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NBS Special Publication 545

Contributions to Color Science

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NOTE TO READER

The following color illustrations:

Figure 9 & 10	page 558
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Figures	page 667 &668

have been reproduced from previously printed articles and do not represent the true colors. For true renditions of the colors (and the accompanying captions for pages 667 & 668), the reader should consult the original article.

Contributions to Color Science

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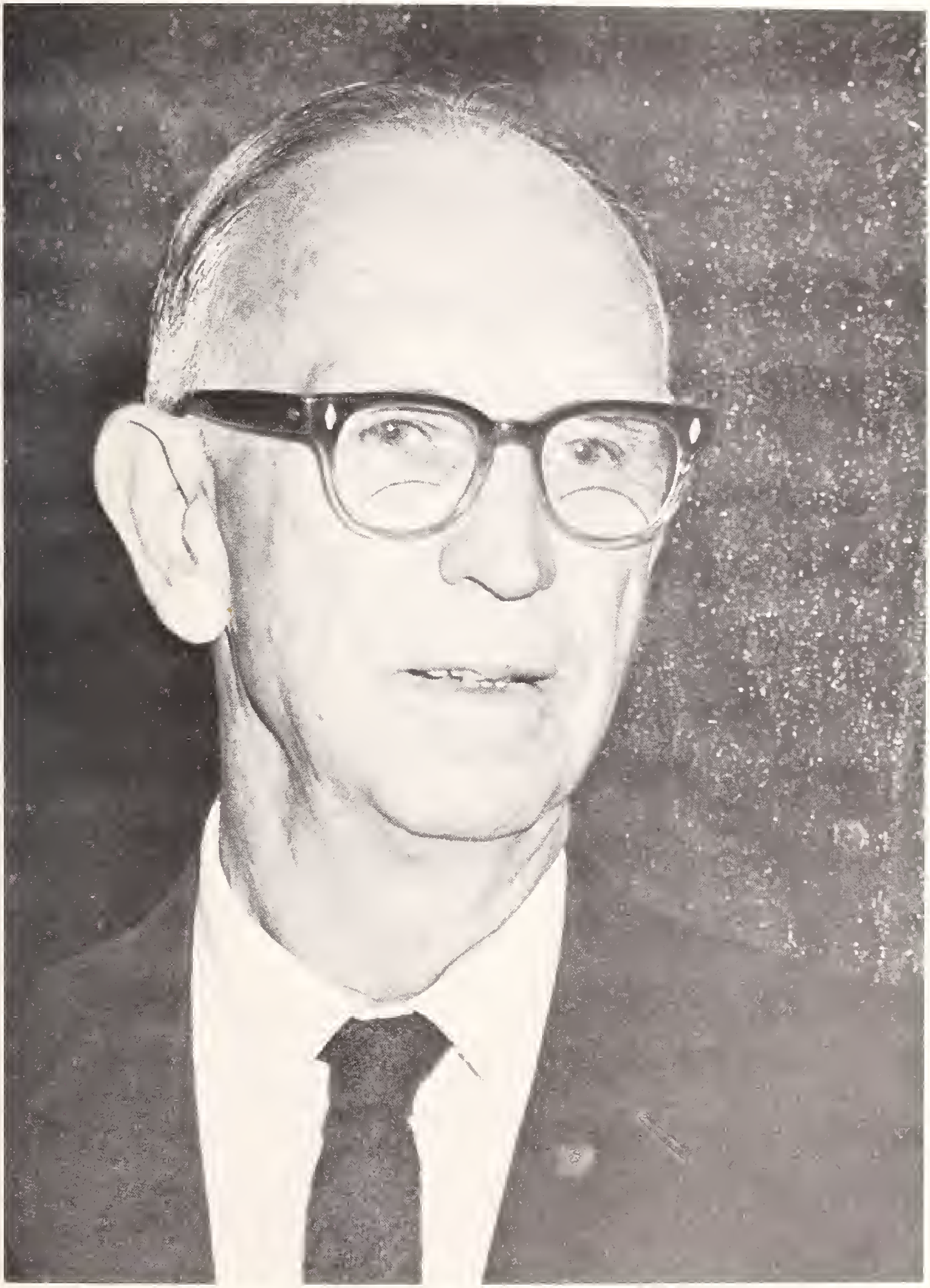
FRONTISPIECE

Deane Brewster Judd
at meeting of CIE Committee on Colorimetry during OSA meeting
Chicago, 24 October 1963
(photo by H. R. Condit)



FOREWORD

Dr. Deane Judd (1900-1972) spent 43 years at the National Bureau of Standards, during which time he made monumental contributions to the science of colorimetry. I doubt that there is a color researcher in the world today that did not know Dr. Deane Judd or his work. It is equally true that all colorimetrists would acknowledge his preeminence in this important field.

His continuing influence is reflected in the constant stream of reprint requests that still comes to NBS. This interest in Judd's work prompted the suggestion that a compilation of his papers should be made available. The sheer volume of his work precluded reproduction of every paper, and we were fortunate to have the assistance of Dr. David MacAdam in selecting the papers reproduced here and explaining their importance.

NBS was privileged to have had such an outstanding person as Deane Judd among its ranks, and we are most pleased to make this compilation available.

E. Ambler.

Ernest Ambler
Director

PREFACE

Colorimetry - the measurement of physical properties of colored materials and presentation of the results in such a manner as to indicate whether the materials appear as intended - was introduced to American industry by Deane B. Judd and greatly advanced by his researches and publications.

This selection from the 200 or more articles written by Dr. Judd is intended to include examples of his major contributions. His lectures and articles were models of lucidity. Very little of consequence has been added to colorimetry since his death in 1972; none of his contributions have been superseded or brought into question. Therefore, this collection of his papers should be useful and instructive, as well as a record and memorial to the dean of colorimetrists.

Each article is preceded by a paragraph, the primary purpose of which is to explain words or expressions that may seem unfamiliar or strange to any readers who were not involved with colorimetry one, two, three, or four decades ago. In some cases, the connotations of some still-current words may have altered since Judd wrote them. Efforts have been made to identify and explain such changes. Some of the introductory paragraphs call attention to related or dependent articles in the collection and mention international agreements that have been based on Judd's innovations or suggestions.

As American representative in colorimetry, at the meetings of the International Commission on Illumination (CIE) in 1931, 1935, 1948, 1951, 1955, 1959, 1963, and 1967, as Director of the Technical Secretariat on Colorimetry of that Commission from 1948 to 1955, and Chairman of its Expert Committee E-1.3.1 (Colorimetry), Judd's influence on the development of modern colorimetry was unique. He initiated and effectively advocated the CIE adoption of the 0,1,0 luminosity coefficients, realizable standards of "daylight" - illuminants "B" and "C", excitation purity, and a uniform method of estimating the spectral distribution of actual daylight of any color temperature (e.g., D_{65}). Many of the articles reproduced in this book are concerned with these and other subjects that are still of interest and under discussion in the CIE and other international forums.

As chairman of the Committee on Uniform Color Scales of the Optical Society of America, Dr. Judd played an indispensable role in devising a practical solution to the major dilemma of colorimetry - that strictly uniform color scales of all kinds are not homologous with euclidean space. This dilemma, the facts of observation that underlie it, and the nature of the compromise with the concept of ideal color space that is necessary, are the subjects of several of Judd's last and most prescient articles, which are included in this book.

A list of articles and books written by Deane B. Judd appears in Appendix 1. This list was based on one compiled by Dorothy Nickerson and published in Die Farbe 23, 243-254 (1974). All corrections that have been called to the attention of Miss Nickerson have been made in Appendix 1. Miss Nickerson and the editor will be grateful to anyone who informs either of them of additional publications or any corrections that are needed in Appendix 1.

Judd's first publication about color was the thesis he presented to The Ohio State University for the M.A. degree. Its title was "Physical characteristics of complementary pigments". Judd thus started his study of color where colorimetry starts - determination of physical data. This also characterized his first published paper on color, "The spectral energy distributions produced by rotary mixing of complementary papers". However, his second paper on color indicated a move beyond physics into the field that, largely by Judd's influence, has come to be called "psychophysics". His next paper on color - the first in this book - signalized his entry into colorimetry - and into the National Bureau of Standards, where he remained 43 years (until 1969). The thesis Judd presented to Cornell University for the Ph.D. indicates a swing of the pendulum towards study of subjective phenomena, to which he returned many times in later years.

The titles of published papers and of abstracts of papers that Judd presented orally at meetings of the Optical Society from 1927 to 1936 indicate that he was busy in the laboratory, doing visual colorimetry and heterochromatic photometry, including color-temperature calibrations. During the same period, however, Judd published some of his most momentous "theoretical" papers concerning the use and interpretation of colorimetric data. His paper on "The 1931 ICI standard observer and coordinate system for colorimetry" introduced colorimetry into American industry. His 1935 paper on "A Maxwell triangle yielding uniform chromaticity scales" is the foundation of nearly all subsequent developments and practice in the interpretation of color differences and determination of color temperatures. Subsequent modifications of Judd's 1935 diagram, by other authors, have been made for the sake of convenience and with far less justification by observational data.

The problem of determining and specifying the whiteness of paper, which engaged Judd's attention about 1936, resulted in several publications. Judd's first paper on color blindness appeared in 1936; subsequent studies led him to propose a modified zone theory of color vision - his last paper on the subject (with G. T. Yonemura) was published in 1971 and is included in this book.

Use of the Munsell system characterized many of Judd's papers, notably those concerned with systematic color names, chromatic adaptation, camouflage, influence of chromatic contrast on visibility, and color rendering. He was President of the Board of Trustees of the Munsell Color Foundation from 1942 to 1972.

Dr. Judd wrote one book, Color in Business, Science, and Industry, which he revised twice in collaboration with Gunter Wyszecki. (The third edition, which was published in 1975, appeared posthumously.) That book, together with this, is a monument to Judd's lasting contributions to colorimetry and to his skill in expounding it.

Judd also made significant contributions to other books, some of which are listed in Appendix 1. One book to which he made indispensable contributions without appearing as author, which is therefore not listed in Appendix 1, was The Science of Color, the report of the Colorimetry Committee of the Optical Society of America, published by T. Y. Crowell in 1953 (republished by the Optical Society in 1963 and subsequently). Dr. Judd served on that committee from its inception in 1932 until it disbanded in 1953. He wrote many letters and memoranda that contained passages which were incorporated, either verbatim or in essence, in many places in the book. Professor Arthur C. Hardy, of the Massachusetts Institute of Technology, and Dr. Judd served as the subcommittee for final review of the manuscript of The Science of Color. Judd was President of the Optical Society of America when the book was published. He was awarded the Frederic Ives Medal by the Optical Society in 1958 and was Editor of the Journal of the Optical Society of America from 1961 through 1963.

Dr. Judd was awarded the Exceptional Service Gold Medal of the U.S. Department of Commerce, 1950; the Godlove Award of the Inter-Society Color Council, 1957; the Gold Medal of the Illuminating Engineering Society, 1961; the Stratton Award of the National Bureau of Standards, 1965; and the Newton Medal of the British Colour Group, 1972.

ABSTRACT

This book is a collection of fifty-seven papers written by Deane B. Judd, a staff member of the National Bureau of Standards from 1926 to 1969, and an internationally recognized authority on color. The contents of this collection include some of the major contributions of Dr. Judd to such areas as the measurement and specification of color, spectrophotometry, color appearance and spacing, and color vision. Each paper is preceded by an introduction which provides general commentary on the article and explains the terminology used. Some introductions also direct the reader to related articles in the collection and point out significant developments, such as international agreements, which were based on Judd's work. A list of more than 200 articles written by Dr. Judd is included in an appendix.

Key Words: Color; color spacing; color vision; colorimetry; Deane Judd; selected writings; spectrophotometry.

CONTENTS

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THE COMPUTATION OF COLORIMETRIC PURITY

J. Opt. Soc. Am. Rev. Sci. Inst. 13, 133-154 (1926)

Colorimetric purity is the ratio of the luminance (called brightness in this paper) of the spectral component, in an additive mixture with the appropriate amount of a standard white light to match a sample color, to the luminance of the sample. The wavelength of the monochromatic component needed to produce the match is the dominant wavelength. During the 1920's and in the early 1930's, dominant wavelengths and colorimetric purities were commonly determined by visual matching. So-called "monochromatic plus white" visual colorimeters were used; they consisted of combinations of monochromators with flicker photometers for determination of the luminances or ratio of luminances of the monochromatic and white components.

Throughout the 1920's and early 1930's, especially at the U.S. National Bureau of Standards, but also at a few academic and industrial research laboratories, programs were pursued to obtain spectrophotometric data, first with visual spectrophotometers and finally by use of photoelectric devices. One purpose of such programs was to implement the use of visual color-matching data for the spectrum, such as were recommended by the OSA Committee on Colorimetry for 1920-21. When automatic, recording spectrophotometers and consequently spectrophotometric data became suitably available, computation of colorimetric coordinates became feasible; the problems of determining dominant wavelength and colorimetric purity from them became pressing. The complexities of those problems are exhibited in this paper. Some of those complexities were eliminated and others were greatly mitigated by the CIE 1931 coordinate system, particularly by the 0,1,0 luminosity coefficients ($\bar{y}_\lambda \equiv V_\lambda$) the possibility of which was demonstrated by Judd.

General use of "excitation purity" instead of colorimetric purity, first suggested by Judd, further simplified the specification of purity. Readers who have come into colorimetry since about 1940 will hardly be able to believe that colorimetrists ever went through such contortions as are described as "routine" at the U.S. National Bureau of Standards in 1926 (see footnote 12, on p. 143). The simplifications of which modern colorimetrists are beneficiaries are, to a very great extent, results of Dr. Judd's contributions in the decade between 1926 and 1936, beginning with this paper.

THE COMPUTATION OF COLORIMETRIC PURITY*

By Deane B. Judd**

ABSTRACT

Formulas expressing colorimetric purity in terms of given trilinear coördinates have been independently derived by Ives and Priest. This paper gives (1) the derivation of still another form of purity formula; (2) a discussion of the computational methods which are, at once, accurate and speedy, together with an analysis of the various formulas with regard to convenience for routine computing; (3) the extension of the new formulas to purity for purples; and, (4) a general solution for the trilinear coördinates (r,g,b) of a color of purity, p, in terms of the coördinates (R,G,B) of the spectrum and the luminosity coefficients (L_r,L_g,L_b).

INTRODUCTION[1]

Formulas for the computation of colorimetric purity from given "trilinear coördinates" and "luminosity coefficients" have been independently derived by H. E. Ives[2] and I. G. Priest. Tuckerman has shown that the formulas of Ives and Priest are mathematical identities provided certain restrictions are placed upon the variables, although the formulas are of very different form. He has derived from Ives' formula, three formulas of which some one is always Priest's formula. The latter thus proves to be a special case of a more general formula. Tuckerman also gives the formula in still another form.[3] I have recently derived by independent argument still other formulas for colorimetric purity which also prove to be convertible into those above mentioned.[4]

* Partial report No. 6 on cooperative investigations in colorimetry by the Bureau of Standards and the Munsell Research Laboratory. Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce and the Director of the Munsell Research Laboratory.

** Munsell Research Associate in Colorimetry at the National Bureau of Standards.

[1] It is presumed that the reader is acquainted with the following previous papers particularly the last named:

1. Ives: "Transformation of Color-Mixture Equations," J. F. I., *180*, pp. 673–701; December, 1915, and J. F. I., *195*, pp. 23–44; January, 1923.

2. Troland: "Report of the Colorimetry Committee of the O. S. A. 1920–21," J.O.S.A. & R.S.I., *6*, pp. 527–596; August, 1922.

3. Priest et al: "The Computation of Colorimetric Purity," J.O.S.A. & R.S.I., *9*, pp. 503-520, November, 1924.

The notation adopted in the last named paper will be followed throughout the present discussion.

[2] See Priest, Loc. cit., p. 510.

[3] Priest, Loc. cit. Bottom of page 511, also Appendix.

[4] Formulas given by Seitz (Zs. für Sinnesphysiol., *54*, Abt. 2, p. 146; 1922) and by Martin (Colour and Methods of Colour Reproduction, p. 133) are presumably intended as purity formulas, but do *not* give purity as measured in "monochromatic analysis." Martin's formula

Following Priest's notation, the formulas in question stand as follows:

$$p=\left(\frac{r-g}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{R-G}\right) \quad \text{[Ives R-G]}$$

$$p=\left(\frac{b-r}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{B-R}\right) \quad \text{[Ives B-R]}$$

$$p=\left(\frac{b-g}{rL_r+gL_g+bL_b}\right)\left(\frac{RL_r+GL_g+BL_b}{B-G}\right) \quad \text{[Ives B-G]}$$

$$p=\frac{1-\dfrac{c}{rL_r+gL_g+bL_b}}{1-\dfrac{C}{RL_r+GL_g+BL_b}} \quad \text{[Priest]}$$

where

$c=$ least of $r,\ g,\ b$

$C=$ least of $R,\ G,\ B$

$$p=\frac{1-\dfrac{r}{rL_r+gL_g+bL_b}}{1-\dfrac{R}{RL_r+GL_g+BL_b}} \quad \text{[Tuckerman R]}$$

$$p=\frac{1-\dfrac{g}{rL_r+gL_g+bL_b}}{1-\dfrac{G}{RL_r+GL_g+BL_b}} \quad \text{[Tuckerman G]}$$

$$p=\frac{1-\dfrac{b}{rL_r+gL_g+bL_b}}{1-\dfrac{B}{RL_r+GL_g+BL_b}} \quad \text{[Tuckerman B]}$$

---

apparently gives what Priest has defined as "absolute purity," which is meaningful only in relation to the particular elementary stimuli the system is referred to. (Priest: Loc. cit. pp. 508–509.) Semi-geometric formulas developed by Froelich (J.O.S.A. & R.S.I., *9*, No. 1, pp. 37–41; 1924) and Guild (Trans. Opt. Soc., *26*, No. 3, p. 162; 1924–25) do give the purity of monochromatic analysis, but involve a term (α-Froelich, ρ Guild) which must be evaluated graphically by measuring two distances on the color triangle; hence these formulas in their published form are not comparable to the purely algebraic formulas. By reducing this term to its analytic equivalent, both Froelich's and Guild's formulas fall readily into identity with any of the forms recorded in the present paper.

$$p=\frac{3-\dfrac{1}{rL_r+gL_g+bL_b}}{3-\dfrac{1}{RL_r+GL_g+BL_b}} \quad \text{[Tuckerman, Symmetrical]}$$

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+\dfrac{r-R}{1-3r}} \quad \text{[Judd R]}$$

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+\dfrac{g-G}{1-3g}} \quad \text{[Judd G]}$$

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+\dfrac{b-B}{1-3b}} \quad \text{[Judd B]}$$

$$p=\frac{rL_r+gL_g+bL_b-\dfrac{r-R}{1-3R}}{rL_r+gL_g+bL_b} \quad \text{[Judd } r\text{]}$$

$$p=\frac{rL_r+gL_g+bL_b-\dfrac{g-G}{1-3G}}{rL_r+gL_g+bL_b} \quad \text{[Judd } g\text{]}$$

$$p=\frac{rL_r+gL_g+bL_b-\dfrac{b-B}{1-3B}}{rL_r+gL_g+bL_b} \quad \text{[Judd } b\text{]}$$

In what follows, these several formulas will be briefly referred to by the designations appended to them in brackets in the above list. The conditions which must be met in order that these various forms be true identities are as follows[5]:

(1) $$R+G+B=1\ ,$$

(2) $$r+g+b=1\ ,$$

(3) $$L_r+L_g+L_b=1\ ,$$

(4) $$\frac{1-3r}{1-3G}=\frac{1-3g}{1-3R}=\frac{1-3b}{1-3B}\ .$$

[5] For the identity of some forms all these conditions are not necessary. However, if all the conditions are satisfied, all the forms are identical.

It is apparent that there is considerable latitude for choice in selecting a particular formula for numerical computation. The purposes of this paper are:

1. To give the derivation of the new formulas.
2. To investigate the formulas from the point of view of convenience and accuracy in numerical computation.
3. To develop and formulate the rules of numerical computation which will insure the smallest possible uncertainty in the computed value of p.[6]
4. To show how the formulas are applied to colors of non-spectral hue (purples) and to discuss the definition of purity for purples.
5. To give a solution for the trilinear coördinates (r, g, b) for any point of given purity (p) in terms of the spectrum scale coördinates (R, G, B).

I. THE DERIVATION OF THE NEW FORMULA

If ρ_λ, γ_λ, β_λ be the respective amounts of the elementary stimuli required to result in the same response as that to homogeneous radiation of wave length (λ), then the relations:

$$R=\frac{\rho_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ ; \qquad G=\frac{\gamma_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ ; \qquad B=\frac{\beta_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ .$$

define R, G, and B. Should neutral heterogeneous radiation be added to the stimulus, each elementary magnitude would be increased by an amount, ω, and the relations:

$$r=\frac{\rho_\lambda+\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda+3\omega}\ ; \quad g=\frac{\gamma_\lambda+\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda+3\omega}\ ; \quad b=\frac{\beta_\lambda+\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda+3\omega}\ .$$

define r, g, and b.

The purity of this mixture is defined as:

$$p=B_\Lambda/(B_\Lambda+B_w)\ , \qquad B_\Lambda+B_w=B_m=B_s\ ;$$

[6]Priest has already pointed out certain practical precautions in the numerical application of these formulas (J.O.S.A. & R.S.I., *9*, pp. 512, 516, and 518), but has not treated the question fully. It may be inferred from his paper that of the three Tuckerman forms (Tuckerman R, Tuckerman G, Tuckerman B) the special case designated as Priest's formula is the most accurate. (J.O.S.A. & R.S.I., *9*, pp. 516–519.) However, no general proof is given nor are the underlying reasons for the numerical discrepancies in his computed results discussed fully. The present study of the formulas has been undertaken at Mr. Priest's request, and much of the material herein developed is the direct result of his suggestions, and of those of Dr. Tuckerman.

where:

B_Λ = brightness of the spectral component of the mixture field,
B_w = brightness of the white component of the mixture field.
B_m = brightness of the mixture field,
B_s = brightness of the sample field.

But B_Λ can be expressed in terms of the elementary stimuli (ρ_λ, γ_λ, β_λ) and their respective luminosity coefficients (L_r, L_g, L_b):

$$B_\Lambda = \rho_\lambda L_r + \gamma_\lambda L_g + \beta_\lambda L_b \ .$$

In the same terms:

$$B_m = (\rho_\lambda + \omega)L_r + (\gamma_\lambda + \omega)L_g + (\beta_\lambda + \omega)L_b = \omega(L_r + L_g + L_b) + \rho_\lambda L_r + \gamma_\lambda L_g + \beta_\lambda L_b .$$

$$= \omega + B_\Lambda \ ; \text{ whence } : B_w = \omega \ .$$

Then, substituting:

$$p = \frac{B_\Lambda}{B_\Lambda + B_w} = \frac{\rho_\lambda L_r + \gamma_\lambda L_g + \beta_\lambda L_b}{\rho_\lambda L_r + \gamma_\lambda L_g + \beta_\lambda L_b + \omega}$$

From the definitions of R, G, and B, the purity may be expressed:

$$p = \frac{RL_r + GL_g + BL_b}{RL_r + GL_g + BL_b + \omega/(\rho_\lambda + \gamma_\lambda + \beta_\lambda)} \ .$$

Also from the definitions of r, g, b, R, G, and B:

$$\frac{\omega}{\rho_\lambda + \gamma_\lambda + \beta_\lambda} = \frac{r - R}{1 - 3r} = \frac{g - G}{1 - 3g} = \frac{b - B}{1 - 3b} \ .$$

The formula may be thrown into the form of the definition for the purity of colors of spectral hue by virtue of the fact that $\rho_\lambda + \gamma_\lambda + \beta_\lambda$ is constant for a given wave length, since ρ_λ, γ_λ and β_λ are separately fixed for a given wave length, regardless of the purity, as the subscripts indicate. Thus:

$$p = b_\Lambda/(b_\Lambda + b_w),$$

where:

$$b_\Lambda = B_\Lambda/(\rho_\lambda + \gamma_\lambda + \beta_\lambda),$$

= brightness of the spectral component of the mixture field in arbitrary units which change as the wave length changes, and

$$b_w = B_w/(\rho_\lambda + \gamma_\lambda + \beta_\lambda),$$

= brightness of the white component of the mixture field in the same units.

From the definitions of r, g, b, R, G, and B:

$$b_\Lambda = RL_r + GL_g + BL_b \ ,$$

$$b_w = \frac{r-R}{1-3r} = \frac{g-G}{1-3g} = \frac{b-B}{1-3b} \ .$$

Substituting these values of b_Λ and b_w in the relation: $p = b_\Lambda/(b_\Lambda + b_w)$, gives the forms designated in the summary (see Introduction) as Judd R, Judd G, and Judd B.

II. COMPUTATIONAL ACCURACY AND CONVENIENCE OF THE FORMULAS

The formulas for the computation of colorimetric purity contain the following quantities:

1. L_r, L_g, and L_b—the chromatic luminosity coefficients, constant for all wave lengths and all purities, and regarded as evaluated once for all by general colorimetric investigation.

2. R, G, and B—the trilinear coördinates of the spectrum, functions of the wave length but not of the purity, and read for each particular case from an interpolation graph.

3. r, g, and b—the trilinear coördinates of the point whose purity (p) it is desired to evaluate. These may be regarded as accurately given. The problem to be solved by any of the formulas is hypothetical, thus: If r, g, and b are the trilinear coördinates of a color in a system whose "luminosity coefficients" and "spectrum scale" for each even 10 mμ are known within the limits of experimental accuracy, what is the purity of that color?

In any particular problem, then, the accuracy of the result depends on the accuracy of the evaluation of (R, G, B) from the spectrum scale. If R, G, and B are correctly determined from the interpolation graph they will meet two conditions, already stated (Page 135):

$$(1) \qquad R+G+B = 1$$

$$(4) \qquad \frac{1-3r}{1-3R} = \frac{1-3g}{1-3G} = \frac{1-3b}{1-3B} \ .$$ [7]

Conditions (2) and (3) of the same group are easily complied with by carrying the computation of the quantities involved out to one more place (say four) than is to be carried through the subsequent calculation,

[7] This condition (4) will be recognized as that which makes the point (r, g, b) and the point (R, G, B) collinear with the neutral or white point (1/3, 1/3, 1/3). It is the analytical expression for the necessity that the color represented by the point (r, g, b) have the same dominant wave length (Λ) as the spectral color represented by the coordinates (R, G, B). Condition (4) will be hereinafter referred to as the collinearity condition.

and then cutting back to three places so that the sum is exactly unity. If R, G, and B are so chosen as to satisfy the conditions (1) and (4), then, all forms of the purity formula are identical and will yield identical results. If there be an error in the determination of R, G, or B so that they do not meet these conditions, none of the forms of purity formula yield the true purity. If, however, for a given error in R, G, or B, one form gives a purity in error by, say, 0.10 (that errors as large as this can occur may be seen from reference to Column 17, Table 2, Appendix) and another, a purity in error by 0.001, the latter form may be said to be computationally accurate in comparison to the former. It is a matter of importance in colorimetric calculation to know which forms are computationally precise, and which are not.

The largeness of the errors in purity for some cases arises from the fact that in all of the formulas differences involving R, G, and B occur.

TABLE 1. *The Convenience of the Purity Formulas*

Designation of Purity Formula	Criterion of Inconvenience	Wave length Region of inconvenience from O. S. A. Elementaries
Ives $R-G$	When $r-g \to 0$	560 mμ to 580 mμ
Ives $B-R$	When $b-r \to 0$	500 mμ to 510 mμ
Ives $B-G$	When $b-g \to 0$	490 mμ to 495 mμ
Tuckerman R	When *$RL_r+GL_g+BL_b \to R$	560 mμ to 580 mμ
Tuckerman G	When $RL_r+GL_g+BL_b \to G$	560 mμ to 584 mμ *and* 440 mμ to 466 mμ
Tuckerman B	When $RL_r+GL_g+BL_b \to B$	494 mμ to 497 mμ
Priest	When**$RL_r+GL_g+BL_b \to C$	None
Judd R or r	When $1-3r \to 0$	520 mμ to 540 mμ
Judd G or g	When $1-3g \to 0$	485 mμ to 490 mμ *and* 585 mμ to 600 mμ
Judd B or b	When $1-3b \to 0$	495 mμ to 498 mμ
Tuckerman Symmetrical	When $RL_r+GL_g+BL_b \to 1/3$	488 mμ to 502 mμ *and* 600 mμ to 720 mμ

* To compare these quantities as the wave length varies, refer to Fig. 1. This shows R, G, B, and $RL_r+GL_g+BL_b$ as functions of the wave length, the excitations and luminosity coefficients involved being those recommended by the O. S. A. Committee on Colorimetry. (Cf. Troland: Loc. cit., 548–551.)

** C is the least of R, G, and B. A glance at Fig. 1 will show that this definition is equivalent to $C=R$, $\lambda<504$ mμ, and $C=B$, $\lambda>504$ mμ. It becomes evident, also, that C never approaches $RL_r+GL_g+BL_b$; hence there is no region of inconvenience.

Whenever one of these differences becomes small in comparison to the values of R, G, and B, the error in purity will be much larger than the uncertainty inherent in R, G, and B.

A glance at the formulas themselves will show the conditions under which they, severally, lose computational precision. Each one of these conditions defines a wave length region (dependent somewhat on the choice of elementaries) in which the formula becomes impractical for computation. These wave length regions approximately evaluated by

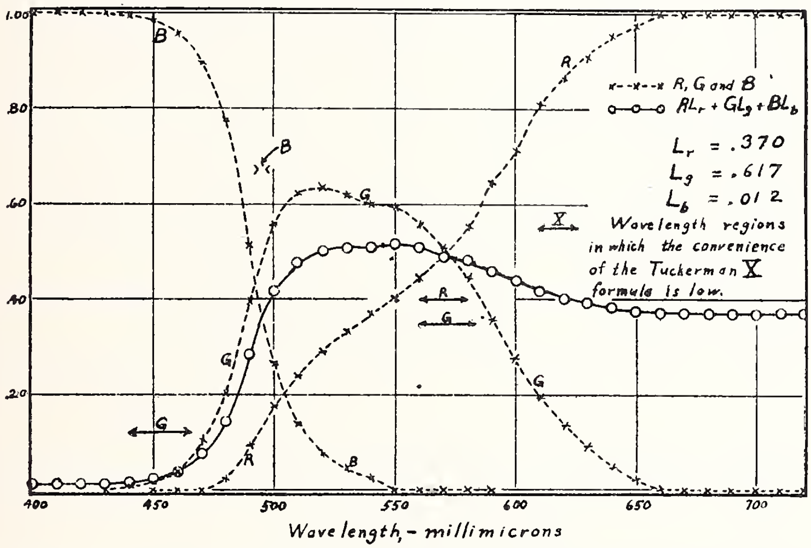

FIG. 1.—*The spectrum coordinates* (R, G, B) *and* $\mathbf{b}_A$ *as functions of the wave length. The wave length regions of inconvenience for the Tuckerman X formulas* (X *being either* R, G, *or* B) *are also indicated.*

reference to the O.S.A.[8] excitations and luminosity coefficients are given in Table 1.

A reference to Table 1 shows that all forms of purity formula save one become inconvenient for practical use in at least one wave length region. The special form of Tuckerman's formula designated by *Priest* so chooses among Tuckerman R, Tuckerman G, and Tuckerman B as to avoid the wave length regions of low computational precision.

While the Tuckerman Symmetrical form has a distinct advantage in numerical computation (to be fully discussed later) it is not applicable

[8] Troland, Loc. cit., pp. 548–551, p. 575.

to the actual computation of all cases of purity as the large region of inconvenience shows.[9]

III. PROCEDURE NECESSARY FOR ACCURATE COMPUTATION

As shown in the preceding section, the only source of error that is troublesome in the solution of our hypothetical problem is the evaluation of R, G, and B. Though these three quantities may be read separately from an interpolation graph (an accurate plot of Fig. 1), still there are two conditions (1 and 4, page 135) which involve them. Thus, if any one of R, G, or B be read from the graph, the other two could be calculated from the conditions themselves. To render as harmless as possible, then, the unavoidable error in R, G, and B; two principles of selection should be followed:

1. Select that spectrum coördinate (R, G, or B) as basis for the calculation of the other two, which can be determined with the least uncertainty.

2. Select some form of purity formula whose "region of inconvenience" does not include the point (r, g, b).

The general method of purity computation embodying these two precautionary selections is:

1. Find the dominant wave length corresponding to the point (r, g, b) from a plot of the spectrum on the color triangle[10] or, better yet, by an accurate method of interpolation, such as a plot of $(R–1/3)/(B–1/3)$ against the wave length.

2. Read from an interpolation graph (an accurate plot of Fig. 1) the value of one of R, G, and B, as follows:

[9] This checks a statement made by Priest (Loc. cit., p. 512) based on actual trial computations. If we set $L_r = L_g = L_b = 1/3$, and substitute in the Tuckerman Symmetrical form, p becomes indeterminate. The usefulness of the formula is, thus, seen to depend on how far the luminosity coefficients deviate from 1/3. The fact that the luminosity coefficients which have been used for the O. S. A. curves by Priest involve an L_r which is close to 1/3 (0.370) accounts for the inconvenience of the formula in the red region. It is readily seen that the Tuckerman Symmetrical form might easily be as convenient as any of the other forms provided only the luminosity coefficients are all appreciably different from 1/3. If, indeed, the luminosity coefficients, $L_r = 0.45$, $L_g = 0.54$, and $L_b = 0.01$ be used for the O. S. A. excitations (Judd, Chromatic Visibility Coefficients by the Method of Least Squares, J.O.S.A. & R.S.I., *10*, No. 6, p. 646, June, 1925,) which fit them better to the visibility than the values used by Priest, the Tuckerman Symmetrical form could be employed in actual computation in a wave length region of extent comparable to that of any other single form of purity formula.

[10] Troland: Loc. cit. p. 575.

For $\Lambda = 380$ mμ to 510 mμ, read R.
For $\Lambda = 510$ mμ to 530 mμ, read G.
For $\Lambda = 530$ mμ to 720 mμ, read B.[11]

3. Calculate the other two coördinates of R, G, B from the equations of condition:

$$(1) \qquad R+G+B=1$$

$$(2) \qquad \frac{1-3r}{1-3R}=\frac{1-3g}{1-3G}=\frac{1-3b}{1-3B}.$$

After this step has been completed, numerical values will be available for r, g, b; R, G, B; and L_r, L_g, L_b, which satisfy all four equations of condition.

4. Substitute these values in any of the forms of the purity formula, and compute p. Since we have complied with the conditions which make all these forms identical, the results will be identical regardless of the form used. However, it will be wise to follow the second principle of selection, and choose a convenient form of purity formula, if the computing of R, G, B (Step 3) to a large number of decimals is to be avoided.

Two alternative methods of computing purity are worthy of mention at this point because they avoid the tedious computation of R, G, B (Step 3) and still yield reliable values for purity by automatically conforming to the two principles of selection as well as to the equations of condition.

[11] The basis underlying this division on the wave length scale is merely the first principle of selection. A reference to Fig. 1 will show that the above choice of spectral coördinate avoids those whose slope with wave length change is highest and whose interpolation error therefore greatest. Since Fig. 1 refers to the O. S. A. excitations, the above division on the wave length scale should be followed only when the choice of color primaries is close to those embodied in the O. S. A. excitations. For other choices, the points of division will be somewhat shifted.

Another justification of this division of wave lengths, scarcely less important, is as follows: The intersection of the hue line defined by (r, g, b) with the spectrum line is being sought. We plan to alter the spectral coordinates (R, G, B) slightly by imposing the collinearity condition (See next step). However, to reduce the purity error to a minimum, we must so alter R, G, B, as to move the point in the general direction of the spectrum line at that wave length. Thus, for long wave lengths, hold B constant as read, and, of necessity, the change by imposing the collinearity condition must consist of varying the R–G ratio, a change which moves the point approximately along the spectrum line. This division of the wave length scale, apparently quite by chance, thus, not only effects a start from the best determined spectral coördinate, but also insures that the change for perfect collinearity does not move the point (R, G, B) from the spectrum line.

A. *Priest's Method.* The procedure in use at the National Bureau of Standards for the calculation of dominant wave length and purity is as follows[12]:

1. From the data for r, g, b the ratio $(r-1/3)/(b-1/3)$ is calculated.

2. The dominant wave length is found by applying this ratio $(r-1/3)/(b-1/3)$, or the ratio $(b-1/3)/(r-1/3)$, to a graph showing the dominant wave length as a function of these two ratios. This graph, itself, was previously prepared from the trilinear coördinates of the spectrum by plotting $(R-1/3)/(B-1/3)$ and $(B-1/3)/(R-1/3)$ against dominant wave length.

3. The whole denominator of Priest's formula, $1-C/(RL_r+GL_g+BL_b)$, corresponding to this dominant wave length is read off a graph showing the denominator as a function of the dominant wave length. (Priest's method in this form does not apply to colors of non-spectral hue.)

4. The numerator is evaluated by inserting the given data, r, g, b, in the form: $1-c/(rL_r+gL_g+bL_b)$, where c is the least of r, g, b; and the numerator is divided by the denominator to find the purity.

By the preparation of the graph referred to in Step 3, the tedious computation of R, G, B, is, in effect, accomplished once for all. Equations of condition (2) and (3) may be easily satisfied as described in Section II. A failure to meet requirements of condition (1) or (4) would appear as an error in plotting or reading the graph of Step 3. However, the graph was so carefully constructed (points plotted every 2 mμ in doubtful regions) as to make the interpolation of the denominator accurate to 0.001 of its value.

It is interesting to note that this method also complies with the two principles of selection almost perfectly. The second principle—the avoidance of "inconvenient wave length regions"—is exactly followed by the choice of the "least of the coördinates." (See Section II, footnote to Table 1.) If the first principle of selection is to be followed exactly, the division of the wave length scale of Step 2, General Method of Computation, must be met by the choice of the "least of the coördinates." A reference to Fig. 1 will show that the region 510 mμ to 530 mμ is the only one in which this is not true. For values of r, g, b which fall in this narrow wave length region, then, the value of purity is not quite

[12] There is no previous published account of the exact procedure followed in routine computation. Mr. Priest states that it is partly (Step 3) due to suggestion of Mr. C. D. Hillman of Keuffel and Esser and requests that this be acknowledged.

so certain by this method as by the rigorous, general method. However, it is still accurate to within 0.001 of its value, since the graph of the denominator may be read to that accuracy. This is several times as accurate as the experimental determination of purity for that wave length region.

B. *Purity by the Judd formulas.* The justification for proposing a new set of formulas for colorimetric purity—an entity that seems already to have adequate functional representation—is two-fold:

1. The derivation is straight-forward and yields a formula easily understandable in terms of the definition of purity.

2. The formulas involve the mixture cöordinates (r, g, b) singly (Judd R, Judd G, and Judd B) or the spectral cöordinates (R, G, B) singly (Judd r, Judd g, Judd b). This property, which is distinctive of no other form, makes possible certain shortcuts in computation, as will appear later.

The steps in the procedure which, like Priest's method, avoids the tedious computation of R, G, and B, are as follows:

1. From r, g, and b find the dominant wave length by the use of an accurate interpolation method (see Step 1, Priest's Method).

2. Read from an interpolation graph (an accurate plot of Fig. 1) $RL_r+GL_g+BL_b$, and the value of one of R, G, or B, corresponding to the dominant wave length, as follows:

For $\Lambda=380$ mμ to 510 mμ, read R.
For $\Lambda=510$ mμ to 530 mμ, read G.
For $\Lambda=530$ mμ to 720 mμ, read B.[13]

3. Calculate the purity from:

$$p=\mathbf{b}_\Lambda/(\mathbf{b}_\Lambda+\mathbf{b}_w),$$

where $\mathbf{b}_\Lambda$ is the value of $RL_r+GL_g+BL_b$ read from the graph. $\mathbf{b}_w$ is figured from $(r-R)/(1-3r)$ or $(g-G)/(1-3g)$ or $(b-B)/(1-3b)$ according as R, G, or B is available from Step 2. The formulas made use of are Judd R, Judd G, or Judd B.

It is readily seen that the first principle of selection (for R, G, or B) is satisfied because of the wave length division of Step 2. The second principle of selection which sidesteps the regions of inconvenience is also followed by this division of the wave length scale as a reference to Table 1 will show.

[13] This division of the wave length scale is the same as that in Step 2, General Method. For colors of non-spectral hue, take Λ as the complementary wave length.

As in Priest's Method, however, there is still a narrow wave length region where the possibility of error due to lack of computational precision must be avoided by extreme care in the drafting of interpolation graphs. This region is 470 mμ to 500 mμ (see Fig. 1) where $RL_r+GL_g+BL_b$ changes rapidly with wave length. This change makes interpolation correspondingly less certain.

It is conceivable that an occasion might arise which would demand the accurate evaluation of a few purities but which did not justify the labor involved in the careful preparation of an interpolation graph (as in Priest's method or the method just described). For this situation, the following modification of the general method is proposed:

1. Steps 1 and 2 are the same. When these are completed, numerical values should be available for r,g,b; L_r, L_g, L_b; and one of R, G, and B.

2. Step 3, the tedious calculation of R, G, and B from the collinearity condition, is omitted.

3. Calculate the purity (p) from Judd r, Judd g, or Judd b, according as R, G, or B is available. These forms of purity formula are easily derived by a method similar to that used for Judd R, Judd G, and Judd B, and may be written in the following form:

$$p=(\boldsymbol{b}_M-\boldsymbol{b}_W)/\boldsymbol{b}_M,$$

where:

$$\boldsymbol{b}_M=rL_r+gL_g+bL_b$$
$$\boldsymbol{b}_W=(r-R)/(1-3R)$$
$$\text{or }=(g-G)/(1-3G)$$
$$\text{or }=(b-B)/(1-3B)$$

It is to be noted that this form of purity formula involves the R, G, and B singly, a property which distinguishes it from all other forms and which is the basis of its claim to greater convenience. On this account the collinearity condition is automatically imposed by the choice of R, G, or B in Step 2 according to the division of the wave length scale. This same division of the wave length scale satisfies the two principles of selection as before. It has, therefore, perfect reliability for all wave lengths but is not particularly rapid in application because $rL_r+gL_g+bL_b$ must be evaluated separately for each purity determination. The exact extent of the unwieldiness is shown in the computation step analysis which follows.

The several procedures here outlined for computing colorimetric purity yield values of the purity which will correspond to the actual results of monochromatic analysis by a given observer only, of course, in so far as the "color excitations" and "luminosity coefficients" used in computation actually correspond to the color vision of that observer. The problem dealt with here is merely that of converting data from the trichromatic form to the terms of homo-hetero analysis, assuming that the fundamental data are accurate.[14]

Although all of the several formulas may be applied in such a way as to yield reliable values for purity, they are not equally convenient for routine computation. We shall here compare the formulas as to simplicity in numerical application assuming that:

1. r, g, and b are given;

2. Λ has been found;

3. Terms and factors which are merely functions of Λ may be found from graphs already prepared in the simplest form.

Listing the operations then required for numerical computation of p in each case we find the following:

	Tuckerman Symmetrical	Priest	Ives	Judd R, G, B	Judd r, g, b
References to graphs	1	1	1	2	1
Products found by mere mental operation (only one figure in one of the factors)	0	0	0	1	1
Products or quotients requiring slide-rule or other assistance	5	5	5	2	5
Differences, or sums of two terms	1	1	1	3	3
Sums of three terms	1	1	1	0	1
Total operations	8	8	8	8	11

[14] The best data extant are to be found as follows:
Troland: J.O.S.A. & R.S.I., *6*, pp. 527–596; August 1922;
Ives: J. F. I., *195*, pp. 23–44; January 1923;
Gibson-Tyndall: Scientific Paper of the Bureau of Standards, *19*, No. 475; August 1923;
Judd: J.O.S.A. & R.S.I., *10*, p. 647; June 1925;
Gibson: J.O.S.A. & R.S.I., *10*, p. 230; February 1925.
Redeterminations of these data are now in course at the Bureau of Standards.

The disadvantage of Judd r, g, b for routine computation is seen at the outset by the three added steps it involves. For the rest, it is apparent that with regard to mere number of separate steps the formulas are on the same footing. The Tuckerman, Priest and Judd R, G, B forms have a slight advantage over the Ives forms in that one of the differences to be found involves only an integer (1 or 3) of one digit as minuend, which makes the mental operation somewhat easier.

The Tuckerman Symmetrical form has the additional slight advantage that one of the divisions is merely the taking of a reciprocal. It is probably the most simple in numerical application where it may be safely used, but as shown in Table 1 it has the larger wave length regions of inconvenience.

The choice thus lies between the Priest formula (either Tuckerman. R, G, or B) and the new formula here developed (Judd R, G, or B) Checking off similar operations of equal difficulty in these two forms we have left:

Operations in Priest's formula not required in Judd's	*Operations in Judd's formula not required in Priest's*
3 slide-rule products	One mental product
1 sum of three terms	2 sums of 2 terms each
	1 reference to a graph

It is seen that there is little to choose between the two formulas on the basis of simplicity in numerical work. The fact that the Judd form may be applied to purity for purples unchanged, while Priest's formula in its original published form cannot, might seem to throw an advantage to the Judd formulas. However, if Priest's formulation (see Introduction) be changed to read C is R, G, or B according as c is r, g, or b, it is then applicable to colors of purple hue also.[15] Hence this advantage is more imaginary than real. Whether the Judd form or the Priest form shall be selected for routine computation seems at this time to be largely a matter of personal inclination.

IV. EXTENSION OF PURITY FORMULAS TO NON-SPECTRAL COLORS

The justification for the concept of purity lies in the experimental fact that many colors (all except purples) can be matched by mixing spectral radiation with neutral radiation.

The relation between the field and component brightnesses is:

$$B_\lambda + B_w = B_m = B_s,$$

[15] Cf. Priest's new paper, J. O. S. A. & R. S. I. **13**, pp. 123-132; Aug. 1926.

where:

B_λ = brightness of the spectral component of the mixture field,

B_w = brightness of the white component of the mixture field,

B_m = brightness of the mixture field.

B_s = brightness of the sample field.

If the concept of purity is to be extended to colors of non-spectral hue (purples) it is evident that its experimental basis will be different from that for colors of spectral hue. This basis is found in the experimental fact that for every non-spectral stimulus there can be found spectral radiation which, when mixed additively with the non-spectral stimulus, will be a color match for the neutral stimulus; again the relation between the field and component brightnesses may be written, thus

$$B_\lambda + B_s = B_m = B_w.$$

It will be noted that in experimentally measuring the purity pertaining to a color of spectral hue, the homogeneous radiation is added to the neutral field:

$$B_\lambda + B_w = B_s.$$

However, when a stimulus yielding a color of non-spectral hue is being considered, the homogeneous radiation is added to the other side—the sample field:

$$B_\lambda + B_s = B_w.$$

These two experimental procedures may be made formally equivalent if we agree to give B_λ a negative sign whenever it refers to homogeneous radiation added to the sample field to produce white. If p is defined as B_λ/B_s for the colors of purple hue as well as for those of spectral hue the values for purity will be negative whenever the color referred to is designated by its complementary. The formal equivalence gained by this convention makes all the formulas that have been derived to apply to colors of spectral hue hold for colors of purple hue also. (See Appendix.) This is the convention adopted in all of Ives' work and will be referred to hereafter as Ives'[16] definition of purity. The perfect formal continuity yielded by this definition has gained preference for it in Great Britain.[17]

[16] Ives: Loc. cit., 1923, p. 39, also letter Ives to Priest, July 1925.

[17] J. Guild: The Geometrical Solution of Colour Mixture Problems, Trans. Opt. Soc., *26*, No. 3, p. 161; 1924–25.

The convention used to represent the purples of monochromatic analysis in the Report of the Committee on Colorimetry[18] amounts to a redefining of purity for colors of purple hue:

$$p = B_\Lambda / B_m = \begin{cases} B_\Lambda / B_s, \text{ for colors of spectral hue,} \\ B_\Lambda / B_w, \text{ for colors of non-spectral hue.} \end{cases}$$

This convention will be hereafter referred to as Troland's definition of purity, B being regarded as always positive. By the use of this definition all existing colors may be represented by purities between zero and plus one, but a separate notation must be made to distinguish spectral from non-spectral hues. Troland does this by adding "C" as suffix to the dominant wave lengths that are complementary to that of the sample field. (E.g., $\Lambda = 520$ mμ, $p = 0.50$, a green, is distinguished from one of its complementaries by $\Lambda = 520\ C$ mμ, $p = 0.50$. According to Ives' definition, these two colors would be: $\Lambda = 520$ mμ, $p = 0.50$, and $\Lambda = 520$ mμ, $p = -1.00$.)

The relation between these two conventional ways of representing colorimetric purity may be easily obtained from the foregoing definitions, thus:

$$p_T = \begin{cases} p_I, \text{ for colors of spectral hue,} \\ p_I/(p_I - 1), \text{ for colors of non-spectral hue.} \end{cases}$$

where:

p_T = purity by Troland's definition,
p_I = purity by Ives' definition.

By Ives' definition for purity, the formula becomes:

$$p = b_\Lambda / (b_\Lambda + b_w),$$

where:

$$b_\Lambda = RL_r + GL_g + BL_b$$
$$b_w = (r - R)/(1 - 3r)$$
$$\text{or } = (g - G)/(1 - 3g)$$
$$\text{or } = (b - B)/(1 - 3b)$$

This formula is general for all colors, the negative purities for the colors of purple hue appear because of the algebraic sign and magnitude of b_w.[19]

[18] L. T. Troland: Loc. cit., p. 575, 583. Also letter Weaver to Priest, August 1925.

[19] b_Λ and b_w are equal to $B_\Lambda/(\rho_\lambda + \gamma_\lambda + \beta_\lambda)$ and $B_w/(\rho_\lambda + \gamma_\lambda + \beta_\lambda)$ only when p refers to colors of spectral hue. The difficulty of attaching any brightness significance to b_Λ and b_w as defined in this section when p refers to purples is readily seen from equations 3 and 6 of the Appendix. It seems preferable, therefore, to regard b_Λ and b_w merely as abbreviations for functions which may be identified as brightness components only when the homogeneous radiation is added to the neutral field.

The formula referring to colors of spectral hue is recognized in each case as identical with that already derived. For formal proof of the formulas given for the colors of purple hue, reference must be had to the Appendix.

V. THE COÖRDINATES (r, g, b) OF THE LINES OF EQUAL PURITY, (p) IN TERMS OF THE COÖRDINATES OF THE SPECTRUM (R, G, B)

The inscription of the lines of equal purity in the color triangle has heretofore been accomplished apparently by graphical methods, making use of the observed visibility[20] which has been regarded as better determined than the visibility dependent on the function $\rho_\lambda L_r+\gamma_\lambda L_g+\beta_\lambda L_b$. It has been shown by the author,[21] however, that with properly chosen luminosity coefficients, the visibility calculated from the color mixture curves can be made to agree with the adopted visibility function to the same degree of approximation as do the experimental means for large numbers of observers from separate set-ups.[22] In view of this agreement, it has been deemed worth while to work out the coördinates of any color in terms of the purity, the luminosity coefficients, and the coördinates of the spectrum scale. The derivation of the formula follows:

Take the Judd R formula for purity, and rewrite:

$$p=\frac{b_\Lambda}{b_\Lambda+(r-R)/(1-3r)},$$

where:

$$b_\Lambda=RL_r+GL_g+BL_b$$

r may be solved for in terms of p, b_Λ, and R. This solution yields:

$$r=\frac{b_\Lambda(1-p)/p+R}{3b_\Lambda(1-p)/p+1}\,.$$

Similarly from Judd G and Judd B, are obtained:

$$g=\frac{b_\Lambda(1-p)/p+G}{3b_\Lambda(1-p)/p+1}\,;\text{ and } b=\frac{b_\Lambda(1-p)/p+B}{3b_\Lambda(1-p)/p+1}\,.$$

If we set $p=1$, we shall find that $r=R$, $g=G$, and $b=B$. This fact may be interpreted to mean that R, G, B are the coördinates of the point

[20] Froelich, Clara L., Algebraic Methods for the Calculation of Color Mixture Transformation Diagrams, J.O.S.A. & R.S.I., *9*, No. 1, pp. 37–41; July 1924.
[21] Judd: Loc. cit., p. 649.
[22] Gibson-Tyndall: Loc. cit., p. 174.

whose purity is unity. The spectral colors have been arbitrarily set upon as of unit purity, and following out this definition (R, G, B) is the point where the line joining (r, g, b) and the center $(1/3, 1/3, 1/3)$ cuts the spectrum line. However, if we interpret R, G, B as the coördinates of the point whose purity is unity, the formulas hold whether that point be defined as on the spectrum line or not.

Since these formulas were derived from the purity formulas, the restrictions there placed on the independent variables, namely:

$$(1) \qquad R+G+B=1 \ ,$$

$$(3) \qquad L_r+L_g+L_b=1 \ .$$

must be imposed here also. If this is done, the r, g, b found by substitution will necessarily satisfy the two conditions:

$$(2) \qquad r+g+b=1 \ ,$$

$$(4) \qquad \frac{1-3r}{1-3R}=\frac{1-3g}{1-3G}=\frac{1-3b}{1-3B} \ .$$

If p is set less than zero and a line inscribed in the triangle from the cöordinates by the above formulas it will be found to course through the region of colors of purple hue, and, of course, to yield a purity agreeing with that by Ives' definition.

If it is desired to inscribe the line of equal purity in the region of purple colors according to Troland's definition, r, g, and b may be solved for from:

$$p=\boldsymbol{b}_\Lambda(1-3r)/(R-r);$$
$$p=\boldsymbol{b}_\Lambda(1-3g)/(G-g);$$
$$p=\boldsymbol{b}_\Lambda(1-3b)/(B-b).$$

This yields:

$$r=(\mathrm{R}-\boldsymbol{b}_\Lambda/p)/(1-3\boldsymbol{b}_\Lambda/p);$$
$$g=(G-\boldsymbol{b}_\Lambda/p)/(1-3\boldsymbol{b}_\Lambda/p);$$
$$b=(\mathrm{B}-\boldsymbol{b}_\Lambda/p)/(1-3\boldsymbol{b}_\Lambda/p).$$

The purity line thus obtained is the same as before but differently numbered. The result may also be obtained from the previous formulas by converting $p_{\text{ Ives}}$ to $p_{\text{ Troland}}$ from the relation $p_T=p_I/(p_I-1)$.

Of course, it is possible to inscribe lines on the color triangle, from either set of formulas, which represent purities not physically realizable, (i.e., lines which fall outside the area bounded by the spectrum scale and the straight line joining its extremes), since any value of r, g, and b, positive or negative, may be obtained from the formulas by substituting the right value for the purity p. Whether the purities represent physical colors or not, however, is beside the point. The formulas, either from Ives' definition or Troland's definition, give the relationship between the trichromatic representation of color data and an arbitrary "monochromatic" method of representation, and they give this relationship whether the objective stimulus invoked is real or imaginary. From the foregoing, it is evident that Ives' definition of purity is the more convenient.

In conclusion, the author wishes to acknowledge his indebtedness to Mr. I. G. Priest for his hearty coöperation.

BUREAU OF STANDARDS,
AUGUST 1925–FEBRUARY 1926.

APPENDIX

IMPORTANCE OF THE COLLINEARITY CONDITION DEMONSTRATED BY ACTUAL COMPUTATION

When any of the forms of purity formula are applied "blindly" (i.e. without regard for the condition equations beyond the accuracy dictated by experimental considerations) the result must be looked upon as so unreliable as to be almost meaningless. Priest[23] has computed from Tuckerman R, Tuckerman G, and Tuckerman B, the purities of a wide range of colors from Weaver's tabulation of "Representative Monochromatic Analyses,"[24] and finds differences in some cases as large as 0.100. In order to show that most of these differences are due to collinearity errors in the *third* place of Weaver's excitation data, the computations of Table 2 have been carried out. The wave length 440 mμ was chosen because: (1) that wave length yields the largest deviations recorded by Priest, and (2) at the same time is sufficiently far distant from the "regions of inconvenience" (Section II, Table 1) that the gross deviations found can not possibly be ascribed to simple rejection error based on the failure to meet exactly the other conditions. A reference to Table 2 and footnote will show how completely the differences are eliminated.

DERIVATION OF THE PURITY FORMULA FOR COLORS OF PURPLE HUE

Let the point (r, g, b) represent the color of the purple hue whose purity, p, is desired. Since this color is of purple hue, there exists homogeneous radiation which, when added to the non-spectral stimulus, will be a color match for the neutral stimulus. Let ρ_λ, γ_λ, β_λ be the excitation values for this homogeneous radiation. Let ρ_c, γ_c, β_c be the excitation values for one of the non-spectral stimuli complementary (in general evoking a purple not referred to by the point, r, g, b, though this point is not excluded) in hue to the spectral color. Then:

$$\rho_\lambda+\rho_c=\gamma_\lambda+\gamma_c=\beta_\lambda+\beta_c=K$$

And, from this:

$$\rho_c=K-\rho_\lambda\,; \qquad \gamma_c=K-\gamma_\lambda\,; \qquad \beta_c=K-\beta_\lambda\,.$$

[23] Priest: Loc. cit., p. 517.
[24] Troland: Loc. cit., pp. 586–587.

TABLE 2. *The Computational Importance of the Collinearity Condition**

p	x'	$3x'$	$1-3x'$	$f'x=\frac{1-3x'}{1-3X}$	$\delta f'x=fx'-fr'$	$\delta x'=\frac{(1-3X)\delta fx}{3}$	$x=x'+\delta x'$	$3x$	$1-3x$	$\frac{1-3x}{1-3X}$	xL_x	ΣxL_x	$\frac{x}{\Sigma xL_x}$	$1-\frac{x}{\Sigma xL_x}$	p	p'
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	R .000	.000	1.000	1.00			.0000	.000	1.000	1.00	.0000		.0000	1.0000		
1.00	G .007	.021	.979	1.00			.0070	.021	.979	1.00	.0043	.0162	.4320	.5680		
	B .993	2.979	1.979	1.00			.9930	2.979	1.979	1.00	.0119		61.30	60.3		
	r .006	.018	.982	.982	.000	.0000	.0060	.0180	.9820	.982	.0022		.2727	.7273	.727	.741
.80	g .015	.045	.955	.976	.006	.0020	.0130	.0390	.9610	.982	.0080	.0220	.5909	.4091	.721	.622
	b .979	2.937	1.937	.979	.003	.0020	.9810	2.9430	1.9430	.983	.0118		44.59	43.59	.723	.683
	r .015	.045	.955	.955	.000	.0000	.0150	.0450	.9550	.955	.0055		.4933	.5067	.507	.521
.60	g .023	.069	.931	.951	.004	.0013	.0217	.0651	.9349	.955	.0133	.0304	.7119	.2881	.507	.467
	b .962	2.886	1.886	.953	.002	.0013	.9633	2.8899	1.8899	.955	.0116		31.69	30.69	.508	.493
	r .034	.102	.898	.898	.000	.0000	.0340	.1020	.8980	.898	.0126		.6995	.3005	.301	.298
.40	g .040	.120	.880	.899	.001	.0003	.0403	.1209	.8791	.898	.0249	.0486	.8290	.1710	.301	.306
	b .926	2.778	1.778	.898	.000	.0003	.9257	2.7771	1.7771	.898	.0111		19.03	18.03	.299	.301
	r .074	.222	.778	.778	.000	.0000	.0740	.2220	.7780	.778	.0274		.8544	.1456	.146	.154
.20	g .081	.243	.757	.773	.005	.0016	.0794	.2382	.7618	.778	.0490	.0866	.9170	.0830	.146	.131
	b .845	2.535	1.535	.776	.002	.0016	.8466	2.5398	1.5398	.778	.0102		9.780	8.780	.146	.144
	r .130	.390	.610	.610	.000	.0000	.1300	.3900	.6100	.610	.0481		.9303	.0697	.070	.053
.10	g .130	.390	.610	.623	.013	.0042	.1342	.4026	.5974	.610	.0828	.1397	.9606	.0394	.069	.092
	b .740	2.220	1.220	.616	.006	.0042	.7358	2.2074	1.2074	.610	.0088		5.268	4.268	.071	.073

$$p=\frac{1-\frac{x}{\Sigma xL_x}}{1-\frac{X}{\Sigma XL_x}},$$

where $\Sigma xL_x=rL_r+gL_g+bL_b$,
$\Sigma XL_x=RL_r+GL_g+BL_b$.
$L_r=0.370$,
$L_g=0.617$,
$L_b=0.012$.

* Columns 1 and 2 were taken from Weaver's data* and represent (1) purity which is one minus one-hundredth the per cent white given in Weaver's table; and (2) the trilinear coordinates (r, g, b) of points on the 440 mμ wave length line. The point corresponding to unit purity is designated as (R, G, B) according to the present notation. The column is headed, x', which may represent either r, g, or b, as indicated. The g-coordinate, not given in Weaver's table, has been calculated from the relation: $g=1-r-b$; which imposes the second condition, (see introduction).

Column 5 is the test for wave length constancy (Condition 4), and the variations within triads show that the condition is not quite met, or $(1-3r)/(1-3R)$ does not quite equal $(1-3g)/(1-3G)$ or $(1-3b)/(1-3B)$.

Each parameter, therefore, defines a distinct point on the 440 mμ wave length line, the point indicated by r being different from that defined by g or by b. It is planned, however, to alter g and b slightly so that they correspond to the point indicated by r, and to show that this slight alteration (in general less than 0.002, see Column 7) is enough to make the calculated purities check.

Column 8 gives the trilinear coordinates of the r-point correct to four places as is shown by the constancy of the triads in column 11.

Column 16 gives the values of purity by all three of the Tuckerman forms, as the headings indicate. The deviations of these purities from those (Column 1) indicated in Weaver's table are quite marked and have already been commented upon by Priest.**

Column 17 gives the purities from the same formulas but by applying them "blindly" to Weaver's uncorrected data (Column 2). These are the same values as are plotted on Priest's comparison graph.***

The very material reduction of the variation within the triads of Column 16 as compared to that in Column 17 shows the computational importance of the fourth equation of condition (condition for collinearity with spectral point and white point).

* J.O.S.A. & R.S.I., *6*, No. 6, p. 586; August 1922. ** Priest: Loc. cit. p. 517. *** Priest: Loc. cit., p. 517.

By adding an amount, ω, (where ω may take positive, negative or zero values) to each of ρ_c γ_c, β_c it will be possible to arrive at the color represented by the point (r, g, b), hence:

$$r=\frac{\omega+K-\rho_\lambda}{3(\omega+K)-(\rho_\lambda+\gamma_\lambda+\beta_\lambda)}\ ;\quad g=\frac{\omega+K-\gamma_\lambda}{3(\omega+K)-(\rho_\lambda+\gamma_\lambda+\beta_\lambda)}\ ; \tag{1}$$
$$B=\frac{\omega+K-\beta_\lambda}{(\omega+K)-(\rho_\lambda+\gamma_\lambda+\beta_\lambda)}\ .$$

The coordinates of the point representing the spectral complementary of (r, g, b) are:

$$R=\frac{\rho_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ ;\qquad G=\frac{\gamma_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ ;\qquad B=\frac{\beta_\lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ . \tag{2}$$

From the experimental procedure in homo-hetero analysis with respect to colors of purple hue: (See Section IV).

$$\boldsymbol{B}_\Lambda+\boldsymbol{B}_s=\boldsymbol{B}_m=\boldsymbol{B}_w\ .$$

From this point, the derivation depends upon what function of the various field brightnesses is chosen to represent the purity of the color (r, g, b). Take Ives' definition first: $p=\boldsymbol{B}_\Lambda/\boldsymbol{B}_s$, $\boldsymbol{B}_\Lambda$ being taken negative. The brightnesses may then be expressed in terms of the luminosity coefficients (L_r, L_g, L_b) of the elementary excitations, as follows:

$$\boldsymbol{B}_\Lambda=-(\rho_\lambda L_r+\gamma_\lambda L_g+\beta_\lambda L_b)\ . \tag{3}$$
$$\boldsymbol{B}_s=(K+\omega-\rho_\lambda)L_r+(K+\omega-\gamma_\lambda)L_g+(K+\omega-\beta_\lambda)L_b,$$
$$=(K+\omega)(L_r+L_g+L_b)-(\rho_\lambda L_r+\gamma_\lambda L_g+\beta_\lambda L_b)\ .$$
$$\boldsymbol{B}_s=K+\omega+\boldsymbol{B}_\Lambda\ . \tag{4}$$
$$\boldsymbol{B}_m=\boldsymbol{B}_w=\boldsymbol{B}_\Lambda+\boldsymbol{B}_s=K+\omega+2\boldsymbol{B}_\Lambda\ . \tag{5}$$

From equations (1):

$$\frac{\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{r-R}{3r-1}+\frac{2\boldsymbol{B}_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{g-G}{3g-1}+\frac{2\boldsymbol{B}_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{b-B}{3b-1}+\frac{2\boldsymbol{B}_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}\ . \tag{6}$$

From equation (5): $p=\boldsymbol{B}_\Lambda/\boldsymbol{B}_s=\boldsymbol{B}_\Lambda/(\boldsymbol{B}_w-\boldsymbol{B}_\Lambda)=\boldsymbol{B}_\Lambda/(K+\omega+\boldsymbol{B}_\Lambda)$. To evaluate K, the constant from which the measurement of the variable, ω, may be said to start, let $p=-1$, when $\omega=0$, then: $-1=\boldsymbol{B}_\Lambda/(K+\boldsymbol{B}_\Lambda)$; and $K=-2\boldsymbol{B}_\Lambda$.
From (5), $\boldsymbol{B}_w=\omega$, whence $p=-\boldsymbol{B}_\Lambda/(\boldsymbol{B}_\Lambda-\omega)$.

Transforming to the arbitrary brightness units of Section I:

$$\boldsymbol{b}_h=\boldsymbol{B}_\Lambda/(\rho_\lambda+\gamma_\lambda+G_\lambda)\ . \tag{7}$$
$$\boldsymbol{b}_h=-(RL_r+GL_g+BL_b)\ ,\ \text{from (2) and (3)}\ , \tag{8}$$

And, from (6):

$$\frac{\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{r-R}{3r-1}+2\boldsymbol{b}_h=\frac{g-G}{3g-1}+2\boldsymbol{b}_h=\frac{b-B}{3b-1}+2\boldsymbol{b}_h\ . \tag{9}$$

Substituting: (7), (8) and (9) in the relation: $p=-\ \boldsymbol{B}_\Lambda/(\boldsymbol{B}_\Lambda-\omega)$

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+(r-R)/(1-3r)} \tag{Judd R}$$

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+(g-G)/(1-3g)} \tag{Judd G}$$

$$=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+(b-B)/(1-3b)} \tag{Judd B}$$

These three forms will be recognized as the Judd R, Judd G, and Judd B forms derived originally to apply to colors of spectral hue. The Ives' definition for purity, therefore, yields the same formula for all colors, spectral or non-spectral.

Taking, in turn, Troland's definition for purity of colors of purple hue: $p=B_\Lambda/B_m$, the brightnesses being taken always positive, we proceed as before. Only the relations which differ from those in the previous derivation are given in the following, the corresponding equations being denoted with the subscript "a."

$$B_\Lambda=\rho_\lambda L_r+\gamma_\lambda L_g+\beta_\lambda L_b\ . \quad (3a)$$

$$B_s=K+\omega-B_\Lambda \quad (4a)$$

$$B_m=B_w=B_s+B_\Lambda=K+\omega-B_\Lambda+B_\Lambda=K+\omega\ . \quad (5a)$$

$$\frac{\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{r-R}{3r-1}-\frac{2B_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{g-G}{3g-1}-\frac{2B_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{b-B}{3b-1}-\frac{2B_\Lambda}{\rho_\lambda+\gamma_\lambda+\beta_\lambda} \quad (6a)$$

From (5a):

$$p=B_\Lambda/B_m=B_\Lambda/(K+\omega)$$

To evaluate K, let $\omega=0$, when $p=\frac{1}{2}$[25]
Then: $\frac{1}{2}=B_\Lambda/K$; and $K=2B_\Lambda$
Hence from (5a): $p=B_\Lambda/(2B_\Lambda+\omega)$.
But from (2) and (3a):

$$b_h=RL_r+GL_g+BL_b\ ,\ \text{therefore} \quad (8a)$$

$$p=\frac{b_h}{2b_h+\omega/(\rho_\lambda+\gamma_\lambda+\beta_\lambda)}\ .$$

$$\frac{\omega}{\rho_\lambda+\gamma_\lambda+\beta_\lambda}=\frac{r-R}{3r-1}-2b_h=\frac{g-G}{3g-1}-2b_h=\frac{b-B}{3b-1}-2b_h\ . \quad (9a)$$

Substituting:

$$p=\frac{RL_r+GL_g+BL_b}{-(r-R)/(1-3r)}$$

$$p=\frac{RL_r+GL_g+BL_b}{-(g-G)/(1-3g)}$$

$$p=\frac{RL_r+GL_g+BL_b}{-(b-B)/(1-3b)}$$

These forms are recognized to be different from the Troland forms for colors of spectral hue. The Troland definition ($p=B_\Lambda/B_m$) does not, therefore, yield formal continuity.

[25] That this is the natural choice may be seen from the fact that $p=\frac{1}{2}$ in Troland's system corresponds to $p=-1$ in Ives'. We cannot set $p=1$, since $p=B_\Lambda/B_m=B_\Lambda/(B_\Lambda+B_s)<1$, if $B_s\neq 0$.

LEAST RETINAL ILLUMINATION BY SPECTRAL LIGHT REQUIRED TO EVOKE THE "BLUE ARCS OF THE RETINA"

J. Res. Nat. Bur. Stand. (U.S.), 2, 441-451 (1929) RP43

This experimental investigation established that the blue arcs that are observed to extend from the macula to the blind spot are caused by activity initiated by rods, not cones.

The term "photon" used throughout this paper refers, not to Planck's quantum of light but, to retinal illuminance measured in the unit now known as the "troland", which is named after the originator of the unit, who called it the "photon". Dr. Judd later played a crucial role in replacing the name "photon" with "troland", both to eliminate confusion with Planck's quantum and to honor L. T. Troland, chairman of the first Optical Society Committee on Colorimetry.

LEAST RETINAL ILLUMINATION BY SPECTRAL LIGHT REQUIRED TO EVOKE THE "BLUE ARCS OF THE RETINA"[1]

By Deane B. Judd

ABSTRACT

Interest in the "blue arcs of the retina," first described by Purkinje, has recently been renewed by Ladd-Franklin, who establishes (in a series of papers) to a high degree of probability the theoretical views of Druault. Druault suggested (and later, independently, Ladd-Franklin also) that the blue arcs are due to the emission of light by the fibers of the optic nerve that pass over the surface of the retina. It has been repeatedly mentioned by Ladd-Franklin and others (Druault, Troland, Amberson, Ellis) that the blue arcs are obtained more easily with red light than with a stimulus of any other color. This suggests that the origin of the nerve activity causing the blue arcs lies in the retinal cones rather than in the rods, since the rods are relatively insensitive to red light.

The blue-arc phenomenon has been tentatively linked by the author with a certain phase (called the Purkinje phase, or Bidwell's "ghost") of the periodic afterimage following brief stimulation by light of the extrafoveal retina. The Purkinje phase undoubtedly depends on the action of the rods; hence, the two phenomena which are quite similar in some respects would differ in origin if the blue arcs were really initiated by cone action.

As a check, then, on the origin of the nerve activity producing them, the blue arcs were aroused by pure spectral light (2° circular field), the retinal illumination being subsequently reduced until the blue arcs no longer appeared. In this way the retinal illumination required to evoke the blue arcs has been determined as a function of wave length for the author's right eye.

CONTENTS

[1] Presented in part before the annual meeting (November, 1927) of the Optical Society of America in Washington.

I. INTRODUCTION

The "blue arcs of the retina," known since Purkinje's account of them in 1825, have attracted considerable recent attention.[2] When the macular[3] region of the retina is stimulated by a patch of light (observers have usually used a circle or vertical band), the subject reports not only the image of this stimulus, but usually also two "blue arcs."[4] These arcs correspond quite exactly to the position of the nerve fibers which extend across the inner surface of the retina from the stimulated area to the exit point of the optic nerve ("blind spot").[5] It is universally agreed[6] that activity of these nerve fibers in some way causes them to become visible as the "blue arcs of the retina." H. Gertz proposed to account for the arcs by secondary stimulation of neighboring retinal structures through (1) an external circuit of, or (2) induction by, the action currents set up by the primary stimulus. This sort of explanation (by secondary electric stimulation) has been accepted by Troland, Amberson, Ellis, and Davis. There is some uncertainty as to which structure is to be thought of as being affected by the secondary electric stimulus, whether it be the nerve fibers parallel to those primarily excited (Amberson),[7] the ganglion cells of those fibers (Amberson),[8] the nerve fibers (bipolar cells) perpendicular to them (Troland),[9] or the rods and cones themselves (H. Gertz; Davis).

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[2] Zeeman, Zs. f. Psych. u. Physiol. d. Sinnesorgane, **6**, p. 233; 1894. Siethoff, Zs. f. Psych. u. Physiol. d. Sinnesorgane, **14**, p. 375; 1897. Tscherning, Klin. Monatsbl. f. Augenheilk., **36**, p. 223; 1898; Ann. de la Policlin. de Paris, 1898; Encyclopédie française d'ophthalmologie, III, p. 228; 1904. Charpentier, Compt. Rend. d. l. Soc. d. Biol. (ser. 2), **8**, p. 765; 1885. Thomsen, Skand. Arch. f. Physiol., **37**, p. 1; 1918. O. Gertz, Hosp.-Tid., Københ. (R), **5**, pp. 7, 1197; 1914. H. Gertz, Ueber autoptische Wahrnehmung der Sehtätigkeit der Netzhaut, Skand. Archiv. f. Physiol., **19**, pp. 381–408; 1907; ibid, **21**, pp. 315–350; 1909. Also Ueber entoptische Wahrnehmung des Actionsstroms der Netzhautfasern, Zentralbl. f. Physiol., **19**, pp. 229–233; 1905. J. C. Hubbard, A Curious Secondary Visual Phenomenon Resulting from a Stimulation of the Macular Region, Psych. Bull., **7**, pp. 196–199; 1910. A. Druault, Sur un Phénomène Entoptique en Rapport avec le trajet des fibres optiques dans la rétine (phénomène des arcs), J. de Physiologie et de Pathologie générale, **16**, p. 649; 1914. L. T. Troland, The "All or None" Law in Visual Response, J. Opt. Soc. Am., **4**, pp. 160–185; 1920. An Entoptic Phenomenon Demonstrating the Optic Impulse, Psych. Bull., **17**, p. 55; 1920. W. R. Amberson, Secondary Excitation in the Retina, Amer. J. Physiol., **69**, pp. 354–370; 1924. Christine Ladd-Franklin, J. Opt. Soc. Am. and Rev. Sci. Inst., **12**, p. 494; 1926; ibid, **14**, p. 474; 1927; Proc. Nat. Acad. of Sci., **12**, p. 413; 1926; Science, **66**, p. 239; 1927; ibid, **67**, p. 162; 1928; Comptes Rendus, **185**, p. 584; 1927; J. Opt. Soc. Am. and Rev. Sci. Inst., **16**, p. 333; 1928. F. W. Ellis, Secondary Excitation of the Retina and the Variation of the Intensity of the Resulting Sensation, Am. J. Physiol., **82**, p. 290; 1927. The Nature of the Stimulus which Excites the Blue Arcs of the Retina and Related Phenomena, Am. J. Physiol., **84**, p. 485; 1928. H. Davis, Theory of "Visible Radiation" from an Excited Nerve Fiber, Science, **67**, p. 69; 1928.

[3] If Amberson's results are correct (loc. cit., p. 359), only those portions of the macular region which contain rods are effective in producing the blue arcs. Stimulation of the central rod-free area does not do it.

[4] Often called "blue" inaccurately as an abbreviation; the color is a slightly reddish blue.

[5] Amberson (loc. cit., p. 361), confirming this conclusion previously based on shape and approximate size only, has shown for 10 observers that the length of the arcs corresponds to the papillo-foveal distance.

[6] Except Siethoff (loc. cit.), whose views are shown to be untenable by Druault (loc. cit., p. 650) and, later, by Amberson (loc. cit., p. 357) also.

[7] Amberson, loc. cit., p. 367.

[8] Amberson, loc. cit., p. 368.

[9] Troland, loc. cit., p. 167.

In 1914 there appeared a paper by Druault, apparently until now unknown in this country,[10] which opposes grave objections to the secondary electric stimulation theory and which proposes another explanation, namely, that the fibers are visible because they emit physical light. Ladd-Franklin (1920–1926) independently formulated similar objections and also independently proposed the same alternative explanation. She added, however, the conclusive argument that the blue arcs must result from action upon the light-sensitive substance in the rods and cones by physical light, because (for many subjects) the blue arcs are followed by a chromatic residual image (dark greenish yellow).[11] Only stimulation of the light-sensitive substance in the cones by physical light yields a chromatic residual image; electric stimulation does not do it nor stimulation by pressure.[12]

Though, to be sure, acceptance of the theory of visible radiation proposed by Druault, even following Ladd-Franklin's definitive argument in its favor, has been far from universal; nevertheless, the writer has been inclined to accept it, persuaded partly by confirmation obtained in connection with the Purkinje phase of the periodic afterimage following momentary stimulation of the extra-foveal retina by light.[13] It was found that a number of the properties of the Purkinje phase, which have hitherto remained unaccounted for, yielded perfectly to the same explanation that had been proposed (by Druault and Ladd-Franklin) for the blue arcs, namely, that active nerve fiber gives off physical light. In the case of the Purkinje phase we assume that light is emitted by the nerve structures (bipolar cells, etc.), leading from the outer surface of the retina (rod-cone layer) to the inner surface.

But there are two outstanding differences between the two phenomena. If we are to ascribe the Purkinje phase entirely or partly to the same sort of bioluminescence, or production of visible radiation to which we ascribe the blue arcs, it would be well to examine these differences.

First, the blue arcs are, at best, very faint, as would befit phenomena of secondary retinal stimulation. The Purkinje phase by the right choice of experimental conditions can be made very vivid and striking. Detailed images nearly as distinct and brilliant as the primary image often appear.[14] The explanation of the brilliance of the Purkinje phase may lie in the fact that the nerve structure to which bioluminescence is attributed lies perhaps one-third to one-fifth as far from the rod-cone layer as the fibers on the inner surface of the retina;

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[10] Sent to Mr. Priest in response to a request for reprints of papers on the subject of color vision.

[11] Ladd-Franklin, Sci., **66**, p. 240; 1927. H. Gertz, Skand. Archiv., **19**, p. 404; 1907.

[12] Ladd-Franklin, Sci., **66**, p. 240; 1927.

[13] D. B. Judd, A Quantitative Investigation of the Purkinje Afterimage., Am. J. Psych., **38**, pp. 507–533; 1927.

[14] Judd, loc. cit., p. 519.

hence, for the same *brightness* of radiating surface (active nerve fiber) the rod-cone illumination would be much higher than that causing the blue arcs, the Purkinje phase appearing, on that account, much more *brilliant*.[15] Another alternative explanation (perhaps less satisfying) is to say that the observed Purkinje phase is the sum of two components: (1) The contribution of physical light from contiguous active nerve fiber and (2) the contribution of a nature similar to that of the other phases (Hering phase, third positive phase, negative phases, etc.).[16] This explanation is favored by the fact that the properties of the Purkinje phase which are particularly suggestive of emission of physical light are observed under conditions which do not give a brilliant image but which give an image of brilliance comparable to that of the blue arcs.

Second, the Purkinje phase seems to arise from initial activity of the rods. It does not appear in the fovea, and it does not appear following a primary stimulus of pure spectral red light (say, of 650 mμ). On the other hand, the blue arcs, according to Ellis,[17] though contrary to Amberson,[18] appear "best defined and most intense when the stimulating image falls within the fovea." Furthermore, a red primary stimulus not only evokes the blue arcs successfully, but a red light is the best for demonstration of the blue arcs.[19] From these facts the conclusion is suggested that the blue arcs are due to nerve activity initiated by the retinal cones.[20]

It is the purpose of the present paper to give the results of an experiment designed to show whether cone activity initiates the nerve activity responsible for the blue arcs (as the facts just presented suggest) or whether rod activity initiates it (as Amberson has concluded and as the connection of the blue arcs with the Purkinje phase in other respects suggest).

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[15] The use of the terms "brightness" and "brilliance" is as given in the report of the colorimetry committee. See J. Opt. Soc. Am. and Rev. Sci. Inst., 6, p. 534; 1922.

[16] Consult in this connection: Hartline (Am. J. of Physiol., 73, p. 600; 1925) who has succeeded in demonstrating that one of the positive afterimage phases (probably the third positive phase) is correlated with a rise in electric potential in the retina.

[17] Ellis, loc. cit., p. 290; 1927.

[18] Amberson, loc. cit., p. 359.

[19] Thus, Druault (loc. cit., p. 649) says, "Le phénomène peut s'observer avec n'importe quelle lumière, mais plus facilement avec la lumière rouge." Ellis says (loc. cit., p. 293; 1927) "Red light is ordinarily employed to excite the blue arcs, and it is especially well adapted for this purpose * * *." Ladd-Franklin (loc. cit. Nat. Acad. Sci.) recommends for demonstration of the blue arcs "A simple band of bright red light thrown upon a screen in a dark room." Hubbard (loc. cit., p. 196) suggests diffuse light from a "ruby lamp" for demonstration of the blue arcs, and further says, "The brushes (arcs) appear very faint at the extreme red. They increase in brightness as the eye travels up the spectrum and are brightest in the orange red * * *."

[20] In spite of this evidence, Amberson (loc. cit., p. 359), from regional considerations alone, concludes that, "* * * the effects under discussion are rod, rather than cone, phenomena, in the sense that the primarily excited and conducting fibers must arise from ganglion cells on the rod pathways in the region of primary stimulation."

As the title of this paper indicates, the experiment consisted of determining the least retinal illumination by spectral light required to evoke the blue arcs. It is apparent that if the rods initiate the nerve activity responsible for the blue arcs a pure spectral stimulus of wave length say, 640 mμ, would have to be at a much higher illumination (measured in photons—a unit embodying "cone" visibility by reason of the method of measurement) than a stimulus of wave length less than, say 550 mμ, because the rods are relatively insensitive to light of wave length 640 mμ. On the other hand, if the cones initiate the nerve activity, a stimulus of a given number of photons would yield a certain definite degree of nerve activity regardless of the wave length of that stimulus (retinal region stimulated remaining constant). If this definite degree of nerve activity were just enough to evoke the blue arcs, it is evident that the retinal illumination required would be a constant independent of wave length. We can determine, then, from the shape of the curve of least retinal illumination against wave length (whether it be a constant illumination or higher for long wave lengths) whether, at the lowest illumination evoking them, the cones or the rods initiate the nerve activity responsible for the blue arcs.

II. APPARATUS

The apparatus used was that designed by Priest [21] for another purpose but which is perfectly adapted without change to the present purpose. It provides (1) a 2° circular field which may be more or less strongly illuminated by light of any wave length from 400 mμ to 660 mμ, and (2) an auxiliary photometer by which the brightness of the field may be measured using the flicker method. It should be noted here that the slits (both collimator and ocular) were opened to a width corresponding to 10 mμ, and that filters reducing stray light were used for wave lengths less than 460 mμ and greater than 630 mμ. Also, the diaphragm and ocular slit were so arranged as to constitute an artificial pupil, roughly rectangular, of about 1.5 by 2.0 mm. The auxiliary photometer was calibrated by comparison with a Holophane light meter,[22] but great care was not taken to insure absolute accuracy, since relative accuracy rather than absolute accuracy is important in this experiment. The absolute values reported (fig. 1) may be in error by as much as 20 per cent, but the relative photometric errors are in the neighborhood of 5 per cent.

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[21] I. G. Priest, Apparatus for the Determination of Color in Terms of Dominant Wave Length, Purity and Brightness, J. Opt. Soc. Am. and Rev. Sci. Inst., 8, pp. 174-198; 1924.

[22] Adjusted and checked by R. P. Teele, photometric section, Bureau of Standards.

The surrounding field illumination for the right (observing) eye was nearly zero, but for the left eye it was somewhat higher, being due to the blackened surface of the eye shield illuminated by the artificial light of the room. The field on which the arcs appeared was therefore not black, but (since no retinal rivalry appeared) a fairly dark gray. No attempt was made to choose conditions highly

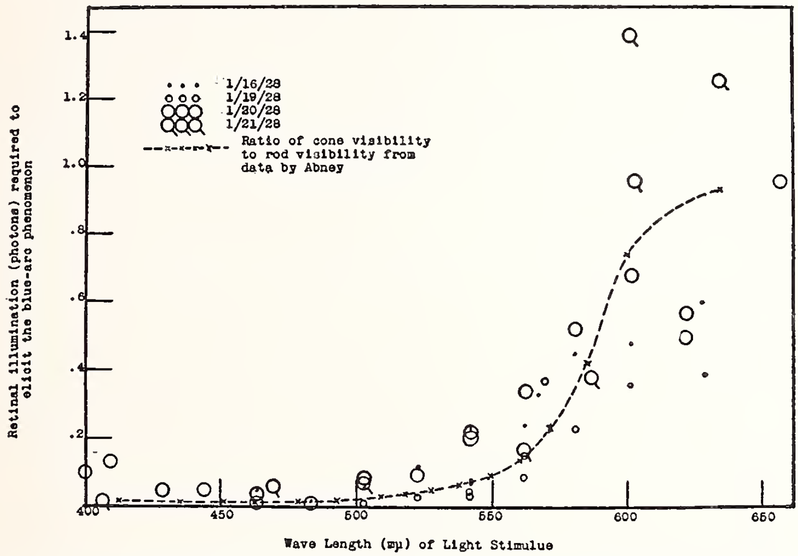

FIG. 1.—*Retinal illumination (photons) required to elicit the blue arc phenomenon as a function of the wave length of the primary light stimulus*

This function agrees well with the ratio of cone visibility to rod visibility from data by Abney

favorable to the production of the blue arcs. They might have been evoked at considerably lower illuminations than those reported if a long narrow vertical slit had been used instead of the circular field, or if they had been projected onto the dark field of the thoroughly dark adapted eyes. The attempt was rather made to approach a uniform and stable state of adaptation.

III. METHOD OF PROCEDURE

The readings were taken on four mornings from 8 to 9 o'clock. The room containing the apparatus was lighted throughout the hour by incandescent lamps. Adaptation was further affected by frequent reading of the illuminated dial of the ammeter attached to the lamp of the auxiliary photometer and by the use of the photometer.

At the start of a determination the wave-length drum was set to the desired reading, a 1 per cent (or 3 per cent or 10 per cent) sector disk was inserted before the collimator slit, and the right eye applied to the eyepiece. The blue arcs were produced by moving the fixation point horizontally away from the stimulus (2° circular field) and were seen to develop from a horizontal band into ever-widening arcs as the distance between the stimulus and fixation point increased. Then, the retinal illumination was decreased (by decreasing the current through the lamp whose ribbon filament was imaged on the collimator slit) to perhaps one-half its value, and the blue arcs again produced by sweeping the fixation point across the right-hand surrounding field. This process was repeated until the blue arcs appeared in only about half the attempts to produce them. The brightness of the field resulting in this condition was then photometered by the flicker method, the 1 per cent sector disk being removed, and the surrounding field illuminated so that its brightness was about 0.5 ml. Since the adjustment of the illumination to the point which just evokes the arcs (that is, a liminal setting) can not be made at all precisely (perhaps a 100 per cent error is not uncommon), only one flicker setting was made. The setting of the wave-length drum was noted on the record sheet. This constituted a single determination.

These determinations were continued without pause until 9 o'clock. No certain evidence of fatigue with respect to the blue arcs, nor of significant change in level of adaptation was found.

The retinal illuminations (I) were calculated from the brightnesses (B) of the surface being viewed according to the formula:

$$I_{\text{(photons)}} = 3.18\, B_{\text{(millilamberts)}}\, A_{\text{(sq. mm)}}$$

or, since the artificial pupil had an area (A) of 3.0 sq. mm:

$$I = 9.54\, B$$

On January 14 the hour was spent in making liminal settings for the sake of practice. Regular observation was commenced on January 16, and all the readings taken thereafter appear plotted in Figure 1, a separate symbol being used for each of the four days.

IV. RESULTS AND DISCUSSION

It is immediately apparent on glancing at Figure 1 that, notwithstanding the low precision (which gives a range of readings at each wave length of about 3 to 1), there is a definite rise [23] with increasing wave length in retinal illumination required to evoke the blue arcs in the author's right eye. Thus, although red light serves best to demonstrate the arcs, still it is true that at the lowest illuminations producing them spectral light of long wave length is definitely less effective than light of short wave length. This situation strongly suggests that the retinal rods initiate the nerve activity responsible for the blue arcs when the retinal illumination is quite low (less than one photon). We shall now proceed to inquire whether the rise in illumination for a stimulus of long wave length is in quantitative agreement with the assumption that the retinal rods initiate the nerve activity responsible for the blue arcs. The derivation of the shape of the curve expected on this assumption may be accomplished in this way:

The liminal setting in each case corresponds to a constant number of impulses per second over the fibers of the optic nerve. The constant number of impulses corresponds to a constant state of activity of the rods (we assume rods, not cones) affected by the primary stimulus. With constant state of adaptation (which has been approximated by the experimental procedure) this constant retinal activity

[23] The points representing the work of each day taken separately all show a very similar rise, but there seems to be a tendency for all points of one day (for example, 1/20/28) to fall above the corresponding points for another day (for example, 1/19/28). This may be due either to (1) a different adaptation level, or (2) a different criterion of appearance or nonappearance of the blue arcs. With respect to (2) it may be remarked that some difficulty was found in adopting a definite and describable criterion for appearance of the arcs. On this account the criterion, although easily remembered throughout a given hour, may not have been the same on the next or any other day. At a liminal illumination a sweep of the fixation point along the horizontal sometimes brought out the moving arcs faintly but with clarity, at other times the suggestion of movement was present without the form of the arcs, and sometimes nothing at all could be detected. There were many gradations between these describable appearances.

It is appropriate at this time to remark that with higher retinal illuminations the arcs are unmistakeably more brilliant than at any of these doubtful stages and take on the pale reddish-blue color that has suggested the name "blue arcs" (at the doubtful stages the arcs appear gray, a fact which is consistent with the "visible radiation" theory). In respect to brilliance variation, my observations substantiate those of Hubbard, who says, "The intensity of the brushes increases with the intensity of the exciting light, * * * " and also those of Ellis (loc. cit., p. 293; 1927) who criticizes Troland's (loc. cit. J. Opt. Soc. Am.) view that the arcs constitute an example of the "all or none law in visual response." While it is true that the observations of Ellis agree with my own, still I do not agree with his criticism of Troland's view because that view is consistent with some variation in brilliance of the arcs even though their brightness (visible radiation theory) or the action current (secondary electric stimulation theory) may be constant. The brilliance should depend, according to that view, somewhat on state of adaptation (and does depend on it, see Amberson, loc. cit., p. 356) and number of nerve fibers activated by the primary stimulus Troland, loc. cit., J. Opt. Soc. Am., p. 168). It seems, on the other hand, reasonable to suppose that Troland himself did not see nearly the variation in brilliance of the arcs that Ellis, Hubbard, or myself have seen, since from measurements he gives (loc. cit., J. Opt. Soc. Am., p. 179) the brilliance of the arcs is only slightly above the threshold. Perhaps Troland might never have been led to cite the blue arcs as an example of the "all or none" law in visual response, if their maximum brilliance for him had been greater. It is my opinion, however, that they should still be thought of as a legitimate example of the "all or none" law even when the observations of Hubbard, Ellis, and myself be taken into account. If Troland's view is to be rejected, evidence must be found by Troland's own technique showing that the brilliance of the blue arcs for other observers is actually as variable as the qualitative observations (just cited) indicate.

is due to the constancy of the product, $E_\lambda(V_r)_\lambda$, where E_λ is the energy (that is, ergs per second per unit visual angle) of the light stimulus of wave length $_\lambda$, and $(V_r)_\lambda$ is the rod visibility for that wave length. We have the relation, then, that the shape of the curve of retinal illumination (I_λ) must be such that

$$E_\lambda(V_r)_\lambda = \text{a constant}$$

But we measure $E\lambda$ by measuring the illumination corresponding to a large multiple (most often here, 100) of it, thus:

$$E_\lambda = KI_\lambda/(V_c)_\lambda$$

where $(V_c)_\lambda$ is the cone visibility of light of wave length λ. (K is the constant which determines the units in which E_λ is expressed.) We believe that only the cones are effective in this photometric measurement by reason of the experimental conditions adopted (2° field of high brightness, bright surrounding field). Solving for I_λ from these two relations, we obtain

$$I_\lambda = (V_c)_\lambda/(V_r)_\lambda \text{ times a constant}$$

On Figure 1 is also shown, therefore, this ratio, $(V_c)(/V_r)_\lambda$ evaluated from data by Abney.[24] It is seen that this ratio (multiplied by a constant arbitrarily selected to secure the best agreement) varies with wave length in such a way as to agree substantially with the measurements taken of the least retinal illumination (I_λ) required to evoke the blue arcs.[25] There are two outstanding departures

[24] Abney, Researches in Colour Vision, pp. 94, 98; London: Longmans-Green; 1913. Column IV, Table IV, p. 94; Table VI, p. 98. This data of Abney is taken because none of the more recent observers apparently used quite the field size which would make their results applicable to this case. It seems clear from Amberson's results (loc. cit., p. 359) that the blue arcs result from a stimulation of the outer zone of the macular region (that is, the macula lutea minus the central rod-free area). Abney's data refer specifically to the macula lutea (yellow spot). To my knowledge, no other investigator gives this specification. There are apparently large individual variations in the size of the yellow spot; Parsons (Introduction to the Study of Colour Vision, Cambridge: University Press, p. 13; 1924) says that the macula may vary from 4° to 12° in different individuals. In Amberson's case, a 6° circular field includes all retinal elements capable of giving rise to the blue arcs. Visibility data taken with a 6° field would, however, be as reliable as that taken (as Abney's data was) specifically for the observer's yellow spot only if the yellow spot of the observer covered the entire 6° field. Otherwise errors would be introduced by absence of information concerning macular pigmentation. Laurens (Am. J. of Physiol., **67,** p. 354; 1924) gives data referring to a 2° field, which is probably too small. This data checks Abney closely, however, giving, as expected, a ratio curve (V_c/V_r) which does not rise quite so steeply as that shown in Figure 1.

[25] It is convenient to note at this point that the results of Troland (loc. cit., Psych. Bull.) on the least retinal illumination required to evoke the blue arcs are not in complete agreement with those referring to the present author's right eye. Just how serious the discrepancies are can not be determined because the account of Troland's results is purely qualitative and is contained in a brief abstract. He says "Measurements of the threshold of the phenomenon with respect to intensity indicate that this is in the neighborhood of one photon, and that there is a distinct minimum in the middle of the spectrum, the curve for the effect corresponding roughly with a reciprocal of the visibility curve." The present results indicate that the threshold is in the neighborhood of one one-hundredth photon (for some wave lengths). There is a minimum (though not distinct) in the middle of the spectrum, and the curve does not (see fig. 1) resemble, even roughly, a reciprocal of the visibility curve (it scarcely rises at all toward short wave lengths). It is possible that these differences are to be expected from the relatively low brilliance (see footnote 23, p. 348) of the blue arcs in Troland's case. Such a brilliance might reasonably necessitate the intensity threshold of one photon which Troland found. This high threshold should account for the rise of the curve toward short wave lengths. (See footnote 26, p. 350.)

from perfect agreement: (1) The rise of the ratio between 550 mμ and 600 mμ is steeper than that of the average I_λ; and (2) the ratio is smaller between 400 mμ and 500 mμ than the average I_λ. Although the agreement is as good as, perhaps better than, can be expected in view of the fact that two different observers are involved, perhaps a part of the failure of the I_λ curve to rise as steeply as the ratio curve may be ascribed to the fact that the slit widths of the spectrometer were quite large (10 mμ). The failure of the I_λ curve to fall to the low values between 400 and 500 mμ reached by the ratio curve may be ascribed to the fact that the primary stimulus, though quite weak (about 0.04 photons), introduced enough scattered light into the field on which the arcs were projected to interfere seriously with their observation.[26] This scattering became more and more troublesome as the wave length of the primary stimulus was decreased. Similar difficulties have been noted by Hubbard,[27] Troland,[28] and Ellis.[29]

It may be concluded, in my opinion, from the work just described, that at low retinal illuminations the blue arcs are evoked from nerve activity initiated by the retinal rods. This conclusion is in agreement with Amberson's results on retinal regions giving rise to the blue arcs. Although other work (summarized in the introduction)[30] suggests that the cones may also initiate the nerve activity responsible for the blue arcs, it is my opinion that this has not yet been established as a fact.[31]

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[26] We should expect Troland's curve resulting from a much higher illumination (about 1 photon) to depart considerably more from the ratio curve on this account. Such a departure is in agreement with the actual curve he found. (See footnote 25, p. 349.)

[27] Hubbard, loc. cit., p. 198.

[28] Troland, loc. cit., J. Opt. Soc. Am., p. 176.

[29] Ellis, loc. cit., p. 292; 1927.

[30] In agreement with this work, I find that a red (630 to 640 mμ) primary stimulus is best in my case also for demonstration of the blue arcs, but it is only slightly better than a green or yellow stimulus. This I ascribe, not to participation of the cones, but to the resulting comparative freedom from scattered light of the field on which the arcs must be projected. This explanation is also given by Ellis (loc. cit., p. 293; 1927).

[31] My view is that probably all nerve fiber, forming rod pathways or cone pathways, retinal or central, active or inactive, is emitting physical light. It seems likely that the rate of emission increases as the degree of activity of the fiber increases (though it is possible that this view conflicts with Troland's undisproved contention of essentially constant brilliance of the blue arcs). It seems significant in this connection to note that the only two cases (blue arcs and Purkinje phase of the periodic afterimage), which demand the postulation of "visible radiation," arise in association with, and only in association with, intense activity of the rod pathways. My view is that the radiation emitted under other conditions (for example, from cone pathways activated by primary stimuli usual for demonstration of the blue arcs—note Amberson's failure to obtain the arcs from purely foveal stimulation—or from rod pathways after a few seconds duration of the stimulus—note transitory character of the blue arcs which has troubled the proponents of the secondary electric stimulation theory since its inception) is too weak to be detected even by the wonderfully sensitive retinal layer only a fraction of a millimeter away. We should not expect the retinal cones to initiate the nerve impulse which causes visible blue arcs because the light scattered from the primary image strong enough to stimulate the cones suitably would be many times as intense as the light emitted by the active fibers of the cone pathways. Furthermore, we should not expect the rods to retain a sufficiently high sensitivity to initiate for more than a few seconds the nerve activity which results in vivid blue arcs. The sensitive substance bleaches out rapidly; and during this rapid bleaching out, and only during this rapid bleaching out, do vivid blue arcs (or vivid Purkinje phases) appear.

It is also true, in my opinion, that the hypothesis of visible radiation from active nerve fiber proposed by Druault and Ladd-Franklin and used by me [32] to account for certain properties of the Purkinje phase of the periodic afterimage is somewhat strengthened by this work, because this work indicates that the nerve activity responsible for the blue arcs sometines (perhaps always) arises from the same origin (retinal rods) as that responsible for the Purkinje phase of the periodic afterimage following brief stimulation of the retina [33] by light. The two phenomena are linked, therefore, by one more common property. Since cogent arguments (Ladd-Franklin) indicate that the blue arcs are due to visible radiation from active nerve fiber, it is, in my opinion, reasonable to suppose that the Purkinje phase is due, at least in part, to the same cause.

Washington, November, 1928.

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[32] Judd, loc. cit., p. 527. The term used here as a substitute for "visible radiation" is "bioluminescence."

[33] In further confirmation of this linkage of the Purkinje phase with the blue arcs, may be mentioned the fact that W. T. M. Forbes obtains them both only with abnormally high retinal illuminations. (See for the Purkinje phase: Judd, loc. cit., p. 521, fig. 2, observer W. T. M. F. requires 56 photons, which is about one hundred times that required by some other observers.) He requires about 10 photons (compare Judd, present work, 0.01 photon) for evoking the blue arcs. This evidence was pointed out by Doctor Forbes himself, who says (letter of November 14, 1928), "You have, as confirmatory, the fact that in my case the blue arcs did not appear with weak stimuli, * * *" and (as just cited) neither did the Purkinje phase.

REDUCTION OF DATA ON MIXTURE OF COLOR STIMULI

J. Res. Nat. Bur. Stand. (U.S.), 4, 515-548 (1930) RP163

This paper, although it used the now-obsolete OSA color-mixture data, established principles and methods that were utilized in the CIE 1931 standard observer and coordinate system. The great freedom of choice of primaries for color matching, and the corresponding freedom of choice of luminosity coefficients (by which the color-mixture curves need be multiplied to sum to the luminosity curve) was so little understood prior to this paper as to be mysterious or even hardly credible. It is still so regarded by many who have not studied and understood this paper or the few secondary sources that have adequately paraphrased it. The term "visibility", used interchangeably with "luminosity" in this paper to refer to the basic spectral data of photometry, has been abandoned in this connection; "luminosity" is the standard term. The adjective "excitation" that appears in quotation marks in this paper was almost immediately abandoned in favor of "mixture", "distribution", or sometimes "matching" to refer to such curves as are the subject of this paper. The term "transmission" used on p. 534 (and elsewhere) has been replaced by "transmittance", in accordance with the principle of nomenclature, which Judd approved, that the "-ion" suffix designates a process and "-ance" designates a quantity. The first set of color-mixture curves for which the luminosity coefficients were 0,1,0 were defined and appeared in this paper. The suggestion that color-mixture curves having those luminosity coefficients be adopted first appeared in this paper. This suggestion led directly to the CIE 1931 color-mixture curves $\bar{x}$, $\bar{y}$, $\bar{z}$. The "alchyne" (lightless line) on which two primaries must be located if the luminosity coefficients are 0,1,0 is defined and discussed in the appendix of this paper.

REDUCTION OF DATA ON MIXTURE OF COLOR STIMULI

By Deane B. Judd

ABSTRACT

Proof is given that any set of distribution curves may be assigned luminosity coefficients giving the same luminosity sum as any other set of distribution curves embodying the same mixture data. Two methods of computing such luminosity coefficients are described and these methods are applied to four specific transformations of the O. S. A. "excitation" curves. By actual computation (for four filters) of dominant wave length, colorimetric purity, and transmission for sunlight it is demonstrated in a practical way that each of these transformations embodies the same mixture data and luminosity data as the O. S. A. "excitation" curves. One of these transformations is proposed for routine computation because it has properties which permit the adoption of simpler methods of computing than the curves now used permit.

CONTENTS

I. INTRODUCTION

Probably the most fundamental set of data relating to color vision is that embodied in the three "excitation"[1] curves or "visual response functions" by means of which the laws of mixture of color stimuli are applied. This set of data is obtained experimentally by introducing into one half of a photometric field light of known spectral composition, and into the other half of the field light of a different known spectral composition, so chosen that the two fields appear matched to the observer. By making a large number of such matches, information is amassed by which it is possible to tell whether any two fields of different spectral composition will appear matched to that observer or not.

It was indicated by Newton [2] and subsequently verified by others [3] that this information, or mixture data, could be embodied in a finite

[1] We say, in this instance, "excitation," because that term is a usual one. Hereafter it will be frequently convenient to substitute the term "distribution" on account of the generality of the mathematical treatment. (See footnote 36, p. 524.)

[2] Isaac Newton, Opticks, London, Innys, pp. 134–137; 1730. We find here a statement of "Newton's law of color mixture." Although this law makes no explicit mention of additive distribution curves, and does not directly state that contributions from different sources to a given primary color process are to be combined by addition, still these concepts are contained implicitly in the law. Our indebtedness to Newton in this respect was acknowledged by J. C. Maxwell (Sci. Papers, Cambridge, p. 149; 1890), who was among the first to state explicitly and use these concepts. Probably Grassman (Pogg. Ann., **89**, pp. 69–84; 1853) first stated them explicitly.

[3] See, for example: A. König, Ueber Newton's Gesetz der Farbenmischung und darauf bezügliche Versuche des Hrn. Eugen Brodhun, Sitz. Akad. Wiss., Berlin, **31**, pp. 311–317; 1887. Newton's law for normal observers has been amply verified by experiment for field brightnesses which are too high to involve the Purkinje effect. We restrict attention, as is usual, to the medium field brightnesses dealt with by the technique of colorimetry for which Newton's law is valid.

set of additive distribution curves.[4] From these curves directly can be obtained the prediction whether two given stimuli of different spectral composition will appear matched or not.[5] If ρ_0, γ_0, and β_0 give the distribution of three color processes [6] (say a red, a green, and a blue process) for an equal energy spectrum, and E the spectral energy distribution of the light source actually used,[7] the condition for a complete color match (hue, saturation, and brilliance) between two stimuli is:

$$\left.\begin{aligned}\int_0^\infty \rho_0 \, E\,(T_1 - T_2)\, d\lambda = 0\\ \int_0^\infty \gamma_0 \, E\,(T_1 - T_2)\, d\lambda = 0\\ \int_0^\infty \beta_0 \, E\,(T_1 - T_2)\, d\lambda = 0\end{aligned}\right\} \qquad (1)$$

where T_1 and T_2 are functions of the wave length, λ; T_1 gives the ratio of the spectral energy at a given wave length of the first stimulus to the spectral energy of the source, E, at that same wave length; T_2 gives the same ratio for the second stimulus.[8] If condition (1) be not satisfied, then the stimuli specified by ET_1 and ET_2 are not a match.

It has long been recognized that if ρ_1, γ_1, and β_1 give the distribution of three primary color processes in the spectrum of energy distribution, E, we can define an infinite number of sets of distribution curves (say, for example, ρ_2, γ_2, and β_2) representing the distribu-

[4] Newton used seven primary colors, which necessitates seven distribution curves. We now know (and Newton seems to have suspected, though he was not sure) that three independent distribution curves giving the distribution of three primary color processes throughout the spectrum are sufficient and necessary to embody the mixture data of a normal observer. More than three distribution curves may be used if desired, but they will not all be independent. The fact that three distribution curves are sufficient was probably first given its proper emphasis by Thomas Young (Lectures on Natural Philosophy, London, Savage, **1**, p. 440; 1807), who made it the basis of his color theory now widely known because of the adoption and elaboration of it by Helmholtz. Experimental evaluations of these distribution curves have been made by König and Dieterici (Sitz. Akad. Wiss., Berlin, **29**, pp. 805–829; 1886), Maxwell (Sci. Papers, Cambridge, pp. 426–444; 1890), Abney (Phil. Trans. Roy. Soc.; 1899), and others.

[5] See Nikolaus Nyberg, Zum Aufbau des Farbenkörpers im Raume aller Lichtempfindungen, Zs. f. Phys., **52**, p. 407; 1928. He gives our relation (1) as (5).

[6] "Color process" is here taken as a name for the activity, retinal and postretinal, which is in one-to-one correspondence with the incidence of radiant energy of a certain range of frequency upon the retina. If the radiant energy is specified by its distribution (E) with respect to frequency (hereafter the wave length in vacuo will be used to specify frequency), then, with the visual mechanism in the "neutral state" (see footnote 61, p. 542), there corresponds to this radiant energy a certain real color process. If E for some wave lengths be negative (which is physically impossible) still it is possible to compute a specification for the "color process" with which this imaginary stimulus is in one-to-one correspondence. Since it is convenient and useful to do this, the term "color process" is taken to refer to such activity, whether that activity actually be possible within the normal visual mechanism (as it very well may be, see footnote 61, p. 542) or not. When any three-color processes, existent or nonexistent, corresponding to any three physical stimuli, real or imaginary, are dignified by adoption, temporarily or permanently, as objects of a set of distribution curves by which the responses of an observer may be predicted, we shall denote their dignity by referring to them as "primary color processes." It is evident that this state of being "primary" either may or may not coincide with the state of being psychologically primary (unitary) or physiologically primary (invariable—independent of intensity, duration, or retinal region stimulated).

[7] $\rho_0 E$, $\gamma_0 E$, and $\beta_0 E$ are the distribution curves of the three primary color processes in the spectrum of the source whose spectral energy distribution is E; they are hereafter designated ρ_1, γ_1, and β_1, respectively. No restrictions as to relative size are placed on the units of ρ_0, γ_0, and β_0; but, for convenience only, ρ_1, γ_1, and β_1, are customarily expressed in comparable units. (See footnote 29, p. 522.)

[8] Note that if the two light stimuli are produced by filters (1 and 2) illuminated by the source whose spectral energy is E, then T_1 and T_2 are merely the spectral transmissions of the filters. The significance of relation (1) may be restated in experimental terms as follows:

Consider a symmetrically divided photometric field, both halves of which are illuminated by a source for which energy as a function of the wave length, λ, is given by E and let the two halves be of equal brightness. Let a filter whose transmission as a function of λ is T_1 be inserted in the path of light which illuminates one half of the field; and let a filter whose transmission as a function of λ is T_2 be inserted in the path of light which illuminates the other half of the field. If the conditions expressed by relation (1) be satisfied, the two halves of the field will then be perfectly matched in quality and brightness (hue, saturation, and brilliance); that is, they will be quite indistinguishable one from the other for the observer to whom ρ_0, γ_0, and β_0 refer.

The reason for introducing the ratios T_1 and T_2 applying to the same source of radiant energy, E, and resulting in the energy distributions ET_1 and ET_2 is that the routine use of the "excitation" curves in computation usually involves such ratios (either spectral transmission or spectral reflection). The products ET_1 and ET_2 would otherwise have more naturally been designated simply as E_1 and E_2 without indication as to the manner of their physical realization.

tions of different sets of primary color processes [9] which would equally well represent the same body of mixture data. We need only to choose at random nine finite constants (K_1 to K_9 subject to the condition:

$$\left.\begin{array}{l} \begin{vmatrix} K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 \\ K_7 & K_8 & K_9 \end{vmatrix} \neq 0^{10} \\ \\ \text{then the expressions:} \\ \rho_2 \equiv K_1\rho_1 + K_2\gamma_1 + K_3\beta_1 \\ \gamma_2 \equiv K_4\rho_1 + K_5\gamma_1 + K_6\beta_1 \\ \beta_2 \equiv K_7\rho_1 + K_8\gamma_1 + K_9\beta_1 \end{array}\right\} \quad (2)$$

define the new set of distribution curves for each wave length.

We say that ρ_2, γ_2, and β_2 represent the same body of mixture data as do ρ_1, γ_1, and β_1 because, whenever T_1 and T_2 are so chosen that relation (1) holds, it is also true that that relation holds when ρ_2, γ_2, and β_2 are substituted for ρ_1, γ_1, and β_1, respectively.[11]

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[9] The new set of primary color processes might be any three different color processes providing the stimulus (real or imaginary) which evokes one of them can not be matched by a mixture of the stimuli which evoke the other two. The new processes might be, for example, any violet, any yellow, and any crimson process. This freedom of choice of primary color process may be stated in another way. If all color processes are represented on a mixture diagram (such as figs. 2 to 5), each process by a point, then any two points may be chosen at random for the first two primaries. The only restriction on the choice of the third primary is that its point must not lie on the straight line connecting the points for the other primaries which were chosen at random.

[10] That this determinant be different from zero is the condition which ensures that ρ_2, γ_2, and β_2 will be three independent functions provided ρ_1, γ_1, and β_1 are. In other words, it ensures that the points representing on the mixture diagram the three new primary color processes do not lie on the same straight line if the points representing the old primaries do not. In actual cases this condition is seldom tested because the collapse of the tristimulus system into a monostimulus or distimulus system is usually prevented by the nature of the problem.

[11] Although the accuracy of this statement has been regarded as so evident that writers following Newton (see footnote 12) who have made use of relation (2) have apparently never bothered to publish a proof of it, still it is, perhaps, well to indicate the proof here for the sake of completeness. We wish to show that $\int_0^\infty \rho_2\,(T_1-T_2)d\lambda=0$. From (2) we may write:

$$\int_0^\infty \rho_2\,(T_1-T_2)d\lambda = \int_0^\infty (K_1\rho_1+K_2\gamma_1+K_3\beta_1)\,(T_1-T_2)d\lambda =$$
$$K_1\int_0^\infty \rho_1\,(T_1-T_2)\,d\lambda + K_2\int_0^\infty \gamma_1\,(T_1-T_2)d\lambda + K_3\int_0^\infty \beta_1\,(T_1-T_2)d\lambda.$$

But from (1) and footnote 7, page 516, each of the three integrals is zero; hence their sum $\int_0^\infty \rho_2\,(T_1-T_2)d\lambda$ equals zero, which is what we wanted to prove. By analogous argument, $\int_0^\infty \gamma_2(T_1-T_2)d\lambda=0$, and $\int_0^\infty \beta_2\,(T_1-T_2)d\lambda=0$; hence ρ_2, γ_2, and β_2 satisfy (1) whenever ρ_1, γ_1, and β_1 do.

This form of proof can also be extended to show that whenever relation (1) is not satisfied by ρ_1, γ_1, and β_1 then neither do ρ_2, γ_2, and β_2 satisfy it. The argument is by reductio ad absurdum: Assume that ρ_2, γ_2, and β_2 do satisfy (1) when ρ_1, γ_1, and β_1 do not. Now it is possible to find constants (k_1 to k_9) such that:

$$\rho_1 = k_1\rho_2 + k_2\gamma_2 + k_3\beta_2$$
$$\gamma_1 = k_4\rho_2 + k_5\gamma_2 + k_6\beta_2$$
$$\beta_1 = k_7\rho_2 + k_8\gamma_2 + k_9\beta_2$$

$$\begin{vmatrix} k_1 & k_2 & k_3 \\ k_4 & k_5 & k_6 \\ k_7 & k_8 & k_9 \end{vmatrix} \neq 0$$

(This is relation (5) of Sec. II; the constants (k_1 to k_9) of this relation may be computed from the constants (K_1 to K_9) of relation (2); see relation (7) in Sec. III.) Hence, as just shown, ρ_1, γ_1, and β_1 satisfy (1) which is contrary to hypothesis.

Hence, if ρ_2, γ_2, and β_2 are used to predict whether the stimulus specified by ET_1 matches that specified by ET_2, the same prediction is obtained as by the use of ρ_1, γ_1, and β_1.[12]

The choice of a particular set of distribution curves out of the triple infinity of sets which can all equally well embody a given set of mixture data is dictated purely by convenience. If, as in this article, the aim is prediction of whether the observer will respond "match" or "not a match" when confronted by two color stimuli of different spectral composition, a set of curves should be chosen that lends itself readily to the computation incident to this prediction.[13] If, on the other hand, the aim is to simplify the basis for speculation of a psychophysiological nature, many other quite different sets of curves would be eligible, the choice depending on the nature of the speculation.[14]

When the technique of heterochromatic photometry was worked out [15] it became convenient to use a method, long known, of specifying a color stimulus. This method is to evaluate for that stimulus three variables—dominant wave length, colorimetric purity, and luminosity.[16] In order to convert specifications of color stimuli from this system into those suggested by relation (1), namely, the

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[12] This has been recognized by nearly all writers following Newton, though they do not always give relation (2) explicitly. It is convenient at this point to introduce a partial list of the publications which have dealt, directly or indirectly, witb transformations of tbis sort:

A. König and C. Dieterici, Die Grundempfindungen und ihre Intensitätsvertheilung im Spectrum, Sitz. Akad. Wiss., Berlin, **29**, pp. 805–829; 1886. (Or see A. König, Gesammelte Abbandlungen zur Physiologischen Optik, Leipzig Barth, pp. 60–87; 1903.) Also Die Grundempfindungen in normalen und anomalen Farbensystemen und ihre Intensitätsvertheilung im Spectrum, Zs. f. Psych. u. Physiol. der Sinnesorgane, **4**, pp. 241–347; 1892. (Or see A. König, Ges. Abh., Leipzig, Barth, pp. 214–321; 1903.)

H. v. Helmholtz. Zs. f. Psych. u. Physiol. der Sinnesorgane, **2**, pp. 1–30; **3**, pp. 1–20. (Or see H. v. Helmboltz, Handbuch der Physiologischen Optik, 2d ed., Leipzig, Voss, pp. 449–458; 1896.)

H. E. Ives, Tbe Transformation of Color-Mixture Equations from One System to Another, J. Frank. Inst., **180**, pp. 673–701; 1915. Also, The Transformation of Color-Mixture Equations from One System to Another. 1l. Graphical Aids, J. Frank. lnst., **195**, pp. 23–44; 1923.

J. Guild, The Transformation of Tricbromatic Mixture Data: Algebraic Metbod, Trans. Opt. Soc., **26**, pp. 95–108; 1924–25. Also, The Geometrical Solution of Colour Mixture Problems, Trans. Opt. Soc., **26**, pp. 139–174; 1924–25.

W. Dziobek, Ueber die Transformation von einem trichromatiscben System auf ein anderes, Zs. für Instrumentenkunde, **46**, pp. 81–84; 1926.

E. Schrödinger, Grundlinien einer Theorie der Farbenmetrik im Tagessehen, Ann. d. Physik (4), **63**, pp. 397–456; 1920.

E. Schrödinger, Ueber das Verhältnis der Vierfarben- zur Dreifarbentheorie, Sitz. Akad. Wiss., Wien, Abt. IIa, **134**, pp. 471–490; 1925.

R. Luther, Aus dem Gebiet der Farbreizmetrik, Zs. f. Techn. Physik, **8**, pp. 540–558; 1927.

1. Runge, Ueber die Ermittlung der Farbkoordinaten aus den Messungen am trichromatischen Kolorimeter, Zs. f. Instrumentenkunde, **48**, pp. 387–396; 1928.

Relation (2) of this paper is given as relation (9) by Helmholtz (Handbuch der Physiologischen Optik, 2d ed., Leipzig, Voss, p. 453; 1896) and as relation (1) by lves (J. Franklin lnst., **180**, p. 678; 1915). A restricted form of the relation is given by König and Dieterici (König, Ges. Abb., pp. 82, 305), and also by Guild as relation (4) (Trans. Opt. Soc., **26**, p. 97; 1924–25), and again as relation (10) (ibid., p. 153).

Compare also relations (12) and (13) given by Schrödinger (Ann. d. Physik, (4), **63**, p. 439; 1920).

[13] Tbe "Elementarempfindungskurven" of König and Dieterici (A. König, Ges. Abh., Leipzig, Barth, pp. 286–288; 1903), the elementary "sensation" curves of Abney (W. de W. Abney, Researches in Colour Vision, London, Longmans-Green, pp. 229–247; 1913), and the elementary "excitations" computed by Weaver using data by König, Dieterici, and Abney (L. T. Troland, Report of tbe Committee on Colorimetry for 1920–21, J. Opt. Soc. Am. and Rev. Sci. lnst., **6**, p. 549; 1922) are examples of distribution curves which are convenient for computation chiefly by reason of tbe fact tbat there are no negative ordinates. The sort of computation for which these curves are used is treated by Troland (J. Opt. Soc. Am. and Rev. Sci. Inst., **6**, pp. 573–592; 1922) and by Priest (J. Opt. Soc. Am. and Rev. Sci. lnst., **9**, pp. 503–520; 1924).

[14] Thus, we might try to choose distribution curves which suggest a simple theoretical account of the facts of dichromasy just as Maxwell (On the Theory of Compound Colours, and the Relations of the Colours of tbe Spectrum, Sci. Papers, Cambridge, pp. 441–444; 1890), and König and Dieterici (A. König, Ges. Abh., Leipzig-Bartb, pp. 303–321; 1903) did. Or we might test the applicability of an extension of Fechner's law for the prediction of sensibility to color differences as Helmholtz (Handb., d. Physiol. Optik, 2d ed., Leipzig-Voss, pp. 449–458; 1896) did. Or we might wish to show that the mixture data could suggest a four-color tbeory of vision as well as a three-color theory as Scbrödinger (Sitz. Akad. Wiss., Wien, Abt. IIa, **134**, pp. 471–490; 1925) did.

[15] Due in large part to tbe research activity of Herbert E. Ives (Studies in the Photometry of Light of Different Colors, Phil. Mag., **24**, pp. 150–188, 352–370, 744–751, 845–863; 1912).

[16] 1. G. Priest, Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity and Brightness, J. Opt. Soc. Am. and Rev. Sci. Inst., 8, pp. 173–200; 1924. (Tbis paper gives referencos to previous publications on tbe same subject.) W. v. Bezold, Ueber das Gestez der Farbenmischung und die pbysiologischen Grundfarben, Pogg. Ann., **150**, pp. 71–78; 1873. We find here (p. 78) probably the first explicit definition of the concept now termed "colorimetric purity." Bezold named it "objective Reinheit."

evaluation for that stimulus of $\int_0^\infty \rho_0 ETd\lambda$, $\int_0^\infty \gamma_0 ETd\lambda$, and $\int_0^\infty \beta_0 ETd\lambda$ (and vice versa), it became necessary to evaluate a set of "luminosity coefficients" L_{r_1}, L_{g_1}, L_{b_1}, such that for each wave length:

$$\rho_1 L_{r_1} + \gamma_1 L_{g_1} + \beta_1 L_{b_1} = L_1 \qquad (3)$$

where L_1 gives the distribution of luminosity of the source for which ρ_1, γ_1, and β_1 give the distribution of the primary color processes.[17] If we are to compute (for example) the colorimetric purity of a stimulus from the tristimulus specification, we must be able to compute from that specification L_1 for the wave length equal to the dominant wave length of the stimulus. The first set of distribution curves to which luminosity coefficients were attached was compiled from König's data by Dr. H. E. Ives. Ives noted that the Grundempfindungskurven of König failed to sum even approximately to the average luminosity function when weighted by any set of positive luminosity coefficients whatever.[18] Consequently, he chose a new set of primary color processes yielding a new set of distribution curves to represent the mixture data of König's observers. He found that this new set of distribution curves summed within experimental error to the average luminosity when weighted by a set of positive luminosity coefficients.

In 1920 Exner [19] independently noted the difficulties which led Ives to make the changes just described. Exner, however, proposed to obtain by purely arbitrary changes a substantial agreement of the mixture data with the luminosity data through the use of positive coefficients. These arbitrary changes (not being a transformation of coordinates) fail to satisfy (2). The resulting set of distribution curves, therefore, does not embody the same set of mixture data as the average König curves, and if used for computation does not give the same numerical results that the König curves give. These curves have attracted the well-deserved criticism of Schrödinger.[20]

It has been shown by the author [21] that one set of distribution curves (the O. S. A. "excitation" curves)[22] will sum, when properly weighted, to a satisfactory approximation of the accepted luminosity function of the source. Throughout the present paper these distribution curves will be used as a basis for computation because they represent in the author's judgment the best reduction of mixture data available. They are somewhat more congruent with the luminosity function than the König-Ives curves, due partly to the

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[17] $\rho_1 \equiv \rho_0 E$, $\gamma_1 \equiv \gamma_0 E$, and $\beta_1 \equiv \beta_0 E$. See footnote 7, p. 516.

[18] J. Frank. Inst., **180**, pp. 693–694; 1915. Ives noted that a negative luminosity coefficient for the "blue" curve would be required; but apparently he regarded a negative coefficient unacceptable because he remarks in italics that "* * * no positive luminosity values for the blue sensation will satisfy the condition" and later (p. 694) implies that the König curves fail to yield consistent luminosity values on this account. Of course, to attach a negative coefficient to the blue curve involves a difficulty on the basis of psychophysiological theory, namely, that the mechanism which is responsible for "blue" must also partially inhibit the "brightness" response; this consideration was probably the reason for Ives's rejection of negative coefficients.

[19] F. Exner, Zur Kenntnis der Grundempfindungen im Helmholtz'schen Farbensystem, Sitz. Akad. Wiss., Wien, Abt. IIa, **129**, pp. 27–46; 1920.

[20] E. Schrödinger, Ueber das Verhältnis der Vierfarben- zur Dreifarbentheorie, Sitz. Akad. Wiss., Wien, Abt. IIa, **134**, p. 479; 1925. He remarks in a footnote, "Von der Korrektur der König'schen Grundempfindungskurven, die Exner weiterhin vornimmt, konnte ich aus dem Grunde keinen Gebrauch machen, weil sie Schönheitsfehler (Konkavitäten nach aussen) der Spectralkurve in der Farbentafel nach sich zieht."

[21] D. B. Judd, Chromatic Visibility Coefficients by the Method of Least Squares, J. Opt. Soc. Am. and Rev. Sci. Inst., **10**, pp. 635–651; 1925.

[22] See footnote 35, p. 524.

more refined method of computing luminosity coefficients but more particularly to the different choice of mixture data they embody and to the fact that more extensive luminsoity data have since become available.

The purposes of the present paper are:

To show that any one of the infinite number of sets of distribution curves representing a given body of mixture data will sum equally well to the luminosity function. (Sec. II.)

To outline two short methods for computing the luminosity coefficients of any of these sets of distribution curves. (Sec. III.)

To present four examples of this problem solved by the methods outlined in Section III. (Sec. IV.)

To present a demonstration that each of the four sets of distribution curves used embody the same mixture data (that of the O. S. A. "excitation" curves) by showing that each of the four sets gives the same results when used for the computation of the dominant wave length, colorimetric purity, and luminosity of color stimuli. (Sec. V.)

To show that one of these sets of distribution curves embodies greater computational convenience than any set heretofore proposed. (Sec. VI.)

II. CHOICES OF DISTRIBUTION CURVES AND THE INDEPENDENCE OF THE LUMINOSITY FUNCTION

Theorem.—Any set of distribution curves may be assigned luminosity coefficients giving the same luminosity sum as any other set of distribution curves embodying the same mixture data.

The theorem restated is: If ρ_1, γ_1, β_1 and ρ_2, γ_2, β_2 are sets of distribution curves which satisfy relation (2) for each wave length, and L_{r1}, L_{g1}, and L_{b1} are three numbers satisfying relation (3) for each wave length, then three numbers, L_{r2}, L_{g2}, and L_{b2}, exist such that, for each wave length:

$$\rho_2 L_{r2} + \gamma_2 L_{g2} + \beta_2 L_{b2} = \rho_1 L_{r1} + \gamma_1 L_{g1} + \beta_1 L_{b1} = L_1 \tag{4}$$

Proof.—Since these two sets are connected by relation (2) it is possible to find constants (k_1 to k_9) such that:

$$\left.\begin{aligned} \rho_1 &= k_1\rho_2 + k_2\gamma_2 + k_3\beta_2 \\ \gamma_1 &= k_4\rho_2 + k_5\gamma_2 + k_6\beta_2 \\ \beta_1 &= k_7\rho_2 + k_8\gamma_2 + k_9\beta_2 \\ &\begin{vmatrix} k_1 & k_2 & k_3 \\ k_4 & k_5 & k_6 \\ k_7 & k_8 & k_9 \end{vmatrix} \neq 0 \end{aligned}\right\} \tag{5}$$

by solving for ρ_1, γ_1, and β_1 from (2).[23] From (3) and (5) we may write:

$$L_{r_1}(k_1\rho_2 + k_2\gamma_2 + k_3\beta_2) + L_{g_1}(k_4\rho_2 + k_5\gamma_2 + k_6\beta_2) + L_{b_1}(k_7\rho_2 + k_8\gamma_2 + k_9\beta_2) = L_1$$

[23] The result of this solution is given as relation (7) in Sec. III. Compare relation (15) given by Schrödinger (Ann. d. Physik, (4), **63**, p. 440; 1920). This solution is also given by Guild (Trans. Opt. Soc., **26**, p. 100; 1924–25) as relation (10), though he does not evaluate the determinant we abbreviate as Δ. (See relation (7).)

or, rearranging terms:

$$\rho_2(L_{r_1}k_1+L_{g_1}k_4+L_{b_1}k_7)+\gamma_2(L_{r_1}k_2+L_{g_1}k_5+L_{b_1}k_8)+\beta_2(L_{r_1}k_3+L_{g_1}k_6+L_{b_1}k_9)=L_1$$

whence it is evident that numbers, L_{r_2}, L_{g_2}, and L_{b_2}, which satisfy (4) do exist because they are:

$$\left.\begin{aligned} L_{r_2}&=L_{r_1}k_1+L_{g_1}k_4+L_{b_1}k_7\\ L_{g_2}&=L_{r_1}k_2+L_{g_1}k_5+L_{b_1}k_8\\ L_{b_2}&=L_{r_1}k_3+L_{g_1}k_6+L_{b_1}k_9\end{aligned}\right\} \qquad (6)^{24}$$

It may easily be shown that the sum of the luminosity coefficients applying to the second set of distribution curves is equal to the sum of the luminosity coefficients applying to the first set of distribution curves provided all distribution curves of both sets have equal areas. It may be seen from relation (6) that if we wish to choose any set of luminosity coefficients whatever which satisfies the restriction just stated, it will be possible to derive a set of distribution curves to which that set of luminosity coefficients apply.[25] It is further evident from relation (6), since we have nine constants out of which to build but three luminosity coefficients whose nature imposes but three added restrictions,[26] that not only will it be possible to find a set of distribution curves embodying the mixture data of the first set of distribution curves to which the arbitrarily chosen set of luminosity coefficients apply, but that it will be possible to find many such sets. We shall have occasion to make use of this possibility in Sections V and VI. (See footnote 52, p. 533, and footnote 74, p. 544.)

It is of interest to indicate the steps in the actual discovery of the truth of our theorem because the discovery involves a special case of relation (2) which is in itself worthy of attention. Consider a set of distribution curves ρ_3, γ_3, and β_3, computed from ρ_1, γ_1, and β_1, by the relations:

$$\left.\begin{aligned} \rho_3&=\rho_1\\ \gamma_3&=\gamma_1\\ \beta_3&=\frac{D\rho_1}{3-D}+\frac{D\gamma_1}{3-D}+\frac{(3-3D)\beta_1}{3-D}\end{aligned}\right\} \qquad (2a)$$

where D is any finite constant except $+1$ and $+3$ [27] independent of wave length; D may be thought of as fractional deficiency of primary color process;[28] here D refers to fractional deficiency of "blue" process since it is attached to β_1, but it may, of course, be attached equally well to ρ_1 or γ_1. It may be noted that relation (2a) is a special case of relation (2); hence ρ_3, γ_3, and β_3 represent the same body of mixture data as ρ_1, γ_1, and β_1.

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[24] I am indebted to Dr. L. B. Tuckerman for this elegantly simple proof.

[25] We have merely to choose the constants (k_1 to k_9) which, according to (6) give the desired values of L_{r2}, L_{g2}, and L_{b2}. These constants (k_1 to k_9) lead, according to (5), from the new distribution curves to the old; hence, they may helpfully be called the "reverse" coefficients. But we can find the direct coefficients (K_1 to K_9) of relation (2) from the reverse coefficients in the same way that the reverse coefficients are found from the direct coefficients. (See relation (7).)

[26] The case of interest is that in which the luminosity coefficients refer to distribution curves of equal area. This condition requires that: $k_1+k_2+k_3=k_4+k_5+k_6=k_7+k_8+k_9=1$.

[27] $D=+3$ makes β_3 infinite. It is ruled out because relation (2) applies to a choice of finite constants only. $D=+1$ is ruled out because the determinant of relation (2) then becomes zero.

[28] Thus, for $D=+1$, we find no trace of the β_1 curve in the β_3 curve because β_3 is simply the sum of half of each of the other two. This corresponds, of course, to the collapse of the tristimulus into a distimulus system in which the orginal "blue" process is missing; hence, we may say that for $D=+1$, the system is wholly "blue" deficient.

It was known that the theorem held for this special case of relation (2) because, by trial and error, the luminosity coefficients, L_{r3}, L_{g3}, and L_{b3}, had been found to be:

$$\left.\begin{aligned} L_{r3}&=L_{r1}-DL_{b1}/3(1-D),\\ L_{g3}&=L_{g1}-DL_{b1}/3(1-D),\\ L_{b3}&=L_{b1}/(1-D)-DL_{b1}/3(1-D)\end{aligned}\right\} \qquad (6a)$$

Examination of (2a) and (6a) shows that ρ_3, γ_3, and β_3 have the following properties: (1) They differ from ρ_1, γ_1, and β_1 in respect to one variable only (here β_1), (2) they yield curves of equal area provided ρ_1, γ_1, and β_1 do, and (3) they have luminosity coefficients which sum to unity provided ρ_1, γ_1, and β_1 have such luminosity coefficients.

After we have made one such transformation with a given value (positive or negative) of D applied to a particular distribution curve (say β_1), we may follow that transformation with another which involves a different value of D applied to the ρ_1 or the γ_1 curve; and so on. But since for each of these transformations it is possible to find luminosity coefficients which satisfy (3) merely by computation according to (6a), and since apparently any set of distribution curves embodying the same mixture data as ρ_1, γ_1, and β_1 (and having equal areas)[29] may be obtained by successive transformations of this kind, it was supposed that the theorem held for all cases.[30]

III. TWO METHODS OF CALCULATING THE LUMINOSITY COEFFICIENTS OF A SET OF DISTRIBUTION CURVES FROM THE LUMINOSITY COEFFICIENTS OF ANOTHER SET OF DISTRIBUTION CURVES EMBODYING THE SAME MIXTURE DATA

If a set of distribution curves is given and the luminosity curve of the source is at hand which the sum of the distribution curves, properly weighted by luminosity coefficients, ought to approximate, the luminosity coefficients which give the best approximation may be computed by the method of least squares.[31] It was apparently assumed by Ives [32] that a new choice of primary color processes giving a new set of distribution curves embodying the same mixture data might improve the goodness of fit of those data to the luminosity data, and such an implication was accepted by the author until the present work showed it to be untenable. The incorrectness of this view can now be conveniently demonstrated by an important corollary of the theorem proven in Section II, namely, that a new choice of distribution curves results in exactly the same agreement with the luminosity data, no better and no worse.[33] For all sets of stimulus primaries,

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[29] It is customary to adjust the ordinates of the three distribution curves so that the areas under the curves are equal. Such an adjustment is convenient because it causes the point representing the color of the light stimulus from the source (whose energy distribution is E) to fall at $r=1/3$, $g=1/3$, $b=1/3$. (See figs. 2 to 5, also relation (9) et seq.)

[30] This argument, as given, does not amount to a proof, as does the one previously given, because it does not show that every set of distribution curves satisfying (2) can be approximated by the successive application of (6a). It has merely shown that many such sets can be obtained.

[31] See footnote 21, p. 519.

[32] See footnote 18, p. 519.

[33] The argument is by reductio ad absurdum: The theorem states that there exist luminosity coefficients which give a new distribution curve sum identical with the old (relation 4). Thus, if a new choice of distribution curves (obtained by a new choice of stimulus primaries) gives a better fit than the best obtainable by the old choice, we can transfer back to the original primaries and find coefficients which give the same sum, a sum which must agree better than the best obtainable, which is impossible.

then, the least square solution results in exactly the same degree of agreement; this method of solving for luminosity coefficients is, of course, always applicable. There are shorter ways, however, of obtaining the best luminosity coefficients (L_{r2}, L_{g2}, L_{b2}) for a set of distribution curves (ρ_2, γ_2, β_2) provided a least square solution for L_{r1}, L_{g1}, and L_{b1} has already been carried out on another set (ρ_1, γ_1, β_1) representing the same mixture data. Two methods will be given; the first method, which requires only that the theorem of Section II be true, involves the following steps:

1. Compute $\rho_1 L_{r1}+\gamma_1 L_{g1}+\beta_1 L_{b1}$ for any three convenient values of the wave length (such as 450, 550 and 650 mμ).[34]

2. Set these numbers equal to $\rho_2 L_{r2}+\gamma_2 L_{g2}+\beta_2 L_{b2}$ for the corresponding three wave lengths and solve for L_{r2}, L_{g2}, and L_{b2}.

By the theorem of Section II, the values thus found will be the luminosity coefficients which give the same luminosity sum ($\rho_2 L_{r2}+\gamma_2 L_{g2}+\beta_2 L_{b2}$) as the original distribution curves gave ($\rho_1 L_{r1}+\gamma_1 L_{g1}+\beta_1 L_{b1}$), not only for the three wave lengths chosen for computation, but for all wave lengths.

The second method is based on relation (6). If the constants k_1 to k_9 be expressed in terms of the constants K_1 to K_9 by the method indicated in Section II (see relation (5) et seq.), there results:

$$\left.\begin{array}{lll}
k_1=\frac{1}{\Delta}\begin{vmatrix}K_5 & K_8\\ K_6 & K_9\end{vmatrix} & k_4=\frac{1}{\Delta}\begin{vmatrix}K_7 & K_4\\ K_9 & K_6\end{vmatrix} & k_7=\frac{1}{\Delta}\begin{vmatrix}K_4 & K_7\\ K_5 & K_8\end{vmatrix}\\
k_2=\frac{1}{\Delta}\begin{vmatrix}K_8 & K_2\\ K_9 & K_3\end{vmatrix} & k_5=\frac{1}{\Delta}\begin{vmatrix}K_1 & K_7\\ K_3 & K_9\end{vmatrix} & k_8=\frac{1}{\Delta}\begin{vmatrix}K_7 & K_1\\ K_8 & K_2\end{vmatrix}\\
k_3=\frac{1}{\Delta}\begin{vmatrix}K_2 & K_5\\ K_3 & K_6\end{vmatrix} & k_6=\frac{1}{\Delta}\begin{vmatrix}K_4 & K_1\\ K_6 & K_3\end{vmatrix} & k_9=\frac{1}{\Delta}\begin{vmatrix}K_1 & K_4\\ K_2 & K_5\end{vmatrix}
\end{array}\right\} \tag{7}$$

where Δ is an abbreviation for: $\begin{vmatrix}K_1 & K_2 & K_3\\ K_4 & K_5 & K_6\\ K_7 & K_8 & K_9\end{vmatrix}$

From (6) we may write:

$$\left.\begin{array}{l}
L_{r2}=\frac{1}{\Delta}\left[L_{r1}\begin{vmatrix}K_5 & K_8\\ K_6 & K_9\end{vmatrix}+L_{g1}\begin{vmatrix}K_7 & K_4\\ K_9 & K_6\end{vmatrix}+L_{b1}\begin{vmatrix}K_4 & K_7\\ K_5 & K_8\end{vmatrix}\right]\\
L_{g2}=\frac{1}{\Delta}\left[L_{r1}\begin{vmatrix}K_8 & K_2\\ K_9 & K_3\end{vmatrix}+L_{g1}\begin{vmatrix}K_1 & K_7\\ K_3 & K_9\end{vmatrix}+L_{b1}\begin{vmatrix}K_7 & K_1\\ K_8 & K_2\end{vmatrix}\right]\\
L_{b2}=\frac{1}{\Delta}\left[L_{r1}\begin{vmatrix}K_2 & K_5\\ K_3 & K_6\end{vmatrix}+L_{g1}\begin{vmatrix}K_4 & K_1\\ K_6 & K_3\end{vmatrix}+L_{b1}\begin{vmatrix}K_1 & K_4\\ K_2 & K_5\end{vmatrix}\right]
\end{array}\right\} \tag{8}$$

and compute L_{r2}, L_{g2}, and L_{b2} directly.

[34] In actual cases, the errors of rejection become troublesome if the values of wave length taken differ only slightly.

98046°—30——5

IV. CALCULATION OF LUMINOSITY COEFFICIENTS FOR FOUR SETS OF DISTRIBUTION CURVES EMBODYING THE SAME MIXTURE DATA AS THE O. S. A. "EXCITATION" CURVES

The mixture data which are embodied in each of the four sets of distribution curves for which luminosity coefficients are to be calculated are those of the O. S. A. "excitation" curves in the form (ρ_0, γ_0, β_0) extrapolated by Priest and Gibson.[35] Table 1 gives (columns 2, 3, and 4) the distribution curves [36] ρ_1, γ_1, and β_1 referring to Abbot-Priest sunlight according to the mixture data embodied in the O. S. A. "excitation" curves. The spectral energy distribution (E) of Abbot-Priest sunlight [37] is given in column 1. The distribution curves of columns 2, 3, and 4 are computed from the O. S. A. "excitation" curves (ρ_0, γ_0, β_0) as follows:

$$\left.\begin{aligned}\rho_1&=0.17547\ \rho_0E\\ \gamma_1&=0.17802\ \gamma_0E\\ \beta_1&=0.17482\ \beta_0E\end{aligned}\right\}\tag{9}$$

[35] J. Opt. Soc. Am. and Rev. Sci. Inst., **10**, p. 230; 1925.

[36] The function of wave length, ρ_1, gives the distribution of a certain sort of "rodness" or "red process" throughout the spectrum of Abbot-Priest sunlight; γ_1 gives the distribution of a certain "green process," and so on. There is no satisfactory proof that these processes have a separate physical existence; hence, the distribution curves given probably do not even approximately characterize any retinal activity (photochemical activity or decomposition of sensitive substance) nor any neural activity (either in the optic nerve, the occipital lobe, or elsewhere); the processes are called primaries by virtue of their position in a mathematical structure (see footnote 6, p. 516) whose present chief justification is that it makes colorimetric computation possible. Although ρ_1, γ_1, and β_1 represent the distribution of certain processes throughout the spectrum of Abbot-Priest sunlight whose claim to the term "primary" is purely mathematical, still it is probable that the distribution of some aspect of the color processes which are really "primary" or "fundamental" may be calculated from ρ_1, γ_1, and β_1 by relation (2). Our present ignorance concerning color vision is such that we know neither the constants of relation (2) nor the aspect of the color processes really "primary" to which these unknown constants refer.

The practice in this paper of referring to ρ, γ, and β briefly as "distribution curves" rather than the more widely used "excitation curves" is due primarily to the fact that the mathematical treatment is general and need not be restricted to the distribution curves of real or imaginary color processes. It is further true that the term "excitation curves" used thus briefly is objectionable to some who wish to reserve that term to apply to processes which actually exist in a physically separated state. Ladd-Franklin has already used "distribution curves" as a substitute term. (See, for example, Appendix to English Edition of the Helmholtz Physiological Optics, **2**, p. 461; 1924.)

[37] I. G. Priest, A Precision Method for Producing Artificial Daylight, Phys. Rev. (2), **11**, p. 502; 1918. J. Opt. Soc. Am. and Rev. Sci. Inst., **12**, p. 479; 1926. Standards Yearbook 1927, p. 204 and Figure 4.

NOTE.—The values of E introduced by Priest into the work of the colorimetry section, Bureau of Standards, some years ago, now in current use, and reproduced in Table 1 of the present paper are not exactly the same as proposed in his earliest publication cited above. Except for change of scale (100 at wave length 590 mμ instead of 100 at wave length 560 mμ) the values given here are as shown in Figure 4 of the Standards Yearbook for 1927. They are given by:

$$E\propto E_{2848}\sin^2\alpha/2$$

where E_{2848} is energy per unit wave length as a function of wave length for the Planckian radiator at 2,848° K. for c_2 at 14,350 micron degrees, and α is the rotation of the plane of polarization of light, as a function of wave length, per millimeter in crystalline quartz.

TABLE 1.—*The distributions of the O. S. A. "Excitations" throughout the spectrum of Abbot-Priest sunlight*

λ in mμ	E (Abbot-Priest sunlight)	ρ_1	γ_1	β_1	$\rho_1 L_{r1}$ ($L_{r1}=0.45$)	$\gamma_1 L_{g1}$ ($L_{g1}=0.54$)	$\beta_1 L_{b1}$ ($L_{b1}=0.01$)	$\rho_1 L_{r1}+\gamma_1 L_{g1}+\beta_1 L_{b1}$
	1	2	3	4	5	6	7	8
380	50.60	----------	----------	442	----------	----------	4.4	4.4
90	56.05	----------	----------	1,274	----------	----------	12.7	12.7
400	61.42	----------	----------	2,717	----------	----------	27.2	27.2
10	66.65	----------	----------	5,045	----------	----------	50.4	50.4
20	71.54	----------	13	7,679	----------	7.0	76.8	83.8
30	76.18	----------	41	12,186	----------	22.1	121.9	144.0
40	80.56	----------	100	14,351	----------	54.0	143.5	197.5
450	84.46	----------	240	14,027	----------	129.6	140.3	269.9
60	88.03	15	595	12,958	6.8	321.3	129.6	457.7
70	91.08	64	1,313	11,098	28.8	709.0	111.0	848.8
80	93.93	231	2,040	7,767	104.0	1,101.6	77.7	1,283.3
90	96.29	693	2,897	3,703	311.8	1,564.4	37.0	1,913.2
500	98.22	1,430	4,546	2,112	643.5	2,454.8	21.1	3,119.4
10	99.77	2,644	6,944	1,517	1,189.8	3,749.8	15.2	4,954.8
20	100.90	4,125	9,160	1,076	1.856.2	4,946.4	10.8	6,813.4
30	101.71	5,479	10,357	765	2,465.6	5,592.8	7.7	8,066.1
40	102.08	6,681	10,958	517	3,006.4	5,917.3	5.2	8,928.9
550	102.20	7,604	11,134	322	3,421.8	6,012.4	3.2	9,437.4
60	101.81	8,325	10,475	196	3,746.2	5,656.5	2.0	9,404.7
70	101.44	8,989	9,336	124	4,045.0	5,041.4	1.2	9,087.6
80	100.93	9,210	7,456	71	4,144.5	4,026.2	.7	8,171.4
90	100.00	9,388	5,269	35	4,224.6	2,845.3	.4	7,070.3
600	98.79	8,841	3,447	17	3,978.4	1,861.4	.2	5,840.0
10	97.45	7,900	1,960	----------	3,555.0	1,058.4	----------	4,613.4
20	95.61	6,291	1,004	----------	2,831.0	542.2	----------	3,373.2
30	93.78	4,690	484	----------	2,110.5	261.4	----------	2,371.9
40	92.38	3,161	164	----------	1,422.4	88.6	----------	1,511.0
650	90.35	1,871	48	----------	842.0	25.9	----------	867.9
60	88.62	1,057	16	----------	475.6	8.6	----------	484.2
70	86.29	606	----------	----------	272.7	----------	----------	272.7
80	84.33	326	----------	----------	146.7	----------	----------	146.7
90	81.99	187	----------	----------	84.2	----------	----------	84.2
700	79.57	105	----------	----------	47.2	----------	----------	47.2
10	77.31	54	----------	----------	24.3	----------	----------	24.3
20	75.21	33	----------	----------	14.8	----------	----------	14.8

The constants in relation (9) are chosen so that (except for errors of rejection of about 5 parts in 100,000 which can not conveniently be avoided): $\Sigma\rho_1\Delta\lambda=\Sigma\gamma_1\Delta\lambda=\Sigma\beta_1\Delta\lambda=1{,}000{,}000$, for $\Delta\lambda=10$ mμ.[38] Although ρ_1, γ_1, and β_1 are specified only at intervals of 10 mμ,[39] other values required may be found by interpolation and extrapolation.[40]

L_{r1}, L_{g1}, and L_{b1} are taken as 0.45, 0.54, and 0.01, respectively;[41] the products $\rho_1 L_{r1}$, $\gamma_1 L_{g1}$, and $\beta_1 L_{b1}$, and the sum of the products appear in columns 5, 6, 7, and 8. In order to show that the differences between this weighted distribution curve sum and standard luminosity data are negligible, these sums[42] divided by the spectral energy, E, of Abbot-Priest sunlight (column 1) are compared in Figure 1 to visibility data taken from the work of Gibson and Tyndall.[43] Visibility via the weighted distribution curve sums is shown by circles with tags; the experimental average visibility reported by Gibson and Tyndall from their data referring to 52 observers is

[38] See footnote 29, p. 522.

[39] This method of specification has thus far been found sufficient; perhaps intervals of 5 mμ (or less) will be justified by subsequent advances in colorimetric technique.

[40] Note that ρ_1, γ_1, and β_1 are to be taken as zero for the 10 mμ intervals for which no value is given in Table 1; a similar remark applies to Tables 2 to 5.

[41] Least square computation shows these to be a reasonable choice for representing luminosity data. (See footnote 21, p. 519.)

[42] Multiplied by the constant, 0.010764, which makes the maximum ordinate approximately unity and, on this account, all the ordinates comparable to the standard visibility data. (See footnote 43.)

[43] K. S. Gibson and E. P. T. Tyndall, The Visibility of Radiant Energy, B. S. Sci. Paper No. 475, pp. 156-159, 174; 1923.

shown by small circles; their adopted or recommended visibility[44] is shown by large circles. It is to be noted that at about half the wave lengths at which this comparison is made, the visibility via the distribution curve sum (small circles with tags) either falls between the Gibson and Tyndall experimental mean (small circles) and the

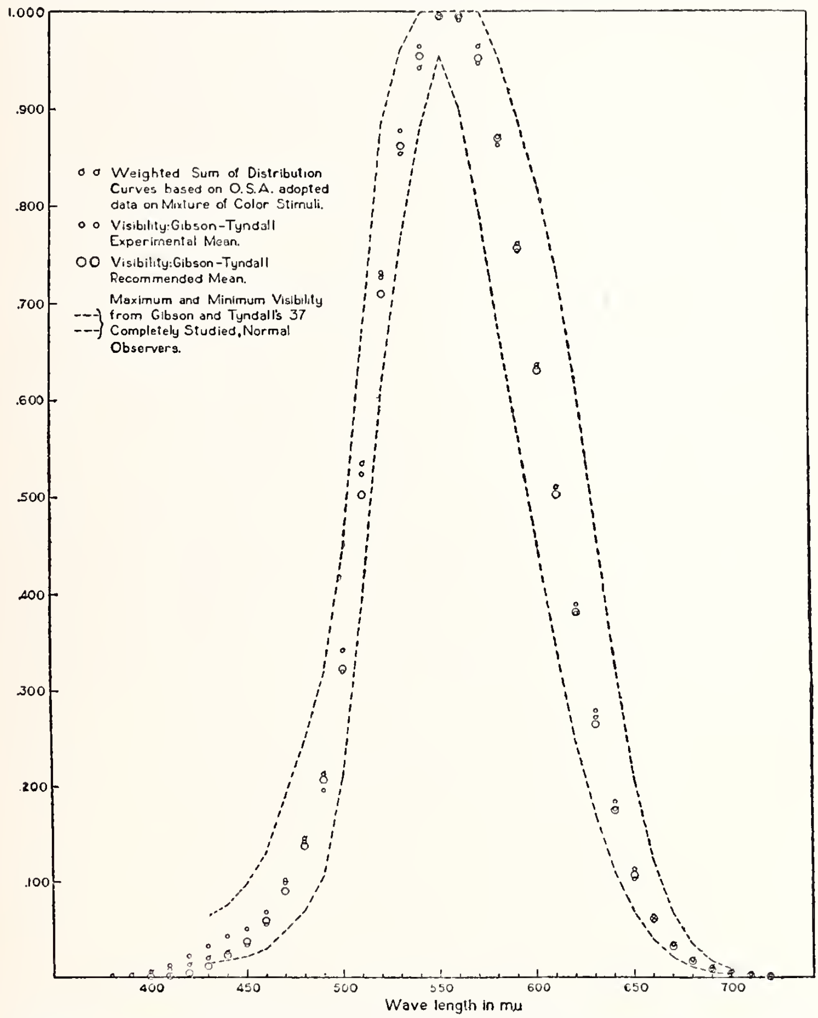

Figure 1.—*Demonstration that the distribution curves expressing mixture data are not inconsistent with standard visibility data*

The weighted distribution curve sum is compared (1) to the Gibson-Tyndall experimental mean visibility (B. S. Sci. Paper No. 475, Table 3, second column), (2) to the recommended mean (ibid, Table 3, last column), and (3) to the highest and lowest visibility found experimentally among the 37 completely studied, normal observers (ibid, Table 2). The weighted distribution curve sum is seen to agree closely with visibility directly determined.

values recommended by them (large circles) or coincides with one of them. To make further evident that the discrepancies which do occur are negligible in comparison to individual differences, the

[44] Subsequently adopted as standard by the International Commission on Illumination, 6th meeting, Geneva, July, 1924; Proceedings, pp. 67 and 232.

maximum and minimum visibilities found by Gibson and Tyndall among their 37 completely studied, normal observers[45] are indicated also (points connected by straight, dotted lines). It may be seen from Figure 1 that the differences which do exist between the values of visibility and the sums of the weighted distribution curve ordinates are negligible in comparison to differences between individual visibilities.[46] On this account the failure of the distribution curves to sum exactly to the standard visibility function need cause no further concern. It is true, of course, that the standard visibility differs significantly for some purposes from the distribution curve sums; but the difference is significant because standard values have been formally adopted, not because sufficient experimental data exists to prove that the distribution curve sums fail to represent normal visibility.[47] For this reason, the luminosity of Abbot-Priest sunlight given by the weighted distribution curve sums (column 8 of Table 1) is taken as "standard" in the present paper for the mixture data of the O. S. A. "excitation" curves; and there need be here no more discussion concerning the "degree of approximation" yielded by a given set of luminosity coefficients. If the values for the luminosity coefficients yield the luminosity given in column 8 of Table 1, they are the correct values for that set of distribution curves and constitute the luminosity coefficients; and if the values do not, they are wrong, and do not constitute the luminosity coefficients.

We now propose to take four different sets of distribution curves representing the distribution of four new sets of "red," "green," and "blue" primary color processes throughout the spectrum of Abbot-Priest sunlight according to the mixture data embodied in the O. S. A. "excitation" curves. These four new sets of distribution curves (ρ_2, γ_2, β_2; ρ_3, γ_3, β_3; ρ_4, γ_4, β_4; and ρ_5, γ_5, β_5) are given by columns 2, 3, and 4 of Tables 2 to 5, and are plotted near the bottom of Figures 2 to 5. The distribution curves have been computed from ρ_1, γ_1, and β_1 (see Table 1) in the following way:

$$\left.\begin{aligned}\rho_2 &\equiv 1.0499\rho_1 - 0.1552\gamma_1 + 0.1054\beta_1\\ \gamma_2 &\equiv 0.2023\rho_1 + 0.7977\gamma_1 + 0.0000\beta_1\\ \beta_2 &\equiv 0.0000\rho_1 + 0.0000\gamma_1 + 1.0000\beta_1\end{aligned}\right\}\qquad(10)$$

$$\left.\begin{aligned}\rho_3 &\equiv 1.0000\rho_1 + 0.0000\gamma_1 + 0.0000\beta_1\\ \gamma_3 &\equiv 0.0000\rho_1 + 1.0000\gamma_1 + 0.0000\beta_1\\ \beta_3 &\equiv 0.3636\rho_1 + 0.3636\gamma_1 + 0.2727\beta_1\end{aligned}\right\}\qquad(11)$$

$$\left.\begin{aligned}\rho_4 &\equiv 0.9783\rho_1 + 0.0000\gamma_1 + 0.0217\beta_1\\ \gamma_4 &\equiv 0.0000\rho_1 + 1.0000\gamma_1 + 0.0000\beta_1\\ \beta_4 &\equiv 0.0000\rho_1 + 0.0000\gamma_1 + 1.0000\beta_1\end{aligned}\right\}\qquad(12)$$

$$\left.\begin{aligned}\rho_5 &\equiv 1.0000\rho_1 + 0.0000\gamma_1 + 0.0000\beta_1\\ \gamma_5 &\equiv 0.4500\rho_1 + 0.5400\gamma_1 + 0.0100\beta_1\\ \beta_5 &\equiv 0.0000\rho_1 + 0.0000\gamma_1 + 1.0000\beta_1\end{aligned}\right\}\qquad(13)$$

[45] One observer was rejected because he was known to be color blind; 14 observers of the 52 were also rejected because they were not studied for the entire visible spectrum, but only for its more luminous portion (490 to 680 mμ). Considering the data referring to the remaining 37 observers for one value of the wave length at a time (at intervals of 10 mμ) the highest value and the lowest value (as recorded in Table 2 of Gibson and Tyndall's paper) for that wave length were plotted.

[46] Comparison has here been only with values of visibility found or adopted by Gibson and Tyndall because that is the most recent work on visibility that has received formal recognition. The same sort of spread among individuals has, however, been found by previous observers; for example, by Coblentz and Emerson (B. S. Sci. Paper No. 303; 1917; see fig. 13).

[47] Note that, since the O. S. A. "excitation" curves are based on so few observers (only three—Koenig, Dieterici, and Abney), they are open to attack as being not truly representative of normal mixture data. It is probable that at some future time the O. S. A. curves will be supplanted by curves more truly representative of normal mixture data. They are not open to attack, however, on the ground of being inconsistent with normal visibility because the differences of the weighted sums of the O. S. A. curves from the standard visibility function (perhaps by good luck) are insignificant relative to individual differences.

TABLE 2.—*The distributions of the König "Grundempfindungen" throughout the spectrum of Abbot-Priest sunlight according to the mixture data embodied in the O. S. A. "excitation" curves*

λ in mμ	ρ_2	γ_2	β_2	$\sigma_2=\rho_2+\gamma_2+\beta_2$	$R_2=\rho_2/\sigma_2$	$G_2=\gamma_2/\sigma_2$	$B_2=\beta_2/\sigma_2$
1	2	3	4	5	6	7	8
380	47	--------	442	489	--------	--------	--------
90	134	--------	1,274	1,408	--------	--------	--------
400	286	--------	2,717	3,003	--------	--------	--------
10	532	--------	5,045	5,577	0.0954	0.0000	0.9046
20	807	10	7,679	8,496	.0950	.0012	.9038
30	1,278	32	12,186	13,496	.0946	.0024	.9029
40	1,497	80	14,351	15,928	.0940	.0050	.9010
450	1,441	192	14,027	15,660	.0920	.0123	.8957
60	1,289	478	12,958	14,725	.0875	.0325	.8800
70	1,033	1,061	11,098	13,192	.0783	.0804	.8413
80	744	1,674	7,767	10,185	.0730	.1644	.7626
90	668	2,451	3,703	6,822	.0979	.3593	.5428
500	1,019	3,916	2,112	7,047	.1446	.5557	.2997
10	1,857	6,074	1,517	9,448	.1965	.6429	.1606
20	3,022	8,142	1,076	12,240	.2469	.6652	.0879
30	4,225	9,370	765	14,360	.2942	.6525	.0533
40	5,369	10,092	517	15,978	.3360	.6316	.0324
550	6,288	10,420	322	17,030	.3692	.6119	.0189
60	7,135	10,040	196	17,371	.4107	.5780	.0113
70	8,001	9,266	124	17,391	.4601	.5328	.0071
80	8,518	7,811	71	16,400	.5194	.4763	.0043
90	9,042	6,102	35	15,179	.5957	.4020	.0023
600	8,748	4,538	17	13,303	.6576	.3411	.0013
10	7,990	3,162	--------	11,152	.7165	.2835	.0000
20	6,449	2,074	--------	8,523	.7567	.2433	.0000
30	4,848	1,335	--------	6,183	.7841	.2159	.0000
40	3,293	771	--------	4,064	.8103	.1897	.0000
650	1,957	417	--------	2,374	.8243	.1757	.0000
60	1,108	226	--------	1,334	.8306	.1694	.0000
70	636	122	--------	758	.8391	.1609	.0000
80	342	66	--------	408	--------	--------	--------
90	196	38	--------	234	--------	--------	--------
700	110	21	--------	131	--------	--------	--------
10	57	11	--------	68	--------	--------	--------
20	35	7	--------	42	--------	--------	--------

TABLE 3.—*The distributions throughout the spectrum of Abbot-Priest sunlight according to the mixture data embodied in the O. S. A. "excitation" curves of primary color processes which suggest a degree of blue-yellow deficiency* [1]

λ in mμ	ρ_3	γ_3	β_3	$\sigma_3=\rho_3+\gamma_3+\beta_3$	$R_3=\rho_3/\sigma_3$	$G_3=\gamma_3/\sigma_3$	$B_3=\beta_3/\sigma_3$
1	2	3	4	5	6	7	8
380	...	...	121	121	...	...	...
90	...	...	347	347	...	...	...
400	...	...	741	741	...	...	...
10	...	...	1, 376	1, 376	0. 0000	0. 0000	1. 0000
20	...	13	2, 099	2, 112	. 0000	. 0062	. 9938
30	...	41	3, 338	3, 379	. 0000	. 0121	. 9879
40	...	100	3, 951	4, 051	. 0000	. 0247	. 9753
450	...	240	3, 913	4, 153	. 0000	. 0578	. 9422
60	15	595	3, 756	4, 366	. 0034	. 1363	. 8603
70	64	1, 313	3, 528	4, 905	. 0130	. 2677	. 7193
80	231	2, 040	2, 944	5, 215	. 0443	. 3912	. 5645
90	693	2, 897	2, 316	5, 906	. 1173	. 4905	. 3921
500	1, 430	4, 546	2, 749	8, 725	. 1639	. 5210	. 3151
10	2, 644	6, 944	3, 901	13, 489	. 1960	. 5148	. 2892
20	4, 125	9, 160	5, 124	18, 409	. 2241	. 4976	. 2783
30	5, 479	10, 357	5, 967	21, 803	. 2513	. 4750	. 2737
40	6, 681	10, 958	6, 555	24, 194	. 2761	. 4529	. 2709
550	7, 604	11, 134	6, 902	25, 640	. 2966	. 4343	. 2692
60	8, 325	10, 475	6, 890	25, 690	. 3241	. 4077	. 2682
70	8, 989	9, 336	6, 697	25, 022	. 3592	. 3731	. 2676
80	9, 210	7, 456	6, 080	22, 746	. 4049	. 3278	. 2673
90	9, 388	5, 269	5, 340	19, 997	. 4695	. 2635	. 2670
600	8, 841	3, 447	4, 473	16, 761	. 5275	. 2057	. 2669
10	7, 900	1, 960	3, 585	13, 445	. 5876	. 1458	. 2666
20	6, 291	1, 004	2, 653	9, 948	. 6324	. 1009	. 2667
30	4, 690	484	1, 881	7, 055	. 6648	. 0686	. 2666
40	3, 161	164	1, 209	4, 534	. 6972	. 0362	. 2667
650	1, 871	48	698	2, 617	. 7149	. 0183	. 2667
60	1, 057	16	390	1, 463	. 7225	. 0109	. 2666
70	606	...	220	826	. 7337	. 0000	. 2663
80	326	...	119	445	...	...	...
90	187	...	68	255	...	...	...
700	105	...	38	143	...	...	...
10	54	...	20	74	...	...	...
20	33	...	12	45	...	...	...

[1] The distribution curves were calculated from the O. S. A. "Excitation" curves according to relation (6a) by setting $D=4/5$.

TABLE 4.—*The distributions of primary color processes such that the "blue" process contributes nothing to the luminosity of the spectrum*

[Mixture data as of O. S. A. "excitation" curves; spectrum of Abbot-Priest sunlight]

λ in mμ	ρ_4	γ_4	β_4	$\sigma_4=\rho_4+\gamma_4+\beta_4$	$R_4=\rho_4/\sigma_4$	$G_4=\gamma_4/\sigma_4$	$B_4=\beta_4/\sigma_4$
1	2	3	4	5	6	7	8
380	10	----------	442	452	----------	----------	----------
90	28	----------	1, 274	1, 302	----------	----------	----------
400	59	----------	2, 717	2, 776	----------	----------	----------
10	110	----------	5, 045	5, 155	0. 0213	0. 0000	0. 9787
20	167	13	7, 679	7, 859	. 0212	. 0017	. 9771
30	265	41	12, 186	12, 492	. 0212	. 0033	. 9755
40	312	100	14, 351	14, 763	. 0211	. 0068	. 9721
450	305	240	14, 027	14, 572	. 0209	. 0165	. 9626
60	296	595	12, 958	13, 849	. 0214	. 0430	. 9357
70	304	1, 313	11, 098	12, 715	. 0239	. 1033	. 8728
80	395	2, 040	7, 767	10, 202	. 0387	. 2000	. 7613
90	758	2, 897	3, 703	7, 358	. 1030	. 3937	. 5033
500	1, 445	4, 546	2, 112	8, 103	. 1783	. 5610	. 2606
10	2, 620	6, 944	1, 517	11, 081	. 2364	. 6267	. 1369
20	4, 059	9, 160	1, 076	14, 295	. 2839	. 6407	. 0753
30	5, 377	10, 357	765	16, 499	. 3259	. 6277	. 0464
40	6, 547	10, 958	517	18, 022	. 3633	. 6080	. 0287
550	7, 446	11, 134	322	18, 902	. 3939	. 5890	. 0170
60	8, 148	10, 475	196	18, 819	. 4330	. 5566	. 0104
70	8, 796	9, 336	124	18, 256	. 4818	. 5114	. 0068
80	9, 011	7, 456	71	16, 538	. 5449	. 4508	. 0043
90	9, 185	5, 269	35	14, 489	. 6339	. 3637	. 0024
600	8, 649	3, 447	17	12, 113	. 7140	. 2846	. 0014
10	7, 728	1, 960	----------	9, 688	. 7977	. 2023	. 0000
20	6, 154	1, 004	----------	7, 158	. 8597	. 1403	. 0000
30	4, 588	484	----------	5, 072	. 9046	. 0954	. 0000
40	3, 092	164	----------	3, 256	. 9496	. 0504	. 0000
650	1, 830	48	----------	1, 878	. 9744	. 0256	. 0000
60	1, 034	16	----------	1, 050	. 9848	. 0152	. 0000
70	593	----------	----------	593	1. 0000	. 0000	. 0000
80	319	----------	----------	319	----------	----------	----------
90	183	----------	----------	183	----------	----------	----------
700	103	----------	----------	103	----------	----------	----------
10	53	----------	----------	53	----------	----------	----------
20	32	----------	----------	32	----------	----------	----------

TABLE 5.—*The distribution throughout the spectrum of Abbot-Priest sunlight of one of the sets of primary color processes for which neither the "red" nor the "blue" primary color process contributes to the luminosity of the spectrum. The distribution of the "green" primary color process is the same as the distribution of luminosity.*

λ in mμ	ρ_5	γ_5	β_5	$\sigma_5 = \rho_5+\gamma_5+\beta_5$	$R_5 = \rho_5/\sigma_5$	$G_5 = \gamma_5/\sigma_5$	$B_5 = \beta_5/\sigma_5$
1	2	3	4	5	6	7	8
380	----------	4	442	446	----------	----------	----------
90	----------	13	1, 274	1, 287	----------	----------	----------
400	----------	27	2, 717	2, 744	----------	----------	----------
10	----------	50	5, 045	5, 095	0. 0000	0. 0098	0. 9902
20	----------	84	7, 679	7, 763	. 0000	. 0108	. 9892
30	----------	144	12, 186	12, 330	. 0000	. 0117	. 9883
40	----------	198	14, 351	14, 549	. 0000	. 0136	. 9864
450	----------	270	14, 027	14, 297	. 0000	. 0189	. 9811
60	15	458	12, 958	13, 431	. 0011	. 0341	. 9648
70	64	849	11, 098	12, 011	. 0053	. 0707	. 9240
80	231	1, 283	7, 767	9, 281	. 0249	. 1382	. 8369
90	693	1, 913	3, 703	6, 309	. 1098	. 3032	. 5869
500	1, 430	3, 119	2, 112	6, 661	. 2147	. 4682	. 3171
10	2, 644	4, 955	1, 517	9, 116	. 2900	. 5435	. 1664
20	4, 125	6, 813	1, 076	12, 014	. 3433	. 5671	. 0896
30	5, 479	8, 066	765	14, 310	. 3829	. 5637	. 0535
40	6, 681	8, 929	517	16, 127	. 4143	. 5537	. 0321
550	7, 604	9, 437	322	17, 363	. 4379	. 5435	. 0185
60	8, 325	9, 405	196	17, 926	. 4644	. 5247	. 0109
70	8, 989	9, 088	124	18, 201	. 4939	. 4993	. 0068
80	9, 210	8, 171	71	17, 452	. 5277	. 4682	. 0041
90	9, 388	7, 070	35	16, 493	. 5692	. 4287	. 0021
600	8, 841	5, 840	17	14, 698	. 6015	. 3973	. 0012
10	7, 900	4, 613	----------	12, 513	. 6313	. 3687	. 0000
20	6, 291	3, 373	----------	9, 664	. 6510	. 3490	. 0000
30	4, 690	2, 372	----------	7, 062	. 6641	. 3359	. 0000
40	3, 161	1, 511	----------	4, 672	. 6766	. 3234	. 0000
650	1, 871	868	----------	2, 739	. 6831	. 3169	. 0000
60	1, 057	484	----------	1, 541	. 6859	. 3141	. 0000
70	606	273	----------	879	. 6894	. 3106	. 0000
80	326	147	----------	473	----------	----------	----------
90	187	84	----------	271	----------	----------	----------
700	105	47	----------	152	----------	----------	----------
10	54	24	----------	78	----------	----------	----------
20	33	15	----------	48	----------	----------	----------

It is to be noted that each of these four new sets of distribution curves represents the same body of mixture data as ρ_1, γ_1, and β_1, because each set satisfies (2).[48] The theorem of Section II must, therefore, hold for these four sets of curves; hence, the methods of computing the luminosity coefficients (Sec. III) which depend on that theorem can be used. We proceed to apply those methods.

The first step in the first method is to compute $\rho_1 L_{r1} + \gamma_1 L_{g1} + \beta_1 L_{b1}$ for three convenient wave lengths. This has already been done for all wave lengths (column 8 of Table 1); we choose 450, 550, and 650 mμ, which give 269.9, 9,437.4, and 867.9, respectively. Then we set up the following four triplets of simultaneous equations:

$$\left.\begin{array}{l} 1{,}441\ L_{r2} + 192\ L_{g2} + 14{,}027\ L_{b2} = 269.9 \\ 6{,}288\ L_{r2} + 10{,}420\ L_{g2} + 322\ L_{b2} = 9{,}437.4 \\ 1{,}957\ L_{r2} + 417\ L_{g2} = 867.9 \end{array}\right\}$$

$$\left.\begin{array}{l} 240\ L_{g3} + 3{,}913\ L_{b3} = 269.9 \\ 7{,}604\ L_{r3} + 11{,}134\ L_{g3} + 6{,}902\ L_{b3} = 9{,}437.4 \\ 1{,}871\ L_{r3} + 48\ L_{g3} + 698\ L_{b3} = 867.9 \end{array}\right\}$$

$$\left.\begin{array}{l} 305\ L_{r4} + 240\ L_{g4} + 14{,}027\ L_{b4} = 269.9 \\ 7{,}446\ L_{r4} + 11{,}134\ L_{g4} + 322\ L_{b4} = 9{,}437.4 \\ 1{,}830\ L_{r4} + 48\ L_{g4} = 867.9 \end{array}\right\}$$

$$\left.\begin{array}{l} 270\ L_{g5} + 14{,}027\ L_{b5} = 269.9 \\ 7{,}604\ L_{r5} + 9{,}437\ L_{g5} + 322\ L_{b5} = 9{,}437.4 \\ 1{,}871\ L_{r5} + 868\ L_{g5} = 867.9 \end{array}\right\}$$

and solve for the luminosity coefficients.[49]

The second method consists merely of the substitution of the values of L_{r1}, L_{g1}, L_{b1} (which are 0.4500, 0.5400, and 0.0100, respectively, in each case) and of the values of the constants, K_1 to K_9 (which are given in relations (10) to (13)) in relation (8).

The results of computation by these two methods appear in Table 6. It may be seen that the first method (which is, perhaps, less provocative of arithmetical blunders, though it takes about ten times as long) yields computational accuracy up to two or three in the fourth decimal place. Rejection error in the second method may readily be made negligible.

TABLE 6.—*Results of solutions for the luminosity coefficients*

Distribution curves	L_r		L_g		L_b	
	Method 1	Method 2	Method 1	Method 2	Method 1	Method 2
$\rho_2, \gamma_2, \beta_2$	0.2872	0.2875	0.7330	0.7327	−0.0203	−0.0203
$\rho_3, \gamma_3, \beta_3$	.4367	.4367	.5268	.5267	.0366	.0367
$\rho_4, \gamma_4, \beta_4$	.4601	.4600	.5401	.5400	.0000	.0000
$\rho_5, \gamma_5, \beta_5$	.0000	.0000	1.0000	1.0000	.0000	.0000

[48] We know, without troubling to integrate with respect to wave length, that each curve has the same area as ρ_1, γ_1, and β_1 because the sum of the coefficients of ρ_1, γ_1, and β_1 in each of the 12 equations given in relations (10), (11), (12), and (13) is unity (except for discrepancies of one in the last place due to errors of rejection in the computation).

[49] The numbers in the left members of the four triplets of equations may be obtained, of course, from columns 2, 3, and 4 of Tables 2, 3, 4, and 5 for wave lengths 450, 550, and 650 mμ.

L_{r2}, L_{g2}, and L_{b2} were further checked by computation of $\rho_2 L_{r2} + \gamma_2 L_{g2} + \beta_2 L_{b2}$ which was found to agree for all wave lengths with $\rho_1 L_{r1} + \gamma_1 L_{g1} + \beta_1 L_{b1}$ in accordance with relation (4); hence, the values found, $L_{r2} = 0.2875$, $L_{g2} = 0.7327$, and $L_{b2} = 0.0203$ are the luminosity coefficients for the distribution curves of the König "Grundempfindungen" according to the mixture data embodied in the O. S. A. "excitation" curves.[50] L_{r3}, L_{g3}, and L_{b3} were checked by computation from relation (6a)[51] and found to be correct. There is no need of checking L_{r4}, L_{g4}, and L_{b4} or L_{r5}, L_{g5}, and L_{b5} because the constants, K_1 to K_9, by which ρ_4, γ_4, and β_4 (relation (12)) and ρ_5, γ_5, and β_5 (relation (13)) were computed, were chosen so as to obtain the luminosity coefficients found. The correctness of this choice can be seen by inspection of relations (12) [52] and (13).

V. DEMONSTRATION THAT THE NEW SETS OF DISTRIBUTION CURVES EMBODY THE MIXTURE DATA OF THE O. S. A. "EXCITATION" CURVES

Since from their derivation (see relations (10), (11), (12), and (13)) the new sets of distribution curves all satisfy relation (2), we know that they must embody the same mixture data as the O. S. A. "excitation" curves. A demonstration more convincing than this formal proof, perhaps, is to compute by each set of curves the dominant wave length, purity, and luminosity of the color stimuli obtained by filters illuminated by Abbot-Priest sunlight, and compare the values obtained. If all five sets of distribution curves yield the same values of dominant wave length, we could say that the identity of their mixture data had been checked. Since luminosity coefficients have been found for each set of distribution curves, we can also compute the colorimetric purity and transmission of the filters (which gives a measure of the luminosity of the color stimulus) for Abbot-Priest sunlight. If all five sets of distribution curves yield the same values of colorimetric purity and transmission for Abbot-Priest sun-

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[50] It is convenient to point out here that the computation of the luminosity coefficients for this approximation to the Grundempfindungskurven demonstrates that the failure reported by Ives (see footnote 18, p. 519) of the König curves to yield consistent luminosity values was due (as he implies but does not state explicitly) to his rejection of a negative value for the "bluo" coefficient. With a negative value, Ives could have obtained as close agreement with König's original Grundempfindungskurven as he did with his transformed curves and positive coefficients. It should be noted that the convexity between 400 and 450 mμ, which the spectrum locus according to the König-Ives curves shows, is not due to the transformation of the coordinates, nor to the avoidance of a negative luminosity coefficient, nor to luminosity considerations at all; it is due to the fact that the original mixture data on which the König-Ives curves are hased is different from that on which the O. S. A. curves are based. The essential difference lies in the inclusion in the König-Ives curves of data by partially color-hlind observers. There is fairly good ground to helieve that such inclusion is justifiable when the König Grundompfindungskurven are heing considered because it was shown by König that the color responses of his protanopic and deuteranopic observers could be predicted closoly from the Grundempfindungskurven of the normal observers merely by the omission of either the red or the green curve. Although it is the author's opinion that such inclusion is still questionable, nevertheless there is some reason to suspect that an improvement was wrought by the introduction of data by the partially color-blind observers because Guild (A Critical Survey of Modern Developments in the Theory and Technique of Colorimetry and Allied Sciences, Proc. Optical Convention, London, Pt. 1, p. 57; 1926) who uses the König-Ives curves for routine computation criticizes the O. S. A. "excitation" curves in these terms, "My own view is that these are in some respects less satisfactory than the König-Ives curves. One serious defect is the absence of any reappearance of red in the violet region, which on Weaver's basis would be all of one color from 440 mμ onwards." This seems to imply that Guild has evidence that the König-Ives curves, which give a marked convexity in the spectrum locus on the mixture diagram from 400 to 450 mμ, agree with experimental results hy his observers hetter than the O. S. A. "excitation" curves which do not have that convexity (compare Guild's fig. 12 with figs. 2 to 5 of the present paper). It is also true that Houstoun (Phil. Mag. (6), **45**, p. 176; 1923) with rather crude apparatus found this convexity, but it is probable that stray light was present in sufficient quantity in the photometric field of Houstoun's apparatus to give his result. (For the effect of stray light under these conditions, see Ahney, on the Change of Hue of Spectrum Colors by Dilution with White Light, Proc. Roy. Soc., A., **83**, 1909.)

[51] Relation (6a) proves, of course, to he a special case of relation (6).

[52] Note that 45/46 is approximately 0.9783, and that 1/46 is approximately 0.0217. See relation (6) et seq.

light, we could say that not only had the identity of their mixture data been checked a second time, but also had we checked (for a third time) the accuracy of the luminosity coefficients, since those coefficients enter into the computation. We proceed to make these computations, and for the purpose of checking we choose four (hypothetical, nonexistent) filters defined by their spectral transmissions which are zero for all wave lengths except as follows:

Filter No. (m)	Spectral transmission (T_λ) [1]	
1	$T_{490}=1.00000$	$T_{610}=0.37680$
2	$T_{640}=1.00000$	$T_{500}=.99291$
3	$T_{590}=1.00000$	$T_{440}=.72571$
4	$T_{460}=1.00000$	$T_{550}=.47877$

[1] Of course, these values of spectral transmission were not chosen at random. The large wave-length regions of zero transmission (all but two intervals of 10 mμ for each hypothetical filter) result in the avoidance of much computation and, at the same time, provide a more rigorous test of the colorimetric identity of the new sets of distribution curves than that afforded by actual filters. The transmissions which are not zero were so chosen that the dominant wave length computed in the usual, routine way from the O. S. A. "excitation" curves for each filter ($m=1$, $m=2$, $m=3$, $m=4$) is as nearly as convenient an exact multiple of 10 (530, 570, 510, and 480, respectively) when expressed in millimicrons. Such dominant wave lengths may be computed with more certainty (say to 0.01 instead of to 0.1 mμ) than values of dominant wave length for which the ordinates of the distribution curves are not specified explicitly but only indirectly by interpolation.

TABLE 7.—*Computation leading up to the evaluation of dominant wave length, colorimetric purity, and transmission of filters ($m=1$ to 4) illuminated by Abbot-Priest sunlight*

Distribution curves	Filter 1 $m=1$			Filter 2 $m=2$			Filter 3 $m=3$			Filter 4 $m=4$		
$\rho_1, \gamma_1, \beta_1$ $n=1$	[1] 693 [1] 2,977	2,897 739	3,703 0	1,420 3,161	4,514 164	2,097 0	0 9,388	73 5,269	10,415 35	15 3,641	595 5,331	12,958 154
	[2] 3,670	3,636	3,703	4,581	4,678	2,097	9,388	5,342	10,450	3,656	5,926	13,112
	[3] 0.3334	0.3303	----------	0.4034	0.4119	----------	0.3728	0.2122	----------	0.1611	0.2611	----------
$\rho_2, \gamma_2, \beta_2$ $n=2$	668 3,011	2,451 1,191	3,703 0	1,012 3,293	3,888 771	2,097 0	1,086 9,042	58 6,102	10,415 35	1,289 3,011	478 4,989	12,958 154
	3,679	3,642	3,703	4,305	4,659	2,097	10,128	6,160	10,450	4,300	5,467	13,112
	0.3337	0.3304	----------	0.3892	0.4212	----------	0.3788	0.2304	----------	0.1879	0.2390	----------
$\rho_3, \gamma_3, \beta_3$ $n=3$	693 2,977	2,897 739	2,316 1,351	1,420 3,161	4,514 164	2,730 1,209	0 9,388	73 5,269	2,867 5,340	15 3,641	595 5,331	3,756 3,304
	3,670	3,636	3,667	4,581	4,678	3,939	9,388	5,342	8,207	3,656	5,926	7,060
	0.3345	0.3314	----------	0.3471	0.3544	----------	0.4093	0.2329	----------	0.2197	0.3561	----------
$\rho_4, \gamma_4, \beta_4$ $n=4$	758 2,912	2,897 739	3,703 0	1,435 3,092	4,514 164	2,097 0	226 9,185	73 5,269	10,415 35	296 3,565	595 5,331	12,958 154
	3,670	3,636	3,703	4,527	4,678	2,097	9,411	5,342	10,450	3,861	5,926	13,112
	0.3334	0.3303	----------	0.4005	0.4139	----------	0.3734	0.2120	----------	0.1686	0.2588	----------
$\rho_5, \gamma_5, \beta_5$ $n=5$	693 2,977	1,913 1,738	3,703 0	1,420 3,161	3,097 1,511	2,097 0	0 9,388	144 7,070	10,415 35	15 3,641	458 4,518	12,958 154
	3,670	3,651	3,703	4,581	4,608	2,097	9,388	7,214	10,450	3,656	4,976	13,112
	0.3329	0.3312	----------	0.4059	0.4083	----------	0.3470	0.2667	----------	0.1681	0.2288	----------

[1] Products. [2] Sums. [3] r, g.

The method of computing dominant wave length, colorimetric purity and transmission follows closely that recommended by the O. S. A. Committee on Colorimetry for 1920–21 [53] except for the computation of purity for which papers by Priest [54] and Judd [55] should be consulted. Table 7 shows (1) the products of the distribution curve ordinates (ρ_n, γ_n, β_n) by transmissions (T_m) [56] of filter, m; (2) the sums of these products which when multiplied by 10 constitute the evaluation of $\int_0^\infty \rho_n T_m d\lambda$, $\int_0^\infty \gamma_n T_m d\lambda$, and $\int_0^\infty \beta_n T_m d\lambda$; and (3) two of the three trilinear coordinates (r, g, b) [57] of the colors whose dominant wave length and colorimetric purity we are computing. The trilinear coordinates of filter, m, via distribution curves ρ_n, γ_n, and β_n are defined as:

$$\left.\begin{aligned} r&=\frac{\int_0^\infty \rho_n T_m d\lambda}{\int_0^\infty \rho_n T_m d\lambda+\int_0^\infty \gamma_n T_m d\lambda+\int_0^\infty \beta_n T_m d\lambda} \\ g&=\frac{\int_0^\infty \gamma_n T_m d\lambda}{\int_0^\infty \rho_n T_m d\lambda+\int_0^\infty \gamma_n T_m d\lambda+\int_0^\infty \beta_n T_m d\lambda} \\ b&=\frac{\int_0^\infty \beta_n T_m d\lambda}{\int_0^\infty \rho_n T_m d\lambda+\int_0^\infty \gamma_n T_m d\lambda+\int_0^\infty \beta_n T_m d\lambda} \end{aligned}\right\} \qquad (14)$$

Figures 2 to 5 show mixture diagrams on which the spectrum locus has been plotted, the r-coordinate of the spectrum, R_n, being plotted against the g-coordinate, G_n. (See columns 6 and 7 of Tables 2 to 5.)[58] On these diagrams, every color is represented by a point; these diagrams possess the property [59] that the points representing the colors which result from the additive mixture of two light stimuli producing any two colors fall upon the straight line connecting the points representing those colors. The "neutral" ($r=1/3$, $g=1/3$) point is connected by straight lines to the points representing the colors of the extremes of the spectrum; these lines serve to divide the diagram into the spectral colors and the nonspectral, or purple, colors. The points representing the colors of filters Nos. 2, 3, and 4 ($m=2$, 3, and 4) are also shown; the color of filter No. 1 ($m=1$) is so close to the "neutral" color that its point has been omitted for the sake of

[53] L. T. Troland, J. Opt. Soc. Am. and Rev. Sci. Inst., **6**, pp. 527–596; 1922.

[54] I. G. Priest, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., **9**, pp. 503–520; 1924. Also The Computation of Colorimetric Purity II, J. Opt. Soc. Am. and Rev. Sci. Inst., **13**, pp. 123–132; 1926.

[55] D. B. Judd, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., **13**, pp. 133–152; 1926.

[56] The subscript, m, here denotes the filter, not (as is usual) the wave length.

[57] r and g are given in Table 7, although b is not. But from relation (14) we know that $r+g+b=1$; hence we may easily find b from r and g since it is $1-r-g$.

[58] The trilinear coordinates (R_n, G_n, B_n) of the spectrum referred to Abbot-Priest sunlight as the "neutral" ($r=1/3$, $g=1/3$) stimulus have been computed only for wave lengths between 410 and 670 mμ, inclusive, because beyond these wave-length limits according to the mixture data of the O. S. A. "excitation" curves in their extrapolated form the trilinear coordinates are constant. If, for example, the computation were carried out for 380, 390, and 400 mμ, the trilinear coordinates found would differ from those of 410 mμ only by reason of computational errors of rejection.

[59] This property constitutes one expression of Newton's law of color mixture; on this account, the diagrams are sometimes called "Newton's mixture diagrams."

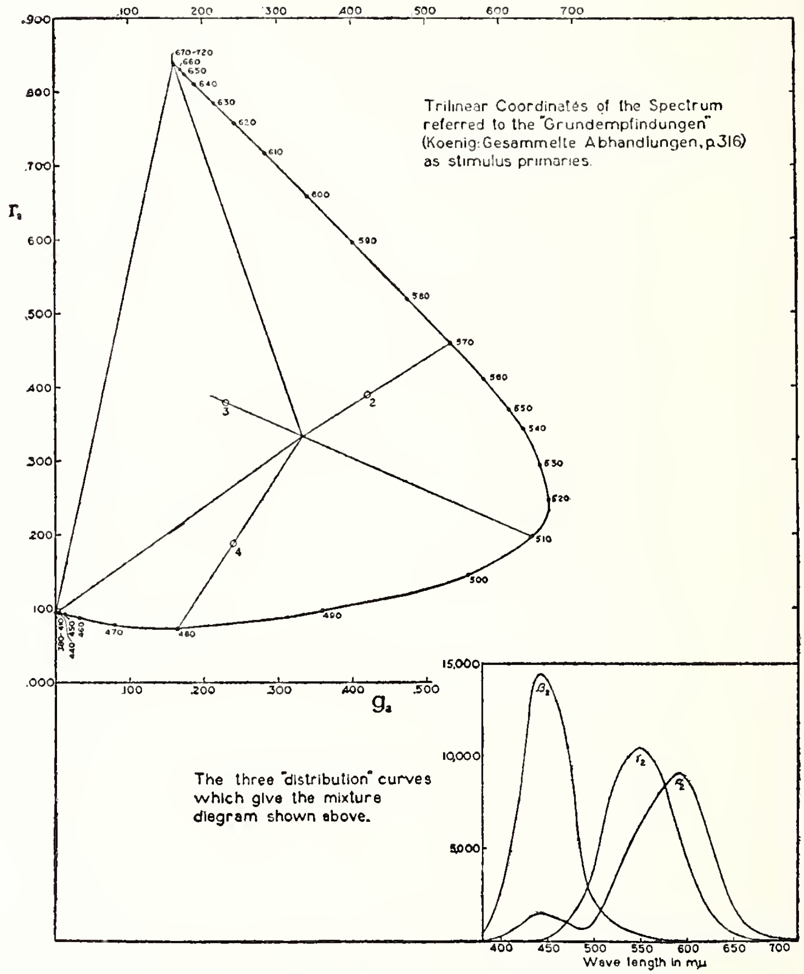

FIGURE 2.—*The distribution curves, ρ_2, γ_2, and β_2, and the mixture diagram showing the spectrum locus given by ρ_2, γ_2, and β_2*

Note that the points representing the colors of filters 2, 3, and 4 (see Table 7, r and g) fall on the straight lines connecting the "neutral" point with 570, 510, and 480 mμ, respectively.

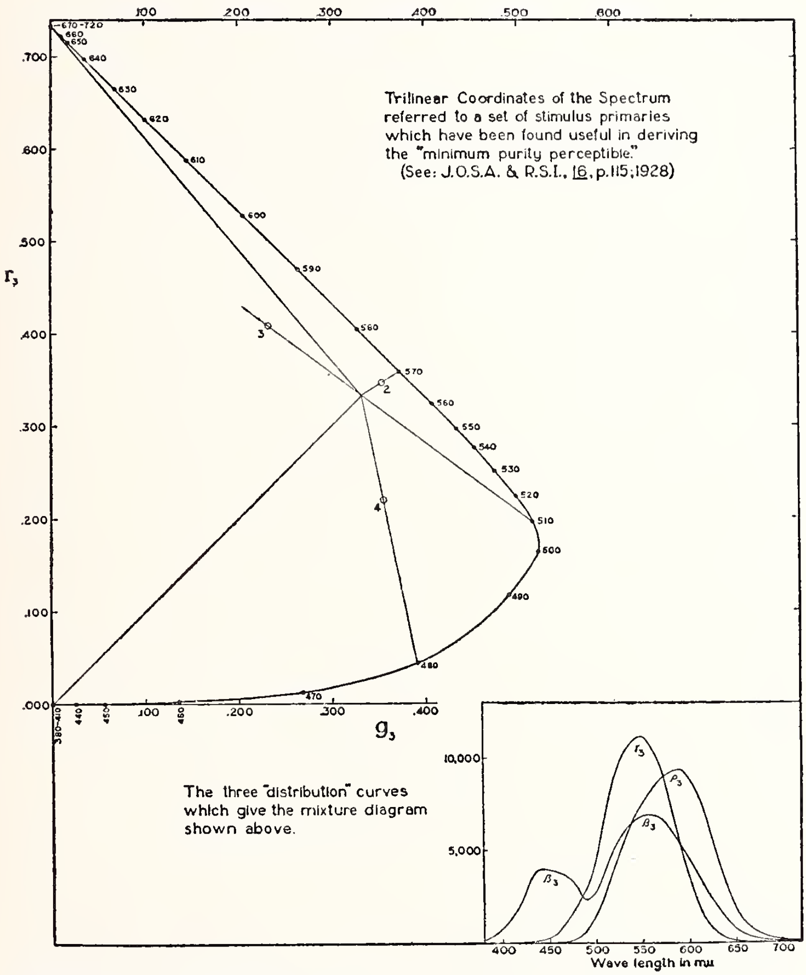

Figure 3.—*The distribution curves, ρ_3, γ_3, and β_3 and the mixture diagram showing the spectrum locus given by ρ_3, γ_3, and β_3*

Note that the points representing the colors of filters 2, 3, and 4 (see Table 7, r and g) fall on the straight lines connecting the "neutral" point with 570, 510, and 480 mμ, respectively.

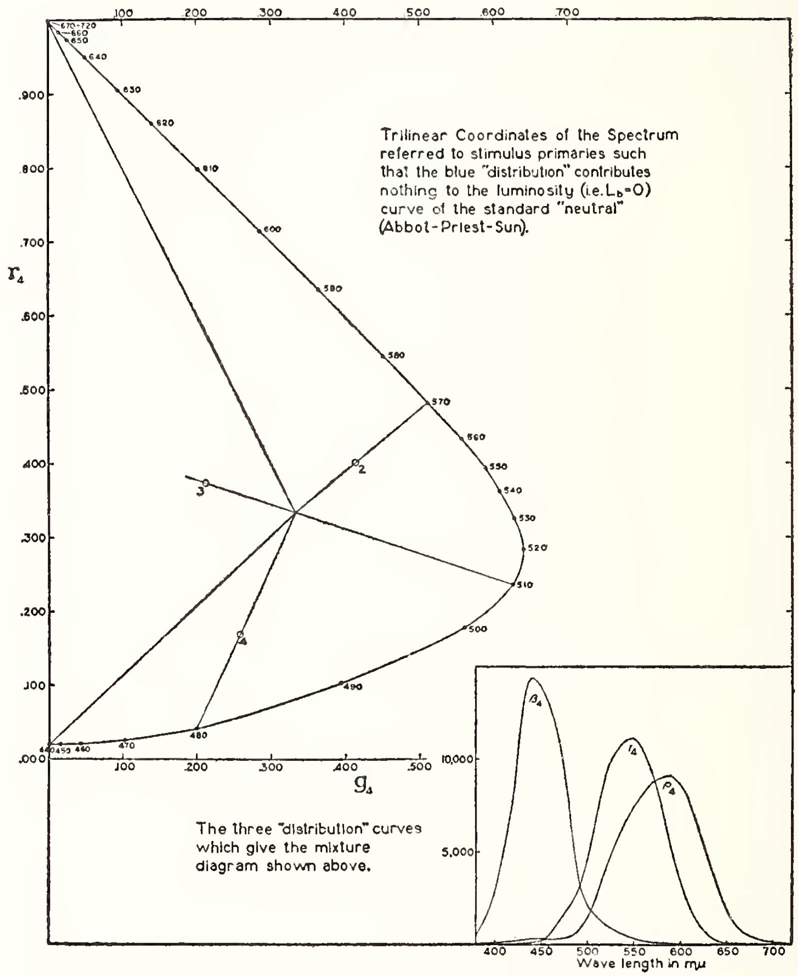

FIGURE 4.—*The distribution curves, ρ_4, γ_4, and β_4, and the mixture diagram showing the spectrum locus given by ρ_4, γ_4, and β_4*

Note that the points representing the colors of filters 2, 3, and 4 (see Table 7, *r* and *g*) fall on the straight lines connecting the "neutral" point with 570, 510, and 480 mμ, respectively.

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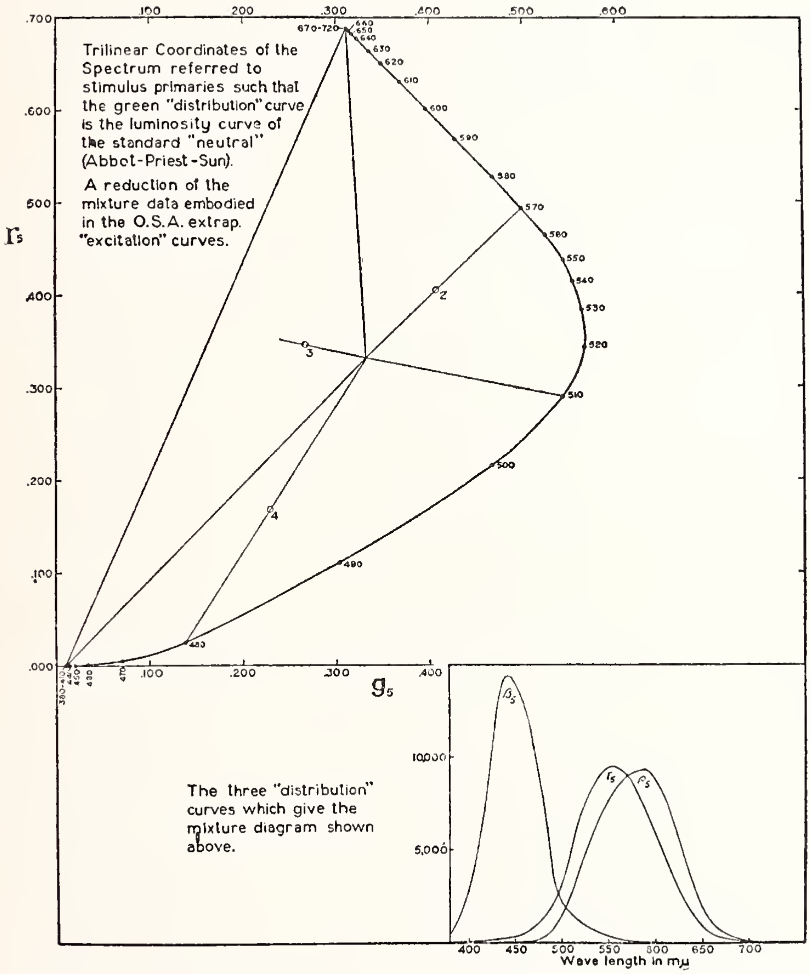

FIGURE 5.—*The distribution curves, ρ_5, γ_5, and β_5, and the mixture diagram showing the spectrum locus given by ρ_5, γ_5, and β_5*

Note that the points representing the colors of filters 2, 3, and 4 (see Table 7, *r* and *g*) fall on the straight lines connecting the "neutral" point with 570, 510, and 480 mμ, respectively.

clarity. The fact that all these apparently different mixture diagrams (figs. 2 to 5), represent the same body of mixture data may be approximately demonstrated by the fact that the points representing the colors of filters Nos. 2, 3, and 4 fall in each case on the straight line connecting the "neutral" point with the point representing the color produced by light of wave length 570, 510, and 480 mμ, respectively.[60] We may also see now what the primary color processes are whose distributions throughout the spectrum are given by ρ_n, γ_n, and β_n ($n=1$ to 5). The "red" primary process for set, n, of distribution curves is that designated on the mixture diagram by $r_n=1$, $g_n=0$; the "green" primary process is that designated by $r_n=0$, $g_n=1$; and the "blue" primary process is that designated by $r_n=g_n=0$.[61]

We proceed with the more exact method for attaining the demonstration that the several mixture diagrams embody identical mixture data (computation of dominant wave length and purity of the filter). The condition that a point (r, g) on the mixture diagram lies on the same straight line with the "neutral" point ($r=1/3$, $g=1/3$) and a point (R, G) of the spectrum locus is that: $(r-1/3)/(g-1/3)=(R-1/3)/(G-1/3)$. We proceed to compute $(r-1/3)/(g-1/3)$ for each of the four filters ($m=1$ to 4) and for each of the five sets of distribution curves ($n=1$ to 5) and to compare it with $(R-1/3)/(G-1/3)$ for the proper wave length and set of distribution curves. Table 8 shows this comparison. It is seen that discrepancies exist in every case; examination shows, however, that these discrepancies may be ascribed to rejection error, since they are smaller than the change introduced by an error of one in the fourth place of the values of r and g.

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[60] It is also true for each diagram (figs. 2 to 5) that the points representing the colors of filters Nos. 1 to 4 ($m=1$ to 4) lie upon the straight lines connecting the points representing the colors produced by light of wave lengths 490 and 610 mμ, 500 and 640 mμ, 440 and 590 mμ, and 460 and 550 mμ, respectively. These straight lines were omitted for the sake of clarity.

[61] Reference to fig. 3 shows that, for $n=3$, the "blue" primary process is that which is evoked by homogeneous radiation of wave length 380 to 410 mμ, when the visual mechanism is in such a state of adaptation that Abbot-Priest sunlight evokes the "white" or "neutral" process. Similarly fig. 4 shows that, for $n=4$, the "red" primary process is that evoked by homogeneous radiation of wave length 670 to 720 mμ. The other primary processes plainly can not be evoked by homogeneous radiation of any wave length for a state of adaptation for which Abbot-Priest sunlight yields the "white" response because the points representing them do not fall on the spectrum locus. It is equally plain that they can not be evoked by a mixture of homogeneous radiation with Abbot-Priest sunlight because they lie outside the area bounded by the spectrum locus and the straight line joining its extremes. Take, for example, the "blue" primary process for $n=4$ (fig. 4); this process could be evoked by a stimulus of dominant wave length about 452 mμ (since the point representing it lies on the same straight line as the points representing the colors evoked by Abbot-Priest sunlight and homogeneous radiation of wave length about 452 mμ) and of purity greater than unity (since the point lies farther from the "neutral" point than does that of the color evoked by homogeneous radiation of wave length 452 mμ). Such a stimulus, of course, does not exist; hence, it is proper to say that this color process which we have happened to choose as a primary process in one case ($n=4$) to place at $r_4=0$, $g_4=0$ of our mixture diagram corresponds to an imaginary stimulus if the visual mechanism of the observer be in the "neutral state"; that is, in such a state of adaptation that Abbot-Priest sunlight evokes the "white" response. It is not proper to say, however, that the color process, itself, is nonexistent, or virtual, because it is possible to evoke colors for the visual mechanism in one state of adaptation which can not be evoked if the visual mechanism is in a different state. We say, therefore, that some of the color processes which we have happened to choose for primaries correspond to imaginary stimuli for the visual mechanism in the "neutral state"; whether these processes, themselves, are real or not, is not definitely known nor is it, for our purposes, important.

TABLE 8.—*Results of computation incident to evaluation of dominant wave length and colorimetric purity of the four filters (m=1 to 4) illuminated by Abbot-Priest sunlight*

Set (n) of distribution curves	Filter 1 m=1			Filter 2 m=2			Filter 3 m=3			Filter 4 m=4		
	$\frac{r-1/3}{g-1/3}$	$\frac{R-1/3}{G-1/3}$	Difference	$\frac{r-1/3}{g-1/3}$	$\frac{R-1/3}{G-1/3}$	Difference	$\frac{r-1/3}{g-1/3}$	$\frac{R-1/3}{G-1/3}$	Difference	$\frac{r-1/3}{g-1/3}$	$\frac{R-1/3}{G-1/3}$	Difference
n=1	−0.0333	−0.0114	0.0219	+0.8919	+0.8911	0.0008	−0.3262	−0.3260	0.0002	+2.3850	+2.3851	0.0001
n=2	−.1379	−.1225	.0154	+.6359	+.6356	.0003	−.4422	−.4419	.0003	+1.5419	+1.5411	.0008
n=3	−.6316	−.5787	.0529	+.6540	+.6508	.0032	−.7570	−.7565	.0005	−4.9825	−4.9914	.0089
n=4	−.0333	−.0251	.0082	+.8337	+.8338	.0001	−.3306	−.3303	.0003	+2.2107	+2.2101	.0006
n=5	+.1905	+.2153	.0248	+.9680	+.9675	.0005	−.2057	−.2060	.0003	+1.5809	+1.5807	.0002

Table 9 shows the final results of computation of dominant wave length, purity, and transmission of the four filters illuminated by Abbot-Priest sunlight. The dominant wave lengths are computed from the differences recorded in Table 8, together with the known rate of variation of dominant wave length with $(r-1/3)/(g-1/3)$ for the wave lengths in question.[62] The purity, p, is computed from:

$$p=\frac{RL_r+GL_g+BL_b}{RL_r+GL_g+BL_b+(g-G)/(1-3g)} \qquad (15)^{63}$$

TABLE 9.—*Computed values of the dominant wave length, colorimetric purity, and transmission of filters (m=1 to 4) illuminated by Abbot-Priest sunlight*

Set (n) of distribution curves	Filter 1 m=1			Filter 2 m=2			Filter 3 m=3			Filter 4 m=4		
	Λ in mμ	p	T_s	Λ in mμ	p	T_s	Λ in mμ	p	T_s	Λ in mμ	p	T_s
n=1	528.33	−0.0153	0.03652	570.01	+0.5521	0.04608	509.99	−0.6461	0.07215	480.00	+0.3235	0.04976
n=2	528.91	−.0156	.03651	570.01	+.5524	.04609	509.98	−.6463	.07213	480.01	+.3235	.04976
n=3	525.18	−.0152	.03652	570.02	+.5511	.04609	509.96	−.6463	.07214	480.01	+.3235	.04976
n=4	529.39	−.0152	.03651	570.00	+.5524	.04608	509.99	−.6459	.07214	480.00	+.3236	.04976
n=5	528.45	−.0158	.03651	570.01	+.5518	.04608	510.01	−.6461	.07214	480.00	+.3236	.04976

The transmission (T_s) for Abbot-Priest sunlight is computed as:

$$T_s=\frac{L_{r_n}\int_0^\infty \rho_n T_m d\lambda+L_{g_n}\int_0^\infty \gamma_n T_m d\lambda+L_{b_n}\int_0^\infty \beta_n T_m d\lambda}{L_{r_n}\int_0^\infty \rho_n d\lambda+L_{g_n}\int_0^\infty \gamma_n d\lambda+L_{b_n}\int_0^\infty \beta_n d\lambda} \qquad (16)$$

where L_{r_n}, L_{g_n}, and L_{b_n} are given in Table 6, method 2; one-tenth of $\int_0^\infty \rho_n T_m d\lambda$, $\int_0^\infty \gamma_n T_m d\lambda$, and $\int_0^\infty \beta_n T_m d\lambda$ are given in Table 7; $\int_0^\infty \rho_n d\lambda$, $\int_0^\infty \gamma_n d\lambda$, and $\int_0^\infty \beta_n d\lambda$ are taken as 1,000,000,[64] and their

[62] This method is applicable because the dominant wave lengths fall so closely on the even 10 mμ where the distribution curve ordinates (and, hence $R-1/3$ and $G-1/3$) are definitely specified. The transmissions of the four filters were chosen with this end in view. See footnote 1 to tables, p. 534.

[63] D. B. Judd, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., **13**, p. 135; 1926. Relation (15) is form "Judd G." In 3 cases out of the 20, the "Judd R" form (Judd, loc. cit.) was used because of its greater convenience.

[64] See footnote 38, p. 525.

weighted sum is 1,000,000, since the luminosity coefficients have been adjusted so as to sum to unity.

It is seen that (with one exception, the dominant wave length of filter No. 1, $m=1$ [65]) to a high degree of accuracy the dominant wave length, colorimetric purity, and transmission for Abbot-Priest sunlight of the four filters is independent of the choice (among the five, $n=1$ to 5) of a set of distribution curves. It has been demonstrated, therefore, that the four new sets ($n=2$ to 5) of distribution curves embody the same mixture data as the O. S. A. "excitation" curves ($n=1$).

VI. CHOICE OF A SET OF DISTRIBUTION CURVES

As has been pointed out in the introduction, convenience in discussion of psychophysiological theory may justify the temporary adoption of any one of a wide range of sets of distribution curves. A set very similar to ρ_2, γ_2, and β_2 was chosen by König and Dieterici because it suggested a simple way of explaining the facts of congenital dichromasy. ρ_2, γ_2, and β_2 have also served to suggest (to the present writer) a simple way of explaining the "hue change" which is observed when "white" light is added to certain portions of the spectrum.[66] ρ_3, γ_3, and β_3 were used by the present writer [67] to represent the experimentally known facts concerning sensibility to purity change.[68] Neither of these sets of distribution curves could be used advantageously in the routine computation of dominant wave length, colorimetric purity, and transmission because both are less convenient for computation than the O. S. A. "excitation" curves (ρ_0, γ_0, β_0) from which they were derived.[69] A set of distribution curves proposed by Luther [70] refers to such a choice of stimulus primaries that $L_b=0$. The computational advantage of this choice of primaries is quite evident. These same advantages may be obtained by the use of ρ_4, γ_4, and β_4, which also have a further advantage over those of Luther because they involve more ordinates having the value zero.

It seems, however, that there exists a choice of stimulus primaries which expresses the mixture data of the O. S. A. "excitation" curves in a way that affords still greater convenience in the routine computation of colorimetric purity, and transmission than any choice of primary color processes heretofore proposed. The set of distribution curves resulting from this choice of primary color processes is ρ_5, γ_5, and β_5 defined in relation (13). The computational superi-

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[65] The explanation for the deviation of the dominant wave length of this filter from 530 mμ by more than 4 mμ lies, of course, in the close approach to zero of the purity of that filter (−0.015). It has already been shown (Table 8) that these discrepancies are ascribable to errors of rejection in computing.

[66] W. de W. Abney, On the Change of Hue of Spectrum Colours on Dilution with White Light, Proc. Roy. Soc., A., **83**, 1909. Also Abney, Researches in Colour Vision, London, Longmans-Green, pp. 255–266; 1913. D. B. Judd, The Empiric Relation between Dominant Wave Length and Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., **14**, p. 475; 1927.

[67] D. B. Judd, Sensibility to Color Change Determined from the Visual Response Functions: Extension to Complete and Partial Dichromasy, J. Opt. Soc. Am. and Rev. Sci. Inst., **16**, p. 115; 1928.

[68] Reference to fig. 3 will show one of the characteristic properties of this set of distribution curves. It is seen that the spectrum locus (at about 572 mμ) approaches much closer to the "neutral" point ($r=1/3$, $g=1/3$) than it does in figs. 2, 4, or 5. This is the dominant wave length of the color stimulus which is, for normal observers, most confusible with the "neutral" stimulus. (See I. G. Priest and F. G. Brickwedde, The Minimum Perceptible Colorimetric Purity as a Function of Dominant Wave Length with Sunlight as Neutral Standard, J. Opt. Soc. Am. and Rev. Sci. Inst., **13**, p. 306; 1926.)

[69] They are less convenient chiefly because they involve fewer ordinates having the value zero.

[70] R. Luther, Aus dem Gebiet der Farbreizmetrik, Zs. f. Techn. Physik, **8**, p. 544; 1927.

ority of this set of distribution curves can best be established by enumerating the conveniences which it involves:

1. No negative ordinates.[71]
2. Twenty out of one hundred and five ordinates specified are zero.[72]
3. $L_r = L_b = 0$.[73]

Although the advantage of having $L_{r_5} = L_{b_5} = 0$ (which makes $L_{g_5} = 1$, and makes γ_5 identical with the luminosity function) [74] will scarcely be denied, still it is of interest to point out exactly how simple the formulas to be used in routine computation of transmission and colorimetric purity become:

$$T_s = \int_0^\infty \gamma_5 T_m d\lambda \Big/ \int_0^\infty \gamma_5 d\lambda \qquad (17)\ [75]$$

$$p = \frac{G(1-3g)}{g(1-3G)} = \frac{3-1/g}{3-1/G} \qquad (18)\ [76]$$

We do not propose that ρ_5, γ_5, and β_5 be adopted forthwith because the confusion of changing from the O. S. A. "excitation" curves in their extrapolated form (ρ_0, γ_0, β_0) would probably more than outweigh the subsequent gain in computational convenience. However, when it does become desirable to adopt a new set of mixture data which is more representative of average, normal vision than that embodied in the O. S. A. "excitation" curves, it is evident that the primaries embodied in ρ_5, γ_5, and β_5 could be taken with advantage for routine computation rather than those embodied in the O. S. A. "excitation" curves.

I wish again to express my indebtedness to Dr. L. B. Tuckerman for his proof of the theorem of Section II. Doctor Tuckerman's suggestions have resulted in a considerably improved presentation of the introduction.

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71 In this respect ρ_5, γ_5, and β_5 are no better than a number of other sets of distribution curves. (See footnote 13, p. 518.)

72 In this respect, these distribution curves are second only to the O. S. A. "excitation" curves themselves (ρ_0, γ_0, β_0), which have 30 out of 105 ordinates specified equal to zero. They are, therefore, somewhat less convenient than the O. S. A. "excitation" curves in their extrapolated form for the computation of dominant wave length.

73 There exists an infinity of sets of distribution curves which have this property; this set is the most advantageous with respect to the two points already mentioned. Another of this infinity of sets was approximately evaluated by Schrödinger (Ueber das Verhältnis der Vierfarben- zur Dreifarbentheorie, Sitz. Akad. Wiss., Wien, Abt. IIa, **134**, pp. 471–490; 1925) in order to demonstrate that the mixture data may suggest a four-color as well as a three-color theory of vision. In fact, a four-color theory is mathematically equivalent to a three-color theory in which one of the three primary color processes is taken as "white." We say the set of distribution curves was "approximately evaluated" because the mixture data embodied in Schrödinger's curves are those of the König Grundempfindungskurven which are not quite the same as those of the O. S. A. "excitation" curves which we have used. Furthermore the luminosity curve which Schrödinger derives from the König Grundempfindungskurven is distinctly too high for short wave lengths. The discrepancies are, in fact, the same ones which led Exner (see footnote 19, p. 519) to make unwarranted arbitrary changes in the König Grundempfindungskurven. Though it was not essential to his purpose, Schrödinger could have derived an acceptable luminosity curve from the König Grundempfindungskurven (see Sec. IV especially footnote 50, p. 533) by using a negative luminosity coefficient (his term, γ, see his relation (1)) of the proper magnitude for the blue curve.

74 It has already been pointed out that if any luminosity coefficients be chosen whose sum is unity, we may find many sets of distribution curves to which they apply (see relation (6) et seq). This set (ρ_5, γ_5, β_5) is one for which $L_r=0$, $L_g=1$ and $L_b=0$, the choice being made purely for the purpose of simplifying computation.

75 Compare with the general formula (16).

76 Compare with the general formula (15). Of course, this form is not always applicable to computation because when G approaches 1/3, it is difficult to avoid serious errors of rejection. In these cases the equally simple form, $p=G(r-g)/g(R-G)$, may be used.

VII. APPENDIX—A GRAPHICAL INTERPRETATION

It has been pointed out [77] that the existence of an infinite number of sets of distribution curves satisfying relation (2) corresponds to the possibility of choosing an infinite number of sets of primary color processes in terms of which to express a given body of mixture data. If all color processes are represented on a mixture diagram (such as figs. 2 to 5), each process by a point, this possibility corresponds to that of choosing any three points in the plane as reference points (with trivial exceptions already noted). It may be easily shown that if the triangle formed by connecting these three points by straight lines incloses the spectrum locus, then the distribution curves of the primary color processes represented by the three reference points will have positive ordinates throughout. Reference to Figures 2 to 5 shows that the four sets of distribution curves we have taken as examples are cases of this kind.[78]

Similarly the freedom of choice for the luminosity coefficients (see relation (6), et seq.) may be interpreted in terms of the selection of reference points on the mixture diagram which represent the primary color processes. We proceed to develop the basis for such an interpretation. The equation

$$rL_r + gL_g + bL_b = 0 \tag{19}$$

defines a straight line on the mixture diagram which represents color processes of zero luminosity; hence, we shall call it after Schrödinger [79] "Alychne" (lightless). Naturally this line does not intersect the spectrum locus; the stimuli corresponding to the color processes represented by points on the Alychne are all imaginary. If one of these points be chosen to represent one of the primary color processes of a three-color system, the luminosity coefficient attached to that color process will be zero.[80] By analogy it is natural to suppose that all points which fall on the same side of the Alychne as the spectrum locus are associated with positive luminosity while those which fall on the opposite side refer to color processes of negative luminosity; hence, we might guess that if one of the first group of points were chosen to represent a primary color process the luminosity coefficient would be positive and if one of the second group of points were chosen the luminosity coefficient would be negative. We proceed to substantiate this guess.

Theorem.[81]—If a luminosity coefficient is positive (negative), the point representing the corresponding primary color process lies on the same (opposite) side of the Alychne as the spectrum locus.

Proof.—As already noted, the Alychne does not intersect the spectrum locus; hence, if we can show that any point lying within the spectrum locus and the straight line joining its extremes lies on the same (opposite) side of the Alychne as the point representing the

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[77] See footnotes 9 and 10, p. 517.

[78] The O. S. A. "excitation" curves, the Elementarempfindungskurven of König, the König-Ives curves, and the "sensation" curves of Abney also satisfy this condition. See footnotes 13 and 18, pp. 518 and 519.

[79] E. Schrödinger, Ueber das Verhältnis der Vierfarben- zur Dreifarbentheorie, Sitz. Akad. Wiss., Wien, Abt. IIa, **134**, p. 476; 1925.

[80] Although this was pointed out by Schrödinger without proof, the accuracy of his statement may readily be checked. Assume (for example) $L_r = 0$. We must see whether the point corresponding to the red primary process falls on the Alychne; that is, we must see whether the coordinates ($r=1$, $g=0$, $b=0$) satisfy relation (19). Substitution shows this to be the case.

[81] For simplicity this theorem is stated in a form applicable only to trilinear diagrams in which the spectrum locus does not pass through infinity. This requires that $\rho+\gamma+\beta$ be not less than zero for any wave length.

primary color process having a positive (negative) luminosity coefficient, the theorem will be proven. We elect for the sake of convenience to show that the "white" point ($r=1/3$, $g=1/3$, $b=1/3$) is so situated.[82]

Assume (for example) L_b greater than (less than) zero; we must show that the point ($r=0$, $g=0$, $b=1$) lies on the same (opposite) side of the Alychne as the "white" point. Now the Alychne may be defined in terms of r, g, L_r, and L_g alone since:

$$r+g+b=1 \qquad \text{(See relation (14))}$$

and

$$L_r+L_g+L_b=1 \qquad \text{(See Table 6)}^{83}$$

thus

$$(2L_r+L_g-1)r+(L_r+2L_g-1)g+1-L_r-L_g=0 \qquad (20)$$

Furthermore, we know that any point (r_1, g_1) is on the same (opposite) side of the line $Ar+Bg+C=0$ as the origin provided:

$$\frac{Ar_1+Bg_1+C}{\pm\sqrt{A^2+B^2}}$$

is positive (negative), where the denominator of the fraction is given the same algebraic sign as C.[84]

Now, from (20), $C=1-L_r-L_g=L_b$, and since, by hypothesis, L_b is greater than (is less than) zero, the theorem is proved (both cases) if for $r_1=g_1=1/3$, Ar_1+Bg_1+C is greater than zero. But, from (20);

$$A=2L_r+L_g-1$$
$$B=L_r+2L_g-1$$
$$C=1-L_r-L_g$$

and substitution shows that when $r_1=g_1=1/3$, $Ar_1+Bg_1+C=1/3$ which is greater than zero.

Therefore, for sets of distribution curves like those of the present paper, it has been proven that the spectrum locus is on the same side of the Alychne as the point representing the primary color process if the corresponding luminosity coefficient is positive, and on the opposite side, if the luminosity coefficient is negative.

In Figure 6 are shown the same trilinear coordinates of the spectrum (R_2, G_2; see columns 6 and 7 of Table 2) that are shown in Figure 2; but the scale is so chosen that the Alychne and the points representing the primary color processes of some of the other sets of distribution curves dealt with may also be shown. From (20) and from the values of the luminosity coefficients found for ρ_2, γ_2, and β_2 (see Table 6), the equation of the Alychne is:

$$r_2=0.066-2.446\ g_2$$

The points representing the primary color processes whose distributions according to wave length are evaluated by ρ_2, γ_2, and β_2 are, of

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[82] See footnote 29, p. 522. Since we make use of the "white" point at ($r=1/3$, $g=1/3$, $b=1/3$) this proof is good for only those sets of distribution curves (like those of the present paper) whose "white" point falls at that place on the mixture diagram. Similar proof for sets of distribution curves not satisfying this restric tion can readily be formulated.

[83] The condition that the luminosity coefficients sum to unity is imposed for the sake of convenience throughout the present paper. It makes the present proof easier to state, but similar proof can readily be given for sets of distribution curves whose luminosity coefficients sum to any positive value.

[84] Ashton, Plane and Solid Analytic Geometry, New York, Scribners, p. 54; 1902.

course ($r=1, g=0, b=0$), ($r=0, g=1, b=0$), and ($r=0, g=0, b=1$), respectively. These points have been labeled R'_2, G'_2, and B'_2, respectively. The analogous points for the O. S. A. "excitation" curves (see Table 1) have been labeled R'_1, G'_1, and B'_1. R'_1 and B'_1 are located at the extremes of the spectrum locus while G'_1 falls at the intersection of the tangents to the spectrum locus at R'_1 and B'_1. The points representing the primary color processes whose distributions throughout the spectrum are given by ρ_3, γ_3, and β_3

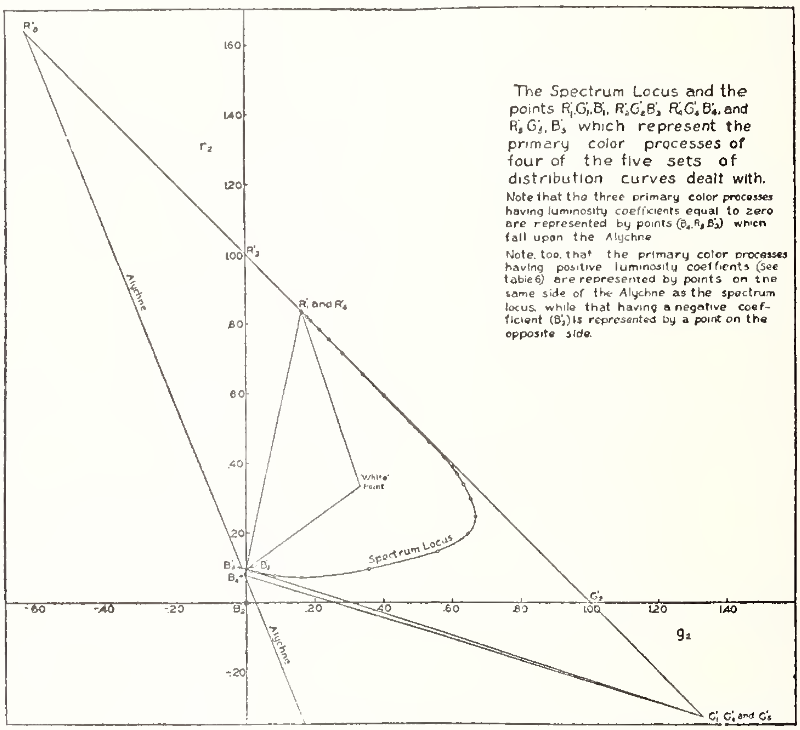

FIGURE 6.—*The mixture diagram and spectrum locus of Figure 2 to such a scale that the Alychne (lightless line) may be shown together with the points representing some of the primary color processes dealt with*

The relation between the algebraic sign of the luminosity coefficient and the position of the point representing the corresponding primary color process with reference to the spectrum locus and the Alychne is exemplified.

are not shown because two of these points (those representing the red and green primary processes) are located too far from the origin to be shown with any convenient scale for r_2 and g_2. The points representing the primary color processes of ρ_4, γ_4, and β_4 (see Table 4) are labeled R'_4, G'_4, and B'_4. R'_4 like R'_1 is located at the long-wave-length end of the spectrum locus. G'_4 does not coincide exactly with G'_1, but it is so nearly coincident that the departure can not be conveniently shown on Figure 6. B'_4 is located at the intersection of the Alychne with the straight line joining the extremes of the spectrum locus. R'_5, G'_5, and B'_5, like R'_1, G'_1, and B'_1 all fall upon the tangents to the extremes of the spectrum locus, the difference being that while R'_1 and B'_1 are the points of tangency

R'_5 and B'_5 are the intersections of the tangents with the Alychne. It is to be noted, of course, that, in accord with the theorem just proven, the points (R'_1, G'_1, B'_1; R'_2, G'_2; R'_4, G'_4; and G'_5) representing primary color processes to which positive luminosity coefficients are attached (see Table 6) fall on the same side of the Alychne as the spectrum locus; also the point (B'_2) representing the only primary color process to which a negative luminosity coefficient is attached is the only point which falls on the side of the Alychne opposite to the spectrum locus.

If, then, we would avoid negative ordinates in the distribution curves of the three primary color processes we must choose processes such that the resulting triangle completely incloses the spectrum locus (as, for example, any of the five sets of distribution curves dealt with in the present paper). If we would make as many ordinates as possible of the distribution curves zero, we should choose processes such that the sides of the resulting triangle are coincident with the spectrum locus for as great a wave-length range as possible (for example, the O. S. A. "excitation" curves, ρ_1, γ_1, and β_1, best, see Table 1; and ρ_5, γ_5, and β_5, next best, see Table 5). If we would avoid negative luminosity coefficients, we must avoid, in our selection of primary color processes, all processes represented by points which fall on the side of the Alychne opposite to the spectrum locus (as has been done, for example, for all choices of primary process shown on Figure 6 except B'_2).[85] If we would make two of the luminosity coefficients equal to zero, we must choose two of the primary color processes such that the points which represent them fall on the Alychne (for example, Schrödinger's Vierfarbentheorie curves see footnote 73, p. 544; and ρ_5, γ_5, and β_5). If two of the luminosity coefficients are zero, the distribution curve of the remaining primary color process coincides with the luminosity curve.[86] ρ_5, γ_5, and β_5 are the distribution curves of a choice of primary color processes which to a considerable degree combine all the properties which have just been enumerated. It is by virtue of these properties that ρ_5, γ_5, and β_5 are advantageous from the standpoint of routine computation.

WASHINGTON, August, 1929.

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[85] We may note that the König-Grundempfindungskurven refer to a "blue" primary process whose point is separated from the spectrum locus by the Alychne. In so far, then, as it is justifiable from the standpoint of psychophysiological interpretation to take the König primary color processes as identical with some aspect of the retinal or post-retinal activity which actually occurs, it is justifiable to conclude that the activity of the "blue" component of the visual mechanism involves an inhibition of the brightness aspect of the response. In other words, we may obtain from König's work a suggestion of the "specific darkening power" of blue light which was advocated by König's theoretical enemies (the Hering school). Ives believed (see footnote 18, p. 519) that König's blue primary process had a sufficiently arbitrary basis to justify another choice of primary that is, a choice which does not involve a negative luminosity coefficient. The König-Ives curves embody a blue primary which falls on the same side of the Alychne as the spectrum locus.

[86] The fact that a certain green process for the sake of computational convenience has been endowed with a distribution curve throughout the spectrum which is identical with the luminosity curve of the spectrum has, of course, no importance from the standpoint of psychophysiological theory whatever. The endowment could be granted any color process represented by a point on the mixture diagram not on the Alychne. If of this infinity of color processes there is one process which seems to have from the psychophysical standpoint a real claim to a distribution curve identical with the luminosity curve, that is the "white" process; and in Schrödinger's Vierfarbentheorie curves (see footnote 73, p. 544) the "white" process is so endowed. These curves embody properties which permit them to be given important theoretical interpretations; but it is not surprising that they should be distinctly inferior to ρ_5, γ_5, and β_5 from the standpoint of convenience in routine computation.

A GENERAL FORMULA FOR THE COMPUTATION OF COLORIMETRIC PURITY

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The term "colorimetric purity" is here used in a generic sense. The ratio of luminance (here called "brightness") of the monochromatic component to that of the sample, previously and subsequently called colorimetric purity, is here assigned the name "spectral brightness purity". The ratio of the distance in any chromaticity diagram between the "white" point and the sample point, to the distance between the white point and the collinear point on the spectrum locus is here assigned the name "spectral excitation purity". This has almost always subsequently been shortened, at least to "excitation purity". Commonly, when determined on the CIE 1931 chromaticity diagram, it is further shortened to "purity". Colorimetric purity is now rarely determined. The most recent attempts to define the quantity here called "saturation" have been called "metric chroma" by the CIE (1976). Metric chroma is, in each version, proportional to excitation purity in a newly defined chromaticity diagram. The factors of proportionality (not explicitly defined or mentioned by the CIE) are functions of wavelength and luminous reflectance. The last sentence in this paper is as true in 1978 as when it was written 47 years ago.

A GENERAL FORMULA FOR THE COMPUTATION OF COLORIMETRIC PURITY

By Deane B. Judd

ABSTRACT

The formula for purity is called general because if the trilinear coordinates of any color referred to any given set of primary color processes and any basic stimulus be given, the colorimetric purity of its stimulus may be computed with respect to any heterogeneous stimulus as the fixed stimulus of homo-heterogeneous analysis. The derivation of the formula is given and five special cases of historical and practical interest are worked out in order to demonstrate that previous formulas for the computation of colorimetric purity and an allied ratio are special cases of this one. The connection is also given between colorimetric purity and the allied ratio variously known as saturation, saturation fraction, and Sättigung.

