NBS TECHNICAL NOTE 1020

U.S. DEPARTMENT OF COMMERCE/ National Bureau of Standards

CALCULATION OF FLUORESCENT EFFICIENCY FROM EXPERIMENTAL DATA BY THE HUYGENS PRINCIPLE

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CALCULATION OF FLUORESCENT EFFICIENCY FROM EXPERIMENTAL DATA BY THE HUYGENS PRINCIPLE

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Issued May 1980

NATIONAL BUREAU OF STANDARDS TECHNICAL NOTE 1020

Nat. Bur. Stand. (U.S.), Tech. Note 1020, 36 pages (May 1980)

CODEN: NBTNAE

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 1980

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402

Stock No. 003-003-02199-7 Price $2.00 (Add 25 percent additional for other than U.S. mailing)

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This paper concerns the situation of a fluorescent semiconducting layer deposited upon a substrate. It is desired to compute the number of quanta of fluorescent radiation per quantum absorbed of the pump radiation. The principal topic which is discussed is the theory of the Huygens Principle method which gives the intensity of the emitted light in terms of the power of an array of point sources in the semiconductor. The method used is a direct application of Huygens Principle to the individual waves that are multiply reflected by the boundary surfaces. The results are given in terms of the constants of the materials, the dimensions, and three quantum numbers, (1) P, the number of two-way trips in the semiconductor, (2) Q, the number of round trips in the substrate, and (3) S, the number of two-way penetrations of the boundary between them. Because of approximations used, the method is mainly useful for radiation emerging nearly normal to the surfaces. For light within 10° to the normal the errors are not more than a few percent. The Huygens Principle method is also developed for use with planar external sources, and this method is compared with the impedance methods. The calculations made with external sources are needed for determining the non-fluorescent optical constants of the materials.

Key Words: Absolute fluorescent efficiency; optical loss constants; index of refraction; Huygens Principle; characteristic wave impedance.

I. INTRODUCTION

This manuscript is concerned with the calculation of internal fluorescent efficiencies of semiconductors, especially amorphous silicon, from appropriate experimental data. The situation is represented by figure 1. A layer of semiconducting material of thickness D2 and complex index of refraction N2, has been deposited upon a substrate of thickness D3 (large compared to D2), and complex index of refraction N3. The sample is irradiated by pump light located in Region 1 (air) with the result that fluorescence is produced by a distribution of sources within the semiconductor. It is the purpose of these calculations to determine the fluorescent efficiency from measurements on the incident beam and beams in both air Region 1 (the "back" direction) at a perpendicular distance X1, and Region 4 (the "forward" direction) at a perpendicular distance X4. The distance of a source within the semiconductor from the air-semiconducting surface is X2.

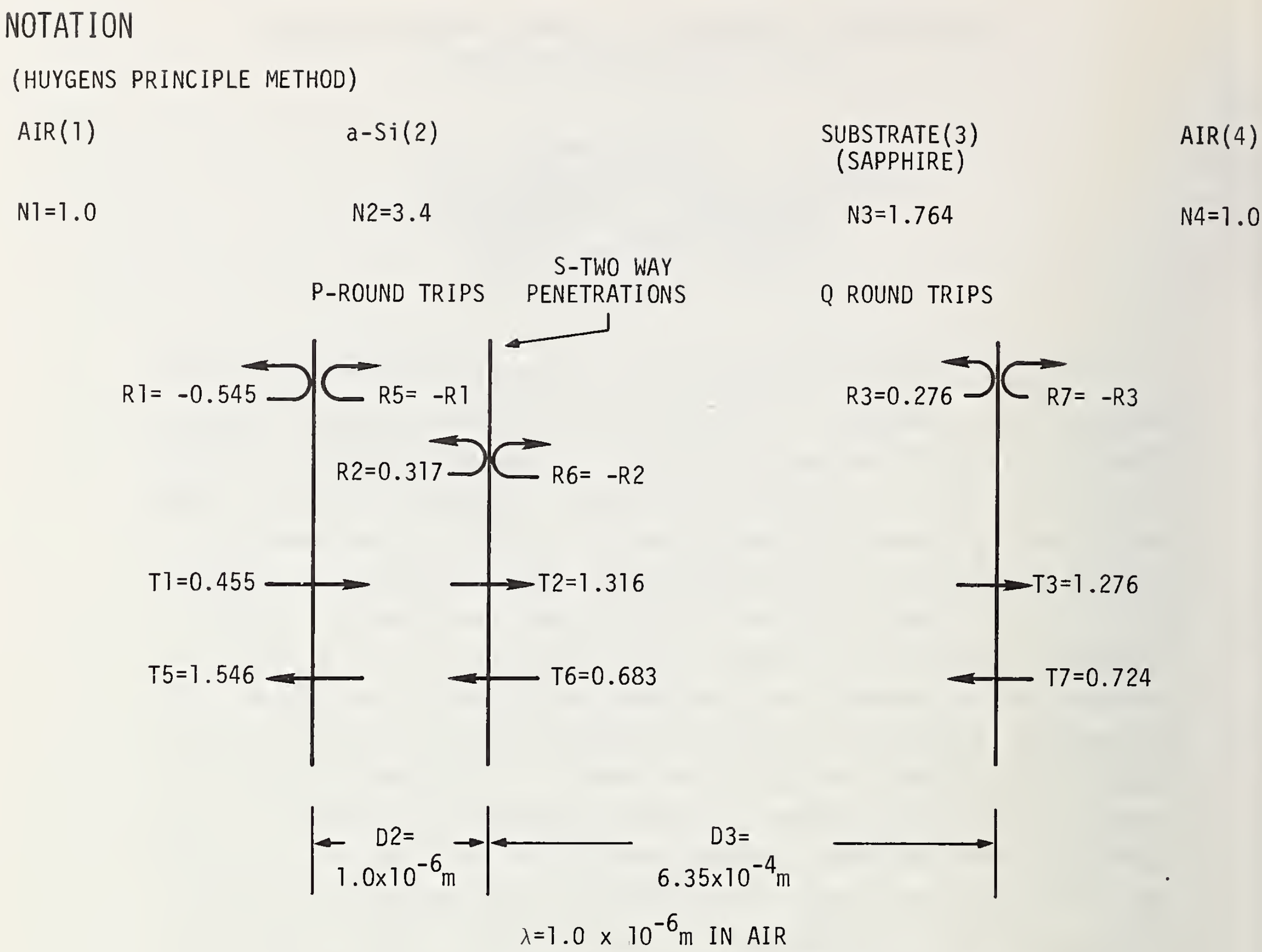


Figure 1. Notation. Geometry of Experiment. Definitions of Amplitude Reflection and Transmission Coefficients.

The subject of absolute yields in photoluminescence has been the subject of a large number of papers. Lipsett [1] has given a summary of the theory and a review of experimental work on solids while Demas and Crosby [2] have given a review of the work on liquids. These reviews contain extensive bibliographies on the subject. A couple of later papers that have come to my attention are listed in References [3] and [4]. However, none of the existing papers seem to be very relevant to the present situation, since other workers have dealt with samples with such thickness and absorption that interference effects did not have to be considered. However, the situation which is of interest here pertains with optically thin samples, where interference effects cannot be neglected.

The various amplitude reflection and transmission coefficients that are employed in the calculations are defined in figure 1, and sample numerical values are given for normal incidence. Later these definitions will be employed with situations of oblique incidence, where their values depend on the angles of incidence and upon the polarization. In principle, therefore, both N2 and N3 are complex. However, there is strong evidence that the losses in the materials at the wavelengths of interest are very small. Therefore, generally the effect of the losses is included only through giving the real parts of the complex propagation constants finite, but small values, while the indices of refraction are considered to be real in calculations of reflection and transmission coefficients. The effect of neglecting the imaginary parts of the indices of refraction in calculating the reflection and transmission coefficients is negligible for the situations which are of interest, as will be justified in the Appendix. A positive reflection coefficient implies no change of phase on reflection, while a minus sign implies a 180° change in phase for the electric intensity.

The chief method of calculation that is used is the "Huygens Principle Method," which employs a detailed description of the propagation of individual waves from a point source in the course of multiple reflections by use of Huygens Principle. The details of this method appear not to be available in the literature.

Unfortunately, the data on the materials which is needed for computing the reflection and transmission coefficients and for computing the absorption is not in as satisfactory state as desired. Therefore, it is also necessary to make measurements on the materials with the use of external sources of light, which can be assumed to give approximately planar wave fronts. In principle, the present method could be used for this purpose, but it is not as convenient as established methods that can be found in various textbooks. However, the comparison of the results from this method and those from one of the established methods to the same plane wave problem can provide a check on the validity of the Huygens Principle method.

All textbooks on optics have some mention of the behavior of plane waves in thin films. The standard text by Born and Wolf [5] has an extensive treatment of interference of light in a thin film deposited on a semi-infinite substrate. This is in the notation that employs only real quantities and therefore can be programmed in BASIC on a small computer, but this treatment is not convenient for more complicated problems. Another method which is more convenient for present purposes and which has been used as a secondary method is the impedance method which is based upon the analogy between waves in media with parallel plane boundaries and infinite cross section with waves on transmission lines [6,7]. Another mathematical approach to this problem is through the use of scattering matrices, wherein the properties of each sample are specified in terms of a scattering matrix [5]. This method is especially useful in the design of a coated lens which may have an arbitrary number of thin films with parallel sides. The properties of the assembly are given by the matrix which is found by multiplying together the matrices representing the various films. This method, despite its different mathematical form, can be shown to be physically equivalent to the impedance method. Here, as a matter of taste, the impedance method is used.

