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Development of a Probability Based Load Criterion for American National Standard A58

Building Code Requirements for Minimum Design Loads in Buildings and Other Structures

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ABSTRACT

Recommended load factors and load combinations are presented which are compatible with the loads in the proposed 1980 version of American National Standard A58, Building Code Requirements for Minimum Design Loads in Buildings and Other Structures. The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load factors were developed using concepts of probabilistic limit states design which incorporate state-of-the-art load and resistance models and available statistical information. Reliabilities associated with representative structural members and elements designed according to current (1979) structural specifications were calculated for reinforced and prestressed concrete, structural steel, cold-formed steel, aluminum, masonry and glued-laminated timber construction. The report presents the rationale for selecting the criterion format and load factors and describes the methodology to be followed by material specification groups for determining resistance factors consistent with the implied level of reliability and the statistical data. The load factors are intended to apply to all types of structural materials used in building construction.

Key words: Aluminum; buildings (codes); design (buildings); concrete (prestressed); concrete (reinforced); limit states; loads (forces); masonry; probability theory; reliability; safety; specifications; standards; statistical analysis; steel; structural engineering; timber.

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EXECUTIVE SUMMARY

American National Standard Committee A58 periodically issues revisions to ANSI Standard A58 - "Building Code Requirements for Minimum Design Loads in Buildings and Other Structures." This document defines magnitudes of dead, live, wind, snow and earthquake loads suitable for inclusion in building codes and other regulatory documents. The A58 Standard Committee is a broad-spectrum group of professionals from the research community, building code groups, industry, professional organizations and trade associations. Their approval of a proposed standard signifies that a consensus of those substantially concerned with its scope and provisions has been reached, in that affected parties have had an opportunity to comment on the standard prior to its implementation and opposing points of view have been treated fairly.

The A58 Standard is concerned solely with structural loadings. The specification of specific allowable stresses or design strengths for materials of construction is outside its scope. The current version of the A58 Standard, ANSI A58.1-1972, is being revised, with a tentative approval and publication date set for 1980.

This report addresses itself to changes to the A58 Standard which may occur subsequent to the 1980 revision. Its purpose is to develop a load criterion, including load factors and load combinations, which would be suitable for limit states design with different materials and methods of construction. The current standard already contains a set of load combinations and probability factors for allowable stress design. This Executive Summary is presented to review briefly the conclusions of the main report, giving an overview of the recommendations and a concise rationale for their development.

Objectives:

1) To recommend a methodology and set of load factors and corresponding load definitions for use in the A58 Standard which would be appropriate for all types of building materials (e.g., structural steel, reinforced and prestressed concrete, heavy timber, engineered masonry, cold-formed steel, aluminum) and, in the future, for building foundations; and

2) To provide a methodology for the various material specification groups to select resistance factors (ϕ) consistent with these load factors and their own specific objectives.

Rationale:

Structural design is a complex process involving iterative cycles of analyzing the performance of idealized structures. Each analysis cycle involves the checking of subassemblies, members, components and connections against various limit states* defined in a structural specification dealing with the particular structural material. Typically this checking process involves satisfying a design criterion of the general form:

Factored Resistance $\geq$ Effect of factored loads.

In the common case where the total load effect is a linear combination of individual loads,

$$\phi R_n \geq \sum_{i=1}^{n} \gamma_i Q_i$$

In this formula the left side reflects the resistance (capacity) of the structural element under consideration, and the right side denotes the forces which the element is expected to support during its intended life (load effects). The term R_n is a nominal resistance corresponding to a limit state (e.g., maximum moment which can be carried by a cross section, buckling load, shear capacity), and ϕ is the "resistance factor," which is less than unity and which reflects the degree of uncertainty associated with the determination of the resistance. The sum γQ is the product of the "load effect" Q (i.e., the force on the member or the element - bending moment, shear force, torque, axial force - or the stress on the component) due to the loading from different structural loads (e.g., dead load, live load due to occupancy, wind load, snow load, earthquake load) and a load factor γ, generally larger than unity, which accounts for the degree of uncertainty inherent in the determination of the forces Q. When nonlinearities in behavior are significant, the load factor should be applied before performing the structural analysis.

In a more general sense ϕR_n may represent a number of limit states (e.g., yielding and tensile strength in a metal tension member) for each element, and $\sum_{i=1}^{n} \gamma_i Q_i$ reflects the largest of several load combinations. A substantial portion of this report is devoted to the determination of values for these γ_i. Using as an example a metal tension member, the following combinations might be checked:

$$\left.\begin{matrix} \phi_y F_y A_n \\ \\ \phi_u F_u A_n \end{matrix}\right\} \geq \left\{\begin{matrix} \gamma_D D_n + \gamma_L L_n \\ \\ \gamma_D D_n + \gamma_{L_1} L_n + \gamma_W W_n \end{matrix}\right.$$

where ϕ_y and ϕ_u are the resistance factors for the yield limit state, F_y, and the tensile strength limit state, F_u, respectively, A_n is the net area, D_n, L_n and W_n are the load

*A glossary of terms is presented in Chapter 9.

effects due to dead, live and wind load, respectively, γ_D, γ_L and γ_W are the load factors for the maximum loads; $\gamma_{L_1} < \gamma_L$ because the live load which is expected on the member at any particular point in time is less than the maximum live load. The load combination which involves the wind load thus reflects the fact that it is not expected that the maximum live load and the maximum wind load will act simultaneously. Traditionally this unexpected simultaneity has been dealt with by multiplying the factor of safety by 3/4 or by increasing allowable stresses by 4/3. The method suggested here is a better reflection of what actually takes place.

The proposed design process thus defines the appropriate limit states, and hence it is often named Limit States Design. Limit states design, in itself, is nothing fundamentally new but is a procedure which, in effect, requires the designer to consider explicitly several different modes of possible structural behavior during design. The particular method above also identifies resistance factors and load factors, and so it is called Load and Resistance Factor Design; it is one (of several) limit states design criteria formats.

Broadly speaking, there are two types of limit states: (1) ultimate limit states under which the structure or component is judged to have failed in its capacity to carry load; and (2) serviceability limit states under which the function of the building is impaired. The recommendations in this report are confined to the ultimate limit states as these are of particular concern in standards and specifications which are intended to protect the public from physical harm.

The recommended load and resistance factor design format which incorporates limit states, resistance factors, load factors and load combinations is a formalization of trends evident in many structural specifications in the United States. It provides a means whereby it is possible to achieve more uniform performance and reliability in structural design than is possible with just one factor of safety. This has long been recognized in reinforced concrete design. Current research in metal structures has also produced tentative rules which apply to steel, cold-formed steel, and aluminum structures. The thesis of this report is that it is also desirable to provide common load combinations and load factors which can be used in connection with all material specifications. This point will be elaborated upon subsequently.

The recommended approach requires that procedures be available to determine values for the resistance factors and the load factors. The development of the load criterion is carried out within the context of probabilistic limit states design. This is because the reliability of a structure or element is defined in a natural way by the probability of not achieving any of its limit states. The procedure used herein is based on modern engineering reliability analysis methods which have been developed, tested and refined over the last decade. The details of the method are described elsewhere in this report. For our purposes here it suffices to say that given a structural member or element designed according to a current structural specification, it is possible to compute the relative reliability of this design from data defining probability distributions and statistics of the resistance, the loads and the load effects. This relative reliability is expressed as a number called the reliability index, β. This index usually varies from 2 to 8, depending on the structure type and loading. By repeatedly determining β for many structural designs, the relative reliability of different structural members built from different structural materials can be compared. If representative values of β are now selected, reflecting the averaged reliability of satisfactory current designs, it is again possible by using reliabilit analysis methods to compute resistance and load factors. It should be clearly pointed out that this process is elaborate, and it is performed as a research operation for use by standard and specification-writing bodies. The designer would only use the standard specified values of ϕ and γ in the structural design operation.

The underlying average reliability β is (1) not necessarily the same for all types of building materials (and there is no reason to force the design profession to adopt a uniform value), and (2) the values of ϕ and γ depend not only on β but also on the load and the resistance statistics. Thus, it is quite likely that if the methodology were applied to each material separately, different values of the load factors γ would be obtained for, say, steel structures and masonry structures. This is an entirely logical consequence of the probabilistic methodology used. However, the use of different load factors for different structural material specifications is undesirable in the design office and results in confusion, especially in structures where the design calls for a mix of materials, say reinforced concrete, structural steel and aluminum (e.g., slabs, frame and curtain walls). It thus was deemed desirable to determine uniform load factors which could be included in the A58 Standard for all structural materials and to provide a means whereby individual material specification writing groups could select suitable nominal resistances and resistance factors corresponding to the load criterion and whatever values of β they desire. The use of common load factors would simplify the design process, particularly when more than one construction material is used in a structure. Various standard groups in the United States agree that the A58 Standard is the logical place for this load criterion inasmuch as it is a national standard and requires consensus approval and public review of the criteria prior to their implementation.

Summary of Procedure:

The details of achieving the objectives discussed above are given in the body of this report, with further details and statistics being provided in the Appendices. The following is only an abbreviated description of the procedure. This consists basically of using a probabilistic safety analysis to guide the selection of load factors that produce desired levels of uniformity in safety which are consistent with existing general practice.

Step 1 Estimate the level of reliability implied by the use of the various current design standards and specifications (e.g. ACI Standard 318, AISC Specifications, etc., and loads from ANSI Standard A58.1-1972) for various common types of members and elements (e.g., beams, columns, beam-columns, walls, fillet welds) using

a) a particular common reliability calculation scheme (Chapter 2);

b) common and realistic best estimates of distribution types and parameters (Chapter 3 and Appendices);

c) the reliability index β as a safety measure for comparison.

Step 2 Observe the β-levels over ranges of material, limit states, nominal load ratios (e.g., live-to-dead, wind-to-dead, snow-to-dead), load combinations, and geographical locations (Chapter 4).

From Steps 1 and 2 it was found that a level of $\beta = 3.0$ was consistent with average current practice for load combinations involving dead plus live or dead plus snow loads, while $\beta = 2.5$ and $\beta = 1.75$ were appropriate for combinations containing wind and earthquake loads, respectively.

Step 3 Based on the observed β levels, determine load factors consistent with the implied safety level and the selected safety checking format. These load factors are compatible with the nominal load definitions in the proposed ANSI A58.1-1980 Standard currently being developed.

From Step 3 the following load combinations and load factors were derived (see Chapter 5 for details)

$1.4\ D_n$

$1.2\ D_n + 1.6\ L_n$

$1.2\ D_n + 1.6\ S_n + (0.5\ L_n \text{ or } 0.8\ W_n)$

$1.2\ D_n + 1.3\ W_n + (0.5\ L_n)$

$1.2\ D_n + 1.5\ E_n + (0.5\ L_n \text{ or } 0.2\ S_n)$

$0.9\ D_n - (1.3\ W_n \text{ or } 1.5\ E_n)$

in which D_n = dead load, L_n = occupancy live load, W_n, S_n = 50-year mean recurrence interval wind and snow loads, and E_n = earthquake load.

Step 4 Display the relationships between the implied β-levels for these load factors and nominal loads for the material statistics (mean resistances, coefficients of variation) against alternate ϕ-factors. These charts are given in Chapter 5, together with example determinations of ϕ for several structural types and materials. This information generally would be sufficient to enable a specification writing group, if it so desires, to select ϕ-factors without further computer operations.

Some Particular Critical Issues

1. The selected load factors do not prevent material specification writing groups from selecting their own ϕ factors together with their own desired values of β. There is no intent here to dictate particular values of ϕ or β to be used in material specifications. Only the load factors are presented along with preliminary resistance variable information and a method by which β can be estimated for any particular ϕ that might be proposed by the material groups for their own specifications. If this procedure is used, material groups do not have to deal with loads to harmonize their own safety levels among their various limit states. If desired, for example, different values of β could be used for bending and shear in concrete structures, or members and connectors in steel structures. The information given also permits the observation of relative safety levels in current practice in several material technologies which may assist material specification groups in selecting their own values of β and ϕ for design.

2. The results of this work, as detailed in the main report, show some differences in β-levels from material to material, limit state to limit state, member type to member type, and especially, from load type to load type. In particular, reliability with respect to wind or earthquake loads appears to be relatively low when compared to that for gravity loads (i.e., dead, live and snow loads), at least according to the methods used for structural safety checking in conventional design. These are methods which are simplified representations

of real building behavior and they have presumably given satisfactory performance in the past. It was decided to propose load factors for combinations involving wind and earthquake loads that will give calculated β values which are comparable to those existing in current practice, and not to attempt to raise these values to those for gravity loads by increasing the nominal loads or the load factors for wind or earthquake loading. Based on the information given here the profession may well feel challenged (1) to justify more explicitly (by analysis or test) why current simplified wind and seismic calculations may be yielding conservative estimates of loads, resistances and safety; (2) to justify why current safety levels for gravity loads are higher than necessary if indeed this is true; (3) to explain why lower safety levels are appropriate for wind and earthquake vis-à-vis gravity loads, or (4) to agree to raise the wind and seismic loads or load factors to achieve a similar reliability as that inherent in gravity loads. While the writers feel that arguments can be cited in favor and against all four options, they decided that this report was not the appropriate forum for what should be a profession-wide debate.

The method of obtaining the load factors and resistance factors presented in this report is general in its applicability. However, the data used herein restrict the utilization of the results to buildings and similar structures. They are not intended for vehicular loads on bridges, transients in reactor containments, and other loads which are considered to be outside the scope of the A58 Standard.

Future Action

The writers expect that the loading criterion presented in this report will be carefully scrutinized by numerous professional organizations and individuals who have interest in or are affected by the scope and provisions of the A58 Standard. The writers feel that a discussion of the recommendations is extremely important, in view of the implications that the adoption of these recommendations would have on structural design in the United States.

The decision as to whether to incorporate the load criterion in a future edition of the A58 Standard lies with the A58 Standard Committee. After an appropriate period of review and public discussion, a draft provision will be prepared containing the load combinations and load factors which will be submitted for ballot by the A58 Standard Committee in accordance with ANSI voluntary consensus standard approval procedures. If approved, the load criterion will become part of the A58 Standard. It will then be up to material specification writing groups to decide whether they wish to adapt their standards to this load criterion in the interest of harmonizing structural design.

1. PROBABILITY-BASED LIMIT STATES DESIGN

This report proposes a series of probability-based load factors for use in the design of building structures. This chapter will define some of the terms used and will discuss why this design process is desirable.

1.1 Limit States Design

When a structure or structural element becomes unfit for its intended purpose it is said to have reached a limit state. For most structures the limit states can be divided into two categories:

Ultimate Limit States are related to a structural collapse of part or all of the structure. Such a limit state should have a very low probability of occurrence since it may lead to loss of life and major financial losses. The most common ultimate limit states are:

a) loss of equilibrium of a part or the whole structure considered as a rigid body (e.g. overturning, uplift, sliding);

b) loss of load-bearing capacity of members due to exceeding the material strength, buckling, fracture, fatigue or fire;

c) Spread of initial local failure into widespread collapse (progressive collapse or lack of structural integrity);

d) very large deformation - transformation into a mechanism, overall instability (e.g. wind flutter, ponding instability).

Serviceability Limit States are related to disruption of the functional use of the structure and/or damage to or deterioration of the structure. Since there is less danger of loss of life, a higher probability of occurrence may be tolerated than in the case of the ultimate limit states. For buildings the following limit states may be important:

a) excessive deflection or rotation affecting the appearance, functional use or drainage of the building or causing damage to non-structural components and their attachment;

b) excessive local damage (cracking or splitting, spalling, local yielding or slip) affecting appearance, use or durability of the structure;

c) excessive vibration affecting the comfort of the occupants or the operation of equipment.

These, in turn, could be divided into groups depending on the load levels to be considered in checking them or the lasting effects of their occurrence.

Limit States Design is a process that involves:

(1) Identification of all modes of failure or ways in which the structure might fail to fulfill its intended purpose (limit states).

(2) Determination of acceptable levels of safety against occurrence of each limit state.

(3) Consideration by the designer of the significant limit states.

In the design of a normal building, Steps 1 and 2 have already been carried out by the standard committee. The design specification lists the limit states to be considered and presents load and resistance factors for use in checking these limit states. For normal structures, the designer carries out Step 3, generally starting with the most critical limit states for the structure in question. The designer of an unusual structure may have to consider all three steps.

The limit states design procedure is, in effect, the traditional engineering design procedure formalized to require specific consideration of the various limit states. Under limit states design, the design of the structure for a bridge or building generally starts with satisfaction of the ultimate limit states followed by checks of the serviceability limit states. The latter checks are either carried out explicitly (by calculating deflections, for example) or by using "deemed to satisfy" clauses such as maximum slenderness ratios, etc. This order of calculation is followed because generally the major functional requirement (major limit state) of the structural components for a building or bridge is to support loads safely. This may not always be true, however. For example, in the design of a water tank or similar sanitary engineering structure, the major functional requirement is that the tank hold water without leaking. Here the order of the design process may well start with consideration of ways to prevent leakage and conclude with checks of whether the resulting strength is adequate.

In this context, then, the strength design procedure presented in the ACI Standard 318 [19]*, and the Load and Resistance Factor Design procedure [9] are limit states design procedures. Ideally, however, the complete limit states design concept should be followed because, all too often in the past, designers and specification writers have given their prime attention to the ultimate limit states and not enough to the factors which might render the building unsatisfactory in everyday use.

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*Numbers in brackets denote references listed in Section 8.

1.2 Methods of Establishing Safety Levels

1.2.1 Allowable Stress or Working Stress Design

Traditionally, structural design has been based on code-specified or service loads and the desired safety has been assumed to exist if the elastically computed stresses did not exceed allowable working stresses which were a preset fraction of the yield strength, crushing strength, modulus of rupture, etc. The loads used in this design process have a high probability of occurrence during the life of the structure. Thus, for example, the dead load is calculated directly from the specified dimensions and assumed densities and is close to the expected dead load. The allowable stresses have been set in an empirical manner to reflect the profession's feeling about the relative variability of various materials. Earlier versions of the ACI Code (for example, the 1951 code) based design on allowable stresses of 0.225 to 0.45 times the concrete strength and 0.5 times the yield strength of the reinforcement; the AISC Specification [26] bases structural steel design on allowable stresses of 0.66 times the yield strength for compact sections in bending; timber specifications base design on 0.2 to 0.25 times the short-term strength of small clear specimens.

The advantages of working stress design are:

(i) Designers are familiar with it and it is simple to apply. The moments or forces from each load are calculated and added together. The resulting sums are multiplied by load combination or probability factors ranging from 1.0 to 0.66 and are used to proportion sections so that the stresses do not exceed the allowable values.

(ii) Structures designed this way are generally believed to behave satisfactorily in service. By keeping stresses low at service loads, deflections, vibrations, crack widths in concrete beams, and the like, were seldom critical. While this was generally true for the types of materials and structures used prior to 1950, the advent of high strength steels and concretes, prestressed concrete and other lightweight structures have made serviceability checks necessary in many more instances.

Working stress design also has some disadvantages:

(i) A given set of allowable stresses will not guarantee a constant level of safety for all structures. Consider two roof structures designed for the same snow load using the same allowable stresses. One structure, a reinforced concrete beam and slab structure, has considerably higher dead load than the other, a reinforced concrete folded plate.

Because the dead load can be estimated with much more precision than the snow load, the roof having the high ratio of dead to live load will have a lower probability of failure than the lighter structure.

(ii) The working stress format may be unsafe when one load counteracts the effects of another. This is especially true when the effect of a relatively predictable dead load counteracts the effect of a highly variable load such as wind. Figure 1.1 shows such a structure designed using working stress design. The tensile and compressive strengths are 200 psi and 1800 psi respectively (1.38 and 12.4 N/mm^2) and, as shown in Fig. 1.1 (c), the dead load has been chosen so that the maximum stresses at service loads (1.0 Dead + 1.0 Wind) are 50 percent of the respective strengths. As shown in Fig. 1.1 (d) an increase of only 20 percent in the wind load is enough to raise the stress at A from half of the tensile strength to the tensile strength. The failure of the Ferrybridge Cooling Towers in England has been attributed to this cause [20].

In summary, then, the main advantage of working stress design is its simplicity; however, it can lead to designs with less safety than normally considered adequate, particularly if loads counteract each other.

1.2.2 Strength Design

Safety provisions in several design standards are based on the ultimate strength of critical member sections (strength design of reinforced concrete in ACI Standard 318, for example) or the load carrying capacity of members and entire frames (Section 2 of the AISC Specifications). In these and similar standards, design is based on factored loads and factored resistances. The loads are amplified or reduced by load factors depending on the type and sense of the load, while the strengths are reduced by resistance factors less than or equal to unity. For example, ACI Standard 318 bases design in flexure against gravity loads on

$$0.9R_n \geq 1.4\ D_n + 1.7\ L_n \qquad (2.1)$$

while Section 2 of AISC Specifications requires that

$$1.0\ R_n \geq 1.7\ (D_n + L_n) \qquad (2.2)$$

Note that these are both load and resistance factor design formats.

Criteria of this type are an attempt to apply partial factors of safety to those variables in the design equation which are known to be unpredictable. Eq. 2.1 attempts to account for the possibility of understrength and overload, while Eq. 2.2 apparently accounts

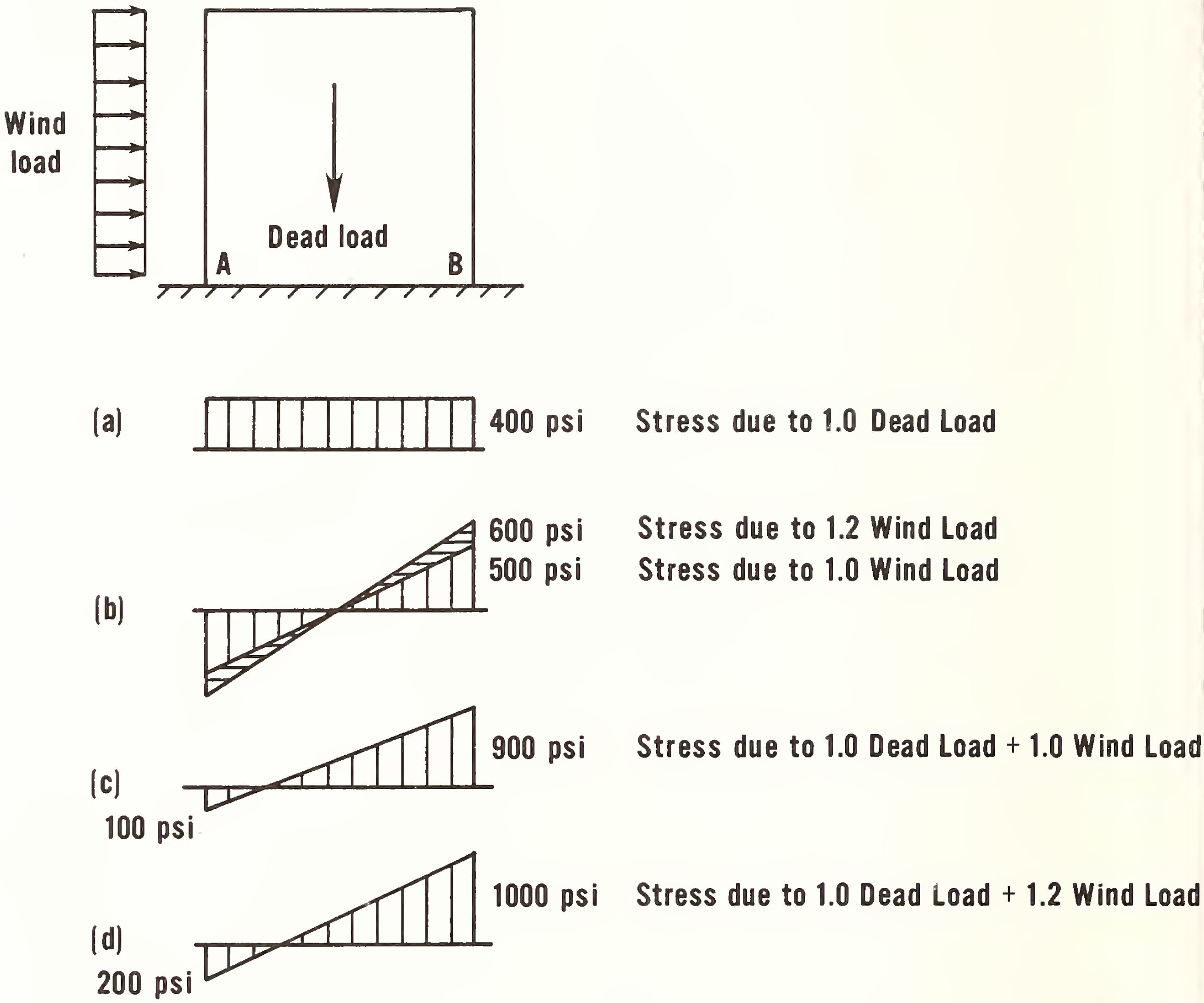


Figure 1.1 - Working Stress Design with Counteracting Loads (1 psi = 6.9 kN/m^2)

only for overload (its resistance factor is unity). Assigning a larger factor to live load than dead load reflects the fact that the variability in live load is known to be larger than dead load and thus is a tacit attempt to make the safety more uniform over the range of likely L_n and D_n values.

However, the load and resistance factors have been selected more or less on the basis of subjective judgment in the past. While they may seem reasonable intuitively, there is no assurance that the design criteria are entirely consistent with the performance objectives of the groups that develop them. In the context of the limit states design process discussed in Section 1.1, Step 2 cannot be completed in a rational manner.

1.2.3 Probability-Based Limit States Design

In Section 1.1, limit states design was defined as being a three stage procedure, the second stage of which involves determination of acceptable levels of safety against the occurrence of each limit state. In probability-based limit states design, probabilistic methods are used to guide the selection of load factors and resistance factors which account for the variabilities in the individual loads and resistances and give the desired overall level of safety. This is described further in Chapter 2. It should be emphasized that the designer deals with load factors and resistance factors similar to those in Eqs. 2.1 and 2.2 and is never required to consider probabilities per se. The particular format adopted in this report is referred to as load and resistance factor design (LRFD).

The principal advantages of probabilistic limit states design are:

(i) More consistent reliability is attained for different design situations because the different variabilities of the various strengths and loads are considered explicitly and independently.

(ii) The reliability level can be chosen to reflect the consequences of failure.

(iii) It gives the designer a better understanding of the fundamental structural requirements and of the behavior of the structure in meeting those requirements.

(iv) It simplifies the design process by encouraging the same design philosophy and procedures to be adopted for all materials of construction.

(v) It is a tool for exercising judgment in non-routine situations.

(vi) It provides a tool for updating standards in a rational manner.

The remainder of this report is devoted to the derivation of load factors that are suitable for a wide range of loadings and structural materials.

2. PROBABILISTIC BASES OF STRUCTURAL RELIABILITY

2.1 Historical Development

Engineering decisions must be made in the presence of uncertainties arising from inherent randomness in many design parameters, imperfect modeling and lack of experience. Indeed, it is precisely on account of these uncertainties and the potential risks arising therefrom that safety margins provided by the specification of allowable stresses, resistance factors, load factors, and the like, are required in design. While strength and load parameters are nondeterministic, they nevertheless exhibit statistical regularity. This suggests that probability theory should furnish the framework for setting specific limits of acceptable performance for design.

The idea that dispersion (or statistical variation) in a parameter such as yield stress or load should be considered in specifying design values is not new, and many standards have recognized this for some time. For example, the design wind speeds and ground snow loads in ANSI Standard A58.1-1972 [2] are determined from the probability distributions for the annual extreme fastest mile wind speed and the annual extreme ground snow load. For ordinary structures, the design value for these parameters is that value which has a probability of being exceeded of 0.02 in any year (the 50-year mean recurrence interval value). Similarly, the acceptance criteria for concrete strength in ACI Standard 318-77 [19] are designed to insure that the probability of obtaining concrete with a strength less than f_c' is less than 10 percent. Other examples could also be cited. An appreciation of the philosophy underlying such provisions is essential: in the presence of uncertainty, absolute reliability is an unattainable goal. However, probability theory and reliability-based design provide a formal framework for developing criteria for design which insure that the probability of unfavorable performance is acceptably small.

While this basic philosophy has been accepted for some time, there have been no standards adopted in the United States which synthesize all the available information for purposes of developing reliability-based criteria for design. The use of statistical methodologies has stopped at the point where the nominal strength or load was specified. Additional load and resistance factors, or allowable stresses, were then selected subjectively to account for unforeseen unfavorable deviations from the nominal values. However, probability theory and structural reliability methods make it possible to select safety factors to be consistent with a desired level of performance (acceptably low probability of unsatisfactory performance). This affords the possibility of more uniform performance in structures and, in some areas where designs appear to be excessively conservative, a reduction in costs.

The remainder of Chapter 2 is devoted to describing the procedures used for analyzing reliabilities associated with existing designs and developing the probability-based load criterion for the A58 Standard.

2.2 Analysis of Reliability of Structures

The conceptual framework for structural reliability and probability-based design is provided by the classical reliability theory described by Freudenthal, Ang, Cornell, and others [1,8]. The loads and resistance terms are assumed to be random variables and the statistical information necessary to describe their probability laws is assumed to be known.

A mathematical model is first derived which relates the resistance and load variables for the limit state of interest. Suppose that this relation is given by

$$g(X_1, X_2, \ldots, X_n) = 0 \tag{2.1}$$

where X_i = resistance or load variable, and that failure occurs when $g < 0$ for any ultimate or serviceability limit state of interest. Failure, defined in a generic sense relative to any limit state, does not necessarily connote collapse or other catastrophic events. Then safety is assured by assigning a small probability p_f to the event that the limit state will be reached, i.e.,

$$p_f = \int \ldots \int f_X(x_1, x_2, \ldots, x_n)\, dx_1 dx_2 \ldots dx_n \tag{2.2}$$

in which f_X is the joint probability density function for X_1, $X_2, \ldots,$ and the integration is performed over the region where $g < 0$.

In the initial applications of this concept to structural safety problems, the limit state was considered to contain just two variables; a resistance R and a load effect Q dimensionally consistent with R. The failure event in this case is $R - Q < 0$ and the probability of failure is computed as,

$$p_f = p(R < Q) = \int_0^\infty F_R(x) f_Q(x) dx \tag{2.3}$$

in which F_R = cumulative probability distribution function (c.d.f.) in R and f_Q = probability density function for Q. If R and Q both have normal distributions, for example, then

$$p_f = \Phi\left[-\frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right] \tag{2.4}$$

where, $\overline{R}$, σ_R = mean and standard deviation (σ_R^2 = variance) for R and similarly for Q; $\Phi[\ ]$ = standard normal probability distribution. If R and Q both have lognormal distributions,

$$p_f \approx \Phi\left[-\frac{\ln(\overline{R}/\overline{Q})}{\sqrt{V_R^2 + V_Q^2}}\right] \tag{2.5}$$

when V_R, V_Q < about 0.30, in which V_R, V_Q = coefficient of variation (c.o.v.) in R and Q. The c.o.v. is a convenient dimensionless measure of variability or uncertainty and will be referred to frequently in the remainder of the report. Other distributions may be specified for R and Q. When this is done, Eq. 2.3 frequently must be evaluated numerically.

This provides a basis for quantitatively measuring structural reliability, such a measure being given by p_f. It is tacitly assumed that all uncertainties in design are contained in the joint probability law f_X and that f_X is known. However, in structural reliability analyses these probability laws are seldom known precisely due to a general scarcity of data. In fact, it may be difficult in many instances to determine the probability densities for the individual variables, let alone the joint density f_X. In some cases, only the first and second order moments, i.e. mean and variance, may be known with any confidence. Moreover, the limit state equation may be highly nonlinear in the basic variables. Even in those instances where statistical information may be sufficient to define the marginal distributions of the individual variables, it usually is impractical to perform numerically the operations necessary to evaluate Eq. 2.2.

2.3 First-Order, Second-Moment Methods

The difficulties outlined above have motivated the development of first-order, second-moment (FOSM) reliability analysis methods, so called because of the way they characterize uncertainty in the variables and the linearizations performed during the reliability analysis [7,15]. In principle, the random variables are characterized by their first and second moments. While any continuous mathematical form of the limit state equation is possible, it must be linearized at some point for purposes of performing the reliability analysis. Linearization of the failure criterion defined by Eq. 2.1 leads to

$$Z \approx g(X_1^*, X_2^*, \ldots X_n^*) + \Sigma\, (X_i - X_i^*) \left(\frac{\partial g}{\partial X_i}\right)_{\underset{\sim}{X}^*} \tag{2.6}$$

where $(X_1^*, X_2^*, \ldots X_n^*)$ is the linearizing point. The reliability analysis then is performed with respect to this linearized version of Eq. 2.1. As might be expected, one of the key considerations is the selection of an appropriate linearizing point.

2.3.1 Mean Value Methods

In earlier structural reliability studies, the point $(X_1^*, X_2^*, \ldots X_n^*)$ was set equal to the mean values $(\overline{X}_1, \overline{X}_2, \ldots \overline{X}_n)$. Assuming the X-variables to be statistically uncorrelated, the mean and standard deviation in Z are approximated by

$$\overline{Z} \approx g(\overline{X}_1, \overline{X}_2, \ldots \overline{X}_n) \tag{2.7}$$

$$\sigma_Z \approx \left[\Sigma \left(\frac{\partial g}{\partial X_i}\right)^2_{\overline{X}_i} \sigma^2_{X_i} \right]^{1/2} \tag{2.8}$$

The extent to which Eqs. 2.7 and 2.8 are accurate depends on the effect of neglecting higher order terms in Eq. 2.6 and the magnitudes of the coefficients of variation in X_i. If g( ) is linear and the variables are uncorrelated, Eqs. 2.7 and 2.8 are exact.

The reliability index β (in some studies, β is termed the safety index) is defined by

$$\beta = \overline{Z}/\sigma_Z \tag{2.9}$$

which is the reciprocal of the estimate of c.o.v. in Z. This is illustrated in Fig. 2.1 which shows the densities of Z for two alternate representations of the simple two-variable problem Z = g(R, Q) = 0 discussed in the previous Section (Eq. 2.2, et. seq.). β is the distance from $\overline{Z}$ to the origin in standard deviation units. As such, β is a measure of the probability that g( ) will be less than zero. Fig. 2.1a shows the probability density function (generally unknown) for Z = R - Q. The shaded area to the left of zero is equal to the probability of failure. Observe that if σ_{R-Q} remains constant, a positive shift in $\overline{R} - \overline{Q}$ will move the density to the right, reducing the failure probability. Thus an increase in β leads to an increase in reliability (lower p_f). Alternatively, if

$$\overline{Z} - \beta\sigma_Z \geq 0 \tag{2.10}$$

the reliability is at least β. Figure 2.1b shows an alternate formulation derived from the failure condition Z = ℓn R/Q < 0.

Since $\overline{R - Q} = \overline{R} - \overline{Q}$ and $\sigma_{R-Q} = \sqrt{\sigma_R^2 + \sigma_Q^2}$, β in Fig 2.1a is defined as

$$\beta = \frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{2.11}$$

Using the alternative formulation of Fig. 2.1b, and using the small-variance approximations $\overline{\ell n\ R/Q} \simeq \ell n\ \overline{R}/\overline{Q}$ and $\sigma_{\ell n\ R/Q} \simeq [V_R^2 + V_Q^2]^{1/2}$,

$$\beta = \frac{\ell n\ \overline{R}/\overline{Q}}{\sqrt{V_R^2 + V_Q^2}} \tag{2.12}$$

Eq. 2.11 was the basis for an early recommendation for a probability-based structural code [22] while Eq. 2.12 was the basis for the development of probability-based load and resistance factor design criteria for steel structures [9].

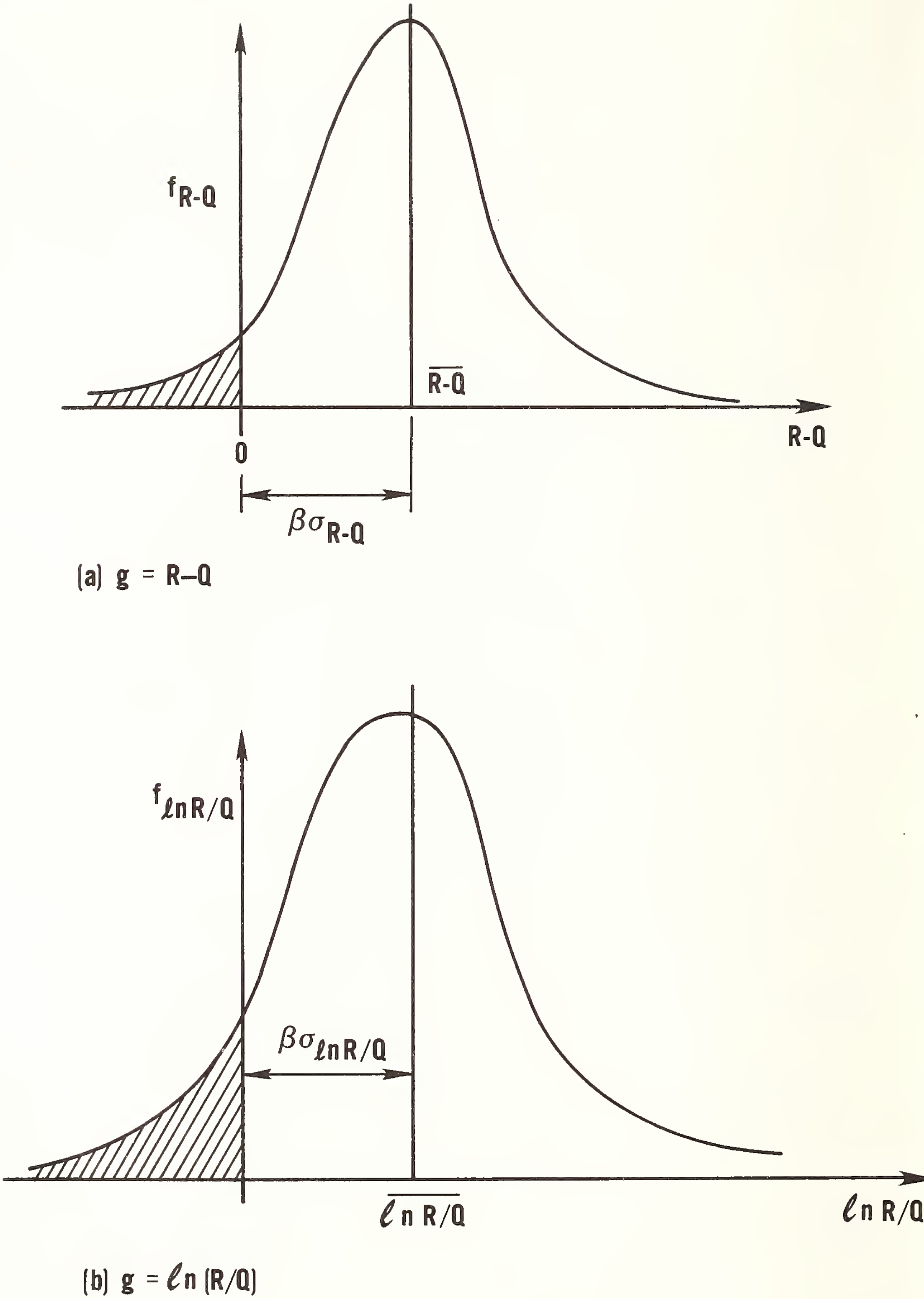


Figure 2.1 - Illustration of the Reliability Index Concept

In this development, no mention has been made of probability distributions; the reliability index β depends only on measures of central tendency ($\bar{Z}$) and dispersion (σ_Z) in the limit state function. However, it is important to realize that if the probability laws governing the variables in the limit state equation are known, there is a relation between β and p_f. In the example just considered, if R and Q are normal and statistically independent, then R - Q is normal with mean $\bar{R} - \bar{Q}$ and variance $\sigma_R^2 + \sigma_Q^2$. The probability of failure is then

$$p_f = P[R - Q < 0] = \frac{1}{\sqrt{2\pi}\,\sigma_{R-Q}} \int_{-\infty}^{0} \exp\left[-1/2\left(\frac{x-(\bar{R}-\bar{Q})}{\sigma_{R-Q}}\right)^2\right]dx$$

$$= \Phi\left[-\frac{\bar{R}-\bar{Q}}{\sqrt{\sigma_R^2+\sigma_Q^2}}\right] \qquad (2.13)$$

Comparing Eqs. 2.13 and 2.11, the reliability index β is related to the percent point function of the standard normal distribution according to,

$$\beta = \Phi^{-1}(1 - p_f) \qquad (2.14)$$

$$p_f = \Phi(-\beta) \qquad (2.15)$$

Even when the probability laws cannot be determined exactly, however, β is a useful comparative measure of reliability and can serve to evaluate the relative safety of various design alternatives, provided that the first and second order statistics are handled consistently. In such cases, the probability of failure computed from Eq. 2.15 is referred to as a "notional" probability, indicating that it should be interpreted, at best, in a comparative sense as opposed to a classical or relative frequency sense.

2.3.2 Advanced Methods

Mean value FOSM methods have two basic shortcomings. First, the g( ) function is linearized at the mean values of the X-variables. When g( ) is nonlinear, significant errors may be introduced at increasing distances from the linearizing point by neglecting higher order terms. In most structural reliability problems, the mean point is, in fact, some distance from g( ) = 0, and thus there are likely to be unacceptable errors in approximating Eq. 2.1 by Eq. 2.6 when g( ) is nonlinear. Second, the mean value methods fail to be invariant to different mechanically equivalent formulations of the same problem. In effect, this means that β depends on how the limit state is formulated. This is a problem not only for nonlinear forms of g( ) but even in certain linear forms as, e.g., when the loads (or load effects) counteract one another. The lack of invariance arises because the linear expansions are taken about the mean value point. This problem may be avoided by linearizing g( ) at some point on the failure surface [7, 14, 15]. This is because g( ) and its partial derivations in Eq. 2.6 are independent of how the problem is formulated only on the surface g( ) = 0.

The selection procedure can be explained as follows. With the limit state and its variables as given in Eq. 2.1, the variables X_i are first transformed to reduced variables with zero mean and unit variance through

$$x_i = \frac{X_i - \overline{X}_i}{\sigma_i} \tag{2.16}$$

In the space of reduced coordinates x_i, the limit state is

$$g_1(x_1,x_2,\ldots,x_n) = 0 \tag{2.17}$$

with failure occurring when $g_1 < 0$. This is illustrated in Figs. 2.2(a) and 2.2(b).

We now define a reliability index β as the shortest distance between the surface $g_1 = 0$ and the origin. The point $(x_1^*,x_2^*,\ldots,x_n^*)$ on $g_1(\ ) = 0$ which corresponds to this shortest distance is referred to as the checking point (some authors call it the design point) and must be determined by solving the system of equations

$$\alpha_i = \frac{\partial g_1/\partial x_i}{[\Sigma(\partial g_1/\partial x_i)^2]^{1/2}} \tag{2.18}$$

$$x_i^* = -\alpha_i\beta \tag{2.19}$$

$$g_1(x_1^*,x_2^*,\ldots,x_n^*) = 0 \tag{2.20}$$

searching for the direction cosines α_i which minimize β. The derivatives are evaluated at the point $(x_1^*, x_2^*,\ldots,x_n^*)$. Note from Fig. 2.2 that this procedure is equivalent to linearizing the limit state equation in reduced variables at the point $(x_1^*,x_2^*,\ldots,x_n^*)$, and computing the reliability associated with the linearized rather than original limit state.

In the original variable space, the checking point variables are given by

$$X_i^* = \overline{X}_i(1 - \alpha_i\beta V_i) \tag{2.21}$$

$$g(X_1^*,X_2^*,\ldots,X_n^*) = 0 \tag{2.22}$$

The set of points $(X_1^*,X_2^*,\ldots,X_n^*)$ will fall in the upper range of the probability distributions for load parameters and the lower range for resistance variables. If necessary, load and resistance factors γ_i for design corresponding to a prescribed reliability index β may then be determined through

$$\gamma_i = X_i^*/X_{n,i} \tag{2.23}$$

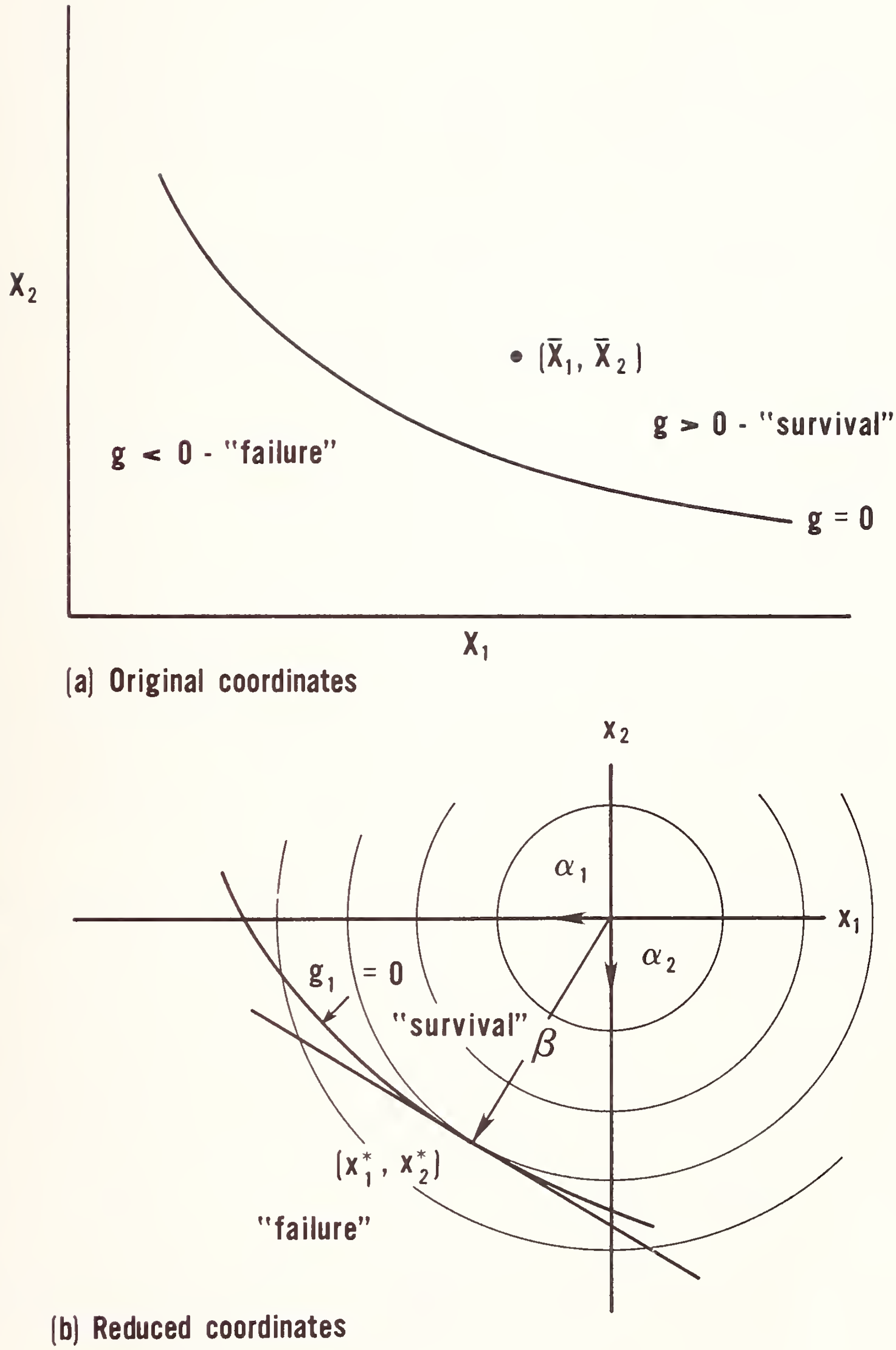


Figure 2.2 - Formulation of Safety Analysis in Original and Reduced Variable Coordinates

in which $X_{n,i}$ is the nominal or design value of the load or resistance parameter specified in the building standard. This may be the load corresponding to a mean recurrence interval of N years, the mean maximum load during a reference period of T years, or any one of a number of other formulations. In the context of American National Standard A58, the $X_{n,i}$ would correspond to the load level in the current or proposed versions of the standard. Thus, the load and resistance factors depend on the way the nominal loads and resistances are specified.

2.4 Approximate Methods for Including Information on Distributions

The first-order, second-moment procedure outlined in the previous section gives values of the reliability index β which may be related to a probability of failure in cases when the variables X_i are normally distributed and when the function g is linear in X_i. In other cases, Eqs. 2.14 and 2.15 are not exact. Many structural problems involve random variables which are clearly non-normal. As examples, instantaneous live loads appear to be modeled more appropriately by Gamma distributions, at least for relatively small loaded areas [4]; recent studies of extreme wind data [16] have shown that the annual extreme wind speed due to extratropical storms is Extreme Value Type I. It seems appropriate that this information be incorporated in the analysis in a way that does not require the multidimensional integration in Eq. 2.2. There are a number of approaches for doing this. The one used in this study [13] currently is also being used for developing reliability-based design procedures in Canada and Europe [3,7,11].

The basic idea is to transform the non-normal variables into equivalent normal variables prior to the solution of Eqs. 2.18 - 2.20. The main advantage of doing this is that sums and differences of independent normal variables are also normal with easily calculated means and variances. The ability to calculate failure probabilities in accordance with Eq. 2.14 and 2.15 is thereby retained. This transformation may be accomplished by approximati the true distribution of variable X_i by a normal distribution at the value X_i^* corresponding to a point on the failure surface. The justification for this is that if the normalization takes place at the point close to that where failure is most likely, (i.e., minimum β), the estimates of the failure probability obtained by the approximate procedure should approximate the true (but unknown) failure probability quite closely.

Following Ref. 13, we determine the mean and standard deviation of the equivalent normal variable such that at the value X_i^*, the cumulative probability and probability density of the actual and approximating normal variable are equal. Thus,

$$\sigma_i^N = \frac{\phi\ (\Phi^{-1}[F_i(X_i^*)])}{f_i(x^*)} \tag{2.24a}$$

$$\overline{X}_i^N = X_i^* - \Phi^{-1}[F_i(X_i^*)]\sigma_i^N \tag{2.24b}$$

in which F_i and f_i = non-normal distribution and density functions of X_i, and ϕ ( ) is the density function for the standard normal variate. Having determined $\overline{X}_i^N$ and σ_i^N of the equivalent normal distributions, the solution proceeds exactly as described in Eqs. 2.16 to 2.20. Inasmuch as the checking point variable X_i^* changes with each iteration, the parameters $\overline{X}_i^N$ and σ_i^N must be recomputed during each iteration cycle also. However, since all calculations are performed by computer, this does not materially add to the complexity of the reliability analysis described earlier.

This approximate technique often yields excellent agreement with the exact solution of Eq. 2.2 [13]. However, it has been noted [15] that the checking point may not correspond exactly to the point where the joint probability density is maximum and failure is most likely. Moreover, this procedure does not reduce the error which is due to the linearization of what may be a generally nonlinear failure boundary at the checking point. Unless the failure boundary is highly nonlinear, however, as is the case in some stability problems, this source of error is small compared to the accuracy with which most of the parameters in engineering reliability analysis can be estimated.

The following summarizes the procedure which is used to compute the reliability index β associated with a particular design or, conversely, a design parameter (such as section modulus) for a prescribed β, probability distributions, and set of means and standard deviations (or c.o.v.):

1. Define the appropriate limit state function: Eq. 2.1.
2. Make an initial guess at the reliability index β (or design parameter).
3. Set the initial checking point values $X_i^* = \overline{X}_i$, for all i.
4. Compute the mean and standard deviation of the equivalent normal distribution for those variables that are non-normal according to Eqs. 2.24.
5. Compute partial derivatives $\partial g/\partial X_i$ evaluated at the point X_i^*.
6. Compute the direction cosines α_i as

$$\alpha_i = (\frac{\partial g}{\partial X_i}\ \sigma_i^N)/[\Sigma(\frac{\partial g}{\partial X_i}\ \sigma_i^N)^2]^{1/2}$$

7. Compute new values of X_i^* from

$$X_i^* = \overline{X}_i^N - \alpha_i \beta \sigma_i^N$$

and repeat steps 4 through 7 until the estimates of α_i stabilize.

8. Compute the value of β necessary for

$$g(X_i^*, X_2^*, \ldots, X_n^*) = 0$$

and repeat steps 4 through 8 until the values of β on successive iterations differ by some small tolerance (say 0.05). Normally, convergence is obtained within 5 cycles or less, depending on the nonlinearity of the limit state equation.

A computer program was developed to perform the calculations leading to the load criterion in this report. This program is described in detail in Appendix F.

<u>Example Calculations</u>

Two examples are presented to illustrate the concepts presented in the previous sections.

As a simple case, consider the two-variable problem which was treated previously;

$$g = R - Q$$

in which R, Q both have normal distributions. Making the transformations

$$r = \frac{R - \overline{R}}{\sigma_R}$$

$$q = \frac{Q - \overline{Q}}{\sigma_Q}$$

The failure criterion becomes,

$$\sigma_R r - \sigma_Q q + \overline{R} - \overline{Q} = 0$$

The failure criteria in the original (R,Q) and reduced (r,q) coordinate systems are shown in Figures 2.3a and 2.3b. The iterative solution is not required here. The checking point variables may be seen to be

$$r^* = -\alpha_R \beta = \frac{-\sigma_R}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \beta$$

$$q^* = -\alpha_Q \beta = \frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \beta$$

leading to a value of reliability index β of

$$\beta = \frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} = \overline{Z}/\sigma_Z$$

The second example, that of calculating β for a steel beam in bending, illustrates the iterative scheme.

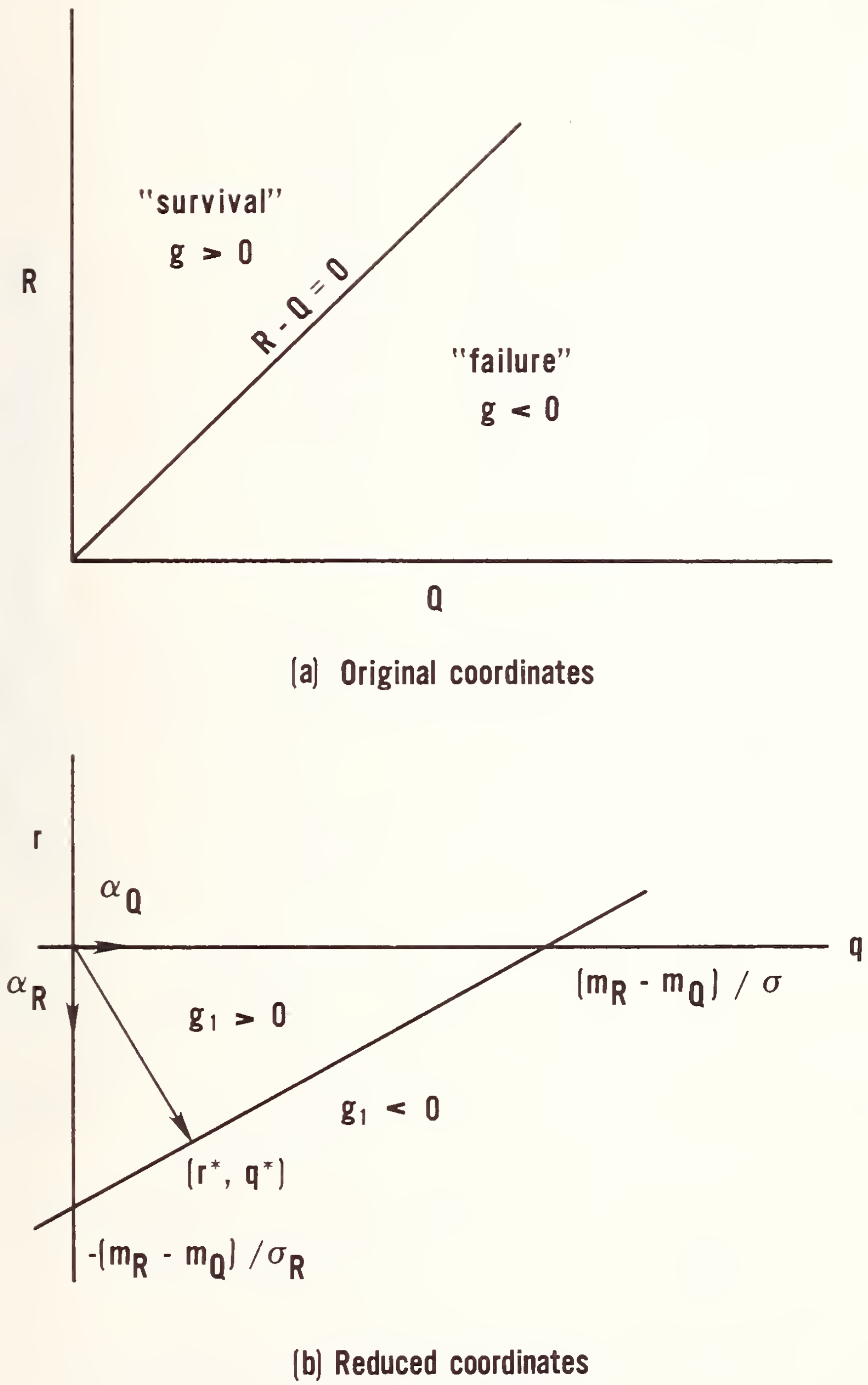


Figure 2.3 - Reliability Calculation for Linear Two-Variable Problem

Suppose the beam is a 16WF31 section with yield stress F_y = 36 ksi (248 N/mm^2) and plastic section modulus Z = 54 in^3 (16387 mm^3) supporting a (deterministic) moment of 1140 in-kip (1.55 N-m). For illustration, statistics of F_y and Z are:

F_y: Lognormal - $\bar{F}_y$ = 38 ksi, V_{F_y} = 0.10

Z: Normal - $\bar{Z}$ = 54 in^3, V_Z = 0.05

and the limit state is

$g(\ ) = F_Y Z - 1140 = 0$

Table 2.1 shows the iterative solution. The initial guess at β was 3.0, and the final solution is β = 5.14. Table 2.2 shows the values that would have been obtained using mean value methods along with strength and stress formulations. Solutions corresponding to Eqs. 2.11 and 2.12 are also displayed graphically in Figure 2.4. The direction along which β is measured clearly depends on the method of formulation when mean value methods are used. This illustrates the invariance problem discussed earlier.

Table 2.1 - Illustration of Reliability Calculations - Iterative Solution Steps

Step 2	β	3.0			5.001		5.136	
Steps 3,7	F_y^*	38	27.64	29.02	23.95	24.46	24.19	24.21
	Z^*	54	50.38	50.17	47.60	47.34	47.14	47.11
Step 4	$\bar{F}_y^N$	37.83	36.31	36.71	34.90	35.12	35.00	35.01
	σ_Y^N	3.80	2.76	2.90	2.40	2.45	2.42	2.42
	$\bar{Z}^N$	54	54	54	54	54	54	54
	σ_Z^N	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Step 5	$\frac{\partial g}{\partial F_y}$	54	50.38	50.17	47.6	47.34	47.14	47.11
	$\frac{\partial g}{\partial Z}$	38	27.64	29.02	23.95	24.46	24.19	24.21
Step 6	α_{F_y}	0.894	0.881	0.880	0.870	0.869	0.868	0.868
	α_{F_Z}	0.447	0.473	0.474	0.493	0.495	0.497	0.497
Step 8	β			5.001		5.136		5.144

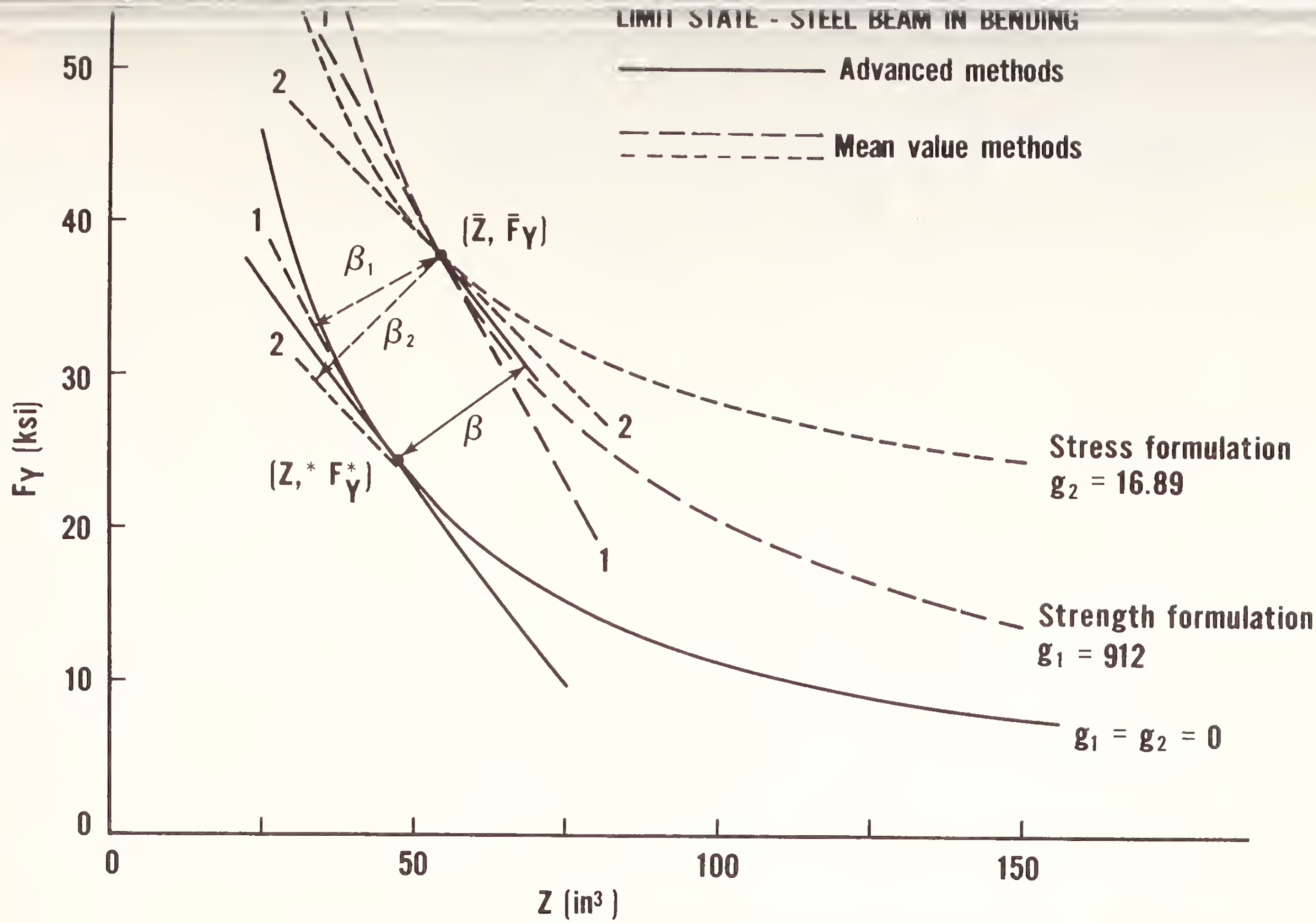


Figure 2.4 - Reliability Calculation for Nonlinear Problem

Table 2.2 - Mean Value FOSM Solutions

Formulation	β	
	Eq. 2.11	Eq. 2.12
Strength (β_1)	3.97	5.25
Stress (β_2)	4.28	5.26

2.5 Load Combinations

Most structural loads vary with time. If a structural element is subjected to only one time-varying load in addition to its dead load, the reliability may be determined simply by considering the combination of the dead load with the maximum time-varying load during some appropriate reference period. It is frequently the case, however, that more than one time-varying load will be acting on a structure at any given time. Conceptually, these load combinations should be dealt with by applying the theory of stochastic processes, which account for the stochastic nature and correlation of the loads in space and time.