CONTENTS

I. INTRODUCTION

The definition of colorimetric purity in American literature and the methods of computing it have been fairly uniform, perhaps due to the comparative recency of its introduction in this country. Abroad, however, a number of allied concepts are variously known by this name, and this concept given, in addition, various other names. Discussion of these concepts, and indeed attempts to provide nomenclature adequate for discussion, have been handicapped by lack of more than fragmentary knowledge of the properties of the concepts, their interrelations and their dependence on choice of coordinate system. It is the purpose of the present paper to supply this lack by deriving a general formula which embraces all these allied concepts in a general system of coordinates.

II. THE PROBLEM

The tristimulus mode of specifying colors has long been recognized as the fundamental one. According to this mode a color is specified by giving the amounts of three stimuli whose mixture is requisite to produce it. The particular three stimuli in terms of which other stimuli are expressed according to the colors they evoke are called primary stimuli and the colors evoked by them are called primary colors. The choice of primary stimuli is virtually unlimited; for example, many groups of three spectrum stimuli may be chosen, or stimuli produced by means of source-and-filter combinations may be substituted for any or all of the spectrum stimuli. The change from one set of primaries to another is merely equivalent to a change in coordinate system. The formula for colorimetric purity to be derived holds for any choice of primaries; that is, the transformation from one coordinate system to another is implied in the formula itself.

Let ρ, γ, β be the O. S. A. "excitation" curves. These functions of wave length specify the spectrum colors in terms of a certain choice of primaries according to an hypothetical observer whose characteristics are known to approximate satisfactorily those of average, normal vision.[1] The O. S. A. "excitation" curves are distinguished by their simplicity from the infinite number of other sets of curves which may represent the characteristics of the same hypothetical observer; that is, none of the ordinates of these distribution curves is less than zero, and many are zero. Any one of these infinite number of other sets of curves, say χ, ψ, ζ, may be computed from ρ, γ, β as follows:

$$\left.\begin{aligned}\chi &\equiv K_1\rho + K_2\gamma + K_3\beta\\ \psi &\equiv K_4\rho + K_5\gamma + K_6\beta\\ \zeta &\equiv K_7\rho + K_8\gamma + K_9\beta\end{aligned}\right\} \qquad (1)$$

where K_1 to K_9 are arbitrary constants such that:

$$\begin{vmatrix} K_1 K_2 K_3\\ K_4 K_5 K_6\\ K_7 K_8 K_9\end{vmatrix} \neq 0 \qquad (2)$$

Define:

$$\left.\begin{aligned}\bar{\chi} &\equiv \int_0^\infty \chi ET d\lambda\\ \bar{\psi} &\equiv \int_0^\infty \psi ET d\lambda\\ \bar{\zeta} &\equiv \int_0^\infty \zeta ET d\lambda\end{aligned}\right\} \qquad (3)$$

where E is the spectral distribution of energy of the source used and T is the spectral transmission (or spectral reflectance in case of a reflecting surface) of the sample which the source illuminates. The product, ET, as a function of wave length is the spectral distribution of energy, therefore, of the stimulus whose colorimetric purity is to be computed[2] by recourse to the mixture data of ρ, γ, β. The numbers, $\bar{\chi}$, $\bar{\psi}$, $\bar{\zeta}$, may be used to specify the color evoked by the stimulus specified by ET because all stimuli for which $\bar{\chi}$, $\bar{\psi}$, $\bar{\zeta}$ are identical produce identical colors within the visual mechanism specified by ρ, γ, β.[3]

Define:

$$\left.\begin{aligned}X &\equiv \chi/(\chi+\psi+\zeta)\\ Y &\equiv \psi/(\chi+\psi+\zeta)\\ Z &\equiv \zeta/(\chi+\psi+\zeta)\end{aligned}\right\} \qquad (4)$$

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[1] For the sake of being specific we take the O. S. A. "excitation" curves in the form extrapolated by Priest and Gibson (J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 10, p. 230, 1925); but we might nearly as well have taken the O. S. A. "excitation" curves in their original form (J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 6, p. 548, 1922) or some other set of curves which yield a satisfactory approximation to average, normal vision.

[2] If the stimulus whose colorimetric purity is to be computed were radiant energy direct from a light source, a single symbol would naturally be used for ET. Since most stimuli whose purities are to be computed are reflected or transmitted energies, it is convenient to use the product, ET.

[3] D. B. Judd, Reduction of Data on Mixture of Color Stimuli, B. S. Jour. Research, vol. 4, pp. 515–548; 1930

X, Y, Z are called the trilinear coordinates of the spectrum; they refer to some set of three primary processes and some basic stimulus [4] which may be inferred from the values of the constants, K_1 to K_9, of relations (1) and (2).

Define:

$$\left.\begin{aligned} x &= \bar{\chi}/(\bar{\chi}+\bar{\psi}+\bar{\zeta}) \\ y &= \bar{\psi}/(\bar{\chi}+\bar{\psi}+\bar{\zeta}) \\ z &= \bar{\zeta}/(\bar{\chi}+\bar{\psi}+\bar{\zeta}) \end{aligned}\right\} \quad (5)$$

The numbers, x, y, z, are called the trilinear coordinates of the color evoked by the stimulus of spectral energy distribution, ET; these trilinear coordinates are referred to the same primary color processes and basic stimulus as X, Y, Z of relation (4).

When the energy of a stimulus is confined to a range of wave lengths, $\lambda-\Delta\lambda$ to $\lambda+\Delta\lambda$, so restricted that reduction of $\Delta\lambda$ makes no difference in the chromaticity of the evoked color, this stimulus is called homogeneous radiant energy of wave length, λ. Homogeneous radiant energy, E_λ, may be colorimetrically specified from relation (3) by the definite integrals:

$$\int_{\lambda-\Delta\lambda}^{\lambda+\Delta\lambda} \chi E_\lambda d\lambda, \int_{\lambda-\Delta\lambda}^{\lambda+\Delta\lambda} \psi E_\lambda d\lambda, \int_{\lambda-\Delta\lambda}^{\lambda+\Delta\lambda} \zeta E_\lambda d\lambda,$$

which for brevity are written hereafter as: $\bar{\chi}_h$, $\bar{\psi}_h$, $\bar{\zeta}_h$. Note that these three values are proportional to: χ, ψ, ζ, for the wave length, λ.

Heterogeneous radiant energy of spectral distribution, E_w, is specified colorimetrically from relation (3) by the definite integrals:

$$\int_0^\infty \chi E_w d\lambda, \int_0^\infty \psi E_w d\lambda, \int_0^\infty \zeta E_w d\lambda,$$

which are abbreviated hereafter as: $\bar{\chi}_w$, $\bar{\psi}_w$, $\bar{\zeta}_w$. The subscript, w, is chosen because it suggests that the heterogeneous radiant energy is, as is usual, such as to evoke a "white" or "neutral" color.

The mixture of homogeneous radiant energy, E_λ, with heterogeneous radiant energy, E_w, is specified colorimetrically by the three sums of these definite integrals:

$$\bar{\chi}_h+\bar{\chi}_w, \bar{\psi}_h+\bar{\psi}_w, \bar{\zeta}_h+\bar{\zeta}_w,$$

that is, for the mixture:

$$\left.\begin{aligned} \bar{\chi} &= \bar{\chi}_h+\bar{\chi}_w \\ \bar{\psi} &= \bar{\psi}_h+\bar{\psi}_w \\ \bar{\zeta} &= \bar{\zeta}_h+\bar{\zeta}_w \end{aligned}\right\} \quad (6)$$

[4] The basic stimulus as defined by Priest is any stimulus whose color is represented at the center of the Maxwell triangle; that is, any stimulus whose color has the trilinear coordinates (see relation (5)), $x=y=z=1/3$. From relation (3) we see that the basic stimulus is any stimulus whose spectral energy distribution E_b, satisfies the condition: $\int_0^\infty \chi E_b d\lambda = \int_0^\infty \psi E_b d\lambda = \int_0^\infty \zeta E_b d\lambda$. To specify a basic stimulus is a convenient way to specify the relative scale magnitudes of χ, ψ, ζ; these magnitudes could be just as rigorously specified by giving the spectral energy distribution, E_x, for which the numbers: $\int_0^\infty \chi E_x d\lambda$, $\int_0^\infty \psi E_x d\lambda$, $\int_0^\infty \zeta E_x d\lambda$, have any given relative values.

Hence, from relation (5), the trilinear coordinates of the color evoked by this mixture are:

$$\left.\begin{aligned} x&=\frac{\bar{\chi}_h+\bar{\chi}_w}{\bar{\chi}_h+\bar{\psi}_h+\bar{\zeta}_h+\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w}\\ y&=\frac{\bar{\psi}_h+\bar{\psi}_w}{\bar{\chi}_h+\bar{\psi}_h+\bar{\zeta}_h+\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w}\\ z&=\frac{\bar{\zeta}_h+\bar{\zeta}_w}{\bar{\chi}_h+\bar{\psi}_h+\bar{\zeta}_h+\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w}\end{aligned}\right\} \quad (7)$$

The trilinear coordinates, x_w, y_w, z_w, of the color evoked by the heterogeneous component of the mixture are:

$$\left.\begin{aligned} x_w&=\bar{\chi}_w/(\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w)\\ y_w&=\bar{\psi}_w/(\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w)\\ z_w&=\bar{\zeta}_w/(\bar{\chi}_w+\bar{\psi}_w+\bar{\zeta}_w)\end{aligned}\right\} \quad (8)$$

Define:

$$L_h \equiv \bar{\rho}_h L_r + \bar{\gamma}_h L_g + \bar{\beta}_h L_b \quad (9)$$

where L_r, L_g, L_b are any constants not all zero. $\bar{\rho}_h$, $\bar{\gamma}_h$, $\bar{\beta}_h$ represent definite integrals analogous to those represented by $\bar{\chi}_h$, $\bar{\psi}_h$, $\bar{\zeta}_h$. We use the symbol, L_h (luminosity), because when $L_r/(L_r+L_g+L_b)=0.45$, $L_g/(L_r+L_g+L_b)=0.54$, and $L_b/(L_r+L_g+L_b)=0.01$, or values not greatly differing from these, L_h varies according to wave length very closely as the luminosity of the spectrum determined experimentally.[5] We refrain from calling L_h the luminosity because L_h deserves such a name only for the special case just mentioned. Although this special case commands the major interest because of the possibility of determining it directly by experiment[6] some attention has been given another case, $L_r=L_g=L_b$, as will appear later.

It has been shown that:[7]

$$L_h=\bar{\chi}_h L_x+\bar{\psi}_h L_y+\bar{\zeta}_h L_z \quad (9a)$$

if:

$$\left.\begin{aligned} L_x&=\frac{1}{\Delta}\left[L_r\begin{vmatrix}K_5 & K_8\\ K_6 & K_9\end{vmatrix}+L_g\begin{vmatrix}K_7 & K_4\\ K_9 & K_6\end{vmatrix}+L_b\begin{vmatrix}K_4 & K_7\\ K_5 & K_8\end{vmatrix}\right]\\ L_y&=\frac{1}{\Delta}\left[L_r\begin{vmatrix}K_8 & K_2\\ K_9 & K_3\end{vmatrix}+L_g\begin{vmatrix}K_1 & K_7\\ K_3 & K_9\end{vmatrix}+L_b\begin{vmatrix}K_7 & K_1\\ K_8 & K_2\end{vmatrix}\right]\\ L_z&=\frac{1}{\Delta}\left[L_r\begin{vmatrix}K_2 & K_5\\ K_3 & K_6\end{vmatrix}+L_g\begin{vmatrix}K_4 & K_1\\ K_6 & K_3\end{vmatrix}+L_b\begin{vmatrix}K_1 & K_4\\ K_2 & K_5\end{vmatrix}\right]\end{aligned}\right\} \quad (9b)$$

where $\Delta \equiv \begin{vmatrix}K_1 & K_2 & K_3\\ K_4 & K_5 & K_6\\ K_7 & K_8 & K_9\end{vmatrix}$, the constants, K_1 to K_9, being those of relations (1) and (2).

[5] D. B. Judd, Chromatic Visibility Coefficients by the Method of Least Squares, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 10, pp. 635-651; 1925.

[6] See footnote 9, p. 831, and footnote 11, p. 832.

[7] See footnote 3, p. 828.

Define:

$$L_m \equiv \bar{\chi} L_x + \bar{\psi} L_y + \bar{\zeta} L_z \qquad (10)$$

The subscript, m, may be read, "of the mixture," and the analogous subscript, h, of relations (9) and (9a) may be read, "of the homogeneous component."

The ratio, L_h/L_m, is of particular interest in colorimetry because in the special case[8] where it is a ratio of luminosities it may be determined experimentally[9] by the methods of heterochromatic photometry, and it has been given a special name, the colorimetric purity. We may now state the problem.

Given: (1) Values of the trilinear coordinates, x, y, z, of a color produced by the mixture of a homogeneous and a heterogeneous component, (2) the trilinear coordinates, x_w, y_w, z_w, of the color evoked by the heterogeneous component alone, (3) the trilinear coordinates, X, Y, Z, of the color evoked by the homogeneous component alone, (4) the constants, K_1 to K_9, specifying the primaries and basic stimulus of the distribution curves as related to the O. S. A. "excitation" curves, and (5) the constants, L_r, L_g, L_b; required: the ratio, L_h/L_m.

III. THE SOLUTION

The derivation of the general formula has been foreshadowed by the derivation of a special case of it.[10]

From relation (9a) and definition (10) we have:

$$\frac{L_h}{L_m} = \frac{\bar{\chi}_h L_x + \bar{\psi}_h L_y + \bar{\zeta}_h L_z}{\bar{\chi} L_x + \bar{\psi} L_y + \bar{\zeta} L_z}$$

which, in virtue of relation (6), becomes:

$$\frac{L_h}{L_m} = \frac{\bar{\chi}_h L_x + \bar{\psi}_h L_y + \bar{\zeta}_h L_z}{\bar{\chi}_h L_x + \bar{\psi}_h L_y + \bar{\zeta}_h L_z + \bar{\chi}_w L_x + \bar{\psi}_w L_y + \bar{\zeta}_w L_z}$$

After the factor, $\bar{\chi}_h + \bar{\psi}_h + \bar{\zeta}_h$, abbreviated as σ, has been divided out, this ratio becomes, from definition (4):

$$\frac{L_h}{L_m} = \frac{XL_x + YL_y + ZL_z}{XL_x + YL_y + ZL_z + \frac{\bar{\chi}_w}{\sigma} L_x + \frac{\bar{\psi}_w}{\sigma} L_y + \frac{\bar{\zeta}_w}{\sigma} L_z} \qquad (11)$$

From definition (4) and relations (7) and (8) it follows at once that:

$$\left.\begin{aligned} \bar{\chi}_w/\sigma &= x_w(x-X)/(x_w-x) \\ \bar{\psi}_w/\sigma &= y_w(y-Y)/(y_w-y) \\ \bar{\zeta}_w/\sigma &= z_w(z-Z)/(z_w-z) \end{aligned}\right\} \qquad (12)$$

[8] See relation (9) et. seq.

[9] I. G. Priest, Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity, and Brightness, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 8, pp. 173–200, 1924.

[10] D. B. Judd, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 13, pp. 136–138; 1926.

By substituting these three values in relation (11) a solution of the problem is obtained; but this solution may be expressed more simply by taking account of the collinearity condition:[11]

$$\frac{x-X}{x_w-x}=\frac{y-Y}{y_w-y}=\frac{z-Z}{z_w-z} \tag{13}$$

which also follows from (4), (7), and (8); it states that the points (X,Y,Z), (x,y,z), and (x_w, y_w, z_w), lie on the same straight line. In virtue of relation (13), relation (11) may be written:

$$\frac{L_h}{L_m}=\frac{XL_x+YL_y+ZL_z}{XL_x+YL_y+ZL_z+F(x_wL_x+y_wL_y+z_wL_z)} \tag{14}$$

where $F=(x-X)/(x_w-x)=(y-Y)/(y_w-y)=(z-Z)/(z_w-z)$. Relation (14) is a solution of the problem since X, Y, Z, x, y, z, x_w, y_w, z_w are known and L_x, L_y, L_z may be found from relation (9b), since the constants, K_1 to K_9, and L_r, L_g, L_b are given.

IV. FIVE SPECIAL CASES

1. For routine computation of colorimetric purity at the National Bureau of Standards it has been customary to choose constants K_1 to K_9, all zero except K_1, K_5, and K_9 (which retains all the simplicity of the O. S. A. "excitation" curves), and so to adjust K_1, K_5, and K_9 that the heterogeneous stimulus is represented at the center of the Maxwell triangle; that is, the heterogeneous stimulus is chosen as the basic stimulus.[12] Hence the constants are taken so that

$$\int_0^\infty \chi E_w d\lambda=\int_0^\infty \psi E_w d\lambda=\int_0^\infty \zeta E_w d\lambda$$

which, from this special case of relation (1), may be written

$$K_1\int_0^\infty \rho E_w d\lambda=K_5\int_0^\infty \gamma E_w d\lambda=K_9\int_0^\infty \beta E_w d\lambda$$

L_x, L_y, and L_z are chosen so that L_h of relation (9a) may be taken to represent the luminosity of the spectrum;[13] hence we may write colorimetric purity, p, instead of L_h/L_m in relation (14) which becomes; since $x_w=y_w=z_w=1/3$:

$$p=\frac{XL_x+YL_y+ZL_z}{XL_x+YL_y+ZL_z+F(L_x+L_y+L_z)/3}$$

Since, as is permissible (see relation (9) et seq.), it is customary to set $L_x+L_y+L_z=1$, a further simplification results:

$$p=\frac{XL_x+YL_y+ZL_z}{XL_x+YL_y+ZL_z+F/3} \tag{15}$$

[11] I. G. Priest, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 9, p. 520; 1924. D. B. Judd, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 13, p. 138; 1926.

[12] See footnote 4, p. 829.

[13] They may be found from relation (9b) if L_r, L_g, and L_b are appropriately chosen (see relation (9) et seq.), or they may be found de novo.

where $F/3=(x-X)/(1-3x)=(y-Y)/(1-3y)=(z-Z)/(1-3z)$. Relation (15) may be recognized as a restatement of the Judd R, Judd G, and Judd B forms of purity formula[14] with obvious changes in symbology. It is also identical with other forms of purity formula previously derived (Ives, Priest, Tuckerman). A formula developed by Froelich [15] is identical with relation (15) providing L_h of relation (9a) be taken equal to her L_λ, and providing her ratio, a, determined graphically by measuring two distances on the mixture diagram, be given its analytic equivalent, $1/(1+F)$. Two formulas developed by Guild [16] also involve ratios of lengths on the mixture diagram, the ratio in the first formula given (Guild's, p. 161) being equal to $1/(1+F)$ and that in the second formula (p. 162) being equal to $1/F$. When these substitutions are made, the first formula can readily be shown to be identical with the Ives form, which has already been proven identical with relation (15); the second formula becomes relation (15) directly.

It may be remarked that the various forms of purity formula with this convention of making the heterogeneous stimulus the basic stimulus of the system arise chiefly because (1) the coordinates of any pair of the three collinear points may be taken to form the luminosity sums, as, for example, in relation (15), which takes the coordinates of the spectrum point and the "white" point; (2) the ratio of lengths may be expressed as the ratio of the segments or as the ratio of either segment to the total, as, for example, in relation (15), which uses F, the ratio of the segments; and (3) this ratio of lengths may be evaluated from any parallel projection of the line, as, for example, in relation (15), which evaluates the ratio, F, from projections onto the axes of a mixture diagram in rectangular coordinates or onto the bisectors of the angles of the equilateral triangle, or as, for another example, in Ives's formulas, which evaluate a ratio of lengths from projections onto the sides of the equilateral triangle, or as in the formulas of Froelich and Guild, which evaluate the ratios directly from the lengths themselves. The various forms of purity formula are all useful, the particular form of purity formula to be chosen depending on the nature of the problem.

2. Another choice of primary processes which tends more toward convenience for those who think in terms of the opponent-colors theory of vision (Hering) is that which takes "neutral" as one of the three primary processes. The chromatic aspect of the response is then described solely by the other two components which have negative as well as positive values, such as a red process, whose negative is green, and a yellow process, whose negative is blue. For such a choice, if further the heterogeneous stimulus be such as to evoke the "neutral" color, and if the "neutral" color be chosen as the third primary whose distribution in the spectrum is given by ζ, the constants, K_1 to K_9, of relation (1) must satisfy the condition:

$$\int_0^\infty \chi E_w d\lambda = \int_0^\infty \psi E_w d\lambda = 0$$

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[14] See footnote 10, p. 831.

[15] Clara L. Froelich, Algebraic Methods for the Calculation of Color Mixture Transformation Diagrams, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 9, p. 39; 1924.

[16] J. Guild, The Geometrical Solution of Colour Mixture Problems, Trans. Opt. Soc., vol. 26, pp. 161-162; 1924-25.

which from relation (1) is equivalent to:

$$K_1\int_0^\infty \rho E_w d\lambda + K_2\int_0^\infty \gamma E_w d\lambda + K_3\int_0^\infty \beta E_w d\lambda = 0$$

$$K_4\int_0^\infty \rho E_w d\lambda + K_5\int_0^\infty \gamma E_w d\lambda + K_6\int_0^\infty \beta E_w d\lambda = 0$$

From relation (8) and the definitions of $\overline{\chi_w}$, $\overline{\psi_w}$, $\overline{\zeta_w}$, it follows that $x_w = y_w = 0$ and $z_w = 1$. Relation (14) becomes:

$$\frac{L_h}{L_m} = \frac{XL_x + YL_y + ZL_z}{XL_x + YL_y + ZL_z + FL_z} \qquad (16)$$

where

$$F = (x - X)/(-x) = (y - Y)/(-y) = (z - Z)/(1 - z)$$

A further simplification is possible if K_7, K_8, and K_9 be chosen equal, respectively, to L_r, L_g, and L_b. From relation (9b) it follows that $L_x = L_y = 0$. And if still further we arbitrarily set $L_z = 1$, as is permissible, we may write:

$$L_h/L_m = Z/(Z + F)$$

If, further, L_r, L_g, and L_b, be chosen so that the ratio may be considered a luminosity ratio as in case (1), this relation may be written as a very simple formula for colorimetric purity:

$$p = (1 - 1/z)/(1 - 1/Z) \qquad (17)$$

Since Schrödinger's "vierfarbentheorie" curves [17] satisfy all the conditions under which relation (17) was derived, this relation may be taken for colorimetric purity if the trilinear coordinates X, Y, Z and x, y, z are computed from those curves. The simplicity of the formula raises the question whether Schrödinger's choice of primary color processes, which were chosen because of their interest for psycho-physiological speculation, might not be a favorable choice purely for the purpose of simplifying the computation of colorimetric purity. This question is discussed presently.

3. Some attention has been given to the ratio, L_h/L_m, in the special case for which $L_x = L_y = L_z$,[18] perhaps chiefly because of the relative ease with which it may be computed. Since, from relation (4), $X + Y + Z = 1$, relation (14) for this special case may be written:

$$L_h/L_m = 1/(1 + F)$$

or

$$L_h/L_m = (x_w - x)/(x_w - X) = (y_w - y)/(y_w - Y) = (z_w - z)/(z_w - Z) \qquad (18)$$

Inspection of relation (18) shows that L_h/L_m for this case is the distance on the mixture diagram from the point (x, y, z) to the "white" point (x_w, y_w, z_w), divided by the distance from the "spectrum" point (X, Y, Z) to the "white" point; hence the ratio may in this case be determined quite easily from the mixture diagram directly.

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[17] E. Schrödinger, Über das Verhältnis der Vierfarben- zur Dreifarbentheorie, Sitz. Akad. Wiss., Wien, IIa, vol. 134, p. 479; 1925.

[18] W. Dziobek and M. Pirani, Normung von Signalgläsern, Proc. International Congress on Illumination, Saranac Inn, pp. 818–833; 1928. I. Runge, Die Einheitsmengen im Maxwell-Helmholtzschen Farbdreieck und die Bestimmung der Farbsättigung, Zs. f. Instrumentenk., vol. 49, pp. 600–603; 1929. S. Rösch, Darstellung der Farbenlehre für die Zwecke des Mineralogen, Fortschritte d. Mineralogie, Kristallographie u. Petrographie, vol. 30, pp. 135–136; 1929.

It should be noted, however, that L_h/L_m in this case is not necessarily the colorimetric purity because L_x, L_y, and L_z have not been especially chosen to make L_h of relation (9a) proportional to the luminosity of the equal-energy spectrum. It should also be noted that this special case loses the characteristic which is the probable cause of its existence if a different choice of primaries or basic stimulus be made; that is, with certain trivial exceptions, the coefficients, L_x, L_y, L_z, in another system of coordinates would no longer be equal; hence this special case of the ratio L_h/L_m would be just as hard to compute as colorimetric purity or any other special case of relation (14).[19] The condition for the exceptions may be seen by inspection of relation (9b).

4. In order to facilitate the computation of colorimetric purity, Runge[20] has suggested another choice of constants, K_1 to K_9. For this choice the simple formula (18) just derived results in the special case of L_h/L_m which has been defined as the colorimetric purity. It is interesting to see how the choice of constants, K_1 to K_9, suggested by Runge insures that relation (18) results in colorimetric purity.

Take L_r, L_g, L_b such that L_h of relation (9) gives the distribution of luminosity in the spectrum. This choice permits us to substitute p for L_h/L_m. Then the constants, K_1 to K_9, are chosen, all zero except K_1, K_5, K_9, which are taken as L_r, L_g, L_b, respectively. From relation (9b) it will be discovered that, for this choice, $L_x = L_y = L_z = 1$; hence it follows that relation (18) in this special case gives colorimetric purity.

According to this plan, however, the basic stimulus of the $\rho\gamma\beta$ system is taken as the heterogeneous stimulus in the $\chi\psi\zeta$ system, necessitating that:

$$\int_0^\infty \rho E_w d\lambda = \int_0^\infty \gamma E_w d\lambda = \int_0^\infty \beta E_w d\lambda$$

which, from relation (1), is equivalent to:

$$(1/L_r)\int_0^\infty \chi E_w d\lambda = (1/L_g)\int_0^\infty \psi E_w d\lambda = (1/L_b)\int_0^\infty \zeta E_w d\lambda$$

whence, from relation (8), it is evident that:

$$x_w = L_r/(L_r + L_g + L_b)$$
$$y_w = L_g/(L_r + L_g + L_b)$$
$$z_w = L_b/(L_r + L_g + L_b)$$

Since L_b in this case is about one-fiftieth of either L_r or L_g, it is evident that the "white" point (x_w, y_w, z_w) falls relatively near one side of the Maxwell triangle.[21] This consequence of Runge's choice of coordinate system is emphasized by setting, as is usual, $L_r + L_g + L_b = 1$, which permits relation (18) for this special case to be written:

$$p = (L_r - x)/(L_r - X) = (L_g - y)/(L_g - Y) = (L_b - z)/(L_b - Z) \qquad (19)$$

[19] Another way of making the same statement is that if L_x, L_y, and L_z in another system be made equal in general, the number defined by such a choice will not be identical with L_h/L_m in this special case.

[20] See footnote 18, p. 834.

[21] See Runge's fig. 2 (footnote 18, p. 834).

5. Another coordinate system has since been proposed by Judd [22] to facilitate the computation of colorimetric purity. This system avoids the close approach of the spectrum locus to the "white" point.

Take $K_2 = K_3 = K_7 = K_8 = 0$, $K_4 = L_r$, $K_5 = L_g$, and $K_6 = L_b$, where L_r, L_g, L_b, as before, are chosen so that L_h of relation (9) gives the distribution of luminosity in the equal-energy spectrum. As in case (2), it is discovered from relation (9b) that two of the three numbers, L_x, L_y, L_z, are zero; that is, in this case, $L_x = L_z = 0$, $L_y = 1$.

K_1, K_9 and $L_r + L_g + L_b$ are chosen so as to make E_w the basic stimulus, which, from relation (1), requires that:

$$K_1 \int_0^\infty \rho E_w d\lambda = K_9 \int_0^\infty \beta E_w d\lambda = L_r \int_0^\infty \rho E_w d\lambda + L_g \int_0^\infty \gamma E_w d\lambda + L_b \int_0^\infty \beta E_w d\lambda$$

Since, by this choice, $x_w = y_w = z_w = 1/3$, relation (14) may be written for $y \neq 1/3$:

$$p = (3 - 1/y)/(3 - 1/Y) \qquad (20)$$

An equally simple form may be written for $y = 1/3$.[22]

Relation (17) is just as simple for routine computation as relation (20) provided the trilinear coordinates (x, y, z) of the mixture are at hand. The distribution curves leading up to relation (17), however, are less simple for routine computation of the coordinates, x, y, z, than those leading up to relation (20) because they embody more ordinates different from zero and because they embody negative as well as positive ordinates; hence relation (20), considering the entire computation of colorimetric purity from spectral distribution of energy, is more advantageous than relation (17).

Relation (20) is not quite so convenient for routine computation as relation (19), since it involves the finding of two reciprocals in addition to the two subtractions and one division indicated in relation (19). However, the ease with which X, Y, Z may be found from x, y, z within the coordinate system of relation (20) is generally considerably greater than for the coordinate system of relation (19) because the coordinate system of relation (20) avoids the close approach to the spectrum locus of the "white" point (x_w, y_w, z_w).[23]

If any coordinate system is to be chosen for the specific purpose of facilitating the computation of colorimetric purity, it would seem that the adoption of that leading to relation (20) would be the most advantageous in routine computation.

V. PURITY AND SATURATION

Discussion of the terms "purity" and "saturation" is facilitated by the introduction of a ratio, L_a/L_m, which is in all respects save one the same as L_h/L_m. Since this ratio is of considerably less importance than L_h/L_m, there is little interest in showing how it may be computed from one set of primaries after having been defined in terms of another, as was done with L_h/L_m. It suffices to write a relation analogous to relation (14) which will serve to define L_a/L_m,

[22] See footnote 3, p. 828.
[23] Compare Runge's fig. 2 (see footnote 18, p. 834) with Judd's fig. 5 (see footnote 3, p. 828).

where the subscript, a, is to be read "absolute" (or "basic," according to Priest) and to be understood to refer to a color represented on the edge of the Maxwell triangle:

$$\frac{L_a}{L_m} \equiv \frac{X_aL_x + Y_aL_y + Z_aL_z}{X_aL_x + Y_aL_y + Z_aL_z + F_a(x_wL_x + y_wL_y + z_wL_z)} \tag{21}$$

where $F_a \equiv (x - X_a)/(x_w - x)$ or $(y - Y_a)/(y_w - y)$ or $(z - Z_a)/(z_w - z)$. Relation (21) differs from relation (14) only by the substitution of X_a, Y_a, Z_a, respectively, for X, Y, Z. Since L_h/L_m expresses by its approach to unity the degree of approach of the color (x, y, z) to the color (X, Y, Z), the ratio, L_a/L_m, similarly gives the degree of approach of the color (x, y, z) to the color (X_a, Y_a, Z_a). The color (X, Y, Z), used as a reference in the one case, may be evoked by some portion of the spectrum; in the second case the color (X_a, Y_a, Z_a), used as a reference, is represented on an edge of the Maxwell triangle; we know, therefore, that one of X_a, Y_a, Z_a is zero.

We have seen that two special cases of the ratio, L_h/L_m, are of interest, the one in which $L_x = L_y = L_z$ and the one in which L_x, L_y, L_z are chosen, so that L_h of relation (9a) may be taken as the luminosity of the spectrum. These two special cases also command all the attention given to the ratio L_a/L_m.

The first case yields a formula analogous to relation (18), which becomes for $x_w = y_w = z_w = 1/3$:

$$L_a/L_m = (1 - 3x)/(1 - 3X_a) = (1 - 3y)/(1 - 3Y_a) = (1 - 3z)/(1 - 3Z_a)$$

which may be written:

$$L_a/L_m = 1 - 3c \tag{22}$$

where c is x or y or z, according as X_a or Y_a or Z_a is zero. This ratio was defined by Exner [24] as "Sättigung," for the coordinate system based on the Grund-Empfindungs-Curven of König and Dieterici.[25] Exner's definition has been followed by a number of European workers. It has been recently restated as a "not wholly unnatural measure of saturation" by Schrödinger.[26]

The second case (for $x_w = y_w = z_w = 1/3$) has been given by Martin[27] as one of three variables for specifying color stimuli; the ratio, L_a/L_m, is defined by a numerical example referring to Abney's "sensation" curves and is called the "saturation fraction." This second case was also mentioned later by Priest [28] incidental to the derivation of a formula for colorimetric purity; he named the ratio, L_a/L_m, for this case the "absolute purity." Still later this ratio was defined by Haschek,[29] who called it "Sättigung."

Probably two reasons may account for choosing as a reference the edges of the Maxwell triangle as in the ratio, L_a/L_m. The first is merely that the resulting formula is simpler because one of X_a, Y_a, Z_a is always zero; this is a logical extension of the motive which prob-

[24] F. Exner, Sitz. Akad. Wiss., Wien, I, vol. 119, p. 233; 1910.
[25] A. König and C. Dieterici, Die Grundempfindungen in normalen und anomalen Farbensystemen und ihre Intensitätsvertheilung im Spectrum, Zs. f. Psych. u. Physiol. d. Sinnesorgane, 4, pp. 241–347; 1893; or see A. König, Ges. Abh., Leipzig, Barth, pp. 214–321; 1903.
[26] E. Schrödinger, Müller-Pouillets Lehrbuch d. Physik, 2d ed., vol. 2, pp. 482–484; 1926.
[27] L. C. Martin, Colour and Methods of Colour Reproduction, London, Blackie, p. 133; 1923.
[28] I. G. Priest, The Computation of Colorimetric Purity, J. Opt. Soc. Am. and Rev. Sci. Inst. vol. 9, p. 509; 1924.
[29] E. Haschek, Quantitative Beziehungen in der Farbenlehre, Sitz. Akad. Wiss., Wien, IIa, vol. 136, pp. 461–468; 1927.

ably led up to relation (18) and applies with particular force to the first case which results in the simple relation (22). The second reason is the suspicion, or the delusion, or, perhaps, the hope that the edges of the Maxwell triangle of the particular coordinate system chosen possess a fundamental theoretical significance by virtue of which they deserve the dignity of being chosen as reference lines. If the primary color processes of the coordinate system were known to be processes functionally distinct somewhere within the visual mechanism, then it could be argued that every color process represented by a point outside the Maxwell triangle would be nonexistent, or imaginary. The edges of the triangle would then represent the colors which differ from an achromatic color of the same brilliance by the maximum amount possible, and hence they would form a rather natural reference locus. However, the argument by which the König Grund-Empfindungs-Curven are taken to give the distributions of the true primary processes is by no means complete. The primaries chosen by König satisfy two criteria: (1) They are a particular "red," "green," and "blue" whose hues are close to three of the four psychologically unitary hues, that is, the hues are nearly Urrot, Urgrün, and Urblau; and (2) the distribution curves for these three primary processes may be used not only to describe the mixture relations of normal vision when used all three together, but also, by disregarding the one or the other of the "red" and "green" curves, the two most common types of partial color-blindness may be described.

Now it is not necessary to assume that one distribution curve must be disregarded to account for partial color-blindness; this is merely the simplest assumption.[30] Furthermore, the König Grund-Empfindungs-Curven are not the only set of curves which satisfy the two conditions given; they are merely the simplest set which satisfy them; that is, the Maxwell triangle which they yield approaches as closely as possible to the spectrum locus as is consistent with the criteria just mentioned. Other sets of distribution curves satisfying the two conditions satisfied in the simplest possible way by the König curves would yield Maxwell triangles whose sides departed to a greater or less extent from the spectrum locus. Now if it should be desired to choose the particular triangle of these possible triangles whose sides represent the colors of the maximum saturation possible, it would be found that so far little evidence has been recorded to guide the choice. It is rather certain, however, that any triangle which touches the spectrum locus is not the right choice, because it is an accepted fact that the colors of the spectrum viewed by a chromatically rested eye are not as saturated as the colors of the spectrum viewed by an eye adapted to the complementary color. But in all the triangles used by those proposing the ratio, L_a/L_m, as of fundamental importance, the spectrum locus does touch at least one side of the triangle. Hence our knowledge of the true physiological primaries, though much too incomplete to determine them, is sufficient to make certain that the coordinate systems thus far used for the ratio, L_a/L_m, fail to give that ratio a single vestige of theoretical importance.

It may be concluded, then, that the interest attached to the ratio, L_a/L_m, like that attached to the ratio, L_h/L_m, is a purely practical

[30] For the general form of dichromasy, see H. v. Helmholtz, Physiol. Optik, 2d ed., Leipsig, Voss. pp. 458–462; 1896.

one; either ratio for any choice of L_x, L_y, L_z will serve as one of three variables by which color stimuli may be classified according to the colors they evoke in the visual mechanism defined by ρ, γ, β. The special case of L_h/L_m when L_x, L_y, L_z are so chosen that it is a ratio of luminosities is, of course, of considerably greater interest than any other special case of either L_h/L_m or L_a/L_m, because this ratio alone can be evaluated by direct measurement.[31] To this special case, therefore, it has seemed expedient to give the name "colorimetric purity," p; and since this ratio serves all the purposes served by any other special case of either L_h/L_m or L_a/L_m, there would seem to be little justification for paying any further attention to them. When the computation of colorimetric purity was not well understood it was natural that allied entities whose derivation seemed simpler should spring up. It should be evident, however, that methods of computation of colorimetric purity are now available which compare so favorably in simplicity with any of those for any of the allied entities that the argument of mere ease of computation is no longer applicable.

It has become customary in some quarters, however, (Runge, Rösch) [32] to speak of purity and saturation (Reinheit and Sättigung) interchangeably and to remark that purity (or saturation) has been computed in four different ways. If we adopt the view that at least these four special cases deserve attention, it would seem conducive to clarity in discussion to adopt separate names for them. Terms proposed by Priest [33] for these four special cases, together with an indication of their derivation, appear in Table 1. It will be noted that, according to this terminology, the most important of the concepts, colorimetric purity, would be renamed "spectral brightness purity." In spite of the fact that previous writers have frequently applied the name "saturation" ("Sättigung") to one or another of these concepts, the suggested terminology does not include that term. This conforms to the recommendations of the O. S. A. committee on colorimetry,[34] which reserve the term "saturation" to apply to the response to a color stimulus rather than to the stimulus itself. The concepts thus far dealt with may be taken as stimulus terms, because they merely serve to group together stimuli which evoke identical colors regardless of the chromatic condition of the visual mechanism.

If, however, we could compute from the trilinear coordinates of a color a number, s, satisfying the following conditions:

(a) $s = 0$ for an achromatic color,

(b) $s_1 = s_2$ for two colors appearing to be equally saturated,

then it would seem reasonable to name this number the "saturation," in which case there would be at hand a quantitative means of evaluating the concept "saturation" which is now only qualitatively defined. Now both the ratios, L_a/L_m and L_h/L_m, satisfy condition (a), providing the heterogeneous stimulus whose color is represented at the point (x_w, y_w, z_w) be so chosen as to evoke an achromatic color; but neither L_a/L_m nor L_h/L_m has been shown to satisfy condition (b). In fact, the failure of one special case of one of these

[31] See footnote 9, p. 831.
[32] See footnote 18, p. 834.
[33] Letter to Dr. M. Pirani, Aug. 21, 1929.
[34] L. T. Troland, Report of Committee on Colorimetry for 1920–21, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 6, pp. 531–538; 1922.

ratios, L_a/L_m, for $L_x=L_y=L_z$ (Exner's "Sättigung," or basic excitation purity referred to the König fundamentals as primaries) to satisfy condition (b) has been proven experimentally by Seitz.[35] Other special cases which have been proposed do not differ from this case in such a way as to repair this deficiency; and, indeed, we may conclude from Seitz's work that no special case of either ratio satisfies condition (b) at all satisfactorily; so neither of these ratios deserve the name "saturation."

A number which may prove to satisfy fairly closely both conditions (a) and (b) is $\int_0^p (dE/dp)dp$, where p is the colorimetric purity [36] of the stimulus and dE/dp is the reciprocal of the least difference in purity perceptible as a function of purity when the change in purity is made at constant brightness along some specified path on the color triangle.[37] $\int_0^p (dE/dp)dp$ may be described as the number of least perceptible differences between a color whose stimulus is of purity, p, and the color of the same brilliance whose stimulus is of purity, $p=0$. Since the saturation of a color depends nearly if not quite as much on the momentary condition of the visual mechanism as upon the stimulus itself, it is evident that we may expect the number, $\int_0^p (dE/dp)dp$, to satisfy conditions (a) and (b) only when the visual mechanism is in such a state that the stimulus for which $p=0$ evokes an achromatic color. Since the saturation of the color evoked by a stimulus is dependent on the brightness as well as the purity of the stimulus, it is further evident that we may hope for $\int_0^p (dE/dp)dp$ to satisfy condition (b) only for the brightness to which dE/dp applies. Whether with these restrictions this number really may be made to satisfy conditions (a) and (b) and deserves the name "saturation" is a matter which is plainly of considerable complexity; it has not yet been decided. An extensive quantity of experimental data has yet to be amassed before this definition of "saturation" may be more than tentatively accepted.

Before this definition, if acceptable, can be of convenient application, a way must be found of computing dE/dp from the trilinear coordinates of the color whose stimulus has a purity, p; but at least a start has already been made in this direction. Schrödinger [38] has derived an expression for dE/dp from theoretical ground; but this expression apparently has not yet been checked against experimental results. Judd [39] has discovered empirically an expression which agrees fairly well with such experimental results as are available, but the agreement is not sufficiently striking to lead to the belief that the expression is of more than temporary theoretical interest.

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[35] W. Seitz, Über die Definition der Sättigung einer Farbe nach Helmholtz und Exner und über das Ostwaldsche Farbensystem, Zs. f. Sinnesphysiol., II, vol. 54, pp. 146-158; 1922; or see Phys. Zs., vol. 23, pp. 297-301; 1922.

[36] We define this number in terms of p, but the definition could be equally well given by substituting for p any special case of either L_a/L_m or L_h/L_m.

[37] For instance, at constant dominant wave length as adopted in defining this number by Jones and Lowry (L. A. Jones and E. M. Lowry, Retinal Sensibility to Saturation Differences, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 13, pp. 25-34; 1926), who identified it with "saturation," or at constant hue, or along such a path that this number is a minimum, as adopted by Schrödinger in defining "Sättigung" (E. Schrödinger, Müller-Pouillets Lehrb. d. Physik, 2d ed., vol. 2, pp. 555-558; 1926). Schrödinger makes the not implausible assumption that this path coincides exactly with the path of constant hue. In general, there is quite a definite difference between the path of constant dominant wave length and the path of constant hue, but we need not here choose between them for the purpose of this definition, because there is no experimental evidence that the number defined on the one basis would differ at all importantly from that defined on the other.

[38] E. Schrödinger, Grundlinien einer Theorie der Farbenmetrik im Tagessehen, Ann. d. Physik (4), vol. 63, pp. 483-520; 1920; see also footnote 37, p. 840.

[39] D. B. Judd, Purity and Saturation, A Saturation Scale for Yellow, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 14, p. 470; 1927; Saturation of Colors Determined from the Visual Response Functions, J. Opt. Soc. Am. and Rev. Sci. Inst., vol. 16, p. 115; 1928.

It is evident, therefore, that our knowledge of vision still falls considerably short of a general quantitative evaluation of the response entity, saturation.

TABLE 1.—*Terminology suggested by Priest*

QUANTITY TO BE NAMED			
L_a/L_m for $L_x=L_y=L_z$	L_h/L_m for $L_x=L_y=L_z$	L_a/L_m for $\bar{\chi}_h L_x+\bar{\psi}_h L_y+\bar{\zeta}_h L_z$ equal to the luminosity of the spectrum	L_h/L_m for $\bar{\chi}_h L_x+\bar{\psi}_h L_y+\bar{\zeta}_h L_z$ equal to the luminosity of the spectrum
DESCRIPTION		DESCRIPTION	
Hueful fraction of excitation relative to the sides of the triangle.	Hueful fraction of excitation relative to the spectrum.	Hueful fraction of brightness relative to the sides of the triangle.	Hueful fraction of brightness relative to the spectrum.
SUGGESTED TERM		SUGGESTED TERM	
Basic excitation purity.	Spectral excitation purity.	Basic brightness purity.	Spectral brightness purity.

WASHINGTON, D. C., May, 1931.

CHROMATICITY SENSIBILITY TO STIMULUS DIFFERENCES

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Figure 1 is the great-grandfather of the CIE 1960 u,v diagram. It was constructed by plotting, on rectangular axes, r and g chromaticity coordinates (in this paper called "trilinear coordinates") derived from color-matching data (here called "excitation curves") that were based on data published by the OSA Colorimetry Committee in 1922. Plotting on rectangular axes was an innovation; since Maxwell's first paper on the subject, such coordinates had been plotted on equilateral triangular axes. Judd found that plotting the OSA r vs g coordinates on rectangular axes gave a diagram that had satisfactorily uniform chromaticity scales, as indicated by Figs. 3 to 11. In 1935, however, Judd plotted transformed values of the CIE 1931 x,y coordinates on equilateral triangular coordinates and obtained a diagram that was even more successful than Fig. 1. With minor modifications, his 1935 diagram ultimately became the CIE 1960 u,v diagram.

CHROMATICITY SENSIBILITY TO STIMULUS DIFFERENCES*

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ABSTRACT

There is described in the present paper a method, discovered empirically, for computing the approximate number of "least perceptible differences" between any two colors of the same brightness whose specifications are available. This method has been stated in simple form as an empirical relation; it is shown to be in substantial agreement with extant sensibility data of the following types: (1) the least wave-length difference perceptible in the pure spectrum as a function of wave length (Steindler, Jones); (2) the least dominant-wave-length difference perceptible at constant purity as a function of purity (Watson, Tyndall); (3) the least purity difference perceptible at constant dominant wave length as a function of purity (Donath); (4) the least purity difference perceptible near zero purity as a function of dominant wave length (Priest, Brickwedde), and (5) the least color-temperature difference perceptible as a function of color temperature (Priest). A mixture diagram is included showing colors specified by their trilinear coordinates and by the dominant wave length and purity of their stimuli. From this diagram the number of "least perceptible differences" separating any two colors of the same brightness may be read with a degree of certainty indicated by the comparisons here presented.

The empirical relation was originally expressed in terms of distribution ("sensation," "excitation") curves which suggest a three-components theory of vision (Young-Helmholtz); but it has been re-expressed in terms of curves which suggest an opponent-colors theory (Hering). Since this re-expression is nearly as convenient as the original expression, it is concluded that such success as has been demonstrated for the empirical relation can not be used as an argument for either form of theory; rather is a theory suggested which is more complex than either, such as that of G. E. Müller.

Contents

* Publication Approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

I. Introduction

1. OBJECT

In the practical comparison and specification of colors a knowledge of the magnitude of the least perceptible difference in color is of much importance. It is especially important in the setting of the limits of tolerance and in deciding whether specifications have been satisfactorily met. The distribution throughout the spectrum of the radiant energy coming from an object can be determined accurately by physical methods, but the color produced by it can be determined only by direct visual observation. The same color may be produced by light stimuli having quite different distributions of energy.

Hence it has been necessary to determine in what way the color evoked in the normal visual mechanism is related to the spectral distribution of the energy of the light stimulus, before the former can be definitely related to the latter, which is immediately measurable. This relation is expressed by a set of spectral distribution curves of some three primary color processes. Special investigations to determine the least perceptible difference in color are also required. This difference, or discriminatory chromatic step, depends upon the color, the conditions of observation, and upon the peculiarities of the observer.

Though many observations of the discriminatory step under given conditions have been made, a large part of the accumulated data have been reported without sufficient details regarding the accompanying experimental conditions; in many cases the stimulus has been described in a way that does not constitute a quantitative specification, and in others the conditions were exceptional, and there is no way in which the results obtained can be related to those obtained under more usual conditions.

In the absence of any suitable and generally applicable expression giving the discriminatory chromatic step, an attempt was made to discover a simple, empirical relation that would apply to all cases in which the brightnesses of the two color stimuli are the same. Such a relation has been found. It is the purpose of this paper to state that relation, and to show in how far it accords with available observations. The data in the literature have been supplemented by previously unpublished material some of which has been obtained specifically to test the relation and some obtained incidentally.

2. DEFINITIONS OF SYMBOLS AND TERMS

In order to avoid misunderstandings and to simplify the following discussion, it is desirable to give here the definitions of certain terms and symbols that will be used, or, at least, to indicate where complete discussions of the terms may be found.

λ,—wave length.

E_λ,—radiant energy per unit wave length as a function of wave length.

Distribution curves,—a short name for a set of three curves giving the spectral distributions of some three primary color processes; these curves express in condensed form data on equivalent color stimuli (40, p. 524).[1]

ρ, γ, β,—a certain set of distribution curves known as the O. S. A. "excitation" curves (18,74, p. 549).

r, g, b,—trilinear coordinates of any color according to the O. S. A. "excitation" curves (74, p. 581; 59, p. 506; 40, p. 536).

R, G, B,—trilinear coordinates of the colors of the spectrum according to the O. S. A. "excitation" curves.

L_r, L_g, L_b—chromatic luminosity coefficients (74, p. 551; 59, p. 506; 36; 40, p. 519).

S,—a short term for $RL_r+GL_g+BL_b$.

Mixture diagram (Maxwell triangle),—a diagram representing colors in such a way that the color corresponding to a mixture of a number of component stimuli is represented according to the center-of-gravity principle with reference to the points representing the colors of the components; for example, the diagram produced by plotting r against g in rectangular coordinates.

Basic stimulus,—any stimulus whose color is represented at the center of the Maxwell triangle; that is, any stimulus whose color is represented at $r=g=b=1/3$.

Prime stimulus,—illuminant of sample giving color (r,g,b).

Prime-basic stimulus,—a basic stimulus used as the illuminant.

c,—either r or g depending on which has changed most for a given color difference.

[1] A number appearing in parentheses sometimes followed by a page reference serves to cite publications listed in the bibliography (Section XI) unless it is accompanied by either *a* or *b*, in which case it cites an equation in this paper.

C,—either R or G depending on which has changed most for a given color difference.

ΔE,—discriminatory step between colors (r_1, g_1) and (r_2, g_2); distinguish E (Empfindung) and E_λ (energy). Various units in terms of which to express ΔE, and the relative sizes of these units, (42, 62), are:

PE_{single},—probable error of a single observation, 1.0;

GID,—greatest imperceptible difference, 1.2;

$(GID+LPD)/2$,—doubtful difference, 3.3;

LPD,—least perceptible difference, 5.4.

Sensibility,—reciprocal of the stimulus increment corresponding to unit discriminatory difference.

Λ, Dominant wave length,—the dominant wave length of a color stimulus with respect to a given heterogeneous stimulus is the wave length of homogeneous light which, when mixed with the proper proportion of the heterogeneous stimulus, produces a match for the color stimulus (58, p. 175).

p, Colorimetric purity,—the ratio of the brightness of the homogeneous component of the mixture just described to the brightness of the mixture (74, p. 538; 58, p. 175).

Achromatic color,—black, white, or gray (74, p. 535).

Chromatic color,—any color which is neither black, white, nor gray (74, p. 535).

Hue,—the attribute of chromatic colors which permits them to be classed as reddish, yellowish, greenish, or bluish (74, p. 534).

Saturation,—the attribute of colors which determines their degree of difference from a gray of the same brilliance; the saturation of an achromatic color is zero (74, p. 535).

Chromaticity,—characterization of a chromatic color determined by its hue and saturation together; an achromatic color has no chromaticity because it does not possess a hue (74, p. 535).

3. CONDITIONS

The conditions assumed in making comparisons with observations are these:

(a) Brightnesses of the color stimuli that are compared have been equalized.

(b) Abbot-Priest sunlight is the heterogeneous stimulus (56, 60, 40, p. 525).

(c) The O. S. A. "excitation" curves (those extrapolated by Priest and Gibson (18) from the ones compiled by the Colorimetry Committee of the Optical Society of America (74, p. 549)) are applicable to the observer.

(d) The chromatic luminosity coefficients, $L_r = 0.370$, $L_g = 0.617$, $L_b = 0.013$ (74, p. 551) apply to the observer; that is, $\rho L_r + \gamma L_g + \beta L_b$ gives for the observer the luminosity of the equal energy spectrum.

II. HISTORICAL SUMMARY

Attempts to derive chromaticity sensibility from the distribution curves expressing data on equivalent color stimuli have been numerous; some of these attempts bear a considerable resemblance to the empirical relation discussed in the present paper. In 1886 König (46, p. 107) suggested that it ought to be easy to construct a mixture diagram such that equal distances in any direction would represent equal discriminatory steps. Later Helmholtz (25) sought to extend Fechner's law for brilliance sensibility to chromaticity sensibility, but his procedure has been criticized by Schrödinger (66) who has given a more elaborate and better supported extension of Fechner's law for the same purpose. Steindler (72) suggested a method of obtaining roughly the chromaticity sensibility for spectrum colors; this method has been modified and extended to all colors by Kohlrausch (48) and has been used with qualitative success by Laurens and Hamilton (49). Guild (23) has constructed a mixture diagram which approximates to some degree the ideal diagram mentioned by König; but Schrödinger's theoretical analysis (67, p. 554) indicates that such an ideal mixture and discrimination diagram does not exist, and, indeed, Houstoun (29) has corroborated this conclusion though his experimental basis is not extensive.

These methods of computing chromaticity sensibility have been derived empirically, with the possible exception of the Helmholtz-Schrödinger method. As far as is known, none have been tested against experimental results except for spectrum colors. The present method resembles the Kohlrausch method, makes use of the mixture diagram suggested by Guild, and of variables similar to those suggested by Houstoun. However, it is simpler in form and application than any of the previous methods and furthermore has been checked against much of the available data on chromaticity sensibility.

III. THE EMPIRICAL RELATION

The empirical relation is this:

$$K\Delta E = |c_1 - c_2| . \tag{1a}$$

In (1a), ΔE is the chromaticity difference between the colors differing in r or g coordinate by $|c_1-c_2|$; $|c_1-c_2|$ being the greater of the two quantities $|r_1-r_2|$ and $|g_1-g_2|$, and r and g denoting, respectively, the value of the red and of the green trilinear coordinates (r, g, b) (these differences are to be taken as essentially positive); K is a positive constant, independent of the chromaticity, the chromaticity difference, and, over the wide range yielding normal daylight vision, of the brightness.

As our tests of the empirical relation are necessarily limited almost completely to small discriminatory differences, it is convenient to express the relation in differential notation. In doing this, we shall adhere to the usual mathematical conventions, removing the essentially positive character of the right member of (1a) and understanding that a negative value of ΔE means a change in the opposite direction to that corresponding to a positive one. Thus we get:

$$KdE = dc. \tag{2a}$$

In so far as the empirical relation is an exact formulation of the phenomena, the ordinary laws of differential calculus regarding a change of variable apply, and we may write:

$$K\frac{dE}{dq} = \frac{dc}{dq}, \text{ or } \frac{dq}{dE} = \frac{K}{(dc/dq)} \tag{3a}$$

where q is any stimulus parameter (dominant wave length, colorimetric purity, angular size of chromatic sector on a rotating disk, color temperature, Lovibond numeral ($N''R$) in the series $35Y+N''R$).

It will be noticed that the empirical relation including relations (2a) and (3a) assumes that dE is an exact derivative. There is no obvious reason for believing that such is the case; rather, the reverse seems more probable. If the empirical relation is the actual one, then relation (1a) will be satisfied whatever the size of the interval ΔE and whatever the terminal chromaticities.

But if dE is not an exact derivative, then relation (3a) can not be an exact expression of the facts; K must be variable, containing the functional factor required to make the left member of (2a) an exact derivative.

IV. Subsidiary Relations

In comparing the demands of the empirical relation with experimental data the following relations will be used as may be required.

When expressed in terms of the colorimetric purity, p, and of the coordinates, R, G, B, of the colors of the spectrum, c takes (37, p. 150) the form:

$$c = \frac{S(1-p)/p + C}{3S(1-p)/p + 1} \tag{4a}$$

where C is either R or G depending on whether c is r or g, and $S \equiv RL_r + GL_g + BL_b$.

It is worth noting that there is a range of dominant wave lengths (approximately 471 to 506 mμ) in which c in the empirical relation for constant purity changes from r to g which necessitates that C of (4a) be R for some purities within this range of dominant wave lengths and G for other purities. At the transition purity for each dominant wave length within this range, $(dr)_p = (dg)_p$; hence, at the transition purity, C may be taken as either R or G. To find the transition purities, differentiate (4a) with respect to λ first with $C=R$, then with $C=G$, equate the two results, and solve for p. Thus (4b) is found:

$$p = \frac{S(dR/d\lambda - dG/d\lambda) - (R-G)(dS/d\lambda)}{(S-\frac{1}{3})(dR/d\lambda - dG/d\lambda) - (R-G)(dS/d\lambda)} \tag{4b}$$

Values of p by (4b) are found to fall between 0 and 1 only for dominant wave lengths between 471 and 506 mμ. Within this dominant-wave-length range, for values of p less than those by (4b), $C=R$; and for values of p greater than those by (4b), $C=G$.

When the observed light is that obtained by reflecting the prime-basic stimulus from a rotating disk divided into two angular sectors of areas P and (1-P), one sector (P) such as to produce, when at rest, the color (r_c, g_c, b_c), and the other such as to produce the color (1/3, 1/3, 1/3) of a non-selective sample of the same reflectance under the same conditions, then the color (r, g, b) evoked by the rotating disk is given by:

$$c = \frac{s_c(1-P)/P + c_c}{3s_c(1-P)/P + 1} \tag{5a}$$

where $s_c \equiv r_cL_r + g_cL_g + b_cL_b$, and c_c stands for r_c or g_c according as c stands for r or g. This is exactly similar and analogous to (4a).

The proof of (5a) follows at once from applying the Talbot-Plateau law (31, 55)—which states that the luminosity of a mixture by alternation is the time-weighted mean of the luminosities of the components—wave length by wave length to the light reflected from the sectors (35).

From Tuckerman's formula for purity (59, p. 520; 37, p. 135), $p_c = (3-1/s_c)/(3-1/S)$, and $p=(3-1/s)/(3-1/S)$, p_c being the purity of the chromatic sector, p being that of the light from the spinning disk, and $s \equiv rL_r + gL_g + bL_b$. Whence, considering the similarity of (4a) and (5a), we can write at once $P=(3-1/s)/(3-1/s_c)$, or the purity of the light from the rotating disk is given by (5b):

$$p = p_c P. \tag{5b}$$

V. Mixture Diagram

Since the empirical relation states that, when the brightnesses of two color stimuli are the same, the difference in the evoked colors produced by them is proportional to the difference in the value of a single one of the trilinear coordinates (r or g) depending on which difference is the greater, it is obvious that the application of that relation to colors differing by large amounts will be facilitated by plotting in rectangular coordinates, the values of r being measured along one axis, and the values of g along the other. (Since $r+g+b=1$, b can be found at once when r and g are known.)

Such a diagram based on the fundamental values used in this paper —the extrapolated O. S. A. "excitation" curves, Abbot-Priest sunlight as basic stimulus, the chromatic luminosity coefficients, $L_r=0.370$, $L_g=0.617$, $L_b=0.013$, and Ives' definition (37, p. 148) of purity for non-spectral stimuli—is shown in Fig. 1; the values of the coordinates corresponding to various dominant wave lengths and purities are given in Table 1; they differ from those published previously (74, p. 585) because of extrapolation (18), change of basic stimulus to Abbot-Priest sunlight, change to Ives' definition of purity, freedom from errors of rejection in computation (59, 37), and a minor change in luminosity coefficients.

In Fig. 1, the lines radiating from the point (1/3, 1/3), corresponding to the basic stimulus, are the loci of unchanged dominant wave length, the appropriate wave length, in mμ, is marked on each; the curved lines are the loci of unchanged purity, each appropriately marked. The field to the left of the lines drawn from (1/3, 1/3) to the ends of the significant portion of the r-axis corresponds to the colors complementary to the indicated dominant wave lengths. In this region the colorimetric purities are negative in accord with Ives' definition.

In this diagram, each point corresponds to a definite chromaticity except the point (1/3, 1/3) which corresponds to achromatic colors. If any two points be regarded as marking the extremities of the hypotenuse of a right-triangle with legs parallel to the axes, then, in so far as the empirical relation is applicable, the discriminatory difference between

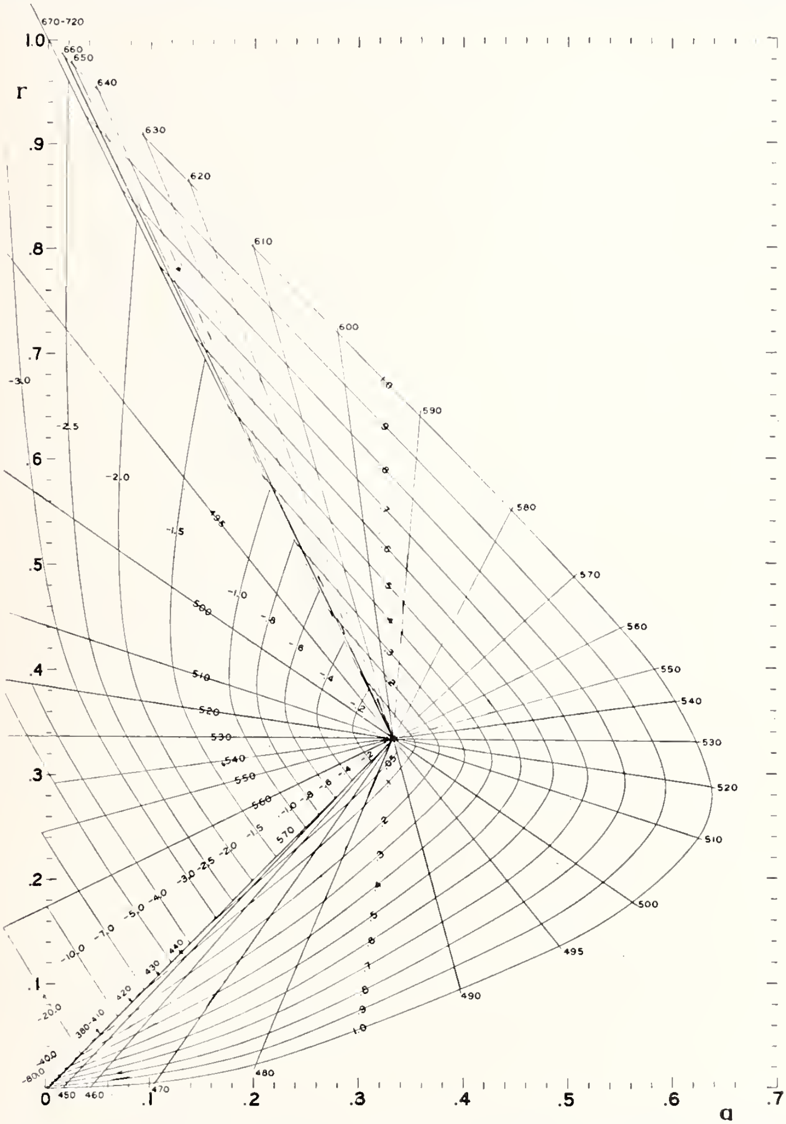

FIG. 1. *The (r, g)-diagram whose use is suggested by the empirical relation (1a). The point representing the chromaticity of any color specified by the dominant wave length and colorimetric purity of its stimulus may be easily found from this diagram because of the loci of constant dominant wave length and constant purity which appear on it. If the empirical relation (1a) were an exact expression of the facts, the chromaticity difference between any two colors represented by points on this diagram,* (r_1, g_1), (r_2, g_2), *would be accurately proportional to the absolute value of* $(r_1 - r_2)$, *or to the absolute value of* $(g_1 - g_2)$, *whichever absolute value is the greater.*

the chromaticities corresponding to these points is proportional to the longer of the two legs of the triangle, the factor of proportionality being the same wherever the points may lie and whatever may be the distance between them.

On this diagram, the graph of least perceptible purity according to

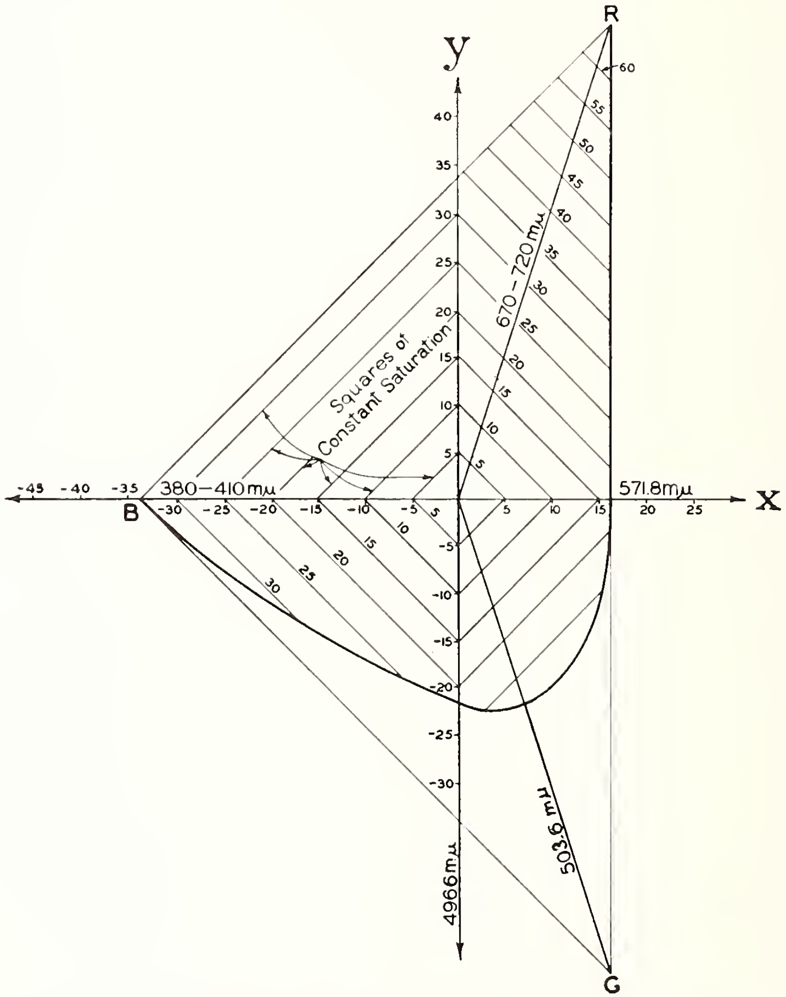

FIG. 2. *A replot of the mixture diagram of Fig. 1 with a change of origin and scale and a rotation of the axes through an angle of 45°. This diagram may be made to serve all the purposes of the (r, g)-diagram of Fig. 1 with but little change in convenience. Since this diagram refers to distribution curves which suggest an opponent-colors hypothesis (Hering) rather than the three-components hypothesis (Young-Helmholtz) suggested by the mixture diagram of Fig. 1, it is concluded that the limited justification of the empiricol relation (1a) demonstrated in the present paper can not be used as an argument in favor of either of these hypotheses as opposed to the other.*

TABLE 1. *Trilinear coordinates (r, g, b) corresponding to various values of dominant wave length and purity; O.S.A. "excitations" extrapolated, Abbot-Priest sun as basic stimulus.*

Section I, Purity positive

p	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*p*
	380–410 mμ			420 mμ			430 mμ			440 mμ			
1.0	.000	.000	1.000	.000	.002	.998	.000	.003	.997	.000	.007	.993	1.0
.9	.001	.001	.998	.002	.004	.994	.002	.005	.993	.002	.009	.989	.9
.8	.003	.003	.994	.004	.006	.990	.004	.007	.989	.004	.011	.985	.8
.7	.005	.005	.990	.006	.008	.986	.006	.009	.985	.007	.014	.979	.7
.6	.008	.008	.984	.009	.011	.980	.010	.013	.977	.011	.017	.972	.6
.5	.012	.012	.976	.013	.015	.972	.014	.017	.969	.016	.023	.961	.5
.4	.018	.018	.964	.020	.021	.959	.021	.024	.955	.024	.031	.945	.4
.3	.028	.028	.944	.030	.032	.938	.032	.034	.934	.036	.042	.922	.3
.2	.045	.045	.910	.048	.050	.902	.051	.053	.896	.057	.063	.880	.2
.1	.087	.087	.826	.092	.093	.815	.096	.098	.806	.106	.111	.783	.1
.05	.142	.142	.716	.148	.149	.703	.154	.155	.691	.165	.169	.666	.05
	450 mμ			460 mμ			470 mμ			480 mμ			
1.0	.000	.017	.983	.001	.044	.955	.005	.105	.890	.023	.203	.774	1.0
.9	.003	.020	.978	.005	.047	.948	.014	.111	.875	.037	.209	.754	.9
.8	.006	.023	.971	.011	.052	.937	.023	.117	.860	.053	.216	.731	.8
.7	.010	.026	.964	.017	.058	.925	.035	.125	.840	.072	.223	.705	.7
.6	.015	.031	.954	.026	.066	.908	.049	.136	.815	.092	.232	.676	.6
.5	.022	.037	.941	.037	.075	.888	.067	.148	.785	.117	.242	.641	.5
.4	.032	.047	.921	.052	.088	.860	.090	.164	.746	.145	.254	.601	.4
.3	.047	.061	.892	.073	.107	.820	.121	.185	.694	.179	.268	.553	.3
.2	.073	.086	.841	.109	.138	.753	.163	.215	.622	.220	.285	.495	.2
.1	.129	.139	.732	.174	.194	.632	.227	.260	.513	.270	.307	.423	.1
.05	.190	.197	.613	.232	.245	.525	.273	.292	.435	.300	.319	.381	.05
	490 mμ			495 mμ			500 mμ			510 mμ			
1.0	.095	.397	.508	.136	.492	.372	.177	.562	.261	.238	.625	.137	1.0
.9	.116	.391	.493	.157	.475	.368	.196	.534	.270	.251	.585	.164	.9
.8	.137	.386	.477	.178	.458	.364	.214	.508	.278	.263	.548	.189	.8
.7	.159	.380	.461	.198	.442	.360	.231	.483	.286	.274	.514	.212	.7
.6	.182	.374	.444	.218	.426	.356	.248	.458	.294	.284	.483	.233	.6
.5	.205	.368	.427	.238	.410	.352	.264	.435	.301	.294	.453	.253	.5
.4	.229	.361	.410	.258	.394	.348	.279	.413	.308	.303	.426	.271	.4
.3	.254	.355	.391	.277	.379	.344	.293	.392	.315	.311	.401	.288	.3
.2	.279	.348	.373	.296	.363	.341	.307	.371	.322	.319	.377	.304	.2
.1	.306	.341	.353	.315	.348	.337	.321	.352	.327	.326	.355	.319	.1
.05	.320	.337	.343										.05
	520 mμ			530 mμ			540 mμ			550 mμ			
1.0	.287	.638	.075	.330	.624	.046	.368	.604	.028	.399	.584	.017	1.0
.9	.294	.594	.112	.331	.582	.087	.363	.564	.073	.389	.548	.063	.9
.8	.300	.555	.145	.331	.544	.125	.358	.530	.112	.381	.515	.104	.8
.7	.305	.519	.176	.331	.509	.160	.354	.497	.149	.373	.485	.142	.7
.6	.310	.486	.204	.331	.478	.191	.351	.467	.182	.366	.458	.176	.6
.5	.315	.455	.230	.332	.449	.219	.347	.440	.213	.359	.433	.208	.5
.4	.319	.427	.254	.332	.422	.246	.344	.415	.241	.353	.410	.237	.4
.3	.323	.401	.276	.333	.397	.270	.341	.393	.266	.348	.388	.264	.3
.2	.327	.377	.296	.333	.374	.293	.338	.372	.290	.343	.369	.288	.2
.1	.330	.354	.316	.333	.353	.314	.336	.352	.312	.338	.350	.312	.1

Table 1, Section I, Purity positive (*continued*)

p	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*p*
	560 mμ			570 mμ			580 mμ			590 mμ			
1.0	.438	.551	.011	.487	.506	.007	.550	.446	.004	.639	.359	.002	1.0
.9	.423	.520	.057	.465	.482	.053	.520	.431	.049	.600	.355	.045	.9
.8	.409	.492	.099	.446	.459	.095	.493	.416	.091	.561	.352	.087	.8
.7	.397	.466	.137	.427	.439	.134	.468	.403	.129	.526	.349	.125	.7
.6	.386	.442	.172	.411	.420	.169	.444	.391	.165	.493	.347	.160	.6
.5	.375	.420	.205	.395	.403	.202	.422	.380	.198	.462	.344	.194	.5
.4	.366	.400	.234	.381	.387	.232	.402	.369	.229	.433	.342	.225	.4
.3	.357	.381	.262	.368	.372	.260	.383	.359	.258	.406	.339	.255	.3
.2	.348	.364	.288	.356	.358	.286	.365	.350	.285	.381	.337	.282	.2
.1	.340	.348	.312	.344	.345	.311	.349	.341	.310	.356	.335	.309	.1
	600 mμ			610 mμ			620 mμ			630 mμ			
1.0	.719	.280	.001	.801	.199	.000	.862	.138	.000	.906	.094	.000	1.0
.9	.669	.287	.044	.743	.216	.041	.799	.161	.040	.839	.122	.039	.9
.8	.623	.293	.084	.689	.231	.080	.739	.183	.078	.776	.148	.076	.8
.7	.580	.299	.121	.637	.246	.117	.681	.205	.114	.713	.175	.112	.7
.6	.539	.305	.156	.588	.260	.152	.626	.225	.149	.654	.199	.147	.6
.5	.500	.310	.190	.540	.274	.186	.572	.245	.183	.596	.224	.180	.5
.4	.463	.315	.222	.495	.287	.218	.521	.264	.215	.540	.247	.213	.4
.3	.428	.320	.252	.452	.299	.249	.472	.282	.246	.486	.270	.244	.3
.2	.395	.325	.280	.411	.311	.278	.424	.300	.276	.443	.292	.275	.2
.1	.363	.329	.308	.371	.322	.306	.378	.317	.305	.382	.313	.305	.1
	640 mμ			650 mμ			660 mμ			670–720 mμ			
1.0	.951	.049	.000	.975	.025	.000	.985	.015	.000	1.000	.000	.000	1.0
.9	.882	.081	.037	.904	.059	.037	.912	.051	.037	.928	.036	.036	.9
.8	.813	.113	.074	.834	.093	.073	.843	.084	.073	.856	.072	.072	.8
.7	.747	.143	.110	.766	.125	.109	.774	.118	.108	.784	.108	.108	.7
.6	.683	.172	.145	.699	.158	.143	.706	.151	.143	.716	.142	.142	.6
.5	.621	.201	.178	.635	.188	.177	.641	.183	.176	.650	.175	.175	.5
.4	.560	.229	.211	.572	.219	.209	.576	.215	.209	.584	.208	.208	.4
.3	.501	.256	.243	.510	.249	.241	.513	.246	.241	.520	.240	.240	.3
.2	.444	.282	.274	.450	.277	.273	.452	.275	.273	.456	.272	.272	.2
.1	.388	.308	.304	.391	.306	.303	.392	.305	.303	.394	.303	.303	.1

Section II, Purity negative

p	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*p*
	492.5 mμ			495 mμ			500 mμ			510 mμ			
−0.2	.380	.310	.310	.370	.304	.326	.358	.298	.344	.346	.295	.359	−0.2
−0.4	.426	.287	.287	.405	.276	.319	.380	.265	.355	.357	.260	.383	−0.4
−0.6	.474	.263	.263	.439	.248	.313	.401	.235	.364	.367	.229	.404	−0.6
−0.8	.522	.239	.239	.472	.222	.306	.420	.207	.373	.376	.202	.422	−0.8
−1.0	.570	.215	.215	.505	.195	.300	.438	.180	.382	.385	.176	.439	−1.0
−1.5	.694	.153	.153	.582	.133	.285	.478	.121	.401	.402	.122	.476	−1.5
−2.0	.826	.087	.087	.654	.075	.271	.513	.070	.417	.417	.078	.505	−2.0
−2.5	.960	.020	.020	.723	.020	.257	.543	.027	.430	.429	.041	.530	−2.5
−3.0	1.106	−.053	−.053	.788	−.032	.244	.569	−.012	.443	.439	.010	.551	−3.0

Table 1, Section 11, Purity negative (*continued*)

p	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*r*	*g*	*b*	*p*
	520 mμ			530 mμ			540 mμ			550 mμ			
−0.2	.339	.295	.366	.334	.297	.369	.329	.300	.371	.325	.303	.372	−0.2
−0.4	.344	.262	.394	.334	.266	.400	.325	.271	.404	.318	.276	.406	−0.4
−0.6	.349	.232	.419	.334	.238	.428	.322	.245	.433	.312	.252	.436	−0.6
−0.8	.353	.206	.441	.335	.213	.452	.319	.222	.459	.306	.230	.464	−0.8
−1.0	.357	.181	.462	.335	.191	.474	.316	.202	.482	.301	.211	.488	−1.0
−1.5	.364	.131	.505	.336	.144	.520	.311	.158	.531	.291	.171	.538	−1.5
−2.0	.370	.091	.539	.336	.107	.557	.306	.124	.570	.282	.138	.580	−2.0
−2.5	.375	.058	.567	.336	.077	.587	.303	.096	.601	.275	.113	.612	−2.5
−3.0	.379	.030	.591	.336	.051	.613	.300	.072	.628	.270	.091	.639	−3.0
−4.0	.386	−.014	.628	.337	.011	.652	.295	.035	.670	.261	.057	.682	−4.0
−5.0	.391	−.046	.655	.337	−.018	.681	.292	.008	.700	.254	.031	.715	−5.0
−7.0										.245	−.004	.759	−7.0
	560 mμ			570 mμ			571.8 mμ						
−0.2	.320	.306	.374	.314	.311	.375	.312	.312	.376				−0.2
−0.4	.309	.282	.409	.296	.292	.412	.294	.294	.412				−0.4
−0.6	.299	.261	.440	.281	.275	.444	.277	.277	.446				−0.6
−0.8	.290	.242	.468	.267	.259	.474	.262	.262	.476				−0.8
−1.0	.281	.225	.494	.255	.245	.500	.249	.249	.502				−1.0
−1.5	.264	.189	.547	.228	.216	.556	.221	.221	.558				−1.5
−2.0	.250	.161	.589	.207	.192	.601	.198	.198	.604				−2.0
−2.5	.239	.137	.624	.190	.172	.638	.179	.179	.642				−2.5
−3.0	.229	.117	.654	.175	.155	.670	.163	.163	.674				−3.0
−4.0	.215	.087	.698	.152	.130	.718	.138	.138	.724				−4.0
−5.0	.203	.063	.734	.135	.111	.754	.120	.120	.760				−5.0
−7.0	.188	.032	.780	.111	.084	.805	.094	.094	.812				−7.0
−10.0	.174	.001	.825	.088	.057	.855	.069	.069	.862				−10.0
−20.0	.153	−.041	.888	.056	.021	.923	.033	.033	.934				−20.0
−40.0				.035	−.002	.967	.012	.012	.976				−40.0
−80.0				.024	−.014	.990	.000	.000	1.000				−80.0

the empirical relation will be a square centered on the point (1/3, 1/3) and having its sides parallel to the axes. One corner will lie on the radial line corresponding to 571.8 mμ and the other on that corresponding to 496.6 mμ, these being the wave lengths at which $|dr|_A = |dg|_A$, and consequently those at which c changes from g to r or from r to g when change in purity at constant dominant wave length is considered. If, as has seemed natural (67, p. 555; 34), we evaluate saturation quantitatively by the number of discriminatory steps from gray,[2] each of the family of squares obtained by a uniform magnification of this one is a locus of colors of constant saturation, provided further that our empiri cal relation expresses the facts. A diagram of these loci, but with the axes rotated through 45°, is shown in Fig. 2.

[2] If, as seems probable, our empirical relation (2a) is in error because dE is not an exact derivative, then the quantitative evaluation of saturation would be defined in this way only when the path from gray to the color in question is specified. This path might reasonably be taken at constant dominant wave length (34) or as the path yielding the fewest discriminatory

VI. Comparison With Experimental Data

1. LIMITATIONS OF DATA, SOURCES OF ERROR

The outstanding limitation of the data to be found in the literature is the almost complete lack of data pertaining either to large color differences or to purple colors.

The only work from which satisfactory data relating to chromaticity sensibility for non-spectral colors—purples, magentas—can be obtained, is that of Donath (11). Smith (71) described his stimuli in such a manner that no quantitative specification of them is possible; Houstoun (28), Hunter (30), and Weissenborn (78) failed to equalize the brightnesses of their intercompared fields; and Jones (33) did not measure the chromaticity sensibility for purples, although one might infer the contrary from a sentence in the Colorimetry Report (74, p. 544). Donath's data for purples refer to a single dominant wave length. They are considered below along with those for other chromaticities.

For an exact comparison of the empirical relation with the available experimental data, it is necessary to know the distribution curves and chromatic luminosity coefficients for each of the several observers, and to know the exact specification of the colors used, which requires, in general, a knowledge of the heterogeneous stimulus ("white") used by the observer. As such information is not available for a single one of the observers, we have assumed the applicability of the extrapolated O. S. A. "excitation" curves and the other conditions listed in section I.

Discrepancies between the observational data and the conclusions drawn from the empirical relation may arise either from errors in the relation or from experimental errors of various kinds. The most likely of the sources of error and discrepancy are listed here for future reference:

(a) A difference between the characteristics of vision of the observer and those defined by the extrapolated O. S. A. "excitation" curves. Differences due to abnormal macular pigmentation (70, 80) or to abnormal spectral transmission of the crystalline lens are not uncommon among observers regarded as possessing normal vision.

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steps (67, p. 555), or it might be taken at constant hue, which differs definitely from the path at constant dominant wave length (1 13, 64, 65, p. 197; 66, p. 512). Exner's definition of "Sättigung" (15), which also assumes dE to be an exact derivative, has been shown experimentally by Seitz (68) to constitute an unsatisfactory quantitative evaluation of saturation. Although in some respects our own tentative definition of saturation by way of the empirical relation closely resembles Exner's definition, it does not necessarily possess the defects found by Seitz.

(b) A difference between $\rho L_r+\gamma L_g+\beta L_b$ and the luminosity of the equal-energy spectrum for the observer.

(c) A difference in field brightness, or surrounding field brightness, or both, giving rise to incomparable states of retinal adaptation.

(d) A difference in the sizes of the fields. This factor is of particular importance when the dominant wave length lies between 460 and 500 mμ, where the transmission of the macular pigment is relatively low.

(e) A difference in the amounts of stray light present and uncorrected for. This factor is of particular importance for spectrum stimuli near the extreme limits (red and violet) of the visible spectrum.

(f) A difference between the heterogeneous stimulus, used by the observer, and Abbot-Priest sunlight. Some difference undoubtedly existed in every one of the cases that we shall consider.

(g) The inherent inability of an observer to maintain, for any extended period, a fixed criterion for the least perceptible difference. His criterion may vary by a factor of two.

2. TYPES OF COMPARISON

The available data suffice for six types of test: 1. Constancy of K. 2. Sensibility to changes in dominant wave length when the colorimetric purity is kept constant. Two cases will be considered: 2a. Sensibility at unit purity. 2b. The ratio of the sensibility at purity p to that at unit purity and the same dominant wave length. 3. Sensibility to change in purity when dominant wave length is kept constant. Here, three cases will be considered: 3a. The sensibility as a function of purity for each of a selected number of dominant wave lengths. 3b. The limiting sensibility as the purity approaches unity for various wave lengths. 3c. The corresponding limit as the purity approaches zero. 4. Sensibility to a change in color temperature. 5. Sensibility to a change in the Lovibond numeral, N'', in the series $35Y+N''R$. 6. The application of the empirical relation to large differences in purity when the dominant wave length is kept constant.

3. DETERMINATION OF K

In the checking of the empirical relation against observational data, it is necessary to assign to K a suitable value. This value will depend, not only upon the unit in which the observations have been expressed, but also upon individual variations and experimental conditions. The last two may introduce factors varying from 1 to 20, even under good conditions.

In this work, the appropriate value of K for a given set of data has been uniformly determined in the following manner. It has been so chosen in each case that the sum of all the ordinates (sensibilities) furnished by the given set of data shall be equal to the sum of the corresponding ordinates defined by the empirical relation. In this way the data given in Tables 2 and 3 for the least perceptible difference were obtained. Observations given in other units were reduced to this one by means of the factors already given (section I).

TABLE 2. *Value of K for monocular observation of a small field.*

Unit of K = change in coordinate per least perceptible difference

Author	Exhibited in	Size of field	Quantity measured	K
Exner	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	0.015
Steindler (12 observers)	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.012
König_1	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.010
Dieterici	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.011
Brodhun (Obs. König_2)	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.014
Uhthoff	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.007
Jones	Fig. 3	Unknown	$(d\Lambda/dE)_{p=1}$	.011
Tyndall	Fig. 3	2° circ.	$(d\Lambda/dE)_{p=1}$	.006
Priest	Fig. 9	4° square	$(dp/dE)_{p\to 0}$	.006
Brickwedde	Fig. 9	4° square	$(dp/dE)_{p\to 0}$	.005
Judd	Table 4	2° circ.	(dp/dE)	.011
Priest	Fig. 10	3° circ.	$(d\theta/dE)$	.009
			Mean	0.010

From Table 2 it is seen that if the observations are made under usual good conditions—field diameter 2 or 3°, monocular vision through an artificial pupil, medium retinal illumination (probably less than 100 photons)—then K may be taken as 0.010 if the unit of ΔE is the least perceptible difference (LPD) and the trilinear coordinates are specified in the manner previously indicated. Under more favorable experimental conditions, the value of K may be much smaller, as in Table 3.

The data of these tables give no indication that the value of K depends either upon the kind of chromaticity sensibility that is under study or upon the actual values of the trilinear coordinates. This is as the empirical relation requires.

4. SENSIBILITY TO CHANGE IN DOMINANT WAVE LENGTH AT CONSTANT PURITY

(a) *Sensibility at unit purity*

The empirical relation (2a) leads through relation (3a) to relation (6a):

$$(d\Lambda/dE)_p = K/(dc/d\Lambda)_p \tag{6a}$$

which at unit purity, in virtue of (4a), becomes:

$$d\lambda/dE = K/(dC/d\lambda)_{p=1} \tag{7a}$$

TABLE 3. *Value of K for binocular observation of a large field.*

Unit of K = change in coordinate per least perceptible difference

Observer	Exhibited in	Size of field	Quantity measured	K
Judd		17×20° rectangular	$d\theta/dE$	0.0013
Donath (4 obs)	Fig. 6	About 8°	$(dp/dE)_{\text{rot}}$	.0024
Donath (4 obs)	Fig. 6	About 8°	$(dp/dE)_{\text{gelb}}$	.0030
Donath (4 obs)	Fig. 7	About 8°	$(dp/dE)_{\text{grun}}$	.0024
Donath (3 obs)	Fig. 7	About 8°	$(dp/dE)_{\text{blau}}$	.0025
Donath (4 obs)	Fig. 8	About 8°	$(dp/dE)_{\text{purpur}}$	.0016
Walker, Brown, and Judd	Fig. 11	6° circ.*	$(dN''/dE)_{\text{Lovibond}}$ $N''R+35Y$	.0008

* Unusually careful monocular observation.

Values of $0.0074/|dC/d\lambda|$, are represented as a function of λ by the continuous line in Fig. 3. In the same figure are plotted the several values of $(0.0074/K)(d\lambda/dE)$ that have been obtained by seven observers (6, 14, 33, 47, 72, 75, 76), the appropriate K in each case being determined in the manner already indicated.

It will be noticed that, although the continuous line does not follow a mean of the experimental results, it gives a satisfactory approximation to the normal sensibility except, possibly, near the violet end of the spectrum, where certain of the sets of data exhibit a definite minimum not duplicated by the continuous line. In this region, the O. S. A. "excitation" curves are somewhat uncertain, on account of the paucity of data, and of the manner in which these data were averaged. More accurate curves might result in a closer fit.

Furthermore, the existence of this minimum in the violet is none too certainly established. A minimum was found by Brodhun (6) and by König$_2$ (6), but the former ascribed it to a failure to equalize the brightnesses; Uhthoff (76), using the same apparatus, but with equalized brightnesses, failed to find it. Steindler found a minimum both in the red and in the violet end of the spectrum. The one at the red end persisted even when she equalized the brightnesses and used a stray light filter (72, p. 50). Both Jones (33), and Laurens and Hamilton (49), using equalized brightnesses, found a minimum at the violet end. Since neither Jones, nor Laurens and Hamilton mention the use of filters for the elimination of stray light, it is possible that their data may be affect-

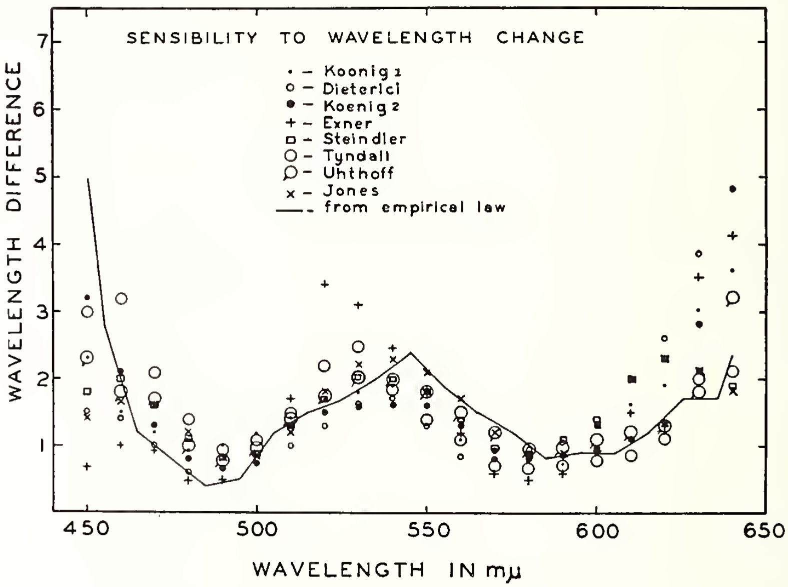

FIG. 3. *Least wave-length difference perceptible in the spectrum. Experimental results from various observers are compared with results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a). The ordinates are* $(0.0074/K)(d\lambda/dE)$*; see p. 88.*

ed by that, as it has been found that the addition of stray light introduces a minimum in the violet similar to that found experimentally for the supposedly pure spectrum. Tyndall (75) who filtered out the stray light, found a slight minimum in the violet.

(*b*) *Ratio of sensibility at purity, p, to that at unit purity.*

The empirical relation (2a) leads through relation (6a) to relation (8a):

$$\frac{(d\Lambda/dE)_p}{(d\Lambda/dE)_{p=1}} = \frac{(dC/d\lambda)_{p=1}}{(dc/d\Lambda)_p} \qquad \text{(8a)}$$

which, in virtue of (4a) becomes relation (9a):

$$\frac{(d\Lambda/dE)_p}{(d\Lambda/dE)_{p=1}} = \frac{[3S(1-p)+p]^2(dC/d\lambda)}{p\{[3S(1-p)+p](dC/d\lambda)+(1-p)(1-3C)(dS/d\lambda)\}} \quad (9a)$$

The values of this expression are represented by the continuous lines in Figs. 4 and 5, the broken lines representing observational data (75, 77). Professor Tyndall very kindly placed at my disposal data not yet published, except in abstract. Not all of Professor Tyndall's results are shown in Fig. 5, but those chosen are typical. The solid dots in Fig. 5 represent values defined by an empirical relation, less satisfactory than (1a), which resembles the Kohlrausch method (48) more closely.

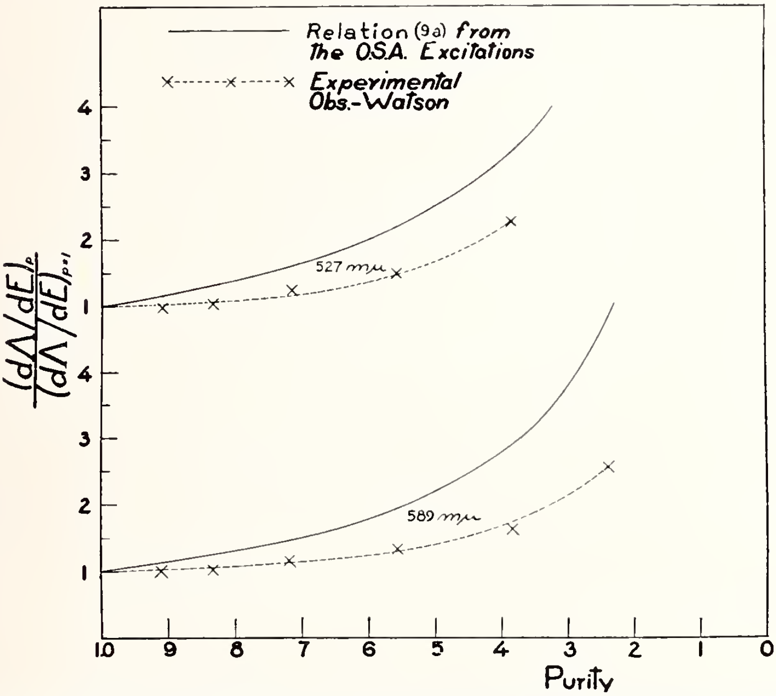

FIG. 4. *Sensibility to wave-length change as affected by the addition of white light. Certain of Watson's experimental results are compared with results computed from the O. S. A. "excitation" curves (Table 1) according to the empirical relation (1a).*

On the whole, the agreement of the computed curve with the observations is good. In each case the observations were made by a single observer, and are affected by his personal characteristics. They may be expected to be affected by the sources of error (a), (b), and (f) (p. 85): the first two apply more particularly to Tyndall (19) and the third to Watson (77).

5. SENSIBILITY TO CHANGE IN PURITY, DOMINANT WAVE LENGTH CONSTANT

(a) Sensibility as a function of purity.

The empirical relation (2a) leads through relations (3a) and (4a) to (10a):

$$(dp/dE)_\Lambda = \frac{K[3S(1-p)+p]^2}{S(3C-1)}. \tag{10a}$$

By differentiating (5b) with respect to E we obtain:

$$(dp/dE)_\Lambda = p_c(dP/dE). \tag{11a}$$

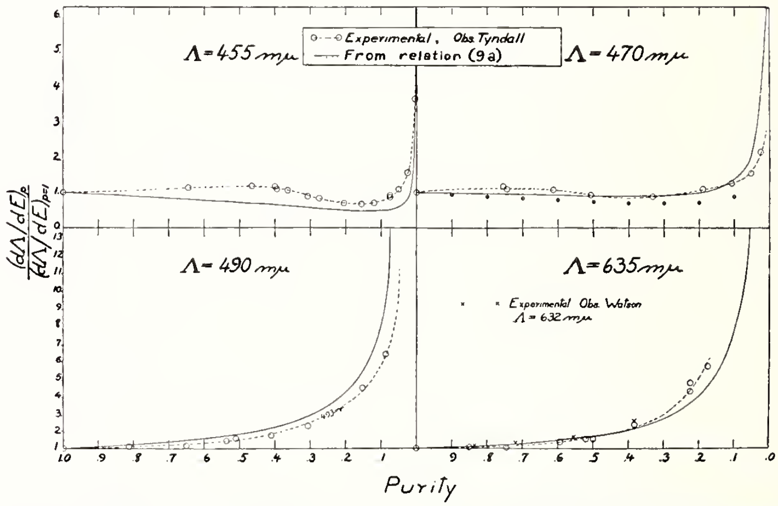

FIG. 5. *Sensibility to wave-length change as affected by the addition of white light. Some of the experimental results by Tyndall and Watson are compared with results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a).*

Values of $(dp/dE)_\Lambda$ from (10a) have been computed for each of a series of wave lengths. They are represented by crosses in Figs. 6, 7, and 8. In the same figures are shown values of the same quantity computed by (11a) from values of dP/dE observed by Donath (11).

The computed graphs agree excellently with the observations for 591 and 606 mμ and for the purple complementary to 494 mμ, but not so well for 480 mμ and still more poorly for 547 mμ, where the computed graph indicates that dp/dE decreases markedly as p increases, while the observations show a slight increase. Examination shows that this same discrepancy exists for each one of Donath's four observers; hence it seems not to be ascribable to individual variation. It has been further found that the slope of the curve according to the empirical relation is nearly independent of L_r, L_g, L_b for any possible values; hence error (b) (p. 86) does not account for this discrepancy.

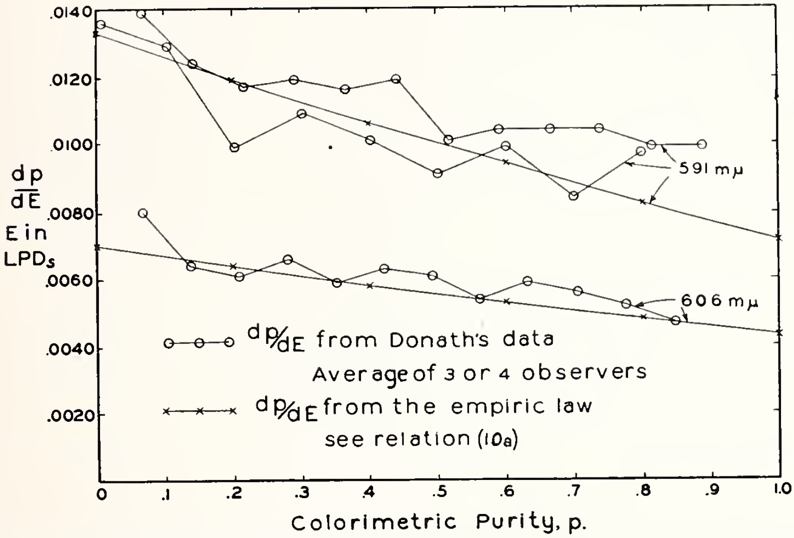

FIG. 6. *Sensibility to purity-change at constant dominant wave length as a function of purity. The experimental results by Donath from rotating disks have been expressed in terms of colorimetric purity by means of relation (11a). With these are compared results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a).*

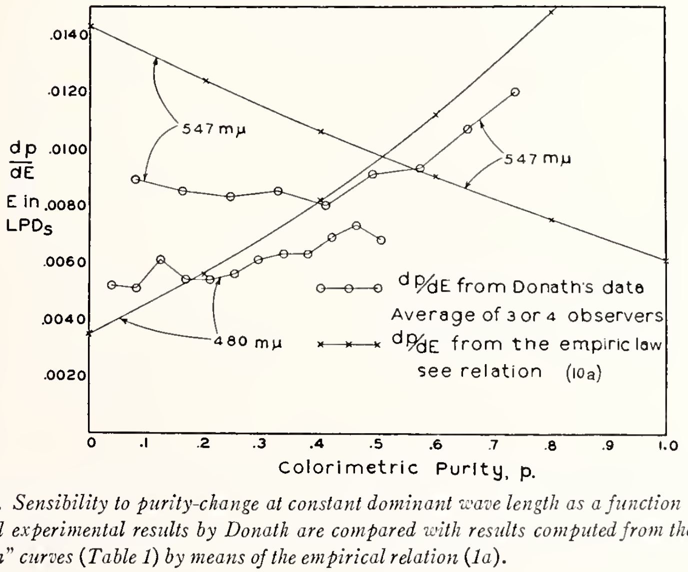

FIG. 7. *Sensibility to purity-change at constant dominant wave length as a function of purity. Additional experimental results by Donath are compared with results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a).*

The discrepancy in the blue (480 mμ) is of little significance. It might well arise from an error in the values assumed for the trilinear coordinates defining the chromaticity of the color used by Donath. His specifications are of such a kind that any estimate of the actual values of the trilinear coordinates, or the dominant wave length and purity, are subject to considerable uncertainty, and the uncertainty in this case is sufficient to account for the discrepancy found.

The discrepancy in the green (547 mμ) can not be dismissed so easily, for the slope of the computed graph is of the same type (negative) for all colors that are likely to be described as green. As it is scarcely probable that a combination of obscure causes would have vitiated the ob-

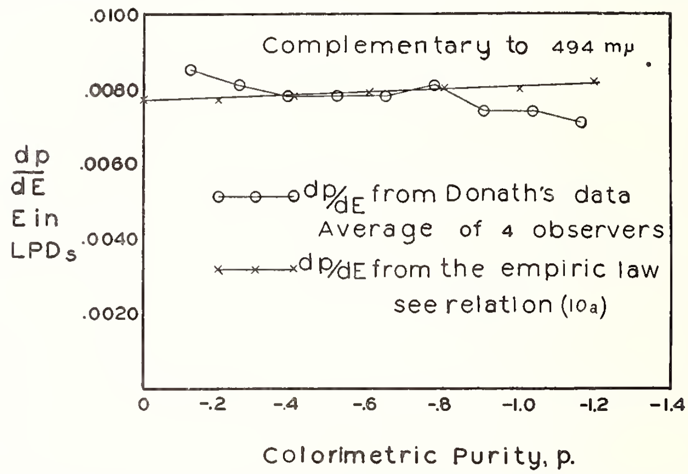

FIG. 8. *Sensibility to purity-change at constant dominant wave length as a function of purity. The remainder of Donath's experimental results are compared with results computed by means of the empirical relation (1a). Note that the purities in this case are less than zero; that is, the colors dealt with are non-spectral or purple colors.*

servations in the green alone, it seems necessary to conclude that the order of approximation of the empirical relation is so low in certain special cases that discrepancies of as much as 50% may arise.

This discrepancy in the green persists even when a vectorial sum of Δr and Δg is used to measure the discriminatory difference between the chromaticities; it is not due to our neglect of one of these coordinates.

Probably, the most convenient way to bring the empirical relation into correspondence with Donath's results in the green, is to express data on equivalent color stimuli in terms of a set of primary colors differing from those used in the O. S. A. "excitation" curves, or in terms of a basic stimulus differing from Abbot-Priest sunlight, or both. It is not profitable to undertake such an adjustment before Donath's results shall have been substantiated by others.

Donath's own conclusion from his observations was that for any one dominant wave length $(dp/dE)_\Lambda$ is independent of the purity. The same conclusion had previously been reached by Munsell (52) and is approximately corroborated by the observations of Gottlieb (22) when account is taken of error (c) (p. 86) but the stimulus was not sufficiently well specified by either of these observers to enable one to compare their observations with values defined by the empirical relation. Aubert (4, p. 132) failed to equalize the brightnesses of his stimuli, and Heymans' data (27) are too fragmentary to be of value as a check upon the empirical relation.

All the experimental data so far considered in this section have been obtained by means of rotating disks. Those obtained by a direct comparison of stationary fields remain to be considered. Unfortunately, they are limited to two published sets (54, 34) of relatively discordant data, and a few, not previously published, check observations by the author. As both of the former disagree with later data by Priest and Brickwedde (61), and in certain important respects with each other, their discordance with the observations of Donath, and with the demands of the empirical relation is of no significance.

TABLE 4. *Saturation sensibility at three colors: Saturated violet-blue, saturated orange-red, and pale violet. Illumination: Abbot-Priest sunlight. Observer: DBJ.*

Case number	Filter	Dominant wave length Λ in mμ	Colorimetric purity p	Least perceptible purity difference dp/dE, dE in *LPDs*
1	$J3654\beta'$	452	0.80	0.16
2	$J4512\beta''$	609	.95	.024
3	None	452	.002	.0038

	Experimental determinations			From the O.S.A. "excitation" curves via relation (10a)
	Nutting and Jones	Jones and Lowry	*DBJ*	
$\frac{(dp/dE)_1}{(dp/dE)_2}$	.045/.047=0.96	.043/.027=1.6	.16/.024=6.7	14.5
$\frac{(dp/dE)_2}{(dp/dE)_3}$	.047/.021=2.2	.027/.025=1.1	.024/.0038=6.3	7.2
$\frac{(dp/dE)_1}{(dp/dE)_3}$	.045/.021=2.1	.043/.025=1.7	.16/.0038=42	104

The results of the check observations made by the author (58) are given in Table 4 together with certain corresponding data, some derived from the results published by Nutting (54) and by Jones and Lowry (34), others computed by means of the empirical relation. The check observations indicate that serious errors, corresponding in some cases to a factor of 20, exist in the data of the other observers, but the check observations, themselves, do not agree satisfactorily with the empirical relation, differing, in some cases, by a factor of 2. Whether this discrepancy is due to errors (a) and (g) (p. 85), or to imperfections in the empirical relation, remains to be seen. The errors in the data published by Nutting, and by Jones and Lowry may have arisen from stray light (error (e)).

(*b*) *Sensibility near unit purity.*

When $p=1$, relation (10a) becomes:

$$(dp/dE)_{\Lambda, p\to 1} = K/S(3C-1). \tag{12a}$$

The only experimental data from which values of the left-hand member of (12a) may be derived are those of Marx and Flieringa (50), and they are not suited for more than a very rough comparison. They may be seriously vitiated by one or more of the sources of error (a) to (g) (p. 85). Furthermore, values of $(dp/dE)_{\Lambda, p\to 1}$ can be inferred from them only on the basis of numerous assumptions.

In spite of these uncertainties their values roughly suggest the general shape of the curve given by (12a). The experimental values differ from those by the empirical relation by a factor of 3.

(*c*) *Sensibility near zero purity.*

When $p=0$, relation (10a) becomes:

$$(dp/dE)_{\Lambda, p\to 0} = 9KS/(3C-1). \tag{13a}$$

The variation of the right-hand member of (13a) with the dominant wave length is shown by the continuous line in Fig. 9. Careful determinations of the left-hand member have been made by Priest and Brickwedde (61), who have very kindly placed their data at my disposal before complete publication. These data are represented by the broken lines.

Except for the maximum at 496.6 mμ, the computed curve agrees with the observations about as well as the two sets of observations agree with one another. The minor discrepancies may well arise from error (a) (p. 85). The maximum at 496.6 mμ, where C for Λ constant changes from R to G (the reverse change occurs at the other maximum), can not be accounted for by error (b) because no reasonable choice of values for the chromatic luminosity coefficients would significantly improve the agreement.

Neither can the absence of this maximum from the curves representing the observed data be accounted for by abnormalities in the vision of the observers (a) (p. 85) because, over a wide range in the choice of primary process and basic stimulus and over the range proper to individual differences, its presence in the computed curve is independent of the form of the distribution curves.

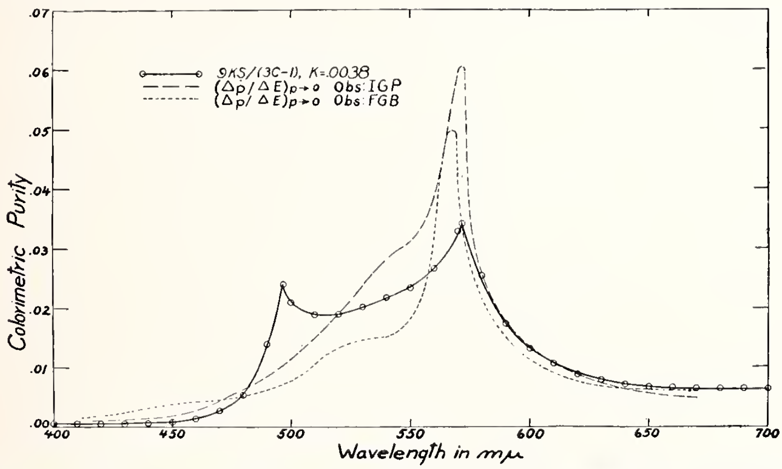

FIG. 9. *Least purity-difference perceptible near zero purity as a function of dominant wave length. Experimental results by Priest and Brickwedde are compared to results computed from the O.S.A. "excitation" curves (Table 1) by means of the empirical relation (1a).*

Hence, it seems that the major portion of the discrepancy near 496.6 mμ seen in Fig. 9 is due to an imperfection in the empirical relation, an imperfection that may in special cases lead to an error of at least 50%.

6. SENSIBILITY TO CHANGE IN COLOR TEMPERATURE

The empirical relation (2a) leads through (3a) to (14a):

$$d\theta/dE = K(d\theta/dc) = K(d\theta/dr) \tag{14a}$$

where θ is the color temperature, and c is the trilinear coordinate that changes most rapidly with θ, which, in this case, is r. It has been discovered by Priest (57) that the observational precision of setting for a chromaticity match between beams of radiant energy of Planckian distribution is approximately a constant when expressed in terms of the spectral centroid of light, λ_c. This discovery may be summarized as an empirical relation: $d\theta/dE = k(d\theta/d\lambda_c)$, where k is a constant. In Fig. 10 this relation is compared with (14a). The ratios $d\theta/d\lambda_c$ (crosses) have been evaluated from results computed by Priest similar to those appearing in his Tables 1 to 4 (57). The ratios, $d\theta/dr$ (circles) were evaluated from computations of r, g, b by Davis and Gibson (10, p. 56), and K was taken (for convenience) as 0.0010. Davis (9) has observed that $d\theta/d\lambda_c$ is quite closely proportional to θ^2; this leads to: $d\theta/dE = k'\theta^2$. Values of $1.30\ \theta^2 \times 10^{-6}$ are represented by dots in Fig. 10.

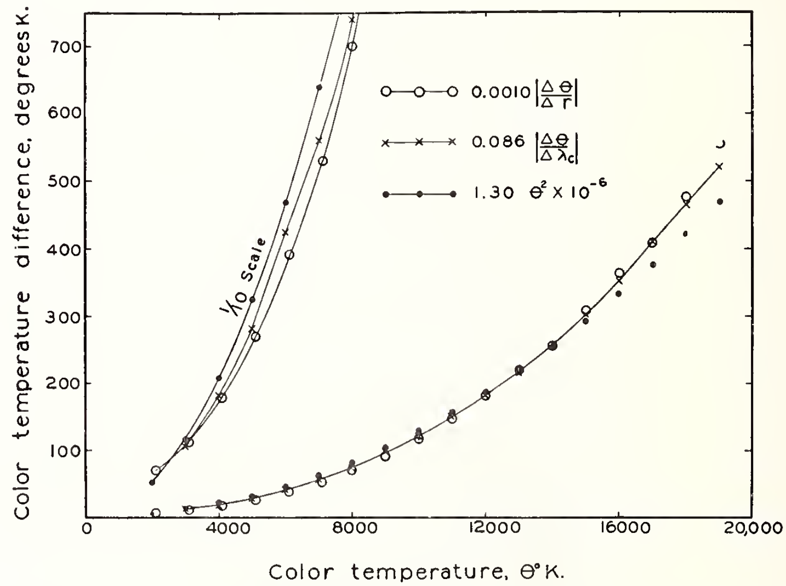

FIG. 10. *Least color-temperature difference perceptible as a function of color temperature. Results according to Priest's empirical, spectral-centroid relation are compared with results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a). A simple function (θ^2) discovered by Davis to be a close representation of Priest's empirical spectral-centroid relation is also shown.*

It may be seen that these three relations give very similar curves; they are probably about equally trustworthy.

7. SENSIBILITY TO CHANGE IN N'' ALONG THE LOVIBOND $35Y+N''R$ LOCUS

The empirical relation (2a) leads through (3a) to (15a):

$$dN''/dE = K(dN''/dc) = K(dN''/dr). \qquad (15a)$$

The variation of $0.00014\ (dN''/dr)$ with N'' is shown by the continuous line in Fig. 11. The circles represent the probable errors of a single observation (PE_{single}) as computed from the direct comparisons, by 3 observers, of 36 Lovibond red glasses with those standardized by Priest and Gibson (63). In this comparison, the Bureau of Standards procedure was followed (43), but in computing the probable errors here used, only such observations as involved the use of identical combinations of standard glasses were used. This restricted the variations to those arising from interpolation between two combinations of standard glasses, eliminating all those that might possibly arise from errors in the N'' values of the standards. As the spectral distribution of the energy is very nearly the same in each of the fields to be compared, it is improbable that individual variations in the vision of the observers need be considered.

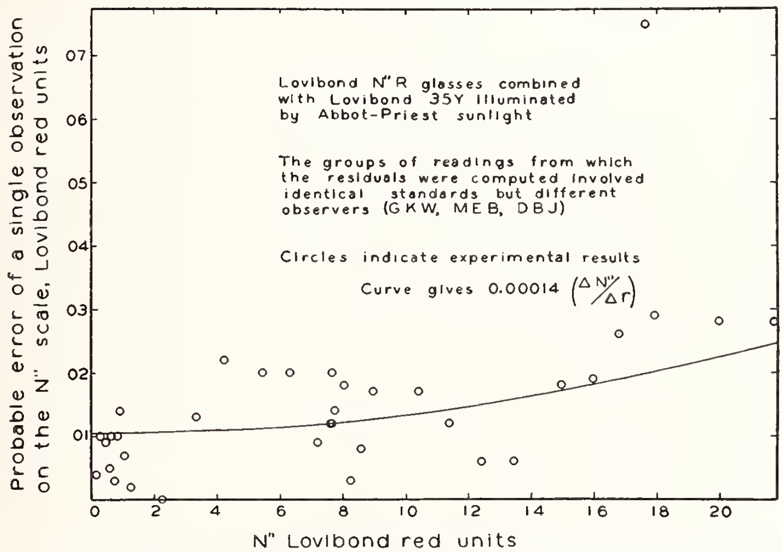

FIG. 11. *Sensibility to change in Lovibond red on the 35Y+N''R locus. Experimental results by three observers (Walker, Brown, Judd) are compared with results computed from the O. S. A. "excitation" curves (Table 1) by means of the empirical relation (1a).*

Although the circles are very scattered (the ordinate of one at 17.7 is 0.075), the curve is a fair representation of their trend. The errors may possibly increase somewhat faster than is indicated by the curve. A small increase might be caused by the decrease in brightness that accompanies an increase in N'', but preliminary experiments have indicated that no important change in chromaticity sensibility is caused by such changes in brightness.

8. APPLICABILITY OF THE RELATION TO LARGE DIFFERENCES IN COLOR

It is exceedingly difficult to judge the equality of two differences in color if these differences exceed a few discriminatory steps. But it is only by reference to such judgments that the applicability of the empirical relation (1a) to large differences in chromaticity—that is, the integrability of (2a), regarding K as a constant—can be tested.

Such judgments of intervals of purity at fixed dominant wave length are available. Using rotary mixing devices and a dominant wave length in the red part of the spectrum, Jacobsohn (32) endeavored to determine the color midway in saturation between two given colors whose stimuli were of the same brightness. Only two of his observers completed the entire series of observations. Their results are given in Table 5, together with data according to the empirical relation (1a). The computation based on relation (5a) assumes that the color of Jacobsohn's chromatic sector—almost pure red, with a slight trace of yellow—has the trilinear coordinates, $r_c = 0.700$, $g_c = 0.150$, $b_c = 0.150$. (Other plausible values lead to a similar conclusion.)

It will be noticed that the subjective estimate of the mean agrees with the mean of the r-coordinates if the difference in the latter does not exceed 0.20, but falls below that mean at greater values of the difference. Incidental observations indicating similar results for yellow have been obtained by the author (38).

Hence we may conclude that the empirical relation, at least as regards changes in purity at fixed dominant wave length, is not strictly applicable to large differences, that (2a) is not integrable when K is regarded as a constant. The maximum demonstrated error of this sort is about 25%.

VII. Conclusions From the Comparisons

From the preceding comparisons we may conclude as follows:

1. K is independent of the type of chromaticity sensibility that is being considered, and for small discriminatory intervals it is also independent of the actual values of the coordinates defining the color. In these respects, the observations accord with the empirical relation.

2. As compared with its value at 510 mμ, the sensibility of the eye to changes in dominant wave length at unit purity in the region 550 to 580 mμ and 400 to 450 mμ (green-yellow and violet) is greater than is demanded by the empirical relation, and in the red and blue-green it is

TABLE 5. *Comparison of the subjective mean determined by Jacobsohn's experimental method with the saturation mean from relation (1a).*

Case No.	First component (carmine) 360P	Second component (carmine plus equivalent gray) 360P	$r_1=r_c$	r_2	r-coordinate of saturation mean $\frac{r_1+r_2}{2}=\bar{r}$*	Subjective mean, experimentally determined: Jacobsohn 360P	Küchler 360P	r_j	r_k	Mean $\frac{r_j+r_k}{2}=\bar{r}_e$	Size of interval between the colors of the components in LPDs† $100\|r_1-r_2\|$	Difference between saturation mean and the experimental subjective mean in LPDs† $100\|\bar{r}-\bar{r}_e\|$
1	360	0	0.700	0.333	0.516	125.4	157.8	0.456	0.488	0.472	36.7	4.4
2	360	90	.700	.421	.560	213.5	205.4	.545	.537	.541	27.9	1.9
3	360	180	.700	.511	.606	270.7	275.8	.605	.610	.608	18.9	0.2
4	360	270	.700	.604	.652	310.3	327.4	.646	.665	.656	9.6	0.4

* By relation (1a) the r-coordinate of the saturation mean is merely the arithmetical mean of the r-coordinates, hence: $\bar{r}=(r_1+r_2)/2$.

† To give the color interval in classical "least perceptible differences" (*LPDs*), K of relation (1a) is taken as 1/100 (see Table 2).

slightly less. These discrepancies might possibly be reduced by using more accurate data on equivalent color stimuli.

3. The ratio of the sensibility of the eye to a change in dominant wave length at purity, p, to that at unit purity varies with p about as is to be expected from the empirical relation (Tyndall), but may increase somewhat less for a given decrease in p than the relation demands (Watson). The apparent discrepancies may arise from individual variations and differences in experimental conditions.

4. The variation, with purity, of the sensibility of the eye to changes in the purity at dominant wave lengths 591, 606, and (complementary to) 494 mμ is the same as is demanded by the relation; that at 480 mμ is nearly the same; but that at 547 mμ is not reconcilable with the relation. These discrepancies mount as high as 50%. Reconciliation may require the use of other primary colors, or another basic stimulus, or both.

5. The limiting value, at unit purity, of the sensibility of the eye to changes in purity seems to vary with the dominant wave length in a way that follows only in a general manner the curve demanded by the empirical relation. The discrepancies, however, are easily ascribable to the unsatisfactory nature of the observations.

6. The limiting value, at zero purity, of the sensibility of the eye to changes in purity varies with the dominant wave length along a curve that gives no indication of the maximum that the empirical relation demands at 496.6 mμ. In this region the empirical relation is in error by as much as 50%.

7. The sensibility of the eye to changes in color temperature, and that to change in N'' along the Lovibond $35Y+N''R$ locus are each about what is to be expected from the empirical relation.

8. Data referring to purity-change indicate that the relation does not hold good for large changes in chromaticity, that relation (2a) is not integrable when K is constant. Errors introduced by the demonstrated lack of integrability mount as high as 25%.

VIII. OTHER RELATIONS TESTED

Besides relation (1a), the following 9 other expressions for $K\Delta E$ were tested against observed data, and were found to be less satisfactory: $\sqrt{(r_1-r_2)^2+(g_1-g_2)^2}$, $\sqrt{(r_1-r_2)^2+(b_1-b_2)^2}$, $\sqrt{(g_1-g_2)^2+(b_1-b_2)^2}$, $|r_1-r_2|+|g_1-g_2|$, $|r_1-r_2|+|b_1-b_2|$, $|g_1-g_2|+|b_1-b_2|$, the greater of $|r_1-r_2|$ and $|b_1-b_2|$, the greater of $|g_1-g_2|$ and $|b_1-b_2|$, and the greatest of $|r_1-r_2|$, $|g_1-g_2|$, and $|b_1-b_2|$. The testing of similar relations in which the coordinates were to be referred to more favorable primary color processes and basic stimuli, if such could be found, and of the relations demanded by Schrödinger's theory (66, 67) were considered, but were for the present rejected on account of the time and the difficulty that would be involved. When it becomes desirable to secure a better agreement than is afforded by the empirical relation (1a), they may be reconsidered.

IX. Improving the Empirical Relation

Many of the discrepancies that we have found to exist between observational data and the demands of the empirical relation may either arise from errors in the assumptions that we have had to make, or have their source in the conditions under which the observations were made. The last includes the characteristics of the observer.

But, some of the discrepancies concerned with the discrimination of differences in purity must be ascribed to a failure of the empirical relation. A large portion of these discrepancies, especially near zero purity, may be described by saying that the eye is relatively less sensitive to changes in purity in the violet and in the green-yellow than is indicated by the empirical relation.

The demands of the relation might be favorably modified either by changing its form, probably making it more complicated, or by changing the chosen primary colors, or basic stimulus, or both, that is, by choosing some homogeneous, linear transformation of the O. S. A. "excitation" curves.

Changing the form of the relation so as to make the change in sensibility proportional to a vector sum of the r- and the g-coordinate differences, as suggested by König (46, p. 107), Kohlrausch (48), and, recently, by Houstoun (28), makes matters slightly worse.

Transfer to other coordinate systems has been suggested by König and Sinden (70, p. 1142) and has been tried by Houstoun (28). If, retaining the form (1a) of the relation, we attempt to secure accord by such a transfer, then it is worth noting that the observational curves in Fig. 9 lie, in large part, between that defined by the empirical relation as applied to normal vision (the unbroken curve) and that defined by the same relation as applied to tritanopia. The latter would rise to infinity at 571.8 mμ (45). Furthermore, König (44) has already noted that the normal observer behaves in some respects like a tritanope provided the field be very small and centrally fixated. A set of curves designed to represent a type of vision intermediate between normal vision for usual field-sizes and tritanopia has been set up (39), and it is found that by their use the discrepancies between the computed curve and Priest's observations near zero purity have been largely eradicated. Further careful tests have not been made, but preliminary comparisons indicate that these curves will not prove satisfactory for chromaticity sensibility to other types of stimulus difference.

X. Theoretical Significance

Since the tests to which we have put the empirical relation have all rested upon the extrapolated O. S. A. "excitation" curves (ρ, γ, β) it might be thought that the degree of success demonstrated for this relation confers upon these curves and upon theories that make use of curves of this type some special claim to being more fundamental than others. Such is not the case. The empirical relation, with relatively slight modification, will apply to any set of distribution curves that gives the same mixture diagram, except for scale and orientation. For example, the distribution curves defined by relations (16a),

$$\left.\begin{aligned} \chi &\equiv 1.00\rho + 1.00\gamma - 2.00\beta \\ \psi &\equiv 3.00\rho - 3.00\gamma \\ \zeta &\equiv -3.94\rho + 2.06\gamma + 2.06\beta \end{aligned}\right\} \tag{16a}$$

where χ refers to a green-yellow primary process ($-\chi$ to the complementary violet process), ψ to a red process ($-\psi$ to the complementary bluish-green process), and ζ to the color process evoked by the basic stimulus ("white") of the $\rho\gamma\beta$-system, are quite different from those of that system. They are of the type that is suggested by the opponent-colors theories (Hering), while the O. S. A. curves are of the type suggested by the three-components theories (Young-Helmholtz). Nevertheless, the two sets of curves lead to identically the same mixture diagram, so that the application of the empirical relation depends only upon the proper orientation of the axes along which the changes called Δr and Δg are to be measured.

This may be easily shown. If x, y, z be the trilinear coordinates in the $\chi\psi\zeta$-system in exactly the same way as r, g, b are in the $\rho\gamma\beta$-system then it follows immediately that:

$$\left.\begin{aligned} x &= 50(r + g - \tfrac{2}{3}) \\ y &= 50(r - g) \end{aligned}\right\}. \tag{17a}$$

The form of (17a) suggests that in so far as curves in the axial planes of x, y and of g, r are concerned there has been no change except a simple transformation of coordinates and a change in scale. The point (x, y) referred to rectangular coordinates lying in the plane of g, r, having their origin at $g=1/3$, $r=1/3$, and rotated in this plane 45° counterclockwise with reference to the (g, r)-axes, will coincide exactly with the point (g, r) if all (g, r)-distances are magnified from 1 to $50\sqrt{2}$. Fig. 2 shows the spectrum locus in the (x, y)-plane. After solving for r and g from (17a) it follows at once that:

$$|r_1 - r_2| = |x_1 - x_2 + y_1 - y_2|/100$$
$$|g_1 - g_2| = |x_1 - x_2 + y_2 - y_1|/100.$$

The largest of $|r_1-r_2|$ and $|g_1-g_2|$ may therefore be written: $(|x_1-x_2|+|y_1-y_2|)/100$, which permits relation (1a) to be expressed simply:

$$K\Delta E = (|x_1 - x_2| + |y_1 - y_2|)/100. \qquad (18a)$$

Relation (18a) with reference to the (x, y)-axes is nearly as simple as (1a) is with reference to the (g, r)-axes; hence, the empirical relation does not depend on the specific form of the distribution curves. A similar independence of form of distribution curves has been pointed out by Brückner (8) for Steindler's method (72). Forbes (16) used (18a) in connection with distribution curves similar to those defined by (16a) in his reduction of Steindler's data (72).

In the preceding transformation one primary process has been retained at the dominant wave length 380 to 410 mμ (violet); this is essential to the applicability of the empirical relation. Unless this is a primary process, then, no matter what type of distribution curves be adopted, the empirical relation fails. To this extent, the relation suggests that violet is a primary color process in vision.

That it is at least practically useful to regard violet as a primary color is also suggested by the Bezold-Brücke phenomenon (5, 7), which is the hue change resulting from a change in brightness, by Allen's observations (2) on visual fatigue as measured by the critical frequency of flicker, and by the existence of tritanopes, that is, of persons having violet-greenyellow blindness (45, 73, 51, p. 51).

On the other hand, blue is universally regarded as psychologically unitary (12, 20, 21, 26, 69, 79), violet being commonly described as redish-blue; one form of change in hue due to admixture of white light (1) can most readily be accounted for by regarding blue, not violet, as primary (41), and there exist tetartanopes, that is, persons having yellow-blue blindness (51, p. 68).

Allen (3) has endeavored to reconcile these conflicting facts, and

Hecht (24) has suggested an interesting partial explanation, but the most satisfactory solution yet offered is G. E. Müller's theory (51) which ascribes primacy to both blue and violet, the latter in the retinal processes, and the former in the optic nerve. Our empirical relation accords with this theory.

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SENSIBILITY TO COLOR-TEMPERATURE CHANGE AS A FUNCTION OF TEMPERATURE

J. Opt. Soc. Am. 23, 7-14 (1933)

This report of experimental settings for greatest imperceptible differences of color temperature, and for least perceptible differences, established the conclusion that visually equally significant differences of color temperature correspond to differences of reciprocal color temperature that are substantially independent of color temperature. This conclusion has been generally used, ever since publication of this paper. It is the basis for the CIE recommendation that reciprocal color temperature should be expressed in reciprocal megakelvins (rMK). One unit of difference, expressed in rMK, is barely noticeable, regardless of color temperature. This unit was formerly called the "micro-reciprocal degree". Because the term "degree" is now deprecated by all international bodies concerned with units (including the CIE), the term "micro-reciprocal degree" and its abbreviations, "mired" and "μrd", are also disapproved and are obsolete. They do not appear in this paper, but were used by Judd in later papers and books.

The relation of color constancy to the constancy of perceptible difference of reciprocal color temperature, to the constancy of effects of color-temperature-conversion filters on reciprocal color temperatures, and to similar constancy of effects of other filters and chromatic reflecting materials, is intriguing and provocative. It has not been seriously considered nor yet been the subject of the investigation that it merits.

Sensibility to Color-Temperature Change as a Function of Temperature*

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Determinations at the same brightness by six normal observers have been made of the color-temperature difference corresponding to a chromaticity difference just doubtfully perceptible over the range of 1800 to 11,000°K. It has been found that this temperature difference corresponds closely to a constant difference in the spectral centroid of light (Priest's empirical relation); it is also closely proportional to the square of the color temperature (Davis' representation of Priest's empirical relation); furthermore it corresponds with good approximation to a constant difference in the "red" trilinear coordinate (Judd's empirical relation). These experimental results have therefore approximately checked three empirical relations previously derived from less complete data. Priest's spectral-centroid relation and Davis' representation of it both agree so closely with the experimental data that the discrepancies are not known to be real, but Judd's empirical relation yields discrepancies about twice as large as the experimental uncertainty. The verification of Priest's spectral-centroid relation confirms the tentative conclusion by Davis and Gibson that between 2000 and 3000°K a given color-temperature difference causes for the normal observer nearly the same size chromaticity difference regardless of whether the radiators be viewed directly or through a blue filter not more highly selective than a "daylight filter." This result has a bearing on the phenomenon of "color constancy"; that is, it helps to explain why the colors of objects are approximately constant for illuminants differing as widely as incandescent-lamplight and natural daylight.

I. INTRODUCTION

When color temperature is used, it is usually important to know the smallest interval in temperature which can be detected under the given experimental conditions. In particular, when color temperature is used for the specification of light sources in colorimetry, it is important to know this temperature interval because the relative distinctions between the various sources may then be determined and tolerances established.

Previous experimental data on this question are somewhat meager. They may be represented as one or another of three empirical relations; first, that the temperature interval corresponding to a given chromaticity difference is about the same as that corresponding to a constant change in the spectral centroid of light; second, that this interval varies approximately as the square of the color temperature; and, third, that it is about the same as that corresponding to a constant change in the r-coordinate of the color represented on a certain Maxwell color-triangle.

The first relation was discovered empirically by Priest[1] from experimental data on the precision of color-temperature settings; he states that his data showed no certain deviation from this relation, but that in individual cases the precision might vary by a factor approaching 2. It is to be noted that the range of color temperature covered by Priest's data is 3000 to 20,000°K, which fails to include most of the usual temperatures produced by incandescent lamps (2000 to 3000°K). The second relation was proposed by Davis[2] as a simpler statement of the spectral-centroid relation; he discovered empirically that the differences between these two relations are scarcely significant. The temperature interval given by this relation is also expressible as that

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* Publication approved by the Acting Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] I. G. Priest, *The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion*, J. Opt. Soc. Am. and Rev. Sci. Inst. 7, 1190–1191 (1923).

[2] R. Davis, *A Correlated Color Temperature for Illuminants*, Bureau Standards J. Research 7, 659–681 (1931). This simpler statement of the spectral-centroid relation might have been deduced by combining two previous findings, one by Gibson (see footnote 10, p. 12) concerning a spectral-centroid relation between incident and transmitted light for daylight filters, the other by Langmuir and Orange (Trans. A.I.E.E., 32, 1944–1946 (1913)) concerning a similar relation involving reciprocal temperature. The mathematical analysis on which this latter finding is based was given later by Foote, Mohler and Fairchild, J. Wash. Acad. Sci. 7, 545–549 (1917), and Gage, Trans. I.E.S. 16, 428–429 (1921) also called attention to this relation.

corresponding to a constant change in reciprocal temperature.

The third relation is a special case of a more general relation discovered empirically by Judd[3] through an examination of all data available on chromaticity sensibility. Data referring to change in color temperature constitute a relatively small part of this whole. The general relation has been found to be in error in some cases by as much as 50 percent. There is no reason to suspect that this special case is any more reliable except that it agrees rather closely with the first two empirical relations which were obtained independent of it.

It is the purpose of the present paper, first, to give an account of a more adequate experimental determination of chromaticity sensibility as a function of temperature, and second, to compare the new experimental results with the three relations enumerated.

II. Apparatus and Procedure

The apparatus used consisted of a color comparator[4] for incandescent lamps combined with a Martens photometer,[5] a Davis-Gibson 2450-to-6500°K filter[6] and suitable screens made up of alternate opaque and transparent lines (half-tone screens) for reducing brightness without introducing any important change in chromaticity. By varying the current through the 400-watt lamps used in the color comparator, color temperatures of about 1800, 2200 and 2900°K were obtained, and the addition of the liquid filter yielded the two other color temperatures investigated: about 5000 and 11,000°K.

The matt surfaces of the color comparator, each surface illuminated by a 400-watt lamp, were viewed through the Martens photometer which, by virtue of a special diaphragm, provided a 2° circular observing field divided along its vertical diameter and surrounded by a dark field. The room containing the apparatus was dimly lighted. By the use of the three half-tone screens and by varying the distances of the lamps from the matt surfaces it was possible to maintain the retinal illumination nearly constant at 30 photons for three of the values of color temperature at which chromaticity sensibility was determined: 2200, 2900 and 11,000°K. At 1800 and 5000°K a converging lens placed in front of each lamp raised the illumination to the same value.

The color temperature of one-half of the field was set by an operator and kept constant by him throughout each group of ten settings; the operator also made adjustments of the half-tone screens and the lamp distances so that the retinal illumination corresponding to the observing field was about 30 photons regardless of the color temperature. The observer controlled the voltage applied to the lamp illuminating the other half of the observing field by operating rheostats with one hand, and, with the other hand, rotated the circle of the Martens photometer in order to eliminate such brightness differences as occurred from varying the voltage of the lamp incident to the settings of color temperature.

Each of six observers made 300 settings, 100 settings on each of three separate days. The general procedure was to make ten "greatest-imperceptible-difference" (GID)[4] settings for one color temperature, then ten more GID settings at another color temperature, and so on for all five color temperatures. Then, after five minutes rest, the observer made ten "least-perceptible-difference" (LPD)[4] settings at each of the five color temperatures in the same order as before. The order of color temperatures for the first observing day was 1800, 2200, 2900, 5000 and 11,000°K; for the second observing day it was 11,000, 5000, 2900, 2200 and 1800°K; and for the final day it was 2900, 2200, 5000, 1800 and 11,000°K.[7] After each setting the operator recorded the voltage of the lamp controlled by the observer.

[3] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. Opt. Soc. Am. **22**, 72–108 (1932).

[4] D. B. Judd, *Precision of Color Temperature Measurements under Various Observing Conditions; A New Color Comparator for Incandescent Lamps*, Bureau Standards J. Research **5**, 1161–1177 (1930). (In Fig. 2, p. 1169 of this paper, a decimal point should be placed before each of the numbers on the ordinate scale.)

[5] F. F. Martens, *Über ein neues Polarisationsphotometer*, Phys. Zeits. **1**, 299–303 (1900).

[6] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub., Bur. Stds., No. 114, 68–69; January 21 (1931).

[7] Later analysis showed that order of presentation of stimuli did not affect sensibility importantly.

III. Results and Discussion

The reduction of these data follows the general plan outlined in a previous paper[4] except that we are not here concerned with median settings. The average between the greatest imperceptible difference and the least perceptible difference, (GID+LPD)/2, is computed for each of the five groups of 20 settings made by each observer on each day at each of the five color temperatures; also the probable error of a single observation, PE_{single}, is computed from each group of ten settings. As before it is found that on the average:

$$3.3\ PE_{single} = 0.5(GID+LPD).$$

We take, therefore, either 3.3 PE_{single} or 0.5(GID+LPD) interchangeably as a measure of the "just doubtful difference" which is the discriminatory unit in terms of which we have chosen to express the experimental results. The final values of the doubtful difference (Table II) are computed as [3.3 PE_{single}+0.5(GID+LPD)]/2. The doubtful difference is a difference sufficiently large to be detected by the observer in about 92 out of each 100 attempts. Note that we deal in this paper only with relative values of the doubtful difference as a function of temperature. For absolute values of the doubtful difference for various observing conditions, reference must be had to a previous paper.[4]

1. Reduction to average doubtful difference

In presenting the average results, it is easier to appreciate the uncertainty arising from individual variations if a reduction is first made to the average size of the doubtful difference; that is, we plan to eliminate that part of the individual variation which can be ascribed either to abnormally high sensibility throughout the whole range of color temperature or to abnormally low sensibility. As a first step in this reduction we compute at each color temperature the ratio for each observer of the doubtful difference to the mean doubtful difference for all observers; the average of these ratios over the five color temperatures is given in Table I. It is to be noted from the second column, headed 0.5(GID+LPD), that some observers set differences as just doubtful nearly twice as large as some others; also from the third column (PE_{single}) that some observers make settings of only about half the precision of some others; and it might therefore be said in a sense that sensibility to color-temperature change varies by nearly a factor of two from individual to individual. It is not to be supposed, however, that a real superiority in vision for some observers compared to certain others has been established. It is quite probable that the manner of manipulating the rheostats and the brightness adjustment of the Martens photometer in conjunction with eye movements is the basis for all the individual variation shown. For example, the arrangement of the observers according to the precision of setting (Table I, third column) corresponds closely to the amount of practise they have had at making chromaticity matches; that is, the smallest probable errors have been obtained by the most experienced observers. This has resulted in spite of the fact that no emphasis was placed on making precise settings; the emphasis was rather on the adoption of a definite technique of manipulating the controls and the maintenance of this technique throughout an entire run of 100 settings.

Table I. *Individual variations of the doubtful difference.*

Observer	Average ratio of the doubtful difference for each observer to the mean doubtful difference 0.5(GID+LPD)	3.3 PE_{single}
DBJ	1.07	0.85
RSH	1.33	1.26
GKW	1.09	1.05
MEB	0.97	1.22
IGP	0.78	0.80
KSG	0.76	0.82

2. Individual variations

Fig. 1 shows by the plotted points the individual results reduced to the average doubtful difference for the six observers. This reduction was made simply by dividing each value of doubtful difference obtained from a given observer by the corresponding average ratio given in Table I, and then averaging the resulting values of 0.5(GID+LPD) and 3.3 PE_{single}. The doubtful differences are shown in terms of three parameters, spectral centroid of light, λ_c, reciprocal color temperature, $1/\theta$, and red coordinate, r; hence any one of the three plots serves to indicate the individual variation. It is to be noted that the points are scattered for each range of color temperature not only with respect to the ordinate (size of the doubtful difference) but also with respect to the abscissa, reciprocal temperature. The first sort of scatter suggests, of course, that some observers respond best to differences in color temperature at one part of the temperature scale, others at another. The amount of this scatter is in each case nearly one-third the mean ordinate. However, this scatter probably refers to experimental error rather than to significant individual variations in characteristics of vision.[8] It may not be clear why there should be any scatter with respect to reciprocal color temperature at all because the lamp illuminating the left half of the field was kept at the same color temperature for all observers. Strictly speaking, however, the abscissa refers to the reciprocal of the color temperature of the right half of the field, that is, to the reciprocal of the color temperature over which the observer had control. This color temperature was by no means constant; it varied from observer to observer in some cases by nearly two doubtful differences. Some of this variation, perhaps about one-third of it, may be ascribed to the uncertainties introduced by the chromatic aberration of the lenses used for two of the color temperatures, to non-uniformities in the half-tone screens, to experimental error, and to the slight aging of the lamps during their 50 hours of use. The major part of these differences are true individual differences, however, and probably refer to difference in yellow-blue sensitivity across the vertical median line of the retina. It has already been shown that such a difference exists for a single observer depending on the field size used;[4] so it is not surprising that different observers with the same field size should show it. These results emphasize the danger of departing from the strict substitution method when light sources are being compared as to color temperature.

3. Comparison with the empirical relations

The three empirical relations mentioned in the introduction may be indicated symbolically:

$$\Delta\lambda_c = K_1, \quad (1)$$

$$\Delta(1/\theta) = K_2, \quad (2)$$

$$\Delta r = K_3, \quad (3)$$

where the symbol, Δ, stands for "doubtful difference," and $\Delta\lambda_c$ is to be read "doubtful difference in spectral centroid of light," $\Delta(1/\theta)$, "doubtful difference in reciprocal color temperature," and so forth; K_1, K_2 and K_3 are constants, independent of color temperature, which characterize the observer and the experimental conditions. These three empirical relations are represented by the three horizontal lines appearing in Fig. 1; the degree of agreement of the

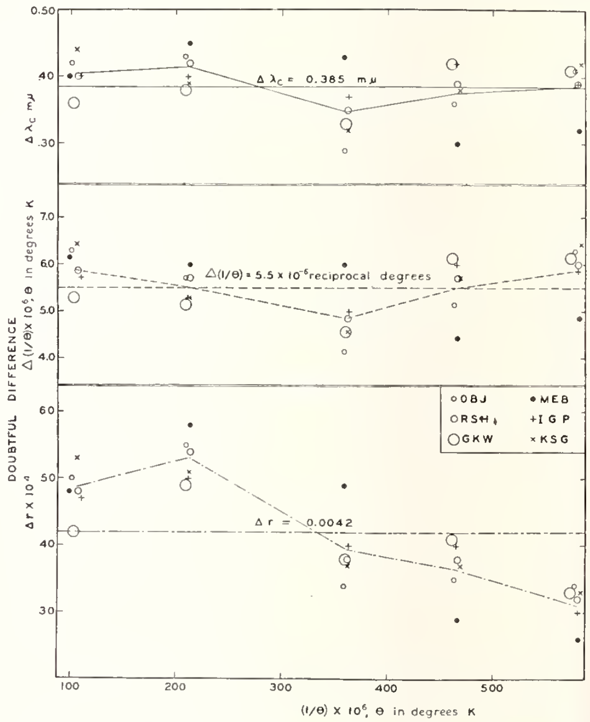

FIG. 1. Individual experimental values of the doubtful difference expressed according to the three parameters suggested by the three empirical relations. If one of the empirical relations represented the experimental results perfectly the corresponding plotted points would not deviate significantly from a horizontal line. Approach to the horizontal indicates, therefore, the degree of approximation. The scales are adjusted so as to afford a fair comparison of the three relations.

experimental results with these empirical relations can be seen from the plotted points which represent individual results and from the broken lines which connect points representing for each color temperature the average for all observers.

It will be seen that all three relations represent fairly well the experimental results. However, the

[8] Analysis shows that variation from day to day accounts for a large part of this scatter.

agreement is particularly good in the case of the spectral-centroid relation discovered by Priest, and in the case of its restatement in terms of color temperature squared (difference of reciprocal color temperature) by Davis. Either of these two relations represents the experimental results so closely that no attempt has been made to find a better law.

In order to show what variation of the doubtful difference in color temperature ($\Delta\theta$) as a function of color temperature is called for by these three empirical relations, it is convenient to write them in other terms:

$$d\theta/dE = -K_1 d\theta/d\lambda_c, \qquad (1a)$$

$$d\theta/dE = -K_2 d\theta/d(1/\theta) = K_2\theta^2, \qquad (2a)$$

$$d\theta/dE = -K_3 d\theta/dr, \qquad (3a)$$

where dE is the chromaticity difference,[9] expressed as the number of doubtful differences, corresponding to the difference in color temperature, $d\theta$. The ratio, $d\theta/dE$, may be read "doubtful difference in color temperature" which is also represented by $\Delta\theta$. If the constants, K_1, K_2 and K_3 be solved for from relations (1a), (2a) and (3a), comparison with relations (1), (2) and (3) indicates that $d\lambda_c/dE$, $d(1/\theta)/dE$, and dr/dE are the same (except for algebraic sign) as $\Delta\lambda_c$, $\Delta(1/\theta)$, and Δr, respectively. The change in sign arises from the fact that λ_c, $1/\theta$, and r are all decreasing functions of θ.

Fig. 2 shows the right-hand members of relations (1a), (2a) and (3a) plotted as ordinate against color temperature, θ, as abscissa. The plotted points (circles) refer to the average experimental values of $d\theta/dE$. Except for the highest temperatures, these curves and points are shown twice, the scale for the upper set being ten times that for the lower. It is again demonstrated that relations (1) and (2) are in good agreement with experiment.

Table II shows the degree of agreement of these three empirical relations with the experimental results more specifically than can be seen from Figs. 1 and 2. It is seen that the average difference expressed in percent between the spectral-centroid relation and the experimental result is 4.9. Relation (2) departs from the experimental results by scarcely more, 5.1 per-

[9] E for "*Empfindung*," a fairly common usage among European writers; see also footnote 3, p. 8, for reference to a paper in which this symbol is used extensively.

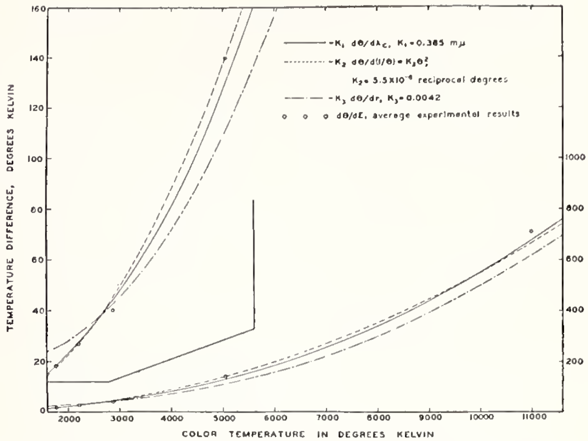

FIG. 2. Mean experimental values of the doubtful difference in color temperature as a function of color temperature compared to the three empirical relations. The rapid variation of the doubtful difference expressed in terms of color temperature compared to its slow variation in the other parameters (see Fig. 1) indicates the relative inconvenience of color temperature for the specification of illuminants. Reciprocal color temperature is suggested as a substitute parameter.

TABLE II. *Check of the three empirical relations against average experimental results.*

Empirical relation	θ	From the experimental results $-d\lambda_c/dE = K_1 = \Delta\lambda_c$	Deviation from mean	Average deviation, percent of mean
$\frac{d\theta}{dE} = -K_1 \frac{d\theta}{d\lambda_c}$	1,764	0.386	0.001	
	2,213	.375	.010	
	2,875	.348	.037	
or $\Delta\lambda_c = K_1$	5,046	.415	.030	
(Priest)	10,990	.404	.019	
	Mean	0.385	0.019	4.9
		$(d\theta/dE)(1/\theta^2) = K_2 = \Delta(1/\theta)$		
$\frac{d\theta}{dE} = +K_2\theta^2$	1,764	5.87×10^{-6}	0.34×10^{-6}	
	2,213	5.50	.03	
	2,875	4.86	.67	
or $\Delta(1/\theta) = K_2$	5,046	5.52	.01	
(Davis)	10,990	5.89	.36	
	Mean	5.53	0.28	5.1
		$-dr/dE = K_3 = \Delta r$		
$\frac{d\theta}{dE} = -K_3 \frac{d\theta}{dr}$	1,764	0.00310	0.00107	
	2,213	.00365	.00052	
	2,875	.00395	.00022	
or $\Delta r = K_3$	5,046	.00531	.00114	
(Judd)	10,990	.00486	.00069	
	Mean	0.00417	0.00073	17.5

cent. Relation (3) differs from the experimental results by an average of 17.5 percent.

Probably these comparisons are sufficient to justify the conclusion that relation (3) fails to represent normal sensibility exactly; that is, the discrepancies are about twice as large as the overall experimental uncertainty; but the comparisons are insufficient to establish that either of the other two relations is in error. Although relation (1) agrees slightly better with the experimental mean doubtful difference than does relation (2), this slightly increased agreement is, of course, not significant. Hence, in the remainder of the paper we take relation (2) to represent the doubtful difference in color temperature because of its greater simplicity; that is, we take:

$$d\theta/dE = -5.5 d\theta/d(1/\theta)\times 10^{-6} = 5.5\theta^2\times 10^{-6} \quad \text{(2b)}$$

in degrees Kelvin per doubtful difference for observation by one eye of a 2° circular field yielding a retinal illumination of 30 photons with dark surroundings. With larger fields, binocular vision, and higher illuminations the doubtful difference may be reduced to one-fifth of these values.

Relation (2b), which has been chosen because of its simplicity, has, it appears, no fundamental significance; it approximates the true relation between the color-temperature limits 1700 and 11,000°K. Relation (3), which is a slightly poorer approximation between these limits, touches more closely on the actual phenomena; for example it indicates correctly the rise to infinity of $d\theta/dE$ as θ approaches zero. This corresponds to the fact that the stimuli defined by Planck's law for very low temperatures are visually identical (saturated red).

The form of relation (2) suggests that reciprocal color temperature would be a more convenient parameter for general use than color temperature, itself, because differences in reciprocal color temperature are proportional to the corresponding chromaticity differences. The use of reciprocal temperature instead of temperature is already fairly prevalent because of the form of the Wien radiation law.

4. Daylight filters and color constancy

From the verification of relation (1) there follows immediately an important relation having to do with blue filters of such transmissive characteristics that they serve to convert radiant energy of low color temperature (2000 to 3000°K) to high color temperatures (3000 to 12,000°K). Certain of these filters are known as "daylight filters," because of their use in producing artificial daylight in its various phases. It was pointed out in 1925 by Gibson[10] that for a given artificial daylight filter a linear relation exists between the spectral centroid of light at various color temperatures and that of the same light transmitted by the filter. He gave a graph demonstrating for seven artificial daylight filters of considerably varying transmissive characteristics not only that this relation is linear but also that the slope of the straight line is nearly unity. From the linearity of the relation we may conclude that a change in the color temperature of an incandescent source corresponding to a given chromaticity interval will produce a change in chromaticity of the light transmitted through such a filter equal to a constant times the given chromaticity interval regardless of the color temperature of the source. Furthermore, from the fact that the slope of the straight line is nearly unity, we may conclude that the chromaticity interval is nearly the same for any color temperature of source regardless of whether the two lamps exhibiting the color-temperature interval be viewed with the filter or without it, provided, of course, that the brightness is still sufficient for daylight vision. This conclusion, contingent on the validity of Priest's spectral-centroid relation, has already been used by Davis and Gibson[11] when they pointed out that the use of their filters in the determination of color temperature would probably yield just as high a precision as could be obtained without them.

As a demonstration of the accuracy of this conclusion there is compared in Fig. 3 the doubtful difference according to the color-temperature-squared relation (2a) with the doubtful difference computed from the assump-

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[10] K. S. Gibson, *Spectral Centroid Relations for Artificial Daylight Filters*, J. Opt. Soc. Am. and Rev. Sci. Inst., 11, 473–478 (1925).

[11] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub. Bur. Stds., No. 114, 79, January, (1931).

tion that the interposition of an artificial daylight filter before two lamps makes no change in the size of the chromaticity difference between them. This latter doubtful difference is found by multiplying a doubtful difference in color temperature of source, $5.5\theta_s^2 \times 10^{-6}$, by the ratio of the change in color temperature of the transmitted light to the change in color temperature of the incident light, $d\theta/d\theta_s$, θ_s being the color temperature of the incident light from the source.

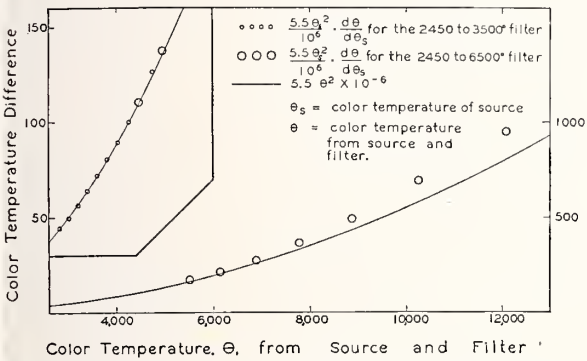

FIG. 3. Comparison of the doubtful difference in color temperature according to relation (2a) with that obtained by the assumption that the interposition of a daylight filter leaves the size of the chromaticity difference between two light sources unchanged. It is seen that this assumption is justified for the Davis-Gibson 2450-to-3500° filter, but not quite for the 2450-to-6500° filter which increases the chromaticity difference slightly.

Fig. 3 shows this comparison for two Davis-Gibson filters, 2450-to-3500°K and 2450-to-6500°K.[12] The curve shows the true variation of the doubtful difference (relation (2a)); the small circles show the variation of the doubtful difference computed from the assumption that the interposition of the 2450-to-3500°K filter leaves the size of the chromaticity difference unchanged; the large circles refer to the 2450-to-6500°K filter.

It is seen from Fig. 3 that the interposition of the less dense blue filter has made no definite change in the size of the chromaticity difference; that is, observation through this filter of two lamps just doubtfully differing in color temperature would result in the lamps still being judged just doubtfully different. The more dense filter slightly magnifies the chromaticity difference although the increase is scarcely significant. This is in conformity with the previous tentative conclusion by Davis and Gibson. We conclude, then, for all artificial daylight filters, and probably for most filters of whatever type that are not more highly selective than these, that the size of the chromaticity interval remains substantially unchanged regardless of whether the two nearly identical stimuli affect the eye immediately or after having passed through the filter.

It should be noted that this conclusion bears directly on a long known phenomenon called color constancy.[13] Color constancy refers to the fact that the colors of the usual objects of vision, which depend largely on selective reflection of light by surfaces, are nearly constant independent of the illuminant throughout a wide range. The classic illustration of color constancy is the fact that "white" objects such as paper, bleached linen and fresh snow are perceived as "white" and universally described as "white" even though illuminated by sources of widely varying spectral distribution of energy. Thus (says Kroh, p. 181), "we name and see the leaves of a book, in which we are reading, white, in spite of the fact that the reflected light may be the reddish-yellow light of a candle, the yellowish-green summer light in the woods, or the bluish-green light of a veranda having a glass roof of that color."

Long discussion led many to the conclusion that color constancy is a question of perception. It is said that a "white" object is seen as a "white" object under various illuminants of widely different spectral distribution of energy largely because the observer recognizes it and knows it to be "white." Although retinal adaptation tends in the same direction, an essential factor in color constancy is said to be a "painting

[12] See footnote 6, p. 8. This paper gives data for finding $d\theta/d\theta_s$.

[13] E. Hering, Grundzüge der Lehre vom Lichtsinn, aus Graefe-Sämisch Handb. d. ges. Augenheilk., XII, 1905 and 1907. H. v. Helmholtz, Handb. d. Physiol. Optik, 3d Ed., **2**, 233, 243 (1911). D. Katz, *Die Erscheinungsweisen der Farben und ihre Beeinflussung durch die individuelle Erfahrung, Ergänzungsband VII*, Zeits. f. Psychol., 1911. E. R. Jaensch, *Über Farbenkontrast und die sog.* Berücksichtigung farbigen Beleuchtung, Zeits. f. Sinnesphysiol. **52**, 165–180 (1921). O. Kroh, *Über Farbenkonstanz und Farbentransformation*, Zeits. f. Sinnesphysiol. **52**, 181–216, 235–273 (1921). A. Gelb, *Die "Farbenkonstanz" der Sehdinge*, in Handb. d. normalen u. pathologischen Physiol. 12, 1st half, Receptionsorgane II, 594–678; Berlin, Springer; 1929.

over of things with memory color" which makes them appear as they were previously experienced in daylight.

Although the present results do not deal directly with the color of objects seen under different illuminants, they may be stated (as is seen presently) as a case in which a change in illuminant has produced little or no change in the size of the color intervals between objects. Let us see what connection this constancy of color intervals may have with color constancy.

The phenomenon of color constancy is commonly described as if colors might be spoken of in an absolute sense, but in the usual descriptions of color constancy (such as that of Kroh, cited above) it is evident that other objects than that expressly referred to are present in the field of view; that is, we are not concerned with the constancy of the color of a uniform object filling the whole field of view, but rather with the color of an object seen against a background. If we had some pale yellow objects lying on a white background, and changes in the spectral distribution of the illuminant resulted in such objects appearing indistinguishable from the background (or even if the distinction became notably less), we would not think of speaking of "color constancy" in such circumstances. Hence, some degree of constancy of the color intervals must accompany every case of "color constancy"; and if it can be shown that the color intervals between objects are constant independent of illuminant because of purely retinal activity, we shall have gone a long way toward establishing the retina as the seat of the processes essential to color constancy.

We may construct from the results presented in Fig. 3 a quantitative example of the constancy of color intervals between the members of a series of yellow samples of a certain type of spectral reflectance. The series of black-body colors from 2000 to 3000°K may be thought of as produced by a series of objects varying from yellow to white illuminated by incandescent-lamplight at a color temperature of 3000°K. Viewing these lights through a daylight filter is, of course, equivalent to illuminating the hypothetical samples with daylight. The fact that the chromaticity distinction between two lamps is very little changed by observation through a daylight filter means for this case that the chromaticity distinction between a pale yellow sample and a white sample will be very little changed by a shift in illuminant from incandescent-lamplight to daylight. Furthermore, it is evident from the manner in which these data were obtained that no question of a possible effect of memory color or recognition of objects can arise; the conclusion holds, therefore, independent of perception or memory color.

This special case strongly suggests that all cases of color constancy involve a high degree of constancy of the size of the color intervals between objects entirely independent of perception or memory color. Another special case of this tentative generality has already been pointed out by A. Kohlrausch.[14] He links the approximate constancy of achromatic colors to the approximate validity of Weber's law concerning sensibility to brightness change. If the tentative conclusion, supported by these two special cases, is actually true for all cases, retinal discrimination combined with retinal adaptation accounts for nearly the whole of color constancy. That retinal processes do not wholly account for color constancy and the allied phenomena of color transformation, is proven by the work of Katz, Jaensch, Kroh and Gelb; but it is likely that the post-retinal effects extend the ability of the observer to judge object colors simply by extending the ability, already provided him to a considerable degree by retinal adaptation, to discount the color of the illuminant. Memory color, if it enters at all, need be invoked for only a very minor and unessential role.

[14] A. Kohlrausch, *Tagessehen, Dämmersehen, Adaptation, I. Allgemeines über Umstimmung und "Farbenkonstanz der Sehdinge,"* in Handb. d. normalen u. pathologischen Physiol. **12**, 2d half, Receptionsorgane II, 1499–1506; Berlin, Springer (1931).

SATURATION SCALE FOR YELLOW COLORS

J Opt. Soc. Am. 23, 35-40 (1933)

The discussion of saturation in "A general formula for the computation of colorimetric purity", J. Res. Nat. Bur. Stand. (U.S.), 7, 827-841 (1931) RP377, should be considered in connection with this paper. The "empirical relation via O.S.A. excitation curves" shown in Fig. 1 is proportional to excitation purity determined in Fig. 1 of Ref. 11. If the results in Fig. 1 of this paper were expressed and plotted in terms of excitation purities thus determined, the "empirical relation" would be straight and the "graphical mean" curve would be correspondingly less convex upwards. It cannot be made straight by use of any projective transformation of Fig. 1 of Ref. 11, or any other color-mixture diagram.

Results that confirm the constancy of saturation scales for large changes of step size, which Judd reported, have recently been published by Marcus and Billmeyer, "Step size in the Munsell color-order system", Colour 73: ... Second Congress of the International Colour Association ... 1973 (Hilger, London), pp. 290-293; J. Opt. Soc. Am. 65, 208-212 (1975); Color 77: ... Third Congress of the International Colour Association ... 1977 (Hilger, London), pp. 488-491.

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Saturation Scale for Yellow Colors*

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The relationship between colorimetric purity of the stimulus and saturation of the color evoked under specified observing conditions has been determined for yellow colors by using the yellow series of Lovibond glasses (dominant wave-length about 575 $m\mu$). Two observers have selected glasses from this series requisite to produce a scale progressing by equal steps from white to yellow. The step size varied from something approaching the least perceptible in one determination to about four times that size in another; it was found that this variation produced no significant change in the shape of the curve connecting purity and saturation. An increase in step size by an additional factor of three was tested for five observers. It was found chiefly to introduce greater individual difference; the average for all observers was not significantly changed. It is concluded that the integral of the sensibility to purity change from zero purity may safely be taken as the saturation of the yellow color evoked under the present observing conditions. The saturation-purity curve resulting from these determinations is compared with that from previous experimental data and with those from two empirical relations. In order to discover whether the discrepancies revealed by this comparison are ascribable to individual difference, groups of glasses representing each of four curves compared were selected and judgments obtained from eight observers with the result that the experimental scale was corroborated. The individual differences shown among these eight observers were too slight to account for the discrepancy. It is concluded that an empirical relation which was previously found fairly well justified by data on other types (wave-length, color-temperature) of chromaticity sensibility, though not perfect, yields a saturation scale for yellow which is close to the true one.

I. INTRODUCTION

SATURATION is defined by Troland[1] as "that attribute of all colors possessing a hue, which determines their degree of difference from a gray of the same brilliance. . . (footnote). It is a subjective attribute of color." This is a purely qualitative definition, but it has been customary to give saturation a quantitative meaning by calling a series of stimuli producing evenly spaced colors of the same hue and brilliance a saturation scale.[2] The saturation of a given color in such a series is measured by the number of steps from the beginning of the series, which is an achromatic color (gray) of the same brilliance. Besides depending upon the accuracy with which the visual intervals between successive members of this series are made identical, the legitimacy of quantifying saturation in this way depends on a number of assumptions, commonly tacit assumptions. The only one which matters here is the assumption

* Publication approved by the Acting Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] L. T. Troland, *Report of the Colorimetry Committee for 1920–21*, J. O. S. A. and R. S. I. **6**, 534–535 (1922).

[2] L. A. Jones and E. M. Lowry, *Retinal Sensibility to Saturation Differences*, J. O. S. A. and R. S. I. **13**, 25–34 (1926).

that the numbers purporting to be saturation are independent of the size of the visual interval between successive members of the series; this assumption would be proved wrong if it were found that a series made up of, say, every fifth stimulus of the original scale produced colors for which the successive differences no longer were equal.

It is the present purpose (1) to set up a saturation scale for yellow, (2) to test whether the size of the interval affects the scale, (3) to compare the saturation scale with previous results both direct and inferred from observations dealing with other hues, and (4) to discover whether discrepancies in published data may be referred to individual difference.

II. Experimental Method

The stimuli making up the saturation scale for yellow were produced by illuminating Lovibond yellow glasses[3] with light from a milk glass illuminated in turn by light chiefly from the overcast north sky. The dominant wave-length of this series of glasses for Abbot-Priest sun as illuminant and heterogeneous stimulus varies from 573 for the low numbers to 575 $m\mu$ for the high numbers, but there is no noticeable regular variation in hue. For increasing Lovibond yellow number, the colors produced increase in saturation and are of nearly constant hue. There exists, of course, also a decrease in brightness; but the effect of this brightness difference in comparing glasses of different Lovibond number was eliminated by using the Martens photometer.[4]

A single determination of the saturation scale involved the following processes: (1) The Lovibond yellow glass corresponding to one extreme of the saturation scale (either "clear glass" or 20Y) was adjusted so that it was seen in one-half of the 6° circular field of the Martens photometer; call this glass, glass (1). (2) A choice of another glass (2) was made such that the initial saturation step that it produced constituted a convenient standard to be maintained throughout the run. This size of step was varied from one to, perhaps, four "least perceptible differences" under these experimental conditions. (3) After scrutinizing, with brightness equalized, the saturation step so produced (1–2), the observer was required to choose a yellow glass (3) which produced a second step (2–3) of saturation equal to the first step (1–2). (4) The result of the third process was checked by requiring the observer to compare again the saturation step (2–3) with the saturation step (1–2). If the observer still judged these steps to be equal, the glass chosen by the third process was accepted. (5) Then the observer was required to select a fourth glass (4) such that step (2–3) was equal to step (3–4), and so on until the other extreme of the saturation scale was reached.

This method may be called the method of equal sense differences; it differs from the more usual method of successive liminal determinations first, and non-essentially, by reason of the larger steps, and second because the observer is enabled to obtain a constant check on the size of step to be chosen.

Six determinations, each on a different day, were made by one observer (DBJ); and three by another (JOR). Of these nine determinations, five started at the lower, four at the upper end of the scale.

III. Comparison with Previous Results

In comparing these results with previous saturation scales for yellow, it is convenient to specify the stimuli in terms of their colorimetric purities.[5] This convenience arises from the fact that if the other attributes of the visual stimulus, dominant wave-length and brightness, are kept nearly constant and within the usual limits, the changes in colorimetric purity of the stimulus produce colors of nearly constant brilliance and hue, but of widely varying saturation. On this account colorimetric purity may be taken as the

[3] The Tintometer (Ltd), The Colour Laboratories, Salisbury, England; 1912. K. S. Gibson and F. K. Harris, *The Lovibond Color System, I. A Spectrophotometric Analysis of the Lovibond Glasses*, B. S. Sci. Paper No. 547; Feb. 17, 1927.

[4] F. F. Martens, *Über ein neues Polarisationsphotometer*, Phys. Zeits. **1**, 299–303 (1900). D. B. Judd and G. K. Walker, *A Study of 129 Lovibond Red Glasses with Respect to the Reliability of Their Nominal Grades*, Oil and Fat Industries **5**, 17–18 (1928).

[5] I. G. Priest, *Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity, and Brightness*, J. O. S. A. and R. S. I. **8**, 173–200 (1924). *The Computation of Colorimetric Purity*, J. O. S. A. and R. S. I. **9**, 503–520 (1924). D. B. Judd, *The Computation of Colorimetric Purity*, J. O. S. A. and R. S. I. **13**, 133–154 (1926).

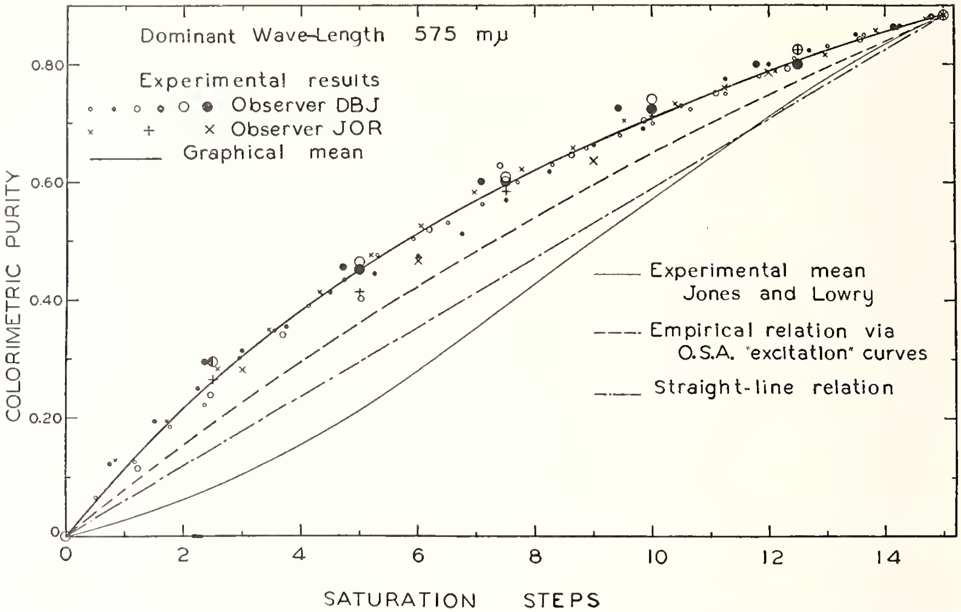

FIG. 1. Colorimetric purity of the stimulus as a function of the saturation of the colors produced. The present experimental results are compared to a previous experimental mean and to two empirical relations.

stimulus analogue of the subjective attribute, saturation. Indeed, so close and striking is the connection between the stimulus entity, colorimetric purity, and the response entity, saturation, that the two terms have frequently been confused.[6]

The purities of the glasses were read from a curve drawn smoothly among points representing the purities of the unit glasses from $N=1$ to $N=20$, also for $N=0.1$ and $N=0.01$. These values were computed from the spectral transmissions of the glasses previously determined by Gibson and Harris.[7] In these computations the O.S.A. "excitation" curves in their extrapolated form[8] were used, and Abbot-Priest sunlight[9] was taken as the heterogeneous stimulus and as the illuminant. Although the light from the overcast north sky passing through the slightly greenish-yellow milk glass which was used is not an accurate duplication of Abbot-Priest sunlight, it is known that the degree of duplication is amply sufficient for this purpose.

Fig. 1 shows the experimental data in terms of colorimetric purity plotted against the number of steps; the total saturation difference between $N=0$ and $N=20$ is taken arbitrarily at fifteen steps; this is a convenient choice because the size of interval is then about equal to one step of the Jones hue scale.[10] It is possible under our experimental conditions to detect more than fifteen differences between $N=0$ and $N=20$; for example, one determination shown in the figure requires twenty-five small circles for its representation. But, since the purpose is merely to determine

[6] D. McL. Purdy, *On the Saturations and Chromatic Thresholds of the Spectral Colours*, Brit. J. Psych. (General Section) **21**, Part 3, 284 (1931). D. B. Judd, *A General Formula for the Computation of Colorimetric Purity*, J. O. S. A. **21**, 741–747 (1931).

[7] See reference 3. Additional data required, and all computations of dominant wave-length and purity, were already available as part of the general standardization of Lovibond glasses.

[8] Spectrophotometry, *Report of the Optical Society of America Progress Committee for 1922–23*, J. O. S. A. and R. S. I. **10**, 230 (1925).

[9] I. G. Priest, *A Precision Method for Producing Artificial Daylight*, Phys. Rev. [2] **11**, 502 (1918). D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, B. S. J. Research **4**, 525 (1930).

[10] L. A. Jones, *The Fundamental Scale of Pure Hue and Retinal Sensibility to Hue Differences*, J. O. S. A. **1**, 63–77 (1917). The Jones "hue scale" also appears in the *Report of the Colorimetry Committee of the O. S. A., 1920–21*, J. O. S. A. and R. S. I. **6**, 545 (1922), under the title, *Spectral Chroma Scale*, which is consistent with the terminology recommended in that report.

the shape of the curve, the particular size of step chosen in this comparison is of no consequence.

It may be seen from Fig. 1 that results by the two observers are in good agreement, and that the variation in size of step by a factor of four does not change the shape of the curve significantly. Later checks by five observers with the same glasses used in a similar, though less direct way has shown that a further increase in size of interval by a factor of three likewise produces no significant effect on the saturation scale; it did show that the use of very large steps, steps 7 or 8 times the size of one step on the Jones hue scale, introduces considerable uncertainty into the determination and increases markedly the individual differences between observers, but the average results of five observers agreed closely with the experimental results indicated in Fig. 1 by the heavy solid curve drawn among the plotted points. A previously reported constant error due to large size of step[11] was proved by this check to be not representative of the average observer. It may be concluded, therefore, that the saturation scale for yellow is independent of the step size used in its determination by an average observer. Hence, saturation of yellow colors may be defined quantitatively as the integral of the purity sensibility from zero up to purity, p. This corroborates for one hue the tacit assumption commonly made for all hues.[12]

There is only one previous determination of a saturation scale for yellow with which to compare our experimental results; this is the determination by Jones and Lowry[2] for dominant wave-length 575 $m\mu$; these results are given in Fig. 1 by the light solid curve.

Sensibility to purity change has been determined for other hues, however, and from these data it has been independently suggested by Munsell[13] and Donath[14] that sensibility to purity change for any given hue is approximately constant independent of purity. If we take, as is usual, the integral of the purity sensibility from zero up to purity, p, as a quantitative expression for saturation, this constancy leads, of course, to a straight-line relation between saturation and purity. This proportionality has doubtless contributed to the confusion, previously mentioned[6] between the two terms. The relation is represented in Fig. 1 by the straight line (dot-dash) connecting the extremes of the other curves.

A second relation has been suggested as a representation of sensibility to purity change, dE/dp, as a function of purity, p:[15]

$$dE/dp = S(3C-1)/K[3S(1-p)+p]^2 \quad (1)$$

where S and C are functions of wave-length simply defined from the O.S.A. excitation curves, and K is a constant depending on the size of the steps in terms of which the color difference, dE, is expressed. (For dE in steps of the size of those in the Jones hue scale, K is approximately 0.01.) This relation has been found to represent Donath's data somewhat more closely than the straight-line relation which he suggested. It is a special case of the general empirical expression which has been found to accord approximately with sensibility data of a number of kinds (wave-length, color-temperature) in addition to sensibility to purity change. According to this expression, the saturation difference between two colors of the same hue is proportional to the distance on the Maxwell triangle between the two points representing the colors. Integration of (1) from zero purity up to purity, p, gives:

$$\int_0^p (dE/dp)dp = p(3C-1)/3K[3S+p(1-3S)]. \quad (2)$$

For dominant wave-length equal to 575 $m\mu$, $C=R=0.51575$,[16] $S=0.48634$; and if we take $K=0.01017$ for the sake of accord with the arbitrarily chosen step size, there results:

$$\text{Number of saturation steps} = 54.7\, p/(4.45-1.40\, p). \quad (3)$$

[11] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. O. S. A. **22**, 99 (1932). The two similar cases mentioned for red may likewise prove to be not representative.

[12] See Introduction.

[13] A. H. Munsell, *On the Relation of the Intensity of Chromatic Stimulus (Physical Saturation) to Chromatic Sensation*, Psych. Bull. **6**, 238–239 (1909).

[14] F. Donath, *Die funktionale Abhängigkeit zwischen Reiz und Empfindung bei der Farbensättigung*, Neue Psych. Stud. **2**, 143–207 (1926).

[15] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. O. S. A. 22, 91 (1932).

[16] D. B. Judd, *Interpolation of the O. S. A. "Excitation" Data by the Fifth-Difference Osculatory Formula*, J. O. S. A. **21**, 531–540 (1931).

This is the equation of the dashed curve of Fig. 1. It will be noted that, although this equation fails significantly to represent our experimental results, it agrees with them better than the determination of Jones and Lowry made under different experimental conditions; it is also a somewhat better representation of them than the straight-line relation.

IV. Dependence on Individual Observer

It is, perhaps, not surprising that the straight-line relation fails to represent our experimental data; we should regard the straight-line relation at best merely as a first approximation.[17] Likewise is the agreement with the experimental results of the empirical relation via the O.S.A. excitation curves about all that should be expected; other data have shown the general empirical relation, from which Eq. (3) was derived, to be unreliable by as much as a factor of two in other special cases. The failure, however, of our experimental results to agree with the direct determinations of Jones and Lowry deserves consideration; as may be judged from the points plotted in Fig. 1, the discrepancy is three or four times too large to be explained by our experimental error. Since only two observers were used in each case, and since it is conceivable that observers may differ importantly in their judgment of the relative sizes of two saturation differences, the suspicion was justified that individual difference might be the source of the discrepancy. Accordingly, a convenient method of checking the importance of individual difference was devised which also has the advantage of introducing experimental conditions resembling those usual in comparing the colors of material objects.

The saturation scales indicated by the four curves of Fig. 1 were set up by Lovibond yellow glasses, eight glasses, placed side by side, in each scale. The four scales were arranged one above the other and illuminated by a large milk glass screen (brightness approximately 50 millilamberts) which, in turn, was illuminated by light from the overcast north sky. The observer's eyes were separated from the vertical plane of the Lovibond glasses by about one meter so that the row of glasses making up the scale subtended about 10 degrees at the observer's eyes, and any two consecutive glasses formed an approximately square field about 2° across and divided vertically. The surrounding field (brightness about 1 ml) was gray and approximately 40° in extent. A similar arrangement was used in comparing three eleven-glass scales, the scale according to Jones and Lowry being omitted.

This arrangement is advantageous in that it permits the observer to use both eyes in a well-accustomed way; on the other hand, noticeable differences in brilliance, as well as saturation differences, were present and had to be discounted by the observers.

The observer was asked to point out the scales in order of their approximation to perfection. The perfect, or ideal, scale was defined as one in which the saturation steps are equal. The observer was also asked to point out the specific faults in his second, third, and fourth choices which resulted in ranking them lower than first. The responses of eight observers were noted both for the eleven-glass and for the eight-glass scales, two of the most experienced observers (IGP, KSG) repeating their criticisms of the eight-glass scales on another day, making 18 separate judgments for three of the scales, and 10 for the fourth. The observers were ignorant of the identities of the scales. A summary of these responses is given in Table I.

An inspection of the table shows that the experimental scale obtained by the method of

Table I. *Distribution of the judgments among the four saturation scales.*

Judgments	Judd and Riley	Eq. (3)	Straight-line	Jones and Lowry
Best scale	15	3	0	0
Second best	3	14	1	0
Third best	0	1	15	2
Fourth best	0	0	2	8
Steps too large at low saturation	4	0	0	0
Equal steps	9	2	0	0
Steps too small at low saturation	5	16	18	10

[17] For some hues, blue and violet, the straight-line relation between saturation and colorimetric purity would seem to be not even approximately correct. Observation by one observer (DBJ) for dominant wave-length 452 $m\mu$ indicates this relation to be in error by a factor of 40 (J. O. S. A. **22**, 94 (1932)).

equal sense differences was preferred; it may be noted, also, that the scale set up according to Eq. (3) is satisfactory to some observers. It may be seen, however, that individual difference is not a source of sufficient uncertainty to account for the discrepancy between our experimental results and those of Jones and Lowry. This discrepancy remains unexplained; it might be due to (1) instrumental error, (2) difference in psychophysical method, or (3) difference in experimental conditions.

The approximate corroboration of the empirical relation based on distances on the Maxwell triangle confirms the use of trilinear coordinates to facilitate the writing of uniform color tolerances.

THE 1931 ICI STANDARD OBSERVER AND COORDINATE SYSTEM FOR COLORIMETRY

J. Opt. Soc. Am. 23, 359-374 (1933)

This, the definitive American presentation of the 1931 recommendations of standard illuminants, color-mixture data, and chromaticity diagram, was prepared and published at the request of the Committee on Colorimetry of the Optical Society. The revision of the 1922 Colorimetry Report, mentioned in the title footnote (b), was not published until twenty years later, so this paper was the principal source of information from which colorimetry developed during two decades. Table VI contains data that are not available elsewhere. The terms "trilinear coordinates" and "trichromatic coefficients" are now rarely used. The term "chromaticity coordinates" is used instead. The terms "distribution coefficients" and "distribution functions" are also now rarely used; "color-mixture" values or functions or curves, are used instead. The plot of x (horizontally) and y (vertically), Fig. 2, became customary, probably as a result of footnote 13 on p. 363. But the approximation mentioned was quickly found to be not good enough. The next paper in this book was the most notable effort to provide a better approximation. Almost all subsequent efforts have grown out of that. The term "mixture diagram" is now rarely used to refer to Fig. 2; "chromaticity diagram" is used instead. "Wavelength" is now written and printed as a single word, without the hyphen used throughout this paper. Colorimetric purity is now rarely used; excitation purity, which for spectral colors is simply the value of 1 -f (where f is defined and computed on p. 367), is used instead. It is usually called simply "purity". In the case of nonspectral colors (purple, magenta, etc.), the same formula is used for f, but the coordinate X, or Y is of the point where the straight line drawn from the white point through the sample point x,y intersects the straight line that connects the short- with the long-wave ends of the spectrum locus. Note that the capital letters X and Y used on pp. 365 to 368 there represent chromaticity coordinates, not tristimulus values. Following the notation used by Judd from p. 369 to p. 374, colorimetrists quickly adopted the convention of using lower-case x and y (without overbars) for chromaticity coordinates, with subscript w to designate those for the white point, subscript s to designate those for a point on the spectrum locus, and subscript c to designate those for a point on the line that connects the short- with the long-wave ends of the spectrum locus. Therefore, pp. 366 to 368 are probably the last place in Judd's writings where capital letters are used to designate chromaticity coordinates. The capital letters X, Y and Z are reserved for tristimulus values, defined by the integrals or summations shown on p. 364. Overbars are used only to designate color-mixture data, as in Tables II to IV, Fig. 1, and the formulas on pp. 364 and 365.

The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry[a, b]

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(Received August 3, 1933)

This report makes available in convenient form the properties of the standard observer recently recommended for colorimetric purposes by the International Commission on Illumination. These data supersede the values published in the 1922 report of the committee on colorimetry known as the O.S.A. excitation data. Forms are given for computing trilinear coordinates (trichromatic coefficients), dominant wave-length, colorimetric purity, and luminous transmission (or reflectance) from spectrophotometric data. Tables of the data needed are included for the 1931 I.C.I. standard illuminants *A*, *B* and *C*.

I. Introduction

Human vision depends on a mechanism that is both complicated and incompletely understood. This mechanism, considered as a detector of radiant energy, is capable of responding to and distinguishing between many different wave-length distributions of energy, and it is adaptable to a wide range of energies. The visual mechanism adapts itself by variations in sensitivity, and on this account quantitatively accurate visual judgments are ordinarily not possible. The mechanism, however, for fields of small angular extent and within the range of energies yielding daylight (photopic) vision, is a satisfactory null instrument. Thus, if two stimuli of different wave-length distribution of energy be found which are once responded to alike by the eye, they will still be seen alike after exposure to another stimulus even if this pre-exposure stimulus is such as to change considerably the appearance of the two equivalent stimuli. For example, if a portion of the spectrum near 640 mμ (red) be superposed on a portion near 550 mμ (yellow green), it will be found possible to obtain the color of this combination from an intermediate portion of the spectrum, say 600 mμ (orange). If the eye is then exposed to a high illumination of wave-length near 640 mμ, and its sensitivity to radiant energy of this wave-length region considerably reduced in this way, it is found that, although neither of the equivalent stimuli appears orange, they still give identical colors, for example, they may yield identical yellows or identical greenish yellows. Hence stimuli once found to be equivalent are always equivalent.

Colorimetry is based on these properties of the normal visual mechanism which make it a satisfactory null instrument. Any color stimulus may be specified for a given observer by finding a convenient second stimulus which is equivalent to it. This second stimulus is most often taken as an additive combination of three stimuli, it being possible to color match nearly any stimulus by the combination of three suitably chosen stimuli. These three stimuli are called primary stimuli, and the specification consists of giving the amounts of each primary stimulus required for the match. This is known as the tristimulus system of color specification.

An outstanding advantage of the tristimulus system of specification lies in the fact that the specification of a color stimulus made up of the sum of any number of component stimuli is obtained by merely adding together the specifications of the components. That is, if $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$, $(\bar{x}_2, \bar{y}_2, \bar{z}_2)$, $\cdots$ $(\bar{x}_n, \bar{y}_n, \bar{z}_n)$, be the respective tristimulus specifications of n stimuli, the specification of the stimulus obtained by the additive combination of these n components is $(\sum_1^n\bar{x}_n,\ \sum_1^n\bar{y}_n,\ \sum_1^n\bar{z}_n)$. Hence it is possible to

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[a] Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[b] Special report prepared at the request of the Committee on Colorimetry for 1933–34. An enlargement and revision of the 1922 Colorimetry Report (J.O.S.A. and R.S.I. **6**, 527–596 (1922)) is in preparation.

[c] U. S. joint representative in colorimetry for the International Commission on Illumination.

compute all equivalent stimuli for an observer from his tristimulus specifications of the spectrum stimuli, because any stimulus may be considered as the sum of a number of spectrum stimuli. The properties of the standard observer are therefore completely defined by giving his tristimulus specifications of the spectrum stimuli with respect to any three primary stimuli whatever. Furthermore, from this information specifications given in terms of one set of primary stimuli may be transferred to any other set of primary stimuli by appropriate transformation equations, this being equivalent to a transfer from one coordinate system to another. There are therefore two considerations in the definition of a standard observer; the first is the determination of a tristimulus specification of spectrum stimuli which shall be representative of normal observers, the second is the selection of a coordinate system which makes the properties of the standard observer conveniently available.

The present report is intended (1) to give specifications for the standard illuminants recommended by the International Commission on Illumination as a result of the Cambridge meetings,[1] (2) to define the 1931 I.C.I. standard coordinate system, (3) to define the properties of the 1931 I.C.I. standard observer in this coordinate system, (4) to give forms and tables facilitating computation of trilinear coordinates (trichromatic coefficients), dominant wavelength, colorimetric purity, and luminous transmission (or reflectance), and (5) to give transformation equations to previous O.S.A. coordinate systems. These standard data supersede those given in the 1922 report of the colorimetry committee.[2] The I.C.I. recommendations deal also with reflection standards, and standard mode of illuminating samples of reflecting materials, and give the properties of the standard observer in another coordinate system. These subjects are not discussed in the present report.

II. Standard Illuminants

"It is recommended[3] that the following three illuminants be adopted as standards for the general colorimetry of materials:

"*A*. A gas-filled lamp of color temperature 2848°K.[4]

"*B*. The same lamp used in combination with a filter composed of a layer one centimeter thick of each of two solutions B_1 and B_2, contained in a double cell made of colorless optical glass. The two solutions are made up as follows:

Solution B_1

Copper sulphate ($CuSO_4\cdot 5H_2O$)	2.452	g
Mannite ($C_6H_8(OH)_6$)	2.452	g
Pyridine (C_5H_5N)	30.0	cc
Distilled water to make	1000	cc

Solution B_2

Cobalt ammonium sulphate ($CoSO_4\cdot(NH_4)_2SO_4\cdot 6H_2O$)	21.71	g
Copper sulphate ($CuSO_4\cdot 5H_2O$)	16.11	g
Sulphuric acid (density 1.835)	10.0	cc
Distilled water to make	1000	cc

"*C*. The same lamp used in combination with a filter consisting of a layer one centimeter thick of each of two solutions, C_1 and C_2, contained in a double cell made of colorless optical glass. The two solutions are made up as follows:

Solution C_1

Copper sulphate ($CuSO_4\cdot 5H_2O$)	3.412	g
Mannite ($C_6H_8(OH)_6$)	3.412	g
Pyridine (C_5H_5N)	30.0	cc
Distilled water to make	1000	cc

Solution C_2

Cobalt ammonium sulphate ($CoSO_4\cdot(NH_4)_2SO_4\cdot 6H_2O$)	30.580	g
Copper sulphate ($CuSO_4\cdot 5H_2O$)	22.520	g
Sulphuric acid (density 1.835)	10.0	cc
Distilled water to make	1000	cc

"It is recognized that for certain special applications (for example, the specification of signal glasses) other luminous sources may be prescribed, but in the absence of special conditions

[1] *Proceedings of the 8th Session, Commission Internationale de l'Éclairage, Cambridge*, pp. 19–29, September, 1931. Note that the initials, C.I.E., which refer to the French form of the title are sometimes used as an abbreviation instead of the initials, I.C.I.

[2] L. T. Troland, Chairman, Report of Committee on Colorimetry for 1920–21, J.O.S.A. and R.S.I. **6**, 547–553 (1922).

[3] *Recommendation 2*. The quotation marks indicate translations of extracts from the official recommendations, which are given in French.

[4] "For calculations of the spectral distribution of energy, the constant, C_2, of Planck is taken equal to 14,350 micron degrees."

one of the three indicated sources should be used."

The liquid filter specified under illuminant *B* is the Davis-Gibson[5] filter for converting color temperature 2848 to 4800°K, and that specified under illuminant *C* is the Davis-Gibson filter for converting 2848 to 6500°K. Illuminant *A* is intended to be typical of light from the gas-filled incandescent lamp, illuminant *B* is an approximate representation of noon sunlight, and illuminant *C* is an approximate representation of average daylight. Table I[6] gives the spectral energy distribution of the three standard illuminants.

TABLE I. *Energy distribution of the three I.C.I. standard illuminants A, B, C for colorimetry.*

Wave-length (mμ)	E_A	E_B	E_C	Wave-length (mμ)	E_A	E_B	E_C
380	9.79	22.40	33.00	580	114.44	101.00	97.80
385	10.90	26.85	39.92	585	118.08	100.07	95.43
390	12.09	31.30	47.40	590	121.73	99.20	93.20
395	13.36	36.18	55.17	595	125.39	98.44	91.22
400	14.71	41.30	63.30	600	129.04	98.00	89.70
405	16.15	46.62	71.81	605	132.70	98.08	88.83
410	17.68	52.10	80.60	610	136.34	98.50	88.40
415	19.29	57.70	89.53	615	139.99	99.06	88.19
420	21.00	63.20	98.10	620	143.62	99.70	88.10
425	22.79	68.37	105.80	625	147.23	100.36	88.06
430	24.67	73.10	112.40	630	150.83	101.00	88.00
435	26.64	77.31	117.75	635	154.42	101.56	87.86
440	28.70	80.80	121.50	640	157.98	102.20	87.80
445	30.85	83.44	123.45	645	161.51	103.05	87.99
450	33.09	85.40	124.00	650	165.03	103.90	88.20
455	35.41	86.88	123.60	655	168.51	104.59	88.20
460	37.82	88.30	123.10	660	171.96	105.00	87.90
465	40.30	90.08	123.30	665	175.38	105.08	87.22
470	42.87	92.00	123.80	670	178.77	104.90	86.30
475	45.52	93.75	124.09	675	182.12	104.55	85.30
480	48.25	95.20	123.90	680	185.43	103.90	84.00
485	51.04	96.23	122.92	685	188.70	102.84	82.21
490	53.91	96.50	120.70	690	191.93	101.60	80.20
495	56.85	95.71	116.90	695	195.12	100.38	78.24
500	59.86	94.20	112.10	700	198.26	99.10	76.30
505	62.93	92.37	106.98	705	201.36	97.70	74.36
510	66.06	90.70	102.30	710	204.41	96.20	72.40
515	69.25	89.65	98.81	715	207.41	94.60	70.40
520	72.50	89.50	96.90	720	210.36	92.90	68.30
525	75.79	90.43	96.78	725	213.26	91.10	66.30
530	79.13	92.20	98.00	730	216.12	89.40	64.40
535	82.52	94.46	99.94	735	218.92	88.00	62.80
540	85.95	96.90	102.10	740	221.66	86.90	61.50
545	89.41	99.16	103.95	745	224.36	85.90	60.20
550	92.91	101.00	105.20	750	227.00	85.20	59.20
555	96.44	102.20	105.67	755	229.58	84.80	58.50
560	100.00	102.80	105.30	760	232.11	84.70	58.10
565	103.58	102.92	104.11	765	234.59	84.90	58.00
570	107.18	102.60	102.30	770	237.01	85.40	58.20
575	110.80	101.90	100.15	775	239.37	86.10	58.50
580	114.44	101.00	97.80	780	241.67	87.00	59.10

III. The Standard Coordinate System

"It is recommended[7] that the standard system of reference for the establishment of colorimetric specifications be a system in which the color of each stimulus is expressed on three scales established by giving values determined for four stimuli chosen so that no two of them can be combined to give a stimulus equivalent to one of the two others."

This is the tristimulus system of specification defined independent of the properties of the observer and also defined so as to permit for purposes of computational convenience the choice of imaginary stimuli as primaries.[8] The four real stimuli are called cardinal stimuli.[9]

"The four stimuli[10] which define the colorimetric scales shall consist of homogeneous radiant energy of wave-lengths 700.0, 546.1 and 435.8 mμ and of standard illuminant *B*. To these stimuli shall be assigned the following values:

	x	y	z
700.0 mμ	0.73467,	0.26533,	0.00000;
546.1 mμ	0.27376,	0.71741,	0.00883;
435.8 mμ	0.16658,	0.00886,	0.82456;
Standard illuminant *B*:	0.34842,	0.35161,	0.29997.

Note: The properties of the standard observer when they are expressed in the system so defined, are given in Table II."

The values assigned to the cardinal stimuli are such that for the standard observer the system has the following properties: (1) the luminosity coefficients of the x, y, and z scales are, respectively, 0, 1, and 0;[11] (2) there are no values less

[5] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Bur. Standards Misc. Pub. No. 114, January, 1931; see also Bur. Standards J. Research **7**, 796 (1931).

[6] This table is identical with Table V in a paper by Smith and Guild, *The C.I.E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 102 (1931–32).

[7] *Recommendation 4.*

[8] Imaginary stimuli are usually chosen for this purpose. One or more of the primary stimuli are imaginary in the tristimulus specifications of the spectrum known as (1) the König elementary sensation curves, (2) the König fundamental sensation curves, (3) the Abney sensation curves, (4) the König-Ives curves, and (5) the O.S.A. excitation curves. The chief gain in computational convenience is the avoidance of negative quantities.

[9] T. Smith and J. Guild, *The C.I.E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 74 (1931–32).

[10] *Recommendation 5.*

[11] D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, Bur. Standards J. Research **4**, 543–544 (1930).

Table II. *The 1931 I.C.I. standard observer.*

Trichromatic coefficients			Wave-length (mμ)	Distribution coefficients for equal energy stimulus			Trichromatic coefficients			Wave-length (mμ)	Distribution coefficients for equal energy stimulus		
x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$	x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$
0.1741	0.0050	0.8209	**380**	0.0014	0.0000	0.0065	0.5125	0.4866	0.0009	**580**	0.9163	0.8700	0.0017
0.1740	0.0050	0.8210	**385**	0.0022	0.0001	0.0105	0.5448	0.4544	0.0008	**585**	0.9786	0.8163	0.0014
0.1738	0.0049	0.8213	**390**	0.0042	0.0001	0.0201	0.5752	0.4242	0.0006	**590**	1.0263	0.7570	0.0011
0.1736	0.0049	0.8215	**395**	0.0076	0.0002	0.0362	0.6029	0.3965	0.0006	**595**	1.0567	0.6949	0.0010
0.1733	0.0048	0.8219	**400**	0.0143	0.0004	0.0679	0.6270	0.3725	0.0005	**600**	1.0622	0.6310	0.0008
0.1730	0.0048	0.8222	**405**	0.0232	0.0006	0.1102	0.6482	0.3514	0.0004	**605**	1.0456	0.5668	0.0006
0.1726	0.0048	0.8226	**410**	0.0435	0.0012	0.2074	0.6658	0.3340	0.0002	**610**	1.0026	0.5030	0.0003
0.1721	0.0048	0.8231	**415**	0.0776	0.0022	0.3713	0.6801	0.3197	0.0002	**615**	0.9384	0.4412	0.0002
0.1714	0.0051	0.8235	**420**	0.1344	0.0040	0.6456	0.6915	0.3083	0.0002	**620**	0.8544	0.3810	0.0002
0.1703	0.0058	0.8239	**425**	0.2148	0.0073	1.0391	0.7006	0.2993	0.0001	**625**	0.7514	0.3210	0.0001
0.1689	0.0069	0.8242	**430**	0.2839	0.0116	1.3856	0.7079	0.2920	0.0001	**630**	0.6424	0.2650	0.0000
0.1669	0.0086	0.8245	**435**	0.3285	0.0168	1.6230	0.7140	0.2859	0.0001	**635**	0.5419	0.2170	0.0000
0.1644	0.0109	0.8247	**440**	0.3483	0.0230	1.7471	0.7190	0.2809	0.0001	**640**	0.4479	0.1750	0.0000
0.1611	0.0138	0.8251	**445**	0.3481	0.0298	1.7826	0.7230	0.2770	0.0000	**645**	0.3608	0.1382	0.0000
0.1566	0.0177	0.8257	**450**	0.3362	0.0380	1.7721	0.7260	0.2740	0.0000	**650**	0.2835	0.1070	0.0000
0.1510	0.0227	0.8263	**455**	0.3187	0.0480	1.7441	0.7283	0.2717	0.0000	**655**	0.2187	0.0816	0.0000
0.1440	0.0297	0.8263	**460**	0.2908	0.0600	1.6692	0.7300	0.2700	0.0000	**660**	0.1649	0.0610	0.0000
0.1355	0.0399	0.8246	**465**	0.2511	0.0739	1.5281	0.7311	0.2689	0.0000	**665**	0.1212	0.0446	0.0000
0.1241	0.0578	0.8181	**470**	0.1954	0.0910	1.2876	0.7320	0.2680	0.0000	**670**	0.0874	0.0320	0.0000
0.1096	0.0868	0.8036	**475**	0.1421	0.1126	1.0419	0.7327	0.2673	0.0000	**675**	0.0636	0.0232	0.0000
0.0913	0.1327	0.7760	**480**	0.0956	0.1390	0.8130	0.7334	0.2666	0.0000	**680**	0.0468	0.0170	0.0000
0.0687	0.2007	0.7306	**485**	0.0580	0.1693	0.6162	0.7340	0.2660	0.0000	**685**	0.0329	0.0119	0.0000
0.0454	0.2950	0.6596	**490**	0.0320	0.2080	0.4652	0.7344	0.2656	0.0000	**690**	0.0227	0.0082	0.0000
0.0235	0.4127	0.5638	**495**	0.0147	0.2586	0.3533	0.7346	0.2654	0.0000	**695**	0.0158	0.0057	0.0000
0.0082	0.5384	0.4534	**500**	0.0049	0.3230	0.2720	0.7347	0.2653	0.0000	**700**	0.0114	0.0041	0.0000
0.0039	0.6548	0.3413	**505**	0.0024	0.4073	0.2123	0.7347	0.2653	0.0000	**705**	0.0081	0.0029	0.0000
0.0139	0.7502	0.2359	**510**	0.0093	0.5030	0.1582	0.7347	0.2653	0.0000	**710**	0.0058	0.0021	0.0000
0.0389	0.8120	0.1491	**515**	0.0291	0.6082	0.1117	0.7347	0.2653	0.0000	**715**	0.0041	0.0015	0.0000
0.0743	0.8338	0.0919	**520**	0.0633	0.7100	0.0782	0.7347	0.2653	0.0000	**720**	0.0029	0.0010	0.0000
0.1142	0.8262	0.0596	**525**	0.1096	0.7932	0.0573	0.7347	0.2653	0.0000	**725**	0.0020	0.0007	0.0000
0.1547	0.8059	0.0394	**530**	0.1655	0.8620	0.0422	0.7347	0.2653	0.0000	**730**	0.0014	0.0005	0.0000
0.1929	0.7816	0.0255	**535**	0.2257	0.9149	0.0298	0.7347	0.2653	0.0000	**735**	0.0010	0.0004	0.0000
0.2296	0.7543	0.0161	**540**	0.2904	0.9540	0.0203	0.7347	0.2653	0.0000	**740**	0.0007	0.0003	0.0000
0.2658	0.7243	0.0099	**545**	0.3597	0.9803	0.0134	0.7347	0.2653	0.0000	**745**	0.0005	0.0002	0.0000
0.3016	0.6923	0.0061	**550**	0.4334	0.9950	0.0087	0.7347	0.2653	0.0000	**750**	0.0003	0.0001	0.0000
0.3373	0.6589	0.0038	**555**	0.5121	1.0002	0.0057	0.7347	0.2653	0.0000	**755**	0.0002	0.0001	0.0000
0.3731	0.6245	0.0024	**560**	0.5945	0.9950	0.0039	0.7347	0.2653	0.0000	**760**	0.0002	0.0001	0.0000
0.4087	0.5896	0.0017	**565**	0.6784	0.9786	0.0027	0.7347	0.2653	0.0000	**765**	0.0001	0.0000	0.0000
0.4441	0.5547	0.0012	**570**	0.7621	0.9520	0.0021	0.7347	0.2653	0.0000	**770**	0.0001	0.0000	0.0000
0.4788	0.5202	0.0010	**575**	0.8425	0.9154	0.0018	0.7347	0.2653	0.0000	**775**	0.0000	0.0000	0.0000
0.5125	0.4866	0.0009	**580**	0.9163	0.8700	0.0017	0.7347	0.2653	0.0000	**780**	0.0000	0.0000	0.0000
										Totals	21.3713	21.3714	21.3715

than zero in the tristimulus specification of the spectrum stimuli, and hence none for any real stimulus, (3) the so-called equal-energy spectrum when additively combined forms a stimulus that is equivalent to the additive combination of equal amounts of the primary stimuli,[12] and (4) relative extension or compression of areas in the Maxwell triangle has been sufficiently avoided that the length of a line is an approximate index of the degree to which the stimuli represented at its extremes differ in chromaticity.[13]

IV. The Standard Observer

The properties of the standard observer are given by his tristimulus specifications of the spectrum stimuli as functions of wave-length. Table II gives this specification for the equal-energy spectrum both in fractions, x, y, z, of the total amount for each wave-length interval of 5 mμ and directly, $\bar{x}$, $\bar{y}$, $\bar{z}$, for each 5 mμ. The fractional values are known as the trilinear coordinates of the spectrum or the trichromatic coefficients of the spectrum; the direct values are known as distribution functions or distribution coefficients. Fig. 1 shows the distribution functions, $\bar{x}$, $\bar{y}$, and $\bar{z}$; it is to be noted that the $\bar{y}$ specification of the equal-energy spectrum is the standard I.C.I. visibility function;[14] this follows from the fact that the observer represented possesses the standard visibility combined with the fact that the coordinate system adopted has luminosity coefficients, 0, 1, and 0. The remaining properties of the standard observer were derived as a slightly smoothed average of ten normal observers investigated by Wright[15] at the University of London and seven normal observers investigated by Guild[16] at the National Physical Laboratory. This average departs significantly from the properties of the observer represented by the excitation curves in the 1922 colorimetry report[2] but the discrepancies are not large compared to established differences between observers commonly called normal.[17]

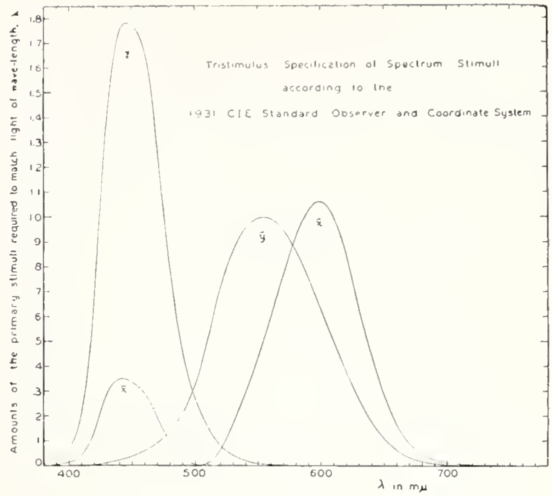

Fig. 1. Tristimulus specifications of spectrum stimuli according to the standard observer and coordinate system recommended by the International Commission on Illumination in 1931. These specifications refer to the equal energy spectrum, and the units of the primary stimuli are adjusted so that their combination in equal proportions matches the equal-energy stimulus; that is, the areas of the three curves have been adjusted to equality by choice of units. These three functions define the properties of the 1931 I.C.I. standard observer.

Fig. 2 is the (x, y)-plot of the Maxwell triangle. It shows the locus of spectrum stimuli as a series of circles, and the locus of Planckian stimuli as a solid line drawn between various computed points which are indicated by crosses. The large circles of the spectrum locus are plotted from the fractional values of Table II, the small circles are

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[12] The equal-energy stimulus is therefore represented at the center of the Maxwell triangle and forms what is termed the basic stimulus of the system (see J. Guild, *On the Fixed Points of a Colorimetric System, Discussion by Priest*, Trans. Opt. Soc. **32**, 25 (1930–31); also footnote 9.

[13] This approximation is fairly good for any of the rectangular plots, (x, y), (y, z), or (x, z), as well as for the equilateral triangle, but the (x, y)-plot yields the best approximation. The (x, y)-plot corresponds most closely to the (r, g)-plot of the 1922 O.S.A. coordinate system. (Cf. D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J.O.S.A. **22**, 72–108 (1932).)

[14] *Proceedings, Commission Internationale de l'Éclairage, 6th Meeting, Geneva*, pp. 67 and 232; July, 1924. The interpolated values are those given in J.O.S.A. **21**, 267–275 (1931).

[15] W. D. Wright, *A Re-Determination of the Trichromatic Coefficients of the Spectral Colours*, Trans. Opt. Soc. **30**, 141–164 (1928–29); *A Re-Determination of the Trichromatic Mixture Data, Medical Research Council*, Reports of the Committee upon the Physiology of Vision, VII, Special Report Series No. 139, London, 1929.

[16] J. Guild, *The Colorimetric Properties of the Spectrum*, Phil. Trans. Roy. Soc. **A230**, 149–187 (1931).

[17] D. B. Judd, *Comparison of Wright's Data on Equivalent Color Stimuli with the O.S.A. Data*, J.O.S.A. **21**, 721 (1931).

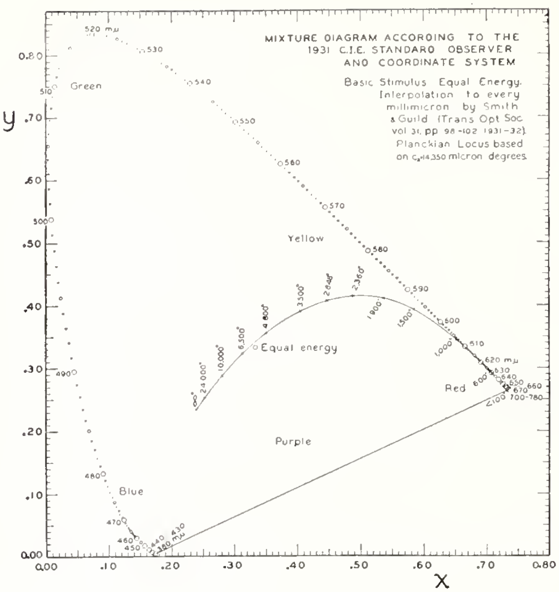

Fig. 2. Mixture diagram according to the standard observer and coordinate system recommended by the International Commission on Illumination in 1931. Additive mixtures of stimuli are represented according to the center-of-gravity principle. The stimuli for nearly achromatic colors are represented near the center (1/3, 1/3) of the diagram, and those for the saturated colors are represented near the spectrum locus and the straight line joining its extremes. The hues of these colors for usual chromatic adaptation are indicated on the diagram.

values interpolated by Smith[18] (see Table VI). The trilinear coordinates of the various Planckian stimuli were computed from summation over wave-length intervals of 10 mμ (see next section). These trilinear coordinates are given in Table V.

The diagram obtained by plotting trilinear coordinates in the triangle or by plotting any two of them in rectangular coordinates (as in Fig. 2) is called a mixture diagram because additive mixtures or combinations of stimuli are represented on the diagram according to the center-of-gravity principle. For example, the straight line joining the extremes of the spectrum locus includes all the points representing the stimuli obtained by additive mixture in various proportions of the extremes of the spectrum; also the area enclosed by this straight line and the spectrum locus includes all the points representing the stimuli obtained by additive combination of the various parts of the spectrum in various proportions, that is, it includes all the points representing real stimuli.

V. Computation Forms

a. Trilinear coordinates

The tristimulus specification of the spectrum stimuli according to the standard observer can be used to test whether or not any two stimuli of known spectral energy distribution are equivalent. If the two stimuli are produced by illuminating two filters of spectral transmission, T_1 and T_2, respectively, by an illuminant of spectral energy distribution, E, the two stimuli are equivalent if:

$$\int_0^\infty \bar{x}ET_1 d\lambda = \int_0^\infty \bar{x}ET_2 d\lambda,$$

$$\int_0^\infty \bar{y}ET_1 d\lambda = \int_0^\infty \bar{y}ET_2 d\lambda,$$

$$\int_0^\infty \bar{z}ET_1 d\lambda = \int_0^\infty \bar{z}ET_2 d\lambda;$$

that is, they are equivalent if they have identical tristimulus specifications. For the majority of practical cases it is sufficient to compute an approximation to the above integrals by summation with intervals of 10 mμ, thus, for a stimulus whose spectral distribution of energy is ET, compute:

$$\sum \bar{x}ET\Delta\lambda,\ \sum \bar{y}ET\Delta\lambda,\ \text{and}\ \sum \bar{z}ET\Delta\lambda,\ \Delta\lambda = 10\ \text{m}\mu.$$

An aid to computations of this sort is a table of the values of $\bar{x}E$, $\bar{y}E$, and $\bar{z}E$ for each 10 mμ interval. Such values are given in Table III for 1931 I.C.I. standard illuminant A, and in Table IV for standard illuminants B and C. The scale magnitudes of these values has in each case been adjusted so that $\sum \bar{y}E\Delta\lambda = 1{,}000{,}000$.[19] This choice of scale shortens the computation of the luminous transmission (or reflectance) of the sample for the respective standard illuminant. The luminous transmission is:

$$(\sum \bar{y}ET\Delta\lambda)/(\sum \bar{y}E\Delta\lambda).$$

[18] T. Smith and J. Guild, *The C.I.E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 98–101 (1931–32).

[19] A similar choice has been made by Smith and Guild (Trans. Opt. Soc. **33**, 103–105 (1931–32)) who give computational tables for wave-length intervals of 5 mμ.

TABLE III. *Computation of x, y, z and* T_w.

Observer: 1931 C.I.E.
Basic stimulus: Equal energy
Illuminant: Planck 2848°K (C_2 = 14,350) (1931 C.I.E. Standard *A*)

Sample: Davis–Gibson 2848-to-4800°K filter
Source of trans. data: B. S. Jour. Research **7**, 796 (1931)

λ	$\bar{x}E$	$\bar{y}E$	$\bar{z}E$	T	$\bar{x}ET$	$\bar{y}ET$	$\bar{z}ET$
380	1		6	0.588	1		4
90	5		23	.666	3		15
400	19	1	93	.721	14	1	67
10	71	2	340	.757	54	2	257
20	262	8	1256	.774	203	6	972
30	649	27	3167	.761	494	21	2410
40	926	61	4647	.723	669	44	3360
450	1031	117	5435	.663	684	78	3603
60	1019	210	5851	.601	612	126	3516
70	776	362	5116	.551	428	199	2819
80	428	622	3636	.507	217	315	1843
90	160	1039	2324	.460	74	478	1069
500	27	1792	1509	.404	11	724	610
10	57	3080	969	.353	20	1087	342
20	425	4771	525	.317	135	1512	166
30	1214	6322	309	.300	364	1897	93
40	2313	7600	162	.290	671	2204	47
550	3732	8568	75	.279	1041	2390	21
60	5510	9222	36	.264	1455	2435	10
70	7571	9457	21	.246	1862	2326	5
80	9719	9228	18	.227	2206	2095	4
90	11579	8540	12	.209	2420	1785	3
600	12704	7547	10	.195	2477	1472	2
10	12669	6356	4	.186	2356	1182	1
20	11373	5071	3	.178	2024	903	1
30	8980	3704		.172	1545	637	
40	6558	2562		.166	1089	425	
650	4336	1637		.162	702	265	
60	2628	972		.157	413	153	
70	1448	530		.151	219	80	
80	804	292		.1440	116	42	
90	404	146		.1360	55	20	
700	209	75		.1284	27	10	
10	110	40		.1210	13	5	
20	57	19		.1134	6	2	
30	28	10		.1063	3	1	
40	14	6		.1007	1	1	
750	6	2		.0964	1	0	
60	4	2		.0938	0		
70	2			.0926			
Sums	109828	100000	35547		24685	24923	21240
x_w, y_w, z_w	0.44759	0.40754	0.14487	x, y, z	0.34842	0.35178	0.29980

TABLE IV. *Computational table for I.C.I. illuminants B and C.*

Illuminant *B*			Wave-length in mμ	Illuminant *C*		
$\bar{x}E$	$\bar{y}E$	$\bar{z}E$		$\bar{x}E$	$\bar{y}E$	$\bar{z}E$
3		14	**380**	4		20
13		60	**90**	19		89
56	2	268	**400**	85	2	404
217	6	1033	**10**	329	9	1570
812	24	3899	**20**	1238	37	5949
1983	81	9678	**30**	2997	122	14628
2689	178	13489	**40**	3975	262	19938
2744	310	14462	**450**	3915	443	20638
2454	506	14085	**60**	3362	694	19299
1718	800	11319	**70**	2272	1058	14972
870	1265	7396	**80**	1112	1618	9461
295	1918	4290	**90**	363	2358	5274
44	2908	2449	**500**	52	3401	2864
81	4360	1371	**10**	89	4833	1520
541	6072	669	**20**	576	6462	712
1458	7594	372	**30**	1523	7934	388
2689	8834	188	**40**	2785	9149	195
4183	9603	84	**550**	4282	9832	86
5840	9774	38	**60**	5880	9841	39
7472	9334	21	**70**	7322	9147	20
8843	8396	16	**80**	8417	7992	16
9728	7176	10	**90**	8984	6627	10
9948	5909	7	**600**	8949	5316	7
9436	4734	3	**10**	8325	4176	2
8140	3630	2	**20**	7070	3153	2
6200	2558		**30**	5309	2190	
4374	1709		**40**	3693	1443	
2815	1062		**650**	2349	886	
1655	612		**60**	1361	504	
876	321		**70**	708	259	
465	169		**80**	369	134	
220	80		**90**	171	62	
108	39		**700**	82	29	
53	19		**10**	39	14	
26	9		**20**	19	6	
12	4		**30**	8	3	
6	2		**40**	4	2	
2	1		**750**	2	1	
2	1		**60**	1	1	
1			**70**	1		
99072	100000	85223	Sums	98041	100000	118103
0.34848	0.35175	0.29977	x_w, y_w, z_w	0.31012	0.31631	0.37357

A worked example of the computation of tristimulus specifications for a stimulus consisting of a lamp and filter is included in Table III. The lamp is taken at a color temperature of 2848°K (standard illuminant *A*) and the filter is the Davis-Gibson 2848-to-4800°K filter,[20] the combination being standard illuminant *B*. The sums of the last three columns are, respectively, $(1/10)\sum\bar{x}ET\Delta\lambda$, $(1/10)\sum\bar{y}ET\Delta\lambda$, and $(1/10)\sum\bar{z}ET\Delta\lambda$, and are found to be 24,685, 24,923, and 21,240. These three numbers, expressed as fractions of their total are given immediately below; they are *x*, *y*, and *z*, respectively, for the lamp-and-filter combination, that is, 0.34842, 0.35178, and 0.29980. The transmission, T_w, of the filter for standard illuminant *A* is $\sum\bar{y}ET\Delta\lambda\times10^{-6}$, or 0.249. This is the method used at the Bureau of Standards for computing trilinear coordinates and luminous transmission for the majority of samples measured spectrophotometrically.

[20] See footnote 5.

The values of *x*, *y*, and *z* computed in this way differ in the fourth decimal from the trilinear coordinates of the cardinal stimulus defined as this lamp-and-filter combination. This difference, due somewhat to rejection error in computation, is chiefly ascribable to our use of summation instead of strict integration. Since, however, a change in trilinear coordinate of one in the fourth decimal corresponds in magnitude to about one one-hundredth of the size of a Jones hue step,[21]

[21] L. A. Jones, *The Fundamental Scale of Pure Hue and Retinal Sensibility to Hue Differences*, J.O.S.A. **1**, 63–77 (1917); see also the *1922 Colorimetry Report*, p. 545.

errors of this magnitude rarely have practical significance. The fifth decimal, however, is sometimes kept for comparison with values similarly computed.

Table V gives the trilinear coordinates for a number of Planckian stimuli computed by summation over wave-length intervals of 10 mμ. The energy distributions were computed from Planck's formula for closed-cavity radiation with the constant, C_2, taken at 14,350 micron degrees.

TABLE V. *Trilinear coordinates for some Planckian stimuli.*

Color temperature (C_2=14,350) θ°K	Trilinear coordinates according to the 1931 I.C.I. coordinate system		
	x	y	z
100 (or less)	0.7347	0.2653	0.0000
300	.7341	.2659	.0000
600	.7090	.2909	.0001
1,000	.6524	.3448	.0028
1,500	.5852	.3934	.0214
1,900	.5372	.4114	.0514
2,360	.4893	.4150	.0957
2,848	.4476	.4075	.1449
3,500	.4049	.3906	.2046
4,800	.3506	.3560	.2933
6,500	.3133	.3235	.3632
10,000	.2806	.2883	.4311
24,000	.2532	.2532	.4936
∞	.2399	.2342	.5259

B. Dominant wave-length and colorimetric purity

The homogeneous-heterogeneous system may also be used to specify color stimuli. In this system the stimulus is specified by giving an equivalent stimulus made up of the additive combination of an homogeneous (spectrum) stimulus with an arbitrarily chosen heterogeneous stimulus, usually one which evokes an achromatic or nearly achromatic color. This equivalent stimulus is specified by giving (1) its brightness, (2) the wave-length of the homogeneous component (called the dominant wave-length of the additive combination), and (3) the fraction of the total brightness of the combination supplied by the homogeneous component (called the colorimetric purity of the combination). The colors of stimuli which can be matched by such a combination are called spectral colors, distinguished from nonspectral (purple) colors whose stimuli cannot be so matched.[22] It is convenient and customary to extend the homogeneous-heterogeneous system to the stimulus of any nonspectral color by giving the wave-length and relative brightness of the spectrum stimulus which must be added to it in order to match the heterogeneous stimulus.[23] In this case, however, the brightness of the spectrum stimulus is given a negative sign since it is added not to the heterogeneous stimulus but to the stimulus to be specified. Hence, colorimetric purities for the stimuli of nonspectral colors have negative signs; but they are continuous at the heterogeneous stimulus with colorimetric purities for spectral colors and may be computed from the same formulas.

The brightness of a stimulus consisting of a source-and-filter combination is computed by multiplying the brightness of the source by the luminous transmission of the filter for the source (see Table III for method of computation). Dominant wave-length and colorimetric purity may be computed from the trilinear coordinates, x, y, z, of the stimulus to be specified provided the trilinear coordinates, x_w, y_w, z_w, of the heterogeneous stimulus be given.

Dominant wave-length may be found approximately from the mixture diagram (such as the (x, y)-plot shown in Fig. 2). The straight line connecting (x, y) with (x_w, y_w) is extended in both directions. If these two points represent real stimuli, the straight line will intersect the spectrum locus either in one place or two, but not more than two because the spectrum locus is nowhere concave. The dominant wave-length is the wave-length given by such an intersection. If there be two dominant wave-lengths, the intersection formed by extending the line through the point (x, y) is found to be associated with a positive value of colorimetric purity, the other intersection with a negative purity, and in this case

[22] I. G. Priest, *Apparatus for the Determination of Color in Terms of Dominant Wave-Length, Purity and Brightness,* J.O.S.A. and R.S.I. **8**, 173–200 (1924). *The Computation of Colorimetric Purity,* J.O.S.A. and R.S.I. **9**, 503–520 (1924). *The Computation of Colorimetric Purity II,* J.O.S.A. and R.S.I. **13**, 123–132 (1926).

[23] H. E. Ives, *The Transformation of Color Mixture Equations from One System to Another. II. Graphical Aids,* J. Frank. Inst. **195**, 39 (1923). The extension to purples was accomplished in the 1922 colorimetry report by giving the brightness of the spectrum component relative to the brightness of the mixture instead of relative to the brightness of the stimulus, itself. This is somewhat less convenient; see D. B. Judd, *The Computation of Colorimetric Purity,* J.O.S.A. and R.S.I. **13**, 147–150 (1926).

the dominant wave-length associated with the positive purity is usually preferred. Stimuli for nonspectral colors, however, must be specified by a dominant wave-length associated with a negative purity because no other exists.

Colorimetric purity, p, is found from the formula:[24]

$$p=(y-fy_w)/y,\quad f=(x-X)/(x_w-X)\quad \text{or}\quad (y-Y)/(y_w-Y)$$

whichever is determinable with the least rejection error, where (X, Y) is the intersection of the straight line with the spectrum locus.

The method used at the Bureau of Standards for computing dominant wave-length and colorimetric purity uses this formula combined with the analytic equivalent of the graphical method just described for obtaining dominant wave-length. This method is illustrated by three worked examples (see Examples 1, 2, 3) each referring to the lamp-and-filter combination whose trilinear coordinates were found as in Table III. In the first case the basic stimulus (equal energy) is taken as the heterogeneous stimulus; this illustrates the computation of a positive purity in which the ratio, f, is evaluated by $(y-Y)/(y_w-Y)$. The dominant wave-length is found to be 575.6 mμ and the purity to be +0.147; that is, the standard observer finds illuminant B to be equivalent to the additive combination of the equal-energy stimulus and light of wave-length 575.6 mμ in the brightness proportion 0.853 to 0.147. In the second and third cases standard illuminant A is taken as the heterogeneous stimulus, the second case showing computation of the dominant wave-length associated with the positive purity, the third, that with the negative purity. In both of these cases the ratio, f, is evaluated by $(x-X)/(x_w-X)$. The dominant wave-length in the second case is found to be 484.7 mμ and the purity +0.146; that is, the standard observer finds illuminant B to be equivalent to the additive combination of illuminant A and light of wave-length 484.7 mμ in the brightness proportion 0.854 to 0.146. The dominant wave-length in the third case is found to be 584.2 mμ and the purity −1.403; that is, the standard observer finds the additive combination of illuminant B with light of wave-length 584.2 mμ in the brightness proportion 1.000 to 1.403 to be equivalent to illuminant A.

[24] This formula combines the advantage of involving the coordinates (X, Y) of the spectrum stimulus singly (D. B. Judd, *The Computation of Colorimetric Purity*, J.O.S.A. and R.S.I. **13**, 145 (1926)) with the advantage of being applicable to any heterogeneous stimulus (x_w, y_w) (D. B. Judd, *A General Formula for Computation of Colorimetric Purity*, J.O.S.A. **21**, 729–747 (1931)).

Example 1.

Guide for computation of dominant wave-length (Λ) *and purity* (p)

Observer:—1931 C.I.E.
Basic stimulus:—Equal energy
Illuminant:—***Planck 2848°K***

Sample:— ***Davis-Gibson 2848— to —4800°K filter*** (***B. S. Jour. Research 7, 796*** (***1931***)).

Formula:

$$p=(y-fy_w)/y,\ f=(x-X)/(x_w-X)\ \text{or}\ (y-Y)/(y_w-Y).$$

x_w and y_w for some common heterogeneous stimuli:

	x_w*	y_w*
Davis-Gibson 6500°K, C.I.E. Illuminant C,	0.31012,	0.31631
Davis-Gibson 4800°K, C.I.E. Illuminant B,	.34848,	.35175
Planck 2848°K, C.I.E. Illuminant A,	.44759,	.40754
Equal energy,	.33338,	.33345

Heterogeneous stimulus:—***Equal energy.***

$$x_w = 0.33338,\qquad y_w = 0.33345$$
$$x = 0.34842,\qquad y = 0.35178$$
$$x_w-x = -0.01504,\qquad y_w-y = -0.01833$$

Of the following two ratios, R, compute the one having the smaller absolute value and continue the corresponding (right or left) side of the optional computation (indicated by question marks (?)); cross off the other side. Λ is read by linear interpolation from table of these ratios as indicated; either X or Y (as is required) is read by linear interpolation from Λ within the intervals of 1 mμ:

$$\Lambda_0=575,\qquad R_0=+\ .77840$$
~~$(y_w-y)/(x_w-x)\ ?=R$~~ $= +0.82051=(x_w-x)/(y_w-y)\ ?=R$
$$\Lambda_1=576,\qquad R_1=+\ .84560$$
$$R-R_0=+0.04211,\qquad R_1-R_0=+0.06720$$
$$\Delta\Lambda=(R-R_0)/(R_1-R_0)=+0.627;\ \Lambda=\Lambda_0+\Delta\Lambda=575\cdot 63\ \text{m}\mu$$
~~$X_0\ ?$~~ $= 0.52020=?\ Y_0$
~~$X_1\ ?$~~ $= 0.51340=?\ Y_1$
~~$X_1-X_0\ ?$~~ $= -0.00680=?\ Y_1-Y_0$
~~$\Delta X=\Delta\Lambda(X_1-X_0)\ ?$~~ $= -0.00426=?\ \Delta\Lambda(Y_1-Y_0)=\Delta Y$
~~$X=X_0+\Delta X\ ?$~~ $= 0.51594=?\ Y_0+\Delta Y=Y$
~~$x-X\ ?$~~ $= -0.16416=?\ y-Y$
~~$x_w-X\ ?$~~ $= -0.18249=?\ y_w-Y$
~~$f=(x-X)/(x_w-X)\ ?$~~ $= +0.89956=?\ (y-Y)/(y_w-Y)=f$
$$fy_w=+0.29996,\qquad y-fy_w=+0.05182$$
$$p=(y-fy_w)/y=+0.1473$$

* Computed with wave-length intervals of 10 mμ.

Example 2.

Guide for computation of dominant wave-length (Λ) *and purity* (p)

Observer:—1931 C.I.E.
Basic stimulus:—Equal energy
Illuminant:—*Planck 2848°K*

Sample:— ***Davis-Gibson 2848— to —4800°K filter (B. S. Jour. Research 7, 796 (1931)).***

Formula:

$$p=(y-fy_w)/y,\ f=(x-X)/(x_w-X) \text{ or } (y-Y)/(y_w-Y).$$

x_w and y_w for some common heterogeneous stimuli:

	x_w*	y_w*
Davis-Gibson 6500°K, C.I.E. Illuminant *C*,	0.31012,	0.31631
Davis-Gibson 4800°K, C.I.E. Illuminant *B*,	.34848,	.35175
Planck 2848°K, C.I.E. Illuminant *A*,	.44759,	.40754
Equal energy,	.33338,	.33345

Heterogeneous stimulus:—***Planck 2848°K.***

$x_w = 0.44759$, $y_w = 0.40754$
$x = 0.34842$, $y = 0.35178$
$x_w-x = +0.09917$, $y_w-y = +0.05576$

Of the following two ratios, R, compute the one having the smaller absolute value and continue the corresponding (right or left) side of the optional computation (indicated by question marks (?)); cross off the other side. Λ is read by linear interpolation from table of these ratios as indicated; either X or Y (as is required) is read by linear interpolation from Λ within the intervals of 1 mμ:

$\Lambda_0=484$, $R_0=+.59430$
$(y_w-y)/(x_w-x)\ ?=R=+0.56227$ ~~$=(x_w-x)/(y_w-y)\ ?=R$~~
$\Lambda_1=485$, $R_1=+.54580$
$R-R_0=-0.03203$, $R_1-R_0=-0.04850$
$\Delta\Lambda=(R-R_0)/(R_1-R_0)=0.660$; $\Lambda=\Lambda_0+\Delta\Lambda=484{\cdot}66$ mμ
$X_0\ ?=0.07340$ ~~$=?\ Y_0$~~
$X_1\ ?=0.06870$ ~~$=?\ Y_1$~~
$X_1-X_0\ ?=-0.00470$ ~~$=?\ Y_1-Y_0$~~
$\Delta X=\Delta\Lambda(X_1-X_0)\ ?=-0.00310$ ~~$=?\ \Delta\Lambda(Y_1-Y_0)=\Delta Y$~~
$X=X_0+\Delta X\ ?=0.07030$ ~~$=?\ Y_0+\Delta Y=Y$~~
$x-X\ ?=+0.27812$ ~~$=?\ y-Y$~~
$x_w-X\ ?=+0.37729$ ~~$=?\ y_w-Y$~~
$f=(x-X)/(x_w-X)\ ?=+0.73715$ ~~$=?\ (y-Y)/(y_w-Y)=f$~~
$fy_w=+0.30042$, $y-fy_w=+0.05136$
$p=(y-fy_w)/y=+0.1460$

Example 3.

Guide for computation of dominant wave-length (Λ) *and purity* (p)

Observer:—1931 C.I.E.
Basic stimulus:—Equal energy
Illuminant:—*Planck 2848°K*

Sample:— ***Davis-Gibson 2848— to —4800°K filter (B. S. Jour. Research 7, 796 (1931)).***

* Computed with wave-length intervals of 10 mμ.

Formula:

$$p=(y-fy_w)/y,\ f=(x-X)/(x_w-X) \text{ or } (y-Y)/(y_w-Y).$$

x_w and y_w for some common heterogeneous stimuli:

	x_w*	y_w*
Davis-Gibson 6500°K, C.I.E. Illuminant *C*,	0.31012,	0.31631
Davis-Gibson 4800°K, C.I.E. Illuminant *B*,	.34848,	.35175
Planck 2848°K, C.I.E. Illuminant *A*,	.44759,	.40754
Equal energy,	.33338,	.33345

Heterogeneous stimulus:—***Planck 2848°K.***

$x_w = 0.44759$, $y_w = 0.40754$
$x = 0.34842$, $y = 0.35178$
$x_w-x = +0.09917$, $y_w-y = +0.05576$

Of the following two ratios, R, compute the one having the smaller absolute value and continue the corresponding (right or left) side of the optional computation (indicated by question marks (?)); cross off the other side. Λ is read by linear interpolation from table of these ratios as indicated; either X or Y (as is required) is read by linear interpolation from Λ within the intervals of 1 mμ:

$\Lambda_0=584$, $R_0=+0.58530$
$(y_w-y)/(x_w-x)\ ?=R=+0.56227$ ~~$=(x_w-x)/(y_w-y)\ ?=R$~~
$\Lambda_1=585$, $R_1=+0.48250$
$R-R_0=-0.02303$, $R_1-R_0=-0.10280$
$\Delta\Lambda=(R-R_0)/(R_1-R_0)=0.224$; $\Lambda=\Lambda_0+\Delta\Lambda=584{\cdot}22$ mμ
$X_0\ ?=0.53850$ ~~$=?\ Y_0$~~
$X_1\ ?=0.54480$ ~~$=?\ Y_1$~~
$X_1-X_0\ ?=+0.00630$ ~~$=?\ Y_1-Y_0$~~
$\Delta X=\Delta\Lambda(X_1-X_0)\ ?=+0.00141$ ~~$=?\ \Delta\Lambda(Y_1-Y_0)=\Delta Y$~~
$X=X_0+\Delta X\ ?=0.53991$ ~~$=?\ Y_0+\Delta Y=Y$~~
$x-X\ ?=-0.19149$ ~~$=?\ y-Y$~~
$x_w-X\ ?=-0.09232$ ~~$=?\ y_w-Y$~~
$f=(x-X)/(x_w-X)\ ?=+2.07420$ ~~$=?\ (y-Y)/(y_w-Y)=f$~~
$fy_w=+0.84532$, $y-fy_w=-0.49354$
$p=(y-fy_w)/y=-1.4030$

This computation form provides for the determination of dominant wave-length, Λ, and of X or Y, by linear interpolation from tabular values given for every millimicron; that is, R_0 and R_1, and either X_0 and X_1 or Y_0 and Y_1 are read from Table VI[25] for the appropriate heterogeneous

* Computed with wave-length intervals of 10 mμ.