A method that has come to my attention after the preparation of the first draft of this manuscript is the expansion of the field of a dipole in terms of plane waves using the Hankel function as a factor in the expansion coefficient [8]. The field is computed by means of the saddle point method. When the method is applied to a stratified medium it is necessary to include a factor which is a function of the reflection coefficient, which is determined by a method equivalent to the impedance method. This integration can be carried out in closed form in the case of a dipole at the interface between two half spaces (what is often called the "Sommerfield Problem" [9]). In more complicated cases such as the dipole at the boundary of a two layer stratified medium, in performing the integration, it is necessary to include contributions from the poles associated with each of the wave modes that are excited, and thus it is necessary to consider the effects of particular waves, in a manner somewhat analagous to the present method. Reference [8] does not consider the still more complicated case, which is considered here, of a dipole located within a layer of a stratified medium. This method may be needed for angles near grazing incidence or when polarization effects become significant.

II. THE HUYGENS PRINCIPLE PROPAGATION METHOD FOR EXTERNAL PLANE PARALLEL SOURCES

The main purpose of treating this topic is tutorial. Any desired numerical results can be obtained by the well-established impedance or scattering matrix methods. It is desirable to make this tutorial presentation in order to introduce some definitions and methods which are required for the much more interesting but more complicated problem of the radiation from fluorescent point sources in the semiconducting slab.

Any radiation incident from the left in figure 1 must emerge either to the left in a "backward" beam in Region 1 or as a "forward" beam in Region 4 if emission from the edges and absorption are neglected. Although it is impossible to identify individual photons that compose these beams, it is convenient to suppose that their paths could be traced, such as in figure 2. The drawing of this figure and others to follow has been simplified by ignoring the changes in direction due to refractions at the boundaries. These paths may be described by the following quantum numbers:

$P \equiv$ Number of two-way transits of Region 2

$Q \equiv$ Number of two-way transits of Region 3

$S \equiv$ Number of two-way penetrations of the boundary between Regions 2 and 3. ($0 \leq S \leq P$, and $0 \leq S \leq Q$).

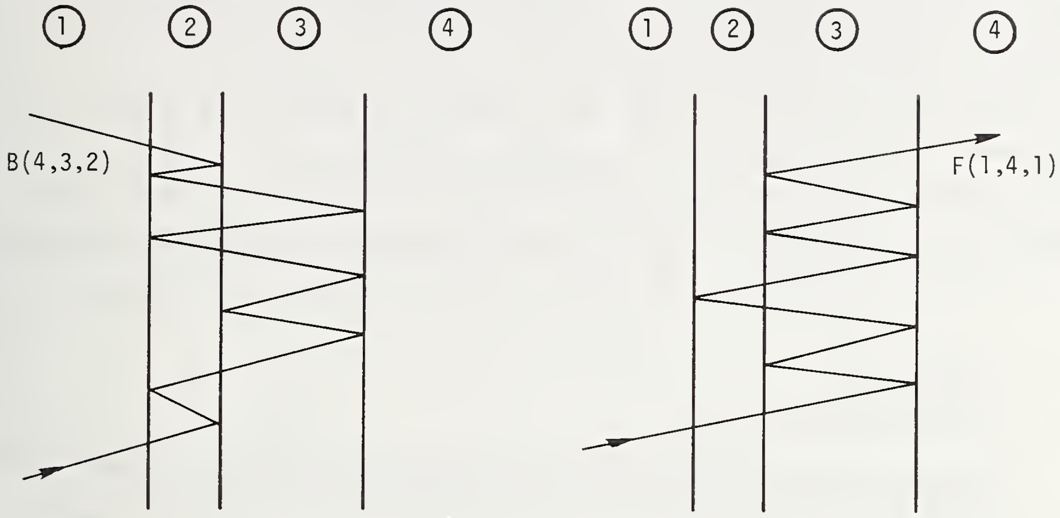
In the definition and evaluation of these quantum numbers, any transits in excess of whole two-way ones are to be ignored. Accordingly, the path in figure 2(a) is denoted as B(4,3,2), while that in figure 2(b) is denoted as F(1,4,1).

Associated with each path is a plane wave, and these waves, having a common source, are coherent. Therefore, the total amplitudes in the backward and forward waves are determined by adding the amplitudes of the individual waves with proper regard to phase.

The incident wave in air is assumed to have unit amplitude and zero phase at the surface of the semiconductor. This assumption does not restrict the generality of the results to be obtained. The amplitudes of individual waves may be seen by reference to the definitions given in figure 1 and by inspection of figure 2 to be given, in general, for a point at a distance X1 in Region 1, with a propagation constant k by

SAMPLE PHOTON PATHS

(DEVIATIONS DUE TO REFRACTIONS ARE IGNORED FOR SIMPLICITY.)



(a) (b)

Figure 2. Sample Photon Paths. (Deviations due to refraction are ignored for simplicity.)

$$B(P,Q,S) = R5^{P-1} \cdot R2^{(P-S)} \cdot R6^{Q-S} \cdot R3^{Q} \cdot T1 \cdot T5 \cdot (T2 \cdot T6)^{S} \cdot EXP\left(- \frac{2 \cdot P \cdot B \cdot D2}{COS\ \Theta 2}\right) \cdot EXP\left(- \frac{2 \cdot Q \cdot C \cdot D3}{COS\ \Theta 3}\right) \cdot EXP\left(\frac{-X1 \cdot k}{COS\ \Theta 1}\right) ,$$

except that

$$B(0,0,0) = R1 \cdot EXP\left(\frac{-X1 \cdot k}{COS\ \Theta 1}\right) , \qquad (1)$$

and for a point at a distance X4 in Region 4 with a propagation constant k, the amplitude is

$$F(P,Q,S) = R5^{P} \cdot R2^{P-S} \cdot R6^{Q-S} \cdot R3^{Q} \cdot T1 \cdot T3 \cdot T2^{S+1} \cdot T6^{S} \cdot EXP\left(\frac{-(2 \cdot P+1) \cdot C \cdot D2}{COS\ \Theta 2}\right) \cdot EXP\left(\frac{-(2 \cdot Q+1) \cdot B \cdot D3}{COS\ \Theta 3}\right) \cdot EXP\left(\frac{-X4 \cdot k}{COS\ \Theta 4}\right) , \qquad (2)$$

where the Θ's are angles of refraction or reflection and where, again B (without indices) and C are respectively the complex propagation constants in Regions 2 and 3. ($B = jk \cdot N2$, and $C = jk \cdot N3$.)

In this manuscript, subscripts are typed on the same lines as the symbols to which they refer. The procedure simplifies typing and conforms to the notation used on computers.

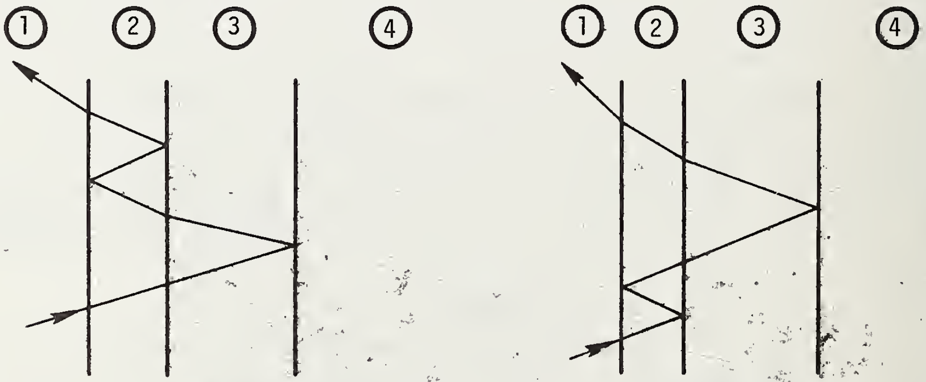
The values for the reflection and transmission coefficients can be obtained by well-known formulas, which are given in a later section dealing with the impedance method. It is to be kept in mind that they are functions of the angles of incidence and of the type of polarization. See Eqs. (35) and (37-41).

Equations (1) and (2) do not take into account the specific order in which the individual reflections and transmissions take place, and there may be more than one wave with the same values of P; Q, and S, and these have the same amplitude. A simple example of such a degeneracy is illustrated by figure 3. In carrying out any summation of amplitudes, one

DEGENERACY

(WAVE PROPAGATION METHOD)

SINCE PRESENT FORMULATION DOES NOT TAKE INTO CONSIDERATION THE ORDER OF THE REFLECTIONS, E(PQS) MAY HAVE A DEGENERACY. FOR A SIMPLE EXAMPLE, CONSIDER THE SITUATION FOR P = 2, Q = 1, & S = 1. WE HAVE TWO DIAGRAMS



FOR THESE QUANTUM NUMBERS, THEREFORE, E1(2, 1, 1) MUST BE MULTIPLIED BY 2 IN ANY SUMMATION, &, IN GENERAL E(P, S, Q) MUST BE MULTIPLIED BY ITS "STATISTICAL WEIGHT."

Figure 3. Example of a Degeneracy.

must multiply any value obtained from Eq. (1) or Eq. (2) by its "statistical weight" M(P,Q,S). In general, I have not derived closed formulas for statistical weights, but I have determined them for a given P, Q, and S by drawing complete sets of the diagrams of the types illustrated in figures 2 and 3 and by counting up the number of diagrams.

