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U.S. DEPARTMENT OF COMMERCE/ National Bureau of Standards

# The Bispectrum and Higher-Order Spectra: A Bibliography

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# The Bispectrum and Higher-Order Spectra: A Bibliography

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The bispectrum or Fourier transform of the 3rd order moments of a time series is useful for the study of nonlinear or non-Gaussian phenomena. This bibliography cites 134 papers covering both theory and application. The entries are classified by content with special effort made to indicate papers that contain material on the computation, display and interpretation of the bispectrum.

Key Words: Bispectrum; cumulant spectra; nonlinear time series; polyspectra; spectrum analysis; statistics; time series.

## INTRODUCTION

The bispectrum or Fourier transform of 3rd order moments is useful for the study of nonlinear structure in time series. It is sensitive to phase coherence between wave components due, for example, to nonlinear wave interactions or to the harmonic structure of periodic functions that are not pure sinusoids. The bispectrum is useful both for detecting the presence of phase coherent structure and measuring the fraction of spectral power due to coherent components.

The bispectrum was first discussed by Tukey (1953). The general idea of harmonic analysis of higher order moments was also introduced by Blanc-Lapierre and Fortet (1953). Magness (1954) used what would now be called the trispectrum (transform of 4th order moments) to compute the spectral response of a quadratic device to non-Gaussian noise. Mazelsky (1954) used higher order spectra to determine non-Gaussian probability functions for the input disturbances and output responses of linear systems. Tick (1961) showed how the cross-bispectrum could be used to estimate the transfer function of a quadratic system. Following the suggestion of Tukey (1963) and Tick (1963), bispectral analysis was first applied to the study of nonlinear phenomena in ocean waves by Hasselmann, Munk and MacDonald (1963). The first rather thorough discussion of the bispectrum and its properties appears in Shaman (1964) which extends Tukey's 1953 unpublished manuscript. Between 1965 and 1967 the general theory of the bispectrum and higher order cumulant spectra (or polyspectra) was intensively developed by Brillinger, Rosenblatt, Van Ness, Godfrey, Parzen, and Akaike. Since 1967 the bispectrum has found a wide variety of applications primarily in the physical sciences. However, the practice of bispectral analysis has received little attention in the statistical literature in more than a decade.

The purpose of this bibliography is to make the literature of the last two decades, now widely scattered in physical sciences journals, more accessible. There may well be other references to harmonic analysis of higher order moments in the literature of stochastic processes. In particular, the vast literature on kernel estimation and identification methods has not been searched for related papers. However, four such papers that are of special interest are included. Hung and Stark (1977) is a review of kernel identification methods with an extensive bibliography (88 entries). Yasui (1979) discusses the application of kernel methods to nonlinear systems analysis, and also has an extensive bibliography (49 entries). Both papers mention the relationship to the bispectrum. Hung and Stark (1979), and Hung, Brillinger, and Stark (1979) use bispectral analysis to compute higher order kernels for nonlinear systems.

The bibliography is arranged alphabetically by first author. There is a chronological listing of first authors and an alphabetical cross reference of second authors with first authors. Finally, the entries are classified by content. Although this is not a review paper, an effort has been made to indicate papers that contain material on the computation, display and interpretation of the bispectrum. Huber, Kleiner, Gasser, and Dummermuth (1971), and Kim and Powers (1979) give nice reviews of the bispectrum and its interpretation.

