrealizable for this case, the situation is rather encouraging because only a small deviation in m_{ex}, m_{ez}, m_{mx}, m_{mz}, ϕ_5, ϕ_6, or θ_{e3} will make $\sin\theta_{m3}$ approach to 1.0, yielding $\theta_{m3} = 90°$.

Thus, from the above considerations, it is more reasonable to choose Case 2 to represent the unknown radiator. Even so, the sum of θ_{mi}, i = 1,2,3 is still approximately 74° off the ideal condition (16b). This amount of deviation is considered too excessive to deduce any meaningful results for other mixed phases. To make a compromise in this regard and in view of the previous comment that the absosolute value for θ_{m2} may be greater or smaller than 90°, we take, for θ_{m2}, the arithmetic average of 81.1028° obtained in (41) with the actual measured difference powers ($P_{d4}>P_{d3}$) and 163.5754° obtained from (23a) by allowing the possibility that the true value for P_{d3} could well be greater than the true value for P_{d4}. That is,

$$\theta_{m2} = 1/2\ (81.1028° + 163.5754°) = 122.3391° \ . \tag{42a}$$

Similarly, for θ_{m3}, we take the arithmetic average of 25.0596° obtained in (41) based on the measured P_{d5} and P_{d6} and 90° obtained from (26a) with a minor adjustment in measured quantities to account for the measurement inaccuracy,

$$\theta_{m3} = 1/2\ (25.0596° + 90°) = 57.5298° \ . \tag{42b}$$

Now, the sum of $\theta_{m1} = 180°$, θ_{m2} in (42a) and θ_{m3} in (42b) almost satisfies (16b).

With this important constraining condition almost satisfied, we then proceed as follows.

From (12d,e,g,j,k,i) we obtain

$$\alpha_1 = 0, \quad \beta_1 = -90^0, \quad \alpha_1' = 0, \quad \beta_1' = 90^0 \ , \tag{43a}$$

$$\psi_{ex} - \psi_{my} = \phi_1 + \alpha_1 - \beta_1 = 57.06^0 \ , \tag{43b}$$

and

$$\psi_{ey} - \psi_{mx} = \phi_1 + \alpha_1' - \beta_1' = -122.94^0 \ . \tag{43c}$$

Note that for this case,

$$\alpha_1 - \alpha_1' = 0 = \theta_{e1}, \quad \beta_1' - \beta_1 = 180^0 = \theta_{m1} , \tag{43d}$$

which exactly satisfy (12m,n).

Equations (14e,i) are useless under the current measurement condition because we do not have a stable measured value for ϕ_2. Therefore, (43b) is the only computed value for $\psi_{ex} - \psi_{my}$, and (43c) for $\psi_{ey} - \psi_{mx}$.

From (22d,e,f,g,c) we have

$$\alpha_3 = -5.1081^0, \ \beta_3 = -43.9804^0, \ \alpha_4 = 4.6576^0, \ \beta_4 = 163.2360^0 , \tag{43e}$$

$$\psi_{ey} - \psi_{mz} = \phi_3 + \alpha_3 - \beta_3 = -154.6277^0 , \tag{43f}$$

and

$$\psi_{ey} - \psi_{mz} = \phi_4 + \alpha_4 - \beta_4 = -172.0784^0 . \tag{43g}$$

The reason that the result in (43f) does not agree exactly with that in (43g) is because of the approximation for θ_{m2} used in (42a). Again, taking the arithmetic average of (43f) and (43g) gives

$$\psi_{ey} - \psi_{mz} = -163.3530° . \tag{43h}$$

From (24d,e,f,g,c), we have

$$\alpha_3' = 113.4279^0, \quad \beta_3' = 78.3587^0, \quad \alpha_4' = 123.1936^0, \quad \beta_4' = -74.4249^0, \tag{43i}$$

$$\psi_{ez} - \psi_{my} = \phi_3 + \alpha_3' - \beta_3' = -158.4308^0 , \tag{43j}$$

and

$$\psi_{ez} - \psi_{my} = \phi_4 + \alpha_4' - \beta_4' = -175.8815^0 . \tag{45k}$$

Taking the arithmetic average of (43j) and (43k) yields

$$\psi_{ez} - \psi_{my} = -167.1562° . \tag{43ℓ}$$

Also, note that

$$\alpha_3 - \alpha_3' = \alpha_4 - \alpha_4' = -118.5360^0 = \theta_{e2}, \quad \beta_3' - \beta_3 = \beta_4' - \beta_4 = 122.3391^0 = \theta_{m2} \tag{43m}$$

which are the results expected from (24h,i).

From (25d,e,f,g,c), we have

$$\alpha_5 = 114.3005^0, \quad \beta_5 = -78.0276^0, \quad \alpha_6 = -56.2228^0, \quad \beta_6 = -99.4802^0 \tag{43n}$$

$$\psi_{ez} - \psi_{mx} = \phi_5 + \alpha_5 - \beta_5 = 0.9881^0 , \tag{43o}$$

and

$$\psi_{ez} - \psi_{mz} = \phi_6 + \alpha_6 - \beta_6 = -5.3426^0 . \tag{43p}$$

Taking the arithmetic average of (43o) and (43p) yields

$$\psi_{ez} - \psi_{mx} = -2.1773° \ . \tag{43q}$$

Finally, from (27d,e,f,g,c), we have

$$\alpha_5' = -4.9569^0, \quad \beta_5' = -20.4978^0, \quad \alpha_6' = -175.4802^0, \quad \beta_6' = -41.9504^0 \ , \tag{43r}$$

$$\psi_{ex} - \psi_{mz} = \phi_5 + \alpha_5' - \beta_5' = -175.7991^0 \ , \tag{43s}$$

and

$$\psi_{ex} - \psi_{mz} = \phi_6 + \alpha_6' - \beta_6' = -182.1298^0 \ . \tag{43t}$$

Taking the arithmetic average of (43s) and (43t) yields

$$\psi_{ex} - \psi_{mz} = -178.9645° \tag{43u}$$

Note that,

$$\alpha_5 - \alpha_5' = \alpha_6 - \alpha_6' = 119.2574^0 = \theta_{e3}, \quad \beta_5' - \beta_5 = \beta_6' - \beta_6 = 57.5298^0 = \theta_{m3} \tag{43v}$$

which are the results expected from (27h,i).

In summary, the results of $\psi_{ex} - \psi_{ey} = 0$, $\psi_{ey} - \psi_{ez} = -118.5360°$, $\psi_{ez} - \psi_{ex} = 119.2574°$, $\psi_{mx} - \psi_{my} = 180°$, $\psi_{my} - \psi_{mz} = 122.3391°$, $\psi_{mz} - \psi_{mx} = 57.5298°$, together with those in (40) and (43b,c,h,ℓ,q,u) are all we need in computing the radiation pattern (5) for the spherical dipole radiator being analyzed.

In addition, the total dipole strengths and orientations can also be determined as follows:

(i) for the electric type,

$$m_{er} = (m_{ex}^2 + m_{ey}^2 + m_{ez}^2)^{1/2} = 2.671865(10^{-4}) \text{ m (actually, amp-m normalized to a unit current)} \ , \tag{43w}$$

$$\theta_e = \cos^{-1}(m_{ez}/m_{er}) = 86.12^0, \quad \phi_e = \tan^{-1}(m_{ey}/m_{ex}) = 44.33^0 \ ; \tag{43x}$$

(ii) for the magnetic type,

$$m_{mr} = 2.220114(10^{-5}) \ \text{m}^2 \text{ (actually, amp-m}^2 \text{ relative to a unit current)} \ , \tag{43y}$$

$$\theta_m = 80.14^0 \quad \phi_m = 38.30^0 \ . \tag{43z}$$

It is significant to note that the electric dipole strength obtained in (43w) agrees very well (within ±1%) with that deduced from another measurement method [4], and that the orientation obtained in (43x) is approximately (within ±2%) equal to that of the spherical dipole radiator (relative to the TEM cell coordinates) placed originally inside the TEM cell to begin with this experiment. This kind of confirmation certainly helps to establish a good degree of confidence in the proposed method. Since there is no a priori knowlege about the equivalent magnetic dipole vector associated with the spherical dipole radiator, we have no way to determine the accuracy for the results obtained in (43y,z).

From (43x) we realize that the electric dipole is approximately located in the plane of $\phi = 45°$ and $\phi = 225°$. The normalized radiation pattern in this particular plane, in accordance with (5), is presented in figure 7. Clearly, the maximum radiation is approximately in the direction perpendicular to the dipole axis as expected. The presence of some magnetic dipole moments causes a minor unsymmetry of the pattern. The total power radiated by this spherical dipole is, in accordance with (6), about 0.283 μw.

The relative phases extracted in this example are relatively consistent, though not perfect, under the practical measurement circumstance. It is felt that a general composite interference source with comparable strengths of electric and magnetic dipoles will give even better results. Corresponding results due to a source of essentially magnetic type are to be reported on in the future.

5. Conclusions

The theoretical development and measurement procedures have been presented to determine the free-space radiation characteristics, both the total radiated power and detailed radiation pattern, for a general unknown emitter. The success of this proposed method relies on the measurements of powers and phases at six different emitter positions when it is placed inside a TEM cell. One of the main requirements is that the emitter is electrically small compared to the wavelength and the TEM cell size. The other requirement is that the frequency is low enough such that only the dominant mode is propagating inside the TEM cell. Both theoretical and experimental examples have been given to demonstrate the usefulness and validity of the method. An error analysis of the final experimental results should be made in the future to gain insight into the accuracy of the method due to background noise and other practical imperfections involved in the measurements.

6. References

[1] Sreenivasiah, I.; Chang, D. C.; Ma, M. T. Characterization of electrically small radiating sources by tests inside a transmission line cell. Nat. Bur. Stand. (U.S.) Tech. Note 1017; 1980.

[2] Sreenivasiah, I.; Chang, D. C.; Ma, M. T. Emission characteristics of electrically small radiating sources from tests inside a TEM cell. IEEE Trans. on Electromag. Compat., EMC-23, No.3: 113-121; August, 1981.

[3] Tippet, J. C. Model characteristics of rectangular coaxial strip line. Ph.D. Thesis, University of Colorado, Boulder; 1978.

