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U.S. DEPARTMENT OF COMMERCE/ National Bureau of Standards

Excitation of a TEM Cell by a Vertical Electric Hertzian Dipole

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Excitation of a TEM Cell by a Vertical Electric Hertzian Dipole

Perry F. Wilson*
David C. Chang*
Mark T. Ma**

*University of Colorado
Boulder, Colorado 80309

**Electromagnetic Fields Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

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FOREWORD

This report describes theoretical and experimental analyses developed by staff of the University of Colorado at Boulder in collaboration with the Electromagnetic Fields Division of the National Bureau of Standards (NBS), under a contract sponsored by NBS. Professor David C. Chang heads the University team. Dr. Mark T. Ma of NBS serves as the technical contract monitor. The period covered by this report extends from July 1979 to July 1980.

The work described in this report represents a further aspect of establishing a theoretical basis for the technical analyses of transverse electromagnetic (TEM) transmission line cells developed at NBS. The general purpose of pursuing theoretical studies is to evaluate the use of TEM cells for (1) measuring the total rf radiated power by a device inserted into the cell for test, or (2) performing necessary susceptibility tests on a small electronic device.

The particular topic addressed herein discusses the manner in which a vertical electrical Hertzian dipole excites a TEM cell under the assumption that the gap between the septum and the side wall is small. This premise is consistent with the cell geometries designed to provide a good impedance match, presently of interest to NBS, and leads to significant simplifications in the mathematical derivation. The formation of the problem also allows a vertical offset for the septum position so that the size of the test area may be increased to accommodate larger pieces of test equipment.

Approximate expressions are found for the field distribution inside a TEM cell and for its characteristic impedance. The latter result consists of a dominant gap dependent logarithmic term, plus a correction series that accounts for the vertical offset of the septum. This vertical offset is allowed to be arbitrary, and therefore the results contained in this report will supplement previous efforts which were restricted to small offsets. In addition, because of the delta function nature of the source considered here, the results lend themselves naturally to an analysis of more complex source configurations such as a practical monopole via Greens function methods. The monopole result may then be used to model probes inserted into a TEM cell to measure or excite fields.

Previous publications under the same effort include:

Tippett, J. C. and Chang, D. C., Radiation characteristics of dipole sources located inside a rectangular coaxial transmission line, NBSIR 75-829 (Jan. 1976).

Tippett, J. C., Chang, D. C., and Crawford, M. L., An analytical and experimental determination of the cut-off frequencies of higher-order TE modes in a TEM cell, NBSIR 76-841 (June 1976).

Tippett, J. C. and Chang, D. C., Higher-order modes in rectangular coaxial line with infinitely thin inner conductor, NBSIR 78-873 (March 1978).

Sreenivasiah, I. and Chang, D. C., A variational expression for the scattering matrix of a coaxial line step discontinuity and its application to an over moded coaxial TEM cell, NBSIR 79-1606 (May 1979).

Tippett, J. C. and Chang, D. C., Dispersion and attenuation characteristics of modes in a TEM cell with a lossy dielectric slab, NBSIR 79-1615 (Aug. 1979).

Sreenivasiah, I., Chang, D. C., and Ma, M. T., Characterization of electrically small radiating sources by tests inside a transmission line cell, NBS Tech Note 1017 (Feb. 1980).

EXCITATION OF A TEM CELL BY A VERTICAL ELECTRIC HERTZIAN DIPOLE

Perry F. Wilson
David C. Chang
and
Mark T. Ma

The excitation of a transverse electromagnetic (TEM) cell by a vertical electric Hertzian dipole is analyzed where the gap between the septum and side wall is assumed to be small. Approximate expressions for the field distribution and characteristic impedance are derived. These expressions are numerically evaluated for some typical geometries, and good agreement with previously published results is shown. The formation also allows a vertical offset for the septum position, thus offering more flexibility of increasing the size of the test area to accommodate larger pieces of test equipment.

Key words: Characteristic impedance; field distribution; Hertzian dipole; integral equation; rectangular coaxial transmission line; TEM cell.

1. INTRODUCTION

Increased levels of electromagnetic radiation in the environment have made desirable the ability to predict the effect of low level interference on an electronic device. Conversely, the device itself may act as a source of low level radiation contributing additional electromagnetic pollution. Thus the electromagnetic interference (EMI) community is interested in testing both the susceptibility, and emission properties of electronic equipment. One approach in use at the National Bureau of Standards (NBS) is the transverse electromagnetic (TEM) cell which provides an isolated, shielded environment to perform such testing. Basically, the TEM cell, as shown in figure 1, consists of a section of rectangular coaxial transmission line (RCTL) coupled at each end to standard 50Ω coaxial line by a tapered section. The results obtained inside a TEM cell may then be related to the free space environment via a result given by Tippet [1, p. 61], and verified experimentally by Crawford [2].

This report is concerned with analyzing the fields excited in the TEM cell by a vertical electric Hertzian dipole. We view this as the first step toward solving more complex problems, such as the excitation of the TEM cell due to a vertical monopole. The Hertzian dipole result may be used as the Green's function in formulating a variational integral for the input impedance of the monopole, an approach employed by Collin [3, pp. 258-261] in treating the excitation of a rectangular waveguide by a coaxial-line probe. The monopole result may then be used to model probes inserted into the cell to measure or excite fields.

Our analysis assumes the RCTL to be of infinite extent with perfectly conducting walls, so in effect we are analyzing the cross section depicted in figure 2.

If we view the cell as two rectangular guides coupled through a pair of apertures, then in each separate guide we expect the final field to be made of the ordinary rectangular waveguide modes, plus a perturbation field due to the presence of the gaps. The method of solution is to formulate integral equations in section 2 for the unknown aperture field. There are two types of mode presentations; namely, the transverse electric (TE or h-type) and the transverse magnetic (TM or e-type). Detailed derivations for the more important TE type are presented in section 2.2 with the corresponding derivations for the TM type given in Appendix A. A simplified approximate expression for the TE integral equation is then given in section 3 by assuming an electrically small gap. The solution of this approximate integral equation is presented in section 4. More specifically, we will assume that $G^2 \ell n(G)$ is much less than unity, where G is some normalized gap size. The TEM cells presently in use at NBS are consistent with this assumption. In formulating the problem we will allow the center conductor, or septum, to be vertically offset but centered horizontally. A horizontal offset could also be treated at the expense of additional burdensome mathematics, but offers no practical advantage. Therefore, it is not treated in this report.

Of special concern is the manner in which the vertical Hertzian dipole excites the TEM mode. We give expressions in section 5 for both the electric field distribution of the TEM mode in the cell, and the transmission line characteristics of an RCTL. Sample numerical results are given in section 6 with the relevant FORTRAN programs listed in Apprndices D and E. These results may be compared to those of Tippet [1], obtained via different methods. Using the Schwarz-Christoffel transformation, he solves for the electric field distribution in an RCTL with a centrally located septum. He also finds expressions for the characteristic impedance of an RCTL using an integral equation approach. In both cases his solutions are expressed in terms of Jacobian elliptic functions, whereas our solutions involve infinite sums of more common functions. Over ranges where both Tippet's expressions and ours are applicable, the agreement is excellent. Together they provide information about TEM cell characteristics for a large class of problems of practical interest. In fact, most situations may be treated using simplified special case solutions (e.g., small gap, small septum width, etc.).

2. FORMULATION OF THE INTEGRAL EQUATION FOR THE UNKNOWN APERTURE FIELDS

2.1. Introduction of the Field Transformations

The RCTL to be analyzed is depicted in figure 2. The outer walls are of lengths 2a and 2b, and the septum is of width 2w. The septum is located a distance g from the side walls, and is displaced vertically by an amount b_1 from the upper wall, and b_2 from the lower wall. The regions above and below the septum will be designated by the superscripts (1) and (2) respectively. The guide is filled with a homogeneous dielectric with permittivity ε_0, and permeability μ_0.

In order to simplify the later analysis, we will remove the z-dependence of field expressions via the Fourier transform pair defined by

$$F(\bar{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}\,(\bar{x}_t) e^{\,i\alpha z}\, d\alpha \tag{1a}$$

$$\tilde{F}(\bar{x}_t) = \int_{-\infty}^{\infty} F(\bar{x}) e^{-i\alpha z} dz \tag{1b}$$

where

$$\bar{x} = (x, y, z) \tag{1c}$$

$$\bar{x}_t = (x, y) \tag{1d}$$

the subscript t refers to the transverse plane, and α is the propagation constant. In addition, we assume time variation according to exp $(-i\omega t)$. Thus we may solve for TE and TM type modes by finding the transformed z-components of the magnetic and electric fields. The TEM mode will then be seen as a special case of each mode type.

In terms of the above transforms we find that $\tilde{H}_z(\bar{x}_t)$ and $\tilde{E}_z(\bar{x}_t)$ satisfy the following wave equations

$$(\nabla_t^2 + \zeta^2)\tilde{H}_z(\bar{x}_t) = -\partial_x \tilde{J}_y\,(\bar{x}_t) \tag{2a}$$

$$(\nabla_t^2 + \zeta^2)\,\tilde{E}_z\,(\bar{x}_t) = \frac{\alpha}{\omega\varepsilon_0}\;\partial_y\,\tilde{J}_y\,(\bar{x}_t) \tag{2b}$$

$$\zeta^2 = k_0^2 - \alpha^2 \tag{2c}$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the free space wave number, $\partial_x = \frac{\partial}{\partial x}$, $\partial_y = \frac{\partial}{\partial y}$, $\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and we have assumed that the current density ($\bar{J}$) has only a vertical component.

For the TE case, the remaining field components are related to $\tilde{H}_z(\bar{x}_t)$ via

$$E_t(\bar{x}_t) = \frac{i\omega\mu_0}{\zeta^2}\;\nabla_t\,\tilde{H}_z\,(\bar{x}_t) \times \bar{a}_z,\ \bar{a}_z = \text{unit vector} \tag{3a}$$

$$\tilde{H}_t(\bar{x}_t) = \frac{i\alpha}{\zeta^2}\,\nabla_t\,\tilde{H}_z(\bar{x}_t) \tag{3b}$$

and for the TM case the transverse field components are given by

$$\tilde{E}_t(\bar{x}_t) = \frac{i\alpha}{\zeta^2}\,\nabla_t\,\tilde{E}_z(\bar{x}_t) \tag{3c}$$

$$\tilde{H}_t(\bar{x}_t) = \frac{-i\omega\varepsilon_0}{\zeta^2}\,\nabla_t\,\tilde{H}_z(\bar{x}_t) \times \bar{a}_z \tag{3d}$$

Boundary conditions for $\tilde{H}_z(\bar{x}_t)$ and $\tilde{E}_z(\bar{x}_t)$ on the guide walls follow from our assumption of perfect conductors. Thus we require that on the guide walls

$$\partial_n\,\tilde{H}_z(\bar{x}_t) = 0,\ \partial_n = \text{normal derivative}\quad \text{(TE)} \tag{4a}$$

$$\tilde{E}_z(\bar{x}_t) = 0 \qquad \text{(TM)} \qquad (4b)$$

These then are the basic equations satisfied by the transformed fields.

2.2. Integral Equation for the TE Gap Field

In order to find expressions for $\tilde{H}_z(\bar{x}_t)$ and $\tilde{E}_z(\bar{x}_t)$ in the cell, our basic approach will be to expand these longitudinal fields in terms of the guide modes in the upper and lower chambers. By guide modes we mean the TE and TM type rectangular waveguide modes which would exist in each region if there were no gap. If we expressed each cell mode as the doubly infinite sum of the corresponding guide modes in both the upper and lower chambers, and then represented the fields excited by the dipole as doubly infinite sums of the cell modes, the resulting expressions for $\tilde{H}_z(\bar{x}_t)$ and $\tilde{E}_z(\bar{x}_t)$ would involve extremely cumbersome infinite quadruple sums. However, by not attempting to exhibit the cell modes explicitly, we avoid a pair of infinite sums, and instead replace them with an infinite integral. This offers some flexibility. The excitation of individual modes will appear as poles, and therefore may be found from residue calculations. However, as it will become apparent later, a field component may be calculated efficiently by numerically evaluating an infinite integral without looking at its modal composition. The TE formulation (h-type) will be given here in some detail. A similar TM derivation is given in Appendix A. Later it will be shown that the TM contribution to the dominant TEM mode is small compared to the TE contribution.

