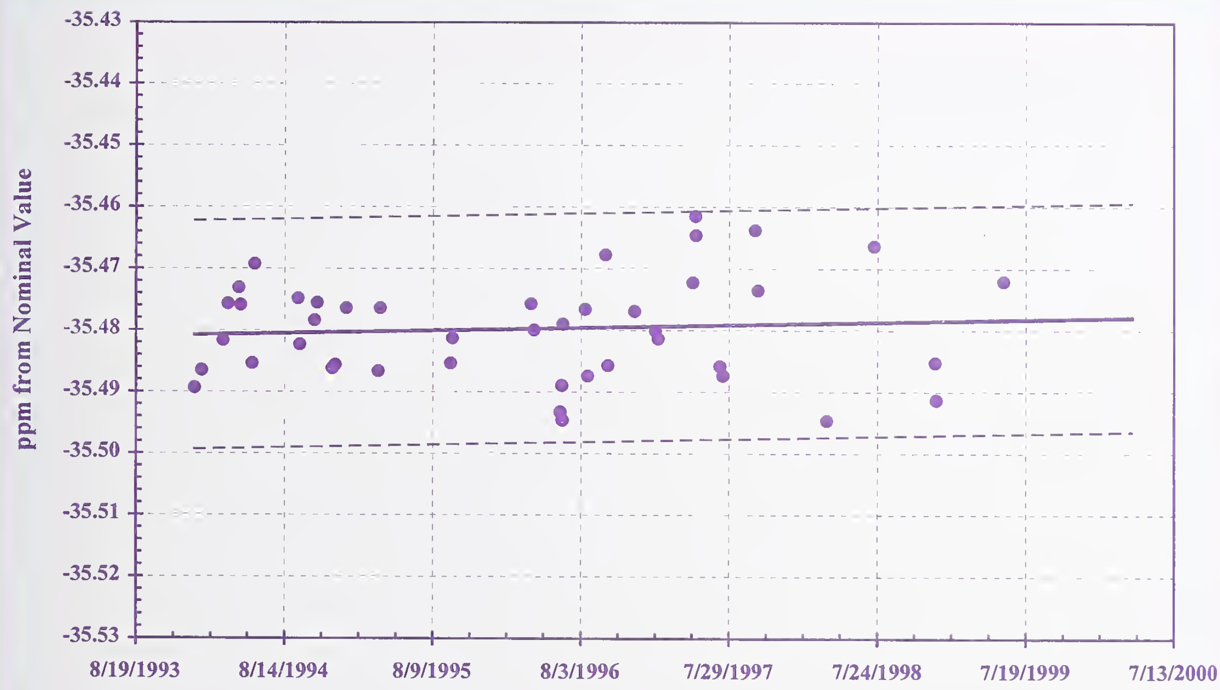
...surement Services:

Error Analysis and Calibration Uncertainty of Capacitance Standards at NIST

NIST Special Publication 250–52



Control Chart for Reference Standards

Y. May Chang

U.S. Department of Commerce
Technology Administration
National Institute of Standards and Technology

The National Institute of Standards and Technology was established in 1988 by Congress to "assist industry in the development of technology . . . needed to improve product quality, to modernize manufacturing processes, to ensure product reliability . . . and to facilitate rapid commercialization . . . of products based on new scientific discoveries."

NIST, originally founded as the National Bureau of Standards in 1901, works to strengthen U.S. industry's competitiveness; advance science and engineering; and improve public health, safety, and the environment. One of the agency's basic functions is to develop, maintain, and retain custody of the national standards of measurement, and provide the means and methods for comparing standards used in science, engineering, manufacturing, commerce, industry, and education with the standards adopted or recognized by the Federal Government.

As an agency of the U.S. Commerce Department's Technology Administration, NIST conducts basic and applied research in the physical sciences and engineering, and develops measurement techniques, test methods, standards, and related services. The Institute does generic and precompetitive work on new and advanced technologies. NIST's research facilities are located at Gaithersburg, MD 20899, and at Boulder, CO 80303. Major technical operating units and their principal activities are listed below. For more information contact the Publications and Program Inquiries Desk, 301-975-3058.

Office of the Director
- National Quality Program
- International and Academic Affairs

Technology Services
- Standards Services
- Technology Partnerships
- Measurement Services
- Technology Innovation
- Information Services

Advanced Technology Program
- Economic Assessment
- Information Technology and Applications
- Chemical and Biomedical Technology
- Materials and Manufacturing Technology
- Electronics and Photonics Technology

Manufacturing Extension Partnership Program
- Regional Programs
- National Programs
- Program Development

Electronics and Electrical Engineering Laboratory
- Microelectronics
- Law Enforcement Standards
- Electricity
- Semiconductor Electronics
- Electromagnetic Fields[1]
- Electromagnetic Technology[1]
- Optoelectronics[1]

Chemical Science and Technology Laboratory
- Biotechnology
- Physical and Chemical Properties[2]
- Analytical Chemistry
- Process Measurements
- Surface and Microanalysis Science

Physics Laboratory
- Electron and Optical Physics
- Atomic Physics
- Optical Technology
- Ionizing Radiation
- Time and Frequency[1]
- Quantum Physics[1]

Materials Science and Engineering Laboratory
- Intelligent Processing of Materials
- Ceramics
- Materials Reliability[1]
- Polymers
- Metallurgy
- NIST Center for Neutron Research

Manufacturing Engineering Laboratory
- Precision Engineering
- Automated Production Technology
- Intelligent Systems
- Fabrication Technology
- Manufacturing Systems Integration

Building and Fire Research Laboratory
- Structures
- Building Materials
- Building Environment
- Fire Safety Engineering
- Fire Science

Information Technology Laboratory
- Mathematical and Computational Sciences[2]
- Advanced Network Technologies
- Computer Security
- Information Access and User Interfaces
- High Performance Systems and Services
- Distributed Computing and Information Services
- Software Diagnostics and Conformance Testing
- Statistical Engineering

[1] At Boulder, CO 80303.
[2] Some elements at Boulder, CO.

NIST Special Publication 250-52

NIST MEASUREMENT SERVICES:
Error Analysis and Calibration Uncertainty of Capacitance Standards at NIST

Y. May Chang

Electricity Division
Electronics and Electrical Engineering Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899-8110

January 2000

U.S. Department of Commerce
William M. Daley, Secretary

Technology Administration
Dr. Cheryl L. Shavers, Under Secretary of Commerce for Technology

National Institute of Standards and Technology
Raymond G. Kammer, Director

National Institute of Standards and Technology Special Publication 250–52
Natl. Inst. Stand. Technol. Spec. Publ. 250–52, 109 pages (Jan. 2000)
CODEN: NSPUE2

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 2000

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402–9325

TABLE OF CONTENTS

List of Tables

List of Figures

List of Symbols

1. Type-2 System

a and b	In-phase and quadrature corrections of the 1:1 ratio.
$\mathbf{C}_1$ to $\mathbf{C}_8$	Dial capacitors of the Type-2 bridge.
$\boldsymbol{C}_1$ to $\boldsymbol{C}_8$	Nominal values of capacitors $\mathbf{C}_1$ to $\mathbf{C}_8$.
C_1 to C_8	Real values of capacitors $\mathbf{C}_1$ to $\mathbf{C}_8$.
$\mathbf{C_s}$, $\mathbf{C_x}$, & $\mathbf{C_d}$	Reference capacitor, unknown capacitor, and total dial capacitors.
$\boldsymbol{C_s}$, $\boldsymbol{C_x}$, & $\boldsymbol{C_d}$	Nominal values of capacitors $\mathbf{C_s}$, $\mathbf{C_x}$, and $\mathbf{C_d}$.
C_s, C_x, & C_d	Real values of capacitors $\mathbf{C_s}$, $\mathbf{C_x}$, and $\mathbf{C_d}$.
$(\Delta C / C)$	Uncertainty in the capacitance due to the lead impedance.
d_1 to d_8	Dial corrections of $\mathbf{C}_1$ to $\mathbf{C}_8$.
d_s	Deviation of the reference capacitor from its nominal value.
DF_c	Dissipation factor of the capacitor.
$\boldsymbol{e}_{\mathbf{mb}}$	Maximum bound of error.
E_s and E_x	Applied voltages to $\mathbf{C_s}$ and $\mathbf{C_x}$.
G_c and ΔG	Parallel conductance of $\mathbf{C}$, and the uncertainty of G_c.
$\mathbf{G_r}$ and g_i	Conductance of the internal dial resistors and admittance of the i^{th} dial.
I_s, I_d, I_x, & I_g	Current through $\mathbf{C_s}$, $\mathbf{C_d}$, $\mathbf{C_x}$, and $\mathbf{G_r}$.
k	Coverage factor for confidence level, normally equal to 2.
L_l	Series inductance of leads.
$\mathbf{M}$ and M	Multiplier of the conductance and its setting.
m	Ratio of measurement, either 1 or 10.
p_5	A constant, either plus or minus 0.5.
r_i	Dial reading of $\mathbf{c}_i$.
$\sum r_k c_k$	Sum of dial readings giving the capacitance value of $\mathbf{c}_k$, for $k > i + 1$.
R_l	Series resistance of leads.
s_a and s_b	Standard uncertainties of the in-phase and quadrature corrections of 1:1 ratio.
s_{cb}	Combined standard uncertainty of the external bridge components.
s_{cd}	Combined standard uncertainty of dial corrections.
s_f	Standard uncertainty of the capacitor due to frequency dependence.
s_i and s_j	Standard deviations of the components of the Type A and Type B standard uncertainties.
s_k	Standard uncertainty of the dial correction of the k^{th} dial.
s_l and s_{l1}	Standard uncertainties of interpolations of the last dial at 1 kHz and 100 kHz.
s_{Ra} and s_{Rb}	Standard uncertainties of the in-phase and quadrature corrections of 1:10 ratio.
s_s	Standard uncertainty of the reference standard, due to its stability.
s_{sc}	Combined standard uncertainty of the reference standard in measurements.
s_{sd}	Equivalent standard deviation (refer as the standard uncertainty) of a component.
s_{tc} and s_{tc1}	Standard uncertainties in temperature corrections of fused-silica and nitrogen dielectric capacitors.
s_{va} and s_{vb}	In-phase and quadrature standard uncertainties of the capacitor due to voltage dependence.

u_c, u_a, & u_b	Combined, Type A and Type B standard uncertainties.
U and U_t	Expanded and total (assigned) uncertainties.
Y_s, Y_x, & y_i	Admittance of $\mathbf{C_s}$, $\mathbf{C_x}$, and $\mathbf{c}_i$.
Y and Z	Effective admittance and impedance of the capacitor, where Y = (1/Z).

2. The Type-12 System

C_1 and C_2	Total capacitance of arm AD and arm CD.
C_a and C_c	Effective internal capacitors on arm AD and arm CD.
C_m and C_r	Capacitance of mica capacitor dial readings and of air capacitor dial reading, *r*.
$\mathbf{C_x}$ and C_x	Unknown capacitor and its value.
$\boldsymbol{C_x}$	Nominal value of capacitor $\mathbf{C_x}$.

For two-terminal measurements:

C_{a0} and C_{c0}	Internal capacitors on arm AD and arm CD without connecting $\mathbf{C_x}$ to arm CD.
C_{a1} and C_{c1}	Internal capacitors on arm AD and arm CD with $\mathbf{C_x}$ connecting to arm CD.
C_{r0} and C_{r1}	Dial readings of air capacitors with and without connecting $\mathbf{C_x}$ to arm CD.

For three-terminal measurements:

C_{a1} and C_{c1}	Internal capacitors on arm AD and arm CD with $\mathbf{C_x}$ connecting to arm AD.
C_{a2} and C_{c2}	Internal capacitors on arm AD and arm CD with $\mathbf{C_x}$ connecting to arm CD.
C_{r1} and C_{r2}	Dial readings of air capacitors with $\mathbf{C_x}$ connecting to arm AD and arm CD.

(For $\boldsymbol{C_x}$ > 0.5 μF)

C_{a1} to C_{a4}	Internal capacitors on arm AD for the four respective measurements.
C_{c1} to C_{c4}	Internal capacitors on arm CD for the four respective measurements.
C_{r1} to C_{r4}	Dial readings of air capacitors for the four respective measurements.
C_{m2} & C_{m3}	Dial readings of mica capacitors for the second and third measurements.

$\mathbf{C_b}$ and C_b	Capacitance decade box and its value.
$\boldsymbol{C_b}$	Nominal value of capacitance decade box $\mathbf{C_b}$.
C_{eff}	Effective capacitance of a capacitor $\mathbf{C_x}$.
$e_{\mathbf{bZ}}$	Maximum error caused by lead impedance.
$e_{\mathbf{ds}}$	Maximum error due to dial switching.
$e_{\mathbf{gc}}$	Maximum error due to residual conductance.
$e_{\mathbf{gr}}$	Maximum error due to contact resistance.
$e_{\mathbf{gs}}$	Maximum error due to conductance standard.
f	Applied frequency.
G_a and G_c	Effective internal conductance on arm AD and arm CD.
G_m and G_r	conductance of mica capacitor dial readings and conductance dial reading, *r*.
$\mathbf{G_s}$ and G_s	Conductance standard and its value.
$\mathbf{G_x}$ and G_x	Conductance of unknown capacitor and its value.
$\mathbf{K_g}$ and K_g	Divider resistor and its value.
L_l	Series inductance of leads.
$\mathbf{R_1}$ and $\mathbf{R_2}$	Ratio resistors on arm AB and arm CB.

R_1 and R_2	Values of ratio resistors $\mathbf{R_1}$ and $\mathbf{R_2}$.
R_{r1} and R_{r2}	Contact resistance of the two balances.
s_{bz}	Standard uncertainty due to lead impedance.
s_{bp2} and s_{bp3}	Standard uncertainties due to corrections of plates P2 and P3.
s_{br}	Standard uncertainty of external bridge components.
s_{dr}	Standard uncertainty in the air capacitor dials due to dial corrections.
s_{ds}	Standard uncertainty due to dial switching.
s_{gc}	Standard uncertainty due to residual conductance.
s_{gm}	Standard uncertainty in conductance corrections of the mica capacitor dials.
s_{gp}	Standard uncertainty in conductance potentiometer.
s_{gpd}	Standard uncertainty of the predicted conductance value of the NIST standard.
s_{gr}	Standard uncertainty due to contact resistance.
s_{grm}	Standard uncertainty due to repeatability of measurements in conductance.
s_{gs}	Standard uncertainty due to conductance standard.
s_m	Combined Standard uncertainty of the internal mica capacitor dials.
s_{m2} and s_{m3}	Standard uncertainties in the mica capacitor dials corresponding the readings of C_{m2} and C_{m3}.
s_{mf}	Standard uncertainty in the mica capacitor dials due to the uncertainty of transfer standard.
s_{mp}	Standard uncertainty in the mica capacitor dials due to temperature variations.
s_{mr}	Standard uncertainty in the mica capacitor dials due to dial corrections.
s_{pd}	Standard uncertainty of the predicted capacitance value of the NIST standard.
s_{ra}	Standard uncertainty contributed by the ratio resistors.
s_{rc}	Combined Standard uncertainty of the internal air capacitor dials.
s_{rc0} to s_{rc4}	Standard uncertainties in the air capacitor dials corresponding the readings of C_{r0} to C_{r4}.
s_{rm}	Standard uncertainty due to repeatability of measurements in capacitance.
s_{tf}	Standard uncertainty in the air capacitor dials due to the uncertainty of transfer standard.
s_{tm}	Standard uncertainty on unknown capacitor due to temperature variations.
s_{tp}	Standard uncertainty in the air capacitor dials due to temperature variations.
u_a and u_b	Type A and Type B standard uncertainties of capacitance measurements.
u_{ga} and u_{gb}	Type A and Type B standard uncertainties of conductance measurements.
U and U_t	Expanded and total (assigned) uncertainties of capacitance measurements.
U_g and U_{gt}	Expanded and total (assigned) uncertainties of conductance measurements.

ERROR ANALYSIS AND CALIBRATION UNCERTAINTY OF CAPACITANCE STANDARDS AT NIST

Y. May Chang
Electricity Division
National Institute of Standards and Technology

ABSTRACT

This document presents the analysis of error sources that contribute to the total uncertainty of capacitance calibrations at the National Institute of Standards and Technology (NIST). Based on considerations of the measuring systems and calibration procedures, and data taken on NIST working and check standards and customer's standards, uncertainties in the calibrations for each model and nominal value of capacitor are estimated. The results of the analysis are expressed as expanded uncertainties using the coverage factor $k = 2$, in accordance with NIST Technical Note 1297. Also included are a detailed description and analysis for each component of error in the evaluation of Type A and Type B standard uncertainties.

1. INTRODUCTION

Since 1960, a transformer-ratio capacitance-measuring system [1], known as the " Type-2 " bridge, has been used in the National Bureau of Standards / National Institute of Standards and Technology (NBS/NIST) impedance calibration laboratory (ICL) to transfer the farad from the primary capacitance laboratory (PCL), via transfer standards, to NIST working standards and, ultimately, to customer's standards. The values of these standards are expressed in terms of measurements of the NIST calculable capacitor, used to realize the farad in SI units. During the past ten years, high quality commercial instruments, such as detectors and function generators, have become available for use in the Type-2 system to improve its resolution. The establishment of a bank of oil-bath-type fused-silica capacitors with predicable temperature corrections for use as reference standards in the ICL has increased the stability of Type-2 system measurements. In 1993, NIST issued a new policy on the expression of uncertainty associated with measurement results, as described in NIST Technical Note (NIST TN) 1297 using the coverage factor $k = 2$ [2], consistent with the international practice [2, 3]. Therefore, based on the above modifications, uncertainties in NIST capacitance calibration have been reevaluated and reestablished for each model and type of capacitance standard measured.

In brief, the combined standard uncertainty of a measured value is the combination of estimates of two types of uncertainties. The first is the Type A standard uncertainty, defined as that which can be evaluated by statistical methods. The other is the Type B standard uncertainty, which has no root in formal statistics, but rather evaluated many times based solely on the experience of the metrologist.

The combined standard uncertainty, u_c, is defined as the "RSS" (root-sum-of-squares) of both types, as:

$$u_c = [u_a^{\ 2} + u_b^{\ 2}]^{1/2} \tag{1}$$

where u_c is the combined standard uncertainty, and
u_a and u_b are Type A and Type B standard uncertainties, respectively.

According to the guidelines recommended by the International Bureau of Weights and Measures [3], the overall, or expanded uncertainty, U, is expressed as:

$$U = k\, u_c, \tag{2a}$$

or

$$U = k\ [\Sigma(s_i)^2 + \Sigma(s_j)^2]^{1/2}, \tag{2b}$$

where U is the expanded uncertainty, k is the coverage factor to be chosen on the basis of the approximate confidence level desired, and s_i and s_j are the standard deviations of the components of Type A and equivalent standard deviations of the components of Type B standard uncertainties, respectively. The coverage factor used at NIST to calculate U is generally $k = 2$, indicating a level of confidence of approximately 95 percent.

Therefore, the total uncertainty, U_t assigned to capacitance calibration at NIST is calculated according to the following equation:

$$U_t = 2 [\Sigma(s_i)^2 + \Sigma(s_j)^2]^{1/2}. \tag{3}$$

In this document, unless otherwise stated, the uniform distribution is used to obtain the equivalent standard deviation of the components of the Type B standard uncertainties from the maximum bounds of error for each source of errors. Thus,

$$s_{sd} = e_{\mathbf{mb}} / \sqrt{3}\,, \tag{4}$$

where s_{sd} is the equivalent standard deviation, as one of the s_j in Eqs. (2b) and (3), (hereafter referred as the standard uncertainty) of a typical component of the Type B standard uncertainty, and $\pm\, e_{\mathbf{mb}}$ are its maximum bounds of error.

2. DESCRIPTION OF SYSTEMS AND MEASUREMENT PROCEDURES

There are two major capacitance measuring systems in the ICL at NIST. One is the Type-2 bridge, which is mainly used to calibrate capacitors of nominal values up to 10 000 pF at frequencies of

100 Hz, 400 Hz, and 1 kHz. The other is the Type-12 bridge, which is a resistance ratio capacitance measuring system for calibration of capacitors of nominal values from 0.001 μF to 1 μF at frequencies of 66 Hz, 100 Hz, 400 Hz, 1 kHz, and 10 kHz. Since 1992, commercial impedance meters have also been used as capacitance comparators, after they have been characterized using NIST standards, in an effort to automate some of the calibration services. A detailed description of the capacitance calibration service at NIST is given in [4].

2.1 Type-2 Capacitance Bridge and Standards

The NIST Type-2 bridge is a transformer-ratio bridge used for comparing unknown capacitors of coaxial connectors with external reference standards whose values are well defined. A set of eight internal air capacitors with values of 100 pF, 10 pF, ... , 10^{-5} pF are used to balance the bridge during measurements. Values of these internal capacitors are selected by means of dial settings to balance the bridge at 1:1, 1:10, or 10:1 ratios. During the transfer of the farad, an NIST-made 10 pF air-bath-type fused-silica capacitor from the PCL is used as the reference to measure a group of 10 pF oil-bath-type fused-silica capacitors in the ICL. These capacitors, which are also NIST-made and have known temperature coefficients, serve as the primary reference in the ICL. Changes in their temperatures are precisely monitored via internal resistive sensors of copper, and are used to adjust their assigned values of capacitance at the time of measurement. The primary reference is employed to measure two secondary reference sets, each having one 10 pF and one 100 pF commercial air-bath-type fused-silica capacitors in a temperature-controlled oven. The secondary references are used for routine calibrations. Also, the primary reference is directly employed to calibrate customer's fused-silica capacitors.

The ICL also has a number of capacitance check standards. Among these are six commercial dry-nitrogen-dielectric, parallel-plate capacitors housed in the Styrofoam containers originally supplied as protectors during shipment. Their nominal values are 1 pF, 10 pF, 100 pF (2), and 1000 pF (2). There is also an NIST-made, 10 000 pF capacitor. These check standards are used to ensure that the measuring system remains under control and are used also as a part of the process of calibrating capacitors having dielectrics other than fused-silica, as shown below.

Calibration procedures for customer's capacitors having nitrogen and air dielectrics have two steps. The first step is to use the secondary reference to measure the check standards. Secondly, the check standards are used as working standards to measure the customers' capacitors. Values of all reference and check standards are kept in a database. They are used in control charts to ensure that the Type-2 bridge is in good operational condition. Figure 1 is a block diagram illustrating the transfer of the farad and the calibration measurement process using the Type-2 system. Prior to performing any measurements, the internal capacitors of the Type-2 bridge are calibrated against the reference standard to obtain dial corrections used in the calculation of that day's measurement results.

2.2 Type-12 Capacitance Bridge

The Type-12 bridge is a resistance ratio bridge having internal capacitors as its reference and is mainly used to calibrate mica-dielectric capacitors of exposed binding-post connectors in the range from 0.001 μF to 1 μF. In general, calibration of customers' capacitors is performed using two measurements (at a 1:1 ratio of the resistance arms) to eliminate the effects of lead impedance. In the two-terminal configuration, the bridge is balanced with and without connecting the unknown capacitor to the capacitance arm. In the three-terminal configuration, the bridge is balanced by connecting the unknown capacitor to each of the capacitance arms. For each configuration, final results are calculated using dial readings from both measurements.

3. UNCERTAINTY ANALYSIS FOR THE TYPE-2 SYSTEM

A complete schematic diagram of the Type-2 bridge is shown in Fig. 2a. Bridge components and operational procedures of the Type-2 system were described in detail by Cutkosky [1]. Figure 2b is a simplified circuit diagram of the Type-2 bridge, where $\mathbf{C_s}$ is the external reference capacitor, $\mathbf{C_x}$ is the unknown capacitor to be measured, $\mathbf{C_d}$ represents the internal dial capacitors, $\mathbf{G_r}$ represents the conductance of the internal dial resistors, and **M** is the multiplier of the conductance settings. At balance, the detector current is equal to zero, and the detector is at virtual ground. The balance is achieved by applying voltages from the appropriate taps of the transformer to the internal capacitors and to the conductance control network. The general balance equation is:

$$I_s + I_d + I_g = I_x, \tag{5}$$

where I_s is the current through the reference capacitor, $\mathbf{C_s}$,
I_d is the current provided by the capacitance network, $\mathbf{C_d}$, as described below,
I_g is the current supplied by the conductance balance divider and network, and
I_x is the current through the unknown capacitor, $\mathbf{C_x}$.

The dial capacitance, $\mathbf{C_d}$, is provided by a set of eight air capacitors whose values range from 100 pF to 10^{-5} pF having a total capacitance up to 111 pF (see Fig. 3). $\mathbf{G_r}$ is a set of four dials providing conductance from 0.001 μS to 1 μS with a multiplier switch to extend the total range from 10^{-8} μS to 1 μS. The dials are switches of a four-decade transformer divider that applies selected voltages to a tuned phase shift network providing conductance balance through a number of ranges (see Fig. 2a). Another switch is included for reversing the sign of the conductance component - for allowing the unknown capacitor to have greater or lesser conductance than the internal capacitors of the Type-2 bridge. It can be seen from Fig. 2a that each capacitor-switch combination provides the equivalent of a decade of capacitance by selection of the voltage applied to the capacitor, and that if an unknown capacitor is connected to the bridge as shown in Fig. 2b and the bridge balanced, the capacitance of $\mathbf{C_x}$ is equal to the value of $\mathbf{C_s}$ plus the sum of the dial settings. The balance equation can be written as:

$$E_s Y_s + E_s (\sum r_i y_i) + p_5 E_s (M \sum g_i) = - E_x Y_x, \tag{6}$$

where E_s and E_x are voltages produced by the transformer windings on the reference, $\mathbf{C_s}$ and the unknown capacitor, $\mathbf{C_x}$ sides of the bridge, respectively,

Y_s and Y_x are admittances of $\mathbf{C_s}$ and $\mathbf{C_x}$, respectively,
r_i is the dial reading of i^{th} dial applying voltage to capacitor $\mathbf{c}_i$, where $0 \le r_i \le 1$, in increments of 0.1,
y_i is the admittance of $\mathbf{c}_i$,
g_i is the effective admittance associated with the settings of the i^{th} conductance dial coupled with the frequency compensation network,
p_5 is a constant whose value can be taken as either plus or minus 0.5, and
M is the multiplier of the conductance dial settings, where $10^{-5} \le M \le 1$.

On the secondary side of the transformer, besides the ±1 taps, a pair of ± 0.1 taps are also externally available for the reference and unknown capacitors to be connected to obtain 1:1, 1:10, and 10:1 ratios.

At present, this bridge is not used to measure the conductance or loss in standard capacitors.

The uncertainty associated with capacitance measurements using the Type-2 bridge is comprised of two types, as given in Eq. (3). The evaluation of the components of the Type B standard uncertainty takes into account errors in

- the transformer ratios,
- the main dial corrections (i.e., values of the $\mathbf{C_d}$ capacitor set),
- frequency dependence of the reference and bridge,
- similar voltage dependence,
- temperature corrections,
- lead impedance, and
- the uncertainty of the reference standards.

The evaluation of the Type A standard uncertainty is based on the variability of the measurement data. A detailed analysis for each type of uncertainty is given in the following sections.

3.1 Transformer Ratios

3.1.1 Corrections and Uncertainties of the 1:1 Ratio

Corrections for errors of the 1:1 ratio of the Type-2 bridge can be determined with two measurements by using two capacitors of nominally equal value. One measurement is taken with $\mathbf{C_x}$ and $\mathbf{C_s}$ connected as shown in Fig. 2b. The other measurement is taken with $\mathbf{C_x}$ and $\mathbf{C_s}$ interchanged. The balance equation of the second measurement can be written as:

$$E_s Y_x + E_s(\sum r_n y_n) + p_5 E_s (M' \sum g_n) = -E_x Y_s. \qquad (7)$$

where parameters with subscript n have the same meanings as those in Eq. (6) with subscript i, and M' is the multiplier of the second balance. M' is normally equal to M.

By combining Eqs. (6) and (7), (with $M = M'$) the result becomes:

$$(E_s + E_x)(Y_s + Y_x) + E_s(\sum r_i y_i + \sum r_n y_n) + p_5 E_s M(\sum g_i + \sum g_n) = 0. \tag{8}$$

Assuming $\boldsymbol{E}_s$ = 1 V as the reference, and by the definition of 1:1 ratio, $(\boldsymbol{E}_x / \boldsymbol{E}_s) \cong -1$, then:

$$E_x = -(1 + a + jb), \tag{9}$$

where a is the in-phase correction to the ratio, and
b is the quadrature correction to the ratio.

The errors in the in-phase and quadrature corrections to the 1:1 ratio that are incurred with the measured data, external capacitance standards, dial readings, interpolation of last dials, load admittance, and lead impedance are shown in Appendix A. The combined standard uncertainties that contribute to the 1:1 ratio correction are estimated, also given in Appendix A, as:

$$s_a < 0.005 \text{ ppm}^1$$

and

$$s_b < 0.03 \text{ ppm}, \tag{10}$$

where s_a and s_b are the standard uncertainties in the in-phase and quadrature corrections of the 1:1 ratio, respectively.

In routine calibrations of capacitors using the Type-2 bridge, only capacitance values are reported even though both capacitance and conductance dials are used to obtain a balance. Therefore, only the in-phase standard uncertainties of the ratio corrections are included in the Type B standard uncertainty.

3.1.2 Calibration and Uncertainties of the 10:1 (and 1:10) Ratio

The calibration method and error analysis related to the 10:1 (and 1:10) ratio transformer are discussed in detail by Cutkosky and Shields [5]. Based on the numerical results contained in [5], standard uncertainties in the corrections at the 1:10 ratio are incurred with voltage variations, temperature variations, and the repeatability of measurements. By combining these components with those at 1:1 ratios, given in Eq. (10), total standard uncertainties that contribute to 10:1 (and 1:10) ratio corrections are estimated as:

$$s_{Ra} \le 42.25 \times 10^{-3} \text{ ppm} < 0.043 \text{ ppm}$$

and

$$s_{Rb} \le 32.49 \times 10^{-3} \text{ ppm} < 0.033 \text{ ppm}, \tag{11}$$

where s_{Ra} and s_{Rb} are the standard uncertainties in the in-phase and quadrature corrections of the

---

[1] The term uncertainty as used in this document refers to the relative standard uncertainty when the unit is expressed in ppm [2].

10:1 (and 1:10) ratio, respectively.

3.2 Dial Corrections

The internal capacitors of the Type-2 bridge are a set of eight three-terminal air capacitors, $\mathbf{C}_1$ to $\mathbf{C}_8$, with adjustable dials, as shown in Fig. 3. One side of each capacitor is connected to the detector, and the other side is connected to the transformer via dial switches, which have the range from −0.1 to +1.0 of full voltage. As shown in Fig. 2a, when a dial is set at any value from +0.1 up to +1.0, the capacitor is connected to the "S" side of the bridge. When a dial is set to −0.1, it is connected to the "X" side of the bridge. The −0.1 positions of the dials are useful in ratio measurements, or measuring capacitors of lower than nominal values. Among these internal capacitors, which range from 100 pF to 10^{-5} pF, only $\mathbf{C}_3$, the 1 pF capacitor is purposely temperature compensated. Since air capacitors vary with the room temperature and humidity, these are calibrated prior to each measurement. All dial capacitors are calibrated at a frequency of 1 kHz, and dial corrections are applied to the measurement results.

3.2.1 Calibration of the Dial Capacitors

The nominal values of the internal capacitors are shown in Fig. 3 and given as:

$$
\begin{aligned}
C_1 &= 100 \text{ pF} = 10\, C_2, \\
C_2 &= 10 \text{ pF} = 10\, C_3, \\
C_3 &= 1 \text{ pF} = 10\, C_4, \\
&\ \ \vdots \\
C_7 &= 10^{-5} \text{ pF} = 10\, C_8, \text{ and} \\
C_8 &= 10^{-6} \text{ pF}.
\end{aligned}
\tag{12}
$$

The first step in the calibration of capacitors $\mathbf{C}_1$ to $\mathbf{C}_8$ is to calibrate $\mathbf{C}_3$ against a 10 pF reference standard using a 1:10 ratio by setting $\mathbf{C}_3$ to the +1.0 switch position and connecting the reference capacitor to the - 0.1 tap on the "X" side of the transformer. Afterward, internal calibrations of dial corrections are performed by using $\mathbf{C}_3$ (at the +1.0 switch position) as reference to calibrate $\mathbf{C}_2$ (at the - 0.1 switch position) and using $\mathbf{C}_3$ (at the - 0.1 switch position) to calibrate $\mathbf{C}_4$ (at the +1.0 switch position). Other internal capacitors can be calibrated similarly by running up and down the range. The balance equation comparing $\mathbf{C}_i$ with $\mathbf{C}_{(i+1)}$ can be written as:

$$C_i + d_i = m\,(C_{(i+1)} + d_{(i+1)} + \sum r_k\, c_k\,), \tag{13}$$

where C_i and $C_{(i+1)}$ are nominal values of $\mathbf{C}_i$ and $\mathbf{C}_{(i+1)}$, respectively,

d_i and $d_{(i+1)}$ are dial corrections of $\mathbf{C}_i$ and $\mathbf{C}_{(i+1)}$, respectively,
$m = 10$ is the ratio, and
$\sum r_k c_k$ is the sum of dial readings giving the capacitance value at balance, for $k > i+1$.

