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Semiconductor Measurement Technology:

A Collection of Computer Programs for Two-Probe Resistance (Spreading Resistance) and Four-Probe Resistance Calculations, RESPAC

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John Albers and Harry L. Berkowitz

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Abstract

This report presents and describes a number of FORTRAN programs which may be used to perform two-probe resistance (spreading resistance) and four-probe resistance calculations for vertically nonuniform resistivity structures. These programs fall into three general categories. They are: 1) programs for calculating the two-probe resistance (spreading resistance) from the resistivity profile, 2) programs for calculating the resistivity profile from the two-probe resistance (the inverse of 1), and 3) programs for calculating the fourprobe resistance from the resistivity profile. Programs in the first and third category are useful for understanding the effects of resistivity variations on the two-probe resistance (spreading resistance) and the four-probe resistance. Programs in the second category are useful for extracting the resistivity profile from spreading resistance data (either measured or calculated).

All of the programs are based upon the Schumann and Gardner solution of the multilayer Laplace equation. As such, local charge neutrality is assumed. The limitations of this assumption are described in the text.

The first part of this report consists of an outline of the derivation of the Schumann and Gardner multilayer Laplace equations. In addition, there is a discussion of the evolution and simplification of this model which has taken place over the past 2 decades. This part of the report is intended to provide the reader with a background so as to make optimal use of the computer programs. The second part of the report contains a discussion of the structure and inner workings of each of the programs. Special attention is paid to the aspects which make the individual programs different from others in the same category. In addition, sample input data used in the programs and the corresponding output data calculated by the programs are also presented. The final parts of the report (the appendices) contain annotated, internally documented listings of the FORTRAN source codes.

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In all, there are ten programs contained in the RESPAC package. The FORTRAN source code (total of about 123 kbytes) and sample input and output data files are available in ASCII format using a number of transfer vehicles. These include: standard 8-track magnetic tape (ASCII, density = 1600, record = 80, block = 1600), 5.25-in. (360-kbyte and 1.2-Mbyte) DOS-formatted floppy disks, and using electronic mail over the Internet. This package is self-contained and is straightforward to run once the FORTRAN is compiled and linked by the user-supplied software. The sample input and output data files are included so that the user can check the programs for proper operation as well as to become acquainted with the setup and use of the codes.

Key words: correction factor; FORTRAN; four-point resistance; Laplace's equation; multilayer analysis; Poisson's equation; probe radius; probe separation; profiles; resistivity; semiconductor devices; semiconductor materials; sheet resistance; spreading resistance; two-point resistance.

INTRODUCTION

The last 2 decades have witnessed the growth and development of a number of atomic and electrical techniques which have been used in the profiling of semiconductor device structures. These developments have tried to keep cadence with the trend in semiconductor technology towards submicrometer devices. The push made by VLSI technology into the submicrometer regime has placed increased demands upon the ability to adequately measure and interpret dopant profiles. The acquisition and analysis of one-dimensional (vertical) profiles form the basis for present research and commercial development. The two-dimensional extension of these techniques has proven to be a formidable problem.

The Secondary Ion Mass Spectrometry (SIMS) technique [1] has become a forefront tool for micrometer and submicrometer chemical analysis, in general, and for the depth-dependent atomic profiling of semiconductor structures, in particular. Other atomic techniques used for submicrometer profiling include the Rutherford Back Scattering (RBS) [2] and Neutron Depth Profiling (NDP) [3]. All of these techniques provide for the indirect measure of the atomic profile. The primary importance of dopants in semiconductors is through the enhancement or diminishing of the electrical conduction. Consequently, a complete picture of the processing of a semiconductor structure requires that the atomic profile be supplemented with the corresponding electrical profile.

In the area of electrical profiling, two-probe spreading resistance [4] and capacitance-voltage [5] are probably the most widely used techniques. Capacitance-voltage is limited by a total carrier concentration above which avalanche breakdown takes place. In addition, four-probe resistance is a useful tool. However, it is limited by the fact that vertical profiling is cumbersome and difficult with this method. Hall measurements [6] also provide useful information, but are also hindered as a vertical profiling technique.

The intelligent use of the spreading resistance technique for micrometer and submicrometer structures requires an understanding of the models which have been developed over the past two decades for the interpretation of the data. This is important as the technique does not provide the carrier profile directly. Rather, the resistivity profile is indirectly obtained from the analysis of the depth-dependent spreading resistance data. This requires a physical model of the measurement in order to connect the data and the underlying resistivity structure. The conversion of the resistivity profile to a carrier density profile takes place through a resistivity-carrier density relation. In addition, the carrier density profile which is obtained from the resistivity profile need not be in one-to-one correspondence with the atomic density. Consequently, it is important to understand the basic model of spreading resistance insofar as its assumptions and possible limitations are concerned. This is especially the case as a number of analysis schemes or algorithms have appeared over the past decade. While they may appear to be quite different, many of them are related to the same basic model. Their differences are mostly due to ways of handling probe current densities, calibration data, and the evaluation of integrals which appear in the relation between the spreading resistance and the resistivity. The purpose of this report is to discuss a number of these algorithms and to present FORTRAN programs based upon them. The intent is to provide a common basis for the interpretation of spreading resistance data. This is motivated in part by the experience of the first author in providing "model" spreading resistance data for an ASTM F.1 effort to compare spreading resistance profiles. The public availability of the RESPAC codes should lead to a common ground for the discussion and interpretation of spreading resistance data.

This report is divided into three parts. The first contains a history of the multilayer Laplace equation model which relates the two- and four-probe resistance to the underlying resistivity structure. This history is not exhaustive but is intended to provide the reader with an indication of the changes and improvements which have taken place over the years since the original calculations of Schumann and Gardner. In addition, the incorporation of several forms of the probe current density, the possible determination of the probe radius and its relation to calibration data, and the effects of the assumption of finite thickness, uniform resistivity "layers" are also considered. The second part of this report contains discussions of the FORTRAN codes developed from the various models discussed in the first part. These codes are contained in the appendices which make up the third portion of this report.

Certain commercial equipment, instruments, or materials are identified in this report in order to specify the procedure adequately. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose. In spite of the authors' experiences that the programs perform correctly on every set of data which has been tried, there can be no assurance that the program will perform equally well on all (possibly anomalous) data. Therefore, both the authors and NIST assume no liability for possible losses resulting from the use of these programs.

HISTORY OF THE SCHUMANN-GARDNER MULTILAYER LAPLACE EQUATION

The primary purpose of the theory of two- and four-probe resistance is to provide a simple

and yet complete model which relates the resistance to the details of the resistivity structure. As is seen in this section, the present theory provides for a well-defined procedure for obtaining the spreading resistance from the resistivity. The reverse or inverse process of obtaining the resistivity from the spreading resistance requires the solution of what is known mathematically as the inverse problem. Simply stated, it is necessary to determine part of an integrand of an integral from the numerical value of the integral. In general, there is no mathematical method for uniquely determining the solution to this problem. The solution to this problem in the case of spreading resistance analysis has tested the ingenuity of numerical analysis.

Over the past two decades, there have been a number of advances which have taken place in the theoretical analysis of two- and four-probe resistance. The starting point of this history is the multilayer Laplace equation analysis of Schumann and Gardner [7,8]. The reader is referred to the book by Koefoed [9] for a comprehensive discussion of Laplace's equation, its solution in cylindrical coordinates, and the multilayer approach applied to geoelectric resistance measurements. The derivation of the recursion relation (discussed later in this report) is also presented in considerable detail by Koefoed. It should be noted that the mathematical description of spreading resistance and geoelectric resistance are exactly the same even though the length scales are vastly different.

The multilayer solution of the Laplace equation provides for the calculation of the resistance on the top surface of a nonuniform resistivity structure. The structure is viewed as being planar and made up of a number of "layers," each of uniform resistivity. For the case of two-probe measurements made on a beveled sample, the results of the planar calculations are used under the assumption that the bevel does not alter the results. As the probes move down the bevel, the definition of the surface changes to that which is presently in contact with the probes. The material which is uphill, so to speak, is neglected. The fundamental assumption of the multilayer analysis is that Laplace's equation is satisfied in each of the "layers" in the material.

There are a number of assumptions which are clearly spelled out and discussed in [7]. It is important to emphasize that Schumann and Gardner were very careful to point out what they thought to be limitations of the model.

First, consider the case of a single uniform layer. The solution to this problem provides a solution which is assumed to describe the potential in each of the "layers." The problem is set up in cylindrical coordinates to emulate the symmetry of a single circular contact on the top surface of the material. The potential is assumed to be independent of the angular variable in this coordinate system. The Laplace equation may then be written as

$$\nabla^2 V(r,z) = \frac{\partial^2}{\partial r^2} V(r,z) + \frac{1}{r} \frac{\partial}{\partial r} V(r,z) + \frac{\partial^2}{\partial z^2} V(r,z) = 0,$$
(1)

where V(r, z) is the potential, r is the radial coordinate, and z is the depth coordinate. This equation may be solved by means of separation of variables, with the result that a particular solution is

$$V(r,z) = \exp(-\lambda z) J_0(\lambda r) + \exp(+\lambda z) J_0(\lambda r), \qquad (2)$$

where $J_0(\lambda r)$ is the Bessel function and λ^2 is the separation of variables constant. The boundary condition on the r part of the solution is that V(r, z) approaches zero as r tends to infinity. This is satisfied by the above for all values of λ . Then, the general solution is an integral of the particular solution with a weighting factor and is of the form

$$V(r,z) = \int_0^\infty \left\{ \left(1 + \theta(\lambda) \right) \exp(-\lambda z) + \psi(\lambda) \exp(+\lambda z) \right\} J_0(\lambda r) d\lambda, \tag{3}$$

where the weighting functions, $\theta(\lambda)$ and $\psi(\lambda)$, are determined from the z-dependent boundary conditions. The above is the general solution of Laplace's equation in cylindrical coordinates for a single layer.

For the case of an *n*-layer structure (pictured in fig. 1), Laplace's equation is assumed to be valid in each of the layers. The single-layer solution as given by eq (3) then provides the basis for the solution in each of the layers. The solution in the *i*-th layer may then be written as

$$V_{i}(r,z) = \int_{0}^{\infty} \left\{ \left(1 + \theta_{i}(\lambda) \right) \exp(-\lambda z) + \psi_{i}(\lambda) \exp(+\lambda z) \right\} J_{0}(\lambda r) d\lambda.$$
(4)

The boundary conditions used to solve the system of equations (determine $\{\theta_i(\lambda)\}\$ and $\{\psi_i(\lambda)\}\$, i = 1, ..., n) are provided by conditions on the top surface, the intermediate interfaces, and the bottom surface. On the top surface, current flow takes place only through the probe which is modeled as a circular plate of radius a. Then the top surface boundary condition is expressed as

$$-\frac{1}{\rho_n}\frac{\partial V_n(r,z)}{\partial z} = J(r),\tag{5}$$

where J(r) is the current density. The original Schumann and Gardner calculation makes use of the current density for the case of a semi-infinite slab in the form

$$J(r) = \begin{cases} I/2\pi a (a^2 - r^2)^{1/2} & \text{if } r \leq a \text{ and } z = 0\\ 0 & \text{if } r > a \text{ and } z = 0 \end{cases}.$$
 (6)

Several other forms of the current density can be used. In particular, the uniform current density in the form

$$J(r) = \begin{cases} I/\pi a^2 & \text{if } r \leq a \text{ and } z = 0\\ 0 & \text{if } r > a \text{ and } z = 0 \end{cases},$$
(7)

would describe the current flow for the case of a layer over a highly conductive layer. On the other hand, the ring delta function form of the current density,

$$J(r) = (I/2\pi r)\delta(r-a), \tag{8}$$



Figure 1. Schematic representation of the multilayer geometry used in the Schumann and Gardner multilayer analysis. In this figure, r and z are the radial and depth coordinate in the cylindrical coordinate system. d is the thickness of the layers and ρ_i and V_i are the resistivity and potential in the *i*-th layer.

where $\delta(r-a)$ is the Dirac delta function, would describe the current flow for a conducting layer over a highly resistive layer. These three forms of the probe current density are pictured in figure 2.

Berkowitz and Lux [10] have discussed how the above forms of the probe current density enter into the final correction factor equations. It should be noted that the above current densities have different patterns but all have the same total current, i.e.,

$$\int_0^{2\pi} \int_0^\infty J(r) d\theta r dr = I.$$
(9)

It might be argued that the "real" current density would be some linear combination of those given by eqs (6 through 8). This is the basis of the variational calculation which is discussed later in this section. For the present time, the probe current density given by eq (6) is used, and the changes introduced by the use of other current densities are presented.

Returning to the boundary conditions, on the bottom surface, the potential is assumed to be well behaved and, more specifically, is assumed to approach zero, i.e.,

$$\lim_{z \to \infty} V_1(r, z) = 0. \tag{10}$$

At the interfaces between the layers, the potentials and the current densities are assumed to be continuous. These are expressed as

$$V_i(r,z) = V_{i-1}(r,z),$$
(11)

and

$$\frac{1}{\rho_i}\frac{\partial V_i(r,z)}{\partial z} = \frac{1}{\rho_{i-1}}\frac{\partial V_{i-1}(r,z)}{\partial z},\tag{12}$$

where the functions and their derivatives are to be evaluated at the interfacial boundaries.

For the case of an *n*-layer structure, the substitution of eq (4) into the boundary conditions given by eqs (5), (10), (11), and (12) gives rise to a set of 2*n* equations in 2*n* unknowns $(\{\theta_i(\lambda)\}, \{\psi_i(\lambda)\}, i = 1, ..., n)$. This simplifies by one equation as $\theta_n(\lambda) = \psi_n(\lambda)$. The analytic solution of this system of equations requires the use of the Cramer's rule of linear algebra. Clearly, this can become rather tedious especially since the expansion coefficients are functions of the continuous variable, λ . Schumann and Gardner worked out the system of equations for the cases up to i = 3. However, it is possible to show that the potential on the top surface of the *n*-layer structure may be written as

$$V_n(r,0) = \frac{I\rho_n}{2\pi a} \int_0^\infty \frac{A_n(\lambda)\sin(\lambda a)J_0(\lambda r)}{\lambda} d\lambda,$$
(13)

where the kernel function, $A_n(\lambda) = 1 + 2\theta_n(\lambda)$, depends upon the resistivities and thicknesses of the "layers" in the multilayer structure (through the solution of the above system



Figure 2. Graphical representation of the three forms of the probe current density. The Schumann and Gardner density is represented in (a), the uniform density in (b) and the ring delta function density in (c).

of simultaneous equations). It is this potential which is of prime importance as the measurement is made on the top surface of the material. Equation (13) represents the potential at a distance r from a single probe. The spreading resistance for a two-probe configuration is defined as the voltage difference between the probes divided by the current, $\Delta V/I$. In order to obtain an expression for the spreading resistance, two things must be done. First, the potential must be averaged over the area of the probe since the probe is not sensitive to the details of the potential, but rather the average potential. Second, the voltage difference between the two probes is calculated assuming that the second probe does not perturb the potential due to the first probe. This is known as superposition and leads to the result that the two-probe spreading resistance on the top surface of an *n*-layer structure may be written as

$$R_n = \frac{\Delta V}{I} = \frac{\rho_n}{2a} C_n,\tag{14}$$

where C_n is the correction factor given by

$$C_n = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda}, \tag{15}$$

where S is the separation between the probes. The Bessel function term, $J_1(\lambda a)$, arises from an averaging of the potential over the area of the probe.

There are three distinct terms in the integrand of the correction factor integral, with each representing different parts of the problem. First, there is the kernel function, $A_n(\lambda)$, which depends upon the resistivities and thicknesses of the layers in the multilayer structure. The second part of the integral, $(J_1(\lambda a)/\lambda a - J_0(\lambda S)/2)$ relates to the two-probe configuration with the probes separated by a distance S. Finally, the term $\sin(\lambda a)/\lambda$ arises from the probe current density (from eq (6)) passing through the probes. In terms of the uniform current density (eq (7)) and the ring delta function current density (eq (8)), the corresponding correction factors would be given by

$$C_n^{un} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} 2J_1(\lambda a) \frac{d\lambda}{\lambda}, \tag{16}$$

and

$$C_n^{rd} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} a J_0(\lambda a) d\lambda, \tag{17}$$

respectively. These forms of the correction factor integral have been discussed by Berkowitz and Lux [10].

Within the confines of the assumptions presented by Schumann and Gardner, eqs (14) and (15) represent a relation between the spreading resistance and the resistivity. This relation is not a simple one as the determination of the resistivity from the spreading resistance, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, requires the inversion of the correction factor integral. For the sake of simplicity and brevity, the notation $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ is used to indicate the calculation of the *n* spreading resistance values from the *n* values of the resistivity,

while the notation $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ is used to indicate the inverse process, i.e., the determination of the *n* values of the resistivity from the *n* spreading resistance values. The former requires single evaluations of the correction factor integral, whereas the latter require inversions of the correction factor integral. There were, in addition, two difficulties associated with the implementation of these equations. First, the determination of the kernel function for a large number of layers required the use of a mainframe computer [11] to carry out the numerical matrix algebra (determination of the set of functions $\{\theta_i(\lambda)\}$, $\{\psi_i(\lambda)\}, i = 1, ..., n$ by numerical evaluation of Cramer's rule). Power-law interpolation [12] techniques were of limited utility due to the large requirement of CPU (still on a mainframe). Also, the partitioning of the (2n by 2n) matrix problem into a smaller set of (2 by 2) matrices had been presented but has not been carried beyond this point [13]. The second problem in the implementation of the correction factor equation has to do with the evaluation of the integral. The last two parts of the integral described above are oscillatory, and a direct evaluation of the integral requires the use of at least a minicomputer.

The first breakthrough in the evaluation of eq (15) effectively removed the numerical difficulty associated with matrix inversion. This was hinted at in the geophysical literature [14] and worked out in detail using the theory of determinants in Koefoed's book [9]. The application of these techniques to the spreading resistance problem culminated with the introduction of a recursion relation by Choo *et al.* [15]. The utility of a recursion relation is that the kernel for an *n*-layer structure can be easily generated from the kernel of an (n-1) layer structure by means of an algebraic relation. In particular, if the kernel is known for the (n-1) layer case, then the kernel for the *n* layer case is given by

$$A_n(\lambda) = \frac{\omega(\lambda)\rho_n + \rho_{n-1}A_{n-1}(\lambda)}{\rho_n + \omega(\lambda)\rho_{n-1}A_{n-1}(\lambda)},$$
(18)

where

$$\omega(\lambda) = \frac{1 - \exp(-2\lambda d)}{1 + \exp(-2\lambda d)},\tag{19}$$

and d is the layer thickness. In practice, the recursion relation is begun with the one-layer case from which the two-layer kernel is generated. This sequence is repeated until the *n*-layer kernel is determined. This method was used in reference [15] with a great reduction in computation time.

At this particular time, the one major problem which remained was the evaluation of the integral in the correction factor equation. D'Avanzo *et al.* [16] made use of a prestored partial integral technique which usually required the use of a minicomputer in its implementation. The focal point of this technique was to use a cubic spline approximation for the kernel of the correction factor integral, i.e.,

$$A(\lambda) = \alpha_i \lambda^3 + \beta_i \lambda^2 + \gamma_i \lambda + A(\lambda_i).$$
⁽²⁰⁾

Then the correction factor integral with the Schumann and Gardner infinite slab current density is given by

$$C = \frac{4}{\pi} \sum_{i=1}^{m-1} \Big\{ \alpha_i \int_{\lambda_i}^{\lambda_{i+1}} \lambda^3 f(\lambda) d\lambda + \beta_i \int_{\lambda_i}^{\lambda_{i+1}} \lambda^2 f(\lambda) d\lambda \Big\}$$

$$+\gamma_i \int_{\lambda_i}^{\lambda_{i+1}} \lambda f(\lambda) d\lambda + \delta_i \int_{\lambda_i}^{\lambda_{i+1}} f(\lambda) d\lambda \Big\},$$
(21)

where the upper integration limit, λ_m , is chosen to ensure the convergence of the integrals, and the function, $f(\lambda)$, is the remaining part of the correction factor integral, i.e.,

$$f(\lambda) = \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \frac{\sin(\lambda a)}{\lambda}.$$
 (22)

In practice, the integrand of the correction factor goes to zero when λ is at most on the order of 100. Then, the $\{\lambda_i\}$ values are chosen as logarithmically equispaced on the interval, $0 \leq \lambda \leq 100$. The integrals in eq (21) are evaluated for specific choices of the probe radius and probe spacing by means of a program called SPINT, and the results of the partial integrals are stored in array form. These arrays need be calculated only once for each value of a and S. The evaluation of the correction factor integral and the extraction of the resistivity from the spreading resistance required the evaluation of the set of 280 (4 by 70) spline coefficients ($\{\alpha_i\}, \{\beta_i\}, \{\gamma_i\}, \{\delta_i\}$). This is carried out in the program called SPRED. Both of these programs are described and listed in reference [16]. They require the use of a minicomputer to carry out the calculation.

At about the same time, Choo *et al.* [17] introduced an integration scheme based upon the Gauss-Laguerre quadrature. They use the uniform current density form of the correction factor integral as given by eq (16). The general philosophy behind quadrature techniques of evaluating integrals is the search for a set of integration points and weights, $\{x_k\}, \{w_k\}, (k = 1, ..., N)$ such that an integral may be evaluated as a finite sum in the form

$$\int_{a}^{b} f(x)dx = \sum_{k=1}^{N} w_{k}f(x_{k}).$$
(23)

It should be clear that knowing the set of integration points and weights vastly simplifies the evaluation of the integral. This simplification may render the integration amenable to evaluation on a microcomputer system. For the case at hand, the Gauss-Laguerre quadrature makes use of the first 33 roots and weights obtained from the 128th order Laguerre polynomial. For the case of the uniform current density, the correction factor may be evaluated as the weighted finite sum,

$$C_n = \frac{8}{\pi} \sum_{k=1}^{33} w_k^{gl} \left\{ \frac{J_1(\lambda_k a)}{\lambda_k a} - \frac{J_0(\lambda_k S)}{2} \right\} \frac{J_1(\lambda_k a)}{\lambda_k} A_n(\lambda_k), \tag{24}$$

where the weights (w_k^{gl}) and integration points $(a\lambda_k)$ are listed in reference [17].

While the above is formulated for the case of the uniform current density, the Schumann and Gardner current density as well as the ring delta function current densities may also be used. It is important to note that the Gauss-Laguerre technique can be used to carry out the calculation $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$. The inverse calculation, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, requires a convergence or bounding criterion. From a completely different point of view, Dickey [18] presented a technique known as the local slope method for the calculation $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$. This method does not require the evaluation of the correction factor integral but rather makes use of a heuristically derived algebraic relation between the correction factor and the local depth derivative of the spreading resistance.

Starting with the trapezoidal Rhomberg integration technique employed by D'Avanzo et al. [16], Albers [19] generated model spreading resistance data, $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$, for a number of resistivity structures. In these calculations, the Schumann and Gardner current density was used with given values of the probe radius and the probe spacing. This technique may be used to carry out this calculation for all three forms of the probe current density. These model spreading resistance data were then used in the D'Avanzo et al. SPINT-SPRED codes as well as the Dickey local slope method to determine the $(\{Ri\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ results for each algorithm. It was found that the SPINT-SPRED codes solved the inversion problem to within 1 percent. The local slope method gave very reasonable semiquantitative results. These are discussed in detail in reference [19]. While it has been shown that the local slope equations cannot be derived from the multilayer equations [20], the speed and quality of the results make it a reasonable program to use.

A typical resistivity structure and the model spreading resistance data generated according to the work of reference [19] are contained in figure 3. There are several things to note from the figure. First, the spreading resistance does arise from a simple scaling of the resistivity. The mapping from the resistivity to the spreading resistance is complex. For example, from the surface down to about 0.2μ m, the resistivity undergoes about a two-order-of-magnitude decrease, whereas the spreading resistance shows only a small decrease. This arises from the fact that the spreading resistance senses the entire resistivity structure. The second point of interest is that the total variation of the resistivity is often large compared with the corresponding variation of the spreading resistance. In practice, small changes in the spreading resistance mirror large changes in the resistivity. This makes the interpretation of spreading resistance particularly sensitive to the presence of noise in the data.

Berkowitz and Lux [21] have carried out an investigation which has shown that it is possible to evaluate the spreading resistance correction factor using a 22-point scheme. The central point of this analysis is the replacement of the correction factor integral by an approximate form as

$$C_{n} = \frac{8}{\pi} \int_{0}^{\infty} A_{n}(\lambda) \left\{ \frac{J_{1}(\lambda a)}{\lambda a} - \frac{J_{0}(\lambda S)}{2} \right\} J_{1}(\lambda a) \frac{d\lambda}{\lambda}$$
$$= \frac{8}{\pi} \int_{L}^{\infty} A_{n}(\lambda) \left(\frac{J_{1}(\lambda a)}{\lambda a} \right)^{2} d\lambda,$$
(25)

where L = 1.12292/S. The choice of the truncated lower limit of integration is dictated by the form of the functions for the case of both insulating and conducting boundaries. The key point is that the approximate integral can be divided into four domains in each of which the Newton-Cotes integration method may be used. The evaluation of the approximate



Figure 3. The resistivity profile and the calculated spreading resistance for a Gaussian implant type structure. This figure is intended to illustrate two important features. First, the variation of the resistivity is always larger than that of the spreading resistance. Second, the mapping from resistivity to spreading resistance is more than just a scaling.

integral in eq (25) is then performed by means of a finite sum as

$$C_n = \sum_{k=1}^{22} w_k^{bl} A_n(\lambda_k), \qquad (26)$$

where the weights, w_k^{bl} , are different from those used in the Gauss-Laguerre technique. Comparison of the correction factor calculated by means of eq (26) and the more extensive technique discussed by Albers [19] shows that the two agree to within better than 1 percent. Equation (26) provides a quick way of carrying out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. Berkowitz and Lux have used the above Newton-Cotes technique to carry out the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation. This makes use of a bounding procedure on the kernel of the integral. This bounding procedure reduces the time required to accurately invert the correction factor integral and is based upon the realization that the recursion relation for the kernel function could be expressed as

$$A_{n}(\lambda) = \frac{\tanh(\lambda d)\rho_{n} + \rho_{n-1}A_{n-1}(\lambda)}{\rho_{n} + \rho_{n-1}A_{n-1}(\lambda)\tanh(\lambda d)}.$$
(27)

For the cases of insulating or conducting boundaries, the kernel function is bounded by the hyperbolic functions, coth and tanh, respectively. This provides for two bounding envelopes between which the kernel must lie. Calculation of $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ using eq (26) followed by the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation using eq (26) in conjunction with the bounding procedure shows that the original resistivities can be obtained to within less than 1 percent. This bounding procedure is extremely general. It may be used in conjunction with the Gauss-Laguerre technique for the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation.

All of the above techniques make use of a single form of the probe current density. Choo et al. [22] have presented a calculation of the spreading resistance correction factor based on a variational technique. This makes use of the uniform current density and the Schumann and Gardner semi-infinite slab current densities as a "basis set," and the amount of each current is varied according to a variational principle. The spreading resistance obtained from this variational method is

$$R_n^v = \frac{\rho_n}{2} C_n^v, \tag{28}$$

where

$$C_n^v = \frac{4}{\pi} \frac{K_1 I_{3s} + K_2 I_{2s}}{K_1 + K_2/2},$$
(29)

where K_1 and K_2 are given by

$$K_1 = \frac{I_2 - I_3/2}{I_1 I_2 - I_3^2},\tag{30}$$

$$K_2 = \frac{I_1/2 - I_3}{I_1 I_2 - I_3^2}.$$
(31)

The integrals I_1, I_2, I_3 are

$$I_1 = \int_0^\infty A_n(\lambda) \left\{ \frac{\sin(\lambda a)}{\lambda a} \right\}^2 a d\lambda,$$
(32)

$$I_2 = \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} \right\}^2 a d\lambda, \qquad (33)$$

$$I_{3} = \int_{0}^{\infty} A_{n}(\lambda) \left\{ \frac{\sin(\lambda a) J_{1}(\lambda a)}{(\lambda a)^{2}} \right\} a d\lambda.$$
(34)

The integrals I_{3S} and I_{2S} are

$$I_{3S} = \int_0^\infty A_n(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a}\right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda,\tag{35}$$

and

$$I_{2S} = \int_0^\infty A_n(\lambda) \left(\frac{J_1(\lambda a)}{\lambda a}\right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda.$$
(36)

Choo et al. [23] have made use of the Berkowitz-Lux approximation [21] to simplify the integrals in eqs (35) and (36) as

$$I_{3S} = \int_{L}^{\infty} A_{n}(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a}\right) \left(\frac{J_{1}(\lambda a)}{\lambda a}\right) d\lambda, \qquad (37)$$

$$I_{2S} = \int_{L}^{\infty} A_{n}(\lambda) \left(\frac{J_{1}(\lambda a)}{\lambda a}\right)^{2} d\lambda.$$
(38)

It should be noted that the integrals given by eqs (32), (33), and (34) contain all of the information which is needed to evaluate eqs (28) and (29). This follows from the fact that the approximate integrals given by eqs (37) and (38) are contained in the corresponding I_3 and I_2 integrals. The various integrals $(I_1, I_2, \text{ and } I_3)$ may be evaluated by means of a segmented Newton-Cotes method similar to that used by Berkowitz and Lux. The coordinates and weights are listed in reference [23]. These equations can be implemented in order to carry out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. The inverse procedure, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, can be carried out by means of a generalization of the Berkowitz-Lux bounding technique where the bounding by the hyperbolic functions is needed for all three of the integrals involved. This analysis shows that the original resistivities can be recovered to within 1 percent.

EFFECTS OF PROBE CURRENT DENSITY

As can be seen from the above discussion, there are three primary current densities which can be used in the calculation of the spreading resistance correction factor. The Schumann and Gardner form seems appropriate to the case of a semi-infinite slab and the uniform current density should be adequate for the case of a layer over a highly conductive layer, while the ring delta function seems appropriate for the case of a layer over a highly resistive layer. The model data calculations presented by Albers [19] (using the trapezoidal Romberg integration scheme) can easily be generalized in order to use any one of the three current densities. The Gauss-Laguerre quadrature [17] may also be used with any one of the above current densities. The Berkowitz-Lux calculations [21] (originally specialized to the uniform current density but, in principle, extendable to the other two current densities) as well as the variational technique employing the Berkowitz-Lux approximate integral [23] can also be used to calculate the spreading resistance from the resistivity. For several structures, the various techniques were used to carry out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. The calculated spreading resistance data obtained from typical diffusion and implant structures show that the differences are typically on the order of 10 to 20 percent. This is in keeping with the results discussed in reference [10].

CALIBRATION DATA AND THE PROBE RADIUS

Having looked at the effects of the probe current density on the results of spreading resistance calculations, it is important to understand that there is a parameter in eqs (14) and (15) which must be measured or at least estimated. This is the probe radius, a. The results of the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation and the inverse $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation are sensitive to the choice of a. One possible way of measuring the probe radius was originally proposed by Dickey [18] as part of a conjecture relating spreading resistance and sheet resistance. In particular, an image calculation of the spreading resistance on the top of an isolated layer of thickness, t, and resistivity, ρ , yields the result

$$R = (\rho/\pi t)\ln(S/a) = (\rho/\pi t)\{\ln(S) - \ln(a)\}.$$
(39)

As the sheet resistance, \mathcal{R} , of this layer is given by $\mathcal{R} = \rho/t$, this would suggest that the spreading resistance may be linear in the logarithm of the probe spacing with a slope related to the sheet resistance and an intercept related to the probe radius. For junction-isolated boron diffusions with junction depths up to 4.3 μ m, Dickey showed that there was good agreement between the sheet resistance measured by the standard four-probe technique and determined from the R versus $\ln(S)$ analysis. This agreement provided justification for the proposed relation between the spreading resistance and the sheet resistance but did not give any indication of the validity of the relation of the intercept of the R versus $\ln(S)$ analysis with the probe radius. The values obtained by Dickey were certainly of the expected size.

In a series of investigations to study the validity of the proposed relation between the spreading resistance and the sheet resistance (as well as the possibility of determining the probe radius), Albers [24,25] used the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation for a number of resistivity structures, probe current densities, a fixed value of the probe radius, and a series of values of the probe separation. The resistivity structures could be used to calculate the sheet resistance directly, and the probe radius was a known input quantity. The model spreading resistance data were used in linear regression analysis in $\ln(S)$ according to eq (39) and the slope and intercept correlated with the sheet resistance and the probe radius. The regression analysis of the model spreading resistance was of general validity, but that the probe radius was not generally obtainable from the regression analysis. Hence, some other technique seemed to be required to determine this quantity.

One possibility was to adopt the point of view that the probe radius is a function of the resistivity. This is hinted at by the form of the calibration data obtained on various resistivity bulk samples. This point of view was incorporated into the Dickey local slope analysis of boron implant data used in the comparison of spreading resistance, SIMS, and CV profiles by Ehrstein *et al.* [26]. The local slope analysis is ideally suited to this as the updating of the probe radius takes place in the algebraic equation relating the correction factor and the local slope. All of the other algorithms and integration schemes would be hampered by the updating of the probe radius which would necessitate the reevaluation of the sampling points and the integration weights.

Piessens et al. [27] have discussed a technique based upon Chebyshev polynomial methods for the evaluation of the Schumann and Gardner equations in the case of a resistivitydependent probe radius. This technique requires a minicomputer system in the evaluation of the resistivity from the spreading resistance. The basis of the assumption of a resistivity-dependent probe radius is the level of agreement obtained by Ehrstein et al. [26] in comparing profiles obtained from spreading resistance, SIMS, and CV. As is seen later in this part of the report, there are a number of reasons why carrier and atomic profiles do not have to agree. Indeed, the use of a resistivity-dependent probe radius appears to allow for one more degree of freedom for every data point taken.

The point of view adopted by Albers and Berkowitz [28,29] is that the probe radius is a single parameter which can best be evaluated by optimizing the agreement between the four-probe resistance obtained from the resistivity profile and measured on the top surface of the structure. The multilayer expression for the in-line four-probe resistance, $Z_n(S)$, has been derived in the form [28]

$$Z_n(S) = 2\rho_n \int_0^\infty A_n(\lambda) \Big\{ J_0(\lambda S) - J_0(2\lambda S) \Big\} \mathcal{I}_\nu(\lambda a) d\lambda, \tag{40}$$

where $\mathcal{I}_{\nu}(\lambda a)$ is the Hankel transform of the generalized probe-current density normalized to unit current [10], and ν is an index which refers to the specific form of the current density. It was shown that the four-probe resistance could always be obtained from the spreading resistance

$$Z_n(S) = R_n(2S) - R_n(S).$$
(41)

The four-probe resistance was shown to be: (i) independent of the probe radius and the probe-current density, and (ii) simply related to the sheet resistance, \mathcal{R}_n , when the distance to an insulating boundary is small compared with the probe spacing. The former observation provided the basis for a calibration procedure for determining the value of the probe radius to be used in spreading resistance profile analysis. This is discussed in detail in reference [28]. The evaluation of the four-probe resistance integral was found to be simplified by the alternative form given by [29]

$$Z_n(S) = \frac{\rho_n}{\pi} \int_{1/2S}^{1/S} A_n(\lambda) d\lambda.$$
(42)

In the above form, the four-probe resistance can easily be calculated using the Newton-Cotes method of evaluation,

$$Z_n(S) = \frac{\rho_n}{\pi} \ln(2) \sum_{i=0}^8 w_{\vartheta,i} \lambda_i A_n(\lambda_i), \qquad (43)$$

where $\lambda_i = 2^{i/8}/2S(0 \le i \le 8), w_{9,i} = C_i/8$, and where C_i are the Newton-Cotes weighting factors. This procedure allows for the variation of the resistivity profile under the requirement that the measured and calculated four-probe resistances agree within given limits.

The general relation between the four-probe resistance and the two-probe resistance as given by eq (41) was derived from the general definitions of these functions. The simplified form for the four-probe resistance integral contained in eq (42) arose from a consideration of the integral contained in eq (40). These two equations have been verified experimentally [30] by means of data obtained from lithographically deposited contacts as well as conventional pressure contacts.

EFFECTS OF FINITE LAYER THICKNESS

The original Schumann and Gardner derivation assumed that the material may be viewed as a series of uniform resistivity layers. These authors realized the possible limitation of this assumption as they showed how the correction factor depended upon the number of layers used in the calculation. This same observation was studied in more detail by Choo et al. [15] by means of the recursion relation presented in eq (18). In order to offset some of the possible difficulties associated with the finite thickness of the "layers," Choo et al. [31] introduced an exponential model in which each of the layers was assumed to have an exponentially varying resistivity. This piecewise exponential was to be a more realistic representation of the continuously varying resistivity. The differences between the correction factor calculated by means of a finite layer picture and a continuum picture were also considered by Albers [32]. Beginning with the recursion relation, he showed how a nonlinear, inhomogeneous differential equation could be rigorously derived for the kernel of the spreading resistance correction factor integral in the limit as the layer thicknesses approach zero. This equation may be transformed to a Riccatti equation which can be solved analytically for an exponentially varying resistivity. When the correction factor calculated from this technique was compared with that obtained from the finite layer picture, it was shown that finite-layer analysis will underestimate the correction factor for the case of a low-resistivity layer over a high-resistivity layer. Over the past 10 years, the beveling techniques which have evolved have allowed for the preparation of very shallow bevels. These shallow bevels have pushed the measurements into the region where the finite layer thickness approximation is reasonably good.

ATOMIC DENSITIES AND CARRIER DENSITIES

In this section of the report, attention is focused on the calculation of carrier densities from atomic densities for planar and beveled structures. The purpose is to provide insight into the validity of the use of the multilayer Laplace equation which assumes charge neutrality in each of the "layers" in the material. In terms of the difference between carrier densities and atomic densities along the bevel cut into the material for spreading resistance profile analysis, Hu [33] has presented a calculation based upon the solution of Poisson's equation for a number of profile types into substrates of the opposite conductivity type, i.e., junction-isolated structures. Using a finite difference solution method, he finds that electrical junctions may be much shallower than metallurgical junctions for structures of the Gaussian and exponential type. As the deviation between carrier densities and atomic densities has also been observed for same conductivity-type substrates, Albers *et al.* [34] have approached the problem from the point of view of the solution of the semiconductor equations using an adaptive finite-element method discussed by Blue and Wilson [35,36]. They find that the deviation of the carrier densities from the atomic densities occurs for both same- and opposite-conductivity type substrates.