In the particular case of the backward plane waves considered here, there is a closed formula for the statistical weight, which is given by the product of two factors M1(P,S) and M2(Q,S). M1(P,S) is just the number of ways of choosing P items S at a time which is given by a formula found in numerous handbooks and textbooks [10].

$$M1(P,S) = \frac{P!}{S!(P-S)!} \quad . \tag{3}$$

M2(Q,S) is the number of ways of choosing Q round trips contained S penetrations, which is the same as the number of ways of placing Q balls in S boxes under the constraints that each box contains at least one ball and that all of the balls are in some box or another. This is given [10] by the binomial coefficient $\binom{Q-1}{S}$, or

$$M2(Q,S) = \frac{(Q-1)!}{S!(Q-1-S)!} \quad . \tag{4}$$

It should be pointed out that B(P,Q,S) and F(P,Q,S) do not necessarily have the same statistical weights for the same values of P, Q, and S. By reference to figure 4, it can be

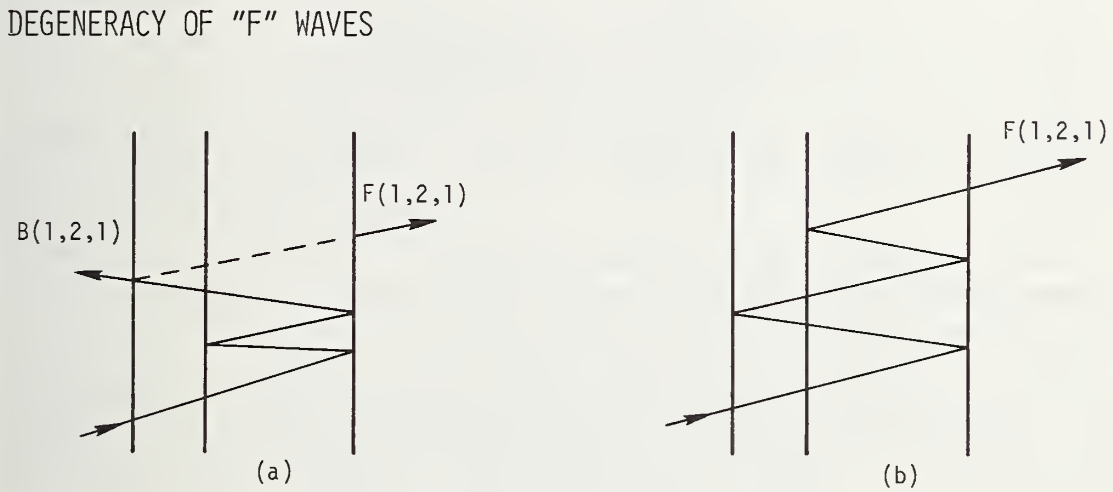


Figure 4. Degeneracy of F Waves.

seen that an "F" path can be generated from a "B" one by extending that path inside the solid material by a one-way transversal of the two slabs, and these have the same quantum numbers. From the construction we note that B(1,2,1) is non-degenerate. However, in figure 4(b) is shown another F(1,2,1) path. Therefore, F(1,2,1) has a statistical weight of 2.

In general, the summation is to be carried numerically with a computer. In principle, the quantum numbers P, Q, and S range from zero to infinity subject to the constraints that $S \leq P$ and $S \leq Q$. In practice, only a finite number of terms are summed. For this purpose it is necessary to rank the waves in accordance with their estimated importance. It is supposed that the relative importance is inversely proportional to the total number of reflections the wave experiences. Table 1 shows a list of the waves that I have employed in

TABLE 1

SAMPLE CALCULATIONS FOR A PLANE WAVE EXTERNAL SOURCE AT NORMAL INCIDENCE

P Q S	Backward Waves M(P,Q,S)	B(P,Q,S)	Forward Waves M(P,Q,S)	F(P,Q,S)	Total Accumulated Intensity
0 0 0	1	- 0.545	1	0.764	0.881
1 0 0	1	0.223	1	0.132	0.948
0 1 0	0	0.0	1	- 5.08 E-2	0.951
1 1 1	1	0.175	1	0.104	0.992
1 1 0	1	- 1.95 E-2	1	- 1.15 E-2	0.993
2 0 0	1	3.85 E-2	1	2.28 E-2	0.995
0 2 0	0	0.0	1	4.45 E-3	0.995
2 1 1	2	3.02 E-2	2	1.79 E-2	0.996
1 2 1	1	- 1.53 E-2	2	- 9.07 E-3	0.996
3 0 0	1	6.65 E-3	1	3.94 E-3	0.996
0 3 0	0	0.0	1	- 3.90 E-4	0.996
1 2 0	1	1.71 E-3	1	1.012 E-3	0.996
2 2 1	2	- 2.64 E-3	4	- 1.57 E-3	0.996
2 1 0	1	- 3.37 E-3	1	- 2.00 E-3	0.996

A statistical weight of zero indicates that there is no wave with the given quantum numbers that exists, as it cannot satisfy all definitions consistently.

calculations with numerical values of the parameters shown in figure 1, arranged approximately in order of decreasing importance, along with their statistical weights. Also are shown the amplitudes. The last column gives the total accumulated intensity (sum of backwards and forwards), and the numbers there give an indication of how the finite sum is converging and how well it is serving as an estimate of the actual infinite sum. It is to be noted that when the waves denoted by the first six entries are combined, only 0.5 percent of the energy is unaccounted for. Therefore, inclusion of from six to ten sets of these waves should give estimates of the true infinite sums which are adequate under practical conditions.

In the calculations summarized in table 1 were carried out in the real notation: losses were assumed to be zero. Also as part of the simplification, all of the exponential phase factors were arbitrarily placed equal to unity, which imples that the two slabs each contained a whole number of half-wavelengths. When such a situation is viewed from the impedance analog, it can be seen that the backward wave should be zero in amplitude and the forward wave should be 1.0 for an incident wave of 1.0. The sums from the computer program were respectively - 8.45E - 2 and 0.987, which are in good agreement with such an expectation.

It should be noted in passing that intensities of the singly degenerate families of waves B(P,0,0) and F(0,Q,0) form geometric progressions, and their sums can be obtained by a well-known formula. However, when I have tried to combine such sums with the amplitudes of a finite number of waves of other types, the convergence has been poorer than by dealing with a finite group of waves, such as in table 1. (Another check on convergence of the method, although for spherical waves, is to be found later in table 3, part A).

If the radiation is observed in either the forward or backward direction as a function of angle with the numerical values suggested by figure 1, there are rapid variations due to interference effects in the substrate superimposed upon slower variations due to interference effects in the semiconductor, the angular separations between a maximum and an adjacent minimum being about 0.6° for the substrate and about 10° for the semiconductor. It is doubtful, especially with non-coherent sources, whether the variations due to the substrate can be resolved under some practical experimental conditions. Therefore, in performing computer calculations it may be desirable to average the substrate variations out by incorporating into the program a DO Loop varying the wavelength over a range corresponding to two or three of these variations in order to get numerical values that are relevant to the experimental data.

III. CALCULATIONS BY HUYGENS PRINCIPLE FOR POINT FLUORESCENT SOURCES

In order to proceed with calculations it is necessary to make a number of assumptions in order to have a concrete problem. It must be conceded at the start that the validities of some of these are subject to debate.

1. Since the fluorescence is generated in a region of the semiconductor with dimensions of the order of fractions of a centimeter, while it is to be observed at distances of about 10 cm with detectors with windows with dimensions of the order of 1 cm, the radiation is to be considered as coming from a distribution of point sources.
2. It is assumed that the absorption of the pump light is small and the distribution of these point sources is uniform in depth within the semiconductor.
3. For simplicity and for the lack of definite information, each point source is assumed to radiate uniformly in all directions. Some discussion of angular distribution of luminescent radiation can be found in the references, although it does not seem to be relevant to the particular situation here. With many optically thick samples of low index of refraction, the radiation more or less accurately complies with Lambert's Law, but with semiconductors the distribution is uniform. Lipsett [1] has summarized this situation and considered briefly other possibilities.
4. The pump source is assumed to be weak enough to not cause super-radiance. Therefore, different point sources are incoherent, and their combined effects at the detector are to be found by adding intensities.

The method which is to be employed is a modification of that developed in the previous section which has led to Eqs. (1) and (2). However, there are a number of complications which require some considerations that were not involved in the derivations of those equations.

1. With external plane wave sources, there are only two groups of waves, backward and forward. However, by reference to figure 5, it can be seen that here are four sets of