The balance equation for calibrating $\mathbf{C_3}$ (where $C_3 = 1$ pF) with the 10 pF reference standard ($\mathbf{C_s}$), according to Eqs. (12) and (13), is:

$$C_s + d_s = 10\,(C_3 + d_3 + \sum r_k c_k), \tag{14}$$

where C_s (= 10 pF) is the nominal value of the reference standard,
d_s is the deviation of the reference from its nominal value, and
$\sum r_k c_k$ is the sum of the dial readings giving the capacitance value at balance, for $k > 3$.

The dial corrections are related similarly by:

$$d_i = 10\,(d_{(i+1)} + \sum r_k c_k), \qquad (k > i+1), \tag{15}$$

and, hence (from Eq. (14)),

$$d_3 = (0.1)\,d_s - \sum r_k c_k . \qquad (k > 3). \tag{16}$$

Therefore, dial corrections for each of the internal capacitors can be determined from a known reference capacitor and dial readings. After the preliminary results of dial corrections are obtained, from Eqs. (15) and (16), the final values of the dial corrections can be determined by including the corrections of those dials used in dial correction measurements.

3.2.2 Uncertainties in Dial Corrections

Rewrite Eqs. (15) and (16) by including the dial corrections of the internal capacitors, d_k as:

$$d_i = 10\,(d_{(i+1)} + \sum r_k \mathbf{c}_k + \sum r_k d_k), \; (k > i+1), \text{ and} \tag{17}$$

$$d_3 = (0.1)\,d_s - (\sum r_k \mathbf{c}_k + \sum r_k d_k), \; (k > 3), \tag{18}$$

where $\mathbf{c}_k$ is the nominal value of the k^{th} capacitor, $\mathbf{c}_k$, and
d_k is the correction to the k^{th} capacitor, $\mathbf{c}_k$.

The standard uncertainties of each internal capacitor can be estimated from Eqs. (17) and (18).

As mentioned in the previous section, (3.2.1), the first step is to calibrate the third dial, $\mathbf{C_3}$ by using a 10 pF reference standard. Therefore, the standard uncertainty of the correction of the third dial, d_3 is determined first. The relationship of the variance of each term in Eq. (18) can be written as:

$$\mathrm{Var}\,(d_3) = (0.1)^2\,(s_{\mathrm{Ra}}^{\;2} + s_s^{\;2}) + \sum r_k^2\,\mathrm{Var}\,(d_k) + s_1^{\;2}, \tag{19}$$

where Var (d_3) is the variance of the dial correction of the third dial, d_3,
s_{Ra} is the in-phase standard uncertainty in the 1:10 ratio, as given in Eq. (11),
s_S is the standard uncertainty of the reference standard,
r_k is the dial reading for the k^{th} dial, where k >3,
Var (d_k) is the variance of the dial correction of the k^{th} dial, where k >3,
s_1 is the standard uncertainty of interpolation of the least significant capacitance dial incurred during measurement, and
$\mathbf{c}_k$ does not have a variance.

Then the standard uncertainty for the correction of the third dial, s_3 can be expressed as:

$$s_3 = [(0.1)^2 s_{Ra}^2 + (0.1)^2 s_S^2 + \sum (r_k^2 s_k^2) + s_1^2]^{(1/2)}, \qquad (20)$$

where s_k is the standard deviation of the dial correction of the k^{th} dial, where k >3.

After s_3 is determined, the standard uncertainties of other dials, s_i, can be estimated from s_3 by using the appropriate expression similar to Eq. (20).

For capacitors $\mathbf{C_4}$ to $\mathbf{C_8}$, the standard uncertainties for the dial corrections can be expressed, according to Eq. (17), as:

$$s_i = [(0.1)^2 s_{Ra}^2 + (0.1)^2 s_{(i-1)}^2 + \sum(r_k^2 s_k^2) + s_1^2]^{(1/2)}, \ (k > i + 1 \text{ and } 4 \le i \le 8). \qquad (21)$$

Similarly, standard uncertainties in dial corrections of $\mathbf{C_1}$ and $\mathbf{C_2}$ are given as:

$$s_i = [(10)^2 s_{Ra}^2 + (10)^2 s_{(i+1)}^2 + (10)^2\sum(r_k^2 s_k^2) + s_1^2]^{(1/2)}, \ (k > i + 1 \text{ and } i = 1 \text{ or } 2). \qquad (22)$$

3.2.3 Analysis of Uncertainties in Dial Corrections

As mentioned previously, only capacitance values are reported. The component of errors from the cross product of quadrature errors in the 10:1 (or 1:10) ratio and the internal capacitors is negligible. Therefore, only the components of the in-phase standard uncertainties of the dial corrections are included in the Type B standard uncertainty. Based upon Eqs. (20), (21), and (22), the standard uncertainty, s_3, for the dial correction d_3 of $\mathbf{C_3}$ is determined first, and the standard uncertainties of other dials, s_i, are obtained from s_3. Estimated values of each term of Eq. (20) are given as follows:

1) According to Eq. (11) in section 3.1.2, the in-phase standard uncertainty for the 10:1 (and 1:10) ratio corrections, s_{Ra}, is estimated to be less than 0.043 ppm. For C_3 = 1 pF, this becomes:

   $s_{Ra} < 0.043 \times 10^{-6}$ pF.

2) The combined standard uncertainty of the reference standard, s_{sc}, is less than 0.72 ppm, which will be discussed in section 3.4, (i.e., s_{sc} < 0.72 ppm). For C_s = 10 pF, then we have:

$s_{sc} < 7.2 \times 10^{-6}$ pF.

3) Table 1 gives a set of typical data of the average values and standard deviations of dial corrections of the Type-2 bridge from 15 measurements taken over a period of one year. Measurement records show that only the three lowest dials are needed during dial correction measurements of $\mathbf{C}_3$. Looking at $r_k = 1.0$ (which is a maximum) in Table 1, the standard deviation of the dial corrections used during the calibration of $\mathbf{C}_3$ is estimated from the standard deviations of the last three dial corrections, (dials number 6, 7, and 8), also given in Table 1. Thus,

$$(\sum r_k^2 s_k^2)^{(1/2)} < 0.08 \times 10^{-6} \text{ pF.}$$

4) The maximum bounds of error due to the interpolation of the last capacitance dial is equal to $\pm (3 \times 10^{-7})$ pF. Therefore, the standard uncertainty of the interpolation of the last dial becomes:

$$s_1 < 0.2 \times 10^{-6} \text{ pF.}$$

According to Eq. (20), the combined standard uncertainty for the third dial can be estimated by using the RSS of the above components, which are also shown in Table 2, as:

$$s_3 = 0.75 \times 10^{-6} \text{ pF} < 10^{-6} \text{ pF.}$$

After s_3 is determined, other s_i can be estimated similarly from Eqs. (21) and (22). Table 2 is the summary of standard uncertainties for each dial correction, s_i, of the Type-2 bridge.

The combined standard uncertainty in the dial corrections during measurements depends on the number of dials being used and is estimated as the RSS of standard uncertainties from those dials. In general, the order of the dial number being used starts from dial number 8 and goes incrementally to dial number 1. Table 3 gives the estimated combined standard uncertainties of dial corrections, s_{cd}, for capacitors of various nominal values, according to the number of dials being used during measurements.

3.3 Uncertainties in External Bridge Components

As mentioned in sections 1 and 2, the transfer of the farad to the impedance calibration laboratory (ICL) is from a group of NIST-made 10 pF fused-silica capacitance standards in the primary capacitance laboratory (PCL). The primary reference in the ICL is a group of the same type of capacitors. The construction and stability of these standards, including results from measurements of their performance, were discussed in detail by Cutkosky and Lee [6]. The analysis of measurement uncertainties when using fused-silica capacitance standards as the reference is mainly based upon the test results from [6]. During routine calibrations when nitrogen dielectric capacitors are used as the reference standards, the uncertainty analysis is based on the performance of such capacitors as published by the manufacturer [7].

3.3.1 Voltage Dependence of Standards

The voltage dependence of a group of ten 10 pF capacitors was observed in [6] by measuring their capacitance and dissipation factors from 100 V to 200 V. Using the average values of the measurements, as well as the systematic and standard errors, the results are:

$$s_{va} = 3.54 \times 10^{-9} < 0.004 \text{ ppm}$$

and

$$s_{vb} = 2.73 \times 10^{-9} < 0.003 \text{ ppm},$$

where s_{va} and s_{vb} are the in-phase and quadrature standard uncertainties, respectively, of the fused-silica capacitance standards due to voltage dependence. This dependency is believed not to change with time.

The uncertainty of measuring the voltage dependence of 100 pF and 1000 pF air capacitors is reported to be in the order of 0.001 ppm [8].

3.3.2 Frequency Dependence of Standards

The results of comparing a group of ten 10 pF fused-silica capacitors with two 10 pF air capacitors at three frequencies (159 Hz, 1592 Hz, and 15 920 Hz) were presented in [6]. Since the transfer of the farad from the PCL to the ICL involves measurements of a transportable 10 pF fused-silica dielectric capacitor at 1592 Hz in the PCL and measurements of that same standard in the ICL at 1 kHz, the uncertainty due to frequency dependence is first estimated between frequencies of 1592 Hz and 1 kHz. The uncertainties due to frequency dependence for frequencies of 400 Hz and 100 Hz are estimated between frequencies of 1 kHz and 400 Hz, and 1 kHz and 100 Hz, respectively. Recently, additional data were obtained from the PCL for three 10 pF fused-silica capacitors (#113, #112, and #125) at frequencies of 1592 Hz, 1 kHz, 400 Hz, and 100 Hz. Using a natural logarithmic fit to all the data of capacitor #113, which has the largest frequency dependence, the maximum values of errors due to frequency dependence at frequencies of 1 kHz, 400 Hz, and 100 Hz are estimated to be 1.17 ppm, 1.50 ppm, and 3.0 ppm, respectively [9].

According to Eq. (4),

$$s_f \leq 0.68 \text{ ppm} < 0.7 \text{ ppm} \quad \text{(between frequencies of 1592 Hz and 1 kHz)}$$

$$s_f \leq 0.87 \text{ ppm} < 0.9 \text{ ppm} \quad \text{(between frequencies of 1 kHz and 400 Hz)}$$

$$s_f \leq 1.73 \text{ ppm} < 1.8 \text{ ppm} \quad \text{(between frequencies of 1 kHz and 100 Hz)}$$

where s_f is the standard uncertainty of the 10 pF fused-silica capacitors due to frequency dependence from the PCL as reference.

The reference standards in the ICL are maintained at a frequency of 1 kHz, and the majority of the calibration measurements, including the determination of dial corrections, are performed at 1 kHz.

Therefore, no additional errors due to frequency dependence are incurred except for calibrations performed at frequencies other than 1 kHz.

3.3.3 Hysteresis Effects in Temperature Corrections of Standards

Each NIST-made 10 pF fused-silica dielectric capacitor has a known temperature coefficient, which is used to obtain a temperature correction during measurements. Errors in the temperature measurements and their effects are negligible ($< 10^{-9}$). However, hysteresis effects of temperature on the capacitance of fused-silica dielectric capacitors are known to exist and will introduce errors in the corrections. As described in [6], changes of capacitance of a group of nine such capacitors were observed when measured at 25 °C before and after being subjected to 50 °C. These results are employed to estimate the uncertainty due to temperature changes in the environment. Utilizing the average change and the standard deviation of the mean, it is estimated to be:

$$s_{tc} \leq 0.084 \text{ ppm} < 0.1 \text{ ppm},$$

where s_{tc} is the standard uncertainty in temperature correction of using fused-silica capacitors as reference.

In routine calibrations of nitrogen and air dielectric capacitors, no temperature corrections are applied. Standard uncertainties associated with the nitrogen dielectric working standards (used as reference standards) due to variations of room temperature during measurements are estimated to be less than 1 ppm, assuming the temperature variation is within ± 0.5 °C. Thus,

$$s_{tc1} \leq 1 \text{ ppm},$$

where s_{tc1} is the standard uncertainty from temperature changes of nitrogen dielectric capacitors used as references.

3.3.4 Lead Impedance

In general, the effects of leads that are used to connect a capacitor, **C**, to a bridge can be represented by the capacitor in series with a resistor and an inductor, as shown in Fig. 4. The expression for the effective impedance in Fig. 4 can be shown to be:

$$Z = \{ R_l + [G_c / (G_c^2 + \omega^2 C^2)] \} + j\,\omega \{ L_l - [C / (G_c^2 + \omega^2 C^2)] \}, \tag{23}$$

where Z is the effective impedance of the capacitor **C**,
R_l is the series resistance of leads,
L_l is the series inductance of leads, and
G_c is the parallel conductance of the capacitor **C**.

The dissipation factors, DF_c, of the fused-silica, nitrogen, and air capacitors are all less than 10^{-5} ; therefore, the magnitude of $(G_c^2 + \omega^2 C^2)$ is approximately equal to $\omega^2 C^2$, since $G_c^2 < (10^{-10})\omega^2 C^2$.

Therefore, Eq. (23) can be written as:

$$Z = [\, R_1 + (\, G_c / \omega^2 C^2 \,)] + \mathrm{j}\,\omega\, [\, L_1 - (\, C / \omega^2 C^2 \,)], \tag{24}$$

Thus, the equivalent equation of admittance, $Y = (1/Z)$ is expressed as:

$$Y = (\, \omega^2 C^2 \,) / [(\, R_1\, \omega^2 C^2 + G_c \,) + \mathrm{j}\, \omega C\, (\, \omega^2 L_1 C - 1 \,)], \tag{25}$$

where Y is the effective admittance of the capacitor.

The value of L_1 is less than 10^{-6} H and G_c is less than 10^{-10} S. By multiplying the numerator and the denominator of Eq. (25) by $[(\, R_1\, \omega^2 C^2 + G_c \,) - \mathrm{j}\, \omega C\, (\, \omega^2 L_1 C - 1 \,)]$ and eliminating all second order terms of L_1 and G_c, Eq. (25) becomes:

$$Y = [(G_c + R_1\, \omega^2 C^2 \,) + \mathrm{j}\, \omega C\, (1 - \omega^2 L_1 C \,)] / [1 + R_1\, \omega^2 C^2 + 2\, R_1\, G_c - 2\, \omega^2 L_1 C\,], \tag{26}$$

For a one meter long coaxial cable, R_1 and L_1 were measured to be approximately 0.1 Ω and 0.5 μH, respectively. For measurements of capacitors of nominal values up to 10^4 pF, at frequencies up to 1592 Hz, Eq. (26) can be reduced to:

$$Y = [(G_c + R_1\, \omega^2 C^2 \,) + \mathrm{j}\, \omega C\, (1 - \omega^2 L_1 C \,)] / [1 - 2\, \omega^2 L_1 C\,]. \tag{27}$$

By multiplying the numerator and the denominator of Eq. (27) by $(\, 1 + 2\, \omega^2 L_1 C\,)$, and eliminating the second order terms of L_1 and G_c, Eq. (27) becomes:

$$Y = (\, G_c + R_1\, \omega^2 C^2 + 2\, R_1\, \omega^4 L_1 C^3 \,) + \mathrm{j}\, \omega C\, (1 + \omega^2 L_1 C\,). \tag{28}$$

Furthermore, the third term in Eq. (28), $(2\, R_1\, \omega^4 L_1 C^3)$ is less than 10^{-15} S, which is negligible as compared with G_c and $(R_1\, \omega^2 C^2)$. Therefore, Eq. (28) becomes:

$$Y = (\, G_c + R_1\, \omega^2 C^2 \,) + \mathrm{j}\, \omega C\, (1 + \omega^2 L_1 C\,). \tag{29}$$

The effective values of capacitance and conductance of the standard with lead impedance are:

$$C_{\mathrm{eff}} = C\,(\, 1 + \omega^2 L_1 C\,) = C\,[\, 1 + (\Delta C / C\,)\,], \text{ and} \tag{30}$$

$$G_{\mathrm{eff}} = (G_c + R_1\, \omega^2 C^2\,) = (G_c + \Delta G\,), \tag{31}$$

where $(\Delta C / C\,)$ and ΔG are uncertainties in the capacitance and conductance values of a capacitor due to the lead impedance.

For measurements using the fused-silica dielectric transfer standard from PCL, $C = 10$ pF, estimated uncertainties in the lead impedance are calculated to be:

$(\Delta C / C\,) \quad = 1.97 \times 10^{-10} \quad \leq \; 0.0002$ ppm, and

$\Delta G \quad = 3.94 \times 10^{-16}\ \mathrm{S} \ \leq\ 4 \times 10^{-7}\ \mathrm{nS}$.

Table 4 gives a summary of the uncertainties due to lead impedance for measurements of various nominal capacitance values at frequencies of 100 Hz, 400 Hz, and 1 kHz.

3.3.5 Uncertainty in the Reference Standard

The relative stability of eleven fused-silica dielectric capacitors was measured over a period of five months in 1964 [6]. With the exception of one capacitor, which exhibited a fairly steady increase in capacitance over time and has a large voltage dependence of capacitance, data taken on the others indicated the maximum bounds of error are ± 0.076 ppm. Therefore, the uncertainty of these standards is taken to be:

$$s_{\mathrm{s}} = 0.044 \ \mathrm{ppm} \ < 0.05 \ \mathrm{ppm},$$

where s_{s} is the standard uncertainty of using the 10 pF fused-silica dielectric capacitors from the PCL as reference.

3.3.6 Combined Uncertainty of the External Bridge Components

The combined uncertainty of the external bridge components is estimated as the RSS of the above five components in sections 3.3.1 to 3.3.5 as:

$$s_{\mathrm{cb}} = (s_{\mathrm{va}}{}^2 + s_{\mathrm{f}}{}^2 + s_{\mathrm{tc}}{}^2 + (\Delta C / C)^2 + s_{\mathrm{s}}{}^2)^{1/2} \quad \mathrm{ppm}, \tag{32}$$

where s_{cb} is the combined standard uncertainty in the external bridge components using the Type-2 bridge to measure fused-silica and nitrogen dielectric capacitors. Table 5 contains a summary of s_{cb} for various nominal values of capacitors measured at 1 kHz.

After the combined Type B standard uncertainty is estimated by using a capacitance standard from the PCL as reference to measure another capacitor, $\mathbf{C_x}$, this value is utilized as the uncertainty (s_{s}) of $\mathbf{C_s}$ when it (previously $\mathbf{C_x}$,) is used as a reference to calibrate other capacitors later on, also shown in Table 5.

3.4 Uncertainties of Fused-Silica and Nitrogen Dielectric Capacitors at 1 kHz

Table 5 is a summary of the calculations for the combined Type B standard uncertainties for measurements of fused-silica and nitrogen dielectric capacitors of different nominal values at a frequency of 1 kHz in the ICL, using the Type-2 bridge. Again, since only capacitance values are reported, only the components in capacitance measurements are used for uncertainty calculations. By taking the RSS value of all the above components, the uncertainty in measurements, using a 10 pF fused-silica capacitance standard from PCL as reference, is estimated to be 0.711 ppm. Therefore, the combined Type B standard uncertainty of measuring an oil-bath type 10 pF fused-silica capacitor in ICL is assigned to be 0.72 ppm, including the components of transformer ratio, dial corrections, and external bridge components. Other calculations using the oil-bath type capacitor as the reference standard utilize s_{sc} = 0.72 ppm as the uncertainty.

The Type A standard uncertainty is estimated from the measurement data on the repeatability of measurements. A calibration of customer's fused-silica dielectric capacitor standard consists of at least five sets of measurements performed over a two-week period, and the pooled standard deviation of within and between days measurements is employed as the Type A standard uncertainty. The expanded uncertainty of such capacitors is calculated from both Type A and Type B standard uncertainties with a coverage factor of $k = 2$ [3], as given in Eq. (3). For nitrogen dielectric capacitor standards, the Type A standard uncertainty is taken from results of calibrations of a large population of standards. In an individual calibration, the capacitor is measured a few times to ensure that the use of the population statistics is valid. In the case where the Type A standard uncertainty is over the limit, the total uncertainty is increased according to the data. Table 6 gives the expanded and assigned uncertainties for calibrations of fused-silica and nitrogen dielectric capacitance standards at 1 kHz, including both Type A and Type B standard uncertainties.

3.5 Uncertainties of Fused-Silica and Nitrogen Dielectric Capacitors at 400 Hz

The error analysis for the calibration of fused-silica and nitrogen dielectric capacitors at a frequency of 400 Hz is similar to that at 1 kHz. Since the assigned value of the oil-bath-type fused-silica reference capacitor in the ICL is measured at 1 kHz, there is an uncertainty of 0.9 ppm in frequency dependent error, when using it as a reference to calibrate other capacitors at 400 Hz.

Table 7 is a summary of the expanded and assigned total uncertainties for calibrations of fused-silica and nitrogen dielectric capacitors at a frequency of 400 Hz.

3.6 Uncertainties of Fused-Silica and Nitrogen Dielectric Capacitors at 100 Hz

The error analysis for the calibration of fused-silica and nitrogen dielectric capacitors at a frequency of 100 Hz is similar to that at 1 kHz with two exceptions:

1) At 100 Hz, due to lower applied voltage, the resolution of the bridge is lower than at 1 kHz, such that an uncertainty exists in the 10^{-5} pF dial, instead of the 10^{-6} pF dial. The maximum bounds of error is estimated to be $\pm$ (3 x10^{-6}) pF. According to the nominal values of the unknown standards, the standard uncertainty to be included in the combined uncertainties at measurement, s_{cb} is :

   $s_{s1} \leq$ 2 ppm (for 1 pF capacitors),
   $s_{s1} \leq$ 0.2 ppm (for 10 pF capacitors),
   $s_{s1} \leq$ 0.02 ppm (for 100 pF capacitors), and
   $s_{s1} \leq$ 0.002 ppm (for 1000 pF capacitors),

   where s_{s1} is the standard uncertainty in the interpolation of the last dial.

2) Since the assigned value of the oil-bath type fused-silica reference capacitor in the ICL is measured at 1 kHz, there is an uncertainty component of 1.6 ppm from its frequency dependency, when using it as a reference to calibrate other capacitors at 100 Hz.

With the above assumptions, the expanded and assigned total uncertainties for capacitance calibrations of fused-silica and nitrogen dielectric types of capacitors at 100 Hz is given in Table 8.

3.7 Uncertainties of Air and Mica Dielectric Capacitors

NIST/Gaithersburg also provides calibration services at a frequency of 1 kHz for air and mica dielectric capacitance standards in three-terminal or two-terminal high-frequency (HF) coaxial arrangements for plugging into a capacitance bridge or an impedance instrument directly. These capacitors are used mainly to calibrate the bridge or instrument at high frequencies, with its reference at 1 kHz. Stabilities of these capacitors are not as high as those with fused-silica and nitrogen dielectrics and they are affected more readily by the environment. Evaluation of the Type B standard uncertainty for these capacitors is based on the manufacturer's published information on sources of instability for these types of capacitors [7, 10] and on the specifications for each nominal value and type of capacitor. Evaluation of the Type A standard uncertainty is estimated from the measurement data; the capacitors are measured a few times to ensure that the standard deviation is within a given limit commensurate with the assigned total uncertainty based on population statistics. Assuming the laboratory conditions during measurements are such that the temperature change is less than ±0.5 °C and the relative humidity change is less than ± 8 %, the combined Type B standard uncertainty and the expanded uncertainties (coverage factor of $k = 2$) for each type of capacitor are estimated. The results are shown in tables 9, 10, 11, and 12.

3.7.1 Three-Terminal Air Capacitance Standards

The terminals of these air dielectric capacitors are arranged to connect directly to the terminals of certain commercial capacitance bridges. Adapters also can be utilized to connect them to other impedance (LRC) meters and instruments. Table 9 gives the expanded and assigned total uncertainties (and their components) of these capacitance standards at 400 Hz and 1 kHz, with nominal values that range from 0.001 pF to 10 000 pF. These capacitors can be used at 100 Hz, as well as at frequencies higher than 1 kHz. The expanded and assigned total uncertainties of three-terminal air capacitors at 100 Hz is shown in table 10.

3.7.2 Two-Terminal HF Coaxial Capacitance Standards and Terminations

The terminals of these capacitors, with the combination of certain commercial precision coaxial connectors, can be connected directly to instruments to obtain low lead inductance and high stability at high frequency (HF) [11, 12]. Therefore, these capacitors are mainly used for measurements in the radio frequency range, with their reference values determined at 1 kHz. There are three types of HF coaxial capacitance standards available, as characterized by their nominal values and dielectric materials.

Table 11 contains the calibration uncertainties for two types of HF coaxial air capacitors. The lower-valued type, with nominal values that range from 1 pF to 20 pF, has a relatively flat frequency response to a few hundred megahertz (see Fig. 3 in Ref. 12). These capacitors can be employed as low-capacitance terminations for any two-port device. The mid-valued type, with nominal values that range from 50 pF to 1000 pF, has negligible changes in effective capacitance with frequency up to the megahertz range (see Fig. 2 in Ref. 11). These capacitors are used mainly to calibrate capacitance bridges and other instruments at high frequencies from their reference values at 1 kHz.

The higher-valued coaxial capacitance standards use mica as a dielectric. The nominal values of these capacitors range from 0.001 μF to 0.1 μF. Like the mid-valued type, these capacitors also have very low changes in effective capacitance below 100 kHz (see Fig. 1 in Ref. 12). The main application for this type of capacitor is to calibrate instruments at higher frequencies from their reference values at 1 kHz. Table 12 gives the various calibration uncertainties for this type of capacitor.

The open-circuit terminations are useful as capacitance standards at low frequency for calibration bridges, and as shield caps for open-circuit lines in establishing initial conditions of line length and signal phase [10]. There are two types of open-circuit terminations, depending on the effective position of termination (plane position). The plane positions of these terminations are 0.26 cm and 4 cm, which have capacitance values of 0.172 pF and 2.670 pF, respectively, at low frequencies. NIST provides calibration services for these terminations at a frequency of 1 kHz with the calibration uncertainties given in Table 12.

4. UNCERTAINTY ANALYSIS FOR THE TYPE-12 SYSTEM

The original unity-ratio admittance bridge, known as the No. 12 type, was developed by Bell Laboratories in 1942 [13] for precision measurements of capacitance up to 1.11 μF and conductance up to 1000 μS. Later on, an improved Type-12 capacitance bridge was designed, also by Bell Laboratories, with a capacitance divider to measure low capacitance values and multi-range, direct reading conductance standards. A complete schematic diagram of the Type-12 bridge is shown in Fig. 5a. Specifications of bridge components and internal standards, and operational procedures of the Type-12 bridge are described in detailed by Wilhelm [14].

There are two mica capacitance dials, three air capacitance dials, and two conductance (resistance) dials, all with ten steps in each dial, in the Type-12 bridge. The mica dials range from 0.1 μF to 1 μF and from 0.01 μF to 0.1 μF, and the air dials range from 0.001 μF to 0.01 μF, 0.0001 μF to 0.001 μF, and 0.00001 μF to 0.0001 μF. A 0.005 μF mica capacitor, measured using the Type-2 bridge, is used as a transfer standard to calibrate the 0.01 μF steps of both the mica capacitance and the air capacitance dials of the Type-12 bridge. All other steps of mica and air capacitance dials are calibrated from the 0.01 μF step, used as a reference. The conductance dials range from 10 μS to 100 μS and from 100 μS to 1000 μS, and are calibrated against resistance standards. Both capacitance and conductance dials are adjusted to obtain a final balance at the time of measurement to provide capacitance and conductance values of the mica capacitors being calibrated by the Type-12 bridge.

4.1 Basic Equations of Capacitance Measurements

Figure 5b is a simplified schematic of the Type-12 bridge showing the components for capacitance measurements, where $\mathbf{R_1}$ and $\mathbf{R_2}$ are resistors with a nominal ratio of 1:1. Using C_1 and C_2 as the total capacitance of each of the other two arms, the basic equation of balance for making capacitance measurements (when the voltage across detector D is zero) is:

$$R_1 C_1 = R_2 C_2, \tag{33}$$

where C_1 and C_2 are the total capacitances of arm AD and arm CD, respectively.

As shown in Fig. 5b, C_1 and C_2 can be expressed as:

$$C_1 = C_x + C_m + C_a \quad \text{(when } \mathbf{C_x} \text{ is connected to arm AD)} \tag{34}$$

and

$$C_2 = C_x + C_c \quad \text{(when } \mathbf{C_x} \text{ is connected to arm CD)}\,, \tag{35}$$

where $\mathbf{C_x}$ is the unknown capacitor to be measured, C_m is the reading of internal mica dial capacitors, and C_a and C_c are effective capacitances of arms AD and CD, respectively, with internal air capacitors coupled so that:

$$C_a + C_c = C \quad \text{(a constant)}. \tag{36}$$

Each dial of the air capacitors of the Type-12 bridge is arranged such that the dial reading indicates the difference of the effective capacitance between arm AD and arm CD, plus a constant capacitance C, as:

$$C_r = (C_a - C_c) + C\,, \tag{37}$$

where C_r is the capacitance value corresponding to the dial reading, r.

Substituting Eq. (36) into Eq. (37) gives:

$$C_r = 2\, C_a\,. \tag{38}$$

The internal air capacitors of the Type-12 bridge simulate three decade capacitors consisting of ten 0.001 µF (1000 pF) , ten 0.0001 µF (100 pF), and ten 0.00001 µF (10 pF) capacitors, and a continuously variable capacitor providing a capacitance range of (11 ± 0.5) pF.

Using the 10 pF/step dial as an example, where $\boldsymbol{C}$ = 50 pF, a set of four capacitors of 5 pF, 10 pF, 15 pF, and 20 pF, are switched back and forth individually from arm CD to arm AD when the dial readings are increased from 0 to 1, 1 to 2, ..., and 9 to 10. The capacitance values corresponding to the dial readings are obtained in accordance with Eq. (36) and with the following procedure for switching the capacitors:

At dial reading $r = 0$, all four capacitors are connected to arm CD, then

$$C_c = 50 \text{ pF and } C_a = 0; \text{ therefore, } C_r = 0.$$

At dial reading $r = 1$, the 5 pF capacitor is connected to arm AD, the other three are connected to arm CD, then

$$C_c = 45 \text{ pF and } C_a = 5 \text{ pF; therefore, } C_r = 10 \text{ pF}.$$

At dial reading $r = 2$, the 10 pF capacitor is connected to arm AD, the other three are connected to arm CD, then

$$C_c = 40 \text{ pF and } C_a = 10 \text{ pF; therefore, } C_r = 20 \text{ pF}.$$

.

.

.

.

.

At dial reading $r = 10$, all four capacitors are connected to arm AD, and then

$$C_c = 0 \text{ and } C_a = 50 \text{ pF; therefore, } C_r = 100 \text{ pF}.$$

Measurements of two- and three-terminal mica capacitors with nominal values up to 0.005 µF are made using internal air capacitors only.

The Type-12 bridge also has a set of internal decade mica capacitors of ten 0.01 µF and ten 0.1 µF which are employed in the arm AD, together with the air capacitors, to measure mica capacitors with nominal values larger than 0.005 µF. As mentioned previously, a 0.005 µF mica capacitor is used as a transfer standard to obtain the dial corrections for the 0.01 µF step of both air and mica dial capacitors from Type-2 bridge measurements. All other steps of air and mica dial capacitors are calibrated against the 0.01 µF step.

Again, the total uncertainty associated with Type-12 bridge capacitance measurements contains both Type A and Type B standard uncertainties. An analysis of each type of uncertainty is contained in the following sections.

4.2 Type B Standard Uncertainty of Capacitance Measurements

The evaluation of the components of the Type B standard uncertainty for the Type-12 bridge is based on uncertainties in the ratio of resistors, $\mathbf{R}_1$ and $\mathbf{R}_2$, in the internal air and mica capacitors, and in the external bridge components. An analysis of each component at various frequencies is discussed in general to cover the entire range of capacitance calibrations. The Type B standard uncertainty for each type of capacitance measurement depends on the measurement procedure and internal standards used for the calibrations.

4.2.1 Ratio Resistors

According to the specifications for the Type-12 bridge [13], the maximum bounds of error in the 1:1 ratio are ± 30 ppm. Thus, according to Eq. (4), the uncertainty in the ratio of 1:1 is estimated as:

$$s_{ra} = 17.3 \text{ ppm} \leq 18 \text{ ppm}, \tag{39}$$

where s_{ra} is the standard uncertainty contributed by the ratio resistors.

4.2.2 Internal Capacitors

The uncertainty components of both air and mica dial capacitors depend on their characteristics and errors due to their calibrations and environmental variations, which are discussed as follows:

1) Dial Corrections

Data and analyses of capacitance dial corrections for the Type-12 bridge at various frequencies (100 Hz, 1 kHz, and 10 kHz) are given in Appendix B. The capacitance correction of each dial setting has been evaluated by fitting a regression line to the corresponding data of 25 years to determine the predicted value. These corrections are updated once a year and are used to measure two sets of check standards of the Type-12 bridge. The data histories of these check standards are analyzed to ensure that the linear models of the dial corrections are valid. Thus, the standard deviations of the predicted values of the dial corrections for a particular date can be obtained. The standard uncertainty of the dial corrections is the RSS of the standard deviations of the predicted values for the dials which depend on the nominal value of the capacitor being measured.

