NBS TECHNICAL NOTE



100-A

# Loading Factors and FDM-FM System Performance Calculations

Supplement to Appendix C of the National Bureau of Standards Technical Note 100 entitled Required Signal-to-Noise Ratios, RF Signal Power, and Bandwidth for Multichannel Radio Communication Systems

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# NBS TECHNICAL NOTE 100-A

ISSUED SEPTEMBER 1967

## LOADING FACTORS AND FDM-FM SYSTEM PERFORMANCE CALCULATIONS

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Power, and Bandwidth for Multichannel
Radio Communication Systems

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For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 20402
Price 20 cents.

# SUPPLEMENT TO APPENDIX C

# OF

# THE NATIONAL BUREAU OF STANDARDS

# TECHNICAL NOTE 100

E. F. FLORMAN

## I N T R O D U C T I O N

This supplement was written for the purpose of extending the scope of the original paper, by providing a more complete set of system-performance design equations, which apply to the Frequency Division Multiplex--Frequency Modulation type of multichannel telecommunication systems; these equations may be used for various types, and combinations of types, of message signals.

The page numbers of this paper correspond to the pages in "The National Bureau of Standards Technical Note 100", on the topic of FDM-FM systems-performance calculations, based primarily on the FM radio receiver transfer function.

The referenced page numbers and equation numbers in this paper refer, of course, to NBS Technical Note 100.

## Factors Which Affect Frequency Deviation of the RF Signal in FDM-FM Systems

For military-service communications, the multichannel system and the FDM-FM equipment should be designed and operated so as to fully utilize the RF spectrum bandwidth, $B_{rf}$, employing a composite baseband noise-type modulating signal having an average power level $P_{mn}$. The output message-channel signal-to-noise ratio, $S_{oc}/N_{oc}$, should be calculated for a composite baseband modulating-signal total average power level $P_{mn}$; and a message-channel modulating-signal average power level $(p_m)_{s,n}$, in the baseband signal spectrum.

Equation (C.5), p. 169, NBS Technical Note 100, applies only to the case of sinusoidal modulating signals. For multichannel systems, we are required to calculate the message (voice) channel output signal-to-noise ratio, $S_{oc}/N_{oc}$, for the case where the composite baseband modulating signal (closely) resembles a noise-type signal; and the message-channel signal may be either a sinusoidal signal or a noise-type signal. Hence, the factors to be substituted in (C.15), p. 172, are obtained from a modified form of (C.5), p. 169.

The general form of (C.5) is based on the following four basic concepts and assumptions:

a. The message-signal output average power, $S_{oc}$, is proportional to $(\Delta F_c)^2_{s,n}$ as indicated by (C.3), p. 168.

b. $(\Delta F_c)_{s,n}$ is dependent upon both the average power level of the modulating message-signal power $(p_m)_{s,n}$ and the "peak factor," $\sqrt{2}$

of the rms voltage associated with $(p_m)_{s,n}$.

c. The maximum bandwidth of the generated RF signal spectrum is a function of the peak frequency deviation, $(\Delta F)_{s,n}$ as shown by (C.9) and (C.10), p. 171. And,

d. $(\Delta F)_{s,n}$ is dependent upon the average power level of the composite total baseband modulating signal, $(P_m)_{s,n}$, and the "peak factor", $(PF_b)_{s,n}$, of the voltage associated with $(P_m)_{s,n}$.

The general form for (C.5) is:

$$\frac{(\Delta F_c)^2_{s,n}}{(\Delta F)^2_{s,n}} = \frac{2}{(PF_b)_{s,n}} \times \frac{(p_m)_{s,n}}{(P_m)_{s,n}}$$

or, $$(\Delta F_c)^2_{s,n} = (\Delta F)^2_{s,n} \times \frac{2}{(PF_b)_{s,n}} \times \frac{(p_m)_{s,n}}{(P_m)_{s,n}} \qquad \text{(C.5a)}$$

where, $(\Delta F_c)_{s,n}$ = Root mean square (rms) deviation of the RF signal due to the message-signal power, $(p_m)_{s,n}$, in the modulating baseband signal; for sinusoidal or noise-type modulating signals, as indicated by the subscript letter.