[25] X_0 and X_1 or Y_0 and Y_1 are read from the last two columns of the table (headed x and y to conform to Table II) which for wave-lengths between 410 and 680 mμ are values interpolated by Smith (see footnote 19. Values of x and y from 380 to 410 and from 680 to 699 mμ have been interpolated to 5 decimals over intervals of 10 mμ by the third-difference osculatory formula (J.O.S.A. 21, 267–275 (1931)) in order to avoid local retrogressions in the ratios, $(x-x_w)/(y-y_w)$ and $(y-y_w)/(x-x_w)$. These values differ from Smith's by less than one in the fourth place.

TABLE VI. *Slopes of the dominant-wave-length lines for various heterogeneous stimuli, and trilinear coordinates of spectrum stimuli for every millimicron.*

Illuminant *A* $x_w=0.4476$, $y_w=0.4075$		Illuminant *B* $x_w=0.3485$, $y_w=0.3517$		Wave-length (mμ)	Illuminant *C* $x_w=0.3101$, $y_w=0.3163$		Equal energy $x_w=0.3334$, $y_w=0.3334$		Wave-length (mμ)	x	y
$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$		$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$			
+0.67950		+0.50303		380	+0.43688		+0.48508		380	0.17410	0.00500
.67954		.50307		381	.43693		.48513		381	.17408	.00499
.67957		.50311		382	.43698		.48517		382	.17406	.00498
.67963		.50319		383	.43706		.48525		383	.17403	.00497
.67968		.50326		384	.43714		.48532		384	.17400	.00496
+ .67972		+ .50330		385	+ .43719		+ .48537		385	.17398	.00495
.67980		.50340		386	.43731		.48548		386	.17394	.00494
.67986		.50347		387	.43739		.48555		387	.17391	.00493
.67991		.50355		388	.43747		.48563		388	.17388	.00492
.68000		.50365		389	.43759		.48574		389	.17384	.00491
+ .68008		+ .50375		390	+ .43770		+ .48584		390	.17380	.00490
.68016		.50385		391	.43782		.48595		391	.17376	.00489
.68024		.50395		392	.43793		.48606		392	.17372	.00488
.68035		.50408		393	.43808		.48620		393	.17367	.00487
.68046		.50421		394	.43822		.48633		394	.17362	.00486
+ .68052		+ .50430		395	+ .43832		+ .48643		395	.17358	.00484
.68066		.50445		396	.43850		.48659		396	.17352	.00483
.68076		.50458		397	.43865		.48673		397	.17347	.00482
.68087		.50471		398	.43879		.48687		398	.17342	.00481
.68102		.50489		399	.43899		.48705		399	.17336	.00481
+ .68115		+ .50504		400	+ .43917		+ .48722		400	.17330	.00480
.68130		.50522		401	.43936		.48740		401	.17324	.00480
.68143		.50538		402	.43954		.48757		402	.17318	.00479
.68157		.50553		403	.43971		.48774		403	.17312	.00478
.68171		.50571		404	.43991		.48792		404	.17306	.00478
+ .68189		+ .50591		405	+ .44013		+ .48813		405	.17299	.00478
.68202		.50607		406	.44031		.48830		406	.17293	.00477
.68222		.50630		407	.44057		.48854		407	.17285	.00477
.68241		.50651		408	.44081		.48877		408	.17278	.00478
.68265		.50679		409	.44111		.48906		409	.17269	.00479
+ .6829		+ .5071		410	+ .4414		+ .4893		410	.1726	.0048
.6831		.5074		411	.4417		.4897		411	.1725	.0048
.6834		.5076		412	.4421		.4900		412	.1724	.0048
.6836		.5079		413	.4424		.4903		413	.1723	.0048
.6839		.5082		414	.4427		.4906		414	.1722	.0048
+ .6841		+ .5085		415	+ .4430		+ .4909		415	.1721	.0048
.6846		.5089		416	.4435		.4913		416	.1720	.0049
.6848		.5092		417	.4438		.4916		417	.1719	.0049
.6855		.5100		418	.4446		.4924		418	.1717	.0050
.6857		.5102		419	.4449		.4927		419	.1716	.0050
+ .6864		+ .5110		420	+ .4457		+ .4935		420	.1714	.0051
.6870		.5117		421	.4465		.4942		421	.1712	.0052
.6877		.5124		422	.4473		.4950		422	.1710	.0053
.6886		.5133		423	.4482		.4959		423	.1708	.0055
.6892		.5140		424	.4490		.4966		424	.1706	.0056
+ .6903		+ .5152		425	+ .4502		+ .4979		425	.1703	.0058
.6914		.5163		426	.4515		.4991		426	.1700	.0060
.6923		.5172		427	.4524		.5000		427	.1698	.0062
.6933		.5184		428	.4537		.5012		428	.1695	.0064
.6944		.5196		429	.4550		.5024		429	.1692	.0066
+ .6957		+ .5209		430	+ .4564		+ .5038		430	.1689	.0069
.6972		.5225		431	.4581		.5055		431	.1685	.0072
.6988		.5241		432	.4598		.5072		432	.1681	.0075
.7000		.5254		433	.4613		.5086		433	.1678	.0078
.7020		.5275		434	.4635		.5108		434	.1673	.0082
+ .7037		+ .5293		435	+ .4654		+ .5126		435	.1669	.0086
.7056		.5314		436	.4676		.5148		436	.1664	.0090
.7074		.5332		437	.4695		.5167		437	.1660	.0094
.7095		.5354		438	.4719		.5190		438	.1655	.0099
.7115		.5375		439	.4742		.5212		439	.1650	.0103
+ .7141		+ .5402		440	+ .4771		+ .5240		440	.1644	.0109
.7165		.5428		441	.4798		.5267		441	.1638	.0114
.7191		.5455		442	.4827		.5296		442	.1632	.0120
.7215		.5481		443	.4855		.5323		443	.1626	.0125
.7244		.5511		444	.4888		.5354		444	.1619	.0131
+ .7277		+ .5546		445	+ .4926		+ .5391		445	.1611	.0138
.7310		.5581		446	.4964		.5428		446	.1603	.0145
.7344		.5617		447	.5002		.5465		447	.1595	.0152
.7382		.5657		448	.5045		.5507		448	.1586	.0160
.7424		.5702		449	.5094		.5555		449	.1576	.0169
+ .7465		+ .5746		450	+ .5141		+ .5600		450	.1566	.0177
.7508		.5791		451	.5190		.5648		451	.1556	.0186
.7556		.5842		452	.5244		.5701		452	.1545	.0196
.7602		.5891		453	.5297		.5753		453	.1534	.0205
.7655		.5947		454	.5358		.5811		454	.1522	.0216
+ .7708		+ .6003		455	+ .5419		+ .5871		455	.1510	.0227
.7766		.6065		456	.5486		.5935		456	.1497	.0239
.7826		.6129		457	.5555		.6003		457	.1484	.0252
.7894		.6201		458	.5633		.6079		458	.1469	.0266
.7963		.6273		459	.5711		.6155		459	.1455	.0281

Table VI.—(*Continued*).

Illuminant *A* $x_w=0.4476$, $y_w=0.4075$ $\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	Illuminant *B* $x_w=0.3485$, $y_w=0.3517$ $\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	Wave-length (mμ)	Illuminant C $x_w=0.3101$, $y_w=0.3163$ $\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	Equal energy $x_w=0.3334$, $y_w=0.3334$ $\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	Wave-length (mμ)	x	y
+ .8036		+ .6351		460	+ .5796		+ .6236		460	.1440	.0297
.8110		.6429		461	.5881		.6319		461	.1425	.0313
.8192		.6516		462	.5975		.6410		462	.1409	.0331
.8281		.6611		463	.6078		.6510		463	.1392	.0351
.8382		.6717		464	.6192		.6622		464	.1374	.0374
+ .8490		+ .6831		465	+ .6317		+ .6743		465	.1355	.0399
.8610		.6958		466	.6455		.6877		466	.1335	.0427
.8747		.7103		467	.6612		.7030		467	.1313	.0459
.8899		.7263		468	.6788		.7200		468	.1290	.0495
.9062		.7435		469	.6976		.7382		469	.1267	.0534
+ .9251		+ .7635		470	+ .7195		+ .7594		470	.1241	.0578
.9455		.7852		471	.7434		.7825		471	.1215	.0626
.9682		.8094		472	.7702		.8084		472	.1187	.0678
.9934	+1.0066	.8364		473	.8002		.8372		473	.1158	.0735
+1.0217	.9788	.8669		474	.8342		.8699		474	.1128	.0798
	+ .9488	+ .9018		475	+ .8736		+ .9075		475	.1096	.0868
	.9168	.9421		476	.9193		.9510	+1.0515	476	.1062	.0945
	.8832	.9879	+1.0122	477	.9719	+1.0289	+1.0009	.9991	477	.1027	.1029
	.8479	+1.0405	.9611	478	+1.0328	.9682		.9449	478	.0991	.1120
	.8107		.9076	479		.9050		.8883	479	.0953	.1219
	+ .7713		+ .8515	480		+ .8391		+ .8290	480	.0913	.1327
	.7296		.7927	481		.7705		.7670	481	.0870	.1444
	.6863		.7322	482		.7002		.7033	482	.0826	.1570
	.6410		.6695	483		.6277		.6374	483	.0780	.1706
	.5943		.6056	484		.5543		.5704	484	.0734	.1851
	+ .5458		+ .5397	485		+ .4789		+ .5013	485	.0687	.2007
	.4953		.4717	486		+ .4015		+ .4302	486	.0640	.2175
	.4433		.4023	487		+ .3227		+ .3577	487	.0594	.2354
	.3899		.3315	488		+ .2428		+ .2838	488	.0547	.2543
	.3353		.2596	489		+ .1619		+ .2089	489	.0500	.2742
	+ .2797		+ .1871	490		+ .0805		+ .1333	490	.0454	.2950
	+ .2224		+ .1127	491		− .0026		+ .0560	491	.0407	.3170
	+ .1638		+ .0371	492		− .0869		− .0225	492	.0361	.3401
	+ .1051		− .0382	493		− .1706		− .1008	493	.0317	.3638
	+ .0464		− .1131	494		− .2537		− .1785	494	.0275	.3880
	− .0123		− .1877	495		− .3364		− .2559	495	.0235	.4127
	− .0708		− .2619	496		.4185		.3329	496	.0198	.4378
	− .1287		− .3350	497		.4993		.4087	497	.0163	.4630
	− .1860		− .4074	498		.5793		.4838	498	.0132	.4883
	− .2423		− .4784	499		.6579		.5574	499	.0105	.5134
	− .2979		− .5486	500		− .7357		− .6304	500	.0082	.5384
	.3519		.6169	501		.8114		.7013	501	.0063	.5628
	.4050		.6842	502		.8863		.7714	502	.0049	.5868
	.4569		.7504	503	−1.0415	.9601		.8403	503	.0040	.6102
	.5075		.8153	504	.9681	−1.0330		.9081	504	.0037	.6328
	− .5574		− .8796	505	− .9046		−1.0252	− .9754	505	.0039	.6548
	.6062	−1.0601	.9433	506	.8490		.9594	−1.0423	506	.0047	.6760
	.6539	.9939	−1.0061	507	.8002		.9021		507	.0061	.6962
	.7006	.9359		508	.7567		.8516		508	.0081	.7154
	.7459	.8850		509	.7178		.8068		509	.0107	.7334
	− .7902	− .8396		510	− .6826		− .7666		510	.0139	.7502
	.8329	.7992		511	.6507		.7304		511	.0178	.7655
	.8742	.7629		512	.6216		.6977		512	.0223	.7793
	.9143	.7298		513	.5947		.6677		513	.0274	.7917
	.9530	.6998		514	.5699		.6403		514	.0329	.8027
−1.0104	− .9897	− .6726		515	− .5471		− .6153		515	.0389	.8120
.9767	−1.0239	.6483		516	.5263		.5928		516	.0453	.8194
.9473		.6262		517	.5072		.5722		517	.0521	.8250
.9208		.6057		518	.4890		.5528		518	.0593	.8292
.8969		.5865		519	.4718		.5347		519	.0667	.8322
− .8757		− .5688		520	− .4557		− .5178		520	.0743	.8338
.8568		.5522		521	.4403		.5019		521	.0821	.8341
.8399		.5368		522	.4258		.4870		522	.0899	.8334
.8244		.5221		523	.4117		.4726		523	.0979	.8317
.8101		.5079		524	.3979		.4587		524	.1060	.8292
− .7963		− .4938		525	− .3842		− .4448		525	.1142	.8262
.7833		.4802		526	.3708		.4313		526	.1223	.8228
.7704		.4664		527	.3572		.4177		527	.1305	.8191
.7583		.4531		528	.3439		.4045		528	.1386	.8150
.7467		.4398		529	.3306		.3913		529	.1467	.8105
− .7352		− .4267		530	− .3174		− .3782		530	.1547	.8059
.7240		.4137		531	.3043		.3652		531	.1625	.8013
.7129		.4008		532	.2913		.3523		532	.1702	.7966
.7021		.3879		533	.2782		.3394		533	.1778	.7918
.6913		.3749		534	.2650		.3264		534	.1854	.7868
− .6808		− .3619		535	− .2519		− .3135		535	.1929	.7816
.6704		.3490		536	.2386		.3005		536	.2003	.7764
.6598		.3357		537	.2252		.2872		537	.2077	.7711
.6493		.3223		538	.2114		.2737		538	.2151	.7656
.6389		.3088		539	.1977		.2602		539	.2224	.7600

Illuminant *A* $x_w=0.4476$, $y_w=0.4075$		Illuminant *B* $x_w=0.3485$, $y_w=0.3517$		Wave-length (mμ)	Illuminant *C* $x_w=0.3101$, $y_w=0.3163$		Equal energy $x_w=0.3334$, $y_w=0.3334$		Wave-length (mμ)	x	y
$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$		$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$	$\frac{(x-x_w)}{(y-y_w)}$	$\frac{(y-y_w)}{(x-x_w)}$			
−.6286		−.2953		540	−.1838		−.2466		540	.2296	.7543
.6179		.2812		541	.1694		.2325		541	.2369	.7485
.6073		.2671		542	.1548		.2182		542	.2441	.7426
.5962		.2523		543	.1397		.2034		543	.2514	.7336
.5851		.2373		544	.1243		.1884		544	.2586	.7305
−.5739		−.2220		545	−.1086		−.1729		545	.2658	.7243
.5625		.2063		546	−.0926		.1573		546	.2729	.7181
.5504		.1899		547	−.0759		.1409		547	.2801	.7118
.5381		.1730		548	−.0586		.1239		548	.2873	.7054
.5257		.1558		549	−.0410		.1067		549	.2944	.6989
−.5126		−.1377		550	−.0226		−.0886		550	.3016	.6923
.4989		−.1189		551	−.0035		−.0698		551	.3088	.6857
.4849		−.0996		552	+.0160		−.0506		552	.3159	.6791
.4700		−.0792		553	+.0365		−.0304		553	.3231	.6724
.4547		−.0583		554	+.0575		−.0096		554	.3302	.6657
−.4387		−.0365		555	+.0794		+.0120		555	.3373	.6589
.4217		−.0133		556	.1025		+.0348		556	.3445	.6520
.4036		+.0109		557	.1265		+.0587		557	.3517	.6451
.3847		+.0359		558	.1512		+.0833		558	.3588	.6383
.3644		+.0626		559	.1774		+.1094		559	.3660	.6314
−.3433		+.0902		560	+.2044		+.1364		560	.3731	.6245
.3210		.1193		561	.2327		.1647		561	.3802	.6175
.2966		.1503		562	.2627		.1949		562	.3874	.6105
.2708		.1826		563	.2938		.2261		563	.3945	.6036
.2433		.2168		564	.3264		.2591		564	.4016	.5966
−.2136		+.2530		565	+.3608		+.2939		565	.4087	.5896
−.1816		.2915		566	.3969		.3307		566	.4158	.5826
−.1469		.3323		567	.4350		.3695		567	.4229	.5756
−.1092		.3757		568	.4752		.4107		568	.4300	.5686
−.0681		.4221		569	.5177		.4544		569	.4371	.5616
−.0238		+.4709		570	+.5621		+.5002		570	.4441	.5547
+.0242		.5227		571	.6086		.5485		571	.4510	.5478
+.0780		.5788		572	.6585		.6005		572	.4580	.5409
+.1377		.6394		573	.7119		.6564		573	.4650	.5339
+.2033		.7039		574	.7679		.7154		574	.4719	.5270
+.2768		+.7733		575	+.8274		+.7784		575	.4788	.5202
.3588		.8479		576	.8904		.8456		576	.4856	.5134
.4521		.9290	+1.0764	577	.9580	+1.0439	.9180		577	.4924	.5066
.5574		+1.0162	.9841	578	+1.0294	.9714	.9952	+1.0048	578	.4991	.4999
.6791			.8996	579		.9039	+1.0788	.9269	579	.5058	.4932
+.8205			+.8226	580		+.8414		+.8554	580	.5125	.4866
.9862	+1.0140		.7521	581		.7833		.7894	581	.5191	.4800
+1.1818	.8462		.6877	582		.7295		.7289	582	.5256	.4735
	.7053		.6285	583		.6793		.6729	583	.5321	.4671
	.5853		.5737	584		.6322		.6207	584	.5385	.4607
	+.4825		+.5232	585		+.5884		+.5724	585	.5448	.4544
	.3936		.4765	586		.5475		.5276	586	.5510	.4482
	.3157		.4332	587		.5091		.4857	587	.5572	.4421
	.2463		.3925	588		.4727		.4463	588	.5633	.4360
	.1859		.3552	589		.4392		.4101	589	.5692	.4301
	+.1309		+.3198	590		+.4070		+.3755	590	.5752	.4242
	+.0817		.2869	591		.3769		.3433	591	.5810	.4184
	+.0381		.2566	592		.3490		.3136	592	.5866	.4128
	−.0021		.2277	593		.3222		.2852	593	.5922	.4072
	−.0380		.2011	594		.2974		.2589	594	.5976	.4018
	−.0708		+.1761	595		+.2739		+.2341	595	.6029	.3965
	−.1004		.1530	596		.2521		.2112	596	.6080	.3914
	−.1270		.1316	597		.2318		.1899	597	.6130	.3865
	−.1516		.1114	598		.2125		.1698	598	.6178	.3817
	−.1744		.0923	599		.1943		.1508	599	.6225	.3770
	−.1951		+.0747	600		+.1773		+.1332	600	.6270	.3725
	.2148		+.0576	601		.1609		.1161	601	.6315	.3680
	.2326		+.0418	602		.1455		.1002	602	.6359	.3637
	.2497		+.0264	603		.1306		.0847	603	.6402	.3594
	.2654		+.0122	604		.1167		.0704	604	.6443	.3553
	−.2797		−.0010	605		+.1038		+.0572	605	.6482	.3514
	.2926		−.0132	606		.0918		+.0449	606	.6520	.3477
	.3051		−.0251	607		.0802		+.0329	607	.6557	.3440
	.3166		−.0360	608		.0693		+.0218	608	.6592	.3405
	.3271		−.0462	609		.0593		+.0115	609	.6625	.3372
	−.3368		−.0558	610		+.0498		+.0018	610	.6658	.3340
	.3461		.0649	611		+.0407		−.0075	611	.6689	.3309
	.3549		.0736	612		+.0321		−.0162	612	.6719	.3279
	.3628		.0815	613		+.0241		−.0243	613	.6747	.3251
	.3703		.0891	614		+.0166		−.0320	614	.6774	.3224
	−.3776		−.0965	615		+.0092		−.0395	615	.6801	.3197
	.3843		.1033	616		+.0024		.0464	616	.6826	.3172
	.3902		.1094	617		−.0037		.0526	617	.6849	.3149
	.3961		.1154	618		−.0098		.0588	618	.6872	.3126
	.4016		.1211	619		−.0156		.0646	619	.6894	.3104

Illuminant A $x_w = 0.4476,\ y_w = 0.4075$ $\frac{(x-x_w)}{(y-y_w)}$	Illuminant A $\frac{(y-y_w)}{(x-x_w)}$	Illuminant B $x_w = 0.3485,\ y_w = 0.3517$ $\frac{(x-x_w)}{(y-y_w)}$	Illuminant B $\frac{(y-y_w)}{(x-x_w)}$	Wave-length (mμ)	Illuminant C $x_w = 0.3101,\ y_w = 0.3163$ $\frac{(x-x_w)}{(y-y_w)}$	Illuminant C $\frac{(y-y_w)}{(x-x_w)}$	Equal energy $x_w = 0.3334,\ y_w = 0.3334$ $\frac{(x-x_w)}{(y-y_w)}$	Equal energy $\frac{(y-y_w)}{(x-x_w)}$	Wave-length (mμ)	x	y
	− .4067		− .1265	620		− .0210		− .0701	620	.6915	.3083
	.4111		.1313	621		.0258		.0750	621	.6935	.3064
	.4157		.1361	622		.0306		.0798	622	.6954	.3045
	.4199		.1405	623		.0351		.0844	623	.6972	.3027
	.4238		.1447	624		.0394		.0886	624	.6989	.3010
	− .4277		− .1488	625		− .0435		− .0929	625	.7006	.2993
	.4313		.1527	626		.0474		.0968	626	.7022	.2977
	.4346		.1562	627		.0511		.1005	627	.7037	.2962
	.4377		.1596	628		.0544		.1038	628	.7051	.2948
	.4409		.1631	629		.0580		.1074	629	.7066	.2933
	− .4437		− .1661	630		− .0611		− .1105	630	.7079	.2920
	.4465		.1691	631		.0641		.1136	631	.7092	.2907
	.4492		.1721	632		.0672		.1167	632	.7105	.2894
	.4517		.1748	633		.0700		.1195	633	.7117	.2882
	.4542		.1776	634		.0727		.1223	634	.7129	.2870
	− .4565		− .1800	635		− .0753		− .1248	635	.7140	.2859
	.4587		.1825	636		.0778		.1273	636	.7151	.2848
	.4607		.1847	637		.0800		.1296	637	.7161	.2838
	.4627		.1869	638		.0823		.1319	638	.7171	.2828
	.4647		.1891	639		.0846		.1341	639	.7181	.2818
	− .4665		− .1911	640		− .0866		− .1362	640	.7190	.2809
	.4682		.1931	641		.0886		.1382	641	.7199	.2800
	.4696		.1947	642		.0903		.1399	642	.7208	.2792
	.4712		.1965	643		.0921		.1417	643	.7216	.2784
	.4725		.1980	644		.0936		.1432	644	.7223	.2777
	− .4739		− .1995	645		− .0952		− .1448	645	.7230	.2770
	.4752		.2010	646		.0967		.1463	646	.7237	.2763
	.4763		.2022	647		.0980		.1476	647	.7243	.2757
	.4775		.2035	648		.0993		.1489	648	.7249	.2751
	.4786		.2048	649		.1006		.1502	649	.7255	.2745
	− .4795		− .2058	650		− .1017		− .1513	650	.7260	.2740
	.4805		.2069	651		.1028		.1524	651	.7265	.2735
	.4814		.2079	652		.1039		.1535	652	.7270	.2730
	.4821		.2088	653		.1047		.1543	653	.7274	.2726
	.4831		.2098	654		.1058		.1554	654	.7279	.2721
	− .4838		− .2106	655		− .1066		− .1562	655	.7283	.2717
	.4845		.2115	656		.1075		.1571	656	.7287	.2713
	.4851		.2121	657		.1081		.1577	657	.7290	.2710
	.4858		.2129	658		.1090		.1586	658	.7294	.2706
	.4864		.2135	659		.1096		.1592	659	.7297	.2703
	− .4869		− .2142	660		− .1103		− .1599	660	.7300	.2700
	.4873		.2146	661		.1107		.1603	661	.7302	.2698
	.4878		.2152	662		.1113		.1609	662	.7305	.2695
	.4882		.2156	663		.1117		.1613	663	.7307	.2693
	.4885		.2160	664		.1122		.1618	664	.7309	.2691
	− .4889		− .2164	665		− .1126		− .1622	665	.7311	.2689
	.4892		.2168	666		.1130		.1626	666	.7313	.2687
	.4896		.2172	667		.1134		.1630	667	.7315	.2685
	.4900		.2176	668		.1139		.1634	668	.7317	.2683
	.4901		.2178	669		.1141		.1637	669	.7318	.2682
	− .4905		− .2183	670		− .1145		− .1641	670	.7320	.2680
	.4907		.2185	671		.1147		.1643	671	.7321	.2679
	.4910		.2189	672		.1151		.1647	672	.7323	.2677
	.4912		.2191	673		.1153		.1649	673	.7324	.2676
	.4916		.2195	674		.1157		.1653	674	.7326	.2674
	− .4918		− .2197	675		− .1159		− .1655	675	.7327	.2673
	.4921		.2201	676		.1164		.1660	676	.7329	.2671
	.4923		.2203	677		.1166		.1662	677	.7330	.2670
	.4925		.2205	678		.1168		.1664	678	.7331	.2669
	.4928		.2209	679		.1172		.1668	679	.7333	.2667
	− .49300		− .22110	680		− .11741		− .16700	680	.73340	.26660
	.49321		.22134	681		.11766		.16725	681	.73352	.26648
	.49343		.22158	682		.11791		.16750	682	.73364	.26636
	.49362		.22180	683		.11814		.16773	683	.73375	.26625
	.49382		.22203	684		.11837		.16796	684	.73386	.26614
	− .49401		− .22225	685		− .11860		− .16819	685	.73397	.26603
	.49419		.22245	686		.11881		.16839	686	.73407	.26593
	.49435		.22263	687		.11899		.16858	687	.73416	.26584
	.49451		.22281	688		.11918		.16877	688	.73425	.26575
	.49465		.22297	689		.11935		.16893	689	.73433	.26567
	− .49477		− .22311	690		− .11949		− .16908	690	.73440	.26560
	.49488		.22324	691		.11962		.16920	691	.73446	.26554
	.49496		.22334	692		.11972		.16931	692	.73451	.26549
	.49503		.22342	693		.11980		.16939	693	.73455	.26545
	.49510		.22350	694		.11987		.16947	694	.73459	.26541
	− .49514		− .22354	695		− .11993		− .16951	695	.73461	.26539
	.49519		.22360	696		.11999		.16957	696	.73464	.26536
	.49521		.22362	697		.12001		.16960	697	.73465	.26535
	.49523		.22364	698		.12003		.16962	698	.73466	.26534
	− .49525		− .22366	699		− .12005		− .16964	699	.73467	.26533

stimulus. Note that dominant wave-length is determined from the slope of the line connecting (x, y) with (x_w, y_w) when the slope is arithmetically less than one, and from the reciprocal of the slope in other cases; this avoids inconveniently large numbers. The values computed by this method are usually uncertain by only one or two in the last figure given. Greater computational certainty could be obtained by carrying the tabular values to more decimal places and by taking account of second differences in the interpolation, but there is little practical need for dominant wave-lengths accurate to hundredths of a millimicron or better, or of purities accurate to the fourth decimal. The error introduced by using linear interpolation between tabular values to only four decimals is considerably smaller than that due to the use of wave-length intervals of 10 mμ in the computation of trilinear coordinates.

VI. Interconversion Between the I.C.I. and Previous O.S.A. Coordinate Systems

Fundamental colorimetric specifications in America have been expressed chiefly in terms of the O.S.A. excitation data[2] in their extrapolated form[26] so adjusted that at the center of the Maxwell triangle there is represented either O.S.A. mean noon sunlight[2] or Abbot-Priest sunlight[27] or Davis-Gibson mean noon sunlight.[28] Specifications given according to these three systems are interconnected as follows:

O.S.A. sunlight to Abbot-Priest sunlight and reverse

$$\bar{r}_0 = 1.00360\ \bar{r}_p, \qquad \bar{r}_p = 0.99641\ \bar{r}_0,$$
$$\bar{g}_0 = 0.98923\ \bar{g}_p, \qquad \bar{g}_p = 1.01089\ \bar{g}_0,$$
$$\bar{b}_0 = 1.00734\ \bar{b}_p, \qquad \bar{b}_p = 0.99271\ \bar{b}_0;$$

O.S.A. sunlight to Davis-Gibson sunlight and reverse

$$\bar{r}_0 = 0.99939\ \bar{r}_d, \qquad \bar{r}_d = 1.00061\ \bar{r}_0,$$
$$\bar{g}_0 = 0.99761\ \bar{g}_d, \qquad \bar{g}_d = 1.00240\ \bar{g}_0,$$
$$\bar{b}_0 = 1.00297\ \bar{b}_d, \qquad \bar{b}_d = 0.99704\ \bar{b}_0;$$

Abbot-Priest sunlight to Davis-Gibson sunlight and reverse

$$\bar{r}_p = 0.99580\ \bar{r}_d, \qquad \bar{r}_d = 1.00421\ \bar{r}_p,$$
$$\bar{g}_p = 1.00847\ \bar{g}_d, \qquad \bar{g}_d = 0.99160\ \bar{g}_p,$$
$$\bar{b}_p = 0.99566\ \bar{b}_d, \qquad \bar{b}_d = 1.00436\ \bar{b}_p,$$

where specifications in the tristimulus system referred to O.S.A. sunlight at the center of the Maxwell triangle are $\bar{r}_0$, $\bar{g}_0$, $\bar{b}_0$, those referred to Abbot-Priest sunlight are $\bar{r}_p$, $\bar{g}_p$, $\bar{b}_p$, and those referred to Davis-Gibson sunlight are $\bar{r}_d$, $\bar{g}_d$, $\bar{b}_d$. The constants of these equations also serve to connect trilinear coordinates in one system with those in another. For example, trilinear coordinates, r_p, g_p, b_p, referred to Abbot-Priest sunlight may be computed from those, r_d, g_d, b_d, referred to Davis-Gibson sunlight, thus:

$$r_p = \frac{0.99580\ r_d}{0.99580\ r_d + 1.00847\ g_d + 0.99566\ b_d},$$
$$g_p = \frac{1.00847\ g_d}{0.99580\ r_d + 1.00847\ g_d + 0.99566\ b_d},$$
$$b_p = \frac{0.99566\ b_d}{0.99580\ r_d + 1.00847\ g_d + 0.99566\ b_d}.$$

No such rigorous connection as this exists between specifications expressed in terms of the O.S.A. excitation data and those expressed in terms of the I.C.I. standard observer and coordinate system because the observers represented are not identical. However, if the O.S.A. coordinate system (apart from the excitation data) be taken as defined by specifications for the four I.C.I. cardinal stimuli, equations of transformation from the one system to the other may be given. These coefficients apply strictly if identical observers are used in the two systems, and they apply approximately to specifications computed from the two sets of standard data. For example, define the coordinate system representing O.S.A. sunlight at the center of the Max-

[26] *Spectrophotometry; Report of O.S.A. Progress Committee for 1922–3,* J.O.S.A. and R.S.I. **10**, 230 (1925).

[27] I. G. Priest, *A Precision Method for Producing Artificial Daylight,* Phys. Rev. (2) **11**, 502 (1918); J.O.S.A. and R.S.I. **12**, 479 (1926). For spectral distribution of energy in Abbot-Priest sunlight and method of adjustment see Bur. Standards J. Research **4**, 525 (1930).

[28] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature,* Bur. Standards Misc. Pub. No. 114, pp. 16, 47; January, 1931.

well triangle by the following values for the I.C.I. cardinal stimuli:

Cardinal stimuli	r_0	g_0	b_0
700.0 mμ	1.0000,	0.0000,	0.0000;
546.1 mμ	0.3899,	0.5891,	0.0210;*
435.8 mμ	0.0000,	0.0049,	0.9951;*
Standard illuminant *B*	0.3432,	0.3271,	0.3297.**

This system is connected to the I.C.I. standard coordinate system by the transformation equations:

I.C.I. to O.S.A. (O.S.A. sunlight) and reverse

$$r_0 = +0.5349\,\bar{x} + 0.6075\,\bar{y} - 0.1146\,\bar{z},$$

$$\bar{g}_0 = -0.5122\,\bar{x} + 1.4183\,\bar{y} + 0.0939\,\bar{z},$$

$$\bar{b}_0 = -0.0124\,\bar{x} + 0.0343\,\bar{y} + 1.1451\,\bar{z};$$

$$\bar{x} = +1.3258\,r_0 - 0.5722\,\bar{g}_0 + 0.1796\,\bar{b}_0,$$

$$\bar{y} = +0.4788\,r_0 + 0.4998\,\bar{g}_0 + 0.0069\,\bar{b}_0,$$

$$\bar{z} = \pm 0.0000\,r_0 - 0.0211\,\bar{g}_0 + 0.8750\,\bar{b}_0.$$

* For interpolated values see J.O.S.A. 21, 531–540 (1931).

** Computed by summation over wave-length intervals of 10 mμ.

Similar transformation equations connecting the I.C.I. coordinate system with the other O.S.A. systems are:

I.C.I. to O.S.A. (Abbot-Priest sunlight) and reverse

$$\bar{r}_p = +0.5330\,\bar{x} + 0.6053\,\bar{y} - 0.1142\,\bar{z},$$

$$\bar{g}_p = -0.5178\,\bar{x} + 1.4338\,\bar{y} + 0.0949\,\bar{z},$$

$$\bar{b}_p = -0.0123\,\bar{x} + 0.0340\,\bar{y} + 1.1368\,\bar{z};$$

$$\bar{x} = +1.3305\,\bar{r}_p - 0.5660\,\bar{g}_p + 0.1809\,\bar{b}_p,$$

$$\bar{y} = +0.4805\,\bar{r}_p + 0.4944\,\bar{g}_p + 0.0070\,\bar{b}_p,$$

$$\bar{z} = \pm 0.0000\,r_p - 0.0209\,\bar{g}_p + 0.8814\,\bar{b}_p.$$

I.C.I. to O.S.A. (Davis-Gibson sunlight) and reverse

$$\bar{r}_d = +0.5352\,\bar{x} + 0.6079\,\bar{y} - 0.1146\,\bar{z},$$

$$\bar{g}_d = -0.5135\,\bar{x} + 1.4217\,\bar{y} + 0.0941\,\bar{z},$$

$$\bar{b}_d = -0.0123\,\bar{x} + 0.0341\,\bar{y} + 1.1418\,\bar{z};$$

$$\bar{x} = +1.3250\,\bar{r}_d - 0.5708\,\bar{g}_d + 0.1801\,\bar{b}_d,$$

$$\bar{y} = +0.4785\,\bar{r}_d + 0.4986\,\bar{g}_d + 0.0069\,\bar{b}_d,$$

$$\bar{z} = \pm 0.0000\,\bar{r}_d - 0.0211\,\bar{g}_d + 0.8776\,\bar{b}_d.$$

A MAXWELL TRIANGLE YIELDING UNIFORM CHROMATICITY SCALES

J. Opt. Soc. Am. 25, 24-35 (1935)

This epoch-making paper is a superb record of research and results on perception of color differences up to its date. Surprisingly little has been added to that information since 1935. Transformation of Fig. 1 to rectangular coordinates that correspond to primaries that have 0,1,0 luminosity coefficients, and rounding to obtain integer coefficients of transformation from the CIE coordinates (x,y) led to the CIE 1960 UCS chromaticity diagram (u,v). The 1960 CIE diagram was the basis of the 1964 CIE U* V* W* color space and color-difference formula. Merely with a 50% increase of the scale of v, the 1964 CIE color space became the CIE 1976 L*u*v* color space, on which the corresponding color-difference formula was based.

The only important matters of terminology encountered in this paper are that "millimicrons" (mμ) are now called "nanometers" (nm) and that "colorimetric purity" is now rarely encountered. The distinction between that and "excitation purity", which is now commonly called "purity", is indicated in "A general formula for the computation of colorimetric purity", J. Res. Nat. Bur. Stand. (U.S.), 7, 827-841 (1931) RP377.

Fig. 1 is a direct descendant of Fig. 1 in the paper entitled "Chromaticity sensibility to stimulus differences", J. Opt. Soc. Am. 22, 72-108 (1932).

A Maxwell Triangle Yielding Uniform Chromaticity Scales*

DEANE B. JUDD, *National Bureau of Standards, Washington, D. C.*

(Received November 30, 1934)

CONTENTS

A colorimetric coordinate system has been found by trial and error whose Maxwell triangle has the useful property that the length of any line on it is a close measure of the chromaticity difference between the stimuli represented at the extremes of the line. Such accurate chromaticity scales may be derived from this triangle merely by stepping off equal intervals on it that it has been called the "uniform-scale triangle." The definition of the system is given, and also a comparison of experimental sensibility data with corresponding data derived from the triangle. An important application of this coordinate system is its use in finding from any series of colors the one most resembling a neighboring color of the same brilliance, for example, the finding of the nearest color temperature for a neighboring non-Planckian stimulus. The method is to draw the shortest line from the point representing the non-Planckian stimulus to the Planckian locus.

I. Introduction

In a previous paper[1] experimental data on chromaticity sensibility were related to the (r, g)-plot of the O. S. A. tristimulus coordinate system. The method of relation was called the "square construction" because by that method the locus of points representing chromatic colors equally distinguishable from any one given color of the same brilliance is a square centered on the point representing the given color.

The present method of relating chromaticity sensibility to the Maxwell triangle is the graphically more convenient "circle construction." When applied to the O. S. A. colorimetric coordinate system, the circle construction resulted in poorer agreement with sensibility data than the analytically simple square construction. A new coordinate system has been derived, however, for the sole purpose of making use of the circle construction to the best advantage on the equilateral Maxwell triangle. This paper gives the definition of the new tristimulus coordinate system, shows the degree of agreement obtained both with the data previously

* Publication Approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. Opt. Soc. Am. **22**, 72 (1932).

assembled and those which have become available since, and points out the uses to which the Maxwell triangle of this coordinate system may be put.

II. Mathematical Analysis

If any two color stimuli specified by points (r_1, g_1, b_1) and (r_2, g_2, b_2) on the equilateral triangle are compared with brightnesses equalized, the circle construction states that the chromaticity difference, ΔE, between them is proportional to the distance, D, between the points representing them, or:

$$K\Delta E = D, \quad (1)$$

where K is a constant of proportionality dependent on the unit (least perceptible difference, probable error and so on) in which ΔE is expressed and on the experimental conditions such as field size and field brightness.[2]

From the geometry of the equilateral triangle the distance, D, may be expressed in terms of the differences between the trilinear coordinates, writing Δr, Δg, Δb as abbreviations for $r_1 - r_2$, $g_1 - g_2$, $b_1 - b_2$, respectively, thus:

$$D = \{2[(\Delta r)^2 + (\Delta g)^2 + (\Delta b)^2]/3\}^{\frac{1}{2}}, \quad (2)$$

where unit D is the altitude of the triangle. This relation may also be written in a form particularly convenient for dealing with a number of distances in the same direction. Let Δv be any one of Δr, Δg, Δb, and Δq be either of the remaining two, then:

$$D = (2\Delta v/3^{\frac{1}{2}})[(\Delta q/\Delta v)^2 + (\Delta q/\Delta v) + 1]^{\frac{1}{2}}, \quad (2a)$$

$(\Delta q/\Delta v)$ constant being the condition for distances in the same direction on the triangle.

In a few cases where the circle construction is analytically inconvenient, the nearly equivalent hexagon construction has been substituted. According to this construction:

$$\begin{aligned} K\Delta E &= \Delta c \\ &= |\Delta r|, |\Delta g|, \text{ or } |\Delta b|, \text{ whichever is the largest;} \\ &= (|\Delta r| + |\Delta g| + |\Delta b|)/2, \end{aligned} \quad (1a)$$

where the absolute value signs (| |) indicate that the differences are to be taken all greater than zero. The hexagon construction differs from the circle construction by a factor varying from $2/3^{\frac{1}{2}}$ to 1 or in maximum by about 15 percent. It may be substituted for the circle construction if differences of this size are not significant.

III. Mixture Diagram

The coordinate system found by trial and error to yield the best agreement with chromaticity sensibility may conveniently be defined as a projective transformation of the 1931 I. C. I. standard coordinate system for colorimetry.[3] If $\bar{x}$, $\bar{y}$, $\bar{z}$ be the tristimulus specifications of a given color stimulus on the I. C. I. system, then $\bar{r}$, $\bar{g}$, $\bar{b}$, the tristimulus specifications on the uniform-scale system, may be found as:

$$\begin{aligned} \bar{r} &= 3.1956\,\bar{x} + 2.4478\,\bar{y} - 0.1434\,\bar{z} \\ \bar{g} &= -2.5455\,\bar{x} + 7.0492\,\bar{y} + 0.9963\,\bar{z} \\ \bar{b} &= 0.0000\,\bar{x} + 0.0000\,\bar{y} + 1.0000\,\bar{z} \end{aligned}$$

The reverse transformation is given by: (3)

$$\begin{aligned} \bar{x} &= 0.24513\,\bar{r} - 0.08512\,\bar{g} + 0.11996\,\bar{b} \\ \bar{y} &= 0.08852\,\bar{r} + 0.11112\,\bar{g} - 0.09802\,\bar{b} \\ \bar{z} &= 0.00000\,\bar{r} + 0.00000\,\bar{g} + 1.00000\,\bar{b} \end{aligned}$$

For convenience in checking arithmetical results the coefficients of these transformation equations are given to at least three more decimal places than would be significant from the trial-and-error adjustment to experimentally known chromaticity sensibility.

It may be noted that the only way in which better agreement with sensibility data may be effected is by a transformation which alters the shape of the spectrum locus. Transformations which merely displace the locus with respect to the triangle, or reorient it, or expand or contract it uniformly do not at all change the agreement. The system defined by (3) is one of many possible systems giving this particular shape to the spectrum locus. The luminosity coefficients are $L_r = 0.08852$, $L_g = 0.11112$, $L_b = -0.09802$.

The uniform scale system is more closely allied to the O. S. A. coordinate system[4] by

[2] Judd, J. Opt. Soc. Am. **22**, 87 (1932).

[3] Proc. 8th Session, International Commission on Illumination, Cambridge, pp. 19–29, September, 1931. D. B Judd, *The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry*, J. Opt. Soc. Am. **23**, 359 (1933). T. Smith and J. Guild, *The C. I. E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 73 (1931–32).

[4] Judd, J. Opt. Soc. Am. **22**, 102 (1932).

TABLE I. *Trilinear coordinates on the uniform-chromaticity-scale triangle. Observer: 1931 I. C. I. Standard.*

Part I. For the spectrum

Wavelength (mμ)	r	g	b	Wavelength (mμ)	r	g	b
380	0.2681	0.2437	0.4881	550	0.3918	0.6072	0.0009
				560	.4404	.5592	.0004
410	.2648	.2461	.4891	570	.4995	.5003	.0002
				580	.5708	.4291	.0002
430	.2575	.2584	.4841	590	.6532	.3467	.0001
440	.2495	.2761	.4744				
				600	.7388	.2611	.0001
450	.2363	.3049	.4588	610	.8169	.1830	.0001
460	.2173	.3493	.4334	620	.8776	.1224	.0001
470	.1961	.4226	.3813	630	.9206	.0793	.0000
480	.1833	.5353	.2814	640	.9521	.0478	.0000
490	.1906	.6467	.1627				
				650	.9728	.0272	.0000
500	.2147	.7092	.0761				
510	.2440	.7249	.0312	700	1.0000	.0000	.0000
520	.2784	.7103	.0113				
530	.3145	.6805	.0050				
540	.3511	.6467	.0022				

Part II. For the Planckian radiator

Color temperature	r	g	b	Color temperature	r	g	b
100	1.0000	0.0000	0.0000	2,848	0.5434	0.4239	0.0327
300	.9981	.0019	.0000	3,500	.5103	.4427	.0470
600	.9237	.0763	.0000	4,800	.4696	.4598	.0706
1,000	.7906	.2086	.0008	6,500	.4409	.4672	.0920
1,500	.6809	.3140	.0051	10,000	.4142	.4699	.1159
1,900	.6242	.3640	.0118	24,000	.3898	.4685	.1417
2,360	.5783	.4001	.0216	∞	.3769	.4663	.1568

Part III. For some miscellaneous stimuli

	r	g	b
Abbot-Priest sunlight	0.4607	0.4665	0.0728
Abbot-Davis-Gibson mean noon sunlight	.4572	.4688	.0740
Sunlight outside atmosphere (Abbot)	.4402	.4713	.0885
Equal energy spectrum recombined	.4583	.4583	.0833

derivation than to the I. C. I. system. The coefficients of transformation between the uniform-scale system and the O. S. A. system are:

O. S. A. to Uniform-Scale	*Uniform-Scale to O. S. A.*	
1, 0, 0.1	1, 0, −0.5	
0, 1, 0.1	0, 1, −0.5	(3a)
0, 0, 0.2	0, 0, 5.0	

These coefficients have been used in transforming data on the O. S. A. basis referring to Lovibond glasses, but in the other comparisons presented, the 1931 I. C. I. standard observer and Eq. (3) have been used.

Fig. 1 shows the spectrum locus and the Planckian locus on the equilateral Maxwell triangle of the new uniform-scale system. A given color stimulus ($\bar{r}$, $\bar{g}$, $\bar{b}$) is represented on this triangle according to its trilinear coordinates (r, g, b) which are fractional parts of the total, $\bar{r}+\bar{g}+\bar{b}$, and refer to distances perpendicular to a side of the triangle. Table I gives the trilinear coordinates from which the spectrum locus and the Planckian locus on Fig. 1 have been plotted; it also gives the trilinear coordinates of the recombined equal-energy spectrum which is the basic stimulus of the I. C. I. system, Abbot-Priest sunlight,[5] mean noon sunlight from Abbot's data as averaged by Davis and Gibson,[6] and sunlight outside the atmosphere from Abbot's data.[7]

On this triangle equal chromaticity intervals are represented by lines of nearly equal length. An outstanding difference between this and the usual Maxwell triangle is the smaller distance between the Planckian locus and the spectrum locus from 550 to 600 mμ.

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[5] I. G. Priest, *A Precision Method for Producing Artificial Daylight*, Phys. Rev. **11**, 502 (1918); J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 479 (1926). D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, Bur. Standards J. Research **4**, 525 (1930); RP163.

[6] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub. Bur. Stand. **114**, 16 (1931).

[7] Reference 6, Table I.

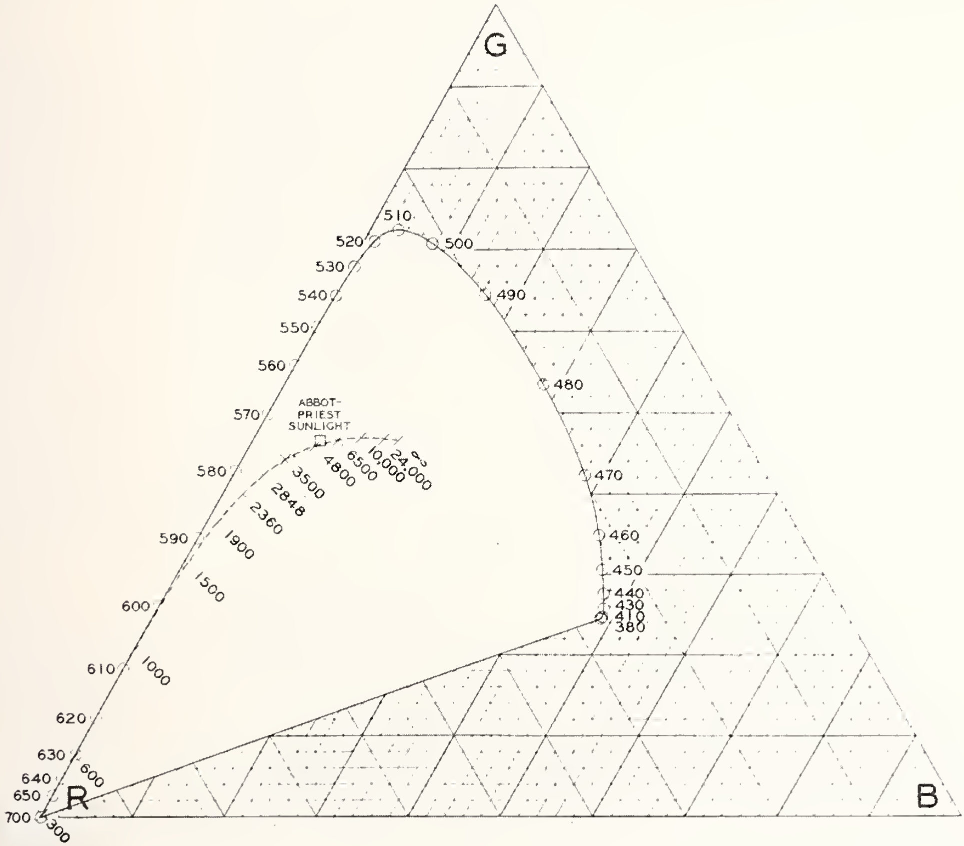

FIG. 1. The uniform-chromaticity-scale triangle. Wavelength along the spectrum locus is indicated in millimicrons; color temperature along the Planckian locus is given in degrees Kelvin.

IV. Comparison with Experimental Data

As in the previous paper the method of comparison is to plot the various types of chromaticity sensibility data on the same graph with similar data derived from the triangle by relations (1) or (1a). The constant, K, is adjusted to a value providing nearly the best agreement in order to facilitate comparison.

In the derivation of the new coordinate system, considerably more weight was given to some chromaticity-sensibility data than to others. These weights were determined in no very systematic or quantitative way; they depended on a critical survey of the whole body of data and on estimates of relative reliability of the various sets of data. Of the rather large proportion of the data not taken into account in the actual derivation of the system, much was found to be in good agreement. All the data yielding sufficiently good agreement to warrant graphical comparison are shown, however, without regard to the weight given them in deriving the system. Data which do not agree well are not shown but are mentioned at appropriate places in the text.

A. Sensibility to change in dominant wavelength at constant purity

1. *Sensibility at unit purity*

By Eq. (1a) the hexagon construction yields for this case:

$$d\lambda/dE = K/(dc_h/d\lambda)_{p=1}, \quad (4)$$

where λ is wavelength and $(dc_h/d\lambda)_{p=1}$ is the

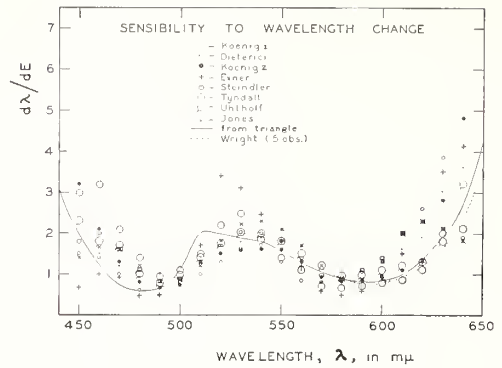

FIG. 2. Chromaticity sensibility to wavelength change in the spectrum.

largest of $|dr/d\lambda|$, $|dg/d\lambda|$, $|db/d\lambda|$ referring to the spectrum locus. Fig. 2 compares experimental values of $d\lambda/dE$ with $0.007/(dc_h/d\lambda)_{p=1}$ which is shown by the solid line. The experimental data indicated by points are those given in Fig. 3 of the previous paper;[8] to these has been added a dotted curve representing the result of a recent careful investigation with modern apparatus by Wright and Pitt.[9]

The degree of agreement shown by Fig. 2 is good but not perfect. The solid curve rises somewhat too high near 510 mμ and stays too low near 470. The curves of results by Laurens and Hamilton[10] showing pronounced secondary minima not duplicated in the solid curve have not been given; a discussion of the reliability of these omitted results is given by Wright and Pitt.[11]

2. Ratio of sensibility at purity, p, to that at unit purity

Analogous to relation (9a) of the previous paper extended to the general choice of heterogeneous stimulus[12] the hexagon construction yields:

$$\frac{(d\Lambda/dE)_p}{(d\Lambda/dE)_{p=1}}=\frac{[s_h(1-p)+s_wp]^2(dc_h/d\lambda)}{s_wp\{[s_h(1-p)+s_wp](dc_h/d\lambda)+(1-p)(c_w-c_h)(ds_h/d\lambda)\}}, \qquad (5)$$

where Λ is dominant wavelength, s_h is an abbreviation for $rL_r+gL_g+bL_b$ for the spectrum at wavelength, Λ, s_w is an abbreviation for the similar sum referring to the heterogeneous stimulus ("white light"), and c as in relation (1a) is either r, g or b. As discussed in the previous paper care must be taken in applying this formula because of the multiple definition of c_h. In the numerator, c_h is taken as r, g or b according as $dr/d\lambda$, $dg/d\lambda$ or $db/d\lambda$ has the greatest absolute value for the spectrum ($p=1$). In the denominator, c_h is chosen as r, g or b according to which choice makes the absolute value of the denominator the greatest. For most values of wavelength, c_h is the same in the numerator and denominator, but not at all wavelengths.

In Fig. 3 are shown the experimental values of this ratio obtained by Tyndall[13] (circles) and those obtained by Watson[14] (crosses). The data by Watson shown on plots marked 530, 585 and 630, refer to wavelengths 527, 589 and 632 mμ, respectively. The curves shown were obtained from Eq. (5) with Abbot-Priest sunlight as the heterogeneous stimulus, that is, for $r_w=0.461$, $g_w=0.466$, $b_w=0.073$. This choice was made because the heterogeneous stimulus used by Tyndall was a color match for Abbot-Priest sunlight; that used by Watson was similar but not definitely known.

It will be noted from Fig. 3 that the agreement at three dominant wavelengths (455, 490 and 630 mμ) is nearly perfect. In the other three cases the experimentally found relation is similar to that yielded by the triangle but not identical.

[8] D. B. Judd, J. Opt. Soc. Am. **22**, 89 (1932).

[9] W. D. Wright and F. H. G. Pitt, *Hue-Discrimination in Normal Colour-Vision*, Proc. Phys. Soc. **46**, 459 (1934).

[10] H. Laurens and W. F. Hamilton, *The Sensibility of the Eye to Differences in Wave Length*, Am. J. Physiol. **65**, 547 (1923).

[11] Wright and Pitt, Proc. Phys. Soc. **46**, 466 (1934).

[12] D. B. Judd, *A General Formula for the Computation of Colorimetric Purity*, J. Opt. Soc. Am. **21**, 729 (1931); see Eq. (14).

[13] E. P. T. Tyndall, *Chromaticity Sensibility to Wave Length Difference as a Function of Purity*, J. Opt. Soc. Am. **23**, 15 (1933).

[14] W. Watson, *Note on the Sensibility of the Eye to Variations of Wave Length*, Proc. Roy. Soc. **B84**, 118 (1911).

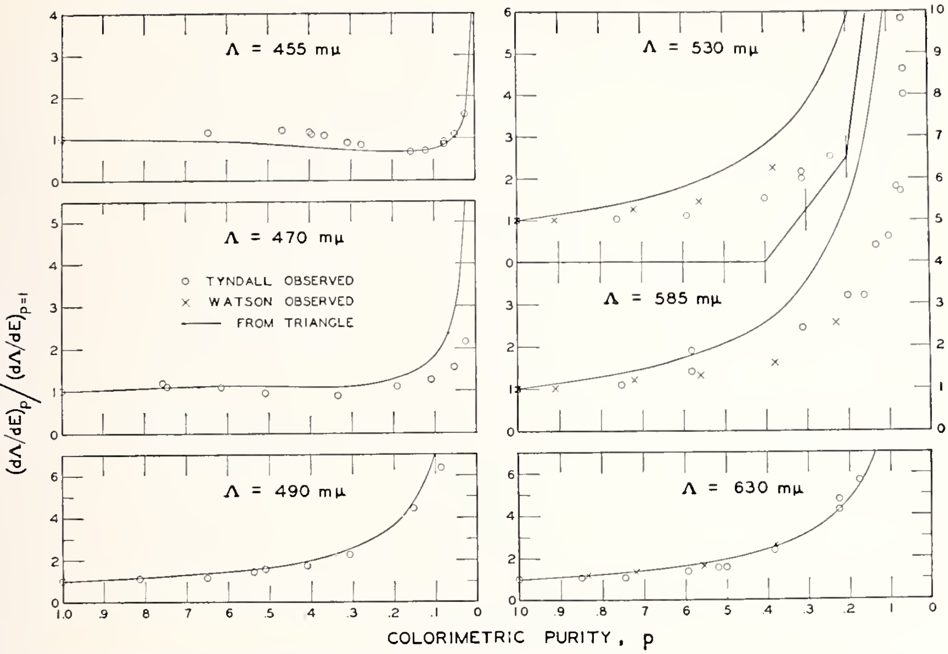

FIG. 3. Chromaticity sensibility to dominant-wavelength change as a function of purity.

B. Sensibility to change in purity, dominant wavelength constant

1. Sensibility as a function of purity

Analogous to relation (10a) of the previous paper extended as was Eq. (5) to the general choice of heterogeneous stimulus it is seen from relations (1) and (2a) that the circle construction yields:

$$\left(\frac{dp}{dE}\right)_{\Lambda\text{ constant}}=\frac{K}{2}\left[\frac{3}{(\Delta q/\Delta v)^2+(\Delta q/\Delta v)+1}\right]^{\frac{1}{2}} \times\frac{[s_h(1-p)+s_wp]^2}{s_hs_w(v_h-v_w)}, \quad (6)$$

where, as in relation (2a), v is any one of r, g, b, and q is either of the remaining two; Δv may be evaluated conveniently here as v_h-v_w, v_h referring to the spectrum at wavelength, Λ, and v_w to the heterogeneous stimulus; Δq similarly may be found as q_h-q_w.

In Fig. 4 are shown a number of experimental determinations of this quantity by Donath[15] (circles) and a determination by Judd[16] (crosses). In Fig. 5 are shown all of the published experimental data on this case in the recent paper by Martin, Warburton and Morgan.[17] The curves in Figs. 4 and 5 refer to wavelengths approximately but not identically those of the experimental results and were obtained from Eq. (6), those in Fig. 4 with K adjusted in each case, those in Fig. 5 with K adjusted to 0.015 throughout. Values of K referring to Donath's work which was done with an improved technic involving rotating disks viewed by both eyes vary from 0.0014 to 0.0023; the value of K for Judd's result referring to a 2° field viewed with one eye is 0.011. The marked dependence of K on observing conditions was discussed in the previous paper.

From Fig. 4 it may be seen that with one exception (550 mμ) the experimental determination of the function agrees with that yielded by the triangle. The data shown in Fig. 5, being results of single settings rather than averages of

[15] F. Donath, *Die funktionale Abhängigkeit zwischen Reiz und Empfindung bei der Farbensättigung*, Neue Psych. Stud. 2, 143 (1926).

[16] J. Opt. Soc. Am. 22, 94 (1932).

[17] L. C. Martin, F. L. Warburton and W. J. Morgan, *Determination of the Sensitiveness of the Eye to Differences in the Saturation of Colours*, Medical Research Council, Reports of the Committee upon the Physiology of Vision, XIII, Special Report Series, No. 188, London, 1933.

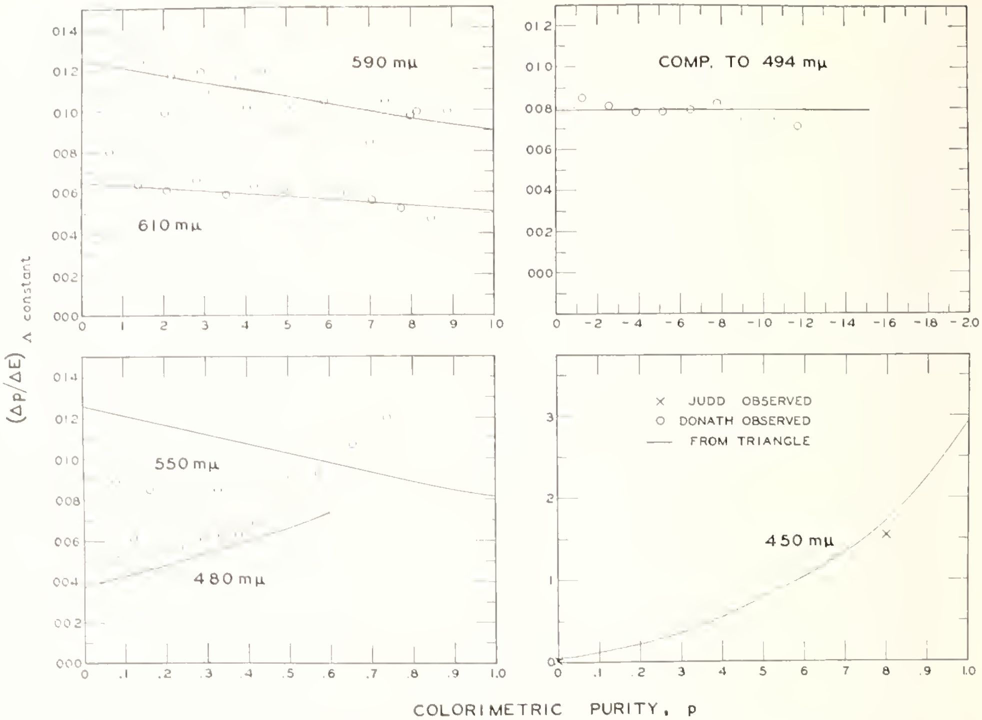

FIG. 4. Chromaticity sensibility to purity change as a function of purity.

from 4 to 10 settings, yield points that are more scattered than those of the previous figures. In general there is fair agreement between the observed function and that obtained from the triangle. Most of the points for 650 and 680 mμ are higher than the curve; perhaps, some of the discrepancy at these wavelengths is ascribable to a low field brightness for this extreme spectral region. There is a general tendency throughout for the experimental data to indicate a rise in dp/dE with purity for low purities. Some of this rise has been ascribed by the authors to a fatigue effect combined with the fact that the majority of the step-by-step settings progressed upward from zero purity; the settings in which the progression was in the opposite direction substantiate this view. It is probably significant that of all wavelengths investigated, that (460 mμ) yielding the most consistent rise for low purities is the only one for which the triangle also indicates a rise. Note also that the observed function for 546 mμ in Fig. 5 is convex upward while that in Fig. 4 for 550 mμ is convex downward, the average being in fair agreement with the function computed from the triangle.

Data by Nutting and Jones[18] and Jones and Lowry[19] deviate considerably from each other and from the functions yielded by the triangle; they are not shown graphically. Possible sources of error in these sets of data have already been discussed.[20]

Data by Judd[21] obtained through a different means (Lovibond yellow glasses) than other sets cited here deviate from the function indicated by the triangle by a factor of about 2.5. Reexamination of these data have shown that an appreciable part of the deviation can be ascribed to the two-millimicron dominant-wavelength variation from one end of the Lovibond-yellow locus to the other which was previously considered negligible. These data, therefore, give a saturation scale along the Lovibond-yellow locus, but not very exactly the scale for dominant

[18] P. G. Nutting, *The Retinal Sensibilities Related to Illuminating Engineering*, Trans. Illum. Eng. Soc. **11**, 16 (1916).

[19] L. A. Jones and E. M. Lowry, *Retinal Sensibility to Saturation Differences*, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 25 (1926).

[20] Judd, J. Opt. Soc. Am. **22**, 95 (1932); see also footnote 17.

[21] D. B. Judd, *Saturation Scale for Yellow Colors*, J. Opt. Soc. Am. **23**, 35 (1933).

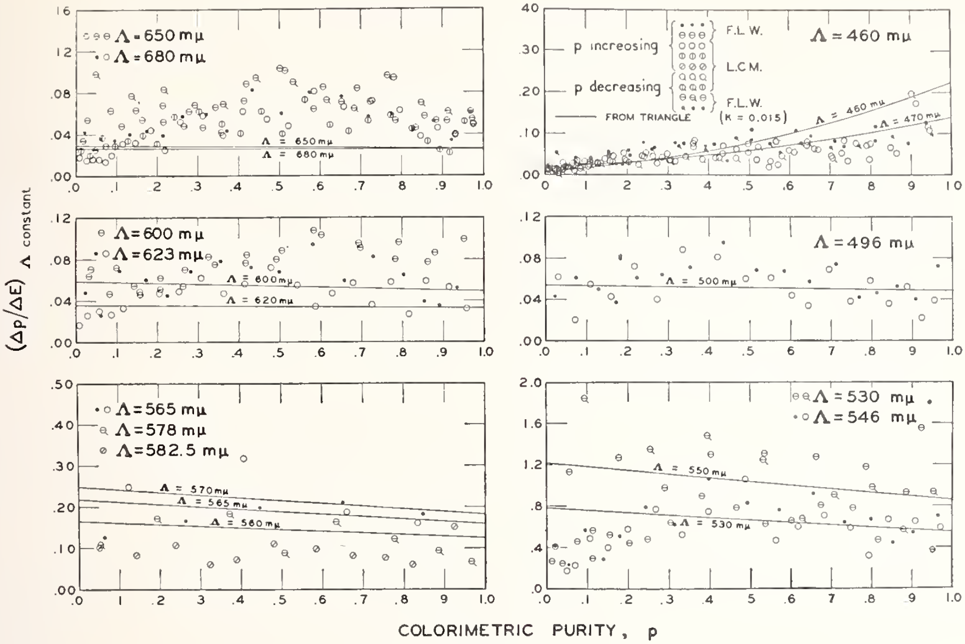

FIG. 5. Chromaticity sensibility to purity change as a function of purity.

wavelength constant at 575 mμ; hence they are not compared with the triangle in the form of dp/dE as a function of p, but as dN''/dE as a function of Lovibond-yellow number, N'' (see later section).

2. Sensibility near zero purity

From Eq. (6) it is seen that the circle construction yields:

$$\left(\frac{dp}{dE}\right)_{p\to 0}=\frac{K}{2}\left(\frac{3}{(\Delta q/\Delta v)^2+(\Delta q/\Delta v)+1}\right)^{\frac{1}{2}} \times\left[\frac{s_h}{s_w(v_h-v_w)}\right]. \quad (6a)$$

In Fig. 6 are shown experimental determinations of this quantity for a number of dominant wavelengths by Priest and Brickwedde[22] and by Purdy;[23] results by Martin, Warburton and Morgan[17] agree well with these and are not shown. These experimental values have been corrected to refer to the visibility function of the 1931 I. C. I. standard observer on the assumption that the true visibility of the three observers is the experimental mean found by Gibson and Tyndall.[24] This assumption is known to be closely correct for Priest, who was one of the 52 observers studied by Gibson and Tyndall; and it is probably nearly correct for the other

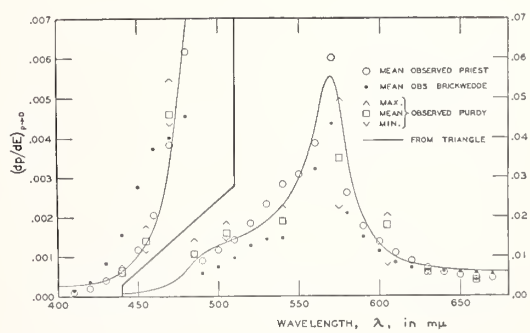

FIG. 6. Chromaticity sensibility to purity change near zero purity as a function of dominant wavelength.

[22] I. G. Priest and F. G. Brickwedde, *The Minimum Perceptible Colorimetric Purity as a Function of Dominant Wave Length with Sunlight as Neutral Standard*, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 306 (1926); see also footnote 1.

[23] D. McL. Purdy, *On the Saturations and Chromatic Thresholds of the Spectral Colours*, Brit. J. Psych. (Gen. Sec.) **21** (Part 3), 283 (1931).

[24] K. S. Gibson and E. P. T. Tyndall, *Visibility of Radiant Energy*, Bur. Stand. Sci. Pap. **19**, 131 (1923–24); S475. See first column of their Table III, p. 174.

two (Brickwedde, Purdy). The correction was made by multiplying the reported purities by the ratio of standard I. C. I. visibility to experimental visibility (V_{ICI}/V_{exp}). This gives the least perceptible colorimetric purity of the field set by the observer as if analyzed photometrically by the standard observer.

In addition to this correction Purdy's values were multiplied by 0.20 to facilitate comparison with the Priest-Brickwedde values. The larger values found by Purdy are attributable to the fact that the size of field used by him (1.5° circular) was less than that used by Priest and Brickwedde (4° square).

The solid curve represents the right-hand member of Eq. (6a) with Abbot-Priest sunlight taken as the heterogeneous stimulus, that is, for $r_w = 0.461$, $g_w = 0.466$, $b_w = 0.073$. This choice was made because Priest and Brickwedde used this heterogeneous stimulus, and because Purdy used one nearly like this. It will be noted that the agreement shown between the corrected experimental results and those from the triangle is good. The uncorrected purities are higher than the plotted values between 400 and 440 $m\mu$ by a factor varying from 3 to 10. The good agreement obtained with the purities corrected on the basis of the Gibson-Tyndall experimental visibility suggests that those values may be more representative than the standard values.

3. *Number of steps between zero and unit purity as a function of wavelength*

According to the circle construction the number of just noticeable chromaticity steps between zero and unit purity is proportional to the distance on the triangle between the point representing the heterogeneous stimulus and the spectrum locus. From relations (1) and (2) this number may be written:

$$\Delta E = (1/K)\{2[(\Delta r)^2 + (\Delta g)^2 + (\Delta b)^2]/3\}^{\frac{1}{2}}, \quad (7)$$

where $\Delta r = r_h - r_w$, $\Delta g = g_h - g_w$, $\Delta b = b_h - b_w$.

Fig. 7 shows this number determined experimentally by Warburton and Martin[17] for a number of dominant wavelengths with the heterogeneous stimulus a close color match for a Planckian radiator at 4800°K.

The curve on Fig. 7 gives the right-hand member of Eq. (7) as a function of dominant wavelength for the heterogeneous stimulus used by Warburton and Martin ($r_w = 0.470$, $g_w = 0.460$, $b_w = 0.070$). As in Fig. 5, the constant, K, is taken as 0.015. The agreement is only fair, but it may be worth noting that Martin and Warburton found more steps than indicated by the curve for the brighter part of the spectrum and fewer steps for the extremes of the spectrum. Perhaps, more steps would have been found at the extremes if examined at the same field brightness as the middle portion.

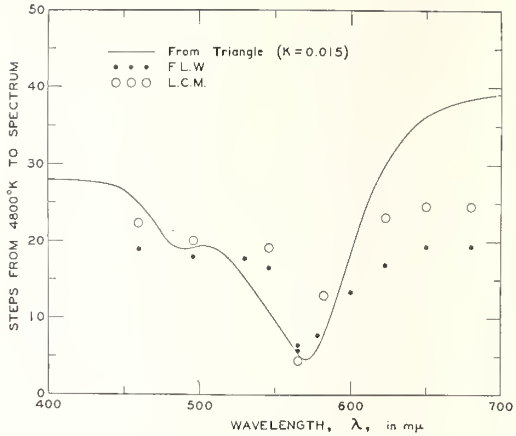

Fig. 7. Number of chromaticity steps from a Planckian radiator at 4800°K to the spectrum as a function of dominant wavelength.

C. Sensibility to change in color temperature

An empirical formula for sensibility to change in color temperature, θ, has already been found and checked[25] over a considerable range (1800 to 11,000°K) of color temperature:

$$d(1/\theta)/dE = \text{a constant.} \quad (8)$$

This relation has been partially responsible for the proposal that reciprocal temperature be used instead of temperature for color specification of illuminants.[26]

From Eq. (1a) it is seen that the hexagon construction yields:

$$d(1/\theta)/dE = Kd(1/\theta)/dc. \quad (9)$$

[25] D. B. Judd, *Sensibility to Color-Temperature Change as a Function of Temperature,* J. Opt. Soc. Am. **23**, 7 (1933).

[26] I. G. Priest, *A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight,* J. Opt. Soc. Am. **23**, 41 (1933).

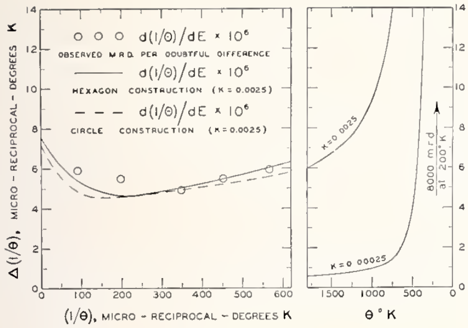

FIG. 8. Sensibility to color-temperature change.

Fig. 8 shows (solid curve) the right-hand member of Eq. (9) as a function of temperature for low temperatures (0 to 1800°K) and as a function of reciprocal temperature for high temperatures (1600°K up, or from zero to 625 micro-reciprocal degrees). Although this curve is far from being horizontal over the entire range, it is nearly horizontal over the range (1800 to 11,000°K or from about 560 to 90 m.r.d.) to which Eq. (8) applies.

Fig. 8 also shows (circles) the data by means of which Eq. (8) was verified. It is seen that the hexagon construction is also closely verified by these data. The agreement is close enough to raise the question whether the approximate hexagon construction should not be replaced by the circle construction which, from Eqs. (1) and (2), yields:

$$\frac{d(1/\theta)}{dE}=\frac{6^{\frac{1}{2}}Kd(1/\theta)}{2[(dr)^2+(dg)^2+(db)^2]^{\frac{1}{2}}}, \qquad (10)$$

where dr, dg and db refer to small differences along the Planckian locus. The right-hand member of Eq. (10) is represented on Fig. 8 by the dotted curve. It is seen that about the same degree of agreement exists with the experimental data.

D. Sensibility to change in Lovibond number

1. *Sensibility to change in Lovibond yellow*

Analogous to Eq. (10) the circle construction yields:

$$\frac{dN''}{dE}=\frac{6^{\frac{1}{2}}KdN''}{2[(dr)^2+(dg)^2+(db)^2]^{\frac{1}{2}}}, \qquad (10a)$$

where N'' refers to Lovibond-yellow number on the Priest-Gibson scale,[27] and dr, dg, db, refer to differences along the Lovibond-yellow locus.

In Fig. 9 are shown experimental determinations of this quantity[28] previously reported as dp/dE on the somewhat inaccurate assumption that the dominant wavelength for the Lovibond yellow glasses is constant. The curve represents the right-hand member of Eq. (10a) with K taken as 0.0045. It is seen that the agreement is fair.

2. *Sensibility to change in Lovibond red combined with 35-yellow*

Eq. (10a) applies to this case also if N'' is taken to refer to Lovibond-red number on the Priest-Gibson scale[29] combined with 35-yellow, and if dr, dg, db, refer to differences along the Lovibond $35Y+N''R$ locus. In Fig. 10 are shown some rather inaccurate determinations of this quantity by way of the probable error of a single setting for chromaticity match in terms of N''. These data were presented and discussed in the previous paper.[30] The curve represents the right-hand member of Eq. (10a) with K taken as 0.00013. The agreement is seen to be about as good as can be expected from data of this low precision (note point at 0.075 for $N''=17.7$). Possibly the circles indicate a curve with a somewhat greater slope than that of the curve derived from the triangle. The decrease in field brightness that accompanies an increase in N'' would account for a small part of such a discrepancy.

E. Conclusions from the comparisons

In every case the agreement previously obtained has been equalled or bettered, and in one case it has been notably improved (see Fig. 6). Agreement with data appearing since the previous paper has also been satisfactory. Considering the rather large individual-observer variation it is estimated that the uniform-chromaticity-scale

[27] I. G. Priest and K. S. Gibson, *Standardizing the Red and Yellow Lovibond Glasses*, J. Opt. Soc. Am. and Rev. Sci. Inst. **16**, 116 (1928).

[28] Judd, J. Opt. Soc. Am. **23**, 35 (1933).

[29] K. S. Gibson and G. W. Haupt, *Standardization of Lovibond Red Glasses in Combination with Lovibond 35 Yellow*, Bur. Standards J. Research, **13**, 433 (1934), RP718.

[30] Judd, J. Opt. Soc. Am. **22**, 98 (1932).

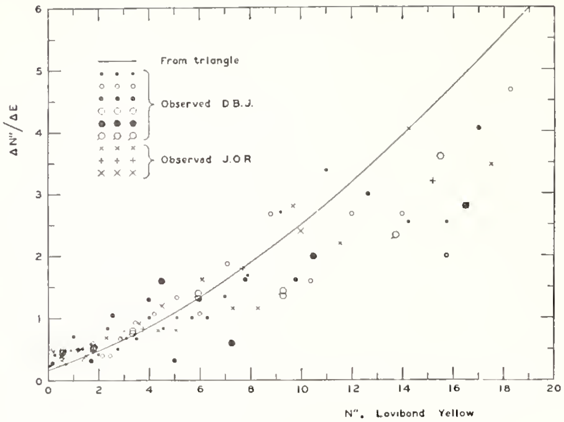

Fig. 9. Sensibility to change in Lovibond yellow.

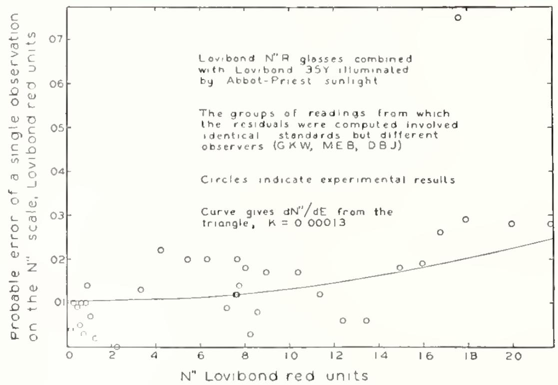

Fig. 10. Sensibility to change in Lovibond red combined with Lovibond 35-yellow.

triangle represents about nine-tenths of available data on chromaticity sensibility within their experimental uncertainty.

V. Applications of the Triangle

The data presented indicate that the system chosen yields, as intended, a Maxwell triangle on which to a good approximation the length of a line is proportional to the chromaticity difference between the stimuli represented at its extremes. Although the data examined nearly all referred to small differences, it is probable that the approximation is of the same order for large differences.[31] The degree of approximation, while by no means sufficient to disprove Schrödinger's[32] theoretical conclusion that such a representation should be made on a spherical surface, is close enough to be of some theoretical interest. Since the coordinate system chosen is only one of an infinity of systems each having the same essential property, it appears possible to fit with certain restrictions the uniform-chromaticity-scale triangle to many of the theories of vision, such as the Hering opponent-colors theory, the Young-Helmholtz three-components formulation, Hecht's[33] modification of it, or the level theory of G. E. Müller.[34] In this way the implications of the uniform-scale triangle might find expression in a variety of theoretical terms. Possibly, too, the theoretical formulation which is found to accomodate the uniform-scale triangle most conveniently may be thought to have some advantage over the others.

Two practical applications of the triangle may be pointed out. One obvious use is the estimation of the chromaticity difference between any two stimuli from their tristimulus specifications. To accomplish this with maximum accuracy the tristimulus specifications must be transformed by appropriate equations such as (3) or (3a). However, the mixture diagram of the standard 1931 I. C. I. coordinate system might be made use of directly to find approximate values of chromaticity difference provided some means were available to take into account the distortion between the two diagrams. A convenient way to do this is to cover the uniform-scale triangle with a number (say 30) of equal circles; then transform by Eq. (3) these circles to the 1931 I. C. I. mixture diagram where they will appear as ellipses. The expansion or contraction of the various parts of the diagram relative to corresponding parts of the uniform-scale diagram would be indicated by the sizes, eccentricities and orientations of these ellipses.

Perhaps, the most important practical application of the uniform-scale triangle is its use in finding from any series of colors the one most

[31] J. Opt. Soc. Am. **23**, 35 (1933).

[32] E. Schrödinger, *Grundlinien einer Theorie der Farbenmetrik im Tagessehen*, Ann. d. Physik (4), **63**, 481 (1920); Die Gesichtsempfindungen, Müller-Pouillets Lehrbuch der Physik, 2nd Ed. **2**, 456 (1926).

[33] Selig Hecht, *The Interrelation of Various Aspects of Color Vision*, J. Opt. Soc. Am. **21**, 615 (1931).

[34] G. E. Müller, *Über die Farbenempfindungen*, Leipzig: Barth, 1930.

resembling any neighboring color of the same brilliance. The method, of course, is to draw the shortest line from the locus of the series to the point representing the neighboring color. This is the geometric equivalent of the method of observation on which many plans of color grading are based. The characteristic of these plans is that they pay attention to chromaticity differences along a series of color standards, and in the usual case that no member of the series gives a perfect chromaticity match, the nearest match is set and the residual difference neglected. In this way a problem which, strictly, is two-dimensional finds a one-dimensional solution that is close enough to be of practical use.

Examples of this plan are the color-grading of illuminants according to color temperature,[35] the grading of cotton-seed oil according to Lovibond red number combined with Lovibond 35-yellow,[36] and the grading of lubricating oil according to the standards of the well-known Union colorimeter.[37] Many uses of a color comparator (such as the Duboscq) having for comparison a column of liquid of variable height are more or less allied to this plan of color-grading; those involving a column producing a relatively large brightness change are little allied, but those involving chiefly changes in chromaticity are good examples of this plan.

In finding nearest color from a series the uniform-scale triangle may be used directly. The locus of the series is plotted together with the point representing the neighboring chromaticity, and the normal to the locus through the point gives the nearest chromaticity of the series. This necessitates transforming the tristimulus specification of the given chromaticity to the uniform-scale triangle each time. It is also possible to carry out the operation once for all by drawing on the uniform-scale triangle a large number of normals to the locus of the series completely covering at sufficiently small intervals the areas of the triangle near the locus. These normals could then be transformed by Eq. (3) to the standard 1931 I. C. I. coordinate system and they would permit nearest chromaticity to be found from trilinear coordinates in the standard system without the necessity for transformation each time to the uniform-scale triangle. It is planned to do this for the Planckian locus to facilitate finding nearest color temperature. Preliminary comparison of nearest color temperatures found by the uniform-scale triangle indicates good agreement with those found by Davis' empirical method.[38]

[35] I. G. Priest, *The Complete Scale of Color Temperature . . .*, Phys. Rev. **20**, 93 (1922); *The Colorimetry and Photometry of Daylight and Incandescent Illuminants . . .*, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1175 (1923).

[36] Report of Color Committee of the American Oil Chemists' Society, Year 1932–1933, Oil and Soap **10**, 114 (1933).

[37] In this instrument no provision at all is made for equalizing the brightnesses of the two fields; hence, the problem, strictly a tridimensional one, is reduced to a single specification by neglecting both small chromaticity and small brilliance differences.

VI. Summary

The uniform-chromaticity-scale triangle is shown to be in good agreement with a large proportion of published data on chromaticity sensibility. The scope, reliability, and consistency of these data are such that it is planned to use the triangle for estimation of chromaticity differences and for the determination of nearest chromaticity from a series for any neighboring chromaticity.

VII. Appendix

If it be desired to avoid the use of triangular coordinate plots, the trilinear coordinates, r, g, b, may be plotted in rectangular coordinates by the transformation:

$$x=(2b+g)/3^{\frac{1}{2}},\quad y=g, \tag{11}$$

where x and y are abscissa and ordinate, respectively, of the usual rectangular coordinates.[39] These transformations place the equilateral triangle (see Fig. 1) in the first quadrant of the (x, y)-plot, with the G-point $(r=0,\ g=1,\ b=0)$ uppermost and the R-point $(r=1,\ g=b=0)$ at the origin $(x=y=0)$.

In rectangular coordinates relation (2) may be written in the familiar form:

$$D=(\overline{\Delta x}^2+\overline{\Delta y}^2)^{\frac{1}{2}} \tag{12}$$

and relation (2a) becomes:

$$D=\begin{cases}\Delta x[(\Delta y/\Delta x)^2+1]^{\frac{1}{2}} & \text{or}\\ \Delta y[(\Delta x/\Delta y)^2+1]^{\frac{1}{2}}.\end{cases} \tag{12a}$$

[38] R. Davis, *A Correlated Color Temperature for Illuminants*, Bur. Standards J. Research **7**, 659 (1931); RP365.

[39] Note distinction from $\bar{x}$ and $\bar{y}$ of relation (3).

A METHOD FOR DETERMINING WHITENESS OF PAPER

Pap. Trade J. 100 (21), 40-42 (1935)

Magnesium oxide, which was used as the working standard, is not much used today. Various preparations of highly purified barium sulfate have replaced it, being more convenient, more durable and less subject to change with age or in various environments. A subcommittee of Technical Committee 1.3 of the CIE worked for many years, attempting to improve on Judd's formula, and on formulas proposed subsequently by other workers throughout the world. The deliberations of that committee were inconclusive. So far as is known, Judd's formula is as useful as any other. The values of r,g,b, the "trilinear coordinates", i.e., the coordinates in the uniform chromaticity-scale diagram (Fig. 1), were computed from the coordinates x,y in the CIE 1931 chromaticity diagram by use of formulas given in Ref. 5, which is included in this collection.

Reprinted from THE PAPER TRADE JOURNAL, *May 23, 1935*

A Method for Determining Whiteness of Paper*

By Deane B. Judd[1]

Abstract

The General Electric reflectance meter ("Brightness" tester) which measures apparent reflectivity for blue light gives results for yellowish papers that are in fair accord with visual whiteness grading, but it cannot be expected to do so for all colors, particularly for bluish papers. The method suggested here is an extension of MacAdam's work on yellowish-white textiles. It makes use of a color diagram on which equal distances in any direction refer to equally perceptible color differences. This color diagram is a transformation of the standard 1931 ICI coordinate system for colorimetry; the colors of the paper samples may be located on this diagram by any fundamental method of colorimetry using the ICI system. There remains yet to be discovered whether graders of paper use essentially the criteria used by MacAdam's observers; but preliminary results indicate that they do.

In cooperation with the Color Committee of the Technical Association of the Pulp and Paper Industry which is developing a standard method for measurement of the color of paper, an attempt is being made to define a suitable reference standard of whiteness and to develop a method of measuring degree of approach to that standard.

Any highly reflecting material (paper, pulp, paint, porcelain, and so on) may depart from white in two characteristic ways; it may fail to be white by being somewhat grayish or it may fail by being reddish, yellowish, greenish, or bluish, that is, by being chromatically colored. If these departures are not pronounced, the material is still called a white or near-white, and it becomes a matter of interest to determine the degree of approach to white, or the *whiteness* of the material.

The ideal standard white is taken here, tentatively, as a perfectly reflecting, perfectly diffusing surface. A convenient working white standard, which approaches to a considerable degree this ideal, may be found in a layer, at least half a millimeter thick, of magneisum oxide.*(1)*

Such a layer when illuminated at about 45 degrees and viewed perpendicular to the surface has as closely as can be determined the same apparent reflectivity as the ideal white standard.*(2)* The apparent reflectivity, furthermore, is constant throughout the spectrum to within one per cent.*(3)*

Whiteness, then, is tentatively to be measured as degree of approach to the appearance of magnesium oxide. Both the chromatic and achromatic departures can be measured separately. The achromatic departure (departure toward gray) is measured by apparent daylight reflectivity, or, more specifically, apparent reflectivity for standard ICI illuminant C.*(4)* The chromatic departure can be measured as distance on the uniform-chromaticity-scale triangle.*(5)* This triangle is a transformation of the standard 1931 ICI coordinate system *(4)* and is shown as Fig. 1, the colors of the spectrum being indicated according to wave length in millimicrons, and the colors of the perfect radiator according to color temperature on the absolute scale. As herein defined the colors of near-white materials illuminated by daylight are represented within the small circle centering on color temperature 6500 deg. K, which represents closely the color of average daylight. The square centers on the point representing average noon sunlight color.

A question to which there seems to be no theoretical answer is, "How much achromatic departure (grayness) is equivalent to a given amount of chromatic departure (say yellowness)?" An observer confronted with an array of near-white materials has to decide this equivalence before he can grade the materials according to whiteness. Much of the disagreement in whiteness grading between observers is ascribable to difference in the way grayness is balanced against, say, yellowness or greenness. The criteria for this judgment are likely to be affected by the training of the observer; and observer trained in grading paper may differ considerably from one trained in grading limestone or laundered white goods. The criterion as between yellowness and grayness has been determined by MacAdam*(6)* for 30 observers trained in judging laundered white goods. It is the purpose of this paper to describe a method by which may be discovered and formulated in a practical way the criteria by means of which experienced graders of paper judge whiteness. It is hoped that there will be found (1) whether magnesium oxide is a satisfactory standard of whiteness. (2) whether the criteria found by MacAdam to apply to the judgment of whiteness of laundered textiles are those used in the visual whiteness grading of paper, and (3) what degree of agreement exists among experienced graders of the whiteness of paper.

Formulation of MacAdam's Results and Extension to All Hues

The near-white textiles studied by MacAdam differed from white in two ways only; they differed by being grayish or by being yellowish. It is assumed in the present work, however, that the significant element indicated by MacAdam's measure of yellowness (colorimetric purity) is perceptibility of the chromatic difference. For the purpose of extending MacAdam's result to samples of other hues (red, green, blue) it is planned to substitute for

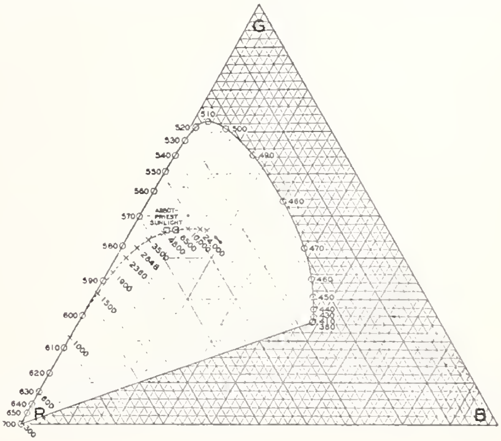

FIG. 1.

The uniform-chromaticity-scale triangle. Wave length along the spectrum locus is indicated in millimicrons; color temperature along the Planckian locus is given in degrees Kelvin. This figure is taken from RP-756 (J. Research NBS, vol. 14, p. 44, 1935); a small circle centered on color temperature 6500 deg. K has been added to indicate the location of near-white samples illuminated by daylight.

* Presented at the annual meeting of the Technical Association of the Pulp and Paper Industry, New York, N. Y., Feb. 18 to 21, 1935.
[1] National Bureau of Standards, Washington, D. C.
* Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

colorimetric purity of yellow samples the perceptibility of the chromatic difference regardless of hue. This can be found with sufficient approximation from color specifications in the ICI system in two ways: (1) substitute for colorimetric purity, p, the ratio, p/p_0 where p_0 is the least perceptible colorimetric purity, or (2) substitute for colorimetric purity, p, the distance on the uniform-chromaticity-scale triangle between the point representing the standard white and the point representing the given sample. Fig. 2 shows how nearly these two procedures are identical; it gives least perceptible colorimetric purity as a function of wave length according to observations by Priest (circles). Brickwedde (dots), and Purdy (squares); and it also gives (solid line) the purities corresponding to the arc of a circle drawn on the equal-chromaticity-scale triangle with the center on the point representing the standard white. The method involving the uniform-chromaticity-scale triangle is to be used because of its greater simplicity.

MacAdam's results were given in his published paper merely as a graph of reflectivity against colorimetric purity, showing contours representing samples judged to possess equal degrees of whiteness. For greater convenience in applying these results we use a formula derived by MacAdam *(7)* as a representation of them:

$$\text{Whiteness} = (R - kp^2)^{1/2}$$

where: R = reflectivity,
p = colorimetric purity,
k = a constant.

The extension to samples of all hues makes the formula take the form:

$$\text{Whiteness} = (R - KS^2)^{1/2} \quad (1)$$

where: S = distance on the uniform-chromaticity-scale triangle between the point representing the standard white and the point representing the sample, and

K = a constant which is found from MacAdam's results to be 6700 when S^2 is measured as $(\Delta r^2 + \Delta g^2 + \Delta b^2)$, Δr, Δg, and Δb being the corresponding differences in the trilinear coordinates.

The use of this formula for yellowish samples gives results in accord with MacAdam's observers, *(6)* and for yellowish samples of nearly constant reflectivity it gives results in accord with VanArsdel's method of computation. *(1)*

Selection of Samples

Thirty samples of paper were selected for measurement from a group of 50 papers composed of 17 samples supplied by John L. Parsons, Chairman, TAPPI Color Committee, of 23 samples supplied by Miss Helen U. Kiely, a member of the committee, and of 10 samples of unknown origin which had been on hand for several years at the National Bureau of Standards. Of these 50 samples 20 were rejected because they duplicated or nearly duplicated in color one of the samples selected for measurement.

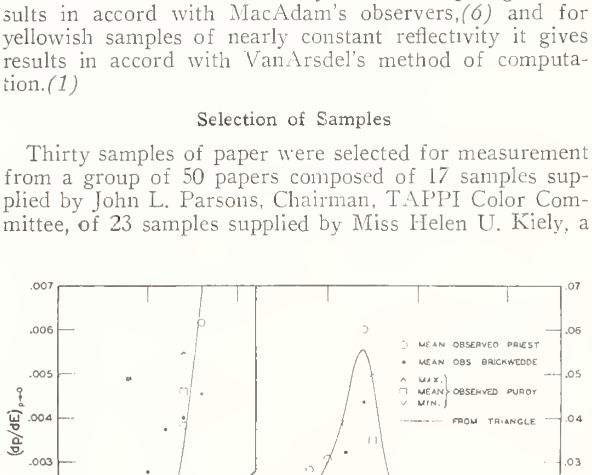

Fig. 2.

Least perceptible colorimetric purity as a function of wave length. This figure is taken from RP-756 (J. Research NBS, vol. 14, p. 50, 1935).

Apparatus and Method

The procedure consisted of measuring (1) the achromatic departure of the sample from the working standard white (magnesium oxide), (2) the chromatic departure, and (3) of combining these two measures into a single index of whiteness according to the formula given above.

The measure of the achromatic departure indicated in the formula is apparent reflectivity, that is, the apparent reflectance of an infinitely thick layer of the sample. For convenience the apparent reflectance of a double thickness of the sample backed with a dark gray backing was measured, it is believed that small differences in reflectance obtained by adding further thicknesses would result in no important change in the arrangement of the samples according to computed whiteness. The felt side of the paper was viewed. This arrangement of the samples was also used in the chromaticity measurements.

The instrument used for the measurement of apparent reflectance relative to magnesium oxide was the Priest-Lange reflectometer. In this instrument the sample is illuminated diffusely from a white-lined sphere, and the amount of light reflected from the sample is compared by means of a Martens photometer to the amount of light reflected from a spot on the sphere wall. Then the magnesium oxide layer is substituted for the sample. The direction of view is perpendicular to the sample (or standard), and the viewing hole is so large that little specularly reflected light enters the photometer. Differences due to finish and gloss are largely avoided in this way. Five settings were made on each of the 30 samples; the resulting means are estimated to be correct to one per cent.

The measurement of the chromatic departure consisted of finding the distance on the uniform-chromaticity-scale triangle between the point representing the sample and the point representing the working standard white. Since the uniform-chromaticity-scale triangle is a transformation of the standard ICI system, any method of measuring the samples in a way sufficiently fundamental to result in location of the point on the standard ICI diagram will serve. There are a number of ways of doing this: (1) spectrophotometry plus computation according to the ICI standard observer, (2) measurement with a tristimulus colorimeter, (3) measurement on a colorimeter which has been specially calibrated in terms of the ICI system.

The method actually used was one of the third type. The double-trapezoid field of the Lummer-Brodhun contrast photometer head was adapted to the viewing of paper samples by substituting for the usual two-sided test plate a holder for paper samples. The illumination is uni-directional and perpendicular to the surface; the angle of view is 45 deg.; these are angular conditions of illuminating and viewing that are reciprocal to the standard ICI conditions. Like those conditions they avoid measurement of the specularly reflected beam, and, as a matter of fact give strictly identical results. The sample and comparison surface were respectively illuminated by 400-watt projection lamps whose color temperatures are known as functions of voltage. The lamp illuminating the sample was set at 2840 deg. K.; at this value of color temperature the lamp combined with a filter of Corning Daylite glass used over the eye-piece gave a close representation of ICI illuminant C. The observer could adjust yellow-blue differences between sample and comparison surface by varying

TABLE I

APPARENT DAYLIGHT REFLECTANCE, TRILINEAR COORDINATES ON THE ICI SYSTEM AND ON THE UNIFORM-CHROMATICITY-SCALE TRIANGLE, AND COMPUTED WHITENESS OF 30 PAPER SAMPLES

Illuminant: Standard 1931 ICI illuminant C

Serial Number	Daylight Reflectance	ICI Trilinear Coordinates			Uniform-chromaticity-scale Trilinear Coordinates			"Whiteness" computed as $(R-KS^2)^{1/2}$
		x	y	z	r	g	b	
1	0.845	0.3156	0.3248	0.3596	0.4429	0.4662	0.0909	0.76
2	.806	.3165	.3262	.3573	.4435	.4664	.0901	.67
5	.713	.3135	.3201	.3664	.4418	.4649	.0934	.79
6	.802	.3159	.3260	.3580	.4429	.4668	.0903	.70
8	.805	.3133	.3251	.3616	.4406	.4681	.0913	.77
9	.751	.3129	.3221	.3651	.4408	.4666	.0927	.81
11	.783	.3124	.3206	.3669	.4406	.4660	.0934	.85
13	.771	.3117	.3204	.3679	.4399	.4664	.0937	.85
15	.803	.3184	.3292	.3524	.4448	.4669	.0884	.45
19	.715	.3081	.3143	.3776	.4375	.4653	.0972	.82
20	.725	.3100	.3167	.3733	.4389	.4654	.0957	.85
21	.782	.3154	.3268	.3578	.4423	.4676	.0901	.67
22	.805	.3165	.3271	.3563	.4433	.4670	.0897	.64
25	.761	.3120	.3204	.3675	.4402	.4662	.0936	.84
26	.724	.3121	.3197	.3682	.4404	.4657	.0939	.83
33	.759	.3115	.3196	.3690	.4398	.4661	.0941	.85
34	.786	.3135	.3219	.3646	.4413	.4661	.0926	.82
35	.794	.3144	.3231	.3625	.4420	.4661	.0919	.80
36	.790	.3150	.3239	.3611	.4424	.4662	.0914	.76
37	.802	.3150	.3237	.3613	.4424	.4660	.0915	.78
38	.813	.3181	.3275	.3544	.4448	.4661	.0892	.55
39	.791	.3108	.3190	.3702	.4393	.4662	.0945	.88
40	.810	.3111	.3220	.3669	.4390	.4679	.0931	.84
41	.806	.3078	.3167	.3755	.4368	.4670	.0962	.86
42	.793	.3339	.3439	.3223	.4565	.4645	.0790	
45	.779	.3317	.3408	.3275	.4551	.4643	.0807	
46	.781	.3294	.3385	.3321	.4534	.4645	.0821	
47	.784	.3267	.3339	.3394	.4518	.4636	.0846	
48	.802	.3259	.3353	.3388	.4507	.4651	.0842	
49	.724	.3220	.3321	.3460	.4476	.4660	.0864	
MgO	1.000	.3101	.3163	.3739	.4392	.4650	.0958	1.00

the voltage on the comparison lamp; and he could adjust red-green differences by introducing a greater or less thickness of a wedge of greenish glass placed before the comparison surface. Then a one-half-millimeter layer of magnesium oxide was substituted for the sample and similar adjustments made. The spectral transmission of the green wedge was measured at the greatest thickness and by computation according to the standard ICI observer the apparatus was calibrated so that from a given voltage on the comparison lamp and a given thickness of wedge required for a color match the position representing the color of the sample either on the ICI coordinate system or on the uniform-chromaticity-scale triangle could be found. These positions could be located with considerable precision because of the ease with which small chromaticity differences could be detected. The complex field combined with the high illuminations given by the 400-watt lamps served to bring out differences estimated at about half those brought out with equal clarity by observation of the doubled sheets by north sky light. Two settings were made on each of the 30 samples selected for measurement; it is estimated that the mean of these two settings is in error by an amount not exceeding in any case 0.0004 in trilinear coordinates of the uniform-chromaticity-scale system.

Results and Discussion

The results of these measurements are given in Table 1 together with the values of "Whiteness" computed from formula (1). The method of computation from trilinear coordinates (r, g, b) and daylight reflectance, R, will be exemplified by the details for one case, sample No. 1, thus:

	r	g	b	
(r, g, b) for sample No. 1	0.4429	0.4662	0.0909	
(r, g, b) for MgO	0.4392	0.4650	0.0958	
$(\Delta r, \Delta g, \Delta b)$	0.0037	0.0012	—0.0049	
$(\Delta r^2, \Delta g^2, \Delta h^2) \times 10^4$	0.137	0.014	0.240	
Sum, $S^2 \times 10^4$				0.391
6700 S^2				0.262
Daylight reflectance, R				0.845
R — 6700 S^2				0.583
"Whiteness" = $(R - 6700\ S^2)^{1/2}$				0.764

It will be noted that values of whiteness for samples 42, 45, 46, 47, 48, and 49 are not given. These samples are not near-whites by the formula; the values of KS^2 for these the samples are greater than R. hence the square-roots of the differences are imaginary. Values of KS^2 for these samples are 3.9, 3.2, 2.6, 1.9, 1.8 and 1.1, respectively. It may be of interest to note that these are the old samples of unknown origin added at the Bureau.

Sets of the samples have been distributed to all those who have signified their willingness to arrange them visually according to whiteness. A few reports of such arrangements show considerable individual difference; however, some degree of agreement with the order of whiteness given by the formula appears to exist. When complete results have been analysed it is hoped to be able to decide: (1) what standard white best agrees with the grading of whiteness by experienced paper graders, (2) how achromatic departures are balanced against chromatic departures from the standard, and (3) what degree of agreement exists among the various graders. There is good prospect that a definition of whiteness can be formulated in practical terms which shall have a logical basis and shall also represent closely the practice of experienced graders of paper.

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(6) MacAdam, D. L., *"The Specification of Whiteness,"* J. Opt. Soc. Am., 24:188-191 (1934)

(7) Letter of Dec. 11, 1934.

A METHOD FOR DETERMINING WHITENESS OF PAPER, II

Pap. Trade J. 103 (8), 38-44 (1936)

Failure to resolve or to agree on a convention to eliminate the ambiguity concerning the chromaticity of the ideal white of paper was largely responsible for the inconclusiveness of the recent report of the CIE subcommittee on whiteness measurement.

Reprinted from the Paper Trade Journal, August 20, 1936

A Method for Determining Whiteness of Paper, II*

By Deane B. Judd[1]

Abstract

In the first paper under this title was proposed a definition of whiteness based on the degree of approach to the appearance of an ideal perfectly reflecting, perfectly diffusing surface. Magnesium oxide was proposed as a working standard white. A plan for testing this definition was outlined; it involved obtaining visual whiteness gradings from 15 observers on a series of 30 papers whose colorimetric specifications were known. The present paper gives the analysis of the results obtained in this way. The general principle of the definition has been corroborated, but it appears that only about one-third of the observers find magnesium oxide an acceptable standard white. About one-third of the observers base their judgments on a standard which is slightly greenish yellow compared to magnesium oxide. This color is called by some of the observers "natural paper white" by which is meant the color of paper made from good grade, well-bleached pulp to which no dye has been added (paper stock is frequently blued with dye). The remaining third of the observers show some agreement with either working standard, but the system or systems of grading used by individuals of this group have been less perfectly discovered. It may be that their criteria for whiteness are inconstant. The definition of whiteness can be written around either working standard with equal convenience, and perhaps a useful purpose would be served by defining both "whiteness" and "natural paper whiteness."

Summary of First Report

In the first communication *(1)* a definition of whiteness was proposed which was based on the degree of approach to the appearance of an ideal perfectly reflecting, perfectly diffusing surface. Magnesium oxide was proposed as a working standard white. A set of 30 papers was collected and their degree of whiteness determined in two ways, first by the proposed method based on the known colorimetric specifications of the papers, and second by direct visual judgment of 15 experienced observers.

From a comparison of the results by these two methods, it was hoped to be able to decide: (1) what standard white best agrees with the grading of whiteness by experienced paper graders, (2) how achromatic departures are balanced against chromatic departures from the standard, and (3) what degree of agreement exists among the various graders.

The present report gives the results of this comparison.

Computed Values Used in the Comparison

The proposed definition of whiteness is:

$$\text{Whiteness} = (R - KS^2)^{1/2} \quad (1)$$

where R = reflectivity

S = distance on the uniform-chromaticity-scale triangle *(2)*, between the point representing the standard white, and the point representing the sample, and

K = a constant which is found, from MacAdam's results on the whiteness grading of laundered white goods *(3)*, to be 6700 when S^2 is measured as $(\overline{\Delta r}^2 + \overline{\Delta g}^2 + \overline{\Delta b}^2)$ Δr, Δg, and Δb being the corresponding differences in the trilinear coordinates.

In testing this formula three tentative working standards of whiteness were used: (1) magnesium oxide ($r = 0.4392$, $g = 0.4650$, $b = 0.0958$), (2) "natural paper white" ($r = 0.4425$, $g = 0.4662$, $b = 0.0913$), and (3) "intermediate" ($r = 0.4402$, $g = 0.4662$, $b = .0936$). The trilinear coordinates specifying the chromaticity of "natural paper white" and "intermediate" were chosen from examination of the results by visual grading. It seemed from inspection of these results that certain groups of observers were using one or another of these three standards of whiteness, and as will presently appear, this is substantially the case.

In the test of the formula for each of these whiteness standards five values of the constant, K, were used: 0, 5400, 6700, 8000, and infinity. It will be noted that the constant, K, determines how the formula balances chromatic departures from the standard against achromatic departures. For K = 0, the formula takes no account whatever of chromatic departures, whiteness becoming equal simply to the square root of the reflectivity. K = 6700 agrees with MacAdam's results on visual whiteness grading of laundered white goods; K = 5400 and K = 8000 are roughly 20 per cent lower and higher, respectively, than MacAdam's value; for very large values of K the formula gives imaginary values of whiteness for all samples of chromaticity different from that of the standard white. The whiteness order (ranking, arrangement) may in these cases be taken as the inverse order of the squares of the right-hand member of Eq. 1. Since K may be made large enough to mask completely the effect of any difference in reflectivity, it may be said that for K equal to infinity the whiteness order by the formula takes no account whatever of reflectivity.

TABLE I

APPARENT DAYLIGHT REFLECTANCE AND TRILINEAR COORDINATES OF THE 29 SAMPLES ON THE ICI SYSTEM AND ON THE UNIFORM-CHROMATICITY-SCALE TRIANGLE

Illuminant: Standard 1931 ICI illuminant C

Serial Number	Daylight Reflectance	ICI Trilinear Coordinates			Uniform-chromaticity-scale Trilinear Coordinates		
		x	y	z	r	g	b
1	0.845	0.3156	0.3248	0.3596	0.4429	0.4662	0.0909
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5	.713	.3135	.3201	.3664	.4418	.4649	.0934
6	.802	.3159	.3260	.3580	.4429	.4668	.0903
9	.751	.3129	.3221	.3651	.4408	.4666	.0927
11	.783	.3124	.3206	.3669	.4406	.4660	.0934
13	.771	.3117	.3204	.3679	.4399	.4664	.0937
15	.803	.3184	.3292	.3524	.4448	.4669	.0884
19	.715	.3081	.3143	.3776	.4375	.4653	.0972
20	.725	.3100	.3167	.3733	.4389	.4654	.0957
21	.782	.3154	.3268	.3578	.4423	.4676	.0901
22	.805	.3165	.3271	.3563	.4433	.4670	.0897
25	.761	.3120	.3204	.3675	.4402	.4662	.0936
26	.724	.3121	.3197	.3682	.4404	.4657	.0939
33	.759	.3115	.3196	.3690	.4398	.4661	.0941
34	.786	.3135	.3219	.3646	.4413	.4661	.0926
35	.794	.3144	.3231	.3625	.4420	.4661	.0919
36	.790	.3150	.3239	.3611	.4424	.4662	.0914
37	.802	.3150	.3237	.3613	.4424	.4660	.0915
38	.813	.3181	.3275	.3544	.4448	.4661	.0892
39	.791	.3108	.3190	.3702	.4393	.4662	.0945
40	.810	.3111	.3220	.3669	.4390	.4679	.0931
41	.806	.3078	.3167	.3755	.4368	.4670	.0962
42	.793	.3339	.3439	.3223	.4565	.4645	.0790
45	.779	.3317	.3408	.3275	.4551	.4643	.0807
46	.781	.3294	.3385	.3321	.4534	.4645	.0821
47	.784	.3267	.3339	.3394	.4518	.4636	.0846
48	.802	.3259	.3353	.3388	.4507	.4651	.0842
49	.724	.3220	.3321	.3460	.4476	.4660	.0864
MgO	1.000	.3101	.3163	.3739	.4392	.4650	.0958

* Presented at the Annual Meeting of the Technical Association of the Pulp and Paper Industry, Waldorf-Astoria Hotel, New York, N. Y., Feb. 17-20, 1936.

Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] National Bureau of Standards, Washington, D. C.

It should be remarked here that the formula has been tested and validated only for samples that are commonly regarded as white or near-white. The formula does not accord with visual whiteness grading of distinctly non-white samples. For example, paper No. 42 as previously reported *(1)* for magnesium oxide as the whiteness standard and K = 6700, comes out according to the formula with imaginary whiteness. This rates it less white than a perfectly absorbing sample (black) which has zero whiteness according to Eq. 1; such a rating is not in accord with visual grading. This defect could be remedied by a somewhat more complicated formulation, and, perhaps, this may be done in the interest of greater academic accuracy; but, for the present, attention will be directed toward discovering whether the present simple formula accords with visual whiteness grading of samples such as those examined here for which there is a practical interest in determining whiteness.

Table I gives the color specifications of the samples used. Trilinear coordinates are given not only in the coordinate system *(2)* referred to by the formula, but also in the coordinate system adopted in 1931 by the International Commission on Illumination *(4)*. Fig. 1 shows as circles most of these 29 samples plotted by means of their trilinear coordinates on the uniform-chromaticity-scale triangle. The three tentative standard whites are indicated by hexagons. Yellow-blue variations on this triangle are represented approximately by horizontal lines on this diagram, yellow being toward the left and blue toward the right; red-green variations are represented by vertical lines, green being toward the top, and red toward the bottom. It will be noted that the chromaticity variation exhibited by this group of paper samples is chiefly a yellow-blue variation, and vertical scatter being only about one-fourth of the horizontal scatter. Furthermore, most of the samples cluster rather closely about a horizontal line representing the yellow-blue variation.

TABLE II

WHITENESS OF THE SAMPLES COMPUTED FROM EQ. 1 FOR THREE DIFFERENT WHITENESS STANDARDS: (1) MAGNESIUM OXIDE, (2) "INTERMEDIATE," AND (3) "NATURAL PAPER WHITE," AND FOR THREE DIFFERENT VALUES OF THE CONSTANT, K: 5400, 6700, AND 8000

Sample	Magnesium oxide			"Intermediate"			"Natural paper white"		
number	5400	6700	8000	5400	6700	8000	5400	6700	8000
1	0.80	0.76	0.73	0.87	0.86	0.85	0.92	0.92	0.92
2	.72	.67	.62	.82	.80	.78	.89	.89	.89
5	.80	.79	.78	.83	.83	.82	.82	.82	.81
6	.74	.70	.65	84	.82	.81	.89	.89	.89
9	.82	.81	.80	86	.86	.86	.85	.85	.84
11	.86	.85	.84	.88	.88	.88	.86	.85	.85
13	.86	.85	.85	.88	.88	.88	.84	.83	.82
15	.57	.45	.28	.73	.69	.64	.85	.84	.83
19	.83	.82	.82	.77	.76	.74	.62	.55	.47
20	.85	.85	.85	.83	.82	.82	.74	.70	.67
21	.72	.67	.62	.82	.81	.79	.87	.87	.87
22	.70	.64	.58	.82	.80	.78	.89	.88	.88
25	.85	.84	.84	.87	.87	.87	.84	.83	.82
26	.83	.83	.82	.85	.85	.85	.81	.80	.79
33	.86	.85	.85	.87	.87	.87	.82	.81	.80
34	.84	.82	.81	.88	.88	.88	.88	.87	.87
35	.82	.80	.77	.87	.87	.86	.89	.89	.89
36	.79	.76	.73	.86	.85	.84	.89	.89	.89
37	.80	.78	.75	87	.86	.85	.90	.90	.90
38	.63	.55	.44	.77	.73	.69	.87	.87	.86
39	.88	.88	.88	.88	88	.88	.82	.80	.79
40	.85	.84	.83	.89	.88	.88	.84	.83	.81
41	87	.86	.85	84	.82	.81	.70	.64	.58
42	1.53i	1.76i	1.97i	1.36i	1.57i	1.76i	1.07i	1.27i	1.44i
45	1.34i	1.56i	1.75i	1.16i	1.37i	1.55i	.85i	1.04i	1.20i
46	1.14i	1.35i	1.53i	.95i	1.15i	1.31i	.58i	.79i	.94i
47	.87i	1.07i	1.23i	.66i	.85i	1.01i	.10	.39i	.58i
48	.79i	1.00i	1.16i	.54i	.74i	.91i	.39	.00	.40i
49	.37i	.59i	.75i	.39	.10	.36i	.68	.63	.57

Apparent daylight reflectance instead of daylight reflectivity as specified by the formula is given in Table I because, as explained before *(1)*, the observations referred to merely a double thickness of the sample rather than to a pile of a large number of sheets.

The specifications given in Table I are the same as previously reported *(1)* except that paper No. 8 has been omitted. In a redetermination of these specifications after an interval of about a year all samples except No. 8 were found to have changed only by insignificant amounts or by amounts scarcely significant. Sample No. 8, however, had become yellowish enough to change considerably its white-

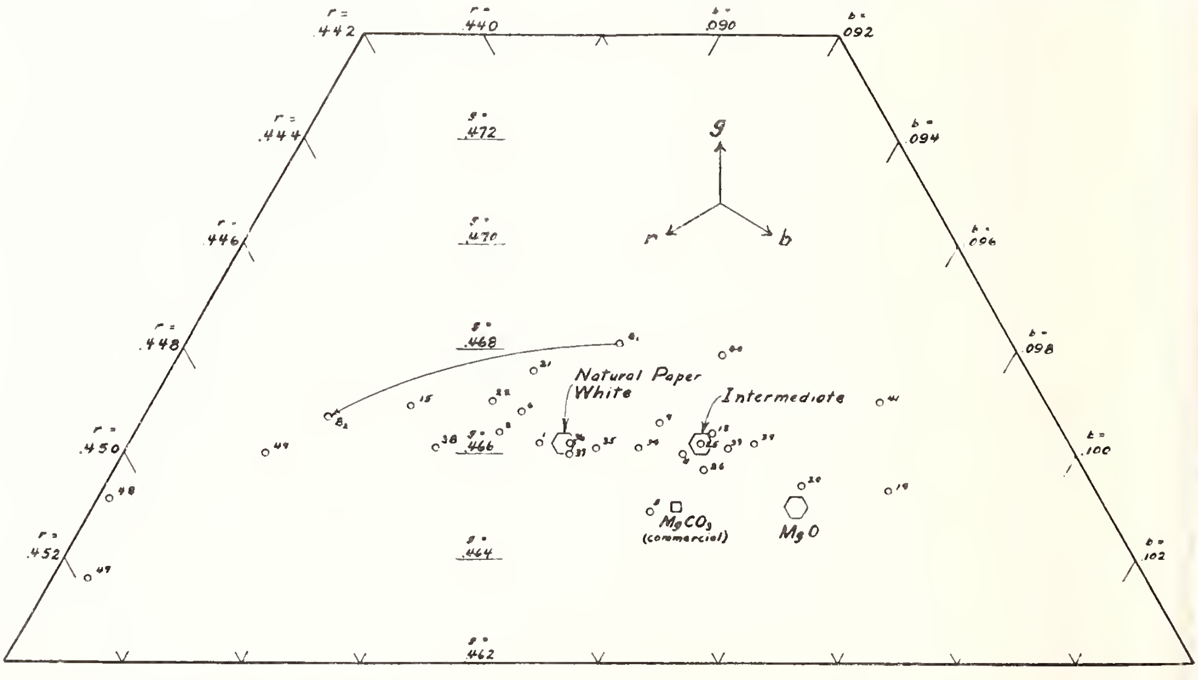

Fig. 1

A portion of the uniform-chromaticity-scale triangle plotted to an enlarged scale showing 27 of the 30 papers. The three tentative standard whites—magnesium oxide, "natural paper white" and "intermediate"—are also represented, together with one sample of commercial magnesium carbonate.

ness rank. On Fig. 1, compare 8_1 with 8_2. Since some of the visual whiteness grading was done nearly three months after the first measurements, the applicability of the first measurement on sample No. 8 is called into serious question; hence this sample was not considered in testing the formula by comparison with visual grading.

Table II gives the whiteness of these 29 samples computed from Eq. 1 for the three tentative standard whites and for K=5400, K=6700 and K=8000, making in all 9 different computed whiteness gradings. Whenever the formula yields imaginary values for whiteness these are recorded by means of the customary symbol, i. which is an abbreviation of $(-1)^{1/2}$; for example, with magnesium oxide as the whiteness standard and for K=6700 sample number 42 has a whiteness of 1.76i which means that for this sample 6700 S^2 (see Table III) exceeded the reflectance (see Table I) by 1.76^2. The interpretation of such an imaginary value is that the sample departs so far from an achromatic color that to give it a whiteness rating has little meaning. Table III gives 6700 S^2 for each of the three tentative standard whites; the inverse rank of these values determines the whiteness order for K equal to infinity.

Table IV gives the whiteness rank of these 29 samples based on the computed values of whiteness given in table 2. The first column gives the identification numbers of the samples arranged nearly in whiteness order for magnesium oxide as the standard white and for K=6700; the remaining columns indicate the whiteness rank of these samples according to the respective whiteness standards and values of K. In the column referring to magnesium oxide as the standard white and to K=6700, the entries would, of course, be in the sequence 1, 2, 3......29 if the first column had been, as intended, in the exact whiteness order computed for these conditions. Inspection of table 2 shows, however, that for the number of figures given (two) there are several ties in whiteness, as for example, papers 13, 20, and 33, all of which have a whiteness of 0.85; these ties were resolved by reference to the original computations which were carried out to the third decimal in spite of the fact that such small differences are probably affected by the experimental uncertainty of the colorimetric specifications. This method of resolving ties was used throughout. Table IV also shows the whiteness rank of these samples for K=0, this considers reflectance alone (see Table I, column 2); and the table shows further the whiteness rank for K equal to infinity, this considers chromatic departures alone (see Table III).

TABLE III

SQUARES OF THE CHROMATIC DEPARTURES (6700 S^2. SEE EQUATION 1). THE WHITENESS ORDER FOR K EQUAL TO INFINITY IS BASED ON THE INVERSE ORDER OF THESE DEPARTURES

Sample number	Magnesium oxide	"Intermediate"	"Natural paper white"
1	0.26	0.10	0.00
2	.36	.16	.02
5	.08	.03	.05
6	.32	.13	.01
9	.10	.01	.04
11	.06	.00	.06
13	.05	.00	.09
15	.60	.33	.10
19	.03	.14	.41
20	.00	.05	.23
21	.33	.13	.02
22	.39	.17	.02
25	.05	.00	.07
26	.04	.00	.08
33	.03	.00	.10
34	.11	.02	.02
35	.16	.04	.00
36	.21	.07	.00
37	.20	.06	.00
38	.51	.28	.06
39	.02	.01	.14
40	.10	.03	.13
41	.07	.13	.40
42	3.90	3.26	2.40
45	3.22	2.65	1.86
46	2.61	2.10	1.40
47	1.92	1.51	.93
48	1.79	1.35	.80
49	1.07	.71	.33

The basis of arrangement of the 29 samples in these several orders is known from the corresponding formulas. If an observer's selections should be in accord with one or another of these arrangements, it might be concluded, therefore, that the criteria used by him had been found.

Visual Whiteness Grading

The visual results were obtained through the cooperation of three members of the TAPPI Color Committee: Dr. John L. Parsons, Miss Helen U. Kiely, and Mr. B. W. Scribner, and through the cooperation of Mr. W. R. Willoughby. The observers were instructed to arrange the samples in the order of whiteness placing the whitest first and the least white last. Since the first communication *(1)* on this subject had already been presented orally, they had a chance to know that magnesium oxide was tentatively proposed as a whiteness standard, and in some cases the observers were supplied with the whiteness of each sample computed on this basis by Eq. 1. On this account some of them may have been prejudiced in favor of the whiteness ratings based on magnesium oxide as the whiteness standard, but such prejudice is not entirely disadvantageous

TABLE IV.—WHITENESS RANK OF THE 29 SAMPLES COMPUTED FROM EQUATION 1

Serial number	Magnesium oxide				"Intermediate"				"Natural paper white"				Reflectance order, K=0
	5400	6700	8000	∞	5400	6700	8000	∞	5400	6700	8000	∞	
39	1	1	1	2	2	2	2	7	18	20	20	20	13
41	2	2	2	9	14	16	16	18	22	22	22	23	5
13	4	4	4	6	5	5	4	2	15	15	15	16	21
11	5	5	6	8	3	1	1	3	11	11	11	12	17
33	3	3	3	4	9	7	7	5	19	18	18	18	23
20	8	6	5	1	17	15	15	12	21	21	21	21	25
25	7	7	7	7	8	6	6	1	16	14	14	14	22
40	6	8	8	12	1	3	3	10	14	16	16	19	3
26	10	9	9	5	13	12	12	4	20	19	19	15	26
34	9	10	11	13	4	4	5	8	8	8	8	7	15
19	11	11	10	3	21	21	21	19	24	24	24	24	28
9	12	12	12	11	11	10	9	6	13	12	12	10	24
35	13	13	14	14	7	8	8	11	5	5	5	4	11
5	14	14	13	10	16	14	14	9	17	17	17	11	29
37	15	15	15	15	10	11	11	13	2	2	2	2	9
1	16	16	17	17	6	9	10	15	1	1	1	3	1
36	17	17	16	16	12	13	13	14	6	4	3	1	14
6	18	18	18	18	15	17	17	16	3	3	4	5	8
21	20	19	19	19	18	18	18	17	9	9	9	8	18
2	19	20	20	20	19	19	19	20	4	6	6	6	4
22	21	21	21	21	20	20	20	21	7	7	7	9	6
38	22	22	22	22	22	22	22	22	10	10	10	13	2
15	23	23	23	23	23	23	23	23	12	13	13	17	7
49	24	24	24	24	24	24	24	24	23	23	23	22	27
48	25	25	25	25	25	25	25	25	25	25	25	25	10
47	26	26	26	26	26	26	26	26	26	26	26	26	16
46	27	27	27	27	27	27	27	27	27	27	27	27	19
45	28	28	28	28	28	28	28	28	28	28	28	28	20
42	29	29	29	29	29	29	29	29	29	29	29	29	12

because it is valuable to know whether observers are sufficiently doubtful of their judgments to be willing to agree to a whiteness grading according to this specific system. The observations were carried out in the observer's own laboratories under the customary illuminant, presumably daylight, and by his own method. Many of the observers used a block of magnesium carbonate as a guide in the grading.

Table V shows the results by the 15 observers. The first column gives the serial number of the samples nearly in order of whiteness according to Eq. 1 for magnesium oxide as the whiteness standard and for K=6700; it is the same as the first column of Table IV. The remaining columns give the whiteness rank of these samples according to the observers identified by the letters A to P, except that ranking D is the average by four observers (G, J, L, O) who were specifically trying to give a whiteness grading based on magnesium oxide and as much in agreement as possible with that indicated in the first column. The columns G, J, L, and O give their individual rankings as free from prejudice as possible. There is also a column giving the average ranking of the columns A to G in which column D is given a weight 4 and the others weight 1; and another column, the final one, giving the average ranking of the columns H to P. As will appear presently, the basis for this division of the observers is their general agreement on whiteness standard.

A considerable amount of individual difference may be noted in Table V. An attempt at this sort of grading by a novice discloses the extreme difficulty of it; many of the color differences are small almost to vanishing, and the color depends by contrast so much on the colors of the neighboring samples that it might easily be suspected that this much individual difference was purely experimental error. Nevertheless, analysis of these data, as will appear presently, and results of repeated gradings indicate that an experienced observer can largely avoid these sources of error.

Correlation Between Observed and Computed Whiteness

It may be seen by inspection of Tables IV and V that there is some degree of agreement between some of the individual whiteness rankings and certain of the computed rankings. For example, observers A, B, and C, give rankings a good deal like those computed for magnesium oxide as whiteness standard, while observers O and P agree somewhat with "natural paper white" as whiteness standard. As a numerical indication of the degree of agreement between these rankings there have been computed coefficients of correlation by the usual methods; thus, correlation coefficient, r, is defined as:

$$r = 1 - 6\ \Sigma d^2/N(N^2 - 1) \qquad (2)$$

where N is the number of samples arranged in order (here 20, 28, or 29), and d is the number by which each sample in one series differs in ranking from the corresponding sample in the other series. It is immediately evident from Eq. 2 that if an observer should give exactly the same ranking as that computed by one of the formulas, values of d for all samples would be zero and the correlation coefficient, r, would be 1.00; this is the value for perfect positive correlation. It is also true that if the observed order were exactly the reverse of the computed order, values of d would be the maximum possible, and $6\,\Sigma\, d^2/N(N^2 - 1)$ would be found to be equal to 2.00, the constant, 6, being of the right size to make this true. The resulting correlation coefficient, r, is therefore, found by Eq. 2 to be —1.00; this is the value for perfect negative correlation. Similarly r = 0 means no connection between the two arrangements whatever. Interpretations of other values of correlation coefficient for N = 29 are:

Correlation Coefficient, r	Interpretation
0.90	Excellent
.80	Good
.70	Fair
.60	Poor
.50	Scarcely significant

For example, observers C and K were asked to repeat their observations a second day and the correlation was computed for each observer with his own previous arrangement. The values of self-correlation so obtained were 0.95 and 0.92, respectively. A correlation of 0.90 is about all that can be expected for observations of the kind with which we are dealing; it indicates that the basis for the judgment has been discovered with a reliability about equal to that of the data themselves.

An example will make clear the method of applying Eq. 2 to the data of Table V and the computed values of Table IV. Compare observer A with the whiteness ranking computed for magnesium oxide as whiteness standard and for K=6700. Both the formula and observer A ranked sample 39 first; for this sample d = 0. The formula placed sample 41 second, but observer A ranked it third; therefore d=1. Sample 13 was ranked fourth by both; so for this sample d = 0. Similarly for samples 11,

TABLE V.—VISUAL WHITENESS RANKINGS BY 15 OBSERVERS AND AVERAGE RANKINGS BY TWO GROUPS OF THEM

Serial number	A	B	C	D * 4 obs.	E	F	G	Avg. A-G	H	I	J	K	L	M	N	O	P	Avg. H-P
39	1	1	3	1	9	13	12	2	8	12	12	15	9	13	..	13	18	14
41	3	2	1	15	16	11	14	10	4	23	13	1	19	14	17	18	23	18
13	4	..	9	2	11	14	6	4	12	11	14	13	18	16	14	17	17	17
11	2	4	10	4	4	5	2	1	7	10	6	12	9	11	2	6	13	8
33	5	5	8	3	7	6	11	3	16	9	17	14	7	13	11	14	15	13
20	8	3	5	13	13	12	5	9	21	13	15	20	22	19	15	22	19	20
25	7	..	15	7	8	9	17	8	14	7	10	17	5	10	13	12	16	12
40	6	6	2	16	15	20	9	15	6	22	18	22	19	15	20	18	12	18
26	15	..	4	12	12	10	11	12	15	8	11	18	8	17	18	16	20	16
34	10	..	14	5	5	1	13	6	11	1	5	11	6	9	10	7	14	10
19	14	7	6	14	14	19	3	14	22	14	16	21	21	18	16	21	22	21
9	11	8	11	8	9	15	22	16	13	16	9	16	16	19	12	15	10	11
35	9	..	7	6	6	4	15	7	4	2	2	10	4	1	9	8	11	6
5	12	..	16	11	17	3	21	17	23	24	19	19	23	21	19	23	21	23
37	19	..	13	9	2	2	1	5	2	4	1	9	2	7	1	1	7	2
1	13	..	20	18	1	8	4	13	1	5	3	2	1	6	5	2	1	1
36	17	..	12	10	3	7	16	11	3	3	4	8	3	2	3	5	8	3
6	18	10	21	19	21	21	8	19	19	19	7	3	13	4	7	3	2	5
21	16	14	19	17	24	18	28	21	20	15	25	23	17	22	21	20	9	22
2	21	9	22	20	18	16	7	18	18	18	21	4	12	3	6	4	3	4
22	22	12	17	23	19	17	18	20	17	17	8	7	11	8	4	10	6	7
38	23	13	18	22	20	23	19	22	9	6	20	6	15	5	8	9	5	9
15	21	11	23	21	24	22	21	23	10	20	22	5	14	25	22	12	4	15
49	24	15	24	24	23	24	29	24	29	21	26	24	29	23	23	29	24	25
48	25	16	25	26	27	25	25	26	24	27	23	25	24	24	25	26	26	24
47	26	17	27	25	24	28	23	25	28	26	24	28	28	28	24	24	25	26
46	27	18	26	27	26	26	24	27	25	25	27	27	27	26	26	25	27	27
45	29	19	28	28	28	27	27	28	26	28	29	26	26	27	27	27	29	28
42	28	20	29	29	29	29	26	29	27	29	28	29	25	29	28	28	28	29

* Average arrangement by 4 observers (G, J, L, O) using as a guide the arrangement indicated in the first column.

TABLE VI

DETAILS OF THE COMPUTATION OF COEFFICIENT OF CORRELATION FOR ONE CASE: OBSERVER A COMPARED TO EQ. 1 WITH MAGNESIUM OXIDE AS WHITENESS STANDARD AND WITH K = 6700

Serial number	d	d^2
39	0	0
41	1	1
13	0	0
11	3	9
33	2	4
20	2	4
25	0	0
40	2	4
26	6	36
34	0	0
19	3	9
9	1	1
35	4	16
5	2	4
37	4	16
1	3	9
36	0	0
6	0	0
21	3	9
2	1	1
22	1	1
38	1	1
15	2	4
49	0	0
48	0	0
47	0	0
46	0	0
45	1	1
42	1	1
Total		131

$$r = 1 - 6\,\Sigma d^2/N(N^2 - 1) = 1 - (6 \times 131)/(29 \times 840) = 1 - 0.032 = 0.968$$

33, and 20, d equals 3, 2, and 2, respectively. Table VI gives these and the remaining details of the computation. The first column gives the serial number of the samples, the second gives the corresponding values of d, and the third gives the squares of d. The coefficient of correlation computed by Eq. 2 is seen to be 0.97.

Table VII summarizes the result of the comparison between observed and computed whiteness arrangements; it gives the correlation coefficients for each observer compared to each of the 13 computed arrangements given in Table IV, except for observer B whose incomplete results were compared with only two of the computed arrangements. Table VII (see last two columns) also gives correlation coefficients between the individual arrangements and one of the two average arrangements recorded in Table V.

Derivation of "Natural Paper White"

The correlations for magnesium oxide as whiteness standard were worked out first and it was noted (see Table VII, column 3) that only four of the sixteen observed arrangements agreed strikingly with the computed arrangement. It was also noted (see Table V) that five observers (L, M, N, O, P) having nearly the poorest agreement with magnesium oxide as whiteness standard agreed fairly well among themselves which suggested that they might be grading according to a different whiteness standard. In an effort to discover what this standard might be the average trilinear coordinates of the first three choices of these five observers was computed, weighting first choice 3, second choice 2 and third choice 1. These trilinear coordinates have been taken as pertaining to "natural paper white," so-called because they refer closely to the chromaticity of paper made from good grade, well-bleached, undyed pulp. The high correlations obtained with Eq. 1 using this whiteness standard (see columns 10 to 13, observers, M, N, O, P) is proof that the effort to discover the basis of grading used by these observers has been successful.

Paper stock is frequently tinted with blue, or red dye, or both, to make it "whiter." This reduces its daylight reflectance somewhat but may make it enough closer in chromaticity to magnesium oxide to give it a higher "whiteness" according to Eq. 1. "Natural paper white" may be taken as a name for the closest approach to "white" possible without the addition of dye.

Separation of Observers Into Groups

As an aid to comprehension of these correlations the observers have been arranged in Table VII either in the order of correlation with magnesium oxide as whiteness standard or in inverse order of the correlation with "natural paper white" whichever correlation was best. It was noted that arrangements A to G gave significant correlation (r greater than 0.60) with magnesium oxide as whiteness standard but failed to give such good correlation with "natural paper white" as standard; while the reverse is true of the remaining observers H to P. This is the basis for the groups taken for averaging purposes in Table V; it is also followed in reporting correlation with other observers in Table VII (see last two columns). Closer analysis of the data show, however, that good agreement with Eq. 1 for magnesium oxide as whiteness standard is confined to the first four observed arrangements (A to D); similarly close correlation with "natural paper white" as standard is confined to only a part of the group chosen (H to P). Accordingly observers E to J have been separated from the others to form an "intermediate" group, although observers I to L are so little different in degree of correlation that to place I and J in the "intermediate" group because their correlation coefficients are less than 0.75 is obviously arbitrary. The members of this intermediate group, however, are characterized in general by significant correlation ranging up to fair with either working standard. The suggestion is that they may be grading according to a whiteness standard which is intermediate between magnesium oxide and "natural paper white."

An attempt has been made to discover this "intermediate" whiteness standard. Since many of these observers used magnesium carbonate as a guide in their judgment of whiteness, the first thought is that this should be tried

TABLE VII.—SUMMARY OF CORRELATION COEFFICIENTS

	Magnesium oxide				"Intermediate"				"Natural paper white"					Experimental mean	
Obs.	5400	6700	8000	∞	5400	6700	8000	∞	5400	6700	8000	∞	R K = O	MgO group	"Natural paper white" group
A	+.97	+.97	+.96	+.88	+.87	+.89	+.89	+.86	+.23	+.23	+.23	+.24	—.13	+.88	
B		+.97								+.41					
C	+.90	+.91	+.91	+.89	+.73	+.73	+.74	+.73	+.16	+.16	+.17	+.19	—.12	+.80	
D (4 obs.)	+.87	+.88	+.87	+.85	+.86	+.91	+.91	+.92	+.36	+.38	+.38	+.44	—.27	+.94	
E	+.71	+.71	+.69	+.68	+.85	+.85	+.84	+.78	+.56	+.59	+.60	+.63	—.02	+.88	
F	+.69	+.69	+.68	+.66	+.76	+.78	+.77	+.77	+.54	+.57	+.58	+.66	—.11	+.84	
G	+.65	+.65	+.64	+.66	+.64	+.61	+.61	+.55	+.46	+.44	+.44	+.39	+.12	+.72	
H	+.56	+.54	+.51	+.40	+.75	+.69	+.68	+.51	+.67	+.67	+.68	+.61	+.50		+.76
I	+.50	+.51	+.49	+.51	+.67	+.67	+.66	+.63	+.65	+.68	+.69	+.70	+.04		+.74
J	+.55	+.55	+.53	+.51	+.74	+.71	+.70	+.64	+.70	+.73	+.74	+.74	+.20		+.83
K	+.33	+.31	+.30	+.22	+.42	+.38	+.36	+.27	+.77	+.77	+.76	+.69	+.56		+.85
L	+.46	+.45	+.43	+.40	+.70	+.68	+.68	+.59	+.77	+.79	+.80	+.80	+.30		+.88
M	+.40	+.38	+.35	+.32	+.60	+.56	+.54	+.43	+.82	+.83	+.83	+.81	+.46		+.92
N	+.43	+.42	+.41	+.40	+.62	+.59	+.59	+.50	+.84	+.85	+.86	+.83	+.32		+.94
O	+.32	+.31	+.31	+.24	+.59	+.54	+.53	+.42	+.91	+.91	+.91	+.85	+.51		+.98
P	+.13	+.11	+.10	+.12	+.41	+.36	+.36	+.27	+.94	+.93	+.92	+.85	+.54		+.86

as a possible whiteness standard. Commercial magnesium carbonate varies somewhat in color presumably because of impurities; furthermore, some samples darken and yellow on exposure to light. However, the trilinear coordinates of a block of commercial magnesium carbonate fairly typical of the samples available here were determined; the corresponding point is indicated by the square on Fig. 1, and suggests that sample No. 5 should be favored if the observers were paying attention to the chromaticity of such a whiteness standard as this. Since inspection of Table V shows that this was true for observer F only, the attempt to correlate observed whiteness gradings with those computed on the basis of magnesium carbonate was given up.

Further similar inspection of Table V and Fig. 1 led to the tentative adoption of a whiteness standard, of the trilinear coordinates labeled on Fig. 1 as "intermediate." Table VII shows the correlations obtained by the use of this whiteness standard in Eq. 1. In four of the seven "intermediate" arrangements (D to J) an improved correlation resulted; these cases may be readily found in Table VII because the highest correlation coefficient for each observer has been put in italics. The improvement is, however, not so striking as that obtained with observers L to P by substituting "natural paper white" for magnesium oxide as whiteness standard; arrangement D, the only instance of excellent correlation with "intermediate" as whiteness standard, has already yielded $r=0.88$ with magnesium oxide.

Arrangement D as mentioned before is the average for 4 observers using magnesium oxide as a guide in their whiteness grading. Since, contrary to the other observed arrangements, the observers making this arrangement were not permitted to grade according to their usual method, arrangement D has been set apart, The fact that better correlation is obtained with the "intermediate" whiteness standard than with magnesium oxide suggests that the observers were accustomed to grade on the basis of a more yellowish standard such as "intermediate" or even "natural paper white." The correlations obtained for the individual observers of this group (G, J, L, O) substantiate this view; only one of them (G) finds his best correlation with magnesium oxide as whiteness standard and his observations are but poorly accounted for on that basis ($r=0.66$).

Check of the Constant K.

For each of the three whiteness standards used in Eq. 1 there appears in Table VII correlation coefficients for each of five values of the constant, K: 0, 5400, 6700, 8000 and ∞. It may be seen that for $K = 0$ (grading on the basis of amount of daylight reflected, no attention paid to chromaticity) only a few instances of significant correlation were found (H, K, O, P) and there were five cases of negative correlation (A, C, D, E, F) apparently indicating that these observers in their whiteness grading penalized a sample reflecting daylight copiously and favored one reflecting less well. Actually, of course, the negative correlation comes in because the samples which approach the chromaticity of magnesium oxide have had their reflectance reduced by dyeing; this is indicated with fair certainty by the high correlation with magnesium oxide and, if necessary, could be shown more elegantly by eliminating this dependence according to the methods of partial correlation. It may be immediately concluded, however, that Eq. 1 describes the method of whiteness grading used by these observers much better for K greater than 5000 than for $K = 0$; that is, these observers pay very definite attention to chromaticity differences and rather tend to overlook reflectance differences when the two criteria conflict. The choice as between $K = 5400$, $K = 6700$, and $K = 8000$ cannot be made from these data; the differences shown are probably not significant, and the small differences which are shown between the high correlations and which might therefore be thought to be significant vary from one observer to another. Even changing K from 8000 to infinity (grading according to chromaticity alone) generally brings only a small decrease in the correlation. Of the 34 cases involving significant correlations (r greater than 0.50) 25 show lower correlation from K equal to infinity, 2 are unchanged, and 7 increase. The average change for these 34 cases is a decrease of 0.04.

It may be concluded, therefore, that the general form of Eq. 1 has been verified by these data and the value of the constant (6700) suggested by MacAdam's data on whiteness of laundered white goods has been shown to be approximately correct for whiteness grading of near white papers. The exact value of the constant is not known, however; probably any value not differing from 6700 by more than a factor of 2 would give about as good agreement as any other such value.

Agreement Between Observers

The last two columns of Table VII show the correlations of the individual arrangements (except for observer B) with one of two average arrangements, A to G, called the magnesium oxide group, and H to P, called the "natural paper white" group. These correlation coefficients are valuable because they give a clue as to how completely Eq. 1 is successful in representing the basis of whiteness grading used by the various observers. Thus, in seven cases out of the 15 (A, C, D, E, H, I, P) the agreement with the respective group is seen to be either less or not significantly greater than that obtained with one or another of the computed arrangements. It may be said, therefore, that the basis for these seven arrangements has probably been successfully revealed by this analysis. For the remaining 8 observers (F, G, J, K, L, M, N, O) there is a small but probably significant superiority in correlation with the average arrangement; that is, it would appear that these observers (particularly O) agree better with their own average than they do with any of the computed arrangements. Thus, it is suggested that these observers must make some use of a common criterion which has so far escaped formulation.

Three possible explanations of this effect apart from random experimental error are (1) error in choice of a whiteness standard, (2) use of special grading practices confined to observers of one business establishment, and (3) undue penalizing of the few reddish and greenish samples (see Fig. 1, samples 5, 21, and 40) because of contrast with the remainder of the samples which show a predominantly yellow-blue variation.

Error in choice of a whiteness standard could be remedied by making further trials similar to those which resulted in "natural paper white" and "intermediate." There seems little possibility of discovering any whiteness standard that will significantly improve the correlation; it is felt that the exploration already concluded has about exhausted the field.

The use of special grading practices might conceivably arise among observers from one business establishment because whiteness is generally taken as an index of quality and there is a natural tendency to rate one's own product favorably in comparison with competitors' products.

To show, however, that this is not the case for the present data, it is merely necessary to give the identification of the observers grouped according to their place of employment. There are four such groups: (1) E, F, H, I, M; (2) G, J, L, O; (3) A, C, P; and (4) C, K. It is obvious that there is no close connection between these groups and those chosen according to basis of their whiteness grading.

The third source of error, undue penalizing of reddish and greenish samples (see Fig. 1, samples 5, 21, and 40) because of contrast, is a more likely one. A sample of the larger group showing chiefly a yellow-blue variation may be placed near another sample not differing very much in color, but sample 5, sample 21, or sample 40 cannot be so placed because there are none such in the group of 20 papers. The result may very well be that they do not seem to belong anywhere in the whiteness series; and if either of the greenish samples (21 or 40) be placed next to the reddish one (5) both the greenish and reddish will gain in saturation and hence will seem to require a lower whiteness grading. Examination of Tables IV and V confirms the view that much of the failure of Eq. 1 to account completely for whiteness grading by direct visual comparison is ascribable to this cause. Note that sample 5 is rated about 15 by Eq. 1, but observers G to J rate it no better than 19; similarly sample 21 is rated 9 by Eq. 1 for "natural paper white" as standard, but observers K to O who otherwise agree with Eq. 1 rate it no better than 17; and similarly again sample 40 is rated 3 by Eq. 1 with the "intermediate" standard, but observed arrangements D, E, F, I, and J which otherwise agree fairly well with this computed arrangement rate it no better than 15. This suggests strongly that further experimental work with a group of samples varying less preponderantly in the yellow-blue sense might result in higher correlation with Eq. 1.

Definition of Whiteness

The results of Table VII suggest that whiteness might be defined in terms of magnesium oxide as a standard without doing great violence to present practice. Observers A, B, and C already grade in good accord with this standard, observer C not significantly less so than A and B in spite of the higher correlation coefficient, because observers A and B were given the whiteness grading by Eq. 1 as a guide which probably accounts for the near perfection of the correlation. Four other observers (G, J, L, O) as a group (arrangement D) are willing to agree substantially to the arrangement found for magnesium oxide as whiteness standard when presented with it in spite of the fact that three of them are accustomed to grade whiteness according to a yellower standard.

The results of this table also suggest that a definition of "natural paper white" following the form of Eq. 1 might also be useful. Five observers obtain correlation coefficients of 0.80 or better with this standard, and it is likely that if the other observers were instructed to grade in accord with degree of resemblance to good grade, well-bleached, undyed stock and were supplied with the computed arrangement according to "natural paper whiteness" as a guide, equally good or better agreement would be found as that with magnesium oxide as whiteness standard.

The agreement found with the tentative "intermediate" standard is probably to be ascribed to whiteness grading according to a criterion varying between the other two standards; no real basis for a separate standard seems to be indicated.

Summary

It may be concluded from the results shown here that much of the trouble and disagreement heretofore arising in whiteness grading of near-white papers is ascribable to the existence among those expert in grading paper of at least two distinct bases for judging whiteness. Judgments by the first basis will be called simply "whiteness" judgments; whiteness is defined as the degree of approach to the appearance of a perfectly reflecting, perfectly diffusing surface. Judgments by the second basis will be called judgments of "natural paper whiteness"; this is defined as the degree of approach to the appearance of paper made from good grade, well-bleached, undyed pulp.

It is, therefore, recommended that magnesium oxide ($r = 0.4392$, $g = 0.4650$, $b = 0.0958$) be taken as a standard white, and that "whiteness" be measured by degree of approach to this standard according to Eq. 1 for $K = 6700$.

It is further recommended that the trilinear coordinates ($r = 0.4425$, $g = 0.4662$, $b = 0.0913$) be taken tentatively as a standard natural paper white, and that "natural paper whiteness" be measured as the degree of approach to this standard according to Eq. 1 for $K = 6700$.

It is also recommended that the relation between reflectance for blue light (measured by the "brightness tester") and both "whiteness" and "natural paper whiteness" be investigated. There is considerable promise that interesting and valuable correlations will be discovered.

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The "solid curve" in Fig. 4, which is there attributed to the I.E.S. (Illuminating Engineering Society), was recommended provisionally in 1924 by the International Commission on Illumination (CIE) and was reaffirmed by that commission in its 1931 recommendations for colorimetry. A proposal by the U.S. committee, under Judd's chairmanship, to revise that curve failed to gain CIE support in 1955. Modern terminology uses nanometer (nm) instead of millimicron (mμ) which is defined on p. 624 and used in Figs. 4 to 7 and Tables I to III. Modern practice (SI, Systeme Internationale) omits the superscript ° after numerical values of temperature in the absolute, i.e., kelvin scale.

"Two-color motion pictures" such as are mentioned on p. 627 have not been made during the past fifty years, and have been rarely, if ever, exhibited since the publication of this article. The two-color photographs occasionally exhibited in lectures by Dr. E. H. Land since 1955 are the best-known recent exemplifications of tritanopia. In usual experience, the effects of the lack of yellow-to-blue discrimination are so slight that they are almost always overlooked or dismissed as trivial, in demonstrations such as Land's.

Judd's statement on p. 619 that rods do not yield chromatic vision has recently been contradicted, with rather convincing demonstrations, by Land's colleagues, John J. McCann and Jeanne L. Benton (see Sci. Am. December 1977, p. 112). Both of the papers by Judd that are included in the references (4 and 7) are included in the present collection.

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COLOR-BLINDNESS AND ANOMALIES OF VISION*

D. B. JUDD**

Summary.—*Normal persons can make visual distinctions of three types: ligh from dark, yellow from blue, red from green; light-dark being the most primitive type of discrimination, and red-green the last acquired. Some otherwise normal persons fail to develop in their organs of sight more than a vestige of the mechanism for red-green discrimination. They are called red-green blind, or partially color-blind. A few persons have only the ability to make light-dark discrimination; they are called totally color-blind. These types of abnormality are discussed and tests for red-green blindness described.*

I. INTRODUCTION

The worker in color is frequently brought face to face with the fact that not everyone sees color as he does. Sometimes he calls two colors nearly identical which another observer will say differ importantly, and at other times the two observers will use color names for a single sample which indicate a very large appearance difference. The color worker has, therefore, to defend his own color judgments against the contrary opinion of others, and many times these differences are not mere language troubles, but are real.

Those who "sell color" in one form or another (paints, dyes, cosmetics, textiles, chromatic motion picture film) to the public will find it to their interest to be sure that they, or those whom they employ

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for their color work, have vision not much different from that of the general public. It is estimated that about five per cent of the total population have color abnormalities of fairly important degree, that is, important enough to disqualify them in many types of color work.

The purpose of this paper is to list the principal types of abnormal chromatic vision, to indicate the various terms used in their description, to discuss briefly the type of abnormality called red-green blindness (inability to distinguish red from green), which causes most

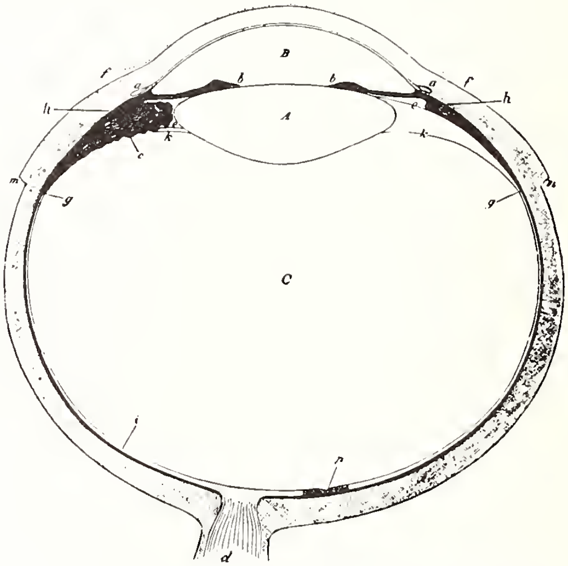

FIG. 1. Horizontal median section of the human eye (*Helmholtz*[10]).

trouble in color work, to describe tests for red-green blindness, and to indicate sources of further information.

II. THE NORMAL OBSERVER

Before describing the symptoms by means of which an abnormal observer may be detected, it is necessary to give some account of the structure and working of the normal visual mechanism.

(1) *Structure of the visual mechanism*

By visual mechanism is meant the eye and its attached nervous and muscular structures (optic nerve, brain, muscles of speech). Fig. 1 is a diagram of a horizontal section through the center of the eye.

Light enters the eye at the tear-film (*f*) covered, curved surface of the *cornea* (*a*), which serves as the principal means of focusing light from distant objects upon the retina (*g*, *i*, *n*, *g*). In its passage from the cornea to the center of the retina (*n*) light passes through the aqueous humour (*B*), the iris diaphragm (*b*, *b*), the crystalline lens (*A*), the vitreous humour (*C*), and the macular pigment near (*n*). The *iris diaphragm* (*b*, *b*) controls the amount of light entering the eye. If the illumination is high, the *pupil* of the diaphragm becomes smaller, restricting the light to the central part of the crystalline lens, which is best suited for the production of sharp images. The focusing of the eye upon near objects is accomplished by thickening of the *crystalline lens* (*A*), the change in shape being permitted by (*B*), a watery fluid called the *aqueous humour*. The *vitreous humour* (*C*), a jelly-like semi-rigid mass, fills the interior of the eye and gives it shape and solidity. The *macular pigment* is an irregular yellow spot covering the central 3 to 12 degrees of the retina, including the *fovea* (*n*), or point of clearest vision, which subtends about 1 degree.

Variation in the amount of macular pigment is a major cause of chromatic disputes among normal observers. Many colorimeters are so designed that the macular region of the retina is used exclusively, and two normal observers will, in general, make identical settings on such colorimeters only when they have equal macular pigmentation. For such colorimeters it has been proposed to supply each observer having a naturally weak macular pigmentation with an external yellow (glass or gelatin) filter to make his color-matches agree with those of the more heavily pigmented observer. Color-matching of the highest precision, however, requires that use be made of a retinal region considerably larger than the extent (4°) of the usual macula; two samples requiring within the macula an adjustment of pigmentation of the observer will for their accurate color-matching not appear equally well matched over a large area of the retina; if they happen to match for the extra-macular retina, the projection of the macular pigment may be seen on either sample when they are side by side, although either will appear perfectly uniform when viewed separately. Thus the macular pigment prevents making, for certain types of samples, a color-comparison of highest precision, and no adjustment by external filters is of assistance. Dark-eyed observers tend to have heavy macular and, indeed, heavy general pigmentation of the retina; albinos have none. Blue-eyed blondes and red-haired observers tend to have light macular and retinal pigmentation.[1]

The *retina* (*g*, *i*, *n*, *g*) contains a number of layers of nerve fibers and cells as well as the layer of receptors, or light-sensitive elements, of vision. The nerve fibers are in contact with the vitreous humour (*C*), and the receptors (rods and cones) are in contact with the retinal pigment (dark brown or black pigment), which is the outermost layer of the retina; so the light has to penetrate the nearly transparent layers of nerve structures before it reaches the rods and cones. Although the nerve structures cast no shadows, the blood vessels of the nerve-fiber layers are opaque and become visible whenever they are so illuminated as to cast their shadows upon unaccustomed rod-cone areas. Blood corpuscles flowing in these capillaries also cast visible shadows and may be seen as dancing specks (called *muscae volitantes* or "flying gnats") by looking at any bright uniform surface such as the blue sky. Opaque structures in the vitreous humour also cast shadows upon the retina which may be seen under these same conditions. If the structure is close to the retina the shadow is sharply defined; such shadows often appear in the form of tangled fibers or strings of beads. Many ill-defined shadows may also be seen which correspond to semi-detached debris drifting about in the liquid parts of the vitreous humour. With a motionless eye, these shadows are rarely seen, but a quick eye-movement will often bring to attention shadows otherwise too ill-defined to be seen. These slow-moving shadows follow the fixation point in a general way, lagging behind a quick eye-movement, then over-shooting when the eye is brought to a stop. They should not be confused with the "flying gnats" which dart rapidly about in every direction all over the visual field when the eye is motionless. These and kindred phenomena are called *entoptic* phenomena. Light not absorbed by the rods and cones is completely absorbed by the retinal pigment.

The *rods* are extremely sensitive to weak light; they are responsible for our vision in twilight. They do not yield chromatic vision, however; hence, twilight vision is achromatic vision; objects in twilight are either white, gray, or black, and of no other color. In strong light such as daylight the rods quickly lose their sensitiveness as if by complete bleaching of the light-sensitive substance which they contain; they have, therefore, little to do with daylight vision.

The *cones* are responsible for chromatic vision and for light-dark discriminations in daylight as well. Each retina, having roughly one square inch of area, is studded with about half a million cones. As stated by Purdy,[2] "The rods and cones have a characteristic distribution on the retina. The fovea and its immediate surroundings contain only cones; they make up the central *rod-free* area of the retina. Just outside this area, a few rods are intermingled with the cones, and the rods become more and more numerous as the edges of the retina are approached. The extreme periphery contains an overwhelmingly large percentage of rods, but is not quite free of cones." The absence of rods from the central 2 degrees including the fovea means that the normal retina has a central blind-spot (*scotoma*) in twilight, which is the basis of Arago's oft-quoted statement, "To see a very weak light, it is necessary not to look at it." Thus, to see a very faint star one must not try to look directly at it, but a little to one side of it. On account of this central blind-spot, twilight vision (rod vision) is often called *scotopia*, or *scotopic vision*. Daylight vision (cone vision) is often called *photopia*, or *photopic vision*. Daylight vision, or chromatic vision, is best at the very center of the retina, and poorest or nearly non-existent at the periphery.

The presence of two separate systems of receptors (rods and cones) in the retina immensely extends the adaptability of our visual organs. The sensitivity of the retina increases fairly rapidly by adaptation to darkness and decreases even more rapidly by exposure to strong light, the total variation in sensitivity being by a factor of more than 50,000. Variation in pupil size gives an additional factor of about 20, making in all a variation of more than a million-fold. If the retinal rods for any reason become non-functional, night blindness (normal for barnyard fowls) results. If the cones become non-functional, *photophobia* (fear of light) or even *hemeralopia* (total or partial day blindness, normal for owls) results.

The visual response from daylight stimulation of any small central area in a normal eye can vary in three different ways; that is, a mixture of at least three lights, each having a different chromaticity (such as a red, a green, and a blue light) is required to match all the colors resulting from activity of a normal eye, each light being independently adjustable in amount. A normal observer is therefore called a *trichromat*. Similarly a partially color-blind observer who can get along with only two of these components in his light mixture is called a *dichromat;* and a totally color-blind observer requiring only one, a *monochromat*.

When the light is absorbed within or near a cone, chemical processes are released which result in a nerve impulse in the fiber which is attached to it. This impulse continues to the end of the short fiber (only a few hundredths of an inch in length) where a connection (*synapse*) is made with another short nerve-fiber or *bipolar cell*. The bipolar cell in turn connects with a nerve-fiber (*axon*) of the optic nerve and a chance is provided also for sidewise communication.

The result is that, in general, outside the fovea several receptors are connected with the same bipolar cell, and many bipolar cells share the same axon of the optic nerve. But in the fovea each cone has its own bipolar cell and optic nerve axon extending to the brain; this makes for relatively acute vision in the fovea. The nerve impulse travels in this way through the retina in a direction opposite that taken by the

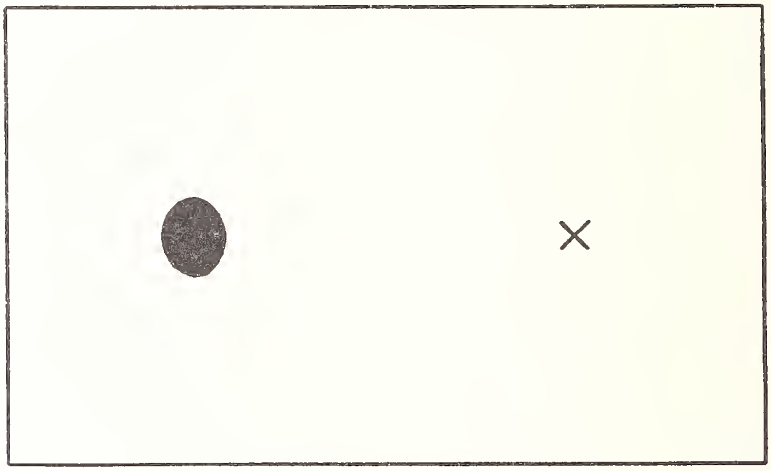

FIG. 2. Demonstration of "blind-spot": hold the page about 8 inches from the left eye and with the right eye closed, look at the cross.

light; and on the innermost surface of the retina, which is in contact with the vitreous humour, the impulse is conducted by the axons of the optic nerve to the exit point of the optic nerve, at which point all the axons or "wires" join to make up a "cable" called the optic nerve (*d*, Fig. 1). It is evident that there must be a hole in the retina to let the optic nerve pass through. Light falling upon this hole does not affect any normal rods or cones; it is called the *blind spot* although recent discoveries[3] indicate that it is relatively insensitive rather than completely blind. The blind spot may be demonstrated by looking at Fig. 2 with the left eye held about eight inches from the page. When the cross is fixated, an adjustment of the distance between the eye and the figure and a slight reorientation of the page will cause the image of the black spot to fall upon the exit point of the optic nerve. In this position the black spot will become invisible provided, of course, that the right eye be closed.

The course of the nerve-fibers across the inner face of the retina is curved, as indicated in Fig. 3, so as to pass around the fovea instead of over it. Since the fovea does not have any nerve structures over it, the retina is thinner here in its central point (see Fig. 1) which is on this account sometimes referred to as the *foveal pit* (*n*). The curved path of the nerve-fibers around the foveal pit may be made visible by stimulating the region to the temporal side of the fovea of the dark-adapted retina of one eye with a moderately intense light. The

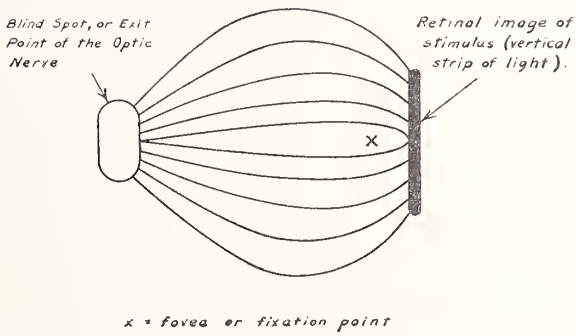

Fig. 3. The course of the long axons of the optic nerve from a region on the temporal side of the retina, around the fovea, to the exit point of the optic nerve.

paths of the nerve-fibers are then seen as reddish blue arcs.[4] Whether the nerve-fibers become visible when active by giving off physical light or by stimulating electrically the underlying structures (bipolar cells, rods, and cones) is a disputed point.

In their path from eye to brain, the fibers of the optic nerves from the two eyes join at the *optic chiasmus*, at which point fibers from corresponding points of the two retinas join and run side by side through various structures of the brain to the *occipital lobe* where, in and about the median and calcarine fissures of the cortex, occurs the cortical projection of the retina. In this region there exists for every pair of corresponding points of the two eyes a single, small area served especially by fibers from those retinal points. If a region of this part of the cortex be destroyed (by a cerebral haemorrhage or gun-shot wound) a disturbance of vision affecting both eyes in the corresponding region occurs. Since these disturbances are frequently only temporary, it is supposed that the cortical projection of the retina gives the location where the coördination between the responses of the two eyes usually occurs, but that many other possible pathways exist which may be used if the accustomed pathways are destroyed.

From the calcarine and median fissures radiate millions of nerve-fibers to all parts of the brain, making possible the coördination of impulses from the eyes with those from other sense organs, and also making possible in response to retinal activity all sorts of muscular activity, the most important of which is speech.

It is to be noted that since the activity of the speech muscles reveals a triadic response to visual stimuli, there must exist not only in the retina, but at every point in the nerve-chain between retina and muscles, a possibility of at least three independent sorts of activity. Hence, it can not be said immediately of a dichromatic observer that he has a defective retinal response; the defect might exist in the nerve connections or even in a very heavy brown pigmentation of the eye media; the fact of dichromatism does not locate the seat of the defect; further evidence is necessary.

This completes the description of the structure of the normal visual mechanism. In summary, it may be noted that several "anomalies" exist in the normal observer. The fovea, point of clearest daylight vision, is blind in twilight; the macula does not respond in the same way as other parts of the retina; each retina has a rather extensive spot that is practically blind at all times; and each eye gives forms and colors (entoptic phenomena, such as the blue arcs of the retina) which correspond to structures of the eye rather than to external objects. None of these "anomalies" is troublesome in daylight vision by both eyes.

(2) *Appearance of the spectrum*

The way the normal visual mechanism works may conveniently be summarized by giving a description of the equal-energy spectrum. The equal-energy spectrum is one in which the radiant energy per unit wavelength is constant. Every visual stimulus which reaches the eye is either a portion of this spectrum or may be regarded as a combination of a number of such portions. We may therefore sketch approximately the properties of an observer by giving his description of the equal-energy spectrum.

To the normal observer the appearance of the spectrum is a series of chromatic colors varying from dim red through orange, yellow, brilliant yellow-green, green, blue, to dim violet. The brightest part of the equal-energy spectrum under usual observing conditions is at a wavelength of 555 millimicrons* (yellowish green), and from this point toward both longer and shorter wavelengths the brightness progressively diminishes. Spectrum red and spectrum blue are more saturated (stronger) colors than spectrum yellow, the least saturated spectrum color occurring at 570 mμ (greenish yellow). Fig. 4 gives for an average observer the brightness of the equal-energy spectrum

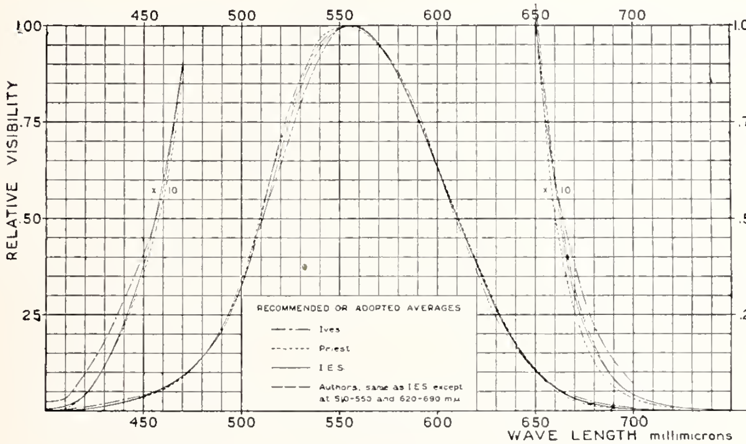

FIG. 4. Brightness of the equal-energy spectrum as a function of wavelength relative to the brightness at 555 mμ, referring to average normal daylight vision (*Gibson and Tyndall*[5]). The solid curve has been officially adopted as representative of the average observer.

as a function of wavelength relative to the brightness at 555 mμ according to several authorities.[5] Fig. 5 gives the number of doubtfully perceptible steps from an approximately achromatic color (color of sunlight or the color which a furnace would have if it could be heated to 4800°K. or 4527°C.) to the spectrum colors as a function of wavelength. The circles and dots were determined by direct observation;[6] the curve was determined by inference from determinations of the numbers of doubtfully perceptible steps between other lights.[7] The number of doubtfully perceptible chromaticity steps

* One millimicron (mμ) = one millionth of a millimeter = approximately four one hundred millionths of an inch.

from an achromatic color is a fairly good indication of saturation; hence, Fig. 5 indicates approximately the saturation of the various parts of the spectrum.

The normal observer is sensitive to radiant energy in such a way that the spectrum has the appearance described above. It is convenient (as will presently appear) also to describe the normal observer

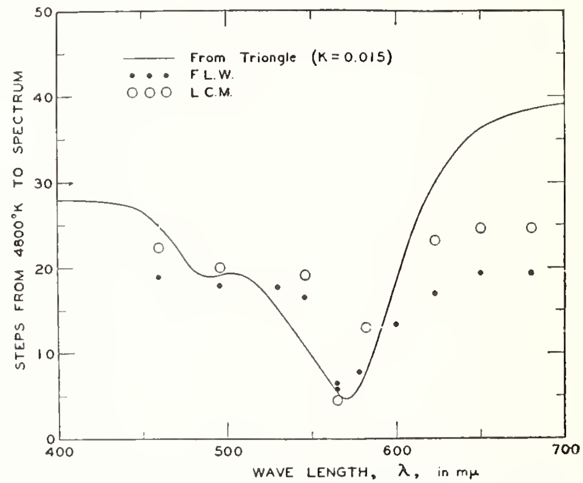

FIG. 5. Number of chromaticity steps from a Planckian radiator at 4800°K. to the spectrum as a function of dominant wavelength, referring to normal chromatic vision (*Judd*[7]).

as one who can make discriminations between light and dark, blue and yellow, and red and green.

III. TYPES OF ABNORMALITY

The chief types of abnormal vision may be discussed under the headings anomalous trichromatism, dichromatism, and monochromatism.

(*1*) *Anomalous trichromatism*

An observer possessing this type of abnormality requires a mixture of three lights to produce all the colors which he is capable of experiencing just as a normal trichromat does, but he requires proportions to produce a given color considerably different from those required by the normal observer. If he is only mildly anomalous, his description of the equal-energy spectrum will agree well with the normal; but if he is extremely anomalous, it will be a good deal like that of a dichromat. Anomalous trichromatism represents a type of vision intermediate between normal vision and red-green blindness. The distinctions made by such an observer are: light from dark, yellow from blue, and red from green; but the ability to make red-green distinctions is relatively weak.

(2) *Dichromatism*

(Partial Color-Blindness)

An observer possessing this type of abnormality requires a mixture of but two lights to produce all the colors which he is capable of experiencing. He can make two kinds of visual discrimination, one achromatic (light-dark) and one chromatic (either blue-yellow or red-green, usually the former).

(*a*) *Red-Green Blindness*

(Dichromatism with Light-Dark and Blue-Yellow Discrimination—Bee Vision)

The spectrum to an observer having red-green blindness appears in two hues only: the short-wave end of the spectrum appears blue; the long-wave end, yellow. These two bands are separated by a neutral point at about 500 mμ which has an achromatic color. From zero at the neutral point the saturation of the spectrum colors increases toward both the long-wave and the short-wave ends. There are two sub-types of red-green blindness: (*1*) abnormal brightness distribution in the spectrum, and (*2*) normal brightness distribution.

(*i*) *Red-Green Blindness with Abnormal Brightness Distribution.*—In this the spectrum is much darker in the long-wave region than it is for normal vision. This type of dichromatism is most commonly called *protanopia* by reference to the three independent processes (red, green, violet) which are usually postulated to account for normal vision. The first process (red sensation) is predominantly released by long-wave light; the second (green sensation) by light of intermediate wavelength; and the third (violet sensation) by short-wave light. Protanopia can be accounted for by considering only the second and third processes; the name, protanopia by its derivation means "first process gone." It is an improvement on the term suggested by the Young-Helmholtz theory of vision, *red-blind*, because such a term implies that protanopes see nothing but green and violet, whereas they really see yellow and blue. Adherents of the Müller theory of vision also use the designation "outer (or retinal) red-green blindness" because the Müller theory places in the retina the structural defect causing this abnormality of vision. The Hering theory does not account for it.

(*ii*) *Red-Green Blindness with Normal Brightness Distribution.*—This type of dichromatism is commonly called *deuteranopia* or "second process gone." It is the most common form of color blindness. The Young-Helmholtz theorists used to call it *green-blindness.* The Müller theorists call this abnormality of vision "inner red-green blindness" because they place in the brain the structural defect causing it. There is some doubt whether the distribution of brightness is strictly normal; some workers believe that to deuteranopes the long-wave part of the spectrum is, on the average, slightly brighter than to the normal.

(*b*) *Yellow-Blue Blindness*

(Dichromatism with Light-Dark and Red-Green Discrimination)

This is a rare form of abnormal vision associated chiefly with diseases of the eye. Objects appear to such an observer a good deal as they do by candle-light to a normal observer. Two-color motion pictures also give mostly red-green differences, and so duplicate fairly well in appearance the world of the yellow-blue blind. The spectrum to an observer having yellow-blue blindness appears in but two hues, red and green; its brightness distribution is normal. There are two types of yellow-blue blindness: (*1*) that with one neutral point, and (*2*) that with two.

(*i*) *Yellow-Blue Blindness with One Neutral Point.*—The neutral point is located at about 570 mμ, the part of the spectrum which is most nearly neutral to the normal observer. This type of dichromatism is commonly called *tritanopia* or "third process gone." The Young-Helmholtz theorists used to call it *violet-lindness.* The Müller theorists call it "outer yellow-blue blindness" as well as tritanopia, in analogy to protanopia or outer red-green blindness. To the tritanope the spectrum is red from the long-wave end to the neutral point and is elsewhere green. The Hering theory does not account for this form of yellow-blue blindness.

(*ii*) *Yellow-Blue Blindness with Two Neutral Points.*—One neutral point (or band) is located at about 588 mμ, the spectrum appearing red from the long-wave end down to this band, then green down to the second neutral point or band at about 465 mμ, the remainder appearing red. This type is so rare that many treatments of color-blindness do not refer to it. Only three or four cases have been studied and reported. It has been called "tetartanopia or inner yellow-blue blindness" by Müller who makes it analogous to deuteranopia or "inner red-green blindness." The Young-Helmholtz theory does not account for it.

(3) *Monochromatism*
(Total Color-Blindness)

An observer possessing this type of abnormality requires but a single light to produce all the colors which he is capable of experiencing; he can match any stimulus with any other stimulus merely by adjusting them to equal brightness; he is capable of light-dark discrimination, and no other kind. His spectrum does not have merely one or two neutral points; it is all neutral. There are two types· (*1*) with central scotoma, and (*2*) without.

(*a*) *Monochromatism with Central Scotoma*
(Owl Vision)

This type of monochromatism is ascribed to non-functioning of the cones. Not only do monochromats of this type have at all times the central scotoma characteristic of normal twilight vision, but they also have photophobia, or fear of light, which is characteristic of owls and of the normal dark-adapted observer, and which is ascribed to rod vision. Further common characteristics of such observers are low visual acuity and *nystagmus* or side-to-side eye-movements as if in an attempt to improve deficient visual acuity by using retinal areas now to the one side, now to the other side of the central blind area.

(*b*) *Monochromatism without Central Scotoma*
(Cat Vision)

This type of monochromatism is characterized by normal visual acuity as well as good foveal vision; it can not be ascribed to the failure of the cones to function, but is ascribed to failure of the chromatic functions either retinal or cortical.

IV. RED-GREEN BLINDNESS

This type of dichromatism deserves special attention because in the first place it is by far the most common, being inherited by about 3 per cent of men and one-tenth of one per cent of women. Thus, a girl has little chance of inheriting incompletely developed eyes, but she has a good chance of inheriting an incomplete ability to transmit normal vision to her sons. The sons of a normal man and a woman of normal vision may thus be dichromats, but it is believed that the daughters will inherit dichromatism only provided that both father and mother be abnormal in those respects.

Another curious thing about congenital red-green blindness is that although it is a sufficiently serious abnormality to disqualify its possessor for color work, it is possessed by otherwise normal, healthy persons who may live their lives through without discovering that their chromatic vision is at all abnormal. Usually, of course, a dichromat gets into all manner of arguments about the appearance of things and begins to suspect something; and, of course, when he learns to drive an automobile on city streets and has to discriminate the red from the green traffic lights, he may conceal his defect from others but can no longer conceal it from himself. Traffic signal red is a yellowish red, seen as yellow by the red-green blind; many of the green traffic signals have purposely been made bluish green so that red-green blind observers would see the signal as weak blue and so be able to tell it from the red signal.

There are in addition to true red-green blind observers, or dichromats, perhaps about an equal number of anomalous trichromats of such extreme degree of anomaly that they are no more eligible for color work than dichromats themselves; that is, they possess the ability to make red-green distinctions only in a slight degree, so that they must have either very strong reddish and greenish lights, or a very pronounced red-green difference, or a very large part of the retina stimulated, or a very long time for decision in order to be sure of a red-green difference. Such extremely protanomalous or deuteranomalous observers have visual systems much more like a protanope or deuteranope than like a normal observer; they can, for example, discriminate the red from the green traffic light when they are close to the semaphore and have a number of seconds in which to make the decision, but are nearly as much at a loss as true dichromats to do it in fast-moving traffic. Or, they might distinguish a red apple from a green one by its color nearly as quickly as a normal observer, but would be unable to tell brass from copper with any assurance. What is to be said about red-green blind observers, therefore, applies nearly as well to these extremely anomalous trichromats.

Fig. 6 shows the relative brightness (luminosity) of the equal-energy spectrum for a protanope, a deuteranope, and a normal observer, according to Pitt.[8] Note that the three curves are very similar except that the curve for the protanope is much lower for wavelengths greater than 560 mμ; that is, the protanope is relatively insensitive to long-wave light.

Fig. 7 shows, as a function of wavelength, the number of chromaticity steps from the neutral point for protanopes and deuteranopes, derived from Pitt's results. This number is a fairly good estimate of the saturation of the spectrum for red-green blind observers. At the neutral point, the saturation of the spectrum color is zero; to the

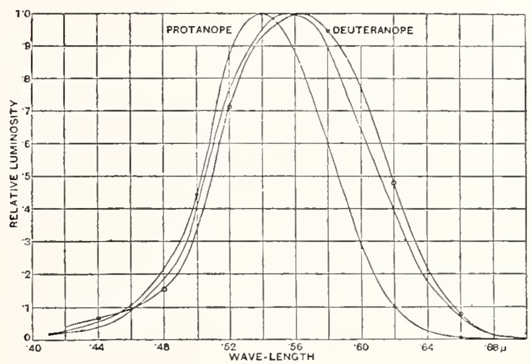

FIG. 6. Relative brightness of the equal-energy spectrum for a protanope, a deuteranope, and a normal observer (*Pitt*[8]).

left of the neutral point the hue is blue, and the saturation rises steadily up to about 430 mμ; to the right of the neutral point the hue is yellow, and the saturation rises rapidly to a smaller maximal value near 540 mμ, above which no further distinct chromaticity change occurs. According to these results the deuteranope has chromatic vision superior to that of the protanope, being able to make about 50 per cent more distinctions. For comparison a part of the curve in Fig. 5 indicating saturation of the spectrum colors for the normal observer is also shown. This comparison indicates that the saturation of the spectrum colors for the normal observer is everywhere superior to that of the red-green blind except possibly between 550 and 590 mμ, where the deuteranope, in particular, seems to have an advantage. We should expect, however, that the normal observer would not be inferior in this respect for any region of the spectrum, and since the size of doubtfully perceptible chromaticity steps depends importantly upon the exact experimental conditions used, the indicated superiority of the red-green blind observers is probably not significant.

These curves (Figs. 6 and 7) indicate what it is believed the red-green blind observers see when they look at the spectrum, but it should be emphasized that they do not indicate the color names which will be used by such observers in describing what they see. The naming of colors by a dichromat gives virtually no clue to what

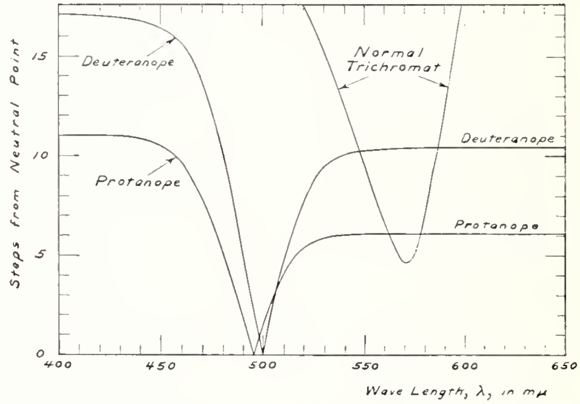

FIG. 7. Number of chromaticity steps from a nearly achromatic color (Planckian radiator at 4800°K) to the various spectrum colors for protanopes, deuteranopes, and normal trichromats.

he is seeing. According to Pitt,* "An interesting demonstration of the fact that color-naming is a bad test was unconsciously given by a protanope. He was shown a colored plate in a book, containing three distinct colors—red, green, and yellow. He was asked what color the red was. His first answer was green, but within a second he had changed his mind and called it red. After his first answer he had allowed his eyes to wander on to the other colors, and, although they looked the same color to him he could discriminate between them by means of their intensity (brightness) difference. The brightest he called yellow, the next brightest green, and the faintest red."

Table I, from Pitt,* shows the descriptions of the spectrum colors

* *Loc. cit.*, p. 33.

given by five dichromats—three deuteranopes and two protanopes. It is evident from the remarks of the dichromats that their color names do not correspond with those used by normal observers, and the question may be raised, "How do we know what the dichromat really sees? Why do we call it red-green blindness?" Strictly speaking, we do not know with certainty what the dichromat sees, any more than

TABLE I

Descriptions of Colors by Five Dichromats—Three Deuteranopes and Two Protanopes

Description by Normal Observer	Wave-length, mμ	Remarks by the Dichromats				
		D_1	D_2	D_3	P_1	P_2
Red	658–780	Yellow	Orange	Yellow-green	Dark color —red?	Orange
Orange	600–658	Yellow (warm)	Yellow	Orange-green	Yellow	Lemon
Golden yellow	583–600	Yellow	Yellow	Orange-green	Yellow	Lemon
Yellow	578–583	Yellow	Yellow (orangy)	Orange-green	Yellow	Yellow
Greenish yellow	567–578	Yellow	Yellow (orangy)		Yellow	Yellow
Green	524–567	Greenish yellow	Yellow-white	Yellow	Yellow nearly gone, may be red?	Light yellow to white
Blue-green	502–524	Whitish	Pink	Yellow	Yellow gone, nearly green	Blue
Blue	431–502	Blue	Mauve	Blue	Green	Royal blue
Violet	390–431	Blue	Blue	Blue	Dark color	Violet

we can be sure what a normal observer sees. Philosophers will tell us that each observer is separate unto himself, and no observer will be certain that any other observer experiences the same sensation called (for example) yellow as he does. Practically speaking, however, no one entertains any serious doubts that all normal observers experience about the same sensations; there is no reason to believe the contrary. And with nearly the same degree of certainty it is believed that the protanope and the deuteranope see light and dark, blue and yellow, the gap between trichromat and dichromat having been bridged by the reports of a few observers who have normal vision by one eye and dichromatic vision by the other.

In Table II there are listed a number of "confusion colors" for protanopes, that is, the color-names by means of which normal observers identify colored samples which can be distinguished by dichromats, if at all, only by their lightness difference. Table III is a similar table for deuteranopes. The information in both tables is taken from Pitt.

TABLE II

Normal Color-Names for Colors of Nearly Identical Chromaticity for Protanopic Observers

Wavelength, mμ	
496	Light blue, gray
499	Green, ivory, fawn, light stone, pink
502	Nile green, cream, brown, purple-brown
505	Mid-Brunswick green, middle stone, deep cream, crimson
509	Sea-green, bronze green, deep stone, primrose, buff
515	Light Brunswick green, red
525	Yellow, orange

TABLE III

Normal Color-Names for Colors of Nearly Identical Chromaticity for Deuteranopic Observers

Wavelength, mμ	
500	Light blue, gray
504	Green, ivory, light stone, fawn, pink
506	Nile green, brown
508	Mid-Brunswick green, cream, middle stone, purple-brown
511	Sea-green, deep cream
514	Light Brunswick green, bronze green, primrose, deep stone
517	Buff, crimson
530	Yellow, red

Each table gives also the wavelength of the equivalent spectrum stimulus, and serves to emphasize a fact often overlooked, that every color experienced by a dichromat is a duplicate of one of his spectrum colors provided the brightness be correctly adjusted. The normal observer requires a mixture of two parts of the spectrum to duplicate the chromaticity of all the colors which he experiences. The color-names given by Pitt are those extensively used in the British paint trade; whenever American usage differs importantly from British, the American equivalent has instead been given in Tables II and III, taking the Maerz and Paul *Dictionary of Color* as the authority.

Tables II and III give a good idea of the wide diversity of color names to be expected from a dichromat in describing the spectrum colors. For example, according to Pitt we would expect a protanope to be able to match a green, an ivory, a fawn, a light stone, and a pink color all with the spectrum color at 499 mμ; so he would be right in describing this spectrum color by any of these color names. Table III is probably more useful for estimating what colors a deuteranope will confuse than Table II is for the protanope, because two samples of the same lightness to a normal observer will appear of nearly the same lightness for the deuteranope. The normal observer can therefore tell whether two colors will be distinguished by the deuteranope because of lightness alone; but he can not make a very reliable estimate for the protanope. As shown in Fig. 6 the protanope responds but slightly to long-wave light which is copiously reflected by red samples; he therefore sees red or reddish samples considerably darker than the normal observer does, and his actual groups of confusion-colors differ more from those of the deuteranope than is indicated by comparison of Tables II and III, which consider chromaticity confusion but not light-dark confusion.

With some limitation Tables II and III serve to indicate also the confusion-colors of anomalous trichromats of extreme type. Thus, an extremely deuteranomalous trichromat will tend to confuse the same colors as a deuteranope, but since his red-green discriminatory power is weak instead of non-existent, he will see pronounced red-green differences invisible to a deuteranope. He might, for example, confuse light green with ivory, or pink with ivory, but still be able to distinguish pink from light green if permitted to make a critical examination of the samples.

A characteristic of most observers having abnormal chromatic vision, be it dichromatism or anomalous trichromatism, is hesitation in making a color judgment; they are accustomed to let the other fellow name the color and then to agree with him.

V. TESTS FOR COLOR BLINDNESS

These tests are designed chiefly to detect protanopes, deuteranopes, and extremely anomalous trichromats, because not only are such observers more numerous and more difficult to discover than other types of abnormal observers, but also these types of vision are particularly unsuited to speedy discrimination of traffic, railway, marine, and aviation signal lights. The successful tests all have the following properties in common: (*1*) they present reds and greens which the subject of the test must distinguish; (*2*) they present them in small areas so that anomalous trichromats as well as dichromats will be discovered; (*3*) they present them intermingled with other colors in such a way as to discourage attempts upon the part of the subject to conceal either defective vision by guessing, or normal vision by making mistakes on purpose; and (*4*) they present reds, greens, confusion and background colors of a variety of lightnesses so that the subject can not get any clue about the colors from his ability to discriminate light from dark. The various successful tests differ essentially only as to the form of response required of the subject. The following tests have been successfully used at the National Bureau of Standards:

(*1*) *The Holmgren Wool Test.*—A number of small varicolored skeins of wool are spread out before the subject, who is required to select those resembling three larger skeins, a red, a green, and a rose.

(*2*) *The Nagel Charts.*—A number of cards are shown to the subject, each card having a series of small circular colored spots arranged in a circular ring. The spots on some of the cards are all weak green, some light and some dark; others are combinations of green with gray, pink, and other confusion-colors, and so on. The subject is required to say which cards bear spots of but one hue and which have more than one hue.

(*3*) *The Stilling Charts.*—The irregular and varicolored spots on these charts are so arranged and colored that a digit can be read by a normal observer on each chart. Some of the charts can not be read by dichromats and extremely anomalous trichromats; and some are hard even for a normal observer to read.

(*4*) *The Ishihara Charts.*—This improved form of Stilling chart supplies both cards which can be read by the normal observer but not by dichromats, and cards which can be read by dichromats but not by normal observers. In addition, there are cards which yield one digit when read by a normal observer and a different digit when read by a dichromat; and there are charts for testing the vision of observers who can not read numbers on which, instead of a digit to be read, there is a continuous path to be traced.

VI. REFERENCES FOR FURTHER READING

The elementary treatment of vision by Purdy[2] gives a readable and more complete account of normal vision than has been included here. *An Introduction to the Study of Colour Vision*, by Parsons,[9] gives a brief summary of nearly all important work up to 1920. In addition to a good chapter on the chief facts of dichromatism, there is a section devoted to the chief theories of vision which have been mentioned here only because of their influence on terminology. Helmholtz's *Treatise on Physiological Optics*[10] is the most authoritative and complete reference work on vision. An important summary and classification of all types of abnormal vision is given in German by G. E. Müller.[11] Unfortunately there is no English translation. Müller's classification has been followed closely in the present discussion of the chief types of abnormal vision. The most thorough quantitative investigation of red-green blindness with modern apparatus is that of Pitt[8] from whose results Fig. 6 and Tables I, II, and III have been taken.

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ESTIMATION OF CHROMATICITY DIFFERENCES AND NEAREST COLOR TEMPERATURE ON THE STANDARD 1931 ICI COLORIMETRIC COORDINATE SYSTEM

J. Opt. Soc. Am. 26, 421-426 (1936)

Circles on Judd's uniform chromaticity diagram [J. Opt. Soc. Am. 25 24-35 (1935)] were transformed to the CIE 1931 chromaticity diagram. They are useful for judging the relative importances of differences of chromaticities represented by points on the CIE diagram. Straight lines perpendicular to the blackbody or planckian locus in Judd's UCS diagram were transformed to the CIE 1931 diagram. Those lines, called iso-temperature lines, are useful for estimating color temperatures of sources whose chromaticities do not exactly match those of blackbodies at any temperatures. As mentioned in the last paragraph of this paper, the wavelength at which each extended iso-temperature line intersects the spectrum locus is the conjunctive wavelength for that temperature. Although the term is not much used (in 1978), it is very useful, because the conjunctive wavelength does not depend upon the choice of chromaticity diagram in which the iso-temperature lines are drawn, and it unambiguously defines their slopes. In a letter dated 4 July 1942 to D. L. MacAdam, Dr. Judd reported three experimental results, averages of observations made in 1934 by H. T. Wensel, D. B. Judd, and W. F. Roeser that may be compared with values interpolated from Table 2 of this paper:

Color temperature	Conjunctive wavelength observed	Conjunctive wavelength interpolated from Table 2	difference
2046 K	583.5 nm	585.3 nm	1.8 nm
2239 K	580.0 nm	583.5 nm	3.5 nm
2727 K	576.0 nm	578.4 nm	2.4 nm

The values attributed to Davis, used for comparison in Table 2, were computed by use of a highly conjectural method, not based on observations or any judgments of nearest color temperature. Because the agreement between results based on direct observation and on the iso-temperature lines of Fig. 2 is not very close, and because no such data based on observations are available for temperatures higher than 2727 K, experiments designed to obtain such data for all color temperatures, up to at least 7000 K, would be very desirable. The modification of the UCS diagram recommended in 1976 by the CIE gives poorer agreement with the results based on observations than does Judd's 1935 UCS diagram, which was the basis of this paper.

Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 ICI Colorimetric Coordinate System*

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(Received September 23, 1936)

Estimation of chromaticity differences has been facilitated by the preparation of a standard mixture diagram showing by a group of ellipses the scales of perceptibility at the various parts of the diagram. The distances from the boundaries of the ellipses to their respective "centers" all correspond approximately to the same number (100) of "least perceptible differences." The estimation of nearest color temperature has been facilitated by the preparation of a mixture diagram on which is shown a family of straight lines intersecting the Planckian locus; each straight line corresponds approximately to the locus of points representing stimuli of chromaticity more closely resembling that of the Planckian radiator at the intersection than that of any other Planckian radiator.

CONTENTS

I. Introduction

SINCE the adoption in 1931 by the International Commission on Illumination of a standard colorimetric coordinate system, by far the majority of fundamental colorimetric specifications have been expressed in that system.[1] A minor defect which is coming to be more keenly felt as the use of the system becomes more extended is the difficulty of interpreting differences in position on the standard mixture diagram in terms of degree and kind of chromaticity difference. The discovery of a projective transformation of the standard system whose Maxwell triangle yields nearly uniform chromaticity scales[2] is a step toward remedying this defect; on this triangle the length of any line is approximately proportional to the perceptibility of the chromaticity difference between the stimuli represented at the extremes of the line.

The substitution of the uniform chromaticity scale (UCS) system for the ICI system immediately suggests itself, but there are several reasons why this should not be done at present. In the first place, although the UCS system was derived from and is a valid representation of about nine-tenths of available data on sensibility to chromaticity differences, and although furthermore many divergent applications of it[3] have shown it to be a definite improvement in spacing over the ICI system, nevertheless the original data are neither sufficiently numerous nor sufficiently self-consistent to be considered definitely final. It is probable that further important improvements in spacing can be made. A second reason for adherence to the present standard ICI system is that the ICI system is more suited to the representation of photometric quantities; this may be seen from the fact that the ICI luminosity coefficients are 0, 1, 0, while the UCS luminosity coefficients are 0.08852, 0.11112, −0.09802. A third reason is that the applicability of data obtained by observing small apertures to observation of chromaticity differences between extended surface colors has not been demonstrated, nor have these data on small differences been shown to apply by simple addition to large chromaticity differences. The optimistic view that such applicability does not require demonstration has been fairly general, but extensive data which are being accumulated do not wholly justify this view. They indicate that the UCS triangle is applicable to small chromaticity differences between surface colors of large as well as small extent; furthermore, the application of the UCS triangle to large as well as small chromaticity differences between point

* Publication approved by the Director of the National Bureau of Standards of the U. S. Department of Commerce.

[1] D. B. Judd, "The 1931 I.C.I. Standard Observer and Coordinate System for Colorimetry," J. Opt. Soc. Am. **23**, 359 (1933).

[2] D. B. Judd, "A Maxwell Triangle Yielding Uniform Chromaticity Scales," Nat. Bur. Stand. J. Research **14**, 41 (1935) RP756; also J. Opt. Soc. Am. **25**, 24 (1935).

[3] See, for example, D. B. Judd, "A Method for Determining Whiteness of Paper," Paper Trade Journal, **100**. No. 21, TS40 (1935); also Tech. Assoc. Papers, Series XVIII, 392 (1935); and D. B. Judd, "A Method for Determining Whiteness of Paper, II," Tech. Assoc. Papers, Series XIX, 359 (1936); also Paper Trade Journal **103**, No. 8, TS38 (1936).

sources (signal lights) fails to result in any discrepancies not explainable by aberrations of the lens system of the eye; but there are data which suggest that the UCS triangle is not applicable to large chromaticity differences between extended surfaces. For these three reasons no departure from the standard 1931 ICI system is proposed at this time.

Accordingly, then, for research purposes many determinations in the colorimetry section of the National Bureau of Standards are being recorded in duplicate, once in the ICI system and once in the UCS system; but there are also many determinations which are expressed in the ICI system alone. For such determinations some of the advantages of the UCS system may be obtained if there is available a graphical representation of the relative scales of the two systems. The present paper serves to give two such representations; one of them shows how the scale of perceptibility varies over the ICI mixture diagram on the assumption that the UCS system gives accurately uniform chromaticity scales as it was intended to do, the other shows, on the same assumption, iso-temperature lines to aid in the estimation of nearest color temperature for near-Planckian colors.[4]

II. Estimation of Chromaticity Differences

The equilateral Maxwell triangle of the ICI system and all three of the mixture diagrams obtained by plotting in rectangular coordinates one of the ICI trilinear coordinates (x, y, z) against another fail to array color stimuli in such a way that equal chromaticity differences are indicated by lines of equal length. It is therefore impossible to give any single reliable perceptibility scale of chromaticity which will apply to the whole diagram as a scale of miles applies to a map. Instead of a single perceptibility scale there is required for a given direction on the mixture diagram a different scale for each different portion of the diagram, and for each point a different scale is, in general, required for each direction. In an effort to provide such a complete set of approximate perceptibility scales for the ICI mixture diagram a group of thirty tangent circles on the UCS system have been transferred to the (x, y) plot of the ICI system where they appear as tangent ellipses of various sizes, eccentricities and orientations; see Fig. 1. The distances from points on the boundaries of these ellipses to the point within them, representing the transformed centers of the original circles, give the desired scales of perceptibility; one one-hundredth of each such distance refers to a chromaticity difference just perceptible with certainty under moderately good experimental conditions (careful monocular observation of a 6° circular field divided along a diameter yielding retinal illuminations between 200 and 1000 photons).

The indicated maximum variation in the scales of perceptibility is about 14 to 1; the distance from point to boundary along the major axis of the largest ellipse ($x=0.23$, $y=0.77$) is about 0.25, while the smallest such distance parallel to a minor axis (see ellipse inclosing $x=0.43$, $y=0.12$) is about 0.018. In most practical uses of the ICI system, however, the maximum variation is only about 5 to 1 because green samples whose colors are so saturated as to fall within the largest ellipses are rare.

The following example will show how Fig. 1 may be used as an aid in the estimation of chromaticity differences. Suppose it be required to know how much the dominant wave-length of a yellow signal light could be changed from 580 mμ and still keep the chromaticity difference no more perceptible than that between the signal at 580 mμ and a bare incandescent lamp at a color temperature of 2360°K. Since signals produced by yellow glasses may be closely matched by one part or another of the spectrum, itself, the spectrum may be used for this estimation. The scale in the direction of 580 mμ from 2360°K is about 0.13 per hundred "least perceptible differences" (LPDs); the scale along the spectrum near 580 mμ is about 0.04 per 100 LPDs. The distance on the diagram along the spectrum locus equivalent to the distance between 2360°K and 580 mμ is: $0.075(0.04/0.13) = 0.023$. This corresponds to slightly less than 3 mμ near 580, hence it would be said that the diagram indicates that both the difference between 2360°K and 580 mμ and that between

[4] For these terms and concepts, see R. Davis, "A Correlated Color Temperature for Illuminants," Nat. Bur. Stand. J. Research **7**, 659 (1931); RP365.

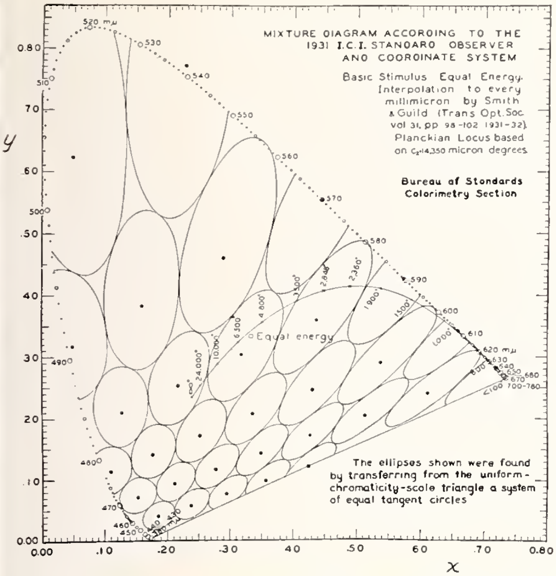

Fig. 1. Perceptibility scales for the standard 1931 ICI colorimetric coordinate system. The distances from points on the boundary of each ellipse to the indicated point within it all correspond approximately to one hundred times the chromaticity difference just perceptible with certainty under moderately good observing conditions.

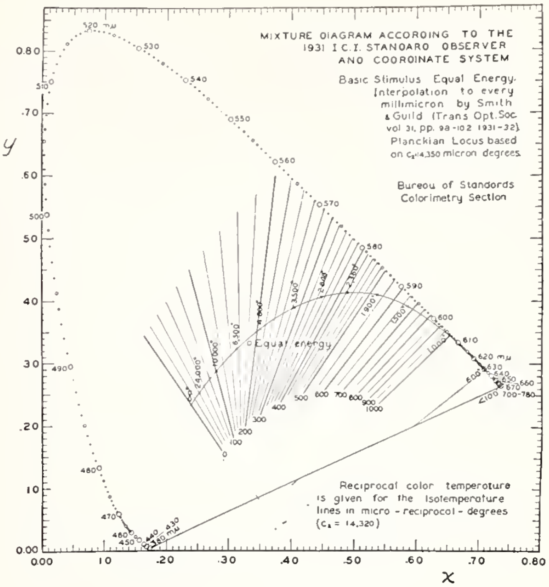

Fig. 2. Iso-temperature lines shown on the standard 1931 ICI colorimetric coordinate system. Color stimuli specified on an iso-temperature line give closer chromaticity matches with the indicated Planckian color than with Planckian colors at neighboring color temperatures.

580 and 583 mμ are about 60 times that least perceptible under good experimental conditions.

It is worth noting that a yellow signal of 580 mμ would resemble a bare vacuum-tungsten lamp sufficiently to be frequently confusible; this accounts for the non-use of such a signal. For example, railway signal yellow is a reddish yellow of dominant wave-length greater than 588 mμ.[5]

This example also serves to emphasize that the size of the LPD varies widely with experimental conditions. The unfavorable conditions often arising in the observation of railway signals (momentary viewing of point source at a retinal illumination of less than 100 photons with no comparison light) require perhaps fifty to one hundred times as large chromaticity differences for discrimination as are required under good laboratory conditions.

III. Nearest Color Temperature

An important practical application of the UCS system is its use in finding from any series of colors the one most resembling any neighboring color of the same brilliance. In this system the method, of course, is to draw the shortest line from the locus of the series to the point representing the neighboring color. This is the geometric equivalent of the method of observation on which many plans of color grading are based. The characteristic of these plans is that they pay attention to chromaticity differences along a series of color standards, and in the usual case that no member of the series gives a perfect chromaticity match, the nearest match is set and the residual difference neglected. In this way a problem which, strictly, is two-dimensional finds a one-dimensional solution that is close enough to be of practical use.

In finding nearest color from a series the UCS system may be used directly. The locus of the series is plotted together with the point representing the neighboring chromaticity, and the shortest line through the point to the locus, usually the normal, gives the nearest chromaticity of the series. It is also possible to carry out the operation once for all by drawing on the UCS triangle a number of normals to the locus of the series completely covering at sufficiently small intervals the areas of the triangle near the

[5] Association of American Railroads, Signal Section Specification 69–35, Signal glasses (exclusive of hand lantern globes). *Manual of Signal Section*, AAR, Part 136 (1935).

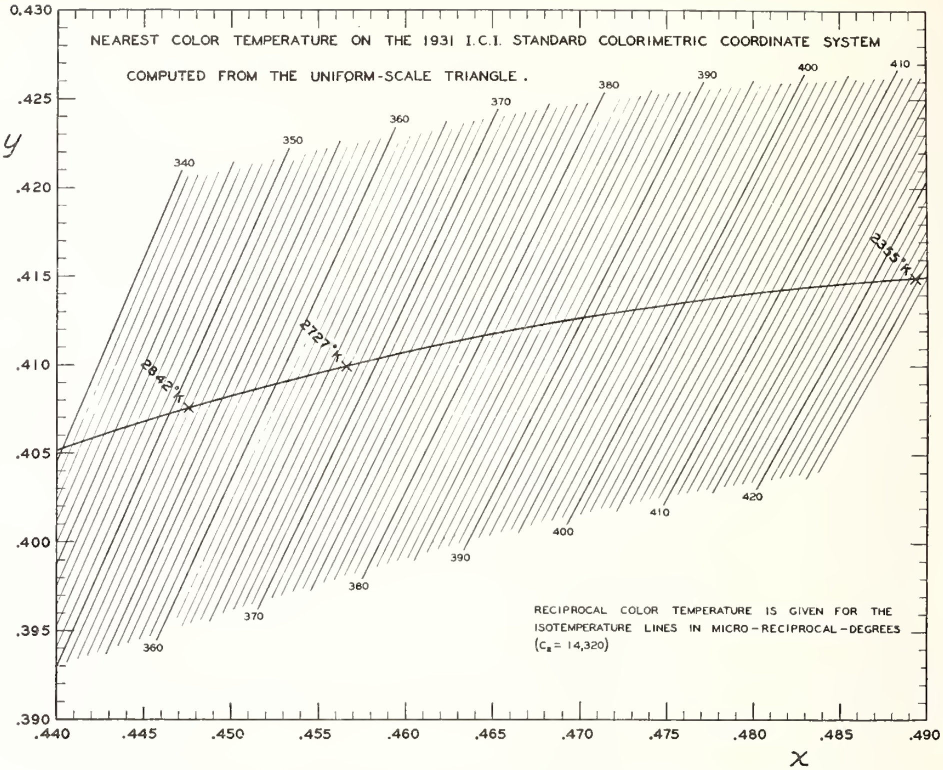

FIG. 3. Iso-temperature lines shown for a small section of Fig. 2. This region includes the most-used portion of the color-temperature scale for incandescent lamps.

locus. These normals can then be transformed to any other desired system and the determination of nearest chromaticity carried on entirely outside the UCS system. This has been done by McNicholas in the estimation of nearest chromaticity of vegetable oils on both the wave-length scale and on the scale composed of combinations of Lovibond 35-yellow with various Lovibond red glasses.[6] Fig. 2 shows a similar series of normals to the Planckian locus transferred to the standard 1931 ICI system.[7] By means of these transferred normals (iso-temperature lines) nearest color temperature estimated according to the UCS system may be approximately read from specifications in the ICI system. Fig. 3 shows a small section of Fig. 2 to an enlarged scale with one normal drawn in for each microreciprocal degree (*μrd*); as pointed out by Priest[8] who proposed the use of reciprocal color temperature, "a difference of one microreciprocal degree is fairly representative of the doubtfully perceptible difference in chromaticity under the most favorable conditions of observation." The large drawing from which this reproduction was made has been used in connection with a check of the present color-temperature scale[9] with blue filters whose use resulted in near-Planckian colors. The original drawing was found to reproduce estimates of nearest color

[6] H. J. McNicholas, "Color and Spectral Transmittance of Vegetable Oils," Nat. Bur. Stand. J. Research **15**, 99 (1935); RP 815; also Oil & Soap **12**, 167 (1935); see his Fig. 8.

[7] Fig. 2 is a reproduction of a figure previously used (see reference 1). Since that time a new color temperature scale has been adopted (see reference 9) and coincidentally the value of C_2 used was changed from 14,350 to 14,320 micron degrees to conform with the International Temperature Scale. This change in C_2 makes only 0.2 percent change in the temperature (or reciprocal temperature) and is too small to be shown in Fig. 2. For this reason the old diagram has been used. In Fig. 3, however, the change (about 0.8 *μrd*) could be easily seen.

In Table II, the values of color temperature listed were taken from Davis' paper in which the older value of 14,350 was used for C_2.

[8] I. G. Priest, "A Proposed Scale for Use in Specifying the Chromaticity of Incandescent Illuminants and Various Phases of Daylight," J. Opt. Soc. Am. **23**, 41 (1933).

[9] H. T. Wensel, D. B. Judd and Wm. F. Roester, "Establishment of a Scale of Color Temperature," Nat. Bur. Stand. J. Research **12**, 527 (1934); RP677.

temperature found directly from the UCS system with a maximum error less than 0.1 μrd. It will be noted that the normals extend only over the region immediately in the neighborhood of the Planckian locus. The experimental setting of nearest color temperature when a considerable chromaticity difference exists between the illuminant in question and the Planckian radiator at any temperature becomes increasingly difficult and ambiguous as the chromaticity difference is increased. It is doubtful whether the solution given by the UCS system is valid much further than the normals have been extended on Fig. 3; and Fig. 2, which shows them extended considerably further, may be criticized because it seems to make the false implication that a definite, unambiguous nearest color temperature can be found experimentally for illuminants having colors very notably non-Planckian.

To facilitate the construction of large-scale plots for accurate work such as Fig. 3 there have been included in Table I the trilinear coordinates of the Planckian locus on the ICI system for the reciprocal color temperatures represented by the transformed normals on Fig. 2, and the reciprocals of the slopes of these transformed normals. The reciprocals of the slopes are given instead of the slopes, themselves, for convenience because, as may be seen from Fig. 2, the slope becomes infinite in the neighborhood of 160 μrd, but is nowhere zero. Interpolation between the values given in Table I will yield points on the Planckian locus and also the slopes of the corresponding iso-temperature lines.

Table II shows a comparison of conjunctive wave-length, λj, referring to correlated color temperature computed by Davis' empirical method with that referring to nearest color temperature from the UCS system. The former

TABLE I. *Trilinear coordinates of Planckian stimuli and reciprocal of slopes of the iso-temperature lines as a function of reciprocal color temperature.*

$1/\theta$ $C_2=14{,}320$ μrd	Trilinear coordinates of Planckian locus		Reciprocal of the slopes of the iso-temperature lines
	x	y	
0	0.2399	0.2342	−0.672
20	.2458	.2429	− .611
40	.2526	.2524	− .536
60	.2605	.2629	− .450
80	.2693	.2743	− .359
100	.2794	.2869	− .266
120	.2909	.3003	− .175
140	.3037	.3141	− .089
160	.3171	.3272	− .012
180	.3306	.3396	+ .057
200	.3445	.3512	+ .120
220	.3583	.3619	+ .175
240	.3723	.3717	+ .227
260	.3865	.3805	+ .275
280	.4006	.3883	+ .319
300	.4142	.3950	+ .361
320	.4273	.4006	+ .400
340	.4403	.4052	+ .437
360	.4526	.4089	+ .472
380	.4646	.4117	+ .505
400	.4760	.4136	+ .535
420	.4868	.4148	+ .563
440	.4973	.4153	+ .590
460	.5072	.4152	+ .615
480	.5167	.4146	+ .639
500	.5256	.4136	+ .661
550	.5460	.4093	+ .713
600	.5638	.4033	+ .762
650	.5797	.3962	+ .808
700	.5941	.3886	+ .849
800	.6183	.3731	+ .923
900	.6370	.3587	+ .983
1000	.6521	.3451	+1.036

TABLE II. *Comparison of conjunctive wave-length referring to nearest color temperature via the UCS system with that referring to Davis' correlated color temperature.*

θ $C_2=14{,}350$ deg K	$1/\theta$ $C_2=14{,}320$ μrd	Conjunctive wave-length, λj, in mμ	
		Nearest color temperature via UCS system	Davis correlated color temperature
1,803	555.8	588.4	586.5
2,011	498.3	585.6	584.2
2,532	395.8	580.2	579.5
3,053	328.2	575.4	575.5
3,585	279.5	570.9	572.9
4,116	243.5	567.2	570.2
4,663	214.9	563.0	568.8
5,222	191.9	559.1	567.0
5,781	173.3	555.5	565.8
6,359	157.6	549.8	564.8
6,945	144.3	546.8	564.0
7,549	132.7	542.0	563.2
8,181	122.5	537.7	562.6
8,826	113.5	533.5	562.1
9,483	105.7	530.0	561.4
10,187	98.4	526.2	561.0
10,876	92.1	524.5	561.2
11,587	86.5	521.7	561.0
13,161	76.1	517.0	560.3
14,817	67.6	514.4	560.3

were taken from Davis' Table VI,[4] the latter were found by extending the iso-temperature lines on the original large-scale plot of Fig. 2 until they cut the spectrum locus. It may be noted that the agreement is good down to 250 μrd (up to 4000°K) but is rather poor at lower reciprocal color temperatures where the validity of the UCS system is best established.[2]

THE PHYSICS OF COLOR TOLERANCE

Am. Dyest. Rep. 28, 441-444 (1939)

This amusing dialogue seems, in retrospect, tongue-in-cheek - even satirical. In the role of a physicist, Judd was in character when he said, "I do not, myself, know how to construct a physical scale giving uniform spacing of color sensation; it's rather out of my line." But we have difficulty separating the actor from the man who five years earlier published the uniform chromaticity scale diagram that was recommended (in an only slightly modified form) in 1960 by the CIE and which after a trivial further modification was resuscitated by the CIE in 1976. Would the physicist, who said "When I see evidence that truly reliable results can be obtained, then I am willing to consider other methods to substitute for the standard methods [CIE 1931]", be ready to accept the 1976 CIE recommendations? Is Judd's exit line "Maybe a psychophysicist can tell me what a hue difference is in terms that mean something" still only a wistful hope?

The Physics of Color Tolerance

DEANE B. JUDD*

ABSTRACT

The physical specification of a reflecting sample requisite to determine its color is the spectrophotometric curve. The spectrophotometer not only gives more complete and fundamental information about such a sample than can be determined by direct visual inspection, but also is generally more sensitive than the human eye, and the results are reproducible. There are, however, pairs of samples which have the same daylight colors which do not have the same spectrophotometric curves. It is, therefore, necessary to take into account some of the properties of the human visual mechanism evaluated by psychophysical methods. These properties are expressed in the response curves of the eye defining the groups of physically dissimilar lights which look alike to the average normal observer. By using these response curves, together with the spectral energy distribution of an illuminant and the spectrophotometric curve of the sample, it is possible to calculate a color specification for the sample. A standard coordinate system for colorimetry has been adopted by an international standardizing body. Graphical, arithmetical, mechanical, and electrical short-cuts have been devised for the reduction of time taken up by the computations. The color specification most commonly takes the form of daylight apparent reflectance, A, of the sample and two trichromatic coefficients, x and y, which indicate chromaticity, apparent reflectance and chromaticity indices taken together make up the color specification. Color differences are evaluated by differences in A, x, and y. A physical quantity can only be defined in terms of the operations which have to be carried out to evaluate it. The above-mentioned operations which lead to differences in A, x, and y are, therefore, the definition of what is meant physically by color difference; this is an unambiguous definition, and by it the color difference between any two samples may be unambiguously evaluated. However, these evaluations often do not agree with estimates of the color differences obtained by direct visual inspection of the samples. Many attempts have been made to devise a series of operations to serve as an alternate definition of color which would yield this agreement, but none of the suggested methods has yet been generally accepted. There is some prospect that a system analogous to the present standard coordinate system will be devised within a few years to give color spacing in accord with perception. The chief obstacle is the paucity of information on the perceptibility of color differences. This information has to be evaluated by psychophysical methods. The physicist awaits the completion of this information with interest because of its bearing on color tolerance, but in the meantime finds many color tolerance problems in which questions of perceptibility are overshadowed by questions of permanence of materials and possible degree of manufacturing control. To these latter questions the present coordinate system is perfectly applicable.

By the physics of color tolerance, one perhaps ought to mean the "pure" physics of it; but it has been found impossible to confine these remarks to pure physics. Attention to other aspects of color tolerance have, however, been reduced to a minimum.

These remarks take the form of a dialogue in two scenes between Mr. Dyer, who dyes fabrics for automobile upholstery, and Mr. Meter, the "pure" physicist. Any resemblance of these characters to real persons, living or dead, is purely coincidental.**

Scene 1, Mr. Meter's Office

Enter Mr. Dyer.

Dyer: Mr. Meter, my firm dyes fabrics for upholstery, mostly for automobiles. The pieces are cut out according to pattern and sewn together. Now if the pieces are all cut out of the same run, they would stand some chance of matching when sewn together and there wouldn't be so much trouble. But our customers produce on a farily large scale; they cut up all the cloth of each large roll into pieces of the same pattern. These pieces are then sewed to those cut from other rolls of goods often from other mills. Sometimes all of the rolls are closely of the same color, but mostly there are differences between rolls which we cannot avoid; so when two pieces are sewed together, the customer is likely to object. It would save us a lot of money if we could tell ahead of time which pieces will match closely enough to be acceptable to the customer. The way it works now, the upholstery has to undergo a whole series of color inspections before it gets to the automobile buyer. The automobile manufacturer is more particular than the ultimate consumer, the upholstery manufacturer is more particular than the automobile manufacturer, and our inspector is still more particular; that's the way it has to be not to have a hitch anywhere along the line. But if we went at this thing scientifically, maybe we could satisfy the ultimate consumer without having to be about four times as particular as he is to provide several margins of safety along the line. This would save everybody a lot of money. So we have decided to measure some samples of our colors, and I was referred to you, Mr. Meter, for advice. They told me that you have an instrument called a color analyzer or spectrophotometer or some such thing that will measure color.

Meter: Yes, we can measure your samples on the spectrophotometer; we can give you a curve analyzing the light reflected by the sample by showing just how much of it falls in the red part of the spectrum, how much in the

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**In the presentation of this dialogue as an Introduction to the Technical Session of the Inter-Society Color Council, February 23, 1939, Dr. Judd read the part of Mr. Meter. The part of Mr. Dyer was read by Dr. I. H. Godlove, Technical Laboratory, Dyestuffs Division of the Du Pont Company.—Editor.

orange, yellow, and so on. Two samples having the same curves have the same color.

Dyer: I'll have to look into these technical details later; but here are my samples, all of them reddish shades of sand brown. I can see the difference between these two all right, but several of these pairs might have been cut from the same roll of cloth as far as I can tell. They were picked out by one of our inspectors, and he is really particular; sometimes I wonder if he can actually see these differences, or is just guessing. But generally one inspector will reject about the same goods as another. Here, what do *you* think about it? I suppose with a trained eye like yours—

Meter: I'm afraid my judgment would be of little use to you; I am accustomed to using the spectrophotometer for the detection of color differences. We find that visual estimation just results in argument and difference of opinion. You are lucky with your somewhat old-fashioned methods (you will pardon the frankness of science, Mr. Dyer) not to have more trouble than you do. The spectrophotometer is more sensitive than the human eye[1] and the results are reproducible.[2] We don't really have to look at samples at all. If we wanted a color match we could run a spectrophotometric curve of the sample, then send the curve to a dyer using our methods, and they could send back the match without having seen the original sample at all.

Dyer: That certainly takes out the human element. If you get two identical curves, you have a color match; and if you get different curves, you have different colors.

Meter: Well, there is more to it than that, Mr. Dyer; as you probably know, you can sometimes get samples that will color match in daylight but fail to do so in lamp light. Such pairs of samples have different spectrophotometric curves.[3]

Dyer: I am getting a little mixed up here. If you get the same spectrophotometric curves, you know you have the same colors, but if you get different curves sometimes you *still* have the same colors?

Meter: Without getting too technical, Mr. Dyer, the whole thing comes down to this. The spectrophotometric curve defines the reflecting properties of the sample, but to deduce the appearance of the sample from its reflecting properties requires some further information. The first additional item is the spectral character of the light source and the angle at which the light is incident upon the sample. Specification of the light source is pure physics and can be accomplished in a definite and satisfactory way.

Dyer: It looks rather involved to me. Do you use light hitting the sample from one direction only?

Meter: Sometimes illumination at 45° is used, and sometimes completely diffused illumination; that is, light striking the sample equally from all directions.

Dyer: I doubt if either one of those kinds of lighting would give us information comparable to what we get from our inspectors. They look at the sample from many angles, and hold it up sometimes so that the light strikes it at a glancing angle, and then again with the light more or less perpendicular to the sample.

Meter: I don't deny that there are difficulties about modes of illumination and viewing; but if you will use *our* methods I am sure you will get along much easier. At least you will know exactly what condition of lighting *was* used. This is specified definitely by physical methods. But to get back to the second item of information required. The appearance of the sample also depends upon what person looks at it; that is, a color-blind person, as you know, does not see these samples the way you and I do. The way we handle this question is to use a standard observer representative of average, normal vision, determined by "psychophysical methods." Of course, the research necessary to evaluate this observer, the psychophysical research, involves something other than pure physics; and we physicists were somewhat reluctant to use the results. But, as you know, color has to do with seeing, so there is no other way out; and I must say that the psychophysicist has done a pretty good job on the standard observer. You would hardly know it isn't physics. This standard observer has been adopted by the International Commission on Illumination[4]; it is used by the National Bureau of Standards, by Massachusetts Institute of Technology, and by many industrial organizations.

Dyer: I am not sure I understand just how I could use this standard observer.

Meter: By using the standard observer we get a color specification from the spectrophotometric curve; it's just a matter of computation.

Dyer: You mean after you get the curve you still have computations to make?

Meter: Yes, there is a regular standardized way of computing the color from the spectrophotometric curve.

Dyer: This is harder than I thought.

Meter: Oh, it's not so bad. We have short cuts of various kinds that reduce the computations for a sample to a matter of minutes.[5]

Dyer: And what do you have when you get through with these computations?

Meter: You have a fundamental color specification according to a method that is coming to be universally recognized.

Dyer: And if I have a sample specified by this method, do I know for sure that another sample with the same specification will match it?

Meter: Positively, — er, unless your observer has abnormal color vision, or unless the illuminant is different from what was put into the computations. This method gives you three numbers for each sample; and if two samples have the same triad of numbers, they have the same color, — subject, of course, to the limitations I just mentioned.

Dyer: What do you call these numbers?

Meter: Well, first, you get the daylight apparent reflectance of the sample; this is high for light samples and low for dark; so it is a specification of the lightness of the color. Then you get two values, x and y, which indicate chromaticity; the indices of lightness and chromaticity, taken together, form a complete specification of the color.

Dyer: Well, Mr. Meter, it's all Greek to me, but I'll take your word for it. Could you tell from all your curves and computations which of my samples is the closest match to the standard, which the next best, and so on?

Meter: Well, no. The psychophysical information required to give the answer to *that* question was never put into the standard observer and coordinate system; so you cannot hope to get it out. As a matter of fact, that information has never been completely worked out.

Dyer: Oh! (disappointed) Then I'm afraid all this curve drawing and computation won't be of so much use to me. If we are to satisfy our customers, we *have* to make judgments of best match all the time. We get our inspectors to do it. I was hoping you could tell us how to do it better.

Meter: The great trouble with your present use of the method of direct estimate is that you have no record of the judgments; whenever similar samples come to hand again, you have to get your inspector to make another guess.

Dyer: Yes, that is true; I begin to see what you mean.

Meter: I suggest that you express all of your samples, standards and rejected samples, in terms of x and y and daylight apparent reflectance, A. Then you will know definitely what the acceptable shades are. And another thing, — there will be many cases when the spectrophotometric curve alone will tell you all you want to know; in fact, that will be true in a majority of cases. If you are working with samples all from the same dye bath, and have records of the limit samples, you can tell at a glance directly from the curve of a new sample whether the customer is going to object.

Dyer: Yes, I see how that might work out.

Meter: But that's not the half of it. Not only will the spectrophotometer assist in establishing, specifying and administering color tolerances, but it will also make possible control of your dyeing process so you won't have any rejections. *You* thought you might reduce expenses by taking full advantage of the customer's tolerance to color difference. I will show you how to reduce your losses from rejections of off-color deliveries to a minimum. I am fairly familiar with the problem of dyeing to a standard sand brown. The trouble comes mainly because the dye bath has to contain at least three dyes. These dyes go onto the fabric at different rates; so after a while the balance of the mixture is significantly disturbed and the fabric comes out off-color. What you have to do is follow the rate of exhaustion of the dyes from the dye bath by means of the spectrophotometer. In this way you will know exactly what color the fabric will be.

Dyer: Well, maybe it's worth a trial. How much does a spectrophotometer cost? One that is accurate enough to pick up these small differences, and fast enough to give us results in time to be of value. Oh yes, — and foolproof enough to keep going under plant conditions.

Meter: I can see that you require an automatic, recording spectrophotometer. And I am sure that you will find it important enough to require the services of a trained operator; these are precision built instruments that cannot be made foolproof.

Dyer: Well, I guess it's worth a trial. Send one up, and recommend somebody to run it. How much will it cost?

Meter: Five thousand dollars. Is that too steep?

Dyer: (earnestly and with great emphasis) My dear Mr. Meter, to a concern doing millions of dollars worth of business, with products sold in every state in this glorious land of ours and exported to all the principal countries of the world, and with payrolls of thousands of workmen in several states, five thousand dollars is but a trifle. (Aside, nervously) I hope it does some good.

Exit Dyer.

End of Scene 1

Scene 2, Six Months Later, Same Place

Enter Dyer.

Meter: Glad you dropped in again, Mr. Dyer. How is the new spectrophotometer working?

Dyer: Doing very nicely, thank you. All you hear around the mill now is "Send it up to the 'spectro' and see who is right." You see we soon got tired of calling it the automatic, recording, photoelectric spectrophotometer; so we call it the 'spectro' for short. It certainly helps out in our sales; the buyer takes one look at the spectro drawing its own curve and forgets to argue about color tolerance. We have a slogan, "With scientific color control, you *know* it's right."

Meter: I hope I didn't encourage you to buy a spectrophotometer just for sales promotion.

Dyer: Far from it. Although I can't say our troubles with color tolerance are over, the spectro has already saved us a lot of money. The other day we dyed fifty rolls of goods to the same color simultaneously in fifty different dye pots and they were accepted without question. We speeded up production and still had no rejections. A few months ago we would have said that was impossible, and without rigid control of the concentration of dyes in the bath it would be impossible today; furthermore, we are only just beginning to learn how to apply the instrument to our problems.

Meter: I am not surprised at your report, but, of course, I am gratified at your success. The methods I recommended to you are past the experimental stage; they have been verified time and again.

Dyer: I hope you won't think I am complaining when I return to you with the same problem I had originally. Although it is not as urgent now as I used to think it was, we still would like to be able to tell more certainly which of two samples is the better match, and to tell ahead of time from our measurements whether the customer is going to object. Taking time for a customer survey is a rather expensive method. We have tried out a few cases to see whether the tolerances in terms of A, x and y which we derived from a customer survey on sand brown would apply to other colors.

Meter: I believe I warned you that this information could not be gotten from our present standard method.

Dyer: I know you did; but I thought maybe we could get approximate information that would be of some value. You were right, though; we found it necessray to set considerably different tolerances depending upon the particular color we were working with. Isn't there some automatic way to relate differences in A, x, and y with size of color difference?

Meter: I do not, myself, know how to construct a physical scale giving uniform spacing of color sensation; it's rather out of my line. There have been proposed some methods for that purpose[7], but none of them has so far been generally adopted; and, I understand, work is still going on directed to that end. I have had such good success, however, with the spectrophotometer alone, and with colorimetric specifications according to the standard system that I have not bothered to try these new methods out very thoroughly. When the psychophysicist has really done a thorough job of it, I won't have to worry about the validity of the method myself. When I see evidence that truly reliable results can be obtained, *then* I am willing to consider other methods to substitute for the standard methods[8].

Dyer: Well, I see that the business of physicists is to make measurements, and I suppose it is the business of psychologists to study the behavior of people. You've helped me a lot to control and measure my fabrics, but if I want to know the effect of these fabrics on my customers (and I certainly do) maybe I ought to consult a psychologist.

Meter: Confidentially, I don't advise it. No psychologist I ever met could talk your language; at least they didn't talk mine. The only kind of quantities that I have found worth talking about are those resulting from physical measurement. A physical quantity is uniquely defined in terms of the operations which have to be carried out to evaluate it[6]. The specification of color difference which I recommended falls almost perfectly under this definition of a physical quantity. I say *almost* perfectly because, as I mentioned before, the standard observer has been determined by methods not strictly physical. But once the observer is adopted arbitrarily, there is no ambiguity about the specification of color in terms of A, x and y; so in this sense they are physical specifications. Psychologists don't deal with any such quantities; they talk about such things as the attributes of color being hue, tint and chroma; but I haven't found out yet how to get a numerical evaluation of any of them.

Dyer: Well, how about consulting one of these psychophysicists, then?

Meter: *That* might not be so bad; and I wouldn't mind coming with you. Maybe a psychophysicist can tell me what a hue difference is in terms that really mean something.

(Exit Meter and Dyer together.)

Acknowledgments

It is a pleasure to acknowledge the assistance of Drs. I. H. Godlove and A. W. Kenney of the du Pont Company, of Drs. L. A. Jones and D. L. MacAdam of Eastman Kodak Company, and of Prof. A. C. Hardy of Massachusetts Institute of Technology, in the preparation of this dialogue.

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METHOD OF DESIGNATING COLORS
(With K. L. Kelly)

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Color names from the vernacular are assigned unambiguously to specific ranges of colors of opaque, nonmetallic reflecting materials, specified in terms of the Munsell notation. The uses of adjectives such as pale, light, brilliant, strong, weak, dark, and deep are systematized so that they are easily comprehended and remembered. The color-name charts (1-34) that accompanied this paper (pp. 367-385) have been omitted, to save space. An improved form of them is contained in "Color - Universal language and dictionary of names" by Kenneth L. Kelly and Deane B. Judd, Nat. Bur. Stand. (U.S.), Spec. Publ. 440 (1976).

Editorial note: Figs. 3, 4 and 5 have also been omitted; the references to them on pp. 362 and 363 should be disregarded.

U. S. Department of Commerce National Bureau of Standards

RESEARCH PAPER RP1239

Part of Journal of Research of the National Bureau of Standards, Volume 23, September 1939

METHOD OF DESIGNATING COLORS [1]

By Deane B. Judd [2] and Kenneth L. Kelly [3]

ABSTRACT

In 1931 the first chairman of the Inter-Society Color Council, E. N. Gathercoal, proposed on behalf of the United States Pharmacopoeial Revision Committee the problem of devising a system of color designations for drugs and chemicals. He said, "A means of designating colors in the United States Pharmacopoeia, in the National Formulary, and in general pharmaceutical literature is desired; such designation to be sufficiently standardized as to be acceptable and usable by science, sufficiently broad to be appreciated and used by science, art, and industry, and sufficiently commonplace to be understood, at least in a general way, by the whole public." With the assistance of the American Pharmaceutical Association, and following plans outlined in 1933 by the Inter-Society Color Council, there has been worked out a solution for this problem, which substantially fulfills the requirements laid down by Dr. Gathercoal.

CONTENTS

[1] Recommended by the Inter-Society Color Council, June 1939, for drugs and chemicals, and tentatively suggested for general use.
[2] Physicist, National Bureau of Standards, and chairman of the Committee on Color Problems of the Inter-Society Color Council.
[3] Research Associate at the National Bureau of Standards, representing the American Pharmaceutical Association.