Consider the structure presented in figure 4. This consists of an *n*-type region and a *p*-type region. A portion of the material is removed producing the beveled surface. The variables of interest to the problem are the electrostatic potential and the electron and hole densities which arise from the atomic donor and acceptor concentrations. From a computational point of view, it is more convenient to formulate the problem in terms of the electrostatic potential and the electron and hole quasi-Fermi levels which are related to the corresponding densities. The model which shall be used to describe the system is based on the basic semiconductor equations [37,38] (steady-state, time-independent):

$$\nabla \cdot (\kappa \nabla V) = -\frac{q}{\epsilon_0} (p - n + N), \tag{44}$$

$$\nabla \cdot (\mu_n \, n \, \nabla \, \phi_n) = R, \tag{45}$$

$$\nabla \cdot (\mu_p \, p \, \nabla \, \phi_p) = -R, \tag{46}$$

where κ is the dielectric constant, V is the electrostatic potential, q is the electronic charge, ϵ_0 is the permittivity of free space, N is the net ionized impurity density, $N = |N_D - N_A|$, N_D is the atomic donor density, N_A is the atomic acceptor density, μ_n and μ_p are the electron and hole mobilities, respectively, and ϕ_n and ϕ_p are the electron and hole quasi-Fermi levels, respectively. The Shockley-Read-Hall recombination is given by:

$$R = \frac{(pn - n_i^2)}{\tau_{n0}(n + n_1) + \tau_{p0}(p + p_1)},$$
(47)

where n_i is the intrinsic carrier density. The electron density, n, and the hole density, p, are related to the electrostatic potential and the corresponding quasi-Fermi levels (for Boltzmann statistics) by:

$$n = n_i \exp(q(V - \phi_n)/kT), \tag{48}$$

$$p = n_i \exp(q(\phi_p - V)/kT).$$
(49)

The solutions of eqs (44), (45), and (46) are obtained by means of an adaptive finiteelement technique. The reader is referred to references [35] and [36] for a more detailed discussion of the solution method and strategies.

For the purpose of investigating the validity of the use of the multilayer Laplace equation, figures 5 through 7 contain the results of the calculations for the cases of Gaussian diffusion type structures ranging in junction depths from 15 μ m down to 0.25 μ m. These



Figure 4. Geometry of the beveled material in which the semiconductor equations are solved by means of a finite element method.



Figure 5. Carrier and atomic distributions for a $15-\mu m$ Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{20} \text{ cm}^{-3}$, and the background concentration is $1.5 \times 10^{15} \text{ cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.



Figure 6. Carrier and atomic distributions for a $1.5 - \mu m$ Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{19} \text{ cm}^{-3}$, and the background concentration is $1.5 \times 10^{15} \text{ cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.



Figure 7. Carrier and atomic distributions for a $0.25 \ \mu m$ Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{19} \text{ cm}^{-3}$, and the background concentration is $1.5 \times 10^{15} \text{ cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.

figures contain the net ionized impurity (atomic) density, the electron and hole densities calculated for a planar material, and the electron and hole densities calculated along the bevel. Figures 5, 6, and 7 contain the results for opposite-conductivity type substrates for a 15- μ m, 1.5- μ m, and 0.25- μ m Gaussian, respectively. In all three cases, the surface concentration is 1.5×10^{20} cm⁻³, and the substrate concentration is 1.5×10^{15} cm⁻³. These deviations represent regions where there is a nonzero charge. This charge separation is due to the repulsion of the mobile carriers in the restraining field due to the immobile ionized impurities. In the regions where there is no deviation, the net local charge is zero. It is extremely important to keep in mind that these represent a best-case scenario where damage, defects, incomplete activation, etc., have not come into play.

STATUS OF SPREADING RESISTANCE ANALYSIS

The calculation of the spreading resistance correction factor using the Schumann and Gardner multilayer Laplace equation has progressed to the point where the calculation of the spreading resistance from the resistivity and the inverse calculation can be performed on microcomputer systems. The interpretation of the probe radius can be done by using a resistivity-dependent probe radius model or by comparing the interpreted profiles in terms of the four-probe resistance. The FORTRAN codes presented in this report provide for the practical ability to perform these calculations. They represent the fruits of the labor of many researchers over the past twenty years. However, there is a question of the validity of the use of the multilayer Laplace equations for the case of shallow layers where the net local charge is nonzero. The user should always be aware of this.

Spreading resistance data can certainly be measured in submicrometer regions, but the interpretation of the data in terms of the multilayer analysis Laplace needs to be very carefully investigated. From a conceptual point of view, it might be argued that the measured data should be related to the physical description provided by the Poisson equation. However, Poisson's equation has not yielded to the simple mathematical structure which the Laplace equation has.

The resolution of this problem for the interpretation of spreading resistance data on shallow VLSI structures requires the investigation of the solution of the Laplace and Poisson equations. The solutions of these two equations should provide an estimate of the differences between the actual carrier distribution and that determined from the Schumann and Gardner analysis. The answer to this problem is actively being pursued by researchers all over the globe.

DISCUSSION OF THE PROGRAMS

The purpose of this portion of the report is to discuss some of the aspects of the various programs and to present examples of the results of the calculations performed by these programs. The reader should keep in mind that the programs listed in the appendices have extensive internal documentation in regard to the background information, input data, and the running logic of the programs. The reader who is familiar with FORTRAN should find

these programs easy to follow and use. This section is intended to supplement the internal documentation by presenting the information outside of the framework of the FORTRAN source code and to discuss typical input and output data.

There are three major subsections of this portion of the report. These are concerned with: 1) programs for calculating the two-probe resistance from the resistivity, 2) programs for calculating the resistivity from the two-probe resistance, and 3) programs for calculating the four-probe resistance from the resistivity. From the discussion of the history of the Schumann and Gardner solution of the multilayer Laplace equation, it should be clear that a conceptually and mathematically efficient representation of the kernel of the integral, $A_n(\lambda)$, is given by eqs (18) and (19) or (27). This same kernel is involved in all twoprobe and four-probe integrals contained in this report. Hence, the discussion of the various programs primarily focuses upon various techniques for numerically evaluating the integrals involved or for carrying out efficient inversions of these integrals.

A special note is in order concerning the units used in the calculations. The resistances (two-probe and four-probe) are in ohms and the resistivities are in ohm cm. All lengths (probe radius, probe spacing, and depths) are in micrometers.

It is also important to note the numerical format of the various output data files. In particular, the format used for the construction of the output data is of a general nature. It has been written to accommodate a wide range of numerical values without loss of generality or data. This was motivated by the fact that datum may be lost on output if it exceeds the format of the write statement. In order to circumvent this, some data may appear to have more significant figures than might be reasonable or needed. This will be invariably the case if the user decides to use the free-field write option of FORTRAN77. This circumvents the loss of data which is always possible when writing in a fixed-field format. The user should keep this in mind when looking at the output. The user should round the output following accepted rules for this purpose. The first group of programs is used to calculate the spreading resistance from the resistivity. Hence, the programs presented in this section represent various methods for evaluating the correction factor integral. In order to familiarize the reader with each of these four programs, they are discussed individually along with sample input and output data. The four programs included in this section are: RGENTR, RGENBL, RGENGL, and RGENVAR. RGENTR

The RGENTR (Resistance GENeration Trapezoidal Romberg) program calculates the twoprobe spreading resistance from eqs (14) to (17).

$$R_n = \frac{\Delta V}{I} = \frac{\rho_n}{2a} C_n,$$

where C_n is the correction factor for the Schumann and Gardner form of the current density given by

$$C_n = rac{4}{\pi} \int_0^\infty A_n(\lambda) iggl\{ rac{J_1(\lambda a)}{\lambda a} - rac{J_0(\lambda S)}{2} iggr\} \sin(\lambda a) rac{d\lambda}{\lambda}.$$

In terms of the uniform current density and the ring delta function current density, the corresponding correction factors would be given by

$$C_n^{un} = rac{4}{\pi} \int_0^\infty A_n(\lambda) iggl\{ rac{J_1(\lambda a)}{\lambda a} - rac{J_0(\lambda S)}{2} iggr\} 2 J_1(\lambda a) rac{d\lambda}{\lambda},$$

and

$$C_n^{rd} = rac{4}{\pi} \int_0^\infty A_n(\lambda) iggl\{ rac{J_1(\lambda a)}{\lambda a} - rac{J_0(\lambda S)}{2} iggr\} a J_0(\lambda a) d\lambda.$$

For the purposes of illustration, the discussion of the code will focus upon the use of the Schumann and Gardner current density. The same program and logic flow follow directly for the uniform current and the ring delta current densities.

The integral is approximated to be over a finite interval as

$$\int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda} \approx \int_{xl}^{xu} A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda}.$$

The correction factor integral is calculated between the limits of xl and xu (both input data) by means of a trapezoidal Romberg integration technique. This particular integration scheme was used previously by D'Avanzo *et al.* [16] in order to calculate the array of partial integrals used in their SPINT-SPRED programs. It was also used by Albers [19] to generate model date for the comparison of correction factor algorithms.

RGENTR reads the input data from logical device 10 (for010) which can be referenced to a data file through an assign statement in the run sequence (see the examples in the tape files). These data must include the following:

npt	number of data points in the structure. Total number of
	points is num=npt+1 (include substrate point)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
subrho	substrate resistivity $(\Omega \cdot cm)$
subth	substrate thickness (micrometers)
rho	array of resistivity values $(\Omega \cdot cm)$
icur	form of the current density
	(Schumann Gardner=1, Choo uniform=2, ring delta=3)
xl	lower limit of correction factor integral (usually zero)
xu	upper limit of correction factor integral

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The trapezoidal Romberg integration scheme is carried out at each depth increment until convergence is achieved to within 0.005 percent. This convergence criterion has been found to give good results with a nominal use of CPU time on a minicomputer system.

The parts of the program are as follows:

rgentr	the main program
intgrl(f,a,b,atemp,iflag)	the subroutine which carries out the integration of the function
	f from the limits of a to b until the predefined convergence
	is attained
tpint(x)	a double precision function which represents the integrand of
	the two-probe resistance integral and includes the kernel and
	the choice of a probe current density
alam(x)	a double precision function which constructs the kernel function
	according to the recursion relation
dbj0(x)	a double precision function which uses a polynomial approxi-
	mation to generate the J_0 Bessel function
dbj1(x)	a double precision function which uses a polynomial approxi-
	mation to generate the J_1 Bessel function

The input data is read from for010. The calculation is performed and the output data are written to for011 and for006. In the example presented here, rgentr.in contains the data to be read in for010, rgentr.out contains the data written to for011, and rgentr.log contains the data written to for006. The rgentr.log file also contains information as to the CPU time used on a VAX 11/785 system. Many workstations, such as SUN and SPARK2, are much faster. Many PCs, especially those operating with 386 or 486 CPU chips and with coprocessor chips, will probably run at about VAX speeds or faster.

Table 1.	Annotated input data file (rgentr.in) for RGENTR program								
50 2.0 50.0	NPT-number of points in structure. Total is NPT + 1 AA-probe radius in micrometers SEP- probe separation in micrometers								
0.015	DELTH-depth increment in micrometers								
1	IBBC - back boundary condition (none=1, cond=2, ins=3)								
126.5	SUBRHO-substrate resistivity in ohm*cm =RHO(1)								
0.015	SUBTH-substrate thickness in micrometers								
1.542	RHO(NPT+1) surface resistivity in ohm*cm								
0.6933	RHO (NPT)								
0.3442	RHO(NPT-1)								
0.19									
0.1159									
0.0769									
0.0543	· · · ·								
0.0401									
0.0307	•								
0.0242									
0.0196	·								
0.0103	•								
0.0131									
0.0125									
0.0124									
0.013									
0.0141									
0.0161									
0.019									
0.0233									
0.0294	•								
0.0382									
0.0514	•								
0.0721	•								
0.1732	•								
0.3081									
0.6095									
1.334									
3.18									
8.036									
20.4									
46.62									
82.75									
109.7	•								
121.4	•								
125.1	•								
120.1									
126.4	•								
126.5									
126.5									
126.5									
126.5									
126.5	•								
126.5	•								
126.5									
120.5									
20.5	TCUR-probe current density (S&G=1 unif=2 ring=3)								
0.0	YL tower limit of integral (usually zoro)								
10.0	XU-upper limit of integral (usually 20/AA)								
Table	2.	Annotated	output	data	file	(raentr.out)) for	RGENTR	program
-------	----	-----------	--------	------	------	--------------	-------	--------	---------

51		total number of points calculated
0.0000E+00	8.1209E+02	depth and spreading resistance
1.5000E-02	7.7709E+02	
3.0000E-02	7.6227E+02	
4.5000E-02	7.5662E+02	
6.0000E-02	7.5634E+02	
7.5000E-02	7.6041E+02	
9.0000E-02	7.6898E+02	
1 0500E-01	7.8279E+02	
1.2000E-01	8.0318E+02	
1 3500E-01	8 3205E+02	
1 5000E-01	8.7224E+02	
1 6500E-01	9.2789E+02	
1 8000E-01	1 0042E+03	
1.9500E-01	1 1090E+03	·
2 1000E-01	1 2528E+03	
2.1000E-01	1 4501E+03	·
2.2000E-01	1 7238E+03	•
2.4000E-01	2 1024E+03	•
2.3500E-01	2.1024L+03	
2.7000E-01	2.03071403	
2.0500E-01	3.3921L+03	
3.0000E-01	4.4000E+03	•
3.1500E-01	0.0/10E+03	·
3.3000E-01	8.4500E+03	·
3.4500E-01	1.2140E+04	•
3.6000E-01	1.8008E+04	•
3.7500E-01	2.7658E+04	•
3.9000E-01	4.3/8/E+04	•
4.0500E-01	7.0412E+04	•
4.2000E-01	1.11/3E+05	•
4.3500E-01	1.6792E+05	•
4.5000E-01	2.2925E+05	•
4.6500E-01	2.7909E+05	•
4.8000E-01	3.0907E+05	•
4.9500E-01	3.2358E+05	•
5.1000E-01	3.2984E+05	•
5.2500E-01	3.3227E+05	•
5.4000E-01	3.3308E+05	
5.5500E-01	3.3331E+05	•
5.7000E-01	3.3338E+05	
5.8500E-01	3.3340E+05	•
6.0000E-01	3.3340E+05	•
6.1500E-01	3.3340E+05	•
6.3000E-01	3.3340E+05	•
6.4500E-01	3.3340E+05	•
6.6000E-01	3.3340E+05	•
6.7500E-01	3.3340E+05	•
6.9000E-01	3.3340E+05	•
7.0500E-01	3.3340E+05	
7.2000E-01	3.3340E+05	
7.3500E-01	3.3340E+05	
7.5000E-01	3.3340E+05	
2.000000000	00000	probe radius in micrometers
50.00000000	00000	probe separation in micrometers
1.5000000000	00000E-02	depth increment in micrometers

CALCULATION OF SPREADING RESISTANCE FROM THE RESISTIVITY USING THE TRAPEZOIDAL ROMBERG INTEGRATION METHOD CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X NO BOUNDARY (INFINITE SLAB) CONDITION USED IN CALCULATION

XL		XU					
0.00000)	10.00000	C				
RADIUS		SEP I	DE	LTH			
2.000000		50.000000		0.015000			
SUBRH0=1.26	55	00E+02: SU	BTI	HICK=1.5000	00E-02		
DEPTH(U)		RHÓ		CORR-FAC	RSP		VOLTAGE
0.00000E+00	1	.54200E+00	2	.10660E-01	8.12093E+02	2	.90961E+00
1.50000E-02	6	.93300E-01	4	.48340E-01	7.77086E+02	2	.89047E+00
3.00000F-02	3	44200F-01	8	.85841F-01	7.62266F+02	2	.88211F+00
4.50000E-02	1	90000F-01	1	.59287F+00	7.56615E+02	$\frac{1}{2}$.87887F+00
6.00000E-02	1	15900E-01	2	61030F+00	7 56335E+02	2	87871F+00
7.50000E-02	7	69000E-02	3	95534F+00	7.60413E+02	$\overline{2}$.88105F+00
9 00000E-02	5	43000E-02	5	66467E+00	7 68979F+02	2	88591E+00
1 05000E-01	4	01000E-02	7	80842E+00	7 82794F+02	2	89365E+00
1 20000E-01	2	07000E-02	1	04648E+01	8 03177E+02	2	90481 F+00
1 35000E-01	2	A2000E-02	1	37520F+01	8 32040F+02	2	02015E+00
1.50000E-01	1	96000E-02	1	78000E+01	8 70040E+02	2	94064E+00
1.50000E-01	1	65000E-02	2	24042E-01	0.722426402	2	06740E.00
1.05000E-01	1	.05000E-02	20	70057E.01	9.27003L+02	2	001945L+00
1.05000E-01	1	21000E-02	2	20600E.01	1 10001E.02	2	044025.00
1.95000E-01	4	.31000E-02	3	.30029E+01	1.109012+03	000	.04493E+00
2.10000E-01	4	.25000E-02	4	.00908E+01	1.25264E+U3	3	16120E-00
2.25000E-01	1	.24000E-02	4	.0//62E+UI	1.45006E+03	3	.10139E+00
2.40000E-01	1	.30000E-02	D	.30387E+01	1.72376E+03	3	.23048E+00
2.55000E-01	Ţ	.41000E-02	5	.96436E+01	2.10244E+03	3	.32272E+00
2.70000E-01	1	.61000E-02	0	.55089E+01	2.636/3E+03	3	.42107E+00
2.85000E-01	1	.90000E-02	1	.14131E+01	3.39212E+03	3	.5304/E+00
3.00000E-01	2	.33000E-02	1	.69232E+01	4.480/8E+03	3	.65135E+00
3.15000E-01	2	.94000E-02	8	.26096E+01	6.0/181E+03	3	./8332E+00
3.30000E-01	3	.82000E-02	8	.85510E+01	8.45662E+03	3	.92720E+00
3.45000E-01	5	.14000E-02	9	.44741E+01	1.21399E+04	4	.08422E+00
3.60000E-01	7	.21000E-02	9	.99057E+01	1.80080E+04	4	.25547E+00
3.75000E-01	1	.07400E-01	1	.03009E+02	2.76578E+04	4	.44182E+00
3.90000E-01	1	.73200E-01	1	.01125E+02	4.37873E+04	4	.64135E+00
4.05000E-01	3	.08100E-01	9	.14151E+01	7.04124E+04	4	.84765E+00
4.20000E-01	6.	.09500E-01	7.	.33241E+01	1.11728E+05	5.	.04816E+00
4.35000E-01	1.	.33400E+00	5	.03497E+01	1.67916E+05	5	.22509E+00
4.50000E-01	3.	.18000E+00	2	.88370E+01	2.29255E+05	5	.36032E+00
4.65000E-01	8	.03600E+00	1	.38921E+01	2.79093E+05	5	.44575E+00
4.80000E-01	2	.04000E+01	6	.06019E+00	3.09070E+05	5	.49006E+00
4.95000E-01	4	.66200E+01	2	.77631E+00	3.23579E+05	5	.50998E+00
5.10000E-01	8	.27500E+01	1	.59440E+00	3.29841E+05	5.	51830E+00
5.25000E-01	1.	.09700E+02	1.	.21158E+00	3.32274E+05	5.	52150E+00
5.40000E-01	1.	21400E+02	1.	.09747E+00	3.33081E+05	5.	52255E+00
5.55000E-01	1.	.25100E+02	1	.06575E+00	3.33314E+05	5	52285E+00
5.70000E-01	1.	.26100E+02	1.	.05750E+00	3.33377E+05	5.	52294E+00
5.85000E-01	1.	26400E+02	1	.05505E+00	3.33395E+05	5	52296E+00
6.00000E-01	1	26400E+02	1	05506E+00	3.33400E+05	5	52296E+00
6.15000E-01	1	26500E+02	1	05424E+00	3.33404E+05	5	52297E+00
6.30000F-01	1	26500E+02	1	05424F+00	3.33404E+05	5	52297F+00
6.45000E-01	1	26500E+02	1	.05424E+00	3.33404E+05	5	52297E+00
6 60000F-01	1	26500E+02	1	05424F+00	3 33404E+05	5	52297E+00
6.75000F-01	1	26500F+02	1	05424F+00	3.33404F+05	5	52297F+00
6.90000E-01	1	26500E+02	1	05424F+00	3.33404F+05	5	52297F+00
7 050000 01	-	26500E.02	4	0E424E.00	2 22404E.OF	5	E2207E.00
7 20000E-01	1	20500E+02	4	05424E+00	2 22404E+05	5.	52291E+00
7 250005-01	1	20500E+02	1	05424E+00	3.33404E+05	Э. Г	52291E+00
7.35000E-01	1.	20500E+02	1.	05424E+00	3.33404E+05	5.	52297E+00
7.50000E-01	Τ.	20500E+02	Τ.	00424E+00	3.33404E+05	э.	92297E+00

CPU time: 00:05:07.72 - this is CPU time for VAX 11/785

RGENGL

The RGENGL (Resistance GENeration Gauss Laguerre) program calculates the two-probe spreading resistance from eqs (14) to (17). These are presented in the text as well as in the previous section where RGENTR was discussed. The original work of Choo *et al.* [17] makes use of the uniform current density form of the correction factor integral. The Schumann and Gardner current density as well as the ring delta function forms may also be treated by this method. The discussion presented here makes use of the uniform current density. The central point of the method is to evaluate the correction factor integral by means of an integration scheme based upon the Gauss-Laguerre quadrature. As noted in the text, quadrature techniques for evaluating integrals are based on knowing a set of integration points and weights, $\{x_k\}, \{w_k\}, (k = 1, ..., N)$ such that an integral may be evaluated as a finite sum in the form

$$\int_a^b f(x)dx = \sum_{k=1}^N w_k f(x_k).$$

The calculation of the correction factor integral is carried out by using a 33-point Gauss-Laguerre method,

$$egin{aligned} &rac{4}{\pi}\int_0^\infty A_n(\lambda)iggl\{rac{J_1(\lambda a)}{\lambda a}-rac{J_0(\lambda S)}{2}iggr\}2J_1(\lambda a)rac{d\lambda}{\lambda}\ &pprox &rac{8}{\pi}\sum_{k=1}^{33}w_k^{gl}iggl\{rac{J_1(\lambda_k a)}{\lambda_k a}-rac{J_0(\lambda_k S)}{2}iggr\}rac{J_1(\lambda_k a)}{\lambda_k}A_n(\lambda_k), \end{aligned}$$

where the weights (w_k^{gl}) and integration points $(\lambda_k a)$ are listed in reference [17] and are used in RGENGL. These are fixed weights and points which are entered through DATA statements. As the points are fixed by the roots of the polynomial and not by the behavior of the integrand, this technique is of limited value as it may not accurately reproduce the probe-spacing dependence of the integrand. It (and the inverse program, GL) is presented here as it represents one of the first attempts to evaluate the integrals and provides the user with the opportunity to run the programs and test the particular quadrature.

RGENGL reads in the input data from logical device 10 (for010). These data include the following:

npt	number of data points in the structure. Total number of
	points is num=npt+1 (include substrate point)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition
	(none=1, conducting=2, insulating=3)
subrho	substrate resistivity $(\Omega \cdot cm)$
subth	substrate thickness (micrometers)

rho	array of resistivity values $(\Omega \cdot \mathbf{cm})$
icur	form of the current density
	(Schumann Gardner=1, Choo uniform=2, ring delta=3)

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The parts of the program are as follows:

rgengl	the main program which carries out the sums for the particular
	structure (the kernel function is constructed by the recursion relation
	inside the main program)
dbj0(x)	a double precision function which uses a polynomial approximation
	to construct the J_0 Bessel function
dbj1(x)	a double precision function which uses a polynomial approximation
	to construct the J_1 Bessel function

After the calculation is finished, the output is written to logical devices for006 and for011. The output may then be assigned to files for storage. Typical results are presented in the subsequent listings of the two files rgengl.log and rgengl.out. The CPU required is typically several seconds on a VAX 11/785. This is down from minutes for RGENTR. Herein lies the very attractive feature of quadrature techniques.

```
Table 4. Annotated input data file (rgengl.in) for RGENGL program
50
                 NPT-number of points in structure. Total is NPT + 1
2.0
                 AA-probe radius in micrometers
50.0
                 SEP- probe separation in micrometers
0.015
                 DELTH-depth increment in micrometers
                 IBBC - back boundary condition (none=1, cond=2, ins=3)
1
                 SUBRHO-substrate resistivity in ohm*cm =RHO(1)
126.5
0.015
                 SUBTH-substrate thickness in micrometers
1.542
                 RHO(NPT+1) surface resistivity in ohm*cm
0.6933
                 RHO (NPT)
                 RHO (NPT-1)
0.3442
0.19
0.1159
0.0769
0.0543
0.0401
0.0307
0.0242
0.0196
0.0165
0.0144
0.0131
0.0125
0.0124
0.013
0.0141
0.0161
0.019
0.0233
0.0294
0.0382
                     .
0.0514
                     .
0.0721
                     .
0.1074
0.1732
                     .
0.3081
                     .
0.6095
                      .
1.334
                     .
3.18
                      .
8.036
                      .
20.4
46.62
82.75
109.7
121.4
                      .
125.1
126.1
126.4
126.4
126.5
126.5
126.5
                      .
126.5
                      .
126.5
                      .
126.5
                      .
126.5
                      .
126.5
                 RH0(2)
126.5
                 ICUR-probe current density (S&G=1,unif=2,ring=3)
2
```

Table 5.	Annotated	output	data	file	(rgengl.out)) for	RGENGL	program

	51
0.000	8.4549E+02
0.015	8.0781E+02
0.030	7.9183E+02
0.045	7.8567E+02
0.060	7.8522E+02
0.075	7.8936E+02
0.090	7.9819E+02
0.105	8.1249E+02
0.120	8.3302E+U2
0.155	0.0507E+02
0 165	9.6303E+02
0.180	1.0423E+03
0.195	1.1510E+03
0.210	1.3003E+03
0.225	1.5051E+03
0.240	1.7893E+03
0.255	2.1825E+03
0.270	2.7374E+03
0.285	3.5223E+03
0.300	4.6539E+03
0.315	0.3090E+03
0.330	0.7925E+03
0.360	1.2034L+04
0.375	2.8891E+04
0.390	4.5895E+04
0.405	7.4171E+04
0.420	1.1844E+05
0.435	1.7905E+05
0.450	2.4531E+05
0.465	2.9900E+05
0.480	3.3121E+05 2.4670E+05
0.495	3.40/9E+05 3.5352E+05
0.525	3.5613E+05
0.540	3.5700E+05
0.555	3.5725E+05
0.570	3.5732E+05
0.585	3.5734E+05
0.600	3.5734E+05
0.615	3.5735E+05
0.630	3.5735E+05
0.045	3.5/35E+05
0.000	3.5735E+05
0.670	3.5735E+05
0,705	3.5735E+05
0.720	3.5735E+05
0.735	3.5735E+05
0.750	3.5735E+05
2.00000	000000000
50.0000	000000000
1.500000	0000000000E-02

2

· · ·

total number of points calculated depth and spreading resistance

probe radius in micrometers probe separation in micrometers depth increment in micrometers current density (SG=1,unif=2,ring=3) Table 6. Annotated log file (rgengl.log) for RGENGL program

CALCULATION OF THE SPREADING RESISTANCE FROM THE RESISTIVITY USING THE 33-POINT GAUSS-LAGUERRE SCHEME NO BOUNDARY AT 0.765 UM. SUBSTRATE AT 0.750UM CHOO UNIFORM CURRENT DENSITY AA= 2.000 SEP= 50.0 DELTH= 0.015 SUBTH= 1.500E-02 SUBRHO= 1.265E+02 #PT= 50

DEPTH		RHO	TNTEGRAL	CORR FAC	RCP	VOL
0.000	1	5420E+00	8 6128E-02	2 1032E_01	8 4540E+02	2 0271 E+00
0.015	ā	0330E-01	1 8302E-01	A 6607E-01	0.4049L402	2.92712+00
0.010	2	4420E-01	2 6126E-01	9.0007E-01	7 0102E.00	2.90732700
0.030	1	0000E 01	5.0130E-01	9.2020E=01	7.9103E+02	2.09000000
0.045	T.	1500E-01	0.4954E-01	1.6540E+00	7.856/E+02	2.8952E+00
0.060	Ŧ	.1590E-01	1.0642E+00	2./100E+00	7.8522E+02	2.8950E+00
0.075	/	.6900E-02	1.6124E+00	4.1059E+00	7.8936E+02	2.8973E+00
0.090	5	.4300E-02	2.3090E+00	5.8798E+00	7.9819E+02	2.9021E+00
0.105	4	.0100E-02	3.1827E+00	8.1046E+00	8.1249E+02	2.9098E+00
0.120	3	.0700E-02	4.2653E+00	1.0861E+01	8.3362E+02	2.9210E+00
0.135	2	.4200E-02	5.6053E+00	1.4274E+01	8.6356E+02	2.9363E+00
0.150	1	9600E-02	7.2551E+00	1.8475E+01	9.0527E+02	2.9568E+00
0.165	1	.6500E-02	9.1680E+00	2.3346E+01	9.6303E+02	2.9836E+00
0.180	1	4400E-02	1.1370E+01	2.8952E+01	1.0423E+03	3.0180E+00
0.195	1	3100E-02	1.3802F+01	3.5146E+01	1 1510E+03	3.0611E+00
0 210	1	2500E-02	1 6341E+01	4 1611E+01	1 3003E+03	3 11/1E+00
0.225	1	2400E-02	1 0066E+01	4.1011L+01	1.50515.02	2 1776E.00
0.225	1. 1	2000E-02	2 1600E+01	4.0001L+01	1.30312+03	2.05075.00
0.240	1. 1	4100E-02	2.10200+01	5.5054E+01	1.7093E+03	3.2327E+00
0.255	1.	4100E-02	2.43140+01	0.1915E+01	2.1825E+03	3.3390E+00
0.270	1.	6100E-02	2.6/08E+01	6.8011E+01	2./3/4E+03	3.43/3E+00
0.285	1.	9000E-02	2.9120E+01	7.4153E+01	3.5223E+03	3.5468E+00
0.300	2	.3300E-02	3.1375E+01	7.9895E+01	4.6539E+03	3.6678E+00
0.315	2.	9400E-02	3.3708E+01	8.5836E+01	6.3090E+03	3.8000E+00
0.330	3.	8200E-02	3.6155E+01	9.2068E+01	8.7925E+03	3.9441E+00
0.345	5.	1400E-02	3.8611E+01	9.8321E+01	1.2634E+04	4.1015E+00
0.360	7.	2100E-02	4.0891E+01	1.0413E+02	1.8769E+04	4.2734E+00
0.375	1.	0740E-01	4.2255E+01	1.0760E+02	2.8891E+04	4.4608E+00
0.390	1.	7320E-01	4.1624E+01	1.0599E+02	4.5895E+04	4.6618E+00
0.405	3.	0810E-01	3.7815E+01	9.6295E+01	7.4171E+04	4.8702E+00
0.420	6.	0950E-01	3.0523E+01	7.7727E+01	1.1844E+05	5.0735E+00
0 435	1	3340E+00	2.1084F+01	5.3689F+01	1.7905F+05	5.2530F+00
0 450	3	1800E+00	1 2117E+01	3 0856E+01	2.4531E+05	5.3897F+00
0 465	8	0360E+00	5 8445E+00	1 4883E+01	2 9900E+05	5 4757E+00
0.480	2	0400E+00	2 5503E+00	6 4043E+00	3 3121 E+05	5 5201 E+00
0.405	2 . A	6600E.01	1 16955.00	2 0755E+00	3 4670E+05	5 5401 E+00
0.495	4.	07505-01	6 7106E 01	2.9755L+00	2 52525.05	5.54012400
0.510	0.	27502+01	5.7100E-01	1.00065.00	3.5552E+05	5.54642+00
0.525	1.	0970E+02	5.0995E-01	1.2960E+00	3.5013E+05	5.55102+00
0.540	1.	2140E+02	4.6192E-01	1.1/03E+00	3.5700E+05	5.55272+00
0.555	1.	2510E+02	4.485/E-01	1.1423E+00	3.5/25E+05	5.5530E+00
0.570	1.	2610E+02	4.4510E-01	1.1334E+00	3.5/32E+05	5.5531E+00
0.585	1.	2640E+02	4.440/E-01	1.1308E+00	3.5/34E+05	5.5531E+00
0.600	1.	2640E+02	4.4408E-01	1.1308E+00	3.5734E+05	5.5531E+00
0.615	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.630	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.645	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.660	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.675	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.690	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.705	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.720	1.	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.735	1	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
0.750	1	2650E+02	4.4373E-01	1.1299E+00	3.5735E+05	5.5531E+00
CPU time		00.00.00	89 - this (Pl	I time is fo	r VAX 11/785	
0.0 0.00		00.00.00				

RGENBL

The RGENBL (Resistance GENeration Berkowitz Lux) program calculates the two-probe spreading resistance starting with eqs (14) to (17). Whereas RGENTR carried out the integration by successive splitting of the subintervals and RGENGL carried out the integration by using fixed sampling points, the RGENBL program makes use of two essential features. The first is to approximate the correction factor integral by removing the probe-spacingdependent Bessel function and to replace its effect by a truncated S-dependent lower limit of integration. In particular, this approximation of the correction factor integral takes the form as

$$C_{n} = \frac{8}{\pi} \int_{0}^{\infty} A_{n}(\lambda) \left\{ \frac{J_{1}(\lambda a)}{\lambda a} - \frac{J_{0}(\lambda S)}{2} \right\} J_{1}(\lambda a) \frac{d\lambda}{\lambda}$$
$$= \frac{8}{\pi} \int_{L}^{\infty} A_{n}(\lambda) \left(\frac{J_{1}(\lambda a)}{\lambda a} \right)^{2} d\lambda,$$

where L = 1.12292/S. The choice of the truncated lower limit of integration is dictated by the form of the functions for the case of both insulating and conducting boundaries. The second feature of the technique is that the approximate integral can be divided into four domains in each of which the Newton-Cotes integration method may be used. The evaluation of the above approximate integral is then performed by means of a finite sum as

$$C_n = \sum_{k=1}^{22} w_k^{bl} A_n(\lambda_k),$$

where the weights, w_k^{bl} , are different from those used in the Gauss-Laguerre technique. The first eight sampling points for this method are dependent upon the probe spacing. The remaining points are fixed. It is the dependence of the first eight sampling points on the probe spacing that allows the technique to have numerical simplicity (of the RGENGL code) and yet reproduce the results of the RGENTR code (with the uniform current density) to within a percent.

RGENBL reads the input data from logical device for010. This can be referenced to a data file through an assign statement. For the purposes of illustration, the input data file, rgenbl.in, is used here. The structure of the input data file is as follows:

npt	number of data points in structure (excluding substrate)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
subrho	substrate resistivity $(\Omega \cdot cm)$
subth	substrate thickness (micrometers)
rho	array of resistivity values $(\Omega \cdot cm)$

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The probe current density is not entered as input data as the analysis is performed with the uniform current density. Application of the technique for the Schumann and Gardner or the ring density requires the analysis of the S-dependence of the integrand. Such analysis (not performed here) should provide the correct representation of the appropriate weights and sampling points.

The RGENBL code consists of just the main program. This contains the fixed points and weights through DATA statements and calculates the S-dependent points from the input. Upon calculating the integrals and the spreading resistance, output is directed to the logical devices for006 and for011. The file rgenbl.in contains the input for the sample run, whereas rgenbl.log and rgenbl.out contain the corresponding output. Typical CPU times are on the order of seconds on a VAX 11/785.