We begin by introducing a Green's function $\tilde{G}^{(h)}(\bar{x}_t, \bar{x}_t')$ which satisfies the following wave equation and boundary condition

$$(\nabla_t^2 + \zeta^2)\, \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') = -\,\delta(\bar{x}_t - \bar{x}_t') \qquad (5a)$$

$$j = \begin{cases} 1 \\ 2 \end{cases} \text{for the } \begin{matrix}\text{upper} \\ \text{lower}\end{matrix} \text{ chamber}$$

$$\partial_n\, \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') = 0 \qquad (5b)$$

The boundary condition for the Green's function applies to the gap as well as the septum and cell walls. In (5), we have used the unprimed coordinates for the field points and the primed coordinates for the source points. The solution to (5) is well known [1,3], and includes the following TE guide mode expansion:

$$\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') = \left(\frac{2}{ab_j}\right) \sum_{m,n=0}^{\infty} \frac{\Delta_m \Delta_n}{(K_{mn}^{(j)2} - \zeta^2)} \cos\frac{m\pi}{2a}(x+a)\, \cos\frac{m\pi}{2a}(x'+a)\, \cos\frac{n\pi y}{b_j}\, \cos\frac{n\pi y'}{b_j}, \qquad (6a)$$

where

$$K_{mn}^{(j)} = \left[\left(\frac{m\pi}{2a}\right)^2 + \left(\frac{n\pi}{b_j}\right)^2 \right]^{1/2} \qquad (6b)$$

$$\Delta_m = \begin{cases} 1/2 & m=0 \\ 1 & m\neq 0 \end{cases} \qquad (6c)$$

Equation (2a) yields a pair of equations for $\tilde{H}_z^{(j)}(\bar{x}_t)$. If we multiply (2a) by $\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t')$, multiply (5a) by $\tilde{H}_z^{(j)}(\bar{x}_t)$, and then subtract, we find

$$\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \nabla_t'^2 \tilde{H}_z^{(j)}(\bar{x}_t) - \tilde{H}_z(\bar{x}_t') \nabla_t'^2 \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t')$$

$$= - \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \partial_{x'}\tilde{J}_y(\bar{x}_t') + \tilde{H}_z^{(j)}(\bar{x}_t') \delta (\bar{x}_t - \bar{x}_t') \tag{7}$$

If we now integrate both sides over the respective guide cross sections, and apply Green's Second Integral identity [4, p. 96], eq (7) yields

$$\int_{\ell'} [\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \partial_{n'}\tilde{H}_z^{(j)}(\bar{x}_t) - \tilde{H}_z^{(j)}(\bar{x}_t') \partial_{n'}\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t')] d\ell'$$

$$= \tilde{H}_z^{(j)}(\bar{x}_t) - \int_{s'} \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \partial_{x'}\tilde{J}_y (\bar{x}_t') ds' \tag{8}$$

where s' is the respective guide cross section, and ℓ' its boundary. Equation (8) may be simplified by applying the boundary conditions for $\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t')$ (5b) and $\tilde{H}_z^{(j)}(\bar{x}_t')$ (4a). An additional application of (5b) allows the integral on the right-hand side of (8) to be integrated by parts. Thus we find (8) reduces to

$$\int_{gaps} \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \partial_n' \tilde{H}_z^{(j)}(\bar{x}_t') d\ell$$

$$= \tilde{H}_z^{(j)}(\bar{x}_t) + \int_{s'} \tilde{J}_y (\bar{x}_t') \partial_{x'} \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') ds' \tag{9}$$

The RCTL is to be excited by an elementary vertical dipole located at a point $\bar{x}_o = (x_o, y_o)$ in the z=0 transverse plane. The source current density is given by

$$\bar{J} (\bar{x}) = \bar{a}_y \, Id\ell \, \delta(\bar{x}_t - \bar{x}_o) \, \delta(z), \tag{10}$$

where $Id\ell$ represents the dipole moment.

If we take the transform (1b) of $\bar{J} (\bar{x})$, and then substitute $\tilde{J}_y(\bar{x}_t')$ into (9), we find

$$\int_{gaps} \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \partial_{n'}\tilde{H}_z^{(j)}(\bar{x}_t') d\ell'$$

$$= \tilde{H}_z^{(j)}(\bar{x}_t) + Id\ell \, \partial_{x'}\tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t')\Big|_{\bar{x}_t' = \bar{x}_o} \tag{11}$$

We now require that the tangential fields be continuous across the gap. From (3a) and (3b) we see that $\tilde{H}_z(\bar{x}_t)$, and $\partial_y\tilde{H}_z(\bar{x}_t)$ must be continuous across the gap. The second condition may be imposed by dropping the superscript in the left-hand side integral, and the first by equating $\tilde{H}_z^{(1)}(x,o)$ to $\tilde{H}_z^{(2)}(x,o)$. Thus noting that $\bar{n} = \pm \bar{a}_y$, $d\ell' = dx'$, and letting the dipole be in the upper chamber, we find

$$\int_{\text{gaps}} \tilde{G}^{(h)}(x,x')\, \partial_{y'} \tilde{H}_z\,(x',o)\, dx' = -Id\ell\; \partial_{x'} \tilde{G}_1^{(h)}(x,o,\bar{x}_t')\Big|_{\bar{x}_t' = \bar{x}_o}\,, \tag{12a}$$

where

$$\tilde{G}^{(h)}(x,x') = \tilde{G}_1^{(h)}(x,o,x',o) + \tilde{G}_2^{(h)}(x,o,x',o) \tag{12b}$$

We may now simplify both the Green's function and the driving term by performing an infinite sum. The Green's function is given by

$$\tilde{G}^{(h)}(x,x') = \sum_{j=1}^{2} \sum_{m,n=o}^{\infty} \left(\frac{2}{ab_j}\right) \frac{\Delta_m \Delta_n}{(K_{mn}^{(j)2} - \zeta^2)} \cos\frac{m\pi}{2a}(x+a) \cos\frac{m\pi}{2a}(x'+a) \tag{13}$$

The summation on n is known [5, p. 40] and yields

$$\sum_{n=o}^{\infty} \frac{\Delta_n}{K_{mn}^{(j)2} - \zeta^2} = -b_j \cot b_j K_m/(2K_m) \tag{14a}$$

where

$$K_m = [\zeta^2 - (\frac{m\pi}{2a})^2]^{1/2} \tag{14b}$$

and thus we may write $\tilde{G}^{(h)}(x,x')$ as follows

$$\tilde{G}^{(h)}(x,x') = -\frac{1}{a} \sum_{j=1}^{2} \sum_{m=o}^{\infty} \frac{\Delta_m \cot b_j K_m}{K_m} \cos\frac{m\pi}{2a}(x+a) \cos\frac{m\pi}{2a}(x'+a) \tag{15}$$

The driving term on the right-hand side of (12a) may similarly be simplified. We notice that

$$-\partial_{x'} \tilde{G}_1^{(h)}(x,o,\bar{x}_t') = \left(\frac{2}{ab_1}\right) \sum_{m,n=o}^{\infty} \frac{\Delta_m \Delta_n}{(K_{mn}^{(1)2} - \zeta^2)} \left(\frac{m\pi}{2a}\right)$$

$$\times \cos\frac{m\pi}{2a}(x+a) \sin\frac{m\pi}{2a}(x'+a) \cos\frac{n\pi y'}{b_1} \tag{16}$$

Again the sum on n is known [5, p. 40]

$$\sum_{n=o}^{\infty} \frac{\Delta_n \cos\frac{n\pi y'}{b_1}}{(K_{mn}^{(1)2} - \zeta^2)} = \frac{-b_1}{2} \left[\frac{\cos K_m (b_1 - y')}{K_m \sin K_m b_1}\right] \tag{17}$$

and thus, after some manipulation, the right-hand side of (12a) may be written

$$-Id\ell\; \partial_{x'} \tilde{G}_1^{(h)}(x,o,\bar{x}_t')\Big|_{\bar{x}_t' = \bar{x}_o} = -Id\ell\; g^{(h)}(x) \tag{18a}$$

$$g^{(h)}(x) = -\frac{1}{a} \sum_{m=1}^{\infty} \frac{\cos K_m (b_1 - y_o)}{K_m \sin K_m b_1} \left(\frac{m\pi}{2a}\right) \cos\frac{m\pi}{2a}(x+a) \sin\frac{m\pi}{2a}(x_o+a) \tag{18b}$$

Defining $f^{(h)}(x') = \partial_{y'} \tilde{H}_z(x',o)$, our integral equation (12) takes the more compact form

$$\int_{gaps} f^{(h)}(x')\, \tilde{G}^{(h)}(x,x')\, dx' = -Id\ell\, g^{(h)}(x) \qquad (19)$$

We next allow the dipole to be centrally located, i.e., $\bar{x}_o = (o,y_o)$, then $g^{(h)}(x)$ given in (18b) reduces to a summation over odd indices m. Therefore $g^{(h)}(x)$ is an odd function in x which implies that $\tilde{G}^{(h)}(x,x')$ is also an odd function in both x and x'. Thus we need only consider the odd component of $f^{(h)}(x')$ for the centrally located dipole. Thus, eq (19) reduces to

$$\int_w^a f^{(h)}(x')\, \tilde{G}^{(h)}(x,x')\, dx' = -\tfrac{1}{2} Id\ell\, g^{(h)}(x) \qquad (20a)$$

$$\tilde{G}^{(h)}(x,x') = -\frac{1}{a} \sum_{j=1}^{2} \sum_{mo} \frac{\cot K_m b_j}{K_m} \sin\frac{m\pi x}{2a} \sin\frac{m\pi x'}{2a} \qquad (20b)$$

$$g^{(h)}(x) = -\frac{1}{a} \sum_{mo} \frac{1}{K_m} \left(\frac{m\pi}{2a}\right) \frac{\cos K_m (b_1-y_o)}{\sin K_m b_1} \sin\frac{m\pi x}{2a} \qquad (20c)$$

where the summation over mo denotes the odd indices (m = 1,3,5···). In order to solve our integral equation (20a), our next step will be to assume the gap is small and then to find an approximate analytic expression for $f^{(h)}(x')$.

3. SMALL GAP APPROXIMATION FOR THE GREEN'S FUNCTION

The integral equation (20a) could be solved in terms of Chebyshev polynominal expansions as was done by Tippet [1]. However, this leads to an infinite system of equations in terms of less familiar special functions. The problem may be simplified considerably by assuming a small gap, and then approximating the Green's function. This will lead to a fairly simple solution in terms of common functions.

We begin by changing variables, letting t = a-x, and t' = a-x'. In terms of these gap parameters, (20) becomes

$$\int_o^g Q^{(h)}(t,t')\, f^{(h)}(t')\, dt' = Id\ell\, g^{(h)}(t), \qquad (21a)$$

where

$$Q^{(h)}(t,t') = -\frac{1}{a} \sum_{j=1}^{2} \sum_{mo} \frac{\cot K_m b_j}{K_m} \cos\frac{m\pi t}{2a} \cos\frac{m\pi t'}{2a} \qquad (21b)$$

$$f^{(h)}(t') \equiv f^{(h)}(x') \text{ when } x' = a-t' \qquad (21c)$$

$$g^{(h)}(t) = -\frac{1}{2a} \sum_{mo} \left(\frac{m\pi}{2a}\right) \frac{\cos K_m (b_1-y_o)}{K_m \sin K_m b_1} \sin \frac{m\pi}{2} \cos \frac{m\pi t}{2a}, \tag{21d}$$

and the upper integral limit in (21a) represents the gap, g = a-w (see figure 2).

Notice that as m becomes very large we find the following asymptotic behavior

$$\frac{\cot K_m b_j}{K_m} \sim \frac{-2a}{m\pi}. \tag{22}$$

Therefore, $Q^{(h)}(t,t')$ is singular as $t \to t'$ since it approaches the harmonic series. We may extract the singular part of $Q^{(h)}(t,t')$ by adding and subtracting the asymptotic limit (22) yielding

$$Q^{(h)}(t,t') = Q_s^{(h)}(t,t') - Q_f^{(h)}(t,t') \tag{23a}$$

$$Q_s^{(h)}(t,t') = \frac{4}{\pi} \sum_{mo} \frac{1}{m} \cos \frac{m\pi t}{2a} \cos \frac{m\pi t'}{2a} \tag{23b}$$

$$Q_f^{(h)}(t,t') = \frac{1}{a} \sum_{j=1}^{2} \sum_{mo} \left[\frac{\cot K_m b_j}{K_m} + \frac{2a}{m\pi}\right] \cos \frac{m\pi t}{2a} \cos \frac{m\pi t'}{2a} \tag{23c}$$

where the subscripts s and f denote the singular and finite terms respectively. The first term inside the brackets in (23c) decays exponentially for large m, and therefore is summable as $t \to t'$. However, $Q_s^{(h)}(t,t')$ has a logarithmic singularity as $t \to t'$, and represents the dominant contribution.