I. HISTORY

The problem was referred by the Inter-Society Color Council to its Committee on Measurement and Specification which, under the chairmanship of I. H. Godlove, presented several reports surveying available methods of color designation. The 1933 annual report of this committe included the outline of a recommended system of color designations. These recommendations were approved by the Council and have been followed by the authors, who have developed this system of color designations, have set the color boundaries, and have worked out methods of applying the system to drugs and chemicals in various forms. The Inter-Society Color Council in 1939 formally approved by letter ballot and recommended to the United States Pharmacopoeial Convention the method now to be described.

Member bodies of the Council whose voting delegates have approved this method of designating the colors of drugs and chemicals are: [4]

American Association of Textile Chemists and Colorists.
American Ceramic Society.
American Psychological Association.
American Society for Testing Materials.
Illuminating Engineering Society.
National Formulary, American Pharmaceutical Association.
Optical Society of America.
Technical Association of the Pulp and Paper Industry.
United States Pharmacopoeial Convention.

II. SCOPE

The recommended color designations apply to powdered drugs and chemicals and to whole crude drugs viewed in daylight. The Council is now engaged in a study of the general applicability of these designations to colors of opaque, nonmetallic surfaces with a view to official adoption for all such surfaces.

The recommended system does not give suitable color designations in its present form for liquids and solids viewed by transmitted light. An extension of the system to cover such samples has been undertaken by the Council. (See also section VIII, 3.)

III. LOGIC OF THE DESIGNATIONS

The designation for all but very grayish colors consists of a hue name (*red*, *green*, *blue*, *purple*, etc.) preceded by appropriate modifiers (such as *weak*, *moderate*, *strong*, *light*, and *dark*). The designation for very grayish colors consists of a noun (*white*, *gray*, or *black*), with modifiers appropriate to the lightness and hue of the colors (such as *dark reddish gray* or *yellowish white*).

1. SURFACE-COLOR SOLID

The relationships between the names can best be understood by a consideration of the psychological color solid. The dimensions of this solid are hue, lightness, and saturation (see fig. 1); the color of any

[4] A majority of the voting delegates representing the individual members of the Council also approved. Textile Color Card Association of the United States (not voting).

matte, opaque surface in daylight is represented by some point in it; hence it is often called the surface-color solid. Lightness starts at zero for black, represented at the bottom of the figure, and is measured by distance from the base plane, being a maximum for white represented at the top of the figure. Hue is represented by angle about the black-white axis, giving the closed series red, yellow, green, blue,

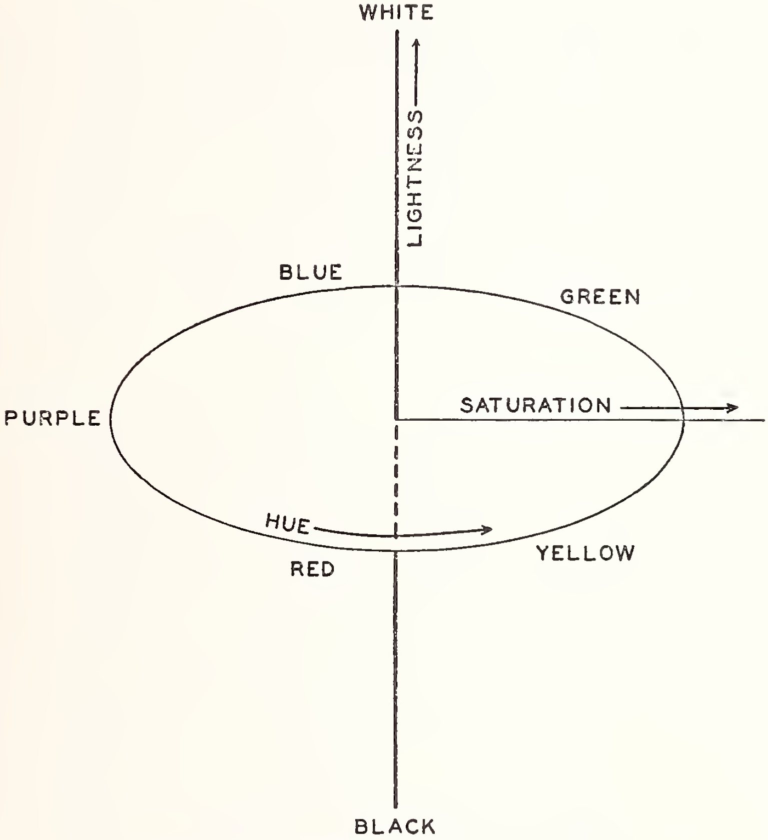

FIGURE 1.—*Dimensions of the surface-color solid.*

purple, red, with their intermediates. Saturation is represented by distance from the black-white axis, being zero for black, white and the intermediate grays, and increasing toward the boundary of the surface-color solid on which would be represented the most vivid colors producible from surfaces [1].[5]

Colors of one hue are therefore represented in the solid by points falling in a single one of the vertical planes intersecting at the black-white axis. Colors of one lightness are represented by points in any one horizontal plane; and colors of one saturation are represented by points in any one of the series of right circular cylinders concentric about the black-white axis.

[5] Figures in brackets indicate the literature references at the end of this paper.

2. BASIC PLAN OF FORMING THE DESIGNATIONS

The hue name of the color designation is intended to indicate a range of hue angle in the color solid, and other words in the designation are to indicate ranges of lightness and saturation for this hue range. The system of modifiers is indicated in figure 2. Deviations

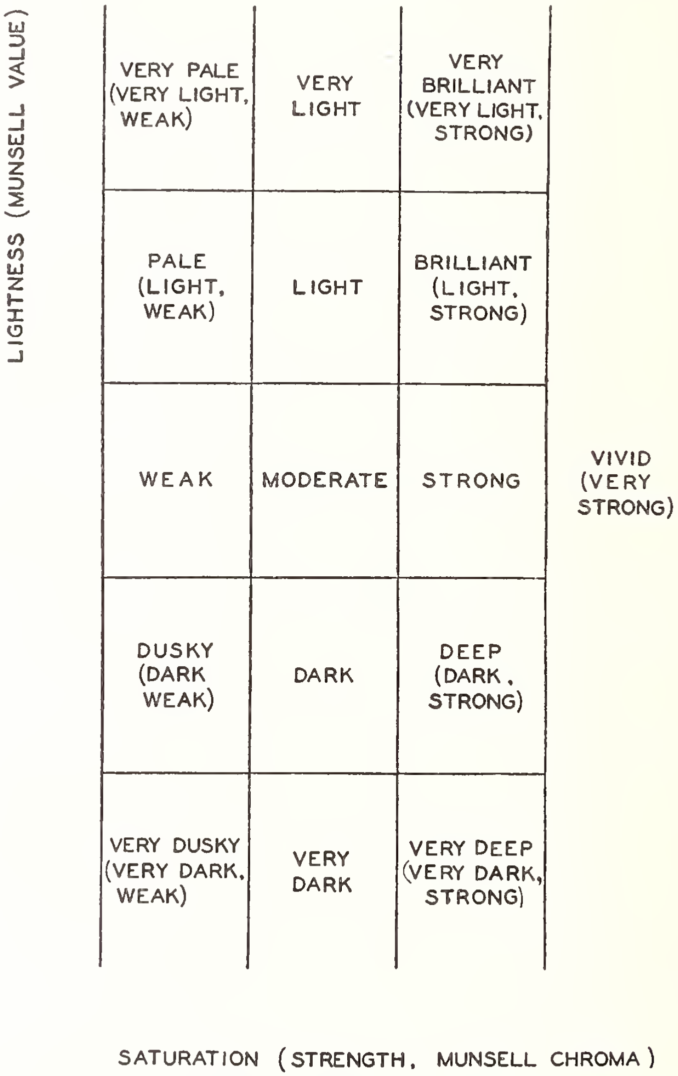

FIGURE 2.—*System of modifiers.*

The color designation, except for very grayish colors, consists of a hue name combined with one of these modifiers.

from the moderate range in lightness are indicated by the terms, *light* and *dark;* deviations in saturation by the terms *weak, strong,* and *vivid;* and deviations in both lightness and saturation by the terms *pale, deep, dusky,* and *brilliant.* The whole color designation, hue name and modifiers, therefore, defines a block of the surface-color solid bounded by vertical planes of constant hue, horizontal planes of constant lightness, and cylindrical sufaces of constant saturation. The surface-color solid is divided into 314 such blocks, with 5 cylindrical blocks for black, grays, and white, making 319 blocks in all.

3. DIVISIONS OF THE HUE CIRCLE

The 1933 recommendations by Dr. Godlove included a 20-point division of the hue circle for colors of moderate saturation, a 10-point division for weak colors and a 5-point division for very weak colors. By adhering to the principle of this recommendation it has been possible to make each color designation refer more nearly to the same-sized color range than would otherwise have been possible. Deviations from the 1933 recommendations have been introduced to make the color designations accord more closely with present usage, particularly usage in the United States Pharmacopoeia and National Formulary. Most of these deviations arise from our introduction of the color terms *pink*, *brown*, and *olive*, and our change to a more restricted color range for *orange*.

4. PINK, ORANGE, BROWN, AND OLIVE

Unlike the terms *green* and *blue*, which are hue names applying to all lightnesses and saturations, the term *yellow* is commonly used to designate not only a certain hue range, but also a high-lightness range within this hue range. Dark colors of the same hue as yellow are commonly called *olive* or *olive brown*. Common usage limits the term *orange* even more strictly; it is taken to refer not simply to a range of yellow-red hues but also to a medium-lightness range and a high-saturation range. Colors of the same hue but of lower lightness and saturation than the orange range are called *browns* or *reddish browns*.

To follow common usage in this respect, there is included a series of hue names applicable to dark colors only, as follows: *reddish brown*, *brown*, *yellowish brown*, *olive brown*, *olive*, and *olive green*. As a further concession to common usage, there is also included the following series of hue names applicable to very light colors only: *purplish pink*, *pink*, and *orange pink*. The chief series of hue names to which these two subsidiary series have been fitted follows closely the 20-point division recommended by Dr. Godlove; the chief series includes the names *red*, *reddish orange*, *orange*, *yellowish orange*, *yellow*, *greenish yellow*, *yellow green*, *yellowish green*, *green*, *bluish green*, *blue green*, *greenish blue*, *blue*, *purplish blue*, *bluish purple*, *purple*, *reddish purple*, *red purple*, and *purplish red*.

5. SOME UNAVOIDABLE DISADVANTAGES

A frequent objection to this system of color designations is that each designation refers to a group of distinguishable colors rather than to a single color. Since there are about ten million surface-colors distinguishable in daylight by the trained human eye and only 319 color designations in this system, it is obvious that the average color range denoted by a single designation must contain about 30,000 distinguishable colors. If it is important to make distinctions among some of these thousands of colors bearing by this system identical designations, resort must be had to one of the many numerical systems of color specification available. Preeminent among these is the colorimetric coordinate system recommended in 1931 by the International Commission on Illumination [2].

A corollary to this objection is that there are many pairs of easily distinguishable colors which receive by this system the same designation, while there are also many pairs that can scarcely be distinguished which receive different designations. This property is, of course, an unavoidable result of dividing the color solid into an arbitrary number of blocks, one for each of the 319 designations. Analogous disadvantages result from identifying the time of events according to date; two events occurring on the same date may be separated by many hours, but on the other hand two scarcely separable midnight events may have to be assigned different dates. Just as identifying the time of an event by giving the date has proved to be useful, so it is anticipated that a system of color designations such as this will find many uses.

IV. DEFINITION OF THE COLOR RANGES

The adjustment of the boundaries defining the 319 color ranges has been carried out by the committees of the Inter-Society Color Council with reference to the color standards of the Munsell Book of Color [3]. These standards are arranged according to Munsell hue, Munsell value, and Munsell chroma, which are intended to be practical evaluations of the surface-color attributes: hue, lightness, and saturation, respectively. That is, all Munsell standards bearing the same Munsell hue notation are intended to have colors of the same hue; likewise, Munsell value is intended to indicate, and does closely, the lightness of the color; and Munsell chroma corresponds well with saturation. For the present, therefore, the definition of the color ranges is given in terms of Munsell hue, value, and chroma. (See the 34 name charts which form the major part of this paper.)

Ultimately, it will be desirable to supplement this practical definition of the color designations by giving the equivalent definition in terms of the 1931 colorimetric coordinate system [2], which does not depend upon the integrity of material standards of color. Any small uncertainty in the boundaries arising from disagreement among the various sets of Munsell color standards, or from their impermanence, will be resolved through spectrophotometric specifications, already partially available, of two sets of master standards deposited at the National Bureau of Standards in 1935 by the Munsell Color Co. A set of smoothed interpolation curves based upon spectrophotometric measurements of a set of Munsell standards by J. J. Glenn and J. T. Killian in A. C. Hardy's laboratory at the Massachusetts Institute of Technology already permits a fairly reliable transfer from Munsell notation to ICI specification to be made. These interpolation curves are available through Dorothy Nickerson, color technologist, Agricultural Marketing Service, Washington, D. C. The Maerz and Paul Dictionary of Color [4] was used to test the suitability of many of the boundaries, and because of its large number (7,056) of colors, well distributed throughout the surface-color solid, it forms the basis for a very satisfactory possible definition of the boundaries for practical use. The determination of the color names of only a few of this large number of samples has so far been completed.

V. HUE BOUNDARIES FOR VARIOUS RANGES OF MUNSELL VALUE AND CHROMA

Although the systematically arranged and graded standards of the Munsell Book of Color have proved to be invaluable aids in the choice of color-designation boundaries to accord with common usage, it has been found that best agreement with such usage is obtained by deviating in many cases from constant Munsell hue. One deviation from constant Munsell chroma has also been made (*3Y* to *8Y*, inclusive, change from 1.5 to 2.0 chroma). These deviations are shown on charts 35 and 36 which consist of the hue boundaries for various chroma ranges. Chart 35 refers to dark and medium colors; chart 36, to all light colors having hue boundaries different from the corresponding dark and medium colors. The hues are indicated by abbreviations such as *pR* for *purplish red*, *R* for *red*, *rO* for *reddish orange*, *r-Br* for *reddish brown*, *Ol-Br* for *olive brown*, *p-Pk* for *purplish pink*, *O-Pk* for *orange pink*, and so on. Munsell hue is indicated on the 100-point scale on the outer circle. These charts show at a glance for all Munsell chromas what ranges of Munsell hue are referred to by the various hue names of this system; this information is of course also obtainable from the name charts (1 to 34), themselves, but less conveniently. The central circle marked *N* for *neutral* refers to Munsell chromas less than 0.5. The 6-point division on chart 35 refers to the chroma range (with the exception noted above) 0.5 to 1.5, and yields color designations involving the hue adjectives, *reddish, brownish, olive, greenish, bluish,* and *purplish,* such as *reddish gray* and *reddish black.* The 5-point division on chart 36 similarly refers to color designations involving the hue adjectives, *pinkish, yellowish, greenish, bluish,* and *purplish.* The 10-point division on chart 35 refers to the chroma range 1.5 to 3.0 and involves the hue names: *red, brown, olive brown, olive green, green, blue green, blue, purplish blue, purple,* and *red purple.* The similar ring on chart 36 has 11 divisions: *pink, orange pink, orange, yellow, yellow green, green, blue green, blue, purplish blue, purple,* and *purplish pink.* The 19-point division on chart 35 refers to chromas greater than 3.0, and it will be noted that within the two chroma ranges, 3.0 to 5.0, and greater than 5.0, the hue boundaries for simplicity are kept at constant Munsell hue. As mentioned above, however, it has been found possible to increase agreement of these color designations with common usage by adopting in 5 out of 19 cases a different hue boundary for the chroma range 3.0 to 5.0 than is used for the chroma range 5.0 up. Many of the hue boundaries for the inner rings are also shifted in a corresponding way for the same reason.

VI. COLOR DESIGNATIONS FOR OPAQUE POWDERS

1. PREPARATION OF SAMPLE

The sample is placed slightly heaped up in a clean aluminum holder at least 2 mm deep. Over it is placed an optical-glass cover of 1 mm thickness, which is pressed down with a rotary motion, and two small rubber bands are snapped across underneath the sample holder between the opposite hooks on the back of the cover glass; see figure 3.

2. LIGHTING AND VIEWING CONDITIONS

The illumination to be used in the color-comparison work is daylight. A table placed by a window so that light reaches the table top from the operator's left or right chiefly from the sky and chiefly at an angle of 45° from the horizontal is recommended. A north window is best because no special precautions are usually required to eliminate direct sunlight, but windows facing in any direction may be used if equipped with suitable diffusing curtains. A canopy of black cloth (preferably black velvet) should be hung above the sample on the side opposite the operator in such a position as to be imaged in the mirror surfaces of the cover glass; such an arrangement eliminates errors from unwanted admixture of light reflected only from the cover glass. The sample and standard placed on the table top are viewed nearly perpendicular to the surfaces, that is, just enough off the perpendicular to avoid having the operator's face mirrored in the cover glass. Illumination at 45° and perpendicular viewing are recommended by the International Commission on Illumination [2]. A skylight or source of artificial daylight located above the sample may also be used, but in such a case the angle of view should be approximately at 45° from the horizontal, and the black cloth should be hung vertically beside the sample opposite the observer. Perpendicular illumination with viewing at 45° gives results equivalent to the recommended ICI method.

It is important that the illumination of sample and working standard be closely the same both in amount and quality; otherwise different Munsell notations will be found by interchanging them. Even with illumination of good uniformity it is best practice to make this interchange as a check during the comparison.

In any computations involving the spectral energy distribution of the illuminant, that of standard illuminant *C* recommended by the International Commission on Illumination [2] as representative of average daylight is to be used.

3. PROCEDURE

Select the two adjacent Munsell constant-hue charts between which the hue of the sample falls. Place these on each side of the sample and cover each with a small gray shield, or if using the large triple-aperture shield (shown in fig. 4), place them under the holes in the side flaps and the sample under the central rectangular opening. The Munsell hue, value, and chroma notations for a sample are found by interpolation among the standards of the Munsell charts; most operators prefer to interpolate first for value, then chroma, and finally hue. For detailed suggestions on this interpolation consult the Munsell Book of Color, Standard edition [3].

Once the Munsell notation is found for the sample, select the color-name chart referring to the hue of the sample (see Munsell hue designations near upper right-hand corner of each name chart). Plot the value and chroma of the sample on this chart, noting that chroma from 0 up to 1.5 has, for convenience, been plotted to a more open scale than the remainder (see vertical double line dividing the two scales on each name chart). Record the name of the block in which this point falls as the color designation of the sample. If, however, the point falls on a value or chroma boundary, or if the hue notation falls exactly between successive charts yielding different color names, the names of all of the blocks touching the point apply to the sample.

4. AN EXAMPLE

Suppose that the hue of the sample falls between the *YR* (*5YR*) and the *YR–Y* (*10YR*) charts, and that its value falls between 7 and 8 value, but nearer 7, and is estimated as 7.2. Suppose, further, that its chroma is found to be closer to 4 than to 6 and is estimated as 4.5. Now compare the sample with the two Munsell 7/4 standards. Suppose that its hue is seen to be nearer to that of the *YR–Y* chart than the *YR* chart, say 4/5 of the hue difference between the charts or four hue steps from the *YR* and one from the *YR–Y* chart. Now interchange the positions of the charts and check the Munsell notation. If these are found to be unchanged, the final notation is *9YR* 7.2/4.5. Now look through the name charts until the one for *9YR* is found (chart 8, see hue designation near upper right-hand corner). Plot 7.2 value and 4.5 chroma on this chart. It falls in the block named *weak orange;* so the color designation of the sample is *a weak orange.*

VII. COLOR DESIGNATIONS FOR WHOLE CRUDE DRUGS

1. COMPARISON WITH MUNSELL COLOR STANDARDS

Hold the sample in the fingers, or in tweezers if the sample is small, a short distance above the chart or charts and move it about for comparison with the Munsell color standards. Care should be taken not to cast a shadow on the standard with which the sample is being compared; on this account the larger samples should be held higher above the charts than the small (see fig. 5). The time required for the comparison, and consequent soil and wear on the standards, will be saved if the charts are arranged in hue sequence on the table top so that each chart covers the 8- and 10-chroma columns on the preceding chart.

2. LIGHTING AND VIEWING CONDITIONS

The samples are to be illuminated at 45° by daylight (see section VI, 2), and viewed along the perpendicular to the surface. Since the samples are held above the plane of the color standards, it is important that the illumination on the two horizontal planes be the same in amount and quality. Care should be taken to hold the surface of the sample as nearly in the horizontal plane as possible; errors in Munsell value by as much as a whole step are possible through inadvertent tilting of the sample surface. If a source of artificial daylight is used giving a diffused even illumination over a large area from above, or if the comparison is made out of doors by diffuse light from a large part of the sky, the angle at which the sample is held with respect to the light source is less critical.

For minimizing the troubles due to uneven illumination on samples (such as roots) having approximately cylindrical surfaces, it is recommended that the axis of the cylinder be held horizontal and pointed in

185952—39——2

the direction of the light source so that neither side of the sample is shaded.

For samples having glossy surfaces the use of a black canopy or curtain will be required as described in section VI, 2.

3. WAYS OF USING THE COLOR DESIGNATIONS

The color of a crude drug may be designated either by giving the name of the average color found in the lot, if the range of color is small, or by giving the color names corresponding to the maximum range of color. This range may be of hue, lightness, or saturation, or a combination of two or all of these. Departures from such a range will frequently be an indication of deterioration or impurity. Color ranges involving chiefly variations in lightness and saturation can often be conveniently indicated by the unmodified hue name, such as *orange*, by which would be meant the color range covered by the designations, *brilliant orange*, *vivid orange*, *strong orange*, *moderate orange*, *weak orange*, *light orange*, *pale orange*, *dark orange*, and *deep orange* (see charts 4 to 6).

VIII. COLOR DESIGNATIONS FOR ANY OBJECT

The main purpose of this paper is to provide information required for obtaining color designations of powdered crude drugs, whole crude drugs, and chemicals of small particle size. To this end, specific instructions have been given in sections VI and VII. A secondary purpose, however, is to facilitate study of the suitability of these color designations to any object. Detailed procedures applicable to this wider use of the system have yet to be formulated; but some general instructions can be given.

1. FOR OPAQUE NONMETALLIC MATERIALS

(a) WITH MATTE SURFACES

Proceed as in section VI or VII. The recommended angles of illumination and viewing need not be strictly followed, because the appearance of a matte surface does not change importantly with small variations in these angular conditions.

(b) WITH GLOSSY SURFACES HAVING NO REGULAR DETAILED STRUCTURE

Samples of vitreous enamel and smooth paint films are often found with glossy surfaces having no regular detailed structure. Proceed as in section VI and VII, giving particular attention to the prescribed angles of illumination and viewing. The characteristic color of the sample is obtained only when specularly reflected light is prevented from reaching the eye of the observer.

(c) WITH GLOSSY SURFACES MADE UP OF CYLINDRICAL ELEMENTS

Samples of satin-finish textiles and glossy brush-marked paint films may be considered as made up of cylindrical elements. Proceed as in section VII, 2 for cylindrical surfaces. It is not always possible to prevent light from being specularly reflected from such glossy surfaces into the eye of the observer; but by so orienting the sample in its own plane that the specular component is reduced to a minimum, the most characteristic color of the sample is usually obtained. Particular attention is being devoted to choice of angular conditions of illuminating and viewing the satin-finish and ribbed-finish standards issued by the Textile Color Card Association of the United States with a view to obtaining designations for the characteristic colors of these standards. Some textiles require more than one angle of view or illumination to bring out the characteristic color or colors; changeable silks are extreme examples of such textiles.

2. FOR METALLIC SURFACES

The characteristic color of a metallic surface is obtained from the specularly reflected light. Proceed, therefore, as in section VI or VII, but obtain, in addition, the hue name for the specularly reflected light. These two names may possibly yield a useful designation of the color of the metallic surface, but they will not correspond well with common usage which involves color terms that apply characteristically to metallic appearance (silver, brass, gold, copper, and so on).

3. FOR MATERIALS WHICH TRANSMIT BUT DO NOT SCATTER LIGHT

Samples of liquids, glass and gelatins are often encountered which transmit but do not scatter light. The transmitted light, of course, yields the characteristic color of such materials, but the color designations of the present system are not all applicable to such materials; for example, *white* is a color designation applying characteristically to materials which *do* scatter light. The Council has undertaken an extension of the present system of color designations to colors of materials viewed by transmitted light. It is planned to compare the appearance of a white surface illuminated by daylight and viewed through a prescribed thickness of the material with the appearance of the Munsell standards similarly illuminated but viewed directly. About 25 of the 319 designations will have to be changed; for example, *white* will have to be supplanted by some such term as *colorless* or *clear*. Similarly, *pinkish white* would have to be changed to some term like *light pinkish* (*color*); and *light gray*, perhaps, to *light smoky* (or to *light smoke* if the noun form is desired). In this way it is hoped that a system in fair accord with common usage will result, but the color terms (*amber*, *claret* and so on) characteristic of materials viewed by transmitted light will not be a part of the system.

4. FOR TRANSLUCENT MATERIALS

Translucent materials both transmit some light and scatter some. The characteristic color for some of such materials is obtained by reflected light, and for others by transmitted light; if in doubt, get both.

IX. SUMMARY

A method of designating the colors of powdered drugs and chemicals and crude whole drugs has been described; this method has, furthermore, been devised with the thought of its applicability to colors of opaque, nonmetallic surfaces generally. Suggestions for its extension to the color designations of transparent and translucent media have also been given. This method is dedicated to everyone who has found it difficult to make his color descriptions intelligible, in the hope that it will eventually be elaborated into a successful system of color designation for the general use of science, art, and industry.

X. REFERENCES

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WASHINGTON, July 14, 1939.

HUE, SATURATION, AND LIGHTNESS OF SURFACE COLORS WITH CHROMATIC ILLUMINATION

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The problem of chromatic adaptation is restated so as to be susceptible of explicit solution. This is the classic paper on the subject. It has not yet been supplanted or equalled in breadth of coverage of the subject. The terminology adopted for use in this paper, after a thorough survey of the prior literature, is still current and standard. The "uniform-chromaticity-scale Maxwell triangle" has a spectrum locus very similar in shape to that of the CIE 1960 u,v chromaticity diagram, which was derived from it.

Hue Saturation and Lightness of Surface Colors with Chromatic Illumination

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The visual mechanism of a normal observer is so constructed that objects keep nearly their daylight colors even when the illuminant departs markedly from average daylight. The processes by means of which the observer adapts to the illuminant or discounts most of the effect of a nondaylight illuminant are complicated; they are known to be partly retinal and partly cortical. By taking into account the various fragments of both qualitative and quantitative information to be found in the literature, relations have been formulated by means of which it is possible to compute approximately the hue, saturation, and lightness (tint, value) of a surface color from the tristimulus specifications of the light reflected from the surface and of the light reflected from the background against which it is viewed. Preliminary observations of 15 surfaces under each of 5 different illuminants have demonstrated the adequacy of the formulation and have led to an approximate evaluation of the constants appearing in it. More detailed and extensive observations have been carried out in the psychological laboratories of Bryn Mawr College and these observations have resulted in an improved formulation.

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I. Introduction

FOR many years it was the view of psychologists that the chromatic adapting power of the eye is unlimited. Constant color stimuli, they said, when acting for a long period on the retina, gradually become less effective and finally fail to produce sensations of chromatic color. When, for example, chromatically tinted glasses are continuously worn, external objects are seen sooner or later nearly in their natural, daylight colors.

This view is a correct one only when applied to moderate departures from daylight in the spectral distribution of energy of the illuminant, but it will readily be understood that there are illuminants (for example, those having all of their energy confined to a narrow wave-length region) by which there is no basis except memory through which all objects can possibly assume their daylight colors. For such illuminants, or for vision by an observer wearing highly selective glasses, the old view is, therefore, quite wrong [27],* and it becomes a matter of some interest, since we know that the eye cannot possibly detect color differences in the absence of a corre-

* Numbers in brackets (sometimes followed by a page number) indicate references in bibliography, Section XII.

sponding stimulus difference, to discover what degree of departure from daylight can be compensated for by chromatic adaptation.

The problem stated in this way was proposed by Professor Harry Helson, Bryn Mawr College, in 1932, and in the course of planning experimental work for its solution, to be carried out jointly at the National Bureau of Standards and at Bryn Mawr College, it was found expedient to restate the problem in the following somewhat broader way: Given the radiant energy of the illuminant as a function of wave-length, and the spectral apparent reflectances of the surfaces in the field of view, it is required to compute the hue, saturation, and lightness of the surface colors.

It may be noted that the latter problem includes the former and is more easily susceptible of explicit solution. Thus, a slight change in illuminant changes the appearance of objects slightly, and the question of how much selectivity of illuminant can be tolerated must be answered in terms of how much change in surface color is to be tolerated. There would also be some commercial interest in the solution of this problem if it could be done cheaply enough for a given sample; thus, a frequent question to be answered by the textile salesman is, "Does this piece of goods change color in artificial light?" Another commercially important question is, "These two color samples harmonize in daylight, but do they go well together in artificial light?" The general problem stated above includes the first of these as a sub-problem and it bears importantly on the other two. The general problem, furthermore, includes that of description of chromaticity differences between large chromatic samples viewed in daylight as by an inspector.

It is the purpose of the present paper to summarize the general principles which have been found in the literature, to present formulas for hue, saturation, and lightness which embody these principles, to describe experiments designed to test in a preliminary way the adequacy of the formulas, and to show the agreement between the formulas and preliminary experiment.

II. Definitions

1. Psychological terms

Aperture color (*Lochfarbe* [7, 48], reduced color [60, p. 42], film color [58, 60], *Flächenfarbe* [60, p. 23; 48])—color such as that experienced as filling a hole in a screen (hence, usually as a small portion of the visual field); it may be either near the plane of the screen or indefinitely far behind it; it is neither concentrated in a single plane nor spread throughout a definite volume; it is nonlocated in depth. Aperture color has the attributes: hue, saturation, and brightness.

Volume color (bulky color [58], body color [92])—color experienced as a property of a volume or bulk.

Surface color [58] (*Oberflächenfarbe* [7, p. 419; 16; 48; 92])*—color experienced as a property of a surface. Surface colors are hard, resist the gaze, take the plane of the object surface, and have its texture [60, p. 23]. We deal here with three of the attributes of surface color: hue, saturation, and lightness.

Hue [87, p. 534]—that attribute of certain colors which permits them to be classed as red, yellow, green, blue and their intermediates.

Chromatic colors [87, p. 535]—colors possessing hue.

Achromatic colors [87, p. 535]—colors not possessing hue. There is, for example, a series of achromatic aperture colors ranging between very dim and very bright; and, a series of achromatic surface colors (grays) ranging between black and white.

Saturation—the attribute of any chromatic color which determines the degree of its difference from the achromatic color most closely resembling it.

Brightness (insistence [48, p. 462; 60, p. 38], *Helligkeit* [7])—the attribute of any aperture color which permits it to be classed as equivalent to some member of the series of achromatic aperture colors ranging between *very dim* and *very bright*.

Lightness [37, p. 213] (brilliance [87, p. 534], brightness [60], whiteness [52, p. 243], *Weisslichkeit* [16, p. 612; 7])†—the attribute of any

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* Wood [92] uses the term, color, only in the restricted sense of chromatic color.

† Bocksch [7, p. 430] uses the terms, *Helligkeit* and *Weisslichkeit*, in accord with our terms, brightness and lightness, respectively; he makes a special point, however, of his belief that the distinction is not connected with that between surface color and aperture color, but applies to all color impressions regardless of mode of appearance. We have adopted the view given by Helmholtz [30, p. 130] as characteristic of ordinary speech; this view is supported by Fiedler's experiments [15], and also agrees with Mintz [63] who considers Fiedler's experiments to be inconclusive

surface color which permits it to be classed as equivalent to some member of the series of grays ranging between *black* and *white.*

2. Psychophysical terms

Luminous reflectance (hereinafter simply reflectance)—the ratio of the quantity of light reflected from a surface to that incident on it.

Luminance [37] (brightness [33, p. 6])—the quotient of the luminous intensity of a surface measured in a given direction by the area of this surface projected on a plane perpendicular to the direction considered.

Apparent reflectance, A, [61, p. 33]—the reflectance which a perfectly diffusing surface, under the same illuminating and viewing conditions, would have to possess in order to yield the same luminance as the surface in question. In the present work Munsell samples were used, and these are so nearly perfect diffusers that the illuminating and viewing conditions need not be carefully controlled provided unidirectional illumination with viewing along the direction of regular reflection be avoided as has always been done in these experiments.

Tristimulus specification [39, p. 359]—specification of the stimulus for an aperture color consisting of the amounts of three primary stimuli required to produce a color match; this specification may be computed from the spectral energy distribution of the stimulus [23, 39] provided the visual characteristics of the observer be known. In the present work the O. S. A. standard observer [18, 87, p. 549] was used, and the coordinate system was the uniform-chromaticity-scale (UCS) triangle [40].

Trilinear coordinates (r,g,b)—the amounts of the three primary stimuli expressed as fractions of their total. In the present work a tristimulus specification is given in the form of the relative luminance of the stimulus (proportional to the apparent reflectance of the sample since we deal here only with groups of equally illuminated samples in the same plane viewed by reflected light) combined with two of the three trilinear coordinates, it being unnecessary to give the third because, taken together, the three sum to unity by definition.

3. Special symbols

$\bar{A}$ ≡ arithmetical average apparent reflectance of the samples in the field of view.

$\bar{A}_{\log}$ ≡ logarithmic average of the same quantities.

A_0 ≡ apparent reflectance of the background against which the samples are viewed.

A_f ≡ apparent reflectance of field, the average apparent reflectance of the samples and background weighted according to proximity in past time [30, p. 268] and in space [56, p. 238] to the central part of the visual field.*

A' ≡ adaptation reflectance, the apparent reflectance at which a sample of trilinear coordinates equal to those of the field (r_f,g_f,b_f) appears most nearly achromatic, a function of the whole viewing situation.

I ≡ illuminance [37, p. 208] of sample plane in footcandles.

(r_0,g_0,b_0) ≡ trilinear coordinates of the illuminant.

(r_f,g_f,b_f) ≡ trilinear coordinates of the field, the parts of the field being weighted in taking an average as in the definition of A_f.

(r_n,g_n,b_n) ≡ trilinear coordinates of the stimulus for an achromatic color; these coordinates are found to be functions of the illuminant, of the particular sample fixated, and of the samples near the fixation point in the immediate past.

$D \equiv [(r-r_n)^2+(g-g_n)^2+(b-b_n)^2]^{\frac{1}{2}}$, distance on the UCS triangle between sample point (r,g,b) and achromatic point (r_n,g_n,b_n).

$D_f \equiv [(r_f-0.44)^2+(g_f-0.47)^2+(b_f-0.09)^2]^{\frac{1}{2}}$, distance from daylight point (0.44, 0.47, 0.09) on the UCS triangle to the field point (r_f,g_f,b_f). This distance has been found to be a fairly good index of the amount of chromatic adaptation induced by a 5-minute exposure of the observer to the field; it enters into the formulas for the trilinear coordinates of the achromatic point.

H ≡ hue estimated by the observer on the following eight-point scale given to each observer as a part of his instructions: red, yellow-red, yellow, yellow-green, green, blue-green, blue, red-blue.† Some of the observers on their own initia-

* Wright [94, p. 7] makes use of a parameter, the integrated light from the scene, in his discussion of adaptation; this variable is proportional to IA_f.

† This scale has also been suggested by Tschermak [91, p. 335].

tive developed a sixteen-point hue-scale formed by the introduction of intermediate hue-names such as bluish red, yellowish red, reddish yellow, and so on. Computed values of hue are given on the eight-point scale; summaries of observed hues, on the sixteen-point scale. The hue notation for an achromatic color is N, meaning "no hue."

$S \equiv$ saturation estimated by the observer on an eleven-point scale running from zero for an achromatic color (black, gray, or white) up to 10 for the most saturated of the 15 daylight surface colors presented.

$L \equiv$ lightness estimated by the observer on an eleven-point scale running from zero for black up to 10 for white, the intermediate values being used to designate a series of grays with uniform visual spacing.

$L' \equiv$ lightness expressed according to a formula derived from one deduced on theoretical grounds and experimentally verified by Adams and Cobb [2], with constants adjusted to make the zero-point of the scale correspond to adaptation reflectance, A', thus: $L' \equiv 10A/(A+A_f)-3.0$, from which it follows that $A'=(3/7)A_f$. As may be seen from the definition of adaptation reflectance, this choice of constants has to do with specification of the sample of trilinear coordinates (r_f,g_f,b_f) which appears most nearly achromatic, and it states that any such sample of apparent reflectance 3/7 that of the field will appear more nearly achromatic than any other such sample.

III. Statement and Explanation of Formulas

1. Assumptions

The formulation is intended to apply to the colors of a group of matte, opaque surfaces viewed against a background in strong uniform illumination; it is based upon the following assumptions:

(*a*) *Color matching.*—If the aperture colors derived from two matte, opaque surfaces are identical (in hue, brightness and saturation) and the surfaces are compared under the same illuminating and viewing conditions, it is assumed that the surface colors will be identical; and, conversely, if the aperture colors are different, the surface colors will be different. This assumption underlies all colorimetry of reflecting surfaces.

(*b*) *Hue.*—It is assumed that the hue of an aperture color is determined by the direction of the straight line in the triangular mixture diagram connecting the point (r,g,b) representing the aperture color and the point (r_n,g_n,b_n) representing an achromatic aperture color of the same brightness. This is the usual interpretation of trilinear-coordinate differences; and by a further assumption, often unwarranted, that a single point can be chosen to represent achromatic colors this assumption has led to the specification of hue by means of dominant wave-length [21; 23, p. 11; 73].

(*c*) *Saturation.*—It is assumed that the length, D, of this line indicates the saturation of the aperture color. This assumption is based upon the spacing of color stimuli yielded by the uniform-chromaticity-scale Maxwell triangle; it is regarded as a first approximation to the truth. The validity of the assumption is limited by possible failure of the UCS system to yield, as intended, substantially a uniform spacing for colors separated by small chromaticity-differences, by possible failure of chromaticity-differences to be arithmetically additive, and by possible important dependence of saturation upon brightness.

(*d*) *Chromatic adaptation.*—It is assumed that hue and saturation of a surface color are equal, respectively, to the hue and saturation of the aperture color derived from it for the same adaptive condition (retinal and cortical) of the observer. This assumption is a usual one among colorimetrists. If the adaptive state is not held constant, this assumption is known to be inadmissible.

(*e*) *Lightness.*—It is assumed that the lightness of a surface color is a function of the apparent reflectance of the surface, A, and of the apparent reflectance of the field, A_f. (A possible slight dependence of lightness upon illuminance is neglected [7, p. 439; 86].) This assumption follows directly from the work of Adams and Cobb [2] by virtue of assumption (*a*); it has been stated explicitly also by Kirschmann [49, p. 363], v. Kries [56, p. 239], Bocksch [7, p. 343], A. Kohlrausch [53, p. 1501], and Kardos [44, p. 188], and is a corollary of the "approximate color constancy of visual objects" [16, 29, 31, 35, 47, 57, 60]. Further references to the principle that object color depends upon the ratios of light

Table I.

H for $r-r_n$ Greater than Zero	$(r-r_n)/(g-g_n)$	H for $r-r_n$ Less than Zero
Red-blue	0.0 to −0.38	Green-yellow
Red	−0.38 to −0.98	Green
Yellow-red	−0.98 to −2.3	Blue-green
Yellow	{−2.3 to −∞; ∞ to 2.0}	Blue
Green-yellow	2.0 to 0.0	Red-blue

reflected from the various parts of the field rather than on the absolute amounts are cited by Helson [24, p. 450]. Bühler [10] stated essentially the same principle by making use of the concept, brightness of illuminated space (*Luftlicht*) instead of apparent reflectance of the field. Katz [48, p. 462] considered that the total brightness (insistence) of the visual field provides the crucial clue to illumination. Pikler [72] concluded from a series of experiments that light penetrating the translucent parts of the eye (iris, cornea) and falling diffusely on the retina supplies crucial clues both to amount and direction of the illumination relative to which lightness of visual objects is determined. Both these views [48, 72] are consistent with this assumption (*e*).

2. Formulas

(*a*) *Hue.*—Hue in accord with assumption (*b*) is indicated by the direction of the line on the UCS mixture diagram connecting the point representing the aperture color (r,g,b) with that representing an achromatic aperture color of the same brightness (r_n,g_n,b_n), and in accord with assumption (*d*) this hue applies also to the surface color corresponding to the aperture color. The direction of this line is expressed by giving its slope, $(r-r_n)/(g-g_n)$, and the algebraic sign of the difference, $(r-r_n)$. The relation is given symbolically in Eq. (1), and Table I supplies the numerical data.

$$H=f[(r-r_n)/(g-g_n), \quad sgn(r-r_n)]. \qquad (1)$$

The adjustment of these limiting values of slopes is based somewhat upon an attempt to fit our preliminary data. The angular extent assigned to each hue name is nearly a constant as would be expected on a mixture diagram of uniform chromaticity-spacing, and it will be noted that the direction on the diagram assigned to red is the exact opposite of that assigned to green; similarly yellow is taken as the exact opposite of blue. It is of interest to discover how this definition of hue agrees with determinations of the stimuli for unitary and intermediate hues. Table II gives the wave-lengths in mμ for the spectrum stimuli for colors of these hues found by various investigators or reported by various authorities [11, 12, 14, 80].* The corresponding values by Eq. (1) were found by taking standard I. C. I. illuminant C [23, 39], representative of average daylight, as the stimulus for an achromatic color; that is, we have taken for this computation: $r_n=0.44$, and $g_n=0.47$. It will be noted that Eq. (1) agrees well with experimentally found stimuli for the unitary hues, yellow, green, and blue, and for their intermediates, but specifies a stimulus for the boundary between red and yellow-red which experiment indicates actually to yield a somewhat bluer hue. This is a result of the simplifying assumption that red and green are exact opposites, and suggests that an assignment of hues without this simplifying restriction might give better agreement.

(*b*) *Saturation.*—In accord with assumption (*c*) saturation, S, is taken proportional to the length, D, of the straight line connecting the point representing the aperture color (r,g,b) with that representing an achromatic aperture color of the same brightness (r_n,g_n,b_n), and in accord with assumption (*d*) this saturation applies also to the surface color corresponding to the aperture color.

$$S=50\,D=50[(r-r_n)^2+(g-g_n)^2+(b-b_n)^2]^{\frac{1}{2}}. \qquad (2)$$

The constant, 50, is adjusted so as to accord approximately with the scale used by the observers for expressing their estimates of saturation.

(*c*) *Achromatic point.*—It will be noted that the formulations for both hue, H, and saturation, S (Eqs. (1) and (2)), are incomplete because the trilinear coordinates of the stimulus for an achromatic or neutral color (r_n,g_n,b_n) are as yet unrelated to the observing situation. By means of these trilinear coordinates, the chromatic adapta-

* For other references to this literature consult [67] and Dimmick and Hubbard [11].

tion (retinal and cortical) of the observer is to be defined. When a daylight-adapted observer enters a room illuminated by light differing chromatically from daylight to an important extent, a rapid chromatic adaptation takes place which is fairly complete in about 5 minutes. A large part of this chromatic adaptation may occur immediately, that is, the observer may immediately react to the surfaces as if they were illuminated by chromatic light, and there is an immediate reverse change either if the observer begins to react again to the surfaces as if illuminated by daylight, or if the samples lose their surface character and are seen as aperture colors [60, p. 10].* This fluctuation occurs with a facility and time character roughly equal to those with which ambiguous pictures (such as the reversible staircase) fluctuate; and, indeed, the situation with which the observer is presented is more than ambiguous, because there are usually not two but many equally valid responses possible depending on the allocation of the chromatic component of the aperture color partly to the surface color and partly to illumina-

* Many observers can shift colors from the surface mode of appearance to the aperture mode at will [60, p. 45]. Thus A. Gelb [16, p. 599] writes, "In this respect it must be noted particularly that under certain conditions *even without the reduction screen* the dark gray paper near the window and the white wall further back can be seen in the way that corresponds with the relations between the physical stimuli. (The same thing holds for other examples.) Of course, to do this requires a characteristic inhibition, more foreign to reality, which we may designate as the 'attitude of pure optics'; we must consider the color of the paper and that of the wall 'critically' (Katz)—contrary to the natural objective way, that is, as Köhler has opportunely remarked, we must detach ourselves from the objectivity of the surface of the objects and degrade them to a kind of light area. Many people achieve this attitude relatively easily, particularly those who have an aptitude for painting; indeed they must use a dark gray pigment to represent a weakly illuminated white (such as the white-painted wall far back from the window). Whoever achieves the mode of seeing referred to, sees, to be sure, the color of the paper and that of the wall in accord with the relative illuminations; however, he does not see 'true objects' (paper, wall) of particular definite colors in different illuminations, but 'color areas' . . . ."

TABLE II. *Spectrum stimuli for various hues.*

	Wave-length in Millimicrons									
Authority	Red	Red to Yellow-red	Yellow-red	Yellow-red to Yellow	Yellow	Green-yellow	Green	Blue-green	Blue	Blue to Red-blue
Helmholtz	687	656					527			431
Bezold		656		589	578	558	532	502	468	432
Donders					582		535		485	
Hess	*PR**				574		495		471	
Hering	*PR**				577		505		470	
Rood	700	621	597	588	581		527	502	473	438
Voeste					577		498		476	
Ladd-Franklin	*PR**				576		505		470	
v. Kries					574		503			
Westphal	*PR**				575		506		479	
Dreher					575		509		477	
Ridgway	644				577		520		473	
Goldytsch	*PR**								467	
Bradley	656		608		579		514		469	421
Goldmann					568		504		468	
Priest	680				583		515		475	
Brückner					578		498		471	
Schubert					574		500		467	
Maerz & Paul	644				585		521		452	
Purdy					576		504		476	
Drever	650				568		508		471	
Weld		622		597		577		492		456
Ornstein			596		575	562	515	492	475	
Terpstra		604	592	585	577	563	510	487	470	440
Verbeek		605	598	587	580	569	530	496		
Dimmick	494*C*†				582		515		475	
Experimental mean			598	589	577	566	512	495	472	436
Eq. (1)	521*C*†	499*C*†	601	583	575	562	521	490	476	461

* *PR* means that the indicated hue is more purple than extreme spectrum red.
† Dominant wave-length of the complementary is indicated by the letter, *C*.

tion color [60, p. 39] in various proportions.* The instantaneous changes are usually considered to be cortical [7, p. 433; 10, p. 115; 35]; some have considered the initial rapid change also to be wholly cortical, but there is considerable evidence for instantaneous retinal adaptation of electrical origin [82, 94, 95]; so it is probable that neural activity, both retinal and cortical, is involved in an essential way [4, 55, 64, p. 46]. The slower progressive changes usually coming to completion in about 5 minutes are thought to be retinal. During this time interval the values of the trilinear coordinates (r_n,g_n,b_n) specifying the chromatic adaptation of the observer undergo corresponding changes, and no attempt has been made in the present work to separate the retinal from the cortical components, or to trace quantitatively either the course of retinal adaptation in time, as has been done by Wright [93], or its definition in terms of these trilinear coordinates (r_n,g_n,b_n). During the period following that of rapid change, an approach to equilibrium often occurs, and it has been found successful to a considerable degree to define this equilibrium state by means of Eq. (3):

$$\left.\begin{aligned} r_n &= r_f - D_f[0.1L'(r_f-0.360) \\ &\qquad -0.018b_fA_f(L')^2\log_{10}2000I] \\ g_n &= g_f - D_f[0.1L'(g_f-0.300) \\ &\qquad -0.030] \end{aligned}\right\}. \quad (3)$$

Formula 3 embodies several ideas that have been widely accepted for many years. When, as was the case in our preliminary experiments, the visual field is filled with objects which, taken together, are spectrally nonselective so that we may write: $r_f=r_0$ and $g_f=g_0$, these formulas could yield strict "color constancy of visual objects" [31] only by the first terms taken alone; the longer second terms (commencing with D_f) may therefore be considered as correction terms indicating necessary departure from strict "color constancy." These correction terms describe variations of the achromatic point (r_n,g_n,b_n) from the field point (r_f,g_f,b_f). Such variations were known as early as 1860, for we read in Helmholtz [30, p. 274], "When a particular color is made dominant in the visual field, a paler shade of the same hue will look white to us, and real white will seem to be the complementary color. Thus the idea of what we mean by white is altered in this case." This is an approximate statement of the facts and would lead to formulas for the achromatic point of the form:

$$\left.\begin{aligned} r_n &= r_f - k(r_f-r_w) \\ g_n &= g_f - k(g_f-g_w) \end{aligned}\right\}, \quad (3a)$$

where (r_w,g_w,b_w) are trilinear coordinates of the original "white" point. Formula (3a) describes excursions of the achromatic point along the straight line connecting the field point with the original "white" point depending on the value of the constant, k, indicating degree of departure from adaptation to the field. These formulas also cover a conclusion by Kravkov [54] concerning the direction of surface-color transformation from a chromatic illuminant. Eq. (3) includes four separate elaborations or corrections of this idea.

The first elaboration is the introduction of the term, L', which extends Eq. (3a) to light and dark parts of the visual field according to the principle that nonselective samples of apparent reflectance considerably greater than the adaptation reflectance, A', take the hue of the illuminant, while those of apparent reflectance considerably less take the hue of the after-image complementary [24]. Helmholtz was familiar with these facts also; he said [30, p. 275], "When we look steadily through a red glass, soon all perfectly dark objects appear to be a vivid green. Thus, alongside the red its complementary color becomes visible. . . ." This account accords with our results except that the after-image complementary is greenish blue instead of green. Troland [88] and Karwoski [45] also called attention to the same phenomenon for stationary fixation point under the name "dimming effect." Hering [13] had previously proposed the dimming technique as a means of differentiating achromatic aperture colors from those of low saturation.

The discrepancy between mixture complementary and after-image complementary gives rise to the second elaboration of Eq. (3a). The after-image complementary departs from the mixture complementary by having more red-blue

* For example, a surface in red light may appear sometimes red and sometimes white or pink according to Bocksch [7, p. 371].

[19; 90; 91, p. 474; 93; 65];* this departure is indicated first by the introduction of the term, $0.030D_f$, as an addition to g_n, second by the use of the trilinear coordinates (0.360, 0.300) different from those (r_w,g_w) suggested by Eq. (3a), and third by the $(L')^2$ term in the expression for r_n. It was found necessary to introduce all three changes to obtain agreement with our preliminary data.

A third elaboration was introduced to take account of the greater prevalence of chromatic after-image or adaptation effects for certain regions of the Maxwell triangle. Thus, for the field point (r_f,g_f,b_f) near the daylight point (0.44, 0.47, 0.09), no chromatic responses not accounted for by the spectral selectivity of the samples were found; hence, the term, D_f, was introduced. Failure of Dittler and Satake [13, p. 245] to find a stimulus for an achromatic color which would also yield on dimming an achromatic color suggests that the correct term is $(k+D_f)$; but our preliminary experiments did not seem to require the introduction of the small constant term, k.

It is of interest to note that Katz [47; 60, p. 37] originally took "normal illumination" to be that found "in the open air under a lightly clouded sky," which has closely the trilinear coordinates (0.44, 0.47, 0.09) relative to which D_f is defined.

The fourth elaboration has to do with an intensification of after-image effects for the blue illuminant for high illuminances, I, and for high field reflectances, A_f. For this reason the term involving the product, $b_fA_f \log_{10} 2000I$, was introduced. This term depends upon information obtained from the extensive data collected by Helson and Jeffers [25, 26] which were made available in advance of their publication through the courtesy of Professor Helson. Our preliminary results refer to a single illuminance for each chromatic illuminant; so they do not afford a check upon this term, although they are consistent with it. If this term were omitted, Eq. (3) would define two field points on the Maxwell triangle for which hue and saturation are independent of lightness, L', instead of one field point (0.44, 0.47, 0.09). The second field point (0.36, 0.30, 0.34) was tentatively named the "blue, hueless point," blue, because it undoubtedly refers to a blue illuminant, and hueless, because the equations suggested that a nonselective sample under an illuminant plotting at that point would generally, independent of lightness, appear gray or hueless. Attempts by Helson to set up a "blue, hueless" illuminant, however, were unsuccessful; the illuminants tried gave after-images of feeble rather than zero chromatic component, and led to the $(L')^2$ term in Eq. (3). This term adds a component of greenish blue to the computed color both for bright and for dim parts of the visual field, the amount of which is proportional to $D_fb_fA_f$. The effect of this term combined with the term, $0.030D_f$, in the expression for g_n is to make surface colors by computation tend toward red-blue for bright visual fields having D_f and b_f greater than zero.

Equation (3) is therefore consistent with much of the data in the literature of visual psychophysics, with the preliminary data to be discussed presently, and with recent experimental data by Helson particularly adapted for testing out certain aspects of the formulation. But there is no basis for believing that it is the only formula meeting these conditions.

(*d*) *Lightness.*—Lightness in accord with assumption (*e*) is computed from the apparent reflectance, A, of the surface in question and from the apparent reflectance of the field, A_f. The Adams and Cobb [2] formula has already been used to define lightness, L', and this is also used for comparison with estimates of our observers after adjustment of the end points of the scale to make zero correspond to the lower limit of apparent reflectance shown by our samples ($A=0.03$) and to make 10 correspond to the upper limit ($A=1.00$); these choices were made in order to give agreement with our observer's estimates of lightness near the ends of the lightness scale, black and white, and they lead to the formula:

$$L=10(L'-L'_{A=0.03})/(L'_{A=1.00}-L'_{A=0.03}). \quad (4)$$

By substituting the definition of L' in Eq. (4), the explicit function of A and A_f quoted by Helson [24, p. 453] is found:

$$L=\frac{10(A-0.03)(A_f+1.00)}{(1.00-0.03)(A_f+A)}. \quad (4a)$$

* Tschermak [91, p. 474] gives eight other references.

(*e*) *Observing situations.—Case 1.* If an observer adapted to daylight enter an enclosure filled with an illumination chromatically different from daylight and is confronted with an array of samples with instructions to report immediately on the aperture color derived from one of them, we would expect to compute the resulting estimates of hue and saturation from Eqs. (1) and (2), in which the trilinear coordinates of the achromatic point (r_n, g_n, b_n) would be set equal to those for the daylight point (0.44, 0.47, 0.09). This is the classical case for which hue can be correlated with dominant wave-length computed relative to the daylight point; the observing situation to which it applies is often very transient, sometimes lasting less than a second.

Case 2. If, however, the observer were instructed to report on a surface color as objectively as possible (that is, with an attempt to discount the illuminant color) his report, except for minor effects from simultaneous contrast, would be expected to correspond with Eqs. (1), (2) and (3) with trilinear coordinates of the achromatic point (r_n, g_n, b_n) taken equal to those for the illuminant point (r_0, g_0, b_0). The period during which such computed hues and saturations could apply would be limited to a few seconds (5 or 10), this duration being usually too small for the development of a chromatic adaptation sufficient to influence the surface color by projection of a negative after-image of the background upon the sample [30, p. 267]. To set the achromatic point equal to the illuminant point was suggested by Priest [74] who said in support of his suggestion, "It is a well-known and significant fact that nonselective diffusely reflecting surfaces are constantly *perceived* as 'white' (or 'gray') for a wide range of variation in the spectral distribution of the illuminant." This suggestion is also implied and justified by Koffka's concept of "shift of level" [51; 52, p. 255]. By arguments based upon chromatic adaptation, which refer to field point instead of illuminant point, a similar suggestion was put forward for longer durations by Ives in 1912 [34], and by Noteboom [71] and Ströble [85] in 1934. This suggestion, intended to apply to extended durations, is allied to the "coefficient law" formulated by v. Kries [30, p. 441; 56, p. 211], and was also implied by Troland [89, p. 182] when he wrote, ". . . it appears that this process (chromatic adaptation) changes the proportions of ordinates between the color-mixture elementaries, so that we must compute the colors on the basis of a distorted triangle."

Case 3. If the observer remain in the enclosure looking at the array of samples against a background for five minutes or more, and if he be asked to report upon the surface color of a sample from its appearance by momentary fixation, his report would be expected to correspond with those for the achromatic point defined by Eq. (3) in which the apparent reflectance of the field, A_f, is an average of the apparent reflectances of the samples and of the background, A_0, against which the samples are viewed. Since about one-half of the sample plane in our preliminary experiments was covered with samples, the other half consisting of the background, we have computed as representative of observation by momentary fixation preceded by moving fixation point:

$$A_f = (A_0 + \bar{A})/2. \qquad (5a)$$

This formulation applies only to momentary observation preceded by a fast-moving fixation point because the surface color by Eqs. (1), (2), (3), and (4) is identified with the hue, saturation and lightness produced by projecting an after-image of the field as a whole (A_f, r_f, g_f, b_f) upon the sample. The observer can easily tell when he fails to comply with these conditions. If his fixation point pauses too long preceding fixation of the sample, he gets a patterned after-image, which by its pattern seems not to belong to the sample [30, p. 269]. If he fixates the sample, itself, too long, the after-image of the field as a whole will die away causing him to change his report; or, perhaps, to note that the first report applies only to the edges of the sample where an after-image of the background has been renewed by small movements of the fixation point within the sample [30, pp. 268, 273]. The color of the center of the sample is dealt with in Case 4.

Case 4. If in Case 3 the observer fixate the sample to be reported upon for more than a second or two,* we would expect the apparent reflectance, A, of the sample, itself, to enter into

* It seems likely from Newhall's work [69] that blinkless fixation for as little as 5 seconds can introduce a significant change.

Table III. *Adaptation reflectance, A′.*

	Expression for A'	Adaptation Reflectance for $\bar{A}=0.33$, $\bar{A}_{\log}=0.24$			
		$A_0=0.03$	$A_0=0.10$	$A_0=0.23$	$A_0=0.80$
Present preliminary experiment, selective samples	$0.6(A_0\bar{A}_{\log})^{\frac{1}{2}}$ $(3/14)(A_0+\bar{A})$ [Eq. (5a)]	0.051 .077	0.093 .092	0.141 .120	0.263 .242
Helson and Jeffers [26], (selective samples)	$0.32(A_0\bar{A}_{\log})^{\frac{1}{2}}$ $(3/22)(A_0+\bar{A})$ [Eq. (5c)]	.027 .049	.050 .059	.075 .076	.140 .154
Helson [24], nonselective samples	$0.8(A_0^3\bar{A}_{\log})^{\frac{1}{4}}$ $(5/28)(3A_0+\bar{A})$	.041 .075	.100 .112	.186 .182	.474 .487

the average for A_f, the longer the fixation, the greater the weight, n, of A, thus:

$$A_f=(nA+A_0+\bar{A})/(n+2). \qquad (5)$$

It will be noted, of course, that Eq. (5a) is the special case of Eq. (5) obtained by setting $n=0$. The special case corresponding to fixation of the sample until equilibrium is reached will be found by setting $n=\infty$; this gives:

$$A_f=A. \qquad (5b)$$

From the definition of L', it will be seen that Eq. (5b) leads, except for $A=0$, to a value of L' constant at $+2$, which, in turn, through Eqs. (1), (2), and (3), leads to hues and saturations characteristic of bright parts of the visual field, that is, to hues and saturations in accord with the principle, discussed by Helson [24] that bright parts arising from nonselective samples take the hue of the illuminant. This result accords with experiments by Helson and Judd [27] on the appearance of a uniform visual field after prolonged adaptation to it; only in a few exceptional cases did the original hue disappear after minutes-long adaptation, and then only in the sense of appearing and disappearing in rhythmic alternation.

Although our observers were instructed to report upon the appearance of the samples by momentary fixation (Case 3), it is of interest to attempt a prediction of the colors which they would obtain by using longer than momentary fixation contrary to instructions; in this way there may be discovered an explanation for some of the individual variations which cannot be explained by the variable allocation of chromatic component between the surface and the illuminant. For this purpose a value of n between zero and infinity must be chosen, and we have taken, from a preliminary examination of Helson's data as well as our own, n equal to 2, which gives:

$$A_f=(2A+A_0+\bar{A})/4. \qquad (5c)$$

It may not be immediately apparent that Eq. (5c) favors the appearance of the illuminant hue more than Eq. (5a); however, adaptation reflectance, A', by Eq. (5a) is $(3/14)(A_0+\bar{A})$ as may be seen from the definition of L', but by Eq. (5c) it is only $(3/22)(A_0+\bar{A})$; so Eq. (5c) gives a considerably larger range of apparent reflectance for which a nonselective sample would take on the illuminant hue. Table III shows a comparison between values of adaptation reflectance, A', computed by Eqs. (5a) and (5c), and those found from an examination (*a*) of our preliminary experimental results represented by the formula: $0.6(A_0\bar{A}_{\log})^{\frac{1}{2}}$, and (*b*) of the experimental results of Helson and Jeffers [26] on selective samples represented by the formula: $0.32(A_0\bar{A}_{\log})^{\frac{1}{2}}$. It will be noted from this table that Eq. (5a) yields values of A' in good agreement with the logarithmic formula previously found for our preliminary experimental results; the logarithmic formula found by Helson and Jeffers for adaptation reflectance in their experiments with the same selective samples does not, however, agree with Eq. (5a), but agrees well with Eq. (5c). This suggests that Helson's observers used longer fixation of sample than ours did. Values of adaptation reflectance found by Helson from his results with nonselective samples [24] represented by the formula: $0.8(A_0^3\bar{A}_{\log})^{\frac{1}{4}}$ are also given; these values are seen not to agree with either of the other two sets. Since, however, the background was more effective in determining adaptation reflectance in these experiments, as is evidenced by the weight 3 given to A_0 in the logarithmic mean, these values have been compared with $(5/28)(3A_0+\bar{A})$ and found to agree fairly well.* The previous view [41] that calculations of apparent reflectance of the field, A_f, should be made by logarithmic mean [24] has therefore been given up in the present treatment. The arithmetical mean is not only adequate, simpler, and applicable to fields involving completely dark areas ($A=0$), but it has also yielded improved agreement with lightness estimates of our observers, as will be seen presently. The improved values of adaptation reflectance which have resulted chiefly from Helson's work [24] have been found to permit the elimination of a factor $(1+10b_f)$ previously considered to be a necessary part of Eq. (3) [41].

These four cases describing different observing situations have been worked out for the samples used in our preliminary experiments, and it is believed that other cases are of little interest at present. One factor, however, in the observing situation which has been shown under some circumstances to be the controlling one has yet to be mentioned; this is the presence in the visual field of objects whose daylight colors are known to the observer. It has been shown [3]

* The constant, 5/28, corresponds to a redefinition of L' as $10A/(A+A_f)-4.2$. Why these data require this minor change in definition of L' is not known.

TABLE IV. *Spectral apparent reflectances for the 15 Munsell samples studied.*

Wave-length in mμ	R 3/7	R 5/10	R 7/6	YR 4/5	Y 5/7	Y 7/8	G 3/4	G 5/7	G 7/7	B 3/5	B 5/6	B 7/4	P 3/6	P 5/6	PB 7/4
380	0.0680	0.084	0.120	0.0502	0.0630	0.0678	0.0684	0.0457	0.080	0.090	0.123	0.098	0.115	0.102	0.113
90	.0672	.123	.341	.0504	.0651	.0710	.0685	.0481	.153	.185	.351	.400	.247	.350	.420
400	.0655	.123	.364	.0507	.0655	.0729	.0687	.0521	.200	.188	.357	.515	.262	.387	.519
10	.0630	.122	.368	.0512	.0655	.0742	.0692	.0597	.232	.187	.355	.558	.264	.405	.551
20	.0601	.120	.366	.0517	.0654	.0754	.0700	.0708	.261	.187	.355	.579	.257	.410	.574
30	.0567	.118	.363	.0524	.0654	.0765	.0711	.0895	.292	.194	.367	.597	.244	.403	.596
40	.0534	.115	.359	.0532	.0655	.0779	.0730	.119	.325	.214	.407	.615	.226	.390	.619
450	.0503	.113	.353	.0542	.0659	.0795	.0770	.152	.362	.235	.455	.633	.207	.371	.637
60	.0476	.111	.346	.0553	.0665	.0819	.0825	.191	.409	.249	.504	.652	.186	.352	.645
70	.0454	.107	.340	.0566	.0675	.0855	.0890	.233	.463	.253	.545	.666	.167	.331	.641
80	.0437	.104	.334	.0582	.0695	.0922	.0955	.278	.517	.248	.555	.675	.149	.310	.630
90	.0426	.101	.329	.0602	.0734	.105	.101	.325	.562	.235	.535	.674	.131	.289	.617
500	.0420	.0980	.328	.0631	.0803	.141	.106	.371	.600	.215	.497	.665	.115	.269	.600
10	.0417	.0955	.330	.0673	.100	.210	.111	.413	.627	.194	.450	.649	.104	.252	.573
20	.0419	.0948	.332	.0741	.148	.307	.114	.435	.639	.169	.401	.628	.0965	.239	.537
30	.0429	.0963	.338	.0850	.206	.424	.114	.432	.632	.145	.351	.604	.0935	.231	.500
40	.0448	.100	.347	.100	.250	.523	.110	.399	.610	.123	.306	.578	.0925	.228	.470
550	.0482	.106	.359	.127	.276	.570	.105	.355	.582	.105	.269	.552	.0934	.228	.448
60	.0545	.118	.377	.158	.289	.588	.0985	.307	.550	.0920	.238	.527	.0959	.231	.430
70	.0648	.143	.410	.188	.294	.595	.0908	.256	.512	.0832	.212	.502	.100	.235	.415
80	.0818	.200	.482	.219	.296	.596	.0839	.212	.475	.0769	.195	.480	.107	.241	.402
90	.114	.293	.599	.245	.295	.594	.0780	.176	.442	.0721	.184	.460	.114	.250	.395
600	.153	.433	.688	.265	.292	.591	.0734	.148	.414	.0684	.175	.440	.119	.264	.390
10	.191	.563	.741	.276	.288	.588	.0702	.127	.390	.0657	.169	.422	.124	.279	.389
20	.221	.639	.763	.285	.283	.585	.0684	.111	.370	.0641	.164	.406	.129	.297	.391
30	.244	.680	.774	.290	.277	.581	.0677	.0992	.353	.0635	.161	.394	.134	.315	.398
40	.263	.701	.778	.295	.272	.578	.0685	.0895	.338	.0634	.159	.384	.140	.336	.408
650	.280	.712	.782	.298	.269	.575	.0705	.0832	.325	.0637	.157	.377	.148	.359	.423
60	.294	.721	.786	.302	.268	.573	.0737	.0789	.315	.0645	.158	.373	.158	.383	.441
70	.305	.728	.790	.305	.268	.575	.0777	.0755	.306	.0657	.160	.370	.170	.408	.462
80	.321	.735	.793	.308	.270	.575	.0824	.0730	.299	.0673	.165	.370	.190	.433	.487
90	.342	.743	.797	.312	.274	.578	.0871	.0713	.294	.0692	.175	.371	.218	.459	.515
700	.368	.750	.800	.315	.280	.582	.0918	.0702	.289	.0712	.191	.374	.253	.484	.546
10	.398	.756	.803	.317	.287	.586	.0965	.0695	.285	.0735	.214	.379	.296	.509	.580
20	.429	.763	.806	.320	.296	.593	.101	.0692	.283	.0762	.247	.384	.351	.529	.620

that the observer may report the "memory color" of the object instead of the color reported by an observer who is ignorant of the daylight color. No precautions were taken in our work to prevent the observers from learning the daylight colors of the samples or to prevent the subsequent identification of the samples by the shapes, which were fairly distinctive; but none of our preliminary results seems to be spurious from this cause. Although memory color may be an important factor influencing the color perceptions of an observer distracted by another task, we found that the surface color for our observing conditions was generated from the situation, itself, with ambiguities so limited as to exclude responses appropriate to memory color except for a few sample-illuminant-background combinations which chanced by the formulation also to yield approximately the daylight color.

IV. Illuminants and Samples Used

The samples used in the preliminary experiments were 15 Munsell papers whose apparent reflectances relative to magnesium oxide as functions of wave-length had been determined in 1926 for another purpose.* These samples were also used by Helson and Jeffers [25, 26]. Some measurements of apparent reflectance were made on each sample for nearly diffuse illumination and normal viewing, and some were made for 45-degree unidirectional illumination and normal viewing. There was no certain difference between the values obtained in these two ways, and the values adopted are based on both; they are given in Table IV.

The illuminants used were natural daylight from a south window and four chromatic illuminants (red, yellow, green, blue) obtained by illuminating the samples with a 500-watt, gas-filled lamp and viewing them through one or another of four Corning glass filters whose spectral transmissions were determined chiefly on a photoelectric spectrophotometer [17] and are given in Table V. A few of the lowest transmissions were checked by means of the König-Martens visual spectrophotometer [62]. The natural daylight supplied an illuminance of about 50 footcandles; the 500-watt lamp, about 700.

The tristimulus specifications of the light reflected by each of the 15 samples when it is illuminated in turn by each of the 5 illuminants were computed from the data of Tables IV and V. Standard I. C. I. illuminant *A* [23, 39] was used in these computations as representative of the 500-watt lamp; and Abbot-Priest sunlight [38, p. 525] was used as representative of natural south daylight. The O. S. A. observer and co-

Table V. *Spectral transmissions of the viewing filters.*

Wave-Length in mμ	Lantern Red	Lantern Yellow	Sextant Green	Lantern Blue
380		0.0002		0.15
90		.0002		.28
400		.0002	<0.0005	.365
10	<0.001*	.0002		.400
20		.0002	.0002	.404
30		.0002	.0003	.400
40	< .00005	.0002	.0004	.384
450		.0002	.0010	.360
60	< .00002	.0003	.0022	.318
70		.0003	.0056	.240
80	< .00002	.0004	.0133	.143
90		.0005	.0280	.070
500	< .00002	.0007	.0580	.033
10		.0011	.0916	.0100
20	< .000005	.0017	.109	.0038
30		.0030	.112	.0017
40	< .000005	.0055	.098	.0010
550		.025	.070	.0013
60	< .000005	.278	.042	.0021
70		.475	.0228	.0014
80	< .00001	.539	.0104	.0004
90	< .00001	.580	.0045	.0001
600	< .00005	.608	.0017	.0001
10	.0002	.630	.0004	.0001
20	.025	.646	< .0002	.0001
30	.273	.660		
40	.548	.668	< .0002	< .0001
650	.645	.672	< .0001	
60	.681	.675		< .0001
70	.699	.676	< .0001	
80	.707	.677		.0001
90	.710	.678	< .0001	.0006
700	.713	.674	< .0001	.0015
10	.714	.669		.0027
20	.714	.656	< .00005	.0025
30	.714	.650		.0023
40	.714	.641	< .00005	.0022
750	.712	.636		.0020
760	.711	.629	< .00005	.0018

* Numbers preceded by the "less-than" (<) sign indicate spectral transmissions greater than those possessed by the respective filters.

* N. B. S. test No. 46045. The samples are identified by Munsell notation in accord with the original Atlas of the Munsell Color System [66].

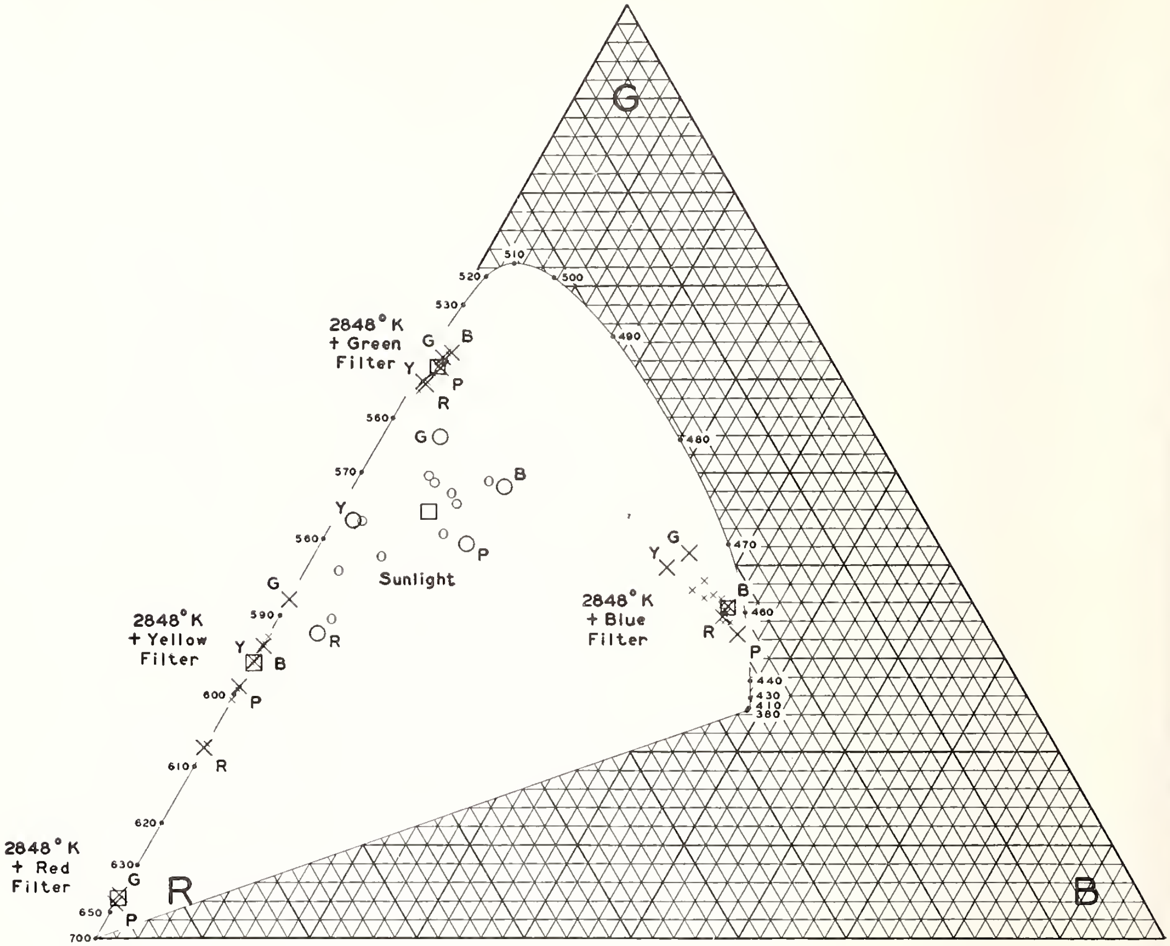

FIG. 1. The 75 illuminant-sample combinations represented on the uniform-chromaticity-scale Maxwell triangle. The samples with Abbot-Priest sunlight as the illuminant are shown by circles; those for the four strongly selective illuminants (red, yellow, green, blue) are shown by crosses. The illuminants are indicated by squares.

ordinate system [18; 87, p. 549] was used for the first computation of tristimulus specifications and these were later transferred to the uniform-chromaticity-scale system [40]. Table VI gives two (r and g) of the trilinear coordinates on this system; the third may be found, if desired, as $b=1-r-g$.* Fig. 1 shows the uniform-chromaticity-scale triangle on which have been plotted points representing (except for the red illuminant) each of the 15 samples under each of the 5 illuminants. The illuminant point is indicated by a square. The samples under sunlight illumination are shown by circles, with large circles for the five samples having the most saturated colors of the 5 Munsell hues, red, yellow, green, blue and purple. For the strongly chromatic illuminants, the samples are shown by crosses, with large crosses for the same 5 samples shown by large circles for sunlight illumination. For the red illuminant only the green (G) and the purple (P) samples are shown; points for the other samples fall on the straight line connecting these two points, and, for the sake of clarity, crosses to indicate them have been omitted. It will be noted that points representing the samples for other illuminants than red do not cluster as closely about the illuminant point. The small cluster of points for the red illuminant is related to the greater selectivity of the red filter (see Table V).

* Computations of trilinear coordinates, r, g, b, and of apparent reflectance, A, for these 75 illuminant-sample combinations were carried out by Miss Mabel E. Brown.

Table VI. *Trilinear coordinates (r,g) of the light reflected from each sample and the apparent reflectances (A) of the samples for each of the five illuminants used.*

Sample	Abbot-Priest Sunlight			Red Illuminant			Yellow Illuminant			Green Illuminant			Blue Illuminant		
	r	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*	*r*	*g*	*A*
MgO	0.458	0.458	1.000	0.958	0.042	1.000	0.705	0.295	1.000	0.372	0.613	1.000	0.237	0.351	1.000
R3/7	.606	.342	.087	.961	.039	.276	.792	.208	.151	.390	.597	.049	.243	.338	.048
R5/10	.628	.327	.221	.959	.041	.706	.796	.204	.404	.391	.595	.110	.241	.344	.110
R7/6	.527	.408	.459	.958	.042	.781	.744	.255	.618	.380	.606	.357	.240	.347	.346
YR4/5	.574	.394	.163	.959	.041	.297	.735	.265	.246	.401	.589	.110	.248	.365	.069
Y5/7	.525	.448	.241	.957	.043	.272	.700	.299	.288	.393	.598	.221	.254	.373	.097
Y7/8	.532	.449	.490	.958	.042	.577	.703	.296	.588	.393	.598	.455	.265	.398	.159
G3/4	.437	.490	.094	.961	.039	.072	.686	.313	.078	.367	.619	.107	.236	.369	.093
G5/7	.407	.539	.285	.953	.047	.087	.637	.362	.171	.362	.624	.377	.237	.413	.267
G7/7	.439	.497	.522	.956	.044	.329	.677	.322	.428	.367	.619	.596	.237	.384	.493
B3/5	.374	.485	.114	.958	.042	.064	.685	.314	.072	.351	.629	.133	.232	.353	.217
B5/6	.386	.491	.277	.959	.041	.161	.684	.315	.185	.354	.627	.324	.232	.363	.472
B7/4	.427	.478	.531	.957	.043	.381	.690	.309	.445	.365	.619	.581	.236	.356	.642
P3/6	.440	.423	.108	.962	.038	.153	.730	.270	.119	.370	.612	.097	.237	.325	.153
P5/6	.456	.434	.253	.961	.039	.359	.731	.269	.273	.370	.614	.235	.236	.337	.310
PB7/4	.428	.467	.458	.960	.040	.428	.704	.294	.405	.364	.619	.485	.236	.350	.600

Table VI also gives the apparent reflectance of each sample computed for each of the 5 illuminants. The 3 numbers (A,r,g) serve to specify the beam reflected from the sample toward the observer relative to that which would be reflected from a surface of magnesium oxide which was the reflectance standard used in the spectrophotometric work [68].

V. Experimental Method

The observer was shown the samples (irregularly shaped papers of about one square inch) spread out in disarray on one of two large cardboards (either a light one, $A_0=0.80$, which appeared white in daylight, or a dark one, $A_0=0.10$, which appeared dark gray). He was asked to arrange them in order of lightness, placing the lightest at the top and the darkest at the bottom. After this task had been done he was asked to estimate the lightness on a scale of 10 equal visual steps such that white would have lightness, $L=10$, and black would have lightness, $L=0$. These estimates were recorded. Then he was asked to arrange the samples in order of saturation, placing the one having the most saturated (strongest) color first and the one most closely resembling an achromatic color last. After this task had been done he was asked to estimate the saturation on a scale of 10 equal visual steps such that the strongest daylight color produced by any of the 15 samples would be assigned the saturation, $S=10$, and such that an achromatic color would be given a saturation, $S=0$. Then he was asked to name the hue of the sample on the 8-point hue scale; see definition of H, Section II-3. These saturation and hue estimates were also recorded. If, as was frequently the case, the observer was not satisfied with one of the 8 or 11 answers suggested by these 8- and 11-point scales, his estimate was recorded on a more finely divided scale. This procedure was carried out for each observer for all 5 illuminants, and for each of the 2 backgrounds. In his manipulation of the samples, the observer was not permitted to place the samples next to one another except in forming estimates of lightness; for hue and saturation estimates the samples were kept about one-half inch apart. Furthermore, the observer was requested to avoid fixating any one sample very long at a time; he was encouraged to look rapidly from one sample to another in order to avoid as much as possible the influence of any one sample on estimates of the color of another.

Although estimates of this sort have sometimes been criticized as meaningless (see, for example, Smith [84] and Guild [22]), they are frequently made and constitute a recognized general psychophysical method called by Boring [8, p. 389] "sense distance, method of fractiona-

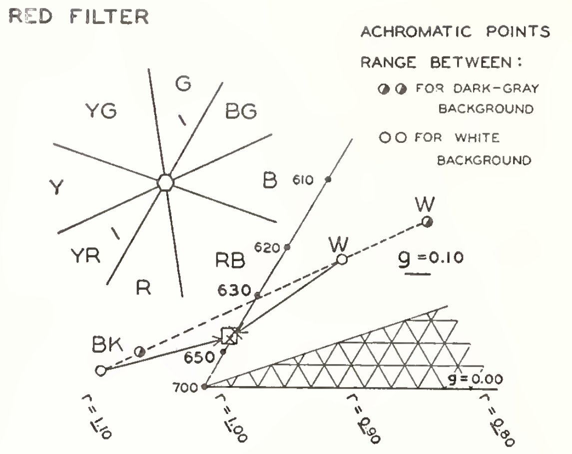

FIG. 2. A large-scale plot of a portion of Fig. 1 showing for the red illuminant (square) the locus of the achromatic points (dotted line) and two of the vectors defining hue and saturation by Eqs. (1), (2) and (3). The definition of hue according to angle by Eq. (1) is indicated in the upper left-hand part of this figure. The vector indicating a blue hue refers to a dark nonselective sample on a light ground; and that indicating a yellow-red hue, to a light nonselective sample on a light ground. Both vectors refer to momentary fixation of the samples by an observer who has been exposed to the field with red illumination for five minutes or more (Case 3, Section III, 2, e).

tion." Richardson and Maxwell [59, 78, 79] and Newhall [70] have applied it to color.

VI. COMPUTED VALUES OF HUE, SATURATION AND LIGHTNESS

Values of hue, saturation and lightness have been computed from Eqs. (1), (2), (3), (4), (5a), and (5c) for the 15 samples used in the preliminary experiments and for 3 hypothetical nonselective samples identified by the appropriate Munsell notations, *N* 2/, *N* 5/, and *N* 9/.* These computations refer to all five illuminants studied and to observing situations described under Cases 3 and 4. Values of hue and saturation have also been computed for Cases 1 and 2.

TABLE VII. *Constants used in computing the trilinear coordinates* (r_n,g_n,b_n) *of the achromatic points for the five illuminants studied (Eq. 3).*

ILLUMINANT	r_f $(=r_0)$	g_f $(=g_0)$	b_f $(=b_0)$	D_f	T_{2848}	I (fc)	$\bar{A}$	A_0	A_f (Eq. (5a))
Abbot-Priest sunlight	0.458	0.458	0.084	0.02	—	50	0.287	0.10 .80	0.194 .544
Red	.958	.042	.000	.68	0.052	36	.330	.10 .80	.215 .565
Yellow	.705	.295	.000	.33	.362	253	.298	.10 .80	.199 .549
Green	.372	.613	.015	.18	.038	27	.282	.10 .80	.191 .541
Blue	.237	.351	.412	.40	.0061	4	.272	.10 .80	.186 .536

In Table VII are given values of the constants used in computing by means of Eq. (3) the trilinear coordinates (r_n,g_n,b_n) of the achromatic points for viewing by Case 3 for the five illuminants. Since the samples and background taken together are nearly nonselective in spectral reflectance, trilinear coordinates (r_f,g_f,b_f) for the field points are taken equal to those for the illuminant points (r_0,g_0,b_0) which, in turn, are taken equal to the trilinear coordinates (r,g,b) for MgO in Table VI by the assumption that MgO is spectrally nonselective [75]. The illuminances (I) supplied by the chromatic illuminants are computed as 700 T_{2848}, where T_{2848} is the transmission of the chromatic filter for light of color temperature 2848°K. Values of T_{2848} were computed in the usual way [23, 39] from values of spectral transmission given in Table V.

Application of the constants in Table VII to Eq. (3) gives the specification of the trilinear coordinates (r_n,g_n,b_n) of the achromatic point as a function of lightness, L' (Table VIII).

Figure 2 shows the locus (dotted line) of achromatic points plotted to a large scale on the Maxwell (UCS) triangle for the red illuminant. Only the extreme lower left-hand corner of the

TABLE VIII.

ILLUMINANT	EQUATION (3)
Abbot-Priest sunlight	$r_n = 0.458 - 0.00022L' + 0.00009(L')^2$ (0.00003)* $g_n = 0.459 - 0.00035L'$
Red	$r_n = 0.958 - 0.0407L'$ $g_n = 0.062 + 0.0175L'$
Yellow	$r_n = 0.705 - 0.0114L'$ $g_n = 0.305 + 0.000165L'$
Green	$r_n = 0.372 - 0.00022L' + 0.000124(L')^2$ (0.000044)* $g_n = 0.618 - 0.0056L'$
Blue	$r_n = 0.237 + 0.0049L' + 0.0063(L')^2$ (0.0022)* $g_n = 0.363 - 0.00204L'$

* This constant refers to the dark background ($A_0 = 0.10$): the one given just above to the light background ($A_0 = 0.80$).

* These hypothetical samples because of their nonselectivity have trilinear coordinates (r,g,b) the same as those given in Table VI for MgO. They have been assigned values of apparent reflectance: 0.036, 0.193, and 0.738 respectively.

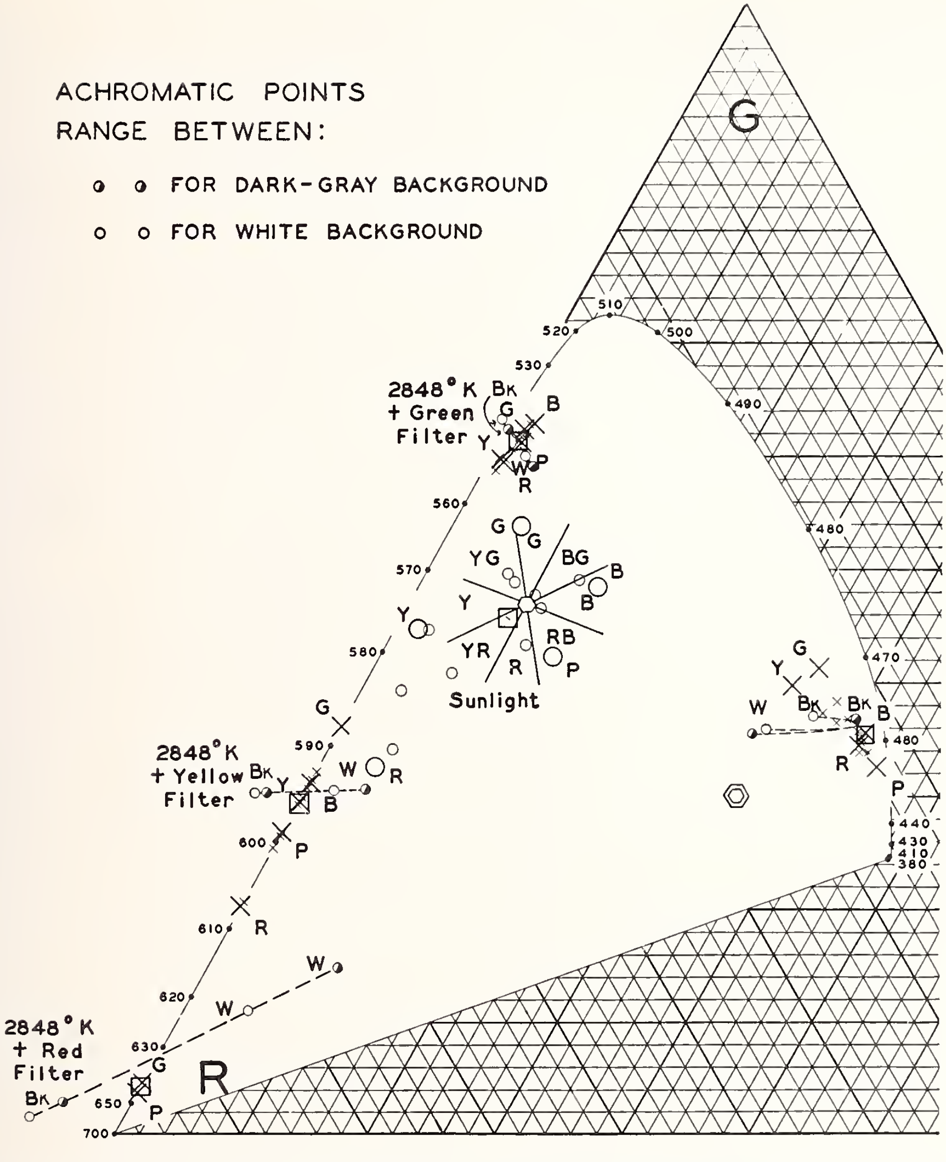

FIG. 3. The loci of achromatic points (dotted lines) on the uniform-chromaticity-scale Maxwell triangle for the five illuminants studied. Note the varying lengths of the loci (determined chiefly by D_f), and the curvature of the loci of achromatic points for the blue illuminant (determined by b_f). Note also how the direction of each nearly straight locus is determined by the position of the point ($r=0.36$, $g=0.30$), indicated by a double hexagon, whose trilinear coordinates appear explicitly in Eq. (3). The definition of hue according to angle by Eq. (1) is indicated graphically by lines intersecting at the daylight point (indicated by a small single hexagon).

triangle (Fig. 1) appears on this plot. Since Eq. (3) reduces for the red illuminant to a first-degree equation (see Table VIII) this locus is a straight line. The limits of the locus for samples viewed by momentary fixation (Case 3) on a dark background ($A_0=0.10$) are shown by shaded circles, one for a dark (Bk) sample ($A=0.03$), and one for a light (W) sample ($A=0.80$); similar limits are indicated for a light background ($A_0=0.80$) by open circles. It will be noted that the achromatic point for the dark sample ($A=0.03$), whether viewed on the light or on the dark background, falls outside the region of the triangle representing mixtures of real stimuli; that is, according to Eq. (3), when an observer has been in a room filled with red light for five

TABLE IX. *Computed hues, lightnesses and saturations for 3 hypothetical nonselective samples for both light and dark backgrounds, for the 5 illuminants, and for 4 observing situations (Cases 1 to 4).*

			Computed for Different Viewing Situations			
			Momentary Fixation of Sample	Fixation of Sample for Several Seconds	Illuminant Color Discounted	Aperture Color for Daylight Adapted Eye
Illuminant	A_0	Sample	(Case 3)	(Case 4)	(Case 2)	(Case 1)
Abbot-Priest sunlight	0.80	N 2/0	RB 0.2/0.2	RB 0.3/0.2	N /0.0	YR /1.1
		N 5/0	RB 3.5/0.1	RB 5.3/0.1	N /0.0	YR /1.1
		N 9/0	N 8.8/0.0	Y 9.4/0.1	N /0.0	YR /1.1
	.10	N 2/0	RB 0.3/0.1	RB 0.6/0.1	N /0.0	YR /1.1
		N 5/0	N 5.2/0.0	N 6.9/0.0	N /0.0	YR /1.1
		N 9/0	YG 9.4/0.1	YG 9.7/0.1	N /0.0	YR /1.1
Red	.80	N 2/0	B 0.2/6.3	B 0.3/5.2	N /0.0	YR /33.9
		N 5/0	RB 3.5/1.8	R 5.3/1.7	N /0.0	YR /33.9
		N 9/0	YR 8.7/6.8	YR 9.4/5.8	N /0.0	YR /33.9
	.10	N 2/0	B 0.3/4.3	B 0.6/2.3	N /0.0	YR /33.9
		N 5/0	YR 5.0/4.6	YR 6.9/5.1	N /0.0	YR /33.9
		N 9/0	YR 9.3/12.0	YR 9.7/7.8	N /0.0	YR /33.9
Yellow	.80	N 2/0	B 0.2/2.3	B 0.3/2.0	N /0.0	YR /16.5
		N 5/0	RB 3.5/0.9	R 5.3/0.6	N /0.0	YR /16.5
		N 9/0	Y 8.8/1.9	Y 9.4/1.6	N /0.0	YR /16.5
	.10	N 2/0	RB 0.3/1.7	RB 0.6/1.1	N /0.0	YR /16.5
		N 5/0	YR 5.1/1.3	Y 6.9/1.4	N /0.0	YR /16.5
		N 9/0	Y 9.4/3.6	Y 9.7/2.2	N /0.0	YR /16.5
Green	.80	N 2/0	RB 0.2/1.3	RB 0.3/1.1	N /0.0	G /8.8
		N 5/0	RB 3.5/0.5	RB 5.3/0.2	N /0.0	G /8.8
		N 9/0	YG 8.8/0.7	YG 9.4/0.6	N /0.0	G /8.8
	.10	N 2/0	RB 0.3/0.9	RB 0.6/0.6	N /0.0	G /8.8
		N 5/0	YG 5.2/0.4	YG 6.9/0.4	N /0.0	G /8.8
		N 9/0	YG 9.4/1.6	YG 9.7/0.9	N /0.0	G /8.8
Blue	.80	N 2/0	RB 0.2/2.5	RB 0.3/1.9	N /0.0	RB /20.0
		N 5/0	RB 3.5/0.9	RB 5.3/0.9	N /0.0	RB /20.0
		N 9/0	B 8.8/4.7	B 9.4/3.4	N /0.0	RB /20.0
	.10	N 2/0	RB 0.3/1.0	RB 0.6/0.9	N /0.0	RB /20.0
		N 5/0	B 5.2/1.7	B 6.9/1.7	N /0.0	RB /20.0
		N 9/0	B 9.4/5.5	B 9.7/2.8	N /0.0	RB /20.0

minutes or more, his visual mechanism is in such a state of chromatic adaptation that no sample exists of apparent reflectance as low as 0.03 such that it will look gray or black against a background of apparent reflectance, $A_0=0.10$, or greater. Fig. 2 also shows the illuminant point (square) and the limits (crosses) of the locus of points representing the 15 samples studied. Two of the vectors whose direction is correlated with hue according to Eq. (2) are also shown; the one associated with a dark sample viewed on a light background indicates a blue hue, the one associated with a light sample on a light ground indicates a yellow-red hue; see graphical representation of Eq. (2) identifying hue with direction on the triangle shown in the upper left-hand portion of the figure.

Figure 3 shows loci of achromatic points for all 5 illuminants studied. Like the locus for the red illuminant, that for the yellow illuminant is a straight line; the other loci are parabolas, those for Abbot-Priest sunlight and the green illuminant being relatively short because of the relatively small chromatic adaptation (measured by D_f) associated with them according to Eq. (3); note particularly the short locus for Abbot-Priest sunlight which falls entirely within the square indicating this illuminant point. Note also how the direction of these loci is controlled by the position of the point, indicated by a double hexagon, whose trilinear coordinates ($r=0.360$, $g=0.300$) appear explicitly in Eq. (3). The graphical representation of Eq. (2) defining hue according to direction on this diagram is centered about the daylight point (0.44, 0.47, 0.09) indicated by a single hexagon.

Table IX gives the computed hues and saturations for the hypothetical nonselective samples for both backgrounds, for the 5 illuminants, and for the 4 observing situations (Cases 1 to 4); it also gives lightness computed for Cases 3 and 4. It should be noted that the hue to be expected for the aperture colors derived from nonselective samples (Case 1) with a chromatic illuminant is approximately the hue of the illuminant, and the saturation is high. This result is independent of the apparent reflectance of the sample. Similarly the surface color with the illuminant color discounted (Case 2) is also expected to be independent in hue and saturation of the apparent reflectance of the sample; that is, as may be seen from Table IX, the colors are all expected to be achromatic (zero saturation, no hue). Cases 3 and 4 yield hues, lightnesses and saturations for nonselective samples which depend upon the relation of the apparent reflectance, A, of the sample to that of the field, A_f. From a comparison of these computations for Case 3 and Case 4 the formulation may be seen to indicate that fixation of the sample for a few seconds contrary to instructions may produce a considerable change in hue and saturation of samples viewed in strongly chromatic illumination, but produces a negligible effect for samples viewed by Abbot-Priest sunlight.

The computed color descriptions for these hypothetical nonselective samples agree, in general, both for Case 3 and for Case 4 with experimental results by Helson [24]. Detailed comparison has yet to be carried out, but evaluations of adaptation reflectance (see Table III) suggest that improved agreement with Helson's results would be obtained by using a definition of apparent reflectance of the field, A_f, slightly different from Eq. (5a), and a slightly different definition of L'.

VII. Comparison of Computation With Experiment

Table X summarizes the experimental results obtained in south-daylight illumination; Table XI, those obtained under the four strongly chromatic illuminants studied. These tables give the average hue, lightness and saturation of the 15 samples estimated by the six observers who completed the series: G. B. Reimann, G. W. Haupt, B. H. Carroll, K. S. Gibson, R. S. Hunter, and D. B. Judd. The individual variation among these observers was considerably larger than the uncertainty of estimates by the two observers who made repeat runs; this individual variation is indicated in Tables X and XI by the ranges of the estimates of hue, lightness and saturation. Similar individual variation is re-

Table X. *Comparison of observed and computed color descriptions. (Hue, lightness and saturation (HL/S) of the 15 selective samples studied. Illuminant: Abbot-Priest sunlight taken as representative of south daylight.)*

Sample	Average Observed Values HL/S	Individual Observer Range $\Delta H \Delta L/\Delta S$	Computed for Different Viewing Situations: Momentary Fixation of Sample (Case 3)	Fixation of Sample for Several Seconds (Case 4)	Illuminant Color Discounted (Case 2)	Aperture Color for Daylight Adapted Eye (Case 1)
Light background (A_0=0.80)						
R 3/7	R 2.6/5.1	R 1–3/2–7	YR 1.6/9.6	YR 2.6/9.6	YR /9.5	YR /10.6
R 5/10	R 5.2/10	R 4–6/10	YR 4.0/10.9	YR 5.8/10.9	YR /10.9	YR /12.0
R 7/6	R 6.9/5.8	RB-YR 5–8/3–8	YR 6.8/4.4	YR 8.2/4.4	YR /4.4	YR /5.5
YR 4/5	YR 3.8/4.8	YR 3–5/1–7	YR 3.0/7.1	YR 4.7/7.1	YR /7.1	YR /8.3
Y 5/7	gY 4.6/4.4	Y-YG 4–5/2–6	Y 4.3/4.4	Y 6.1/4.4	Y /4.4	Y /5.4
Y 7/8	Y 6.7/9.4	Y 6–8/9–10	Y 7.1/4.9	Y 8.3/4.9	Y /5.0	Y /5.9
G 3/4	yG 2.0/2.9	YG-G 1–3/1–6	G 1.7/1.9	G 2.8/1.9	G /2.0	YG /1.3
G 5/7	G 5.3/9.8	G 4–6/9–10	G 4.9/5.0	G 6.7/5.0	G /5.0	G /4.2
G 7/7	yG 7.3/5.2	YG-G 7–8/4–8	G 7.4/2.4	G 8.5/2.4	G /2.4	YG /1.9
B 3/5	B 3.0/6.2	B 2–4/5–7	B 2.1/5.3	B 3.4/5.3	B /5.2	B /4.2
B 5/6	B 4.9/7.9	B 4–6/7–9	BG 4.8/4.4	BG 6.6/4.4	BG /4.4	B /3.3
B 7/4	gB 7.3/3.4	BG-B 7–8/1–8	BG 7.4/1.9	BG 8.6/1.9	BG /1.9	BG /0.8
P 3/6	RB 2.9/6.8	RB 2–4/6–8	RB 2.0/3.4	RB 3.3/3.4	RB /3.3	RB /3.3
P 5/6	RB 5.1/5.3	RB 5–6/4–8	RB 4.5/1.8	RB 6.3/1.8	RB /1.8	R /2.2
PB 7/4	B 6.8/3.6	rB-B 6–7/1–8	B 6.8/1.9	B 8.1/1.9	B /1.9	RB /1.0
Dark background (A_0=0.10)						
R 3/7	R 3.1/5.1	R 1–5/3–8	YR 2.7/9.6	YR 4.2/9.6	YR /9.5	YR /10.6
R 5/10	R 6.0/10	RB-R 5–7/10	YR 5.7/11.0	YR 7.3/10.9	YR /10.9	YR /12.0
R 7/6	R 7.4/6.8	RB-YR 7–9/3–9	YR 8.1/4.4	YR 8.9/4.4	YR /4.4	YR /5.5
YR 4/5	YR 4.1/5.9	YR 2–5/2–9	YR 4.7/7.1	YR 6.4/7.1	YR /7.1	YR /8.3
Y 5/7	Y 4.6/4.5	Y 2–6/2–6	Y 6.0/4.5	Y 7.5/4.4	Y /4.4	Y /5.4
Y 7/8	Y 7.4/9.9	Y 6–9/9–11	Y 8.3/4.9	Y 9.0/5.0	Y /5.0	Y /5.9
G 3/4	G 2.4/2.2	G 1–4/2–4	G 2.8/1.9	G 4.5/1.9	G /2.0	YG /1.3
G 5/7	G 5.7/9.0	G 4–8/8–10	G 6.6/5.0	G 7.9/5.0	G /5.0	G /4.2
G 7/7	G 7.4/4.7	YG-BG 6–9/3–8	G 8.5/2.4	G 9.2/2.4	G /2.4	YG /1.9
B 3/5	B 3.2/7.2	B 2–4/7–8	B 3.3/5.3	B 5.1/5.3	B /5.2	B /4.2
B 5/6	B 5.4/6.8	B 4–6/5–8	BG 6.4/4.3	BG 7.9/4.3	BG /4.4	B /3.3
B 7/4	gB 7.6/1.8	BG-B 5–10/1–4	BG 8.5/1.9	BG 9.2/1.9	BG /1.9	BG /0.8
P 3/6	RB 3.1/5.7	RB 2–4/4–9	RB 3.2/3.4	RB 4.9/3.4	RB /3.3	RB /3.3
P 5/6	RB 5.2/3.8	RB 4–6/1–7	RB 6.1/1.7	RB 7.6/1.7	RB /1.8	R /2.2
PB 7/4	rB 7.2/1.7	RB-B 5–8/1–3	B 8.6/1.9	B 8.9/1.8	B /1.9	RB /1.0

TABLE XI. *Comparison of observed and computed color descriptions. Hue, lightness and saturation (HL/S) of the 15 selective samples studied under the 4 chromatic illuminants; see Table V.*

Sample	Average Observed Values HL/S	Individual Observer Range $\Delta H \Delta L/\Delta S$	Computed for Different Viewing Situations: Momentary Fixation of Sample (Case 3)	Fixation of Sample for Several Seconds (Case 4)	Illuminant Color Discounted (Case 2)	Aperture Color for Daylight Adapted Eye (Case 1)
Red illuminant, light background (A_0=0.80)						
R 3/7	bR 4.0/1.2	RB-YR 3–5/0–4	R 4.7/1.7	YR **6.6**/3.0	YR /0.2	YR /**34.1**
R 5/10	rYR 9.2/1.4	R-rY 8–10/0–4	YR 8.5/**6.6**	YR 9.3/**5.9**	YR /0.1	YR /**33.9**
R 7/6	YR 9.7/1.3	yR-rY 8–10/0–4	YR 9.0/**7.0**	YR 9.5/**6.1**	N /0.0	YR /**33.9**
YR 4/5	bR 4.2/1.1	RB-YR 3–5/0–4	R 5.0/1.9	YR 6.8/3.1	YR /0.1	YR /**33.9**
Y 5/7	bR 3.9/1.0	RB-YR 3–5/0–4	R 4.6/1.5	YR 6.5/2.7	BG /0.1	YR /**33.8**
Y 7/8	yR 7.5/1.0	R-YR 6–9/0–2	YR 7.7/**5.3**	YR 8.8/**5.3**	N /0.0	YR /**33.9**
G 3/4	B 1.1/1.9	BG-BR 0–2/0–6	B 1.1/4.9	B 2.1/3.3	YR /0.2	**YR** /**34.1**
G 5/7	B 1.4/1.9	BG-RB 1–2/0–6	B 1.5/4.8	B 2.5/3.0	BG /0.4	**YR** /**33.6**
G 7/7	yR 4.6/1.1	R-YR 4–6/0–4	R 5.4/2.0	YR **7.1**/3.3	BG /0.1	YR /**33.7**
B 3/5	gB 1.1/1.8	BG-B 0–2/0–6	B 0.9/5.3	B 1.6/3.6	N /0.0	**YR** /**33.9**
B 5/6	RB 2.3/1.4	B-R 1–4/0–6	B 2.9/2.6	RB **4.6**/1.5	YR /0.1	YR /**33.9**
B 7/4	yR 5.2/1.0	R-YR 4–6/0–4	YR 6.0/2.7	YR **7.6**/3.8	BG /0.1	YR /**33.8**
P 3/6	RB 2.3/1.5	B-YR 1–4/0–6	B 2.7/2.7	RB **4.4**/1.4	YR /0.3	YR /**34.2**
P 5/6	bR 5.0/1.0	RB-YR 4–6/0–4	YR 5.8/2.8	YR **7.4**/3.9	YR /0.2	YR /**34.1**
PB 7/4	R 5.8/1.3	RB-YR 5–7/0–4	YR 6.5/3.6	YR **7.9**/4.3	YR /0.1	YR /**34.0**
Red illuminant, dark background (A_0=0.10)						
R 3/7	YR 5.5/3.4	YR 5–6/2–5	YR 6.3/**7.0**	YR **7.8**/**6.3**	YR /**0.2**	YR /**34.1**
R 5/10	YR 9.7/6.3	YR 7–11/3–12	YR 9.2/11.8	YR 9.6/7.8	YR /**0.1**	YR /**33.9**
R 7/6	YR 9.8/6.3	YR 7–11/3–12	YR 9.4/12.2	YR 9.8/7.8	**N** /**0.0**	YR /**33.9**
YR 4/5	YR 6.3/4.1	YR 5–7/3–6	YR 6.6/**7.1**	YR **8.0**/6.1	YR /**0.1**	YR /**33.9**
Y 5/7	YR 6.0/3.4	YR 5–8/2–5	YR 6.2/**6.5**	YR 7.8/**6.0**	**BG** /**0.1**	YR /**33.8**
Y 7/8	YR 9.0/6.0	YR 7–10/3–11	YR 8.6/10.7	YR 9.3/7.5	**N** /**0.0**	YR /**33.9**
G 3/4	RB 1.3/0.4	RB-R 1–2/0–2	RB 1.9/1.9	R **3.5**/2.0	**YR** /0.2	**YR** /**34.1**
G 5/7	R 1.8/0.6	RB-R 1–3/0–4	RB 2.4/1.5	**YR 4.2**/2.0	BG /0.4	**YR** /**33.6**
G 7/7	YR 6.7/3.9	YR 6–8/3–6	YR 6.9/**7.6**	YR 8.3/6.2	**BG** /**0.1**	YR /**33.7**
B 3/5	RB 1.0/0.6	B-RB 0–2/0–4	B 1.6/2.3	R **3.0**/1.5	N /0.0	**YR** /**33.9**
B 5/6	YR 3.7/1.7	R-YG 3–4/1–4	YR 4.4/3.5	YR **6.3**/4.4	YR /**0.1**	YR /**33.9**
B 7/4	YR 7.5/4.6	YR 6–8/3–8	YR 7.4/8.4	YR **8.6**/6.7	**BG** /**0.1**	YR /**33.8**
P 3/6	rYR 3.8/2.0	R-YR 3–4/1–4	YR 4.2/3.5	YR **6.1**/4.4	YR /**0.3**	YR /**34.2**
P 5/6	YR 6.9/4.2	YR 5–8/2–8	YR 7.2/8.4	YR 8.5/6.8	YR /**0.2**	YR /**34.1**
PB 7/4	YR 7.9/5.0	YR 6–9/3–10	YR 7.8/9.4	YR 8.8/7.2	YR /**0.1**	YR /**34.0**
Yellow illuminant, light background (A_0=0.80)						
R 3/7	rBR 3.8/10.0	RB-R 3–6/4–16	R 2.8/6.3	R 4.4/6.5	**YR** /6.2	**YR** /**22.4**
R 5/10	yR 6.8/9.7	R-YR 5–9/5–16	YR 6.3/7.3	YR 7.8/7.5	YR /6.4	YR /**22.7**
R 7/6	rYR 8.3/6.1	R-Y 8–10/3–10	YR 8.0/4.2	YR 9.0/4.1	R /2.8	YR /**19.1**
YR 4/5	bR 4.4/3.3	RB-Y 3–5/1–4	R 4.3/2.6	R **6.2**/2.8	YR /2.1	YR /**18.5**
Y 5/7	YG 4.8/0.2	Y-G 4–6/0–1	RB 4.9/0.4	YR **6.7**/0.5	BG /0.3	**YR** /**16.3**
Y 7/8	gY 8.3/0.4	Y-YG 8–10/0–1	Y 7.8/1.4	Y 8.8/1.4	BG /0.1	**YR** /**16.4**
G 3/4	bRB 1.5/4.2	B-RB 1–2/0–8	B 1.3/2.6	B 2.1/1.9	**BG** /1.3	**YR** /**15.2**
G 5/7	gBG 3.8/6.1	G-B 3–6/3–8	BG 3.1/4.8	BG 4.8/4.4	BG /4.8	**YR** /**12.1**
G 7/7	yG 6.4/4.3	YG-G 5–7/2–8	G 6.5/**1.1**	G **7.9**/**1.1**	**BG** /1.9	**YR** /**14.7**
B 3/5	bRB 1.5/4.7	B-RB 1–2/0–10	B 1.2/2.6	B 1.9/2.1	**BG** /1.4	**YR** /**15.2**
B 5/6	gB 3.1/1.4	BG-RB 2–4/0–3	B 3.3/1.7	BG **5.1**/1.0	BG /1.4	**YR** /**15.1**
B 7/4	yG 6.7/2.7	YG-G 5–7/1–4	YG 6.6/**0.4**	YG **8.0**/0.5	**BG** /1.0	**YR** /**15.5**
P 3/6	rRB 2.5/5.5	RB-YR 2–3/2–10	RB 2.2/1.8	R 3.5/2.1	YR /1.8	YR /**18.2**
P 5/6	bR 4.7/3.3	RB-Y 4–6/1–4	R 4.7/2.3	YR 6.5/2.6	YR /1.8	YR /**18.2**
PB 7/4	YG 6.2/0.2	Y-G 5–7/0–1	**YR** 6.3/0.9	**YR 7.8**/1.1	RB /0.1	**YR** /**16.5**

Table XI (*Continued*).

Sample	Average Observed Values HL/S	Individual Observer Range $\Delta H \Delta L/\Delta S$	Computed for Different Viewing Situations: Momentary Fixation of Sample (Case 3)	Fixation of Sample for Several Seconds (Case 4)	Illuminant Color Discounted (Case 2)	Aperture Color for Daylight Adapted Eye (Case 1)
Yellow illuminant, dark background ($A_0 = 0.10$)						
R 3/7	bR 4.9/8.4	RB-R 4–6/7–12	**YR** 4.2/7.0	**YR** 6.1/7.2	**YR** /6.2	**YR** /22.4
R 5/10	rYR 7.7/10.4	R-YR 6–10/8–16	YR 7.6/8.5	YR 8.7/8.0	YR /**6.4**	YR /22.7
R 7/6	yYR 8.6/8.9	R-Y 7–10/6–16	YR 8.9/5.6	YR 9.4/**4.6**	R /2.8	YR /**19.1**
YR 4/5	YR 5.3/5.2	YR 3–6/4–7	YR 5.9/3.6	YR **7.6**/3.5	YR /2.1	YR /18.5
Y 5/7	gY 5.8/3.5	Y-YG 5–6/1–6	Y 6.5/1.8	Y **8.0**/3.6	**BG /0.3**	**YR /16.3**
Y 7/8	gY 8.4/6.6	Y-yYG 7–10/2–14	Y 8.8/3.1	Y 9.4/2.0	**BG** /0.1	**YR /16.4**
G 3/4	yG 1.5/0.6	YG-G 1–2/0–2	**B** 2.2/1.4	**BG 3.6**/0.7	**BG** /1.3	**YR /15.2**
G 5/7	G 4.7/8.6	G 4–6/7–12	G 4.7/**3.8**	G 6.5/**3.7**	**BG** /4.8	**YR /12.1**
G 7/7	gYG 7.2/6.6	YG-G 6–8/4–10	YG 7.8/**1.9**	YG **8.8/1.4**	**BG /1.9**	**YR /14.7**
B 3/5	yG 1.4/0.4	YG-G 1–2/0–2	**B** 2.0/1.4	**BG 3.4**/0.9	**BG** /1.4	**YR /15.2**
B 5/6	gYG 4.2/4.3	YG-G 3–5/3–6	YG 5.0/**0.7**	YG **6.8/0.7**	**BG** /1.4	**YR /15.1**
B 7/4	YG 7.2/5.3	Y-G 6–8/2–8	Y 7.9/2.2	Y **8.9/1.4**	**BG** /1.0	**YR /15.5**
P 3/6	rYR 3.1/3.4	R-YR 2–5/2–6	R 3.4/2.4	YR 5.2/2.6	YR /1.8	YR /18.2
P 5/6	YR 5.3/4.9	YR 4–6/3–6	YR 6.3/3.6	YR **7.8**/3.3	YR /1.8	YR /18.2
PB 7/4	Y 7.0/4.7	Y-gY 6–8/2–8	Y 7.6/2.6	Y **8.7**/1.9	**RB** /0.1	**YR** /16.5
Green illuminant, light background ($A_0 = 0.80$)						
R 3/7	RB 1.0/3.3	RB 1–2/0–8	**R** 0.5/2.1	**R** 1.0/1.8	**YR** /1.2	G /7.8
R 5/10	R 2.3/2.4	RB-yG 2–3/0–6	R 2.0/1.9	R 3.2/1.7	YR /1.3	**G** /7.7
R 7/6	rY 5.5/1.4	R-G 4–7/0–4	YR 5.8/0.5	YR 7.4/0.5	YR /0.5	**G** /8.4
YR 4/5	YR 2.5/2.8	R-YG 2–3/0–6	R 2.0/2.4	R 3.2/2.2	YR /1.9	YG /7.4
Y 5/7	gY 4.1/2.8	YR-YG 3–5/1–4	YR 4.0/1.5	YR **5.8**/1.3	YR /1.3	YG /7.9
Y 7/8	gY 7.1/5.1	Y-YG 6–8/3–8	**YR** 6.7/**1.3**	**YR** 8.1/1.3	**YR** /1.3	YG /7.9
G 3/4	B 2.3/1.8	G-RB 2–3/0–6	RB 2.0/0.7	RB 3.2/0.4	G /0.4	G /**9.1**
G 5/7	G 5.8/2.4	G 4–8/0–4	G 6.0/0.8	G 7.6/0.9	G /0.7	G /**9.4**
G 7/7	G 8.3/0.9	G 7–9/0–2	G 7.9/0.8	G 8.9/0.9	G /0.4	G /**9.1**
B 3/5	B 2.8/3.8	B 2–4/2–6	B 2.4/**1.3**	**BG** 3.9/**1.3**	**BG** /1.3	**G** /**9.8**
B 5/6	B 5.1/3.9	B 3–6/1–7	**BG** 5.4/1.1	**BG 7.1**/1.2	**BG** /1.2	**G** /**9.6**
B 7/4	B 8.0/0.5	BG-B 6–9/0–2	**G** 7.8/0.8	**G** 8.8/0.9	BG /0.5	**G** /9.1
P 3/6	bRB 2.3/2.1	B-RB 2–3/0–6	RB 1.7/1.1	RB 2.8/0.8	B-RB /0.2	**G** /8.7
P 5/6	B 4.1/0.8	BG-RB 2–5/0–4	RB 4.2/0.4	B **6.1**/0.1	BG /0.1	**G** /8.8
PB 7/4	B 7.2/1.0	B 6–8/0–2	**G** 7.0/0.7	**G** 8.3/0.7	**BG** /0.5	**G** /9.1
Green illuminant, dark background ($A_0 = 0.10$)						
R 3/7	bR 1.0/2.2	RB-R 0–2/0–6	R 1.0/1.7	R 1.9/1.4	**YR** /1.2	G /**7.8**
R 5/10	yYR 2.9/2.4	YR–Y 2–6/1–4	YR 3.2/1.4	YR 4.9/1.3	YR /1.3	G /**7.7**
R 7/6	gY 7.7/3.6	Y-YG 6–10/1–10	YG 7.2/0.9	Y 8.4/0.8	**YR** /0.5	G /**8.4**
YR 4/5	yYR 3.6/3.8	YR-Y 3–6/2–6	YR 3.2/2.0	YR 4.9/1.9	YR /1.9	**YG** /**7.4**
Y 5/7	gY 6.1/4.8	Y-YG 4–8/2–10	Y 5.6/**1.4**	Y 7.3/**1.3**	**YR** /1.3	**YG** /**7.9**
Y 7/8	Y 8.5/6.7	Y 8–12/3–16	Y 8.0/**1.6**	Y 8.9/**1.4**	**YR** /1.3	**YG** /7.9
G 3/4	G 2.8/1.2	G 2–4/0–4	BG 3.2/0.3	G **4.8**/0.4	G /0.4	G /**9.1**
G 5/7	yG 7.7/4.4	YG-G 7–10/2–10	G 7.4/1.6	G 8.6/1.3	G /0.7	G /9.4
G 7/7	gYG 9.3/4.6	YG-G 9–12/1–12	YG 8.8/1.7	YG 9.4/1.2	G /0.4	G /9.1
B 3/5	gB 3.7/3.2	BG-B 3–4/2–6	BG 3.8/**1.3**	BG **5.6/1.4**	BG /1.3	**G** /**9.8**
B 5/6	gB 7.2/4.0	G-B 6–10/2–10	G 7.0/1.7	G 8.2/1.5	BG /**1.2**	G /9.6
B 7/4	yG 9.2/4.2	YG-G 8–12/1–12	YG 8.7/1.6	YG 9.4/1.2	**BG** /0.5	G /9.1
P 3/6	RB 2.4/0.5	RB 1–4/0–4	RB 2.8/0.4	B-RB **4.6**/0.2	B-RB /0.2	**G** /8.7
P 5/6	bG 5.9/2.0	G-BG 4–8/0–6	**YG** 5.7/0.6	**YG** 7.5/0.6	BG /0.1	G /8.8
PB 7/4	G 8.7/6.1	YG-BG 8–12/2–12	YG 8.2/1.5	G 9.0/**1.0**	BG /**0.5**	G /9.1

Table XI (*Continued*).

Sample	Average Observed Values HL/S	Individual Observer Range $\Delta H \Delta L/\Delta S$	Computed for Different Viewing Situations: Momentary Fixation of Sample (Case 3)	Fixation of Sample for Several Seconds (Case 4)	Illuminant Color Discounted (Case 2)	Aperture Color for Daylight Adapted Eye (Case 1)
Blue illuminant, light background (A_0=0.80)						
R 3/7	rRB 1.4/1.8	RB-R 1–2/0–6	RB 0.5/2.7	RB 1.0/2.1	R /0.8	RB /**20.3**
R 5/10	bR 3.0/3.6	RB-R 3/2-6	RB **2.0**/1.6	RB 3.3/**1.3**	R /**0.4**	RB /**20.1**
R 7/6	RB 5.9/3.1	RB-rRB 5–7/1–5	RB 5.7/1.3	RB 7.3/1.8	**R** /**0.3**	RB /**20.0**
YR 4/5	R 2.4/2.5	RB-YR 2–3/1–4	RB **1.1**/**0.2**	**Y** 1.8/0.6	**YG** /1.5	RB /**18.5**
Y 5/7	rYR 3.5/2.8	RB-Y 3–4/1–4	**YG 1.7**/1.0	**YG** 2.9/1.6	**YG** /2.4	RB /**17.6**
Y 7/8	yYR 5.4/4.6	R-Y 4–6/1–6	**YG 2.9**/3.8	**YG** 4.6/3.9	**YG** /4.6	**B** /15.6
G 3/4	bRB 2.2/0.5	B-RB 2–3/0–2	B 1.6/0.5	YG 2.8/0.3	**YG** /1.2	B /19.0
G 5/7	yG 5.1/3.8	YG-BG 4–6/2–5	YG 4.7/3.5	YG 6.5/3.4	YG /4.4	**B** /16.7
G 7/7	yG 7.6/1.5	Y-G 6–9/0–4	**BG** 7.1/1.9	**BG** 8.4/2.1	YG /2.3	**B** /18.2
B 3/5	B 3.8/2.2	B 3–4/0–4	**RB** 3.9/0.9	**RB 5.7**/1.2	B /0.3	**RB** /20.2
B 5/6	gB 6.2/4.4	BG-B 5–7/2–6	B 6.9/2.1	B **8.2**/2.3	**G** /**0.7**	RB /**19.6**
B 7/4	B 8.3/2.2	G-B 7–9/1–4	B 8.2/3.5	B 9.1/3.1	**YG** /**0.3**	RB /**19.7**
P 3/6	rB 3.0/3.4	B-RB 2–4/0–4	RB 2.8/2.8	RB 4.4/2.6	RB /1.8	RB /**21.4**
P 5/6	rB 4.8/2.8	B-rRB 4–5/0–5	RB 5.3/2.0	RB **6.9**/2.5	RB /1.0	RB /**20.8**
PB 7/4	B 7.9/2.3	B 6–9/1–4	B 7.9/3.5	B 8.9/3.1	**RB** /**0.1**	RB /**20.1**
Blue illuminant, dark background (A_0=0.10)						
R 3/7	bR 1.2/2.0	RB-R 0–2/0–4	RB 1.0/1.7	RB 1.8/1.6	R /0.8	RB /**20.3**
R 5/10	rRB 3.3/4.2	RB-R 2–4/1–6	RB 3.2/1.3	RB **4.9**/1.4	R /**0.4**	RB /**20.1**
R 7/6	rRB 7.2/2.5	RB-R 6–8/0–4	**B** 7.2/3.1	**B** 8.4/2.3	R /0.3	RB /**20.0**
YR 4/5	YR 2.7/3.3	rYR-YR 2–4/2–5	**Y** 2.0/**1.0**	**Y** 3.3/**0.8**	**YG** /1.5	**RB** /**18.5**
Y 5/7	yYR 3.9/4.2	YR-Y 3–5/3–5	**YG** 2.9/**1.7**	**YG** 4.6/**1.5**	**YG** /**2.4**	**RB** /**17.6**
Y 7/8	rY 6.2/6.2	YR-Y 5–7/4–8	**YG** 4.5/**3.4**	**YG** 6.2/**3.3**	**YG** /4.6	**B** /15.6
G 3/4	yG 2.7/1.7	YG-G 2–4/1–3	YG 2.8/**0.4**	G 4.4/0.5	YG /1.2	**B** /19.0
G 5/7	yG 6.4/5.1	YG-G 6–7/4–8	G 6.3/**3.4**	G **7.8**/**3.4**	YG /4.4	**B** /16.7
G 7/7	gBG 8.3/2.1	G-B 8–9/1–8	BG 8.3/3.7	BG 9.1/2.2	**YG** /2.3	B /18.2
B 3/5	B 5.1/5.1	gB-B 4–6/4–8	B 5.6/**2.2**	B **7.2**/**2.0**	B /**0.3**	**RB** /20.2
B 5/6	B 7.3/5.5	B 7–8/4–10	B 8.2/4.1	B **9.0**/**2.4**	**G** /**0.7**	**RB** /**19.6**
B 7/4	B 9.0/3.4	B 8–10/1–10	B 9.0/5.1	B 9.5/2.6	**YG** /**0.3**	**RB** /**19.7**
P 3/6	bRB 4.1/4.2	B-RB 3–5/2–8	RB 4.3/2.9	RB **6.1**/3.1	RB /1.8	RB /**21.4**
P 5/6	rB 6.3/4.4	B-RB 6–7/3–8	B 6.8/3.7	RB **8.1**/2.9	RB /1.0	RB /**20.8**
PB 7/4	B 8.7/3.9	B 8–10/2–10	B 8.9/5.2	B 9.4/2.8	**RB** /**0.1**	**RB** /**20.1**

ported by Bocksch [7, p. 366]. In parallel columns are also given the results of computations of hue, lightness and saturation for the four observing situations (Cases 1 to 4).

It may be seen from Table X that individual saturation estimates for most of the samples viewed in south daylight are notably less certain than those for hue and lightness. For example, PB 7/4 observed on the light background was called blue by 5 of the 6 observers, and reddish-blue by the sixth; hence the hue range is indicated as rB-B; the lightness estimates ranged from 6 to 7 with an average of 6.8; but the saturation estimates range all the way from 1 to 8. For each of 9 of the 15 samples viewed on the light background all 6 observers gave the same hue name; this is indicated in Table X by expressing the hue range for each of these 9 samples with a single hue symbol (R, YR and so on).

Whenever the computed hue, lightness or saturation falls within the corresponding individual-observer range, we have taken it as an indication that the formulation is substantially correct; and similarly, whenever the computed saturation is 0.5 or less, and one observer reports zero saturation we have taken the computed hue to be correct even though it does not agree with reports by the other observers. Computed hues, lightnesses and saturations which do not agree with our experimental results are given in boldface type for easy identification.

1. South daylight

It will be noted from Table X that Eq. (2) is fairly successful in duplicating the hue estimates, there being only 3 samples of the 15 for which the computed result by any of the cases (1 to 4) falls outside the observed range; and indeed these 3 discrepancies are by but one division of the 8-point hue scale. Two of them (R 3/7, R 5/10) could have been eliminated by abandoning the simplifying principle of making red by Eq. (2) the exact opposite of green.

Computation of lightness by Case 3 is in even closer agreement, no discrepancy for the light background, and only one for the dark, being shown. Case 4 (Eq. (5c)) yields consistently higher values of lightness, and for nearly half of the samples yields values too high to be acceptable. It would appear, therefore, that our observers followed instructions to judge lightness without fixating the sample in question more than a few seconds at a time; that is, Eq. (5a) rather than Eq. (5c) describes the observing habits of our observers in south daylight during their judgment of lightness.

In spite of the large individual variation in the estimates of saturation, Eq. (3) fails for about half of the samples to yield acceptable values of saturation. This failure indicates that the UCS triangle does not yield as uniform a spacing of surface colors viewed in a field subtending visual angles large relative to the macular pigment as it does for aperture colors of small angular extent (assumption *c*, Section III, 1). For improved agreement, the red end of the spectrum locus must be moved closer to the daylight point (0.44, 0.47, 0.09) and the yellow part must be moved further away. Experiments on relative size of small color differences between large surfaces have already yielded the same conclusion [42, 43].

It may also be noted from Table X that the computed hues and saturations are much alike for the observing situations (Cases 1 to 4) evaluated; that is, it makes little difference according to the formulation whether an observer (*a*) fixates the sample momentarily, or (*b*) for several seconds, or (*c*) discounts the faint yellow-red color of south daylight compared to average daylight, or (*d*) is even adapted to average daylight and judges the samples merely as color areas or patches of chromatic light. A far different conclusion applies, however, to strongly chromatic illuminants as may be seen from Table XI.

2. Strongly chromatic illuminants

Table XI shows that the formulation is as successful for strongly chromatic illuminants as for south daylight. As before, Case 3 is seen to yield nearly perfect agreement with estimated lightnesses, while Case 4 yields higher lightnesses which for about 40 percent of the sample-illuminant-background combinations are too high to be acceptable. Fig. 4 compares the average estimates of lightness with the lightnesses computed by Eqs. (4) and (5a) (Case 3) for the 15 samples both on the light and the dark background for all 5 illuminants. It is seen that the Adams and Cobb formula renders correctly the effect on lightness of changing from light to dark background provided the apparent reflectance of the field, A_f, be computed by Eq. (5a), an arithmetical mean. (The quadratic mean recommended by Adams and Cobb [2] was found not to give agreement with our experimental results; neither does Eq. (5c) give agreement.) It would seem, therefore, that our observers in their estimates of lightness followed instructions not to fixate the sample being judged too long; and, indeed, since light and dark samples such as could exemplify approximately the end points (black and white) of the lightness scale, were always present in the field, there was an incentive to shift the fixation point often from the sample being judged to those representative of the end points. For estimates of hue and saturation, however, no fixed points were exemplified in the field of view, and Table XI shows that Case 4 yields agreement with our observers which is not significantly different from that yielded by Case 3. It is possible, therefore, that some of our observers fixated the sample whose color was being judged long enough to make Case 4 apply. Case 2 gives fairly good predictions for samples on the light background, but for the red and

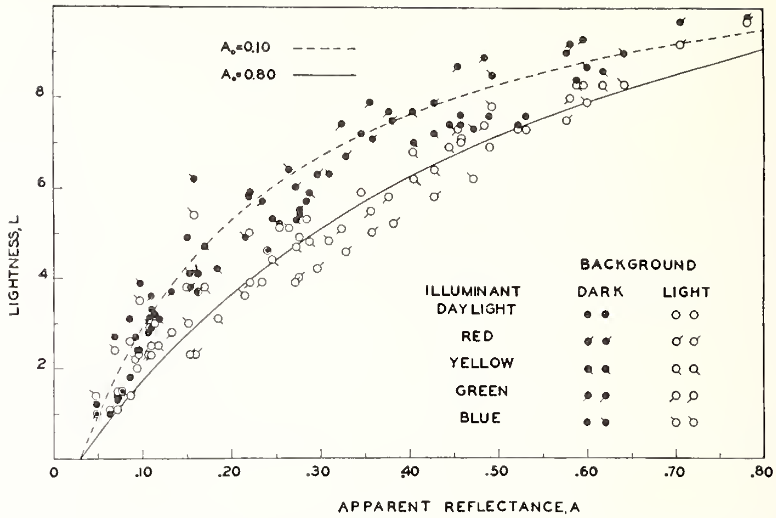

FIG. 4. Comparison of estimated lightness, L, with that computed from Eqs. (4) and (5a). The agreement is generally good with the exception of dark samples on a light background for nonred illuminants. See Section VIII for discussion of this discrepancy.

yellow illuminants is definitely inapplicable to those on the dark. Case 1 yields hues in agreement with many of those actually reported, but as might be expected, the saturations are much higher than those reported.

Table XII summarizes the success of the formulation for the four observing situations.

TABLE XII. *Number of sample-illuminant-background combinations correct.*

Illuminant	Apparent Reflectance of Background A_0	Number Correct Case 3	Case 4	Case 2	Case 1
South daylight	0.80	12 15/7	12 10/7	12 /7	11 /5
	.10	12 14/7	12 8/7	12 /7	11 /6
Red	.80	15 15/12	15 7/12	15 /15	11 /0
	.10	15 15/11	14 7/13	9 /3	12 /0
Yellow	.80	14 15/13	14 9/14	10 /15	5 /0
	.10	12 15/12	12 6/10	5 /3	5 /0
Green	.80	10 15/13	9 12/13	10 /13	7 /2
	.10	14 15/12	14 12/10	10 /8	8 /8
Blue	.80	11 11/14	10 12/14	7 /10	8 /0
	.10	11 15/9	11 9/9	8 /8	6 /0
Total	.80	62 71/59	60 50/60	54 /60	42 /7
	.10	64 74/51	63 42/49	44 /29	42 /14
Grand total correct		126 145/110	123 92/109	98 /89	84 /21
Percent correct		84 97/73	82 61/73	65 /59	56 /14

Each computed result correct in the sense that it is within the range of the individual estimates is given in Tables X and XI in plain (not boldface) type. The total numbers correct in this sense out of the 15 samples are given in Table XII for each illuminant-background combination. If the total should be listed 15 15/15, perfect agreement between formulation and preliminary experiment would be indicated for hue, lightness and saturation; but if the total should be 0 0/0, no correspondence at all would be indicated. Totals for all illuminants are given for each background, and grand totals for each observing situation. The grand totals are also expressed in percent of the total number (150) of sample-illuminant-background combinations. It will be seen that the formulation for Case 3 yields 97 percent of these combinations in agreement with experiment in regard to lightness, 84 in regard to hue, and 73 percent in regard to saturation. The superiority of Case 3 over Cases 2 and 4 is gratifying because it indicates that the attempt to take into account the actual observing situation has been largely successful. On the other hand, there is need for further work before it can be said that this success is complete.

Out of the 56 combinations for which Case 3 failed to yield acceptable color descriptions, all but 4 can apparently be adjusted by a more careful choice of constants in the definition of L' and in Eq. (3), by a redefinition of hue (Eq. (2)) without the simplifying restriction that red shall be taken as the exact opposite of green, and by the use of a coordinate system giving improved chromaticity spacing. The preliminary data do not afford a very reliable basis for such revaluation of the constants; and it is planned next to examine carefully the more complete data of Helson and Jeffers [26] which have already served as guides in several aspects of the formulation. The 4 combinations which are still a puzzle are 2 on the light background (B 7/4, PB 7/4) under the green illuminant, and 2 under the blue illuminant, both involving Y 7/8, one for the light, the other for the dark background. It is interesting to note that the observers reported more nearly the daylight colors of these samples than apparently can be justified from the computations, but we do not believe that the observers remembered the daylight colors. Helson and Jeffers [25, 26] find color descriptions in accord with our computations for samples B 7/4 and PB 7/4 under the green illuminant; this finding suggests an unknown source of error in our preliminary experiments. But they find the same colors as our observers did for sample Y 7/8 under the blue illuminant; this finding suggests that the formulation may have to be revised.

The large individual variations in estimates of saturation and the smaller variations in hue can be explained, first by the ambiguity of the observing situation which permits allocation by the observer of any chromatic component of the illumination hue in indeterminate proportion between the illumination and the surface color,* and second by variations in eye movements such as distinguish Case 3 from Case 4. The need for these explanations in regard to saturation would be more acute, however, if our observers had agreed better in their saturation estimates in south daylight (Table X). A large part of the individual variation is to be ascribed to uncertainty inherent in our experimental method.

* Katz [48, p. 219] ascribes the large individual differences principally to differences in attitude. Gelb [16, p. 677] likewise mentions attitude which he found to be "fluctuating and unstable" corresponding to different organizations of the visual field. This view is close to ours in Section III, 2, c.

TABLE XIII. *Effect on the color of a sample caused by substitution of a dark for a light background.*

	Hue		
	Number of sample-illuminant combinations		
	Observed to change in the RYGBR sense by more than 1/16 of the hue circuit	Observed to change by less than 1/16 of the hue circuit	Observed to change in the RBGYR sense by more than 1/16 of the hue circuit
Computed to change in the RBGYR sense	0	5	7
Same hue on the 8-point hue scale	6	32	3
Computed to change in the RYGBR sense	13	8	1
	Lightness		
	Number of sample-illuminant combinations		
	Observed to decrease by more than 0.5	Observed to change by 0.5 or less	Observed to increase by more than 0.5
Computed to increase by more than 0.5	0	28	44
Computed to change by 0.5 or less	0	3	0
Computed to decrease by more than 0.5	0	0	0
	Saturation		
	Number of sample-illuminant combinations		
	Observed to decrease by more than 0.5	Observed to change by 0.5 or less	Observed to increase by more than 0.5
Computed to increase by more than 0.5	3	1	34
Computed to change by 0.5 or less	8	5	14
Computed to decrease by more than 0.5	8	1	1

3. Substitution of a dark for a light background

The comparison between computed and experimentally obtained hues and saturations has so far dealt only with the ranges in the experimental values. One might readily suppose that the agreement between computation and experiment for both light and dark backgrounds arises from an experimental uncertainty sufficiently large to cover any differences between them. The success of the formulation is more detailed and complete, however, than has yet been demonstrated, and to show this success we have compared the direction of hue differences computed for each sample-illuminant combination with that observed, taking the average observed values in Tables X and XI. Similar comparisons have been made for the direction of lightness and saturation differences and have been summarized in Table XIII. It will be noted that for about 65 percent of the sample-illuminant combinations the computed change agrees with that indicated by the average of the actual estimates in the sense that for these combinations either the direction of the difference agrees, when both observed and computed differences are significant, or both computed and observed differences are too small to be significant. About 33 percent of the combinations show disagreement in the limited sense either that no significant difference was observed even though predicted by the computation, or the reverse. There are 28 combinations (37 percent) which showed limited disagreement of this sort in lightness; for these combinations no significant difference was observed from substitution of a dark for a light background, but Eq. (4) indicates for each combination an increase in lightness greater than 0.5. This may also be seen from Fig. 4. It is probably significant that nearly all (13 out of 15) of the samples viewed by south daylight fall in this category. Helson [25, 26] has discussed the conditions under which the expected increase in lightness is found by observation.

The combinations showing direct contradiction between computation and experiment are few; 1 in hue, none in lightness, and 4 in saturation. The contradiction in hue is found for sample PB 7/4 under the yellow illuminant (see Table XI); the computed hue change is from YR to Y; the observed, from YG to Y. There is some doubt as to the significance of this apparent contradiction because of the low saturation (0 to 1) of the reported color. The 4 contradictions in saturation are found for samples R 3/7, G 5/7 and P 3/6 under the yellow illuminant, and R 7/6 under the blue (see Table XI). These discrepancies are by about 2 steps on the saturation scale-of-10; considering the large individual variation in estimates of saturation a few such contradictions are to be expected even in an average of estimates by 6 observers. At least one observer corroborated each computed result. It may be concluded that the formulation is successful in specifying the observed effect on color of the sample caused by substitution of a dark for a light background.

VIII. Influence of Light-Scattering Elements Between Sample and Retina

If the optical media between the sample plane and the retinal image of that plane contain light-scattering elements, it is to be expected that the aperture colors corresponding to all surfaces will be modified in the sense corresponding to admixture of light reflected from the field as a whole. The amount of the admixture would correspond to the number of light-scattering elements; these elements might be in the space between the observer and the samples (air molecules, dust or fog), on the surface of the filters through which the observers looked (imperfect polishing of the surface, dust or grease), or in the optical media of the eye, itself (incipient cataract, floating particles in the aqueous humor, inclusions or haze in the vitreous humor). Although such elements were present in all these places, no specific account of them has been taken, first, because it has been assumed that the amount of light scattered by them is too small to produce a significant effect, and second because such light would chiefly serve as a clue to the illumination [10] rather than influence the estimates of hue, lightness and saturation of the surface color.

Certain aspects of the formulation and comparison with experiment indicate, however, that the influence of light-scattering elements may not be negligible. The situation in which such influence would make itself most evident is the observation of dark samples on a light background. Light from the background would be scattered onto the retinal image of the dark sample, and even though amounting to a very small fraction of the retinal illuminance corresponding to the background, might constitute a considerable fraction of that corresponding to the dark sample, itself. In such a case the chromaticity of the color would be altered toward that of the scattered light and the observed lightness would be higher than could be accounted for by Eq. (4). Reference to Fig. 4 shows that all samples of apparent reflectance less than 0.18 were observed to have lightnesses higher than those computed from Eq. (4); the discrepancies being particularly large for the blue illuminant. It may therefore be significant that in Table XI the only sample-illuminant-background combinations yielding computed values too low to fall within the individual variation are 4 samples of low apparent reflectance on the light background under the blue illuminant. These discrepancies suggest the presence of light-scattering elements in the eye media so small that they are selective in their scattering power, scattering more short wave than long wave light somewhat in accord with the Rayleigh law that such scattering is inversely proportional to the fourth power of the wave-length. Light-scattering elements of this sort were postulated by Holladay [32] to explain certain of his measurements on the amount of stray light in the eye. Further studies of this quantity have been carried out by Bartley and Fry [5], by le Grand [20] and by Schouten [81]. Helmholtz [30, p. 274, 278] also noted that dark areas surrounded by a blue field take on a blue color and suggested that fluorescence of the eye media as well as scattering of light might be a contributing cause.

It should be noted in this connection that a part of the effect of the $(L')^2$ term in the formula for r_n (Eq. (3)) is to contribute to a red-blue component for dark samples on a light ground. This component is proportional to the apparent reflectance, A_f, and blue trilinear coordinate, b_f, of the field; so it is already closely of the form required to take account of Rayleigh scattering by the eye media. Since this term combined with $0.030D_f$ in the expression for g_n agrees qualitatively with Karwoski's findings [46] of the prevalence of red-blue in the after-images produced by intense stimuli, we have so far viewed it as an approximate representation of some photochemical or neurological fact of the retina or central nervous system. Perhaps, however, it is to be considered as partly based upon Rayleigh scattering and fluorescence by the eye media. A better approximation to Rayleigh scattering could be obtained by the substitution of the term, $1+10b_f$, for b_f; and it might also be worth while to modify Eq. (4) by the appropriate introduction of such a term. Appraisal of the value of such modifications will be easier after a detailed check of the formulation against the extensive experimental results of Helson and Jeffers [26] has been carried out.

IX. Bezold-Brücke Phenomenon and the Hue Change by Admixture of Achromatic Light

It was observed by v. Bezold [6] and by Brücke [9] that an increase in luminance not only increases the brightness of the aperture color but also introduces a change in hue. Red, yellow-red, and yellow-green shift toward yellow; red-blue and blue-green toward blue. By reverse application of assumption (*d*) (Section III) it will be noted that the present formulation should apply to this hue change, often called the "Bezold-Brücke" phenomenon. Inspection of Eq. (3) shows that the formulation agrees in a broad way with the Bezold-Brücke phenomenon provided the formulation be rewritten to apply to aperture colors by substitution of luminance for apparent luminous reflectance, A. There have been some quantitative studies of this phenomenon notably by Janicki and Lau [36] and by Purdy [76]* against which it is possible to check the present formulation, and, perhaps, obtain better values for the constants; but computations by Eqs. (1) and (3) for comparison with these quantitative results have not yet been made.

The hue change by admixture of achromatic light has been studied by Müller [77] both for dark and for light backgrounds, and by Abney [1] for dark background. Abney found red, yellow-red, and yellow-green to shift toward yellow, and red-blue to shift toward red. The hue change for a light background is analogous to that for dark with colors of red-blue hue, but reverses for red, yellow-red and yellow-green which shift away from yellow. It may be shown from Eq. (3) that the present formulation is in qualitative agreement with these known facts, but quantitative comparison has yet to be made.

These hue changes by change of luminance and by admixture of achromatic light make the concept of dominant wave-length an unreliable indication of hue. The present formulation may therefore lead to a substitute for dominant wave-length by means of which colorimetric results (such as obtained by the spectrophotometer) may be interpreted in accord with the actual hue

* Consult Purdy [76] for 14 other references to the Bezold-Brücke phenomenon, experimental and theoretical.

of the surface color. Aids to computation of hue by Eq. (1) which would make it readily applicable in practice have yet to be worked out.

X. Relation to Visual Theory

The present formulation giving approximately the hue, saturation and lightness of a surface color in any of a wide variety of illuminants for both light and dark backgrounds is, of course, in no sense a theory of vision. It does not separate photochemical activity from neural activity in the retina, nor retinal activity from cortical, nor even photochemical effects from fluorescence and scattering of light by optical media of the eye. The primaries of the coordinate system used are not intended to represent colors produced by structurally or functionally separated parts of the visual mechanism; they merely compose one of an infinite number of sets giving approximately uniform chromaticity spacing. It is true that the $(L')^2$ term is confined in Eq. (3) to the expression for r_n; similarly the term $0.030D_f$ in the expression for g_n finds no counterpart in the expression for r_n. This simplicity does not, however, indicate a fundamental significance to be attached to the primaries of the coordinate system so much as it indicates the preliminary character of Eq. (3) which has been derived empirically. Probably, further work will show the necessity of a small $(L')^2$ term in the equation for g_n, and of a term composed of D_f multiplied by a small constant in that for r_n. The sole reason for choice of the coordinate system used is its degree of approach to uniform chromaticity spacing and it would be logical to transfer the formulation to some other coordinate system if one could be devised to yield a more uniform chromaticity spacing.

The validation of the present empirical formulation does, however, clear the ground for construction of a theory. The relation of the chromatic components (hue and saturation) of the surface color to the corresponding aperture color and the different character of the correspondence between the achromatic components (lightness and brightness) foreshadowed by the work of Bocksch [7, p. 395] has been formulated and verified. The variables defining the viewing situation (A_f, r_f, g_f, b_f) have been formulated and their effect upon hue, saturation and lightness of the surface color have been quantitatively evaluated within a coordinate system defined in terms of the standard I. C. I. system [23, 39]. Various components of these effects have been well known, but the total amounts of them have not been evaluated till now.

From the variables found to be pertinent in defining the viewing situation it may be concluded that the more familiar theories of vision (Helmholtz, Hering, Ladd-Franklin) require elaboration before they are applicable to this problem. These theories refer with some adequacy to aperture color with a dark surrounding field, but not to the more complicated situation of objects in an illuminated space. The suggestion by Ives [34] for an extension of the Helmholtz theory [30, pp. 235, 276] to this case is seen (Section III, 2, e) to be an approximation to the truth for the limited groups of samples for which $A \doteq A_f$, for illuminants that are not too selective, and probably also for durations less than 10 seconds. It may be seen from Figs. 2 and 3, however, that some strongly selective illuminants give rise to achromatic points which are outside the region corresponding to real stimuli; we may conclude, therefore, that to follow out Ives' suggestion would require a coordinate system in which the spectrum locus falls well inside the Maxwell triangle, such as those suggested by Helmholtz [28] and Sinden [83] on other grounds.

The theory of G. E. Müller [64] goes far toward allocation of the various color effects to activities of one or another portion of the visual mechanism, and it is furthermore elaborated to take account of light and dark surrounding fields. The information supplied by our formulation might be used to quantify the Müller theory. An initial difficulty arising from lack of any distinction in the Müller theory between brightness and lightness has been discussed by Bocksch [7, p. 436ff] who suggested a way of elaborating the Müller theory in this respect. To extend the Müller theory, however, to the perception of object and illumination color (Katz [48], Gelb [16]) in such a way as to embrace the present formulation, including the appearance of surface-color hues nearly complementary to the illuminant hue by after image projection would be a long, involved task; but an attempt might be worth while because any resultant quantitative color theory would stimulate further visual research by being amenable to quantitative verification.

XI. Summary

The stimulus for an aperture color can be expressed as a function of three variables. Surface color, however, includes the idea of a surface viewed in the presence of other surfaces in an illuminated space; it requires at least six variables. The observer may perceive the visual field yielded by such surfaces as a pattern of juxtaposed color areas, or aperture colors; or he may perceive this field organized into objects in an illuminated space. The surfaces of the objects possess surface colors and the space possesses an illumination color; surface color and illumination color are both related to the pattern of aperture colors, illumination color being derived from clues obtained from the whole visual field and its organization. Simple visual fields, such as those studied here, can be produced by more than one combination of objects with illumination, and accordingly, it is usual to find that observers will report more than one organization of them. This ambiguity generates large individual variations. Other variations arise because the eye-movements of the observer are uncontrolled; that is, the surface color may depend somewhat upon whether he has just previously been looking at the darkest or the lightest object in the field.

From a study of the literature, formulas for hue, lightness and saturation of surface colors have been set up for all illuminants (daylight and chromatic), for all backgrounds (black, gray, white, and chromatic), and for 4 viewing situations. These formulas for the most common viewing situation are based upon the principle known in Helmholtz' time [30, p. 267] that the hue and saturation of the surface color seen at any given instant is the result of projecting upon the field provided by the surface an after image of the average field seen for the previous 15 or 20 seconds. It has been found impractical to represent changes in retinal sensitivity which produce these after-images by changes in coordinate system so as to make achromatic colors correspond to equal stimulation of the red, green and blue processes as in the Young-Helmholtz theory. Instead, the same coordinate system has been used for all states of chromatic adaptation, a central part of the formulation being the definition of the trilinear coordinates of the point in this system representing for each state of adaptation the stimulus for an achromatic color. This achromatic point is found to wander about in the neighborhood of the point representing the average of the visual field. There is, therefore, scarcely any real stimulus which cannot under some viewing situation give rise to an achromatic color, and, indeed, many of the achromatic points are to be found outside of the area representing real stimuli (that is, outside of the area of the Maxwell triangle bounded by the spectrum locus and the straight line joining its extremes).

The formulas have been checked and their constants approximately evaluated by means of preliminary observations by 6 observers viewing 15 samples of known spectral apparent reflectance under 5 illuminants (daylight and four strongly chromatic illuminants) of known spectral energy distribution with two spectrally nonselective backgrounds (in daylight, dark gray and white) of known apparent reflectance. The formulas have been improved by a preliminary study of more careful and extensive observations of the same samples under the same illuminants by Helson [24, 25, 26]. The hues, lightnesses and saturations computed by the improved formulas have been compared with the preliminary results and have been shown to be in agreement with estimated lightness in 97 percent of the 150 sample-illuminant-background combinations studied, to be in agreement with estimated hue in 84 percent, and to be in agreement with estimated saturation in 73 percent of the combinations studied. The connection of the formulas to theories of vision has been pointed out, and there has been given a brief discussion of their application to visual phenomena known by other names (the "dimming effect," light-scattering by the optical media of the eye, Bezold-Brücke phenomenon, and the hue change by admixture of achromatic light).

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THE MUNSELL COLOR SYSTEM - FOREWORD

J. Opt. Soc. Am. 30, 574 (1940)

This brief statement is significant because it places in perspective not only the five papers that it preceded but also the development of the Munsell concept over four decades. The distinction between psychological and psychophysical definitions and systems that underlies this *Foreword* is that between verbal descriptions of sense impressions and quantitative determinations of physical circumstances under which identical sense impressions are produced. Quantitative specifications of physical conditions or procedures constitute operational definitions, such as were the foundation of the reconstruction of physics that occurred during the third and fourth decades of the twentieth century. Adoption of the operational (i.e., psychophysical) definition of color resulted in the modern development of colorimetry.

The Munsell Color System

Foreword

AN obvious purpose is served by arranging this series of five papers on the Munsell Color System together as a unit. The various steps in the development of the ideas of the originator, Albert H. Munsell, may in this way be presented together with the technical data resulting from each step so that for the first time it is possible for a reader to trace this development in complete detail.

But there is a second, less obvious, though no less important purpose served by this series of papers. It arises from the fact that Mr. Munsell's original idea for color specification had two aspects: First, that the color notation should indicate color as perceived by the observer; second, that the assignment of the notations should be based upon an accurately reproducible system of measurement. That is, the ideal color notation should conform to a purely psychological requirement and at the same time enjoy a rigorous psychophysical definition.[1]

It is a common first impression of many students of colorimetry that this double requirement is easy to meet; in fact, it is often erroneously supposed by them that there is no need for the distinction between psychological and psychophysical systems. More thorough consideration reveals this error and often gives rise to the opinion that the two kinds of system are so utterly different in their concepts that there is no possibility of correspondence between them. To one who holds this opinion, Mr. Munsell's original idea seems to be based upon ignorance and impossible to fulfill.

[1] Color is defined as a psychophysical quantity in the report of the Committee on Colorimetry now in preparation. In this series of papers on the Munsell Color System, the psychological definition of color given in the 1922 report (J. Opt. Soc. Am. **6**, 531 (1922)) has, therefore, been superseded by a psychophysical definition.

There are possible, however, many psychophysical color systems. Some of them, such as the tristimulus system, yield variables having no obvious relation to the psychological attributes of color as perceived by the observer. Others, such as the dominant wave-length and purity system, yield variables having some degree of correspondence to them. Mr. Munsell knew well that the search for a psychophysical color system whose notation should correspond with what an observer sees is far from hopeless because he discovered a psychophysical system which fulfills this condition to a surprising degree, better than any yet found of comparable simplicity.

The second purpose of arranging these papers into a series is, therefore, to bring out the much misunderstood relations and distinctions between psychological and psychophysical color systems. The second paper of this series, that by J. E. Tyler and A. C. Hardy, analyzes the psychophysical color system discovered by A. H. Munsell. The third paper, that by K. S. Gibson and D. Nickerson, shows the degree to which the color standards of the 1915 *Atlas of the Munsell Color System* accord with this psychophysical color system. The fourth paper, that by J. J. Glenn and J. T. Killian, defines psychophysically the color system previously defined only by the material color standards of the 1929 *Munsell Book of Color*. The final paper of the series, "Preliminary report of the O.S.A. subcommittee on the spacing of the Munsell colors," by S. M. Newhall, Chairman, presents extensive data on which is to be based a psychophysical color system intended to fulfill as closely as possible the psychological ideal of A. H. Munsell.

D. B. Judd, *Associate Editor*

INTRODUCTION TO COLOR

Paper in <u>Symposium on Color</u>, pp. 1-13 (American Society for Testing and Materials, Philadelphia, Pa., 1941)

The story about the man on the desert island who found a lot of colored papers in a trunk washed up on the beach inspired the profusely illustrated but more prosaic explanation of how order can be made out of color chaos in <u>The Science of Color</u> of the Optical Society Committee on Colorimetry. Judd was a very active member of that committee.

The initials I.C.I. of the English name of the International Commission on Illumination have been abandoned because of the likelihood of confusion of them with the designation of a large British company that is prominent in connection with color. By international agreement, the initials CIE of the French name, Commission Internationale de l'Éclairage, are now used in all English- and French-speaking countries. The initials IBK of the German form of the name are sometimes encountered, as are MKO of the Russian form.

All of the Judd papers included in the list of references are included in the present collection.

Errata: P. 4, line 37, second word should be "thoughts"; p. 9, line 10, insert "non-" before "negative".

INTRODUCTION TO COLOR

By Deane B. Judd[1]

I am somewhat embarrassed at being referred to as a "color expert." One of the difficulties of a so-called color expert is that nearly everyone knows a good deal about some phase or other of color and expects the expert to know more. He is supposed to discuss chromophores with the chemist, mordants with the dyer, spectrophotometric curves with the physicist, color specifications with the engineer, and color sensation and perception with the psychologist.

Color has many different meanings. The chemist and dyer may refer to a pigment or a dye as a color, the physicist may mean a spectrophotometric curve, the psychologist may mean a sensation, but to the man in the street, and the woman in the store, color is a property of objects and lights themselves.

All of these meanings for color make sense to those who use them, and there is a good deal of sense to all of them. In our discussion of the specification and use of color in evaluating the appearance of materials it would be desirable to take some one meaning for the term "color." It may as well be admitted now, however, that there is small hope of achieving a uniform usage for the term "color" however desirable it might be from the standpoint of avoiding confusion. Let us therefore see first what can be done toward defining color from the commonsense point of view, and later see what departures from this definition are required for color specification and measurement of materials.

Psychological Definition of Color

We shall not be too far from the commonsense definition if we say that color is everything that is seen by the eye. That takes in too much territory, of course, but it gets across the general idea. The eye also sees shape, texture, and flicker. Maybe it would be more accurate to say that color is the nonshape, nontemporal, nontextural aspect of appearance. This is a rather negative definition; and it would be embarrassing if we discovered later that we should have made other exceptions. So let us try a more positive form of definition. *Color is that aspect of the appearance of objects and lights which depends upon the spectral composition of the radiant energy reaching the retina of the eye and upon its temporal and spatial distribution thereon.* Let us not stop to argue over every word of this definition, however amusing it might be to do so. Suffice it to say that this definition is the best I can do to put the commonsense idea of color into words of as many as four syllables.

Of course, everybody sees color, and knows about it much more thoroughly through his own eyes than he ever is likely to from a definition composed of four-syllable words. Everybody organizes his own color experience and can describe it in more or less understandable terms. The difficulty comes

[1] Physicist, National Bureau of Standards, Washington, D. C.

about from the fact that each one's color terms refer to his own special problems. The textile dyer says that these two colors differ in strength, because one looks to him as if produced by more dye than the other. The artist says they differ in warmth because he is interested in producing a warm or a cool effect.

So let us get away from all special interests and purposes. Let us see how a man on a desert island would sort out a batch of colored papers washed up on the beach in a trunk.

Attributes of Color

First he spreads them out on the sand to look at them and finds a purely chance arrangement or disorder. Then he notices that a certain group of the colors lack an important characteristic possessed by the others; that is, they lack *hue*. He separates the colors into two groups—chromatic colors, those that possess hue, and achromatic colors, those that do not possess hue. He notes that the achromatic colors can be arranged in a single series extending from black at the bottom through a series of dark grays, middle grays, and light grays to white at the top; this series varies in lightness alone. Furthermore, for each chromatic color he notes that there can be found a gray of the same lightness, called the equivalent gray.

Then he turns his attention to the chromatic colors and sorts them out, putting all the reds together, then the yellows, greens and blues; and he also notes that there are others falling intermediate to these groups so that a whole circle can be set up, each part of which differs from the neighboring part by only a small step. This classification is by hue.

Of course, some of the colors of a given hue are found to be dark colors while others are light; he has already noticed this fact by comparison with the lightness scale of achromatic colors. But by sorting out all colors of the same hue (say red) and same lightness (say medium lightness) he discovers that there still are variations. The mode of variation is more subtle than the other attributes; it is called saturation. He finds that saturation supplies the basis for his very first step toward order; for achromatic colors (black, grays, white) saturation is zero; for chromatic colors, greater than zero. Saturation is proportional to the color departure from the equivalent gray.

Since there are three attributes of object-color, our hypothetical desert islander finds that he cannot arrange all of the colors in the trunk in a single plane diagram and still keep them in order. He can make an orderly arrangement of all colors of the same lightness, however; so he divides up the colors into groups, each group of about the same lightness; then arranges each group separately, and by placing the darkest colors on a disk, the next lighter group on a higher disk, as in a layer cake, he builds up a solid representation of all three attributes of color. Lightness is represented by distance above the base plane; hue, by angle about the central or black-gray-white axis; and saturation by distance from the axis. This space representation, in which each point represents a color, is called a color solid.

Modes of Appearance of Object Colors

Now suppose that our lonely color expert had found his trunk filled, not with colored papers, but with dye solutions in test tubes. If he had been a dye chemist in his laboratory, he would have classified them, perhaps, first according to molecular structure, then according to concentration or strength, and he would have paid no attention to their appearance. But, he has nothing but his eyes to guide him, so he proceeds just as he did with the colored papers, and arrives at exactly the same attributes of color for transparent volumes, as he did for opaque surfaces. He can describe the difference between the colors of two dye solutions in terms of hue, lightness, and saturation just as aptly as that between the colors of two papers. But there is one distinguishing mark in the color solid for transparent volumes; the topmost point in the solid, occupied by white in the solid representing the colors of surfaces, is now occupied by a color known paradoxically as colorless, see Fig. 1. This is the name by which the color of perfectly transparent objects (plastics, glasses, crystals) and media (water, alcohol, oils) which do not absorb any light has come to be known.

USEFULNESS OF THE COLOR SOLIDS

What has been gained by recourse to the man on the desert isle? Have we found a system less useful because it is not adapted perfectly to use by dyers, painters, chemists, botanists, physicists, artists or any special use? No, as a matter of fact, description of color by hue, lightness and saturation is used to some extent by all of them and above all by our friends, the man in the street and the woman in the store. Hue is the most prominent attribute of color, and is usually the first thought of. Color and color differences are universally described by such terms as red, yellow, green, blue and purple. The adjectives suggested by the attribute *lightness* are *light* and *dark*, universally used to describe color; and those suggested by the attribute *saturation* are *strong* (intense) and *weak* (dull). The term *strong* means saturated, and *weak* means not saturated. But it should be admitted right away that the dyer and dye chemist do not

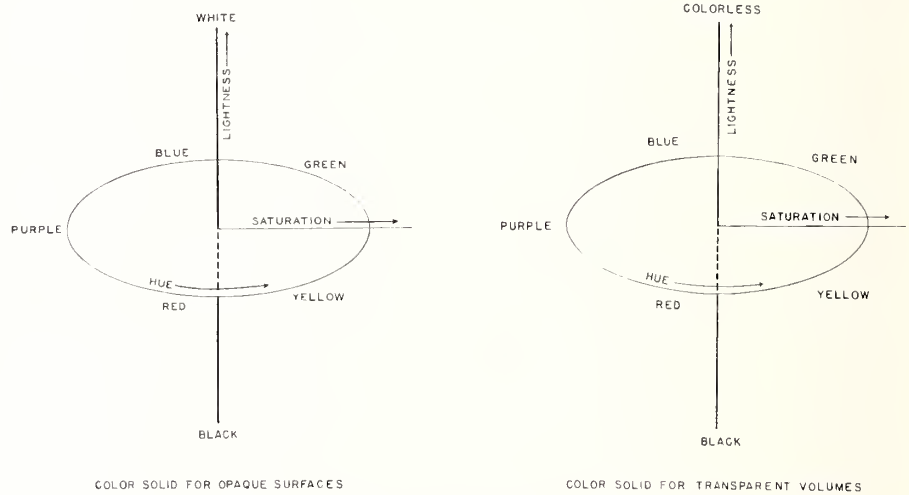


FIG. 1.—Dimensions and Fixed Points of Two Object-Color Solids.

The attributes of colors perceived as belonging to opaque surfaces are the same as those of colors perceived as belonging to transparent volumes. However, lightness for opaque surfaces varies from black to white, that for transparent volumes between black and colorless.

use the terms *strong* and *weak* in this sense, nor indeed are they used as strictly color terms by the dyer since the precise meaning of stronger varies for him from one dye to another even though the same color may be dyed by means of each. The substitute terms, *intense* and *dull* have sometimes been used.

A recent important study has been carried out by one of the du Pont Laboratories to see whether those trained in the detection of color differences for special purposes and the description of them in special terms could make use of the simple terms suggested by the object-color solids. It was discovered that not only was this possible, but there was actually a significant gain in agreement among the descriptions; and it was considered reasonable to ascribe this gain to the greater simplicity of the terms. This result is the more surprising because it appears likely that the maximum benefit of the simplified terms would be achieved only after the observers had become thoroughly familiar with the concepts so as to experience the color differences directly in accord with them. Many of the observers in this experiment were forced by unfamiliarity to judge the differences in terms of their previous training, then to translate them more or less accurately into hue, lightness, and saturation.

The various color solids not only organize in the simplest possible way, our throughts about color and color differences, but they also permit us to understand special color terms which would otherwise be obscure. Such terms as paler, deeper, cleaner, muddier, blacker and whiter, refer to paths or directions in the color solid. Thus, *paler* means both lighter and weaker, and its opposite *deeper* means both darker and stronger. Blacker means closer to black in the color solid, and so on. The term, *depth* of color is used in a number of A.S.T.M. specifications. The color solids have been used as guides in the selection of terminology for the I.S.C.C.-N.B.S. method of designating colors (11).[2]

TECHNICAL DEFINITION OF COLOR

We have seen how the commonsense definition of color leads to the various forms of object-color solid, in which each point represents a single color characterized by its particular hue, lightness, and saturation. One might readily suppose that measurement of the color of a given object consists merely of locating the proper point in the appropriate color solid. There are several reasons why this will not work. In the first place hue, lightness, and saturation are psychological quantities and can be determined by visual estimate only. There are long techniques for carrying out such estimates with considerable accuracy, but any rapid estimate is much too imprecise for engineering use. Secondly, no two observers would get the same result; and gross discrepancies would be frequent. And lastly, and by far the worst, each object has not one but many colors depending upon the amount and spectral composition of the illumination and upon the other colors in the visual field at the same time. An object in a given situation has a certain color or appearance, but in another situation a quite different appearance. This fact is well known, but frequently overlooked. It is well recognized by milady in the purchase of an evening gown. She must know the colors used in the decoration of the ballroom, and, if possible, the colors of the other gowns to be present. If everyone else is wearing a vivid red dress her own moderate red gown might be re-

[2] The boldface numbers in parentheses refer to the reports and papers appearing in the list of references appended to this paper, see p. 13.

duced to mauve by simple retinal fatigue. So it is not possible to determine exactly what point in the color solid represents the color of an object no matter how much we know about the object itself by physical measurement.

Now it is very awkward to be forced to have a different color specification for a given specimen for each different viewing situation. So practical considerations dictate a restriction of conditions. Color measurement is therefore customarily carried out so as to refer first to a standard background, second for each illuminant separately, third for a standard observer adopted to be representative of observers of normal color vision, and fourth by psychophysical quantities which correlate more or less well with the psychological attributes, hue, lightness and saturation.

The quantity that is measured in modern colorimetry is therefore somewhat too restricted to correlate perfectly with the psychological definition of color, the most important restriction often not recognized being that the measurements give a reliable indication of appearance for one background only. So valuable has modern colorimetry proved, however, that it is very tempting to redefine color as the quantity measured by colorimetry. The basis for this redefinition is that we perceive objects only by means of the light which comes from them; therefore let us define color as an aspect of light, only, somewhat as follows: *Color is that aspect of the appearance of light which depends upon its spectral composition.* Formulation of the strict technical definition of color has been accomplished only recently by the Committee on Colorimetry of the Optical Society of America whose report is now in preparation. To trace the full implications of this definition including the complete argument by which an object may be said to have color in spite of the fact that color is defined as an appearance aspect of light, only, takes up many pages of the report; so no attempt will be made to elaborate the argument further at present. Suffice it to say that the work has been carefully done, and deserves to be studied thoroughly by anyone writing color specifications because it will necessarily exert a profound influence on colorimetric practice for many years to come.

The technical definition of color is called a psychophysical definition. Color by this definition may be evaluated by physical measurements taken on the illuminant and object combined with information obtained by the psychological act known as color matching. Color is specified by reference to a point in some psychophysical color system which correlates more or less closely with the purely psychological arrangements given by the color solids. All of the color systems to be mentioned hereafter will be psychophysical systems.

PHYSICAL INSTRUMENTS AND APPEARANCE OF OBJECTS

Physical measurements of objects which correlate with color and its mode of appearance are made with the spectrophotometer and the goniophotometer. The spectrophotometer compares in spectral composition the radiant energy leaving an object with that incident upon it. The two beams of radiant energy are first decomposed into their spectral components, and then compared photometrically wave length by wave length. The spectrophotometer yields primary information relative to color and is used in the A.S.T.M. Standard Method of Test for Spectral Apparent Reflectivity of Paints (D 307 – 39).[3] The goniophotometer determines

[3] 1939 Book of A.S.T.M. Standards, Part II, p. 815.

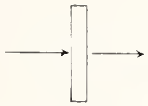
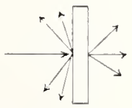
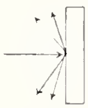
the angular distribution of radiant energy leaving an object. It yields primary information relative to glossiness and transparency which are aspects of appearance closely associated with color and with its mode of appearance.

Table I shows the correspondence between physical characteristics of the object, the geometry of the measurement, the instrument, the way its color is judged, and the color solid appropriate to representation of its color.

energy are seldom used. But in the development and testing of the optical theory on which practice is based, frequent use of the spectrophotometer and goniophotometer have been made. The quantity of primary interest is transparency; this quantity is not an attribute of color but determines its mode of appearance.

Opaque materials also require both the spectrophotometer and the goniophotometer. The use of the spectrophotometer for determining the color of opaque materials is discussed by Parker.[6] This instrument is used not only for the study of working standards of color discussed by Godlove,[7] but also increasingly for the specimens themselves. The goniophotometer yields primary information relative to glossiness which, although not an attribute of color, is so important an aspect of appearance that color and glossiness have often to be considered together.

TABLE I.—CORRELATION BETWEEN PHYSICAL MEASUREMENT OF THE OBJECT AND ITS COLOR.

	Transparent Sample, Measured on the Spectrophotometer	Translucent Sample, Measured on the Spectrophotometer and Goniophotometer	Opaque Sample, Measured on the Spectrophotometer and Goniophotometer
Geometry of the measurement			
Quantity measured	Amount and spectral distribution of transmitted energy	Amount, spectral and angular distribution of emitted energy	Amount, spectral and angular distribution of reflected energy
Color judged by	Transmitted light	Transmitted and reflected light	Reflected light
Appropriate color solid	Volume-color solid	....	Surface-color solid

Transparent materials require the use of the spectrophotometer only. In practice, specimens are most frequently color matched with material standards prepared for that purpose; but the standards themselves are usually specified by spectrophotometric measurements. These standards are discussed by Scofield.[4]

Translucent materials require the use of both spectrophotometer and goniophotometer. The practical methods of handling translucent materials are described by Sawyer.[5] Here again the instruments giving a complete analysis of the angular and spectral distribution of the transmitted and reflected radiant

COLOR SPECIFICATION BY COMBINATIONS OF LIGHTS

We come now to the methods of expressing spectrophotometric results.

[4] See p. 18.
[5] See p. 23.
[6] See p. 47.
[7] See p. 37.

We have already seen that a standard illuminant must be chosen. In this way we can characterize all objects in accord with the reflected or transmitted energy which leaves them. There are two chief ways of specifying such energy: first, by color match with an additive combination of three fixed lights or primaries; and second, by color match with an additive combination of two lights, one fixed such as daylight, the other variable, such as the various parts of the spectrum. The first way is known as tristimulus specification and is important because it permits an easy reduction of spectrophotometric data. The best-known example of the fixed-light, variable-light way is by dominant wave length and purity; this way is important because it yields variables which correlate to a degree with the psychological color attributes, hue and saturation. The fixed light is usually the same as the illuminant and is represented by the origin of a system of polar coordinates.

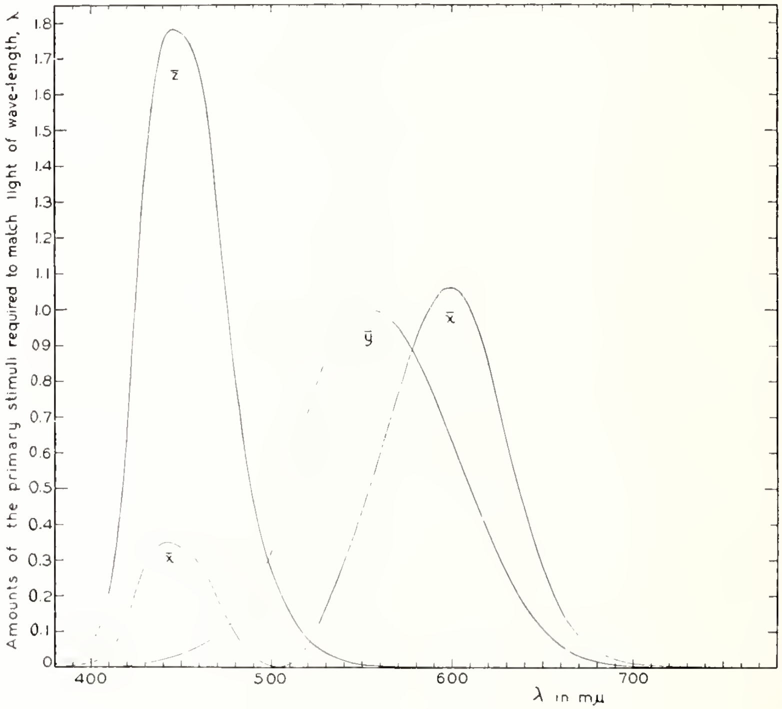


FIG. 2.—Tristimulus Specifications of the Various Parts of the Spectrum According to the 1931 I.C.I. (or C.I.E.) Standard Observer and Coordinate System; see also Table II.

Tristimulus Specification.—We have already seen that object colors viewed under constant conditions by our desert islander vary in three independent ways. We should therefore not be surprised that three lights are required in an additive mixture to color match the beams of light leaving such objects. The amounts of the three fixed lights or primaries constitute the specification. If the photometric field be illuminated by two components of tristimulus specification, X_1, Y_1, Z_1, and X_2, Y_2, Z_2, respectively, the specification of the resultant is simply: $X_1 + X_2$, $Y_1 + Y_2$,

$Z_1 + Z_2$. The same principle applies, of course, to additive combinations of any number of lights. This simple law has been repeatedly verified for a large middle range of brightnesses, and when have to know the tristimulus specifications of each part of the spectrum. These specifications vary somewhat from observer to observer and may be given for any one of a wide number of pri-

TABLE II.—THE 1931 I.C.I. STANDARD OBSERVER.

Trichromatic Coefficients			Wave Length, (mμ)	Tristimulus Specifications of the Equal-Energy Spectrum			Trichromatic Coefficients			Wave Length, (mμ)	Tristimulus Specifications of the Equal-Energy Spectrum		
x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$	x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$
0.1741	0.0050	0.8209	380	0.0014	0.0000	0.0065	0.5125	0.4866	0.0009	580	0.9163	0.8700	0.0017
0.1740	0.0050	0.8210	385	0.0022	0.0001	0.0105	0.5448	0.4544	0.0008	585	0.9786	0.8163	0.0014
0.1738	0.0049	0.8213	390	0.0042	0.0001	0.0201	0.5752	0.4242	0.0006	590	1.0263	0.7570	0.0011
0.1736	0.0049	0.8215	395	0.0076	0.0002	0.0362	0.6029	0.3965	0.0006	595	1.0567	0.6949	0.0010
0.1733	0.0048	0.8219	400	0.0143	0.0004	0.0679	0.6270	0.3725	0.0005	600	1.0622	0.6310	0.0008
0.1730	0.0048	0.8222	405	0.0232	0.0006	0.1102	0.6482	0.3514	0.0004	605	1.0456	0.5668	0.0006
0.1726	0.0048	0.8226	410	0.0435	0.0012	0.2074	0.6658	0.3340	0.0002	610	1.0026	0.5030	0.0003
0.1721	0.0048	0.8231	415	0.0776	0.0022	0.3713	0.6801	0.3197	0.0002	615	0.9384	0.4412	0.0002
0.1714	0.0051	0.8235	420	0.1344	0.0040	0.6456	0.6915	0.3083	0.0002	620	0.8544	0.3810	0.0002
0.1703	0.0058	0.8239	425	0.2148	0.0073	1.0391	0.7006	0.2993	0.0001	625	0.7514	0.3210	0.0001
0.1689	0.0069	0.8242	430	0.2839	0.0116	1.3856	0.7079	0.2920	0.0001	630	0.6424	0.2650	0.0000
0.1669	0.0086	0.8245	435	0.3285	0.0168	1.6230	0.7140	0.2859	0.0001	635	0.5419	0.2170	0.0000
0.1644	0.0109	0.8247	440	0.3483	0.0230	1.7471	0.7190	0.2809	0.0001	640	0.4479	0.1750	0.0000
0.1611	0.0138	0.8251	445	0.3481	0.0298	1.7826	0.7230	0.2770	0.0000	645	0.3608	0.1382	0.0000
0.1566	0.0177	0.8257	450	0.3362	0.0380	1.7721	0.7260	0.2740	0.0000	650	0.2835	0.1070	0.0000
0.1510	0.0227	0.8263	455	0.3187	0.0480	1.7441	0.7283	0.2717	0.0000	655	0.2187	0.0816	0.0000
0.1440	0.0297	0.8263	460	0.2908	0.0600	1.6692	0.7300	0.2700	0.0000	660	0.1649	0.0610	0.0000
0.1355	0.0399	0.8246	465	0.2511	0.0739	1.5281	0.7311	0.2689	0.0000	665	0.1212	0.0446	0.0000
0.1241	0.0578	0.8181	470	0.1954	0.0910	1.2876	0.7320	0.2680	0.0000	670	0.0874	0.0320	0.0000
0.1096	0.0868	0.8036	475	0.1421	0.1126	1.0419	0.7327	0.2673	0.0000	675	0.0636	0.0232	0.0000
0.0913	0.1327	0.7760	480	0.0956	0.1390	0.8130	0.7334	0.2666	0.0000	680	0.0468	0.0170	0.0000
0.0687	0.2007	0.7306	485	0.0580	0.1693	0.6162	0.7340	0.2660	0.0000	685	0.0329	0.0119	0.0000
0.0454	0.2950	0.6596	490	0.0320	0.2080	0.4652	0.7344	0.2656	0.0000	690	0.0227	0.0082	0.0000
0.0235	0.4127	0.5638	495	0.0147	0.2586	0.3533	0.7346	0.2654	0.0000	695	0.0158	0.0057	0.0000
0.0082	0.5384	0.4534	500	0.0049	0.3230	0.2720	0.7347	0.2653	0.0000	700	0.0114	0.0041	0.0000
0.0039	0.6548	0.3413	505	0.0024	0.4073	0.2123	0.7347	0.2653	0.0000	705	0.0081	0.0029	0.0000
0.0139	0.7502	0.2359	510	0.0093	0.5030	0.1582	0.7347	0.2653	0.0000	710	0.0058	0.0021	0.0000
0.0389	0.8120	0.1491	515	0.0291	0.6082	0.1117	0.7347	0.2653	0.0000	715	0.0041	0.0015	0.0000
0.0743	0.8338	0.0919	520	0.0633	0.7100	0.0782	0.7347	0.2653	0.0000	720	0.0029	0.0010	0.0000
0.1142	0.8262	0.0596	525	0.1096	0.7932	0.0573	0.7347	0.2653	0.0000	725	0.0020	0.0007	0.0000
0.1547	0.8059	0.0394	530	0.1655	0.8620	0.0422	0.7347	0.2653	0.0000	730	0.0014	0.0005	0.0000
0.1929	0.7816	0.0255	535	0.2257	0.9149	0.0298	0.7347	0.2653	0.0000	735	0.0010	0.0004	0.0000
0.2296	0.7543	0.0161	540	0.2904	0.9540	0.0203	0.7347	0.2653	0.0000	740	0.0007	0.0003	0.0000
0.2658	0.7243	0.0099	545	0.3597	0.9803	0.0134	0.7347	0.2653	0.0000	745	0.0005	0.0002	0.0000
0.3016	0.6923	0.0061	550	0.4334	0.9950	0.0087	0.7347	0.2653	0.0000	750	0.0003	0.0001	0.0000
0.3373	0.6589	0.0038	555	0.5121	1.0002	0.0057	0.7347	0.2653	0.0000	755	0.0002	0.0001	0.0000
0.3731	0.6245	0.0024	560	0.5945	0.9950	0.0039	0.7347	0.2653	0.0000	760	0.0002	0.0001	0.0000
0.4087	0.5896	0.0017	565	0.6784	0.9786	0.0027	0.7347	0.2653	0.0000	765	0.0001	0.0000	0.0000
0.4441	0.5547	0.0012	570	0.7621	0.9520	0.0021	0.7347	0.2653	0.0000	770	0.0001	0.0000	0.0000
0.4788	0.5202	0.0010	575	0.8425	0.9154	0.0018	0.7347	0.2653	0.0000	775	0.0000	0.0000	0.0000
0.5125	0.4866	0.0009	580	0.9163	0.8700	0.0017	0.7347	0.2653	0.0000	780	0.0000	0.0000	0.0000
Totals											21.3713	21.3714	21.3715

it is recalled that any light may be considered as the sum of the various parts of the spectrum, it is evident that the color specification corresponding to any spectrophotometric curve can be computed by simple addition. Of course we maries. It is almost universal practice, however, to use tristimulus specifications of the spectrum according to the standard observer and primaries recommended in 1931 by the International Commission on Illumination (3, 4, 8). Figure 2 shows these specifications for the equal-energy spectrum; and Table II gives them.

The advantage of having all colorimetric specifications given in the same system, and so immediately comparable, is very great; furthermore, the coordinate system chosen has advantages in routine computation; so it was adopted in Great Britain, Germany, and France as well as in this country and its use is constantly spreading. The I.C.I. system is used in the A.S.T.M. Tentative Method of Test for Color of Lubricating Oil and Petrolatum by Means of A.S.T.M. Union Colorimeter (D 155 – 39 T),[8] and it will undoubtedly be used in many A.S.T.M. specifications yet to be drafted.

Although the theory of tristimulus specification of color is simple, the choice of primaries to accord with easy routine reduction of spectrophotometric results has introduced features which are somewhat puzzling. The following questions and answers about the 1931 I.C.I. standard observer and coordinate system may be helpful. Of course, from a practical standpoint these questions might just as well not be brought up; you can go ahead and use the system on faith, but most of us would rather know some of the answers even though they do not affect the practical use of the system.

Question 1.—What are the primary lights of the I.C.I. system? Where can I purchase them?

Answer.—You cannot purchase them; they are all imaginary. You can easily draw a graph of energy distributions of these imaginary lights having the requisite colorimetric characteristics, but these distributions will be found to have negative ordinates for some portions of the spectrum (6, p. 516).

Question 2.—Why were not actual lights that can be set up and used taken instead of fictitious lights?

Answer.—Actual lights would lead to tristimulus specifications of the spectrum negative for some portions of the spectrum. It is better to have non-negative specifications than to have negative lights. You have to use the specifications in computation all the time but you could not use the primary lights even if they were real.

Question 3.—Why could not real lights be used?

Answer.—Because we do not have a real standard observer either. He is nothing but a column of figures obtained by averaging.

Question 4.—How do you know that the imaginary lights selected are the correct ones?

Answer.—Any three lights, real or imaginary, may be selected, that is, provided only that no two of them can be combined to color match the third. There is, therefore, no "correct" selection. The choice is based on convenience only (6, p. 544).

Question 5.—What is so convenient about the I.C.I. system?

Answer.—Well, for one thing you can easily find the candlepower of a light from its tristimulus specification.

Question 6.—How can I get candlepower of a light from its tristimulus specification?

Answer.—The candlepower is directly proportional to the amount of the second primary, Y, (1). This comes about from the fact that the second curve, the $\bar{y}$-curve, is the standard luminosity function (6, p. 544).

Question 7.—If the luminosity comes from the Y light, what is the luminosity of the X light and the Z light?

[8] 1939 Book of A.S.T.M. Standards, Part III, p. 598.

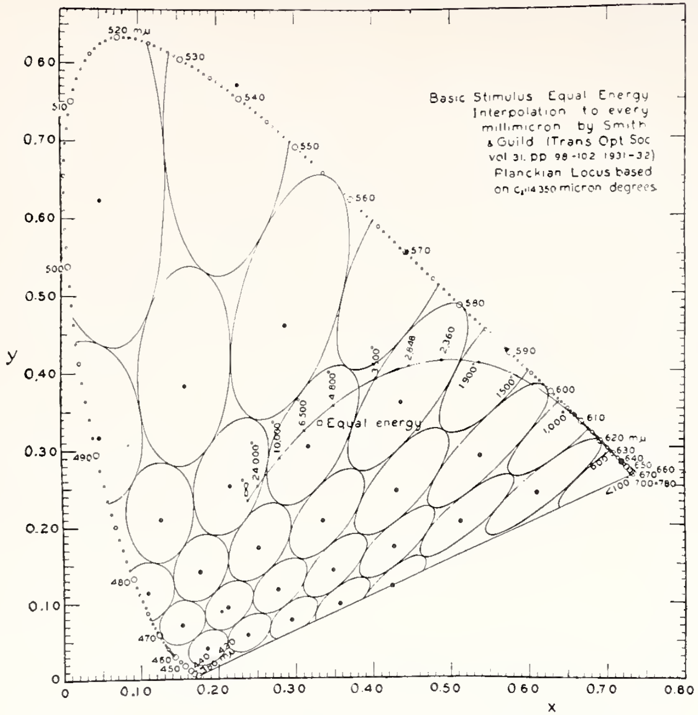


FIG. 3—Chromaticity (or Mixture) Diagram Formed by Plotting in Rectangular Coordinates the Trichromatic Coefficient, x, Against the Trichromatic Coefficient, y.

These trichromatic coefficients refer to the 1931 I.C.I. (or C.I.E.) standard observer and coordinate system. The small circles indicate the chromaticities of the various parts of the spectrum; see Table II.

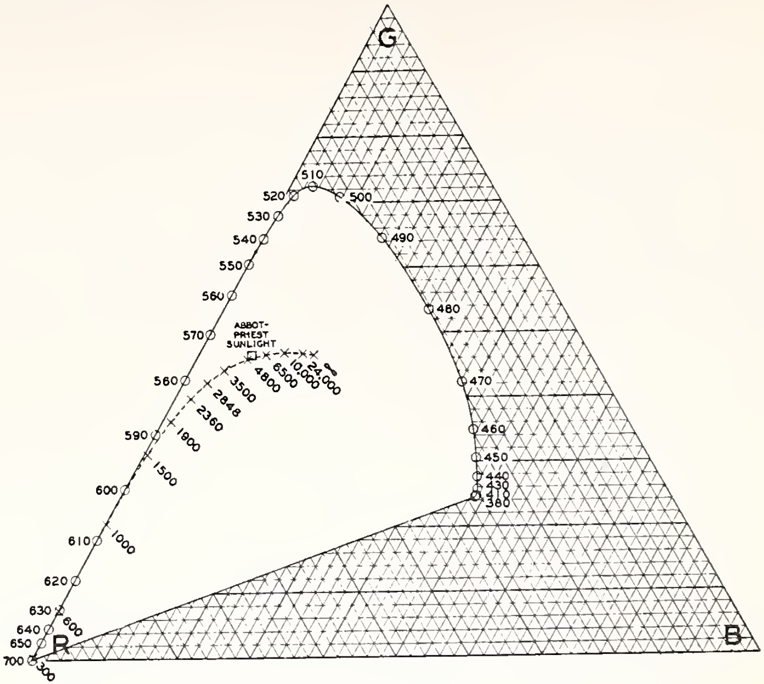


FIG. 4.—Uniform-Chromaticity-Scale Triangle According to Judd.

Other color systems having substantially the same chromaticity spacing have been devised by MacAdam, by Breckenridge and Schaub, and by Scofield.

Answer.—They have zero luminosity.

Question 8.—How can you call anything that has zero luminosity a light? How can there be a nonluminous light?

Answer.—There cannot be any such thing as a nonluminous light; as already stated, these lights are imaginary.

Well, that is enough of a catechism. The outstanding disadvantage of this colorimetric system is that the specifications resulting from it fail to suggest the color readily. It is nearly always necessary to represent chromaticity separately. This is done by use of a chromaticity diagram obtained by plotting $X/(X + Y + Z)$ against $Y/(X + Y + Z)$; see Fig. 3. These fractions are called trilinear coordinates or trichromatic coefficients and are designated (x, y, z), the z-coordinate being an abbreviation for $Z/(X + Y + Z)$. By plotting a point (x, y) on such a chromaticity diagram or map to represent a specimen, the chromaticity relationship with other specimens is made clear. Table II gives the trichromatic coefficients of the spectrum colors.

Recently there has been an effort to supplement the standard coordinate system by choice of other primaries which yield a chromaticity spacing of colors more in accord with that in the constant-lightness planes of the surface-color solid. Such systems, known as uniform-chromaticity-scale systems, have been proposed by Judd (9, 10), MacAdam (12), Breckenridge and Schaub (2), and by Scofield (16). The Scofield proposal, made to A.S.T.M. Committee D-1 on Paint, Varnish, Lacquer and Related Products bears a further resemblance to the constant-lightness planes of the surface-color solid because it places gray at the origin of the coordinates. Figure 4 shows the Judd uniform-chromaticity-scale triangle. A family of equal tangent circles on this triangle has been transferred to the (x, y)-diagram of the Standard I.C.I. system (Fig. 3) and the resulting ellipses serve to indicate the chief distortions of that diagram. If the improved spacing of the various uniform-chromaticity-scale triangles proves to be of practical importance, we may expect to see color specifications increasingly given both according to the standard system and according to some uniform-chromaticity-scale system. Attempts to explore the practical advantages of such alternate color systems should be encouraged.

Polar-Coordinate Specification.—The great advantage of the fixed-light, variable-light method of color specification is that it lends itself to the use of polar coordinates. The variables may therefore be made to correlate fairly well with the psychological attributes hue and saturation which are also plotted in polar coordinates, hue being plotted as the angle, saturation as the radius.

The most widely used variables are dominant wave length and purity. Dominant wave length (3, 8, 13, 14) is the wave length of the spectrum light required to be added to the fixed light to color match the specimen; purity (3, 5, 7, 13, 14, 15) is the ratio of the amount of the spectrum light to the total amount of the two-part combination. This definition of purity is a convenient one, but the choice is not too happy for the production of good correlation with saturation. By this definition, the colors of the spectrum have constant (unit) purity; but they are usually perceived as of rather widely varying saturation. In spite of this handicap, it is fairly easy to learn how to visualize the color of a surface from its reflectance, dominant wave length, and purity. Oftentimes a color specification is given both in tristimulus terms and by dominant wave length and purity, and thereby becomes more easily understood. Either form may easily be computed from the other.

Color Specification by Material Standards

By far the great majority of color specifications are administered by material standards of color, and often they bear the whole brunt of the specification, no more fundamental specification being given. Sometimes it is feasible to define them so that they may be reproduced from easily obtainable materials, as in solutions of inorganic salts, many of which are used in A.S.T.M. tests. Preparation of opaque color standards is more difficult, but in the standard magnesium-oxide surface on which present spectrophotometric practice discussed by Parker[9] is based, these difficulties have largely been overcome.

Since color specification by material standards is discussed in some detail by Scofield[10] and Godlove,[11] it will be sufficient here to point out why material standards of color are so practical in spite of the difficulties of impermanence.

We have just seen that technical colorimetry based upon additive combinations of lights requires four restrictions: (*a*) standard background, (*b*) accurate accounting of the illuminant, (*c*) use of a standard observer, and (*d*) use of psychophysical quantities which correlate with the psychological attributes, hue, lightness and saturation. The use of a material color standard makes immediately plain that the essential act required of the observer is simply that of color matching. Note first that the specification succeeds regardless of surroundings provided only that both sample and standard are viewed against the same surroundings. Second, accurate specification of the illuminant is often not required because it is found that change in illuminant produces an effect of the second order. Similarly in the third place, small variations in color vision from one normal observer to another produce only a second-order effect; so a standard observer is not required. And finally, the only use of the psychological attributes, hue, lightness and saturation arises in the description of the differences between specimen and standard. This simplicity accounts for the success of material standards of color.

Many A.S.T.M. specifications embody material color standards devised for one particular test. Reference to the index to A.S.T.M. standards and tentative standards shows in fact that the Society has made wide use of such standards. Many of these are mentioned by Scofield[10] and Godlove.[11]

Summary

A distinction has been drawn between the psychological definition of color and the technical definition. Color standards based upon combinations of lights and those based upon collections of material standards have been discussed. This discussion is intended to prepare the ground for the more detailed accounts of color specification and use in evaluating the appearance of materials which follow.

[9] See p. 47.
[10] See p. 18.
[11] See p. 37.

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(11) D. B. Judd and K. L. Kelly, "Method of Designating Colors," *Journal of Research*, Nat. Bureau Standards, Vol. 23, p. 355 (1939) (*Research Paper 1239*).

(12) D. L. MacAdam, "Projective Transformations of I.C.I. Color Specifications," *Journal*, Optical Soc. America, Vol. 27, p. 294 (1937).

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THE DEFINITION OF BLACK AND WHITE

Am. J. Psychol. 54, 289-294 (1941)

By way of contrast with Mr. Meter in "The physics of color tolerance", here is the real Judd - the psychophysicist _par excellence_. Identifying psychological concepts and propositions that can be tested, and restating them in terms of operations, is the task of a psychophysicist. Judd undertook that task for white and black, and the proposition that they are surface colors, in this article.

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THE DEFINITION OF BLACK AND WHITE

Professor Dimmick courteously submitted to me in advance of publication a copy of the preceding note. I have consequently an opportunity for an immediate reply. My views on black and white have accordingly been written out more completely than before and I have in turn submitted them to him for a rejoinder.

Tentative definition. Black and white are the two extremes of the series of achromatic surface colors, of zero glossiness and transparency, which differ in lightness alone.

Sufficient conditions for appearance, illumination uniform. A surface color is color perceived as belonging to a surface. If an opaque object be viewed among other objects in a space sufficiently illuminated to provide photopic vision, the normal observer (O) will generally perceive that it is an object with surfaces possessing color.[1] If it be granted that the achromatic color seen by O takes on the surface mode of appearance, the following formula for lightness, L, of the surface color, derived from a formula by Adams and Cobb,[2] serves to describe stimulus conditions for black and white, in which, for $L = 0$, the color is black, and for $L = 1$, the color is white.[3]

$$L = A(A_f + 1)/(A_f + A),\ A_f > 0 \ldots\ldots\ldots\ldots\ldots [1]$$

where: A = luminous apparent reflectance of the surface under consideration,[4] and A_f = luminous apparent reflectance of the field, an average of the luminous apparent reflectances of the surfaces in the field, weighted according to proximity in space and in past time to the fixation point.[5] The surfaces must be, and be per-

[1] Factors tending to prevent such perception are objects providing visual fields insufficiently elaborated in gross structure, or unsupplied with visible microstructure. Cf. R. B. MacLeod, An experimental investigation of brightness constancy, *Arch. Psychol.,* 21, 1932, (no. 135), 1-101.

[2] E. Q. Adams and P. W. Cobb, The effect on foveal vision of bright (and dark) surroundings, *J. Exper. Psychol.,* 5, 1922, 41

[3] No case of failure of these conditions to produce black and white colors is known; other conditions have, however, been known to produce black and white colors, and these will be discussed presently.

[4] H. J. McNicholas, Absolute methods in reflectometry, *Bur. Stand. J. Res.,* 1, 1928, 33.

[5] D. B. Judd, Hue, saturation and lightness of surface colors with chromatic illumination, *J. Opt. Soc. Amer.,* 30, 1940, 4; also *J. Res. Nat. Bur. Stand.,* 24, 1940, 296.

ceived as if, uniformly illuminated. A_f must be greater than zero because for $A_f = 0$ there exists no illumination which will yield photopic vision.

It will be noted that the only way for a surface to yield a black color according to Equation [1] is to have a luminous apparent reflectance $A = 0$; and the only way for it to yield a white color is to have $A = 1$. Intermediate values of A correspond to surface colors of lightness intermediate between 0 and 1; values of A higher than 1 correspond to glossy surface colors; and there is, of course, no surface for which A is less than zero.

It is interesting to note how lightness by [1] varies for $A = A_f$ as both approach zero. For $A = A_f$, we have:

$$L = (A + 1)/2, A > 0 \ldots\ldots\ldots\ldots [1a]$$

which approaches 0.5 as A approaches zero. The interpretation is that a surface of luminous apparent reflectance the same as that of the field of which it forms a part always yields a lightness higher than 0.5; and if it is a gray, it approaches middle gray through lighter grays as the luminous apparent reflectance of the field approaches zero.

This interpretation bears a superficial resemblance to the Hering mid-gray corresponding to the intrinsic light of the retina.[6] If A be taken equal to zero, however, instead of equal to A_f, the value of lightness by Equation [1] is always zero. This result bears a superficial resemblance to the Ladd-Franklin view that black is "the psychical correlate of a cortical condition of inactivity in correspondence to a non-stimulated retinal area."[7] The Ladd-Franklin view has been shown to be inadequate by Knox.[8] Equation [1] does not really suggest either the Hering or the Ladd-Franklin view, first because it refers to photopic vision only, and secondly because analysis shows that the limiting value of lightness varies according to Equation [1] between 0 and 1 depending on the ratio of A to A_f as they approach zero. The requirement in Equation [1] that A_f be greater than zero suggests on the contrary that perception of neither black nor white nor, indeed, of a surface color of any intermediate lightness can result from zero stimulus. This impossibility should be related to the difficulty of obtaining the perception of a surface from a field which does not supply sufficient light to the observer to yield photopic vision, and to the impossibility of obtaining surface perception from a field supplying no light whatever.[9]

Necessary and sufficient conditions for surfaces, illumination uniform. We have said that Equation [1] describes sufficient conditions for the appearance of black or white in a uniformly illuminated scene; that is, whenever $A = 1$, the corresponding surface color, if achromatic, matte, and opaque, is always reported as white, and whenever $A = 0$ (A_f sufficiently greater than zero to yield photopic vision), the

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[6] E. Hering, *Grundzüge der Lehre vom Lichtsinn,* 1920, 214.

[7] M. R. Neifeld, The Ladd-Franklin theory of the black-sensation, *Psychol. Rev.,* 31, 1924, 498-502, also *C. Ladd-Franklin, Colour and Colour Theories,* 1929, 241-246.

[8] G. W. Knox, A contradiction to the Ladd-Franklin theory of blackness, *J. Psychol.,* 8, 1939, 13-21.

[9] J. Ward, Is 'black' a sensation?, *Brit. J. Psychol.,* 1, 1905, 408.

corresponding surface color is always reported as black.[10] If, however, there are no surfaces in the field having luminous apparent reflectances as high as 1 or as low as 0, it is common, though not inevitable, for an *O* to report black or white for the terminal members of the series of achromatic colors actually experienced in the scene, provided the maximum luminous apparent reflectance, A_x, of any surface present in the field, at the time of observation or not too much previously, exceed the minimum, A_n, by a factor sufficiently large (probably between 10 and 100 for most *O*s). It is possible to expand Equation [1] into a more general form which embraces these common conditions for obtaining black and white. This general statement of the stimulus conditions for a given lightness yields the necessary and sufficient conditions for the appearance of black and white colors perceived as belonging to uniformly illuminated surfaces in accord with the tentative definition given,

$$L = \frac{(A - A_n)\ (A_f + A_x)}{(A_x - A_n)\ (A_f + A)},\ A_f > 0,\ 0 < A_x \leqq 1,\ A_n < kA_x \ldots\ldots [2]$$

where *k* is a constant, perhaps characteristic of every *O* probably contained within the limits of 0.10 and 0.01. Knox has suggested the value of 1/60 for *k*,[11] basing his suggestion on the work of Gelb, Koffka, and Harrower.[12]

Examination of Equation [2] shows that it is in accord with the tentative definition of black and white given at the beginning of this note. The only way for L to assume the value of zero (corresponding to black) is to have the surface take on the luminous apparent reflectance, A_n, that is, to be the darkest surface in the field of view. Similarly white, $L = 1$, corresponds to the lightest matte, opaque surface color in the field of view provided the luminous apparent reflectance, A_x, of the corresponding surface exceeds A_n by the factor of more than $1/k$; that is, provided $A_n < kA_x$.

Equation [1] becomes the special case of Equation [2] obtained by setting $A_n = 0$ and $A_x = 1$. Another special case of Equation [2], that obtained by setting $A_n = 0.03$ and $A_x = 1$, has been compared with the visual estimates of 8 *O*s.[13] The reports of some of these *O*s would have agreed better with Equation [2] if the value of A_x had been set closer to that actually fulfilled by the experimental conditions ($A_x = 0.80$); that is, some *O*s called surfaces white which had luminous apparent reflectances about equal to 0.80; others called them light gray.

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[10] If the surface does not give rise to a surface color, but, as is frequently the case for $A = 0$, to an aperture color, the descriptive term, black, is less likely to be used; in fact it is exceptional. Thus, E. B. Titchener says, "The distinction drawn in everyday life between black and dark, so far as it is drawn at all, seems rather to suggest the psychological distinction of superficial and roomy colors; black is superficial, like the color of colored paper; dark is roomy, like the color of a transparent liquid. (A further word on black, *J. Philos., Psychol., & Sci. Methods*, 13, 1916, 649.)

[11] Knox, *op. cit.*, 15.

[12] Kurt Koffka, *Principles of Gestalt Psychology*, 1935, 251-252.

[13] Judd, *op. cit.*, *J. Opt. Soc. Amer.*, 30, 1940, 24; also *J. Research Nat. Bur. Stand.*, 24, 1940, 319.

Scenes involving surfaces perceived as unequally illuminated. A normal *O* in an unknown room filled with unknown objects illuminated by an unknown light is nevertheless able to report on the amount and approximate direction of the maximum illumination and, therefore, upon the amount of illumination pertaining to surfaces in various orientations relative to this direction. The lightness of the colors of such surfaces is primarily dependent upon the impression of illumination pertaining to them. The factors entering into the impression of illumination are complicated and are probably too poorly understood to justify an immediate attempt at formulation (compare Katz,[14] Koffka,[15] Pikler,[16] and Bühler[17]).

Equation [2] succeeds without specific reference to amount of illumination because in the simple case of a number of surfaces perceived as equally illuminated the average brightness of the scene (total insistence according to Katz[18]) is almost the sole determinant of the impression of illumination. The average brightness of the scene is not mentioned explicitly in Equation [2], but it is measured by the average luminous apparent reflectance of the field, A_f. This principle has been stated by Kirschmann,[19] Von Kries,[20] Bocksch,[21] Kohlrausch,[22] and Kardos.[23] Attention should be drawn to an aspect of the definition of A_f (and A_n and A_x also) which may be overlooked; this definition refers to surfaces which have been in the scene before observation as well as those at the time of observation. Often surfaces previously but no longer present in the field have supplied the information upon which the impression of illumination depends. In such cases the *O*s are sometimes said to have a mental criterion or standard of white which is subject to complicated instantaneous changes. Equation [2] shows the pertinent variables, and its hyperbolic form essentially approximates the true relation; but there is no guarantee that a solution can be found for every *O* even for the simple case of colors of surfaces perceived in uniform illumination, because it is frequently impossible to evaluate A_n and A_x for a given *O* or to know how to weight the various surfaces in computing the average luminous apparent reflectance of the field, A_f. It is probable that the most important considerations bearing on such weighting relate to retinal adaptation. Values of A_n and A_x, on the other hand, may refer to surfaces which have been presented in the scene too far in the past to bear at all

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[14] D. Katz, *Der Aufbau der Farbwelt,* 1930, 460 ff.

[15] Koffka, *op. cit.,* 258 ff.

[16] J. Pikler, Das Augenhüllenlicht als Mass der Farben, *Zsch. f. Psychol.,* 120, 1931, 189; 125, 1932, 90.

[17] K. Bühler, *Die Erscheinungsweisen der Farben,* 1922, 1-209.

[18] Katz, *op. cit.,* 462.

[19] A. Kirschmann, Die psychologisch-ästhetische Bedeutung des Licht und Farbenkontrastes, *Philos. Stud.,* 7, 1891, 363.

[20] J. von Kries, Die Gesichtsempfindungen, in W. A. Nagel's *Handb. d. Physiol. des Menschen,* 3, 1905, 239.

[21] H. Bocksch, Duplizitätstheorie und Farbenkonstanz, *Zsch. f. Psychol.,* 102, 1927, 343.

[22] A. Kohlrausch, Allgemeines über Umstimmung und 'Farbenkonstanz der Sehdinge,' in A. Bethe's *Handb. d. norm. u. pathol. Physiol.* 12/2, Receptionsorgane II, 1931, 1501.

[23] L. Kardos, Versuch einer mathematischen Analyse von Gesetzen der Farbensehens. Nähere Bestimmung des funktionalen Verhältnisses zwischen Farbenerlebnis und Reizgesamtheit, *Zsch. f. Sinnesphysiol.,* 66, 1935, 188.

importantly on retinal adaptation. For very marked local variations in illumination, such as provided by a spotlight in exact registration with a surface, the impression of the illumination according to Gelb is determined by the surfaces actually present.[24] For such cases, at least, there is some hope of writing a definition of lightness by extension of Equation [2].

Black or white from nonsurface modes of appearance. This note started with a tentative definition of black and white as qualities of surface color; as far as can be ascertained, no serious attempt has ever been made to deny that surface colors may have these qualities. It is now pertinent to inquire whether black and white are commonly or ever experienced as qualities of color in other modes of appearance. That is, can there be a black or white volume color, a black or white film (aperture) color, or a black or white luminous color?

From casual observation it seems safe to conclude that volumic blacks and whites are exceptional but not impossible, that some trained *O*s experience black and white film colors while naïve *O*s commonly experience them only as dim or bright in various degrees, and that a white luminous color is similarly possible; but a black luminous color is self-contradictory. I venture to suggest without being able to prove it that these reports for other modes of appearance are derived from the surface mode which characteristically yields the black and white quality. Other writers have adopted similar views.[25] Helmholtz has asserted that black and white in common parlance refer only to the colors of objects.[26] Troland has stated, "Thus, good blacks and other dark colors, such as browns and olives, are found only as surface colors or in a surface color environment."[27] Neifeld, even in explaining the Ladd-Franklin view that black is a sensation, states, "Black is the psychic correlate of the absorption, *by objects* (italics mine), of all the visible light-ray frequencies."[28]

Others have adopted the divergent view that it is not only possible but useful to abstract from these different kinds of percepts the black and white quality. They assert that there is a black sensation and a white sensation.[29] It would be impossible within a reasonable space to review the various arguments that have been

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[24] Adhémar Gelb, Die 'Farbenkonstanz' der Sehdinge, in A. Bethe's *Handb. d. norm. u. pathol. Physiol.,* 12/1, Receptionsorgane, II, 1929, 674.

[25] A. Fick, Gesichtsinn, in L. Hermann's *Handb. d. Physiol. d. Sinnesorgane,* 1879, 204 f.; A. Kirschmann, Der Metallglanz und die Farbe der Metalle, *Arch. f. d. ges. Psychol.,* 41, 1921, 94; J. Ward, Is 'black' a sensation?, *Brit. J. Psychol.,* 1, 1905; A further note on the sensory character of black, *Brit. J. Psychol.,* 8, 1916, 212; K. Fiedler, Das Schwarz-Weiss Problem, *Neue Psychol. Stud.,* 2, 1926, 343-408.

[26] H. v. Helmholtz, *Physiological Optics,* Eng. trans., 2, 1924, 130.

[27] L. T. Troland, Visual phenomena and their stimulus correlations, in C. Murchisons' *Handbook of General Experimental Psychology,* 1934, 657.

[28] M. R. Neifeld, The Ladd-Franklin theory of the black-sensation, *Psychol. Rev.,* 31, 1924, 498; also C. Ladd-Franklin, *Colour and Colour Theories,* 1929, 241.

[29] E. Hering, *Zur Lehre vom Lichtsinn,* 1874, 88. G. E. Muller, Über die Farbenempfindungen, *Zsch. f. Psychol.* 17, 1930, 16 f. E. B. Titchener, A note on the sensory character of black, *J. Phil. Psychol. and Sci. Methods,* 13, 1916, 113; A further word on black, *J. Phil. Psychol. and Sci. Methods,* 13, 1916, 649; Visual intensity, this JOURNAL 34, 1923, 310. C. Stumpf, Die Attribute der Gesichtsempfindungen, *Abh. d. kgl. preuss. Akad. d. Wiss., phil. -hist. Klass,* 8, 1917; see also this JOURNAL, 32, 1921, 155. W. Wundt, *Grundzüge der physiologischen Psychologie,* 5th ed., 2, 1905, 147-193. A. Tschermak, Licht und Farbensinn, in A. Bethe's *Handb. d. norm. u. pathol. Physiol.,* 12/1, Receptionsorgane II, 1929, 296. H.

framed, and it would be difficult to add anything appreciable to them. It is interesting to note, however, that Titchener,[30] having reviewed the evidence and taken his stand, says that we would all welcome a crucial experiment.

It would be worthwhile, I believe, to study the methods of describing color as a function of the mode of appearance, and to search for pertinent stimulus-variables and functions of them correlating with reports of the colors by direct visual estimation. Such a study would constitute a whole series of crucial experiments. Let all abstractions, including those of black and white sensations, be tested for usefulness in color description for all modes of appearance. On this account, we should watch with particular interest the further development of Professor Dimmick's proposal to specify color in all modes of appearance by the seven primaries: red, yellow, green, blue, white, gray, and black, taken four at a time, two chromatic and two achromatic.[31] In the meantime, the tentative definition of black and white with which this note was introduced may as well be taken. Its usefulness is exemplified by the psychophysical relation given in Equation [2], and its validity has not been questioned by anyone. Whether it be possible to extend with similar validity and usefulness the definition of black and white to modes of appearance other than surface remains to be decided by further study.

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Bocksch, Duplizitatstheorie und Farbenkonstanz, *Zsch. f. Psychol.*, 102, 1927, 343 ff, 430. G. M. Michaels, Black: A non-light sensation, *Psychol. Rev.*, 32, 1925, 248. M. R. Neifeld, The Ladd-Franklin theory of the black-sensation, *Psychol. Rev.*, 31, 1924, 498-502; also C. Ladd-Franklin, *Colour and Colour Theories*, 1929, 241-246. F. L. Dimmick, The series of blacks, grays, and whites, *Psychol. Rev.*, 32, 1925, 334; A reinterpretation of the color-pyramid, *Psychol. Rev.*, 36, 1929, 83; Black and white, this Journal, *supra*, 286-289; F. L. Dimmick and C. H. Holt, Gray and the color pyramid, this Journal, 39, 1928, 284.

[30] Titchener, A note on the sensory character of black, *J. Philos.*, 13, 1916, 120.

[31] F. L. Dimmick and M. R. Hubbard, The spectral location of psychologically unique yellow, green, and blue, this Journal, 52, 1939, 242-254; The spectral components of psychologically unique red, *ibid.*, 52, 1939, 348-353.

A PROPOSED METHOD OF DESIGNATING COLOR
(with F. S. Scofield and R. S. Hunter)

Am. Soc. Test. Mater. Bull., No. 110, 19-24 (May 1941)

The α, β chromaticity diagram recommended for use in recording colors, and for evaluating yellowing, fading and color differences, has distinctly different geometric proportions than the UCS diagram introduced by Judd in 1935. It is said to be more appropriate for colors that subtend large field angles. Its use with results obtained with photoelectric (3-filter) colorimeters is stressed. The "trilinear coordinates" that are mentioned in the text that follows Eq. (1) are now usually called chromaticity coordinates. The color-difference formula briefly sketched in this paper became the basis for the NBS unit of color difference, which was often called the "judd".

A Proposed Method of Designating Color

By Francis Scofield,[1] Deane B. Judd,[2] and Richard S. Hunter[2]

SYNOPSIS

A proposed tristimulus method of designating color is described together with some of its applications. The proposed method is used for designating numerically the colors of opaque objects and transparent materials. Combined in the new system are a number of desirable properties: (1) relation by transformation equations to the standard I.C.I. system, (2) choice of a chromaticity diagram yielding approximately uniform chromaticity scales and having an origin at the neutral point so that it is easy to visualize the appearance of a specimen from the coordinates of its color, and (3) the opportunity to compute coordinates of colors rapidly from settings on the respective samples taken by means of any one of several of the recently developed photoelectric tristimulus colorimeters.

At the recent A.S.T.M.-I.S.C.C. joint Symposium on Color—Its Specification and Use in Evaluating the Appearance of Materials,[3] attention was called to the commercial importance of the colors of many of the different materials with which the A.S.T.M. deals. The science of colorimetry has advanced rapidly in recent years so that greatly improved methods of measuring and numerically designating the colors of materials are now coming into use. The present paper deals with one of these new methods which it is felt may prove particularly well suited to some of the problems of color testing and specification with which the A.S.T.M. deals. In the work of the Society's Committee D-1 on Paint, Varnish, Lacquer, and Related Products, a system of the type briefly described below could be used to indicate the depth of color permissible in oils, varnishes, and other liquids, to evaluate the tinting strength of white and colored pigments, to measure the amount of fading of tinted paints, and to determine the degree of yellowing of white paints and enamels.

The method of designating color which is described in the present paper was brought to the attention of the Society in a paper given as part of the Symposium on Paint Testing (*1*)[4] at its 1939 annual meeting. Although only described orally at this meeting, there developed from this description a considerable interest in the method. Because of this interest, the authors were asked by the chairman of Subcommittee XVIII on Physical Properties of Materials of Committee D-1 to prepare a short account of the method which would include some of its possible uses, chiefly in the paint field.

Before describing the new tristimulus method for designating colors, it seems desirable to give a brief explanation of what a tristimulus method is. Such an explanation was prepared a few months ago (*2*), and in the following three paragraphs this explanation is repeated.

"Color may be specified either by identifying a particular combination of lights requisite to produce a match, or by identifying a material standard which, in conjunction with a known illuminant, will produce a match. Since any uniform portion of the visual field near its center may be seen by a normal observer to vary in three independent ways, there must always be three independent variables in a color specification.

"The most convenient way to specify color by identifying a particular combination of lights is to give the amounts of three primaries required to produce a color match by additive combination. This is called the tristimulus specification of color, and because of its convenience it is used as a common denominator by means of which all other systems of color specification may be interrelated. The colorimetric coordinate system recommended in 1931 by the International Commission on Illumination (the I.C.I. system) (*3*) is now the most widely used tristimulus system. Linear, homogeneous transformations of the I.C.I. system have been proposed for various purposes, some for greater simplicity in computation, some for providing uniform chromaticity scales, and others for quantifying theories of color vision.

"Another form of identifying a combination of lights to specify a color is to give the brightness of one light of fixed quality (such as average daylight) and the brightness of a light of variable quality separately identified (as by wave length in the spectrum). This form of identification leads naturally to a separation of the three variables into a brightness and two chromaticity variables. The two chromaticity variables, a length and an angle, yield a polar coordinate system. The angle is identified variously as dominant wave length, hue wave length, hue angle and hue number; it varies with the wave length of a spectrum light and correlates well with the hue of the perceived color (provided the fixed light appears achromatic). The length is known variously as colorimetric purity or spectral brightness purity, spectral excitation purity, basic brightness purity, and basic excitation purity; with proper choice of fixed light it correlates fairly well with the saturation of the perceived color."

The method described herein is a tristimulus method, but the coordinates given can be readily converted to numbers which respectively designate the hue, lightness, and saturation of the color perceived as belonging to any given object.

Definition of the Coordinate System

It is proposed to specify by three numbers the color of any object illuminated by I.C.I. illuminant C (a standard illuminant similar in spectral character to average daylight) (*3*). One of these three numbers will, as suggested, serve to indicate lightness and is either the luminous ap-

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[3] Symposium available as separate publication, June, 1941.

[4] The italic numbers in parentheses refer to the reports and papers appearing in the list of references appended to this paper.

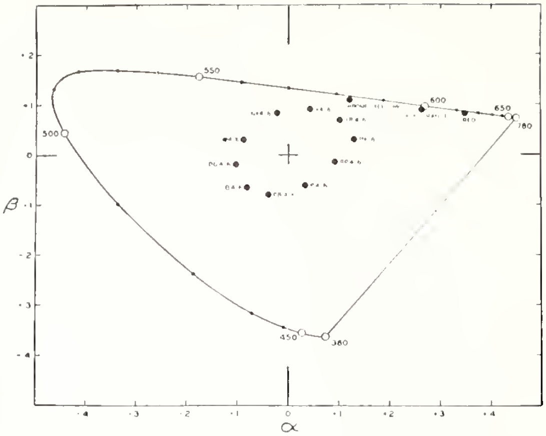

Fig. 1.—The (α, β)-Diagram Showing the Locus of Spectrum Colors, the Points Representing Ten Munsell Colors, the Points Representing Three Federal-Specification Colors, and the Origin, or Neutral Point.

parent reflectance, A_{θ_I,θ_V}, of a specimen viewed by reflected light, or the luminous transmission, T, of a specimen viewed by transmitted light. The other two numbers, α and β, will serve to specify the chromaticity of the color and are related to values in the standard I.C.I. system (*3*) by the following equations of transformation:

$$\left.\begin{aligned} \alpha &= \frac{2.4266x - 1.3631y - 0.3214}{1.0000x + 2.2633y + 1.1054} \\ \beta &= \frac{0.5710x + 1.2447y - 0.5708}{1.0000x + 2.2633y + 1.1054} \end{aligned}\right\} \quad \ldots\ldots\ldots(1)$$

where x and y and α and β are trilinear coordinates of the sample (x and y on the I.C.I. system, α and β on the proposed system). The reverse transformation is given by the equations:

$$\left.\begin{aligned} x &= \frac{0.5583\alpha + 0.1631\beta + 0.2466}{0.0100\alpha - 1.4347\beta + 0.7951} \\ y &= \frac{-0.2515\alpha + 0.6285\beta + 0.2515}{0.0100\alpha - 1.4347\beta + 0.7951} \end{aligned}\right\} \quad \ldots\ldots\ldots(2)$$

When α is plotted against β in rectangular coordinates, a Maxwell triangle or mixture diagram results on which equal chromaticity-differences between surface colors are represented by approximately equal distances. The uniformity of the chromaticity spacing afforded by this (α, β)-diagram is thought to be superior for colors subtending large visual angles to that of the Judd U.C.S. triangle (*4*). In deriving the new diagram, use has been made of preliminary conclusions from studies of the applicability of the U.C.S. triangle to the specification of uniform color tolerances for surface colors of large area (*5*, *6*, *7*). Although planned chiefly for surface colors subtending large visual angles, the (α, β)-diagram does not differ so markedly from the earlier U.C.S. triangle that it cannot be used for the colors of clear liquids and other specimens usually observed in smaller areas.

Figure 1 shows the spectrum locus of the (α, β)-diagram. Compared to the U.C.S. triangle, the distance from the long-wave (red) extreme of the spectrum locus to the illuminant point has been shortened, that from the middle (green) part of the spectrum locus has been lengthened, and the length of the line connecting the extremes of the spectrum locus has been decreased. This latter change accords with a conclusion drawn from studies with Lovibond glasses (*8*). In addition to the spectrum locus, points representing the ten Munsell[5] hues at value 4 and chroma 6 are plotted in Fig. 1 together with points representing MgO illuminated by I.C.I. illuminant C (plotted at the origin), medium chrome yellow (Federal Specification TT-P-53, 2/12/37), international orange (Federal Specification TT-P-59, 6/17/37) and water-resistant red enamel (Federal Specification TT-E-531*a*, 6/4/35). The values of α and β, which are given in Table I, were all computed from x and y according to Eq. 1.

Advantages of the Coordinate System

The above tristimulus system for designating colors possesses a number of properties which make it potentially useful to the Society. Although differing from those in the I.C.I. system, values in the new coordinate system are numerically related by Eq. 1 to those in the internationally recognized system. Since it is thus possible to derive values of α and β from x and y, which in turn can be computed from the spectral curves of the samples represented, values of α and β can also be computed from the spectral curves of samples. It will be remembered that A.S.T.M. Standard Method of Test for Spectral Apparent Reflectivity of Paints (D 307–39)[6]

[5] Trilinear coordinates read from smoothed curves by Nickerson (*9*), see Table II.

[6] 1939 Book of A.S.T.M. Standards, Part II, p. 815.

TABLE I.—THE TRILINEAR COORDINATES, IN BOTH x AND y AND α AND β, WHICH ARE REPRESENTED BY POINTS IN FIG. 1 TO SHOW THEIR RELATIVE POSITIONS IN THE (α, β)-DIAGRAM.

Color	x	y	α	β
Toluidine Red, Federal Specification TT-E-531*a*	0.668	0.318	+0.3471	+0.0828
International Orange Federal Specification TT-P-59	0.610	0.362	+0.2625	+0.0900
Medium Chrome Yellow Federal Specification TT-P-53	0.519	0.153	+0.1210	+0.1092
Munsell colors				
R 4/6	0.430	0.317	+0.1287	+0.0308
YR 4/6	0.451	0.387	+0.1009	+0.0692
Y 4/6	0.429	0.448	+0.0427	+0.0910
GY 4/6	0.367	0.459	−0.0225	+0.0837
G 4/6	0.261	0.393	−0.0992	+0.0299
BG 4/6	0.226	0.323	−0.1034	−0.0193
B 4/6	0.214	0.261	−0.0827	−0.0648
PB 4/6	0.233	0.233	−0.0394	−0.0792
P 4/6	0.289	0.233	+0.0324	−0.0603
RP 4/6	0.362	0.269	+0.0917	−0.0141

TABLE II.—COLORS OF TWELVE GLASS COLOR STANDARDS OF THE A.S.T.M. UNION COLORIMETER.

Standard Color	Luminous Transmission, T	I.C.I. Trilinear Coordinates		(α, β) Trilinear Coordinates	
		x	y	α	β
No. 1	0.751	0.3488	0.3815	+0.0022	+0.0446
No. 1½	0.654	0.3995	0.4460	+0.0159	+0.0845
No. 2	0.443	0.4724	0.4765	+0.0660	+0.1099
No. 2½	0.365	0.4985	0.4570	+0.1006	+0.1071
No. 3	0.287	0.5252	0.4402	+0.1344	+0.1054
No. 3½	0.211	0.5561	0.4234	+0.1721	+0.1046
No. 4	0.096	0.5908	0.3995	+0.2183	+0.1014
No. 4½	0.065	0.6199	0.3758	+0.2603	+0.0974
No. 5	0.036	0.6528	0.3467	+0.3107	+0.0918
No. 6	0.017	0.6764	0.3234	+0.3497	+0.0867
No. 7	0.0066	0.6841	0.3155	+0.3629	+0.0849
No. 8	0.0020	0.7140	0.2860	+0.4141	+0.0782

covers the method of obtaining the spectral curves of paint samples.

There are several valuable properties of the new system which are not possessed by the I.C.I. standard system. In the first place, the (α, β)-diagram yields fairly uniform chromaticity-scales which are especially well suited for representing the colors of surfaces and other stimuli covering large areas in the field of view. Values of α and β can be used to designate the chromaticities of transparent liquids and other nonopaque objects as well as of surfaces.

Secondly, the chromaticity diagram of the new system is plotted in rectangular coordinates with the origin at the neutral point. By transforming values of α and β for opaque surfaces to polar coordinates, numbers are obtained which can be combined with the square roots of the apparent reflectances of the respective surfaces to give numerical records of the colors of the surfaces in variables which the layman will recognize. The polar angle gives a number correlating well with hue; the square root of apparent reflectance, a number correlating closely with lightness (or value); and the polar radius, a number correlating fairly well with saturation (or chroma). Thus numbers may be assigned to a surface which will correspond to the attributes, hue, lightness, and saturation, which the layman thinks of when he describes the color perceived as belonging to a surface as "deep red" or "pale blue-green."

The third advantage has to do with the speed with which values of α and β may be determined on the recently developed, relatively inexpensive photoelectric tristimulus colorimeters. From the viewpoint of the Society, it is important that the techniques of measuring the samples and obtaining the desired numerical designations of color be not too complicated. That these techniques are not too complicated will be shown below. Only three or four readings need be obtained from each sample and these are quickly converted to suitable values of α, β, and A_{θ_I,θ_V} or T by a few steps of computation.

Measurement of the Coordinates α and β

As was noted above, a sample can be measured with a spectrophotometer, then values of x, y, and A_{θ_I,θ_V} or T can be computed from the spectral curve, and finally values of α and β can be derived from x and y by using Eq. 1. However, this procedure is long and tedious and will, in practice, probably be followed only when it is necessary to calibrate a standard or to settle an argument requiring greater accuracy than that obtainable from measurements with photoelectric tristimulus instruments.

Instruments of this latter type are distinguished by the fact that each employs a source, filters, and a photocell which make the device as nearly equivalent to the I.C.I. standard observer as practicable. Of the several devices which have been described, only those designed by Hunter (*10*) and Perry (*11*) are suited for the measurement of samples under I.C.I. illuminant C. However, it is understood that two new instruments for the photoelectric tristimulus colorimetry of opaque and transparent objects will shortly appear on the instrument market. One will be a goniophotometer (American Instrument Co.) the geometric features of which were described by Wetlaufer and Scott (*12*); the other will be an apparent-reflectance and transmission meter (Photovolt Corp.).

Some photoelectric tristimulus colorimeters use three filters, others use four. With four filters, it is possible, as van den Akker has shown (*13*), to duplicate more closely than with three the spectral sensitivity of the I.C.I observer. With three filters, however, one fewer setting is required for every sample; moreover the accuracy possible seems wholly adequate for many problems in which similar samples are compared for color. Three filters are used in the multipurpose reflectometer described by Hunter, and also, it is believed, in the instrument now being designed by the Photovolt Corp. Four filters are used in Perry's Blancometer, and four are planned, according to information supplied by the designers, for the goniophotometer.

The chief advantage of all these tristimulus colorimeters is the speed with which measurements of color can be made. Only a minute or less is required to obtain the readings of a sample for each of the three or four filters. Conversion of these settings to values of α, β, and A_{θ_I,θ_V} or T will usually take a little more time than was required for the settings, but when compared with the times required to obtain tristimulus values from other sources of data, it is very brief (6, p. 259). Actually the (α, β)-system was originally chosen so that it would be possible to convert quickly the readings from the multipurpose reflectometer to values of chromaticity in a uniform-chromaticity-scale mixture diagram. If readings relative to a standard with the amber, green, and blue filters of that instrument are designated by A, G, and B, respectively,

$$\left.\begin{aligned} \alpha &\doteq \frac{A - G}{A + 2G + B} \\ \beta &\doteq \frac{0.4\,(G - B)}{A + 2G + B} \end{aligned}\right\} \qquad (3)$$

Other photoelectric tristimulus colorimeters probably employ, or will employ, source-filter-photocell combinations which are spectrally almost identical to those used in the multipurpose reflectometer. Consequently it should be possible to convert settings from these other instruments to values of α and β as easily as settings from the multipurpose instrument.

One potentially serious source of error must always be thought of when tristimulus measurements are used to give values of α and β. The source-filter-photocell combinations used in the various photoelectric instruments fail to be spectrally equivalent to the I.C.I. standard observer. The errors which result from this spectral disparity vary with the amount of the disparity in the particular instrument and with the degree of spectral difference between the calibrated standard used and the sample compared with it for color (*14*). Thus the effect of this spectral disparity may be largely overcome, as was noted by Perry (*11*), by obtaining for the measurement of any sample a calibrated standard spectrally similar to it. By spectrally similar, it is meant that the spectral curves of the sample and standard must either approach identity or approach a condition in which they differ by a constant factor throughout the spectrum.

In spite of this limitation, photoelectric tristimulus colorimeters can be widely used to make color measurements of the materials with which the A.S.T.M. deals. Perry has pointed out, for instance, that practically all white and near-white specimens can be considered spectrally similar to one another and can therefore all be measured against any calibrated white standard. Because of this simplification for the measurement of whites, Perry called the device designed by him the Blancometer (*11*).

Frequently the samples in a group to be intercompared for color are spectrally similar because of similar origin. When the specimens differ from each other only in the amounts of fading or bleaching which have occurred, in the concentrations of the pigment or dye present, or in the processing, aging, or heating treatments used to prepare the separate samples, they are very frequently similar in spectral character. When, on the other hand, the specimens of a group of similarly colored samples have been obtained by the use of different dye or pigment mixtures, they are likely to be spectrally dissimilar because of the probable different spectral properties of the different colorants used. The colors of the samples in a group of this latter type cannot be accurately intercompared with a photoelectric tristimulus colorimeter.

Applications of the Method for Designating Color

Experience with the proposed method for designating colors has demonstrated its usefulness for a number of purposes. There follow several examples which indicate how α and β, indicating chromaticity, and sometimes A_{θ_I,θ_V} or T, indicating lightness, are obtained and may be used to record various color characteristics of different types of specimens.

Finding Values of α and β from Values of x and y:

In published records, the chromaticities of specimens are usually designated by x and y because of the widespread acceptance of the I.C.I. observer and coordinate system as standard. When it is desired to find values of α and β in the new uniform-chromaticity-scale diagram from values of x and y, Eq. 1 is used. As already noted, to find, for use in Fig. 1, the values of α and β for the ten Munsell colors, the chrome yellow, the international orange, the toluidine red, and also the various spectrum colors, these equations were used with the values of x and y already known. Except for the spectrum colors, the values of x and y, and the values of α and β computed therefrom and used for Fig. 1 are given in Table I.

Recording the Colors of Oils:

It is thought that the new method may prove useful for describing the colors of oils. To indicate what might be accomplished in this respect, use of the new method for designating the colors of lubricating oils is demonstrated.

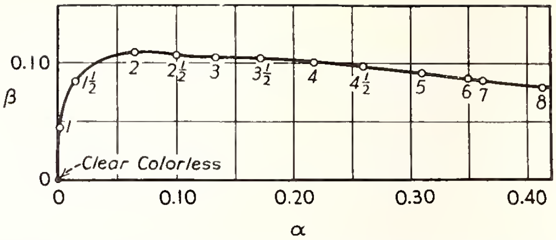

Fig. 2.—Part of (α, β)-Diagram Showing Chromaticity of the 12 Glass Combinations Used as Color Standards for the A.S.T.M. Union Colorimeter in Tentative Standard D 155 – 39 T.