Table 7.	Annotated input data file (rgenbl.in) for RGENBL program
50 2.0 50.0 0.015	NPT-number of points in structure. Total is NPT + 1 AA-probe radius in micrometers SEP- probe separation in micrometers DELTH-depth increment in micrometers
1 126.5 0.015	IBBC - back boundary condition (none=1, cond=2, ins=3) SUBRHO-substrate resistivity in ohm∗cm =RHO(1) SUBTH-substrate thickness in micrometers
0.6933	RHO (NPT-1) SUFFACE FESTSURVICY TH ONIN*CIA RHO (NPT-1)
0.19	
0.1159	
0.0769	
0.0543	
0.0401	
0.0307	•
0.0196	
0.0165	
0.0144	
0.0131	
0.0125	
0.0124	
0.013	•
0.0161	
0.019	
0.0233	
0.0294	
0.0382	
0.0514	
0.0721	•
0.1732	
0.3081	
0.6095	
1.334	
3.18	
8.036	
46 62	
82.75	
109.7	
121.4	
125.1	
126.1	•
120.4	•
126.5	
126.5	
126.5	
126.5	
126.5	
120.5	•
126.5	
126.5	RH0(2)

ladie o. Annotated output data tile (rgendl.out) for KGENBL proc	Table	8.	Annotated	output	data	file	(rgenbl.out)	for	RGENBL	progra	am
--	-------	----	-----------	--------	------	------	--------------	-----	--------	--------	----

0.000 0.015 0.030 0.045 0.060 0.075 0.090 0.105 0.120 0.135 0.150 0.255 0.210 0.225 0.240 0.255 0.240 0.255 0.300 0.315 0.300 0.315 0.300 0.345 0.300 0.405 0.405 0.405 0.405 0.405 0.405 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.555 0.570 0.575 0.600 0.615 0.600 0.755 0.720 0.755 0.720 0.755 0.720 0.755 0.750 0	51 8.1512E+02 7.7918E+02 7.5313E+02 7.5813E+02 7.5776E+02 7.5776E+02 7.6180E+02 8.0460E+02 8.3353E+02 8.7382E+02 9.2961E+02 1.0062E+03 1.1112E+03 1.2554E+03 1.4532E+03 1.4532E+03 1.7278E+03 2.6440E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.4024E+03 3.26440E+04 1.2180E+04 1.8053E+04 1.2180E+04 1.8053E+04 2.7680E+04 4.3721E+04 7.0159E+04 1.1122E+05 1.6716E+05 2.2830E+05 3.2259E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05 3.3254E+05
0.750	3.3254E+05
2.00000	000000000000000000000000000000000000000
50.0000	000000000000000000000000000000000000000
1.500000	000000000000E-02

total number of points calculated depth and spreading resistance

probe radius in micrometers probe separation in micrometers depth incrrement in micrometers

5

Table 9. Annotated log file (rgenbl.log) for RGENBL program

CALCULATION OF SPREADING RESISTANCE FROM THE RESISTIVITY BERKOWITZ-LUX APPROXIMATE INTEGRAL WITH THE UNIFORM CURRENT DENSITY NO BOUNDARY AT 0.765 UM. SUBSTRATE AT 0.750 UM RADIUS=2.0000 SEP= 50.0 DELTH= 0.015 SUBTH=1.500E-02 SUBRH0=1.265E+02 #PTS=50

DEPTH	RHO	INTEGRAL	CORR.FAC	RSP	VOL
0.000	1.5420E+00	8.3034E-02	2.1144E-01	8.1512E+02	2.9112E+00
0.015	6.9330E-01	1.7654E-01	4.4955E-01	7.7918E+02	2.8916E+00
0.030	3.4420E-01	3.4864E-01	8.8781E-01	7.6396E+02	2.8831E+00
0.045	1.9000E-01	6.2677E-01	1.5961E+00	7.5813E+02	2.8797E+00
0.060	1.1590E-01	1.0270E+00	2.6152E+00	7.5776E+02	2.8795E+00
0.075	7.6900F-02	1.5561E+00	3.9626E+00	7.6180E+02	2.8818E+00
0.090	5 4300E-02	2 2285E+00	5 6748E+00	7,7036F+02	2 8867F+00
0 105	4 0100E-02	3 0718E+00	7.8223E+00	7.8419F+02	2.8944F+00
0 120	3 0700E-02	4 1168E+00	1 0483E+01	8 0460E+02	2 9056E+00
0 135	2 4200E-02	5 4104E+00	1 3777E+01	8 3353E+02	2.9000E+00
0 150	1 9600F-02	7 0030E+00	1 7833E+01	8 7382E+02	2 9414E+00
0.165	1.5000E-02	8 8498F+00	2 2536E+01	9 2961E+02	2.0414E+00
0.100	1 4400E-02	1 0075E+01	2.2000E+01	1 0062E+02	3 0027E+00
0.100	1 3100E-02	1 3324E+01	2.7349E+01	1 1112E+03	3 0458E+00
0.130	1.3100E-02	1.5524C+01	4 0174E-01	1.2554E+03	3 0099E+00
0.210	1.2300E-02	1.3770L+01	4.0174L+01	1.2004E+00	3.0908L+00
0.225	1.24000-02	0.0077E.01	4.0070LTOI	1.40020400	2 0275E.00
0.240	1.3000E-02	2.00//E+01	5.3103E+01	1./2/0E+03	3.23/3E+00
0.235	1.4100E-02	2.3401E+U1	5.9/95E+01	2.10/0E+03	3.3230E+00
0.270	1.0100E-02	2.5790E+UI	0.3090E+01	2.0440E+03	3.4223E+00
0.285	1.9000E-02	2.8129E+01	7.1029E+01	3.4024E+03	3.5318E+00
0.300	2.3300E-02	3.0307E+01	7.7176E+UI	4.4955E+03	3.0528E+00
0.315	2.9400E-02	3.2555E+01	8.2900E+01	0.0932E+03	3.7848E+00
0.330	3.8200E-02	3.4899E+01	8.8869E+01	8.48/0E+03	3.9288E+00
0.345	5.1400E-02	3.7223E+01	9.4/88E+01	1.21805+04	4.0857E+00
0.360	7.2100E-02	3.9330E+01	1.0015E+02	1.8053E+04	4.2565E+00
0.375	1.0/40E-01	4.0484E+01	1.0309E+02	2.7680E+04	4.4422E+00
0.390	1./320E-01	3.9652E+01	1.009/E+02	4.3/21E+04	4.640/E+00
0.405	3.0810E-01	3.5769E+01	9.1085E+01	7.0159E+04	4.8461E+00
0.420	6.0950E-01	2.8664E+01	7.2992E+01	1.1122E+05	5.0462E+00
0.435	1.3340E+00	1.9684E+01	5.0124E+01	1.6/16E+05	5.2231E+00
0.450	3.1800E+00	1.1277E+01	2.8717E+01	2.2830E+05	5.3585E+00
0.465	8.0360E+00	5.4349E+00	1.3840E+01	2.7804E+05	5.4441E+00
0.480	2.0400E+01	2.3718E+00	6.0396E+00	3.0802E+05	5.4886E+00
0.495	4.6620E+01	1.0869E+00	2.7678E+00	3.2259E+05	5.5086E+00
0.510	8.2750E+01	6.2436E-01	1.5899E+00	3.2891E+05	5.5171E+00
0.525	1.0970E+02	4.7451E-01	1.2083E+00	3.3139E+05	5.5203E+00
0.540	1.2140E+02	4.2984E-01	1.0946E+00	3.3221E+05	5.5214E+00
0.555	1.2510E+02	4.1743E-01	1.0630E+00	3.3244E+05	5.5217E+00
0.570	1.2610E+02	4.1420E-01	1.0547E+00	3.3251E+05	5.5218E+00
0.585	1.2640E+02	4.1324E-01	1.0523E+00	3.3253E+05	5.5218E+00
0.600	1.2640E+02	4.1324E-01	1.0523E+00	3.3253E+05	5.5218E+00
0.615	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.630	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.645	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.660	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.675	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.690	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.705	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.720	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.735	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
0.750	1.2650E+02	4.1292E-01	1.0515E+00	3.3254E+05	5.5218E+00
CPLL + im	e 00.00.07	21 - this is	CPU time fo	r VAX 11/785	

ties. These provide a "basis set," where the amount of each current is varied according to a variational principle. The spreading resistance obtained from this variational method is

RGENVAR

The program RGENVAR (Resistance GENeration VARiational) is based upon the simplified implementation of the variation scheme discussed by Choo *et al.* [22,23]. There are three aspects of this calculation and the code which are of interest. First, the calculation does not make use of a single-probe current density, but rather makes use of a "mix" of the uniform current density and the Schumann and Gardner semi-infinite slab current densi-

$$R_n^v = \frac{\rho_n}{2} C_n^v,$$

where

$$C_n^v = \frac{4}{\pi} \frac{K_1 I_{3s} + K_2 I_{2s}}{K_1 + K_2/2},$$

where K_1 and K_2 are given by

$$K_1 = \frac{I_2 - I_3/2}{I_1 I_2 - I_3^2},$$
$$K_2 = \frac{I_1/2 - I_3}{I_1 I_2 - I_2^2}.$$

 $I_1 = \int_0^\infty A_n(\lambda) \left\{ \frac{\sin(\lambda a)}{\lambda a} \right\}^2 a d\lambda,$

 $I_2 = \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} \right\}^2 a d\lambda,$

The integrals I_1, I_2, I_3 are

$$I_3 = \int_0^\infty A_n(\lambda) \bigg\{ \frac{\sin(\lambda a) J_1(\lambda a)}{(\lambda a)^2} \bigg\} a d\lambda.$$
 Finally, the integrals I_{3s} and I_{2s} are

$$I_{3s} = \int_0^\infty A_n(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a}\right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda,$$

and

$$I_{2s} = \int_0^\infty A_n(\lambda) \left(\frac{J_1(\lambda a)}{\lambda a}\right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda.$$

The second and third aspects of this calculation are appropriate generalizations of the RGENBL analysis to this problem. In particular, the Berkowitz-Lux approximation [21] is used to simplify the integrals (and to remove the S-dependent Bessel function with an S-dependent lower limit of integration) as

$$I_{3s} = \int_{L}^{\infty} A_{n}(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a}\right) \left(\frac{J_{1}(\lambda a)}{\lambda a}\right) d\lambda,$$

$$I_{2s} = \int_L^\infty A_n(\lambda) igg(rac{J_1(\lambda a)}{\lambda a} igg)^2 d\lambda.$$

It should be noted that the integrals, I_1 , I_2 , and I_3 , contain all of the information which is needed to evaluate the correction factor and the spreading resistance. This follows from the fact that the approximate integrals, I_{3s} and I_{2s} are contained in the corresponding I_3 and I_2 integrals. The third aspect of the calculation is the evaluation of the various integrals (I_1 , I_2 , and I_3) by means of a segmented Newton-Cotes method similar to that used by Berkowitz and Lux. The coordinates and weights of the fixed points as well as the form of the S-dependent points are listed and discussed in reference [23]. These are incorporated in RGENVAR through DATA statements and the appropriate computations.

RGENVAR reads the input data from the logical device for010. These include:

n	number of resistivity, spreading resistance values
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
ibbc	boundary condition (none=1, conducting=2, insulation=3)
subrho	substrate resistivity $(\Omega \cdot \mathbf{cm})$
${\tt subth}$	substrate thickness (micrometers)
rho(i)	<i>i-th</i> resistivity relative to the surface $(\Omega \cdot cm)$

As originally formulated, the integration scheme is not able to handle an insulating boundary. In the present implementation, this has been circumvented by starting the integration one point away from the singularity at the origin when there is an insulating boundary. This has been found to lead to very good results with no loss of generality.

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The file rgenvar.in contains the input data to be read in for010. The files rgenvar.log and rgenvar.out contain the output data written to for006 and for011. The computation also takes several CPU seconds much like RGENGL and RGENBL.

Table 10.	Annotated input data file (rgenvar.in) for RGENVAR program
Table 10. 50 2.0 50.0 0.015 1 126.5 0.015 1.542 0.6933 0.3442 0.19 0.1159 0.0769 0.0543 0.0401 0.0307 0.0242 0.0196 0.0165 0.0144 0.0131 0.0125 0.0124 0.013 0.0124 0.013 0.0141 0.019 0.0233 0.0294 0.0382 0.0514 0.0721 0.1074 0.732 0.3081 0.695 1.334 3.18 8.036 20.4 46.62 82.75 109.7 121.4 126.1 126.4 126.5 126.5	Annotated input data file (rgenvar.in) for RGENVAR program NPT-number of points in structure. Total is NPT + 1 AA-probe radius in micrometers DELTH-depth increment in micrometers IBBC - back boundary condition (none=1, cond=2, ins=3) SUBRHO-substrate thickness in micrometers RH0(NPT+1) surface resistivity in ohm*cm RH0(NPT) RH0(NPT-1)
126.5	
126.5 126.5	
126.5 126.5	
126.5	RH0(2)

Table 11.	Annotated output	data file	(rgenvar.out) for RGENVAR program
	51		total number of points calculated
0.000	8.0688E+02		depth and spreading resistance
0.015	7.6620E+02		
0.030	7.4523E+02		
0.045	7.3371E+02		
0.060	7.2839E+02		
0 075	7 2830E+02		
0.090	7 3334E+02		·
0.105	7 4399E+02		·
0.120	7 6136E+02		·
0.120	7 8711E+02		
0.150	9 2392E+02		•
0.150	9 7534E+02		•
0.105	0.46515.00		•
0.100	9.40310+02		•
0.195	1.0445E+03		•
0.210	1.1/94E+03		•
0.225	1.3046E+U3		•
0.240	1.621/E+03		•
0.255	1.97/5E+03		•
0.270	2.4/95E+03		•
0.285	3.1891E+03		
0.300	4.2110E+03		
0.315	5.7027E+03		
0.330	7.9339E+03		
0.345	1.1367E+04		
0.360	1.6804E+04		
0.375	2.5663E+04		
0.390	4.0285E+04		
0.405	6.4054E+04		
0.420	1.0034E+05		
0.435	1.4903E+05		
0.450	2.0237E+05		
0.465	2.4762E+05		
0.480	2.7753E+05		
0.495	2.9401E+05		
0 510	3 0209E+05		
0.525	3_0550E+05		·
0 540	3 0667E+05		·
0.555	3 0701E+05		•
0.570	3 0710E+05		•
0.585	3 0713E+05		•
0.000	3 0713E+05		·
0.600	2.0714E+05		•
0.015	2.0714E+05		·
0.630	3.0714E+05		•
0.045	3.0714E+05		•
0.000	3.0/14E+05		•
0.675	3.0/14E+05		•
0.690	3.0/14E+05		•
0.705	3.0/14E+05		•
0.720	3.0/14E+05		
0.735	3.0714E+05		•
0.750	3.0714E+05		
2.00000	000000000		probe radius in micrometers
50.0000	000000000		probe separation in micrometers
1.500000	0000000000E-02		depth increment in micrometers

Table 12. Annotated log file (rgenvar.log) for RGENVAR program

CALCULATION OF SPREADING RESISTANCE FROM THE RESISTIVITY USING CHOO ET AL VARIATIONAL TECHNIQUE-BL NO BOUNDARY AT 0.765 UM. SUBSTRATE AT 0.750 UM RADIUS=2.0000 SEP= 50.0 DELTH= 0.015 SUBTH=1.500E-02 SUBRH0=1.265E+02 #PTS=50

DEPTH	RHO	CORR.FAC	RSP	VOL
0.000	1.5420E+00	2.0931E-01	8.0688E+02	2.9068E+00
0.015	6.9330E-01	4.4206E-01	7.6620E+02	2.8843E+00
0.030	3.4420E-01	8.6604E-01	7.4523E+02	2.8723E+00
0.045	1.9000E-01	1.5446E+00	7.3371E+02	2.8655E+00
0.060	1.1590E-01	2.5139E+00	7.2839E+02	2.8624E+00
0.075	7.6900E-02	3.7883E+00	7.2830E+02	2.8623E+00
0.090	5.4300E-02	5.4021E+00	7.3334E+02	2.8653E+00
0.105	4.0100E-02	7.4214E+00	7.4399E+02	2.8716E+00
0.120	3.0700E-02	9.9200E+00	7.6136E+02	2.8816E+00
0.135	2.4200E-02	1.3010E+01	7.8711E+02	2.8960E+00
0.150	1.9600E-02	1.6813E+01	8.2382E+02	2.9158E+00
0.165	1.6500E-02	2.1220E+01	8.7534E+02	2.9422E+00
0.180	1.4400E-02	2.6292E+01	9.4651E+02	2.9761E+00
0.195	1.3100E-02	3.1894E+01	1.0445E+03	3.0189E+00
0.210	1.2500E-02	3.7741E+01	1.1794E+03	3.0717E+00
0.225	1.2400E-02	4.4018E+01	1.3646E+03	3.1350E+00
0.240	1.3000E-02	4.9898E+01	1.6217E+03	3.2100E+00
0.255	1.4100E-02	5.6098E+01	1.9775E+03	3.2961E+00
0.270	1.6100E-02	6.1602E+01	2.4795E+03	3.3944E+00
0.285	1.9000E-02	6.7138E+01	3.1891E+03	3.5037E+00
0.300	2.3300E-02	7.2292E+01	4.2110E+03	3.6244E+00
0.315	2.9400E-02	7.7588E+01	5.7027E+03	3.7561E+00
0.330	3.8200E-02	8.3077E+01	7.9339E+03	3.8995E+00
0.345	5.1400E-02	8.8460E+01	1.1367E+04	4.0556E+00
0.360	7.2100E-02	9.3225E+01	1.6804E+04	4.2254E+00
0.375	1.0740E-01	9.5578E+01	2.5663E+04	4.4093E+00
0.390	1.7320E-01	9.3036E+01	4.0285E+04	4.6051E+00
0.405	3.0810E-01	8.3161E+01	6.4054E+04	4.8065E+00
0.420	6.0950E-01	6.5849E+01	1.0034E+05	5.0015E+00
0.435	1.3340E+00	4.4080E+UI	1.4903E+05	5.1/33E+00
0.450	3.1800E+00	2.5455E+UI	2.0237E+05	5.3001E+00
0.405	8.0300E+00	1.2320E+UI	2.4/02E+05	5.3930E+00
0.400	2.0400E+01	3.4417E+00	2.7753L+05	5.4433L+00
0.495	4.0020E+01	2.5220E+00	2.9401E+05	5.4004E+00
0.510	1 0070E+02	1.4003E+00	3.0209E+05	5.4850E+00
0.540	1.2140E+02	1.0104E+00	3.0667E+05	5 4867E+00
0.555	1.2510E+02	9 8165E-01	3.0701E+05	5 4872E+00
0.570	1 2610E+02	9 7415E-01	3 0710E+05	5 4873E+00
0 585	1 2640E+02	9 7192E-01	3 0713E+05	5 4873E+00
0 600	1 2640E+02	9 7194F-01	3 0713E+05	5 4873E+00
0.615	1 2650E+02	9 7120E-01	3.0714E+05	5.4873E+00
0.630	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.645	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.660	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.675	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.690	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.705	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.720	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.735	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
0.750	1.2650E+02	9.7120E-01	3.0714E+05	5.4873E+00
CPU time	: 00:00:07.4	19 - this is	CPU time for	VAX 11/785

CALCULATION OF THE RESISTIVITY FROM THE TWO-PROBE RESISTANCE

The second group of programs is used to calculate the resistivity from the spreading resistance. The three programs included in this section are: BL, GL, and VAR. These are related to the programs RGENBL, RGENGL, and RGENVAR which were discussed in the previous section. For example, the calculation of the spreading resistance from the resistivity is carried out with RGENBL, then the inversion or calculation of the resistivity from the spreading resistance is accomplished with the program BL. All three inversion programs have very similar architecture and program logic. The program BL is discussed first because of historical reasons. It was the first program to make use of a bounding procedure in order to carry out the spreading resistance correction factor integral inversion. The procedure is used until the calculated spreading resistance agrees to within given limits with the input spreading resistance. This bounding procedure is extremely general and is not dependent upon the quadrature method for evaluating the correction factor integral. Hence, it is easily used in a straightforward way in the Gauss-Laguerre program, GL, and is readily generalized in the variational program, VAR, where several forms of the current density necessitate the application of the bounding procedure to several integrals.

It is of interest to note that there are a number of other programs which can be used to calculate the resistivity from the spreading resistance. These include the SPINT-SPRED programs presented by D'Avanzo *et al.* [16] and the LOCAL SLOPE program presented by Dickey [18]. Both of the programs have been discussed and documented in the appropriate references. The SPINT-SPRED programs represent the inversion of the spreading resistance following the calculation of the integrals involved in the RGENTR program. The use of a earlier version of RGENTR in the discussion of the SPINT-SPRED and the LOCAL SLOPE programs has been discussed elsewhere [19].

The BL (Berkowitz-Lux) program is based upon the Berkowitz-Lux approximation for the correction factor integral. This is discussed and used in the RGENBL program. The program calculates the resistivity by varying the resistivity needed to obtain a predetermined level of agreement between the input spreading resistance and the calculated spreading resistance. In this, the bounding procedure is very important. This is based upon the observation that the hyperbolic functions, tanh and coth, provide the limits of the form of the kernel of the correction factor integral for the cases of conducting and insulating boundaries. The correct kernel must then lie between these two extremes which then provide bounding envelopes for the kernel function. This assists the code in the seeking of the best value of the resistivity for a given value of the spreading resistance.

The BL program reads the input data from for011 (bl.in is the example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	<i>i-th</i> value of depth, in micrometers (relative to surface)
	and i-th value of two-probe resistance (relative to surface)
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)

The BL program uses the approximate correction factor, the 22-point Newton-Cotes quadrature, and the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in bl.out and bl.log. Notice that the CPU needed to carry out the inversion is on the order of seconds. This puts it into the time frame of real-time analysis which is possible on a PC-based system.

Table 13.	Annotated input This is the out	data file (bl.in) for BL program put data file (rgenbl.out) from RGENBL program.
	E 1	NPT surplus of escanding societance values
0.000	9 1510E+00	DEPTH(1) RSP(1)
0.000	7 7018F+02	DEPTH(2) $RSP(2)$
0.015	7.7910L+02	
0.030	7 50125.02	
0.045	7.50136+02	· ·
0.000	7.5770E+02	· ·
0.075	7.0180E+02	· ·
0.090	7.7036E+02	• •
0.105	7.8419E+02	
0.120	8.0460E+02	• •
0.135	8.3353E+02	
0.150	8.7382E+02	•
0.165	9.2961E+02	· · ·
0.180	1.0062E+03	
0.195	1.1112E+03	
0.210	1.2554E+03	
0.225	1.4532E+03	
0.240	1.7278E+03	
0.255	2.1078E+03	
0.270	2.6440E+03	
0.285	3.4024E+03	
0.300	4.4955E+03	
0.315	6.0932E+03	
0.330	8 4870E+03	
0 345	1 2180E+04	· ·
0.360	1 8053E+04	
0.300	2 7690E+04	· ·
0.375	2.7000E+04	· ·
0.390	4.37210+04	• •
0.405	7.0159E+04	• •
0.420	1.1122E+05	· ·
0.435	1.6/16E+05	• •
0.450	2.2830E+05	• •
0.465	2.7804E+05	· ·
0.480	3.0802E+05	
0.495	3.2259E+05	· ·
0.510	3.2891E+05	
0.525	3.3139E+05	
0.540	3.3221E+05	
0.555	3.3244E+05	
0.570	3.3251E+05	
0.585	3.3253E+05	
0.600	3.3253E+05	
0.615	3.3254E+05	
0.630	3.3254E+05	
0.645	3.3254E+05	
0.660	3.3254E+05	
0.675	3.3254E+05	
0.690	3.3254E+05	
0.705	3.3254E+05	
0.720	3.3254E+05	
0 735	3.3254F+05	
0 750	3.3254E+05	DEPTH (NPT) RSP (NPT)
2 000000	000000000	AA-probe radius in micrometers
50 00000	0000000000	SEP-probe separation in micrometers
1 500000	00000000000000000000000000000000000000	DELTH-depth increment (laver thickness)
1.000000		DECHI-dopon meromono (rayer onrekness)

51	number of corrected points
0.000 1.545E+00	depth (micrometers) and resistivity (ohm*cm)
0.015 6.951E-01	
0.030 3.449E-01	
0.045 1.904E-01	
0.060 1.160E-01	
0.075 7.691E-02	
0.090 5.431E-02	
0.105 4.012E-02	
0.120 3.070E-02	
0.135 2.420E-02	
0.150 1.960E-02	
0.165 1.649E-02	
0.180 1.441E-02	•
0.195 1.310E-02	
0.210 1.250E-02	
0.225 1.240E-02	
0.240 1.300E-02	
0.255 1.410E-02	
0.270 1.610E-02	•
0.285 1.900E-02	•
0.300 2.330E-02	•
0.315 2.940E-02	
0.330 $3.6212-020.345$ $5.141E-02$	•
0.345 $5.1412-020.360$ $7.213E-02$	•
0.375 1.074E-01	•
0.390 1.732E=01	•
0.405 3.081F-01	
0.420 6.091E-01	
0.435 1.336E+00	
0.450 3.180E+00	
0.465 8.035E+00	
0.480 2.039E+01	
0.495 4.542E+01	
0.510 8.225E+01	
0.525 1.098E+02	
0.540 1.263E+02	
0.555 1.264E+02	
0.570 1.265E+02	
0.585 1.265E+02	
0.600 1.265E+02	
0.615 1.265E+02	•
0.030 1.265E+02	•
0.045 1.205E+02	•
0.000 1.205E+02	•
0.675 1.205E+02	
0.090 1.205E+02	•
0.705 1.205E+02	•
0.720 1.205E+02	
0.750 1.205E+02	·
	•

CALCULATION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE USING THE BERKOWITZ-LUX APPROXIMATE INTEGRAL RADIUS= 2.0 SEP= 50.0 DELTH= 0.015 # PTS= 51 R5= 0.412922 R6= 0.003736 R7= 115.745620

NPT	DEPTH	RMEAS	1-RCAL/R	RHO	SENS	ITER
1	0.000	8.151E+02	-9.882E-05	1.545E+00	2.182E+01	2
2	0.015	7.792E+02	-5.763E-05	6.951E-01	4.398E+01	2
3	0.030	7.640E+02	-3.414E-05	3.449E-01	7.286E+01	2
4	0.045	7.581E+02	-2.080E-05	1.904E-01	8.813E+01	2
5	0.060	7.578E+02	-1.301E-05	1.160E-01	8.021E+01	2
6	0.075	7.618E+02	-8.485E-06	7.691E-02	6.329E+01	2
7	0.090	7.704E+02	-5.816E-06	5.431E-02	4.790E+01	2
8	0.105	7.842E+02	-4.177E-06	4.012E-02	3.611E+01	2
9	0.120	8.046E+02	-3.173E-06	3.070E-02	2.746E+01	2
10	0.135	8.335E+02	-2.580E-06	2.420E-02	2.110E+01	2
11	0 150	8 738E+02	-2.189E-06	1.960E-02	1.640F+01	2
12	0 165	9 296F+02	-2 044E-06	1 649E-02	1 301E+01	2
13	0 180	1 006E+03	-2 087E-06	1 441E-02	1 052E+01	2
14	0.105	1 111E+03	-2.007E-00	1 3105-02	9 671E+00	2
1.6	0.195	1.0555.02	-2.230L-00	1.0500-02	7 2075.00	2
10	0.210	1.255E+03	-2.702E-00	1.2500-02	7.3276+00	2
10	0.225	1.453E+03	-3.235E-06	1.240E-02	6.280E+00	2
17	0.240	1.728E+03	-4.600E-06	1.300E-02	5.539E+00	2
18	0.255	2.108E+03	-5.931E-06	1.410E-02	4.926E+00	2
19	0.270	2.644E+03	-9.154E-06	1.610E-02	4.483E+00	2
20	0.285	3.402E+03	-1.389E-05	1.900E-02	4.111E+00	2
21	0.300	4.496E+03	-2.436E-05	2.330E-02	3.813E+00	2
22	0.315	6.093E+03	-4.523E-05	2.940E-02	3.547E+00	2
23	0.330	8.487E+03	-8.382E-05	3.821E-02	3.303E+00	2
24	0.345	1.218E+04	-1.473E-04	5.141E-02	3.084E+00	2
25	0.360	1.805E+04	-1.694E-04	7.213E-02	2.896E+00	2
26	0.375	2.768E+04	2.557E-04	1.074E-01	2.763E+00	2
27	0.390	4.372E+04	4.929E-06	1.732E-01	2.725E+00	3
28	0.405	7.016E+04	6.893E-05	3.081E-01	2.830E+00	3
20	0 420	1 112E+05	4 325E-04	6 091F-01	3 174E+00	3
30	0 435	1 672E+05	-6 103E-04	1 336E+00	4 010E+00	Å
21	0.450	2 283E+05	8 160E-07	3 180E+00	5 946E+00	5
30	0.450	2.200E+05	1 110E-06	9 035E+00	1 065E+01	5
22	0.400	2.7000000	2 960E 06	2 020E.01	1.005L+01	5
24	0.400	3.000E+05	2.002E-00	2.039E+01	2.1495+01	2
34	0.495	3.220E+05	7.309E-04	4.542E+UI	3.881E+UI	3
35	0.510	3.289E+05	1.206E-04	8.225E+01	5.284E+01	3
30	0.525	3.314E+05	1.709E-05	1.098E+02	5.628E+01	3
37	0.540	3.322E+05	-6./51E-04	1.263E+02	5.660E+01	1
38	0.555	3.324E+05	-2.054E-04	1.264E+02	5.660E+01	1
39	0.570	3.325E+05	-5.860E-05	1.265E+02	5.660E+01	1
40	0.585	3.325E+05	5.080E-07	1.265E+02	5.660E+01	1
41	0.600	3.325E+05	-2.954E-05	1.265E+02	5.660E+01	1
42	0.615	3.325E+05	0.000E+00	1.265E+02	5.660E+01	1
43	0.630	3.325E+05	2.776E-17	1.265E+02	5.660E+01	1
44	0.645	3.325E+05	0.000E+00	1.265E+02	5.660E+01	1
45	0.660	3.325E+05	2.776E-17	1.265E+02	5.660E+01	1
46	0.675	3.325E+05	0.000E+00	1.265E+02	5.660E+01	1
47	0.690	3.325E+05	2.776E-17	1.265E+02	5.660E+01	1
48	0.705	3.325E+05	0.000E+00	1.265E+02	5.660E+01	1
49	0.720	3.325E+05	2.776E-17	1.265E+02	5.660E+01	1
50	0.735	3.325F+05	0.000F+00	1.265E+02	5.660F+01	1
51	0.750	3.325E+05	0.000F+00	1.265E+02	0.000F+00	Ô
CPII	time: 00.0	0.06 69 - +1	his is (PII	time for VAY	11/785	Ŭ
U U	NUME NUT	$v_{1}v_{2}v_{3}v_{3}v_{3}v_{3}v_{3}v_{3}v_{3}v_{3$	UND IN VEU	VIDE LOC VAA	111103	

The GL (Gauss-Laguerre) program is based upon the Gauss-Laguerre quadrature for calculating the correction factor integral. This is discussed and used in the RGENGL program. The present implementation of this program makes use of the same bounding procedure used in the BL code.

The GL program reads the input data from for011 (gl.in is an example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	<i>i-th</i> value of depth relative to surface and
	<i>i-th</i> value of two-probe resistance relative to surface
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
icur	current density (Schumann Gardner=1, Choo uniform=2,
	ring delta=3)

The code, like the RGENGL code, consists of the main program, GL, along with the two double precision functions, dbj0(x) and dbj1(x), needed to construct the Bessel functions involved in the calculations.

The GL program uses the Gauss-Laguerre quadrature and the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in gl.out and gl.log. Like BL, the CPU needed to carry out the inversion is on the order of seconds. The limitation of the paired set of programs, RGENGL and GL, is that the use of the fixed sampling points may not adequately represent the S-dependence of the integral and may consequently give results which are inaccurate relative to RGENTR.

Table 16.	Annotated inpu	t data file	(gl.in) for	GL program	
	This is output	data file	(rgengl.out)	from RGENGL	program

\

	51	NPT-number of	f spreading resist	tance values
0 000	8 4549E+02	DEPTH(1)	RSP(1)	
0.015	8 0781E+02	DEPTH(2)	RSP(2)	
0.030	7 0183E+02	DEPTH(3)	RSP(3)	
0.045	7 8567E+02			
0.040	7.9507E+02	•	•	
0.000	7.0026E+02	•	•	
0.075	7.09302+02	•	•	
0.090	9 10/0E+02	·	•	
0.103	0.1249L+02	•	•	
0.120	0.3302E+02	•	•	
0.155	0.0330E+02	•	•	
0.150	9.0327L+02	•	•	
0.105	9.0303E+02	•	•	
0.100	1.04236+03	·	•	
0.195	1.1510E+03	•	•	
0.210	1 50035+03	•	•	
0.225	1.5051E+03	•	•	
0.240	1./893E+03	•	•	
0.255	2.1825E+03	•	•	
0.270	2./3/4E+U3	•	•	
0.285	3.5223E+03	•	•	
0.300	4.6539E+03	•	•	
0.315	6.3090E+03	•	•	
0.330	8.7925E+03	•	•	
0.345	1.2634E+04	•	•	
0.360	1.8/69E+04	•	•	
0.375	2.8891E+04		•	
0.390	4.5895E+04		•	
0.405	7.4171E+04	•	•	
0.420	1.1844E+05	•	•	
0.435	1.7905E+05		•	
0.450	2.4531E+05	•	•	
0.465	2.9900E+05	•	•	
0.480	3.3121E+05	•	•	
0.495	3.4679E+05		•	
0.510	3.5352E+05	•	•	
0.525	3.5613E+05	•		
0.540	3.5700E+05	•	•	
0.555	3.5725E+05	•	•	
0.570	3.5732E+05	٠	•	
0.585	3.5734E+05	•	•	
0.600	3.5734E+05	•		
0.615	3.5735E+05	•	•	
0.630	3.5735E+05	•	•	
0.645	3.5735E+05	•	•	
0.660	3.5735E+05	٠	•	
0.675	3.5735E+05	٠	•	
0.690	3.5735E+05		•	
0.705	3.5735E+05	•		
0.720	3.5735E+05			
0.735	3.5735E+05			
0.750	3.5735E+05	DEPTH (NPT)	RSP (NPT)	
2.00000	000000000	AA-probe radi	us in micrometers	5
50.0000	000000000	SEP-probe sep	aration in micron	neters
1.500000	0000000000E-02	DELTH-depth i	ncrement (layer t	hickness)
	2	ICUR-probe cu	rrent density (S&	G=1 unif=2 ring=3)

Table 17. Annotated output data file (gl.out) for GL program

	51	number of points		
0.000	1.543E+00	depth (micrometers) a	ind resistivity	(ohm*cm)
0.015	6.940E-01	•		
0.030	3.446E-01	•		
0.045	1.902E-01			
0.060	1.159E-01	•		
0.075	7.692E-02	•		
0.090	5.431E-02			
0.105	4.010E-02			
0.120	3.071E-02			
0.135	2.420E-02			
0.150	1.960E-02			
0.165	1.650E-02	•		
0.180	1.441E-02			
0.195	1.310E-02			
0.210	1.250E-02			
0.225	1.240E-02			
0.240	1.300E-02			
0.255	1.410E-02			
0.270	1.610E-02			
0.285	1.900E-02			
0.300	2.330E-02			
0.315	2.941E-02			
0.330	3.822E-02			
0.345	5.142E-02			
0.360	7.213E-02			
0.375	1.074E-01			
0.390	1.732E-01	•		
0.405	3.081E-01			
0.420	6.092E-01			
0.435	1.336E+00			
0.450	3.180E+00			
0.465	8.038E+00			
0.480	2.040E+01			
0.495	4.560E+01			
0.510	8.249E+01	•		
0.525	1.097E+02	٠		
0.540	1.263E+02			
0.555	1.264E+02			
0.570	1.265E+02			
0.585	1.265E+02	•		
0.600	1.265E+02			
0.615	1.265E+02			
0.630	1.265E+02			
0.645	1.265E+02	•		
0.660	1.265E+02			
0.675	1.265E+02			
0.690	1.265E+02			
0.705	1.265E+02			
0.720	1.265E+02			
0.735	1.265E+02			
0.750	1.265E+02			

CALCULATION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE USING THE CHOO ET AL GAUSS-LAGUERRE QUADRATURE CHOO UNIFORM CURRENT DENSITY RADIUS= 2.0 SEP= 50.0 DELTH= 0.015 # PTS= 51 R5= 0.443730 R6= 0.003903 R7= 119.978679

NPT	DEPTH	RMEAS	1-RCAL/R	RHO	SENS	ITER
1	0.000	8.455E+02	-3.794E-05	1.543E+00	2.172E+01	2
2	0.015	8.078E+02	-2.407E-05	6.940E-01	4.380E+01	2
3	0.030	7.918E+02	-1.552E-05	3.446E-01	7.259E+01	2
4	0.045	7.857E+02	-1.022E-05	1.902E-01	8.792E+01	2
5	0.060	7.852E+02	-6.917E-06	1.159E-01	8.010E+01	2
6	0.075	7.894E+02	-4.963E-06	7.692E-02	6.328E+01	2
7	0.090	7.982E+02	-3.854E-06	5.431E-02	4.789E+01	2
8	0.105	8.125E+02	-3.258E-06	4.010E-02	3.610E+01	2
9	0.120	8.336E+02	-3.070E-06	3.071E-02	2.747E+01	2
10	0.135	8.636E+02	-3.183E-06	2.420E-02	2.111E+01	2
11	0.150	9.053E+02	-3.440E-06	1.960E-02	1.641E+01	2
12	0.165	9.630E+02	-3.988E-06	1.650E-02	1.302E+01	2
13	0.180	1.042E+03	-4.799E-06	1.441E-02	1.052E+01	2
14	0.195	1.151E+03	-5.773E-06	1.310E-02	8.674E+00	2
15	0.210	1.300E+03	-7.528E-06	1.250E-02	7.329E+00	2
16	0.225	1.505E+03	-9.049E-06	1.240E-02	6.284E+00	2
17	0.240	1.789E+03	-1.286E-05	1.300E-02	5.543E+00	2
18	0.255	2.183E+03	-1.610E-05	1.410E-02	4.929E+00	2
19	0.270	2.737E+03	-2.367E-05	1.610E-02	4.485E+00	2
20	0.285	3.522E+03	-3.348E-05	1.900E-02	4.112E+00	2
21	0.300	4.654E+03	-5.371E-05	2.330E-02	3.813E+00	2
22	0.315	6.309E+03	-8.934E-05	2.941E-02	3.544E+00	2
23	0.330	8.793E+03	-1.461E-04	3.822E-02	3.295E+00	2
24	0.345	1.263E+04	-2.275E-04	5.142E-02	3.071E+00	2
25	0.360	1.877E+04	-2.447E-04	7.213E-02	2.875E+00	2
26	0.375	2.889E+04	2.09/E-04	1.0/4E-01	2./34E+00	2
27	0.390	4.590E+04	4.046E-06	1./32E-01	2.688E+00	3
28	0.405	/.41/E+04	5.862E-05	3.081E-01	2.787E+00	3
29	0.420	1.184E+05	3.782E-04	6.092E-01	3.12/E+00	3
30	0.435	1./91E+05	-5.083E-04	1.336E+00	3.959E+00	4
31	0.450	2.453E+05	5.049E-07	3.180E+00	5.905E+00	5
32	0.465	2.990E+05	1 4495 06	0.038E+00	1.004E+01	5
24	0.460	3.312E+03	I.440E-00	2.040E+01	2.13/E+UI 2.021E.01	2
34	0.495	3.406E+03	5.099E-04	4.500E+01	5.931E+01	ు స
25	0.510	3.535E+U5	9.708E-05	0.249E+UI	5.361E+UI	2
30	0.525	2 570E.05	1.434E-03	1.0976+02	5.740E+01	1
38	0.540	3.570E+05	-0.830L-04	1.203L+02	5.707E+01	1
30	0.535	3.573E+05	-1.912L-04	1.204C+02	5.707E+01	1
40	0.585	3.573E+05	-3.455L-05	1.205L+02	5.707E+01	1
40	0.000	3 573E+05	-2 750E-05	1 265E+02	5 787E+01	1
42	0.615	3 574E+05	0.000E+00	1.265E+02	5 787E+01	1
42	0.630	3 574E+05	-2 776F-17	1.265E+02	5.787E+01	1
40	0 645	3 574E+05	0.000F+00	1 265E+02	5 787E+01	1
45	0.660	3 574E+05	-2 776E-17	1 265E+02	5 787E+01	1
46	0.675	3.574E+05	0.000F+00	1 265E+02	5 787E+01	1
47	0.690	3.574E+05	-2.776F-17	1.265E+02	5.787E+01	1
48	0.705	3.574F+05	0.000F+00	1.265E+02	5.787E+01	1
49	0.720	3.574E+05	-2.776E-17	1.265E+02	5.787F+01	1
50	0.735	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
51	0.750	3.574E+05	0.000E+00	1.265E+02	0.000E+00	0
						-

CPU time: 00:00:07.14 - this is CPU time for VAX 11/785

VAR

The VAR (VARiational) program is based upon the variational technique for calculating the correction factor integral. This is discussed and used in the RGENVAR program. The present implementation of this program makes use of the bounding procedure used in the BL code. As there are several integrals involved (coming from the mixing of the two current forms), the bounding procedure must be generalized to include all of the integrals.

The VAR program reads the input data from for011 (var.in is an example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	<i>i-th</i> value of depth (relative to surface)
	and <i>i</i> -th value of two-probe resistance (relative to surface)
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)

The code, like the RGENVAR code, consists of just the main program, VAR. This program uses the variational generalization of the Berkowitz-Lux approximation for the integrals, the extended Newton-Cotes quadrature, and the above-mentioned generalization of the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in var.out and var.log. Like BL, the CPU needed to carry out the inversion is on the order of seconds.