Having split the Green's function we may now take advantage of a small gap assumption. The series (23b) may be summed [6, pp. 96-97] giving

$$Q_s^{(h)}(t,t') = \frac{1}{\pi} \ln \left[\cot \left(\frac{\pi}{4a} |t-t'|\right) \cot \left(\frac{\pi}{4a} |t+t'|\right)\right] \tag{24}$$

Note that both $|t+t'|$ and $|t-t'|$ are bounded by 2g. If we assume $g\pi/2a \ll 1$, the cotangents may be replaced by the reciprocal of their arguments giving

$$Q_s^{(h)}(t,t') \approx -\frac{1}{\pi} \ln \left[\left(\frac{\pi}{4a}\right)^2 |t^2-t'^2|\right] \tag{25}$$

Next consider $Q_f^{(h)}(t,t')$ for low frequencies. The term in brackets decays rapidly and only a few initial terms need to be considered for which the cosine terms remain near unity under the same assumption of a small gap. Thus $Q_f^{(h)}(t,t')$ is approximately independent of t' and t

$$Q_f^{(h)}(t,t') \approx \frac{1}{a} \sum_{j=1}^{2} \sum_{mo} \left[\frac{\cot K_m b_j}{K_m} + \frac{2a}{m\pi}\right] \equiv Q_f^{(h)} \tag{26}$$

If we let

$$A^{(h)} = \int_0^g f^{(h)}(t')dt' \tag{27}$$

then our approximate Green's function yields the following integral equation for (21a)

$$-\frac{1}{\pi}\int_{0}^{g} f^{(h)}(t')\ln\left[\left(\frac{\pi}{4a}\right)^{2}|t^{2}-t'^{2}|\right]dt' = A^{(h)}Q_f^{(h)} + Id\ell\, g^{(h)}(t) \tag{28}$$

We may extend the definition of $f^{(h)}(t')$ according to $f^{(h)}(t') = f^{(h)}(-t')$ which corresponds to introducing an image field. Noting that our solution is only valid for $o \leq t \leq g$, we may split the logarithmic term into the sum of two terms, let $t' \to -t'$ in the $|t+t'|$ term, and rewrite (28) as

$$-\frac{1}{\pi}\int_{-g}^{g} f^{(h)}(t')\ln\left[\frac{\pi}{4a}|t-t'|\right]dt' = A^{(h)}Q_f^{(h)} + Id\ell\, g^{(h)}(t) \tag{29}$$

This is the approximate integral equation to be solved. For notational purposes we define the following quantities

$$g^{(h)}(t) = \sum_{mo} g_m^{(h)}(y_o)\cos\frac{m\pi t}{2a} \tag{30a}$$

$$g_m^{(h)}(y_o) = \frac{1}{2a}\left(\frac{m\pi}{2a}\right)\frac{\cos K_m(b_1-y_o)}{K_m\sin K_m b_1}\sin\frac{m\pi}{2} \tag{30b}$$

Then our integral equation (29) may be written as

$$\frac{1}{\pi}\int_{-g}^{g} f^{(h)}(t')\ln\left[\frac{\pi}{4a}|t-t'|\right] = A^{(h)}Q_f^{(h)} + Id\ell\sum_{mo} g_m^{(h)}(y_o)\cos\frac{m\pi t}{2a} \tag{31}$$

Having formulated an approximate integral equation, we next proceed to solve it via a Fourier series approach.

4. SOLUTION OF THE APPROXIMATE INTEGRAL EQUATION

Our particular problem may be solved directly by taking advantage of a Fourier expansion for the logarithmic kernel. By expanding the other relevant functions in a similar manner, and applying orthogonality, the coefficients may then be evaluated.

We begin with the following result given in [7, p. 171]

$$\ln|\cos\theta-\cos\phi| = \ln\tfrac{1}{2} - 2\sum_{n=1}^{\infty}\frac{1}{n}\cos n\theta\cos n\phi \tag{32}$$

With the substitution of $t = g\cos\theta$, and $t' = g\cos\phi$, (31) and (32) combine to give

$$\frac{1}{\pi}\int_{0}^{\pi} f^{(h)}(g\cos\phi)\sin\phi\left[\ln\left(\frac{\pi g}{8a}\right) - 2\sum_{n=1}^{\infty}\frac{1}{n}\cos n\theta\cos n\phi\right]d\phi$$

$$= \frac{1}{g}\left[A^{(h)}Q_f^{(h)} + Id\ell\sum_{mo} g_m^{(h)}(y_o)\cos\left(\frac{m\pi g}{2a}\cos\theta\right)\right] \tag{33}$$

If we now expand $f^{(h)}(g\cos\phi)\sin\phi$, and $\cos\left(\frac{m\pi g}{2a}\cos\theta\right)$ in cosine series as follows

$$f^{(h)}(g\cos\phi)\sin\phi = \sum_{p=o}^{\infty} a_p^{(h)}\cos p\phi \tag{34a}$$

$$\cos\left(\frac{m\pi g}{2a}\cos\theta\right) = \sum_{n=o}^{\infty} c_{nm}^{(h)}\cos n\theta \tag{34b}$$

then orthogonality gives

$$a_o^{(h)}\ln\left(\frac{\pi g}{8a}\right) = \frac{1}{g}\left[A^{(h)}\,Q_f^{(h)} + Id\ell \sum_{mo} g_m^{(h)}(y_o)\, c_{om}^{(h)}\right] \tag{35a}$$

$$a_n^{(h)}\,\frac{1}{n} = -\frac{1}{g}\, Id\ell \sum_{m} g_m^{(h)}(y_o)\, c_{nm}^{(h)} \qquad (n\geqslant 1) \tag{35b}$$

It remains to evaluate $c_{nm}^{(h)}$ given by

$$c_{nm}^{(h)} = \frac{2\Delta_n}{\pi}\int_0^{\pi} \cos\left(\frac{m\pi g}{2a}\cos\theta\right)\cos n\theta\, d\theta \tag{36}$$

This integral is well known [5, p. 402], and we find

$$c_{nm}^{(h)} = 2\Delta_n \cos\frac{n\pi}{2}\, J_n\left(\frac{m\pi g}{2a}\right) \tag{37}$$

which gives

$$a_o^{(h)} = \frac{1}{g\,\ln\left(\frac{\pi g}{8a}\right)}\left[A^{(h)}Q_f^{(h)} + Id\ell \sum_{mo} g_m^{(h)}(y_o)\, J_o\left(\frac{m\pi g}{2a}\right)\right] \tag{38a}$$

$$a_n^{(h)} = -\frac{2n}{g}\cos\frac{n\pi}{2}\, Id\ell \sum_{mo} g_m^{(h)}(y_o)\, J_n\left(\frac{m\pi g}{2a}\right), \tag{38b}$$

where J_n is the Besel function of the first kind of order n.

Because of the quick exponential decay of $g_m^{(h)}(y_o)$ at low frequencies, and the small argument of the Bessel functions for the first few terms, a reasonable zeroth order solution may be achieved by retaining only $a_o^{(h)}$. This may be seen more easily by asymptotically estimating $a_n^{(h)}$. We have, near the septum where convergence is slowest, that

$$g_m^{(h)}(y_o) \sim e^{-mY_o}\sin\frac{m\pi}{2} \tag{39}$$

where $Y_o = \pi y_o/2a$. We need to look at

$$\cos\frac{n\pi}{2}\sum_{mo} g_m^{(h)}(y_o)\, J_n\left(\frac{m\pi g}{2a}\right) \sim (-1)^n\, S_{2n} \tag{40a}$$

$$S_{2n} = \sum_{mo} e^{-mY_o}\sin\frac{m\pi}{2}\, J_{2n}\left(\frac{m\pi g}{2a}\right) \tag{40b}$$

The sum S_{2n} is evaluated approximately in Appendix B making use of the small gap assumption. If we now compare $a_o^{(h)}$ to $a_{2n}^{(h)}$ we find

$$\frac{a_{2n}^{(h)}}{a_o^{(h)}} \approx 2n \ \ln \left(\frac{8a}{\pi g}\right) \left(\tfrac{1}{2} k \frac{\pi g}{2a}\right)^{2n} \left(1 - \frac{1}{2k^2}\right) \tag{41a}$$

$$k^2 = \frac{4e^{-2Y_o}}{(1 + e^{-2Y_o})^2} < 1 \tag{41b}$$

This bound shows that neglecting $a_n^{(h)}$ is consistent with our earlier simplification of the finite kernel component $Q_f^{(h)}$. Thus our zeroth order approximate solution is given by

$$f^{(h)}(g\cos\phi) \sin\phi = a_o^{(h)} \tag{42}$$

where $a_o^{(h)}$ is given by (38a). We may now solve for $A^{(h)}$. Substituting t' = gcosϕ into (27) gives

$$A^{(h)} = g \int_0^{\pi/2} f^{(h)}(g\cos\phi) \sin\phi \, d\phi = g \frac{\pi}{2} a_o^{(h)} \tag{43}$$

and thus we find that $a_o^{(h)}$ is given by

$$a_o^{(h)} = \frac{Id\ell}{g} \left[\ln \left(\frac{\pi g}{8a}\right) - \frac{\pi}{2} Q_f^{(h)}\right]^{-1} \sum_{mo} g_m^{(h)}(y_o) \ J_o \left(\frac{m\pi g}{2a}\right) \tag{44}$$

In terms of our gap parameter t' we find

$$f^{(h)}(t') = \frac{Id\ell}{\sqrt{g^2 - t'^2}} \left[\ln \left(\frac{\pi g}{8a}\right) - \frac{\pi}{2} Q_f^{(h)}\right]^{-1} \sum_{mo} g_m^{(h)}(y_o) \ J_o \left(\frac{m\pi g}{2a}\right) \tag{45}$$

Notice that $f^{(h)}(t')$ is singular according to $(g-t')^{1/2}$ at the septum edge, t'=g, which is as expected since $f^{(h)}(t')$ is proportional to E_x in the gap and therefore must satisfy the appropriate edge condition. We will now use this approximate gap field expression to examine how the Hertzian dipole excites the cell, and specifically the TEM mode.

5. GAP EXCITATION OF THE RCTL MODES

5.1 Electric Field Distribution Due to TE and TM Type Modes

We begin with the expression given in (11) for $\tilde{H}_z(\bar{x}_t)$, which generates the TE modes. For convenience of presentation, it is rewritten as follows:

$$\tilde{H}_z^{(j)}(\bar{x}_t) = \int_{gaps} \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \, \partial_{n'} \tilde{H}_z^{(j)}(\bar{x}_t') d\ell'$$

$$- \delta_{1j} \ Id\ell \ \partial_{x'} \tilde{G}_1^{(h)}(\bar{x}_t, \bar{x}_t') \Big|_{\bar{x}_t' = \bar{x}_o}, \tag{46a}$$

where δ_{1j} is the Kronecker delta.

The corresponding expression for $\tilde{E}_z$ $(\bar{x}_t)$, which generates the TM modes, can also be obtained by using steps similar to those leading to (11),

$$\tilde{E}_z^{(j)}(\bar{x}_t) = \int_{gaps} \partial_{n'}\tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t') \tilde{E}_z^{(j)}(\bar{x}_t') \, d\ell'$$

$$+ \delta_{1j} \frac{\alpha}{\omega\varepsilon_o} I d\ell \, \partial_{y'}\tilde{G}_1^{(e)}(\bar{x}_t, \bar{x}_t')\Big|_{\bar{x}_t' = \bar{x}_o} \tag{46b}$$

In each expression the first term represents the fields excited by the gap, and the second term represents the usual rectangular guide modes excited in each chamber by the vertical Hertzian dipole. If there were no gap, then the RCTL would not support a TEM mode, therefore we expect the TEM mode information to be contained in the gap pertubation term. In Appendix C we show explicitly that as $\alpha \to k_o$, E_y and E_x, due to the superposition of the unperturbed TE and TM components, tend toward zero. Thus in order to analyze the TEM mode we need only consider the perturbed terms.