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First authors by year

1953
Blanc-Lapierre, A.
Tukey, J. W.

1954
Magness, T.A.
Mazelsky, B.

1959
Tukey, J. W.

1960
Shiryaev, A. N.

1961
Tick, L. J.

1963
Hasselmann, K.
MacDonald, G.
Sinai, Ya. G.
Tick, L. J.
Tukey, J. W.

1964
Leonov, V. P.
Madden, T.
Rosenblatt, M.
Shaman, P.

1965
Brillinger, D. R.
Godfrey, M. D.
Haubrich, R. A.
Rosenblatt, M.

1966
Akaike, H.
Aubry, M. P.
Hasselmann, K.
Rosenblatt, M.
Van Ness, J. W. (2)

1967
Aubry, M. P.
Bartlett, M. S.
Brillinger, D. R. (2)
Parzen, E.

1968
Bozzi Zadro, M.
Cartwright, D. E.
Hinich, M. J.
Korein, J.
Marussi, A.

1970
Alekseyev, V. G.
Brillinger, D. R.
Dumermuth, G.
Van Atta, C. W.
Westcott, M.

1971
Barnett, T.
Dumermuth, G.
Huber, P. J.
Kleiner, B.
Rosenblatt, M.

1972
Brillinger, D. R.
Dalzell, J. F. (2)
Feuerverger, A.
Gasser, T.
Kiriyama, K.
Mitsuishi, A.
Murty, T. S.
Sasaki, K.

1973
Brillinger, D. R.
Roden, G. I.
Tachi, S.
Yeh, T. T.

1974
Brillinger, D. R. (2)
Mager, P. P.
Van Atta, C. W.
Yao, N. C.

1975
Akaike, H.
Brillinger, D. R.
Gasser, T.
Kedem-Kimelfeld, B.
Mager, P. P.
Neshyba, S
Ohta, M.
Sasaki, K
Sato, T.
Tachi, S.
Yao, N. C.
Zhurbenko, G.

1976
Akaike, H.
Briscoe, M. G.
Helland, K. N.
Kim, Y. C.
Lii, K. S.
Lumley, J. L.
Ten Hoopen, M.
Ueno, T.
Yamakawa, S.

1977
Armstrong, J. W.
Brillinger, D. R.
Davies, R. B.
Helland, K. N.
Hung, G.
Kim, Y. C.
Murata, T.
Ohta, M.
Powers, E. J.
Rao, S. T.
Sasaki, K.
Sato, T. (3)
Ten Hoopen, M.
Ueno, T.
Yao, N. C.

1978
Barresen, R.
Dubkov, A. A.
Dumermuth, G.
Gerzon, M. A.
Hasselmann, D. E.
Kim, Y. C.
McComas, C. H. III
Ohta, M.
Rosenblatt, M.
Sasaki, K.
Sato, T. (3)
Sclove, S. L.
Shimizu, H.
Tanaka, K.

1979
Fried, D. L.
Gabrielli, C.
Helland, K. N.
Hinich, M. J.
Hung, G. (2)
Kim, Y. C.
Sasaki, K.
Sato, T.
Van Atta, C. W.
Yasui, S.

1980 (Partial)
Herring, J.
Kim, Y. C.
McComas, C. H.
Rosenblatt, M.
Sasaki, O.