Table 19.	Annotated input This is output o	data file (var.in) for VAR program data file (rgenvar.out) from RGENVAR program.
	51	NPT, number of encoding registeres values
0.000	0 0666E*00	NETTH(1) PSP(1)
0.000	7 6620E.02	
0.015	7.45225.02	
0.030	7 22715.00	DEF (11(5) K31 (5)
0.045	7.00205.00	· ·
0.000	7.2039E+02	• •
0.075	7.203UE+UZ	· ·
0.090	7.3334E+02	· ·
0.105	7.4399E+02	• •
0.120	7.0130E+02	· ·
0.135	7.8/11E+02	· ·
0.150	8.2382E+02	· ·
0.165	8.7534E+02	• •
0.180	9.4651E+02	
0.195	1.0445E+03	· ·
0.210	1.1794E+03	· ·
0.225	1.3646E+03	· ·
0.240	1.6217E+03	· ·
0.255	1.9775E+03	
0.270	2.4795E+03	
0.285	3.1891E+03	
0.300	4.2110E+03	
0.315	5.7027E+03	
0.330	7.9339E+03	
0.345	1.1367E+04	
0.360	1.6804E+04	
0.375	2.5663E+04	
0.390	4.0285E+04	
0.405	6.4054E+04	
0.420	1.0034E+05	
0.435	1.4903E+05	
0.450	2.0237E+05	
0.465	2.4762E+05	
0.480	2.7753E+05	
0.495	2.9401E+05	
0.510	3.0209E+05	
0.525	3.0550E+05	
0.540	3.0667E+05	
0.555	3.0701E+05	
0.570	3.0710E+05	
0.585	3.0713E+05	
0.600	3.0713E+05	
0.615	3.0714E+05	
0.630	3.0714E+05	
0.645	3.0714E+05	
0.660	3.0714E+05	
0.675	3.0714E+05	
0.690	3.0714E+05	
0.705	3.0714E+05	
0.720	3.0714E+05	
0.735	3.0714E+05	
0.750	3.0714E+05	DEPTH (NPT) RSP (NPT)
2,00000	000000000	AA-probe radius in micrometers
50.0000	000000000	SEP-probe separation in micrometers
1.500000	0000000000E-02	DELTH-depth increment (laver thickness)

Table 20. 7	Annotated	output	data	file	(var.out)	for	VAR	program

51		number of points		
0.000E+00	1.536E+00	depth (micrometers)	and resistivity	(ohm∗cm)
1.500E-02	6.835E-01			
3.000E-02	3.563E-01			
4.500E-02	1.873E-01			
6 000E-02	1.110E-01			
7 500E-02	7 565E-02	•		
9.000E-02	5 400E-02	•		
9.000L-02	3.400L-02	•		
1.050E-01	4.000E-02	•		
1.200E-01	3.00/E-02	•		
1.350E-01	2.419E-02	•		
1.500E-01	1.960E-02			
1.650E-01	1.650E-02			
1.800E-01	1.440E-02			
1.950E-01	1.310E-02			
2.100E-01	1.250E-02			
2.250E-01	1.240E-02			
2 400E-01	1.300E-02			
2.550E-01	1 410E-02			
2.330E-01	1.610E-02	•		
2.700E-01	1.0101-02	•		
2.8502-01	1.900E-02	•		
3.000E-01	2.330E-02	•		
3.150E-01	2.940E-02	•		
3.300E-01	3.820E-02			
3.450E-01	5.140E-02			
3.600E-01	7.220E-02			
3.750E-01	1.072E-01			
3.900E-01	1.732E-01			
4 050E-01	3 079E-01			
4.200E-01	6 084E-01	•		
4.2505 01	1 2225+00	•		
4.350E-01	2.1775.00	•		
4.500E-01	3.1//E+00	•		
4.650E-01	8.018E+00	•		
4.800E-01	2.014E+01	•		
4.950E-01	4.650E+01	٠		
5.100E-01	8.210E+01			
5.250E-01	1.088E+02			
5.400E-01	1.216E+02	•		
5.550E-01	1.264E+02			
5 700E-01	1.265E+02			
5 850E-01	1 265E+02			
6.000E-01	1.265E+02			
6 1505 01	1.2050-02	۰		
6.150E-01	1.205E+02	•		
0.300E-01	1.205E+02	•		
6.450E-01	1.265E+02	•		
6.600E-01	1.265E+02	•		
6.750E-01	1.265E+02			
6.900E-01	1.265E+02			
7.050E-01	1.265E+02			
7.200E-01	1.265E+02			
7.350E-01	1.265E+02			
7.500E-01	1.265E+02			

Table 21. Annotated log file (var.log) for VAR program

CALCULATION OF THE RESISTIVITY FROM SPREADING RESISTANCE USING CHOO ET AL VARIATIONAL TECHNIQUE-BL AA= 0.000 SEP= 25.000 DELTH= 0.00750 NPT= 51 R5= 0.7628E+00 R6= 0.7473E-02 R7= 0.2314E+03

NPT	DEPTH	RMEAS	1-RCAL/R	RHO	ITER
1	0.000E+00	8.069E+02	1.472E-04	1.536E+00	3
2	1.500E-02	7.662E+02	5.933E-04	6.835E-01	3
3	3.000E-02	7.452E+02	-9.894E-04	3.563E-01	4
4	4.500E-02	7.337E+02	5.037E-05	1.873E-01	5
5	6.000E-02	7.284E+02	8.414E-04	1.110E-01	3
6	7.500E-02	7.283E+02	3.534E-04	7.565E-02	3
7	9.000E-02	7.333E+02	1.571E-04	5.400E-02	3
8	1.050E-01	7.440E+02	7.414E-05	4.000E-02	3
9	1.200E-01	7.614E+02	3.881E-05	3.067E-02	3
10	1.350E-01	7.871E+02	2.187E-05	2.419E-02	3
11	1.500E-01	8.238E+02	1.193E-05	1.960E-02	3
12	1.650E-01	8.753E+02	7.087E-06	1.650E-02	3
13	1.800E-01	9.465E+02	4.265E-06	1.440E-02	3
14	1.950E-01	1.045E+03	2.452E-06	1.310E-02	3
15	2.100E-01	1.179E+03	1.613E-06	1.250E-02	3
16	2.250E-01	1.365E+03	8.666E-07	1.240E-02	3
17	2.400E-01	1.622E+03	6.332E-07	1.300E-02	3
18	2.550F-01	1.978E+03	3.445E-07	1.410E-02	3
19	2.700E-01	2.480E+03	2.576E-07	1.610E-02	3
20	2.850F-01	3.189F+03	1.759E-07	1.900E-02	3
21	3.000E-01	4.211F+03	1.600E-07	2.330E-02	3
22	3 150E-01	5 703E+03	1.669E-07	2.940F-02	3
23	3 300E-01	7 934F+03	1 904E-07	3 820E-02	3
24	3 450F-01	1 137E+04	2 112E=07	5 140E-02	3
25	3 600E-01	1 680E+04	-7 855E-04	7 220E-02	2
26	3 750E-01	2 566E+04	9 868F-04	1 072E-01	2
27	3 900E-01	A 020E+04	1 204E-05	1 732E-01	2
28	4 050E-01	6 405E+04	1 208E-04	3 070F-01	3
20	4.200E-01	1 003E+05	6 416E-04	6 084E-01	3
20	4.350E-01	1 400E+05	5 601E-07	1 333E+00	5
31	4.500E-01	2 024E+05	4 305E_06	3 177E+00	5
30	4.650E-01	2.024L+05	4.609E-05	8 018E+00	5
32	4.000E-01	2 775E+05	7 224E-04	2 014E+00	5
34	4.050E-01	2.775E+05	5 300E-05	4 650E+01	7
35	5 100E-01	2.940L405	2 038E-04	9.210E+01	5
36	5.250E-01	3.021L+05	2.030L-04	1 099E+02	3
27	5.250E-01	3.055L+05	2.009L-04	1.000L+02	3
20	5.400L-01	2 070E-05	-2 919E-04	1.2100+02	1
20	5.550L-01	2 071E.05	-2.0100-04	1.2046402	1
39	5.700L-01	2 071E+05	-9.420L-03	1.2051+02	1
40	5.050E-01	2 071E+05	2 161E OF	1.2036+02	1
41	6 150E 01	2 071E+05	-3.101E-05	1.2036+02	1
42	6 200E 01	2 071E+05	-2.776E-17	1.2056702	1
43	6.300E-01	2 071E+05	-2.776E-17	1.205L+02	1
44	6.430E-01	2 071E+05	-2.776E 17	1.205L+02	1
40	6 750E 01	2 071E+05	-2.776E 17	1.2036+02	1
40	6.750E-01	3.071E+05	-2.770E-17	1.205E+02	1
47	7 0505 01	3.071E+05 2.071E.05	-2.110E-11	1.205E+02	1
40	7.050E-01	3.0/1E+05	-2.110E-11	1.203E+02	4
49	7.2000-01	3.0/1E+05 2.071E.0F	-2.110E-11	1.205E+02	1
50	7.35UE-UI	3.0/1E+05	-2.110E-11	1.205E+02	1
10	1.500E-01	3.0/1E+05		1.205E+V2	0
			- INTI 1 100 - 4/		

CPU time: 00:00:07.12 - this is CPU time for VAX 11/785

CALCULATION OF THE FOUR-PROBE RESISTANCE FROM THE RESISTIVITY

The third and final group of programs is used to calculate the four-probe resistance from the resistivity. The three programs included in this section are: PROBE24, FOURCAL, and SHAZAM. PROBE24 and FOURCAL are included for the calculation of the four-probe resistance by means of fixed point and trapezoidal Romberg techniques, respectively. On the other hand, SHAZAM is based upon an approximate form of the four-probe resistance integral which can be fairly easily calculated using a Newton-Cotes quadrature.

PROBE24

The multilayer expression for the in-line four-probe resistance, $Z_n(S)$, has been derived in the form [28]

$$Z_n(S) = 2\rho_n \int_0^\infty A_n(\lambda) \Big\{ J_0(\lambda S) - J_0(2\lambda S) \Big\} \mathcal{I}_{\nu}(\lambda a) d\lambda,$$

where $\mathcal{I}_{\nu}(\lambda a)$ is the Hankel transform of the generalized probe-current density normalized to unit current [10], and ν is an index which refers to the specific form of the current density.

The PROBE24 code calculates the four-probe resistance from the resistivity using a 1250or 10000-point sum to replace the above integral. In its present form, PROBE24 makes use of the uniform current density. As discussed in the FORTRAN source code, this can be changed with no difficulty. However, the results of the previous work [28,29] indicate that the four-probe resistance is independent of both the current density and the probe radius. Hence, the current density need not be altered.

The program reads the input data from for005. Probe24.in contains an example of the data file in the form containing the following information.

npt	number of data points
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
aa	probe radius (micrometers)
subrho	substrate resistivity $(\Omega \cdot cm)$
subth	substrate thickness (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
rho	array of resistivity values $(\Omega \cdot cm)$
nmany	number of integration points to be used (either 1250 or 10000)

The PROBE24 code consists of the main program along with the double precision functions dbj0(x) and dbj1(x) which are used to construct the Bessel functions which appear in the integral. The kernel function is evaluated directly inside the program. Once the calculation has been completed, the output data are written to for006, for011, for012, for013. Not all of the information contained in these files may be of interest. The user may wish to comment out the lines involving writing to logical devices 11, 12, and 13.

Table 22.	Annotated input data file (probe24.in) for PROBE24 program
2 0	AA - probe radius in micrometers
50 0	
50.0	SEP - probe separation in micrometers
0.015	DELTH - depth increment in micrometers
50	NPT - number of points
0.015	SUBT - substrate thickness in micrometers
126 5	SITTRHO - substrate resistivity in obm*cm
1	IPPC - substrate boundary condition (none=1 con=2 ins=3)
1 540	
1.542	
0.6933	RHU (NPT)
0.3442	
0.19	
0.1159	
0 0769	
0.0543	
0.0040	•
0.0401	
0.0307	
0.0242	
0.0196	
0.0165	
0.0144	
0.0131	
0.0125	
0.0123	
0.0124	•
0.013	
0.0141	
0.0161	
0.019	
0.0233	
0.0294	
0.0382	
0.0514	
0.0721	•
0.0721	•
0.10/4	
0.1/32	•
0.3081	
0.6095	
1.334	
3.18	
8.036	
20.4	
46.62	
82 75	
100 7	
101 4	•
121.4	•
125.1	•
126.1	
126.4	
126.4	
126.5	
126.5	
126.5	
126.5	
126.5	
126 5	
126 F	
120.5	
120.5	
120.5	NU(ANV
1250	MMMMI - number of integration points

Table 23. Annotated log file (probe24.log) for PROBE24 program

FOUR-PROBE RESISTANCE CALCULATIONS USING THE BERKOWITZ NMANY= 1250 POINT INTEGRATOR STRUCTURE OVER UNIFORM LAYER (SUBRHO,SUBTH) ALL OVER (NONE=1,CON=2,INS=3) 1

AA	SEP DI	ELTH	SUBTI	H	SUBRI	10
2.00	50.0 0.0	D15	1.50E	-02	1.27E4	⊦02
2.00 DEPTH (UM) 0.0000E+00 1.5000E-02 3.0000E-02 4.5000E-02 4.5000E-02 9.0000E-02 1.0500E-01 1.2000E-01 1.3500E-01 1.5000E-01 1.5000E-01 1.6500E-01 2.5500E-01 2.4000E-01 2.5500E-01 3.0000E-01 3.0000E-01 3.0000E-01 3.4500E-01 3.0000E-01 3.4500E-01 3.0000E-01 3.0000E-01 3.0000E-01 4.0500E-01 4.0500E-01 4.0500E-01 4.5000E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.500E-01 5.000E-01 6.1500E-01 6.000E-01 6.000E-01	50.0 0.0 RH0 1.5420E+00 6.9330E-01 3.4420E-01 1.9000E-01 1.1590E-01 1.590E-01 7.6900E-02 5.4300E-02 4.0100E-02 3.0700E-02 1.4400E-02 1.3100E-02 1.4400E-02 1.3100E-02 1.2500E-02 1.2400E-02 1.3000E-02 1.3000E-02 1.2400E-02 1.3000E-02 1.2500E-02 1.2500E-02 1.2500E-01 1.3340E+00 3.1800E+00 8.0360E+00 2.0400E+01 4.6620E+01 8.2750E+01 1.0970E+02 1.2650E+02 1.26	FOUR- 1.439 1.440 1.442 1.446 1.454 1.454 1.456 1.486 1.298 1.696 2.196 2.000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4.0000 4	1.505F PR0+02 6E+02 6E+02 6E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 6E+02 8E+02 8E+02 6E+02 8E+02 8E+02 8E+02 6E+02 8E+03 8E+03	-02 DERIVAT 6.6443E 6.6206E 6.6206E 6.6206E 6.6207E 6.6207E 6.6207E 6.6207E 6.6207E 6.7183E 6.7183E 6.7183E 6.9035E 7.1199E 7.3761E 7.7291E 8.2148E 8.8776E 9.7822E 1.1016E 1.2695E 1.5000E 1.8143E 2.2491E 2.8469E 3.6759E 4.8234E 6.4145E 8.6069E 1.1548E 1.5248E 1.5248E 1.5248E 1.5248E 2.2663E 2.4545E 2.2663E 2.4545E 2.5663E 2.6922E 2.6923E	1.27E+ IVE +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +02 +03 +03 +03 +03 +03 +03 +03 +03 +03 +03 +03 +03 +03 +03 +04 <t< td=""><td>SHEET 5.7505E+02 5.7549E+02 5.7549E+02 5.7648E+02 5.7648E+02 5.8213E+02 5.8213E+02 5.8213E+02 5.8213E+02 5.9757E+02 7.1128E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.45778E+02 7.3306E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.2858E+03 1.2466E+03 3.1680E+03 3.2434E+03 3.2434E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 5.8028E+03 3.2246E+03 5.8028E+03 5.8028E+03 5.2246E+03 5.8028E+03 5.2246E+03 5.8028E+03 5.2246E+03 5.2026E+06 5.2026E+06 5.0263E+06 5.0263E+06 5.0263E+06 5.0269E+06</td></t<>	SHEET 5.7505E+02 5.7549E+02 5.7549E+02 5.7648E+02 5.7648E+02 5.8213E+02 5.8213E+02 5.8213E+02 5.8213E+02 5.9757E+02 7.1128E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.3072E+02 7.45778E+02 7.3306E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.474E+03 1.2858E+03 1.2466E+03 3.1680E+03 3.2434E+03 3.2434E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 3.2246E+03 5.8028E+03 3.2246E+03 5.8028E+03 5.8028E+03 5.2246E+03 5.8028E+03 5.2246E+03 5.8028E+03 5.2246E+03 5.2026E+06 5.2026E+06 5.0263E+06 5.0263E+06 5.0263E+06 5.0269E+06
7.3500E-01	1.2650E+02	4.030	1E+03	2.6923E	+04 4	.2167E+07
7.5000E-01	1.2650E+02	4.030	1E+03	2.6923E	+04 8	.4333E+07

CPU time: 00:00:11.87 - this is CPU time for VAX 11/785

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FOURCAL

The FOURCAL code makes use of the trapezoidal Romberg numerical integration to evaluate the integral in the four-probe resistance,

$$Z_n(S) = 2
ho_n \int_0^\infty A_n(\lambda) \Big\{ J_0(\lambda S) - J_0(2\lambda S) \Big\} \mathcal{I}_
u(\lambda a) d\lambda$$

This technique has been discussed previously in the RGENTR program.

FOURCAL performs the calculation for any of the three forms of the probe current density. Calculations using FOURCAL were originally employed to show that the four-probe resistance is independent of the probe current density and the probe radius. The program reads the input data from for005. Fourcal.in contains an example of the data file in the form containing the following information.

npt	number of data points
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none =1, conducting =2, insulating =3)
subrho	substrate resistivity $(\Omega \cdot cm)$
${\tt subth}$	substrate thickness (micrometers)
rho	array of resistivity values $(\Omega \cdot cm)$
icur	current density (Schumann Gardner=1, Choo uniform=2, ring delta=3)
xl	lower limit of integration (usually zero)
xu	upper limit of integration (from 10 on up)

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The FORTRAN source code consists of the following parts:

fourcal	the main program	
intgrl(f,a,b,atemp,iflag)	the subroutine which carries out the numerical integration	
fpint(x)	the double precision function which constructs the integrand	
	of the four-probe resistance integral	
deint(x)	the double precision function which constructs the integrand	
	of the logarithmic derivative function (related to the	
	Dickey conjecture)	
alam(x)	the double precision function which constructs the kernel	
	of the integral by means of the recursion relation	
dbj0(x) and $dbj1(x)$	the two double precision functions generate the two Bessel	
	functions, J_0 and J_1	

Once the calculation has been completed, the output data are written to for006, for011, for012, for013. Not all of the information contained in these files may be of interest. The user may wish to comment out the lines involving writing to logical devices 11, 12, and 13.

Table 24.	Annotated input data file (fourcal.in) for FOURCAL program
50 2.0 50.0 0.015 1 126.5 0.015 1.542 0.6933 0.3442	NPT - number of points in the structure AA - probe radius in micrometers SEP - probe separation in micrometers DELTH - depth increment in micrometers IBBC - boundary condition (none=1,cond=2,ins=3) SUBRHO - substrate resistivity in ohm*cm SUBTH - substrate thickness in micrometers RHO (NPT+1) RHO (NPT) RHO (NPT-1)
0.1900 0.1159 0.0769 0.0543 0.0401 0.0307	
0.01242 0.0196 0.0165 0.0144 0.0131 0.0125	
0.0124 0.013 0.0141 0.0161 0.019 0.0233	
0.0294 0.0382 0.0514 0.0721 0.1074 0.1732 0.3081	
0.6095 1.334 3.18 8.036 20.4 46.62	
82.75 109.7 121.4 125.1 126.1 126.4	
126.4 126.5 126.5 126.5 126.5 126.5 126.5 126.5 126.5 126.5	
126.5 2 0.0 10.0	ICUR - current density (S&G=1, unif=2, ring=3) XL - lower limit of integral (usually zero) XU - upper limit of integral

Table 25. Annotated log file (fourcal.log) for FOURCAL program

CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X NO BOUNDARY CONDITION USED IN CALCULATION

XL= 0.00	10000; XU = 10.	.000000		
RADIUS=	2.000000; SEP=	= 50.000000;	DELTH= (0.015000
SUBRH0=1.2	6500E+02; SUB1	HICK=1.50000)E-02	
DEPTH(U)	RHÓ	FOUR-PROBE	DERIV	SHEET
0.000E+00	1.542E+00	1.437F+02	6.686F+02	6.750F+02
1 500E=02	6 933E-01	1 438E+02	6 638E+02	6 755E+02
2 000E-02	2 4425-01	1 4405.02	6 605E.02	6 765E.00
3.000E=02	1 000E 01	1.4400+02		0.705E+02
4.500E-02	1.900E-01	1.444E+02	0.033E+02	6.785E+02
6.000E-02	1.159E-01	1.451E+02	6.662E+02	6.821E+02
7.500E-02	7.690E-02	1.464E+02	6.716E+02	6.882E+02
9.000E-02	5.430E-02	1.483E+02	6.803E+02	6.976E+02
1.050E-01	4.010E-02	1.511E+02	6.932E+02	7.113E+02
1.200E-01	3.070E-02	1.551E+02	7.115E+02	7.307E+02
1.350E-01	2.420E-02	1.606E+02	7.370E+02	7.578E+02
1.500E-01	1.960E-02	1.682F+02	7.723E+02	7.951E+02
1 650E-01	1 650E-02	1 786E+02	8 207E+02	8 466E+02
1 0005-01	1 4405-02	1 0205.02	0.2072402	0.1705.00
1.0002-01	1.440E=02	1.920E+02	0.0092+02	9.1/20+02
1.950E-01	1.3106-02	2.122E+U2	9.7726+02	1.014E+03
2.100E-01	1.250E-02	2.385E+02	1.100E+03	1.14/E+03
2.250E-01	1.240E-02	2.742E+02	1.268E+03	1.331E+03
2.400E-01	1.300E-02	3.228E+02	1.498E+03	1.586E÷03
2.550E-01	1.410E-02	3.887E+02	1.812E+03	1.941E+03
2.700E-01	1.610E-02	4.789E+02	2.246E+03	2.446E+03
2.850E-01	1.900E-02	6.013E+02	2.843E+03	3.168E+03
3.000F-01	2.330F-02	7.680E+02	3.671E+03	4.225E+03
3 150E-01	2 940E-02	9 933E+02	4 818E+03	5 803E+03
3 3005-01	3 920E_02	1 2055+02	6 100E+03	9 243E+02
2 4505 01	5.020L-02	1.6025.02	0.409L+03	1 0105.04
3.450E-01	5.140E-02	1.093E+03	8.002E+03	1.2190-04
3.000E-01	7.210E-02	2.195E+03	1.155E+04	1.892E+04
3.750E-01	1.0/4E-01	2.773E+03	1.525E+04	3.120E+04
3.900E-01	1.732E-01	3.338E+03	1.929E+04	5.528E+04
4.050E-01	3.081E-01	3.750E+03	2.268E+04	1.061E+05
4.200E-01	6.095E-01	3.950E+03	2.457E+04	2.193E+05
4.350E-01	1.334E+00	4.012E+03	2.527E+04	4.765E+05
4.500E-01	3.180E+00	4.025E+03	2.558E+04	1.026E+06
4.650E-01	8.036E+00	4.028E+03	2.596E+04	1.989E+06
4.800F-01	2.040E+01	4.028F+03	2.660F+04	3.164E+06
A 050E-01	4 662E+01	1.028E+03	2 7345+04	4 123E+06
5 100E-01	9 075E-01	4.020E+03	2.7042+04	A 7545.06
5.1002-01	0.2/3E+VI	4.0202+03	2.7090+04	4.754E+00
5.2500-01	1.0976+02	4.0200+03	2.0100+04	5.202E+00
5.400E-01	1.2146+02	4.028E+03	2.825E+04	5.601E+06
5.550E-01	1.251E+02	4.028E+03	2.828E+04	6.01/E+06
5.700E-01	1.261E+02	4.028E+03	2.829E+04	6.485±+06
5.850E-01	1.264E+02	4.028E+03	2.829E+04	7.027E+06
6.000E-01	1.264E+02	4.028E+03	2.829E+04	7.666E+06
6.150E-01	1.265E+02	4.028E+03	2.829E+04	8.433E+06
6.300E-01	1.265E+02	4.028E+03	2.829E+04	9.370E+06
6.450E-01	1.265E+02	4.028E+03	2.829E+04	1.054E+07
6.600E-01	1.265E+02	4.028F+03	2.829F+04	1.205E+07
6.750E-01	1.265E+02	4.028E+03	2 829F+04	1 406E±07
6 900F-01	1 265E+02	4 028F+03	2 820F+04	1 687E+07
7 0505-01	1 2655.02	1 0285+02	2 8205.04	2 1005.07
7 2005 01	1 0655.00	A 000E.02	0 0005.04	2.1V0E+U/
7 2505 01	1.203E+02	4.020E+U3	2.0295+04	4.017E.07
7.350E-01	1.205E+U2	4.028E+03	2.8292+04	4.21/E+0/
7.500E-01	1.265E+02	4.028E+03	2.829E+04	8.433E+07
CPU time:	00:31:25.85 -	this is CPU	time for V	AX 11/785
SHAZAM

Using the PROBE24 and FOURCAL programs, extensive numerical investigations have shown that the four-probe resistance is: (i) independent of the probe radius and the probecurrent density, and (ii) simply related to the sheet resistance, \mathcal{R}_n , when the distance to an insulating boundary is small compared with the probe spacing. The evaluation of the four-probe resistance integral was found to be simplified by the alternative form given by

$$Z_n(S) = \frac{\rho_n}{\pi} \int_{1/2S}^{1/S} A_n(\lambda) d\lambda.$$

In the above form, the four-probe resistance can easily be calculated using the Newton-Cotes method of evaluation,

$$Z_n(S) = \frac{\rho_n}{\pi} \ln(2) \sum_{i=0}^8 w_{9,i} \lambda_i A_n(\lambda_i),$$

where $\lambda_i = 2^{i/8}/2S(0 \le i \le 8), w_{9,i} = C_i/8$, and where C_i are the Newton-Cotes weighting factors.

The SHAZAM code carries out the above calculation of the four-probe resistance using the approximate four-probe integral and the nine-point Newton-Cotes quadrature. The program reads the input data from for009. Shazam.in contains an example of the data file in the form containing the following information.

n	number of resistivity, sheet resistance values
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
ibbc	<pre>boundary condition (none=0; conducting=1; insulating=2)</pre>
subrho	substrate resistivity $(\Omega \cdot cm)$
subt	substrate thickness (micrometers)
rho(i)	i-th resistivity relative to the surface

Once the calculation has been completed, the output data are written to for006, for011, for012. Not all of the information contained in these files may be of interest.

Table 26.	Annotated input data file (shazam.in) for SHAZAM program
50.0 0.015 50 126.5	SEP - probe separation in micrometers DELTH - depth increment in micrometers N - number of points SUBRHO - substrate resistivity in ohm*cm
0.015 1 1 542	IBBC - boundary condition (none-1,con-2,ins-3) RH0(NPT+1)
0.6933	RH0 (NPT) RH0 (NPT-1)
0.19	
0.1159	
0.0769	
0.0543	•
0.0307	·
0.0242	
0.0196	
0.0165	
0.0144	
0.0131	
0.0125	•
0.0124	·
0.0141	
0.0161	
0.019	
0.0233	
0.0294	
0.0382	•
0.0721	
0.1074	
0.1732	
0.3081	
0.6095	
1.334	•
8.036	•
20.4	
46.62	
82.75	
109.7	
121.4	•
125.1	·
126.4	
126.4	
126.5	
126.5	
120.5	•
120.5	•
126.5	
126.5	
126.5	
126.5	

Table 27. Annotated log file (shazam.log) for SHAZAM program

FOUR-PROBE RESISTANCE CALCULATED BY SIMPLIFIED INTEGRAL ALSO CALCULATED IS THE EDUCATED GUESS OF THE LN(S) DERIVATIVE FORM NO BOUNDARY AT 0.765 UM. SUBSTRATE AT 0.750 UM SEP= 50. DELTH= 0.015 SUBTH=1.500E-02 SUBRH0=1.265E+02

LOWER LIMIT= 0.01000000 UPPER LIMIT= 0.02000000

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	DEPTH	RHO	FOUR-PROBE	DERIVATIVE
$\begin{array}{llllllllllllllllllllllllllllllllllll$	0.000	1.5420E+00	1.4347E+02	4.2657E-02
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.015	6.9330E-01	1.4355E+02	9.4924E-02
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.030	3.4420E-01	1.4374E+02	1.9146E-01
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.045	1.9000E-01	1.4415E+02	3.4783E-01
0.075 7.6900E-02 1.4514E+02 8.7138E-01 0.090 5.4300E-02 1.4805E+02 1.2504E+00 0.105 4.0100E-02 1.5085E+02 1.7255E+00 0.120 3.0700E-02 1.6330E+02 3.0407E+00 0.135 2.4200E-02 1.6786E+02 3.9338E+00 0.165 1.6500E-02 1.7824E+02 4.9659E+00 0.180 1.4400E-02 1.9237E+02 6.1480E+00 0.195 1.3000E-02 2.1159E+02 7.4446E+00 0.210 1.2500E-02 2.7299E+02 1.0197E+01 0.240 1.3000E-02 3.2105E+02 1.4432E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.275 1.4100E-02 3.8586E+02 1.2790E+01 0.300 2.3300E-02 7.5279E+02 1.5548E+01 0.315 2.9400E-02 1.5994E+03 1.6030E+01 0.330 3.8200E-02 1.2436E+03 1.6030E+01 0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.345 5.1400E-01 3.2321E+03 1.5046E+01 0.375 <td>0.060</td> <td>1.1590F-01</td> <td>1.4490E+02</td> <td>5 7320E-01</td>	0.060	1.1590F-01	1.4490E+02	5 7320E-01
0.090 5.4300E-02 1.4805E+02 1.2504E+00 0.105 4.0100E-02 1.5085E+02 1.7255E+00 0.120 3.0700E-02 1.5481E+02 2.3137E+00 0.135 2.4200E-02 1.6030E+02 3.938E+00 0.150 1.9600E-02 1.6786E+02 3.9338E+00 0.165 1.6500E-02 1.7224E+02 4.9659E+00 0.180 1.4400E-02 1.9237E+02 6.1480E+00 0.210 1.2500E-02 2.7299E+02 1.0197E+01 0.240 1.3000E-02 3.2105E+02 1.4432E+01 0.241 1.3000E-02 3.2105E+02 1.4482E+01 0.255 1.4100E-02 3.8586E+02 1.2790E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.300 2.3300E-02 7.5279E+02 1.6070E+01 0.303 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 2.0331E+03 1.6030E+01 0.345 5.1400E-02 2.0331E+03 1.6030E+01 0.390 1.7320E-01 3.0135E+03 1.6037E+01 0.405	0.075	7 6900E=02	1 4614E+02	8 7138E-01
0.105 0.100E-02 1.500E+02 1.7255E+00 0.120 3.0700E-02 1.5481E+02 2.3137E+00 0.135 2.4200E-02 1.6030E+02 3.0407E+00 0.165 1.6500E-02 1.6786E+02 3.9338E+00 0.165 1.6500E-02 1.7824E+02 4.9659E+00 0.180 1.4400E-02 1.9237E+02 6.1480E+00 0.210 1.2500E-02 2.3769E+02 8.7827E+00 0.210 1.2500E-02 2.3769E+02 1.0197E+01 0.240 1.3000E-02 3.2105E+02 1.483E+01 0.255 1.4100E-02 3.8586E+02 1.2790E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.300 2.3300E-02 7.5279E+02 1.6070E+01 0.315 2.9400E-02 9.6519E+02 1.6030E+01 0.330 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 2.0331E+03 1.6030E+01 0.345 5.1400E-01 3.4296E+03 6.6577E+00 0.405 3.0810E-01 3.4296E+03 1.6297E+01 0.345	0.000	5 4300E-02	1 49055-02	1 25045.00
0.120 3.0700E-02 1.548E+02 2.3137E+00 0.135 2.4200E-02 1.6030E+02 3.0407E+00 0.150 1.9600E-02 1.6786E+02 3.9338E+00 0.165 1.6500E-02 1.7824E+02 4.9659E+00 0.180 1.4400E-02 2.3769E+02 8.7827E+00 0.195 1.3100E-02 2.3769E+02 8.7827E+00 0.225 1.2400E-02 2.7299E+02 1.0197E+01 0.240 1.3000E-02 3.2105E+02 1.4438E+01 0.255 1.4100E-02 3.8586E+02 1.2790E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.300 2.3300E-02 7.5279E+02 1.6070E+01 0.315 2.9400E-02 9.6519E+02 1.6070E+01 0.345 5.1400E-02 2.0331E+03 1.6030E+01 0.345 5.1400E-02 2.0331E+03 1.6030E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+001 0.455 <td>0.105</td> <td>A 0100E-02</td> <td>1 50955.02</td> <td>1 70555.00</td>	0.105	A 0100E-02	1 50955.02	1 70555.00
0.120 3.0100E-02 1.6340E+02 2.3137E+00 0.135 2.4200E-02 1.6786E+02 3.9338E+00 0.165 1.6500E-02 1.7824E+02 4.9659E+00 0.180 1.4400E-02 1.9237E+02 6.1480E+00 0.195 1.3100E-02 2.1159E+02 7.4446E+00 0.210 1.2500E+02 2.7299E+02 1.0197E+01 0.225 1.2400E-02 3.2105E+02 1.4432E+01 0.255 1.4100E-02 3.8586E+02 1.2790E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.300 2.3300E-02 7.5279E+02 1.6070E+01 0.300 2.3300E-02 1.5994E+03 1.6030E+01 0.300 2.3300E-02 1.5994E+03 1.6030E+01 0.345 5.1400E-02 2.0331E+03 1.5046E+01 0.330 3.8200E-02 2.0331E+03 1.0057E+01 0.345 5.1400E-01 3.5231E+03 1.6030E+01 0.350 1.7320E-01 3.0135E+03 1.0057E+01 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.445 <td>0.100</td> <td>2.0700E-02</td> <td>1.50052402</td> <td>2 21275.00</td>	0.100	2.0700E-02	1.50052402	2 21275.00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.120	3.0700E=02	1.54010+02	2.31376+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.135	2.42002-02	1.6030E+02	3.0407E+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.150	1.9600E-02	1.6786E+02	3.9338E+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.165	1.6500E-02	1./824E+02	4.9659E+00
0.195 1.3100E-02 2.1159E+02 7.4446E+00 0.210 1.2500E-02 2.3769E+02 8.7827E+00 0.225 1.2400E-02 2.7299E+02 1.0197E+01 0.240 1.3000E-02 3.2105E+02 1.1483E+01 0.255 1.4100E-02 4.7411E+02 1.3860E+01 0.270 1.6100E-02 4.7411E+02 1.3860E+01 0.285 1.9000E-02 5.9290E+02 1.4827E+01 0.300 2.3300E-02 7.5279E+02 1.6070E+01 0.330 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.360 7.2100E-02 2.031E+03 1.3045E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.455 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.455	0.180	1.4400E-02	1.9237E+02	6.1480E+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.195	1.3100E-02	2.1159E+02	7.4446E+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.210	1.2500E-02	2.3769E+02	8.7827E+00
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.225	1.2400E-02	2.7299E+02	1.0197E+01
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.240	1.3000E-02	3.2105E+02	1.1483E+01
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.255	1.4100E-02	3.8586E+02	1.2790E+01
0.285 1.9000E-02 5.9290E+02 1.4827E+01 0.300 2.3300E-02 7.5279E+02 1.5548E+01 0.315 2.9400E-02 9.6519E+02 1.6070E+01 0.330 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.360 7.2100E-02 2.0331E+03 1.5046E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.455 1.3340E+00 3.8870E+03 1.8097E+00 0.456 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.480 2.0400E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0063E-02 0.555 1.2610E+02 4.0266E+03 2.0016E-02 0.560 <td>0.270</td> <td>1.6100E-02</td> <td>4.7411E+02</td> <td>1.3860E+01</td>	0.270	1.6100E-02	4.7411E+02	1.3860E+01
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.285	1.9000E-02	5.9290E+02	1.4827E+01
0.315 2.9400E-02 9.6519E+02 1.6070E+01 0.330 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.360 7.2100E-02 2.0331E+03 1.5046E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0016E-02 0.560 1.2640E+02 4.0266E+03 2.0000E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.615 <td>0.300</td> <td>2.3300E-02</td> <td>7.5279E+02</td> <td>1.5548E+01</td>	0.300	2.3300E-02	7.5279E+02	1.5548E+01
0.330 3.8200E-02 1.2436E+03 1.6297E+01 0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.360 7.2100E-02 2.0331E+03 1.5046E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.455 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+01 4.0179E+03 1.2366E-01 0.465 8.0360E+01 4.0255E+03 2.0839E-02 0.510 8.2750E+01 4.0255E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0063E-02 0.570 1.2610E+02 4.0266E+03 2.0006E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.600 <td>0.315</td> <td>2.9400E-02</td> <td>9.6519E+02</td> <td>1.6070E+01</td>	0.315	2.9400E-02	9.6519E+02	1.6070E+01
0.345 5.1400E-02 1.5994E+03 1.6030E+01 0.360 7.2100E-02 2.0331E+03 1.5046E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.455 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+01 4.0179E+03 1.2366E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0255E+03 3.0562E-02 0.510 8.2750E+01 4.0255E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0063E-02 0.540 1.2610E+02 4.0266E+03 2.0016E-02 0.570 1.2610E+02 4.0266E+03 2.0000E-02 0.585 <td>0.330</td> <td>3.8200E-02</td> <td>1.2436E+03</td> <td>1.6297E+01</td>	0.330	3.8200E-02	1.2436E+03	1.6297E+01
0.360 7.2100E-02 2.0331E+03 1.5046E+01 0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+01 4.0179E+03 1.2366E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0255E+03 3.0562E-02 0.510 8.2750E+01 4.0255E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0224E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 <td>0.345</td> <td>5.1400E-02</td> <td>1.5994E+03</td> <td>1.6030E+01</td>	0.345	5.1400E-02	1.5994E+03	1.6030E+01
0.375 1.0740E-01 2.5231E+03 1.3045E+01 0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.485 2.0400E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0024E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 <td>0.360</td> <td>7 2100F-02</td> <td>2.0331E+03</td> <td>1.5046F+01</td>	0.360	7 2100F-02	2.0331E+03	1.5046F+01
0.390 1.7320E-01 3.0135E+03 1.0057E+01 0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.540 1.2140E+02 4.0266E+03 2.0063E-02 0.555 1.2510E+02 4.0266E+03 2.0016E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.600 1.2650E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 <td>0 375</td> <td>1 0740E-01</td> <td>2 5231E+03</td> <td>1.3045E+01</td>	0 375	1 0740E-01	2 5231E+03	1.3045E+01
0.405 3.0810E-01 3.4296E+03 6.6577E+00 0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.455 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.540 1.2140E+02 4.0266E+03 2.0063E-02 0.555 1.2510E+02 4.0266E+03 2.0016E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.690 <td>0.390</td> <td>1 7320E-01</td> <td>3 0135E+03</td> <td>1.0057E+01</td>	0.390	1 7320E-01	3 0135E+03	1.0057E+01
0.420 6.0950E-01 3.7199E+03 3.7388E+00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.485 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0266E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0000E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 <td>0 405</td> <td>3 0810E-01</td> <td>3 4296E+03</td> <td>6 6577E+00</td>	0 405	3 0810E-01	3 4296E+03	6 6577E+00
0.420 0.0300E-01 0.01050E-03 0.000E-00 0.435 1.3340E+00 3.8870E+03 1.8097E+00 0.450 3.1800E+00 3.9685E+03 7.8036E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.540 1.2140E+02 4.0266E+03 2.0063E-02 0.555 1.2510E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.690 <td>0 420</td> <td>6 0050E-01</td> <td>3 7100F+03</td> <td>3 7388E+00</td>	0 420	6 0050E-01	3 7100F+03	3 7388E+00
0.450 3.1800E+00 3.9685E+03 7.8036E+00 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.485 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0063E-02 0.555 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 <td>0.435</td> <td>1 3340E+00</td> <td>3 8870E+03</td> <td>1 8007E+00</td>	0.435	1 3340E+00	3 8870E+03	1 8007E+00
0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.465 8.0360E+00 4.0038E+03 3.1246E-01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0024E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 <td>0.450</td> <td>1.3340E+00</td> <td>3 0685E+03</td> <td>7 8036E-01</td>	0.450	1.3340E+00	3 0685E+03	7 8036E-01
0.485 0.3030E+00 4.0038E+03 3.1240E+01 0.480 2.0400E+01 4.0179E+03 1.2366E-01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0265E+03 2.0839E-02 0.540 1.2140E+02 4.0266E+03 2.0024E-02 0.555 1.2510E+02 4.0266E+03 2.0038E-02 0.555 1.2510E+02 4.0266E+03 2.0016E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 <td>0.450</td> <td>9.0260E.00</td> <td>4 0028E+03</td> <td>2 1246E-01</td>	0.450	9.0260E.00	4 0028E+03	2 1246E-01
0.480 2.0400E+01 4.0179E+03 1.2380E+01 0.495 4.6620E+01 4.0233E+03 5.4210E-02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0263E+03 2.3060E-02 0.540 1.2140E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0024E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 <td>0.405</td> <td>0.0300E+00</td> <td>4.00302+03</td> <td>1 22665 01</td>	0.405	0.0300E+00	4.00302+03	1 22665 01
0.495 4.0620E+01 4.0233E+03 5.4210E+02 0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0263E+03 2.0839E-02 0.540 1.2140E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0024E-02 0.570 1.2610E+02 4.0266E+03 2.0016E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 <td>0.480</td> <td>2.0400E+01</td> <td>4.01795+03</td> <td>1.2300E-01</td>	0.480	2.0400E+01	4.01795+03	1.2300E-01
0.510 8.2750E+01 4.0255E+03 3.0562E-02 0.525 1.0970E+02 4.0263E+03 2.3060E-02 0.540 1.2140E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0224E-02 0.570 1.2610E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0000E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 <td>0.495</td> <td>4.6620E+01</td> <td>4.02336+03</td> <td>5.4210E-02</td>	0.495	4.6620E+01	4.02336+03	5.4210E-02
0.525 1.0970E+02 4.0263E+03 2.3060E+02 0.540 1.2140E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0224E-02 0.570 1.2610E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 <td>0.510</td> <td>8.2750E+01</td> <td>4.0255E+03</td> <td>3.05020-02</td>	0.510	8.2750E+01	4.0255E+03	3.05020-02
0.540 1.2140E+02 4.0265E+03 2.0839E-02 0.555 1.2510E+02 4.0266E+03 2.0224E-02 0.570 1.2610E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 <td>0.525</td> <td>1.0970E+02</td> <td>4.0263E+03</td> <td>2.3060E-02</td>	0.525	1.0970E+02	4.0263E+03	2.3060E-02
0.555 1.2510E+02 4.0266E+03 2.0224E-02 0.570 1.2610E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.540	1.2140E+02	4.0265E+03	2.0839E-02
0.570 1.2610E+02 4.0266E+03 2.0063E-02 0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.555	1.2510E+02	4.0266E+03	2.0224E-02
0.585 1.2640E+02 4.0266E+03 2.0016E-02 0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.570	1.2610E+02	4.0266E+03	2.0063E-02
0.600 1.2640E+02 4.0266E+03 2.0016E-02 0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.585	1.2640E+02	4.0266E+03	2.0016E-02
0.615 1.2650E+02 4.0266E+03 2.0000E-02 0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.600	1.2640E+02	4.0266E+03	2.0016E-02
0.630 1.2650E+02 4.0266E+03 2.0000E-02 0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.615	1.2650E+02	4.0266E+03	2.0000E-02
0.645 1.2650E+02 4.0266E+03 2.0000E-02 0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.630	1.2650E+02	4.0266E+03	2.0000E-02
0.660 1.2650E+02 4.0266E+03 2.0000E-02 0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.645	1.2650E+02	4.0266E+03	2.0000E-02
0.675 1.2650E+02 4.0266E+03 2.0000E-02 0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.660	1.2650E+02	4.0266E+03	2.0000E-02
0.690 1.2650E+02 4.0266E+03 2.0000E-02 0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.675	1.2650E+02	4.0266E+03	2.0000E-02
0.705 1.2650E+02 4.0266E+03 2.0000E-02 0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.690	1.2650E+02	4.0266E+03	2.0000E-02
0.720 1.2650E+02 4.0266E+03 2.0000E-02 0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.705	1.2650E+02	4.0266E+03	2.0000E-02
0.735 1.2650E+02 4.0266E+03 2.0000E-02 0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.720	1.2650E+02	4.0266E+03	2.0000E-02
0.750 1.2650E+02 4.0266E+03 2.0000E-02	0.735	1.2650E+02	4.0266E+03	2.0000E-02
	0.750	1.2650E+02	4.0266E+03	2.0000E-02