Through the kindness of H. M. Hancock, Chairman of Subcommittee VI on Color of the Society's Committee D-2 on Petroleum Products and Lubricants, the spectrophotometric curves of the twelve glass combinations used as color standards for the A.S.T.M. Union Colorimeter[7] were placed at the disposal of the authors.[8] The luminous transmissions, T, and x, y, α, and β for each of the standards were computed from the corresponding spectral-transmission curves and entered in Table II. The point representing the chromaticity of each of these standards was then plotted in a section of the (α, β)-diagram, Fig. 2.

This diagram has been prepared to show the more accurate designation of colors possible with the tristimulus system. Not only would it be possible by the use of the new method to choose the best designation of the color of an oil on the Union Colorimeter scale, but it would be further possible to identify quantitatively any abnormal greenness or pinkness of the sample. Such a greenness or pinkness would be indicated by a location of the point representing the sample either above or below, respectively, the line of points of average oil colors plotted in Fig. 2. Incidentally, the spacing of the points in Fig. 2 shows that small adjustments of the colors of the standards would provide a scale of more uniform chromaticity intervals.

In addition to their use for the more accurate designation of colors of lubricating oils, values of α, β, and T could be used to represent the colors of shellacs, varnishes and other transparent materials.

Evaluation of Yellowing:

The values of α and β of panels of two white paints before and after exposure are used here to illustrate the

[7] Tentative Method of Test for Color of Lubricating Oil and Petrolatum by Means of A.S.T.M. Union Colorimeter (D 155 – 39 T), 1939 Book of A.S.T.M. Standards, Part III, p. 598.

[8] From test report 57968 of the National Bureau of Standards to A.S.T.M. Committee D-2 on Petroleum Products and Lubricants.

TABLE III.—COLOR OF TWO WHITE PAINTS BEFORE AND AFTER FOUR-MONTHS EXPOSURE.

	Paint No. 1				Paint No. 2			
	Exposed in Light		Exposed in Dark		Exposed in Light		Exposed in Dark	
	Original	Exposed	Original	Exposed	Original	Exposed	Original	Exposed
Settings with multipurpose reflectometer ($A_{45°,0°}$): Blue (B)	0.7414	0.7269	0.7358	0.6991	0.7789	0.7360	0.7830	0.7304
Amber (A)	0.8557	0.8498	0.8578	0.8579	0.8691	0.8578	0.8661	0.8627
Green (G)	0.8333	0.8256	0.8339	0.8256	0.8488	0.8301	0.8480	0.8335
Change in $A_{45°,0°}$		−0.0077		−0.0083		−0.0187		−0.0145
$A - G$	+0.0224	+0.0242	+0.0239	+0.0323	+0.0203	+0.0277	+0.0181	+0.0292
$G - B$	0.0919	0.0987	0.0981	0.1265	0.0699	0.0941	0.0650	0.1031
$0.4\ (G - B)$	0.0368	0.0395	0.0392	0.0506	0.0280	0.0376	0.0260	0.0412
$A + 2G + B$	3.2637	3.2279	3.2614	3.2082	3.3456	3.2540	3.3451	3.2601
α	0.0069	0.0075	0.0073	0.0101	0.0061	0.0085	0.0054	0.0090
β	0.0113	0.0122	0.0120	0.0158	0.0084	0.0116	0.0078	0.0126

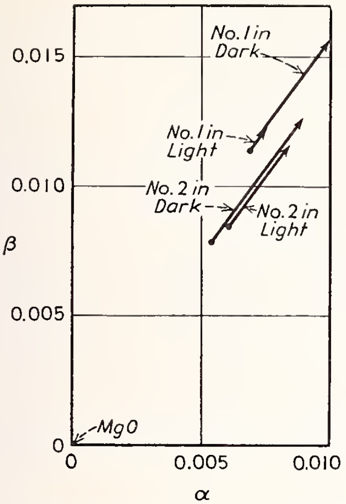

Fig. 3.—Part of (α, β)-Diagram Showing the Chromaticity Change Accompanying Yellowing of Two White Paints Both in the Light and in the Dark.

evaluation of amount of yellowing. The values of α and β were obtained from settings with the Hunter multi-purpose reflectometer (*10*) and Eq. 3. Table III gives the instrument settings for two panels of each of the two paints both before and after the exposure for yellowing. It also shows the different quantities computed in order to find the values of α and β.

The changes of chromaticity of the paints are shown graphically on the plot of part of the (α, β)-diagram in Fig. 3. In this figure, an arrow connects the two points representing each panel before and after exposure; thus the length of arrow is a measure of amount of yellowing. One panel of each paint was kept four months in the dark, the other was exposed for four months in the light. With each paint, the two initial panels failed to match each other in color exactly. Each paint yellowed less when exposed to light than when kept in the dark; in fact the yellowing of paint No. 1 in the light was almost negligible. Paint No. 1 was initially yellower than paint No. 2, but it yellowed less on exposure. The chromaticity change accompanying yellowing, it may be noted, is represented on the (α, β)-diagram by nearly parallel vectors pointing toward the yellow region of the diagram.

Fading of Tinted Paints:

Fading of tinted paints may be numerically measured by the amount of migration toward the origin of the point in the (α, β)-diagram representing the paint. An example of the use of this method was given by Scofield and Cornthwaite (*15*) who tinted a number of paints with Prussian blue and exposed them outdoors at Washington, D. C. Inspection of the paints revealed that the colors became less saturated with exposure and it proved convenient to express the fading or loss of saturation by

$$F \text{ (in per cent)} = 100 \left(\frac{D_b - D_a}{D_b} \right)$$

where D_b and D_a are the distances from the origin to the points representing the paint before and after exposure, respectively. Data showing the changes in color of one paint as it faded are reproduced in Table IV. The reflectometer settings for this paint (A, G, and B), α and β, the distance of the point in the (α, β)-diagram from the origin ($\sqrt{\alpha^2 + \beta^2}$), and the "percentage fading" for the ten different days on which this paint was measured are given.

This method of measuring amount of fading would have to be modified for specimens which change markedly in hue angle or reflectance during fading. However, materials which change markedly in hue angle or reflectance during fading are in the minority and the proposed numerical method of measuring fading is therefore applicable to all but a small proportion of colored specimens. When this method proves inadequate, the complete color-difference formula referred to in the next section may be used.

Possible Use of the (α, β)-Tristimulus System for the Numerical Measurement of Amounts of Color Difference and the Quantitative Representation of Color Tolerances:

One of the most promising fields for application of a method of designating color such as that described herein is in the description and numerical measurement of color differences. Where there exists a suitable relation for measuring amounts of color difference between specimens from tristimulus values of the specimens, it is possible to measure and express color tolerances quantitatively.

Starting with the method of measuring color tolerances devised by Judd (*7*), methods which appear to be practical have been worked out for use with the tristimulus system described above. However, the explanation of these methods would require more space than is available here. They will instead be described in forthcoming papers (*14*, *16*).

Conclusion

It should be understood that the α, β, and A_{θ_I,θ_V} or T tristimulus system which has been described is not offered as the best ultimate system for recording the colors of the materials with which the A.S.T.M. deals. However, it is con-

TABLE IV.—FADING OF BLUE PAINTS EXPOSED AT WASHINGTON. AVERAGE OF 24 PANELS.

Age, days	Reflectometer Readings			Coordinates		Distance from Origin	Fading, F, per cent
	Blue	Amber	Green	α	β	$(\sqrt{\alpha^2 + \beta^2})$	
0	0.3135	0.0600	0.1225	−0.1011	−0.1235	0.1596	...
1	0.3090	0.0716	0.1322	−0.0940	−0.1096	0.1444	9.5
4	0.3138	0.0772	0.1359	−0.0886	−0.1074	0.1392	12.8
7	0.3138	0.0812	0.1391	−0.0860	−0.1038	0.1348	15.5
14	0.3122	0.0883	0.1452	−0.0824	−0.0967	0.1270	20.4
21	0.3347	0.1097	0.1716	−0.0785	−0.0828	0.1142	28.4
28	0.3374	0.1184	0.1800	−0.0755	−0.0772	0.1080	32.3
35	0.3488	0.1313	0.1953	−0.0735	−0.0705	0.1018	36.2
42	0.3563	0.1448	0.2108	−0.0715	−0.0631	0.0954	40.2
50	0.3719	0.1576	0.2264	−0.0700	−0.0591	0.0916	42.6

sidered to represent a considerable advance in the science of colorimetry and is therefore described for the benefit of those who wish to record color by a system more apt and convenient than any heretofore available. There is no reason why the Society should not take advantage of present developments rather than wait for the ultimate solution. The method of recording colors by α, β, and A_{θ_I,θ_V} or T possesses properties which merit its consideration and possible adoption by the Society as a tentative method for recording colors.

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COLORIMETRY OF PULP AND PAPER WITH SPECIAL REFERENCE TO "BRIGHTNESS" AND "WHITENESS"

Pulp Pap. Mag. Can. 43, 92-98 (1942)

The role of bleaching in increasing the luminous reflectance and whiteness of paper is discussed in terms of spectrophotometry, not colorimetry. Moderate amounts of blue dye also increase whiteness, although they decrease luminous reflectance. Fluorescent whiteners, which are frequently used in 1978 but which are not mentioned in this paper, greatly complicate spectral measurements of white paper and the evaluation of whiteness.

Editorial note: P. 92 (p. 2 of reprint), Fig. 2, and the first column and the first two lines of the second column of p. 94 (p. 3 of reprint) should be disregarded.

COLORIMETRY OF PULP AND PAPER WITH SPECIAL REFERENCE TO "BRIGHTNESS" AND "WHITENESS"

Deane B. Judd [1]

ABSTRACT—The measurement of the color of pulp and paper may be accomplished in several ways. The most fundamental method is by the spectrophotometer followed by computations in accord with a standard observer and colorimetric coordinate system. Measurements of high precision may also be carried out by means of a three-component photoelectric colorimeter, the accuracy of which depends upon the degree with which the three photoelectric components approach in spectral character those of the standard observer. And, finally, color specification and control is also possible by direct, visual comparison with material standards of color. From any fundamental specification of color may be found an estimate of the "whiteness" of pulp or paper. Whiteness is the perceptual aspect of pulp color most directly related to its degree of bleaching. Definitions of this term and of the term "brightness," often misapplied in this connection, are given and discussed.

I. Introduction

Colorimetry of pulp and paper embraces a considerable fraction of the whole technic of colorimetry. It is the purpose of this discussion to outline in a broad way the various methods of colorimetry applicable to pulp and paper, and to tell something of the particular uses of each. Since this discussion cannot be exhaustive, the effort will be made at least to define the terms used so as to make clear the ideas required in the assessment of the various methods of colorimetry.

We start forthwith by defining colorimetry. *Colorimetry* is the measurement of color. *Color,* in turn, is that aspect of the appearance of objects and lights which is dependent upon the amount and spectral composition of radiant energy incident on the retina of the eye. Color perception depends not only upon the color of the object perceived but also on the colors of the surroundings.

II. Spectrophotometry:—The Fundamental Basis

The color of an object depends upon the changes which it introduces in the amount and spectral composition of the incident radiant energy within the visible spectrum. A black paper reflects very little of the incident visible energy; a white paper reflects nearly all of it. A red paper reflects little of the incident short-wave energy and much of the energy in the long-wave extreme of the visible spectrum. The spectrophotometer measures these characteristics and therefore provides a way of evaluating the physical properties of paper which determine its color. The spectrophotometer consists of two essential parts, a prism or diffraction grating to disperse the light according to wavelength, and a photometer to measure the distribution of it according to wavelength. The accepted standard in the spectrophotometry of reflecting materials such as pulp and paper is a layer at least half a millimeter thick of magnesium oxide freshly prepared by collecting on a suitable surface the smoke of magnesium ribbon or shavings burning in air (51). This layer of MgO is a suitable standard because of its reproducibility, and it is a natural choice because it introduces only a negligible change in the amount and spectral composition of incident visible radiant energy. The spectrophotometer analyses the radiant energy reflected from a paper specimen in some certain group of directions and for some certain angular condition of incident radiant energy. Strictly speaking, these measurements relate to the appearance of a pulp or paper specimen only when it is illuminated and viewed from these same angles. We say, therefore, that the quantity evaluated by the spectrophotometer is the spectral apparent reflectance (13, 38, 47), or spectral directional reflectance (31) relative to that for magnesium oxide. If the spectrophotometer collected all of the reflected radiant energy and evoluated it in terms of the incident energy, we would say that it evaluated spectral reflectance. *Spectral reflectance* of a specimen is the ratio of reflected to incident homogeneous radiant energy. *Apparent reflectance* is the reflectance which a perfectly diffusing surface would need to have in order to produce, under the same angular conditions of irradiation and viewing, the same instrumenetal effect as the specimen actually measured. The distinction between reflectance and apparent reflectance is necessary because of failure of all actual surfaces to diffuse perfectly the reflected energy. Sometimes the diffusing characteristics of pulp and paper specimens under consideration are such that this distinction has little practical importance; for other specimens it is important. Application of a spectrophotometer to actual samples yields, therefore, a curve showing the variation with wavelength of spectral apparent reflectance relative to magnesium oxide, if, as is usual, an opaque layer of paper or pulp is measured, the term, spectral apparent reflectivity, is used. Reflectance characterizes a particular sample or specimen which may or may not be opaque; reflectivity (38, 47) characterizes a material.

The spectrophotometer is used for the evaluation of primary material standards of color as a check on their permanence, for research on the effect of variations in mill practice on the color of paper (16), for choice of dyes (11), as a basis for checking conformity to a color tolerance (39, 45), and for general trouble-shooting in color. There are automatic spectrophotometers rapid enough to be used as the basis for routine colorimetry (13, 20, 22, 43, 48), and there are manually operated spectrophotometers (13) and abridged spectrophometers (27, 70) sufficiently inexpensive to compete with photoelectric colorimeters not pretending to provide a spectral analysis. But the spectrophotometer is not a colorimeter, it does not measure color; it only measures properties which are responsible for color.

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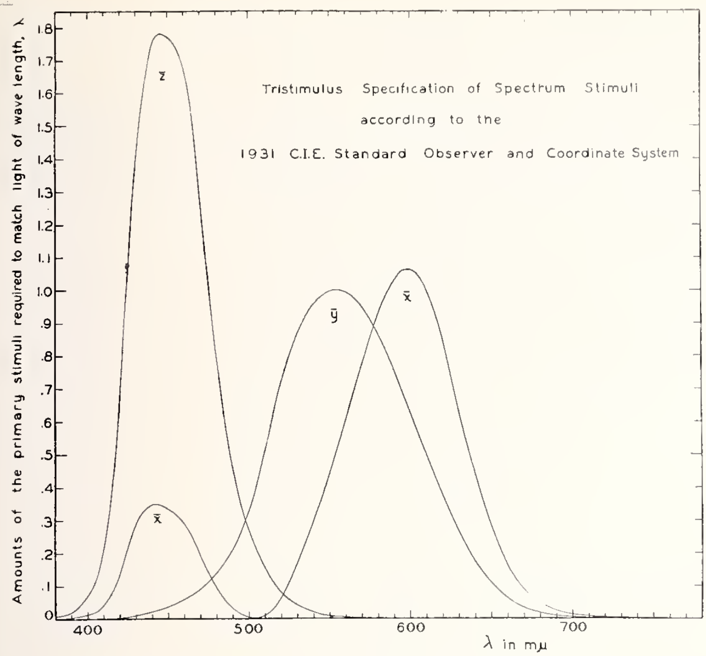


Fig. 1. Tristimulus specifications of a spectrum of unit energy per unit wavelength, the specifications being in accord with the 1931 ICI standard observer and coordinate system.

III. The 1931 ICI Standard Observer and Coordinate System for Colorimetry

If the spectrophotometer is to be used as the basis for a color measurement, the spectral analysis of a beam of reflected light must be put together again in a way analogous to the synthesis performed by the human eye. This synthesis is based upon the tristimulus method of specifying color. Tristimulus specification of color is accomplished by setting up a color match with an additive combination of three fixed lights called primaries or primary stimuli. The amounts of the three fixed lights or primaries required to make the match constitutes the specification. If a photometric field be illuminated by components of tristimulus specification, X_1, Y_1, Z_1, and X_2, Y_2, Z_2, respectively, the specification of the resultant is obtained simply by adding together the specifications of the components. The great names to be associated with this method are Thomas Young (74), von Helmholtz (23), and Maxwell (46). Grassman, a mathematician, formulated the law (15) on which it is based somewhat in these terms: Equivalent lights mixed with equivalent lights produce equivalent mixtures. This is the colorimetric equivalent of the axiom: equals added to equals, the results are equal.

The synthesis of the component spectral parts of a light beam is accomplished by successive application of Grassman's law. We go to the various parts of the curve of spectral apparent reflectance in succession at convenient intervals, say 10 mμ, and read the amount of energy in the light beam; then we find the amounts of the three primaries appropriate to that amount at that wavelength. The total amount of the three primaries required to duplicate the recombined light beam is obtained simply by adding the amounts required for each wavelength interval separately. Of course, to carry out this computation we have to know the tristimulus specifications of each part of the spectrum. These specifications vary somewhat from observer to observer and may be given for any one of a wide number of primary stimuli. It is almost universal practice, however, to use tristimulus specifications of the spectrum according to the standard observer and primaries recommended in 1931 by the International Commission on Illumination at the meeting in Cambridge, England (7, 21, 34, 63). The standard observer was evaluated by averaging the properties of 17 normal observers studied in England (17, 72). The coordinate system was based upon suggestions put forward in America (32).

Fig. 1 shows the tristimulus specifications of *the so-called equal-energy spectrum,* a spectrum having unit energy per unit wavelength, the specifications being in accord with the 1931 ICI standard observer and coordinate system. This coordinate system was chosen for the express purpose of simplifying the reduction of spectrophotometric data. The central curve, the y curve, is the well-known luminosity function of the normal eye, formerly known as the visibility function (12, 30, 32, 68).

Arithmetical, graphical, mechanical and electrical aids for reducing spectrophotometric data by the ICI system have been devised (5, 19, 21, 59, 62, 65, 69).

The ICI system specifies the color of every known light beam, be it emitted or reflected, by giving the amounts of the three primaries required to produce a color match for the standard observer. The outstanding disadvantage of this colorimetric system is that the specifications resulting from it fail to suggest the color perception at all readily. It is common practice to express the amounts of the primaries required to produce a given color as fractions of their total, and to represent the color by a point in a diagram obtained by plotting any one of these trilinear coordinates against any other, usually by plotting $Y/(X+Y+Z)$ against $X/(X+Y+Z)$ to form what is known as a mixture diagram or chromaticity diagram. The ratios are also called *trichromatic coefficients* (63). The color perceived is judged by the position of the point defined by the trichromatic coefficients relative to points representing known colors. The various parts of the spectrum constitute important colorimetric landmarks for this purpose also the lights produced by heating an enclosure to various temperatures (the so-called Planckian colors),

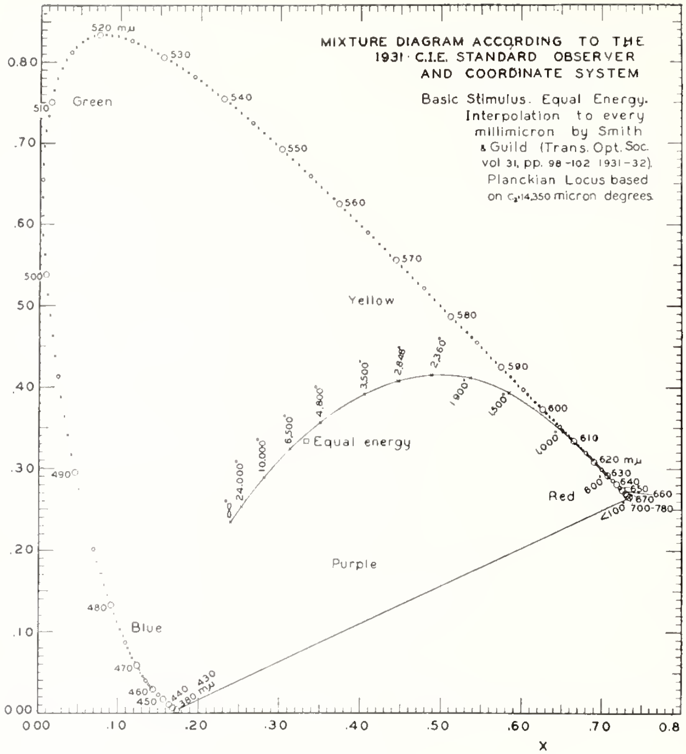


Fig. 2. The (x, y)-chromaticity or mixture diagram of the standard ICI colorimetric system. Points representing the colors of the various parts of the spectrum are indicated in millimicrons; points along the Planckian locus are indicated by the absolute temperatures in degrees Kelvin.

and for the colors of surfaces, such as pulp and paper, the color of the illuminant is an important landmark. Fig. 2 shows the most used chromaticity diagram of the ICI system, that formed by plotting Y/(X+Y+Z) abbreviated as y, against X/(X+Y+Z) abbreviated as x. On this diagram are points representing the various parts of the spectrum identified by wavelength in millimicrons, and also the locus of Planckian colors identified by the absolute temperature in degrees Kelvin.

One common way of making use of the various parts of the spectrum as colorimetric landmarks is to determine from the point representing the object color and that representing the illuminant color what is known as the dominant wavelength (21, 34, 57, 58). This wavelength is found graphically from the chromaticity diagram by extending the straight line connecting the two points until it cuts the spectrum locus, the wavelength of the intersection being known as the dominant wave length. *Dominant wavelength* of a luminous area is defined as the wavelength of that part of the spectrum which has to be added to a fixed light, such as daylight, to produce a color match for the luminous area.

The degree of approach of the object-color point to the spectrum point is measured by a ratio called purity, or, more exactly, spectral excitation purity (21, 33). *Spectral excitation purity* is defined as the distance between the object-color point and the illuminant-color point on the ICI mixture diagram divided by the distance between the spectrum-color point and the illuminant-color point. Dominant wavelength and purity are often found to be descriptive of the color perceived as belonging to the object because dominant wavelength indicates what part of the spectrum the object color is to be associated with, and purity indicates how close the association is. To anyone who knows the appearance of the various parts of the spectrum and their mixtures with common illuminants, say daylight, dominant wavelength and purity give useful information.

The color of a surface is specified in the ICI system by three numbers: luminous reflectance, Y, and the trichromatic coefficients, x and y. Or, alternatively, the color of a surface may be specified by luminous reflectance, dominant wavelength, and purity.

The ICI system is used in conjunction with the spectrophotometer in the fundamental colorimetry of primary material standards of color; and it is used in determinations of the gamut of colors producible in paper by various concentrations of a dye or combination of dyes in a given furnish (11, 16). It is possible, and under some circumstances profitable, to evaluate within the ICI system the color effect of the chief variations possible in the furnish. The beater-room man would in this way know ahead of time what change in dye concentration would be required for a given variation in, say, pulp color, pH, or freeness of stock (16). The ICI system is sometimes used in conjunction with spectrophotometric measurements to determine conformity to a color tolerance (39, 40, 45). Linear transformations (2, 3, 6, 25, 31, 35, 44, 56, 60, 61, 73) have been proposed which are more convenient than the ICI coordinate system for expressing color tolerances. By means of such transformations it is possible to use the ICI standard observer to compute estimates of "whiteness" to be discussed presently (25, 36, 37).

IV. Photoelectric Colorimeters

It is possible to obtain by means of a photo-electric cell or a barrier-layer cell used in conjunction with a light source and three suitably chosen optical filters, a sufficient approximation to ICI color specifications for many purposes. If no attempt is made to choose optical filters to duplicate the properties of the standard observer, the photoelectric comparator may still be a useful instrument, but ought not to be called a colorimeter since it then does not measure color.

The sources of error in photoelectric colorimetry for general purposes are legion (10, 18, 24, 25, 28, 55, 71), and it is not within the scope of this discussion even to enumerate them. However, the effect of failure of the source-filter-photocell combination to duplicate perfectly the properties of the standard observer may be rendered negligible by the use of a material standard of color of known ICI specifications having spectral characteristics similar to those of the unknown specimens whose color it is desired to evaluate (25). Since all white and near-white papers differ but slightly in spectral characteristics, no very striking degree of duplication of the standard observer is required to make a photoelectric colorimeter useful for comparing the colors of such paper with that of the standard magnesium-oxide layer (56). Indeed, this fact renders many photoelectric comparators useful, even though they make no pretense of duplicating the standard observer (27, 69). Such instruments may prove to be very valuable, for example, in control of the color of paper provided there is only one essential cause for color variation. Such comparators have also introduced a good deal of confusion because of attempts to give names to the quantities which they measure. Since these quantities depend essentially on the use of the instrument to a restricted class of specimens, and can often not be measured in any other way than by that one instrument, they usually do not deserve any special name except the name of the instrument.

The photoelectric colorimeter, however, in conjunction with calibrated standards, provides a precise, rapid, and reasonably accurate way of evaluating luminous reflectance, Y, the trichromatic coefficients, x and y, and estimates of the perceptual attributes to be discussed presently which can be computed from Y, x and y. Even without calibrated standards, a trichromatic colorimeter provides probably the most convenient moment to moment mill check upon the constancy of pulp color and paper color, and is therefore a very convenient way to check for conformity to a color tolerance (25), regardless of whether they are expressed in fundamental terms, or simply in terms of the instrument itself.

V. Material Standards of Surface Color

Probably the most used method of maintaining the correct concentration of dye in the beater furnish is by the experienced eye of the color man. He can compare the color of the paper on the machine with that of a sample approved by the customer, and through his years of experience gathered by means of more or less costly mistakes can make adjustments of the furnish to correct any color errors. He may often be able to tell from the look of the furnish whether the paper color will be off far enough to bring a complaint. This method is measurement of a sort, and the evaluation of the color is directly in terms of the information desired.

The method of direct comparison may be made more formally quantitative by resort to a system of material color standards defining attributes by means of which color may be specified. Examples of such systems are the Munsell system (49, 52) giving evaluation of Munsell hue, value, and chroma, by direct visual interpolation among the members of the system, and the Ostwald system (53, 54, 66) yielding Ostwald hue, black content and white content.

The precision possible by such direct visual interpolation is insufficient for most color control of paper, but approximate color specifications by such systems can conveniently be cabled, and the use of such systems by commercial designers (4, 29) using dyed paper suggests a possible advantage of them to the papermaker or, at least, to his salesman.

VI. Estimates of the Perceptual Attributes of Color

It is probably fair to say that the excuse for all colorimetry of pulp and paper is to make sure that the customer will perceive that the paper has a certain desired color. Let us inquire what are the attributes of color perception. Let us get away from modern colorimetry. Consider a man on a desert isle,—no spectrophometer. Suppose he finds a trunk full of colored papers washed up on the beach. How would he sort them according to color?

1. *Hue, Lightness and Saturation.*

First he spreads them out on the sand to look at them (chart 1), and finds a purely chance arrangement or disorder. His next step is to put all papers of the same color together. In doing this, he notices that a certain group of the colored papers lacks an important characteristic possessed by the other papers; that is, some of the papers lack *hue*. He separates the colored papers into two groups (chart 2), chromatic, those perceived as having hue, and achromatic, those perceived to be without hue. He notes that the achromatic papers can be arranged in a single series ranging from black at the bottom through a series of dark grays, middle grays and light grays to white at the top (chart 3); this series varies in lightness alone. Furthermore, for each chromatic paper he notes that there can be found a gray of the same lightness called the equivalent gray.

Then he turns his attention to the chromatic papers and sorts them out, putting all the red ones together, then the yellow ones, green ones and blue ones (chart 4); and he also notes that there are others falling intermediate to these groups (chart 5) so that a whole circle can be set up, each part of which differs from the neighboring part by only a small step. This classification is by hue. Lightness and hue are the most obvious attributes of color perception.

But by sorting out all papers of the same hue (say red) and the same lightness (say medium lightness) he discovers that there are still variations in color (chart 6). The mode of variation is more subtle than the other attributes; it is called saturation. He finds that saturation supplies the basis for the division of the papers into a chromatic group and an achromatic group. For achromatic papers (black, grays, white) saturation is zero; for chromatic papers, greater then zero. Saturation is proportional to the color departure from the equivalent gray (chart 6). *Hue* is the attribute of a color perception which permits it to be classed as red, yellow, green, blue, purple, or an intermediate (41, 42). *Lightness* is the attribute of a color perception which permits it to be classed as equivalent to one of a series of grays ranging from black to white.

Saturation is the attribute of a color perception which expresses the degree of its difference from the equivalent gray.

A natural question is, "Why don't we measure hue, lightness, and saturation directly, if that is the way a customer judges color?" And the answer is that it can't be done. The perception of an object color depends not only upon the light reaching the eye of the observer from the object, but also upon that from the surroundings (1, 14, 52). An object may be perceived as having any one of a considerable range of colors depending on the surroundings. Ask a jewelry salesman why he displays diamonds on black velvet.

We may, however, estimate from measurements of color, what the color perception will be under ordinary conditions of viewing. Thus, an estimate of hue may be found from dominant wave length which is sometimes on that account confusingly called hue wavelength. Munsell hue and Ostwald hue are also useful for such estimates. Hue estimates are particularly valuable because a color departure in which the hue of the sample is the same as the hue of the standard are less likely to be objectionable than equal departures involving a hue difference.

Lightness estimates on a perceptually nearly uniform scale for ordinary observing conditions are given by the square root of the luminous reflectance, Y (40). One of the most important optical properties of paper is to form a suitable background for printed characters. The principal conditions for legibility of characters is that the lightness difference between letters and background be large. This estimate of lightness serves to evaluate quantitatively the legibility of print. Munsell value gives closely the same estimate (14, 50).

Approximate saturation estimates are given by purity. Munsell chroma gives essentially the same estimate on a basis which permits more reliable intercomparison between colors of different dominant wavelength. If a customer perceives that the delivered paper fails to match the standard by being less saturated, he will feel inclined to register a complaint because it will seem to him that the papermaker tried to skimp on the dye. But if he perceives it to be of higher saturation, he is less likely to be critical. These remarks, of course, do not apply to white papers.

2. *"Whiteness" and "Brightness"*

There are many other perceptual attributes of surface color in addition to hue, lightness and saturation; these subsidiary attributes refer to alternate ways by means of which our color preceptions may be organized, and they are subsidiary only in the sense that they are less common. One of these is "whiteness" which is allied to Ostwald "white content." The meaning of whiteness of pulp and paper is fairly well agreed upon. It is commonly taken to mean the degree of approach to the appearance of an ideal white. The ideal white surface on which judgments of whiteness are based is frequently taken as the ideal perfectly reflecting, perfectly diffusing surface, satisfactorily approximated for practical purposes by an opaque layer of magnesium oxide. Some observers have been found, however, to make similar judgments based upon the color of paper made from well-bleached, undyed pulp as a standard white (37). The perceptual character of whiteness is, I think, very evident to anyone who has attempted to make estimates of it. A paper perceived to have nearly a perfect white color at one moment will often be perceived as very much off-white when viewed later under only slightly different circumstances. However, if a satisfactory standard white is at hand, paper graders can learn to make these difficult estimates with good consistency as has been demonstrated by obtaining repeat estimates after a lapse of several days (37). It is probable that an experienced grader could also learn to make similar estimates of "pinkness" or "blueness," in accord with degree of resemblance to the appearance of a standard pink or blue.

A much less satisfactory situation exists with respect to the term, "brightness". Several more or less related subsidiary color attributes are now referred to by this name. Perhaps the most common attribute is that which suggests the phrases "bright red, bright green, or bright yellow" in the sense of being the kind of color produced by clean stock and dyes. Brightness in this sense would be defined as the degree of departure from a dark gray, or some other standard of the color of dirt. In this sense it would be logical to speak of one white as brighter than another, and we could even have one black brighter than another because it was a more jet black. Brightness of color in this sense is synonymous with cleanness; another name for the same idea which has been approved in psychological circles (64, 67) is "intensity." This use of the term, brightness, is not recommended.

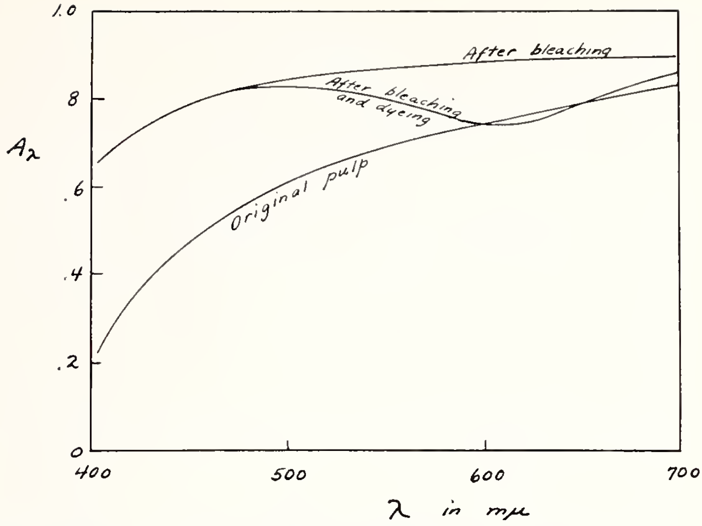
Another fairly common usage of the term, brightness, is to designate the Y tristimulus value of the ICI system. For reflecting specimens, such as pulp and paper, this usage would make brightness synonymous either with luminous apparent reflectance, or with luminous apparent reflectivity, and is, of course, to be avoided.

Still another more specialized meaning for the term, brightness, particularly as applied to pulps is that it is the aspect of the appearance of pulp which indicates the whiteness of the whitest paper that can be made from that pulp by addition of a suitable blue dye (8, 9, 43). Both this rather complicated idea, and "whiteness" are related to degree of bleach.

Fortunately we do not have to depend upon anything as complicated as this idea, or as uncertain as an estimate of whiteness for evaluating the degree of bleach. Luminous reflectance, Y, itself, is a good measure of the degree of bleach, as is also the Z value of the ICI system and the closely allied quantity measured by the Appleton-GE reflection meter. It is unfortunate that quantities of this sort have as just mentioned sometimes been given the name, brightness, among paper manufacturers; for example, the Appleton-GE reflection meter (89, 27) was called by its designers the "brightness tester," and its widespread use because of the many good features of its design and the thoroughness of the standardization work have caused this misuse of the term, brightness, to increase its foothold in the industry. The quantity actually measured by the Appleton-GE reflection meter is apparent reflectivity for a certain kind of blue light, and the short name, bluflect (43), has been proposed but did not catch on. This quantity is certainly not brightness. Brightness has two well-recognized and closely connected meanings, both of which characterize a self-luminous area, or the combination of a non-self-luminous surface with an illuminant. Photometric brightness is defined by the Illuminating Engineering Society (26) as "the luminous intensity of any surface in a given direction per unit of projected area of the surface as viewed from that direction." There are many recognized units of photometric brightness; international practice is to express it in candles per unit area of surface; other units are the stilb, the lambert, and the footlambert (26). The term, brightness, is also commonly used to denote the perceptual evaluation of photometric brightness, and in this usage depends upon the photometric brightness of the surroundings, though, of course, photometric brightness itself does not.

It may be instructive to trace the connection between lightness, whiteness, and apparent reflectivity for blue light relative to magnesium oxide for a yellowish pulp which is first bleached, then dyed approximately to maximum whiteness with blue dye; see Fig. 3. The effect

Fig. 3. *Typical curves of spectral apparent reflectivity first for an undyed yellowish pulp, second, for the pulp after bleaching, and third for the pulp after both bleaching and admixture of blue dye.*



of bleaching the pulp is to raise its spectral apparent reflectance, A, throughout the visible spectrum (43), as indicated, but the greatest increase occurs in the short-wave portion of the spectrum. Therefore, reflectivity for blue light is strikingly increased, and daylight apparent reflectivity is moderately increased; correspondingly, the lightness of the perceived color is moderately increased. Whiteness of pulps such as this depends predominantly on degree of approach to nonselectivity of spectral apparent reflectivity, and since there has been a moderate increase in this degree of approach, the whiteness of the pulp is also moderately increased. Now consider the effect of adding blue dye. The curve of spectral apparent reflectivity is left nearly unchanged in the short-wave portion of the spectrum; so apparent reflectivity for blue light is only slightly reduced. In other portions of the spectrum, however, there is a marked reduction in spectral apparent reflectivity (43). Therefore, luminous apparent reflectivity is markedly reduced as is also the lightness of the perceived color. The addition of blue dye has, however, reduced the spectral apparent reflectivity in the portions of the spectrum where it was previously highest. It has therefore produced a much closer approach to nonselectivity with consequent marked increase in whiteness of the perceived color.

It will be noted that apparent reflectivity for blue light is not a very good measure of the whiteness of the color perceived to belong to a pulp. By blue dye, whiteness may be increased markedly with but little effect on apparent reflectivity for blue light. But apparent reflectivity for blue light is a good indication of the whiteness of the whitest paper that can be made from the pulp by addition of suitable blue dye, and chiefly to this property it owes what advantage it has over apparent reflectivity of other kinds of light as a measure of the degree to which the bleaching process has been carried.

Recent restrictions on the use of chlorine for a bleach because of war-time requirements have made more urgent the need for an industry-wide recognition of a measure of the degree of bleach. It is to be hoped that such a measure will be defined in fundamental terms, entirely divorced from any one measuring device, and that the optical quantity by means of which degree of bleach is measured be not called brightness.

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FRESNEL REFLECTION OF DIFFUSELY INCIDENT LIGHT

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The caveat expressed in the last two sentences, concerning internal reflection of incompletely diffuse light, is relevant in almost all Kubelka-Munk analyses of pigmented layers. Because of incomplete diffusion and microscopic deviations of the surface from perfect planeness, internal reflectance is always less than 60% and often less than 50%.

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FRESNEL REFLECTION OF DIFFUSELY INCIDENT LIGHT

By Deane B. Judd

ABSTRACT

The reflection factor of a plane boundary between two media has been computed by the Fresnel formula for unpolarized, perfectly diffused incident light as a function of the relative index of refraction of the media. Because of total internal reflection, the factors depend importantly upon whether the diffuse flux is incident externally or internally. For example, diffuse light in air incident on the plane surface of glass of index 1.5 is 9.2 percent reflected; but if the perfectly diffuse light is incident internally, the reflection factor is 60 percent.

In the optical specification of light-scattering materials[1] by means of the absorption and scattering coefficients of the Kubelka-Munk theory,[2] it is frequently necessary to consider the effect of both external and internal reflection of light diffusely incident on a plane boundary between two media of different indices of refraction.[3] The reflectance at such a boundary has therefore been computed by application of the well-known Fresnel formula. The results are shown in table 1 as a function of the ratio, m, of the index of refraction of the denser medium to that of the rarer. The reflectance for perpendicular incidence, r, computed from the formula

$$r=(m-1)^2/(m+1)^2 \tag{1}$$

is also shown in the table.

The reflectance for completely diffused unpolarized light incident externally was computed from the approximate formula given by McNicholas:[4]

$$r=\sum_0^{\pi/2} r_\phi \sin 2\phi\Delta\phi \Big/ \sum_0^{\pi/2} \sin 2\phi\Delta\phi, \tag{2}$$

where r_ϕ is the reflectance by the Fresnel formula for unpolarized light incident on the surface from the rarer medium at an angle, ϕ, from the perpendicular to the surface. The angular interval, $\Delta\phi$, used in these summations was 0.04 radian.

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TABLE 1.—*Reflectance of unpolarized light at a plane boundary between two media as a function of their relative index of refraction, m*

m	Reflectance for perpendicular incidence	Reflectance for completely diffuse incidence		*m*	Reflectance for perpendicular incidence	Reflectance for completely diffuse incidence	
		External reflection	Internal reflection			External reflection	Internal reflection
1.00	0.00000	0.0000	0.000	1.31	0.01801	0.0628	0.454
1.01	.00002	.0028	.022	1.32	.01902	.0644	.463
1.02	.00010	.0055	.044	1.33	.02006	.0660	.472
1.03	.00022	.0082	.064	1.34	.02111	.0676	.480
1.04	.00038	.0108	.084	1.35	.02218	.0692	.489
1.05	.00059	.0134	.103	1.36	.02327	.0707	.497
1.06	.00085	.0158	.122	1.37	.02437	.0723	.505
1.07	.00114	.0183	.140	1.38	.02549	.0738	.513
1.08	.00148	.0206	.158	1.39	.02663	.0754	.520
1.09	.00185	.0230	.175	1.40	.02778	.0769	.528
1.10	.00227	.0252	.192	1.41	.02894	.0784	.536
1.11	.00272	.0274	.208	1.42	.03012	.0800	.543
1.12	.00320	.0294	.224	1.43	.03131	.0815	.550
1.13	.00372	.0314	.240	1.44	.03252	.0830	.557
1.14	.00428	.0334	.254	1.45	.03373	.0845	.564
1.15	.00487	.0353	.269	1.46	.03497	.0860	.571
1.16	.00549	.0371	.283	1.47	.03621	.0875	.577
1.17	.00614	.0389	.296	1.48	.03746	.0890	.584
1.18	.00682	.0407	.309	1.49	.03873	.0904	.590
1.19	.00753	.0425	.322	1.50	.04000	.0919	.596
1.20	.00826	.0443	.335	1.51	.04129	.0934	.602
1.21	.00903	.0461	.347	1.52	.04258	.0948	.608
1.22	.00982	.0478	.359	1.53	.04389	.0963	.614
1.23	.01064	.0496	.371	1.54	.04520	.0977	.619
1.24	.01148	.0513	.382	1.55	.04652	.0992	.624
1.25	.01235	.0530	.393	1.56	.04785	.1006	.630
1.26	.01323	.0546	.404	1.57	.04919	.1020	.635
1.27	.01415	.0563	.414	1.58	.05054	.1035	.640
1.28	.01508	.0579	.424	1.59	.05189	.1049	.645
1.29	.01604	.0596	.434	1.60	.05325	.1063	.650
1.30	.01701	.0612	.444				

The reflectance for completely diffused unpolarized light incident internally was computed from the similar formula:

$$r=\sum_{0}^{\pi/2} r_\theta \sin 2\theta\Delta\theta \Big/ \sum_{0}^{\pi/2} \sin 2\theta\Delta\theta, \tag{3}$$

where r_θ is the reflectance by the Fresnel formula for unpolarized light incident on the surface from the denser medium at an angle, θ, from the perpendicular to the surface. The angular interval, $\Delta\theta$, used in these summations was 0.04 radian except for the interval of 0.04 radian containing the critical angle ($\theta_c=\sin^{-1} 1/m$) for which it was taken as 0.0005 radian.

Values of r_ϕ were read directly from tables published by Moon.[5] Values of r_θ for θ less than the critical angle were found by reading the value of r_ϕ from Moon's table for ϕ equal to the angle whose sin is $m \sin\theta$. For θ greater than the critical angle, r_θ is, of course, equal to 1.

The summations were carried out for $m=1.1$, 1.2, 1.3, 1.4, 1.5, and 1.6. Intermediate values were found by third-difference osculatory interpolation [6] with the terminal intervals filled in by extrapolation of the third-differences. The uncertainty is estimated to be less than 2 in the last figure given. The values of reflectance for external

[5] Parry Moon, *A table of Fresnel reflection*, J. Math. Phys. **19**, 1 (1940).

[6] J. W Glover, *Derivation of the United States Mortality Table by osculatory interpolation*, Quart. Pub. Am. Statistical Assoc. **12**, 90 (1910).
D. B. Judd, *Extension of the standard visibility function to intervals of 1 millimicron by third-difference osculatory interpolation*, BS J. Research **6**, 465 (1931) RP280.

diffuse incidence agree well with those computed by Ryde and Cooper (see footnote 3) from a formula derived by Walsh.[7]

Figure 1 shows as a function of *m*, reflectance for perpendicular incidence, reflectance for external diffuse incidence, and reflectance for internal diffuse incidence. It may be seen that there is consider-

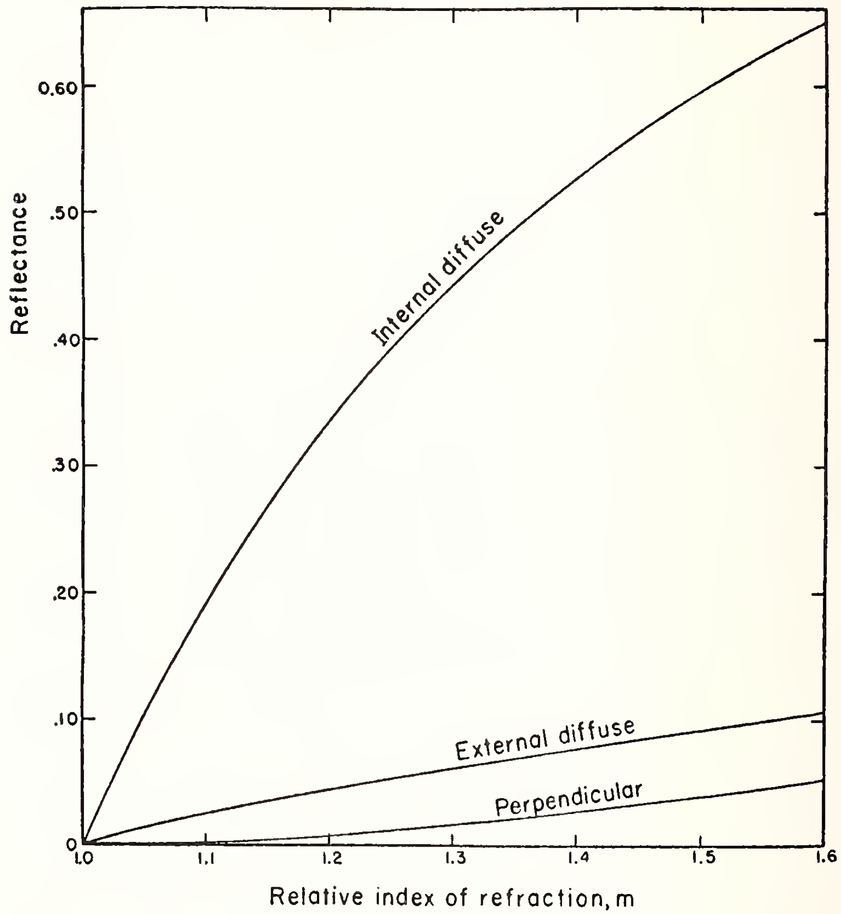

FIGURE 1.—*Fraction of incident light reflected from a plane interface for three angular conditions of illumination:* (*a*) *perpendicular incidence,* (*b*) *perfectly diffuse incidence from the less dense medium* (*external*), *and* (*c*) *perfectly diffuse incidence from the denser medium* (*internal*).

Note that internal reflection of diffusely incident light takes place to a considerable degree even at boundaries between media of only slightly differing index of refraction (*m* approaching 1.0).

able difference between these functions not only in amount but also in the shapes of the curves which represent them.

Preston,[3] Ryde,[1] and Duntley [8] have correctly warned that internal reflectance is sharply dependent on the degree of approach to complete diffusion of the incident light. For the same reasons it is

[7] J. W. T. Walsh, *The reflection factor of a polished glass surface for diffused light,* Dept. Sci. Ind. Res. (Brit.), Illumination Research Tech. Pap. **2**, 10 (1926).

[8] S. Q. Duntley, *The optical properties of diffusing materials,* J. Opt. Soc. Am. **32**, 61 (1942).

also sharply dependent upon the degree of approach of the surface to a perfect plane. A slightly wavy or scratched surface bounding a diffusing medium, or a surface exhibiting an "orange-peel" texture should be expected to have values of internal reflectance considerably lower than those for a perfect plane.

Acknowledgment is made to Margaret M. Balcom for carrying out the computations.

WASHINGTON, August 1942.

FACTS OF COLOR BLINDNESS

J. Opt. Soc. Am. 33, 294-307 (1943)

Little comment is needed for this classic paper. The terminology used has not been changed in the subsequent third of a century, perhaps because this summary of the facts was so complete and the exposition so lucid.

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Facts of Color-Blindness*

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1. EARLY HISTORY

COLOR-BLINDNESS is a subject requiring statistical treatment; there are types of color-blindness and a greater or lesser adherence of groups of observers to these types. These facts are disclosed in a literature which is sufficiently extensive itself to require statistical treatment. Figure 1 shows the distribution of titles according to decade for a fairly complete bibliography on color-blindness. To present some semblance of historical perspective, it is convenient to summarize the discoveries about color-blindness up to 1855 before attempting a summary of present-day knowledge.

According to this bibliography there appeared in 1684 in the *Philosophical Transactions of the Royal Society* an account by D. Tuberville of "Several Remarkable Cases in Physick Relating Chiefly to the Eyes."[51] This is perhaps the first published indication that there is such an abnormality as color-blindness. Ninety-three years later the indication is definite. J. Huddart published in the same transactions (1777) an account "Of a Person Which Could Not Distinguish Colours,"[18] and Whisson in 1778 published "An Account of a Remarkable Imperfection of Sight."[57]

However, not until Dalton's account of his own case appeared in 1798 in the *Memoirs of the Literary and Philosophical Society of Manchester*[2] could it be said that a lucid description of color-blindness had been given. Dalton said of the spectrum (p. 31), "I found that persons in general distinguish six kinds of colour in the solar image, namely, red, orange, yellow, green, blue, and purple. Newton, indeed, divides the purple in indigo and violet; but the difference between him and others is merely nominal. To me it is quite otherwise. I see only two, or at most three, distinctions. These I should call yellow and blue, or yellow, blue and purple. My yellow comprehends the red, orange, yellow, and green of others; and my blue and purple coincide with theirs." And elsewhere (p. 34), "Woolen yarn, dyed crimson, or dark blue is the same to me."

Dalton's explanation for his abnormal vision was that his eye media must absorb the red end of the spectrum strongly and so prevent him from seeing as others did. In commenting on this

* Paper presented at the Symposium on Color-Blindness at the meeting of the Optical Society of America, New York, March 5–6, 1943.

explanation, Thomas Young[61] said, ". . . it is much more simple to suppose the absence or paralysis of those fibers of the retina which are calculated to perceive red." This is the original statement of the Young theory of color-blindness, which has probably attracted more attention than any other explanation and exerts a profound influence today.

The question was immediately raised whether Dalton was really blind to the extreme red end of the spectrum and Herschel (p. 60),[58] after experimenting with Dalton as a subject told him, "It is clear to me that you, and all others so affected, perceive *as light* every ray which others do. . . . It seems to me that we (the normal-eyed) have three primary sensations where you have only two. We refer or can refer in imagination all colours to three, ... All other colours we think we perceive to be mixtures of these, and can produce them by actual mixtures of powders of the primary hues, ... Now, to eyes of your kind, it seems to me that all your tints are referable to two." However, Dalton persisted in his own explanation, and was not proven to be wrong until after his death in 1844 when posthumous examination of his eye media showed them to be normally transparent for red light (p. 54).[58]

Color-blindness aroused much interest during the early part of the nineteenth century, and was called Daltonism by European writers of those times over the protests of British scientists, who thought Dalton should be known as the originator of the atomic theory of matter rather than the arch-type of a defect of vision. A continuing controversy raged concerning whether color-blind observers are really blind to red. The answer was given by Seebeck[48] in 1837 that some color-blinds are relatively insensitive to red, but most are not.

In 1852 Helmholtz[13] gave an account of compound colors on the basis of the Young theory, and in 1855 Maxwell[31] pointed out that according to this theory it is possible to define the confusion colors of color-blinds in terms of the color triangle now known as the Maxwell triangle. He said, "If we find two combinations of colours which appear identical to a color-blind person, and mark their positions on the triangle of colours, then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two."

Thus, within 60 years of Dalton's description of color-blindness many of the chief questions had been raised and answered correctly. Color-blindness was found to have a number of forms, chief of which correspond to a reduction from the normal three-component visual system to a two-component system as in Dalton's case of partial color-blindness, or to a one-component system as

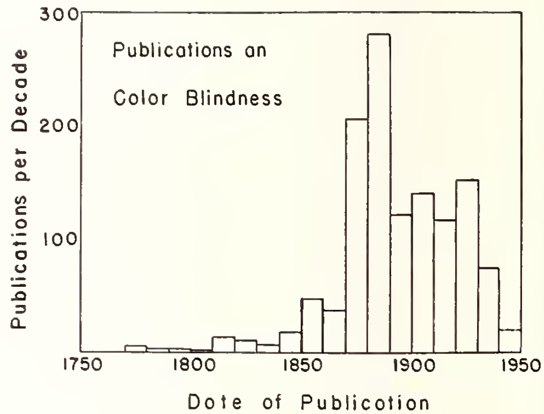

Fig. 1. Distribution of 1200 publications on color-blindness by decade. Note the great activity between 1870 and 1890, and the continuing steady interest since that time.

in cases of total color-blindness. The chief characteristic of color-blindness, that of chromaticity confusion, was recognized and Maxwell had pointed out how these confusions could be represented accurately on the Maxwell triangle. Furthermore, Dalton had correctly analyzed his own sensations as yellow and blue. It was known that there are degrees of chromaticity confusion as well as different kinds. Permanent or inherited color-blindness was found in some degree or other in about 10 percent of the male population, and in a considerably smaller fraction of the female population. Cases of color-blindness associated with injury and disease had also been established.

Thus in 1855 it would have been possible for a clairvoyant to make an outline of the chief facts of color-blindness from the then existing literature. But there was considerable contradictory evidence and many conclusions drawn from faulty or insufficient ground. Thus, because of lack of photometric equipment, Herschel failed to establish that Dalton was less than normally sensitive to the long-wave end of the spectrum in spite of the fact that both Young's explanation and Dalton's own explanation required it. It had been correctly pointed out that, in spite of a proven connection between color vision and the pigmentation of the eye media, the color of eyes and hair is no reliable guide to color-blindness; but the question was still open whether color-blind observers could be detected by a flatness of the skull just beneath the eyebrows. One eminent British authority prescribed test methods foreshadowing the Holmgren wool test, but also suggested (p. 152)[58] that the detection of color-blind observers might also be assisted by "a singular expression of their eyes, either a startled expression as if they were alarmed" or "an aimless eager glance, as if seeking to perceive something but unable to find it." This point was raised again in 1881.[19] Almost the first theory of color-blindness, the Young theory, still commands respect, but there were other theories to choose from, the eye-pigment theory, the choroid-coat coloration theory, and various phrenological theories, none at all elaborated so as to be amenable to check. Thus we may say that in 1855 the important questions about color-blindness had been raised and qualitatively correct answers had been supplied, but the qualitative facts had not been proven, they had merely been suggested; and of the quantitative facts almost none had been established.

The year 1855 is a convenient one for taking stock not only because the ground-work had been laid by Helmholtz and Maxwell for the evaluation of the precise properties of color-blind vision, but also because the importance of excluding color-blind observers from certain occupations was beginning to be generally recognized. The bibliography lists about 100 titles up to 1855 or less than 10 percent of the total of 1200 titles; see Fig. 1. The period of greatest activity in the study of color-blindness was between 1870 and 1890 and the interest has continued steadily up to the present time. From this extensive literature, the facts of color-blindness have been drawn. It is the purpose of this paper to present a summary of these facts, and it is hoped that they will be given essentially their proper emphasis and perspective in spite of the fact that only 80 of the 1200 publications have been studied in any detail. This summary has four parts, (a) classification of visual systems, (b) causes of color-blindness, (c) luminosity functions of red-green-blind and red-green-weak observers, and (d) chromaticity confusions of red-green-blind observers.

2. CLASSIFICATION OF VISUAL SYSTEMS

Seebeck's classification of vision into normal and two types of abnormal vision has been corroborated by scores of studies since 1837. In addition to these three classes many more have been established. Table I summarizes the characteristics of the chief classes of vision which are now fairly well established as well as several which are still subjects of controversy.

Normal vision, and indeed any type of visual system, may be conveniently summarized by giving a description of an equal-energy spectrum as it appears to an observer of that type. The equal-energy spectrum is one having constant radiance per unit wave-length. Every stimulus affecting the eye is either a portion of this spectrum or may be regarded as a combination of a number of such portions. We may therefore sketch approximately the properties of an observer by giving his description of the spectrum just as Dalton did for his own vision.

A. Normal Vision

To the normal observer the spectrum appears as a series of chromatic colors varying from dim red through orange, yellow, brilliant yellow-green, green, blue, to dim violet. The brightest part of the equal-energy spectrum under usual observing conditions is between wave-lengths 540 and 570 millimicrons (yellowish green), and from this point (average 555 mμ) toward both greater and smaller wave-lengths the brightness progressively diminishes. These facts are represented by the luminosity function of the average eye whose maximum is at 555 mμ. The colors seen by a normal observer can be duplicated with trivial exceptions by mixtures of three lights, a red light, a green light, and a blue light, each light being independently adjustable in amount; two lights are insufficient and more than three not necessary. On this account normal vision is classed as trichromatic. A corollary of trichro-

TABLE I. Classification and characteristics of the various visual systems.

Designation of type according to number of components	Discriminations possible by this type	Wave-length of the maximum of luminosity function, mμ	Wave-length of neutral points in spectrum, mμ	Non-theoretical designation (v. Kries)[1]	Theoretical designations: Young-Helmholtz	Theoretical designations: Hering	Theoretical designations: G. E. Müller
Trichromatism	Light-dark Yellow-blue Red-green	555	None	Normal system	Normal system	Normal system	Normal system
	Light-dark Yellow-blue Red-green (weak)	540	None	Protanomaly[2]	Abnormal red function	——	Alteration[5] system
	Light-dark Yellow-blue Red-green (weak)	560	None	Deuteranomaly[2]	Abnormal green function	Red-green weakness	Alteration[5] system
	Light-dark Red-green Yellow-blue (weak)	560	None	Tritanomaly[3]	Abnormal violet function	Yellow-blue weakness	Alteration[5] system
Dichromatism	Light-dark Yellow-blue	540	493	Protanopia[7]	Red blindness	——	Outer red-green blindness
	Light-dark Yellow-blue	560	497	Deuteranopia[7]	Green blindness	Red-green blindness	Inner red-green blindness
	Light-dark Red-green	560	572	Tritanopia	Violet blindness	——	Outer yellow-blue blindness
	Light-dark Red-green	560	470 580	Tetartanopia[4]	——	Yellow-blue blindness	Inner yellow-blue blindness
Monochromatism	Light-dark	510	All	Congenital total color-blindness[8]	——	——	Cone blindness[6]
	Light-dark	560	All	Acquired total color-blindness[8]	——	Total color-blindness	Inner total color-blindness, Type I
	Light-dark	540	All	——	——	——	Inner total color-blindness, Type II

[1] See literature reference 28, p. 402.
[2] König (reference 25) proposed the designation, anomalous trichromatism, for the classes later designated separately by Nagel (reference 37) in an extension of the v. Kries terminology.
[3] This extension of the v. Kries terminology was proposed by Engelking (reference 4).
[4] This extension of the v. Kries terminology was proposed by Müller (reference 34, p. 68).
[5] The term, alteration system, was proposed originally by v. Kries (references 27, 28, p. 408); Müller also uses Nagel's terminology; see note 2.
[6] The term, cone blindness, is based upon the duplicity theory of v. Kries and Parinaud, now widely accepted (reference 33).
[7] Protanopic observers are sometimes called scoterythrous (reference 47), and deuteranopic, photerythrous.
[8] Total color-blindness is often called achromatopsia (reference 49) or achromatopia.

matism is that three kinds of discrimination are possible. It is convenient to classify the discriminations of a normal observer as light-dark, yellow-blue, and red-green; that is, a normal observer can tell light colors from dark ones, yellowish colors from blue ones, and greenish colors from red ones. For example, he can tell greenish grays from reddish grays, greenish yellows from reddish yellows, and greenish blues from reddish blues; these are all examples of red-green discriminations.

The types of abnormal vision may be classed under anomalous trichromatism, dichromatism, and monochromatism.

B. Anomalous Trichromatism

An observer possessing anomalous trichromatic vision requires a mixture of three lights to produce all the colors which he is capable of experiencing just as a normal trichromat does, but to produce a given color he requires proportions significantly different from those required by the normal observer. If he is only mildly anomalous, his description of the equal-energy spectrum will agree somewhat with the normal; but if he is extremely anomalous, it will scarcely differ from that of a dichromat. The distinctions made by such an observer are light-dark, yellow-blue, and red-green; but the ability to make one of the chromatic distinctions (usually red-green) is relatively weak. There are two fairly prevalent types of anomalous trichromat, called protanomaly and deuteranomaly, and a rare type, tritanomaly, recently discovered by Engelking.[4] Observers of these types are intermediate in ability to make chromatic distinctions between normal observers and dichromatic observers of the protanopic, deuteranopic and tritanopic types respectively, to be discussed presently. Thus, the anomalous observer can make with difficulty the distinctions which the corresponding dichromat cannot make at all. However, in other respects anomalous trichromatism is not intermediate between normal trichromatism and dichromatism. For example, the luminosity function of protanomalous observers of all degrees is deficient at the long-wave end like that of the protanope. This deficiency is indicated roughly in Table I by the average wave-length (540 mμ) of the brightest part of the equal-energy spectrum. The deuteranomalous and tritanomalous observers on the other hand have luminosity functions well within normal limits.

C. Dichromatism

An observer possessing a dichromatic visual system requires a mixture of but two lights to produce all the colors which he is capable of experiencing. He can make two kinds of visual discrimination, one achromatic (light-dark) and one chromatic (either yellow-blue or red-green, usually the former).

The spectrum to an observer having red-green blindness appears in two hues only; the short-wave end of the spectrum appears blue; the long-wave end, yellow (just as Dalton described it in 1798). These two bands are separated by a region at about 495 mμ, which like average daylight has no hue at all, called the neutral point. From zero at the neutral point the saturations of the spectrum colors increase toward both the long-wave and the short-wave ends. There are two subtypes of red-green blindness; one characterized by abnormally low luminosity of the long-wave portion of the spectrum, the other by a substantially normal luminosity function.

The first subtype is that possessed by Dalton and called by Helmholtz (p. 360)[14] *red-blindness* following Young's explanation. The nontheoretical designation of this type, protanopia, proposed by v. Kries,[27] is probably the most widely used.* This designation is an improvement over the theoretical designation, red-blindness, because such a term taken from Young's original hypothesis suggests that protanopes see nothing but green and violet, whereas cases of monocular protanopia have shown that they really see yellow and blue.[3] Adherents of the Müller theory of vision also use the designation "outer (or retinal) red-green blindness" because the Müller theory,[34] places in the retina the defect causing this abnormality of vision. The Hering theory does not account for it. An alternate descriptive term, scoterythrous,[47] is sometimes substituted for protanopic. In Table I the shortening of the long-wave end of the spectrum for protanopes is indicated by the average wave-length (540 mμ) of the maximum of the protanopic luminosity function. This wave-length is definitely, though only slightly, beyond the normal limits.

The second subtype of dichromatic vision is known as deuteranopia, formerly called green blindness by Helmholtz following Young's explanation. The Müller theorists call this abnormality of vision "inner, or neural, red-green blindness" because they place in the optic nerve the structural defect causing it. The luminosity functions of deuteranopes fall well within normal

* By derivation the terms, protanopia and deuteranopia, are rather closely allied to the theoretical terms, red-blind and green-blind, which they were intended to replace. That v. Kries wished them to refer only to experimental facts is indicated by the following discussion (reference 28, p. 402) written by him in 1911:

"The connection thus established and experimentally verified between the two types of dichromats and between both of them and normal trichromats is identical with what Helmholtz had already supposed to be probably the case. In order to have a name for it that expresses simply the experimental results without involving any theoretical consequences, the writer speaks of the two colour systems of dichromats as being *reduction forms* of normal vision. At any rate the simplest and most natural explanation of these abnormalities is to regard them as being *deficiency effects*, in the sense that each of these types lacks one of the component factors of the visual organ as assumed in the Helmholtz theory. This was just the idea that was conveyed by the terms red-blind and green-blind that were formerly in use. And yet these expressions have been the source of much misunderstanding, for which the words themselves are perhaps partly responsible. As a matter of fact, green-blind persons are not really blind to green light, nor red-blind persons to red light; nor can it be assumed that the former lack the sensation of green and the latter the sensation of red. (Moreover, simply because the dichromat is supposed not to have some one of the three components of the organ of vision, we have no right at all to infer that ordinary heterogeneous daylight which has no colour for a normal eye looks blue-green to one kind of dichromat and purple to the other kind.) In order to have brief descriptive terms for the relation that has been found to exist here, without expressing any theoretical bias, the writer suggests the names *protanopes* and *deuteranopes* to describe the two kinds of dichromats, that is, persons who lack the first component or the second component, respectively, of the normal visual organ."

limits, but there is some indication that on the average they are higher than normal on the long-wave side; compare, in Table I, the maximum of the normal luminosity function (555 mμ) with that for deuteranopes (560 mμ). Deuteranopic observers are also sometimes known as photerythrous.[47]

Dichromatic vision may also take the form of yellow-blue blindness which is a rare form associated chiefly with diseases of the eye. Objects appear to such an observer a good deal as they do by candlelight to a normal observer. The spectrum to an observer having yellow-blue blindness appears in but two hues, red and green; his luminosity function is normal or nearly so, a possible slight defect on the short-wave end being hard to separate from the effects of yellow pigmentation of the eye media, which is very common. There are two subtypes of yellow-blue blindness.

The first subtype is called tritanopia. A tritanopic observer sees one neutral point in the spectrum (at about 570 mμ) which divides it into a red (long-wave) and a bluish green (short-wave) portion. The Young-Helmholtz theory gives it the name of violet blindness. The Müller theory calls it "outer yellow-blue blindness" as well as tritanopia, in analogy to protanopia or outer red-green blindness. The Hering theory does not account for it.

The second subtype is called tetartanopia by Müller who also refers to it as "inner yellow-blue blindness" and makes it analogous to deuteranopia or inner red-green blindness. Tetartanopia is characterized by two neutral points in the spectrum, one at about 580 mμ, the other at about 470 mμ. To a tetartanope the spectrum appears red from the long-wave end down to the first neutral point, green between the neutral points and red again at the short-wave end. This type is so rare that many classifications fail to include it. Only 5 cases have been studied and reported. It is associated with impaired ability to distinguish red from green and therefore resembles to a considerable extent total color-blindness.[49]

D. Monochromatism

An observer possessing this type of abnormality requires but a single light to produce all the colors which he is capable of experiencing; he can match any two stimuli merely by adjusting them to equal brightness; he is capable of light-dark discrimination, but of no chromatic discrimination whatsoever. His spectrum does not have merely one or two neutral points; it is all neutral. On this account monochromatism is rather aptly called achromatopsia or achromatopia.[49]

The most common form of monochromatism is often called cone blindness since it is universally ascribed to non-functioning of the retinal cones; the most common designation is congenital total color-blindness. Not only do monochromats of this type have at all times at least a trace of the central scotoma characteristic of normal twilight vision, but they also tend to have photophobia, or fear of light, which is characteristic of owls and of the normal dark-adapted observer, and which is ascribed to rod vision. Further common characteristics of such observers are low visual acuity and nystagmus or side-to-side eye-movements as if in an attempt to improve deficient visual acuity by using retinal areas now to the one side, now to the other side of a central blind area. The luminosity function of a cone-blind observer is the same as that of a normal dark-adapted observer; compared to the normal luminosity function it is greatly deficient on the long-wave side, and has a maximum at 510 instead of 555 mμ.

There are two less common forms of monochromatism both of which are thought to arise solely from disease of the eye and the optic nerve. Unlike cone blindness these forms are characterized by normal visual acuity as well as good foveal vision. The first type is characterized by a normal or deuteranopic luminosity function with maximum at about 555 mμ or 560 mμ; the second by a protanopic luminosity function whose maximum is at about 540 mμ. The second form is regarded as the result of an observer being born with protanopic vision and then acquiring tritanopia as a result of disease. Both are known as acquired total color-blindness.

3. CAUSES OF COLOR-BLINDNESS*

A. Congenital Color-Blindness

All forms of visual system except the two atypical forms of monochromatism just discussed

* Prepared in collaboration with Dr. LeGrand H. Hardy.

may be inherited; the abnormal forms may therefore be considered as congenital defects. Protanopia, protanomaly, deuteranopia, and deuteranomaly are regarded as sex-linked characteristics.[43, 55, 59] According to this view, a mother with normal color sense may transmit any of these defects to her sons, but her daughters by a normal father are either normal or become themselves transmitters of the defect. Color-blind daughters result either from the union of a color-blind man with a woman who is a transmitter of the same type of defect, or from parents both of whom have the same defect. By this view congenital color-blindness should be much more prevalent among males than among females. Table II, based chiefly upon the study of 5000 school children by v. Planta[43] and 18,000 school children by Waaler,[55] shows how closely the statistical facts agree with this view. Eight percent of the boys were found to possess a color defect, while less than one-half of one percent of the girls were color-blind. It is obvious, however, that by this view at least 8 percent of women must be transmitters of color-blindness.[55]

Total color-blindness is only slightly more prevalent among males than females. Estimates of the frequency of occurrence of total color-blindness have also been included in Table II, and are based upon Köllner's[24] summary of studies by Göthlin[8] and Peter.[40] Total color-blindness is regarded as a simple recessive characteristic; total color-blinds are frequently found to have parents having close blood relationships.

The estimates of one ten-thousandths of one percent for the occurrence of tritanomaly and tetartanopia correspond to three cases of tritanomaly,[4, 5, 11] and five cases of tetartanopia reported in the literature.[1, 15, 16, 34, 45, 46, 52, 53] All three cases of tritanomaly were closely related males; so there is some evidence that this defect is also sex-linked. Congenital tritanopia is but rarely studied and reported. Perhaps such cases are less seldom detected because the practically important red, yellow, and green colors can be distinguished from each other by them, though they cannot distinguish yellow from light gray. At any rate, a reliable estimate of the frequency of occurrence of congenital tritanopia cannot be made. The figure given in Table I (one out of a million males) corresponds to five cases reported in the literature[17, 23, 29, 41, 56] some of which might have been acquired tritanopia rather than congenital.

B. Acquired Color-Blindness

Acquired color-blindness (central or peripheral defect or scotoma for colors) occurs in all diseases affecting the central retina or its tracts in the optic nerve, the optic tract, or the optic cortex.

TABLE II. Frequency of occurrence of inherited visual systems.

Designation by number of components	Non-theoretical designation (v. Kries)	Percentage of the population that have these visual systems: Male		Female	
Anomalous trichromatism	Protanomaly	1.0		0.02	
	Deuteranomaly	4.9		0.38	
	Tritanomaly	0.0001		0.0000	
			5.9		0.40
Dichromatism	Protanopia	1.0		0.02	
	Deuteranopia	1.1		0.01	
	Tritanopia	0.0001		0.0000	
	Tetartanopia	0.0001		0.0000	
			2.1		0.03
Monochromatism	Total color-blindness	0.003		0.002	
Abnormal systems			8.0		0.43
Normal system			92.0		99.57

The most common disease of the central nervous system which causes central field defects (both for form and color) is multiple sclerosis. Several generalized non-infectious diseases cause optic neuritis with attendant loss of color and form discrimination. The most common are blood dyscrasias (pernicious anemia, secondary anemia, or leukemia) and deficiency diseases (B1 deficiency, so called "diabetic amblyopia," and optic neuritis of pregnancy). The most common occurrences of central color field defect are in connection with toxic amblyopia, and those toxins which most commonly depress the central (as opposed to the peripheral) field are:

Carbon disulfide—used in preparation of rayon, rubber, explosives, hides, insecticides, and wall paper.

Lead poisoning found in painters, plumbers, riveters, storage battery workers, and compositors.

Spinal anesthesia.

Sulfanilamide—may cause optic neuritis or hemorrhages into optic nerve.

Snuff, iodoform, and stramonium—rare cases.

Thallium—used in industry as a rat poison and a depilatory.

Tobacco-alcohol—the most common of all the agents producing amblyopia.

Toxic agents and disease affecting the conducting (transmissive) elements of the optical apparatus (nerve fiber layer of retina, optic nerve, and tract) are classically supposed to cause red-green weakness distinguished from deuteranomaly only by having poorer light-dark and yellow-blue discriminations. This defect becomes progressively worse until red-green blindness is reached which is distinguished from deuteranopia chiefly by its poorer light-dark discrimination. Finally total color-blindness results.

On the other hand, injuries (such as produced by syphilitic infections[26]) which primarily affect the receptive (rod-cone) mechanism show first a depression of the ability to make blue-yellow discriminations. Acquired tritanopia, unlike acquired red-green blindness, may become stabilized and be distinguished from congenital tritanopia chiefly by the memory possessed by the subject of his former yellow and blue sensations. Acquired tritanopia leads to total color-blindness only when progressive red-green blindness ensues; the latter, however, may of itself lead to total color-blindness[22] as noted above.

There is no therapy for acquired color-blindness since this is merely a symptom. The therapy is directed towards the underlying cause. The return of normal vision is usually by stages the reverse of those which characterized the disease.

In addition to acquired color-blindness which prevents the subject from seeing colors which are normally present, some mention should be made of chromatopsia which refers to the vision of colors which are normally absent. Erythropsia, or red vision, is the most frequent. It occurs normally after the eyes have been exposed to a very bright light, such as sunlight reflected from snow. Since in erythropsia the bright portions of the visual field are tinged with bluish red and the dark portions with yellowish green, the name is only half accurate. Erythropsia, however, is often a source of anxiety to a patient on the sudden access of light to the retina after a cataract extraction, iridectomy, or even when the pupils have been dilated by atropine. It is probably due to the prolonged and predominant red phase of the after-image.[54, 60] Cyanopsia, blue vision, may form a fleeting symptom after cataract extraction, probably owing to the previous influence of adaptation to the yellow color of the lens, since before extraction the short-wave light had all been absorbed. It also occasionally is a rare symptom in poisoning (as by digitalis) or in diseases of the retina and choroid, as also is (very rarely) ianthopsia, violet vision, and chloropsia, green vision. Xanthopsia, yellow vision, is said to be a somewhat more common symptom of jaundice and also occurs in picric-acid poisoning.

C. Color-Blindness of the Trichromatic Observer

Lest it be forgotten that chromatic vision is normal only under very special conditions, the following circumstances are listed under which normal eyes are color-blind:

I. *Indirect vision*,—vision in the retinal regions somewhat removed from the fovea differs from deuteranomaly only by reduced light-dark, blue-yellow, and form discrimination; vision in a more peripheral zone differs from deuteranopia only in these same ways; and vision in the periphery differs from acquired total color-blindness only by reduced ability to make form and light-dark discriminations.

II. *Insufficient size*,—a luminous area of a given brightness perceived by the para-foveal region may, by reducing its area sufficiently, be made to fail to elicit any chromatic color although still easily visible. For such areas the normally trichromatic para-foveal region becomes characterized by a monochromatic visual system, the type being indistinguishable from acquired total color-blindness.

III. *Insufficient brightness*,—the normal dark-adapted eye is in every respect functionally identical with that of the congenitally total color-blind.

IV. *Insufficient time*,—a luminous area may be perceived as brightness when exposed for so short a time that no chromatic color is experienced. Under these conditions, as in flicker photometry, the normal eye makes light-dark discriminations only, the type of vision being closely like acquired total color-blindness.[62]

V. *Chromatic fatigue*,—a form of artificial color-blindness can be produced temporarily (for a few minutes) by flooding the eye with a high luminous flux of chromatic light. In addition to rendering the eye insensitive to red, green, blue or violet light, whichever is the color of the fatiguing light, there is a shorter positive after-image of this light. This could be described as temporary chromatopsia.

The ability to overcome these handicaps to chromatic vision renders one normal observer much more valuable than another. For many tasks, particularly war-time tasks, that observer is more valuable who is not troubled by chromatic fatigue, and who can distinguish correctly the colors of small, dim, briefly visible objects even though he "catches them only in the corner of his eye." It has been found that on the average anomalous trichromats are much more susceptible to impairment of their chromatic vision by these circumstances than are normal observers. For example, they are said to experience chromatic contrast effects much greater than normal which interfere with their correct perception of colors. Because of this average susceptibility to handicaps, there has been a disposition to disqualify all anomalous trichromats for tasks to which dichromats are unfitted. Thus, it is stated by v. Kries (p. 409),[28] "We are indebted especially to Nagel's investigations[36,38] for light on this subject. Donders had long ago expressed the view that such persons had a 'dim sense of colour.' This indeed has now been thoroughly established; that is, the anomaly of the colour system (in the sense of having to make a different setting for the Rayleigh equation)[44] can be considered as an important and sure sign of 'colour infirmity.' "

And again (p. 421),[28] "From the practical standpoint, the thing of most importance, . . . , is that not only dichromats but persons with anomalous vision also are below par in a number of respects, and they are particularly unsuitable for railway or marine service. This, therefore, to a certain extent is the chief reason for improving the methods of investigation. The examination of these people is the right way of going at the problem of finding out all persons with abnormal colour vision and eliminating them."

There is a question, however, whether to paint all anomalous trichromats with the same brush may not unjustly disqualify certain of them for tasks which they are perfectly capable of performing. Thus, Köllner remarks (p. 515),[24] "From the foregoing it follows that red-green imbalance (rotgrünungleichheit), by which anomalous trichromats are characterized, plainly has nothing to do with the red-green sensation. 'Anomalous trichromasy' and 'red-green weakness' do not mean the same thing; by the former we have supersensitivity at least for one color, and on the other hand in spite of the absence of red-green imbalance there can occur a deficiency of the red-green sensation. Anomalous trichromasy is characterized by a 'constancy' of the red-green imbalance (see above) and is sharply separated from the normal proportions; the degree of red-green sensation on the other hand can vary widely and all steps from color-blindness up to the normal average can exist, indeed it can exceed that average. As to the color vision of anomalous observers, one can say in general with respect to their decreased red-green sensation relative to the normal that under favorable observing conditions they see practically as well as the normal, but under unfavorable circumstances they can approach the condition of the red-green blind very closely or even equal it."

These views are largely borne out by the findings of McKeon and Wright[32] who say, "The family of hue discrimination curves shows clearly how successive degrees of the defect are revealed in an increasing inability to distinguish different colours. The curves form a continuous series between the curve for the normal trichromat and that for the protanope as reported by Pitt.[42]. . The luminosity and mixture curves . . . show comparatively little variation among themselves, and there is some justification for deriving a mean curve for the 11 observers."

Köllner's statement is also borne out by Nelson (p. 687)[39] who states, "If the differences between the normal and anomalous hue-discrimination curves are taken as a measure of the degree of deficiency of the observer, then it can be seen that the other characteristics cannot always be treated as an obvious guide to the extent of the

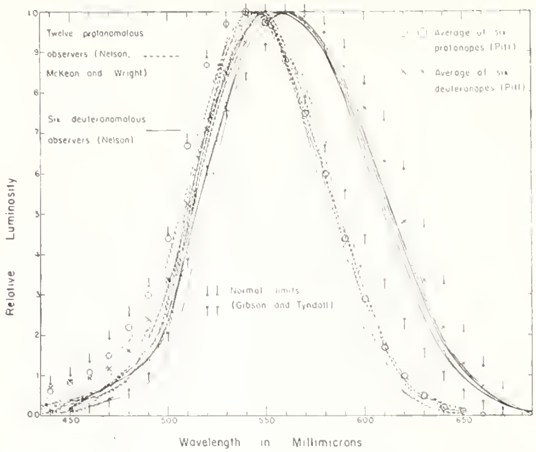

FIG. 2. Relative luminosity functions of anomalous trichromats (6 deuteranomalous, 12 protanomalous) compared to averages for dichromats and to limits for normal trichromats. Deuteranopic and deuteranomalous luminosity functions fall generally within normal limits; but protanopic and protanomalous luminosity functions fall outside of normal limits.

defect. . . . This apparent discrepancy between the various characteristics is particularly noticeable as regards the position on the Nagel distribution curve (Rayleigh equation). For example, in the case of observers *A* and *D* . . . , *A* is farther from the normal on the Nagel curve, but his hue-discrimination is the more nearly normal."

The facts of protanomaly and deuteranomaly are therefore such as to raise the question whether this classification has very much practical value. After a subject has been found to be an anomalous trichromat, the question still arises as to what he is good for. This question is further touched upon in the discussions of tests, present and proposed, which are to follow.*

4. LUMINOSITY FUNCTIONS OF RED-GREEN-BLIND AND RED-GREEN-WEAK OBSERVERS

Figure 2 shows by dotted lines the luminosity functions of eleven protanomalous observers studied by McKeon and Wright[32] and one studied by Nelson.[39] It also shows by solid lines the luminosity functions of six deuteranomalous observers studied by Nelson.[39] Although at the short-wave end of the spectrum these groups of functions overlap and are not significantly different, the two groups divide quite sharply at the long-wave end. In this respect they are like the groups of functions representing the two corresponding types of dichromatic observer, the deuteranope, and the protanope. The circles represent the average of six protanopic luminosity functions evaluated by Pitt,[42] and the crosses represent his average of six deuteranopic luminosity functions. The arrows indicate the maximum and minimum values of relative luminosity found by Gibson and Tyndall[6] among their 37 completely studied normal observers. Although because of differences in experimental conditions no highly precise intercomparison is possible between the data shown for anomalous and dichromatic observers and these normal limits, nevertheless Fig. 2 indicates that the deuteranopic and deuteranomalous observers alike possess luminosity functions generally well within normal limits; but both protanomalous observers and protanopic observers possess luminosity functions that are abnormally low at the long-wave end.

Figure 2 tends to indicate also that protanomalous observers possess abnormally narrow luminosity functions even compared to protanopes; but because of the small number of cases this indication may fail to be significant. On this account in Table I the maximum of the protanomalous luminosity function, like that of the protanope, is given as 540 mμ, although Fig. 2 indicates that 545 mμ might be a slightly better value. The experimental conditions (field size, field luminance, surrounding-field luminance) and technics (adaptation, control of fixation, length of observing time) govern the participation of the rod mechanism in determining these functions. In any one study the luminosity curves may be thrown more or less to the short-wave side because of partial participation of the rods, either intended or inadvertent. The intertwining of the curves at the short-wave end is probably to be ascribed chiefly to individual variation of the amount of yellow or brown pigmentation of the eye media, such as degree of macular pigmentation.

Since from Fig. 2 both deuteranopic and deuteranomalous luminosity functions fall generally within normal limits, it is just as valid to

* See the other papers of this Symposium on Color-Blindness.

predict the brightness judgments of these abnormal observers from the standard luminosity function as it is those of a normal observer. Thus, for normal, deuteranopic and deuteranomalous observers alike, the relative luminosities of two colors specified by (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) in the standard ICI colorimetric coordinate system,[10, 20] may be judged by a comparison of Y_1 with Y_2.

For protanopic and protanomalous observers, however, colors which are red to the normal observer are significantly darker relative to other colors. Figure 3 shows that it is possible to deduct from normal luminosity a correction for the redness of the color and so arrive at a protanopic and protanomalous evaluation of luminosity. Instead of comparing the Y values, use should accordingly be made of the values of the function: $0.086Z+1.160Y-0.393X$. Figure 3 shows this function for the equal-energy spectrum compared to the upper and lower limits of 18 protanopic and protanomalous luminosity functions (6 protanopic functions determined by Pitt, 12 protanomalous functions determined by Nelson, and by McKeon and Wright). Since the tristimulus values, X, Y, Z, are known for many colors, the function $0.086Z+1.160Y-0.393X$ is quite useful; for example, the tristimulus values of the Munsell colors are known,[7, 21] hence, by this function there can be determined with good validity which of any two surface colors of given Munsell notation will appear lighter either to a protanope

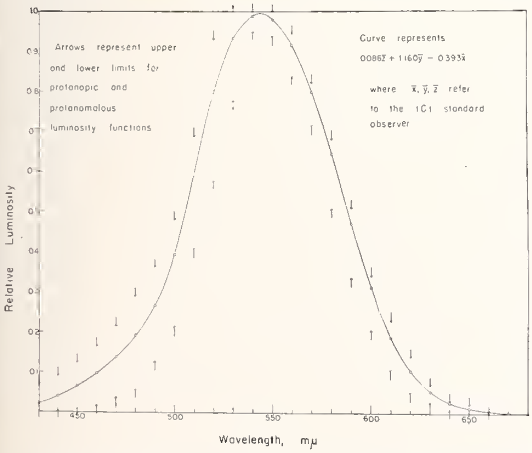

FIG. 3. Demonstration that a representative protanopic and protanomalous luminosity can be computed from the normal specification of any color. Instead of normal luminosity, Y, use $0.086Z+1.160Y-0.393X$.

or to a protanomalous observer. Such information is of assistance in the design of color-blindness tests.

Thus, it is seen that the facts of color-blindness are such that the brightness judgments of abnormal observers of the most common types may be conveniently predicted from the same color specification used for the normal observer.

5. CHROMATICITY CONFUSIONS OF RED-GREEN-BLIND OBSERVERS

The outstanding characteristic of a dichromatic observer which permits him to be distinguished from an anomalous trichromat is that although he makes important chromaticity confusions, he does not object to color matches set up by a normal observer between two fields of different spectral composition. On this account dichromatic visual systems are called reduction systems, and it is possible to derive from the normal specification of a color an adequate specification of the color for a dichromatic observer. With anomalous trichromatic observers, it is, of course, quite otherwise.

In 1855 Maxwell[30] pointed out from the Young-Helmholtz theory, how to define the chromaticity confusions of the color-blind in terms of the color triangle, now known as a chromaticity diagram. By this theory a dichromatic observer differs from the normal trichromatic observer by lacking one of the three primary color responses. The chromaticity for a dichromatic observer is therefore wholly determined by the ratio of the two remaining responses. For this ratio constant, there is defined on the normal chromaticity diagram, a straight line passing through the point representing the missing primary. A family of straight lines on the Maxwell triangle all passing through this point, therefore, represent by this theory all of the chromaticity confusions of the dichromatic observer.

Although the Young-Helmholtz theory is no longer regarded as a completely adequate basis for explaining color-blindness, this particular prediction from the theory was verified in a very satisfactory way in 1935 by Pitt.[42] Pitt found that the lines representing the chromaticity confusions converge closely on a single point, both for the protanopic observer and for the deuter-

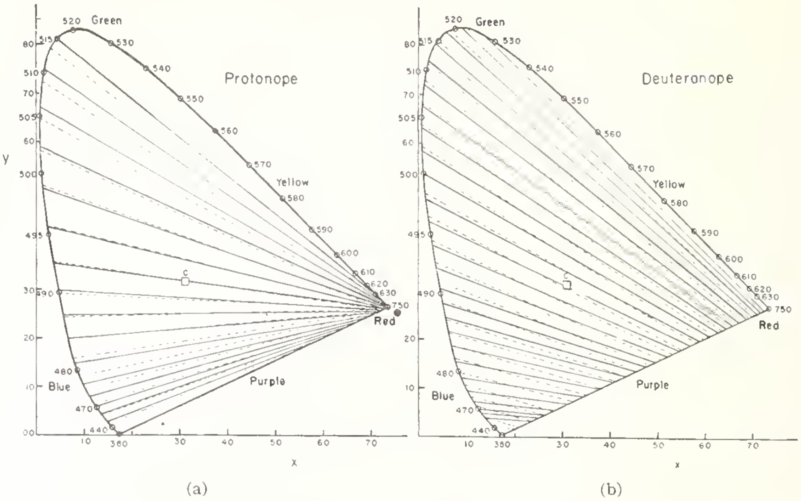

FIG. 4. Chromaticity confusions of the protanope and deuteranope (after Pitt). Note how closely the chromaticity-confusion lines (dotted) intersect at a single point for each of the two common types of red-green blindness. The solid lines included for comparison are co-punctal (protanope at $x=0.747$, $y=0.253$; deuteranope at $x=1.000$, $y=0.000$).

anope. Figure 4a shows as dotted lines on the (x,y)-chromaticity diagram of the ICI colorimetric coordinate system[10, 20] the results of Pitt's determination for protanopic observers; and Fig. 4b shows similarly the results for deuteranopic observers. The families of solid lines drawn in for comparison pass through a common point in each case (protanope: $x=0.747$, $y=0.253$; deuteranope: $x=1.000$, $y=0.000$). The spacing of the lines has been adjusted by Pitt from his determination of chromaticity sensibility to wavelength change for dichromats so that the chromaticity represented by one line is just distinguishable from that represented by its neighbor. It may be seen that the failure of the dotted lines to converge to a single point in the case of the protanope is not significant, and for the deuteranope is scarcely significant.

As remarked by Pitt (p. 44)[42] these results together with the luminosity data just presented "are of the most fundamental kind for dichromats, and on such measurements are based, either knowingly or unknowingly, all colour-blind tests." It will be worthwhile, therefore, to inquire further into the interpretation of Fig. 4.

In Fig. 4a there is a confusion line passing nearly through the point representing ICI standard illuminant C (average daylight). This line passes through the spectrum locus at about 493 mμ. From this line it may be concluded that Pitt's average protanope will not be able to distinguish the chromaticity of illuminant C from that of the spectrum at about 493 mμ; that is, his neutral point is at 493 mμ as is indicated in Table I. However, the position of the neutral point is subject to rather large individual variations as is shown by Hecht and Shlaer.[12] These variations are to be expected because of the dependence of the chromaticity of illuminant C on degree of macular pigmentation, which does not affect the chromaticity of spectrally homogeneous light at all.

Similarly, from Fig. 4b it is seen that the average deuteranopic neutral point for Pitt's six observers occurs at about 497 mμ.

From this neutral point the red-green-blind observer sees yellow of increasing saturation up to a wave-length of about 530 mμ after which there is no further chromaticity change. In the direction of decreasing wave-length the dichromatic observer sees blue of saturation increasing to the end of the spectrum. Note how apt was Dalton's description of the spectrum: "My yellow comprehends the red, orange, yellow, and green of others; and my blue and purple coincide with theirs." We must take Dalton's purple to mean saturated blue.

This description of the spectrum fits both protanope and deuteranope, and we may ask how Seebeck[48] could deduce that Dalton was a protanope. This deduction follows from Dalton's further statement, "Woolen yarn, dyed crimson, or dark blue is the same to me." Assuming that the names, crimson and dark blue, have retained their meaning over the intervening 100 years, we may use Fig. 4 to check this deduction. Harvard Crimson[50] is found by way of the Munsell color system to have $x=0.48$, $y=0.25$; dark blue (Navy 1)[50] is similarly found to have $x=0.27$, $y=0.25$. These points are indicated on Figs. 4a and 4b by crosses. From Fig. 4a it may be noted that an average protanope is expected to confuse these chromaticities, since the points representing them lie on the same line; but from Fig. 4b it should be anticipated that to the average deuteranope crimson would appear as a brownish gray quite distinct from the appearance of dark blue which would be seen correctly.

Thus, it is seen that the facts of red-green blindness are such that the chromaticities experienced by such observers can be predicted from the same standard color specifications used by the normal observer. For example, the points representing on the (x,y)-diagram the standards of the *Munsell Book of Color*[35] are known.[7, 21] Therefore, from the Munsell notation of a color may be obtained a reliable indication of its chromatic appearance to a red-green-blind observer.

6. SUMMARY

Color-blindness is of various types, and there is a greater or lesser adherence of groups of observers to these types. The most common are the two types of red-green blindness, protanopia and deuteranopia, and the corresponding types of anomalous vision, protanomaly and deuteranomaly. Although red-green-blind observers should obviously be excluded from occupations requiring the ability to make red-green distinctions, there is doubt what to do with an observer proved to be anomalous because in spite of the fact that many anomalous observers have virtually no ability to distinguish red from green, others do so nearly or quite as well as normal observers.

Color-blindness is usually congenital, but it may also be acquired as a result of injury or disease. Congenital red-green blindness and anomalous vision are much more prevalent among males than among females, and are regarded as sex-linked characteristics. Total color-blindness, however, is thought to be a simple recessive characteristic.

Red-green blindness is designated as a reduction form of normal vision because two lights of whatever spectral compositions are required to produce a match for the normal observer will also be indistinguishable by a red-green-blind observer. Red-green weakness, or anomalous vision, on the other hand, is called an alteration form of vision, because such matches by the normal are frequently quite unacceptable to the anomalous observer. In spite of this it is possible to predict the brightness judgments of the anomalous observer with good validity from the normal specifications of the colors being dealt with, the chromaticity alone being unpredictable. Methods of predicting precisely the appearance of colors to red-green-blind observers are described with a view to their possible usefulness in the design and development of color-blindness tests.

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COLOR BLINDNESS AND THE DETECTION OF CAMOUFLAGE

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This paper answers the old question and assertions concerning the utility of color blindness for special purposes. So far as is known, even today, color blindness is a disability. The suggestion that reduction of confusion by random patterns of equally light reds and greens may endow red-green blind observers with an advantage for detection of equally light embedded objects is novel. If true, the case is so rarely encountered that it is of no practical significance.

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COLORBLINDNESS AND THE DETECTION OF CAMOUFLAGE

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ACCORDING to newspaper reports, colorblind observers have frequently been successful in spotting otherwise perfectly camouflaged positions. In order to show whether these reports can be believed, a brief analysis of the ways by which a normal observer can detect off-color camouflage must first be given.

NORMAL VISION

A normal observer can make color discriminations of three kinds: light-dark, blue-yellow and red-green. If a camouflaged position appears neither lighter nor darker, neither bluer nor yellower and neither redder nor greener than the surrounding terrain, the observer with his naked eye can not detect it because of its color; it is therefore perfectly colored and matches its background perfectly.

RED-GREEN BLINDNESS

The two most common forms of colorblindness are called deuteranopia and protanopia. Deuteranopes and protanopes are called colorblinds because they can not make red-green discriminations. To hide a position from such an observer as these it is sufficient to make it neither lighter nor darker, and neither bluer nor yellower than the background. It is not necessary to worry about whether the position is redder or greener than the surrounding terrain. These observers find it hard to pick out ripe strawberries from green or to pick out a rotten apple from a barrel of red apples, since the color differences involved are chiefly red-green differences. Since they can make yellow-blue discriminations quite as well as the normal observers, they are sometimes said to be only partially colorblind.

RED-GREEN WEAKNESS

There are two other forms of abnormal vision which have to be considered. They are forms of vision intermediate between normal vision and deuteranopia and protanopia, respectively. The form intermediate between normal vision and deuteranopia is known as deuteranomalous vision, that tending toward protanopia as protanomalous vision. There are more anomalous observers of these types than there are partial colorblinds. About 2 per cent. of the male population would be classed as partially colorblind, and another 6 per cent. as anomalous, making altogether 8 per cent. abnormal. The protanomalous and deuteranomalous observers can make the three types of color discrimination possible for the normal observer, but their ability to make red-green discrimination is less than normal.

Other forms of colorblindness are relatively rare and associated with diseases of the eye; such eyes can not possibly compete with normal eyes for the detection of camouflage and can therefore be passed over. Likewise the fact that protanopes are distinguished from deuteranopes by being less sensitive to the long-wave (red) end of the spectrum than the normal observer has no separate special bearing on the detection of camouflage. But there is an important distinction between red-green blind and red-green weak observers, that is, between partially colorblind and anomalous observers which can conveniently be brought out by reference to an analysis of the ways by which a camouflaged position can be imperfectly colored.

COLOR FOR CAMOUFLAGE

If a position is concealed by being covered with material having the same reflecting properties throughout the spectrum as the surroundings, then it can not be detected. Such concealment can be approximated, for example, by using the actual vegetation or dirt of the surrounding terrain, and these methods are recommended by the Engineers Field Manual wherever practicable. Cut branches, however, rapidly change color from wilting of the leaves, and dirt after a rainstorm because of difference in rate of drying out may be a giveaway, although the spectral character of the material is very similar to that of the surrounding terrain. These are examples of the most common way by which a camouflaged position becomes imperfectly colored; this way may be called the imperfect use of spectrally correct materials.

Because of the impermanence of natural materials for camouflage, paint containing more permanent colorants is widely used for camouflage. Various branches of the War and Navy Departments have issued color standards for such paints. In the formulation of such paints it is desirable to choose pigments having spectral characteristics similar to those of the elements of terrain which have to be matched, but whenever the pigments are different from those coloring the terrain, some differences in spectral reflectance have to be tolerated. Therefore, if a paint is prepared to match an element of terrain for a normal observer, there will be portions of the spectrum at which the reflectances nevertheless differ. By viewing the two through a selective filter transmitting such a portion of the spectrum, the normal observer, or any observer, would be able to see one lighter than the other, although viewed with the naked eye they would match perfectly. Color matches involving such invisible spectral differences are sometimes called pseudo matches, but a preferred terminology suggested by Ostwald is to refer to them as metamers or metameric pairs. Such pairs as exhibit marked spectral differences are called highly metameric, and are said to exhibit a high degree of metamerism. Whenever paint is used for camouflage some degree of metamerism must be tolerated; such camouflage, however accurately adjusted by the naked eye, is always open to detection by the use of selective filters, or by photographic means, the conspicuousness of the installation being proportional to the degree of the metamerism.

Thus, there are two ways in which camouflage paint may be wrong; the components may be combined in improper proportion, or they may themselves be spectrally inappropriate.

Reduction Systems of Vision

If a red-green-blind observer be shown a pair of samples which match to the normal observer, he will be unable to discriminate them. That is, normal metamers are also metamers for the red-green blind. On this account protanopia and deuteranopia are called reduction systems of vision. A red-green-blind observer fails to discriminate many pairs which are conspicuously different for the normal observer; and if the normal observer can not tell the color difference between two samples, neither can he. Therefore, if the camouflaged position be imperfectly colored solely because of choice of a spectrally imperfect material, there is no basis for expecting a red-green-blind observer to detect it.

The red-green weak observers, however, do not possess reduction systems. A metameric pair set up by a normal observer will usually be more or less off-match for a protanomalous or deuteranomalous observer. There is therefore a chance that an anomalous observer could with his naked eye detect a camouflaged position which would be undetectable by a normal or colorblind observer. But give the normal observer the correct spectral filter, and he could also detect the difference.

Effect of the Background

We have seen that a colorblind observer can not detect camouflage, which is at fault solely because of spectrally imperfect materials. Any advantage in substituting a colorblind for a normal observer must therefore rest in the detection of positions whose colors are imperfectly adjusted to that of the surrounding terrain. Let us inquire if there are likely kinds of blunders in applying camouflage which would be easier for the colorblind observer to detect than the normal.

A fairly common scene within which it is required to conceal a position is made up of patches of reddish-brown earth and yellowish-green foliage. The variegated pattern composed of these patches is well adapted to the concealment of a position from a normal observer. Even though it be somewhat too light or too bluish, the normal observer could fail to detect it because of the larger red-green differences in the scene. But consider the appearance of the scene to a red-green blind. The normal green of foliage to him appears dark-yellowish brown; the normal reddish brown of earth also appears dark-yellowish brown to him. He is not sensitive to the red-green differences which for the normal produce a variegated pattern; instead he may see a nearly uniform yellowish-brown field. Any element of terrain which is too light or too bluish could be quite conspicuous to such an observer. It is therefore possible to believe that a colorblind observer may detect camouflaged positions not detectable by the normal observer.

Can Colorblindness Be Produced by Filters?

It is a natural question to raise whether this possible advantage of the colorblind can be duplicated by giving a suitable viewing filter to a normal observer. The filter required to suppress normal red-green discrimination is, of course, one which transmits only in the blue and yellow portions of the spectrum. If a filter could be found, for example, which transmits the double band 450 to 490 mμ and 560 to 585 mμ, it would render the red-green differences between grass and earth about one fifth as prominent and at the same time preserve about the same prominence of any yellow-blue differences. However, such a filter would transmit less than 10 per cent. of incident daylight, probably much less. It is a question whether any improvement in detection of lightness differences or yellow-blue differences would be obtained by a normal observer in this way even against a highly variegated red-green background. It should be noted that such a filter, although it would render a normal observer relatively blind to red-green differences, by no means makes him equivalent to either a protanope or a deuteranope. Such a filter would endow the subject of the experiment with a luminosity function having two separate maxima, one at about 470 mμ, the other at about 570 mμ, whereas the deuteranope has a nearly normal luminosity function whose maximum is at 555 mμ, and the protanope a similar function with the maximum shifted to about 540 mμ. It is possible to produce the phenomena of color blindness separately by means of filters, but they can not all be bestowed in this way upon a normal observer at the same time.

STANDARD RESPONSE FUNCTIONS FOR PROTANOPIC AND DEUTERANOPIC VISION

J. Opt. Soc. Am. 35, 199-221 (1945) and J. Opt. Soc. Am. 39, 505 Lett. (1949)

The seeming tautology, "copunctal point", used in line 23 of the first column on p. 211, and elsewhere in this and other papers is merely an elision of "point common to a set of copunctal lines". The seeming inconsistency of the point ($x_d = 1.08$, $y_d = -0.08$) of intersection of Pitt's confusion lines for deuteranopes with Maxwell's idea that deuteranopic confusions result from the absence or ineffectiveness of green receptors is an artifact of the CIE (here called ICI) 1931 chromaticity diagram. When transformed to the CIE 1960 diagram, that point is at $u_d = -36$, $v_d = 4$, which is clearly in the green sector, although remote from the gamut of physically realizable colors. The point of convergence for deuteranopes ($x_d = 1.75$, $y_d = -0.75$) reported more recently by Yustova (Proc. Symposium, Nat. Phys. Lab. 1957) corresponds to a less-remote point, $u_d = -2.8$, $v_d = 1.8$, in the green sector of the CIE 1960 diagram. Every point that is below the line $12y - 2x + 3 = 0$ and on the tangent to the long-wavelength end of the spectrum locus is transformed to a point in the green sector of the CIE 1960 (u,v) diagram, on the tangent to the long-wavelength end of the spectrum locus in that diagram. Therefore, criticisms of Maxwell's version of Young's theory that are based on seemingly incredible locations in the CIE 1931 chromaticity diagram of points of convergence of confusion lines for deuteranopes, are fallacious. However, the point $x_d = 1$, $y_d = 0$, which Judd adopted for convenience in this paper, corresponds to the point $u = 4$, $v = 0$ in the CIE 1960 diagram. Whether this point represents a green depends on the credibility of the CIE 1960 diagram. (The CIE 1976 u',v' diagram does not differ from the 1960 diagram in any way relevant to this discussion.)

Standard Response Functions for Protanopic and Deuteranopic Vision

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I. INTRODUCTION

When the two halves of a photometric field are illuminated to the same degree with light of the same spectral composition, they cannot be distinguished; that is, they produce a color match. But it is possible also to produce a color match between lights of different spectral compositions; such lights are called metamers,[22, 51] and are said to form metameric pairs. The functions which give the conditions to be met by the spectral composition of any two lights in order that they shall form a metameric pair for a given observer serve to define the characteristics of that observer; and in 1931 the International Commission on Illumination adopted such functions which define the standard observer.[23, 27]

Protanopic and deuteranopic vision are the two most common forms of dichromatic vision, each form occurring in about one percent of otherwise normal human males.[31] They are known as reduction forms because neither a protanope nor a deuteranope ordinarily distinguishes the members of any pair of normal metamers; that is, a color match set up by a normal observer will not be objected to by a dichromat unless the pigmentation of his eye media, including the macula, be grossly different from that of the normal observer. The standard color specifications are therefore generally valid for dichromats in the sense that if two lights have the same standard color specifications, no average dichromat will be able to distinguish them.

These specifications are, however, unnecessarily complicated for such observers. In addition to normal metamers, dichromats find many other pairs which they can not distinguish. These additional metamers are often called confusion colors; they are colors which are distinct to the normal although identical to the dichromat. The normal color specification consists of three numbers, because the normal visual system is capable of three independent modes of variation and on that account is often called trichromatic. The dichromatic specification need consist of but two numbers. These two numbers may be derived from the standard color specification in a simple way. The connection between trichromatic and dichromatic specifications was indicated in complete detail by Maxwell in 1855[44] by appeal to the Young theory of vision. The original simple form of this theory has been disproved, the extensions of it vary with the phenomena to be explained, and there is doubt whether the theory, however extended, can yield more than a partial explanation of the complicated facts of vision. However, the connection between dichromatic and trichromatic color specifications first worked out by Maxwell on theoretical ground does not depend upon the portions of Young's hypothesis now given up; on the contrary this connection has been repeatedly proven to be correct. It is the purpose of this paper to derive, by this principle, response functions for protanopic and deuteranopic vision, to show how they may be used to find whether or not two colors will be confused by an average protanope or average deuteranope, and finally to point out a few of the theoretical implications of the functions chosen.

II. THEORY

Maxwell wrote in his letter of January 4, 1855, to Dr. George Wilson:[44] "If we find two combinations of colours which appear identical to a Colour-Blind person, and mark their positions on the triangle of colours (now called the Maxwell triangle), then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two.

"We may in the same way find other lines passing through the series of colours which appear alike to the Colour-Blind. All these lines either pass through one point or are parallel, according to the standard colours which we have assumed, . . . Knowing this law of Colour-Blind vision, we may predict any number of equations which will be true for eyes having this defect.

"The mathematical expression of the difference between Colour-Blind and ordinary vision is that colour to the former is a function of two independent variables, but to an ordinary eye, of three; and that the relation of the two kinds of vision is not arbitrary, but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement, which forms one-third of the apparatus by which we receive sensations of colour."

The straight lines on the Maxwell triangle serve to indicate the chromaticity confusions of the color-blind observer. The point of intersection of these lines indicates the normal chromaticity of the primary process not possessed by the color-blind. Suppose for the protanope that the chromaticity-confusion lines are copunctal at (x_p, y_p) on the (x, y)-plane of the standard I.C.I. colorimetric coordinate system,[23, 27] and suppose for the deuteranope that the corresponding point is (x_d, y_d). Let us inquire how to derive from the functions (of wave-length) (X, Y, Z) defining the standard normal observer, three new functions (W_d, W_p, K) such that all three taken together represent the normal observer, such that (W_d, K) taken together represent average deuteranopic vision, and such that (W_p, K) taken together represent average protanopic vision. This terminology (W_d, W_p, K) is taken from v. Kries* who followed König[37] closely. The symbol W is intended to suggest "warm;" and K, "cold" (kalt), corresponding to whatever warm color (orange, yellow, greenish yellow) and whatever cold color (blue or violet) is sensed by red-green-blind observers.

The derivation consists of making new choices of primary processes such that two of them correspond to (x_p, y_p) and (x_d, y_d), respectively. It was pointed out by König in 1886[37] for this very purpose, and by many others since,[13, 24, 25, 43, 54] for other purposes, that such choices result from defining the new functions (W_d, W_p, K) as weighted sums of the old, thus:

$$\begin{aligned} W_d &= k_1X+k_2Y+k_3Z, \\ W_p &= k_4X+k_5Y+k_6Z, \\ K &= k_7X+k_8Y+k_9Z, \end{aligned} \tag{1}$$

where k_1 to k_9 are constants which may be given any arbitrary values whose determinant differs from zero:

$$\begin{vmatrix} k_1, & k_2, & k_3 \\ k_4, & k_5, & k_6 \\ k_7, & k_8, & k_9 \end{vmatrix} \neq 0. \tag{1a}$$

Since the point (x_p, y_p) corresponds to the primary W_d not possessed by the protanope, $W_p=K=0$, for $X/(X+Y+Z)=x_p$, and $Y/(X+Y+Z)=y_p$, and we may write from Eq. (1):

$$\begin{aligned} k_4x_p+k_5y_p+k_6z_p &= 0, \\ k_7x_p+k_8y_p+k_9z_p &= 0, \end{aligned} \tag{2a}$$

where z is defined as $Z/(X+Y+Z)$ and is equal to $1-x-y$.

Similarly, since the point (x_d, y_d) corresponds to the primary not possessed by deuteranopes we may write:

$$\begin{aligned} k_1x_d+k_2y_d+k_3z_d &= 0, \\ k_7x_d+k_8y_d+k_9z_d &= 0. \end{aligned} \tag{2b}$$

The four conditions expressed by Eqs. (2a) and (2b) are the only conditions that have to be met to insure that W_p and W_d represent the primary color processes not possessed by the deuteranope and protanope, respectively. Since nine conditions are required to determine the nine constants of Eq. (1), it may be seen that many coordinate systems can serve both for normal trichromatic visual systems and for dichromatic systems by neglect of the one or the other of two of the three normal components.

Two of the remaining five degrees of freedom are required to insure, as is convenient, that the equal-energy stimulus be kept as the basic stimulus[14] of the system, that is, that it be represented at the center of the Maxwell triangle of the new coordinate system as well as in the standard I.C.I. system. This requirement is satisfied if the distribution curves of the new

* Reference 39, p. 164.

primary color processes throughout the equal-energy spectrum be adjusted, like those of the I.C.I. system, to equal areas; that is, if $k_1+k_2+k_3=k_4+k_5+k_6=k_7+k_8+k_9$. Another degree of freedom must be expended to set the arbitrary units in which the distribution curves are expressed. The three conditions together may be expressed conveniently as:

$$k_1+k_2+k_3=1,$$
$$k_4+k_5+k_6=1, \quad (3)$$
$$k_7+k_8+k_9=1.$$

König[37] used the other two conditions to fix the third primary at an imaginary color of dominant wave-length near to that usually corresponding to unitary blue, that is, a stimulus which is perceived under ordinary observing conditions as a blue which is neither reddish nor greenish. In this way König derived the "fundamental sensation" curves incorporated by Ladd-Franklin[41] into her theory of color vision. Following further studies of dichromatic vision, however, König gave up the attempt to make any of the primaries correspond to a unitary hue.[38] In this similar reduction of present-day data on the vision of dichromats, it has likewise seemed advisable to pay no attention to the color perception ordinarily corresponding to the primaries adopted, but rather to strive for the simplest possible adequate representation of the data. Thus, it is possible to satisfy conditions (2) and (3) in such a way as to yield wave-length functions for the primaries each consisting of a curve possessing the single-peak shape of the luminosity function. To avoid functions for W_d and W_p having two maxima it is sufficient simply to require:

$$k_3=-0.22k_1,$$
$$k_6=-0.22k_4. \quad (4)$$

These requirements prevent W_d and W_p from being large near 440 mμ by utilizing only enough of the Z function to cancel approximately the secondary maximum of the X function in that region.

III. CHROMATICITY CONFUSION OF DICHROMATS

As first pointed out by Maxwell, the chromaticity confusions of either type of dichromat may be represented on the chromaticity diagram, or Maxwell triangle, for normal trichromatic vision by a family of straight lines passing through a single point, (x_p, y_p) for the protanope, (x_d, y_d) for the deuteranope. The first determination of these points was that by König and Dieterici[37] based upon two protanopes and two deuteranopes. From these points were derived the "fundamental-sensation" curves (Grund-Empfindungs-Curven) R', G', and B'; and from the approximate relation[30] between these curves and those of the present standard coordinate system for colorimetry, an estimate of the coordinates of the points may be made. From the previously evaluated transformation equations* from standard color specifications (X, Y, Z) to (R', G', B') may be found the reverse transformations†

$$\begin{aligned} X&=0.244R'-0.058G'+0.014B',\\ Y&=0.056R'+0.150G'-0.005B',\\ Z&=0.000R'+0.000G'+0.200B'. \end{aligned} \quad (5)$$

By setting $G'=B'=0$ in Eq. (5), we find $X=0.244R'$, $Y=0.056R'$, $Z=0$, whence:

$$x_p=0.244R'/(0.244R'+0.056R')=0.81$$
$$y_p=0.056R'/(0.244R'+0.056R')=0.19.$$

Similarly by setting $R'=B'=0$, we find $x_d=-0.63$, $y_d=1.63$.

Of course, no very great dependence can be placed on these results of this pioneer work. The connection with the present standard coordinate system is uncertain not only because it is based upon but two partially dark-adapted normal observers (König and Dieterici, who incidentally differed importantly from each other), but also because the basic stimulus of the system ("sunlight reaching the earth's surface through atmosphere of highest transmission") is essentially undefined. If we assume $x=0.33$ and $y=0.34$ for this basic stimulus[46] the protanopic and deuteranopic "neutral" points in the spectrum are found graphically on the (x, y) plot of the standard I.C.I. system to be 496 and 511 mμ, respectively. This agrees only approximately with the wave-lengths (495 and 504 mμ, respectively) read from the König-Dieterici triangle;‡

* See reference 30, Eq. (7b).
† See reference 30, Eq. (2).
‡ See reference 37, Fig. 7.

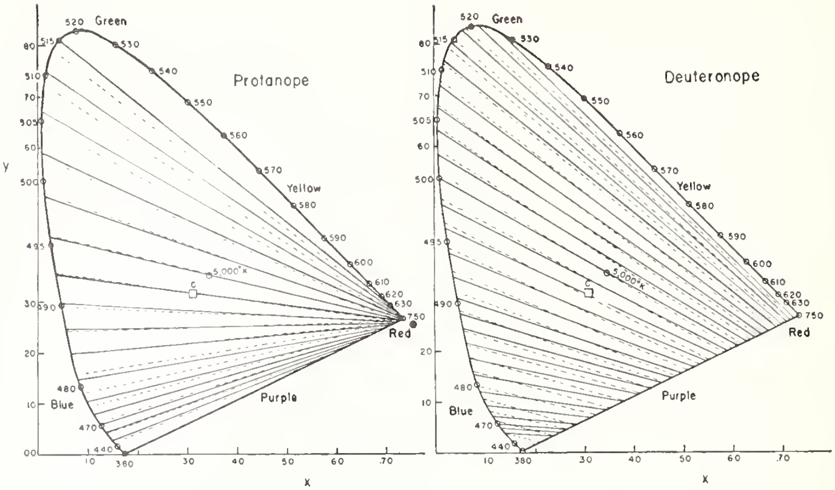

FIG. 1. Chromaticity confusions of the protanope and deuteranope (after Pitt). Note how closely the chromaticity-confusion lines (dotted) intersect at a single point for each of the two recognized types of red-green blindness. The solid lines included for comparison are copunctal (protanope at $x=0.747$, $y=0.253$; deuteranope at $x=1.000$, $y=0.000$).

and indicates that the possibilities for considerable error have been realized.

In 1935, however, Pitt[52] published data giving the average chromaticity confusions of 8 deuteranopes and 7 protanopes in terms of the *WDW* coordinate system proposed by Wright[59] for visual research because it takes account in a simple way of variations in ocular pigmentation (macula lutea, crystalline lens, humors). From the primaries of this system (spectrum lights at 460, 530, and 650 mμ, respectively) and from the fact that I.C.I. standard Illuminant B[23, 27] has in this system for the standard observer the chromaticity coordinates $r=0.249$, $g=0.399$, equations, which hold for the I.C.I. standard observer, may be derived to connect these chromaticity coordinates (r, g) with those (x, y) of the I.C.I. standard coordinate system:

$$x=\frac{0.874r-0.023g+0.144}{0.402r-0.222g+1.000},\quad y=\frac{0.354r+0.597g+0.030}{0.402r-0.222g+1.000}. \tag{6}$$

By means of these equations the chromaticity-confusion lines shown on Pitt's Figs. 15 and 16 have been transferred to the I.C.I. system and are shown as dotted lines in Figs. 1a and 1b. It will be noted that the protanope chromaticity confusion lines plotted in Fig. 1a all run very closely through a single point ($x_p=0.747$, $y_p=0.253$) in conformity to the principle enunciated by Maxwell from the Young theory. The deuteranope chromaticity-confusion lines (Fig. 1b) are not so perfectly copunctal as this, but most of them come fairly close to the point $x_d=1.08$, $y_d=-0.08$.

In 1936, Hecht and Shlaer[15] published complete data for two dichromats, one protanope, and one deuteranope. These data include luminosity functions, wave-length discrimination, and color-mixture data. The color-mixture data take the form of the energy ratio of two spectral primaries (458.7 and 570.0 mμ) required to produce a chromaticity match for each part of the spectrum. This energy ratio can be accurately defined by the dichromatic observer for the middle portion of the spectrum, but near both 460 mμ and 570 mμ a given energy ratio is found to apply to a considerable spectral range which continues to widen as the spectral extremes are approached. Table I gives average values of these energy ratios found from the Hecht-Schlaer data taking due account of the rapid decline in the ability of these observers to discriminate wave-length change near 460 and 570 mμ. These data may be made independent of the degree of ocular-media pigmentation by transforming them to the red and blue primaries of the *WDW*[59] system. In this system spectral primaries (460 and 650 mμ) are used, but, instead of energy units, arbitrary units such as to make the amounts of the primaries equal for 494 mμ are adopted. By the usual methods,[26] equations of transformation have been found connecting trichromatic coefficients (r, b) in the *WDW*

TABLE I. Comparison of the Hecht-Shlaer color-mixture data for one protanope and one deuteranope with Pitt's data for 8 protanopes and 7 deuteranopes.

Wave-length in mμ	Hecht-Shlaer protanope HJ		Pitt average protanope	Hecht-Schlaer deuteranope AWG		Pitt average deuteranope
	f_p	$b=1-r$	$b=1-r$	f_d	$b=1-r$	$b=1-r$
450			1.03			1.03
460	150.0	1.00	1.00	150	1.00	1.00
470	6.4	.93	.93	28	.92	.93
480	2.1	.81	.80	11.2	.79	.80
490	.77	.61	.57	5.0	.62	.57
500	.294	.37	.34	1.5	.32	.34
510	.100	.17	.17	.526	.14	.14
520	.033	.06	.09	.182	.05	.06
530	.0113	.02	.06	.062	.02	.03
540	.0040	.01	.04	.025	.01	.02
550	.0005	.001	.02	.010	.003	.01
560			.01	.0025	.001	.00
570			.00			.00
580			.00			.00
590			.00			.00

system to the energy ratios f_d and f_p for the protanope *HJ* and the deuteranope *AWG*:

$$r=(0.492-0.003f_p)/(0.492+0.997f_p),\quad b=f_p/(0.492+0.997f_p), \tag{7a}$$

$$r=(31.85-0.021f_d)/(31.85+0.979f_d),\quad b=f_d/(31.85+0.979f_d). \tag{7b}$$

Table I shows the color-mixture data for the protanope *HJ* and the deuteranope *AWG* transformed by Eqs. (7a) and (7b) to the red and blue primaries of the *WDW* system, and for comparison it also shows the average values found by Pitt[52] for 8 protanopes and 7 deuteranopes, respectively. It is seen that the Hecht-Shlaer color-mixture data resemble those of Pitt considerably, and comparison of the discrepancies with the known wave-length discrimination of dichromats,[15, 52] and with the individual differences among the dichromats studied by Pitt* shows that the corroboration is wholly satisfactory.† It is to be concluded, therefore, that the Hecht-Schlaer data when referred to the same degree of ocular pigmentation as is included in the I.C.I. standard observer indicate the same values of (x_p, y_p) and (x_d, y_d) as the Pitt data.

From Fig. 1 it may be seen that a source of color temperature 5000°K ($x=0.344$, $y=0.351$) would possess for an average protanope the same chromaticity as the spectrum at 496 mμ; and for an average deuteranope, the neutral point would be at 500 mμ. As pointed out by v. Kries[38b] a dichromat having ocular-pigmentation heavier than that of the standard observer would have his neutral point shifted toward the long wave end of the spectrum; for a lightly pigmented dichromat a displacement toward the short wave

* Reference 52, p. 15.

† Although the color-mixture data for these two dichromats were not put by Hecht and Shlaer into a form suited to prediction of dichromat color matches between heterogeneous stimuli and spectrum stimuli, such a reduction has been carried out recently by Fry (reference 10). From this reduction it would be possible to determine independently the chromaticity coordinates (x_p, y_p) and (x_d, y_d) of the primary processes not possessed by the two forms of dichromat, instead of relying for this purpose upon Pitt's transformation from the dichromatic to trichromatic diagram of the *WDW* system by determination of the dichromatic luminosities of the primaries. This reduction indicates, however, that the complete radiator at 5000°K should have had the chromaticity of the spectrum at 498 mμ for protanope *HJ* and 508.5 mμ for deuteranope *AWG*, but by direct observation these wave-lengths were found to be 491.5 mμ and 495 mμ, respectively. These discrepancies amount to 5 or 10 times the just noticeable difference for these observers. Although the major part of the discrepancy for protanope *HJ* can be eliminated by more exact methods of reducing the data and by filling in the wave-length region 380 to 470 mμ from Pitt's data, the Hecht-Shlaer data being uncertain in that region, still it would seem that no great reliance is to be placed upon the predictions of dichromat color matches from these data. These discrepancies suggest that the experimental conditions under which the luminosity functions of these observers were obtained may have brought into play a smaller retinal region than that for the color-mixture data although ostensibly the conditions were identical. Perhaps the mere fact that the luminosity and color-mixture determinations were made separately is a sufficient explanation of the discrepancy. It is doubtful therefore whether separate determination of the copunctal points (x_p, y_p) and (x_d, y_d) from these data would add appreciably to the information obtained by way of the *WDW* coordinate system.

TABLE II. Individual variation in wave-length of neutral point compared to the expected influence of ocular pigmentation based upon 35 normal observers studied by Wright (reference 59).

Source of data	Number and type of observers	Wave-length in mμ of the spectral region having the dichromatic chromaticity of a source of color temperature 5000°K Protanope Min.	Protanope Max.	Protanope Average	Deuteranope Min.	Deuteranope Max.	Deuteranope Average
Fig. 1	35 normals	489	504	496	494	508	500
König (1884)	7 protanopes 7 deuteranopes	495	499	497	500	509	503
Pitt (1935)	5 protanopes 6 deuteranopes	494	497	496	495	507	500
Hecht-Shlaer (1936)	10 protanopes 12 deuteranopes	492	508	498	491	525	510

end is expected. Wright[59] found that, among the 35 normal observers studied, variations in pigmentation made the point representing 5000°K vary from ($x=0.29$, $y=0.27$) for no ocular pigmentation to ($x=0.39$, $y=0.43$) for the most heavily pigmented observer studied. Table II shows the wave-lengths of the neutral points to be expected from protanopes and deuteranopes having these extremes of pigmentation, and it also shows for comparison the average and extreme wave-lengths of neutral points found by König,[35] Pitt,[52] and Hecht and Shlaer.[15] The König wave-lengths have been increased by 3 mμ for protanopes and 4 mμ for deuteranopes to take account of the chromaticity difference between the average daylight used by him and color temperature 5000°K. Pitt used Illuminant *B* for comparison in determining neutral points; the corrections for the chromaticity difference between this source and one at 5000°K are negligible and have not been applied. It will be noted from Table II that the König and Pitt data not only agree in average values with the neutral points read from Fig. 1, but also fall generally within the limits corresponding to those expected from the pigmentation range of Wright's 35 normal observers. The single deuteranope studied by König who falls slightly outside the pigmentation limits of Wright's 35 normal observers probably corresponds to nothing more than the error of random sampling.

One of the Hecht-Shlaer protanopes and 7 of their deuteranopes, however, have neutral points considerably higher than the maximum values corresponding to the most heavily pigmented of Wright's 35 normal observers. That 7 out of 12 deuteranopes are found to have heavier ocular pigmentation than any of 35 normal observers is too much to explain by the error of random sampling. This fact suggests strongly that the populations from which the observers were drawn are significantly different in ocular pigmentation. By this view we must be prepared to accept rather wide individual variations in pigmentation. For example, the deuteranope, verified by Hecht and Shlaer as having a neutral point at 525 mμ, must have an ocular pigmentation 8 or 9 times as heavy as the average found by Ludvigh and McCarthy[42] for ocular pigmentation of 62-year-old eyes, excluding the macular pigment. Table II also suggests the possibility that deuteranopes tend to have heavier pigmentation than protanopes or normals. It is concluded that individual variations in neutral point are ascribable to variations in ocular pigmentation (macula, lens, humors).

IV. DERIVATION OF RESPONSE FUNCTIONS

It is worth noticing that the point ($x_p=0.747$, $y_p=0.253$) found from Pitt's data as the common crossing point of the protanopic chromaticity-confusion lines is very close to the long wave extreme of the spectrum ($x=0.735$, $y=0.265$) used as a primary in the OSA coordinate system.[57] Reference to Fig. 1a shows immediately that in spite of this close approach, the long wave extreme of the spectrum is not eligible to represent (x_p, y_p). For example, the line connecting the extremes of the spectrum cuts all of the protanopic chromaticity-confusion lines (dotted); that is, the protanopic chromaticity of the long wave extreme differs maximally from that of the short wave extreme. But if the long wave extreme of the spectrum locus were itself taken as the point (x_p, y_p), one interpretation would be that the two extremes of the spectrum have the same protanopic chromaticity. On this account the solid lines of Fig. 1a drawn in for comparison have been made copunctal at $(x_p=0.747,\ y_p=0.253)$. Although the choice of the long wave extreme of the spectrum to represent (x_p, y_p) flatly contradicts the facts, it is of interest to inquire what response functions are generated by this choice because it bears on a question raised in 1798 by Dalton[3] and argued many times since, namely: Are protanopes red-blind?

Similarly it is worth noting that the point $(x_d=1.08,\ y_d=-0.08)$ estimated from Pitt's data as the most probable common crossing point of the deuteranopic chromaticity-confusion lines is fairly close to the X primary $(x=1.00,\ y=0.00)$ of the standard I.C.I. coordinate system. The solid lines of Fig. 1b have been drawn through this point, and it is seen that with one exception the solid lines are either closely parallel to the nearest dotted line or cross a single dotted line. Since the separation of the dotted lines indicates the smallest chromaticity difference perceptible to Pitt's observers, it may be concluded that to set $x_d=1$ and $y_d=0$ is not inconsistent with these data; and, indeed, from a set of lines drawn through the point $(x_d=1.08,\ y_d=-0.08)$ it may be concluded that the former choice is scarcely less apt than the latter which was chosen as the best estimate possible. It will be of interest, therefore, to study both choices.

It is convenient to start this study by investigating the implications of setting $x_p+y_p=x_d+y_d=1$, or, stated another way, since $z=1-x-y$, the implications of setting $z_p=z_d=0$. All four points under consideration meet this requirement.

If $z_p=z_d=0$, from Eqs. (2a) and (2b), we find $k_7=k_8=0$, provided $x_p/y_p\neq x_d/y_d$, that is, provided (x_p, y_p) and (x_d, y_d) are different points; and from Eq. (3), we find that $k_9=1$. From Eq. (1) we may therefore write: $K=Z$. This conclusion is important. It says that since both the protanopic chromaticity-confusion lines and the deuteranopic-confusion lines are copunctal at points lying on the tangent to the spectrum locus at the long wave extreme, the Z function of the standard observer is adequate to represent the K function used in describing the chromaticity confusions of both deuteranope and protanope. König[37] from somewhat incomplete data[6] drew this conclusion, and v. Kries* deduced it from the identity of the K function of protanope and deuteranope on the assumption that both are reduction forms of normal vision.

A further conclusion is possible from setting $z_p=z_d=0$. From (2b), (3), and (4), we may write three simultaneous equations with only three unknowns, thus:

$$k_1x_d+k_2y_d=0,\quad k_1+k_2+k_3=1,$$
$$k_3=-0.22k_1,$$

from which the coefficients, k_1 to k_3, defining the W_d function by Eq. (1), are found to be:

$$\begin{aligned} k_1&=-y_d/(x_d-0.78y_d),\\ k_2&=x_d/(x_d-0.78y_d),\\ k_3&=0.22y_d/(x_d-0.78y_d). \end{aligned} \tag{8a}$$

Thus it is seen that the W_d function (warm curve of deuteranopes) is determined wholly by the coordinates of the point (x_d, y_d) provided $z_d=z_p=0$, as they do for the four choices of interest.

Similarly, from Eqs. (2a), (3), and (4), we find that:

$$\begin{aligned} k_4&=-y_p/(x_p-0.78y_p),\\ k_5&=x_p/(x_p-0.78y_p),\\ k_6&=0.22y_p/(x_p-0.78y_p). \end{aligned} \tag{8b}$$

Thus it is seen that the W_p function (warm curve of protanopes) is determined wholly by the coordinates of the point (x_p, y_p) again provided $z_d=z_p=0$.

The next step is to find the response functions resulting from setting $x_d=1,\ y_d=z_d=0$, as is justified from available data on chromaticity confusions of deuteranopes. From Eq. (8a), we find immediately that: $k_1=k_3=0$, and $k_2=1$. Hence from Eq. (1), $W_d=Y$. This conclusion is important. It says that since the deuteranopic chromaticity-confusion lines may be taken as copunctal at $x_d=1,\ y_d=z_d=0$, then the Y function of the standard observer is adequate to

* Reference 39, p. 164.

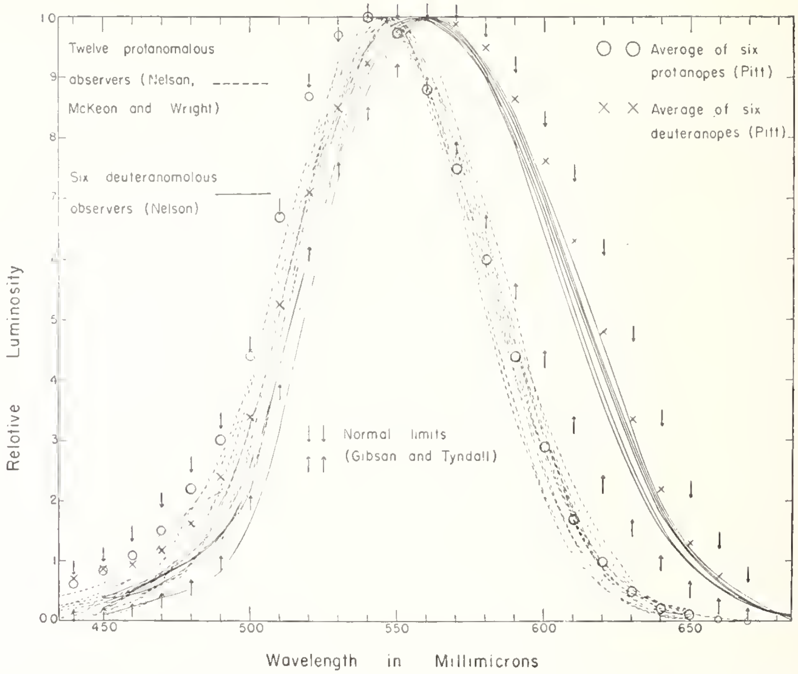

FIG. 2. Relative luminosity functions of anomalous trichromats (6 deuteranomalous, 12 protanomalous) compared to averages for dichromats and to limits for normal trichromats. Deuteranopic and deuteranomalous luminosity functions fall generally within normal limits; but protanopic and protanomalous luminosity functions fall outside of normal limits.

represent the W_d function (warm curve of the deuteranope) for describing the chromaticity confusions of the deuteranope. The great convenience of being able to use two (Z and Y) of the three functions representing the standard observer directly in deuteranopic chromaticity specifications should be emphasized. It means that the chromaticity of any color stimulus already specified in terms of the standard observer may be specified for an average deuteranope merely by leaving the X specification out of account. That is, any two stimuli for which the Y specification and the Z specification bear the same ratio will be chromaticity matches for an average deuteranope.

The W_p function is found from the values of $x_p=0.747$, $y_p=0.253$, through Eqs. (8b) and (1) to be:

$$W_p=-0.460X+1.359Y+0.101Z. \quad (9)$$

The W_p function generated from the theoretically interesting but factually inadmissible values of $x_p=0.735$, $Y_p=0.265$ is found in a similar way to be:

$$W_p'=-0.503X+1.392Y+0.111Z. \quad (9a)$$

The W_d function agreeing as closely to the Pitt average data as is possible ($x_d=1.08$, $y_d=-0.08$, $z_d=0.00$) is found from Eqs. (8a) and (1) to be:

$$W_d'=0.079X+1.061Y-0.017Z. \quad (10)$$

V. LUMINOSITY FUNCTIONS OF RED-GREEN-BLIND OBSERVERS COMPARED TO THOSE OF NORMAL AND ANOMALOUS TRICHROMATS

We may now inquire into the relationship between the functions, W_d, W_p, W_d', and W_p', and the luminosity functions of dichromatic and trichromatic observers. Figure 2 shows by dotted lines the luminosity functions of eleven protanomalous observers studied by McKeon and Wright[45] and one studied by Nelson.[49] It also shows by solid lines the luminosity functions of six deuteranomalous observers studied by Nelson.[49] Although at the short wave end of the spectrum these groups of functions overlap and are not significantly different, the two groups divide quite sharply at the long wave end. In this respect they are like the groups of functions representing the two corresponding types of dichromatic observer, the deuteranope and the protanope. The circles represent the average of six protanopic luminosity functions evaluated by Pitt,[52] and the crosses represent his average of six deuteranopic luminosity functions. The arrows indicate the maximum and minimum values of relative luminosity found by Gibson and Tyndall[11] among their 37 completely studied normal observers. Although because of differences in experimental conditions no highly precise intercomparison is possible between the data shown for anomalous and dichromatic observers and these normal limits, nevertheless Fig. 2 indicates that the deuteranopic and deuteranomalous observers alike possess luminosity functions generally well within normal limits; but both protanomalous observers and protanopic observers possess luminosity functions that are abnormally low at the long wave end.

Figure 2 tends to indicate also that protanomalous observers possess abnormally narrow luminosity functions even compared to protanopes, and that deuteranopes possess luminosity functions abnormally high between 570 and 610 mμ with deuteranomalous observers intermediate in this respect; but because of the small number of abnormal observers tested, these indications may fail to be significant. The experimental conditions (field size, field luminance, surrounding-field luminance) and techniques (adaptation, control of fixation, length of observing time) govern the participation of the rod mechanism in determining these functions. In any one study the luminosity curves may be thrown more or less to the short wave side because of partial participation of the rods, either intended or inadvertent. The intertwining of the curves at the short wave end is probably to be ascribed chiefly to individual variation in the amount of yellow or brown pigmentation of the eye media, such as degree of macular pigmentation.

Since from Fig. 2 both deuteranopic and deuteranomalous luminosity functions fall generally within the limits of 37 normal observers, it is just as valid to predict the brightness judgments of these abnormal observers from the standard luminosity function as it is those of an

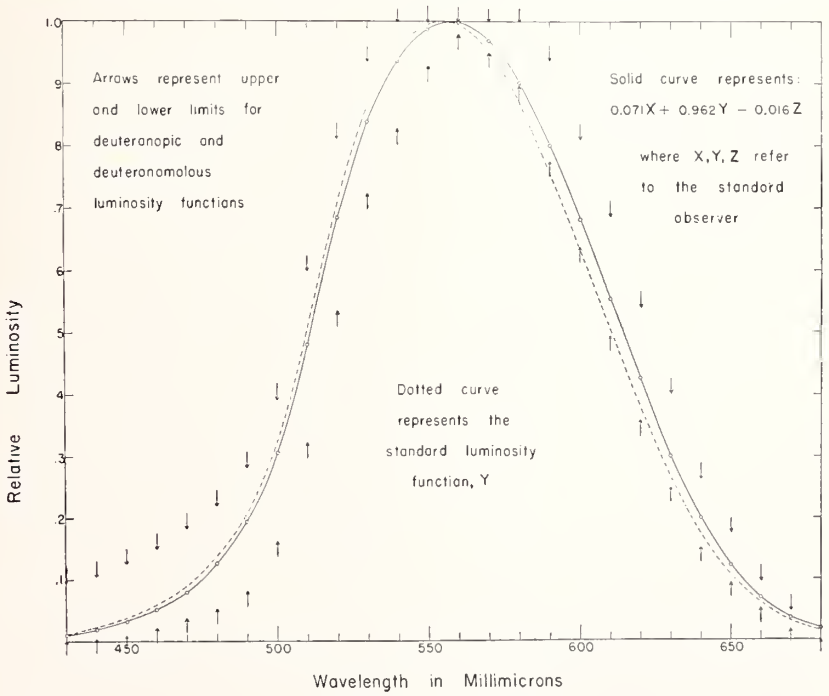

FIG. 3. Comparison of the functions W_d (standard luminosity function Y) and W_d' (solid curve) with upper and lower limits for deuteranopic and deuteranomalous luminosity functions (arrows). The standard luminosity function is seen to fall satisfactorily within these limits except for the wave-length region between 570 and 610 mμ. The function W_d' yields slightly better agreement.

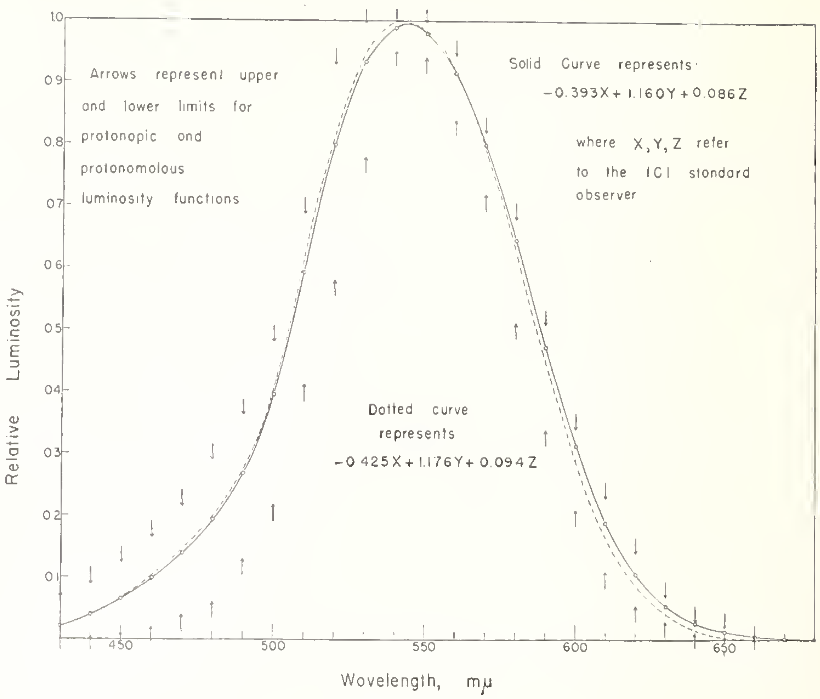

FIG. 4. Comparison of the functions W_p (solid curve) and W_p' (dotted curve) with upper and lower limits for protanopic and protanomalous luminosity functions (arrows). Both functions are seen to fall satisfactorily within these limits except that W_p' is perhaps too low for wave-lengths greater than 650 mμ.

observer chosen at random from the large population group to be classed as of normal color vision. Figure 3 compares the standard luminosity function $Y=W_d$ (dotted line) with the upper and lower limits (arrows) for the deuteranopic and deuteranomalous luminosity functions referred to in Fig. 2. The standard luminosity function is seen to fall satisfactorily within these limits except for the wave-length region between 570 and 610 mμ. Studies by Sloan[56] corroborated by the recent work of Walters and Wright[58] show that the shape of the luminosity function within this wave-length region is particularly subject to variation with experimental conditions. It is therefore perhaps reasonable to ascribe the whole of this small discrepancy to variation in experimental conditions.

Figure 3 also shows the function W_d' [Eq. (10)] adjusted to unit maximum as a solid line. This function represents available data on deuteranopic luminosity slightly better than the standard luminosity function $Y=W_d$, just as, combined with the K function, it represents deuteranopic chromaticity data slightly better. There is therefore some basis for arguing, as in the most common form of Young-Helmholtz theory, that the normal luminosity function should be thought of as consisting of three components, a red, a green, and a violet.* Whether this possible justification of the usual Young-Helmholtz view would be borne out by a study of an adequate sample of normal and deuteranopic observers remains to be decided by further extensive experiment. The two functions, W_d and W_d', differ so little, however, that it would rarely be of practical importance to use one rather than the other. Because of its greater simplicity it seems better to take $W_d=Y$ as the function representing deuteranopic and deuteranomalous luminosity.

Figure 4 compares the functions $0.854W_p$ and $0.845W_p'$ with the upper and lower limits (arrows) for the protanopic and protanomalous luminosity functions referred to in Fig. 2. The constants 0.854 and 0.845 serve to adjust the functions approximately to unit maximum, and

* An account of dichromatic vision following this view has recently been worked out by Pitt (reference 53).

from Eqs. (9) and (9a), respectively, yield:

$$0.854W_p = -0.393X + 1.160Y + 0.086Z \quad \text{(solid curve)},$$

$$0.845W_p' = -0.425X + 1.176Y + 0.094Z \quad \text{(dotted curve)}.$$

It is seen that both functions fall satisfactorily between the limits at least for wave-lengths shorter than 650 mμ. The function W_p, therefore, not only represents adequately, when combined with the K function, the chromaticity confusions of the average protanope, but also, taken by itself, his brightness judgments. The function W_p', although it grossly fails, when combined with the K function, to represent the chromaticity confusions of the protanope, nevertheless succeeds in representing adequately his brightness judgments except for wave-lengths greater than 650 mμ. This rather minor difference in luminosity function is all that differentiates red-blind vision (luminosity zero for wave-lengths greater than 700 mμ) from the experimentally determined facts of protanopia. The old name, red blindness, is therefore nearly but not quite accurate.

By way of contrast it may be pointed out that the corresponding old name, green blindness, for the type of defect known since v. Kries[46] as deuteranopia is almost wholly wrong. Since the standard luminosity function represents the brightness judgments of deuteranopic observers just as closely as those of a considerable fraction of normal observers, and is far from being zero anywhere within the spectrum region which ordinarily appears green (490 to 550 mμ), deuteranopes are far from being insensitive to the green part of the spectrum. It is true that protanopia and deuteranopia can logically be described as red blindness and green blindness, respectively, in terms of the Young-Helmholtz theory, but in a purely descriptive nontheoretical sense, no observer has ever been found to be quite red-blind, and none even remotely deserving the name, green-blind.

The following functions therefore may be taken to be representative of normal, deuteranopic and protanopic vision:

$$K = Z,$$
$$W_p = -0.460X + 1.359Y + 0.101Z, \quad (11)$$
$$W_d = Y,$$

K and W_d for deuteranopic vision, K and W_p for protanopic vision, and all three for normal vision. It is also true, although no adjustments other than Eq. (4) have been made for it nor any detailed quantitative comparisons presented, that W_p and W_d taken together closely represent tritanopic vision.[31, 33, 38, 47] Table III gives these three functions for 10-mμ intervals of the visible spectrum. The function W_p is taken as the luminosity function of a typical protanope, the function W_d is taken as the luminosity function of a typical deuteranope as well as the

TABLE III. Response functions for protanopic (W_p, K) and deuteranopic (W_d, K) vision.

Wave-length in mμ	W_d	W_p	K
380	0.00004	0.00006	0.0065
390	.00012	.00024	.0201
400	.0004	.00082	.0679
410	.0012	.00257	.2074
420	.0040	.00883	.6456
430	.0116	.0251	1.3856
440	.0230	.0476	1.7471
450	.0380	.0759	1.7721
460	.0600	.1163	1.6692
470	.0910	.1638	1.2876
480	.1390	.2270	.8130
490	.2080	.3150	.4652
500	.3230	.4642	.2720
510	.5030	.6953	.1582
520	.7100	.9437	.0782
530	.8620	1.0997	.0422
540	.9540	1.1650	.0203
550	.9950	1.1537	.0087
560	.9950	1.0791	.0039
570	.9520	.9434	.0021
580	.8700	.7610	.0017
590	.7570	.5568	.0011
600	.6310	.3690	.0008
610	.5030	.2224	.0003
620	.3810	.1248	.0002
630	.2650	.0646	.0000
640	.1750	.0318	.0000
650	.1070	.0150	.0000
660	.0610	.0070	.0000
670	.0320	.0033	.0000
680	.0170	.00157	.0000
690	.0082	.00070	.0000
700	.0041	.00035	.0000
710	.0021	.00018	.0000
720	.00105	.000089	.0000
730	.00052	.000044	.0000
740	.00025	.000021	.0000
750	.00012	.000010	.0000
760	.00006	.000005	.0000
770	.00003	.000003	.0000

normal observer (and perhaps also that of a typical tritanope),[7, 8, 34] and the function K is unassociated with luminosity.

The representation of normal vision by K, W_d, and W_p is just as adequate as that afforded by the functions Z, Y, and X of the standard I.C.I. coordinate system, and indeed functions closely like these were proposed by the author in 1931 for international adoption through the American representative, Mr. Irwin G. Priest, but were rejected because the corresponding chromaticity diagram gave considerably less uniform chromaticity scales than the coordinate system (X, Y, Z) finally adopted. There is therefore nothing to be gained by supplanting the present X function by W_p to represent normal vision, and this substitution is not recommended.

The representation of deuteranopic vision by K and W_p, and protanopic vision by K and W_p, is valid in the same way as representation of normal vision by X, Y, and Z. No single normal observer, or protanope, or deuteranope, can be found except by accident who conforms to the standard representation of the respective group, but the departures of each single individual from the standard may be expected to be smaller than some of the individual differences within the group.

TABLE IV. Chromaticity coordinates for deuteranopic and protanopic vision.

Wave-length in mμ	Protanopia w_p	Protanopia k_p	Deuteranopia w_d	Deuteranopia k_d
380	0.012	0.988	0.006	0.994
430	.018	.982	.008	.992
450	.041	.959	.021	.979
460	.065	.935	.035	.965
470	.113	.887	.066	.934
475	.156	.844	.098	.902
480	.218	.782	.146	.854
485	.301	.699	.216	.784
490	.404	.596	.309	.691
495	.518	.482	.423	.577
500	.631	.369	.543	.457
505	.730	.270	.657	.343
510	.815	.185	.761	.239
515	.881	.119	.845	.155
520	.923	.077	.901	.099
530	.963	.037	.953	.047
540	.982	.018	.979	.021
550	.992	.007	.991	.009
600	.998	.002	.999	.001
650	1.000	.000	1.000	.000
750	1.000	.000	1.000	.000

VI. USE OF THE RESPONSE FUNCTIONS

The response functions K, W_p, and W_d have been used by the ISCC committee on color-blindness* to find whether or not two colors will be confused by an average protanope or an average deuteranope. Such use will be illustrated by solution of three problems; first, what is the influence of the choice of standard neutral source on the dichromatic neutral points in the spectrum, second, how much darker to an average protanope are normally reddish colors, and third, what arrangements of the colors of the Farnsworth dichotomous test B-20[9] are characteristic of average deuteranopic and protanopic vision.

A. Wave-Length of Neutral Point

The determination of the neutral point in the spectrum of a dichromatic observer involves setting up a chromaticity match between the standard neutral source (I.C.I. Illuminant A, I.C.I. Illuminant C, blackbody at 5000°K, and so forth) and some part of the spectrum; see Table II. Deuteranopic and protanopic chromaticity is conveniently specified by chromaticity coordinates w_d, k_d and w_p, k_p, the first pair for deuteranopic vision, the second for protanopic. The definitions of these coordinates and their equivalents in terms of X, Y, Z, from Eq. (11) are as follows:

$$w_d = W_d/(W_d+K) = Y/(Y+Z),\quad k_d = K/(W_d+K) = Z/(Y+Z); \tag{12a}$$

$$w_p = W_p/(W_p+K) = \frac{-0.460X+1.359Y+0.101Z}{-0.460X+1.359Y+1.101Z}, \tag{12b}$$

$$k_p = K/(W_p+K) = Z/(-0.460X+1.359Y+1.101Z).$$

* Committee on color-blindness tests, Inter-Society Color Council, Co-Chairmen: Dr. LeGrand H. Hardy, and the author.

TABLE V. Protanopic and deuteranopic neutral points in the spectrum derived from the dichromatic chromaticity coordinates w_p and w_d.

Standard source	I.C.I. chromaticity coordinates x	y	Dichromatic chromaticity coordinates w_p	w_d	Wave-length of the neutral point, mμ λ_p	λ_d
I.C.I. Illuminant *A*	0.4476	0.4075	0.7144	0.7377	504.0	508.9
I.C.I. Illuminant *B*	.3484	.3516	.5369	.5396	495.8	499.8
Complete radiator at 5000°K	.3445	.3512	.5346	.5358	495.7	499.7
I.C.I. Illuminant *C*	.3101	.3163	.4652	.4585	492.7	496.4
Illuminant *D**	.2999	.3120	.4559	.4457	492.3	495.8
Illuminant *S*†	.2319	.2318	.3286	.3018	486.4	489.6

* Illuminant D is produced by an incandescent lamp and Corning Daylite glass filter (reference 32, p. 60), and has a nearest color temperature (reference 28) of about 7500°K.
† Illuminant S was found (reference 32, p. 58) by weighting Abbot's "sun-outside-atmosphere" energy data by the inverse λ^4 scattering relation, and has been designated as "limit blue sky."

Table IV shows these coordinates for the various parts of the spectrum, particularly the portions near 495 mμ where chromaticity for these types of dichromatic vision varies most rapidly with wave-length. Since from their definitions the two coordinates sum to unity, only one of them is required to specify the dichromatic chromaticity of a stimulus.

In order to find the neutral point in the spectrum of, say, an average protanope for a standard neutral source specified in the standard I.C.I. system by X_0, Y_0, Z_0, it is necessary only to compute w_p from Eq. (12b) and find by interpolation among the values of w_p for the spectrum given in Table IV the wave-length for the corresponding value. This wave-length corresponds to the same chromaticity as the standard neutral source and is therefore the desired neutral point. This method is the analytic equivalent of a graphical method using the chromaticity diagram of the standard system; see Fig. 1. The graphical method is to draw a straight line from the copunctal point through the point representing the chromaticity of the standard neutral source and extend it until it cuts the spectrum locus. The wave-length corresponding to this intersection is the desired neutral point in the dichromatic spectrum.

Table V gives the standard chromaticity specifications, x and y, for several sources that might be taken as standard neutrals, the values of w_p and w_d for these sources, and the corresponding neutral points found by the analytical method from Table IV. It will be noted that the change in neutral point depending upon choice of standard neutral source is considerably greater than the difference between the neutral points of the typical protanope and typical deuteranope for any one source.

B. Protanopic Luminous Reflectance

The second problem is to find how much darker to an average protanope are normally reddish colors. For spectrum colors the solution is given simply by comparing W_p with $Y=W_d$ in Table III. It is seen, for example, that relative to an equal-energy source, an average protanope finds the spectrum from 700 to 770 mμ to have less than 10 percent of the normal luminosity. It could be said therefore in a sense that he is more than 90 percent blind to the long wave end of the spectrum; but we would also have to say in the same sense that he is supersensitive by a factor of 2 for the spectral region near 450 mμ. It is more convenient to say that the luminosity function of the average protanope is shifted about 10 mμ toward the short wave end of the spectrum relative to normal.

If the color is that of a surface it is customary to estimate its lightness or darkness from the luminous reflectance of the surface for I.C.I. Illuminant *C* relative to magnesium oxide. For magnesium oxide, $X=0.9804$, $Y=1.0000$, and $Z=1.1812$, and from Eq. (9), $W_p=1.0274$. Protanopic luminous reflectance relative to magnesium oxide is therefore equal to $W_p/1.0274$ or $0.9733W_p$. It is convenient to represent on the (x, y) chromaticity diagram the dependence of protanopic luminous reflectance of a surface on the normal chromaticity of its color.

If Q be the ratio of the protanopic luminous reflectance of a surface to its normal luminous

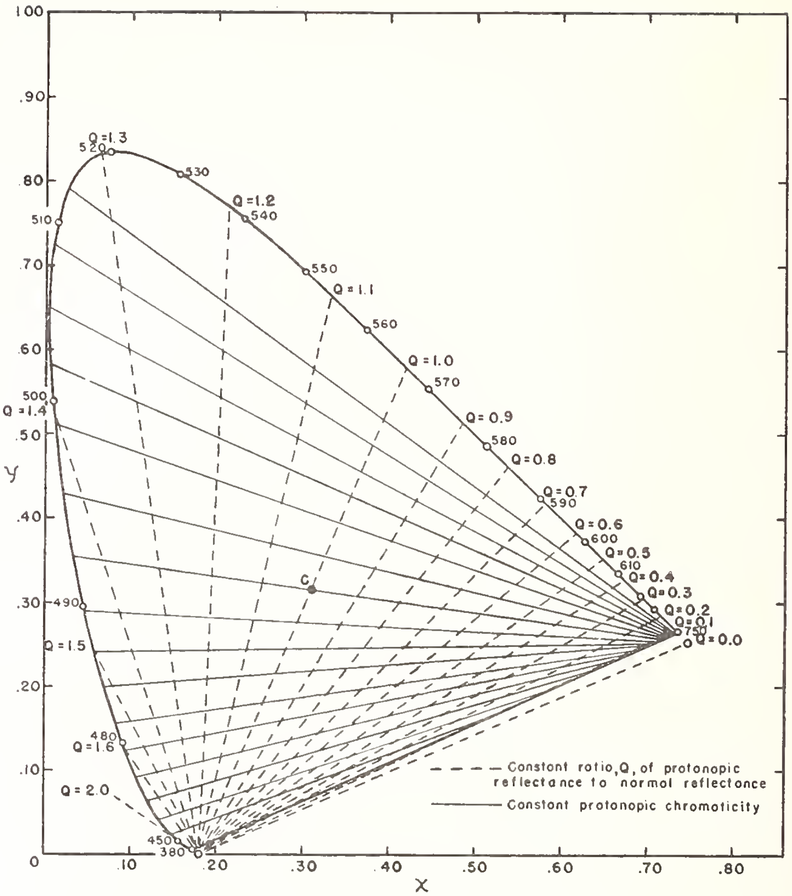

FIG. 5. Relation of protanopic to normal vision. The solid lines (as in Fig. 1) represent constant protanopic chromaticity; the dotted lines represent constant values of the ratio Q of protanopic luminous reflectance to normal luminous reflectance. Both reflectances are taken relative to that of magnesium oxide and refer to I.C.I. standard Illuminant C. Note that the line (dotted) for $Q=1$ passes through the point, $x=0.310$, $y=0.316$, representing Illuminant C. Note also that the line for $Q=0$ passes through the copunctal point, $x=0.747$, $y=0.253$; this line could be called the protanopic alychne (lightless line), just as the line for $Q=\infty$ [$y=0$, see Eq. (13)] has sometimes been called the normal alychne.

reflectance Y, we may write:

$$Q=0.9733W_p/Y$$
$$=0.9733(-0.460X+1.359Y+0.101Z)/Y$$
$$=0.9733(-0.460x+1.359y+0.101z)/y,$$

whence it is found:

$$x=y(Q-1.224)/(-0.546)+0.180. \quad (13)$$

Examination of Eq. (13) shows that it represents a family of straight lines on the (x, y) chromaticity diagram passing through the point ($x=0.180$, $y=0.000$). For $Q=1$, the line passes through the point representing Illuminant C ($x=0.3101$, $y=0.3163$). All surfaces whose chromaticities under Illuminant C are represented on this line have identical protanopic and normal luminous reflectances. For $Q=2.0$, the line passes close to 450 mμ; all surfaces represented on this line have protanopic luminous reflectance twice the normal luminous reflectance. For $Q=0.10$, the line passes close to 650 mμ; all surfaces represented on this line have protanopic luminous reflectance equal to 10 percent of the normal luminous reflectance. Figure 5 shows a number of these lines of constant Q (dotted) on the (x, y) chromaticity diagram; it indicates completely and quantitatively how much darker to an average protanope are normally reddish colors. It also shows the lines of protanopic chromaticity confusion; it therefore gives a convenient, concise and quantitative summary of the connection between average protanopic vision and the I.C.I. standard observer.

Two of the series of straight lines of constant Q have special interest. One of them ($Q=\infty$) corresponds to $y=0$, sometimes called the alychne[25, 55] because on it are represented imaginary chromaticities which are unassociated with luminosity. The only way to make the quotient Q become indefinitely large is to make the denominator Y equal zero; that is, by choosing a chromaticity represented on the alychne. The other line of special interest is found by setting $Q=0$, which defines the line:

$$x=y(1.224/0.546)+0.180. \qquad (14)$$

This line could well be called the protanopic alychne. Note that it passes through the copunctal point ($x=0.747$, $y=0.253$) of the lines representing the chromaticities confused by the average protanope, and that it does not cross any portion of the chromaticity diagram representing real surfaces.

The copunctal point for the lines of constant Q is found from Eq. (11) also to be the point for which $W_d=W_p=0$. If, as seems at least approximately correct, W_d and W_p taken together represent average tritanopic vision, then it can be said that the lines of constant ratio between protanopic and normal reflectance are also chromaticity-confusion lines for tritanopic vision.

C. Farnsworth Dichotomous Test (B-20)

The third problem is to find the characteristic deuteranopic and protanopic arrangement of the colors of the Farnsworth dichotomous test B-20. Table VI gives the Munsell hue of the colors used, Munsell value and chroma being constant at 5/2, their tristimulus specifications (X, Y, Z) for I.C.I. Illuminant C, one of the distimulus specifications W_p multiplied by the constant 0.973_3 to make it correspond to protanopic reflectance relative to MgO, and the dichromatic chromaticity coordinates w_p and w_d. The fraction $W_d/(W_p+W_d)$, identified by the notation w_t, is also given in order to test whether this fraction correlates with the arrangement of the colors found by Farnsworth to be characteristic of a tritanope. It is first to be noted from the relative constancy of the daylight reflectance, $W_d=Y$, and the protanopic reflectance, $0.9733W_p$, for these colors that the papers used in the Farnsworth dichotomous test have nearly the same reflectances for Illuminant C not only for the standard observer and typical deuteranope

TABLE VI. Tristimulus and distimulus specifications of the colors of the Farnsworth dichotomous test for color vision.

Serial No.	Munsell hue (at 5/2)	Tristimulus specifications X	Y	Z	Protanopic reflectance $0.9733\ W_p$	Distimulus chromaticity coordinates w_p	w_d	w_t
1	10*B*	0.2121	0.2229	0.3292	0.2321	0.4201	0.4037	0.4831
2	5*B*	.1749	.1917	.2831	.2030	.4242	.4037	.4789
3	10*BG*	.1820	.2043	.2790	.2162	.4432	.4227	.4791
4	5*BG*	.1782	.2077	.2538	.2199	.4709	.4501	.4790
5	5*G*	.1618	.1888	.2017	.1972	.5011	.4835	.4824
6	10*GY*	.1766	.2017	.1873	.2061	.5307	.5185	.4878
7	5*GY*	.1908	.2097	.1804	.2096	.5442	.5376	.4933
8	10*Y*	.2135	.2292	.1798	.2253	.5628	.5604	.4975
9	5*Y*	.2057	.2117	.1795	.2056	.5406	.5412	.5006
10	10*YR*	.2170	.2143	.1780	.2038	.5405	.5463	.5058
11	5*YR*	.2029	.1927	.1755	.1813	.5149	.5234	.5084
12	10*R*	.2364	.2185	.2055	.2034	.5042	.5153	.5111
13	5*R*	.2158	.1954	.2041	.1818	.4779	.4891	.5113
14	10*RP*	.2426	.2229	.2453	.2103	.4684	.4761	.5077
15	5*RP*	.2304	.2138	.2559	.2048	.4512	.4552	.5040
16	10*P*	.2256	.2092	.2751	.2027	.4309	.4320	.5011
17	5*P*	.2070	.1959	.2818	.1942	.4145	.4101	.4954
18	10*PB*	.2199	.2118	.3301	.2140	.3998	.3908	.4906
19	7.5*PB*	.2055	.2035	.3036	.2071	.4121	.4013	.4888
20	5*PB*	.2052	.2087	.3277	.2164	.4042	.3891	.4842

($W_d = Y$), but also for the typical protanope 0.9733 (W_p), the variation being between 0.2321 and 0.1818 which corresponds to only 0.55 of a Munsell value step.[50] Since these papers are exhibited with a dark gray border which interferes with perception of lightness differences more than it does chromaticity differences,* the basis for arranging the colors in order must be chiefly according to chromaticity. The characteristic protanopic arrangement is therefore to be found in accord with w_p, and the characteristic deuteranopic arrangement according to w_d.

Table VII shows the serial order according to w_p compared to that for a protanope reported by Farnsworth. That is, color number 8 was picked by the Farnsworth protanope as the warmest of the 20 colors and it also has the highest value of w_p (0.5628 from Table V); so number 8 heads both the second and third column of Table VI. A similar comparison is given between the arrangements according to w_d and a deuteranope reported by Farnsworth, and between w_t and the tritanope reported by Farnsworth. It is evident

TABLE VII. Comparison of the serial orders of the colors of the Farnsworth dichotomous test according to w_p, w_d, and w_t with the orders reported by Farnsworth for a protanope (red-bluegreen confuser), and deuteranope (green-redpurple confuser), and a tritanope (violet-greenishyellow confuser).

Normal order (based on hue)	Serial numbers in order from warmest to coldest dichromatic color according to:					
	w_p	Farnsworth protanope	w_d	Farnsworth deuteranope	w_t	Farnsworth tritanope
1	8	8	8	10	13	13
2	7	10	10	9	12	14
3	9	7	9	8	11	15
4	10	6	7	7	14	12
5	6	9	11	11	10	11
6	11	11	6	12	15	16
7	12	5	12	6	16	10
8	5	12	13	13	9	9
9	13	13	5	14	8	17
10	4	4	14	5	17	18
11	14	3	15	15	7	8
12	15	15	4	16	18	19
13	3	14	16	4	19	7
14	16	2	3	17	6	20
15	2	16	17	3	20	1
16	1	17	2	2	1	6
17	17	1	1	18	5	2
18	19	19	19	20	3	5
19	20	20	18	19	4	4
20	18	18	20	1	2	3

* Reference 29, p. 425.

from Table VII that the correlations indicated by these comparisons are high; that is, the results chosen by Farnsworth to be typical of protanopia, deuteranopia, and tritanopia agree well though not perfectly with the chromaticity arrangements according to w_p, w_d, and w_t, respectively.

Failure of the agreement to be perfect might be caused by failure of the tristimulus specifications reported for certain samples of the Munsell papers to apply exactly to the particular samples used in the Farnsworth test B-20, to careless arrangement of the colors by the dichromatic observer, or to arrangement on a basis other than chromaticity, or to significant difference between the visual system of the dichromat and that represented by the pairs of functions, K, W_p, and W_d taken here to represent the average protanope and deuteranope. By plotting the orders given in Table VII according to w_p, w_d, and w_t on the Farnsworth chart for analyzing results on the dichotomous test (see his Fig. 15), it is discovered that most of the minor differences are ascribable to a real difference between the typical Farnsworth dichromats and the corresponding hypothetical observers defined by the functions K, W_p, and W_d. The axes of confusion for the hypothetical observers fall, however, within the limits indicated by Farnsworth (see his Fig. 15) as embracing the three types of dichromat delineated by the dichotomous test.

Although this corroboration was expected for protanopic and deuteranopic vision because of the firm basis for choosing the functions K, W_p, and W_d, it is interesting to note that substantially the same agreement is found for tritanopic vision. The discrepancies are of the sort to be ascribed to individual differences. It is evident that the Farnsworth test is well adapted for surveys of the characteristics of dichromats; a statistical treatment of an extended body of data found by this test would serve to determine whether the functions proposed here for dichromatic vision describe, as was intended, visual systems intermediate to actual observers of the corresponding type; also whether the functions W_p and W_d, taken together, describe a type of tritanopic vision intermediate to actual tritanopes.

(a)

(b)

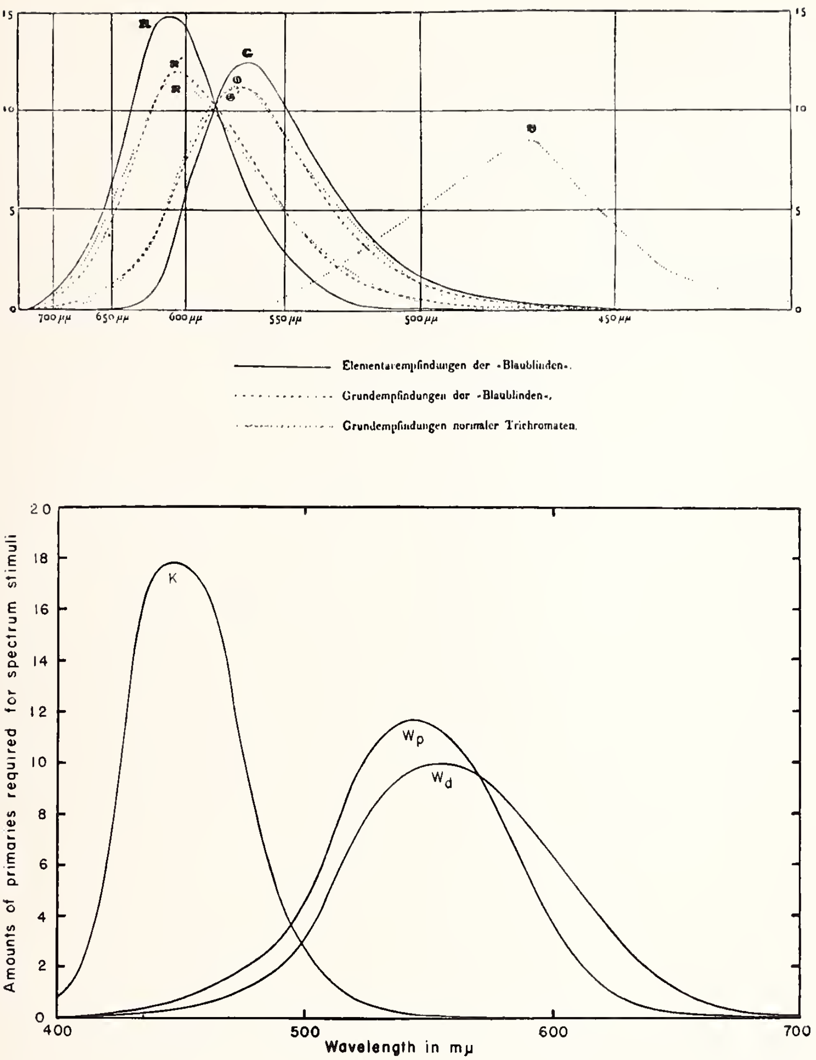

FIG. 6. Comparison of the functions W_d, W_p, and K with the "fundamental-sensation" curves (dotted) derived by König in 1897. If allowance is made for the descending and expanding wave-length scale and for the energy distribution of gas light used by König, it will be noted that there is no essential difference between the two sets of functions.

VII. THEORETICAL IMPLICATIONS OF THE RESPONSE FUNCTIONS

The functions W_d, W_p, and K defined in Eq. (11) have theoretical implications the validity of which in no way interferes with the practical applications just discussed. It is convenient to bring out these implications by indicating the relation of these functions to the later theoretical views of Arthur König, to the recent dominator-modulator theory of Granit, and to the zone-theory of G. E. Müller.

A. The König Theories

In 1886 and in 1893 Arthur König published accounts of his monumental work on the color systems of normal and abnormal observers.[36,37] The coordinate system chosen by him to represent his experimental results satisfies Eqs. (1) and (2) but does not satisfy Eq. (4) since one of the "fundamental sensations" was taken as unitary blue. In 1897, however, König published an important paper on tritanopia or "blue-blindness" in which choice of unitary blue as a "fundamental sensation" was given up. He says in a footnote,* "I will immediately remark here that regarding the quality of this third fundamental sensation of the Young-Helmholtz color theory—whether it be blue or violet—I can still pass no definite judgment. In any case, however, I am now inclined to move it far closer to violet than was the case in the year 1886 (compare Sitz. Akad. Wiss. Berlin, July 29, 1886)."

As a result of this change in choice of third primary the "fundamental-sensation" curves bear a close resemblance to the functions W_d, W_p, and K. Figure 6a is a reproduction of König's Fig. 2 shown for comparison with Fig. 6b which is a plot of the functions W_d, W_p, and K. The dotted curves of Fig. 6a (the fundamental-sensation curves) are plotted against a descending and expanding wave-length scale, the curves are adjusted to equal area on this non-uniform scale, and refer to the dispersion spectrum of gaslight. Apart from these differences, however, it could be said that the functions W_d, W_p, and K are modern evaluations of the functions taken in 1897 by König to represent the "fundamental-sensation" curves of the eye. From the standpoint of the König theory, therefore, it is but to be expected that W_d and W_p taken together describe the color vision of a tritanope. It is no small tribute to König that the simplest representation of normal and dichromatic vision possible at the present time so closely resembles the picture given by him more than forty years ago.

These later König views have so far received much less attention than his earlier evaluation of the "fundamental-sensation" curves which were included in the second edition of Helmholtz' *Physiological Optics*,[16] made the basis of a new color theory by Ladd-Franklin, and reinserted in that form in the English translation of the third German edition of Helmholtz.[18] Although he apparently did not draw attention to it in any publication, König immediately recognized that the distribution of the fundamental "red" sensation throughout the spectrum was closely the same as the luminosity distribution, and he discussed this consequence of his new view with, among others, his brilliant former pupil, Christine Ladd-Franklin, and J. v. Kries.* The then current view of the Young-Helmholtz theory was that luminosity should be thought of as made up of the sum of three components, a red, a green, and a blue or violet component. König's new view that all or nearly all of the luminosity might reside in a single one (red) of the components met with no favor at that time. This view as well as his previous views were omitted entirely from the third edition of Helmholtz[17] prepared in 1911 by Gullstrand, Nagel, and v. Kries. However, the completeness with which König in 1897 anticipated the present description of dichromatic and trichromatic vision is strikingly shown in the following footnote which appears in a note specially prepared in 1924 by v. Kries for insertion in the English edition of Helmholtz,** "König long ago called attention incidentally to the curious fact that the luminosity values of the colours . . . turn out to be very nearly the same function of the wave-length for deuteranopes and persons with normal vision. Since in the case of deuteranopes light of long wave-lengths has no effect except on the red component, we must suppose also that this action depends on the wave-length in the same way. But then the further result is that the distribution of luminosity is not affected, or at least almost inappreciably affected, by the addition of the green component by which the deuteranopic visual organ is converted into a normal organ; in other words, that in the case of normal vision also the luminosity goes practically hand in hand with the action of the red component."

B. Dominator-Modulator Theory of Granit

In 1943 Granit[12] gave a brief outline of a physiological theory of color perception based upon his oscillographic studies of the responses of isolated fibers from the optic nerves of animals (cat, guinea pig, frog, snake, rat). In the light-adapted eyes of these animals the simple spectral sensitivity curves recorded with his micro-electrode technique were found to be of two types: (a) broad absorption bands, called *dominators*,

* Reference 38, p. 716; reference 38a, p. 396.

* Communicated privately to the author by Dr. Ladd-Franklin in 1928.

** Reference 18, p. 412.

and (b) narrow bands, called *modulators*. The dominator of the light-adapted animal eye was found to have its maximum in the spectral region around 560 mμ. The form and spectral location of the dominator were found to be practically identical with the average curve obtained from massed receptors in the light-adapted eyes of the same species. This and its good correspondence with respect to form and location with the luminosity curve of the light-adapted human eye led Granit to the conclusion that the dominator is responsible for the sensation of brightness, which thus is taken as our dominant impression. Modulation of the dominant impression of brightness to color would seem to be the task of the much rarer modulators which occupy narrow bands of sensitivity in three preferential regions around 580 to 600 mμ, 520 to 540 mμ, and 450 to 470 mμ.

It will be noted from Fig. 6b that the functions W_d, W_p, and K proposed here to represent dichromatic and normal trichromatic vision conform fairly well with the dominator-modulator theory. The K function is the narrowest of the three and since it is unassociated with luminosity could be thought of as a modulator for normal, protanopic, and deuteranopic vision. The W_p function is somewhat narrower than the W_d function and could be thought of as a modulator in the case of normal and tritanopic vision. The spectral location of these functions does not, however, conform perfectly with any of the three preferential regions found by Granit in his study of the eyes of animals.

Granit's view of deuteranopia differs somewhat from that suggested by the functions W_d and K. He says,* "Colour-blindness need not, but *can* be possible without parallel change of the photopic luminosity curve. A colour-blindness of this type would be the common form of red-green blindness known as deuteranopia, to be interpreted as absence of the 'red' and 'green' modulators, with the remaining dominator alone giving the normal luminosity curve." The view suggested by the functions W_d and K is that deuteranopia consists of the absence of only the green modulator, W_p.

Granit attacks the classical trichromatic theory of color blindness as follows:* "The trichromatic theory regards white as the result of the summed effects of, chiefly, the 'red' and the 'green' sensitivity curves. This forces the theory to accept the consequence that removal of 'red' and/or 'green' should cause removal of the perception of luminosity in the same region of the spectrum. Hence there can be no colour-blindness without profound changes in the form and locus of the luminosity curve. It is an admission of failure to have to explain so important a phenomenon as deuteranopia by pushing it aside to be taken care of by the 'higher centres.' " It is interesting to note that this criticism of classic three-components theory does not apply to the later König form of this theory.

It may be concluded that the functions W_d, W_p, and K are nearly, if not perfectly, consistent with Granit's dominator-modulator theory, and also that this theory was foreshadowed by König's later views. König, himself, however, began to be interested in the theoretical possibilities of an hypothetical post-receptor center of the visual mechanism apparently first suggested by Donders.[5] He states,† "This is not the place to go into any further discussion of the hypothesis of the existence of a second more centrally located structure of the color system. I wish only to call special attention to the fact that with its assumption many difficulties still existing in the explanation of color vision in the extrafoveal and peripheral parts of the retina, for what I have called the blueblindness of the fovea, for the apparent achromaticity of sensations in the lowest brightness levels, and so forth, would be removed. Through assumption of pathological processes in this more centrally located color apparatus there could be explained further many cases of erythropsia, chloropia and so forth and finally perhaps color sense disturbances due to hysteria and so forth."

This view had been favored for some years by v. Kries‡ and was adopted later by Adams[1] and G. E. Müller.[47,48] Lest it appear that the concise account of dichromatic and normal trichromatic vision afforded by the functions W_d, W_p, and K establish the correctness of the three-components

* Reference 12, p. 13.

* Reference 12, p. 14.
† Reference 38, p. 730; reference 38a, p. 414.
‡ Reference 39, p. 269.

theory of vision according to König and Granit, and render further theorizing useless, two of the shortcomings of this form of three-components theory may be mentioned, and the way the zone theory of G. E. Müller overcomes these deficiencies may be indicated.

In the long wave end stretch of the spectrum from 700 to 770 mμ the chromaticity is constant; that is, any part of this region of the spectrum can be matched perfectly for a normal observer by any other part provided only that the luminances of the two be equalized. This constancy of chromaticity is indicated in the standard I.C.I. coordinate system by the fact that X and Y bear the same ratio over this wave-length range, and the same thing is indicated from the functions W_d, W_p, and K from the fact that W_d and W_p also bear a constant ratio over the same range. If, as in König's later view, we regard W_d and W_p as the spectral distributions of the fundamental red and green sensations, the question arises, "What could account for the fact that while varying by a factor of 100 these two distributions still manage to bear exactly the same ratio?" There is only one answer at all plausible; the only way we would expect two processes to bear a constant ratio is that they are both functions of some other single process. Perhaps, for example, W_d and W_p are proportional to excitation of the fibers of the optic nerve in the red and green sense, resulting from receptors (retinal cones, say) containing preponderantly a red-sensitive and a green-sensitive substance, respectively. If the green-sensitive substance were entirely insensitive for wave-lengths greater than 700 mμ, that is, suppose its sensitivity curve corresponded to W_p (see Fig. 4), the explanation for constant chromaticity for wave-lengths longer than 700 mμ would be very simple and believable; that is, only one photosensitive substance, the red sensitive, is affected by radiant energy of this wave-length range. The dependence of the fundamental red sensation upon this kind of radiant energy is customarily ascribed to leakage of a small fairly constant fraction of the green-sensitive substance into the predominantly red-sensitive cones. Thus it is seen that the functions W_d, W_p, and K though they give a concise account of the connection between normal and dichromatic vision at the level of, perhaps, the fibers of the optic nerve, do not suffice to explain plausibly the constant chromaticity of the long wave end stretch of the spectrum for the normal observer. To provide a satisfactory explanation these functions have to be supplemented by another three-components system referring to the photosensitive substances, themselves.

Although formulation of normal and dichromatic vision in terms of the functions W_d, W_p, and K gives a clear and satisfactory definition of the samples of radiant energy confused both by an average normal observer and average dichromatic observers, it does not provide any mechanism to explain the usual sensations. That is, these functions describe what the observers confuse but not what they see. Thus for the normal we have the fundamental sensations of red, green, and violet in equal amount summing to neutral. But when the red sensation is missing as in protanopia instead of having green and violet left, we find that blue and yellow are seen. Similarly when the green sensation is missing as in deuteranopia or as in certain extra-foveal portions of the normal retina, we see blue and yellow rather than red and violet as might seem reasonable from the formulation of deuteranopia from W_d and K taken without W_p. And finally, when both green and violet are missing, we see neutral colors only as in acquired total color-blindness, rather than the red which might be expected. This failure of the three-components theory to indicate correctly the sensations derivable from the various presumed combinations of the fundamental sensations has been pointed out innumerable times, chiefly by psychologists, who could scarcely be expected to be satisfied with a formulation giving false implications of visual experience even though it did predict correctly which samples of radiant energy are found to be identical in appearance. This failure led to the Hering and Ladd-Franklin theories of vision, and set König, Donders[5] and v. Kries to thinking about higher zones in the color apparatus. This lack is also felt by Granit who remarks,* "The experiments with the cone-eye of the snake suggested that the dominator itself is composed of modulators joined together in such a fashion—either photochemically or by

* Reference 12, p. 13.

connections in the retinal synapses—as to operate as a *functional* unit. This assumption . . . would explain why stimulation of all modulators together also causes an impression of white, and not of all colours confused. . . . Alternatively, the modulators could be coupled in antagonistic pairs which simultaneously neutralized each other at the retinal or some higher level." The first suggestion accords closely with the Adams theory[1,2] which is probably the first to follow explicitly the dominator-modulator form. The latter suggestion corresponds somewhat to the zone theory of G. E. Müller[47] which by 1924 had been elaborated so as to deal satisfactorily in a qualitative way with all known forms of dichromatic and monochromatic vision.

C. Zone Theory of G. E. Müller

The Müller theory includes three separate color systems, one in the zone of the initial photosensitive substances, a second in the zone of sensory retinal processes aroused by action of the initial photosensitive substances, and a third in the zone of excitations of optic-nerve fibers. The first color system has red, green and violet primaries (R, G, V) and accords with the classical Young-Helmholtz theory. The second system includes two pairs of antagonistic chromatic processes: yellowish red-greenish blue (yR-gB), and greenish yellow-reddish blue (gY-rB). The first named of each pair has a lightening effect; the last named, a darkening effect. And, finally, the third system includes two pairs of antagonistic chromatic excitations and one non-antagonistic pair of achromatic excitations: red-green (r-g), yellow-blue (y-b) and white-black (w-s). White does not cancel black, but combines with it to give gray.

Failure of the yR-gB process corresponds to protanopia. Yellowish red and greenish blue are the hues of the colors confused by the protanope with gray. Since the normal lightening effect of the yR process is missing the theory indicates that the luminosity function of the protanope is shifted toward the short wave end, which corresponds to the facts.

Failure of the gY-rB process corresponds to tritanopia. Greenish yellow and reddish blue are the hues of the colors confused by the tritanope with gray. Since the normal lightening effect of the gY process is missing the theory indicates that the luminosity function of the tritanope is higher than normal at the short wave end. The few data available[7,8,33,34] are inconclusive concerning this indicated deviation from the normal luminosity function.

Failure of the r-g excitation corresponds to deuteranopia. Purplish red and green are the hues of the colors confused by the deuteranope with gray. The theory indicates that the deuteranopic luminosity function corresponds to the normal, which is the same indication given by the functions W_d, W_p, and K [see Eq. (11)].

If, as seems likely, it is possible to quantify the Müller theory so as to make it embody the same dichromatic confusion colors as those indicated by the functions W_d, W_p, and K, a much more complete account of visual phenomena would be so achieved. The shortcomings of the König-Granit three-components theory outlined above would be overcome. One could then take his choice between a single color system yielding conveniently all of the dichromatic color confusions, or a three-system theory yielding the same confusions together with a prediction of dichromatic sensations.

The major theoretical point of difference between these two views lies in the account of deuteranopia. By the König-Granit theory, deuteranopia and protanopia are retinal defects of similar character. By the Müller theory, deuteranopia is a defect of the optic-nerve fibers; protanopia and tritanopia, defects of the cones. Though there is no conclusive evidence pointing to which, if either, view is correct, it may be significant first that diseases of the nerve-fiber layer of the retinal optic nerve and tract always produce the deuteranopic form of red-green blindness[33] never the protanopic form, and second, there have been several observers found who have tritanopia or protanopia in one eye[4,19,20,21] and normal vision in the other, but never a case of congenital monocular deuteranopia.*

VIII. SUMMARY

There have been derived, chiefly from the work of Pitt, three functions of wave-length which permit easy definition of the samples of radiant energy found to be identical (metameric)

* Reference 4, p. 78.

not only by the I.C.I. standard observer but also by the two most common types of red-green-blind observers, protanopes and deuteranopes. These three functions W_d, W_p, and K may be taken together to indicate the metamers of the normal observer; W_d and K, taken together indicate the metamers of the average deuteranopic observer; and W_p and K, those of the average protanopic observer. The functions W_d and W_p taken together also yield a close approximation to the less well determined metamers of the average tritanopic observer. Methods of using these functions to derive the variation in wave-length of neutral point for dichromatic observers with choice of neutral source are given together with a convenient graphical solution of the ratio of protanopic reflectance to normal reflectance as a function of normal chromaticity. A derivation of the average protanopic, deuteranopic, and tritanopic responses to the Farnsworth dichotomous test for color blindness is also given by way of these three functions. The functions W_d, W_p, and K, therefore, serve to make conveniently accessible the color confusions of dichromatic observers. These three functions also conform to the type of three-components color theory outlined in 1897 by Arthur König, and they are consistent with the dominator-modulator theory proposed by Granit in 1943. They do not, however, form the basis of a complete theory of vision such as that of G. E. Müller.

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Letters to the Editor

Standard Response Functions for Protanopic and Deuteranopic Vision

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IT has been suggested by Le Grand[1] that the standard response functions, proposed by me[2] to represent protanopic and deuteranopic vision, introduce contradictions in the calculation of luminous quantities because two of the three primaries are unassociated with luminosity. He points out that if we explain protanopia by assuming that the primary carrying all the luminosity is suppressed, this would seem to require the observer to be not merely red-green blind, but completely blind. He recommends that another coordinate system, equally or perhaps better in accord with the color confusions of deuteranopes and protanopes, whose W_p process has a luminosity coefficient of about 13 percent, be used "in order to avoid contradictions."

Leaving out of account possible deviations from the law of additivity of luminances on which photometry is based (a law by no means certainly established),[3] a consistent photometric scale may be derived by computation based on the luminosity function. If the luminosity function is correct for the observer, he will make photometric matches that do not contradict the computed values. Since the standard luminosity function, and those proposed by me for protanopic and deuteranopic vision alike, all fail precisely to represent any actual observer, the use of the scales resting on these functions all will disclose contradictions, some not at all negligible. The validity of the standard scale of luminance is simply of an arbitrary sort based on definition. The usefulness of the definition depends upon the fact that the standard luminosity function is intermediate to those of observers having normal vision. The usefulness of the luminosity functions proposed by me to represent dichromatic vision, and that suggested by Le Grand alike, also arises from the fact that they all lie intermediate to the luminosity functions of observers having the corresponding type of vision. No contradictions will be avoided by using the less convenient function proposed by Le Grand.

It might seem from the foregoing that there is no point to the note written by Le Grand; but that is not true either. The question of how the luminosity process in protanopic eyes manages to be transferred from receptor cells responding in accord with the normal luminosity function, to those responding in accord with the protanopic luminosity function, is a difficulty in visual theories assuming separate receptors for the luminous and chromatic aspects of the response (the so-called dominator-modulator type of theory suggested by the researches of Granit).[4] In addition to the suppression of the process yielding the normal luminosity function, it is necessary to assume that the W_p process takes over the role of dominator, in order to account for protanopic vision. This is an *ad hoc* assumption detracting from the plausibility of this particular form of dominator-modulator theory.

If, however, we take instead of the normal luminosity function, W_d, to apply to deuteranopic vision, the function, W_d', preferred by Le Grand, no such *ad hoc* assumption is required. The suppression of the W_d' function leaves the system with 13 percent of its normal luminosity distributed correctly according to the protanopic luminosity functions determined experimentally. This does, indeed, remove the difficulty spoken of by Le Grand as a "contradiction."

The difference between the two luminosity functions, W_d and W_d' is much smaller than the spread of luminosity functions for normal observers. My view is that the question will only be settled by measuring the luminosity functions of hundreds of normal and deuteranopic observers under identical conditions, and the application of careful statistical treatment to the results. It may be that there is a significant difference between the normal and deuteranopic luminosity functions, but until this is proven the computationally simpler choice, W_d, may as well be used.

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THE COLOR PERCEPTIONS OF DEUTERANOPIC AND PROTANOPIC OBSERVERS

J. Opt. Soc. Am. 39, 252-256 (1949)

This article provides an unambiguous general basis for design and production of tests for protanopia and deuteranopia. However, to conserve space, the original publication and this reproduction of it greatly abridged the tables that resulted from the work reported. The complete tables are in an article that has the same title in J. Res. Nat. Bur. Stand. (U.S.), 41, 247-271 (1949) RP1922.

The Color Perceptions of Deuteranopic and Protanopic Observers

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It is well established that about 2 percent of otherwise normal human males are from birth confusers of red and green. There is considerable interest in the question: What do red-green confusers see? From a knowledge of the normal color perceptions corresponding to deuteranopic and protanopic red and green, we may not only understand better why color-blindness tests sometimes fail, and so be in a position to develop improved tests, but also the color-deficient observer may understand better the nature of his color-confusions and be aided to avoid their consequences. If an observer has normal trichromatic vision over a portion of his total retinal area and dichromatic vision over another portion, he may give valid testimony regarding the color perceptions characteristic of the particular form of dichromatic vision possessed by him. Preeminent among such observers are those born with one normal eye and one dichromatic eye. A review of the rather considerable literature on this subject shows that the color perceptions of both protanopic and deuteranopic observers are confined to two hues, yellow and blue, closely like those perceived under usual conditions in the spectrum at 575 mμ and 470 mμ, respectively, by normal observers. By combining this result with standard response functions recently derived for protanopic and deuteranopic vision, it has been possible to give quantitative estimates of the color perceptions typical of these observers for the whole range of colors in the Munsell Book of Color. These estimates take the form of protanopic and deuteranopic Munsell notations, and by using them it is possible not only to arrange the Munsell papers in ways which presumably appear orderly to red-green confusing dichromats but also to get immediately from the notations an accurate idea of the colors usually perceived in these arrangements by deuteranopes and protanopes much as the ordinary Munsell notations serve to describe the visual color perceptions of a normal observer.

I. INTRODUCTION

THE question, "What colors do color blind observers confuse?" is a very practical one, capable of objective solution by putting each color-blind observer to trial and noting his mistakes. This question has been given a fairly satisfactory general answer for protanopic and deuteranopic observers. The question, "What colors do color blind observers see?" is a more subtle one, involving a fine point in the theory of knowledge arising out of the fact that there is no common color language between the color blind and the normal eyed. Yet the essential facts have been ferreted out. It is one purpose of the present paper to review briefly the evidence by which these color perceptions have become qualitatively known. A second purpose is to indicate the basis of a comprehensive intertranslation, which is being published elsewhere,[1] between normal perceptions of surface colors on the one hand and protanopic and deuteranopic surface-color perceptions on the other. This intertranslation provides a method of expressing the color perceptions of the average red-green confuser in terms that are immediately comprehensible both to those trained in the interpretation of colorimetric coordinate systems and to the untrained alike. For most of the tabular and bibliographic material, reference must be had to the complete account.[1]

II. REVIEW OF THE LITERATURE

Persons born with one normal eye and one color-blind eye give us our most direct evidence of the color perceptions of the colorblind. Less reliable evidence may be obtained from persons who confuse red with green because of diseases of the eyes or optic nerves and who are therefore familiar with normal colors through past experience before becoming afflicted. And, finally, some information may be obtained from the peripheral parts of the normal retina which respond with confusions similar to those of deuteranopia. Any observer a portion of whose retinas yield normal color vision while another portion yields either the kind of red-green confusion characteristic of protanopia or that characteristic of deuteranopia can give valid testimony.

It is now a fairly well accepted view that red-green confusers see neutral colors normally and see chromatic colors of two hues approximately what normal observers call pure yellow and pure blue with little of an admixture of red and green. The literature has been reviewed to see whether this rough indication can be made more precise.

It is well known that the normal retina has a zone (perhaps 20 to 50°) from the fovea within which yellow, blue, black, and white are perceived much as at the fovea, but red-green distinctions are scarcely, if at all, possible. The luminosity function characterizing this zone is substantially normal, and therefore the properties of the zone approach those of deuteranopia. The precise hues of the yellowish and bluish colors seen by means of this zone are hard to determine. However, results by Hess, Baird, Dreher, and Goldman indicate that the yellow hue seen in the periphery corresponds to that of a spectrum stimulus of 567 to 575 mμ viewed foveally under customary observing conditions, and the blue hue corresponds to 460 to 471 mμ.

[1] D. B. Judd, "The color perceptions of deuteranopic and protanopic observers," J. Research Nat. Bur. of Stand. 41, 247 (1948); RP1922.

Toxic agents and disease affecting the conducting elements of the visual mechanism (nerve elements and connections, optic nerve, and tract) cause a progressive lessening of the ability to distinguish red from green. The luminosity function is not essentially different from normal. Progressive red-green blindness therefore also corresponds to deuteranopia. According to Köllner,[2] a patient in the dichromatic state sees only yellow and blue in the spectrum, the yellow corresponding to the hue elicited in the normal eye by light of wave-length 575 mμ, the blue to the hue corresponding to that of wave-length 471 mμ.

A review of 37 cases of unilateral defects in color vision reported in the literature shows nine reports (Hippel, Holmgren, Hering, Hess, Nagel, Hayes, v. Kries, Goldschmidt, and Sloan) giving quantitative information regarding the hues perceived by protanopic and deuteranopic observers. The yellow hues reported correspond to spectrum stimuli ranging between 573 and 584 mμ; the blues range between 452 and 480 mμ. There is no significant difference between the protanopic and deuteranopic reports of hue. It is concluded that the wave-lengths 575 mμ and 470 mμ, respectively, represent these reports with no indication of significant individual variations. The most divergent report[3] of 584 and 452 mμ, respectively, was shown by auxiliary studies by means of Munsell papers[3] to be ascribable to an abnormally heavy ocular pigmentation.

In summary, it may be stated that evidence of three separate kinds combines to indicate that the hues perceived by protanopic and deuteranopic observers correspond closely to those perceived by normal-sighted observers to belong to spectrum stimuli of wave-length 575 mμ (yellow) and 470 mμ (blue).

III. AGREEMENT WITH THEORIES OF COLOR VISION

The evidence just reviewed that red-green confusers see yellow and blue of a certain kind does not prove that all red-green confusers have these perceptions of hue. The absence of conflicting evidence merely renders the validity of the conclusion highly probable. It is pertinent, therefore, to trace the influence exerted on color theories by these relatively few reports.

According to the original simple form of the three-components theory (red, green, violet), red-green blindness must come from a failure of either the red component (protanopia), or the green component (deuteranopia), from which it must follow that protanopes must see only mixtures of green and violet, and deuteranopes, only mixtures of red and violet. Printing-ink reproductions of the colors of the spectrum perceived by these types of red-green confusers, according to this theoretical view, have been published by Abney. Largely because of the failure of this simple theory to give a good explanation of the colors perceived in the peripheral portions of the normal retina or of those reported by observers having unilateral defects, the original form of the three-component theory is now generally regarded as insufficient. The modifications proposed

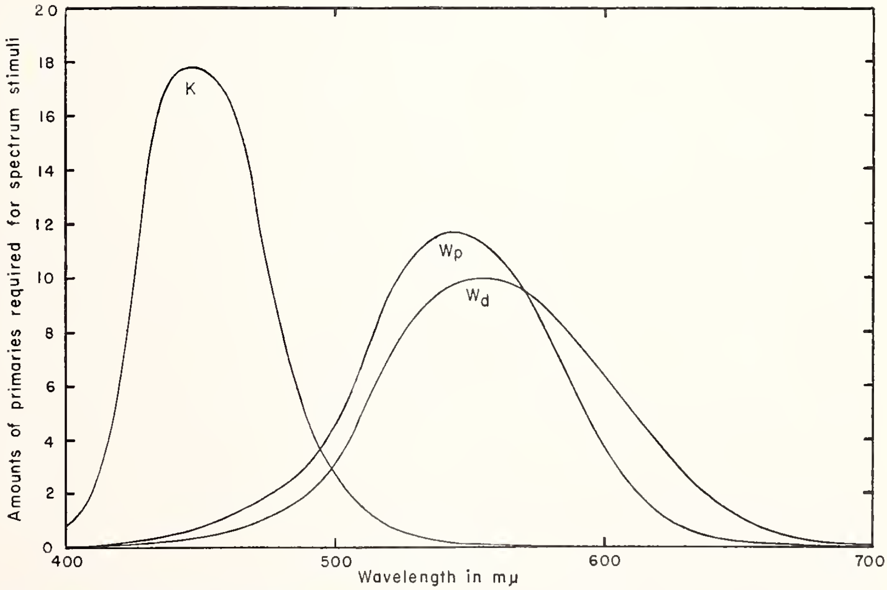

FIG. 1. Wave-length functions (K, W_p, W_d, see Eq. (1)) by means of which protanopic and deuteranopic red-green confusions may be computed. Any two colors having identical values of K and W_p are identical to the average protanope, and those having identical values of K and W_d are identical to the average deuteranope.

[2] H. Köllner, *Die Störungen des Farbensinnes* (Karger, Berlin, 1912).
[3] L. L. Sloan and L. Wollach, "Case of unilateral deuteranopia," J. Opt. Soc. Am. 38, 502 (1948).

TABLE I. Deuteranopic and protanopic chromaticity coordinates for Munsell renotations of hues 5Y and 5PB.

Munsell renotation	Chromaticity coordinates w_d	w_p	Munsell renotation	Chromaticity coordinates w_d	w_p
5Y 9/12	0.8510	0.8433	N 1/ to 9/	0.4585	0.4652
5Y 9/10	0.7911	0.7827			
5Y 9/8	0.7296	0.7218			
5Y 9/6	0.6628	0.6569			
5Y 9/4	0.5955	0.5929			
5Y 9/2	0.5291	0.5305	5PB 9/2	0.4362	0.4457
5Y 8/12	0.8805	0.8731			
5Y 8/10	0.8181	0.8093			
5Y 8/8	0.7494	0.7408			
5Y 8/6	0.6764	0.6696	5PB 8/6	0.3615	0.3810
5Y 8/4	0.6025	0.5993	5PB 8/4	0.3973	0.4119
5Y 8/2	0.5325	0.5335	5PB 8/2	0.4325	0.4425
5Y 7/12	0.9125	0.9061			
5Y 7/10	0.8552	0.8466	5PB 7/10	0.2927	0.3214
5Y 7/8	0.7788	0.7693	5PB 7/8	0.3246	0.3489
5Y 7/6	0.7007	0.6927	5PB 7/6	0.3570	0.3768
5Y 7/4	0.6184	0.6139	5PB 7/4	0.3913	0.4065
5Y 7/2	0.5379	0.5383	5PB 7/2	0.4272	0.4377
5Y 6/12	0.9425	0.9377	5PB 6/12	0.2507	0.2856
5Y 6/10	0.8935	0.8859	5PB 6/10	0.2804	0.3107
5y 6/8	0.8231	0.8134	5PB 6/8	0.3096	0.3354
5y 6/6	0.7346	0.7251	5PB 6/6	0.3426	0.3638
5Y 6/4	0.6373	0.6314	5PB 6/4	0.3823	0.3984
5Y 6/2	0.5471	0.5468	5PB 6/2	0.4208	0.4319
5Y 5/12	0.9903	0.9894	5PB 5/12	0.2299	0.2675
5Y 5/10	0.9336	0.9279	5PB 5/10	0.2577	0.2909
5Y 5/8	0.8655	0.8562	5PB 5/8	0.2891	0.3176
5Y 5/6	0.7783	0.7677	5PB 5/6	0.3242	0.3476
5Y 5/4	0.6667	0.6588	5PB 5/4	0.3662	0.3838
5Y 5/2	0.5569	0.5556	5PB 5/2	0.4106	0.4226
			5PB 4/12	0.2017	0.2426
			5PB 4/10	0.2282	0.2653
5Y 4/8	0.9153	0.9081	5PB 4/8	0.2596	0.2919
5Y 4/6	0.8200	0.8092	5PB 4/6	0.2997	0.3261
5Y 4/4	0.7061	0.6962	5PB 4/4	0.3442	0.3642
5Y 4/2	0.5774	0.5741	5PB 4/2	0.3956	0.4091
			5PB 3/12	0.1606	0.2065
			5PB 3/10	0.1886	0.2305
			5PB 3/8	0.2223	0.2594
5Y 3/6	0.8839	0.8745	5PB 3/6	0.2605	0.2919
5Y 3/4	0.7373	0.7261	5PB 3/4	0.3104	0.3346
5Y 3/2	0.5899	0.5854	5PB 3/2	0.3729	0.3890
			5PB 2/12	0.1213	0.1709
			5PB 2/10	0.1459	0.1925
			5PB 2/8	0.1793	0.2215
			5PB 2/6	0.2247	0.2604
5Y 2/4	0.8380	0.8266	5PB 2/4	0.2833	0.3107
5Y 2/2	0.6149	0.6182	5PB 2/2	0.3564	0.3740
			5PB 1/10	0.0998	0.1506
			5PB 1/8	0.1314	0.1785
			5PB 1/6	0.1739	0.2157
			5PB 1/4	0.2337	0.2671
5Y 1/2	0.7392	0.7267	5PB 1/2	0.3127	0.3355

by Fick and by Ladd-Franklin both provide an explanation for protanopic and deuteranopic vision in which the hues of the colors perceived are yellow and blue, respectively.

According to the opponent-colors theory proposed by Hering, there can be only one form of red-green confusion, that resulting from the failure of the red-green process. This leaves the yellow-blue and the black-white processes intact. The form thus predicted is deuteranopia, and the hues are indicated correctly.

More recent theories of vision take account of the possibility of regroupings of elementary processes at more than one stage of the visual mechanism and are called stage theories. The most prominent of these theories (v. Kries, Schrödinger, Adams, Müller) have the property in common that the final stage is very like the Hering "opponent-colors" formulation.

In summary, all theories of vision with any pretense of being complete provide for the perception of yellow and blue by red-green confusers.

IV. DERIVATION OF DEUTERANOPIC AND PROTANOPIC MUNSELL NOTATIONS

Since there is no experimental or theoretical evidence against the view that yellow and blue are the hues perceived by red-green confusers, it is proposed to build a method of designating protanopic and deuteranopic perceptions of surface colors on this basis. It has been shown[4] that the color confusions of red-green-blind observers can be found from three numbers (K, W_p, W_d) related to the tri-stimulus values (X, Y, Z) of the ICI standard observer by the transformation equations

$$\left.\begin{aligned} K &= Z \\ W_p &= -0.460X+1.359Y+0.101Z \\ W_d &= Y \end{aligned}\right\}. \quad (1)$$

Any two colors having identical values of K and W_p are identical to the average protanope; those having identical values of K and W_d are identical to the average deuteranope. Figure 1 shows in arbitrary units the values of K, W_p, and W_d for all parts of a spectrum of unit irradiance per unit wave-length.

For the ICI observer luminous directional reflectance is customarily expressed relative to that (Y_0) of magnesium oxide illuminated and viewed under geometrical conditions identical to those used for the specimen itself. By analogy we take for deuteranopic luminous directional reflectance

$$W_d/(W_d)_0 = Y/Y_0, \quad (2)$$

and for protanopic luminous directional reflectance

$$\frac{W_p}{(W_p)_0} = \frac{-0.460X+1.359Y+0.101Z}{-0.460X_0+1.359Y_0+0.101Z_0}. \quad (3)$$

But since for magnesium oxide illuminated by standard source C (representative of average day-

[4] D. B. Judd, "Standard response functions for protanopic and deuteranopic vision," J. Research Nat. Bur. of Stand. **33**, 407 (1944); RP1618; also J. Opt. Soc. Am. **35**, 199 (1945).

light) it is customary to take $X_0=0.9804$, $Y_0=1.0000$, and $Z_0=1.1512$, protanopic luminous directional reflectance for this illuminant may be written simply as $0.9733W_p$.

Similarly, by analogy to the customary practice of specifying the chromatic aspect of a color by chromaticity coordinates (x, y, z), we define deuteranopic chromaticity coordinates (w_d, k_d) as

$$\begin{aligned} w_d &\equiv W_d/(W_d+K)=Y/(Y+Z) \\ &=y/(y+z)=y/(1-x), \\ k_d &\equiv K/(W_d+K)=Z/(Y+Z)=z/(y+z) \\ &=(1-x-y)/(1-x). \end{aligned} \tag{4}$$

And we define protanopic chromaticity coordinates (w_p, k_p) as

$$\begin{aligned} w_p &\equiv \frac{W_p}{W_p+K}=\frac{-0.460X+1.359Y+0.101Z}{-0.460X+1.359Y+1.101Z} \\ &=\frac{-0.561x+1.258y+0.101}{-1.561x+0.258y+1.101}, \\ k_p &\equiv \frac{K}{W_p+K}=\frac{Z}{-0.460X+1.359Y+1.101Z} \\ &=\frac{1.000-x-y}{-1.561x+0.258y+1.101}. \end{aligned} \tag{5}$$

Since each pair of dichromatic coordinates sums to unity by definition, only one of each need be used. We accordingly select w_d and w_p as indications of the deuteranopic and protanopic "warm" quality, respectively. According to the data and views already summarized, it is legitimate to regard them more precisely as indications of "yellowness." Values of w_d and w_p together with the reflectance $W_d/(W_d)_0$, and $W_p/(W_p)_0$ have been computed from the known tristimulus values for all of the chips of the 1929 Munsell Book of Color[5] by means of Eqs. (2), (3), (4), and (5) (see complete paper[1]).

Evaluation of these quantities serves to indicate which chips of the Munsell Book of Color would be nearly of the same appearance when viewed by an average protanope or deuteranope, but they do not give an easily understood indication of the color perceptions themselves. The basic premises for which evidence has already been summarized are, however, that red-green confusers perceive black, gray, and white normally, that their yellow corresponds closely to what an observer having normal color vision sees in the spectrum at 575 mμ, and that their blue corresponds closely to 470 mμ. These facts may be conveniently expressed in the

TABLE II. Deuteranopic arrangement.

Munsell book notation	Deuteranopic reflectance $W_d=Y$	Deuteranopic chromaticity coordinate w_d	Munsell renotations: Deuteranopic	Munsell renotations: Normal
5Y 8 12	0.5706	0.8911	5Y 7.9 12.2	5.5Y 7.9 12.3
5Y 8 10	.5784	.8578	5Y 7.9 11.0	5.5Y 7.9 11.1
10Y 8 8	.6035	.7918	5Y 8.1 9.2	0.5GY 8.1 9.1
5Y 8 8	.5794	.7660	5Y 7.9 8.2	5.5Y 7.9 8.2
5Y 8 8	.5732	.7606	5Y 7.9 8.1	5.5Y 7.9 8.3
5GY 8 8	.6022	.7464	5Y 8.1 7.9	4.5GY 8.1 3.7
10YR 8 8	.6049	.7064	5Y 8.1 6.9	0.5Y 8.1 7.6
5Y 8 6	.5784	.6681	5Y 7.9 5.7	4.5Y 7.9 5.8
10Y 8 6	.5996	.6664	5Y 8.0 5.7	0.5GY 8.0 5.7
5GY 8 6	.5929	.6605	5Y 8.0 5.5	5.5GY 8.0 6.2
10YR 8 6	.6114	.6263	5Y 8.1 4.7	9.0YR 8.1 5.5
5Y 8 4	.5651	.5851	5Y 7.8 3.4	4.0Y 7.8 3.5
10Y 8 4	.5997	.5827	5Y 8.0 3.4	1.0GY 8.0 3.4
10GY 8 6	.5793	.5720	5Y 7.9 3.1	9.5GY 7.9 5.1
5GY 8 4	.6041	.5719	5Y 8.1 3.2	6.0GY 8.1 3.6
10YR 8 4	.5972	.5579	5Y 8.0 2.8	9.0YR 8.0 3.4
10GY 8 4	.6181	.5406	5Y 8.2 2.3	0.5G 8.2 4.0
5YR 8 4	.5970	.5311	5Y 8.0 2.0	4.0YR 8.0 3.5
10R 8 4	.6115	.5156	5Y 8.1 1.6	10.0R 8.1 3.7
5Y 8 2	.5570	.5141	5Y 7.8 1.5	3.5Y 7.8 1.6
5GY 8 2	.5851	.5107	5Y 8.0 1.5	6.0GY 8.0 1.6
5G 8 6	.5595	.4964	5Y 7.8 1.1	6.5G 7.8 4.7
5YR 8 2	.5917	.4949	5Y 8.0 1.0	4.0YR 8.0 2.0
5G 8 4	.5881	.4899	5Y 8.0 0.9	5.5G 8.0 3.3
5G 8 2	.5694	.4847	5Y 7.9 0.8	4.5G 7.9 2.1
5R 8 4	.5862	.4834	5Y 8.0 0.7	3.5R 8.0 3.6
10G 8 2	.5965	.4806	5Y 8.0 0.6	5.0G 8.0 2.0
N 8	.5751	.4700	5Y 7.9 0.4	2.5GY 7.9 0.3
5R 8 2	.6021	.4699	5Y 8.1 0.4	2.0R 8.1 2.2
10RP 8 6	.6251	.4693	5Y 8.2 0.4	8.5RP 8.2 3.6
10RP 8 4	.6107	.4678	5Y 8.1 0.3	9.5RP 8.1 2.7
5BG 8 2	.6082	.4656	5Y 8.1 0.3	1.0BG 8.1 2.0
5RP 8 2	.6435	.4633	5Y 8.3 0.2	8.0RP 8.3 1.9
5RP 8 6	.6058	.4611	5Y 8.1 0.1	6.5RP 8.1 3.4
5RP 8 4	.6050	.4588	5Y 8.1 0.1	5.5RP 8.1 2.8
10BG 8 2	.6258	.4557	5PB 8.2 0.4	5.0BG 8.2 1.9
5B 8 2	.6179	.4361	5PB 8.2 1.7	4.0B 8.2 2.2
5P 8 2	.5897	.4330	5PB 8.0 1.8	5.0P 8.0 2.5
10P 8 4	.6182	.4289	5PB 8.2 2.2	7.5P 8.2 3.5
5B 8 4	.6025	.4272	5PB 8.1 2.3	4.0B 8.1 2.9
10B 8 2	.6514	.4170	5PB 8.3 3.1	1.0PB 8.3 3.0
5PB 8 2	.6380	.4139	5PB 8.3 3.2	5.0PB 8.3 3.2
10PB 8 2	.6499	.4099	5PB 8.3 3.4	7.5PB 8.3 3.4
P 8 4	.5940	.4004	5PB 8.0 3.9	4.5P 8.0 4.8

(For continuation of this table see reference 1.)

Munsell color system by giving for each set of values of W/W_0 and w the Munsell equivalent of the purple-blue (5PB) or yellow (5Y) hue; that is, we identify each color perceived by a red-green confuser by giving the Munsell notation of the color of Munsell hue 5Y or 5PB with which he would confuse it. These Munsell hues correspond closely to those of the spectrum at 470 and 575 mμ, respectively, and since the exact hues corresponding to these portions of the spectrum depend somewhat on the observing conditions, more precise fitting (as by 6PB and 7Y) is probably not warranted.

The deuteranopic and protanopic Munsell renotations based in this way resulted from the following steps: (1) From Table I of the OSA subcommittee report on the spacing of the Munsell colors[6] were read the chromaticity coordinates (x, y) for enough colors of the hues 5Y and 5PB to cover the range of the Munsell papers. (2) From

[5] *Munsell Book of Color* (Library and Pocket Editions) (Munsell Color Company, Baltimore, 1929).

Table III. Protanopic arrangement.

Munsell book notation	Protanopic reflectance $0.9733\ W_p$	Protanopic chromaticity coordinate w_p	Munsell renotations Protanopic	Munsell renotations Normal
10BG 8/2	0.6464	0.4705	5Y 8.3/0.2	5.0BG 8.2/1.9
5B 8/2	.6391	.4511	5PB 8.3/1.3	4.0B 8.2/2.2
10B 8/2	.6732	.4317	5PB 8.4/2.8	1.0PB 8.3/3.0
5PB 8/2	.6543	.4267	5PB 8.3/3.2	5.0PB 8.3/3.2
10PB 8/2	.6608	.4205	5PB 8.4/3.5	7.5PB 8.3/3.4
10Y 8/8	.5773	.7889	5Y 7.9/9.3	0.5GY 8.1/9.1
5GY 8/8	.5991	.7505	5Y 8.0/8.3	4.5GY 8.1/8.7
5GY 8/6	.5914	.6659	5Y 8.0/5.9	5.5GY 8.0/6.2
10Y 8/6	.5817	.6657	5Y 8.0/5.8	0.5GY 8.0/5.7
5Y 8/6	.5451	.6609	5Y 7.7/5.7	4.5Y 7.9/5.8
10YR 8/6	.5632	.6134	5Y 7.8/4.3	9.0YR 8.1/5.5
10GY 8/6	.5987	.5866	5Y 8.0/3.6	9.5GY 7.9/5.1
10Y 8/4	.5873	.5842	5Y 8.0/3.5	1.0GY 8.0/3.4
5GY 8/4	.6044	.5786	5Y 8.1/3.4	6.0GY 8.1/3.6
10GY 8/4	.6362	.5545	5Y 8.2/2.7	0.5G 8.2/4.0
10YR 8/4	.5656	.5512	5Y 7.9/2.5	9.0YR 8.0/3.4
5YR 8/4	.5614	.5225	5Y 7.8/1.6	4.0YR 8.0/3.5
5G 8/6	.5954	.5187	5Y 8.0/1.5	6.5G 7.8/4.7
5GY 8/2	.5851	.5174	5Y 8.0/1.5	6.0GY 8.0/1.6
5G 8/4	.6141	.5075	5Y 8.1/1.3	5.5G 8.0/3.3
10R 8/4	.5725	.5059	5Y 7.9/1.2	10.0R 8.1/3.7
5G 8/2	.5854	.4984	5Y 8.0/0.9	4.5G 7.9/2.1
10G 8/2	.6133	.4943	5Y 8.1/0.8	5.0G 8.0/2.0
5YR 8/2	.5731	.4937	5Y 7.9/0.8	4.0YR 8.0/2.0
5BG 8/2	.6281	.4803	5Y 8.2/0.5	1.0BG 8.1/2.0
5R 8/4	.5565	.4772	5Y 7.8/0.4	3.5R 8.0/3.6
N 8/	.5741	.4764	5Y 7.9/0.3	2.5GY 7.9/0.3
5R 8/2	.5857	.4698	5Y 8.0/0.2	2.0R 8.1/2.2
5RP 8/2	.6317	.4654	N 8.2/	8.0RP 8.3/1.9
10RP 8/4	.5905	.4662	N 8.0/	9.5RP 8.1/2.7
10RP 8/6	.5958	.4641	5PB 8.0/0.1	8.5RP 8.2/3.6
5RP 8/6	.5839	.4586	5PB 8.0/0.6	6.5RP 8.1/3.4
5RP 8/4	.5874	.4582	5PB 8.0/0.7	5.5RP 8.1/2.8
5B 8/4	.6310	.4452	5PB 8.2/1.8	4.0B 8.1/2.9
5P 8/2	.5877	.4388	5PB 8.0/2.2	5.0P 8.0/2.5
10P 8/4	.6081	.4316	5PB 8.1/2.8	7.5P 8.2/3.5
5P 8/4	.5903	.4054	5PB 8.0/4.5	4.5P 8.0/4.8
5Y 8/12	.5208	.8848	5Y 7.6/11.8	5.5Y 7.9/12.3
5Y 8/10	.5292	.8501	5Y 7.6/10.8	5.5Y 7.9/11.1
5Y 8/8	.5393	.7579	5Y 7.7/8.3	5.5Y 7.9/8.3
5Y 8/8	.5339	.7525	5Y 7.7/8.2	5.5Y 7.9/8.2
10YR 8/8	.5451	.6902	5Y 7.7/6.4	0.5Y 8.1/7.6
5Y 8/4	.5426	.5818	5Y 7.7/3.4	4.0Y 7.8/3.5
5Y 8/2	.5455	.5156	5Y 7.7/1.5	3.5Y 7.8/1.6

(For continuation of this table see reference 1.)

these chromaticity coordinates the dichromatic coordinates w_d and w_p were computed, according to Eqs. (4) and (5). (3) Families of curves were then plotted from these data on each of two graphs, one with w_d as the abscissa, the other with w_p as the abscissa, the ordinate being Munsell chroma, each curve then showing the variation of Munsell chroma for hue 5Y or 5PB with dichromatic coordinate (w_d or w_p) for some one Munsell value. (4) From the reflectance relative to magnesium oxide, W/W_0, (found from Eqs. (2) and (3)) the Munsell value was read from Table II of the subcommittee report.[6] (5) Finally, by interpolation among the curves of the corresponding family according to this Munsell value, the Munsell hue (whether 5Y or 5PB) was determined, and the Munsell chroma corresponding to the dichromatic coordinate, w_d or w_p, was read with an uncertainty of about 0.1 chroma step.

Munsell dichromatic renotations (hue value/chroma) were found in this way for the chips (about 400) of the 1929 Munsell Book of Color. Table I shows the chromaticity coordinates w_d and w_p obtained in step (b); and Tables II and III show the Munsell dichromatic renotations for the chips of value 8/ compared to the normal renotations. Similar data for other Munsell values are given in the complete paper.[1] Comparison of the dichromatic Munsell renotations with the corresponding normal Munsell renotation indicates in a precise and detailed way the difference between the surface-color perceptions of the normal observer and those of the average deuteranope and average protanope, respectively.

It is suggested that persons who have been found to have either protanopic or deuteranopic vision by the usual tests might find it interesting and possibly instructive to lay out the Munsell papers in the order given in Tables II or III, as the case may be, and see to what degree the Munsell dichromatic notation accords with their own perceptions. To the extent that the accord is good, the person will discover how well his type of vision is known, and he may be encouraged to study the basis of the Munsell dichromatic notation and so gain an additional insight into the relation of his own vision to that possessed by the majority. If the Munsell dichromatic notation fails to accord with his perceptions in ways not to be corrected by viewing the papers through bluish or brownish goggles, it is likely that the observer will find that he has been mistyped by careless administration or interpretation of a routine test, and a retest would be in order. Should the observer find that he is indeed a red-green confusing dichromat, he would have a chance to contribute to the knowledge of color perceptions by studying and reporting the nature and degree of any contradictions.

[6] S. M. Newhall, D. Nickerson, and D. B. Judd, "Final report of the O.S.A. subcommittee on the spacing of the Munsell colors," J. Opt. Soc. Am. **33**, 385 (1943).

RESPONSE FUNCTIONS FOR TYPES OF VISION ACCORDING TO THE MÜLLER THEORY

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This is the definitive publication of Judd's version of the Müller zone theory of color vision. In Doc. Ophthalmol. 3, 251-288 (1949), the third of Eqs. (9) on p. 270 is a repetition of the second equation; the correct third equation is missing. It should be the third of Eqs. (1) on p. 4 of the present paper.

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Response Functions for Types of Vision According to the Müller Theory

By Deane B. Judd

According to the Müller theory of vision there are three stages in the visual process, an initial photochemical stage, an intermediate chemical stage relating to the chromatic aspect, and a final stage of excitations of the optic-nerve fibers. By taking advantage of recent precise information regarding the metamers characteristic of normal, protanopic, and deuteranopic vision there have been derived the spectral variations of the responses for each stage as functions of wavelength. These response functions account precisely for the same normal metamers as the ICI standard observer, and closely for the same confusions by color-blind observers as the simpler König theory. Furthermore these functions describe chromatic thresholds of the normal eye (Abney, Priest-Brickwedde) as a gradual approach to tritanopic vision as field size and luminance are decreased.

I. Introduction

In the ninteenth century, two rival theories of vision monopolized most of the interest of investigators. One of these is the Young-Helmholtz three-components formulation; the other is the Hering opponent-colors theory. The discovery of the facts of red-green blindness dealt fatal blows to the then current forms of both of these simple theories, though proponents of the respective theories continued to pump a semblance of life into them with wordy battles. The opponent-colors theory in its original simple form can be made to yield but a single form of red-green blindness, that known as deuteranopia. It must overlook the established fact of a second type of red-green blindness, protanopia, in which the luminosity function is deficient in the long-wave portion of the spectrum and in which the chromaticity confusions are consistently different from those of deuteranopia. The three-components theory explains the confusions made by both types perfectly but in its original simple form has to predict that deuteranopic vision consists of mixtures of red and violet and protanopic vision consists of mixtures of green and violet. When cases of unilateral red-green blindness showed consistently that the perceptions of red-green-blind observers have the hues blue and yellow and no others, the original simple three-components formulation became obsolete. Some advocates of this simple theory took refuge in a suggestion by Fick [1] [1] that red-green confusion is the result, not of the nonfunctioning of either the red or the green receptor system, but rather of the two receptor systems having identical photosensitive substances, either that for red (deuteranopia) or that for green (protanopia). By this suggestion, the responses from the red cones combine with those from the green, regardless of the photosensitive substance in either, to give yellow. This combination can take place in the postretinal portion of the nervous system, as emphasized by Hecht [2], for binocular fusion of colors, and it is permissible to assume that it always takes place there even in binary stimulation of one eye alone. From this view it is only a step to the theoretical position originally proposed by Donders [3] and later espoused by König [4], von Kries [5], and Adams [6] that the three-components formulation holds for processes in one stage of the visual mechanism (perhaps the photosensitive-substance stage), while the opponent-colors theory holds for

[1] Figures in brackets indicate the literature references at the end of this paper.

processes in a later stage (perhaps the optic nerve). This view may be called the stage or "zone" theory of vision. Furthermore, a very able advocate of the opponent-colors theory, G. E. Müller, adopted a theoretical view [7] that, although divergent in detail and elaborated to include an additional stage, was essentially in agreement with the stage theories favored by Donders, König, von Kries, and Adams.

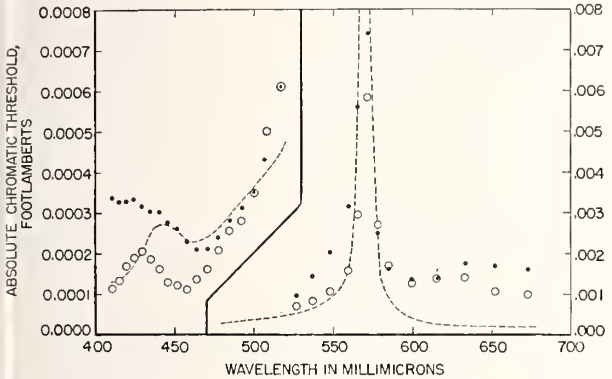

FIGURE 1. *Absolute chromatic thresholds for homogeneous light as a function of wavelength.*

The dotted curve is based on the Müller theory; see section VII. ●, Observed Abney; ○, observed Watson; — — — —, $\frac{0.0020\ (w-s)}{[(yR)^2+(0.04gY)^2]^{1/2}}$.

It was shown by Abney [8] in 1910, by Priest and Brickwedde [9] in 1926, by Guild [10] in 1928, by Holmes [11] in 1941, by MacAdam [12] in 1942, and probably by others, that the nearly achromatic color of noon sunlight is more confusible with the greenish yellow color of spectrum light at 570 mμ by a normal observer than with any other nearby portion of the spectrum, definitely more than with the yellow portion (575 to 585 mμ). Figure 1 shows the brightness in foot lamberts found by Abney to be required to produce the perception of a chromatic color noticeably distinct from the achromatic color of light from the carbon arc. Figure 2 shows the Priest-Brickwedde determination of minimum perceptible colorimetric purity. In both of these figures the maximum near 570 mμ is outstanding. Similar results were found by Guild, Holmes, and MacAdam. This outstanding maximum might suggest that pigmentation of the eye media of the normal eye absorbs a large fraction of the short-wave (violet) portion of the spectrum, or for small fields it could mean that because of the chromatic aberration of the eye, the short-wave portion of the sunlight spectrum is out of focus and largely lost. But the most likely explanation is that the normal eye, at least in the fovea, has some of the characteristics of a tritanopic eye; a tritanope has a neutral point in the spectrum near 570 mμ where the normal observer has this quasi-neutral point. Furthermore, for very small fields subtending 20′ or less, it has been shown by König [4], Willmer [13], Hartridge [15], and Wright [14] that the fovea is tritanopic.

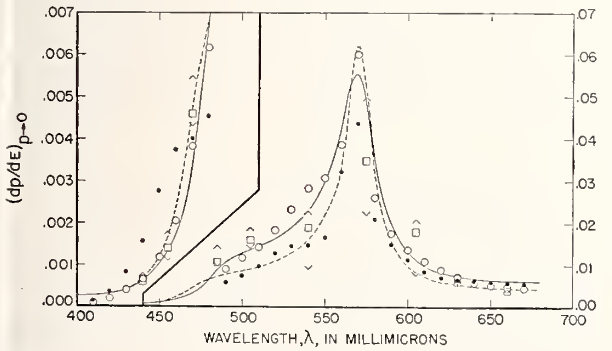

FIGURE 2. *Least fraction of spectrum light detectable as a chromatic difference in a mixture with sunlight, 2° observing field.*

The solid line is an empirical representation of these data based on the uniform-chromaticity-scale triangle (Judd); the dotted curve is based on the Müller theory; see section VII.

○, Mean observed Priest; ●, mean observed Brickwedde; ∧, □, ∨, observed Purdy; ————, from triangle; — — — —, $\frac{0.05\ (w-s)}{[(yR)^2+(0.5gY)^2]^{1/2}}$.

In an attempt to describe the chromaticity sensibility of the normal observer in terms of an approach to tritanopia, Judd [16] derived a transformation of the OSA "excitations" corresponding to an 80-percent dilution of the violet excitation with red and green. Figure 3 shows the resulting excitation curves and Maxwell triangle, and Figure 4 shows how this formulation corresponds with Priest's data on minimum perceptible colorimetric purity. This formulation corresponds to a theoretical suggestion similar to Fick's proposal to account for red-green confusion; it suggests that in the fovea the red and green substance from the red and green cones has leaked into the violet cones to a serious degree (80% leakage). A similar degree of success was demonstrated by Hecht [17] in another development of the Young-Helmholtz theory. Both explantions suffer, however, from a failure to permit an account of dichromatic vision, as do various coordinate systems empirically derived to represent in a simple way the facts of chromaticity sensibility [18, 19, 20].

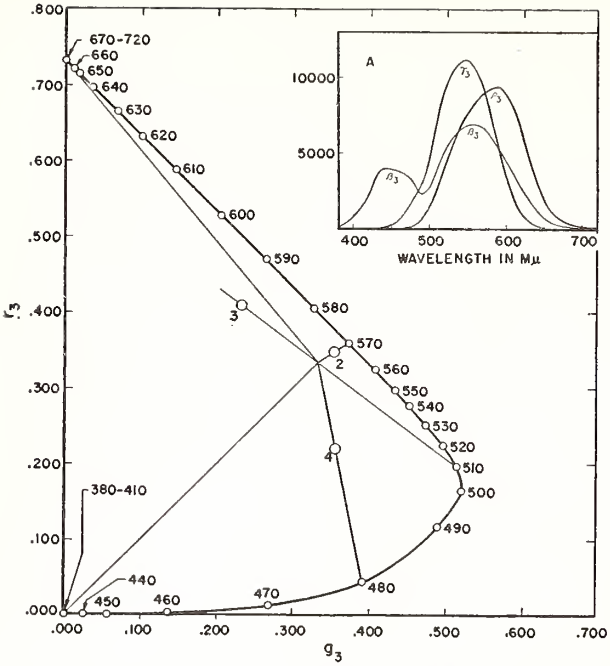

FIGURE 3. *The response functions and Maxwell triangle corresponding to a three-components explanation (Judd) of the data shown in figure 2.*

The suggested explanation is that segregation of photosensitive substances specific for long-wave energy is poor, violet cones having nearly as much (80%) as the red and green cones. Chromaticity coordinates of the spectrum referred to a set of stimulus primaries that have been found useful in deriving the "minimum purity perceptible." (See BS J. Research 4, 515 (1930) RP163 also J. Opt. Soc. Am. and Rev. Sci. Instr. 16, 115 (1928)). A, The three "distribution" curves which give the mixture diagram shown.

There are, however, two accounts of chromaticity sensibility that do seem also to permit good explanations of dichromatic vision, that by Adams [21] and the recent excellent treatment of chromaticity sensibility by Stiles [22]. An account of protanopia and tritanopia by the Adams theory has not yet been worked out in detail.

An outstanding defect of the three-component accounts of chromaticity sensibility is that there is no satisfactory explanation of the primary character of the spectrum in the neighborhood of 475 mμ. Most normal observers (though not all) see this portion of the spectrum as blue, and they see the short-wave extreme as binary in character, a mixture of red and blue. In commenting on this difficulty, it was remarked by Judd [23] in 1932, "The most satisfactory solution yet offered is Müller's theory which ascribes primacy to both blue and violet, the latter in the retinal processes, and the former in the optic nerve."

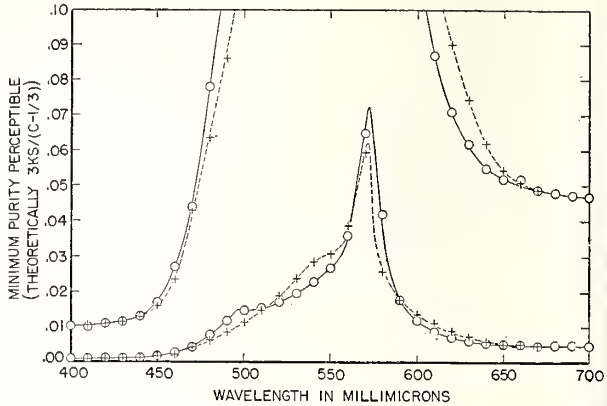

FIGURE 4. *Least fraction of spectrum light detectable as a chromatic difference in a mixture with sunlight, 2° observing field.*

The dotted line represents a part of the data shown in figure 2; the solid line is based on a three-components explanation (Judd, see fig. 3) of the tendency of the normal eye under these conditions to make tritanopic confusions. ○ $3KS/(c-1/3)$; $K=0.0024$, 0.8 blue deficient; +, $(dp/dE)_{p\to 0}$. Irwin G Priest, observer.

As a prerequisite to a quantitative explanation in terms of the Müller theory for the confusibility of sunlight with the spectrum at 570 mμ and for chromaticity sensibility generally, there must be derived the colorimetric coordinate systems corresponding to the two additional stages, the retinal and the optic nerve stages, of the Müller theory. They have so far been described only qualitatively, or at least semiquantitatively. Since the theory has been adjusted to correspond qualitatively with the facts of colorblindness, and since those facts have recently become known quantitatively (chiefly through the work of Pitt [33]), it is now possible to evaluate these coordinate systems, and so lay the ground work for a possible explanation of chromaticity sensibility based on the Müller theory.

