NBS TECHNICAL NOTE 1004

U.S. DEPARTMENT OF COMMERCE/National Bureau of Standards

Calibrating Two 6-Port Reflectometers With Only One Impedance Standard

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+ Technical Note

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CALIBRATING TWO 6-PORT REFLECTOMETERS WITH ONLY ONE IMPEDANCE STANDARD

Cletus A. Hoer

This paper describes a technique for calibrating a pair of 6-port reflectometers for measuring the reflection coefficient of 1-port devices, or the scattering parameters of reciprocal 2-port devices. The operations in the calibration consist of connecting the two 6-ports together, connecting each 6-port to a calibration circuit consisting of two terminations of unknown impedance and a leveling loop, and then connecting the standard. The standard can be one termination whose complex impedance is known, or a precision length of transmission line whose cross-sectional dimensions are known. The length and loss of the line are not required. The solution for the constants which characterize each 6-port is closed, requiring no iteration.

Key Words: Calibration; current; impedance; network analyzer; power; reflection coefficient; scattering parameters; six-port; reflectometer; voltage.

I. Introduction

A 6-port reflectometer can be used to measure the impedance or reflection coefficient of 1-port devices connected to its reference plane [1-8]. It will also measure voltage, current, or power at that reference plane [9]. Two 6-port reflectometers can be used to measure all of the scattering parameters of a 2-port device which is inserted between the two reflectometers as shown in figure 1 [10]. This paper describes a technique for calibrating a pair of 6-port reflectometers for making all of these measurements.

Each 6-port reflectometer can be characterized by a square matrix containing 16 real constants. The objective of the calibration is to determine these two matrices which will be called $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ corresponding to 6-port #1 and 6-port #2.

The calibration procedure is first described without referring to any equations. This is followed by a review of 6-port reflectometer equations which are expressed in matrix notation. The solution for the constants contained in $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ is then given in detail using matrix algebra whenever possible. An appendix outlines a computer program for taking and processing the calibration data.

The calibration technique described in this paper is similar to that described by Allred and Manney for calibrating two 4-port couplers [11]. The solution is a closed solution, requiring no iteration.

II. Outline of Calibration Procedure

The calibration consists of two parts. The first part uses no standards and obtains enough of the constants in $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ to make impedance ratio measurements at either 6-port reference plane. The second part of the calibration requires the use of one impedance

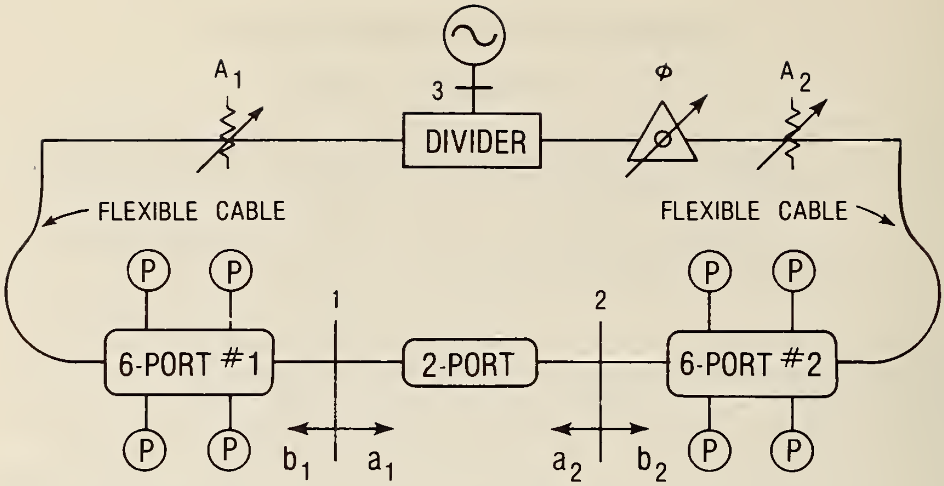


Figure 1. Using two 6-port reflectometers to measure all of the scattering parameters of a 2-port device.

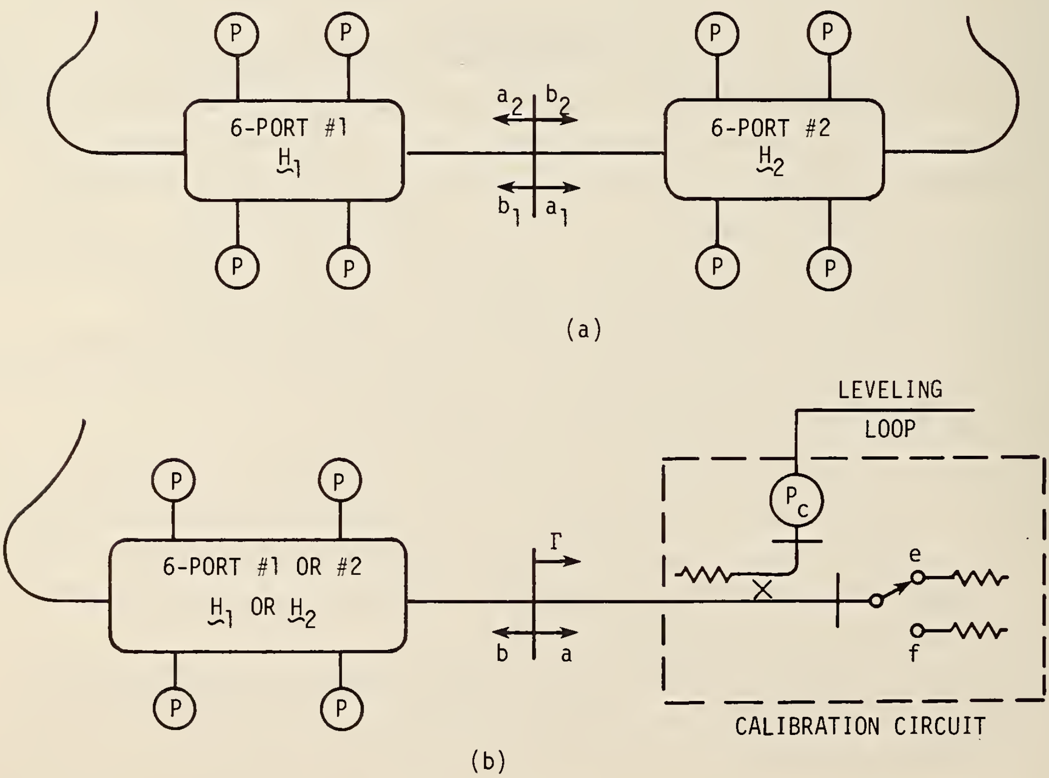


Figure 2. The first part of the calibration consists of connecting the two 6-port reference planes together (a), and then connecting each 6-port to a calibration circuit (b). The output P_c of the directional coupler is either measured or held constant by leveling the generator. The two different terminations at e and f need not be known.

standard to find the remaining constants needed in measuring an absolute impedance or reflection coefficient. The first part of the calibration consists of the following three operations which are illustrated in figure 2.

1. The two reference planes are connected together and all 6-port sidearm power readings are recorded at four or more different settings of A_1, A_2 and ϕ. The values of attenuation A_1, A_2 and the value of phase ϕ do not need to be known. This set of measurements is sufficient to determine $\underset{\sim}{H}_2$ in terms of $\underset{\sim}{H}_1$, thereby reducing the number of unknowns from 32 to 16. If 6-port #1 is already calibrated so that $\underset{\sim}{H}_1$ is known, then this set of measurements is sufficient to calibrate any other 6-port.
2. Six-port #1 is connected to the calibration circuit shown in figure 2b and its sidearm power readings are recorded at each of the two switch positions e and f within the calibration circuit. Power level, P_c, is either recorded for both switch positions or held constant by leveling the generator.
3. Six-port #2 is connected to the calibration circuit and its sidearm power readings are recorded as in step 2. P_c is again either recorded or held constant by leveling the generator.

To make the connections in steps 1 to 3 requires that all connectors be of the same type and sexless. The term "calibration circuit" is not meant to imply that this circuit contains any standards, but only that it is a convenient circuit to use in the calibration procedure. The impedance of the two terminations in the calibration circuit need not be known. It is assumed only that the two values of impedance seen looking into the calibration circuit are the same when either 6-port is connected to it.

Steps 2 and 3 are sufficient to reduce the number of unknowns from 16 to 8. Half of the constants in $\underset{\sim}{H}_1$ are given in terms of the other half. The remaining 8 unknown constants are reduced to just 3 unknowns by taking advantage of the dependent relationship between the constants. Only 12 of the 16 constants in either $\underset{\sim}{H}_1$ or $\underset{\sim}{H}_2$ are independent. It will be shown that these three remaining unknown constants are not needed for calculating the impedance ratio of two terminations. Therefore the set of measurements described in steps 1 to 3 which uses no standards is sufficient to calibrate each 6-port for making impedance ratio measurements.

The second part of the calibration uses one impedance standard to find two of the three remaining unknown constants. These two constants are needed to calculate absolute impedances or reflection coefficients from which S-parameters of 2-port devices are calculated. The standard can be a uniform length of transmission line which is inserted between the two 6-ports as shown in figure 3. With the line inserted, all 6-port sidearm power readings are recorded at two or more different settings of A_1, A_2, and ϕ. As in step 1 above, the values of A_1, A_2, and ϕ do not need to be known. This set of measurements is sufficient to determine the two needed constants and also $\gamma\ell$ of the line, where γ is the complex propagation constant of the line and ℓ is its length. The length and loss of the line do not need to be known, only its cross-sectional dimensions need to be known.

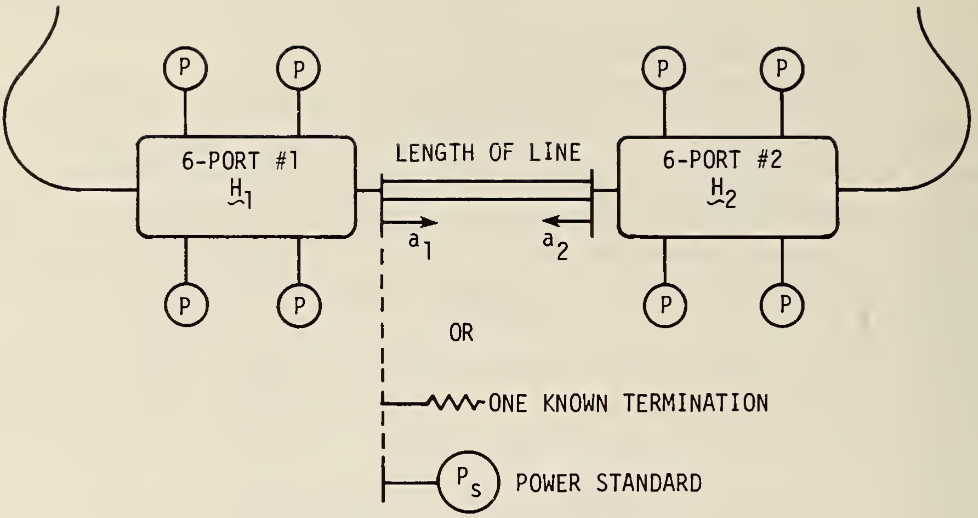


Figure 3. A uniform length of transmission line or one known termination is the only impedance standard needed. A power standard is connected to one of the 6-port reflectometers to calibrate them for making absolute power measurements.

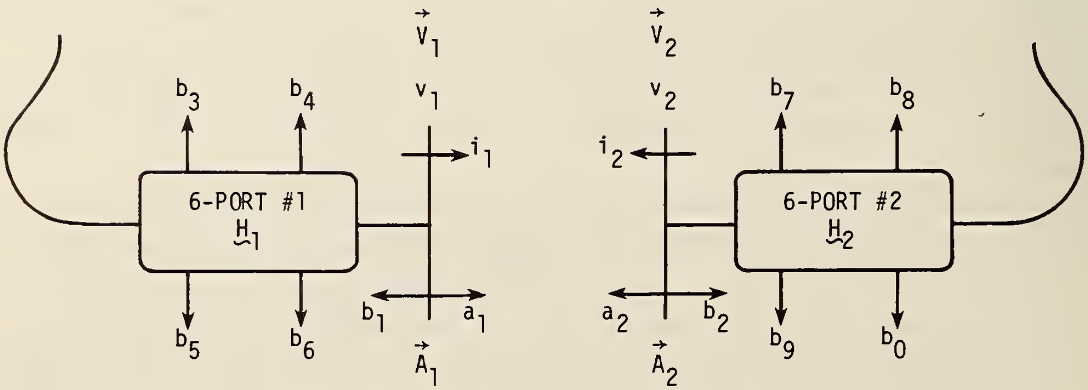


Figure 4. Defining the incident and reflected waves at the different ports.