$(\Delta F)_{s,n}$ = Peak RF-signal frequency deviation due to the composite baseband modulating signal, for sinusoidal or noise-type modulating signals, as indicated by the subscript letters.

$(PF_b)_{s,n}$ = Peak factor of the composite baseband signal (see p. 17) either sinusoidal or noise-type signals, as indicated by the subscript letters; at the modulator input terminals.

$(p_m)_{s,n}$ = Average power level of the message signal, in the message-signal-channel bandwidth, at the modulator input terminals; for sinusoidal or for noise-type signals, as indicated by the subscript letters.

$(P_m)_{s,n}$ = Average power level of the composite baseband signal, at the modulator input terminals; for sinusoidal or for noise-type signals, as indicated by the subscript letters.

The peak factor, PF, is defined as: The ratio of the signal voltage peak amplitude which is exceeded for only (approximately) 1/10 percent of the time-to-the rms voltage of the composite signal. For a discussion of the effects of noise-type modulating signals see pp. 11-22, inclusive. For noise-type signals, the voltage-ratio peak factor is $\sqrt{20}$ or, 13 db in terms of a power ratio. For sinusoidal signals the voltage-ratio peak factor is $\sqrt{2}$. Composite baseband modulating-signal voltage peaks determine the peak RF deviations and thereby determine the (effective or operating) maximum bandwidth of the RF-signal spectrum.

Hence, equation (C.5) assumes the following forms, depending upon the type of message-channel signal, the type of baseband signal loading, and the peak factor of the (modulating) composite baseband signal:

| Conditions | Form of Equation | |
|---|---|---|
| Voice-channel tone modulating signal, and baseband tone-signal loading | $\dfrac{(\Delta F_c)_s^2}{(\Delta F)_s^2} = \dfrac{2 \times p_{ms}}{2 \times P_{ms}}$ | |
| or, | $(\Delta F_c)_s^2 = (\Delta F)_s^2 \times \dfrac{p_{ms}}{P_{ms}}$ | (C.5b) |
| Voice-channel white-noise modulating signal and baseband white-noise signal loading | $\dfrac{(\Delta F_c)_n^2}{(\Delta F)_n^2} = \dfrac{2 \times p_{mn}}{20 \times P_{mn}}$ | |
| or, | $(\Delta F_c)_n^2 = (\Delta F)_n^2 \times \dfrac{p_{mn}}{10P_{mn}}$ | (C.5c) |
| Voice-channel tone modulating signal, and baseband white-noise signal loading. | $\dfrac{(\Delta F_c)_s^2}{(\Delta F)_n^2} = \dfrac{2 \times p_{ms}}{20 \times P_{mn}}$ | |
| or, | $(\Delta F_c)_s^2 = (\Delta F)_n^2 \times \dfrac{p_{ms}}{10P_{mn}}$ | (C.5d) |

Equations (C.5b), (C.5c), or (C.5d) are used to derive equations (C.20) to (C.22a), inclusive, see pp. 173j to 173l, inclusive.

## Baseband-Signal Loading Factors

The three "loading factors", $P_{ms}/p_{ms}$, $P_{mn}/p_{mn}$, and $P_{mn}/p_{ms}$, used in equations (C.5b), (C.5c), and (C.5d), respectively, are considered separately in the following discussion.

The loading factor $P_{ms}/p_{ms}$ applies only to the case where the baseband modulating signals are sinusoidal and are limited to only two or three separate signals, with a total power of $P_{ms}$; hence this factor does not apply and may not be used for frequency division multiplex multichannel systems. Note also that $P_{ms}$ is much greater than $p_{ms}$.