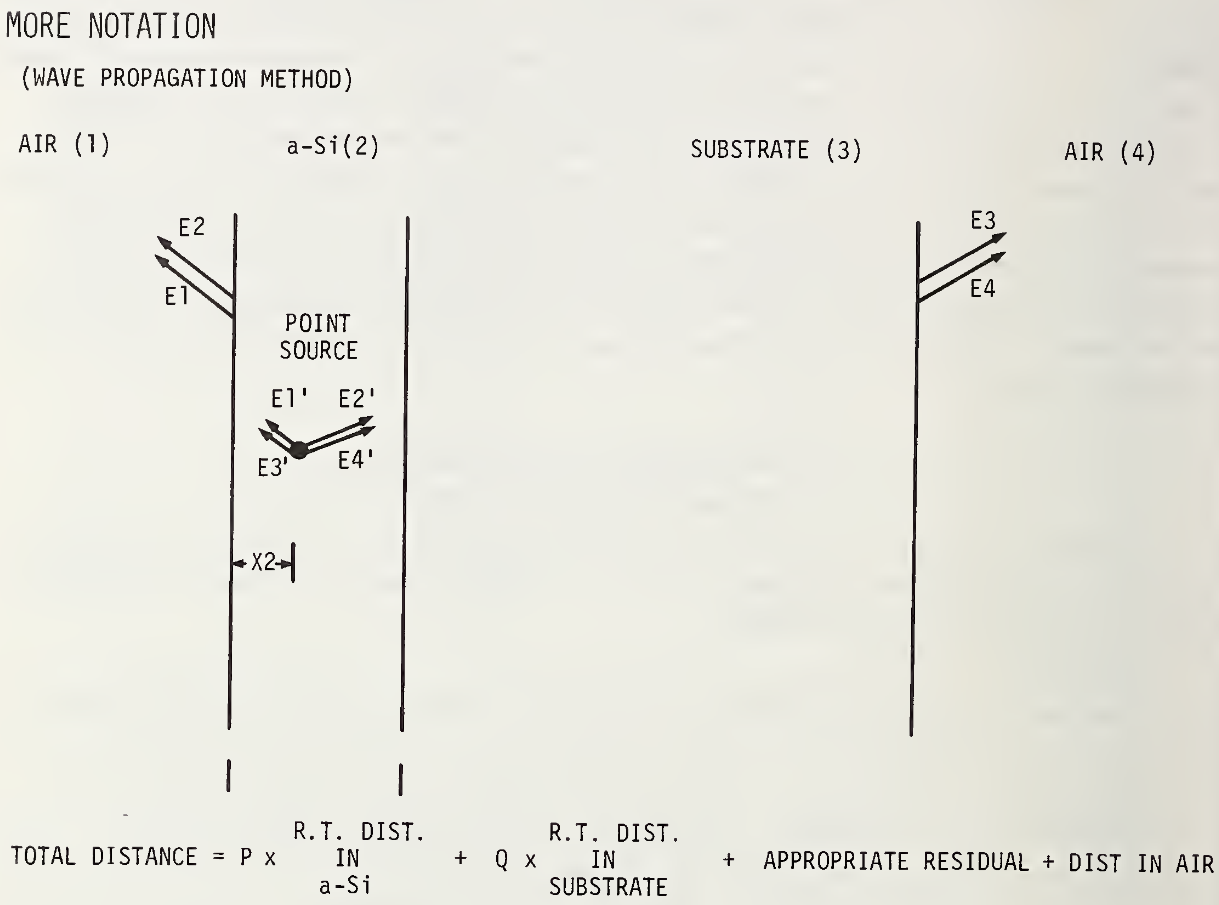


Figure 5. More Notation. Designation of Waves from Point Fluorescent Sources.

spherical waves, and therefore, it is necessary to derive four, rather than two, equations analogous to Eqs. (1) and (2). The E1 wave is one which leaves the source as E1' in a direction towards the left and, after a number of reflections (not shown in the diagram), ultimately emerges from the semiconductor air surface into Region 1. Also emerging into Region 1 are E2 waves which left the source as E2' to the right. Similarly, there are two sets of waves E3 and E4 emerging from the substrate air surface into Region 4, E3 having originated as E3' to the left and E4 having originated as E4' to the right. The individual waves in these four groups are to be labeled by the same quantum numbers P, Q, and S that have been defined previously. The total path length of any wave in the solid materials is an integral number of transversals of the solid layers plus a "residual." The E1 and E3 waves, originating to the left, have one residual, $X2/\cos\Theta 2$, while the E2 and E4 waves, starting to the right, have another residual $(D2-X2)/\cos\Theta 2$.

2. With planar external sources, the interest is comparing the amplitude (or intensity) of an emerging beam with the amplitude (or intensity) with the incident beam. Thus the input and output are specified in the same units, and what unit system is used is arbitrary as long as the same units are used for input and output. However, in the present situation, the sources are to be specified by power (in watts) while the emerging beams are to be specified in amplitude (volts per meter) or in terms of intensity (watts per square meter). Therefore, it is necessary to introduce and evaluate a normalization constant U to make the units consistent.
3. With planar external sources, the detector and the source are in the same medium (air), and therefore the ratio of the intensity of an emerging beam to the incident one is given by the square of the ratio of the amplitudes. However, in the present case, the source is in semiconductor while the detector is in air. Therefore, any comparison of the intensity of an emerging beam with its intensity in the close vicinity of the source must consider the ratio of the intrinsic impedances of the two media as well as the square of the ratio of amplitudes.
4. With point sources the area of a wave front expands as it moves away from the source. Thus any formula for the amplitude must contain a factor to account for the spreading with the wave. For spherical waves, in a single medium this factor, of course, is the reciprocal of the distance.

For evaluating Items (2), (3) and (4), it is expedient to set aside the previous notation and the geometry of figures 1 and 5, and refer to the simpler situation of figure 6. A point source located at Sa in a medium of index of refraction Na radiates 1 W. The circle represents a wave front that is tangent to the planar boundary with a medium of index of refraction Nb.

The electric intensity Ea at a point on this wave front may be determined from the fact that the power of 1 W is equal to the area of the spherical wave front multiplied by the Poynting vector, assuming that the radius is large compared to a wavelength.

The magnitude of the Poynting vector is given by

$$W = (Ea)^2/Za \ , \tag{5}$$

where Za is the impedance of the medium a. According to well-known theory,

$$Za = (\mu/\varepsilon)^{1/2} = 376.6/Na \ , \tag{6}$$

where μ equals the magnetic permeability of the medium (which here is assumed to be that of a vacuum), and ε equals the electric permittivity of the medium. 376.6 Ω is to be recognized as the intrinsic impedance of empty space [6,7].

This reasoning has the result that

$$Ea = \frac{1}{ra}\sqrt{\frac{376.6}{4\pi Na}} = \frac{5.475}{ra \cdot \sqrt{Na}} \ . \tag{7}$$

GEOMETRY FOR EVALUATING NORMALIZATION FACTOR

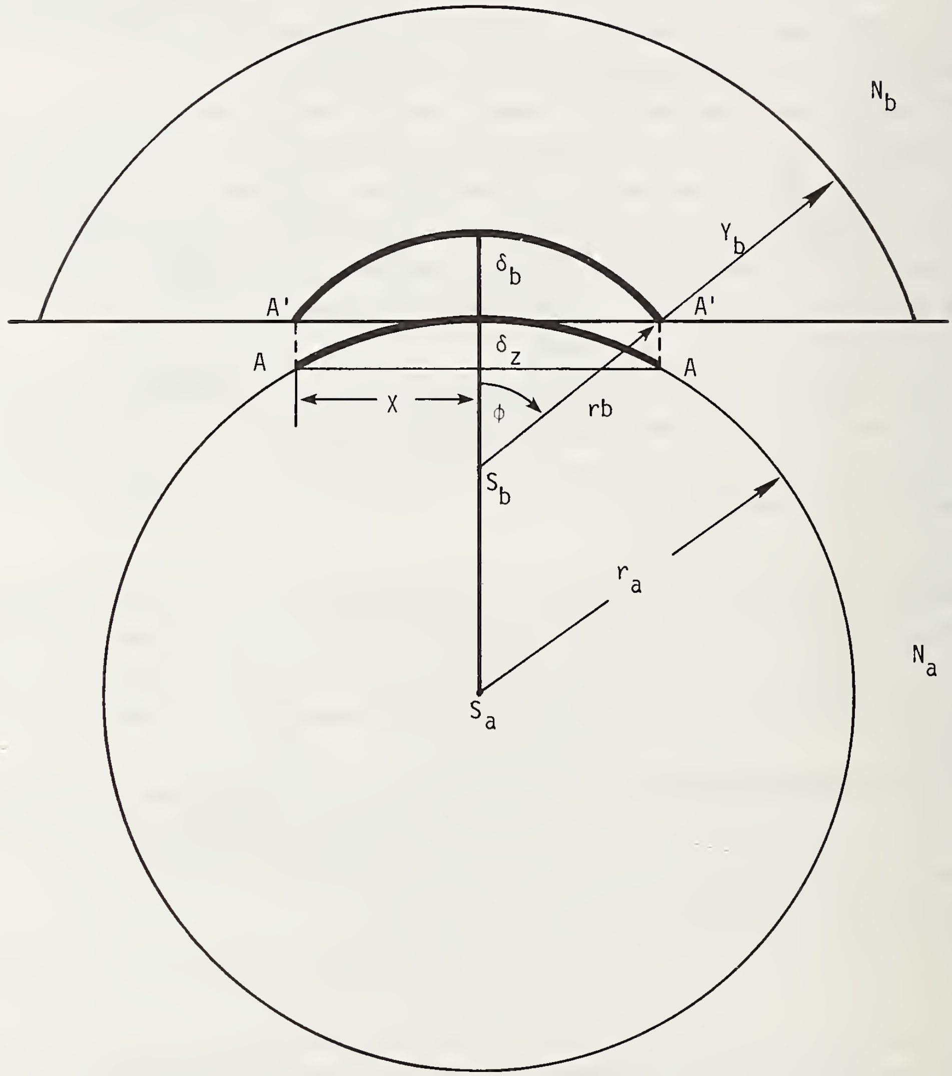


Figure 6. Geometry for Evaluation Normalization Factor.

On the other hand, since the source is a point one, Ea is given by Ea = U/ra, where U is the normalization constant mentioned earlier. Therefore,

$$U = 5.475/Na^{1/2} \; . \tag{8}$$

In figure 6, the portion of the wave front AA that is tangent to the surface is shown as A'A' a short time later, after it has entered medium b. The shape of this wave front is approximately that of a sphere of radius rb. (The approximation is most valid for the central portion of the wave front, for which the δ's are small compared to the r's.) The amplitude on A'A' is Ea multiplied by the amplitude transmission coefficient T. By the use of a well-known elementary argument using the sagitta approximation formula ($X^2 = 2rS$), and by the definition of the index of refraction, it may be shown that

$$ra/Na = rb/Nb \; . \tag{9}$$

As the wave progresses further into b, it acts as though it had been emitted by a virtual point source Sb, and the intensity varies inversely with the distance from this point. Therefore, when the wave has progressed a distance yb into the medium, the intensity is given by

$$Eb = \frac{rb}{yb + rb} \cdot \frac{UT}{ra} = \frac{UT}{Na \cdot yb/Nb + ra} \; , \tag{10}$$

by the use of Eq. (9). In practice, ra is small compared with yb, and therefore the second term in the denominator can be neglected.