The standard can also be a termination (not a short or an open) whose complex reflection coefficient is known. With this standard connected to either 6-port, the sidearm power readings for that 6-port are recorded. The two needed constants are calculated from this set of readings and the previously known value of the reflection coefficient.

The last remaining unknown constant is determined only if each 6-port is to be used for measuring voltage, current, or power as well as reflection coefficient and S-parameters. To determine this last constant, a power standard is connected to either 6-port reference plane as indicated in figure 3. The sidearm power readings of that 6-port as well as the reading of the power standard are recorded. The last constant is calculated from this set of readings. All constants in $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ are now known so that either 6-port can be used to measure active as well as passive circuit parameters.

III. Review of 6-port Reflectometer Equations

A. a and b Parameters.

Using the notation shown in figure 4, let the emergent wave at sidearm #i of 6-port #1 be [1,2]

$$b_i = A_i' a_1 + B_i' b_1 , \quad i = 3 \cdots 6 \tag{1}$$

where A_i' and B_i' are functions of the scattering parameters of the 6-port junction and also of the reflection coefficient of each sidearm detector. The power, P_i, indicated by the detector on sidearm #i is

$$P_i = c_i |b_i|^2 = c_i |A_i' a_1 + B_i' b_1|^2 \tag{2}$$

where c_i is a real constant that does not need to be known. It will be convenient to combine c_i with A_i' and B_i' so that eq. (2) becomes

$$P_i = |A_i a_1 + B_i b_1|^2 , \qquad i = 3 \cdots 6 \tag{3}$$

where A_i and B_i are new constants which will be determined in the calibration procedure described in this paper. Expanding eq. (3),

$$\begin{aligned} P_i = {} & |A_i a_1|^2 + 2|A_i B_i| \cos \zeta_i \, |a_1 b_1| \cos \psi_1 \\ & + |B_i b_1|^2 - 2|A_i B_i| \sin \zeta_i \, |a_1 b_1| \sin \psi_1 , \quad i = 3 \cdots 6 \end{aligned} \tag{4}$$

where

$$\psi_1 \equiv \arg (b_1/a_1) , \ \zeta_i \equiv \arg(B_i/A_i)$$

In matrix notation, the four equations in eq. (4) can be written

$$\begin{pmatrix} P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix} = \begin{pmatrix} |A_3|^2 & |B_3|^2 & 2|A_3B_3| \cos \zeta_3 & -2|A_3B_3| \sin \zeta_3 \\ |A_4|^2 & |B_4|^2 & 2|A_4B_4| \cos \zeta_4 & -2|A_4B_4| \sin \zeta_4 \\ |A_5|^2 & |B_5|^2 & 2|A_5B_5| \cos \zeta_5 & -2|A_5B_5| \sin \zeta_5 \\ |A_6|^2 & |B_6|^2 & 2|A_6B_6| \cos \zeta_6 & -2|A_6B_6| \sin \zeta_6 \end{pmatrix} \begin{pmatrix} |a_1|^2 \\ |b_1|^2 \\ |a_1b_1| \cos \psi_1 \\ |a_1b_1| \sin \psi_1 \end{pmatrix} \tag{5}$$

or

$$\vec{P}_1 = \underset{\sim}{B}_1 \vec{A}_1 \tag{6}$$

where ($\rightarrow$) indicates a column vector and ($\sim$) indicates a square matrix. From eq. (6),

$$\vec{A}_1 = \underset{\sim}{B}_1^{-1} P_1 \equiv \underset{\sim}{G}_1 \vec{P}_1 \tag{7}$$

where

$$\underset{\sim}{G}_1 \equiv \underset{\sim}{B}_1^{-1} \equiv \begin{pmatrix} \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \\ \beta_3 & \beta_4 & \beta_5 & \beta_6 \\ c_3 & c_4 & c_5 & c_6 \\ s_3 & s_4 & s_5 & s_6 \end{pmatrix} . \tag{8}$$

Conditions for the existence of $\underset{\sim}{B}^{-1}$ are given in reference [4]. Since all elements in eq. (5) are real, $\underset{\sim}{B}_1$ and hence $\underset{\sim}{G}_1$ are real matrices. Expanding eq. (7),

$$\begin{aligned} |a_1|^2 &= \sum \alpha_i P_i \\ |b_1|^2 &= \sum \beta_i P_i \\ |a_1b_1| \cos \psi_1 &= \sum c_i P_i \\ |a_1b_1| \sin \psi_1 &= \sum s_i P_i \quad . \end{aligned} \tag{9}$$

The ratio $\rho_1 = b_1/a_1$ can be written

$$\rho_1 = \frac{|a_1b_1|}{|a_1|^2} (\cos \psi_1 + j \sin \psi_1) \tag{10}$$

which from eq. (9) becomes

$$\rho_1 = \frac{\Sigma\, c_i P_i + j\, \Sigma\, s_i P_i}{\Sigma\, \alpha_i P_i} \quad , \qquad i = 3 \cdots 6 \; . \tag{11}$$

The symbol ρ is used instead of Γ because the ratio b_1/a_1 is not a reflection coefficient in the usual sense during much of the calibration process to be described. For a passive termination, ρ is the usual reflection coefficient Γ.

B. v and i Notation.

The voltage v_1 and the current i_1 at the reference plane are related to the waves a_1 and b_1 by

$$v_1 = a_1 + b_1 \tag{12}$$

$$i_1 Z_{01} = a_1 - b_1 \tag{13}$$

where Z_{01} is the characteristic impedance. Using eqs. (12) and (13), a matrix equation similar to that for $\vec{A}$ can be written in terms of v_1 and $i_1 Z_{01}$;

$$\begin{pmatrix} |v_1|^2 \\ |i_1 Z_{01}|^2 \\ |v_1 i_1 Z_{01}| \cos \theta_1 \\ |v_1 i_1 Z_{01}| \sin \theta_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} |a_1|^2 \\ |b_1|^2 \\ |a_1 b_1| \cos \psi_1 \\ |a_1 b_1| \sin \psi_1 \end{pmatrix} \tag{14}$$

or

$$\vec{V}_1 = \underset{\sim}{K} \vec{A}_1 \tag{15}$$

where

$$\theta_1 \equiv \theta_{v_1} - \theta_{i_1} - \theta_{Z_{01}} \quad . \tag{16}$$

If Z_{01} is real, then $\theta_{Z_{01}} = 0$ and θ_1 is the phase angle of the impedance Z_1 connected at the reference plane. Using eq. (7) in eq. (15) gives

$$\vec{V}_1 = \underset{\sim}{K} \underset{\sim}{G}_1 \vec{P}_1 \equiv \underset{\sim}{H}_1 \vec{P}_1 \tag{17}$$

where

$$\underset{\sim}{H}_1 \equiv \underset{\sim}{K} \underset{\sim}{G}_1 \equiv \begin{pmatrix} \nu_3 & \nu_4 & \nu_5 & \nu_6 \\ \mu_3 & \mu_4 & \mu_5 & \mu_6 \\ q_3 & q_4 & q_5 & q_6 \\ r_3 & r_4 & r_5 & r_6 \end{pmatrix} \quad . \tag{18}$$

Expanding eq. (17) yields a set of equations similar to eq. (9);

$$|v_1|^2 = \sum \nu_i P_i$$

$$|i_1 Z_{01}|^2 = \sum \mu_i P_i \qquad i = 3\cdots 6 \tag{19}$$

$$|v_1 i_1 Z_{01}| \cos \theta_1 = \sum q_i P_i$$

$$|v_1 i_1 Z_{01}| \sin \theta_1 = \sum r_i P_i \quad .$$

The ratio Z_1/Z_{01} can be written

$$\frac{Z_1}{Z_{01}} = \frac{|v_1 i_1 Z_{01}|}{|i_1 Z_{01}|^2} (\cos \theta_1 + j \sin \theta_1) \tag{20}$$

$$= \frac{\Sigma\, q_i P_i + j\, \Sigma\, r_i P_i}{\Sigma\, \mu_i P_i} \quad , \quad i = 3\cdots 6 \quad . \tag{21}$$

A set of equations similar to eqs. (1) - (21) can be written for 6-port #2. Subscripts 1 or 2 on the matrices will indicate 6-port #1 or #2.

For converting between a and b parameters and v and i parameters, it is useful to know that

$$\underset{\sim}{K}^{-1} = \frac{1}{4} \underset{\sim}{K} \quad . \tag{22}$$

Then

$$\underset{\sim}{G}_1 = \underset{\sim}{K}^{-1} \underset{\sim}{H}_1 = \frac{1}{4} \underset{\sim}{K}\, \underset{\sim}{H}_1 \quad . \tag{23}$$

The object of the calibration is to determine either $\underset{\sim}{G}$ or $\underset{\sim}{H}$ since one can easily be obtained from the other. In the procedure to be described, it is easier to obtain $\underset{\sim}{H}$ using v and i notation than it is to obtain $\underset{\sim}{G}$ using a and b notation.

IV. Reference Planes Together

The first step in the calibration procedure is to connect the two reference planes #1 and #2 together. Referring to figure 2a it is apparent that

$$a_1 = b_2 \qquad\qquad b_1 = a_2 \tag{24}$$

$$|a_1| e^{j\psi_{a_1}} = |b_2| e^{j\psi_{b_2}} \qquad\qquad |b_1| e^{j\psi_{b_1}} = |a_2| e^{j\psi_{a_2}} \quad . \tag{25}$$

Then

$$\psi_1 \equiv \psi_{b_1} - \psi_{a_1} = \psi_{a_2} - \psi_{b_2} = -\psi_2 \quad . \tag{26}$$

The matrices $\vec{A}_1$ and $\vec{A}_2$ are then related by

$$\vec{A}_1 = \underset{\sim}{N}_a \vec{A}_2 \tag{27}$$

where

$$\underset{\sim}{N}_a \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \underset{\sim}{N}_a^{-1} \tag{28}$$

The relation between $\vec{V}_1$ and $\vec{V}_2$ can be obtained by using eq. (15) in eq. (27).

$$\underset{\sim}{K}^{-1}\vec{V}_1 = \underset{\sim}{N}_a \underset{\sim}{K}^{-1}\vec{V}_2 \tag{29}$$

or

$$\vec{V}_1 = \underset{\sim}{N}_v \vec{V}_2 \tag{30}$$

where

$$\underset{\sim}{N}_v \equiv \underset{\sim}{K}\underset{\sim}{N}_a\underset{\sim}{K}^{-1} = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} \underset{\sim}{I} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & -\underset{\sim}{I} \end{array}\right) = \underset{\sim}{N}_v^{-1} \tag{31}$$

and where $\underset{\sim}{I}$ is a 2 by 2 identity matrix and $\underset{\sim}{0}$ is a 2 by 2 zero matrix. The fact that $\underset{\sim}{N}_v$ is diagonal and can be so partitioned turns out to make eq. (30) a more useful relation to use than eq. (27) later in the derivation.

A. $\underset{\sim}{H}_2$ in terms of $\underset{\sim}{H}_1$.

To get an expression relating $\underset{\sim}{H}_2$ and $\underset{\sim}{H}_1$, use (17) in (30) to obtain

$$\underset{\sim}{H}_1 \vec{P}_1 = \underset{\sim}{N}_v \underset{\sim}{H}_2 \vec{P}_2 \quad . \tag{32}$$

Four or more measurements are taken with the reference planes together by changing A_1, A_2, and ϕ to different but unknown settings. Let these measurements be designated by subscripts a, b, c,···. These measurements which each generate an equation like eq. (32) can be combined into the single matrix equation

$$\underset{\sim}{H}_1 [\vec{P}_{1a}\ \vec{P}_{1b}\ \vec{P}_{1c}\ \vec{P}_{1d} \cdots] = \underset{\sim}{N}_v \underset{\sim}{H}_2 [\vec{P}_{2a}\ \vec{P}_{2b}\ \vec{P}_{2c}\ \vec{P}_{2d} \cdots] \tag{33}$$

or simply

$$\underset{\sim}{H}_1\underline{P}_1 = \underset{\sim}{N}_v\underset{\sim}{H}_2\underline{P}_2 \tag{34}$$

where underlining indicates a matrix that is formed from a set of column matrices, and is usually not square. If only four measurements are taken, $\underline{P}_1$ and $\underline{P}_2$ are square and eq. (34) gives

$$\underset{\sim}{H}_2 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underset{\sim}{J} \tag{35}$$

where

$$\underset{\sim}{J} \equiv \underset{\sim}{P}_1 \underset{\sim}{P}_2^{-1} \quad . \tag{36a}$$

When more than four measurements are taken, eq. (35) is a least squares solution of (34) where J is then given by $\underline{P}_1$ times the pseudo-inverse of $\underline{P}_2$ [12];

$$\underset{\sim}{J} \equiv \underline{P}_1 \, \underline{P}_2^T \, (\underline{P}_2 \, \underline{P}_2^T)^{-1} \quad . \tag{36b}$$

If $\underset{\sim}{H}_1$ is known by some previous calibration, $\underset{\sim}{H}_2$ can be calculated from eq. (35).