The loading factor $P_{mn}/p_{mn}$ applies to the "operating" case where both the composite baseband signal power, $P_{mn}$, and the voice-channel message-signal power, $p_{mn}$, are composed of noise-type signals. Note that the voice-channel signal may also be composed of a number of randomly-phased sinusoidal signals, such as independent teletype signals; however, in this particular case, the voice-channel signal cannot be a single sinusoidal signal. For the operating case, where the message signals are of various types, such as voice, teletype, digital data, etc., and the message-signal power levels, $p_{mni}$, are unequal, the "operating loading factor" is derived as follows:

$$P_{mn} = N_{a1}\, p_{mn1} + N_{a2}\, p_{mn2} + \cdot \cdot \cdot$$

where, $N_{ai}$ is the number of "active" channels each of which transmit a particular type of message signal and a particular message-channel signal power, $p_{mni}$, therefore,

$$\text{Operating Loading Factor} = \left[\frac{P_{mn}}{p_{mni}}\right]_{op} = \frac{N_{a1}\, p_{mn1} + N_{a2}\, p_{mn2}}{p_{mni}} \qquad \text{(C.5e)}$$

Hence, the "operating loading factor" is defined as the ratio of the composite modulating (noise type) baseband signal power, $P_{mn}$, to a particular message-signal power level, $p_{mni}$, in the baseband-signal spectrum.

The number of active channels, $N_{ai}$, associated with each particular type of message signal, depends upon the "activity factor" which in turn depends upon the characteristics of the message signal. For voice signals, the activity factor is approximately 0.25, that is, if N' channels are allocated for voice messages, then $N_{av} = 0.25\ N'$. For data or noise-type signals (in the voice channel) the activity factor may be assumed to be 1.0, and if N" channels are allocated for data messages, then $N_{ad} = N''$. This difference between the activity factor for voice signals and for data signals is the basic reason for the comparatively "heavy" loading effects of data signals. The total number of active channels is:

$$N_a = N_{a1} + N_{a2} + \cdots \qquad \text{(C.5f)}$$

In (C.5f), $N_a$ is the maximum number of simultaneously active channels, of the N available channels, for "peak message load" conditions; where the peak-message load is exceeded for less than (approximately) one percent of the time.

When all channels are noise loaded at the same power level, $p_{mn}$, the loading factor, $P_{mn}/p_{mn}$, is equal to the total number of available channels, N. The standard method of conducting "noise-power-ratio" (NPR) measurements requires that all channels, except one, be noise loaded; therefore, the "NPR loading factor" is given as:

$$\text{NPR Loading Factor} = \left[\frac{P_{mn}}{p_{mn}}\right]_{npr} = N \qquad \text{(C.5g)}$$

The loading factor $P_{mn}/p_{ms}$ applies to the case where the composite baseband signal, having an average power level of $P_{mn}$, is composed of a large number of randomly-phased sinusoidal signals and/or white-noise type signals; and the voice-channel signal, having an average power level, $p_{ms}$, is a single test-tone sinusoidal signal. The channel test-tone signal is used as a convenient means of properly adjusting the equipment, this test-tone signal is also used as a standard reference power level, and may be set approximately 10 db higher than the allowable power level of the channel noise-type signals, $p_{mn}$. Hence the "test-tone or standard loading factor", $P_{mn}/p_{ms}$, will differ from the loading factor $P_{mn}/p_{mn}$, given by (C.5e) or (C.5g), and will be inversely proportional to the power level $p_{ms}$ and directly proportional to the power level $p_{mni}$, therefore:

$$\text{Test-Tone or Standard Loading Factor} \left\{ \begin{aligned} &= \frac{P_{mn}}{p_{ms}} = \left[\frac{P_{mn}}{p_{mni}}\right]_{op} \times \frac{p_{mni}}{p_{ms}} = \left[\frac{P_{mn}}{p_{mn}}\right]_{npr} \times \frac{p_{mn}}{p_{ms}} \\ &= N_{a1}\frac{p_{mn1}}{p_{ms}} + N_{a2}\frac{p_{mn2}}{p_{ms}} + \ldots = N\frac{p_{mn}}{p_{ms}} \end{aligned} \right\} \quad \text{(C.5h)}$$

It is evident from (C.5f) and (C.5h) that if any two of the three parameters: (1) the "standard loading factor", $P_{mn}/p_{ms}$; (2) the channel message-signal "loading ratio", $p_{mni}/p_{ms}$; and (3) the number of "active" channels, $N_a$; are specified, the third parameter is determined.