With the aid of Eq. (8),

$$Eb = 5.475 \; Nb \cdot T/(yb \cdot Na^{3/2}) \; . \tag{11}$$

Now suppose that a thin parallel sided slab of some other material, c, is placed between a and b. By one application of the reasoning which leads to Eq. (9), ra/Na = rc/Nc, and by a second application, rc/Nc = rb/Nb, and Eq. (9) still holds. It can be inferred, then, if an arbitrary number of such thin slabs are put between a and b, r/N is an invariant of the system, and Eqs. (9-11) still hold, except that T must be replaced by a product of transmission coefficients. The only practical effect of the introduction of these slabs is to displace the location of the virtual source Sb.

This presentation, for simplicity, has neglected the phase of Eb. In the presentation that follows the effect of phase is included by replacing the real notation by a complex one and introducing appropriate exponential phase factors for the media.

It is now possible to see, in returning to the original geometry of figures 1 and 5, how to write down by inspection valid expressions for E1(P,Q,S) . . . E4(P,Q,S). The index "a" is to be identified with "2" for the semiconductor, and the index "b" corresponds to air, not only in Region 1 adjacent to the semiconductor but also, because of the invariance of r/N, in

Region 4, adjacent to the substrate. Therefore, Nb = 1, and yb is to be replaced by the detector distance $\frac{X1}{\cos\Theta 1}$ or $\frac{X4}{\cos\Theta 4}$ (X1 and X4 are perpendicular distances). The single factor of T is to be replaced by a product of transmission and reflection coefficients raised to various powers, and to take in to account the phase, the complex notation is to be used and the final expressions are to contain appropriate exponential phase factors. The intensity of any wave is given by the absolute magnitude of the square of its amplitude divided by the intrinsic wave impedance of empty space (376.6 Ω).

The expressions for the desired amplitudes are written by inspection by the use of Eq. (11) as follows (for a source of 1 W):

$$E1(P,Q,S) = \frac{5.475\cdot\cos\Theta 1}{X1\cdot(N2)^{3/2}}\cdot R5^{P}\cdot R2^{P-S}\cdot R6^{Q-S}\cdot R3^{Q}\cdot T5\cdot T2^{S}\cdot T6^{S}$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot N2\cdot(2\cdot P\cdot D2+X2)}{\cos\Theta 2}\right)\cdot\mathrm{EXP}\left(-\frac{k\cdot N3\cdot 2\cdot Q\cdot D3}{\cos\Theta 3}\right)$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot X1}{\cos\Theta 1}\right)\ . \tag{12}$$

$$E2(P,Q,S) = \frac{5.475\cdot\cos\Theta 1}{X1\cdot(N2)^{3/2}}\cdot R5^{P}\cdot R2^{P-S+1}\cdot R6^{Q-S}\cdot R3^{Q}\cdot T5\cdot T2^{S}\cdot T6^{S}$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot N2\cdot[(2P+2)\cdot D2-X2]}{\cos\Theta 2}\right)\cdot\mathrm{EXP}\left(-\frac{k\cdot N3\cdot 2\cdot Q\cdot D3}{\cos\Theta 3}\right)$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot X1}{\cos\Theta 1}\right)\ . \tag{13}$$

$$E3(P,Q,S) = \frac{5.475\cdot\cos\Theta 4}{X4\cdot(N2)^{3/2}}\cdot R5^{P+1}\cdot R2^{P-S}\cdot R6^{Q-S}\cdot R3^{Q}\cdot T2^{S+1}\cdot T6^{S}\cdot T3$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot N2\cdot[(2P+1)\cdot D2+X2]}{\cos\Theta 2}\right)\cdot\mathrm{EXP}\left(-\frac{k\cdot N3\cdot(2Q+1)\cdot D3}{\cos\Theta 3}\right)$$
$$\cdot\mathrm{EXP}\left(-\frac{A\cdot X4}{\cos\Theta 4}\right)\ . \tag{14}$$

$$E4(P,Q,S) = \frac{5.475\cdot\cos\Theta 4}{X4\cdot(N2)^{3/2}}\cdot R5^{P}\cdot R2^{P-S}\cdot R6^{Q-S}\cdot R3^{Q}\cdot T2^{S+1}\cdot T5^{S}\cdot T3$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot N2\cdot[(2P+1)\cdot D2-X2]}{\cos\Theta 2}\right)\cdot\mathrm{EXP}\left(-\frac{k\cdot N3\cdot(2Q+1)\cdot D3}{\cos\Theta 3}\right)$$
$$\cdot\mathrm{EXP}\left(-\frac{k\cdot X4}{\cos\Theta 4}\right)\ , \tag{15}$$

where k is propagation constant in air.

$$k = 2\pi j/\lambda\ , \tag{16}$$

where λ is the wavelength and $j = (-1)^{1/2}$.

Equations (12-15) have been derived on the assumption of the invariance of N/r, which was derived by making the assumption that r is given by the approximate formula $r = X^2/2\delta$. The errors resulting from these assumptions result in the fact that the radii of curvature of the emerging waves are not exactly $X1/\mathrm{COS}\Theta 1$ and $X4/\mathrm{COS}\Theta 4$, respectively. Now the magnitude of these errors will be estimated.

The exact formula is given by

$$r_b = \frac{X^2}{2\delta}\left(1 + \frac{\delta^2{}_b}{X^2}\right) .$$

The second term in the parentheses is to be considered as a correction term. Its meaning can be made more apparent by expressing it approximately in terms of the angle ϕ, the angle between S_bA' and the normals to the surfaces in figure 6.

$$\tan \phi = X/\left(r_b - \delta_b\right).$$

If ϕ is small, $\tan \phi$ may be approximated as ϕ, δ_b can be ignored in comparison with r_b, and r can be expressed approximately as $r_b = X^2/2\delta_b$, with the result that

$$\phi = 2\delta_b/X$$

and, to a second approximation,

$$r_b = \frac{X^2}{2\delta_b}\left(1 + \frac{\phi^2}{4}\right) .$$

It can be seen that the correction approaches zero for $\phi = 0$ (normal incidence) and increases with the square of ϕ. It is about one percent for ϕ equal to about 10 degrees. For small angles, the angle of refraction is approximately equal to ϕ.

For waves that undergo more than one refraction, in general the value of ϕ changes with each refraction, but since the error depends upon the square of ϕ the errors are cumulative. Table 1 indicates that more than 99.5 percent of the intensity is carried by waves of low order. Therefore subjectively, it may be estimated that for waves emerging with angles of 10 degrees or less, the errors in the radii curvature, and therefore the errors in the amplitude are less than about 3 percent, and they decrease with the square of ϕ, as ϕ decreases.

The effect of losses in the media may be included by allowing the index of refraction to become complex, such that N is replaced by NR - j NI, where NR and NI are real, and NR is "the index of refraction" as listed in handbooks. Accordingly, the propagation constant

$$\begin{aligned} \gamma &= \alpha + j\beta \\ &= k\cdot N = j2\pi N/\lambda \\ &= 2\pi NI/\lambda + j2\pi NR/\lambda \quad . \end{aligned} \tag{17}$$

The total amplitude for waves emerging from semiconductor to air from a single 1 W source is given by

$$BT = \sum_{P,Q,S} [M1(P,Q,S) \cdot E1(P,Q,S) + M2(P,Q,S) \cdot E2(P,Q,S)] \tag{18}$$

while the total amplitude emerging from substrate into air is given by

$$FT = \sum_{P,Q,S} [M3(P,Q,S) \cdot E3(P,Q,S) + M4(P,Q,S) \cdot E4(P,Q,S)] \ , \tag{19}$$

where the M's are statistical weights to be found by drawing diagrams. Values for waves with quantum numbers up to and including 3 are found in table 2. It is to be noted for all cases that have been studied, $M3(P,Q,S) = M4(P,Q,S)$.

The summations are carried out in principle for P, Q, and S ranging from zero to infinity subject to the constraints that $S \leq P$ (for E1, E3 and E4), $S \leq (P+1)$ (for E2) and $S \leq Q$ (for a

The intensities of the two sets of waves are then given by $BT \cdot BT^*$ and $FT \cdot FT^*$. The intensities computed this way for individual waves are to be added. In practice, by computer it appears most practical to compute the amplitudes from Eqs (15-19), and then perform the remaining calculations numerically. Now it will be shown that, because of the simple way in which the coordinate of the source X2 appears in these equations, it is possible, in computing the total intensity due to a continuous distribution of sources, to avoid integrating over the distribution by replacing the factors containing X2 by their average values.

It is to be noted in carrying out the summations of Eq. (18), each of the E1 terms depends on X2 only through the same factor $EXP(-\alpha 2 \cdot X2 - j\beta 2 \cdot X2)$ while each E2 term contains a factor $EXP(+\alpha 2 \cdot X2 + j\beta 2 \cdot X2)$. Therefore, formally, Eq. (18) can be written as

$$BT = A1 \cdot EXP\left[(-\ \alpha 2 \cdot X2 - j\beta 2 \cdot X2)/COS\Theta 2\right] + A2 \cdot EXP\left[(\alpha 2 \cdot X2 + j\beta 2 \cdot X2)/COS\Theta 2\right] \ , \tag{20}$$

and the corresponding intensity

$$BT \cdot BT^* = A1 \cdot A1^*\ EXP(-\ 2 \cdot \alpha 2 \cdot X2/COS\Theta 2) + A2 \cdot A2^* \cdot EXP(2 \cdot \alpha 2 \cdot X2/COS\Theta 2)$$

$$+ A1 \cdot A2^* \cdot EXP(-\ 2j \cdot \beta 2 \cdot X2/COS\Theta 2) + A1^* A2 \cdot EXP(2j \cdot \beta 2 \cdot X2/COS\Theta 2) \ , \tag{21}$$

where A1 and A2 are not functions of X2. Explicit expressions for them will be found in Eqs. (25) and (26).