B. Conditions on a_2/a_1.

When taking four or more measurements with the reference planes together, the values of a_2/a_1 at the common reference plane must not all have the same magnitude or all have the same phase. If so, the inverse in (36a) or (36b) will not exist. In the present NBS system, 4 to 6 measurements are taken with A_1, A_2, and ϕ set so that a_2/a_1 at the common reference plane is approximately equal to 1 at -90°, 1 at 180°, 1 at 0°, 0.3 at 90°, 1 at 90°, 0.3 at -90°.

V. Calibration Circuit

Two different terminations of unknown impedance can be used to complete the calibration of each 6-port for impedance ratio measurements. Each termination is preceded by some circuit for measuring or controlling the relative power into the two terminations such as shown in figure 2b. Let $\Gamma = \Gamma' + j\Gamma''$ be the reflection coefficient looking into this "calibration circuit." When 6-port #1 is connected to the calibration circuit, $\vec{A}_1$ defined by (5) and (6) can be written

$$\vec{A}_1 = \begin{pmatrix} |a_1|^2 \\ |b_1|^2 \\ |a_1 b_1| \cos \psi_1 \\ |a_1 b_1| \sin \psi_1 \end{pmatrix} = |a_1|^2 \begin{pmatrix} 1 \\ |\Gamma|^2 \\ \Gamma' \\ \Gamma'' \end{pmatrix} \equiv |a_1|^2 \, \vec{\Gamma} \tag{37}$$

where $\vec{\Gamma}$ is a new column vector which is a function only of the reflection coefficient Γ. Now add a subscript e or f to (37) to indicate which termination is being used;

$$\vec{A}_{1e} = |a_{1e}|^2 \, \vec{\Gamma}_e \tag{38}$$

$$\vec{A}_{1f} = |a_{1f}|^2 \, \vec{\Gamma}_f \quad . \tag{39}$$

A similar set of equations is obtained when 6-port #2 is connected to the calibration circuit;

$$\vec{A}_{2e} = |a_{2e}|^2 \, \vec{\Gamma}_e \tag{40}$$

$$\vec{A}_{2f} = |a_{2f}|^2 \, \vec{\Gamma}_f \quad . \tag{41}$$

Comparing (38) and (40) shows that when termination e is used,

$$\vec{A}_{1e} = \frac{|a_{1e}|^2}{|a_{2e}|^2} \, \vec{A}_{2e} \quad . \tag{42}$$

Likewise when termination f is used, (39) and (41) lead to

$$\vec{A}_{1f} = \frac{|a_{1f}|^2}{|a_{2f}|^2} \, \vec{A}_{2f} \quad . \tag{43}$$

A. Measurement Sequence.

Equation (42) assumes that $\vec{\Gamma}_e$ in (38) is the same as $\vec{\Gamma}_e$ in (40), and equation (43) assumes that $\vec{\Gamma}_f$ in (39) is equal to $\vec{\Gamma}_f$ in (41). To assure that this assumption is valid, consider the two measurement sequences shown in Table 1. In the first sequence of measurements the switch must be repeatable so that $\vec{\Gamma}_e$ in measurement #1 is the same as $\vec{\Gamma}_e$ in measurement #4. The $\vec{\Gamma}_f$ in measurement #2 will be the same as $\vec{\Gamma}_f$ in measurement #3 because the switch position is not changed. In the second sequence of measurements the switch need not be repeatable. However, this sequence requires that 6-port #1 be connected to the calibration circuit twice instead of just once. In either sequence the connectors at each 6-port reference plane and at the calibration circuit reference plane must be repeatable.

TABLE 1.

Measurement number	Switch must be repeatable		Switch need not be repeatable	
	6-Port Connected	Termination used	6-Port Connected	Termination used
1	1	e	1	e
2	1	f	2	e
3	2	f	2	f
4	2	e	1	f

Table 1. Two possible sequences of measurement when using the calibration circuit.

B. Using P_c.

For any one switch position, the calibration circuit can be considered as a 3-port junction. It can be shown that for any such 3-port junction, the ratio of two incident powers at the input port, say $|a_1|^2/|a_2|^2$, is equal to the ratio of the two corresponding values of P_c, P_{c1}/P_{c2}, for a given termination at the third port. Therefore (42) becomes

$$\vec{A}_{1e} = \frac{P_{c1e}}{P_{c2e}} \vec{A}_{2e} \tag{44}$$

where a subscript e has been added to the power ratio to indicate which termination is used. This equation shows the advantage of leveling the generator with the output from the power detector measuring P_c so that the power ratio in (44) is unity and need not be measured.

C. Relating $\underset{\sim}{H}_2$ to $\underset{\sim}{H}_1$.

Using (15) in (44) gives

$$\vec{V}_{1e} = \frac{P_{c1e}}{P_{c2e}} \vec{V}_{2e} \tag{45}$$

which can be expressed in terms of $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ using (17);

$$\underset{\sim}{H}_1 \vec{P}_{1e} = \frac{P_{c1e}}{P_{c2e}} \underset{\sim}{H}_2 \vec{P}_{2e} \quad . \tag{46}$$

Starting with equation (43) leads to a similar expression for termination f;

$$\underset{\sim}{H}_1 \vec{P}_{1f} = \frac{P_{c1f}}{P_{c2f}} \underset{\sim}{H}_2 \vec{P}_{2f} \quad . \tag{47}$$

These last two equations can be combined into a single matrix equation

$$\underset{\sim}{H}_1 [\vec{P}_{1e} \vec{P}_{1f}] = \underset{\sim}{H}_2 [\frac{P_{c1e}}{P_{c2e}} \vec{P}_{2e} \quad \frac{P_{c1f}}{P_{c2f}} \vec{P}_{2f}] \tag{48}$$

or simply

$$\underset{\sim}{H}_1 \underline{D}_1 = \underset{\sim}{H}_2 \underline{D}_2 \tag{49}$$

where $\underline{D}_1$ and $\underline{D}_2$ are each 4 by 2 matrices containing power readings.

D. Determining half of $\underset{\sim}{H}_1$.

We now proceed to eliminate $\underset{\sim}{H}_2$ from (49) and reduce the number of unknowns in $\underset{\sim}{H}_1$ from 16 to 8. Using (35) in (49) gives

$$\underset{\sim}{H}_1 \underline{D}_1 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underset{\sim}{J} \underline{D}_2 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underline{E} \tag{50}$$

where $\underline{E}$ is a 4 by 2 matrix

$$\underline{E} \equiv \underset{\sim}{J} \underline{D}_2 \quad . \tag{51}$$

Now partition $\underset{\sim}{H}_1$, $\underline{D}_1$, $\underset{\sim}{N}_v$, and $\underline{E}$ into 2 by 2 matrices so that eq. (50) becomes

$$\left(\begin{array}{c|c} \underset{\sim}{h}_1 & \underset{\sim}{h}_2 \\ \hline \underset{\sim}{h}_3 & \underset{\sim}{h}_4 \end{array}\right)\left(\begin{array}{c} \underset{\sim}{d}_1 \\ \hline \underset{\sim}{d}_2 \end{array}\right) = \left(\begin{array}{c|c} \underset{\sim}{I} & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & -\underset{\sim}{I} \end{array}\right)\left(\begin{array}{c|c} \underset{\sim}{h}_1 & \underset{\sim}{h}_2 \\ \hline \underset{\sim}{h}_3 & \underset{\sim}{h}_4 \end{array}\right)\left(\begin{array}{c} \underset{\sim}{e}_1 \\ \hline \underset{\sim}{e}_2 \end{array}\right) \quad . \tag{52}$$

Each symbol in eq. (52) indicates a 2 by 2 matrix. This equation expands into the two matrix equations

$$\underset{\sim}{h}_1 \underset{\sim}{d}_1 + \underset{\sim}{h}_2 \underset{\sim}{d}_2 = \underset{\sim}{h}_1 \underset{\sim}{e}_1 + \underset{\sim}{h}_2 \underset{\sim}{e}_2 \tag{53}$$

$$\underset{\sim}{h}_3 \underset{\sim}{d}_1 + \underset{\sim}{h}_4 \underset{\sim}{d}_2 = -\underset{\sim}{h}_3 \underset{\sim}{e}_1 - \underset{\sim}{h}_4 \underset{\sim}{e}_2 \tag{54}$$

Solving eq. (53) for $\underset{\sim}{h}_2$,

$$\underset{\sim}{h}_2 (\underset{\sim}{d}_2 - \underset{\sim}{e}_2) = \underset{\sim}{h}_1 (\underset{\sim}{e}_1 - \underset{\sim}{d}_1) \tag{55}$$

or

$$\underset{\sim}{h}_2 = \underset{\sim}{h}_1 \underset{\sim}{\alpha} \tag{56}$$

where

$$\underset{\sim}{\alpha} \equiv (\underset{\sim}{e}_1 - \underset{\sim}{d}_1)(\underset{\sim}{d}_2 - \underset{\sim}{e}_2)^{-1} \quad . \tag{57}$$

Solving eq. (54) for $\underset{\sim}{h}_3$,

$$\underset{\sim}{h}_3 (\underset{\sim}{d}_1 + \underset{\sim}{e}_1) = -\underset{\sim}{h}_4 (\underset{\sim}{e}_2 + \underset{\sim}{d}_2) \tag{58}$$

or

$$\underset{\sim}{h}_3 = \underset{\sim}{h}_4 \underset{\sim}{\beta} \tag{59}$$

where

$$\underset{\sim}{\beta} \equiv - (\underset{\sim}{e}_2 + \underset{\sim}{d}_2)(\underset{\sim}{d}_1 + \underset{\sim}{e}_1)^{-1} \quad . \tag{60}$$

Using eqs. (56) and (59), matrix $\underset{\sim}{H}_1$ can now be written

$$\underset{\sim}{H}_1 = \left(\begin{array}{c|c} \underset{\sim}{h}_1 & \underset{\sim}{h}_1\underset{\sim}{\alpha} \\ \hline \underset{\sim}{h}_4\underset{\sim}{\beta} & \underset{\sim}{h}_4 \end{array}\right) = \left(\begin{array}{c|c} \underset{\sim}{h}_1 & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{h}_4 \end{array}\right)\left(\begin{array}{c|c} \underset{\sim}{I} & \underset{\sim}{\alpha} \\ \hline \underset{\sim}{\beta} & \underset{\sim}{I} \end{array}\right) \tag{61}$$

or

$$\underset{\sim}{H}_1 = \underset{\sim}{h} \underset{\sim}{\gamma} \tag{62}$$

where $\underset{\sim}{\gamma}$ is known but $\underset{\sim}{h}$ is yet to be determined. Using eqs. (62) in (17), any measurement with 6-port #1 can be written

$$\vec{V}_1 = \underset{\sim}{H}_1\vec{P}_1 = \underset{\sim}{h}\underset{\sim}{\gamma}\vec{P}_1 \quad . \tag{63}$$

Since $\underset{\sim}{\gamma}$ and $\vec{P}$ are known, let

$$\vec{\delta} \equiv \gamma\vec{P}_1 \equiv \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} \quad . \tag{64}$$

Then

$$\vec{V}_1 = \underset{\sim}{h}\,\vec{\delta} \tag{65}$$

where $\vec{\delta}$ is known. Expanding eq. (65) gives four equations

$$\begin{pmatrix} |v_1|^2 \\ |i_1Z_{01}|^2 \\ |v_1i_1Z_{01}|\cos\theta_1 \\ |v_1i_1Z_{01}|\sin\theta_1 \end{pmatrix} = \begin{pmatrix} \nu_3 & \nu_4 & 0 & 0 \\ \mu_3 & \mu_4 & 0 & 0 \\ 0 & 0 & q_5 & q_6 \\ 0 & 0 & r_5 & r_6 \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} \tag{66}$$

or

$$|v_1|^2 = \nu_3\delta_1 + \nu_4\delta_2 \tag{67a}$$

$$|i_1Z_{01}|^2 = \mu_3\delta_1 + \mu_4\delta_2 \tag{67b}$$

$$|v_1i_1Z_{01}|\cos\theta_1 = q_5\delta_3 + q_6\delta_4 \tag{67c}$$

$$|v_1i_1Z_{01}|\sin\theta_1 = r_5\delta_3 + r_6\delta_4 \quad . \tag{67d}$$

E. Exploiting redundancy.

It appears that this is as far as one can go in the derivation with simple matrix manipulations. There are still 8 unknown constants in (67) to be determined. To reduce the number of unknowns further, the nonlinear relationship between these 4 equations is used. Note that the square of equation (67c) plus the square of equation (67d) must equal the product of equations (67a) and (67b);

$$|v_1i_1Z_{01}|^2\cos^2\theta_1 + |v_1i_1Z_{01}|\sin^2\theta_1 = |v_1|^2|i_1Z_{01}|^2 \ . \tag{68}$$

A similar but more useful relation between the remaining 8 constants will now be used to determine 5 new constants, 3 of which allow each 6-port to make impedance ratio measurements with no further calibration.