Beginning with the y-component of the electric field due to TE-type modes, we have

$$\delta\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{i\omega\mu_o}{\zeta^2} \int_{gaps} \partial_x \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') \, \partial_{y'}\tilde{H}_z(x',o) d\ell' \tag{47}$$

where in the gap

$$\partial_x \tilde{G}_j^{(h)}(\bar{x}_t, \bar{x}_t') = \frac{1}{a} \sum_{mo} \left(\frac{m\pi}{2a}\right) \frac{\cos K_m(b_j-y)}{K_m \sin K_m b_j} \cos\frac{m\pi x}{2a} \sin\frac{m\pi x'}{2a} \tag{48}$$

If we let x' = a-gcosϕ and perform the integration [5, p. 402], the TE-type contribution to the y-component of the perturbed electric field takes the following form

$$\delta\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{i\omega\mu_o}{\zeta^2} g \, a_o^{(h)} \frac{\pi}{2} \tilde{e}_y^{(h)}(\bar{x}_t) \tag{49a}$$

$$\tilde{e}_y^{(h)}(\bar{x}_t) = \left(\frac{2}{a}\right) \sum_{mo} \left(\frac{m\pi}{2a}\right) \frac{\cos K_m(b_j-y)}{K_m \sin K_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{49b}$$

A similar analysis of the x-component of the perturbed electric field due to TE-type contributions yields

$$\delta\tilde{E}_x^{(j)}(\bar{x}_t) = \frac{i\omega\mu_o}{\zeta^2} g \, a_o^{(h)} \frac{\pi}{2} \tilde{e}_x^{(h)}(\bar{x}_t) \tag{50a}$$

$$\tilde{e}_x^{(h)}(\bar{x}_t) = \left(\frac{2}{a}\right) \sum_{mo} \frac{\sin K_m(b_j-y)}{\sin K_m b_j} \sin\frac{m\pi}{2} \sin\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{50b}$$

In order to evaluate the perturbed electric field due to TM contributions, we again begin with the y-component given by

$$\delta\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{i\alpha}{\zeta^2} \int_{gaps} \partial_y \partial_{y'} \tilde{G}_j^{(e)}(\bar{x}_t,\bar{x}_t') \, \tilde{E}_z^{(j)}(\bar{x}_t')d\ell' \qquad (51)$$

where in the gap

$$\partial_y \partial_{y'} \tilde{G}_j^{(e)}(\bar{x}_t,\bar{x}_t') = -\frac{1}{a} \sum_{mo} \frac{K_m \cos K_m (b_j-y)}{\sin K_m b_j} \cos\frac{m\pi x}{2a} \cos\frac{m\pi x'}{2a} \qquad (52)$$

If we combine the previous two equations and integrate by parts, $\delta\tilde{E}_y^{(j)}(\bar{x}_t)$ may be given as follows

$$\delta\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{2i\alpha}{a\zeta^2} \sum_{mo} \left(\frac{2a}{m\pi}\right) \frac{K_m \cos K_m (b_j-y)}{\sin K_m b_j} \cos\frac{m\pi x}{2a} \int_w^a f^{(e)}(x') \cos\frac{m\pi x'}{2a} dx' \qquad (53)$$

Letting $x' = a - g\cos\phi$ and integrating [5, p. 403], we find

$$\delta\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{i\alpha^2}{\zeta^2\omega\varepsilon_o} g \, a_1^{(e)} \frac{\pi}{2} \tilde{e}_y^{(e)}(\bar{x}_t) \qquad (54a)$$

$$\tilde{e}_g^{(e)}(\bar{x}_t) = \left(\frac{8}{\pi g}\right) \sum_{mo} \frac{1}{m} \left(\frac{2a}{m\pi}\right) \frac{K_m \cos K_m (b_j-y)}{\sin K_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_1\left(\frac{m\pi g}{2a}\right), \qquad (54b)$$

where $a_1^{(e)}$ is defined by (a-17) in Appendix A.

A similar calculation gives the following result for $\delta\tilde{E}_x^{(j)}(\bar{x}_t)$ due to TM-type modes

$$\delta\tilde{E}_x^{(j)}(\bar{x}_t) = \frac{i\alpha^2}{\zeta^2\omega\varepsilon_o} g \, a_1^{(e)} \frac{\pi}{2} \tilde{e}_x^{(e)}(\bar{x}_t) \qquad (55a)$$

$$\tilde{e}_x^{(e)}(\bar{x}_t) = \left(\frac{8}{\pi g}\right) \sum_{mo} \left(\frac{1}{m}\right) \frac{\sin K_m (b_j-y)}{\sin K_m b_j} \sin\frac{m\pi}{2} \sin\frac{m\pi x}{2a} J_1\left(\frac{m\pi g}{2a}\right), \qquad (55b)$$

Clearly the total field distribution is the superposition of TE and TM type contributions. We next examine the manner in which the above expressions contribute to the TEM mode.

5.2 TEM Mode Excitation

In order to characterize the TEM mode, we need to take the transform (1a), and then analyze the poles at $\alpha = \pm k_0$ which represent the forward and backward travelling waves. A TEM mode component arises both from TE and TM type contributions. The TEM mode has an associated gap voltage which may be determined by integrating the x-component of the electric field across the gap. Both $f^{(h)}(x')$ and $f^{(e)}(x')$ are proportional to $\tilde{E}_x(x',o)$, thus the result of the gap integration is already known for both cases. The total voltage is the sum of the two results. Before analyzing the electric field distribution we will examine further these gap voltages.

We begin with the TE case by referring to the x-component of the electric field given in (3a). The voltage due to the TE type contribution is

$$\tilde{V}^{(h)} = \frac{-i\omega\mu_o}{\zeta^2} \int_w^a \partial_{y'} \tilde{H}_z (x',o)dx' \qquad (56)$$

Recalling that $f^{(h)}(x') = \partial_{y'} \tilde{H}_z(x',o)$, making the substitution $x' = a\text{-}g\cos\phi$, and using (42), we obtain

$$\tilde{V}^{(h)} = \frac{-i\omega\mu_o}{\zeta^2} g \int_0^{\pi/2} f^{(h)}(g\cos\phi) \sin\phi \, d\phi = \frac{-i\omega\mu_o}{\zeta^2} g \frac{\pi}{2} a_o^{(h)} \tag{57}$$

Thus, after evaluating the appropriate residue, we find that the forward and backward voltages are given by

$$V^{(h)}_{TEM} = \frac{\pi}{4} g \eta_o a_o^{(h)} \Big|_{\alpha = k_o} \tag{58}$$

where η_o is the characteristic impedance of the free space TEM wave. A similar analysis of the TM contribution presented by (a-25) in Appendix A gives

$$V^{(e)}_{TEM} = \frac{\pi}{4} g \eta_o \left(\frac{\omega\varepsilon_o}{2\alpha} a_1^{(e)}\right)\Big|_{\alpha = k_o} \tag{59}$$

The gap voltage excited by the TEM mode is thus given by

$$V_{TEM} = \frac{\pi}{4} g \eta_o \left(a_o^{(h)} + \frac{\omega\varepsilon_o}{2\alpha} a_1^{(e)}\right) \Big|_{\alpha = k_o} \tag{60}$$

where we have suppressed the z-dependence.

The electric field distribution of the TEM mode may be readily found from the previous expressions, (49a), (50a), (54a) and (55a). If we normalize each expression by the appropriate voltage expression we find

$$\left(\frac{b}{V^{(h)}_{TEM}}\right) E_y^{(j)}(\bar{x}_t) = 2\left(\frac{b}{a}\right) \sum_{mo} \frac{\cosh \hat{K}_m (b_j - y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{61a}$$

$$\left(\frac{b}{V^{(e)}_{TEM}}\right) E_y^{(j)}(\bar{x}_t) = \left(\frac{8b}{\pi g}\right) \sum_{mo} \frac{1}{m} \frac{\cosh \hat{K}_m (b_j - y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_1\left(\frac{m\pi g}{2a}\right) \tag{61b}$$

$$\left(\frac{b}{V^{(h)}_{TEM}}\right) E_x^{(j)}(\bar{x}_t) = -2\left(\frac{b}{a}\right) \sum_{mo} \frac{\sinh \hat{K}_m (b_j - y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \sin\frac{m\pi x}{2a} J_o\left(\frac{m\pi y}{2a}\right) \tag{61c}$$

$$\left(\frac{b}{V^{(e)}_{TEM}}\right) E_x^{(j)}(\bar{x}_t) = -\left(\frac{8b}{\pi g}\right) \sum_{mo} \frac{1}{m} \frac{\sinh \hat{K}_m (b_j - y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \sin\frac{m\pi x}{2a} J_1\left(\frac{m\pi g}{2a}\right) \tag{61d}$$

where

$$\hat{K}_m = \left(\frac{m\pi}{2a}\right) \tag{61e}$$

The x- and y-components arising from the TE and TM derivations look quite similar. We may show that, up to the order of our previous approximations, both are equivalent. We make use of the following Bessel function recurrance relationship

$$2\,J_1(z) = z\,(J_o(z) + J_2(z)) \tag{62}$$

which allows us to write the TM contribution to the y-component of the electric field as follows

$$\left(\frac{b}{V_{TEM}^{(e)}}\right) E_y^{(j)}(\bar{x}_t) = 2\left(\frac{b}{a}\right) \sum_{mo} \frac{\cosh \hat{K}_m (b_j-y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} \left(J_o\left(\frac{m\pi g}{2a}\right) + J_2\left(\frac{m\pi g}{2a}\right)\right) \tag{63}$$

As we did in justifying a zeroth order solution, we may compare the two series asymptotically and find

$$\left(\frac{b}{V_{TEM}^{(e)}}\right) E_y^{(j)}(\bar{x}_t) \sim 2\left(\frac{b}{a}\right) S_o \left\{1 + \frac{S_2}{S_o}\right\} \tag{64}$$

where S_{2n} has been defined previously in eq (40b). As was shown in Appendix B, the ratio S_2/S_o is on the order of $(\pi g/2a)^2$, and therefore may be neglected. Thus (63) reduces to

$$\left(\frac{b}{V_{TEM}^{(e)}}\right) E_y^{(j)}(\bar{x}_t) = 2\left(\frac{b}{a}\right) \sum_{mo} \frac{\cosh \hat{K}_m (b_j-y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{65}$$

which is the same as we found for the TE excitation. An analogous treatment of the x-components shows they also are equivalent up to the order of our approximations. Thus, we find that the electric field distribution of the TEM mode is given by

$$E_y^{(j)}(\bar{x}_t) = (V_{TEM}^{(h)} + V_{TEM}^{(e)})\, e_y^{(j)}(\bar{x}_t) \tag{66a}$$

$$E_x^{(j)}(\bar{x}_t) = (V_{TEM}^{(h)} + V_{TEM}^{(p)})\, e_x^{(j)}(\bar{x}_t) \tag{66b}$$

where

$$e_x^{(j)}(\bar{x}_t) = -\left(\frac{2}{a}\right) \sum_{mo} \frac{\sinh \hat{K}_m (b_j-y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \sin\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{66c}$$

and

$$e_y^{(j)}(\bar{x}_t) = \left(\frac{2}{a}\right) \sum_{mo} \frac{\cosh \hat{K}_m (b_j-y)}{\sinh \hat{K}_m b_j} \sin\frac{m\pi}{2} \cos\frac{m\pi x}{2a} J_o\left(\frac{m\pi g}{2a}\right) \tag{66d}$$

We need to know how strongly $V_{TEM}^{(h)}$ and $V_{TEM}^{(e)}$ are excited. If we evaluate $V_{TEM}^{(h)}$ given by (58), and $V_{TEM}^{(e)}$ given by (59) at $\alpha = k_o$, and normalize the result, we find

$$\frac{aV_{TEM}^{(h)}}{Id\ell n_o} = \frac{\frac{\pi}{8} \sum_{mo} \frac{\cosh \hat{K}_m (b_1-y_o)}{\sinh \hat{K}_m b_1} \sin\frac{m\pi}{2} J_o\left(\frac{m\pi g}{2a}\right)}{\left[\ell n\left(\frac{8a}{\pi g}\right) + \sum_{mo} \frac{1}{m} \sum_{j=1}^{2} \left(\coth\frac{m\pi b_j}{2a} - 1\right)\right]} \tag{67a}$$

$$\frac{V_{TEM}^{(e)}}{Id\ell n_o} = \frac{\frac{\pi}{8}\sum_{mo} m \frac{\cosh \hat{K}_m (b_1-y_o)}{\sinh \hat{K}_m b_1} \sin \frac{m\pi}{2} J_1 (\frac{m\pi g}{2a})}{[\frac{2a}{\pi g} + \sum_{mo}\sum_{j=1}^{2} (\coth \frac{m\pi b_j}{2a} - 1) J_1 (\frac{m\pi g}{2a})]} \tag{67b}$$

If the septum is not offset too radically, both the series in the demoninators will be much less than the gap dependent term. Thus, in forming the ratio of the voltages they may be neglected. As before in seeking bounds, we look at the asymptotic terms of the numerator series and find

$$\frac{V_{TEM}^{(e)}}{V_{TEM}^{(h)}} \approx -G \ \ell n \ (\frac{G}{4}) \frac{T_1}{S_o} \tag{68}$$

where $G = \pi g/2a$, T_1 is defined by (a-19b) in Appendix A, and S_o is defined by (40b). What we find is

$$\left| \frac{V_{TEM}^{(e)}}{V_{TEM}^{(h)}} \right| = \tfrac{1}{2} G^2 \ \ell n \ (\frac{4}{G}) \tag{69}$$

The ratio tends toward zero as $G \to 0$, but more importantly, we see that neglecting $V_{TEM}^{(e)}$ in comparison to $V_{TEM}^{(h)}$ is consistent with our previous approximation (small gap). The bound is quite good for reasonably small gaps. Even if $g/a = .25$, no longer very small, we find that the bound is still less than 18%.