The "standard loading factor", (A + B Log N), is defined by the CCIR Recommendation 294 (IXth Plenary Assembly, Los Angeles, 1959) as the average power level of the composite modulating baseband signal, $P_{mn}$, referred to the average power level of the voice-channel test-tone signal, $p_{ms}$, therefore,

$$\text{Standard Loading Factor} = 10 \log_{10} \frac{P_{mn}}{p_{ms}} \equiv (A + B \log_{10} N), \text{ db} \qquad \text{(C.5i)}$$

where, A and B have particular assigned values, depending upon the total number of available message-signal or voice channels, N, and the types of messages, for example: for N = 12 to 240 channels, A = -1db and B = 4;
for N 240 channels, A = -15db and B = 10.

Note that the CCIR definition of the Loading Factor is simply the ratio of the composite modulating (noise type) baseband signal power, $P_{mn}$, to the test-tone signal power, $p_{ms}$; both signal powers are measured at the same point in the composite baseband-signal channel.

The standard loading factor, $P_{mn}/p_{ms}$; may be specified and/or determined either in terms of the "maximum" number of simultaneously active channels, $N_a$, (not exceeded for approximately one percent of the time) and the ratio of the message-signal power, $p_{mni}$, to the test-tone signal power, $p_{ms}$, in the message-signal channels, see, (C.5f) and (C.5h); or, the Loading Factor $P_{mn}/p_{ms}$ may be specified in terms of the standard type, $A + B \text{ Log}_{10}N$, see (C.5i).

The system output signal-to-noise ratio depends upon the relative power levels of the individual message signals, as is evident from the above loading factors. However, in order to avoid signal distortion within the equipment, the absolute power levels of the individual message signals, $(p_m)_{s,n,}$ and also the composite baseband-signal power, $P_{mn}$, must be adjusted so that the voltage peaks of these signals do not overload the circuts in the equipment.

If the maximum bandwidth of the RF-signal spectrum, $B_{rf}$, is fixed, an increase in the value of the Standard Loading Factor, $A + B \text{ Log}_{10}N$, or an equivalent increase in the number of "active" channels, $N_a$, will increase the Operating Loading Factor, $P_{mn}/p_{mni}$, and cause a reduction of the output signal-to-noise ratio, $S_{oc}/N_{oc}$, and consequently, will degrade the system performance.

In summary, the above types of loading factors are used in equations (C.20) to (C.22a), (p. 173j to p. 173l) inclusive, to calculate the output message-signal and test-tone signal-to-noise ratio, and the output noise-power-ratio. Detailed instructions regarding the correct loading factor to be used with each of the above equations are given on pp. 173j to 173l.

The Loading Factor ($A + B \, Log_{10}N$) is usually in terms of the "hybrid" unit, dbm0. This unit is confusing because of the fact that the power-ratio term, $10 \, Log_{10} \, (P_{mn}/p_{ms})$, is by definition, in db units, and not in dbm units; also, the factor 0 is often interpreted to mean that the test-tone signal power level, $p_{ms}$, is 0 dbm--in general, this interpretation is not correct.

Composite Baseband-Signal Power, $P_m$,

and RF-Signal Spectrum Bandwidth, $B_{rf}$

In order to modify (C.15), p. 172, to apply to the cases where either sinusoidal or noise-type modulating signals are considered, we note that equal values of baseband modulating signal powers $P_{ms}$ and $P_{mn}$, do not generate the same effective RF spectrum bandwidths, $(B_{rf})_s$ and $(B_{rf})_n$, respectively. This fact implies that $(\Delta F)s$ is not equal to $(\Delta F)_n$, for the same (particular) value of the generated RF spectrum bandwidth, $B_{rf}$. The relationship between $(\Delta F)_s$ and $(\Delta F)_n$, for particular values of $B_{rf}/f_m$, may be obtained from Figure 3-1, p. 12.