As the 1 W source moves between the limits of $X2 = 0$ to D2, these quantities vary only in accordance with the exponential factors that are visible. Therefore, if the 1 W point source is replaced by a 1 W source distributed continuously and uniformly in this region, the intensity is given by substituting into Eq. (21) average values for the exponential factors. The factors in the last two terms are periodic in X2, and therefore if D2 is large compared to one wavelength, they are negligible compared to other terms, or if D2 corresponds to a whole number of periods, they will be identically zero even if D2 is small. Therefore, these

TABLE 2

STATISTICAL WEIGHTS OF FLUORESCENT WAVES WITH QUANTUM NUMBERS OF 3 OR LESS

P	Q	S	M1	M2	M3	M4
0	0	0	1	1	1	1
1	0	0	1	1	1	1
0	1	0	0	0	1	1
1	1	0	0	0	1	1
0	1	1	0	1	0	0
1	1	1	1	2	1	1
2	2	2	1	3	1	1
1	2	0	0	0	1	1
2	0	0	1	1	1	1
0	2	0	0	0	1	1
2	1	1	2	3	2	2
1	2	1	1	2	2	2
2	1	0	0	0	1	1
1	2	2	0	1	0	0
0	2	1	0	1	0	0
3	0	0	1	1	1	1
0	3	0	0	0	1	1
3	3	3	1	3	1	1
3	1	0	0	0	1	1
3	1	1	3	4	3	3
1	3	1	1	2	3	3
1	3	0	0	0	1	1
3	2	0	0	0	1	1
3	2	1	3	4	3	3
3	2	2	3	6	3	3
2	3	2	2	3	3	3
2	3	3	0	1	0	0
0	3	1	0	1	0	0

terms are neglected. The average values of the other terms may be computed by integrating with respect to X2 from 0 to D2, and dividing the result by D2. For $EXP(-\ 2\cdot\alpha 2\cdot X2/COS\Theta 2)$, the result is, by use of Eq. (17)

$$G1 = \frac{\lambda COS\Theta 2}{4\pi\cdot NI2\cdot D2}\left[1 - EXP(-\ 4\pi\cdot NI2\cdot D2/\lambda COS\Theta 2)\right] , \tag{22}$$

and for $EXP(+\ 2\cdot\alpha 2\cdot X2/COS\Theta 2)$, the result is

$$G2 = \frac{\lambda COS\Theta 2}{4\pi\cdot NI2\cdot D2}\left[EXP(4\pi\cdot NI2\cdot D2/\lambda COS\Theta 2) - 1\right] . \tag{23}$$

Therefore, it can be concluded that the total backward intensity

$$BT\cdot BT^* = G1\cdot A1\cdot A1^* + G2\cdot A2\cdot A2^* , \tag{24}$$

where

$$A1 = \frac{5.475\cdot COS\Theta 1}{X1\cdot (N2)^{3/2}} \sum_{P,Q,S} \left[R5^{P}\cdot P2^{P-S}\cdot R6^{Q-S}\cdot R_3{}^{Q}\cdot T5\cdot T2^{S}\cdot T6^{S}\right.$$
$$\cdot EXP\left(-\ \frac{k\cdot N2\cdot 2\cdot P\cdot D2}{COS\Theta 2}\right)\cdot EXP\left(-\ \frac{k\cdot N3\cdot 2\cdot Q\cdot D3}{COS\Theta 3}\right)$$
$$\left.\cdot EXP\left(-\ \frac{k\cdot X1}{COS\Theta 1}\right)\right] . \tag{25}$$

$$A2 = \frac{5.475\cdot COS\Theta 1}{X1\cdot (N2)^{3/2}} \sum_{P,Q,S} \left[R5^{P}\cdot R2^{P-S+1}\cdot R6^{Q-S}\cdot R3^{Q}\cdot T5\cdot T2^{S}\cdot T6^{S}\right.$$
$$\cdot EXP\left(-\ \frac{k\cdot N2\cdot (2P+2)\cdot D2}{COS\Theta 2}\right)\cdot EXP\left(-\ \frac{k\cdot N3\cdot 2\cdot Q\cdot D3}{COS\Theta 3}\right)$$
$$\left.\cdot EXP\left(-\ \frac{k\cdot X1}{COS\Theta 1}\right)\right] , \tag{26}$$

by use of Eqs. (12) and (13).

In a similar way the total intensity emerging from the substrate into Region 4 can be shown to be

$$FT\cdot FT^* = G1\cdot A3\cdot A3^* + G2\cdot A4\cdot A4^* \tag{27}$$

where, by use of Eqs. (14) and (15),

$$A3 = \frac{5.475 \cdot COS\Theta 4}{X4 \cdot (N2)^{3/2}} \sum_{P,Q,S} \Bigg(R5^{P+1} \cdot R2^{P-S} \cdot R6^{Q-S} \cdot R3^{Q} \cdot T2^{S+1} \cdot T6^{S} \cdot T3$$

$$\cdot EXP\left(- \frac{k \cdot N2 \cdot (2P+1) \cdot D2}{COS\Theta 2}\right) \cdot EXP\left(- \frac{k \cdot N3 \cdot (2Q+1) \cdot D3}{COS\Theta 3}\right)$$

$$\cdot EXP\left(- \frac{k \cdot X4}{COS\Theta 4}\right)\Bigg) , \qquad (28)$$

and

$$A4 = \frac{5.475 \cdot COS\Theta 4}{X4 \cdot (N2)^{3/2}} \sum_{P,Q,S} \Bigg(R5^{P} \cdot R2^{P-S} \cdot R6^{Q-S} \cdot R3^{Q} \cdot T2^{S+1} \cdot T5^{S} \cdot T3$$

$$\cdot EXP\left(- \frac{k \cdot N2 \cdot (2P+1) \cdot D2}{COS\Theta 2}\right) \cdot EXP\left(- \frac{k \cdot N3 \cdot (2Q+1) \cdot D3}{COS\Theta 3}\right)$$

$$\cdot EXP\left(- \frac{k \cdot X4}{COS\Theta 4}\right)\Bigg) . \qquad (29)$$

Usually the losses are small or neglected completely ($NI2 \to 0$). In this limit both Eqs. (22) and (23) become indeterminant (zero/zero). In such a case, the exponentials are expanded in a power series, and if the first two non-vanishing terms are retained,

$$G1 = 1 - 2\pi \cdot NI2 \cdot D2/(\lambda \cdot COS\Theta 2) , \qquad (30)$$

and

$$G2 = 1 + 2\pi \cdot NI2 \cdot D2/(\lambda \cdot COS\Theta 2) . \qquad (31)$$

The first computer calculations based upon this theory and upon the geometry specified by figure 1 employed only three sources, and the amplitudes were printed out separately. The three sources gave nearly identical amplitudes even though they were in very different locations. When the calculations were repeated with 19 sources, a large variation was found, but there was a quasi periodicity in the amplitude when position changed, and apparently the three original ones happened by accident to be approximately whole periods apart. On the other hand, when the calculations were extended to apply to 99 equally spaced sources, the total intensities were found to be only slightly different for those found with 19 sources. Shown in table 3 are some results obtained with 99 sources and with a continuous source, and the agreements between the average values are excellent, to better than one percent. In Part A, the thickness D2 is varied from 1.0×10^{-6} m to 2×10^{-6} m in steps of 0.1×10^{-6} m (0.38 wavelengths). Because of interference effects, the intensities vary. Shown in the table are minimum and maximum values and the averages of the intensities and also the ratios of the average intensities in the two directions. Also, for the 99 discrete sources are

TABLE 3

SAMPLE CALCULATIONS OF FLUORESCENT EMISSION

Geometry of Fig. 1 — No Losses — Normal Incidence

A. Semiconductor Thickness D2 varied in ten steps, starting at 1.0×10^{-6} m (0.38 wavelength)

Number Sets of Waves	Sources	Step Size	Intensity through Semiconductor (W/m^2) Minimum	Maximum	Average	Intensity through Substrate (W/m^2) Minimum	Maximum	Average	Ratio (FF*/BB*) (Substrate/Semiconductor) Minimum	Maximum	Average	Average Deviation
6	99	1×10^{-7} m	0.3251	1.0081	0.6591	0.3233	1.0859	0.6988	0.9833	1.0980	1.0495	0.0432
15	99	1×10^{-7}	0.3936	1.1406	0.6669	0.4170	1.2168	0.7061	1.0349	1.0721	1.0565	0.0110
28	99	1×10^{-7}	0.3723	1.1775	0.6679	0.3936	1.2565	0.7082	1.0432	1.0691	1.0571	0.0079
28	cont.	1×10^{-7}	0.3709	1.803	0.6684	0.3960	1.2605	0.7101	1.0481	1.0690	1.0609	0.0080

B. Substrate thickness varied in ten steps, starting at 6.35×10^{-4} m

Number Sets of Waves	Sources	Step Size	Intensity through Semiconductor (W/m^2) Minimum	Maximum	Average	Intensity through Substrate (W/m^2) Minimum	Maximum	Average	Ratio (FF*/BB*) (Substrate/Semiconductor) Minimum	Maximum	Average	Average Deviation
28	99	1.9×10^{-7} m	0.3714	0.4745	0.4137	0.3802	0.8551	0.5799	1.0385	1.8022	1.3770	0.2761
28	cont.	1.9×10^{-7}	0.3708	0.4792	0.4148	0.3825	0.8601	0.5834	1.0381	1.7953	1.3825	0.2815
28	99	1.9×10^{-6}	0.3723	0.4745	0.4144	0.3936	0.8551	0.5767	1.0558	1.8022	1.3655	0.2852
28	cont.	1.9×10^{-6}	0.3709	0.4792	0.4162	0.3807	0.8603	0.5802	1.0064	1.8145	1.3686	2.849

shown the effect of using differing numbers of waves. In the first line are used six sets of waves (those listed in the first six lines of table 2, with P = 0 and 1, and Q = 0 and 1), while for the second line, with 15 sets, those with either P = 2 or Q = 2, or both, have been added. Those entries with 28 sets employ all of the waves listed in table 2. It is to be noted that the convergence is excellent. The average values change by only about a percent at most from 6 sets to 28 sets. Thus just the first six sets are quite a good approximation.