5.3 Characteristic Impedance of the Cell

The characteristic impedance of the cell may now be found using our gap voltage expression. The power in the forward wave is given by

$$P = \frac{V^2}{2z_o} \tag{70}$$

An alternate expression given by Tippet [1, p. 59] is

$$P = \frac{z_o}{2} \left[\frac{Id\ell E_o}{2V_o}\right]^2 \tag{71}$$

where V_o is the gap voltage that excites a field E_o at the source point. Neglecting $V_{TEM}^{(e)}$, we may solve for z_o, insert (67a) for V, substitute (61a) at $\vec{x}_o$ for E_o/V_o, and we find

$$\frac{z_o}{n_o} = \frac{\pi}{8} \left[\ell n \ (\frac{8a}{\pi g}) - \sum_{mo} \frac{1}{m} \sum_{j=1}^{2} (1-\coth \frac{m\pi b_j}{2a})\right]^{-1} \tag{72}$$

For normal geometries the series term will be much smaller than the logarithmic term. Thus, we have a very simple formula which demonstrates that the dominant effect on the characteristic impedance comes from the gap size, and not the vertical offset. Some numerical results on the field distribution and the characteristic impedance for various cell dimensions are presented in the next section.

6. NUMERICAL RESULTS

Tippet [1] has done much work in characterizing an RCTL. In order to check the validity of our approximate analysis, we may compare our results with his for some typical geometries. For the special case of a symmetrically located septum, Tippet [1, p. 52] found an expression for the normalized magnitude of the electric field given by

$$\left(\frac{b}{V}\right) E_o = \left[\frac{m'b}{K(\alpha')}\right] \left[\frac{dn^2\ (m'z)}{sn^2\ (m'w) - sn^2\ (m'z)}\right]^{1/2} \tag{73}$$

where dn and sn are Jacobian elliptic functions. The x- and y-components of the electric field are found from the real and imaginary parts of E_o, and the remaining notation is defined in his derivation. Two basic differences appear between the above expression and our result (61). First, our expression is in terms of more common functions, and secondly (61) allows for a vertical offset. A FORTRAN program to evaluate (61) was written, and is listed in Appendix D. In Tables 1 and 2 our result is compared to numbers given by Tippet [1, p. 54] for a typical geometry given by $b_1 = b_2 = b$, $b/a = 1.0$ and $g/a = 0.17$. As we see, there is a large area of excellent agreement, which represents a large working area for analyzing equipment. The slow convergence of the series near the septum is treated by adding and subtracting the asymptotic term. In the case of the y-component of the electric field (61a), we find

$$\frac{\cosh \hat{K}_m (b_j - y)}{\sinh \hat{K}_m b_j} \sim 1 + 2e^{-2mB_j} \tag{74}$$

for $y=o$, where $B_j = \pi b_j/2a$. By adding and subtracting this term and evaluating the resulting asymptotic series, we can get faster convergence near the septum. However, we still do not get good results at the gap, as evidenced by the results in Table 1.

Our expression for the characteristic impedance of the cell (72) may also be compared to a similar result found by Tippet [1, p. 35] and given by

$$\frac{z_o}{\eta_o} = 1/4 \left\{ \frac{K\ [1 + A_1\ (2-\beta^2)] - 2A_1 E}{K'\ (1 - \beta^2\ A_1) + 2A_1\ E'} \right\}^{-1} \tag{75}$$

where again his expression is in terms of Jacobian elliptic functions of different kinds K and E. The modulus and notation are that of his thesis. His solution assumes that

$$e^{-m\pi b_j/a} \sim o, \qquad j = 1 \text{ and } 2 \tag{76}$$

for m>1, and is therefore not applicable for a septum with a large vertical offset. The expression given in this report (72) is not subject to this restriction, although the series will converge more slowly as the offset grows. A large offset might be useful to increase the size of the test area to accommodate larger pieces of equipment. Again, a program was written to evaluate (72), and is given in Appendix E. In figures 3 and 4 we compare our result to Tippet's for gaps of $g/a = .1$ and $g/a = .2$. As expected, we find good agreement for a nearly centered septum. However, the curves separate more as the offset grows. It should be noted that if small argument expressions are substituted for the elliptic functions in (75), it reduces to

$$\frac{Z_o}{\eta_o} = \frac{\pi}{8}\left[\ell n\left(\frac{8a}{\pi g}\right) - \sum_{j=1}^{2}\left(1 - \coth\frac{\pi b_j}{2a}\right)\right]^{-1} \tag{77}$$

A comparison of the above expression to (72) shows that they are similar, except that only the first term of our correction series appears in Tippet's result. The correction series becomes more important as the offset increases. Tippet also provides an expression for the normalized characteristic impedance when the septum width 2w is small (large gap),

$$\frac{Z_o}{\eta_o} = \frac{1}{2\pi}\left[\ell n\left(\frac{8a}{\pi w}\right) + 2A_1\right], \tag{78a}$$

$$A_1 = \frac{\cosh\frac{\pi(b_1+b_2)}{2a} - \cosh\frac{\pi(b_1-b_2)}{2a}}{\sinh\frac{\pi(b_1+b_2)}{2a}} - 1 \tag{78b}$$

where again the condition (76) is assumed. Together (78) and (72) provide good approximate solutions for a wide range of geometries.

7. CONCLUDING REMARKS

This report examines the excitation of a TEM cell with a small gap by a vertical Hertzian dipole. Numerical results on the detailed field distribution inside the cell and the characteristic impedance of the cell have been obtained. The results are applicable to the TEM cells now in use at NBS. The analysis allows a large vertical offset of the center conductor which might be useful to increase the size of the test area to accommodate larger pieces of equipment. The problem of a narrow septum could be solved in an analagous fashion by formulating an integral equation for the current on the septum. It is believed that the results given here will prove useful in solving for the monopole excitation of a TEM cell. Once the method of solution has been established, other similar problems, such as a probe in a shielded strip line, could perhaps be analagously treated.

8. REFERENCES

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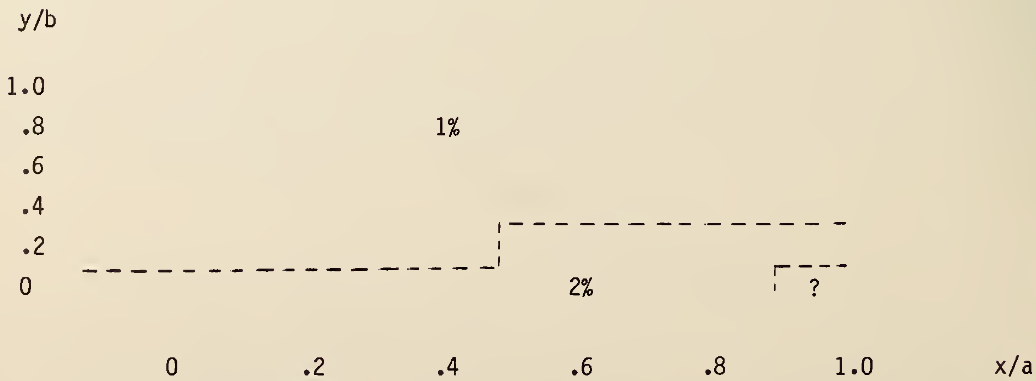
Table 1. Sample Results of Field Distribution Inside an RCTL (x-component)

y/b						
1.0	.000	.000	.000	.000	.000	.000
.8	.000	.060	.129	.208	.279	.308
.6	.000	.108	.244	.422	.601	.682
.4	.000	.126	.310	.618	1.030	1.244
.2	.000	.090	.246	.640	1.678	2.316
0	.000	.000	.000	.000	.000	.000
	0	.2	.4	.6	.8	1.0 x/a

1.a. The normalized x-component of the electric field with $b_1 = b_2 = b$, $b/a = 1.0$, and $g/a = 0.17$ (Wilson).

y/b						
1.0	.000	.000	.000	.000	.000	.000
.8	.000	.060	.129	.208	.278	.307
.6	.000	.108	.245	.422	.600	.680
.4	.000	.127	.311	.620	1.029	1.237
.2	.000	.090	.248	.647	1.684	2.285
0	.000	.000	.000	.000	.000	3.603
	0	.2	.4	.6	.8	1.0 x/a

1.b. The normalized x-component of the electric field with $b_1 = b_2 = b$, $b/a = 1.0$, and $g/a = 0.17$ (Tippet)



1.c. The percentage difference between the results given in Tables 1.a. and 1.b.

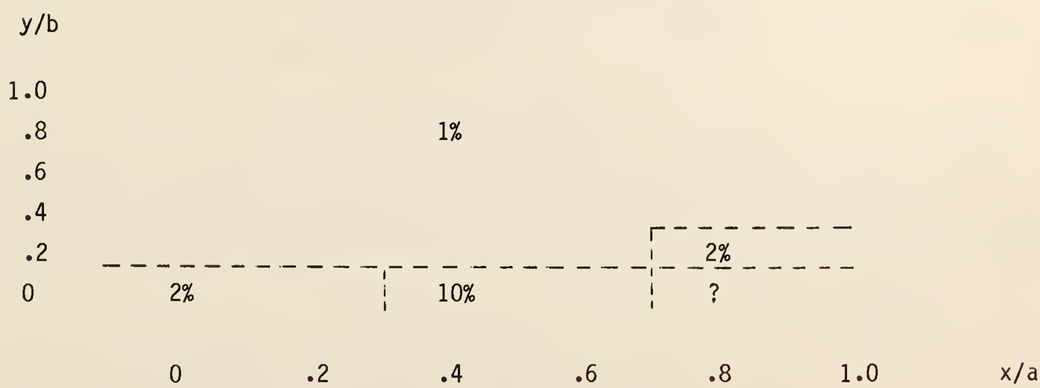
Table 2. Sample Results of Field Distribution Inside an RCTL (y-component)

y/b							
1.0	.824	.794	.699	.531	.289	.000	
.8	.853	.826	.737	.569	.316	.000	
.6	.935	.918	.853	.701	.412	.000	
.4	1.049	1.052	1.051	.979	.657	.000	
.2	1.153	1.185	1.296	1.499	1.364	.000	
0	1.180	1.223	1.382	1.808	3.292	.000	
	0	.2	.4	.6	.8	1.0	x/a

2.a. The normalized y-component of the electric field with $b_1 = b_2 = b$, b/a = 1.0, and g/a = 0.17 (Wilson).

y/b							
1.0	.824	.793	.698	.530	.289	.000	
.8	.853	.825	.736	.568	.315	.000	
.6	.935	.917	.852	.699	.410	.000	
.4	1.049	1.052	1.051	.977	.652	.000	
.2	1.153	1.186	1.298	1.499	1.343	.000	
0	1.196	1.245	1.431	1.986	6.640	.000	
	0	.2	.4	.6	.8	1.0	x/a

2.b. The normalized y-component of the electric field with $b_1 = b_2 = b$, b/a = 1.0, and g/a = 0.17 (Tippet)



2.c. The percentage difference between the results given in Tables 2.a. and 2.b.

APPENDIX A

APPROXIMATE TM GAP FIELD DERIVATION

The major steps in the derivation of the approximate gap field expression for the TM (or e-type) case are similar to those taken in the TE case. Thus, most of the intermediate derivations will be omitted. We again begin by introducing an associated Green's function $\tilde{G}_j^{(e)}(\bar{x}, \bar{x}_t')$ which satisfies the following wave equation and boundary conditions (gap as well as walls)

$$(\nabla_t^2 + \zeta^2)\, \tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t') = -\delta\,(\bar{x} - \bar{x}_t') \tag{a-1a}$$

$$\tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t') = 0, \tag{a-1b}$$

and is given by

$$\tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t') = \left(\frac{2}{ab_j}\right) \sum_{m,n=1}^{\infty} \frac{1}{(K_{mn}^{(j)2} - \zeta^2)}$$

$$\sin\frac{m\pi}{2a}(x+a)\,\sin\frac{m\pi}{2a}(x'+a)\,\sin\frac{n\pi y}{b_j}\,\sin\frac{n\pi y'}{b_j} \tag{a-2}$$

We next multiply the wave equations for $\tilde{E}_z^{(j)}(\bar{x}_t')$ given in (2b) and $\tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t')$, by $\tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t')$ and $\tilde{E}_z^{(j)}(\bar{x}_t')$, respectively, subtract the two results, and integrate over the respective chamber cross sections S'. An application of Green's Second Integral identity and the boundary conditions satisfied by $\tilde{G}_j^{(e)}$ and $\tilde{E}_z^{(j)}$ will considerably simplify the integral equation. Inserting the current density distribution transform of (10) will yield

$$\begin{aligned} &- \int_{gaps} \partial_{n'} \tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t')\, \tilde{E}_z^{(j)}(\bar{x}_t')\, d\ell' \\ &= \tilde{E}_z^{(j)}(\bar{x}_t) - \frac{\alpha}{\omega\varepsilon_o}\, I d\ell\, \partial_{y'} \tilde{G}_1^{(e)}(\bar{x}_t, \bar{x}_t')\big|_{\bar{x}_t' = \bar{x}_o} \end{aligned} \tag{a-3}$$