The power level of the baseband sinusoidal modulating signal, $P_{ms}$, which is required to generate a particular RF signal spectrum having a bandwidth $B_{rf}$, depends upon the characteristics of the Modulator and the pre-emphasis circuit; the same "effective" RF signal bandwidth would be generated by a noise-type modulating signal having a power level $P_{mn}$, which is somewhat lower than the power level $P_{ms}$. The ratio, $P_{mn}/P_{ms}$, of the noise-type baseband (modulating) signal power, $P_{mn}$, to the sinusoidal baseband (modulating) signal power, $P_{ms}$, each of which will generate the same effective RF signal spectrum, $B_{rf}$, may be obtained from Figure 3-4, p. 20, NBS Technical Note 100.

System-Performance Design Equations

Equation (C.15) applies only to the case of sinusoidal modulating signals. The following work covers the cases where noise-type modulating signals and white-noise signal loading techniques are used. The formats of the following equations, derived from (C.15) are of the following two types:

(1) The antenna noise-power temperature is assumed to be approximately equal to the "standard" temperature, $T_o$; under this condition, the radio receiver "noise figure", $F_r$, may be used instead of the term "$T_A + T_{er}$"; however, this form of equation would not be accurate for low-noise antenna systems. Also, this (type of) equation differs in format from (C.15)

(2) The second type of equation, also derived from (C.15), has the same form as (C.15), includes the term "$T_A + T_{er}$", is accurate for the case of low-noise antenna systems, and can be used directly with the design curves in this report.

To obtain a general form of the system-performance design equation, we substitute (C.5a), (C.6d), (C.10), and (C.14) in (C.4) and obtain:

$$\frac{(S_{oc})_{s,n}}{(N_{oc})_{s,n}} \times \frac{B_c}{f_m} \times \frac{(PF_b)_{s,n}(P_m)_{s,n}}{2(p_m)_{s,n}} = \left(\frac{(\Delta F)_{s,n}}{f_c}\right)^2 \times \frac{P_r}{2K\,(T_A + T_{er})\, f_m} \qquad (C.15a)$$

The receiver-input effective noise-power temperature, $T_{er}$, is:

$$T_{er} = (F_r - 1)\, T_o \qquad (C.16)$$

where, $F_r$ = the receiver noise figure, as a ratio

and, $T_o$ = "standard" temperature in degrees Kelvin, (≈ 290 deg. K)

Under conditions where $T_A \approx T_o$ = 290 deg K, we have:

$$T_A + T_{er} = T_A + T_oF_r - T_o = T_oF_r \qquad (C.17)$$

Also, $$f_c = f_m \qquad (C.18)$$

and, $$B_{rf} = B_{if} \qquad (C.19)$$

Substituting (C.5b), (C.17), (C.18), and (C.19) in (C.15a), and rearranging terms, we obtain:

$$\frac{(S_{oc})_s}{(N_{oc})_s} = \frac{P_r}{kT_oB_{if}F_r} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_s}{f_m}\right)^2 \times \frac{P_{ms}}{2P_{ms}}$$

or,

$$= \frac{P_r}{k\,(T_A + T_{er})\, B_{if}} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_s}{f_m}\right)^2 \times \frac{P_{ms}}{2P_{ms}} \qquad (C.20)$$

Equation (C.20) may also be written with the same terms and in the same form as (C.15a), and is,

$$\frac{(S_{oc})_s}{(N_{oc})_s} \times \frac{B_c}{f_m} \times \frac{2P_{ms}}{p_{ms}} = \left(\frac{(\Delta F)_s}{f_m}\right)^2 \times \frac{P_r}{k\,(T_A + T_{er})\,f_m} \qquad \text{(C.20a)}$$

Note that (C.20) and (C.20a) apply to the case where both the baseband modulating signals and the channel modulating signals are sinusoidal and are limited to not more than two or three separate signals; also, $p_{ms} << P_{ms}$. Hence this form of equation may not be used for frequency division multiplex systems, employing composite noise-type baseband signals.