Loads (or load effects) acting on structural elements typically are represented by various combinations of load process models such as those in Fig. 2.5. Permanent loads (Fig. 2.5a) such as dead loads change very slowly and maintain a relatively constant (albeit random) magnitude. Sustained loads (Fig. 2.5b) may change at discrete times but in between changes remain relatively constant. They may be absent entirely for certain periods. Occupancy live loads fall in this category. Finally, transient loads of short duration (Fig. 2.5c) occur relatively infrequently. Since their durations are so small relative to permanent and sustained loads, they are modeled as impulses. Extreme wind and earthquake loads are examples.

The terminology "arbitrary-point-in-time" load is used frequently in later sections. It is simply the load that would be measured if the load process were to be sampled at some time instant, e.g., in a load survey. The probability densities of the arbitrary-point-in-time loads are shown in Figs. 2.5a - 2.5c. The impulse at zero represents the probability that the load magnitude is zero at the time samples are taken.

In this report, the analysis of reliability associated with ultimate limit states requires that the maximum total load during a reference period taken as 50 years be characteri: When more than one time-varying load acts, it is extremely unlikely that each load will reach its peak lifetime value at the same moment. This is illustrated in Fig. 2.5d.

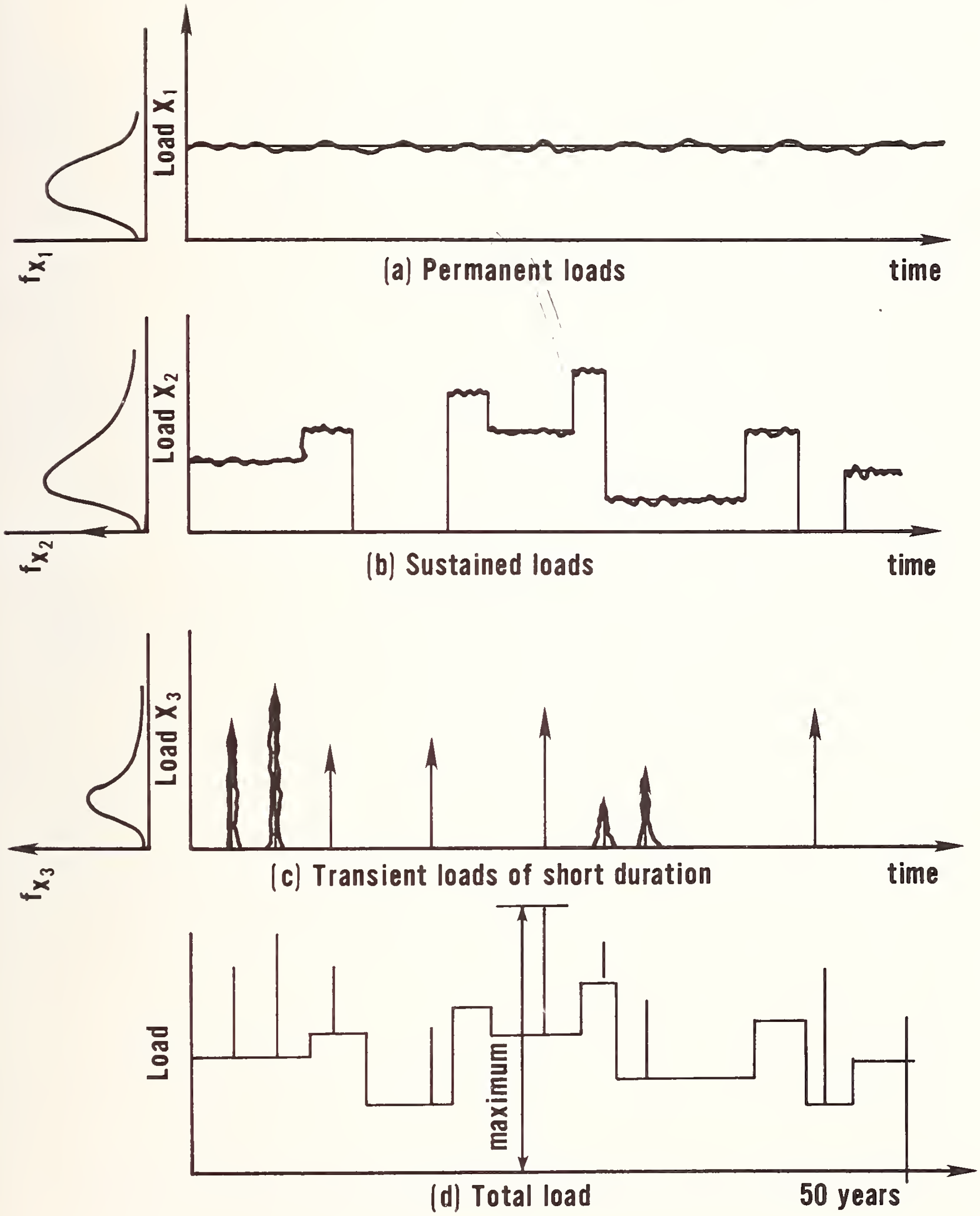


Figure 2.5 - Typical Load Process Models

Consequently, a structural component could be designed for a total load which is less than the sum of the peak loads, and in fact this is recognized in Section 4.2 of ANSI Standard A58.1-1972 [2]. The probability factors in that section have evolved on the basis of experience rather than a thorough consideration of the underlying nature of the loads, however.

For practical reliability analyses, it is necessary to work with random variable representations of the load rather than random process representations. One such procedure is a generalization [14] of a model first proposed by Ferry Borges [6]. It is first assumed that for each time-varying load X_i, the life T may be divided into a number of elementary time intervals, τ_i, such that the value of load X_i is constant within τ_i and values of X_i within successive time intervals are statistically independent. The probability of a nonzero value of X_i within each time interval is p_i. The load histories are then arranged in order of decreasing basic time interval (increasing number of load changes) as shown in Figure 2.6. Given that r_i nonzero values of X_i occur within interval τ_{i-1}, the distribution of the maximum of X_i within interval τ_{i-1} is given by

$$F_{max}(x) = F_i(x)^{r_i} \tag{2.25}$$

in which $r_i = \tau_{i-1}/\tau_i$ (termed the repetition number) and F_i = distribution of X_i within the elementary interval τ_i. Using the theorem of total probability and the binomial theorem, the distribution of maximum load within T is given by

$$F_{max}(x) = [1 - p_i(1-F_i(x))]^{r_i} \tag{2.26}$$

Beginning with variable X_n, the maximum of X_n within interval τ_n is found using Eq. 2.26. The distribution of the sum $Z_{n-1} = [\max X_n + X_{n-1}]$ can be found through convolution. Using the procedure for normalizing non-normal random variables explained earlier, this calculation can be handled quite easily. Working down through the set of load histories, the distribution of $Z_{n-2} = \max Z_{n-1} + X_{n-2}$ is computed using Eq. 2.26, and the process is repeated until all loads have been summed.

Although this represents a sophisticated approach to load combinations, there are a number of difficulties with its use. The assumption that each peak value of load remains constant within its basic time interval is a conservative one but probably is not unduly so if the basic time intervals are chosen to be reasonably short. A more serious shortcoming is the necessity of making assumptions regarding the number of basic intervals and the probability of a nonzero load value within each one. Information regarding τ_i or r_i and p_i generally is not available or is not easily recoverable from available load data and as

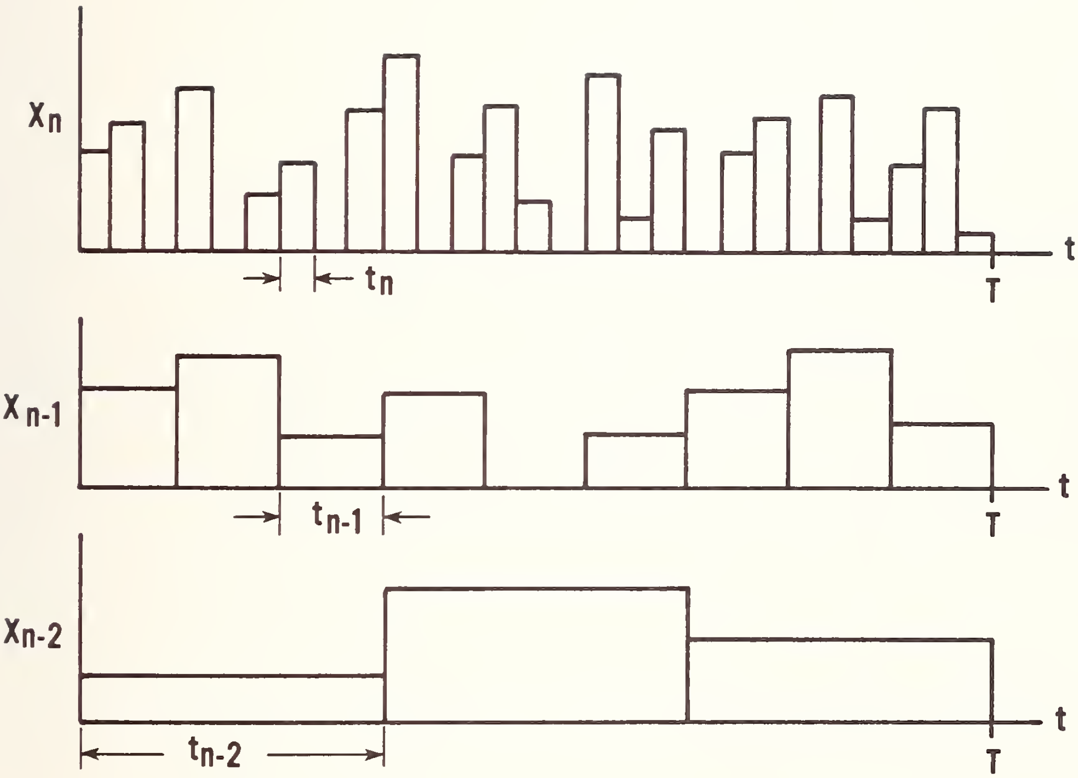


Figure 2.6 - Idealized Model of Three Time-Varying Load Processes

a consequence r_i and p_i must be determined artificially. The safety criteria are quite sensitive to the selection of these parameters. In short, the quality of data and our knowledge of the various load processes may not be sufficient to warrant the use of this model in practical reliability analysis and design work.

An alternate way to handle load combinations is through the use of "Turkstra's rule" [17]. This says, in effect, that the maximum of a combination of load effects will occur when one of the loads is at its lifetime maximum value while others assume their instantaneous values. In other words, if

$$Z(t) = X_1(t) + X_2(t) + \dots + X_n(t) \tag{2.27}$$

then max Z is given by

$$\max Z \simeq \max_i \left[\max_T X_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{n} X_j(t)\right] \tag{2.28}$$

If there are n time-varying loads in the limit state equation, in general it is necessary to consider n distinct load combinations in computing the associated reliability. This tends to be unconservative in certain instances where the probability of a joint occurrence of more than one maximum value is not negligible or in the situation where the maximum combined effect occurs when two variables simultaneously attain "near maximum" values. Nevertheless, recent research on load combinations based on the concept of up-crossing rates of random processes show that Turkstra's rule is a good approximation in many practical cases [10,18]. This model will be used for the load combination work in this study because of its simplicity and because it is consistent with the observation that failures frequently occur as a consequence of one load attaining an extreme value.

The following is an example of load combination analysis according to Eq. 2.28. Assume that the loads of interest are dead, live and wind load. As discussed in the following section, the load effects are,

D = permanent or dead load (duration = lifetime T)

L_{apt} = arbitrary-point-in-time live load

L = maximum live load during T

W_{apt} = arbitrary-point-in-time wind load

W = maximum wind load during T

According to Eq. 2.28, the calculation of β for reference period T would require the following load combinations to be considered:

$$D + L + W_{apt} \tag{2.29a}$$

$$D + L_{apt} + W \tag{2.29b}$$

Accordingly, the reliability calculations require the means, variances (or c.o.v.) and probability distributions for the variables in Eqs. 2.29. For example, for Eq. 2.29b, $\overline{X}_i$, V_i and F_i would have to be determined for the three variables D, L_{apt} and W.

The minimum value of β calculated from these combinations would provide a lower bound on the reliability of the element.

3. LOAD AND RESISTANCE DISTRIBUTIONS AND PARAMETER VALUES

3.1 General

The preceding chapter demonstrated that the determination of reliability indices or load and resistance factors for a prescribed β depends on the estimates of the mean and variance and probability distributions of the random variables in the limit state equation. Data on the resistance and load variables required in order to develop the reliability based load criterion are summarized in this chapter. Appendices A - E contain detailed information.

Reliability studies conducted over the past decade have not always used the same means, variances and distributions due to availability of data and the continually changing state of knowledge. By and large, the statistical descriptions used in this report are a synthesis of values reported in numerous previous studies on structural loads and load models, behavior of structural members, and reliability based design. In a sense, they are a consensus of the specialists who have published in these areas. We have relied on published, professionally accepted data insofar as possible. Although we recognize that knowledge of structural loads and the behavior of materials is continually evolving, we have opted not to employ load and resistance models which are developmental or speculative in nature, even when those models show considerable potential. It is our judgment that models and data which provide the technical basis for standard provisions should be thoroughly validated prior to their incorporation.

The sources for statistics and distributions for individual loads are primarily the load subcommittees within ANSI Committee A58 that have expertise in and responsibility for the loads in the current version and projected revisions of the A58 Standard. Similarly, data on resistance of structural members and components is obtained from the numerous research reports and papers published by individual researchers, industrial groups and trade associations. In the following section, we summarize load and resistance distributions and parameter values used in the reliability analysis and loading criteria development.

3.2 Characterization of Load and Resistance Variables

The basic information required is the probability distribution of each load or resistance variable and estimates of its mean and standard deviation (or coefficient of variation) or equivalent distribution parameters (e.g., mode and shape factor for extreme value distributions). The mean and c.o.v. of these basic variables should be representative of values that would be expected in actual structures in situ. While there frequently are sufficient data to obtain a reasonable estimate of the probability distribution, in other cases this must be assumed on the basis of physical argument or intuition. We have emphasized the use of two-parameter distributions because, with few exceptions, the quantity of data necessary to estimate higher order statistics with any confidence does not exist in structural reliability problems.

In the context of the first-order, second-moment approach to reliability, the concept of uncertainty, exemplified by variability or scatter in the variable, is conveyed through its variance or coefficient of variation (c.o.v.). The uncertainties used in the reliability analysis should include all imponderables which may affect design reliability. These would include "inherent" statistical variability in the basic strength or load parameter. Additional sources of uncertainty arise due to modeling and prediction errors and incomplete information; included in these "modeling uncertainties" would be errors in estimating the parameters of the distribution function, idealizations of the actual load process in space and time, uncertainties in calculation, and deviations in the application of the A58 load standard or material specification from the idealized cases considered in their development. While occasionally there may be some data available with which to estimate these latter uncertainty measures, frequently they must be estimated on the basis of professional judgment and experience. The key test in differentiating between the "inherent" and "modeling" uncertainties is in whether the acquisition of additional information would materially reduce their estimated magnitude. If the variability is intrinsic to the problem, additional sampling is not likely to reduce its magnitude, although the confidence interval on the estimate would contract. In contrast, uncertainties due to "modeling" should decrease as improved models and additional data become available.

Let X denote a basic resistance or load design variable. Although the true mean and c.o.v. of X, $\overline{X}$ and V_X, should be employed when evaluating reliability, these generally are not known precisely in structural engineering problems owing to insufficient data and information. What are available instead are estimates $\hat{\overline{X}}$ and $\hat{V}_X$ of the mean and c.o.v. of X which are usually computed from idealized models and data gathered under carefully controlled conditions. Therefore, while $\hat{V}_X$ reflects basic statistical variability, it fails to encompass all sources of uncertainty that contribute to the total variability in X. If the bias and uncertainty measure (c.o.v.) attributed to these additional factors are given by $\overline{B}$, and V_B, then according to procedures described in detail by Ang and Cornell [1], $\overline{X}$ and V_X are evaluated as,

$$\bar{X} = \bar{B}\hat{\bar{X}} \tag{3.1}$$

$$V_X = [\hat{V}_X^2 + V_B^2]^{1/2} \tag{3.2}$$

That is, one increases the variability according to the uncertainty in one's ability to estimate the parameter. (If the model is unbiased, $\bar{B} = 1$.) Frequently, V_B can be broken down into several parts, in which case $V_B = [V_1^2 + V_2^2 + \ldots]^{1/2}$. It is implicit in this formulation that V_B measures primarily the uncertainty in predicting the true mean of X by $\hat{\bar{X}}$ [1].

When data are available, $\hat{\bar{X}}$ and $\hat{V}_X$ can be calculated from the samples using classical statistical analysis techniques. In cases where the data are limited, the c.o.v. may be estimated from knowing the range over which it is felt, on the basis of past experience, the data should lie. If it is assumed, for example, that values in the midrange are more likely than those near the extremes (X has a "bell-shaped" density) and that roughly 95 percent of the values fall within x_1 and x_2, then

$$\hat{\bar{X}} \simeq \frac{x_1 + x_2}{2} \tag{3.3}$$

$$\hat{V}_X \simeq \frac{1}{2}\left(\frac{x_2 - x_1}{x_2 + x_1}\right) \tag{3.4}$$

Similar techniques may be used to estimate V_B, provided that information on the range of the means is available.

In the following sections of this chapter, the means or characteristic extremes have been normalized with respect to their nominal values. This is done for convenience and makes the statistics applicable to a wide range of design situations. The statistics of the load or resistance variable can easily be computed for each <u>design situation</u> that is defined by <u>nominal</u> load and resistances, since if

$$X = (X/X_n) \cdot X_n \tag{3.5}$$

then

$$\bar{X} = \overline{(X/X_n)} \cdot X_n \tag{3.6}$$

$$V_X = V_{(X/X_n)} \tag{3.7}$$

In most instances the basic resistance variable is taken as the strength of the structural member in question, and the basic load variable is the load effect (moment, shear, etc.) dimensionally consistent with the resistance. These can be used directly when the limit state is formulated as a linear combination of resistance and load variables. The linear formulation is quite common in practice and was used for most of the studies described in later sections.

3.3 Specifications and Standards

Current design practice in the United States for the various material technologies is governed by standards and specifications which are kept current by standard committees. These standards are then adopted (occasionally with modifications) by local or regional building authorities as the official basis for design. The following standards and specifications were used in the present study to define nominal parameter values:

Reinforced and Prestressed Concrete Structures:

ACI Standard 318, "Building Code Requirements for Reinforced Concrete," American Concrete Institute, 1977 [19].

Steel Building Structures

"Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," American Institute of Steel Construction, 1978 [24].

"Specification for the Design of Cold-Formed Steel Structural Members" American Iron and Steel Institute, 1968 [28].

Aluminum Structures

"Specifications for Aluminum Structures," The Aluminum Association, Third Edition, April 1976 [29].

Masonry Structures

"Specifications for the Design and Construction of Load Bearing Concrete Masonry" National Concrete Masonry Association, 1968 [25].

"Building Code Requirements for Engineered Brick Masonry" Brick Institute of America, 1969 [26].

Wood Structures

"Standard Specifications for Structural Glued-Laminated Timber of Douglas Fir, Western Larch, Southern Pine and California Redwood," American Institute of Timber Construction, 1974 [27].

3.4 Load Distributions and Parameters

The development of the probability distributions and estimates of their parameters are described in detail in Appendix A. The loading information is summarized in Table 3.1. D, L, S, W, E refer to dead and the maximum values of live, snow, wind and earthquake load effects* over a reference period of 50 years. The annual and arbitrary-point-in-time-

*The distinction between load and load effect and their analysis is discussed in Appendix A.

values of the load effect are denoted through the subscripts "ann" and "apt." With the exception of E, the nominal loads are all defined by the values specified in the ANSI A58.1-1972 load standard. The nominal snow and wind are the 50-year mean recurrence interval values. The nominal earthquake load

Table 3.1 - Load Distributions and Parameters

Load	$\overline{X}/X_n$	V_X	cdf
D	1.05	0.10	Normal
L	Eqs. 3.9 or 3.10	0.25	Type I
L_{apt}	Eq. 3.11	Table A.2	Gamma
W	0.78	0.37	Type I
W_{ann}	0.33	0.59	Type I
W_{apt}	(-0.021)	(18.7)	Type I
S	0.82	0.26	Type II
S_{ann}	0.20	0.73	Lognormal
E	(Site dependent) Appendix A	(2.3)	Type II

E_n is the value from the 1976 edition of the Uniform Building Code. Values given in parentheses are characteristic extreme and shape parameters of extreme value distributions rather than mean and c.o.v. V_X includes uncertainties due to inherent variability, load modeling and analysis.

Two values of the nominal live load L_n are of interest in this study. The first is the value in ANSI A58.1-1972, which was used to determine the values of β which correspond to existing accepted practice. The corresponding L_n is,

$$L_n = [1 - \min \{ 0.0008A_T,\ 0.6,\ 0.23(1 + \frac{D_n}{L_o})\}]\ L_o \tag{3.9}$$

in which A_T = tributary area (see glossary, Chapter 9) and L_o = basic (unreduced) live load given in Table 1 of ANSI A58.1-1972. The second nominal live load is that proposed for the 1980 version of the A58 Standard,

$$L_n = [0.25 + 15/\sqrt{A_I}]\ L_o \tag{3.10}$$

in which A_I = influence area. This nominal value happens to equal the 50-year mean value, $\overline{L}$. The live load factor in the new load criterion is derived so as to be compatible with the 1980 nominal live load. Similarly, for the arbitrary point-in-time live load,

$$\overline{L}/L_n = \frac{0.24}{1 - \min \{0.0008A_T,\ 0.6,\ 0.23(1 + \frac{D_n}{L_0})\}} \quad \text{(A58.1-1972 Standard)} \tag{3.11a}$$

$$\bar{L}/L_n = \frac{0.24}{0.25 + 15/\sqrt{A_I}} \quad \text{(proposed 1980 A58 Standard)} \qquad (3.11b)$$

(proposed 1980 A58 Standard)

The environmental loads are site-dependent. The values given in Table 3.1 are representative; the variation with site is illustrated in Appendix A.

3.5 Resistance Distributions and Parameters

The resistance of structural members, cross sections, cross-sectional elements and connectors is generally expressed by an analytical formula which has been derived from theory or experiment. In most cases of importance to structural design specifications, a clearly defined analytical model exists which has its origin in structural mechanics theory and which has been verified by experiment. It is possible, however, to cite cases where the basis of the model is purely theoretical or solely experimental. While it is evident that many types of analytical models exist in the design specifications of the various structural material groups, only a representative sample of them could be considered within the scope of this report. Enough models were considered, however, to arrive at representative parameters for the development of load factors. Detailed descriptions of these models are presented in the Appendices (B for reinforced and prestressed concrete, C for metals, D for masonry, and E for glulam and heavy timber), together with the collection of the available statistical information.

In most cases, the resistance was assumed to take the following product form:

$$R = R_n(PMF) \qquad (3.12)$$

$$\bar{R}/R_n = \bar{P}\,\bar{M}\,\bar{F} \qquad (3.13)$$

$$V_R = \sqrt{V_P^2 + V_M^2 + V_F^2} \qquad (3.14)$$

R_n in these equations is the nominal resistance based on the model used to best predict the resistance, and on the nominal material properties and the nominal ("handbook") geometric properties. For example, for a "compact" steel beam $R_n = F_y Z$, where F_y is the specified yield stress and Z is the plastic section modulus.

The factor P is the ratio of test capacities, representing actual in-situ performance, to the prediction according to the model used. The modeling of the capacity is thus defined by P (P standing for "professional"). Similarly, M and F (M defining "material" and F "fabrication") denote ratios of actual to nominal material properties and cross-sectional properties.

For example, for a "compact" steel beam,

$$\overline{P} = \frac{(\overline{M}_p)_{Test}}{Z F_y} \; ; \; \overline{M} = \overline{F}_y / F_y ; \; \overline{F} = \overline{Z}/Z \qquad (3.15)$$

where $(\overline{M}_p)_{test}$ = the mean plastic moment obtained from tests of beams, $\overline{F}_y$ = the mean static yield stress and $\overline{Z}$ = the mean plastic section modulus. In Appendix C it was found that

$\overline{P} = 1.02$, $V_p = 0.06$

$\overline{M} = 1.05$, $V_M = 0.10$

$\overline{F} = 1.00$, $V_F = 0.05$

and thus

$$\overline{R} = F_y Z\ (1.02 \times 1.05 \times 1.00) = 1.07\ F_y Z$$

and $V_R = \sqrt{0.06^2 + 0.10^2 + 0.05^2} = 0.13$

The simple resistance model of Eq. 3.12 suffices for most cases which we have considered, although more complex models were used also (see especially reinforced concrete beam-columns in Appendix B and masonry walls in Appendix D).

The rationale for selecting the material statistics for each particular structural material is discussed in detail in the Appendices, where the origin and the significance of the data is also considered. Most of this material for reinforced concrete structures and for metal structures has been previously treated quite extensively in the literature. However, little has been previously presented for masonry and wood structures.

3.5.1 Resistance Statistics for Reinforced and Prestressed Concrete Structures

Table 3.2 presents representative statistical data (from Appendix B) for reinforced and prestressed concrete members. The probability distributions are assumed to be normal; $\overline{R}/R_n$ and V_R were obtained by fitting a normal distribution to the lower tail of the simulated distribution.

Table 3.2
Typical Resistance Statistics for Concrete Members

Designation	$\overline{R}/R_n$	V_R
Flexure, Reinforced Concrete, Grade 60	1.05	0.11
Flexure, Reinforced Concrete, Grade 40	1.14	0.14
Flexure, Cast-in-Place Pretensioned Beams	1.06	0.08
Flexure, Cast-in-Place Post-Tensioned Beams	1.04	0.095
Short Columns, Compression Failure, f'_c = 3 ksi	1.05	0.16
Short Columns, Tension Failure, f'_c = 3 and 5 ksi	1.05	0.12

Table 3.2 (Continued)

Designation	$\bar{R}/R_n$	V_R
Slender Columns, kL/h = 20, f' = 5 ksi, compression failure	1.10	0.17
Slender Columns, kL/h = 20, f'_c = 5 ksi, tension failure	0.95	0.12
Shear, Beams with a/d > 2.5, ρ_w = 0.008		
no stirrups	0.93	0.21
minimum stirrups	1.00	0.19
$\rho_V f_y$ = 150 psi	1.09	0.17

Note: 1 ksi = 6.9 N/mm^2

3.5.2 Resistance Statistics for Metal Structural Members

Following are some representative samples of resistance statistics for metal members and components (from Appendix C). Probability distributions were assumed to be lognormal in each case.

Table 3.3

Typical Resistance Statistics for Metal Structural Members

Designation	$\bar{R}/R_n$	V_R
Structural Steel		
Tension members, limit state - yielding	1.05	0.11
Tension member, limit state - tensile strength	1.10	0.11
Compact Beam, uniform moment	1.07	0.13
Beam-Column	1.07	0.15
Plate Girders, flexure	1.08	0.12
A325 HS Bolts, tension	1.20	0.09
Axially Loaded Column	1.08	0.14
Cold-Formed Steel		
Braced Beams with stiffened flanges	1.17	0.17
Columns with stiffened flanges	1.07	0.20
Aluminum		
Beams, laterally braced	1.10	0.08
Beams, unbraced	1.03	0.13

3.5.3 Resistance of Engineered Brick and Concrete Masonry

Statistical characteristics of unreinforced masonry walls in compression plus bending are derived from data on full size walls tested in the laboratory, augmented by a factor to account for differences between fabrication and curing conditions in situ and in the laboratory (Appendix D).

The strength of brick and concrete masonry walls in compression plus bending appears to be modeled satisfactorily by a lognormal distribution. The mean and c.o.v. of strength, measured in terms of vertical load, are summarized in Table 3.4 for two common wall slendernesses. The mean values depend on eccentricity ratio, e/t, and on slenderness, h/t. Variations in these estimates among individual sets of data naturally are to be expected; however, these values are representative and are suitable for the reliability analyses leading to the load criterion development.

Table 3.4

Resistance of masonry walls in compression plus bending

Type	Slenderness h/t	Brick Masonry $\overline{R}/R_n$ e/t = 0	Brick Masonry $\overline{R}/R_n$ e/t = 1/6	Brick Masonry V_R	Concrete Masonry $\overline{R}/R_n$	Concrete Masonry V_R
Inspected	10	5.3	6.0	0.18	4.2	0.19
	15	5.6	6.3	0.18	4.8	0.19
Uninspected	10	3.2	3.6	0.21	2.5	0.21
	15	3.4	3.8	0.21	2.9	0.21

As discussed in Appendix D, there is some question as to whether $\overline{R}/R_n$ and V_R referred to vertical load capacity are the most realistic statistical parameters for characterizing resistance when e/t becomes large. Calibrations were also performed for pure flexure, which provides an estimate of the reliability at very large eccentricities. In pure bending, $\overline{R}/R_n \approx 3.9$ and $V_R \approx 0.24$.

3.5.4 Glulam Members in Bending, Tension and Compression

The behavior of glued-laminated (glulam) structural members in bending, tension and compression has been determined from laboratory tests of large specimens, adjusted for load duration and, in the case of flexural members, for size. These data are discussed in detail in Appendix E, along with some problems in analyzing reliability of wood structures. Dimension lumber and light frame construction have not been included in this study.

Estimates of means and c.o.v. are presented in Table 3.5 for flexure. Additional data is presented in Table E.4 in the Appendix. One factor which influenced the decision to emphasize glulam data was that the current strength-load duration relation appears to be more suitable for glulam than for other timber members containing more imperfections. As discussed in Appendix E, $\overline{R}/R_n$ depends on the load combination because the load duration effect for each maximum load is different. Minor variations in the statistics with species have been ignored. There is conflicting evidence on whether the cumulative probability distributions are Weibull or lognormal. In the reliability analysis of existing designs in the following chapter, both distributions are used to demonstrate the sensitivity to assumptions regarding distributions.

Table 3.5

Resistance Statistics of Glulam Beams

Maximum Load in Combination	D	L	S	W,E
$\overline{R}/R_n$	1.75	1.97	1.62	1.80
V_R	0.18	0.18	0.18	0.18

4. CALIBRATION WITH EXISTING STANDARDS

4.1 General Considerations

Previous parts of this report have established the basis for a probability based design methodology and have summarized statistical data on loads and resistances. For the development of a probability based load criterion, it is necessary to establish target reliability indices, β. In order to do this it is first required to establish β values inherent in present design practice. This chapter will review these β's from the details presented in the Appendices which deal specifically with the material technologies of reinforced and prestressed concrete, metals, wood and masonry. The reliability indices typical of present design will be used as a guide in establishing targets for the new load criterion.

4.2 Gravity Loads

The prevalent load combinations involving gravity loads are: (1) dead plus maximum occupancy live loads on floors (D + L) and (2) dead plus maximum snow load for roofs (D + S). Each design situation is defined by a set of nominal resistance and load values. In present allowable stress design specifications,

$$R_n/FS = D_n + L_n \tag{4.1}$$

In plastic design of steel structures,

$$R_n = 1.7\ (D_n + L_n) \tag{4.2}$$

In concrete structures,

$$\phi R_n = 1.4\ D_n + 1.7\ L_n \tag{4.3}$$

The gravity load cases govern in many practical design situations and are considered to be of fundamental importance in the calibration work.

Typical representative variations of β with L_o/D_n and S_n/D_n are given in Fig. 4.1 for reinforced concrete and steel beams. L_o, recall, is the basic live load in Table 1 of Ref. 2, e.g., 50 psf (2.39 N/m^2) in offices. From this figure it is evident that the variation of β for such beams is remarkably similar. In each case β decreases as L_o/D_n or S_n/D_n increases. When viewing the similarity it should be kept in mind, however, that reinforced concrete beams have practical ranges of L_o/D_n or S_n/D_n of 0.5 to 1.5, while for steel beams this range is from 1 to 2. As shown in Fig. 4.1, the significant load ratios for steel beams are thus shifted to the right with regard to concrete beams. Representative values for β are thus 2.8 and 3.1 for concrete beams, and 2.5 and 2.9 for steel beams for, respectively, the D + L and the D + S combination.

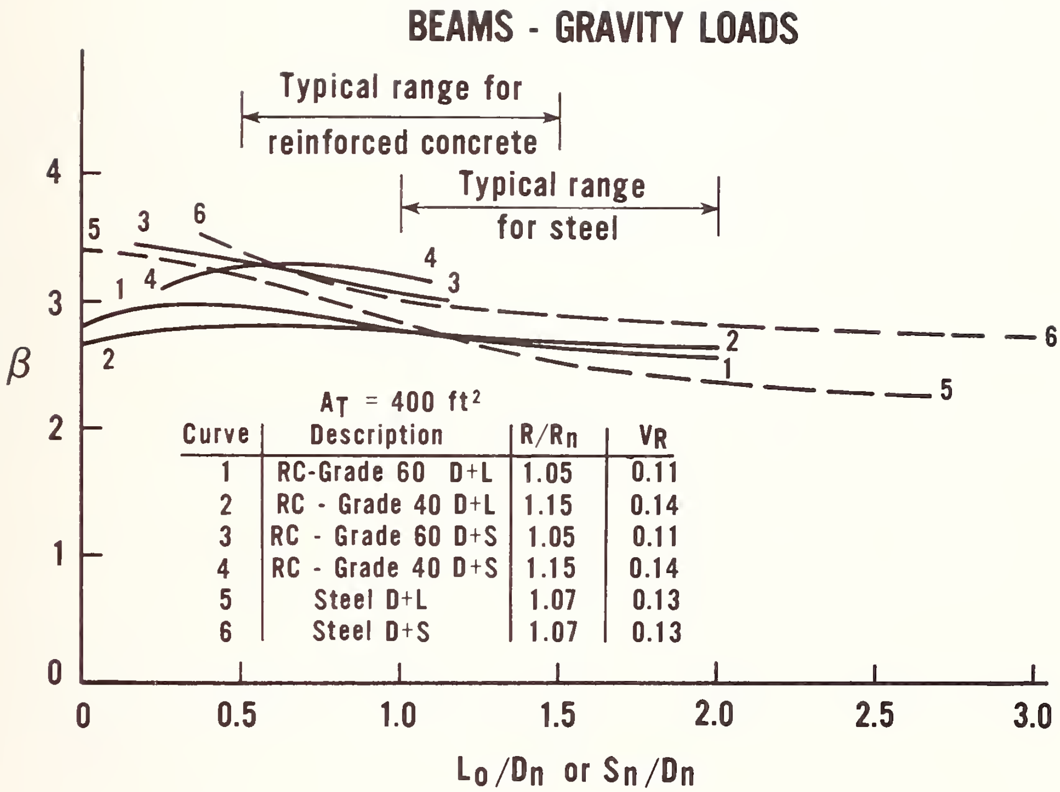


Figure 4.1 - Reliability Index for Steel and Reinforced Concrete Beams Conforming to Current Criteria - Gravity Loads (100 ft^2 = 9.3 m^2)

Other typical values of β for the D + L combination for metal structures are summarized (see Appendix C for a detailed tabulation) as follows (1 ft^2 = 0.093 m^2):

Tension members, limit state yield, (AISC), $L_o/D_n = 2$, $\beta = 2.5$

Compact simple beams, (AISC), $A_T = 1000$ ft^2, $L_o/D_n = 2$, $\beta = 3.1$

Steel columns (AISC), $A_T = 2500$ ft^2, $L_o/D_n = 1$, $\beta = 3.1$ and 2.8, respectively, for typical major axis and minor axis buckling

Cold-formed steel and aluminum members have typically high L_o/D_n ratios (around 5), and so β's for these elements are usually around 2.5.

Typical values of β for the D + L combination for concrete structures are (from Appendix B):

Cast-in-place postensioned beams, $A_T = 400$ ft^2, $L_o/D_n = 1$, $\beta = 3.0$

Plant-precast pretensioned beams, $A_T = 400$ ft^2, $L_o/D_n = 1$, $\beta = 3.6$

Tied columns, Compression failures, $A_T = 1200$ ft^2, $L_o/D_n = 1$, $\beta = 3.4$

Spiral columns, Compression failures, $A_T = 1200$ ft^2, $L_o/D_n = 1$, $\beta = 3.1$

Shear, beam with minimum stirrups, $A_T = 400$ ft^2, $L_o/D_n = 1$, $\beta = 2.0$.

While the reliability index for typical steel and concrete structures under dead and live loads is in the vicinity of 3, β for typical brick and concrete masonry walls and columns under compression and bending appearsto be considerably higher (see Figure 4.2 and Appendix D, figures D.7 - D.10). For example, for walls in compression built with inspected workmanship with a typical live-to-dead load ratio $L_o/D_n = 0.5$, a tributary area of 400 ft^2 (37 m^3) and a height-to-thickness ratio of 10, $\beta = 7.4$ for brick masonry and $\beta = 6.2$ for concrete masonry. The reliability for uninspected masonry, with its higher c.o.v. and lower $\overline{R}/R_n$, is considerably less; uninspected workmanship causes β for the same brick wall to decrease to 4.7. At high eccentricities (e/t in excess of 1/6), β begins to diminish, falling to about 3 when e/t reaches the maximum allowable value of 1/3. Reliability indices for reinforced masonry columns in compression are between 6 and 7.

Reliabilities calculated for glued-laminated timber members are quite similar to those for the lighter metal structures (see Figure 4.3 and Appendix E, Figures E.2 - E.4); β varies from 2.2 to 2.6 for a beam with a typical $L_o/D_n = 3$, and β is in the range 2.1 - 2.5 for a beam with $S_n/D_n = 3$. The sensitivity of β to probability distribution for resistance becomes less pronounced at larger and more typical ratios of L_o/D_n and S_n/D_n.

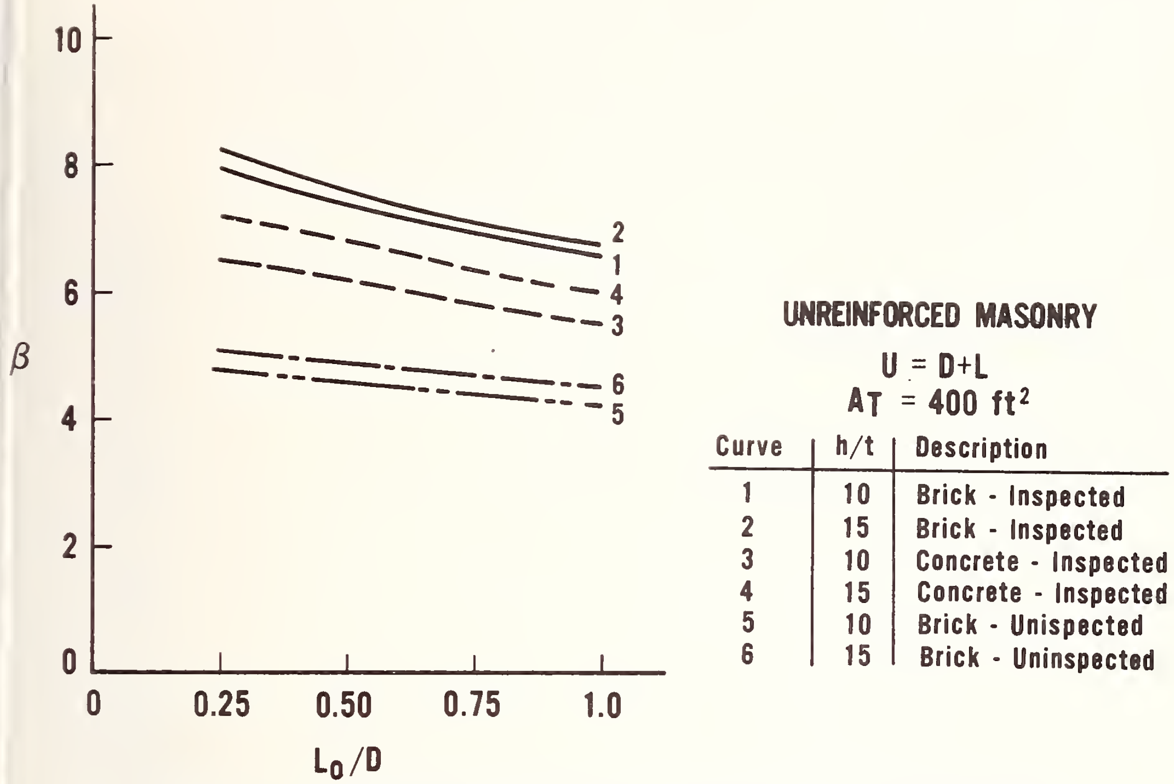


Figure 4.2 - Reliability Index for Nonreinforced Brick and Concrete Masonry Walls Conforming to Current Criteria (100 ft^2 = 9.3 m^2)

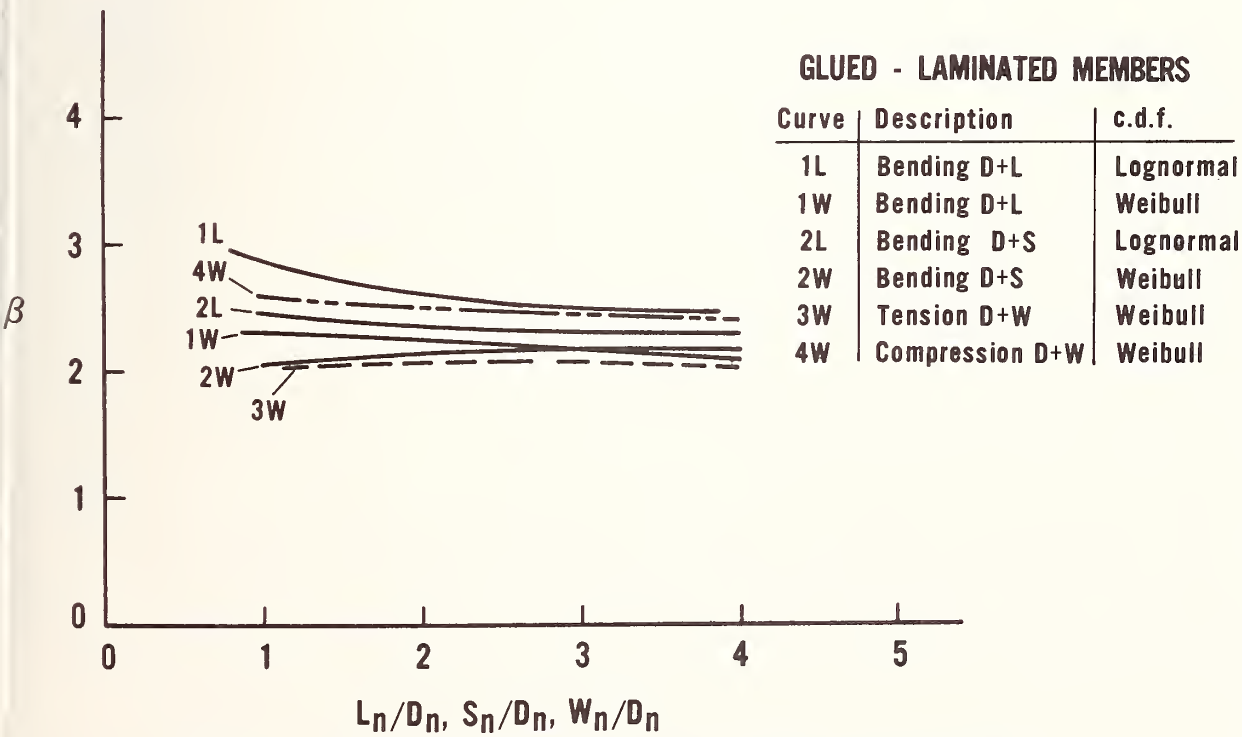


Figure 4.3 - Reliability Index for Glulam Members Conforming to Current Criteria

4.3 Gravity and Environmental Loads

The major load combinations are dead, live and wind (D + L + W) and dead, live and earthquake (D + L + E). Discussion will first focus on the combinations $D + L_{apt} + W$ and $D + L + W_{apt}$, where L and W define the maximum magnitudes and L_{apt} and W_{apt} are the "arbitrary-point-in-time" values.

The variation of β with various L_o/D_n and W_n/D_n ratios is shown in Fig. 4.4 and 4.5, respectively, for steel and reinforced concrete beams. In the calibration, R_n is determined for each design situation, from

Allowable stress design:

$$R_n = (FS)(D_n + L_n + W_n)\ (3/4) \tag{4.4}$$

Plastic design in steel:

$$R_n = 1.3(D_n + L_n + W_n) \tag{4.5}$$

Reinforced concrete:

$$R_n = 0.75(1.4D_n + 1.7L_n + 1.7W)/\phi \tag{4.6}$$

The effect of the rate of loading has been included in the calibration by multiplying $\overline{R}/R_n$ by 1.10 for steel members and 1.05 for reinforced concrete members. This difference accounts for the relatively higher dead load component of the total load effect in the concrete structures as compared with steel structures. Assuming the time needed for the wind load effect to reach a limit state value is the same for both types of beam, the rate is higher for steel beams since the wind component of the total load effect is greater. The strain rate effect for steel structures was estimated to be of the same order as in the standard ASTM coupon test, giving essentially the mill test yield stress rather than the static yield stress as the basic material variable (i.e., $\overline{R}$ is multiplied by 1.10). For concrete structures the effect was cut in half, i.e., to 1.05.

From Figs. 4.3 to 4.5 it can be seen that β decreases as W_n/D_n increases, and that β increases as L_o/D_n increases. While the curves in Figs. 4.4 and 4.5 are for beams, the results would be similar for other types of members for which the resistance statistics are similar. It can be seen from Figs. 4.4 and 4.5 and the data presented in the Appendices for the various material technologies that β for wind approaches a value of 2 in cases where wind is the major load component. With greater live and dead load, the value of β increases to that of the D + L case. In general the wind load combinations result in a somewhat lower reliability in current practice than the D + L and the D + S combinations. This is due to the 1/3 increase in allowable stress (or the use of 3/4 of the total factored

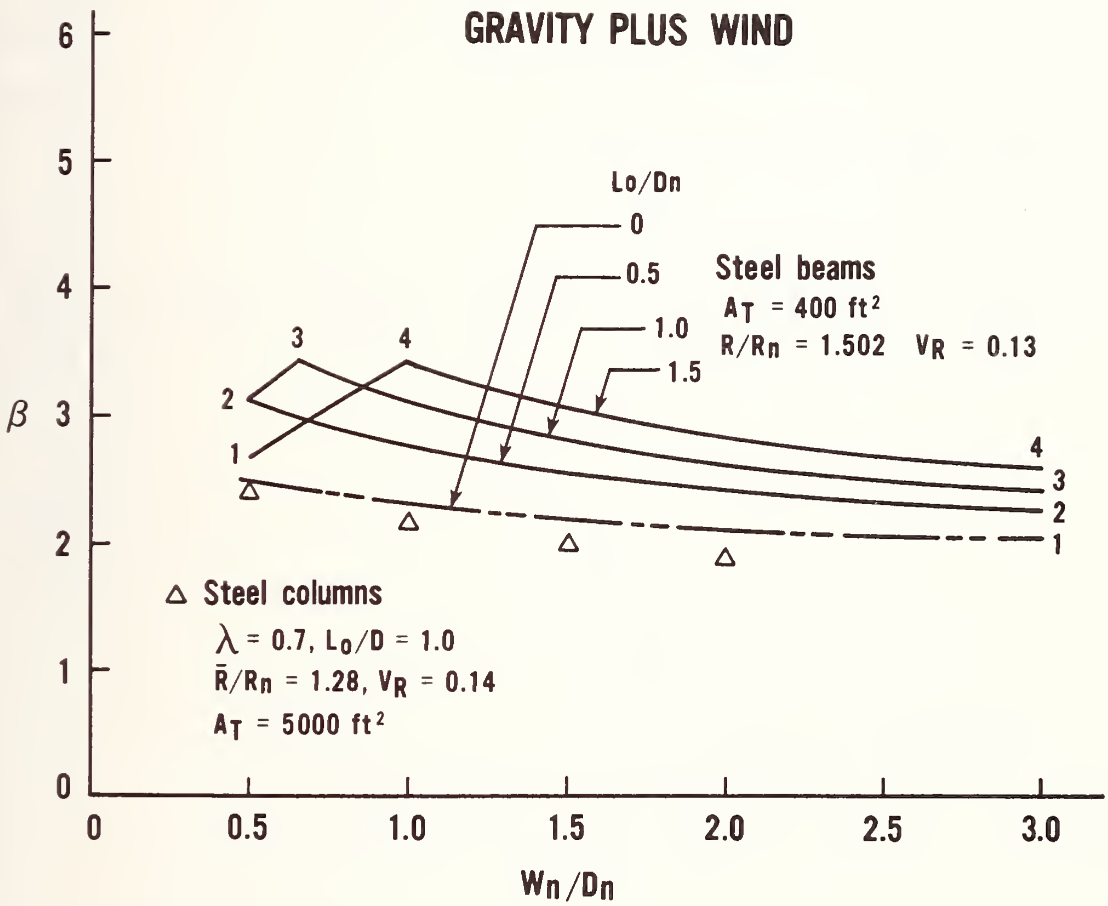


Figure 4.4 - Reliability Index for Steel Members Conforming to Current Criteria - Gravity Plus Wind Loads (100 ft^2 = 9.3 m^2)

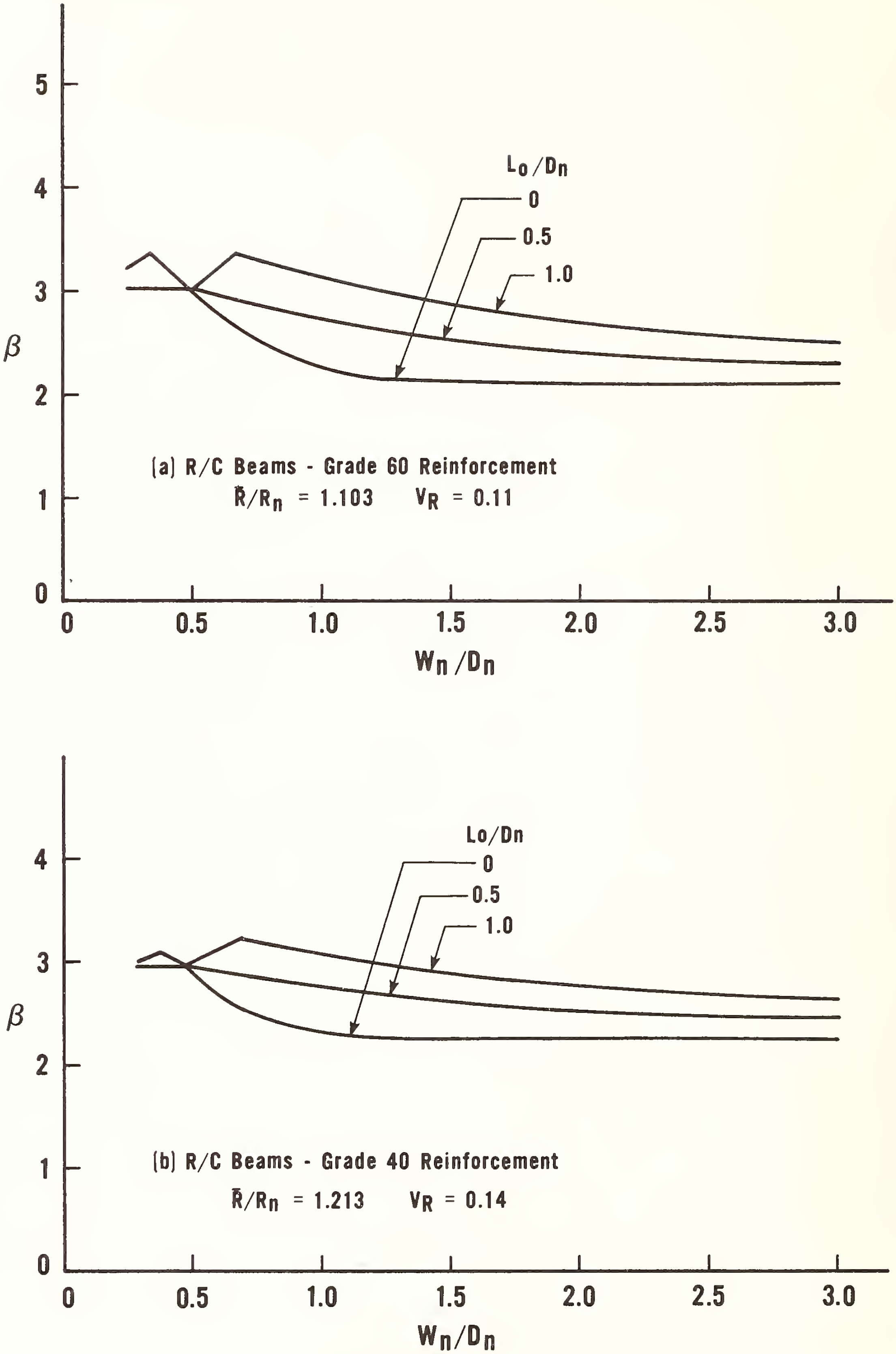


Figure 4.5 - Reliability Index for Reinforced Concrete Beams Conforming to Current Criteria - Gravity Plus Wind Load - A_T = 400 ft^2 (37 m^2)

loads) in all of the current codes used for calibration in this report. It is possible that the reliability of structures under wind is only apparently less because of such factors as load-sharing by cladding and load redistribution among members. The fact that a number of members share the load, not all of which will be equally understrength, provides a mitigating effect that has not been directly included in the analysis.

Typical values of β for the earthquake loading case D + L + E, are given in Fig. 4.6 for two locations; Boston and Los Angeles, and for steel and concrete beams and columns, respectively. Strain rate effects have been incorporated in this analysis. Due to the high variability of the earthquake loads (see Appendix A) as compared to the variability of dead loads, the β-versus-E_n/D_n curves flatten out rapidly to values which reflect essentially only the contribution of the earthquake load effect. Reliability indices for D + L + E are lower than for D + L + W. While the difference between beams and columns is small the effect of geographic location on β is pronounced. Values of β for steel tend to be somewhat lower than for concrete. Typical values of β for D + L + E from Fig. 4.6 are:

Material	E_n/D_n	Location	β
Steel beam	2	Boston	2.0
Steel beam	2	Los Angeles	1.5
Concrete beam	1	Boston	2.1
Concrete beam	1	Los Angeles	1.6

When the effects of wind or earthquake counteract the effect of gravity loads, the reliability indices tend to be somewhat lower than when the loads are additive, as indicated in Fig. 4.7 for the W - D combination. The discrepancy is especially pronounced in allowable stress formats where, as indicated in Chapter 1, it is difficult to treat the combinations in which loads are added and subtracted consistently from a safety viewpoint. The descending branch of the curves in Fig. 4.7 is a result of the minimum strength that the member has even if it is not specifically designed to resist counteracting load effects, e.g.,

Allowable stress design:

$$R_n = \max \begin{cases} (D_n + L_n)FS \\ 0.75\ (W_n - D_n)FS \end{cases}$$

Concrete structures:

$$\phi R_n = \max \begin{cases} 1.4\ D_n + 1.7\ L_n \\ 1.3\ W_n - 0.9\ D_n \end{cases}$$

The reliability for concrete beams and columns generally is closer to the additive load cases because the individual load (effects) are factored.

This section has examined the notional probability of exceeding a limit state, as characterized by the reliability index β, for various load combinations and for various

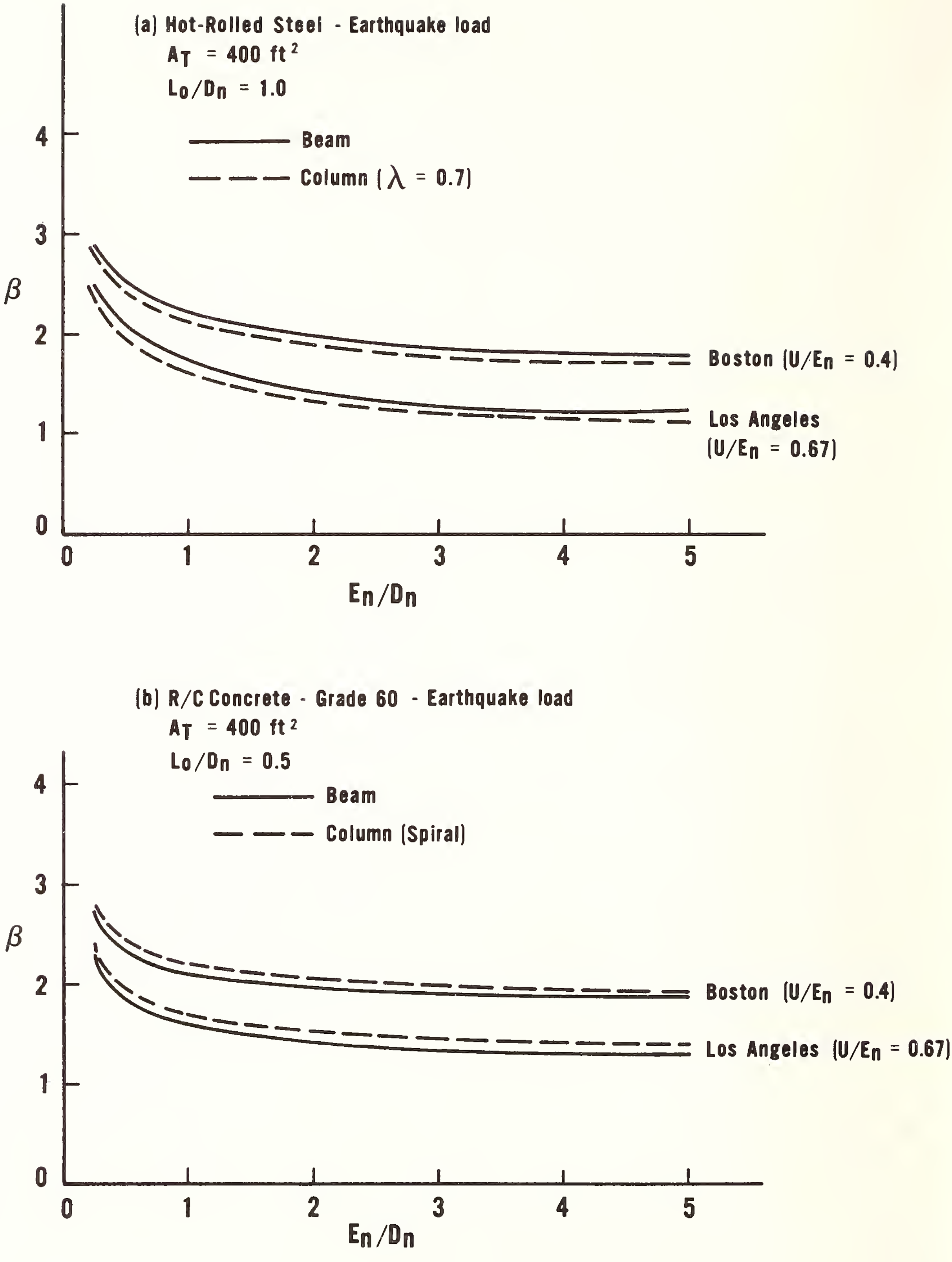


Figure 4.6 - Reliability Index for Steel and Reinforced Concrete Beams - Gravity Plus Earthquake Load (100 ft^2 = 9.3 m^2)

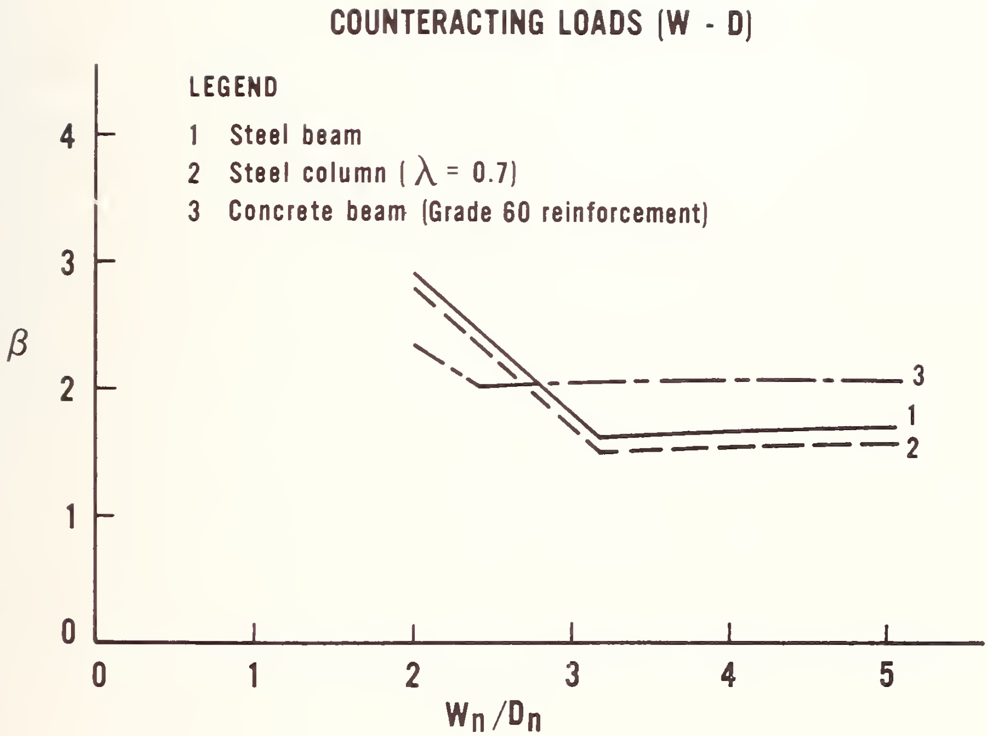


Figure 4.7 - Current Reliability Index for Steel and Reinforced Concrete Members - Counteracting Loads

material technologies. It appears that β inherent in current design is smaller for load combinations which include load effects due to wind or earthquake than for load combinations with gravity loads only. This conclusion may be only apparently true. In the case of live loads, the load consists of multiple discrete sources and the effect on the structure is generally local. This is quite different from wind and earthquake loads which affect the entire structure. Many mitigating effects cannot be directly translated into rationally definable quantities, and since structures do not seem to experience problems due to this apparent reduction of reliability, it was decided to allow the smaller reliability indices under the load combinations involving wind and earthquake to carry over in setting the target reliabilities, β_o. This is done in Chapter 5 of this report.