Equations (67b) - (67d) can be used to get an expression for the impedance Z_1 connected to the reference plane of 6-port #1;

$$\frac{Z_1}{Z_{01}} = \frac{|v_1 i_1 Z_{01}|}{|i_1 Z_{01}|^2} (\cos \theta_1 + j \sin \theta_1) \tag{69}$$

$$= \frac{\left(q_5 \delta_3 + q_6 \delta_4\right) + j \left(r_5 \delta_3 + r_6 \delta_4\right)}{\mu_3 \delta_1 + \mu_4 \delta_2} \tag{70}$$

$$= \frac{\left(q_5 + j r_5\right) \delta_3 + \left(q_6 + j r_6\right) \delta_4}{\mu_3 \delta_1 + \mu_4 \delta_2} \tag{71}$$

$$= K_0 \frac{\delta_3 + (x + j y) \delta_4}{\delta_1 + \mu \delta_2} \tag{72}$$

where

$$K_0 \equiv \frac{q_5 + j r_5}{\mu_3} \tag{73}$$

$$x \equiv \frac{q_6 q_5 + r_6 r_5}{q_5^2 + r_5^2} \tag{74}$$

$$y \equiv \frac{r_6 q_5 - q_6 r_5}{q_5^2 + r_5^2} \tag{75}$$

$$\mu \equiv \mu_4 / \mu_3 \tag{76}$$

Note that if the three real parameters x, y, and μ can be determined, eq. (72) can be used to measure the ratio of two impedances since $K_0 Z_{01}$ will cancel. Another expression involving Z_1 can be obtained from the ratio of equation (67a) to (67b);

$$\left|\frac{Z_1}{Z_{01}}\right|^2 = \left|\frac{v_1}{i_1 Z_{01}}\right|^2 = \frac{\nu_3 \delta_1 + \nu_4 \delta_2}{\mu_3 \delta_1 + \mu_4 \delta_2} \quad . \tag{77}$$

The square of the magnitude of Z_1/Z_{01} calculated from (72) must equal that calculated from (77);

$$\frac{\nu_4(\delta_2 + \nu \delta_1)}{\mu_3(\delta_1 + \mu \delta_2)} = |K_0|^2 \frac{|\delta_3 + (x+jy) \delta_4|^2}{|\delta_1 + \mu \delta_2|^2} \tag{78}$$

or

$$(\delta_2 + \nu \delta_1)(\delta_1 + \mu \delta_2) = K \left[(\delta_3 + x \delta_4)^2 + (y \delta_4)^2\right] \tag{79}$$

where

$$K \equiv \frac{|K_0|^2 \mu_3}{\nu_4} = \frac{q_5^2 + r_5^2}{\mu_3 \nu_4} \tag{80}$$

$$\nu \equiv \nu_3/\nu_4 \quad . \tag{81}$$

F. Five new constants.

Equation (79) is a nonlinear equation with five new real constants to be determined: ν, μ, x, y, and K. So far at least six measurements have been made with 6-port #1, four with the reference planes together plus two when connected to the calibration circuit. These six measurements can be used to solve eq. (79) for the five constants. Expanding eq. (79) leads to

$$(1+\mu\nu)\delta_1\delta_2 = K\ \delta_3^2 + 2Kx\delta_3\delta_4 + K(x^2+y^2)\delta_4^2 - \nu\delta_1^2 - \mu\delta_2^2 \tag{82}$$

or

$$\delta_1\delta_2 = x_1\delta_3^2 + x_2\delta_3\delta_4 + x_3\delta_4^2 - x_4\delta_1^2 - x_5\delta_2^2 \tag{83}$$

where eq. (83) is linear in the new variables $x_1 \cdots x_5$ which are defined by

$$(1+\mu\nu)x_1 \equiv K \tag{84a}$$

$$(1+\mu\nu)x_2 \equiv 2Kx \tag{84b}$$

$$(1+\mu\nu)x_3 \equiv K(x^2+y^2) \tag{84c}$$

$$(1+\mu\nu)x_4 \equiv \nu \tag{84d}$$

$$(1+\mu\nu)x_5 \equiv \mu \quad . \tag{84e}$$

The six measurements on 6-port #1 each give an equation like eq. (83) which can be combined into one matrix equation

$$\begin{pmatrix} \delta_3^2 & \delta_3\delta_4 & \delta_4^2 & -\delta_1^2 & -\delta_2^2 \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ \multicolumn{5}{c}{\text{6 rows or measurements}} \\ \multicolumn{5}{c}{\text{from 6-port \#1}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} \delta_1\delta_2 \\ - \\ - \\ - \\ \\ \\ \end{pmatrix} \tag{85}$$

or simply

$$\underline{R}\vec{X} = \vec{F} \quad . \tag{86}$$

If only five of the six measurements are used, $\underline{R}$ is square and $\vec{X}$ is given by

$$\vec{X} = \underset{\sim}{R}^{-1}\vec{F} \quad . \tag{87}$$

Using six or more measurements, $\vec{X}$ is calculated from

$$\vec{X} = (\underline{R}^T\underline{R})^{-1}\underline{R}^T\vec{F} \tag{88}$$

Finally the desired parameters ν, μ, x, y and K are calculated from the elements $x_1 \cdots x_5$ of $\vec{X}$ using eq. (84).

The value of $\mu\nu$ is first calculated from the product of eqs. (84d) and (84e) which leads to

$$\mu\nu = \frac{2x_4x_5}{1 - 2x_4x_5 \pm \sqrt{1-4x_4x_5}} \quad . \tag{89}$$

It can be shown that $1/\mu\nu$ is given by this same equation. Therefore one of the two values given by eq. (89) is $\mu\nu$ and the other is $1/(\mu\nu)$. To separate the two roots we need to know only if $|\mu\nu|$ is greater than or less than 1. It is usually possible to choose and number the 6-port sidearms so that ideally $|\mu\nu| << 1$. For example, the choice and numbering of the sidearms in figure 5 ideally gives $\mu = \nu = 0$. Then in the non-ideal case, $|\mu\nu| < 1$ and the smaller of the two values given by eq. (89) is clearly the correct value of $\mu\nu$. Two more examples are shown in figure 6 which shows two circuits suggested by Engen [6]. If the sidearms are numbered as shown, both circuits have the same $\underset{\sim}{G}_1$ and $\underset{\sim}{H}_1$ matrices, and both have $\mu = \nu = 0$ in the ideal case.

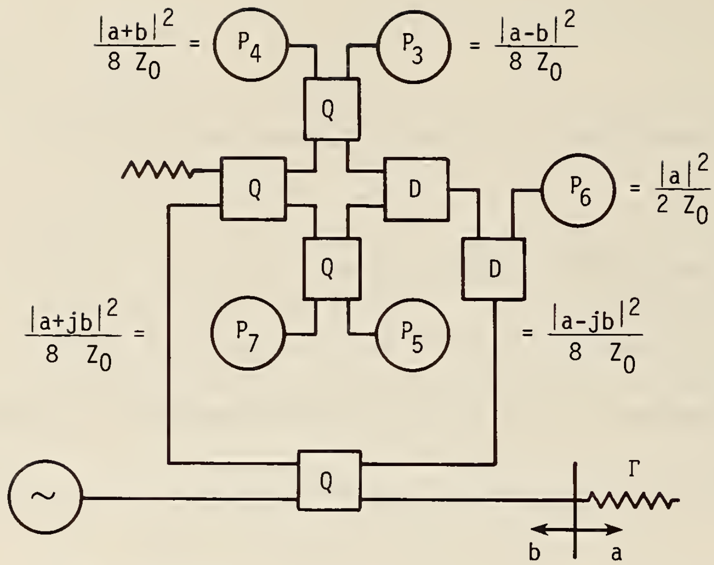
With $\mu\nu$ known, K, ν, and μ are calculated from eqs. (84a), (84d), and (84e). The values of x and y are independent of the value of $\mu\nu$;

$$x = \frac{x_2}{2x_1} \tag{90}$$

$$y = \pm\sqrt{\frac{x_3}{x_1} - x^2} \quad . \tag{91}$$

The sign of y can be determined from a knowledge of what ideal components would give. For the choice of sidearms in figure 6, y is negative.

Another way to determine or to check the value of $\mu\nu$ and the sign of y is to calculate the value of the ratio of the two impedances seen looking into the calibration circuit using eq. (72). This ratio can be compared with that obtained by some other measuring device used to measure the two impedances.



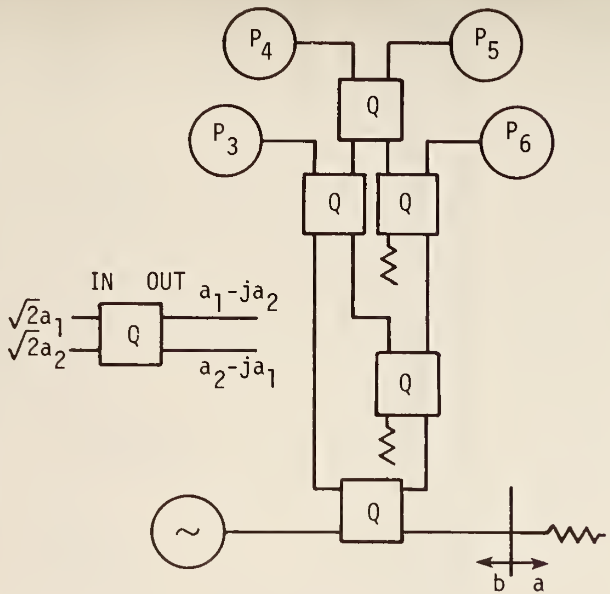
USING DETECTORS:

$$\underset{\sim}{G}_1 = \alpha_6 \begin{array}{c} \begin{matrix} 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 2 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \end{bmatrix} \end{array} \text{ OR } \alpha_6 \begin{array}{c} \begin{matrix} 3 & 4 & 7 & 6 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 2 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix} \end{array}$$

$$\underset{\sim}{H}_1 = \frac{\mu_3}{2} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & 2 & 0 \end{bmatrix} \text{ OR } \frac{\mu_3}{2} \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$\nu = \mu = x = 0$	$\nu = \mu = x = 0$
$y = -0.5,\ K = 1$	$y = +0.5,\ K = 1$
$K_0 = j$	$K_0 = -j$

Figure 5. An example of choosing and numbering the 6-port sidearms so that $|\mu\nu| \ll 1$. The left matrices apply when using detectors 3, 4, 5, and 6 (ignoring P_7). The right matrices apply when using detectors 3, 4, 7, and 6 (ignoring P_5). Q is a quadrature hybrid and D is an in-phase power divider as defined in figure 6. The parameters ν, μ, x, y, K, and K_o are calculated from equations (81), (76), (74), (75), (80), and (73).