Proper values for $(\Delta F)_s/f_m$ in (C.20) and (C.20a) may be obtained from Figure 3-1, p. 12, for particular given or assumed values of $B_{if}/f_m$. The ratio, $p_{ms}/P_{ms}$, may be calculated from measured or assumed values of $p_{ms}$ and $P_{ms}$.

If both the composite (baseband) modulating signal and the message-channel signal are noise-type signals, we have, using (C.5c):

$$\left.\begin{aligned} \frac{(S_{oc})_n}{(N_{oc})_n} &= \frac{P_r}{kT_o B_{if} F_r} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{p_{mn}}{20P_{mn}} \\ \text{or,} \quad &= \frac{P_r}{k\,(T_A + T_{er})\,B_{if}} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{p_{mn}}{20P_{mn}} \end{aligned}\right\} \text{(C.21)}$$

Equation (C.21) may also be written in the same form as (C.15a), and is:

$$\frac{(S_{oc})_n}{(N_{oc})_n} \times \frac{B_c}{f_m} \times \frac{20P_{mn}}{p_{mn}} = \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{P_r}{k\,(T_A + T_{er})\,f_m} \qquad \text{(C.21a)}$$

The ratio, $p_{mn}/P_{mn}$, may be calculated from (C.5e), p. 173e, or (C.5h), p. 173f. These equations are used to calculate the operating loading factor $(P_{mn}/p_{mn})_{op}$, which is then substituted in (C.21) or (C.21a) to calculate the baseband output message signal-to-noise ratio, $(S_{oc})_n/(N_{oc})_n$. Equations (C.21) and (C.21a) yield the output baseband-signal "Noise-Power Ratio," NPR, provided that the ratio $p_{mn}/P_{mn}$ is obtained from (C.5g), or is made equal to 1/N. The NPR in the output baseband signal is defined and may be measured by the same methods used for the output voice-channel measurements; except that the latter measurements include the noise contributed by the (receive) multiplex equipment.

Again, the factor $(\Delta F)_n/f_m$, in (C.21), and (C.21a), may be obtained from Figure 3-1, p. 12, for given or assumed values for $B_{if}/f_m$.

If the composite (baseband) modulating signal is a white-noise type signal, and the message-channel signal is sinusoidal, we have, using (C.5d):

$$\left.\begin{aligned}\frac{(S_{oc})_s}{(N_{oc})_n} &= \frac{P_r}{kT_oB_{if}F_r} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{p_{ms}}{20P_{mn}} \\ \text{or,} \quad &= \frac{P_r}{k\,(T_A + T_{er})\,B_{if}} \times \frac{B_{if}}{B_c} \times \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{p_{ms}}{20P_{mn}}\end{aligned}\right\} \quad (C.22)$$

Equation (C.22) may also be written in the same form as (C.15a), and is:

$$\frac{(S_{oc})_s}{(N_{oc})_n} \times \frac{B_c}{f_m} \times \frac{20P_{mn}}{p_{ms}} = \left(\frac{(\Delta F)_n}{f_m}\right)^2 \times \frac{P_r}{k\,(T_A + T_{er})\,f_m} \qquad (C.22a)$$

Proper values for $(\Delta F)_n/f_m$, in (C.22) and (C.22a), may be obtained from Figure 3-1, p. 12, for particular given or assumed values of $B_{if}/f_m$. The ratio $p_{ms}/P_{mn}$ may be calculated from (C.5h), p. 173f, or (C.5i),, p. 173g.