[4] Crawford, M. L.; Workman, J. L. Predicting free-space radiated emissions from electronic equipment using TEM cell and open-field site measurements. IEEE Internat. Synp. on Electromag. Compat. Record 80-85; 1980.

```
100 REM....THIS ROUTINE WILL COLLECT AND ANALYZE OR SIMPLY ASSIST IN THE
105 REM....ANALYSIS OF POWER AND PHASE DATA  AS OUTLINED IN THIS NBS TECH NOTE.
110 REM....THE RADIATION PATTERN FOR THE EQUIPMENT UNDER TEST
115 REM....IS ALSO DEVELOPED.  THIS PROGRAM IS WRITTEN ON THE HP 9830 WITH
120 REM....THE HP 9862A PLOTTER, 8568A SPECTRUM ANALYZER; THE INPUT/OUTPUT
125 REM....COMMANDS MUST BE MODIFIED TO ACTUAL EQUIPMENT USED.  THE PHASE
130 REM....IS INPUT MANUALLY WHEN PROMPTED BY THE COMPUTER.
135 REM....
140 REM....
145 DEF FNA(X)=ATN(X/SQR(1-X*X+1E-99))
150 DEF FNB(X)=ATN(SQR(1-X*X)/(X+1E-99))+2*ATN1E+99*(X<0)
155 REM INITIALIZE ALL MATRICIES...............................................
160 DIM A$[10],B$[10],C$[90]
165 DIM P[6],Q[6],C[3,6],D[3,6],E[3],F[3],M[3],N[3]
170 DIM S[185,2],U[100,2],W[6],Z[6,6]
175 MAT S=ZER[185,2]
180 MAT U=ZER[100,2]
185 MAT W=ZER[6]
190 MAT Z=ZER[6,6]
195 DATA 1,1,-1,-1,1,1,1,1,1,1,-1,-1,-1,-1,1,1,1,1
200 DATA 1,-1,0,0,0,0,0,0,1,-1,0,0,0,0,0,0,1,-1
205 MAT READ C[3,6],D[3,6]
210 MAT P=CON[6]
215 MAT Q=CON[6]
220 MAT E=CON[3]
225 MAT F=CON[3]
230 MAT M=CON[3]
235 MAT N=CON[3]
240 REM....IF POWER AND PHASE DATA ARE TO BE COLLECTED USE THE NEXT SUBROUTINE
245 REM....AND SKIP THE READ DATA COMMANDS.  IF DATA IS TO BE STUDIED AFTER
250 REM....COLLECTION, USE DATA READ WITH UPDATED INFORMATION. (E0,F0 ALSO).
255 REM....AND SKIP THE DATA COLLECTION SUBROUTINE. (USE OR DELETE LINE 260)
260 GOTO 275
265 GOSUB 4200
270 GOTO 300
275 REM....READ IN DATA FROM THE DATA COMMANDS AT THE END................
280 GOSUB 4910
285 E0=11.83
290 F0=3E+07
295 REM....END DATA INPUT....
300 FIXED 4
305 PRINT "FREQUENCY="(F0/1E+06);"MHZ.  CELL FACTOR E0=";E0
310 PRINT
315 Q0=E0/1.414213562
320 P0=Q0
325 K0=(2*PI*F0)/3E+08
330 K2=1/(2*E0↑2)
335 K1=1/(2*K0↑2*E0↑2)
340 FLOAT 4
345 MAT E=C*P
350 MAT E=(K2)*E
355 MAT M=C*Q
360 MAT M=(K1)*M
365 MAT F=D*P
370 MAT F=(K2)*F
375 MAT N=D*Q
380 MAT N=(K1)*N
385 FOR C=1 TO 3
390 IF E[C]>0 THEN 400
395 E[C]=0
400 IF M[C]>0 THEN 410
405 M[C]=0
410 NEXT C
415 PRINT
420 PRINT
425 PRINT "THE SUM POWERS ARE;"
430 PRINT "PS1=";P[1];"PS2=";P[2];"PS3=";P[3]
435 PRINT "PS4=";P[4];"PS5=";P[5];"PS6=";P[6]
440 PRINT
```

```
445 PRINT
450 PRINT "THE DIFFERENCE POWERS ARE;"
455 PRINT "PD1=";O[2];"PD2=";O[1];"PD3=";O[4]
460 PRINT "PD4=";O[3];"PD5=";O[6];"PD6="Q[5]
465 PRINT
470 FIXED 4
475 PRINT
480 PRINT "THE PHASE ANGLES, SUM TO DIFFERENCE POWERS ARE (1-6);"
485 PRINT W[1];W[2];W[3];W[4];W[5];W[6];"DEGREES"
490 FOR C=1 TO 6
495 W[C]=(W[C]/180)*PI
500 NEXT C
505 PRINT W[1];W[2];W[3];W[4];W[5];W[6];"RADIANS"
510 PRINT
515 PRINT
520 FLOAT 4
525 PRINT "THE ELECTRIC DIPOLE MOMENTS ARE;"
530 PRINT "MEX=";SQR(E[1]);"MEY=";SQR(E[2]);"MEZ=";SQR(E[3])
535 PRINT
540 PRINT
545 PRINT "THE MAGNETIC DIPOLE MOMENTS ARE;"
550 PRINT "MMX=";SQR(M[1]);"MMY=";SQR(M[2]);"MMZ=";SQR(M[3])
555 PRINT
560 PRINT
565 PRINT "THE ELECTRIC DIPOLE MOMENT CROSS TERMS ARE;"
570 PRINT "MEXY=";F[1];"MEYZ=";F[2];"MEZX=";F[3]
575 PRINT
580 PRINT
585 PRINT "THE MAGNETIC DIPOLE MOMENT CROSS TERMS ARE;"
590 PRINT "MMXY=";N[1];"MMYZ=";N[2];"MMZX=";N[3]
595 A1=E[1]+(K0↑2)*M[1]
600 A2=E[2]+(K0↑2)*M[2]
605 A3=E[3]+(K0↑2)*M[3]
610 A4=F[1]+(K0↑2)*N[1]
615 A5=F[2]+(K0↑2)*N[2]
620 A6=F[3]+(K0↑2)*N[3]
625 P9=(10*K0↑2)*(A1+A2+A3)
630 PRINT
635 PRINT
640 PRINT "AX↑2=";A1;"AY↑2=";A2;"AZ↑2=";A3
645 PRINT
650 PRINT "A XY=";A4;"A YZ=";A5;"A ZX=";A6
655 PRINT
660 PRINT
665 PRINT
670 PRINT "THE TOTAL RADIATED POWER IN FREE SPACE IS"P9"WATTS "
675 PRINT
680 PRINT
685 PRINT
690 REM GO DEVELOP THE E TO M CROSS TERMS................................
695 GOSUB 1890
700 REM....DEVELOP THE B TERMS FOR THE PATTERN.............................
705 GOSUB 3125
710 REM ...THE RADIATION PATTERN IS DEVELOPED HERE...........................
715 REM....THETA=I, PHI=J, LAMDA=L
720 REM....ALL ANGLES IN DEGREES...
725 DEG
730 CFLAG 2
735 L=3E+08/F0
740 DISP "RADIAL DISTANCE IN METERS????";
745 BEEP
750 INPUT R
755 DISP "VARY THETA OR PHI? (T/P)";
760 BEEP
765 INPUT A$
770 IF A$[1]="P" THEN 795
775 IF A$[1]="T" THEN 785
780 GOTO 755
785 GOSUB 1545
```

```
790 GOTO 800
795 GOSUB 1755
800 GOSUB 940
805 GOSUB 1200
810 PRINT
815 FIXED 2
820 IF A$[1]="P" THEN 855
825 PRINT "THE RADIATION PATTERN FOR PHI="J-180"DEG AND R="R"METERS"
830 PRINT "HAS THE FOLLOWING MAXIMUM AND RELATIVE MAXIMUMS:"
835 PRINT
840 FORMAT 5X,F8.0,6X,F12.2
845 PRINT "THETA DEGREES       RADIATED POWER (DB WATTS BELOW MAX)"
850 GOTO 875
855 PRINT "THE RADIATION PATTERN FOR THETA="I"DEG. AND R="R"METERS"
860 PRINT "HAS THE FOLLOWING MAXIMUM AND RELATIVE MAXIMUMS:"
865 PRINT
870 PRINT "  PHI DEGREES       RADIATED POWER (DB WATTS BELOW MAX)"
875 PRINT
880 FOR Z=1 TO Q
885 WRITE (15,840)U[Z,1],U[Z,2]
890 NEXT Z
895 PRINT
900 PRINT
905 DISP "REPEAT RADIATION PATTERN? (Y/N)";
910 BEEP
915 INPUT B$
920 IF B$[1]="Y" THEN 740
925 DISP "THATS ALL FOLKS!";
930 STOP
935 END
940 REM....THIS SUBROUTINE WILL FIND THE MAXIMUM VALUE IN THE RADIATION PATTERN
945 REM....THEN IT WILL REFERENCE ALL VALUES TO THIS MAXIMUM, CONVERT EVERY
950 REM....ENTRY TO DB BELOW THE MAXIMUM AND FINALLY FIND ALL RELATIVE MAXIMUMS
955 REM....THAT EXIST FOR THE PATTERN CALCULATED IN MATRIX  S..................
960 REM......
965 IF  NOT FLAG2 THEN 995
970 DISP "REFER TO NEW MAXIMUM? (Y/N).";
975 BEEP
980 INPUT B$
985 IF B$[1]="Y" THEN 995
990 GOTO 1030
995 D2=S[1,1]
1000 FOR C=2 TO K
1005 IF S[C,2]<D2 THEN 1015
1010 D2=S[C,2]
1015 NEXT C
1020 SFLAG 2
1025 FLOAT 4
1030 PRINT "THE MAXIMUM POWER RADIATED FOR ONE DIRECTION=";D2;"WATTS/M↑2"
1035 PRINT
1040 PRINT
1045 FOR C=1 TO K
1050 S[C,2]=(1/D2)*S[C,2]
1055 IF S[C,2]>1E-50 THEN 1065
1060 S[C,2]=1E-50
1065 S[C,2]=10*LGT(S[C,2])
1070 NEXT C
1075 IF S[K,2]<S[1,2] AND S[2,2]<S[1,2] THEN 1090
1080 Q=1
1085 GOTO 1105
1090 U[1,1]=S[1,1]
1095 U[1,2]=S[1,2]
1100 Q=2
1105 FOR C=2 TO K-1
1110 G=C-1
1115 D=C+1
1120 IF S[G,2]<S[C,2] AND S[D,2]<S[C,2] THEN 1130
1125 GOTO 1145
1130 U[Q,1]=S[C,1]
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```
1135 U[Q,2]=S[C,2]
1140 Q=Q+1
1145 NEXT C
1150 REM ...ALL MAXIMUMS FOR 0 TO 179 DEG ARE FOUND..
1155 G=K-1
1160 IF S[G,2]<S[K,2] AND S[1,2]<S[K,2] THEN 1175
1165 Q=Q-1
1170 RETURN
1175 U[Q,1]=S[K,1]
1180 U[Q,2]=S[K,2]
1185 RETURN
1190 STOP
1195 END
1200 REM....THIS ROUTINE WILL PLOT AND LABEL THE RADIATION PATTERN.............
1205 REM......
1210 REM......
1215 DISP "INSTALL PAPER AND SET BOUNDS";
1220 BEEP
1225 STOP
1230 SCALE -30,370,-34,6
1235 FOR C=1 TO K
1240 PLOT S[C,1],S[C,2]
1245 NEXT C
1250 PEN
1255 DISP " LABEL CURVE  (Y/N)??";
1260 BEEP
1265 INPUT B$
1270 IF B$[1,1]="Y" THEN 1285
1275 PEN
1280 RETURN
1285 XAXIS -30,20,0,360
1290 YAXIS 350,+2,-30,2
1295 XAXIS 2,-20,360,0
1300 YAXIS 0,-2,2,-30
1305 PEN
1310 FORMAT "FREQ =",F7.2," MHZ  PHI=",F6.1,",",F6.1," DEG.  MAX=",E9.2," W/M↑2"
1315 FORMAT F4.0
1320 FORMAT "FREQUENCY=",F8.2,"MHZ.  THETA=",F6.1," DEG.  MAX=",E9.2,"WATTS/M↑2"
1325 FOR C=-28 TO 0 STEP 4
1330 PLOT -25,C-0.3,-1
1335 LABEL (1315)C
1340 NEXT C
1345 PEN
1350 PLOT -30,-15,-1
1355 LABEL (*)"DB"
1360 PLOT 150,-33,-1
1365 IF A$[1]="P" THEN 1470
1370 LABEL (*)"THETA DEGREES"
1375 FORMAT "PHI=",F6.1
1380 PLOT 40,-33,-1
1385 LABEL (1375)(J-180)
1390 PLOT 280,-33,-1
1395 LABEL (1375)J
1400 Q9=180
1405 FOR C=20 TO 340 STEP 40
1410 IF C>180 THEN 1430
1415 PLOT C-12,-31,-1
1420 LABEL (1315)C
1425 GOTO 1445
1430 Q9=Q9-40
1435 PLOT C-12,-31,-1
1440 LABEL (1315)Q9
1445 NEXT C
1450 PLOT 10,3,-1
1455 LABEL (1310)(F0/1E+06),J-180,J,D2
1460 PEN
1465 GOTO 1510
1470 FOR C=0 TO 360 STEP 40
1475 PLOT C-12,-31,-1
```

```
1480 LABEL (1315)C
1485 NEXT C
1490 PEN
1495 LABEL (*)"PHI DEGREES"
1500 PLOT 10,3,-1
1505 LABEL (1320)(F0/1E+06),I,D2
1510 DISP "INPUT TITLE.  60 CHAR MAX.";
1515 INPUT C$
1520 PLOT 10,5,-1
1525 LABEL (*)C$
1530 RETURN
1535 STOP
1540 END
1545 REM....THIS ROUTINE CALCULATES THE RADIATION PATTERN FOR A FIXED PHI
1550 REM....WITH THETA VARIED FROM 0 TO 180 DEGREES IN STEPS OF 2 DEGREE........
1555 REM....THE DATA IS DEPOSITED IN MATRIX S
1560 REM......
1565 REM......
1570 DEG
1575 DISP "INPUT ANGLE PHI (DEGREES)";
1580 BEEP
1585 INPUT J
1590 J1=COS(J)
1595 J2=SIN(J)
1600 PRINT "PLEASE BE PATIENT WHILE THE RADIATION PATTERN IS CALCULATED......"