We next wish to impose the continuity of the tangential fields across the gap, which requires that $\tilde{E}_z(\bar{x}_t)$ and $\partial_y \tilde{E}_z(\bar{x}_t)$ be continuous at the gap. However, a slight difficulty arises. The y-derivative may not be taken inside the integral because $\partial_{n'} \tilde{G}_j^{(e)}(\bar{x}_t, \bar{x}_t')$ involves asymptotic behavior according to

$$\sim \frac{m\pi}{2a}\, e^{-y\frac{m\pi}{2a}} \tag{a-4}$$

which will diverge as $y \to o$. This problem may be avoided by first integrating the left-hand side of eq (a-3) by parts. We then find the following result

$$-\int_{\text{gaps}} \partial_{n'} \tilde{G}^{(e)}(\bar{x}_t,\bar{x}_t') \, \partial_{x'} \tilde{E}_z(\bar{x}) \, d\ell'$$

$$= \frac{\alpha}{\omega\varepsilon_o} \, Id\ell \, \partial_y \partial_{y'} \tilde{G}_1^{(e)}(\bar{x}_t, o, \bar{x}_t')|_{\bar{x}_t' = \bar{x}_o} \tag{a-5}$$

where

$$\tilde{G}^{(e)} = \tilde{G}_1^{(e)}(x,o,x',o) + \tilde{G}_2(x,o,x',o).$$

Analagous to the TE case, the Hertzian dipole is centrally located, the summations over n are performed to simplify the Green's functions, and the integral equation (a-5) takes the following form

$$\int_w^a f^{(e)}(x') \, \tilde{G}^{(e)}(x, x') \, dx' = -\tfrac{1}{2} \, Id\ell g^{(e)}(x) \tag{a-6a}$$

where

$$f^{(e)}(x') = \partial_{x'} \tilde{E}_z^{(j)}(x', o) \tag{a-6b}$$

$$\tilde{G}^{(e)}(x,x') = -\frac{2}{\pi} \sum_{j=1}^{2} \sum_{mo} \frac{K_m}{m} \cot K_m b_j \cos \frac{m\pi x}{2a} \sin \frac{m\pi x'}{2a} \tag{a-6c}$$

$$g^{(e)}(x) = \frac{\alpha}{\omega\varepsilon_o a} \sum_{mo} \frac{K_m \cos K_m (b_1 - y_o)}{\sin K_m b_1} \cos \frac{m\pi x}{2a} \tag{a-6d}$$

Next, the singular component of $\tilde{G}^{(e)}(x, x')$ is extracted and a small gap is assumed. In terms of the gap variables t and t' we find after some manipulation

$$-\frac{1}{\pi} \int_{-g}^{g} f^{(e)}(t') \frac{dt'}{t-t'} = A^{(e)} \, Q_f^{(e)}(t) + Id\ell \, \frac{2\alpha}{\omega\varepsilon_o} \, g^{(e)}(t) \tag{a-7}$$

where

$$f^{(e)}(t') = f^{(e)}(-t') \tag{a-8a}$$

$$A^{(e)} = \int_o^g f^{(e)}(t') \, dt' \tag{a-8b}$$

$$Q_f^{(e)}(t) = \sum_{mo} Q_{fm}^{(e)} \sin \frac{m\pi t}{2a} \tag{a-8c}$$

$$Q_{fm}^{(e)} = \frac{2}{\pi} \sum_{j=1}^{2} \left[\frac{K_m}{m} \cot K_m b_j - \frac{\pi}{2a}\right] \tag{a-8d}$$

$$g^{(e)}(t) = \sum_{mo} g_m^{(e)}(y_o) \sin \frac{m\pi t}{2a} \tag{a-8e}$$

$$g_m^{(e)}(y_o) = -\frac{1}{4a}\frac{K_m \cos K_m (b_1-y_o)}{\sin K_m b_1} \sin\frac{m\pi}{2} \tag{a-8f}$$

Finally, if we let

$$-h_m^{(e)}(y_o) = A^{(e)}Q_{fm}^{(e)} + Id\ell\, \frac{2\alpha}{\omega\varepsilon_o}\, g_m^{(e)}(y_o) \tag{a-9}$$

our integral equation may be written

$$\frac{1}{\pi}\int_{-g}^{g} f^{(e)}(t')\,\frac{dt'}{t-t'} = \sum_{mo} h_m^{(e)}(y_o)\sin\frac{m\pi t}{2a} \tag{a-10}$$

Again a Fourier series approach will be used to solve (a-10). We make the change of variables $t = g\cos\theta$, and $t' = g\cos\phi$, and (a-10) becomes

$$\frac{1}{\pi}\int_{0}^{\pi} f^{(e)}(g\cos\phi)\,\frac{\sin\phi\, d\phi}{\cos\theta - \cos\phi} = \sum_{mo} h_m^{(e)}(y_o)\sin\left(\frac{m\pi g}{2a}\cos\theta\right) \tag{a-11}$$

If we now differentiate (32) with respect to ϕ we find

$$\frac{\sin\phi}{\cos\theta - \cos\phi} = 2\sum_{n=1}^{\infty}\cos n\theta \sin n\phi \tag{a-12}$$

which gives a Fourier expansion to be used analagously to (32). We next expand $f^{(e)}(g\cos\phi)$ in a sine series, and $\sin\left(\frac{m\pi g}{2a}\cos\theta\right)$ in a cosine series

$$f^{(e)}(g\cos\phi) = \sum_{p=1}^{\infty} a_p^{(e)}\sin p\phi \tag{a-13a}$$

$$\sin\left(\frac{m\pi g}{2a}\cos\theta\right) = \sum_{n=1}^{\infty} C_{nm}^{(e)}\cos n\theta \tag{a-13b}$$

then orthogonality gives

$$a_n^{(e)} = \sum_{mo} C_{mn}^{(e)} h_m^{(e)}(y_o) \tag{a-14}$$

It remains to evaluate $C_{nm}^{(e)}$ given by

$$C_{nm}^{(e)} = \frac{2}{\pi}\int_{0}^{\pi}\sin\left(\frac{m\pi g}{2a}\cos\theta\right)\cos n\theta\, d\theta \tag{a-15}$$

which is a known integral given in [5, p. 402]

$$C_{nm}^{(e)} = 2\sin\frac{n\pi}{2}\, J_n\left(\frac{m\pi g}{2a}\right) \tag{a-16}$$

Thus, we find

$$a_n^{(e)} = 2\sin\frac{n\pi}{2}\sum_{mo} h_m^{(e)}(y_o)\, J_n\left(\frac{m\pi g}{2a}\right) \tag{a-17}$$

Again, as with the TE case, we wish to base our solution on the initial term $a_1^{(e)}$, and neglect the higher coefficients. Referring to $h_m^{(e)}(y_o)$, we find

$$h_m^{(e)}(y_o) \sim m e^{-mY_o} \sin\frac{m\pi}{2} \tag{a-18}$$

where $Y_o = \pi y_o/2a$. Thus, we need to look at

$$\sin\frac{n\pi}{2} \sum_{mo} h_m^{(e)}(y_o)\, J_n\,(mG) \sim (-1)^n\, T_{2n+1} \tag{a-19a}$$

$$T_{2n+1} = \sum_{mo} e^{-mY_o}\, m \sin\frac{m\pi}{2}\, J_{2n+1}\,(mG) \tag{a-19b}$$

where $G = \pi g/2a$. We may make use of the following two Bessel function relationships [8, p. 519]

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z) \tag{a-20a}$$

$$J_{n-1}(z) - J_{n+1}(z) = 2\, J_n'(z) \tag{a-20b}$$

to generate the following pair of relationships between T_{2n+1} and S_{2n} [which is given in (40b)],

$$T_{2n-1} + T_{2n+1} = \frac{4n}{G} S_{2n} \tag{a-21a}$$

$$T_{2n-1} - T_{2n+1} = 2\, S_{2n}' \tag{a-21b}$$

and thus we find

$$T_{2n+1} = \frac{2n}{G} S_{2n} - S_{2n}' \tag{a-22}$$

where derivatives are with respect to G. Substituting this in the result (b-19) found in Appendix B gives

$$T_{2n+1} \approx \frac{-e^{-Y_o}}{(1-p)} (-1)^n \left(\tfrac{1}{2}k\right)^{2n} (k^2-1)\, G^{2n+1} \tag{a-23a}$$

$$T_1 \approx \frac{-e^{-Y_o}}{(1-p)} \left(k^2 - \tfrac{1}{2}\right) G \tag{a-23b}$$

Therefore, comparing $a_{2n+1}^{(e)}$ to $a_1^{(e)}$ we find the following

$$\frac{a_{2n+1}^{(e)}}{a_1^{(e)}} \approx \left(\tfrac{1}{2}k\right)^{2n} \frac{(k^2-1)}{(k^2-\tfrac{1}{2})}\, G^{2n} \tag{a-24}$$

so neglecting higher order terms in the TM solution is consistent with the small gap approximation made in the TE derivation. Thus, our approximate gap field is given by

$$f^{(e)}(g\cos\phi) = a_1^{(e)}\sin\phi \tag{a-25}$$

Next we may solve for the undetermined constant $A^{(e)}$. Substituting $t' = g\cos\phi$ into (a-8b) gives

$$A^{(e)} = g\, a_1^{(a)} \frac{\pi}{4} \tag{a-26}$$

and thus we find that

$$a_1^{(e)} = \frac{Id\ell}{g}\,\frac{2\alpha}{\omega\varepsilon_o} \sum_{mo} g_m^{(e)}(y_o)\, J_1\left(\frac{m\pi g}{2a}\right) \left[\frac{1}{g} - \frac{\pi}{2} \sum_{mo} Q_{fm}^{(e)}\, J_1\left(\frac{m\pi g}{2a}\right)\right]^{-1} \tag{a-27}$$

In terms of our gap parameter t' we find

$$f^{(e)}(t') = \frac{Id\ell}{g^2}\,\frac{2\alpha}{\omega\varepsilon_o}\sqrt{g^2 - t'^2} \sum_{mo} g_m^{(e)}(y_o)\, J_1\left(\frac{m\pi g}{2a}\right) \left[\frac{1}{g} - \frac{\pi}{2} \sum_{mo} Q_{fm}^{(e)} J_1\left(\frac{m\pi g}{2a}\right)\right]^{-1} \tag{a-28}$$

and this expression, which is proportional to $\tilde{E}_x$, will be used in analyzing the TEM mode characteristics.

APPENDIX B

EVALUATION OF THE SERIES S_{2n}

We wish to evaluate the series S_{2n} defined by (40b). If we let $G = \pi g/2a$, and $p = -e^{-2Y_o}$, our series may be written in the following form

$$S_{2n} = e^{-Y_o} \sum_{m=o}^{\infty} p^m J_{2n} ((2m+1) G) \tag{b-1}$$

If we replace the Bessel function by an integral representation given in [8, p. 360],

$$J_{2n}(z) = \frac{1}{\pi} \int_{o}^{\pi} \cos (z\sin\theta - 2n\theta) \, d\theta \tag{b-2}$$

S_{2n} takes the following form

$$S_{2n} = \frac{e^{-Y_o}}{\pi} \int_{o}^{\pi} d\theta \sum_{m=o}^{\infty} p^m \cos [(2m+1) G \sin\theta - 2n\theta] \tag{b-3}$$

We next expand the cosine function which generates two summable series

$$S_{2n} = \frac{e^{-Y_o}}{\pi} \int_{o}^{\pi} d\theta \quad \cos (z-2n\theta) \sum_{m=o}^{\infty} p^m \cos 2mz - \sin (z-2n\theta) \sum_{m=o}^{\infty} p^m \sin 2mz \tag{b-4}$$

where we have let $z = G\sin\theta$. These two series are given in [5, p. 40] and S_{2n} becomes

$$S_{2n} = \frac{e^{-Y_o}}{\pi} \int_{o}^{\pi} d\theta \; \frac{\cos (z-2n\theta) - p\cos (z+2n\theta)}{1 - 2p \cos 2z + p^2} \tag{b-5}$$

Because $z = G\sin\theta \ll 1$ according to our small gap assumption, we may approximate the numerator and denominator as follows

$$\begin{aligned} \cos (z-2n\theta) - p\cos (z + 2n\theta) &= \\ &\approx (1-p) \left(1 - \frac{z^2}{2}\right) \cos 2n\theta + (1+p) \, z \sin 2n\theta \end{aligned} \tag{b-6}$$

$$1 - 2p \cos 2z + p^2 \approx (1 - p)^2 \, (1 - k^2 z^2) \tag{b-7a}$$

where

$$k^2 = {}^{-4p}/ \, (1-p)^2 \tag{b-7b}$$

This reduces S_{2n} to

$$S_{2n} = \frac{e^{-Y_o}}{\pi \, (1-p)} \left[I_{2n}^{(1)} + \frac{(1+p)}{(1-p)} I_{2n}^{(2)} \right] \tag{b-8a}$$

$$I_{2n}^{(1)} = \int_0^{\pi} \frac{(1 - \frac{1}{2} z^2) \cos 2n\theta \, d\theta}{1 - k^2 z^2} \tag{b-8b}$$

$$I_{2n}^{(2)} = \int_0^{\pi} \frac{z \sin 2n\theta \, d\theta}{1 - k^2 z^2} \tag{b-8c}$$

It remains to evaluate these two integrals.