5. DEVELOPMENT OF DESIGN CRITERIA

5.1 Scope

The loading criterion developed in this report is intended for use in the design of buildings and other structures. It has been developed to be compatible with the loads presented in the 1980 version of the A58 Standard. The numerical values of the factors will generally need adjustment if used with loads which have been developed on the basis of different assumptions (e.g., 30 yr. reference period versus the 50 year period specified herein) or loads of different character (e.g., vehicle loads on bridges) from the loads discussed in this report. However, the methodology of arriving at load factors still applies, and enough information and instruction is contained herein to generate them in a manner consistent with the load factors presented here.

The load criterion presented applies only to the ultimate limit states. Load criteria governing serviceability limit states currently are under study. It is possible that an LRFD format may not be appropriate in all instances for serviceability checks.

5.2 Selection of Format

Probability based limit states design is based on loads or load effects which are multiplied by load factors which are generally greater than unity and resistances which are multiplied by resistance factors, less than unity, according to the equation:

$$\text{Factored resistance} \geq \text{Effect of factored loads} \qquad (5.1)$$

The characteristics of a number of different formats for presenting this equation will be reviewed in this section prior to choosing the format proposed in this report. The final choice of format must balance theoretical appeal, computational ease, accuracy and user acceptance.

5.2.1 Load Factors

The National Building Code of Canada [21] uses the probability factor format given in Eq. 5.2 to specify the basic loading cases:

$$\text{Factored Load Effects} = U\,\{\gamma_D D_n + \Psi(\gamma_L L_n + \gamma_W W_n + \gamma_T T_n)\} \qquad (5.2)$$

where U refers to the load effects due to loads in the brackets and D_n, L_n, etc. are the loads; γ_D, γ_L, etc. are load factors; and Ψ is a load combination probability factor equal to 1.0, 0.7 and 0.6 if one, two or three loads are included in the bracket. The dead load factor γ_D may have values of 1.25 when D_n and L_n, etc. are additive and 0.85 where D_n counteracts L_n, etc.

In this format and all others discussed, the terms γ_D, γ_L, etc. account for variations in the dead or other loads themselves plus variations in the load effects due to uncertainties in the load models and the structural analysis. The factor Ψ in this format accounts for the reduced probability that maximum dead, live, wind, etc. loads act simultaneously.

The European Concrete Committee Model Code [3] uses Eq. 5.3 to define the basic factored load effects:

$$\text{Factored Load Effects} = U\ \{\gamma_D \overline{D} + \gamma_Q\ [Q_{1k} + \sum_{i>1} (\Psi_{oi} Q_{ik})]\} \qquad (5.3)$$

where U refers to the load effects due to all the loads in the brackets; Q_{1k} and Q_{ik} are the characteristic values of the principal variable load (Q_1) and some other less important variable load; Ψ_{oi} is the ratio of the frequent or arbitrary point-in-time value of the i<u>th</u> load to the characteristic value of that load; and γ_Q is the load factor on the combination of variable loads. The characteristic value of a load is a moderately high fractile of the arbitrary-point-in-time distribution of that load, roughly comparable to the loads specified in the ANSI A58 Standard. In computing the maximum factored load effect for a problem involving several variable loads, it may be necessary to consider several combinations with each of the loads considered as the principal variable load in turn. In situations involving P-Δ moments, the right hand side of Eq. 5.3 is multiplied by an analysis factor, γ_{f_3}

The LRFD format proposed by Ravindra and Galambos [9] involves a set of several load factor equations which include the most common load combinations. In simplified form, these are:

For dead load and live load:

$$\text{Factored Load Effects} = \gamma_D \overline{D} + \gamma_L \overline{L} \qquad (5.4)$$

where $\overline{D}$ is the load effect due to the mean dead load and $\overline{L}$ is the load effect due to the mean of the maximum live loads anticipated on structures during their lifetimes (mean lifetime maximum live load).

For dead load plus arbitrary point-in-time live load and lifetime maximum wind load:

$$\text{Factored Load Effects} = (\gamma_D \overline{D} + \gamma_{apt} \overline{L}_{apt} + \gamma_W \overline{W}) \qquad (5.5)$$

where $\overline{L}_{apt}$ is the load effect due to the mean arbitrary point-in-time live load which, as explained in Chapter 3, is different from (lower than) $\overline{L}$ in Eq. 5.4, and $\overline{W}$ is the mean lifetime maximum wind load.

For dead plus arbitrary point-in-time live load plus lifetime maximum snow load:

$$\text{Factored Load Effects} = (\gamma_D\overline{D} + \gamma_{apt}\overline{L}_{apt} + \gamma_S\overline{S} \qquad (5.6)$$

For lifetime maximum wind load minus dead load:

$$\text{Factored Load Effects} = (\gamma_W\overline{W} - \gamma_{Dmin}\overline{D}) \qquad (5.7)$$

Equation 5.5 is actually a restatement of Eq. 5.3 in which wind load is the principal variable load and live load is the only other variable load that is significant. The term $\gamma_{apt}\overline{L}_{apt}$ in Eq. 5.5 is equivalent to $U\{\gamma_Q\ \Psi_{oi}\ Q_{ik}\}$ in Eq. 5.3, the major difference being that the load is given as a multiple of the maximum load ($\Psi_{oi}Q_{ik}$) in Eq. 5.3, but as a separate loading case with its own load factors in Eq. 5.5. In general it would seem that the advantages of the computational simplification attained by expressing the arbitrary-point-in-time live load as $\Psi_{oi}\overline{L}$ in Eq. 5.3 will more than offset any advantages due to the increased accuracy attained by considering a separate loading case, L_{apt} (Eq. 5.5). The same could be said in comparing Eq. 5.3 and Eq. 5.6.

Equations 5.3, 5.5 and 5.6 appear to express the true combination of loads in a better fashion than Eq. 5.2. In an interior column of a symmetrical sway frame, for example, the vertical loads are due to D and L while the moments are primarily due to the wind load. If the critical loading condition involves both axial force and moment due to D, L and W, Eq. 5.2 would base design on 70 percent of the wind load moment while Eqs. 5.3 and 5.5 would use the entire wind load moment.

If the methodology of Eq. 5.2 were applied to loadings consisting of dead, live, wind and snow, a total of 14 loading combinations conceivably could be considered (including all cases involving γ_D = 0.85 and 1.25). If Eq. 5.3 is applied to these loadings a total of 32 combinations can be postulated. If, however, the methodology of Eqs. 5.4 to 5.7 is applied to these loadings, only four combinations need to be considered. Clearly, if computational simplicity is considered important, a few fundamental load combinations must be explicitly stated for design, as is done in Eq. 5.4 to 5.7.

The set of load factors recommended in this report will combine the best features of Equations 5.3 and LRFD. In general, the load factors should be applied to the load prior to performing the analysis which transforms the load to a load effect. Provided that the relation between load and load effect is linear or nearly so, it makes no difference when the load factors are applied. However, for certain nonlinear problems, it is unconservative to factor the load effect.

5.2.2 Resistance Factors

The left hand side of Eq. 5.1, factored resistance, can also be expressed in several ways. The most familiar of these to North American designers is the use of resistance factors based on structural action. In this format, the left hand side of Eq. 5.1 is expressed as ϕR_n where ϕ is a strength reduction factor or resistance factor which applies to a particular structural action such as flexure, shear, bond, axial compression, etc. This design format is used in the ACI Standard 318 [19] and the Load and Resistance Factor Design of steel structures [9].

The ACI Code ϕ factors represent an early attempt to account for the possibility of understrength as well as the consequences of failure and mode of failure. The history of these provisions and a brief discussion of their statistical derivation is presented by MacGregor [23]. Based on a reliability analysis model, Ravindra and Galambos [9] have proposed load and resistance factors for structural steel design. The resistance factors, ϕ, differ for each ultimate limit state. Essentially, the factors proposed do not reflect the mode of failure, except that the very serious consequences of a connection failure relative to its cost are reflected by lower ϕ factors based on a target probability of failure that was arbitrarily set at 2 1/2 orders of magnitude lower than for members.

The other important method of specifying resistance factors uses material partial safety factors. In the Comité Euro-International du Béton (CEB) Model Code [3], the strength of a cross-section is computed using design material strengths equal to f'_c/γ_c and f_y/γ_s where γ_c and γ_s are material partial safety factors or material understrength factors for concrete and steel respectively. These partial safety factors are the same for all limit states. For average construction quality these terms have values of 1.5 and 1.15 which correspond roughly to 1 in 1000 understrengths of concrete and steel. If the anticipated dimensional tolerances exceed normal practice, the designer is asked to reduce the effective depths, etc. used in calculations by the difference between the anticipated and normal tolerances. Although there is provision in this system to recognize the consequences of failure or mode of failure, the CEB has no intention at present of including these effects in normal design.

The major factors to be considered in deriving resistance factors include:

(1) Variability in member strength due to variability of material properties in the structure. In the case of a composite material two or more material variabilities may have to be considered.

(2) Variability in member strength due to variability of dimensions.

(3) Variability in member strength due to simplifying assumptions in the resistance equations (e.g., the use of a rectangular stress block in concrete design). This is referred to as variability due to model error.

(4) Increased risk to building occupants if failure occurs without warning and the post-failure strength is much less than original strength.

(5) Importance of member in structure.

(6) Designers' familiarity with method used.

Table 5.1 compares the manner in which the two resistance factor formats listed earlier respond to these factors. A rating of 0 is given if this factor is not included in the method as normally used, a rating of 1 is given if the factor is considered and a rating of 2 is given if the particular factor is treated particularly well. Only the format has been considered in Table 5.1. The validity of the statistical analyses used to derive the existing resistance factors has been ignored since the derivation of factors for future codes will presumably be more up-to-date.

Based on the ratings given in Table 5.1 the structural action resistance factor or ϕ factor is recommended for use in material standards in the United States.

5.3 Target Reliability Indices

It is not the purpose of this report to make specific recommendations to material specification groups as to precisely what reliabilities their strength criteria should be targeted upon. As discussed in the executive summary, it is the writers' feeling that decisions of this nature fall outside the scope of the A58 Standard; instead, they are the responsibility of the material specification committees where the necessary expertise on material performance exists. Nonetheless, it is necessary to have an idea of the range that these target reliabilities are likely to fall within, so as to make it possible to perform the necessary calculations leading to specific load factors. It should be emphasized that this actually places little restriction upon individual material specification groups, since once the load factors are determined, the actual design reliabilities may be adjusted through an appropriate selection of ϕ-factors. Indeed, some simple graphs of ϕ vs. β are provided in a later section that specification committees can use to assist in making these decisions.

The target reliabilities selected here, denoted β_o, then, are chosen solely for the purpose of enabling the load factors to be selected intelligently. We feel strongly that

TABLE 5.1

COMPARISON OF RESISTANCE FACTOR FORMATS

Reasons for Resistance Factors	Structural Action Resistance Factor, ϕ		Material partial Safety Factors, γ_m	
	Rating	Remarks	Rating	Remarks
1. Variability of Material Properties	1	Derivation of ϕ factor only considers variability of materials in member with average proportions, ρ, etc.	2	Derivation of γ_m factor can consider variability of material alone. In members made of two materials the safety variability can then be considered for all proportions.
2. Variability of Dimensions	1	Derivation of ϕ factor considers dimensional variability of average size members.	0	Not normally considered. This could be considered in part by basing design on characteristic dimensions.
3. Variability due to Model Error	2	By differentiating between structural actions, the assumptions and approximations inherent in each action can be considered.	0	Since same γ_m factor is used for all actions, there is no easy way to include model error. This could be done by multiplying the computed resistance by a ϕ factor.
4. Mode of Failure	1	The average mode of failure can be reflected in setting the ϕ factors.	1	The mode of failure of the constituent materials could be considered in the derivation of the corresponding material factors.
5. Importance of Member	1	Only treats an average column or an average beam.	0	This could be done by multiplying the computed resistance by a ϕ factor.
6. Designers' Familiarity with Method	1	Currently used in North America	0	Rejected by ACI Committee 318 in 1963.

the new probability-based load criterion should lead to designs which are essentially the same, in an overall sense, as those obtained using current acceptable practice. This is because of the evolutionary nature of codes and standards, which requires changes to be made cautiously and deliberately. This is not to say that all designs would remain the same in every instance; if that were so, there would be no reason or motivation for using the new criterion. One of the advantages of reliability-based design is that it enables inconsistencies within a particular specification to be eliminated and more uniform reliabilities to be attained over a range of situations.

The target β-values selected for deriving the load factors are representative of those associated with existing designs. As shown in Chapter 4 and the Appendices, these span the range from 1.5 for some metal tension members to over 7 for certain masonry walls with very small vertical load eccentricities. However, many flexural and compression members tend to fall within the range $\beta = 2.5$ to 3.0 for the D + L, D + S, and D + L + W load combinations. These are among the most common combinations governing designs in large parts of the United States, and there is general professional agreement that present designs in these cases are satisfactory. It seems appropriate, then, that the target β_o chosen for purposes of deriving the load criterion fall within this range. In the following, the target β_o for D + L and D + S is 3.0; for D + L + W, $\beta_o = 2.5$; and for D + L + E, $\beta_o = 1.75$. Generally speaking, these values are slightly more conservative than indicated by current practice when the transient load (L_n, W_n, S_n, E_n) is large in comparison with the permanent load and less conservative when the permanent load is a major component.

5.4 Reliability-Based Design

While several levels of sophistication for reliability-based design can be identified, two of particular current interest are referred to as Level II and Level I methods. Level II methods are primarily of interest to technical committees. For a given limit state, they employ safety checks at a number of discrete points, e.g., at selected values of L_n/D_n W_n/D_n, etc. The basic design variables in the limit state equation are specified in advance. Reliability is measured either by the reliability index or a notional probability of failure. Level I methods involve the selection of one set of load factors to be applied to all designs, regardless of L_n/D_n, W_n/D_n, etc., and a resistance factor which depends on the material and limit state. Levels I and II can be made equivalent if the load and resistance factors in the Level I format are allowed to vary.

For operational convenience, practical design criteria in the United States will be of Level I type in the foreseeable future. It is instructive, however, to examine how the load and resistance factors corresponding to prescribed values of β_o vary for different limit states and load situations. The reader will then be in a better position to appreciate some of the considerations which guide the selection of the material-independent load criterion. In this section, Level II design criteria are presented for selected cases. The format selected for the criteria is the load and resistance factor format presented in Section 5.2.

Load and resistance factors corresponding to $\beta_o = 3$ for steel beams are shown in Fig. 5.1 for D + L, and in Fig. 5.2 for D + S. Factors derived with $\beta_o = 2.5$ for D + L + W are shown in Fig. 5.3. Similar relations are presented in Figs. 5.4 through 5.6 for concrete beams with Grade 60 and Grade 40 reinforcement. These factors are compatible with nominal loads specified in the 1980 version of the A58 Standard.

Several points are worth noting about these figures. First of all, the resistance factor is relatively insensitive to the time-varying load(s) in the combination (e.g., live, snow, wind, as appropriate) when that load is very small. Similarly, the load factors do not appear to be especially sensitive to the resistance statistics. Although a certain amount of coupling between the resistance and load factors exists, the fact that this coupling appears relatively weak has some important implications for the general load criterion to be developed in the next section. The load factor for dead load (effect) is much lower than in any existing or proposed standard that the writers are aware of. This is because the variability in D is quite small compared to other load variabilities. The magnitude of γ_D appears to be virtually independent of the magnitude of the time-varying load(s) in the equation. The live load factor in the D + L + W combinations in Figs. 5.3 and 5.6 is less than unity because L_{apt} is much less than L_n. A comparison of Figs. 5.1 to 5.3 and 5.4 to 5.6 shows that the resistance factor is in the same range for the D + L and D + L + W combinations.

These observations indicate that choosing γ_D to be constant and uncoupling the resistance and load factors will not cause significant deviations from the target reliability in Level I design. On the other hand, the load factor on the time-varying load in the combination increases as that load increases because its higher variability becomes increasingly more important in determining the total load effect. It follows that if the load factors for time-varying loads are specified as constant, as is done in current design procedures,

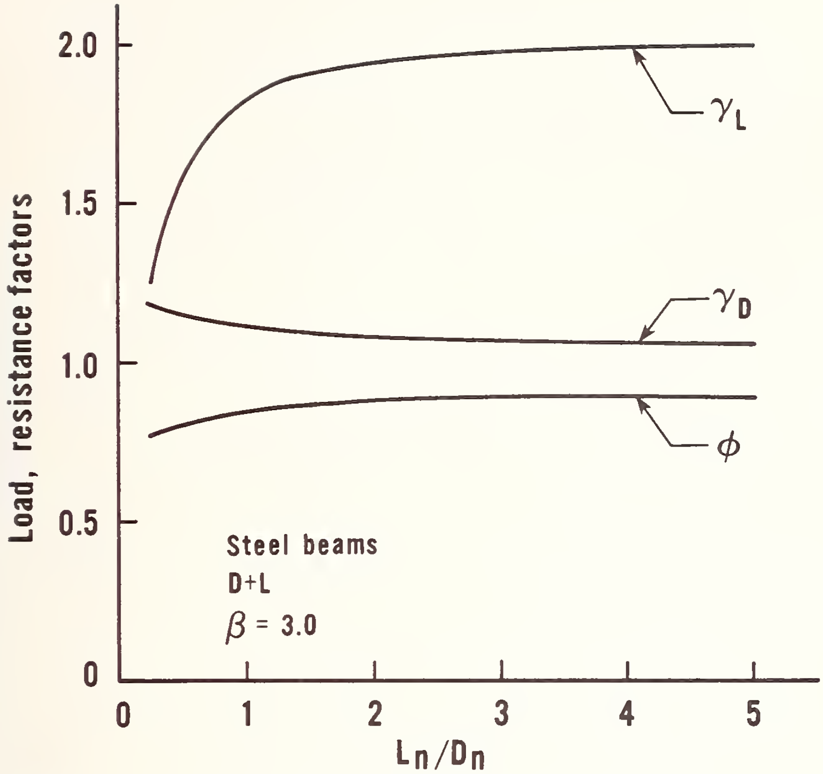


Figure 5.1 - Load and Resistance Factors for Flexure in Steel Beams (D + L)

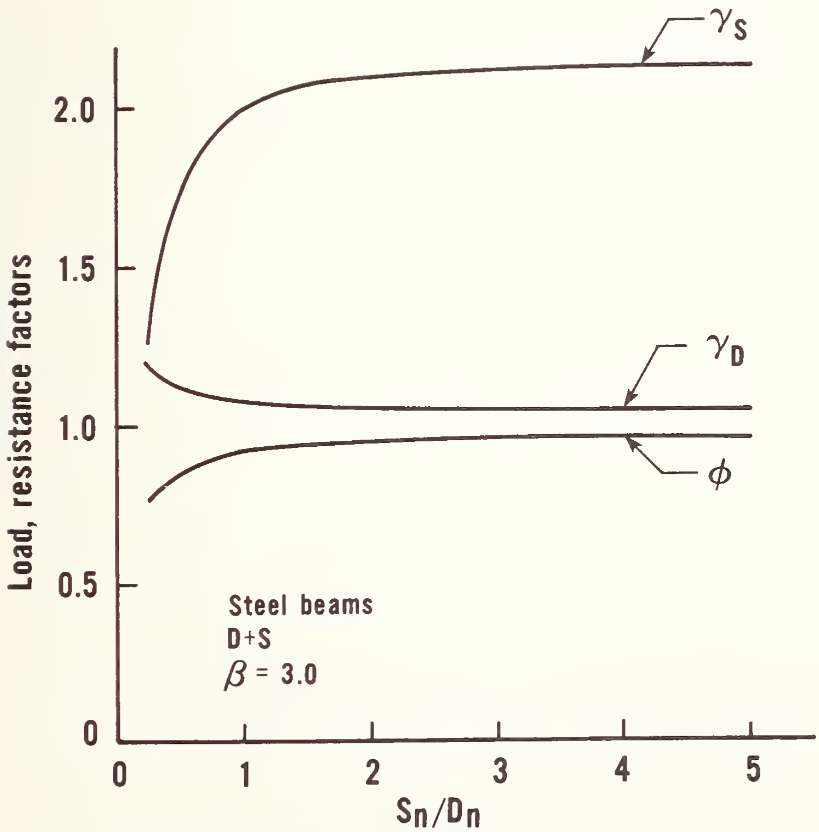


Figure 5.2 - Load and Resistance Factors for Flexure in Steel Beams (D + S)

Figure 5.3a

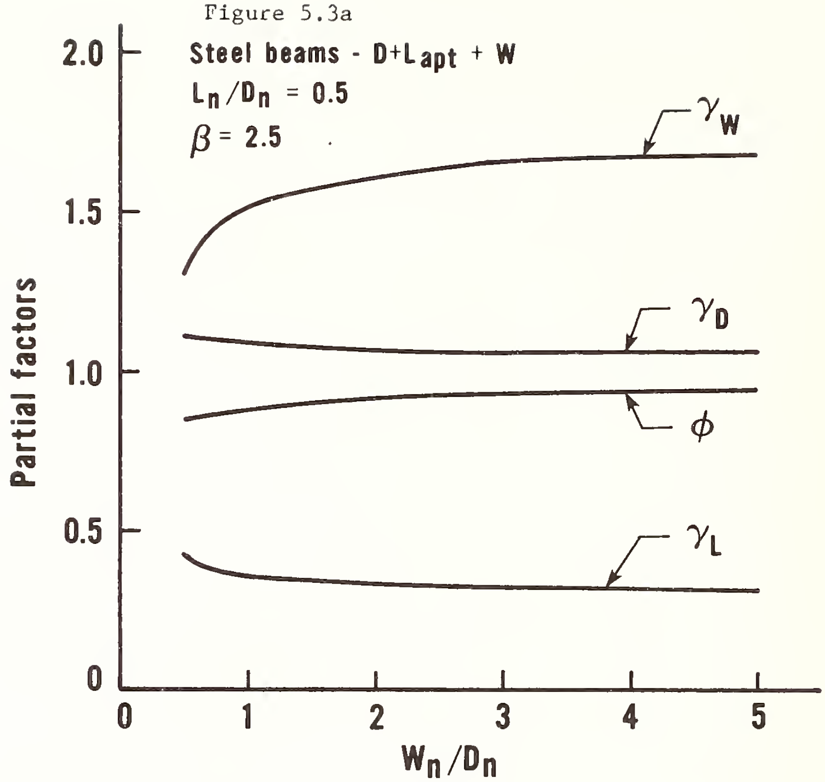


Figure 5.3b

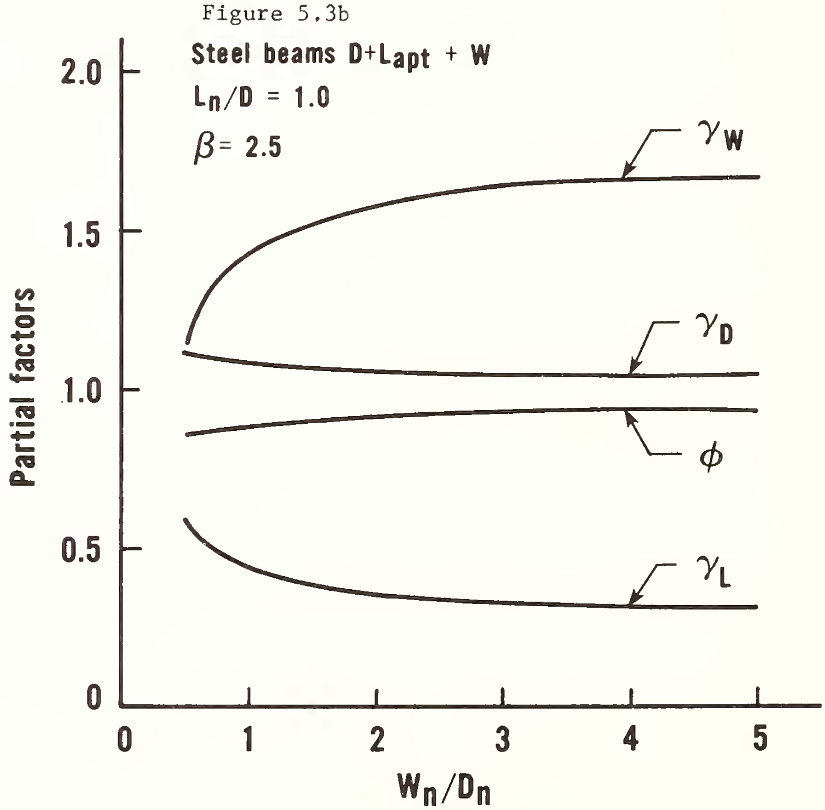


Figure 5.3 - Load and Resistance Factors for Flexure in Steel Beams (D + L + W)

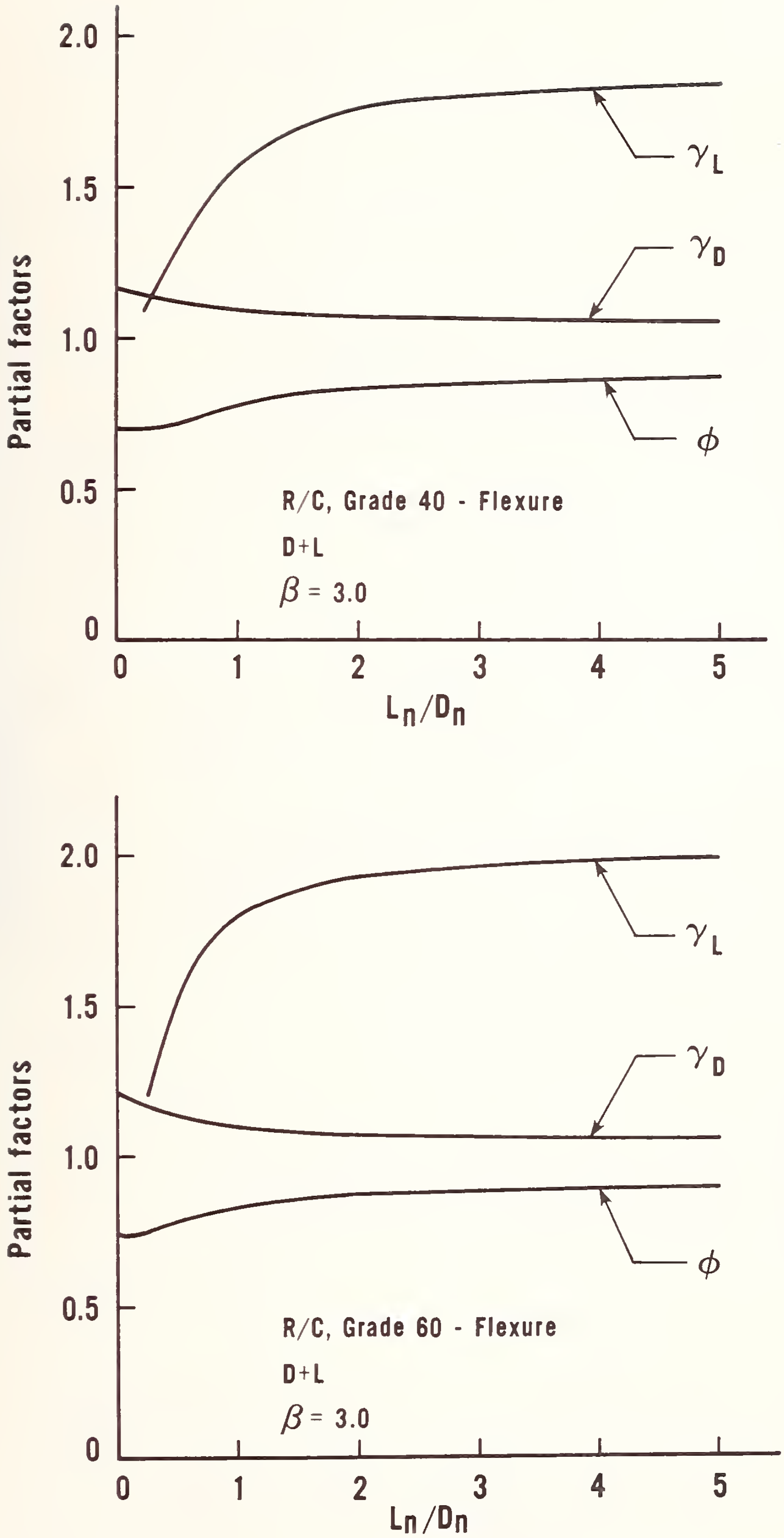


Figure 5.4 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Live Loads)

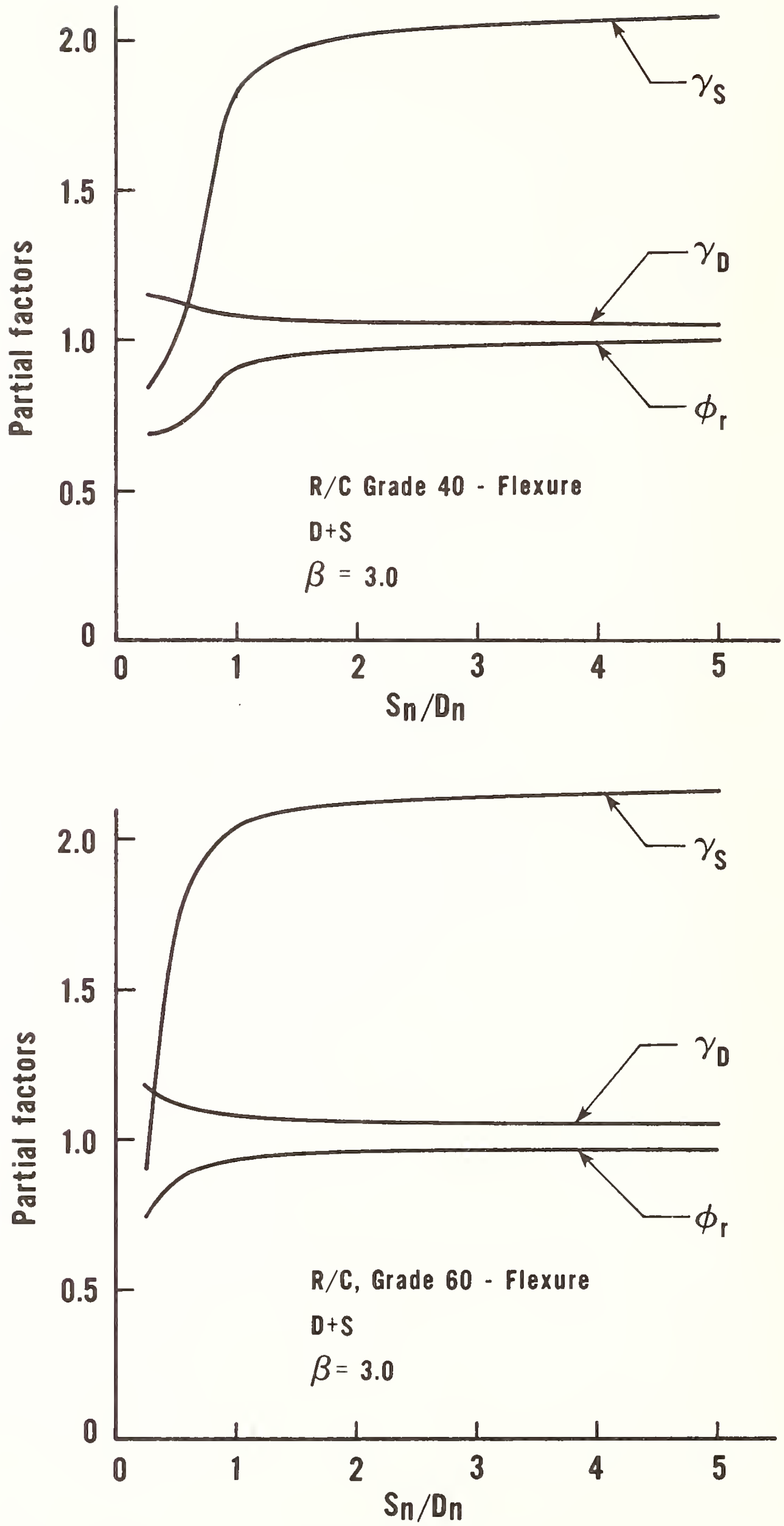


Figure 5.5 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Snow Loads)

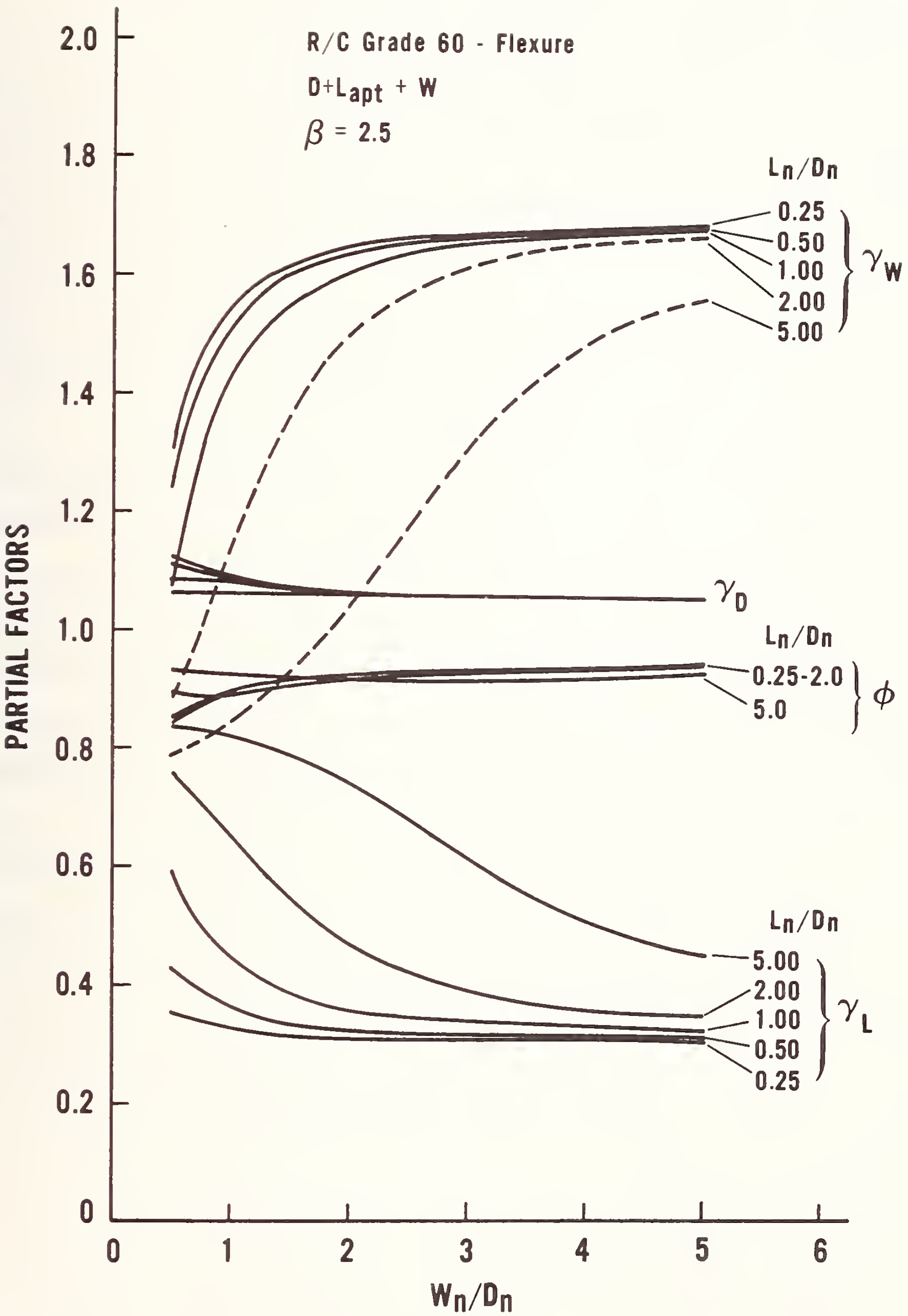


Figure 5.6 - Load and Resistance Factors for Flexure in Reinforced Concrete Beams (Gravity plus Wind Load)

there will be some deviation from the ideal constant reliability for certain load situations. It should be noted that if the dead load factor is fixed, the time varying load factor would not drop quite so rapidly for small load ratios in Figs. 5.1 - 5.6.

5.5 Selection of Load Factors

Section 5.4 has shown that for the reliability requirement to be fulfilled, (ϕ, γ_i) must depend on the particular load combination, strength, and on the mean, variance and c.d.f. of all variables in the limit state equation. If a constant set of ϕ and γ's are prescribed, the associated reliabilities will deviate from the target reliabilities for certain design situations. However, it is possible to select one set of load factors that minimizes the extent of this deviation when considered over all likely combinations of load. While the resistance factors will depend on the material and limit state of interest, the load factors will be independent of these considerations.

In general, an optimal set of load factors can be selected by (1) defining some function which measures the "closeness" between the target reliability and the reliability associated with the proposed load and resistance factor set, and (2) selecting γ_i so as to minimize this function. The choice of an appropriate function is not unique, and some of these are quite sophisticated. It is possible to select the function so as to heavily penalize unconservatism (or vice versa) or to include such economic factors as total life cost (in which case, a discount rate must be estimated). However, for first-generation reliability load criteria it seems most appropriate to use a simple function.

We first observe that associated with β_o and a given set of nominal loads, there is some corresponding required nominal resistance, R_n^{II}; this may be calculated from the Level II load and resistance factors in Section 5.4 that are functions of the load ratio and load combination. On the other hand, a design equation which prescribes a set of load factors that are constant for all load ratios will also lead to a nominal resistance, R_n^I that may differ from R_n^{II}. For example, if the factored resistance and dead, live, and wind loads are linearly related,

$$R_n^I = (\gamma_D D_n + \gamma_L L_n + \gamma_W W_n)/\phi \tag{5.8}$$

We then select a set of γ and ϕ to minimize,

$$I(\phi,\gamma_i) = \sum_i [R_{n_i}^{II} - R_{n_i}^I]^2 \, p_i \tag{5.9}$$

over a predefined set of combinations of dead, live, snow, wind and earthquake loads, wherein p_i = the relative weight assigned to the ith load combination. The implication of minimizing the square of the difference between R_n^{II} and R_n^{I} is that deviations from β_o which are conservative and those which are unconservative are penalized equally.

Minimization requires the selection, a priori, of a particular criterion format. In principle, this could range from an equation with one overall safety factor to some of the complex formats being considered by standards organizations in Europe. As with multiple regression analysis, the more independent factors that are assigned, the closer the criterion will come to achieving the target β_o over all possible design situations. The format discussed in Section 5.2 appears to be the best compromise between the conflicting needs of minimizing deviations from the ideal and of having a criterion simple enough for everyday design use.

The nominal load ratios L_n/D_n, S_n/D_n, etc. and the relative frequency of different common load situations vary for different construction materials. The weights assigned for the D + L and D + S combinations in Tables 5.2a and 5.2b represent our best estimates for the likelihood of different load situations, but it should be noted that they are not based on extensive empirical data. Studies of the sensitivity of the optimal safety factors to various assumptions showed that they were considerably more sensitive to the range of L_n/D_n, etc., than to the distribution of p_i within that range. Note that with reinforced concrete and masonry structures, the dead load contributes a significant component to the total load effect. For load combinations involving wind and earthquake, it was assumed that values of W_n/D_n and E_n/D_n of 0.5, 1.0, 3.0 and 5.0 were equally likely. For lower ratios, gravity loads would tend to govern. The optimization was performed over each load combination separately.

Gravity Loads We considered first the D + L and D + S combinations for the different construction materials, determined R_n^{II} for $\beta_o = 3.0$, and determined the optimal ϕ, γ_L,γ_S for selected situations using Eq. 5.9. A restriction placed on the process was that γ_D = 1.2; while the results in Section 5.4 showed that the best value of γ_D would be about 1.10, it is doubtful that the profession would accept this low a value. A portion of the results of this first phase is shown in Table 5.3. Of course, the optimal ϕ and γ_i depend on the load combination and material. The second stage was to select one γ-factor which could be used with both live and snow load, an additional constraint placed on the process to simplify the final load criteria. This γ-factor should be as close to the load factors listed in column 4 of Table 5.3 as possible; at the same time, ϕ should fall close to the desirable range of 0.80 - 0.85 for flexure in steel and concrete beams. This is to allow material specification writing groups some leeway to adjust ϕ for different quality control procedures, minor changes in target reliability, etc. It was found that $\gamma_L = \gamma_S$ = 1.6 or 1.7 both fulfilled these requirements and 1.6 was chosen to allow specificatio writers a little additional flexibility in selecting ϕ. Eq. 5.9 can then be used to compute the optimal ϕ corresponding to $\gamma_D = 1.2$ and $\gamma_L = \gamma_S = 1.6$; these are shown in the final column of Table 5.3. The gravity load case is, thus,

$$U = 1.2\ D_n + 1.6\ L_n \tag{5.10a}$$

$$U = 1.2\ D_n + 1.6\ S_n \tag{5.10b}$$

An additional condition $U \geq 1.4\ D_n$ prevents U from becoming too small as L_n approaches zero; this condition governs when $L_n/D_n < 0.12$.

Wind The next step was to derive optimal load factors for the D + L + W combination. Using the load combination rule discussed in Section 2.5, Eq. 2.25 et. seq., this actually requires two checking equations (see Eqs. 2.26). The maximum of the two governs design:

$$U = \max \begin{cases} \gamma_D D_n + \gamma_{L_1} L_n + \gamma_W W_n & (5.11a) \\ \gamma_D D_n + \gamma_L L_n + \gamma_{W_1} W_n & (5.11b) \end{cases}$$

in which $\gamma_{L_1} L_n$ and $\gamma_{W_1} W_n$ are equal to the factored arbitrary point-in-time live and wind loads, respectively, as discussed in Section 5.2.1.

Table 5.2a - Weights for D + L

	L_n/D_n						
Material	0.25	0.5	1.0	1.5	2.0	3.0	5.0
Steel	0	10	20	25	35	7	3
R/C	10	45	30	10	5	0	0
Light Gage & Aluminum	0	0	6	17	22	33	22
Glulam	0	5	26	26	26	12	5
Masonry	36	36	20	6	2	0	0

Table 5.2b - Weights for D + S

	S_n/D_n						
Material	0.25	0.50	1.0	1.5	2.0	3.0	5.0
Steel	0	10	20	25	35	7	3
R/C	30	40	20	5	5	0	0
Light Gage & Aluminum	0	0	6	17	22	33	22
Glulam	0	2	16	32	32	18	0
Masonry	36	36	20	6	2	0	0

Table 5.3 - Optimal Load and Resistance Factors for Gravity Loads

Material	Combination	Optimum Values		Optimum ϕ for
		ϕ	γ_L, γ_S	$\gamma_D = 1.2$, $\gamma_L = 1.6$
Steel Beam ($\beta_o = 3$)	D + L	0.96	2.10	0.78
	D + S	1.05	2.32	0.79
R/C Beam, Gr. 60 ($\beta_o = 3$)	D + L	0.87	1.83	0.81
	D + S	0.93	1.93	0.84
R/C Beam, Gr. 40 ($\beta_o = 3$)	D + L	0.82	1.61	0.81
	D + S	0.85	1.56	0.86
Glulam Beam* ($\beta_o = 2.5$)	D + L	0.59	1.38	0.66
	D + S	0.59	1.08	0.77
Brick Masonry Wall* ($\beta_o = 7.5$)	D + L	0.38	4.10	0.22
Brick Masonry Wall* ($\beta_o = 5.0$)	D + L	0.52	2.45	0.41
Concrete Masonry Wall* ($\beta_o = 6.5$)	D + L	0.41	3.28	0.27
Concrete Masonry Wall* ($\beta_o = 5.0$)	D + L	0.49	2.38	0.40

* $\overline{R}/R_n$ assumed to equal to 1.0 for illustration.

Optimal load and resistance factors were determined by first calculating R_n^{II} correspondin to $\beta_o = 2.5$ for Eq. 5.11a and $\beta_o = 3.0$ for Eq. 5.11b and then minimizing Eq. 5.9; γ_D = 1.2 as before. A portion of the results for steel and concrete beams with Grade 60 reinforcement are shown in Table 5.4, in which $A_I = 1000$ ft^2 (93 m^2) to determine the statistics of L_{apt}.

Table 5.4 - Load and Resistance Factors for Gravity Plus Wind Loads

		Optimum Values			Optimum ϕ when $\gamma_W = 1.3$		
Material	Eq.	ϕ	γ_L	γ_W	$\gamma_{L_1} = 0.3$	$\gamma_{L_1} = 0.4$	$\gamma_{L_1} = 0.5$
Steel Beam	5.11a	1.11	0.61	1.71	0.85	0.87	0.89
	5.11b	0.93	1.97	0.08	-	0.81	-
Concrete Beam	5.11a	1.06	0.49	1.76	0.82	0.83	0.84
	5.11b	0.86	1.63	0.14	-	0.81	-

Note that the ϕ-factors tend to be too high in comparison with the D + L and D + S combination and some reduction in ϕ, γ_L and γ_W appears necessary. Clearly, if the limit state is the same (e.g., flexure), the ϕ-factor should not depend on the load combination. Moreover, γ_L in Eq. 5.11b should be 1.6, since this equation should approach Eq. 5.10a as W_n becomes small. It was found that by making $\gamma_L = 1.6$, $\gamma_{L_1} = 0.4$ or 0.5, $\gamma_W = 1.3$, and $\gamma_{w_1} = 0.10$, the optimal ϕ-factors were close to the desired range (0.8 - 0.85) and were within a few percent of those for the D + L and D + S combinations. These are listed in the final columns of Table 5.4. Considering other influence areas, it was found that $\gamma_{L_1} = 0.5$ was more satisfactory, particularly at larger areas, and this value was adopted. Eqs. 5.11 become

$$U = \max \begin{cases} 1.2\ D_n + 0.5\ L_n + 1.3\ W_n & (5.12a) \\ 1.2\ D_n + 1.6\ L_n + 0.10\ W_n & (5.12b) \end{cases}$$

In most practical cases, the term 0.10 W_n in Eq. 5.12b could be ignored, which would reduce this criterion to Eq. 5.10a and make the Eq. 5.12a the relevant wind load safety check.

Earthquake An optimal load factor for earthquake loads was determined similarly. Values of R_n^{II} corresponding to $\beta_o = 1.75$ were calculated for steel and reinforced concrete beams loaded in the combinations D + L + E for Boston and Los Angeles. $A_I = 1000$ ft^2 (93 m^2) for purposes of computing statistics of L_{apt}. If the minimization is performed for the two sites separately, γ_E is site-dependent. In order to compute one load factor, the two sets of R_n^{II} were combined and the optimization was performed over both sets of data. The optimal ϕ, γ_L and γ_E for steel and concrete beams assuming $\gamma_D = 1.2$ are shown in the first three columns of Table 5.5.

Table 5.5

Load Factors for Earthquake Loads

Material	Combinations	ϕ	γ_L	γ_E	Optimal ϕ when $\gamma_E = 1.5$		
					$\gamma_{L_1} = 0.0$	$\gamma_{L_1} = 0.2$	$\gamma_{L_1} = 0.5$
Steel Beam	D + L + E	1.25	0.39	2.31	0.82	0.85	0.90
R/C Beam	D + L + E	1.21	0.38	2.37	0.80	0.82	0.85

The γ_E factor then was adjusted so as to force the ϕ down to the same range as for the other load combinations. It was found that by making $\gamma_{L_1} = 0.2$ and $\gamma_E = 1.5$, the corresponding optimal ϕ (listed in the last columns of Table 5.5) is about the same as for the other load combinations. However the factored load 0.2 L_n would be less than the mean of L_{apt} in many instances, and it was decided to raise γ_{L_1} to 0.5. The alternative combination in which $\gamma_E = 1.3$ also was carefully considered because of the consistency with the treatment of wind loads. With this alternative, the optimal ϕ factors were much less than 0.80; conversely, if the same ϕ used with the gravity and wind load combinations were to be used in combinations with earthquake load the reliability indices would be less than $\beta = 1.75$. There is simply too great a difference in c.o.v. in wind load (0.30 - 0.40) and earthquake load (greater than 1.00) to warrant the same load factor for each.

A similar analysis with the combination D + S + E showed that the necessary snow load factor was close to zero, implying that snow and earthquake loads in combination could be neglected. Nevertheless, it seems sensible to specify $\gamma_{S_1} = 0.2$ for conservatism in areas subject to heavy snow and to earthquake hazards.

Counteracting Loads Common instances in which loads counteract one another include cases where load effects due to wind or earthquake act in a sense opposing gravity load effects. This case is extremely difficult to handle using mean-value reliability analysis methods but is relatively straightforward using the advanced procedure. The two cases U = W - D and U = E - D are considered.

Constraints placed on the minimization simplify the problem. First, since the probability density function of dead load is symmetrical about $\bar{D}/D_n = 1.05$ and since $\gamma_D = 1.2$ when loads are additive, it is reasonable that $\gamma_D = 0.9$ when loads counteract. Second, the ϕ-factor for a particular material and limit state should be the same, regardless of the load combination.

Accordingly, γ_W (and γ_E) are selected by first computing R_n^{II} for $\beta_o = 2.0$, fixing $\gamma_D = 0.9$ and $\phi = 0.85$, and selecting the γ_W(and γ_E) which minimizes Eq. 5.9. The same characterizations of the wind and earthquake environments are used here as for the combinations where the load effects are additive. It was assumed that values of W_n/D_n, E_n/D_n between 2 and 5 were equally probable. The optimal value of γ_W (and γ_E) depends on the choice of ϕ; for example, γ_W varied from 1.22 to 1.26 for steel beams as ϕ was increased from 0.85 to 0.90. In the interest of consistency with the additive combinations involving these loads, the load combinations are,

$$U = 0.9\ D_n - 1.3\ W_n \tag{5.14}$$

$$U = 0.9\ D_n - 1.5\ E_n \tag{5.15}$$

It is interesting to note that if γ_W is selected to best achieve $\beta_0 = 2.5$, the same as for the additive combination D + W, then $\gamma_W = 1.5$. This would result in additional conservatism against counteracting forces over existing practice.

Other combinations may be treated similarly. For example, a combination of live plus snow load may be important in design of upper story columns. Similarly, a combination of wind and snow load may be important for certain roof structures. These cases involve considering the combinations

$$D + L_{apt} + S$$

$$D + W_{ann} + S$$

The load factors on L_n and W_n that lead to values of ϕ in the desired range are $\gamma_{L_1} = 0.5$ and $\gamma_{W_1} = 0.8$ (cf Eqs. 5.11).

In sum, the load combinations and load factors recommended for use by the individual material specification writers in their design specifications are:

$$U = \text{maximum of} \begin{cases} 1.4\ D_n \\ 1.2\ D_n + 1.6\ L_n \\ 1.2\ D_n + 1.6\ S_n + (0.5\ L_n \text{ or } 0.8\ W_n) \\ 1.2\ D_n + 1.3\ W_n + 0.5\ L_n \\ 1.2\ D_n + 1.5\ E_n + (0.5\ L_n \text{ or } 0.2\ S_n) \\ 0.9\ D_n - (1.3\ W_n \text{ or } 1.5\ E_n) \end{cases} \tag{5.16}$$

It should be noted that the designer may have to consider other loading combinations in certain unusual situations. While this could be done using the methodology described in this report if data on the individual loads were available, appropriate factors also could be estimated by noting whether any similarities exist between the load in question and the loads in Eqs. 5.16. For example, it might be appropriate to select a factor of 1.6 for rain loads.

In Fig. 5.7, the resulting β's for various combinations of the ratios L_o/D_n, S_n/D_n, W_n/D_n are given for an influence area A_I = 1000 ft^2 (93 m^2) and for the case of compact steel beams for which $\bar{R}/R_n$ = 1.07 and V_R = 0.13. This case represents a representative structural type which is performing satisfactorily in current design. The ranges of β-values inherent in current design practice (AISC Specification, Part 1) are given in Tables C-7.2 and C-7.3. Following is a representative set of values:

Loading	Tributary Area	L_o/D_n	S_n/D_n	W_n/D_n	β
D + L	200 ft^2	1.0	0	0	2.6
	1000 ft^2	1.0	0	0	3.1
D + S	-	0	2.0	0	2.8
D + W	-	0	0	2.0	2.1
D + L + W	400 ft^2	1.0	0	2.0	2.6

According to the new design procedure with the proposed load factors (Fig. 5.7), the values of β are much more condensed. These values are, for ϕ of 0.85, equal to (for an influence area of 1000 ft^2)

Loading	L_o/D_n	S_n/D_n	W_n/D_n	β
D + L	1.5	0	0	2.8
D + S	0	2.0	0	2.9
D + L + W	1	0	1	3.0
	1	0	2	2.8
	1	0	5	2.5

Figs. 5.8 and 5.9 show the variation in β with L_o/D_n, S_n/D_n and W_n/D_n for concrete beams with Grade 60 reinforcement (ϕ = 0.85) and for reinforced concrete columns (ϕ = 0.65). In the most practical range of load ratios, β is close to 3 for beams and is about 3.25 for columns. The values of β are considerably more uniform for different design situations than is the case with current criteria.

5.6 Recommendations to Material Specification Groups

It is anticipated that material specification groups will want to experiment in selecting resistance factors to use along with the load criterion in the previous section.

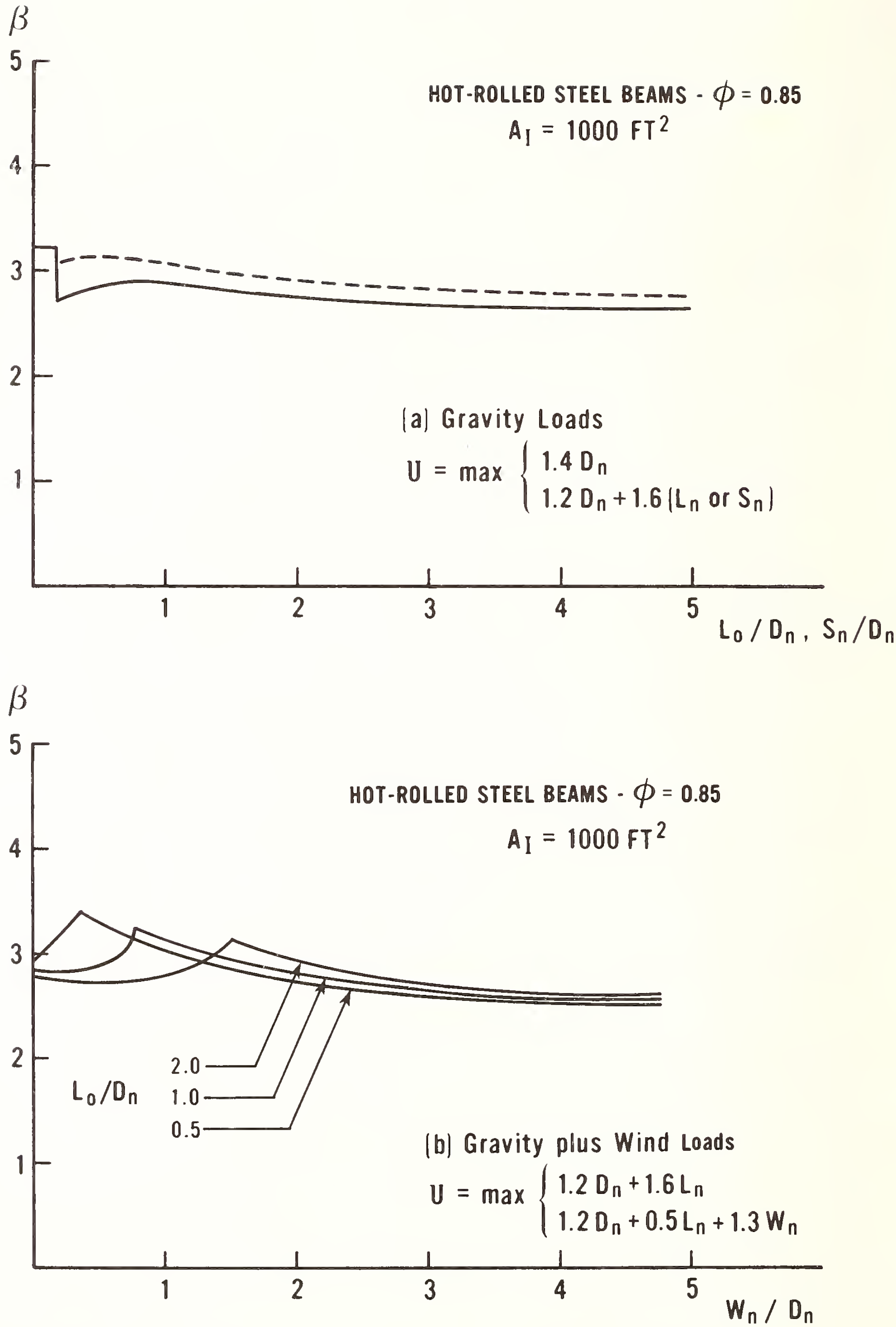


Figure 5.7 - Reliability Index for Steel Beams Using Proposed Load Criterion

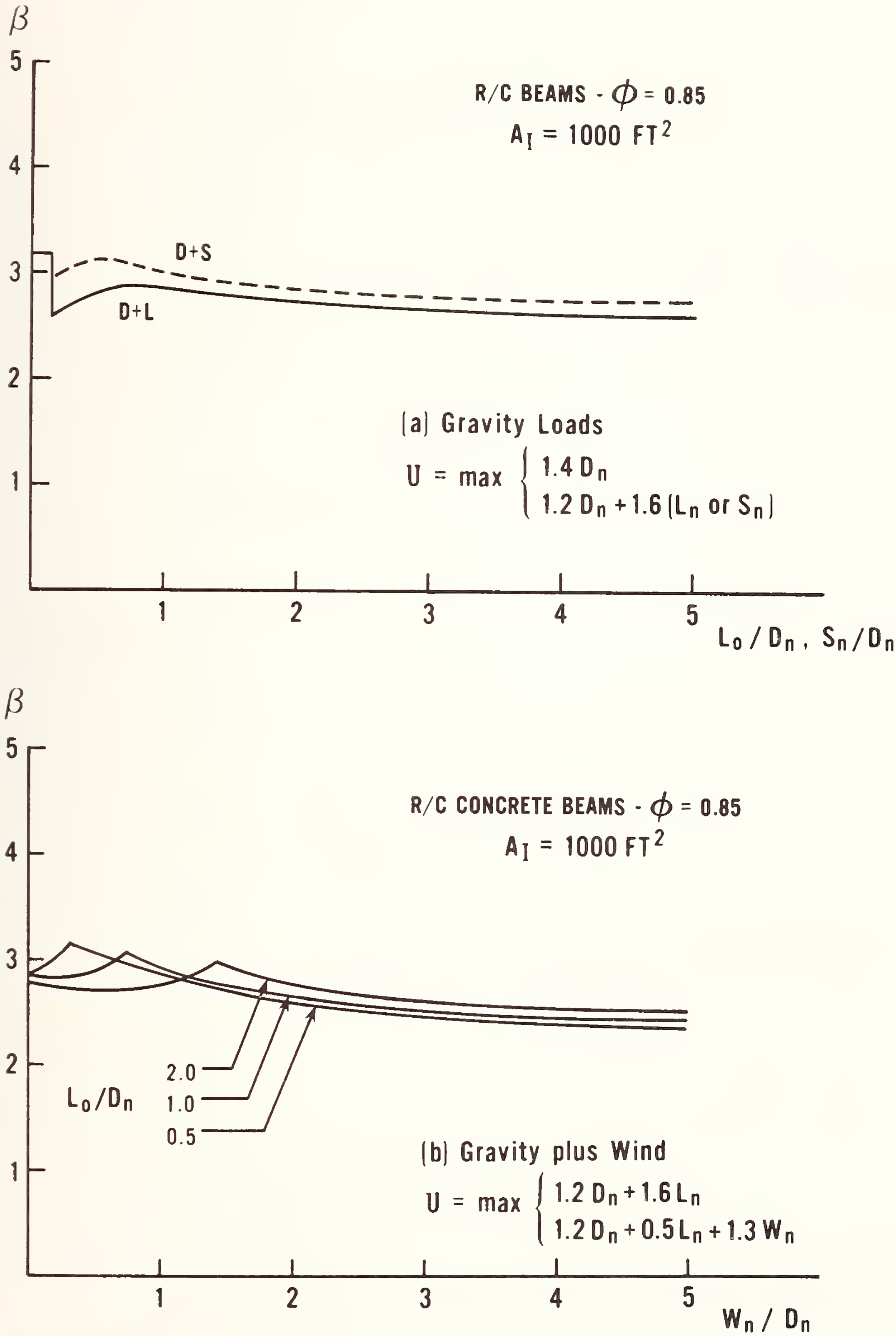


Figure 5.8 - Reliability Index for Reinforced Concrete Beams Using Proposed Load Criterion

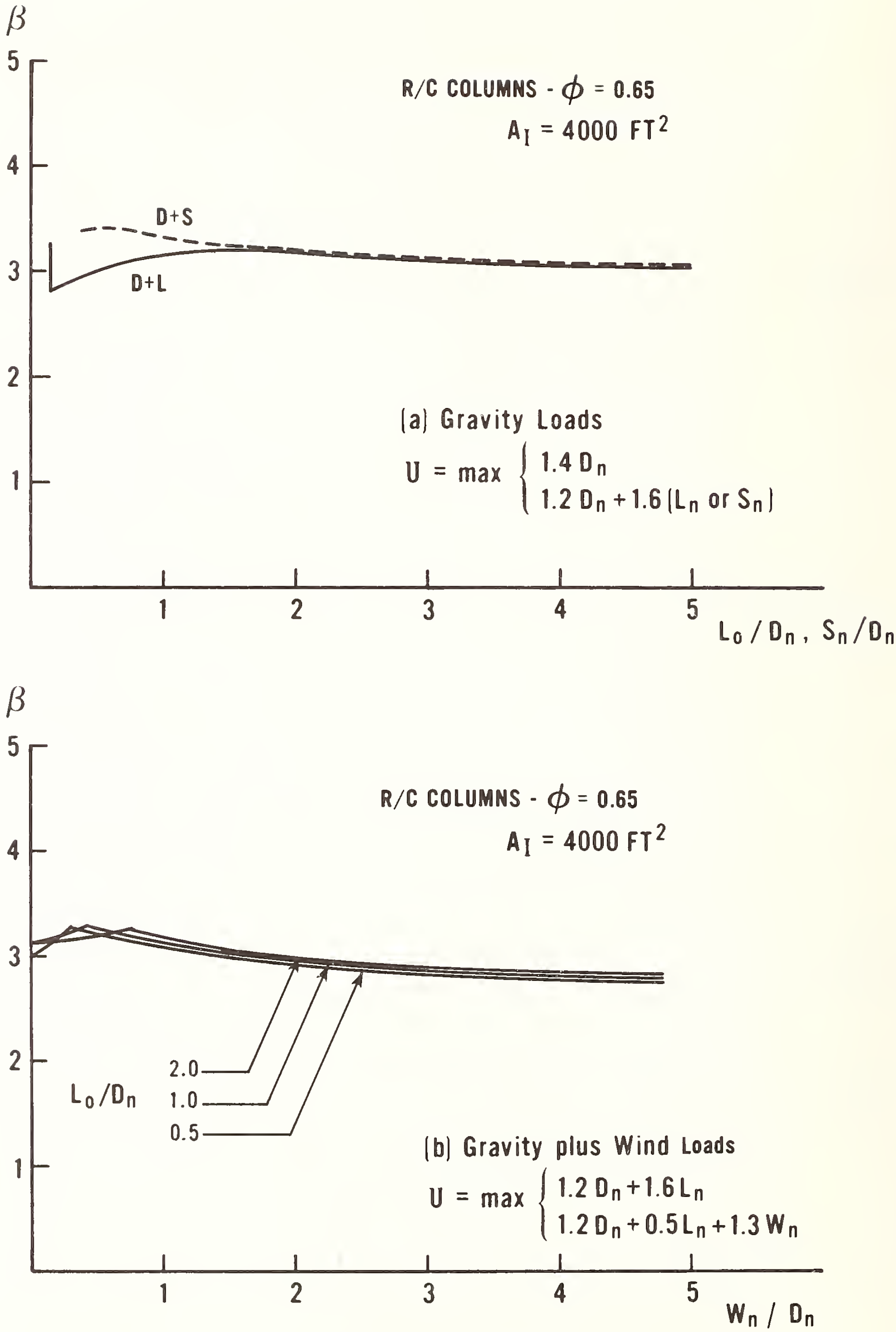


Figure 5.9 - Reliability Index for Reinforced Concrete Columns Using Proposed Load Criterion

With the load factors fixed, the reliability β can still be adjusted by varying the ϕ-factor and the specification of nominal resistance for different materials and limit states. Chapter 4 and Appendices B - E on materials provide some indication as to where current specifications stand in terms of comparative reliabilities. While these results may be used as a guide, specification committees may very well feel that some relative adjustments are warranted within their provisions. Additional material data can be used to refine and to increase the confidence in the resistance factors selected. The choice of β to be used in selecting resistance criteria should consider, among other factors, the ductility associated with each mode of resistance, the effect of loading rate in enhancing the strength of certain materials, the relative frequency of occurrence of different design situations, and the consequence of failure.