In Part B, the substrate thickness is varied in ten steps starting from an original thickness of 6.35×10^{-4} m, either in steps of 1.9×10^{-7} m (0.38 wavelength, as above) or of 1.9×10^{-6} m (3.8 wavelengths). Again there is excellent agreement between the results with 99 sources and a continuous source. The average values are somewhat different in Parts A and B. The values are not very different for the two step sizes. However, other calculations with D3 changing by orders of magnitude do show a change in the ratio, especially as D3 gets comparable with the detector distance X1 or X4, where the approximations that have been made are not valid.

It remains to say how the theory developed here is to be applied to experiment. An optical pump beam is incident upon the system of figure 1. Its power is measured, and with an appropriate filter the reflected and the transmitted powers are measured. From these data it can be deduced how many watts are absorbed. Then, with a filter that passes the fluorescent radiation in use with the detector, the intensity of the fluorescent radiation is measured at some chosen angle (which may be in either the back or forward direction). From this measurement and by application of the theory it can be deduced how many watts of fluorescent radiation are produced per watt absorbed of the pump. By knowledge of the wavelengths this result can be expressed as the number of fluorescent quanta per pump quantum, the absolute quantum efficiency, which is the desired result. The consistency of the method can be checked by repeating the process by observing the fluorescent radiation at other angles, especially comparing the forward and backward directions. If there should be an apparent systematic variation of efficiency with angle, it is necessary to look for an appropriate modification of the method or for some error that needs correction. One such error might be the assumption that the angular radiation pattern of the point fluorescent sources is uniform.

IV. THE IMPEDANCE METHOD

Since the impedance method has been described in many textbooks, two of which are cited as reference [6] and [7], only those features relevant to the present work will be discussed, and many details are left to be found in the references. The method is applicable to the propagation of radiation in slabs of material with infinite cross sections and plane parallel sides. The wave impedance Z of a medium is the ratio of the component electric intensity in the medium parallel to a boundary plane to the component of magnetic intensity parallel to a boundary plane.

For oblique incidence, it is necessary to consider two cases: (1) Transverse Electric (TE), when the total electric intensity is parallel to a boundary plane (while the magnetic intensity has a component perpendicular to the plane), and (2) the Transverse Magnetic (TM) case, which is the converse.

In the theory each medium is represented by its characteristic impedance Zo, which is defined as the value of the impedance of a traveling wave in that medium. In the special case of normal incidence (when both fields are parallel to the reference plane), some times called Transverse Electromagnetic (TEM), the characteristic impedance is the intrinsic impedance that was given in Eq. (6). For oblique incidence the characteristic impedance is the intrinsic impedance multiplied by a function of Θ, the angle of incidence or refraction. For non-magnetic media,

$$Zo = 376.6/(NCOS\Theta), \quad (TE) \tag{32}$$

and

$$Zo = 376.6COS\Theta/N \ . \quad (TM) \tag{33}$$

Both revert to Eq. (6) for $\Theta = 0$.

In the equations that are to follow, it is implied that all waves are monochromatic and that their time variations are given by a factor EXP(+ j·angular frequency·time). Then progressing a distance +X in a direction normal to the wave fronts the amplitude is modified by multiplication by a factor EXP(-kNX) = EXP($-\gamma$X), by reference to Eq. (17).

The usefulness of these definitions is because they lead to an essentially perfect analogy with situation in a transmission line with the electric intensity corresponding to the voltage and the magnetic intensity corresponding to the current. The characteristic impedance is defined as the ratio of the voltage to the current for a traveling wave in a semi-infinite line. Therefore, many equations and methods of graphical analysis which were developed for transmission lines are directly applicable to the solution of plane wave problems.

Some of the features of this analogy are illustrated by figure 7. For simplicity normal incidence is assumed. In the upper portion (a), YY represents the plane boundary between two homogeneous isotropic linear media, P and Q, having characteristic impedances ZP and ZQ, and X is the distance from the boundary, positive towards the left. A source of plane waves is somewhere to the left resulting in an incident wave with electric intensity Ei(X) incident upon the boundary, giving rise to the reflected wave Er(X) and the transmitted wave Et(X), where these are complex quantities. In medium P the total intensity EP(X) = Ei(X) + Er(X). Associated with these electric intensities (E) are related magnetic intensities (H). At the boundary there are well-known conditions relating the field in the two media, one of which (continuity of the tangential components of E) results in the condition that

$$Ei(0) + Er(0) = Et(0) \ ,$$

or

$$Ei(0)(1 + R) = Et(0) \ . \tag{34}$$

ANALOGY BETWEEN PLANE WAVES IN SEMI-INFINITE MEDIA & TRANSMISSION LINE

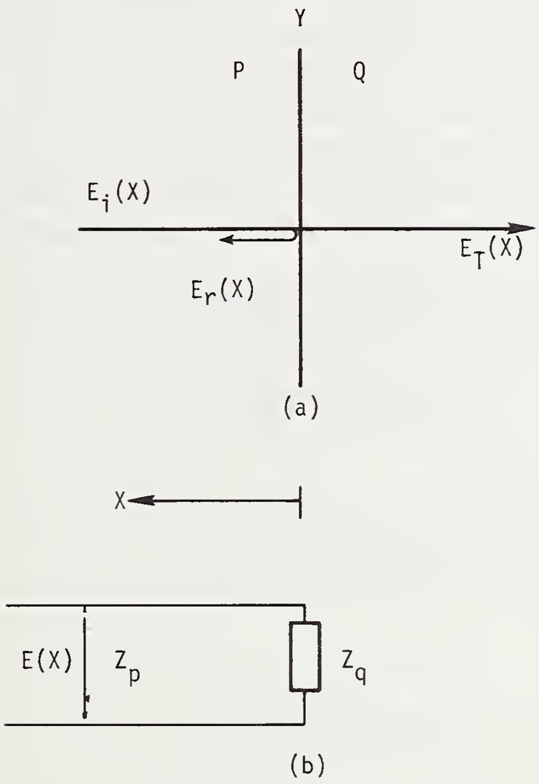


Figure 7. Analogy between Plane Waves in Semi-Infinite Media and Loaded Transmission Line.

Application of the boundary conditions also determines the value of the reflection coefficient for the electric intensity

$$R = (ZQ - ZP)/(ZQ + ZP) \tag{35}$$

With use of the sign convention that has been introduced, in general

$$EP(X) = Ei(0)\cdot[EXP(+\gamma X) + R\cdot EXP(-\gamma X)] \ , \tag{36}$$

with X being measured from the interface. (It should be remarked that there is not a universal agreement upon a sign convention. Some references, such as reference [6], measure X from the source rather than from the boundary. Therefore, in those references, the signs of the exponents in Eq. (36) are reversed.)

If the transmission coefficient T is defined as T = Et(0)/Ei(0), it can be seen from Eqs. (34) and (36) that

$$T = 2ZQ/(ZQ + ZP) \;. \tag{37}$$

With the notation and sign convention used here, T = 1 + R.

By an extension of this reasoning to oblique incidence, it is found that Eqs. (35) and (37) still apply if the Z's are interpreted as appropriate characteristic impedances with Snell's law providing a relation between the Θ's. In detail, for TE waves,

$$R = \frac{NP\cdot COS\Theta P - NQ\cdot COS\Theta Q}{NP\cdot COS\Theta P + NQ\cdot COS\Theta Q} \;, \tag{38}$$

$$T = \frac{2\cdot NP\cdot COS\Theta P}{NP\cdot COS\Theta P + NQ\cdot COS\Theta Q} \tag{39}$$

while for TM waves,

$$R = \frac{NP\cdot COS\Theta Q - NQ\cdot COS\Theta P}{NP\cdot COS\Theta Q + NQ\cdot COS\Theta P} \;, \tag{40}$$

and

$$T = \frac{2\cdot NP\cdot COS\Theta P}{NP\cdot COS\Theta Q + NQ\cdot COS\Theta P} \;. \tag{41}$$

If the ratio of NP to NQ is eliminated from Eqs. (38-41) by the use of Snell's law, it can be shown that they are identical to the Fresnel equations. Therefore, in spite of the differences in physical concepts and mathematical form, Eqs. (35) and (37) are equivalent to the Fresnel equations.

In the transmission line analog shown in figure 7(b), a uniform characteristic impedance ZP is terminated by an impedance ZQ, and a generator, not specified in detail, is causing a wave to go down the line and be reflected with change in amplitude and phase at the end terminated by ZQ. Propagation, phase and constants may be defined in a manner that is identical with the previous ones. There is the boundary condition that the ratio of the total (incident plus reflected, taken with regard to phase) voltage to the total current must be equal to ZQ. When circuit theory and this boundary condition are applied, it is found that Eqs. (34-37) hold, provided that the E's are defined as voltages and the H's as currents.

It is of interest to define that quantity Z(X) as the ratio of the total voltage E(X) to the total current H(X). It may be shown that

$$Z(X) = ZP\cdot\frac{(ZP+ZQ)\cdot EXP(\gamma X) - (ZP-ZQ)\cdot EXP(-\gamma X)}{(ZP+ZQ)\cdot EXP(\gamma X) + (ZP-ZQ)\cdot EXP(-\gamma X)} \;, \tag{42}$$

For brevity, the expression for H(X), which is of no direct interest, is not given here. It may be found in the references.