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|---|---|
| Arahata, E. | Akaike, H. (2) |
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| Bendiner, D. J. | Roden, G. I. |
| Bozzi Zadro, M. | Marussi, A. |
| Brillinger, D. R. | Hung. G. |
| Briscoe, M. G. | McComas, C. H. |
| Caputo, M. | Bozzi Zadro, M. |
| Clay, C. S. | Hinich, M. J. |
| Czapski, U. | Rao. S. T. |
| Fortet, R. | Blanc-Lapierre, A. |
| Gasser, T. | Dumermuth, G. |
| Hatakeyama, K. | Ohta, M. |
| Henry, R. F. | Murty, T. S. |
| Huber, P. J. | Dumermuth, G. (2) |
| Inoue, T. | Shimizu, H. |
| Johnson, L. C. | Barnett, T. |
| Keddam, M. | Gabrielli, C. |
| Kikkawa, S. | Tanaka, K. |
| Kim, Y. C. | Powers, E. J. |
| Kishimoto, T. | Sato, T. |
| Kleiner, B. | Huber, P. J. |
| Lii, K. S. | Helland, K. N. (2) |
| Malakhov, A. N. | Dubkov, A. A. |
| Munk, W. | Hasselmann, K. |
| Nakajima, T. | Ueno, T. |
| Nakashima, S. N. | Mitsuishi, A. |
| Neshyba, S. | Yao, N. C. (2) |
| Ohara, H. | Murata, T. |
| Powers, E. J. | Kim, Y. C. (4) |
| Rosenblatt, M. | Brillinger, D. R. (2) |
| Rosenblatt, M. | Lii, K. S. |
| Sasaki, K. | Sato, T. (5) |
| Sasaki, O. | Sato, T. (2) |
| Sato, T. | Kiriyama, K. |
| Sato, T. | Sasaki, K. (5) |
| Sato, T. | Sasaki, O. |
| Sobey, E. J. C. | Neshyba, S. |
| Stark, L. | Hung, G. (2) |
| Tachi, S. | Ueno, T. |
| Takeuchi, K. | Lumley, J. L. |
| Tick, L. J. | Korein, J. |
| Van Atta, C. W. | Helland, K. N. |
| Van Atta, C. W. | Yeh, T. T. |
| Van Ness, J. W. | Rosenblatt, M. |
| Yamaguchi, S. | Ohta, M. (2) |
| Yeh, T. T. | Van Atta, C. W. |
| Zandt, P. A. | Ten Hoopen, M. (2). |
| Zuev, N. M. | Zhurbenko, G. |

Brief Discussion or Close Relationship

Tukey (1959) (1963)
Tick (1963)
Bartlett (1967)
Hinich and Clay (1968)
Westcott (1970)
Dumermuth, Huber, Kleiner and Gasser (1970)
Rosenblatt (1971)
Brillinger (1974)
Magar (1974) (1975)
Van Atta (1974)
Ohta, Hatakeyama, Hiromitsu and Yamaguchi (1975)
Kedem-Kimelfeld (1975)
Helland and Van Atta (1976)
Hung and Stark (1977)
Ohta, Yamaguchi, and Hiromitsu (1978)
Sclove (1978)
Yasui (1979)
Gabrielli, Keddam, and Raillon (1979)

General Theory

Blanc-Lapierre and Fortet (1953)
Shiryaev (1960)
Tick (1961)
Hasselmann, Munk and MacDonald (1963)
MacDonald (1963)
Sinai (1963)
Shaman (1964)
Brillinger (1965)
Rosenblatt and Van Ness (1965)
Rosenblatt (1966)
Van Ness (1966)
Akaike (1966)
Parzen (1967)
Brillinger and Rosenblatt (1967a, b)
Huber, Kleiner, Gasser, and Dumermuth (1971)
Sasaki, Sato, and Yamahita (1975)
Brillinger (1975)

Estimation/Computation

Tick (1961)
Hasselmann, Munk and MacDonald (1963)
Shaman (1964)
Brillinger (1965)
Rosenblatt and Van Ness (1965)
Godfrey (1965)
Haubrich (1965)
Rosenblatt (1966)
Van Ness (1966)
Brillinger and Rosenblatt (1967a, b)
Alekseyev (1970)
Van Atta and Yeh (1970)
Huber, Kleiner, Gasser and Dumermuth (1971)
Sasaki, Sato and Yamashita (1975)
Akaike, Arahata, Ozaki (1975)
Dumermuth and Gasser (1978)
Kim and Powers (1979)

Display/Interpretation

Hasselmann, Munk and MacDonald (1963)
MacDonald (1963)
Shaman (1964)
Godfrey (1965)
Haubrich (1965)
Brillinger and Rosenblatt (1967b)
Cartwright (1968)
Marussi, Bozzi Zadro and Manzoni (1968)
Bozzi Zadro and Caputo (1968)
Huber, Kleiner, Gasser and Dumermuth (1971)
Roden and Bendiner (1973)
Neshyba and Sobey (1975)
Lii, Rosenblatt and Van Atta (1976)
Helland, Lii, and Rosenblatt (1977), (1979)
Dumermuth and Gasser (1978)
Kim and Powers (1979)
Kim, Beall, Powers, and Miksad (1980)