For the air capacitor dials, the standard deviations of the predicted values of the dial corrections of those dials being used during measurements for the two-terminal and three-terminal configurations range between 0.018 pF and 0.076 pF, as given in the first rows of Tables 13a and 13b[2]. According to the nominal values of the unknown capacitors, the respective standard uncertainties (in ppm) of the dial corrections of the air capacitors, s_{dr}, for both the two-terminal and three-terminal measurements are estimated and also given in Tables 13a and 13b (second rows).

For the mica capacitor dials, the standard uncertainties are estimated in the same manner as in the air capacitor dials for the two-terminal and three-terminal measurements, respectively. According to the dials being used during measurements and the nominal values of the unknown capacitors, the standard uncertainty of the corrections of the mica capacitor dials, s_{mr}, are included in Tables 14a and 14b, for the two-terminal and three-terminal

[2] Starting from Table 13, the Tables are arranged that Table xxa and Table xxb are using the "a" and the "b" to refer to the components of standard uncertainties for the two-terminal and three-terminal measurements, respectively.

measurements, respectively.

2) Temperature variations

The temperature coefficients of air and mica capacitors are 10 ppm/°C and 45 ppm/°C, respectively. The temperature in the ICL is always maintained at (23 ± 1) °C. Therefore, the maximum bounds of error owing to temperature effects for the readings of the air and mica capacitance dials are ±10 ppm and ± 45 ppm, respectively. According to Eq. (4),

$$s_{tp} = 5.77 \text{ ppm} \leq 6 \text{ ppm}, \quad \text{and} \quad s_{mp} = 25.98 \text{ ppm} \leq 26 \text{ ppm}, \tag{40}$$

where s_{tp} and s_{mp} refer to the standard uncertainties in the air and mica dial capacitors, respectively, due to temperature variations, which are included in Tables 13a to 14b.

3) Dial switching

The maximum bounds of error due to the switching of dials, $\pm\, \boldsymbol{e}_{ds}$ are estimated to be ± 0.01 pF, which will affect only the measurements of low capacitance. Accordingly,

$$\begin{aligned}
s_{ds} &= 5.77 \text{ ppm} \leq 6 \text{ ppm} && (\text{for } \boldsymbol{C} = 0.001\ \mu\text{F}),\\
s_{ds} &= 0.58 \text{ ppm} \leq 0.6 \text{ ppm} && (\text{for } \boldsymbol{C} = 0.01\ \mu\text{F}),\\
s_{ds} &= 0.06 \text{ ppm} \leq 0.1 \text{ ppm} && (\text{for } \boldsymbol{C} = 0.1\ \mu\text{F}), \quad \text{and}\\
s_{ds} &= 0.006 \text{ ppm} \leq 0.01 \text{ ppm} && (\text{for } \boldsymbol{C} = 1\ \mu\text{F}).
\end{aligned} \tag{41}$$

where s_{ds} is the standard uncertainty due to the switching of dials, also included in Tables 13a to 14b.

4) Uncertainty of transfer standard

From numerous measurements, the calibrated value of the transfer standard, a 0.005 μF mica capacitor has an uncertainty of less than 0.03 pF, which will cause an uncertainty of less than 6 ppm in the dial corrections of both air and mica dial capacitors. Thus,

$$s_{tf} \leq 6 \text{ ppm}, \quad \text{and} \quad s_{mf} \leq 6 \text{ ppm}, \tag{42}$$

where s_{tf} and s_{mf} are the standard uncertainties in the air and mica capacitor dial corrections, respectively, due to the uncertainty of the transfer standard, which are given in Tables 13a to 14b.

For measurements of capacitors with nominal values up to 0.005 μF, (only air-dial capacitors are needed), s_{tf} is used as the standard uncertainty. However, for capacitors of nominal values larger than 0.005 μF, the RSS of s_{tf} and s_{mf} is used to estimate the standard uncertainty from the dial corrections in measurements.

The combined uncertainty of the internal capacitors is defined as the RSS of the above components, which are estimated for each type of measurement as:

$$s_{rc} = (s_{dr}^{\,2} + s_{tp}^{\,2} + s_{ds}^{\,2} + s_{tf}^{\,2})^{(\frac{1}{2})} \text{, and} \tag{43}$$

$$s_{m} = (s_{mr}^{\,2} + s_{mp}^{\,2} + s_{ds}^{\,2} + s_{mf}^{\,2})^{(\frac{1}{2})} , \tag{44}$$

where s_{rc} and s_{m} are the combined uncertainties in the air and mica capacitor dials, respectively, which are also included in Tables 13a to 14b.

4.2.3 External Bridge Components

1) Capacitance Between Connectors

Two special plates, P2 and P3, have been constructed to connect to the Type-12 bridge so that it can perform direct measurements on two-terminal and three-terminal mica capacitors, respectively, without using cables. Each plate has been evaluated to obtain its series inductance and resistance, and parallel capacitance. After these corrections are applied, the maximum bounds of error introduced by connections are found to be ± 0.02 pF and ± 0.01 pF for plates P2 and P3, respectively. These values imply that, according to Eq. (4), the uncertainty when using plate P2 is in the range from 0.012 ppm (in measuring 1 μF capacitors) to 12 ppm (in measuring 0.001 μF capacitors), and the uncertainty when using plate P3 is from 0.006 ppm to 6 ppm. Hence,

$$0.012 \le s_{bp2} \le 12 \text{ ppm} \qquad \text{and} \qquad 0.006 \le s_{bp3} \le 6 \text{ ppm}, \tag{45}$$

where s_{bp2} and s_{bp3} are the standard uncertainties due to the applied corrections of plates P2 and P3, respectively.

2) Series Lead Impedance

The effective capacitance of an unknown capacitor, C_x, including series lead inductance is taken to be (see derivation of Eq. (30a) in sect. 3.3.4):

$$C_{eff} = C_x (1 + \omega^2 L_l C_x), \tag{46}$$

where C_{eff} is the effective capacitance, L_l is the series lead inductance, and $\omega = 2\pi f$ in which f is the applied frequency. The lead inductance of the Type-12 bridge is estimated to be less than 0.2 μH, and the maximum error caused by lead impedance, e_{bZ} is less than $(2\pi f)^2 (0.2)10^{-6} (C_x)$, which is $8f^2 (C_x)$ ppm. Accordingly,

$$s_{bZ} = 4.6 f^2 (C_x) \quad \text{ppm}, \tag{47}$$

where s_{bZ} is the standard uncertainty due to lead impedance, f is the applied frequency in kilohertz (kHz), and C_x is nominal value of the unknown capacitor in microfarad (μF).

The combined uncertainty of the external bridge components, s_{br}, is defined as the RSS of the above two components, which are estimated from each type of measurement, as:

$$s_{br} = (s_{bp}^{\ 2} + s_{bZ}^{\ 2})^{(½)} , \tag{48}$$

where s_{bp} is equal to either s_{bp2} or s_{bp3}, depending on which plate (either P2 or P3) is being used. These are given in Tables 15a and 15b.

The Type B component of standard uncertainty for capacitance measurements made with the Type-12 bridge, U_b, is defined as the RSS of the four components, s_{ra}, s_{rc}, s_m, and s_{br} in Eqs. (39), (43), (44), and (48). This uncertainty is analyzed in detail for each type of measurement in the following section and the numerical results are given in Tables 15a and 15b for the two-terminal and three-terminal measurements, respectively.

4.3 Type B Standard Uncertainty for Various Types of Capacitance Measurement

4.3.1 Two-Terminal Measurements of Low-Capacitance Values (0.001 μF ≤ C_x ≤ 0.005 μF)

4.3.1.1 Basic Equations of Balance

This type of measurement covers the range of capacitors with nominal values from 0.001 μF to 0.005 μF, where only internal air-dial capacitors are used to balance the bridge. Using internal air capacitors, the bridge is balanced twice, with and without the unknown capacitor connected to the arm CD, as shown in Fig. 6. According to Eq. (33), the equations of balance are given as:

$$R_1 C_{a0} = R_2 C_{c0} \tag{49}$$

and

$$R_1 C_{a1} = R_2 (C_{c1} + C_x), \tag{50}$$

where C_{a0} and C_{c0} are capacitance values of internal air capacitors, $\mathbf{C_a}$ and $\mathbf{C_c}$, at balance without connecting the unknown capacitor $\mathbf{C_x}$, and C_{a1} and C_{c1} are the corresponding capacitance values at balance with $\mathbf{C_x}$ connected to arm CD.

From Eqs. (49) and (50), the value of the unknown capacitor can be written as:

$$C_x = [1 + (R_1 / R_2)] (C_{a1} - C_{a0}) , \tag{51}$$

By substituting Eq. (38) into Eq. (51), C_x can be expressed in terms of dial readings as:

$$C_x = (½) [1 + (R_1 / R_2)] (C_{r1} - C_{r0}) , \tag{52}$$

where $C_{r1} = 2\ C_{a1}$ and $C_{r0} = 2\ C_{a0}$ are dial readings of the internal air capacitors at balance, with and without connecting $\mathbf{C_x}$ to arm CD, respectively.

The ratio resistors are always set to be equal, ($R_1 = R_2$) so that Eq. (52) becomes:

$$C_x = (C_{r1} - C_{r0}), \tag{53}$$

which is the equation used to calculate the value of the unknown capacitor.

4.3.1.2 Analysis

The relationship among the variances of the terms of Eq. (52) can be estimated by:

$$\mathrm{Var}(C_x) = (½)^2 (C_{r1} - C_{r0})^2 s_{ra}^{\ 2} + (½)^2 ([1 + (R_1 / R_2)]^2 (s_{rc1}^{\ 2} + s_{rc0}^{\ 2}), \tag{54}$$

where s_{ra}, s_{rc1}, and s_{rc0} are the standard uncertainties in the ratio (contributed by the resistors) and the air capacitor dials of both measurements, respectively.

By substituting Eq. (53) and ($R_1 = R_2$) into Eq. (54), it becomes:

$$\mathrm{Var}(C_x) = [(½) C_x s_{ra}]^2 + s_{rc}^{\ 2}, \tag{55}$$

where $s_{rc}^{\ 2} = s_{rc1}^{\ 2} + s_{rc0}^{\ 2}$.

From Eq. (39), the uncertainty contributed by the ratio resistors for two-terminal low-capacitance measurements, [(½) $C_x s_{ra}$], is 9 ppm (see the first three columns of Table 15a). The capacitance uncertainties associated with each dial reading, s_{rc0} and s_{rc1}, can be estimated from the corresponding terms of Eq. (43), as:

$$s_{rc0} = (s_{dr0}^{\ 2} + s_{tp0}^{\ 2} + s_{ds0}^{\ 2} + s_{tf0}^{\ 2})^{(½)}, \text{ and}$$
$$s_{rc1} = (s_{dr1}^{\ 2} + s_{tp1}^{\ 2} + s_{ds1}^{\ 2} + s_{tf1}^{\ 2})^{(½)}. \tag{56}$$

The combined uncertainty of the internal air dial capacitors, s_{rc}, is estimated to be between 18 ppm and 62 ppm. (See the first three columns of Tables 13a and 15a).

At frequencies of 100 Hz and 1 kHz, the uncertainties due to lead impedance are negligible for capacitor values up to 0.005 µF. The uncertainty of evaluating plate P2, 0.012 pF, is found to be between 3 ppm and 12 ppm.

At a frequency of 10 kHz, the lead impedance including series inductance is calculated from Eq. (47), and the combined uncertainties in the bridge components range from 4 ppm to 12 ppm.

The Type-B standard uncertainties for two-terminal low capacitance value measurements, as well as their individual uncertainty components, are included in the first three columns of Table 15a.

4.3.2 Two-Terminal Measurements of High-Capacitance Values ($0.005\ \mu F < C_x \leq 1\ \mu F$)

4.3.2.1 Basic Equations of Balance

Measurement procedures of this type are the same as those for low-capacitance values, except that the mica dial capacitors are needed when the unknown is connected to arm CD, as shown in Fig. 7. The equations of balance, corresponding to Eqs. (49) and (50) become:

$$R_1 C_{a0} = R_2 C_{c0} \tag{57}$$

and

$$R_1 (C_{a1} + C_m) = R_2 (C_{c1} + C_x), \tag{58}$$

where C_{a0} and C_{c0} are the values of the air capacitors at balance without connecting the unknown capacitor $\mathbf{C_x}$; C_{a1} and C_{c1} are the values of the air capacitors, and C_m is the value of the mica capacitors at balance with $\mathbf{C_x}$ connected to arm CD.

The value of the unknown capacitor can be expressed as:

$$C_x = (R_1 / R_2) C_m + (½)\ [1 + (R_1 / R_2)] (C_{r1} - C_{r0}), \tag{59}$$

where C_m is the mica dial reading, and $C_{r1} = 2\ C_{a1}$ and $C_{r0} = 2C_{a0}$ are air-dial readings of internal capacitors at balance, with and without connecting $\mathbf{C_x}$ to arm CD, respectively.

The ratio resistors are always set to be equal, ($R_1 = R_2$) so that Eq. (59) becomes:

$$C_x = C_m + (C_{r1} - C_{r0}), \tag{60}$$

which is the equation used to calculate the value of the unknown capacitor.

4.3.2.2 Analysis

The relationship among the variances of the terms of Eq. (59) can be estimated by:

$$\begin{aligned} \mathrm{Var}(C_x) = {} & [C_m + (½)(C_{r1} - C_{r0})]^2 s_{ra}^{\ 2} + (R_1 / R_2)^2 s_m^{\ 2} \\ & + (½)^2 [1 + (R_1 / R_2)]^2 (s_{rc1}^{\ 2} + s_{rc0}^{\ 2}), \end{aligned} \tag{61}$$

where s_{ra}, s_m, s_{rc1}, and s_{rc0} are the standard uncertainties in the ratio (contributed by the resistors), the mica capacitor dial, and the air capacitor dials of both measurements, respectively.

According to Eq. (60), the first term of Eq. (61) can be expressed as:

$$[C_m + (½)(C_{r1} - C_{r0})]^2 s_{ra}^{\ 2} \leq (C_x)^2 s_{ra}^{\ 2}, \tag{62}$$

By substituting Eq. (62) and ($R_1 = R_2$) into Eq. (61), it becomes:

$$\mathrm{Var}(C_x) \le (C_x\, s_{ra})^2 + s_m^{\,2} + s_{rc}^{\,2}, \tag{63}$$

where $s_{rc}^{\,2} = s_{rc1}^{\,2} + s_{rc0}^{\,2}$.

From Eq. (39), the uncertainty contributed by the ratio resistors, s_{ra}, for the two-terminal high-capacitance measurements is 18 ppm. (See Table 15a).

The uncertainty of the internal mica capacitors is estimated from the dials being used. According to Eq. (60), the value of the mica-dials correspond to the nominal value of the unknown capacitor. Therefore, the combined uncertainty of the internal mica capacitors, s_m, is estimated to range between 28 ppm and 35 ppm, from Eq. (44). (See Table 14a). The combined uncertainty of the internal air-dial capacitors for the two-terminal high capacitance measurements, s_{rc}, is estimated to be less than 9 ppm, from Eq. (43). (See Tables 13a, columns 4 to 10 and 15a).

Uncertainties due to the capacitance of plate P2 are ± 0.02 pF and uncertainties in lead impedance are functions of both frequency and capacitance values, as shown in Eq. (47). At frequencies of 100 Hz and 1 kHz, the combined uncertainties are less than 5 ppm. However, at a frequency of 10 kHz, these uncertainties increase substantially for high valued capacitors, and are calculated to be as large as 475 ppm.

The Type-B standard uncertainties for two-terminal high-capacitance value measurements, as well as their individual uncertainty components, are included in Table 15a.

4.3.3 Three-Terminal Measurements of Low-Capacitance Values (0.001 µF ≤ C_x ≤ 0.005 µF)

4.3.3.1 Basic Equations of Balance

This type of measurement is similar to those of two-terminal low-capacitance measurements, except that the unknown capacitor, $\mathbf{C_x}$, is connected to arm AD in the first balance and to arm CD in the second balance, as illustrated in Fig. 8. In this case, the equations of balance are:

$$R_1 (C_{a1} + C_x) = R_2\, C_{c1} \tag{64}$$

and

$$R_1\, C_{a2} = R_2 (C_{c2} + C_x), \tag{65}$$

where C_{a1} and C_{c1} are the values of the selected internal capacitors at balance with $\mathbf{C_x}$ connected to arm AD, and C_{a2} and C_{c2} are the values at balance with $\mathbf{C_x}$ connected to arm CD.

From Eqs. (64) and (65), the value of the unknown capacitor can be expressed as:

$$C_x = (½) (C_{r2} - C_{r1}), \tag{66}$$

where $C_{r1} = 2\ C_{a1}$ and $C_{r2} = 2\ C_{a2}$ are dial readings of internal air capacitors at balance with $\mathbf{C}_x$ connected to arm AD and to arm CD, respectively.

4.3.3.2 Analysis

The relationship among the variances of the terms of Eq. (66) can be estimated by:

$$\mathrm{Var}(C_x) = (\tfrac{1}{2})^2\, s_{rc1}^{\ 2} + (\tfrac{1}{2})^2\, s_{rc0}^{\ 2}\,, \tag{67}$$

where s_{rc1} and s_{rc0} are the standard uncertainties in the air capacitor dials of both measurements, respectively.

Eq. (67) can also be expressed as:

$$\mathrm{Var}(C_x) = [(\tfrac{1}{2})\, s_{rc}\,]^2, \tag{68}$$

where $s_{rc}^{\ 2} = s_{rc1}^{\ 2} + s_{rc0}^{\ 2}$.

According to Eq. (66), C_x is independent of R_1 and R_2, so uncertainties contributed by the ratio resistors will not affect this type of measurement. The uncertainties in the air dial readings are estimated to be between 8 ppm and 31 ppm. (See the first three columns of Table 15b).

Uncertainties due to external bridge components are similar to uncertainties of two-terminal measurements of the same nominal values, except that the capacitance of the plate, P3, is 0.006 pF, and these are found to be less than 6 ppm.

The Type-B standard uncertainties of three-terminal low-capacitance value measurements are included in Table 15b, as well as their individual uncertainty components for each capacitance value at 100 Hz, 1 kHz, and 10 kHz .

4.3.4 Three-Terminal Measurements of High-Capacitance Values up to 0.5 μF
(0.005 μF < $\boldsymbol{C}_x \leq$ 0.5 μF)

4.3.4.1 Basic Equations of Balance

Measurement procedures for capacitors of this type are similar to those of three-terminal low-capacitance value measurements, except that internal mica capacitors are needed for the second balance, and a dummy capacitor (a mica decade capacitance box) is connected into the arm CD for both balances, as shown in Fig. 9. The equations of balance, as in Eqs. (64) and (65), become:

$$R_1\,(C_{a1} + C_x) = R_2\,(C_{c1} + C_b\,) \tag{69}$$

and

$$R_1\,(C_{a2} + C_m) = R_2\,(C_{c2} + C_x + C_b\,), \tag{70}$$

where C_{a1} and C_{c1} are values of the internal air capacitors at balance with the unknown capacitor, $\mathbf{C_x}$ connected to arm AD, C_{a2} and C_{c2} are values of the internal air capacitors at balance with $\mathbf{C_x}$ connected to arm CD, C_m are values of the internal mica capacitors at the second balance, and C_b is the value of the dummy capacitance box.

From Eqs. (69) and (70), the value of the unknown capacitor can be expressed as:

$$C_x = [R_1/(R_1+R_2)]C_m + (½)[R_1/(R_1+R_2)](C_{r2} - C_{r1}). \tag{71}$$

The ratio resistors are always set to be equal ($R_1 = R_2$), so Eq. (71) becomes:

$$C_x = (½)\,C_m + (¼)\,(C_{r2} - C_{r1}), \tag{72}$$

which is the equation used to calculate the unknown capacitor.

4.3.4.2 Analysis

Rewrite Eq. (71) as:

$$C_x = \{(R_1/R_2)/[1+(R_1/R_2)]\}C_m + (½)\,\{(R_1/R_2)/[1+(R_1/R_2)]\}(C_{r2} - C_{r1}). \tag{73}$$

The relationship among the variances of the terms of Eq. (71) can be estimated by:

$$\mathrm{Var}(C_x) = [1+(R_1/R_2)]^{-4}\;[C_m + (½)\,(C_{r1}-C_{r0})]^2\, s_{ra}^{\;2} + \{(R_1/R_2)/[1+(R_1/R_2)]\}^2\, s_m^{\;2}$$
$$+ (½)^2\,\{(R_1/R_2)/[1+(R_1/R_2)]\}^2\,(s_{rc1}^{\;2} + s_{rc0}^{\;2}), \tag{74}$$

where s_{ra}, s_m, s_{rc1}, and s_{rc0} are the standard uncertainties in the ratio (contributed by the resistors), the mica capacitor dial, and the air capacitor dials of both measurements, respectively.

By substituting Eq. (72) and ($R_1 = R_2$) into Eq. (74), it becomes:

$$\mathrm{Var}(C_x) = [(½)\,C_x\, s_{ra}]^2 + [(½)\,s_m]^2 + [(¼)\,s_{rc}]^2, \tag{75}$$

where $s_{rc}^{\;2} = s_{rc1}^{\;2} + s_{rc0}^{\;2}$.

As in the two-terminal high-capacitance value measurements, the uncertainty contributed by the ratio resistors in the three-terminal case, [(½) $C_x\, s_{ra}$], is also 9 ppm. (See Table 15b).

Using an analysis similar to that in the case of two-terminal high-capacitance measurements, one finds that the uncertainties in the internal mica capacitors for the three-terminal measurements, [(½)s_m], are estimated to be less than 23 ppm. The uncertainties in the air-dial capacitors, [(¼) s_{rc}], are estimated to be less than 3 ppm. These are also given in Table 15b.

Uncertainties due to external bridge components are similar to those in making two-terminal measurements of the same nominal values, except that the capacitance of the plate, P3, is 0.006 pF, and these are estimated to be less than 3 ppm at frequencies of 100 Hz and 1 kHz. However, at a frequency of 10 kHz, these uncertainties increase substantially for high-valued capacitors, and are calculated to be as large as 235 ppm.

The Type-B standard uncertainties of three-terminal high-capacitance value measurements are included in Table 15b, as well as their individual uncertainty components for each capacitance value at 100 Hz, 1 kHz, and 10 kHz.

4.3.5 Three-Terminal Measurements of Capacitors Larger Than 0.5 µF

4.3.5.1 Basic Equations of Balance

Due to range limitations of the internal mica capacitors of the Type-12 bridge, four balances are needed to measure unknown capacitors larger than 0.5 µF, and measurements are performed at frequencies only up to 1 kHz. The dummy capacitor box, $\mathbf{C_b}$, is connected across the arm CD and its value is set to either 0 or the nominal value of the unknown capacitor, $\mathbf{C_x}$, as shown in Fig. 10. Using $C_x = 1$ µF as an example, the measurement sequence and equations of balance in accordance to Fig.10 are described as follows:

1) Connect $\mathbf{C_x}$ across arm AD, set the mica dials to zero, and the value of C_b to 1 µF. Use the air dials to obtain a balance, and record the reading as C_{r1}. The equation of balance is:

$$R_1 (C_{a1} + C_x) = R_2 (C_{c1} + C_b). \tag{76}$$

2) Disconnect $\mathbf{C_x}$, and leave C_b at 1 µF. Use both the air and mica dials to obtain a balance, and record the readings as C_{r2} and C_{m2}, respectively. The equation of balance is:

$$R_1 (C_{a2} + C_{m2}) = R_2 (C_{c2} + C_b). \tag{77}$$

3) Connect $\mathbf{C_x}$ across arm CD, and set C_b to zero. Use both the air and mica dials to obtain a balance, and record the readings as C_{r3} and C_{m3}, respectively. The equation of balance is:

$$R_1 (C_{a3} + C_{m3}) = R_2 (C_{c3} + C_x). \tag{78}$$

4) Disconnect $\mathbf{C_x}$, set both mica dials and C_b to zero. Use the air dials to obtain a balance, and record the reading as C_{r4}. The equation of balance is:

$$R_1 C_{a4} = R_2 C_{c4}. \tag{79}$$

The numerical subscripts of Eqs. (76) to (79) indicate the values of mica and air capacitors in the corresponding measurements.

The value of the unknown capacitor, $\mathbf{C_x}$ can be expressed as:

$$C_x = [\,R_1 / (R_1 + R_2\,)]\,(\,C_{m2} + C_{m3}\,) + (½)\,[(\,C_{r2} - C_{r1}\,) + (\,C_{r3} - C_{r4}\,)], \tag{80}$$

where C_{r1} to C_{r4} and C_{m2} to C_{m3} are air and mica dial readings, respectively, corresponding to each measurement.

The ratio resistors are always set to be equal ($R_1 = R_2$), so Eq. (80) becomes:

$$C_x = (½)\,(\,C_{m2} + C_{m3}\,) + (½)\,[(\,C_{r2} - C_{r1}\,) + (\,C_{r3} - C_{r4}\,)], \tag{81}$$

which is the equation used to calculate the value of the unknown capacitor.

4.3.5.2 Analysis

Rewrite Eq. (80) as:

$$C_x = \{(R_1/R_2) / [1 + (R_1/R_2\,)]\}(\,C_{m2} + C_{m3}\,) + (½\,)\,[(\,C_{r2} - C_{r1}\,) + (\,C_{r3} - C_{r4}\,)]. \tag{82}$$

The relationship among the variances of the terms of Eq. (82) can be estimated by:

$$\begin{aligned} \mathrm{Var}(C_x) = {} & [1 + (\,R_1 / R_2\,)]^{-4}\;(C_{m2} + C_{m3}\,)^2\, s_{ra}^{\;2} \\ & + \{(\,R_1 / R_2\,) / [1 + (\,R_1 / R_2\,)]\}^2\,(\,s_{m2}^{\;2} + s_{m3}^{\;2}\,) \\ & + (½)^2\,(\,s_{rc1}^{\;2} + s_{rc2}^{\;2} + s_{rc3}^{\;2} + s_{rc4}^{\;2}\,), \end{aligned} \tag{83}$$

where s_{ra}, s_{m2}, s_{m3}, s_{rc1}, s_{rc2}, s_{rc3} and s_{rc4} are the standard uncertainties in the ratio (contributed by the resistors), the mica capacitor dials, and the air capacitor dials of the four measurements, respectively.

According to Eq. (81), the first term of Eq. (83) can be expressed as:

$$(C_{m2} + C_{m3}\,)^2\, s_{ra}^{\;2} \;\le\; (2C_x\,)^2\, s_{ra}^{\;2}, \tag{84}$$

By substituting Eq. (84) and ($R_1 = R_2$) into Eq. (83), it becomes:

$$\mathrm{Var}(C_x) \;\le\; [(½)\,C_x\, s_{ra}\,]^2 + [(½)\,s_m]^2 \;+ [(½)\,s_{rc}\,]^2\,, \tag{85}$$

where $s_m^{\;2} = s_{m2}^{\;2} + s_{m3}^{\;2}$ and $s_{rc}^{\;2} = s_{rc1}^{\;2} + s_{rc2}^{\;2} + s_{rc3}^{\;2} + s_{rc4}^{\;2}$.

In Eq. (85), the first term represents uncertainties in the ratio; the second and third terms represent uncertainties in the mica and air dial readings, respectively.

As in the three-terminal high-capacitance value measurements, the uncertainty contributed by the ratio resistors, $[(½)\,C_x\, s_{ra}]$, is also 9 ppm. (See Table 15b). According to Eq. (81), two balances using the internal mica capacitors and four balances using the internal air capacitors are required to measure the unknown capacitor of value equal to 1 μF. The combined uncertainty in the mica capacitor dial readings, [(½) s_{m}], is estimated to be less than 16 ppm. The uncertainty in the air capacitance dial readings, [(½) s_{rc}], is estimated to be 5 ppm. (See the last column of Table 15b).

For C_x = 1 μF, the uncertainty due to the external bridge components is estimated to be 5 ppm at frequencies of 100 Hz and 1 kHz.

The Type-B standard uncertainties for measurements of three-terminal mica capacitors of value equal to 1 μF, as well as their individual uncertainty components, at both 100 Hz and 1 kHz, are also included in Table 15b.

4.4 Type A Standard Uncertainty of Capacitance Measurements

The evaluation of the Type A standard uncertainty for the Type-12 system in making capacitance measurements is based on the repeatability and the effect of temperature variations on unknown capacitors during measurements. Also involved are the stabilities of the NIST standards that are used with an impedance meter as a comparator. An uncertainty analysis of each component that contributes to the Type A standard uncertainty for both two- and three-terminal measurements is given in the following sections.

4.4.1 Repeatability of Measurements

The procedure for using the Type-12 bridge to calibrate mica capacitors at NIST is to perform two complete measurements on the unknown capacitor and to report the average of these two results as the calibrated value. The range of these two measured values is compared with a given "limit" to ensure that it is within this limit. In the case where the range is over the limit, additional measurements are required to observe the variability of the unknown and to obtain an estimate of the component of uncertainty due to repeatability. From NIST historical data of a large population of standards, the range is 60 ppm, except for capacitors of 0.001 μF, for which the range is 100 ppm. These limits correspond to the maximum bounds of error in repeatability of ±30 ppm for all capacitors except the 0.001 μF capacitors where the limits are ±50 ppm. According to Eq. (4),

$$s_{rm} = 17.3 \text{ ppm} \le 20 \text{ ppm}, \quad (\text{for } C_x > 0.001\ \mu\text{F}),$$

$$s_{rm} = 28.9 \text{ ppm} \le 30 \text{ ppm}, \quad (\text{for } C_x = 0.001\ \mu\text{F}), \tag{86}$$

where s_{rm} is the standard uncertainty due to repeatability of results of measurements of capacitance. The values of s_{rm} are included in Tables 16a and 16b.

4.4.2 Effect of Temperature Variations on Unknown Capacitors

As stated previously, the temperature coefficients of air and mica dielectric capacitors are approximately 10 ppm/°C and 45 ppm/°C, respectively, and the temperature of the ICL is always maintained at 23 °C ± 1 °C. Uncertainties in unknown capacitors due to temperature changes are dependent on the length of time needed to perform measurements and to connect additional components. If an external decade capacitance box is needed (such as in three-terminal calibrations of capacitors with values from 0.01 μF and up), the temperature coefficient of the box will also contribute additional uncertainties in measurements. The maximum bounds of error due to temperature changes are equal to ±TC*(ΔT). According to Eq. (4),

$$s_{tm} = TC^*(\Delta T) / \sqrt{3} \ \text{ppm}, \tag{87}$$

where s_{tm} is the standard uncertainty component owing to the effect of temperature changes in the ICL on the values of the unknown capacitor, TC is the temperature coefficient (in ppm/°C), and ΔT is the maximum change in temperature (in °C) during the measurement.

In two-terminal measurements, it takes less than 10 min to complete the measurement on one unknown capacitor. The maximum change in temperature during measurement is 0.15 °C, which corresponds to an uncertainty, s_{tm}, of 4 ppm in capacitance, as shown in Table 16a.

The time required to complete three-terminal measurements on one unknown capacitor is less than 20 min, which corresponds to a maximum change in temperature of less than 0.3 °C, and introduces an uncertainty, s_{tm}, of 10 ppm in the low-value unknown capacitors ($\boldsymbol{C}_{\mathbf{x}} \leq 0.005$ μF). For 0.005 μF < $\boldsymbol{C}_{\mathbf{x}} \leq 0.5$ μF, the values of a capacitance box are needed for balances. Although the value of the capacitance box does not enter into the calculation of unknown capacitors, the variation in its capacitance values during measurements is reflected in the balance readings and the final value of the unknown capacitor, $\boldsymbol{C}_{\mathbf{x}}$, as shown in Eqs. (68) and (69). The combined uncertainty due to temperature changes that occur during measurement, s_{tm}, is estimated to be 15 ppm, which is the RSS of uncertainties in the unknown capacitor and in the capacitance box. For $\boldsymbol{C}_{\mathbf{x}} > 0.5$ μF, it may take as long as 30 min to complete one measurement, which corresponds to a maximum change in temperature up to 0.4 °C. The combined uncertainty due to temperature changes that occur during measurement, s_{tm}, is estimated to be 20 ppm, which is the RSS of uncertainties for the unknown capacitor and for the capacitance box. The values of s_{tm} are included in Table 16b.

4.4.3 Stabilities of NIST Standards

Since 1993, the calibration procedures for measuring mica capacitors at 1 kHz at NIST have been modified by using commercial impedance meters to compare a customer's standard with a NIST standard of the same type over a short time. Then, the value for a customer's standard is calculated from the predicted value of the NIST standard by using the "substitution method". This method is similar to the procedures used for inductance standards calibration, and is described in detail in [15]. The Type-12 bridge is used to measure the NIST standards (for updating data bases from which predicted values of NIST standards are derived), and to calibrate customers' standards at frequencies other than 1 kHz. From three to five years of data on NIST standards at 1 kHz, the standard uncertainties of the predicted values of both two- and three-terminal measurements of NIST standards, s_{pd}, are estimated to be between 10 ppm and 25 ppm, which are included in the Tables 16a and 16b.

Similar measurements have also been performed on four NIST standards at frequencies of 100 Hz and 10 kHz, and standard uncertainties of their predicted values are also given in the Tables 16a and 16b. The Type-12 bridge is employed to perform measurements on mica capacitors with nominal values other than these four values at frequencies other than 1 kHz.