1605 PRINT
1610 PRINT
1615 PRINT
1620 K=0
1625 FOR I=0 TO 180 STEP 2
1630 K=K+1
1635 I1=COS(I)
1640 I2=SIN(I)
1645 P8=A1*(I1↑2*J1↑2+J2↑2)+A2*(I1↑2*J2↑2+J1↑2)+(A3*I2↑2)+B1*I1
1650 P7=2*A4*I2↑2*J1*J2+2*A5*I1*I2*J2+2*A6*I1*I2*J1-B2*I2*J1-B3*I2*J2
1655 P6=((15*PI)/(R↑2*L↑2))*(P8-P7)
1660 S[K,1]=I
1665 S[K,2]=P6
1670 NEXT I
1675 J=J+180
1680 J1=COS(J)
1685 J2=SIN(J)
1690 Q9=180
1695 FOR I=178 TO 0 STEP -2
1700 Q9=Q9+2
1705 K=K+1
1710 I1=COS(I)
1715 I2=SIN(I)
1720 P8=A1*(I1↑2*J1↑2+J2↑2)+A2*(I1↑2*J2↑2+J1↑2)+(A3*I2↑2)+B1*I1
1725 P7=2*A4*I2↑2*J1*J2+2*A5*I1*I2*J2+2*A6*I1*I2*J1-B2*I2*J1-B3*I2*J2
1730 P6=((15*PI)/(R↑2*L↑2))*(P8-P7)
1735 S[K,1]=Q9
1740 S[K,2]=P6
1745 NEXT I
1750 RETURN
1755 REM....THIS ROUTINE CALCULATES THE RADIATION PATTERN FOR A FIXED THETA
1760 REM....WITH PHI VARIED FROM 0 TO 360 DEGREES IN STEPS OF 2 DEGREE..........
1765 REM.... DATA IS DEPOSITED IN MATRIX S
1770 REM......
1775 REM......
1780 DISP "INPUT ANGLE THETA (DEGREES)";
1785 BEEP
1790 INPUT I
1795 I1=COS(I)
1800 I2=SIN(I)
1805 PRINT "PLEASE BE PATIENT WHILE THE RADIATION PATTERN IS CALCULATED....."
1810 PRINT
1815 PRINT
1820 PRINT
```

```
1825 K=0
1830 FOR J=0 TO 360 STEP 2
1835 K=K+1
1840 J1=COS(J)
1845 J2=SIN(J)
1850 P8=A1*(I1↑2+J1↑2+J2↑2)+A2*(I1↑2*J2↑2+J1↑2)+(A3*I2↑2)+B1*I1
1855 P7=2*A4*I2↑2*J1*J2+2*A5*I1*I2*J2+2*A6*I1*I2*J1-B2*I2*J1-B3*I2*J2
1860 P6=((15*P1)/(R↑2*L↑2))*(P8-P7)
1865 S[K,1]=J
1870 S[K,2]=P6
1875 NEXT J
1880 RETURN
1885 STOP
1890 REM THIS ROUTINE CALCULATES THE E TO M CROSS TERMS.........................
1895 FIXED 4
1900 FOR C=1 TO 3
1905 E[C]=SQR(E[C])
1910 M[C]=SQR(M[C])
1915 NEXT C
1920 REM THIS ROUTINE IS DESIGNED TO LAUNDRY FAULTY DATA
1925 FOR C=1 TO 3
1930 IF E[C]#0 THEN 1940
1935 E[C]=1E-20
1940 IF M[C]#0 THEN 1950
1945 M[C]=1E-20
1950 NEXT C
1955 E1=F[1]/(E[1]*E[2])
1960 IF ABS(E1) <= 1 THEN 1975
1965 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (E1)=";E1
1970 E1=E1/ABS(E1)
1975 E1=FNB(E1)
1980 M1=N[1]/(M[1]*M[2])
1985 IF ABS(M1) <= 1 THEN 2000
1990 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (M1)=";M1
1995 M1=M1/ABS(M1)
2000 M1=FNB(M1)
2005 E2=F[2]/(E[2]*E[3])
2010 IF ABS(E2) <= 1 THEN 2025
2015 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (E2)=";E2
2020 E2=E2/ABS(E2)
2025 E2=FNB(E2)
2030 M2=N[2]/(M[2]*M[3])
2035 IF ABS(M2) <= 1 THEN 2050
2040 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (M2)=";M2
2045 M2=M2/ABS(M2)
2050 M2=FNB(M2)
2055 E3=F[3]/(E[3]*E[1])
2060 IF ABS(E3) <= 1 THEN 2075
2065 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (E3)=";E3
2070 E3=E3/ABS(E3)
2075 E3=FNB(E3)
2080 M3=N[3]/(M[3]*M[1])
2085 IF ABS(M3) <= 1 THEN 2100
2090 PRINT "ERROR***INCOMPATIBLE DATA POINTS REQUIRE ACOS OF (M3)=";M3
2095 M3=M3/ABS(M3)
2100 M3=FNB(M3)
2105 PRINT "E1,M1=";(E1/PI)*180;(M1/PI)*180
2110 PRINT "E2,M2=";(E2/PI)*180;(M2/PI)*180
2115 PRINT "E3,M3=";(E3/PI)*180;(M3/PI)*180
2120 PRINT
2125 FOR C=1 TO 6
2130 IF ABS(W[C])<1.5706 OR ABS(W[C])>1.571 THEN 2170
2135 S=W[C]/ABS(W[C])
2140 W[C]=ABS(W[C])
2145 IF W[C] <= 1.570796 THEN 2160
2150 W[C]=1.571
2155 GOTO 2165
2160 W[C]=1.5706
2165 W[C]=S*W[C]
```

```
2170 NEXT C
2175 DISP "BELIEVE E1, M1 OR BOTH?";
2180 INPUT B$
2185 IF B$[1]="B" THEN 2345
2190 T1=TAN(W[1]-W[2])
2195 U=1
2200 V=2
2205 IF B$[1,2]="M1" THEN 2280
2210 DISP "SIGN E1, +1,-1?";
2215 INPUT S1
2220 E1=S1*E1
2225 T0=SIN(E1)
2230 S2=SGN(O[1]-O[2])
2235 GOSUB 2885
2240 IF  NOT FLAG1 THEN 2260
2245 PRINT "NO SOLUTION FOR M1 FROM E1=";(E1/PI)*180;"  SIN(M1)=";W1
2250 E1=S1-E1
2255 GOTO 2175
2260 PRINT "FOR E1=";(E1/PI)*180;"  M1="
2265 GOSUB 2980
2270 M1=T4
2275 GOTO 2345
2280 DISP "SIGN M1, +1,-1?";
2285 INPUT S1
2290 M1=S1*M1
2295 T0=SIN(M1)
2300 S2=SGN(P[1]-P[2])
2305 GOSUB 2940
2310 IF  NOT FLAG1 THEN 2330
2315 PRINT "NO SOLUTION FOR E1 FROM M1=";(M1/PI)*180;"  SIN(E1)=";W1
2320 M1=S1*M1
2325 GOTO 2175
2330 PRINT "FOR M1=";(M1/PI)*180;"  E1="
2335 GOSUB 2980
2340 E1=T4
2345 DISP "BELIEVE E2, M2 OR BOTH?";
2350 INPUT B$
2355 IF B$[1]="B" THEN 2515
2360 T1=TAN(W[3]-W[4])
2365 U=2
2370 V=3
2375 IF B$[1,2]="M2" THEN 2450
2380 DISP "SIGN E2, +1,-1?";
2385 INPUT S1
2390 E2=S1*E2
2395 T0=SIN(E2)
2400 S2=SGN(O[3]-O[4])
2405 GOSUB 2885
2410 IF  NOT FLAG1 THEN 2430
2415 PRINT "NO SOLUTION FOR M2 FROM E2=";(E2/PI)*180;"  SIN(E2)=";W1
2420 E2=S1*E2
2425 GOTO 2345
2430 PRINT "FOR E2=";(E2/PI)*180;"  M2="
2435 GOSUB 2980
2440 M2=T4
2445 GOTO 2515
2450 DISP "SIGN M2, +1,-1?";
2455 INPUT S1
2460 M2=S1*M2
2465 T0=SIN(M2)
2470 S2=SGN(P[3]-P[4])
2475 GOSUB 2940
2480 IF  NOT FLAG1 THEN 2500
2485 PRINT "NO SOLUTION FOR E2 FROM M2=";(M2/PI)*180;"  SIN(E2)=";W1
2490 M2=S1*M2
2495 GOTO 2345
2500 PRINT "FOR M2=";(M2/PI)*180;"  E2="
2505 GOSUB 2980
2510 E2=T4
```

```
2515 DISP "BELIEVE E3, M3 OR BOTH?";
2520 INPUT B$
2525 IF B$[1]="B" THEN 2685
2530 T1=TAN(W[5]-W[6])
2535 U=3
2540 V=1
2545 IF B$[1,2]="M3" THEN 2620
2550 DISP "SIGN E3, +1,-1?";
2555 INPUT S1
2560 E3=S1*E3
2565 T0=SIN(E3)
2570 S2=SGN(O[5]-O[6])
2575 GOSUB 2885
2580 IF  NOT FLAG1 THEN 2600
2585 PRINT "NO SOLUTION FOR M3 FROM E3=";(E3/PI)*180;"  SIN(M3)=";W1
2590 E3=S1*E3
2595 GOTO 2515
2600 PRINT "FOR E3=";(E3/PI)*180;"  M3="
2605 GOSUB 2980
2610 M3=T4
2615 GOTO 2685
2620 DISP "SIGN M3, +1,-1?";
2625 INPUT S1
2630 M3=S1*M3
2635 T0=SIN(M3)
2640 S2=SGN(P[5]-P[6])
2645 GOSUB 2940
2650 IF  NOT FLAG1 THEN 2670
2655 PRINT "NO SOLUTION FOR E3 FROM M3=";(M3/PI)*180;"  SIN(E3)=";W1
2660 M3=S1*M3
2665 GOTO 2515
2670 PRINT "FOR M3=";(M3/PI)*180;"  E3="
2675 GOSUB 2980
2680 E3=T4
2685 REM....NEW E AND M TERMS CALCULATED.
2690 PRINT "E1,M1=";(E1/PI)*180;(M1/PI)*180
2695 PRINT "E2,M2=";(E2/PI)*180;(M2/PI)*180
2700 PRINT "E3,M3=";(E3/PI)*180;(M3/PI)*180
2705 PRINT
2710 DISP "SELECT OTHER CHOICE? Y/N";
2715 INPUT B$
2720 IF B$[1]="N" THEN 2860
2725 DISP "E1,E2,E3,M1,M2,M3: 1,2,3,4,5,6?";
2730 INPUT Z6
2735 GOTO Z6 OF 2740,2760,2780,2800,2820,2840
2740 DISP "INPUT E1";
2745 INPUT E1
2750 E1=(E1/180)*PI
2755 GOTO 2685
2760 DISP "INPUT E2";
2765 INPUT E2
2770 E2=(E2/180)*PI
2775 GOTO 2685
2780 DISP "INPUT E3";
2785 INPUT E3
2790 E3=(E3/180)*PI
2795 GOTO 2685
2800 DISP "INPUT M1";
2805 INPUT M1
2810 M1=(M1/180)*PI
2815 GOTO 2685
2820 DISP "INPUT M2";
2825 INPUT M2
2830 M2=(M2/180)*PI
2835 GOTO 2685
2840 DISP "INPUT M3";
2845 INPUT M3
2850 M3=(M3/180)*PI
2855 GOTO 2685
```

```
2860 DISP "REPEAT E AND M CALCULATIONS? Y/N  ";
2865 INPUT B$
2870 IF B$[1]="Y" THEN 1955
2875 RETURN
2880 STOP
2885 REM....CALCULATE M FROM E TERMS.
2890 CFLAG 1
2895 W=-T1*((E[U]*M[V])↑2-(E[U]*M[U])↑2-(E[V]*M[V])↑2+(E[V]*M[U])↑2)
2900 W=W+(2*(M[U]*M[U]-M[V]*M[V])*E[U]*E[V]*T0)
2905 W=W/(4*T0*T1*E[U]*E[V]*M[U]*M[V]-2*M[U]*M[V]*(E[U]*E[U]-E[V]*E[V]))
2910 IF ABS(W) <= 1 THEN 2930
2915 W1=W
2920 W=SGN(W)
2925 SFLAG 1
2930 RETURN
2935 STOP
2940 REM....CALCULATE E FROM M TERMS.
2945 CFLAG 1
2950 W=-T1*((E[U]*M[V])↑2-(E[U]*M[U])↑2-(E[V]*M[V])↑2+(E[V]*M[U])↑2)
2955 W=W+(2*(E[U]*E[U]-E[V]*E[V])*M[U]*M[V]*T0)
2960 W=W/(4*T0*T1*E[U]*E[V]*M[U]*M[V]-2*E[U]*E[V]*(M[U]*M[U]-M[V]*M[V]))
2965 GOTO 2910
2970 RETURN
2975 STOP
2980 REM....DETERMINE CORRECT QUAD FOR INVERSE SINE FUNCTION.
2985 T2=FNA(W)
2990 IF W<0 THEN 3005
2995 T3=PI-T2
3000 GOTO 3010
3005 T3=-(PI+T2)
3010 IF S2<0 THEN 3025
3015 T4=T2
3020 GOTO 3030
3025 T4=T3
3030 PRINT "POSSIBLE ANGLES";(T2/PI)*180;",";(T3/PI)*180;" CHOSEN=";(T4/PI)*180
3035 PRINT
3040 RETURN
3045 STOP
3050 END
3055 REM....DETERMINE CORRECT QUAD FOR INVERSE TANGENT FUNCTION.
3060 IF ABS(B)>1E-25 THEN 3070
3065 B=(SGN(B))*1E-25
3070 IF B <= 0 THEN 3085
3075 Z4=ATN(A/B)
3080 RETURN
3085 Z4=PI+ATN(A/B)
3090 RETURN
3095 STOP
3100 REM....TAKE ANGLES BETWEEN +- 180 DEGREES.
3105 IF ABS(Z5) <= PI THEN 3115
3110 Z5=Z5-(SGN(Z5)*2*PI)
3115 RETURN
3120 STOP
3125 REM....TEST EQUATIONS FOR (EX-MY).
3130 A=Q0*E[2]*SIN(E1)
3135 B=P0*E[1]+Q0*E[2]*COS(E1)
3140 GOSUB 3055
3145 Z[1,2]=Z4
3150 A=Q0*M[1]*COS(M1)-P0*M[2]
3155 B=Q0*M[1]*SIN(M1)
3160 GOSUB 3055
3165 Z[1,3]=Z4
3170 Z5=W[1]+Z[1,2]-Z[1,3]
3175 GOSUB 3100
3180 Z[1,1]=Z5
3185 Z0=(W[1]/PI)*180
3190 Z1=(Z[1,2]/PI)*180
3195 Z2=(Z[1,3]/PI)*180
3200 Z3=(Z[1,1]/PI)*180
```

```
3205 WRITE (15,3210)"EX-MY=";Z0;" +";Z1;" -";Z2;" =";Z3
3210 FORMAT 4F9.3
3215 A=-P0*E[2]*SIN(E1)
3220 B=Q0*E[1]-P0*E[2]*COS(E1)
3225 GOSUB 3055
3230 Z[1,5]=Z4
3235 A=-(P0*M[1]*COS(M1)+Q0*M[2])
3240 B=-P0*M[1]*SIN(M1)
3245 GOSUB 3055
3250 Z[1,6]=Z4
3255 Z5=W[2]+Z[1,5]-Z[1,6]
3260 GOSUB 3100
3265 Z[1,4]=Z5
3270 Z0=(W[2]/PI)*180
3275 Z1=(Z[1,5]/PI)*180