CPU time: 00:00:06.69 - this is CPU time for VAX 11/785

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AVAILABILITY OF RESPAC SOFTWARE PACKAGE

The source codes for the RESPAC software package are written in FORTRAN77 and have been run on a VAX 11/785 minicomputer system. These codes should run on other systems with little, if any, need for modification.

In all, there are ten programs contained in the RESPAC software package. The FOR-TRAN77 source code (total of about 123 kbytes) and sample input and output data files are available in ASCII format using a number of transfer vehicles. These include: standard 8-track magnetic tape (ASCII, density = 1600, Record = 80, block = 1600), 5.25-in. (360-kbyte and 1.2-Mbyte) DOS-formatted floppy disks, and using electronic mail over the Internet. This package is self-contained and is straightforward to run once the FORTRAN is compiled and linked by the user-supplied software. The sample input and output data files are included so that the user can check the programs for proper operation as well as to become acquainted with the setup and use of the codes.

For more information or to receive a copy of the code and the report, please contact:

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In requesting a copy of the RESPAC software package, please specify the transfer mode and include an 8-track tape or a DOS-formatted floppy disk. Requests over Internet will be answered by return e-mail in ASCII mode.

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C***** C	**************************************
	THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT TWO-PROBE (SPREADING) RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. THE CORRECTION FACTOR INTEGRAL IS CALCULATED OVER THE INTERVAL FROM XL TO XU BY MEANS OF A TRAPEZOIDAL ROMBERG METHOD USED ORIGINALLY BY D'AVANZO ET AL. IN THEIR "SPINT" PROGRAM FOR CALCULATING THE ARRAY OF PARTIAL INTEGRALS.
	REFERENCES: THE TRAPEZOIDAL ROMBERG METHOD WAS USED BY D'AVANZO ET AL. FOR CALCULATING THE CORRECTION FACTOR INTEGRAL FOR THE PURPOSE OF SPREADING RESISTANCE PROFILE ANALYSIS. THIS IS DESCRIBED IN THE REPORT "SPREADING RESISTANCE FOR IMPURITY PROFILES" BY DONALD C. D'AVANZO, ROBERT D. RUNG, AND ROBERT W. DUTTON, TECHNICAL REPORT NO. 5013-2 (FEBRUARY 1977), STANFORD UNIVERSITY, STANFORD, CALIFORNIA. THIS IS ALSO DESCRIBED IN THE PAPER "HIGH SPEED IMPLEMENTATION AND EXPERIMENTAL EVALUATION OF MULTILAYER SPREADING RESISTANCE ANALYSIS" BY DONALD C. D'AVANZO, ROBERT D. RUNG, ARNON GAT, AND ROBERT W. DUTTON, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 125, PP.1170-1176 (1978). THE CALCULATION OF THE SPREADING RESISTANCE FROM THE RESISTIVITY USING THE TRAPEZOIDAL ROMBERG MEHTOD IS DISCUSSED IN THE PAPER "COMPARISON OF SPREADING RESISTANCE CORRECTION FACTOR ALGORITHMS USING MODEL DATA" BY JOHN ALBERS, SOLID-STATE ELECTRONICS, VOL. 23, PP. 1197-1205 (1980).
C C C C	THE CALCULATION MAY BE CARRIED OUT FOR THE CHOO ET AL. UNIFORM CURRENT DENSITY, THE SCHUMANN AND GARDNER CURRENT DENSITY, OR THE RING DELTA FUNCTION CURRENT DENSITY.
с с с с с с с с с с	THE PROGRAM READS (FROM FORO10) THE NUMBER OF POINTS IN THE STRUCTURE, THE PROBE RADIUS, THE PROBE SEPARATION, DEPTH INCREMENT (IN MICROMETERS), THE SUBSTRATE RESISTIVITY, SUBSTRATE THICKNESS, THE BACKSURFACE BOUNDARY CONDITION, THE RESISTIVIES OF THE STRUCTURE (FROM THE TOP SURFACE), THE PROBE CURRENT DENSITY, AND THE LOWER AND UPPER LIMITS OF INTEGRATION.
C C C	THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C C C	THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES AS WELL AS THE RESULTS OF THE CALCULATION.
C C	IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, PLOTTING, OR FURTHER ANALYSIS.
C****	***************************************

IMPLICIT DOUBLE PRECISION (A-H, 0-Z) EXTERNAL TPINT DIMENSION RH0 (300), CF (300), RSP (300), DEPTH (300), VOL (300) COMMON DELTH, AA, SEP, XL, XU, RHO, NUM1, ICUR, IBBC, SUBRHO, SUBTH DATA PI/3.141592654D0/ С С ----INPUT SECTION---С THE FOLLOWING VARIABLES ARE READ BY THE PROGRAM FROM FORO10 С NPT----NUMBER OF DATA POINTS IN THE STRUCTURE. TOTAL NUMBER OF С POINTS IS NUM=NPT+1 (INCLUDE SUBSTRATE POINT) С AA----PROBE RADIUS (MICROMETERS) С SEP----SEPARATION BETWEEN THE PROBES (MICROMETERS) С DELTH--DEPTH INCREMENT (MICROMETERS) С IBBC---BACKSURFACE BOUNDARY CONDITION С (NONE=1, COND=2, INS=3) С SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM) С SUBTH--SUBSTRATE THICKNESS (MICROMETERS) С RHO----ARRAY OF RESISTIVITY VALUES (OHM*CM) С ICUR---FORM OF THE CURRENT DENSITY C C (SCHUMANN GARDNER=1, CHOO UNIFORM=2, RING DELTA =3) XL----LOWER LIMIT OF CORRECTION FACTOR INTEGRAL (USUALLY ZERO) С XU-----UPPER LIMIT OF CORRECTION FACTOR INTEGRAL С С TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY, С TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET С SUBTH=DELTH AND SUBRHO=RHO(1). С C* ****** С READ THE TOTAL NUMBER OF POINTS OR SUBLAYERS IN STRUCTURE. READ (10,*) NPT NUM=NPT+1 С READ THE EFFECTIVE ELECTRICAL PROBE RADIUS READ (10,*) AA С READ THE SEPARATION BETWEEN THE PROBES READ (10, *) SEP READ THE DISTANCE BETWEEN POINTS OR THE SUBLAYER THICKNESS. С READ (10,*) DELTH С READ THE BOUNDARY CONDITION (NONE=1, COND=2, INS=3) READ (10,*) IBBC С READ THE SUBSTRATE RESISTIVITY READ (10, *) SUBRHO С READ THE SUBSTRATE THICKNESS READ (10,*) SUBTH

С NEXT LOOP READS THE RESISTIVITIES FROM THE C SURFACE TO THE SUBSTRATE. DO 10 I=NUM,2,-1 READ (10,*) RHO(I) 10 CONTINUE RHO(1) = SUBRHOС READ THE FORM OF THE CURRENT DENSITY C (SCHUMANN GARDNER=1, CHOO UNIF=2, RING=3) READ (10,*) ICUR READ THE LOWER LIMIT OF INTEGRATION (USUALLY ZERO) С READ (10,*) XL С READ THE UPPER LIMIT OF INTEGRATION (20/AA USUALLY GOOD FOR XU) READ (10,*) XU С С THE 1200 DO LOOP IS THE MAJOR LOOP WHICH CALCULATES THE CORRECTION С FACTOR INTEGRALS (OVER THE INTERVAL FROM XL TO XU) FROM THE SUBSTRATE С (NUM1=NUM) TO THE SURFACE (NUM1=1) BY INCREMENTING NUM1 BY -1. С THE RELEVANT INFORMATION IS PASSED THROUGH AND CONTAINED IN THE С COMMON STATEMENT IN THE MAIN PROGRAM AND THE SUBROUTINES. С IFLAG IS SET EQUAL TO ZERO AS XL IS USUALLY ZERO AND THE INTEGRAND IS ZERO AT XL=O. HOWEVER, THE CURRENT DENSITY FUNCTIONS MAY GIVE С ARTIFICIAL SINGULARITIES AT THE ORIGIN. FOR EXAMPLE, THE SCHUMANN С С AND GARDNER CURRENT DENSITY FUNCTION, SIN(X)/X, GOES TO ZERO AS X С GOES TO ZERO BUT THE COMPUTER WOULD SIGNAL AN OVERFLOW DUE TO THE С 1/X. THIS IS REMOVED BY THE SETTING OF IFLAG=0. C D0 1200 NUM1=1, NUM IFLAG=0 CALL INTGRL (TPINT, XL, XU, CF (NUM1), IFLAG) CF(NUM1) = 4.0D0 * CF(NUM1)/PIRSP(NUM1) = CF(NUM1) * RHO(NUM1) / (2.0D-4 * AA)DEPTH (NUM1) = DELTH* (NUM-NUM1) VOL(NUM1)=DLOG10(RSP(NUM1)) 1200 CONTINUE

WRITE (6,7777) 7777 FORMAT(1H1) WRITE (6,1300) 1300 FORMAT (2X, 'CALCULATION OF SPREADING RESISTANCE FROM THE 1 RESISTIVITY',/,2X, 2 'USING THE TRAPEZOIDAL ROMBERG INTEGRATION METHOD') GO TO (1400,1420,1440), ICUR 1400 WRITE(6,1410) 1410 FORMAT (2X, 'SCHUMANN AND GARDNER CURRENT DENSITY=SIN(AX)/X') GO TO 1453 1420 WRITE (6,1430) 1430 FORMAT(2X, 'CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X') GO TO 1453 1440 WRITE (6,1450) 1450 FORMAT(2X, 'RING CURRENT DENSITY=A*DBJO(AX)') 1453 GO TO (1455,1465,1475), IBBC 1455 WRITE (6,1460) 1460 FORMAT (2X, 'NO BOUNDARY (INFINITE SLAB) CONDITION USED 1 IN CALCULATION'./) GO TO 1500 1465 WRITE (6,1470) 1470 FORMAT (2X, 'PERFECTLY CONDUCTING BOUNDARY A1=TANH(LD)',/) GO TO 1500 1475 WRITE (6,1480) 1480 FORMAT (2X, 'PERFECTLY INSULATING BOUNDARY A1=COTH(LD)',/) 1500 WRITE (6,1550) 1550 FORMAT (2X, ' XL XU') WRITE (6,1600) XL,XU 1600 FORMAT(2X,2(F11.6,1X)) WRITE (6,1700) 1700 FORMAT (2X, ' RADIUS SEP DELTH') WRITE (6,2200) AA, SEP, DELTH 2200 FORMAT (1X,3(F11.6,1X)) WRITE (6,2250) SUBRHO, SUBTH 2250 FORMAT (2X, 'SUBRHO=', 1PE11.5, '; SUBTHICK=', 1PE11.5) WRITE (6,2300) 2300 FORMAT (2X, 'DEPTH (U) ', 8X, 'RHO', 5X, 'CORR-FAC', 8X, 'RSP',7X,'VOLTAGE') 1 WRITE (11.*)NUM D0 2500 I=NUM,1,-1 WRITE (6,2400) DEPTH(I),RHO(I),CF(I),RSP(I),VOL(I) 2400 FORMAT(1X,6(1PE11.5,1X)) WRITE (11,2405) DEPTH(I), RSP(I) 2405 FORMAT(1X,2(1PE12.4,1X)) 2500 CONTINUE WRITE (11,*)AA WRITE (11,*)SEP WRITE (11,*)DELTH STOP FND

	SUBROUTINE INTGRL(F,A,B,ATEMP,IFLAG)
C****	*******
С	
С	D'AVANZO'S ET AL. TRAPEZOIDAL ROMBERG INTEGRATION SUBROUTINE.
C	THIS ROUTINE NUMERICALLY INTEGRATES F BETWEEN THE LIMITS A AND B
č	LISTNG THE TRAPEZOTDAL AND ROMBERG METHODS AND RETURNS THE INTEGRAL
ĉ	EDON & TO R TN ATEND AND ACCIMUMATES THE DESIMITS OF SEVERAL
C	CHOCEGGETVE CALLS THE ACC
C	TE TELACO THEN E (A) TE SET TO A (THIS OPVILATES OVEDELOW DOOD ENS
C	IF IFLAGED THEN F(U) IS SET TO U (THIS UDVIATES UVERFLUW FRUDLEMS
C	DUE TU FUNCTIONS LIKE SIN(X)/X
C	
(****	******
С	IMPLICIT DOUBLE PRECISION (A-H,O-Z)
	EXTERNAL F
	DOUBLE PRECISION AI (30,30), MI, A, B, H, ACC, ATEMP, FO, F, CONV
С	DIMENSION AI(30,30)
	CONV=5.0D-5
	J=30
	M=1
	K=1
	F0=0.D0
	IF ((A.NE.O.DO).OR.(IFLAG.EQ.1)) FO=F(A)
	AI(M,K) = .5DO*(B-A)*(FO+F(B))
	D0 30 I=1.J
	MT=0.0D0
	H = (B - A) / (2 ODO * * (M - 1))
	I P(NT=2**(M-1))
	D0 10 N=1 LPCNT
	MT = MT + F(A + (N + H - 5DO + H))
10	
10	MT-MT+H
	$AT(M_{+}1 K) = FDO_{+}(AT(M K) MT)$
	NDONE = N.1
	MFUNE = M+1
	$\frac{1}{2} \nabla \nabla$
00	AI(M+1, K+1) = ((4.0D0**K)*AI(M+1, K) - AI(M, K)) / (4.0D0**K-1.D0)
20	
	$A = M^{2} = A = (M^{2} + 1, M^{2})$
0	IF (I.LE.6) GU IU 60
C	STUP INTEGRATING WHEN INTEGRAL CUNVERGES TO WITHIN CONV (.005%)
	IF (DABS(ATEMP-A1(M,M))/DABS(ATEMP).GT.CUNV) GU TU 60
53	ACC=ACC+ATEMP
	RETURN
60	M=M+1
	K=1
30	CONTINUE
	RETURN
	END
C****	END OF INTGRL SUBROUTINE************************************

C****	DOUBLE PRECISION FUNCTION TPINT(X)
000000000000	TPINT IS THE INTEGRAND OF THE SCHUMANN AND GARDNER MULTILAYER LAPLACE EXPRESSION FOR TWO-PROBE RESISTANCE INTEGRAL WITH A CHOICE OF ONE OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY. LINE 10 CONTAINS THE PROBE-TO-PROBE CONTRIBUTION WHILE THE CHOICE OF THE PROBE CURRENT DENSITY IS PROVIDED ON THE FOLLOWING LINES: 100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY 200-THE CHOO ET AL. UNIFORM CURRENT DENSITY 300-THE RING DELTA FUNCTION CURRENT DENSITY
C****	**********
C C 10 100 200	IMPLICIT DOUBLE PRECISION (A-H,O-Z) DIMENSION RH0(300) DOUBLE PRECISION X,DBJ0,DBJ1,AA,SEP,ALAM,DELTH,RH0(300),XL,XU DOUBLE PRECISION BESINT,SUBRH0,SUBTH COMMON DELTH,AA,SEP,XL,XU,RH0,NUM1,ICUR,IBBC,SUBRH0,SUBTH PRTPR=(DBJ1(AA*X)/(AA*X)-DBJ0(SEP*X)/2.DO) G0 T0 (100,200,300),ICUR TPINT=ALAM(X)*PRTPR*DSIN(X*AA)/X RETURN TPINT=ALAM(X)*PRTPR*2.DO*DBJ1(AA*X)/X
300	RETURN TPINT=ALAM(X)*PRTPR*AA*DBJO(AA*X) RETURN END
C****	<pre>*END OF FTPINT DOUBLE PRECISION FUNCTION************************************</pre>
C****	***************************************
	THIS FUNCTION CALCULATES THE VALUE OF THE RESISTIVITY DEPENDENT PORTION OF THE VARIOUS INTEGRALS USING THE RECURSION RELATION OF CHOO ET AL. AND EXTENDED BY BERKOWITZ AND LUX.
	REFERENCES: "ON THE CALCULATION OF SPREADING RESISTANCE CORRECTION FACTORS" BY S. C. CHOO, M. S. LEONG AND K. L. KUAN, SOLID-STATE ELECTRONICS, VOL. 19, PP. 561-565 (1976). "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES" BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 128, PP. 1137-1141 (1981).
C****	*******
	IMPLICIT DOUBLE PRECISION (A-H,O-Z) DIMENSION RH0(300) COMMON DELTH,AA,SEP,XL,XU,RH0,NUM1,ICUR,IBBC,SUBRH0,SUBTH DTXSB=DTANH(X*SUBTH) DTXDL=DTANH(X*DELTH) ALAM=1.0D0 IF (IBBC.EQ.2) ALAM=DTXSB IF (IBBC.EQ.3) ALAM=1.0D0/DTXSB IF (NUM1.EQ.1) GO TO 110
	ALAM = (DTXDL * RHO (I) + RHO (I-1) * ALAM) / (RHO (I) + RHO (I-1) * ALAM * DTXDL)

100 110	CONTINUE RETURN
C****	END *END OF ALAM DOUBLE PRECISION FUNCTION************************************
C****	**************************************
C	DOLYNONTAL ADDDOVTNATTON TO THE DECCEL EUNCTION DIO
C	PULYNUMIAL APPRUXIMATION TO THE BESSEL FUNCTION BJO
с с с с с	REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.1 IS USED FOR X < 3 AND EQUATION 9.4.3 IS USED FOR X > 3
C****	***************************************
C	IMPLICI! DOUBLE PRECISION $(A-H, 0-Z)$ DOUBLE PRECISION X X1 X2 FO THETA
C	DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C	DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7
C	DUUBLE PRECISIUN CI,C2,C3,C4,C5,C6,C7 DATA A1/1 000000000/ A2/-2 249999700/ A3/1 265620800/
	DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
	DATA A7/0.0002100D0/
	DATA B1/.79788456D0/,B2/00000077D0/,B3/00552740D0/
	DATA B4/-:0000931200/,B3/:0013723700/,B0/-:0007280300/
	DATA C1/78539816D0/,C2/04166397D0/,C3/00003954D0/
	DATA C4/.00262573D0/,C5/00054125D0/,C6/00029333D0/
	IF (X.GT.3.0D0) G0 T0 10
	X2=X*X/9.0D0
	DBJO=((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
10	CONTINUE
10	X1=3.D0/X
	F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
	IHE IA = X + ((((((/*XI+C0)*XI+C0)*XI+C4)*XI+C3)*XI+C2)*XI+C1) $DB.IO = FO * DCOS (THE TA) / DSORT(X)$
	RETURN
Carrier	END
U****	*FIND OL NDIO NOODFE LUECTOTON LONCITON************************************

DOUBLE PRECISION FUNCTION DBJ1(X) C**** ****** ***** С С POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1 С С REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ С AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED С MATHEMATICS SERIES 55. EQUATION 9.4.4 IS USED FOR X < 3 С AND EQUATION 9.4.6 IS USED FOR X > 3 С IMPLICIT DOUBLE PRECISION (A-H.O-Z) С DOUBLE PRECISION X, X1, X2, F1, THETA С DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7 С DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7 С DOUBLE PRECISION C1.C2.C3.C4.C5.C6.C7 DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/ DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/ DATA A7/.00001109D0/ DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/ DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/ DATA B7/-.00020033D0/ DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/ DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/ DATA C7/-.00029166D0/ IF (X.GT.3.0D0) G0 T0 10 X2=X*X/9.D0 DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1) RETURN 10 X1=3.0D0/X F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+((((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJ1=F1*DCOS(THETA)/DSQRT(X) RETURN END

C****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF RGENTR V1.0 PROGRAM****

C**** C	**************************************
000000000000000000000000000000000000000	THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING) RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE CHOO ET AL. 33-POINT INTEGRATION METHOD BASED UPON THE GAUSS-LAGUERRE QUADRATURE. THE 33 SAMPLING POINTS AND WEIGHTS ARE INITIALIZED BY DATA STATEMENTS. THE FINAL SET OF 33 WEIGHTS USED IN THE CALCULATIONS ARE CALCULATED FROM THE INITIAL SET OF WEIGHTS AND THE PROBE CURRENT DENSITY. THE DISCUSSION OF THE 33-POINT GAUSS-LAGUERRE QUADRATURE MAY BE FOUND IN THE REFERENCE.
C C C C C	REFERENCE: "SPREADING RESISTANCE CALCULATIONS BY THE GAUSS-LAGUERRE QUADRATURE," BY S. C. CHOO, M. S. LEONG, H. L. HONG, L. LE, AND L. S. TAN, SOLID STATE ELECTRONICS VOL 21, PP. 769-774, 1978.
	THE CURRENT DENSITIES WHICH MAY BE USED ARE THE SCHUMANN AND GARDNER, THE CHOO ET AL. UNIFORM CURRENT DENSITY, AND THE RING DELTA FUNCTION CURRENT DENSITY.
	THE PROGRAM READS (FROM FORO1O) THE PROBE RADIUS, PROBE SEPARATION, DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE RESISTIVITY, SUBSTRATE THICKNESS, THE RESISTIVITY ARRAY (FROM THE TOP SURFACE), AND THE PROBE CURRENT DENSITY.
	THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
	THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES AS WELL AS THE RESULTS OF THE CALCULATION.
	IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, PLOTTING, OR FURTHER ANALYSIS.
C****	**************************************

C*************************************
C THE SET OF 33 FIXED SAMPLING POINTS, KA(I), AND THE INITIAL SET OF C WEIGHTS, W(I), I=1,33, ARE ENTERED THROUGH THE DATA STATEMENTS C
C*************************************
DATA KA(1)/.011251388D0/,KA(2)/.059284741D0/
DATA KA(3)/.145707967D0/,KA(4)/.270553179D0/
DATA KA(5)/.433841408D0/,KA(6)/.635597666D0/
DATA KA(7)/.875852385D0/,KA(8)/1.154641702D0/
DATA KA(9)/1.472007563D0/,KA(10)/1.827997778D0/
DATA KA(11)/2.222666072D0/,KA(12)/2.65607213D0/
DATA KA(13)/3.128281655D0/,KA(14)/3.63936642D0/
DATA KA(15)/4.18940433D0/,KA(16)/4.778479488D0/
DATA KA(17)/5.406682272D0/,KA(18)/6.074109403D0/
DATA KA(19)/6.78086404D0/,KA(20)/7.527055861D0/
DATA KA(21)/8.312801165D0/,KA(22)/9.1382229/1D0/
DATA KA(23)/10.003451129D0/,KA(24)/10.908622439D0/
DATA KA(25)/11.853880/69D0/,KA(26)/12.8393//192D0/
DATA KA(27)/13.80527012300/,KA(28)/14.93172540500/
DATA KA(29)/10.036910/0600/, KA(30)/17.10/02339200/
DATA KA(31)/10.37024000100/, KA(32)/19.00070014000/
DATA W(1) / 028874906D0 / W(2) / 067219473D0 /
DATA W(3) / 10563085D0 / W(4) / 14406297D0 /
DATA W(5) / 18251763D0 / W(6) / 22099994D0 /
DATA $W(7)/.25951551D0/.W(8)/.2980701D0/$
DATA W(9)/.33666957D0/.W(10)/.3753198D0/
DATA W(11)/.41402672D0/.W(12)/.45279633D0/
DATA W(13)/.49163467D0/,W(14)/.53054783D0/
DATA W(15)/.56954199D0/,W(16)/.60862338D0/
DATA W(17)/.6477983DO/,W(18)/.69707314DO/
DATA W(19)/.7264544D0/,W(20)/.7659486D0/
DATA W(21)/.80556248D0/,W(22)/.84530276D0/
DATA W(23)/.88517632D0/,W(24)/.9251902D0/
DATA W(25)/.96535161D0/,W(26)/1.00566765D0/
DATA W(27)/1.04614585D0/,W(28)/1.086/93/5D0/
DATA $W(29)/1.12/61908D0/, W(30)/1.168629/2D0/$
DATA $W(31)/1.2098337700/, W(32)/1.2512394600/$
DATA W(33)/1.292855300/,P1/3.1415920500/

C****	***************************************
	<pre>INPUT SECTIONTHE FOLLOWING VARIABLES ARE READ FROM FORO10 NNUMBER OF POINTS (TOTAL NUMBER IS N+1) AAPROBE RADIUS (MICROMETERS) SEPPROBE SEPARATION (MICROMETERS) DELTHSUBLAYER THICKNESS (MICROMETERS) IBBCBOUNDARY CONDITION (NONE=1, CONDUCTING=2, INSULATING=3) SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM) SUBTHSUBSTRATE THICKNESS (MICROMETERS) RHO()-RESISTIVITY ARRAY (OHM*CM) ICURCURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3)</pre>
C C C C	TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY, TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET SUBTH=DELTH AND SUBRHO=RHO(1).
C****	
C	READ THE NUMBER OF DATA PUINTS IN THE STRUCTURE (EXCLUSIVE OF SUBSTRATE) READ (10.*) N
С	READ THE PROBE RADIUS (MICROMETERS)
C	READ (10,*) AA READ THE PROBE SEPARATION (MICROMETERS)
C	READ (10,*) SEP
С	READ THE DEPTH INCREMENT (MICROMETERS)
С	READ (10,*) DELIN READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
-	READ (10,*) IBBC
С	READ THE SUBSTRATE RESISTIVITY (OHM*CM)
С	READ THE SUBSTRATE THICKNESS (MICROMETERS)
	READ (10,*) SUBTH
C	DO TO T=1 N
10 C C	READ (10,*) RH0(I) CONTINUE RH0(N+1)=SUBRH0 READ THE PROBE CURRENT DENSITY (SCHUMANN AND GARDNER=1, CH00 UNIFORM=2, RING DELTA=3) READ (10,*) ICUR
	DU 11 1=1,N+1

WRITE(6,7777)

7777 FORMAT (1H1)

WRITE(6,12)

- 12 FORMAT(2X, 'CALCULATION OF THE SPREADING RESISTANCE FROM THE 1 RESISTIVITY',/,2X,'USING THE 33-POINT GAUSS-LAGUERRE SCHEME') G0 T0 (15,20,25), IBBC
- 15 WRITE(6,17)SUBTH+N*DELTH,N*DELTH
- 17 FORMAT (2X, 'NO BOUNDARY AT', F10.3,' UM. SUBSTRATE AT', F8.3,'UM') GO TO 29
- 20 WRITE(6,21)SUBTH+N*DELTH,N*DELTH
- 21 FORMAT (2X, 'CONDUCTING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT', 1 F8.3,' UM')

GO TO 29

- 25 WRITE(6,26)SUBTH+N*DELTH,N*DELTH
- 26 FORMAT(2X,'INSULATING BOUNDARY AT',F10.3,' UM. SUBSTRATE AT', 1 F8.3,' UM')
- 29 CONTINUE
 - GO TO (30,40,50), ICUR
- 30 WRITE(6,35)
- 35 FORMAT(2X,'SCHUMANN AND GARDNER CURRENT DENSITY') GO TO 60
- 40 WRITE(6,45)
- 45 FORMAT(2X, 'CHOO UNIFORM CURRENT DENSITY') GO TO 60
- 50 WRITE(6,55)
- 55 FORMAT(2X, 'RING DELTA FUNCTION CURRENT DENSITY')
- 60 WRITE (6, 65) AA, SEP, DELTH, SUBTH, SUBRHO, N
- 65 FORMAT(2X,'AA=',F6.3,' SEP=',F7.1,' DELTH=',F6.3,' SUBTH=', 1 1PE11.3,' SUBRHO=',1PE11.3,' #PT=',I3) D0 70 I=1,33 (T) (AA)
 - L(I) = KA(I) / AA
 - T(I)=DTANH(L(I)*DELTH)
- 70 CONTINUE

THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3) IS INCORPORATED IN THE CALCULATION OF THE KERNEL FUNCTION FOR THE FIRST POINT IN THE CALCULATION, THE SUBSTRATE POINT. FOR THE INFINITE SLAB BOUNDARY CONDITION, THE KERNEL FUNCTION IS UNITY. FOR THE CONDUCTING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE TANH FUNCTION FOR THE INSULATING BOUNDARY CONDITION, THE KERNEL IS THE COTH FUNCTION. THESE THREE FORMS FOLLOW FROM THE FORM OF THE KERNEL FUNCTION PRESENTED IN THE REFERENCE.
<pre>************************************</pre>
THE FINAL SET OF WEIGHTS, W(I) (I=1,33) ARE NOW CONSTRUCTED FROM THE INITIAL SET OF WEIGHTS AND THE TWO-PROBE CONFIGURATION FUNCTION (IN LINE 90) AND THE FORM OF THE CURRENT DENSITY FUNCTION (IN THE APPROPRIATE NEXT LINE DEPENDING UPON THE VALUE OF ICUR).
<pre>************************************</pre>

THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION FACTOR INTEGRAL FOR THE SUBSTRATE POINT. THE CORRECTION FACTOR, THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.