Some simple aids have been prepared to assist material specification writing groups in making their selections. It has been assumed that the load combination of primary interest to standard committees is the D + L combination. This combination governs design in many practical instances. Even when it does not, it is frequently used for preliminary sizing of members, which are then checked against lateral load effects. Accordingly, Figs. 5.10a through 5.10e present curves relating the reliability β, $\overline{R}/R_n$, V_R, and ϕ for the design criterion,

$$\phi R_n \geq 1.2 D_n + 1.6 L_n$$

The curves are presented in terms of basic live load L_o (e.g., L_o = 50 psf in offices) because many designers find it more convenient to think in terms of L_o than L_n, which may incorporate a reduction. The curves were computed for a basic influence area of A_I = 1000 ft^2 (93 m^2) and therefore $L_n = 0.724\ L_o$ from Eq. 3.10. In all cases, however $\overline{L}/L_n$ = 1.0. Thus, the corresponding values of L_o for any other influence area of interest can be calculated by multiplying the L_o in Figs. 5.10 by the factor $0.724\ [0.25 + 15/\sqrt{A_I}]^{-1}$. Each of these figures describes the relation between β, $\overline{R}/R_n$ and V_R for a prescribed ϕ, values of which range from 0.6 - 0.9. For problems outside the range presented here, the computer analysis in Appendix F must be used; however, Figs. 5.10a through 5.10e cover most practical cases.

As an example of their use, suppose we are dealing with a material and limit state in which the capacity is described by $\overline{R}/R_n$ = 1.10 and V_R = 0.15 (this case seems quite common). The ranges in β corresponding to the range in L_o/D_n and several candidate ϕ values are:

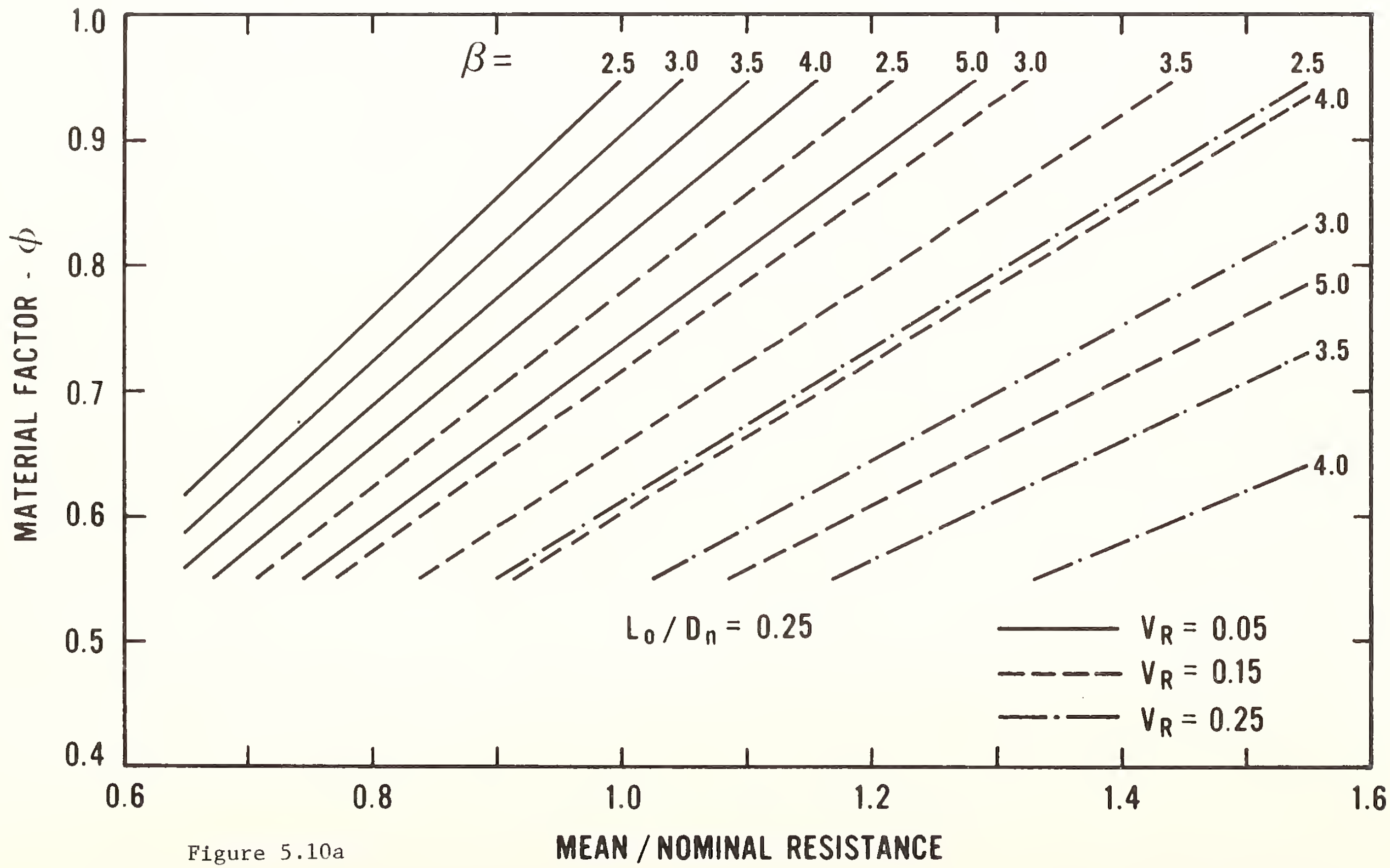
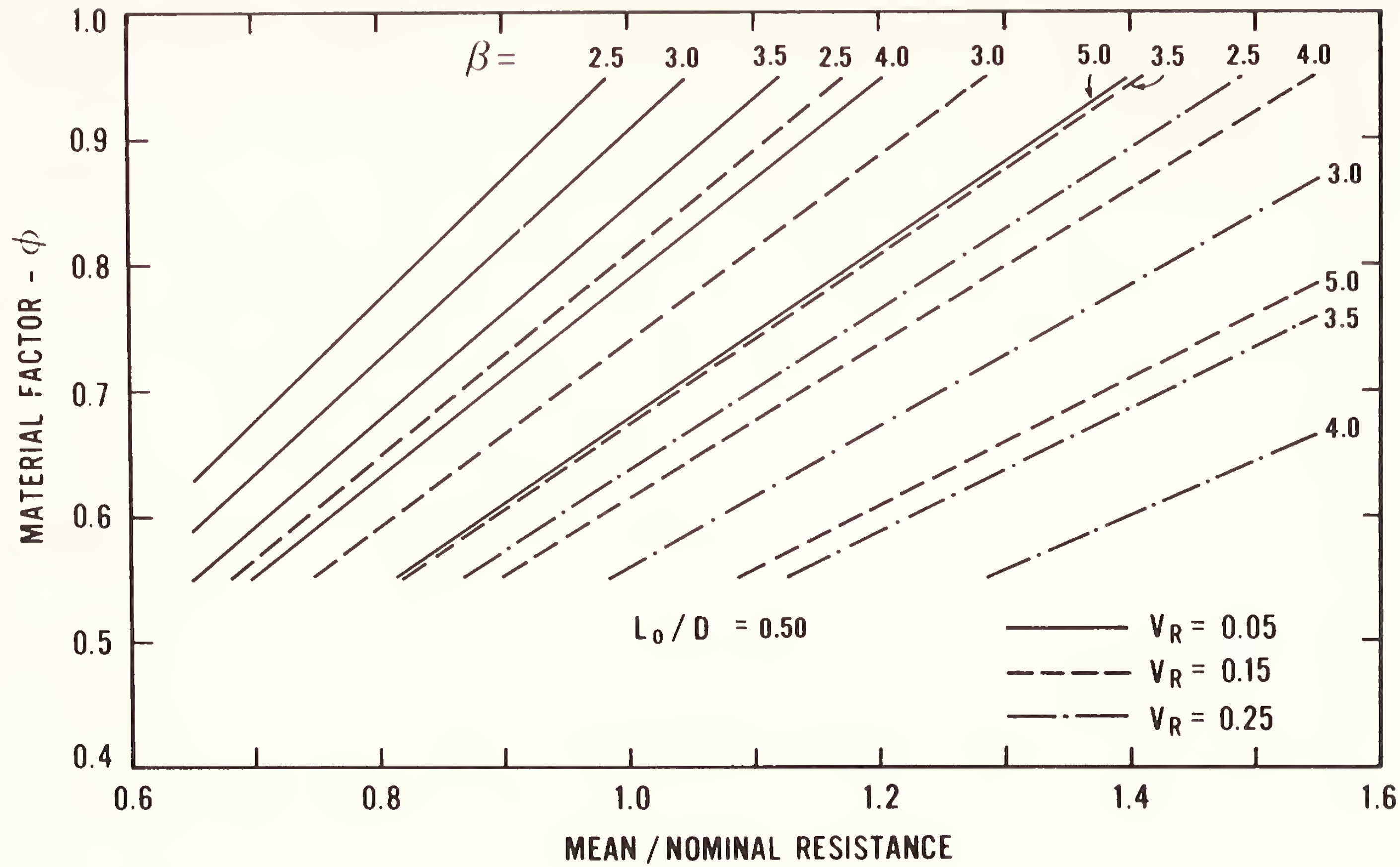


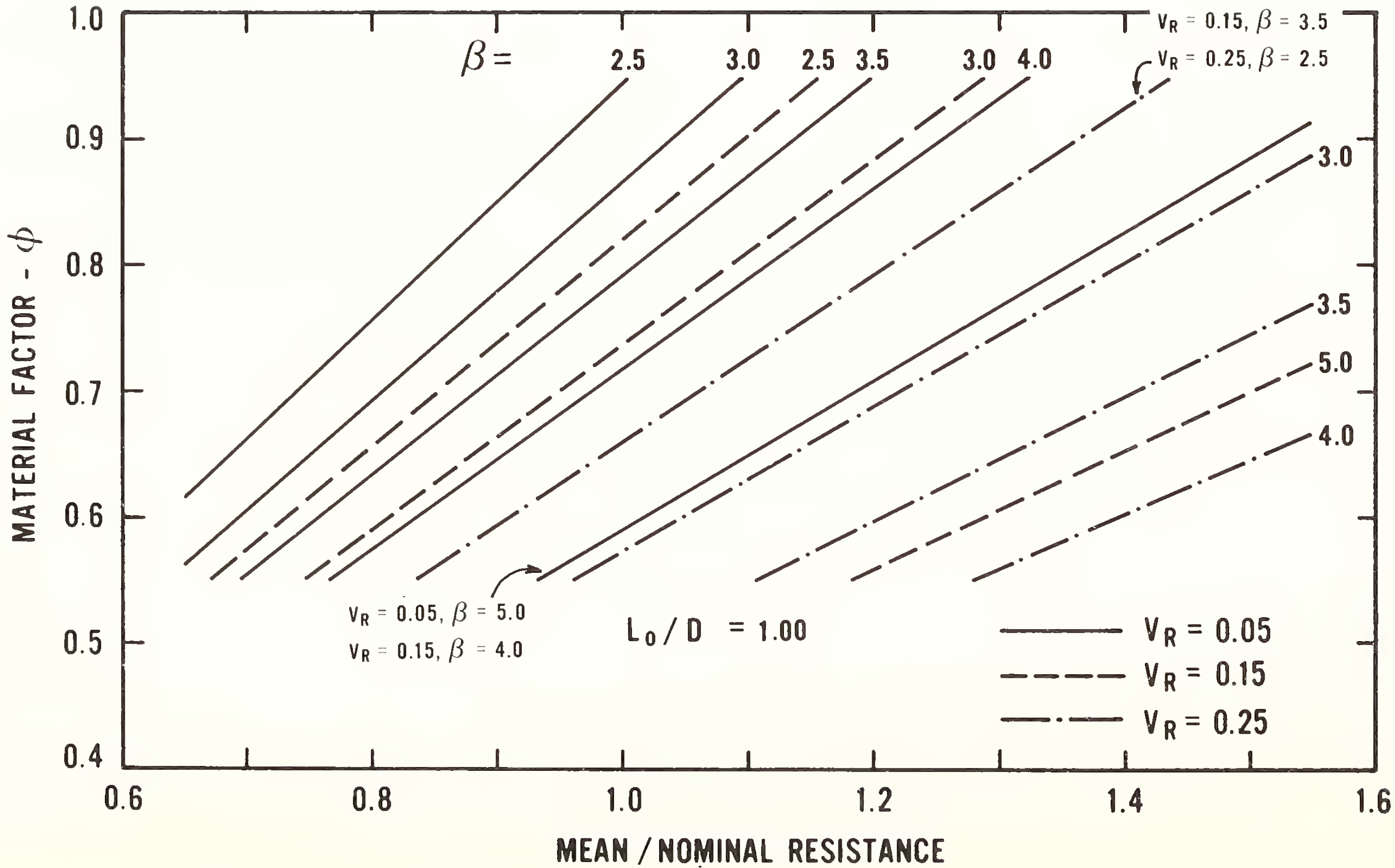
Figure 5.10a

Figure 5.10 - Aids to the Selection of Resistance Factors



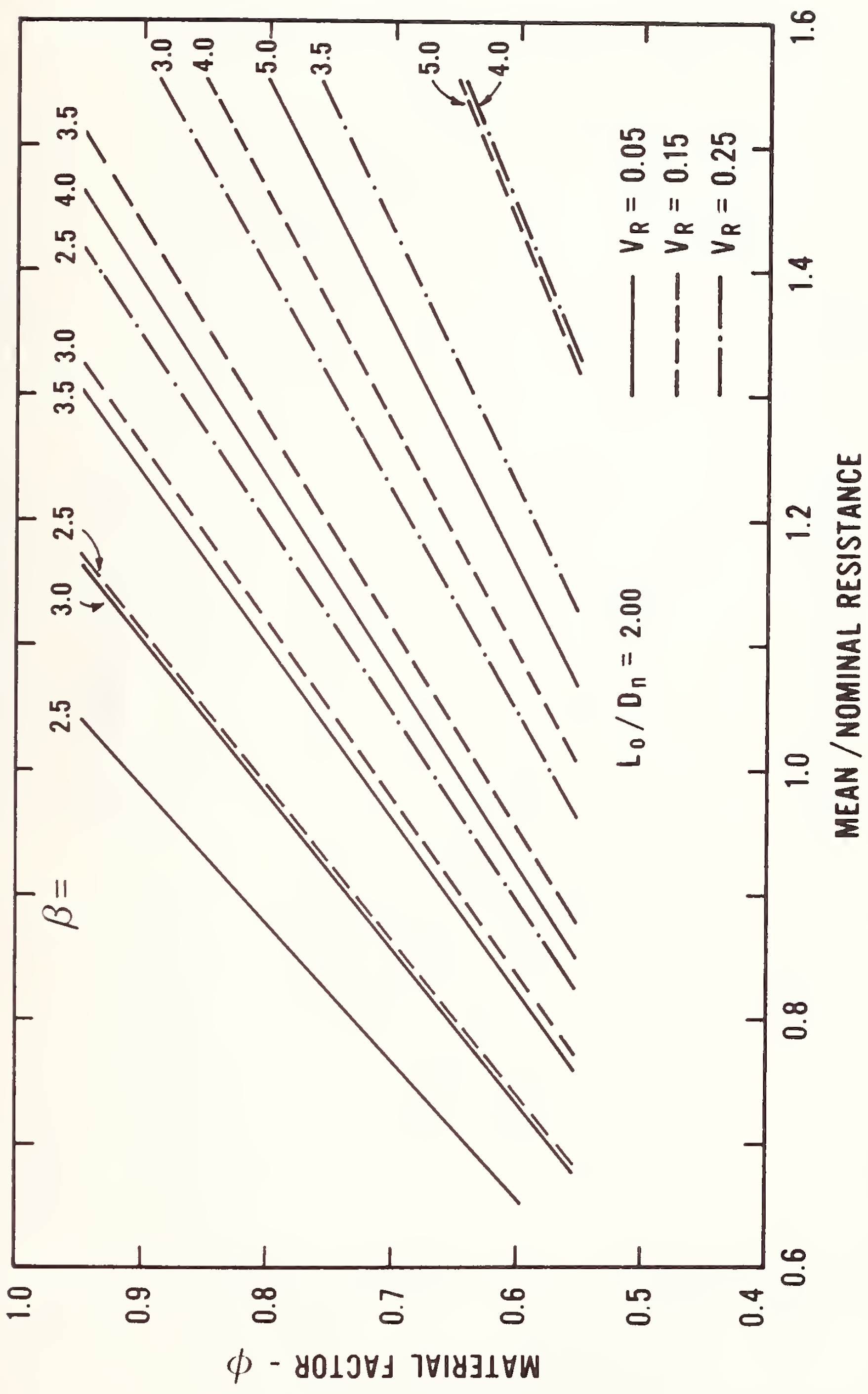
β =
2.5
3.0
3.5
2.5
4.0
3.0
5.0
3.5
2.5
4.0
3.0
5.0
3.5
4.0
L_o/D = 0.50
V_R = 0.05
V_R = 0.15
V_R = 0.25
MATERIAL FACTOR - ϕ
MEAN / NOMINAL RESISTANCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Figure 5.10b



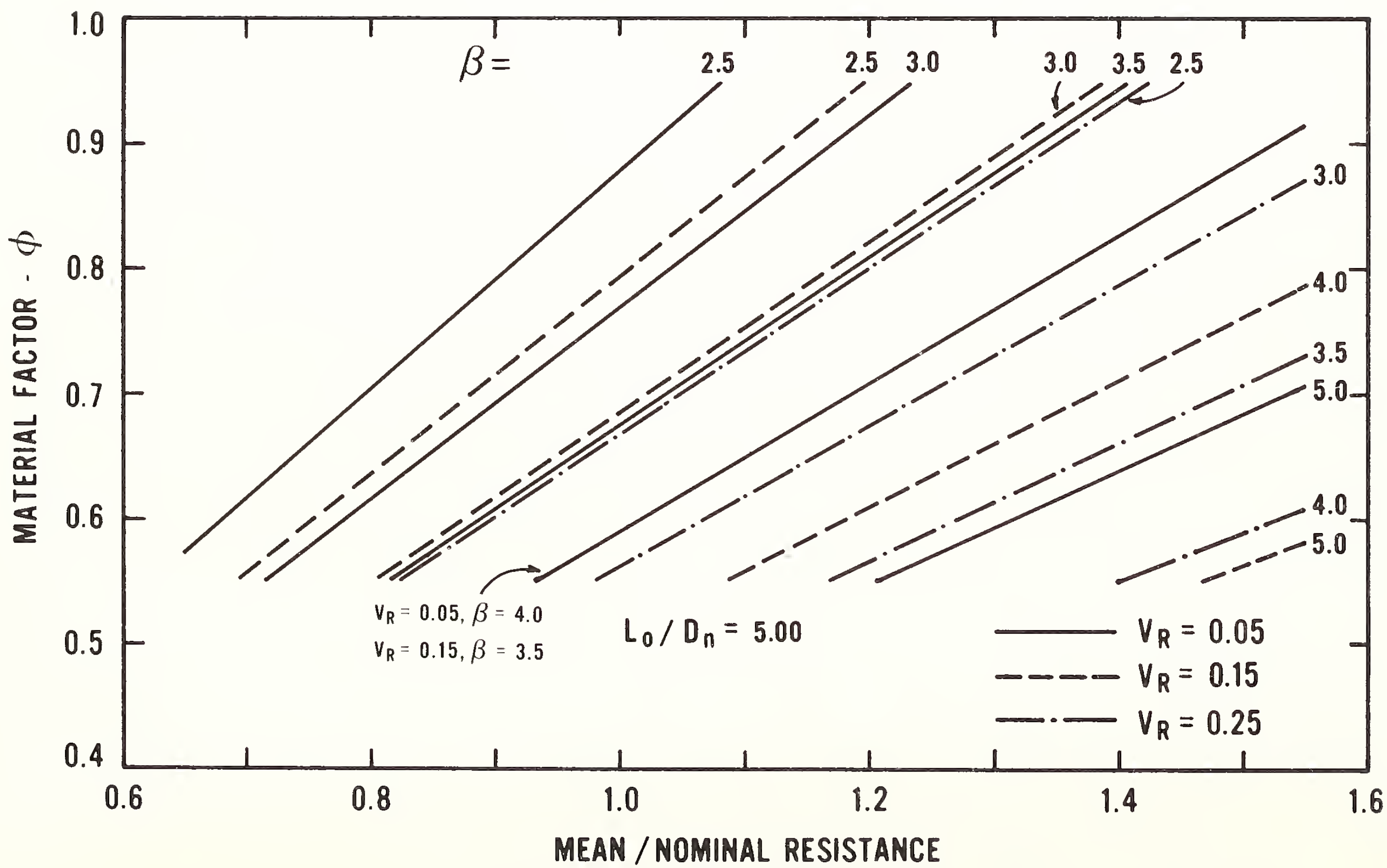
MATERIAL FACTOR - ϕ
MEAN / NOMINAL RESISTANCE
β =
2.5
3.0
2.5
3.5
3.0
4.0
V_R = 0.15, β = 3.5
V_R = 0.25, β = 2.5
3.0
3.5
5.0
4.0
V_R = 0.05, β = 5.0
V_R = 0.15, β = 4.0
L_o / D = 1.00
V_R = 0.05
V_R = 0.15
V_R = 0.25

Figure 5.10c



MATERIAL FACTOR - φ
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.6
0.8
1.0
1.2
1.4
1.6
MEAN/NOMINAL RESISTANCE
β =
2.5
3.0
2.5
3.5
3.0
2.5
4.0
3.5
3.0
4.0
5.0
3.5
5.0
4.0
$L_o/D_n = 2.00$
$V_R = 0.05$
$V_R = 0.15$
$V_R = 0.25$

Figure 5.10d



MATERIAL FACTOR - ϕ
MEAN / NOMINAL RESISTANCE
β =
2.5
2.5 3.0
3.0 3.5 2.5
3.0
4.0
3.5
5.0
4.0
5.0
V_R = 0.05, β = 4.0
V_R = 0.15, β = 3.5
L_o / D_n = 5.00
V_R = 0.05
V_R = 0.15
V_R = 0.25

Figure 5.10e

ϕ	0.70	0.75	0.80	0.85
β	3.3-3.8	3.0-3.4	2.8-3.1	2.6-2.8

One would need either some idea of the prevalent L_o/D for this situation or the relative frequency of each L_o/D_n. The value of ϕ corresponding to the desired β could then be found.

When the c.o.v. or ϕ values are between those which are presented in Figs. 5.10a through 5.10e linear interpolation is perfectly acceptable. Further resolution in ϕ (e.g. ϕ = 0.83 rather than 0.80 or 0.85) may not be warranted.

A comprehensive example of the selection of ϕ is presented in the following section.

5.7 Resistance Factors Compatible with Selected Load Factors

The following discussion will focus on the methods by which material specification writing bodies can arrive at resistance factors compatible with the load factors presented in this report. The ϕ-factors discussed below are presented for purposes of illustrating concepts and should not be considered as being recommendations by American National Standard Committee A58. The final choice of reliability indices and resistance factors rests with the specification writing groups.

5.7.1 Metal Structures

The following data illustrate two kinds of information that may be developed by a specification writing committee. The case considered is a steel beam: $\bar{R}/R_n$ = 1.07, V_R = 0.13.

Resistance factors for a given $\bar{R}/R_n$ and V_R can be obtained by interpolation from the charts relating β, ϕ, $\bar{R}/R_n$, V_R and L_o/D_n (Figs. 5.10a - e). For illustration, values of β for a given ϕ, L_o/D_n, $\bar{R}/R_n$, V_R are:

L_o/D_n	ϕ	β	
1	0.82	3.0	ϕ for given β
2	0.79	3.0	
1	0.8	3.1	β for given ϕ
1	0.85	2.8	
1	0.9	2.5	
2	0.8	3.0	
2	0.85	2.7	
2	0.9	2.5	

From such a tabulation, considering a desired level of β, the committee might choose ϕ = 0.80 or, perhaps ϕ = 0.85, as the basis for designing steel beams. Similar data are given in Table 5.6 for other types of structural elements.

The committee might next want to consider typical designs to compare current design practice with the future design practice based on the new load factors. Parametric studies of the type discussed below might be performed, where the ratio R_{nf}/R_{nc} (subscripts f and c refer to "future" and "current," respectively) is determined from the relationships

$$R_{nf} = (1.2\ D_n + 1.6\ L_n)/\phi$$

$$R_{nf} = (1.2\ D_n + 1.6\ S_n)/\phi$$

$$R_{nf} = (1.2\ D_n + 0.5\ L_n + 1.3\ W_n)/\phi$$

in which L_n is evaluated according to Eq. 3.10, and

$$R_{nc} = (FS)\ (D_n + L_n)$$

$$R_{nc} = (FS)(D_n + S_n)$$

$$R_{nc} = (FS)(D_n + L_n + W_n)(0.75)$$

in which L_n is evaluated according to Eq. 3.9.

Table 5.7 and Fig. 5.11 give the results for steel beams. If, for example, ϕ = 0.85 is selected, the required section modulus for the new design will be 1.04 times the value for the current design for S_n/D_n = 2 (typical roof beam); it will be 0.96 times the current value for L_o/D_n = 1.5 and A_T = 1000 ft^2 (93 m^2) (typical floor beam). For D + L + W the ratio will be somewhat larger than unity if A_T = 1000 ft^2 and the live load reduction is permitted; in other instances, it may be less. Should the committee decide that ϕ = 0.9, with a corresponding β of approximately 2.5, is desirable for beams, then the ratios of R_{nf}/R_{nc} would reduce, as shown in Figure 5.11a.

5.7.2 Reinforced and Prestressed Concrete Structures

The first step in selecting ϕ factors for concrete members is to select a target β. In the calibrations presented in Appendix B, current reliability levels calculated for D + L were

Reinforced concrete beams in flexure, current β = 2.6 to 3.2.

Plant Produced Pretensioned Beams in Flexure, Current β = 3.2 to 4.0.

Tied Columns, compression failures, current β = 3.0 to 3.5.

Spiral Columns, compression failures, current β = 2.6 to 3.3.

Shear, beams with stirrups, current β = 1.9 to 2.4.

Table 5.6

Resistance Factors for Metal Structures

Type Element	L_o/D_n	Target β	$\overline{R}/R_n$	V_R	ϕ
Compact Steel beam	1	3	1.07	0.13	0.82
Tension member, F_y	1	3	1.05	0.11	0.83
Tension member, F_u	1	4	1.10	0.11	0.71
Continuous beam	1	3	1.11	0.13	0.85
Elastic beam, LTB	1	3	1.03	0.12	0.80
Inelastic beam, LTB	1	3	1.11	0.14	0.83
Beam-Columns	1	3	1.07	0.15	0.79
Plate Girders, Flexure	1	3	1.08	0.12	0.84
Plate Girders, Shear	1	3	1.14	0.16	0.82
Composite Beams	1	3	1.04	0.14	0.78
Columns, λ = 0.5	1	3	1.08	0.14	0.83
Columns, λ = 0.5	1	3.5	1.08	0.14	0.75
Fillet Welds	1	4.5	1.47	0.18	0.71
HSS Bolts, A325, tension	1	4.5	1.20	0.09	0.73
HSS Bolts, A325, Shear	1	4.5	1.00	0.10	0.59
HSS Bolts, A325, shear	1	4.0	1.00	0.10	0.65
CF beams, stiffened flanges	5	3.0	1.17	0.17	0.77
Aluminum beams	5	3.0	1.10	0.08	0.82

Table 5.7

Comparison of Proposed and Current Designs

Steel Beams FS = 5/3

A_T	ϕ	L_o/D_n	S_n/D_n	W_n/D_n	L_n/L_o	R_{nf}/R_{nc}
-	0.85	0	1	0	0	0.99
-	0.85	0	1.5	0	0	1.02
-	0.85	0	2	0	0	1.04
400 ft^2	0.80	1	0	0	0.780	1.09
400 ft^2	0.85	1	0	0	0.780	1.03
400 ft^2	0.90	1	0	0	0.780	0.97
400 ft^2	0.85	1.5	0	0	0.780	1.07
1000 ft^2	0.85	0.5	0	0	0.585	0.98
1000 ft^2	0.85	1	0	0	0.585	0.98
1000 ft^2	0.85	1.5	0	0	0.585	0.96
1000 ft^2	0.85	2	0	0	0.585	0.94
1000 ft^2	0.85	2.5	0	0	0.585	0.93
1000 ft^2	0.85	0.5	0	1	0.585	1.13
		0.5	0	2	0.585	1.16
		0.5	0	3	0.585	1.18
		1	0	1	0.585	1.04
		1	0	2	0.585	1.09
		1	0	3	0.585	1.12

Note: 100 ft^2 = 9.3 m^2

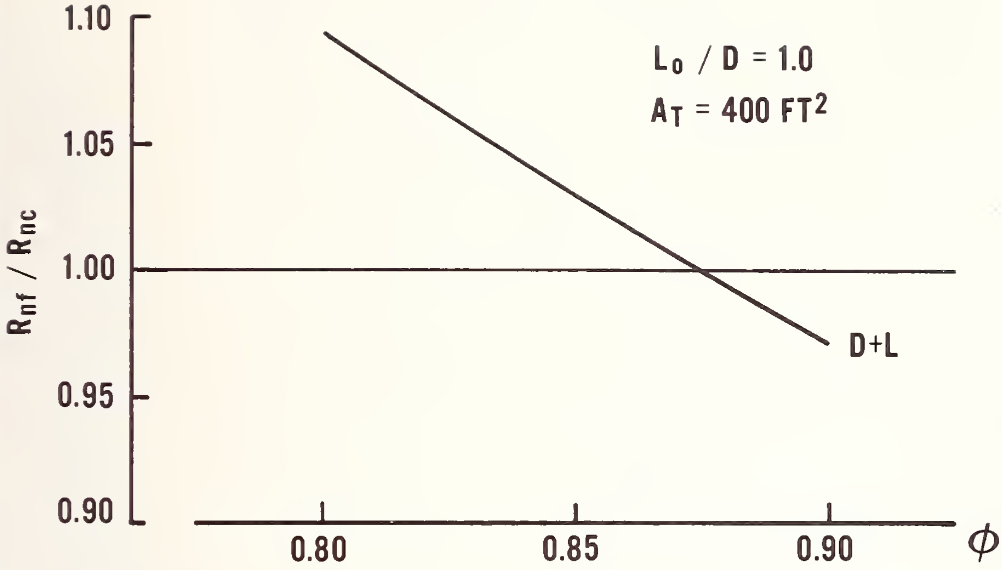


Figure 5.11a

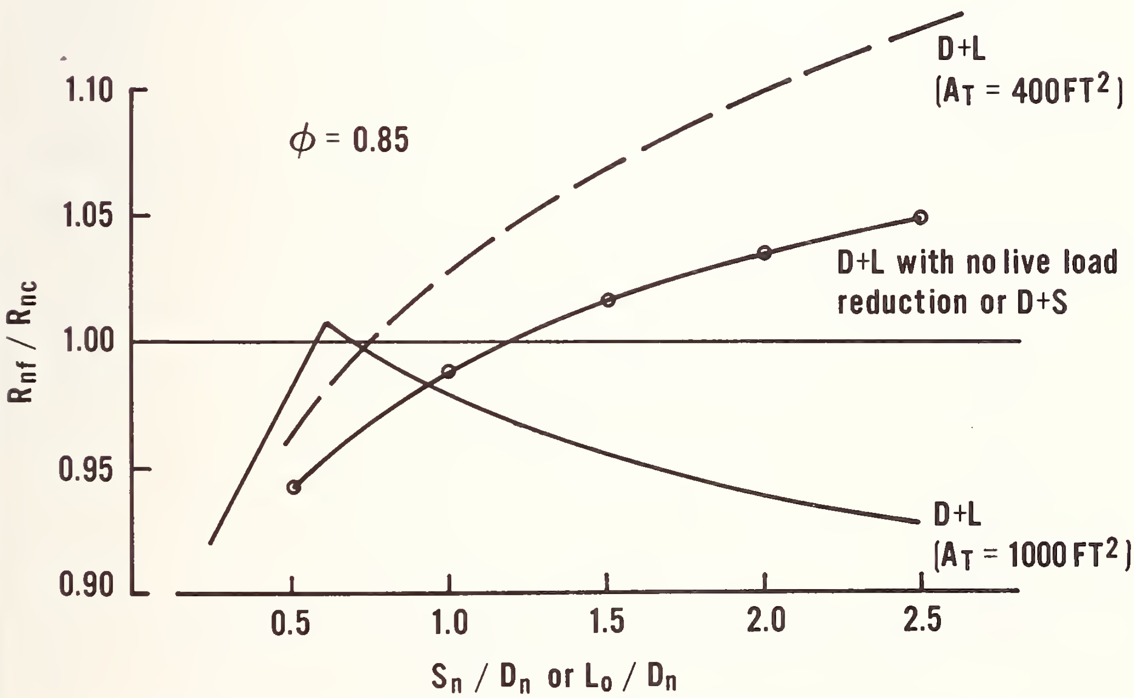


Figure 5.11b **HOT-ROLLED STEEL BEAMS - GRAVITY LOADS**

Figure 5.11 - Comparison of Designs Using Existing and Proposed Criteria for Steel Beams (100 ft^2 = 9.3 m^2)

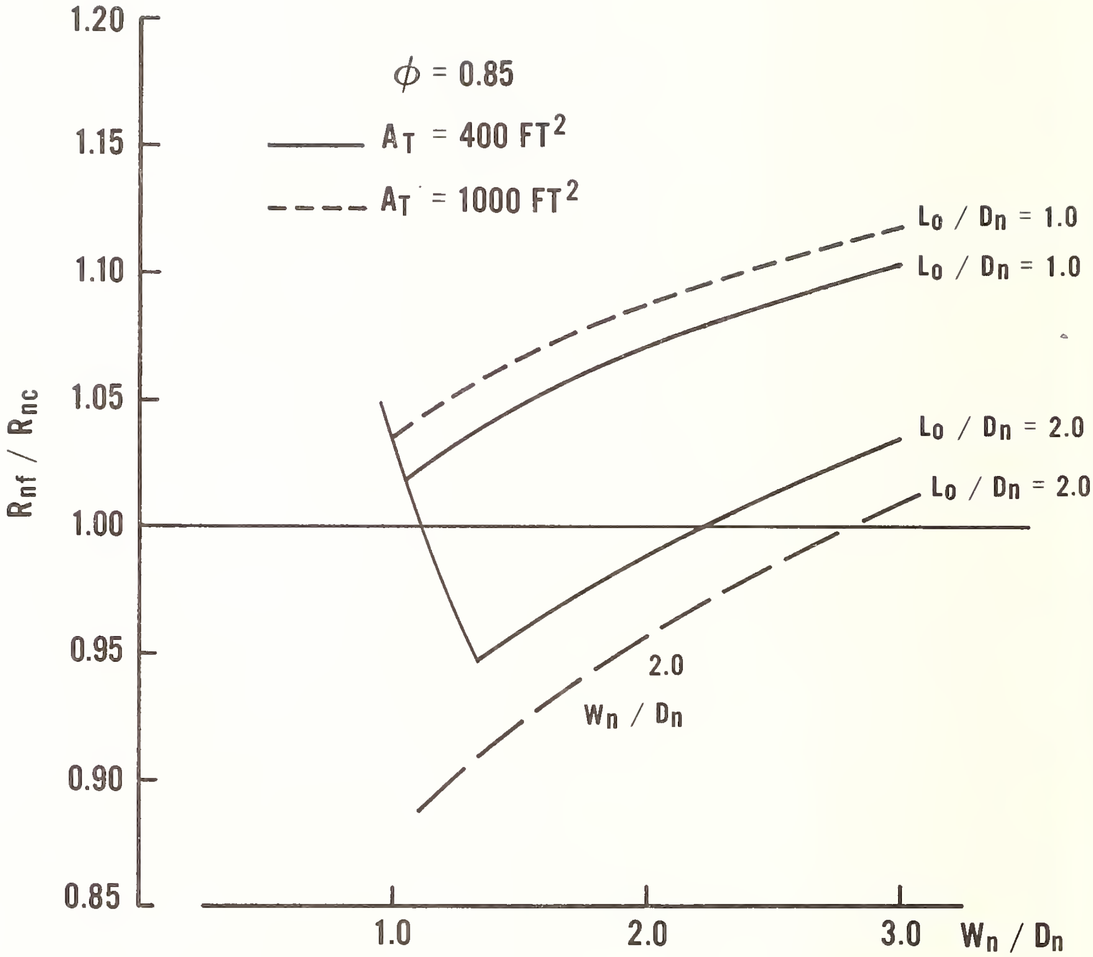


Figure 5.11c **HOT-ROLLED STEEL BEAMS - GRAVITY PLUS WIND LOADS**

Figure 5.11 (Continued)

A comparison of the current β values shows a wide range in the apparent reliabilities of various types of concrete members. Two significant areas are the higher reliabilities for pretensioned beams than for reinforced concrete beams and the very low reliability indices obtained for shear.

Using the charts presented in Fig. 5.10, ϕ values have been computed for a wide range of reinforced and prestressed members. These are summarized in Table 5.8, and are based on β = 3 for ductile failures such as would occur in under-reinforced beams and in spiral columns, and β = 3.5 for brittle failures expected in shear and tied columns. A higher reliability may be desirable for brittle failures in which failure occurs with little previous warning and in which load redistribution may not occur.

Finally, a comparison of existing designs (R_{nc}) and designs using the new load criterion (R_{nf}) is present in Fig. 5.12. It may be observed that it is possible to achieve essential conformity between them with an appropriate selection of ϕ factors, if in fact such conformity is desirable.

5.7.3 Glulam and Other Heavy Timber Structures

Additional research may be desirable before ϕ-factors for glulam members and other heavy timber construction can be specified. Following are some general observations for consideration by timber specification groups.

The ϕ factor will depend on the way the nominal design resistance is specified. This is clear from Eq. 2.23; it is the product ϕR_n rather than the two terms separately that determines reliability. If R_n is computed on the current basis of a 10-year total load duration, and if it is assumed that similar levels of reliability are desirable in the probability-based limit states criterion, the ϕ-factor will exceed unity. From an practical viewpoint, it would be desirable to have ϕ in the range 0.75 - 0.85 for glulam beams in flexure, and corresponding values for tension, compression and shear. Experience has shown that ϕ values in this range allow room for future adjustments on the part of the specification committee for changes in reliability and improvements in manufacturing and quality control. Values of ϕ in excess of 0.90 leave very little room for such adjustments.

Second, R_n should reflect the effects of cumulative load duration in some way. Since the purpose of ϕ is to account for uncontrollable deviations from the predicted strength, it would be highly inappropriate to lump the load duration effect in with ϕ; the variability in load duration effect, however, should be included in ϕ. Knowledge regarding the effect

Table 5.8

Values of Resistance Factors for Reinforced Concrete Members

Action / Type of Member	$\overline{R}/R_n$	V_R	Range of ϕ for $L_o/D_n = 0.25 - 2.0$
Flexure, Reinforced Concrete, $\beta = 3.0$			
Beam, Grade 40, $\rho = 0.35\rho_b$	1.14	0.14	0.82 - 0.84
Beam, Grade 60, $\rho = 0.57\rho_b$	1.05	0.11	0.80 - 0.85
Beam, Grade 60, $\rho = 0.73\rho_b$	1.01	0.12	0.76 - 0.80
Two way slabs, Grade 60	1.16	0.15	0.83 - 0.86
Continuous, one-way slabs	1.22	0.16	0.85 - 0.88
Flexure, Plant Produced Pretensioned Concrete, $\beta = 3.0$			
Double T $\omega_p = 0.054$	1.06	0.057	0.86 - 0.95
Beam $\omega_p = 0.228$	1.06	0.083	0.83 - 0.90
Beam $\omega_p = 0.295$	1.04	0.10	0.80 - 0.86
Flexure, Cast-in-Situ Post-Tensioned Concrete $\beta = 3.0$			
$\omega_p = 0.228$	1.03	0.11	0.78 - 0.83
$\omega_p = 0.295$	1.05	0.14	0.76 - 0.79
Tied Columns, Compression Failures, $\beta = 3.5$			
3000 psi Concrete, short	1.05	0.16	0.65 - 0.69
5000 psi Concrete, short	0.95	0.14	0.61 - 0.66
5000 psi Concrete, $\ell/h = 20$	1.10	0.17	0.66 - 0.70
Spiral Columns, Compression Failures, $\beta = 3.0$			
3000 psi Concrete, short	1.05	0.16	0.74 - 0.76
5000 psi Concrete, short	0.95	0.14	0.69 - 0.72
Shear, $\beta = 3.5$			
Beams without stirrups	0.93	0.21	0.50 - 0.52
Beams with minimum stirrups	1.00	0.19	0.60 - 0.64
Beams with $\rho_v f_y = 150$	1.09	0.17	0.66 - 0.70

Note: 1 psi = 6895 N/m^2

of duration of load on strength is in a state of flux (see Appendix E). Regardless of how sophisticated theoretical models become, however, the results will have to be reduced to the LRFD format for design office use, since structural designers in the United States appear to be unwilling to work with anything more complicated than this.

Third, the data presented in Appendix E is insufficient to determine whether any statistically significant differences in $\overline{R}/R_n$ and V_R (upon which ϕ depends) exist among species. If possible, it would appear desirable to allow any differences to be ironed out in the determination of R_n so that different ϕ values would not be needed for, e.g., Douglas Fir and Southern Pine beams in flexure.

Fourth, it should be decided whether ϕ should depend on whether the timber members or laminating stock is visually or machine graded.

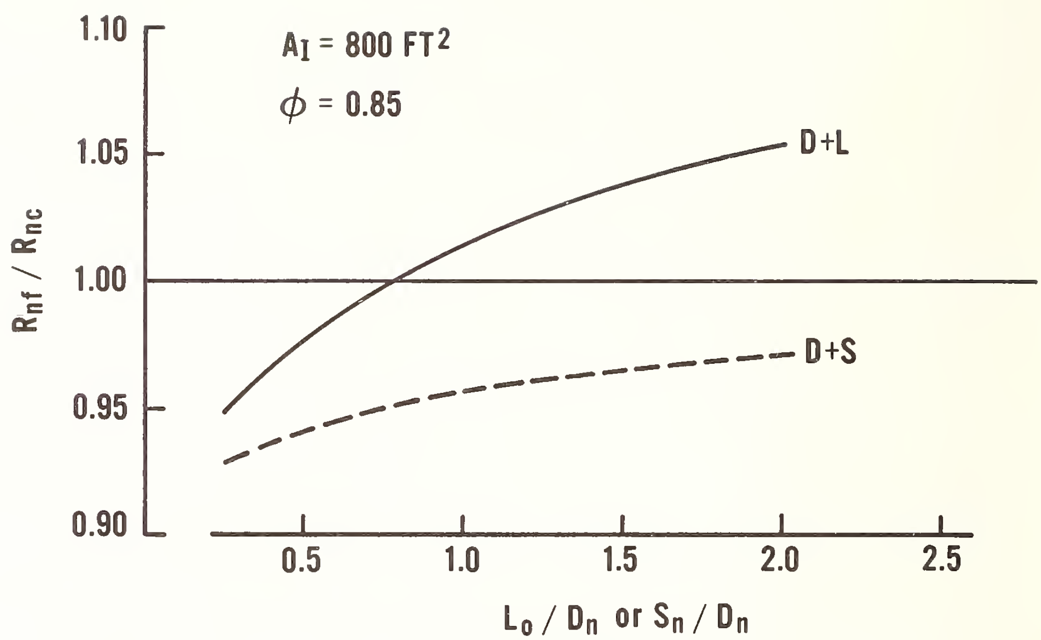
5.7.4 Masonry Structures

Current design of engineered brick and concrete masonry structures uses working stress principles. Masonry specification writing groups moving toward limit states design have almost complete flexibility in choosing their strength criteria. The following points should be considered.

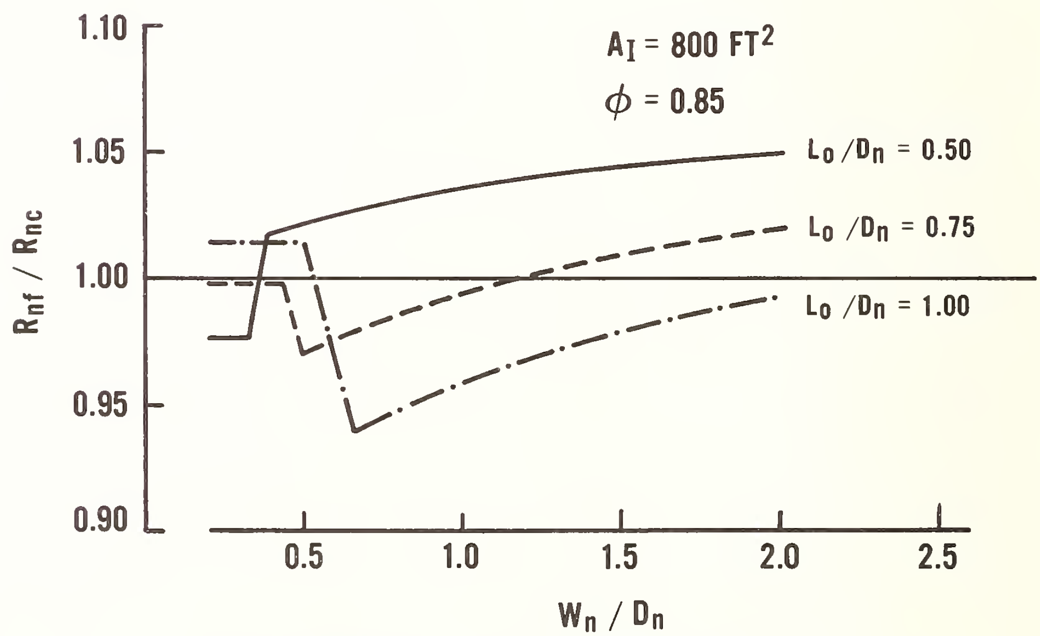
First, the specification of the ϕ factor and nominal resistance R_n for different members and limit states are interrelated, as discussed in connection with wood structures in Section 5.7.3.

Second, the substantial reduction in β which occurs in unreinforced masonry walls as the load eccentricity increases, discussed in Chapter 4 and Appendix D, is of concern. Such a large variation does not appear to be desirable. If the mode (ductile or brittle) and the consequences of failure of such a wall are relatively uniform for all eccentricities, then β should also be relatively uniform and some relative adjustments should be made in methods of computing R_n. It seems that some reduction in conservatism would be possible at small eccentricities, and that perhaps an increase in conservatism could be desirable at large eccentricities. Such adjustments could be made either by modifying the manner in which R_n depends on load eccentricity or by allowing ϕ to depend on eccentricity. If the failure mode and consequences are relatively uniform, the adjustments should probably be made to R_n.

Third, the standard governing engineered brick masonry distinguishes between inspected and uninspected workmanship. When the workmanship is inspected, wall alignment, thickness



(a) R/C BEAMS - GRAVITY LOADS



(b) R/C BEAMS - GRAVITY PLUS WIND LOADS

Figure 5.12 - Comparison of Designs Using Existing and Proposed Criteria for Reinforced Concrete Beams (100 ft^2 = 9.3 m^2)

of joints, effects of partially filled joints and other factors which would reduce the probable strength and increase its variability are more carefully controlled. It appears desirable that this distinction be made in a limit states criterion. Data on the effect of inspection on R_n and V_R and on the variability in construction practice across the United States would be useful. The upsurge in the use of engineered masonry and in masonry research may well provide additional data on this aspect. The specification writing group has a choice as to whether workmanship should be reflected in ϕ or in R_n.

6. SUMMARY AND CONCLUSIONS

This report has described the development of a set of recommended load factors and load combinations for use with loads in the proposed 1980 version of American National Standard A58, Building Code Requirements for Minimum Design Loads in Buildings and Other Structures. The scope of the resulting recommended load criterion is the same as that of the A58 Standard, which covers dead, live, wind, snow and earthquake loads. The criterion does not apply to vehicle loads on bridges, transients in reactor containments, and other loads which are considered outside the scope of the A58 Standard. A series of aids to material specification writing groups to assist them in their selection of resistance factors is also presented.

The method of arriving at the resulting load factors is an advanced reliability analysis procedure. Earlier versions of this method have been used in the development of the Canadian Limit States Design specifications for steel structures for buildings, the Ontario Bridge Code, and the proposed Load and Resistance Factor Design criteria for structural steel in the United States. The method used in this work employs information on the probability distributions of the random variables, while the earlier methods only considered mean values and standard deviations. It was reassuring to find that the less sophisticated process gave results which are similar to those from the more advanced method.

The procedure by which the load factors were developed consisted of:

1) Collecting and evaluating statistical and probabilistic information on various types of structural loads (dead, live, snow, wind, earthquake) and structural capacities (resistances). Much of this material was already available in the literature, but additional data evaluation and probabilistic analysis was necessary for the environmental loads (wind, snow, earthquake), for glulam members, and for masonry walls. The input from the load subcommittees of American National Standard Committee A58 was especially helpful, as was the previous research of the authors. The details of the data evaluation are presented in the Appendices.

2) Evaluating the relative reliability implied in current design. The measure of reliability was the reliability index β. This is consistent with previous work in this field. Values of β were determined using a computer program. The basis of the method is described in Chapter 2 and the description of the program is presented in Appendix F.

3) Selecting target reliabilities and developing load factors consistent with these target reliabilities.

It was not surprising that values of the reliability index β varied a great deal, depending on the type of structural load (e.g., gravity versus wind), the type of structural material, the limit state and the kind of element within a structure. In selecting the target reliability it was decided, after carefully examining the resulting reliability indices for the many design situations, that $\beta_o = 3$ is a representative average value for many frequently used structural elements when they are subjected to gravity loading, while $\beta_o = 2.5$ and $\beta_o = 1.75$ are representative values for loads which include wind and earthquake, respectively.

The recommended load combinations and load factors are as follows:

$1.4\ D_n$

$1.2\ D_n + 1.6\ L_n$

$1.2\ D_n + 1.6\ S_n + (0.5\ L_n \text{ or } 0.8\ W_n)$

$1.2\ D_n + 1.3\ W_n + 0.5\ L_n$

$1.2\ D_n + 1.5\ E_n + (0.5\ L_n \text{ or } 0.2\ S_n)$

$0.9\ D_n - (1.3\ W_n \text{ or } 1.5\ E_n)$

The load combinations assume that the simultaneous occurrence of maximum values of snow, wind, earthquake and live loads is not likely. The smaller load factors in these combinations are a reflection of the fact that the factored arbitrary-point-in-time load is less than the nominal load.

It was felt that while the determination of the resistance factor ϕ in the design criterion

$$\phi R_n \geq \Sigma\ \gamma_i Q_{ni}$$

was not within the purview of the A58 Standard, it would be helpful to specification writing groups if a method was given that they would find relatively easy to apply. Accordingly, charts are presented which permit the determination of values of ϕ, given a desired β-level and material statistics, which are consistent with the load factors recommended in this report. Material specification writing groups can thus select their own target β values reflecting the particular situation of interest to them, and can determine a ϕ consistent with the selected β; conversely, they can choose ϕ and determine the resulting β. The computer program given in Appendix F may, of course, also be used for this operation.

No attempt is made to enforce common levels of β for all materials and member types, and enough information is given to the specification writers to accommodate their needs. This freedom is especially helpful if, say, $\beta = 3$ is used for member design while it is required that $\beta = 4.5$ for connectors. Sufficient data on resistance variables is presented in Appendices B through E that material specification groups can make such decisions intelligently.

The load factors and load combinations recommended herein apply to the loads explicitly covered in the proposed 1980 version of the A58 Standard. There are other types of loads, of course, such as ponding loads, temperature loads, construction loads, etc. The methodology presented here may be employed to develop load factors for them if the statistical information is first determined.

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The American Iron and Steel Institute

The Brick Institute of America

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The National Forest Products Association

The National Research Council of Canada

The Western Forest Products Laboratory

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9. GLOSSARY

Allowable Stress Design or Working Stress Design: A method of proportioning structures such that the computed elastic stress does not exceed a specified limiting stress.

Arbitrary-Point-in-Time Load: loading which is on the structure at any instant in time.

Building Standard: a document defining minimum standards for design.

Calibration: a process of adjusting the parameters in a new standard to achieve approximatel the same reliability as exists in a current standard or specification.

Coefficient of Variation: the ratio of the standard deviation to the mean of a random variable.

Dead Load: load due to structural self weight and the permanent features on the building.

Environmental Loads: loads on a structure due to wind, snow, earthquake or temperature.

Factor of Safety: a factor by which a designated limit state force or stress is divided to obtain a specified allowable value.

Format of design checking procedure: an ordered sequence of products of load factors and load effects which must be checked in the design process.

First-Order Second-Moment (FOSM) Reliability Methods: Methods which involve (1) linearizing the limit state function through a Taylor series expansion at some point (first-order), and (2) computing a notional reliability measure which is a function only of the means and variances (first and second moments) of the random variables rather than their probability distributions.

Failure: a condition where a limit state is reached. This may or may not involve collapse or other catastrophic occurrences.

Influence Area: That area over which the influence function for load effect (beam shear, column thrust, etc.) is significantly different from zero. For columns, this is four times the traditional tributary area; for beams, twice; and for a slab, they are equal.

Limit States: criteria beyond which a structure or structural element is judged to be no longer useful for its intended function (serviceability limit state) or beyond which it is judged to be unsafe (ultimate limit state).

Limit States Design: a design method which aims at providing safety against a structure or structural element being rendered unfit for use.

Load Combinations: loads which are likely to act simultaneously.

Load Effect: the force in a member or an element (axial force, shear force, bending moment, torque) due to the loading.

Load Factors: a factor by which a nominal load effect is multiplied to account for the uncertainties inherent in the determination of the load effect.

Load and Resistance Factor Design: a design method which uses load factors and resistance factors in the design format.

Maximum Load: the maximum load that acts on a structure during some reference period, herein taken as 50 years.

Mean Recurrence Interval (MRI): The average time between occurrences of a random variable which exceed its MRI value. The probability that the MRI value will be exceeded in any occurrence is 1/(MRI).

Nominal Load Effect: calculated using a nominal load; the nominal load frequently is defined with reference to a probability level; e.g. 50 year mean recurrence interval wind speed used in calculating the wind load.

Nominal Resistance: Calculated using nominal material and cross- sectional properties and a rationally developed formula based on an analytical and/or experimental model of limit state behavior.

Probability Distribution: a mathematical law which describes the probability that a random variable will assume certain values; either a cumulative distribution function (cdf) or a probability density function is used.

Probabilistic Design: a design method which explicitly utilizes probability theory in the safety checking process.

Probability of Failure: the probability that the limit state is exceeded or violated.

Probability of Survival (Reliability): the probability that the limit state is not attained.

Reliability Index: a computed quantity defining the relative reliability of a structure or structural element.

Resistance: the maximum load carrying capacity as defined by a limit state.

Resistance Factor: a factor by which the nominal resistance is multiplied to account for the uncertainties inherent in its determination.

Target Reliability: a desired level of reliability in a proposed design method.

10. NOMENCLATURE

The following nomenclature defines the major symbols used in this report. The symbols used are those generally used in the literature. Special care was taken to retain the familiar symbols particular to each branch of technology which was encountered, and no attempt was made to unify symbols from the various material technologies. Thus, it occasionally happens that several symbols are used for the same quantity, or, that several quantities are defined by the same symbol. The notation is also defined where it occurs, so the context will aid in defining the particular quantity.

A: peak ground acceleration

A: cross-sectional area

A: generalized structural load

A_g: gross cross-sectional area

A_I: influence area

A_n: net cross-sectional area

A_T: tributary area

B: generalized modeling parameter

C: base shear coefficient

C_p: pressure coefficient

c: generalized influence coefficient

d: cross-sectional dimension

D: dead load intensity or load effect; $\overline{D}$ and D_n are mean and nominal values respectively

E: earthquake load effect; $\overline{E}$ and E_n are mean and nominal values, respectively

E: tensile modulus of elasticity

e: load eccentricity

E_Z: exposure factor

F: generalized variable denoting cross-sectional parameters; $\overline{F}$ is mean value

FS: factor of safety

F_X, f_x: cumulative distribution function (cdf) and probability density function for random variable X, respectively

F_a, f_a: allowable and computed axial stress

F_b, f_b: allowable and computed axial stress

f'_c: 28-day concrete strength

f_c; compressive stress

F_{cr}; critical stress

F_{Exx}; tensile strength of weld metal

f_m: compressive strength of prism tests

F_n; 10 yr nominal design stress for wood

f_r: bending stress

F_r: modulus of rupture

f_t: tensile stress

F_u: yield stress

F_{ys}: static yield stress

f_y: yield stress

G: gust factor

G: elastic modulus in shear

g: generalized design function

h: cross-sectional dimension

I: moment of inertia

I: importance factor

K: building factor

k: effective length factor

L: length

L: live load intensity or load effect; $\overline{L}$ and L_n are mean and nominal values, respectively; L_{apt} is arbitrary-point-in-time value

L_o: basic code-specified live load

M: bending moment

M_u: ultimate bending moment

M: generalized material factor; $\overline{M}$ is mean value

M_p: plastic moment

P: generalized professional factor; $\overline{P}$ is mean value

P: axial force

P_f: probability of failure

Q: generalized load effect; $\overline{Q}$ is mean value

Q: form factor

R: generalized resistance; $\overline{R}$ and R_n are mean and nominal values, respectively

R: system factor

r: radius of gyration

S: soil factor

S: snow load effect; $\overline{S}$ and S_n are mean and nominal values, respectively

S: elastic section modulus

S_{V_o}: spectral amplification factor

t: cross-sectional dimension

T,t: time

V: coefficient of variation

V: wind velocity

V_u: ultimate shear capacity

W: weight of structure

X: generalized parameter

Z: zone factor

Z: plastic section modulus

α: direction cosine

β: reliability index

γ: load factor

λ: slenderness parameter

ϕ: resistance factor

σ: standard deviation

σ_{cr}: critical stress

σ_u: tensile stress

APPENDIX A ANALYSIS OF STRUCTURAL LOADS

General Remarks

The load effect Q_i is related to the structural load through the relation

$$Q_i = c_i B_i A_i \tag{A.1}$$

in which c_i = influence coefficient, B_i = modeling parameter, and A_i = structural load. It is assumed that the transformation from load to load effect is linear, and that c_i, B_i and A_i are statistically independent.