Tables 16a and 16b provide summaries of the uncertainty components of Type A evaluation of standard uncertainties for the Type-12 bridge in measuring two-terminal and three-terminal mica dielectric capacitors, respectively, at frequencies of 100 Hz, 1 kHz, and 10 kHz. The Type A standard uncertainties for capacitance measurements using the Type-12 system are defined as the RSS of these components (s_{rm}, s_{tm}, and s_{pd}), and also are included in Tables 16a and 16b.

4.5 Expanded Uncertainty for Capacitance Measurement Using the Type-12 System

The expanded uncertainty for each nominal value of mica capacitor is calculated from both Type A and Type B components of uncertainty with a coverage factor of $k = 2$ [2]. Tables 17a and 17b give the expanded and assigned total uncertainties for two-terminal and three-terminal capacitance measurements, respectively, using the Type-12 system.

4.6 Type B Standard Uncertainty of Conductance Measurements

The Type-12 bridge, which has a conductance standard as well as mica and air capacitance standards, was designed to be direct-reading for capacitance and conductance. The analysis of the uncertainties in conductance measurements is based on considerations taken into account by the designers of the Type-12 bridge [14].

During measurements, both capacitance and conductance dials are adjusted to obtain a balance from which to determine the capacitance and conductance values of the unknown capacitor. As shown in Fig. 5a, the conductance standard, $\mathbf{G_s}$, of the Type-12 bridge consists of two decades with ten 100 µS steps, and ten 10 µS steps, plus a continuously variable standard with a range of -1 µS to 11 µS. Similar to the internal air capacitors, $\mathbf{G_s}$ is a differential-type standard, independently connected to arms AD and CD such that the sum of the conductance in both arms is a constant. There is a divider resistor, $\mathbf{K_g}$, which serves as a range shifter, so that the reading of the conductance standard can be divided by 1, 10, 100, 1000, or 10 000. Specifications and operational procedures for the conductance standard of the Type-12 bridge are described in detail by Wilhelm [14]. Figure 11 is a simplified diagram of the Type-12 bridge including the conductance components in each arm, where (G_a/K_g) and (G_c/K_g) are effective conductance adjustments to arm AD and arm CD, respectively, and the quantity of $(G_a + G_c)$ is equal to a constant. G_s provides readings in both decades and the variable dial to cover the required range and keeps the sum of G_a and G_c a constant. Equations used to calculate the conductance of unknown capacitors are similar to those used to calculate capacitance with C_m, C_{r1}, and C_{r2} being replaced by G_m, (G_{r1}/K_g), and (G_{r2}/K_g), respectively. For example, in the two- terminal high-capacitance measurements, from Eq. (60), the conductance value, G_x, of the unknown capacitor $\mathbf{C_x}$, is calculated from the two balances as:

$$G_x = G_m + (G_{r1} - G_{r2})/K_g \,, \tag{86}$$

where G_m is the conductance of the internal mica capacitors with $\mathbf{C_x}$ connected to arm CD,
G_{r1} is the conductance dial readings without connecting $\mathbf{C_x}$,
G_{r2} is the conductance dial readings with C_x connected to arm CD, and
K_g is the divider setting.

For the cases where mica capacitance dials are not used to perform conductance measurements, the corresponding term of G_m is equal to zero.

In general, the Type-12 bridge is used for high-Q capacitance measurements, and conductance is the minor component. Therefore, the uncertainty analysis of conductance measurement for each nominal value of mica capacitor is based on a general measurement procedure for both two- and three-terminal measurements at frequencies of 1 kHz, 100 Hz, and 10 kHz. There are five components included in the Type B evaluation of standard uncertainty of conductance measurements. These are discussed in the following sections.

4.6.1 Conductance Standard

According to [13], the conductance standard, which was devised by Young [16], has a deviation of ±0.2 % from its nominal value. Since there are two balances required for most measurements except for an unknown $\boldsymbol{C_x} = 1\ \mu\text{F}$, which required four balances, the maximum bounds of error due to the limit of accuracy of the conductance standard are estimated to be:

$$\pm \boldsymbol{e_{gs}} = \pm 0.002\ (2)^{(1/2)}\ G_s \quad \mu\text{S} \quad (\text{for } \boldsymbol{C_x} < 1\ \mu\text{F}), \text{ and}$$

$$\pm \boldsymbol{e_{gs}} = \pm 0.002\ (4)^{(1/2)}\ G_s \quad \mu\text{S} \quad (\text{for } \boldsymbol{C_x} = 1\ \mu\text{F}), \tag{87}$$

where $\pm \boldsymbol{e_{gs}}$ are the maximum bounds of error in conductance measurements due to the limit of accuracy in the conductance standard, G_s (in μS). Accordingly, the uncertainties in conductance standards are expressed as:

$$s_{gs} = 0.0016\ G_s \quad \mu\text{S} \quad (\text{for } \boldsymbol{C_x} < 1\ \mu\text{F}), \text{ and}$$

$$s_{gs} = 0.0023\ G_s \quad \mu\text{S} \quad (\text{for } \boldsymbol{C_x} = 1\ \mu\text{F}), \tag{88}$$

where s_{gs} are the standard uncertainties due to the limit of accuracy in the conductance standard, G_s (in μS).

According to the conductance values of the check standards, the values of s_{gs} for each nominal value of capacitor at frequencies of 100 Hz, 1 kHz, and 10 kHz in the two-terminal and three-terminal configurations are estimated and included in Tables 18a and 18b, respectively.

4.6.2 Contact Resistance

The contact resistance in the capacitance standard decade switches was estimated to be 0.005 Ω [14].

This resistance is considered as being in series with the internal capacitance of the decade switches. Since there are two or more balances required for each measurement, and results are calculated from the difference of these readings, the maximum bounds of error for each nominal value of $\boldsymbol{C}_{\mathbf{x}}$ are estimated from the difference of the contact resistance of each balance as:

$$\pm e_{\mathbf{gr}} = \pm\omega^2 \boldsymbol{C}_{\mathbf{x}}^{\;2} (R_{\mathrm{r1}} - R_{\mathrm{r2}}), \tag{89}$$

where $\pm e_{\mathbf{gr}}$ are the maximum bounds of error due to contact resistance, and R_{r1} and R_{r2} are contact resistance in the capacitance decade switches at the first and second balances, respectively.

Assuming the differences between R_{r1} and R_{r2} are less than 0.002 Ω, Eq. (89) can be expressed as:

$$\pm e_{\mathbf{gr}} = \pm 0.08 f^2 \boldsymbol{C}_{\mathbf{x}}^{\;2}. \tag{90}$$

Accordingly, the standard uncertainty in contact resistance, s_{gr}, is estimated as:

$$s_{\mathrm{gr}} = 0.046 f^2 \boldsymbol{C}_{\mathbf{x}}^{\;2}. \tag{91}$$

The values of s_{gr} for each nominal value of capacitor at frequencies of 100 Hz, 1 kHz, and 10 kHz in the two-terminal and three-terminal configurations are included in Tables 18a and 18b, respectively.

4.6.3 Conductance Potentiometer

According to [14], the limited resolution of the conductance potentiometer has an error in reading the continuous adjusted dial of ±0.1 uS, plus ±0.1 μS in calibration, all divided by the conductance divider $\boldsymbol{K}_{\mathbf{g}}$. Accordingly, the standard uncertainty in conductance measurements as well as in conductance calibrations due to the limited resolution of the conductance potentiometer is estimated to be 0.058 μS. When two (or four) balances are needed for routine measurements the combined uncertainty, defined as the RSS of the above three (or five) terms, (two (or four) in measurements and one in calibration, all are 0.058 μS), is expressed as:

$$s_{\mathrm{gp}} = 0.10 / K_{\mathrm{g}} \quad \mu\mathrm{S}, \qquad (\text{for } \boldsymbol{C}_{\mathbf{x}} < 1\ \mu\mathrm{F}), \quad \text{and}$$

$$s_{\mathrm{gp}} = 0.13 / K_{\mathrm{g}} \quad \mu\mathrm{S}, \qquad (\text{for } \boldsymbol{C}_{\mathbf{x}} = 1\ \mu\mathrm{F}), \tag{92}$$

where s_{gp} is the standard uncertainty in conductance potentiometer, due to its limited resolution, and K_{g} is the setting of the conductance divider.

According to the conductance divider, $\boldsymbol{K}_{\mathbf{g}}$, being used in measurements, the values of s_{gp} for each nominal value of capacitors at frequencies of 100 Hz, 1 kHz, and 10 kHz in the two-terminal and three-terminal configurations are estimated and included in Tables 18a and 18b, respectively.

4.6.4 Residual Conductance

According to [14], the maximum bounds of error due to the limit of resolution in residual conductance in arms AD and CD are functions of frequency and given as:

$$\pm e_{gc} = \pm(0.02) f / K_g \quad \mu\text{S}, \tag{93}$$

where $\pm e_{gc}$ are the maximum bounds of error due to the residual conductance. Accordingly,

$$s_{gc} = (0.0115) f / K_g \quad \mu\text{S}, \tag{94}$$

where s_{gc} is the standard uncertainty due to the residual conductance,
f is the applied frequency in kHz, and
K_g is the setting of the conductance divider.

The values of s_{gc} for each nominal value of capacitor at frequencies of 100 Hz, 1 kHz, and 10 kHz in the two-terminal and three-terminal configurations are included in Tables 18a and 18b, respectively.

4.6.5 Dial Corrections

Data and analyses of conductance corrections of internal mica dial capacitors for the Type-12 bridge at frequencies of 100 Hz, 1 kHz, and 10 kHz are given in Appendix C. As discussed in section 4.2.2, the standard uncertainty of the dial corrections is estimated from the predicted values of the dials being used during measurements, depending on the nominal value of the capacitor being measured. The standard uncertainty in conductance corrections of the internal mica capacitors, s_{gm}, for the two-terminal and three-terminal measurements of capacitors $C_x \geq 0.01$ µF at frequencies of 100 Hz, 1 kHz, and 10 kHz are estimated and given in Table 18a and 18b, respectively.

The Type B standard uncertainties for the Type-12 bridge measurement of conductance at frequencies of 1 kHz, 100 Hz, and 10 kHz, defined as the RSS of the above uncertainties, are also given in Tables 18a and 18b.

4.7 Type A Standard Uncertainty of Conductance Measurements

The evaluation of the Type A standard uncertainty for the Type-12 system in making conductance measurements is similar to that in making capacitance measurements. It is based on the repeatability in measurement results, the variation of conductance corrections of the capacitance dials during measurements, and the stability of the NIST conductance standards used with an impedance meter as a comparator. An analysis of each term is given in the following sections.

4.7.1 Repeatability of Measurements

As pointed out in section 4.4.1 for repeatability of capacitance measurements, the range of a given "limit" for conductance measurements is also contained in NIST internal documents for each nominal value of capacitor at each frequency. Accordingly, the maximum bounds of error of individual capacitor are obtained, and the standard uncertainty in conductance is estimated as:

$$s_{grm} = (\text{maximum bounds of error}) / \sqrt{3}\ \ \mu\text{S}, \tag{95}$$

where s_{grm} is the standard uncertainty due to repeatability of results of measurements of conductance and is given in Table 19a and 19b.

4.7.2 Stabilities of NIST Standards

Using the same method as in section 4.4.4, conductance values of unknown capacitors measured at 100 Hz, 1 kHz, and 10 kHz can be calculated by utilizing the "substitution method" [15] from data obtained by employing an impedance meter as a comparator. The uncertainties of the predicted values of conductance in unknown capacitors at a frequency of 100 Hz, 1 kHz, and 10 kHz, s_{gpd} are given in Tables 19a and 19b as well. The "substitution method" is not used on some capacitors at frequencies other than 1 kHz; therefore, these uncertainties are not applied for those frequencies.

The Type A standard uncertainty of conductance measurements is defined as the RSS of s_{grm} and s_{gpd}, and these are all summarized in Table 19a and 19b.

4.8 Expanded Uncertainty for Conductance Measurement Using the Type-12 System

The expanded uncertainties of conductance measurements (for each nominal value of mica capacitor) are calculated from both Type A and Type B standard uncertainties with a coverage factor of $k = 2$ [2], and the results are provided in Table 19a and 19b for the two-terminal and three-terminal configurations, respectively.

5. CONCLUSION

The revised uncertainties of NIST capacitance calibrations have been established, according to NIST's policy on expression of uncertainty [2] and on the modification of measurement procedures in Impedance Calibration Laboratory. Numerical results are also included in the NIST Calibration Services Users Guide (SP 250, 1998 edition) [17].

6. ACKNOWLEDGMENTS

The author would like to acknowledge the members of the MCOM Subcommittee for Capacitance Calibration Services; Carroll Croarkin (Chairman), Barry Bell, Norman Belecki, Andrew Koffman, and John Mayo-Wells for their valuable suggestions and contributions of the analysis. Special thanks to Robert Palm for his support in the graphs of the manuscript.

7. REFERENCES

[1] R. D. Cutkosky, "Capacitance Bridge -- NBS Type 2," National Bureau of Standards Report 7103, March, 1963.

[2] B. N. Taylor and C. E. Kuyatt, "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," Natl. Inst. Stand. Technol. Technical Note 1297, September, 1994.

[3] International Bureau of Weights and Measures (BIPM) Working Group on the Statement of Uncertainties, Metrologia, vol. 17, pp. 73-74, 1981.

[4] Y. M. Chang and S. B. Tillett, "NIST Calibration Service for Capacitance Standards at Low Frequencies," Natl. Inst. Stand. Technol. Special Publication 250 Series (NIST SP250-47), April, 1998.

[5] R. D. Cutkosky and J. Q. Shields, "The Precision Measurement of Transformer Ratios," IRE Trans. Instrum., vol. I-9, pp. 243-250, September, 1960.

[6] R. D. Cutkosky and L. H. Lee, "Improved Ten-Picofarad Fused Silica Dielectric Capacitor," J. Res. Nat. Bur. Stand., vol. 69C (Eng. and Instr.) no. 3, pp. 173-179, July-September, 1965.

[7] "A Highly Stable Reference Standard Capacitor, "The General Radio Experimenter, vol. 37, no. 8, August, 1963.

[8] J. Q. Shields, "Voltage Dependence of Precision Air Capacitors," J. Res. Nat. Bur. Stand., vol. 69C (Eng. and Instr.), no. 4, pp. 265-274, Oct.-Dec. 1965.

[9] Y. M. Chang and S. B. Tillett, "Recent Developments in the Capacitance Calibration Services at the National Institute of Standards and Technology," Proceedings of NCSL, July, 1998.

[10] "General Radio Catalog 73" General Radio Company, 1973.

[11] "Capacitance Standards with Precision Connectors," The General Radio Experimenter, vol. 41, no. 9, September, 1967.

[12] "More Coaxial Capacitance Standards," The General Radio Experimenter, vol. 42, no. 5, May, 1968.

[13] W. D. Voelker, "An Improved Capacitance Bridge for Precision Measurements," Bell Labs Record, vol. 20, pp. 133-137, Jan., 1942.

[14] H. T. Wilhelm, "The Type-12 Capacitance Bridge," Bell Labs Memorandum, MM 70-4735-9, Case 70052-213, File 37954-15, May, 1970.

[15] Y. M. Chang and S. B. Tillett, "Calibration Procedures for Inductance Standards using a Commercial Impedance Meter as a Comparator," Natl. Inst. Stand. Internal Report (NISTIR) 4466, Nov., 1990.

[16] C. H. Young, Conductance Standard, U. S. Patent 2-326-274.

[17] "NIST Calibration Services Users Guide," Natl. Inst. Stand. Technol. Special Publication 250, 1998 edition, U. S. Government Printing Office, Washington, DC 20402.

Table 1. Dial Corrections of the Type-2 Bridge

[All values are in (10^{-6} pF), except nominal values of dials]

Dial No.	1	2	3	4	5	6	7	8
Nominal Value (pF)	100	10	1	0.1	0.01	0.001	0.0001	0.00001
4/25/94	5160.1	-890.59	3.741	-11.426	1.657	0.566	0.357	0.336
5/27/94	5169.1	-870.49	7.951	-11.405	1.560	0.556	0.356	0.336
6/1/94	5903.3	-756.47	13.353	-10.365	1.664	0.566	0.357	0.336
8/4/94	7587.3	-478.27	34.973	-7.803	1.900	0.592	0.359	0.436
8/19/94	8078.1	-372.19	36.181	-6.982	2.002	0.600	0.360	0.436
9/16/94	6801.0	-634.24	39.876	-8.712	1.829	0.583	0.358	0.336
10/31/94	7270.2	-517.18	36.682	-8.132	1.887	0.589	0.459	0.446
11/28/94	6932.2	-549.18	22.982	-8.702	1.830	0.583	0.358	0.336
12/30/94	7051.8	-620.02	23.898	-8.910	1.929	0.591	0.459	0.446
1/17/95	6209.7	-733.73	11.127	-10.087	1.691	0.569	0.457	0.446
2/27/95	6769.5	-692.15	14.485	-9.752	1.725	0.572	0.457	0.446
3/17/95	6174.9	-738.51	7.149	-9.985	1.801	0.680	0.468	0.447
4/28/95	6117.6	-742.64	7.235	-10.276	1.672	0.567	0.357	0.436
5/19/95	6442.8	-680.12	7.688	-9.731	1.727	0.573	0.357	0.436
Mean Value	6547.7	-662.56	19.094	-9.448	1.777	0.585	0.394	0.404
Standard Deviation	839.0	146.07	13.052	1.302	0.126	0.030	0.051	0.053

Table 2. Standard Uncertainties in Dial Corrections of the Type-2 Bridge

[All values are in (10^{-6} pF), except nominal values of dials]

Dial No.	3		4	5	6	7	8		2	1
Nominal Value C_i (pF)	1		0.1	0.01	0.001	1E-04	1E-05		10	100
Mean Value of Dial Correction	19.094		-9.448	1.777	0.585	0.394	0.404		-662.56	6547.69
Standard Dev.	13.052		1.302	0.126	0.030	0.051	0.053		146.07	839.0
No. of Dials Used	3		3	3	1	1	1		4	5
Components of Standard Uncertainty										
$(0.1)s_{Ra}$	0.0043	$(0.1)s_{Ra}$	4E-04	4E-05	4E-06	4E-07	4E-08	$(10)s_{Ra}$	4.3	43
$[\Sigma(r_k^2 s_k^2)]^{(1/2)}$	0.08	$[\Sigma(r_k^2 s_k^2)]^{(1/2)}$	0.08	0.08	0.05	0.05	0.05	$10[\Sigma(r_k^2 s_k^2)]^{(1/2)}$	1.48	13.1
s_1	0.2	s_1	0.2	0.2	0.2	0.2	0.2	s_1	0.2	0.2
$(0.1)s_s$	0.75	$(0.1)s_{(i-1)}$	0.100	0.025	0.023	0.022	0.022	$(10)s_{(i+1)}$	10	130
SSQ	0.609		0.056	0.047	0.043	0.043	0.043		120.72	18920.7
RSS	0.780		0.237	0.217	0.207	0.207	0.207		10.99	137.6
Assigned Uncertainty (s_i)	1		0.25	0.23	0.22	0.22	0.22		13	150

Remarks :

s_{Ra}	:	Standard uncertainty of the 1:10 ratio
$[\Sigma(r_k^2 s_k^2)]^{(1/2)}$	:	Standard Deviation of the dial corrections for the k^{th} dial of reading r_k
s_1	:	Standard uncertainty due to the interpolation in last dial
s_s	:	Uncertainty of reference standard

Table 3. Combined Uncertainties in Dial Corrections Using the Type-2 Bridge to Measure Capacitors of Nominal Values of 1 pF, 10 pF, 100 pF, and 1000 pF

Nominal Value of Capacitor (pF)	1	10	100	1000
No. of Dials Used Routinely	4	5	5	6
Assigned Combined Uncertainty				
s_{cd} (10^{-6} pF) *	0.45	0.51	0.51	1.12
s_{cd} (ppm)	0.45000	0.05100	0.00510	0.00112

Remark :

* From number of dials used to perform measurements, as given in the following table:

No.of dials used	1	2	3	4	5	6	7	8
Dial No.	8	7 & 8	6 to 8	5 to 8	4 to 8	3 to 8	2 to 8	1 to 8
Assigned Combined Uncertainty s_{cd} (10^{-6} pF)	0.22	0.31	0.38	0.45	0.51	1.12	13.05	150.57

Table 4. Uncertainties in Lead Impedance Using the Type-2 Bridge to Measure Capacitors at Frequencies of 100 Hz, 400 Hz, and 1 kHz

Frequency	1 kHz	400 Hz	100 Hz
Nominal Value of Capacitor (pF)	$\Delta C / C$ (ppm)	$\Delta C / C$ (ppm)	$\Delta C / C$ (ppm)
1	<< 0.0002	<< 0.0002	<< 0.0002
10	0.0002	<< 0.0002	<< 0.0002
100	0.00197	0.00032	<< 0.0002
1000	0.0197	0.0032	0.0002
10 000	0.197	0.032	0.00197

Table 5. Summary of Components of Type B Standard Uncertainties Using the Type-2 Bridge to Measure Fused-Silica and Nitrogen Dielectric Capacitors at 1 kHz

(All values are in ppm - except nominal values of standards)

Reference STD C_s (pF) Serial No. Dielectric Temperature Control	10 (#125) Fused-Silica (Oil Bath) (Prim.Cap.Lab)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#102) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#517) Nitrogen	1000 (#1717) Nitrogen
Unknown STD C_x (pF) Serial No.(or Customer) Dielectric Temperature Control	10 (#121) Fused-Silica (Oil Bath) (Imp.Cal.Lab)	1 (AHcap-CTM) Fused-Silica (Air Bath)	10 (#102-CTM) Fused-Silica (Air Bath)	100 (#131-CTM) Fused-Silica (Air Bath)	10 (#871-CTM) Nitrogen	100 (#493-#517) Nitrogen	1000 (#1717-#1865) Nitrogen	100 (CTM) Nitrogen	1000 (CTM) Nitrogen
Ratio (C_x/C_s)	(1:1)	(1:10)	(1:1)	(10:1)	(1:1)	(1:1)	(10:1)	(1:1)	(1:1)
Components of Standard Uncertainties									
Transfermer Ratio (s_{Ra})	0.005	0.043	0.005	0.043	0.005	0.005	0.043	0.005	0.005
Dial Correction (s_{cd})	0.051	0.450	0.051	0.051	0.051	0.005	0.005	0.005	0.001
External Bridge Components:									
Voltage Dependence (s_{va})	0.004	0.005	0.005	0.005	0.001	0.001	0.001	0.001	0.001
Temperature Correction (s_{tc} or s_{tc1})	0.1	0.1	0.1	0.1	0.1	0.1	0.1	1	1
Frequency Dependence (s_f)	0.7	0	0	0	0	0	0	0	0
Lead Impedance ($\Delta C/C$)	0.0002	0.0002	0.0002	0.002	0.0002	0.002	0.02	0.002	0.02
Uncertainty of Ref Std (s_s)	0.05	0.72	0.72	0.72	0.737	0.737	0.737	0.901	0.901
Combined Uncertainty (s_{cb})	0.709	0.727	0.727	0.727	0.744	0.744	0.744	1.346	1.346
Type-B Standard Uncertainty (u_b)	0.711	0.856	0.729	0.730	0.745	0.744	0.745	1.346	1.346
Assigned u_b	0.72	0.9	0.73	0.73	0.75	0.75	0.75	1.35	1.35
Estimated Uncertainties in Unknown Standard			(#102) 0.1	(#131) 0.1		(#517) 0.5	(#1717) 0.5		

Table 6. Expanded and Assigned Uncertainties Using the Type-2 Bridge to Measure Fused-Silica and Nitrogen Dielectric Capacitors at 1 kHz

(All values are in ppm - except nominal values of standards)

Reference STD C_s (pF) Serial No. Dielectric Temperature Control	10 (#125) Fused-Silica (Oil Bath) (Prim. Std. Lab)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#102) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#517) Nitrogen	1000 (#1717) Nitrogen
Unknown STD C_x (pF) Serial No. (or Customer's STD) Dielectric Temperature Control	10 (#121) Fused-Silica (Oil Bath) (Imp. Cal. Lab)	1 (AHcap-CTM) Fused-Silica (Air Bath)	10 (#102-CTM) Fused-Silica (Air Bath)	100 (#131-CTM) Fused-Silica (Air Bath)	10 (#871-CTM) Nitrogen	100 (#493-#517) Nitrogen	1000 (#1717-#1865) Nitrogen	100 (CTM) Nitrogen	1000 (CTM) Nitrogen
Type-A Standard Uncertainty (u_a)									
From Measurements	0.05	0.4	0.15	0.15					
Limits of Variability					1.6	0.5	0.5	1.2	1.2
Between the Two Bridges					0.7			0.5	0.5
Type-B Standard Uncertainty (u_b)	0.72	0.9	0.73	0.73	0.75	0.75	0.75	1.35	1.35
Expanded Uncertainty (U)	1.4435	1.9698	1.4905	1.4905	3.8013	1.8028	1.8028	3.7483	3.7483
Assigned Total Uncertainty (U_t)	1.45	2	1.5	1.5	4	2	2	4	4

Table 7. Expanded and Assigned Uncertainties Using the Type-2 Bridge to Measure Fused-Silica and Nitrogen Dielectric Capacitors at 400 Hz

(All values are in ppm - except nominal values of standards)

Reference STD C_s (pF) Serial No. Dielectric Temperature Control	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#102) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#517) Nitrogen	1000 (#1717) Nitrogen
Unknown STD C_x (pF) Serial No.(or Customer's STD) Dielectric Temperature Control	1 (AHcap-CTM) Fused-Silica (Air Bath)	10 (#102-CTM) Fused-Silica (Air Bath)	100 (#131-CTM) Fused-Silica (Air Bath)	10 (#871-CTM) Nitrogen	100 (#493-#517) Nitrogen	1000 (#1717-#1865) Nitrogen	100 (CTM) Nitrogen	1000 (CTM) Nitrogen
Ratio (C_x/C_s)	(1:10)	(1:1)	(10:1)	(1:1)	(1:1)	(10:1)	(1:1)	(1:1)
Components of Standard Uncertainties								
Transfermer Ratio (s_{Ra})	0.043	0.005	0.043	0.005	0.005	0.043	0.005	0.005
Dial Correction (s_{cd})	0.450	0.051	0.051	0.051	0.005	0.005	0.005	0.001
External Bridge Components:								
Voltage Dependence (s_{va})	0.005	0.005	0.005	0.001	0.001	0.001	0.001	0.001
Temperature Correction (s_{tc} or s_{tc1})	0.06	0.06	0.06	0.06	0.06	0.06	1	1
Frequency Dependence (s_f)	0.9	0.9	0.9	0	0	0	0	0
Lead Impedance ($\Delta C/C$)	0	0	0.0003	0	0.0003	0.003	0.0003	0.003
Uncertainty of Ref Std (s_s)	0.72	0.72	0.72	1.204	1.204	1.204	1.346	1.346
Combined Uncertainty (s_{cb})	1.154	1.154	1.154	1.206	1.206	1.206	1.677	1.677
Type-B Standard Uncertainty (u_b)	1.240	1.155	1.156	1.207	1.206	1.206	1.677	1.677
Assigned u_b	1.25	1.2	1.2	1.25	1.25	1.25	1.7	1.7
Type-A Standard Uncertainty (u_a) From Measurements Limits of Variability	0.6	0.2	0.2	2	0.5	0.5	1.6	1.6
Expanded Uncertainty (U)	2.773	2.433	2.433	4.717	2.693	2.693	4.669	4.669
Assigned Total Uncertainty (U_t)	3	2.5	2.5	5	3	3	5	5

Table 8. Expanded and Assigned Uncertainties Using the Type-2 Bridge to Measure Fused-Silica and Nitrogen Dielectric Capacitors at 100 Hz

(All values are in ppm - except nominal values of standards)

Reference STD C_s (pF) Serial No. Dielectric Temperature Control	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#121) Fused-Silica (Oil Bath)	10 (#102) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#131) Fused-Silica (Air Bath)	100 (#517) Nitrogen	1000 (#1717) Nitrogen
Unknown STD C_x (pF) Serial No.(or Customer's STD) Dielectric Temperature Control	1 (AHcap-CTM) Fused-Silica (Air Bath)	10 (#102-CTM) Fused-Silica (Air Bath)	100 (#131-CTM) Fused-Silica (Air Bath)	10 (#871-CTM) Nitrogen	100 (#493-#517) Nitrogen	1000 (#1717-#1865) Nitrogen	100 (CTM) Nitrogen	1000 (CTM) Nitrogen
Ratio (C_x/C_s)	(1:10)	(1:1)	(10:1)	(1:1)	(1:1)	(10:1)	(1:1)	(1:1)
Components of Standard Uncertainties								
Transfermer Ratio (s_{Ra})	0.043	0.005	0.043	0.005	0.005	0.043	0.005	0.005
Dial Correction (s_{cd})	0.450	0.051	0.051	0.051	0.005	0.005	0.005	0.001
External Bridge Components:								
Voltage Dependence (s_{va})	0.005	0.005	0.005	0.001	0.001	0.001	0.001	0.001
Temperature Correction (s_{tc} or s_{tc1})	0.06	0.06	0.06	0.06	0.06	0.06	1	1
Frequency Dependence (s_f)	1.6	1.6	1.6	0	0	0	0	0
Lead Impedance ($\Delta C/C$)	0	0	0	0	0	0.0002	0	0.0002
Uncertainty of Ref Std (s_s)	0.72	0.72	0.72	1.803	1.803	1.803	1.916	1.916
Interpolation of Last Dial (s_{sl})	2	0.2	0.02	0.02	0.02	0.002	0.02	0.002
Combined Uncertainty (s_{cb})	2.661	1.767	1.756	1.804	1.804	1.804	2.162	2.162
Type-B Standard Uncertainty (u_b)	2.699	1.768	1.757	1.805	1.804	1.804	2.162	2.162
Assigned u_b	2.7	1.8	1.8	1.85	1.85	1.85	2.2	2.2
Type-A Standard Uncertainty (u_a) From Measurements Limits of Variability	0.8	0.2	0.2	2	0.6	0.6	1.8	1.8
Expanded Uncertainty (U)	5.632	3.622	3.622	5.449	3.890	3.890	5.685	5.685
Assigned Total Uncertainty (U_t)	6	4	4	6	4	4	6	6

Table 9. Expanded and Assigned Total Uncertainties Using the Type-2 Bridge to Measure Three-Terminal Air Capacitors at Frequencies of 400 Hz and 1 kHz

(All values are in ppm, except nominal values of capacitors)

Nominal Value of Capacitors (pF)	0.001	0.01	0.1	1 to 1000	10 000
Components of Standard Uncertainties					
Temperature	20	20	15	10	10
Relative Humidity	16	16	16	16	16
Hysterises	20	20	20	20	20
Voltage	5	5	5	5	5
Interpolation of Last Dial (2×10^{-7} pF)	200	20	2	0.2 to 0.0002	0.00002
Type B Standard Uncertainty (u_b)	202.68	38.48	30.17	27.95	27.95
Assigned u_b	210	40	32	30	30
Type A Standard Uncertainty (u_a)	950	90	30	30	60
Expanded Uncertainty (U)	1945.87	196.98	87.73	84.85	134.16
Assigned Total Uncertainty (U_t)	2000	200	100	100	150

Table 10. Expanded and Assigned Total Uncertainties Using the Type-2 Bridge to Measure Three-Terminal Air Capacitors at 100 Hz

(All values are in ppm, except nominal values of capacitors)

Nominal Value of Capacitors (pF)	0.01	0.1	1	10 to 1000
Components of Standard Uncertainties				
Temperature	20	15	10	10
Relative Humidity	16	16	16	16
Hysterises	20	20	20	20
Voltage	5	5	5	5
Frequency	1	1	1	1
Interpolation of Last Dial (2×10^{-6} pF)	200	20	2	0.2 to 0.002
Type B Standard Uncertainty (u_b)	202.69	36.15	28.04	27.97
Assigned u_b	220	40	30	30
Type A Standard Uncertainty (u_a)	600	100	30	30
Expanded Uncertainty (U)	1278.12	215.41	84.85	84.85
Assigned Total Uncertainty (U_t)	1300	230	100	100

Table 11. Expanded and Assigned Total Uncertainties Using the Type-2 Bridge to Measure Two-Terminal Coaxial Air Capacitors at 1 kHz

(All values are in ppm, except nominal values of capacitors)

Nominal Value of Capacitors (pF)	(Lower-Valued) 1	2	5	10	20	(Mid-Valued) 50 to 1000
Components of Standard Uncertainties						
Temperature	100	40	20	40	20	10
Relative Humidity	400	200	80	0	0	16
Hysterises	20	20	20	20	20	20
Voltage	5	5	5	5	5	5
Type B Standard Uncertainty (u_b)	412.83	205	85	45	28.72	27.95
Assigned u_b	415	205	85	45	30	28
Type A Standard Uncertainty (u_a)	35	35	35	20	20	10
Expanded Uncertainty (U)	832.95	415.93	183.85	98.49	72.11	59.46
Assigned Total Uncertainty (U_t)	840	420	200	100	75	60

Table 12. Expanded and Assigned Total Uncertainties of Using the Type-2 Bridge to Measure Two-Terminal Coaxial Mica Capacitors and Open-Circuit Terminations at 1 kHz