3280 Z2=(Z[1,6]/PI)*180
3285 Z3=(Z[1,4]/PI)*180
3290 WRITE (15,3210)"EX-MY=";Z0;" +";Z1;" -";Z2;" =";Z3
3295 PRINT
3300 REM....TEST EQUATIONS FOR (EY-MX).
3305 A=-P0*E[1]*SIN(E1)
3310 B=P0*E[1]*COS(E1)+Q0*E[2]
3315 GOSUB 3055
3320 Z[2,2]=Z4
3325 A=Q0*M[1]-P0*M[2]*COS(M1)
3330 B=P0*M[2]*SIN(M1)
3335 GOSUB 3055
3340 Z[2,3]=Z4
3345 Z5=W[1]+Z[2,2]-Z[2,3]
3350 GOSUB 3100
3355 Z[2,1]=Z5
3360 Z0=(W[1]/PI)*180
3365 Z1=(Z[2,2]/PI)*180
3370 Z2=(Z[2,3]/PI)*180
3375 Z3=(Z[2,1]/PI)*180
3380 WRITE (15,3210)"EY-MX=";Z0;" +";Z1;" -";Z2;" =";Z3
3385 A=-Q0*E[1]*SIN(E1)
3390 B=Q0*E[1]*COS(E1)-P0*E[2]
3395 GOSUB 3055
3400 Z[2,5]=Z4
3405 A=-(P0*M[1]+Q0*M[2]*COS(M1))
3410 B=Q0*M[2]*SIN(M1)
3415 GOSUB 3055
3420 Z[2,6]=Z4
3425 Z5=W[2]+Z[2,5]-Z[2,6]
3430 GOSUB 3100
3435 Z[2,4]=Z5
3440 Z0=(W[2]/PI)*180
3445 Z1=(Z[2,5]/PI)*180
3450 Z2=(Z[2,6]/PI)*180
3455 Z3=(Z[2,4]/PI)*180
3460 WRITE (15,3210)"EY-MX=";Z0;" +";Z1;" -";Z2;" =";Z3
3465 PRINT
3470 REM....TEST EQUATIONS FOR (EY-MZ).
3475 A=Q0*E[3]*SIN(E2)
3480 B=P0*E[2]+Q0*E[3]*COS(E2)
3485 GOSUB 3055
3490 Z[3,2]=Z4
3495 A=Q0*M[2]*COS(M2)-P0*M[3]
3500 B=Q0*M[2]*SIN(M2)
3505 GOSUB 3055
3510 Z[3,3]=Z4
3515 Z5=W[3]+Z[3,2]-Z[3,3]
3520 GOSUB 3100
3525 Z[3,1]=Z5
3530 Z0=(W[3]/PI)*180
3535 Z1=(Z[3,2]/PI)*180
3540 Z2=(Z[3,3]/PI)*180
3545 Z3=(Z[3,1]/PI)*180
```

```
3550 WRITE (15,3210)"EY-MZ=";Z0;" +";Z1;" -";Z2;" =";Z3
3555 A=-P0*E[3]*SIN(E2)
3560 B=Q0*E[2]-P0*E[3]*COS(E2)
3565 GOSUB 3055
3570 Z[3,5]=Z4
3575 A=-(P0*M[2]*COS(M2)+Q0*M[3])
3580 B=-P0*M[2]*SIN(M2)
3585 GOSUB 3055
3590 Z[3,6]=Z4
3595 Z5=W[4]+Z[3,5]-Z[3,6]
3600 GOSUB 3100
3605 Z[3,4]=Z5
3610 Z0=(W[4]/PI)*180
3615 Z1=(Z[3,5]/PI)*180
3620 Z2=(Z[3,6]/PI)*180
3625 Z3=(Z[3,4]/PI)*180
3630 WRITE (15,3210)"EY-MZ=";Z0;" +";Z1;" -";Z2;" =";Z3
3635 PRINT
3640 REM....TEST EQUATIONS FOR (EZ-MY).
3645 A=-P0*E[2]*SIN(E2)
3650 B=P0*E[2]*COS(E2)+Q0*E[3]
3655 GOSUB 3055
3660 Z[4,2]=Z4
3665 A=Q0*M[2]-P0*M[3]*COS(M2)
3670 B=P0*M[3]*SIN(M2)
3675 GOSUB 3055
3680 Z[4,3]=Z4
3685 Z5=W[3]+Z[4,2]-Z[4,3]
3690 GOSUB 3100
3695 Z[4,1]=Z5
3700 Z0=(W[3]/PI)*180
3705 Z1=(Z[4,2]/PI)*180
3710 Z2=(Z[4,3]/PI)*180
3715 Z3=(Z[4,1]/PI)*180
3720 WRITE (15,3210)"EZ-MY=";Z0;" +";Z1;" -";Z2;" =";Z3
3725 A=-Q0*E[2]*SIN(E2)
3730 B=Q0*E[2]*COS(E2)-P0*E[3]
3735 GOSUB 3055
3740 Z[4,5]=Z4
3745 A=-(P0*M[2]+Q0*M[3]*COS(M2))
3750 B=Q0*M[3]*SIN(M2)
3755 GOSUB 3055
3760 Z[4,6]=Z4
3765 Z5=W[4]+Z[4,5]-Z[4,6]
3770 GOSUB 3100
3775 Z[4,4]=Z5
3780 Z0=(W[4]/PI)*180
3785 Z1=(Z[4,5]/PI)*180
3790 Z2=(Z[4,6]/PI)*180
3795 Z3=(Z[4,4]/PI)*180
3800 WRITE (15,3210)"EZ-MY=";Z0;" +";Z1;" -";Z2;" =";Z3
3805 PRINT
3810 REM....TEST EQUATIONS FOR (EZ-MX).
3815 A=Q0*E[1]*SIN(E3)
3820 B=P0*E[3]+Q0*E[1]*COS(E3)
3825 GOSUB 3055
3830 Z[5,2]=Z4
3835 A=Q0*M[3]*COS(M3)-P0*M[1]
3840 B=Q0*M[3]*SIN(M3)
3845 GOSUB 3055
3850 Z[5,3]=Z4
3855 Z5=W[5]+Z[5,2]-Z[5,3]
3860 GOSUB 3100
3865 Z[5,1]=Z5
3870 Z0=(W[5]/PI)*180
3875 Z1=(Z[5,2]/PI)*180
3880 Z2=(Z[5,3]/PI)*180
3885 Z3=(Z[5,1]/PI)*180
3890 WRITE (15,3210)"EZ-MX=";Z0;" +";Z1;" -";Z2;" =";Z3
```

```
3895 A=-P0*E[1]*SIN(E3)
3900 B=Q0*E[3]-P0*E[1]*COS(E3)
3905 GOSUB 3055
3910 Z[5,5]=Z4
3915 A=-(P0*M[3]*COS(M3)+Q0*M[1])
3920 B=-P0*M[3]*SIN(M3)
3925 GOSUB 3055
3930 Z[5,6]=Z4
3935 Z5=W[6]+Z[5,5]-Z[5,6]
3940 GOSUB 3100
3945 Z[5,4]=Z5
3950 Z0=(W[6]/PI)*180
3955 Z1=(Z[5,5]/PI)*180
3960 Z2=(Z[5,6]/PI)*180
3965 Z3=(Z[5,4]/PI)*180
3970 WRITE (15,3210)"EZ-MX=";Z0;" +";Z1;" -";Z2;" =";Z3
3975 PRINT
3980 REM....TEST EQUATIONS FOR (EX-MZ).
3985 A=-P0*E[3]*SIN(E3)
3990 B=P0*E[3]*COS(E3)+Q0*E[1]
3995 GOSUB 3055
4000 Z[6,2]=Z4
4005 A=Q0*M[3]-P0*M[1]*COS(M3)
4010 B=P0*M[1]*SIN(M3)
4015 GOSUB 3055
4020 Z[6,3]=Z4
4025 Z5=W[5]+Z[6,2]-Z[6,3]
4030 GOSUB 3100
4035 Z[6,1]=Z5
4040 Z0=(W[5]/PI)*180
4045 Z1=(Z[6,2]/PI)*180
4050 Z2=(Z[6,3]/PI)*180
4055 Z3=(Z[6,1]/PI)*180
4060 WRITE (15,3210)"EX-MZ=";Z0;" +";Z1;" -";Z2;" =";Z3
4065 A=-Q0*E[3]*SIN(E3)
4070 B=Q0*E[3]*COS(E3)-P0*E[1]
4075 GOSUB 3055
4080 Z[6,5]=Z4
4085 A=-(P0*M[3]+Q0*M[1]*COS(M3))
4090 B=Q0*M[1]*SIN(M3)
4095 GOSUB 3055
4100 Z[6,6]=Z4
4105 Z5=W[6]+Z[6,5]-Z[6,6]
4110 GOSUB 3100
4115 Z[6,4]=Z5
4120 Z0=(W[6]/PI)*180
4125 Z1=(Z[6,5]/PI)*180
4130 Z2=(Z[6,6]/PI)*180
4135 Z3=(Z[6,4]/PI)*180
4140 WRITE (15,3210)"EX-MZ=";Z0;" +";Z1;" -";Z2;" =";Z3
4145 PRINT
4150 PRINT
4155 B1=2*K0*(E[1]*M[2]*SIN(Z[1,1])-E[2]*M[1]*SIN(Z[2,1]))
4160 B2=2*K0*(E[2]*M[3]*SIN(Z[3,1])-E[3]*M[2]*SIN(Z[4,1]))
4165 B3=2*K0*(E[3]*M[1]*SIN(Z[5,1])-E[1]*M[3]*SIN(Z[6,1]))
4170 WRITE (15,4175)"BXY-BYX=";B1;"   BYZ-BZY=";B2;"   BZX-BXZ=";B3
4175 FORMAT 3E12.4
4180 PRINT
4185 PRINT
4190 RETURN
4195 STOP
4200 REM BEGIN THE MEASUREMENT PROCEDURE AS OUTLINED IN THIS NBS TECH NOTE.
4205 DISP "PLACE EUT IN TEM CELL AT CENTER"
4210 BEEP
4215 STOP
4220 DISP "INPUT TEM CELL FACTOR (E0)";
4225 BEEP
4230 INPUT E0
4235 REM ...INITIALIZE SPECTRUM ANALYZER AND SET OPERATING FREQUENCY........
```

```
4240 GOSUB 4550
4245 DISP "ALIGN X-X',Y-Y',Z-Z'";
4250 BEEP
4255 STOP
4260 DISP "ROTATE 45 DEG CCW ABOUT Z (Z')"
4265 BEEP
4270 STOP
4275 REM READ THE SPECTRUM ANALYZER AND GET PS1, PD1, AND PHASE 1.
4280 GOSUB 4690
4285 P[1]=P1
4290 O[2]=P2
4295 W[1]=P3
4300 DISP "ROTATE 90 DEG CCW ABOUT Z (Z')"
4305 BEEP
4310 STOP
4315 REM READ THE SPECTRUM ANALYZER AND GET PS2, PD2, AND PHASE 2.
4320 GOSUB 4690
4325 P[2]=P1
4330 O[1]=P2
4335 W[2]=P3
4340 DISP "ALIGN X-Y',Y-Z',Z-X'"
4345 BEEP
4350 STOP
4355 DISP "ROTATE 45 DEG CCW ABOUT Z (X')"
4360 BEEP
4365 STOP
4370 REM READ THE SPECTRUM ANALYZER AND GET PS3, PD3, AND PHASE 3
4375 GOSUB 4690
4380 P[3]=P1
4385 O[4]=P2
4390 W[3]=P3
4395 DISP "ROTATE 90 DEG CCW ABOUT Z (X')"
4400 BEEP
4405 STOP
4410 REM READ THE SPECTRUM ANALYZER AND GET PS4, PD4, AND PHASE 4.
4415 GOSUB 4690
4420 P[4]=P1
4425 O[3]=P2
4430 W[4]=P3
4435 DISP "ALIGN X-Z',Y-X',Z-Y'"
4440 BEEP
4445 STOP
4450 DISP "ROTATE 45 DEG CCW ABOUT Z (Y')"
4455 BEEP
4460 STOP
4465 REM READ THE SPECTRUM ANALYZER AND GET PS5, PD5, AND PHASE 5.
4470 GOSUB 4690
4475 P[5]=P1
4480 O[6]=P2
4485 W[5]=P3
4490 DISP "ROTATE 90 DEG CCW ABOUT Z (Y')"
4495 BEEP
4500 STOP
4505 REM READ THE SPECTRUM ANALYZER AND GET PS6, PD6, AND PHASE 6.
4510 GOSUB 4690
4515 P[6]=P1
4520 O[5]=P2
4525 W[6]=P3
4530 REM......
4535 REM THIS CONCLUDES THE GATHERING OF DATA FOR THE EMISSIONS CHARACTERIZATION
4540 RETURN
4545 END
4550 REM THIS ROUTINE INITIALIZES THE SPECTRUM ANALYZER.........................
4555 REM THIS IS WHERE THE NECESSARY FREQUENCY INFORMATION IS GATHERED..........
4560 CMD "?U2"
4565 FORMAT 3B
4570 OUTPUT (13,4565)256,4,512
4575 CMD "?U2","VB 10 KZ FA 2 MZ FB 200 MZ M2 EK"
4580 WAIT 500
```

```
4585 CMD "?U2","E1"
4590 DISP "MOVE MARKER TO DESIRED FREQUENCY";
4595 STOP
4600 CMD "?U2","MT1"
4605 WAIT 500
4610 CMD "?U2","SP 2 MZ RB 30 KZ HD"
4615 WAIT 500
4620 CMD "?U2","MF"
4625 CMD "?R5"
4630 ENTER (13,*)F0
4635 CMD "?U2","M2,E1"
4640 FIXED 2
4645 PRINT "FREQUENCY=";(F0/1E+06);" MHZ"
4650 FLOAT 2
4655 DISP "INPUT AMPLITUDE THRESHOLD (DBM)";
4660 BEEP
4665 INPUT Z1
4670 PRINT
4675 RETURN
4680 STOP
4685 END
4690 REM READ THE AMPLITUDE INFORMATION FROM THE SPECTRUM ANALYZER.............
4695 REM......
4700 REM......
4705 DISP "SET COAX SWITCH TO THE SUM PORT";
4710 STOP
4715 CMD "?U2"," E1,MA"
4720 CMD "?R5"
4725 ENTER (13,*)P1
4730 DISP "SET COAX SWITCH TO DIFFERENCE";
4735 STOP
4740 CMD "?U2","E1 MA"
4745 CMD "?R5"
4750 ENTER (13,*)P2
4755 DISP "INPUT PHASE (SUM-DIFF)";
4760 INPUT P3
4765 FIXED 4
4770 PRINT "PS=";P1;"DBM.    PD=";P2;"DBM  THRESHOLD=";Z1
4775 PRINT
4780 PRINT "WITH A CORRECTION FOR LOSSES, THE POWER LEVELS ARE;"
4785 IF P1<Z1 THEN 4800
4790 P1=P1+3.6845
4795 GOTO 4805
4800 P1=0
4805 IF P2<Z1 THEN 4820
4810 P2=P2+3.8345
4815 GOTO 4825
4820 P2=0
4825 PRINT
4830 IF P1=0 THEN 4840
4835 P1=10↑((P1/10)-3)
4840 IF P2=0 THEN 4850
4845 P2=10↑((P2/10)-3)
4850 FLOAT 10
4855 PRINT "PS=";P1;"W         PD=";P2;"W"
4860 PRINT
4865 FIXED 6
4870 PRINT "PHASE =";P3;"DEG      ";(P3/180)*PI;" RAD"
4875 P3=(P3/180)*PI
4880 PRINT
4885 PRINT
4890 PRINT
4895 RETURN
4900 STOP
4905 END
4910 REM....READ IN POWER AND PHASE DATA, SUM 1-6, DIFFERENCE 1-6, PHI 1-6.
4915 DATA 4.8978E-06,3.4674E-08,7.0795E-06
4920 DATA 4.1687E-07,7.9433E-06,2.2387E-07
4925 DATA 9.7724E-09,3.02E-09,1.2023E-08
```

```
4930 DATA 3.3113E-10,1.4125E-08,1.5488E-09
4935 READ P[1],P[2],P[3],P[4],P[5],P[6]
4940 READ Q[2],Q[1],Q[4],Q[3],Q[6],Q[5]
4945 DATA 96.4976,-102.7023,103.6022
4950 DATA -144.5,-80.4027,-91.1973
4955 READ W[1],W[2],W[3],W[4],W[5],W[6]
4960 RETURN
4965 END
```