It is very important to note that equation (4.19), (p. 69, NBS Technical Note 100) differs from the above equations (C.21a) and (C.22a), by a factor of 10; due to the difference between the peak factor of 3 db for a sinusoidal (modulating) signal, and the peak factor of 13 db for a noise type (modulating) signal. The net result, when using the system-design curves of NBS Technical Note 100, is that the factor, $P_r/f_m\ (T_A + T_{er})$, must be <u>increased</u> by 10 db (in absolute value) above the value which is obtained from these design curve charts.

Note that in the above derivations, no allowance was made for pre-emphasis of the modulating signal--proper allowance for the effect of this technique can be made by using a multiplying factor; however, the value of this pre-emphasis factor depends upon the baseband-signal frequency, being a maximum for the highest frequency, $f_m$, in the baseband signal. The primary purpose of pre-emphasis (plus de-emphasis) is to equalize the $S_{oc}N_{oc}$ ratio, and the NPR across the output baseband-signal spectrum.

Also, the message-channel output noise power, $N_{oc}$, is assumed to be flat weighted, "F1A" weighting reduces the noise power, and increases the output test-tone signal-power to noise-power ratio, $S_{oc}/N_{oc}$, by 3 db. "C" weighting increases the output test-tone $S_{oc}/N_{oc}$ by 1.5 db. Noise-power-ratio measurements are not affected by weighting techniques, because the weighting affects both the signal and the noise equally.

Proper allowances are made for diversity effects on the $S_{oc}N_{oc}$ ratio, in the system-performance design equations and curves in NBS Technical Note 100.

Application of Equations (C.20) to (C.22a), Inclusive

In conclusion, the output signal-to-noise ratio, $S_{oc}/N_{oc}$, has been considered for the following three cases:

(a) Sinusoidal modulating signals only; use (C.20) or (C.20a); with the explicit understanding that $P_{ms} >> p_{ms}$, and that these equations are not to be used for FDM-FM systems employing noise-type signals in the composite baseband signal.

(b) For noise type signals in the active or operating channels, calculate $(P_{mn}/p_{mni})_{op}$ from (C.5e) or (C.5h), and substitute in (C.21) or (C.21a) to calculate the output message signal-to-noise ratio, $(S_{oc})_n/(N_{oc})_n$. For noise-type signal loading in all channels except for "slots"; use (C.21) or (C.21a), and $(P_{mn}/p_{mni})_{npr} = N$, to calculate the NPR.

(c) For test-tone signal in a voice channel and noise-type signal loading of some, or all, of the remaining channels; obtain $P_{mn}/p_{ms}$ from (C.5h) or (C.5i) and insert in (C.22) or (C.22a), to calculate the channel output test-tone $S_{oc}N_{oc}$ ratio.

The above system-performance equations apply primarily to the transfer function of the FM radio receiver, in terms of the receiver thermal noise and the power level of the receiver-input RF signal; path-intermodulation noise and intermodulation distortion effects in the RF Modulator, the frequency-division Multiplex, and other (extraneous) equipment-noise effects are not included* The system-performance design equations, (C.20) to (C.22a), can be used interchangeably with (4.19), p. 69, provided that the proper forms are used for the factors: $(P_m)_{s,n}/(p_m)_{s,n}$, $(\Delta F)_{s,n}/f_m$; and the correct peak factor, 2 or 20 is used with the term $P_{mn}$.

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*To optimize equipment performance, thermal noise may be assumed to be equal to the equipment-intermodulation distortion. Path intermodulation noise has the effect of limiting the bandwidth, $B_{rf}$, which can be transmitted over a tropospheric radio path without serious distortion effects.

## Discussion and Conclusions

A great deal of confusion would be avoided in the interpretation of telecommunication system performance by the use of the following guidelines:

(A) The "message"-channel signal power, $(p_m)_{s,n}$, and the composite baseband signal power, $P_{mn}$, should be adjusted to their proper operating levels in the Multiplex equipment; for the maximum number of simultaneously-active channels during peak load conditions, (pp. 173e, 173f). This procedure establishes the Loading Factors $P_{mn}/p_{mn}$ and $P_{mn}/p_{ms}$.