It is convenient from a conceptual point of view to delineate the various factors which contribute to the overall uncertainty in the load effect on a member. In addition to the basic variability in the load, uncertainty arises from the load model which transforms the actual spatially and temporally varying load into a statically equivalent uniformly distributed load (EUDL) which can be used for design purposes. The effects of this load modeling are reflected in the parameter B_i in Eq. A.1, which may be assumed to have mean of unity and a c.o.v. V_{B_i} which reflects the uncertainty in the load modeling. Finally, uncertainties arise from the analysis which transforms the EUDL to a load effect, reflected in parameter c_i. These would include two-dimensional idealizations of three dimensional structures, fixity of supports, rigidity of connections, continuity and so forth. Thus, V_{c_i} would, in general, depend on the load as well as the structure.

The mean and c.o.v. of the load effect are then,

$$\overline{Q}_i = \overline{c}_i \ \overline{B}_i \ \overline{A}_i \tag{A.2}$$

$$V_{Q_i} = [V^2_{c_i} + V^2_{B_i} + V^2_{A_i}]^{1/2} \tag{A.3}$$

In the absence of information to the contrary, $\overline{B}_i = 1.0$; V_{c_i} and V_{B_i} represent best professional estimates of the uncertainty due to load modeling and analysis.

When several loads act, the load effect on a member would be,

$$Q = c[c_1 \ B_1 \ A_1 + c_2 \ B_2 \ A_2 + \ldots] \tag{A.4a}$$

in which all variables are assumed to be statistically independent. In this model, c_1, c_2 ... reflect structural analysis effects which are unique to a particular load (effect) while the factor c reflects those features of the structural analysis which are common to all loads (effects). One would obtain the same representation through the model,

$$Q = c_1 \ B_1 \ A_1 + c_2 \ B_2 \ A_2 + \ldots \tag{A.4b}$$

if it were assumed that c_1, c_2, ... are correlated. In the load analysis used in this study, it was found that because of the magnitudes of the c.o.v., this correlation could be ignored. This simplifies the analysis of uncertainties in Q.

In the remaining sections of Appendix A, the statistical descriptions of the dead, live, wind, snow and earthquake loads used in developing the load criterion are presented. The first and second order statistics for each of the loads are analyzed in accordance with Eqs. A.2 and A.3. It should be noted that the reliability analysis uses the load effects Q_i as the basic variable in the limit state equation.

In evaluating the load statistics, the basic sources of information were the load subcommittees within American National Standard Committee A58. This information was supplemented by additional published data, where appropriate.

Dead Load

The dead load is assumed to remain constant throughout the life of the structure. The dead load results from the weight of elements comprising the structure and includes permanent equipment, partitions and installations, roofing, floor coverings, etc. Most investigators feel that the probability distribution is normal or close to it. Many have assumed that the ratio of mean load to nominal load is unity and that the coefficient of variation V_D = 0.06 - 0.15, with a typical value of 0.10. Some of the values used in recently published reliability based design work are listed in Table A.1.

Table A.1

Statistics of Dead Load ($D = C \cdot B \cdot A_D$)

Reference	$\overline{D}/D_n$	$[V_B^2 + V_{A_D}^2]^{1/2}$	V_D
A.9	1.00	0.06	0.08
A.2	1.0	0.07	0.10
A.7	1.0	0.09	0.10
A58 Live Load Subcommittee	1.05	-	0.07
A.11	1.05	-	0.09
A.13	1.0	-	0.05
Appendix B of this report	1.03	0.09	0.10

It may be argued that the variability in dead load should depend on the construction material. Strictly speaking, this is true; however, the dependence of V_D on material is very weak because much of the variability in permanent loads is caused by the weights of non-structural items such as roofing, partitions, etc. There is a feeling on the part of many design professionals that there is a tendency on the part of designers to underestimate the total dead load. Accordingly it is assumed that $\overline{D}/D_n$ = 1.05 and V_D = 0.10 for all construction materials considered in this study.

Live Load

Live loads include the weight of people and their possessions, furniture, moveable partitions and other portable fixtures and equipment. The total live load on a floor area may be thought of conveniently as consisting of a sustained component which remains relatively constant within a particular occupancy, referred to as the "arbitrary point-in-time live load," and an extraordinary component which arises from infrequent clustering of people above and beyond normal personnel load, or from activities such as remodeling. The load combination analysis procedure described in Chapter 2 requires knowledge of statistical characteristics of both the maximum live load L during a 50 year reference period and the arbitrary point-in-time live load, L_{apt}.

(a) Arbitrary Point-in-Time Live Load - L_{apt}

Characteristics of L_{apt} may be obtained directly from the results of load surveys which are analyzed using probabilistic load models [A.12, A.14, A.8]. Numerous load surveys have been conducted in recent years in the U.S. and Europe. Although most of these have focussed on office buildings, some data on residence, retail establishments and other occupancies are also available. A summary of results from analyses of load survey data is presented in Table A.2.

Table A.2

Arbitrary-Point-in-Time Live Loads in Offices ($L = c \cdot B \cdot A$)

Reference	A_I = 200		1000		5000		10000	
	$\bar{L}_{apt}/L_n$	V_A	$\bar{L}_{apt}/L_n$	V_A	$\bar{L}_{apt}L_n$	V_A	$\bar{L}_{apt}/L_n$	V_A
A.12	0.24	0.89	Varies	0.52	Varies	0.41	Varies	0.40
A.8	0.23	0.85	Varies	0.55	Varies	0.46	Varies	0.45
A.4	0.22	0.70	Varies	0.40	Varies	0.26	Varies	0.20
A.2	0.16	0.70	Varies	0.48	Varies	0.38	Varies	0.36
A.16	0.15	0.59	Varies	0.26	Varies	0.20	Varies	0.18
V_L	0.8		0.5		0.45		0.4	

The presentation in terms of <u>influence</u> area A_I rather than tributary area A_T ($A_I = 2\ A_T$ for a beam, 4 A_T for columns, and panel area for two-way slabs) has been found to give more consistent reliability for the various load effects. The statistical estimates in Table A.2 include the effects of furnishings and normal personnel loads. L_{apt} appears to be fitted best by a Gamma probability distribution [A.6].

The mean $\overline{L}_{apt}$ is about 12 psf (575N/m^2) for office occupancies and appears to be independent of influence area. On the other hand, the nominal live load L_n in Table A.2 is the value specified in ANSI Standard A58.1-1972 [A.3];

$$L_n = L_o \{ 1 - \min [0.0008 A_T, 0.6, 0.23 (1 + D_n/L_o)]\} \qquad (A.5)$$

in which L_o = the basic unreduced live load (Table 1 of A58.1-1972). Thus, the ratio $\overline{L}_{apt}/L_n$ in Table A.2 varies in those cases where the current A58 standard allows a reduction to be applied. The draft A58 load standard for 1980 currently under review uses a different live load reduction procedure, namely

$$L_n = L_o \left[0.25 + \frac{15}{\sqrt{A_I}}\right] \qquad (A.6)$$

which will affect $\overline{L}_{apt}/L_n$. In calibrating to existing practice, the current nominal live load, Eq. A.5, is used; however, when computing reliabilities for the proposed load criterion, the new nominal live load, Eq. A.6, is used.

While $\overline{L}_{apt}$ appears constant for all influence areas, V_A clearly decreases as the influence area increases. This is a consequence of the load averaging which occurs over large areas.

The c.o.v. in live load effect must incorporate uncertainties in the load modeling and in the analysis which transforms the EUDL to a load effect; V_B in this case is assumed to be 0.10 and V_C = 0.05. Considering these variabilities along with those in the load in Table A.2, V_L was described by a curve passing through the points given in the last line of Table A.2. The ratio $\overline{L}_{apt}/L_o$ is taken as = 12/50 = 0.24; $\overline{L}_{apt}/L_n$ may then be computed using Eq. A.5 or A.6, as appropriate. While the above analysis was performed using data derived from surveys of offices, results for several other occupancies (e.g., residences, retail establishments) are similar enough that these statistics may be applied to them also.

(b) Maximum Live Load - L

While load surveys describe the loads acting on a structure at any point in time, they are insufficient to determine the maximum load which may be expected to act on the structure during a 50-year reference period. Changes in occupancy may cause increases (or decreases) in the load supported by a structural member. In addition, extraordinary load events usually are not reflected in load survey data.

Probability models are available [A.4, A.12, A.14], which can be used to estimate the statistical characteristics of the maximum live load L. In addition to the survey data and distribution of L_{apt} described above, one needs to know (or estimate) the frequency of occupant changes and of extraordinary load events and the loads induced by extraordinary events. Once the upper fractiles of F_L are computed (numerically), a Type I extreme value distribution of largest values is fitted to them and the mean and c.o.v. are back-calculated.

Some results of recent studies are presented in Table A.3 for office occupancies, in which L_n is determined according to Eq. A.5.

Table A.3

Statistics of Maximum Live Load ($L = c \cdot B \cdot A$)

Reference	A_I = 200		1000		5000		10000	
	$\bar{L}/L_n$	V_A	$\bar{L}/L_n$	V_A	$\bar{L}/L_n$	V_A	$\bar{L}/L_n$	V_A
A.12	1.38	0.14	Varies	0.13	Varies	0.15	Varies	0.15
A.8	1.11	0.19	Varies	0.16	Varies	0.16	Varies	0.16
A.4	1.18	0.18	Varies	0.13	Varies	0.10	Varies	0.09
A.16	-	0.23	Varies	0.18	Varies	0.14	Varies	0.12

A comparison of $\bar{L}$ to L_n is also shown in Fig. A.1 as a function of area. Note that the L_n proposed for the 1980 version of the A58 Standard is equivalent to the 50-year mean value, and that the values of L_n given in ANSI A58.1-1972 underestimate the 50-year mean live load for areas in the range of 500 - 2000 ft^2 (46 - 186 m^2).

The total variability V_L in maximum live load effect is obtained by augmenting the data-based variability in Table A.3 with modeling and analysis uncertainties, as discussed previously. V_B should reflect uncertainty in the modeling of the 50-yr maximum load. This would include uncertainties in the description of the arbitrary point-in-time live load process with time and in the modeling of the extraordinary load events. Since these considerations are not at issue in the analysis of variability in L_{apt}, it is logical that V_B should be greater for the maximum live load, and V_B has been taken equal to 0.20 in this study and others [A.7,A.9]. V_c is taken as 0.05. Considering the basic variabilities in Table A.3, Eq. A.3 yields $V_L \simeq 0.25$. Although there is a very slight tendency for V_L to vary with A_I, the variation is insignificant and will be ignored.

By way of comparison, several Canadian studies have used $\bar{L}/L_n = 0.70$ and $V_L = 0.30$. These statistics are based on a 30-year reference period. Since the Canadian live load reduction procedure is quite different from Eq. A.5, one would not expect the $\bar{L}/L_n$ values to be comparable. It should be noted, however, that if L has a Type I distribution of largest values, it may be shown that the c.o.v. for a 50-year reference period would be

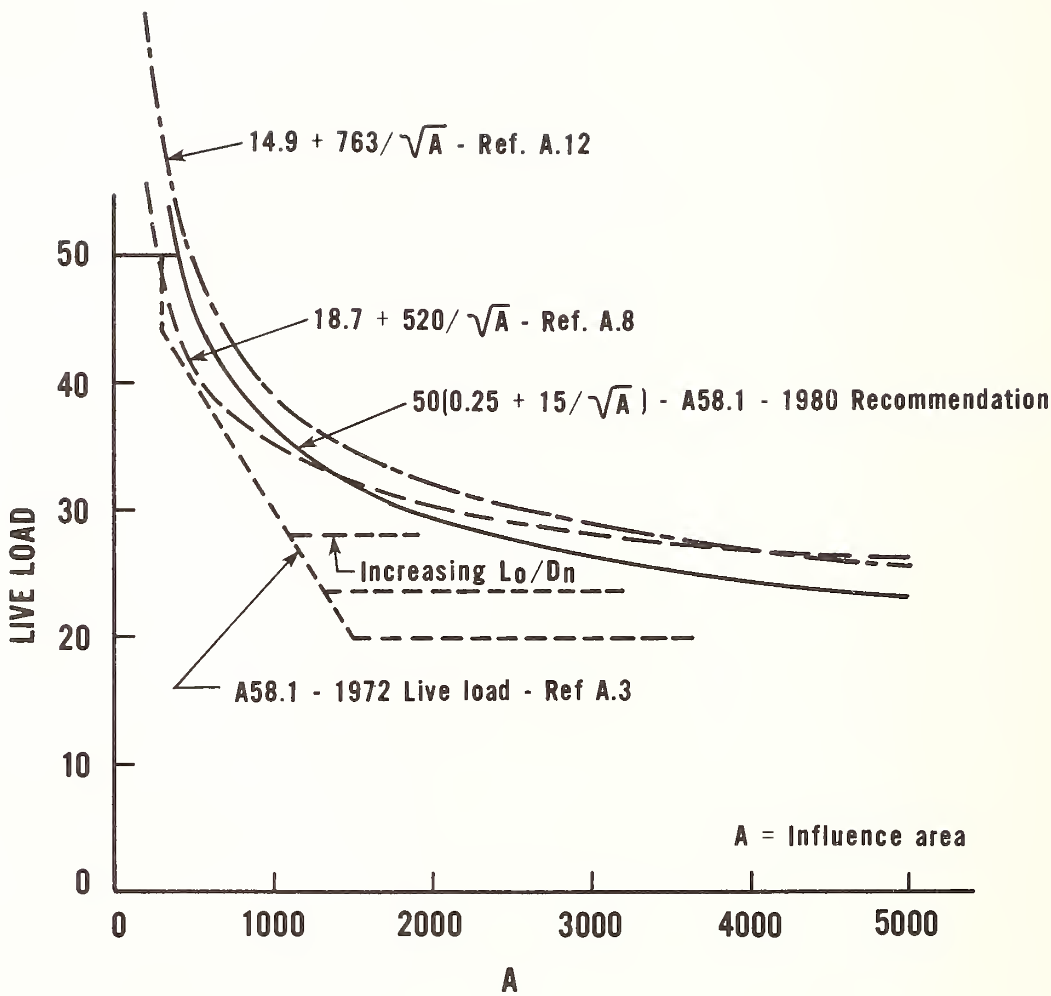

Figure A.1 - Reduction in Live Load with Area approximately 85-90% of that for a 30-yr reference period, assuming a constant rate of occupancy changes, or $V_L = 0.30 \times 0.85 = 0.26$. Thus, at least in terms of overall variability, the results are comparable.

While the results in Table A.2 and Fig. A.1 were derived for offices, examination of data for several other occupancies including multistory residences and retail establishments shows similar variabilities and reduction in load with area. It appears reasonable to assume that the ratio $\bar{L}/L_n$ and V_L are essentially independent of occupancy type for many occupancies. Naturally, the values of $\bar{L}$ or of L_n would depend on occupancy through the value of L_o which enters into the calculation of both and which is specified for different occupancy types in Table 1 of the A58 Standard. One known exception to this rule is the warehouse occupancy, where the reduction in load with increasing area appears much less pronounced than would be indicated by Eq. A.6. There may be other similar occupancies where the reduction is different that will be identified by additional load surveys. Meanwhile, the reduction factor in Eq. A.6 may be assumed to be general enough to be applied to numerous occupancies in which the basic live load L_o is less than 100 psf (4.8 kN/m^2).

Wind Load

Wind loads are derived using statistical data on wind speeds, pressure coefficients, parameters related to exposure and wind speed profile, and a gust factor which incorporates the effects of short gusts and the dynamic response of the structure. For the load combination studies contemplated, the important random variables characterizing the wind load include the daily maximum, annual maximum, and the 50-year maximum wind speed; the latter can be derived from the annual maximum using the relation

$$F_{V_{50}}(v) = [F_V(v)]^{50} \qquad \text{(A.7)}$$

in which V and V_{50} = annual extreme and 50-year maximum wind speeds, and F_V denotes the cumulative distribution function (c.d.f.) of random variable V.

The wind load acting on a structure or component may be written as,

$$W = c\, C_p\, E_Z\, G\, V^2 \qquad \text{(A.8)}$$

in which c = constant, C_p = pressure coefficient, E_Z = exposure coefficient, G = gust factor and V = wind speed referenced to a height of 10 m. C_p depends on the geometry of the structure, E_Z depends on its location (e.g., urban area, open country), and G depends on the turbulence of the wind and the dynamic interaction between the structure and wind. Because velocity enters the equation in terms of its squared value, its statistics are especially important. However, uncertainties in the estimation of the pressure coefficients, the exposure factor, and the gust factor (which includes turbulence, damping and natural frequency) also contribute to the overall variability in wind load. It should be noted that the uncertainty in modeling the effect of wind on the structure is reflected by uncertainties in C_p and G; thus these serve essentially the same purpose as V_B in the previous sections.

(a) Maximum Wind Load - W

Most of the statistical data available are for the annual extreme fastest mile wind speed; the pressure coefficients and gust factor in Eq. A.8 are consistent with the fastest mile specification. Recent analysis [A.17] of this data has shown that the appropriate probability distribution of the annual extreme for extratropical winds is Extreme Value Type I. The same analyses show that the mean and c.o.v. are dependent on geographical locations. These estimates are based on typically 30-40 years of record.

Since it obviously is impractical to perform reliability analyses separately for the more than one hundred sites for which wind speed data are available, seven sites were selected from Ref. A.17 which span the range of data reported and which provide broad geographical representation. These sites and the annual and 50-yr wind speed data are presented in Table A.4.

Table A.4

Wind Load Data

Site	Annual			50-yr Max.			V_n	W/W_n	
	m	$\overline{V}$	V_V	$\overline{V}_{50}$	$V_{V_{50}}$	c.o.v	A58.1-1972	u	α
Baltimore, MD	29	55.9	0.12	76.9	0.09	0.11	75	0.96	5.48
Detroit, MI	44	48.9	0.14	69.8	0.10	0.12	80	0.51	5.31
St. Louis, MO	19	47.4	0.16	70.0	0.11	0.14	70	0.62	3.18
Austin, TX	35	45.1	0.12	61.9	0.09	0.11	80	0.43	8.03
Tucson, AZ	30	51.4	0.17	77.6	0.11	0.14	70	0.69	2.52
Rochester, NY	37	53.5	0.10	69.3	0.08	0.09	70	0.71	4.83
Sacramento, CA	29	46.0	0.22	77.3	0.13	0.16	65	0.65	1.77

Data on the annual extremes is taken directly from Ref. A.17. Since V is Type I, V_{50} is also Type I, with mean and c.o.v. obtained from

$$\overline{V}_{50} = \overline{V}\,(1 + \frac{\sqrt{6}}{\pi}\,V_V\,\ln 50) \qquad (A.9)$$

$$\overline{V}_{50} \cdot V_{V_{50}} = \overline{V} \cdot V_V \qquad (A.10)$$

The total c.o.v. in 50-year wind speed, given in column 6, includes uncertainties due to sampling and observation, defined as $3.8\,\overline{V}\,V_V/(\sqrt{m} \cdot \overline{V}_{50})$ [A.18] and 0.02, respectively.

While the probability distributions for the wind speed are assumed to be Extreme Value Type I, it is not immediately clear what the probability distribution for the wind load should be. The square of a Type I variable does not have a Type I distribution. The fact that C_p, E_Z and G are also random makes it difficult to determine the distribution of W in closed form.

The approach taken in this study was to compute the c.d.f. of wind load, F_W, numerically. This requires knowledge of the c.d.f. and statistics of C_p, G and E_Z in addition to those for V. It was assumed that C_p, G and E_Z each may be described by a normal distribution; the means $\overline{C}_p$, $\overline{G}$ and $\overline{E}_Z$ are defined by the values in ANSI Standard A58.1-1972 [A.3]. V_{C_p}, V_G and V_{E_Z} were obtained to be representative of values used in recent studies [A.7, A.15, A.18]; $V_{C_p} = 0.12$, $V_G = 0.11$ and $V_{E_Z} = 0.16$. V_{E_Z} is largest due to the relative uncertainty regarding building exposure, which includes effects of surface roughness, nearby obstructions in the wind stream and other factors. In comparison with these effects, $V_C \simeq 0.05$ is very small and can be ignored. The distribution F_W then was determined by Monte Carlo simulation and, as an independent check, by numerical integration. A portion of these results is shown in Fig. A.2. These c.d.f. also incorporate a reduction factor of 0.85 to account for the reduced probability that the maximum wind speed will occur in a direction most unfavorable to the response of building. Inspection of these and similar distribution functions for other sites revealed that F_W could be fitted very well by a Type I distribution over the range of the distribution above its 90th percentile. This is the region of particular interest in structural reliability work. This procedure is illustrated in Fig. A.2. The characteristic extreme u and shape α of the fitted Type I distribution for W/W_n at each of the sites are listed in the last two columns of Table A.4. The nominal wind load W_n is defined as that corresponding to the 50-year mean recurrence interval (MRI) load according to ANSI Standard A58.1-1972.

Having performed this analysis for the seven sites, a composite set of statistical estimates was drawn in order to keep the calibration and design work at a manageable level. W/W_n has a Type I distribution, with $u = 0.65$ and $\alpha = 4.45$; the implied mean and

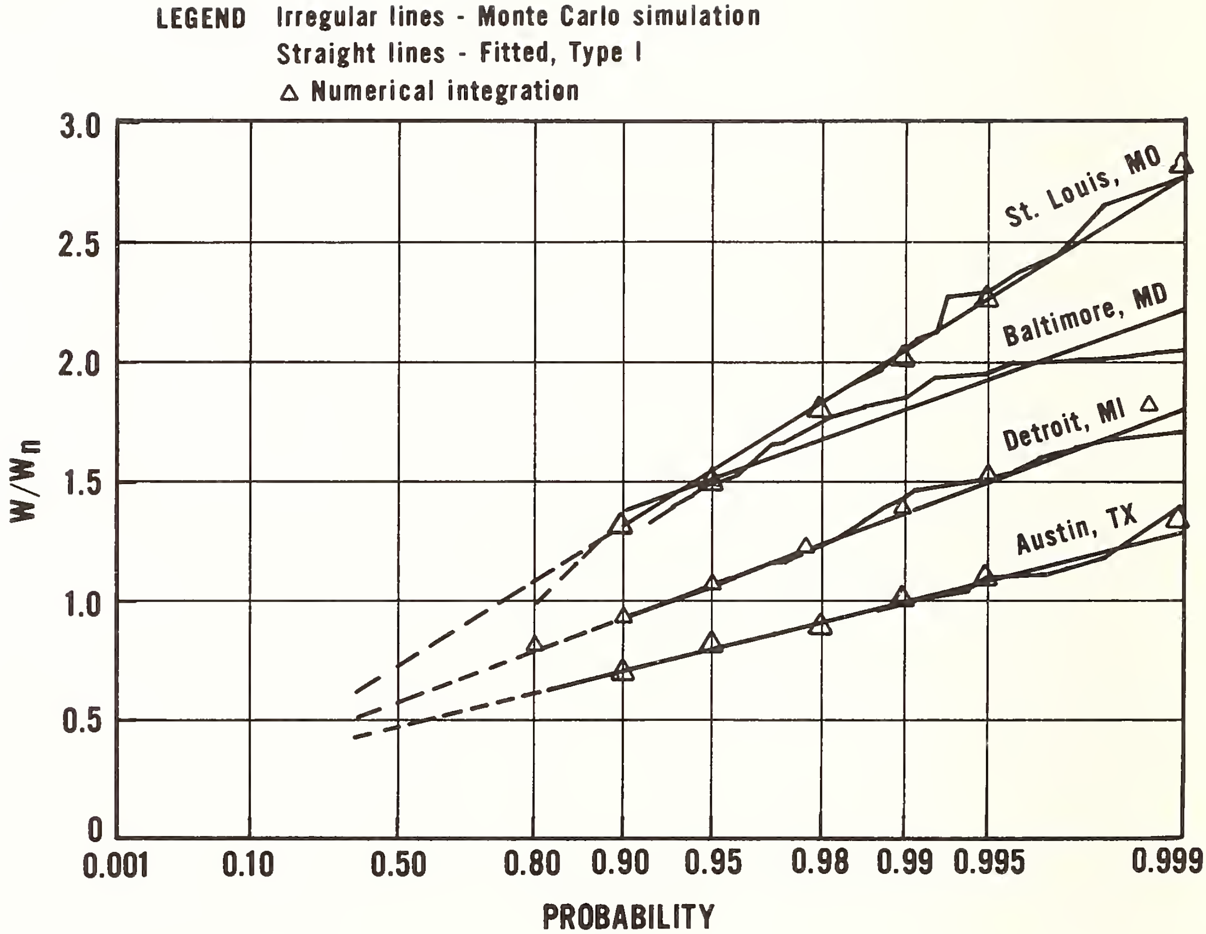

Figure A.2 - Probability Distribution of 50-year Maximum Wind Load c.o.v. are $\overline{W}/W_n = 0.78$ and $V_{W/W_n} = 0.37$. These correspond to a c.d.f. which has been fitted to the true $F_W(\ )$ in the 90th percentile and above. The composite $\overline{W}/W_n$ using the wind loads in the proposed 1980 edition of the A58 Standard is nearly identical. It should be noted that these statistical estimates may be quite different than those obtained through a classical mean value FOSM analysis of Eq. A.8, viz. $\overline{W} = C\,\overline{C}_p\,\overline{G}\,E_Z\,\overline{V}^2$ and $V_W = [V_{C_p}^2 + V_G^2 + V_{E_Z}^2 + V_V^2]^{1/2}$, using the same basic information.

(b) Yearly Maximum Wind Load - W_{ann}

Parameters for the c.d.f. for annual maximum wind load may be determined similarly, utilizing the site-dependent data in Table A.4. The shape and characteristic extreme for the Type I distribution of W_{ann}/W_n fitted to the 90th percentile and above of the true distribution are $u = 0.24$ and $\alpha = 6.65$. The implied mean and c.o.v. of this fitted distribution are $\overline{W}_{ann}/W_n = 0.33$ and $V_{W_{ann}} = 0.59$.

(c) Daily Maximum Wind Load - W_{apt}

Data on daily maximum wind speeds are stored at the National Climatic Center, Asheville, NC. Most of these data have not been published in the open literature and generally are recoverable only with considerable effort and expense. A thorough analysis of data at selected sites of interest to the US aerospace program [A.20] indicated that the daily maximum fastest mile wind speed is Type I. Analysis of daily maximum wind speeds at 13 metropolitan areas across the US in 1974 [A.15] showed that, on the average, $\overline{V}_{daily}/V_n = 0.23$ with a c.o.v. of 0.35 (V_n is the 50-year MRI value). Having determined the c.d.f. and statistics of wind speed, the determination of the statistical characteristics of the daily maximum wind load proceeds as before: the cumulative distribution function of W_{apt}/W_n is computed numerically and a Type I distribution is fitted to its 90th percentile and above. The characteristic extreme and shape of this fitted distribution are $u = -0.021$, $\alpha = 18.7$.

Snow Load

Snow loads are derived using climatological data and field studies which relate the snow load on the roof of a structure to the ground snow load and the roof exposure, geometry and thermal characteristics. This results in an estimate of the roof snow load which can be given as

$$S = C_s q \qquad \text{(A.11)}$$

in which q = ground snow load, C_s = snow load coefficient relating the ground to roof loads; C_s depends on roof exposure, geometry and thermal factors. The factor C_s serves much the same purpose as factor B in Eq. 1.

In the study of snow loads in combination with other loads, the important random variables are the annual maximum and the 50-year maximum snow loads. The 50-year maximum ground load can be derived from the annual maximum ground snow load through Eq. A.7.

The bulk of the statistical data on snow loads is for the annual extreme ground snow load q_{an}, for which there are numerous meteorological records. The annual extreme ground snow load is also the basis for the current and proposed A58 snow load provisions; in what follows the nominal snow load q_n is the 50-year MRI value. A recent analysis of these data has been performed by the US Army Cold Regions Research and Engineering Laboratory [A.22]. Included in this analysis are water-equivalent loads at some 180 first order weather stations and snow depths at some 9000 additional sites which are then converted to loads through density-depth relations. The data are taken from the winters of 1952 - 1978 and usually include 26 or 27 years of record. This analysis forms the basis for the proposed revisions to the A58 Standard for 1980.

It was decided to work directly with the water-equivalent load data in the reliability analysis. The CRREL analysis of these data indicates that the c.d.f. for annual extreme ground snow load is lognormal with parameters that vary from site to site. As with the wind data, a number of sites across the US were selected for more detailed analysis. These sites and the parameters $\lambda = E[\ell n\ q_{an}]$ and $\zeta = \sqrt{Var[\ell n\ q_{an}]}$ of the lognormal c.d.f. for the annual extreme are listed in Table A.5. Cities were selected in which there was measurable snow accumulation in each of the years of record.

Table A.5

Water-Equivalent Ground Snow Load Data

Site	Annual Extreme Ground Load			A58.1-1972	50-yr Maximum Roof Load	
	Years of Record	λ	ζ	q_n	u	α
Green Bay, WI	26	2.01	0.70	28	0.87	5.07
Rochester, NY	26	2.49	0.56	34	0.83	6.16
Boston, MA	25	2.28	0.51	30	0.70	6.63
Detroit, MI	20	1.63	0.58	18	0.69	5.97
Omaha, NB	25	1.60	0.69	25	0.62	5.20
Cleveland, OH	26	1.50	0.58	19	0.60	6.30
Columbia, MO	25	1.21	0.84	20	0.69	4.05
Great Falls, MT	26	1.77	0.49	15	0.80	7.16

Additional uncertainties in the roof load arise from the randomness in the snow load coefficient C_s which translates the ground load to a roof load. The C_s factors in the current and proposed A58 Standards have been selected on the basis of field surveys, augmented to a considerable degree by professional judgment, and have been chosen to be conservative. The only roof configuration for which there is sufficient survey data to estimate the statistical variation in C_s is a flat roof with normal insulation in a normal setting. In ANSI Standard A58.1-1972, $C_s = 0.8$ for this situation. The best estimate of the distribution of C_s in this case is that it is symmetrical (assumed normal) with $\overline{C}_s = 0.5$ and $V_{C_s} = 0.23$, (Wayne Tobiasson of CRREL, private communication). The effect of the analysis factor V_c is inconsequential in comparison.

(a) Maximum Roof Snow Load - S

With the probability distribution functions of C_s and q (either the annual or 50-year maximum) defined, the distribution of S/S_n may be computed by numerical quadrature. The resulting distribution for the 50-year maximum is not fitted over its entire range by any of the common two-parameter distributions. However, since the limiting distribution for a series of lognormally distributed variates is the Type II extreme value distribution of largest values, it would be expected that the c.d.f. for the 50-year maximum would approach a Type II. Accordingly, a Type II distribution was fitted to the computed distribution over the 90th percentile and above, as shown in Fig. A.3. The characteristic extreme and shape (u, α) listed by site in Table A.5 are for the fitted Type II distribution.

A composite set of parameters describing S/S_n was developed from the results presented in Table A.5, which were used in the reliability analysis: $u = 0.72$ and $\alpha = 5.82$. These correspond to $\overline{S}/S_n = 0.82$ and $V_S = 0.26$. Some substantiation for these estimates is found in Ref. A.10, where the snow loads on roofs were predicted using Monte Carlo simulation and a sophisticated snow accumulation model.

(b) Annual Extreme Roof Snow Load - S_{apt}

The probability distribution for the annual roof snow load may be computed similarly. Since the c.o.v. in annual extreme ground load typically is much larger than V_{C_s} (0.65 vs. 0.23), it would be expected that the distribution of S_{apt}/S_n could be approximated by a lognormal distribution, at least in the upper percentiles. Composite values $\overline{S}_{apt}/S_n = 0.20$ and $V_S = 0.73$ were obtained for the reliability analysis.

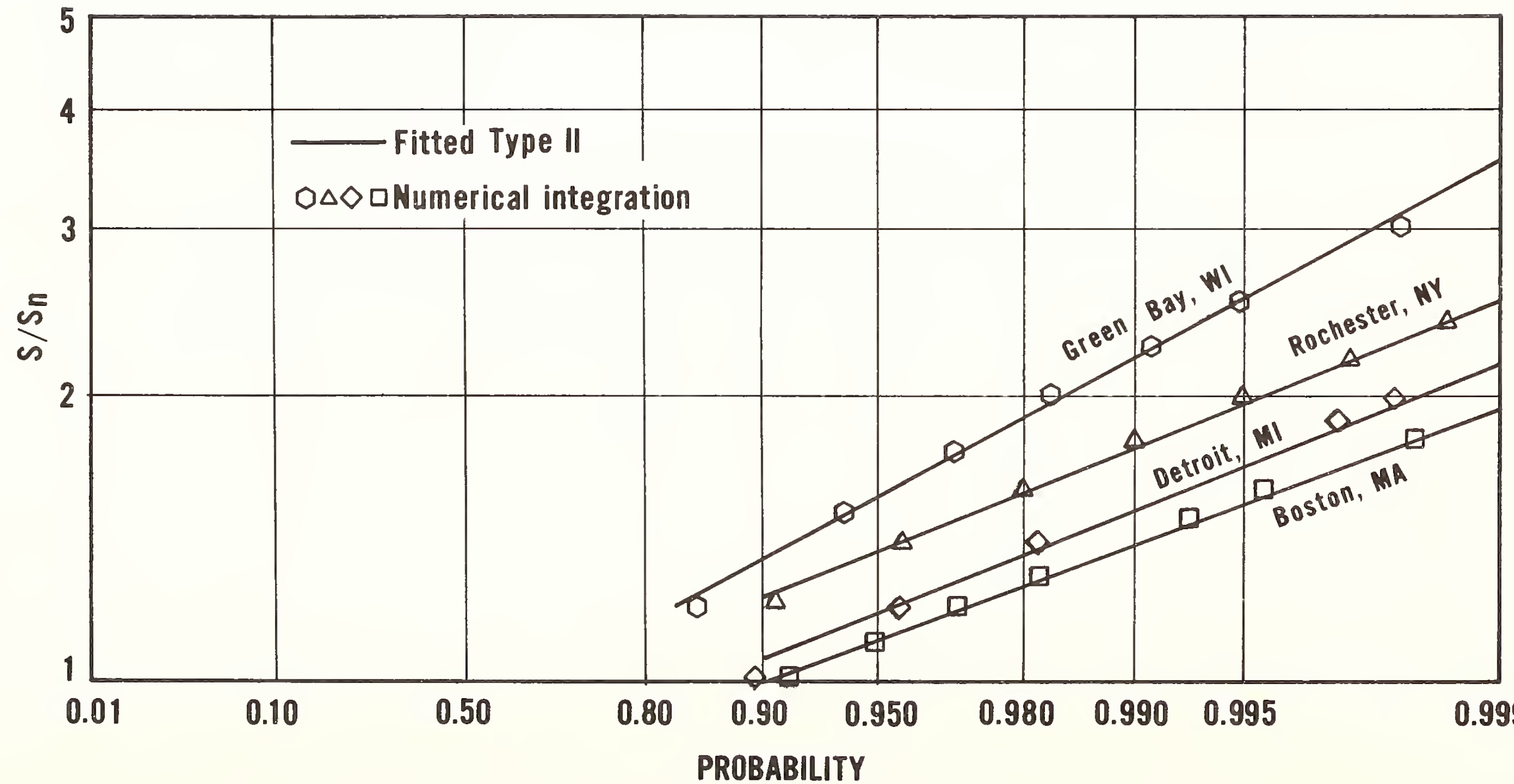

Figure A.3 - Probability Distribution of 50-Year Maximum Snow Load

Earthquake Load

(a) Introduction

The philosophy of seismic design does not lend itself well to the development of a material-independent load criterion, and the problem of how earthquake loads should be treated in load combination work is one that has not yet been completely resolved. We hope our difficulties will encourage the seismic engineering community in the future to attempt to express their problem in terms more compatible with other loads. The fundamental issues are familiar to the community: the codified seismic provisions for the loads required to be used for static, linearly elastic member strength design are not well related to the building behavior anticipated under design ground accelerations. The limit state implicitly addressed is not first yield of a cross-section but some less well defined building-wide behavior such as life threatening damage to elements or even collapse.

Lacking the time and expertise to solve problems that the earthquake engineering community has been studying actively for decades, the recent ATC-3 effort [A.19] was taken as the basis for translating member yield to "building failure."* The link is what is termed the R factor. It is used to reduce base shears to design values, reflecting primarily the ductility of members under dynamic loads, the toughness of the entire structure, as well as resistance elements ignored in conventional structural engineering. We use it here to reduce predicted loads (base shears) to reflect the same phenomena. We assume, naively perhaps, that the values for R arrived at by ATC-3 are best-estimates (mean) values**.

(b) Seismic Environment

The hazard is described in terms of the 50-year maximum peak ground acceleration, A. Techniques for estimating the probability distribution of this random variable for a given site are well established and widely used. They have been applied systematically to the entire continental U.S. by Algermissen and Perkins [A.1]. Even though different investigators may very well produce different estimates, the Algermissen-Perkins results will be used here because they are already a basic element in seismic zoning proposals for the U.S. [A.19]. They provide a map of peak ground accelerations associated with a 10% probability

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*Attempts to restrict the calibration to the member-yield limit state fail because the implied reliability of existing practice is simply too low.

**There is some empirical damage evidence in the commentary to suggest that the R values given in the text are conservative (lower than mean) values.

of being exceeded in 50 years. They also provide estimates of the dependence of probability of acceleration near that 10% point. Their statements are consistent with the assumption that A follows a Type II extreme value distribution (an assumption confirmed by elementary, theoretical seismic hazard analysis [A.5]):

$$F_A(a) = e^{-(a/u)^{-k}} \qquad a \geq 0 \qquad \text{(A.12)}$$

with parameters u and k. Algermissen-Perkins' statement that for all cities the mean return period increases (approximately) by a factor of 5 for a doubling of a is consistent with* the value

$$k = 2.3$$

The mapped value a_{10} has a 10% probability of being exceeded, implying that**

$$u = a_{10} \left[\ln \left(\frac{1}{1-0.1}\right)\right]^{1/2.3} = 0.38\ a_{10} \qquad \text{(A.13)}$$

The conclusion is that the probability distribution of the 50-year maximum peak ground acceleration of any city with Algermissen-Perkins mapped acceleration a_{10} is

$$F_A(a) = \exp\left[-\left(\frac{a}{0.38a_{10}}\right)^{-2.3}\right] \qquad a \geq 0 \qquad \text{(A.14)}$$

For example, Massachusetts and much of New England have a mapped value of $a_{10} = 0.09g$. Therefore the modal (most likely) value is u = (0.38)(0.09) or 0.034g. For the Type II distribution the mean and coefficient of variation are

$$\bar{A} = u\,\Gamma\left(1 - \frac{1}{k}\right) = 1.58\ u = 0.60\ a_{10} \qquad \text{(A.15a)}$$

(or 0.054g for Massachusetts) and

$$V_A = \sqrt{\frac{\Gamma\left(1 - \frac{2}{k}\right)}{\Gamma^2\left(1 - \frac{1}{k}\right)} - 1} = 138\% \qquad \text{(A.15b)}$$

For Los Angeles, $a_{10} \simeq 0.4g$, $u = 0.15g$, and $\bar{A} = 0.24g$, with $V_A = 138\%$. (In fact, in such highly seismic areas, it may well be that the k value is larger owing to magnitude and acceleration "saturation" effects. This would imply a higher mean and a lower coefficient of variation for the same mapped value. Because the safety analysis "checking point" may well be approximately the mapped value, the error in continuing to use the same k (and V) for these higher seismicity areas may not be much in error.)

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*For large values of a: $\ln\left(\frac{1}{1-F(a)}\right) \simeq k\ \ln(a/u)$

** $1 - 0.1 = \exp\left[-(a_{10}/u)^{-2.3}\right]$

(c) Seismic Loading

Load effects due to seismic ground shaking are normally determined for conventional buildings by methods based on static analyses of the structures for loads which are proportional to the base shear, Q, which explicitly (or implicitly) is calculated from an equation of the form*:

$$Q = (B)\ A\ S_{V_o}\ S\ \frac{1}{R}\ W \qquad \text{(A.16)}$$

in which

A = peak ground acceleration

S_{V_o} = spectral amplication factor (a function of period and damping)

S = soil factor (assumed here to equal 1 for calibration purposes)

W = weight of structure

R = system factor

and

B = a random factor with mean equal to one introduced here to account for load modeling and other uncertainties.

The factor R accounts for ductility of materials, members, and the structural system as well as for elements of resistance normally ignored in structural calculations (in this we follow the ATC-3 outline as the "best" current view of seismic behavior). We use for calibration ordinary steel or concrete framed structures (as distinct from special, moment-resisting frames) for which an R of 5 will be used.** For $\overline{S}_{V_o}$ we use (consistent with ATC-3): $1.2/T^{2/3}$, and for calibration we adopt $T = 0.3$, yielding $\overline{S}_{V_o} = 2.7$ or

$$\overline{Q} = \frac{2.7}{5}\ \overline{A}\ \overline{W} = 0.54\ \overline{A}\ \overline{W}$$

The implied mean base shear coefficient, $\overline{Q}/\overline{W}$, will be $0.54\overline{A}$, e.g., about 0.029 for Massachusetts and 0.13 for Los Angeles.

To relate this mean to nominal values we use the procedure in the 1976 Uniform Building Code in which [21]

$$Q_n = ZKCISW \qquad \text{(A.17)}$$

in which

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*For example, in the UBC 76 code the product $\frac{AS_{V_o}}{R}$ corresponds to the product ZCK. The notation here is closer to that of ATC-3.

**ATC-3 recommends 4 1/2 for buildings with reinforced masonry shear walls, 5 1/2 for concrete shear walls, and 5 for steel braced frames.

Z = zone factor (1.0, 3/4, 3/8, and 3/16) for Zones IV, III, II, and I, respectively)

K = building factor (here, 1 for ordinary frames)

C = base shear coefficient (0.12 for $T \leq 0.3$)

I = importance factor (here 1 for calibration)

S = soil factor (again 1 for calibration)

W = weight of structure.

Therefore for calibration

$$Q_n = 0.12\ Z\ W$$

For example in most of Massachusetts, Z = 3/8; therefore,

$$\overline{Q}/Q_n = \frac{0.029\ W}{(0.12)(0.375)W} = 0.64$$

whereas in Los Angeles (Z = 1.0)

$$\overline{Q}/Q_n = \frac{0.13}{0.12} = 1.08$$

The uncertainty in Q will be overwhelmingly dominated by that in A; therefore the values of coefficients of variation of the other factors need not be given special care. For reference, however, we estimate that V_W might be about 0.07 to 0.1, consistent with dead and arbitrary point-in-time live loads averaged over large areas. The uncertainty represented by B includes that due to load modeling and static-for-dynamic analyses (e.g., errors in the c.d.f. of A, superposition of modal responses, deviations from the code-implied mode shapes, the approximate distribution of the static force over the height of the structure) and the usual V_C (for static analysis uncertainties). This value could be 0.2 or somewhat higher. S_{V_o} has a c.o.v. of about 0.3 for the implied periods and dampings. The uncertainty in R may be very large given the limited physical test verification. However, we believe that variation is less than the plus-50% level that would be necessary to materially increase the variability of Q relative to the 138% due to A alone. For this same reason the shape of the CDF of Q will be effectively Type II as well.

In conclusion, it is assumed that Q has a Type II Extreme Value distribution with k = 2.3 (i.e, V_Q = 138%) and mean to nominal ratio of

$$\overline{Q}/Q_n = 0.54\ \overline{A}/0.12Z$$

or

$$= 0.32\ a_{10}/0.12Z$$

The ratio of u_Q to Q_n is

$$\frac{u_Q}{Q_n} = \frac{0.34\overline{A}}{0.12Z} = \frac{0.20\ a_{10}}{0.12Z}$$

in which a_{10} is the Algermissen-Perkins mapped acceleration and Z the corresponding 1976 UBC zone factor for any particular city.

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APPENDIX B - REINFORCED AND PRESTRESSED CONCRETE MEMBERS

B.1 Introduction

The strength of a reinforced concrete member may vary from the calculated or "nominal strength" due to variations in the material strengths and the dimensions of the member as well as variabilities inherent in the equations used to compute member strengths. This appendix briefly reviews each of these sources of variability and documents the statistics which were used in determining the reliability levels for concrete structures.

B.2 Basic Variables

The basic variables affecting the strength of concrete members are the concrete strength in compression and tension, the yield strength of the reinforcement and the dimensions of the cross-sections. The variability of these quantities was based primarily on the data summarized in references B.1, B.2 and B.3.

Three major assumptions were made in determining the strengths to be used in the code calibrations.

1. The variabilities of the material properties and dimensions correspond to average quality construction.

This assumption was made because the results were intended to represent the overall variability of North American construction practice rather than the variability of a particular job which may be done well or poorly. In a similar manner, the reinforcement was assumed to be drawn from a population representing all sources of reinforcement in the United States and Canada rather than from a specific mill or area.

2. The material strengths were assumed to be representative of relatively slow loading rates for load combinations of dead, live and snow loads. The yield strength of steel was based on a so-called "static" loading rate [B.2] and the crushing and tensile strengths of concrete were based on a 1 hour loading to failure.

The strengths of concrete and reinforcement tend to increase at rapid rates of loading. In the case of wind or earthquake loads the concrete and reinforcement strengths were assumed to increase by 5 percent.

3. Long time strength changes of the concrete and steel due to increasing maturity of the concrete and possible future corrosion of the reinforcement were ignored. Washa and Wendt [B.4] reported an average strength ratio of 2.39 comparing the compressive strength at age 25 years to the strength at age 28 days. In tests of concrete strength

at 99 points in a 22 year old concrete building, the average strength was found to be 8050 psi (56 N/mm^2) with a standard deviation of 500 psi (3.5 N/mm^2) compared to an average 28 day cylinder strength for the same project of 3780 psi (26 N/mm^2) and a specified strength of 3000 psi (21 N/mm^2) [B.5]. In this case the ratio of average 22-year strength to average 28-day strength was 2.13.

Thus, relating the concrete strength to the 28-day test cylinder strength leads to a conservative estimate of member strengths, particularly in the cases of shear and bond or for columns.

(a) Concrete

The compressive and tensile strengths of concrete in structures were based on the assumption of a slow rate of loading, corresponding to failure in a test lasting one hour [B.1]. The mean compression strength of concrete in structures was taken as 2760 psi (19 N/mm^2) and 4028 psi (28 N/mm^2) for 3000 and 5000-psi concrete, respectively. This compares with the value of 0.85 f'_c (2550 and 4250 psi for 3000 and 5000 psi concrete, respectively) used as the maximum compressive stress in the ACI Code. The coefficient of variation, V_c, of the in-situ compression strength was taken as

$$V_c = (V^2_{ccyl} + 0.0084)^{1/2} \qquad \text{(B.1)}$$

where V_{ccyl} is the coefficient of variation of the cylinder tests. For average control V_{ccyl} is about 15 and 12 percent for 3000 and 5000 psi concrete, respectively, and V_c can be taken as 18 percent and 15 percent [B.1]. In an independent study, Ellingwood [B.6] estimated the coefficient of variation of the in-situ strength as 0.207 for average control.

Bond strength and shear strength involve tensile failures of the concrete in essentially biaxial compression-tension stress fields. Again the strength of concrete in a structure subjected to a slow rate of loading was considered critical. This strength is best represente by the splitting strength of concrete which, following relationships given in Ref. B.1, gives mean in-situ tensile strengths of 306 and 366 psi (2.11 and 2.5 N/mm^2) for 3000 and 5000-psi concrete respectively. The coefficient of variation of the in-situ tensile strength was taken equal to 18 percent which is the value assumed for the compressive strength. Other studies [B.6] have also concluded that the c.o.v. in tensile and compressive strengths of concrete could be assumed to be equal.

Both the tensile and compressive strengths were assumed to follow a normal distribution. The assumed values are given in Table B.1.

Table B.1

Basic Variables

Property	Mean	V*	σ**	Ref.
Concrete Normal Control				
Compressive strength in structure loaded to failure in one hour.				
f'_c = 3000 psi	2760 psi	0.18	-	B.1
= 4000 psi	3390 psi	0.18	-	B.1
= 5000 psi	4028 psi	0.15	-	B.1
Tensile strength in structure, loaded to failure in one hour.				
f'_c = 3000 psi	306 psi	0.18	-	B.1
= 4000 psi	339 psi	0.18	-	B.1
= 5000 psi	366 psi	0.18	-	B.1
Reinforcement				
Grade 40, Static Yield	45.3 ksi	0.116	5.3 ksi	B.2
Grade 60, Static Yield	67.5 ksi	0.098	6.6 ksi	B.2
Grade 270 Prestressing Strand, Tensile Strength in Static Test	281 ksi	0.025	7.0 ksi	B.9
Dimensions				
Overall depth - Nominal				
Slab (1696 Swedish Slabs)	+0.03 in	-	0.47 in	B.3
(99 Slabs)	+0.21 in	-	0.26 in	B.5
Beam (108 beams)	-0.12 in	-	0.25 in	B.3
(24 beams)	+0.81 in	-	0.55 in	B.5
Effective depth - Nominal				
One-way Slab; Top Bars				
(1696 Swedish Slabs)	-0.75 in	-	0.63 in	B.3
(99 Slabs)	-0.04 in	-	0.37 in	B.5
Values Used	-0.40 in	-	0.50 in	
One-way Slab; Bottom Bars				
(2805 Swedish Slabs)	-0.13 in	-	0.34 in	B.3
(96 Slabs)	-0.16 in	-	0.35 in	B.5
Values Used	-0.13 in	-	0.35 in	
Beam, Top Bars	-0.22 in	-	0.53 in	B.3
Beam Stem Width - Nominal Width	+0.10 in	-	0.15 in	B.3
Column width, breadth - Nominal	+0.06 in	-	0.25 in	B.3
Cover, bottom steel in beams	+0.06 in	-	0.45 in	B.3
	-0.35 in	-	0.28 in	B.5

1 psi = 6895 Pa ; 1 in = 25.4 mm

*Coefficient of variation

**Standard deviation

(b) Reinforcement

Based on studies of the statistics of the strength of Grade 40 and Grade 60 reinforcing bars [B.2], the means and coefficients of variation of the static yield strengths were taken as 45.3 ksi (312 N/mm^2) and 0.116, respectively, for Grade 40 hot-rolled deformed bars and 67.5 ksi (465 N/mm^2) and 0.098 for Grade 60 bars. The beta c.d.f. was used to model the yield stress and ultimate strengths [B.2].

For Grade 40 bars, Allen [B.7] assumed a normal distribution of yield stress with a mean of 1.072 times the specified or 42.9 ksi (296 N/mm^2) and a coefficient of variation of 9 percent. Ellingwood [B.8] assumed a lognormal distribution for Grade 40 steel with a mean strength of 47.7 ksi (329 N/mm^2) and a coefficient of variation of 9 percent; in a later publication [B.6], this mean was reduced by 3 ksi (21 N/mm^2) and the coefficient of variation was increased to 11 percent to account for variabilities due to bar size effects and strain rate effects.

The ultimate static tensile strength of prestressing wires and strands with a nominal tensile strength of 270 ksi (1862 N/mm^2) was taken as 281 ksi (1938 N/mm^2) with a coefficient of variation of 0.025 [B.9]. This strength was assumed to have a normal distribution.

The assumed distribution parameters for the reinforcement are given in Table B.1. It is interesting to note that the standard deviation of the yield strengths of reinforcing bars and the tensile strength of the prestressing strands are almost the same, increasing from about 5 ksi for Grade 40 steel to about 7 ksi for the prestressing steels. This explains the very small coefficient of variation given in Table B.1.

(c) Dimensions

The differences between the nominal and as-built dimensions are best characterized by the mean and the standard deviation of the error. Since these standard deviations are roughly independent of beam size the coefficients of variation decrease as the member sizes increase. As a result, the overall variability of the strengths of columns or beams was found to be size dependent.

The most important dimensional variations are summarized in Table B.1. For slabs, Ref. B.3 contains data from Swedish studies reported in 1953 and 1968. The mean errors and standard deviations of the 1968 data were roughly half as large as those in the 1953 study. The data in Ref. B.5, although limited to measurements taken in one building in St. Louis, was considered important since this building was designed and built to conform to the 1953 ACI Code. For slab dimensions in particular, the St. Louis data was significantly different from the Swedish data, as shown in Table B.1. The data in Ref. B.5 has been considered in selecting the distributions for the effective depths of top bars in slabs since the variabilities reported in Ref. B.3 seemed excessive in this case.

For purposes of comparison, Allen [B.7] assumed the average effective depth, d, to equal the specified value with a coefficient of variation of 0.025 + 0.20/d in his studies of flexural capacity. Ellingwood [B.6] suggested that the coefficient of variation of concrete member dimensions is $0.4/h_n$ while that for effective depth of reinforcement in flexural members is $0.68/h_n$ where h_n = nominal member dimension. These give values similar to the values in Table B.1.

B.3 Properties of Members for Use in Reliability Studies

B.3.1 Calculation of Statistics of Resistance

The probability distributions and statistics for the capacities of reinforced and prestressed concrete members were studied using a Monte Carlo technique and were spot checked using direct calculations of the means and standard deviations of the resistances. The steps in the Monte Carlo procedure included:

1. A series of relatively accurate methods of calculating member resistances in flexure, shear, bond, etc. was obtained from the literature or were derived. In general these procedures were more comprehensive than the normal design procedures. By comparison to tests, the bias and variability of the computational procedure itself was obtained. This term, referred to as "model error," will be discussed more fully later.

2. A series of representative cross-sections or members were chosen, each defined by a set of nominal material strengths and nominal dimensions. For each particular member the following calculations were carried out.

3. The nominal resistance, R_n, was computed based on the nominal material strengths and dimensions and the ACI Code [B.10] calculation procedures (with $\phi = 1.0$).

4. A set of material strengths and dimensions was generated randomly from statistical distributions of each variable. This set of strengths, etc. plus a randomly generated value of the model error was used with the accurate calculation procedure to estimate the theoretical capacity R of a member having this particular combination of strengths and dimensions. The strength ratio R/R_n was calculated. The mean of this ratio and its coefficient of variation were evaluated.

5. By repeating steps 2 to 5 for a series of nominal cross-sections a measure of the mean and c.o.v. in R/R_n was obtained. In most cases those were expressed in terms of the mean $\overline{R}/R_n$ and the coefficient of variation, V_R of a normal distribution fitted to the portion of the lower range below the 5th percentile of the strength distribution.

B.3.2 Calculation of Model Error

To determine the "model error" the accurate calculation procedure was compared to tests to get the mean and coefficient of variation of the ratio of test strength divided by calculated strength. The variability determined in this way was assumed to result from three causes [B.11]:

$$V_{T/C} = \sqrt{V_m^2 + V_{test}^2 + V_{spec}^2} \qquad (B.2)$$

where $V_{T/C}$ is the coefficient of variation obtained directly from the comparison of the measured and calculated strengths; V_m represents the variability of the model itself, V_{test} represents the uncertainties in the measured loads due to such things as the accuracies of the gages, errors in readings, definitions of failure and V_{spec} represents errors introduced by such things as differences between the strengths in the test specimen and in control cylinders, variations in actual specimen dimensions from those measured. The values of V_{spec} were calculated using the Monte Carlo procedure assuming variabilities representative of in-batch variations in concrete strength, yield strength and possible errors in dimensions. Typically V_{spec} was found to be about 4 percent and V_{test} about 2 to 4 percent. Thus if the coefficient of variation of the measured to calculated capacity was 6.4 percent the variability of the model error would be [B.11]:

$$V_m = \sqrt{V_{T/C}^2 - V_{test}^2 - V_{spec}^2} \qquad (B.3)$$

$$= \sqrt{0.064^2 - 0.04^2 - 0.04^2}$$

$$= 0.046$$

A random variable having a mean value equal to the average value of R_{test}/R_{calc} and a coefficient of variation of V_m was included in step 4 of the Monte Carlo calculations described above.

B.3.3 Flexure and Combined Flexure and Axial Load

(a) Model Used in Calculation of Statistics

For a given axial load the flexural capacity of a reinforced concrete member was computed by deriving a moment-curvature diagram for the cross-section. The maximum moment capacity for that particular axial load was then taken as the highest point on the moment-curvature diagram. This approach allowed either compression or tension failures to be detected in cracked or uncracked members without a change in calculation procedures or equations. For beams, the axial load was set equal to zero in all cases. For columns, a sufficient number of axial load levels was considered to develop an interaction diagram which was used to determine the strengths at various eccentricity ratios.

The calculations were based on the assumption of plane strains remaining plane, a modified Hognestad stress-strain curve for concrete with the maximum concrete stress equal to the value given in Section B.1 (a), and on an elastic-plastic stress-strain curve for the reinforcement. Of all the assumptions made, the latter had the greatest effect on the accuracy of the solutions. Selected calculations based on stress-strain curves which included a strain-hardening branch suggested that inclusion of strain hardening would increase the ultimate moment by amounts ranging from less than 5 percent for steel ratios representative of beams, to as much as 25 percent for very lightly reinforced slabs. For column sections, the moment capacities at very low axial loads were increased by about 15 percent when strain-hardening was included but no significant effect was noted for most other eccentricities. The effect of strain hardening of the reinforcement was ignored in this study because the deformations required to utilize strain-hardening are very large and are accompanied by a risk of failures due to bond, shear, etc. before a complete hinge system develops. An exception has been made in the case of thin lightly reinforced slabs in which yield-line failures are possible.

The computational model was compared to tests of hinged-ended reinforced concrete columns [B.11] and simply-supported reinforced and prestressed concrete beams [B.9]. For columns, the mean ratio of test to calculated load was 1.01 with a coefficient of variation of 0.064. A study of the experimental data suggested that uncertainties in the loading procedures and measuring apparatus could introduce a coefficient of variation V_{test} = 0.02, while possible differences between the actual dimensions and material strengths at the failure section and those measured in control specimens could introduce a coefficient of variation V_{spec} = 0.04. Following Eq. B.3, the resulting model error was calculated as 0.046.

For prestressed concrete beams the mean ratio of calculated to test load was also 1.01 and had a coefficient of variation of 0.054. Using V_{test} = 0.02 and V_{spec} = 0.025, the coefficient of variation of the model error was computed as 0.043. Similar results were obtained for reinforced concrete beams although the comparison was limited to beams which did not develop significant strain hardening.

In all calculations of the variability of the flexural strength of beams or the combined axial load and moment capacity of columns, the model error was assumed to have a mean of 1.01 and a coefficient of variation of 0.046 and has been incorporated in all the distribution data for flexure or combined flexure and axial loads in the remainder of this Appendix.

(b) Reinforced Concrete Flexural Members

(i) Effects of Continuity

The ACI Code requires that continuous beams be designed for checkerboard live loadings. The effect of this is to require up to 1.10 times the statical moment, $w\ell^2/8$, in end spans and up to about 1.23 times the statical moment in interior spans. Assuming the maximum redistribution of moments allowed by the ACI Code occurred, the total moment capacities decrease to about 1.05 and 1.12 times the statical moment in end and interior spans, respectively. The ACI Code does not allow redistribution if the steel ratio exceeds half of the balanced steel ratio.

In this study it will be assumed that in the case of beams the reinforcement ratios in negative moment regions and/or the bar cut-off locations are such that redistribution cannot be counted on and a beam will be assumed to fail if one section reaches its moment capacity. On the other hand, redistribution will be considered in the case of one or two-way slabs which almost invariably are very lightly reinforced and continuous.

(ii) One-way Slabs

One-way reinforced slabs typically vary from 4 to 8 inches (102 - 203 mm) in thickness with reinforcement ratios from about 0.004 to about 0.008. Such slabs are typically continous at one or both ends and are designed for positive and negative moments which total 1.1 to 1.23 times the statical requirements. Assuming strain hardening will cause an increase of 10 percent in the mean capacity, the strength distribution of simply supported one-way slabs can be represented by:

$\overline{R}/R_n = 1.12 \qquad V_R = 0.19$

The high coefficient of variation is a result of the relatively thin sections considered. For continuous slabs, the effect of moment redistribution increases the mean to at least

1.12 x 1.05 = 1.18. On the other hand, two sections must fail for the slab to fail. As a result, the effective overall c.o.v. decreases. Typical values are given in Table B.2. The values given in this table are a composite of a number of values for various thicknesses and reinforcement ratios.

Typical ratios of specified live load to dead load range from 0.5 to 2.5. No live load reduction is allowed in one-way slabs. For snow loading, ratios of snow to dead load would range from about 0.25 to 1.25. Wind loading is not a critical design problem in such slabs.

(iii) Two-way Slab Systems

Flat plate and flat slab floors typically range from 5 inches (127 mm) thick for 15 ft (4.6 mm) square bays for apartment loadings to 9 or 10 inches (229 - 254 mm) thick on 25 ft (7.6 mm) square bays for industrial loadings. In contrast to one-way slabs, two-way slabs are designed for 1.0 times the statical moment and are always continuous. The steel ratios in a flat plate range from about 0.003 in positive moment regions to as high as 0.02 in negative moment regions near columns. In most cases, however, the steel ratios are 0.01 or less. The flexural failure mode of two-way slab structures is highly ductile. A representative description of the strength of two-way slabs reinforced with Grade 60 steel would be:

$\overline{R}/R_n = 1.12 \quad V_R = 0.14$

Typical specified live to dead load ratios range from 0.7 to 2.0. The latter ratio corresponds to industrial or storage loadings. Snow to live load ratios range from 0.25 to 1.25. Although two-way slab structures are generally braced to resist the wind loads, unbraced flat plate buildings do occur. In unbraced structures, wind load moments will range from 0 to 0.5 times the dead load moments. Typical influence areas (equal to tributary areas in this case) range from 250 to 600 ft^2 (23 - 56 m^2).