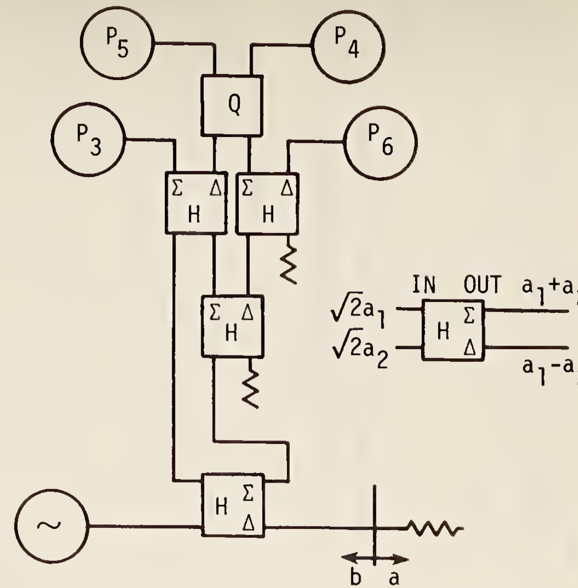


$$P_3 = \frac{|a-b|^2}{4Z_0}$$

$$P_4 = \frac{|a(1-j)+b|^2}{8Z_0}$$

$$P_5 = \frac{|a(1+j)+b|^2}{8Z_0}$$

$$P_6 = \frac{|a|^2}{4Z_0}$$



$$\underset{\sim}{G}_1 = \frac{\alpha_6}{4}\begin{bmatrix} 0 & 0 & 0 & 4 \\ 4 & 4 & 4 & -6 \\ -2 & 2 & 2 & -1 \\ 0 & -4 & 4 & 0 \end{bmatrix} \quad , \quad \underset{\sim}{H}_1 = \frac{\mu_3}{2}\begin{bmatrix} 0 & 2 & 2 & -1 \\ 2 & 0 & 0 & 0 \\ -1 & -1 & -1 & 2.5 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$$\nu = \mu = 0,\ x = 0.5,\ y = -1,\ K = 1.25,\ K_0 = -1 + j2$$

Figure 6. More examples of choosing and numbering the 6-port sidearms so that $|\mu\nu| << 1$. Both 6-port designs have the same matrix parameter if the sidearms are numbered as shown. The boxes labeled Q and H are quadrature hybrids and 180° hybrids, defined on the sides of the figure. The parameters ν, μ, x, y, K, and K_o are calculated from equations (81), (76), (74), (75), (80), and (73).

After the five constants ν, μ, x, y, and K are determined, eq. (72) can be used to calculate complex impedance ratios, or eqs. (67a) and (67b) can be used to calculate voltage and current ratios. Power ratios or reflection coefficient ratios cannot be calculated without further calibration involving an impedance standard. Note that no standards have been used in the calibration process so far.

G. Six-Port #2 is also calibrated.

When 6-port #1 is calibrated for making ratio measurements of voltage, current, and impedance, so is 6-port #2. To show this, first note that for any measurement on 6-port #2 eq. (17) gives

$$\vec{V}_2 = \underset{\sim}{H}_2 \vec{P}_2 \quad . \tag{92}$$

Using eq. (35) in eq. (92),

$$\vec{V}_2 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underset{\sim}{J} \vec{P}_2 \quad . \tag{93}$$

But from eq. (62), $\underset{\sim}{H}_1 = \underset{\sim}{h} \underset{\sim}{\gamma}$ so that eq. (93) becomes

$$\vec{V}_2 = \underset{\sim}{N}_v \underset{\sim}{h} \vec{T} \tag{94}$$

where T is known;

$$\vec{T} \equiv \underset{\sim}{\gamma} \underset{\sim}{J} \vec{P}_2 \equiv \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} \tag{95}$$

Equation (94) for 6-port #2 is identical in form to eq. (65) which is for 6-port #1 except for $\underset{\sim}{N}_v$ which only has the effect of changing the sign of some of the equations. Expanding eq. (94) gives the four equations

$$|v_2|^2 = \nu_3 t_1 + \nu_4 t_2 = \nu_4(t_2 + \nu t_1) \tag{96a}$$

$$|i_2 Z_{02}|^2 = \mu_3 t_1 + \mu_4 t_2 = \mu_3(t_1 + \mu t_2) \tag{96b}$$

$$|v_2 i_2 Z_{02}| \cos \theta_2 = - q_5 t_3 - q_6 t_4 \tag{96c}$$

$$|v_2 t_2 Z_{02}| \sin \theta_2 = - r_5 t_3 - r_6 t_4 \tag{96d}$$

The last three equations give an expression for the impedance Z_2 connected to the reference plane of 6-port #2. Following the same steps as in eqs. (69)--(72) leads to

$$\frac{Z_2}{Z_{02}} = - K_0 \frac{t_3 + (x+jy)t_4}{t_1 + \mu t_2} \tag{97}$$

where K_0, x, y and μ are the same constants used in eq. (72). Therefore, when x, y, and μ are determined, either 6-port can be used to make impedance ratio measurements. Also since ν as well as μ is known, eqs. (96a) and (96b) can be used to calculate voltage or current ratios with 6-port #2.

H. Choosing the two terminations.

One purpose of the measurements made when each 6-port is connected to the calibration circuit is to define the measurement reference planes. For this reason, the magnitude of the two values of reflection coefficient Γ_e and Γ_f seen looking into the calibration circuit input port should be fairly large. Therefore, a short and an open were initially chosen for the two terminations in the NBS system. However, the short and open combination led to trouble at certain frequencies in inverting the matrices in eqs. (57) and (60). When $\arg(\Gamma_e) = 0$ and $\arg(\Gamma_f) = 180°$ (or vice versa), the matrix being inverted in (57) becomes singular. When $\arg(\Gamma_e) = 90°$ and $\arg(\Gamma_f) = -90°$ (or vice versa), the matrix in (60) becomes singular. Either of these two sets of phase angles will occur at certain frequencies when using a short and an open as the two terminations. At angles other than these two combinations, the solution appears to be well behaved even though $\arg(\Gamma_e) - \arg(\Gamma_f) = \pm 180°$.

The optimum set of terminations has not been determined. However, it appears from computer simulations that two terminations whose phase angles are 90° apart do not create any singularities in the solution. One way of creating two terminations whose reflection coefficients are 90° apart over a broad frequency range is shown in the calibration circuit in figure 7. Assuming ideal components, $\Gamma_e = 0.25$ at $\theta_e + 90°$, and $\Gamma_f = 0.5$ at θ_f where θ_e and θ_f are unknown residual phase shifts in the lengths of line. The length of line in switch position f is chosen so that $\theta_f = \theta_e$. The difference in phase is then 90° over the frequency range for which the quadrature hybrid and the power divider are designed to operate.

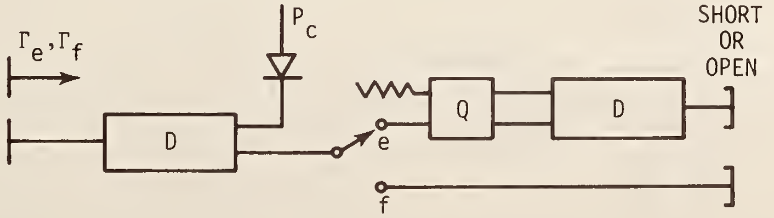


Figure 7. A calibration circuit with two terminations at e and f which have a 90° phase difference. D is an in-phase power divider and Q is a quadrature hybrid.

VI. Adding the Standard

The constant K_0 in eqs. (72) and (97) has not yet been determined. To find K_0 we can use a uniform length of line or a standard termination whose impedance Z_s or reflection coefficient Γ_s is known. It is convenient at this time to rewrite eqs. (72) and (97) in the form

$$Z_1/Z_{01} = K_0 z_1 \tag{98}$$

$$Z_2/Z_{02} = K_0 z_2 \tag{99}$$

where the small z's, z_1 and z_2, are the coefficients of K_0 in eqs. (72) and (97). The accuracies of the values of z_1 and z_2 are independent of the accuracy of any standard used. Their accuracies are functions only of the precision with which the previous measurements have been made.

A. Standard Termination.

If a standard termination of known impedance Z_{1s} is measured on 6-port #1, eq. (98) gives

$$K_0 Z_{01} = \frac{Z_{1s}}{z_{1s}} \tag{100}$$

so that 6-port #1 is now calibrated for impedance measurements. Since normally $Z_{01} = Z_{02}$, 6-port #2 is calibrated also. Or using Γ_s,

$$K_0 = \frac{1}{z_{1s}} \frac{Z_{1s}}{Z_{01}} = \frac{1}{z_{1s}} \frac{1+\Gamma_s}{1-\Gamma_s} \quad . \tag{101}$$

Examination of (100) and (101) shows that the standard cannot be a short or an open since K_0 is finite and not zero. A value near Z_{01} would be quite adequate.

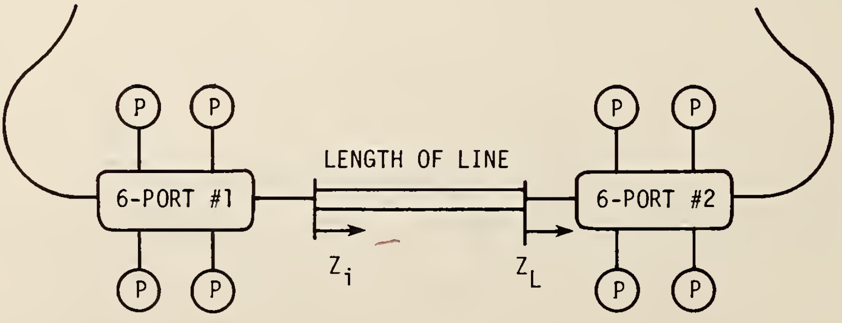


Figure 8. Defining the direction of Z_i and Z_L.

B. Standard Line.

When using a uniform length of line as the standard, define the input impedance Z_i and the load impedance Z_L as shown in figure 8. The two impedances are related by [13]

$$\frac{Z_i}{Z_0} = \frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell} \tag{102}$$

where Z_0 is the characteristic impedance of the line, γ is the complex propagation constant, and ℓ is the length of the line. Connect 6-port #1 to the input end of the line and 6-port #2 to the load end so that

$$Z_1 = Z_i \ , \quad Z_2 = - Z_L \quad . \tag{103}$$

Then eq. (102) becomes

$$\frac{Z_1}{Z_0} \left(1 - \frac{Z_2}{Z_0} \tanh \gamma\ell\right) = - \frac{Z_2}{Z_0} + \tanh \gamma\ell \tag{104}$$

Assuming that $Z_{01} = Z_{02} = Z_0$, eqs. (98) and (99) in eq. (104) give

$$K_0(z_1+z_2) = K_0^2 z_1 z_2 \tanh \gamma\ell + \tanh \gamma\ell \quad . \tag{105}$$

This equation can be solved for K_0 if $\gamma\ell$ is known. If $\gamma\ell$ is not known, K_0 can still be determined but more measurements are required as discussed below.

1) Determining $\gamma\ell$ of the line: By taking two or more measurements with the 6-port reflectometers when the line is inserted it is possible to solve for K_0 without knowing $\gamma\ell$. In fact, $\gamma\ell$ can be determined along with K_0. First divide eq. (105) by K_0 to get

$$z_1 + z_2 = z_1 z_2 K_0 \tanh \gamma\ell + \frac{\tanh \gamma\ell}{K_0} \quad . \tag{106}$$

This equation is linear in the two unknowns

$$u \equiv K_0 \tanh \gamma\ell \tag{107}$$

$$v \equiv \frac{\tanh \gamma\ell}{K_0} \quad . \tag{108}$$

Two or more measurements are taken with the line inserted by changing A_1, A_2, or ϕ to different but unknown settings. In the present NBS system, two to four measurements are taken by changing ϕ only. ϕ is such that $a_2/a_1 \approx 1$ at 90°, -90°, 0°, and 180°. Each measurement gives an equation like eq. (106) which can be combined into the single matrix equation

$$\begin{pmatrix} z_1 z_2 & 1 \\ - & - \\ - & - \\ & \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} z_1 + z_2 \\ - \\ - \\ \end{pmatrix} \tag{109}$$

or

$$\underline{U}\ \vec{V} = \vec{W} \quad .$$

If only two measurements are taken, $\underline{U}$ is a square 2 by 2 matrix and the solution is

$$\vec{V} = \underset{\sim}{U}^{-1}\ \vec{W} \quad . \tag{110a}$$

If three or more measurements are taken, a least squares solution is obtained using the pseudoinverse for complex matrices [14].

$$\vec{V} = (\underline{U}^*\underline{U})^{-1}\underline{U}^*\vec{W} \quad , \tag{110b}$$

where $\underline{U}^*$ is the transpose of the complex conjugate of $\underline{U}$. Knowing u and v, K_0 and tanh $\gamma\ell$ are calculated from eqs. (107) and (108);

$$K_0 = \pm\sqrt{\frac{u}{v}} \tag{111}$$

$$\tanh \gamma\ell = K_0\ v = u/K_o \quad . \tag{112}$$

One way of choosing the sign of K_0 is by comparing the value from eq. (111) with an estimate calculated from the 6-port circuit assuming all components are ideal. For example, in the ideal circuit shown in figure 5, using detectors 3, 4, 5, and 6, the phase angle of K_0 calculated from eq. (73) is

$$\arg\ (K_0) = \tan^{-1} \frac{r_5}{q_5} = 90° \quad .$$

The sign of K_0 from eq. (111) is chosen so that the phase angle is closest to 90°. The sign can also be chosen so that the impedances calculated from eq. (72) for the two terminations in the calibration circuit are consistant with their estimated or previously calibrated values. Or eqs. (107) and (108) can be used to write

$$\tanh \gamma\ell = \pm \sqrt{uv} \tag{113}$$

$$K_0 = \frac{1}{v} \tanh \gamma\ell \tag{114}$$

in which case the sign is chosen so that tanh $\gamma\ell$ is consistent with that calculated from an estimate of $\gamma\ell$ determined from the dimensions of the line. In either case $\gamma\ell$ is determined along with K_0. Each 6-port is now calibrated for measuring impedance or reflection coefficients relative to the characteristic impedance of the line.