D0 200 I=1,33 R2=R2+W(I)*A(1,I) CONTINUE INTGRL(N+1)=R2 CF(N+1)=(8.0D0*R2)/PI RSP(N+1)=RH0(N+1)*CF(N+1)*1D4/(2.0D0*AA) V0L(N+1)=DL0C10(RSP(N+1))

C****	**********
с с с с с с с с с с с с	THE 500 LOOP IS THE MAJOR LOOP WHICH CONTAINS THE CALCULATION OF THE CORRECTION FACTOR INTEGRAL FOR THE N-POINT LAYERED STRUCTURE STARTING FROM THE DEEPEST POINT TO THE TOP SURFACE. THE CORRECTION FACTOR, THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C****	***********
	D0 500 KK=N,1,-1
	R2=0.000 P2_PH0(KK.1) (PH0(KK)
	D0 250 T=1.33
	A(2,I) = (R3*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R3)
250	CONTINUE
	D0 275 I=1,33
075	R2=R2+W(I)*A(2,I)
275	
	CE(KK) = (8,000*TNTGRI(KK))/PT
	RSP(KK)=RH0(KK)*CF(KK)*1D4/(2.0D0*AA)
	VOL (KK) = DLOGIO (RSP (KK))
	D0 350 I=1,33
250	A(1,1)=A(2,1)
330	
500	CONTINUE
500 C****	CONTINUE ************************************
500 C**** C	CONTINUE ************************************
500 C**** C C	CONTINUE ************************************
500 C**** C C C	CONTINUE ************************************
500 C**** C C C C C ****	CONTINUE ************************************
500 C**** C C C**** 550	CONTINUE ************************************
500 C**** C C C C C **** 550	CONTINUE ************************************
500 C**** C C C C C **** 550	CONTINUE ************************************
500 C**** C C C C **** 550	CONTINUE ************************************
500 C**** C C C C **** 550	CONTINUE ************************************
500 C**** C C C**** 550 650	CONTINUE TEND OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION TORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RHO(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11 675)DEPTH(I) RSP(I)
500 C**** C C C**** 550 650 675	CONTINUE TEND OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION TORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RHO(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4)
500 C**** C C C**** 550 650 675 600	CONTINUE TEND OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION TORMAT(/,3X, 'DEPTH',6X, 'RHO',6X, 'INTEGRAL',4X, 'CORR.FAC',6X, 'RSP',8X, 'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RH0(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4) CONTINUE
500 C**** C C C**** 550 650 675 600	CONTINUE END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION WRITE(6,550) FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 1 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RH0(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4) CONTINUE WRITE(11,*)AA
500 C**** C C 550 650 675 600	CONTINUE ************************************
500 C**** C C C**** 550 650 675 600	CONTINUE END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION WRITE(6,550) FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 1 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RHO(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4) CONTINUE WRITE(11,*)AA WRITE(11,*)DELTH WRITE(11,*)DELTH WRITE(11,*)CUP
500 C**** C C C**** 550 650 675 600	CONTINUE END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION WRITE(6,550) FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 1 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RH0(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4) CONTINUE WRITE(11,*)AA WRITE(11,*)SEP WRITE(11,*)DELTH WRITE(11,*)ICUR STOP
500 C**** C C C**** 550 650 675 600	CONTINUE END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION WRITE(6,550) FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X, 1 'RSP',8X,'VOL') WRITE(11,*)N+1 D0 600 I=1,N+1 WRITE(6,650)DEPTH(I),RH0(I),INTGRL(I),CF(I),RSP(I),VOL(I) FORMAT(F8.3,5(1PE12.4)) WRITE(11,675)DEPTH(I),RSP(I) FORMAT(2X,F7.3,2X,1PE12.4) CONTINUE WRITE(11,*)SEP WRITE(11,*)DELTH WRITE(11,*)ICUR STOP END

DOUBLE PRECISION FUNCTION DBJO(X)
C*************************************
C
C POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJO C REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ C AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED C MATHEMATICS SERIES 55. EQUATION 9.4.1 IS USED FOR X < 3 C EQUATION 9.4.3 IS USED FOR X > 3
C DOUBLE PRECISION X,X1,X2,F0,THETA C DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7 C DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7 C DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7 DATA A1/1.000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/ DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/ DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/ DATA A7/0.0002100D0/ DATA B1/.79788456D0/,B2/00000077D0/,B3/00552740D0/
DATA B4/00009512D0/,B5/.00137237D0/,B6/00072805D0/ DATA B7/.00014476D0/ DATA C1/78539816D0/,C2/04166397D0/,C3/00003954D0/ DATA C4/.00262573D0/,C5/00054125D0/,C6/00029333D0/ DATA C7/.00013558D0/ TE (X GT 3 0D0) G0 T0 10
X2=X*X/9.0D0 DBJ0=((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
RETURN 10 CONTINUE X1=3.DO/X
FO=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJO=FO*DCOS(THETA)/DSQRT(X) RETURN END
C*****END OF DBJO DOUBLE PRECISION FUNCTION************************************

DOUBLE PRECISION FUNCTION DBJ1(X) С С POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1 Ċ REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ Ċ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED C C MATHEMATICS SERIES 55. EQUATION 9.4.4 IS USED FOR X < 3 AND EQUATION 9.4.6 IS USED FOR X > 3 С C* ****** IMPLICIT DOUBLE PRECISION (A-H, 0-Z) С DOUBLE PRECISION X, X1, X2, F1, THETA С DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7 С DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7 С DOUBLE PRECISION C1, C2, C3, C4, C5, C6, C7 DATA A1/0.500000D0/, A2/-.56249985D0/, A3/.21093573D0/ DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/ DATA A7/.00001109D0/ DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/ DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/ DATA B7/-.00020033D0/ DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/ DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/ DATA C7/-.00029166D0/ IF (X.GT.3.0DO) GO TO 10 X2=X*X/9.D0 DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1) RETURN 10 X1=3.0D0/X F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+((((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJ1=F1*DCOS(THETA)/DSQRT(X) RETURN **FND**

C*****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF RGENGL V1.0 PROGRAM*****

C*****	**************************************
	THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING) RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE BERKOWITZ AND LUX APPROXIMATE INTEGRAL AND THEIR 22-POINT INTEGRATION METHOD BASED UPON THE NEWTON-COTES QUADRATURE.
	REFERENCE: "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES", BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL 128, PP. 1137-1141 (1981).
	THE SET OF SAMPLING POINTS AND WEIGHTS ARE SPECIALIZED TO THE CHOO ET AL. UNIFORM CURRENT DENSITY (ALTHOUGH THE SETS OF SAMPLING POINTS AND WEIGHTS MAY BE CONSTRUCTED FOR THE SCHUMANN AND GARDENER CURRENT DENSITY OR THE RING DELTA FUNCTION CURRENT DENSITY BY FOLLOWING THE REASONING PRESENTED IN THE REFERENCE).
	THE PROGRAM READS (FROM FORO1O) THE NUMBER OF POINTS IN THE STRUCTURE, THE PROBE RADIUS, THE PROBE SEPARATION, THE DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE RESISTIVITY, THE SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY (FROM THE TOP SURFACE).
	THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
	THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES AS WELL AS THE RESULTS OF THE CALCULATION.
	IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, PLOTTING OR FURTHER ANALYSIS.
C****	*******
	IMPLICIT DOUBLE PRECISION (A-H,O-Z) DIMENSION RSP(500),RH0(500),DEPTH(500),CF(500),VOL(500) DIMENSION A(2,22),C(9),W(22),T(22) DOUBLE PRECISION LAMDA(22),KA(22),INGRL(500)

C***	*************************
	THE SET OF FIXED (PROBE-SPACING INDEPENDENT) SAMPLING POINTS, KA(I), AND WEIGHTS, W(I), (I=9,,22) ARE ENTERED BY THE DATA STATEMENTS. THE 9 POINT NEWTON-COTES COEFFICIENTS ARE ENTERED BY DATA STATEMENTS.
C***	<pre>DATA KA(9)/0.1D0/,KA(10)/0.177828D0/,KA(11)/0.316228D0/ DATA KA(12)/0.562341D0/,KA(13)/1.0D0/,KA(14)/1.70793D0/ DATA KA(15)/2.41586D0/,KA(16)/3.12378D0/,KA(17)/5.23853D0/ DATA KA(18)/8.48093D0/,KA(19)/11.6681D0/,KA(20)/14.829D0/ DATA KA(18)/8.48093D0/,KA(19)/11.6681D0/,KA(20)/14.829D0/ DATA KA(21)/17.9861D0/,KA(22)/39.3D0/,PI/3.14159265D0/ DATA W(9)/0.00446607D0/,W(10)/0.03611D0/,W(11)/0.0236709D0/ DATA W(9)/0.106292D0/,W(13)/0.0773292D0/,W(14)/0.115397D0/ DATA W(12)/0.106292D0/,W(13)/0.0773292D0/,W(14)/0.115397D0/ DATA W(15)/0.0172741D0/,W(16)/0.0087751D0/,W(17)/0.00668606D0/ DATA W(18)/0.00163407D0/,W(19)/0.000628701D0/,W(20)/0.0003062D0/ DATA W(18)/0.001717D0/,W(22)/0.000412D0/ DATA C(1)/0.0348854D0/,C(2)/0.20769D0/,C(3)/-0.0327337D0/ DATA C(4)/0.370229D0/,C(5)/-0.160141D0/ C(6)=C(4) C(7)=C(3) C(8)=C(2) C(9)=C(1)</pre>
C**** C C C C C C C C C C C C C C C C C	INPUT SECTIONTHE FOLLOWING VARIABLES ARE READ BY THE PROGRAM NPTNUMBER OF DATA POINTS IN STRUCTURE (EXCLUDING SUBSTRATE) AAPROBE RADIUS (MICROMETERS) SEPSEPARATION BETWEEN THE PROBES (MICROMETERS) DELTHDEPTH INCREMENT (MICROMETERS) IBBCBACKSURFACE BOUNDARY CONDITION (NONE=1,COND=2,INS=3) SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM) SUBTHSUBSTRATE THICKNESS (MICROMETERS) RHOARRAY OF RESISTIVITY VALUES (OHM*CM)
	TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY, TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET SUBTH=DELTH AND SUBRHO=RHO(1).
C****	**************************************
С	READ (10,*) N READ THE PROBE RADIUS (MICROMETERS)
С	READ THE PROBE SPACING (MICROMETERS)
С	READ (10,*) SEP READ THE DEPTH INCREMENT (MICROMETERS)
C	READ (10,*) DELTH READ THE BACK SURFACE BOUNDARY CONDITION (NONE=1 COND=2 TNS=3)
6	READ (10,*) IBBC
C	READ (10,*) SUBRHO
С	READ THE SUBSTRATE THICKNESS (MICROMETERS) READ (10.*) SUBTH
С	READ THE N DEPTH-DEPENDENT VALUES OF THE STRUCTURE RESISTIVITY (OHM*CM)

```
D0 70 I=1.N
      READ (10,*) RHO(I)
 70
      CONTINUE
     RHO(N+1) = SUBRHO
     WRITE(6,7777)
 7777 FORMAT(1H1)
     WRITE(6,80)
     FORMAT (2X, 'CALCULATION OF SPREADING RESISTANCE FROM THE
80
     1 RESISTIVITY',/,2X,'BERKOWITZ-LUX APPROXIMATE INTEGRAL WITH
    2 THE UNIFORM CURRENT DENSITY')
     GO TO (90, 100, 110), IBBC
     WRITE(6,95)SUBTH+N*DELTH,N*DELTH
90
     FORMAT(2X, 'NO BOUNDARY AT', F10.3,' UM. SUBSTRATE AT', F8.3.' UM')
95
     GO TO 120
100 WRITE (6.105) SUBTH+N*DELTH.N*DELTH
105 FORMAT(2X, 'CONDUCTING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT',
    1 F8.3, 'UM')
     GO TO 120
110 WRITE (6, 115) SUBTH+N*DELTH, N*DELTH
115 FORMAT (2X, 'INSULATING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT',
    1 F8.3.' UM')
120 WRITE (6, 130) AA, SEP, DELTH, SUBTH, SUBRHO, N
130 FORMAT (2X, 'RADIUS=', F6.4, 2X, 'SÉP=', F6.1, 2X, 'DELTH=', F7.3, 2X,
1 'SUBTH=', 1PE9.3, 2X, 'SUBRHO=', 1PE9.3, ' # PTS= ', I3)
     D0 140 I=1.N+1
     DEPTH(I)=DELTH*(I-1)
140 CONTINUE
     Q=1.12292DO*(AA/SEP)
     QQ=DLOG(0.1/Q)
С
С
     HERE THE SAMPLING POINTS, KA (I), AND WEIGHTS, W(I), FOR THE FIRST
С
     NINE POINTS (WHICH DEPEND UPON THE PROBE SPACING) ARE CALCULATED.
С
DO 150 I=1.8
     KA(I) = Q * (0.1DO/Q) * * ((I-1.0)/8.0)
 150 CONTINUE
     DO 155 I=1,22
     LAMDA(I) = KA(I) / AA
 155 CONTINUE
     DO 160 I=1,8
     W(I) = 0.25D0 * QQ * KA(I) * C(I)
160 CONTINUE
     W(9) = W(9) + 0.249376D0 * QQ * KA(9) * C(9)
     DO 180 I=1,22
     R1=LAMDA(I) *DELTH
     T(I) = DTANH(R1)
180 CONTINUE
```

C****	************
	THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3) IS INCORPORATED IN THE CALCULATION OF THE KERNEL FUNCTION FOR THE FIRST POINT IN THE CALCULATION, THE SUBSTRATE POINT. FOR THE INFINITE SLAB BOUNDARY CONDITION, THE KERNEL FUNCTION IS UNITY. FOR THE CONDUCTING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE TANH FUNCTION. FOR THE INSULATING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE COTH FUNCTION. THESE THREE FORMS FOLLOW FROM THE FORM OF THE KERNEL FUNCTION PRESENTED IN THE REFERENCE
Č	
C****	*********
190	D0 190 I=1,22 R1=LAMDA(I)*SUBTH A(1,I)=1.0D0 A(2,I)=1.0D0 IF(IBBC.EQ.2) A(1,I)=DTANH(R1) IF(IBBC.EQ.3) A(1,I)=1.0D0/DTANH(R1) CONTINUE
C****	******
с с с с с с с	THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION FACTOR INTEGRAL FOR THE SUBSTRATE POINT. THE CORRECTION FACTOR, THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C****	***************************************
200	R2=0.0D0 D0 200 I=1,22 R2=R2+W(I)*A(1,I) CONTINUE INGRL(N+1)=R2 CF(N+1)=(8.0D0*R2)/PI RSP(N+1)=RH0(N+1)*CF(N+1)*1.0D4/(2.0D0*AA) VOL(N+1)=DL0G10(RSP(N+1))

C****	*****
C	THE 500 LOOP IS THE MAJOR LOOP WHICH CONTAINS THE CALCULATION OF THE CORRECTION FACTOR INTEGRAL FOR THE N-POINT LAYERED STRUCTURE STARTING
C C C	THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C****	******
	D0 500 KK=N,1,-1
	R2=0.0D0
	R3=RHU (KK+1)/RHU (KK)
	$\frac{1}{2} \frac{1}{2} \frac{1}$
250	A(2,1) = (R3*A(1,1)+I(1))/(1.000+I(1)*A(1,1)*R3) $CONTINUE$ $D0 275 I=1 22$
	$P_{2} = P_{2} = W(T) + \Delta(2 T)$
275	CONTINUE
2.0	INGRL(KK)=R2
	CF(KK) = (8.0*INGRL(KK))/PI
	RSP(KK)=RH0(KK)*CF(KK)*1.D4/(2.DO*AA)
	VOL (KK)=DLOG10 (RSP (KK))
	DU 350 1=1,22
250	A(1,1)=A(2,1)
500	CONTINUE
C****	***************************************
C	
C C	END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C****	***********
550	WRITE $(6, 550)$
550	FURMAT(/,3X,'DEPTH',6X,'RHU',6X,'INTEGRAL',4X,'CURR.FAC',6X,
	WRTTE(11 \pm)N \pm 1
	WRITE(13.*)N+1
	D0 600 I=1,N+1
	WRITE(6,650)DEPTH(I),RH0(I),INGRL(I),CF(I),RSP(I),VOL(I)
650	FORMAT(F8.3,5(1PE12.4))
	WRITE(11,675)DEPTH(I),RSP(I)
675	WRIE(13,675) DEPIH(1), RHU(1)
600	
000	WRTTE(11 *)AA
	WRITE(11.*)SEP
	WRITE(11,*)DELTH
	STOP
	END

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С RGENVAR VERSION 1.0 С С THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING) С RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND С GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. С С THE CALCULATIONS ARE BASED UPON THE VARIATIONAL METHOD DEVELOPED BY С CHOO AND COWORKERS. THIS MAKES USE OF A VARIATIONALLY DETERMINED С MIXING OF THE SCHUMANN AND GARDNER CURRENT DENSITY AND THE CHOO ET AL. С UNIFORM CURRENT DENSITY. С C C THE CALCULATION OF THE VARIOUS INTEGRALS IS CARRIED OUT BY AN ADAPTATION OF THE BERKOWITZ-LUX APPROXIMATE INTERGAL CONCEPT AS WELL С AS A SEGMENTED NEWTON-COTES QUADRATURE. С C REFERENCES: *SPREADING RESISTANCE CALCULATIONS BY THE VARIATIONAL С METHOD," BY S. C. CHOO, M. S. LEONG, AND L. S. TAN, SOLID STATE ELECTRONICS, VOL. 24, PP. 557-562, 1981. С С "AN EFFICIENT NUMERICAL SCHEME FOR SPREADING RESISTANCE С CALCULATIONS BASED ON THE VARIATIONAL METHOD. " BY С S. C. CHOO, M. S. LEONG, AND J. H. SIM, C C SOLID STATE ELECTRONICS, VOL. 26, PP. 723-730, 1983. Ĉ THE PROGRAM READS (FROM FORO10) THE PROBE RADIUS, PROBE SEPARATION, С DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE С RESISTIVITY, SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY (FROM С THE TOP SURFACE). С С THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE Ċ ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL. С С THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES С AS WELL AS THE RESULTS OF THE CALCULATION. С IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, С PLOTTING OR FURTHER ANALYSIS. С С IMPLICIT DOUBLE PRECISION (A-H, 0-Z) DIMENSION RSP (500), RH0 (500), DEPTH (500), CF (500), VOL (500) DIMENSION A(2,55), CN9(9), W(3,55), T(55) DOUBLE PRECISION KA(55), L, K1, K2 DOUBLE PRECISION I1 (500), I2 (500), I3 (500), I2S (500), I3S (500) С THE SET OF PROBE-SPACING INDEPENDENT SAMPLING POINTS, KA(I), (I=27,55) С AND WEIGHTS, W(J,I), (J=1,3 I=27,55) ARE ENTERED HERE BY MEANS OF С DATA STATEMENTS. THE NEWTON-COTES COEFFICIENTS ARE ALSO ENTERED С С HERE IN DATA STATEMENTS. С

DATA KA(27)/0.12D0/,KA(28)/0.203885D0/,KA(29)/0.346410D0/ DATA KA(30)/0.588566D0/,KA(31)/1.0D0/,KA(32)/1.53540D0/ DATA KA(33)/2.07080D0/,KA(34)/2.60619D0/,KA(35)/3.14159D0/ DATA KA(36)/3.31412DO/,KA(37)/3.48665DO/,KA(38)/3.65917DO/ DATA KA(39)/3.83171D0/,KA(40)/6.28319D0/,KA(41)/7.01558D0/ DATA KA(42)/9.42478DO/,KA(43)/10.1735DO/,KA(44)/12.5664DO/ DATA KA(45)/13.3237D0/,KA(46)/15.7080D0/,KA(47)/16.4701D0/ DATA KA(48)/18.8496D0/,KA(49)/19.6159D0/,KA(50)/39.24D0/ DATA KA(51)/49.87D0/,KA(52)/56.32D0/,KA(53)/311.0D0/ DATA KA(54)/983.5D0/,KA(55)/1100.0D0/,W(1,27)/1.96943D-2/ DATA W(2,27)/4.92949D-3/,W(3,27)/9.85305D-3/ DATA W(1,28)/1.51585D-1/,W(2,28)/3.80282D-2/ DATA W(3,28)/7.59243D-2/,W(1,29)/9.40755D-2/ DATA W(2,29)/2.37573D-2/,W(3,29)/4.72756D-2/ DATA W(1,30)/3.94777D-1/,W(2,30)/1.01659D-1/ DATA W(3,30)/2.00331D-1/,W(1,31)/2.34711D-1/ DATA W(2,31)/6.41889D-2/,W(3,31)/1.22743D-1/ DATA W(1,32)/3.22596D-1/,W(2,32)/1.02248D-1/ DATA W(3,32)/1.81618D-1/,W(1,33)/5.12837D-2/ DATA W(2,33)/2.17231D-2/,W(3,33)/3.33773D-2/ DATA W(1,34)/2.91802D-2/,W(2,34)/2.46688D-2/ DATA W(3,34)/2.68298D-2/,W(1,35)/4.03186D-2/ DATA W(2,35)/1.80765D-3/,W(3,35)/-9.61334D-4/ DATA W(1,36)/0.0D0/,W(2,36)/1.03120D-3/ DATA W(3,36)/0.0D0/,W(1,37)/0.0D0/ DATA W(2,37)/1.54721D-4/,W(3,37)/0.0D0/ DATA W(1,38)/0.0D0/,W(2,38)/9.17871D-5/ DATA W(3,38)/0.0D0/,W(1,39)/0.0D0/ DATA W(2,39)/3.84395D-3/,W(3,39)/9.19809D-3/ DATA W(1,40)/4.73102D-2/,W(2,40)/0.0D0/ DATA W(3,40)/8.01205D-3/,W(1,41)/0.0D0/ DATA W(2,41)/3.91858D-3/,W(3,41)/2.52148D-3/ DATA W(1,42)/1.90479D-2/,W(2,42)/0.0D0/ DATA W(3,42)/2.32728D-3/,W(1,43)/0.0D0/ DATA W(2,43)/1.08897D-3/,W(3,43)/1.08692D-3/ DATA W(1,44)/1.03669D-2/,W(2,44)/0.0D0/ DATA W(3,44)/1.02754D-3/,W(1,45)/0.0D0/ DATA W(2,45)/4.57291D-4/,W(3,45)/5.80787D-4/ DATA W(1,46)/6.53592D-3/,W(2,46)/0.0D0/ DATA W(3,46)/5.56264D-4/,W(1,47)/0.0D0/ DATA W(2,47)/2.25498D-4/,W(3,47)/3.52250D-4/ DATA W(1,48)/8.71084D-3/,W(2,48)/0.0D0/ DATA W(3,48)/3.40149D-4/,W(1,49)/0.0D0/ DATA W(2,49)/8.28909D-5/,W(3,49)/3.09148D-4/ DATA W(1,50)/0.0D0/,W(2,50)/4.12225D-4/ DATA W(3,50)/0.0D0/,W(1,51)/0.0D0/,PI/3.14159265D0/ DATA W(2,51)/0.0D0/,W(3,51)/1.92433D-3/ DATA W(1,52)/1.78441D-2/,W(2,52)/0.0D0/,W(3,52)/0.0D0/ DATA W(1,53)/9.02512D-4/,W(2,53)/0.0D0/,W(3,53)/0.0D0/ DATA W(1,54)/1.52804D-3/,W(2,54)/0.0D0/,W(3,54)/0.0D0/ DATA W(1,55)/0.0D0/,W(2,55)/0.0D0/,W(3,55)/9.40768D-6/

C****	C09=4.0/14175.0D0 CN9(1)=C09*989.0D0 CN9(2)=C09*5888.0D0 CN9(3)=-C09*928.0D0 CN9(4)=C09*10496.0D0 CN9(5)=-C09*4540.0D0 CN9(6)=CN9(4) CN9(7)=CN9(3) CN9(8)=CN9(2) CN9(9)=CN9(1)
C	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
с с с с с с с с с с с с с с с с с с с	<pre>INPUT SECTIONDEFINITION OF SYMBOLS N = NUMBER OF RESISTIVITY, SPREADING RESISTANCE VALUES AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) IBBC = BOUNDARY CONDITION (NONE=1, CONDUCTING=2, INSULATION=3) SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM) SUBTH = SUBSTRATE THICKNESS (MICROMETERS) RHO(I) = I-TH RESISTIVITY RELATIVE TO THE SURFACE</pre>
	AS ORIGINALLY FORMULATED THE INTEGRATION SCHEME OF CHOO ET AL. IS NOT ABLE TO HANDLE AN INSULATING BOUNDARY. WE HAVE CIRCUMVENTED THIS PROBLEM BY STARTING THE INTEGRATION ONE POINT AWAY FROM THE SINGULARITY AT THE ORIGIN WHEN THERE IS AN INSULATING BOUNDARY. WE HAVE FOUND THAT THIS LEADS TO VERY GOOD RESULTS WITH NO LOSS OF GENERALITY.
	TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET SUBTH=DELTH AND SUBRHO=RHO(1).
C****	****
С	READ THE NUMBER OF DATA POINTS IN THE STRUCTURE (EXCLUSIVE OF SUBSTRATE)
_	READ (10, *) N
С	READ THE PROBE RADIUS (MICROMETERS)
C	READ THE PROPE CRACING (NICONETERS)
C	READ THE FRUDE SPACING (MICROMETERS)
C	READ THE DEPTH INCREMENT (MICROMETERS)
C	READ (10. *) DELTH
С	READ THE BACK SURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
	READ(10,*) IBBC
С	READ THE SUBSTRATE RESISTIVITY (OHM*CM)
	READ(10,*) SUBRHO
C	READ THE SUBSTRATE THICKNESS (MICKUMETERS)
С	READ (10,*) SOBTH READ THE RESISTIVITIES (OHM*CM) OF THE STRUCTURE D0 70 I=1,N
7.6	READ(10,*) RHO(I)
70	CUNTINUE

```
RHO(N+1) = SUBRHO
     L=1.122918967D0*(AA/SEP)
     WRITE (6,7777)
7777 FORMAT(1H1)
     WRITE(6,80)
     FORMAT (2X, 'CALCULATION OF SPREADING RESISTANCE FROM THE
80
    1 RESISTIVÍTY',/,2X,'USING CHOO ET AL VARIATIONAL TECHNIQUE-BL')
GO TO (90,100,110), IBBC
     WRITE(6.95)SUBTH+N*DELTH.N*DELTH
90
95
     FORMAT (2X.'NO BOUNDARY AT', F10.3.' UM. SUBSTRATE AT', F8.3.' UM')
     GO TO 120
100 WRITE(6,105)SUBTH+N*DELTH,N*DELTH
105 FORMAT(2X, 'CONDUCTING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT',
    1 F8.3, 'UM')
     GO TO 120
110 WRITE(6,115)SUBTH+N*DELTH,N*DELTH
115 FORMAT (2X, 'INSULATING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT',
    1 F8.3, 'UM')
120 WRITE (6.130) AA, SEP, DELTH, SUBTH, SUBRHO, N
130 FORMAT (2X, 'RADIUS=', F6.4, 2X, 'SÉP=', F6.1, 2X, 'DELTH=', F7.3, 2X,
1 'SUBTH=', 1PE9.3, 2X, 'SUBRHO=', 1PE9.3, ' # PTS= ', I3)
     D0 140 I=1.N+1
     DEPTH(I) = DELTH * (I-1)
140 CONTINUE
С
С
     BEGIN CONSTRUCTION OF SAMPLING POINTS AND WEIGHTS
С
     REMEMBER TO ADD WEIGHTS AT BOUNDARY OF SAMPLING REGIONS.
     THE REGION DESIGNATIONS FOLLOW THE NOTATION OF THE SECOND REFERENCE.
С
С
С
С
     REGION A (0 \leq KA \leq 1D-5) (I=1,9)
С
DO 142 I=1,9
     KA(I)=1.D-5*(I-1)/8.D0
     W(1,I)=1.D-5*CN9(I)/8.D0
     W(2,I) = W(1,I)/4.DO
     W(3,I) = W(1,I)/2.D0
142 CONTINUE
     WEND1=W(1,9)
     WEND2=W(2,9)
     WEND3=W(3,9)
```

```
C*********
           *******
С
С
    REGION B (1D-5<=KA<=L) (I=9,17)
С
DO 144 I=9,17
    KA(I) = 1.D - 5 * (L * 1.D5) * * ((I - 9.)/8.)
    W(1, I) = CN9(I-8) * KA(I) * DLOG(L*1.D5) / 8.
    W(2,I) = W(1,I)/4.DO
    W(3, I) = W(1, I) / 2.DO
    CONTINUE
144
    W(1,9) = W(1,9) + WEND1
    W(2,9) = W(2,9) + WEND2
    W(3,9) = W(3,9) + WEND3
*******
С
С
    REGION C (L\leq=KA\leq=.12) (I=18,26)
С
DO 146 I=18.26
    KA(I) = L * (0.12DO/L) * * ((I-18.)/8.)
    W(1, I) = CN9(I-17) * KA(I) * DLOG(0.12/L)/8.
    W(2,I) = W(1,I)/4.DO
    W(3,I) = W(1,I)/2.DO
146 CONTINUE
С
С
    REGION D, E, AND F
С
    THE VALUES OF THE KA(I) AND W(I) HAVE BEEN ENTERED BY THE
С
    DATA STATEMENTS AT THE BEGINNING OF THE PROGRAM
С
TT=1
    IF(IBBC.EQ.2) II=2
    DO 180 I=II,55
    R1=KA(I) *DELTH/AA
    T(I) = DTANH(R1)
    CONTINUE
180
    DO 190 I=II,55
    R1=KA(I)*SUBTH/AA
    A(1,I) = 1.000
    A(2,I)=1.0D0
    IF(IBBC.EQ.2) A(1,I)=DTANH(R1)
    IF(IBBC.EQ.3) A(1,I)=1.0D0/DTANH(R1)
190 CONTINUE
    RI1=0.0D0
    RI2=0.0D0
    RI3=0.0D0
    DO 200 I=18,55
    RI1=RI1+W(1,I)*A(1,I)
    RI2=RI2+W(2,I)*A(1,I)
    RI3=RI3+W(3,I)*A(1,I)
```

200	CONTINUE
	I2S(N+1)=RI2
	I3S(N+1)=RI3
	D0 210 I=II,17
	RI1=RI1+W(1,I)*A(1,I)
	RI2=RI2+W(2,I)*A(1,I)
	RI3=RI3+W(3,I)*A(1,I)
210	CONTINUE
	I1(N+1)=RI1
	I2(N+1)=RI2
	I3(N+1)=RI3
	DEN=(I1(N+1)*I2(N+1)-I3(N+1)*I3(N+1))
	K1 = (I2(N+1) - 0.5D0 * I3(N+1)) / DEN
	$K_{2}=(0.5D0*I1(N+1)-I3(N+1))/DEN$
	CFDÈN=4.0D0/(PI*(K1+0.5D0*K2))
	CF(N+1) = CFDEN*(K1*I3S(N+1)+K2*I2S(N+1))
	RSP(N+1) = RHO(N+1) * CF(N+1) * 1.D4/(2.ODO*AA)
	VOL(N+1) = DLOG1O(RSP(N+1))
C****	*****
C	
Č	LAYERED STRUCTURE CALCULATION
Č	
- C****	******
•••••	D0 500 KK=N 1 -1
	RT1=0.0D0
	RT2=0.0D0
	RT3=0 0D0
	$R_3 = R_{H0} (K_{K+1}) / R_{H0} (K_{K})$
	D0 250 T = TT 55
	$A(2 T) - (R_{3} + A(1 T) + T(T)) / (1 OD + T(T) + A(1 T) + R_{3})$
250	CONTINUE
200	D0 275 I-18 55
	DU = 275 = 10,55 DT1 = DT1 = W(1 = T) = A(2 = T)
	$DTO_DTO_W(0, T) \neq A(0, T)$
	$\frac{T}{T} = \frac{T}{T} + \frac{T}$
075	(2, 1) = (2, 1) = (2, 1)
215	
	$\frac{125(KK) = K12}{125(KK) = D12}$
	135(RK) = R13
	DU = 280 I = II, I7 DI = DI = W(1 I) + A(0 I)
	KII=KII+W(I,I)*A(2,I)
	K12=R12+W(2,1)*A(2,1)
000	RI3=RI3+W(3,1)*A(2,1)
280	
	$\prod_{i=1}^{n} (KK) = K \prod_{i=1}^{n} (KK) = K \prod_{i=1}$
	12(KK) = K12
	13(KK) = K13
	DEN=(11(KK)*12(KK)-13(KK)*13(KK))
	$K1 = (12(KK) - 0.5D0 \times 13(KK))/DEN$
	K2=(0.5D0+11(KK)-13(KK))/DEN
	CFDEN=4.0D0/(P1*(K1+0.5D0*K2))
	CF(KK) = CFDEN*(K1*13S(KK)+K2*12S(KK))
	KSP(KK)=KHU(KK)*CF(KK)*1.D4/(2.0D0*AA)
	VUL(KK)=DLUGIO(KSP(KK))
	DU 350 1=11,55
	A(1,1)=A(2,1)

350	CUNTINOE
500	CONTINUE
C****	********
Č	END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C****	***************************************
	WRITE(6,550)
550	FORMAT (/, 3X, 'DEPTH', 6X, 'RHO', 6X, 'CORR.FAC', 6X, 'RSP', 6X, 'VOL')
	WRITE(11,*)N+1
	D0 600 I=1,N+1
	WRITE(6,650)DEPTH(I),RH0(I),CF(I),RSP(I),VOL(I)
650	FORMAT(F8.3,5(1PE12.4))
	WRITE(11,675)DEPTH(I),RSP(I)
675	FORMAT (2X, F7.3, 2X, 1PE12.4)
600	CONTINUE
	WRITE(11,*)AA
	WRITE(11.*)SEP
	WRITE (11.*) DELTH
	STOP
	FND
C****	*END OF RGENVAR V1.0 PROGRAM************************************

APPENDIX E -- BL LISTING

(****	***************************************
С	BL VERSION 1.0
	THIS PROGRAM CALCULATES THE RESISTIVITY PROFILE FROM THE SPREADING RESISTANCE BY USING: 1) THE BERKOWITZ-LUX APPROXIMATE INTEGRAL, 2) THE 22-POINT INTEGRATION SCHEME BASED UPON THE NEWTON-COTES TECHNIQUE, AND 3) A BOUNDING PROCEDURE WHICH SIMPLIFIES THE INVERSION OF THE CORRECTION FACTOR INTEGRAL. THIS PROGRAM INVERTS THE CALCULATION PERFORMED IN RGENBL VERSION 1.0
	THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT RESISTIVITY FROM THE DEPTH-DEPENDENT TWO-PROBE (SPREADING) RESISTANCE USING THE SCHUMANN AND GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE BERKOWITZ AND LUX APPROXIMATE INTEGRAL AND THEIR 22-POINT INTEGRATION METHOD BASED UPON THE NEWTON-COTES QUADRATURE. AS THE EXTRACTION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE INVOLVES THE SOLUTION OF THE INVERSION PROBLEM, THE CODE MAKES USE OF AN ITERATION METHOD BASED UPON A BOUNDING PROCEDURE FIRST USED BY BERKOWITZ AND LUX.
	THE SET OF SAMPLING POINTS AND WEIGHTS ARE SPECIALIZED TO THE CHOO ET AL. UNIFORM CURRENT DENSITY (ALTHOUGH THE SETS OF SAMPLING POINTS AND WEIGHTS MAY BE CONSTRUCTED FOR THE SCHUMANN AND GARDENER CURRENT DENSITY OR THE RING DELTA FUNCTION CURRENT DENSITY BY FOLLOWING THE REASONING PRESENTED IN THE REFERENCE).
	REFERENCE: "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES", BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL 128, PP. 1137-1141 (1981).
	THE PROGRAM READS (FROM FORO1O) THE PROBE RADIUS, PROBE SEPARATION, DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE RESISTIVITY, THE SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY (FROM THE TOP SURFACE)
	THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
	THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES AS WELL AS THE RESULTS OF THE CALCULATION.
	IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, PLOTTING, OR FURTHER ANALYSIS.
C*****	<pre>************************************</pre>
C****	*******
--	--
с с с с с с с с с с	HERE THE FIXED VALUES OF THE SAMPLING POINTS (KA) AND WEIGHTS (W) ARE ENTERED BY DATA STATEMENTS. THE REMAINING SAMPLING POINTS AND WEIGHTS (THE FIRST 8) ARE CALCULATED FROM AA AND SEP. THE COEFFICIENTS FOR THE NINE-POINT NEWTON COTES SCHEME ARE ALSO ENTERED HERE BY MEANS OF DATA STATEMENTS.
C****	*******
	DATA KA(9)/0.1D0/,KA(10)/0.177828D0/,KA(11)/0.316228D0/ DATA KA(12)/0.562341D0/,KA(13)/1.0D0/,KA(14)/1.70793D0/ DATA KA(15)/2.41586D0/,KA(16)/3.12378D0/,KA(17)/5.23853D0/ DATA KA(18)/8.48093D0/,KA(19)/11.6681D0/,KA(20)/14.829D0/ DATA KA(21)/17.9861D0/,KA(22)/39.3D0/,PI/3.14159265D0/ DATA KA(21)/17.9861D0/,KA(22)/39.3D0/,PI/3.14159265D0/ DATA W(9)/0.00446607D0/,W(10)/0.03611D0/,W(11)/0.0236709D0/ DATA W(12)/0.106292D0/,W(13)/0.0773292D0/,W(14)/0.115397D0/ DATA W(15)/0.0172741D0/,W(16)/0.0087751D0/,W(17)/0.00668606D0/ DATA W(18)/0.00163407D0/,W(19)/0.000628701D0/,W(20)/0.0003062D0/ DATA W(21)/0.0001717D0/,W(22)/0.000412D0/ DATA C(1)/0.0348854D0/C(2)/0.20769D0/,C(3)/-0.0327337D0/
	DATA C(4)/0.370229D0/,C(5)/-0.160141D0/
	C(6) = C(4)
	C(7) = C(3)
	C(8) = C(2)
	(0) - (1)
C****	······································
C**** C	·*************************************
C**** C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
C***** C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
C***** C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES
C***** C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE)
C***** C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE)
C***** C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS)
C***** C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS)
C***** C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ THE NUMBER OF TWO-PROBE RESISTANCES FROM THE SURFACE
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE DO 70 I=1,NPT
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE DO 70 I=1,NPT READ (11,*)DEPTH(I),RSP(I)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ (11,*)DEPTH(I),RSP(I) CONTINUE
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ (11,*)DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ (11,*)DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ (11,*)DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS) READ (11,*)AA READ THE PROBE SEPARATION (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11, *) NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ THE DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS) READ (11, *) AA READ THE PROBE SEPARATION (MICROMETERS) READ (11, *) SEP
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE DO 70 I=1,NPT READ THE DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS) READ (11,*)AA READ THE PROBE SEPARATION (MICROMETERS) READ (11,*)SEP READ THE DEPTH INCREMENT (MICROMETERS)
C***** C C C C C C C C C C C C C C C C	BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS NPT = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE) RSP(I) = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE) AA = PROBE RADIUS (MICROMETERS) SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES READ (11,*)NPT READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE D0 70 I=1,NPT READ THE DEPTH(I),RSP(I) CONTINUE READ THE PROBE RADIUS (MICROMETERS) READ (11,*)AA READ THE PROBE SEPARATION (MICROMETERS) READ (11,*)SEP READ THE DEPTH INCREMENT (MICROMETERS) READ (11,*)DELTH

```
WRITE (6,7777)
7777 FORMAT(1H1)
     WRITE(6,80)
     FORMAT (2X, 'CALCULATION OF THE RESITIVITY FROM THE SPREADING
80
    1 RESISTANCE', /, ' USING THE BERKOWITZ-LUX APPROXIMATE INTEGRAL')
120 WRITE (6,130) AA, SEP, DELTH, NPT
130 FORMAT(2X, 'RADIUS=', F6.1, 2X, 'SEP=', F6.1, 2X, 'DELTH=', F7.3, 2X,
    1 ' # PTS=',I3)
     AA=AA*1D-4
     SEP=SEP*1D-4
     DELTH=DELTH*1D-4
     SEP=SEP/AA
     DELTH=DELTH/AA
     R1=1.12292D0*(1.D0/SEP)
     R2=DLOG(0.1/R1)
     R3=DEXP(R2/8.)
С
С
     HERE THE SAMPLING POINTS, KA (I), AND WEIGHTS, W(I), FOR THE FIRST
     NINE POINTS (WHICH DEPEND UPON THE PROBE SPACING) ARE CALCULATED.
С
С
     THESE DEPEND UPON THE PROBE SPACING THROUGH THE BERKOWITZ-LUX
С
     APPROXIMATE FORM OF THE CORRECTION FACTOR INTEGRAL
С
DO 140 I=8,1,-1
     KA(I) = KA(I + 1) / R3
140
     CONTINUE
     DO 150 I=1,8
     W(I) = .25D0 * R2 * KA(I) * C(I)
150
     CONTINUE
     W(9) = W(9) + .249376D0 * R2 * KA(9) * C(9)
     DO 155 I=1,22
     R4=KA(I) * DELTH
     T(I) = DTANH(R4)
     CONTINUE
155
     R5=0.0D0
     R6=0.0D0
     R7=0.0D0
```