Pan-joist floors typically vary from 11 to 24.5 inches (279 - 622 mm) in total depth and span 15 to 40 feet (4.6 - 12.2 m). The reinforcement ratios vary from 0.0006 to 0.004 in positive moment regions and 0.005 to 0.013 in negative moment regions. Although such structures are typically continuous, the amount of moment redistribution which can be accommodated is variable due to the possibility of shear failures or shifts in the points of contraflexure. The first section to yield would generally be at the supports where the steel percentages are frequently high enough that strain hardening would not significantly affect the failure moment. A representative description of the distribution of strengths of joist floors reinforced with Grade 60 reinforcement would be;

Table B.2

Resistance Statistics

Action	Type of Member	Details	$\bar{R}/R_n$	V_R
Flexure Reinforced Concrete	Continuous one-way slabs	5 in. thick, Grade 40	1.22	0.16
		5 in. thick, Grade 60	1.21	0.15
	Two-way slabs	5 in. thick, Grade 60	1.16	0.15
		7 in. thick, Grade 60	1.12	0.14
	One-way pan joists	13 in. overall depth, Grade 60	1.13	0.135
	Beams, Grade 40, f'_c = 5 ksi	$\rho = 0.005 = 0.09\ \rho_b$	1.18	0.14
		$\rho = 0.019 = 0.35\ \rho_b$	1.14	0.14
	Beams, Grade 60, f'_c = 5 ksi	$\rho = 0.006 = 0.14\ \rho_b$	1.04	0.08
		$\rho = 0.015 = 0.31\ \rho_b$	1.09	0.11
		$\rho = 0.027 = 0.57\ \rho_b$	1.05	0.11
		$\rho = 0.034 = 0.73\ \rho_b$	1.01	0.12
Flexure, Reinforced Concrete - Overall Values			1.05	0.11
Flexure Prestressed Concrete	Plant Precast Pretensioned	$\omega_p = 0.054$	1.06	0.057
		$\omega_p = 0.122$	1.05	0.061
		$\omega_p = 0.228$	1.06	0.083
		$\omega_p = 0.295$	1.04	0.097
	Cast-in-Place Post-tensioned	$\omega_p = 0.054$	1.02	0.061
		$\omega_p = 0.122$	1.05	0.083
		$\omega_p = 0.228$	1.03	0.111
		$\omega_p = 0.295$	1.05	0.144
Flexure, Plant Precast Pretensioned, Overall Value			1.06	0.08
Cast-in-Place Post-tensioned, Overall Value			1.04	0.095
Axial Load and Flexure	Short Columns, Compression Failures	f'_c = 3 ksi	1.05	0.16
		f'_c = 5 ksi	0.95	0.14
	Short Columns, Tension Failures	f'_c = 3 and 5 ksi	1.05	0.12
	Slender Columns, $k\ell/h$ = 20, Compression Failures	f'_c = 5 ksi	1.10	0.17
	Slender Columns, $k\ell/h$ = 20, Tension Failures	f'_c = 5 ksi	0.95	0.12
Shear	Beams with $a/d \geq 2.5$, $\rho_w = 0.008$	No stirrups	0.93	0.21
		Min stirrups	1.00	0.19
		$\rho_v f_y$ = 150 psi	1.09	0.17

1 ksi = 6.9 N/mm^2; 1 in = 25.4 mm

$\overline{R}/R_n = 1.13 \qquad V_R = 0.135$

Typical live to dead load ratios range from 0.5 to 1.5. Wind to dead load combinations are generally not considered.

(iv) Reinforced Concrete Beams

The range of properties of reinforced concrete beams is almost infinite. For the purpose of this study, beams will be assumed to have effective depths in excess of 10 inches (254 mm) and no moment redistribution or strain hardening will be considered. Representative beam cross-sections would have the strength distributions listed in Table B.2. Clearly, the means and coefficients of variation vary considerably. Two cases have been emphasized in the reliability studies:

$\overline{R}/R_n = 1.05 \qquad V_R = 0.11$ (Beam, Grade 60 reinforcement)

$\overline{R}/R_n = 1.14 \qquad V_R = 0.14$ (Beam, Grade 40 reinforcement)

In addition, a third special case should be considered when setting ϕ factors for reinforced concrete. This is

$\overline{R}/R_n = 1.01$ and $V_R = 0.125$ (Beams, high ρ)

Significant loading ratios range from about 0.25 to 1.50 for live to dead load and from 0 to 0.5 for wind to dead load ratios. Typical influence areas (equal to 2 times the tributary area) range from about 400 to 8000 ft^2 (37 - 743 m^2).

For comparison, Allen [B.7] reported $\overline{R}/R_n = 1.06$ to 1.25 depending on rate of loading and reinforcement ratio and $V_R = 0.09$ to 0.21, the higher values being for shallow members and poor workmanship. Ellingwood [B.6] suggested a representative value $\overline{R}/R_n = 1.12$ for moderately reinforced concrete members with Grade 40 reinforcement and $V_R = 0.13$ to 0.16, the higher values being for shallow members.

(v) Prestressed Concrete The coefficient of variation in the strength of prestressing tendons is quite small and because of this, the coefficient of variation in the moment capacity of prestressed concrete is considerably smaller than that of reinforced concrete [B.9]. In addition, the coefficient of variation of plant produced pretensioned concrete members is reduced by the better quality control for such members. Typical strength properties of prestressed beams are given in Table B.2. These can be summarized as:

Plant-produced, precast, pretensioned concrete beams:

Normal steel percentages,

$\overline{R}/R_n = 1.06 \qquad V_R = 0.08$

Maximum steel percentages normally allowed,

$\overline{R}/R_n = 1.04 \qquad V_R = 0.10$

Post-tensioned beams:

Normal steel percentages,

$\overline{R}/R_n = 1.04 \qquad V_R = 0.095$

Maximum steel percentages normally allowed,

$\overline{R}/R_n = 1.05 \qquad V_R = 0.14$

Typical live to dead load ratios for pretensioned beams range from 0.5 to 1.75. Wind load is seldom a design factor for plant produced pretensioned concrete members.

The loading ratios expected for post-tensioned beams should be the same as for comparable reinforced concrete beams.

(c) Reinforced Concrete Columns

(i) Typical Loading Ratios

Columns are subjected to combinations of axial load and moment ranging, in theory, from pure axial load to pure moment. The ratio of moment to load can be expressed using the eccentricity ratio $e/h = M/Ph$, where h is the overall depth of the column. This ratio equals zero for pure axial load and infinity for pure moment. The variability of columns tends to be greater for compression failures, initiated by crushing of the concrete, than for tension failures in which failure is initiated by yielding of the steel [B.8, B.11].

For columns supporting the roof of a concrete building, eccentricity ratios of 0.65 or so, corresponding to tension failures, are experienced. Live to dead load ratios or snow to dead load ratios between 0.25 and 0.75 are most typical. Based on ratios of axial load effects, wind to dead load ratios are generally less than 0.25. If, however, the ratios of wind load moments to dead load moments are considered, typical ratios for top story columns range from 0.25 to 5.0. In calibration studies for the tension failure range, top story columns were considered and the loading ratios and the variabilities were based on moments rather than axial loads. The influence area (four times the tributary area) of a top story column was assumed to be 1600 ft^2 (149 m^2) based on a 20 ft (6.1 m) bay size.

Columns supporting one to three floors plus a roof typically would fail in compression with eccentricity ratios of about 0.25. For columns supporting more than three floors typical e/h ratios approach 0.10. Typical live to dead load ratios range from 0.25 to

1.25 in terms of code live loads or 0.15 to 0.75 in terms of nominal or reduced live loads. The wind to dead load ratios, based on axial load effects, ranged from 0 to 0.50. In calibration studies for columns failing in compression, the variability along lines of constant e/h was considered and the calculations were based on an influence area of 4800 ft^2 (446 m^2) based on three floors each having 20 ft (6.1 m) bays.

(ii) Variability in Strength of Short Columns

The variability in strength of reinforced concrete tied columns has been studied by Ellingwood [B.6, B.8] and by Grant et. al. [B.11]. Ellingwood suggests somewhat higher variabilities than Grant for essentially two reasons:

(a) Ellingwood has assumed a coefficient of variation of model error of 0.061 while Grant took the coefficient of variation of the model error to be 0.046 as discussed in Section B.3.2;

(b) Ellingwood increased the data-based estimates of variability to account for uncertainties due to data sampling and observation errors, while Grant used representative average data-based estimates.

Despite these differences, the c.o.v. in short column capacity shown in Fig. 3 of Ref. B.11, Fig. 4 of Ref. B.8 and Fig. A.5 of Ref. B.6 are very close to one another, falling in the range 0.10 - 0.17, depending on eccentricity and reinforcement ratios. These references suggest the following means and c.o.v. in strength of short columns:

Compression Failures:	3000 psi concrete	$\overline{R}/R_n = 1.05$	$V_R = 0.16$
	5000 psi concrete	$\overline{R}/R_n = 0.95$	$V_R = 0.14$
Tension Failures:	$\overline{R}/R_n = 1.05$	$V_R = 0.12$	

The values for tension failure are close to those proposed for flexure in reinforced concrete beams, as expected.

(iii) Slender Columns

Slender reinforced concrete columns are relatively rare. In a group of 22000 columns surveyed in the late 1960's [B.12], 94 percent had h/ℓ less than or equal to 10, 5 percent has h/ℓ between 10 and 20 and the remaining 1 percent had h/ℓ between 20 and 30. In this sample the loading ratios and eccentricity ratios appeared to be similar to those for short columns.

The variability in the strength of hinged-end tied columns bent in single curvature was studied using a Monte Carlo technique. In this study a moment-curvature diagram was generated and used to compute the deflected shape of the column and the resulting maximum moments for any slenderness ratio, axial load and eccentricity of load. The results, expressed in terms of the ratio of theoretical strength to ACI code strength [B.10] based on $\phi = 1.0$ were used to select the representative distribution properties given below. These are for a 12 inch square column with $f'_c = 5000$ psi, $f_y = 60000$ psi and 2.2 percent reinforcement and $\ell/h = 20$.

Compression failure - $e/h = 0.1$, $\overline{R}/R_n = 1.10$, $V_R = 0.17$

Tension failure - $e/h = 0.7$, $\overline{R}/R_n = 0.95$, $V_R = 0.11$

B.3.4 Shear in Reinforced Concrete Beams

The words "shear strength of reinforced concrete" refer to a family of failure modes, some of which are related only in that shear forces are present. No completely satisfactory mechanical model exists for predicting shear strength and this complicated the study of the variability of the shear strength of reinforced concrete.

The study of the variability of shear strength of reinforced concrete was limited to two cases: beams with a/d greater than or equal to 2.5 with or without stirrups. The limitation on the type of beams considered corresponds to the limits on the normal design equations for shear in beams in Sections 11.3 and 11.5 of the ACI Code (See also ACI Section 11.8.1).

The shear strength variability was studied in Ref. B.13 by comparing theoretical strengths computed using the shear strength regression equation developed by Zsutty [B.14] to design strengths computed using Eq. 11-2, 11-3, and 11-17 of the ACI Code with ϕ set equal to 1.0. When compared to 62 tests of beams with stirrups and 96 beams without, all of which failed in shear and had a/d ratios from 2.3 to 4.9, the overall ratio of test to theoretical strength had a mean of 1.09 and a coefficient of variation of 12.5 percent. For beams without stirrups the mean and coefficient of variation were 1.12 and 8.7 percent, for beams with relatively low amounts of stirrups they were 1.085 and 13.7 percent and for beams with large amounts of stirrups they were 1.13 and 8.2 percent, respectively. In this study the model error was based on a mean of 1.09 and a coefficient of variation of 12.5 percent. The latter value was decreased to 11.5 percent to allow for in-test and in-specimen variations as explained in Section B.3.2.

The shear strength model used in the calculations was dependent on the longitudinal steel percentage in the beam while the ACI Code equations are essentially independent of steel percentage. This led to very low mean strength ratios for low longitudinal steel ratios, ρ. In the calibrations the following mean strength ratios and coefficients of variation were used. These are based on $\rho = 0.008$ which is close to the minimum expected for a beam susceptible to shear failures and on beams with no stirrups, minimum stirrups, and moderate stirrups:

No stirrups	$\overline{R}/R_n = 0.93$	$V_R = 0.21$
Minimum stirrups	$\overline{R}/R_n = 1.00$	$V_R = 0.19$
Moderate stirrups	$\overline{R}/R_n = 1.09$	$V_R = 0.17$

An earlier independent evaluation of variability in shear strength [B.6] using the truss analogy rather than Zsutty's equation led to estimates of V_R in the range 0.20 - 0.23, depending on the amount of web reinforcement. In that study, the model error was primarily based on data from several sources reported in Chapter 6 of Ref. B.15. Most of these data were obtained from beams in which a/d was less than 2.5. Analysis of these data, source by source, led to a c.o.v. in model error of 0.15 rather than the 0.115 cited above; this is sufficient to account for the difference in V_R.

B.3.5 Overall Summary

The means and variabilities of individual cross sections considered in the preceding sections are listed in Table B.2. For flexure, representative values are also suggested in this table.

B.4 Results of Calibrations for Concrete Members

B.4.1 Reinforced and Prestressed Concrete Beams in Flexure

Tables B.3 and B.4 present values of β for reinforced and prestressed concrete beams for various combinations of dead, live, wind and snow loads. In all cases the calculations have been carried out for a tributary area of 400 ft^2 (37.2 m^2) and a nominal total dead load of 100 psf (4.8 kN/m^2). Live load reduction factors were calculated using ANSI A58.1-1972 Section 3.5.1 for all levels of live load [B.16]. In addition, values of β are given for L/D ratios of 1.0 and 1.5 based on no live load reduction as would be applicable if L exceeds 100 psf. The true value of β generally would lie somewhere between the two values given.

B.4.2 Reinforced Concrete Columns

The values of β in Table B.5 have been computed for two cases. Compression failures are assumed to occur in columns supporting two floors and a roof with a total influence area of 4800 sq. ft. Tension failures are assumed to occur in columns supporting a roof,

Table B.3

Values of β for Flexure, Reinforced Concrete Beams

Factored loads taken as the larger of ACI Code Equations 9.1 and 9.2 with the load factor for snow taken as 1.7 and $\phi = 0.9$.

Live Load Reduction Factors from ANSI A58.1-1972 Sections 3.5.1 and 3.5.2 for a tributary area of 400 ft^2 (37 m^2).

Case	LLRF	$\overline{R}/R_n$	V_R	$\frac{Lo}{Dn}$	$\frac{Sn}{Dn}$	$\frac{Wn}{Dn}$	β
Beam - Grade 60 Low ρ	-	1.09	0.115	0	-	-	2.92
	Yes			0.50	-	-	3.09
	Yes			1.00	-	-	2.90
	No			1.00	-	-	3.74
	Yes			1.50	-	-	2.74
	No			1.50	-	-	3.67
Beam - Grade 60 Med ρ	-	1.05	0.11	0	-	-	2.80
	Yes			0.50	-	-	2.98
	Yes			1.00	-	-	2.78
	No			1.00	-	-	3.65
	Yes			1.50	-	-	2.62
	No			1.50	-	-	3.58
Beam - Grade 60 Very Low ρ	Yes	1.04	0.08	1.00	-	-	3.02
Beam - Grade 60 Very High ρ	Yes	1.01	0.12	1.00	-	-	2.61
Beam - Grade 40, Low ρ	Yes	1.18	0.14	1.00	-	-	2.93
Beam - Grade 40, Med ρ	Yes	1.14	0.14	1.00	-	-	2.77
Beam - Grade 60, Med ρ	-	1.05	0.11	-	0.50	-	3.33
	-			-	1.00	-	3.08
Beam - Grade 60, Med ρ	Yes	1.05	0.11	0.50	-	0.25	3.28*
				0.50	-	0.25	2.98**
				0.50	-	0.50	2.74*
				0.50	-	0.50	3.34**
				0.50	-	1.00	2.50*
				0.50	-	1.00	4.55**
				1.00	-	0.25	3.90*
				1.00	-	0.25	2.78**
				1.00	-	0.50	3.24*
				1.00	-	0.50	2.78**
				1.00	-	1.00	2.90*
				1.00	-	1.00	3.76**

*Based on Dead Load + Arbitrary Point-in-Time Live Load + Maximum Wind Load

**Based on Dead Load + Maximum Live Load + Arbitrary Point-in-Time Wind Load

Table B.4

Values of β for Flexure, Prestressed Concrete Beams

Factored loads taken as the larger of ACI Code Equations 9.1 and 9.2 with the load factor for snow taken as 1.7 and ϕ = 0.9.

Live Load Reduction Factors from ANSI A58.1-1972 Sections 3.5.1 and 3.5.2 for tributary area of 400 ft^2 (37 m^2).

Case	LLRF	$\bar{R}/R_n$	V_R	$\frac{Lo}{Dn}$	$\frac{Sn}{Dn}$	$\frac{Wn}{Dn}$	β
Cast-in-Place Post-tensioned Very Low ρ	Yes	1.02	0.061	1.00	-	-	3.05
Cast-in-Place Post-tensioned Low ρ	-	1.05	0.083	0	-	-	3.40
	Yes			0.50	-	-	3.47
	Yes			1.00	-	-	3.04
	No			1.00	-	-	3.98
	Yes			1.50	-	-	2.81
	No			1.50	-	-	3.82
Cast-in-Place Post-tensioned High ρ	Yes	1.03	0.111	1.00	-	-	2.68
Cast-in-Place Post-tensioned Very High ρ	Yes	1.05	0.144				2.40
Plant Precast, pre-tensioned Very Low ρ	Yes	1.06	0.057	1.00	-	-	3.72
Plant Precast, Pre-tensioned Low ρ	Yes	1.05	0.061	0	-	-	3.77
	Yes			0.5	-	-	4.04
	Yes			1.0	-	-	3.60
	Yes			1.5	-	-	3.27
Plant Precast, Pre-tensioned High ρ	Yes	1.06	0.083	1.0	-	-	3.39
Plant Precast, Pre-tensioned Very High ρ	Yes	1.04	0.097	1.0	-	-	3.09

Table B.5

Values of β for Columns

Factored loads taken as larger of ACI Code Equations 9.1 and 9.2 and ϕ = 0.7 or 0.75 for compressive failures and 0.7 or 0.9 for tension failures. Live load reduction factors from ANSI A58.1-1972 Sections 3.5.1 and 3.5.2. ($100\ ft^2 = 9.3\ m^2$)

Case	ℓ/h	f'_c	$A_I\ (ft^2)$	$\bar{R}/R_n$	V_R	ϕ	$\frac{Lo}{DN}$	$\frac{Wn}{Dn}$	β
Compression Failures, Tied Columns	Short	3	4800	1.05	0.16	0.7	0	-	2.98
							0.5	-	3.07
							1.0	-	3.41
	Short	5	4800	0.95	0.14	0.7	0	-	2.97
							0.5	-	3.09
							1.0	-	3.49
	20	5	4800	1.10	0.17	0.7	0.5	-	3.04
	Short	3	4800	1.05	0.16	0.7	0.5	0.5	2.74
	Short	5	4800	0.95	0.14	0.7	0.5	0.5	2.69
Tension failures, Tied Columns	Short	3 - 5	1600	1.05	0.12	0.7	0	-	3.85
						0.9	0	-	2.62
	Short	3 - 5	1600	1.05	0.12	0.7	0.5	-	4.25
						0.9	0.5	-	3.11
						0.7	1.0	-	4.23
						0.9	1.0	-	3.17
						0.7	0.5	1.0	3.40
						0.9	0.5	1.0	2.43
						0.7	0.5	5.0	2.76
						0.9	0.5	5.0	2.02
Compression failures, Spiral Columns	Short	3	4800	1.05	0.16	0.75	0	-	2.75
							0.5	-	2.85
							1.0	-	2.99
	Short	5	4800	0.95	0.14	0.75	0	-	2.69
							0.5	-	2.81
							1.0	-	3.24

Table B.6

Values of β for Shear

Factored loads taken as greater of ACI Code Equations 9.1 and 9.2 and ϕ = 0.85. Live load reduction factors from ANSI A58.1-1972 Section 3.5.1. Tributary area = 400 sq. ft. Concrete strength 4 ksi, stirrup strength 40 ksi. ($100\ ft^2 = 9.3\ m^2$; 1 ksi = 6.89 N/mm^2)

Case	$\rho_v f_y$	$\bar{R}/R_n$	V_R	$\frac{Lo}{Dn}$	$\frac{Wn}{Dn}$	β
No stirrups, ϕ = 0.85/2	0	0.93	0.21	0	-	3.09
Minimum stirrups, ϕ = 0.85	50	1.00	0.19	0	-	1.85
				0.5	-	1.99
				1.0	-	2.01
				0.5	0.5	1.97
Two times minimum stirrups, ϕ = 0.85	100	1.07	0.17	0	-	2.24
				0.5	-	2.39
				1.0	-	2.39
				0.5	0.5	2.32
Three times minimum stirrups, ϕ = 0.85	150	1.09	0.17	0	-	2.31
				0.5	-	2.45
				1.0	-	2.45
				0.5	0.5	2.38

only. In this case the influence area was taken as 1600 ft^2 (149 m^2). For tension failures the ϕ factor will fall between 0.7 and 0.9. Separate calculations have been carried out for each of these values. Values of β are also given in Table B.5 for spiral columns. For these cases the values of $\overline{R}/R_n$ and V_R were taken equal to those for tied columns but ϕ was set equal to 0.75.

B.4.3 Shear

Table B.6 gives values of β for shear in beams with and without stirrups. The value given for beams without stirrups was computed using $\phi = 0.85/2$ since ACI Code, Section 11.5.5, requires stirrups if V_u exceeds $0.5\ \phi\ V_c$.

B.5 Variability of Dead Load of Concrete Structures

A survey of literature indicated concrete densities ranging from 137 to 149 pcf for 3/4 inch aggregate concrete and from 141 to 155 pcf for 1 1/2 inch aggregate concrete. Because insufficient data was available to calculate a meaningful mean and standard deviation the mean density was computed as (137 + 155)/2 or 146 pcf. This was rounded off to the accepted value of 145 pcf. Similarly the standard deviation was computed as (155 - 137)/4 which gave a coefficient of variation of 3%.

Based on the dimensional variations reported in Ref. B.3 and the variability of the density, the ratio of mean to nominal dead load and its coefficient of variation were computed as 1.00 and 0.08 for a 6 inch slab, 1.00 and 0.07 for a slab and beam floor, and 1.04 and 0.04 for an 18 inch square column. For a beam, slab and column structure the values were 1.00 and 0.06. The average weight per square foot was 100 psf.

The dead load of a building includes both the self weight of the structure and superimposed dead loads. As an extreme case, the superimposed dead load was arbitrarily assumed to have a nominal value of 40 psf with a mean $\overline{D}/D_n = 1.10$ and the high coefficient of variation of 0.15. Using these values the variability of the total dead load can be computed to have $\overline{D}/D_n = 1.03$ and $V_D = 0.059$. To this must be added the variabilities in the analysis factor c and the load model B. If each of these is assumed to have a mean 1.0 and V = 0.05, the overall variability of dead load is found to be

$$\overline{D}/D_n = 1.03 \text{ and } V_D = 0.093.$$

References to Appendix B

B.1 Mirza, S. A., Hatzinikolas, M., MacGregor, J.G., "Statistical Descriptions of the Strength of Concrete," Journal of the Structural Division, ASCE, Vol. 105, No. ST6, June 1979, pp. 1021 - 1037.

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APPENDIX C - DATA FOR METAL MEMBERS AND COMPONENTS

C.1 Introduction

This appendix will present basic data on metal members and components for use in developing probability-based common load factors for metal, concrete (reinforced and prestressed), timber and masonry structures. The kind of data presented here will consist of expressions for the analytical model used in the design equations and the mean value and the coefficient of variation of the resistance. For metal structures the resistance can usually be expressed by the simple relationship

$$R = R_n\,(PMF) \tag{C.1}$$

where R_n is the nominal resistance based on the analytical model accepted by the structural engineering profession for the design of the particular element under consideration for minimum specified material properties and "Handbook" sectional properties. R, P, M, and F are random parameters denoting the resistance (R), the accuracy of the model ("professional" factor, P), the material properties (M) and the sectional properties ("fabrication" factor, F). The parameters P, M and F are typically ratios of actual-to-nominal values. The statistical properties of interest are the mean resistance:

$$\overline{R} = R_n\,\overline{P}\,\overline{M}\,\overline{F} \tag{C.2}$$

where the property with a bar defines the mean, and the coefficient of variation (C.O.V.), which is defined by

$$V_R = \sqrt{V_P^2 + V_M^2 + V_F^2} \tag{C.3}$$

C.2 Sources of Data for Metal Structural Elements

The data used herein were specifically produced for projects which had as their aim the development of Load and Resistance Factor Design criteria for three different kinds of metal structures:

1. Steel structures produced from hot-rolled shapes, plates or bars*.
2. Steel structures produced from cold-formed members**.
3. Aluminum structures***.

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*The design of these structures is governed by the "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings", American Institute of Steel Construction. Research for developing the data was sponsored by the American Iron and Steel Institute (AISI) at Washington University in St. Louis (1970-1975).

**The design of these structures is governed by the "Specification for the Design of Cold-Formed Steel Structural Members", American Iron and Steel Institute. Research was sponsored by AISI at the University of Missouri at Rolla and at Washington University (1976-1979).

***The design of these structures is governed by the "Specifications for Aluminum Structures," The Aluminum Association. Research was sponsored by the Aluminum Association at Washington University (1978-1980).

The data was developed by evaluating information obtained from manufacturers, from catalogs and Handbooks, and from original research publications. The information is presented in part in the open technical literature and in part in research reports. The following sections of this Appendix present a summary of the data used in the study to develop load factors for the ANSI-A58 Standard. Obviously it was not possible to use all of the available data. Only representative groupings of data are used to obtain background for the load-factor work. The remainder of the information is, of course, very relevant to the specification writing bodies of the individual material groups as they develop the various ϕ-factors. These data are given in great detail in the original references.

C.3 Steel Structures Produced From Hot-Rolled Elements

The resistance statistics for hot-rolled elements are published in Refs. C.1 through C.9 and in Ref. C.11. For steel structures it was found that the Handbook sectional properties were equal to the mean values, with a C.O.V. of 0.05, and thus,

$\overline{F} = 1.00$ and $V_F = 0.05$

is used throughout, except for fillet welds for which $V_F = 0.15$ was used to reflect the variability of the weld throat area.

The material property statistics are summarized in Table C.1. It should be noted that the yield stress values were all adjusted for the static level of loading, recognizing that most loads on structures are static loads. However, for load combinations involving wind it was assumed that the strength given by $\overline{M}$ in Table C.1 is multiplied by 1.1 to account for the rate of loading, as explained in more detail in the main part of the report.

Table C.2 lists the modeling statistics as obtained from the literature for tension members, beams, connectors, plate girders, composite beams and beam-columns. No variability was assumed for tension members since the model is the same as the tension coupon (the variability is all in the material and the cross sectional properties, M and F). No modeling error was assumed for connectors. Underlying this assumption is the inherent ductility of the connection and the validity of the Lower Bound Theorem of Plasticity. For the other members the analytical model is the ultimate strength of the element or the member. For compact simple beams this is the plastic moment, for continuous beams it is the plastic mechanism, for laterally unsupported beams it is the elastic or the inelastic buckling load, and for compact composite beams it is the plastic capacity of the composite

Table C.1

Material Property Statistics for Hot-Rolled Steel Elements

Property	Ref.	Mean	$\overline{M}$	V_M
Static Yield Stress, Flanges	C8	$1.05\ F_y$	1.05	0.10
Static Yield Stress, Webs	C8	$1.10\ F_y$	1.10	0.11
Moduli of Elasticity	C8	E or G	1.00	0.06
Static Yield Stress in Shear	C8	$1.11\ F_y/\sqrt{3}$	1.11	0.10
Poisson's Ratio	C8	0.3	1.00	0.03
Tensile Strength of Steel	C9	$1.10\ F_u$	1.10	0.11
Tensile Strength of Weld, σ_u/F_{EXX}	C6	$1.05\ F_{EXX}$	1.05	0.04
Shear Stress of Weld, τ_u/σ_u	C6	$0.84\ \sigma_u$	0.84	0.10
Tensile Strength of HSS Bolts, A325	C6	$1.20\ F_u$	1.20	0.07
Tensile Strength of HSS Bolts, A490	C6	$1.07\ F_u$	1.07	0.02
Shear Strength of HSS Bolts,	C6	$0.625\ \sigma_u$	0.625	0.05

F_y : Specified yield stress

F_u : Specified tensile strength

σ_u : Tensile strength

τ_u : Shear strength

F_{EXX} : Specified tensile strength of weld metal

Table C.2

Modeling Statistics for Hot-Rolled Steel Elements

Type Element	Model	Ref.	$\overline{P}$	V_P
Tension Members	A_nF_y and A_nF_u	C9	1.00	0
Compact W-Beams				
uniform moment	M_p	C2	1.02	0.06
continuous	Mechanism	C2	1.06	0.07
Elastic W-beams, LTB	S_xF_{cr}	C2	1.03	0.09
Inelastic W-beams, LTB	Straight line transition	C2	1.06	0.09
Connectors (Welds, HSS Bolts)	-	C6	1.00	0
Beam-Columns	Interaction Equations	C3	1.02	0.10
Plate Girders in Flexure	M_u	C4	1.03	0.05
Plate Girders in Shear	V_u	C4	1.03	0.11
Compact Composite Beams	M_u	C5	0.99	0.08

A_n : Net area

M_p : Plastic moment

S_x : Elastic section modulus

F_{cr} : Critical stress

M_u : Ultimate moment capacity

V_u : Ultimate shear capacity

LTB : Lateral-torsional buckling

section. The ultimate strength of plate girders is based on the Basler theory, and for beam-columns it is defined by the SSRC interaction equation [Ref. C.10]. The ratio $\overline{P}$ is thus the mean ratio of test-to-prediction.

Table C.3 summarizes the information on steel members, giving the mean resistance ratio $\overline{R}/R_n$ and the corresponding C.O.V., V_R. For high-strength steel bolts in shear the effect of joint length is included. This effect was not included in Ref. C.6, but it is included here as a result of the realization that the AISC Specification allowable stresses are based on a 50 in long connection [C.12].

Table C.4 presents the resistance statistics for centrally loaded pinned-end columns. These statistics apply for steel columns of SSRC Column Curve 2 [C.10] which represents a subset of the steel compression members used in practice. It should be pointed out that in the development of the ϕ-factors the SSRC column curves 1 and 3 must also be analyzed. The mean column strength $\overline{\sigma_{cr}/F_{ys}}$ as given in column 2 of Table C.4 [from Ref. C.11] is approximated by SSRC Column Curve 2 by the relationships

$$\overline{\sigma_{cr}/F_{ys}} = \begin{cases} 1.0 \quad \text{for} \quad \lambda \leq 0.15 \\ 1.035 - 0.202\,\lambda - 0.222\,\lambda^2 \quad \text{for } 0.15 \leq \lambda \leq 0.10 \\ -0.111 + 0.636/\lambda + 0.087/\lambda^2 \quad \text{for } 1.0 \leq \lambda \leq 2.0 \end{cases} \qquad \text{(C.4)}$$

where σ_{cr} is the critical stress, P_{cr}/A

P_{cr} is the critical load

F_{ys} is the static yield stress

and

$$\lambda = \frac{KL}{r} \cdot \frac{1}{\pi}\sqrt{\frac{F_{ys}}{E}} \qquad \text{(C.5)}$$

In Eq. C.5, KL/r is the governing effective slenderness ratio and E is the modulus of elasticity.

The mean resistance of the column is thus

$$\overline{R} = \overline{\sigma_{cr}/F_{ys}}\ \overline{P}\ \overline{M}\ \overline{F} \qquad \text{(C.6)}$$

C.4 Steel Structures Produced From Cold Formed Elements

The resistance statistics given in Table C.5 were taken from unpublished research reports issued by the Civil Engineering Department of the University of Missouri at Rolla in January 1979 and authored by W.-W. Yu, T.V. Galambos and T.-N. Rang. The nominal resistance in each case is the allowable resistance according to the 1968 AISI Specification for cold-formed structures times the factor of safety. The factor of safety is equal to

Table C.3

Resistance Statistics for Hot-Rolled Steel Elements

Type Element	$\overline{P}$	V_p	$\overline{M}$	V_M	$\overline{F}$	V_F	$\overline{R}_y/R_n$	V_R
Tension member, yield	1.00	0	1.05	0.10	1.00	0.05	1.05	0.1
Tension member, ultimate	1.00	0	1.10	0.10	1.00	0.05	1.10	0.1
Compact beam, uniform moment	1.02	0.06	1.05	0.10	1.00	0.05	1.07	0.1
Compact beam, continuous	1.06	0.07	1.05	0.10	1.00	0.05	1.11	0.1
Elastic beam, LTB	1.03	0.09	1.00	0.06	1.00	0.05	1.03	0.1
Inelastic beam, LTB	1.06	0.09	1.05	0.10	1.00	0.05	1.11	0.1
Beam-Columns	1.02	0.10	1.05	0.10	1.00	0.05	1.07	0.1
Plate-girders in flexure	1.03	0.05	1.05	0.10	1.00	0.05	1.08	0.1
Plate-girders in shear	1.03	0.11	1.11	0.10	1.00	0.05	1.14	0.1
Compact composite beams	0.99	0.08	1.05	0.10	1.00	0.05	1.04	0.1
Fillet welds	1.00	0	0.88*	0.11**	1.00	0.05	0.88	0.1
ASS bolts in tension, A325	1.00	0	1.20	0.07	1.00	0.05	1.20	0.0
ASS bolts in tension, A490	1.00	0	1.07	0.02	1.00	0.05	1.07	0.0
HSS bolts in shear, A325	0.79[a)]	0	0.75+	0.09++	1.00	0.05	0.60	0.1
HSS bolts in shear, A490	0.78[a)]	0	0.67x	0.05xx	1.00	0.05	0.52	0.0

a) effect of joint length included. For A325 bolts this is 1/1.26 = 0.79 and for A490 bol is 1/1.28 = 0.78 (Ref. C.12)

* 1.05 x 0.84

** $\sqrt{0.04^2 + 0.10^2}$

\+ 1.20 x 0.625

++ $\sqrt{0.07^2 + 0.05^2}$

x 1.07 x 0.625

xx $\sqrt{0.02^2 + 0.05^2}$

See Table C.1

Table C.4

Resistance Statistics for Hot-Rolled Steel Columns

λ	$\overline{\sigma_{cr}}/F_{ys}$ Theory	V_{theory}	$\overline{P}$	V_p	$\overline{M}$	V_M	$\overline{F}$	V_F	$\overline{P}\ \overline{M}\ \overline{F}$	V_R
0.3	0.936	0.02	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.12
0.5	0.849	0.04	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.13
0.7	0.749	0.06	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.14
0.9	0.646	0.08	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.15
1.1	0.539	0.08	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.15
1.3	0.439	0.07	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.14
1.5	0.355	0.06	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.14
1.7	0.290	0.06	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.14
1.9	0.239	0.05	1.03	0.05	1.05	0.05	1.00	0.05	1.08	0.13

Table C.5

Resistance Statistics for Cold-Formed Steel Members

Type Member	R_n	$\overline{R}/R_n$	V_R
1. Tension Member	$A_n F_y$	1.10	0.11
2. Braced Beams in Flexure, Flanges Stiffened	$S_{eff} F_y$	1.17	0.17
3. Braced Beams in Flexure, Flanges Unstiffened	$S_{eff} F_y$	1.60	0.28
4. Laterally Unbraced Beams	$S_x F_{cr}$	1.15	0.17
5. Columns, Flexural Buckling, Elastic	$\pi^2 EI/L^2$	0.97	0.09
6. Columns, Flexural Buckling, Inelastic, compact	*	1.20	0.13
7. Columns, Flexural Buckling, Inelastic, stiffened	**	1.07	0.20
8. Columns, Flexural Buckling, Inelastic, unstiffened	**	1.68	0.26
9. Columns, Flexural Buckling, Inelastic, cold work	***	1.21	0.14
10. Columns, Torsional-Flexural Buckling, Elastic	+	1.11	0.13
11. Columns, Torsional-Flexural Buckling, Inelastic	++	1.32	0.18

A_n = Net section

S_{eff} = Effective elastic section modulus

F_{cr} = Lateral-torsional buckling stress from AISI Specification

S_x = Elastic Section Modulus

I = Moment of inertia; Q = from factor

* $AF_y(1 - \lambda^2/4)$; $\lambda = (L/r)(1/\pi)\sqrt{F_y/E}$

** $AF_yQ(1 - \lambda^2/4)$; $\lambda = (L/r)(1/\pi)\sqrt{Q F_y/E}$

*** $AF_{ya}(1 - \lambda^2/4)$; $\lambda = (L/r)(1/\pi)\sqrt{Q F_{ya}/E}$

\+ Elastic critical load from AISI Specification

++ Inelastic critical load from AISI Specification

5/3 for lines 1, 2, 3 and 4 in Table C.5, it is equal to 23/12 for lines 5, 7, 8, 10 and 11, and is equal to

$$F.S. = 5/3 + (\lambda/8\sqrt{2})\ (3 - \lambda^2/2) \qquad (C.7)$$

for lines 6 and 9.

C.5 Aluminum Structures

The resistance statistics in Table C.6 were taken from an unpublished research report issued by the Civil Engineering Department of Washington University in St. Louis, Mo., in May 1979, authored by T.V. Galambos. The nominal resistance in each entry is the allowable resistance according to the Specifications of the Aluminum Association, multiplied by the indicated factor of safety.

C.6 Calibration to Existing Codes for Metal Structures

C.6.1 General Definitions

Calibration is performed by using the method described in Chapter 2 of this report. The calibration process determines a reliability index β, given the mean and the coefficient of variation of a member designed according to a current code and the applicable statistical information concerning the loads. The theory and the methodology of the operation is given in the body of this report; here only the specific details as they relate to the metal structures are explained.

The mean resistance is

$$\bar{R} = (\bar{R}/R_n)\ R_n \qquad (C.8)$$

where R_n is the nominal resistance determined according to accepted theoretical models which may, or may not, also be the same model which is used in the existing structural specification. $\bar{R}/R_n$ is the information tabulated in Tables C.3 through C.6. R_n is defined by

$$R_n = (r)\ (FS)\ R_c \qquad (C.9)$$

where r = 1 if the nominal and the code resistance are based on the same analytical model, and

$$r = \frac{R_n}{(FS)\ R_{ns}} \qquad (C.10)$$

if they are not. In Eq. C.9, FS is the code-specified factor of safety or load factor, and R_{nc} is the code specified resistance. Combining Eqs. C.9 and C.8, we obtain

$$\bar{R} = (\bar{R}/R_n)\ r\ (FS)\ R_{nc} \qquad (C.11)$$

Table C.6

Resistance Statistics for Aluminum Structures

Type Member	F.S.	$\bar{R}/R_n$	V_R
1. Tension members, limit state yield	1.65	1.10	0.08
2. Tension members, limit state ultimate	1.95	1.10	0.08
3. Beams, limit state yield	1.65	1.10	0.08
4. Beams, limit state lateral buckling	1.65	1.03	0.13
5. Beams, limit state inelastic local buckling	1.65	1.00	0.09
6. Columns, limit state yield	1.82	1.10	0.08
7. Columns, limit state local buckling	1.95	1.0	0.09
8. Columns, limit state overall buckling, $\lambda = 1$	1.95	0.92	0.14
9. Columns, limit state overall buckling, $\lambda = 1.6$	1.95	0.87	0.13
10. Columns, limit state overall buckling, $\lambda = 2.0$	1.95	0.91	0.14

$\lambda = (L/r)(1/\pi)\sqrt{F_y/E}$

where

$$R_{nc} = D_n + L_n \quad \text{(C.12)}$$

$$R_{nc} = (D_n + L_n + W_n)(3/4) \quad \text{(C.13)}$$

$$R_{nc} = D_n + S_n \quad \text{(C.14)}$$

depending on the particular load combination used. Subscripts n define the nominal, code specified, load effects (moments, shears, axial forces, etc.), and

D = dead load

L = load due to occupancy

S = snow load

W = wind load (or, alternately, earthquake load)

The 3/4 factor in Eq. C.13 signifies the allowable increase of one-third for wind or earthquake loads which is permitted by each of the three codes under consideration here. The nominal live load effect L_n includes the live load reduction factor as per ANSI Standard A.58.1-1972.

C.6.2 <u>Discussion of the Results Presented in Table C.7</u>

Tables C.7.1 through C.7.8 present the reliability index values (β's) which were computed for the data given in Tables C.3 through C.6. The combination and ranges of loads used in computing β's were chosen to cover a broad range including typical values encountered in metal structures. The purpose of calculating the β's was to arrive at an overview of what values of the reliability index underlie present design practice so as to aid in the selection of target β's. Following is a brief list of observations from this exercise.

1) There is a considerable spread of β's for the various applications of metal structures, varying from a low of β = 1.1 (Table C.7.2, W-D) to a high of β = 7.0 (Table C.7.8). One of the purposes of choosing one (or several, as it will appear later) target β's is to reduce this spread in reliability.

2) It is evident from all of the Tables C.7 that β falls off as the ratios L_o/D_n, S_n/D_n or W_n/D_n increase. This is to be expected because (a) the C.O.V.'s of the live, snow, and wind loads are all greater than the C.O.V. of the dead loads, and (b) all three of the metal codes considered here have only one factor of safety or load factor for each load combination set. Therefore β decreases as the dead load component of the total load effect decreases. This is especially so for cold-formed steel and aluminum structures.

Table C.7.1

Safety Indices for Current Design

a) Tension Members, AISC Specification Part 1. Section 1.5.1.1

Limit State: yield; R_m/R_n = 1.05; V_R = 0.11; r = 1

F.S. = 5/3

Tributary area: 500 ft^2 (46 m^2)

Loading	L_o/D_n	W_n/D_n	S_n/D_n	β_{Yield}	$\beta_{Ultimate}$
D + L	0.5	-	-	3.2	4.6
	1	-	-	2.7	3.9
	2	-	-	2.5	3.4
	3	-	-	2.4	3.3
	4	-	-	2.4	3.2
	5	-	-	2.4	3.2
D + W*	-	1	-	1.8	2.8
	-	3	-	1.7	2.4
	-	5	-	1.6	2.3
	-	7	-	1.6	2.3
	-	10	-	1.6	2.2
D + S	-	-	1	3.0	3.5
	-	-	2	2.8	3.3
	-	-	3	2.7	3.2
	-	-	4	2.6	3.1
	-	-	5	2.6	3.0

Limit State: Ultimate; R_m/R_n = 1.10; V_R = 0.11; r = 1

F.S. = 2

Tributary Area: 500 ft^2 (46 m^2)

* β was determined without strain-rate increase on the yield stress.

Table C.7.2 Continued

a) Compact Simple Beams, Uniform Moment

AISC Specification Section 1.5.1.4.1

$\overline{R}/R_n = 1.07$; $V_R = 0.13$; $r = 1$; F.S. = 1.70

b) Continuous Beams, Plastic Design

AISC Specification Part 2

$R_m/R_n = 1.11$; $V_R = 0.13$; $r = 1$; F.S. = 1.70

Loading	Tributary Area	L_o/D	W_n/D_n	S_n/D_n	β_a	β_b
D + L	200 ft^2 ($19\ m^2$)	0.5	-	-	3.0	3.2
		1	-	-	2.6	2.7
		1.5	-	-	2.3	2.5
		2	-	-	2.2	2.3
		3	-	-	2.0	2.2
	1000 ft^2 ($93\ m^2$)	0.5	-	-	3.1	3.3
		1	-	-	3.1	3.3
		1.5	-	-	3.1	3.3
		2	-	-	3.1	3.3
		3	-	-	3.1	3.2
D + S	-	-	-	1	3.0	3.1
		-	-	2	2.8	2.9
		-	-	3	2.7	2.8
		-	-	4	2.7	2.8
		-	-	5	2.6	2.7
W - D*	-	-	1.2	-	1.1	1.2
		-	1.5	-	1.2	1.3
		-	2	-	1.4	1.4
		-	3	-	1.5	1.6
		-	5	-	1.5	1.7

* β was determined without strain-rate increase on the yield stress

Table C.7.3 Continued

a) <u>Compact Simple Beams, Uniform Moment</u>

AISC Specification Part 2

$\overline{R}/R_n = 1.07$; $V_R = 0.13$; $r = 1$; L.F. = 1.30

Strain rate effect included: A; multiply $\overline{R}/R_n$ by 1.10

Strain rate effect excluded: B

Loading	L_o/D_n	W_n/D_n	β_A	β_B
$D + L_{apt} + W$	0.5	0.5	3.2	2.7
		1	2.8	2.4
		1.5	2.5	2.2
		2	2.4	2.1
		3	2.2	-
	1.0	0.5	3.6	3.2
		1	3.1	2.8
		1.5	2.8	2.5
		2	2.6	2.3
		3	2.4	-
	1.5	0.5	3.8	3.3
		1	3.4	3.0
		1.5	3.1	2.7
		2	2.8	2.5
		3	2.6	-
$D + L + W_{apt}$	0.5	0.5	3.9	3.4
		1.0	5.2	4.7
	1.0	0.5	3.1	2.7
		1.0	4.0	3.6
		1.5	4.8	4.4
	1.5	0.5	2.7	2.3
		1.0	3.4	3.0
		1.5	4.0	3.7

Loading	W_n/D_n	β_A
$D + W$	0.5	2.5
	1.0	2.3
	1.5	2.2
	2.0	2.1
	2.5	2.1
	3.0	2.0

Table C.7.4 Continued

Centrally Loaded Columns AISC Specification Section 1.5.1.3 (1 ft^2 = 0.093 m^2)

A_T	λ	$(\bar{R}/R_n)$ r (FS)	V_R	L_o/D_n	β	L_o/D_n	β	L_o/D_n	β	L_o/D_n	β
2500 ft^2	0.3	1.81	0.12	-	-	1	3.3	-	-	-	
	0.5	1.76	0.13	-	-	1	3.1	-	-	-	
	0.7	1.70	0.14	0.5	2.6	1	2.8	1.5	2.9	2	2.9
	0.9	1.64	0.15	-	-	1	2.5	-	-	-	
	1.1	1.59	0.15	-	-	1	2.3	-	-	-	
	1.3	1.57	0.14	-	-	1	2.3	-	-	-	
	1.5	1.66	0.14	-	-	1	2.6	-	-	-	
	1.7	1.74	0.14	-	-	1	2.9	-	-	-	
	1.9	1.79	0.13	-	-	1	3.2	-	-	-	
1250 ft^2	0.7	1.70	0.14	1	2.3	-	-	-	-	-	

A_T	λ	L_o/D_n	W_n/D_n	β^*
2500 ft^2	0.7	1	0.5	2.4
	0.7	1	1	2.2
	0.7	1	1.5	2.0
	0.7	1	2	1.9
	0.3	0.5	2	1.9
	0.3	1	1	2.6
	0.3	2	0.5	3.8
	0.5	0.5	2	1.8
		1	1	2.4
		2	0.5	3.5
		0.5	0.5	2.1
		1	1	2.4
		1.5	1.5	2.5
		2	2	2.5

A_T	λ	L_o/D_n	W_n/D_n	β^*
5000 ft^2	0.3	0.5	2	1.9
	0.5	0.5	2	1.8
	0.7	0.5	2	1.6
	0.9	0.5	2	1.5
	1.1	0.5	2	1.4
	1.3	0.5	2	1.4
	1.5	0.5	2	1.5
	1.7	0.5	2	1.7
	1.9	0.5	2	1.9

* β was determined without strain-rate increase on the yield stress.

Table C.7.5 Continued

Cold Formed Steel Members, AISI Specification. $r = 1.0^{*}$

Item in Table C.5	$\overline{R}/R_n$	F.S.	V_r	L_n/D_n **	β	L_n/D_n **	β	L_n/D_n **	β
1	1.10	5/3	0.11	5	2.5	8	2.4	10	2.4
2	1.17	5/3	0.17	5	2.4	8	2.4	10	2.3
3	1.60	5/3	0.28	5	2.8	8	2.7	10	2.7
4	1.15	5/3	0.17	5	2.4	8	2.3	10	2.3
5	0.97	23/12	0.09	5	2.7+	8	2.6+	10	2.5+
7	1.07	23/12	0.20	5	2.5+	8	–	10	–
8	1.68	23/12	0.26	5	3.5+	8	–	10	–
10	1.11	23/12	0.13	5	3.0+	8	–	10	–
11	1.32	23/12	0.18	5	3.4+	8	–	10	–

+Determined from lognormal model, $\beta = \dfrac{\ell n\ \overline{R}/\overline{Q}}{\sqrt{V_R^2 + V_Q^2}}$

*Model for resistance is same as model used as basis for the specification

**Based on the stipulation that $L_n = \overline{L}$, where L_n is the code live load reduced due to tributary area and $\overline{L}$ is the mean live load.

Table C.7.6 Continued

Aluminum Members, Aluminum Association Specifications, $r = 1.0^{*}$

Item in Table C.6	L_n/D_n **	β	L_n/D_n **	β	L_n/D_n	β
1	5	2.5	8	2.4	10	2.4
2	5	3.1	8	3.0	10	3.0
3	5	2.5	8	2.4	10	2.4
4	5	2.1	8	2.1	10	2.0
5	5	2.2+	8	2.1+	10	2.0+
6	5	2.9	8	2.8	10	2.8
7	5	2.8	8	2.7	10	2.6
8	5	2.3+	8	2.2+	10	2.2+
9	5	2.1	8	2.0	10	2.0
10	5	2.2	8	2.2	10	2.1

+Determined from lognormal model

*Model for resistance is same as model used as basis for the specification

**Based on the stipulation that $L_n = \bar{L}$

Table C.7.7 Continued

Fillet welds, AISC Specification Section 1.5.3

$\overline{R} = 0.88\ A_W\ F_{EXX}$ (Table C.3); $R_n = 0.3\ A_W\ F_{EXX}$; $\overline{R}/R_n = 0.88/0.30 = 2.93$;

$V_R = 0.18$; Tributary Area: 500 ft^2 (46 m^2)

L_n/D_n	W_n/D_n	β
0	-	5.0
0.5	-	4.8
1	-	4.3
2	-	3.9
5	-	3.7
10	-	3.7
2	0.5	4.9
2	1	4.6
2	2	4.0
2	3	3.6
2	5	3.3

Table C.7.8 Continued

H.S.S. Bolts, AISC Specification Section 1.5.2 (1 ksi = 6.9 N/mm^2)

	Type Bolt	Type Load	AISC Spec.	$\bar{R}$	V_R	F_u	F_t	$\bar{R}/R_n$	L_o/D_n	β	L_o/D_n	β
1	A325	Tension	1969	$1.20\ A_T\ F_u$	0.09	120 ksi	40 ksi	2.70	1	5.0	2	4.4
2	A325	Tension	1978	$1.20\ A_T\ F_u$	0.09	120 ksi	44 ksi	2.45	1	4.6	2	4.0
3	A490	Tension	1969	$1.07\ A_T\ F_u$	0.05	150 ksi	54 ksi	2.22	1	4.4	2	3.8
4	A490	Tension	1978	$1.07\ A_T\ F_u$	0.05	150 ksi	54 ksi	2.22	1	4.4	2	3.8
5	A325	Shear	1969	$0.60\ A_s\ F_u$	0.10	120 ksi	22 ksi	3.27	1	7.0*	2	6.1*
6	A325	Shear	1978	$0.60\ A_s\ F_u$	0.10	120 ksi	30 ksi	2.40	1	5.0*	2	4.4*
7	A490	Shear	1969	$0.52\ A_s\ F_u$	0.07	150 ksi	32 ksi	2.44	1	5.8*	2	4.9*
8	A490	Shear	1978	$0.52\ A_s\ F_u$	0.07	150 ksi	40 ksi	1.95	1	4.1*	2	3.5*

A_T = Tension area

A_s = Shear area

F_u = Tensile strength of bolt

A_g = Gross area

$A_t = 0.75\ A_g$

F_t = AISC allowable stress

$R_n = A_g\ F_t$

$A_s = A_g$ (shear plane through shank)

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* β determined using lognormal model

Table C.8

Representative β's, Current Design (1 ft^2 = 0.093 m^2)

Member or Element	Code	L_o/D_n	S_n/D_n	W_n/D_n	β
Tension Member, Yield	AISC	2	0	0	2.5
Tension Member, Ultimate	AISC	2	0	0	3.4
Compact Simple Beam, A_T = 1000 ft^2	AISC	2	0	0	3.1
Compact Simple Beam, A_T = 1000 ft^2	AISC	0	2	0	2.8
Compact Simple Beam, A_T = 1000 ft^2	AISC	0.5	0	2	2.4
Column, A_t = 2500 ft^2, λ = 0.5	AISC	1	0	0	3.1
Column, A_T = 2500 ft^2, λ = 0.7	AISC	1	0	0	2.8
Column, A_T = 2500 ft^2, λ = 0.7	AISC	1	0	1	2.2
C.F. Beams, Braced, Stiffened Flanges	AISI	5	0	0	2.4
C.F. Columns, Stiffened Flanges	AISI	5	0	0	2.5
Aluminum Beams	AA	5	0	0	2.5
Aluminum Columns	AA	5	0	0	2.8
Fillet Welds	AISC	2	0	0	3.9
A325 Bolts, Tension	AISC	2	0	0	4.0
A325 Bolts, Shear	AISC	2	0	0	4.4

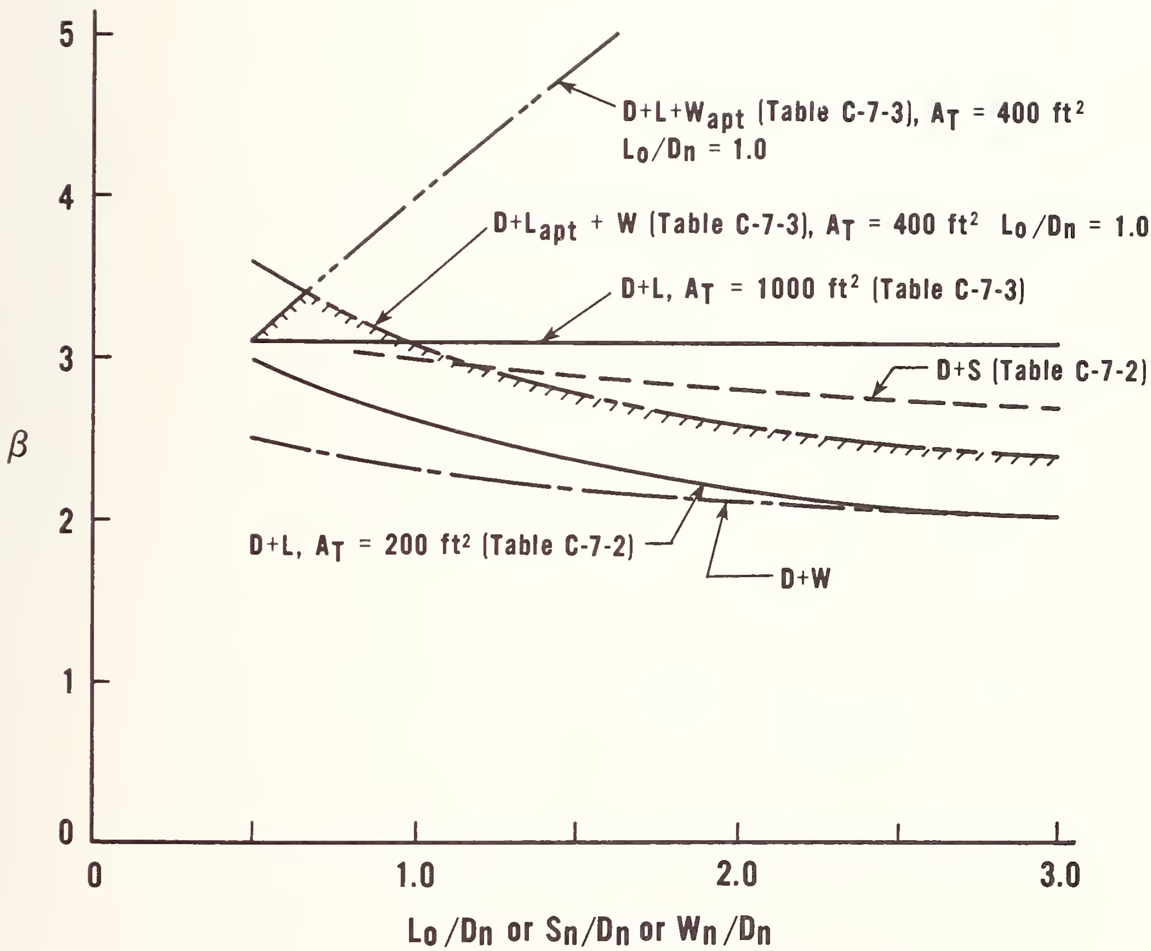

Figure C.1 - Reliability Index for Tension Members (AISC Specifications)

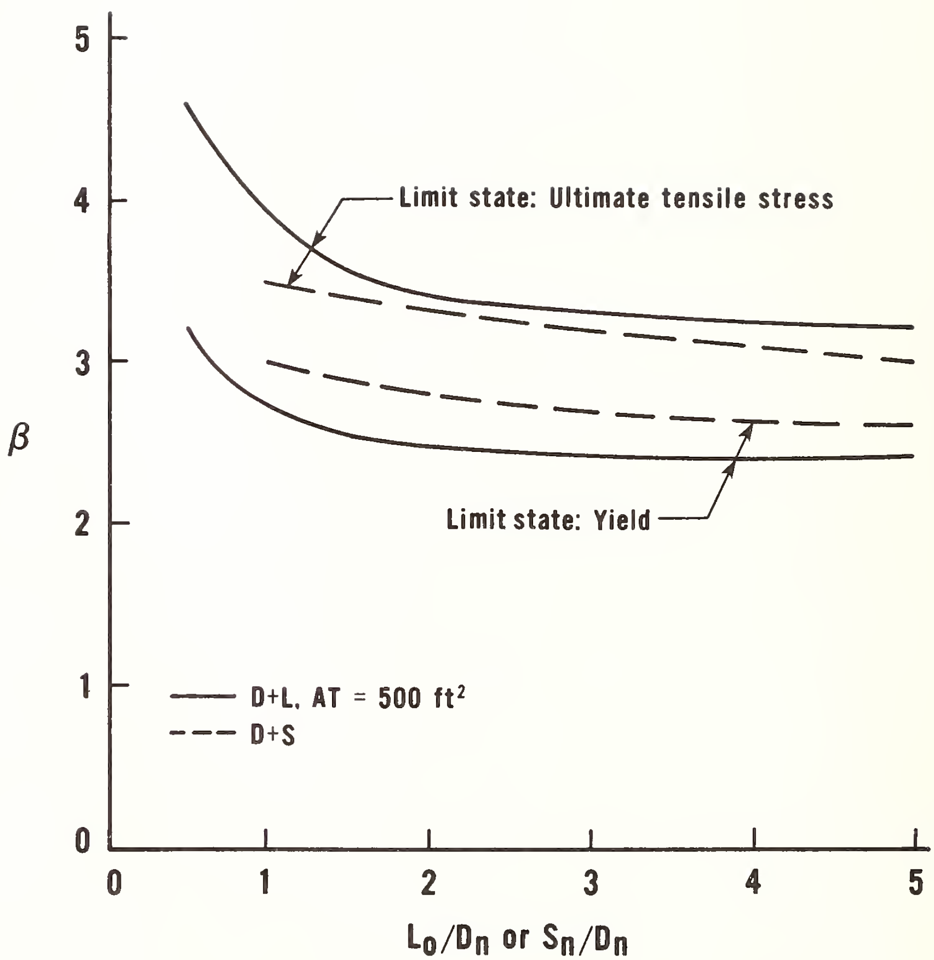

Figure C.2 - Reliability Index for Flexural Members (AISC Specifications)

3) In the case of tension members the specifications require the satisfaction of two limit states: yield and ultimate tensile strength. The reliability for these two limit states is different, as can be seen by the following examples:

AISC Specification: $\beta_Y = 2.7$, $\beta_u = 3.9$ (Table C.7.1, $L_o/D_n = 1.0$)

Aluminum Specification: $\beta_Y = 2.5$, $\beta_u = 3.1$ (Table C.7.6, $L_o/D_n = 5.0$)

Thus it is evident that a higher value of the target β should be set for the ultimate limit state (see also Fig. C.1).

4) Tributary area has an effect on β (see Fig. C.2) because of the differences between the live load reduction models in ANSI Standard A58.1-1972 and the mean live load intensity. The two curves in Fig. C.2 illustrate the expected extremes of this effect (i.e., A_T = 200 ft and 1000 ft^2 (19 and 93 m^2) where A_T is the tributary area).

5) In the wind-load calibration in Table C.7.3 (steel beams) allowance was made for the increased yield strength due to the rate of loading. It was assumed that for steel structures the rate of straining under wind gusts is equal to the strain rate during testing steel coupons in a testing machine. This increases $\overline{M}$ to 1.1 $\overline{M}$ [C.8]. The strain rate effect increases β (compare β_A and β_B in Table C.7.3). For example, for $L_o/D_n = 1$ and $W_n/D_n = 1$, β changes from 2.8 to 3.1.

Two combinations involving wind must be considered: $D + L_{apt} + W$ and $D + L + W_{apt}$, where L_{apt} is the abritrary-point-in-time live load (e.g., 12 psf for offices) and W_{apt} is the daily wind. The terms L and W signify maximum lifetime values. The interrelationship between these two cases is shown in Fig. C.2.

6) Values of β for connectors are considerably higher than for members (see Tables C.7.7 and C.7.8) and so a higher target β should be selected for connectors.

7) Typical representative β's for a number of cases, presented as an aid to selecting target β's, are given in Table C.8. From this sample it is evident that the selected target β's of 3.0 for members under gravity loading and 2.5 for combined wind and gravity loading is reasonable. Also, target values of β = 4 or 4.5 are indicated for connectors.

<u>References for Appendix C</u>

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C.2 Yura, J.A., Galambos, T. V., Ravindra, M.K., "The Bending Resistance of Steel Beams," <u>Journal of the Structural Division</u>, ASCE, Vol. 104, No. ST9, Proc. Paper 14015, Sept. 1978, pp. 1355-1370.

C.3 Bjorhovde, R., Galambos, T.V., Ravindra, M.K., "LRFD Criteria for Steel Beam-Columns," Journal of the Structural Division, ASCE, Vol. 104, No. ST9, Proc. Paper 14016, Sept. 1978, pp. 1371-1387.