2) Determining Z_0 of the line: Since $\gamma\ell$ is known, it can be used in determining Z_0 of the line. For a coaxial transmission line [13]

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \tag{115}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \tag{116}$$

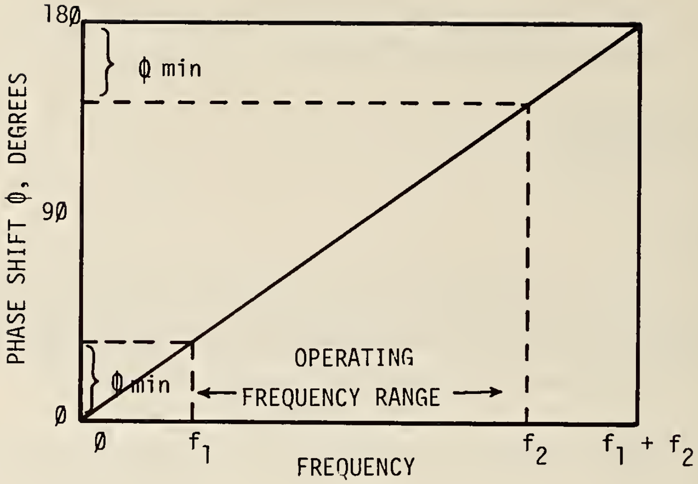
where R and L are the series resistance and inductance per unit length of the line, and G and C are the parallel conductance and capacitance per unit length of the line. For an air line $|G| << |\omega C|$. Then eqs. (115) and (116) combine to give

$$Z_0 = \frac{\gamma}{G+j\omega C} \simeq \frac{\gamma\ell}{j\omega C\ell} \quad . \tag{117}$$

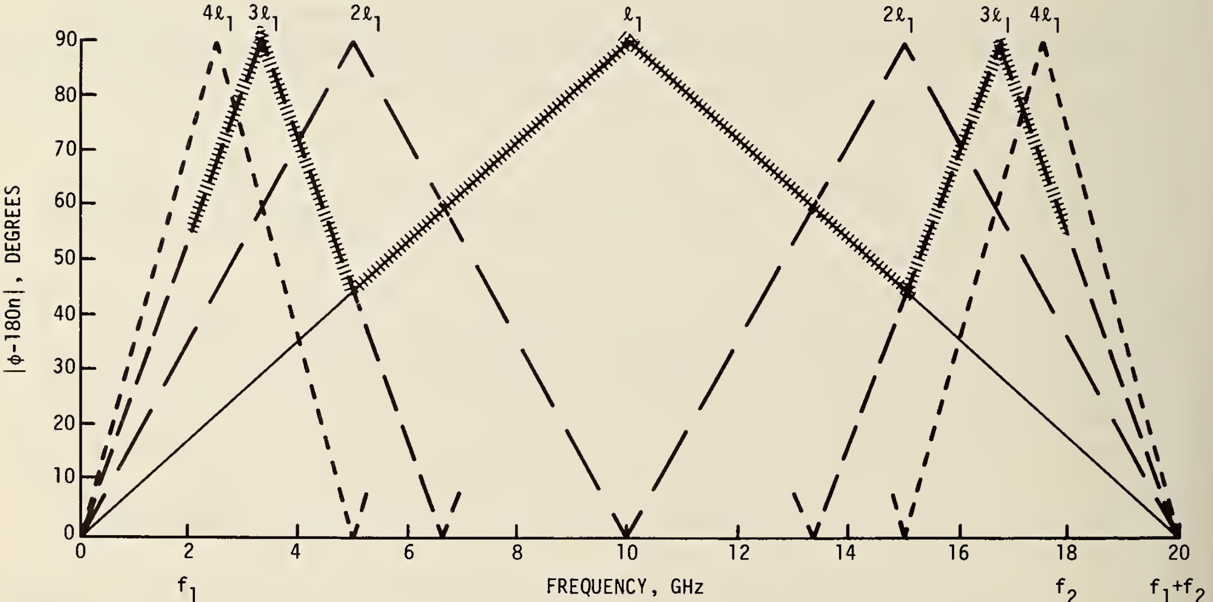
Since $\gamma\ell$ is known, only $C\ell$ needs to be determined. This can be calculated from the cross-sectional dimensions of the line or measured with a low frequency capacitance bridge. Since C is independent of frequency, $C\ell$ is the total line capacitance at low frequency and can be accurately measured at say 1 KHz. Thus $\gamma\ell$ and $C\ell$ and hence Z_0 can all be determined without knowing the length or series loss of the line when shunt losses are negligible.

3) Choosing the line length: When using a length of line as the standard, its length must not be too near a multiple of half wavelengths long at the frequencies of operation, or the solution will become ill-conditioned. One way to avoid this condition is to choose the length as illustrated in figure 9a. If the 6-ports are designed to operate over the frequency range f_1 to f_2, the length is chosen so that it is a half-wavelength long at the frequency $f_1 + f_2$. Then the effective minimum phase shift will be the same at both ends of the band of operation, and will be 90° at the center of the band. If the effective phase shift at the band edges is too small, or if the line length becomes physically too small, then multiples of this length can be used as illustrated in figure 9b. In figure 9b, $|\phi-180n|$ has been plotted instead of ϕ, choosing n so that all values fall within 0 to 90°.

For example, to operate over the 2 to 18 GHz range, $f_1 + f_2 = 20$ GHz. For a coaxial system, a length of air line 0.75 cm long is a half wavelength at 20 GHz. So 0.75 cm would be a preferred length. This line would have a minimum effective phase shift of 18° at both 2 and 18 GHz. If this phase shift is considered to be too small, a second line three times as long as the first (2.25 cm) would increase the minimum effective phase shift to 45° as shown by the shaded accent in figure 9b. The shorter line would be used from 5 to 15 GHz, and the longer one outside of this range.



(a)



(b)

Figure 9. Effective phase shift through standard transmission lines as a function of frequency where ϕ is the phase shift through the line and n = 0, 1, 2,.... In (b), n has been chosen so that $|\phi-180n|$ is between 0 and 90°. The line lengths are chosen so that they are multiples of a half wavelength at $f_1 + f_2$, where f_1 and f_2 are the frequency limits of operation of the 6-ports. For coaxial air lines,

$$\phi = 12 f_{GHz} \ell_{cm}, \text{ degrees.}$$

$$\ell_1 = \frac{15}{f_1 + f_2}, \text{ cm for f in GHz.}$$

VII. Filling in Calibration Matrices

Now that K_0 has been determined, the calibration matrices $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ can be completed except for one constant which is not required unless absolute measurements of power, voltage, or current are desired. Let the components of K_0 be

$$K_0 \equiv K_1 + j\, K_2 \quad . \tag{118}$$

Then from eq. (73),

$$q_5 = \mu_3\, K_1 \tag{119}$$

$$r_5 = \mu_3\, K_2 \quad . \tag{120}$$

From eqs. (74) and (75),

$$\begin{aligned} q_6 &= xq_5 - yr_5 \\ &= \mu_3(xK_1 - yK_2) \end{aligned} \tag{121}$$

$$\begin{aligned} r_6 &= yq_5 + xr_5 \\ &= \mu_3(yK_1 + xK_2) \quad . \end{aligned} \tag{122}$$

And from eq. (80)

$$\nu_4 = \mu_3 \frac{|K_0|^2}{K} \quad . \tag{123}$$

The elements of the 4 by 4 matrix $\underset{\sim}{h}$ which was defined in eqs. (61) and (62) and expanded in eq. (65) are now all known except for μ_3. Factoring μ_3 out of $\underset{\sim}{h}$ it becomes

$$\underset{\sim}{h} \equiv \begin{pmatrix} \nu_3 & \nu_4 & 0 & 0 \\ \mu_3 & \mu_4 & 0 & 0 \\ 0 & 0 & q_5 & q_6 \\ 0 & 0 & r_5 & r_6 \end{pmatrix} = \mu_3 \begin{pmatrix} \frac{|K_0|^2}{K}\nu & \frac{|K_0|^2}{K} & 0 & 0 \\ 1 & \mu & 0 & 0 \\ 0 & 0 & K_1 & xK_1-yK_2 \\ 0 & 0 & K_2 & yK_1+xK_2 \end{pmatrix} = \mu_3 \underset{\sim}{H} \tag{124}$$

where $\underset{\sim}{H}$ is the matrix $\underset{\sim}{h}$ with the unknown parameter μ_3 factored out. The matrix $\underset{\sim}{H}_1$ is calculated from eq. (62)

$$\underset{\sim}{H}_1 = \underset{\sim}{h}\,\underset{\sim}{\gamma} = \mu_3\, \underset{\sim}{H}\,\underset{\sim}{\gamma} \tag{125}$$

and H_2 is calculated from eq. (35)

$$\underset{\sim}{H}_2 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underset{\sim}{J} = \mu_3 \underset{\sim}{N}_v \underset{\sim}{H} \underset{\sim}{\gamma} \underset{\sim}{J} \quad . \tag{126}$$

The parameter μ_3 will cancel in calculating impedance or ratios of voltage, current, or power. It is only needed to calculate absolute values of voltage, current, or power.

To calculate reflection coefficients or S-parameters, $\underset{\sim}{G}_1$ and $\underset{\sim}{G}_2$ are calculated from $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ using (23).

$$\underset{\sim}{G}_1 = \frac{1}{4} \underset{\sim}{K}\underset{\sim}{H}_1 = \frac{\mu_3}{4} \underset{\sim}{K}\underset{\sim}{H}\underset{\sim}{\gamma} \tag{127}$$

$$\underset{\sim}{G}_2 = \frac{1}{4} \underset{\sim}{K}\underset{\sim}{H}_2 = \frac{\mu_3}{4} \underset{\sim}{K}\underset{\sim}{N}_v\underset{\sim}{H}\underset{\sim}{\gamma}\underset{\sim}{J} \quad . \tag{128}$$

The parameter μ_3 also cancels in calculating reflection coefficient or S-parameters.

VIII. Calibrating for Power, Voltage, and Current

To calibrate both 6-port reflectometers for measuring power, a power standard is connected to one of them, say #1. Then eq. (19c) gives*

$$P_s Z_{01} = |v_1 i_1 Z_{01}| \cos \theta_1 = \mu_3 \sum \frac{q_i}{\mu_3} P_i \,, \qquad i = 3\ldots6 \tag{129}$$

where the parameters q_i/μ_3 are known in $\underset{\sim}{H}\underset{\sim}{\gamma}$, and P_s is the net power into the standard power meter. The constant μ_3/Z_{01} can be calculated from (129). Then the net power into any termination connected to reference plane #1 is

$$P_1 = \frac{\mu_3}{Z_{01}} \sum \frac{q_i}{\mu_3} P_i \quad . \quad i = 3\ldots6$$

Note that Z_{01} does not have to be known to calibrate the 6-port for making power measurements. However, Z_{01} does have to be known to find μ_3 which is needed for absolute measurements of voltage or current. From eqs. (19a) and (19b)

$$|v_1|^2 = \mu_3 \sum \frac{\nu_i}{\mu_3} P_i \tag{130}$$

$$i = 3\ldots6$$

$$|i_1|^2 = \frac{\mu_3}{|Z_{01}|^2} \sum \frac{\mu_i}{\mu_3} P_i \tag{131}$$

where the parameters ν_i/μ_3 and μ_i/μ_3 are known in $\underset{\sim}{H}\underset{\sim}{\gamma}$. When μ_s and $Z_0 = Z_{01} = Z_{02}$ are known, both 6-ports are completely calibrated.