II. Formulation for Normal Vision

According to the Müller theory,[2] light stimuli can elicit three different primary sensitizing processes (P-processes) in the cone mechanism, whose strengths are determined according to wavelength of the incident radiant energy according to functions similar to those defining the three components of the Young-Helmholtz theory. The P_1-process is aroused by the spectral region 475

[2] Acknowledgment is made to Michael J. Zigler, Department of Psychology, Wellesley College, who kindly supplied a very helpful translation into English of these parts of Müller's discussion of color-blindness [7].

mμ up to the long-wave visible extreme. The P_2-process is aroused by the spectral region between the long-wave end stretch (770 mμ to a wavelength greater than 655 mμ) and the short-wave end stretch (380 mμ to a wavelength less than 450 mμ). The P_3-process is aroused by the spectral region between 540 mμ and the short-wave visible extreme. From this description, the distribution curves of the P-processes are seen to resemble closely the OSA excitation curves [24]. The best modern evaluation of these distribution curves based upon the ICI standard observer [25] is to be obtained [26] by the following transformation:

$$\left.\begin{aligned} P_1&=3.1956X+2.4478Y-0.6434Z \\ P_2&=-2.5455X+7.0492Y+0.4963Z \\ P_3&=0.0000X+0.0000Y+5.0000Z \end{aligned}\right\}, \qquad (1)$$

which are graphed in figure 5. The reverse transformation [27] is given by

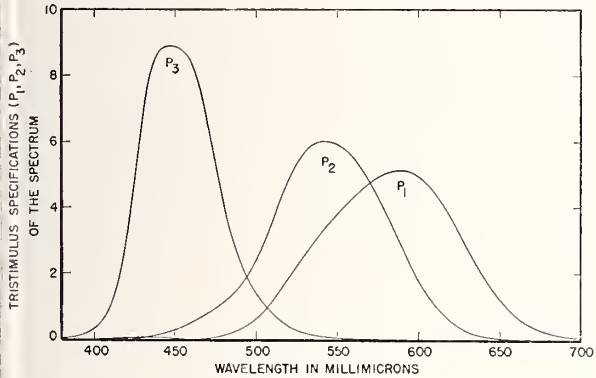

FIGURE 5. *Rates of decomposition of the photosensitive substances of the three-components theory as functions of wavelength.*

These functions are suited to the first stage of the Müller theory.

$$\left.\begin{aligned} X&=0.24513P_1-0.08512P_2+0.03999P_3 \\ Y&=0.08852P_1+0.11112P_2+0.00036P_3 \\ Z&=0.00000P_1+0.00000P_2+0.20000P_3 \end{aligned}\right\}. \qquad (1a)$$

Except for the small secondary maximum of the P_1 curve in the neighborhood of 430 mμ, these functions conform essentially to the description by Müller of the primary sensitizing processes. These processes contribute immediately to excitation of a black-white "substance" of the optic-nerve fibers in the sense of producing white. They also act upon certain assumed chromatic sensory substances in the cones of the normal eye as follows: the P_1-process acts to produce a major yellowish red (yR) process in a yellowish red-bluish green (yR-bG) substance and a minor greenish yellow (gY) process in a greenish yellow-reddish blue (gY-rB) substance. The P_2-process acts to produce a major bluish green (bG) process in the yR-bG substance and a minor greenish yellow (gY) process in the gY-rB substance. The P_3-process acts to produce a reddish blue (rB) process in the gY-rB substance. It is further assumed that the processes within each of these chromatic sensory substances are antagonistic so that a yR-process cancels completely a bG-process of equal strength; and a gY-process may cancel completely an rB-process. The wavelengths at which stimulation by homogeneous radiant energy would produce these cancellations are between 560 and 570 mμ for yR to cancel bG, and near 495 mμ for gY to cancel rB.

From this description it would seem that the transformation from the amounts of the primary sensitizing processes to the amounts of the chromatic sensory processes might take on the simple form:

$$\left.\begin{aligned} yR&=-bG=a_1P_1-a_2P_2 \\ gY&=-rB=b_1P_1+b_2P_2-b_3P_3 \end{aligned}\right\}, \qquad (2)$$

where a_1, a_2, b_1, b_2, and b_3 are constants greater than zero, with a_1 greater than b_1, and a_2 greater than b_2.

In addition to the primary sensitizing processes (P_1, P_2, P_3) and the chromatic sensory processes (yR-bG, gY-rB), there are six different excitations of the optic nerve (w, s, r, g, y, b), which correlate with the introspectively pure white, black, red, green, yellow, and blue sensations, respectively. The chromatic excitations are assumed to arise from the chromatic sensory processes alone, the yR-process arousing a major r-excitation and a minor y-excitation as indicated by the notation yR. Similar major and minor excitations are aroused by the bG, gY, and rB processes. The white-excitation of the optic nerve comes from the immediate effect of the primary sensitizing processes (P_1, P_2, P_3) to which secondary contributions from the yR- and gY-processes are added. The black-excitation comes chiefly by induction [7a, p. 85], from a white surrounding field or from a white preexposure field, but a secondary contribution comes from the bG and rB processes. Like the chromatic sensory processes, the four chromatic excitations of the optic nerve make antagonistic pairs, an amount of r-excitation cancelling a like amount of g-excitation, and the same cancellations for y- and b-excitation. For stimulation by homogeneous radiant energy, the one cancellation occurs at the wavelength for arousing unitary yellow, which is probably between 575 and 582 mμ under usual observing conditions for an average normal observer. A second (r,g)-cancellation occurs at the wavelength for arousing unitary blue, and a (y,b)-cancellation occurs at the wavelength for unitary green.

For self-luminous areas with a neutral surrounding field the s-excitation acts as negative w-excitation; however, they do not cancel, but combine to give gray. From this description it would appear that the excitations of the optic nerve could be found for the normal observer by the following transformations:

$$\left.\begin{aligned} r&=-g=c_1yR+c_2rB=-c_1bG-c_2gY\\ y&=-b=d_1gY+d_2yR=-d_1rB-d_2bG\\ w&=e_1P_1+e_2P_2+e_3P_3+e_4yR+e_5gY\\ s&=e_4bG+e_5rB \end{aligned}\right\}, \qquad (3)$$

where the luminance of the area is given by the difference, $w-s$, between the white and black excitation, and the symbols with subscripts represent constants evaluated so far only by the conditions that c_1 is greater than d_2, and d_1 is greater than c_2.

Equation 3 is similar to those set up by Schrödinger [28] in accord with the theoretical views of von Kries [5], and by Adams [21] in accord with his own theory [6]. Adams has, moreover, pointed out the advantages and theoretical plausibility of the view that the various stages are not linearly connected. For simplicity in the present derivation, attention will be confined to the assumption represented in eq 2 and 3 that the connection is linear and homogeneous.

III. Dichromatic Vision

According to the Müller theory, protanopia corresponds to the failure of the (yR, bG)-chromatic substance; and on this account, it is called by him outer red-green blindness. Thus, for protanopia, $yR=bG=0$, and we may write from eq 3:

$$\left.\begin{aligned} r_p&=-g_p=c_2rB=-c_2gY\\ y_p&=-b_p=d_1gY=-d_1rB\\ w_p&=e_1P_1+e_2P_2+e_3P_3+e_5gY\\ s_p&=e_5rB \end{aligned}\right\}. \qquad (4)$$

From eq 4 one might think that the Müller theory predicts for protanopia simply the sensations of black and white plus the two chromatic sensations greenish yellow and reddish blue, since a given amount of blue excitation is always bound up inextricably with the same minor red excitation. This is indeed the simplest prediction from the formulation and corresponds fairly well with reports of unilaterally protanopic observers. Müller, however, points out that although failure of the (yR, bG) chromatic process is sufficient to produce the symptoms of protanopia completely, this failure could be accompanied by a failure of some of the chromatic processes in the optic nerve, which combination of circumstances could give rise to an observer having protanopic vision by all tests actually sensing only the hues yellow and blue, or even only the hues red and green. Such observers could be distinguished from each other only if one eye had trichromatic, and the other, protanopic vision.

Deuteranopia, on the other hand, is ascribed by Müller to failure of the (r, g)-sense of the optic nerve and is called inner red-green blindness. Thus, for deuteranopia, $r_d=-g_d=0$, and we may write from eq 3:

$$\left.\begin{aligned} y_d&=-b_d=d_1gY+d_2yR\\ w_d&=e_1P_1+e_2P_2+e_3P_3+e_4yR+e_5gY\\ s_d&=e_4bG+e_5rB \end{aligned}\right\}. \qquad (5)$$

From eq 5 it is plain that the predicted sensations of deuteranopes must be black, white, yellow, and blue; there are no alternatives.

Tritanopia, like protanopia, is ascribed to a retinal defect. It corresponds to failure of the (gY, rB)-chromatic substance and is called outer yellow-blue blindness. Thus, for tritanopia, $gY=rB=0$, and we may write from eq 3:

$$\left.\begin{aligned} r_t &= -g_t = c_1 yR = -c_1 bG, \\ y_t &= -b_t = d_2 yR = -d_2 bG, \\ w_t &= e_1 P_1 + e_2 P_2 + e_3 P_3 + e_4 yR, \\ e_t &= e_4 bG \end{aligned}\right\}. \qquad (6)$$

The sensations of tritanopes are seen to be predicted as black, white, and either yellow and blue, or red and green, or some fixed combination such as yellowish red and bluish green. The latter hues correspond to the simplest prediction, since the (yR, bG) chromatic sensory process is unaffected. These hues agree well with the reports of tritanopes who have acquired the defect through a disease of the retina.

IV. Evaluation of the Constants

Response functions for normal and dichromatic vision according to the Müller theory can be evaluated from eq 1, 2, and 3, for all three stages of excitation, provided the 14 constants of eq 2 and 3 be evaluated. For the normal mechanism adapted to a stimulus yielding an achromatic color, the stimuli for the unitary hues, red, yellow, green, and blue, are known [29, 30] within limits; these stimuli must excite only the respective r-, y-, g-, and b-processes of the optic nerve. The stimulus yielding the achromatic color, itself, must cause the chromatic sensory processes yR, gY, bG, and rB to vanish, and also reduce to zero the chromatic r-, y-, g-, and b-processes of the optic nerve. The colors confused with gray by the typical protanope, deuteranope, and tritanope must conform to the yR-process, the g-process, the rB-process, and their complements, respectively. The difference between the spectral luminosity function of the typical protanope and the same function for the normal observer must be a constant fraction of the (yR, bG)-process in order to conform to Müller's proposal; and similarly the difference between normal and tritanopic luminosity must be a constant fraction of the (gY, rB)-process.

There are many more than 14 conditions to be satisfied, including several that are set down by Müller only in qualitative terms, and some that relate to sensibility to chromaticity differences. In evaluating these constants, the best determined conditions (marked by asterisks in the next sections) have been satisfied perfectly; and from the resulting excitation curves it may be seen to what extent the less well-determined conditions are satisfied. For example, of the data for the normal stimuli for the unitary hues, only those for unitary yellow have been used, since they were specifically mentioned by Müller as indicating the stimulus to be between 575 and 582 μ. Unitary blue has necessarily to be taken as the complement of unitary yellow relative to the stimulus for white or gray; and unitary red and green were taken as the confusion colors for typical deuteranopia.

1. Hueless Point and Unitary Yellow

For an achromatic color, both the chromatic processes of the optic nerve and the chromatic sensory processes must cancel to zero. It is known [31] that stimulation of the normal eye by a source of equal energy results under ordinary conditions of observation in a closely achromatic, or hueless color. For such a source, $X=Y=Z$, and from eq 1, $P_1=P_2=P_3$; hence we may write with sufficient accuracy from eq 2

$$yR = a_1 P_1 - a_2 P_2 = 0,$$

whence:

$$a_1/a_2 = 1.000; \qquad (7)^*$$

and similarly

$$gY = b_1 P_1 + b_2 P_2 - b_3 P_3 = 0,$$

whence we find

$$b_1 + b_2 = b_3. \qquad (8)^*$$

By setting $r=g=y=b=0$ in eq 3, and substituting eq 7 and 8, the expressions vanish, and no further relation is found.

For the r-g cancellation point between 575 and 582 mμ, we may take somewhat arbitrarily for simplicity the crossing point of X and Y at 578.1 mμ. For stimulation by homogeneous energy of this wavelength we may write

$$\begin{aligned} r = -g &= c_1 yR + c_2 rB = 0 \\ &= c_1(a_1 P_1 - a_2 P_2) - c_2(b_1 P_1 + b_2 P_2 - b_3 P_3) = 0 \\ &= (a_1 c_1 - b_1 c_2)(3.1956X + 2.4478Y - 0.6434Z) - \\ &\quad (a_2 c_1 + b_2 c_2)(-2.5455X + 7.0492Y + 0.4963Z) + \\ &\quad b_3 c_2 (5.0000Z) = 0 \end{aligned}$$

By substituting $X=Y=0.4996$, $Z=0.0008$, which refer closely to 578.1 mμ, we obtain

$$2.8189(a_1c_1-b_1c_2)-2.2505(a_2c_1+b_2c_2)+0.0040b_3c_2=0. \quad (9)^*$$

2. Dichromatic Copunctal Points

The chromaticity confusions of dichromatic vision may be represented on the Maxwell triangle by families of straight lines, all lines of one family intersecting at a common point, known as the copunctal point [32]. These copunctal points have been evaluated from recent determinations, chiefly by Pitt [33], and expressed in terms of the ICI standard coordinate system. They embody the essential information regarding the chromaticity confusions of dichromats and lead to a convenient expression of four conditions affecting the unknown constants.

According to the Müller theory, protanopia corresponds to the failure of the (yR,bG)-chromatic substance. For certain stimuli the (gY,rB)-process is also reduced to zero. These stimuli are the equal-energy stimulus and all of those confused with it by the protanope. These chromaticity confusions are indicated on the Maxwell triangle by a straight line passing through the equal-energy point, and for every point on this line, $gY=-rB=0$. In particular, since all of the chromaticity confusion lines pass through a single point, these conditions must hold for the protanopic copunctal point defined by $x=X/(X+Y+Z)=0.747$, $y=Y/(X+Y+Z)=0.253$, $z=Z/(X+Y+Z)=0$. Hence we may write from eq 2

$$b_1P_1+b_2P_2-b_3P_3=$$
$$b_1(3.1956X+2.4478Y-0.6434Z)+$$
$$b_2(-2.5455X+7.0492Y+0.4963Z)-$$
$$b_3(5.0000Z)=0;$$

whence we find

$$b_1/b_2=0.0393. \quad (10)^*$$

Similarly, from the less well-determined tritanopic copunctal point, $x=0.18$, $y=0.00$, $z=0.82$, we may write

$$yR=-bG=a_1P_1-a_2P_2=0,$$
$$=a_1(3.1956X+2.4478Y-0.6434Z)-$$
$$a_2(-2.5455X+7.0492Y+0.4963Z)=0;$$

whence we find $a_1/a_2=-0.0512/0.0476=-1.07$, a value contradictory to eq 7 based upon th achromatic stimulus for the normal observer which is much more reliably established than th tritanopic copunctal point. By setting $a_1=a$ in conformity with eq 7, we find that a condition for the tritanopic copunctal point by the Mülle formulation is that $P_1=P_2$, which from eq 1 is equivalent to $y=1.987x-0.327$. This line intersects the x-axis of the chromaticity diagram a $x=0.165$, $y=0.000$, which is in as good agreement with the facts as the approximate evaluation o the copunctal point ($x=0.18$, $y=0.00$) estimated with the help of the König theory [32]. The latter accords with Müller's view that tritanopia is characterized by a single neutral point in the spectrum (near 570 mμ); the former places a second neutral point near the violet extreme o the spectrum (430 mμ) and arises from application of the Müller theory to the properties of the standard observer. As already noted, P_1 evaluated by this means has a small secondary maximum in the neighborhood of 430 mμ, causing it to cross the P_2-curve at this point, a result quite unanticipated by Müller. Actual reports o tritanopic vision are fairly well divided in this respect; for example, the cases reported by König [4] and Köllner [34] yielded a single neutra point; those by Collin and Nagel [35] and Piper [36] had neutral points or areas in the neighborhood of 430 mμ. Willmer and Wright [14] found an indication of such a neutral region for small fields in the normal fovea, and Pitt [37] considers this to be typical of tritanopia. The difference in chromaticity between 430 mμ and the short-wave end of the spectrum is small and it is possible that individual variations among tritanopes can account for the slight discrepancy in the report between no short-wave neutral point and one near 430 mμ. On the other hand, it seems to be fairly frequent that tritanopes have ocular media pigmented heavily with brown pigment, and it is also possible that this pigmentation would cause the spectrum to become invisible to many tritanopes at a wavelength greater than 430 mμ as in a case reported by Farnsworth [38]. The formulation could be made to accord strictly with the Müller view on this point by choosing constants in eq 1, so that the representation of P_2 is everywhere higher than P_1 for wavelengths less than 500 mμ, such as that found by Stiles [22], but such a wavelength disribution of P_2 is itself contrary to Müller's view. t seems more useful to proceed with an account f tritanopia involving a second neutral point ear 430 mμ. Take therefore $a_1=a_2$ as in eq 7.

By the same argument given for protanopia, e may set $y=-b=0$, for the deuteranopic opunctal point defined by $Y=Z=0$. For eq 1, , and 3 we may write

$$=-b=d_1gY+d_2yR=d_1(b_1P_1+b_2P_2-b_3P_3)+d_2(a_1P_1-a_2P_2)=0,$$

$$=(b_1d_1+a_1d_2)(3.1956X+2.4478Y-0.6434Z)+(b_2d_1-a_2d_2)(-2.5455X+7.0492Y+0.4963Z)-b_3d_1(5.0000Z);$$

hence we find, since $Y=Z=0$,

$$3.1956(b_1d_1+a_1d_2)-2.5455(b_2d_1-a_2d_2)=0. \qquad (11)^*$$

Two other conditions may be derived from the rotanopic and deuteranopic copunctal points. t has been shown [32] that the deuteranopic opunctal point lies on the deuteranopic alychne, hat is, the line on the Maxwell triangle associated ith zero deuteranopic luminosity; and the rotanopic copunctal point lies on the protanopic ychne. Hence for the respective copunctal oints we may set $(w-s)_d$ and $(w-s)_p$ equal to ero, and from eq 1, 2, and 3 there are found:

$$e_1-0.0393e_2+2e_5(b_1-0.0393b_2)=0, \qquad (12)^*$$

nd

$$3.1956(e_1+2e_4a_1+2e_5b_1)-2.5455(e_2-2e_4a_2+2e_5b_2)=0. \qquad (13)^*$$

3. Dichromatic Luminosity Functions

It has been shown that the luminosity functions red-green-blind observers can be expressed as nctions of X, Y, and Z [39] by eq 14 and 15, in hich W_p is the protanopic luminosity and W_d, e deuteranopic

$$W_p=0.460X+1.359Y+0.101Z, \qquad (14)$$

$$W_d=Y. \qquad (15)$$

rom eq 1a, these functions may be written in rms of the primary processes, P_1, P_2, and P_3:

$$W_p=0.00754P_1+0.19017P_2+0.00229P_3, \qquad (14a)$$

$$W_d=0.08852P_1+0.11112P_2+0.00036P_3. \qquad (15a)$$

But from eq 2, 4, and 5, we may write

$$W_p=(w-s)_p=(e_1+2b_1e_5)P_1+(e_2+2b_2e_5)P_2+(e_3-2b_3e_5)P_3, \qquad (14b)$$

$$W_d=(w-s)_d=(w-s)=(e_1+2a_1e_4+2b_1e_5)P_1+(e_2-2a_2e_4+2b_2e_5)P_2+(e_3-2b_3e_5)P_3. \qquad (15b)$$

By equating the coefficients of P_1, P_2, and P_3 in eq 14a and 14b, we obtain three additional conditions to be satisfied by the constants:

$$e_1+2b_1e_5=0.00754, \qquad (16)$$

$$e_2+2b_2e_5=0.19017, \qquad (17)$$

$$e_3-2b_3e_5=0.00229. \qquad (18)$$

Similarly, by equating the coefficients in eq 15a and 15b we obtain three more conditions:

$$e_1+2a_1e_4+2b_1e_5=0.08852, \qquad (19)$$

$$e_2-2a_2e_4+2b_2e_5=0.11112, \qquad (20)^*$$

$$e_3-2b_3e_5=0.00036. \qquad (21)^*$$

Equations 16 to 21 are not entirely independent of the conditions previously found, nor are they all congruent. From eq 16 and 17 there may be derived eq 12; and from eq 19 and 20 there may be derived eq 13. Furthermore, eq 7, 10, 12, 19, and 20 combine to give eq 16 and 17. Equations 18 and 21 are contradictory. Since it is an essential part of the Müller theory that the deuteranopic luminosity function be the same as the normal, we must accept eq 21 and reject eq 18. This choice will prevent $(w-s)_p$ in eq 14b from being exactly equal to W_p in eq 14. It remains to be found whether the resulting evaluation of $(w-s)_p$ is as representative of available data on the protanopic luminosity function as is W_p. Thus, we have obtained only two additional independent conditions from protanopic and deuteranopic luminosity functions, eq 20 and 21.

If it be assumed for the moment that tritanopic luminosity is the same as deuteranopic and normal luminosity, as it may well be judging from the available information [7a, pp 53 and 102; 34, 36, 40, 41, 42, 43], then from eq 2 and 6 we obtain eq 22:

$$(w-s)=(w-s)_t=e_1P_1+e_2P_2+e_3P_3+2e_4\ (a_1P_1-a_2P_2). \quad (22)$$

By comparing the coefficients of P_1, P_2, and P_3 in eq 15b and 22 we see from each of the three comparisons that e_5 must be zero; that is, there can be no darkening effect from the gY-process, such as implied by eq 3. The Müller theory thus cannot abide having equality between tritanopic and normal luminosity. Müller was well aware that his theory required the tritanopic luminosity function to be different from normal and remarks [7a, p. 63] "In regard to spectral luminosity distribution in tritanopia, there must be, if no complications exist, because of the absence of the w value of the gY process, a decrease in the luminosity of yellow and yellowish lights in comparison to normal. . . . Unfortunately there have been up to now no investigations of the spectral luminosity characteristic of tritanopia." Since the present purpose is to find the coordinate systems implied by the Müller theory, we must disregard the rather inconclusive indications that there is no difference between tritanopic and normal luminosity; hence no attention can be paid to eq 22 in evaluating the constants, and e_b must be given a positive, though small value. Take arbitrarily, then:

$$e_5=0.03\ e_4. \quad (23)^*$$

4. Chromaticity Sensibility and Theory

We may now take stock of the conditions that must be satisfied by the 14 constants:

Criterion	Resulting condition
Hueless point	Eq 7 and 8
Unitary yellow	Eq 9
Protanopic copunctal point	Eq 10 and 12
Deuteranopic copunctal point	Eq 11 and 13
Dichromatic luminosity	Eq 20, 21, and 23.

These 10 equations have been marked with asterisks to show that they were used in the derivation of the constants.

There remain four conditions to be set up before the 14 constants can be evaluated. Three of the four conditions refer to the relative sizes of a_1 and b_3, c_1 and d_1, and a_1 and c_1.

The first ratio, a_1/b_3, has to do with the relative sensibility of the eye to yellowish red-bluish green differences on the one hand and greenish yellow-reddish blue differences on the other. Empirical studies on large fields [20, 21] indicate that a_1/b_3 is about 2.5, that is, the normal eye detects (yR, bG)-differences more readily by about a factor of 2.5 than would be judged from the relatively great overlap of P_1 and P_2. By setting $c_1/d_1=1.0$, this greater sensibility to (yR, bG)-differences is preserved in the optic-nerve stage.

The ratio of a_1/c_1 has to do with the comparative amounts of the chromatic sensory process and the chromatic excitations of the optic nerve. There does not seem to be any fundamental meaning to this comparison. It has merely to do with a relation between the units expressing the rate of a chemical process in the retinal receptors and those expressing the frequency of the resulting impulses in the fibers of the optic nerve. This ratio may be set arbitrarily, and for simplicity we set $a_1/c_1=1$.

The final condition refers to the size of the arbitrary units in which the chromatic responses are to be expressed; for simplicity take $a_1=1$.

Solution of these 14 equations simultaneously gives the values of the constants:

$a_1=1.0000, a_2=1.0000,$

$b_1=0.0151, b_2=0.3849, b_3=0.40000,$

$c_1=1.0000, c_2=0.6265,$

$d_1=1.0000,\ d_2=0.1622,$

$e_1=0.0075, e_2=0.1912, e_3=0.0013, e_4=0.0405,$
$e_5=0.0012$

It will be noted that, as required by eq 2, a_1 is greater than b_1, and a_2 is greater than b_2. Furthermore, as required by eq 3, c_1 is greater than d_2, and d_1 is greater than c_2. This correspondence with the Müller description indicates how thorough was his grasp of the facts from purely qualitative data, though probably $c_2=0.6265$ is not as small compared to $d_1=1.0000$ as would be expected from Müller's designation of d_1gY and

$c_2 gY$ as a major y-excitation and a minor g-excitation, respectively, resulting from the gY process. The value of c_2 would be reduced somewhat by taking a higher value for the wavelength of the spectrum stimulus for unitary yellow, say 582 mμ instead of 578 mμ; see eq 9.

V. Definition of the Coordinate Systems

We may now insert these constants into eq 2 and 3, and so give explicit definitions of the two new coordinate systems implied in the Müller theory. The coordinate system applying to the chromatic sensory processes of the retinal receptors is defined by eq 2a:

$$\left.\begin{aligned} yR&=-bG=P_1-P_2. \\ gY&=-rB=0.0151P_1+0.3849P_2-0.4000P_3 \end{aligned}\right\} \quad (2a)$$

The coordinate system applying to the processes in he optic nerve fibers is defined by eq 3a;

$$\left.\begin{aligned} r&=-g=yR+0.6265rB=-bG-0.6265gY, \\ y&=-b=gY+0.1622yR=-rB-0.1622bG, \\ w&=0.0075P_1+0.1912P_2+0.0013P_3+0.0405\,yR+0.0012gY, \\ s&=0.0405bG+0.0012rB. \end{aligned}\right\} \quad (3a)$$

These two coordinate systems may also be defined in terms of the standard 1931 ICI coordinate ystem for colorimetry from eq 1. Equation 2b ives the definition of the colorimetric coordinate ystem for normal observers corresponding to the hromatic sensory processes combined with the uminosity function Y; and eq 2c gives the reverse transformation from this coordinate system o the standard ICI system:

$$\left.\begin{aligned} yR=-bG&=5.741X-4.601Y-1.140Z, \\ gY=-rB&=-0.932X+2.750Y-1.819Z, \\ Y&=1.000Y; \end{aligned}\right\} \quad (2b)$$

$$\left.\begin{aligned} X&=0.1581yR-0.0991gY+Y, \\ Y&=Y, \\ Z&=-0.0810yR-0.4991gY+Y, \end{aligned}\right\} \quad (2c)$$

Equation 3b gives the definition of the colorimetric coordinate system for normal observers corresponding to the excitations of the optic nerve, and eq 3c gives the reverse transformation from this coordinate system to the standard ICI system:

$$\left.\begin{aligned} r=-g&=6.325X-6.325Y \\ y=-b&=2.004Y-2.004Z \\ (w-s)&=1.000Y \end{aligned}\right\} \quad (3b)$$

$$\left.\begin{aligned} X&=0.1581r+(w-s) \\ Y&=(w-s) \\ Z&=-0.4991y+(w-s) \end{aligned}\right\} \quad (3c)$$

Table 1 gives the response functions of the normal, protanopic, deuteranopic, and tritanopic types of vision derived from the ICI standard observer according to the Müller theory (eq 1, 2b, 3b, 4, 5, and 6).

The very simple transformation equations between the chromatic excitations of the optic nerve according to the Müller theory and the standard 1931 ICI coordinate system for colorimetry arise, of course, from the fact that the X-primary of the ICI system corresponds to a stimulus for unitary red, and the Z-primary corresponds to unitary blue. The r,y $(w\text{-}s)$ system corresponds to the central stage of the Adams theory [6], and coordinate systems closely resembling that described by eq 3b have been used by Adams with considerable success to explain chromaticity spacing for large fields, chiefly studies of the spacing of the Munsell colors [21].

The coordinate system set up by Schrödinger [44] in 1925 resembles closely that defined by eq 3b except that it was not adjusted to correspond to the same balance between (y,b)-excitation and the (r,g)-excitation ($a_1/b_3=2.5$, $c_1/d_1=1.0$). Schouten [45] made use of conditions derived from the hueless point (eqs 7 and 8), the deuteranopic neutral point (eq 11) and the unitary yellow point (eq 9) to compute response functions for assumed central r-, y-, g-, and b-processes. These functions bear a considerable resemblance to r and y evaluated from eq 3b. Thus it is seen that the essence of eq 3b is neither new nor confined to the Müller theory; it arises from the opponent-colors theory of Hering and has been used in at least three theoretical studies since 1925.

Figure 6 shows as functions of wavelength P_1, P_2, P_3; yR and gY; r and y; and finally in the

TABLE 1. *Response functions of the normal, protanopic, deuteranopic, and tritanopic types of vision derived from the ICI standard observer according to the Müller theory*

[Spectrum: equienergy]

Wave length	Primary sensitizing processes (possessed by all types of vision)				Chromatic sensory processes		Optic-nerve excitation				
	P_1	P_2	P_3	Normal vision Protanopic vision Deuteranopic vision Tritanopic vision	yR-bG ------- $(yR$-$bG)_d$ $(yR$-$bG)_t$	gY-rB $(gY$-$rB)_p$ $(gY$-$rB)_d$ -------	r-g ([a]) ------- ([a])	y-b ([a]) $(y$-$b)_d$ ([a])	w-s ----- $(w$-$s)_d$ -----	----- $(w$-$s)_p$ ----- -----	----- ----- ----- $(w$-$s)_t$
$m\mu$											
380	-----	-----	32		-------	−13	+8	−13	-----	-----	-----
390	1	-----	100		+1	−40	+26	−40	0. 1	-----	0. 2
400	3	-----	340		+3	−136	+88	−135	. 4	0. 1	. 6
410	8	1	1, 037		+8	−414	+267	−413	1. 2	. 5	2. 1
420	24	6	3, 228		+17	−1, 288	+824	−1, 286	4. 0	2. 5	7. 0
430	44	47	6, 928		−3	−2, 752	+1, 722	−2, 753	11. 6	12	18. 1
440	45	143	8, 736		−97	−3, 439	+2, 057	−3, 454	23	31	31. 1
450	27	292	8, 860		−264	−3, 432	+1, 886	−3, 475	38	59	46
460	2	511	8, 346		−509	−3, 142	+1, 459	−3, 224	60	100	67
470	19	783	6, 438		−764	−2, 274	+660	−2, 398	91	152	96
480	123	1, 140	4, 065		−1, 017	−1, 185	−275	−1, 350	139	220	142
490	312	1, 616	2, 326		−1, 304	−304	−1, 113	−515	208	312	208
500	631	2, 399	1, 360		−1, 768	+389	−2, 012	+102	323	464	322
510	1, 159	3, 600	791		−2, 441	+1, 087	−3, 122	+691	503	699	501
520	1, 890	4, 883	391		−2, 993	+1, 751	−4, 090	+1, 266	710	949	706
530	2, 612	5, 676	211		−3, 064	+2, 140	−4, 405	+1, 643	862	1, 107	857
540	3, 205	5, 996	102		−2, 746	+2, 316	−4, 197	+1, 871	954	1, 174	948
550	3, 815	5, 915	44		−2, 100	+2, 317	−3, 552	+1, 976	995	1, 163	990
560	4, 333	5, 503	20		−1, 170	+2, 176	−2, 533	+1, 986	995	1, 089	990
570	4, 764	4, 772	10		−8	+1, 904	−1, 201	+1, 903	952	953	948
580	5, 057	3, 801	8		+1, 255	+1, 536	+293	+1, 740	870	770	866
590	5, 132	2, 724	6		+2, 408	+1, 124	+1, 704	+1, 514	757	564	754
600	4, 938	1, 745	4		+3, 194	+744	+2, 728	+1, 263	631	376	629
610	4, 435	994	2		+3, 441	+449	+3, 160	+1, 003	503	228	502
620	3, 663	510	1		+3, 152	+252	+2, 994	+763	381	129	380
630	2, 702	233	-----		+2, 469	+130	+2, 387	+531	265	68	265
640	1, 860	94	-----		+1, 766	+64	+1, 726	+351	175	34	175
650	1, 168	33	-----		+1, 135	+30	+1, 116	+214	107	16	107
660	676	10	-----		+666	+14	+657	+122	61	8	61
670	358	3	-----		+354	+7	+350	+64	32	4	32
680	191	1	-----		+190	+3	+188	+34	17	2	17
690	93	-----	-----		+93	+1	+92	+16	8. 2	1	8
700	46	-----	-----		+46	+1	+46	+8	4. 1	-----	4. 1
710	24	-----	-----		+24	-------	+23	+4	2. 1	-----	2. 1
720	12	-----	-----		+11	-------	+11	+2	1. 0	-----	. 9
730	6	-----	-----		+6	-------	+6	+1	. 5	-----	. 4
740	3	-----	-----		+3	-------	+3	-------	. 3	-----	. 3

[a] For protanopic and tritanopic vision the Müller theory does not state rigidly that the optic nerve excitation must follow eq 4 and 6, though this is the simplest prediction. Either *r-g* or *y-b*, but not both, may be zero. If they are not zero, they have wavelength distributions proportional to the chromatic sensory processes.

lower left quadrant the deuteranopic luminosity according to eq 3b, the protanopic luminosity according to eq 4 together with the luminosity contributions of the chromatic sensory processes yR and gY.

VI. Protanopic Luminosity Function

In the ICI system the standard luminosity function is represented by the second function, Y; and from eq 3b it may be seen that the Müller

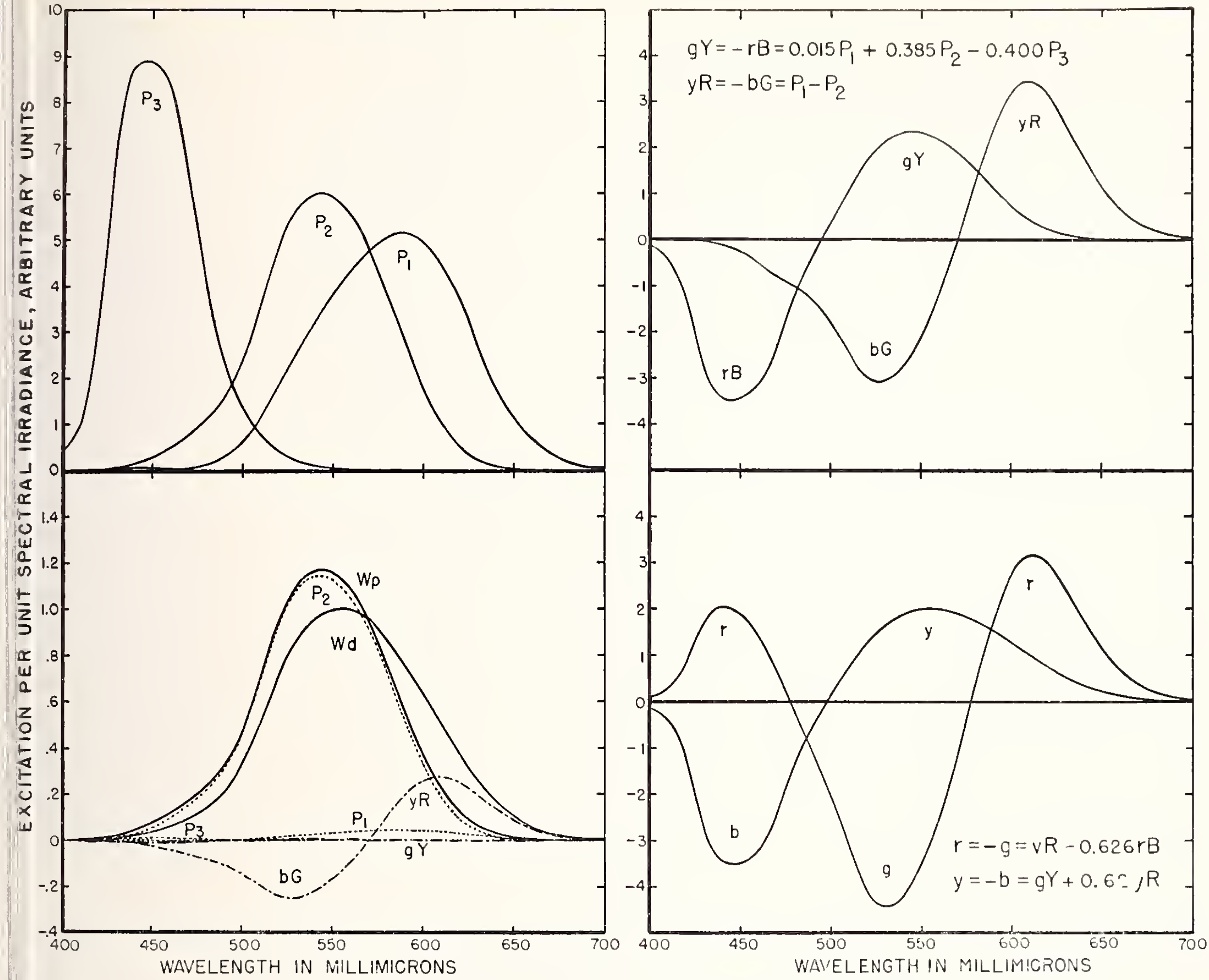

Figure 6. *Response functions according to the three stages of the Müller theory.*

Upper left: Processes in the initial photosensitive stage (same as Young theory, see eq 1 and fig. 5); lower left: components in the luminosity function $(w-s)$ ›oth for normal and deuteranopic, W_d, and protanopic, W_p, vision (see eq 3a); upper right: chromatic retinal sensory processes (see eq 2a); lower right: hromatic processes in the optic-nerve fiber stage (same as the Hering theory, see eq 3a).

heory can be formulated, as he claimed, in such way that the difference between the w-excitation nd the s-excitation gives the normal luminosity unction. This function has already been shown o be as satisfactory a representation of deuter-nopic luminosity as it is for some normal lu-ninosity functions, because the deuteranopic lu-ninosity functions fall within normal limits [32]. n these two respects this formulation of the Müller theory conforms exactly to that previously vorked out in accord with the König theory [4, 32]. However, it was noted previously that eq 21 ontradicts eq 18; so it remains to be seen whether he prediction of protanopic luminosity by this ormulation of the Müller theory is as acceptable as that by the König theory. By inserting the constants in eq 4 it is found that this formulation of the Müller theory requires protanopic luminosity to be given by:

$$(w\text{-}s)_p = 0.0075\ P_1 + 0.1921\ P_2 + 0.0003\ P_3. \quad (4a)$$

The previous formulation of the König theory yielded the equation:

$$W_p = 0.0075\ P_1 + 0.1902\ P_2 + 0.0023\ P_3. \quad (14a)$$

Figure 7 is a plot of these functions adjusted approximately to unit maximum, together with upper and lower limits of available data on luminosity functions of protanopic and protanom-

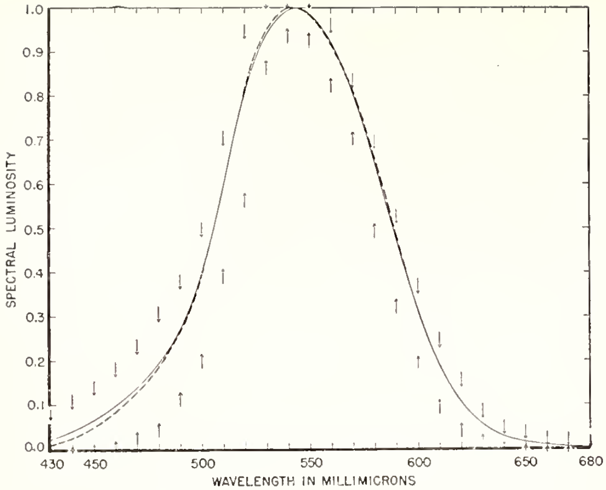

FIGURE 7. *Protanopic and protanomalous luminosity functions.*

The solid curve, W_p, corresponds to a wavelength function derived in accord with the later König form of three-components formulation for protanopic luminosity [32]; the dotted curve, $(w-s)_p$, is based on the Müller theory. The arrows indicate maximum and minimum luminosities of 12 protanomalous and six protanopic observers. Note that these data support both functions about equally well.

alous observers [32]. It will be seen that these data support both functions about equally well.

VII. Chromatic Thresholds, Normal and Tritanopic

We are now in position to inquire whether the Müller theory offers a basis for explaining the chromatic-threshold data of Abney [8] and Priest Brickwedde [9] referred to earlier. According to the Müller theory the ability of an observer to detect a slight variation in chromaticity from a central chromaticity, such as that of the light from a carbon arc, would depend upon the excitation of the chromatic sensory processes. The amounts of the excitation of these processes corresponding to any color specified in terms (X, Y, Z) of the 1931 ICI standard observer can be found from eq 2b. And, in particular, they have been found for the spectrum colors for unit irradiance and are plotted in the upper right quadrant of figure 6. Both of the above sets of data are given, however, in luminous units (luminance of the field just yielding a chromatic difference from carbon-arc light in Abney's work, or luminance fraction required to be mixed with sunlight to produce a color just noticeably different from sunlight in the work of Priest and Brickwedde). We should expect to compare with them, therefore, the excitations o the chromatic sensory processes corresponding t the colors of a spectrum of constant luminance that is, we should expect to find the chromati thresholds in luminance terms to correspond t the reciprocal of some combination of $yR/(w-s)$ and $gY/(w-s)$. The exact form of combinatio would seem to be expressible in terms of th probability of a chromaticity difference bein discriminated as a function of the probabilities o each of the two independent chromatic processe becoming effective considered separately. For th present purpose it is sufficient to take tentativel the combination as the square root of the sum o the squares; that is, assume, for the moment that the effective chromatic excitation for larg fields and high luminance is proportional to $[(yR)^2+(gY)^2]^{1/2}/(w-s)$. For experimental con ditions, such as restricted angular size of field o low luminance, that make the normal eye respon more or less like a tritanopic eye, the effectiv chromatic excitation may be assumed to be pro portional to $[(yR)^2+f^2\ (gY)^2]^{1/2}/(w-s)$, where f i the relative effectiveness of the gY-rB proces compared to the yR-bG process. In general, w would compare to the chromatic threshold dB/dE, expressed in luminous terms, the reciprocal of these assumed effective chromatic excitation so as to study the validity of the relation

$$dB/dE=k(w-s)/[(yR)^2+f^2\ (gY)^2]^{1/2}, \qquad (24)$$

where k is the constant required to adjust th theoretical function to the units in which th chromatic threshold is expressed.

Abney's data have been found to agree fairl well with eq 24 for $k=0.0020$ and $f=0.04$; se dotted curve of figure 1. The course of the experimentally determined function is followed well except for wavelengths greater than 590 mμ where the predicted threshold is considerabl lower than that found experimentally. As far a is known, no explanation of these data has previously been suggested. It should be pointed out also that more recent determinations of th chromatic threshold by Purdy [46], and Otero Plaza, and Casero [47] are quite at variance wit these data, and indeed with each other. The show neither the sharp peak at 570 mμ nor th decline to small values near 450 mμ. Needless t say, they are quite unexplainable by the Mülle theory. The data by Abney and Watson, however, may be summarized by saying that they conform fairly well to the Müller theory for a retinal region in which the normal gY, rB process is 96 percent ineffective. Tritanopia corresponds to complete ineffectiveness of this process.

The data of Priest and Brickwedde have been found to agree well with eq 24 for $k=0.05$, and $f=0.5$; see dotted curve on figure 2. The degree of agreement is quite comparable to that obtained by a coordinate system adjusted empirically to represent such data [18]; see solid curve. In this case, the less complete data by Purdy are in substantial agreement and are also shown. It should be pointed out, however, that these data have been corrected to refer to the standard luminosity function by multiplying them by the ratio of the standard luminosity function to that found by Gibson and Tyndall. It is probable that an improvement in the theoretical account of other psychophysical data by means of the Müller theory would result from revaluation in terms of an observer based on the Gibson-Tyndall experimental mean [18] luminosity function instead of on the standard observer. However, we may say that the Priest-Brickwedde data correspond well to the Müller theory for a retinal region in which the normal gY-rB-process is 50 percent effective.

It is concluded that the Müller theory affords a good explanation of chromatic thresholds in terms of a gradual approach to tritanopic vision. A thorough study of the implications of the Müller theory for chromaticity sensibility of all types such as that carried out by Stiles [22] for the three-components theory would seem to be worth while.

VIII. Summary and Conclusion

By taking into account the metamers known to be characteristic of protanopic, deuteranopic, and normal vision as well as data on the stimulus for a neutral color and the stimulus for a color of unitary yellow hue, the spectral variations of the responses for each of the three stages of the Müller theory of vision have been evaluated as functions of wavelength.

These response functions are shown to yield an account of normal, protanopic, and deuteranopic vision that differs in no essential respect from the simpler explanation yielded by the König form of three-components theory. They differ in their explanation of tritanopic vision by requiring the tritanopic luminosity function to be slightly higher in the short-wave end of the spectrum than normal; the three-components explanation requires it to be slightly lower in this part of the spectrum.

The chromatic response functions of the second stage of the Müller theory are shown to lead to a satisfactory and convenient account of the approach to tritanopia exhibited by the normal eye in viewing small fields or fields of luminance near the chromatic threshold.

It is concluded that the qualitative ideas of Müller lead to admissible and consistent coordinate systems. The Müller theory shows how the three-components formulation of Young, Helmholtz, and König (first stage) and the opponent-colors formulation of Hering (third stage) may both be accepted, and the explaining power of both be simultaneously utilized. The intermediate stage is also a promising and powerful theoretical tool. The quantitative consistency of the Müller ideas and the success demonstrated in accounting for tritanopic confusions made by normal observers does not, of course, prove the Müller theory to be completely, or even basically, correct. Alternate explanations are possible. There are important gaps in our knowledge of retinal chemistry and conduction and integration of nerve impulses that, if filled, might disprove the Müller theory and require adoption of an alternate account. Furthermore, several aspects of the Müller explanation, though admissible in the present state of our knowledge, seem implausible and unlikely to be born out by future work. At the very least, however, the Müller theory must be viewed as a forward step, and the coordinate system suggested by the second stage has practical value regardless of any of these future theoretical developments.

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[47] J. M. Otero, L. Plaza, and L. Casero, El intervalo acromatico y las sensaciones fundamentales en la zona media del espectro, Anales. Fisica Quimica[A], **16** (1948).

Washington, July 9, 1948.

TRITANOPIA WITH ABNORMALLY HEAVY OCULAR PIGMENTATION
(with L. Plaza and D. Farnsworth)

J. Opt. Soc. Am. 40, 833-841 (1950)

This is an unprecedented, detailed diagnosis of an atypical case of a rare kind of anomalous color vision. The reader should be aware that the term "co-punctal point", which is used many times, is a contraction that means the point in a chromaticity diagram at which all of the confusion lines of an observer converge, i.e., are "co-punctal". The original, literally correct, usage "lines co-punctal" appears only once in this paper, nine lines below Fig. 4 on p. 835.

The term "ocular pigment" used in the paper may be a source of some confusion because, when first used in the text, on p. 839, "ocular" refers collectively to pigments in the crystalline lens and in the macula (central spot) of the retina. However, Fig. 6 and its caption indicate that "ocular" refers specifically to absorption and pigment in the refractive media of the eye. This is the sense in which "ocular" should be understood throughout the remainder of the paper. As mentioned on p. 839, the absorption in the refractive media is due chiefly to pigmentation of the crystalline lens. The authors evidently had lens pigment in mind whenever they wrote "ocular pigment" from p. 840 to the end of the paper.

Tritanopia with Abnormally Heavy Ocular Pigmentation

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The chromaticity confusions characteristic of tritanopia may be represented on the (x, y)-chromaticity diagram by a family of straight lines intersecting at a copunctal point near the shortwave extreme of the spectrum locus. A case of congenital tritanopia is reported that departs from typical tritanopia, first by having a luminosity function abnormally curtailed on the shortwave end, second by having chromaticity confusions among object colors describable by straight lines on the (x, y)-plot intersecting in an area surrounding the spectrum locus at 460 mμ instead of near the shortwave extreme, and third by confusing incandescent lamp light at a color temperature of 2900°K with the spectrum at 586 mμ instead of the typical tritanopic value of 579 mμ. It has been found that all three of these disagreements with typical tritanopia are to be expected from a tritanope possessing normal macular pigmentation combined with an ocular pigment five times normal. We believe, therefore, that this case of atypical tritanopia departs from typical tritanopia because of abnormally heavy ocular pigmentation.

I. INTRODUCTION

TRITANOPIA is a form of abnormal vision frequently encountered in diseases of the retina such as inflammation of the retina or separation of the retina from contact with the choroid coat. The inflamed or detached portion of the retina yields color perceptions of red and bluish green or mixtures thereof,[1] but is incapable of distinguishing between gray and what appears to the healthy part of the eye either as greenish yellow or as bluish purple. The tritanopic area of the retina can yield an exact match for any part of the spectrum (and indeed any color whatever) from a mixture in suitable proportions of the extreme long-wave end of the spectrum (red) with a part of the spectrum near 470 mμ (blue). Tritanopia is therefore said to be a dichromatic form of vision; that is, the color matches set up by means of a tritanopic retinal area can be expressed as a function of two variables. This distinguishes it from normal vision, said to be trichromatic since it requires three variables.

If a sufficiently large portion (say 2° subtense) of a healthy retina, however, be used to distinguish the colors of two sufficiently bright fields (luminance of, say, 1 millilambert or more), it is very rare that tritanopic mistakes are made. The one observer in a million having such retinas with no previous history of retinal disease is called a congenital tritanope, and congenital tritanopes are eagerly sought after because by studying their color matches and perceptions we ought to be able to obtain more precise information regarding tritanopia than in any other way. A case of what appeared to be congenital tritanopia was reported by one of us[2] in 1943. It is the purpose of this paper to compare this case with the accepted view of typical tritanopia, to report the results of our calculations on the influence of variations in pigmentation of the eye media on color matches made by a tritanope, and from these results to give an interpretation of this case of congenital tritanopia.

Interest in tritanopia has increased in recent years because of rediscovery of the fact announced by König in 1894[3] that when the normal eye is used to detect the difference between small colored spots by central fixation it makes tritanopic mistakes, and is, in fact, tritanopic.[4] We now know that the normal eye tends to make tritanopic errors whenever it is used to compare

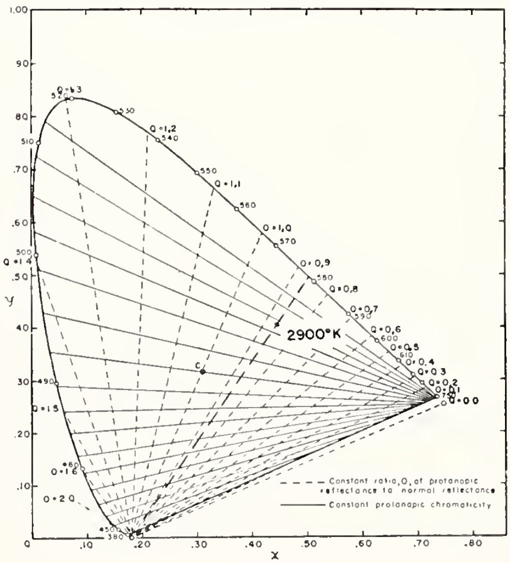

FIG. 1. Chromaticity confusions (dotted lines) of the classical tritanope on the (x, y)-chromaticity diagram (see reference 10). Note that the classical tritanope sees the spectrum at about 567 mμ as having the same chromaticity as standard source C (representative of average daylight), and that he sees the spectrum at about 579 mμ as having the same chromaticity as an incandescent lamp at a color temperature of 2900°K.

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[1] H. Köllner, *Die Störungen des Farbensinnes* (Karger, Berlin, 1912).

[2] D. Farnsworth, J. Opt. Soc. Am. **33**, 350 (1943).

[3] A. König, *Ueber den menschlichen Sehpurpur und seine Bedeutung für das Sehen* (Sitz. Akad. Wiss. Berlin, June, 1894), p. 577; also *Gesammelte Abhandlungen* (Leipzig, Barth, 1903), p. 338.

[4] E. N. Willmer and W. D. Wright, Nature **156**, 119 (1945).

TABLE I.

		Tritanomalous Figure	Tritanomalous Background
Normal	Figure	5 *GY* 7/6	7.5 *Y* 6/6
		5 *GY* 6/4	7.5 *Y* 7/6
	Background	7.5 *PB* 7/6	2.5 *P* 7/6
		7.5 *PB* 6/6	5 *P* 6/4

the colors of two fields either too dimly lit or too small to be correctly seen.[5–7]

II. CLASSICAL TRITANOPIA

The accepted view of tritanopia is based largely upon measurements and reports by nine pathological cases studied by König.[8] He found that color matches between various parts of the spectrum and a two-part mixture of spectrum lights (apparently 480 with 650 mμ) was set by these tritanopes in such a way that they would find matched any metameric pair set by an observer of normal vision. On this account tritanopia is called a reduction form of normal vision, and we can show the chromaticity confusions characteristic of tritanopia on the same chromaticity diagram used for normal vision. Figure 1 shows (dotted lines) on the (x, y)-chromaticity diagram of the standard ICI coordinate system for colorimetry[9] the chromaticity confusions characteristic of the nine cases of acquired tritanopia studied by König.[8] This figure was taken from a paper by Judd[10] and shows protanopic confusions (solid lines) as well as tritanopic (dotted). The tritanopic chromaticity-confusion lines intersect at a co-punctal point ($x=0.18$, $y=0.00$) near the shortwave end of the spectrum locus. This choice of co-punctal point is consistent with the matches set by the six of the nine subjects so studied and with the reports of all of them that the spectrum appeared greenish blue, bluish green, or green down to the shortwave end. The color matches and reports of color perceptions made by these observers do not determine the location of their co-punctal points exactly, but all must fall very near to $y=0.00$ and between $x=0.16$ and 0.20. In his theoretical account of tritanopia König[8] set the tritanopic co-punctal point to correspond closely to what we now know as $x=0.18$, $y=0.00$ as was shown by Judd.[10]

Subsequent studies of acquired tritanopia have confirmed the observations by König's observers.[11–15] Some of them,[11–13] show only one neutral point in the spectrum which falls in the neighborhood of 570 mμ, the longwave portion appearing red; the shortwave, bluish green. This corresponds to the location of the copunctal point at $x=0.18$, $y=0.00$; note on Fig. 1 that the dotted line passing through the point representing illuminant C (close to average daylight chromaticity) intersects the spectrum locus at only one point, that corresponding to about 567 mμ. Other studies[14,15] show

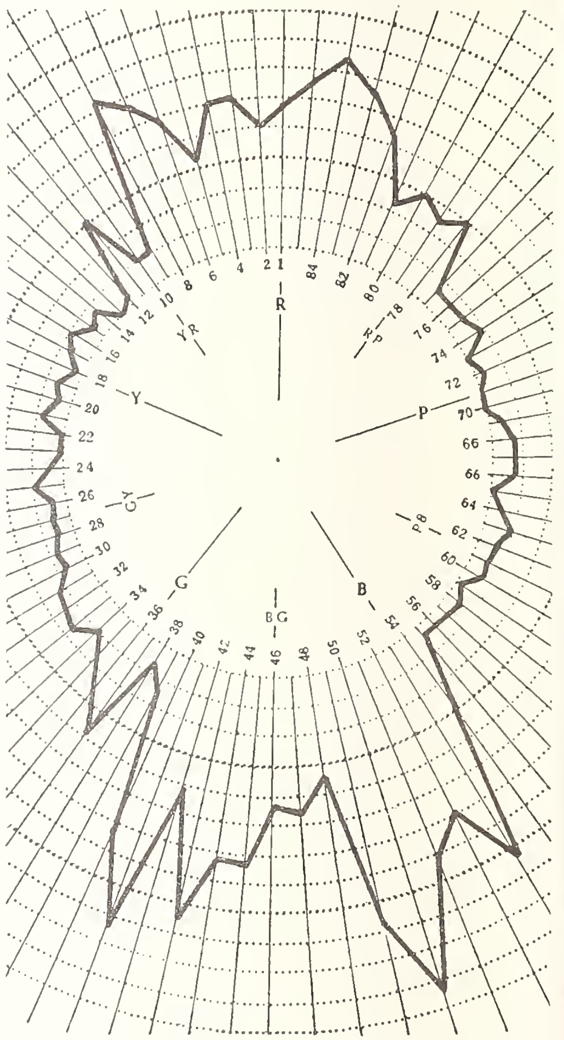

FIG. 2. Score made by Mr. V. on F-M 100-hue test, average of four trials, shown on the Farnsworth profile diagram. (See reference 28.) Mr. V's ability to distinguish hues in the yellow to green and purple to blue ranges is about equal to that of the normal eye, but he cannot distinguish yellowish reds from purplish reds, nor greens from blues.

[5] D. Farnsworth and J. D. Reed, Retention of discriminable hue of ten colors at small subtense, Color Vision Report No. 7, Med. Res. Lab., U. S. N. Submarine Base, 20 April 1944.

[6] H. Hartridge, Phil. Trans. Roy. Soc., London (B) **232**, 535 (1947); Nature **155**, 391, 657 (1945).

[7] W. E. K. Middleton, J. Opt. Soc. Am. **39**, 582 (1949).

[8] A. König, Ueber "Blaublindheit," Sitz. Akad. Wiss. Berlin, p. 718 (July 8, 1897); also *Gesammelte Abhandlungen*, p. 396 (Barth, Leipzig, 1903).

[9] Commission international de l'Éclairage, Proceedings of the 8th Session, Cambridge, England, 19–29 (September, 1931). A. C. Hardy, *Handbook of Colorimetry* (Technology Press, Cambridge, Massachusetts, 1936). D. B. Judd, J. Opt. Soc. Am. **23**, 359 (1933).

[10] D. B. Judd, J. Research Nat. Bureau of Stand. **33**, 407 (1944); *RP*1618; also J. Opt. Soc. Am. **35**, 199 (1945).

[11] H. Köllner, Zeits. f. Augenheilk. **19**, Ergänzungsheft (1908).

[12] H. Köllner, Zeits. f. Augenheilk. **17**, 234 (1907).

[13] H. Köllner, Zeits. Sinnesphysiol. **42**, 281 (1907).

[14] Collin and W. A. Nagel, Zeits. Sinnesphysiol. **41**, 74 (1907).

[15] H. Piper, Zeits. Psychol. **38**, 153 (1905).

a second neutral point or region in the neighborhood of 430 mμ. This corresponds to location of the copunctal point at $x=0.165$, $y=0.000$, as is possible also for one of König's nine tritanopes. Willmer and Wright[4] likewise found an indication of such a neutral region for small fields in the normal fovea, and Pitt[16] considers this to be typical of tritanopia though the cases cited[8] by him in support of the view showed only the neutral point near 570 mμ. G. E. Müller[17] in his detailed analysis of the various types of color blindness recognizes the fact that individual cases of tritanopia may show either a neutral region near 430 mμ or no second neutral point at all, and suggests explanations for these individual differences in terms of his stage theory of color vision. It has been shown by Judd[18] that the Müller theory evaluated according to modern data on additive color mixture[9] accords best with a tritanopic co-punctal point at $x=0.165$, $y=0.000$.

There are also observers[19–26] having dichromatic vision and ability to distinguish red from green who are further characterized by confusion of yellow colors with blue. These observers have two neutral points in the spectrum, one in the neighborhood of 582 mμ, the other in the neighborhood of 470 mμ. Some students of colorblindness class these observers as tritanopes,[27] but Müller[17] has proposed a new name for them—tertartanopes. Because of the paucity of cases on record, because of the rather large individual differences between cases, and because of the fact that the ability to distinguish red from green is usually far below normal, there is doubt what the exact characteristics of this class are, and indeed doubt whether it is useful to have this class at all.

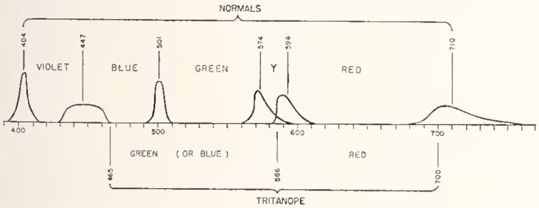

FIG. 3. Division of the spectrum according to hue by the normal eye (violet, blue, green, yellow, red) compared to that by Mr. V's tritanopic eye (green, red). The division points are subject to individual variation. Those for the normal eye are indicated here by approximate distribution curves derived from settings of 141 observers with normal color vision.

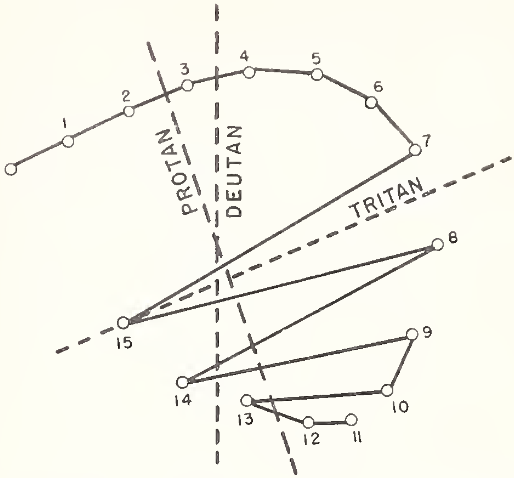

FIG. 4. Sequence of colors found by Mr. V. on the dichotomous test (see reference 28). The normal eye sees these colors (various Munsell hues at value 5/, chroma /4) in serial order (0 to 15). The kinds of confusion made by the three types of dichromat and the corresponding anomalous trichromat are indicated by dotted lines (protan, deutan, tritan).

It may be concluded that the classic view of tritanopia derived from pathological cases is that it is a reduction form of vision whose chromaticity confusions therefore may be derived from the chromaticity diagram of the standard ICI colorimetric coordinate system. These confusions are indicated by a family of straight lines co-punctal either at $x=0.18$, $y=0.00$, or at $x=0.165$, $y=0.000$, or at some intermediate point. Thus, from Fig. 1 it is seen that a retinal region rendered tritanopic by inflammation or detachment should find the spectrum between 567 and 568 mμ to have the chromaticity of standard illuminant C, that is, this part of the spectrum that appears normally greenish yellow cannot be distinguished from daylight by a tritanopic retinal area. Similarly, the yellowish color of a gas-filled incandescent lamp at a color temperature of 2900°K would be confused by a classical tritanope with the color of the spectrum at about 578 to 579 mμ. And, in general, if two color chips of known chromaticity coordinates (x, y) are represented by points on the same line of any of the family of lines passing through the copunctal point, we would expect a classical tritanope to find them to be of the same chromaticity.

III. DESCRIPTION OF A CASE OF ATYPICAL CONGENITAL TRITANOPIA

The subject of this report, Mr. V., was the father of two children, sons, one deuteranomalous and the other protanomalous. Examination of the two sons revealed only traits which are characteristic in these types. Their mother has normal color vision. The family traditions contain but one item of evidence related to color facility in the ancestry; the mother of the subject was

[16] F. H. G. Pitt, Proc. Roy. Soc. (B) **132**, 101 (1944).
[17] G. E. Müller, *Darstellung und Erklärung der verschiedenen Typen der Farbenblindheit* (Göttingen, Vandenhoeck, & Ruprecht, 1924), p. 51.
[18] D. B. Judd, J. Research Nat. Bureau of Stand. 42, 1 (1949); *RP*1946.
[19] S. Alrutz, Psyke **10**, 1, 130 (1915).
[20] E. Hering, Pflügers Arch. f. d. ges. Physiol. **57**, 308 (1894).
[21] G. Hermann, *Ein Beitrag zur Casuistik der Farbenblindheit*, Inaug. Diss. (Dorpat, 1882).
[22] D. B. Judd, J. Opt. Soc. Am. **33**, 294 (1943).
[23] F. Richardson, Psych. Bull. 8, 55 (1911).
[24] F. Richardson-Robertson, Amer. J. Psych. 34, 157 (1923).
[25] E. Uhry, *Beitrag zur Casuistik der Blau-Gelbblindheit*, Diss. (Strassburg, 1894).
[26] M. von Vintschgau, Pflügers Arch. f. d. ges. Physiol. **57**, 191 (1894).
[27] G. F. Göthlin, Acta Ophthal. **21**, 88 (1943).

TABLE II. Relative spectral luminosities for atypical congenital tritanope.

λ in mμ	(a) Relative lumi-nosities for 2900°K source	(b) Lumi-nosities relative to unity at 580 mμ for equal-energy source	λ in mμ	(a) Relative lumi-nosities for 2900°K source	(b) Lumi-nosities relative to unity at 580 mμ for equal-energy source
400	0	0.000	575	691	0.994
425	0	0.000	600	636	0.807
450	2	0.010	625	385	0.423
475	12	0.043	650	165	0.161
500	66	0.173	675	33	0.030
525	175	0.370	700	2	0.002
550	396	0.687	725	0	0.000

noted for her ability as a pearl matcher. Two of his cousins are anomalous trichromats.

The congenital nature of his tritanopia is affirmed by incidents from his history as a jeweler. At the age of 21 while apprenticed to a clockmaker he could find no difference between the green and blue enamel tiles which he had been assigned to set alternately in the minute positions on a clock face. In later life his children remember being called in from play to help him distinguish emeralds from sapphires.

The subject has had no severe illnesses or accidents in his lifetime and carried on his work as a jeweler and watch repairer up to the age of 66. Regular examinations (one son is an optometrist) revealed no ocular abnormalities up to the time of our last test sessions. His lenses became cataractous two years later. Tests were made at ages of 61, 63, and 66.

1. Tests on Polychromatic Plates

The subject gave normal responses on all Ishihara, Rabkin, and American Optical Company Plates. He made no errors on a replica of the Royal Canadian Navy Lantern or the 1940 Edition of the Inter-Society Color Council Color Aptitude Test. Indeed, his performance on all tests for the detection of red-green blindness was above average.

His responses to the 1929 Stilling plates intended to detect tritanomaly were ambiguous (as they are for many normals) and indicate that the colors in these and the Rabkin tritanomalous plates were ineffectively chosen for detecting color deficiency of this subject's type. However, a polychromatic plate of the reversible figure type was constructed based on an apparent co-punctal point at 460 mμ which was effective. Two other plates, based on apparent co-punctal points at 400 and at 475, were not effective. The Munsell designations of the colors used in the successful plate are given in Table I.

2. F-M 100-Hue Test

The average of four examinations made in 1942 and 1943 with the Farnsworth-Munsell 100-Hue Test[28] is shown in Fig. 2. It indicates near-normal color discrimination between red and blue-green through neutral (medians of the error scores are at 1 and 46), between yellow and green and between purple-blue and red-purple. Confusions are indicated between greens and blues, between red-purples and yellow-reds, and between greenish yellow, neutral and bluish purple.

[28] D. Farnsworth, J. Opt. Soc. Am. 33, 568 (1943).

3. Spectral Settings

Spectral divisions were made with a high intensity projection spectroscope which used a 1000-watt projection lamp as the source and was burned at approximatley 2900°K. A spectrum was projected on a white screen along which a pointer could be moved; the transilluminated background had a luminance of 8 foot-lamberts, of the same color temperature as the source; spectral luminance at 570 mμ was about 12-foot-lamberts. Observers were asked to "set the needle at points between the colors." Figure 3 shows the averages and approximate distributions of settings made by 141 normal observers in comparison with the settings made by the subject. All spectral stimuli below 585 mμ were called "blue" by the subject (or, "blue or green- some people say one thing and some the other") and all above 588 mμ were called "red or reddish." The 586-mμ region matched the background color. The shortest wave-length at which the subject was certain of chromatic color by this method was at about 465 mμ. Special observing conditions with a dark surround gave an absolute limit of sensation at 447 or 450 mμ. This is the point at which the average normal divides blue from violet. The brilliant violet region was quite invisible to the subject. No neutral point was found as far as the subject could see in the shortwave end of the spectrum when light adapted.

4. Dichotomous Test

The subject's pattern on the Dichotomous test is that given on page 6 of the Manual[29] and is shown in

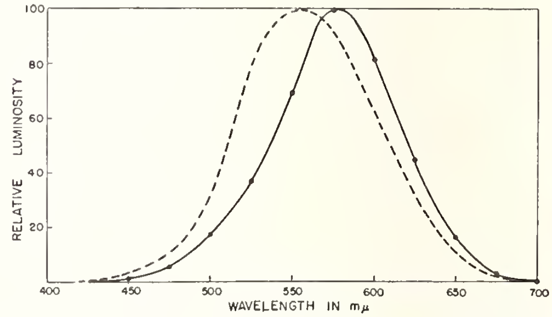

FIG. 5. Comparison of Mr. V's relative spectral luminosity function (solid curve) with that of the standard observer (dotted curve). Mr. V finds the brightest part of the equal-energy spectrum at about 580 mμ instead of the usual 555 mμ, and the spectrum seen by him is considerably cut off on the short-wave end compared to that seen by an observer with normal vision.

[29] Dean Farnsworth, *Manual of the Farnsworth Dichotomous Test for Color Blindness* (The Psychological Corporation, New York, 1947).

Table III. Munsell chips found by atypical congenital tritanope to have identical chromaticities in daylight.

Greenish yellow chip: Munsell production number	Munsell hue	Chromaticity coordinates (illuminant *C*) *x*	*y*	Purplish chip of closest chromaticity: Munsell production number	Munsell hue	Chromaticity coordinates (illuminant *C*) *x*	*y*	*x*-intercept of the straight line connecting the points representing the two chromaticities
				Color matches found in 1943				
143	7*G*	0.277	0.364	162	6*B*	0.243	0.276	0.14
142	6*G*	0.280	0.370	163	7*B*	0.244	0.274	0.14
142	6*G*	0.280	0.370	164	8*B*	0.244	0.272	0.14
141	5*G*	0.283	0.373	165	9*B*	0.248	0.272	0.15
136	10*GY*	0.315	0.398	169	3*PB*	0.257	0.265	0.14
134	8*GY*	0.326	0.407	169	3*PB*	0.257	0.265	0.13
132	6*GY*	0.340	0.413	170	4*PB*	0.259	0.262	0.12
131	5*GY*	0.343	0.411	173	7*PB*	0.266	0.259	0.14
129	3*GY*	0.361	0.425	173	7*PB*	0.266	0.259	0.12
126	10*Y*	0.380	0.425	176	10*PB*	0.279	0.261	0.12
123	7*Y*	0.401	0.431	178	2*P*	0.289	0.266	0.11
122	6*Y*	0.406	0.431	179	3*P*	0.295	0.269	0.11
121	5*Y*	0.411	0.434	180	4*P*	0.296	0.267	0.11
118	2*Y*	0.425	0.425	184	8*P*	0.316	0.273	0.12
117	1*Y*	0.426	0.421	185	9*P*	0.321	0.275	0.12
116	10*YR*	0.427	0.413	188	2*RP*	0.334	0.282	0.13
115	9*YR*	0.429	0.407	187	1*RP*	0.330	0.278	0.12
114	8*YR*	0.431	0.401	185	9*P*	0.321	0.275	0.08
111	5*YR*	0.423	0.382	190	4*RP*	0.350	0.290	0.12
110	4*YR*	0.428	0.376	192	6*RP*	0.354	0.291	0.10
107	1*YR*	0.418	0.357	194	8*RP*	0.363	0.295	0.10
								Mean 0.12
								PE Mean 0.0025
				Color matches found in 1945				
141	5*G*	0.283	0.373	165	9*B*	0.248	0.272	0.15
140	4*G*	0.290	0.383	164	8*B*	0.244	0.272	0.13
139	3*G*	0.297	0.386	164	8*B*	0.244	0.272	0.12
137	1*G*	0.308	0.393	163	7*B*	0.244	0.274	0.10
137	1*G*	0.308	0.393	164	8*B*	0.244	0.272	0.10
137	1*G*	0.308	0.393	165	9*B*	0.248	0.272	0.11
136	10*GY*	0.315	0.398	165	9*B*	0.248	0.272	0.10
134	8*GY*	0.326	0.407	167	1*PB*	0.252	0.267	0.11
133	7*GY*	0.333	0.409	168	2*PB*	0.254	0.266	0.11
133	7*GY*	0.333	0.409	169	3*PB*	0.257	0.265	0.12
132	6*GY*	0.340	0.413	170	4*PB*	0.259	0.262	0.12
131	5*GY*	0.343	0.411	170	4*PB*	0.259	0.262	0.11
131	5*GY*	0.343	0.411	171	5*PB*	0.263	0.265	0.12
129	3*GY*	0.361	0.425	172	6*PB*	0.265	0.263	0.11
129	3*GY*	0.361	0.425	173	7*PB*	0.266	0.259	0.12
127	1*GY*	0.373	0.428	174	8*PB*	0.271	0.261	0.11
126	10*Y*	0.380	0.425	175	9*PB*	0.277	0.263	0.11
126	10*Y*	0.380	0.425	176	10*PB*	0.279	0.261	0.12
124	8*Y*	0.394	0.427	177	1*P*	0.286	0.268	0.10
123	7*Y*	0.401	0.431	178	2*P*	0.289	0.266	0.11
121	5*Y*	0.411	0.434	179	3*P*	0.295	0.269	0.10
119	3*Y*	0.422	0.428	181	5*P*	0.302	0.271	0.10
119	3*Y*	0.422	0.428	182	6*P*	0.309	0.274	0.11
116	10*YR*	0.427	0.413	183	7*P*	0.313	0.271	0.10
116	10*YR*	0.427	0.413	184	8*P*	0.316	0.273	0.10
116	10*YR*	0.427	0.413	185	9*P*	0.321	0.275	0.11
115	9*YR*	0.429	0.407	186	10*P*	0.323	0.275	0.10
114	8*YR*	0.431	0.401	187	1*RP*	0.330	0.278	0.10
113	7*YR*	0.429	0.392	187	1*RP*	0.330	0.278	0.09
113	7*YR*	0.429	0.392	188	2*RP*	0.334	0.282	0.09
112	6*YR*	0.427	0.386	189	3*RP*	0.340	0.285	0.09
								Mean 0.11
								PE Mean 0.0015

Fig. 4 on a projective transformation of the ICI chromaticity diagram.

The serial order, 14, 9, 10, 13, 12, 11, produces a pattern somewhat counterclockwise of what would be expected from classical tritanopia[10] if no allowance were made for such individually variable factors as ocular pigmentation.

5. Luminosity Function

The subject's luminosity data were taken in 1948. By using the same instrument, the luminance of a test patch of the source was equated to those of stimuli of selected wave-lengths from 400 to 725 mμ. The surround of the test patches was black, room illumination dim. The patches were small, each subtending 1° by 1.5° visual angle. Variable instrument errors could have amounted to as much as 3 mμ, and no corrections were applied for possible variations of transmittance of the spectroscope with wave-length. Luminance reduction was accomplished by the use of Wratten filters which are accurate to about five percent of their densities. Values were obtained by hetero-chromatic matching. When the data are examined these sources of possible error should be remembered.

Table II gives the average of four readings for relative luminance per unit wave-length in column (*a*), calculated for an equal-energy source and adjusted to unity at the maximum value in column (*b*). The latter figures are plotted in Fig. 5 in comparison with the standard luminosity curve.

Regardless of uncertainties in the data there were three unequivocal results outside the range of experimental error, (a) a decided depression of the short-wave side of the curve was found, (b) the luminosity peak shifted about 20 mμ in the long-wave direction compared to the standard luminosity curve and (c) a perfect chromatic match was obtained for the background near the peak of the curve. This latter was repeatedly verified; the subject was very sensitive to a 2-mμ change in either side of the 586-mμ region, always recognizing one as bluish, the other as reddish.

6. Contiguous Matches

Matches were made with circular disks of Munsell papers under Macbeth daylight lamps approximating illuminant *C*, with color temperatures averaging about 6400°K. The visual angle subtended by each disk was approximately 3°. One of the matching disks could be tilted to the incident light until a brightness match was obtained. Many sets of Munsell papers were used, embracing both high and low chromas at several value levels and all gave comparable results. However, the 100-hue series gave the most reliable data because the small color interval between steps supplied an almost continuous hue series against which to match; see Table III.

When the matching chromaticities from the 5/5 series were plotted on (x, y) diagrams and the points connected by straight lines the extensions intersected closely, in 1943 near $x=0.180$, $y=0.100$ and in 1945 near $x=0.155$, $y=0.075$. Other sets of matches gave similar results but with more scatter of the intersections.

Although these reports by Mr. V. correspond in an approximate way to typical acquired tritanopia, they differ from classical tritanopia in three respects. In the first place, plot of the pairs of points specified as near chromaticity matches in Table III and connection of the members of each pair by a straight line on the (x, y)-chromaticity diagram fail to yield a family of straight lines intersecting at a point intermediate be-

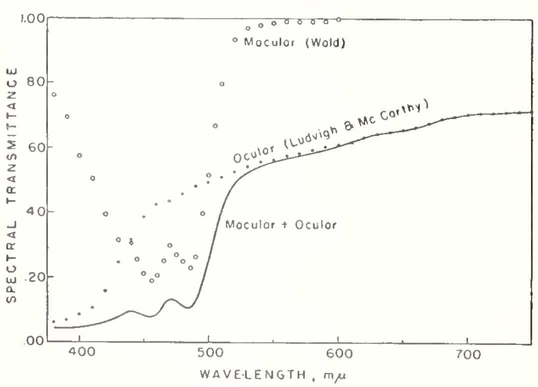

FIG. 6. Spectral transmittance of the ocular media (cornea, lens, humours) according to Ludvigh and McCarthy (see reference 31) (dots), and spectral internal transmittance of the macula lutea (circles) according to Wald. (See reference 30.) The curve shows the product of these two spectral transmittances and for the normal eye is our best estimate of the ratio of the radiant flux incident upon the fovea to that incident on the cornea so as to pass through the pupil from a source at the fixation point.

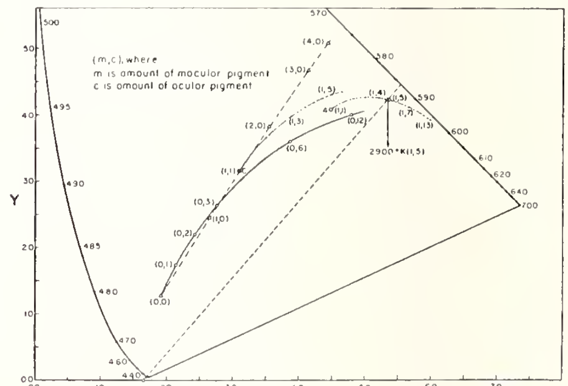

FIG. 7. Changes in chromaticity produced by various multiples of the normal macular and normal ocular pigmentation. The point (0, 0) corresponds to the chromaticity of standard source *C* viewed by a pigment-free but otherwise normal eye. The point (1, 1) corresponds to source *C* for the standard observer; the point *A*(1, 1) corresponds to source *A* for the standard observer. Note from the dotted straight line that the classical tritanope with normal macular pigment and 5 times normal ocular pigmentation would find an incandescent lamp at a color temperature of 2900°K to have the same chromaticity as the spectrum at 586 mμ.

tween $x=0.165$, $y=0.000$ and $x=0.18$ and $y=0.00$. Instead the x-intercepts average 0.12 for the 1943 matches (see last column, Table III) and 0.11 for the 1945 matches. In the second place the part of the spectrum having the same chromaticity as a tungsten-filament lamp of color temperature 2900°K is not between 578 and 579 mμ, but is at 586 mμ. And in the third place, Mr. V. could not see the spectrum at the short-wave end much below 450 mμ; but an area of the retina rendered tritanopic by disease has a luminosity function not significantly different from normal and in the usual spectroscope will easily serve to detect light of wave-length less than 420 mμ. It is concluded, therefore, that Mr. V., if tritanopic at all, must be classed as atypically tritanopic. Furthermore, there is an indication from the x-intercepts (odds 50 to 1 against a pure chance explanation), that Mr. V's color-vision system changed in a two-year interval still further from classical tritanopia.

IV. INFLUENCE OF OCULAR PIGMENTATION ON TRITANOPIC CONFUSIONS

The first two ways in which Mr. V's reports depart from classical tritanopia suggest that his eye media have more than the usual yellow pigmentation. His failure to respond to the short-wave end of the spectrum might be expected because of absorption of an abnormally high proportion of such energy by a heavy pigmentation of the ocular media. Furthermore, the chromaticity of an incandescent lamp at a color temperature of 2900°K, represented for the ICI standard observer at $x=0.444$, $y=0.406$, would for a heavily pigmented eye be shifted toward the long-wave end of the spectrum and might be representable by a point falling on the line connecting the tritanopic co-punctal point with that representing 586 mμ as found by Mr. V. However, the changes in the chromaticity confusions between object colors caused by making the classical tritanope view them through an amber filter, such as one corresponding to some multiple of the normal pigmentation of the eye media, are complicated. They cannot be determined precisely unless the spectral transmittances of the amber filter, and the spectral reflectances of the objects viewed are both known.

There are two principal pigments in the ocular media. One of these is a melanin type pigment that gathers gradually in the normal crystalline lens as the subject grows older. The other is the macular pigment suffusing the retina in a central spot subtending 3 or 4° identified by Wald with a high degree of probability as xanthophyll.[30] Figure 6 shows (circles) the spectral internal transmittance of the macular pigment according to Wald, and it also shows (dots) the spectral transmittance of the refractive media of the eye according to the measurements of Ludvigh and McCarthy for an average age of 21.5 years.[31] Even at this comparatively low age the absorption of light by the refractive media of the eye is due chiefly to the pigmentation of the crystalline lens. The solid curve on Fig. 6 is the product of the spectral internal transmittance of the macular pigment by the spectral transmittance of the refractive media of the eye, and it represents the spectral transmittance of the macular and ocular pigments combined.

Figure 7 shows on the (x, y)-chromaticity diagram of the standard ICI system the chromaticity changes

TABLE IV. Chromaticity coordinates of 18 of the Munsell 5/5 papers illuminated by standard illuminant C and viewed through an amber filter equivalent to 5 times the normal pigmentation of the refractive media of the eye (see Fig. 6).

Munsell production number	Nominal Munsell hue	Chromaticity coordinates (Illuminant C combined with 5 times normal lens pigment) x	y
110	4*YR*	0.532	0.418
114	8*YR*	0.524	0.432
117	1*Y*	0.513	0.447
121	5*Y*	0.497	0.462
126	10*Y*	0.474	0.469
130	4*GY*	0.451	0.479
136	10*GY*	0.422	0.476
138	2*G*	0.410	0.477
141	5*G*	0.398	0.468
164	8*B*	0.375	0.422
166	10*B*	0.385	0.419
169	3*PB*	0.397	0.415
172	6*PB*	0.409	0.415
175	9*PB*	0.428	0.409
180	4*P*	0.454	0.403
183	7*P*	0.473	0.400
188	2*RP*	0.493	0.398
193	7*RP*	0.513	0.393

TABLE V. Standard relative luminosity function, $\bar{y}_\lambda$, for equal-energy spectrum adjusted to accord with ocular pigmentation 6 times that found by Ludvigh and McCarthy for young normally pigmented eyes.

Wave-length in mμ λ	Standard relative luminosity (*ICI* obs.) $\bar{y}_\lambda$	Spectral transmittance of normal ocular media T_λ	Relative spectral luminosity: Standard eye with 6 times normal ocular pigment $T_\lambda{}^6\bar{y}_\lambda/(T_\lambda{}^6\bar{y}_\lambda)_{max}$	Relative spectral luminosity: Observed for Mr. V (From Table II)
400	0.0004	0.086	0.000	0.000
425	0.0073	0.204	0.000	0.000
450	0.0380	0.388	0.005	0.010
475	0.1126	0.448	0.031	0.043
500	0.3230	0.495	0.150	0.173
525	0.7932	0.534	0.533	0.370
550	0.9950	0.566	0.889	0.687
575	0.9154	0.588	0.994	0.994
600	0.6310	0.610	0.820	0.807
625	0.3210	0.636	0.515	0.423
650	0.1070	0.657	0.203	0.161
675	0.0232	0.683	0.053	0.030
700	0.0041	0.705	0.011	0.002

[30] G. Wald, Science **101**, 653 (June 29, 1945).
[31] E. Ludvigh and E. F. McCarthy, Arch. Ophthal. **20**, 37 (July, 1938).

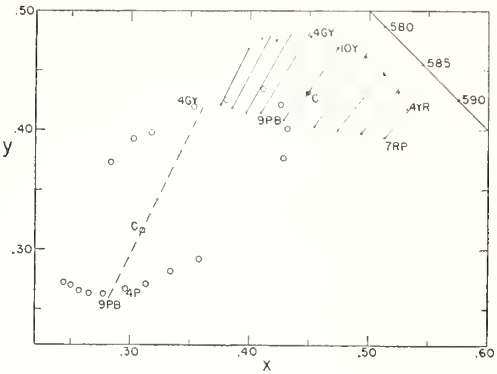

FIG. 8. Effect of increasing ocular pigmentation by a factor of 5. The chromaticities of certain of the Munsell papers (value 5/, chroma /5) illuminated by standard source C are shown on the (x, y)-chromaticity diagram both for direct observation by the standard observer (circles), and for observation through a filter having 4 times the yellowish pigmentation of the normal ocular media (dots). The straight lines coursing among these points are drawn so as to pass through the co-punctal point ($x=0.165$, $y=0.000$) of the classical tritanope. Note that interposition of the yellow filter causes different pairs of Munsell papers to be confused by the classical tritanope; for example, the pair *9PB-4GY* is changed nearly to *9PB-10Y*.

produced by viewing standard illuminant C (representative of average daylight) through various multiples of the normal amounts of macular and ocular pigmentation represented in Fig. 6. The chromaticity coordinates of these points have been computed by a routine method (summation over wave-length intervals of 10 mμ). The usual location of the point representing illuminant C ($x=0.3101$, $y=0.3163$) is considered to correspond to the normal amount both of macular and of ocular pigmentation. It is identified by the pigment notation (1, 1). The chromaticity corresponding to removal of both these amounts is notated (0, 0); this corresponds to the pigment-free or albino eye. The chromaticity corresponding to the normal amount of macular pigment and zero amount of ocular pigment is notated (1, 0). And, in general, the notation is (m, c), where m is the multiple of the normal macular pigment to which the computations apply, and c is the multiple of the normal ocular pigment.

It will be noted from Fig. 7 that the chromaticity changes corresponding to increasing the amount of macular pigment change the long-wave tritanopic neutral point only slightly. This may be seen from the fact that the line connecting them is nearly parallel to the straight lines passing through the tritanopic copunctal point shown on Fig. 7 as $x=0.165$, $y=0.000$. The chromaticity changes corresponding to increase in the amount of lens pigment, however, are such as to increase considerably the long-wave tritanopic neutral point. This may be seen from the curvature of the solid line on Fig. 7. These chromaticity differences between the influence of the macular and ocular pigments arise, of course, from their differing spectral characteristics (see Fig. 6). Oddly enough, the influence of the normal amount of macular pigment is closely the same as the influence of three times the normal amount of ocular pigment, but thereafter there is an increasing divergence between the two loci; compare the solid line on Fig. 7 with the dot-dash line. We cannot look for an explanation of the abnormally high wave-length of Mr. V's neutral point (586 mμ for 2900°K instead of 578 mμ) in terms of abnormally heavy macular pigment. A plausible explanation could rest on abnormally heavy lens pigment, since it is believed that the greatest change in pigmentation with age occurs in the lens and Mr. V's age was over 60 when first tested.

The dotted line on Fig. 7 passing through the point representing standard illuminant A ($x=0.4476$, $y=0.4075$) shows the results of similar computations with normal macular pigmentation for increasing amounts of ocular pigment. It is seen that at five times normal ocular pigment, the classical tritanope would expect to find illuminant A (color temperature of 2854°K) identical in chromaticity to the spectrum at about 586.5 mμ, and a lamp at a color temperature of 2900°K would correspond to 586 mμ; see point x close to the intersection of the curved dotted line with the straight dotted line passing through the co-punctal point and the spectrum locus at 586 mμ. We have, therefore, sought an explanation of Mr. V's atypical chromaticity matches between the various Munsell chips in terms of classical tritanopia modified by five times the normal ocular pigment of a young eye.

The chromaticities of the Munsell papers of value 5/ and chroma /5, used for testing Mr. V. have already been published for standard illuminant C by Granville,

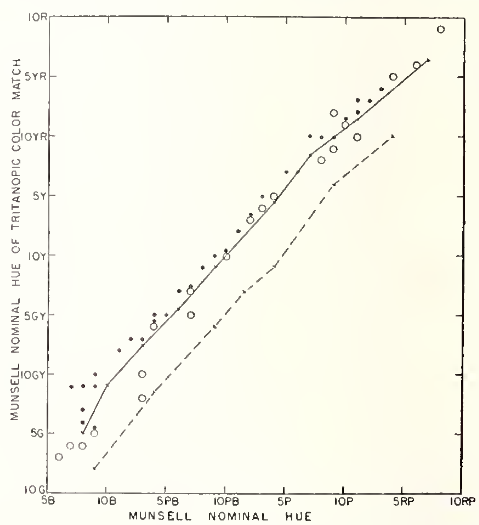

FIG. 9. Comparison of Mr. V's chromaticity matches (circles and dots) with those of the classical tritanope (dotted line) and with those of the classical tritanope having 5 times normal ocular pigmentation (solid line). Note that Mr. V's 1943 observations (circles) correspond to the classical tritanope with about 5 times normal ocular pigmentation, and his 1945 observations correspond to about 6 times normal ocular pigmentation.

Nickerson and Foss;[32] see their Table III. The chromaticities of 18 of the same papers have been computed for illuminant *C* combined with 5 times normal lens pigment by 10 mμ summation from unpublished spectral reflectances of those papers supplied to us through the courtesy of Granville, Nickerson and Foss. Table IV gives these chromaticity coordinates (x, y).

Figure 8 compares the chromaticities of 18 of these Munsell papers of value 5/ and chroma /5 both for illuminant *C* (circles) and for illuminant *C* screened through 5 times the normal lens pigment (dots). The influence of interposing a filter of five times the normal lens pigment (spectral transmittances shown in Fig. 6) on the chromaticity of illuminant *C* is shown by comparing the location of the open square with that of the solid square. This is the same comparison shown on Fig. 7 between the notations (1,1) and (1,5). The chromaticity points of the Munsell papers all undergo fairly similar shifts except that the points representing blue and purple colors shift somewhat more than the others. The result is that the Munsell 5/5 locus is not only shifted toward the reddish yellow but shortened in the blue-yellow direction and rotated slightly counterclockwise. Because of this rotation straight lines through the copunctal point ($x=0.165$, $y=0.000$) connect different pairs of Munsell papers; see straight lines drawn on Fig. 8. For example, the dotted lines on Fig. 8 indicate that a classical tritanope should find the paper of nominal Munsell hue 9*PB* nearly the same chromaticity as 4*GY* if they are both illuminated by illuminant *C*, but if the classical tritanope views them through an amber filter corresponding to five times the normal lens pigment he would instead find 9*PB* closely similar to 10*Y*.

V. COMPARISON OF ATYPICAL TRITANOPE WITH A HEAVILY PIGMENTED CLASSICAL TRITANOPE

Table V gives values for the standard luminosity curve for an equal-energy spectrum and for the same values adjusted to six times the absorbance of normal ocular pigment which corresponds to the prediction for Mr. V's eyes at the time his luminosity curve was taken in 1948. Comparison of these predicted values with those in Table II found by actual observation shows good agreement.

Mr. V. found that a lamp of color temperature 2900°K had the same chromaticity as the spectrum at 586 mμ. Since we found the multiple of normal lens pigment to be 5 by this criterion, in this respect there is perfect agreement between the actual and theoretical observer.

The pairs of Munsell 5/5 papers that yield equivalence for the theoretical observer can be read with an uncertainty of less than one Munsell hue step from the straight lines drawn on Fig. 8 so as to pass through the tritanopic copunctal point ($x=0.165$, $y=0.000$). These can be compared conveniently with the chromaticity matches found by Mr. V. by plotting for each pair of papers the Munsell nominal hue of the paper found to match against the Munsell nominal hue of the first paper of the pair; see Fig. 9. The circles represent the pairs found by Mr. V. in 1943; the dots, those found in 1945, both taken from Table III. The chromaticity equivalences for the theoretical observer (classical tritanope with five times the normal lens pigment) are shown by the solid line, and those for the classical tritanope with normal macular and lens pigment are shown by the dotted line. It is seen that the chromaticity matches found among the Munsell 5/5 papers by Mr. V. do depart significantly from those of the classical tritanope with normal pigment, but that the 1943 matches are closely what would be expected from a classical tritanope with five times the normal lens pigment. In other words if lines were drawn on Fig. 8 to indicate Mr. V's chromaticity confusions the x-intercepts based on the circles would average 0.12 as shown in Table III but the x-intercepts based on the theoretical removal of 5 times normal lens pigment (dashed line) would average close to 0.165 in agreement with classical tritanopia. The 1945 matches, as was seen from the x-intercepts in Table III, depart from the classical tritanope even more than the 1943 matches, and apparently (from Fig. 9) correspond closely to what would be expected of the classical tritanope with six times normal lens pigment.

It is our interpretation, therefore, that our congenital tritanope is atypical almost wholly because of an abnormally heavy pigmentation of the lens. We would classify Mr. V. as a case of tritanopia with abnormally heavy ocular pigmentation.

VI. DISCUSSION

It is worth pointing out that although we have used the tritanopic co-punctal point ($x=0.165$, $y=0.000$) required by the Müller theory of vision[17,18] as the basis for this analysis, the case could have been analysed with equal success on the basis of the tritanopic copunctal point indicated by König's observations on nine cases of acquired tritanopia.[8] This basis would have resulted in finding somewhat more than five times normal lens pigment for the 1943 measurements by Mr. V., and somewhat more than six times normal lens pigment for the 1945 measurements. Therefore, we cannot decide between these two theoretical evaluations of the co-punctal point from our study of Mr. V. To yield an improved determination of the co-punctal point characteristic of tritanopia will require the reports of tritanopes whose ocular media are normally transparent.

[32] W. C. Granville, D. Nickerson, and C. E. Foss, J. Opt. Soc. Am. 33, 376 (1943).

OBJECT-COLOR CHANGES FROM DAYLIGHT TO INCANDESCENT FILAMENT ILLUMINATION
(with H. Helson and M. H. Warren)

Illum. Eng. 47, 221-233 (1952)

Section 3, Theory, of the paper is one of the most complete published reviews of the history and experimental basis for the von Kries coefficient law and the rediscoveries and elaborations of it.

Examination of p. 600 of the Evans (1943) reference reveals that the sentence quoted by Judd in the second paragraph of the second column on p. 230 was quoted by Evans from a memorandum submitted to him by D. L. MacAdam. That memorandum, which contained a proof that the coefficient law can be valid for only one set of primaries, persuaded Evans to rewrite his discussion of the law in terms of Wright's primaries. Evans indicates that he had originally written it in terms of the CIE (ICI) primaries.

The term "co-punctal point" that is used in Table IV and on pp. 229 and 230 signifies the common point of intersection of all the lines that are drawn through chromaticities that are confused by a dichromatic observer.

Object-Color Changes From Daylight to Incandescent Filament Illumination

By HARRY HELSON
DEANE B. JUDD
MARTHA H. WARREN

IT IS KNOWN that the eye adapts to a considerable extent to counteract changes in the quantity and quality of different sources of illumination but its powers of adaptation are not unlimited. The new state of adaptation itself, along with the change in spectral energies reaching the eyes, results in changes in the colors of objects as the source is changed. The present report is based upon a study of the colors of Munsell samples under Macbeth daylight and incandescent filament light (color temperature 2854°K) with a view toward formulating a quantitative theory for predicting the changes in passing from one to the other. A simplified Young-Helmholtz theory predicts fairly accurately the colors of the samples under source *A* from a knowledge of their chromaticity coordinates for that source. The theory does not attempt to predict color changes due to the influence of backgrounds of different reflectances but data for white, gray, and black backgrounds are presented. The quantitative theory is based upon the changed chromatic adaptation of the eye under source *A* as compared with its adaptation under source *C*.

When an observer takes an array of colored objects from natural daylight into a room illuminated by an incandescent filament source, he notices immediately that the colors perceived to belong to the objects have changed to a marked degree. The blues have become darker and much less saturated; the greens, yellower; and the purples much redder. These changes can be predicted by computation from the spectral reflectances of the objects by means of the C.I.E. standard observer by changing from standard source *C* (representative of average daylight) to standard source *A* (representative of an incandescent filament source). These predictions hold good for the daylight adapted eye. Unfortunately, the observer's eye begins to change almost immediately on entering the artificially illuminated room, and after five minutes or so the eye increases its sensitivity to the short-wave part of the spectrum so that many of the objects are perceived to have approximately the same colors that they had in daylight. This is known as color constancy. Thus, the blue objects during this period of adaptation to the reddish-yellow of the incandescent filament illumination regain some of the saturation that they had in daylight. The greens change back from yellowish-greens to greens or even bluish-greens; but the purples lose only a small part of the red that they gained in the transition from daylight to incandescent filament lighting.

The perception of the colors of objects in a room lighted by chromatic illumination is thus a combination of two effects; the first arising from the changed spectral character of the radiant energy leaving the objects, complicated, but known; the second arising from the changed state of adaptation of the eye, complicated, and not fully known.

The present study, carried out with the support of a grant from the I.E.S. Research Fund, is intended to evaluate the influence on color perception of the changed state of chromatic adaptation in passing from daylight to incandescent filament illumination. If this influence can be evaluated, the results will also be applicable to fluorescent lamps, such as the warm-white, producing about the same chromatic adaptive state as the incandescent filament source. Of course, the spectral composition of the warm-white fluorescent lamp will cause the radiant energy leaving the objects to be markedly different from that leaving objects illuminated by the incandescent filament source, but as noted above, this part of the problem has already been solved by means of the C.I.E. standard observer and coordinate system.

1. Method

In order to obtain reliable estimates of the colors of a standard set of objects such as the Munsell color chips, it is necessary to train a group of observers having normal color vision to report in

A paper presented at the National Technical Conference of the Illuminating Engineering Socity, August 27-30, 1951, Washington, D. C. Authors: Harry Helson, Professor of Psychology, University of Texas; Deane B. Judd, Physicist, National Bureau of Standards; Martha H. Warren, Research Assistant, Brooklyn College. This study was made at Brooklyn College during the 1950-51 academic year. Work done under Project No. 11 of the I.E.S. Research Fund.

terms of some color system with which they have become familiar. The color system chosen for this purpose was a modification of the Munsell system. It consisted of a description of color in terms of hue, lightness, and saturation as usually defined. Lightness and saturation (Munsell value and chroma) were estimated on an eleven-point scale ranging from 0 to 10 with 5 denoting medium lightness or saturation. Observers were permitted to make their estimates in half or quarter unit steps and often did so, so that in actual use the lightness-saturation scale was more than an eleven-step scale. Hues were denoted in terms of the four unitary (psychologically primary) qualities of red, yellow, green, or blue and their combinations. Thus a hue which seemed to have equal parts of red and yellow was called yellow-red (*YR*). A hue with equal parts of red and yellow-red was called red yellow-red (*RYR*) while equal amounts of yellow and yellow-red were called yellow yellow-red (*YYR*). Binary hues in the other regions of the color circle (or color "mixtures") such as yellow and green, green and blue, blue and red were similarly described. The continuum extending from one primary to the adjacent primary on the hue circle (red to yellow, yellow to green, green to blue, blue to red) was divided into eight parts, each part representing an equal amount of the next primary. Thus, to compute the average hue from a number of estimates of the same chip, a yellowish red was considered as having one part of yellow to six parts of red. The next step, a red yellow-red, was regarded as having two parts of yellow to six parts of red. This notation provided 32 hue steps from red through yellow to green to blue and back to red. The numerical notation was for purposes of computation only since the observers' estimates of hue were always in terms of the qualitative categories described above. This numerical scale also made it possible to translate observers' reports into Munsell notation which differs from ours in that the Munsell system employs purple (P) as a fifth primary hue whereas in the system used by the observers purple is regarded as a binary hue with equal amounts of red and blue. Furthermore, our observers' unitary blue did not correspond with the Munsell 5.0 blue which was almost always regarded as somewhat on the greenish side while Munsell 5.0 PB was almost always reported as pure, unitary blue. Thus our observers' estimates were based on an intuitive apprehension of the unitary hues, red, green, blue, yellow, and their mixtures.

Since the observers were inexperienced to begin with, it was necessary to instruct them regarding the meaning of the terms hue, lightness, and saturation and their specification in terms of the system just described. For this purpose the Munsell Book of Color was employed both to acquaint them with these three basic attributes of color and to demonstrate how they could be ordered on scales of equal-appearing intervals. Thus it was demonstrated how the difference in lightness or saturation between scale steps 3 and 4 was equal (or supposed to be equal) to the difference between steps 5 and 6 or any other two adjacent steps. The observers were then required to place Munsell color chips on the Student Charts and to compare their ordering with the Munsell Book of Color. This procedure was most effective in teaching the observers not only how to order colors with respect to hue, lightness and saturation but also served to direct their attention to differences in colors. Next the observers were required to place a single chip on a chart without the aid of a range of chips to establish a continuum. This meant that the observer had to be able to recognize a chip's Munsell value and chroma without regard to lighter or darker, more or less saturated chips to guide him. Finally, the observers were required to estimate the hue, lightness and saturation of the chips without reference to the Munsell Book of Color, thus anticipating what would be required in actual observations. When the observers were able to designate the chips to within one step of Munsell hue, value, and chroma, they were considered sufficiently trained to begin regular observations. It required about 8 hours of training and practice as just described for the observers to reach the criterion demanded for participation in this study. Training was conducted in a room having a good northern exposure giving a good approximation to standard source *C*. Out of the 9 college students trained in judging the colors, the 6 best were chosen as observers in the final experiments. It should be emphasized that our observers were not trained to try to remember how the chips looked in daylight or what they had called them during their practice periods. They were instructed, after they had attained proficiency in estimating the three Munsell attributes, to make the best estimates of the color chips as they looked under the conditions of this study. That memory played some part in the final

TABLE I.—Reflectance and Photometric Brightness of the Backgrounds.

Background Reflectance		Photometric Brightness (footlamberts)	
		Source *C*	Source *A*
Black	0.03	2.2	1.7
Gray	0.21	15.4	12.0
White	0.78	57.2	44.2

TABLE II.—Hue, Lightness and Saturation of 60 Munsell Samples on White and Black Backgrounds Under Sources C and A

Munsell renotation		Judgment Under Source C						Judgment Under Source A					
		White Background			Black Background			White Background			Black Background		
H	V/C	H	$S.D._M$	L/S	H	$S.D._M$	L/S	H	$S.D._M$	L/S	H	$S.D._M$	L/S
3.5R	2.3/2.2	8.7R	4.5	1.8/1.2	4.6R	1.8	3.0/3.4	2.5RP	13.8	2.1/2.4	10.0R	1.1	2.9/3.5
3.5R	4.06/4.8	8.7R	1.4	3.8/4.0	4.6R	1.5	4.8/3.9	4.6R	2.2	3.9/4.5	4.6R	1.5	5.3/4.2
5.0R	4.20/12.8	4.6R	0.4	5.0/11.8	5.4R	0.4	5.5/12.0	5.8R	0.5	5.5/12.7	5.4R	0.8	6.7/11.8
3.5R	6.96/7.0	0.8R	1.3	5.3/4.9	6.5R	1.4	6.5/5.1	8.3R	1.4	5.4/4.8	8.7R	1.4	7.0/5.0
3.5R	7.97/3.4	1.7R	1.0	7.5/4.0	2.9R	1.4	7.5/4.0	5.8R	0.9	7.1/4.5	5.4R	0.8	7.6/3.7
9.5R	5.06/4.9	2.5YR	2.1	4.7/4.7	1.7YR	2.4	6.0/4.3	1.7YR	1.7	4.6/3.5	2.9YR	1.8	5.8/4.3
4.5YR	2.43/1.7	8.5Y	9.9	1.5/2.0	9.6YR	2.8	3.1/3.9	3.8YR	5.3	1.8/1.6	6.7YR	2.1	3.6/3.6
4.5YR	4.92/4.6	8.7YR	2.9	4.1/3.9	2.1YR	2.4	6.2/4.2	6.2YR	1.7	4.5/3.8	7.1YR	1.4	5.4/4.3
6.0YR	6.20/11.5	5.0YR	1.7	5.0/9.5	8.7YR	2.1	6.1/8.5	4.6YR	1.6	5.2/9.0	5.4YR	1.2	5.9/9.3
4.5YR	7.03/8.2	5.0YR	1.7	5.5/5.3	7.9YR	2.6	5.8/6.2	5.4YR	1.2	6.1/0.5	8.3YR	1.1	6.3/6.9
4.0YR	8.03/3.4	0.8YR	2.1	7.1/3.7	2.5YR	3.1	7.7/3.3	5.0YR	2.2	7.0/3.7	2.5YR	2.6	7.2/4.0
10.0YR	5.08/4.4	9.2YR	3.0	4.4/3.7	2.9Y	1.7	5.6/4.0	5.8Y	4.1	4.4/4.0	0.8Y	2.0	5.2/4.8
5.0Y	2.53/1.4	9.6YR	5.9	1.8/2.2	4.6GY	1.9	3.0/3.5	8.0Y	4.6	1.2/1.4	6.2Y	4.5	3.6/3.8
5.5Y	4.07/3.9	5.8GY	2.0	3.6/4.8	4.6Y	2.5	4.7/4.5	7.1GY	1.5	3.8/5.4	7.9Y	3.2	4.5/4.5
5.5Y	7.88/12.3	5.6Y	1.2	5.8/8.7	4.6Y	1.0	6.0/9.8	8.3Y	0.9	5.7/7.3	5.8Y	1.3	6.7/8.2
5.5Y	7.90/8.3	7.1Y	0.8	6.4/6.8	5.8Y	1.1	6.6/7.2	5.8Y	1.3	5.6/6.2	4.6Y	0.4	6.6/8.2
5.0Y	7.80/5.5	5.4Y	1.4	5.7/4.6	5.4Y	0.8	6.7/5.2	7.1Y	1.4	5.8/4.8	7.5Y	1.6	7.1/5.7
10.0Y	4.37/3.7	8.3GY	1.5	4.6/5.8	4.6GY	1.9	5.0/4.9	9.2GY	1.4	5.0/4.7	5.8GY	1.7	5.2/4.5
5.5GY	2.5/2.0	2.1G	0.9	1.5/2.3	0.8G	0.5	3.8/4.4	1.9G	1.0	1.3/1.5	0.4G	0.4	3.3/3.9
5.0GY	5.10/5.4	10.0GY	0.0	4.8/6.1	6.7GY	1.8	5.7/6.0	10.0GY	0.6	4.8/6.0	7.5GY	1.1	5.6/5.7
4.0GY	6.97/9.5	7.5GY	1.9	5.5/7.8	4.2GY	1.7	6.4/8.2	9.6GY	0.8	5.4/8.2	4.6GY	1.8	6.7/7.2
4.5GY	8.06/8.6	5.0GY	1.7	6.1/8.0	2.1GY	1.7	7.0/6.6	5.8GY	1.7	6.4/6.7	2.5GY	2.4	6.9/7.2
6.0GY	8.07/3.6	1.2G	0.9	7.0/3.8	1.7GY	1.7	7.6/3.7	1.2 G	0.5	6.9/4.0	7.9GY	2.3	7.7/3.1
10.0GY	5.0/4.3	1.7G	0.5	4.6/4.5	10.0GY	0.6	5.8/5.2	2.9G	1.2	4.5/5.9	2.5G	1.1	5.7/4.7
7.0G	2.22/2.4	3.3G	1.0	1.9/2.0	3.3G	0.5	3.7/5.7	8.0BG	5.1	0.9/0.7	3.3G	0.5	3.0/5.2
5.0G	4.86/4.7	4.2G	0.6	4.8/5.2	4.6G	1.0	5.5/4.5	5.0G	0.9	4.5/6.2	5.0G	1.3	5.6/5.7
5.0G	4.89/8.1	5.0G	0.6	5.2/8.3	7.0G	1.2	6.0/5.9	4.6G	0.8	5.0/7.9	3.7G	0.5	5.7/7.2
7.0G	7.05/6.2	3.3G	1.1	6.0/5.1	7.0G	1.2	7.0/5.0	1.5BG	3.5	5.5/6.2	3.7G	1.6	5.6/5.1
6.0G	8.66/2.7	4.6G	1.7	8.2/3.4	3.6BG	1.9	9.2/1.8	2.5BG	2.5	7.7/4.3	5.8GY	3.1	8.2/2.7
10.0G	3.21/4.7	7.0G	0.9	3.4/5.6	8.4G	0.7	5.9/7.3	7.0G	1.4	3.4/6.0	8.4G	0.7	4.7/7.5
4.5BG	2.24/2.5	10.0GY	7.9	1.7/2.2	5.0G	1.1	3.9/5.9	8.5BG	7.5	1.0/0.8	8.4G	1.5	3.3/4.2
5.5BG	4.31/4.0	1.0BG	2.3	4.6/5.0	4.0BG	0.8	5.2/4.6	1.0B	3.8	4.5/5.7	3.0BG	3.2	5.2/4.7
5.0BG	5.13/6.4	2.5BG	2.2	5.0/7.2	5.0BG	1.7	5.3/6.6	3.6BG	1.5	4.5/6.4	5.0BG	2.5	5.8/5.8
2.0BG	6.37/4.5	5.0G	2.6	4.9/5.3	3.0BG	2.5	6.8/4.7	3.0BG	6.0	5.4/5.9	7.0G	2.5	6.7/4.6
4.5BG	7.22/3.3	9.0BG	5.0	6.1/4.7	2.3B	5.3	8.1/2.8	5.0BG	4.5	5.7/4.8	3.0BG	5.8	7.5/.37
9.0BG	4.14/4.0	5.5BG	5.0	4.2/5.2	3.8B	4.7	5.8/4.6	8.0BG	4.2	4.1/5.0	8.0BG	4.5	5.1/4.2
5.0B	3.13/2.3	2.0B	7.0	3.1/3.0	4.0B	3.8	4.5/3.3	7.0B	4.7	2.0/2.5	4.8B	3.3	4.2/2.8
5.5B	4.98/5.1	7.6B	1.0	4.7/5.2	7.6B	1.8	6.2/5.0	7.0B	0.8	4.9/5.7	8.0B	1.3	5.8/4.8
5.0B	4.96/6.8	7.6B	2.3	5.0/7.5	6.2B	2.8	5.7/6.8	8.0B	1.3	5.0/6.0	8.5B	1.8	5.7/6.2
5.5B	6.11/6.4	6.0B	3.0	5.7/6.0	6.0B	3.3	6.2/6.3	8.5B	1.0	5.3/6.5	7.0B	3.0	6.6/5.2
4.0B	8.15/2.2	10.0B	0.8	7.0/3.2	7.6B	9.3	9.0/1.6	8.0B	2.0	7.6/3.7	7.3PB	11.3	8.7/1.7
10.0B	4.15/4.0	10.0B	1.5	4.4/4.0	10.0B	3.3	5.7/4.3	1.6PB	1.3	3.9/4.2	10.0B	1.7	5.3/3.3
4.5PB	2.15/2.6	3.0PB	0.8	1.5/2.8	7.0PB	4.8	2.9/3.4	6.4PB	2.8	1.5/1.5	7.5PB	2.3	3.0/4.0
4.5PB	3.26/9.3	2.5PB	1.3	4.7/5.2	7.5PB	3.8	5.1/4.8	5.0PB	0.7	3.8/3.4	5.0PB	1.3	5.2/3.2
5.0PB	4.23/4.2	2.5PB	0.7	4.7/9.8	2.0PB	0.3	5.3/8.7	2.5PB	0.7	4.7/9.7	1.6PB	0.0	4.7/7.8
5.0PB	6.27/7.2	3.0PB	0.8	6.0/5.8	4.8PB	0.8	5.4/5.7	4.6PB	0.8	5.3/5.1	6.4PB	1.3	6.8/4.2
5.0PB	8.26/3.0	6.0PB	1.0	7.7/4.3	6.2PB	5.2	9.2/θ.9	4.8PB	0.8	7.8/4.0	0.5B	17.2	9.3/0.8
10.0PB	4.13/4.6	9.5PB	0.7	4.2/5.3	1.2P	1.8	5.8/5.3	1.2P	0.8	4.0/4.5	1.2P	0.8	5.0/4.2
5.0P	2.24/3.3	1.0PB	8.3	1.8/2.7	5.5P	3.0	3.6/4.1	5.0P	3.5	2.0/2.4	4.0P	1.8	2.9/4.2
5.0P	4.95/4.7	2.8P	1.5	5.2/4.7	5.5P	1.8	6.2/4.7	3.3P	2.0	4.6/5.4	7.4P	2.5	6.1/4.3
5.5P	4.08/12.6	5.0P	2.0	4.8/8.6	8.8P	2.0	5.0/7.0	3.5RP	1.5	5.0/9.5	5.0RP	1.5	5.7/7.8
5.5P	7.05/6.4	2.5P	2.2	6.1/5.5	0.8RP	2.8	7.5/4.6	1.8RP	2.2	5.7/5.0	7.2RP	1.8	7.3/5.3
5.0P	8.02/4.7	3.3P	2.7	7.3/4.7	3.4RP	3.3	8.3/2.1	1.5RP	2.2	7.0/4.7	9.2RP	2.0	8.1/3.2
10.0P	3.16/4.3	10.0P	3.3	3.2/4.8	10.0P	2.8	4.2/5.7	10.0P	2.2	3.7/5.9	4.5RP	2.2	4.0/4.7
6.5RP	2.27/2.3	5.0PB	12.5	1.4/1.6	3.5RP	1.8	3.2/3.3	2.6RP	3.3	2.5/3.2	8.8RP	0.8	3.0/3.8
4.0RP	4.32/4.4	8.0P	2.7	4.1/3.6	6.6RP	2.0	5.0/5.1	3.5RP	2.0	3.9/3.7	8.0RP	1.5	5.4/3.2
5.0RP	4.29/10.7	4.5RP	0.8	4.7/7.2	4.5RP	1.5	5.4/8.3	8.8RP	1.5	5.5/9.6	9.2RP	0.7	5.7/8.7
4.5RP	6.32/8.4	5.5RP	1.8	5.2/6.1	7.2RP	1.5	6.4/6.3	8.8RP	0.8	5.5/5.5	1.0R	2.3	6.3/6.5
5.5RP	8.08/2.6	3.5RP	2.5	7.2/3.3	10.0RP	1.2	9.2/1.8	8.0RP	3.3	6.7/3.7	0.8YR	2.3	8.3/3.0
10.0RP	5.30/4.2	0.5R	5.8	4.2/3.1	1.0R	1.8	6.6/4.0	0.5R	2.1	4.7/3.2	2.0R	3.8	5.2/4.2

estimates cannot be denied, but a naive, phenomenological approach was encouraged during the final observations.

The observations were made in a light-tight booth lined with a good non-selective white cardboard throughout. The light sources were suspended from the ceiling by means of a pulley so that they could be adjusted to give the desired illumination on a shelf upon which the color chips were displayed. Macbeth Daylight lamp installations containing two 1000-watt incandescent lamps with appropriate blue filters to approximate standard source C (6700°K) and a standard 500-watt incandescent lamp operated at the correct current to yield standard source A (2854°K) served as the two sources of illumination. Observers were fully adapted to the source of illumination under which the chips were viewed by remaining in the booth for five minutes before the observations began. The chips were viewed successively on each of three non-selective backgrounds having reflectances of 3, 21 and 78 per cent respectively.

Table I gives the six conditions under which the observations were made. The illumination levels were 73 footcandles with the source approximating daylight and 57 footcandles with the source ap-

TABLE III.—Observed and Theoretical Hues and Saturations of Munsell Samples on Gray Backgrounds Under Sources *C* and *A*.

Munsell renotation		Observer's Estimate		Departure from Color Constancy *A* - *C*		Theoretical Estimate	Theoretical Departure *A* - *C*	
		Under *C*	Under *A*					
H	V/C	H/S	H/S	ΔH	ΔS	H/S	ΔH	ΔS
3.5R	2.3/2.2	6.5R / 3.2	6RP / 3.5	−10.5	.3	4.8R / 3.0	1.3	.8
3.5R	4.06/4.8	5.7R / 4.1	7.8R / 1.7	2.1	−2.4	5.3R / 5.7	1.8	.9
5.0R	4.20/12.8	5.0R /12.5	5.3R /13.6	.3	1.1	6.3R /16.1	1.3	3.3
3.5R	6.96/7.0	4.7R / 5.3	8.4R / 5.5	3.7	.2	6.2R / 8.0	2.7	1.0
3.5R	7.97/3.4	1.0R / 4.2	6.1R / 3.6	5.1	−.6	5.4R / 3.9	1.9	.5
9.5R	5.06/4.9	0.4YR / 3.8	2.4YR / 4.3	2.0	.5	1.6YR / 5.8	.4	.7
4.5YR	2.43/1.7	8.7YR / 3.3	9YR / 3.2	.3	−.1	3.8YR / 2.1	.4	.7
4.5YR	4.92/4.6	6.7YR / 3.8	6YR / 4.3	−.7	.5	4.8YR / 5.2	.3	.6
6.0YR	6.20/11.5	7.1YR / 9.7	5.6YR / 9.8	−1.5	.1	6.4YR /12.2	.7	.5
4.5YR	7.03/8.2	6.9YR / 6.3	5.7YR / 6.3	1.2	.0	4.9YR / 8.9	2.1	.9
4.0YR	8.03/3.4	0.2YR / 3.3	4.3YR / 3.7	4.1	.4	4.8YR / 3.7	.8	.3
10.0YR	5.08/4.4	0.3Y / 3.6	3.7Y / 3.6	3.4	.0	2.5Y / 4.6	2.5	.2
5.0Y	2.53/1.4	5.4GY / 2.8	1.9GY / 2.9	−3.5	.1	2.3Y / 1.6	−2.3	.2
5.5Y	4.07/3.9	4.3GY / 4.4	5.7GY / 5.5	1.4	.1	9.0Y / 3.8	3.5	−.1
5.5Y	7.88/12.3	5.5Y /10.7	5.5Y / 9.4	.0	−1.3	9.5Y /11.9	4.0	−.4
5.5Y	7.90/8.3	4.9Y / 8.4	4.9Y / 8.1	.0	−.3	7.4Y / 7.8	1.9	−.4
5.0Y	7.80/5.5	5.4Y / .57	5.3Y / 6.5	−.1	.8	7.9Y / 5.5	2.9	.0
10.0Y	4.37/3.7	7.5GY / 4.5	8.2GY / 5.1	.7	.6	3.8GY / 4.2	3.8	.5
5.5GY	2.5/2.0	0.6G / 3.3	0.5G / 3.7	−.1	.4	9.0GY / 2.4	3.5	.4
5.0GY	5.10/5.4	8.1GY / 5.5	0.4G / 6.2	2.3	.7	8.2GY / 6.4	3.2	1.0
4.0GY	6.97/9.5	3.2GY / 8.2	7.1GY / 7.2	3.9	−1.0	7.3GY /10.6	3.3	1.1
4.5GY	8.06/8.6	2.2GY / 7.7	4.6GY / 6.7	2.4	−1.0	7.3GY / 9.2	2.8	.6
6.0GY	8.07/3.6	2.9GY / 3.8	5.3GY / 4.3	2.4	.5	7.7GY / 3.8	1.7	.2
10.0GY	5.0/4.3	0.9G / 4.5	1.9G / 5.6	1.0	1.1	2.7G / 5.3	2.7	1.0
7.0G	2.22/2.4	2.8G / 4.2	1BG / 3.9	8.2	−.3	2.5BG / 2.8	5.5	.4
5.0G	4.86/4.7	3.1G / 4.2	4.3G / 5.9	1.2	.8	9.7G / 6.2	4.7	2.5
5.0G	4.89/8.1	3.6G / 8.0	4.6G / 8.3	1.0	.3	10.1G /10.1	5.0	2.0
7.0G	7.05/6.2	4.3G / 5.4	5.0G / 5.2	.7	−.2	2.5BG / 7.8	5.5	1.6
10.0G	3.21/4.7	4.6G / 6.8	3.3G / 6.2	−1.3	−.6	4.0BG / 5.4	4.0	.7
4.5BG	2.24/2.5	6G / 4.3	8.5BG / 3.5	12.5	−.8	8.7BG / 2.5	4.2	.0
5.5BG	4.31/4.0	3BG / 4.8	7BG / 5.5	4.0	.7	8.8BG / 4.8	3.3	.8
5.0BG	5.13/6.4	5BG / 7.2	5BG / 6.9	.0	−.3	8.9BG / 7.5	3.9	1.1
2.0BG	6.37/4.5	2.5BG / 5.3	3BG / 5.0	.5	−.3	5.1BG / 5.9	3.1	1.4
4.5BG	7.22/3.3	9BG / 4.2	6BG / 4.5	−3.0	.3	7.8BG / 4.2	3.3	.9
9.0BG	4.14/4.0	10BG / 4.7	1BG / 5.0	−9.0	.3	2.5B / 4.7	3.5	.7
5.0B	3.13/2.3	4B / 3.7	4B / 4.1	0.0	.4	7.1B / 2.7	2.1	.4
5.5B	4.98/5.1	9B / 5.4	9B / 5.9	0.0	.5	7.2B / 6.0	1.7	.9
5.0B	4.96/6.8	9B / 6.7	10B / 7.5	1.0	.8	8.2B / 8.1	3.2	1.3
5.5B	6.11/6.4	.5PB / 6.3	7B / 6.4	−3.5	.1	7.4B / 7.6	1.9	.8
4.0B	8.15/2.2	8B / 2.9	7.5B / 2.4	−.5	−.5	5.0B / 3.1	1.0	.9
10.0B	4.15/4.0	10B / 5.0	1PB / 5.0	1.0	.0	2.5PB / 4.1	2.5	.1
4.5PB	2.15/2.6	1.5PB / 3.5	2.5PB / 3.4	1.0	−.1	6.0PB / 2.8	1.5	.2
5.0PB	4.23/4.2	4PB / 5.4	4.5PB / 4.9	.5	−.5	6.3PB / 4.6	1.3	.4
4.5PB	3.26/9.3	2.5PB / 9.5	2.5PB / 9.5	.0	.0	6.8PB /11.4	2.3	2.1
5.0PB	6.27/7.2	4PB / 5.7	5.5PB / 5.5	−1.5	−.2	6.5PB / 8.0	1.5	.8
5.0PB	8.26/3.0	1.5PB / 2.5	2.5PB / 1.3	−1.0	−1.2	6.6PB / 3.5	1.6	.5
10.0PB	4.13/4.6	9.5PB / 5.4	0.5P / 5.4	1.0	.0	3P / 4.9	3.0	.3
5.0P	2.24/3.3	10P / 3.8	1RP / 4.4	1.0	.6	9.7P / 3.7	4.7	.4
5.0P	4.95/4.7	3P / 5.0	7P / 4.9	4.0	−.1	7.9P / 5.6	2.9	.9
5.5P	4.08/12.6	8P / 7.8	3.5RP / 7.2	5.5	−.6	9.5P /14.5	4.0	1.9
5.5P	7.05/6.4	9.5P / 4.9	5.5RP / 5.6	6.0	.7	8.8P / 7.9	3.3	1.5
5.0P	8.02/4.7	8P / 4.2	6RP / 4.0	8.0	−.2	7.1P / 5.4	2.1	.7
10.0P	3.16/4.3	8P / 4.5	2RP / 5.7	4.0	1.2	2.4RP / 5.4	2.4	1.1
6.5RP	2.27/2.3	5P / 3.3	3.5RP / 3.7	8.5	.4	9.4RP / 3.0	2.9	.7
4.0RP	4.32/4.4	3.5RP / 4.5	5RP / 4.5	1.5	.0	7.2RP / 5.6	3.2	1.2
5.0RP	4.29/10.7	6RP / 8.2	9RP / 9.0	3.0	.8	7.9RP /13.3	2.9	2.6
4.5RP	6.32/8.4	6RP / 6.4	10RP / 6.3	4.0	−.1	7.5RP /10.4	3.0	2.0
5.5RP	8.08/2.6	7.5RP / 3.0	5.8R / 3.6	8.3	.6	7.5RP / 3.2	2.0	.6
10.0RP	5.30/4.2	6.5RP / 3.4	1R / 4.0	4.5	.6	2.7R / 5.2	2.7	1.0

proximating source *A*. The difference of about 28 per cent between the two types of illumination being studied is not great enough to be responsible for the color differences observed. Previous studies have shown that it takes changes in illumination levels of the order of 100 to 1 to cause appreciable shifts in hue under the conditions of this study (Helson and Jeffers, 1940).

The samples measured 1″ x 1″ and were mounted in cardboard holders similar to the background. They were placed a few inches apart on a shelf in the booth and 11 were exposed in random order during each observation. This was for the purpose of providing a variety of colors in the field of view both to simulate natural conditions of vision and to enable the observers to make their judgments with a number of samples serving as frames of reference. The samples were selected as follows: from the Munsell Book of Color, ten major hues were selected at high value and high chroma, ten at high value and medium chroma, ten at medium value and high chroma, ten at medium value and medium chroma, and ten at low value and low chroma. Ten equally spaced intermediate hues were selected at medium value and medium chroma. Three nearly non-selective samples were also observed but they are not included in the results because according to the simple theory tested here they should remain nearly achromatic in the two sources. Actually they were reported as having hues of very low saturation, depending largely on the background against which they were viewed.

2. Results and Discussion

Table II shows the average results of the six observers for the 60 Munsell samples judged under sources *C* and *A* on white and black backgrounds. Standard deviations of the means of the hue judgments are also given. It will be noted from this presentation of data on the two backgrounds representing extremes in reflectance and therefore giving maximal background effects, that the differences in color due to background are as great as those corresponding to the change from daylight to incandescent filament illumination. This is in keeping with previous findings (Helson, 1938) and shows that hue and saturation of an object depend importantly upon the relation of its reflectance to adaptation-reflectance. Adaptation-reflectance is a weighted mean of the reflectance of all objects in the field of view and is largely determined by the background (Judd, 1940). Since we are primarily interested in the effect of the source in this study we pass over further consideration of background effects at this time.

Table III shows for the gray background the hues and saturations reported under sources *C* and *A* and the differences caused by changing the adaptation of the observers from daylight (*C*) to incandescent filament illumination (*A*). If it were true that the colors of objects remain completely independent of the source (so-called perfect color constancy) these differences would fail to differ significantly from zero. Statistical test of the differences between the hues reported under the two sources reveals that they are very highly significant, the t-value for the obtained differences being 6.7 which is nearly three times the value of 2.66 necessary for significance at the one per cent level of confidence. While many of the differences are small because some colors remain practically unchanged under the two sources, the changes in the rest of the samples are sufficiently great relative to individual-observer differences to yield overall significance statistically. Note particularly how the purple samples change to red-purple.

3. Theory

It is known that the complete theory of object-color perception is very complicated (Judd, 1940) and must take account of the background as well as the object and the spectral composition of the illuminant if the adaptation of the eye is to be included in the theory. There is considerable experimental evidence to support the view that the state of chromatic adaptation of the observer's eye may be taken at least approximately into account by means of the three-components, or Young-Helmholtz, theory of vision. By this theory there are supposed to be three independent sets of photosensitive elements in the retina, one preponderantly sensitive to the short-wave part of the spectrum, and reporting violet or blue; a second, sensitive to the middle of the spectrum, and reporting green; and the third, sensitive to the long-wave part, and reporting red. By this theory the relative gain in sensitivity to blue experienced by the observer adapted to reddish-yellow light comes from the relatively low stimulation from the short-wave part of the spectrum produced by that kind of light, or alternatively, stimulation by reddish-yellow light desensitizes the red receptors and to a somewhat lesser extent, the green. The object colors perceived under reddish-yellow light are therefore the resultants of relatively increased stimulation from the long-wave portion of the spectrum combined with decreased sensitivity of the red and green receptors affected most by it.

Now a spectrally nonselective diffusing surface (10 to 40 per cent reflectance) usually appears closely gray to an observer adapted to the chromatic illuminant, even though the illuminant departs considerably from daylight quality (Judd, 1940; Helson and Michels, 1948). By the Young-Helmholtz theory the perception of gray corresponds to some fixed proportion (about equal) between the three responses (violet, green, red). This experimental fact may be invoked to yield by means of the theory at least an approximate prediction of the influence of chromatic adaptation by correcting the red, green, and violet responses for each viewed object by the same factor required to make the non-selective objects correspond to the fixed proportion of responses characteristic of gray for any given source of illumination.

3.1 Early Work. To show what experimental support this view has, we shall trace the development and test of it. Helmholtz (1860) early indicated the connection conceived by him between chromatic adaptation and the three independent mechanisms of Young's theory by saying (page 362): ". . . the nervous substance in question is less sensitive to new reacting light falling on it than the rest of the retina that was not previously stimulated. . . ." Following this enunciation of the principle, many observations were made of local color changes due to a locally applied stimulus called the adapting light. The observer then looked at a uniform field called the reacting light and saw an after-image of the adapting light. The qualitative reports of the observer on the color of the after-image were then checked against the predictions of the theory. If the reacting light is gray, the color of the after-image is usually complementary to that of the adapting light, and this can easily be explained by the Young-Helmholtz theory. For example, Helmholtz (1860) says (page 367): ". . . Thus the after-image of red is blue-green; that of yellow is blue; that of green is pink-red, and vice versa. . . Thus an eye which has been acted on by yellow light, say, is thereafter in a condition in which the blue components of white light affect it more than yellow does. Accordingly, the effect of fatiguing the retina is not uniformly extended to every kind of stimulation, but chiefly to stimulation similar to the primary stimulation. This fact is explained very simply by Young's assumption of three different kinds of sensory nerves for the different colors. For since colored light does not stimulate these three kinds of nerves all to the same extent, different degrees of fatigue must also be the result of different degrees of stimulation. When the eye has been exposed to red, then the red-sensitive nerves are strongly stimulated and much fatigued; whereas the green-sensitive and violet-sensitive nerves are feebly stimulated and not much fatigued. If afterwards white light falls on the eye, the green-sensitive and violet-sensitive nerves will be relatively more affected by it than the red-sensitive nerves; and hence the impression of blue-green, which is complementary to red, will predominate in the sensation."

If we are going to predict quantitatively by the three-components theory the influence of chromatic adaptation, we must evaluate precisely the spectral sensitivity curves of the three components. It was already known in 1860 that in no part of the spectrum are two of these sensitivity curves equal to zero.

3.2 Persistence of the Optical Matches. To make a precise evaluation of the particular sensitivity curves applicable to prediction of the influence of chromatic adaptation required establishment of the central principle on which modern colorimetry is based, namely: that metameric color matches are not dependent on chromatic adaptation. The early work on this vital principle is well summarized by von Kries (1905) who names it *the persistence of the optical matches*, and states (page 209): "It is now established that for small and directly fixated fields optical matches show no change regardless of whether this retinal region has its adaptation changed by means of any light whatsoever. For example, a homogeneous yellow, and a yellow mixed from red and green, that under the usual conditions appear centrally equal, are seen after previous exposure to yellow light both more pale, and are seen after blue illumination both more saturated yellow, appear nevertheless always exactly equal to each other. Similarly the equality of a white undecomposed spectrally and a white mixed from two spectral complementaries is not destroyed if both, because of a previous chromatic illumination, appear strongly colored (in the complementary of the adapting light).

"The validity of this law has been tested recently in a systematic way by Mr. Bühler (1903), without a discrepancy ever having been found greater than the uncertainty of the observation. One may therefore take the independence of the optical matches, not, to be sure, for the adaptation of the whole visual mechanism, but rather for the central retinal region, or, stated theoretically, for the isolated portion of the visual mechanism functioning trichromatically.

"Hering (1893) has for a long time stated the independence of optical matches with adaptation with particular vigor and entirely generally. Later, however, Tschermak (1898) as has already been discussed above, in a study carried out under Hering's direction has reported a dependence of particular matches on adaptation, and has extended this statement (in my opinion wrongly) also to the center of the retina. Our law is therefore at many times (v. Kries and Nagel, 1896, 1900) and on different sides affirmed and denied, without the special conditions under which it holds having been determined."

More recently it has been shown by Wright (1934, 1936) that the optical matches also fail to persist if the fields are too bright. The breakdown occurs for fields yielding retinal illuminances in the range of 10,000 to 20,000 trolands. The special conditions under which this law supplying the basis for modern colorimetry holds are now considered to be those yielding vision by the retinal cones illuminated to less than 10,000 trolands. By virtue of this law, we can use the standard observer to compute the change in object-color when any source of known spectral-energy distribution (such as a fluorescent lamp) is substituted for any other known source of the same chromaticity.

3.3 The Coefficient Law. Before the precise evaluation of the spectral sensitivity curves of the three components presumed by the Young-Helmholtz theory could be carried out by colorimetric methods, another important principle had to be verified. This principle arose from the use of a retinal area near that pre-exposed to the adapting light as a control on the adaptive state of the former. The light, called the comparison light, falling on this control area was adjusted until it matched the reacting light. The objective difference between the reacting light and the comparison light color matching it served as a quantitative measure of the degree and kind of chromatic adaptation caused by the adapting light.

It was presumed by von Kries that the adaptive effect of the adapting light could be adequately expressed as a reduction of sensitivity by a constant fraction. In the words of von Kries (1905) it is reasonable (p. 211) "to assume that so far as the effect of an outer stimulus comes in question, the adaptation, whether of the visual mechanism in toto or of a particular part, makes itself effective as a greater or lesser sensitivity relative to each stimulus, and to be sure so that the result occurs always pretty much in accord with a product, aR, where R is the stimulus value, a, however, the disposition for this kind of stimulus. The simplification lies, as one sees, in the assumption that the modification which a stimulus-result undergoes through the existing sensitivity, allows itself to be represented for all chromaticities and photometric brightnesses simultaneously through the use of a coefficient. If this is the case, there exist several very simple laws, which also appear to be specially adapted for experimental test. Namely, it must be that, if L_1 on one retinal region causes the same result as L_2 on another, and similarly M_1, working on the first, causes the same effect as M_2 on the other, in every case also $L_1 + M_1$ must have here the same effect as $L_2 + M_2$ there. Especially must the equality of stimulus-results of two lights working on differently adapted retinal regions always remain fixed by proportional increase or decrease of the same. I wish to designate for short this assumption as the coefficient law. . . . A strict validity of the coefficient law can come into question only when the trichromatic apparatus functions in an isolated way, and the experimental test should be restricted on purpose to small directly fixated fields. . . .

"The extended studies of Wirth (1900-1903) show that the law can be considered as nearly valid for reacting lights that are not too weak. If this is the case, then we can say that each adaptation of the visual mechanism (more exactly of the color-engendering apparatus) compared with a definite, perhaps considered as standard, state is completely characterized if for three different lights (or light mixtures) is given the change in appearance they undergo because of the adaptation. Then, from this the change of any other light mixture whatsoever allows itself to be deduced."

3.4 Fundamental Colors. The coefficient law stated by von Kries and approximately substantiated by Wirth is an extension of Helmholtz' original conception that "the nervous substance in question is less sensitive to new reacting light falling on it than the rest of the retina that was not previously stimulated." The importance of this law was recognized by Wright (1934) who made some further checks on it in connection with his determination of the primary processes for his own eyes. Wright (1934) states (p. 63), "Now suppose R', G' and B' are hypothetical stimuli that produce responses along A, B, and C, . . . three independent sets of fibers to the brain. Then a reduction in sensitivity produced by light adaptation will, for a test colour that stimulates A alone produce an intensity depression of R' but no colour change; similarly if B or C are stimulated alone. Then if, in a given state of adaptation, R' were reduced to r' and G' to g', as each response is independent, $(R' + G')$ would be reduced to $(r' + g')$, $(2R' + G')$ would be reduced to $(2r' + g')$ and so on. In other words the geometry of the colour triangle holds equally well for the depressed eye as for the eye in its normal state. . . . Hence if the effect of a particular colour adaptation on the colours of the three instrument primaries R, G, B is known, then the effect on any colour in the colour triangle can be calculated.

"This deduction is also given by von Kries (1905), but it seemed desirable to check it experimentally and the results of one test are given below. . . . The agreement is remarkably close. Similar experiments have confirmed that result, although the margin of experimental error has usually been greater than here.

"The effect of varying the test colour is thus reduced to a matter of colorimetric additions and subtractions. If the intensity of the test colour is reduced, the recovery curves are reduced by the same amounts throughout; if the colour is changed, the change can be calculated provided the effect of the adaptation on three arbitrarily selected primaries is known. These conclusions must not be regarded as absolutely rigid; they appear to hold at least very approximately over a mean range of intensity.

"Determination of the Fundamental Response Curves: It has been mentioned above that if we could find a test colour that stimulated only one of the responses then no matter how the sensitivity

TABLE IV.—Fundamental Colors of the Young-Helmholtz Theory.

Method of Determination	Fundamental Color					
	Red		Green		Blue or Violet	
	x	y	x	y	x	y
Chromatic adaptation						
Wright	0.737	0.264	2.913	—1.768	0.134	0.022
Walters	.749	.253	14.39	—12.47	....	....
Dichromatic co-punctal points	.747	.253	1.00 to 1.20	0.00 to —.20	0.16 to .18	0.00

of the eye was depressed, no colour change could be evoked; for a colour change necessitates a change in the proportions of at least two of the responses and since only one is being stimulated, no qualitative change can be produced.

"Further if the positions in the colour triangle of these three fundamental stimuli were known, then the fundamental response or excitation curves could be computed by combination with the trichromatic mixture curves and the luminosity curve. Hence to determine these response curves it is only necessary to find those theoretical stimuli that, no matter how the sensitivity of the eye has been depressed, evoke sensations that are modified in intensity but not in colour.

"A direct investigation of the spectral radiations for this property can be made very easily. The patches in both the left and right eye can be illuminated with the same spectral radiation and the right eye can then be adapted with any coloured light. For a comprehensive test, red, green and blue adaptations should all be used. At the end of the adaptation the two monochromatic patches are again viewed and if no colour change has been induced in the right eye, it should be possible to obtain a match merely by reducing the intensity of the light in the left eye.

"If such a series of experiments is conducted, it is found that in two regions of the spectrum there are radiations in which only very small changes are induced. The first of these is in the extreme red, from 0.70 μ to the end of the spectrum. With this test colour, with any wavelength of adapting light, only an insignificant colour change is induced. . . . It has been shown above that once the effect on three primaries is known for a particular adaptation, the change produced on any given point in the colour triangle can be computed. It has now to be shown that the inverse process, the location of a point that has a given colour change, for our purposes zero change, is also possible."

By using this inverse process Wright determined for his own eye the color specifications of each of the three hypothetical stimuli required to stimulate separately the presumed independent processes by which the influence of adaptation is to be taken into account. These stimuli were specified in terms of the WDW coordinate system, but the specifications have been transformed to chromaticity coordinates (x, y) in the standard C.I.E. system by the transformation equations given by Judd (1944) and are given in Table IV.

In conformity to an early conclusion by Helmholtz, all three of these hypothetical stimuli correspond to colors more saturated than the spectrum colors. Thus, there is no part of the spectrum capable of stimulating one of the fundamental responses without at the same time stimulating at least one other of them. Extreme spectrum red comes the closest to doing this, fundamental red being only slightly beyond. The other two fundamentals are considerably super-spectral green and blue. Wright's results for his own eyes indicate definitely, moreover, that the third fundamental must be blue; it cannot be violet. He states (page 70), "On one point of previous dispute it is possible to come to a definite conclusion. It is quite certain that blue and not violet is the fundamental stimulus. It is only necessary to adapt the eye to a red radiation and observe the great change that is produced in a test colour at, say, 0.43 μ to prove that this radiation stimulates at least two responses."

Two of the colors (red and green) corresponding to the independent processes of the three-components theory were again determined by this method by Walters (1942) for his own eye. The chromaticity coordinates (x, y) of these colors in the standard C.I.E. system are also given in Table IV. The checks obtained for the coefficient law by Walters are not so good as those previously obtained by Wright, and this is ascribed to individual observer difference. Although the agreement between Walters' and Wright's determination of the fundamental red and green colors is fair, no determination of the third primary color was made by Walters, and he was uncertain whether it is blue or violet. He suggests that his results indicate that the three processes of the Young-Helmholtz theory are not entirely independent.

A considerably more reliable way to evaluate the fundamental colors of the Young-Helmholtz theory would seem to be by means of the well-established facts of partial color blindness. There are three types of dichromatic vision, protanopia, deuteranopia, and tritanopia; and one of the great triumphs of the three-components theory is its simple, straightforward account of the colors confused by these types of dichromatic observers. The confusion colors for each type are found to lie along a family of straight lines on the chromaticity diagram intersecting closely at a single point. If the fundamental colors of the three-components theory are taken at the co-punctal points of protanopic, deuteranopic, and tritanopic vision, respectively, the three-components theory explains all three types precisely and in detail as being the result of a non-functioning of one of the three fundamental processes. Table IV shows for comparison the chromaticity coordinates (Judd, 1944) of the three co-punctal points. The agreement on the red primary is very good; but that for the other two primaries indicates the incomplete nature of the explanation already foreshadowed by the discovery that the coefficient law holds only approximately. It must be remembered, however, that these checks were obtained by colorimetric methods between a pre-exposed and a rested portion of the visual field. Although the defects in the coefficient law are easy to demonstrate by quantitative colorimetric methods, the central fact that the law holds approximately offers a promise of a simple approximate account of the changes in object-color perception due to chromatic adaptation of the whole visual field. These changes cannot be determined by methods of color matching; they must be estimated by relatively imprecise methods like the one used to obtain the present results.

3.5 Predictions Based on Three-components Theory. Many qualitative studies of chromatic adaptation of the whole visual field have been found to be satisfactorily summarized by the coefficient law though the authors of the accounts of these studies have not used that name for it, nor have they cited the previous work. Instead the ideas seem to have been developed independently from the general conception of the three-components theory. Thus Ives (1912) says: "Before leaving this topic a subjective element may be touched upon. After working for a time under a carbon lamp or other similar artificial light one ceases to notice that it is yellow. One apparently forms a new scale of color with unsaturated yellow as one's white. In this way compensation is introduced for the really enormous changes in color, which actually occur. Just how to represent this compensation is a matter probably yet to be determined. It might tentatively be assumed, however, that the three *sensations* are somehow re-weighted. . . . If now this same compensating process is applied to all the colors seen under this light, their distorted position is to some extent overcome. But not entirely, for the two different processes of color-change, one by change in spectral intensity distribution, the other by change in relative sensation intensities, never quite match. . . ."

Similarly, Noteboom (1935) describing a study of the adaptation produced by wearing chromatic goggles states: "In general, however, the lenses of goggles are held close to the eye so that the light from the surroundings, the room and so forth, is also filtered. Under these conditions the eye after a short time does not sense a white blotting paper as giving the impression of a green or a blue, or whatever the color of the lenses may be, but perceives it just as white as before putting on the goggles. There remains, therefore, only a reduction of brightness, the effect of the colored glass on perception being no different from that of a neutral gray glass on a white surface. This so-called adaptation of the eye to the color of the glasses takes place very quickly and completely for weakly colored glasses. With strongly colored glasses the eye takes a longer time, and also in extreme cases there is a remnant of perceived color on looking at a white surface. The change from daylight to artificial tungsten- or kerosene-lamp light without change in the white sensation is an example of this adaptation of the eye.

"In order to take account in the color triangle of this impression which the eye, or better to say, consciousness has, one must again equalize the color values belonging to the shifted white point, therefore to multiply them by factors easy to determine. In this way the color point again falls on the white point corresponding to the actual sensation.

"With these factors the values of the other color points are also to be multiplied, and in this way one obtains a representation of the sensations in terms of the unadapted eye. Whether this method corresponds to the observed facts is, to be sure, not known, but, on the one hand, the agreement with typical practical observations is quite good, and, on the other hand, there is scarcely a more plausible assumption to be made; above all, studies along this line are lacking."

It is evident that Noteboom was unfamiliar both with the early work summarized by von Kries (1905) and with the work of Wright (1934) which had appeared just a year before. Nevertheless he made use of exactly the ideas justified by that work.

The same thing is true of Ströble (1937) who writes: ". . . if one, for example, suddenly darkens a room illuminated by bright daylight and at the same time turns on the electric light, all objects during the first moment regardless of color (except very strongly saturated greens, blues and purples) appear with hues between yellow-green and red; but after a short time (of the order of a second) appear again in the original hues and saturations—though of course slightly shifted.

"The eye adjusts itself therefore to the changed illumination by the exact amount that this illumination happens to differ from the color of daylight. This adaptation of the eye does not show up in the color triangle, but it is possible to transform the chromaticity coordinates so that the point representing the illumination falls at the center of the color triangle. The question naturally arises whether such a transformation is physiologically justified. As was pointed out earlier, the fundamental sensation curves are experimentally determined, but the units of the fundamental sensations are arbitrarily adjusted so that the existence of a white color impression corresponds to equal amounts of the three fundamental sensations. If now this white color impression corresponds to another color, the units must again be chosen so that now this color falls at the center of the color triangle. . . .

"The computation of the above-described transformation is well established. The determination of the color coordinates of any given color in the new system consists, as is immediately obvious, merely in this that the known color coordinates in the old system be divided by the corresponding color coordinates of the light source in the old system, and that from the quotients the percentages be computed. . . ."

None of these authors states explicitly that the transformation to be valid must apply to the fundamental colors of the three-components theory, and to no other set of primary colors. Perhaps they tacitly assumed that the coordinate system they used was the fundamental system. Ströble used the König-Ives coordinate system (Ives, 1915); Noteboom, the system corresponding to the OSA excitation curves (Troland, 1922). Neither of these systems have primaries in good agreement with those specified in Table IV.

Attention was explicitly called by Evans (1943, 1948) to the importance of choice of coordinate system for this transformation. It is plain from Helmholtz' early accounts, as we have seen, that his conception applies only to the fundamental processes of the three-components theory; but this restriction has tended to be forgotten, or at least neglected, perhaps because no generally accepted determination of the colors corresponding to these processes has yet been made. Evans (1943) states (page 590), "Wright's work indicates that the relative responses of the eye receptors under a given set of conditions can be obtained by dividing the integrals of the stimulus with respect to each receptor, by the integral of the adapting luminant with respect to the same receptor. On careful consideration, however, it is apparent that there is only one possible set of sensitivity distributions for three primary receptors for which this result will hold. Wright (1934) determined such a set and referred to them as the 'fundamental' sensitivities of the eye. Walters (1942) has since redetermined them in the red and green regions and has thrown some doubt on Wright's values in the blue. . . ."

Evans (1943) further points out (page 600) that "because only one system can meet the requirements laid down, it is not likely that the I.C.I. primaries constitute such a system since they were chosen merely for convenience in computation." Nevertheless, our first theoretical computations were based on the C.I.E. (or I.C.I.) system because the C.I.E. tristimulus values for the Munsell samples used were already known (Kelly, Gibson, Nickerson, 1943; Granville, Nickerson, Foss, 1943) both for standard sources A and C, and it was hoped that this convenient and well-known international system of specifying colors could be used to give a valid first approximation of the influence of chromatic adaptation. Evans' conjecture proved, however, to be correct. The predictions based upon the C.I.E. primaries were found to be no more valid than the still simpler first approximation that object-color perception remains independent of changes in quality of the illumination (so-called color constancy).

3.6 Theory Tested. Our next theoretical computations were based upon the comparatively well-established dichromatic co-punctal chromaticities. The coordinate system based on these chromaticities (Table IV) as primaries may be defined by giving the connection between the tristimulus values, R, G, V, of any color in terms of the tristimulus values X, Y, Z, of that color in the C.I.E. system, which is closely (Judd, 1944) the following:

$$\begin{aligned} R &= 1.00Y \\ G &= -0.46X + 1.36Y + 0.10Z \\ V &= 1.00Z \end{aligned} \qquad (1)$$

For standard source C,

$$X_c = 0.980,\ Y_c = 1.000,\ Z_c = 1.181,$$

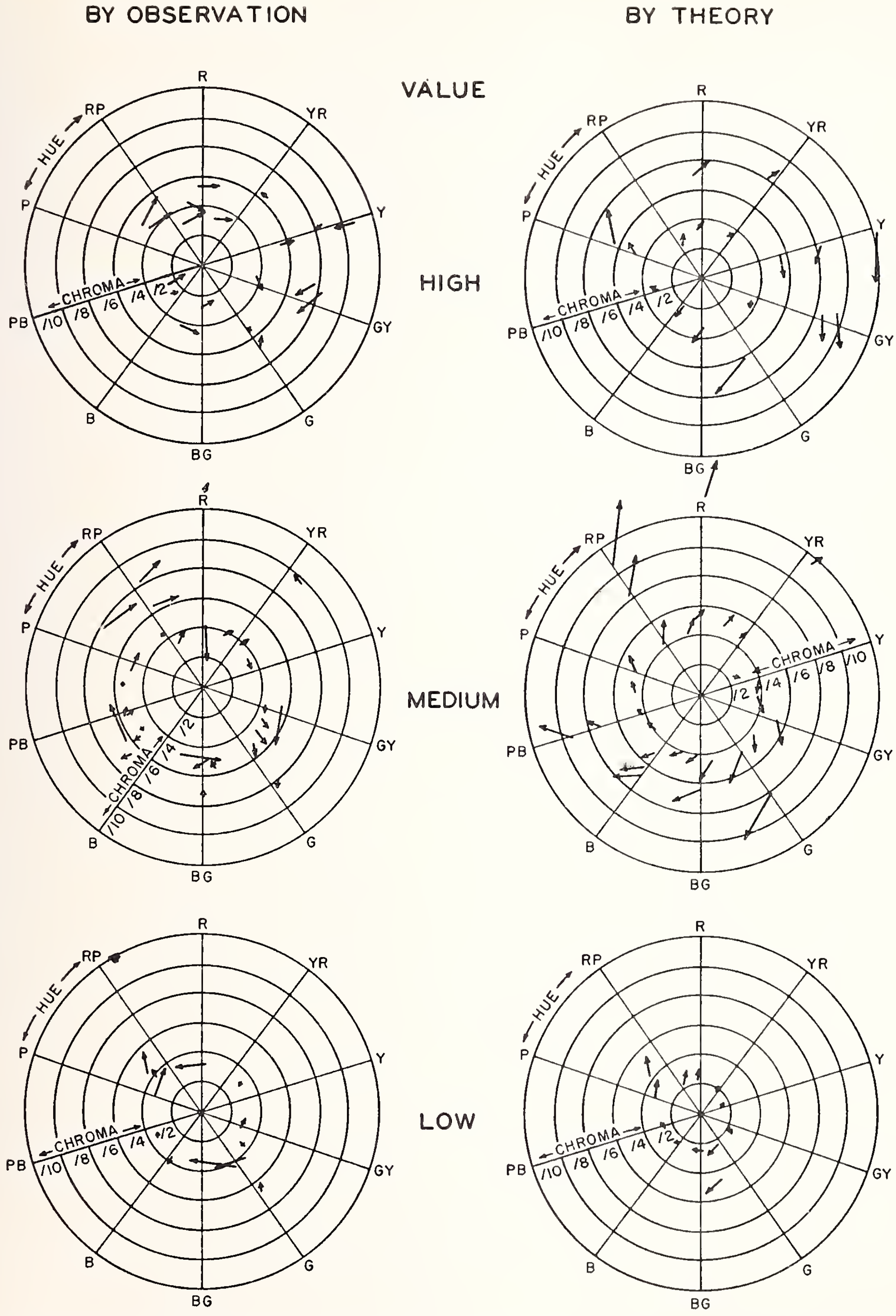

Figure 1. Changes in hue and saturation in passing from daylight (source *C*) to incandescent filament illumination (source *A*) according to observation and according to theory. Samples have been grouped according to high, medium and low reflectance and the observed data are for gray background.

and for source A:

$$X_a = 1.098,\ Y_a = 1.000,\ Z_a = 0.355.$$

From equation 1 we find:

$$R_c = 1.000,\ G_c = 1.027,\ V_c = 1.181$$

$$R_a = 1.000,\ G_a = 0.890,\ V_a = 0.355$$

To find the red, green and violet responses (R', G', V') of an observer adapted to incandescent-lamp light viewing an object under incandescent-lamp light, we have merely to write, in accord with the proposals of Ives, Wright, Notcboom, Ströble, and Evans:

$$\begin{aligned} R' &= R(R_c/R_a) = R(1.000/1.000) = 1.000\,R \\ G' &= G(G_c/G_a) = G(1.027/0.890) = 1.154\,G \\ V' &= V(V_c/V_a) = V(1.181/0.355) = 3.327\,V \end{aligned} \quad (2)$$

The coefficients, 1.154 and 3.327, indicate the sensitivities of the presumed green and violet mechanisms relative to that of the red caused by adaptation to standard source A; that is, adaptation to incandescent lamp light has made the green mechanism slightly more responsive and the violet mechanism more than three times as responsive relative to the red mechanism than was the case for adaptation to daylight. These changes in sensitivity are those required to account for the experimental fact that spectrally nonselective specimens are seen as gray by the observer adapted to standard source A. It remains to apply these same sensitivities to compute R', G', V' for each of the chips studied. The responses, R, G, V, for these chips are computed by equation (1) from the known tristimulus values (X, Y, Z) of the chip under source A; these responses apply to an observer adapted to source C. Then the responses, R', G', V', are computed from equation (2); these responses refer to an observer adapted to source A.

The next step is to find the corresponding Munsell renotation defined (Newhall, Nickerson, Judd, 1943) in terms of daylight reflectance, Y, and chromaticity coordinates, x, y, where:

$$x = X/(X+Y+Z)\,;\ y = Y/(X+Y+Z) \quad (3)$$

if X', Y', Z' are the tristimulus values of the responses of the observer corrected for adaptation to source A, they may be written from equations (1) and (2) as:

$$\begin{aligned} X' &= 1.154X - 0.458Y + 0.473Z \\ Y' &= 1.000Y \\ Z' &= 3.327Z \end{aligned} \quad (4)$$

The actual calculations for X', however, were made according to the equivalent expression:

$$X' = -2.509G + 2.954Y + 0.724Z \quad (4a)$$

the values of G having all been computed for the Munsell specimens previously from equation (1).

3.7 Theoretical Results Compared to Observations. From equations (3) and (4) the chromaticity coordinates, x' and y', corrected for chromatic adaptation to standard source A may be computed for each chip studied, and from Y', x' and y', may be read the Munsell renotation of the chips under source A corrected for adaptation by this theory. These are given in Table III, and the differences between the Munsell renotation of the chips seen under source C and the Munsell renotations of the chips seen under source A corrected for adaptation to that source are also given.

Fig. 1 shows by vectors in the polar-coordinate plots of hue and saturation the changes in the color perceptions of the chips caused by changing from standard source C to standard source A. It also shows for comparison the changes predicted by the theory. It is evident that there is general agreement between observation and theory, but in isolated cases the hue changes found are very different in amount and sometimes even in direction from those predicted. Furthermore Fig. 1 shows that the saturations perceived under source A should by this theory generally exceed those perceived under source C; but this is not borne out by the observations.

The discrepancies between the theoretical and observed saturations are statistically significant for all three backgrounds. The average difference between theoretical estimates and observers' estimates of saturation are 1.12 for white background, 0.67 for gray background, and 1.14 for black background with t-values of 5.71, 2.90, and 5.95 respectively, which are highly significant and therefore show significant departure of theory from observations. The differences between the theoretical and observed saturations are small for the most part, amounting to less than one step in the 0 to 10 scale of saturation employed by the observers. But because the predicted saturations tend to be greater than the observed saturations fairly consistently even this small difference proves to be statistically significant.

A similar statistical analysis of the differences between the theoretical and observed hues shows that the discrepancies are not statistically significant for any background. The average difference between theoretical and observed hues is 0.54 for white background, 0.58 for gray background, and 1.11 for black background with t-values of 1.24, 1.07, and 1.71 respectively. All three t-values lie between the 10 and 50 per cent confidence levels and therefore do not denote significant departure of the theoretieal from the observed hues. Henee we ean eonelude that the partieular form of trichromatie theory tested here prediets adequately the hues pereeived to belong to objects under souree *A* from a knowledge of their C.I.E. ehromatieity eoordinates under this souree.

In view of the approximate nature of the simple theory there is some question whether it is worthwhile to make eomputations based on other eoordinate systems such as that indieated by Wright's work (see Table IV). Perhaps sueh ealeulations would improve the agreement between the observed and predieted saturation ehanges. One important result of the present study. however. is the establishment of the faet that people differ greatly with respeet to ehromatie adaptation. Further work* with other sourees of illumination might yield a reliable basis for deeiding between the various eoordinate systems supposed to eonform to the fundamental eolors of the three-eomponents theory. However, the goodness of fit of the theory to the data already obtained indieates that the assumptions underlying it are essentially correet for the eonditions of this study, and that tables based on this theory and giving the ehanges predieted for various light sourees of eommereial interest might be of eonsiderable praetieal value to the illuminating engineer.

Acknowledgments

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*Research is being continued at the University of Texas to include a study of colors under the cool white (4500K) fluorescent lamp.

SPECIFICATION OF COLOR-RENDERING PROPERTIES OF FLUORESCENT LAMPS
(with C. W. Jerome)

Illum. Eng. 48, 259-267 (1953)

Although the CIE-recommended method for evaluating the color rendering properties of light sources has largely superseded such methods as are described in this paper, the nature of the problem and its fundamental parameters are more clearly presented here than in any later papers.

Errata: In the title line, "Specification" is misspelled; p. 261, first column, line 36 "monochromatic" is misspelled.

Specifcation of Color-Rendering Properties Of Fluorescent Lamps

By CHARLES W. JEROME
DEANE B. JUDD

IN RECENT years, increasing attention has been paid to the color-rendering properties of light sources; that is, how objects of various colors appear under them. Prior to the advent of the fluorescent lamp, the measure of the color of a light source also, to a large extent, specified its color-rendering properties. Such lamps were incandescent and as such had spectral energy distributions (*SED*'s) similar to black-body radiators at relatively low color temperatures as modified by any filters which might be used in conjunction therewith. Therefore, the establishment of the color of a light source also established its spectral energy distribution, and thereby the appearance of colors under it.

This is not the case with fluorescent lighting. These lamps make possible relatively high levels of illumination of almost any desired color and the large selection of usable phosphors also makes possible considerably different spectral energy distributions for any given color. This latter variable may give rise to a considerable change in the subjective color of an object in going from an area lighted by one type of lamp to one lighted by another. The most common example of this change is found in going from a room lighted by incandescent lamps to one lighted by warm-white fluorescent. These lamps are a reasonably good color match but most deep reds will appear darker and grayer under the fluorescent lamp due to the deficiency of the latter in far-red emission. By the same token, some greens and blues may appear lighter and more saturated by this change due to higher density of radiant flux from the fluorescent lamp in these regions. It is this obvious change in colors, coupled with the advanced state of development of the sciences of photometry and colorimetry, which has been largely responsible for the increased interest in color-rendition measurements.

The present study was undertaken by the authors as members of a sub-committee on color of the American Standards Association's C-78 Committee investigating this subject. The results reported herein are by no means the complete story but merely represent a start in the amassing of the large amount of data which will be necessary for the final solution of this problem. It is hoped that these data will serve to indicate the intricacies of this problem, and serve as a springboard for future investigations.

As stated generically above, color-rendition of a light source is determined by the appearance of colors under it. This suggests that color rendition can be estimated by viewing objects of various colors under several light sources and judging which source gives the best color-rendition. This method is unsatisfactory for four reasons:

1. A random choice of test objects will usually not include those required for a critical test.
2. No criterion is set up for the correct color of the test objects.
3. The comparison leads to a qualitative answer only; that is, no strictly quantitative measure can be made.
4. It is subject to personal preferences as to the appearance of colors. These preferences are different for each individual, and may even be different for the same individual at different times.

Therefore, the first problem in this study is the establishment of a criterion for judging colors; in other words, setting up a standard light source under which the true colors of samples can be determined. Then the color-rendering properties of a test lamp can be judged on the basis of how nearly alike in color the same sample looks under it and under the standard.

Fluorescent lamps were first developed to have the same visual color as a black-body radiator at a particular temperature. This temperature became an integral part of the color nomenclature of these lamps, for example, 6500K, Daylight; 4500K White (Cool White); 3500K White; 3000K White (Warm White). Since these various "white" lamps differ widely in color, any object color will look different under one than under another. Consequently, a standard light source should be set up for each "white" fluorescent lamp.

In the rendition of colors, energy is emitted by the illuminating source and is reflected or absorbed in varying proportions by the test object depending

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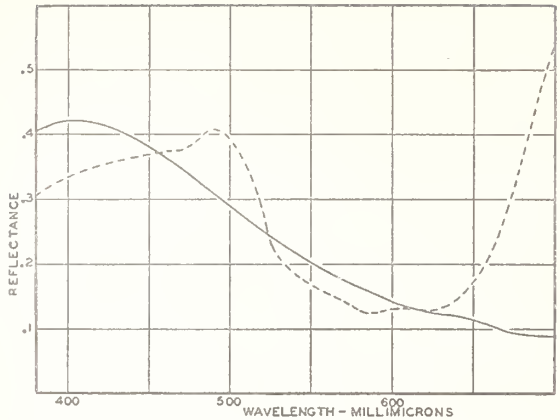


Figure 1a. Spectral reflectance of a metameric pair of blue colors.

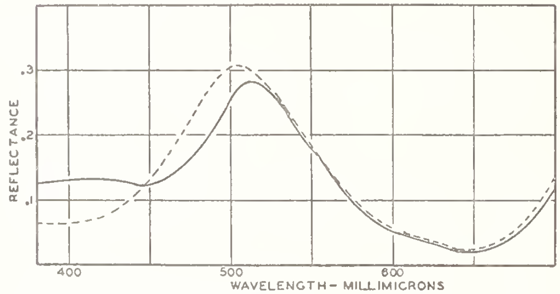


Figure 1b. Spectral reflectance of a metameric pair of green colors.

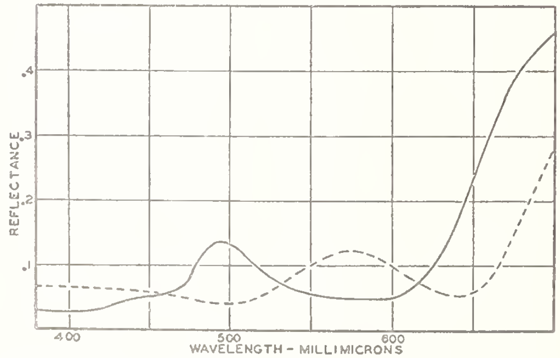


Figure 1c. Spectral reflectance of a metameric pair of olive colors.

on the wavelength of the energy and spectral selectivity of the object. Then, for the color of the object to be properly rendered, the source must emit energy within every one of a set of 5 to 10 wave bands in the visible spectrum in order that the selectivity of the test object in this band may properly alter the incident energy to give rise to the proper color sensation. Broad bands are specified here because if very narrow bands were used, certain discontinuous sources that have been found

TABLE I.—Spectral Reflectances of Three Pairs of Specimens Used To Test the Color-Rendering Of One Light Source Relative to Another.

Wavelength (mu)	Blue Specimens		Green Specimens		Olive Specimens	
400	0.419	0.333	0.130	0.066	0.029	0.066
410	.420	.343	.131	.068	.030	.066
420	.414	.351	.130	.073	.032	.065
430	.406	.358	.128	.086	.039	.063
440	.393	.363	.124	.106	.046	.061
450	.380	.368	.125	.132	.051	.058
460	.362	.371	.132	.163	.054	.055
470	.346	.376	.151	.200	.065	.050
480	.328	.386	.180	.242	.101	.046
490	.309	.408	.215	.281	.132	.042
500	.289	.391	.256	.303	.131	.040
510	.270	.348	.279	.301	.117	.043
520	.251	.277	.274	.282	.093	.052
530	.233	.221	.247	.252	.075	.068
540	.217	.183	.211	.213	.065	.085
550	.201	.167	.177	.182	.058	.101
560	.188	.153	.145	.148	.054	.114
570	.174	.141	.110	.116	.053	.123
580	.163	.129	.083	.087	.051	.124
590	.153	.126	.064	.068	.050	.115
600	.142	.132	.052	.056	.050	.100
610	.134	.132	.044	.048	.057	.083
620	.128	.130	.036	.039	.079	.070
630	.122	.131	.029	.032	.116	.058
640	.120	.144	.024	.026	.166	.053
650	.116	.173	.022	.024	.229	.055
660	.105	.217	.026	.030	.295	.078
670	.098	.279	.036	.043	.358	.118
680	.093	.367	.054	.064	.405	.166
690	.091	.462	.079	.096	.435	.220
700	.090	.546	.116	.140	.460	.280

to give good color rendition would be disqualified. The best example of this is probably daylight itself wherein the Fraunhofer lines make this a discontinuous source but do not interfere with the rendition of object colors. The CO_2 gaseous discharge tube is another example of a discontinuous source that gives very satisfactory color rendition.

A blackbody radiator fills the requirement of a suitable standard source; that of having energy in every broad wavelength band of the visible spectrum. Its spectral energy distribution is also well defined. Therefore, it seems logical to take a blackbody radiator at the same nominal color temperature as the lamps to be investigated as the standard for that color.

The establishment of a standard source overcomes the objection (2) of having no criterion for the correct color. If a standard observer is used to compute color specifications of the test objects for the standard and test sources, quantitative values may be obtained which provide a basis for overcoming objection (3). If, further, the object-color differences caused by passing from standard to test source are expressed in terms of number of just perceptible differences, objection (4) is overcome provided the test objects chosen are critical (objection 1).

Such a procedure has been used by the National Bureau of Standards[1] to test the degree of duplication of natural daylight by various artificial light sources. This procedure takes the form of computations of chromaticity coordinates, x, y, in the C.I.E. system[2] for three metameric pairs of colors that are so chosen that one pair (green) serves to detect deviations in the shortwave end of the spectrum between the source being investigated and the standard source; one pair (blue) similarly tests for the longwave and the third pair (olive) tests the middle of the spectrum. The spectral reflectances of these specimens are listed in Table I and are shown in Fig. 1. The chromaticity coordinates are then expressed in terms of a uniform-chromaticity-scale diagram, such as that of Breckenridge and Schaub,[3] so that equal distances anywhere in the diagram will represent approximately equally perceptible chromaticity differences, and the coordinate differences ($\Delta_1 x''$ and $\Delta_1 y''$) between the members of the pairs are determined. This is done for the standard and each source to be compared to it and the change in these differences (second differences, $\Delta_2 x''$ and $\Delta_2 y''$) in going from the standard to the test source is determined. That is, $\Delta_2 x'' = \Delta_1 x''_{\text{(Std.)}} - \Delta_1 x''_{\text{(Fluorescent Lamp)}}$ with a similar expression for $\Delta_2 y''$.

These data are reduced to a single deviation, D, for each test source from the standard by averaging the values of $\sqrt{\Delta_2 x''^2 + \Delta_2 y''^2}$ for the three metameric pairs. D in turn is converted to a "Duplication Index," I, by the transformation:

$$I = \frac{1 - 10D}{1 + 10D} \times 100.$$

This latter transformation is employed to arrive at an easily recognized scale of values. For example, by this transformation, an index of 100 represents exact duplication; an index of 50 is approximately the degree of duplication of gas-filled incandescent lamp light for natural daylight; and the index for monochromic light of wavelength 700 millimicrons for natural daylight is approximately zero.

This method of assessing the degree to which a test source duplicates the color rendition of the standard source is strictly valid for these three pairs of test objects only. It has general validity only to the degree that these test objects adequately represent objects to be viewed under the test sources. Furthermore, even with the number of test objects reduced to six, the computations are laborious and the method is quite unsuited for any quick routine check such as might be employed with a lamp factory. However, for all of the considerable number of sources of artificial daylight to which it has been applied, this method has yielded results in good agreement with the qualitative results of the subjective method; so it is proposed in this paper to use this method as a yard-

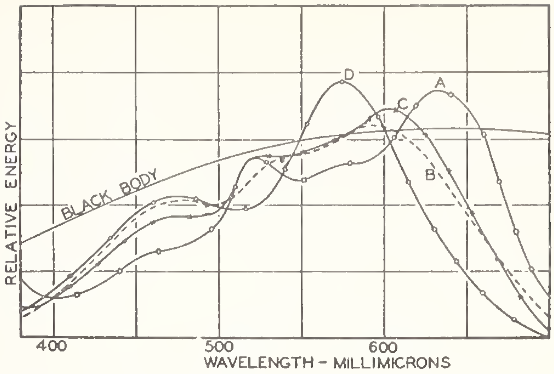


Figure 2. Spectral energy distributions — cool whites.

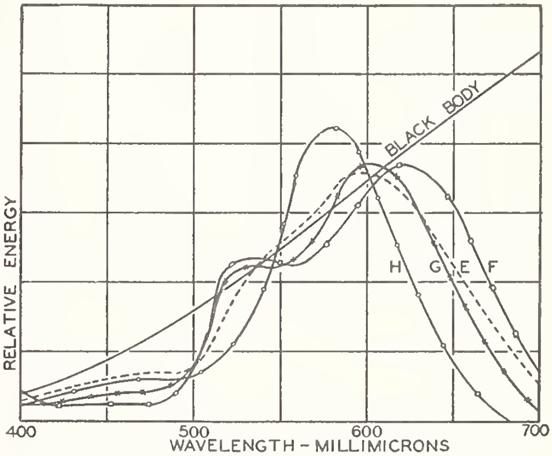


Figure 3. Spectral energy distributions — warm whites

stick of the efficacy of shorter methods devised for the same purpose. Accordingly, by this method, the duplication indices, I, of four spectral distributions of cool white (4500K) for black-body radiation of 4500K, and four spectral energy distributions of warm white for blackbody radiation of 2854K (C.I.E. source A) have been computed with the following results.

Cool Whites		Warm Whites	
	I		I
A	69	E	62
B	75	F	69
C	76	G	54
D	58	H	45

The spectral energy distributions of these light sources without the mercury-lines are shown in Figs. 2 and 3 and are given with the mercury-lines in Table II.

In all the computations in this paper, the weighted-ordinate method of integration is used. The effect of the mercury lines energy of the fluorescent lamps is added to that of the phosphor emis-

TABLE II.—Spectral Energy Distributions.

	COOL WHITES					WARM WHITES				
λ	Std. (4500K)	A	B	C	D	(2854K)	E	F	G	H
400	54.8	4.0	7.6	5.8	8.7	7.9	3.9	4.0	2.0	3.1
*405.0	——	14.5	21.0	16.8	17.3	——	16.0	19.8	19.8	18.3
410	58.8	3.9	10.1	7.2	11.2	9.5	4.4	2.8	2.2	3.8
420	62.7	4.1	12.6	8.9	13.3	11.3	5.1	2.1	2.8	4.8
430	66.6	5.0	16.0	10.7	16.0	13.3	5.9	2.1	3.2	5.8
*435.8	——	37.5	51.5	40.0	44.3	——	38.5	48.3	46.5	43.5
440	70.4	6.0	18.8	12.5	18.8	15.5	6.6	2.0	3.6	6.5
450	73.9	7.0	21.9	14.7	21.4	17.8	7.5	1.9	4.0	7.3
460	77.2	7.7	24.0	16.1	23.3	20.4	7.9	1.9	4.4	7.7
470	80.4	7.9	24.7	16.4	24.2	23.1	8.0	2.1	4.4	7.9
480	83.3	8.4	25.0	17.0	24.3	26.0	8.2	2.2	4.6	7.9
490	86.1	9.2	24.7	17.2	24.0	29.1	8.6	3.6	5.8	8.2
500	88.6	10.4	24.4	18.2	22.7	32.3	9.5	6.8	8.2	8.6
510	91.0	13.5	26.0	21.5	22.3	35.6	13.5	12.4	14.4	10.5
520	93.3	16.2	29.0	25.2	22.7	39.1	20.0	20.0	21.4	13.7
530	95.2	15.6	31.8	25.5	24.7	42.7	25.0	21.0	22.4	17.0
540	97.0	14.9	33.2	24.8	29.2	46.4	28.5	20.8	22.4	24.2
*546.1	——	21.8	26.5	22.0	22.0	——	20.3	25.0	26.8	22.5
550	98.6	14.1	33.6	24.0	35.0	50.1	31.0	20.0	22.4	33.0
560	100.0	14.6	34.6	24.2	40.4	53.9	34.0	20.2	24.2	46.6
570	101.1	15.0	36.2	26.2	44.2	57.8	37.3	21.4	27.2	54.5
*578.0	——	4.1	6.2	5.0	5.2	——	4.8	4.8	5.2	4.4
580	102.2	15.5	37.8	28.3	44.1	61.7	40.6	23.4	31.2	57.9
590	103.0	15.9	38.8	30.5	40.8	65.6	43.4	26.4	35.8	54.5
600	103.6	16.9	38.8	32.4	36.0	69.6	44.0	29.2	38.0	48.5
610	104.1	18.8	37.1	31.8	29.0	73.5	42.4	31.8	37.2	39.0
620	104.6	20.7	35.0	30.2	23.4	77.3	40.3	33.0	35.0	30.4
630	104.8	22.2	32.0	27.5	18.5	81.3	36.5	32.8	31.0	23.4
640	104.9	21.6	26.8	23.0	14.6	85.1	32.4	30.6	26.0	16.8
650	104.9	20.9	22.2	18.5	10.6	89.0	27.2	27.2	20.6	10.8
660	104.8	18.2	18.9	14.8	8.0	92.6	23.5	23.4	15.6	6.8
670	104.4	14.0	14.5	10.5	5.2	96.3	18.8	18.2	10.4	3.0
680	104.1	10.3	10.9	7.5	3.0	100.0	14.8	14.0	7.4	0.8
690	103.6	6.3	7.0	3.4	1.2	103.3	10.2	10.0	4.4	——
700	103.1	3.8	4.0	1.3	——	106.9	6.5	6.6	0.6	——

*Mercury lines. Energy values are those in excess of energy per 10 mμ interval of the continuum.

sion after multiplying their measured energy by the factor: effective slit width of monochromator divided by the wavelength interval used in the weighted-ordinate integration as described elsewhere by one of us.[4]

As brought out in the above discussion, the spectral energy distribution of a light source is the determining factor in its color-rendering properties. This suggests the *SED* curve as a specification for this property. Two lamps having identical *SED* curves will have identical color-rendering properties and also identical color. Therefore, the establishment of a standard *SED* curve suffices to specify the color-rendering properties. However, there are practical limitations to such a specification. Lamps from different manufacturers may use similar blends of phosphors but may differ considerably in color. Also, there is a variation in color from lamp to lamp of a single manufacturer, the amount of the variation depending on the strictness of quality control measures employed. Consequently, concurrently with the establishment of a standard *SED* curve, tolerances on either side of it must also be established. These tolerances must be made loose enough so that maximum deviation from the standard at any one point will be allowed. When this is done for all points, the tolerances tend to lose significance since they would seem to allow maximum deviation at several, or all, points, whereas such an occurrence is obviously unsuitable. This objection could be overcome by making a single figure of merit similar to the "duplication index" described above, which would average the deviations from the standard, together with suitable tolerances thereon, as a supplementary part of the specification.

Another drawback in the use of *SED* curves as a complete measure of color-rendition is the difficulty in their interpretation, especially when lamps of considerably different *SED*'s are to be compared. This is indicated by the curves in Figs. 2 and 3. The comparison of any two such curves can lead only to qualitative estimates of the rendition of specific colors. For quantitative results recourse must be made to laborious computations similar to those used in the determination of the "duplication indices" above. What is most desired, then, is some simpler specification.

The *SED* curve implies that measurements of the energy in a number of narrow bands have been made. These bands are narrow and of sufficient number so that a continuous curve can be drawn. For example, the radiometer on which the *SED* curves of Figs. 2 and 3 were determined used bands 5 millimicrons wide throughout. Therefore, one simplification possible is to measure the energy in a smaller number of wider bands. This is the basis of a measure of color rendition of fluorescent lamps developed in Europe.[5, 6, 7] In their procedure, the per cent luminous energy (radiant energy weighted by the spectral luminous efficiency of the normal eye) in eight spectral bands, and tolerable limits to deviations therefrom are specified. The bands chosen as of equal importance in color rendition, and the tolerances set to accord with fluorescent-lamp control attainable in England in 1947 are:

Band No. 1	from 380 to 420 mu	±20%
Band No. 2	from 420 to 440 mu	±10%
Band No. 3	from 440 to 460 mu	±10%
Band No. 4	from 450 to 510 mu	±10%
Band No. 5	from 510 to 560 mu	± 5%
Band No. 6	from 560 to 610 mu	± 5%
Band No. 7	from 610 to 660 mu	±10%
Band No. 8	from 660 to 760 mu	±20%

These band limits are shown superimposed on a spectral energy distribution curve of a typical fluorescent lamp in Fig. 4.

Table III shows the distributions of luminous energy in per cent for the two standards (4500K and 2854K) and for the eight fluorescent lamps (see Table II and Figs. 2 and 3). The deviations from standard are also shown for the eight lamps.

A sample computation sheet for one of the lamps is shown as Table IV.

It is evident from the deviations from the standard given in Table III and the tolerances used by the British that a fluorescent lamp must be the standard used in the latter specification. This may be convenient for routine checks when such a standard has been established, but it seems to us desirable to try to establish a standard which will represent the best color-rendering light source available. This is one of the factors influencing our choice of blackbody radiation for this purpose. The greatest apparent drawback to the British procedure is the specification of eight different tolerances and the resultant difficulty in properly evaluating the overall effect of the deviations.

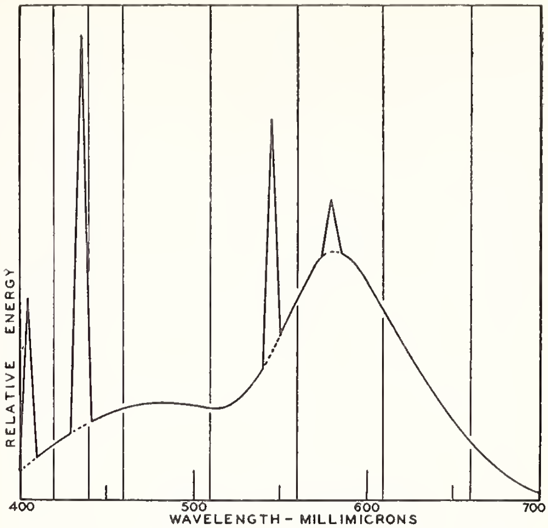


Figure 4. Spectral band limits in British specification.

We have, therefore, tried averaging the deviations in order to arrive at a single numerical index which would be a measure of the color-rendering properties of the lamp in question and which could

TABLE III.—Luminous Energies and Deviations from Standard in Eight Spectral Bands.

COOL WHITES

Band	4500K	Lamp A		Lamp B		Lamp C		Lamp D	
	L%	L%	ΔL	L%	ΔL	L%	ΔL	L%	ΔL
1	.02	.01	.01	.01	.01	.01	.01	.01	.01
2	.16	.42	.26	.35	.19	.33	.17	.33	.17
3	.56	.30	.26	.46	.10	.40	.16	.47	.09
4	8.63	5.68	2.95	6.75	1.88	6.41	2.22	6.61	2.02
5	39.17	44.92	5.75	42.27	3.10	42.21	3.04	40.26	1.09
6	38.50	34.81	3.69	39.76	1.26	39.23	.73	45.34	6.84
7	12.05	13.20	1.15	10.06	1.99	11.09	.96	6.84	5.21
8	.91	.66	.25	.34	.57	.32	.59	.13	.78
		ΔL:	1.79		1.14		.99		2.03

WARM WHITES

Band	2854K	Lamp E		Lamp F		Lamp G		Lamp H	
	L%	L%	ΔL	L%	ΔL	L%	ΔL	L%	ΔL
1	.01	.01	—	.01	—	.01	—	.01	—
2	.06	.24	.18	.35	.29	.31	.25	.24	.18
3	.25	.17	.08	.06	.19	.11	.14	.15	.10
4	5.47	2.91	2.56	2.56	2.91	2.89	2.58	2.40	3.07
5	33.45	37.67	4.22	42.43	8.98	40.15	6.70	33.56	.11
6	42.59	45.89	3.30	39.19	3.40	43.47	.88	55.21	12.62
7	16.64	12.64	4.00	14.77	1.87	12.74	3.90	8.35	8.29
8	1.53	.47	1.06	.63	.90	.32	1.21	.08	1.45
		ΔL:	1.93		2.32		1.96		3.23

TABLE IV.—Computation of Radiant and Luminous Energy Distribution.

Lamp: B

Band System: British Specification

NOTE: The energies of the mercury lines are separately recorded at 405.0, 435.8, 546.1, and 578.0 millimicrons.

Wavelength	E	y	L = Ey
400 mu	7.6	.0004	.0030
405.0	21.0	.0006	.0126
410	10.1	.0012	.0121
420	12.6	.0040	.0504
430	16.0	.0116	.1856
435.8	51.5	.0178	.9167
440	18.8	.0230	.4324
450	21.9	.038	.8322
460	24.0	.060	1.4400
470	24.7	.091	2.2477
480	25.0	.139	3.4650
490	24.7	.208	5.1376
500	24.4	.323	7.8812
510	26.0	.503	13.0780
520	29.0	.710	20.5900
530	31.8	.862	27.4116
540	33.2	.954	31.6728
546.1	26.5	.984	26.0760
550	33.6	.995	33.4320
560	34.6	.995	34.4270
570	36.2	.952	34.4624
578.0	6.2	.889	5.5118
580	37.8	.870	32.8860
590	38.8	.757	29.3716
600	38.8	.631	24.4828
610	37.1	.503	18.6613
620	35.0	.381	13.3350
630	32.0	.265	8.4800
640	26.8	.175	4.6900
650	22.2	.107	2.3754
660	18.9	.061	1.1529
670	14.5	.032	.4640
680	10.9	.017	.1853
690	7.0	.0082	.0574
700	4.0	.0041	.0164
Totals:	863.2		385.4462

Band No.	Wavelength Limits	E E%	L L%
1	380-	45.0	.0529
	420	5.2%	.01%
2	420-	83.2	1.3437
	440	9.6%	.35%
3	440-	43.3	1.7684
	460	5.0%	.46%
4	460-	123.8	26.0005
	510	14.4%	6.75%
5	510-	184.4	162.9349
	560	21.4%	42.27%
6	560-	193.6	153.2587
	610	22.4%	39.76%
7	610-	144.0	38.7875
	660	16.7%	10.06%
8	660-	45.9	1.2996
	760	5.3%	.34%
		863.2	385.4462
		100.0%	100.00%

then be compared to its duplication index, *I*. For example, the magnitude of the deviations given in Table III have been averaged with the results as indicated. The correlation between these averages and the computed duplication indices is shown in Fig. 5. These data are shown in another way in Fig. 6 wherein the correlation between computed indices (*I*) and average differences between per cent luminous energy of blackbody and per cent luminous energy of the fluorescent lamp in each of the bands expressed as per cent of that of the standard is shown. This correlation is not as good as that shown in Fig. 5.

A further disadvantage of the British specification is the extremely low luminous energy in the

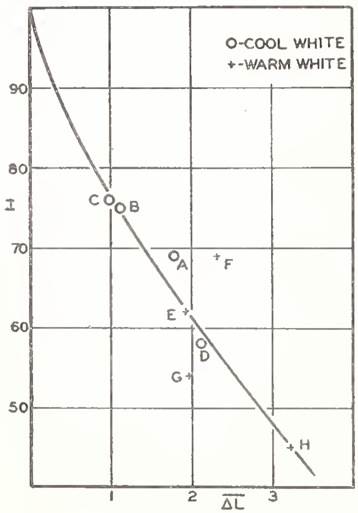
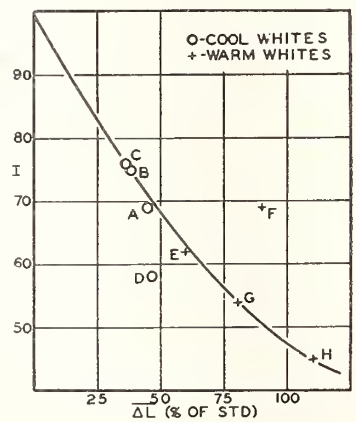


Figure 5. (left) Average luminous energy differences in British bands vs. duplication indices.

Figure 6. (right) Average luminous energy differences (per cent of standard) in British bands vs. duplication indices.



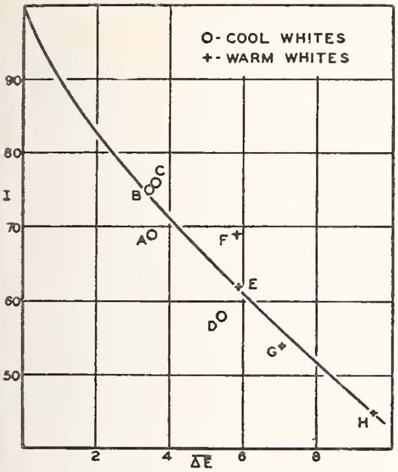
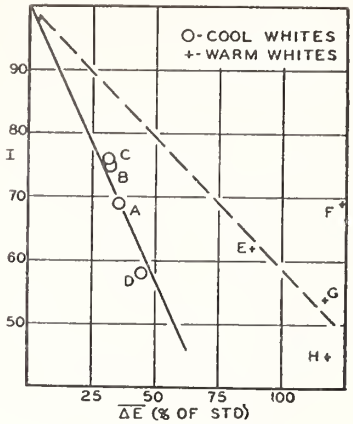


Figure 7. (left) Average radiant energy differences in British bands vs. duplication indices.

Figure 8. (right) Average radiant energy differences (per cent of standard) in British bands vs. duplication indices.



bands at the end of the spectrum due to the low sensitivity of the eye in these regions. Since the fundamental controlling factor in color rendition is the radiant energy emitted by the light source, a survey was made of this property of the test lamps in the spectral bands of the British specification. The results are given in Table V and shown in Figs. 7 and 8. In Fig. 7, the abscissa is the average difference between per cent radiant energy of the test lamp and per cent radiant energy of the standard. Fig. 8 shows the correlation between *I* and the average deviation from the standard per cent radiant energy in each band, the deviations being expressed as percentages of the per cent radiant energy of the standard in that band.

Fig. 8 indicates a possible correlation within each white although in each case there are deviations which are nearly as large as when a single correlation is assumed. Fig. 7 shows a much better correlation between duplication indices and average radiant energy differences.

Figs. 5, 6, 7, and 8 point up an observation which has been corroborated in the other studies reported herein. That is, the average of the actual differences in the magnitude of either luminous energy or radiant energy in a number of bands leads to a better correlation with the duplication indices than when these differences are expressed as a percentage of the standard values in each band and then averaged. Therefore, in the other results reported, it is the actual magnitude of the differences which are considered.

In view of the disadvantage of low luminous energies in the British bands at the ends of the spectrum another set of 10 bands was set up such that for a light source of equal spectral energy

TABLE V.—Radiant Energies and Deviations from Standard in Eight Spectral Bands.

COOL WHITES

Band	4500K E%	Lamp A E%	Lamp A ΔE	Lamp B E%	Lamp B ΔE	Lamp C E%	Lamp C ΔE	Lamp D E%	Lamp D ΔE
1	5.1	5.3	.2	5.2	.1	5.2	.1	5.8	.7
2	4.7	10.3	5.6	9.6	4.9	9.3	4.6	10.2	5.5
3	5.2	3.0	2.2	5.0	.2	4.4	.8	5.7	.5
4	15.0	10.1	4.9	14.4	.6	13.3	1.7	15.7	.7
5	16.9	21.0	4.1	21.4	4.5	21.9	5.0	22.0	5.1
6	18.1	18.3	.2	22.4	4.3	22.8	4.7	27.4	9.3
7	18.5	22.6	4.1	16.7	1.8	18.6	.1	11.4	7.1
8	16.5	9.4	7.1	5.3	11.2	4.5	12.0	1.8	14.7
		ΔE:	3.55		3.45		3.63		5.45

WARM WHITES

Band	2854K E%	Lamp E E%	Lamp E ΔE	Lamp F E%	Lamp F ΔE	Lamp G E%	Lamp G ΔE	Lamp H E%	Lamp H ΔE
1	1.4	3.7	2.3	4.8	3.4	4.3	2.9	4.2	2.8
2	1.6	6.9	5.3	9.2	7.6	8.9	7.3	8.4	6.8
3	2.2	2.0	.2	.7	1.5	1.4	.8	2.2	—
4	8.5	6.2	2.3	3.8	4.7	5.5	3.0	6.4	2.1
5	13.7	20.5	6.8	21.5	7.8	22.8	9.1	21.4	7.7
6	19.5	28.7	9.2	23.0	3.5	28.4	8.9	40.3	20.8
7	25.4	23.4	2.0	26.4	1.0	23.5	1.9	16.0	9.4
8	27.7	8.6	19.1	10.6	17.1	5.2	22.5	1.1	26.6
		ΔE:	5.90		5.81		7.05		9.53

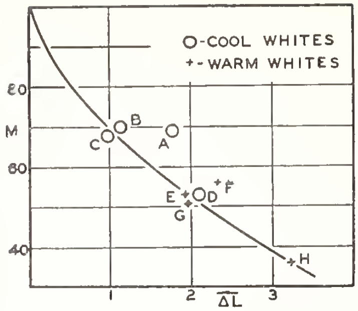
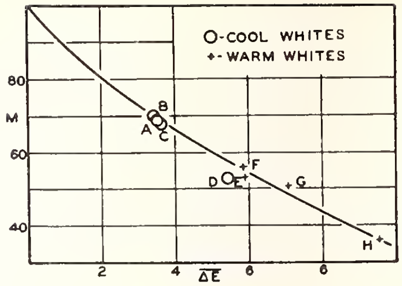


Figure 9. (left) Average luminous energy differences in British bands vs. modified indices.

Figure 10. (right) Average radiant energy differences in British bands vs. modified indices.



distribution approximately equal luminous energy would be obtained in each. The band limits are:

Band No. 1	Below 497.5 mu
Band No. 2	from 497.5 to 517.5
Band No. 3	from 517.5 to 532.5
Band No. 4	from 532.5 to 547.5
Band No. 5	from 547.5 to 557.5
Band No. 6	from 557.5 to 567.5
Band No. 7	from 567.5 to 582.5
Band No. 8	from 582.5 to 597.5
Band No. 9	from 597.5 to 617.5
Band No. 10	Above 617.5

Since other than established band systems are being considered, still another set of bands has been investigated. These are of equal width, each 50 millimicrons wide (400-450, 450-500, 500-550, 550-600, 600-650, and 650-700 millimicrons). The average deviations from the standard in each of the sets of bands tested of the luminous and radiant energies of our test lamps are listed in Table V.

These data would seem to indicate no particular advantage of one band system over another. The British band system has been used for several years in Europe and although there have been rumors of dissatisfaction with the system, considerable data and experience with it have been obtained and special photometers built for simplified determination of the luminous energies in the separate bands. For these reasons there has been a great reluctance to change of the system and until a band system that can show decided advantages over the old system can be developed, there will be no point in making a change.

Up to this point, the discussion has been predicated on the duplication indices derived from the use of metameric pairs as explained earlier in this paper. This index has obvious advantages for the specific application for which it was developed, that is, as a measure of the efficacy of light sources to be used for color matching of filters, painted surfaces, and so on. In that application, the occurrence of metameric pairs of colors can be commonplace. In the more general application for overall illumination, the appearance of individual colors is more important than the comparison of pairs of colors. This has led some investigators[8] to study the color changes evinced by individual objects in going from one kind of illumination to another. It has also been suggested that commonplace objects should be used for this determination, such as butter, lettuce, coffee, and meat to mention a few. In view of these suggestions, a modified duplication index (M) has been developed and the correlation of our radiant and luminous energy data for the several band systems investigated above correlated thereto.

The modified index is derived similarly to the duplication index, I, using the same six test objects

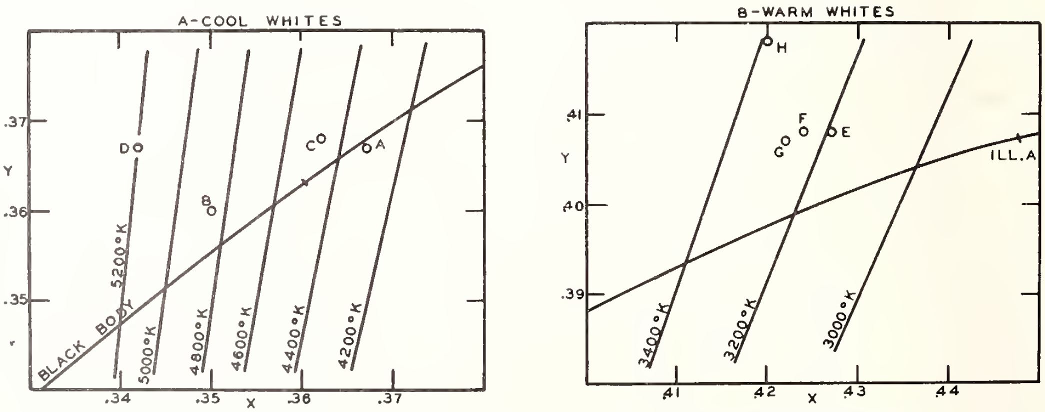


Figure 11. Chromaticities of lamps A to H on the C.I.E. color mixture diagram.

TABLE VI.—Average Luminous and Radiant Energy Differences in Three Sets of Band Systems.

SED	I	M	British Spec. L	British Spec. E	10 Bands L	10 Bands E	6 Bands of 50 mu L	6 Bands of 50 mu E
A	69	69	1.79	3.55	2.02	1.10	2.57	3.97
B	75	70	1.14	3.45	1.48	2.70	1.23	4.08
O	76	68	.99	3.63	1.54	2.48	1.26	4.68
D	58	53	2.03	5.45	2.49	4.42	2.77	6.77
E	62	53	1.93	5.90	1.82	5.16	2.21	7.15
F	69	56	2.32	5.81	2.12	3.32	3.18	7.27
G	54	51	1.96	7.05	2.89	5.34	2.42	8.80
H	45	36	3.23	9.53	3.59	7.50	4.86	11.33

comprising the metameric pairs. However, in the modification the changes in the color parameters in the rectangular uniform chromaticity scale are used instead of the second differences ($\Delta_2 x''$ and $\Delta_2 y''$). That is:

$$\Delta x'' = x''_{\text{(Std.)}} - x''_{\text{(fluorescent lamps)}}$$

and $$\Delta y'' = y''_{\text{(Std.)}} - y''_{\text{(fluorescent lamps)}}$$

Then, as with I, a single deviation, D, for each illuminant is obtained by averaging the six values of $\sqrt{\Delta x''^2 + \Delta y''^2}$. M is then derived from D by the transformation:

$$M = \frac{1 - 10D}{1 + 40D} \times 100.$$

The modified index, M, for our eight fluorescent lamps for their respective blackbody standards are computed to be:

Cool Whites *SED*	*M*	Warm Whites *SED*	*M*
A	69	E	53
B	70	F	56
C	68	G	51
D	53	H	36

The correlation between these indices and the luminosity and energy analyses of the several band systems show that the correlation in the British band system is fully as good as that in the others. These relationships are shown in Figs. 10 and 11.

Before closing, it should be mentioned that the fluorescent lamps used in this study were chosen at random from several different production lots and from different manufacturers. Consequently, they are not all of the same color, nor do they match the color of the blackbody radiators chosen as the standards. The position of the lamps on the C.I.E. Color Mixture Diagram is shown in Fig. 11. The duplication indices of these lamps (both I and M) might be raised somewhat if the standard chosen in each case were the blackbody radiator of the nearest color temperature.

In summary, the conclusions arising from the present study are:

1. The spectral energy distribution of a light source is the only complete specification of its color-rendering properties.

2. A single index is required to supplement the *SED* curve to give an overall quantitative measure of the color rendition. When two *SED* curves coincide throughout, there is no need for the index since the color rendition of the two light sources will also be identical. The need for the single index arises to evaluate the relative merits of two lamps whose *SED* curves do not coincide.

3. Equipment for the precise determination of *SED* curves is too complicated and expensive for universal use. Therefore, a simpler procedure is required.

4. The measurement of relative energies in a finite number of spectral bands fulfills the requirements of number 3 above.

5. The use of the eight wavelength bands proposed by the British is recommended until such time as a selection of bands showing superior utility is developed.

6. The use of radiant energies in the spectral bands is recommended rather than the use of luminous energies. Radiant energy is the fundamental parameter in color rendition and the data contained herein show better correlation between duplication indices and energy relationships.

7. The single index suggested for color rendition of a lamp is the average difference between lamp and standard in per cent radiant energy within the eight British bands. Indices based on differences expressed as per cents of energies of the standard within the same wavelength band show poorer correlation with the duplication indices.

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PROGRESS REPORT BY OPTICAL SOCIETY OF AMERICA COMMITTEE ON UNIFORM COLOR SCALES

J. Opt. Soc. Am. 45, 673-676 (1955)

This report states the original intention of the committee - to obtain 500 color chips with which to judge visually the uniformity of a three-dimensional color space that was derived from data available in 1953. A subsequent revision of that plan, to use a much smaller number of colors for visual scaling and for improvement of the color space before making the full number of color chips, delayed the specification of the 500 colors until early 1972. Production of those colors was not completed until 1977. Visual examination of the several hundred color scales that can be assembled from them remains to be done. It is doubtful whether the surviving committee members can obtain sufficient data to reach consensus so as to enable them to report significant improvement over the color space used to design the 1977 colors. See MacAdam, J. Opt. Soc. Am. 64, 1691-1702 (1974).

Tetrahedral is an inappropriate description of the space lattice used, because the lines that connect all nearest neighbors define tetrahedra that enclose only one-third of the space. The excluded two-thirds of the space is enclosed by regular octahedra. The lattice is properly described as regular rhombohedral. Six of the eight vertices of each regular rhombohedral unit cell are vertices of one of the octahedra. The other two vertices of the regular rhombohedral unit cell are vertices of regular tetrahedra whose bases are opposite faces of the octahedron. The description "face-centered cubic" mentioned on p. 674 is also erroneous.

Progress Report by Optical Society of America Committee on Uniform Color Scales

Deane B. Judd, *Chairman*

Summary.—We have progressed in our laboratory studies to the point where we need to prepare a set of about 500 color chips, equally spaced according to the laboratory results, to be studied by the committee as a closing check before preparation of a final report.

History.—In the report of the Committee on Colorimetry [J. Opt. Soc. Am. **34**, 688 (1944)] it is stated: "Color differences consisting solely of chromaticity differences with no luminance differences are rare, and in this case there is no known way of evaluating the noticeability of combined luminance and chromaticity differences. The complete experimental clarification of this problem is one of the major programs yet to be undertaken in the field of colorimetric research. Influences of field size and adaptation to various luminances and chromaticities also await thorough quantitative study."

Two informal conferences on ways to carry on this program were held, one in conjunction with a meeting of our Society (February 21, 1947), Hotel Pennsylvania, New York City), and one in conjunction with a meeting of the Inter-Society Color Council (April 25–26, 1947, Medical Research Laboratories of the U. S. Submarine Base, New London, Connecticut), and finally a formal meeting was called at the suggestion of Dr. R. C. Gibbs, Chairman, Division of Physical Sciences, National

Research Council, for June 20, 1947 at the National Academy of Sciences, Washington, D. C. Those present were Balinkin, Bellamy, Dimmick, Foss, Gibbs, Godlove, Granville, Judd, MacAdam, Nickerson, and Newhall. The usefulness of more comprehensive and accurate information regarding the perceptibility of color differences was discussed at considerable length, and it was brought out that:

(1) The National Bureau of Standards would use this information to improve the "NBS unit of color difference" used in the setting and administration of color tolerances.

(2) The Munsell Color Company would use this information to improve the spacing of the color scales making up the Munsell Color System.

(3) The plastics industry would use this information because of its application to the problem of setting up color tolerances for the production of colored plastics and to the problem of evaluating color differences between production batches and the accepted standard.

(4) The photographic industry would use the information, if it were available, as a tool in its studies and tests of the fidelity of color reproduction of a scene by a given photographic process; and it is to be presumed that this information would likewise be useful in studies of television in color, and in three-color process printing.

(5) The Department of Agriculture would use the information in its studies of the colorimetry of agricultural products.

(6) Others were interested in this information because it would show under what conditions and to what approximation it is possible to define a "homogeneous isotropic color space" (Balinkin, 1941). Even a somewhat imprecise approximation to such a space would seem to open up a new possibility in assembling collections of color chips for various purposes. Thus, a collection of color chips might be assembled each one of which differed from each of its nearest neighbors by amounts of nearly equal perceptibility. Such a collection would not only cover a given color gamut with the fewest possible number of chips, but would also be maximally useful in setting color tolerances.

(7) The Navy Department would use information regarding the perceptibility of color differences in solving unnumbered problems arising in the design of fighting equipment.

(8) It was also considered that more precise information regarding uniformity of color scales would aid somewhat in color coordination, or selection of groups of colors to be used together for some specified purpose in which the question of color harmony is important. Possible fields of application lie in interior decoration, exterior painting of prefabricated houses forming a planned community, and in choice of color for hospital rooms with the attendant question of color therapeutics.

A comprehensive and detailed research program was drawn up involving the determination of color difference ellipsoids in CIE space for a systematic sampling of the chromaticity twofold, and evaluating of the influences of field size, field luminance, surrounding-field luminance and chromaticity, angular separation of the fields, and individual-observer differences. It was voted unanimously that, *subject to the approval of the Optical Society of America* and the Inter-Society Color Council, that the National Research Council be requested to set up a committee on uniform color scales.

On recommendation by Dr. L. A. Jones, Chairman of the OSA Committee on Colorimetry, the OSA Board of Directors (October, 1947) voted not to approve this project for the National Research Council, but rather that it should be undertaken as a project of the Optical Society of America. The committee was appointed as follows: Balinkin, Bellamy, Dimmick, Foss, Godlove, Granville, Judd (Chairman), MacAdam, Newhall, Nickerson, Saunderson, and in 1953 there were added Davidson, Ingle, Smith, and Hunter.

Summary of Researches by Members of the Committee.—A meeting of the committee was held on October 21, 1948 at the Hotel Fort Shelby, Detroit, and a tentative parceling out of the extensive program was made. For several years no further meetings were held, but those of the committee who could get support from their own organizations went ahead with their tasks, individually.

(1) Direct experimental attack.—The major advance in determination of rates of change of color perception in the CIE system was made by Brown and MacAdam (1949). The rates of change in all directions from 38 well-distributed starting colors were determined by analysis of the errors of setting for a color match, and were expressed by means of the color-difference ellipsoid for each starting color. They also made a study of the influence of luminance level on these rates of change for three starting colors. Brown (1951) extended this study to 5 other colors and found the change rates to be relatively constant for luminances above one foot-lambert. Below that level the rates of color-perception change decrease particularly in the directions of tritanopic confusion. Burnham and Newhall (1953) studied the influence of decreasing the field size on color perception, and corroborated the previous finding of Middleton and Holmes (1949) that vision of the normal observer for small fields tends toward the tritanopic. This implies that the color-perception change rates should be influenced by decrease of field size much as found by Brown (1951) for decrease in luminance. MacAdam's (1949) work on the influence of color contrast work on acuity likewise showed the change rates to decrease sharply with decrease in field size but did not show clearly the tritanopic tendency toward preferential decrease in the purplish blue–greenish yellow sense. Brown (1952) also made a preliminary study of the influence of surrounding field color. Traub and Balinkin (1954) studied the influence of angular separation between the fields to be compared and found that the change rates toward lighter and darker colors were markedly decreased but those for constant luminance were decreased but little.

(2) Check of color-difference formulas.—A number of formulas defining a distance element in some transformation of CIE space had been previously devised (Nickerson, 1936; Balinkin, 1939; Judd, 1939; Adams,1942; Hunter, 1942; Saunderson and Milner, 1946; Judd, 1952) and found to give moderately good correlation with perceptibility of the color difference by direct visual estimate [Judd, (a)1939; Wright, 1941, 1943; Balinkin, 1941; Bellamy and Newhall, 1942; MacAdam, (b)1943; Nickerson and Stultz, 1944]. For a comparison of these formulas with regard to representation of Munsell spacing, see Burnham (1949). Some members of the committee (and some nonmembers) pushed forward with further experimental checks of these formulas (Nickerson, 1950; Boyd, 1951; Webber and Billmeyer, 1953), with development of methods to apply the formulas more quickly (Nickerson, 1950; Opler, Meikle, and Charlesworth, 1953), and with development of improved distance elements and transformations of CIE space (Godlove, 1951, 1952). Others showed that use of formulas so far developed, though convenient and moderately successful, do not correlate as well with visual estimates as direct appeal to the experimentally determined change rates [Davidson (a), 1951; Ingle and Rudick, 1953; Davidson and Friede, 1953], and Davidson [(b) 1951, 1954] has shown how to use the experimental results of Brown and MacAdam (1949) without too much loss of time.

(3) Preparation of color chips to form uniform scales.—The ultimate proof that the problem of producing uniform color scales has been solved is, of course, the actual production of chips exemplifying such uniform spacing for any skeptic to look at. There has been some progress in this direction by members of the committee. Foss (1949) explained in more detail his plan to sample the color solid by the most regular of the space lattices generated by the close packing of uniform spheres. This space lattice has been called variously tetrahedral (Foss, 1947), three-layer close packing (Judd, 1952), face-centered cubic (by crystallographers), and regular rhombohedral (by mathematicians). Foss has made a preliminary sampling of Munsell renotation space according to this space lattice. Davidson and Luttringhaus (1951) produced a Munsell color book in textiles in which Munsell

renotation space is sampled uniformly along the scales of Munsell hue, value, and chroma. Hemmendinger and Davidson (1954) have produced local samplings of the color solid for use in further studies of color-perception-change rates.

Meeting of October, 1953.—This meeting was called to permit an assessment of the progress made since 1948, and to see whether further coordination of the research efforts of the committee members was needed. Those who had been concerned with the direct experimental attack held the view that the experimental facts have now been established in their essentials, and said that they did not expect to make any further studies. It is now possible, they said, by interpolation among the 38 colors studied to assess reliably the perceptibility of any color difference from the tristimulus values of the two colors, and, by appeal to the auxiliary studies (field size, field luminance, color of background, separation of the two fields), to do it for any experimental conditions of interest. Those who had been working with color-difference formulas pointed out correctly that the experimental results are extremely complicated [MacAdam (a), 1943], not suited for convenient engineering applications, and anyway seemed to prove that a truly uniform set of tridimensional color scales in accord with sampling by the regular rhombohedral space lattice is not possible (Silberstein, 1942; Silberstein and MacAdam, 1945; Judd, 1952). The achievement of most practical value, they said' would be the development of a color-difference formula, simple enough to be convenient and at the same time giving a satisfactory reliable representation of the essential experimental facts. It developed that no one had been able to form a conception of the nature and importance of the compromise involved in such a formula. Those primarily interested in the production of uniformly spaced color chips took the position that the time is now ripe to apply everything that has been learned by the numerous psychophysical studies to the central problem of producing a set of chips of uniform tridimensional spacing. In this way, they pointed out, the nature of the compromises that have to be made will become evident merely by looking at the chips. We will see, furthermore, whether it is true that interpolations made among the 38 starting colors in the Brown and MacAdam study (1949) are reliable, and can more easily in this way make any last refinements that may be required. This view finally won the unanimous approval of the committee and a plan for producing such a set of chips for use by the committee in a final check prior to issuance of a report was drawn up.

Production of a 500-Chip Uniform Tridimensional Color Scale.—It was agreed that the colors should be derived by a regular rhombohedral sampling either of Munsell renotation color space (Newhall, 1940; Newhall, Nickerson, and Judd, 1943), or of this space corrected by Davidson to take account of the chief discrepancies between it and the MacAdam (1942) and the Brown and MacAdam (1949) results, whichever seemed better to Davidson who undertook to make the formulations of paint for the 500 chips. The Munsell Color Company will supply the base paints from which the samples will be formulated, and also the basic formulation graphs from which the formulations for any desired Munsell renotation within the gamut of the base paints can be read. A set of these chips is to be supplied to each member of the committee for study.

Progress Since the October, 1953, Meeting.—Early in 1954, Dr. Günter Wyszecki, Fulbright Scholar from Berlin stationed as a guest worker at the National Bureau of Standards, computed the Munsell renotations of a suitable 500-chip regular rhombohedral sampling of this color space (Wyszecki, 1954). Dr. Davidson has made a start toward a tentative revision of the definition of the Munsell renotations so as to make the spacing agree as closely as possible with the data of MacAdam (1942) and MacAdam and Brown (1949). From this tentative revision the daylight reflectances (Y) and chromaticity coordinates (x,y) may be read for the renotations corresponding to the regular rhombohedral sampling computed by Dr. Wyszecki.

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CHROMATICITIES EXHIBITING MAXIMAL CONTRAST

Nat. Bur. Stand. (U.S.) Report No. 4408 (1955)

Figure 1 of this report, which does not actually appear in it, is substantially the same as Fig. 1 in the paper entitled "A Maxwell triangle yielding uniform chromaticity scales", which is included in this book. However, estimates for very small fields would require use of the Breckenridge and Schaub diagram (Ref. 5), in the manner specified in the last sentence of this report.

CHROMATICITIES EXHIBITING

MAXIMAL CONTRAST

By

Deane B. Judd

I. REQUEST:

1. Which two hues will give the greatest contrast visible to the eye?

2. Which will give the least contrast?

3. Is there any table, chart, or graph, published or unpublished, which would give this information in terms of the CIE chromaticity diagram?

4. Could we have a copy of such a diagram showing areas of color contrast?

II. Introduction

Color contrast may be defined as the perceptual size of the difference between two colors. As shown in NBS Report 3773, Determination of Color of Maximum Contrast, [1]* color contrast may be broken down into a lightness component and two chromatic components, and the relative importance of the lightness component depends upon the observing conditions, particularly upon the angular subtense of the elements to which the colors are perceived to belong. For fields of small angular subtense (0.01°) the lightness component determines the color contrast, and, indeed, no chromatic component is perceptible. For fields of large angular subtense (10°), the chromatic component of the color contrast assumes an importance at least as great as that of the lightness component. The question, "Which two hues will give the greatest contrast visible to the eye?", is interpreted as having reference only to the chromatic components of contrast. It might be rephrased: "What two colors of identical luminance are perceived as maximally different?", or "What two chromaticities are perceived as maximally different?"

As in NBS Report 3773, the basic information is to be found in formulas already derived and verified within about a factor of 2 for various specific observing conditions; see formulas 28, 29, 30, 31, 32, 33, 34, and 35, given by Judd (1952) [2]. The determination of the two maximally different chromaticities is especially convenient

*Figures in brackets indicate the index references at the end of this report.

from formula 30, which expresses the difference ΔE, between two colors of equal luminance as:

$$\Delta E = k_2 \Delta S \qquad (1)$$

where ΔS is proportional to the distance between the two chromaticity points plotted on a uniform-chromaticity-scale diagram, such as that by Judd (1935) [3], MacAdam (1937) [4], or Breckenridge and Schaub (1939) [5].

III. Maximally different chromaticities for 2° fields

The method of finding maximally different chromaticities suggested by equation 1 is by inspection of the uniform-chromaticity-scale (UCS) diagram valid for the particular field size in question. The Judd-UCS diagram (see Fig. 74, reference 2) applies to field sizes of about 2°. The area inclosed by the spectrum locus and the straight line joining its extremes is the locus of points representing chromaticities producible by all additive mixtures of spectrum lights, that is, all possible chromaticities. From the shape of this locus it is evident that one of the maximally different chromaticities must be that of the long-wave (red) extreme of the spectrum. The second chromaticity to be paired with this is not sharply defined. What is evident from inspection of this locus is that any spectrum chromaticity from about 510 (bluish green) to 465 mμ (blue) is separated from extreme spectrum red by about the maximum amount. The Breckenridge-Schaub-RUCS diagram (Figure 1 of this report, or Fig. 77, reference 2) gives the same indication, as does, indeed, the MacAdam-UCS diagram (reference 4).

IV. Maximally different chromaticities for 0.1° fields

If one of the two chromaticities whose contrast must be maximized subtends an angle of the order of 0.1° the spacing indicated for 2° fields does not apply. The perceptibility of chromaticity differences from the gray point in the purplish blue to greenish yellow direction becomes vanishingly small. This phenomenon is known as small-field tritanopia (Judd, 1949 [6], Wright, 1949 [7], Middleton and Holmes, 1949 [8]). Under these conditions all chromaticities represented by any straight line passing through the tritanopic co-punctal point ($x = 0.165$, $y = 0.000$ or thereabouts), which is near the short-wave (violet) extreme of the spectrum locus, become indistinguishable from each other. A measure of the chromaticity contrast between two colors viewed with an angular subtense of about 0.1° is the distance between the projections of the corresponding two chromaticity points from the tritanopic co-punctal point to the straight line representing the long-wave branch of the spectrum locus. By this measure the maximum chromaticity contrast would be found between spectrum red and spectrum blue at about 465 mμ, but nearly as great contrast is indicated between extreme spectrum red and any spectrum chromaticity of wavelength between 450 and 510 mμ. The indication derived from the 2° condition thus holds fairly well for smaller field sizes as well.

V. Answers to the specific questions

1. Maximally contrasting pairs of chromaticities are produced by extreme spectrum red paired with the spectrum bluish green to spectrum blue.

2. Minimally contrasting chromaticities are those yielding a perfect match, or zero contrast. These pairs may be produced by colors of any hue.

3. The UCS diagrams show approximately the perceptibility of the difference between any two chromaticities by the distance between the points representing them when they are presented in fields of about 2° subtense.

4. Fig. 1 is the RUCS diagram developed by Breckenridge and Schaub [5]. It is a projection of the CIE chromaticity diagram such that for viewing in fields of about 2° subtense the distance between the two points is approximately proportional to the contrast between the two chromaticities represented. From this diagram there may be read off immediately a measure (linear distance on diagram) of the chromaticity contrast between any two spectrum colors. Since it is also a mixture diagram, the chromaticities of any mixture of two colors fall on the straight line connecting the points representing them. This diagram thus yields indirectly a measure of the contrast between any two colors viewed in fields of about 2° subtense. It may also be extended to yield estimates of chromaticity contrast between colors viewed in fields of about 0.1° subtense by distance between projections of the two chromaticity points from the tritanopic co-punctal point ($x'' = -0.391$, $y'' = 0.014$) onto the straight line $x'' = 0.075$, as explained above.

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Figure 1.

The spectrum locus and Planckian (color temperature) locus plotted in rectangular-uniform-chromaticity-scale (RUCS) coordinates by F. C. Breckenridge and W. R. Schaub [5].

EXTENSION OF THE MUNSELL RENOTATION SYSTEM TO VERY DARK COLORS
(with G. Wyszecki)

J. Opt. Soc. Am. 46, 281-284 (1956)

This lucid account of a straightforward attack on a clearly defined problem does not require any clarification or elaboration, for anyone who has read the earlier articles in this collection. The nomenclature and procedures used have been explained there. They are meticulously used in this article. The data employed are readily available, in Refs. 1 and 8. The results should be useful to anyone who has to do with very dark colors, or even ordinary colored objects that are viewed in environments very much more brightly illuminated than the objects of concern.

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Extension of the Munsell Renotation System to Very Dark Colors

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By using the same principles by which the OSA Subcommittee on the Spacing of the Munsell Colors extended the definition of the ideal Munsell system beyond the gamut of the colors actually observed, this definition has been further extended to Munsell renotation values less than 1/. These principles conform to the logical requirement that the constant-chroma locus for any chroma, however small, must extend beyond the spectrum locus (and the straight line connecting its extremes) as Munsell renotation value approaches 0/ if the Munsell renotation chroma is to correlate with the saturation of the color perception of the specimen viewed against a neutral background of Munsell value greater than 5/.

I. INTRODUCTION

THE final report of the OSA Subcommittee on the Spacing of the Munsell Colors[1] gives a definition of the ideal Munsell system in terms of daylight reflectance, Y, and chromaticity coordinates, x,y. This definition extends in Munsell value from 1/ to 9/, and in chroma out to the MacAdam[2] limits; and notations by this ideal system are customarily distinguished from those found by visual interpolation along the color scales of the *Munsell Book of Color*[3] (called book notations) by taking the name, Munsell renotations.

Extension of this definition to very light colors (Munsell value 9/ to 10/) can be carried out with fair satisfaction by extrapolation because the constant-chroma loci change only slightly from value 7/ to 9/. Extension of the definition to very dark colors (Munsell value 1/ to 0/), on the other hand, requires much more care because the constant-chroma loci on the (x,y)-diagram expand indefinitely as Munsell value approaches zero. This may be demonstrated by the fact that a color of any chromaticity whatever, even that of any spectrum color, cannot be distinguished from black (Munsell value equal to zero) under the standard conditions (background of neutral gray of Munsell value 5/ or greater) for which Munsell notations correlate with color perception. Consider a hypothetical specimen of spectral reflectance or transmittance zero except for a narrow wavelength range. As the width of this wavelength range is allowed to approach zero, the daylight reflectance or transmittance, Y, must approach zero, and the color at some Munsell value greater than zero must become indistinguishable from black. But the chromaticity coordinates, x,y, of the hypothetical color remain constant and specify a point on the spectrum locus. It follows that as Munsell value approaches zero, the locus of chroma constant at any chroma, however small, must intersect the spectrum locus. The beginning of this rapid expansion of the chroma loci may be seen by comparing the constant-chroma loci for Munsell value 2/ with those for 1/.

It must be admitted that the need for this extension is not very important if attention be confined to specimens viewed by reflected light. Since their definition in 1943 and subsequent wide use for specifying the colors of specimens viewed by reflected light, only a few specimens of Munsell value less than 1/ have aroused any practical interest, and most of these are nominally black. On the other hand, there are many specimens viewed by transmitted light, such as those among the standards of the DIN-system,[4] having Munsell values less than 1/ and having considerable practical importance.

It is the purpose of the present paper to extend the definition of the ideal Munsell system to Munsell values 0.8/, 0.6/, 0.4/, and 0.2/ by application of the same principles as were used by the OSA Subcommittee to extend it to value 1/.

II. BASIC PLAN OF THE MUNSELL RENOTATIONS

The Munsell renotations were derived to express the average of about three million observations made by 41 subjects as described in the Preliminary Report of the OSA Subcommittee on the Spacing of the Munsell Colors, by S. M. Newhall.[5] This derivation was considerably facilitated by the discovery that the constant-chroma loci so found varied only by negligible amounts from one single basic shape independent of chroma and value when plotted on the Adams chromatic-value diagram,[6] formed by plotting $0.4(V_z-V_y)$ against V_x-V_y. The V-function is the Munsell value function.[5] On this diagram the constant-chroma loci are ovoids not too different from circles centering roughly on the chromaticity point for the illuminant (CIE source C),[7] but the exact placing of the ovoids relative to this point varied linearly from one value level to another at the rate of 0.02 in V_x-V_y per value step, and 0.016 in $0.4(V_z-V_y)$ per value step. All diameters of the ovoids

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TABLE I. The CIE (Y,x,y) equivalents of the recommended Munsell renotation for 40 hues and 4 values (V=0.8/, 0.6/, 0.4/, and 0.2/), and 6 chromas (C=/1, /2, /3, /4, /6, and /8) up to the theoretical pigment maximum.

Reds

V/C	Y	2.5R x	2.5R y	5.0R x	5.0R y	7.5R x	7.5R y	10.0R x	10.0R y
0.8/8	0.00944	0.483	0.195	0.536	0.214	0.584	0.234	0.635	0.259
6		0.455	0.219	0.496	0.237	0.534	0.255	0.578	0.280
4		0.421	0.245	0.450	0.261	0.477	0.276	0.508	0.296
3		0.400	0.259	0.423	0.275	0.441	0.288	0.461	0.304
2		0.381	0.272	0.399	0.286	0.411	0.297	0.423	0.309
1		0.348	0.294	0.357	0.302	0.362	0.308	0.367	0.314
0.6/8	0.00699	0.489	0.176	0.551	0.197	0.604	0.214	0.660	0.235
6		0.464	0.200	0.514	0.221	0.558	0.240	0.605	0.261
4		0.432	0.227	0.469	0.246	0.502	0.264	0.537	0.284
3		0.412	0.244	0.440	0.261	0.467	0.278	0.493	0.296
2		0.391	0.260	0.411	0.274	0.431	0.290	0.447	0.305
1		0.356	0.286	0.365	0.294	0.375	0.305	0.382	0.314
0.4/6	0.00467	0.477	0.170	0.537	0.190	0.588	0.208	0.649	0.229
4		0.450	0.198	0.498	0.219	0.539	0.238	0.582	0.258
3		0.430	0.218	0.469	0.238	0.503	0.256	0.537	0.275
2		0.411	0.236	0.441	0.255	0.466	0.272	0.490	0.289
1		0.371	0.270	0.386	0.283	0.399	0.294	0.409	0.305
0.2/3	0.00237	0.470	0.162	0.527	0.183	0.581	0.203	0.637	0.226
2		0.451	0.183	0.501	0.204	0.543	0.224	0.592	0.246
1		0.404	0.230	0.435	0.249	0.458	0.265	0.484	0.284

Yellow-reds

V/C	Y	2.5YR x	2.5YR y	5.0YR x	5.0YR y	7.5YR x	7.5YR y	10.0YR x	10.0YR y
0.8/6	0.00944	0.637	0.320						
4		0.558	0.330	0.612	0.376				
3		0.495	0.334	0.529	0.372	0.554	0.409		
2		0.445	0.333	0.463	0.361	0.475	0.386	0.481	0.411
1		0.376	0.327	0.384	0.342	0.386	0.351	0.386	0.360
0.6/6	0.00699	0.693	0.303						
4		0.603	0.322						
3		0.542	0.330	0.601	0.372				
2		0.474	0.332	0.505	0.367	0.526	0.397	0.551	0.444
1		0.394	0.328	0.403	0.345	0.408	0.359	0.410	0.374
0.4/4	0.00467	0.665	0.298						
3		0.606	0.314						
2		0.534	0.324	0.585	0.367				
1		0.428	0.327	0.448	0.354	0.462	0.379	0.471	0.407
0.2/2	0.00237	0.679	0.290						
1		0.526	0.317	0.584	0.366				

Yellows

V/C	Y	2.5Y x	2.5Y y	5.0Y x	5.0Y y	7.5Y x	7.5Y y	10.0Y x	10.0Y y
0.8/2	0.00944	0.479	0.439	0.465	0.457	0.434	0.460	0.397	0.448
1		0.381	0.370	0.372	0.375	0.359	0.375	0.346	0.371
0.6/2	0.00699							0.432	0.501
1		0.404	0.388	0.388	0.394	0.374	0.392	0.356	0.385
0.4/1	0.00467	0.468	0.432	0.445	0.444	0.411	0.436	0.379	0.422

Green-yellows

V/C	Y	2.5GY x	2.5GY y	5.0GY x	5.0GY y	7.5GY x	7.5GY y	10.0GY x	10.0GY y
0.8/6	0.00944							0.150	0.791
4						0.305	0.613	0.254	0.537
3		0.418	0.564	0.363	0.524	0.313	0.481	0.280	0.447
2		0.363	0.425	0.336	0.410	0.314	0.394	0.298	0.381
1		0.335	0.364	0.322	0.357	0.312	0.351	0.306	0.346
0.6/4	0.00699							0.208	0.652
3						0.304	0.561	0.263	0.499
2		0.377	0.468	0.342	0.442	0.315	0.420	0.292	0.399
1		0.338	0.376	0.325	0.367	0.314	0.359	0.304	0.351
0.4/3	0.00467							0.204	0.645
2				0.358	0.528	0.312	0.482	0.277	0.445
1		0.350	0.404	0.331	0.391	0.315	0.379	0.299	0.365
0.2/2	0.00237							0.185	0.676
1		0.394	0.522	0.349	0.485	0.308	0.449	0.285	0.423

Greens

V/C	Y	2.5G x	2.5G y	5.0G x	5.0G y	7.5G x	7.5G y	10.0G x	10.0G y
0.8/6	0.00944	0.102	0.660	0.082	0.553	0.073	0.476	0.070	0.408
4		0.225	0.488	0.205	0.447	0.191	0.414	0.178	0.382
3		0.262	0.424	0.247	0.403	0.236	0.385	0.224	0.366
2		0.287	0.371	0.280	0.363	0.272	0.355	0.265	0.346
1		0.300	0.341	0.296	0.338	0.293	0.335	0.289	0.332
0.6/4	0.00699	0.175	0.561	0.152	0.493	0.137	0.440	0.124	0.389
3		0.241	0.465	0.221	0.431	0.204	0.400	0.190	0.370
2		0.281	0.388	0.270	0.376	0.259	0.363	0.247	0.349
1		0.300	0.348	0.294	0.343	0.289	0.338	0.283	0.332
0.4/3	0.00467	0.166	0.564	0.143	0.499	0.126	0.442	0.112	0.390
2		0.258	0.423	0.239	0.399	0.226	0.380	0.213	0.361
1		0.292	0.360	0.283	0.351	0.276	0.344	0.270	0.338
0.2/2	0.00237	0.144	0.584	0.117	0.516	0.097	0.458	0.080	0.397
1		0.266	0.403	0.255	0.390	0.241	0.375	0.229	0.358

Blue-greens

V/C	Y	2.5BG x	2.5BG y	5.0BG x	5.0BG y	7.5BG x	7.5BG y	10.0BG x	10.0BG y
0.8/6	0.00944	0.070	0.341	0.072	0.275	0.077	0.233	0.086	0.199
4		0.163	0.342	0.150	0.299	0.145	0.264	0.146	0.237
3		0.209	0.338	0.196	0.308	0.187	0.281	0.183	0.258
2		0.253	0.332	0.241	0.315	0.230	0.296	0.223	0.280
1		0.283	0.325	0.276	0.316	0.270	0.308	0.266	0.300
0.6/4	0.00699	0.117	0.341	0.112	0.284	0.113	0.254	0.116	0.221
3		0.177	0.339	0.164	0.299	0.160	0.275	0.160	0.249
2		0.236	0.334	0.221	0.311	0.213	0.295	0.206	0.276
1		0.277	0.326	0.269	0.316	0.264	0.309	0.258	0.300
0.4/4	0.00467							0.074	0.187
3		0.103	0.335	0.102	0.278	0.106	0.247	0.116	0.217
2		0.196	0.332	0.180	0.298	0.173	0.275	0.169	0.249
1		0.259	0.326	0.248	0.310	0.242	0.300	0.236	0.284
0.2/2	0.00237	0.068	0.332	0.066	0.261	0.072	0.226	0.085	0.195
1		0.210	0.330	0.191	0.295	0.183	0.275	0.176	0.251

Blues

V/C	Y	2.5B x	2.5B y	5.0B x	5.0B y	7.5B x	7.5B y	10.0B x	10.0B y
0.8/6	0.00944	0.094	0.181	0.106	0.163	0.115	0.153	0.128	0.145
4		0.149	0.222	0.154	0.207	0.160	0.196	0.168	0.187
3		0.182	0.246	0.184	0.231	0.187	0.221	0.192	0.212
2		0.220	0.271	0.218	0.258	0.220	0.249	0.222	0.241
1		0.264	0.295	0.262	0.289	0.262	0.283	0.263	0.278
0.6/6	0.00699			0.088	0.145	0.099	0.136	0.115	0.128
4		0.123	0.202	0.134	0.187	0.143	0.178	0.153	0.172
3		0.162	0.233	0.167	0.217	0.172	0.206	0.178	0.197
2		0.202	0.260	0.202	0.245	0.204	0.235	0.209	0.227
1		0.255	0.291	0.252	0.282	0.252	0.275	0.254	0.268
0.4/4	0.00467	0.087	0.172	0.102	0.159	0.113	0.151	0.126	0.145
3		0.123	0.203	0.133	0.190	0.141	0.180	0.151	0.172
2		0.169	0.236	0.172	0.223	0.176	0.213	0.183	0.203
1		0.233	0.275	0.232	0.267	0.232	0.259	0.234	0.251
0.2/3	0.00237					0.097	0.133	0.112	0.127
2		0.097	0.177	0.111	0.164	0.121	0.157	0.133	0.149
1		0.175	0.239	0.178	0.226	0.182	0.216	0.188	0.206

Purple-blues

V/C	Y	2.5PB x	2.5PB y	5.0PB x	5.0PB y	7.5PB x	7.5PB y	10.0PB x	10.0PB y
0.8/8	0.00944	0.117	0.105	0.139	0.102	0.179	0.104	0.220	0.112
6		0.142	0.138	0.160	0.132	0.194	0.131	0.229	0.137
4		0.178	0.181	0.192	0.174	0.216	0.170	0.242	0.170
3		0.200	0.205	0.212	0.200	0.231	0.194	0.252	0.194
2		0.225	0.234	0.234	0.226	0.247	0.221	0.263	0.219
1		0.265	0.273	0.269	0.268	0.275	0.264	0.283	0.262
0.6/8	0.00699			0.131	0.088	0.176	0.092	0.216	0.098
6		0.131	0.122	0.152	0.118	0.188	0.117	0.225	0.124
4		0.166	0.165	0.182	0.160	0.208	0.155	0.237	0.157
3		0.188	0.190	0.201	0.185	0.222	0.180	0.246	0.178
2		0.215	0.221	0.223	0.215	0.239	0.208	0.257	0.204
1		0.257	0.263	0.260	0.260	0.268	0.254	0.278	0.250
0.4/8	0.00467					0.165	0.072	0.206	0.078
6		0.113	0.098	0.135	0.095	0.175	0.095	0.212	0.100
4		0.141	0.139	0.161	0.134	0.192	0.130	0.223	0.131
3		0.163	0.165	0.179	0.158	0.204	0.153	0.230	0.151
2		0.190	0.196	0.202	0.188	0.220	0.180	0.241	0.176
1		0.238	0.246	0.244	0.239	0.253	0.234	0.265	0.228
0.2/6	0.00237					0.159	0.061	0.206	0.064
4		0.109	0.094	0.133	0.090	0.171	0.087	0.213	0.088
3		0.129	0.121	0.150	0.115	0.181	0.108	0.219	0.106
2		0.147	0.143	0.165	0.136	0.192	0.130	0.227	0.126
1		0.196	0.200	0.207	0.193	0.224	0.186	0.248	0.180

Purples

V/C	Y	2.5P x	2.5P y	5.0P x	5.0P y	7.5P x	7.5P y	10.0P x	10.0P y
0.8/8	0.00944	0.248	0.120	0.275	0.127	0.291	0.132	0.308	0.137
6		0.255	0.144	0.279	0.151	0.294	0.156	0.309	0.162
4		0.264	0.174	0.283	0.179	0.298	0.184	0.310	0.189
3		0.270	0.196	0.286	0.202	0.301	0.206	0.312	0.210
2		0.277	0.220	0.292	0.224	0.304	0.228	0.312	0.232
1		0.291	0.262	0.300	0.264	0.307	0.266	0.312	0.269
0.6/8	0.00699	0.244	0.104	0.270	0.110	0.288	0.115	0.304	0.119
6		0.250	0.129	0.274	0.136	0.292	0.141	0.306	0.145
4		0.258	0.160	0.280	0.166	0.296	0.170	0.308	0.174
3		0.264	0.181	0.282	0.184	0.298	0.189	0.310	0.193
2		0.272	0.205	0.287	0.207	0.301	0.211	0.311	0.214
1		0.286	0.250	0.295	0.251	0.306	0.254	0.312	0.256
0.4/8	0.00467	0.233	0.082	0.260	0.087	0.280	0.091	0.298	0.095
6		0.238	0.104	0.265	0.110	0.284	0.114	0.302	0.119
4		0.246	0.134	0.272	0.138	0.289	0.142	0.304	0.146
3		0.252	0.153	0.276	0.157	0.292	0.161	0.306	0.165
2		0.259	0.177	0.281	0.182	0.296	0.185	0.309	0.189
1		0.276	0.226	0.291	0.228	0.303	0.230	0.312	0.234
0.2/8	0.00237	0.232	0.052	0.264	0.056	0.277	0.058	0.291	0.060
6		0.236	0.067	0.266	0.072	0.280	0.074	0.293	0.075
4		0.241	0.090	0.269	0.093	0.283	0.094	0.296	0.097
3		0.245	0.106	0.272	0.109	0.285	0.111	0.298	0.113
2		0.250	0.127	0.275	0.129	0.288	0.131	0.300	0.134
1		0.266	0.180	0.283	0.181	0.295	0.183	0.305	0.185

TABLE I.—*Continued.*

		Red-purples							
		2.5*RP*		5.0*RP*		7.5*RP*		10.0*RP*	
V/*C*	*Y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
0.8/8	0.00944	0.329	0.144	0.362	0.154	0.397	0.165	0.435	0.177
6		0.328	0.168	0.355	0.179	0.384	0.190	0.415	0.203
4		0.326	0.195	0.347	0.206	0.369	0.217	0.393	0.230
3		0.324	0.216	0.342	0.224	0.360	0.234	0.379	0.246
2		0.322	0.236	0.336	0.243	0.350	0.251	0.365	0.261
1		0.317	0.272	0.325	0.276	0.332	0.281	0.339	0.287
0.6/8	0.00699	0.326	0.125	0.359	0.135	0.397	0.146	0.434	0.158
6		0.325	0.151	0.354	0.159	0.387	0.170	0.419	0.182
4		0.324	0.179	0.347	0.189	0.373	0.200	0.399	0.211
3		0.323	0.198	0.343	0.207	0.364	0.217	0.386	0.229
2		0.322	0.218	0.337	0.226	0.355	0.236	0.372	0.247
1		0.318	0.259	0.327	0.264	0.336	0.271	0.346	0.278
0.4/8	0.00467	0.320	0.100	0.350	0.106	0.391	0.117	0.437	0.128
6		0.320	0.123	0.348	0.131	0.384	0.141	0.423	0.153
4		0.320	0.151	0.344	0.158	0.374	0.169	0.406	0.181
3		0.320	0.170	0.341	0.177	0.368	0.188	0.394	0.200
2		0.320	0.193	0.337	0.199	0.360	0.209	0.381	0.220
1		0.319	0.237	0.328	0.242	0.343	0.251	0.355	0.259
0.2/6	0.00237	0.312	0.078	0.342	0.084				
4		0.313	0.100	0.341	0.106	0.381	0.115	0.424	0.125
3		0.314	0.116	0.340	0.122	0.376	0.131	0.415	0.143
2		0.315	0.137	0.337	0.143	0.370	0.152	0.404	0.164
1		0.316	0.188	0.331	0.194	0.353	0.203	0.375	0.214

are directly proportional to chroma independent of Munsell value, but the position of the constant-hue loci is a function of value.

