The Bispectrum and Higher-Order Spectra: A Bibliography
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2Some divisions within the center are located at Boulder, CO 80303.
The Bispectrum and Higher-Order Spectra: A Bibliography

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, Secretary

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

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The Bispectrum and Higher-Order Spectra: A Bibliography

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The bispectrum or Fourier transform of the 3\textsuperscript{rd} order moments of a time series is useful for the study of nonlinear or non-Gaussian phenomena. This bibliography cites 134 papers covering both theory and application. The entries are classified by content with special effort made to indicate papers that contain material on the computation, display and interpretation of the bispectrum.

Key Words: Bispectrum; cumulant spectra; nonlinear time series; polyspectra; spectrum analysis; statistics; time series.

INTRODUCTION

The bispectrum or Fourier transform of 3\textsuperscript{rd} order moments is useful for the study of nonlinear structure in time series. It is sensitive to phase coherence between wave components due, for example, to nonlinear wave interactions or to the harmonic structure of periodic functions that are not pure sinusoids. The bispectrum is useful both for detecting the presence of phase coherent structure and measuring the fraction of spectral power due to coherent components.

The bispectrum was first discussed by Tukey (1953). The general idea of harmonic analysis of higher order moments was also introduced by Blanc-Lapierre and Fortet (1953). Magness (1954) used what would now be called the trispectrum (transform of 4\textsuperscript{th} order moments) to compute the spectral response of a quadratic device to non-Gaussian noise. Mazelsky (1954) used higher order spectra to determine non-Gaussian probability functions for the input disturbances and output responses of linear systems. Tick (1961) showed how the cross-bispectrum could be used to estimate the transfer function of a quadratic system. Following the suggestion of Tukey (1963) and Tick (1963), bispectral analysis was first applied to the study of nonlinear phenomena in ocean waves by Hasselmann, Munk and MacDonald (1963). The first rather thorough discussion of the bispectrum and its properties appears in Shaman (1964) which extends Tukey's 1953 unpublished manuscript. Between 1965 and 1967 the general theory of the bispectrum and higher order cumulant spectra (or polyspectra) was intensively developed by Brillinger, Rosenblatt, Van Ness, Godfrey, Parzen, and Akaike. Since 1967 the bispectrum has found a wide variety of applications primarily in the physical sciences. However, the practice of bispectral analysis has received little attention in the statistical literature in more than a decade.
The purpose of this bibliography is to make the literature of the last two decades, now widely scattered in physical sciences journals, more accessible. There may well be other references to harmonic analysis of higher order moments in the literature of stochastic processes. In particular, the vast literature on kernel estimation and identification methods has not been searched for related papers. However, four such papers that are of special interest are included. Hung and Stark (1977) is a review of kernel identification methods with an extensive bibliography (88 entries). Yasui (1979) discusses the application of kernel methods to nonlinear systems analysis, and also has an extensive bibliography (49 entries). Both papers mention the relationship to the bispectrum. Hung and Stark (1979), and Hung, Brillinger, and Stark (1979) use bispectral analysis to compute higher order kernels for nonlinear systems.

The bibliography is arranged alphabetically by first author. There is a chronological listing of first authors and an alphabetical cross reference of second authors with first authors. Finally, the entries are classified by content. Although this is not a review paper, an effort has been made to indicate papers that contain material on the computation, display and interpretation of the bispectrum. Huber, Kleiner, Gasser, and Dummermuth (1971), and Kim and Powers (1979) give nice reviews of the bispectrum and its interpretation.

I would like to thank Vicki Schneller of the Department of Commerce Boulder Library for her assistance with the literature search, and Lorna Buhse of the Statistical Engineering Division for her help in organizing and typing the bibliography.
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Mazelsky, B.

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NOTE: The principal publication outlet for the foregoing data is the Journal of Physical and Chemical Reference Data (JPCRFD) published quarterly for NBS by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements available from ACS, 1155 Sixteenth St., NW, Washington, DC 20036.

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