We begin with $I_{2n}^{(1)}$ which in terms of θ is given by

$$I_{2n}^{(1)} = \int_0^{\pi} \frac{(1 - \frac{1}{2} G^2 \sin^2\theta) \cos 2n\theta \, d\theta}{1 - k^2 G^2 \sin^2\theta} \tag{b-9}$$

The integral may alternately be written as

$$I_{2n}^{(1)} = 2 \int_0^{\pi/2} \frac{(1 - \frac{1}{2} G^2 \sin^2\theta) \cos 2n\theta \, d\theta}{1 - k^2 G^2 \sin^2\theta} \tag{b-10}$$

We now expand the numerator in terms of cosines as follows

$$\begin{aligned} &(1 - \tfrac{1}{2} G^2 \sin^2\theta) \cos 2n\theta \\ &= \frac{1}{8} G^2 \cos 2(n-1)\theta + (1 - \tfrac{1}{4} G^2) \cos 2n\theta + \frac{1}{8} G^2 \cos 2(n+1)\theta \end{aligned} \tag{b-11}$$

Thus our integral may be expressed as follows

$$I_{2n}^{(1)} = \frac{1}{8} G^2 K_{n-1} + (1 - \tfrac{1}{4} G^2) K_n + \frac{1}{8} G^2 K_{n+1} \tag{b-12a}$$

where

$$K_n = 2 \int_0^{\pi} \frac{\cos 2n\theta \, d\theta}{1 - k^2 G^2 \sin^2\theta} \tag{b-12b}$$

This integral is given in Gradshteyn and Ryzhik [5, p. 368] and we find

$$K_n = \frac{(-1)^n \pi}{\sqrt{1 - k^2 G^2}} \left(\frac{1 - \sqrt{1 - k^2 G^2}}{kG} \right)^{2n} \tag{b-13}$$

Recalling the definition of k in (b-7b), the small gap assumption gives $kG \ll 1$, thus the square root may be approximated by:

$$\sqrt{1 - k^2 G^2} \approx 1 - \tfrac{1}{2} k^2 G^2 \tag{b-14}$$

and K_n reduces to

$$K_n \approx \frac{(-1)^n \pi \, (\frac{1}{2} kG)^{2n}}{(1 - \frac{1}{2} k^2 G^2)} \tag{b-15}$$

Thus we find $I_{2n}^{(1)}$ approximately given by

$$I_o^{(1)} \approx \frac{\pi}{(1 - \frac{1}{2} k^2 G^2)} (1 - \tfrac{1}{4} G^2) \qquad \text{(b-16a)}$$

$$I_{2n}^{(1)} \approx \frac{\pi \, (-1)^n}{(1 - \frac{1}{2} k^2 G^2)} (\tfrac{1}{2} kG)^{2n} (1 - \frac{1}{2k^2} - \tfrac{1}{4} G^2) \qquad n > o \qquad \text{(b-16b)}$$

where approximations of the order of (b-14) have been made.

We next need to evaluate $I_{2n}^{(2)}$ given by

$$I_{2n}^{(2)} = G \int_o^\pi \frac{\sin 2n\theta \, \sin\theta \, d\theta}{1 - k^2 G^2 \sin^2\theta} \qquad \text{(b-17)}$$

If we split the integration path and let $\theta' = \pi - \theta$ over $(\pi/2, \pi)$, as we did in generating (b-10), we find that the integrand is the negative of (b-17). Thus we find

$$I_{2n}^{(2)} = 0 \qquad \text{(b-18)}$$

This finally gives us a small gap approximation for S_{2n}

$$S_o \approx \frac{e^{-Y_o}}{(1-p)} \frac{1}{(1 - \frac{1}{2} k^2 G^2)} (1 - \tfrac{1}{4} G^2) \qquad \text{(b-19a)}$$

$$S_{2n} \approx \frac{e^{-Y_o}}{(1-p)} \frac{(-1)^n}{(1 - \frac{1}{2} k^2 G^2)} (\tfrac{1}{2} kG)^{2n} (1 - \frac{1}{2k^2} - \tfrac{1}{4} G^2) \qquad n > 1 \qquad \text{(b-19b)}$$

APPENDIX C

EVALUATION OF THE UNPERTURBED EXCITATION TERMS AS $\alpha \to k_o$

The y-component of the electric field from both types of modes is given by

$$\tilde{E}_y(\bar{x}_t) = \frac{-i\omega\mu_o}{\zeta^2} \partial_x \tilde{H}_z(\bar{x}_t) + \frac{i\alpha}{\zeta^2} \partial_y \tilde{E}_z(\bar{x}_t) \qquad \text{(c-1)}$$

which upon substitution of the unperturbed expressions for $\tilde{H}_z(\bar{x}_t)$ given in (11) and $\tilde{E}_z(\bar{x}_t)$ in (a-3) gives

$$\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{iId\ell}{\zeta^2} \delta_{1j} \left[\omega\mu_o \partial_x \partial_{x'} \tilde{G}_j^{(h)}(\bar{x}_t,\bar{x}_t') + \frac{\alpha^2}{\omega\varepsilon_o} \partial_y \partial_{y'} \tilde{G}_j^{(e)}(\bar{x}_t,\bar{x}_t')\right]_{\bar{x}_t' = \bar{x}_o} \qquad \text{(c-2)}$$

We next examine the Green's functions and find

$$\partial_x \partial_{x'} \tilde{G}_j^{(h)}(\bar{x}_t,\bar{x}_t') = \sum_{m=1,n=0}^{\infty} \left(\frac{m\pi}{2a}\right)^2 \frac{1}{(K_{mn}^{(j)2} - \zeta^2)} g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t') \qquad \text{(c-3a)}$$

$$\partial_y \partial_{y'} \tilde{G}_j^{(e)}(\bar{x}_t,\bar{x}_t') = \sum_{m,\ n=1}^{\infty} \left(\frac{n\pi}{b_j}\right)^2 \frac{1}{(K_{mn}^{(j)2} - \zeta^2)} g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t') \qquad \text{(c-3b)}$$

$$g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t') = \left(\frac{2}{ab_j}\right) \sin\frac{m\pi}{2a}(x+a) \sin\frac{m\pi}{2a}(x'+a) \cos\frac{n\pi y}{b_j} \cos\frac{n\pi y'}{b_j} \qquad \text{(c-3c)}$$

Thus, we find that $\tilde{E}_y^{(j)}(\bar{x}_t)$ may be expressed as follows:

$$\tilde{E}_y^{(j)}(\bar{x}_t) = \frac{iId\ell\delta_{1j}}{\zeta^2} \sum_{m=1,n=0}^{\infty} \frac{\omega\mu_o}{(K_{mn}^{(j)2} - \zeta^2)} \left[\left(\frac{m\pi}{2a}\right)^2 + \frac{\alpha^2}{k_o^2}\left(\frac{n\pi}{b_j}\right)^2\right] g_{mn}^{(j)}(\bar{x}_t,\bar{x}_o) \qquad \text{(c-4)}$$

For $\alpha = k_o$ (or $\zeta = 0$) and with $K_{mn}^{(j)}$ defined in (6b), our expression for $\tilde{E}_y^{(j)}(\bar{x}_t)$ as $\alpha \to k_o$ may be simplified to

$$\tilde{E}_y^{(j)}(\bar{x}_t)\Big|_{\alpha=k_o} = \lim_{\alpha\to k_o} \frac{i\omega\mu}{\zeta^2} Id\ell\delta_{1j} \sum_{m=1,n=0} g_{mn}^{(j)}(\bar{x}_t,\bar{x}_o) \qquad \text{(c-5)}$$

Notice that if we expand the delta function $\delta(\bar{x}_t-\bar{x}_t')$ in a double Fourier series over a chamber cross section, we find

$$\sum_{m=1,n=0} g_{mn}^{(j)}(\bar{x}_t,\bar{x}_t') = \left(\frac{\pi}{2}\right)^2 \delta(\bar{x}_t-\bar{x}_t') \qquad \text{(c-6)}$$

Thus we see that the unperturbed terms do not contribute to the y-component of the electric field of the TEM mode.