## Applications

### Acoustics

Ohta, Yamaguchi, and Iwashige (1977)
Sasaki, Sato, and Nakamura (1977)
Gerzon (1978)
Sato and Sasaki (1979)

### Astrophysics

Aubry (1966), (1967)
Armstrong (1977)
Fried (1979)

### Biomedicine

Korein, Tick, Zeitlin, and Randt (1968)
Dumermuth, Huber, Kleiner and Gasser (1970), (1971)
Huber, Kleiner, Gasser, and Dumermuth (1971)
Barnett, Johnson, Naitoh, Hicks and Nute (1971)
Magar (1974)
Tachi (1975)
Ten Hoopen and Zandt (1976), (1977)

### Economics

Godfrey (1965)

### Fluid Mechanics - Turbulence

Van Atta and Yeh (1970)
Yeh and Van Atta (1973)
Van Atta (1974), (1979)
Helland and Van Atta (1976)
Lumley and Takeuchi (1976)
Lii, Rosenblatt and Van Atta (1976)
Helland, Lii, and Rosenblatt (1977), (1979)
Rosenblatt (1978)
Herring (1980)

### Geophysics

Brillinger (1973)
Haubrich (1965)
Hinich and Clay (1968)
Bozzi Zadro and Caputo (1968)
Madden (1964)
Marussi, Bozzi Zadro and Manzoni (1968)

Goodness-of-Fit Tests

Gasser (1975)

Hydromechanics

Hasselmann (1966)
Dalzell (1972a, b)

Image Processing

Sato and Sasaki (1977)
Sato, Sasaki, and Nonaka (1978)
Sasaki, Sato and Nakamura (1978)

Kernel Estimation and Identification

Hung and Stark (1977), (1979)
Hung, Brillinger and Stark (1979)
Yasui (1979)

Mechanical Engineering

Mazelsky (1954)
Yamakawa (1976)
Sato, Sasaki and Nakamura (1977)
Sato and Sasaki (1977)
Ueno and Nakajima (1977)
Sato, Sasaki, and Nonaka (1978)
Shimizu and Inoue (1978)

Meteorology

MacDonald (1963)
Sato, Sasaki, and Mori (1975)

Nonlinear Prediction

Van Ness (1966)

Oceanography

Hasselmann, Munk and MacDonald (1963)
Cartwright (1968)
Murty and Henry (1972)
Roden and Bendiner (1973)
Yao (1974)
Neshyba and Sobey (1975)
Yao, Neshyba and Crew (1975), (1977)
Briscoe (1976)
Rao, Czapski, and Sedefian (1977)
McComas (1978)
Hasselmann (1978)
McComas and Briscoe (1980)

Optics

Sato, Kishimoto and Sasaki (1978)
Sato and Sasaki (1978)
Sasaki, Sato and Oda (1980)

Plasma Physics

Kim and Powers (1976), (1977), (1978), (1979)
Powers and Kim (1977)
Kim, Beall, Powers, and Miksad (1980)

Transfer Function Estimation

Magness (1954)
Tick (1961)
Brillinger (1970)
Tachi (1975)
Kedem-Kimelfeld (1975)
Brillinger (1977)
Borresen (1978)
Hinich (1979)

Signal Processing

Parzen (1967)
Tachi (1973)
Murata and Ohara (1977)
Tanaka, Kikkawa, and Ohara (1978)
Dubkov and Malakhov (1978)

Simulation

Sasaki and Sato (1978)

Stochastic Processes

Rosenblatt (1964), (1980)
Brillinger (1972), (1974)
Zhurbenko and Zuev (1975)
Davies (1977)

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