C****	*******
C	THE THETHETE CLAD CONDUCTING DOUNDADY AND THEM ATTNO DOUNDADY
C	THE INFINITE SLAB, CUNDUCTING BUUNDARY, AND INSULATING BUUNDARY
	LIGED TO THIVEDT THE CODDECTION EACTOR THECODAL
C	OSED TO INVERT THE CONNECTION PACTOR INTEGRAL.
C (****	****
0	D0 160 I=1.22
	R5=R5+W(I)
	R6=R6+W(I)*T(I)
	R7=R7+W(I)/T(I)
160	CONTINUE
	WRITE(6,165)R5,R6,R7
165	FORMAT(2X, 'R5=', F11.6, ' R6=', F11.6, ' R7=', F11.6,/)
	R8=1.2/324D0/AA
	SDIP((NPT)=0.000
	RH0(NPT) = 1 ODO/R9
	ITER(NPT)=0
	SENS (NPT) =0.0D0
	D0 170 I=1,22
	B(I)=1.0D0
170	CONTINUE
	D0 190 I=1,2
	DU = 180 K = 1,22
100	
100	
190	D0 1000 K=NPT-1.11
	ITER (K)=0
	R10=0.D0
	R11=R8/RSP(K)/COND(K+1)
	R12=RSP(K+1)/RSP(K)
	R13=R12
	GOTO 1450
1060	CUNV=DABS(1.0D0-R14*R11/R12)
	IF (CUNV.LI.ID-3) GU TU 1200 TE (P10 E0 1) P15-0 0D0
	$R_{16}-R_{15}$
	R15=R12
	R17 = (R14 - R6) / (R7 - R14) / R12
	R18=R7*R17*R11-1.0
	R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
	IF (R18.GT.0.0) R13=R13/2.0/R17
	IF (R18.LT.0.0) R13=2.0*R11*R6/R13
	VAR1 = (R15 - R13) * (R15 - R16)
	$\frac{1}{1} \left(\frac{1}{1} \right) = \frac{1}{1} \left(\frac{1}{1} \left(\frac{1}{1} \right) = \frac{1}{1} \left(\frac{1}{1} \left(\frac{1}{1} \right) = \frac{1}{1$
	$VAR_2 = R_{10} - 2.* JUDINI (R_{10}/2.)$ TE (VAR2 EQ 0 0) R13= (R15*R15-R13*R16) / (2*R15-R13-R16)
1170	R12=R13
11/0	GO TO 1450
1200	R19=R11*R14*COND(K+1)
	R20=0 0

```
DO 200 I=1,22
     SEP=A(1,I)*(1.0-T(I)*T(I))
     DELTH = (R12*T(I)*A(1,I)+1.0)**2
     R20=W(I)*SEP/DELTH+R20
     A(1,I) = A(2,I)
 200 CONTINUE
     SENS(K)=1.0/(1.0-R20*R13/R14)
     COND(K) = R19
     RHO(K) = 1.0/R19
     SDIF(K)=R14*R8/R13/COND(K+1)
     SDIF(K) = 1.0 - SDIF(K) / RSP(K)
     ITER(K) = R10
 1000 CONTINUE
 1001 FORMAT(2X,I3,2X,F7.3,2X,4(1PE11.3),3X,I5)
 1002 FORMAT (4X, 'NPT', 4X, 'DEPTH', 5X, 'RMEAS', 6X, '1-RCAL/R', 4X, 'RHO', 8X
    1 ,'SENS',6X,'ITER',/)
 1003 FORMAT(2X, F7.3, 2X, 1PE11.3)
 1004 FORMAT(2X, 1PE11.4)
     WRITE(6,1002)
     WRITE(12,*)NPT
     D0 1020 I=1.NPT
     WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),SENS(I),ITER(I)
     WRITE (12, 1003) DEPTH (I), RHO (I)
 1020 CONTINUE
     STOP
С
С
     THE LINES BELOW ARE USED AS PART OF THE INVERSION PROCEDURE BASED
С
     UPON THE BOUNDING METHOD.
C
1450 D0 1475 I=1.22
     A(2,I) = (R12*A(1,I)+T(I)) / (1.0+T(I)*A(1,I)*R12)
     B(I) = A(2, I)
1475 CONTINUE
     R14=0.0
     DO 1480 I=1,22
     R14=R14+W(I)*B(I)
1480 CONTINUE
     R10=R10+1.
     GO TO 1060
     END
```

C**** C	**************************************
000000	THIS PROGRAM CALCULATES RESISTIVITY FROM SPREADING RESISTANCE. THE CALCULATION MAKES USE OF THE 33-POINT GAUSS-LAGUERRE EVALUATION OF THE CORRECTION FACTOR INTEGRAL ALONG WITH THE BERKOWITZ-LUX BOUNDING METHOD. THIS PROGRAM IS THE INVERSE OF THE RGENGL VERSION 1.0 PROGRAM
	THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT RESISTIVITY FROM THE TWO-PROBE (SPREADING) RESISTANCE USING THE SCHUMANN AND GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
	THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE CHOO ET AL. 33-POINT INTEGRATION METHOD BASED UPON THE GAUSS-LAGUERRE QUADRATURE. THE BERKOWITZ AND LUX BOUNDING METHOD IS USED TO CARRY OUT THE INVERSION OF THE CORRECTION FACTOR INTEGRAL. THE 33 SAMPLING POINTS AND WEIGHTS ARE INITIALIZED BY DATA STATEMENTS
C C C	THE FINAL SET OF 33 WEIGHTS USED IN THE CALCULATIONS IS CALCULATED FROM THE INITIAL SET OF WEIGHTS AND THE PROBE CURRENT DENSITY.
с с с с с с с с	THE CURRENT DENSITIES WHICH MAY BE USED ARE THE SCHUMANN AND GARDNER, THE CHOO ET AL. UNIFORM CURRENT DENSITY, AND THE RING DELTA FUNCTION CURRENT DENSITY. THE DISCUSSION OF THE 33-POINT GAUSS-LAGUERRE QUADRATURE MAY BE FOUND IN THE REFERENCE.
00000	REFERENCE: "SPREADING RESISTANCE CALCULATIONS BY THE GAUSS-LAGUERRE QUADRATURE," BY S. C. CHOO, M. S. LEONG, H. L. HONG, L. LE, AND L. S. TAN, SOLID STATE ELECTRONICS VOL 21, PP. 769-774, 1978.
	THE PROGRAM READS (FROM FORO1O) THE PROBE RADIUS, PROBE SEPARATION, DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE RESISTIVITY, SUBSTRATE THICKNESS, THE RESISTIVITY ARRAY (FROM THE TOP SURFACE), AND THE PROBE CURRENT DENSITY.
	THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
	THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES AS WELL AS THE RESULTS OF THE CALCULATION.
	IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, PLOTTING, OR FURTHER ANALYSIS.
C*****	<pre>IMPLICIT DOUBLE PRECISION (A-H,0-Z) DIMENSION T(33),W(33),A(2,33),RH0(500),RSP(500),DEPTH(500) DIMENSION CF(500),VOL(500),COND(500),B(33),SDIF(500),SENS(500) DIMENSION ITER(500) DOUBLE PRECISION KA(33)</pre>

C****	***************************************
	THE SET OF 33 FIXED SAMPLING POINTS, KA(I), AND THE INITIAL SET OF WEIGHTS, W(I), I=1,33, ARE ENTERED THROUGH THE DATA STATEMENTS. THESE ARE THE FIRST 33 POINTS AND WEIGHTS OF THE GAUSS-LAGUERRE METHOD OBTAINED FROM A 128-ORDER LAGUERRE POLYNOMIAL
C****	***************************************
	DATA KA(1)/.011251388D0/,KA(2)/.059284741D0/
	DATA KA(3)/.145707967D0/,KA(4)/.270553179D0/
	DATA KA(5)/.433841408D0/,KA(6)/.63559/666D0/
	DATA $KA(7)/.87585238500/, KA(8)/1.15404170200/$
	DATA $KA(9)/1.47200750500/, KA(10)/1.02799777000/$
	DATA KA(13)/3 128281655D0/ KA(14)/3 63936642D0/
	DATA KA(15)/4.18940433D0/ KA(16)/4.778479488D0/
	DATA KA(17)/5.406682272D0/.KA(18)/6.074109403D0/
	DATA KA(19)/6.78086404D0/,KA(20)/7.527055861D0/
	DATA KA(21)/8.312801165D0/,KA(22)/9.138222971D0/
	DATA KA(23)/10.003451129D0/,KA(24)/10.908622439D0/
	DATA KA(25)/11.853880769D0/,KA(26)/12.839377192D0/
	DATA KA(27)/13.865270123D0/,KA(28)/14.931725465D0/
	DATA KA(29)/16.038916768D0/,KA(30)/17.187025392D0/
	DATA KA(31)/18.376240681D0/,KA(32)/19.606760148D0/
	DATA $KA(33)/20.07070900700/$
	DATA W(3) / 10563085D0 / W(A) / 14406297D0 /
	DATA W(5) / 18251763D0 / W(6) / 22099994D0 /
	DATA W(7) / .25951551D0 / .W(8) / .2980701D0 /
	DATA W(9)/.33666957D0/,W(10)/.3753198D0/
	DATA W(11)/.41402672D0/,W(12)/.45279633D0/
	DATA W(13)/.49163467D0/,W(14)/.53054783D0/
	DATA W(15)/.56954199D0/,W(16)/.60862338D0/
	DATA W(17)/.6477983D0/,W(18)/.69707314D0/
	DATA W(19)/.7264544D0/,W(20)/.7659486D0/
	DATA $W(21)/.80556248D0/, W(22)/.84530276D0/$
	DATA $W(23)/.8851/032D0/, W(24)/.9251902D0/$
	DATA W(27)/1 0A61A585D0/ W(28)/1 08670375D0/
	DATA W(29)/1 12761908D0/ W(30)/1 16862972D0/
	DATA W(31)/1.20983377D0/.W(32)/1.25123946D0/
	DATA W(33)/1.2928553D0/,PI/3.14159265D0/

С С BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS С С NPT = NUMBER OF RESISTIVITY, SPREADING RESISTANCE VALUES С DEPTH(I) = DEPTH RELATIVE TO SURFACE (MICROMETERS) С RSP(I) = TWO-PROBE RESISTANCE RELATIVE TO SURFACE (OHMS) С AA = PROBE RADIUS (MICROMETERS) C C SEP = PROBE SEPARATION (MICROMETERS) DELTH = SUBLAYER THICKNESS (MICROMETERS) С ICUR = CURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3) С READ THE NUMBER OF POINTS IN THE STRUCTURE С READ(11, *)NPT С READ THE DEPTH AND TWO-PROBE RESISTANCE VALUES RELATIVE TO SURFACE D0 10 I=1.NPT READ(11,*)DEPTH(I),RSP(I) 10 CONTINUE С READ THE ELECTRICAL PROBE RADIUS (MICROMETERS) READ(11, *)AAREAD THE SEPARATION BETWEEN THE PROBES (MICROMETERS) С READ(11, *) SEP С READ THE DEPTH INCREMENT (MICROMETERS) READ(11.*)DELTH READ THE PROBE CURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3) С READ(11,*)ICUR WRITE (6,7777) 7777 FORMAT (1H1) WRITE(6,20) FORMAT (2X, 'CALCULATION OF THE RESITIVITY FROM THE SPREADING 20 1 RESISTANCE',/,2X, 2 'USING THE CHOO ET AL GAUSS-LAGUERRE QUADRATURE') GO TO (30, 40, 50), ICUR WRITE(6,35) 30 FORMAT(2X, 'SCHUMANN AND GARDNER CURRENT DENSITY') 35 GO TO 60 40 WRITE(6,45) FORMAT (2X. 'CHOO UNIFORM CURRENT DENSITY') 45 GO TO 60 50 WRITE(6.55) FORMAT(2X, 'RING DELTA FUNCTION CURRENT DENSITY') 55 WRITE(6,65)AA,SEP,DELTH,NPT 60 FORMAT(2X, 'RADIUS=', F6.1, 2X, 'SEP=', F6.1, 2X, 'DELTH=', F7.3, 2X, 65 1 ' # PTS=',I3)

```
С
     THE FINAL SET OF WEIGHTS, W(I) (I=1,33) ARE NOW CONSTRUCTED FROM THE INITIAL SET OF WEIGHTS AND THE TWO-PROBE CONFIGURATION FUNCTION (IN
С
С
С
     LINE 90) AND THE FORM OF THE CURRENT DENSITY FUNCTION (IN THE
     APPROPRIATE NEXT LINE DEPENDING UPON THE VALUE OF ICUR).
С
С
DO 70 I=1,33
     T(I) = DTANH(DELTH * KA(I) / AA)
     A(1,I)=1.0D0
     A(2, I) = 1.000
90
     W(I) = W(I) * (DBJ1(KA(I))/KA(I) * 2-DBJO(KA(I) * SEP/AA)/(2.DO * KA(I)))
     IF (ICUR.EQ.1) W(I) = W(I) * DSIN(KA(I))/2.DO
     IF (ICUR.EQ.2) W(I) = W(I) * DBJ1(KA(I))
     IF (ICUR.EQ.3) W(I)=W(I)*KA(I)*DBJO(KA(I))/2.DO
70
     CONTINUE
C
     THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION
С
С
     FACTOR INTEGRAL FOR THE SUBSTRATE POINT. THE CORRECTION FACTOR. THE
     SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
С
C
R5=0.0
     R6=0.0
     R7=0.0
     DO 80 I=1,33
     R5=R5+W(I)
     R6=R6+W(I)*T(I)
     R7=R7+W(I)/T(I)
80
     CONTINUE
     WRITE(6,165)R5,R6,R7
    FORMAT (2X, 'R5=', F11.6, ' R6=', F11.6, ' R7=', F11.6,/)
165
     R8=1.27324D4/AA
     R9=R8/RSP(NPT) *R5
     SDIF(NPT) = 0.0
     COND (NPT) = R9
     RHO(NPT) = 1.0/R9
     ITER (NPT) =0
     SENS (NPT) =0.0
     DO 170 I=1,33
     B(I) = 1.0
170 CONTINUE
```

```
D0 1000 K=NPT-1,1,-1
     ITER(K)=0
     R10=0.
     R11=R8/RSP(K)/COND(K+1)
     R12=RSP(K+1)/RSP(K)
     R13=R12
     GOTO 1450
1060 CONV=DABS(1.0-R14*R11/R12)
     IF (CONV.LT.1D-3) GO TO 1200
     IF (R10.EQ.1.) R15=0.0
     R16=R15
     R15=R12
     R17 = (R14 - R6) / (R7 - R14) / R12
     R18=R7*R17*R11-1.0
     R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
     IF (R18.GT.0.0) R13=R13/2.0/R17
     IF (R18.LT.0.0) R13=2.0*R11*R6/R13
     VAR1 = (R15 - R13) * (R15 - R16)
     IF (VAR1.LE.O.O) GO TO 1170
     VAR2=R10-2.*JIDINT(R10/2.)
     IF (VAR2.EQ.0.0) R13=(R15*R15-R13*R16)/(2*R15-R13-R16)
1170 R12=R13
     GO TO 1450
1200 R19=R11*R14*COND(K+1)
     R20=0.0
     DO 200 I=1,33
     SEP=A(1,I)*(1.0-T(I)*T(I))
     DELTH=(R12*T(I)*A(1,I)+1.0)**2
     R20=W(I) *SEP/DELTH+R20
     A(1,I) = A(2,I)
200 CONTINUE
     SENS(K) = 1.0/(1.0 - R20 * R13/R14)
     COND(K) = R19
     RHO(K) = 1.0/R19
     SDIF(K)=R14*R8/R13/COND(K+1)
     SDIF(K) = 1.0 - SDIF(K) / RSP(K)
     ITER(K)=R10
1000 CONTINUE
1001 FORMAT(2X, I3, 2X, F7.3, 2X, 4(1PE11.3), 3X, I5)
1002 FORMAT (4X, 'NPT', 4X, 'DEPTH', 5X, 'RMEAS', 6X, '1-RCAL/R', 4X, 'RHO', 8X
    1 ,'SENS',6X,'ITER',/)
1003 FORMAT(2X, F7.3, 2X, 1PE11.3)
1004 FORMAT(2X, 1PE11.4)
     WRITE(6,1002)
     WRITE(12,*)NPT
     DO 1020 I=1,NPT
     WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),SENS(I),ITER(I)
     WRITE(12,1003)DEPTH(I),RHO(I)
1020 CONTINUE
     STOP
```

```
С
С
   THE REMAINING LINES ARE PART OF THE INVERSION METHOD BASED UPON THE
С
   BERKOWITZ AND LUX BOUNDING TECHNIQUE.
C
1450 D0 1475 I=1,33
   A(2,I) = (R12*A(1,I)+T(I))/(1.0+T(I)*A(1,I)*R12)
   B(I) = A(2, I)
1475 CONTINUE
   R14=0.0
   DO 1480 I=1,33
   R14=R14+W(I)*B(I)
1480 CONTINUE
   R10=R10+1.
   GO TO 1060
   END
```

C	DOUBLE PRECISION FUNCTION DBJO(X)
C****	***************************************
	POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJO REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.1 IS USED FOR X < 3 EQUATION 9.4.3 IS USED FOR X > 3
(****	TMPI TCTT DOUBLE PRECISION (A-H.O-7)
С	DOUBLE PRECISION X,X1,X2,F0,THETA
С	DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7
С	DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7
С	DOUBLE PRECISION C1, C2, C3, C4, C5, C6, C7
	DATA A1/1.000000000/,A2/-2.249999/D0/,A3/1.2656208D0/
	DATA A7/0.0002100D0/
	DATA B1/.79788456D0/.B2/00000077D0/.B3/00552740D0/
	DATA B4/00009512D0/.B5/.00137237D0/.B6/00072805D0/
	DATA B7/.00014476D0/
	DATA C1/78539816D0/,C2/04166397D0/,C3/00003954D0/
	DATA C4/.00262573D0/,C5/00054125D0/,C6/00029333D0/
	DATA C7/.00013558D0/
	1F(X.GI.3.0D0) GU IU IU 10
	$DB_{0} = ((((A_{7} \times 2 + A_{6}) \times 2 + A_{5}) \times 2 + A_{4}) \times 2 + A_{3}) \times 2 + A_{2}) \times 2 + A_{1}$
	RETURN
10	CONTINUE
	X1=3.D0/X
	F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
	HE A = X + ((((((/*X1+L6)*X1+L6)*X1+L4)*X1+L3)*X1+L2)*X1+L1)
	FND
C****	*END OF THE DBJO DOUBLE PRECISION FUNCTION************************************

DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************
C POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1 C REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ C AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.4 IS USED FOR X < 3 C AND EQUATION 9.4.6 IS USED FOR X > 3
C*************************************
<pre>IMPLICIT DOUBLE PRECISION (A-H, 0-Z) C DOUBLE PRECISION X, X1, X2, F1, THETA C DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7 C DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7 C DOUBLE PRECISION C1, C2, C3, C4, C5, C6, C7 DATA A1/0.500000D0/, A2/56249985D0/, A3/.21093573D0/ DATA A4/03954289D0/, A5/.00443319D0/, A6/00031761D0/ DATA A4/03954289D0/, A5/.0000156D0/, B3/.01659667D0/ DATA B1/.79788456D0/, B2/.00000156D0/, B3/.01659667D0/ DATA B1/.79788456D0/, B5/00249511D0/, B6/.00113653D0/ DATA B4/.00017105D0/, B5/00249511D0/, B6/.00113653D0/ DATA B7/00020033D0/ DATA C1/-2.35619449D0/, C2/.12499612D0/, C3/.00005650D0/ DATA C4/00637879D0/, C5/.00074348D0/, C6/.00079824D0/ DATA C7/00029166D0/</pre>
IF (X.GT.3.0D0) G0 T0 10
X2=X*X/9.D0
DBJ1=X [*] ((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1) RETURN
10 X1=3.0D0/X
F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJ1=F1*DCOS(THETA)/DSQRT(X) RETURN END
C****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF THE GL VERSION 1.0****

С VAR VERSION 1.0 С С THIS PROGRAM CALCULATES THE RESISTIVITY PROFILE FROM THE С SPREADING RESISTANCE USING THE VARIATIONAL TECHNIQUE OF С CHOO ET AL. AND A GENERALIZATION TO THE VARIATIONAL TECHNIQUES С OF THE BERKOWITZ AND LUX BOUNDING METHOD FOR INVERTING THE CORRECTION С FACTOR INTEGRAL. С THIS PROGRAM IS THE INVERSE OF THE RGENVAR VERSION 1.0 PROGRAM С С THE CALCULATION OF THE VARIOUS INTEGRALS IS CARRIED OUT BY AN С ADAPTATION OF THE BERKOWITZ-LUX APPROXIMATE INTERGAL AS WELL AS THE С SEGMENTED NEWTON-COTES QUADRATURE. С С REFERENCES: "SPREADING RESISTANCE CALCULATIONS BY THE VARIATIONAL С METHOD," BY S. C. CHOO, M. S. LEONG, AND L. S. TAN, SOLID STATE ELECTRONICS, VOL. 24, PP. 557-562, 1981. С С "AN EFFICIENT NUMERICAL SCHEME FOR SPREADING RESISTANCE С CALCULATIONS BASED ON THE VARIATIONAL METHOD," BY С S. C. CHOO, M. S. LEONG, AND J. H. SIM, С SOLID STATE ELECTRONICS, VOL. 26, PP. 723-730, 1983. С С THE PROGRAM READS (FROM FORO10) THE PROBE RADIUS, PROBE SEPARATION, С DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE С RESISTIVITY, SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY (FROM С THE TOP SURFACE). С С THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE С ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL. С С THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES С AS WELL AS THE RESULTS OF THE CALCULATION. С С IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE, С PLOTTING, OR FURTHER ANALYSIS. С IMPLICIT DOUBLE PRECISION (A-H, 0-Z) DIMENSION RSP(1000), RH0(1000), DEPTH(1000), CF(1000) DIMENSION A(2,55), CN9(9), W(3,55), T(55), VOL(1000) DIMENSION COND (1000), B (55), SDIF (1000), SENS (1000), ITER (1000) DOUBLE PRECISION KA(55), L, K1, K2, I1(1000), I2(1000) DOUBLE PRECISION I3 (1000), I2S (1000), I3S (1000) С THE SET OF PROBE-SPACING INDEPENDENT SAMPLING POINTS, KA(I), (I=27,55) С AND WEIGHTS, W(J,I), (J=1,3 I=27,55) ARE ENTERED HERE BY MEANS OF С DATA STATEMENTS. THE NEWTON-COTES COEFFICIENTS ARE ALSO ENTERED С HERE IN DATA STATEMENTS. С C

DATA KA(27)/0.12D0/,KA(28)/0.203885D0/,KA(29)/0.346410D0/ DATA KA(30)/0.588566D0/,KA(31)/1.0D0/,KA(32)/1.53540D0/ DATA KA(33)/2.07080D0/,KA(34)/2.60619D0/,KA(35)/3.14159D0/ DATA KA(36)/3.31412DO/,KA(37)/3.48665DO/,KA(38)/3.65917DO/ DATA KA(39)/3.83171D0/,KA(40)/6.28319D0/,KA(41)/7.01558D0/ DATA KA(42)/9.42478DO/,KA(43)/10.1735DO/,KA(44)/12.5664DO/ DATA KA(45)/13.3237D0/,KA(46)/15.7080D0/,KA(47)/16.4701D0/ DATA KA(48)/18.8496D0/,KA(49)/19.6159D0/,KA(50)/39.24D0/ DATA KA(51)/49.87D0/,KA(52)/56.32D0/,KA(53)/311.0D0/ DATA KA(54)/983.5D0/,KA(55)/1100.0D0/,W(1,27)/1.96943D-2/ DATA W(2,27)/4.92949D-3/,W(3,27)/9.85305D-3/ DATA W(1,28)/1.51585D-1/,W(2,28)/3.80282D-2/ DATA W(3,28)/7.59243D-2/,W(1,29)/9.40755D-2/ DATA W(2,29)/2.37573D-2/,W(3,29)/4.72756D-2/ DATA W(1,30)/3.94777D-1/,W(2,30)/1.01659D-1/ DATA W(3,30)/2.00331D-1/,W(1,31)/2.34711D-1/ DATA W(2,31)/6.41889D-2/,W(3,31)/1.22743D-1/ DATA W(1,32)/3.22596D-1/,W(2,32)/1.02248D-1/ DATA W(3,32)/1.81618D-1/,W(1,33)/5.12837D-2/ DATA W(2,33)/2.17231D-2/,W(3,33)/3.33773D-2/ DATA W(1,34)/2.91802D-2/,W(2,34)/2.46688D-2/ DATA W(3,34)/2.68298D-2/,W(1,35)/4.03186D-2/ DATA W(2,35)/1.80765D-3/,W(3,35)/-9.61334D-4/ DATA W(1,36)/0.0D0/,W(2,36)/1.03120D-3/ DATA W(3,36)/0.0D0/,W(1,37)/0.0D0/ DATA W(2,37)/1.54721D-4/,W(3,37)/0.0D0/ DATA W(1,38)/0.0D0/,W(2,38)/9.17871D-5/ DATA W(3,38)/0.0D0/,W(1,39)/0.0D0/ DATA W(2,39)/3.84395D-3/,W(3,39)/9.19809D-3/ DATA W(1,40)/4.73102D-2/,W(2,40)/0.0D0/ DATA W(3,40)/8.01205D-3/,W(1,41)/0.0D0/ DATA W(2,41)/3.91858D-3/,W(3,41)/2.52148D-3/ DATA W(1,42)/1.90479D-2/,W(2,42)/0.0D0/ DATA W(3,42)/2.32728D-3/,W(1,43)/0.0D0/ DATA W(2,43)/1.08897D-3/,W(3,43)/1.08692D-3/ DATA W(1,44)/1.03669D-2/,W(2,44)/0.0D0/ DATA W(3,44)/1.02754D-3/,W(1,45)/0.0D0/ DATA W(2,45)/4.57291D-4/,W(3,45)/5.80787D-4/ DATA W(1,46)/6.53592D-3/,W(2,46)/0.0D0/ DATA W(3,46)/5.56264D-4/,W(1,47)/0.0D0/ DATA W(2,47)/2.25498D-4/,W(3,47)/3.52250D-4/ DATA W(1,48)/8.71084D-3/,W(2,48)/0.0D0/ DATA W(3,48)/3.40149D-4/,W(1,49)/0.0D0/ DATA W(2,49)/8.28909D-5/,W(3,49)/3.09148D-4/ DATA W(1,50)/0.0D0/,W(2,50)/4.12225D-4/ DATA W(3,50)/0.0D0/,W(1,51)/0.0D0/,PI/3.14159265D0/ DATA W(2,51)/0.0D0/,W(3,51)/1.92433D-3/ DATA W(1,52)/1.78441D-2/,W(2,52)/0.0D0/,W(3,52)/0.0D0/ DATA W(1,53)/9.02512D-4/,W(2,53)/0.0D0/,W(3,53)/0.0D0/ DATA W(1,54)/1.52804D-3/,W(2,54)/0.0D0/,W(3,54)/0.0D0/ DATA W(1,55)/0.0D0/,W(2,55)/0.0D0/,W(3,55)/9.40768D-6/

```
C09=4.0D0/14175.0D0
     CN9(1) = C09 * 989.000
     CN9(2)=C09*5888.0D0
     CN9(3) = -C09 * 928.000
     CN9(4) = C09 * 10496.000
     CN9(5) = -C09 + 4540.000
     CN9(6) = CN9(4)
     CN9(7) = CN9(3)
     CN9(8) = CN9(2)
     CN9(9) = CN9(1)
C**********
                                              *****
                 *****
С
С
     BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
С
С
     NPT
              = NUMBER OF RESISTIVITY & SPREADING RESISTANCE VALUES
С
     AA
              = PROBE RADIUS (MICROMETERS)
     SEP
С
              = PROBE SEPARATION (MICROMETERS)
С
     DELTH
             = SUBLAYER THICKNESS (MICROMETERS)
С
     DEPTH(I) = DEPTH (MICROMETERS)
С
     RSP(I) = SPREADING RESISTANCE (OHMS)
С
С
     READ THE NUMBER OF POINTS IN THE STRUCTURE
     READ(11.*)NPT
С
     READ THE DEPTH AND TWO-PROBE RESISTANCE FROM THE SURFACE
     D0 70 I=1,NPT
     READ(11, *)DEPTH(I), RSP(I)
     CONTINUE
70
С
     READ THE ELECTRICAL PROBE RADIUS (MICROMETERS)
     READ(11, *)AA
     READ THE PROBE SEPARATION (MICROMETERS)
С
     READ(11, *)SEP
С
     READ THE DEPTH INCREMENT (MICROMETERS)
     READ(11,*)DELTH
     SUBRSP=RSP(NPT)
     FORMAT(8X, 'AA=', F10.3,' SEP=', F8.3,' DELTH=', F8.5,' NPT=', I5)
75
     AA=AA*1D-4
     SEP=SEP*1D-4
     DELTH=DELTH*1D-4
     SEP=SEP/AA
     DELTH=DELTH/AA
     L=1.122918967D0*(1.0/SEP)
     WRITE (6,7777)
 7777 FORMAT(1H1)
     WRITE(6,80)
     FORMAT (8X, 'CALCULATION OF THE RESISTIVITY FROM SPREADING
 80
    1 RESISTANCE')
     WRITE(6,81)
     FORMAT (8X, 'USING CHOO ET AL VARIATIONAL TECHNIQUE-BL')
 81
      WRITE(6,75)AA,SEP,DELTH,NPT
```

```
С
С
    BEGIN CONSTRUCTION OF SAMPLING POINTS AND WEIGHTS
С
    REMEMBER TO ADD WEIGHTS AT BOUNDARY OF SAMPLING REGIONS.
С
    THE REGION DESIGNATIONS FOLLOW THE NOTATION OF THE SECOND REFERENCE.
С
С
С
    REGION A (0 \leq KA \leq 1D-5) (I=1,9)
С
DO 142 I=1.9
    KA(I) = 1.D - 5 * (I - 1) / 8.D0
    W(1, I) = 1.D - 5 * CN9(I) / 8.D0
    W(2,I) = W(1,I)/4.DO
    W(3,I) = W(1,I)/2.D0
142
    CONTINUE
    WEND1=W(1,9)
    WEND2=W(2,9)
    WEND3=W(3.9)
C
С
    REGION B (1D-5\langle =KA \langle =L \rangle) (I=9.17)
C
C*****
     ******
    DO 144 I=9,17
    KA(I)=1.D-5*(L*1.D5)**((I-9.)/8.)
    W(1, I) = CN9(I-8) * KA(I) * DLOG(L*1.D5) / 8.DO
    W(2,I) = W(1,I)/4.DO
    W(3, I) = W(1, I) / 2.DO
    CONTINUE
144
    W(1,9) = W(1,9) + WEND1
    W(2,9) = W(2,9) + WEND2
    W(3,9) = W(3,9) + WEND3
С
С
    REGION C (L\leq=KA\leq=.12) (I=18,26)
С
D0 146 I=18,26
    KA(I) = L * (0.12DO/L) * * ((I-18.)/8.)
    W(1, I) = CN9(I-17) * KA(I) * DLOG(0.12/L)/8.DO
    W(2,I) = W(1,I) / 4.DO
    W(3,I) = W(1,I)/2.D0
146 CONTINUE
```

C****	**************
С	
C	REGION D E AND E
č	THE VALUES OF THE VALUE AND WATE HAVE BEEN ENTEDED BY THE
C	THE VALUES OF THE RA(1) AND W(1) HAVE DEEN ENTERED BT THE
C	DATA STATEMENTS AT THE BEGINNING OF THE PROGRAM
С	
C****	*********
•	D0 155 T=1 55
	I(1) = DIANH(R4)
	A(1, I) = 1.000
	A(2, I) = 1.000
	B(T) = 1.000
155	CONTINUE
100	
	R611=0.0D0
	R7I1=0.0D0
	R5I2=0.0D0
	R6I2=0.0D0
	R7I2=0.0D0
	R5T3=0 0D0
	R713=0.0D0
	DU 200 1=18,55
	R5I1=R5I1+W(1,I)
	R5I2=R5I2+W(2,I)
	R5I3=R5I3+W(3.I)
	B6T1 - B6T1 + W(1, T) + T(T)
	Pero-Pero, W(2, T) + T(T)
	$NOIZ = NOIZ + \pi(2, 1) + 1(1)$
	R613=R613+W(3,1)*I(1)
	R7I1=R7I1+W(1,I)/I(I)
	R7I2=R7I2+W(2,I)/T(I)
	R7I3=R7I3+W(3,I)/T(I)
200	CONTINUE
200	B5T2S-B5T2
	DET2C-DET2
	R6125=R612
	K6135=K613
	R7I2S=R7I2
	R7I3S=R7I3
	D0 210 I=2,17
	R5I1=R5I1+Ŵ(1.I)
	R5T2=R5T2+W(2,T)
	$PCT1_PCT1_W(1_T)_{+T}(T)$
	ROII=ROII+M(I,I)+I(I)
	K012=K012+W(2,1)*I(1)
	R6I3=R6I3+W(3,1)*I(1)
	R7I1=R7I1+W(1,I)/T(I)
	R7I2=R7I2+W(2,I)/T(I)
	R7I3=R7I3+W(3,I)/T(I)
210	CONTINUE

118

```
R5I1=R5I1+W(1,1)
     R5I2=R5I2+W(2,1)
     R5I3=R5I3+₩(3,1)
     R6I1=R6I1+W(1,1)*T(1)
     R6I2=R6I2+W(2,1)*T(1)
     R6I3=R6I3+W(3,1)*T(1)
     DEN5=(R5I1*R5I2-R5I3*R5I3)
     R5K1=(R5I2-0.5D0*R5I3)/DEN5
     R5K2=(0.5D0*R5I1-R5I3)/DEN5
     CFDEN5=1.0D0/(R5K1+0.5D0*R5K2)
     CFINT5=CFDEN5*(R5K1*R5I3S+R5K2*R5I2S)
     R5=CFINT5
     DEN6=(R6I1*R6I2-R6I3*R6I3)
     R6K1=(R6I2-0.5D0*R6I3)/DEN6
     R6K2=(0.5D0*R6I1-R6I3)/DEN6
     CFDEN6=1.0D0/(R6K1+0.5D0*R6K2)
     CFINT6=CFDEN6*(R6K1*R6I3S+R6K2*R6I2S)
     R6=CFINT6
     DEN7=(R7I1*R7I2-R7I3*R7I3)
     R7K1=(R7I2-0.5D0*R7I3)/DEN7
     R7K2=(0.7D0*R7I1-R7I3)/DEN7
     CFDEN7=1.0D0/(R7K1+0.5*R7K2)
     CFINT7=CFDEN7*(R7K1*R7I3S+R7K2*R7I2S)
     R7=CFINT7
     WRITE(6,165)R5,R6,R7
165 FORMAT (8X, 'R5=', E11.4, ' R6=', E11.4, ' R7=', E11.4)
     R8=1.27324/(2.0*AA)
     R9=R8/SUBRSP*R5
     SDIF(NPT)=0.0D0
     COND(NPT) = R9
     RHO(NPT) = 1.0DO/R9
     ITER(NPT)=0
     SENS (NPT) =0.0D0
```

```
С
С
     LAYERED STRUCTURE CALCULATION
С
C****
    D0 1000 KK=NPT-1,1,-1
     ITER(KK)=0
     IR10=0
     R11=R8/RSP(KK)/COND(KK+1)
     R12=RSP(KK+1)/RSP(KK)
     R13=R12
     GOTO 1450
1060 CONV=DABS(1.0D0-R14*R11/R12)
     IF (CONV.LT.1D-3) GO TO 1200
     IF (IR10.EQ.1) R15=0.0D0
     R16=R15
     R15=R12
     R17 = (R14 - R6) / (R7 - R14) / R12
     R18=R7*R17*R11-1.0D0
     R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
     IF (R18.GT.0.0) R13=R13/2.0D0/R17
     IF (R18.LT.0.0) R13=2.0D0*R11*R6/R13
     VAR1 = (R15 - R13) * (R15 - R16)
     IF (VAR1.LE.O.ODO) GO TO 1170
     VAR2=IR10-2.D0*INT(IR10/2.D0)
     IF (VAR2.EQ.0.0) R13=(R15*R15-R13*R16)/(2*R15-R13-R16)
1170 R12=R13
     GO TO 1450
1200 R19=R11*R14*COND(KK+1)
     R20=0.0D0
     DO 1059 I=1,55
     SEP=A(1,I)*(1.0-T(I)*T(I))
     DELTH=(R12*T(I)*A(1,I)+1.0)**2
     R20=W(1,I)*SEP/DELTH+R20
     A(1,I) = A(2,I)
1059 CONTINUE
     SENS(KK)=1.0D0/(1.0D0-R20*R13/R14)
     COND(KK)=R19
     RH0(KK)=1.0D0/R19
     SDIF(KK)=R14*R8/R13/COND(KK+1)
     SDIF(KK)=1.0D0-SDIF(KK)/RSP(KK)
     ITER(KK)=IR10
1000 CONTINUE
1001 FORMAT(8X, I5, 2X, 4(1PE12.3), 3X, I5)
1002 FORMAT (12X, 'NPT', 6X, 'DEPTH', 5X, 'RMEAS', 8X, '1-RCAL/R', 4X, 'RHO ',
    1 8X,'ITER',/)
1003 FORMAT(8X, 1PE11.3, 2X, 1PE11.3)
1004 FORMAT(8X, 1PE11.4)
     WRITE(6,1002)
     WRITE(12,*)NPT
     D0 1020 I=1,NPT
     WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),ITER(I)
     WRITE(12,1003) DEPTH(I), RHO(I)
1020 CONTINUE
     STOP
```

C****	*******	****
C C C	THE REMAINING LINES ARE PART OF THE GENERALIZATION TO THE V	ARIATIONAL RSION OF
C	THE CORRECTION FACTOR INTEGRAL (FOR WHICH THERE ARE SEVERAL IN THE PRESENT CASE)	INTEGRALS
C	IN THE TRESENT CASE).	
C****	****	****
1450	D0 1475 I=1,55	
	A(2,I) = (R12*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R12) B(I)=A(2,I)	
1475	CONTINUE	
	R14=0.0D0	
	R5I1=0.0D0	
	R5I2=0.0D0	
	R5I3=0.0D0	
	D0 1480 I=18,55	
	R5I1=R5I1+W(1,I)*B(I)	
	R5I2=R5I2+W(2,I)*B(I)	
	R513=R513+W(3,1)*B(1)	
1480	CONTINUE	
	R512S=R512	
	K5135=K513	
	DU 1485 1=1,17	
	K511=K511+W(1,1)*B(1)	
	R512=R512+W(2,1)*B(1)	
1405	K513=K513+W(3,1)*D(1)	
1465	CUNTINUE DEN_(DET1,DET0,DET2,DET2)	
	DEN=(RS11*RS12-RS13*RS13)	
	RSN1=(RS12-0.5D0*RS15)/DEN	
	$CEDEN-1 ODO/(BEK1_0 EDO*REK2)$	
	(FINT-(FDFN*(R5K1*R5T3S*R5K2*R5T2S))	
	R14=CEINT	
	TR10=TR10+1	
	G0 T0 1060	
	END	
C****	END OF THE VAR VERSION 1.0 PROGRAM************************************	****