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C.7 Ravindra, M.K., Cornell, C.A., Galambos, T.V., "Wind and Snow Load Factors for Use in LRFD", Journal of the Structural Division, ASCE, Vol. 104, No. ST9, Proc. Paper 14006, Sept. 1978, pp. 1443-1457.

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C.9 American Iron and Steel Institute, "Proposed Criteria for Load and Resistance Factor Design of Steel Building Structures," AISI Bulletin No. 27, Jan. 1978.

C.10 Johnston, B.G., Editor, "Guide to Stability Design Criteria for Metal Structures" 3rd ed., Wiley-Interscience, New York, New York, 1976.

C.11 Bjorhovde, R., "A Probabilistic Approach to Maximum Column Strength," in Safety and Reliability of Metal Structures, ASCE, New York, 1972.

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PENDIX D - MASONRY STRUCTURES

troduction

Design criteria in the U.S. for engineered masonry construction have been developed the Brick Institute of America (BIA) (formerly the Structural Clay Products Institute) .4] for the use of clay brick and by the National Concrete Masonry Association (NCMA) .29] for solid and hollow concrete block. These, or similar criteria, presently used in st parts of the U.S. rely on linear working stress design principles. Factors of safety r unreinforced masonry construction traditionally have been quite large for static ertical loads which cause primarily compression [D.27], values of 4.5 - 7 being common D.14, D.25, D.32]. The factors of safety decrease when the load eccentricity and the oment increase.

Strength design and the use of load criteria based on probabilistic limit states esign principles are relatively new concepts in the masonry area. This Appendix describes he statistics and probability distributions (where possible) used to calculate reliabilities f masonry structural components. Most of the available data are for unreinforced masonry oad bearing walls in compression plus bending. Since this is a prevalent design condition, t appears reasonable to tie reliability based design to this limit state.

urrent Design Practice for Brick Masonry

In the current BIA Standard for engineered brick masonry [D.4], allowable vertical oads on nonreinforced walls* are computed from

$$P_n = C_e C_s (0.20 f'_m) A_g \tag{D.1}$$

hen $e/t \leq 1/3$, in which

e = vertical eccentricity, M/P;

h, t = height, thickness of wall;

C_e = eccentricity coefficient, which depends on e/t and e_1/e_2;

C_s = slenderness coefficient, which depends on h/t and e_1/e_2;

A_g = gross cross sectional area;

f'_m = compressive strength from prism tests (28 day strength) or brick tests;

e_1 and e_2 = ,respectively, the smaller and larger virtual eccentricities at lateral supports.

*In the context of present masonry design specifications, a wall is defined as a vertical member whose width is at least three times its thickness.

The BIA standard specifies that f'_m be determined on the basis of the average of 5 prisms in which h/t = 5 or greater. e_1/e_2 is positive when the member is bent in single curvature and negative when in double curvature, and thus reflects the end conditions and type of bending. The slenderness for walls is limited to $h/t < 10(3 - e_1/e_2)$. This study will focus on walls in which $e/t \leq 1/3$.

The requirements for nonreinforced brick masonry columns are very much the same; the allowable compressive axial stress is $0.16\ f'_m$ rather than $0.2\ f'_m$ (Equation D.1) and the slenderness is limited to $h/t \geq 5(4 - e_1/e_2)$.

Current Design Practice for Concrete Masonry

The NCMA standard [D.29] requires that nonreinforced concrete masonry walls subject to eccentric load be proportioned so that

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \tag{D.2a}$$

in which f_a, f_b = computed axial and flexural compressive stresses according to linear stress distribution theory and

$$F_b = 0.30\ f'_m = \text{allowable flexural compressive stress;} \tag{D.2b}$$

$$F_a = 0.2\ f'_m\ [1 - (h/40t)^3] = \text{allowable axial stress.} \tag{D.2c}$$

The bracketed term in equation D.2c is a slenderness factor and serves much the same purpose as C_s in equation D.1. Slenderness h/t is limited to 20 or less. The maximum virtual eccentricity is limited to one-third the thickness in solid units and to the value which produces tension in hollow units.

The requirements for columns are similar; the allowable compressive stress is $0.18\ f'_m\ [1 - (h/30t)^3]$ in Eq. D.2c.

Data Analysis - General Observations

Available masonry data falls into essentially three categories; unit (brick or concrete block) and mortar strength, masonry prism (small assemblages of masonry) strength, and structural element strength. Strength properties of the constituents are not directly helpful in determining means and variabilities to be used in reliability based design because of the composite nature of masonry. Although there is a correlation between unit strength and full size element strength (e.g., Ref. D.28), the element strength is substantial higher than would be predicted on the basis of the strength of the mortar. While data on prisms are somewhat more useful, full size structural elements usually behave more uniformly than prisms and wallettes [D.15] and are less affected by small variations in workmanship and materials.

In the absence of a definitive model to relate the strength of masonry walls to rengths of their constituents, the approach taken here is to determine the statistical ata for the reliability analysis on the basis of test data on full size walls. These ata, referred to vertical load-carrying capacity P, or stress $f = P/A$, are presented in e following paragraphs.

est Data on Brick Masonry Walls in Compression and Bending

Chapter 5 of Ref. D.25 summarizes the results of numerous tests conducted by the ructural Clay Products Institute to determine strength of single wythe walls in compression lus bending. Parameters varied included wall slenderness, eccentricity of load and end estraint. All specimens were built with Type S mortar and inspected workmanship. Bricks ad compressive strengths which ranged from 10,000 to 13,5000 psi (69-93 N/mm^2). As with ost other masonry testing programs, there are seldom more than 5 replicates of any one onfiguration.

Some of the test data where there were sufficient replicates to obtain meaningful stimates are summarized in Table D.1. In some instances, it was necessary to group test ata together because of the small number of replicates; for example, for the designation $h/t \simeq 23$, the actual h/t was between 21.9 and 24.1; the variability attributable to this grouping is inconsequential.

In attempt to obtain a larger data sample, P/P_n was plotted as a function of h/t for $e/t = 0$, 1/6, and 1/3, as shown in Figures D.1, D.2 and D.3. Regression analysis of these data revealed that $\overline{P}/P_n$ depends on slenderness and eccentricity. The scatter in the data appears nearly constant with respect to h/t. Parameter e_1/e_2 was not included as a factor in the regression. Examination of the data revealed no significant dependence on e_1/e_2; moreover, e_1/e_2 is not well controlled in the field, due to the randomness in applied loads, and as a result may contribute to the variability in strength in situ. In all cases, the standard errors were quite close: 753, 730 and 656 psi, (5.2, 5.0 and 4.5 N/mm^2), respectively, for $e/t = 0$, 1/6 and 1/3. The implied coefficients of variation in story height wall strength ($h/t \simeq 12$) are 0.14, 0.12, and 0.12, respectively. These are very close to the values reported in Table D.1 for samples of smaller size but in which the walls were (nearly) replicates.

Probability distributions for the load carrying capacity were investigated using the data in Table D.1 from Ref. D.25 in which $h/t > 10$. Probability plots of P/P_n and $\ell n\, P/P_n$ revealed that the lognormal and Weibull distribution each were best in three cases

Table D.1

Brick Masonry Walls in Compression Plus Bending

Source	Description of Wall	n	$\overline{P}/P_n^*$	c.o.v.
Ref. D.25	$h/t = 20.5$, $e_2/t = 1/3$, $e_1/e_2 = -1$	12	4.94	0.073
	$h/t \simeq 23$, $e_2/t = 0$, $e_1/e_2 = 0$	12	6.93	0.10
	$h/t \simeq 23$, $e_2/t = 1/6$, $e_1/e_2 = 0$	12	6.96	0.12
	$h/t \leq 7$, $e_2/t = 1/6$, $e_1/e_2 = 1$	9	6.21	0.10
	$h/t \leq 7$, $e_2/t = 1/3$, $e_1/e_2 = 1$	9	5.87	0.14
	$h/t \simeq 22$, $e_2/t = 1/6$, $e_1/e_2 = -1/2$	6	6.92	0.15
	$h/t \simeq 10$, $e_1/e_2 = -1$	10	4.60	0.11
	$h/t = 21$, $e_1/e_2 = -1$	16	6.27	0.10
	$h/t = 26$, $e_1/e_2 = -1$	15	6.02	0.16
Ref. D.24	$h/t = 20$, $e/t = 0$	15	9.88	0.12
	$h/t = 20$, $e/t = 1/6$	14	>9	0.13
Ref. D.33	2 ft by 8 ft-4in brick - Pure compression	2	5.78	-
	2 ft by 8 ft-4in brick - Pure compression	2	7.54	-
	2 ft by 8 ft-4in brick - Pure compression	2	8.00	-
	2 ft by 8 ft-4in brick - Pure compression	2	6.58	-
Ref. D.12	32" x 96" x 4" - Fixed ends	2	7.40	-
Ref. D.28	Story height, pure compression	47	7.34	0.15
Ref. D.2	Story height, Axial load, Type M, 9" wall	37	3.18	0.11
	Story height, Axial load, Type M	15	6.23	0.17
	Story height, Axial load, Type N	15	6.58	0.19
	Story height, $e/t = 1/8$, Type M	15	4.34	0.18
	Story height, $e/t = 1/8$, Type N	15	4.24	0.19
Ref. D.8	Story height, axial load, Type N	14	8.17	0.22

$^*P_n$ computed in accordance with BIA requirements. Ref. 4.

1 ft = 0.3048 m

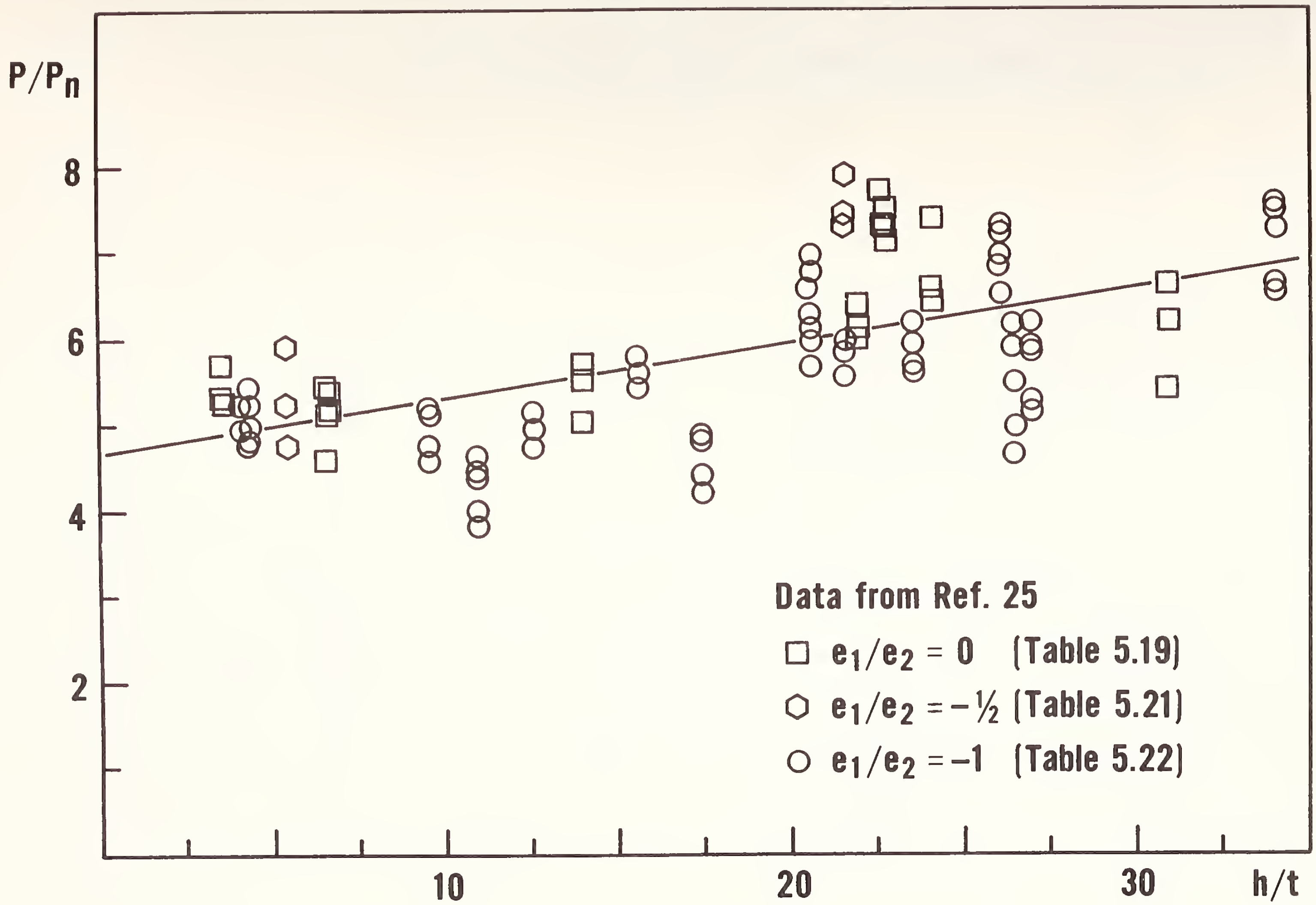


Figure D.1 - Strength of Brick Masonry Walls in Compression Plus Bending ($e_2/t = 0$)

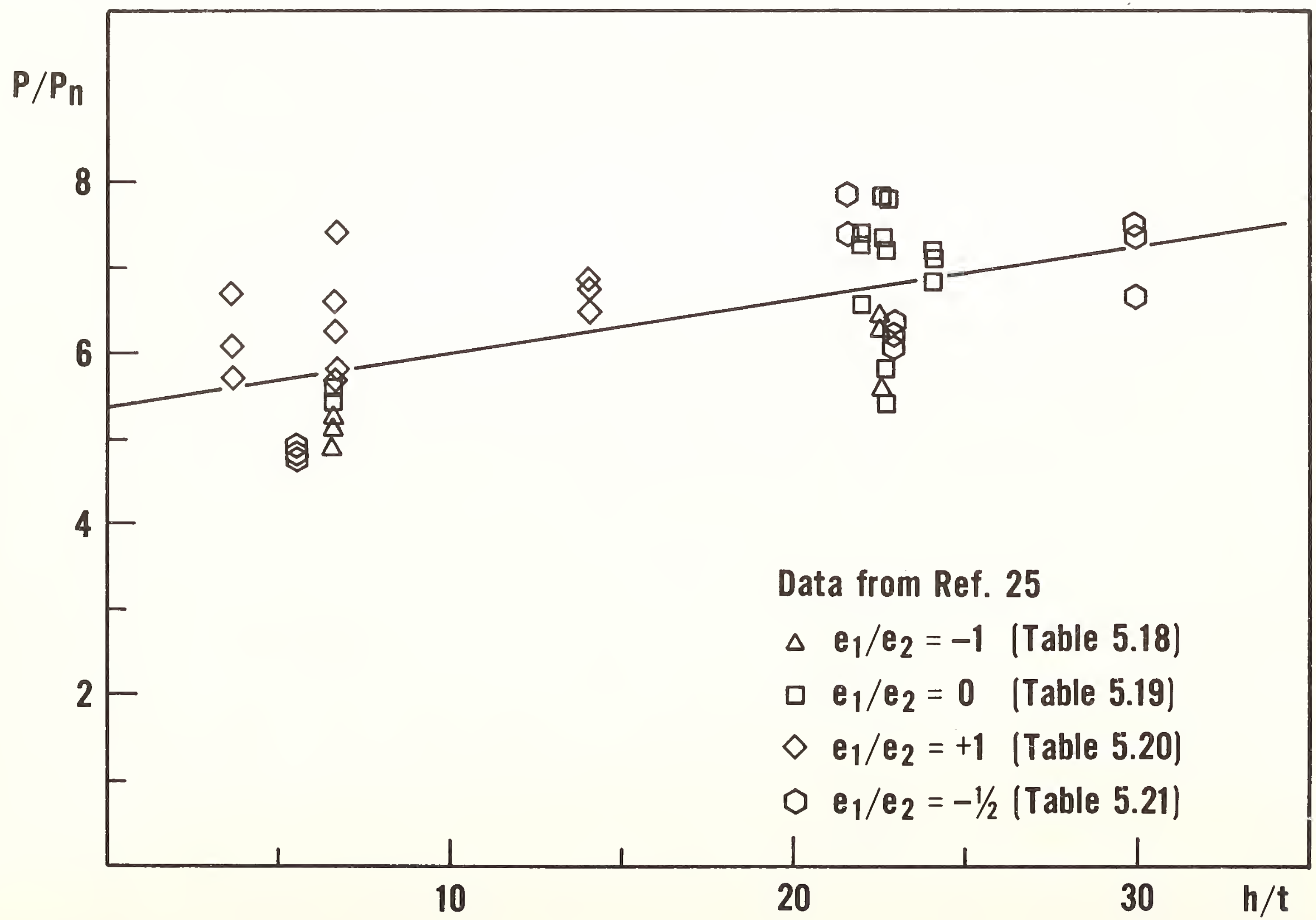


Figure D.2 - Strength of Brick Masonry Walls in Compression Plus Bending ($e_2/t = 1/6$)

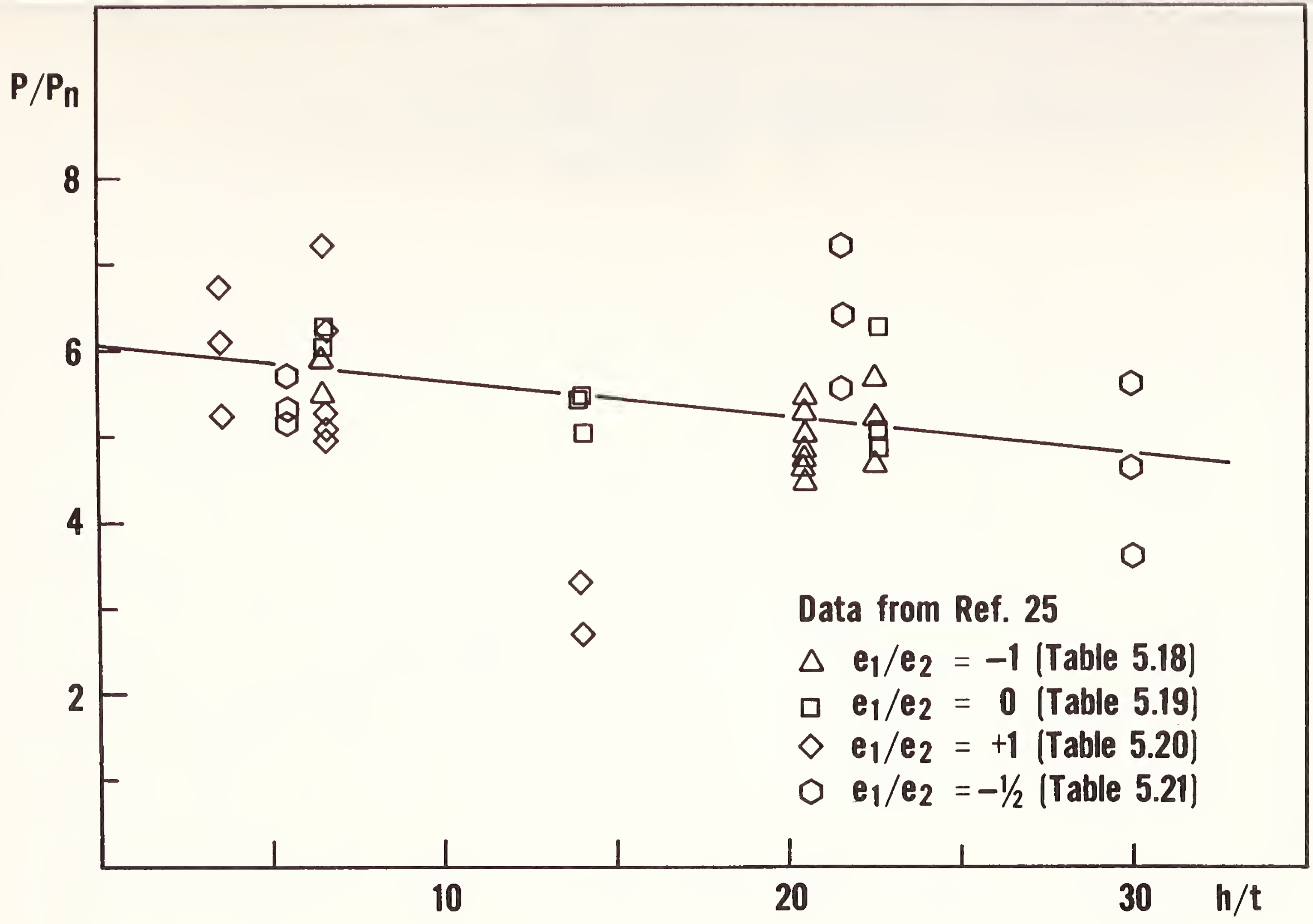


Figure D.3 - Strength of Brick Masonry Walls in Compression Plus Bending ($e_2/t = 1/3$)

and the normal distribution was best in one case. However, the Weibull distribution was also worst in four of the seven cases. Accordingly, the probability distribution for brick masonry wall strength in compression and bending may be assumed to have a lognormal distribution.

Masonry research conducted at the National Bureau of Standards (NBS) provided additional data on the strength of masonry walls. In one program [D.33] a number of walls 8 ft (2.4 m) in height were tested. All were unreinforced and were constructed using techniques representative of good workmanship. The walls were loaded in compression only, flexure only, or a combination of compression and flexure. Only in the cases of pure bending and pure compression were replicates tested. Walls were pinned at the top and partially restrained at the bottom. Additional brick walls were tested in various combinations of compression and flexure in a subsequent program [D.12]. Specimens were tested with both pinned and flat end conditions but unfortunately all of the pinned ended walls exceeded the BIA slenderness limitations [D.4]. Some of the test data from the NBS test programs are presented in Table D.1.

Numerous large masonry walls have been tested in research programs in the U.K. which, when analyzed according to U.S. standards [D.4, D.29], should be indicative of masonry performance in the U.S. In one program [D.25], a number of story height walls with lengths 4-6 ft (1.2-1.8 m) and thicknesses 4-9 in (102-229 mm) were loaded in pure compression. In order to enlarge the data sample, a regression analysis of ultimate stress in the wall upon unit strength was performed for solid and cored brick walls, as shown in Figure D.4. Since one acceptable way of determining P_n is from the unit strength [D.4], the results should be comparable to Figures D.1 - D.3. The standard error of regression was 268 psi (1.9 N/mm^2); the implied coefficient of variation in ultimate strength of a wall using 6000 psi units is 0.15. None of the walls failed in buckling, and it was concluded that slenderness effects were negligible in walls of this height (h/t about 20). According to the BIA standard, the allowable stress in such walls using Type N mortar and inspected workmanship (Table 2, Ref. D.4) would be 250 psi (2.2 N/mm^2); thus, $\overline{P}/P_n$ = 7.34.

In a second program [D.2], a series of tests was conducted on story height brick panels in pure compression and with eccentric axial load. Estimates of variability in wall strength obtained from regression analysis of wall strength on brick cube strength are given in Table D.1. In a third program [8], walls 100 in (2540 mm) in height by 36

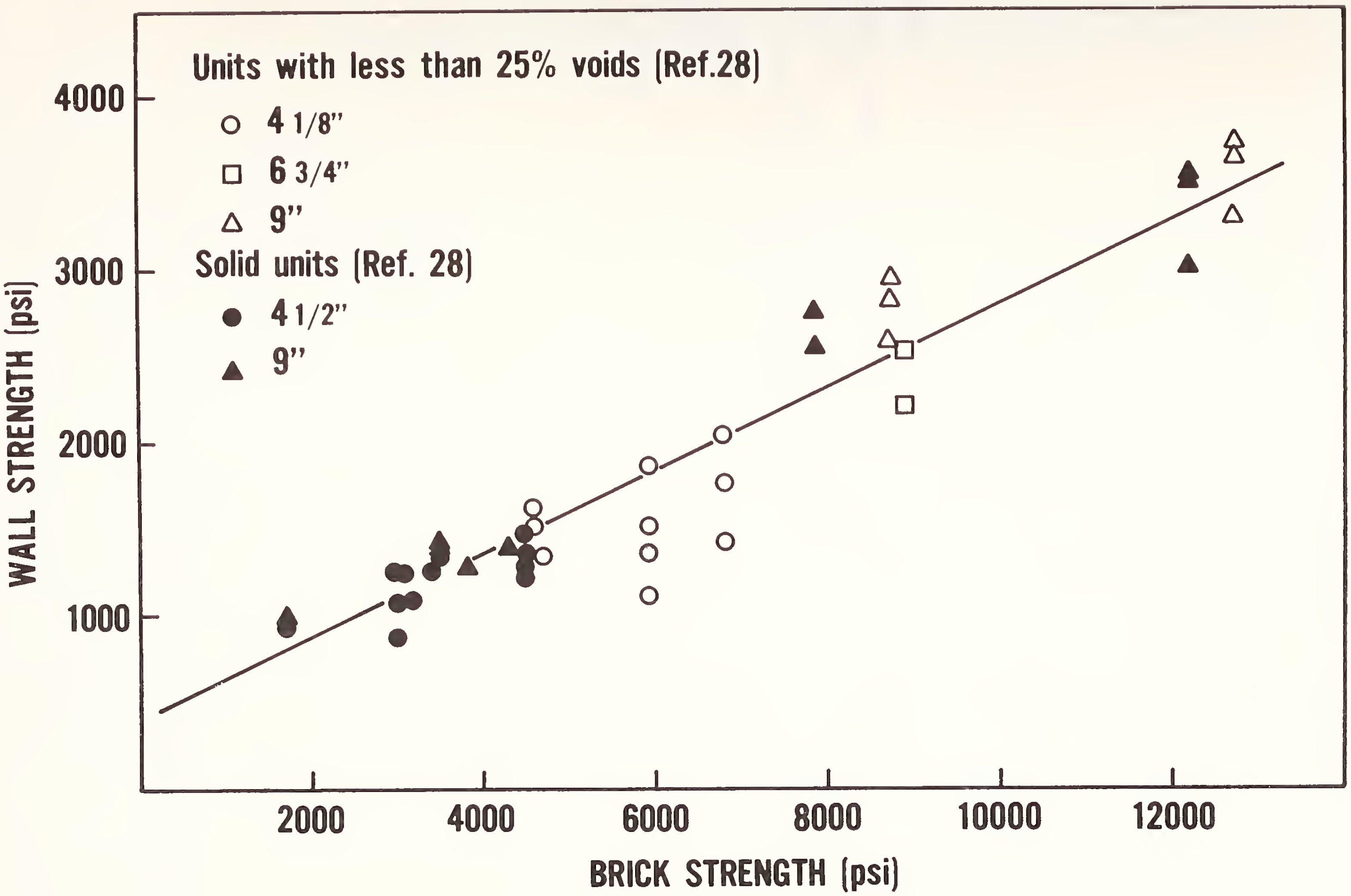


Figure D.4 - Correlation of Wall Strength to Brick Strength inches (914 mm) in length and 4.5 inches (114 mm) thick were loaded at a virtual eccentricity of less than t/17; the results are presented in Table D.1.

Other scattered sources of data on the behavior of masonry walls tend to confirm the results presented in Table D.1. In Macchi's [D.21] review of load factors implied by various brick masonry design criteria in Europe, a histogram of 151 tests of piers and walls in the U.K. was shown which implied a c.o.v. in load capacity of $V_p = 0.22$. Included in this, however, are the effects of a multitude of slenderness ratios and eccentricities which were not factored out, so this estimate serves as an upper bound.

Less data exist for masonry columns in compression than for walls. Some data on concentrically loaded reinforced masonry columns from Ref. D.25 are shown in Figure D.5. All columns were about 12.5 inches (318 mm) square and varied in height. According to a recent Monte Carlo study on reinforced concrete columns [D.9], variability in strength is independent of reinforcement ratio ρ when $\rho > 0.01$. It was assumed that slenderness could be neglected. For those 5 brick columns where $\rho = 0.0067$, $\overline{P}/P_n = 5.04$ and $V_p = 0.19$. For the remaining 24 columns in which $\rho > 0.01$, $\overline{P}/P_n = 4.24$ and $V_p = 0.11$. These variabilities are very similar to those for reinforced concrete compression members [D.9]. Additional data on the strength of brickwork piers is provided by Brettle [D.3] whose study of strength followed ACI ultimate strength principles quite closely. From 13 tests of eccentrically loaded masonry piers, $\overline{P}/P_n = 0.96$, where P_n is derived from ultimate strength principles, and $V_p = 0.11$.

Test Data on Brick Masonry Walls in Flexure and Shear

The strength of prisms and walls loaded in flexure depends, in part, on the tensile bond of the brickwork. Compared to compressive strength, the modulus of rupture (MOR) of brick prisms and walls generally exhibits considerably higher variability and depends on whether loading occurs parallel or perpendicular to the bed joints. Regression analysis of wall flexural strength perpendicular to joints vs. prism strength of the data provided in Ref. D.24 showed that $V_r = 0.26$ in single wythe masonry walls 48 in by 90 inches (1219 x 2286 mm) in size; for a prism strength of 4000 psi, $\overline{f}/f_n = 3.89$. Flexural tests of walls spanning 7 1/2 ft (2.3 m) in a direction perpendicular to bed joints reported in Chapter 5 of Ref. [D.25] suggest that $\overline{f}/f_n = 3.64$ and $V_r = 0.20$. Hendry [D.16] has suggested that coefficients of variation for lateral resistance are of the order 0.20 in walls relying primarily on tensile bond. Data on walls in flexure are summarized in Table D.2.

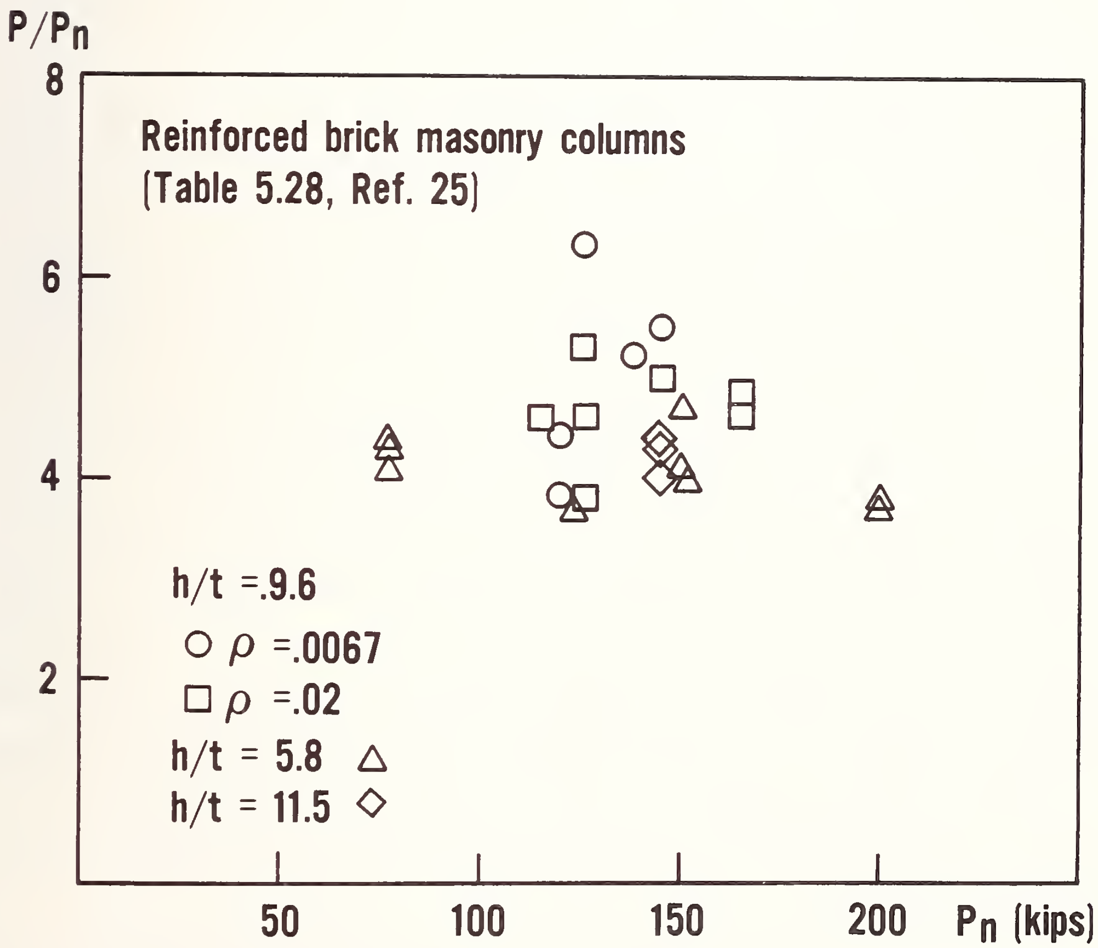


Figure D.5 - Strength of Reinforced Brick Masonry Columns

Table D.2

Brick Masonry Walls in Flexure

Source	Description	n	$\overline{f}/f_n$	c.o.v.
Ref. D.25	Single wythe, Type S mortar, Inspected	29	3.64	0.20
	Multi wythe, Type S, Inspected	21	4.26	0.18
	Multi wythe, Uninspected	6	6.25	0.25
Ref. D.24	Single wythe, Type S, Inspected	15	3.89	0.26

Variability in strength in shear and diagonal tension depends, in part, on the joint properties and, as with flexure, is quite large. For example, Ref. [D.24] presents test data on fifteen-4 ft square (1.2 m square) walls tested in diagonal tension. A regression analysis of the ultimate shear stress vs. prism strength resulted in V_s = 0.24 and $\overline{f}/f_n$ = 4.38 for walls in which f'_m = 4000 psi (27.6 N/mm^2).

Concrete Masonry in Compression and Bending

As with brick masonry, data on concrete masonry is available in terms of unit, prism and structural component strength, the latter of which is of particular interest.

Tests were conducted at the National Bureau of Standards on 6-in (152 mm) reinforced and 8-in (203 mm) unreinforced hollow core concrete masonry walls of varying heights (10 - 20 ft or 3.05 - 6.1 m) and load eccentricities [D.32]. The walls were restrained at the bottom and free to rotate at the top. The walls were built, cured, and tested in the laboratory and are representative of excellent workmanship. Type S mortar was used in their fabrication. Although the walls were tested at different ages, all data have been lumped; as noted earlier, the wall strength is not solely dependent on the mortar strength.

Some of the test data are summarized in Table D.3. Interestingly, slenderness effects were not apparent for the 8-inch (203 mm) unreinforced walls in which e/t = 0 and, accordingly, the data for different wall heights have been lumped. The variability in capacity does not appear to depend on either e/t or h/t in these tests. The allowable load P_n decreases as h/t increases [D.29]; the ratio $\overline{P}/P_n$ shown in Table D.3 is computed for h = 10 ft (3.05 m).

Table 3.1 of Ref. D.11 summarizes test data in a recent literature review on various 4 ft by 8 ft (1.2 by 2.4 m) unreinforced concrete masonry walls in compression; $\overline{P}/P_n$ from an analysis of these test data is given in Table D.3 . Additional concrete masonry walls

Table D.3

Concrete Masonry Walls in Compression Plus Bending

Source	Description	n	$\overline{P}/P_n^*$	c.o.v.
Ref. D.32	8in Unreinforced, Story height	12	4.28	0.17
	6in Reinforced, Story height	9	5.62	0.13
Ref. D.12	6-in Unreinforced, h/t = 17	10	6.05	0.10
Ref. D.11	8-in hollow block	2	4.50	-
	8-in solid block	1	4.28	-
	4-in Block-Block cavity	2	3.85	-
	8-in Hollow block	2	4.81	-
	8-in Hollow block	2	3.48	-
Ref. D.14	8-in block, 4 ft by 8 ft walls	7	6.4	-
Ref. D.26	200 mm hollow block, Story height, M	9	4.54	0.15
	100 mm solid block, Story height, M	6	-	0.20
	100 mm hollow block, Story height, M	9	-	0.15
	All data - f'_m assumed as 3000 psi	38	4.25	0.17

P_n^* = allowable computed in accordance with NCMA requirements (Ref. 29)

were tested in replicates of two under various vertical load eccentricities and transverse load conditions [D.12]. Ten of the specimens tested in vertical compression satisfied current NCMA design requirements which limit the virtual eccentricity in unreinforced hollow unit walls to that which causes tension on the cross section. Table D.3 gives $\overline{P}/P_n$ and V_p.

The conservatism in the allowable vertical load for unreinforced masonry tends to increase as h/t increases, as can be seen in Figure D.6. These data correspond to a number of different eccentricities and end restraint conditions. A regression analysis of $\overline{P}/P_n$ on h/t yielded a standard error of 0.96 and $\overline{P}/P_n$ = 4.9 for a story height wall; the c.o.v. would be 0.20.

Additional data generated in masonry research in the U.K. may also be indicative of performance of masonry structures in the U.S. In one program [D.26], 38 wall panels 2.6 m high and 1.8 m wide were tested. Slenderness was not considered to be a factor in the tests. Data for replicate wall tests within the test series are presented in Table D.3. In order to enlarge that data sample, a regression analysis of wall strength on companion masonry couplet strength was performed. The results for the 38 tests were $\overline{P}_{wall}/P_{couplet}$ = 0.82 and V_p = 0.17. The magnitude in the variability is very similar to that for lightly reinforced concrete columns in pure compression [D.9].

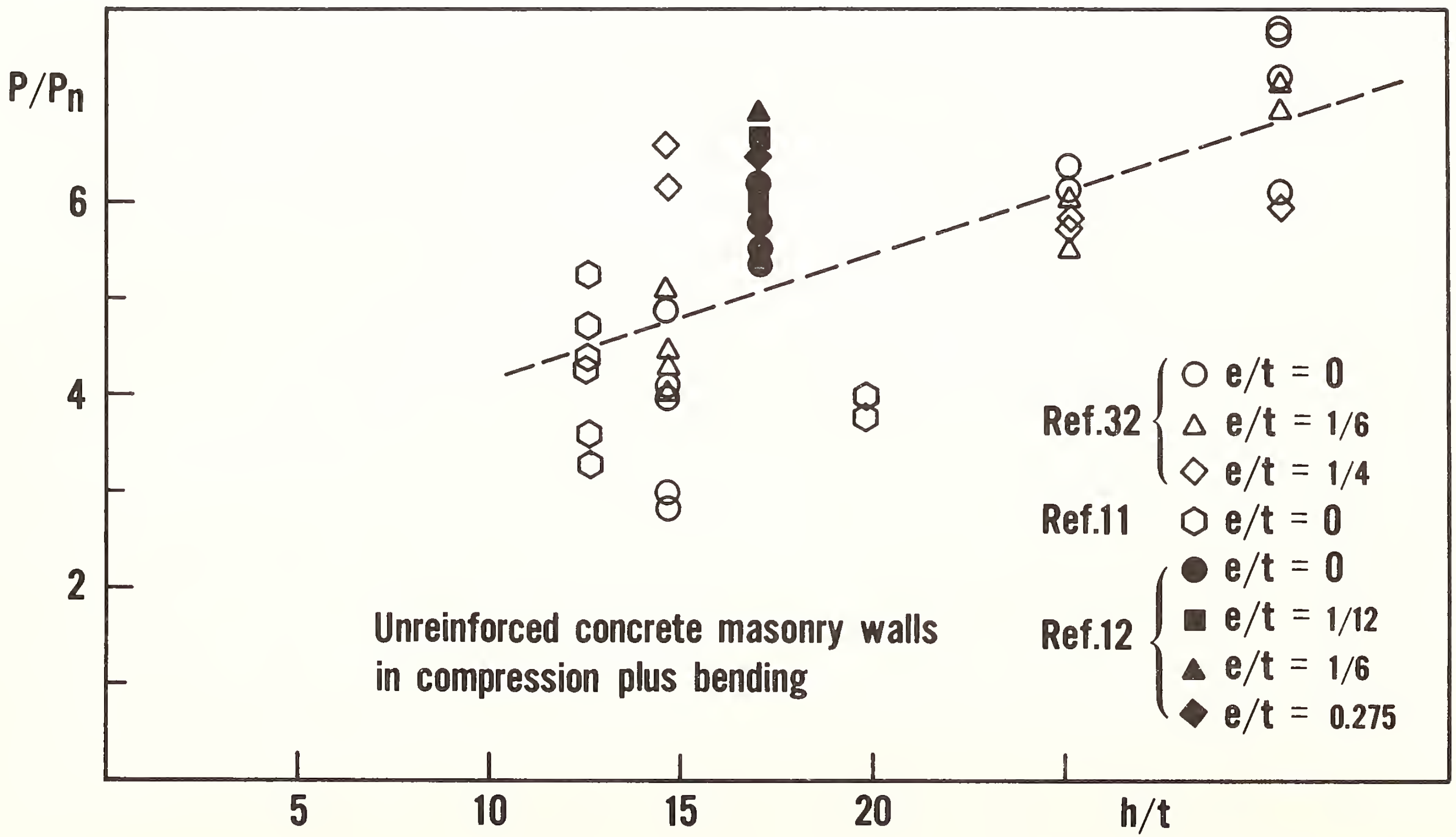


Figure D.6 - Strength of Concrete Masonry Walls in Compression Plus Bending

Very little data on strength of walls in flexure (zero axial load) are available .1, D.11]. As examples of what might be expected, two hollow block walls tested in exure [D.12] developed flexural tensile strength of 18 and 40 psi (0.12 and 0.28 N/mm^2). e allowable stress for Type S mortar is 23 psi (0.16 N/mm^2) [D.29]. In connection with e strength of reinforced concrete masonry beams, it has been found that the ACI ultimate rength equations are good predictors for strength. Based on 38 tests reported in Ref. 7, $\overline{M}_u/M_{un}$ = 1.14 and V_m = 0.16.

alysis of Masonry Structure Reliability

The laboratory test data on full size structural members is insufficient, in itself, r calculating the reliability of masonry structural elements in situ. While the data on P_n presented in the previous sections incorporates variabilities from many of the same urces which cause strength variability in situ, laboratory tests, with their carefully ntrolled workmanship, curing conditions, etc., would tend to exhibit less variability in rformance. Alignment of walls, thicknesses of mortar joints and completeness of joints, rticularly in hollow core units, are simply more difficult to control in the field [D.7. 17].

Accordingly, the basic resistance variable to be used in the reliability analysis for lls with low vertical load eccentricities (e/t < 1/6) and for flexure is defined as,

$$R = P_{lab} \cdot (P_{field}/P_{lab}) = P_{lab} \cdot B \tag{D.3}$$

which P_{lab} is the capacity measured in the laboratory and B is a random variable to count for differences in fabrication and curing between the laboratory and field. The ean and c.o.v. in R are

$$\overline{R} = \overline{P}_{lab} \cdot \overline{B} \tag{D.4}$$

$$V_R = [V_P^2 + V_B^2]^{1/2} \tag{D.5}$$

$\overline{P}_{lab}$ and V_P have been presented in Tables D.1 - D.3 and Figures D.1 - D.6. In walls aded in compression plus bending, V_p is typically about 0.14 for brick masonry and 0.15 r concrete masonry. In pure flexure, $\overline{f}_r/f_n \simeq 3.6$ and $V_r \simeq 0.21$. $\overline{B}$ and V_B are estimated follows. The basic masonry unit strengths are the same in the field as in the laboratory. e strength of masonry walls in compression is not strongly affected by the quality of e mortar [D.17]. The mean compressive prism strength may be used as a basis for design .4, D.29]; tests at NBS indicate that this is a good predictor of the ultimate strength companion walls loaded in compression. On the other hand, it is clear that $\overline{B}$ and V_B

depend on whether or not the workmanship is inspected. In engineered masonry structures in which quality control procedures set forth in the standards are followed, the element performance may approach that observed in laboratory tests. However, when the work is uninspected, the ultimate strength of walls tends to be about 60 percent of the strength in inspected walls [D.17]. The effect of inspection becomes more important for large load eccentricities in which the mortar joints are subjected to tension.

Accordingly, it is assumed that $\overline{B}$ = 1.0 for inspected workmanship and $\overline{B}$ = 0.6 when workmanship is uninspected. V_B is determined by fabrication and curing conditions in the field; this would include alignment, thickness of joints, the effect of partial joints, and so forth. A study comparing strength of concrete in situ to standard cylinder strength indicated that $V_B \simeq 0.11$ for average cure and $V_B \simeq 0.15$ for poor cure [D.10]. These are a reflection of the difference between field and lab placement and may be used, pending the acquisition of data that would shed additional light on the problem.

Table D.4 summarizes the statistics used in the analyses of the reliability of unreinfor masonry walls in compression and in bending. The c.d.f. of resistance is lognormal.

Table D.4

Statistics of Resistance of Masonry Walls in Compression and Bending

Type		Brick Masonry	Concrete Masonry
Compression Plus Bending - inspected	$\overline{R}/R_n$	Figs. D.1 - 2	Fig. D.6
	V_R	0.18	0.19
Compression Plus Bending - Uninspected	$\overline{R}/R_n$	0.6 x Inspected Value	0.6 x Inspected Value
	V_R	0.21	0.21
Pure Flexure Inspected	$\overline{R}/R_n$	3.90	-
	V_R	0.24	-

When the vertical load eccentricity becomes large (e/t in excess of 1/6), $\overline{R}/R_n$ and V_R referred to vertical load may not always be the best statistical parameters for characteri resistance. Consideration of the thrust-moment interaction diagram describing masonry wall strength shows that any reasonable loading path which could lead to a failure at eccentricities in excess of, say, e/t = 1/6 would involve a large increase in moment in comparison with the increase in axial load. The reliability in compression plus bending actually depends on the orientation of the (P, M) load vector with respect to the interaction diagram which describes strength.

For eccentricity ratios in excess of 1/6, the reliability analysis may be formulated in terms of a limit state equation which describes the P,M interaction relationship rather than the linear form.

The capacity of short masonry walls in compression and bending can be predicted accurately using a strength analysis based on a linear distribution of stresses at failure, provided that the compressive strength is af'_m , the factor a being the ratio of the compressive strength when there is a strain gradient to the compressive strength under uniform compression [D.12, D.33]. The factor a depends on the load eccentricity. The tensile strength of the masonry wall may be ignored; this has a negligible effect [D.12, D.33] except when the stress state in the wall approaches a state of pure flexure.

Accordingly, the limit state equations for a wall or column built with solid units are,

$$af_m - (Pt + 6M)/bt^2 = 0; \ 0 < M/Pt < 1/6 \qquad \text{(D.6a)}$$

$$af_m - \frac{2}{3}\ \frac{P}{b}/(\frac{t}{2} - \frac{M}{P}) = 0; \ 1/6 < M/Pt < 1/3 \qquad \text{(D.6b)}$$

in which af_m = apparent flexural compressive strength of masonry in the presence of a strain gradient; b,t = width, thickness of the element; P = axial thrust; parameter a is a function of e/t, increasing from unity at e/t = 0 to approximately 1.4 at e/t = 1/3; M = moment. Slenderness effects may be accounted for by amplifying the moment as suggested for reinforced concrete design [D.5]. However, tests conducted at the National Bureau of Standards on unreinforced concrete masonry walls [D.33] indicate that the slenderness effect (and moment amplification) can be neglected when h/t < 14; that is, the strength of masonry walls of story height or less can be predicted from their short wall capacity. Other test results [D.28, D.30] substantiate this observation. The extension of these equations to encompass hollow core units is relatively straightforward.

Reliability indices for brick and concrete masonry walls in various combinations of compression and bending are shown in Figures D.7 through D.9. In Figs. D.7 and D.8, the resistance and loads are assumed to be linearly related. The larger variability and reduced mean associated with uninspected workmanship have a pronounced effect on β. Reliabilities of brick and concrete masonry walls in compression in which the eccentricity in vertical load is small are similar. Reliabilities in pure bending are considerably less than in compression.

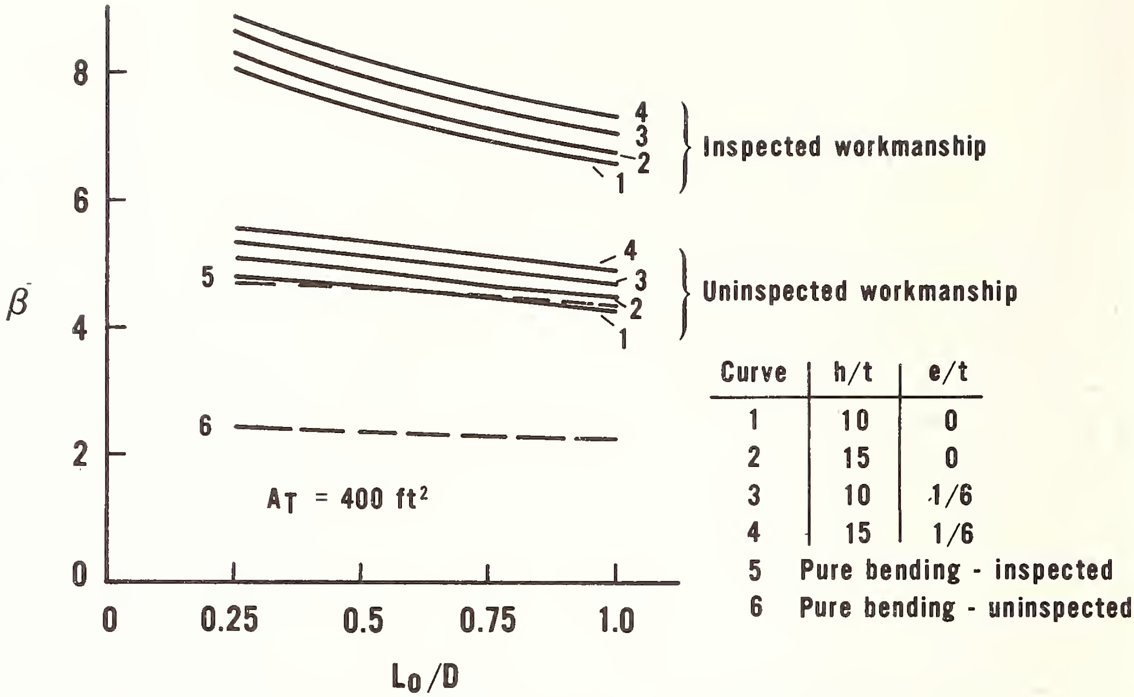


Figure D.7 - Reliability Index for Nonreinforced Brick Masonry Walls

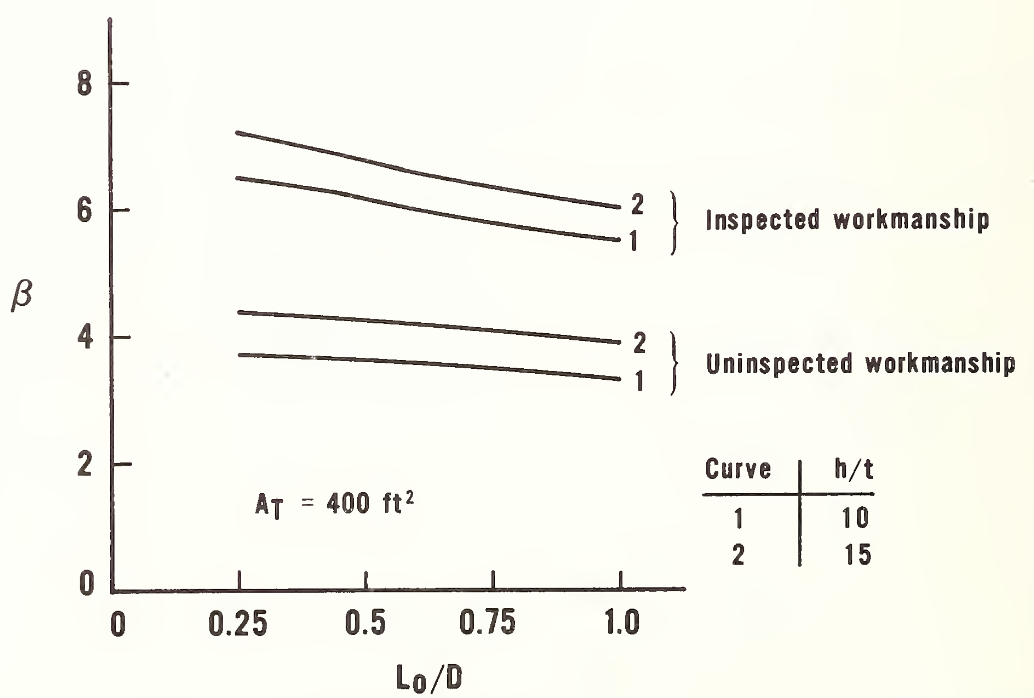
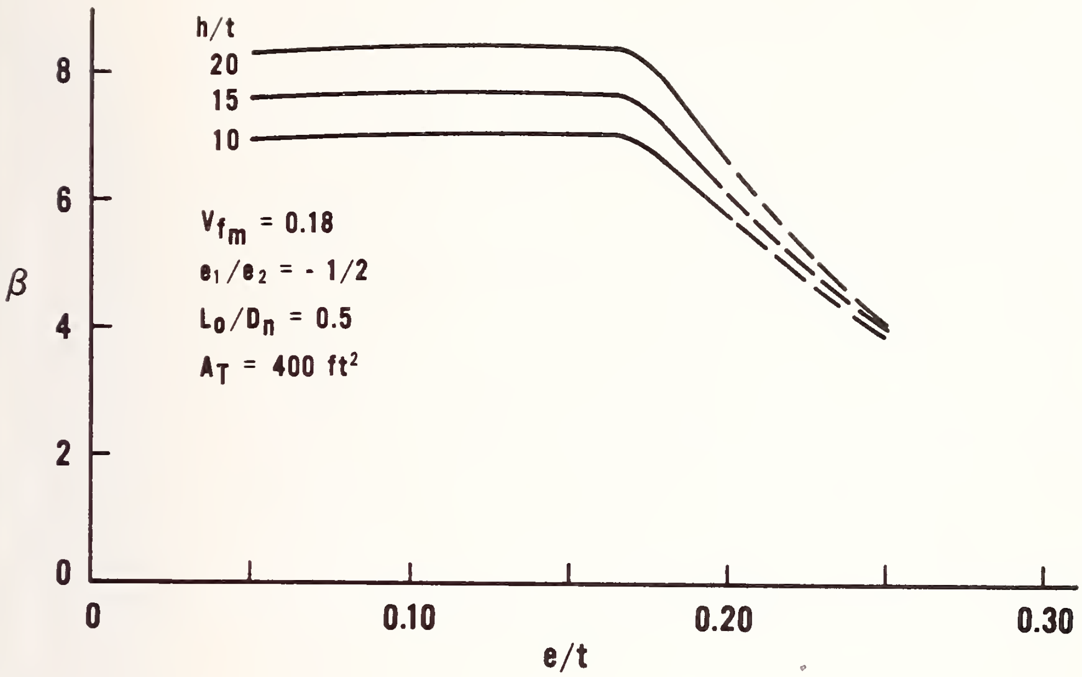


Figure D.8 - Reliability Index for Nonreinforced Concrete Masonry Walls



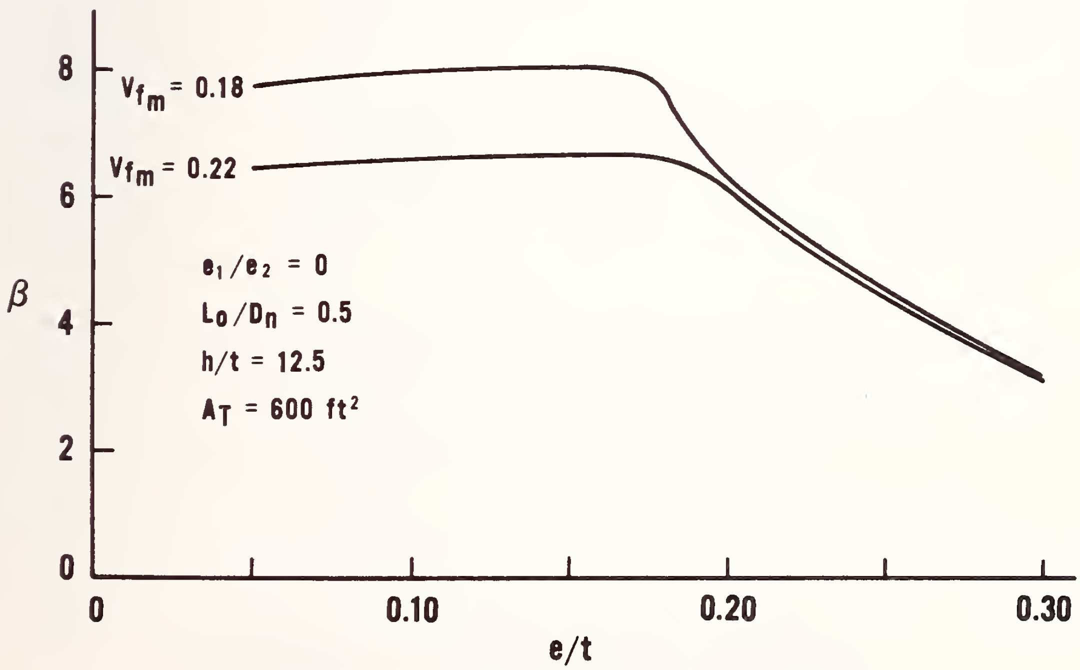


Figure D.9 - Effect of Virtual Eccentricity on Reliability of Nonreinforced Masonry Walls

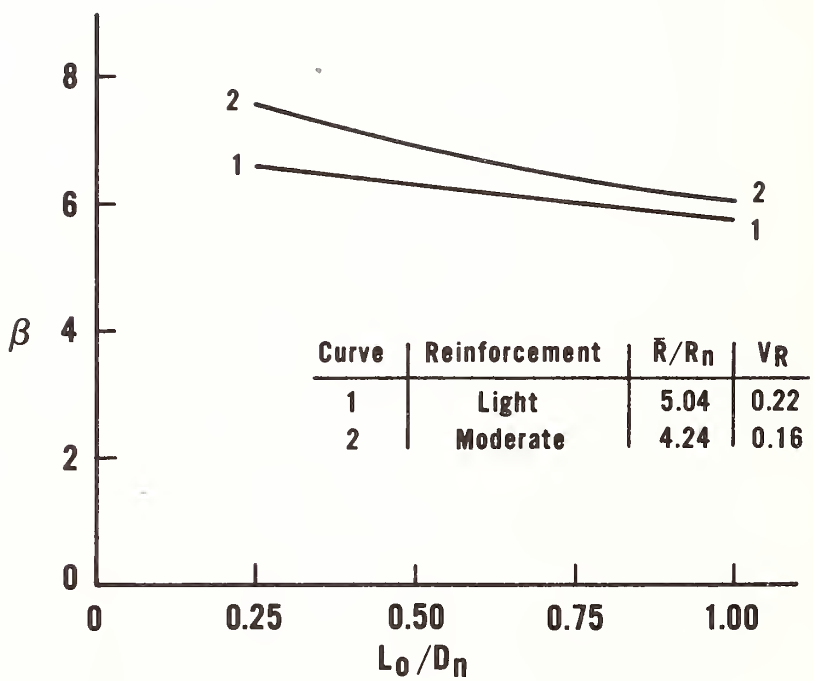


Figure D.10 - Reliability Index for Reinforced Brick Masonry Columns

Figure D.9 illustrates β as a function of load eccentricity, computed according to Eqs. D.6 rather than the linear formulation for g( ). V_{f_m} was assumed to be equal to V_p. It is interesting to note that when $e/t < 1/6$ this analysis leads to a similar result as the linear formulation. As the eccentricity increases past the point where tensile stresses are induced on the section, β drops precipitously, to the point where at $e/t = 1/3$ it is actually somewhat lower than the pure bending case. The decrease in safety margin with increasing eccentricity has been remarked upon in previous studies [D.32]. In reliability terms, it occurs because β is no longer measured radially from the design point as e/t increases beyond about 1/6.

Figure D.10 illustrates β for reinforced brick masonry columns in compression. Separate curves are presented for lightly reinforced and moderately reinforced columns. The data-based variability (Figure D.4 and accompanying discussion) has been augmented by $V_B = 0.11$ for average quality control. The values of β appear to be of the same order of magnitude as for unreinforced masonry walls in which the vertical load eccentricity is small.

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APPENDIX E - GLUED-LAMINATED AND OTHER HEAVY TIMBER STRUCTURES

Introduction

Current standards by which timber structures are designed include the National Design Standard for stress graded lumber [E.16] and the standard published by the American Institute of Timber Construction [E.22] for glued-laminated (glulam) construction. Timber structural systems currently are designed according to linear working stress design principles. Adjustments in allowable stress are permitted for certain load combinations. Numerous recent studies have been directed toward placing timber engineering on a strength basis in which the factors of safety will be derived probabilistically [E.3, E.10, E.17, E.21, E.23, E.26]. It is the purpose of this Appendix to present those data which currently are available to assist in establishing reliability benchmarks for timber design and in selecting a load criterion for timber and other construction materials.

Understanding of the behavior of wood structures, particularly under sustained load conditions, is rapidly evolving [E.5]. This analysis shows where information gaps exist and indicates where additional work may be desirable.

Data Analysis - General Observations

Most timber structures in the US are of light frame construction and utilize dimension lumber. Extensive test data on the in-grade strength of dimension lumber of various species are available in the literature. These data show that, among other factors, the strength of dimension lumber as received by the contractor depends on the grading procedure and grade, the species and in some cases the geographical location in which it is grown, its moisture content, the rate at which the load is applied and the duration of the load. Testing in-grade insures that the effects of all factors, except rate and duration of load, are reflected in the determination of strength. Each of these factors contributes uncertainty to the prediction of engineering properties. However, the light frame structure is not really designed in the same sense that a steel or reinforced concrete structure is designed. Standard wall and floor systems generally do not rely on single member design. Thus, statistics on the strength of individual pieces of dimension lumber are not an entirely satisfactory basis for reliability analysis.

The analysis of reliability of structural elements which utilize dimension lumber repetitiously, such as shear walls, roof trusses and floor diaphragms, would have more meaning. However, the only test data available which is suitable for the present analysis re for floor systems with uniform and concentrated loads and floor diaphragms. In other ases, such as racking of walls and compression plus bending in walls, there are limited est data and in no case are there sufficient replicates that estimates of coefficients of ariation or probability distributions can be made. Analytical models to describe the erformance of walls or floor systems in terms of the properties of their components are eginning to be developed [E.4, E.20] which would enable the strength of structural omponents to be predicted by Monte Carlo simulation.