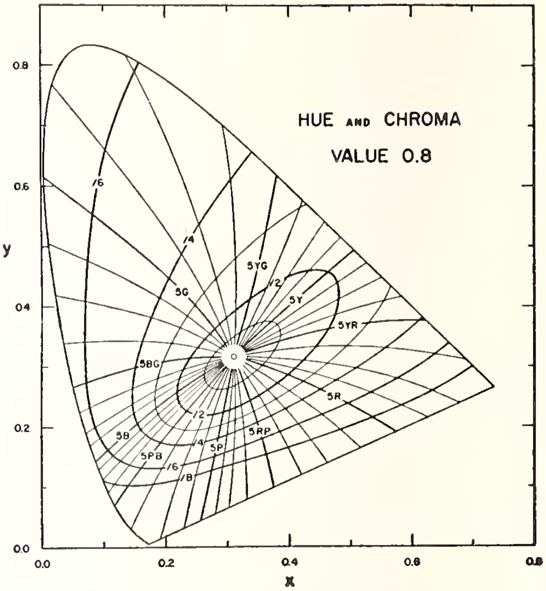

FIG. 1. Loci of constant hue and constant chroma in CIE (x,y)-coordinates, at value 0.8/.

III. EXTRAPOLATION OF MUNSELL RENOTATIONS TO VALUES 0.8, 0.6, 0.4, AND 0.2

The simple relation between Munsell renotations and the Adams chromatic-value diagram facilitates extrapolation to Munsell values lower than 1/. The steps in this extrapolation are as follows:

1. *Basic ovoids for value 1/.*—Each ovoid is defined in Table I of the Final Report of the OSA Subcommittee on the Spacing of the Munsell colors[1] by forty chromaticity (x,y) points, one for each interval of 2.5 Munsell hue steps. The corresponding tristimulus values, X,Y,Z, were found as $Y=1.210$ from Table II for value $V=1/$, and as $X=1.210\ x/y$, $Z=1.210\ z/y$. The corresponding values of V_x,V_y,V_z were then read from the Nickerson tables,[8] and V_x-V_y and $0.4(V_z-V_y)$ computed.

2. *Corresponding ovoids for values 0.8/, 0.6/, 0.4/, and 0.2/.*—The Adams chromatic values for the corresponding ovoids for values less than 1/ were found by adding the following amounts:

Munsell value	Decentering from value 1/	
	V_x-V_y	$0.4(V_z-V_y)$
1.0	0.0000	0.0000
0.8	0.0040	0.0032
0.6	0.0080	0.0064
0.4	0.0120	0.0096
0.2	0.0160	0.0128

By this method 40 points on each constant-chroma locus were found in terms of V_x-V_y and $0.4(V_z-V_y)$.

3. *Extension to chromas /1 and /3.*—The Adams chromatic values for the chroma /1 locus were found by dividing by two the chromatic values, V_x-V_y and $0.4(V_z-V_y)$, for chroma /2. Those for the chroma /3 locus were found by taking the arithmetical average of those for chromas /2 and /4.

4. *Conversion from Adams chromatic value to CIE chromaticity coordinates, x,y.*—The values of V_x were found from (V_x-V_y) by the obvious formula: $V_x=V_y+(V_x-V_y)$, where V_y is either 0.8/, 0.6/, 0.4/, or 0.2/, depending on the value level for which the definition of Munsell hue and chroma is desired. Similarly values of V_z were found from $0.4(V_z-V_y)$ by the formula: $V_z=V_y+2.5[0.4(V_z-V_y)]$. The tristimulus values,

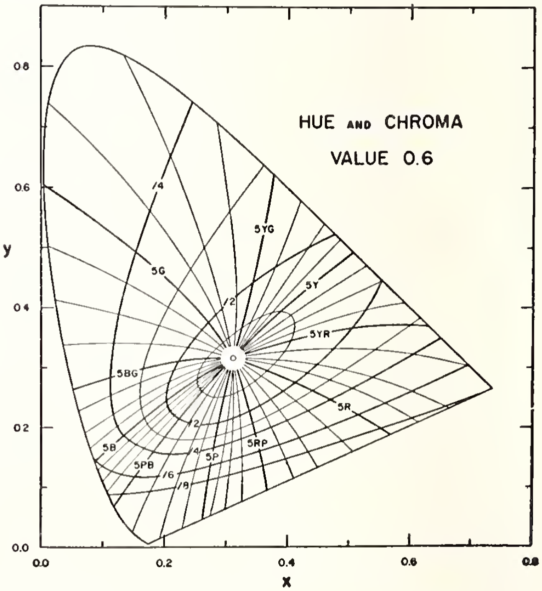

FIG. 2. Loci of constant hue and constant chroma in CIE (x,y)-coordinates, at value 0.6/.

[8] D. Nickerson, Am. Dyestuff Reptr. 39, 541 (August 21, 1950).

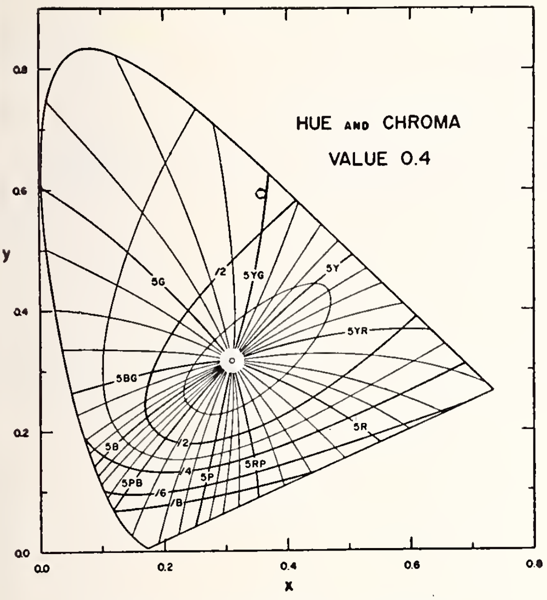

FIG. 3. Loci of constant hue and constant chroma in CIE (x,y)-coordinates, at value 0.4/.

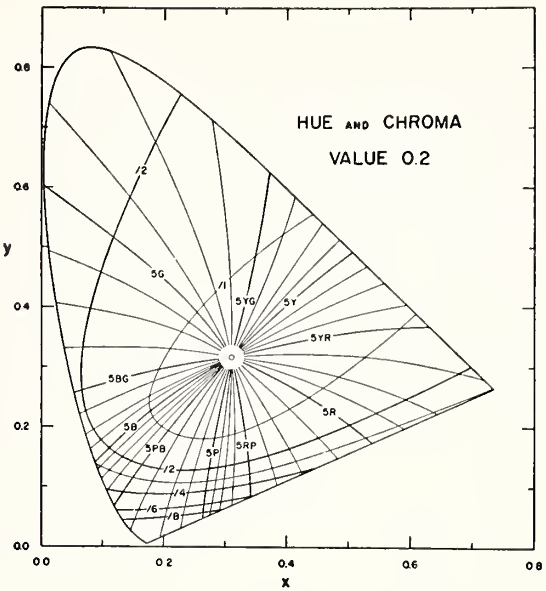

FIG. 4. Loci of constant hue and constant chroma in CIE (x,y)-coordinates, at value 0.2/.

X,Y,Z, were then read by interpolation from tables of V_x,V_y,V_z, respectively.[8] Chromaticity coordinates, x,y, were computed from their definitions: $x=X/(X+Y+Z)$ and $y=Y/(X+Y+Z)$. From these chromaticity coordinates the constant-chroma loci for values 0.8/, 0.6/, 0.4/, and 0.2/ were plotted on large-scale (x,y)-chromaticity diagrams.

5. Extrapolation of the constant-hue loci.—Reference to Fig. 13 of the Final Report of the OSA Subcommittee on the Spacing of the Munsell Colors will show that, except for Munsell hue 10Y, there is a progressive fanning out of the constant-hue loci on the (x,y)-diagram. The constant-hue loci were found for values 0.8/, 0.6/, 0.4/, and 0.2/ by continuing this fanning out at the rate shown between values 2/ and 1/. The results are shown on Figs. 1, 2, 3, and 4.

6. Chromaticity coordinates, x,y, for all hues and chromas up to /8 at values 0.8/, 0.6/, 0.4/, and 0.2/.—These chromaticity coordinates were read from Figs. 1, 2, 3, and 4, and are given to the nearest third decimal in Table I.

IV. VISUAL CHECK OF THE EXTRAPOLATED MUNSELL RENOTATIONS

The extension of the definition of the ideal Munsell system by extrapolation, both by us and by the OSA subcommittee, is, of course, subject to check by direct visual comparison of specimens of Munsell value less than 2/ with those of value greater than 2/, all specimens being viewed against a light gray background (neutral 5/ and higher), much as was done by the OSA subcommittee[5] for the specimens in the *Munsell Book of Color*. A start has been made in the collection and spectrophotometric measurement of a choice of specimens suitable for this study, and it is planned to make a separate report of these results.

COLOR RENDITION WITH FLUORESCENT SOURCES OF ILLUMINATION
(with H. Helson and M. Wilson)

Illum. Eng. 51, 329-346 (1956)

This investigation, which predates the development of the current CIE method for evaluating the color rendering of lamps, consisted of a great number of subjective judgments of the colors of Munsell samples, when illuminated by fluorescent lamps. Each perceived color was reported in terms of the Munsell sample that was remembered to have the same appearance in natural daylight. The reports were compared with predictions of the equivalent colors obtained by use of the von Kries coefficient law. The invariant points of the three types of dichromatic vision were used as primaries. As with tungsten-lamp light, the observed results agreed reasonably well with the predictions.

Editorial note: The color chart (Fig. 4) has been omitted. The notes about the color chart in the abstract and on pp. 334 and 336 should be disregarded.

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Color Rendition with Fluorescent Sources of Illumination

By HARRY HELSON
DEANE B. JUDD
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Color and form are partners in producing the artistry of light. An outstanding feature of modern interiors is the striking use of color. It, together with new functional forms, sets the atmosphere or "feeling" of present day architecture. The former universal use of incandescent lamps greatly emphasized the yellows and reds. Fluorescent sources are now bringing emphasis to the blues and greens for richer, more satisfying color harmony. Distortion of anticipated appearance through use of certain light sources is to be avoided.

Dr. Harry Helson and his colleagues have completed a study of a method of predicting the changes in appearance as one goes from one light source to another. A chart showing maximum changes due to different observers has been made up.

The equations developed through this project enable one to predict the color of any object, viewed under any illuminant, as perceived by an observer adapted to that illuminant. This theory does not attempt to predict color changes due to the influence of backgrounds of different reflectances but experimental data for white, gray and black backgrounds were presented in an earlier paper. Tables based on the theory giving the color changes predicted for various light sources of commercial interest should be of considerable practical value to the illuminating engineer in helping the architect and interior designer to visualize the actual appearance of his chosen colors.

1. Introduction

It is commonly recognized that objects have different colors in different source of illumination due to changes in the spectral energies of the light leaving the objects and to changes in sensitivity of the eye arising from alteration in the quantity or quality of light affecting the visual mechanism. In earlier studies of colors of objects in strongly chromatic illuminants (Helson, 1938; Helson and Jeffers, 1940; Judd, 1940), and of object colors in incandescent (tungsten) illuminants (Helson and Grove, 1947; Helson, Judd, and Warren, 1952), it was shown that good first-order predictions of hue, lightness, and saturation could be made by taking into account the changed visual adaptation to the non-daylight viewing conditions as compared with daylight adaptation. The present study extends the former investigations into the field of fluorescent illumination partly because of the widspread use of fluorescent sources in daily life and partly because of the peculiar nature of their spectral-energy distributions as compared with either daylight or incandescent sources. It is not known if formulations based on sources of fairly smooth spectral distributions of energy, such as tungsten or daylight, carry over to sources of the irregular energy distributions found with fluorescent lamps.

This investigation is concerned with the application of formulas previously found adequate in specifying the colors of objects in incandescent illumination (Helson, Judd, and Warren, 1952) to three representative types of fluorescent illumina-

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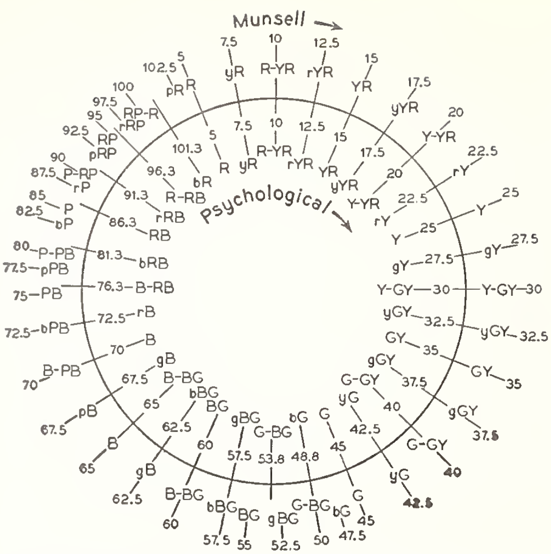


Figure 1. Showing the relation between the Munsell and psychological color circles. The Munsell hue circle employs five primary hues; the psychological hue circle has four primary hues.

tion. These formulas take into account the changed state of adaptation of the visual mechanism in passing from daylight to the fluorescent illuminants. This approach is important in the development of quantitative predictions of the colors of objects in various illuminants and in the determination of the primaries necessary to account for the results of changes in visual adaptation arising from altered spectral energy distributions of sources, from effects of backgrounds against which objects are viewed, or from other changes in viewing conditions which affect the adaptive state of the eye. Here only the spectral energy distribution of the source is the variable under study.

2. Method

In order to obtain reliable estimates of colors under different sources, observers were trained in a modified Munsell system of color nomenclature similar to the one used in a previous study (Helson, Judd, Warren, 1952). The modified system employs the four psychological primaries, red, yellow, green and blue and mixtures of the four. The hue circle given in Fig. 1 shows how a simple correspondence was established between this system, which may be called the psychological system, and the Munsell system so that the final judgments could be expressed in Munsell renotation (Newhall, Nickerson and Judd, 1943). The psychological system takes account of the observers' intuitive notion of blue which does not correspond with the Munsell hue for blue, and treats the Munsell purple region of the hue circle as a combination of red and blue rather than as a fifth primary. Thus, while the two systems agree in the red, yellow, and green regions, they diverge in the blue and purple regions since one hue step in the psychological system must cover two hue steps in the Munsell system as shown in Fig. 1. For example, in the modified system, the term red-blue covers the Munsell designations of purple and reddish-purple. To take care of this difference the Munsell numerical notations for these hues, 85.0 and 87.5, were averaged to give 86.3 and this number was taken as the numerical notation for red-blue in the psychological system. The hue circle in the modified system was divided into thirty-two steps and observers gave judgments in terms of this qualitative scheme which was then translated into the Munsell numerical notation for computational purposes. The Munsell system was used in training observers to report on lightness and saturation. Lightness was judged on a scale from 0 to 10 and saturation on a scale from 0 to 15.

The observers were five psychology students at the University of Texas. They required an average of eight hours of training to reach the criterion demanded for this study, *viz.,* estimating colors within 2.5 Munsell hue steps and one Munsell value and chroma step. Preliminary observations were also made by a sixth observer, who, however, failed to qualify, and whose results were therefore omitted from the tabulated averages. The observers were trained by methods described in an earlier study of color changes in passing from daylight to incandescent-lamp illumination (Helson, Judd, Warren, 1952). The training sessions took place in a room with a northern exposure to approximate the conditions of standardization by which the Munsell renotations were determined.

The observations under fluorescent sources were made in a light-tight booth supplied with a standard two-lamp fluorescent fixture. The walls and shelf in the booth were lined with non-selective gray cardboard. A fan was placed in the booth to maintain a fairly constant ambient temperature. A current of between 0.80 and 0.82 amperes was supplied to the primary of the ballast of the fixture. The rated primary current is 0.84 ampere. The spectral energy distributions of the three fluorescent sources are shown in Fig. 2. The observers made their estimates of the samples (described later) successively under the 3500K, 6500K and 4500K fluorescent sources. For judgments of daylight colors the fluorescent fixture was replaced by two standard Macbeth daylight lamps approxi-

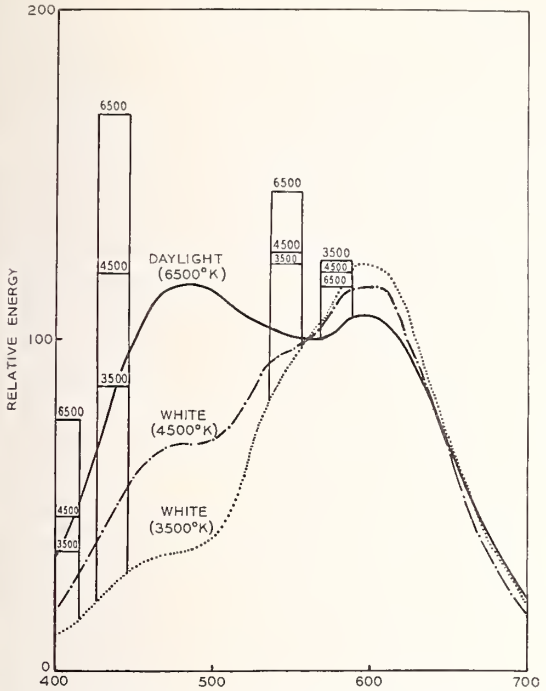


Figure 2. Spectral energy distributions of the fluorescent sources.

mating 6500K and the observations were repeated. Illuminations on the sample plane from the four sources are given in Table I.

TABLE I — Illuminations on the Sample Plane.

Source	Illumination (ft-c)
3500K fluorescent	32.0
4500K fluorescent	30.0
6500K fluorescent	27.0
Macbeth daylight (incandescent lamp plus Corning Daylite filter)	37.0

The observers adapted to the illumination in the booth for five minutes before the observations were made. Two sessions of about 45 minutes each were necessary to complete the observations for each source by one subject. One hundred thirty standard Munsell samples were chosen to give the maximum range of lightness and saturation in various combinations for the ten major Munsell hues given in Tables II, III, and IV. The samples were 1¼ by 1¼ inches and were mounted on gray cardboard similar to that which lined the walls of the booth. The 130 samples were randomized for each subject under each illuminant to minimize contrast effects. Twenty samples were exposed at one time. The tristimulus values of the 130 Munsell samples in the three illuminants are given in a paper by Nickerson and Wilson (1950). The observed hues given in Tables II, III and IV in Munsell terms have been corrected to take account of the difference between the observer's own scale of hues and the Munsell renotated hue scale. The five estimates of each sample as judged under Macbeth daylight were averaged to give the mean judgment. This mean judgment was compared with the Munsell renotation for source C to find the difference by which the fluorescent judgments would be expected to depart from Munsell renotation derived for the fluorescent sources. Thus, if the Munsell renotation of a given sample were 5.0R and the observers' mean judgment under Macbeth daylight were 6.0R, a correction of −1.0 would be applied to the mean judgments of that sample under the fluorescent sources. Corrections were calculated in this way for the hue, lightness and saturation of each sample, and were applied to the mean judgments obtained under the fluorescent sources. The extreme judgments showing the range of observer estimates were corrected by applying the same average corrections. The corrections did not always improve the agreement between observed mean hues and the predicted hues but in most cases the rationale seemed warranted since it resulted in better agreement between theory and observation. A similar procedure was followed by Bouma and Kruithof (1947/1948).

3. Results and Discussion

In Tables II, III and IV are given the average and inter-observer range of estimates of hue, lightness and saturation by five observers and the theoretical predictions of 130 Munsell samples viewed against gray background with 6500K, 4500K, and 3500K fluorescent sources of illumination. In general, the hue changes, as summarized for five observers in Table V and as shown for six observers in Fig. 3, are in clockwise direction around the hue circle as one goes from daylight (here taken as Munsell renotation) to each of the fluorescent sources. Thus the red and yellow-red hues become yellower, the green-yellows become greener, the greens and blue-greens become bluer, and the blues, purple-blues, purples and red-purples become redder, except for the blues in 6500K which become slightly greener. The hue shifts are remarkably small on the average considering the common belief that fluorescent light results in considerable distortion of colors. They are generally less than 2.50 in the numerical Munsell hue scale. The average observed hue shift from daylight to 6500K is 0.70, to 4500K it is 0.61, and to 3500K, it is 1.84. It is seen that these changes represent differences less than the change from red to yellowish red, or from

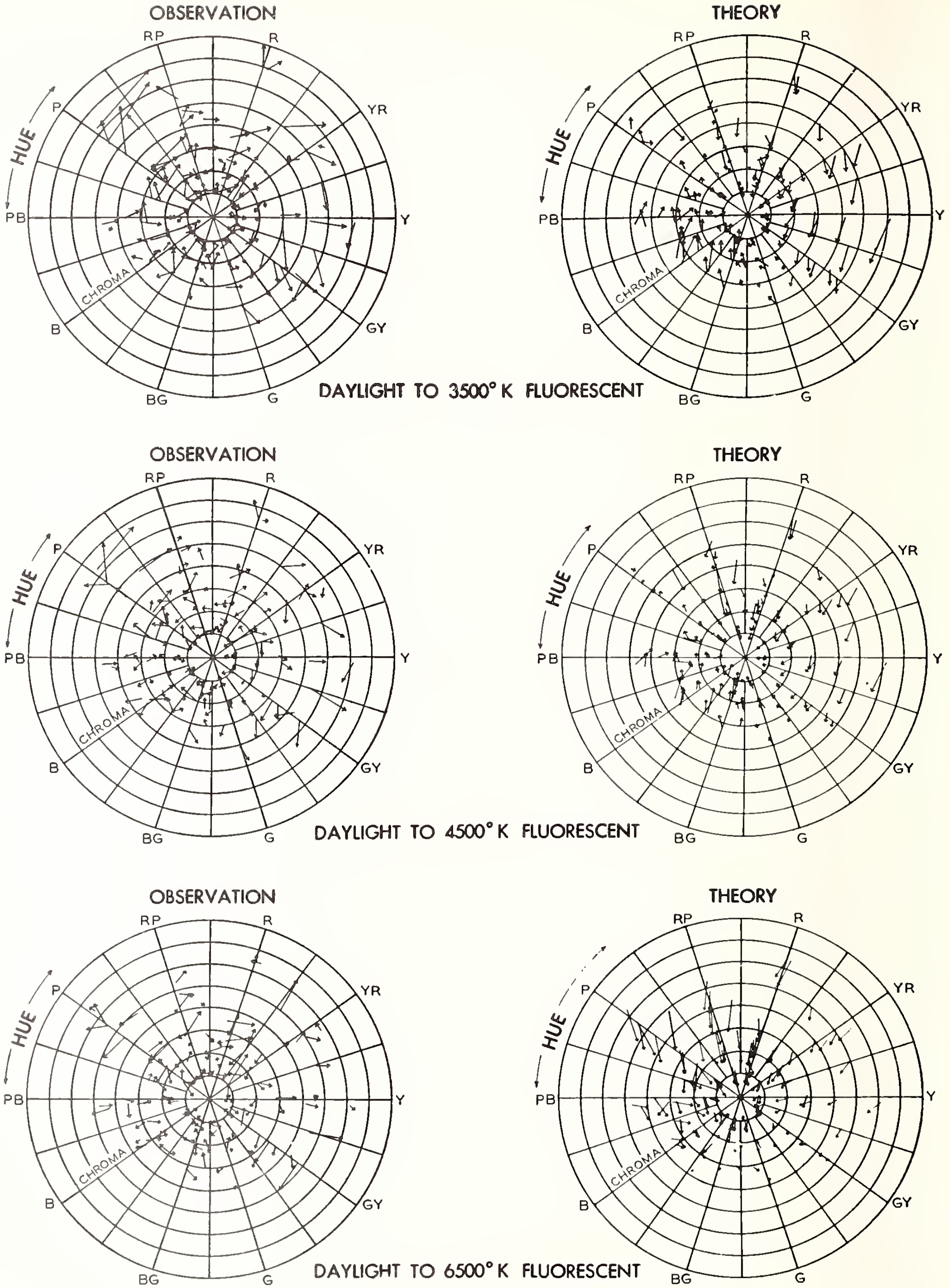


Figure 3. Changes in hue and saturation of 130 Munsell samples in passing from daylight (initial points of vectors) to fluorescent illumination (terminal points of vectors) according to observation (average of six observers) and according to theory.

TABLE II — Observed and Computed Hue, Lightness and Saturation, and Observers' Range under 6500K Fluorescent Illumination.

Munsell Renotation H	V/C	Observed H	L/S	Observers' Range H	S	Computed H	L/S
3.5R	2.3/2.2	10.0R	1.9/2.5	6.0R — 3.5YR	1.0 — 4.0	6.3R	2.3/1.2
3.0R	2.5/4.8	6.2R	2.6/5.1	3.2R — 0.7YR	3.9 — 5.9	2.0R	2.5/3.2
5.0R	3.2/7.4	4.2R	3.3/6.8	1.2R — 8.7R	4.8 — 9.8	4.4R	3.1/5.3
4.0R	5.0/2.8	2.3R	4.4/3.2	6.3RP — 8.8R	2.1 — 4.1	3.3R	5.0/2.5
3.5R	4.1/4.7	8.8RP	3.8/4.2	1.8RP — 3.0R	3.2 — 5.2	3.3R	4.0/3.9
4.0R	4.9/5.3	8.7R	4.6/5.6	4.7R — 2.2YR	4.8 — 6.8	3.8R	4.9/4.9
5.0R	4.2/12.6	4.8R	4.4/13.4	3.5R — 7.3R	9.8 — 15.8	5.0R	4.1/10.6
5.5R	5.0/12.8	4.5R	5.0/12.8	4.0R — 6.5R	10.8 — 15.8	4.7R	4.9/10.9
2.5R	7.2/2.6	0.8R	7.6/2.1	4.8RP — 3.5R	1.1 — 4.1	2.5R	7.1/2.3
3.5R	8.0/3.6	3.0R	8.2/3.6	1.5R — 4.0R	2.6 — 4.6	3.1R	8.0/3.3
3.0R	7.1/5.2	2.5R	7.6/5.1	6.8RP — 8.0R	4.6 — 5.6	2.5R	7.1/4.8
3.5R	7.0/7.0	8.2R	7.3/7.6	5.7R — 0.7YR	5.2 — 10.2	2.8R	6.9/6.3
10.0R	3.3/5.4	9.0R	3.7/7.2	9.0R — 9.0R	6.2 — 8.2	9.3R	3.3/5.5
9.5R	5.1/4.9	8.3R	4.9/3.6	4.5R — 2.0YR	2.6 — 5.1	10.0R	5.0/4.3
0.5YR	6.2/10.1	10.0R	6.7/13.1	6.5R — 4.0YR	9.5 — 16.5	2.2YR	6.2/9.0
4.5YR	2.4/1.8	1.0YR	2.5/2.9	7.5R — 2.5Y	0.6 — 4.6	4.4YR	2.4/1.1
5.0YR	3.3/4.1	6.5YR	2.9/3.6	4.5YR — 4.5Y	2.4 — 5.4	4.9YR	3.3/3.1
4.0YR	4.9/2.1	2.5YR	5.2/1.5	9.5R — 4.5YR	0.1 — 3.1	3.8YR	4.9/1.8
4.5YR	4.9/4.6	4.5YR	4.7/4.5	0.5YR — 0.5Y	3.1 — 6.6	4.4YR	4.9/4.0
5.5YR	5.1/9.0	5.0YR	5.4/10.2	2.5YR — 7.5YR	8.4 — 12.4	5.7YR	5.1/7.6
3.0YR	7.0/2.1	3.5YR	7.6/2.0	7.5R — 10.0YR	1.2 — 2.2	3.0YR	7.0/2.1
4.0YR	8.0/3.5	10.0R	7.9/2.5	6.0R — 3.5YR	1.9 — 4.9	4.0YR	8.0/3.2
3.5YR	6.9/5.9	4.0YR	7.6/4.2	9.5R — 7.0YR	3.9 — 4.9	4.2YR	6.9/5.5
4.5YR	7.0/8.2	5.0YR	6.9/8.4	0.5YR — 8.0YR	5.6 — 13.6	5.0YR	7.0/7.5
5.5YR	6.9/10.1	7.0YR	7.3/11.1	5.0YR — 10.0YR	7.9 — 12.9	9.6YR	6.9/9.8
6.0YR	6.2/11.5	5.0YR	5.8/11.5	5.0YR — 5.0YR	9.7 — 14.7	6.2YR	6.2/10.0
10.0YR	4.2/4.6	1.5Y	4.8/5.6	8.5YR — 3.5Y	3.3 — 8.3	9.9YR	4.2/3.8
10.0YR	5.1/4.4	2.0Y	5.1/4.8	4.5YR — 7.0Y	3.8 — 6.8	0.3Y	5.1/3.9
10.0YR	6.5/9.8	0.5Y	6.5/9.9	8.5YR — 1.0Y	4.7 — 15.7	1.0Y	6.5/8.8
5.0Y	2.5/1.4	2.0Y	2.6/1.7	3.5YR — 1.0GY	0.6 — 3.6	4.4Y	2.5/0.9
4.5Y	5.2/1.7	7.0Y	5.0/1.9	9.5YR — 9.5GY	1.3 — 2.3	5.4Y	5.1/1.5
5.0Y	4.1/3.6	7.5Y	3.9/3.6	9.5YR — 4.5GY	3.0 — 4.5	5.5Y	4.1/3.1
6.0Y	5.0/6.0	8.0Y	5.0/6.5	2.0Y — 2.0GY	4.3 — 9.3	6.4Y	5.0/5.2
3.5Y	7.8/1.6	7.0Y	8.3/1.4	9.0R — 6.5G	1.4 — 1.4	3.9Y	7.8/1.7
4.5Y	7.9/5 8	4.5Y	7.4/7.4	2.5Y — 5.0Y	4.6 — 9.6	5.5Y	8.0/5.5
5.5Y	7.9/12.3	6.0Y	7.6/12.7	5.0Y — 7.5Y	9.7 — 15.7	6.5Y	7.9/11.2
5.5Y	7.9/8.3	6.0Y	7.9/9.5	6.0Y — 6.0Y	8.7 — 12.7	6.3Y	7.9/7.7
0.5GY	3.3/1.8	1.0GY	2.5/1.7	3.5Y — 8.5GY	0.3 — 2.3	0.7GY	3.3/1.5
10.0Y	4.3/3.7	1.5GY	4.3/3.6	4.5Y — 9.5GY	2.4 — 4.4	0.4GY	4.3/3 2
0.5GY	8.1/9.1	9.5Y	8.3/12.1	8.0Y — 3.0GY	8.7 — 15.7	0.8GY	8.1/8.8
5.5GY	2.5/2.2	5.0GY	2.3/2.5	9.0Y — 9.0GY	0.6 — 4.6	5.4GY	2.5/1.5
5.0GY	5.1/2.0	4.0GY	5.1/1.5	7.5Y — 2.5G	1.3 — 2.3	5.1GY	5.1/1.8
5.0GY	5.1/5.4	8.0GY	4.9/5.8	4.0GY — 9.0GY	4.4 — 8.4	5.5GY	5.1/5.0
5.0GY	5.1/7.1	6.5GY	4.9/7.3	3.5GY — 8.5GY	5.7 — 11.7	5.3GY	5.2/6.4
5.5GY	7.0/1.8	3.0G	7.3/2.4	8.0Y — 9.3G	1.4 — 4.4	5.0GY	7.0/1.7
4.0GY	7.0/9.5	9.5GY	6.6/10.0	5.5GY — 3.0G	8.0 — 11.5	4.3GY	7.0/9.1
5.0GY	7.1/6.4	10.0GY	6.9/7.7	4.0GY — 4.0G	6.7 — 8.7	5.2GY	7.1/6.4
6.0GY	8.1/3.6	9.0Y	8.8/3.1	7.0Y — 2.0GY	2.9 — 3.9	5.8GY	8.1/3.5
4.5GY	8.1/8.7	7.0GY	8.1/9.4	5.0GY — 2.5G	6.6 — 11.6	5.0GY	8.1/8 4
10.0GY	4.2/5.4	9.0GY	4.6/6.0	3.0GY — 0.5G	4.0 — 10.0	0.1G	4.2/4.7
10.0GY	5.0/4.3	1.0G	4.9/3.7	8.5GY — 3.5G	2.1 — 5.1	10.0GY	5.1/4.0
10.0GY	7.1/6.5	1.0G	7.1/7.0	6.5GY — 4.0G	5.2 — 8.2	10.0GY	7.2/6.5
7.0G	2.2/2.4	9.3G	1.9/1.9	8.5G — 2.3BG	1.1 — 3.1	6.7G	2.2/1.5
4.0G	3.3/4.8	3.5G	3.2/4.8	10.0GY — 5.0G	3.2 — 6.2	3.9G	3.3/3.7
6.5G	4.9/2.4	4.0G	4.7/2.6	8.7GY — 2.5BG	1.6 — 4.1	5.8G	4.9/2.1
5.0G	4.9/4.8	2.3G	5.2/3.8	0.5G — 6.8G	2.2 — 5.2	4.1G	4.9/4.2
5.0G	4.9/8.2	5.5G	5.1/8.0	3.5G — 6.0G	6.6 — 9.6	4.4G	4.9/6.9
6.5G	7.0/2.1	6.5G	7.6/1.9	5.5GY — 0.5B	0.9 — 2.9	5.0G	7.0/2.0
7.0G	7.0/6.1	8.2G	6.9/6.4	0.2G — 7.7BG	5.2 — 8.2	6.1G	7.2/5.5
6.0G	8.7/2.7	9.0G	8.7/3.2	10.0GY — 5.0BG	2.7 — 4.2	5.0G	8.7/2.0
10.0G	3.2/4.7	1.7BG	3.5/3.5	9.2G — 8.0BG	2.3 — 4.3	9.7G	3.2/3.1
0.5BG	4.1/4.5	10.0G	4.1/4.3	5.7G — 2.0BG	3.9 — 4.9	10.0G	4.1/3.7
1.0BG	6.4/6.0	1.8BG	6.8/4.8	1.8BG — 1.8BG	4.2 — 6.2	9.8G	6.4/5.3
4.5BG	2.2/2.5	3.8BG	2.2/3.7	3.0BG — 6.8BG	2.9 — 5.9	4.6BG	2.2/1.4
5.0BG	2.3/3.7	4.0BG	2.5/5.1	2.5BG — 6.3BG	3.9 — 6.9	5.4BG	2.2/2.1
5.5BG	4.3/2.5	10.0BG	4.5/2.4	3.0BG — 6.7B	1.0 — 3.5	5.0BG	4.2/2.0
5.5BG	4.3/4.0	6.3BG	3.9/3.9	8.0G — 5.5B	2.7 — 4.7	5.2BG	4.3/3.2
5.0BG	5.1/6.4	2.5BG	5.7/7.1	5.7G — 3.2B	3.7 — 12.7	5.1BG	5.1/5.7
2.0BG	6.4/4.5	5.2BG	6.9/4.3	6.2G — 6.2B	2.9 — 5.9	1.4BG	6.3/3.9
3.5BG	7.3/2.1	9.0BG	8.0/2.0	8.5GY — 6.0B	1.3 — 2.8	2.7BG	7.3/1.8
4.5BG	7.2/3.4	7.7BG	7.3/3.0	9.7GY — 4.7B	1.0 — 4.0	4.8BG	7.2/2.8
4.5BG	5.1/2.5	3.3BG	3.9/2.7	6.5G — 9.0BG	1.8 — 4.8	3.8BG	5.1/2.3
9.5BG	3.2/5.8	3.0B	3.3/5.8	3.0BG — 9.2B	2.8 — 9.8	0.5B	3.1/3.9
9.5BG	4.1/4.0	8.2BG	4.3/4.2	4.2G — 4.2B	3.2 — 5.2	0.1B	4.1/3.1
10.0BG	6.3/5.8	9.0BG	6.8/6.6	5.0BG — 2.5B	3.6 — 8.6	0.2B	6.3/5.0
2.0B	2.2/2.2	6.5BG	2.2/2 6	3.2BG — 2.0B	1.6 — 4.6	2.2B	2.1/1.6
5.0B	3.1/2.3	5.3B	3.2/1.4	2.0B — 3.3PB	0.2 — 2.8	7.5B	3.2/1.7
6.5B	4.9/2.6	5.5B	4.8/2.2	10.0PB 2.2/5.5 10.0B	1.4 — 4.4	7.5B	4.9/2.3
5.5B	5.0/5.1	4.0B	4.8/5.7	6.5BG — 9.0B	4.5 — 8.5	6.6B	5.0/4.3
6.5B	4.0/6.9	4.2B	4.1/7.3	4.2B — 4.2B	4.7 — 9.7	9.9B	3.9/5.1

(Table II continued on next page.)

TABLE II *(Continued)*.

Munsell Renotation H	V/C	Observed H	L/S	Observers' Range H	S	Computed H	L/S
5.0B	5.0/6.8	5.5B	5.2/7.6	4.0B — 6.5B	5.6 — 10.6	6.3B	4.9/5.7
5.5B	6.1/6.4	4.0B	6.9/4.8	7.0BG — 7.0B	3.8 — 5.8	6.3B	6.1/5.6
4.0B	8.2/2.2	6.0B	8.5/2.0	6.0B — 6.0B	1.8 — 2.8	5.3B	8.2/2.0
5.0B	7.1/4.7	5.0B	7.2/4.6	3.0B — 5.5B	2.9 — 7.9	6.1B	7.1/4.1
9.5B	3.2/5.6	9.0B	3.3/6.7	2.5B — 2.5PB	3.5 — 11.5	1.1PB	3.2/4.5
0.5PB	4.2/4.1	10.0B	4.1/4.0	7.5B — 2.5PB	2.6 — 5.6	1.9PB	4.1/3 4
9.5B	7.2/5.7	8.0B	7.8/4.6	6.0B — 8.5B	2.4 — 5.9	0.6PB	7.2/5.3
4.5PB	2.2/2.6	4.0PB	2.4/3.5	2.0PB — 4.5PB	1.9 — 5.9	4.7PB	2.1/1.7
4.0PB	2.3/5.9	1.5PB	2.1/5.9	7.0B — 4.5PB	4.3 — 9.3	5.3PB	2.2/4.0
4.0PB	3.3/9.4	4.0PB	3.4/8.4	3.5PB — 6.0PB	5.8 — 12.8	5.9PB	3.2/8.0
5.0PB	4.2/4.3	6.0PB	4.4/3.6	2.5PB — 8.8PB	2.6 — 4.6	5.6PB	4.2/3.9
5.0PB	5.1/3 1	5.8PB	5.1/3.9	4.2PB — 8.0PB	3.3 — 5.3	5.9PB	5.1/2.9
4.0PB	5.3/6.2	4.5PB	5.4/5.2	4.5PB — 4.5PB	4.0 — 7.0	5.5PB	5.2/5.6
3.5PB	5.3/8.6	3.0PB	5.2/10.2	2.0PB — 4.5PB	6.6 — 11.6	4.6PB	5.2/7.9
5.0PB	8.3/3.2	6.3PB	8.9/3.2	1.7PB — 8.0P	2.8 — 3.8	5.5PB	8.2/3.0
4.0PB	7.2/5.6	4.5PB	7.6/5.2	2.5PB — 5.0PB	5.0 — 7.0	5.1PB	7.1/5.3
5.0PB	6.3/7.4	4.5PB	6.2/6.5	3.5PB — 6.0PB	4.7 — 9.2	5.0PB	6.2/6.2
10.0PB	2.2/4.5	1.5P	1.9/5.4	7.2PB — 1.0RP	3.4 — 7.4	0.1P	2.1/3.8
10.0PB	4.1/4.6	8.3PB	4.3/2.8	6.0PB — 9.8PB	2.1 — 3.6	0.5P	4.1/4.4
10.0PB	7.0/5.3	1.4P	7.8/5.0	7.1PB — 5.9P	4.2 — 6.2	1.5P	7.0/6.0
5.5P	2.1/2.9	8.2P	2.1/3.5	1.2P — 5.0RP	2.7 — 3.7	5.4P	2..1/1.9
5.0P	2.2/4.6	0.2P	2.7/5.7	6.2PB — 5.0P	3.9 — 8.9	4.2P	2.1/3.1
5.0P	3.3/11.2	4.0P	3.4/13.2	9.0PB — 9.0P	11.2 — 16.2	3.8P	3.1/8.9
5.0P	5.0/2.5	9.0P	4.2/1.8	10.0PB — 5.0RP	1.3 — 3.3	5.0P	5.0/2.4
5.0P	5.0/6 8	9.5P·	6.0/6.5	3.5P — 3.5RP	5.1 — 8.1	5.0P	4.9/6.3
5.0P	5.1/10.9	3.3P	5.3/12.3	7.3PB — 2.3RP	10.9 — 15.9	3.9P	5.0/8.7
5.5P	4.1/12.6	5.5P	4.2/13.2	1.5P — 1.5RP	10.4 — 16.4	5.0P	4.0/11 5
4.5P	6.9/2.7	8.8PB	8.0/1.4	6.7B — 0.5RP	0.6 — 2.6	4.9P	6.8/2 8
5.0P	7.1/4.8	1.0P	7.3/3.6	6.0B — 7.3P	2.9 — 3.9	5.2P	7.1/4.5
5.5P	7.0/6.4	2.3P	7.2/6 0	1.3PB — 1.3RP	4.6 — 7.6	5.6P	7.0/6.4
5.0P	6.1/9.1	1.0P	5.7/9.9	3.0PB — 8.0P	7.1 — 13.1	4.6P	6.1/8.6
4 5P	5.0/4.7	5.7P	4.4/5.7	2.8P — 2.8RP	4.0 — 9.0	4.8P	4.9/3.4
9.0P	3.0/9.8	6.0P	3.0/10.2	3.0P — 8.0P	7.8 — 12.8	7.9P	2.9/7.9
10.0P	3.2/4.5	7.0P	2.6/4.1	7.0PB — 7.0RP	2.1 — 5.1	7.5P	3.1/3.2
9.5P	7.3/6.0	3.3P	7.6/6.1	1.3PB — 1.3RP	5.3 — 7.3	9.5P	7.2/5.7
6.0RP	2.3/2.3	1.0RP	2.8/2.2	5.2PB — 2.7YR	1.0 — 5.0	5.3RP	2.2/1.5
4.0RP	2.4/5.9	8.3RP	3.1/7.7	7.0P — 8.2YR	6.1 — 11.1	2.7RP	2.3/3.8
5.0RP	3.2/8.4	7.0RP	3.1/9.4	4.0RP — 9.0RP	7.4 — 12.4	3.5RP	3.1/6.2
4.0RP	4.3/4.4	1.5RP	4.4/3.8	5.5P — 5.5RP	2.8 — 4.8	3.7RP	4.3/3.9
5.0RP	5.2/2.3	2.0R	5.7/2.3	3.0RP — 8.0R	1.7 — 3.7	5.2RP	5.1/2.2
4.5RP	5.1/6.2	2.0RP	5.1/6.0	7.0P — 7.0RP	5.2 — 7.2	4.3RP	5.1/5.7
5.0RP	4.3/10.7	6.5RP	4.3/11.5	2.5RP — 10.0RP	8.7 — 15.7	3.8RP	4.2/8.9
5.5RP	5.3/9.7	7.3RP	5.4/9.3	4.3RP — 9.3RP	7.1 — 11.1	4.6RP	5.3/8.6
4.5RP	6.3/8.5	3.0RP	6.5/8.2	9.5P — 4.5RP	5.8 — 10.8	3.9RP	6.3/7.7
6.5RP	7.3/2.3	5.3RP	8.2/1.9	10.0P — 8.7RP	1.1 — 2.6	6.4RP	7.3/2.2
5.5RP	8.1/2.8	7.5RP	5.4/2.4	3.5RP — 6.0R	1.9 — 3.4	5.3RP	8.1/2.7
4.5RP	7.4/4.9	3.5RP	7.9/5.2	3.3RP — 4.5RP	2.9 — 6.9	4.1RP	7.3/4.8
8.5RP	3.2/8.5	8.5RP	3.4/8.3	6.5RP — 1.5R	7.1 — 11.1	8.3RP	3.1/6.2
10.0RP	5.3/4.2	9.1RP	5.4/3.5	3.3RP — 7.0 R	2.3 — 3.8	9.2RP	5.3/3.9
9.0RP	7.1/5.8	0.8R	7.8/5.1	7.8RP — 1.5R	3.2 — 6.7	9.0RP	7.0/5.2

reddish-yellow to yellow, or from green-yellow green to yellowish green, each of which involves a change of 2.50 in the Munsell scale. Note that the observers were trained to report only within 2.50 Munsell hue steps. The greatest change from daylight is found with the lowest color temperature source 3500K, where the average hue change is over twice that found for either of the other two sources.

The differences among observers which appear in the inter-observer range of estimates in Tables II, III and IV are in line with what is known about individual differences in color vision from many types of study and are only in part due to the method of report employed in this investigation. Observers differ in the amounts of red, green and blue light necessary to produce white as well as in matching the chromatic gamut of colors. Colorimetric studies of the effects of adaptation as well as matches made while viewing samples under different sources of illumination either alternately or simultaneously reveal individual differences in the effects of chromatic adaptation of the sort found here. Thus 5.0R 4.2/12.8 in 6500K fluorescent illumination is reported as 3.5R (slightly on the bluish side) by one extreme observer and 7.3R (slightly

COLOR CHART

Figure 4. Approximate range (maximum among individual observers) of changes in color appearance from daylight to fluorescent sources (earlier type lamps).

TABLE III — Observed and Computed Hue, Lightness and Saturation, and Observers' Range under 4500K Fluorescent Illumination.

Munsell Renotation H	Munsell Renotation V/C	Observed H	Observed L/S	Observers' Range H	Observers' Range S	Computed H	Computed L/S
3.5R	2.3/2.2	6.3R	2.8/3.4	9.8RP — 8.5YR	1.0 — 8.0	3.3R	2.3/1.5
3.0R	2.5/4.8	5.5R	2.5/5.1	7.0RP — 0.7YR	3.9 — 6.9	2.7R	2.5/3.3
5.0R	3.2/7.4	4.2R	3.3/8.4	3.7R — 6.2R	5.8 — 12.8	4.8R	3.2/5 6
4.0R	5.0/2.8	5.0R	5.0/2.9	1.3R — 7.5R	2.1 — 4.1	4.5R	5.0/2.5
3.5R	4.1/4.7	0.5R	4.5/4.6	8.0RP — 5.5R	4.2 — 6.2	4.1R	4.1/4.1
4.0R	4.9/5.3	6.5R	4.6/6.8	8.5RP — 2.2YR	4.8 — 10.8	5.0R	5.0/4.9
5.0R	4.2/12.6	4.0R	4.0/14.4	3.5R — 6.0R	8.8 — 17.8	5.5R	4.2/11.0
5.5R	5.0/12.8	6.0R	4.4/13.0	4.0R — 6.5R	8.8 — 15.8	5.6R	5.1/11.2
2.5R	7.2/2.6	10.0RP	7.2/1.9	4.8RP — 3.5R	1.1 — 3.1	3.7R	7.2/2.4
3.5R	8.0/3.6	6.0R	8.2/3.3	1.5R — 1.5YR	1.6 — 4.6	5.0R	8.0/3.1
3.0R	7.1/5.2	3.8R	8.3/4.6	6.8RP — 5.5R	2.6 — 6.6	4.0R	7.1/4.5
3.5R	7.0/7.0	6.0R	7.2/8.4	3.2R — 0.7YR	7.2 — 10.2	4.2R	7.1/6.4
10.0R	3.3/5.4	1.0YR	3.4/7.4	9.0R — 4.0YR	6.2 — 11.2	0.8YR	3.3/4.0
9.5R	5.1/4.9	8.0R	5.4/4.2	4.5R — 2.0YR	3.6 — 5.1	1.0YR	5.1/4.4
0.5YR	6.2/10.1	2.5YR	6.2/11.5	9.0R — 4.0YR	7.5 — 16.5	2.0YR	6.4/9.3
4.5YR	2.4/1.8	6.5YR	2.6/2.2	7.5R — 7.5GY	0.4 — 6.6	4.6YR	2.4/1.2
5.0YR	3.3/4.1	8.5YR	3.0/5.2	4.5YR — 7.0Y	4.4 — 7.4	7.3YR	3.3/2.9
4.0YR	4.9/2.1	3.5YR	5.0/1.3	9.5R — 7.0YR	0.6 — 2.1	5.0YR	5.0/1.8
4.5YR	4.9/4.6	6.0YR	4.6/4.4	3.0YR — 0.5Y	3.6 — 5.6	5.8YR	5.0/4.0
5.5YR	5.1/9.0	4.5YR	5.9/10.6	10.0R — 7.5YR	7.4 — 15.4	6.9YR	5.2/7.7
3.0YR	7.0/2.1	6.0YR	7.1/1.9	10.0R — 10.0Y	1.2 — 2.2	5.0YR	7.1/2.0
4.0YR	8.0/3.5	1.5YR	8.3/3.6	8.5R — 6.0YR	1.9 — 6.9	5.9YR	8.1/3.0
3.5YR	6.9/5.9	4.0YR	7.2/5.4	9.5R — 7.0YR	2.9 — 11.9	5.0YR	7.0/5.5
4.5YR	7.0/8.2	8.0YR	7.2/6.9	5.5YR — 3.0Y	4.6 — 9.1	6.2YR	7.2/7.5
5.5YR	6.9/10.1	6.5YR	7.4/10.5	5.0YR — 7.5YR	7.9 — 13.9	7.0YR	7.1/9.2
6.0YR	6.2/11.5	7.0YR	6.0/10.9	5.0YR — 10.0YR	7.7 — 13.7	7.4YR	6.3/10.0
10.0YR	4.2/4.6	1.0Y	4.2/4.9	6.0YR — 3.5Y	4.3 — 6.3	1.6Y	4.3/3.9
10.0YR	5.1/4.4	4.5Y	5.2/4.3	9.5YR — 9.5Y	3.8 — 5.3	1.9Y	5.2/3.8
10.0YR	6.5/9.8	2.5Y	6.1/11.7	1.0Y — 6.0Y	7.7 — 14.7	2.4Y	6.6/8.8
5.0Y	2.5/1.4	3.5Y	2.3/1.1	8.5YR — 6.0GY	0.4 — 3.6	6.3Y	2.5/1.0
4.5Y	5.2/1.7	5.0Y	4.7/1.9	4.5Y — 7.0Y	1.3 — 2.3	7.3Y	5.2/1.6
5.0Y	4.1/3.6	8.5Y	4.1/3.9	4.5Y — 4.5GY	2.5 — 5.5	7.8Y	4.1/3.2
6.0Y	5.0/6.0	7.5Y	5.2/6.9	9.5YR — 2.0GY	5.3 — 11.3	8.6Y	5.1/5.4
3.5Y	7.8/1.6	2.5Y	7.6/2.1	6.5R — 6.5Y	1.4 — 3.4	6.3Y	7.8/1.6
4.5Y	7.9/5.8	5.5Y	7.7/6.4	5.0Y — 7.5Y	3.6 — 8.6	7.9Y	8.0/5.6
5.5Y	7.9/12.3	8.0Y	8.1/11.7	5.0Y — 10.0Y	8.7 — 15.7	9.2Y	8.1/11.5
5.5Y	7.9/8.3	5.5Y	7.7/9.9	3.5Y — 6.0Y	6.7 — 12.7	8.1Y	8.0/7.9
0.5GY	3.3/1.8	0.5GY	2.8/2.2	3.5Y — 6.0GY	1.3 — 4.3	1.8GY	3.3/1.6
10.0Y	4.3/3.7	3.5GY	4.3/4.0	7.0Y — 7.0GY	2.4 — 4.4	2.2GY	4.3/3.5
0.5GY	8.1/9.1	1.0GY	8.1/12.7	8.0Y — 3.0GY	7.7 — 15.7	2.3GY	8.2/9.1
5.5GY	2.5/2.2	2.0GY	2.1/2.0	4.0Y — 9.0GY	0.6 — 5.6	6.3GY	2.4/1.7
5.0GY	5.1/2.0	4.5GY	4.5/1.9	2.5Y — 10.0GY	1.3 — 3.3	6.1GY	5.2/2.0
5.0GY	5.1/5.4	5.5GY	5.1/5.6	1.5GY — 9.0GY	1.9 — 10.4	6.5GY	5.2/5.4
5.0GY	5.1/7.1	7.0GY	4.7/7.9	3.5GY — 1.0G	4.7 — 12.7	6.5GY	5.2/7.0
5.5GY	7.0/1.8	9.0GY	6.6/1.6	5.5GY — 5.5G	1.4 — 2.4	6.4GY	7.0/1.8
4.0GY	7.0/9.5	8.0GY	6.7/10.9	5.5GY — 3.0G	8.5 — 13.5	5.7GY	7.0/9.7
5.0GY	7.1/6.4	9.5GY	7.5/6.9	4.0GY — 6.5G	4.7 — 9.7	6.1GY	7.2/6.8
6.0GY	8.1/3.6	6.0GY	8.5/3.4	7.0Y — 4.5G	1.9 — 4.4	6.5GY	8.0/3.8
4.5GY	8.1/8.7	7.0GY	8.1/7.4	5.0GY — 5.0G	4.6 — 12.6	5.8GY	8.1/8.9
10.0GY	4.2/5.4	0.5G	3.9/6.2	8.0GY — 3.0G	4.0 — 10.0	0.7G	4.2/5.0
10.0GY	5.1/4.3	0.5G	5.2/5.0	8.5GY — 3.5G	3.1 — 6.6	0.2G	5.0/4.2
10.0GY	7.1/6.5	2.0G	7.0/8.1	6.5GY — 4.0G	5.2 — 10.2	0.7G	7.1/6.7
7.0G	2.2/2.4	8.5G	2.3/2.7	8.5G — 8.5G	0.1 — 4.1	8.5G	2.1/1.5
4.0G	3.3/4.8	4.5G	3.0/4.4	2.5G — 5.0G	3.2 — 5.2	5.0G	3.2/3.8
6.5G	4.9/2.4	0.7BG	5.1/2.8	1.2G — 6.2B	1.6 — 4.6	6.5G	4.9/2.2
5.0G	4.9/4.8	3.3G	5.0/5.2	0.5G — 6.8G	3.2 — 7.2	5.3G	4.8/4.4
5.0G	4.9/8.2	4.5G	5.7/9.6	1.0G — 6.0G	6.6 — 13.6	5.3G	4.8/7.3
6.5G	7.0/2.1	3.0G	7.5/1.8	3.0GY — 8.0BG	0.9 — 2.4	6.3G	6.9/2.1
7.0G	7.0/6.1	8.0G	6.8/7.0	7.7GY — 4.0BG	6.2 — 9.2	7.1G	7.0/6.0
6.0G	8.7/2.7	2.5G	8.9/2.6	7.5GY — 5.0BG	2.2 — 2.7	6.3G	8.6/2.3
10.0G	3.2/4.7	1.7BG	3.2/5.1	9.2G — 8.0BG	4.3 — 6.3	1.0BG	3.1/3.5
0.5BG	4.1/4.5	0.7BG	3.6/3.1	8.2G — 7.0BG	2.9 — 3.9	1.0BG	4.0/3.7
1.0BG	6.4/6.0	1.8BG	6.9/5.4	5.5G — 6.8BG	3.2 — 9.2	0.8BG	6.3/5.4
4.5BG	2.2/2.5	4.5BG	2.4/2.7	3.0BG — 6.8BG	0.9 — 4.9	7.1BG	2.1/1.4
5.0BG	2.3/3.7	4.5BG	2.3/3.7	3.0BG — 6.8BG	2.9 — 4.9	7.9BG	2.1/1.9
5.5BG	4.3/2.5	8.0BG	4.2/2.5	3.0BG — 1.7B	0.5 — 3.5	5.9BG	4.2/2.1
5.5BG	4.3/4.0	2.5B	4.4/3.7	4.3BG — 0.5PB	1.7 — 4.7	6.6BG	4.2/3.2
5.0BG	5.1/6.4	2.5BG	5.2/7.6	5.7G — 8.2BG	5.7 — 12.7	6.5BG	5.0/5.5
2.0BG	6.4/4.5	2.5BG	7.1/3.6	3.7G — 8.7BG	1.9 — 4.4	2.6BG	6.2/4.0
3.5BG	7.3/2.1	8.5BG	8.0/2.0	1.0G — 6.0B	1.3 — 3.3	1.6BG	7.2/2.0
4.5BG	7.2/3.4	0.2BG	7.2/3.0	7.2GY — 9.7BG	2.0 — 5.0	5.1BG	7.1/3.1
4.5BG	5.1/2.5	5.3BG	5.0/2.4	4.0G — 1.5B	1.8 — 2.8	5.5BG	5.1/2.2
9.5BG	3.2/5.8	3.0B	3.0/5.2	8.0BG — 4.2B	2.8 — 9.8	2.4B	3.0/3.8
9.5BG	4 1/4.0	9.0BG	4.2/4.4	3.0BG — 4.2B	3.2 — 6.2	1.6B	4.0/3.1
10.0BG	6.3/5.8	8.0BG	6.3/5.2	5.0BG — 2.5B	3.6 — 6.6	1.5B	6.1/5.1
2.0B	2.2/2.2	6.5B	2.4/2.0	3.2BG — 5.7PB	0.4 — 4.6	7.4B	2.1/1.2
5.0B	3.1/2.3	4.0B	3.3/2.0	2.0B — 7.0B	1.8 — 2.8	8.9B	3.1/1.8
6.5B	4.9/2.6	7.0B	5.2/2.0	5.0B — 10.0B	1.4 — 2.9	9.3B	4.9/2.5
5.5B	5.0/5.1	4.0B	5.0/5.3	6.5BG — 6.5B	4.5 — 6.5	7.8B	4.8/4.5
6.5B	4.0/6.9	5.2B	4.4/8.3	4.2B — 6.7B	6.7 — 12.7	0.2PB	3.7/5.7

(Table III continued on next page.)

TABLE III *(Continued).*

Munsell Renotation		Observed		Observers' Range		Computed	
H	V/C	H	L/S	H	S	H	L/S
5.0B	5.0/6.8	4.0B	4.5/8.0	9.0BG — 6.5B	5.5 — 12.5	7.7B	4.7/5.9
5.5B	6.1/6.4	3.5B	6.1/5.2	4.5BG — 7.0B	3.8 — 7.8	7.5B	5.9/5.9
4.0B	8.2/2.2	4.0B	8.0/1.6	6.0BG — 6.0B	1.3 — 1.8	5.0B	8.1/2.2
5.0B	7.1/4.7	5.0B	7.5/4.2	3.0B — 5.5B	1.9 — 5.9	7.5B	7.0/4.5
9.5B	3.2/5.6	9.0B	2.7/6.7	5.0B — 2.5PB	4.5 — 10.5	2.9PB	3.0/4.6
0.5PB	4.2/4.1	8.5B	3.7/3.8	2.5B — 10.0B	1.6 — 4.6	2.9PB	4.0/3.6
9.5B	7.2/5.7	8.5B	7.6/5.5	8.5B — 8.5B	3.9 — 6.9	1.9PB	7.0/5.5
4.5PB	2.2/2.6	4.5PB	2.4/3.2	4.5PB — 4.5PB	1.9 — 5.9	6.3PB	2.1/1.8
1.0PB	2.3/5.9	2.5PB	2.2/6.7	2.0PB — 4.5PB	4.3 — 10.3	3.3PB	2.1/4.6
4.0PB	3.3/9.4	4.0PB	2.7/8.2	3.5PB — 6.0PB	4.8 — 10.8	6.4PB	3.0/8.0
5.0PB	4.2/4.3	5.8PB	4.1/4.4	5.0PB — 8.8PB	3.6 — 5.6	6.4PB	4.1/4.2
5.0PB	5.1/3.1	5.5PB	5.1/2.7	4.2PB — 0.5P	2.3 — 3.3	6.3PB	5.1/3.1
4.0PB	5.3/6.2	5.5PB	5.6/6.6	2.0PB — 8.3PB	5.0 — 9.0	5.9PB	5.1/5.9
3.5PB	5.3/8.6	3.0PB	5.3/8.0	2.0PB — 4.5PB	6.6 — 10.6	5.7PB	5.1/8.5
5.0PB	8.3/3.2	6.8PB	8.6/2.8	5.9B — 7.2RP	1.8 — 3.8	6.9PB	8.2/3.3
4.0PB	7.2/5.6	5.3PB	7.5/4.8	2.5PB — 8.8PB	3.0 — 7.0	5.9PB	7.1/6.0
5.0PB	6.3/7.4	6.0PB	5.8/6.4	6.0PB — 6.0PB	4.2 — 8.2	5.9PB	6.1/6.6
10.0PB	2.2/5.5	1.5P	2.2/7.2	7.2PB — 6.0P	5.4 — 8.4	1.4P	2.1/3.9
10.0PB	4.1/4.6	8.5PB	3.9/3.1	6.0PB — 4.8P	1.6 — 4.6	1.3P	4.1/4.5
10.0PB	7.0/5.3	4.9P	7.2/5.3	0.9P — 0.9RP	3.2 — 8.2	1.6P	7.0/5.7
5.5P	2.1/2.9	8.2P	2.3/3.8	1.2P — 5.0RP	2.2 — 5.7	6.0P	2.1/2.1
5.0P	2.2/4.6	3.3P	2.6/5.8	6.2PB — 10.0P	4.4 — 6.9	5.0P	2.1/3.3
5.0P	3.3/11.2	6.0P	3.7/13.8	9.0PB — 9.0RP	10.2 — 16.2	5.0P	3.1/9.7
5.0P	5.0/2.5	6.0P	4.7/1.6	10.0PB — 10.0P	0.8 — 2.3	5.6P	5.0/2.7
5.0P	5.0/6.8	8.5P	5.2/7.1	8.5PB — 8.5RP	6.1 — 8.1	5.6P	4.9/6.7
5.0P	5.1/10.9	3.3P	4.9/12.7	2.3P — 7.3P	10.9 — 16.9	5.4P	5.0/11.1
5.5P	4.1/12.6	1.5RP	4.6/13.4	1.5P — 6.5RP	11.4 — 16.4	5.6P	4.0/12.2
4.5P	6.9/2.7	0.8P	7.7/1.8	6.7B — 5.5RP	0.6 — 2.6	5.5P	6.8/3.0
5.0P	7.1/4.8	7.3P	7.3/5.5	2.3P — 2.3RP	3.4 — 10.9	5.6P	7.1/5.0
5.5P	7.0/6.4	9.3P	7.0/6.8	1.3P — 1.3RP	5.6 — 10.6	6.1P	7.0/6.9
5.0P	6.1/9.1	5.0P	6.1/10.1	8.0PB — 3.0RP	8.1 — 14.1	5.5P	6.1/9.2
4.5P	5.0/4.7	7.7P	4.4/6.2	7.8PB — 7.8RP	5.0 — 7.0	5.4P	4.9/4.8
9.0P	3.0/9.8	4.3P	2.8/10.2	9.2B — 3.0RP	5.8 — 12.8	8.3P	3.0/7.9
10.0P	3.2/4.5	7.0P	3.1/4.3	7.0PB — 7.0RP	2.1 — 7.1	0.2RP	3.1/3.6
9.5P	7.3/6.0	7.3P	7.3/6.7	1.3P — 1.3RP	5.3 — 8.3	10.0P	7.3/6.2
6.0RP	2.3/2.3	1.7R	2.7/2.4	9.0P — 2.7YR	1.0 — 5.0	5.8RP	2.2/1.6
4.0RP	2.4/5.9	7.0RP	2.9/7.5	2.0P — 7.0R	5.1 — 11.1	3.0RP	2.4/4.1
5.0RP	3.2/8.4	6.0RP	3.2/8.4	9.0P — 9.0RP	6.4 — 11.4	4.1RP	3.1/6.1
4.0RP	4.3/4.4	5.5RP	4.2/4.6	0.5RP — 0.5R	3.8 — 5.8	4.0RP	4.3/4.1
5.0RP	5.2/2.3	1.0R	5.8/2.3	3.0RP — 8.0R	1.7 — 3.7	5.5RP	5.2/2.3
4.5RP	5.1/6.2	3.0RP	4.8/5.2	7.0P — 7.0RP	4.2 — 6.2	4.3RP	5.2/6.0
5.0RP	4.3/10.7	5.5RP	4.4/10.9	2.5RP — 10.0RP	8.7 — 12.7	4.0RP	4.3/9.5
5.5RP	5.3/9.7	7.0RP	5.4/9.9	4.3RP — 0.5R	8.1 — 13.1	5.3RP	5.3/8.9
4.5RP	6.3/8.5	3.5RP	6.6/7.8	9.5P — 8.2RP	5.8 — 8.8	4.5RP	6.3/8.2
6.5RP	7.3/2.3	2.5RP	7.7/2.1	5.0P — 6.2RP	1.6 — 3.1	5.9RP	7.3/2.1
5.5RP	8.1/2.8	1.0RP	7.7/3.1	7.3PB — 6.0RP	1.9 — 4.9	6.3RP	8.1/2.7
4.5RP	7.4/4.9	3.2RP	7.9/5.8	8.3P — 5.2RP	4.9 — 6.4	4.7RP	7.4/4.9
8.5RP	3.2/8.5	7.7RP	3.5/9.7	6.5RP — 2.7R	7.1 — 11.1	7.5RP	3.1/6.4
10.0RP	5.3/4.2	7.3RP	4.8/4.3	3.3RP — 2.0R	2.8 — 5.8	0.1R	5.3/3.9
9.0RP	7.1/5.8	8.3RP	7.7/6.0	2.8RP — 1.5R	4.2 — 7.2	9.9RP	7.1/5.4

on the yellowish side) by another extreme observer while the average estimate is 4.8R. This type of result has also been found with a daylight source (Helson, 1939). The range of hues reported by the five observers for the 130 samples used in this study is no greater than has been found previously both with incandescent lamp light and daylight sources. Fig. 4 is a reproduction in color of 10 of these 130 samples as judged by the average observer and by the two extreme observers in each of the fluorescent sources.

The small hue changes found in this study make it appear that the differences between fluorescent and daylight illumination commonly reported must be sought in something other than color changes of single objects unless these objects are very familiar (such as complexions and foods). In the latter case even small changes in hue may influence consumer acceptance of light sources. Various comments from observers apart from their reports of specific colors reveal that they recognized the fluorescent illumination was different from "normal daylight." Thus one observer reported that "Everything looks distorted in some way but I can't tell just how"; another reported that "All the samples seem to fluoresce"; and still another: "The samples glow — it's like looking at a powdery pastel." Sometimes the change in hue could not be adequately described in conventional terms as in the following report: "This sample looks like a red gone wrong." Small hue changes in many samples coupled with unusual appearance of well-known colors may cumulate to produce an over-all unfavorable impression of the quality of illumination. Whether or not the total effect is acceptable must be determined by aesthetic studies beyond the scope of an investigation concerned with the color-rendering properties of sources of illumination.

TABLE IV — Observed and Computed Hue, Lightness and Saturation, and Observers' Range Under 3500K Fluorescent Illumination.

Munsell Renotation H	V/C	Observed H	L/S	Observers' Range H	S	Computed H	L/S
3.5R	2.3/2.2	0.8YR	2.3/3.5	9.8RP — 3.5YR	3.0 — 4.0	4.0R	2.3/1.5
3.0R	2.5/4.8	1.3R	2.4/5.0	7.0RP — 5.7R	3.9 — 5.9	3.1R	2.5/3.4
5.0R	3.2/7.4	6.2R	3.4/7.8	3.7R — 8.7R	5.8 — 12.8	5.2R	3.3/5.9
4.0R	5.0/2.8	10.0R	5.6/2.5	7.5R — 5.0YR	1.1 — 4.1	5.4R	5.1/2.6
3.5R	4.1/4.7	3.5R	4.5/4.2	0.5R — 5.5R	2.2 — 5.2	5.0R	4.2/4.1
4.0R	4.9/5.3	**7.0R**	**4.6/6.7**	8.5RP — 2.2YR	5.8 — 9.8	5.4R	5.1/5.4
5.0R	4.2/12.6	4.5R	4.1/15.4	3.5R — 6.0R	11.8 — 17.8	5.9R	4.3/11.3
5.5R	5.0/12.8	6.5R	6.1/13.1	4.0R — 4.0YR	9.3 — 15.8	5.8R	5.2/11.5
2.5R	7.2/2.6	1.8R	7.6/2.9	9.8RP — 6.0R	2.1 — 4.1	5.5R	7.2/2.4
3.5R	8.0/3.6	6.5R	9.1/3.8	1.5R — 4.0YR	2.6 — 5.6	5.5R	8.1/3.0
3.0R	7.1/5.2	6.3R	7.9/5.1	1.8R — 0.5YR	2.6 — 8.6	4.9R	7.2/4.6
3.5R	7.0/7.0	8.7R	6.5/9.2	3.2R — 3.2YR	7.2 — 11.2	5.0R	7.1/6.4
10.0R	3.3/5.4	10.0R	3.2/6.4	9.0R — 1.5YR	5.2 — 8.2	1.0YR	3.4/4.2
9.5R	5.1/4.9	0.5YR	5.5/4.1	9.5R — 2.0YR	2.1 — 6.1	3.0YR	5.2/4.2
0.5YR	6.2/10.1	4.0YR	6.6/12.3	1.5YR — 6.5YR	9.5 — 14.5	2.1YR	6.5/8.8
4.5YR	2.4/1.8	1.0YR	**2.5/2.4**	7.5R — 2.5YR	1.6 — 3.6	5.8YR	2.4/1.2
5.0YR	3.3/4.1	5.5YR	2.9/4.6	4.5YR — 9.5YR	2.4 — 7.4	7.3YR	3.4/3.2
4.0YR	4.9/2.1	8.5YR	5.3/1.8	4.5YR — 7.0Y	1.1 — 3.1	7.5YR	5.0/1.8
4.5YR	4.9/4.6	4.0YR	5.2/4.5	3.0YR — 5.5YR	4.1 — 4.6	5.1YR	5.1/4.0
5.5YR	5.1/9.0	4.5YR	4.4/8.0	2.5YR — 7.5YR	3.4 — 12.4	8.3YR	5.3/7.7
3.0YR	7.0/2.1	9.5YR	8.5/2.3	10.0R — 10.0Y	1.2 — 3.2	6.3YR	7.1/1.9
4.0YR	8.0/3.5	4.0YR	8.1/2.8	1.0YR — 1.0Y	1.9 — 4.9	7.4YR	8.2/3.2
3.5YR	6.9/5.9	3.5YR	7.6/4.9	2.0YR — 4.5YR	3.9 — 5.9	6.1YR	7.1/5.6
4.5YR	7.0/8.2	8.0YR	8.6/8.4	3.0YR — 0.5Y	7.6 — 10.6	7.5YR	7.3/7.6
5.5YR	6.9/10.1	8.5YR	6.5/11.7	5.0YR — 2.5Y	8.9 — 14.9	8.3YR	7.2/9.2
6.0YR	6.2/11.5	5.5YR	5.9/11.1	2.5YR — 10.0YR	6.7 — 15.7	8.2YR	6.5/10.1
10.0YR	4.2/4.6	1.5Y	4.1/4.5	8.5YR — 6.0Y	3.3 — 6.3	3.7Y	4.3/3.9
10.0YR	5.1/4.4	2.5Y	5.4/3.6	4.5YR — 9.5Y	2.8 — 4.8	3.4Y	5.2/3.8
10.0YR	6.5/9.8	3.8Y	6.0/9.9	2.3Y — 6.0Y	7.7 — 14.7	3.8Y	6.8/8.9
5.0Y	2.5/1.4	1.5Y	2.3/1.5	6.0YR — 1.0GY	0.6 — 2.6	7.3Y	2.5/1.1
4.5Y	5.2/1.7	1.5GY	4.7/2.8	4.5Y — 2.0G	2.3 — 4.3	9.4Y	5.2/1.7
5.0Y	4.1/3.6	5.5Y	3.5/3.7	7.0YR — 7.0GY	1.5 — 8.5	10.0Y	4.2/3.4
6.0Y	5.0/6.0	5.0GY	5.1/7.7	2.0GY — 9.5GY	7.3 — 9.3	0.6GY	5.2/5.6
3.5Y	7.8/1.6	5.5Y	7.3/2.8	4.0Y — 6.5Y	1.4 — 6.4	8.7Y	7.8/1.6
4.5Y	7.9/5.8	4.5Y	8.7/7.4	2.5Y — 5.0Y	3.6 — 13.6	0.3GY	8.1/5.7
5.5Y	7.9/12.3	8.5Y	7.9/11.9	7.5Y — 10.0Y	8.7 — 16.7	1.1GY	8.2/11.5
5.5Y	7.9/8.3	6.0Y	7.6/11.7	6.0Y — 6.0Y	7.7 — 15.7	9.9Y	8.1/8.0
0.5GY	3.3/1.8	4.3GY	2.6/1.6	6.0Y — 9.8G	0.3 — 3.3	3.5GY	3.3/1.8
10.0Y	4.3/3.7	4.5GY	3.7/3.8	2.0GY — 7.0GY	2.4 — 5.4	4.1GY	4.3/3.7
0.5GY	8.1/9.1	4.0GY	8.3/11.3	0.5GY — 3.0G	7.7 — 15.7	3.9GY	8.2/9.4
5.5GY	2.5/2.2	4.0GY	2.3/2.0	9.0Y — 6.5GY	1.1 — 2.6	7.0GY	2.4/1.6
5.0GY	5.1/2.0	4.5GY	5.1/2.2	5.0Y — 7.5GY	1.3 — 3.3	7.0GY	5.1/2.2
5.0GY	5.1/5.4	8.0GY	4.7/6.0	6.5GY — 9.0GY	2.4 — 9.4	7.5GY	5.2/6.0
5.0GY	5.1/7.1	7.0GY	4.8/9.3	1.0GY — 8.5GY	4.7 — 13.7	7.5GY	5.2/7.6
5.5GY	7.0/1.8	8.5Y	8.3/2.8	8.0Y — 0.5GY	1.4 — 6.4	7.0GY	7.0/1.9
4.0GY	7.0/9.5	8.0GY	6.9/10.1	5.5GY — 3.0G	6.5 — 12.5	6.6GY	7.1/10.4
5.0GY	7.1/6.4	7.5GY	6.7/7.9	4.0GY — 4.0G	4.7 — 9.7	7.0GY	7.2/7.2
6.0GY	8.1/3.6	2.0GY	9.2/4.3	9.5Y — 7.0GY	2.9 — 5.9	7.1GY	8.1/4.0
4.5GY	8.1/8.7	5.0GY	7.9/11.2	5.0GY — 5.0GY	5.6 — 14.6	6.4GY	8.1/9.4
10.0GY	4.2/5.4	1.0G	4.0/6.2	8.0GY — 3.0G	4.0 — 8.0	1.0G	4.1/5.5
10.0GY	5.0/4.3	0.5G	4.8/4.7	3.5GY — 3.5G	3.1 — 6.1	1.2G	5.0/4.6
10.0GY	7.1/6.5	10.0GY	7.5/8.4	4.0GY — 4.0G	6.2 — 10.2	1.4G	7.1/7.2
7.0G	2.2/2.4	10.0G	1.7/2.1	8.5G — 2.3BG	1.1 — 3.1	9.1G	2.1/1.5
4.0G	3.3/4.8	4.5G	3.4/4.0	2.5G — 5.0G	2.2 — 6.2	5.8G	3.1/3.8
6.5G	4.9/2.4	2.2BG	4.3/3.6	7.5G — 6.2BG	2.6 — 5.6	7.4G	4.8/2.3
5.0G	4.9/4.8	2.5G	5.6/4.8	0.5G — 3.0G	3.2 — 6.2	6.2G	4.7/4.6
5.0G	4.9/8.2	3.5G	4.6/10.2	1.0G — 6.0G	7.6 — 12.6	5.7G	4.6/7.3
6.5G	7.0/2.1	1.0G	8.0/2.3	0.5GY — 5.5BG	1.9 — 2.9	7.4G	6.9/2.2
7.0G	7.0/6.1	3.7G	7.6/6.4	0.2G — 5.2G	4.2 — 8.2	7.4G	6.9/6.2
6.0G	8.7/2.7	10.0GY	9.1/3.4	10.0YR — 2.5B	2.7 — 5.7	6.8G	8.6/2.4
10.0G	3.2/4.7	2.3BG	3.4/3.3	9.2G — 3.0BG	1.3 — 4.3	2.2BG	3.0/3.3
0.5BG	4.1/4.5	7.5G	3.5/3.1	5.7G — 2.0BG	1.9 — 4.9	2.0BG	4.0/3.8
1.0BG	6.4/6.0	3.3BG	6.8/5.4	5.5G — 0.5B	4.2 — 7.2	1.4BG	6.2/5.6
4.5BG	2.2/2.5	6.3BG	1.6/2.1	3.0BG — 1.8B	0.9 — 3.9	8.1BG	2.1/1.3
5.0BG	2.3/3.7	5.0BG	1.5/4.9	2.5BG — 1.3B	0.9 — 11.9	9.1BG	2.1/1.8
5.5BG	4.3/2.5	0.2B	3.6/2.3	1.7G — 9.2B	1.5 — 3.5	7.1BG	4.1/2.0
5.5BG	4.3/4.0	0.5B	3.8/4.3	0.5BG — 8.0B	2.7 — 6.7	7.1BG	4.1/3.2
5.0BG	5.1/6.4	4.2BG	5.8/7.3	9.5G — 8.2BG	6.7 — 8.7	7.6BG	4.8/5.2
2.0BG	6.4/4.5	3.5BG	7.1/4.0	3.7G — 3.7B	2.9 — 4.9	3.3BG	6.2/4.2
3.5BG	7.3/2.1	7.2BG	8.4/1.9	8.5GY — 6.0B	1.3 — 3.3	4.0BG	7.2/2.0
4.5BG	7.2/3.4	5.0BG	8.3/2.5	7.2GY — 7.2B	1.0 — 5.0	5.9BG	7.1/3.0
4.5BG	5.1/2.5	3.8BG	4.9/3.0	1.5G — 6.5B	1.8 — 3.8	6.0BG	5.0/2.3
9.5BG	3.2/5.8	4.2B	2.1/4.3	1.7B — 9.2B	1.8 — 7.8	4.9B	2.7/3.6
9.5BG	4.1/4.0	2.2B	4.3/3 6	9.2BG — 6.7B	3.2 — 4.2	2.8B	3.9/3.1
10.0BG	6.3/5.8	2.0B	6.6/7.0	5.0BG — 7.5B	4.6 — 10.6	2.5B	6.0/5.0
2.0B	2.2/2.2	9.5B	1.9/1.4	2.0B — 8.2PB	0.4 — 2.6	7.8B	2.1/1.4
5.0B	3.1/2.3	5.0B	2.4/0.9	2.0B — 7.0B	0.2 — 2.8	10.0B	3.0/1.8
6.5B	4.9/2.6	5.5B	5.2/1.9	2.5B — 7.5B	0.4 — 3.4	9.6B	4.8/2.5
5.5B	5.0/5.1	6.5B	4.4/4.3	4.0B — 9.0B	3.5 — 4.5	9.2B	4.7/4.5
6.5B	4.0/6.9	5.7B	3.6/6.9	1.7B — 9.2B	3.7 — 9.7	2.0PB	3.5/5.7

(Table IV continued on next page.)

TABLE IV *(Continued).*

Munsell Renotation		Observed		Observers' Range		Computed	
H	V/C	H	L/S	H	S	H	L/S
5.0B	5.0/6.8	6.5B	4.3/6.6	4.0B — 9.0B	5.6 — 7.6	9.6B	4.6/4.6
5.5B	6.1/6.4	6.0B	6.5/5.4	4.5B — 7.0B	3.8 — 7.8	9.2B	6.1/5.8
4.0B	8.2/2.2	4.0B	9.0/1.1	4.0B — 4.0B	0.8 — 1.8	6.2B	8.1/2.2
5.0B	7.1/4.7	4.0B	7.4/4.3	0.5B — 5.5B	2.9 — 6.9	8.5B	6.9/4.6
9.5B	3.2/5.6	0.5PB	2.7/6.7	7.5B — 2.5PB	3.5 — 8.5	4.7PB	2.9/4.7
0.5PB	4.2/4.1	1.0PB	3.5/4.0	10.0B — 2.5PB	2.6 — 4.6	3.9PB	4.0/3.8
9.5B	7.2/5.7	9.0B	8.5/3.9	8.5B — 1.0PB	2.9 — 4.9	2.5PB	6.9/5.6
4.5PB	2.2/2.6	3.5PB	1.4/2.9	9.5B — 4.5PB	1.9 — 4.9	7.5PB	2.0/1.8
4.0PB	2.3/5.9	3.0PB	1.1/4.4	2.0PB — 4.5PB	3.3 — 5.3	7.5PB	2.0/4.6
4.0PB	3.3/9.4	4.5PB	3.1/8.1	3.5PB — 6.0PB	5.3 — 10.8	7.5PB	2.9/8.6
5.0PB	4.2/4.3	4.8PB	3.9/3.4	2.5PB — 8.8PB	2.6 — 5.6	7.3PB	4.1/4.4
5.0PB	5.1/3.1	5.3PB	5.2/3.4	1.7PB — 8.0PB	2.3 — 4.3	7.5PB	5.0/3.1
4.0PB	5.3/6.2	6.8PB	5.1/6.2	2.0PB — 8.3P	5.0 — 8.0	7.1PB	5.1/6.3
3.5PB	5.3/8.6	3.0PB	5.4/8.8	2.0PB — 4.5PB	4.6 — 13.6	6.1PB	5.0/9.0
5.0PB	8.3/3.2	7.1PB	9.4/2.2	9.6B — 5.9P	1.8 — 2.8	7.3PB	8.1/3.4
4.0PB	7.2/5.6	8.3PB	7.3/6.0	5.0PB — 3.8P	4.0 — 8.0	7.0PB	7.0/5.8
5.0PB	6.3/7.4	6.3PB	5.5/6.4	3.5PB — 9.8PB	4.2 — 8.2	6.9PB	6.0/7.0
10.0PB	2.2/5.5	4.0P	2.6/6.4	1.0P — 1.0RP	5.4 — 9.4	2.0P	2.1/4.3
10.0PB	4.1/4.6	3.8P	4.1/4.8	9.8PB — 9.8P	2.6 — 6.6	2.1P	4.0/4.9
10.0PB	7.0/5.3	1.2RP	6.9/5.4	7.1PB — 5.9R	3.2 — 10.2	2.2P	6.9/5.8
5.5P	2.1/2.9	0.3RP	2.5/3.9	1.2P — 10.0RP	2.7 — 5.7	6.3P	2.1/2.3
5.0P	2.2/4.6	6.0P	2.3/6.6	10.0PB — 10.0P	4.9 — 8.9	5.4P	2.1/3.6
5.0P	3.3/11.2	1.0RP	3.4/13.0	4.0P — 9.0RP	10.2 — 15.2	5.7P	3.1/10.4
5.0P	5.0/2.5	2.0RP	5.4/1.8	10.0P — 10.0RP	1.3 — 2.3	6.4P	5.0/2.9
5.0P	5.0/6.8	4.5RP	5.1/7.5	8.5P — 8.5RP	7.1 — 8.1	6.3P	4.9/7.4
5.0P	5.1/10.9	6.3P	5.0/12.7	2.3P — 2.3RP	10.9 — 16.9	5.9P	5.0/11.8
5.5P	4.1/12.6	3.5RP	4.0/14.1	6.5P — 6.5RP	12.4 — 16.4	6.4P	4.0/13.3
4.5P	6.9/2.7	1.5RP	8.1/2.6	5.5P — 5.5RP	0.6 — 4.6	5.9P	6.8/3.1
5.0P	7.1/4.8	6.3P	7.5/5.5	2.3PB — 2.3RP	3.9 — 7.4	6.1P	7.0/5.2
5.5P	7.1/6.4	9.3P	8.3/6.0	1.3P — 1.3RP	4.6 — 7.6	8.2P	7.0/8.0
5.0P	6.1/9.1	9.0P	6.1/12.5	3.0P — 3.0RP	8.1 — 16.1	6.0P	6.0/9.9
4.5P	5.0/4.7	7.7P	3.8/6.2	2.8P — 2.8RP	5.0 — 8.0	6.1P	4.9/5.2
9.0P	3.0/9.8	9.0P	3.1/11.6	3.0P — 3.0RP	8.8 — 14.8	9.1P	2.9/8.6
10.0P	3.2/4.5	3.0RP	2.7/5.1	2.0P — 7.0RP	4.1 — 7.1	0.6RP	3.2/4.3
9.5P	7.3/6.0	5.3P	7.8/6.7	1.3P — 1.3RP	5.3 — 9.3	0.5RP	7.3/6.5
6.0RP	2.3/2.3	3.0R	2.6/2.4	4.0RP — 2.7YR	1.5 — 5.0	6.2RP	2.2/1.6
4.0RP	2.4/5.9	4.0RP	2.5/7.5	7.0P — 2.0R	7.1 — 8.1	3.4RP	2.4/4.4
5.0RP	3.2/8.4	7.0RP	2.8/9.0	9.0P — 9.0RP	6.4 — 11.4	4.4RP	3.2/7.0
4.0RP	4.3/4.4	4.5RP	4.6/3.5	5.5P — 0.5R	1.8 — 5.8	4.4RP	4.3/4.2
5.0RP	5.2/2.3	2.8R	5.6/2.3	8.0RP — 1.7YR	0.7 — 3.7	6.2RP	5.2/2.3
4.5RP	5.1/6.2	5.0RP	4.6/6.4	2.0RP — 7.0RP	5.2 — 8.2	5.2RP	5.2/6.3
5.0RP	4.3/10.7	3.0RP	4.3/12.3	5.0P — 5.0RP	8.7 — 16.7	4.7RP	4.3/9.9
5.5RP	5.3/9.7	8.3RP	6.5/9.1	4.3RP — 9.3RP	6.1 — 12.1	5.6RP	5.4/9.6
4.5RP	6.3/8.5	5.5RP	6.9/10.0	9.5P — 0.7R	8.8 — 12.8	4.8RP	6.4/8.7
6.5RP	7.3/2.3	2.8RP	8.6/2.4	10.0B — 8.7R	1.1 — 3.1	7.1RP	7.3/2.2
5.5RP	8.1/2.8	5.5RP	9.1/3.0	3.5RP — 6.0RP	1.9 — 4.4	6.3RP	8.1/2.9
4.5RP	7.4/4.9	6.3RP	8.1/6.5	3.3RP — 7.0RP	4.9 — 8.9	5.0RP	7.4/5.0
8.5RP	3.2/8.5	0.5R	3.4/8.4	6.5RP — 1.5R	6.1 — 11.1	7.9RP	3.2/6.8
10.0RP	5.3/4.2	7.0RP	5.0/3.8	3.3RP — 2.0R	2.8 — 4.8	0.9R	5.4/4.0
9.0RP	7.1/5.8	1.0R	8.3/6.1	9.0RP — 4.0R	3.2 — 9.2	10.0RP	7.2/5.5

Turning now to saturation we find from the summary in Table VI that, in general, the samples were estimated slightly more saturated in the fluorescent illuminants than in daylight as checked by observations under Macbeth daylight illumination (incandescent lamp plus Corning Daylite filter). The average observed gain in saturation as compared with daylight is 0.13 in 6500K, 0.25 in 4500K, and 0.33 in 3500K. Only two hues, green-yellow and purple in 3500K, change more than one Munsell chroma step on the average as shown in Table VI. In view of the large inter-observer variations when estimating saturation these small increases cannot be regarded as either practically important or statistically significant. Saturation can therefore be said to change very little from source to source within the limits of this study.

No extended discussion of lightness is necessary because of the excellent agreement between observed and theoretical estimates of this dimension, not only for the present study (Tables II, III and IV), but also in a previous study (Judd, 1940) providing a more severe test.

4. Theory

The chief purpose of the present investigation was to test whether or not a relatively simple quantitative formulation found adequate to predict the colors of objects seen under incandescent-filament sources of light (Helson, Judd, and Warren, 1952) would be adequate to predict the colors of objects seen under fluorescent sources of illumination. As in the previous paper, the theoretical computations were based on a three-components theory whose primaries are at the co-punctal points of the three types of dichromatic vision (protan-

TABLE V — Mean Hue Changes of 130 Munsell Samples in Passing from Daylight (Munsell Renotation) to Three Fluorescent Sources.

Daylight Munsell Renotation		Fluorescent Sources 6500°K	4500°K	3500°K
Red	Observed	0.32 yellower	0.59 yellower	2.07 yellower
	Calculated	0.09 bluer	0.65 yellower	1.35 yellower
Yellow-Red	Observed	0.14 bluer	1.43 yellower	1.45 yellower
	Calculated	0.46 yellower	1.58 yellower	2.91 yellower*
Yellow	Observed	0.86 greener	0.95 greener	2.75 greener
	Calculated	0.48 greener	2.48 greener*	4.39 greener
Green-Yellow	Observed	1.37 greener	1.25 greener	0.05 yellower
	Calculated	0.01 greener	0.91 greener	1.68 greener
Green	Observed	0.30 bluer	0.05 bluer	0.74 yellower
	Calculated	0.73 yellower	0.41 bluer	1.17 bluer
Blue-Green	Observed	1.08 bluer	0.75 bluer	2.10 bluer
	Calculated	0.02 yellower	1.28 bluer	2.45 bluer
Blue	Observed	0.92 greener	0.40 greener	0.76 redder
	Calculated	1.44 redder**	2.91 redder**	4.10 redder**
Purple-Blue	Observed	0.09 redder	0.76 redder	2.11 redder
	Calculated	0.86 redder	1.49 redder*	2.62 redder
Purple	Observed	1.71 bluer	0.32 redder	3.62 redder
	Calculated	0.54 bluer	0.29 redder	0.96 redder**
Red-Purple	Observed	0.27 redder	0.08 redder	1.14 redder
	Calculated	0.64 bluer	0.27 bluer	0.21 redder

*Difference between observation and theory statistically significant between 1% and 5% levels.
**Difference between observation and theory statistically significant at or beyond the 1% level.

opia, deuteranopia, tritanopia). The coordinate system based on these particular primaries may be defined by giving the connection between the tristimulus values, R, G, V, of any object color and the tristimulus values X, Y, Z (Y = luminous reflectance) of that color in the C.I.E. system, which is closely (Judd, 1944) the following:

$$\left.\begin{aligned} R &= 1.00Y \\ G &= -0.46X + 1.36Y + 0.10Z \\ V &= 1.00Z \end{aligned}\right\} \quad (1)$$

The reverse transformation is obtained by solving explicitly for X, Y, Z, in terms of R. G, V, thus:

$$\left.\begin{aligned} X &= 2.957R - 2.174G + 0.217V \\ Y &= 1.000R \\ Z &= 1.000V \end{aligned}\right\} \quad (1a)$$

It is assumed that the tristimulus values, R, G, V, also correspond to the responses of an observer adapted to C.I.E. source C (representative of average daylight). To find the red, green, and violet responses, R′, G′, V′, of an observer adapted to a source of some other chromaticity, the tristimulus values, G and V, are adjusted relative to the R-response to take account of the changed sensitivities of the red, green, and violet receptor mechanisms in accord with equation 2:

$$\left.\begin{aligned} R' &= (R_c/R_s)\,R \\ G' &= (G_c/G_s)\,G \\ V' &= (V_c/V_s)\,V \end{aligned}\right\} \quad (2)$$

where R_c, G_c, V_c are the tristimulus values of C.I.E. source C, and R_s, G_s, V_s those for the source being compared to it. From equation (1) the tristimulus values, X′, Y′, Z′, corresponding to R′, G′, V′, may be expressed in terms of the unadjusted tristimulus values X, Y, Z, as in equation 3:

$$\left.\begin{aligned} X' &= (G_c/G_s)\,X + 2.957\,(R_c/R_s - G_c/G_s)\,Y \\ &\quad + 0.217\,(V_c/V_s - G_c/G_s)\,Z \\ Y' &= (R_c/R_s)\,Y \\ Z' &= (V_c/V_s)\,Z \end{aligned}\right\} \quad (3)$$

*This theory, with somewhat different primaries, was also found by Bouma and Kruithof (1947/1948) to be successful for the prediction of the hues of object-color perceptions both for the incandescent lamp as source, and for the low-pressure mercury lamp, which yields a discontinuous spectrum. They made no attempt to apply the theory to prediction of the saturations of object-color perceptions.

TABLE VI — Mean Change in Saturation of 10 Munsell Major Hues from Daylight to Fluorescent Illumination.

Munsell Renotation (Daylight)	Fluorescent Sources 6500K Obs.	6500K Comp.	4500K Obs.	4500K Comp.	3500K Obs.	3500K Comp.
R	0.3	−0.9	0.6	−0.9	0.7	−0.8
YR	0.1	−0.7	0.1	−0.8	0.0	−0.7
Y	0.6	−0.5	0.7	−0.3	0.9	−0.2
GY	0.3	−0.3	0.3	0.0	1.0	0.3
G	−0.3	−0.8	0.1	−0.6	0.0	−0.5
BG	0.3	−0.8	−0.1	−0.8	0.0	−0.9
B	−0.1	−0.8	0.0	−0.6	−0.6	−0.7
PB	−0.2	−0.7	−0.1	−0.4	−0.2	−0.2
P	0.3	−0.9	0.7	−0.3	1.1	0.2
RP	0.0	−0.9	0.2	−0.7	0.4	−0.4
Average	0.13	−0.73	0.25	−0.54	0.33	−0.39

TABLE VII — Computations for Calculating Hue and Saturation of Munsell Sample 3.5R 2.3/2.2 in Three Fluorescent Illuminants.

Tristimulus values X	Y	Z	$Z'=2.343Z+.024$	$Y'=Y+.020$	$X'=1.093X-.275Y+.271Z+.020$	$X'+Y'+Z'$	x'	y'	z'
3500K Fluorescent									
0516	.0421	.0184	.0671	.0621	.0698	.1990	.351	.312	.337
4500K Fluorescent									
Tristimulus values X	Y	Z	$Z'=1.538Z+.024$	$Y'=Y+.020$	$X'=1.046X-.133Y+.107Z+.020$	$X'+Y'+Z'$	x'	y'	z'
.0489	.0410	.0277	.0666	.0610	.0687	.1963	.350	.311	.339
6500K Fluorescent									
Tristimulus values X	Y	Z	$Z'=1.082Z+.024$	$Y'=Y+.020$	$X'=.996X+.012Y+.019Z+.020$	$X'+Y'+Z'$	x'	y'	z'
.0458	.0397	.0391	.0663	.0597	.0668	.1928	.346	.310	.344

The tristimulus values of source C and of the three fluorescent sources used in the observations (Nickerson and Wilson, 1950) are as follows:

	X	Y	Z
Source C	0.980	1.000	1.181
3500K fluorescent	1.021	1.000	0.504
4500K fluorescent	.988	1.000	0.768
6500K fluorescent	.952	1.000	1.091

The sensitivities of the green and violet receptor mechanisms for an observer adapted to one of the several sources studied relative to those for the observer adapted to an equal luminance of source C are found in accord with equation 2 as follows:

	R_c/R_s	G_c/G_s	V_c/V_s
C.I.E Source A (2854K)	1.000	1.154	3.327
3500K fluorescent	1.000	1.093	2.343
4500K fluorescent	1.000	1.046	1.538
6500K fluorescent	1.000	0.996	1.082
C.I.E. Source C (6740K)	1.000	1.000	1.000

It will be noted that the degrees of chromatic adaptation that formula 3 must take into account for the fluorescent sources are intermediate to that for source A, previously studied (Helson, Judd, Warren, 1952), and source C, taken as the standard for normal adaptation. In particular, the chromatic adjustment required for 6500K fluorescent is quite minor.

The tristimulus values, X′, Y′, Z′, corresponding to the responses adjusted for the changed sensitivities by this form of three-component theory may be found from equation 3 as follows:

3500K fluorescent

$$\left.\begin{aligned} X' &= 1.093X - 0.275Y + 0.271Z \\ Y' &= 1.000Y \\ Z' &= 2.343Z \end{aligned}\right\} \quad (3a)$$

4500K fluorescent

$$\left.\begin{aligned} X' &= 1.046X - 0.133Y + 0.107Z \\ Y' &= 1.000Y \\ Z' &= 1.538Z \end{aligned}\right\} \quad (3b)$$

6500K fluorescent

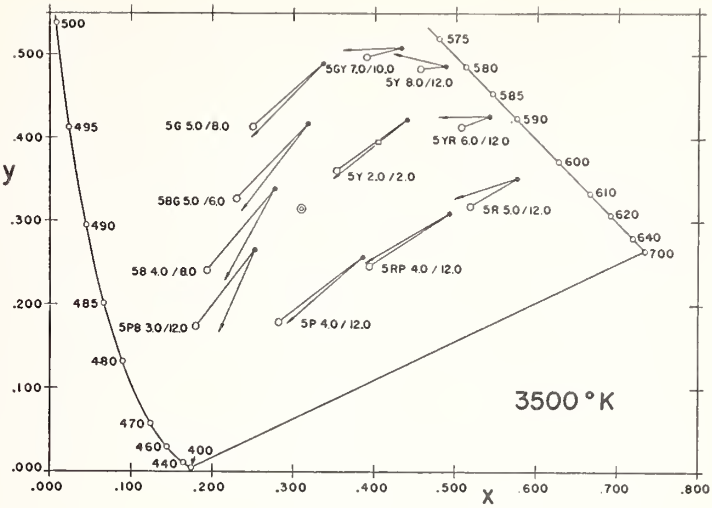
$$\left.\begin{aligned} X' &= 0.996X + 0.012Y + 0.019Z \\ Y' &= 1.000Y \\ Z' &= 1.082Z \end{aligned}\right\} \quad (3c)$$

To find the Munsell renotations of the colors corresponding to these adjusted responses it is only necessary to compute chromaticity coordinates (x′, y′) in the usual way for X′, Y′, Z′ and read the renotations by interpolation on the charts defining the ideal Munsell system (Newhall, Nickerson, Judd, 1943). We deemed it worthwhile, however, to depart from this simple theory by introducing three additive constants, $0.020X_c$, $0.020Y_c$, $0.020Z_c$ into the expressions for X′, Y′, Z′, respectively, given above. These constants were determined empirically from the results of the previous study (Helson, Judd, Warren, 1952) which showed that the simple theory predicted generally higher saturations (on average higher by 0.85 Munsell step) under source A than under source C, a prediction not borne out by the observations.

The method of calculating the hue and saturation to be expected of any sample under any source when its tristimulus values, X, Y, Z, for that source are known, according to the present formulation, is illustrated in Table VII wherein the theoretical X′, Y′, Z′ for samples in the three given sources are determined from equations 3a, 3b, and 3c modified by the additive constants.

As seen from Table VII and the discussion of the formula, the tristimulus values, X, Y, Z, of the sample to be predicted in any illumination must be known for that illumination and new tristimulus

Figure 5. Predicted departures from daylight chromaticness for eleven Munsell samples viewed under the 3500K fluorescent source. The circles indicate the chromaticity points in the (x,y)-diagram of the 1931 CIE system for the samples illuminated by CIE source C (representative of average daylight). The dots indicate the corresponding chromaticity points for the samples illuminated by the fluorescent source. The arrow heads indicate corresponding chromaticity coordinates (x′, y′) adjusted in accord wi'h modified equation 3 to take account of the chromatic adaptive state of the observer. Double circle refers to source C; square to fluorescent source.



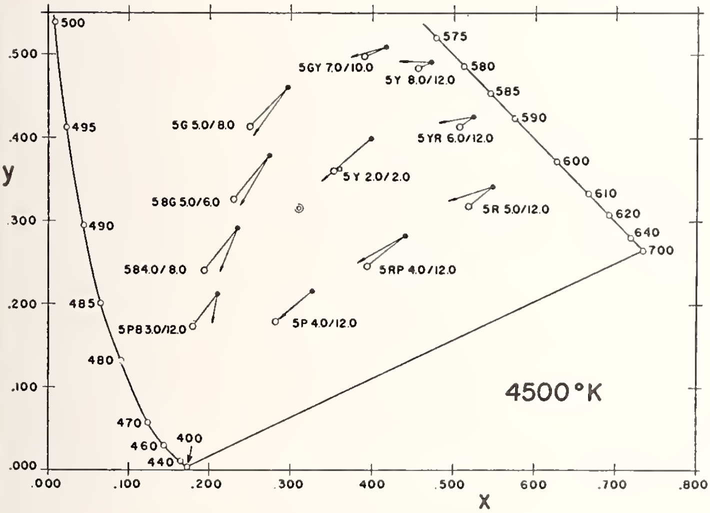
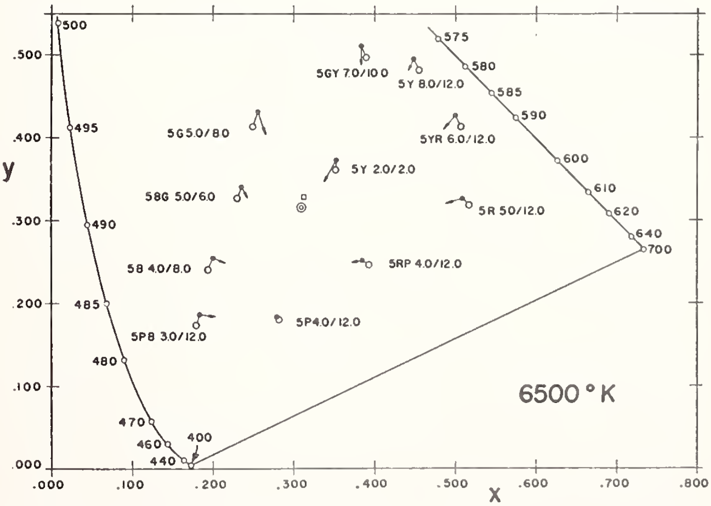


Figure 6. Predicted departures from daylight chromaticness for eleven Munsell samples viewed under the 4500K fluorescent source. Circles, dots, arrow heads, and square have same meaning as in Fig. 5

Figure 7. Predicted departures from daylight chromaticness for eleven Munsell samples viewed under the 6500K fluorescent source. Circles, dots, arrow heads, and square have same meaning as in Fig. 5



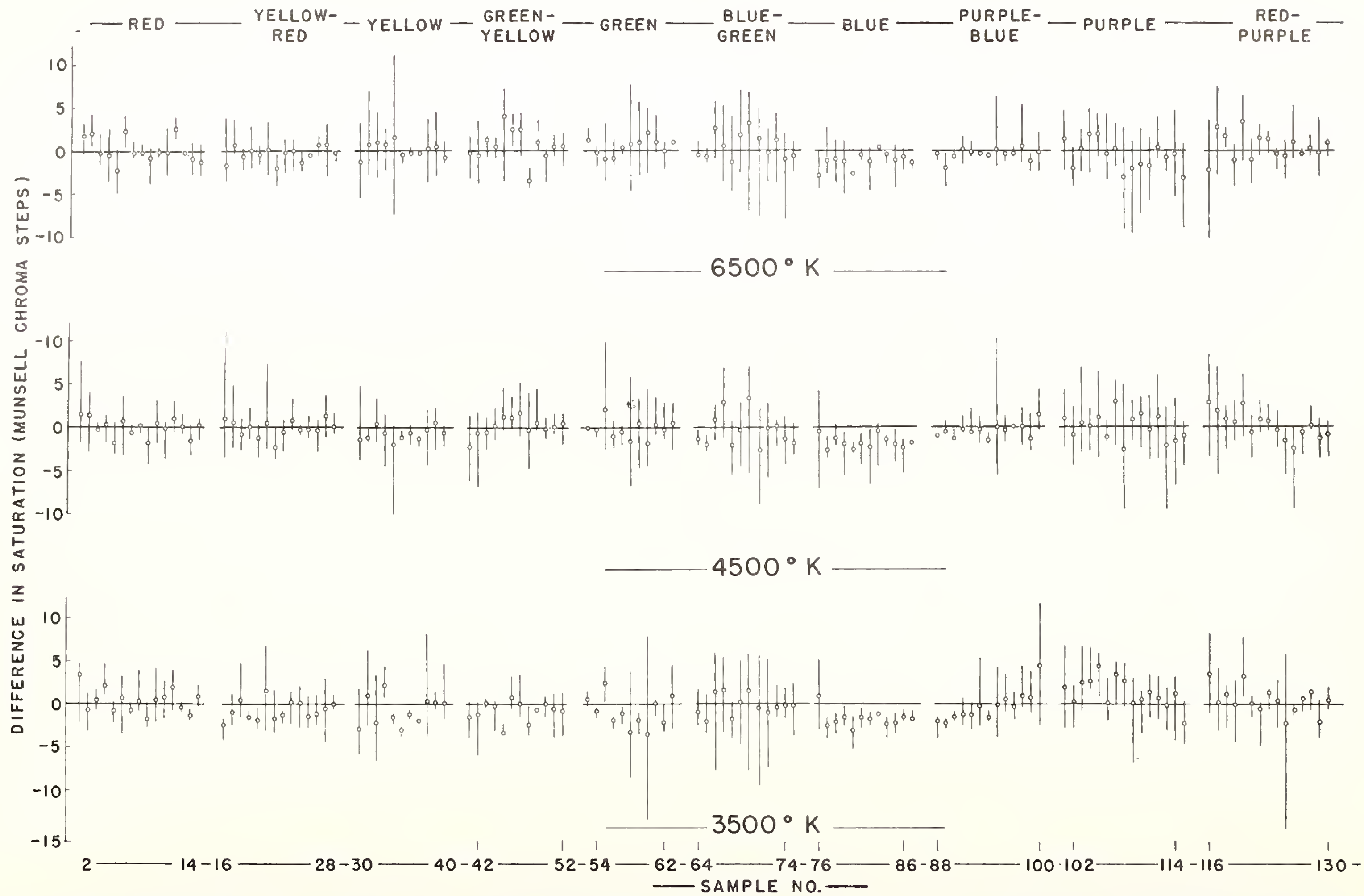


Figure 8. Departures of the experimental means from the predicted hues of the colors perceived to belong to the 130 Munsell samples illuminated by three fluorescent sources. The abscissa shows the serial numbers of the samples in the order used for Tables II, III and IV and grouped as in those tables according to Munsell hue. The circles represent the differences on the Munsell 100-hue scale between the experimentally found hues (means of 5 observers) and the hues predicted in accord with computations as in Table VII. The vertical solid lines, indicate by their length the extent of the individual-observer ranges in hue estimates, and by their positions relative to the horizontal base lines (representing predicted hues) whether the predicted hues fall within the individual-observer ranges. Note that the majority of these vertical lines intersect the base line, indicating that the predicted hues do fall within the inter-observer ranges. Note also that for the specimens in the blue hue range, there is a small, but regular and apparently significant, exception to this general rule for all three fluorescent sources.

values, X′, Y′, Z′, must be computed according to the equation indicated in Table VII to take account of the changed state of adaptation of the eye in that illumination as compared with daylight (here taken as C.I.E. source C). After X′, Y′, and Z′ have been determined, chromaticity coordinates, x′, y′, z′, are determined in the usual manner from the tristimulus values and the predicted color is found from a plot of the renotated Munsell colors in the C.I.E. color mixture diagram (Newhall, Nickerson, and Judd, 1943). Since the Munsell grid is by value level it is necessary in general to interpolate between adjacent value levels for Munsell hue and chroma. Munsell hue and chroma, so found, are the predictions for hue and saturation of the color perceived to belong to the specimen under the fluorescent source.

The method of determining the changes in hue and saturation in passing from daylight to any other type of illumination, according to the theory underlying the present study, is graphically illustrated in Figs. 5, 6, and 7 wherein are plotted 11 Munsell samples (1) according to their C.I.E. specifications in daylight, (2) in each of the three fluorescent illuminants, and (3) according to the correction factors applied to take into account the altered state of visual adaptation to the fluorescent illuminant.

The 11 samples are identified by Munsell book notation. The corresponding renotations (see Tables III, IV, and V) are as follows: 5.5R 5.0/12.8, 6.0YR 6.2/11.5, 5.5Y 7.9/12.3, 5.0Y 2.5/1.4, 4.0GY 7.0/9.5, 5.0G 4.9/8.2, 5.0BG 5.1/6.4, 6.5B 4.0/6.9, 4.0PB 3.3/9.4, 5.5P 4.1/12.6, and 5.0RP 4.3/10.7.

From the plots in Figs. 5, 6, and 7 it is seen that the corrected C.I.E. coordinates of the samples are for the most part quite different from their corresponding uncorrected coordinates, though, in general, fairly close to the daylight coordinates of the specimen. In this way, these differences are expressed wholly in terms of the C.I.E. color mixture diagram. The predicted hues and saturations can be read in Munsell terms from a plot of the renotated Munsell colors in the C.I.E. color mixture diagram (Newhall, Nickerson, and Judd, 1943).

5. Agreement Between Calculated And Observed Results

To test the adequacy of the formulas in predicting the changes in hue from daylight to the three fluorescent lights employed in this study a graph (Fig. 8) showing departures from the predictions relative to the individual-observer ranges has been prepared, and Wilcoxon's test of significance of differences between paired replicates, a non-parametric test, was used (Wilcoxon, 1949). In Fig. 8, the abscissa is the serial number of the sample in the order listed in Tables II, III, and IV. The plotted circles indicate the differences on the Munsell numerical hue scale between the estimated hues (means of five observers) and the hues computed as in Table VII. The upper ends of the vertical solid lines similarly represent the differences between the highest hue estimate among the five observers and the hues computed as in Table VII; and the lower ends refer to the lowest estimate; so the length of the line indicates the inter-observer hue range. The three horizontal base lines represent the predicted hues for the three fluorescent sources. Note the markedly greater individual-observer differences, so far unexplained, for the blue-green, purple and red-purple samples compared to most samples of other hues. Most of the occasional large ranges for these other hues are ascribable to the samples being rendered nearly gray (3.5Y 7.8/1.6 and 6.5G 7.8/2.1 for 6500K source); but other large ranges are unaccounted for (5.5P 5.0/6.7 for 4500K source, 5.0BG 5.1/6.4 for 6500K source and 6.0G 8.7/2.7 for 3500K source). Note also that the vertical lines in most cases intersect the base lines; that is, in general, the predicted hues fall within the inter-observer ranges. Furthermore the circles are scattered in more or less random fashion about the base lines; that is, there is generally no regular trend in the indicated departures of experiment from theory. An outstanding exception is formed by the blue samples. Fig. 8 shows not only that a preponderance of the hue-estimate averages in the blue region are greener than the predicted hues for all three sources, but in addition, so many of the ranges are entirely on the green side of the predicted hues that a significant difference between theory and experiment is strongly indicated.

To obtain an evaluation of the significance of such trends, the Wilcoxon non-parametric test was employed. The 130 Munsell samples were originally chosen to divide about equally among the Munsell 10 major hues given in Table V and shown separated in Tables II, III, and IV and on Fig. 8. The significance of the differences between the theoretically computed hues and the average estimates of the hues (corrected for the departure of the observers' hue estimates in Macbeth daylight from the Munsell renotations) was determined for each of the ten major Munsell hues, making 30 distinct statistical tests (Table V). As might have been expected from Fig. 8, the agreement between formula and observation is, on the whole, shown

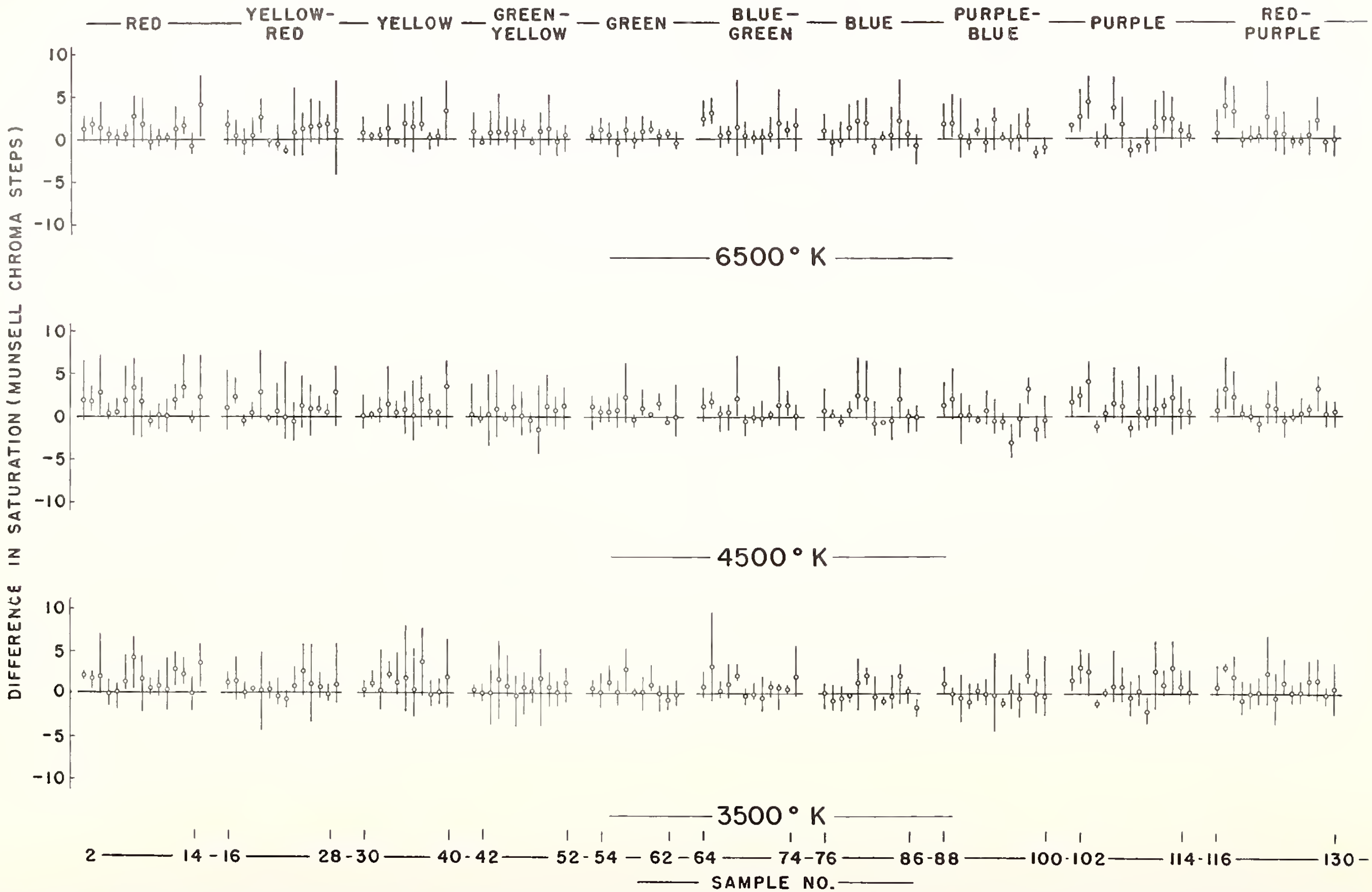


Figure 9. Departures of the experimental means from the predicted saturations of the color perceived to belong to the 130 Munsell samples illuminated by three fluorescent sources. The abscissa shows the serial numbers of the samples in the order used for Tables II, III and IV and grouped as in those tables according to Munsell hue. The circles represent the differences on the Munsell chroma scales between the experimentally found saturations (means of five observers) and the saturations predicted in accord with computations as in Table VII. The vertical solid lines indicate by their length the extent of the individual-observer ranges in saturation estimates, and by their positions relative to the horizontal base lines (representing predicted saturations) whether the predicted saturations fall within the individual-observer ranges. Note that the majority of these vertical lines intersect the base line, indicating that the predicted saturations do fall within the individual-observer ranges. Note also that the five-observer averages of estimated saturation tend to be higher than the predicted saturations by about one Munsell step for each of the three types (6500, 4500, 3500) of fluorescent lamp studied.

by the Wilcoxon test to be excellent, the *direction* of hue shift being correctly predicted in 20 out of the 30 comparisons. Only two of the ten discrepancies between observed and predicted directions of hue shift are statistically significant (blue in 6500K and 4500K). The Wilcoxon test corroborates quantitatively the qualitative indication of Fig. 8 that there is a significant discrepancy between observation and theory for the blue samples viewed under all three sources studied, but since this discrepancy is substantially the same for the 6500K fluorescent lamp causing slight chromatic-adaptation change from daylight as for the 3500K fluorescent lamp involving considerable chromatic-adaptation change, the explanation of the discrepancy should probably be looked for elsewhere than in our formulation for the effect of chromatic-adaptation change, perhaps in failure of the C.I.E. standard observer to represent the five observers actually used. Of the seven statistically significant discrepancies in Table V between formula and observation, five concern differences in the *amount* of hue shift only, the calculated shift in four cases being greater than that observed. We can therefore regard the present formulation as adequate to predict the changes in hue with change in illumination found in this study in the sense that the significant discrepancies are of the order of magnitude of the individual-observer differences.

Fig. 9 is a plot of observed saturations minus the predicted saturations against the serial number of the Munsell sample. It shows for saturation the average and the range compared to the prediction just as Fig. 8 shows these for hue. Table VI shows for samples of each of the 10 Munsell hues the average observed saturation change from daylight to fluorescent illumination and the average predicted saturation change. We find from Table VI that the formulas predict loss of saturation for almost all of the hues, but in most cases an actual, though slight, gain in saturation was observed for the three sources as compared with daylight. Since the observed gain in saturation is on the average less than one-quarter of a Munsell step it must be concluded that saturation remains practically constant with change in quality of illumination of the order investigated here. This result agrees well with that found in the previous study comparing the color rendition of incandescent-lamp light (C.I.E. source A) with daylight (C.I.E. source C) and indicating an average rise in saturation of one-tenth of a Munsell step. The predicted changes in saturation are less than a Munsell step on the average and since the observers were trained to respond to saturation differences corresponding to only one step of Munsell chroma, most of the discrepancies between formula and observation can be regarded as well within the discrimination tolerance of the observers. On the other hand, in view of the predominant number of predicted losses in saturation as compared with the observed gains, suspicion may well be directed to the empirical modifications, $0.020X_c$, $0.020Y_c$, $0.020Z_c$, introduced on the basis of the previous study of incandescent-lamp light (Helson, Judd, Warren, 1952). These modifications reduce the predicted saturations in every case, and computation for a representative group of samples indicates that on average they reduce the predicted saturations by a little less than one Munsell step. In other words, if the modifications had not been introduced the predicted saturation changes would have averaged slightly plus in agreement with the observations. It may be concluded that the present observations support the simple theory (equation 3) better than the empirically modified theory actually used in the computations (Table VII).

In view of the well-established correlation (Judd, 1940) between lightness of the perceived color and luminous directional reflectance (Y in the C.I.E. system) of the object for the source used, which forms the basis of the theoretical estimates here, it is not necessary to test the agreement of theory and observation statistically to determine goodness of fit for lightnes. The theoretical values for lightness can be seen by inspection to be in excellent agreement with the observed, the average difference (Tables II, III, and IV) being less than 40 per cent of the uncertainty (one Munsell value step) of each observation.

6. Conclusion

The rendition of object colors by a light source depends upon the spectral reflectances of the objects, the spectral distribution of the radiant energy emitted by the source, and the adaptive state of the eye. Previous studies have shown that a simple three-components theory of vision provides formulas by means of which predictions of object-color perceptions under incandescent-lamp light may be made from these determining factors. The present study employed three fluorescent sources of chromaticity intermediate to incandescent-lamp light and daylight, and included many more specimen objects (130) than the previous studies. It is shown that the average judgments of color perception by five observers are satisfactorily accounted for by the same formula approximately validated in the previous study (equation 3). This formula differs from that actually used in the present computations by omission of the empirical modification based on the previous study and inserted as a refinement of the simple theory. The present observations support the simple theory better than the supposed refinement. To obtain the predictions of hue, lightness, and saturation from equation 3 in terms of the Munsell renotation system it is necessary to find the Munsell renotation corresponding to X', Y', Z' by tri-dimensional interpolation in the charts defining the ideal Munsell system (Newhall, Nickerson, Judd, 1943).

An outstanding result of this study is that, while the averages of five individual judgments on each of the colors are satisfactorily accounted for by this formula, the individual judgments themselves show wide variation. This result was also found in the study of incandescent-lamp light. We do not know whether the disparities are ascribable chiefly to the uncertainty, admittedly rather large, inherent in the difficulty of the absolute judgments of hue, lightness, and saturation required of the observers, or whether the sources used actually produced rather widely varying adaptive effects on the different observers. We incline to the view that the individual variations arise from a combination of these, and perhaps other factors.

An important consequence of the imprecise nature of the method and of the large individual differences is that the formulas shown not to be significantly in error by the observations have still not been given the strictest tests possible. This may be seen from the many disparities between the computed and observed results in Figs. 3, 8 and 9 and in Tables II, III, and IV. We do not know whether these disparities are due to a wrong form of theory, such as failure to take account of the reflectance ratio of specimen to background which is known to infleunce color perception (Helson, 1938; Judd, 1940), or to other error sources, such as failure of the light sources to emit radiant flux of the spectral energy distributions used in the calculations, or the failure of the color specimens to have the spectral reflectances used in the calculations, or failure of the C.I.E. standard observer to represent the actual observers.

To settle these questions would seem to require a procedure involving a higher order of precision than our method of absolute judgment of hue, lightness, and saturation; for example, a procedure such as the binocular matching technic (Wright, 1934; Schouten and Ornstein, 1939; Walters, 1942; Hunt, 1950; Winch and Young, 1951; Burnham, Evans and Newhall, 1952; Brewer, 1954). This technic is also better adapted than ours to determine precisely the primaries of the three-components system giving optimum predictions, and to discover formulas, perhaps not based on the three-components theory, that will yield precise predictions for each observer rather than approximate predictions for an average group of observers.

The present study, in spite of its imperfections, has shown that a quantitative formulation designed to take account of the effects of adaptation applies not only to fairly smooth spectral energy distributions but also to such irregular energy distributions as are yielded by fluorescent lamps. This finding supports the belief that the effects of adaptation of moderate degree can be accounted for by the proper set of primaries and a quantitative theory which is applicable to all types of spectral energy distribution.

Acknowledgments

The present investigation was made possible by a grant from the I.E.S. Research Fund. We are also indebted to Miss Dorothy Nickerson of the Production and Marketing Administration of the U. S. Department of Agriculture for providing the measured fluorescent sources and the Munsell samples used in this study and for her active cooperation in all phases of the problem.

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This short account of the rationale of the ISCC-NBS method of designating colors concludes with statistics on the sources of some 7500 color names. Fifty categories of sources of those names were found to be appropriate, from flowers (which contributed 528 of the principal color names of the 7500), through dyestuffs (252, of which 108 refer to synthetic dyes), jewels (125), moods (82), mythology (46), to organizations (19, such as Academy Blue, Harvard Crimson, Princeton Orange, Red Cross and Vassar Rose). However obscure the name of a color, if a specimen is available, a precise description of it in ordinary words can be obtained by use of the ISCC-NBS method.

DESCRIPTION OF COLOR

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I. *Introduction*—Color is an aspect of the appearance of objects and lights; that is, color is part of how objects and lights look to us. Sir William Henry Perkin, whom we honor at this Perkin centennial, has showed us how to control the colors of objects; so in deference to Perkin I will confine my remarks to object colors.

Now I suppose that everyone reading this paper knows the particular aspect of appearance that is called color, but a definition is useful anyway. Color is the aspect of appearance that depends chiefly on the spectral composition of the light reaching the eye. If the light reaching the eye from the object is composed predominantly of long-wave light, the object is usually seen to have a red color; if predominantly of short-wave light, a blue color; and if predominantly of middle-wave light, a green color. This is where Perkin's discovery of the first synthetic dye comes in. By applying a dye to an object, the spectral composition of the light reaching the eye from the object can be changed; and so the color can be varied. Perkin's discovery has led to many more, and much better dyes, thus making possible many more object colors; and in this sense we may say that Perkin has also presented us with an aggravation of the problem of describing colors.

Before tackling this problem we should understand one final complexity. What we see when looking at an object depends not only on the light coming from the object. It depends also on light reaching our eyes from the surroundings, and even on light reaching our eyes before we look at the object. This is more than pedantic hair-splitting; color changes induced by the surroundings are large. Take a purple plaque, having the official color description, Bathroom Orchid; and I think that you would agree to the description, at least, the last part of it, when viewed by incandescent-lamp light with gray surroundings. But if we used vivid red-purple surroundings, the color of the plaque would no longer be correctly described as orchid, but would instead be some kind of a gray. We do not yet know how to deal precisely with the influence of surroundings on color; so I expect to deal with it by side-stepping it. All of the color descriptions that I will discuss will apply to objects viewed with gray surroundings by observers with normal color vision adapted to daylight.

II. *Systematic Color Description in its Own Terms*—Although we usually describe colors of objects by mentioning other objects such as fruits (orange, lemon, raspberry), flowers (lilac, rose, buttercup), or jewels (emerald, ruby, sapphire), it is possible to describe color in its own terms, that is, of course, psychological terms. To make perfectly clear what I mean by description in purely color terms, I propose to repeat a classic experiment which was old 30 years ago when I first became interested in color and is known as the "desert island experiment."

Imagine a cast-away on a desert island who finds a trunkful of chips of all different colors floating up to the beach on a raft. On the island he has no colorimeters, or spectrophotometers, or samples of dyes or pigments. The whole island is made of light-gray coral with little vegetation; so he does not even have the usual large array of natural objects with which to compare the color chips but he does have coral gray, sea blue, cloud white, sky blue, sunset orange, sunset red, foliage green, and a few others. After his usual breakfast of fish, having nothing better to do, he spreads the color chips out on the sand to see what he can make of them. The chips, thus in disarray, make no sense at all, but he does notice one thing.* Some of the chips are like the background, coral gray, some lighter, some darker, while the others have something special. He separates the chips into two groups. One group has black, white, and grays in it; the other group of colors all have this special property, hue, that the first group does not have. Hue is the first attribute of color that he has discovered.

He goes to work on the small group because that is simpler and discovers that they can be arranged in an orderly sequence ranging from black at the bottom to white at the top. The variable shown by this series is lightness; and this is the second of the attributes of color.

The larger group of hueful colors is more complicated. First he goes to work to divide them into groups of the different kinds of hue. There are reds, yellows, greens, and blues; but each group is still rather mixed up. There are light and dark reds, light and dark greens, and so on.

He decides that he will never get anywhere unless he confines his attention to groups of colors of the same lightness. So he picks out a group of colors each one of which differs from white about as much as it differs from black. These are the colors of middle lightness. From this group of colors he chooses two sub-groups. The bottom sub-group are all more grayish than the top sub-group. This is his first inkling of the third attribute of color called saturation, or departure from the nearest gray.

To see what can be made out of this new ordering principle, he selects groups of reds, yellows, greens, and blues of the same lightness, and finds that these groups can now be put into orderly, one-dimensional series, extending from a nearly gray color to a comparatively vivid color quite different from any gray. In each of these series the only variation is in saturation.

Maybe he can find hue series, too; so out of the top sub-group of more saturated colors of the same lightness, he picks an amazing sub-sub-group. Each one of these colors differs by the same amount from gray, and so all have identical saturations as well as lightnesses. He finds that these chips can be arranged in an orderly closed curve, and this proves that hue is a cyclic variable. If it starts from red, and goes through yellow, green, blue, and purple, it comes back to red again.

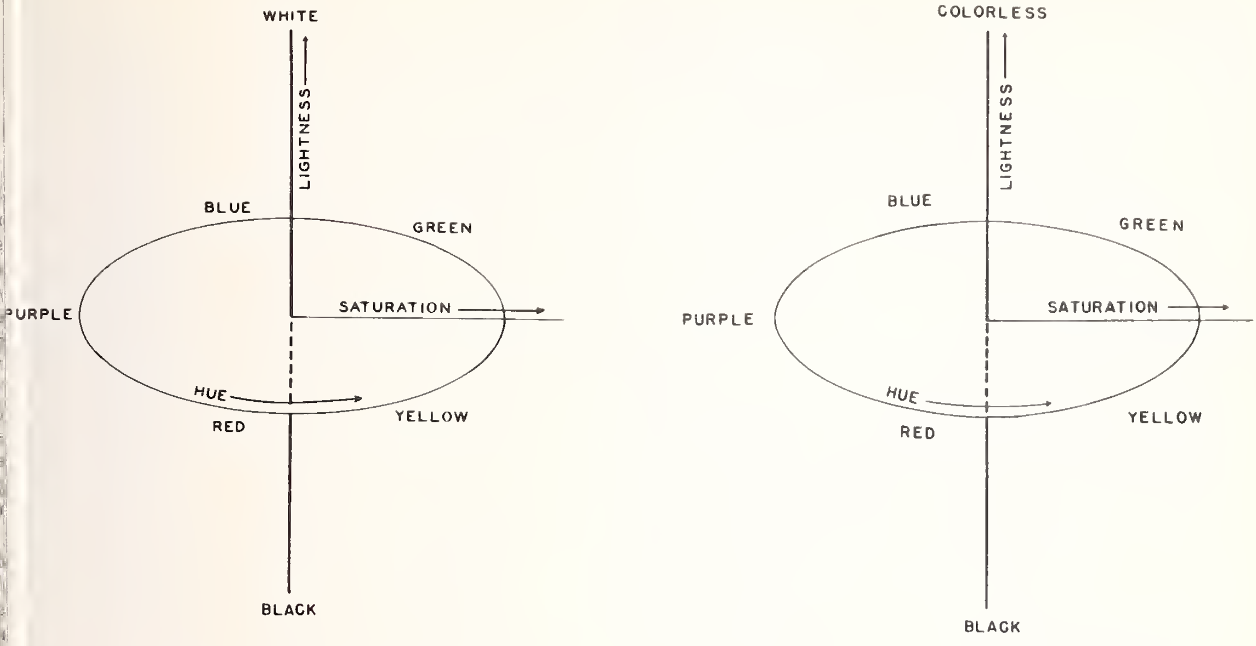
To summarize, lightness extends from black to white; saturation extends outward from each gray; and hue is cyclic.

From this point on our desert islander goes from one triumph to another. All of the saturation series for colors of equal lightness start from the same gray, and make the spokes of a wheel. The hue circuit can be added to these spokes, and for colors of middle lightness, a completely orderly classification has been achieved. The same treatment can be given to colors of any constant lightness.

Finally if attention be confined to the colors of any one hue, and its opposite, an orderly classification by lightness and saturation is possible.

By using the ideas known as hue, lightness, and satura-

*It has not been feasible to reproduce in color the ten demonstration charts which accompanied the presentation of this paper; but certain of the color plates published in "The Science of Color" (Committee on Colorimetry, Optical Society of America, New York: Thomas Y. Crowell Co., 1953) illustrate the same points. The reader may refer to Plates 8, 10, 13, 1[illegible] 17, 18, and 16 of "The Science of Color."



COLOR SOLID FOR OPAQUE SURFACES

COLOR SOLID FOR TRANSPARENT VOLUMES

Figure 1

Color Solids.

on, color may be described in its own terms. And simply by tempts to classify colors on an appearance basis, without ny measurement, we have shown that no two-dimensional assification can succeed in being orderly; we come out with succession of orderly two-dimensional classifications which ombine to form an orderly three-dimensional, or space, repre- ntation of colors. Such an orderly space diagram is called e color solid; its dimensions for object colors are hue, light- ess, and saturation as shown in Figure 1. Lightness for rface colors extends from black to white; for colors of trans- rent volumes it extends from black to perfectly clear or lorless, sometimes called "water-white." If I had not prom- ed not to discuss description of the colors of light-sources I ould point out that the vertical dimension of the light-source lor solid is brightness varying from invisible to dazzling, stead of lightness varying from black to white. But since I omised to discuss only object colors, I cannot tell you this.

III. *The ISCC-NBS Method of Designating Colors*—The ter-Society Color Council was organized in 1931 and now nsists of delegates from more than 20 national societies in- rested in color, and in addition has about 300 individual embers. The problem of describing color in an orderly and ientific way came to us from the US Pharmacopoeia and e National Formulary of the American Pharmaceutical Assn., o books describing the properties of drugs and chemicals ed in medicine, but it is easy to see that nearly all of the ember bodies (American Association of Textile Chemists and olorists, American Ceramic Society, American Institute of rchitects, American Institute of Decorators, American ychological Association, Illuminating Engineering Society, ptical Society of America, Color Association of the United ates, and so on) are concerned with this problem. A operative attack on this problem was organized, and der the chairmanship of the late Dr. I. H. Godlove, e ground work for a solution was laid. The basic idea is to divide up the color solid into two or three hundred pockets and assign to each pocket a designation, indicating the approximate hue, lightness, and saturation. Figure 2 shows the pockets tentatively chosen for colors of purple hue; other hue names chosen were red, orange, yellow, green, blue, violet, purple, and to these have been added the hybrid names pink, brown, and olive. Lightness is indicated by the terms white, light, dark, and black. Saturation is indicated by the terms, grayish, moderate, strong, and vivid. Some abbreviations have been introduced such as pale for "light, grayish," deep for "dark, strong," and the adverb "very" added to give designations for the 267 pockets finally chosen. These designations are combinations of 2 to 4 of what we intended should be pure color terms, such as light red, or dark grayish yellowish brown. In deference to common usage we have included a few slightly "impure color terms," two fruits (orange, olive) and two flowers (pink, violet).

The definition of the pockets to which the designations apply has been set up in terms of the Munsell color system[1, 2] because that color system was intended to show, and has succeeded to a considerable degree, in showing in daylight to an observer of normal color vision, adapted to daylight, uniform scales of hue, lightness, and saturation. Figure 3 shows how the pockets are defined, and is called a color-name chart. The first such chart was published in National Bureau of Standards Research Paper, RP1239,[3] which gives a preliminary choice of these boundaries, now somewhat revised, as in Figure 3. Note that for each of the color designations the boundaries are given in terms of Munsell chroma (Munsell correlate for saturation) and Munsell value (Munsell correlate for lightness). Note also that the color term, brown, refers only to the darker and more grayish colors of orange hue; it is thus not purely a hue name, and for this reason I referred to it as a hybrid name denoting certain ranges of lightness and saturation as well as hue, as do also olive and pink.

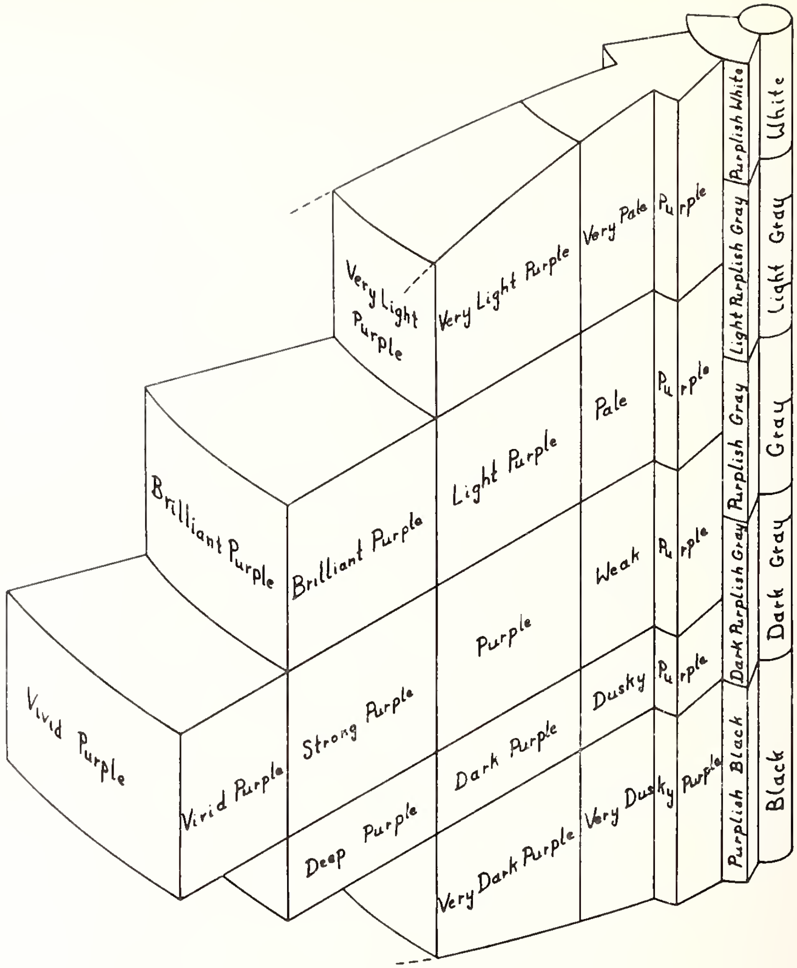


Figure 2

Division of the Purple Section of the Color Solid.

The usual way to find the ISCC-NBS color designation of a specimen is to compare it to the color scales of the Munsell Book of Color.[4] The pocket edition is convenient for obtaining the Munsell notation (hue value/chroma) of a specimen. Figures 4 and 5 show how this is done for a powdered-drug specimen. The powder is put in a holder under a cover glass (Figure 4) and compared with charts of adjacent hue from the pocket edition behind a gray mask with three holes in it (Figure 5); the central one for the specimen, the outer two for the Munsell chips mounted on the charts. Figure 6 shows how this comparison is made for a crude drug, in this case a fragment of leaf held by tweezers. Figures 7 and 8 show how this is done for a solution. The solution is put into a flat-bottomed tube (Figure 7) and a white card is viewed through the solution by looking down from the top (Figure 8), the Munsell chips being viewed through the two black-surrounded holes at the sides.

The colors of microscopic structures are compared with the Munsell color scales by means of a comparison microscope, or by a camera lucida, bringing an unmagnified image of the Munsell chip in juxtaposition with the magnified image of the structure; see Figure 9. The ISCC-NBS designation of the color is then found by turning to the name chart of the corresponding Munsell hue, and reading the designation co responding to the Munsell value and chroma found for th specimen.

These are the usual ways to find the ISCC-NBS colo designation of a specimen, but the most precise way is t measure the reflectance or transmittance of the specimen a function of wave-length by means of a spectrophotomete compute its tristimulus values, X, Y, Z, on the international recognized CIE color system, read the Munsell renotation b interpolation from the diagrams defining the ideal Munse system in the final report of the OSA Subcommittee on th Spacing of the Munsell Colors,[2] and then get the color designa tion from the name charts. This longer method is sometim used for a specimen for which it is particularly important know that it has some one color designation, and not a neigh boring designation.

The revised name charts are given in NBS Circular 55 The ISCC-NBS Method of Designating Colors and a Dictio ary of Color Names, by K. L. Kelly and myself, which h just appeared in its second printing. This 150-page volum may be obtained from the US Government Printing Offic postpaid, for the modest sum of $2.00. It contains also alphabetical list of about 7,500 color names used for chi

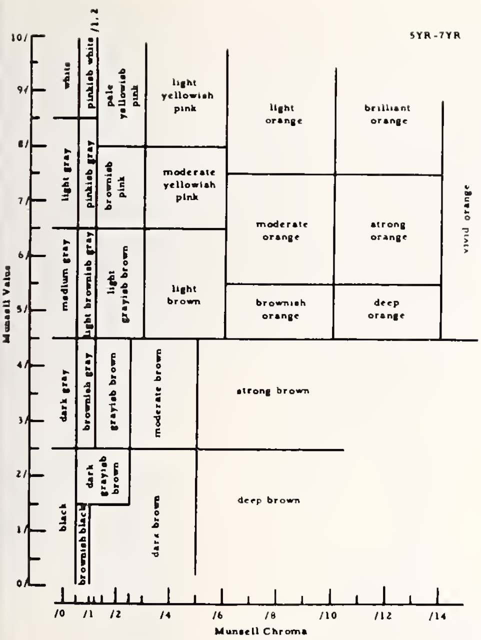


Figure 3

Color Name Chart.

Figure 4

Powder in Holder for Determination of Color Designation.

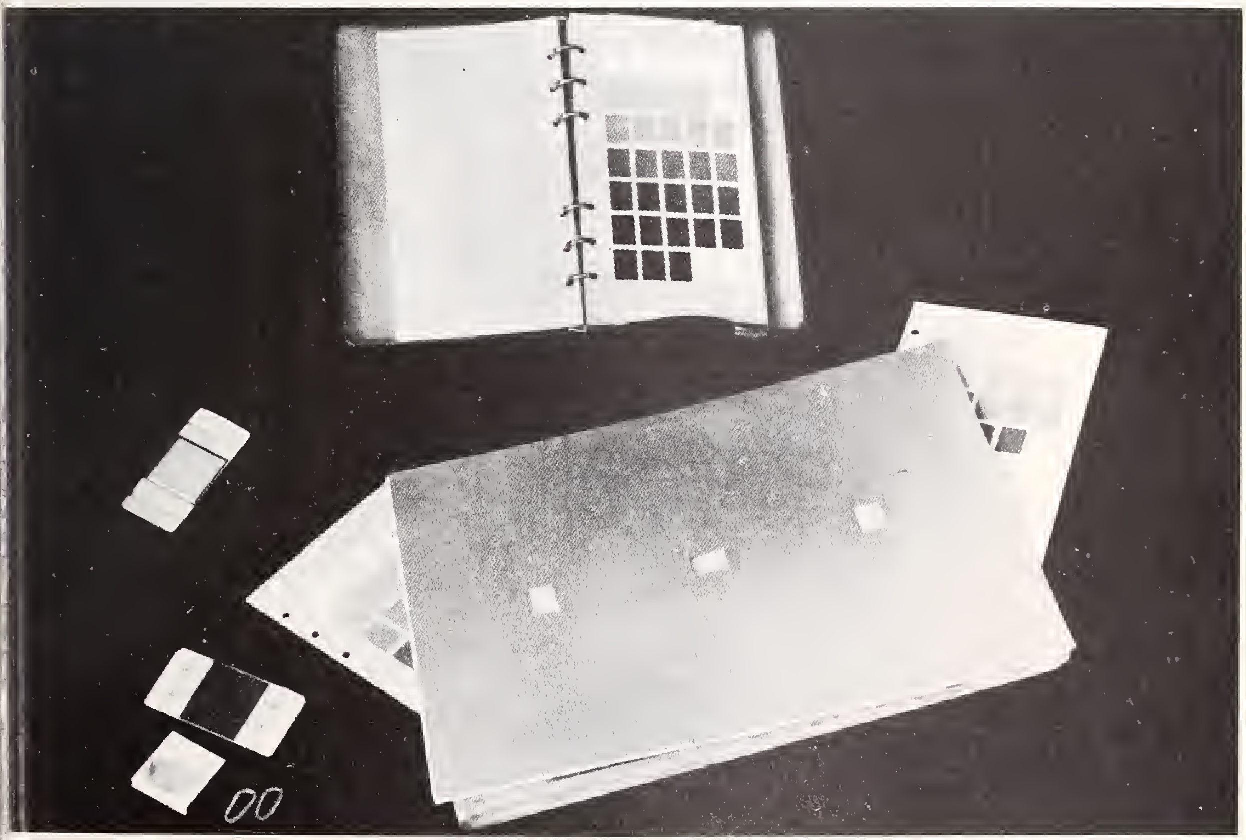


Figure 5

Comparison of Powder with Charts of Adjacent Hue from the Munsell Book of Color.

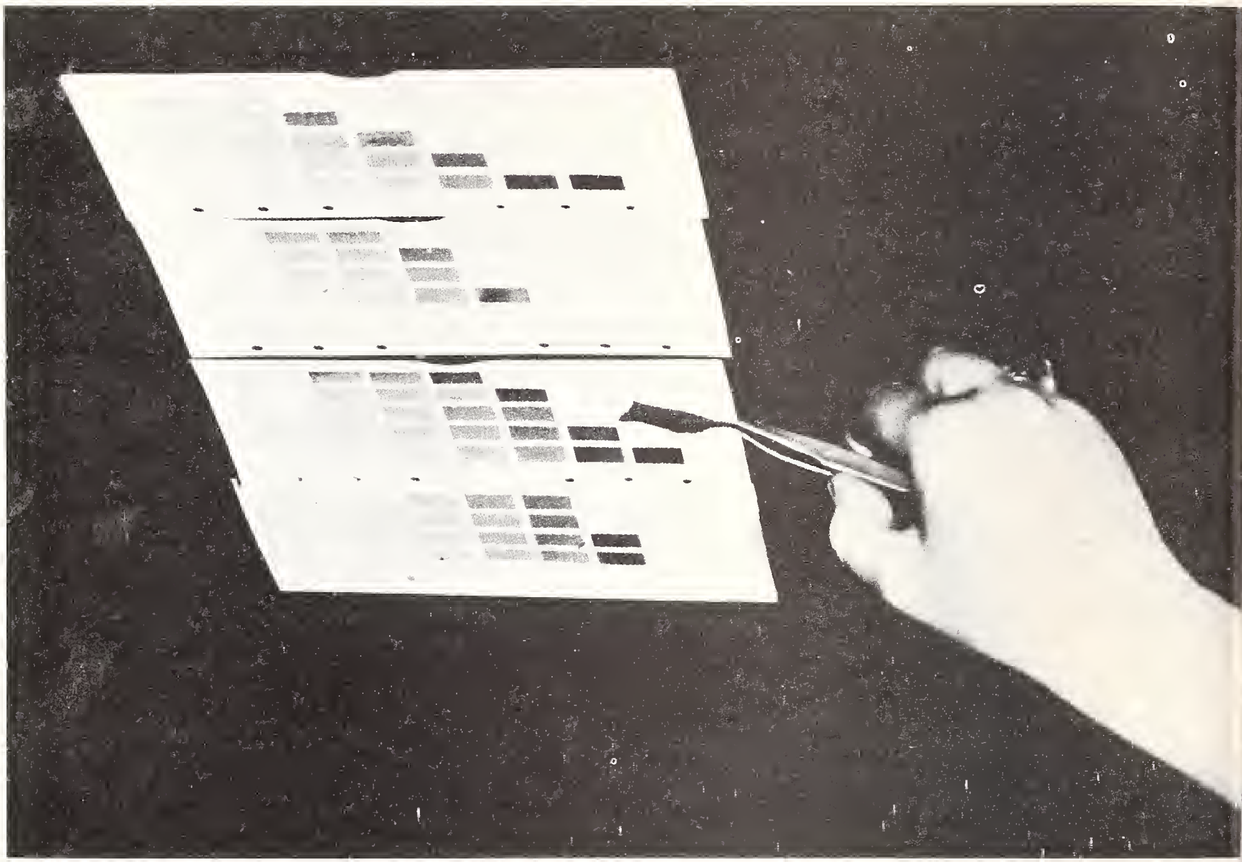


Figure 6

Comparison of a Leaf with Charts from the Munsell Book of Color.

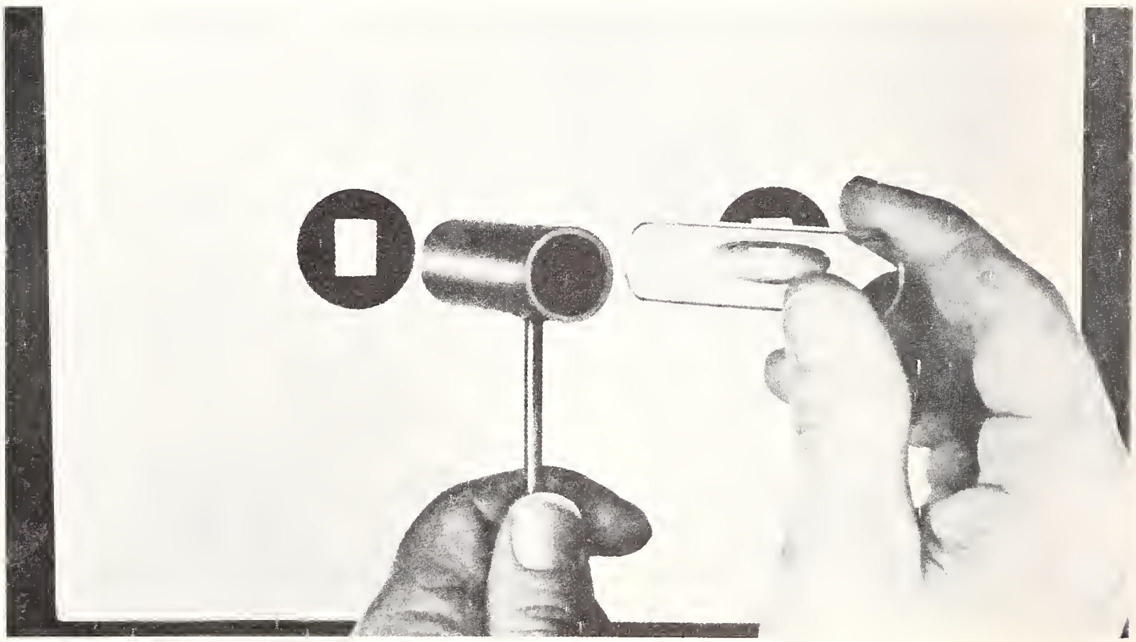


Figure 7

Equipment for Use in Determination of Color Designations of a Liquid.

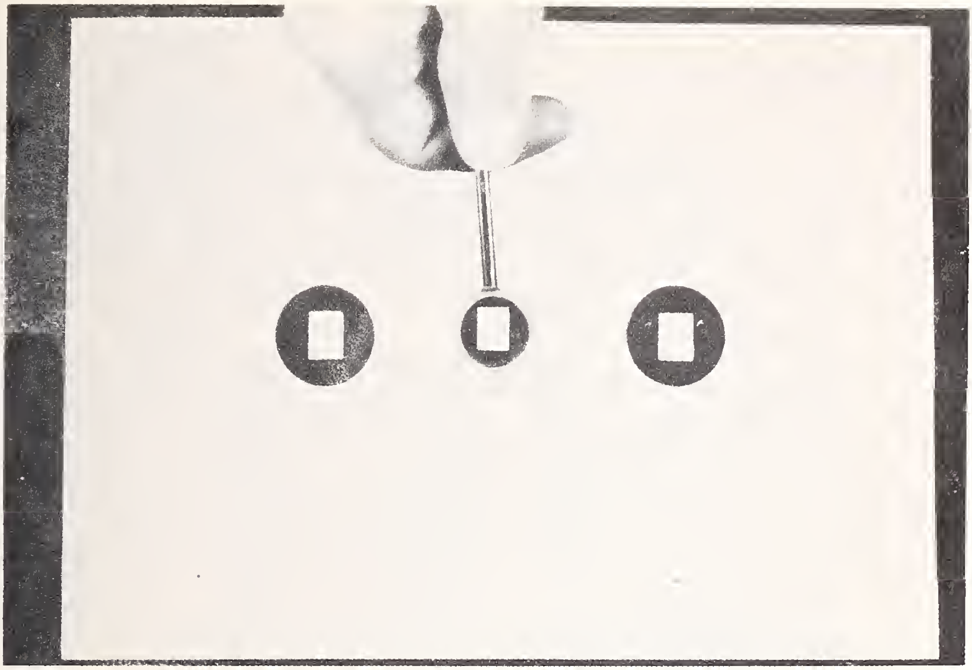


Figure 8

Comparison of a Liquid with Charts from the Munsell Book of Color.

14 collections of color chips, including the well-known aerz and Paul Dictionary, Ridgway, Plochère, Standard olor Card of America, and the Taylor, Knoche, Granville escriptive Color Names Dictionary. Table I shows the kind information given in this alphabetical list. Exact synonyms e shown indented after the name; note 11 exact synonyms r Acacia. The second column indicates the source of the me and its defining color chip; M for Maerz & Paul, R for dgway, P for Plochère, and so on. The third column shows abbreviation of the ISCC-NBS designation of the color of e chip defining the name: s.V for strong violet, m.YG for oderate yellow-green, m.yBr for moderate yellowish brown, d so on. These designations were obtained by the methods dicated above by various volunteers from the Inter-Society olor Council. The third column also identifies by number the CC-NBS color designation so that the reader may easily rn to a second listing of all of the color names, this time on e basis of color, and find there all other color names of the me designation.

Figure 10 shows all of the color names used for chips found correspond to the ISCC-NBS color designation: *Pale Blue.* om Maerz and Paul come such names as Alice Blue, Baby ue, and Persian Blue. From Plochère: Cuddle Blue, Iceberg reen and Zephyr Blue. From Ridgway: Light Dull Glaucous-ue and Light Violet-Plumbeous. From the Standard Color ard of America: Blue Flower and Sky Blue. And from Taylor, noche and Granville: Light Aqua, Powder Blue, and Wedg-ood Blue. All of these are pale blues. This second listing ves synonyms and near-synonyms for each of the 7,500 color ames listed alphabetically in the dictionary proper.

By means of these two tables, we of the Inter-Society olor Council believe that we have introduced a degree of rder into the identification of colors by names. We have done by giving to each color name a convenient designation dicating in ordinary language the approximate hue, light-ess, and saturation of the color that will be perceived by an bserver of normal color vision adapted to daylight when he looks at the corresponding color chip itself illuminated by daylight.

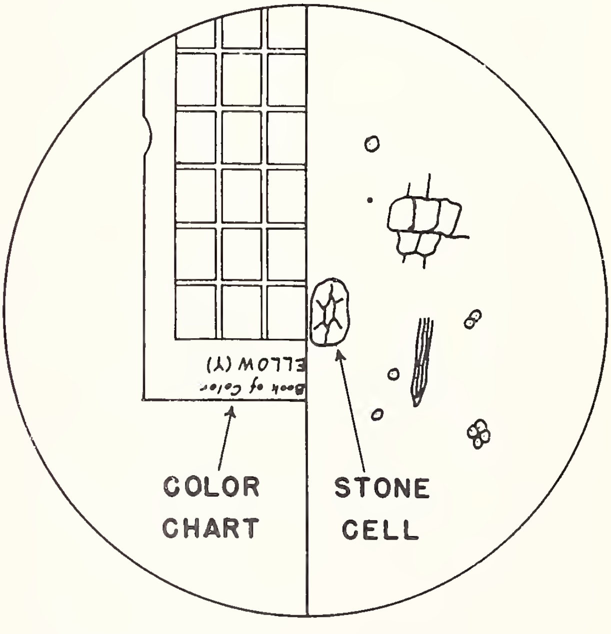


Figure 9

Schematic Representation of Method Used with Microscopic Samples.

IV. *Sources of Color Names*--It is easy to trace by a study of this Dictionary of Color Names, the influence of Perkins' discovery of mauve on color description. Mr. Kelly and I have counted up all of the color names out of the 7,500 listed that

TABLE I

Excerpts from Dictionary of Color Names

Color name	Source	ISCC-NBS color designation with serial number from section 14
Abbey	M	s.V 207
Absinthe (Green) Foliage Green	M	m.YG 120
Absinthe Green	R	m.YG 120
Absinthe Yellow	M	gy.gY 105
Absinthe Yellow	P	m.Y 87
Abstract Blue	P	v.p.B 184
Abyss	P	p.G 149
Acacia Gaude, Luteous, Olde, Oold, Wald, Wau, Weld, Wield, Woald, Wod, Wold	M	1.gY 101, m.gY 102
Acacia	P	gy.G 150
Acacia Gray	P	gGy 155
Acacia Green	P	gGy 155
Academy Blue	M	m.gB 173
Acajou (same as Laurel Oak)	M	m.rBr 43
Acajou Red	R	gy.R 19
Acanthe Butternut, Siberian Brown	M	m.01Br 95
Acetin Blue	R	gy.pB 204
Acier (same as Grey Drab)	M	1.gy.01 109
Ackermann's Green	M	d.yG 137
Ackermann's Green	R	m. G 145
Aconite Violet	H	d. V 212
Aconite Violet Datura	M	m.P 223
Aconite Violet	R	m.P 223
Acorn Meadowlark	M	gy.yBr 80
Adamia (same as Eminence)	M	d.P 224
Adelaide (same as Prunella)	M	deep V 208
Aden	M	1.G 144
Aden	P	1.G 144
Admilar Logwood	M	blackish P 230, pBlack 235
Admiral	P	d.gy.P 229
Adobe	M	m.yBr 77
Adobe Brown Cinnamon Brown	T	m.yBr 77

refer to dyestuffs. We found 252. A slight majority (144) of these come from non-synthetic dyes (Bastard Saffron, Cochineal, Indigo, Madder, Tanbark, and so on). But 108 refer to synthetic dyes (Acetin Blue, Anthracene Green, Naphthalene Yellow, and so on). So I think it is fair to say that Perkin's discovery of mauve is responsible for the addition of many more than 100 color names to the English language. I say

185. PALE BLUE

Maerz and Paul

Alice Blue	35G5	Light Blue 6	34F2
Aquamarine	35I3	Lucky Stone	34G6
Baby Blue	35E2	Myosotis Blue	36G4
Bambino	36G2	Old Blue	36F3
Bleu Passé	36F3	Persian Blue	33C4
Burn Blue	34B4	Pompeian Blue	36E4
Celestial Blue	35D3	Poudre	36H2
Columbia Blue	35D6	Poudre Blue	36H2
Endive Blue	43B5	Powder Blue	36H2
Forget-me-not [Blue]	36G4	Russian Blue	35D4
Iris	43B5	Sistine	35H5
Starlight	34E6		

Plochere

Blue Horizon	766 Bgg 6–f	Prudence	911 Gb 6–g
Blue Mist	853 Gbb 5–e	Retreat	668 B 6–d
Cuddle Blue	767 Bgg 6–g	Rio Blue	757 Bgg 5–
Debonair	902 Gb 5–f	Russian Blue	813 BG 6–e
Dreamy Blue	909 Gb 6–e	Saga Blue	708 Bg 5–d
Dusky Blue	709 Bg 5–e	Shadow Blue	862 Gbb 6–
Fall Blue	660 B 5–d	Sleepy Hollow	910 Gb 6–f
Gray Mist	669 B 6–e	Smoke	716 Bg 6–d
Holland Blue	758 Bgg 5–f	Smoke Ring	717 Bg 6–e
Horizon	861 Gbb 6–e	Static	806 BG 5–f
Iceberg Green	957 G 6–e	Sterling Blue	718 Bg 6–f
Magic Moon	854 Gbb 5–f	Sublime	652 B 4–d
Melodious	661 B 5–e	Thames River	814 BG 6–f
Moonlit Water	701 Bg 4–e	Translucent Blue	805 BG 5–e
Platonic	765 Bgg 6–e	Zephyr Blue	710 Bg 5–f

Ridgway

Alice Blue	XXXIV 45''b	Pale Amparo Blue	IX 51f
Cadet Gray	XLII 45'''b	Pale Cadet Blue	XXI 49'd
Clear Green-Blue Gray	XLVIII 45''''d	Pale Dull Glaucous-Blue	XLII 41'''f
Dutch Blue	XLIII 49'''b	Pale Forget-me-not Blue	XXII 51*f
French Gray	LII 49''''f	Pale Green-Blue Gray	XLVIII 45'''
Lt. Alice Blue	XXXIV 45''d	Pale Mazarine Blue	IX 49f
Lt. Dull Glaucous-Blue	XLII 41'''d	Pale Medici Blue	XLVIII 41'''
Lt. King's Blue	XXII 47*d	Pale Payne's Gray	XLIX 49''''f
Lt. Neropalin Blue	XXII 49*d	Pale Violet-Plumbeous	XLIX 53''''f
Lt. Payne's Gray	XLIX 49''''d	Parula Blue	XLII 45'''
Lt. Sky Blue	XX 47'f	Persian Blue	XX 45'f
Lt. Violet-Plumbeous	XLIX 53''''d	Plumbeous	LII 49''''b
Russian Blue	XLII 45'''d		

Taylor, Knoche, Granville

Aqua Gray gm	19 fe	Lt. Aqua gm	18
Cadet Gray m	13 ge	Lt. Aqua Blue gm	16
Chalk Blue gm	16 ec	Lt. Gray Blue gm	15
Dawn Blue gm	15 dc	Mist Blue gm	15
Dusk m	13 fe	Pastel Blue gm	14
Dusty Aqua gm	18 ge	Pearl Gray gm	13
Dusty Aqua Blue gm	17 ge	Pewter m	13
Dusty Blue gm	15 ge	Powder Blue gm	14
Haze Blue g	13 ec	Sky Blue gm	15
Horizon Blue gm	15 gc	Smoke m	13
Wedgwood Blue gm	14 gc		

Textile Color Card Association

Blue Flower	70011	Old Blue	701
Dustblu	70195	Sistine	70C
Sky Blue	70010		

Other Sources

Caeruleo-Glaucus	B	Gray Blue	S
Caeruleo-Griseus	B	Griseo-Caeruleus	B
Caeruleus	B	Griseo-Lazulinus	B
Caesius	B	Hull Gray (Lt. Gray) (MA)	F 26
Chalky Blue	S	Lavendulo-Griseus	B
Dull Blue	A	Lineus	B
Dull Greenish Blue	A	Pale Blue	RC
Dull Reddish Blue	A	Plumbeus	B
Glauco-Griseus	B	Sublazulinus	B
Ultramarine	S		

Figure 10

Color Names Corresponding to the ISCC-NBS Color Designati Pale Blue.

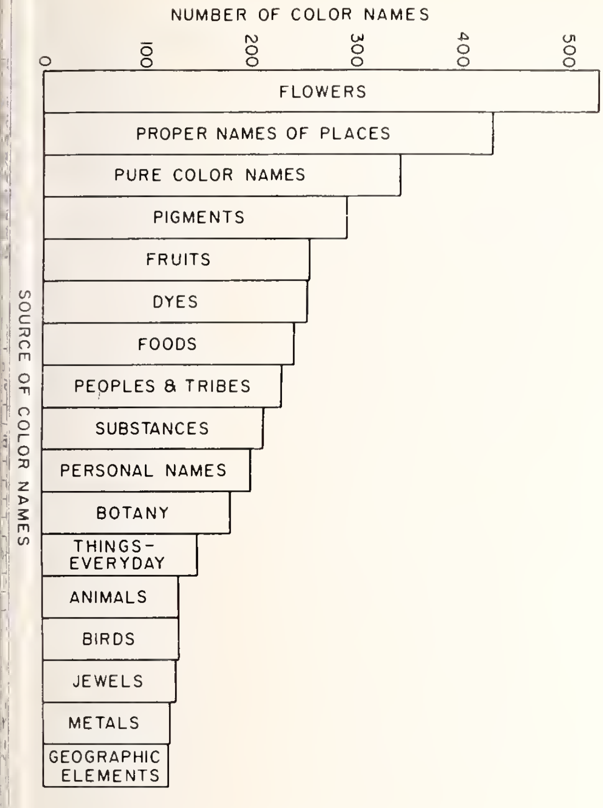


Figure 11

ome Categories of Sources of Color Names with the Number of Names in Each.

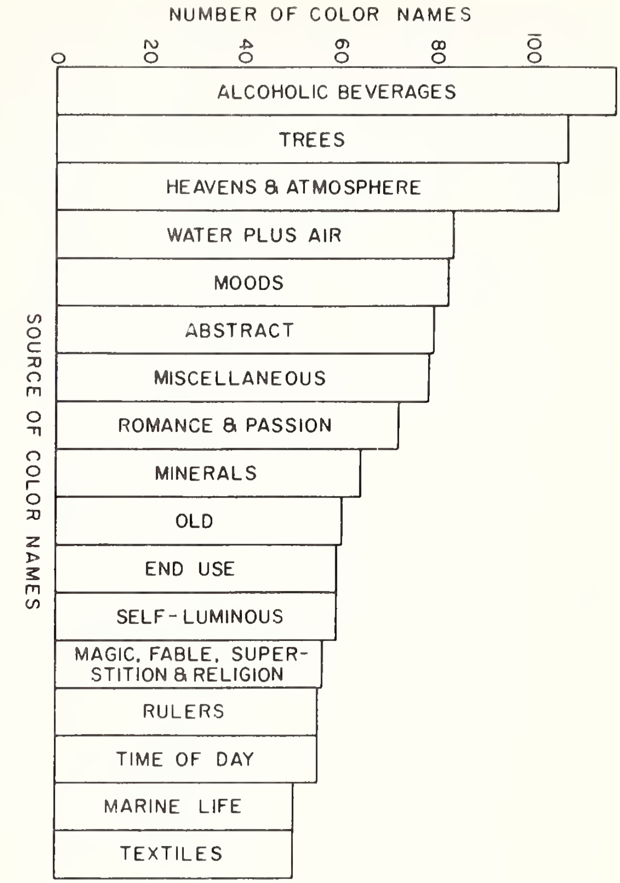


Figure 12

Further Categories of Color-Name Sources.

iany more because NBS Circular 553 contains but a small action of all color names. It includes simply the color names iat have been found to be important enough to be defined by measured specimen of the color intended.

Once having started this minor study into the sources of olor names, we found it hard to stop. Mr. Kelly and I have lassified the principal color names of the 7,500 listed (minor ariants being omitted). The classification was according to ource by means of categories invented for this purpose as we ent along. I hope that you will be as interested and intrigued s we were to discover that, so great is the diversity of source, 0 categories are needed. These categories indicate not only e sources of coloring materials, and industries founded on olor, but also what people have expected color to do for em. Figure 11 shows some of these 50 categories arranged in rder of the number of color names put into each.

1. *Flowers* (528), from Ageratum and Amarylis down to Vistaria and Zinnia. This is the most copious source of color ames.

2. *Proper Names of Places* (427), from Antwerp Brown nd Brittany Blue down to Paris Mud and Zanzibar (a dark rayish reddish brown).

3. *Pure color Names* (340), such as Black, Blue, Red, Vhite, Yellow, and combinations like Deep Dull Yellow Green, to say nothing of Latin equivalents like Subpurpureo-riseus (purplish gray). We put all names involving *pink* in is category, but we don't know which came first, the color, ink, or the flower, pink. In any case horticulturists can be roud of their great influence on color names.

4. *Pigments* (290) such as Bister, Chrome Green, and Umber.

5. *Fruits* (254) from Apricot and Banana to Prune and Tangerine.

6. *Dyes* (252), already discussed.

7. *Foods* (239) from Bran and Brown Sugar to Suet and Yolk Yellow.

8. *Peoples and Tribes* (221) from Chinese Red, Dutch Blue, and English Vermilion to Tyrian Purple and Zuni Brown.

9. *Substances* (214) from Amber and Asphalt through Ivory, to Slate and Sulphur Yellow.

10. *Personal Names* (200) such as Alice Blue, Bismarck Brown, Botticelli (a deep reddish purple), Byron, Cellini, Cleopatra (a vivid blue), Du Barry Blue, Eve Green (maybe the color she wore), Isabella Color (a dark grayish yellow), Kelly Green, Marco Polo, Marie Antoinette, Monet Blue, Napoleon Blue, Pinocchio, Pygmalion, Raphael, Rembrandt, Robinhood Green, Salome Pink, Sappho, Sinbad, Titian, Vandyke Brown, Victoria Violet, and Watteau. To have a color become known by your name is fame indeed.

11. *Botany* (183), from Acacia and Amaranth to Zedoary Wash.

12. *Everyday Things* (149), such as Brick Red, Paper White, and Sunburn.

13. *Animals* (133) from Beaver and Buff (for Buffalo), to Chamois, Fawn, Kitten's Ear, and Mouse Grey.

14. *Birds* (133) from Bluejay to Parrot Green and Turkey Red.

15. *Jewels* (125) from Amethyst to Topaz.

16. *Metals* (123) from Brass to Zinc.

17. *Geographic elements* (121) from Beach Tan through Glacier Blue to Surf Green.

Figure 12 shows a continuation of the categories. Note that the number scale is more open, being about 100 maximum instead of 500.

18. *Alcoholic Beverages* (117) from Absinthe Green through Champagne and Claret to Sauterne.

19. *Trees* (107) from Blue Spruce to Red Mahogany and Willow Green.

20. *Heavens and Atmosphere* (105) such as Air Blue, Aurora Yellow, and Zephyr Green.

21. *Water Plus Air* (83) such as Cloud, Fog, Froth, Smog, Smoke, and Spray.

22. *Moods* (82), such as Caprice Lavender, Cheerful Yellow, Deep Reverie (a dark purplish gray), Delight (a very pale purple), Intimate Mood (a light grayish purplish red), Joy (a light yellowish green), Languid Lavender, Lilting Green, Peace (a yellowish white), Quiet (greenish white), Repose (a grayish yellow-green), Vanity (a moderate yellowish pink), and Whimsical (a light purple). It is plain that color is supposed to influence mood.

23. *Abstract* (79) from Fortune, Freedom, Illusion, and Inspiration, to Triumph Blue and Wan Blue.

24. *Miscellaneous* (78), that is, 78 failures to classify source. Some of these simply failed to fit into any of the 49 categories like Caravan, Doubloon, Seered Green, and Silver Lining; others stumped us completely, like Adamia, Aloma, Jaffi, Kabistan, Kris Kilim, Nikko, and Persenche. Maybe some were invented out of thin air by a sales promoter, and maybe some are just misprints that were copied.

25. *Romance and Passion* (72). This list is almost worth giving in its entirety, but we sample it freely: Aphrodite, Bewitch, Blue Tease, Blush, Charm, Coquette, Crystals of Venus, Cupid Pink, Dream Stuff, Enchantress, Flirtation, Folly, Garter Blue, Golden Rapture, Irresistible, Love Light, Surrender, Temptation, Vamp, and Venus.

26. *Minerals* (64) from Agate to Travertine.

27. *Old* (60) such as Antique Brown, Colonial, and Renaissance.

28. *End Use* (59) from Battleship Gray and Black Boottopping to Government Wall Green and National School Bus Chrome.

29. *Self-luminous* (59) such as Blaze, Ember, Fire Red, Plenty Bright (a strong bluish green), and Sunburst.

30. *Magic, Fable, Superstition and Religion* (56) from Amulet, Angel Red, and Atonement through Elf, Goblin Scarlet and Magic Moon to Pearly Gates, Sorcerer, and Tabu.

31. *Rulers* (55) such as Baronet, Mandarin Red, Mogul, Nabob, Royal Blue, Sheik and Tzarine.

32. *Time of Day* (55) from Blue Hour, Cold Morn, and Dusk, to Midnight Blue, Night Horizon, and Twilight Blue.

33. *Marine Life* (50) from Coral to Lobster Red and Shrimp Pink.

34. Textiles (50) undyed, of course, from Beige and Ecru to Khaki, Pongee, and Russet.

Figure 13 shows the remaining categories for sources of color names. The number scale has again been enlarged, the maximum being about 50 instead of 100 or 500.

35. *Mythology* (46) from Ambrosia, Atlantis and Bacchus, through Daphne Pink, Dryad, Naiad, and Nectar to Nymph Green, Ondine, Pan, Psyche, and Titania.

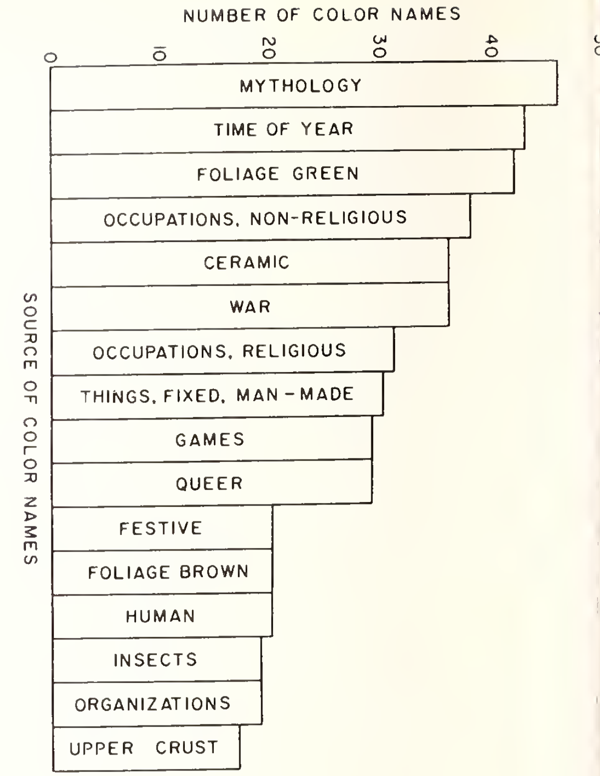


Figure 13

Further Categories of Color-Name Sources.

36. *Time of Year* (43) such as April Sky, Fall Yellov June Green, and Spring.

37. *Foliage Green* (42). These 42 names are just differe ways of saying practically the same thing: Greenery, Gra Green, Foliage, Herbaceus, and so on.

38. *Occupations,* Non-Religious (38), such as Ballerin Brigand, Buccaneer, Cadet, Cowboy, Fakir, Geisha, Peasar Pirate, Sailor, Troubador Red, Vagabond and Viking. Tl more exotic occupations seem to have a better chance to ha a color named for them than the more prosaic. Many of the could have been included under *Romance.*

39. *Ceramics* (36) from Celadon and Delft Blue to Ti Red and Wedgewood Blue.

40. *War* (36), from Army Brown and Navy Blue to Poi and Tommy Red.

41. *Occupations, Religious* (31), such as Bishop, Cardin Friar, Lama, Nuncio and Prelate. In ancient times, particular before Perkin's discovery of mauve, only persons of high ra could afford garments dyed to have some of these colors.

42. *Things, Fixed and Man Made* (30) such as Barn Re Opera Pink, and Palace Blue.

43. *Games* (29) such as Casino Pink, Chukker Brow Golf Green, Hunter Green, Hockey, Jockey, Polo Tan, Sl and Tennis.

44. *Queer* (29) such as Best Effort, Black Knight (a da bluish gray), Conclude, Dream Cream, Nil (a white), Pi Y Rose Breath and Rubaiyat (yes, it's a dark red).

45. *Festive* (20) from Carnival Red to Merry Green.

46. *Foliage Brown* (20), twenty different ways to say color of a dead leaf: Dried Leaf, Withered Leaf, Feuille rte, Filemot, Phyllamort, and so on.

47. *Human* (20) from Baby Blue, Bambino and Flesh, Green Eyes, Hair Brown, Nude, and Pickaninny.

48. *Insects* (19) such as Beetle, Fire Fly, Flea, Grass-pper, Moth, and Puce.

49. *Organizations* (19) such as Academy Blue, Big 4 low, Cambridge Blue, Devon Brown, Eton Blue, Harvard mson, Princeton Orange, Red Cross, Vassar Rose and e Blue.

50. *Upper Crust* (17) such as Blasé Blue, Fashion Gray, de Beige, and Swank (a very pale violet).

I hope that by listing these categories together with a few examples from each I have given you some impression of the ways color names get into the language. However obscure the derivation of a color name, we can give a definition of it in ordinary language provided we have a specimen of it. We have only to determine its ISCC-NBS designation. This method of designating colors gives a more precise description of color in words than has been possible heretofore.

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A NEW LOOK AT THE MEASUREMENT OF LIGHT AND COLOR

Illum. Eng. 53, 61-71 (1958)

The problem posed by this paper, inconsistency between the only color-mixture functions ever determined directly by normal observers, and the luminous efficiency function adopted in 1924 by the CIE, has not been squarely faced, much less solved, by any standardizing organization. The proposal set forth on the final page of this paper has not been seriously considered. The problem is evaded by the casuistry that the CIE has adopted only the color-mixture data for large-field observation, whereas the luminous efficiency function is appropriate for small fields typical of visual photometers. Furthermore, that lights of different colors have obviously different brightnesses, even in small fields, as well as in practical applications, when their luminances are equal according to the CIE luminous efficiency function, has not yet forced reconsideration of the luminous efficiency function. The first step needed for solution of the problem is a redefinition of a luminous efficiency function as a linear combination of some directly determined color-mixture functions, be they Stiles's 2° data or results of repetitions of Stiles's work with more-modern apparatus and more observers. Because of the Helmholtz-Kohlrausch effect described in this article, no revision of the luminous efficiency data can, alone, solve the entire problem. Compensation for the effects of chromatic differences of lights, according to departures from additivity of luminances, will require use of consistent color-mixture and luminous-efficiency data.

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Preprint No. 7

A new look at the . . .

Measurement of Light and Color

By DEANE B. JUDD

Since 1935 the standard definition of light has been based on a luminous-efficiency function first adopted provisionally by the International Commission on Illumination (CIE) in 1924. This function was intended to give the reciprocal of the spectral radiance required to make each part of the spectrum appear equally bright to the average normal eye, and new determinations during the last 30 years indicate that the standard luminous-efficiency function succeeds very well in doing this, except perhaps that it may be too low in the short-wave portion (less than 460 mμ). At the 1955 Congress of the CIE in Zürich, Dr. W. S. Stiles presented preliminary results of determinations of the color-mixture functions that suggested an entirely new basis for the definition of light. To make clear this new basis, and the advantages that it may have, the basic principles of the measurement of light and color are examined briefly.

Measurement of Light

Basic problem in the measurement of light is to determine how many units of light are emitted by an unknown source even though this source has a color quality, or chromaticity, different from that of the standard of light with which it must be compared. Sometimes this problem is called the problem of heterochromatic photometry. Heterochromatic photometry has been carried out in three ways; one is by direct visual comparison, the so-called equality-of-brightness method; another is to apply the equality-of-brightness method in succession to a series of lights of intermediate chromaticities, called the cascade or step-by-step method; and the third is to present the two lights to be compared alternately in the same photometric field with a frequency of about five alterations per second, the so-called flicker method.

The first method, that is, by direct visual comparison, is the most direct and fundamental. The observer sees the two lights to be compared, side by side, filling the two halves of a photometer field. He must adjust the radiant flux of the comparison field until the two halves of the photometer field appear to him equally bright. This method is also the most difficult. If, as is usual, the standard is light from an incandescent lamp, and the light to be evaluated is, for example, the orange-red light from a neon lamp, there may be a range by a factor of two or more, within which the observer cannot decide which half of the field is the brighter. The larger the chromaticity difference between the test side of the field and the standard side, the greater is the range of uncertainty.

The second, or cascade method, substitutes a series of less difficult comparisons for the single direct comparison. Each step of the cascade method is less difficult because the chromaticity differences between the two sides of the photometer field is smaller.

In the third, or flicker method, the observer adjusts the radiant flux of the comparison field until the perception of flicker is a minimum. The reason why this adjustment is supposed to correspond to equal brightness of the two lights, is that the perception of chromaticness requires more time to develop in the visual mechanism of the observer than is required for perceiving brightness. The speed of alternation is set so as to eliminate, or nearly eliminate, any flicker caused by a chromaticity difference between the two lights, and then an adjustment for a minmum of flicker is supposed to correspond to the condition that the luminous flux of the one light is the same as that from the other.

These three methods of comparing an unknown light with a standard of light unfortunately yield rather discordant results. They all depend importantly on the angular size of the photometric field, and indeed this might be expected because the spectral sensitivity of the human eye is simply the average of the receptors (rods and cones) of the portion stimulated, and it is known histologically

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(from the work of Polyak, 1941, for example) that the proportion of rods increases from zero at the fovea centralis to ever higher fractions as the periphery of the retina is approached. Since the rods have their maximum sensitivity at about 510 mμ, and the cones at about 560 mμ, it is plain that a different appraisal of radiant flux may be obtained from a small field than from a large. Furthermore, the central three or four degrees of the normal human retina is covered with an irregular spot of yellow pigment, the macular pigment, and this too would lead us to expect a different appraisal from a field of large angular extent than from a small one.

These three methods likewise depend differently on the magnitude of the areal density of radiant flux coming per unit solid angle from the photometric fields. If this radiant flux density, or radiance, is sufficiently high, the retinal rods reach a maximum in their response (as shown by Aguilar and Stiles, 1953), while the retinal cones give increasing responses even though the radiance be increased by a thousand fold beyond this amount. The change in eye sensitivity from that of the cones at high radiance to that of the rods at low, has been known since 1825 as the Purkinje effect. Photometry by the direct-comparison method, or by the cascade method, is greatly influenced by the Purkinje effect. Photometry by the flicker method also depends on the radiance of the photometric field, but the direction of change is opposite. For want of a better name, this dependence is called the inverse Purkinje effect. The existence of an inverse Purkinje effect in flicker photometry makes us wonder whether flicker photometry really measures the same aspect of radiant flux as is measured by equality-of-brightness photometry. The fact is that we know relatively little about what goes on in the visual mechanism during a setting by flicker photometry except that the phenomena must be very complicated.

If the field size be kept small, say less than two degrees, the radiance be kept high, and the photometric fields be surrounded by a bright field, the participation of the rods is minimized, and the three methods of heterochromatic photometry yield results that are much less discordant. The early years of the International Commission on Illumination were spent in trying to define a standard photometric procedures along these lines that could be used by all countries in setting up and administering their standards of light, but no satisfactory method could be developed.

The requirements that a satisfactory definition of light must meet are:

1. Equal amounts of light by this definition must appear equally bright under the standard observing conditions of field size and of photometric brightness of field and surrounding field.

2. The amount of light from two or more sources shall be the sum of the amounts from the separate sources.

The first requirement points toward the equality-of-brightness method, or at least toward the cascade method. In so far as the flicker method disagrees under the standard observing conditions with these more fundamental methods, the flicker method cannot form the basis of a satisfactory definition of light.

The second requirement, that the photometric scale be additive, is imposed because a nonadditive definition of light is judged to be too complicated to be of any practical use in industry. (The inverse-square law, for example, would not apply to light.) Unfortunately, the evaluation of radiant flux by our eyes is not always as simple as this. If the two lights, standard and test, have the same chromaticities, even though they may be spectrally very different, the photometric scale by direct visual comparison is strictly additive. The more complicated facts of heterochromatic photometry may be best described in terms of an actual experiment.

If one half of a photometric field be filled with one unit of light from the standard source of incandescent-lamp light of color temperature 2042° K, and enough red light be introduced into the other half of the field to make the two half-fields appear equally bright, by requirement (1) this amount of red light must also be one unit.

Secondly, if green light be substituted for the red in the comparison half of the photometric field, similarly there may be found one unit of green light.

Thirdly, if now the one unit of red light and the one unit of green light both be introduced simultaneously into the comparison field, this mixture ought to appear as bright as two units of the standard light, if the second requirement, additivity, is to be fulfilled. Experimentally, however, the mixture of the red and green lights usually appears darker than the two units of the standard incandescent light. In order to obtain the appearance of equal brightness it is necessary to reduce the amount of light from the standard source by amounts ranging from 0 to 50 per cent. This type of failure of additivity was first noticed by Helmholtz and has been studied in detail by Kohlrausch (1923, 1937). The effect has often been called the Helmholtz-Kohlrausch effect since J. Urbanek and E. Ferencz (1939) gave it that brief name in their brilliant and complete analysis of visual heterochromatic photometry. It is discussed by H. König (1947) in his detailed analysis of the concept of brightness under the name "Farbenglut."

To be sure, the setting for one unit of red light, and that for one unit of green light, is uncertain by about a factor of two. (The third setting, that between the standard incandescent-lamp light and the mixture of one unit of red light with one unit of green light, is, of course, much more certain because the chromaticity difference is much smaller.) The Helmholtz-Kohlrausch effect is thus not certainly demonstrable by a single trial at these three operations, but has statistical meaning only. Nevertheless it is not possible to devise a definition of light that meets both requirements strictly. If the definition includes strict additivity, it must imply failure by at least 25 per cent in some cases to have equal amounts of light appear equally bright.

On the other hand, if the comparison be made by flicker photometry, the Helmholtz-Kohlrausch effect, though not completely absent, is much smaller (Jaggi, 1939; Dresler, 1938). This experimental fact suggests basing a definition of light on flicker photometry.

The Helmholtz-Kohlrausch effect corresponds roughly to the theoretical idea that most of our brightness perception results directly from receptors in the retina that serve this purpose only, but some of it is associated with the red-sensing and green-sensing mechanisms. Thus, when the red and green lights are mixed and the redness and greenness cancel to produce yellow, some of the brightness perception is lost. This theoretical idea is supported by the fact that deuteranomalous observers, who have an ability to discriminate red from green much less than that possessed by observers of normal vision, show almost no Helmholtz-Kohlrausch effect (Kohlrausch, 1937). This experimental fact has led to the rather paradoxical proposal that a definition of light be based on equality-of-brightness photometry by deuteranomalous observers.

The question of defining light in a practically useful way has been argued at great length in the International Commission on Illumination ever since its formation early in this century. In 1935 they resolved the dilemma by rejecting all three methods of heterochromatic photometry, and substituting for them the idea of luminous efficiency. Each sample of radiant flux is said to be associated with a fixed amount of luminous flux, the ratio between the two being luminous efficiency in lumens per watt. Luminous efficiency is taken to be solely a function of the wavelength composition of the radiant flux, the particular function being defined by adopting values for the relative luminous efficiencies of each part of the visible spectrum, these values having been adopted provisionally by the CIE in 1924. This definition of light was reaffirmed (CIE, 1939) as follows:

"Every result of photometric measurement shall be placed on the unique fundamental base defined by the relation:

$$B = K_m \int E_\lambda V_\lambda d\lambda \qquad (1)$$

where B means luminous brightness (now called luminance), K_m the maximum coefficient of visibility (now called maximum luminous efficiency), E_λ the spectral radiances per unit wavelength, V_λ the relative visibility factors (now called relative luminous efficiencies) defined by the CIE, λ the wavelength."

It is evident that if every photometric measurement is placed on this base, the photometric scale used will be strictly additive, for additivity is written into the definition itself. It may not be too clear, however, how to place every photometric measurement on this base unless each source is measured spectroradiometrically to determine its spectral radiance as a function of wavelength, E_λ, and then luminance, B, is computed directly by the defining equation (1). This fundamental method is too time-consuming, and too beset with difficulty, to be used except occasionally. The practical method recommended in assigning a number of candelas to specify the luminous intensity of an unknown source is the so-called filter method. A filter is selected which may be applied to the standard source to produce a close chromaticity match for the unknown source. The spectral transmittance of this filter as a function of wavelength is then determined with a spectrophotometer. The transmittance of the filter for the standard light source whose spectral composition is known is then computed by means of the above specified relative spectral luminous efficiencies. Then the comparison is made by direct visual comparison in a two-degree field of adequate luminance. If a reasonably close duplication in spectral composition is attainable, but not a sufficiently close duplication of chromaticity, the flicker method is recommended. The result of either method takes into account the influence of the filter by using the transmittance found for it by computation.

We may summarize the definition of light, which has lived now for 20 years, by saying that it is technically convenient because it makes light an additive quantity by definition, but does not conform strictly to the first requirement that equal amounts of light shall appear equally bright in all cases. The curve of spectral luminous efficiency has been chosen as an average of determinations by the cascade and flicker methods so that equal amounts of light from whatever two parts of the spectrum appear equally bright to the average observer, and, of course, it was the intention so to define light that this requirement should be met as closely as possible for light sources of mixed wavelength composition as well. Because of this intention, luminous flux has been officially defined as the characteristic of radiant flux which expresses its capacity to produce a luminous sensation as evaluated from the standard spectral luminous efficiencies. This has been an official attempt to eat the cake and have it too, that is, to claim to have satisfied the two basic requirements which we have seen cannot both be satisfied simultaneously. At the Zürich meeting of the CIE in 1955, it was proposed to drop from the definition of luminous flux the phrase, *in accord with its ability to produce luminous sensation.* This proposal recognizes the facts of life. It is not enough to intend to evaluate radiant flux in accord with luminous sensation; the fact is that the definition given fails to do so, as must also any definition that makes light an additive quantity. We have therefore the definition of a technical quantity that is practically useful because it satisfies the second requirement perfectly and the first requirement approximately. It is, however, not the only definition that does this, and to see how another proposal has arisen for a still less contradictory definition of light, we must introduce a new subject, the measurement of color.

Measurement of Color

Measurement of color is based on the experimental fact well known in the time of Thomas Young (1805) that it is possible to produce nearly every color by a mixture of lights of but three colors, red, green, and blue. First set up a spotlight (say incandescent lamp light) to shine on a white screen. This appears to us as a bright spot having a reddish yellow color of low saturation. Then let a neighboring spot on the screen be illuminated simultaneously by light from three lamps, one red, one green, and the other blue. The radiant flux entering the eye of the observer is the sum of the separate fluxes that would enter the eye from each lamp separately if the other two were turned off. The observer sees the second spot as red, green, blue, or some intermediate color depending on the relative amounts of radiant flux received from the red, green, and blue lights. By varying these amounts we discover in general that we can produce almost any other color. For example, by mixing red light with green light we can produce yellow colors; by mixing red light with blue light we can produce purple colors; and from blue and green lights, cyan colors. In particular, we discover that we can produce by mixture of much red, nearly as much green, and a small amount of blue, an exact match for the color of the first spotlight, incandescent-lamp light. By giving the amounts of red, green, and blue light required for the color match, known as tristimulus values, this light can be specified, and we can say that we have measured its color.

The colors that cannot be produced by such a three-light mixture are those that appear more saturated than any of these mixtures, for example, the colors of the pure spectrum. Even if we choose our primary red, green, and blue lights directly from the spectrum, it is still found that not all of the other spectrum colors can be produced by mixture of any one triad of primary colors. But even these colors can be specified by tristimulus values. It is necessary only to add to the light to be specified some of the primary differing most from it, thus reducing its purity so that the mixture can be matched by some combination of the other two primaries. For example, let the color to be specified be that of the spectrum at about 490 mμ, which is seen as a saturated color of bluegreen hue. By mixing the blue and green primaries in the correct proportion we can produce a mixture of this bluegreen hue, but by comparison with the pure spectrum color, this mixture appears grayish, or less saturated. Then we add a sufficient amount of the red primary to the light (480 mμ), whose color is to be specified, to produce a match to some proportion of the blue and green primaries. In this way, the spectrum color (490 mμ) has been specified in terms of the three primaries, red, green, and blue. As before, these amounts are said to be the tristimulus values of the color to be specified except that the amount of the primary (red) added to the test color is recorded with a negative algebraic sign. The specification might be $R = -20$; $G = 60$; $B = 60$.

The usual theoretical interpretation of this finding that no selection of three primary lights can be found whose combination in positive amounts produces all other colors is that there exist three light-sensitive substances in the normal retina whose spectral sensitivities overlap. That is, nearly all parts of the visible spectrum serve to excite more than one of the three independent receptor systems comprising the visual mechanism. Conversely, if it were possible to find parts of the spectrum that yield excitation of each receptor system to the exclusion of the other two, then, and only then, could we expect to duplicate all colors with mixtures of these three spectrum lights.

Usefulness of the method of color measurement by means of tristimulus values is limited first by the fact that the choice of red, green, and blue lights to be used as primaries is arbitrary and has still to be standardized, and second its usefulness is limited by the fact that another observer, even though also possessing normal color vision, would find somewhat different tristimulus values for the same test color; so the observer, too, has to be standardized. Finally we would like to know what amounts of any three specified primaries would be required by an average observer to match this same test color. The basis for transferring tristimulus values from one set of primaries to another can be discovered by experiments with the simple apparatus just described. To summarize the results of these experiments it is hard to do better than to take the summary written out by Grassmann (1853), now known as Grassmann's laws:

1. The eye can distinguish only three kinds of color differences or variations (expressible as variations in dominant wavelength, luminance, and purity.

2. If, of a two-component mixture, one component is steadily changed (while the other remains constant), the color of the mixture steadily changes.

3. Lights of the same color (that is, the same dominant wavelength, same luminance, and same purity, or alternatively, the same tristimulus values) produce identical effects in mixtures regardless of their spectral composition.

All modern colorimetry is based upon the principle stated in the third Grassmann law. It means that we can deal with lights on the basis of their colors alone, without regard for their spectral composition. It generates the following important corollaries:

3a. Two lights of the same color, added to two other lights of the same color, produce mixtures that likewise have the same color; or stated in terms of tristimulus values, if X_1, Y_1, Z_1 are the amounts of red, green, and blue light required to match one light, and X_2, Y_2, Z_2 are the amounts for a second light, then the tristimulus values for the color produced by adding these two lights together are simply: $X_1 + X_2$; $Y_1 + Y_2$; $Z_1 + Z_2$.

3b. Two lights of the same color, each subtracted respectively from mixtures of equal color, leave remainders that color match. This corollary justifies the use of a negative tristimulus value to specify colors outside the gamut of the three primaries chosen.

3c. If one unit of one light has the same color as one unit of another light, any number of units (or fraction of a unit) of the one light has the same color as the same number of units (or same fraction of a unit) of the other. That is, increasing or decreasing by any factor the radiance of two lights of the same color, even though of different spectral compositions, will not destroy the color match.

Grassmann's laws have been subjected to repeated experimental tests (Blottiau, 1947; Trezona, 1953, 1954; Stiles, 1955), and no certain failures of these laws have been established for observation within a large middle range of luminance (about 1 to 1000 millilamberts) by using observing fields of small angular extent (those subtending 2 degrees or less). For fields less bright than one millilambert, departures from Grassmann's laws have been found that seem to be ascribable to the intrusion of rod vision. Wright (1936) also has shown that the color match might break down if the radiance of the fields be increased unduly. Such a breakdown may occur at luminances greater than 1000 millilamberts.

Grassmann's laws permit us to specify color by means of three additive scales, a red, a green, and a blue. They do more. Every light may be specified physically by the spectral composition of its radiant flux. If we knew for any observer the tristimulus values of unit radiant flux for each part of the spectrum, we could by Grassmann's law (3c) compute how much red, green, and blue is required to match each part of the spectrum of any known light, then by Grassmann's law (3a) we find simply by adding up these amounts for the whole spectrum a prediction of the tristimulus values that would be found by that observer for the light itself. To set up a standard observer and colorimetric coordinate system it is necessary, first to decide on some set of primary colors to be taken as standard, second to decide on the units by which to express the amounts of these primaries, and third to evaluate in these units for the average, normal eye, the tristimulus values of each part of the visible spectrum.

The 1931 CIE Standard Observer And Coordinate System for Colorimetry

In 1931 the International Commission on Illumination (CIE) recommended a standard system of the kind just described for international use in the measurement of color.

1. *Choice of primaries.* In order to permit the tristimulus values of the spectrum to be specified by a number not less than zero, three imaginary primaries were chosen, a blue theoretically more pure than any spectrum blue, a green theoretically more pure than any spectrum green, and a purplish red theoretically more pure than any producible by mixing spectrum red with spectrum violet. These primaries are imaginary in the sense that energy distributions corresponding to them must have negative values for some parts of the spectrum; but in terms of the facts of color-mixture summarized

by Grassmann's laws they are just as meaningful as primaries corresponding to real lights, that is, lights definable by energy distributions nowhere in the spectrum less than zero.

2. *Choice of units.* Arbitrary units were chosen for the scales on which the amounts of these primaries are expressed such that equal amounts correspond to the chromaticity of the so-called equal energy source, that is, a source such that the amount of energy between any two wavelength limits is proportional to the wavelength difference between the limits.

3. *Choice of standard tristimulus values of the* spectrum. Since no direct determinations of the tristimulus values of the equal-energy spectrum had been made at that time, the standard tristimulus values were deduced from measurements of spectrum chromaticities, x_λ, y_λ, z_λ, by 17 British observers (10 studied by Wright, 1928-1929; 7 by Guild, 1931) combined with the standard luminous-efficiency function, V_λ, at that time already adopted provisionally. The derivation of standard tristimulus values of the spectrum was such that these values, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$ satisfied the two conditions (2 and 3):

$$\begin{aligned} \bar{x}_\lambda / (\bar{x}_\lambda + \bar{y}_\lambda + \bar{z}_\lambda) &= x_\lambda \\ \bar{y}_\lambda / (\bar{x}_\lambda + \bar{y}_\lambda + \bar{z}_\lambda) &= y_\lambda \\ \bar{z}_\lambda / (\bar{x}_\lambda + \bar{y}_\lambda + \bar{z}_\lambda) &= z_\lambda \end{aligned} \quad (2)$$

$$\bar{x}_\lambda L_x + \bar{y}_\lambda L_y + \bar{z}_\lambda L_z = V_\lambda \quad (3)$$

Condition (3) states that the luminous efficiency of each part of the spectrum shall be obtained from the tristimulus values of that part by weighting each primary with its own luminous efficiency, L_x, L_y, L_z. This condition is a special case of equation (1) giving the standard definition of light. $\bar{x}_\lambda L_x$ is the amount of light associated with $\bar{x}$-amount of the red primary, and $\bar{y}_\lambda L_y$ and, $\bar{z}_\lambda L_z$, similarly are those associated with the other two primaries. Since, as has already been pointed out, the standard definition of light is strictly additive, the total amount of light associated with unit radiance for each part of the spectrum, specified by $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, must equal the luminous efficiency, V_λ, of that part.

This special case of the general statement (1) of the additivity of light is known as Abney's law. Abney's law is prevented by the Helmholtz-Kohlrausch effect from being in strict accord with the facts. But in 1931 it was considered more important to make the standard observer accord with the strictly additive definition of light about to be adopted as standard than to try to take account of the Helmholtz-Kohlrausch effect, at that time not thoroughly understood.

Fig. 1 shows the tristimulus values, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, of the equal-energy spectrum derived in accord with equations (2) and (3). Note that none of these values is less than zero; this property results from the choice of primary colors. It is also true that the areas beneath the three curves are equal; this property results from the choice of units in terms of which to express the amounts X, Y, Z, of the primaries. Furthermore it is true that the sum of the tristimulus values at each wavelength expressed in luminous terms, is the luminous efficiency at that wavelength, V_λ, in accord with condition (3). Because of a rather surprising property, not yet mentioned, introduced by the choice of primaries, the red and blue primaries (X and Z) are completely unassociated with any luminous value (Judd, 1930, 1933); that is, $L_x = L_z = 0$; therefore all of the luminous value is associated with the amount of the green primary (Y), and the $\bar{y}_\lambda$-curve shown in Fig. 1 is identically the same function of wavelength as the luminous efficiency (V_λ) function.

For the last twenty years measurements of light and color have been made in accord with tristimulus values, X, Y, Z, defined in terms of the tristimulus values of the spectrum, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, and the spectral radiance, E_λ of the light to be specified, thus:

$$\begin{aligned} X &= \int E_\lambda \bar{x}_\lambda \, d\lambda \\ Y &= \int E_\lambda \bar{y}_\lambda \, d\lambda = \int E_\lambda V_\lambda \, d\lambda = B/K_m \\ Z &= \int E_\lambda \bar{z}_\lambda \, d\lambda \end{aligned} \quad (4)$$

The Y-tristimulus value gives the amount of the light (directly in lumens if multiplied by K_m), and all three tristimulus values, X, Y, Z, taken together, specify its color. The system is completely self-consistent, logical, and convenient to use in practice; indeed, it has been found to be of very great practical value. We have next to inquire into how well these definitions of light and color have been found to accord with what actual observers see.

Does the Standard of Light Agree With Actual Observations?

Of course, the Helmholtz-Kohlrausch effect prevents equal amounts of light from always appearing equally bright. What it really tells us is that light is not strictly an additive quantity. But since light has been defined as a strictly additive quantity, rediscoveries of the Helmholtz-Kohlrausch effect have taken the form of finding special cases in which lights of equal luminance fail to appear equally bright. MacAdam (1948), Dresler (1953), and Chapanis and Halsey (1953) have found that, to produce equal brightness, lights of equal luminance should be used only for chromaticities between that of daylight and the yellow part of the spectrum (570 to 580 mμ). As the chromaticity of

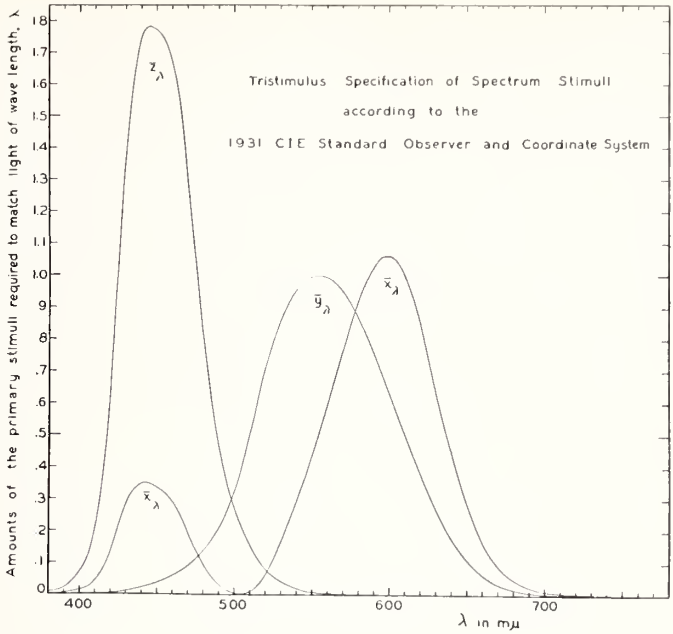

Figure 1. Tristimulus specifications of spectrum stimuli according to the standard observer and coordinate system recommended by the International Commission on Illumination in 1931. These specifications refer to the equal energy spectrum, and the units of the primary stimuli are adjusted so that their combination in equal proportions matches the equal-energy stimulus; that is, the areas of the three curves have been adjusted to equality by choice of units. These three functions define the properties of the 1931 C.I.E. standard observer.

the light departs from this daylight-to-yellow range, less and less luminance is required to produce equal brightness. The answer to the question: Does the standard of light agree with actual observations?, is therefore, *no*. Furthermore, discrepancies of this kind should have been expected from the work of Kohlrausch (1923, 1937) and Dresler (1938). The real meaning of these later studies is that more people are finding out about the Helmholtz-Kohlrausch effect, and, to be sure, are finding that the amount of the effect is large enough to be of some practical importance.

Does the Standard of Color Agree With Actual Observations?

This question may be expressed as follows: If any two colors differing in spectral composition are found by the 1931 CIE standard to have identically the same triad of tristimulus values, X, Y, Z, do these colors look alike when viewed by an observer of normal color vision Almost immediately after the standard definition of color was recommended in 1931, such tests were made and almost invariably the answer was found to be, *no*. In comparing a mixture of spectrum red plus spectrum green to match a spectrum yellow, for example, most observers might say that the standard mixture is too green but others would say it is too red. As long as we can find a good number of observers to cast a minority vote, we say that the standard definition of color has been fairly well supported. Hardly anyone has exactly standard color vision, just as hardly anyone has exactly average height.

Not until 1948 did there appear results of tests in which all actual observers disagreed with the standard in the same sense. Jacobsen (1948) and Judd (1949) found the standard definition of color inapplicable to the colorimetry of rutile and anatase forms of titanium-dioxide pigment. As a result of these and similar checks on near-white surfaces and nearly transparent media (kerosene and other refined petroleum products compared to yellow tinted glasses), some segments of American industry commenced to reduce their spectrophotoemtric data by means of a revised Z-function, higher in the neighborhood of 400 mμ than the standard by about a factor of 5 (CIE, 1951).

Redetermination of the Color-Mixture Functions at the NPL

As a result of an American attempt to have the CIE revise the definition of color by changes in the short-wave end of the spectrum, important for colorimetry, though relatively unimportant for photometry, a redetermination of the color-mixture functions was undertaken by Dr. W. S. Stiles at the National Physical Laboratory in Great Britain. Perhaps, *redetermination* is not quite the correct word. Guild and Stiles have pointed out that the color-mixture functions basic to the standard definition of color have never really been determined directly. As we have seen, they were merely inferred from the chromaticities of the spectrum colors found by Wright (1928-1929) and Guild (1931) combined with the standard luminous-efficiency function, V_λ, on the assumption that Abney's law is strictly valid. What has been undertaken by Stiles is therefore a careful determination, first in the history of color measurement, of the amount of spectrum red, spectrum green, and spectrum blue radiant flux required by a sufficiently large and representative group of observers (50 or 60) to color match the other parts of the spectrum. Preliminary results for 10 observers for both a 2-degree and a 10-degree field have already been published (Stiles, 1955).

Whether Stiles' final results will be found worthy of adoption as the basis of a new definition of color remains to be seen. This will depend on how successfully the new color-mixture functions predict actual color matches in titanium-pigment colorimetry and in other tests involving light of complicated wavelength composition. Nevertheless, in spite of their preliminary character, when Stiles presented his results at the 1955 Congress of the CIE in Zürich, it was recognized that a new look might have to be taken at our standards for light and color. The need for a change in the basic philosophy of these standards was suggested by Stiles' attempt to derive the standard luminous-efficiency function from his preliminary 2-degree color-mixture functions by means of Abney's law. He found that by weighting these functions by the standard luminous efficiencies of the spectrum primaries used, the sum was far from agreement with the standard luminous-efficiency function, V_λ, in the other parts of the spectrum. He found that by weighting them with the factors derived from photometric settings by the actual observers used, the disagreement was still present. In short, no matter how the color-mixture functions are weighted they will not sum to the standard luminous-efficiency function.

Fig. 2 compares the standard luminous-efficiency function ($V_\lambda = \bar{y}_\lambda$) shown, as a solid line, with the sum of Stiles' preliminary 1955 color-mixture functions, both for the 2-degree and the 10-degree field when these functions are weighted by factors producing nearly the best agreement possible. We need not pay attention to the 10-degree field data (large circles) because they ought not to sum to the standard luminous-efficiency function developed to apply to 2-degree observation. The sum of the 2-degree color-mixture functions, however, is seen to be higher, not only in the short-wave part of the spectrum, but, when reduced as shown to unit maximum, substantially higher in nearly all parts of the spectrum. These results suggest, therefore, not merely a minor revision for the short-wave part of the spectrum, unimportant in photometry, but a major revision throughout the whole spectrum, in the function to be used in the definition of light.

New Look

There are several views that might be taken of the preliminary results from the National Physical Laboratory:

(1) Let us wait until the final results are available before we think what bearing these color-matching data have on the standard of light; maybe these preliminary data are misleading.

(2) Let the standard of color eventually be defined in whatever way it has to be defined, but let the standard definition of light remain the same as it has been for the last 20 years, that is, based on the principle that the standard luminous-efficiency function is the reciprocal of the radiance required

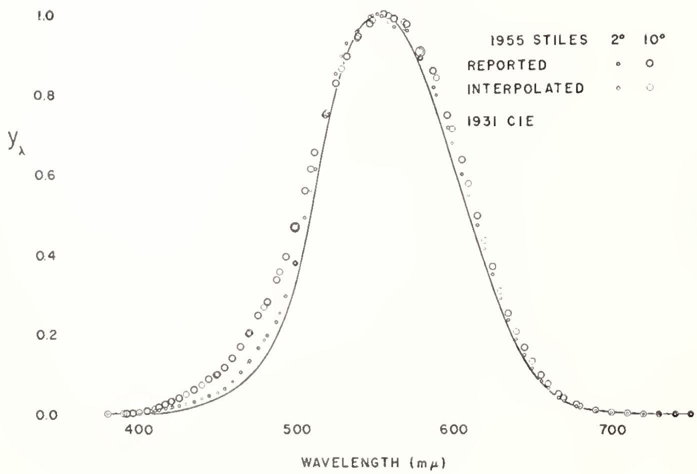

Figure 2. A comparison of the C.I.E. standard luminous-efficiency function (solid line) with the sum of Stiles' color-mixture functions weighted to produce nearly the best agreement possible and then reduced to unit maximum.

at each wavelength to produce fields appearing equally bright for the average, normal eye.

(3) Let the standard of light be revised to accord with a weighted sum of whatever color-mixture functions are finally adopted in a revised definition of color.

Not many of the delegates at the Zürich Congress of the CIE took the view that the preliminary color-mixture functions reported by Stiles are misleading in respect to their bearing on the standard definition of light. It was recognized that most, if not all, of the discrepancy is ascribable to the Helmholtz-Kohlrausch effect. Furthermore, there were also presented at the Zürich Congress of the CIE some confirmatory measurements by Sperling at the U. S. Naval Medical Research Laboratory, New London. Sperling determined directly for the three observers, not only color-mixture functions, but also luminous-efficiency functions by the method of direct visual comparison. Two of the three sets of data showed the color-mixture functions expressed in luminous terms to sum to a wavelength function broader than the luminous-efficiency function for that observer.

The objection to the second view, that independent definitions of light and color eventually be established, is that the contradictory character of the two definitions would be very evident. Thus, generally, any two samples of radiant flux of different spectral compositions found to have exactly the same colors by a definition based upon actual determination of color-mixture functions for the average eye would, on applying the present definition of light, be found to correspond to different amounts of light. By this view, therefore, we would continually be placed in the untenable position of saying that pairs of lights that cannot be distinguished by the average eye, nevertheless, by the average eye have different luminances.

On this account, many of the delegates at the Zürich Congress (for example, Stiles, 1955; Judd, 1955; and MacAdam, 1955) have taken the third view, which looks forward to the eventual establishment of new definitions of light and color, perfectly congruent, just as the present definitions are. The present definitions are congruent because the definition of color has been made to conform to a previously formulated definition of light. The new definitions of light and color would be made congruent by taking for the new definition of light a new luminous-efficiency function computed as some weighted mean of the color-mixture functions to be adopted, the particular weights assigned to be chosen so as best to accord with the needs of practical photometry.

The advantage of this third view is not that it yields congruent definitions of light and color; the present definitions also yield perfect congruence. Nor is it that the third view defines light in such a way that equal amounts of light always appear equally bright to the average observer; as we have seen, no additive definition of light can possibly meet this condition; the Helmholtz-Kohlrausch effect prevents it. No, the advantage of the third view is that it promises to meet the requirement that when lights are found to be equal in color by the new definition, an observer of average, normal color-vision will find them really to be equal, and for this condition of match (under which precision of photometric setting is at a maximum), the standard definition of light will indicate the two lights to be identical in amount.

The disadvantage of the new look is that the required luminous-efficiency function will define as equal amounts of light, such amounts of radiant energy for the various parts of the spectrum as careful photometry by an average, normal observer would show do not appear equally bright. Since it would take a long photometric study to show this discrepancy, it would seem that this disadvantage might be tolerable.

Time alone will tell whether or not this new point of view will lead to new and more satisfactory definitions of light and color.

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SOME COLOR DEMONSTRATIONS I HAVE SHOWN

J. Opt. Soc. Am. 49, 322-328 (1959)

This lucid description of demonstrations that constituted Judd's Ives Medal lecture does not need any clarification here. The terminology is still current and, for the most part, explained for the casual reader, in the text itself. The subject matter ranges from one of the earliest subjects (blue arcs of the retina) to which Judd directed his scientific attention, to those for which he is best known - chromatic adaptation and color rendering.

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Some Color Demonstrations I Have Shown

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The Ives Medal Address consisted mostly of demonstrations of phenomena of color perception. The following demonstrations were attempted: blue arcs of the retina, Maxwell's spot shown with Miles filter (macular pigment as an entoptic phenomenon), attributes of color perception (Desert Island experiment with Priest charts), color perceptions of protanopes and deuteranopes, metamerism (Stearns textiles, Granville grays), chromatic adaptation (simultaneous and successive contrast), and color rendition of light sources (object-color perception as the projection of the after-image of the background onto the object).

INTRODUCTION

THIS is a short account of the Ives Medal Address presented on Friday, October 10, 1958, at the 43rd Annual Meeting of the Society, Detroit, Michigan. As the title indicates this address consisted mostly of demonstrations of various phenomena of color perception.

BLUE ARCS OF THE RETINA

Figure 1 is a diagram of a horizontal cross section of the human eye. It is well known that the foveal pit, shown at *r*, is the part of the retina giving us our most acute vision, and that the contributing anatomical facts are first that the retinal cones are here most closely packed and second that here, and here only, can the image forming light flux impinge on the retinal cones after penetrating a single layer of nerve fibers, instead of several layers as in other parts of the retina. Indeed at the fovea the retina is thinnest of all places, and this is what makes it a pit. The necessary result of this absence of nerve fibers conducting signals across the inner face of the retina on the way to the exit point of the optic nerve (blind spot), shown at *d*, is sometimes forgotten, namely: that signals from the retina on the side of the fovea opposite to the exit point of the optic nerve must pass around the fovea, above and below, the nerve fibers coursing above and below the fovea in arc shaped patterns. When these nerve fibers are conducting signals they become entoptically visible provided the retinal areas through which they pass are otherwise relatively unstimulated. The name for this entoptic phenomenon, apparently first described by Purkinje[1] in 1825, is the "blue arcs of the retina".

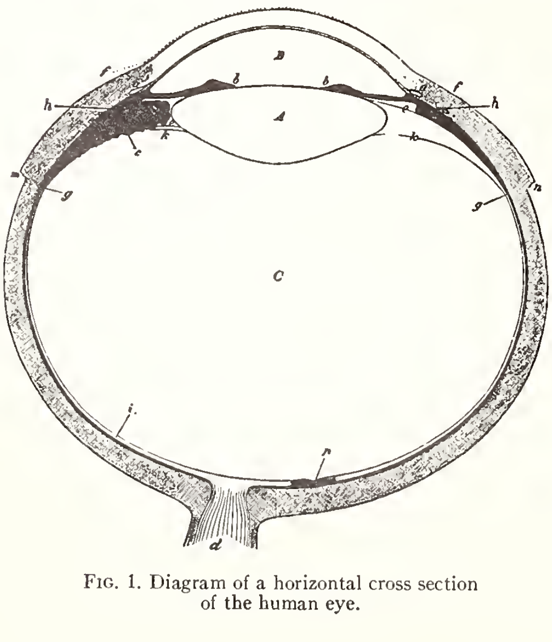

FIG. 1. Diagram of a horizontal cross section of the human eye.

A vertical band of red light was shown on the screen and the members of the audience were instructed to close the left eye and look with the right eye slightly to the right of the red band. Then to close the right eye and look with the left eye slightly to the left of the vertical red band. For any given fixation point two reddish-blue arcs are often reported to appear for a second or two. It is not known by what means the excited nerve fibers manage to stimulate the cones over which they pass; one view is that the known electrical transients accompanying each nerve impulse do it, another view[2] is that excited nerve fiber gives off radiant energy which excites the underlying cones in the usual way.

The "blue arcs of the retina" have no bearing on color measurement, nor on anything practical, yet discovered. This is science in its purest form.

MAXWELL'S SPOT SHOWN WITH MILES FILTER

Light entering the pupil, *bb* in Fig. 1, on its way to form an image on the retina, *g i r g*, may pass through two structures containing yellow pigment, the lens *A*, and the macula (shown as a shaded area around the foveal pit *r*). The spectral transmittance of the ocular media (cornea, aqueous humor, lens, vitreous humor) according to Ludvigh and McCarthy[3] is shown by dots in Fig. 2, and may be taken with little error to refer to the lens, the other media being relatively transparent. These data indicate that the pigment in the lens is a melanin type of pigment, like that found in the skin.

[1] J. E. Purkinje, Beobachtungen und Versuche zur Physiologie der Sinne 2, 74 (1825).
[2] D. B. Judd, Bur. Standards J. Research 2, 441 (1929).
[3] E. Ludvigh and E. F. McCarthy, Arch. Ophathalmol. **20**, 37 (1938).

The macular pigment according to Wald[4] is xanthophyll with a spectral transmittance estimated from absolute thresholds of light measured foveally and extra foveally as shown by the circles in Fig. 2.

An entoptic phenomenon, now known as Maxwell's spot, was reported by J. C. Maxwell[5] in 1856, and was ascribed by him to absorption of light by the macular pigment. To demonstrate Maxwell's spot, a two-part slide, half gray and half purple, was shown. The left half was made of exposed photographic film transmitting about one percent of the incident light. The right half was made of a filter[6] suggested by Miles who has made extensive studies of Maxwell's spot.[7] This filter likewise transmits about one percent of incident incandescent-lamp light, but the light transmitted is confined to two spectral bands, 400 to 500 $m\mu$, and 670 to 750 $m\mu$. The members of the audience were instructed to look at the gray area for about 10 sec and then to shift to the purple area. Under these conditions the appearance of an irregular spot subtending about 3° is often reported. The color of the spot is usually reported as red or pink, and it lasts about 10 or 15 sec. Then if the gray area be fixated, a yellowish green spot, brighter than the surrounding gray, is often reported of the same shape as the pink spot seen previously on the purple field. Walls and Mathews[8] have found that a report of "blue" instead of "red" or "pink" usually means that the subject has protanopic or protanomalous vision, and that a report of no Maxwell's spot under controlled favorable conditions with the Miles filter usually means that the subject has deuteranopic or deuteranomalous vision.

Since it is sometimes contended that the macular pigment is not really present in the living eye, but is only a post mortem artifact, there is no universal acceptance of Maxwell's explanation of the spot as an entoptic appearance of macular pigment. It has been shown,[9] however, that the colors reported for both normal and color-defective vision can be accounted for solely from Wald's values of spectral transmittance of the macular pigment combined with the v. Kries coefficient law of chromatic adaptation. Maybe differences between foveal and extra-foveal vision unconnected to macular pigment contribute to it.

DESERT ISLAND EXPERIMENT WITH PRIEST CHARTS

To make perfectly clear what is meant by description of color perceptions by such terms as red, pink, blue, yellowish green, lighter, and darker, the charts constructed by Irwin G. Priest early in the 1920's as a part of his contribution to color terminology[10] were shown. Image a castaway on a desert island who finds a trunk full of chips of all different colors floating up to the beach on a raft. Having nothing better to do, he spreads the color chips out on the sand to see what he can make of them. The chips, thus in disarray, make no sense at all, but he does notice one thing[11]; some of the chips are like the background, coral gray, some lighter, some darker, while the others have something special. He separates the chips into two groups. One group has black, white, and grays in it; the other group of colors all have this special property, hue, that the first group does not have. Hue is the first attribute of color perception that he has discovered.

He goes to work on the small group because that is simpler and discovers that they can be arranged in an orderly sequence ranging from black at the bottom to white at the top. The variable shown by this series is lightness; and this is the second of the attributes of color perception.

The larger group of hueful color chips is more complicated. First he goes to work to divide them into groups of the different kinds of hue. There are reds, yellows, greens, and blues; but each group is still rather mixed up. There are light and dark reds, light and dark greens, and so on.

He decides that he will never get anywhere unless he confines his attention to groups of colors of the same lightness. So he picks out a group of chips perceived to differ from white about as much as it differs from black. These are the chips perceived to have colors of middle lightness. From this group of chips he chooses two subgroups. The bottom subgroup is all more grayish than

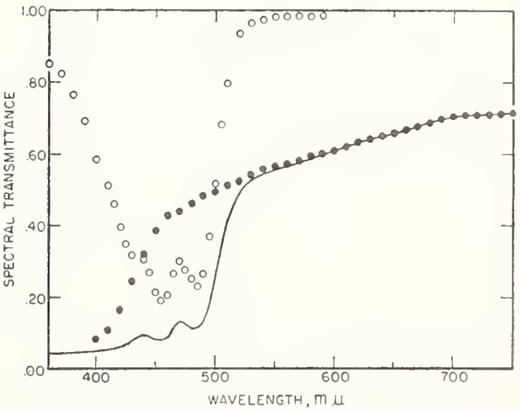

Fig. 2. Spectral transmittance of ocular media (cornea, aqueous humor, lens, vitreous humor) shown as dots, and the macular pigment shown as circles. The continuous curve shows the product of these two spectral-transmittance functions.

[4] G. Wald, Science **101**, 653 (1945).

[5] J. C. Maxwell, "On the unequal sensibility of the foramen centrale to light of different colours," Reports British Association, p. 12 (1856).

[6] Obtainable from Eastman Kodak Company by the designation: "Experimental Wratten Filter No. 2389."

[7] W. R. Miles, Science **109**, 441 (1949).

[8] G. L. Walls and R. W. Mathews, University of California Publication in Psychology **7**, 1–172 (1952).

[9] D. B. Judd, *Coloquio sobre Problemas Opticos de la Vision* (Madrid, Bermejo, 1953), Vol. 2.

[10] Charles Bittinger, J. Opt. Soc. Am. **10**, 291(*t*) (1925).

[11] Committee on Colorimetry, Optical Society of America, See color plates 8, 10, 13, 11, 17, 18, and 16 in *The Science of Color* (Thomas Y. Crowell Company, New York, 1953).

the top subgroup. This is his first inkling of the third attribute of color perception called saturation, or departure from the nearest gray.

To see what can be made out of this new ordering principle, he selects groups of reds, yellows, greens, and blues of the same lightness, and finds that these groups can now be put into orderly, one-dimensional series, extending from a nearly gray color to a comparatively vivid color quite different from any gray. In each of these series the only variation is saturation.