A SAMPLE OUTPUT FROM THE ABOVE PROGRAM.

```
FREQUENCY= 30.0000    MHZ.  CELL FACTOR E0= 11.8300


THE SUM POWERS ARE;
PS1= 4.8978E-06    PS2= 3.4674E-08    PS3= 7.0795E-06
PS4= 4.1687E-07    PS5= 7.9433E-06    PS6= 2.2387E-07


THE DIFFERENCE POWERS ARE;
PD1= 9.7724E-09    PD2= 3.0200E-09    PD3= 1.2023E-08
PD4= 3.3113E-10    PD5= 1.4125E-08    PD6= 1.5488E-09


THE PHASE ANGLES, SUM TO DIFFERENCE POWERS ARE (1-6);
 96.4976    -102.7023    103.6022    -144.5000    -80.4027    -91.1973
DEGREES
 1.6842  -1.7925   1.8082  -2.5220  -1.4033  -1.5917  RADIANS


THE ELECTRIC DIPOLE MOMENTS ARE;
MEX= 1.4149E-04    MEY= 1.2339E-04    MEZ= 1.9580E-04


THE MAGNETIC DIPOLE MOMENTS ARE;
MMX= 1.2075E-05    MMY= 9.2589E-06    MMZ= 1.1742E-05


THE ELECTRIC DIPOLE MOMENT CROSS TERMS ARE;
MEXY= 1.7375E-08    MEYZ= 2.3804E-08    MEZX= 2.7579E-08


THE MAGNETIC DIPOLE MOMENT CROSS TERMS ARE;
MMXY=-6.1108E-11    MMYZ=-1.0581E-10    MMZX=-1.1381E-10


AX↑2= 2.0077E-08    AY↑2= 1.5260E-08    AZ↑2= 3.8394E-08

A XY= 1.7351E-08    A YZ= 2.3762E-08    A ZX= 2.7535E-08


THE TOTAL RADIATED POWER IN FREE SPACE IS 2.9107E-07    WATTS


E1,M1= 5.6240   123.1319
E2,M2= 9.8623   166.7139
E3,M3= 5.4354   143.3872
```

```
FOR E1=-5.6240     M1=
POSSIBLE ANGLES 4.5135   , 175.4865     CHOSEN= 175.4865

FOR E2= 9.8623     M2=
POSSIBLE ANGLES 15.6733     , 164.3267     CHOSEN= 164.3267

FOR E3=-5.4354     M3=
POSSIBLE ANGLES-0.1456   ,-179.8544     CHOSEN=-179.8544

E1,M1=-5.6240   175.4865
E2,M2= 9.8623   164.3267
E3,M3=-5.4354  -179.8544

E1,M1=-5.6240   4.5135
E2,M2= 9.8623   164.3267
E3,M3=-5.4354  -179.8544

EX-MY=   96.498 +   -2.620 -   71.122 =   22.756
EX-MY= -102.702 +   32.903 -  267.445 =   22.756

EY-MX=   96.498 +    3.004 -   75.635 =   23.866
EY-MX= -102.702 +   38.527 -  -88.041 =   23.866

EY-MZ=  103.602 +    6.052 -  -83.096 = -167.250
EY-MZ= -144.500 +  205.754 -  228.504 = -167.250

EZ-MY=  103.602 +   -3.810 -   81.231 =   18.561
EZ-MY= -144.500 +  195.892 -   32.831 =   18.561

EZ-MX=  -80.403 +   -2.280 -  269.928 =    7.389
EZ-MX=  -91.197 +   13.706 -  -84.880 =    7.389

EX-MZ=  -80.403 +    3.156 -   90.074 = -167.321
EX-MZ=  -91.197 +   19.142 -   95.265 = -167.321


BXY-BYX= -1.2081E-10   BYZ-BZY= -1.1270E-09   BZX-BXZ=  8.4037E-10

PLEASE BE PATIENT WHILE THE RADIATION PATTERN IS CALCULATED......


THE MAXIMUM POWER RADIATED FOR ONE DIRECTION= 3.8429E-09    WATTS/M↑2


THE RADIATION PATTERN FOR PHI= 45.00   DEG AND R= 3.00    METERS
HAS THE FOLLOWING MAXIMUM AND RELATIVE MAXIMUMS;

THETA DEGREES       RADIATED POWER (DB WATTS BELOW MAX)

          134             -0.01
          314              0.00


THE END
```