---

*Equation (129) assumes that Z_{01} is real so that $\theta_1 = \arg(v_1/i_1)$.

IX. Evaluating the Calibration

When taking more measurements during the calibration than are needed, the redundant equations can be used to determine the consistency and precision of the measurements. When the reference planes are together, each measurement made by the two 6-port relectometers should satisfy

$$P_1 = -P_2 \tag{132}$$

$$Z_1 = -Z_2 \tag{133}$$

$$\Gamma_1 = 1/\Gamma_2 \tag{134}$$

$$S_{12} = S_{21} = 1 = \sqrt{\rho_1 \rho_2} \ . \tag{135}$$

When connected to the calibration circuit, the 6-port reflectometer measurements should give

$$P_1 = P_2 \tag{136}$$

$$Z_1 = Z_2 \tag{137}$$

$$\Gamma_1 = \Gamma_2 \ . \tag{138}$$

Lastly, when the standard line is inserted, each measurement should satisfy

$$\Gamma_1 = e^{-2\gamma \ell}/\Gamma_2 \tag{139}$$

$$S_{12} = S_{21} = e^{-\gamma \ell} = \sqrt{\rho_1 \rho_2} \ . \tag{140}$$

Another useful check on the calibration is to calculate the matrix $\underset{\sim}{B}$ in (5) using (8);

$$\underset{\sim}{B} = \underset{\sim}{G}^{-1} \ . \tag{141}$$

Note that the elements of $\underset{\sim}{B}$ in (5) are not all independent since

$$4|A_i|^2|B_i|^2 = (2|A_iB_i| \cos \zeta_i)^2 + (2|A_iB_i| \sin \zeta_i)^2 \ . \tag{142}$$

In words, four times the product of the first two elements in any row of $\underset{\sim}{B}$ should equal the square of element 3 plus the square of element 4 in that row.

The extent to which the calibration data satisfy equations (132)--(142) is a measure of the quality of the calibration.

If a uniform length of transmission line is used as the standard, another check is to calculate $\alpha \ell / \sqrt{f}$ and $\beta \ell / f$ at each frequency f, where $\gamma \ell \equiv \alpha \ell + j \beta \ell$. Both of these parameters should be constant with frequency.

X. Acknowledgment

The author wishes to acknowledge the earlier unpublished work of Charles Manney in which he showed that self-calibration techniques can be applied to two 6-port reflectometers.

References

[1] Hoer, C. A., and Engen, C. A., Analysis of a six-port junction for measuring v, i, a, b, z, Γ, and phase, Presented at the Proc. IMEKO Symp. Acquisition and Proc. of Measurement Data for Automation, Dresden, Germany (June 17-23, 1973). Published in ACTA IMEKO 1973, Vol. 1, pp. 213-222.

[2] Hoer, C. A., Using six-port and eight-port junctions to measure active and passive circuit parameters, NBS Tech. Note 673, 23 pp. (Sept. 1975).

[3] Cronson, H. M., and Susman, L., Automated six-port microwave calibration system, Sperry Research Center Final Technical Report No. SCRC-CR-76-32, 72 pp. (April 1977).

[4] Engen, G. F., The six-port reflectometer: An alternative network analyzer, IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 1075-1080 (Dec. 1977).

[5] Cronson, H. M., and Susman, L., A six-port automatic network analyzer, IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 1086-1091 (Dec. 1977).

[6] Engen, G. F., An improved circuit for implementing the six-port technique of microwave measurements, IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 1080-1083 (Dec. 1977).

[7] Komarek, E. L., An application of the six-port reflectometer to precision measurement of microwave one-port parameters, 1977 International Microwave Symposium, San Diego, CA (June 21-23, 1977). Digest pp. 56-57.

[8] Weidman, M. P., A semi-automated six-port for measuring millimeterwave power and complex reflection coefficient, IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 1083-1085 (Dec. 1977).

[9] Engen, G. F., and Hoer, C. A., Application of an arbitrary six-port junction to power measurement problems, IEEE Trans. Instrum. Meas., Vol. IM-21, pp. 470-474 (Nov. 1972).

[10] Hoer, C. A., A network analyzer incorporating two six-port reflectometers, IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 1070-1074 (Dec. 1977).

[11] Allred, C. M., and Manney, C. H., The calibration and use of directional couplers without standards, IEEE Trans. Instrum. Meas., Vol. IM-25, pp. 84-89 (March 1976).

[12] Greville, T. N. E., The pseudoinverse of a rectangular or singular matrix and its application to the solution of systems of linear equations, SIAM Review, Vol. 1, No. 1, pp. 38-43 (Jan. 1959).

[13] Ramo, Whinnery, and Van Duzer, Fields and Waves in Communication Electronics, New York: Wiley, Table 1.23, p. 46 (1965).

[14] Penrose, R., On best approximate solutions of linear matrix equations, Proc. Cambridge Philos. Soc., Vol. 52, pp. 17-19 (1956).

APPENDIX

This appendix gives an outline of a program for taking the calibration data, a description of how the data can be stored, and finally an outline of a second program for calculating the matrices $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ which characterize the two 6-ports. Equations given in the second outline refer to the equation numbers in the body of this paper.

A. Taking the Calibration Data.

This section contains an outline of a program for taking the calibration data using a precision length of transmission line as the impedance standard. It is assumed that when each 6-port is connected to the calibration circuit the generator is leveled with the output, P_c, of this circuit so that P_c is constant and need not be measured.

```
Connect reference planes together
     For measurement #a, b, c,...corresponding to four or more settings of
     a2/a1 such as 1 at -90°, 1 at 180°, 1 at 0°, 0.3 at 90°, 1 at 90°, 0.3 at -90°.
          For each frequency
               For each sidearm on 6-port #1
                    Read sidearm power
               Next sidearm
               For each sidearm on 6-port #2
                    Read sidearm power
               Next sidearm
          Next frequency
     Next measurement (next setting of a2/a1)
Connect 6-port #1 to calibration circuit
     Set calibration circuit switch to position "e"
          For each frequency
               For each sidearm of 6-Port #1
                    Read sidearm power
               Next sidearm
          Next frequency
     Set calibration circuit switch to position "f"
          For each frequency
               For each sidearm of 6-Port #1
                    Read sidearm power
               Next sidearm
          Next frequency
```

```
Connect 6-Port #2 to calibration circuit
          For each frequency
               For each sidearm of 6-Port #2
                    Read sidearm power
               Next sidearm
          Next frequency
     Set calibration circuit switch to position "e"
          For each frequency
               For each sidearm of 6-Port #2
                    Read sidearm power
               Next sidearm
          Next frequency
Insert standard line between the two 6-Ports.
     For measurement # g, h,...corresponding to two or more settings of a2/a1
     such as 1 at 90°, 1 at -90°, 1 at 180°, 1 at 0°.
          For each frequency
               For each sidearm of 6-Port #1
                    Read sidearm power
               Next sidearm
               For each sidearm of 6-Port #2
                    Read sidearm power
               Next sidearm
          Next frequency
     Next measurement
End
```

B. Data Sequence.

In the above sequence for taking the data, all sidearm power readings are recorded at all frequencies for any one setting of A_1, A_2, and ϕ. This sequence of taking the data minimizes wear on the electromechanical switches in the components A_1, A_2, and ϕ.

C. Data Array.

All of the calibration data is taken before any computations are done. One way of storing the calibration data is shown in figure 10. As the 6-port sidearm power readings are taken, they are stored in one large data array called $\underline{P}$. Each element P in $\underline{P}$ represents a power reading. Each row in $\underline{P}$ contains all of the power readings taken at one frequency, so that $\underline{P}$ contains as many rows as frequencies. The number of columns in $\underline{P}$ is at least 64, but it can be larger depending on how many redundant measurements are made.

To save on computer memory, $\underline{P}$ can be broken into two equal parts as shown in figure 10 and each part stored separately. All of the power readings for the first part of $\underline{P}$ with the reference planes together are taken before any of the readings are taken for the second half of $\underline{P}$.

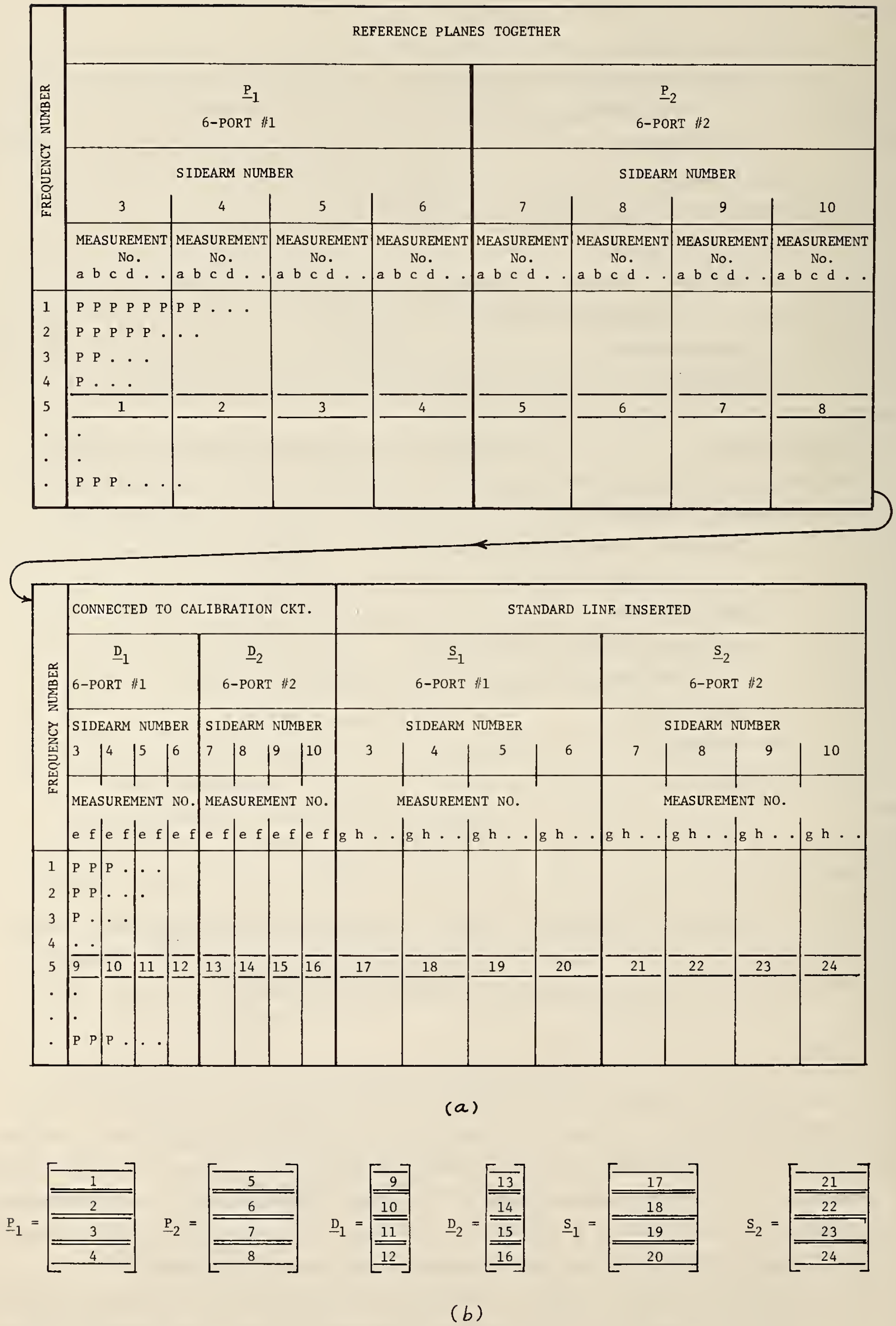


Figure 10. One way of storing the sidearm power readings, P, so that all readings taken at a given frequency are in one row (a). After all the data is taken, the matrices $\underline{P}_1$, $\underline{P}_2$, $\underline{D}_1$, $\underline{D}_2$, $\underline{S}_1$, and $\underline{S}_2$ are formed from any one row as indicated by the example in (b) for row (frequency) #5.