(B) The proper Loading Factors $P_{mn}/p_{mn}$ and $P_{mn}/p_{ms}$ should be used in the system-performance design equations, (C.20) to (C.22a), inclusive, and these loading factors should be specified and measured at the input terminals of the RF Modulator.

(C) The absolute power level of the composite baseband modulating signal, $P_{mn}$, should be adjusted, at the input terminals of the Modulator, so as to produce an RF-signal spectrum bandwidth, $B_{rf}$, equal to the effective IF-signal bandwidth, $B_{if}$, of the radio receiver. Note that this required modulating-signal power level, $P_{mn}$, is independent of the "normal" operating power level of the composite baseband signal of the Multiplex equipment, and hence either an attenuator or an amplifier is required to adjust the modulating-signal power at the input terminals of the Modulator. The "optimum" value of $P_{mn}$ may be obtained from NPR measurements, made with the (entire) equipment arranged in a back-to-back configuration, including operating values of pre-emphasis and de-emphasis.

(D) The system-performance criterion should be the ratio:

$$\frac{\text{message-channel output average message-signal power}}{\text{message-channel output average noise power}} \; ; \; \frac{S_{oc}}{N_{oc}}$$

this ratio should be specified and measured at the system output terminals. More specifically, the time-varying statistical characteristics of this output signal-to-noise ratio is an accurate measure of the system-performance, and may be combined with the performance characteristics of the message-signal decoding units, to obtain an estimate of the message error rate.

The output message-channel average noise power level cannot be used as a system-performance criterion, without a corresponding measurement of the message-signal or the test-tone signal average power. In other words, system performance can not be specified or measured simply in terms of an absolute output noise-power level; the system must be specified and measured in terms of the output signal-to-noise ratio.

Equations (C.20) to (C.22a), inclusive, were derived for multichannel FDM-FM systems, and apply to the message-channel output signal-to-noise ratio in the output baseband-signal spectrum; however, proper allowances must be made for the "acceptance bandwidth" of the instrument used to measure the message-signal power and the noise-signal power, in the baseband signal spectrum; this bandwidth must be somewhat less than the message-signal spectrum bandwidth. These equations may also be used to calculate and to define the measurement of the signal-to-noise ratio in the output message channel, provided that proper allowance is made for the degradation of the signal-to-noise ratio due to the multiplexing equipment.

The following list of factors and terms, used in this paper, is provided for convenient reference:

$S_{oc}$ = Average message-signal power level, per message channel, at the radio receiver output, in watts.

$N_{oc}$ = Radio receiver output average noise power level, per message-signal spectrum bandwidth, in watts.

$B_c$ = Message-signal spectrum bandwidth, in Hz.

$f_m$ = Maximum or highest frequency in the baseband modulating signal.

$(p_m)_{s,n}$ = (See p. 173c)

$(P_m)_{s,n}$ = (See p. 173c)

$(\Delta F)_{s,n}$ = (See p. 173b)

$(\Delta F_c)_{s,n}$ = (See p. 173b)

$P_r$ = Total available carrier-signal power at the input terminals of the FM radio receiver, in watts.

$k$ = Boltzman's constant = $1.3804 \times 10^{-23}$ Joules per degree Kelvin

$T_A$ = Receiving antenna system output noise temperature, degrees Kelvin, see Appendix A, NBS TN 100.

$T_{er}$ = Radio receiver effective input noise temperature (IRE Comm. on Noise, 1960), degrees Kelvin, see (C.16), p. 173j.

$T_o$ = Standard temperature (290 degrees, Kelvin)

$\frac{\Delta F}{f_m}$ = Deviation ratio, or (frequency) modulation index.

$(PF_b)_{s,n}$ = (See p. 173c).

$kT_o$ = $4.00316 \times 10^{-21}$ Joules = -204 dbw x sec = -174 dbm x sec

$F_r$ = Radio receiver noise figure, see p. 173j.

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