However, the unperturbed terms will contribute a reactance at the source point. A similar analysis of the x-component of the electric field yields the same result, except that there is no reactive contribution at the source.

```
000000B         SUBROUTINE GUIDE(A,B,G)
000000B
          C     THIS SUBROUTINE CALCULATES THE X-, AND Y-COMPONENTS OF THE
          C     ELECTRIC FIELD IN A CORNER OF THE GUIDE. THE INPUTS ARE,
          C       A  THE HALF WIDTH OF THE GUIDE.
          C       B  THE CHAMBER HEIGHT NORMALIZED BY A.
          C       G  THE GAP WIDTH NORMALIZED BY A.
          C     THE SUBROUTINE RETURNS THE FIELD VALUES AT THE LATTICE
          C     POINTS OF A SIX BY SIX GRID.
000000B
000000B         REAL EXX(6,6),EYY(6,6),XX(6),YY(6)
000000B         PRINT 9
000127B    9    FORMAT(1X,*ITERATIONS*,8X,*FINAL STEP VALUES*,11X,*X*,9X,*Y*/)
000127B         DO 11 I=1,6
000130B         DO 10 J=1,6
000132B           X=(J-1.0)/5.0
000134B           Y=(6.0-I)/5.0
000137B           XX(J)=X*A
000141B           YY(I)=Y*A
000142B           CALL NORMFLD(X,Y,G,B,EX,EY)
000150B           EXX(I,J)=EX
000153B           EYY(I,J)=EY
000154B   10      CONTINUE
000157B   11      CONTINUE
000161B         PRINT 12
000165B   12    FORMAT(5(/),1X,*THE X-COMPONENT OF THE ELECTRIC FIELD*/)
000165B         PRINT 13,(XX(JJ),JJ=1,6)
000174B   13    FORMAT(1X,6F10.7,12X,*X/Y*/)
000174B         DO 15 II=1,6
000175B           PRINT 14,(EXX(II,K),K=1,6),YY(II)
000213B   14      FORMAT(1X,6F10.7,5X,F10.7)
000213B   15      CONTINUE
000215B         PRINT 16
000221B   16    FORMAT(5(/),1X,*THE Y-COMPONENT OF THE ELECTRIC FIELD*/)
000221B         PRINT 17,(XX(JJ),JJ=1,6)
000230B   17    FORMAT(1X,6F10.7,12X,*X/Y*/)
000230B         DO 19 II=1,6
000231B           PRINT 18,(EYY(II,K),K=1,6),YY(II)
000247B   18      FORMAT(1X,6F10.7,5X,F10.7)
000247B   19      CONTINUE
000251B         RETURN
000253B         END
```

```
000000B         SUBROUTINE NORMFLD(X,Y,G,B,EX,EY)
000000B
          C     THIS SUBROUTINE COMPUTES THE ELECTRIC FIELD COMPONENTS AT
          C     A SPECIFIED GRID POINT. THE INPUTS ARE,
          C       X,Y  THE X AND Y COORDS OF THE GRID POINT NORMALIZED BY A.
          C       G,B  THE GAP, AND CHAMBER HEIGHT NORMALIZED BY A.
          C     THE OUTPUTS ARE EX, AND EY, THE X-, AND Y-COMPONENTS OF THE
          C     ELECTRIC FIELD AT THE SPECIFIED POINT.
000000B
000000B         REAL PI,MPI
000000B         PI=4.0*ATAN(1.0)
```

```
000004B          EX=0.0
000005B          EY=0.0
000006B          DO 10 M=1.30
000010B            MPI=((2.0*M)-1.0)*PI/2.0
000014B            X1=MPI*X
000015B            X2=MPI*G
000016B            X3=-MPI*Y
000020B            X4=-MPI*(2.0*B-Y)
000024B            X5=-MPI*2.0*B
000025B            CALL BJO(X2,BJ)
000031B            ST1=FLOAT(M)
000032B            ST2=(EXP(X3)-EXP(X4))/(1.0-EXP(X5))
000044B            ST3=(EXP(X3)+EXP(X4))/(1.0-EXP(X5))
000057B            IF (Y.GT.0.1) GO TO 5
000062B            ST2=ST2-EXP(X3)
000065B            ST3=ST3-EXP(X3)-2.0*EXP(X5)
000072B    5       ST4=SIN(MPI)*SIN(X1)*BJ*ST2
000102B            ST5=SIN(MPI)*COS(X1)*BJ*ST3
000111B            EX=EX+ST4
000114B            EY=EY+ST5
000115B            IF (ST3.LT.0.0001) GO TO 20
000115B    10      CONTINUE
000122B    20    PRINT 21,ST1,ST2,ST3,X,Y
000133B    21    FORMAT(1X,F10.7,5X,2F10.7,5X,2F10.7)
000133B          IF (Y.GT.0.1) GO TO 30
000135B          IF (X.GT.0.95) GO TO 30
000137B          XPI=PI*X/2.0
000141B          ST6=EXP(-PI*B)
000145B          ST7=2.0*COS(XPI)*ST6*(1.0+ST6*ST6)
000152B          STB=1.0+2.0*COS(2.0*XPI)*ST6*ST6
000160B          EY=EY+(0.5/COS(XPI))+(ST7/STB)
000170B    30    EX=2.0*B*EX
000172B          EY=2.0*B*EY
000174B          RETURN
000176B          END


000000B          SUBROUTINE BJO(X,BJ)
000000B
         C       THIS SUBROUTINE CALCULATES THE BESSEL FUNCTION JO(X) AND
         C       RETURNS THE VALUE THROUGH BJ.
000000B
000000B          T=X/3.
000001B          Z=3./X
000002B          Y=T*T
000003B          IF (X.GE.3.) GO TO 10
000004B          BJ=1.-Y*(2.2499997-Y*(1.265620B-Y*(.3163866-Y*(.0444479-
                1Y*(.0039444-Y*.0002100)))))
000015B          RETURN
000020B    10    W=SQRT(X)
000023B          AF=.797BB456-Z*(.00000077+Z*(.00552740+Z*(.00009512-Z*(.00137237
                1-Z*(.00072B05-Z*.00014476)))))
000034B          THETA=X-.7B539B16-Z*(.04166397+Z*(.00003954-Z*(.00262573
                1-Z*(.00054125+Z*(.00029333-Z*.0001355B)))))
000047B          BJ=AF*COS(THETA)/W
000053B          RETURN
000056B          END
```

```
000000B           SUBROUTINE ZO(B1,B2,G,X)
000000B
           C     THIS SUBROUTINE CALCULATES THE CHARACTERISTIC IMPEDANCE OF AN
           C     RCTL NORMALIZED BY NO, THE FREE SPACE IMPEDANCE OF THE TEM
           C     MODE. THE INPUTS ARE
           C       B1,B2  THE DISTANCES FROM THE SEPTUM TO THE UPPER, AND LOWER
           C       CHAMBER WALLS NORMALIZED BY A.
           C       G  THE GAP LENGTH NORMALIZED BY A.
           C     THE NORMALIZED CHARACTERISTIC IMPEDANCE IS OUTPUT VIA X.
000000B
000000B           REAL PI,QO
000000B           PI=4.0*ATAN(1.0)
000004B           ST1=0.0
000005B           DO 20 M=1,20
000010B             X1=-PI*B1*((2.0*M)-1.0)
000015B             X2=-PI*B2*((2.0*M)-1.0)
000021B             R1=2.0*EXP(X1)/(1.0-EXP(X1))
000027B             R2=2.0*EXP(X2)/(1.0-EXP(X2))
000034B             CHECK1=ABS(R1)
000035B             CHECK2=ABS(R2)
000037B             CHECK=AMAX1(CHECK1,CHECK2)
000041B            ST1=ST1+(R1+R2)/(2.0*M-1.0)
000047B             IF (CHECK.LT.0.0001) GO TO 21
000047B     20    CONTINUE
000054B     21    QO=2.0*ST1
000055B           X3=8.0/G/PI
000060B           X4=2.0*ALOG(X3)
000063B           X=PI/4.0/(X4+QO)
000066B          PRINT 30
000073B     30   FORMAT(//,4X,*B1*,8X,*B2*,8X,*G*,9X,*ZO*,8X,*QO*,8X,*FINAL STEP*/)
000073B           PRINT 22,B1,B2,G,X,QO,CHECK
000104B     22    FORMAT(1X,6F10.7)
000104B           RETURN
000106B           END
```

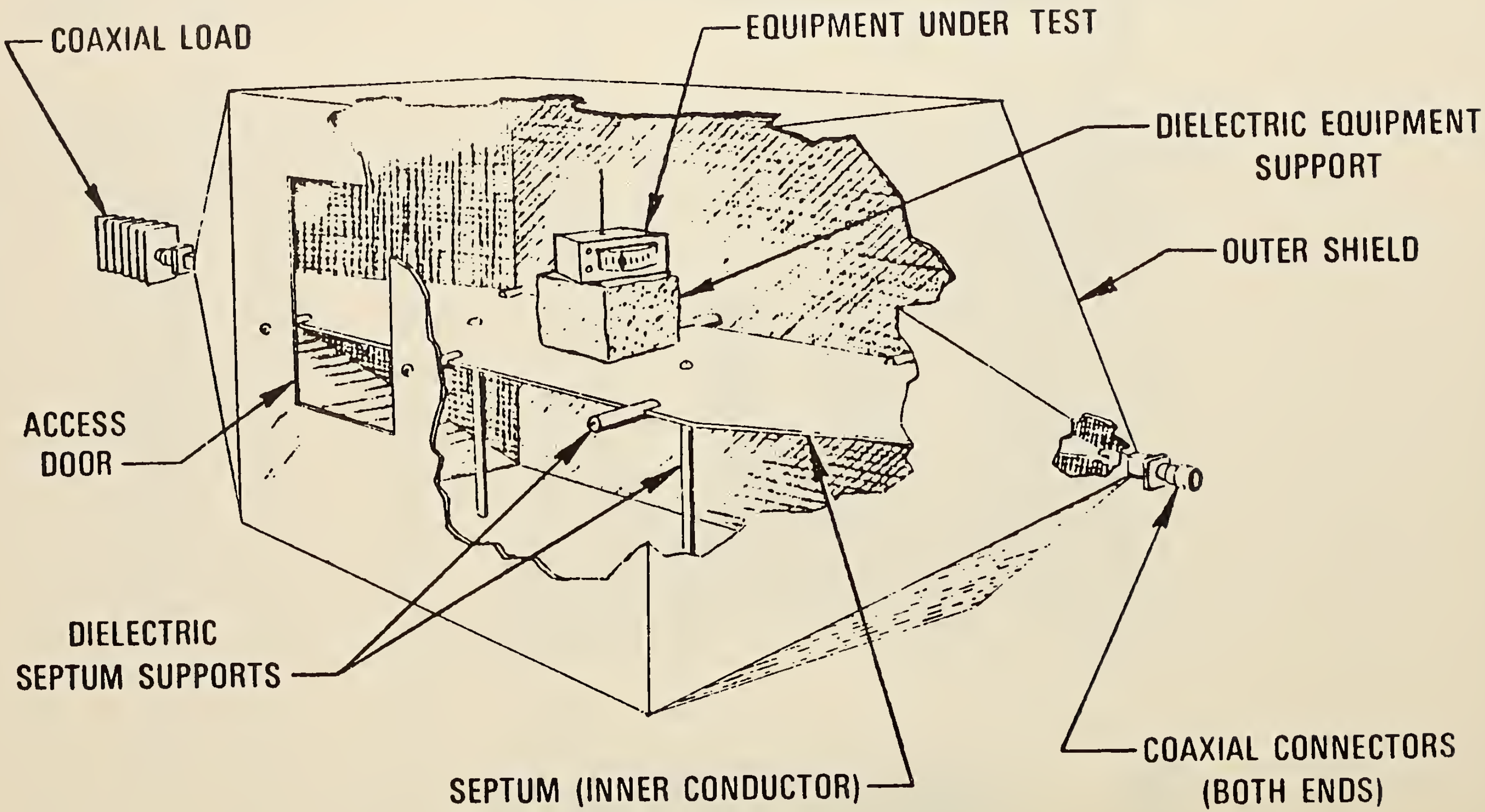


Figure 1. Diagram of an NBS TEM cell.

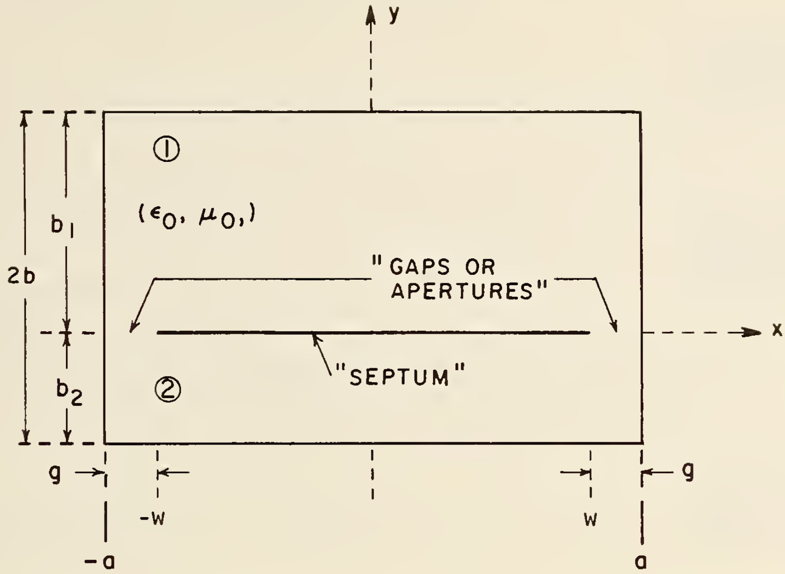


Figure 2. RCTL cross section

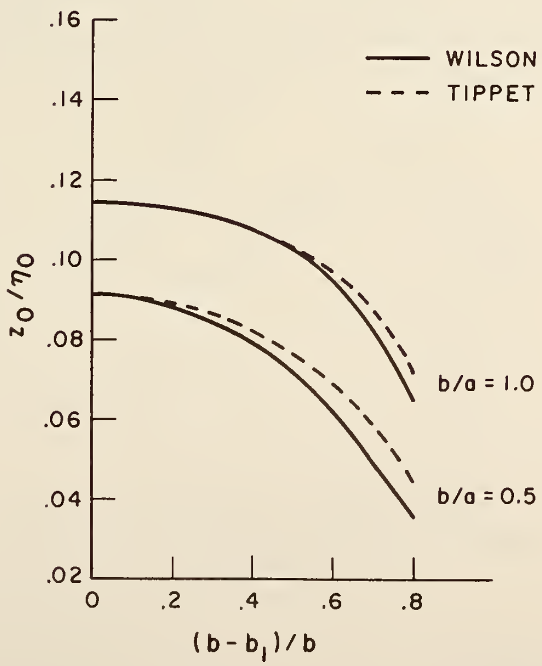


Figure 3. Normalized characteristic impedance z_0/η_0 for $g/a = 0.1$

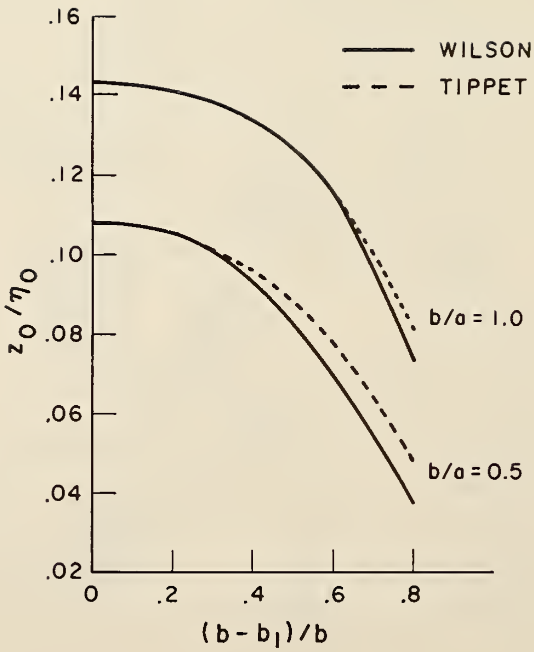


Figure 4. Normalized characteristic impedance z_0/n_0 for g/a = 0.2

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

he excitation of a transverse electromagnetic (TEM) cell by a vertical electric lertzian dipole is analyzed where the gap between the septum and side wall is assumed o be small. Approximate expressions for the field distribution and characteristic mpedance are derived. These expressions are numerically evaluated for some typical eometries, and good agreement with previously published results is shown. The ormation also allows a vertical offset for the septum position, thus offering more lexibility of increasing the size of the test area to accommodate larger pieces of est equipment.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

haracteristic impedance; field distribution; Hertzian dipole; integral equation; ectangular coaxial transmission line; TEM cell.

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