C*************************************
C THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, FOUR-PROBE RESISTANCE, C Z(X,S), THE DERIVATIVE OF THE TWO-PROBE (SPREADING) RESISTANCE WITH C RESPECT TO THE PROBE SPACING, D(X,S), AS WELL AS THE FOUR-PROBE SHEET C RESISTANCE, SH(X).
C THE FOUR-PROBE RESISTANCE AND THE DERIVATIVE FUNCTION ARE CALCULATED C FROM THE MULTILAYER SOLUTION OF LAPLACE'S EQUATION (SEE REFERENCE). C THE FOUR-PROBE SHEET RESISTANCE IS CALCULATED FROM THE TRADITIONAL C LAMINAR FLOW MODEL.
C THE INTEGRALS INVOLVED IN Z(X,S) AND D(X,S) ARE EVALUATED BY MEANS C OF A 1250- (OR 10000-) POINT INTEGRATION METHOD USED BY BERKOWITZ.
C REFERENCES: "THE RELATION BETWEEN TWO-PROBE AND FOUR-PROBE RESISTANCES ON NONUNIFORM STRUCTURES", C BY JOHN ALBERS AND H. L. BERKOWITZ, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 131, PP. 392-398 (1984).
C THIS PROGRAM WAS ORIGINALLY WRITTEN C FOR THE CALCULATION OF THE FOUR-PROBE FUNCTION Z(X). WE HAVE C USED THE INTEGRATION SCHEME TO ALSO CALCULATE THE FUNCTION S(X), C THE DERIVATIVE OF SPREADING RESISTANCE WITH RESPECT TO LN(S). IN C ADDITION, THE SHEET RESISTANCE IS ALSO CALCULATED BY MEANS OF THE C STANDARD LAMINAR FLOW MODEL. THESE THREE FUNCTION ARE COMPARED, C AND EACH IS STORED IN APPROPRIATE DATA FILES.
C NOTE-THE EQUATIONS USED MAKE USE OF THE CHOO UNIFORM CURRENT C DENSITY. THIS CAN BE CHANGED IF DESIRED BY ALTERING THE LINES WHERE W(1,I) AND W(2,I) ARE DEFINED.
C C ***********************************
IMPLICIT DOUBLE PRECISION (A-H,0-Z) DOUBLE PRECISION COND(300), OUT(6,300), ROL(300), W5(5) DOUBLE PRECISION HTAN(10000), F(10000), W(2,10000)
C*************************************
C DESCRIPTION OF VARIABLES AND ARRATS
C NPT = NUMBER OF DATA POINTS C SEP = SEPARATION BETWEEN THE PROBES (MICROMETERS) C DELTH = DEPTH INCREMENT (MICROMETERS) C AA = PROBE RADIUS (MICROMETERS) C SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM) C SUBTH = SUBSTRATE THICKNESS (MICROMETERS)
C (NONE = 1, COND = 2, INS = 3) C DEAL OF DESTSTITUTE VALUES (OUN CM)
C NMANY = NUMBER OF INTEGRATION POINTS TO BE USE
C (EITHER 1250 UR 10000 (ALL BEIS UFF FUR UTHERS)) C
C*************************************

С	READ THE ELECTRICAL PROBE RADIUS (MICROMETERS)
C	READ (5,*) AA
C	READ THE FRUDE SEPARATION (MICROMETERS) READ (5 *) SEP
С	READ THE DEPTH INCREMENT (MICROMETERS)
_	READ (5,*) DELTH
С	READ THE NUMBER OF POINTS IN THE STRUCTURE
~	READ (5,*) NPT
C	READ THE SUBSTRATE THICKNESS (MICRUMETERS)
C	READ THE SUBSTRATE RESSISTIVITY (NHM*(M)
C	READ (5,*) SUBRHO
С	READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
	READ (5,*) IBBC
c	NUM=NPT
C	DO 7 T-1 NUM
	READ (5.*) ROL(I)
	COND(I)=1.ODO/ROL(I)
7	CONTINUE
C	READ THE NUMBER OF INTEGRATION POINTS TO BE USED
C	THIS MAY BE EITHER 1250 UR 10000 (ALL BETS UFF FUR UTHERS)
C	TE(NMANY NE 1250 OR NMANY NE 10000) GOTO 666
Č	D0 5 I=1.2
	DO 2 J=1, NMANY
_	W(I, J) = 0.0D0
2	CONTINUE
5	$W_5(1) = 7$ DO
	W5(2) = 32.00
	W5(3)=12.D0
	W5 (4)=32.D0
	W5(5)=7.D0
	TEMP2=0.0
	TEMP3=0.0
	DX=500.03/NMANY
	D0 10 I=1,5
10	W5(1) = W5(1) + 2.000 + 0X/45 PT = 3.14150265400
	D0 20 I=2.5
	II=I-1
	W(1,II)=W5(I)
20	W(2,II)=W5(I)
	A=AA/SEP T=DFI TH/SEP
	SUB=SUBT/SEP
	S1=SEP*1.D-4

```
DO 30 I=4,1246,4
     D0 30 J=0.4
     IJ=I+J
     J1=J+1
     W(1, IJ) = W(1, IJ) + W5(J1)
     W(2,IJ) = W(2,IJ) + W5(J1)
30
     DO 11 I=1.NMANY
     X=DX*I
     W(1, I) = W(1, I) * (DBJO(X) - DBJO(2*X)) * 2*DBJ1(A*X) / (A*X)
     W(2,I) = W(2,I) * 2 * DBJ1(X) * DBJ1(A * X)
 11
     DO 13 I=1,NMANY
     X=DX*I
     XT=X*T
     IF (XT.GT.40) HTAN(I)=1.0D0
     IF (XT.GT.40) G0 T0 17
     HTAN(I) = (EXP(X*T) - EXP(-X*T)) / (EXP(X*T) + EXP(-X*T))
17
     XSUB=X*SUB
     IF (XSUB.GT.40) F(I)=1.0D0
     IF (XSUB.GT.40) GO TO 375
     F(I) = (EXP(X*SUB) + EXP(-X*SUB)) / (EXP(X*SUB) - EXP(-X*SUB))
375 GO TO (400,500,600), IBBC
400 F(I)=1.0D0
     GO TO 600
500 F(I) = 1.0DO/F(I)
600 TEMP1=TEMP1+W(1,I)
     TEMP=TEMP+W(1,I)*F(I)
     TEMP2=TEMP2+W(2,I)
     TEMP3=TEMP3+W(2,I)*F(I)
 13 CONTINUE
     CORR=0.5DO*(1+7./64.*A*A)-TEMP1
     IF (A.GT..01) CORR=0.0
     NUM1=NUM+1
     COND(NUM1)=1.0D0/SUBRH0
     OUT(1,NUM1)=NUM*T*SEP
     OUT (2, NUM1) = SUBRHO
     OUT(3, NUM1) = COND(NUM1)
     OUT (4, NUM1) = (TEMP+CORR) /PI/S1*SUBRHO
     OUT (5, NUM1) = COND (NUM1) * SUB*S1
     OUT(6,NUM1)=(TEMP3+CORR)/PI/AA/1.OD-4*SUBRHO
     DO 14 I=NUM.1.-1
     OUT(1,I) = (I-1) *T*SEP
     OUT(2,I) = ROL(I)
     OUT(3, I) = COND(I)
     TEMP=0.0
     TEMP3=0.0
     I1=I+1
     OUT(5, I) = OUT(5, I1) + COND(I) * T * S1
```

```
DO 15 J=1, NMANY
      F(J) = (COND(I) * F(J) + HTAN(J) * COND(I1))
     1 / (COND(I1) + COND(I) * F(J) * HTAN(J))
      TEMP = TEMP + W(1, J) * F(J)
  15 TEMP3=TEMP3+W(2, J) * F(J)
      OUT(4,I) = (TEMP+CORR) / (PI*S1*COND(I))
      OUT(6, I) = (TEMP3+CORR) / (PI*AA*1.OD-4*COND(I))
  14 CONTINUE
      DO 16 I=1,NUM1
      OUT(5,I) = 1.0DO/OUT(5,I)
С
       OUT(4,I) = OUT(4,I) * PI/DLOG(2.0D0)
  16 OUT(6,I)=OUT(6,I)*PI
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE (6,100)
      FORMAT (2X, 'FOUR-PROBE RESISTANCE CALCULATIONS USING THE')
 100
      WRITE (6,125) NMANY
     FORMAT (2X, 'BERKOWITZ NMANY=', I7, ' POINT INTEGRATOR ')
125
      WRITE (6,150)
      FORMAT (2X, 'STRUCTURE OVER UNIFORM LAYER (SUBRHO, SUBTH)')
150
      WRITE (6,155) IBBC
155
      FORMAT (2X, 'ALL OVER (NONE=1, CON=2, INS=3)', I4, /)
      WRITE (6,200)
      FORMAT (4X, 'AA', 7X, 'SEP', 7X, 'DELTH', 5X, 'SUBTH', 8X, 'SUBRHO')
200
      WRITE (6,250) AA, SEP, DELTH, SUBT, SUBRHO
      FORMAT (2X, F5.2, 2X, F9.1, 2X, F7.3, 2(2X, 1PE11.2))
250
      WRITE (6,275)
275 FORMAT (/, 2X, 'DEPTH (UM)', 6X, 'RHO', 7X, 'FOUR-PRB', 4X, 'DERIVATI', 5X
     1 ,'SHEET')
      D0 300 I=1,NUM1
      WRITE (6,350) OUT(1,I),OUT(2,I),OUT(4,I),OUT(6,I),OUT(5,I)
350 FORMAT (1X,5(1PE11.4,1X))
      WRITE (11,800) OUT(1,I),OUT(4,I)
      WRITE (12,800) OUT(1,I),OUT(6,I)
      WRITE (13,800) OUT (1,I), OUT (5,I)
800 FORMAT(1X,2(1PE11.3,1X))
300 CONTINUE
      WRITE (11,900)
      WRITE (12,950)
      WRITE (13,975)
900 FORMAT(2X, 'RESULTS FOR DIFFERENCE FUNCTION USING BL NMANY=', I7.
     1' INT')
950 FORMAT(2X, 'RESULTS FOR DERIVATIVE FUNCTION USING BL NMANY=', I7,
     1' INT')
975 FORMAT(2X, 'CALCULATED SHEET RESISTANCE FROM LAMINAR MODEL')
      STOP
      FND
```

DOUBLE PRECISION FUNCTION DBJO(X)	
C*************************************	×.
C POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJO	
C REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.1 IS USED FOR X < 3 C EQUATION 9.4.3 IS USED FOR X > 3 C	7
C*************************************	i.
IMPLICIT DOUBLE PRECISION (A-H,O-Z) C DOUBLE PRECISION X,X1,X2,F0,THETA C DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7 C DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7 C DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7	
DATA A1/1.0000000D0/,A2/-2.24999997D0/,A3/1.2656208D0/ DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/ DATA A7/0.0002100D0/ DATA B1/.79788456D0/,B2/00000077D0/,B3/00552740D0/	
DATA B4/00009512D0/,B5/.00137237D0/,B6/00072805D0/	
DATA B7/.00014476D0/ DATA C1/78539816D0/,C2/04166397D0/,C3/00003954D0/ DATA C4/.00262573D0/,C5/00054125D0/,C6/00029333D0/ DATA C7/.00013558D0/ IF (X.GT.3.0D0) G0 T0 10	
X2=X*X/9.0D0 DBJ0=((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1 RETURN	
10 CUNTINGE X1=3.D0/X F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJ0=F0*DC0S(THETA)/DSQRT(X) RETURN END	
C*****END OF DOUBLE PRECISION FUNCTION DBJO************************************	ŧ

DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************
C
C POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ C AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED C MATHEMATICS SERIES 55. EQUATION 9.4.4 IS USED FOR X < 3 C AND EQUATION 9.4.6 IS USED FOR X > 3 C
C*************************************
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C DOUBLE PRECISION X,X1,X2,F1,THETA
C DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7
C DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
DATA A1/0.500000D0/,A2/56249985D0/,A3/.21093573D0/
DATA A4/03954289D0/,A5/.00443319D0/,A6/00031761D0/
DATA A7/.00001109D0/
DATA B1/.79788456D0/.B2/.00000156D0/.B3/.01659667D0/
DATA B4/.00017105D0/.B5/00249511D0/.B6/.00113653D0/
DATA B7/00020033D0/
DATA C1/-2.35619449D0/.C2/.12499612D0/.C3/.00005650D0/
DATA $(4/-0.0637879D0/(5/0.0074348D0/(6/0.00079824D0/))$
DATA $(7/-0.002916600)$
IF (X GT 3 ODO) GO TO 10
$X_{2} X_{*} X_{/9} DO$
DB [1 - Y + (((((A7 + Y) + A6) + Y) + A5) + Y) + A(A) + Y) + A(A) + Y) + A(A) + Y) + A(A) + Y + A(A) + Y) + A(A) + Y) + A(A) + Y + A(A) + Y) + A(A) + A(A) + Y) + A(A) +
$10 \times 1-2 0 0 0 / Y$
$10 \ A1 = 3.000 / A$ E1 = ((((P7 + Y1 + P6) + Y1 + P6) + Y1 + P4) + Y1 + P2) + Y1 + P2) + Y1 + P1
T = (((((() / * AI + D0) * AI + D3) * AI + D4) * AI + D3) * AI + D2) * AI + D1 $T = T = (((((() / * AI + D0) * AI + D3) * AI + D3) * AI + D3) * AI + D1$
$\frac{11}{1} = \frac{1}{1} = 1$
ENV
C*****END UF DUUBLE PRECISIUN FUNCTION DBJI END UF PRUBE24 VERSION 1.0****

FOURCAL VERSION 1.0

THIS PROGRAM CALCULATES THE FOLLOWING DEPTH-DEPENDENT QUANTITIES: 1) FOUR-PROBE RESISTANCE, Z(X,S),

- 2) THE DERIVATIVE OF THE TWO-PROBE (SPREADING) RESISTANCE WITH
- RESPECT TO THE LOGARITHM OF THE PROBE SPACING, D(X,S), AND
- 3) THE FOUR-PROBE SHEET RESISTANCE, SH(X).

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THE FOUR-PROBE RESISTANCE AND THE DERIVATIVE FUNCTION ARE CALCULATED FROM THE SCHUMANN AND GARDNER MULTILAYER SOLUTION OF LAPLACE'S EQUATION (SEE REFERENCES) WHILE THE FOUR-PROBE SHEET RESISTANCE IS CALCULATED FROM THE TRADITIONAL LAMINAR FLOW MODEL.

THE INTEGRALS INVOLVED IN Z(X,S) AND D(X,S) ARE EVALUATED OVER THE INTERVAL FROM XL TO XU BY MEANS OF A TRAPEZOIDAL ROMBERG METHOD USED ORIGINALLY BY D'AVANZO ET AL. IN THEIR SPINT PROGRAM FOR CALCULATING THE ARRAY OF PARTIAL INTEGRALS.

REFERENCES: THE TRAPEZOIDAL ROMBERG METHOD WAS USED BY D'AVANZO ET AL. FOR CALCULATING THE CORRECTION FACTOR INTEGRAL FOR THE PURPOSE OF SPREADING RESISTANCE PROFILE ANALYSIS. THIS IS DESCRIBED IN THE REPORT "SPREADING RESISTANCE FOR IMPURITY PROFILES" BY DONALD C. D'AVANZO, ROBERT D. RUNG, AND ROBERT W. DUTTON, TECHNICAL REPORT NO. 5013-2 (FEBRUARY 1977), STANFORD UNIVERSITY, STANFORD, CALIFORNIA. THIS IS ALSO DESCRIBED IN THE PAPER "HIGH SPEED IMPLEMENTATION AND EXPERIMENTAL EVALUATION OF MULTILAYER SPREADING RESISTANCE ANALYSIS" BY DONALD C. D'AVANZO, ROBERT D. RUNG, ARNON GAT, AND ROBERT W. DUTTON, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 125, PP.1170-1176 (1978).

> THE CALCULATION OF THE SPREADING RESISTANCE FROM THE RESISTIVITY USING THE TRAPEZOIDAL ROMBERG MEHTOD IS DISCUSSED IN THE PAPER "COMPARISON OF SPREADING RESISTANCE CORRECTION FACTOR ALGORITHMS USING MODEL DATA" BY JOHN ALBERS, SOLID-STATE ELECTRONICS, VOL. 23, PP. 1197-1205 (1980).

THE PRESENT CALCULATION IS DESCRIBE IN THE PAPER "THE RELATION BETWEEN TWO-PROBE AND FOUR-PROBE RESISTANCES ON NONUNIFORM STRUCTURES", BY JOHN ALBERS AND H. L. BERKOWITZ, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 131, PP. 392-398 (1984).

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С
С
     DESCRIPTION OF THE VARIABLES AND ARRAYS
С
С
         NPT = NUMBER OF DATA POINTS
С
         AA
               = PROBE RADIUS (MICROMETERS)
С
         SEP
               = SEPARATION BETWEEN THE PROBES (MICROMETERS)
С
         DELTH = DEPTH INCREMENT (MICROMETERS)
С
         IBBC
               = BACKSURFACE BOUNDARY CONDITION
С
                  (NONE = 1, COND = 2, INS = 3)
С
         SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM)
С
         SUBTH = SUBSTRATE THICKNESS (MICROMETERS)
С
         RHO
               = ARRAY OF RESISTIVITY VALUES (OHM*CM)
С
         ICUR
                = CURRENT DENSITY
С
                  (SCHUMANN & GARDNER = 1, CHOO UNIFORM = 2, RING DELTA = 3)
С
         XL = LOWER LIMIT OF INTEGRATION (USUALLY ZERO)
С
         XU = UPPER LIMIT OF INTEGRATION (FROM 10 ON UP)
С
С
     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY.
С
     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
С
     SUBTH=DELTH AND SUBRHO=RHO(1).
С
С
     READ THE TOTAL NUMBER OF POINTS OR SUBLAYERS IN STRUCTURE.
     READ (5,*) NPT
     NUM=NPT+1
С
     READ THE EFFECTIVE ELECTRICAL PROBE RADIUS
     READ (5, *) AA
С
     READ THE SEPARATION BETWEEN THE PROBES
     READ (5, *) SEP
     READ THE DISTANCE BETWEEN POINTS OR THE SUBLAYER THICKNESS.
C
     READ (5,*) DELTH
     READ THE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
С
     READ (5,*) IBBC
С
     READ THE SUBSTRATE RESISTIVITY
     READ (5,*) SUBRHO
     READ THE SUBSTRATE THICKNESS
С
     READ (5,*) SUBTH
     RESISTIVITY ARRAY READ FROM SURFACE TO SUBSTRATE.
С
     DO 10 I=NUM,2,-1
     READ (5,*) RHO(I)
     COND(I) = 1.0DO/RHO(I)
    CONTINUE
 10
C
     READ THE FORM OF THE CURRENT DENSITY
     READ(5,*) ICUR
С
     READ THE LOWER LIMIT OF INTEGRATION
     READ (5,*) XL
С
     READ THE UPPER LIMIT OF INTEGRATION
     READ (5,*) XU
     RHO(1) = SUBRHO
     COND(1) = 1.0DO/SUBRHO
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C****	*******
с с с с с с с с с с с с с с	THE 1200 D0 LOOP IS THE MAJOR LOOP WHICH CALCULATES THE VARIOUS INTEGRALS (OVER THE INTERVAL FROM XL TO XU) FROM THE SUBSTRATE (NUM1=NUM) TO THE SURFACE (NUM1=1) BY INCREMENTING NUM1 BY -1. THE RELEVANT INFORMATION IS PASSED THROUGH AND CONTAINED IN THE COMMON STATEMENT IN THE MAIN PROGRAM AND THE SUBROUTINES. IFLAG IS SET EQUAL TO ZERO AS XL IS USUALLY ZERO AND THE INTEGRAND IS ZERO AT XL=0. HOWEVER, THE CURRENT DENSITY FUNCTIONS MAY GIVE ARTIFICIAL SINGULARITIES AT THE ORIGIN. FOR EXAMPLE, THE SCHUMANN AND GRADNER CURRENT DENSITY FUNCTION, SIN(X)/X, GOES TO ZERO AS X GOES TO ZERO BUT THE COMPUTER WOULD SIGNAL AN OVERFLOW DUE TO THE 1/X. THIS IS REMOVED BY THE SETTING OF IFLAG=0.
C****	***************************************
	D0 1200 NUM1=1,NUM
	IFLAG=0
	CALL INTGRL (FPINT, XL, XU, ZEX (NUM1), IFLAG)
	ZEX(NUM1) = ZEX(NUM1) * RHO(NUM1) / (PI*AA*1.OD-4)
	CALL TNTGRI (DETNT XI, XU DEX (NUM1), TELAG)
	DEX(NIM1) = DEX(NIM1) * RH0(NIM1) / (AA*1 OD-4)
1000	
1200	
	SHEET (1) = $CUND(1) * 30D I I * 1.00 - 4$
	DU 1300 NUM1=2, NUM
	SHEET $(NUM1) = 1.0DO/SHEET (NUM1-1) + CUND (NUM1) * DELTH*1.0D-4$
	SHEET (NUM1)=1.0D0/SHEET (NUM1)
1300	CONTINUE
	WRITE (6,7777)
7777	FORMAT(1H1)
	GO TO (1400,1420,1440), ICUR
1400	WRITE(6,1410)
1410	FORMAT (2X, 'SCHUMANN AND GARDNER CURRENT DENSITY=SIN(AX)/X')
	GO TO 1453
1420	WRITE(6,1430)
1430	FORMAT (2X, 'CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X')
	GO TO 1453
1440	WRITE (6 1450)
1/50	EORMAT(2) 'RING CURRENT DENSITY= $A*DR.IO(AX)$ ')
1450	CO TO (14EE 146E 147E) TBBC
1455	GU = U (1455, 1465, 1475), IDDC WDITE (6, 1460)
1455	$\frac{1}{100} \frac{1}{100} \frac{1}$
1460	GO TO 1500
1465	WRITE (6,1470)
1470	FORMAT (2X, 'PERFECTLY CONDUCTING BOUNDARY A1=TANH(LD)',/)
	G0 T0 1500
1475	WRITE (6.1480)
1480	FORMAT (2X, 'PERFECTLY INSULATING BOUNDARY A1=COTH(LD)',/)
2100	

```
1500 WRITE (6,1550) XL,XU
 1550 FORMAT (2X, 'XL=', F11.6, '; XU=', F11.6)
WRITE (6,1700) AA, SEP, DELTH
 1700 FORMAT (2X, 'RADIUS=', F11.6,'; SEP=', F11.6,'; DELTH=', F11.6)
      WRITE (6,2250) SUBRHO, SUBTH
 2250 FORMAT (2X, 'SUBRHO=', 1PE11.5, '; SUBTHICK=', 1PE11.5)
      WRITE (6,2300)
 2300 FORMAT (2X, 'DEPTH (U)', 8X, 'RHO', 5X, 'FOUR-PROBE', 6X, 'DERIV', 8X,
          'SHEET')
     1
      WRITE(11,*)NUM
      WRITE(12,*)NUM
      WRITE(13,*)NUM
      D0 2500 I=NUM,1,-1
      WRITE (6,2400) DEPTH(I), RH0(I), ZEX(I), DEX(I), SHEET(I)
 2400 FORMAT(1X,5(1PE11.3,1X))
      WRITE (11,2405) DEPTH(I), ZEX(I)
      WRITE (12,2405) DEPTH(I), DEX(I)
      WRITE (13,2405) DEPTH(I), SHEET(I)
 2405 FORMAT(1X,2(1PE11.3,1X))
 2500 CONTINUE
      WRITE (11,1550) XL,XU
      WRITE (11,1700) AA, SEP, DELTH
      WRITE (11,2250) SUBRHO, SUBTH
      WRITE (12,1550) XL,XU
      WRITE (12,1700) AA, SEP, DELTH
      WRITE (12,2250) SUBRHO, SUBTH
      STOP
      END
```

	SUBROUTINE INTGRL (F, A, B, ATEMP, IFLAG)
C****	***********
000000	D'AVANZO'S ET AL. TRAPEZOIDAL ROMBERG INTEGRATION SUBROUTINE. THIS ROUTINE NUMERICALLY INTEGRATES F BETWEEN THE LIMITS A AND B USING THE TRAPEZOIDAL AND ROMBERG METHODS AND RETURNS THE INTEGRAL FROM A TO B IN ATEMP AND ACCUMULATES THE RESULTS OF SEVERAL SUCCESSIVE CALLS IN ACC.
C C	DUE TO FUNCTIONS LIKE SIN(X)/X
C****	******
	EXTERNAL F DOUBLE PRECISION AI (30,30), MI, A, B, H, ACC, ATEMP, FO, F, CONV
C	CALCOLATE THE TRAPEZUIDAL APPRUXIMATION TO THE AREA CONV=5.0D-5 J=30 M-1
	K-1
С	SET F(0)=0 F0=0.D0
	IF ((A.NE.O.DO).OR.(IFLAG.EQ.1)) FO=F(A) AI(M,K)=.5DO*(B-A)*(FO+F(B)) DO 30 I=1,J MI=0.0DO H=(B-A)/(2.0DO**(M-1)) LPCNT=2**(M-1)
	D0 10 N-1 I PCNT
	MT - MT + E(A + (N + H - 5DO + H))
10	
10	MT-MT+H
	AT(M+1, K) = 5DO * (AT(M, K) + MT)
С	INCREASE THE ACCURACY OF THE TRAPEZOIDAL APPROXIMATIONS
C	BY USING THE ROMBERG METHOD
	MPONE = M+1
	DO 20 K=1, MPONE
	AI(M+1,K+1) = ((4.0D0**K)*AI(M+1,K)-AI(M,K))/(4.0D0**K-1.D0)
20	CONTINUE
	ATEMP=AI(M+1,M+1)
	IF (I.LE.6) GO TO 60
С	STOP INTEGRATING WHEN INTEGRAL CUNVERGES TO WITHIN .005%
С	NOTE-CONVERGENCE CRITERIUN IS UN NEXT PRUGRAM LINE
50	IF (DABS(ATEMP-AT(M,M))/DABS(ATEMP).GT.CUNV) GU TU OU
53	
60	M=M+1
00	K=1
30	CONTINUE
	RETURN
	END
C****	*END OF SUBROUTINE INTGRL************************************

/

DOUBLE PRECISION FUNCTION FPINT(X) С С FPINT IS THE INTEGRAND OF THE INTEGRAL INVOLVED IN THE FOUR-PROBE С RESISTANCE. THE EXPRESSION MAY BE DERIVED FROM THE SCHUMANN AND С GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION. С THE IS AN OPTION OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY: С 100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY С 200-THE CHOO ET AL. UNIFORM CURRENT DENSITY С 300-THE RING DELTA FUNCTION CURRENT DENSITY С IMPLICIT DOUBLE PRECISION (A-H, 0-Z) DIMENSION RH0(300) COMMON DELTH, AA, SEP, XL, XU, RHO, NUM1, ICUR, IBBC, SUBRHO, SUBTH PRTPR=DBJO(SEP*X)-DBJO(SEP*X*2.D0) G0 T0 (100,200,300), ICUR 100 FPINT=ALAM(X)*PRTPR*DSIN(X*AA)/X RETURN 200 FPINT=ALAM(X)*PRTPR*2.DO*DBJ1(AA*X)/X RETURN FPINT=ALAM(X)*PRTPR*AA*DBJO(AA*X) 300 RETURN FND DOUBLE PRECISION FUNCTION DEINT(X) С С DEINT IS THE INTEGRAND OF THE INTEGRAL INVOLVED IN THE DERIVATIVE С OF THE TWO-PROBE (SPREADING) RESISTANCE WITH RESPECT TO THE NATURAL LOGARITHM OF THE PROBE SPACING, D(X,S) = d R(X,S) / d lnS. С С THE EXPRESSION MAY BE DERIVED FROM THE SCHUMANN AND GARDNER SOLUTION С OF THE MULTILAYER LAPLACE EQUATION. С THERE IS AN OPTION OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY: С 100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY С 200-THE CHOO ET AL. UNIFORM CURRENT DENSITY C 300-THE RING DELTA FUNCTION CURRENT DENSITY C IMPLICIT DOUBLE PRECISION (A-H, 0-Z) DIMENSION RH0(300) COMMON DELTH, AA, SEP, XL, XU, RHO, NUM1, ICUR, IBBC, SUBRHO, SUBTH PRTPR=X*SEP*DBJ1(SEP*X) GO TO (100,200,300), ICUR 100 DEINT=ALAM(X) *PRTPR*DSIN(X*AA)/X RETURN DEINT=ALAM(X)*PRTPR*2.DO*DBJ1(AA*X)/X 200 RETURN DEINT=ALAM(X) *PRTPR*AA*DBJO(AA*X) 300 RETURN END

	DOUBLE PRECISION FUNCTION ALAM(X)
C****	***************************************
	THIS FUNCTION CALCULATES THE VALUE OF THE RESISTIVITY DEPENDENT PORTION OF THE VARIOUS INTEGRALS USING THE RECURSION RELATION OF CHOO ET AL. AND EXTENDED BY BERKOWITZ AND LUX.
	REFERENCES: "ON THE CALCULATION OF SPREADING RESISTANCE CORRECTION FACTORS" BY S. C. CHOO, M. S. LEONG AND K. L. KUAN, SOLID-STATE ELECTRONICS, VOL. 19, PP. 561-565 (1976).
	"AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES" BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 128, PP. 1137-1141 (1981).
C****	*****
	IMPLICIT DOUBLE PRECISION (A-H, 0-Z)
	DIMENSION RH0(300) COMMON DELTH, AA, SEP, XL, XU, RH0, NUM1, ICUR, IBBC, SUBRH0, SUBTH DTXSB=DTANH(X*SUBTH) DTXDL=DTANH(X*DELTH) ALAM=1.0D0
	IF (IBBC.EQ.2) ALAM=ALAM*DTXSB IF (IBBC.EQ.3) ALAM=ALAM/DTXSB IF (NUM1.EQ.1) GO TO 110
100	ALAM=(DTXDL*RH0(I)+RH0(I-1)*ALAM)/(RH0(I)+RH0(I-1)*ALAM*DTXDL)
110	RETURN
C****	*DOUBLE PRECISION FUNCTION ALAM***********************************

DOUBLE PRECISION FUNCTION DBJO(X)		
C*************************************		
C		
C POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJO		
C REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.1 IS USED FOR X < 3 AND EQUATION 9.4.3 IS USED FOR X > 3 C		
C*************************************		
IMPLICIT DOUBLE PRECISION (A-H, U-Z) C DOUBLE PRECISION X,X1,X2,F0,THETA C DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7 C DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7 C DOUBLE PRECISION C1.C2.C3,C4.C5,C6,C7		
DATA A1/1.000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/ DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/ DATA A7/0.0002100D0/ DATA B1/.79788456D0/,B2/00000077D0/,B3/00552740D0/ DATA B4/00009512D0/,B5/.00137237D0/,B6/00072805D0/ DATA B7/.00014476D0/		
DATA C1/78539816D0/,C2/04166397D0/,C3/00003954D0/ DATA C4/.00262573D0/,C5/00054125D0/,C6/00029333D0/ DATA C7/.00013558D0/ IF (X.GT.3.0D0) G0 T0 10 X2=X*X/9.0D0 DBJO=(((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1 RETURN		
10 CONTINUE X1=3.DO/X		
FO=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJO=FO*DCOS(THETA)/DSQRT(X) RETURN END		
C****DOUBLE PRECISION FUNCTION DBJO************************************		

	DOUBLE PRECISION FUNCTION DBJ1(X)
C****	**************
CCC	POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
	REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS SERIES 55. EQUATION 9.4.4 IS USED FOR X < 3 AND EQUATION 9.4.6 IS USED FOR X > 3
C****	
СССС	IMPLICIT DUBLE PRECISION (A-H, U-2) DOUBLE PRECISION X, X1, X2, F1, THETA DOUBLE PRECISION A1, A2, A3, A4, A5, A6, A7 DOUBLE PRECISION B1, B2, B3, B4, B5, B6, B7 DOUBLE PRECISION C1, C2, C3, C4, C5, C6, C7 DATA A1/0.500000D0/, A2/56249985D0/, A3/.21093573D0/ DATA A4/03954289D0/, A5/.00443319D0/, A6/00031761D0/ DATA A4/03954289D0/, B2/.00000156D0/, B3/.01659667D0/ DATA B1/.79788456D0/, B2/.00000156D0/, B3/.01659667D0/ DATA B1/.79788456D0/, B5/00249511D0/, B6/.00113653D0/ DATA B4/.00017105D0/, B5/00249511D0/, B6/.00113653D0/ DATA B7/00020033D0/ DATA C1/-2.35619449D0/, C2/.12499612D0/, C3/.00005650D0/ DATA C4/00637879D0/, C5/.00074348D0/, C6/.00079824D0/ DATA C7/00029166D0/ IF (X.GT.3.0D0) G0 T0 10 X2=X*X/9.D0 DBJ1=X*(((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
10	RETURN
10	F1=3.000/X F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1 THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1 DBJ1=F1*DC0S(THETA)/DSQRT(X) RETURN END

C*****END OF DOUBLE PRECISION FUNCTION DBJ1 & END OF FOURCAL VERSION 1.0****

APPENDIX J -- SHAZAM LISTING

С SHAZAM VERSION 1.0 С С THIS PROGRAM CALCULATES THE FOUR-PROBE RESISTANCE (FOR IN-LINE С CONFIGURATION) FROM THE RESISTIVITY BY USING THE 1/2S AND 1/S LIMITS IN THE SIMPLIFIED INTEGRAL DISCUSSED BY ALBERS С С AND BERKOWITZ. ALSO CALCULATED IS THE BEST EDUCATED GUESS AS TO THE С APPROXIMATE FORM FOR THE LN(S) SLOPE. С С REFERENCE: "AN ALTERNATIVE APPROACH TO THE CALCULATION OF FOUR-PROBE С RESISTANCES ON NONUNIFORM STRUCTURES". С BY JOHN ALBERS AND H. L. BERKOWITZ, JOURNAL OF THE С ELECTROCHEMICAL SOCIETY, VOL. 132, PP. 2453-2456, (1985). С DIMENSION RH0 (500), DEPTH (500), ZAP (500), DAP (500) DIMENSION A(2,9), C(9), W(9), T(9)REAL L(9) С С THE NINE NEWTON-COTES COEFFICIENTS ARE ENTERED IN THE С FOLLOWING DATA STATEMENT С DATA C(1)/0.0348854/,C(2)/0.20769/,C(3)/-0.0327337/ DATA C(4)/0.370229/,C(5)/-0.160141/,PI/3.14159265/ C(6) = C(4)C(7) = C(3)C(8) = C(2)C(9) = C(1)С С BEGINNING OF INPUT SECTION: DEFINITION OF SYSBOLS С С = NUMBER OF RESISTIVITY, SHEET RESISTANCE VALUES N = PROBE SEPARATION (MICROMETERS) С SEP С DELTH = SUBLAYER THICKNESS (MICROMETERS) С IBBC = BOUNDARY CONDITION (NONE = 0; CONDUCTING = 1; INSULATING = 2) SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM) С С SUBT = SUBSTRATE THICKNESS (MICROMETERS) С RHO(I) = I-TH RESISTIVITY RELATIVE TO THE SURFACEС
```
READ THE PROBE SEPARATION (MICROMETERS)
С
      READ(9, *)SEP
С
      READ THE DEPTH INCREMENT (MICROMETERS)
      READ(9, *)DELTH
      READ THE NUMBER OF POINTS IN THE STRUCTURE (EXCLUDING SUBSTRATE POINT)
С
      READ(9,*)N
С
      READ THE SUBSTRATE RESISTIVITY (OHM*CM)
      READ (9, *) SUBRHO
С
      READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ (9, *) SUBT
С
      READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ(9,*)IBBC
С
      READ THE SUBSTRATE RESISTIVITIES (OHM*CM)
      DO 70 I=1,N
      READ(9,*)RHO(I)
      CONTINUE
 70
      WRITE(6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
      FORMAT(2X, 'FOUR-PROBE RESISTANCE CALCULATED BY SIMPLIFIED
 80
     1 INTEGRAL')
      WRITE(6,81)
      FORMAT(2X, 'ALSO CALCULATED IS THE EDUCATED GUESS OF THE
 81
     1 LN(S) FORM')
      GO TO (90, 100, 110) IBBC
      WRITE(6,95)SUBT+N*DELTH,N*DELTH
 90
      FORMAT(2X, 'NO BOUNDARY AT', F10.3,' UM. SUBSTRATE AT', F8.3,' UM')
 95
      GO TO 120
 100 WRITE(6,105)SUBT+N*DELTH,N*DELTH
 105 FORMAT(2X, 'CONDUCTING BOUNDARY AT', F10.3, 'UM. SUBSTRATE AT',
     1 F8.3,' UM')
      GO TO 120
 110 WRITE(6,115)SUBT+N*DELTH,N*DELTH
 115 FORMAT(2X, 'INSULATING BOUNDARY AT', F10.3,' UM. SUBSTRATE AT',
     1 F8.3,' UM')
 120 WRITE (6,130) SEP, DELTH, SUBT, SUBRHO
 130 FORMAT (2X, 'SEP=', F6.0, 2X, 'DELTH=', F7.3, 2X, 'SUBTH=', 1PE9.3, 2X,
     1 'SUBRHO=',1PE9.3,/)
      DO 140 I=1,N+1
      DEPTH(I) = DELTH * (I-1)
 140 CONTINUE
      Q=1.0/(2.0*SEP)
      DO 150 I=1,9
      L(I) = Q * 2.0 * * ((I - 1.0) / 8.0)
 150 CONTINUE
      DO 160 I=1.9
      W(I) = L(I) * C(I)
 160 CONTINUE
      WRITE(6,170)L(1),L(9)
 170 FORMAT(2X, 'LOWER LIMIT= ', F10.8, 2X, 'UPPER LIMIT= ', F10.8)
      DO 180 I=1.9
      R1=L(I)*DELTH
      T(I) = TANH(R1)
 180 CONTINUE
```

APPENDIX J -- SHAZAM LISTING

```
С
С
    CALCULATION FOR THE SUBSTRATE POINT
С
DO 190 I=1.9
    R1=L(I)*SUBT
    A(1,I)=1.0
    A(2,I)=1.0
    IF(IBBC.EQ.2) A(1,I)=TANH(R1)
    IF(IBBC.EQ.3) A(1,I)=1.0/TANH(R1)
190
    CONTINUE
    RH0(N+1)=SUBRH0
    R2=0.0
    R3=1.0
    WW=SUBT/SUBRHO
    DO 200 I=1,9
    R2=R2+W(I)*A(1,I)
200 CONTINUE
    ZAP(N+1) = R2*ALOG(2.0)*RHO(N+1)/PI
    DAP(N+1) = L(9) * A(1,9)
    R5=SUBT
С
С
    CALCULATION FOR THE N-LAYER STRUCTURE
С
DO 500 K=N,1,-1
    R4=0.0
    R3=RH0(K+1)/RH0(K)
    D0 250 I=1,9
    A(2,I) = (R3*A(1,I)+T(I))/(1.0+T(I)*A(1,I)*R3)
250 CONTINUE
    R2=0.0
    DO 275 I=1,9
    R2=R2+W(I)*A(2,I)
275
   CONTINUE
    D0 300 II=N,K,-1
    R4=R4+1.0/RH0(II)
300 CONTINUE
    ZAP(K)=R2*AL0G(2.0)*RH0(K)/PI
    DAP(K) = L(9) * A(2,9)
    DO 350 I=1,9
    A(1,I) = A(2,I)
350 CONTINUE
500 CONTINUE
```

APPENDIX J -- SHAZAM LISTING

```
С
С
   END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
С
WRITE(6,550)
550 FORMAT(/, 3X, 'DEPTH', 6X, 'RHO', 6X, 'FOUR-PROBE', 4X, 'DERIVAT', /)
   WRITE(11, *)N+1
   WRITE(12,*)N+1
   DO 600 I=1,N+1
   WRITE(6,650)DEPTH(I),RH0(I),1e4*ZAP(I),DAP(I)
650 FORMAT(F8.3,3(2X,1PE12.4))
   WRITE (11,700) DEPTH (I), ZAP (I)
   WRITE(12,700)DEPTH(I),DAP(I)
700 FORMAT(1X,2(1PE11.3,1X))
600 CONTINUE
   STOP
   END
```



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