(All values are in ppm, except nominal values of capacitors)

	Coaxial Mica Capacitors		Open-Circuit Terminations	
Nominal Value of Capacitors (pF)	1000	5000 to 100 000	0.172	2.67
Components of Standard Uncertainties				
Temperature	5	15	150	150
Relative Humidity	16	16	400	100
Hysteresis	20	20	20	20
Voltage	2	2	5	5
Type B Standard Uncertainty (u_b)	26.17	29.75	427.70	181.45
Assigned u_b	27	30	450	200
Type A Standard Uncertainty (u_a)	10	10	200	60
Expanded Uncertainty (U)	57.58	63.25	984.89	417.61
Assigned Total Uncertainty (U_t)	60	65	1000	450

Table 13a. Summary of Components of Standard Uncertainties in Air Capacitor Dial Corrections for Two-Terminal Measurements

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Std Dev.of Pred.Values of Dial Correction (pF)	0.0601	0.0672	0.0759	0.0208	0.0208	0.0208	0.0208	0.0208	0.0276	0.0213
Std Dev.of Pred.Values of Dial Correction (ppm)	60.136	33.584	15.184	2.085	1.042	0.417	0.208	0.104	0.055	0.021
Components of Standard Uncertainties (ppm)										
Pred.Values of Dial Corr't (s_{dr})	60.136	33.584	15.184	2.085	1.042	0.417	0.208	0.104	0.055	0.021
Temperature Variation (s_{tp})	6	6	6	6	6	6	6	6	6	6
Dial Switching (s_{ds})	6	3	1.5	0.6	0.3	0.15	0.1	0.05	0.02	0.01
Transfer Standard (s_{tf})	6	6	6	6	6	6	6	6	6	6
RSS	61.027	34.769	17.459	8.758	8.554	8.497	8.488	8.486	8.485	8.485
Assigned Standard Uncertainties in Air Cap. Dial Corrections (s_{rc})	62	35	18	9	9	9	9	9	9	9

Table 13b. Summary of Components of Standard Uncertainties in Air Capacitor Dial Corrections for Three-Terminal Measurements

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Std Dev.of Pred.Values of Dial Correction (pF)	0.0607	0.0749	0.0636	0.0224	0.0241	0.0183	0.0266	0.0191	0.0264	0.0324
Std Dev.of Pred.Values of Dial Correction (ppm)	60.744	37.448	12.723	2.238	1.205	0.365	0.266	0.095	0.053	0.032
Components of Standard Uncertainties (ppm)										
Pred.Values of Dial Corr't (s_{dr})	60.744	37.448	12.723	2.238	1.205	0.365	0.266	0.095	0.053	0.032
Temperature Variation (s_{tp})	6	6	6	6	6	6	6	6	6	6
Dial Switching (s_{ds})	6	3	1.5	0.6	0.3	0.15	0.1	0.05	0.02	0.01
Transfer Standard (s_{tf})	6	6	6	6	6	6	6	6	6	6
RSS	61.627	38.515	15.366	8.796	8.576	8.494	8.490	8.486	8.485	8.485
Assigned Standard Uncertainties in Air Cap. Dial Corrections (s_{rc})	62	39	16	9	9	9	9	9	9	9

Table 14a. Summary of Components of Standard Uncertainties in Mica Capacitor Dial Corrections for Two-Terminal Measurements

Unknown Standard C_x (μF)	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties (ppm)							
(At all Frequencies)							
Temperature Variation (s_{mp})	26	26	26	26	26	26	26
Dial Switching (s_{ds})	0.6	0.3	0.15	0.1	0.05	0.02	0.01
Transfer Standard (s_{mf})	6	6	6	6	6	6	6
(At Frequency = 100 Hz)							
Std Dev.of Pred.Values of Dial Correction (pF)	.1585	.2331	.4412	1.0096	2.5727	7.7378	11.8816
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	15.850	11.655	8.824	10.096	12.864	15.476	11.882
RSS	31.042	29.119	28.105	28.530	29.622	30.846	29.209
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	32	30	29	29	30	31	30
(At Frequency = 1 kHz)							
Std Dev.of Pred.Values of Dial Correction (pF)	0.0702	0.1208	0.3019	0.6075	1.413	4.4166	6.5888
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	7.020	6.040	6.038	6.075	7.065	8.833	6.589
RSS	27.598	27.360	27.358	27.366	27.603	28.107	27.485
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	28	28	28	28	28	29	28
(At Frequency = 10 kHz)							
Std Dev.of Pred.Values of Dial Correction (pF)	0.0627	0.1087	0.275	0.4965	1.288	3.991	21.8635
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	6.270	5.435	5.500	4.965	6.440	7.982	21.864
RSS	27.417	27.233	27.245	27.142	27.450	27.852	34.497
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	28	28	28	28	28	28	35

Table 14b. Summary of Components of Standard Uncertainties in Mica Capacitor Dial Corrections for Three-Terminal Measurements

Unknown Standard C_x (μF)	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties (ppm)							
(At all Frequencies)							
Temperature Variation (s_{mp})	26	26	26	26	26	26	26
Dial Switching (s_{ds})	0.6	0.3	0.15	0.1	0.05	0.02	0.01
Transfer Standard (s_{mf})	6	6	6	6	6	6	6
(At Frequency = 100 Hz)							
Std Dev.of Pred.Values of Dial Correction (pF)	0.2331	0.3617	1.0096	2.5727	7.1997	11.8816	16.8030
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	23.310	18.085	20.192	25.727	35.999	23.763	16.803
RSS	35.436	32.236	33.463	37.066	44.810	35.731	31.533
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	36	33	34	38	45	36	32
(At Frequency = 1 kHz)							
Std Dev.of Pred.Values of Dial Correction (pF)	0.1208	0.2414	0.6075	1.4130	4.1541	6.5888	9.3179
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	12.080	12.070	12.150	14.130	20.771	13.178	9.318
RSS	29.297	29.288	29.320	30.194	33.814	29.760	28.263
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	30	30	30	31	34	30	29
(At Frequency = 10 kHz)							
Std Dev.of Pred.Values of Dial Correction (pF)	0.1087	0.2229	0.4965	1.2880	3.2637	21.8635	NA
Std Dev.of Pred.Values of Dial Correction (ppm) (s_{mr})	10.870	11.145	9.930	12.880	16.319	43.727	NA
RSS	28.819	28.919	28.472	29.629	31.278	51.225	NA
Assigned Standard Uncertainties in Mica Cap. Dial Corrections (s_m)	29	29	29	30	32	52	NA

Table 15a. Summary of Components of Type B Standard Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Two-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in ppm - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(At all Frequencies)										
Ratio Resistor Arms (s_{ra})	9	9	9	18	18	18	18	18	18	18
Air Capacitor Dials (s_{rc})	62	35	18	9	9	9	9	9	9	9
(Frequency = 100 Hz)										
Mica Capacitor Dials (s_m)	-	-	-	32	30	29	29	30	31	30
External Bridge Component (s_{br})	12	6	3	2	1	1	1	1	3	5
Type-B Standard Uncertainty (U_b)	63.79	36.63	20.35	37.85	36.14	35.31	35.31	36.14	37.08	36.47
Assigned U_b	65	40	25	40	40	40	40	40	40	40
(Frequency = 1 kHz)										
Mica Capacitor Dials (s_m)	-	-	-	28	28	28	28	28	29	28
External Bridge Component (s_{br})	12	6	3	2	1	1	1	1	3	5
Type-B Standard Uncertainty (U_b)	63.79	36.63	20.35	34.54	34.50	34.50	34.50	34.50	35.43	34.84
Assigned U_b	65	40	25	40	40	40	40	40	40	40
(Frequency = 10 kHz)										
Mica Capacitor Dials (s_m)	-	-	-	28	28	28	28	28	28	35
External Bridge Component (s_{br})	12	7	4	5	10	25	48	95	235	470
Type-B Standard Uncertainty (U_b)	63.79	36.81	20.52	34.84	35.90	42.59	59.10	101.06	237.52	471.73
Assigned U_b	65	40	25	40	40	45	65	105	240	475

Table 15b. Summary of Components of Type B Standard Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Three-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in ppm - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(At all Frequencies)										
Ratio Resistor Arms (s_{ra})	-	-	-	9	9	9	9	9	9	9
Air Capacitor Dials (s_{rc})	31	20	8	3	3	3	3	3	3	5
(Frequency = 100 Hz)										
Mica Capacitor Dials (s_m)	-	-	-	18	17	17	19	23	18	16
External Bridge Component (s_{br})	6	3	2	1	1	1	1	1	3	5
Type-B Standard Uncertainty (U_b)	31.58	20.22	8.25	20.37	19.49	19.49	21.26	24.90	20.57	19.67
Assigned U_b	35	25	15	25	25	25	25	25	25	25
(Frequency = 1 kHz)										
Mica Capacitor Dials (s_m)	-	-	-	15	15	15	16	17	15	15
External Bridge Component (s_{br})	6	3	2	1	1	1	1	1	3	5
Type-B Standard Uncertainty (U_b)	31.58	20.22	8.25	17.78	17.78	17.78	18.63	19.49	18.00	18.87
Assigned U_b	35	25	15	20	20	20	20	20	20	20
(Frequency = 10 kHz)										
Mica Capacitor Dials (s_m)	-	-	-	15	15	15	15	16	26	NA
External Bridge Component (s_{br})	6	4	3	5	10	25	48	95	235	NA
Type-B Standard Uncertainty (U_b)	31.58	20.40	8.54	18.44	20.37	30.66	51.18	96.80	236.62	NA
Assigned U_b	35	25	15	20	25	35	55	100	240	NA

Table 16a. Summary of Components of Type A Standard Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Two-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in ppm - except nominal values of unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(Frequency = 100 Hz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	20
Temperature Variation (s_{tm})	4	4	4	4	4	4	4	4	4	4
Predicted Values (s_{pd})	34	-	-	32	-	-	10	-	-	10
Type-A Standard Uncertainty (u_a)	45.52	20.40	20.40	37.95	20.40	20.40	22.72	20.40	20.40	22.72
Assigned u_a	46	25	25	38	25	25	25	25	25	25
(Frequency = 1 kHz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	20
Temperature Variation (s_{tm})	4	4	4	4	4	4	4	4	4	4
Predicted Values (s_{pd})	25	25	20	15	20	15	10	10	10	10
Type-A Standard Uncertainty (u_a)	39.26	32.26	28.57	25.32	28.57	25.32	22.72	22.72	22.72	22.72
Assigned u_a	40	35	30	30	30	30	25	25	25	25
(Frequency = 10 kHz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	20
Temperature Variation (s_{tm})	4	4	4	4	4	4	4	4	4	4
Predicted Values (s_{pd})	15	-	-	10	-	-	15	-	-	15
Type-A Standard Uncertainty (u_a)	33.78	20.40	20.40	22.72	20.40	20.40	25.32	20.40	20.40	25.32
Assigned u_a	35	25	25	25	25	25	28	25	25	28

Table 16b. Summary of Components of Type A Standard Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Three-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in ppm - except nominal values of unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(Frequency = 100 Hz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	20
Temperature Variation (s_{tm})	10	10	10	15	15	15	15	15	15	20
Predicted Values (s_{pd})	35	-	-	15	-	-	15	-	-	10
Type-A Standard Uncertainty (u_a)	47.17	22.36	22.36	29.15	25.00	25.00	29.15	25.00	25.00	30.00
Assigned u_a	48	25	25	30	25	25	30	25	25	30
(Frequency = 1 kHz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	20
Temperature Variation (s_{tm})	10	10	10	15	15	15	15	15	15	20
Predicted Values (s_{pd})	25	20	25	15	15	15	10	10	10	10
Type-A Standard Uncertainty (u_a)	40.31	30.00	33.54	29.15	29.15	29.15	26.93	26.93	26.93	30.00
Assigned u_a	45	30	45	30	30	30	30	30	30	30
(Frequency = 10 kHz)										
Repeatability (s_{rm})	30	20	20	20	20	20	20	20	20	NA
Temperature Variation (s_{tm})	10	10	10	15	15	15	15	15	15	NA
Predicted Values (s_{pd})	35	-	-	5	-	-	35	-	-	
Type-A Standard Uncertainty (u_a)	47.17	22.36	22.36	25.50	25.00	25.00	43.01	25.00	25.00	NA
Assigned u_a	48	25	25	30	25	25	45	25	25	NA

Table 17a. Summary of Expanded and Assigned Total Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Two-Terminal Mica Capacitors at 100 Hz, 1kHz, and 10 kHz

(All values are in ppm - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
(Frequency = 100 Hz)										
Type-A Standard Uncertainty (u_a)	46	25	25	38	25	25	25	25	25	25
Type-B Standard Uncertainty (u_b)	65	40	25	40	40	40	40	40	40	40
Expanded Uncertainty (U)	159.26	94.34	70.71	110.34	94.34	94.34	94.34	94.34	94.34	94.34
Assigned Total Uncertainty (U_t)	160	120	100	120	120	120	120	120	120	120
(Frequency = 1 kHz)										
Type-A Standard Uncertainty (u_a)	40	35	30	30	30	30	25	25	25	25
Type-B Standard Uncertainty (u_b)	65	40	25	40	40	40	40	40	40	40
Expanded Uncertainty (U)	152.64	106.30	78.10	100.00	100.00	100.00	94.34	94.34	94.34	94.34
Assigned Total Uncertainty (U_t)	160	120	100	120	120	120	120	120	120	120
(Frequency = 10 kHz)										
Type-A Standard Uncertainty (u_a)	35	25	25	25	25	25	28	25	25	28
Type-B Standard Uncertainty (u_b)	65	40	25	40	40	45	65	105	240	475
Expanded Uncertainty (U)	147.65	94.34	70.71	94.34	94.34	102.96	141.55	215.87	482.60	951.65
Assigned Total Uncertainty (U_t)	160	120	100	120	120	120	150	250	500	1000

Table 17b. Summary of Expanded and Assigned Total Uncertainties in Capacitance Measurements Using the Type-12 Bridge to Measure Three-Terminal Mica Capacitors at 100 Hz, 1kHz, and 10 kHz

(All values are in ppm - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
(Frequency = 100 Hz)										
Type-A Standard Uncertainty (u_a)	48	25	25	30	25	25	30	25	25	30
Type-B Standard Uncertainty (u_b)	35	25	15	25	25	25	25	25	25	25
Expanded Uncertainty (U)	118.81	70.71	58.31	78.10	70.71	70.71	78.10	70.71	70.71	78.10
Assigned Total Uncertainty (U_t)	120	100	100	100	100	100	100	100	100	100
(Frequency = 1 kHz)										
Type-A Standard Uncertainty (u_a)	45	30	35	30	30	30	30	30	30	30
Type-B Standard Uncertainty (u_b)	35	25	15	20	20	20	20	20	20	20
Expanded Uncertainty (U)	114.02	78.10	76.16	72.11	72.11	72.11	72.11	72.11	72.11	72.11
Assigned Total Uncertainty (U_t)	120	100	100	100	100	100	100	100	100	100
(Frequency = 10 kHz)										
Type-A Standard Uncertainty (u_a)	48	25	25	30	25	25	45	25	25	NA
Type-B Standard Uncertainty (u_b)	35	25	15	20	25	35	55	100	240	NA
Expanded Uncertainty (U)	118.81	70.71	58.31	72.11	70.71	86.02	142.13	206.16	482.60	NA
Assigned Total Uncertainty (U_t)	120	100	100	100	100	100	150	250	500	NA

Table 18a. Summary of Components of Type B Standard Uncertainties in Conductance Measurements Using the Type-12 Bridge to Measure Two-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in μS - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(Frequency = 100 Hz)										
Accuracy of G_s (s_{gs})	4.8E-07	6.4E-07	1.3E-06	1.6E-06	3.2E-06	3.2E-06	2.4E-05	3.2E-05	1.6E-04	2.8E-04
Contact Resistance (s_{gr})	4.6E-10	1.8E-09	1.2E-08	4.6E-08	1.8E-07	1.2E-06	4.6E-06	1.8E-05	1.2E-04	4.6E-04
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.3E-04
Residu. Conduct. (s_{gc})	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-04
Mica Capacitor Dials (s_{gm})	-	-	-	1.2E-04	1.4E-04	1.5E-04	3.9E-04	7.6E-04	2.9E-03	4.7E-03
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.6E-04	1.7E-04	1.8E-04	4.1E-04	7.7E-04	2.9E-03	4.8E-03
(Frequency = 1 kHz)										
Accuracy of G_s (s_{gs})	2.4E-06	3.2E-06	9.6E-06	1.6E-05	3.2E-05	3.2E-05	9.6E-05	1.6E-04	5.6E-04	2.3E-03
Contact Resistance (s_{gr})	4.6E-08	1.8E-07	1.2E-06	4.6E-06	1.8E-05	1.2E-04	4.6E-04	1.8E-03	1.2E-02	4.6E-02
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.3E-04
Residu. Conduct. (s_{gc})	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05
Mica Capacitor Dials (s_{gm})	-	-	-	9.0E-05	1.7E-04	2.6E-04	8.0E-04	2.9E-03	8.7E-03	1.5E-02
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.4E-04	2.0E-04	3.0E-04	9.3E-04	3.4E-03	1.4E-02	4.8E-02
(Frequency = 10 kHz)										
Accuracy of G_s (s_{gs})	1.6E-05	1.9E-05	3.2E-05	4.8E-05	9.6E-05	9.6E-05	1.6E-03	3.2E-03	1.6E-02	1.4E-01
Contact Resistance (s_{gr})	4.6E-06	1.8E-05	1.2E-04	4.6E-04	1.8E-03	1.2E-02	4.6E-02	1.8E-01	1.2E+00	4.6E+00
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-03	1.0E-03	1.3E-01
Residu. Conduct. (s_{gc})	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-03	1.2E-03	1.2E-03	1.2E-02
Mica Capacitor Dials (s_{gm})	-	-	-	1.4E-03	3.1E-03	6.9E-03	2.7E-02	6.5E-02	2.6E-01	1.2E+00
Type-B Std. Uncertainty (u_{gb})	1.5E-04	1.6E-04	1.9E-04	1.5E-03	3.6E-03	1.3E-02	5.3E-02	2.0E-01	1.2E+00	4.8E+00

Table 18b. Summary of Components of Type B Standard Uncertainties in Conductance Measurements Using the Type-12 Bridge to Measure Three-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in μS - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components of Standard Uncertainties										
(Frequency = 100 Hz)										
Accuracy of G_s (s_{gs})	3.2E-07	3.2E-07	8.0E-07	1.3E-06	3.2E-06	3.2E-06	3.2E-05	8.0E-05	1.9E-04	3.5E-04
Contact Resistance (s_{gr})	4.6E-10	1.8E-09	1E-08	4.6E-08	1.8E-07	1.2E-06	4.6E-06	1.8E-05	1.2E-04	4.6E-04
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.3E-04
Residu. Conduct. (s_{gc})	1.2E-06	1.2E-06	1E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-06	1.2E-04
Mica Capacitor Dials (s_{gm})	-	-	-	1.4E-04	1.5E-04	3.9E-04	7.6E-04	2.6E-03	4.7E-03	6.7E-03
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.7E-04	1.8E-04	4.1E-04	7.7E-04	2.6E-03	4.7E-03	6.7E-03
(Frequency = 1 kHz)										
Accuracy of G_s (s_{gs})	1.6E-06	1.6E-06	8.0E-06	1.3E-05	3.2E-05	3.2E-05	1.1E-04	1.6E-04	5.6E-04	3.5E-03
Contact Resistance (s_{gr})	4.6E-08	1.8E-07	1E-06	4.6E-06	1.8E-05	1.2E-04	4.6E-04	1.8E-03	1.2E-02	4.6E-02
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.3E-04
Residu. Conduct. (s_{gc})	1.2E-05	1.2E-05	1E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05	1.2E-05
Mica Capacitor Dials (s_{gm})	-	-	-	1.7E-04	2.4E-04	8.0E-04	2.9E-03	8.4E-03	1.4E-02	2.0E-02
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	2.0E-04	2.7E-04	8.2E-04	2.9E-03	8.6E-03	1.8E-02	5.1E-02
(Frequency = 10 kHz)										
Accuracy of G_s (s_{gs})	4.8E-06	8.0E-06	1.6E-05	3.2E-05	8.0E-05	8.0E-05	1.6E-03	3.2E-03	1.6E-02	NA
Contact Resistance (s_{gr})	4.6E-06	1.8E-05	1.2E-04	4.6E-04	1.8E-03	1.2E-02	4.6E-02	1.8E-01	1.2E+00	NA
Conductance Pot. (s_{gp})	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-03	1.0E-03	NA
Residu. Conduct. (s_{gc})	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-03	1.2E-03	1.2E-03	NA
Mica Capacitor Dials (s_{gm})	-	-	-	3.1E-03	4.9E-03	2.4E-02	6.5E-02	1.6E-01	1.2E+00	NA
Type-B Std. Uncertainty (u_{gb})	1.5E-04	1.5E-04	1.9E-04	3.1E-03	5.3E-03	2.6E-02	7.9E-02	2.4E-01	1.7E+00	NA

Table 19a. Summary of Components of Type A, Expanded, and Assigned Uncertainties in Conductance Measurements Using the Type-12 Bridge to Measure Two-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in μS - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components										
(Frequency = 100 Hz)										
Repeatability (s_{grm})	2.3E-05	2.3E-05	2.3E-05	2.9E-05	4.0E-05	7.2E-05	1.2E-04	1.8E-04	4.6E-04	1.2E-03
Predicted Values (s_{gpd})	1.0E-04	-	-	1.2E-04	-	-	2.1E-04	-	-	5.0E-03
Type-A Std. Uncertainty (u_{ga})	1.0E-04	2.3E-05	2.3E-05	1.2E-04	4.0E-05	7.2E-05	2.4E-04	1.8E-04	4.6E-04	5.1E-03
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.6E-04	1.7E-04	1.8E-04	4.1E-04	7.7E-04	2.9E-03	4.8E-03
Expanded Uncertainty (U_g)	0.00029	0.00021	0.00021	0.00040	0.00035	0.00039	0.00094	0.00158	0.00590	0.01399
Assigned Total Uncertainty (U_{gt})	0.0003	0.0002	0.0002	0.0004	0.0004	0.0004	0.001	0.002	0.006	0.02
(Frequency = 1 kHz)										
Repeatability (s_{grm})	4.0E-05	4.6E-05	7.5E-05	1.2E-04	2.3E-04	5.2E-04	1.2E-03	2.3E-03	7.5E-03	2.3E-02
Predicted Values (s_{gpd})	1.0E-04	2.0E-04	2.0E-04	3.0E-04	1.0E-03	2.0E-03	6.0E-03	1.5E-02	5.5E-02	9.5E-02
Type-A Std. Uncertainty (u_{ga})	1.1E-04	2.1E-04	2.1E-04	3.2E-04	1.0E-03	2.1E-03	6.1E-03	1.5E-02	5.6E-02	9.8E-02
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.4E-04	2.0E-04	3.0E-04	9.3E-04	3.4E-03	1.4E-02	4.8E-02
Expanded Uncertainty (U_g)	0.00030	0.00046	0.00047	0.00070	0.00209	0.00418	0.01236	0.03112	0.11473	0.21824
Assigned Total Uncertainty (U_{gt})	0.0003	0.0005	0.0005	0.001	0.003	0.005	0.02	0.04	0.12	0.3
(Frequency = 10 kHz)										
Repeatability (s_{grm})	1.8E-04	2.3E-04	5.8E-04	1.2E-03	2.3E-03	8.1E-03	2.3E-02	6.4E-02	3.5E-01	1.2E+00
Predicted Values (s_{gpd})	1.0E-04	-	-	3.3E-03	-	-	3.5E-02	-	-	1.0E+00
Type-A Std. Uncertainty (u_{ga})	2.0E-04	2.3E-04	5.8E-04	3.5E-03	2.3E-03	8.1E-03	4.2E-02	6.4E-02	3.5E-01	1.5E+00
Type-B Std. Uncertainty (u_{gb})	1.5E-04	1.6E-04	1.9E-04	1.5E-03	3.6E-03	1.3E-02	5.3E-02	2.0E-01	1.2E+00	4.8E+00
Expanded Uncertainty (U_g)	0.0005	0.0006	0.0012	0.0076	0.0085	0.0314	0.1356	0.4116	2.4667	10.0424
Assigned Total Uncertainty (U_{gt})	0.001	0.001	0.005	0.01	0.02	0.05	0.2	0.5	3.0	10.0

Table 19b. Summary of Components of Type A, Expanded, and Assigned Uncertainties in Conductance Measurements Using the Type-12 Bridge to Measure Three-Terminal Mica Capacitors at 100 Hz, 1 kHz, and 10 kHz

(All values are in μS - except nominal values of the unknown capacitance standards)

Unknown Standard C_x (μF)	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.5	1.0
Components										
(Frequency = 100 Hz)										
Repeatability (s_{grm})	2.3E-05	2.3E-05	2.3E-05	2.9E-05	4.0E-05	7.2E-05	1.2E-04	1.8E-04	4.6E-04	1.2E-03
Predicted Values (s_{gpd})	1.5E-05	-	-	7.4E-05	-	-	1.9E-04	-	-	4.6E-03
Type-A Std. Uncertainty (u_{ga})	2.7E-05	2.3E-05	2.3E-05	7.9E-05	4.0E-05	7.2E-05	2.2E-04	1.8E-04	4.6E-04	1.2E-03
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	1.7E-04	1.8E-04	4.1E-04	7.7E-04	2.6E-03	4.7E-03	6.7E-03
Expanded Uncertainty (U_g)	0.00021	0.00021	0.00021	0.00037	0.00037	0.00083	0.00161	0.00528	0.00950	0.01359
Assigned Total Uncertainty (U_{gt})	0.0002	0.0002	0.0002	0.0004	0.0004	0.0009	0.002	0.006	0.010	0.02
(Frequency = 1 kHz)										
Repeatability (s_{grm})	4.0E-05	4.6E-05	7.5E-05	1.2E-04	2.3E-04	5.2E-04	1.2E-03	2.3E-03	7.5E-03	2.3E-02
Predicted Values (s_{gpd})	1.0E-04	2.0E-04	2.0E-04	3.0E-04	1.0E-03	2.0E-03	6.0E-03	1.5E-02	5.5E-02	9.5E-02
Type-A Std. Uncertainty (u_{ga})	1.1E-04	2.1E-04	2.1E-04	3.2E-04	1.0E-03	2.1E-03	6.1E-03	1.5E-02	5.6E-02	9.8E-02
Type-B Std. Uncertainty (u_{gb})	1.0E-04	1.0E-04	1.0E-04	2.0E-04	2.7E-04	8.2E-04	2.9E-03	8.6E-03	1.8E-02	5.1E-02
Expanded Uncertainty (U_g)	0.00030	0.00046	0.00047	0.00076	0.00212	0.00444	0.01354	0.03487	0.11700	0.22018
Assigned Total Uncertainty (U_{gt})	0.0003	0.0005	0.0005	0.001	0.003	0.005	0.02	0.04	0.12	0.3
(Frequency = 10 kHz)										
Repeatability (s_{grm})	1.8E-04	2.3E-04	5.8E-04	1.2E-03	2.3E-03	8.1E-03	2.3E-02	6.4E-02	3.5E-01	NA
Predicted Values (s_{gpd})	3.5E-05	-	-	2.3E-03	-	-	3.2E-02	-	-	NA
Type-A Std. Uncertainty (u_{ga})	1.8E-04	2.3E-04	5.8E-04	2.6E-03	2.3E-03	8.1E-03	3.9E-02	6.4E-02	3.5E-01	NA
Type-B Std. Uncertainty (u_{gb})	1.5E-04	1.5E-04	1.9E-04	3.1E-03	5.3E-03	2.6E-02	7.9E-02	2.4E-01	1.7E+00	NA
Expanded Uncertainty (U_g)	0.0005	0.0006	0.0012	0.0081	0.0115	0.0548	0.1773	0.5001	3.4532	NA
Assigned Total Uncertainty (U_{gt})	0.001	0.001	0.005	0.01	0.02	0.06	0.2	0.5	4.0	NA

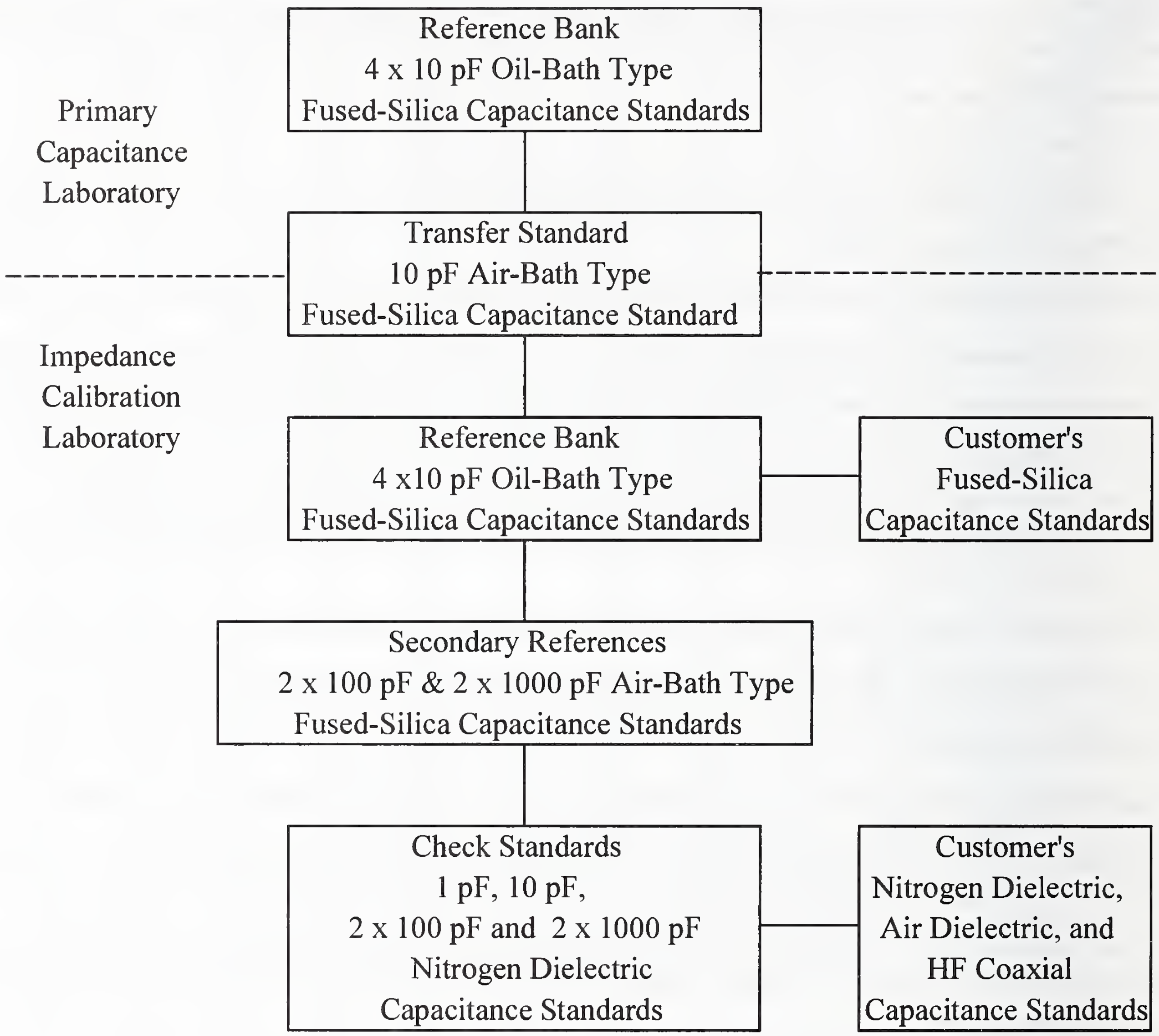


Figure 1. Block Diagram of the Farad Transfer and Calibration Process Using the Type-2 Bridge.