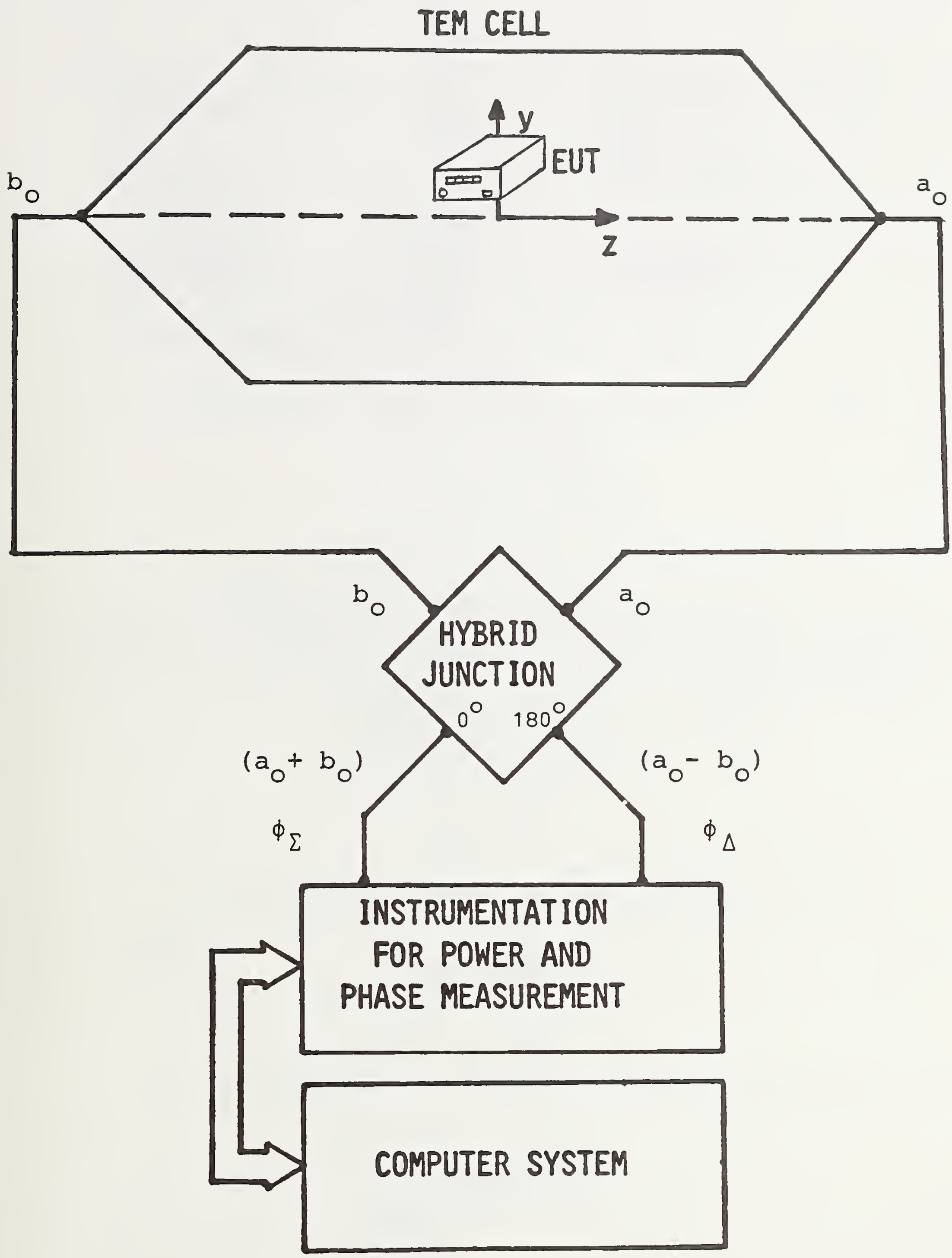

Figure 1. Emissions testing measurement system.