After all of the data is taken, the elements of $\underline{P}$ are used a row at a time to calculate the matrices $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ for the frequency corresponding to that row. The elements in any one row of $\underline{P}$ are first sent to six different smaller matrices as indicated in figure 10b which shows an example using row #5. In general, the data in any one row is transfered as follows:

1. The readings taken when the reference planes are together are sent to $\underline{P}_1$ and $\underline{P}_2$;

$$\underline{P}_1 = [\vec{P}_{1a}\ \vec{P}_{1b}\ \vec{P}_{1c}\ \vec{P}_{1d}..], \qquad \underline{P}_2 = [\vec{P}_{2a}\ \vec{P}_{2b}\ \vec{P}_{2c}\ \vec{P}_{2d}..] \qquad (33)\ (34)$$

2. The readings taken when each 6-port is connected to the calibration circuit are sent to $\underline{D}_1$ and $\underline{D}_2$;

$$\underline{D}_1 = [\vec{P}_{1e}\ \vec{P}_{1f}], \qquad \underline{D}_2 = [\vec{P}_{2e}, \vec{P}_{2f}] \qquad (48)\ (49)$$

3. The readings taken when the standard line is inserted are sent to $\underline{S}_1$ and $\underline{S}_2$;

$$\underline{S}_1 = [\vec{P}_{1g}\ \vec{P}_{1h}..], \qquad \underline{S}_2 = [\vec{P}_{2g}\ \vec{P}_{2h}..]$$

A subscript 1 or 2 refers to 6-port #1 or #2, and subscripts a, b, c,...h refer to the different measurement conditions. Each of the vectors in $\underline{P}_1 \ldots \underline{S}_2$ contains four power readings. For example, $\underline{D}_1$ is

$$\underline{D}_1 = [\vec{P}_{1e}\ \vec{P}_{1f}] = \begin{bmatrix} P_{3e} & P_{3f} \\ P_{4e} & P_{4f} \\ P_{5e} & P_{5f} \\ P_{6e} & P_{6f} \end{bmatrix}$$

where subscript 3, 4, 5, or 6 on P refers to the sidearm number, and e and f refer to the measurement condition where termination e or f was used in the calibration circuit.

D. Calculating $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$.

This section contains an outline of a program for calculating the matrices $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ which characterize the two 6-ports. The choice of roots and signs corresponds to the 6-port design shown in figure 5 using detectors 3, 4, 5, and 6.

For each frequency

Fill in matrices $\underline{P}_1$, $\underline{P}_2$, $\underline{D}_1$, $\underline{D}_2$, $\underline{S}_1$ and $\underline{S}_2$ from the $\underline{P}$ array.

Calculate matrix $\underset{\sim}{J}$.

If 4 measurements are taken with ref planes together,

$$\underset{\sim}{J} = \underset{\sim}{P}_1\ \underset{\sim}{P}_2^{-1} . \qquad (36a)$$

If more than 4 measurements are taken,

$$\underset{\sim}{J} = \underline{P}_1\underline{P}_2^T\ (\underline{P}_2\underline{P}_2^T)^{-1} . \qquad (36b)$$

Calculate $\underline{E} = \underset{\sim}{J}\, \underline{D}_2$. (51)

Partition $\underline{D}_1$ and $\underline{E}$.

$$\underline{D}_1 = \begin{pmatrix} \underset{\sim}{d}_1 \\ \underset{\sim}{d}_2 \end{pmatrix} \quad , \quad \underline{E} = \begin{pmatrix} \underset{\sim}{e}_1 \\ \underset{\sim}{e}_2 \end{pmatrix} \tag{52}$$

Calculate $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ matrices.

$$\underset{\sim}{\alpha} = (\underset{\sim}{e}_1 - \underset{\sim}{d}_1)(\underset{\sim}{d}_2 - \underset{\sim}{e}_2)^{-1} \tag{57}$$

$$\underset{\sim}{\beta} = -(\underset{\sim}{e}_2 + \underset{\sim}{d}_2)(\underset{\sim}{d}_1 + \underset{\sim}{e}_1)^{-1} \tag{60}$$

Fill in $\underset{\sim}{\gamma}$ matrix.

$$\underset{\sim}{\gamma} = \begin{pmatrix} \underset{\sim}{I} & \underset{\sim}{\alpha} \\ \underset{\sim}{\beta} & \underset{\sim}{I} \end{pmatrix} \qquad (61)\ (62)$$

Calculate $\vec{\delta}$ vectors for 6-port #1, store in $\underline{\delta}$. *

$$\underline{\delta} = \gamma[\underline{P}_1\ \underline{D}_1] \tag{64}$$

Set up and solve the matrix equation

$$\underline{R}\, \vec{X} = \vec{F} \tag{86}$$

for $\vec{X}$. Each column of $\underline{\delta}$ is used to fill in one row of $\underline{R}$ and $\vec{F}$.

Calculate $\mu\nu$ from elements 4 and 5 of $\vec{X}$ assuming $|\mu\nu| < 1$.

$$\mu\nu = \frac{2\, x_4 x_5}{1 - 2x_4x_5 + \sqrt{1-4x_4x_5}} \tag{89}$$

Calculate K, ν, μ, x, and y from the elements of $\vec{X}$ and $\mu\nu$.

$$K = x_1(1 + \mu\nu) \tag{84a}$$

$$\nu = x_4(1 + \mu\nu) \tag{84d}$$

$$\mu = x_5(1 + \mu\nu) \tag{84e}$$

$$x = x_2/2x_1 \tag{90}$$

$$y = -\sqrt{\frac{x_3}{x_1} - x^2} \tag{91}$$

*See section C (Data Array) for definition of $\underline{P}_1$ and $\underline{D}_1$. $\underline{\delta}$ is a new matrix formed from 6 or more $\vec{\delta}$ vectors.

Using the standard line data, set up and solve

$$\underline{U}\vec{V} = \vec{W} \quad . \tag{109}$$

Calculate

$$\underline{\delta}_s = \underset{\sim}{\gamma}\underline{S}_1 \tag{64}$$

$$\underline{T}_s = \underset{\sim}{\gamma}\underset{\sim}{J}\underline{S}_2 \quad . \tag{95}$$

For each measurement g, h,..

calculate z_1 and z_2 for each column of $\underline{\delta}_s$ and $\underline{T}_s$.

$$z_1 = \frac{\delta_3 + (x+jy)\delta_4}{\delta_1 + \mu\delta_2} \tag{72, 98}$$

$$z_2 = -\frac{t_3 + (x+jy)t_4}{t_1 + \mu t_3} \quad . \tag{97, 99}$$

Fill in corresponding row of $\underline{U}$ and $\vec{W}$.

$$\underline{U} = \begin{pmatrix} z_1 z_2 & 1 \\ - & - \\ - & - \\ - & - \end{pmatrix} , \qquad \vec{W} = \begin{pmatrix} z_1 + z_2 \\ - \\ - \\ - \end{pmatrix} \quad . \tag{109}$$

Next measurement

Solve $\underline{U}\vec{V} = \vec{W}$ for $\vec{V} = \begin{pmatrix} u \\ v \end{pmatrix}$.

If two measurements are taken with the standard line inserted,

$$\vec{V} = \underset{\sim}{U}^{-1}\vec{W} \quad . \tag{110a}$$

If more than two measurements are taken with the line inserted,

$$\vec{V} = (\underline{U}^*\underline{U})^{-1}(\underline{U}^*\vec{W}) \quad . \tag{110b}$$

Calculate K_o and $\gamma\ell$ from the elements of $\vec{V}$.

$$K_o = K_1 + jK_2 = \sqrt{\frac{u}{v}} \tag{111}$$

choose the sign of K_o such that

$$0° < \arg(K_o) < 180° \tag{112b}$$

$$\tanh \gamma \ell = \frac{u}{K_o} \equiv x+jy \tag{112}$$

$$\alpha \ell = 0.25 \ \ell n \ (1 + \frac{4x}{1-2x+x^2+y^2}) \quad \dagger$$

$$\beta \ell = 0.5 \tan^{-1}(\frac{2y}{1-x^2-y^2}) \quad \dagger$$

Fill in matrix $\underset{\sim}{H}$.

$$\underset{\sim}{H} = \begin{pmatrix} \frac{|K_o|^2}{K}\nu & \frac{|K_o|^2}{K} & 0 & 0 \\ 1 & \mu & 0 & 0 \\ 0 & 0 & K_1 & xK_1-yK_2 \\ 0 & 0 & K_2 & yK_1+xK_2 \end{pmatrix} \tag{124}$$

Calculate $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$, the desired calibration matrices, setting $\mu_3 = 1$.

$$\underset{\sim}{H}_1 = \underset{\sim}{H}\underset{\sim}{\gamma} \tag{125}$$

$$\underset{\sim}{H}_2 = \underset{\sim}{N}_v \underset{\sim}{H}_1 \underset{\sim}{J} \tag{126}$$

where N_v is given by (31). (This completes the calibration. The remainder of the program evaluates the calibration.)

Evaluate the calibration.

Calculate $\underset{\sim}{G}_1 = \frac{1}{4}\underset{\sim}{K}\underset{\sim}{H}_1$ (127)

$$\underset{\sim}{G}_2 = \frac{1}{4}\underset{\sim}{K}\underset{\sim}{H}_2 \tag{128}$$

$$\underset{\sim}{B}_1 = \underset{\sim}{G}_1^{-1} \quad (8)$$

$$\underset{\sim}{B}_2 = \underset{\sim}{G}_2^{-1} \quad (8)$$

where $\underset{\sim}{K}$ is given by equations (14) and (15).

Calculate inconsistency in calibration constants.

For each row in $\underset{\sim}{B}_1$ calculate and print

$$\Delta_i = \frac{B(i,1)B(i,2)}{[B(i,3)^2 + B(i,4)^2]/4} - 1 \tag{5}$$

where i = row number.

†See Dwight, "Tables of Integrals and Other Mathematical Data," Macmillian Co., 4th ed., p. 168, #722.2.

Next row in $\underset{\sim}{B}_1$

Repeat the calculation of Δ_i using $\underset{\sim}{B}_2$.

For data taken with reference planes together

calculate $\underline{A}_1 = \underset{\sim}{G}_1\underline{P}_1$ (7)

$\underline{A}_2 = \underset{\sim}{G}_2\underline{P}_2$. (7)

For each column of $\underline{A}_1$ and $\underline{A}_2$ corresponding to measurement #a, b, c,...calculate and print

$$\Delta\rho = \rho_1 - 1/\rho_2 \tag{134}$$

$$\Delta S_{21} = \sqrt{\rho_1\rho_2} - 1 \tag{135}$$

$$\Delta P = \frac{P_1+P_2}{|a_1|^2} \quad . \tag{132}$$

Next column.

For data taken with standard line inserted

calculate $\underline{A}_1 = \underset{\sim}{G}_1\underline{S}_1$ (7)

$\underline{A}_2 = \underset{\sim}{G}_2\underline{S}_2$. (7)

For each column of $\underline{A}_1$ and $\underline{A}_2$ corresponding to measurement #g, h,...calculate and print

$$\Delta\rho = \rho_1 - e^{-2\gamma\ell}/\rho_2 \tag{139}$$

$$\Delta S_{21} = \sqrt{\rho_1\rho_2} - e^{-\gamma\ell} \quad . \tag{140}$$

Next column.

End of evaluation.

Store $\underset{\sim}{H}_1$ and $\underset{\sim}{H}_2$ (or $\underset{\sim}{G}_1$ and $\underset{\sim}{G}_2$) on tape or disk.

Next frequency.

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This paper describes a technique for calibrating a pair of 6-port reflectometers for measuring the reflection coefficient of 1-port devices, or the scattering parameters of reciprocal 2-port devices. The operations in the calibration consist of connecting the two 6-ports together, connecting each 6-port to a calibration circuit consisting of two terminations of unknown impedance and a leveling loop, and then connecting the standard. The standard can be one termination whose complex impedance is known, or a precision length of transmission line whose cross-sectional dimensions are known. The length and loss of the line are not required. The solution for the constants which characterize each 6-port is closed, requiring no iteration.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Calibration; current; impedance; network analyzer; power; reflection coefficient; scattering parameters; six-port; reflectometer; voltage.

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