Glulam and heavy timber structures are of greater interest in reliability based design work in the sense that they are engineered in a similar manner as steel and concrete structures and may compete directly with steel and reinforced concrete as alternate structural systems. Thus, in this case, comparative reliability estimates for the different construction materials have some relevance. (Another reason for focussing on glulam is the load duration problem, discussed later.)

The following sections summarize the strength data that are available for glulam and heavy timber construction. Differences in species, fabrication and testing procedure militate against the pooling of data, and statistics are presented for individual data samples. The purpose here will be primarily to establish the range over which the mean and variance in strength might be expected to vary. The strength data are all based on a standard 5-minute load test. Load duration effects and their incorporation in the reliability analysis are considered in a separate section.

Test Data on Glulam Members

Most of the available data for glulam members are derived from flexural tests of simply supported beams. Since many glulam beams are designed to be simply supported in practice, statistics obtained from tests of full scale beams in the laboratory (with appropriate adjustment for load duration) should provide an excellent indication of behavior in situ. The flexural strength is defined by the modulus of rupture F_r; the load-deflection curves are essentially linear and the failure mode is brittle.

The comprehensive testing program conducted by the Forest Products Laboratory (FPL) [E.2, E.12-15] on beams with Douglas Fir and Southern Pine laminating stock is the primary source of data on behavior of glulam beams. Test results are summarized in Table E.la. Nominal design stresses F_n for glulam beams commonly are referred to a uniformly loaded beam with a depth of 12 in (305 mm) and a span-to-depth ratio of 21:1, and all FPL data in Table la are presented on this basis. It should be noted that the tension laminating

Table E.1
Test Data on Glulam Beams

Data based on 12" depth, 21:1 span: depth, uniformly loaded unless otherwise noted.

(a) Flexural Data

Source	Beam Series	No.	Description	Strength Statistics	
				Mean/nominal	V_{F_r}
Ref. E.15	A	15	D. Fir/E. Spruce	2.31	0.24
	B	15	D. Fir/E. Spruce	2.60	0.14
	C	15	D. Fir/Wane	2.75	0.17
	D	15	Hem Fir	2.34	0.12
	E	15	D. Fir	2.59	0.16
	F	15	S. Pine	2.77	0.17
	G	15	Hem Fir	2.41	0.15
	H	15	Western Wood	2.63	0.19
Ref. E.14	86-90	5	D. Fir/L. Pine L3	3.17	0.081
	91-95	5	D. Fir/L. Pine L2	2.65	0.156
	96-105	10	D. Fir/L. Pine L1	2.52	0.086
Ref. E.13	61-70	10	S. Pine/I	2.82	0.114
	71-80	10	S. Pine/II	2.48	0.094
Ref. E.12	36-40	5	S. Pine	3.50	0.09
	41-45	5	D. Fir	2.67	0.13
	46,48-50	4	D. Fir	2.64	0.13
Ref. E.2	1-5,21-23	7	D. Fir/301	2.38	0.11
	11-15,24-26	7	S. Pine/301	2.09	0.10
	6-10	5	D. Fir/301+	2.72	0.07
	16-20	5	S. Pine/301+	2.19	0.25
Ref. E.9	Lumped	86	D. Fir (12% Moist)	2.80	0.14
Ref. E.21		56	D. Fir	2.59	0.17
Ref. E.24		19	Lodge Pine	-	0.24
Ref. E.8		6	S. Pine	3.18	-
		6	S. Pine	2.75	-
		6	S. Pine	2.91	-
		6	S. Pine	3.04	-
		6	Hem Fir	2.73	-
		6	Hem Fir	2.82	-
		6	D. Fir/Hem Fir	3.20	-
		6	Lodge Pine	2.94	-
		6	Lodge Pine	2.72	-
Ref. E.6		16	Hem Fir	2.88	0.12

b) Glulam Members in Tension

Source	No.	Description	Strength Statistics	
			Mean/nominal	V_{F_t}
Ref. E.19	180	D. Fir, L3	2.45	0.23
	20	D. Fir, L3	2.75	0.16
	20	D. Fir, L3	2.92	0.09
	20	D. Fir, L3	2.33	0.14
	95	D. Fir, L3	3.0	0.20
	20	D. Fir, L3	2.67	0.15

c) Glulam Members in Compression

Source	No.	Description	Mean/nominal	V_{F_c}
Ref. E.9	26	D. Fir, L3	2.73	0.12
	25	D. Fir, L2	2.62	0.12
	25	S. Pine No. 3	2.42	0.12
	25	S. Pine No.2	2.74	0.12

Table E.2

Heavy Timber

(a) Flexure

Source	Species	Size*	No.	$\overline{F}_r$	V_{F_r} (est)
Ref. E.25	Longleaf pine Green - Dense	6x12, 8x16	13	7260	0.24
	D. Fir - Green S 1	8x16	36	7070	0.11
	D. Fir - Green S 2	8x16	66	6240	0.16
	Sitka spruce - both S2	8x16	12	5040	0.33
	W Hem - Both S 2	8x16	35	5300	0.20

(b) Compression

Source	Species	Size*	No.	$\overline{F}_c$	V_{F_c} (est)
Ref. E.25	Southern Pine Select	12 x12	68	4610	0.22
	D. Fir select	12 x12	48	4020	0.26

*All dimensions in inches (1 in = 25.4 mm) stock used in the fabrication of these beams often was of minimum quality. The reason was that most of the test programs were aimed at better utilizing laminating stock and the tests were conducted to show that the allowable stresses were comparable to existing glulam technology. Some of the data in Ref. E.15 do not correspond to any particular stress grade; in such cases, the allowable bending stress which most closely corresponds to the grade and arrangements of laminations was selected for $\overline{F}_r/F_n$. Therefore, these data have inherent bias, i.e., the average strength of beams in service in which the laminations were selected randomly from a particular grade stock might be somewhat higher than indicated by the FPL tests. This is partially offset by the fact that the commercial fabricators, knowing in some instances that the beams were to be used in laboratory test programs, could have exercised more stringent quality control than normal. Unfortunately, the effect of these factors on the statistics of F_r cannot be ascertained. The mean in situ strength of similar populations of glulam beams could be approximately 10 to 15 percent higher than indicated by the test series; however, the test data presented in Table E.1a have not been corrected for this factor.

It should also be noted in this regard that some lumber producers skim the top of the grades from their production for use in millwork, ladder rails, etc. This means that the test samples in research reports may not be entirely representative of what the engineer gets when he orders, say, Grade 2 or better dimension lumber or L1 laminating stock.

Several analyses of the data in Ref. [E.15] summarized in Table E.1a were performed in this study using a maximum probability plot correlation method to determine the best probability distribution for F_r. The normal, lognormal and Weibull distributions were considered to be candidate distributions. When each sample of 15 beams was considered separately the normal distribution was best for series F and G; the lognormal for A, D, E, H; and the Weibull for B and C. However, the lognormal was worst in 3 cases (B, C, G) and the Weibull was worst in 4 (D, E, F, H). When Douglas Fir series A, B, and C are pooled (45 beams) by normalizing the data to a 12 inch (305 mm) depth and 21:1 span/depth ratio, the Weibull distribution offers the best fit; these data are plotted on Weibull probability paper in Fig. E.1. The distribution parameters are a function of load duration and are presented later in this Appendix.

Additional data from other research programs are also summarized in Table E.1a. Knab and Moody [E.9] pooled all FPL data for beams which had a stress rating of 2400 psi (16.6

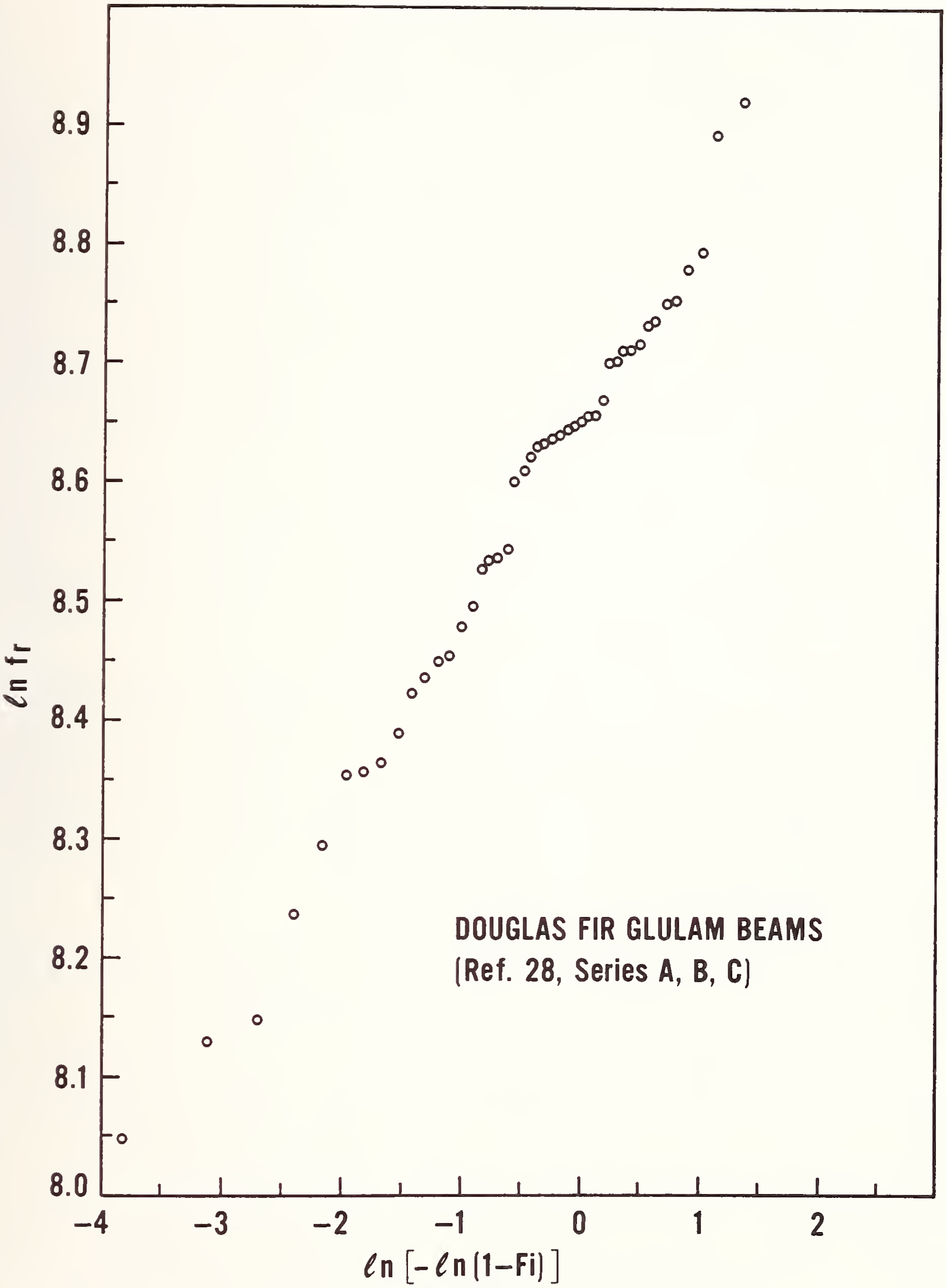

Figure E.1 - Probability Distribution for Modulus of Rupture in Glued-Laminated Beams

N/mm^2) to obtain their estimate of mean and coefficient of variation. Sexsmith and Fox [E.21] created an expanded sample by taking several smaller sets of data on beams of different sizes and normalizing all test data to a reference beam volume. Analysis of their data shows that the Weibull distribution provides a somewhat better fit to the data than the lognormal distribution. Johnson's [E.8] test results are presented in the form of average and minimum F_r for each group of 6 beams; the c.o.v. is estimated to average about 0.15-0.16 for all tests. The data summarized in Table E.1a showing the ratios of F_r to allowable stress tend to confirm the values reported in the Forest Product Laboratory test series. As with the latter tests, the tension laminations were selected from low-line stock. Interestingly, there appears to be no correlation in Johnson's data between $\overline{F}_r/F_n$ and F_n, which suggests that reliabilities for different stress grades would be about the same.

Data for strength of glulam members in tension and compression is presented in Tables E.1b and E.1c. Relatively speaking, the c.o.v. in tensile strength tends to be larger than for flexure, averaging about 0.20; in compression, $V_c = 0.12$. Shear failures are not a common problem in glulam beams [E.14].

Table E.2 presents some data on the strength of heavy sawn timbers in flexure and in compression [E.25]. The grading procedure used at the time of these tests was different from the procedure used now, and it is difficult to relate the test MOR to a design allowable stress. The estimated coefficients of variation are higher than observed for glulam, which is not surprising in view of the lower quality control.

Effects of Load Duration and Rate of Loading

The strength of wood members is known to be affected significantly by the rate of loading and the duration of the load, so much so that most standards permit these factors to be incorporated in some way in design. Failure in wood structural members under sustained loads appears to be a creep-rupture phenomenon [E.5]. Most standards in the past have recognized this implicitly by permitting the allowable stresses to be increased for loads with shorter duration than the standard duration of 10 years. However, there is presently a considerable amount of controversy over what the actual load rate and duration effects are [E.5, E.7, E.10] and how these factors should be taken into account in reliability analysis.

A summary of world literature on effects of load rate and duration has recently been published [E.7], which traces the developments which led to the curve which purports to relate the strengths of wood to a 10-year, or normal, load duration. This curve has been used in the National Design Specification [E.16] since 1952, and is the basis for the increases in allowable stresses permitted by most codes and standards in the US. The mean strength level, as a percentage of the standard 5 minute test, is approximately

$$SL = 108.4/D^{0.04635} + 18.03 \qquad (E.1)$$

(the "Madison curve") in which time D (in seconds) includes the uploading and sustained load times. Eq. E.1 is based on small clear specimen tests; nevertheless, it is used in the U.S. in computing design stresses for dimension lumber, glulam and heavy timber construction.

More recent work separates constant loading and rate of loading effects because different loading conditions are involved. After reanalyzing the available data, Gerhards [E.7] found that the mean strength reduction is described by

$$SL = 87.8 - 5.8\log_{10}D \text{ (D in hr)} \qquad (E.2)$$

Here, D does not include the uploading time. The scatter about this equation indicates a c.o.v. of 0.07. When the period at sustained load is long relative to the uploading time, Eqs. E.1 and E.2 lead to about the same result.

An analytical model has recently been developed [E.5] to predict the dependence of wood strength on load history which treats failure as a creep rupture phenomenon. This model predicts a dependence of strength of clear wood on load duration which is approximately the same as Eq. E.1. It remains to be validated for wood members with imperfections.

Since laminating stock generally is of higher quality than dimension lumber, it is reasonable to expect that the load duration effect in glulam members would be similar to Eqs. E.1 and E.2. The real problem is not the establishment of a load duration - strength relation, per se, at least for "almost clear" wood. If failure is, in fact, a creep rupture phenomenon, the <u>entire load history</u> must be known as a function of time in order to predict failure. The duration of any one load is of secondary importance. This would require a stochastic process (rather than a random variable) description of each load, and the profession is years away from being able to do this. In order to treat structural reliability of wood members as a random variable problem, then, "equivalent" load durations must be specified. This is discussed subsequently.

<u>Size Effects</u>

While test data are inconclusive regarding effect of member size on tension and compression strength, most data show a definite size effect for flexure, with the modulus of rupture tending to decrease with member size. Most codes and standards require that the allowable stress be reduced in beams which are over 12 in (305 mm) in depth.

This size effect has been studied using statistical strength theories based on the weakest link hypothesis [E.1], in which F_r is dependent on the volume and type of loading. If F_r is independent of beam width, and the span-to-depth ratio is constant, then on the basis of tests on small clear Douglas Fir beams,

$$\frac{F_{r_2}}{F_{r_1}} = \left(\frac{d_1}{d_2}\right)^{1/9} \tag{E.3}$$

in which F_i, d_i = modulus of rupture and depth of beam i. This is the formula used to correct for beam depth in Refs. E.16 and E.22.

The exponent 1/9 is related to the c.o.v. in F_r as a consequence of the Weibull strength theory [E.1]. It would be expected that the exponent in Eq. E.3 would increase for glulam and dimension lumber. In fact, data analyzed by Sexsmith and Fox [E.21] show this to be the case, where a relation between strength and volume $F_r = aV^{-1/10.6}$ was presented for glulam beams. Making the same assumptions as those leading to Eq. E.3 with these data, we would obtain

$$\frac{F_{r_2}}{F_{r_1}} = (d_1/d_2)^{1/5.3} \tag{E.4}$$

The exponent implies that the variability in F_r for large members is approximately 1.5 that for small clear specimens; interestingly, this is approximately the same ratio cited by Wood [26]. The scatter about the regression line underlying Eq. E.4 implies a variability of approximately 0.08 due to size effect.

Reliability Analysis for Glulam Members

The basic resistance variable used in the reliability studies for glulam beams is defined by,

$$R = (F_r \cdot S) \cdot SL \cdot Size \tag{E.5}$$

in which $F_r \cdot S$ = basic 5-minute test value, the statistics of which have been described in Table E.1. This must be modified by size and load duration parameters. The mean and c.o.v. in R are defined by,

$$\overline{R} = (\overline{F_r \cdot S}) \cdot \overline{SL} \cdot \overline{Size} \tag{E.6}$$

$$V_R = [V^2_{F_r S} + V^2_{SL} + V^2_{size}]^{1/2} \tag{E.7}$$

Data in Table E.1 show that most values of $\overline{F}_r$ fall between 2.5 F_n and 2.9 F_n (recall F_n = 10-yr nominal design stress), with an average value of 2.72 F_n. In computing this value, the FPL data were not arbitrarily increased to account for marginal quality tension laminating stock because the data from Ref. E.21 (which presumably was unbiased) indicated that such an increase may not be warranted. However, the reliability calculations will be performed over a range of values to show the sensitivity to this parameter. A representative value of V_{F_r} is 0.14. Statistical variations in the section modulus S are inconsequential in comparison with those in the other parameters and as a consequence, S is treated as deterministic.

In practice, beams are corrected for depth using Eq. E.3, while the true correction is given by Eq. E.4. The size adjustment factor in Eq. E.5, which accounts for the difference between the size adjustment required (Eq. E.3) and that needed (Eq. E.4) is ,

$$\overline{Size} = \left(\frac{d_{test}}{d}\right)^{\left(\frac{1}{5.3} - \frac{1}{9}\right)} \qquad (E.8)$$

in which d_{test} = depth of beams upon which the statistical analysis of strength was based (Table E.1) and d = depth of beam in service. Most of the beams in Table E.1 are from 12 in - 24 in (305 - 611 mm) in depth, while beams in service would vary from 12 -36 in (305 - 915 mm) in depth. Corresponding to (d_{test}/d) = 0.6 - 1.0, the size parameter varies in mean value from 0.96 - 1.00. For this study, it will be taken as 0.98, reflecting the fact that most glulam beams in service are somewhat larger than those used in the test series; V_{size} = 0.08.

The mean of the load duration parameter depends on the transient load in the load combination (live, snow, wind, etc.). The variability V_{SL} = 0.07.

Live Load The maximum live load on the structure occurs due to the superposition of an extraordinary load upon the sustained load. For the lighter occupancies, the extraordinary load is a substantial percentage of the total live load. The duration of this load would usually be less than a day or two per event and cumulatively would account to a month or less during a 50-yr reference period. Strength levels calculated from Eqs. E.1 and E.2 for a range of cumulative load durations are shown in Table E.3. For extraordinary live loads, the range of interest would be 7-30 days.

Table E.3

Cumulative Duration	Strength Level (Percent)	
	Eq. E.1	Eq. E.2
7 days	76.5	74.9
30 days	72.7	71.2
2 months	71	69.5
3 months	70	68.5
6 months	68	66.7

Because of the relative insensitivity of SL to duration, a value of $\overline{SL} = 0.74$ is assumed for maximum live load.

The mean and c.o.v. of the basic resistance variable used in reliability analysis for live load are (existing standards allow no increase in resistance R_n for live loads)

$$\overline{R}/R_n = \frac{(2.72\ F_n S)\ (0.74)\ (0.98)}{F_n\ S} = 1.97 \tag{E.9}$$

$$V_R = [(0.14)^2 + (0.07)^2 + (0.08)^2]^{1/2} = 0.18 \tag{E.10}$$

The probability distribution for R is very close to Weibull because of the relative magnitudes of V_{F_r}, V_{size} and V_{SL}. While the characterization of flexural capacity is felt to be representative of what would be expected in practice, the sensitivity of the reliability analysis to $\overline{R}/R_n$ and c.d.f. will be investigated.

Snow Load Snow loads remain on glulam-supported roofs long enough for a certain amount of cumulative damage to occur. The strength level shown in Table E.3 for cumulative durations of from 1 to 6 months appears insensitive to the exact cumulative duration. Refs. E.16 and E.25 allow a 15 percent increase in allowable stress. Accordingly a value $\overline{SL} = 0.70$ is selected for snow load and $\overline{R}/R_n$ becomes

$$\overline{R}/R_n = \frac{(2.72\ F_n S)\ (0.70)\ (0.98)}{1.15\ F_n S} = 1.62$$

with $V_R = 0.18$ as before.

Dead Load only For permanent loads, Eq. E.1 yields $\overline{SL} = 0.59$; Refs. E.16 and E.25 require that the allowable stress be reduced by 10 percent. Accordingly,

$$\overline{R}/R_n = \frac{(2.72\ F_n S)\ (0.59)\ (0.98)}{0.9\ F_n S} = 1.75$$

Wind and Earthquake The duration of the 50-year maximum wind or earthquake load certainly is less than 5 minutes. On the other hand, some cumulative damage may already have occurred through static fatigue. For cumulative duration of strong wind over a 50 year reference period ranging from 30 minutes to 4 hrs, the strength reduction in Eq. E.1 decreases from 0.95 to 0.88. Refs. E.16 and E.25 allow a 33 percent increase in nominal resistance. Accordingly,

$$\bar{R}/R_n \simeq \frac{(2.72\ F_n S)\ (0.90)\ (0.98)}{1.33\ F_n S} = 1.80$$

Statistics for tension and compression members are handled similarly, except the size effect is not included as in Eq. E.5. A summary of strength of glulam members is given in Table E.4.

Table E.4

Statistics of Strength of Glulam Members

Maximum Load in Combination	Bending		Tension		Compression	
	$\bar{R}/R_n$	V_R	$\bar{R}/R_n$	V_R	$\bar{R}/R_n$	V_R
D	1.75	0.18	1.74	0.21	1.73	0.14
L	1.97	0.18	1.96	0.21	1.95	0.14
S	1.62	0.18	1.61	0.21	1.60	0.14
W	1.80	0.18	1.79	0.21	1.78	0.14
E	1.80	0.18	1.79	0.21	1.78	0.14

Calculated values of β for beams loaded with the D + S and D + L load combinations are presented in Figures E.2 and E.3. $\bar{R}/R_n$ and V_R have been varied from the summary statistics in Table E.4 to illustrate the sensitivity of β to these parameters. $\bar{L}/L_n$ = 1.0 in Fig. E.2. When L_n/D_n and S_n/D_n are small, β is quite sensitive to the assumed distribution for R. When the dead load acts alone, β = 2.45 if R is lognormal and β = 1.91 if R is Weibull. Since V_D is small, the characteristics of R dominate the reliability analysis. For more realistic values of L_n/D_n and S_n/D_n, however, in the range of 2 - 4, this sensitivity is much less pronounced. The reliability apparently is somewhat less for the D + S combination than for the D + L combination. Figure E.4 illustrates the reliability index for tension and compression members subjected to wind. The large c.o.v. in tension strength causes β for this case to be somewhat less than for either compression or bending members.

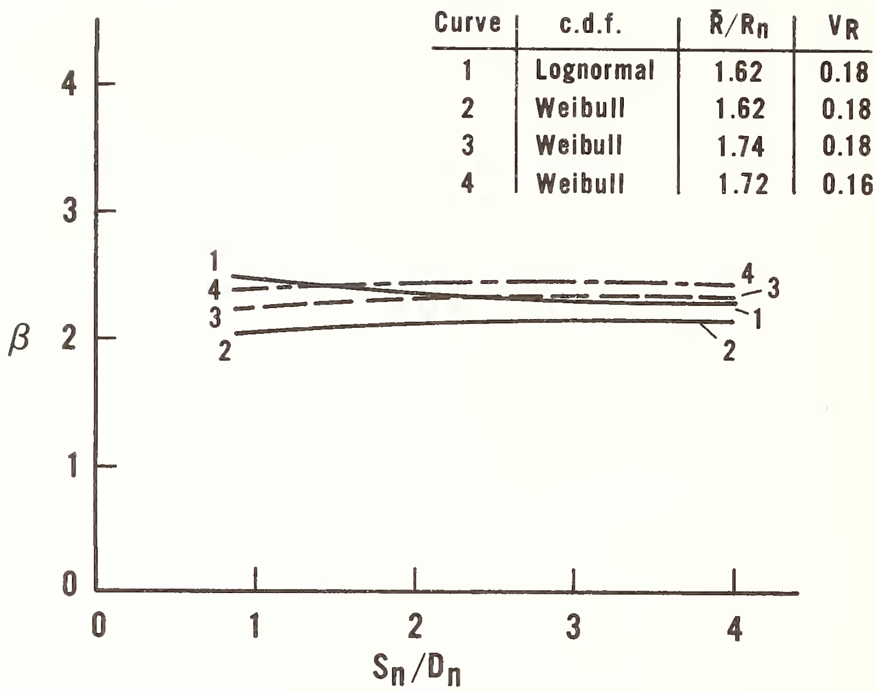

Figure E.2 - Reliability Index for Glulam Beams - Snow Load

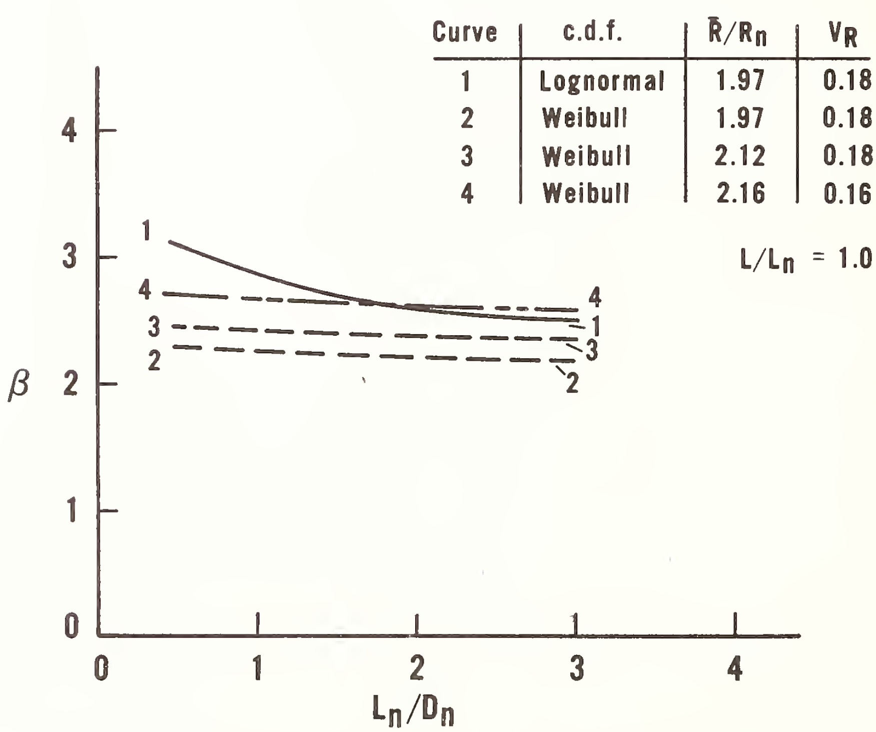

Figure E.3 - Reliability Index for Glulam Beams - Live Load

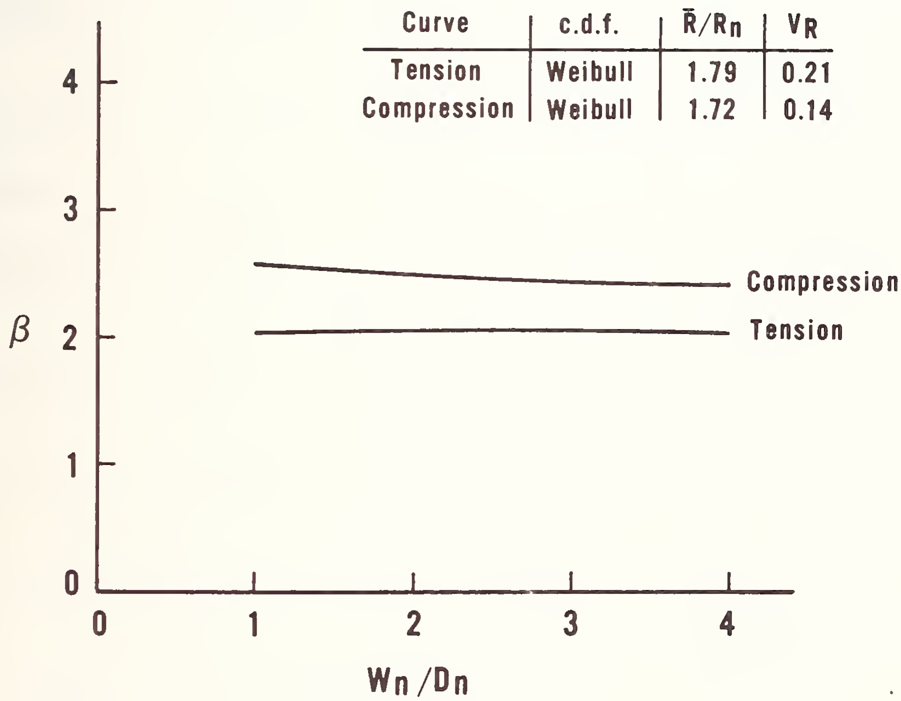

Figure E.4 - Reliability Index for Tension and Compression Members - Wind Load

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APPENDIX F - COMPUTER PROGRAM

The purpose of this appendix is to describe the computer program used in the reliability analyses. Two separate problems can be handled with this program: (1) for a given design situation defined by a set of nominal load and resistance variables, calculate β (analysis), and (2) for a prescribed β and set of nominal loads, calculate the required nominal resistance and partial factors to be applied to the nominal value of each basic variable in the limit state equation (Level II design). The analysis procedure is summarized in Chapter 2 where an example calculation is also given.

The computer program can work with the following two-parametered probability distributions

Normal

Lognormal

Gamma

Gumbel (Extreme Value Type I)

Frechet (Extreme Value Type II)

Weibull (Extreme Value Type III)

Additional distribution functions may be added if desired. In addition, several different forms of the limit state equation are allowed in the present version:

$$X_1 \mp X_2 \mp X_3 \ldots \mp X_n = 0 \qquad \text{(F.1)}$$

$$bX_1X_2 - X_3 = 0 \qquad \text{(F.2)}$$

$$X_1 - (X_2t + 6X_3)/bt^2 = 0;\ 0 \leq X_3/X_2t \leq 1/6 \qquad \text{(F.3a)}$$

$$X_1 - \frac{2}{3}\frac{P}{b}\left(\frac{t}{2} - \frac{X_3}{X_2}\right) = 0;\ 1/6 < X_3/X_2t \leq 1/2 \qquad \text{(F.3b)}$$

in which X_i = basic variables and b,t = constants. Eq. F.1 is the common linear form of the limit state equation. Eq. F.2 is an alternate description of the limit state for a simple tension or bending member, in which X_1,X_2 = yield stress and section property, respectively, and X_3 = total load effect. Eqs. F.3a and F.3b describe the strength of an unreinforced masonry wall in compression plus bending, and were used to determine β at large vertical load eccentricities. Additional limit states could be added, if desired.

The linear form of the limit state equation was used for most of the calibrations and all of the Level II design calculations. The program assumes that X_1 is the resistance variable; X_1 may have a normal, lognormal, or Weibull probability distribution. When the design option is selected, the iteration is performed on $\overline{X}_1$. Variables $X_2 - X_6$ may have any of the six distributions listed previously. There is no restriction as to which X - variable describes which load.

The basic information required for either analysis or design options is the probability distribution for each variable, the ratio of mean to nominal value, $\overline{X}/X_n$, and the coefficient of variation V_X. For extreme value distributions I and II, the ratio of characteristic extreme to nominal, u/X_n, and the shape parameter k may be specified instead. There are additional input variables that describe the size of the problem, number of analysis or design situations to be considered, descriptors to assist in interpreting output, and so forth. Design situations may then be specified by a set of nominal loads and resistances. The means are then computed as $\overline{X} = (\overline{X}/X_n)\ X_n$ and the solution proceeds as described in Chapter 2. A detailed description of input data follows.

(1) NCASES

(2) HEADING FOR PROBLEM - ARBITRARY - MAXIMUM OF 72 CHARACTERS

(3) PROB N NG NLRFD BTA B T

(4) $(TYPE)_i$ $(DIST)_i$ $(\overline{X}/X_n)_i$ $(c.o.v.)_i$ γ_i

⋮

One card for each of "N" X-Variables.

⋮

(5) X_{n_1} X_{n_2} X_{n_3} . . . . .

⋮

One card for each of "NLRFD" design situations

⋮

Card sets (2) - (5) may be repeated "NCASES" times.

The above parameters are defined as follows.

NCASES = number of problems - a problem is defined by a set of X-variables and their statistics (no limit).

PROB = ANALYS - calculate β for design situation

DESIGN - calculate partial factors for fixed β (Level II design).

N = number of X-variables in limit state equation

NG = designation of limit state;

= 1 - Eq. F.1, = 2 - Eq. F.2, = 3 - Eq. F.3

NLRFD = number of design situations in the problem (no limit)

BTA = reliability index β. If PROB = ANALYS, BTA is the initial guess at solution for $g(\beta) = 0$; if PROB = ANALYS, BTA is the target reliability for which Level II partial factors are sought.

B,T = constants. If NG = 1, they are not referenced. If NG = 2, B is an appropriate constant in Eq. F.2. If NG = 3, B,T = width, thickness of masonry wall.

$(\mathrm{TYPE})_i$ = user - defined description of X_i, e.g., "resist," "wind," etc.; maximum of 6 characters

$(\mathrm{DIST})_i$ = Probability distribution of X_i

= NORMAL - Normal distribution

= LOGNOR - Lognormal distribution

= GAMMMA - Gamma distribution

= GUMBEL - Type I Extreme value

= FRECHE - Type II Extreme value

= WEIBUL - Type III Extreme value

$(\overline{X}/X_n)_i$, $(\mathrm{c.o.v.})_i$ = mean-to-nominal, coefficient of variation

γ_i = partial safety factor for $(X_n)_i$. If PROB = ANALYS, γ_i is not needed as input.

$X_{n_1}, X_{n_2}, \ldots, X_{n_N}$ = nominal load and resistance variables which define each design situation. When the design option is selected, $X_{n_1} \cdot (\overline{X}_1/X_{n_1})$ is the initial guess at the solution for $g(\overline{X}_1) = 0$.

Table F.1 shows the input data used to calculate β's for existing reinforced concrete beams under the D + L + W combination. Two values of L_0/D were selected: 0.5 and 1.0. Four values of W_n/D were considered at each L_o/D. Since $A_T = 400\ \mathrm{ft}^2$, $L_n = 0.68L_o$; $\overline{L}/L_n = 1.147$ for $D + L + W_{apt}$ and $\overline{L}/L_n = 0.353$ for $D + L_{apt} + W$. The statistics for maximum wind are $\overline{W}/W_n = 0.78$, $V_W = 0.37$, while for arbitrary-point-in-time wind, $u/W_n = -0.021$ and $k = 18.7$, the characteristic extreme and shape, respectively. The program is able to make the distinction by testing the magnitude of $(\mathrm{c.o.v.})_i$ in card (4); if the value in this location exceeds 1.0, the program assumes that u/X_n and K were given.

A listing of the program follows. The addition of other limit states would require changes to subroutine GDGDX. Other distribution functions would require additional statements in subroutines CALC and PARAME. Separate subroutines must also be added to compute $F_{X_i}(X_i^*)$, $f_{X_i}(X_i^*)$, $\overline{X}_i^N$, $\sigma_{X_i}^N$ in accordance with Eqs. 2.24 (cf. subroutine FRECHE, which performs these operations for the Extreme Value Type II c.d.f.).

The program was written in Standard Fortran for a UNIVAC 1108 Exec 8 system. Several functions and subroutines from the UNIVAC scientific package were used which would have to be changed if the program were to run on another system:

TINORM ( ) - Inverse of standard normal distribution function: $X = \Phi^{-1}(p)$

GAMIN ( ) - Incomplete gamma function, necessary to evaluate gamma probability distribution function $= \int t^{n-1} e^{-t} dt/\Gamma(n)$

GAMMA ( ) - Complete gamma function.

Two cautionary notes are in order. First of all, the entire program (including UNIVAC - supplied routines) is written in single precision. When β becomes large (say, 5 or greater) round off errors may occur when quantities such as $1 - p_f$ are computed. Second, convergence problems were encountered in the cases where $\beta\, \sigma^N_{X_i}/\overline{X}^N_i \simeq 1.0$. This difficulty appears to be inherent to this particular reliability method, which replaces the actual non-normal variables with fitted normal variables prior to performing the reliability analysis. Consider, for example, the simple two-variable problem,

$$X_1 - X_2 = 0$$

The reliability analysis leads to a value of β:

$$\beta = \frac{\overline{X}^N_1 - \overline{X}^N_2}{[(\sigma^N_{X_1})^2 + (\sigma^N_{X_2})^2]^{1/2}}$$

where $\overline{X}^N_i$, $\sigma^N_{X_i}$ = mean, standard deviation in accordance with Eqs. 2.24. Conversely, the central factor of safety is,

$$\overline{X}^N_1/\overline{X}^N_2 = \frac{[V_1^2 + V_2^2\,(1 - \beta^2 V_1^2)]^{1/2}}{(1 - \beta^2 V_1^2)}$$

in which $V_i = \sigma^N_{X_i}/\overline{X}^N_i$. It is clear that as $\beta V_1 \rightarrow 1$, the central factor of safety increases without bound. There is no obvious way of circumventing this problem, and users of the method should be aware that it might occur. This was encountered in some of the analyses of masonry walls in "nearly pure" compression and of some connections where both variability in behavior and conservatism in practice are high.

Table F.1 - Sample Data Preparation

```
4
 P/C - FLEXURE - GRADE 40 - MED RHO - D + LI + WMAX   AI = 800 FT**2
   ANALYS         4          1          8         3.0
   RESIST     NORMAL     1.213      0.145        0.90
     DEAD     NORMAL     1.050       0.10        1.05
     LIVE     GAMMMA     0.353       0.55       1.275
     WIND     GUMBEL     0.780      0.370       1.275
    2.198        1.0      0.34       0.25
    2.357        1.0      0.34       0.50
    2.712        1.0      0.34       0.75
    3.066        1.0      0.34       1.00
    2.841        1.0      0.68       0.25
    2.841        1.0      0.68       0.50
    3.193        1.0      0.68       0.75
    3.543        1.0      0.68       1.00
 R/C - FLEXURE - GRADE 40 - MED RHO - D + LMAX + WI   AI = 800 FT**2
   ANALYS         4          1          8         3.0
   RESIST     NORMAL     1.213      0.145        0.90
     DEAD     NORMAL     1.050       0.10        1.05
     LIVE     GUMBEL     1.147       0.25       1.275
     WIND     GUMBEL    -0.021      18.70      1.275
    2.198        1.0      0.34       0.25
    2.357        1.0      0.34       0.50
    2.712        1.0      0.34       0.75
    3.066        1.0      0.34       1.00
    2.841        1.0      0.68       0.25
    2.841        1.0      0.68       0.50
    3.193        1.0      0.68       0.75
    3.543        1.0      0.68       1.00
 R/C - FLEXURE - GRADE 60 - MED RHO - D + LI + WMAX   AI = 800 FT**2
   ANALYS         4          1          8         3.0
   RESIST     NORMAL     1.103      0.110        0.90
     DEAD     NORMAL     1.050       0.10        1.05
     LIVE     GAMMMA     0.353       0.55       1.275
     WIND     GUMBEL     0.780      0.370       1.275
    2.198        1.0      0.34       0.25
    2.357        1.0      0.34       0.50
    2.712        1.0      0.34       0.75
    3.066        1.0      0.34       1.00
    2.841        1.0      0.68       0.25
    2.841        1.0      0.68       0.50
    3.193        1.0      0.68       0.75
    3.543        1.0      0.68       1.00
 R/C - FLEXURE - GRADE 60 - MED RHO - D + LMAX + WI   AI = 800 FT**2
   ANALYS         4          1          8         3.0
   RESIST     NORMAL     1.103      0.110        0.90
     DEAD     NORMAL     1.050       0.10        1.05
     LIVE     GUMBEL     1.147       0.25       1.275
     WIND     GUMBEL    -0.021      18.70       1.275
    2.198        1.0      0.34       0.25
    2.357        1.0      0.34       0.50
    2.712        1.0      0.34       0.75
    3.066        1.0      0.34       1.00
    2.841        1.0      0.68       0.25
    2.841        1.0      0.68       0.50
    3.193        1.0      0.68       0.75
    3.543        1.0      0.68       1.00
```

Table F.2 - Computer Program Listing

```
C   PROGRAM A58LF CALCULATES SAFETY INDEX BETA FOR GIVEN DESIGN
C   OR COMPUTES PARTIAL FACTORS FOR GIVEN BETA.
C   TYPE(I) = VARIABLE IDENTIFIER
C   DIST(I) = PROBABILITY DISTRIBUTION
C   U1(I), U2(I) = FIRST AND SECOND MOMENT PARAMETERS OF PROBABILITY
C   DISTRIBUTION, I.E., U1 = MEAN, OR CHARACTERISTIC EXTREME.
C   R(I), MX(I), CVX(I) = MEAN/NOMINAL RATIO, MEAN, COEFFICIENT
C   OF VARIATION.
C   XN(I) = NOMINAL DESIGN VALUES.
C   PF(I) = PARTIAL SAFETY FACTORS
      REAL MX, MXN, K
      DIMENSION TYPE(6), HEADER(12)
      COMMON/INSTAT/DIST(6), R(6), MX(6), CVX(6), K(6), U(6),
     1U1(6), U2(6)
      COMMON/CONSTS/N, NAL, NNR, NITAL, EPS, NG, B, T
      COMMON/NOMINL/XN(6), PF(6)
      COMMON/METRIC/X(6),MXN(6),SDXN(6),AL(6), BETA, BTA
      DATA/EPS,NAL,NNR,NITAL/0.001,100,20,20/
      READ 905, NCASES
905   FORMAT( )
      DO 1000 ICASE = 1, NCASES
C
C   READ IN BASIC PROGRAM VARIABLES
C
      READ 900, (HEADER(I), I = 1, 12)
      PRINT 902, (HEADER(I), I = 1, 12)
      READ 901, PROB, N, NG, NLRFD, BTA,  B, T
      IF(PROB .EQ. 'ANALYS') PRINT 802
      IF(PROB .EQ. 'DESIGN') PRINT 806, BTA
      DO 10 I = 1, N
10    READ 903, TYPE(I),DIST(I),U1(I),U2(I),PF(I)
      PRINT 800, (TYPE(I),I=1,N)
      PRINT 801, (DIST(I),I=1,N)
      IF(PROB .EQ. 'ANALYS') PRINT 803, (PF(I),I=1,N)
C
C   PERFORM ANALYSIS OR DESIGN CALCULATIONS FOR EACH OF FOLLOWING
C   NLRFD LOADING SITUATIONS.
C
      DO 1000 JJ = 1, NLRFD
      READ 904, (XN(I), I=1,N)
C   CIMPUTE DISTRIBUTION PARAMETERS OF PROGRAM VARIABLES FROM
C   NOMINAL DATA INPUT.
      CALL PARAME
C
C   BEGIN RELIABILITY CALCULATIONS.  ITERATIONS PERFORMED WITHIN
C   SUBROUTINE CALC.
      CALL CALC(PROB)
      PRINT 807, (XN(I),I=1,N)
      PRINT 804, (R(I),I=1,N)
      PRINT 805, (CVX(I),I=1,N)
      PRINT 808, (X(I),I=1,N)
      PRINT 809, (AL(I),I=1,N)
      IF(PROB .EQ. 'ANALYS') PRINT 810, BETA
      IF(PROB .EQ. 'DESIGN') PRINT 803, (PF(I),I=1,N)
1000  CONTINUE
C
```

```
900    FORMAT(12A6)
901    FORMAT(4X,A6,3I10,3F10.0)
902    FORMAT(////12A6//)
903    FORMAT(2(4XA6),3F10.0)
800    FORMAT(//' VARIABLE IN G( ) = 0',6(4X,A6))
801    FORMAT('          DISTRIBUTION',6(4X,A6))
802    FORMAT(//20X,'ANALYSIS'//)
803    FORMAT('      PARTIAL FACTORS',6F10.3)
804    FORMAT('          MEAN/NOMINAL',6F10.3)
805    FORMAT('                C.O.V.',6F10.3)
806    FORMAT(//20X,'DESIGN - BETA =',F6.3//)
807    FORMAT(//'                 XN(I)',6F10.2)
808    FORMAT('                  X(I)',6F10.3)
809    FORMAT('              ALPHA(I)',6F10.3)
810    FORMAT(15X,'***** BETA =',F6.2,' *****')
904    FORMAT(6F10.0)
       STOP
       END
       SUBROUTINE CALC(PROB)
C   CALC IS THE MAIN ROUTINE PERFORMING THE ITERATIONS OF STEPS 4 - 10
       EXTERNAL FBETA
       REAL MX, MXN, K
       DIMENSION XP(6), DGDX(6), A(6)
       COMMON/INSTAT/DIST(6), R(6), MX(6), CVX(6), K(6), U(6)
       COMMON/CONSTS/N, NAL, NNR, NITAL, EPS, NG
       COMMON/NOMINL/XN(6),PF(6)
       COMMON/METRIC/X(6),MXN(6),SDXN(6),AL(6), BETA, BTA
C   SET INITIAL CHECKING POINT VALUES EQUAL TO MEANS
       ITAL = 1
       BETA = BTA
       DO 10 I = 1, N
10     X(I) = MX(I)
99     IAL = 1
C   COMPUTE PARTIAL DERIVATIVES AT CHECKING POINT
100    CALL GDGDX(X,G,DGDX)
C
C   COMPUTE MEAN, STANDARD DEVIATION OF EQUIVALENT NORMAL DISTRIBUTION
C HAVING SAME CUMULATIVE AND DENSITY AT THE CHECKING POINT
       DO 17 I = 1, N
       IF(DIST(I) .EQ. 'NORMAL') GO TO 11
       IF(DIST(I) .EQ. 'LOGNOR') GO TO 12
       IF(DIST(I) .EQ. 'GAMMMA') GO TO 13
       IF(DIST(I) .EQ. 'GUMBEL') GO TO 14
       IF(DIST(I) .EQ. 'FRECHE') GO TO 15
       IF(DIST(I) .EQ. 'WEIBUL') GO TO 16
11     MXN(I) = MX(I)
       SDXN(I) = CVX(I)*MX(I)
       GO TO 17
12     CALL LOGNOR(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
13     CALL GAMMAL(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
14     CALL GUMBEL(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
15     CALL FRECHE(X(I),U(I),K(I),MXN(I),SDXN(I))
       GO TO 17
16     CALL WEIBUL(X(I),U(I),K(I),MXN(I),SDXN(I))
17     CONTINUE
C
C   COMPUTE DIRECTION COSINES FOR EACH VARIABLE
C
       SUM = 0.
       DO 20 I = 1, N
```

```
      A(I) = DGDX(I)*SDXN(I)
20    SUM = SUM + A(I)*A(I)
      SUM = SQRT(SUM)
      DO 21 I = 1, N
21    AL(I) = A(I)/SUM
C  COMPUTE NEW CHECKING POINT VALUES
      DO 22 I = 1, N
      XP(I) = X(I)
22    X(I) = MXN(I) - AL(I)*BETA*SDXN(I)
C  TEST WHETHER INTERIM ESTIMATES OF X(I) HAVE STABILIZED
      DO 24 I = 1, N
24    IF(ABS((X(I)-XP(I))/X(I)) .GT. 0.005) GO TO 23
      IF(PROB .EQ. 'ANALYS') GO TO 30
      IF(PROB .EQ. 'DESIGN') GO TO 31
23    IAL = IAL + 1
      IF(IAL .LE. NAL) GO TO 100
      GO TO 43
C
C  ANALYSIS PROBLEM.
C  COMPUTE VALUE OF BETA SUCH THAT G(    ) = 0.
C
30    BST = BETA
      CALL   NI(BETA,FBETA,BST,EPS,NNR,IER)
      IF(IER .EQ. 0) GO TO 25
      GO TO 41
C  TEST FOR CONVERGENCE OF SOLUTION
25    IF(ABS((BETA-BST)/BETA) .LT. 0.005) RETURN
      ITAL = ITAL + 1
      IF(ITAL .LE. NITAL) GO TO 99
      GO TO 42
C
C  DESIGN PROBLEM.
C  MODIFY MX(1) SO AS TO ACHIEVE G( ) = 0.
C
31    IF(ITAL .EQ. 1) GO TO 26
      DTHDG = (MX(1) - TH)/(G - GTH)
      TH = MX(1)
      GTH = G
      MX(1) = MX(1) - G*DTHDG
      IF(ABS((MX(1)-TH)/MX(1)) .LT. 0.005) GO TO 28
      GO TO 32
26    TH = MX(1)
      GTH = G
      IF(G) 27,28,29
27    MX(1) = 1.1*MX(1)
      GO TO 32
29    MX(1) = 0.9*MX(1)
32    ITAL = ITAL + 1
      IF(ITAL .GT. NITAL) GO TO 42
      CALL PARAMR
      GO TO 99
C
C  COMPUTE PARTIAL FACTORS FOR NOMINAL LOADS AND RESISTANCES.
C
28    XN(1) = MX(1)/R(1)
      DO 33 I = 1, N
33    PF(I) = X(I)/XN(I)
      RETURN
C
C  ERROR MESSAGES
C
41    PRINT 101
101   FORMAT(' SOLUTION OF G( ) = 0 NONCONVERGENT')
      CALL EXIT
```

```
42      PRINT 102
102     FORMAT(' SOLUTION OF N+1 EQUATIONS NONCONVERGENT')
        CALL EXIT
43      PRINT 103
103     FORMAT(' INTERIM SOLUTION FOR AL(I) NONCONVERGENT')
        CALL EXIT
        END
-----------------------------------------------------------------
        SUBROUTINE FBETA(XX,F,DERF)
        REAL MXN
C   SUBROUTINE EVALUATES G( ) AND ITS DERIVATIVES WITH RESPECT TO BETA
        COMMON/CONSTS/N
        COMMON/METRIC/X(6),MXN(6),SDXN(6),AL(6),BETA,BTA
        DIMENSION X1(6), DGDX(6)
        DO 20 I = 1, N
20      X1(I) = MXN(I) - AL(I)*XX*SDXN(I)
        CALL GDGDX(X1,G,DGDX)
        F = G
        DERF = 0.0
        DO 21 I = 1, N
        DXDB = -AL(I)*SDXN(I)
21      DERF = DERF + DGDX(I)*DXDB
        RETURN
        END
-----------------------------------------------------------------
        SUBROUTINE FRECHE(X,U,AL,MXN,SDXN)
        REAL MXN
        A = (U/X)**AL
        FC = EXP(-A)
        FD = FC*A*AL/X
        CALL XNORM(X,FC,FD,MXN,SDXN)
        RETURN
        END
-----------------------------------------------------------------
        SUBROUTINE FXX(X,F,DERF)
        COMMON/FXNORM/FC1
        PHIX = 0.398942*EXP(-X*X/2.)
        F = PHIX*((1./X)-(1./X**3)+(3./X**5))-FC1
        DERF = -PHIX*(1.+15./X**6)
        RETURN
        END
-----------------------------------------------------------------
        SUBROUTINE GAMMAL(X,LAM,K,MXN,SDXN)
        REAL LAM, K, MXN
        XX = LAM*X
        CALL GAMMA(K,GK,$21,$22)
        FC = GAMIN(XX,K)
        FD = LAM*XX**(K-1)*EXP(-XX)/GK
        CALL XNORM(X,FC,FD,MXN,SDXN)
        RETURN
21      WRITE(6,200)
200     FORMAT('***LOG10(GX) HAS BEEN COMPUTED***')
        GO TO 23
22      WRITE(6,201)
201     FORMAT('***ARGUMENT IS ZERO OR NEGATIVE***')
23      CALL EXIT
        END
-----------------------------------------------------------------
        SUBROUTINE GDGDX(X,G,DGDX)
C   EVALUATE G( ) AND ITS DERIVATIVES AT POINT X(I).
        DIMENSION X(1),DGDX(1)
        COMMON/CONSTS/N,NAL,NNR,NITAL,EPS,NG,B,T
        COMMON/NOMINL/XN(6),PF(6)
        GO TO (1,2,3)     , NG
C   LIMIT STATE FUNCTION LINEAR IN BASIC VARIABLES.
1       G = X(1)
        DGDX(1) = 1.
```

```
      DO 22 I = 2, N
      IF(XN(I) .LT. 0.) GO TO 23
      DGDX(I) = -1.
      GO TO 22
23    DGDX(I) = 1.
22    G = G + DGDX(I)*X(I)
      RETURN
C
C  INSERT OTHER LIMIT STATES NEEDED
C
2     G = B*X(1)*X(2) - X(3)
      DGDX(1) = B*X(2)
      DGDX(2) = B*X(1)
      DGDX(3) = -1.
      RETURN
C
C  MASONRY WALL INTERACTION CURVE.
3     R = X(3)/(X(2)*T)
      IF(R .LT. 0.) GO TO 99
      IF(R .GT. 0.5) GO TO 99
      IF(R .GT. 0.166667) GO TO 31
C  FAILURE SURFACE 1 - UNCRACKED SECTION
      G = X(1) - (X(2)*T+6.*X(3))/(B*T*T)
      DGDX(1) = 1.
      DGDX(2) = -1./(B*T)
      DGDX(3) = -6./(B*T*T)
      RETURN
C  FAILURE SURFACE 2 - CRACKED SECTION.
31    A = .5*T - X(3)/X(2)
      C1 = 2./(3.*B*A)
      G = X(1) - C1*X(2)
      DGDX(1) = 1.
      DGDX(2) = -C1*(1. - X(3)/(A*X(2)))
      DGDX(3) = -C1/A
      RETURN
C
99    PRINT 101, R
101   FORMAT('  X(3)/(X(2)*T) =',F10.5,'  IS OUT OF RANGE')
      CALL EXIT
C
      END
```

```
      SUBROUTINE GUMBEL(X,U,AL,MXN,SDXN)
      REAL MXN
      A = EXP(-AL*(X-U))
      IF ( A.GT.7.5E-07 ) GO TO 1
      FC = A-A*A/2.
      FD = (AL*A)*(1.-FC)
      GO TO 2
    1 FC = EXP(-A)
      FD = AL*A*FC
    2 CALL XNORM(X,FC,FD,MXN,SDXN)
      RETURN
      END
```

```
      SUBROUTINE LOGNOR(X,U,AL,MXN,SDXN)
      REAL MXN
      SDXN = AL*X
      MXN = X*(1. - ALOG(X) + U)
      RETURN
      END
```

```
      SUBROUTINE PARAME
      REAL MX, K
      COMMON/INSTAT/DIST(6),R(6),MX(6),CVX(6),K(6),U(6),U1(6),U2(6)
      COMMON/CONSTS/N
      COMMON/NOMINL/XN(6),PF(6)
```

```
C   SUBROUTINE COMPUTES THE DISTRIBUTION PARAMETERS FOR THOSE
C   VARIABLES WHICH ARE NON-NORMAL
C   FROM THE MEANS, COEFFICIENTS OF VARIATION INPUT
C
      DO 9 I = 1, N
      R(I) = U1(I)
      CVX(I) = U2(I)
9     MX(I) = ABS(XN(I)*R(I))
C
C   LOAD VARIABLE PARAMETERS
      DO 20 I = 2, N
      IF(DIST(I) .EQ. 'NORMAL') GO TO 20
      IF(DIST(I) .EQ. 'LOGNOR') GO TO 12
      IF(DIST(I) .EQ. 'GAMMMA') GO TO 13
      IF(DIST(I) .EQ. 'GUMBEL') GO TO 14
      IF(DIST(I) .EQ. 'FRECHE') GO TO 15
      IF(DIST(I) .EQ. 'WEIBUL') GO TO 16
C
12    U(I) = ALOG(MX(I)/SQRT(1.+CVX(I)*CVX(I)))
      K(I) = SQRT(ALOG(1.+CVX(I)*CVX(I)))
      GO TO 20
C
13    K(I) = 1./(CVX(I)*CVX(I))
      U(I) = K(I)/MX(I)
      GO TO 20
C
14    IF(U2(I) .GT. 1.0) GO TO 140
      SDX = MX(I)*CVX(I)
      K(I) = 1.282/SDX
      U(I) = MX(I) - 0.5772/K(I)
      GO TO 20
140   U(I) = U1(I)*XN(I)
      K(I) = ABS(U2(I)/XN(I))
      MX(I) = U(I) + 0.5772/K(I)
      CVX(I) = 1.282/(K(I)*MX(I))
      R(I) = ABS(MX(I)/XN(I))
      GO TO 20
C
15    IF(U2(I) .GT. 1.) GO TO 150
      K(I) = 2.33/(CVX(I)**0.677)
      C1 = 1. - 1./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      U(I) = MX(I)/GC1
      GO TO 20
150   U(I) = U1(I)*XN(I)
      K(I) = U2(I)
      C1 = 1. - 1./K(I)
      C2 = 1. - 2./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      CALL GAMMA(C2,GC2,$21,$22)
      MX(I) = U(I)*GC1
      CVX(I) = SQRT(GC2/(GC1**2) - 1.)
      R(I) = ABS(MX(I)/XN(I))
      GO TO 20
C
16    K(I) = 1./(CVX(I)**1.08)
      C1 = 1. + 1./K(I)
      CALL GAMMA(C1,GC1,$21,$22)
      U(I) = MX(I)/GC1
20    CONTINUE
C
```

```
C   COMPUTE PARAMETERS FOR RESISTANCE VARIABLE
      ENTRY PARAMR
      IF(DIST(1) .EQ. 'NORMAL') GO TO 31
      IF(DIST(1) .EQ. 'LOGNOR') GO TO 32
      IF(DIST(1) .EQ. 'WEIBUL') GO TO 33
32    K(1) = SQRT(ALOG(1.+CVX(1)*CVX(1)))
      U(1) = ALOG(MX(1)/SQRT(1.+CVX(1)*CVX(1)))
      GO TO 31
33    K(1) = 1./(CVX(1)**1.08)
      C1 = 1. + 1./K(1)
      CALL GAMMA(C1,GC1,$21,$22)
      U(1) = MX(1)/GC1
31    RETURN
C
21    WRITE(6,200)
200   FORMAT('***LOG10(GX) HAS BEEN COMPUTED***')
      GO TO 23
22    WRITE(6,201)
201   FORMAT('***ARGUMENT IS ZERO OR NEGATIVE***')
23    CALL EXIT
      END
```

```
      SUBROUTINE   NI(X,FCT,XST,EPS,IEND,IER)
      IER = 0
      X = XST
      TOL = X
      CALL FCT(TOL,F,DERF)
      TOLF = 100.*EPS
      DO 6 I = 1, IEND
      IF(F)1,7,1
1     IF(DERF)2,8,2
2     DX = F/DERF
      XP = X
      X = X - DX
C
C   PREVENT NEGATIVE ROOT OR OVERSHOOTING
      IF(X .LE. 0.0) X = XP/10.
C
      TOL = X
      CALL FCT(TOL,F,DERF)
      TOL = EPS
      A = ABS(X)
      IF(A-1.)4,4,3
3     TOL = TOL*A
4     IF(ABS(DX) - TOL) 5,5,6
5     IF(ABS(F) - TOLF) 7,7,6
6     CONTINUE
      IER = 1
7     RETURN
8     IER = 2
      RETURN
      END
```

```
      SUBROUTINE WEIBUL(X,U,AL,MXN,SDXN)
      REAL MXN
      A = (X/U)**AL
      FC = EXP(-A)
      FD = AL*A*FC/X
      FC = 1. - FC
      CALL XNORM(X,FC,FD,MXN,SDXN)
      RETURN
      END
```

```
      SUBROUTINE XNORM(X,FC,FD,MXN,SDXN)
      EXTERNAL  FXX
      COMMON/CONSTS/N,NAL,NNR,NITAL,EPS
      COMMON/FXNORM/FC1
      FC1 = FC
      REAL  MXN
      IF ( FC.GT.7.5E-07 ) GO TO 1
      XST = 4.8
      CALL RTNI (XX,FXX,XST,EPS,NNR,IER)
      GO TO 2
    1 XX = TINORM(FC,$21)
    2 SDXN = 0.398942*EXP(-XX*XX/2.)/FD
      MXN = X - XX*SDXN
      RETURN
21    WRITE(6,100) FC
100   FORMAT(10X,'****EXIT CALLED FROM XNORM - FC =',E15.5)
      CALL EXIT
      END
```

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Recommended load factors and load combinations are presented which are compatible with the loads recommended in the proposed 1980 version of American National Standard A58, Building Code Requirements for Minimum Design Loads in Buildings and Other Structures (ANSI A58.1-1980 D). The load effects considered are due to dead, occupancy live, snow, wind and earthquake loads. The load factors were developed using concepts of probabilistic limit states design which incorporate state-of-the-art load and resistance models and available statistical information. Reliabilities associated with representative structural members and elements designed according to current (1979) structural specifications were calculated for reinforced and prestressed concrete, structural steel, cold-formed steel, aluminum, masonry and glued-laminated timber construction. The report presents the rationale for selecting the criterion format and load factors and describes the methodology to be followed by material specification groups for determining resistance factors consistent with the implied level of reliability and the statistical data. The load factors are intended to apply to all types of structural materials used in building construction.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Aluminum; buildings (codes); design (buildings); concrete (prestressed); concrete (reinforced); limit states; loads (forces); masonry; probability theory; reliability; safety; specifications; standards; statistical analysis; steel; structural engineering; timber.

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