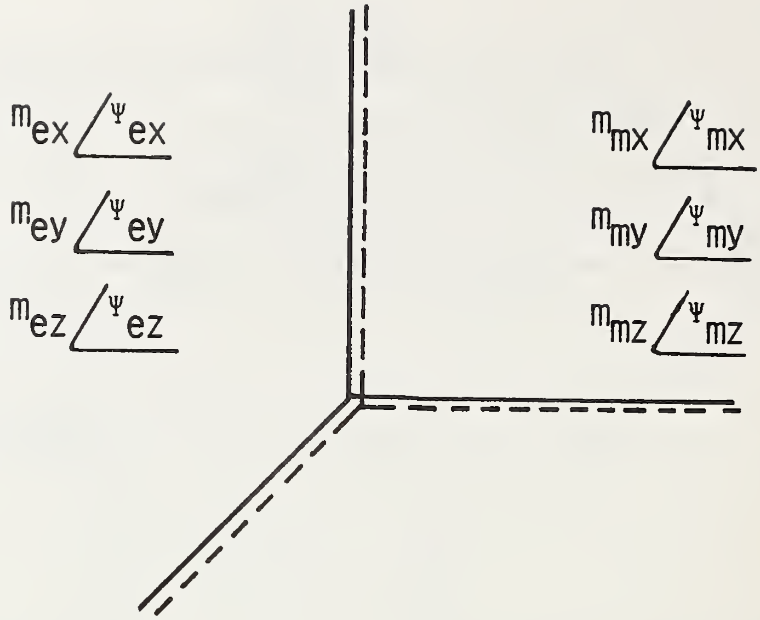

Figure 2. Unknown equipment under test (EUT) is made of equivalent three orthogonal electric and three orthogonal magnetic dipoles.

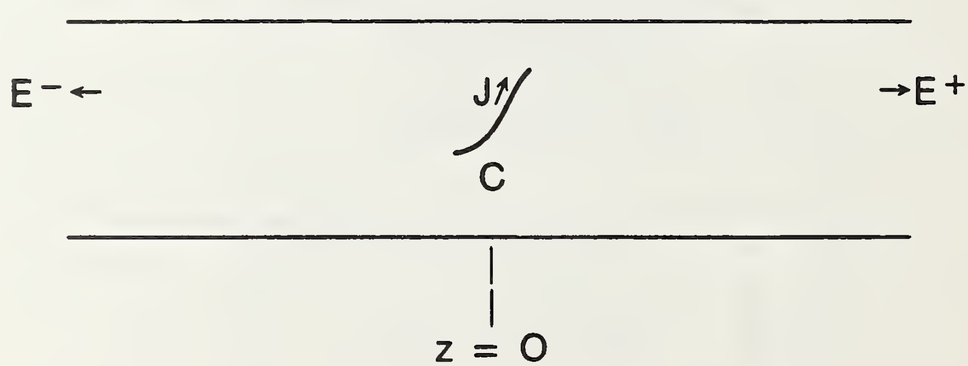

Figure 3. An arbitrary current source inside one half of a TEM cell.

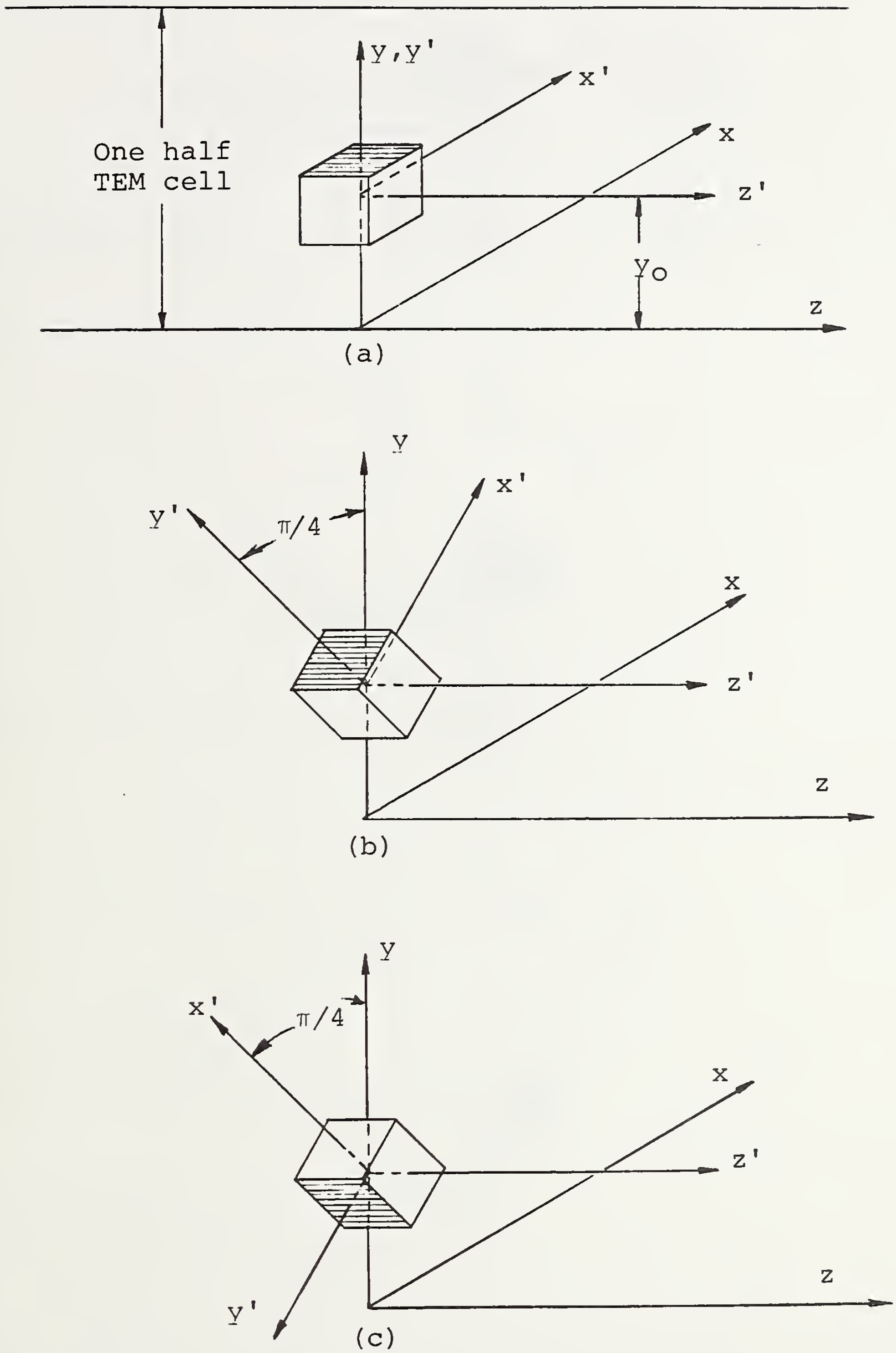

Figure 4. Two EUT orientations in the TEM cell.

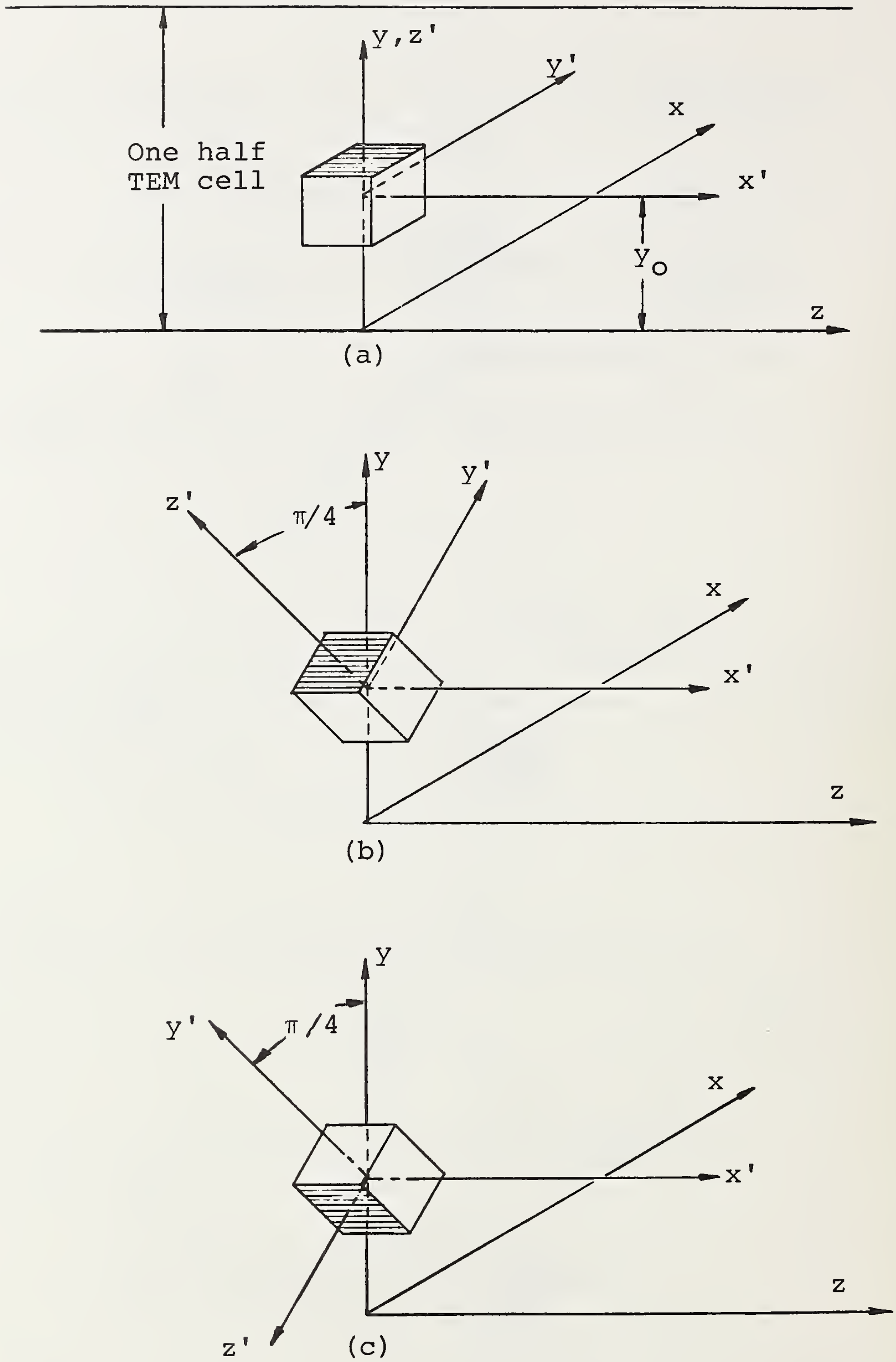

Figure 5. Another two EUT orientations in the TEM cell.