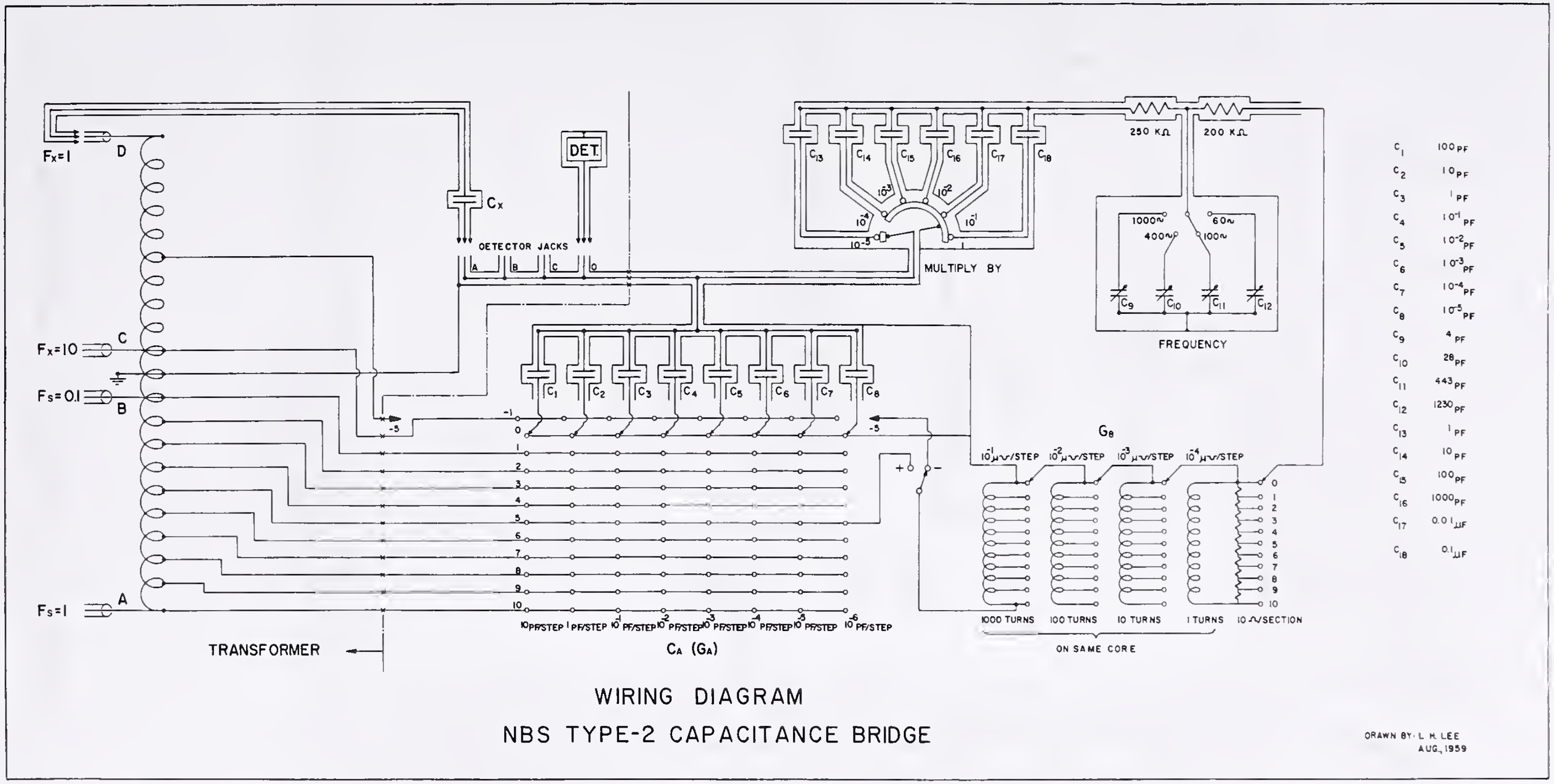


Figure 2a. Schematic Diagram of the NIST Type-2 Capacitance Bridge.

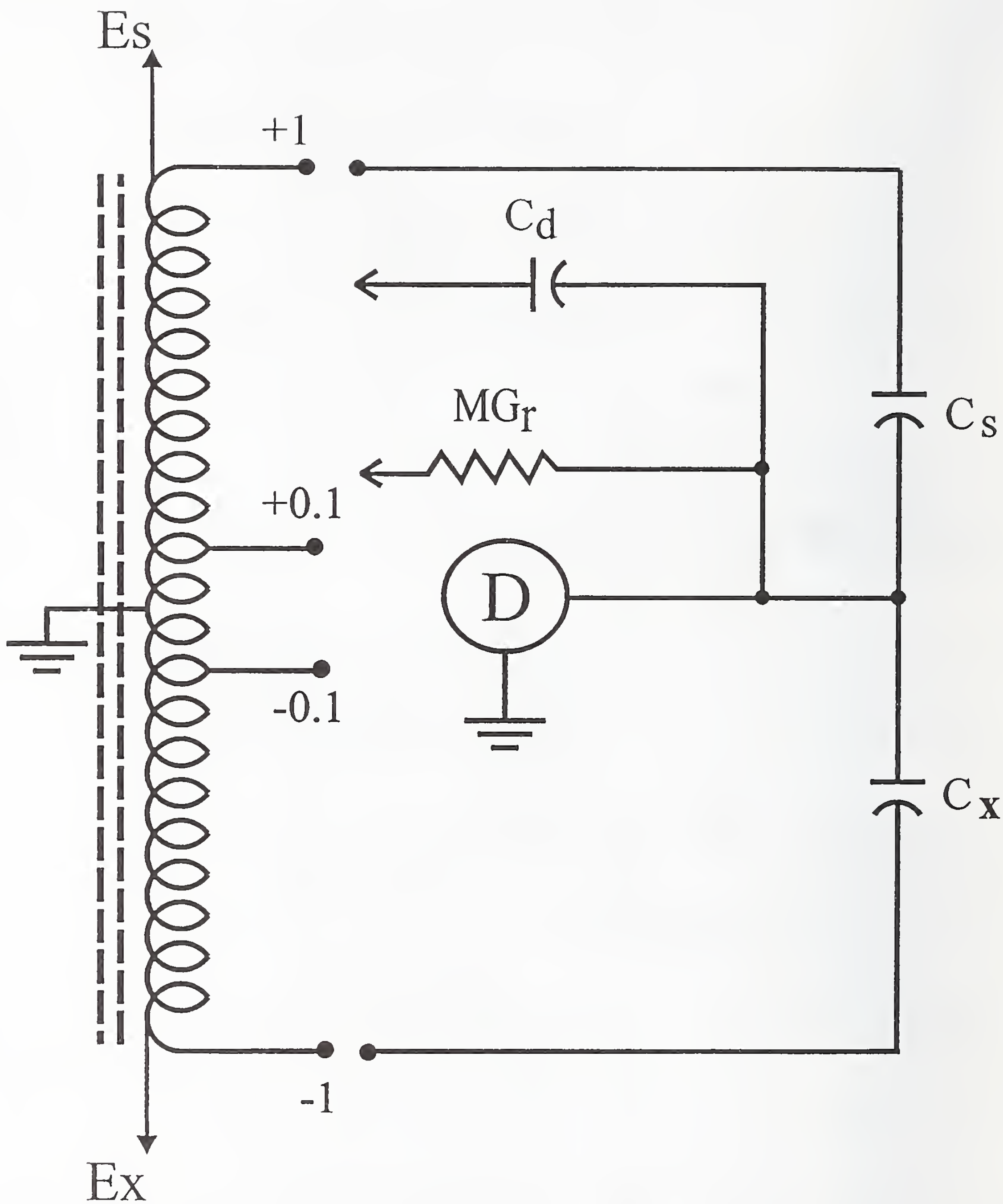


Figure 2b. Simplified Circuit Diagram of the NIST Type-2 Capacitance Bridge.

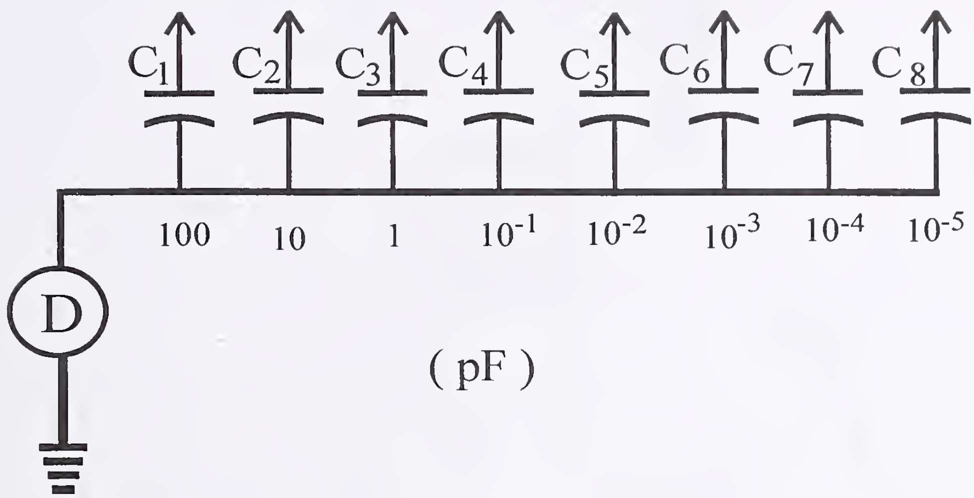


Figure 3. Internal Capacitors of the Type-2 Bridge.

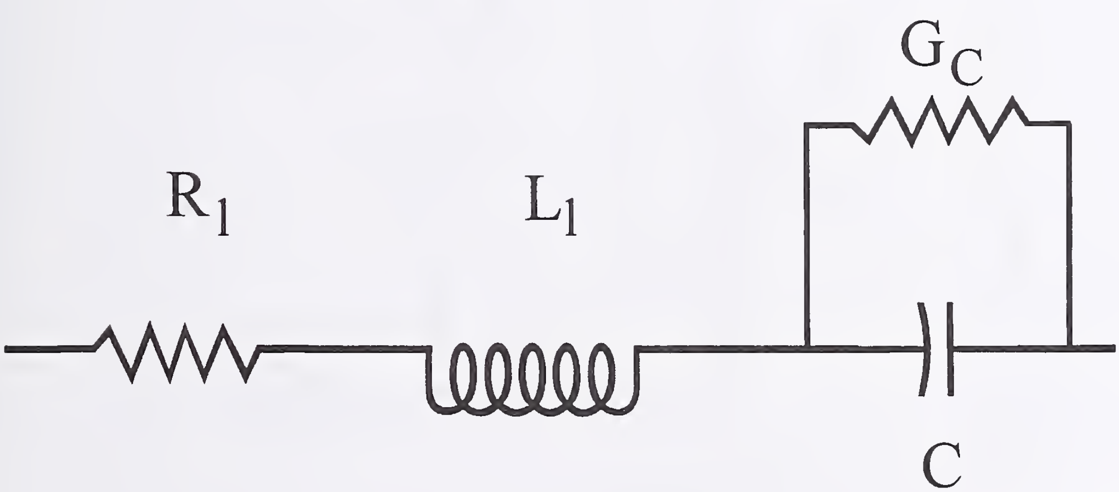


Figure 4. Lead Impedance of a Capacitor Connected to the Bridge.

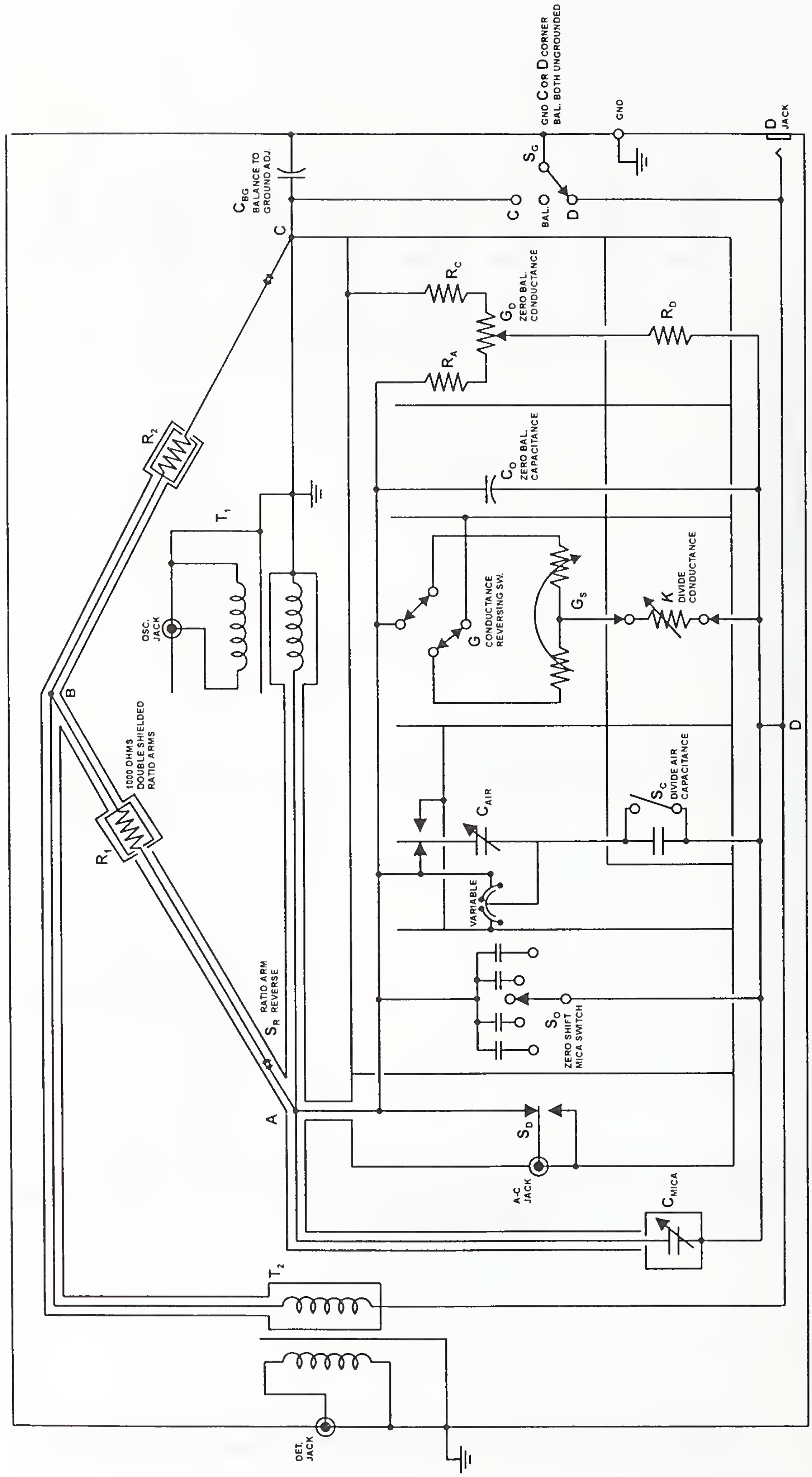


Figure 5a. Schematic Diagram of the Type-12 Capacitance Bridge.

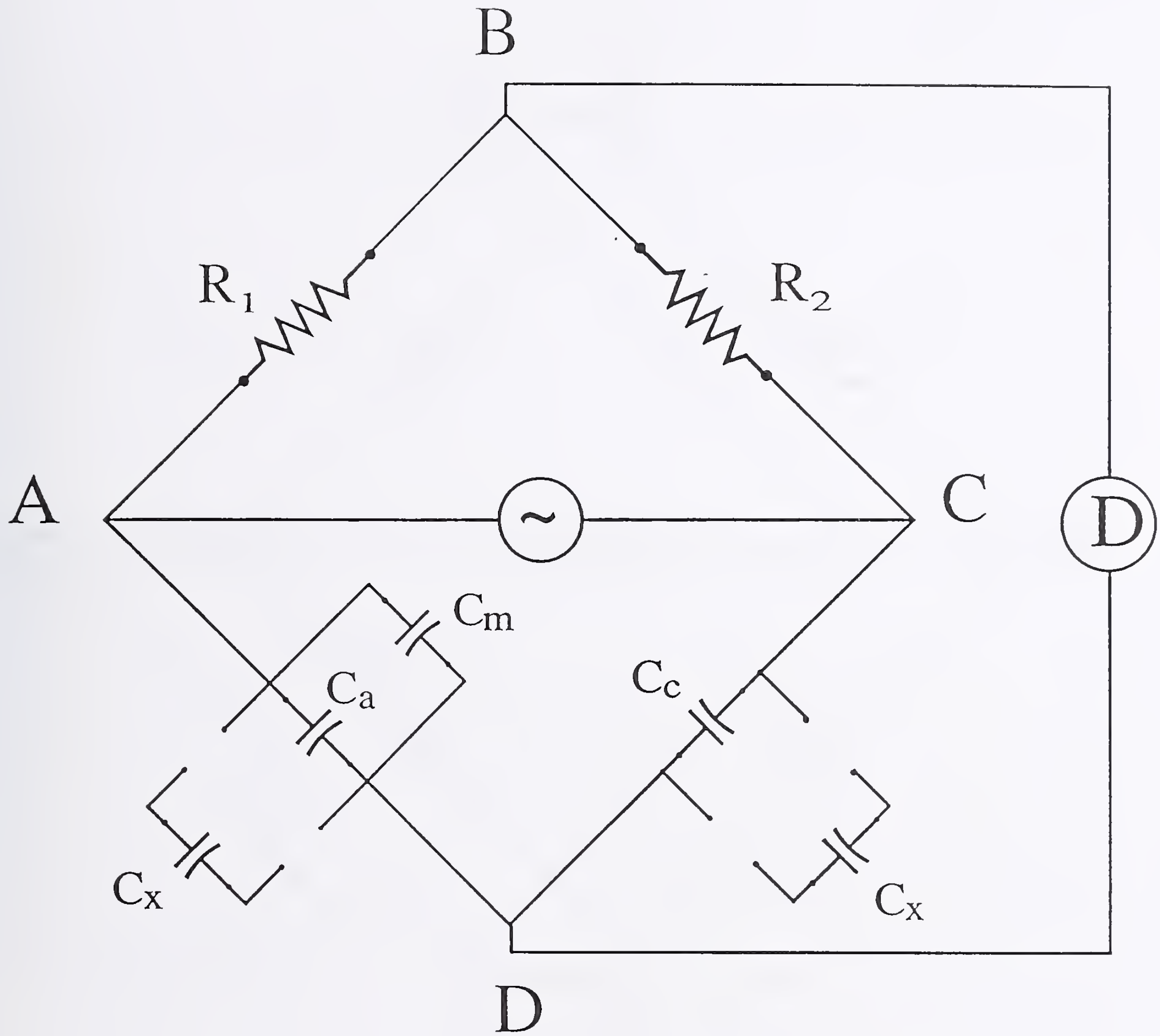


Figure 5b. Simplified Circuit Diagram of the Type-12 Capacitance Bridge.

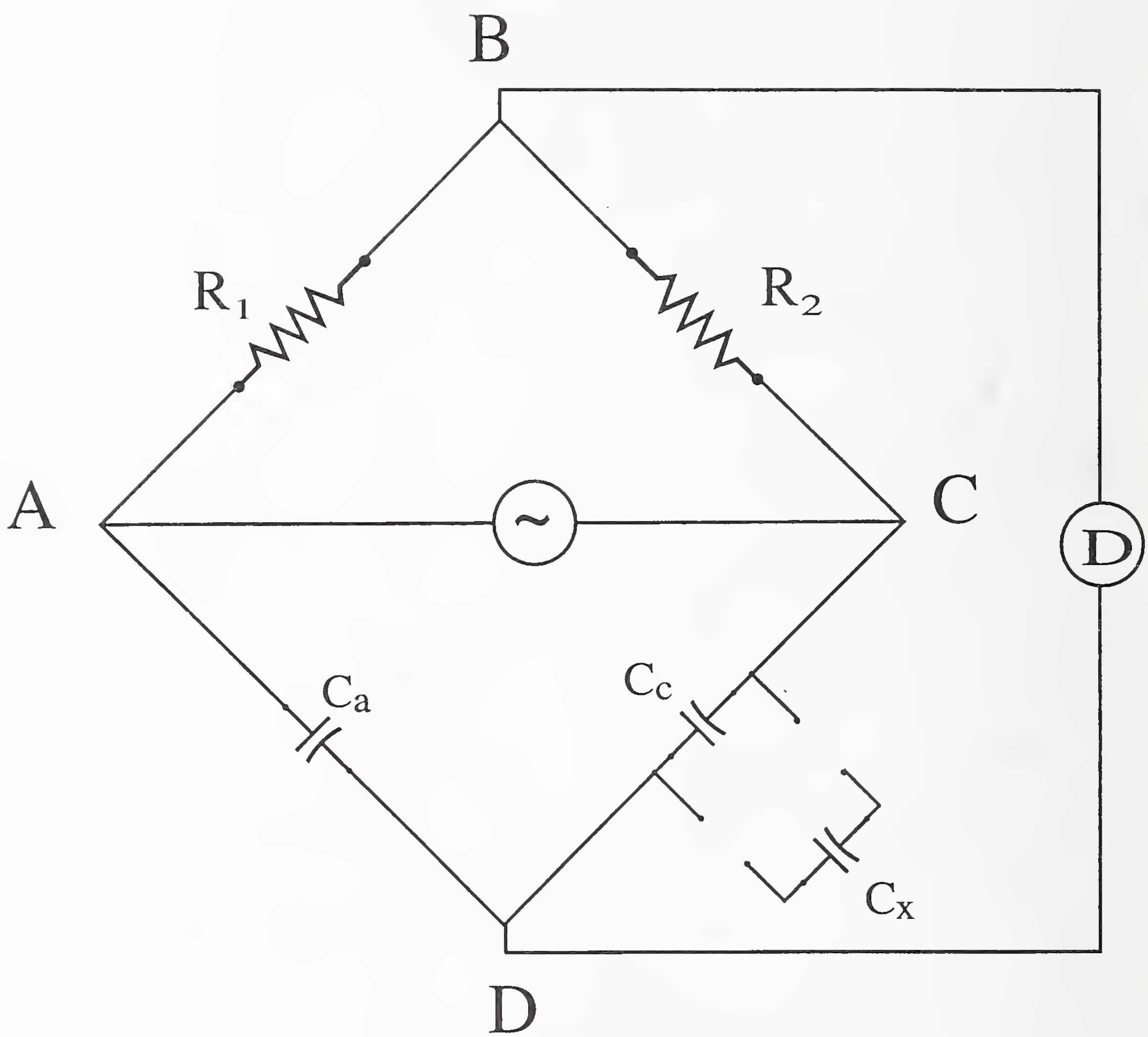


Figure 6. Components of the Type-12 Bridge for Two-Terminal Low-Capacitance Values Measurements.

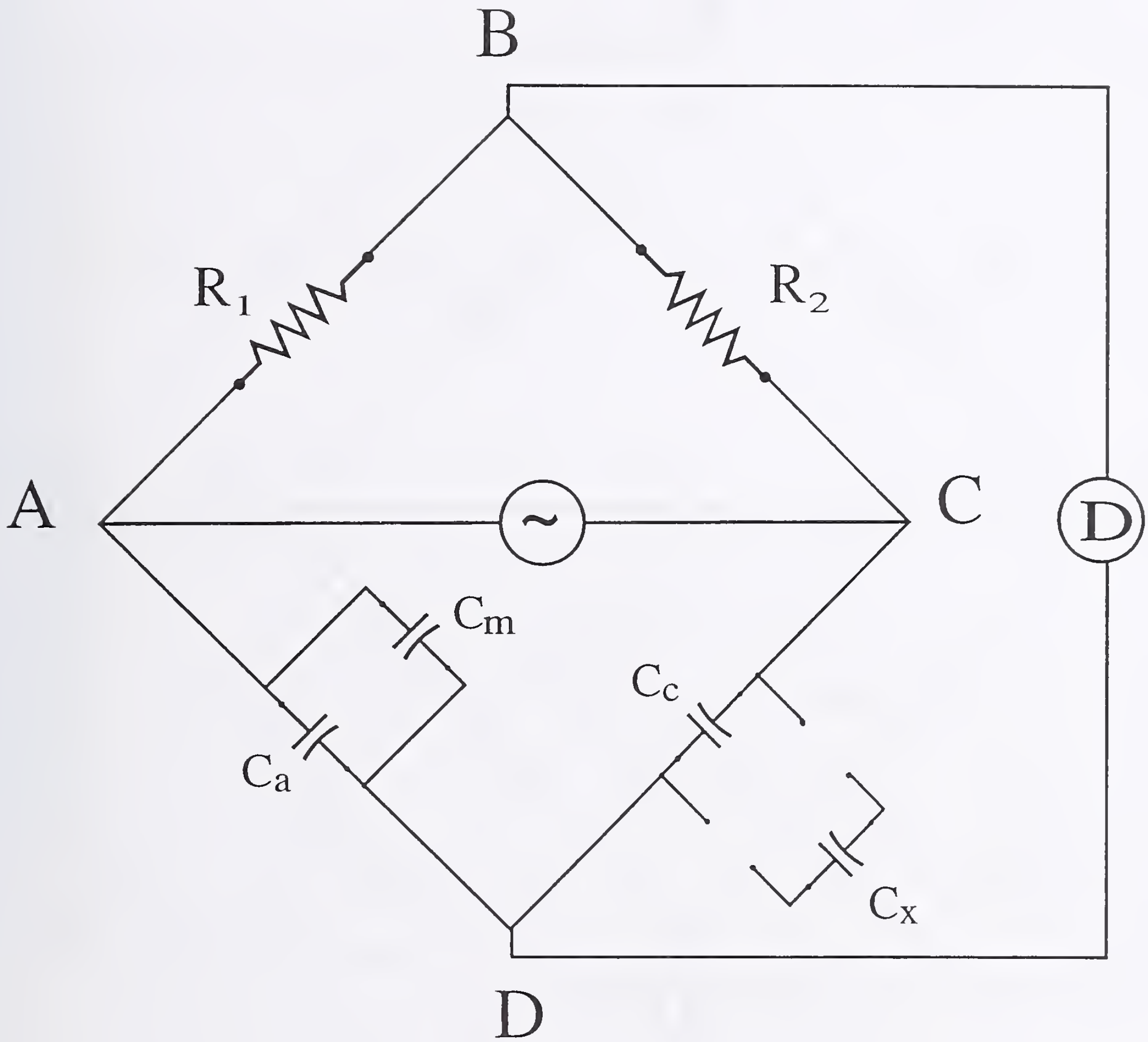


Figure 7. Components of the Type-12 Bridge for Two-Terminal High-Capacitance Values Measurements.

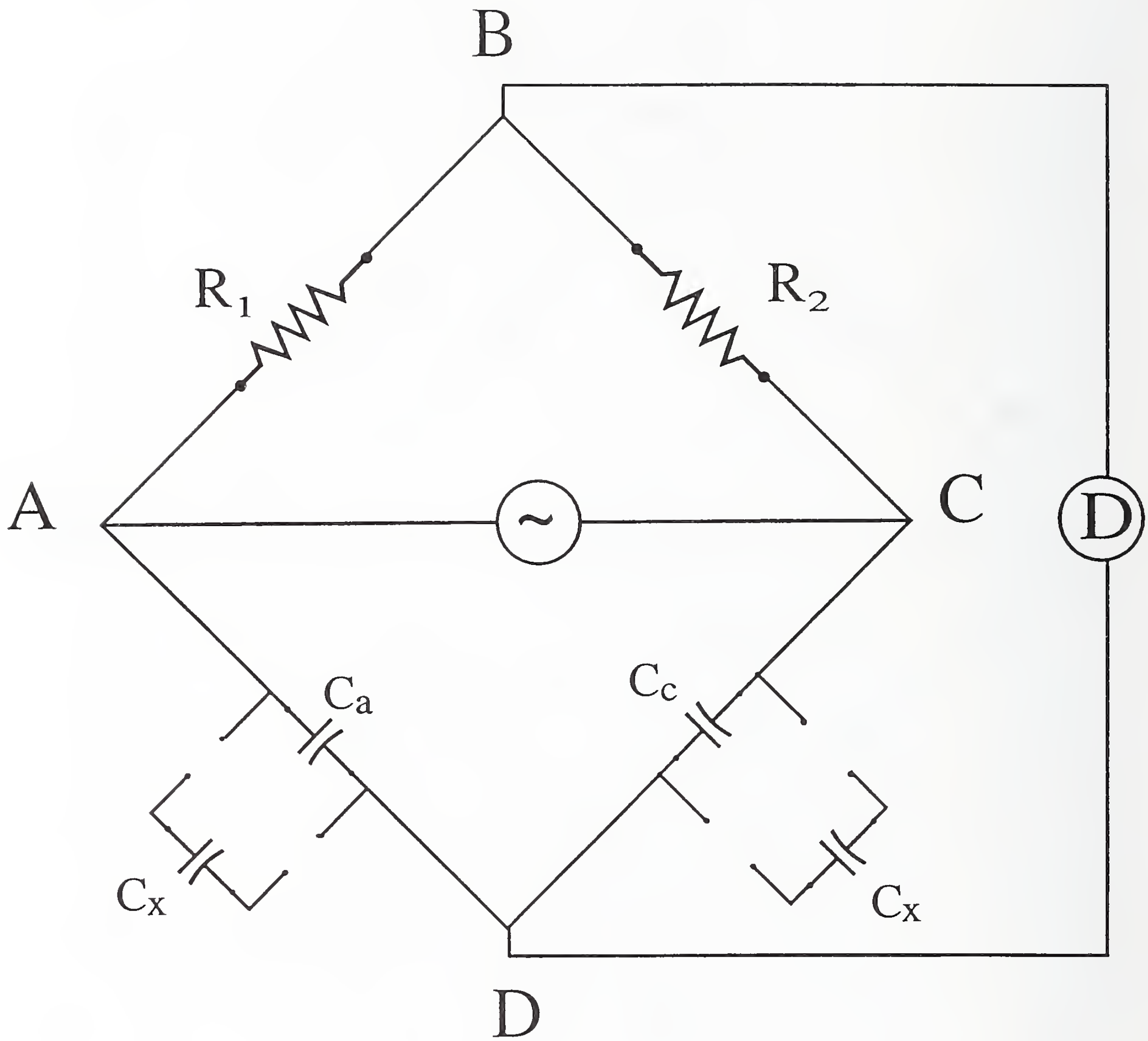


Figure 8. Components of the Type-12 Bridge for Three-Terminal Low-Capacitance Values Measurements.

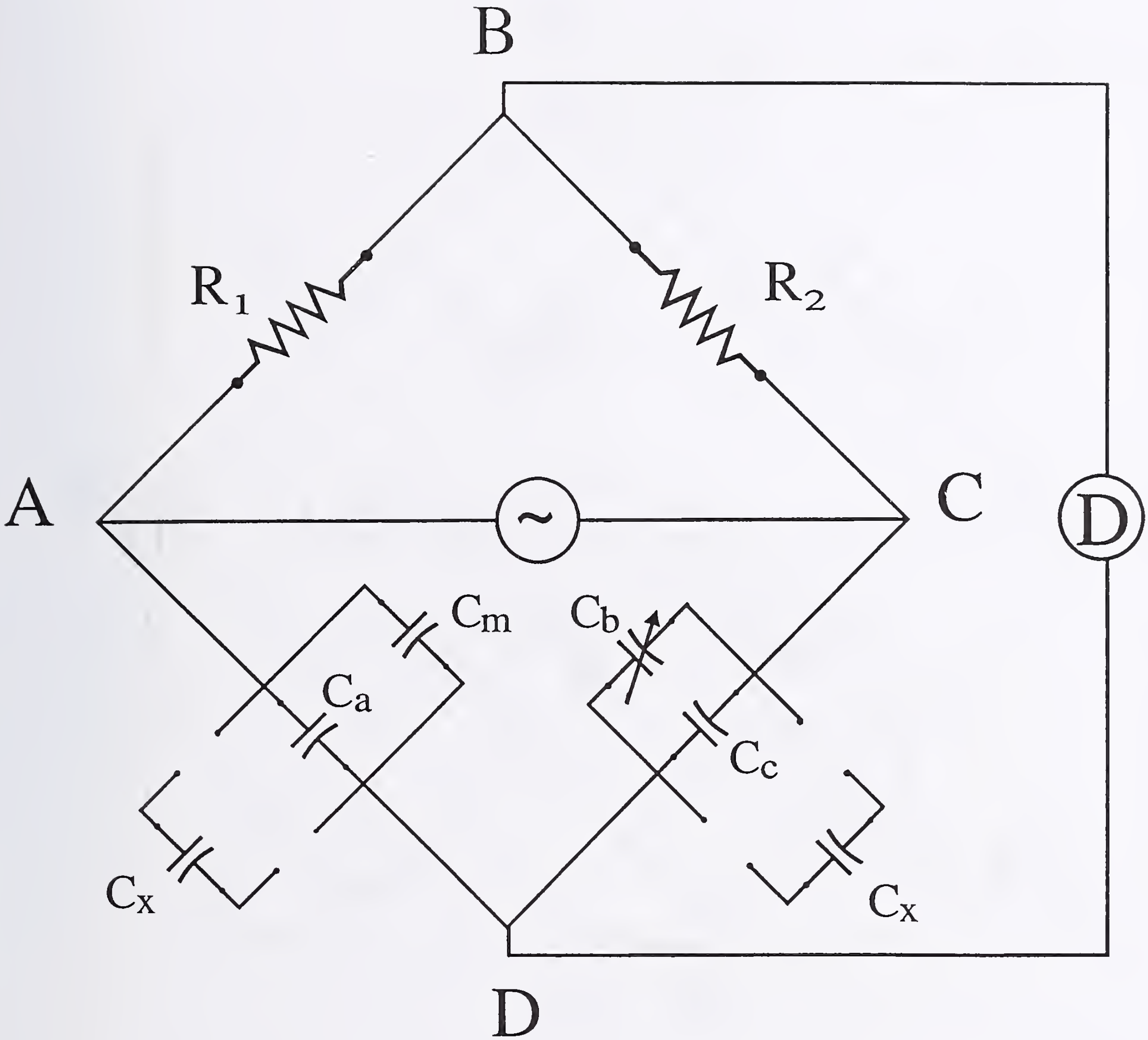


Figure 9. Components of the Type-12 Bridge for Three-Terminal Measurements of Capacitors up to 0.5 μF.