This important Eq. (42), can be put into several different forms. The terms may be regrouped so that the exponentials can be expressed in terms of hyperbolic functions, and if the line is lossless, it is appropriate to express these, in turn, in terms of trigonometric functions. Also, ZQ can be expressed in terms of the reflection coefficient R, and one may define a reflection coefficient R(X) by solving Eq. (35) for ZQ and substituting the result for ZQ in Eq. (42) and then by substituting Z(X) into Eq. (35). Also Eq. (42) has several graphical representations, the most well-known and most useful of which is the Smith Chart [11], which is a plot based upon the reflection coefficient version of Eq. (42). The Smith Chart can be converted into a slide rule for solving problems based upon Eq. (42).

In most of the discussions of Eq. (42), the termination of the left end of the transmission line is unspecified. However, Harnwell [12] has considered the situation in which the left end of a finite length of line is connected to a Thévnin generator with an internal impedance ZG, and he has defined a sending-end reflection coefficient RG by replacing ZQ in Eq. (35) by ZG. He has then derived an equation for the voltage as a function of distance with the two reflection coefficients as parameters. He has then expanded the exponential factors that appear by power series and arranged the terms so that they may be interpreted as (1) an incident wave going to the right, (2) a wave reflected at the right end, which (3) in turn is reflected at the left end giving rise to a wave (4) reflected at the left end and going to the right, and so ad infinitum. Thus, the formal equivalence between the Huygen's principle method and impedance method can be established.

There are countless applications of Eq. (42) and of the Smith Chart that are highly important and interesting in other contexts which must be ignored here for the sake of brevity. The interest here is in recognizing that, by use of the analogy that has been described, Eq. (42), which originally derived for transmission lines, may be used to give the answers to problems pertaining to plane waves in planar stratified media, such as in figure 1.

This method of analysis is now applied to the problem of a planar source located in Region 1. No important loss in generality results in the simplifying assumption that this is located an infinitesimal distance to the left of the boundary between Regions 1 and 2. Then the equivalent transmission line circuit is given in figure 8, where in the four regions are represented by transmission lines of appropriate lengths and intrinsic impedances. The line of characteristic impedance Z3 (substrate) is terminated by the impedance Z4 (air). Z3', the impedance at the input to Z3, is calculated from Z3 and Z4 by means of Eq. (42). Z3' serves as the termination for Z2, whose input impedance Z2' is calculated by a second application of Eq. (42). By successive uses of Eq. (35) or one of its modifications, the reflection coefficients at the sets of interfaces may be calculated. The reflection coefficient at the interface between Regions 1 and 2 is one of the quantities of prime interest, since this is the amplitude reflection coefficient of the entire system. The square of its absolute magnitude is the reflection coefficient of the system, and this is to be compared to that calculated by the wave propagation method and that determined by experiment.

Also by successive applications of Eqs. (34) and (36), the field developed in Region 4 at the boundary with Region 3 can be computed in terms of the field strength incident from Region 1 on to the boundary with Region 2. The ratio of these field strengths is the

IMPEDANCE METHOD

MAKE USE OF TRANSMISSION LINE ANALOG & EMPLOY TRANSMISSION LINE THEORY (PLANAR SOURCES)

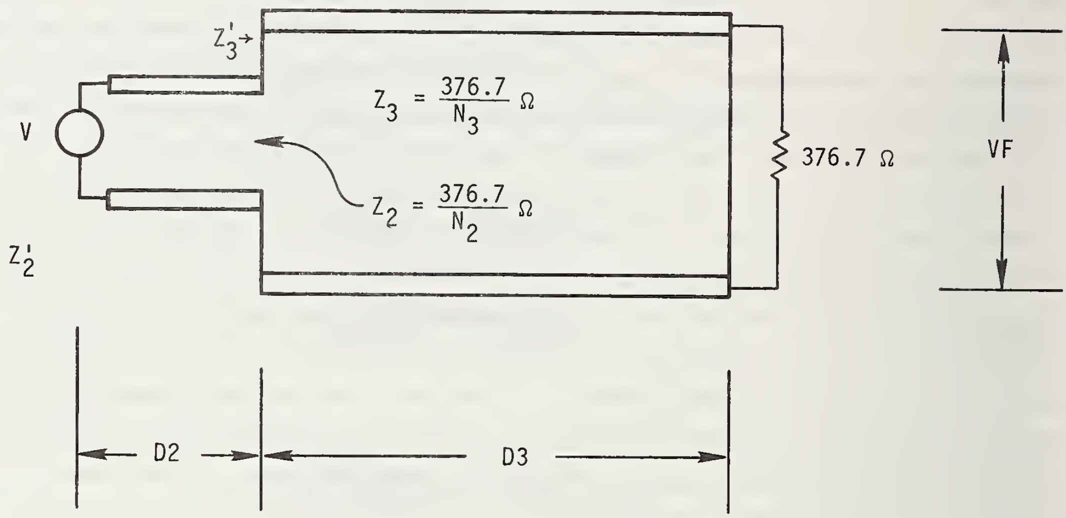


Figure 8. Equivalent Circuit for Use with a Planar External Source in Region 1.

amplitude transmission coefficient of the system. The square of its absolute magnitude is the transmittance of the system.

In principle, the equations pertaining to these various steps could be combined algebraically to give a single equation for the reflectance and a single one for the transmittance. However, it is apparent that these equations would be very cumbersome and complicated, and they have not been derived. In the computer programs that I have written, I have carried out the indicated steps one at a time numerically, and many of the steps in the program have been merely to write down these various equations several times with various subscripts.

One special case of Eq. (42) that is of interest here is when the medium is lossless and when the thickness is a whole number of half wavelengths. Then the exponential factors are both equal to +1. Then Eq. (42) reduces to Z(X) = ZQ. This result was used in the discussion of table 1. As stated earlier, the input impedance of the system is the intrinsic impedance of empty space, and therefore the transmission line model gives the reflectance as 0 and the transmittance as +1.

There remains the question of how these calculations are to be used with experimental results to obtain optical (non-fluorescent) data on the semiconductor and on the substrate. At the start one works with the substrate alone, with no semiconductor deposited on it. A computer program intended to work with the combination may be used for the substrate alone by placing N2 = 1 and D2 = 0.

If the reflectance and transmittance are measured as a function of angle with a monochromatic source or at a fixed angle, and if the surfaces are flat and parallel, there are large variations in intensity due to interference effects (i.e., "fringes"). The index of refraction can be found by knowledge of the geometrical thickness and the angular or wavelength shift per fringe. The computer program is run with various assumed values of NI3 or $\alpha 3$, and average values of the reflectance and transmittance are computed and compared with experiment. The value which agrees with experiment can be taken as a measure of the losses. Another indication of the losses is given by comparing the maximum transmittance (or reflectance) with the minimum. Ideally the minimum should be zero for no losses. However, because of the roughness of the surface or non-monochromaticity of the source, the minimum is likely to be less pronounced than it should be. Therefore, a maximum to the minimum ratio is likely to give an overestimate of the losses. Nevertheless, such a value is useful as a check, although it is not completely reliable.

Once the properties of the substrate have been determined, the same method may be applied to the full system. Now the response in transmission and reflection consists of narrow substrate fringes superimposed upon broader semiconductor fringes, since the thickness of the semiconductor is much less. In the use of the computer program, it may be desirable to average out the effects of the substrate fringes by applying a DO Loop of proper amplitude to the wavelength or angle. Then the properties of the semiconductor can be found by the procedure used with the substrate alone.

The writer wishes to thank Eric G. Johnson, Jr. for helpful suggestions, and Robert J. Phelan for suggesting the problem and for his support.

APPENDIX

Error in Reflection and Transmission Coefficient From Neglect of Losses

A rough experimental estimate of the upper limit in the attenuation coefficient of a-Si is 10^6/m, while an upper limit to the attenuation coefficient of sapphire is 10^2/m. By reference to Eq. (17), it may be seen that, for a wavelength of 10^{-6} m, the upper limit to the imaginary part of the index of refraction of a-Si is $1/2\pi$, while that of a sapphire is four orders of magnitude smaller and can be neglected in comparison. By inspection it appears the reflection coefficients that are affected most are R2 and R6 = -R2. By use of Eq. (38) or Eq. (40) with $\cos\Theta = 0$, it may be shown that the magnitude is 0.31696 if the imaginary part is neglected and 0.31830 if it is given the value of $1/2\pi$. The difference is negligible. There is also a phase shift of about 7.4°, which is unimportant with the low order reflections which are listed in tables 1 and 2.

The transmission coefficient most likely to be affected is T5. It has the magnitude of 1.545 if the imaginary part is zero and the value of 1.5482 if it is $1/2\pi$. The shift in phase angle is 0.61°. These differences are negligible. Therefore, it can be concluded that the neglect of losses in the calculation of reflection and transmission coefficients is unimportant under the conditions of the present calculations.

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5. Publication Date: May 1980

6. Performing Organization Code

AUTHOR(S)

Yardley Beers

8. Performing Organ. Report No.

PERFORMING ORGANIZATION NAME AND ADDRESS

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This paper concerns the situation of a fluorescent semiconducting layer deposited upon a substrate. It is desired to compute the number of quanta of fluorescent radiation per quantum absorbed of the pump radiation. The principal topic which is discussed is the theory of the Huygens principle method which gives the intensity of the emitted light in terms of the power of an array of point sources in the semiconductor. The method used is a direct application of Huygens Principle to the individual waves that are multiply reflected by the boundary surfaces. The results are given in terms of the constants of the materials, the dimensions, and three quantum numbers, (1) P, the number of two-way trips in the semiconductor, (2) Q, the number of round trips in the substrate, and (3) S, the number of two-way penetrations of the boundary between them. Because of approximations used, the method is mainly useful for radiation emerging nearly normal to the surfaces. For light within 10° to the normal the errors are not more than a few percent. The Huygens Principle method is also developed for use with planar external sources, and this method is compared with the impedance methods. The calculations made with external sources are needed for determining the non-fluorescent optical constants of the materials.

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