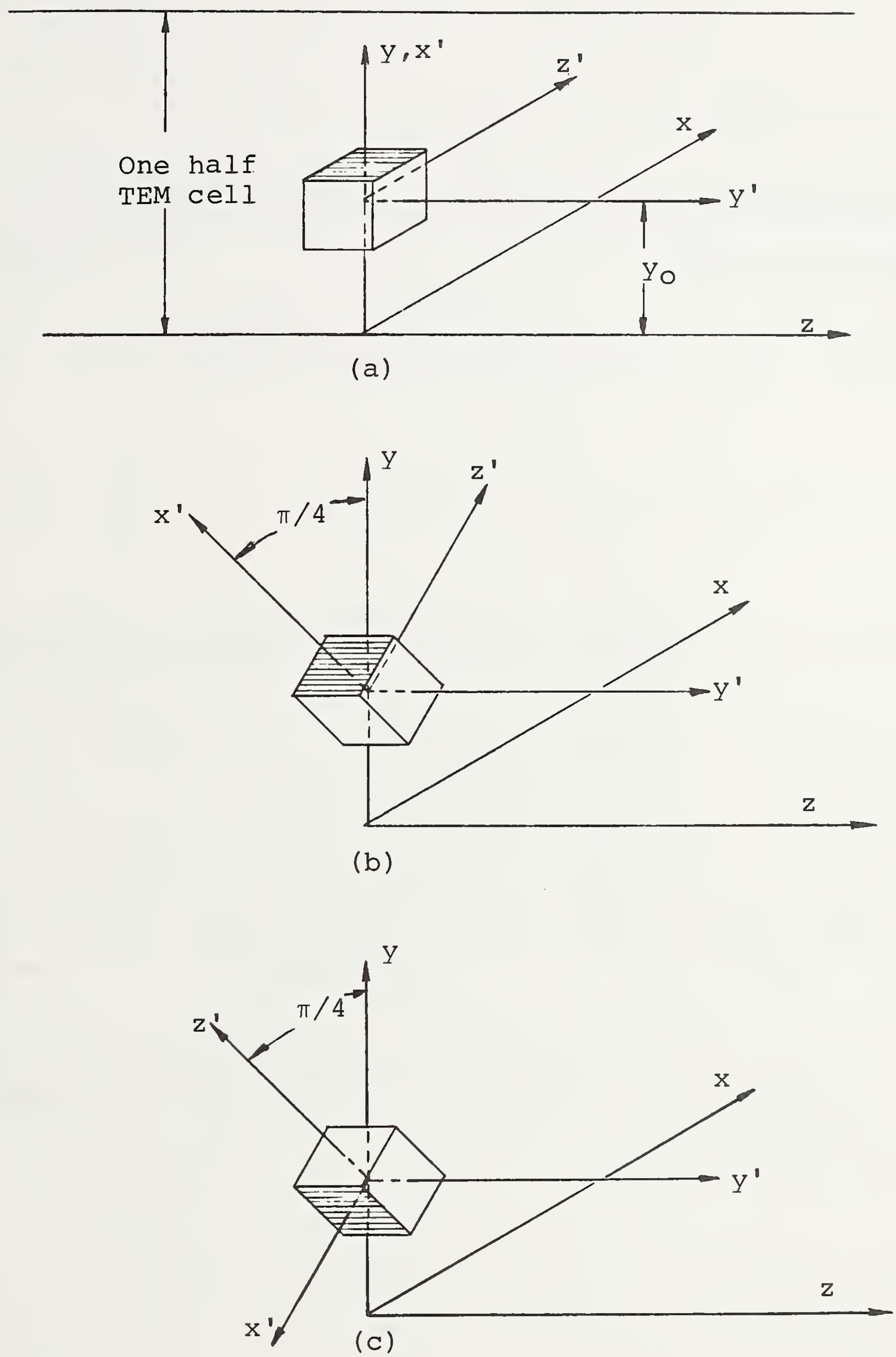

Figure 6. Final two EUT orientations in the TEM cell.

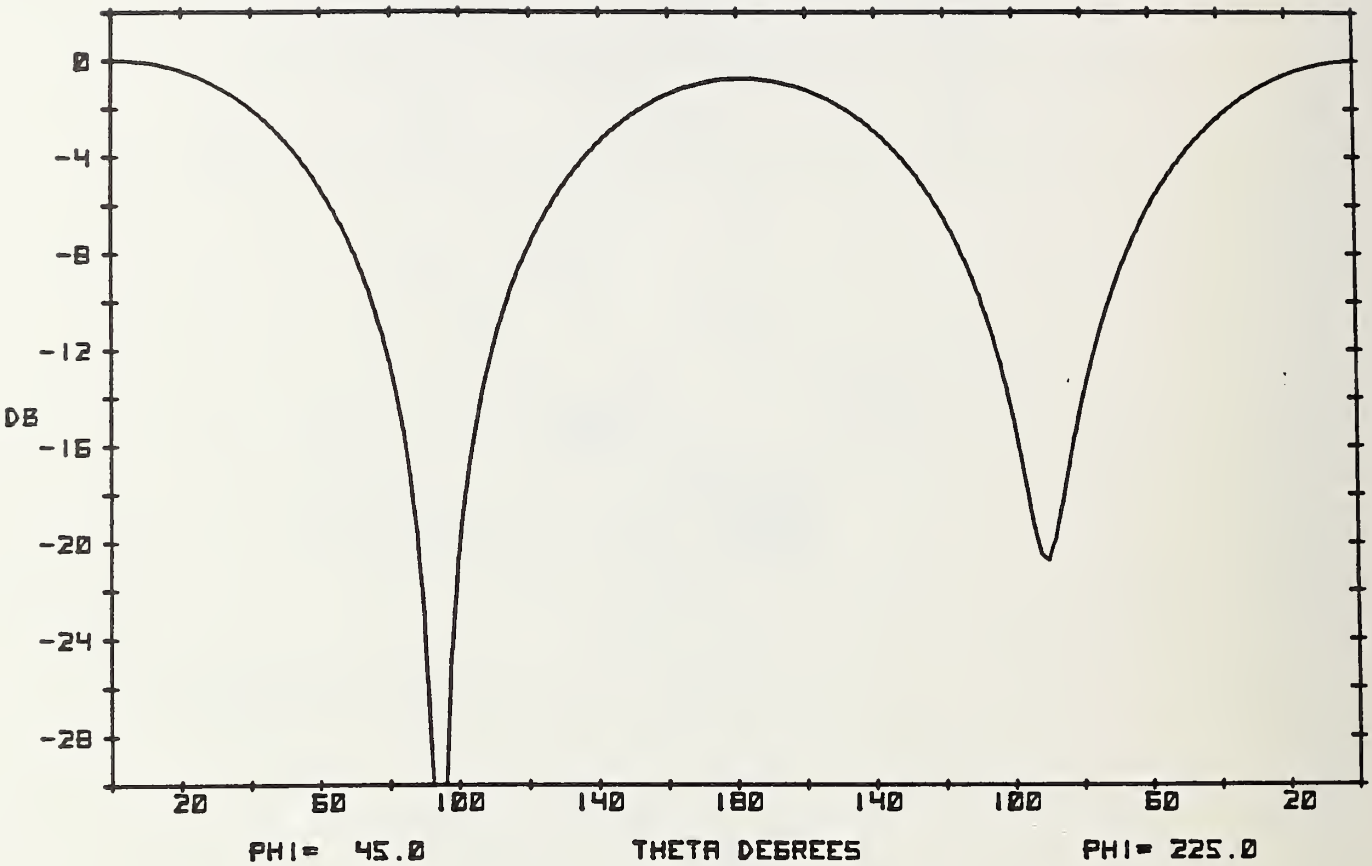

Figure 7. Calculated power pattern in the phi = 45°, 225° plane for a spherical dipole radiator, based on measured data.

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A Method to Quantify the Radiation Characteristics of an Unknown Interference Source

5. AUTHOR(S)

Mark T. Ma and Galen H. Koepke

6. PERFORMING ORGANIZATION *(If joint or other than NBS, see instructions)*

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10. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

11. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)*

A new method for determining the radiation characteristics of leakage from electronic equipment for interference studies is described in this report. Basically, an unintentional leakage source is considered to be electrically small, and may be characterized by three equivalent orthogonal electric dipole moments and three equivalent orthogonal magnetic dipole moments. When an unkonown source object is placed at the center of a transverse electromagnetic (TEM) cell, its radiated energy couples into the fundamental transmission mode and propagates toward the two output ports of the TEM cell. With a hybrid junction inserted into a loop connecting the cell output ports, one is able to measure the sum and difference powers and the relative phase between the sum and difference outputs. Systematic measurements of these powers and phases at six different source object positions, based on a well-developed theory, are sufficient to determine the amplitudes and phases of the unknown component dipole moments, from which the detailed free-space radiation pattern of the unknown source and the total radiated power can be determined. Results of simulated theoretical examples and an experiment using a spherical dipole radiator are given to illustrate the theory and measurement procedure.

12. KEY WORDS *(Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)*

dipole moments; electrically small; interference source; leakage; phase measurements; power measurements; radiation pattern; TEM cell; total radiated power.

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