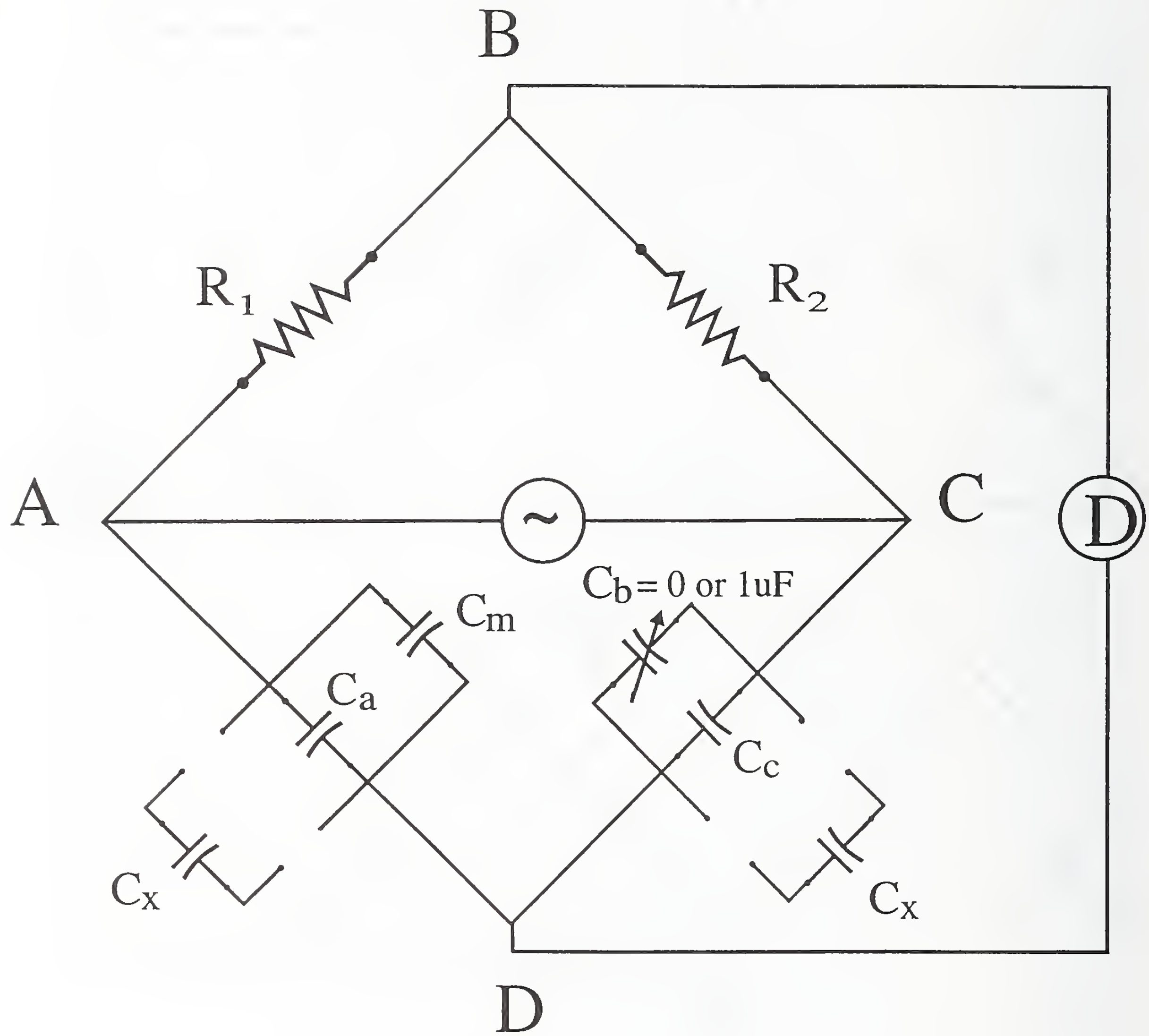


Figure 10. Components of the Type-12 Bridge for Three-Terminal Measurements of Capacitors Larger Than 0.5 μF.

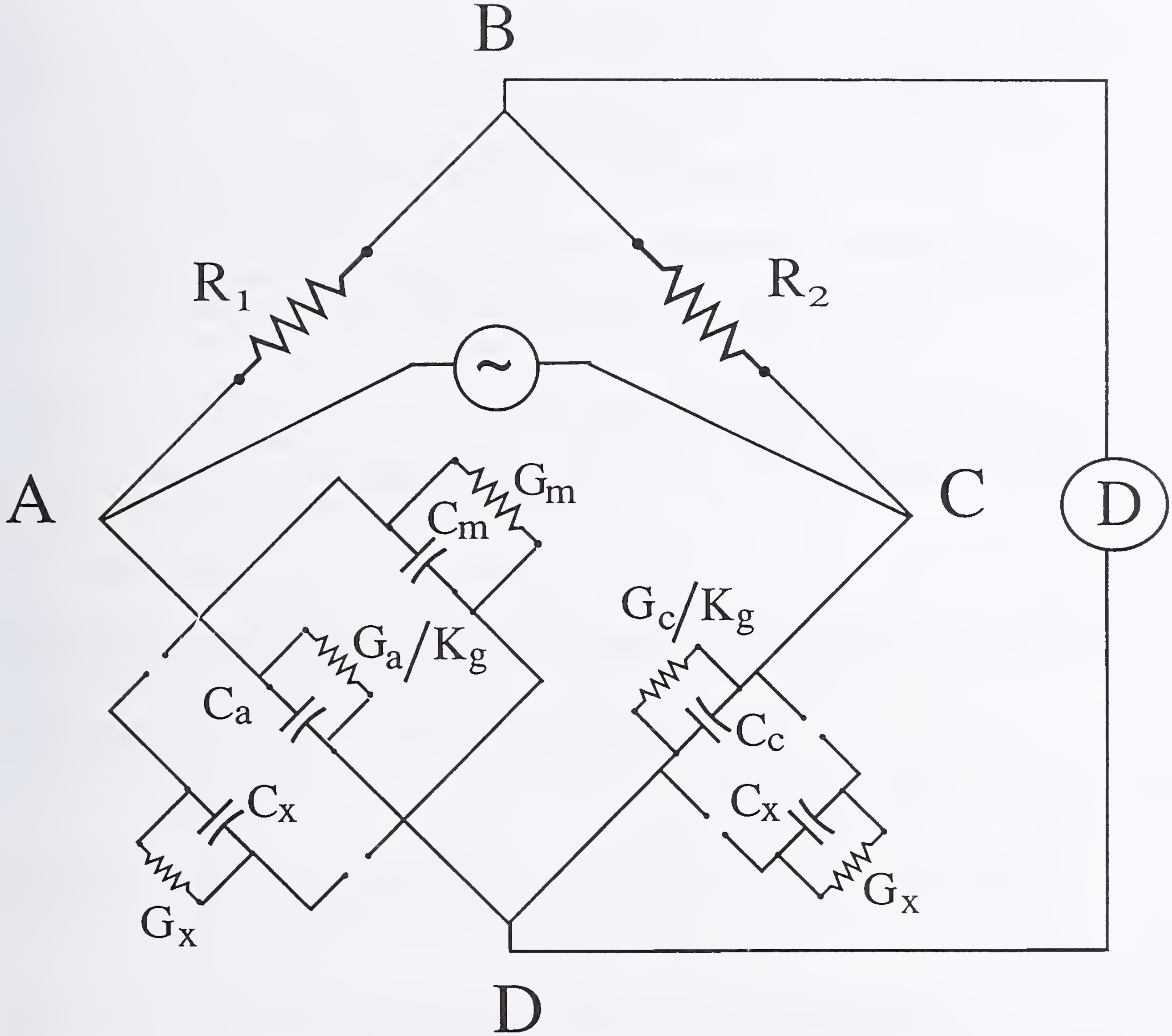


Figure 11. Simplified Circuit Diagram of the Type-12 Capacitance Bridge Including the Conductance Components.

APPENDIX A. UNCERTAINTY ANALYSIS OF THE CORRECTIONS TO THE 1:1 RATIO

Rewrite the balance equation and the corrections to the 1:1 ratio of the Type-2 bridge as shown in Eqs. (6) to (9) of the text as Eqs. (A1) to (A4) as:

$$E_S\, Y_S + E_S\,(\sum r_i y_i) + p_5\, E_S\,(M \sum g_i) = -\, E_X\, Y_X\,, \qquad \text{(A1)}$$

where E_S and E_X are voltages produced by the transformer windings on the reference, $\mathbf{C_s}$ and the unknown capacitor, $\mathbf{C_x}$ sides of the bridge, respectively,

Y_S and Y_X are admittances of $\mathbf{C_s}$ and $\mathbf{C_x}$, respectively,

r_i is the dial reading of i^{th} dial applying voltage to capacitor $\mathbf{c}_i$,

y_i is the admittance of $\mathbf{c}_i$,

g_i is the effective admittance associated with the settings of the i^{th} conductance dial coupled with the frequency compensation network,

p_5 is a constant whose value can be taken as either plus or minus 0.5, and

M is the multiplier of the conductance dial settings, where $10^{-5} \leq M \leq 1$.

Corrections for errors of 1:1 ratio of the Type-2 bridge can be determined in two measurements by using two capacitors of nominally equal value. One measurement is taken with $\mathbf{C_x}$ and $\mathbf{C_s}$ connected as shown in Fig. 2b. The other measurement is taken with $\mathbf{C_x}$ and $\mathbf{C_s}$ interchanged. The balance equation of the second measurement can be written as:

$$E_S\, Y_X + E_S\,(\sum r_n y_n) + p_5\, E_S\,(M' \sum g_n) = -\, E_X\, Y_S \qquad \text{(A2)}$$

where parameters with subscript n have the same meanings as those in Eq. (A1) with subscript i, and M' is the multiplier of the second balance. M' is normally equal to M.

By combining Eqs. (A1) and (A2), (with $M = M'$) the result becomes:

$$(E_S + E_X)(Y_S + Y_X) + E_S\,(\sum r_i y_i + \sum r_n + y_n) + p_5\, E_S\, M\,(\sum g_i + \sum g_n) = 0. \qquad \text{(A3)}$$

Assuming $\boldsymbol{E_s}$ = 1 V as the reference, and by the definition of 1:1 ratio, $(\boldsymbol{E_x} / \boldsymbol{E_s}) \cong -1$, then:

$$E_X = -\,(1 + a + jb), \qquad \text{(A4)}$$

where a and b are the in-phase and quadrature corrections to the ratio, respectively.

Substitution of Eq. (A4) into Eq. (A3) gives:

$$(a + jb) = \{[(\sum r_i y_i + \sum r_n y_n) + p_5\, M\,(\sum g_i + \sum g_n)] \,/\, (Y_S + Y_X)\}. \qquad \text{(A5)}$$

Let $Y_k = j\omega C_k\,(1 - jDF_k)$, $y_k = j\omega c_k\,(1 - jdf_k)$, and $g_k = g_k\,(1 + jx_k)$, where DF_k and df_k are the dissipation factors of the respective capacitors $\mathbf{C_k}$ and $\mathbf{c_k}$, and x_k is the phase correction of the resistor network used in the conductance dials. The magnitudes of DF_k, df_k, and x_k are all less than

10^{-5}. Therefore, by eliminating these terms, the approximate equation for the ratio corrections is:

$$a_1 + jb_1 = [\,(\sum r_i c_i + \sum r_n c_n) + j(-1/\omega)\, p_5 M (\sum g_i + \sum g_n)\,] / (C_s + C_x), \qquad \text{(A6)}$$

where a_1 and b_1 are in-phase and quadrature corrections applied to the 1:1 ratio.

The design of performing the above two measurements to obtain ratio corrections is an attempt to eliminate errors in non-unity ratios. However, the error sources which occurred during both measurements are similar and affect the extent to which the offset cannot be eliminated. Therefore, the exact values of ratio corrections in Eq. (A5) can be expressed as:

$$(a + jb) = a_1 + e_a + jb_1 + je_b, \qquad \text{(A7)}$$

where e_a and e_b are the errors in the in-phase and quadrature corrections, respectively, of the 1:1 ratio, and these can be derived by combining Eqs. (A5), (A6), and (A7) as follows.

By including the in-phase and quadrature components, the first term in the numerator of the right hand side of the Eq. (A5) can be written as:

$$(\sum r_i y_i + \sum r_n y_n) = \sum r_i (j\omega c_i)(1 - jdf_i) + \sum r_n (j\omega c_n)(1 - jdf_n)$$

$$= \omega\, [(\sum r_i c_i\, df_i + \sum r_n c_n\, df_n) + j(\sum r_i c_i + \sum r_n c_n)\,]. \qquad \text{(A8)}$$

The second term in the numerator of the right hand side of the Eq. (A5) can be written as:

$$p_5 M (\sum g_i + \sum g_n) = p_5 M [\sum g_i (1 + jx_i) + \sum g_n (1 + jx_n)\,]$$

$$= p_5 M [(\sum g_i + \sum g_n) + j(\sum g_i x_i + \sum g_n x_n)\,]. \qquad \text{(A9)}$$

The numerator of the right hand side of the Eq. (A5) is the summation of Eqs. (A8) and (A9), which is:

$$(\sum r_i y_i + \sum r_n y_n) + p_5 M (\sum g_i + \sum g_n) =$$

$$\omega\, [(\sum r_i c_i\, df_i + \sum r_n c_n\, df_n) + (p_5 M / \omega)(\sum g_i + \sum g_n)]$$

$$+ j\omega\, [\,(\sum r_i c_i + \sum r_n c_n) + (p_5 M / \omega)(\sum g_i x_i + \sum g_n x_n)\,]. \qquad \text{(A10)}$$

The denominator of the right hand side of the Eq. (A5) can be written as:

$$(Y_s + Y_x) = j\omega Cs\,(1 - jDF_s) + j\omega Cx\,(1 - jDFx) = \omega\,[(Cs\, DF_s + Cx\, DF_x) + j(Cs + Cx)].$$

Thus,

$$[1 / (Y_s + Y_x)] = (1/\omega)\,\{1 / [(C_s DF_s + C_x DF_x) + j(C_s + C_x)]\}. \qquad \text{(A11)}$$

Multiply both the numerator and the denominator of the right hand side of Eq. (A11) by $[(C_s Df_s + C_x DF_x) - j(C_s + C_x)]$ and eliminate the second order terms of DF_s and Df_x. Eq. (A11) becomes:

$$[1 / (Y_s + Y_x)] = (1/\omega) \{ [(C_s DF_s + C_x DF_x) / (C_s + C_x)^2] - j[1/(C_s + C_x)]\}.$$

In measurements using the 1:1 ratio, $C_s \cong C_x$, and

$$[(C_s DF_s + C_x \mathrm{DF}_x) / (C_s + C_x)] \cong [(DF_s + DF_x) / 2]. \text{ Then,}$$

$$[1 / (Y_s + Y_x)] = (1 / \omega)\{(DF_s + DF_x) / [2 (C_s + C_x)]\} - j\{1 / [\omega (C_s + C_x)]\}. \qquad \text{(A12)}$$

The expression of Eq. (A5) can be obtained by the multiplication of Eq. (A10) and (A12). After the separation of the real and imaginary parts, and the elimination of the second order terms of DF_s, DF_x, df_i, df_n, x_i, and x_n, the in-phase correction, a, and the quadrature correction, b of the 1:1 ratio can be expressed as:

$$a = \{ (p_5 M / \omega)[(½) (\sum g_i + \sum g_n) (DF_s + DF_x) + (\sum g_i x_i + \sum g_n x_n)]$$

$$+ (\sum r_i c_i + \sum r_n c_n)\} / (C_s + C_x),$$

and

$$b = \{[(½) (\sum r_i c_i + \sum r_n c_n) (DF_s + DF_x) - (\sum r_i c_i df_i + \sum r_n c_n df_n)]$$

$$- (p_5 M / \omega)(\sum g_i + \sum g_n)\} / (C_s + C_x). \qquad \text{(A13)}$$

According to Eq. (A7),

$$e_a = a - a_1$$

and

$$e_b = b - b1. \qquad \text{(A14)}$$

By substituting a_1 and b_1 from Eq. (A6) and a and b from Eq. (A13) into Eq. (A14), the errors in the in-phase and quadrature corrections of the 1:1 ratio become:

$$e_a = (p_5 M /\omega)[(½) (\sum g_i + \sum g_n) (DF_s + DF_x) + (\sum g_i x_i + \sum g_n x_n)] / (C_s + C_x), \qquad \text{(A15)}$$

and

$$e_b = [(½) (\sum r_i c_i + \sum r_n c_n) (DF_s + DF_x) - (\sum r_i c_i df_i + \sum r_n c_n df_n)] / (C_s + C_x). \qquad \text{(A16)}$$

In general, the capacitance and conductance dial readings of the two measurements when $\mathbf{C_x}$ and $\mathbf{C_s}$ are interchanged are in opposite signs, i.e., one is positive and the other is negative. Therefore, the magnitude of summations of the dial readings of the two measurements (i and n) in Eqs. (A15) and (A16) is usually less than each individual reading, and these can be estimated from the measured data.

In order to estimate the standard uncertainties in the ratio corrections, a few assumptions and approximations are needed to apply to Eqs. (A15) and (A16). In general, all values of DF_s, DF_x, df_i, df_n, x_i, and x_n are less than 10^{-5}, and ratios of $[x_i / (DF_s + DF_x)]$ and $[x_n / (DF_s + DF_x)]$ are less than 1. Maximum bounds of errors $\pm\, e_a$ and $\pm\, e_b$, in ratio corrections are estimated from the following sources and related assumptions; and the standard uncertainties, s_a and s_b, are obtained in accordance to Eq. (4) of the text.

1) Uncertainties estimated from the measured data

According to the data from measurements performed on capacitors of various nominal values, it was found that:

$|(\sum g_i + \sum g_n)| \leq 10^{-8}$ S and $|(\sum r_i c_i + \sum r_n c_n)| \leq 10^{-5}$ pF (for $C_s = C_x =$ 10 pF),
$|(\sum g_i + \sum g_n)| \leq 10^{-7}$ S and $|(\sum r_i c_i + \sum r_n c_n)| \leq 10^{-4}$ pF (for $C_s = C_x =$ 100 pF),
$|(\sum g_i + \sum g_n)| \leq 10^{-6}$ S and $|(\sum r_i c_i + \sum r_n c_n)| \leq 10^{-3}$ pF (for $C_s = C_x =$ 1000 pF), and
$M \leq 10^{-3}$.

By appropriate multiplication in Eqs. (A15) and (A16), the maximum bounds of errors are estimated, and the standard uncertainties are found to be:

$s_{a1} \leq 0.001$ ppm and $s_{b1} << 0.001$ ppm,

where s_{a1} and s_{b1} are the standard uncertainties from measured data in the in-phase and quadrature corrections of the 1:1 ratio, respectively.

2) Uncertainties due to the inaccuracies of the external capacitance standards

Assume the external capacitors, $\mathbf{C_s}$ and $\mathbf{C_x}$, are inaccurate by ± 0.01%, the maximum bounds of errors are estimated by substituting $C_s = C_s\,(1 \pm 10^{-4})$ and $C_x = C_x\,(1 \pm 10^{-4})$ into Eqs. (A15) and (A16). Accordingly,

$s_{a2} << 0.001$ ppm and $s_{b2} << 0.001$ ppm,

where s_{a2} and s_{b2} are the standard uncertainties of the external capacitance standards, in the in-phase and quadrature corrections of the 1:1 ratio, respectively.

3) Uncertainties due to the dial readings

Assume the capacitance and conductance dials are inaccurate by ± 1%, the maximum bounds of errors are estimated by substituting $c_i = c_i\,(1 \pm 0.01)$, $c_n = c_n\,(1 \pm 0.01)$, $g_i = g_i\,(1 \pm 0.01)$, and $g_n = g_n\,(1 \pm 0.01)$ into Eqs. (A15) and (A16). Accordingly,

$s_{a3} << 0.001$ ppm and $s_{b3} << 0.001$ ppm,

where s_{a3} and s_{b3} are the standard uncertainties of the dial readings, in the in-phase and quadrature corrections of the 1:1 ratio, respectively.

4) Uncertainties estimated from the interpolation of last dials

The sensitivity of the conductance dials is such that changes of the ± 1 step on the fourth dial do not affect the balance. This introduces a standard uncertainty to the in-phase correction of:

$s_{a4} \leq 0.002$ ppm,

where s_{a4} is the standard uncertainty in the in-phase correction of the 1:1 ratio from the interpolation of the last dial.

The sensitivity of the capacitance dials is such that the difference between two steps of the last dial can be estimated to gain another digit of accuracy. Such interpolation introduces the maximum bounds of error of ± (3 x 10^{-7}) pF to the dial readings, which affects the quadrature correction according to the nominal values of test capacitors:

$s_{b4} \leq$ 0.02 ppm (for $C_s = C_x =$ 10 pF),
$s_{b4} \leq$ 0.002 ppm (for $C_s = C_x =$ 100 pF), and
$s_{b4} \leq$ 0.0002 ppm (for $C_s = C_x =$ 1000 pF),

where s_{b4} is the standard uncertainty in the quadrature correction of the 1:1 ratio from the interpolation of the last dial.

3) Uncertainties due to load admittance and lead impedance

An accurate method to analyze and measure the components of loading errors and lead impedance was discussed and calculated in [5]. The maximum bounds of errors in ratio corrections owing to loading effect and lead impedance are estimated. Accordingly,

$s_{a5} \leq 0.004$ ppm and $s_{b5} \leq 0.019$ ppm,

where s_{a5} and s_{b5} are the standard uncertainties due to load admittance and lead impedance, in the in-phase and quadrature corrections of the 1:1 ratio, respectively.

In routine calibration of capacitors using the Type-2 bridge, only capacitance values are reported, even though both capacitance and conductance dials are used to obtain a balance. Therefore, only the in-phase standard uncertainties of the ratio corrections are to be included in the Type B standard uncertainty. The combined standard uncertainties that contribute to 1:1 ratio corrections, s_a and s_b, are defined as the square root of the sum-of-squares (RSS) of each component, which are estimated to be:

$$s_a = (s_{a1}^2 + s_{a2}^2 + s_{a3}^2 + s_{a4}^2 + s_{a5}^2)^{(1/2)} < 0.005 \text{ ppm}, \quad \text{(A17)}$$

and

$$s_b = (sb_1^2 + sb_2^2 + sb_3^2 + sb_4^2 + sb_5^2)^{(1/2)} < 0.03 \text{ ppm}. \quad \text{(A18)}$$

APPENDIX B. CAPACITANCE DIAL CORRECTIONS OF THE TYPE-12 BRIDGE

Data and analyses of capacitance corrections for each capacitor dial of the Type-12 bridge are included in the following tables:

Table B-1. Data and Analysis of the (0.1 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 100 Hz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	19.701	41.053	97.318	88.878	108.694	129.700	183.656	204.267	225.604	242.514
11/67	20.437	44.477	98.434	95.788	116.348	140.055	191.927	213.334	237.464	255.166
5/68	21.847	45.248	101.395	96.267	118.168	141.237	195.189	217.845	241.382	260.284
9/69	20.758	44.866	99.509	100.527	121.418	145.294	197.802	219.432	243.710	261.716
12/70	20.664	45.464	100.178	98.560	119.320	143.764	196.294	217.848	242.717	260.552
9/72	20.817	45.897	96.451	106.001	127.050	151.815	199.822	221.694	246.911	265.385
6/74	21.012	44.577	96.139	99.552	120.756	143.991	193.474	215.618	239.273	257.560
11/75	21.435	46.830	103.188	105.313	126.560	151.373	205.533	227.700	253.190	271.885
1/77	21.693	45.409	104.405	100.598	122.436	145.847	202.762	225.370	249.734	268.604
4/78	19.716	41.565	99.931	93.988	113.837	135.478	191.719	212.355	234.427	251.531
11/80	19.338	42.613	99.161	98.911	118.419	141.564	195.577	215.905	239.642	255.905
8/82	20.726	45.563	104.109	109.993	130.962	155.511	211.717	233.171	257.810	275.475
9/83	20.642	46.292	98.191	111.796	132.763	158.175	208.107	229.569	255.559	273.476
1/85	20.874	48.402	104.174	117.362	138.158	164.957	218.316	240.047	267.163	284.982
3/85	21.416	46.567	103.820	107.933	129.529	154.597	209.513	231.897	257.120	275.731
2/91*	20.367	43.857	99.149	106.297	126.824	150.029	201.141	222.516	246.128	263.833
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	20.7152	44.9175	100.347	102.360	123.203	147.087	200.159	221.786	246.115	264.037
SD (pF)	0.7072	1.9211	2.836	7.365	7.628	8.928	8.843	9.131	10.375	10.701
Slope (pF/day)	-1.5E-05	1.9E-04	3.9E-04	2.0E-03	2.0E-03	2.2E-03	2.3E-03	2.3E-03	2.5E-03	2.4E-03
(pF/year)	-0.006	0.071	0.143	0.721	0.718	0.796	0.829	0.823	0.897	0.886
SDfit (pF)	0.731	1.916	2.732	5.363	5.763	7.045	6.706	7.147	8.361	8.850
SD(2/01) (pF)	0.981	2.573	3.668	7.200	7.738	9.459	9.004	9.595	11.225	11.882

* See Section 4.2.2

Table B-2. Data and Analysis of the (0.1 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 1 kHz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	9.522	20.953	34.653	43.932	53.654	64.848	76.408	87.139	98.631	106.029
11/67	9.705	22.182	34.121	46.703	56.383	68.800	78.367	89.047	101.917	109.401
5/68	10.244	22.408	35.507	46.347	56.786	68.780	79.704	91.092	103.511	111.678
10/69	9.533	22.286	34.772	49.342	59.030	71.561	81.847	92.502	105.488	112.861
12/70	9.542	22.709	35.576	48.770	58.502	71.422	82.079	92.883	106.127	113.514
9/72	9.897	23.147	31.106	53.148	63.368	76.185	82.202	92.999	106.621	114.550
6/74	10.752	23.997	33.094	52.792	63.621	76.659	83.574	95.366	108.613	116.993
11/75	10.588	24.618	39.073	55.295	66.103	79.755	92.148	103.905	118.263	126.865
1/77	10.213	22.744	37.782	50.548	60.833	73.164	86.462	92.568	110.491	118.474
4/78	9.394	21.063	36.221	48.260	57.992	69.640	83.019	93.405	105.327	112.628
11/80	9.435	22.353	36.393	52.559	62.166	75.404	88.194	98.502	111.187	118.503
8/82	9.446	22.688	37.819	56.923	66.559	79.701	93.087	103.668	117.187	124.176
9/83	9.592	23.459	33.441	58.878	68.678	82.418	90.272	101.104	115.226	122.771
1/85	9.724	24.240	38.454	61.502	71.393	85.677	97.786	108.552	123.278	130.692
3/85	9.171	21.622	34.525	52.493	61.847	74.082	87.226	97.457	110.102	116.931
2/91	9.857	22.480	33.344	56.339	66.521	79.029	88.098	98.671	111.643	118.968
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	9.7884	22.6843	35.3676	52.1144	62.0898	74.8203	85.6546	96.1788	109.601	117.190
SD (pF)	0.4470	1.0358	2.1787	4.8477	4.9138	5.6067	5.8077	5.8919	6.494	6.523
Slope (pF/day)	-3.8E-05	7.5E-05	1.2E-04	1.4E-03	1.4E-03	1.5E-03	1.7E-03	1.6E-03	1.8E-03	1.7E-03
(pF/year)	-0.014	0.027	0.045	0.526	0.517	0.564	0.624	0.601	0.643	0.617
SDfit (pF)	0.451	1.053	2.230	3.094	3.286	3.970	3.767	4.100	4.674	4.908
SD(2/01) (pF)	0.606	1.413	2.994	4.154	4.412	5.330	5.058	5.505	6.275	6.589

Table B-3. Data and Analysis of the (0.1 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 10 kHz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	16.633	40.841	49.829	109.127	133.238	173.159	197.804	234.249	267.953	334.641
11/67	17.537	43.049	51.619	113.617	138.727	180.194	193.636	232.127	265.649	333.446
5/68	17.580	42.757	51.198	112.780	137.699	178.588	192.175	229.817	263.211	331.357
9/69	17.023	42.341	51.152	113.782	137.885	179.716	195.162	230.105	263.758	330.093
12/70	17.577	43.579	53.528	116.540	141.480	183.752	198.649	236.748	271.662	338.677
7/72	17.200	43.264	47.719	116.766	140.603	183.300	192.219	233.629	268.911	338.315
6/74	17.242	42.382	46.622	113.742	137.876	179.084	190.984	214.931	247.191	316.558
11/75	17.373	43.618	53.638	117.938	143.700	185.088	200.033	234.903	264.293	333.768
1/77	17.433	43.129	53.667	117.288	141.523	182.779	197.197	234.093	268.804	336.119
4/78	16.649	41.610	52.254	114.918	138.599	180.768	194.839	224.878	252.769	321.523
11/80	15.938	42.038	52.608	116.776	140.616	180.812	213.659	251.150	290.010	352.910
8/82	16.626	42.233	52.724	120.318	145.889	187.436	200.815	239.526	264.850	328.029
9/83	16.262	42.149	47.889	118.916	143.913	186.143	202.890	238.802	281.866	349.791
1/85	16.406	39.757	49.519	114.702	139.098	178.572	195.466	225.324	251.888	306.074
3/85	16.218	41.479	52.377	115.708	138.774	178.980	195.691	231.559	265.150	330.821
2/91	16.440	41.815	50.474	114.584	137.736	178.309	197.036	232.733	265.200	381.138
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	16.8836	42.2526	51.0511	115.469	139.835	181.043	197.391	232.786	265.823	335.204
SD (pF)	0.5486	1.0266	2.2112	2.681	3.050	3.570	5.433	7.740	10.413	16.652
Slope (pF/day)	-1.5E-04	-1.7E-04	-3.4E-06	4.9E-04	3.9E-04	2.8E-04	7.0E-04	4.2E-04	2.5E-04	2.1E-03
(pF/year)	-0.054	-0.061	-0.001	0.177	0.141	0.103	0.255	0.153	0.091	0.749
SDfit (pF)	0.397	0.959	2.289	2.431	2.973	3.612	5.288	7.928	10.756	16.285
SD(2/01) (pF)	0.532	1.288	3.073	3.264	3.991	4.850	7.100	10.644	14.441	21.863

Table B-4. Data and Analysis of the (0.01 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 100 Hz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	2.558	4.926	7.174	9.840	12.623	14.896	14.297	17.852	20.346	20.623
11/67	2.625	5.143	7.624	10.009	12.915	15.365	14.988	18.629	21.272	21.610
5/68	2.869	5.312	7.746	10.342	13.396	15.802	15.491	19.284	21.900	22.447
9/69	2.748	5.161	7.616	10.066	12.994	15.354	14.925	18.615	21.250	21.668
12/70	2.613	5.186	7.619	9.907	12.835	15.398	15.018	18.656	21.449	21.824
9/72	2.531	4.978	7.249	9.696	12.602	15.066	14.710	18.554	21.358	21.820
6/74	2.700	5.340	7.391	9.638	12.584	15.011	14.454	18.104	20.857	21.272
11/75	2.733	5.162	7.615	9.838	12.801	15.117	14.655	18.411	21.117	21.615
1/77	2.694	5.178	7.534	9.940	12.989	15.458	14.922	18.643	21.339	21.753
4/78	2.373	4.774	6.707	8.948	11.758	14.134	13.794	16.750	19.338	19.506
11/80	2.462	4.778	6.880	8.789	11.673	14.044	12.596	16.207	18.829	19.183
8/82	2.573	5.109	7.083	9.094	12.029	14.328	13.611	17.282	19.888	20.209
9/83	2.560	5.155	7.132	9.092	11.990	14.445	13.767	17.450	20.127	20.537
1/85	2.667	5.210	7.462	9.317	12.252	14.854	14.049	17.711	20.489	20.781
3/85	2.671	5.264	7.200	9.133	11.992	14.577	13.778	17.519	20.262	20.551
2/91	2.589	5.152	6.584	8.570	11.486	14.342	13.384	16.967	19.256	18.972
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	2.6229	5.1143	7.2885	9.5137	12.4324	14.8869	14.2774	17.9146	20.5673	20.8982
SD (pF)	0.1190	0.1678	0.3499	0.5232	0.5589	0.5261	0.7566	0.8420	0.9011	1.0326
Slope (pF/day)	-1.3E-05	1.6E-07	-8.5E-05	-1.7E-04	-1.7E-04	-1.4E-04	-2.1E-04	-2.2E-04	-2.3E-04	-2.8E-04
(pF/year)	-0.005	0.000	-0.031	-0.063	-0.063	-0.052	-0.078	-0.081	-0.086	-0.101
SDfit (pF)	0.118	0.174	0.277	0.269	0.329	0.380	0.516	0.622	0.676	0.752
SD(2/01) (pF)	0.159	0.233	0.371	0.362	0.441	0.510	0.693	0.836	0.907	1.010

Table B-5. Data and Analysis of the (0.01 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 1 kHz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	1.165	2.741	3.516	4.629	6.035	7.545	5.495	7.686	9.454	8.387
11/67	1.185	2.851	3.739	4.507	5.937	7.535	5.583	7.754	9.534	8.500
5/68	1.244	2.875	3.726	4.625	6.098	7.662	5.693	7.922	9.720	8.719
10/69	1.218	2.808	3.583	4.466	5.914	7.429	5.315	7.550	9.295	8.261
12/70	1.178	2.888	3.664	4.449	5.870	7.553	5.513	7.704	9.584	8.522
9/72	1.234	2.895	3.524	4.283	5.730	7.354	5.238	7.454	9.338	8.300
6/74	1.278	2.905	3.696	4.466	5.981	7.546	5.482	7.754	9.557	8.587
11/75	1.251	2.957	3.760	4.493	5.999	7.652	5.648	7.921	9.774	8.805
1/77	1.184	2.818	3.416	4.205	5.654	7.228	5.032	7.256	9.097	8.093
4/78	1.093	2.646	3.084	3.855	5.193	6.704	4.337	6.450	8.216	7.066
11/80	1.099	2.706	3.205	3.801	5.138	6.687	4.348	6.460	8.181	7.128
8/82	1.086	2.674	3.068	3.659	4.999	6.570	4.159	6.302	8.055	6.906
9/83	1.125	2.850	3.245	3.872	5.237	6.877	4.507	6.657	8.492	7.375
1/85	1.137	2.835	3.332	3.682	5.134	6.792	4.494	6.666	8.501	7.429
3/85	1.028	2.652	2.860	3.233	4.534	6.110	3.503	5.562	7.375	6.188
2/91	1.054	2.715	2.881	3.465	4.801	6.407	3.621	5.685	7.494	6.362
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	1.1599	2.8010	3.3937	4.1056	5.5159	7.1032	4.8730	7.0489	8.8542	7.7893
SD (pF)	0.0747	0.0986	0.3064	0.4498	0.5031	0.5043	0.7345	0.7935	0.8052	0.8622
Slope (pF/day)	-2.1E-05	-1.8E-05	-9.7E-05	-1.6E-04	-1.7E-04	-1.7E-04	-2.5E-04	-2.7E-04	-2.7E-04	-2.8E-04
(pF/year)	-0.008	-0.006	-0.035	-0.057	-0.063	-0.061	-0.091	-0.097	-0.097	-0.102
SDfit (pF)	0.052	0.090	0.172	0.180	0.225	0.253	0.335	0.379	0.407	0.453
SD(2/01) (pF)	0.070	0.121	0.231	0.241	0.302	0.339	0.450	0.509	0.547	0.608

Table B-6. Data and Analysis of the (0.01 μF/step) Mica Capacitor Dial Corrections of the Type-12 Bridge at 10 kHz

[All values are in pF except the dials which are in (μF/step)]

Date \ Dial	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	0.269	1.491	1.829	2.800	4.312	6.523	5.180	8.255	11.291	11.987
11/67	0.354	1.670	2.239	2.971	4.562	6.850	5.692	8.821	11.925	12.710
5/68	0.321	1.610	2.148	2.905	4.511	6.784	5.583	8.742	11.855	12.634
9/69	0.325	1.568	1.946	2.729	4.289	6.519	5.175	8.282	11.338	12.113
12/90	0.353	1.701	2.211	2.942	4.530	6.833	5.616	8.754	11.908	12.699
9/72	0.339	1.623	1.994	2.631	4.182	6.451	5.130	8.254	11.316	12.092
6/74	0.360	1.613	1.998	2.716	4.319	6.509	5.092	8.164	11.097	11.857
11/75	0.352	1.692	2.082	2.686	4.229	6.510	5.135	8.271	11.389	12.153
1/77	0.306	1.608	1.916	2.585	4.164	6.428	5.024	8.081	11.224	12.018
4/78	0.223	1.421	1.559	2.215	3.693	5.876	4.327	7.313	10.383	11.121
11/80	0.167	1.386	1.420	1.749	3.128	5.337	3.741	6.542	9.719	10.403
8/82	0.211	1.432	1.530	2.054	3.519	5.703	4.101	7.140	10.170	10.871
9/83	0.240	1.425	1.485	1.860	3.362	5.592	3.935	6.997	9.975	10.657
1/85	0.220	1.480	1.475	1.902	3.382	5.627	3.907	7.016	10.054	10.676
3/85	0.158	1.452	1.302	1.713	3.242	5.427	3.568	6.527	9.552	10.156
2/91	0.179	1.380	1.089	1.412	2.876	5.095	3.141	6.192	9.239	9.917
N. Val. (pF)	1.0E+05	2.0E+05	3.0E+05	4.0E+05	5.0E+05	6.0E+05	7.0E+05	8.0E+05	9.0E+05	1.0E+06
Mean (pF)	0.2736	1.5345	1.7639	2.3669	3.8938	6.1290	4.6467	7.7094	10.7772	11.5040
SD (pF)	0.0734	0.1126	0.3547	0.5138	0.5648	0.5893	0.8161	0.8759	0.8937	0.9458
Slope (pF/day)	-1.4E-05	-1.6E-05	-7.6E-05	-1.2E-04	-1.3E-04	-1.3E-04	-1.9E-04	-2.0E-04	-2.0E-04	-2.1E-04
(pF/year)	-0.005	-0.006	-0.028	-0.045	-0.048	-0.049	-0.070	-0.073	-0.073	-0.077
SDfit (pF)	0.047	0.081	0.162	0.166	0.205	0.224	0.274	0.335	0.341	0.370
SD(2/01) (pF)	0.062	0.109	0.217	0.223	0.275	0.301	0.368	0.449	0.458	0.496

Table B-7. Data and Analysis of the (1000 pF/step) Air Capacitor Dial Corrections of the Type-12 Bridge

(All values are in pF)

Date \ Dial	1000	2000	3000	4000	5000	6000	7000	8000	9000	10 000
6/67	0.060	0.010	0.189	0.109	0.189	0.154	0.334	0.403	0.353	0.403
11/67	0.075	0.017	0.253	0.118	0.204	0.156	0.388	0.463	0.392	0.460
5/68	0.055	0.009	0.184	0.113	0.185	0.145	0.317	0.379	0.336	0.391
9/69	0.069	0.021	0.232	0.136	0.214	0.172	0.381	0.454	0.400	0.463
12/70	0.074	0.025	0.244	0.150	0.241	0.196	0.406	0.487	0.431	0.503
9/72	0.072	0.040	0.208	0.146	0.231	0.211	0.394	0.473	0.434	0.499
6/74	0.069	0.040	0.198	0.183	0.259	0.239	0.396	0.465	0.428	0.485
11/75	0.081	0.050	0.244	0.213	0.305	0.279	0.461	0.545	0.509	0.588
1/77	0.071	0.049	0.184	0.219	0.298	0.286	0.425	0.491	0.466	0.531
4/78	0.070	0.064	0.192	0.235	0.313	0.307	0.432	0.502	0.481	0.543
11/80	0.071	0.062	0.242	0.233	0.330	0.318	0.509	0.591	0.563	0.642
8/82	0.086	0.071	0.241	0.248	0.336	0.338	0.510	0.593	0.580	0.653
9/83	0.090	0.070	0.259	0.256	0.357	0.347	0.535	0.629	0.607	0.695
1/85	0.087	0.071	0.244	0.261	0.366	0.361	0.529	0.624	0.605	0.690
3/85	0.094	0.080	0.231	0.277	0.380	0.375	0.523	0.613	0.587	0.671
2/91	0.109	0.098	0.226	0.255	0.352	0.366	0.523	0.629	0.631	0.724
N. Val. (pF)	1000	2000	3000	4000	5000	6000	7000	8000	9000	10 000
Mean (pF)	0.0771	0.0486	0.2232	0.1970	0.2850	0.2656	0.4414	0.5213	0.4877	0.5588
SD (pF)	0.0135	0.0269	0.0263	0.0595	0.0671	0.0834	0.0725	0.0828	0.0970	0.1086
Slope (pF/day)	4.4E-06	1.0E-05	3.3E-06	2.1E-05	2.4E-05	3.0E-05	2.5E-05	2.9E-05	3.5E-05	3.9E-05
(pF/year)	0.002	0.004	0.001	0.008	0.009	0.011	0.009	0.011	0.013	0.014
SDfit (pF)	0.007	0.005	0.026	0.020	0.022	0.022	0.028	0.032	0.026	0.032
SD(2/01) (pF)	0.010	0.007	0.034	0.027	0.029	0.029	0.037	0.042	0.035	0.042

Table B-8. Data and Analysis of the (100 pF/step) Air Capacitor Dial Corrections of the Type-12 Bridge

(All values are in pF)

Date \ Dial	100	200	300	400	500	600	700	800	900	1000
6/67	-0.003	0.125	0.092	0.075	0.067	0.200	0.167	0.165	0.302	0.300
11/67	-0.001	0.130	0.096	0.077	0.076	0.211	0.176	0.125	0.307	0.309
5/68	-0.002	0.127	0.096	0.083	0.071	0.204	0.166	0.165	0.298	0.297
9/69	0.000	0.132	0.098	0.082	0.076	0.209	0.173	0.175	0.309	0.308
12/70	-0.004	0.129	0.095	0.077	0.075	0.204	0.174	0.174	0.310	0.311
9/72	-0.004	0.131	0.097	0.083	0.078	0.211	0.180	0.172	0.312	0.314
6/74	0.000	0.133	0.096	0.089	0.086	0.215	0.179	0.179	0.308	0.305
11/75	-0.002	0.130	0.097	0.083	0.084	0.217	0.185	0.184	0.316	0.315
1/77	-0.002	0.130	0.093	0.085	0.079	0.211	0.184	0.176	0.313	0.308
4/78	-0.003	0.126	0.096	0.088	0.087	0.213	0.180	0.180	0.311	0.309
11/80	0.000	0.136	0.103	0.092	0.091	0.228	0.199	0.206	0.321	0.320
8/82	0.014	0.143	0.108	0.094	0.093	0.224	0.189	0.188	0.326	0.322
9/83	0.001	0.134	0.103	0.093	0.091	0.223	0.192	0.190	0.329	0.327
1/85	-0.001	0.131	0.107	0.102	0.101	0.221	0.189	0.188	0.322	0.325
3/85	0.001	0.129	0.101	0.092	0.093	0.226	0.192	0.193	0.318	0.321
2/91	0.013	0.138	0.097	0.093	0.081	0.223	0.210	0.208	0.318	0.339
N. Val. (pF)	100	200	300	400	500	600	700	800	900	1000
Mean (pF)	0.0004	0.1315	0.0984	0.0868	0.0831	0.2150	0.1834	0.1793	0.3138	0.3144
SD (pF)	0.0053	0.0046	0.0047	0.0074	0.0093	0.0085	0.0116	0.0191	0.0084	0.0108
Slope (pF/day)	1.3E-06	9.9E-07	1.1E-06	2.4E-06	2.7E-06	2.8E-06	4.0E-06	5.8E-06	2.6E-06	3.7E-06
(pF/year)	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.001	0.001
SDfit (pF)	0.004	0.004	0.004	0.004	0.006	0.004	0.005	0.012	0.005	0.005
SD(2/01) (pF)	0.006	0.005	0.005	0.005	0.008	0.006	0.006	0.016	0.007	0.007

Table B-9. Data and Analysis of the (10 pF/step) Air Capacitor Dial Corrections of the Type-12 Bridge

(All values are in pF)

Date \ Dial	10	20	30	40	50	60	70	80	90	100
6/67	0.007	-0.057	0.005	0.047	0.059	-0.015	0.047	0.059	-0.004	0.007
11/67	0.007	-0.060	0.006	0.051	0.059	-0.006	0.060	0.068	0.003	0.012
5/68	0.006	-0.056	0.009	0.055	0.065	-0.001	0.060	0.067	0.000	0.009
9/69	0.006	-0.061	-0.001	0.054	0.061	-0.003	0.062	0.069	0.003	0.014
12/70	0.006	-0.059	0.004	0.050	0.062	-0.005	0.059	0.072	0.005	0.011
9/72	0.004	-0.062	0.001	0.047	0.060	-0.005	0.056	0.069	0.004	0.013
6/74	0.006	-0.058	0.007	0.056	0.067	0.003	0.069	0.072	0.005	0.011
11/75	0.005	-0.059	0.006	0.053	0.058	-0.006	0.059	0.063	-0.001	0.011
1/77	0.006	-0.057	0.009	0.048	0.053	-0.008	0.054	0.071	0.007	0.013
4/78	0.009	-0.053	0.004	0.048	0.054	-0.008	0.056	0.072	0.010	0.017
11/80	0.006	-0.061	-0.001	0.051	0.057	-0.008	0.055	0.066	0.014	0.018
8/82	0.011	-0.053	0.010	0.061	0.061	-0.005	0.070	0.081	0.016	0.019
9/83	0.009	-0.052	0.007	0.056	0.064	0.006	0.073	0.079	0.010	0.018
1/85	0.010	-0.053	0.004	0.046	0.057	0.000	0.064	0.070	0.008	0.019
3/85	0.015	-0.052	0.008	0.050	0.069	0.005	0.064	0.068	0.009	0.016
2/91	0.013	-0.037	0.041	0.079	0.092	0.041	0.106	0.101	0.031	0.015
N. Val. (pF)	10	20	30	40	50	60	70	80	90	100
Mean (pF)	0.0079	-0.0556	0.0074	0.0533	0.0624	-0.0009	0.0634	0.0717	0.0075	0.0139
SD (pF)	0.0031	0.0060	0.0095	0.0080	0.0090	0.0124	0.0131	0.0094	0.0082	0.0037
Slope (pF/day)	8.6E-07	1.7E-06	1.9E-06	1.5E-06	1.6E-06	3.0E-06	3.3E-06	2.4E-06	2.6E-06	1.1E-06
(pF/year)	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
SDfit (pF)	0.002	0.004	0.008	0.007	0.008	0.010	0.010	0.007	0.005	0.002
SD(2/01) (pF)	0.003	0.006	0.011	0.010	0.011	0.013	0.014	0.010	0.006	0.003

APPENDIX C. CONDUCTANCE DIAL CORRECTIONS OF THE TYPE-12 BRIDGE

Data and analyses of conductance corrections for each mica capacitance dial of the Type-12 bridge are included in the following tables:

Table C-1. Data and Analysis of Conductance Corrections of (0.1 μF/step) Mica Capacitance Dial of the Type-12 Bridge at 100 Hz

(All values are in 0.001 μS, except the dial which is 0.1 μF/step)

DIAL	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	15.422	29.480	50.507	51.425	66.792	80.853	101.950	117.432	131.360	147.055
11/67	15.573	30.235	50.848	52.920	68.485	83.173	103.873	119.520	134.050	149.875
5/68	15.743	30.173	51.198	52.458	68.240	82.723	103.808	119.660	134.058	150.110
9/69	15.693	30.395	51.225	54.055	69.765	84.570	105.435	121.320	136.035	152.025
12/70	15.715	30.563	51.328	53.610	69.340	84.240	105.033	120.908	135.635	151.625
9/72	15.915	30.885	51.800	55.570	71.485	86.500	107.485	123.513	138.455	154.538
6/74	15.965	30.625	51.873	54.035	70.050	84.778	106.178	122.265	136.845	153.253
11/75	15.710	30.388	51.223	53.695	69.333	84.185	105.025	120.893	135.780	151.678
1/77	15.858	30.230	51.670	52.705	68.508	82.953	104.418	120.410	134.738	151.003
4/78	15.490	29.505	50.995	51.713	67.243	81.280	102.760	118.395	132.310	148.010
11/80	15.250	29.470	50.200	51.913	67.203	81.570	102.523	117.965	132.483	148.273
8/82	15.398	29.625	50.568	48.255	63.630	77.965	99.140	114.585	128.813	144.710
9/83	15.485	30.228	50.773	54.513	70.013	84.783	105.533	121.128	135.940	151.943
1/85	15.400	30.315	50.933	55.215	70.668	85.605	106.255	121.870	136.780	152.570
3/85	15.518	29.993	51.063	53.140	68.680	83.713	104.778	120.455	134.840	150.700
2/91*	14.625	28.197	48.720	49.175	63.505	76.885	97.022	111.475	124.565	139.132
Mean (nS)	15.5475	30.0192	50.9328	52.7748	68.3088	82.8610	103.826	119.487	133.918	149.781
SD (nS)	0.3191	0.6465	0.7473	1.9816	2.2459	2.6261	2.697	3.027	3.451	3.773
Slope(nS/day)	-7.8E-05	-1.3E-04	-1.5E-04	-2.2E-04	-3.1E-04	-3.5E-04	-4.0E-04	-4.9E-04	-5.4E-04	-6.2E-04
Slope(nS/year)	-0.0285	-0.0469	-0.0563	-0.0798	-0.1126	-0.1295	-0.1449	-0.1772	-0.1989	-0.2267
SDfit	0.2512	0.5690	0.6478	1.9616	2.1653	2.5382	2.5701	2.8362	3.2443	3.5147
SD(2/01) (nS)	0.3373	0.7639	0.8697	2.6335	2.9070	3.4077	3.4506	3.8078	4.3556	4.7188

* See Section 4.6.5

Table C-2. Data and Analysis of Conductance Corrections of (0.1 μF/step) Mica Capacitance Dial of the Type-12 Bridge at 1 kHz

(All values are in 0.001 μS, except the dial which is 0.1 μF/step)

DIAL	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	47.305	98.877	264.330	224.372	271.098	326.986	500.838	558.528	600.240	676.897
11/67	46.940	100.430	263.095	229.102	276.858	335.962	506.872	563.892	607.910	685.677
5/68	47.805	99.947	266.455	226.535	274.364	330.679	505.564	563.470	606.765	684.400
9/69	47.680	101.745	265.495	235.212	282.337	341.020	513.247	571.053	615.033	693.146
12/70	48.053	103.612	266.732	233.383	280.935	340.675	513.425	573.657	619.455	696.497
9/72	48.202	102.982	266.970	241.348	288.850	347.110	521.205	580.365	622.810	702.902
6/74	48.520	101.800	268.048	233.778	282.025	339.182	514.950	575.485	616.887	695.442
11/75	48.898	103.762	270.010	237.880	286.902	345.262	520.682	581.692	635.092	705.292
1/77	49.190	102.397	270.695	231.745	281.047	338.747	516.430	577.637	622.715	702.235
4/78	47.653	98.143	266.622	225.987	274.072	328.845	504.327	562.692	602.840	680.885
11/80	49.485	104.973	272.487	241.390	290.853	349.962	525.185	583.917	631.317	711.745
8/82	49.868	105.278	272.753	243.995	294.873	352.798	530.568	588.847	637.482	715.372
9/83	50.190	107.250	272.320	249.523	299.703	360.835	535.808	593.048	645.178	727.558
1/85	49.015	108.588	271.920	251.525	302.480	363.743	536.508	598.165	647.708	728.600
3/85	49.383	104.635	271.035	237.648	288.115	348.248	522.268	582.600	629.233	708.830
2/91	49.877	104.365	272.027	238.170	288.995	347.640	521.587	581.903	624.710	705.013
Mean (nS)	48.629	103.049	268.812	236.35	285.219	343.606	518.092	577.309	622.836	701.281
SD (nS)	1.0010	2.8719	3.2009	7.9901	9.0811	10.5141	10.7484	11.3459	14.3714	15.2700
Slope(nS/day)	3.3E-04	7.5E-04	1.1E-03	2.0E-03	2.5E-03	2.8E-03	3.0E-03	3.2E-03	3.8E-03	4.2E-03
Slope(nS/year)	0.1187	0.2750	0.3878	0.7221	0.9075	1.0133	1.0896	1.1755	1.3767	1.5399
SDfit	0.5267	2.1373	1.5763	6.2415	6.4693	7.7757	7.5324	7.7394	10.6895	10.7667
SD(2/01) (nS)	0.70714	2.86942	2.11632	8.37964	8.68548	10.4394	10.1127	10.3907	14.3514	14.4551

Table C-3. Data and Analysis of Conductance Corrections of (0.1 μF/step) Mica Capacitance Dial of the Type-12 Bridge at 10 kHz

(All values are in 0.001 μS, except the dial which is 0.1 μF/step)

DIAL	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.X
6/67	492.308	1332.120	3119.400	4189.440	4649.620	5955.480	8594.100	10269.700	9946.360	13599.400
11/67	494.150	1350.415	3135.955	4212.320	4746.648	6035.588	8716.003	10398.200	10250.800	13701.600
5/68	494.148	1332.136	3127.273	4148.161	4627.199	5926.961	8596.424	10266.500	10104.200	13631.000
9/69	486.015	1327.123	3093.330	4176.263	4637.970	5938.353	8662.910	10264.940	10127.130	13492.130
12/70	503.190	1387.243	3148.095	4202.098	4643.225	5896.328	8723.505	10416.780	10026.540	13419.540
9/72	488.310	1347.368	3120.208	4302.298	4807.788	6188.928	9006.043	10757.383	10395.990	13851.110
6/74	441.985	1194.242	2898.130	3874.418	4197.330	5365.218	7934.030	9787.693	9423.110	13694.270
11/75	509.012	1361.352	3197.980	4337.532	4874.010	6246.212	9003.290	10564.867	10717.750	15029.550
1/77	498.798	1333.823	3044.683	4012.843	4292.203	5370.488	7791.873	9196.882	8557.492	11750.903
4/78	487.290	1322.732	3090.928	4127.648	4585.618	5815.363	8360.133	9562.302	9014.248	11810.493
11/80	518.512	1381.355	3205.027	4311.050	4810.923	5885.795	8535.668	9385.540	9035.413	12330.785
8/82	509.590	1353.365	3176.718	4185.795	4634.123	5873.650	8586.728	9899.805	9608.808	13451.135
9/83	498.020	1373.128	3057.235	4127.068	4719.675	6011.483	8954.090	11046.698	11089.305	14631.913
1/85	488.340	1319.033	3133.200	4134.693	4883.835	6264.903	9145.970	10427.038	10508.105	14019.673
3/85	523.880	1418.883	3164.990	4217.098	4887.130	6133.763	8723.120	10486.753	10250.585	13853.518
2/91	514.700	1352.630	3168.623	4187.365	4862.783	6208.600	8941.918	10604.585	10574.028	14193.370
Mean (nS)	496.766	1342.93	3117.61	4171.63	4678.76	5944.82	8642.24	10208.48	9976.867	13528.77
SD (nS)	18.750	47.883	74.158	112.207	199.359	266.430	369.292	509.454	686.186	894.493
Slope(nS/day)	3.0E-03	4.4E-03	5.7E-03	2.7E-05	2.6E-02	2.5E-02	3.2E-02	1.1E-02	3.3E-02	3.6E-02
Slope(nS/year)	1.077	1.602	2.082	0.010	9.398	9.024	11.757	4.157	11.984	13.074
SDfit	17.639	48.080	75.149	116.146	193.896	267.313	371.903	526.409	704.537	920.661
SD(2/01) (nS)	23.6814	64.5503	100.892	155.934	260.319	358.886	499.305	706.7409	945.8904	1236.052

Table C-4. Data and Analysis of Conductance Corrections of (0.01 uF/step) Mica Capacitance Dial of the Type-12 Bridge at 100 Hz

(All values are in 0.001 μS, except the dial which is 0.01 μF/step)

DIAL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	1.507	2.643	5.015	6.565	8.083	9.217	11.580	13.102	14.257	15.782
11/67	1.520	2.680	5.098	6.655	8.165	9.320	11.723	13.273	14.450	16.035
5/68	1.543	2.710	5.138	6.698	8.285	9.465	11.898	13.465	14.628	16.193
9/69	1.603	2.765	5.165	6.760	8.315	9.463	11.888	13.448	14.590	16.160
12/70	1.575	2.743	5.193	6.720	8.280	9.465	11.885	13.460	14.658	16.205
9/72	1.478	2.660	5.143	6.700	8.255	9.450	11.973	13.573	14.785	16.365
6/74	1.588	2.725	5.248	6.830	8.360	9.560	11.993	13.585	14.798	16.320
11/75	1.518	2.683	5.218	6.750	8.245	9.503	11.850	13.440	14.570	16.088
1/77	1.430	2.555	5.048	6.598	8.228	9.430	11.855	13.440	14.590	16.115
4/78	1.490	2.628	4.980	6.478	7.950	9.103	11.465	13.023	14.185	15.728
11/80	1.465	2.635	4.993	6.490	8.065	9.130	11.440	12.985	14.130	15.593
8/82	1.478	2.563	4.895	6.410	7.923	9.118	11.438	13.038	14.180	15.648
9/83	1.583	2.855	5.125	6.513	8.060	9.178	11.705	13.228	14.263	15.790
1/85	1.613	2.730	5.145	6.523	8.070	9.195	11.603	13.125	14.293	15.825
3/85	1.520	2.728	5.128	6.563	8.080	9.220	11.613	13.158	14.313	15.838
2/91	1.857	2.987	5.337	6.717	8.070	8.880	11.190	12.730	13.397	14.665
Mean (nS)	1.5480	2.7056	5.1168	6.6231	8.1521	9.2936	11.6937	13.2546	14.3804	15.8969
SD (nS)	0.0981	0.1062	0.1110	0.1217	0.1321	0.1907	0.2307	0.2452	0.3420	0.4072
Slope(nS/day)	1.5E-05	1.6E-05	4.0E-06	-1.9E-05	-3.0E-05	-5.1E-05	-5.8E-05	-6.1E-05	-9.0E-05	-1.1E-04
Slope(nS/year)	0.0056	0.0057	0.0015	-0.0071	-0.0108	-0.0186	-0.0213	-0.0222	-0.0330	-0.0402
SDfit	0.0923	0.1013	0.1144	0.1143	0.1099	0.1391	0.1775	0.1910	0.2525	0.2938
SD(2/01) (nS)	0.12393	0.13601	0.15356	0.15347	0.14756	0.18677	0.23836	0.25649	0.33897	0.39448

Table C-5. Data and Analysis of Conductance Corrections of (0.01 uF/step) Mica Capacitance Dial of the Type-12 Bridge at 1 kHz

(All values are in 0.001 μS, except the dial which is 0.01 μF/step)

DIAL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	5.743	9.678	16.308	22.865	28.638	32.697	39.490	45.435	49.357	55.620
11/67	5.747	9.888	16.678	23.072	28.872	33.133	40.125	45.860	49.870	55.987
5/68	5.778	9.775	16.505	23.045	28.835	32.940	39.813	45.813	49.832	56.142
9/69	5.798	9.818	16.533	23.013	28.865	33.032	39.927	45.962	50.045	56.322
12/70	5.785	9.995	16.883	23.275	29.055	33.432	40.415	46.432	50.720	56.987
9/72	5.840	10.003	16.707	23.185	28.975	33.157	40.120	46.087	50.302	56.652
6/74	5.808	9.813	16.593	23.195	29.058	33.179	40.043	46.030	50.045	56.470
11/75	5.658	9.728	16.508	22.978	28.873	33.115	40.135	46.252	50.412	56.935
1/77	5.700	9.758	16.370	22.943	28.693	33.050	39.928	46.060	50.310	56.882
4/78	5.638	9.615	16.178	22.620	28.350	32.490	39.077	44.880	48.905	55.043
11/80	5.765	10.050	16.935	23.273	29.130	33.418	40.260	46.403	50.460	56.905
8/82	5.760	9.863	16.560	22.878	28.853	33.140	40.085	46.375	50.520	57.110
9/83	5.758	10.023	16.825	23.138	28.860	33.345	40.420	46.643	51.475	57.865
1/85	5.578	9.870	16.720	22.760	28.635	33.143	40.200	46.363	50.548	56.990
3/85	5.743	9.990	16.765	22.905	28.693	33.098	40.068	46.043	50.440	56.763
2/91	5.780	9.920	16.473	22.868	28.738	33.160	39.875	45.707	49.972	56.815
Mean (nS)	5.7424	9.8617	16.5963	23.0008	28.8202	33.0956	39.9988	46.0216	50.2008	56.5930
SD (nS)	0.0680	0.1303	0.2094	0.1851	0.1949	0.2389	0.3358	0.4336	0.5799	0.6626
Slope(nS/day)	-7.4E-06	1.7E-05	1.3E-05	-2.3E-05	-1.6E-05	2.2E-05	2.1E-05	3.8E-05	8.0E-05	1.3E-04
Slope(nS/year)	-0.0027	0.0062	0.0049	-0.0083	-0.0058	0.0082	0.0077	0.0138	0.0293	0.0460
SDfit	0.0674	0.1267	0.2136	0.1812	0.1969	0.2396	0.3427	0.4367	0.5583	0.5923
SD(2/01) (nS)	0.09046	0.17008	0.28677	0.24331	0.26441	0.32166	0.46009	0.58627	0.74954	0.79517

Table C-6. Data and Analysis of Conductance Corrections of (0.01 uF/step) Mica Capacitance Dial of the Type-12 Bridge at 10 kHz

(All values are in 0.001 μS, except the dial which is 0.01 μF/step)

DIAL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.0X
6/67	36.761	73.168	128.885	183.211	227.610	275.876	352.813	414.849	453.899	552.168
11/67	36.918	74.121	130.590	183.374	227.313	276.509	355.158	417.483	457.254	556.228
5/68	37.018	73.790	129.590	183.465	227.440	275.523	353.373	414.815	455.213	551.212
9/69	37.225	68.573	124.640	179.008	222.140	272.695	350.403	412.085	445.650	544.008
12/70	37.393	75.473	132.605	186.113	230.045	280.003	357.448	419.518	460.445	559.753
9/72	35.573	72.020	125.523	178.633	220.370	267.068	341.600	401.893	442.503	538.615
6/74	35.543	70.820	125.855	175.913	215.863	263.580	334.605	392.242	428.757	496.662
11/75	37.330	74.835	131.513	187.008	230.460	280.780	358.585	424.585	464.905	563.802
1/77	35.370	71.388	125.915	179.930	221.373	269.292	341.165	404.885	444.090	548.910
4/78	35.638	71.273	125.816	179.858	222.605	269.015	342.115	400.530	441.767	542.245
11/80	37.193	77.985	132.345	186.200	230.512	282.745	358.795	417.550	468.235	574.872
8/82	37.413	75.218	133.318	187.315	231.468	280.923	361.620	423.888	466.238	563.078
9/83	35.608	72.873	129.023	177.423	217.978	270.463	349.908	406.953	440.248	542.608
1/85	39.015	76.540	133.590	184.778	231.168	283.430	360.645	415.523	463.850	556.068
3/85	35.303	73.620	129.383	181.688	223.143	273.448	351.205	413.033	455.255	563.633
2/91	36.652	73.875	128.665	181.350	223.655	273.513	348.088	410.108	450.418	551.593
Mean (nS)	36.6221	73.4733	129.204	182.204	225.196	274.679	351.095	411.871	452.42	550.341
SD (nS)	1.0318	2.3405	2.9816	3.5414	4.9637	5.8793	7.8936	8.7564	11.173	17.2627
Slope(nS/day)	-9.2E-06	2.8E-04	2.8E-04	2.7E-05	-5.8E-05	2.6E-04	2.5E-04	-6.5E-05	4.8E-04	1.2E-03
Slope(nS/year)	-0.0034	0.1029	0.1015	0.0100	-0.0213	0.0953	0.0912	-0.0237	0.1739	0.4550
SDfit	1.0678	2.2960	2.9904	3.6649	5.1355	6.0434	8.1419	9.0620	11.4910	17.5384
SD(2/01) (nS)	1.43354	3.08257	4.01479	4.92038	6.89472	8.11363	10.931	12.1664	15.4275	23.5465

☆U.S. GOVERNMENT PRINTING OFFICE: 2000-460-746-17124

THE SP 250 SERIES ON NIST MEASUREMENT SERVICES*

* Entries containing a stock number (SN003–003–) and price can be purchased from the Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402-9325. GPO will accept checks, money orders, VISA, and MasterCard. For more infomation, or to place an order, call (202) 512-1800. Be sure to cite the stock number on all orders.

Entries containing PB numbers can be purchased from the National Technical Information Service, Springfield, VA 22161. NTIS will accept American Express in addition to the payment methods listed for GPO. For more information call (703)487-4650; to place an order call (800) 553–6487. Fax: (703) 321–8547. Be sure to cite the PB number on all orders.

Entries without stock or PB numbers are in preparation.

SP 250–45 Radiation Processing Dosimetry Calibration Services: Manual of Calibration Procedures
SN003-003-03514-9 $4.00

SP 250–46 NIST Multifunction Calibration System
SN003-003-03515-7 $2.75

SP 250–47 NIST Calibration Service for Capacitance Standards at Low Frequencies
SN003-003-03549-1 $7.00

SP 250–48 Spectral Reflectance
SN003-003-03545-9 $14.00

SP 250–49 NIST Calibration Services for Gas Flow Meters: Piston Prover and Bell Prover Gas Flow Facilities
SN003-003-03560-2 $6.00

SP 250–51 Calibration Service of Optoelectronic Frequency Response at 1319 nm for Combined Photodiode/rf Power Sensor Transfer Standards
SN003-003-03623-4

SP 250–52 Error Analysis and Calibration Uncertainty of Capacitance Standards at NIST

SP 250–53 Calibration Service for Spectral Responsivity of Laser and Optical-Fiber Power Meters at Wavelengths Between 0.4 μm and 1.8 μm
SN003-003-03624-2

* Entries containing a stock number (SN003–003–) and price can be purchased from the Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402-9325. GPO will accept checks, money orders, VISA, and MasterCard. For more infomation, or to place an order, call (202) 512-1800. Be sure to cite the stock number on all orders.

Entries containing PB numbers can be purchased from the National Technical Information Service, Springfield, VA 22161. NTIS will accept American Express in addition to the payment methods listed for GPO. For more information call (703)487-4650; to place an order call (800) 553–6487. Fax: (703) 321–8547. Be sure to cite the PB number on all orders.

Entries without stock or PB numbers are in preparation.

U.S. Department of Commerce
National Institute of Standards
and Technology
Gaithersburg, MD 20899–0001