Maybe he can find hue series, too; so out of the top subgroup of chips perceived to have colors of higher saturation and of the same lightness, he picks an amazing sub-subgroup. Each one of these colors differs by the same amount from gray, and so all have identical saturations as well as lightness. He finds that these chips can be arranged in an orderly closed curve, and this proves that hue is a cyclic variable. If it starts from red, and goes through yellow, green, blue, and purple, it comes back to red again.

To summarize: lightness extends from black to white; saturation extends outward from each gray; and hue is cyclic.

From this point on our desert islander goes from one triumph to another. All of the saturation series for colors of equal lightness start from the same gray, and make the spokes of a wheel. The hue circuit can be added to these spokes, and for colors of middle lightness, a completely orderly classification has been achieved. The same treatment can be given to colors of any constant lightness.

Finally, if attention be confined to the color perceptions of any one hue, and its opposite, an orderly classification by lightness and saturation is possible.

By using the ideas known as hue, lightness, and saturation, color perceptions may be described in their own terms. And simply by attempts to classify colors on an appearance basis, without any measurement, we have shown that no two-dimensional classification can succeed in being orderly for anyone of normal color vision. We come out with a succession of orderly two-dimensional classifications which combine to form an orderly three-dimensional, or space, representation of color perceptions. The Munsell system of color notation is based upon this three-dimensional representation of color perceptions; and Mr. Priest's charts were made of color chips taken from the original *Atlas of the Munsell Color System* and revisions thereof.[12]

COLOR PERCEPTIONS OF PROTANOPES AND DEUTERANOPES

About 2% of otherwise normal human males (and about 0.04% of human females) have visual systems of two degrees of freedom, instead of the normal three degrees of freedom, and are called dichromats. Dichromats carrying out the desert-island experiment find it quite possible to place the color chips in a two-dimensional array that they perceive to be perfectly in order. To accomplish this they place as identical in color whole groups of chips perceived as different in color by observers of normal trichromatic vision. Figure 3 shows on the CIE (x,y)-chromaticity diagram the loci of chromaticities perceived as identical by the two chief groups of dichromats, known as protanopes and deuteranopes. These loci of confusion chromaticities have been plotted in accord with work by Pitt,[13] the dotted lines being experimental; the solid lines, theoretical. The colors perceived by protanopes and deuteranopes are the grays and all kinds of yellow and blues. We know this from reports of observers who have dichromatic vision in one eye and normal trichromatic vision in the other.

To show the nature of the chromaticity confusions by protanopes a gray card on which were mounted a red chip and a blue-green chip were exhibited in artificial daylight. The chromaticities of these three colors (gray, red, blue-green) plot on the protanopic confusion line passing through the C point; see left half of Fig. 3. Another gray card was shown with a purplish red chip and a green chip mounted on it. The chromaticities of these three colors plot on the deuteranopic confusion line passing through the C point; see right half of Fig. 3. For an observer possessing the corresponding type of vision, these chips would all appear gray closely of the same lightness. No member of the audience, however, volunteered to assent to this description of the colors shown.

To show further how a group of colors arranged in an orderly two-dimensional way for normal color vision would be put into an orderly one-dimensional array, a set of 12 color chips, all of about 40 percent reflectance, 4 orange, 3 yellow, and 5 yellowish green, were shown in three separate arrays. The array appearing orderly for normal vision arranged them in three saturation series radiating from a central gray. The protanopic and deuteranopic arrays consisted of single series ranging from saturated yellow to gray, the orange and yellowish green chips being inserted in accord with dichromatic color matching. One member of the audience who knew his vision to be abnormal volunteered the statement that the deuteranopic series appeared to have yellow hues, but some of the yellows tended to the orange side and some to the green side. Subsequent application of the Ishihara chart test of color perception showed him to have deuteranomalous vision, which is a form of anomalous trichromatism in which the ability to distinguish deuteranopic confusion colors is reduced, but is not absent.

[12] A. H. Munsell, *Atlas of the Munsell Color System* (Wadsworth-Howland and Company, Malden, Massachusetts, 1915). *Munsell Book of Color* (Munsell Color Company, Baltimore, Maryland, 1929).

[13] F. H. G. Pitt, "Characteristics of dichromatic vision," Medical Research Council, Report of the Committee on the Physiology of Vision, XIV, Special Report Series No. 200 (London 1935).

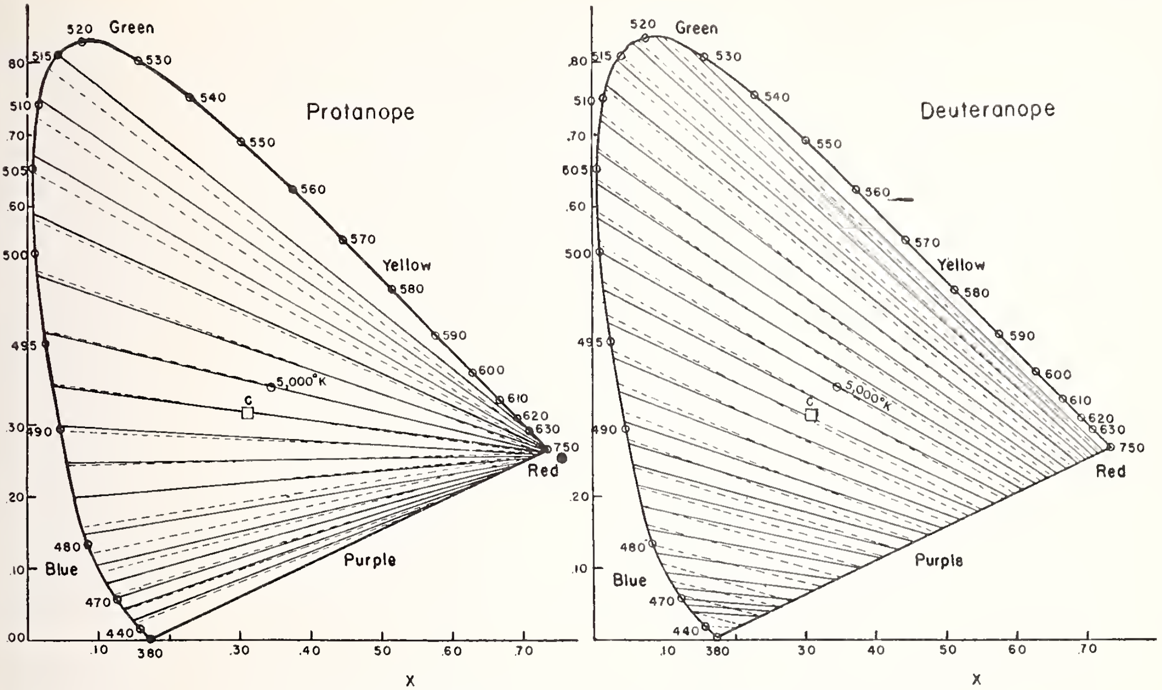

FIG. 3. Chromaticity confusions of the protanope and deuteranope, from Pitt.[13]

METAMERISM SHOWN BY STEARNS TEXTILES AND GRANVILLE GRAYS

Just as dichromats fail to distinguish chips that differ in spectral reflectance (we know that they differ simply from the fact that trichromats perceive them as of markedly different color), so also do trichromats fail to distinguish chips of markedly different spectral character. A pair of chips seen to be identical in color though the light entering the observer's eye from one is known to be spectrally different from that reflected by the other are said to form a metameric pair. Two such pairs were shown, one the well-known pair of green textiles prepared by Stearns,[14] the other, the scarcely less well-known pair of gray paints formulated by Granville[15]; one from white, black, and brown pigments yielding a paint of nearly nonselective spectral reflectance; the other from white, yellow, green, and purple pigments yielding a paint of highly selective spectral reflectance (see Fig. 4).

In artificial daylight these pairs were perceived each to consist of chips of nearly the same color. Then the different spectral character of the two members of each pair was demonstrated by showing them by incandescent-lamp light where the chips of each pair were then perceived as having colors rather markedly different.

Color matches between chips of this sort depend on the amount of lens and macular pigment possessed by the observer. The Maxwell spot may be seen by looking from one member to the other of either of these metameric pairs. Furthermore, chiefly because of variations in lens pigment, no two observers require quite the same kind of artificial daylight to make these pairs color match viewed at close range where the Maxwell spot is easily overlooked because it occupies but a small fraction of the total area of the chips. The problem, basic to colorimetry, of precisely which spectrally different lights are to be regarded as identical in color is solved arbitrarily by the adoption of standard color-mixture functions; and the Granville grays have been used by Kelly[16] in a check of color-mixture functions against the observations of a group of 39 observers who were known by color-perception tests (such as the

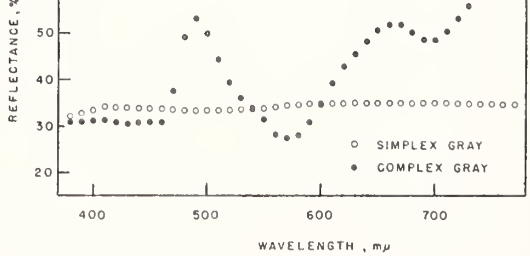

FIG. 4. Spectral reflectance of the Granville grays, from Kelly.[16]

[14] F. C. Dexter and E. I. Stearns, J. Opt. Soc. Am. 38, 816 (1948).

[15] W. C. Granville and D. B. Judd, J. Opt. Soc. Am. 39, 632A (1949).

[16] K. L. Kelly, J. Research Natl. Bur. Standards 60, 97 (1958); RP2825.

Ishihara charts) to be normal trichromats. Figure 4 is is taken from Kelly's report of this work.

CHROMATIC ADAPTATION (SIMULTANEOUS AND SUCCESSIVE CONTRAST)

The normal use of the eye is to provide signals by means of which the observer becomes aware of the objects and lights that surround him, and the normal way that successful signals are produced is by the roving eye, fixating now one and now another object or light in the environment. In this normal use of the eye, its chromatic sensitivity undergoes rapid and continuous changes of primary influence on color perception. To show how important these changes may be, a slide was projected that consisted of red, yellow, green, and blue quadrants. The members of the audience were requested to fixate the center of the slide for about 15 sec, then the slide was removed, and after a second or two there appeared (apparently to each member of the audience) the after-image complementary of the four hues; that is, by means of the retinal area previously exposed to the red quadrant, the white screen appeared blue-green; similarly, other segments of the white screen appeared purplish blue, red-purple, and orange. This is the classic demonstration of successive contrast, or the projection of a negative after-image onto a white surface. This after-image requires fixation for several seconds for its appearance. It moves with the eye, and has no object character.

To show that there is an instantaneous type of adjustment in chromatic sensitivity of the visual mechanism, an orchid plaque was shown against a gray background under which conditions the color perception was that of light purplish red of moderate saturation. Then it was presented against a brilliant magenta background. The color perception was then reported by many in the audience as gray. It was noted that the transition from orchid to gray took place without perceptible time delay, and that the color was stable. Furthermore, the color was perceived to belong to the plaque in the same sense that the light purplish red color of moderate saturation was perceived to belong to it when viewed against a gray background. Note that the Munsell notation of the plaque (about 5RP 6/4) is the same regardless of background; the color perception depends on background. Only for objects viewed in daylight by an observer with normal vision adapted to daylight does the Munsell notation yield the correlation with color perception that gives the Munsell notation its unique significance.

COLOR RENDITION OF LIGHT SOURCES

The problem of what you see when you look at an object viewed in a room lit with an unknown kind of light is the problem of color perception, itself. The observer looks from one object to another in the room and obtains color perceptions not only of the objects but also of the light by which they are lit. Exactly how these color perceptions arise is not yet known. The primary information must come from the field of view, but even if the tristimulus values of each element in the visual field are known, it is impossible to predict reliably what the color perception of any element will be. The previous experience of the observer comes into play, and one well established way that experience makes itself felt is by what is known as memory color; that is, a familiar object tends to be perceived again in accord with its daylight color previously perceived.

With memory color ruled out, prediction becomes possible though difficult. An example of object-color perception not easily predictable from the tristimulus values of the object and its surround was shown. A sample, actually appearing blue in daylight but unknown to the audience, was presented in red light, first with a surround of black velvet, then with a surround of white cardboard. The red light used was incandescent-lamp light filtered by a selenium red glass such as used in traffic signals (plus an unavoidable small amount of stray incandescent-lamp light). The chromaticity points for the unknown chip, and for the black velvet and white cardboard used as surrounds, illuminated by this red light are negligibly different from that for the source, itself. The reflectance of the unknown chip is about 4 percent. The problem is to predict what the color perception of the chip will be with the black-velvet surround (reflectance 0.4 percent) and what it will be with the white surround (reflectance about 80 percent). The first part of the problem is easy; the expected perception is dark red. But what color will be seen with the white surround?

The audience report for the black velvet surround was, as expected, dark red. For the white cardboard surround it was first black, then after several minutes of looking, some members reported blackish blue.

To show how blackish blue might be predicted it was suggested that the way to think of color perception is the projection of an after-image of what the observer had last looked at (the white cardboard illuminated by red light) onto the object whose color is perceived (unknown sample illuminated by red light). There is both a long-term chromatic adaptation such as produces the classical negative after-image, and there is a rapid adjustment of chromatic sensitivity of the visual mechanism such as produces the classical contrast phenomena. The rapid adjustment prevented the unknown object from appearing red (even though its chromaticity coordinates were identical to those of the red light source) and gave rise to the color perception black, or dark gray. The long-term adjustment yielded the hue of the after-image complementary.

A fairly successful attempt has been made to write formulas for hue, lightness, and saturation of the color perception from the tristimulus values of the object being viewed and the tristimulus values of the average of the colors in the immediate vicinity.[17] This formula

[17] D. B. Judd, J. Opt. Soc. Am. 30, 2 (1940).

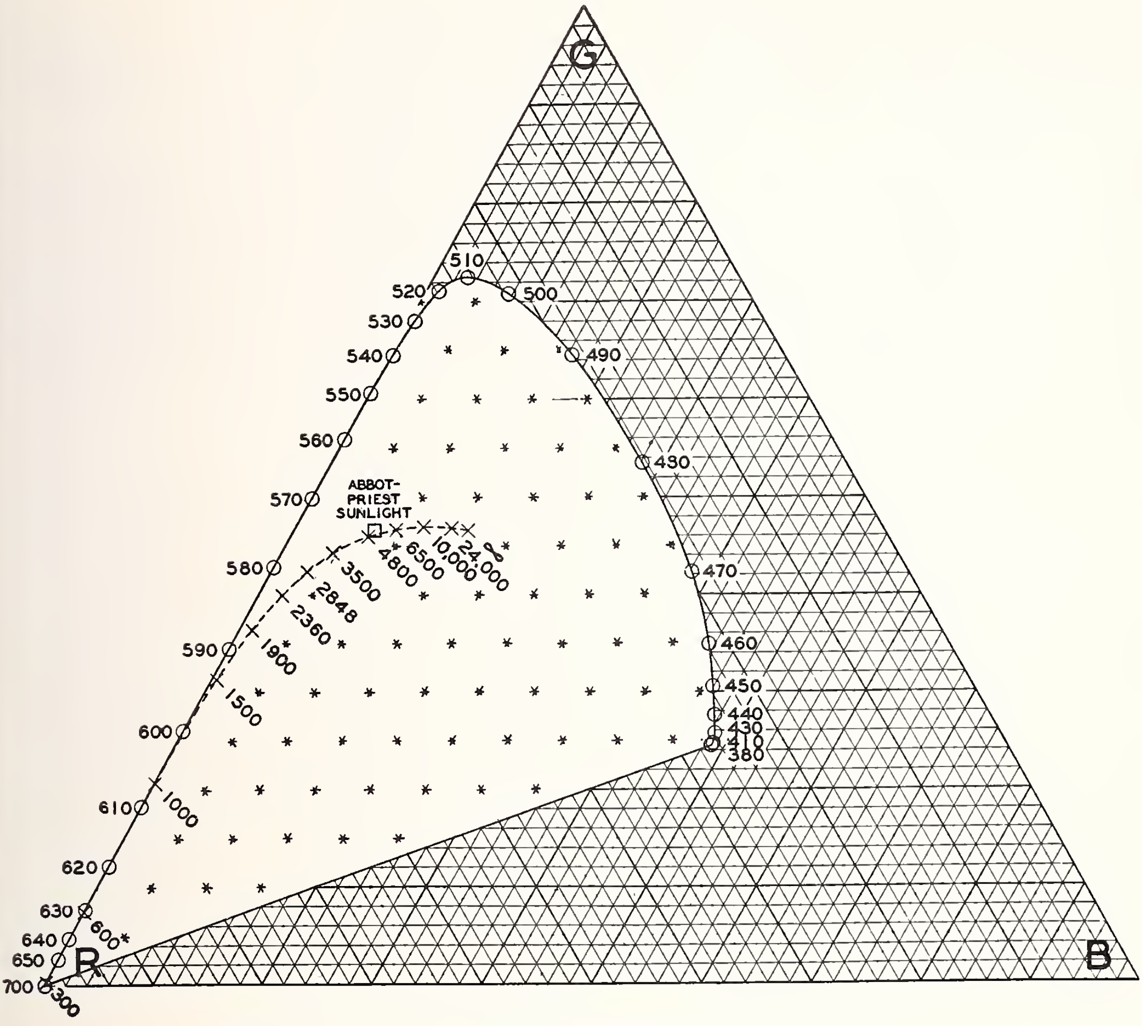

FIG. 5. Maxwell triangle yielding approximately uniform chromaticity scales.

is based on the idea that the chromatic state of the eye effective in a given color perception can be described by specifying the chromaticity coordinates of the point in the UCS triangle (see Fig. 5) that would correspond to the perception of an achromatic color such as gray. For an observer adapted to daylight, the achromatic point would be the point representing the chromaticity of daylight. For an observer looking here and there at objects in a field averaging to a color different from that of daylight, the achromatic point would be somewhere near to the point representing the average chromaticity of the field. It is of interest to note that the reference point tentatively suggested by Land[18] to explain object-color perceptions from two-color projection is precisely the average chromaticity of the field. This suggestion would yield a prediction of "black" or "dark gray" for the unknown sample viewed with the white cardboard surround, which agrees with most of the reports by the audience.

[18] E. H. Land, J. Opt. Soc. Am. 48, 865A (1958).

In the formulas developed, hue is determined by the direction of the vector on the UCS triangle starting from the achromatic point and ending with the sample point, and saturation is determined by the length of this vector. The achromatic point is defined so as to fall in the vicinity of the field point along a locus defined by the ratio of the sample luminance to the field luminance; see dotted line on Fig. 6. The unknown sample point is shown by the square, as is also the field point with either the black velvet or white cardboard surround. The achromatic point for the black velvet surround would fall close to the open circle marked *W*, and the vector, interpreted by the correlation of hue name with angle shown in the left side of the figure, corresponds to yellowish red. For the white cardboard surround the achromatic point would fall somewhere in the neighborhood of the half-closed circle marked *Bk*. The vector from this point to the sample point (square) corresponds to a prediction of "blue".

It should be emphasized that these formulas are

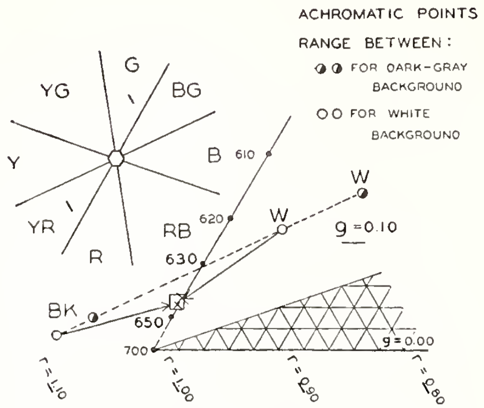

FIG. 6. Prediction of color perceptions of objects seen by red light.

empirical, and do not indicate the processes by which the visual mechanism manages to yield the predicted color perceptions. The formulas are based on the general idea that the projection an after-image of any color onto the same color at a luminance lower by a factor of about 2 produces closely the color perception of gray. Projection of the after-image of this color onto any other color produces perceptions of chromatic colors whose hues and saturations can be computed from the deviation of that color from the color corresponding to the perception of gray.

Just because a set of formulas based on the idea of projection of after-images is reasonably successful does not mean that this is the only mechanism by which object-color perceptions are produced. Marked changes in object-color perception may be produced within time intervals too short for the effect to be ascribed to projection of negative after images. Examples of these effects were shown by Land[18] with two-color projection of slides in which the object-color perceptions were obtained instantaneously with only minor variations being seen on prolonged examination.

Another example of these instantaneous changes constitutes the final demonstration. A white and yellow chip were presented on a black background first by artificial daylight, then by yellow light. The color perceptions were by daylight, white and yellow; then, by yellow light, both chips were seen as yellow. This result might have been predicted by computation with a standard set of color-mixture functions such as the 1931 CIE standard observer which shows the two chips to have identical tristimulus values in yellow light. Then a white cardboard mask was put over the black surround. Immediately both chips were perceived as white. This prediction is given by the empirical formulas[17] based on after-image projection, but no time elapsed such as would permit the sensitivity of the eye to yellow to be reduced nearly to zero. Indeed, the yellow color was still seen, but with the white background the yellow was perceived to belong not to the chips, but rather to the illuminant, and in this way, the object-color, itself, was perceived as white. The change in report would seem to be due to a post-retinal, interpretive process, and this view is supported by reports of rapid changes between the two admissible interpretations: yellow objects illuminated by daylight, or white objects illuminated by yellow light, much as the reversible staircase pictures change from the top view to the bottom view. This interpretive process permits us to see immediately in a high proportion of cases what we will perceive minutes later after chromatic adaptation has had time to reach equilibrium.

APPRAISAL OF LAND'S WORK ON TWO-PRIMARY COLOR PROJECTIONS

J. Opt. Soc. Am. 50, 254-268 (1960)

This critical review of the literature about object-color percep-ion and the relation of the effects demonstrated by E. H. Land to ffects reported and discussed by earlier workers, is another example f Judd's knack of making clear a complicated and poorly understood ubject. No terminological difficulties are present - all specialized erms are clearly explained when first introduced; potentially confusing onnotations are pointed out. The present (1978) understanding of the ffects demonstrated by Land is essentially as presented here - otwithstanding extensive, interesting, related experiments by Land and is associates in more recent years.

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Appraisal of Land's Work on Two-Primary Color Projections

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An analysis of the results of Land's experiments with two-primary color projections has been carried out in terms of the known phenomena of object-color perception. It is shown that no new theory is required for the prediction of Land's result that two-primary color projections can produce object-color perceptions of all hues; nor for his result that many choices of pairs of primaries yield substantially the same object-color perceptions. Land's hypothesis that when the colors of the patches of light making up a scene are restricted to a one-dimensional variation of any sort, the observer usually perceives the objects in that scene as essentially without hue, is new; several special cases of it are supported by previous work as well as Land's. This hypothesis deserves the serious attention of research workers in object-color perception.

I. INTRODUCTION

THE most complete qualitative account ever published of the color perceptions producible by projecting in register two color-separation positives as a function of the colors of the projecting lights was published a few months ago[1] by Land of the Polaroid Corporation, and this paper has been followed by numerous popular accounts, such as that in *Fortune*.[2]

The color-separation positives are produced by taking two photographs, one through a filter transmitting the long-wave third of the visible spectrum (585 to 700 mμ), the other through a filter transmitting the middle third (490 to 600 mμ). The exposures used are such as to produce identical records of the "gray scale" (a series of spectrally nonselective specimens covering in convenient steps the reflectance range from zero to one).

The image of one of these positives (usually the long-wave positive) is ordinarily projected in red light (590 to 700 mμ); the other ordinarily by incandescent-lamp light; but other projecting lights are also used. The two images on the screen are carefully adjusted to be in register.

II. SUMMARY OF LAND'S EXPERIMENTS

Land's experiments involving object-color perceptions together with the results reported may be summarized as follows:

3.[3] By combining the long-wave picture shown in red light with the middle-wave picture shown in incandescent-lamp light results in the perception of full color in a portrait of a blond girl (blond hair, pale blue eyes, red coat, blue-green collar, and strikingly natural flesh tones) in the sense that the rendition is acceptable as a lovely color photograph, faithful rendition not being claimed.

4. These color perceptions arise with no visible time lapse after presentation.

5. These color perceptions are largely independent of the color of the surrounding field.

6. By using these same projecting lights a pair of photographic records of objects on a desk yielded simultaneous perceptions of a yellow pencil, a white pencil, a white marmalade jar, a green-blue book, a blue tax stamp on a package of cigarettes, a pale blue color on a map representing an ocean area, a brown wooden box, a green blotter, a black ink stand, and red lettering on a magazine, it being admitted that addition of a third picture (projected in blue?) yields color perceptions of somewhat higher saturation.

7. By using these same projecting lights (red and incandescent lamp) the color perceptions of auburn hair and green sweater are shown to be largely independent of the angle subtended at the eye of the observer by the picture.

8. By reversing the projecting lights of experiment 7 so that the long-wave picture is projected by incandescent-lamp light and the middle-wave picture by red light, the sweater is perceived to be reddish, the hair greenish, and the lips saturated blue-green.

9. By showing separation positives of twelve objects photographed against a gray background, only one of which (an orange) was of known color, marked consistency in reports of names (including red, gray, and dark blue) for the colors perceived to belong to the objects was found for naive observers having normal color vision.

10. These reports are largely independent of the ratios of luminous flux (flux of red light to flux of incandescent-lamp light) by which the separation positives are projected (sometimes over ranges as high as 100 to 1 in the ratios).

11. These qualitative reports are also largely independent of variations in contrast of the separation positive projected in incandescent-lamp light with contrast of the other separation positive held constant.

12. Substituting yellow-green light (480 to 600 mμ) for incandescent-lamp light for the projection of the middle-wave separation positive of grocery packages changed the color perceived to belong to the background

[1] Edwin H. Land, Proc. Natl. Acad. Aci. 45, 115, 636 (1959).
[2] Francis Bello, Fortune 59, 144 (1959).
[3] The numbers are those used by Land to identify the individual experiments. The experiments omitted from this summary are those in which no objects were perceived.

slightly, but the colors perceived to belong to the objects were not qualitatively changed at all.

13. The long-wave separation positive of a bowl of fruit was projected with extreme red light (650 to 700 mμ); and the middle-wave positive, with yellow light (560 to 600 mμ). It is implied that the colors perceived to belong to the fruit in the projected picture are in reasonable accord with those of the kind of fruit shown.

14. The same color-separation positives were projected with yellow light (560 to 600 mμ) for the long-wave record and blue-green light (400 to 490 mμ) for the middle-wave record, and though the color of the background was perceived as slightly altered, the color perceptions of the fruit remained qualitatively unchanged.

15. The projecting lights were then changed to incandescent-lamp light and green-blue light (380 to 495 mμ), and the foregoing remarks on the color perceptions were found still to apply.

16. It was found that yellow light (560 to 600 mμ) could be used for projecting the middle-wave separation positive (combined with red light for the long-wave positive) or for projecting the long-wave positive (combined with green light for the middle-wave positive), the object-color perceptions remaining substantially the same.

17. Three identical black-and-white transparencies, two in superposition in one projector, one in the other projector, are projected in register on the screen, one with red light, the other with green light. The objects were perceived as substantially black, gray, or white.

18. The middle-wave positive is projected by incandescent-lamp light and on the resulting projected image is superposed a uniform field of adjustable luminance of red light produced by placing no positive at all in the red-light projector. For all luminance ratios (red light to incandescent-lamp) "there appears only a uniform wash of the colors in their classical mixtures."

20. The two separation-positives of objects on a desk (see experiment 6) are projected far out of register. The gamut of color perceptions is strongly restricted; while going beyond the "classically expected" color perceptions, there is only the smallest suggestion of the new colors which here are few and unsaturated. Then the images are slowly brought into registration. At the instant of perfect registration, the "full gamut of color snaps into appearance."

22. A positive is projected by red light (590 to 700 mμ) and a negative of the same scene is projected by incandescent-lamp light. In spite of the fact that the various parts of the scene cover substantially all ratios of red light to incandescent-lamp light, the objects are perceived as essentially without hue.

III. WHAT LAND MEANS BY "CLASSICALLY EXPECTED COLORS"

It should be noted that, in reporting the results of experiments 18 and 20, Land speaks of "classical mixtures" and "classically expected" color perceptions. When a color is perceived as belonging to a patch of light, that is (following the terminology developed from the work of Katz[4]), is perceived in the aperture mode of appearance, there is good correlation between the color of the light (determined, for example, by its dominant wavelength, purity, and luminance) and the perception of it (characterized by hue, saturation, and brightness) when viewed against a dark surround. When Land speaks of the "classical expectation" it would seem that he is referring to this well-known good correlation, as summarized, for example, by Kelly.[5] This is made fairly clear by his account of experiment 1, "The double projector is turned on, a filter transmitting red (Wratten No. 24) is placed over the lens of one projector and the polarizers are rotated to change the observed color of the screen from full white to full red, traversing the various shades of pink in between. Thus the screen is seen in the colors expected on the basis of classical color theory."

A more accurate description of the color perceptions usually reported by a trained observer for the mixtures of red light and incandescent-lamp light viewed against a dark surround would be that they vary from a reddish yellow of low saturation through orange to red with increasing saturation, but since this is precisely what is indicated by the tristimulus values of the mixture colors (or by their dominant wavelengths and purities), no great point need be made of the less accurate description as from "white through pink to red" which is indeed what any unpracticed observer would be likely to report if, as is often done, the incandescent-lamp light, is inaccurately said to be "white" light. The important conclusion is that what Land means by "classical expectations" is the accepted good correlation between the color of the light patch and the perception of its color viewed against a dark surround.

Thus, any report of colors perceived as having hues outside of the range, reddish yellow through orange to red, is contrary to the "classical theory" so defined, and Land's experiments 3, 6, and 9 are quite sufficient to demonstrate that the "classical theory" fails spectacularly to account for reports of color perceptions when the observer is confronted with a complex pattern of the lights obtainable by mixture of incandescent-lamp light with red light. Land's conclusion that "there is a discrepancy between the conclusions that one would reach on the basis of the standard theory of color mixing and the results we obtain in studying total images, and this departure from what we expect on the basis of colorimetry is not a small effect but is complete. . ." does indeed follow from his experiments, and is quite correct. His further remark "that the factors in color vision hitherto regarded as determinative are significant only

[4] D. Katz, Z. Psychol. Ergänzungsbd. 7, (1911); *Der Aufbau der Farbwelt* (Barth, Leipzig, 1930).

[5] K. L. Kelly, J. Research Natl. Bur. Standards 31, 271 (1943), RP1565.

in a certain special case," has, I think, to be qualified. It is true that the color specification of lights obtained by colorimetry in terms of tristimulus values, or in terms of dominant wavelength, purity, and luminance usually accords with the perceptions of those colors only when the patch of light is viewed with a dark surround, and it is also true that this condition is a special case of no great practical interest, but the phrase "hitherto regarded as determinative" implies too much. It implies that nobody has ever before noticed that the color perceived to belong to a patch of light or an object depends on factors other than the radiant flux coming from that patch or that object.

IV. COMPARISON OF LAND'S RESULTS WITH PREVIOUS RESULTS

It is convenient to review what is known regarding color perception under two categories: (1) color perceptions in the aperture mode (that is colors perceived to belong to a hole in a screen and thus nonlocalized in depth); and (2) color perceptions in the object mode (that is, colors perceived to belong to a nonself-luminous object and thus completely localized).

1. Aperture Mode

It has long been known that by looking at a patch of one color for some 10 to 20 sec and then looking at a large uniform gray field, the shape of the first patch is seen in a color roughly complementary to the original. This is known as projecting a negative afterimage, or successive contrast. When two contiguous patches of light are viewed simultaneously with fixation point held steady, the color perceived to belong to each patch is influenced in the direction that would be obtained more plainly by viewing them successively. This is known as simultaneous contrast. Both simultaneous and successive contrast will be recognized as departures from what Land has called the "classical expectation."

The importance of successive contrast in determining the colors perceived to belong to one or another patch of light in a pattern of such patches was known long ago. It is sufficient to quote from Helmholtz,[6] "Even in comparing colored areas with each other that lie side by side in the visual field, successive contrast, that is, contrast caused by after-images, is a very important factor, as anyone can easily verify. It has generally been supposed that in these cases it was simply a matter of simultaneous contrast, because hitherto in the theory of contrast little account has been taken of a certain characteristic of human vision. Under ordinary circumstances, we are accustomed to let our eyes roam slowly about over the visual field continuously, so that the point of fixation glides from one part of the observed object to another. This wandering of the eye occurs involuntarily, and we are so used to it that it requires extraordinary effort and attention to focus the gaze perfectly sharply on a definite point of the visual field even for 10 or 20 seconds. . . . This wandering of the gaze serves to keep up on all parts of the retina a continual alternation between stronger and weaker stimulation, and between different colors, and is evidently of great significance for the normality and efficiency of the visual mechanism. . . . Now let us consider what happens when the eye wanders in this way over a field where there are different colours or areas of different luminosity. If we observe a limited coloured field with the eye accurately focused on some point of it, a sharply defined after-image will be developed, which is therefore easily recognized. If two different points of the object in the same line of sight have been observed for a long time, two well defined after-images will be formed partly overlapping each other; but without special attention they are not now easily recognized as being copies of the object. But if the gaze has moved slowly over the object, without being held on any point naturally the after-image will be simply a faded spot, and it is no longer so easy to recognize, although it is actually there for the attentive observer. Now if the look is transferred to an adjacent field of another colour, this colour of course will be altered by the influence of the after-image, exactly as if we had had these different colours one after the other in the field of vision. Accordingly, in a case like this we do not have simultaneous contrast, at least not by itself; but we have here also successive contrast, and the phenomena are entirely, or in large part, identical with those described in the preceding chapter. . . . Now the phenomena will be described that belong partly to simultaneous, but mainly to successive contrast, as they are manifested under ordinary natural conditions of vision. The colour changes that occur in these circumstances are exactly the same as those already described for pure successive contrast. In general, they are much more distinct and striking than those of pure simultaneous contrast; and when the two might cause different results, those of successive contrast invariably predominate in the natural use of the eye; and when both evoke the same effects, the alterations of colour always become much more considerable when the gaze ceases to be steady and begins to wander."

It is clear that departures from the "classically expected" color perceptions were known many years ago. Helmholtz cites, for example, an account of afterimages by Peiresc in 1634. These departures from any expectation derived solely from the radiant flux leaving the area, itself, were thus known even before the data on which colorimetry is based were foreshadowed by Maxwell's pioneer work[7]; that is, it has long been established that a prediction of the color perceived to belong to a patch of light must be based not only on the color of that patch but also on the colors surrounding it and on the colors previously viewed by the observer, and

[6] H. v. Helmholtz, *Physiological Optics* (Optical Society of America, 1924), Vol. 2, p. 265 ff.

[7] J. C. Maxwell, Proc. Roy. Soc. (London) **10**, 404, 484 (1860).

this fact was known even before a basis for the measurement of the colors of the various light patches had been established. This early qualitative work has recently been elaborated in an impressive series of quantitative studies by Wright,[8] Hunt,[9] MacAdam,[10] Burnham, Evans and Newhall,[11] and Jameson and Hurvich.[12]

Land's position is that chromatic adaptation effects are minor compared to the results of his experiments 14, 15, and 16, and that the color perceptions reported by him appear immediately (see experiment 4) so that there is no time for the development of distortions due to chromatic adaptation. The position is most clearly stated in Land's[1] footnote 1 which states in part, "The work in this paper should not be confused with experiments such as those of Hess which describe how a stimulus of a particular wavelength looks differently colored when viewed after the eye has been exposed to another wavelength for a significant period of time. Nor should it be confused with those theories which explain the phenomenon which Hess measured, and other allied phenomena, as the result of fatigue of the eye to various states of chromatic adaptation. The colors described in our experiments appear immediately, and do not alter appreciably with time, and we do not seek to explain them as effects of adaptation."

Now the position that chromatic adaptation has nothing to do with the perception of the colors of a pattern of light patches is very hard to maintain because adaptation in foveal vision is known to be very rapid (see, for example, the direct determination by Hecht[13]). Just because the observers do not report any large qualitative changes after the first second or so, does not prove that chromatic adaptation caused by the eye roving from one patch of light to another within the first second is not a significant factor. Furthermore, Helmholtz' view that the roving eye is a determinative factor in color perception has received dramatic support by recent studies made with a stabilized retinal image (that is, an image which remains always on the same set of retinal receptors). Riggs[14] and Ditchburn[15] have independently found that light patterns presented in this way (which excludes the possibility of any successive contrast, even that due to small tremors of the eye) become invisible within a time interval somewhere between a few seconds and one minute. There is no doubt, therefore, that color perception depends importantly on the intermittent stimulation of the retinal receptors which is characteristic of normal use of the eyes. Adaptation, either achromatic, chromatic, or both, is an essential part of the act of seeing; there is no vision without adaptation. It is thus very doubtful whether the color perceptions reported by Land are uninfluenced by chromatic adaptation, but it is quite possible that other factors are more important, and perhaps it is in this sense that we should understand Land's statement just quoted that "we do not seek to explain them as effects of adaptation." This brings us to a summary of what is known about color perceptions in the object mode.

[8] W. D. Wright, Proc. Roy. Soc. (London) **B115**, 49 (1934).
[9] R. W. G. Hunt, J. Opt. Soc. Am. **40**, 362 (1950).
[10] D. L. MacAdam, J. Soc. Motion Picture Television Engrs. **65**, 455 (1956).
[11] R. W. Burnham, R. M. Evans, and S. M. Newhall, J. Opt. Soc. Am. **47**, 35 (1957).
[12] D. Jameson and L. M. Hurvich, J. Opt. Soc. Am. **46**, 405 (1956).
[13] Selig Hecht, J. Gen. Physiol. 4, 113 (1921).
[14] L. Riggs, F. Ratliff, J. C. Cornsweet, and T. N. Cornsweet, J. Opt. Soc. Am. **43**, 495 (1953).
[15] R. W. Ditchburn, in *Symposium on Visual Problems of Colour, National Physical Laboratory* (Her Majesty's Stationery Office, London, 1958), Vol. II, p. 415.

2. Object Mode

It is generally supposed that the tristimulus values of a color are relatable to the rate at which radiant energy is absorbed by the three photopigments in the retinal receptors. Furthermore, it is often supposed that when the effects of chromatic adaptation are taken into account the resultant aperture-color perception corresponds to the signals sent out from the retina to the cortex by way of the optic nerve. When, however, a color is perceived to belong to an object, information from a number of light patches has to be integrated in the cortex even to yield the perception of an object; so it is generally assumed that object-color perception may also be an important function of the way the signals received from the retina are processed in the central nervous system. This view permits us to anticipate rapid, almost instantaneous, effects corresponding to different modes of processing. The key to the complicated phenomena observed is that simultaneously with the perception of the object color there must be a perception of the color of light by which the object is seen to be illuminated.

The qualitative facts of object-color perception were well known in Helmholtz' time, and it is hard to summarize them better than to quote from his explanation of them (see reference 6, p. 286 f).

"Colours have their greatest significance for us in so far as they are properties of bodies and can be used as marks or identifications of bodies. Hence in our observations with the sense of vision we always start out by forming a judgment about the colours of bodies, eliminating the differences of illumination by which a body is revealed to us. In Sec. 20 it was noticed that in this sense we make a plain distinction between a dimly illuminated white surface and a highly illuminated grey one. Therefore, we have a certain difficulty about realizing that brightly lighted grey is the same as dimly illuminated white. By some device the intense light must be confined strictly to the grey field, so that we cannot infer from the sense impression that the grey is more highly illuminated than the rest of the field of vision. It is then only that we recognize its identity with white. Just as we are accustomed and trained to form a judgment of colours of bodies by eliminating the different brightness of illumination by which we see them, we eliminate the colour of the illumination also. There is plenty of opportunity of investigating these same corporeal colours in sunshine outdoors, in the blue light of the clear sky, in the weak white light of the overcast sky, in the red-yellow light of the setting sun, and by red-yellow candle light. . . . By seeing objects of the same colour under these various illuminations, in spite of the difference of illumination, we learn to form a correct idea of the colours of bodies, that is, to judge how such a body would look in white light; and since we are interested only in the colour that the body retains permanently, we are not conscious at all of the separate sensations which contribute to form our judgment."

Our ability to perceive with some accuracy the color of the light illuminating the object, to discount this color with remarkable success and so to perceive quite closely the color of the object regardless of the color of the illuminant has been termed *object-color constancy.* This phenomenon has been studied and analyzed in detail by scores of research workers, by Bocksch,[16] by Gelb,[17] and by Jaensch,[18] to mention a few. The chief objections to the Helmholtz explanation are that he spoke of discounting the illuminant color as a judgment instead of treating it as the automatic essential characteristic of object-color perception, and, in general, he seemed to be trying to explain away the difference between object-color perception and aperture-color perception as a systematic and understandable error rather than taking the object-color perception as the phenomenon of central interest. Since, however, Helmholtz (reference 6, p. 285) says that "the acts of judgment here spoken of are always executed unconsciously and involuntarily," the difference in explanation may be taken as chiefly a matter of choice of words. Any "discounting of the illuminant color" is, of course, a direct departure of the color perception from what Land refers to as the "classically expected" result.

The beautiful demonstration described by Land as experiment 20 would be explained according to Helmholtz' views somewhat as follows: When the two separation positives are presented far out of register, the viewer does not perceive any objects, but merely a pattern of light patches; so his report is influenced by successive and simultaneous contrast alone and goes beyond the "classically expected" color perceptions only to a slight extent. When the images are brought into registration, the pattern of lights is perceived as representing objects illuminated by some kind of light. The perceived color of this light is discounted and the resultant object-color perceptions depart very markedly from the colors previously perceived to belong to the individual patches of light, or as Land puts it "the full gamut of color snaps into appearance." Troland[19] gave closely the same explanation, in 1926.

The phrase, *discounting of the illuminant color*, long ago fell into disfavor because it seemed to imply a deliberate intellectual judgment, and the name *color transformation* has frequently been used to indicate the difference between the colors perceived to belong to the patches of light making up a scene and those perceived to belong to the objects depicted by combinations of those patches. Much of the extensive German work on object-color perception is described by this rather special term, such as that by Haack,[20] by Jaensch and Wiegand,[21] by Kravkov,[22] by Kroh,[23] and by Marzynski.[24] Although many of the studies were aimed at clarifying the relation between simultaneous and successive contrast on the one hand, and color transformation on the other, some attempts were made to develop quantitative laws by which object-color perceptions could be predicted such as by Kravkov,[25] by Kardos,[26] and by Pikler.[27] Pikler made a plausible case for the view that the color of the light scattered within the eyeball is directly sensed and is the color discounted in the formation of object-color perceptions. Others argued for the view that the separation of the color of the illumination arises from perception of light scattered by dust particles in the air. A more usual view is that the perception of the illumination color arises from the scene, itself, and should be estimated from the colors of any high lights from glossy objects depicted in the scene, or in the absence of high lights, as some kind of average of the colors of the light received by the observer's eye from the whole scene; this latter view (some kind of average) was tentatively suggested by Land in his talk before the Optical Society of America at the October, 1958 meeting.[28] A good modern summary of the facts of object-color perception was published by Helson,[29] in 1943. He criticizes the term, *color transformation*, on the sound ground that it unnecessarily implies the existence of a true object-color perception (that perceived to belong to the object illuminated by daylight), where the fact is that the object-color perceptions arise from each scene independently, and the set of object-color perceptions derived from any scene is just as valid as that derived from any other. He proposes, instead of *color transformation*, the term *color conversion* to denote any change in color perception of a given

[16] H. Bocksch, Z. Psychol. **102**, 338 (1927).
[17] A. Gelb, Die "Farbenkonstanz der Sehdinge," in Bethe *Handbuch der normalen und pathologischen Physiologie, Receptionsorgane II* (Verlag Julius Springer, Berlin, 1929), **12**/1, p. 594.
[18] E. R. Jaensch, Z. Sinnesphysiol. **52**, 165 (1921).
[19] T. L. Troland, Am. J. Physiol. Opt. **7**, 375 (1926).
[20] T. Haack, Z. Psychol. **112**, 93 (1929).
[21] E. R. Jaensch and F. Wiegand, Arch. ges. Psychol. **85**, 95 (1932).
[22] S. W. Kravkov, Psychol. Forsch. **10**, 20 (1927).
[23] O. Kroh, Z. Sinnesphysiol. **52**, 181, 235 (1921).
[24] G. Marzynski, Z. Psychol. **87**, 45 (1921).
[25] S. W. Kravkov, Psychol. Forsch. **16**, 160 (1932).
[26] L. Kardos, Z. Sinnesphysiol. **66**, 182 (1935).
[27] J. Pikler, Z. Psychol. **120**, 189 (1931); **125**. 90 (1932).
[28] D. B. Judd, J. Opt. Soc. Am. **49**, 322 (1959); Land's tentative suggestion is mentioned on p. 327.
[29] H. Helson, J. Opt. Soc. Am. **33**, 555 (1943).

object due to any circumstances whatever with no implication that any illuminant or particular conditions of viewing are standard.

It may not be evident from this brief summary of more than a century of work on object-color perception that the departure of the object-color perception from the color perception of the patches of light of which it is composed is sufficiently great to justify in a sense the description already quoted from Land that "the departure from what we expect on the basis of colorimetry is not a small effect but is complete. . . ." As a matter of fact, it is easily possible to devise a scene in which an object reflecting light of any chromaticity whatsoever is perceived as gray. The first careful experimental proof that has come to my attention is a part of a basic experiment reported by Helson[30] in which 27 trained observers viewed a gray scale (a series of spectrally nonselective specimens covering in convenient steps the reflectance range from zero to one) by red, yellow, green, and blue light each approximating in color some part of the spectrum. The observers reported some member or members of the scale as yielding the color perception gray regardless of the light-source used. Thus, every part of the Maxwell triangle has under some circumstances to be labeled "gray," and it is certainly true that a knowledge of the color of each light patch making up a scene does not give directly any prediction of the colors perceived to belong to the objects depicted therein. If this is what Land means by saying that "the departure from what we expect on the basis of colorimetry is not a small effect but is complete . . .", we must certainly agree with him, and so would Helmholtz simply from his explanation of object-color perception in terms of discounting the illumination color. The statement can even be extended to aperture-color perceptions by virtue of the comparatively recent work of Helson and Michels[31] who showed that by successive contrast alone a large fraction of the Maxwell triangle can yield aperture-color perceptions without hue. But it would seem that Land's conclusion is intended to mean much more than this. He seems to mean that the tristimulus specification of the colors of the light patches by which an object is depicted has no bearing whatsoever on the colors perceived to belong to the object, and that the facts of color mixture are quite without significance for object-color perception. He restates the first part of his hypothesis (reference 1, p. 637) as follows, "Color in images cannot be described in terms of wavelength and, in so far as the color is changed by alteration of wavelength, the change does not follow the rules of color-mixing theory." This is quite wrong. Color in images can be described in terms of wavelength by making use of the facts of color mixture; and the method was published nearly 20 years ago. Let us return to a consideration of Helson's basic experiment.[30]

Helson did not stop with the observation that a gray object-color perception can be produced by light of any color. His observers also reported their description of the colors perceived to belong to every member of the gray scale in terms of hue, lightness, and saturation, and they did this for each of three spectrally nonselective backgrounds, white, gray, and black. The regularity of the reports showed clearly the underlying principle stated by Helson, as follows, "non-selective samples in chromatic illumination exhibit the color of the illuminant, the color of the after-image complementary to the illuminant, or achromaticity, depending upon the relation of the reflectance of the sample to the adaptation reflectance."

Rather crude empirical formulas for the prediction in terms of hue, lightness, and saturation of the color perception of any object viewed under any kind of light were developed by Judd[32] to take account of the "discounting of the illuminant color" and the principle enunciated by Helson.[30] The discounting of the illuminant color is achieved by defining on the Maxwell triangle a point intended to correspond to the perception of gray in the object mode. By its definition this achromatic point is close to the point representing the chromaticity of the illuminant; so the idea is not too far from Helmholtz' view of "discounting the illuminant color." By Helson's principle, however, this point is also a function of the luminance of the object relative to the average luminance of the scene. The prediction of hue was derived from the direction of the vector extending from the achromatic point on the Maxwell triangle to the point representing the chromaticity of the light coming from the object. The prediction of saturation was taken from the length of the vector. The predictions by these formulas were checked against the reports of six observers on 15 Munsell papers in four widely different illuminations and found[32] to be only moderately successful in duplicating the estimates made in quantitative terms, and they were checked by Kruithof and Bouma[33] against the reports of two observers on the hues perceived to belong to the 100 Ostwald hue-circuit color chips illuminated by incandescent-lamp light, and again the formulas failed to a degree to predict precisely the reports given in quantitative terms. It is still to be expected, however, that the formulas would be successful in predicting qualitative results such as those reported by Land.

It will probably be recongized immediately that an empirical construction which relates the hue of the object-color perception to a direction on the Maxwell triangle relative to an achromatic point that is defined so as to be close to the chromaticity of the light from

[30] H. Helson, J. Exptl. Psychol. 23, 439 (1938).
[31] H. Helson and W. C. Michels, J. Opt. Soc. Am. 38, 1025 (1948).
[32] D. B. Judd, J. Research Natl. Bur. Standards 24, 293 (1940); also J. Opt. Soc. Am. 30, 2 (1940).
[33] A. A. Kruithof and P. J. Bouma, Physica 9, 957 (1942).

TABLE I. Predicted hue, lightness, and saturation of object-color perceptions producible by two-primary projection (red light and incandescent-lamp light).

Density of spot long-wave exposure red light projection							
	0.0	YR 8/11	YR 8/11	YR 8/10	YR 8/9	YR 8/7	Y 9/4
	0.4	YR 6/10	YR 6/10	YR 6/8	YR 6/6	Y 7/3	GY 8/3
	0.8	YR 4/9	YR 4/7	YR 4/5	YR 5/1	G 6/3	G 8/5
	1.2	R 2/8	R 2/4	RB 3/1	BG 4/3	G 6/5	G 8/6
	1.6	R 1/4	RB 1/1	BG 2/4	BG 4/6	G 6/7	G 8/7
	2.0	B 1/2	BG 1/5	BG 2/7	BG 4/8	G 6/7	G 8/7
		2.0	1.6	1.2	0.8	0.4	0.0
		Density of spot, middle-wave exposure incandescent-lamp projection					

the whole scene yields the conclusion independently announced by Land (reference 1, p. 125) that "the color in an image is indeed independent of the over-all flux in the individual component images (in a two-primary color projection), and second, that the color at a point in an image is independent of the wave-lengths of the radiation at that point." What is not so clear is whether these formulas developed 20 years ago will successfully predict the appearance of object-colors of all hues in a two-primary color projection (see, for example, experiments 3, 6, and 9).

Before presenting such predictions it is worthwhile to point out that any reasonable attempt to describe the changes in object-color perceptions in quantitative terms as a function of the time of viewing after presentation of the scene will show that significant changes do occur, and that these changes depend (as Helmholtz stated) on the particular eye movements by which the scene is viewed. I agree with Land, however, that compared to the very gross departures between the colors perceived to belong to the patches of light by which the object is depicted and the color perceptions of the object, itself, the time variations of the latter are small and may as well be neglected in discussion of any purely qualitative results. In this connection it may be worthwhile to quote from my 1940 paper (reference 32, pp. 300–301), "When a daylight-adapted observer enters a room illuminated by light differing chromatically from daylight to an important extent, a rapid chromatic adaption takes place which is fairly complete in about 5 minutes. A *large part* of this chromatic adaptation may occur *immediately*; that is, the observer may immediately react to the surfaces as if they were illuminated by chromatic light, and there is an immediate reverse change either if the observer begins to react again to the surfaces as if illuminated by daylight, or if the samples lose their surface character and are seen as aperture colors." It should be remarked that with a detailed scene depicting several three-dimensional objects, it is all but impossible to see the scene simply as a pattern of light patches, and virtually the only possible perception corresponds to the object mode keyed to whatever illuminant-color perception is generated by the scene.

The colors predicted by the formulas[32] to be perceived to belong to objects depicted in two systems of two-primary color projections have been computed.[34] The first system is closely like that used in experiment 3, red light (610 mμ, $r=0.8169$, $g=0.1830$, $b=0.0001$) for the long-wave record, and incandescent-lamp light (color temperature 2854°K, $r=0.5434$, $g=0.4239$, $b=0.0327$) for the middle-wave record. The second system is fairly close to that used in experiment 15, incandescent light (2854°K) for the long-wave record, and green light ($r=0.3600$, $g=0.6200$, $b=0.0200$) for the middle-wave record. The luminance of the screen by the projectors without slides was assumed to be 10 ft-L. Predictions were computed for 36 objects depicted by a systematic sampling of all possible combinations of optical density in the long-wave and middle-wave records. The chromaticities of these combinations all plot, of course somewhere on the straight line connecting on the Maxwell triangle the chromaticity points of the two primaries. The objects were assumed to be displayed on a dark gray background (luminance 1.0 ft-L) and the average luminance of the scene was taken at 1.5 ft-L. The hue, lightness, and saturation predicted for the object depicted by each of the 36 combinations of the two primaries are shown in Tables I and II. The choice of these combinations accords with uniform square sampling of Land's very useful coordinate system produced by plotting the optical density of the spot on the long-wave record as ordinate against that of the spot on the middle-wave record as abscissa. The rows and columns in Tables I and II thus correspond to horizontal and vertical lines on Land's Fig. 13, thus facilitating comparison.

It will be noted that the predictions in Tables I and II are substantially identical. This accords with what Land reported in his experiments 3 and 15. It may also be noted that these predictions are in good agreement with the summary of Land's results presented in his Fig. 13. The color perceptions predicted along the diagonal (lower left to upper right) corresponding to equal densities in the long-wave and middle-wave records are most of them near neutrals (10 of the 12 predictions are for saturations of 2 or less) much as reported by Land. The predicted hues cover the entire hue circuit

[34] I am glad to acknowledge the assistance of my colleague, Dr. Gerald L. Howett, who carried out the computations.

Table II. Predicted hue, lightness, and saturation of object-color perceptions producible by two-primary projection (incandescent-lamp light and green light).

Density of spot long-wave exposure incandescent lamp projection	0.0	YR 8/7	YR 8/7	YR 8/6	YR 8/5	YR 8/3	GY 9/1
	0.4	YR 6/7	YR 6/6	YR 6/5	YR 6/3	GY 7/1	G 8/4
	0.8	R 4/7	R 4/6	YR 4/3	N 5/	G 6/4	G 8/6
	1.2	R 2/6	R 2/3	N 3/	G 4/3	G 6/6	G 8/7
	1.6	R 1/4	N 1/	BG 2/3	BG 4/6	G 6/7	G 8/8
	2.0	RB 1/1	BG 1/3	BG 2/6	BG 4/7	G 6/8	G 8/8
Density of spot, middle-wave exposure green-light projection		2.0	1.6	1.2	0.8	0.4	0.0

(red, yellow-red, yellow, green-yellow, green, blue-green, blue, and red-blue) just as reported by Land.

The outstanding discrepancy between the prediction and the reports by Land is his report of "light blue" where the object-color perception predicted by the Helson-Judd formulation is G 6/3 or G 6/4 (lightgreen). We will return to this later.

It may also be noted that Table I predicts object-color perceptions of higher saturation in the yellow-red hue (up to 11) than Table II (up to 7), and of higher saturation in the green hue in Table II (up to 8) than in Table I (up to 7); that is, by using a red projection light with incandescent-lamp light more vivid red color perceptions may be produced, and by using a green projection light with incandescent-lamp light more vivid green color perceptions are produced. Qualitative trials by my colleague McCamy seem to confirm this and leads to the suspicion that the formulas derived 20 years ago for all kinds of scenes not only give more detailed information but also are more reliable in some respects for predicting object-color perceptions producible by two-primary color projections than the new coordinate system proposed by Land particularly for this special case.

The agreement between Land's experimental results and computed results such as those given in Tables I and II disproves the first part of Land's hypothesis that color in images cannot be described in terms of wavelength. They can be so described in detail, in more detail than given by Land's coordinate system derived for one particular set of two-primary color projections, and with equal, or perhaps greater reliability. Furthermore, the assessment of the wavelength composition of the light patches by which the object is depicted is carried out by the "classical theory of color mixture." This description cannot be achieved from the facts of color mixture alone; the Helmholtz principle that the illuminant color be discounted, and the Helson principle involving ratio of the luminance of the light patches by which the object is depicted to the average luminance of the scene, have both to be taken into account. It would seem that lack of familiarity with the Helmholtz and Helson principles has led Land to conclude erroneously that the facts of color mixture play no role in object-color perception. The fact that the tristimulus values of the color of a light patch correlate well with the color perceived to belong to that patch when viewed with a dark surround is but a minor factor in their merit. Their chief usefulness lies in the fact that by means of tristimulus values all the infinity of light patches of different wavelength composition that look alike are identified by a single triad of numbers. Whatever perceptions may be aroused by one of these light patches in a given set of viewing conditions will be aroused by all. To ignore tristimulus values of the light patches in a study of object-color perception is to make the task far harder than necessary, and the results much less general.

Nevertheless, ignoring tristimulus values, Land has carried out (reference 1, p. 636) a comprehensive study of the hues of the object-color perceptions found to be producible in two-primary color projections for all possible combinations of spectral primaries. The results of these studies are summarized[1] in Land's Fig. 3, p. 638. The main features of these results can be derived from the Helson-Judd formulation.[30,32]

Note first that the most striking feature of this summary, the "short-wave reversal," is very directly explained by this formulation. Since Land's experiments were based on two records, a long-wave record and a middle-wave record, the chief determinant of whether "the sensations in the synthetic situation are in the same color-order as they were in the original situation" is whether original reds are rendered as reds in the image and original greens as greens. Since by the Helson-Judd formulation the determinant of hue is direction on the UCS projection of the Maxwell triangle (see Fig. 1, same as Fig. 5 of reference 28), and since the direction defined in Fig. 2 (same as Fig. 6 of reference 28) to accord with zero red to green difference (that is, yellow to blue) is closely the horizontal direction, the correct hue order (red to green) will accord with the order: long-wave projecting light to short-wave projecting light when those lights are chosen from the long-wave part of the spectrum throughout the wavelength region in which the direction of the spectrum locus has any vertical component. Figure 1 shows this wavelength region to be 700 to 510 mμ. In the remainder of the spectrum (510 to 380 mμ) there is also a vertical component, but in this portion decreasing the wavelength

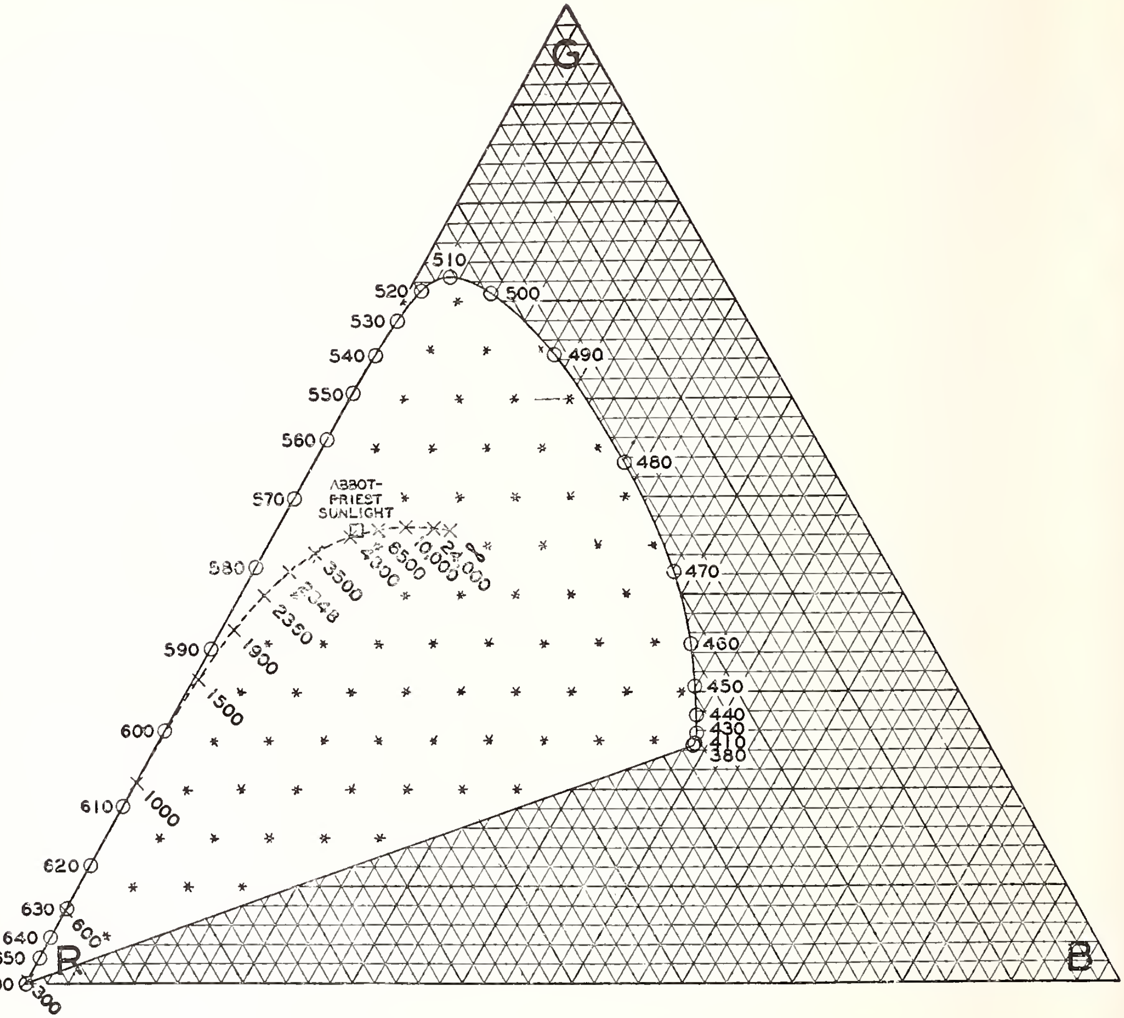

FIG. 1. The uniform-chromaticness-scale UCS triangle [same as Fig. 1, J. Research Natl. Bur. Standards, 14, 41 (1935), RP756; also J. Opt. Soc. Am. 25, 24 (1935)]. This is a projective transformation of the 1931 CIE chromaticity diagram such that the distance between any two points is approximately proportional to the perceptual importance of the chromatic difference between the colors represented by those points.

corresponds to increasing redness instead of decreasing redness. The Helson-Judd formulation thus predicts concordance with the correct hue-order for projecting lights chosen from the wavelength region 700 to 510 mμ, and the reversal of that order from 510 to 380 mμ. Compare this prediction to Land's report (reference 1, p. 639), "When a wave-length from 400 to 435 is used as a short stimulus, with one from 440 to 510 as the long, there is the reversal noted in the lower left hand corner of the diagram; the objects which were red in the original scene are a vivid green, and the green objects appear to be a somewhat purplish red." Land's results are seen to confirm this prediction of the Helson-Judd formulation in a very striking way. By noting the range of slopes of the spectrum locus on the UCS triangle within the wavelength range 510 to 380 mμ and comparing them with the definition of hue given in Fig. 2, it is seen that the prediction is that red objects will be rendered with yellow-green to green hues and green objects will be rendered in red to red-blue hues. The Helson-Judd formulation thus not only predicts correctly the wavelength range producing reversal of the color order but also predicts correctly that the original greens will be rendered as reds inclining toward blue.

Note also that the wavelength difference corresponding to the "achromatic" wash also corresponds closely to a fixed length along the spectrum locus of the UCS triangle.

It may also be worthwhile to point out what the prediction of the Helson-Judd formulation is for the wavelength of the spectrum primary between 570 and 590 mμ to be combined with incandescent-lamp light (taken as CIE source *A*, color temperature 2854°K. $C_2 = 14\ 380$) as a second primary to make the red objects depicted in the two-primary color projection be perceived as neither reddish nor greenish. From the definition of hue by angle on the UCS triangle (see Fig. 2), the direction identified with a yellow neither reddish nor greenish is, as just noted, very close to the horizontal. By drawing such a nearly horizontal line on the UCS triangle (see Fig. 1) through the point for CIE source A (here identified as of color temperature 2848, $C_2 = 14\ 350$) and extending it to the spectrum locus, we obtain a close estimate of this wavelength, a more precise estimate requiring a statement of the luminance ratio between object and average of the scene. We see that this approximate estimate is 581 mμ. According to Land's report, "The remarkable fact is that many observers of various ages and races, given the simple instruction to find the point where the red object turns to green, have all set the wavelength dial to within one or two millimicrons of 588. From this consistency and precision we have learned that the eye must have a fantastic mechanism for finding a balance point within a band of wavelengths." From the discrepancy between the predicted wavelength (581 mμ) and that (588 mμ) reported by Land, it would seem likely that either Land's incandescent-lamp light was set at a color temperature lower than 2854°K, or else the definition of a yellow hue as corresponding to a nearly horizontal direction on the UCS diagram is open to a minor revision. I agree that the eye must have a fantastic mechanism for finding this balance point, and I hold the view (or, at least, entertain the hope) that the Helson-Judd formulation describes its basic character.

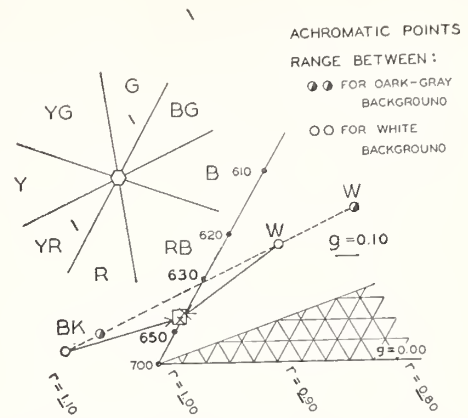

Fig. 2. The definition of hue according to the Helson-Judd formulation in terms of direction of a vector on the UCS triangle [same as Fig. 2 of J. Research Natl. Bur. Standard, **24**, 293 (1940), RP1285; also J. Opt. Soc. Am. **30**, 2 (1940)]. The vector originates at the chromaticity point corresponding to an achromatic object-color perception, and terminates at the chromaticity point corresponding to the object whose hue is to be predicted. The locus of achromatic points defined for red illumination by the Helson-Judd formulation is shown as a dotted line. The illuminant point is shown by a square near 645 mμ on the spectrum locus, and the extreme chromaticity range of the test objects is indicated by two crosses within this square. Two vectors, both referring to white surroundings, are shown. One corresponds to an object (daylight color: white, yellow, light pink, or brilliant red) reflecting almost all long-wave incident light, and predicts a yellow-red hue. The other corresponds to an object (daylight color: black, deep blue, or deep green) reflecting very little long-wave incident light, and predicts a blue hue.

V. MEMORY COLOR

We come now to a consideration of another well established factor in object-color perception, known as memory color. As the name implies, the phenomenon is that when a familiar object is depicted in a scene the color perception of it tends to be changed in the direction of the color previously perceived to belong to that object. This phenomenon has been studied with some care by Adams.[35]

Land alludes to memory color in connection with his experiment 8 by pointing out that the experiment demonstrates that these reported colors (red sweater, green hair, and blue-green lips) are independent of what the observer expects. He recognizes the pertinence of memory color also in his description of experiment 9 by pointing out that one of the objects (an orange) was such that the observers had prior association of the color with the object. Whenever there is a report distinctly at variance from what would be expected from the pattern of colors presented (see Table I), it is legitimate to inquire whether memory color might not have contributed significantly to the perception. Perhaps, the report of "light blue" already mentioned instead of the expected "light green" may be explained by memory color. Perhaps this report came from experiment 6 in which one of the objects shown was a blue tax stamp on a package of cigarettes, or perhaps it came from the pale blue color on a map representing an ocean area. Tests made with the scene consisting of an array of unknown color swatches would settle this point.

[35] G. K. Adams, Am. J. Psychol. 34, 359 (1923).

VI. WHAT NEW FACTS HAVE BEEN DISCOVERED BY LAND?

The foregoing comparison of Land's results with previous results indicates that most of the phenomena reported by him agree with previous reports. The first part of Land's hypothesis (that "color in images cannot be described in terms of wavelength") is shown to be wrong, and the second part (that we need chiefly to consider the information in the long-wave and middle-wave records to arrive at a prediction of the object-color perceptions) obviously yields less general and less detailed predictions than a previously published method (the Helson-Judd formulation). The predictions are less detailed because they are qualitative rather than quantitative and say nothing about the saturation of the chromatic object-color perceptions. The predictions are less general becaues they refer only to a particular set of two-primary color projections, and because they do not indicate the effect of choosing pairs of primaries not within this set. It is nevertheless remarkable that in a few years of intensive study Land, and his associates, should have been able to rediscover independently so large a fraction of the known phenomena of object-color perception, and it is not too surprising that he should have been led by them to the false hypothesis that color in images cannot be described in terms of wavelength. The literature of object-color perception is studded with hypotheses that have later been found unacceptable. The second part of the hypothesis (that we need chiefly to consider the information in the long-wave and middle-wave records) is not too dissimilar from the old, disproved hypothesis of the constancy of object-color perceptions regardless of the color and amount of the illumination. The color-constancy hypothesis really states simply that it is hard to fool an observer even though incomplete information is provided him for the object-color perception; similarly Land's hypothesis that we need chiefly to consider the information in the long-wave and middle-wave records states that it is hard to fool an observer even though no short-wave information is given to him. The hypothesis that middle-wave and long-wave information provides *all* the information necessary to determine the object-color perception is obviously doomed to failure; but like the equally untenable hypothesis of absolute color constancy of visual objects it has obviously been very fruitful in the sense of suggesting valuable experiments, and we are indebted to Land for pointing out how much can be learned about object-color perception simply from two-color projections of middle-wave and long-wave records. We are indebted to him even more for presenting the phenomena so clearly and dramatically in a series of demonstrations involving simply two-primary color projection. By these demonstrations thousands of people have been introduced to the fascinating facts of object-color perception previously hidden from all but a few score students of this specialized subject, and scores of researches that otherwise would never have been undertaken at all will undoubtedly utilize Land's technique. His oral and written presentations are to be criticized chiefly because of lack of any serious attempt to relate the results of his experiments to previous studies of the same phenomena. I hope that the present appraisal makes up in some measure for this lack in Land's presentation.

But is it true that no new facts have been disclosed by this extensive re-examination of object-color perception by a quite novel approach? I would like to summarize now the facts that I have learned about for the first time from Land's demonstrations and publications. Perhaps these facts, too, have already been discovered, and others, more familiar than I with the literature of object-color perception, will say that these, too, have been reported before, but this does not diminish my debt to Land.

1

Land has discovered that astonishingly satisfactory color pictures in which are perceived object colors of all hues can be produced by a wide variety of choices of projecting lights by two-primary color projection, and the object-color perceptions are substantially independent of this choice. The fact that certain two-color systems produce such pictures has been publicized repeatedly since 1897. Ducos du Hauron[36] wrote, "During my last researches I have discovered a marvelous law by virtue of which an image, composed of only two monochromes, is capable of producing on the visual organs, under certain conditions, a colored sensation as complete as the trichromatic images, which I have already obtained. . . . Observers actually believe that they see the three colors where they ought to be, although they know that they are actually absent." In 1926 Troland[37] wrote, "It has long been a matter of surprise to physicists that very satisfactory photographs in natural colors could be obtained by the use of only two primary colors. Nevertheless, two-color reproductions frequently give an impression of rendering all of the colors of the scene. . . . Regardless of what our intellectual processes may be, in viewing a two-color photograph under the given conditions we nevertheless have the perception of certain colors which are not actually represented in the physical stimuli."

On March 5, 1943, it was demonstrated before the Optical Society of America by Evans[38] that by using a red primary and a yellow primary a two-color additive picture could be produced that yielded object-color perceptions of greenish blue and orange. The fact that such pictures can be produced without substantial change by a wide variety of choices of pairs of projecting lights can be derived from the accepted principles of object-color perception, and, indeed, by means of formulas which I myself developed; but I did not recognize this implication of my own formulas, and would never have thought to search for it within them, had I not seen the Land demonstrations.

2

Land has discovered that in evaluating the illuminant color to be discounted so as to arrive at a valid predic-

[36] Ducos du Hauron, *La Triplice Photographique* (Paris, 1897), p. 214.

[37] See reference 19. That Troland had anticipated Land's experiment 20 by more than 30 years is shown by this quotation (p. 376 ff). "A very interesting breakdown of the illusion occurs when the color pictures are badly out of register. . . . When this lack of register becomes sufficiently bad, the entire picture assumes the aspect of a patchwork of red and green without any important variations in hue. . . . The out-of-register picture represents nothing with which we are familiar and consequently we cannot draw upon the color correcting and stabilizing mechanisms which have to be established in the brain by past experience. The observation is one of the most convincing ones which the writer has ever seen to demonstrate the very important part which central activities play in perception."

[38] R. M. Evans, J. Opt. Soc. Am. 33, 592 (1943).

tion of object-color perception, only the scene in which the object is observed should be assessed; other scenes within the visual field are irrelevant. This conclusion is proved by Land's experiment 5, and is pointed out by Land with this very penetrating comment, "This experiment gives the first premonition that multiple color universes can coexist side by side, or one within another."

I am prepared to accept this conclusion. I see that it follows from Helmholtz' view that an essential basis of object-color perception is discounting of the illuminant color, and on rereading Helmholtz[6] (p. 287, "Hence, with all coloured surfaces without distinction, *wherever they are in the sphere of the coloured illumination*, we get accustomed to subtracting the illuminating colour from them in order to find the colour of the object," italics mine) I find almost a previous statement of this conclusion; but I never appreciated this meaning, though perhaps others have. This is an important principle, and I anticipate that its application to previous and future experimental work will be most clarifying to me. It shows immediately, for example, that Pikler's[27] explanation of object-color perception, already cited, in terms of the directly sensed color of light scattered within the eye-ball must be rejected, likewise the explanation based on light scattered by dust particles in the air.

3

Land has discovered that in a scene depicting objects shown by two-primary color projection, the objects will be perceived as having essentially no hue if the amounts, C_1 and C_2, of the primaries in all portions of the scene conform to the relation: $\log C_1 = a \log C_2 + b$, regardless of the choice of the values assigned to the constants, a and b. This sweeping generalization, though perhaps not proved, is made very plausible by the results of Land's experiments 17, 18, and 22. This is a very exciting generalization and deserves the serious attention of students of object-color perception.

Some special cases of this generalization are well known. One group of such cases is found by restricting C_1 in such a way that it bears a constant ratio k to C_2; then $\log C_1 = \log C_2 + \log k$, and we see that this group of cases is obtained by setting $a=1$. In these cases all parts of the scene have the same chromaticity, and by discounting the average color of the scene as the illuminant color, we arrive at the prediction that, in scenes so presented as to minimize the effects of successive contrast, the objects should be perceived essentially as having no hue. That is, although the whole scene may be perceived as chromatic, the signals arriving through the optic nerve are so processed as to ascribe the chromatic character entirely to the illuminant, leaving no chromatic character to the object-color perceptions. By this prediction, the reports by Helson already cited[30] of chromatic perceptions attached to objects in field of constant chromaticity have to be ascribed to the influence of successive contrast, which Helson's experimental conditions certainly did not minimize. (The influence of successive contrast at constant chromaticity has been studied by Troland[39] and is sometimes referred to as the "dimming effect.") Note that here, like Land, "we do not seek to explain these phenomena as effects of adaptation."

A second group of special cases of Land's generalization is found by holding C_1 constant while permitting C_2 to vary over a wide range, and is described by the general formula by setting $a=0$. This corresponds to viewing a black-and-white picture through a glass reflecting a uniform luminance of color C_1 as for example, in viewing objects in a show window with a sunlit red brick wall behind the observer on the other side of the street. Any reports of object-color perceptions all having no hue would probably be explained by pointing out that these scenes may be perceived in two ways: (1) as if viewed directly in which case the dark objects would be perceived as of different chromaticness than the light objects, and (2) as if viewed through a film of colored light, in which case the objects would be perceived uniformly as without hue. My colleague McCamy has made a preliminary study of such scenes by the method used in Land's experiment 18. Most of the reports agree with the second kind of perception much as reported by Land, "One sees the wash of red light over the image from the short projector." Some report the dark objects perceived as redder than the light objects, which accords with the first kind of perception. How can we determine which of a number of interpretations of an ambiguous visual field will be most commonly perceived? The best answer that has been available heretofore is that ascribed to a most gifted student of visual perception, Dr. Adelbert Ames, "What the eye sees is the mind's best guess as to what is out front." Perhaps Land's generalization will prove to be a reliable guide as to what the "mind's best guess" will be.

The crucial experiment which led Land to propose his generalization is experiment 22, which corresponds to setting $a=-1$. The projection in red light of a positive of a complicated scene in register with a negative of the same scene projected in incandescent-lamp light yields various kinds of perceptible regularity depending on the flux ratios of red light to incandescent-lamp light by which the image of the scene is formed. If the luminous flux ratio is close to unity, the picture is robbed of its principal light-dark variations. Any picture with greatly reduced brightness contrast is known to be incapable of showing any distinct object character. See, for example, a demonstration published by Evans[40] showing a picture with full chromatic variation but greatly reduced brightness variation. This picture is hard to see as representing a girl sitting in front of a

[39] L. T. Troland, Am. J. Psychol. 28, 497 (1917).
[40] R. M. Evans, *An Introduction to Color* (New York, John Wiley & Sons, 1948), Plate VI, opposite p. 144.

haystack, but it is easy to see as a pattern of colored areas. As Evans puts it, "It is evident that greater perception of depth is given by variations in relative brightness than by variations in color with brightness differences decreased." Thus, if the luminous flux ratio is close to unity, the accepted explanation of Land's report that "there are only reds, and whites, and a little pinkness" would be that this picture does not give object-color perception with its discounting of the illumination color, but only aperture-color perception.

On the other hand, if the luminous flux of the red light considerably exceeds that of the incandescent-lamp light, experiment 22 would yield considerable brightness variation, and this explanation does not apply. Why does the scene not show light objects as red and dark objects as green as indicated by Land's Fig.13, or, indeed, as indicated by the Helson-Judd formulation? The student of object-color perception would have to say that perhaps this correlation of all the light parts of the scene in red light and all dark parts in incandescent-lamp light is perceived and causes the perception of the dark objects as green to be the "mind's second best guess."

It is interesting to note that the accepted explanation of the substantial absence of chromatic object-color perceptions for $a=1$, $a=0$, and $a=-1$, are all different: $a=1$ corresponds to discounting of the color of the illumination, $a=0$ corresponds to the perception of an intervening film of colored light, and $a=-1$ corresponds either to the absence of perceived objects, or to the perception of a transcending regularity among the colors of the light patches making up the scene. All of these might be summed up by saying that it is very hard to fool an observer. Land's generalization in the more sweeping nonlinear form may perhaps be legitimately restated by saying that when the colors of the patches of light making up a scene are restricted to a one-dimensional variation of any sort, the observer usually perceives that regularity instead of the object-color chromaticness otherwise to be expected. Whether or not this generalization is finally verified, it will surely be the stimulus and guide for many future researches in object-color perception.

VII. CAN A TWO-PRIMARY COLOR PROCESS YIELD PERFECT COLOR RENDITION?

Regarding two-primary color processes, Land has posed the two-part question (p. 637), "How does the sensation elicited by the stimuli at a given place in the image compare with the sensation the observer would have had when he was looking at the subject being photographed; are the sensations in the synthetic situation in the same color-order as they were in the original situation?" No answer is offered for the first part, but the second part is answered in the affirmative for a large variety of choices of pairs of primaries taken from the spectrum. This answer has apparently caused a number of science writers in popular magazines to mention Land's work as providing the basis for perfectly faithful two-color photography, two-color printing, and two-color television, and to ask how it has happened that color technology has unnecessarily saddled us with three-color photography, three-color printing, and three-color television.

It is easy to show that two-primary color processes must fail to yield faithful color rendition to an extent essentially greater than the all too large departures from reality afflicting current three-primary color processes. By taking a long-wave record and a middle-wave record of a scene, each element or light patch making up the scene has its color specified by two numbers only, which we may take as D_l and D_m, the density of the positive taken by long-wave energy, and that of the positive taken by middle-wave energy, respectively. But since the colors of light patches require three numbers for their complete specification, there exist for each pair of numbers, D_l and D_m, a whole series of light patches whose colors are perceived to be different, many of them grossly different. For example, long-wave and middle-wave records of a scene might easily be taken so that a dark gray tree trunk would correspond to $D_l=D_m=1$ (that is, this patch of both records transmits 10%). But there are many other colors that might occur in the original scene that would be represented by identical patches in the records. For example, a yellow-green leaf might photograph both through the long-wave filter and through the middle wave filter to produce spots on the positives transmitting 10% of the incident flux. Also the petal of a purple flower might be such as to be represented by precisely identical spots, $D_l=D_m=1$. Now it is evident that the distinctions between these three original colors (dark gray, yellow-green, purple) have been lost by taking only these two records, the long-wave record and the middle-wave record. Nothing done by choice of projecting lights, or the processing of the signals in the cortex of the observer can make these spots viewed in similar surroundings appear different unless the observer recognizes the objects depicted and has his perceptions changed by memory color. Except for memory color, the observer must perceive the colors of the treetrunk, the leaf, and the flower as identical. They all correspond to the same tristimulus values; so the Helson-Judd formulation has to predict identical object-color perceptions of them. They all correspond to one and the same point on the Land[1] coordinate system (his Fig. 13); so according to Land's own reports they produce identical dark grays. If the leaf and petal were shown against the tree-trunk as a background, even the shapes of leaf and petal would be indistinguishable. This is a handicap that does not apply to three-primary color processes. If the scene includes a reasonably complete sampling of the triple manifold of colors perceptible by an observer with normal color vision, two-primary color processes cannot produce a faithful color reproduction, and Land has never stated that they do.

Let us now answer by means of the Helson-Judd formulation the first part of Land's question which he did not answer. "How does the sensation elicited by the stimuli at a given place in the image compare with the sensation the observer would have had when he was looking at the subject being pnotographed?" Table I shows the range of object-color perceptions possible in two-primary color projections when red light and incandescent-lamp light are used in the projection. As pointed out by Land these include perceptions of all hues. If the scene being photographed includes only these colors, the answer to Land's so far unanswered question is that the observer may indeed obtain from the projected image object-color perceptions closely like those produced by looking at the scene being photographed. In spite of the fact that these perceptions include all hues, the fact that the whole range can be systematically laid out on a two-dimensional graph proves that this range of possible object-color perceptions is only a two-dimensional array as would be expected from the fact that only two records are used to project it. If the scene to be photographed includes any color outside this range, this color will be perceived in the scene, if at all, only by virtue of memory color. For example, if the scene shows a man in an olive-drab uniform (Y 4/4), next to a bed of purple iris (RB 4/6), with green grass (GY 5/8) in the foreground, and blue sky (B 9/4) in the background, about the best that this scene could be rendered by all possible mixtures of red light with incandescent-lamp light would be to show a man in a brown uniform (YR 4/4), next to a bed of purplish black (RB 1/1) iris, with blue-green (BG 5/8) grass in the foreground, and pale green (G 7/4) sky in the background.

Other choices of pairs of primaries corresponding to substantially different directions on the UCS diagram might render this scene satisfactorily, but must necessarily yield a two-dimensional array of object-color perceptions; so each of these choices would have its own characteristic limitations. A two-dimensional array of object-color perceptions of all hues is by no means equivalent to the full three-dimensional array.

Land has reported beautiful renditions of certain scenes involving all hues as producible by two-primary color projection. He mentioned a certain scene (experiment 6) whose rendition was only slightly improved by adding a third picture. All of this may be correct. Land never claimed faithful color reproduction of all scenes by two-primary color projection, nor to my knowledge application of two-primary color processes to photography, printing, or television.

Perhaps it is worth noting that the scenes studied by Land are all interior scenes. If such scenes are viewed by candlelight, object-color perceptions possessing all hues may be obtained. These scenes involve light patches essentially varying only in the long-wave to middle-wave sense, the importance of the middle-wave to short-wave variations being very minor because of the relatively small fraction of short-wave radiant flux emitted by a candle. What Land has showed us is that the object-color perceptions of scenes viewed by candlelight do not depend essentially on the small amount of middle-wave to short-wave information present; they can be closely duplicated by presenting the long-wave to middle-wave information alone. The faithfulness of color reproduction possible from a long-wave and a middle-wave record of a scene is close to, but cannot exceed, what is perceptible by viewing the scene by candlelight.

VIII. SUMMARY

An analysis of the results of Land's experiments with two-primary color projections has been carried out in terms of the known phenomena of object-color perception. It is shown that no new theory is required for the prediction of Land's result that two-primary color projections can produce object-color perceptions of all hues; nor for his result that many choices of pairs of primaries yield substantially the same object-color perceptions. It is true that measurement of the colors of the light patches depicting objects in a scene does not yield directly a valid prediction of the colors perceived to belong to the objects; but by taking into account principles developed during the last 100 years, color measurements can be made to yield valid qualitative predictions of object-color perceptions.

By two-primary color processes, some scenes may be reproduced showing objects perceived to have colors of any hue. This was known as early as 1897, and has been repeatedly pointed out since.

Projections from the same two photographic records may result in images producing substantially the same object-color perceptions regardless of choice of pairs of projecting lights within a wide range. This was discovered by Land, but is derivable from previously developed principles which also indicate which pairs of projecting lights give substantially identical results.

The object-color perceptions derivable from two-primary color projections constitute, as might be expected, two-dimensional arrays. Scenes showing reasonably complete sampling of the triple manifold of perceived object colors must necessarily include pairs of object colors very different in appearance that will be rendered alike by any two-primary color process.

When the colors of the patches of light making up a scene are restricted to a one-dimensional variation of any sort, the observer usually perceives the objects in that scene as essentially without hue. Land enunciated this principle and his work has gone far toward proving it. This principle deserves the serious attention of research workers in object-color perception.

ACKNOWLEDGMENTS

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A FIVE-ATTRIBUTE SYSTEM OF DESCRIBING VISUAL APPEARANCE

Am. Soc. Test. Mater. Spec. Tech. Publ. 297, 1-15 (1961)

This little monograph established terminology that has become customary in both science and technology. The only obsolete detail that can be improved is the third example of illuminant mode, on p. 12. A modern example, to replace radium paint, might be the light-emitting diode (LED) that is used in hand calculators, electric watches and other electronic display devices.

A FIVE-ATTRIBUTE SYSTEM OF DESCRIBING VISUAL APPEARANCE

By Deane B. Judd[1]

Synopsis

This is an extension of the three-attribute system of color description based on the perceptual studies of Katz. The nonshape, nontextural aspects of visual appearance are classified according to four modes of appearance: aperture, illuminant, volume, and surface modes. The aperture mode has three attributes: hue, saturation, and brightness; the illuminant mode, four: hue, saturation, brightness, and transparency; the volume mode, four: hue, saturation, lightness, and transparency; and the surface mode, five: hue, saturation, lightness, transparency, and glossiness. The relations between the attributes are discussed, and a short list of appearance descriptions is included as a demonstration of the convenience and inclusiveness of the system.

A common usage of the term, color, is one that distinguishes between colors and grays; but the distinction is seldom rigidly kept. No one says that a man wears a colorless suit because it is a black, white, or gray suit. No one says that a light red is the same color as a dark red, even though the two may be identical in hue and saturation; instead, color names (such as pink and maroon) identifying them as different colors are used. It has been recognized accordingly that colors perceived in the nonself-luminous object mode have the three attributes, hue, saturation, and lightness [*1*].[2] The corresponding attributes of color perceptions in the self-luminous modes are hue, saturation, and brightness.

Common parlance also recognizes appearance distinctions not to be described in terms of these triads of color-perception attributes. It is said, for example, that the appearance of one solution is a transparent blue as distinguished from a murky blue. Likewise, the appearance of a painted surface is commonly described as a glossy black as distinguished from a mat black. Those who refuse to class white, black, and grays as colors are likely to consider that water is colorless. This position is not particularly helpful, however, because a mirror surface, and air, which may equally well be called colorless, plainly are both different in appearance from water. The three-attribute system of color-appearance description is, of course, insufficient to deal with these distinctions because they are not color distinctions.

As a final example of the inadequacy of the three-attribute system of color-appearance description may be cited the distinction between gold and yellow, and that between copper and red-

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[2] The italic numbers in brackets refer to the list of references appended to this paper.

dish brown. These distinctions are not wholly a matter of gloss, because a gold surface may be depolished until it has the same gloss as that of a glossy yellow paint of the same hue, saturation, and lightness, and yet the appearance of the two surfaces will be very different—it will be said that they have different colors. It is equally impossible to duplicate the color of copper by means of glossy reddish-brown paint even though the three attributes of color-appearance are duplicated.

These difficulties have long been recognized and investigated by psychologists [*2–8*]. One result of these investigations is the drawing of the distinction, already mentioned, between the various modes of visual appearance. A classification of these modes and the nonshape, nontextural attributes possessed by each mode are listed below.

Modes of Visual Appearance:

1. Aperture (nonlocated) mode
2. Located modes
   (*a*) Illuminant (self-luminous object) mode
   (*b*) Nonself-luminous object modes
      (*1*) Volume mode
      (*2*) Surface mode

Nonshape, Nontextural Attributes of Visual Appearance:

1. Magnitude attributes: Possessed by all modes
   (*a*) Brightness: Possessed by the aperture and illuminant modes, varies from very dim to very bright
   (*b*) Lightness: Possessed by the object modes (volume and surface) and varies from black to white (surface), or from black to perfectly clear (volume)
2. Quality attributes: Possessed by all modes
   (*a*) Hue
   (*b*) Saturation
3. Transparency: Possessed by all located modes
4. Glossiness: Possessed by the surface mode

SYSTEMATIC CLASSIFICATION OF APPEARANCE

Classification is achieved by listing for each of the modes of appearance its nonshape, nontextural attributes:

1. Aperture (nonlocated) mode
   Attributes: Hue, saturation, brightness
2. Located (nonaperture) modes
   (*a*) Illuminant (self-luminous, volume) mode
      Attributes: Hue, saturation, brightness, transparency
   (*b*) Object (nonself-luminous) modes
      (*1*) Volume (nonsurface) mode
         Attributes: Hue, saturation, lightness, transparency
      (*2*) Surface (nonvolume) mode
         Attributes: Hue, saturation, lightness, transparency, glossiness

DEFINITIONS

Aperture mode (of color appearance) refers to color perceived simply as filling a hole in a screen.

A color perceived in the aperture mode has no definite location, because it may be either in the plane of the screen or indefinitely far behind it; such a color is perceived neither as concentrated in a single plane nor spread into a volume, but is nonlocated in depth. Viewing a surface at a distance of several feet through a hole in a screen a few inches in front of the surface often results in the perception of color in the aperture mode, and the screen, on this account, is sometimes called a *reduction* screen, because it may succeed in reducing the color perception from the surface mode to the aperture mode. If, because of the presence of such microstructure as brush marks, or for no determinable reason at all, the color is perceived as belonging to the surface, the reduction screen has failed of its purpose; that is, it has failed to reduce the mode of appearance from surface to aperture. The distinction between surface mode and aperture mode is thus not dependent on the particular devices in front of the observer, but is dependent on the facts of his visual perception of those devices at the moment.

Illuminant mode (of color appearance) refers to color perceived to belong to a volume or bulk that seems to be emitting light (distinction from a volume that seems to be scattering or transmitting light). A color perceived in the illuminant mode is always volumic, never confined to a surface [*6*].

Volume mode (of color appearance) refers to color perceived to belong to a volume or bulk.

Surface mode (of color appearance) refers to color perceived to belong to a surface.

Object mode (of color appearance) refers to color perceived to belong to an object, that is, to a definite location in visual space with definite boundaries.

The object mode is either the volume mode or the surface mode. Although the view of a scene from a single direction often results in the perception of colors in the object mode, the use of two sets of angular conditions of viewing the scene, such as, for example, is provided by viewing binocularly, greatly enhances the possibility of perceptions in the object mode and makes them less ambiguous. The color perceived to belong to any one nonself-luminous object is often found to be nearly constant, regardless of whether the spectral distribution of the source varies within rather wide limits. The term 'color constancy" refers to this approximate constancy of the color perceptions of any given object viewed under different amounts and kinds of light.

Brightness is the attribute of a color perceived in the aperture or illuminant mode by which the observer judges whether the corresponding area is emitting light at a greater or lesser rate. Brightness varies from very dim to very bright.

Lightness is the attribute of a color perceived in the object mode by which the observer judges whether the object is transmitting or reflecting a greater or lesser fraction of the incident light. Lightness varies from black to white for colors perceived in the surface mode. It varies from black to perfectly transparent (or colorless) for nonself-luminous object colors perceived in the volume mode.

Hue is the attribute of certain color perceptions that permits them to be classified as reddish, yellowish, greenish, bluish, or their intermediates.

TABLE I.—END POINTS OF TRANSPARENCY

Mode of Appearance	Lower Limit	Upper Limit
Illuminant....	just greater than zero	just less than perfect
Volume......	just greater than zero	perfectly transparent
Surface.......	zero	just less than perfect

(An) achromatic color perception is one having no hue. White, gray, black, the color perceived to belong to air, to water, or to a nonselectively reflecting mirror, are examples of achromatic color perceptions.

(A) chromatic color perception is one having a hue.

Saturation is the attribute of any chromatic color perception that determines the degree of its difference from the achromatic color perception most nearly like it. The achromatic color perception most nearly like a given chromatic color perception will be the one perceived in the same mode of appearance and neither lighter nor darker (nonself-luminous modes), or neither brighter nor dimmer (self-luminous modes).

Transparency is the attribute of appearance in the volume mode which determines the degree to which colors are seen behind this volume. The end points of transparency for the three modes of appearance possessing this attribute are indicated in Table I.

Luminance (at a point on an object or aperture and for a given line of sight) is the ratio of the luminous flux per unit solid angle leaving an infinitesimal area containing the point under consideration to the product of the infinitesimal area by the cosine of the angle between the line of sight and the normal to the infinitesimal area. Luminance is the stimulus correlate for the brightness of the color perception.

Luminous directional reflectance (of a light-scattering object) is the ratio of the luminous flux reflected from the object in the direction of view to the

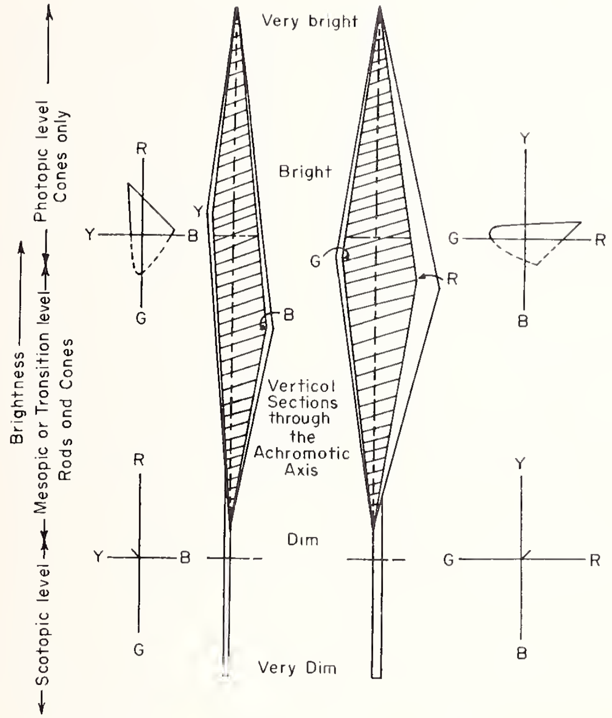

FIG. 1.—Sections of the Solid Representing Color Perceived at 4-deg Subtense in the Aperture and Illuminant Modes with Dark Background.

The two orthogonal vertical sections through the axis, representing achromatic color perceptions, are cross-hatched. One is a section through the plane representing colors perceived as having yellow and blue hues; the other, through the plane representing colors perceived as having red and green hues. There are four horizontal sections, two at moderately high brightness, two at low brightness. The brightnesses at which the sections are taken are indicated by dot-dash lines on the vertical sections. The part of the solid cut away in each vertical section is indicated by a dotted line on the corresponding horizontal section at the higher brightness. Most of the color perceptions in the scotopic level of brightness result from rod action and are achromatic, but the yellowish red hues experienced from threshold and near-threshold spectral stimuli of wavelength 620 mμ, and higher, result from cone action. On this account the horizontal cross-section at low brightness is simply a short straight line.

luminous flux reflected in the same direction from the ideal, perfectly reflecting, perfectly diffusing surface illuminated in the same way as the object. Luminous directional reflectance is the stimulus correlate for the lightness of the color perception of a light-scattering object.

Luminous transmittance (of a nonlight-scattering object) is the ratio of the luminous flux transmitted by the object to the luminous flux incident upon it. Luminous transmittance is the stimulus correlate for the lightness of the color perception of a nonlight-diffusing object.

Glossiness is the attribute of appearance in the surface mode that determines the degree of its difference from the appearance of a mat surface perceived to have the same hue, saturation, lightness, and transparency. Glossiness determines the degree to which colors in front of a surface may be seen as if behind it.

Gray is a general name for achromatic colors perceived in the object mode, and is also sometimes less aptly used to designate achromatic colors of not too high brightness perceived in the aperture and illuminant mode. For the object mode, saturation is the attribute of any chromatic color perception that determines the degree of its difference from the nearest gray.

INTERRELATIONS AMONG THE ATTRIBUTES

Aperture and Illuminant Modes:

Some general relations among the attributes, hue, saturation, and brightness of colors of moderate angular subtense (say 4 deg) perceived in the aperture and illuminant modes with a dark surround have long been recognized; they have been expressed by drawing the color "pyramid" or color solid.[3] In this solid, hue and saturation are shown in polar coordinates, hues being arranged according to angle about the vertical axis, and saturation being proportional to distance from the achromatic axis of the solid. The boundaries of the solid are irregular and approximate somewhat a long, relatively thin double pyramid or cone (Fig. 1) converging to a vertex toward high brightness, and to a short line toward low brightness. These boundaries represent the limits beyond which our experience of colors in the aperture and illuminant modes cannot extend; they express the limitations of the visual mechanism. The axis shown in each vertical section of the figure, for example, represents achromatic color perceptions; it extends from very dim (absolute threshold) to very bright (dazzling). Its extent is finite because the number of distinguishable brightness steps is finite. Although the luminance of the stimulus may be indefinitely increased, the higher luminances fail to produce higher brightnesses because the visual mechanism has a certain limited capacity for perceiving brightness that cannot be exceeded. There are indicated in Fig. 1, along with the sketch of the various cross-sections of the color solid, the approximate extents of the photopic (cone) state and the mesopic (rod and cone) state according to brightness of the color perception. The lowest brightnesses correspond chiefly to the rod state. These are the brightnesses of the photochromatic interval between the absolute threshold and the chromatic threshold [*9–11*].

[3] The explanatory value of the color solid has frequently been impaired because the terms *white* and *black* have been used to describe the colors represented at the upper and lower extremities of the achromatic axis. These terms apply strictly to colors perceived in the surface mode only and, as will appear presently, the color solid referring to surface colors has a very different shape.

If it were not for the fact that there is no photochromatic interval for spectrum stimuli of wavelength greater than 620 mμ, the color solid would come to a point at the bottom of the mesopic range and would extend throughout a purely scotopic range simply as an extension of the achromatic axis representing the hueless colors perceived purely by action of the retinal rods. For long-wave spectrum stimuli, however, the color perceived at the absolute threshold of light is a yellowish red of threshold saturation [*11*]; so the lower terminal of the color solid is a line extending in the direction corresponding to a yellowish-red hue.

The surface of the hue-saturation-brightness solid shows the colors perceived to belong to self-luminous areas either sending single-frequency light to the eye, or sending mixtures in various proportions of light from the two extremes of the visible spectrum. The part of the surface representing greenish-yellow color perceptions of maximum saturation is closer to the axis of the figure throughout a medium range of brightnesses than that of other hues because the number of distinguishable saturation steps from daylight to greenish yellow spectrum light (570 mμ) is less than that obtainable from spectrum stimuli and mixtures thereof yielding color perceptions of other hues [*12*]. The tapering of the figure both toward higher and lower brightnesses expresses the fact that the chromatic part of the visual mechanism works best with stimuli of intermediate luminance. As the luminance is lowered the chromatic response of the cone mechanism is progressively swamped by increased rod action and by increased importance of the self-light of the retina, both of which yield achromatic perceptions. As the luminance is increased, the chromatic response is swamped by achromatic excitation from the cones themselves [*13*, *14*]. The consequence of these two saturation-decreasing influences, one for increasing luminance, the other for decreasing luminance, is that there is a luminance at which for increasing luminance the increase in saturation with brightness gives way to a decrease. This turning point of saturation [*13*] occurs at a higher brightness for color perceptions of yellow hue than for other hues [*9*, *15*]. All of these experimental facts are shown schematically in Fig. 1.

Surface Mode:

The solid representing appearances of zero transparency and glossiness belonging to surfaces of large angular extent (say 4 to 10 deg) illuminated by daylight and viewed against a light background (middle gray to white) has boundaries that are determined not only by the limitations of the visual mechanism but also by the limitations arising from the physical surface and its illumination by daylight. One of these limitations is the fact that a nonfluorescing surface cannot reflect more radiant energy of a given wavelength than falls upon it. The axis of this figure represents grays, ranging in lightness from black to white. Instead of being like a double cone or pyramid sharply pointed at the ends, this figure may be approximately described as a rounded rhomboid; the height is not much different from the greatest horizontal thickness (Fig. 2).

Since this solid represents the color perceptions of surfaces illuminated by daylight, it refers to the photopic range of brightness only and depends for its shape on action of the retinal cones as opposed to the retinal rods. There is no swamping of the chromatic responses by rod action or by the self-light of the retina near zero lightness (black), nor is there any achromatic response of the cones due to overstimulation to swamp the chromatic responses near maximum lightness (white).

There is a ridge extending from the white point toward the area representing saturated surface-color perceptions of yellow hue, and this ridge bends to pass through orange and red hues as it progresses to higher saturations and lower lightnesses [*16–20*]. This ridge (say the $\bar{z}$-function of the CIE system) can be confined to the short-wave portion of the visible spectrum where the luminous-efficiency is relatively low. The blue ridge is partly due to this cause also, and partly due to the antichromatic influence of the bright surrounding field, which prevents the chromatic aspect of the response to areas of much lower luminance from being perceived. In this surface-color-perception solid, all of the color perceptions of surfaces whose reflectances are confined to a single

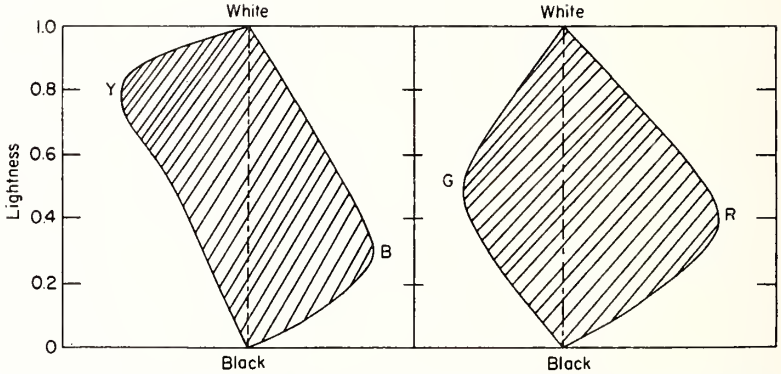

FIG. 2.—Vertical Orthogonal Sections of the Solid Representing Colors Perceived in the Surface Mode as at Zero Glossiness and Transparency with White Background.

expresses the fact that there are surfaces perceived to have saturated yellow colors whose lightness is nearly as high as that of white.

It is also true that saturated blue surface colors exist whose lightness is very low, almost as low as that of black. The possibility of these surface-color perceptions is represented in the lower boundary of the surface color solid, which has an inverted ridge roughly symmetrical to the upper ridge representing yellow color perceptions [*18*].

As an explanation for the yellow ridge may be cited the fact that one of the three color-matching functions waveband of infinitesimal width are represented at precisely one point, the black point.

The shape of this surface-color solid is not very different from that which just encloses the Munsell color tree, and the charts of constant hue and those of constant value in the "Munsell Book of Color" [*21*] represent approximately vertical half-sections and horizontal sections, respectively, of the surface-color-perception solid. Attempts have been made [*17,18*] to determine by computation the exact shape of the limiting surface of this solid. The series of Ostwald "Vollfarben" [*22*] are represented by a closed curve on the surface of this solid [*18*]; that is, the "Vollfarben" or "full colors" yield surface-color perceptions whose saturation is close to the maximum derivable from real surfaces illuminated by daylight for the given hue and lightness. Other surface-

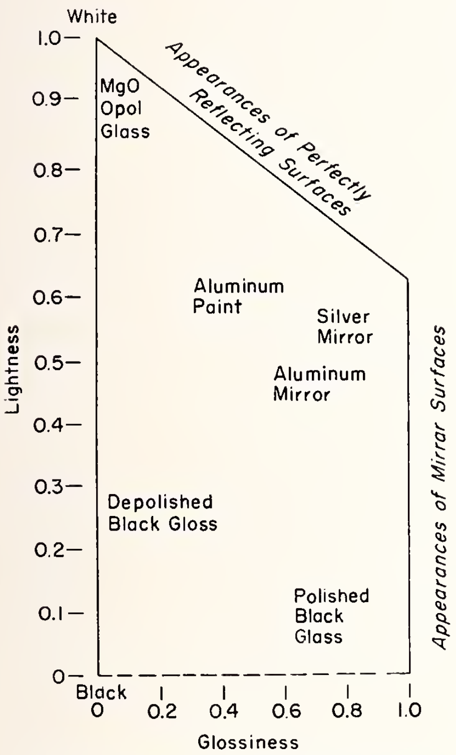

FIG. 3.—Diagram Representing Opaque Achromatic Appearances in the Surface Mode.

color perceptions derivable from real surfaces and having the same hue as, but a different lightness from, a given full color would likewise be represented on the boundary of this surface-color-perception solid; but whether the saturations of such perceptions are necessarily less than those yielded by the full color has yet to be determined.

Just as there is a solid representing surface appearances of zero transparency and glossiness in accord with the attributes of hue, saturation, and lightness, there must also be a solid representing achromatic appearances of surfaces in terms of lightness, glossiness, and transparency. In this case Cartesian coordinates rather than cylindrical coordinates are convenient. The vertical coordinate, as in Fig. 2, is lightness, one horizontal coordinate is glossiness, and the other horizontal coordinate is transparency. The contours of this three-dimensional figure are likewise determined both by the limitations of the visual mechanism and by the limitations arising from the fact that we are dealing with the appearances of physical surfaces viewed in daylight. Unlike the previous case, however, these contours have not been worked out even approximately. There is shown in Fig. 3, however, a rough sketch of one face of this solid, that at zero transparency.

The vertical line at the left of the figure represents grays, ranging in lightness from black to white; it is coincident with the axis of the hue-saturation-lightness solid, just discussed, which represents mat, opaque appearances. The sloping line at the top of the figure is taken to represent the appearances of perfectly reflecting surfaces; these appearances may be arranged in a series of light grays extending from an opaque white appearance to a perfectly glossy, gray appearance. Whether this line is straight is not definitely known, but that it slopes downward cannot be doubted because the appearance of a perfect mirror is not as light as the white color of the ideal perfectly reflecting and perfectly diffusing surface.

The vertical line at the right of the figure represents the appearances of perfect mirror surfaces of various reflectances from unity down to, but not including, zero. Such appearances are approximately equal (precisely equal according to Jones [*23*]) in glossiness; they are accordingly represented as equally distant from the black-white axis which refers to zero glossiness, though, perhaps, the line should incline somewhat toward black.

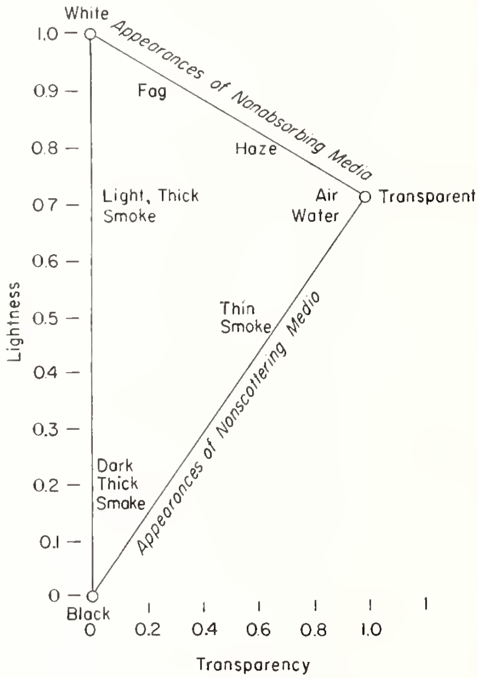

FIG. 4.—Diagram of Achromatic Appearance in the Volume Mode.

The dotted line at the bottom of the figure expresses the fact that there is no series of appearances extending from mat black to the appearance of a perfect mirror of zero reflectance. As soon as the reflectance of the mirror becomes zero, visually its surface ceases to exist. There is, however, a series of appearances extending from mat black to perfectly glossy black, which would correspond to a perfect mirror of reflectance just sufficiently different from zero to yield a visible image of a small light source. The appearance of polished black glass would be represented at glossiness somewhat less than 1.0 (perfect), and lightness somewhat greater than zero. The lightness and glossiness of the appearances of various other opaque surfaces have been estimated and are indicated in Fig. 3. It is to be noted that the appearances of surfaces of constant reflectance may be represented along lines roughly parallel to the top line of the figure.

Because of their transparency, it is possible to see either surfaces, volumes, apertures, or lights through surfaces. The case of the appearance of a volume of one color seen through a somewhat transparent surface of another is a frequent one; it is dealt with under object mode.

Volume Mode:

Since appearances perceived to belong to volumes possess only four attributes, the relations are more easily represented than for surface appearances, which possess five. All achromatic object appearances (except for shape and texture) may be represented on a plane diagram (Fig. 4) by plotting lightness against transparency. Since we are dealing with the appearances of actual physical media viewed in daylight, the boundaries of this diagram are determined partly by physical limitations and partly by limitations introduced by the visual mechanism. Figure 4 shows a rough sketch of the boundaries.

The vertical line at the left of the figure represents zero transparency. The colors here represented are not in the volume mode of appearance, but rather in the surface mode, because only surface appearances may have zero transparency. But we have volume appearances approaching zero transparency, hence this line is a boundary of the area representing achromatic volume appearances, but no point on it represents a volume appearance.

The sloping line extending from white to transparent presents the appearances of media that do not absorb light. Whether the line is straight is not known, but that it slopes downward is certain because the volume appearance of a transparent, nonabsorbing medium (vacuum, air, water) is unmistakably darker than white.

The sloping line at the bottom of the figure from transparent to black represents the colors of nonlight-scattering volumes. It expresses the fact that a medium of dark color cannot be seen through as easily as one of lighter color even though both media transmit light in a rectilinear way. Whether this line is straight or curved is not known.

The appearances of various media having achromatic colors have been estimated in terms of lightness and transparency; these are indicated in Fig. 4. Some of these media (smoke, water) yield colors in the surface mode as well as in the volume mode; but, of course, only the appearance in the volume mode is represented by a point in this figure.

Object Modes:

Some nonself-luminous objects yield color perceptions in the surface mode only; these are objects which the incident light seems not to penetrate. It is also possible to experience color in the volume mode alone, although in these cases the possessor of the color is termed a medium rather than an object (fog, haze, water to a diver). A more complicated case, however, is the class of objects that yield color perceptions both in the volume and in the surface mode (water, glass, transparent and translucent objects). This case deserves some separate discussion.

Consider first the case of a piece of nonlight-absorbing, nonlight-scattering optical glass with polished surfaces immersed in nonlight-absorbing, nonlight-scattering media of various indices of refraction. The appearance of the glass in the volume mode remains constant independent of the medium; it is a perfectly transparent, achromatic appearance of moderate lightness (see appearance of air or water in Fig. 4). The appearance of the glass surface is always glossy but varies somewhat in lightness and transparency. The lightness decreases slightly and the transparency approaches perfection as the index of the medium approaches that of the glass. When the two indices become precisely equal, the surface disappears; that is, there is no color in the surface mode because visually there is no surface.

Consider next the colors of a piece of nonlight-absorbing optical glass viewed in air as a function of the state of polish of the surfaces. As before, the volume appearance of the glass remains constant; it is perfectly transparent. The surface appearance varies in lightness, glossiness, and transparency. As the surface is depolished, the lightness of the color perceived to belong to the surface increases, the glossiness and transparency perceived to belong to the surface decrease. The relation of lightness to glossiness is approximately indicated in Fig. 3 by a line parallel to the top line of the figure. Large decreases in glossiness due to roughening are accompanied by perceptually somewhat smaller increases in lightness. The glossiness of the surface may be reduced to zero by completely depolishing the surface;

but the transparency does not fall completely to zero.

Consider, finally, the colors of polished, nonlight-absorbing blocks of glass whose only difference is the presence of small, uniformly distributed air bubbles of uniform size in varying numbers. The surface appearance is constant; it is a glossy, rather highly transparent appearance of achromatic color of medium lightness. As the number of air bubbles increases, the appearance of the glass in the volume mode varies both in lightness and transparency. The lightness increases nearly to that of white; the transparency decreases to zero, at which point the glass ceases to yield any appearance in the volume mode at all. Such a block is seen only by the bounding surfaces with the moderately glossy opaque appearance of near-white color characteristic of polished milk glass.

Transition States Between the Modes:

The division lines between the various modes of appearance have been made definite in the interest of logic, and it is true that our visual experience justifies these distinctions in a rather striking way and makes them useful. There are, however, transition states between the modes. They arise out of the complexity of our visual responses which have so far resisted complete classification and require further careful study. Many appearances in a transition state are due to visual fields whose structure is insufficient to yield a definite determination of appearance. Such fields are ambiguous, permitting two or more interpretations or perceptions; the appearances may fluctuate from one mode to another according to interpretation or observer attitude. Some of the transition states are sufficiently common to deserve discussion.

Volume-to-Surface Mode.—When an object yields a volume appearance of medium to low transparency behind a surface appearance of high transparency and glossiness, the surface and volume appearances frequently tend to merge into a single glossy surface appearance of zero transparency and changed lightness. Conversely, any very glossy surface appearance of zero transparency (metallic mirror surface) frequently tends to resolve into a glossy highly transparent surface appearance overlying a volume appearance.

Surface-to-Aperture Mode.—A surface appearance may be changed into an aperture color by a failure to fixate the surface accurately. This change is particularly likely to occur with uniform, mat, surface appearances, because the structure of the physical surface ordinarily furnishing many of the clues by means of which the appearance takes on surface character is imperceptible. All nontextured, very dark, mat, surface appearances have something of the indefinite localization of aperture colors. Conversely, a dim aperture color with a bright surrounding field has something of this kind of surface character, and for this case the surface-color term, black, is likely to seem appropriate.

Aperture-to-Illuminant Mode.—An aperture color with a dark surrounding field takes on the character of a somewhat imperfectly localized illuminant color. Such aperture colors, if dim, do not at all seem to be black, any more than a very dim source could be called black. Conversely, a source surrounded by simple structures often loses its localization and appears to be filling a hole in a screen; that is, the illuminant appearance frequently changes to an aperture color.

Surface to Illuminant Mode.—A

surface appearance is the more likely to change into the illuminant mode the closer the point representing it in the surface-color solid (Fig. 2) approaches the upper boundary. This high probability of the illuminant mode seems to relate to a judgment on the part of the observer that the color perceived is too luminous to result from a non-self-luminous object; so the perception is of a color belonging to a self-luminous object, and the brightness attribute applies instead of the lightness attribute. Conversely, a rather dim color in the illuminant mode may shift to the surface mode, as when a glowing coal illuminated by sunlight may shift as it cools down to the appearance of a surface coated by red pigment. Surface colors near the upper boundary of the surface-color solid are particularly easy to produce by fluorescent colorants. The conditions normally required to achieve the transition from surface to illuminant mode have been studied in detail recently by Evans [24], and his results indicate that this transition stage usually involves the judgment that the appearance corresponds to that of a fluorescent specimen. Accordingly he has proposed to call this transition stage the "fluorence" mode. In the fluorence mode, the attributes of appearance are hue, saturation, brightness, and transparency, but Evans' results indicate that luminance is a poor correlate for brightness in the fluorence mode, much poorer than has been indicated by experimental results with the illuminant mode. In some "fluorence" appearances luminance gives spectacularly false indications of brightness.

DESCRIPTIONS OF SOME COMMON APPEARANCES

Aperture Mode:

1. The color of the completely overcast sky in daytime is an achromatic color of medium brightness in the aperture mode. At night the brightness is nearly zero.
2. The color of the clear sky in daytime is a color of blue hue, low to medium saturation, and medium brightness in the aperture mode.

Illuminant Mode:

1. The appearance of the sun is a very bright yellow of very low saturation and transparency in the illuminant mode.
2. The appearance of the neon glow tube is in three modes; it consists of a bright yellowish-red illuminant appearance of high saturation and low transparency surrounded by the volume and surface colors of the glass tube.
3. In the dark, the appearance of radium paint (such as once used on watch dials) is a greenish-yellow of low brightness, saturation, and transparency in the illuminant mode.

Volume Mode of Appearance of Objects Viewed by Daylight:

1. The appearance of the air in a room (or of a vacuum) is light, achromatic, and perfectly transparent.
2. The appearance of the air in a valley is often a light blue of low saturation and medium transparency.
3. The appearance of fog in a room is a light gray of medium transparency.
4. The appearance of cumulus clouds varies from near-white surface colors to gray of low transparency and a wide range of lightness in the volume mode.

Surface Mode of Appearance of Objects Viewed by Daylight:

1. The usual appearance of depolished chalk (magnesium oxide, magnesium carbonate) is white of zero glossiness and transparency.
2. The appearance of mercury (pol-

ished chromium, polished steel, polished nickel, polished aluminum, and so forth) is achromatic of medium lightness, zero transparency, and nearly perfect glossiness.

3. The appearance of aluminum paint differs from that of polished aluminum by being lighter and less glossy.

4. The appearance of polished gold (or brass) is moderately saturated yellow of medium lightness, zero transparency, and nearly perfect glossiness.

FIG. 5.—Ambiguous Grays.

This illustration may be seen: (1) as six areas, two white, two black, and two different grays, all opaque, or (2) as three areas, an opaque white and black behind a uniform transparent gray.

5. The appearance of polished copper is a dark yellowish-red of medium saturation, zero transparency, and nearly perfect glossiness.

6. The appearance of polished black glass is usually black, perfectly glossy, and perfectly opaque; sometimes it has the more complex appearance of a mat black behind a glossy, transparent, achromatic surface.

7. The appearance of depolished black glass is a dark gray of zero transparency and glossiness.

8. The middle band of Fig. 5 may be seen in two ways; it may be a uniform gray of medium lightness, glossiness, and transparency in front of a near white and a near black; or it may consist of two grays of medium glossiness and zero transparency, the left-hand gray being dark between two near blacks, the right-hand gray being light between two near whites.

9. The appearance of tracing cloth is usually a blue surface of low saturation and of medium lightness, transparency, and glossiness.

10. The appearance of clear cellophane is a very glossy, very transparent surface whose color is achromatic of medium lightness.

11. The appearance of nonlight-absorbing waxed paper usually differs from clear cellophane simply by being lighter, less glossy, and less transparent.

12. The appearance of sheer nylon hosiery is frequently a dark yellowish-red of moderate saturation and transparency, and low glossiness.

Complex Object Appearances in Daylight:

1. The appearance of a block of clear optical glass (small bulks of water viewed in air, air bubbles in water) has two parts: it consists of a light, perfectly transparent, achromatic volume enclosed by a highly transparent, perfectly glossy, achromatic surface of medium lightness.

2. The appearance of pencil yellow (representative of glossy paints) has two parts: it consists of a light, saturated, mat, opaque, yellow surface behind a glossy, moderately light, transparent, gray surface.

3. The appearance of a polished surface of solid methyl violet (representative of highly absorptive pigments) has two parts: it consists of a dark, unsaturated, mat, opaque, reddish-blue surface behind a glossy, trans-

parent surface whose color is a light, saturated yellowish-green.

4. The appearance of a polished block of canary glass has three parts: it consists of a moderately light, glossy, transparent, gray surface overlying a transparent, light, saturated yellow volume and a moderately transparent volume of a saturated green color.

Summary

It should be remarked in conclusion that our knowledge of visual experience is considerable but far from complete. The foregoing system of classifying the nonshape, nontextural aspects of appearance accordingly is adequate to the description of a considerable portion of visual experience, but not to all; furthermore, other systems are possible, each one having different, but equally unavoidable, arbitrary features. Since students of visual perception are not in perfect agreement, this system cannot be consistent with all views; and, as a matter of fact, it is not in perfect agreement with any published views. This system does not cover all aspects of visual appearance (size, shape, texture, mottling), nor, indeed, all modes of appearance. No mention has been made of the important, but little studied, and rather mysterious illumination mode of appearance. A color perception in the illumination mode always accompanies the perception of an object color, yet it is not referred to a definite volume in the illuminant mode, nor is it the perception of the volume color of the space in which the object color is perceived. It is a color perceived to belong to the illumination of the object based on clues from the scene within which the object is perceived instead of being based on any view of the source itself.

Acknowledgments:

The author wishes to express his appreciation to H. Helson and D. McL. Purdy for an introduction to the psychological literature on modes of appearance and for constructive criticism, and especially to K. S. Gibson for aid in working out and testing the system and for the following comment:

When I was young and life was bright
I used to say the sun was white;
But now I'm told in terms emphatic,
The sun is really achromatic.

It used to be correct to say
The autumn skies were bleak and gray;
But now I've learned—Oh, thought ecstatic,
Gray skies are really achromatic!

And when it rained and made a mess
The drops to me were colorless;
But now in scenes so hydrostatic
Those drops are simply achromatic!

And thus it was, and thus it is,
I think by gosh and then gee whiz,
My mind is warped and quite erratic;
I'm sure my thoughts are achromatic!

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MAXWELL AND MODERN COLORIMETRY

J. Phot. Sci. 9, 341-352 (1961)

The only problems of terminology encountered in this paper arise because so much of it is quoted from Maxwell's writings. In most cases, the quotations are clearly indicated or easily recognized. Judd's insertion of modern terminology, parenthetically or otherwise, is helpful and correct as of 1978.

In some instances careful reading is required to avoid confusion between Maxwell's terminology and Judd's translations.

Maxwell and Modern Colorimetry

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ABSTRACT. The methods used by Maxwell to reduce colour-matching data from rotating sector discs (Maxwell discs) so as to represent the chromaticity of the unknown colour on a chromaticity diagram (Maxwell triangle) form the basis of modern colorimetry. Maxwell's determination of the colour-matching functions for two observers carried out by these methods is found, when expressed in terms of flux units instead of the slit-width units used by Maxwell, to be in essential agreement with the 1931 CIE standard observer defining the colour scales currently used internationally.

SUMMARY OF MAXWELLIAN COLORIMETRY*

ALL vision is colour vision, for it is only by observing differences of colour that we distinguish the forms of objects. I include differences of brightness among differences of colour (II-267).

All light is composed of the rays of the spectrum taken in different proportions (II-268). Objects which we call coloured when illuminated by daylight make a selection of these rays, and our eyes receive from them only a part of the light which falls on them. This is the optical explanation of the colours of bodies when illuminated by daylight. They separate the daylight into its component parts absorbing some and scattering others.

To bring a quality (such as colour) within the grasp of exact science we must conceive it as depending on the values of one or more variable quantities, and the first step is to determine the number of these variables which are necessary and sufficient to determine the quality of a colour (II-271). We do not require any elaborate experiments to prove that the quality of colour can vary in three and only in three independent ways.

Now colour depends on three things (II-272). If we call these the amounts of the three primary colours, and if we are able in any way to measure these three amounts (tristimulus values) we may consider the colour as specified by these three measurements. If I wish to specify the position of a point in a (usual) room, I may do so by giving the measurements of three distances—namely, the height above the floor, and the distances from any two touching walls (II-271). If the amounts of the primary colours are taken as these distances, we may say, by a useful geometrical convention, that the colour is represented to our mathematical imagination by the point so found in the room. If there are several colours, represented by several points, the chromatic relations of the colours will be represented by the geometrical relations of the points (II-272).

There is a still more convenient method of representing the relations of colours by means of Young's colour triangle (now known as the Maxwell triangle). It is impossible to represent on a plane piece of paper every conceivable colour; to do this requires space of three dimensions. If, however, we consider only colours of the same total amount—that is, colours in which the sum of the tristimulus values is the same, then the variations in chromaticity of all such colours may be represented by points on a plane. For this purpose we must draw a plane cutting off equal lengths from the three lines representing the primary colours. The part of this plane within the space in which we have been distributing our colours will be an equilateral triangle. The three primary colours will be at the three angles, white or grey will be in the middle, the degree of purity of any colour will be expressed by its distance from the middle point, and its hue (dominant or complementary wavelength) will depend on the angular position of the line which joins it with the middle point.

Paper read at a three-day conference to mark the centenary of James Clerk Maxwell's demonstration of trichromatic colour reproduction, held on 16, 17 and 18 May 1961, and organized by the Colour Group, in collaboration with The Institute of Physics and The Physical Society, and the Inter-Society Colour Council of America, in London.
MS. received 11 August 1961.

*Except for parenthetical expressions and substitution of modern terminology, this summary is composed entirely of quotations from the writings of James Clerk Maxwell. The page numbers cited are from reference 1, and are preceded by "II" to indicate that they refer to volume II.

Thus the ideas of purity and hue can be expressed geometrically on the (Maxwell) triangle. To understand what is meant by brightness (luminance) we have only to suppose the illumination of the whole triangle increased or diminished, so that by means of this adjustment of illumination the triangle may be made to exhibit every variety of colour (II-273). If we now take any two colours in the triangle and mix them in any proportions, we shall find the resultant colour in the line joining the component colours at the point corresponding to their centre of gravity.

I have said nothing about the nature of the three primary colours. In order to lay down on paper the relations between actual colours, it is not necessary to know what the primary colours are. We may take any three colours, provisionally, as the angles of a triangle, and determine the positions of any other observed colour with respect to these so as to form a kind of chart of colours. In studying mixtures of colours, we must either mix the rays of light themselves, or we must combine the impressions of colours within the eye by the rotation of coloured papers on a disc (Maxwell disc).

We can make a mixture of any three of the colours of the spectrum, and vary the colour of the mixture by altering the amount of any of the three components. If the observer looks at a prism illuminated by daylight from each of three suitably disposed slits (with a lens to image the slits at the pupil of his eye) he sees (by Maxwellian view) this compound colour. If we place this compound colour side by side with any other colour, we can alter the compound colour till it appears exactly similar to the other. When the match is pronounced perfect, the positions of the slits are registered, and the breadth of each slit is carefully measured by means of a gauge (II-274). The records of these breadths asserts that a mixture of three spectrum colours is, in the opinion of the observer, identical with the fourth colour. In order to make a survey of the spectrum we select three points for purposes of comparison, and we call these the three standard colours. The standard colours are selected on the same principles as those which guide the engineer in selecting stations for a survey. They must be conspicuous and invariable and not in the same straight line. In the chart of the spectrum colours we may see the relations of the various colours of the spectrum to the three (working) standard colours and to each other.

Experiments on colour indicate very considerable differences between the vision of different persons, all of whom are of the ordinary type (II-278). These differences are exactly of the same kind as would be observed if one of the persons wore yellow spectacles. In fact, most of us have near the middle of the retina a yellow spot through which the rays must pass before they reach the sensitive organ. When a mixture of red and bluish green light falls on the ordinary surface of the retina, it is of a neutral tint, but when it falls on the yellow spot only the red light reaches the optic nerve and we see a red spot (Maxwell spot) floating like a rosy cloud over the illuminated field (II-279).

INTRODUCTION

The author would like to emphasize at the outset that the above summary is composed of quotations from the writings of James Clerk Maxwell. It is thus a summary of Maxwellian colorimetry. Each one may estimate for himself just how adequate a summary this is for modern colorimetry. The author can find no essential theoretical point in modern colorimetry that is not covered by Maxwell; and is forced to the humbling admission that we have spent the last hundred years simply in applying the Maxwell theory. Our debt to Maxwell is recorded in the language, itself; in the phrases, Maxwellian view, Maxwell disc, Maxwell triangle, and Maxwell spot. Yet in spite of the clarity of his statements about colour measurement, the author found his treatment of colour-matching data obscure, and would like to lead you through some of the steps that he has taken to discover whether this obscure treatment is, in fact, in accord with modern colorimetry.

Location of a Point on the Maxwell Triangle from Measurements by Means of the Maxwell Disc

To make sure that all of you know what Maxwell discs are, and what they are used for, the author cannot do better than show Maxwell's own illustration (Fig. 1), and quote from his description (p. 122)*:

"The coloured paper is cut into the form of discs, each with a small hole in the centre, and divided along a radius, so as to admit of several of them being placed on the same axis, so that part of each is exposed. By slipping one disc over another, we can expose any given portion of each colour. These discs are placed on a little top or teetotum, consisting of a flat disc of tin plate and a vertical axis of ivory. This axis passes through the centre of the discs, and the quantity of each colour exposed is measured by a graduation on

*Whenever, in the present text, page numbers are mentioned, they are the page numbers in reference 1, volume I.

FIG. 6

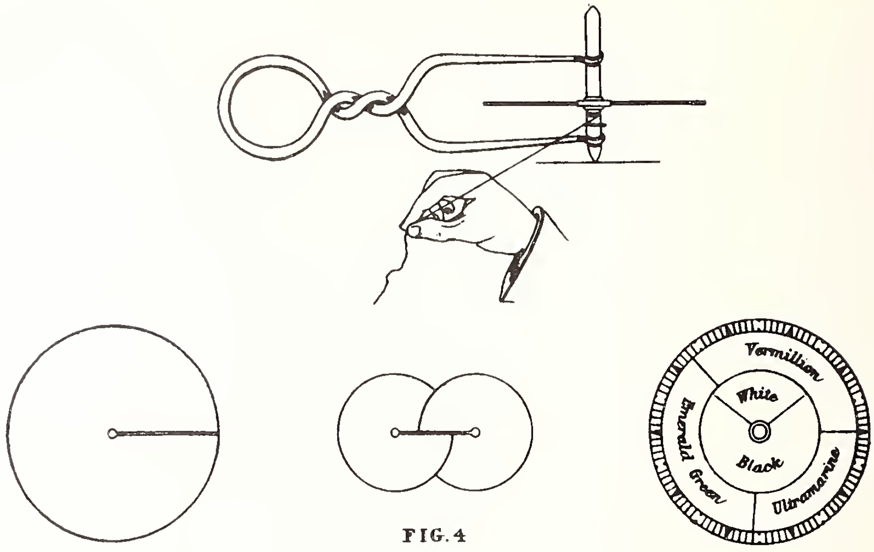

FIG. 4

FIG. 3

FIG. 5

Fig. 1. Maxwell's Figs. 3, 4, 5 and 6 (see reference 1, p. 122) showing construction of the Maxwell disc and the scale dividing the circumference of the disc into 100 parts for measuring the sector fractions required for the colour match.

the rim of the disc, which is divided into 100 parts. By spinning the top, each colour is presented to the eye for a time proportional to the angle of the sector exposed, and I have found by independent experiments, that the colour produced by fast spinning is identical with that produced by causing the light of the different colours to fall on the retina at once."

Maxwell discs continue to be used for colour measurement to the present day. The Munsell Color Company, for example, uses them along with spectrophotometry, thousands of times each year. The discs are spun by an electric motor, or, left stationary, are viewed through a spinning wedge, but the principle is the same.

Unknown Colour within Gamut

Fig. 2 shows the basic modern determination of tristimulus values R_w, G_w, B_w of an unknown near-white colour W with the red, green, and blue colours of the outer disc serving as working primaries. The tristimulus values of these primary colours are, of course, (1, 0, 0), (0, 1, 0), (0, 0, 1) for red, green and blue, respectively. The centre-of-gravity principle is applied by writing the following simple formula[2] for the tristimulus values R_m, G_m, B_m, of a mixture of three sectors occupying factions, f_1, f_2, f_3, of a Maxwell disc:

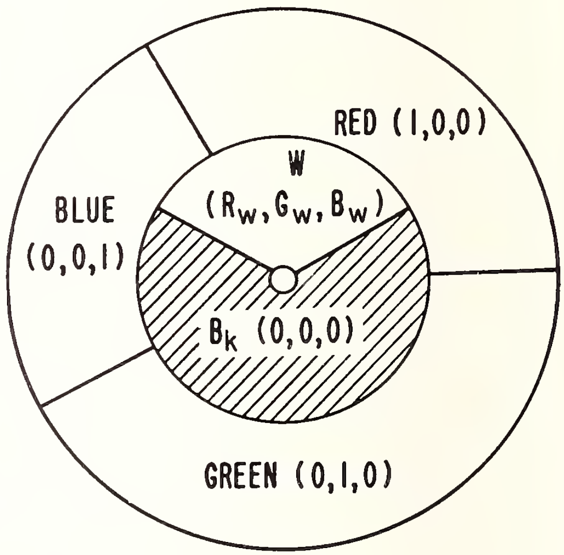

Fig. 2. Maxwell disc set up to determine the tristimulus values, Rw, Gw, Bw, of a near-white colour, W, in terms of the working primaries, red, green, and blue. This arrangement is that of Maxwell's Fig. 5 shown in our Fig. 1. The tristimulus values of the colours are shown in parentheses.

$$\left.\begin{aligned} R_m &= f_1R_1 + f_2R_2 + f_3R_3 \\ G_m &= f_1G_1 + f_2G_2 + f_3G_3 \\ B_m &= f_1B_1 + f_2B_2 + f_3B_3 \\ 1 &= f_1 + f_2 + f_3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1)$$

and the same form applies, not only to three sectors, but also to any number greater than zero.

The condition for a colour match between two such mixtures, M and M′, is that:

$$R_m = R'_m,\ G_m = G'_m,\ B_m = B'_m \quad \ldots\ldots\ldots\ldots (2)$$

For the outer disc, equation (1) becomes:

$$\begin{aligned} R_m &= f_rR_r + f_gR_g + f_bR_b = f_r \\ G_m &= f_rG_r + f_gG_g + f_bG_b = f_g \\ B_m &= f_rB_r + f_gB_g + f_bB_b = f_b \end{aligned}$$

since $R_r=1$, $G_r=B_r=0$; $G_g=1$, $R_g=B_g=0$; $R_b=1$, $R_b=G_b=0$, for the red, green, and blue colours taken as primaries.

For the inner disc, equation (1) becomes:

$$\begin{aligned} R'_m &= f_wR_w + f_oR_o = f_wR_w \\ G'_m &= f_wG_w + f_oG_o = f_wG_w \\ B'_m &= f_wB_w + f_oB_o = f_wB_w \end{aligned}$$

since $R_o=G_o=B_o=0$, for the black sector.

Since by equation (2) the condition for a match is that these tristimulus values be individually equal, one may write:

$$\begin{aligned} f_r &= f_wR_w \\ f_g &= f_wG_w \\ f_b &= f_wB_w \end{aligned}$$

whence it is evident that the tristimulus values of the unknown colour W are:

$$R_w = f_r/f_w,\ G_w = f_g/f_w,\ B_w = f_b/f_w \quad \ldots\ldots\ldots (3)$$

and its chromaticity coordinates ($r=R/(R+G+B)$, $g=G/(R+G+B)$, $b=B/(R+G+B)$) are:

$$r_w = f_r,\ g_w = f_g,\ b_w = f_b. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Note that with three outer sectors, there are only two degrees of freedom, since $f_1+f_2+f_3=1$; and with two inner sectors, there is added only one degree of freedom; so there are altogether three degrees of freedom. This implies that there is only one adjustment of the five sectors that will yield a colour match for any given observer having trichromatic vision. Equation (4) shows that the disc proportions are themselves the coordinates of the unknown colour mixed with black (0, 0, 0) on the inner disc.

If one wishes to locate a point on the Maxwell triangle from these measurements by a Maxwell disc, one simply plots on coordinate paper (either rectangular or triangular) the point (f_r, f_g, f_b), any two of these disc fractions being sufficient to determine the point.

Maxwell's Solution

For Maxwell the reduction of such data was so simple and obvious that he never dignified it by writing any algebraic formulas. Instead, he described an arithmetic and geometry for dealing with such data, and the description was given simply by a numerical example. To quote from his account except that terminology and algebraic symbols consistent with those introduced above are substituted in Maxwell's numerical example:

"Red, blue, and green, being taken (for convenience) as standard colours, are conceived to be represented by three points, taken (for convenience) at the angles of an equilateral triangle. Any colour compounded of these three is to be represented by a point found by conceiving masses proportional to the several components of the colour placed at their respective angular points, and taking the centre of gravity of the three masses. In this way each colour will indicate by its position the proportions of the elements of which it is composed. The total intensity of the colour is to be measured by the whole number of divisions (R+B+G) of red, blue, and green of which it is composed. This may be indicated by a number or coefficient appended to the name of the colour, by which the number of divisions it occupies must be multiplied to obtain its mass in calculating the results of new combinations."

"This will be best explained by an example on the diagram (Fig. 3). We have by experiment (1)

$$f_r(R)+f_b(B)+f_g(G) \text{ matches } f_w(W)+f_o(Bk)$$

"To find the position of the resultant neutral tint, we must conceive a mass of f_r at R, of f_b at B, and of f_g at G, and find the centre of gravity; see Fig. 3. This may be done by taking the line $\overline{BR}$, and dividing it in the proportion of f_r to f_b at the point *a*, where

$$\overline{aR} : \overline{aB} : : f_b : f_r.$$

Then, joining *a* with G, divide the joining line in W in the proportion of f_g to (f_b+f_r), W will be the position of the neutral tint required, which is not white but f_w of white, diluted with f_o of black, which has hardly any effect whatever, except in decreasing the amount of the other colour ($R_o=G_o=B_o=0$). The total intensity of our white paper will be represented by $1/f_w$; so that, whenever white enters into an equation, the number of divisions must be multiplied by the coefficient, $1/f_w$, before any true results can be obtained."

The coefficient, $1/f_w$, mentioned by Maxwell is seen from equations (3) and (4) to be equal to $R/r=G/g=B/b$, that is, it is the factor by which chromaticity coordinates are converted into tristimulus values, and is, indeed, the same as Maxwell's definition of it as R+G+B. Furthermore, the instructions given by Maxwell from Newton's law of colour mixture for finding the point on the Maxwell triangle corresponding to the unknown colour W, does indeed lead to the point (f_r, f_g, f_b). One may ask why Maxwell bothered

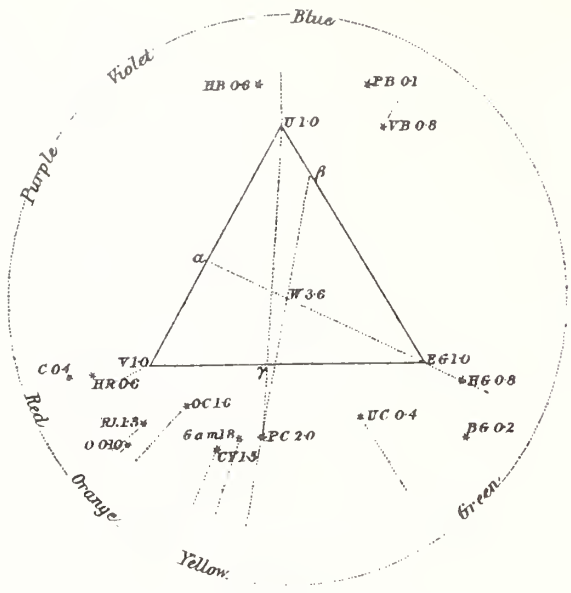

Fig. 3. The Maxwell triangle (see reference 1, p. 122) illustrating Maxwell's method of locating the chromaticity points for the unknown near-white colour, W, inside the triangle, and the unknown yellow colour, PC (pale chrome), outside the triangle. The chromaticity point for the red primary is marked V for Vermilion; that for the green primary, EG for emerald green; and that for the blue primary, U for ultramarine. Newton's centre-of-gravity law of colour mixture is used here directly.

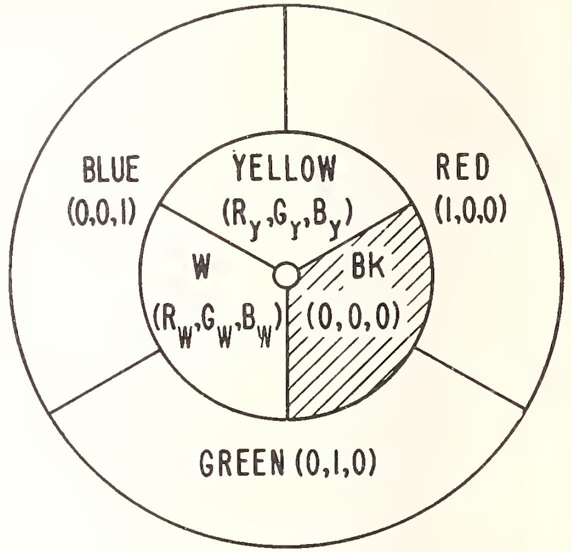

Fig. 4. Maxwell disc set up to determine the chromaticity co-ordinates of an unknown (yellow) colour outside the Maxwell triangle by dilution with an adjustable fraction of white, W. The tristimulus values of the colours of the six sectors are shown in parentheses. This method is in accord with current practice, but was not used by Maxwell.

to give detailed instructions for locating this point on a sheet of paper without coordinate lines printed on it. The answer seems to be that nobody had coordinate paper in those days.

Unknown Colour Outside of Gamut

If the unknown colour cannot be matched by positive amounts of the primaries, the modern method is to desaturate it, such as by the addition of a known amount of white.

Let an unknown yellow colour Y be chosen so that the mixture of the red and green primaries in the proportion required to produce the hue of the unknown produces a colour less saturated than the unknown. The modern tactic to evaluate the tristimulus values (R_y, G_y, B_y) by means of the Maxwell disc would be to combine a sector of the unknown with a sector of the known white (R_w, G_w, B_w) and a sector of the ideal black (O, O, O) to form a three-sector inner disc; see Fig. 4. The three-sector outer disc provides two degrees of freedom, and the three-sector inner disc provides two more, making four in all. There will thus be not one single adjustment of the discs resulting in a colour match but rather a whole series of adjustments. The solutions for the tristimulus values and chromaticity coordinates follow from equations (1) and (2) and result in equations (3a) and (4a) analogous to (3) and (4):

$$\left.\begin{aligned} R_y &= (f_r - f_w R_w)/f_y \\ G_y &= (f_g - f_w G_w)/f_y \\ B_y &= (f_b - f_w B_w)/f_y \end{aligned}\right\} \quad \text{(3a)}$$

$$\left.\begin{aligned} r_y &= (f_r - f_w R_w)/D \\ g_y &= (f_g - f_w G_w)/D \\ b_y &= (f_b - f_w B_w)/D \end{aligned}\right\} \quad \text{(4a)}$$

where $D = 1 - f_w(R_w + G_w + B_w)$.

Note that if the amount (f_w) of desaturating white is zero, equation (4a) becomes: $r_y = f_r$, $g_y = f_g$, $b_y = f_b$, in strict analogy to equation (4). Note also that B_y is less than zero provided f_b is less than $f_w B_w$, and in this case the point, y, representing the chromaticity on the unknown colour Y on the Maxwell diagram will fall outside of the Maxwell triangle.

Maxwell's Solution for Colour Outside of Gamut

Maxwell did not use the modern plan of desaturating the unknown to bring it within the gamut of the primaries. Instead he substituted the unknown

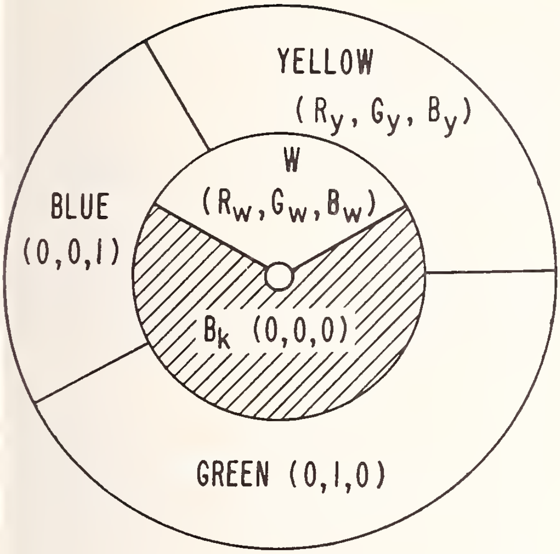

Fig. 5. Moxwell disc set up to determine the chromaticity co-ordinates of on unknown (yellow) colour outside the Moxwell triongle by substitution of it for one (red) of the primories. This method wos used by Maxwell; it requires determinotion of the tristimulus volues (R_w, G_w, B_w) of the white sector by a previous determination; see Fig. 2.

colour Y for one of the primaries and obtained a match for a mixture of the known white (R_w, G_w, B_w) with black (O, O, O); see Fig. 5. There are only three degrees of freedom; so the settings will be unique. He says (p. 132):

"We may take, as the next example, the method of representing the relation of unknown colour Y to the standard colours on our diagram, by making use of an experiment in which the unknown, the standard blue, and the standard green, produce a neutral grey. The resulting equation is:

$$f'_y(Y)+f'_b(B)+f'_g(G) \text{ matches } f'_w(W)+f'_o(Bk)$$

"In order to obtain the total intensity of white, we must multiply the number of divisions, f'_w by the proper coefficient, which is $1/f_w$. The result is f'_w/f_w, which therefore measures the total intensity on both sides of the equation.

"Subtracting the intensity of $f'_b(B)+f'_g(G)$, or $f'_b+f'_g$ from f'_w/f_w, we obtain $(f'_w/f_w)-f'_b-f'_g$ as the corrected value of $f'(Y)$."

Maxwell then outlines the geometrical procedure by which, by the centre-of-gravity principle, the point *y* is located on the Maxwell triangle; see Fig. 3. By checking this through in detail it may be shown that this procedure does indeed locate precisely the point (r_y, g_y, b_y). The question still remains whether Maxwell's expression, $(f'_w/f_w)-f'_b-f'_g$, for what he called intensity "measured by R+G+B," is really the factor by which to multiply chromaticity coordinates to convert them into tristimulus values. If this is correct, Maxwell's "intensity" is the same as the "number of trichromatic units" defined by Guild[3] without any acknowledgment to Maxwell. There may be those who see immediately from Maxwell's instructions that the rules leading to this evaluation are obviously correct, but the author is not one of them.

From equation (1) and (2), the values of the disc fractions, f'_y, f'_g, f'_b and f'_w, required for a colour match must satisfy the following conditions:

$$\begin{aligned} f'_yR_y &= f'_wR_w \\ f'_g + f'_yG_y &= f'_wG_w \\ f'_b + f'_yB_y &= f'_wB_w \end{aligned}$$

from which the tristimulus values (f'_yR_y, f'_yG_y, f'_yB_y) of the fraction f'_y of the unknown colour Y are found as:

$$\begin{aligned} f'_yR_y &= f'_wR_w \\ f'_yG_y &= f'_wG_w - f'_g \\ f'_yB_y &= f'_wB_w - f'_b \end{aligned}$$

But, from equation (3), the tristimulus values (R_w, G_w, B_w) of the white may be written in terms of disc fractions in the first experiment as f_r/f_w, f_b/f_w, and f_b/f_w, respectively, which results in the expressions:

$$\left.\begin{aligned} f'_yR_y &= f_rf'_w/f_w \\ f'_yG_y &= f_gf'_w/f_w - f'_g \\ f'_yB_y &= f_bf'_w/f_w - f'_b \end{aligned}\right\} \quad \ldots\ldots\ldots (5)$$

If Maxwell's expression, $(f'_w/f_w)-f'_g-f'_b$, for the "intensity" of the fraction, f'_y, of the unknown colour Y is correct, this must be the sum of the tristimulus values evaluated in equation (5). Since $f_r+f_g+f_b=1$, it will be seen that this is indeed the case. It must therefore be admitted that Maxwell gave a correct method of finding by means of the Maxwell disc not only the chromaticity coordinates but also the tristimulus values of an unknown colour, regardless of whether it falls within or outside of the gamut, by substitution of it for one of the working primaries. As far as the author is concerned, although Maxwell obviously knew all about it, he did not give a proof of the method, probably because it seemed to him too obvious to require proof. I think that his presentation by numerical example is sufficiently obscure that no one could criticize Guild for failing to notice that his "trichromatic units" had previously been used by Maxwell. This obscure presentation by Maxwell nevertheless had led to modern colorimetry, and the only contribution to it that we have managed to make in the 100 years since Maxwell (except to make coordinate paper available) is to supply convenient analytical expressions. Maxwell cannot even be criticized for obtaining the solution expressed in Fig. 5

indirectly by matching to a mixture of black and white, instead of directly as in Fig. 4. His defence (p. 133) of doing it the hard way has merit. He says that "it has been observed that experiments, in which the resulting tint is neutral, are more accurate than those in which the resulting tint has a decided colour . . . owing to the effect of accidental colours produced in the eye in the latter case." If, as is likely, the surroundings of the observer averaged to a colour approximating neutral grey, we would have to grant that Maxwell's practice of mixing always to a neutral grey may have an advantage. It is well recognized that the colour of the surrounding field most favourable to detection of a difference between two test colours is the average of the two colours to be compared; see, for example, Schönfelder[4].

COLOUR-MATCHING FUNCTIONS

It is generally recognised that Maxwell in his paper *On the Theory of Compound Colours, and the Relations of the colours of the Spectrum* (Phil. Trans., 1860) published the first determinations of the colour-matching functions, though these first determinations are considered to be crude. Let us see how these determinations compare with the 1931 CIE standard observer, the currently accepted basis for international comparisons of colours.

Maxwell constructed two specimen colorimeters, one a laboratory instrument used for the determinations reported, the other, based on the same principles, a portable instrument "to use in obtaining equations from a greater variety of observers." Fig. 6 shows a diagram of the portable instrument. Sunlight reflected from a piece of white paper enters the instrument at C, is reflected by mirrors, M, M′, and e, and fills one half of the photometric field viewed by the observer at E. Sunlight reflected from the same piece of white paper also enters three slits, X, Y, Z, passes through prisms P and P¹ twice after reflexion from mirror S, and after reflexion at mirror e, fills the other half of the photometric field. By closing slits X and Y, this half of the field is filled with spectrum blue light, and by altering the slit widths any colour within the gamut provided by the spectrum colours chosen for primaries may be made to appear in this mixture half of the field.

As in his experiments with Maxwell discs, Maxwell always mixed two of his primaries with the unknown spectrum colour at a wavelength different from those of the primaries to produce a match for sunlight. The evaluation of the tristimulus values of spectrum colour, λ, is therefore accomplished in two steps: first, to match sunlight with all three primaries, then to match sunlight with the unknown part of the spectrum mixed with two of the primaries. Maxwell expressed the tristimulus values in terms of slit width, one unit being one two-hundredth of an inch. If

Fig. 8.

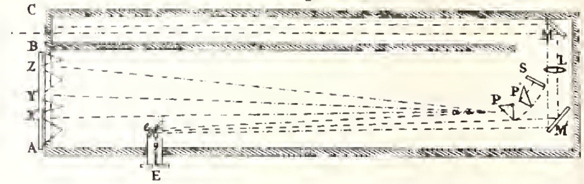

Fig. 6. Maxwell's Fig. 8 (see reference 1, p. 433) showing a diagram of his portable colorimeter. This museum piece is preserved at Cambridge University.

S'_r, S'_g, S'_b are the slit widths of the primaries required to match sunlight, and if S_λ, S_g, S_b, are the slit widths required to match sunlight where λ is some part of the spectrum of unknown tristimulus values R_λ, G_λ, B_λ, substituted for the red primary, then the condition for match is:

$$\left.\begin{aligned} S_\lambda R_\lambda &= S'_r \\ S_\lambda G_\lambda + S_g &= S'_g \\ S_\lambda B_\lambda + S_b &= S'_b \end{aligned}\right\} \quad (6)$$

Maxwell commenced his reduction of the data by computing the tristimulus values for the amount S_λ of the unknown, thus:

$$\left.\begin{aligned} S_\lambda R_\lambda &= S'_r \\ S_\lambda G_\lambda &= S'_g - S_g \\ S_\lambda B_\lambda &= S'_b - S_b \end{aligned}\right\} \quad (7)$$

Then to compute "intensities of the three standard colours at different points of the spectrum" he says "The intensities are found by dividing every colour-equation by the coefficient (S_λ) of the colour on the left-hand side." Now if this is done in (7) one obtains:

$$\left.\begin{aligned} R_\lambda &= S'_r/S_\lambda \\ G_\lambda &= (S'_g - S_g)/S_\lambda \\ B_\lambda &= (S'_b - S_b)/S_\lambda \end{aligned}\right\} \quad (8)$$

whence we see that what Maxwell called "the intensities of the three standard colours" are precisely what would now be called the tristimulus values for the spectrum colour of wavelength λ. If one plots these tristimulus values as a function of wavelength one should obtain precisely the colour-matching functions expressed relative to three spectral primaries. At the wavelengths of these primaries one must have (1, 0, 0), (0, 1, 0) and (0, 0, 1) respectively for red, green, and blue. This merely says for example, that the red primary matches itself, and this result can be obtained from Maxwell's own method of reduction by taking the red primary to substitute for itself in which case $S = S'_r$, $S_g = S'_g$, $S_b = S'_b$, and from (8), $R = 1$, $G = O$, and $B = O$.

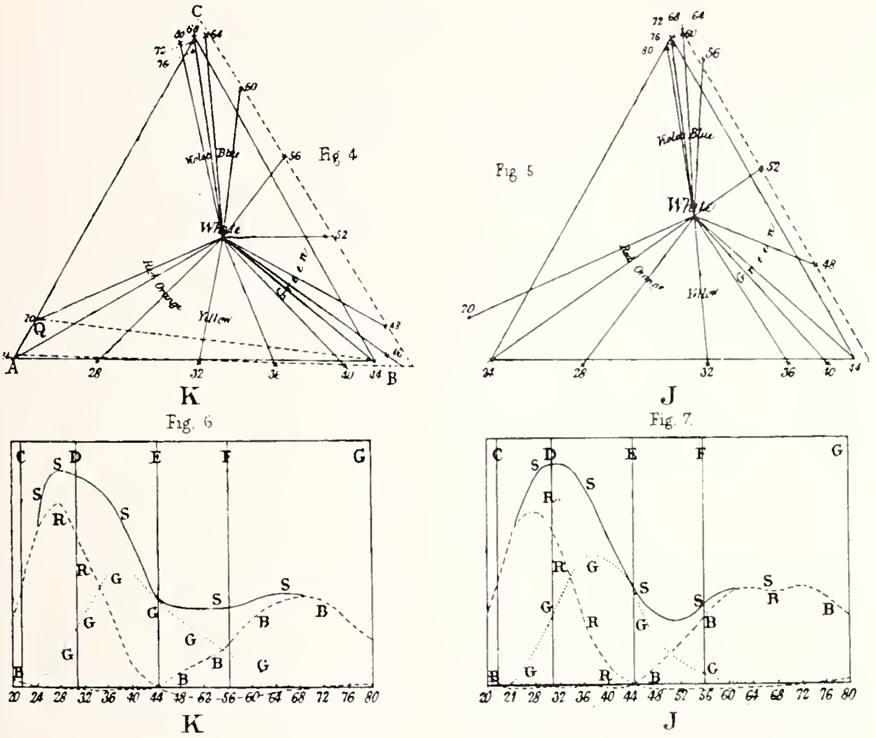

Fig. 7. Maxwell's Figs. 4, 5, 6 and 7 (see reference 1, p. 433) showing for two observers (K and J) the spectrum locus on the Maxwell triangle (Maxwell Figs. 4 and 5) relative to working primaries taken from the spectrum at Maxwell's scale units: 24, 44 and 68. The curves shown in Maxwell's Figs. 6 and 7 are intended to be the "intensities" of the primaries (R, G, B) and their sum (S) in slit-width units. What these curves actually are is not known. They do not correspond to the tristimulus values of the spectrum colours derived from Maxwell's data by the method that he described.

A glance at Maxwell's Figs. 6 and 7 (our Fig. 7), supposed to show the "intensities of the three standard colours" for observers K and J is sufficient to demonstrate that these curves do not have unit value at the primary wavelengths (scale readings 24, 44, 68 for red, green, and blue, respectively). They do not result from Maxwell's correct reduction of his data, and no further attention will be paid to them. Only Maxwell's numerical data reduced by the method given by him will be considered.

If R_λ, G_λ, B_λ, found in accord with equation (8), are taken to be the widths of the slits centered on the primary wavelengths (630·2, 528·1, 456·9nm) required to match the flux corresponding to unit slit-width centered on wavelength λ, and if $(P/dS)_\lambda$ is the flux per unit slit width, then the flux of one of the primaries (say red) required to match the flux passing through a slit of width R_λ centered on the wavelength, λ, is $R_\lambda\ (P/dS)_{630\cdot 2}$. To convert this to the flux of the red primary required to match unit flux at

Table 1

Derivation of the Factors for Converting the Tristimulus Values, R_λ, G_λ, B_λ, from Slit-Width Units to Flux Units

Maxwell Scale S	*Wavelength nm*	*Maxwell's name for the colour*	$d\lambda/dS$	$(P/d\lambda)_\lambda$ *for sunlight air mass 2*	$(P/dS)_\lambda$	*Correction factors (see equation 16)* 3351 $(P/dS)_\lambda$ *Red*	1952 $(P/dS)_\lambda$ *Green*	853 (P/dS) *Blue*
20	663·2	Red	36·0	116	4176	0·80	0·47	0·20
24	630·2	Scarlet	28·4	118	3351	1·00	0·58	0·25
28	606·4	Orange	23·6	117	2761	1·21	0·71	0·31
32	583·1	Yellow	22·0	117	2574	1·30	0·76	0·33
36	562·5	Yellow-green	19·1	118	2254	1·49	0·87	0·38
40	544·9	Green	17·2	119	2046	1·63	0·95	0·41
44	528·1	Green	16·4	119	1952	1·72	1·00	0·44
48	508·6	Bluish Green	15·0	120	1800	1·86	1·08	0·47
52	499·7	Blue-green	12·8	121	1549	2·16	1·26	0·55
56	486·4	Greenish Blue	11·4	119	1357	2·46	1·43	0·62
60	475·1	Blue	10·2	116	1183	2·84	1·65	0·72
64	465·9	Blue	9·1	111	1010	3·32	1·93	0·84
68	456·9	Blue	8·2	104	853	3·94	2·29	1·00
72	449·4	Indigo	7·8	101	788	4·24	2·47	1·08
76	441·2	Indigo	7·6	91	692	4·86	2·83	1·23
80	434·2	Indigo	7·4	84	622	5·40	3·14	1·37

wavelength λ, one must divide by $(P/dS)_\lambda$. The ormulas to convert R_λ, G_λ, B_λ in slit-width units to R_λ, G_λ, B_λ, in the flux units are thus:

$$\left.\begin{aligned} &R_\lambda \text{ (flux units)} \\ &= R_\lambda \text{ (slit-width units)} \times (P/dS)_{630\cdot 2}/(P/dS)_\lambda \\ &G_\lambda \text{ (flux units)} \\ &= G_\lambda \text{ (slit-width units)} \times (P/dS)_{528\cdot 1}/(P/dS)_\lambda \\ &B_\lambda \text{ (flux units)} \\ &= B_\lambda \text{ (slit-width units)} \times (P/dS)_{456\cdot 9}/(P/dS)_\lambda \end{aligned}\right\} \quad (9)$$

Now Maxwell did not measure the flux per unit slit width as a function of wavelength reaching the observing field from the sunlight reflected from white paper; so he did not know, nor do we, what it was. We are, however, better documented than he to make an estimate. The flux per unit wavelength from direct sunlight at air mass 2 may be taken from Parry Moon's compilation[5] as our estimate of $P/d\lambda$, and one may take from Maxwell's own measurements of the wavelengths in Paris inches for his scale settings a good evaluation of $d\lambda/dS$, and the product of these with the wavelengths converted to nm gives the evaluation of the correction factors (P/dS) defined in (9). Table 1 shows the derivation of these correction factors in detail.

There are several points worthy of comment in Table 1.

The wavelengths corresponding to the various scale settings are given by Maxwell by numbers whose units are unspecified, thus: Red 2450, Scarlet 2328, Orange 2240, and so on. It is presumed that the units are the same as those used in his previous paper[6] which states: Red 2425, Orange 2244, and so on, and further that "The wavelengths are expressed in millionths of a Paris inch." On the assumption that a Paris inch is 25·4mm, this implies that a certain red part of the spectrum has a wavelength of $0{\cdot}002425 \times 25{\cdot}4 = 0{\cdot}06160$mm $=$ 61,600nm. An inquiry directed to the National Bureau of Standards Office of Weights and Measures disclosed that one Paris inch is not 25·4 but 27·07mm, and this value would place this red part of the spectrum at 65,545nm. It seems likely that when Maxwell stated that the wavelengths are expressed in millionths of a Paris inch, he meant that the unit was one Paris inch $\times 10^{-8}$. This assumption has been used to find the wavelengths listed in Table 1 in nm. It will be noted that, by this assumption, the colour names given to the various parts of the spectrum by Maxwell accord well with the names currently given to them. Furthermore, the wavelengths of the red, green, and blue primaries used by Maxwell are by this assumption 630·2, 528·1, and 456·9nm, respectively. These numbers accord well with an evaluation by Guild[7] who said that the wavelengths of the primaries used by Maxwell were approximately 630·7, 528·6, and 457·3nm.

The values of $d\lambda/dS$ are found by differencing the numbers in the wavelength column and are the number of nanometers in four scale units. These values decline regularly with decreasing wavelength in a way corresponding to a prismatic spectrum as expected, but with some irregularities suggesting that Maxwell's interferometric method of measuring wavelength had a rather low precision.

The correction factors for changing R_λ from slit-width units to flux units vary from 1·72 to 3·94 for

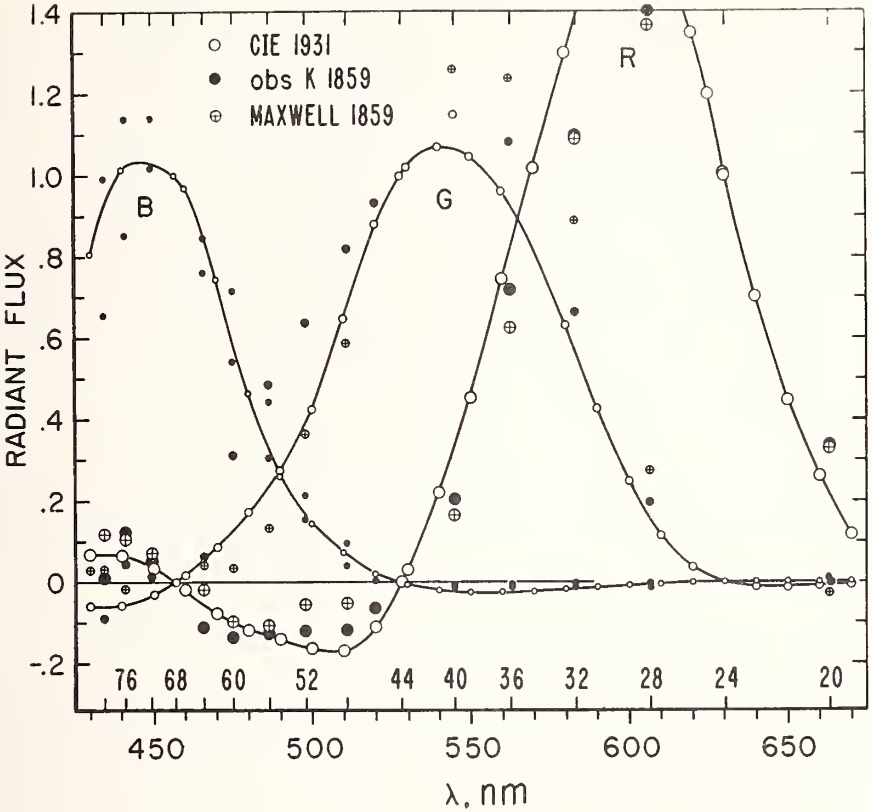

Fig. 8. Rodiont flux of the spectrum primories, 630.2, 528.1, 456.9 nm, required in on odditive mixture to motch unit rodiont flux of the spectrum:

1931 CIE stondord observer—open circles connected by solid lines

Moxwell's observer K—solid circles

Moxwell (observer J)—crossed circles

Note thot Moxwell's doto reduced by the method developed by him ond expressed in units of rodiont flux ore in substontiol ogreement with modern colour-matching functions.

wavelengths between the green and blue primaries, where the values of R_λ are less than zero. On Maxwell's plot, these negative values appear quite small compared to modern data.

The values of R_λ, G_λ, B_λ for Maxwell's observer K expressed in flux units are shown as solid circles on Fig. 8, and those for Maxwell, himself, are shown as cross-circle combinations. These points appear at the wavelengths corresponding to every four of Maxwell's scale units shown on the wavelength scale at the bottom.

The curves plotted through the open circles correspond to the 1931 CIE standard observer expressed relative to the spectrum primaries (630·2, 528·1 and 456·9nm respectively) used by Maxwell. If $\bar{x}$, $\bar{y}$, $\bar{z}$ are the tristimulus values of the equal-energy spectrum in the CIE colorimetric coordinate system, the values $\bar{r}$, $\bar{g}$, $\bar{b}$, plotted in Fig. 8 are those found from:

$$\left.\begin{aligned}\bar{r} &= +1\cdot6785\,\bar{x} - 0\cdot2716\,\bar{y} - 0\cdot2933\,\bar{z}\\ \bar{g} &= -0\cdot5275\,\bar{x} + 1\cdot2813\,\bar{y} + 0\cdot0559\,\bar{z}\\ \bar{b} &= +0\cdot0144\,\bar{x} - 0\cdot0352\,\bar{y} + 0\cdot5787\,\bar{z}\end{aligned}\right\} \quad \ldots (10)$$

The curves thus represent what our modern colorimetric standards would predict, or should one say postdict, for the result of Maxwell's experiment. They indicate the radiant flux of each of Maxwell's primaries required to produce a match for unit radiant flux for each part of the spectrum. Note that the prediction for $\bar{b}$ falls between the results by Maxwell's observer K and Maxwell himself, except for $\bar{b}$ nearly zero. The $\bar{g}$-curve is somewhat narrower than found by Maxwell, and the $\bar{r}$-curve peaks at a wavelength about 10nm shorter than that found by Maxwell. The negative values of $\bar{r}$ between 460 and 520nm which appear negligible on Maxwell's plots using slit-width units are seen on this plot to be the currently accepted order of magnitude.

It may be concluded that Maxwell not only devised a correct method for determining the tristimulus values of the spectrum, but also carried out reliable determinations except for the spectrum extremes. By putting the results of observer K and Maxwell (observer J) on the same plot in Fig. 8, the large individual difference is emphasized. Maxwell's explanation of this difference was "that the yellow spot at the foramen centrale of Soemmering will be found to be the cause of this phenomenon. It absorbs the rays between E and F, and would, if placed in the path of the incident light, produce a corresponding dark band in the spectrum formed by a prism." Except that one would now speak of individual difference in macular pigmentation rather than the "yellow spot at the foramen centrale of Soemmering" the current explanation would be much the same. One would probably also mention degree of lens pigmentation.

DEPENDENCE OF COLOUR-MATCHING FUNCTIONS ON ANGULAR SUBTENSE

Considerable effort has recently been expended in an attempt to derive colour-matching functions for a representative group of observers with normal colour vision making the comparisons in an observing field of large angular extent (10°). Although Maxwell seems not to have supplied information by means of which one can evaluate the angular subtense of the fields observed in his researches, he knew that field subtense was important. He states (p. 433) "This blindness of my eyes to the parts of the spectrum between the fixed lines E and F (527 and 486nm) appears to be confined to the region surrounding the axis of vision, as the field of view, when adjusted for my eyes looking directly at the colour, is decidedly out of adjustment when I view it by indirect vision, turning the axis of my eye towards some other point. The prism then appears greener and brighter than the mirror, showing that the parts of my eye at a distance from the axis are more sensitive to this blue-green light than the parts close to the axis."

Maxwell's view that this discrepancy between small-field and large-field colour matching is ascribable to the absorption of the macular pigment has been largely corroborated by the recent work of Stiles and Burch[8]. About two-thirds of the difference between 2° and the 10° colour-matching functions is ascribable to macular pigment. The current view of the wavelength range within which the macular pigment absorbs strongly[9], however, is 440 to 490nm rather than 486 to 527nm.

MAXWELL, THE FATHER OF MODERN COLORIMETRY

I close this comparison of Maxwellian colorimetry with modern colorimetry by quoting a few passages from Maxwell's papers as further evidence that Maxwell might be called the father of modern colorimetry.

Trivariance of Colour (p. 131)

"The nature of a colour may be considered as dependent on *three* things, as, for instance, redness, blueness, and greenness. This is confirmed by the fact that any tint may be imitated by mixing red, blue, and green alone, provided that tint does not exceed a certain brilliancy.

"Another way of showing that colour depends on *three* things is by considering how two tints, say two lilacs, may differ. In the first place, one may be lighter or darker than the other, that is, the tints differ in *shade* (lightness). Secondly, one may be more blue or more red than the other, that is, they may differ in hue. Thirdly, one may be more or less decided in its colour; it may vary from purity on the one hand, to neutrality on the other. This is sometimes expressed by saying that they may differ in *tint* (saturation)."

Pigment Mixture and Tinting Strength (p. 142)

"By grinding coloured powders together, the differently-coloured particles may be so intermingled that the eye cannot distinguish the colours of the separate powders, but receives the impression of a uniform tint, depending on the nature and proportions of the pigments used. In this way, Newton mixed the powders of orpiment, purple, bise, and viride aeris, so as to form a grey, which, in sunlight, resembled white paper in the shade. (Newton's Opticks, Book I. Part II., Exp. XV.) This method of mixture, besides being adopted by all painters, has been employed by optical writers as a means of obtaining numerical results.... There are two objections, however, to this method of exhibiting colours to the eye. When two powders of unequal fineness are mixed, the particles of the finer powder cover those of the coarser, so as to produce more than their due effect in influencing the resultant tint. For instance, a small quantity of lamp-black, mixed with a large quantity of chalk, will produce a mixture which is nearly black. Although the powders generally used are not so different in this respect as lamp-black and chalk, the results of mixing given weights of any coloured powders must be greatly modified by the mode in which these powders have been prepared.

"Again, the light which reaches the eye from the surface of the mixed powders consists partly of light which has fallen on one of the substances mixed without being modified by the other, and partly of light, which, by repeated reflexion or transmission, has been acted on by both substances. The colour of these rays will not be a mixture of those of the substances, but will be the result of absorption due to both substances successively. Thus, a mixture of yellow and blue produces a neutral tint tending towards red, but the remainder of white light, after passing through both, is green; and this green is generally sufficiently powerful to overpower the reddish grey due to the separate colours of the substances mixed."

Maxwellian View (p. 144)

"The mode of viewing the beam of light directly, without first throwing it on a screen, was not much used by the older experimenters, but it possesses the advantage of saving much light, and admits of examining the rays before they have been stopped in any way."

Frequency vs. Wavelength (p. 149)

"With respect to Newton's construction, we now know that the proportions of the colours of the spectrum vary with the nature of the refracting medium. The *only absolute* index of the kind of light is the *time* of its vibration. The length of its vibration depends on the medium in which it is; and if any proportions are to be sought among the wavelengths of the colours, they must be determined for those tissues of the eye in which their physical effects are supposed to terminate."

Metamerism (p. 129)

"That these experiments (colour matches by Maxwell discs) are really evidence relating to the constitution of the eye, and not mere comparisons of two things which are in themselves identical, may be shown by observing these resultant tints through coloured glasses, or by using gas-light instead of daylight. The tints which before appeared identical will now be manifestly different, and will require alteration, to reduce them to equality. . . ."

CONCLUSION

This comparison of Maxwellian with modern colorimetry has shown that Maxwell in a few year's study of what must for him have been merely an interesting sideline not only developed the essential concepts of colour measurement, but also carried out a determination of the colour-matching functions basic to modern colorimetry that compares favourably with the best modern determinations.

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THE UNSOLVED PROBLEM OF COLOUR PERCEPTION

Palette 10, 30-35 (1962)

No problems of terminology occur in this straightforward little paper, except the question "what is measurement?". An answer cannot be expected in a short article, nor in a commentary. Perceptions can only be experienced: description is only a faint reminder of prior experience; can physical procedures (measurement) be expected to do any more - or as much?

For "millimicrons" (in Fig. 1) read "nanometers".

The Unsolved Problem of Colour Perception

Deane B. Judd

Introduction

To get at the problem of colour perception, let us try to answer the question: Can we measure colour? In one sense we can certainly measure it, but in a more fundamental sense we cannot.

Tristimulus colorimetry

There is a respected science of colour measurement. It starts with Newton, Grassmann, Maxwell, and von Helmholtz, continues with König and Abney, and through the contributions of Wright and Guild has culminated in an internationally agreed upon method of measuring colour by means of the 1931 C.I.E. standard observer. If a patch of light is known physically by its spectral radiance, $\varphi(\lambda)$ (radiant flux per unit solid angle, unit orthogonally projected area, and unit wavelength), we find a specification of its colour by determining its tristimulus values, X, Y, Z, as follows:

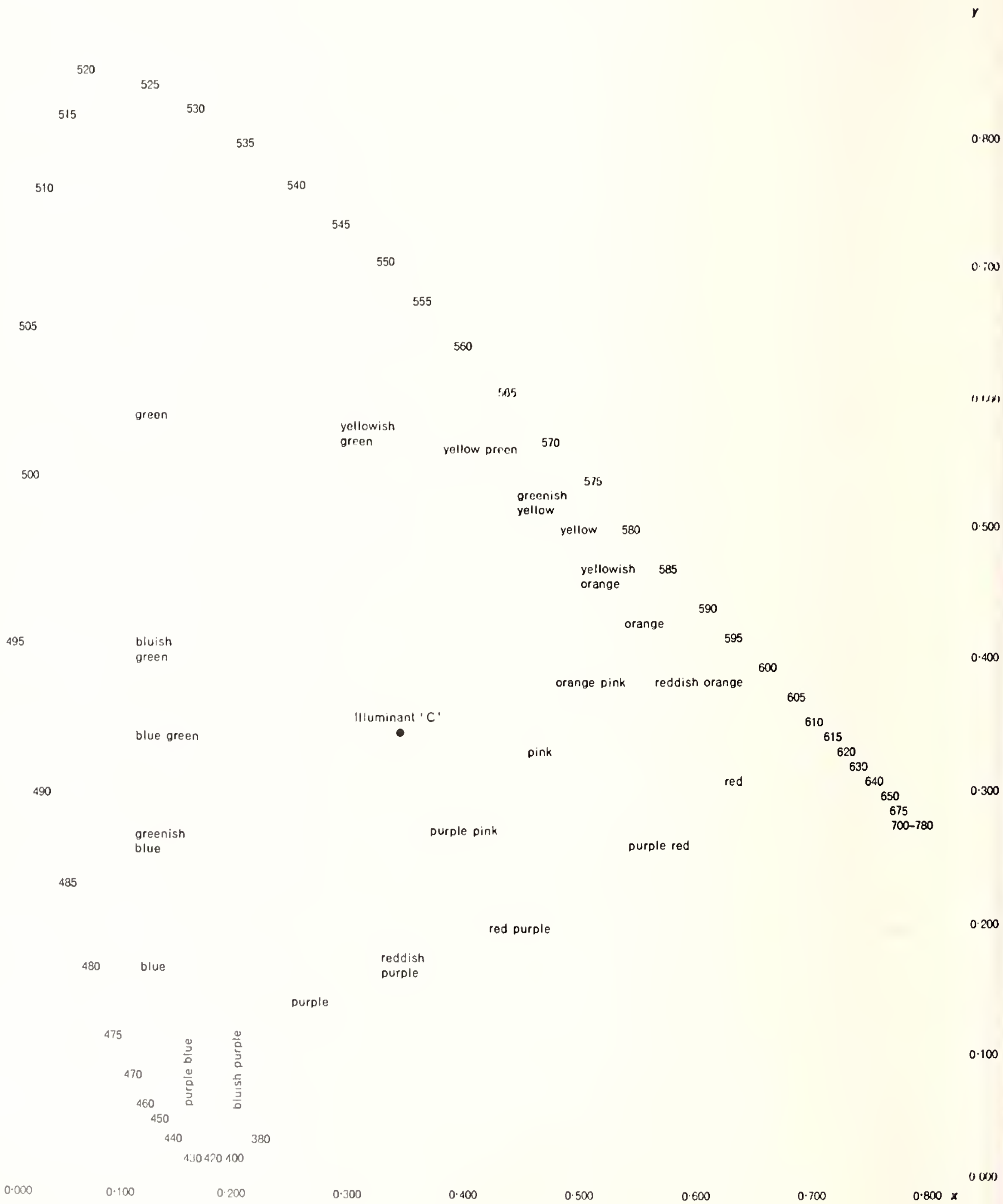
$$X = \int_K^L \varphi(\lambda)\,\bar{x}(\lambda)\,d\lambda\,,$$

$$Y = \int_K^L \varphi(\lambda)\,y(\lambda)\,d\lambda\,,$$

$$Z = \int_K^L \varphi(\lambda)\,\bar{z}(\lambda)\,d\lambda\,.$$

In what sense do the tristimulus values of a patch of light amount to a measurement of its colour? Leaving aside the differences between observers with so-called normal vision, such as caused by the presence of more or less yellow pigment in their eye media (lens, macula), the tristimulus values measure colour only in the sense that if there are two patches of light characterised by X_1, Y_1, Z_1, and X_2, Y_2, Z_2, respectively, and if $X_1 = X_2$, $Y_1 = Y_2$, and $Z_1 = Z_2$, then the patches will look alike in colour. In this sense, the answer to the question: Can we measure colour? is that we certainly can. Colour measurement in this sense is very useful in science and technology. It permits the establishment of

Fig. 1. The hues perceived to belong to patches of light (1 to 4° in angular subtense, luminance between 0·1 and 100 millilamberts) viewed either with a dark surround, or with a surround of near-daylight quality whose luminance is somewhat less than that of the patch itself, as a function of the chromaticity of the patch. This is the C.I.E. (x, y)-chromaticity diagram formed by plotting $Y/(X+Y+Z)$ against $X/(X+Y+Z)$. Spectrum chromaticities are shown, the wavelengths being indicated in millimicrons (after Kelly).



colour standards firmly based on physical measurements, and it permits scientists experimenting in colour perception to record conveniently and with satisfactory precision an essential characteristic of the stimuli that they use. It permits this even though these stimuli of identical colour may differ enormously in spectral composition. If we define colour as the property of light by which an observer of normal colour vision can distinguish between two structure-free patches of light of the same size and shape, then we may say that we can measure colour. If the spectral radiances of two patches of light are known, then we can predict with some confidence whether or not they will look alike.

Estimation of colour perception

Anyone responding to the complicated scenes of daily life, whether they be the interiors of our living and work spaces, the landscapes and seascapes presented to us by nature, or the carefully contrived scenes recorded in paint by artists, can estimate in purely perceptual terms the colour of each individual patch making up the scene. Some of these terms are red, yellow, green, blue, grayish, and vivid. Object colours are also described in terms of light and dark; self-luminous areas, as bright and dim. These estimations may be made systematically on scales of hue, saturation and lightness (ranging from black to white) for object colours, and in terms of hue, saturation and brightness (ranging from dim to dazzling) for areas perceived as self-luminous. These estimates, furthermore, under suitably controlled conditions, show a considerable degree of agreement from one practised observer of normal colour vision to another. It can be argued that these estimates constitute measurement in a sense, and perhaps they do; but I choose to call them only estimates. In any case, the ability to make such estimates does not permit us to predict from physical measurements of a scene what colours will be seen there; it merely makes sure that any such prediction can be put to a test that is at least semi-quantitative.

The unsolved problem of colour perception is: Given the spectral radiance of every patch of light making up the scene; required to predict the hue, lightness, and saturation of every object colour that would be perceived in the scene by an observer with normal colour vision.

The ability to specify the tristimulus values of each such patch of light is far from a solution to this problem. It is merely the first step. So far is this ability from being a solution that Goethe regarded with impatience any study of colour based on the torturing of light through a prism, Hering devoted little attention to such studies, and recently Land has stated that colour perception has almost nothing to do with the wavelength of the light reaching the eye of the observer. In this sense of predicting from physical measurements of a scene what colours will be perceived there, we cannot yet measure colour.

Analytical approach to colour perception

It is fairly common practice in explaining the tristimulus system of colour measurements to point out that the colour-matching functions, x_λ, y_λ, z_λ, shown in Figure 4 of the preceding article, may be thought of as sensitivity functions. Thus the $x(\lambda)$-function is said to express the wavelength variation of the sensitivity of the eye to a certain kind of redness; the $y(\lambda)$-function, that to greenness; and the $z(\lambda)$-function, that to blueness. The tristimulus values, X, Y, Z, of a stimulus by this view correspond to evaluations of the redness, greenness, and blueness responses, respectively, of the eye to the stimulus. Equal redness and greenness responses correspond to the yellowness response; and equal response to all three, to the whiteness-response. One of the most used devices establishing a correlation between tristimulus values of a patch of light and the colour perceived to belong to that patch is the chromaticity diagram. This diagram is formed by plotting the ratio of one of the tristimulus values to their sum against the similar ratio of another, say $Y/(X+Y+Z)$ against $X/(X+Y+Z)$. The resulting diagram is then used as a map of perceived colour quality. Figure 1 shows an allocation of the names of these colour qualities within the (x, y)-chromaticity diagram published by Kelly. Of course, those who say that the colour-matching functions may be thought of as sensitivities must hasten to say that the sensitivity of the eye is constantly changing within wide limits by chromatic adaptation; so $x(\lambda)$ $y(\lambda)$, $z(\lambda)$ have to refer to sensitivities only for an eye adapted to daylight or to darkness. Furthermore, the names for the colour qualities of Figure 1 apply only to patches of light viewed against a darker surround. The analytical approach to colour perception within more complicated scenes then proceeds by pointing out and evaluating the various factors that prevent the tristimulus values from correlating with the colour perception.

General chromatic adaptation

When the whole visual field is filled with chromatic light long enough for adaptation to be nearly complete, that kind of light tends to appear achromatic, or neutral, and lights of other chromaticities undergo corresponding changes. For example, to the red-adapted eye, red light may appear nearly neutral; and in the so-called cigarette illusion, reviewed in detail by Helson, the glowing tip of the cigarette appears to the red-adapted eye, not orange red, but greenish yellow. General chromatic adaptation is a special case of successive contrast. It has received a great deal of study with simple scenes, for example, by von Kries, Wright, Judd, Helson, MacAdam, Burnham, Evans and Newhall, and Jameson and Hurwitz. To a first approximation, the effect of chromatic adaptation is that of changing by a factor independent of wavelength the relative redness, greenness, and blueness sensitivities of the eye. This is known as the von Kries coefficient law. A completely unstimulated region of the retina may, however, by general chromatic adaptation yield a chromatic perception. This is known as chromatic induction.

Local chromatic adaptation

The appearance of an element of the visual field depends not only on the tristimulus values of that element but also on those of neighbouring elements. Scores of studies of this phenomenon have been carried out under the names of simultaneous contrast, and retinal interaction. It has been generally supposed (Helson, Judd) that nearby elements exert greater influence on the perceived colour than distant elements, and a valuable quantitative corroboration has recently been supplied by Jameson and Hurwitz.

Discounting of illumination colour

In scenes presented too briefly (say a few seconds) for general chromatic adaptation to approach any-
33 where near to completion, and depicting objects illuminated by some kind of light, the colours perceived to belong to the objects depend markedly upon the colour perceived to belong to the illumination. Thus, a scene depicting objects illuminated by red light may result in the perception of very non-red objects long before the redness has disappeared by general chromatic adaptation. The redness is still there, but it is perceived to belong, not to the objects, but to the illumination. This phenomenon was mentioned by Helmholtz and has been studied in detail by Jaensch under the name of colour transformation.

Memory colour

In scenes depicting familiar objects, the perceived colour tends markedly to approach the colour perceived previously to belong to those objects. The classic study of memory colour is that by Adams, who found large and consistent influences. McCamy has recently demonstrated repeatedly in lectures that familiar chromatic objects are perceived by many in his audiences as having a tinge of the correct hues even when presented in a black-and-white picture.

The analytical approach to colour perception thus starts from its almost complete success for the simple field consisting of a patch of light with a neutral surround viewed by a daylight- or dark-adapted observer, and then proceeds to evaluate the various perturbations (chromatic adaptation, local and general; discounting of illumination colour; memory colour) arising from scenes somewhat more complicated. The danger in this very sensible and fruitful approach is that success in identifying the various factors involved in colour perception be mistaken for success in solving the problem of colour perception. It is easy to fall into the error of believing that the essential truth resides in the chromaticity diagram, that perturbing influences are known and accounted for, and that these are minor. The fact is that the perturbing influences are quite large, and nobody knows how they fit together.

Configurational approach to colour perception

To illustrate how inadequate it is to consider, for example, that the regions of the chromaticity diagram labelled 'red' (see Fig. 1) really correspond to light that is invariably, because of its wavelength composition, perceived as red, it is sufficient to cite two findings. Helson showed to his observers a gray scale (a set of painted chips having reflectances, non-selective with wavelength, ranging between zero and one) on a white surround illuminated by red light. According to Figure 1, the chips should have appeared as a scale of reds ranging from light to dark, but Helson's observers reported pale red, purplish gray, and dark saturated green-blues. Land presented scenes formed by projecting a black-and-white picture, taken with red light and projected in red light, in superposition with a similar picture taken with blue-green light but projected by incandescent-lamp light. According to Figure 1, the colours to be seen would be confined to those of orange, reddish orange, and red hues. The observers of this two-primary projection, however, reported colours of all hues—saturated reds and greens, pale yellow, and dark grayish blue. As a matter of fact, Helson and Michels have shown by experiment that light of nearly any chromaticity (that is, nearly any location on Figure 1) can be made to appear achromatic (gray or black) by choosing a surround of the proper luminance and chromaticity. A similar conclusion can be drawn from Troland's experiment with the 'dimming effect'.

The configurational approach to colour perception is based on the premise that to neglect the interaction of the parts of the visual field may be throwing away the essence of the problem. This approach requires, at some stage of the study, the use of experimental visual fields, such as those used by Land, approaching in complexity those of daily life. It is not concerned with saying that such and such report is an example of, say, discounting of the illumination colour, or simultaneous colour contrast, or some combination of these with other effects, but addresses itself to the central problem: Given any scene made up of elements, each of known tristimulus values; required to predict the colours perceived to belong to the objects depicted in the scene. This is the problem of colour perception, as yet unsolved. The configurational approach, however, has yielded some basic principles. The mechanism of colour perception works by perceiving colour differences. For each element of any scene, there is a stimulus (definable by a set of tristimulus values) which if substituted for that element would result in the perception of an achromatic colour. This stimulus is a function of the totality of all elements making up the scene, and may be expressed as some function of the tristimulus values of those elements. The colour perceived to belong to the actual element of the scene is determined by the difference between this element and the achromatic stimulus associated with it.

For scenes perceived to be uniformly illuminated, it has been found by Judd that the luminance and chromaticity coordinates of this basic, achromatic-appearing element, or adaptation level, as it is called by Helson, may be approximately evaluated in a simple way. The chromaticity coordinates of the basic element are close to the average of the totality of elements comprising the scene, and its luminance is about one-half that of the average luminance of the scene.

For scenes depicting objects illuminated by more than one kind or amount of light, that is, any scene depicting cast shadows, this simple rule does not work; and a successful rule has yet to be formulated. Artists since the time of Leonardo have dealt successfully with this problem by painting such pictures. Either they find by trial and error which paint to use to depict, for example, the partially shadowed part of a red object, or they know from long experience. This is a routine problem with which art students must cope, but it no longer attracts the serious attention of mature artists. Perhaps in another fifty years there will be a scientific answer; so far, the science of colour has provided little more than the concepts in terms of which a general answer may be expressed.

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RELATION BETWEEN NORMAL TRICHROMATIC VISION AND DICHROMATIC VISION REPRESENTING A REDUCED FORM OF NORMAL VISION

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Dichromatic vision is generally assumed to differ from normal trichromatic vision merely by lack of one of the three primary-color sensations of normal vision. This paper provides, for the first time, an analytical proof of the validity of a method originated in 1860 by J. C. Maxwell for identifying the color of the sensation lacking for any particular dichromatic observer.

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RELATION BETWEEN NORMAL TRICHROMATIC VISION AND DICHROMATIC VISION REPRESENTING A REDUCED FORM OF NORMAL VISION

DEANE B. JUDD*

In 1860 Maxwell subtracted the tristimulus values of a color set by a dichromatic observer from the tristimulus values of the same color set by an observer with normal color vision, and stated that the three differences specified " that color, the lack of sensation of which forms a defect of the dichromatic eye." Nuberg and Yustova, in 1955, gave a vectorial proof of this relation, and in the present paper an analytical proof is given.

INTRODUCTION

In an important paper, investigation of color vision of dichromats, Nuberg and Yustova [1] drew attention to the method used by Maxwell [2] to evaluate " that color, the lack of sensation of which forms a defect of the dichromatic eye." Maxwell's method was to obtain the tristimulus values set by the dichromatic observer to match any color and to subtract them from the tristimulus values of the same color set by an observer having normal trichromatic vision. The differences specify the color for the perception of which no mechanism exists in the dichromatic observer.

Nuberg and Yustova [1] further pointed out that prior to their own work no further use of Maxwell's method had been made. Instead, the chromaticity coordinates of the missing primary for various types of dichromatic vision have been determined by the more complicated and less precise method of finding two pairs of chromaticities, C_1, C_2, and C_3, C_4 (See Fig. 1) such that the two chromaticities of each pair are indistinguishable to the dichromat but as different as possible to the observer of normal color vision. The two chromaticity points of each pair define a straight line on the chromaticity diagram on which the chromaticity point for the missing primary must be located; so this chromaticity point is determined as the intersection I of the two lines defined by the two pairs of chromaticities.

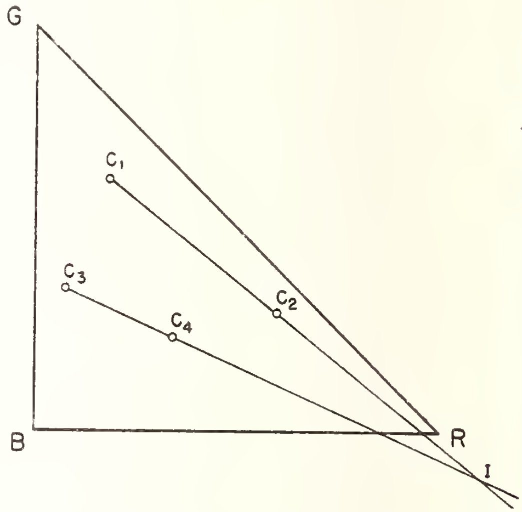

Fig. 1. Chromaticity diagram for any system of working primaries, R, G. B. If the dichromatic observer finds chromaticities C_1 and C_2 indistinguishable, and also C_3 indistinguishable from C_4, the chromaticity point for the missing primary is given by the intersection point I.

Nuberg and Yustova [1] stated the condition that must be satisfied for either method to be valid, and gave a vectorial proof of Maxwell's method. The condition is that all matches set by the trichromatic observer must hold for the dichromatic observer (that is, the vision of the dichromatic observer must be what is

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called a reduction form of that of the trichromatic observer). It is the purpose of the present paper to give an analytical proof of Maxwell's method.

Given: An observer with trichromatic vision defined by color-matching functions, $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$, expressed in any arbitrarily chosen tristimulus system, and an observer with dichromatic vision for whom all metameric matches set by the trichromatic observer hold. Let R_1, G_1, B_1 be the tristimulus values of any arbitrarily chosen color set by the trichromatic observer and let R_2, G_2, B_2 be the tristimulus values of the same color set by the dichromatic observer such that $R_1 \neq R_2$, $G_1 \neq G_2$, or $B_1 \neq B_2$, that is, set so that the trichromatic observer sees a definite mismatch between the arbitrarily chosen color and the match set by the dichromatic observer.

Required to Prove

That the intersection point on the chromaticity diagram of the arbitrarily chosen tristimulus system for the straight lines each of which is a locus of chromaticities confused by the dichromat has coordinates r_I, g_I, equal to the ratios of the differences, R_1-R_2, G_1-G_2, B_1-B_2, to their sum; that is, to prove that:

$$\left.\begin{aligned} r_I &= (R_1-R_2)/[(R_1-R_2)+(G_1-G_2)+(B_1-B_2)],\\ g_I &= (G_1-G_2)/[(R_1-R_2)+(G_1-G_2)+(B_1-B_2)]. \end{aligned}\right\} \quad (1)$$

Proof

Let $L_{\lambda,1}$ and $L_{\lambda,2}$ be the spectral radiances of any two patches of light. The conditions that must be satisfied if these two patches are to appear identical to the trichromatic observer whose color-matching functions are $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$, are

$$\left.\begin{aligned} \int_0^\infty (L_{\lambda,1}-L_{\lambda,2})\,\bar{r}(\lambda)\,d\lambda &= 0,\\ \int_0^\infty (L_{\lambda,1}-L_{\lambda,2})\,\bar{g}(\lambda)\,d\lambda &= 0,\\ \int_0^\infty (L_{\lambda,1}-L_{\lambda,2})\,\bar{b}(\lambda)\,d\lambda &= 0. \end{aligned}\right\} \quad (2)$$

If the two patches of light are spectrally identical, that is, if $L_{\lambda,1}-L_{\lambda,2}=0$ throughout the visible spectrum, Eq. (2) is satisfied regardless of what form the color-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ may take, and the match is what is known as a nonmetameric, or isomeric, match. On the other hand, if $L_{\lambda,1}-L_{\lambda,2}$ is different from zero somewhere within the visible spectrum, the match depends on the color-matching functions of the trichromatic observer, and is known as a metameric match. A dichromatic observer requires but two color-matching functions, $\bar{w}(\lambda)$ and $\bar{k}(\lambda)$, and if this dichromatic observer finds all metameric matches set by the trichromatic observer also to be matches for him, then the two color-matching functions of the dichromat must be weighted sums of the three color-matching functions of the trichormat; that is, $\bar{w}(\lambda)$ and $\bar{k}(\lambda)$ must have the form:

$$\left.\begin{aligned} \bar{w}(\lambda) &= c_{11}\,\bar{r}(\lambda)+c_{12}\,\bar{g}(\lambda)+c_{13}\,\bar{b}(\lambda),\\ \bar{k}(\lambda) &= c_{21}\,\bar{r}(\lambda)+c_{22}\,\bar{g}(\lambda)+c_{23}\,\bar{b}(\lambda), \end{aligned}\right\} \quad (3)$$

in order to ensure that

$$\int_0^\infty (L_{\lambda,1}-L_{\lambda,2})\,\bar{w}(\lambda)\,d\lambda = \int_0^\infty (L_{\lambda,1}-L_{\lambda,2})\,\bar{k}(\lambda)\,d\lambda = 0.$$

The tristimulus values of the first light patch are defined in the usual way as:

$$R_1 = \int_0^\infty L_{\lambda,1}\,\bar{r}(\lambda)\,d\lambda, \quad G_1 = \int_0^\infty L_{\lambda,1}\,\bar{g}(\lambda)\,d\lambda,$$
$$B_1 = \int_0^\infty L_{\lambda,1}\,\bar{b}(\lambda)\,d\lambda,$$

and R_2, G_2, B_2, W_1, K_1, W_2, K_2 are defined similarly.

Chromaticity coordinates (r, g, b) for the trichromatic observer and those (w, k) for the dichromatic observer are defined in the usual way as:

$$r = R/(R+G+B), \quad g = G/(R+G+B),$$
$$b = B/(R+G+B),$$
$$w = W/(W+K), \quad k = K/(W+K),$$

so that

$$r+g+b = 1, \quad \text{and} \quad w+k = 1.$$

Chromaticity Coordinates of the Intersection Point

The chromaticity coordinates, r_I, g_I, of the intersectom point of the dichromatic chromaticity-confusion lines depend on the weights (c_{11}, c_{12}, and so on from Eq. (3)) by which the dichromatic color-matching functions are related to the trichromatic. For each chromaticity confusion line, the dichromatic chromaticity coordinates, w, k, become fixed at, say, w_0, k_0, and this condition may be expressed as:

$$w_0 = W/(W+K),$$

or, solving for W/K:

$$W/K = w_0/(1-w_0),$$

and from Eq. (3) this condition may be written as:

$$\frac{c_{11}R+c_{12}G+c_{13}B}{c_{21}R+c_{22}G+c_{23}B} = \frac{w_0}{1-w_0}.$$

By dividing numerator and denominator by $R+G+B$, we obtain:

$$\frac{c_{11}r+c_{12}g+c_{13}b}{c_{21}r+c_{22}g+c_{23}b} = \frac{w_0}{1-w_0}.$$

By substituting for b its equivalent, $1-r-g$, we obtain:

$$\frac{(c_{11}-c_{13})r+(c_{12}-c_{13})g+c_{13}}{(c_{21}-c_{23})r+(c_{22}-c_{23})g+c_{23}} = \frac{w_0}{1-w_0},$$

which may take the form of:

$$(c_1r+c_2g+c_3)-\frac{w_0}{1-w_0}(c_4r+c_5g+c_6) = 0, \quad (4)$$

where $c_{11}-c_{13}$ is abbreviated as c_1 and $c_{12}-c_{13}$ as c_2; and so forth. Since Eq. (4) is a linear equation in r and g, it defines a straight line on the (r, g)-chromaticity diagram for any chosen fixed value of w_0; and if w_0 be given a series of fixed values, Eq. (4) defines a family of straight lines intersecting at a common point. To find the intersection point, solve the equations:

$$\left.\begin{aligned} c_1r+c_2g+c_3 &= 0, \\ c_4r+c_5g+c_6 &= 0, \end{aligned}\right\} \quad (5)$$

simultaneously, and substitute the resulting values, r_I and g_I, into Eq. (4). It is seen that these particular values (r_I, g_I) of r and g satisfy Eq. (4) regardless of what fixed value may be chosen for w_0. The values of the chromaticity coordinates of the intersection point are:

$$\left.\begin{aligned} r_I &= \frac{c_2c_6-c_3c_5}{c_1c_5-c_2c_4} \\ &= (c_{12}c_{23}-c_{13}c_{22})/(c_{12}c_{23}-c_{13}c_{22} \\ &\quad +c_{13}c_{21}-c_{11}c_{23}+c_{11}c_{22}-c_{12}c_{21}), \\ g_I &= \frac{c_3c_4-c_1c_6}{c_1c_5-c_2c_4} \\ &= (c_{13}c_{21}-c_{11}c_{23})/(c_{12}c_{23}-c_{13}c_{22} \\ &\quad +c_{13}c_{21}-c_{11}c_{23}+c_{11}c_{22}-c_{12}c_{21}). \end{aligned}\right\} \quad (6)$$

Specification of a Color Pair Indistinguishable in Dichromatic Vision but Different in Trichromatic Vision

If the tristimulus values of any color be taken as R_1, G_1, B_1, the tristimulus values, R_2, G_2, B_2, of any other color indistinguishable in dichromatic vision must be such that:

$$W_1 = W_2 \quad \text{and} \quad K_1 = K_2,$$

and these conditions by virtue of Eq. (3) can be written:

$$\left.\begin{aligned} c_{11}R_1+c_{12}G_1+c_{13}B_1 \\ = c_{11}R_2+c_{12}G_2+c_{13}B_2, \\ c_{21}R_1+c_{22}G_1+c_{23}B_1 \\ = c_{21}R_2+c_{22}G_2+c_{23}B_2. \end{aligned}\right\} \quad (7)$$

With no loss of generality we can ensure that the color R_2, G_2, B_2 is different in trichromatic vision from color R_1, G_1, B_1 simply by taking R_2 arbitrarily at any value different from R_1. Let G_2 and B_2 be expressed as unknown fractions, f_g and f_b, of G_1 and B_1, respectively; that is:

$$G_2 = f_gG_1, \quad \text{and} \quad B_2 = f_bB_1.$$

By making these substitutions in Eq. (7) we obtain two equations with the two unknowns, f_g and f_b. Solution of these two equations simultaneously results in:

$$\left.\begin{aligned} f_g &= 1+(c_{11}c_{23}-c_{13}c_{21})(R_1-R_2)/ \\ &\quad (c_{12}c_{23}-c_{13}c_{22})G_1, \\ f_b &= 1+(c_{12}c_{21}-c_{11}c_{22})(R_1-R_2)/ \\ &\quad (c_{12}c_{23}-c_{13}c_{22})B_1. \end{aligned}\right\} \quad (8)$$

The tristimulus values of the second color different in trichromatic vision but identical in dichromatic vision are thus:

$$R_2, \quad f_g G_1, \quad f_b B_1,$$

for f_g and f_b as given in Eq. (8). The differences whose ratios to their sums we must prove (see Eq. (1)) to be equal to the coordinates of the intersection point r_I, g_I, b_I are thus:

$$\begin{aligned} R_1-R_2 &= R_1-R_2, \\ G_1-G_2 &= (R_1-R_2)(c_{13}c_{21}-c_{11}c_{23})/ \\ &\quad (c_{12}c_{23}-c_{13}c_{22}), \\ B_1-B_2 &= (R_1-R_2)(c_{11}c_{22}-c_{12}c_{21})/ \\ &\quad (c_{12}c_{23}-c_{13}c_{22}). \end{aligned}$$

The ratios of these numbers to their sum are:

$$\left.\begin{aligned} &(R_1-R_2)/[(R_1-R_2)+(G_1-G_2) \\ &\quad +(B_1-B_2)] \\ &\quad = (c_{12}c_{23}-c_{13}c_{22})/(c_{12}c_{23}-c_{13}c_{22} \\ &\quad +c_{13}c_{21}-c_{11}c_{23}+c_{11}c_{22}-c_{12}c_{21}), \\ &(G_1-G_2)/[(R_1-R_2)+(G_1-G_2) \\ &\quad +(B_1-B_2)] \\ &\quad = (c_{13}c_{21}-c_{11}c_{23})/(c_{12}c_{23}-c_{13}c_{22} \\ &\quad +c_{13}c_{21}-c_{11}c_{23}+c_{11}c_{22}-c_{12}c_{21}). \end{aligned}\right\} \quad (9)$$

Comparison with Eq. (6) shows these ratios also to be equal to the chromaticity coordinates of the intersection point, r_I, g_I, which was to be proved (Eq. (1)).

SUMMARY

If, in any arbitrarily chosen tristimulus system, color R_2, G_2, B_2 appears in dichromatic vision to be like color R_1, G_1, B_1 but in trichromatic vision to be different, and if every color match set in accord with this trichromatic vision also appears to be a match for this kind of dichromatic vision, then the differences R_1-R_2, G_1-G_2, B_1-B_2 and the chromaticity coordinates, r_I, g_I, b_I of the intersection point of the dichromatic confusion lines are alike proportional to:

$$c_{12}c_{23}-c_{13}c_{22}, \quad c_{13}c_{21}-c_{11}c_{23}, \quad c_{11}c_{22}-c_{12}c_{21},$$

where the color-matching functions $\bar{w}(\lambda)$, $\bar{k}(\lambda)$ defining this kind of dichromatic vision in the same arbitrarily chosen tristimulus system are related to the color-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ defining this kind of trichromatic vision in accord with:

$$\begin{aligned} \bar{w}(\lambda) &= c_{11}\bar{r}(\lambda)+c_{12}\bar{g}(\lambda)+c_{13}\bar{b}(\lambda), \\ \bar{k}(\lambda) &= c_{21}\bar{r}(\lambda)+c_{22}\bar{g}(\lambda)+c_{23}\bar{b}(\lambda). \end{aligned}$$

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[1] Nuberg, N. D. and Yustova, E. N. *Trudy Gos. Opticheskogo Instituta*, **24**, (1955), 33.

[2] Maxwell, J. C. *Phil. Trans.*, **150**, (1860), 57.

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SPECTRAL DISTRIBUTION OF TYPICAL DAYLIGHT AS A FUNCTION OF CORRELATED COLOR TEMPERATURE

(with D. L. MacAdam and G. Wyszecki)

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The method for computing the spectral distribution of daylight for any color temperature, recommended in 1964 by the CIE, is based on the data and formulas given in this paper. The CIE-recommended distributions for D_{55}, D_{65}, and D_{75} were computed by use of that method and are not significantly different from those given in Table V and shown in Fig. 3.

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Spectral Distribution of Typical Daylight as a Function of Correlated Color Temperature

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Spectral distributions of 622 samples of daylight (skylight, and sunlight plus skylight) have been subjected to characteristic vector analysis, as composite data and in three subgroups (99 distributions measured by Budde; 249, by Condit; and 274, by Henderson and Hodgkiss). The chromaticity coordinates (x,y) computed from these distributions have been compared with direct visual determinations of chromaticity coordinates of daylight by Nayatani and Wyszecki, and by Chamberlin, Lawrence, and Belbin. It was found that the chromaticities indicated by the spectral distributions and by direct visual colorimetry cluster about the curve: $y=2.870x-3.000x^2-0.275$. This curve of typical daylight chromaticities falls slightly on the green side of the Planckian locus. From the mean and the first two characteristic vectors of the composite data, spectral distribution curves have been reconstituted by choice of scalar multiples of the vectors such that the chromaticity points fall on the curve of typical daylight chromaticities at places corresponding to correlated color temperatures of 4800°, 5500°, 6500°, 7500°, and 10 000°K. The representative character of these reconstituted spectral-distribution curves has been established by comparison with the measured curves from each subgroup yielding the closest approximation to the same chromaticities. The agreement so found suggests that this family of curves is more representative of the various phases of daylight between correlated color temperatures 4800° and 10 000°K than any previously derived distributions.

INTRODUCTION

EVER since light sources B and C were recommended in 1931 by the International Commission on Illumination at its meeting in Cambridge, England, they have served as standard sources for the colorimetry of materials.[1] These sources are combinations of a gas-filled lamp at a color temperature of 2854°K with Davis–Gibson[2] liquid filters intended to approximate within the visible spectrum the spectral distributions of the complete radiator (blackbody) at temperatures of 4800° and 6500°K, respectively. Source B (4800°K) has served in the colorimetric laboratory as a representative of noon sunlight, and source C (6500°K) has served as a representative of average daylight. This laboratory use of these standard sources has steadily diminished in recent years because of the increasing dependence placed on indirect colorimetry by means of the spectrophotometer; but the spectral energy distributions of sources B and C have been widely and increasingly used in the reduction of spectrophotometric data to the colorimetric variables (Y,x,y) of the 1931 CIE standard-observer system.[1]

The development of the Davis–Gibson liquid filters, and their use to realize standard sources for colorimetry, was an important step toward an internationally recognized general method of colorimetry, but standard sources so realized have limitations that prevent them from leading to a completely general method. For example, the ultraviolet content[3] is too low to permit these sources to be an adequate representation of sunlight or daylight in the colorimetry of fluorescent materials. Furthermore, the standard sources cannot yield the high luminous flux required for the most precise inspection of industrial products because the liquids in the filters would become heated above the boiling point. Because of these limitations, because of minor inconveniences arising from the use of liquid filters (difficulties of preparation according to the careful prescription given, need to replace the filters every few months because of drift in their spectral character, temperature coefficient of spectral transmittance), and because improved glass filters could be made available,[4]

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‡ Thorn Electrical Industries Limited, Great Cambridge Road, Enfield, Middlesex, England.
[1] *Proceedings of the International Commission on Illumination, 8th Session, Cambridge, 1931* (Cambridge University Press, Cambridge, England, 1932) p. 19.
[2] R. Davis and K. S. Gibson, "Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature," Natl. Bur. Std. (U. S.), Misc. Pub. 114 (1931).
[3] R. Davis, K. S. Gibson, and G. W. Haupt, J. Res. Natl. Bur. Std. **50**, 31 (1953); J. Opt. Soc. Am. 43, 172 (1953).
[4] G. N. Rautian, N. V. Lobanova, and M. A. Znamenskaya, Z. Tech. Fisiki **26**, 193 (1956). D. B. Judd, Farbe **10**, 31 (1961).

a proposal was made by the Russian delegate to the CIE Committee on Colorimetry in 1957 to define standard sources solely as blackbody radiators and to substitute glass filters for liquid filters in their realization.[5]

As a part of the studies of this Russian proposal, undertaken at the request of the CIE Committee on Colorimetry, a questionnaire devised by one of us (GW) was circulated by the Canadian Committee on Colorimetry, and a similar questionnaire devised by D. Nickerson was circulated by the U. S. Technical Committee on Colorimetry.

The replies to the Canadian questionnaire have been summarized[6] as follows:

"Question 4.—Do you suggest that the present CIE standard sources should be changed? If so, which sources do you recommend? Please give references and reasons for your choice."

The replies received can be separated into three groups, each of which is about equal in size:

"(i) No comments; leaves it up to the CIE to investigate the problem and come forward with recommendations."

"(ii) No change recommended; emphasizes the possible confusion a change may bring about and the considerable effect it may have on existing tabulations and instrumentation."

"(iii) New sources or changes are desirable; it would be desirable to have one or more standard daylight sources which are of more practical use than the present Source C and thus encourage the use of the standard source in inspection work. A new standard source should provide a high level of illuminance over a large area of the visual field. The ultraviolet content should be comparable with that of natural daylight both in amount and spectral distribution."

Nickerson[6] summarized the replies to the questionnaire as follows: "No change should be made in CIE standard light sources until a 300–800 nm definition for a target standard for daylight can be agreed upon, and a sufficiently close match to this can be obtained in an actual lamp, or in a lamp–filter combination, that can be agreed upon for use as a standard source for colorimetric and visual use. Further, that any series adopted for light-source standards should include one that in color temperature is at least as high as 7400°K. This is for use as a daylight substitute in colorimetry and for visual use, to be adopted either in replacement for, or in addition to, the present source C."

[5] *Proceedings of the International Commission on Illumination, 14th Session, Brussels, 1959* (Bureau Central de la Commission Internationale de l'Eclairage, 57, rue Cuvier, Paris, 5e, France, 1960), Vol. A, p. 93.

[6] *Proceedings of the International Commission on Illumination, 15th Session, Vienna, 1963* (Bureau Central de la Commission Internationale de l'Eclairage) (to be published).

A meeting of U. S. Technical Committee on Colorimetry was convened in Rochester in October 1962 to consider the results of these questionnaires and to explore what next steps ought to be taken in the development of standard sources for colorimetry. All members of the committee were present (I. A. Balinkin, University of Cincinnati; D. B. Judd, National Bureau of Standards; D. L. MacAdam, Eastman Kodak Company; D. Nickerson, U. S. Department of Agriculture; and W. T. Wintringham, Bell Telephone Laboratories). Present also as guests at one time or another were W. E. K. Middleton and G. Wyszecki, National Research Council of Canada; Norman Macbeth, Macbeth Corporation; and H. R. Condit, O. E. Miller, and J. L. Simonds, Eastman Kodak Company. Macbeth criticized existing standards of daylight, including Abbot–Gibson daylight[7] and Middleton's computed distributions[8] for the overcast sky with an urban terrain suggested by the committee in 1955 as a target standard,[9] as well as CIE sources B and C, as being too pink to be acceptable representations of typical daylight. These recommended spectral distributions correspond to chromaticities on the pink side of the Planckian locus on the chromaticity diagram. He called attention to the fact that over a period of several decades his company has sold thousands of artificial daylighting units not only with a color temperature range, but also with a pink–green range. Many units yielding colors on the pink side of the Planckian locus, and even those falling very slightly on the green side, have been returned by customers with the complaint that they were too pink to give color renditions characterizing natural daylight.[10]

Examination of a chromaticity diagram supplied by Nickerson comparing the chromaticities corresponding to the spectral distributions of sunlight and daylight by Abbot[11] as summarized by Moon,[12] by Taylor and Kerr,[13] and by Henderson and Hodgkiss supplied in advance of publication[14] was found to support Macbeth's indirect finding that typical daylight is more greenish than existing standards of daylight.

MacAdam reported that extensive spectral distribution data for various phases of daylight had been obtained in Rochester in recent years independently in

[7] K. S. Gibson, J. Opt. Soc. Am. **30**, 88 (1940).

[8] W. E. K. Middleton, J. Opt. Soc. Am. **44**, 793 (1954).

[9] *Proceedings of the International Commission on Illumination, 13th Session, Zürich, 1955* (Bureau Central de la Commission Internationale de l'Eclairage, 57, rue Cuvier, Paris 5e, France, 1955), Vol. I, p. 1.3.1 U-9.

[10] Norman Macbeth and W. B. Reese, "Some practical notes on standard illumination practices for color matching in the U. S. A.—past, present, and future," 7th International Conference on Color; Florence, Prato, and Padua, Italy; 2–7 May 1963.

[11] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, "The distribution of energy in the spectra of the sun and stars," Smithsonian Miscellaneous Collections **74**, No. 7, Publ. No. 2714 (1923).

[12] P. Moon, J. Franklin Inst. **230**, 583 (1940).

[13] A. H. Taylor and G. P. Kerr, J. Opt. Soc. Am. **31**, 3 (1941).

[14] S. T. Henderson and D. Hodgkiss, Brit. J. Appl. Phys. **14**, 125 (1963).

two projects of the Eastman Kodak Company, one (Condit)[15] by spectroradiometer, the other (Miller) by interference filters, and that some of the data gathered by Condit had been subjected to characteristic vector analysis by Simonds.[16] Wyszecki said that a project of the National Research Council was about to get under way to measure the spectral distribution of north skylight in Ottawa simultaneously with visual determinations of chromaticity as a check. In view of the fact that data recently taken, or about to be taken, were far more numerous, and sampled much more completely the range of spectral distributions of daylight than previously published data, it was decided that the technical colorimetry committees of Canada and the United States would jointly undertake to evaluate from these recent data (Ottawa, Rochester, and Enfield, England), a series of related spectral distributions of typical daylight extending over a considerable range of correlated color temperatures. It is the purpose of this paper to describe the method of evaluation, to present the series of spectral distributions, and to present evidence supporting the view that each individual spectral distribution of the series is typical of daylight of a particular correlated color temperature.

COLLABORATION OBTAINED

Condit, Henderson, and Budde supplied spectral distribution data relative to 560 nm for various samples of daylight in the form of relative spectral irradiance for each 10-nm interval between 330 and 700 nm in advance of publication. Condit supplied distributions for 249 samples of daylight; Henderson, 274; and Budde, 99.

Simonds supervised derivation of the first four characteristic vectors not only for the composite data (622 distributions), but also for the three subgroups (Condit, Henderson, Budde) over the same spectral range, and for the Henderson data over the range 330 to 780 nm. The program and computer time for computing the chromaticity coordinates for each input distribution, for evaluation of the characteristic vectors, for obtaining reconstituted distributions from the means and the first two characteristic vectors, and for checking these against the corresponding individual input distributions were supplied by the Eastman Kodak Company.

CHARACTERISTIC VECTOR METHOD

A mathematical statement of the method, adapted from Simonds,[16] might be given as follows:

Response data E_λ (spectral irradiance) are available for $r(38)$ levels of the variable λ (wavelength). For each experimental condition, then, the r values of E_λ constitute a one-row r-column vector of response data.

[15] H. R. Condit and F. Grum, J. Opt. Soc. Am. 53, 1340 (1963); 54, 937 (1964).

[16] R. H. Morris and J. H. Morrissey, J. Opt. Soc. Am. 44, 530 (1954); J. L. Simonds, *ibid.* 53, 968 (1963).

For n (622, 249, 274, or 99) sample sets of data, the response vectors can be arrayed to form a data matrix of n rows and r columns.

It is possible to find a set of characteristic vectors, which, when added in the proper amounts to the mean response vector, will adequately approximate any of the original family of n response vectors. Details of the computational procedure are given by Simonds.[16] The procedure involves the computation of the variance–covariance matrix S from the original n-by-r data matrix of response vectors. Characteristic vectors of this matrix are obtained corresponding to the latent roots of the determinantal equation

$$|S-LI|=0, \tag{1}$$

where I is an r-by-r unit matrix and L is a diagonal matrix (r-by-r) of the latent roots. The characteristic vectors, like the response vectors themselves, are sets of r numbers. Mathematically stated, the sample responses at each value of wavelength are given by Eq. (2):

$$\left.\begin{aligned} E_1&=\bar{E}_1+M_1V_{1,1}+M_2V_{2,1}\\ &\qquad+M_3V_{3,1}+\cdots M_pV_{p,1},\\ E_2&=\bar{E}_2+M_1V_{1,2}+M_2V_{2,2}\\ &\qquad+M_3V_{3,2}+\cdots M_pV_{p,2},\\ E_r&=\bar{E}_r+M_1V_{1,r}+M_2V_{2,r}\\ &\qquad+M_3V_{3,r}+\cdots M_pV_{p,r}.\end{aligned}\right\} \quad p\leq r. \tag{2}$$

The M's are the amounts of the characteristic vectors which must be added to the mean response vector in order to produce the sample response vector. The characteristic vectors V_p are uniquely determined for a given family of response curves. The same characteristic vectors apply to all response vectors belonging to the original family from which the vectors are derived. Only the values of M_1, M_2, $\cdots$, M_p vary from one response curve to another. The M's, therefore, are a complete specification of the response vector to which they apply. Together with the uniquely determined characteristic vectors and the mean response vector, the M's are sufficient information with which to reconstruct the entire response vector from which they were derived.

The number p of characteristic vectors required to explain all the differences among a family of curves, each represented by r responses, will be equal to, or less than r. For a perfect fit to all the response vectors, r characteristic vectors may be required. The power of the tool, however, comes from the fact that a large percentage of the variability among the family of homologous response vectors may be explained by using only a few characteristic vectors.

The p vectors, when graphically presented, are basis curves or vectors. The following statements are given without proof:

(1) The p characteristic vectors are orthogonal; that is, they represent independent types of response

variability. Mathematically stated,

$$\sum_{i=1}^{r} V_{a,i}V_{b,i}=0, \quad a\neq b.$$

(2) The M's, called scalar multiples, can be determined for each sample response vector as a simple linear combination of the response data at the r values of wavelength. The weighting coefficients are uniquely determined for a particular set of vectors.

(3) The derivation of the vectors ensures that the first vector accounts for the largest amount of the total response variability; the second vector accounts for the second largest amount of variability; and so forth.

EXPERIMENTAL DATA

The 249 spectral distributions of Rochester daylight supplied by Condit were obtained by comparing the light reflected from a magnesium oxide test plate illuminated by incandescent lamp light of known color temperature with that reflected from a barium sulfate test plate tilted 15° off the vertical and illuminated either by light from the sky and light from the sun at various altitudes in the plane perpendicular to the test plate, or simply by light from the sky with the sun being back of the plate or obscured by clouds. The instrument used was the Beckman DK spectrophotometer whose slitwidths are automatically adjusted to maintain the output of the photodetector constant. The spectral bands transmitted varied from about 1 to 3 nm at half-height. The scanning time was about 1 min. The measured spectral distribution curves extended from 330 to 700 nm, and were read at somewhat irregular intervals dictated by an attempt to obtain values of relative spectral irradiance at the middle of each major absorption or transmission band detected. The input data were derived at intervals of 10 nm by interpolation of these data. Small corrections were applied for the difference in spectral reflectance between magnesium oxide and barium sulfate. The measurements refer to sky conditions not yielding any visual evidence of industrial contamination.

The 274 spectral distributions of daylight in Enfield, England, supplied by Henderson, were obtained partly by light reflected from a horizontal diffusing white plate of known spectral reflectance relative to magnesium oxide, consisting of mat finish Vitrolite receiving light from nearly the whole hemisphere including direct sunlight if it was not obscured by clouds. The remainder of the observations refer to the north sky at 45° with an acceptance angle of about 6°. The Hilger and Watts D290 grating monochromator was used, and the scan was divided between two photocells (Mazda 27M3 from 300 to 580 nm, RCA 1P22 from 540 to 780 nm). The resulting spectroradiometer was calibrated by light from an incandescent lamp at a color temperature of 2854°K reflected from a magnesium oxide screen.[14] Narrow equal slits gave a spectral bandwidth of 1.5 nm. "With the instrument's acceptance angle of about 6°, a comparatively small area was viewed and consequently the signal was sensitive to changes in this small region of the sky. . . . With readings taken every 100 Å . . . a run could be completed in about 10 min, . . . Attempts to compensate for the variation of intensity with time by using a second monochromator set at a fixed wavelength throughout the scan gave some improvement but this could not be carried out regularly. It was decided that a program involving many measurements over an extended period would compensate for the random variations during any one run. With this direct-view arrangement it was desirable to make the measurements at times of apparently steady sky conditions. On a good day the variation over a period of 10 min would be within 3% of the mean value at any given wavelength, but often it was worse than this, up to 10% variation being common. . . . The majority of the observations were made on the laboratory roof and the white plate was used to give an average distribution for most of the hemisphere. The measurement procedure was as before, with some reduction in variability during a run on account of the integrating effect of the white plate."

The 99 spectral distributions of Ottawa daylight were obtained by Budde by measuring the light received at wavelength λ either from the total sky, or from the north sky, relative to that at wavelength 560 nm. This measurement relative to the irradiance at 560 nm eliminated fluctuations of the total irradiance during scanning of the spectrum. The apparatus consisted basically of an integrating sphere, a Hilger quartz prism double monochromator (D191) and a 1P28 RCA photomultiplier tube. The spectrum was scanned with a bandwidth ranging from 1 nm in the ultraviolet to approximately 7 nm in the red. The spectral range covered was from 300 to 720 nm. Simultaneous measurements of the same daylight by means of a Donaldson six-primary colorimeter[17] were made by two observers as a closing check on the accuracy of the measurements of spectral distribution. The agreement in chromaticity coordinates, calculated from the visual and the spectroradiometric measurements, was better than 0.006 in x and y. The data supplied for analysis were those derived from averaging the original distribution curves over bands of 10 nm width.

The composite input data consisted of 622 spectral distributions E_λ (Condit 249, Henderson 274, Budde 99) weighted in inverse proportion to the residual variances $V(E_\lambda/E_{560})$ when the data from each source were separately analyzed and reconstituted with four vectors. The variance for each set of data was computed in the usual way as the sum of the squares of the differences between each input data and the corre-

[17] R. Donaldson, Proc. Phys. Soc. (London) 59, 554 (1947).

TABLE I. Mean and first four characteristic vectors for composite data on spectral irradiance of daylight.

Wavelength (nm)	Mean	V_1	V_2	V_3	V_4
330	553	420	85	91	12
340	573	406	78	78	21
350	618	416	67	66	19
360	615	380	53	52	19
370	688	424	61	43	11
380	634	385	30	4	−41
390	658	350	12	−2	−40
400	948	434	−11	−22	−32
410	1048	463	−5	−35	−28
420	1059	439	−7	−50	−28
430	968	371	−12	−51	−11
440	1139	367	−26	−50	2
450	1256	359	−29	−60	15
460	1255	326	−28	−63	18
470	1213	279	−26	−65	21
480	1213	243	−26	−59	20
490	1135	201	−18	−58	20
500	1131	162	−15	−47	23
510	1108	132	−13	−45	21
520	1065	86	−12	−33	22
530	1088	61	−10	−20	17
540	1053	42	−5	−15	13
550	1044	19	−3	−9	8
560	1000	0	0	0	0
570	960	−16	2	11	−7
580	951	−35	5	−10	−5
590	891	−35	21	−3	−7
600	905	−58	32	−5	−7
610	903	−72	41	−10	−7
620	884	−86	47	−20	−9
630	840	−95	51	−22	−8
640	851	−109	67	−36	−5
650	819	−107	73	−48	−3
660	826	−120	86	−55	−2
670	849	−140	98	−61	2
680	813	−136	102	−65	2
690	719	−120	83	−57	3
700	743	−133	96	−64	5

sponding value reconstituted from the mean and the first four characteristic vectors by Eq. (2) divided by the number of input data. On this basis, each of the monitored distributions supplied by Budde received full weight, the unmonitored distributions supplied by Condit and Henderson receiving, as expected, somewhat lower weights.

MEANS AND CHARACTERISTIC VECTORS

Table I gives, for the composite data, the means and the first four characteristic vectors. The analogous means and vectors derived from the three subsets of data (Condit, Henderson, Budde) for the same spectral range are available from the authors on request as are also means and characteristic vectors derived from the Henderson data for the range 330 to 780 nm.

It will be noted from Table I that, at the normalizing wavelength 560 nm, the mean of the spectral distributions is set at 1000, and the values of the characteristic vectors are all identically zero there. Reconstitutions of spectral distributions of daylight obtained by Eq. 2 will thus also have values of 1000 at the normalizing wavelength 560 nm.

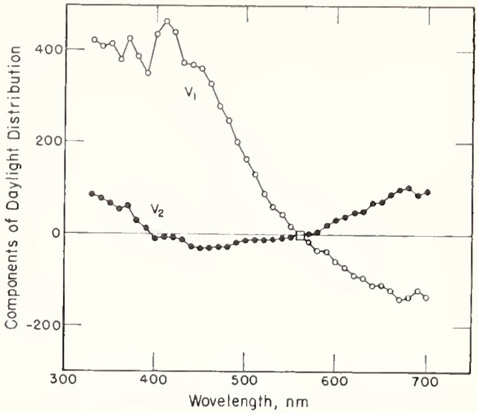

FIG. 1. Vectors V_1 (open circles) and V_2 (solid circles) derived from the composite data (622 spectral distributions).

It will also be noted that the values of the first characteristic vector are greater than zero for wavelengths less than 560 nm, and less than zero for wavelengths greater than 560 nm. The most common variation from one measured curve of daylight spectral distribution to another is thus such as to correspond chiefly to a change in correlated color temperature of the daylight. This yellow-to-blue variation corresponds to presence or absence of clouds in the sky, and to the inclusion or exclusion of direct sunlight.

Note also that the second vector has low values from 380 to 560 nm and higher values in the longwave (red) part of the visible spectrum. This second most common variation thus corresponds to a pink–green variation such as might be caused by the presence of little or much water in the form of vapor and haze.

Figure 1 is a plot of the first and second vectors V_1 and V_2 derived from the composite data. By adding various scalar multiples M_1 and M_2 of these vectors to the mean curve, it is evident that variations either in the yellow–blue sense or the pink–green sense may be introduced into the reconstituted curves. Thus, by adjustment of the scalar multiples M_1 and M_2, reconstituted daylight distribution curves would be generated to correspond to any chromaticity within the daylight range; in particular, values of these scalar multiples exist which make the chromaticity point corresponding to the reconstituted curve fall at any desired point on a locus of typical daylight chromaticities. Any two linear combinations of characteristic vectors would similarly yield an unique distribution curve for each chromaticity point. It was found, however, that reconstituted curves satisfactorily reproducing the measured curves were generated by using only the first two vectors. The remainder of this paper describes the derivation of such a locus on the chromaticity diagram, the derivation of the corresponding scalar multiples for correlated color temperatures 4800°, 5500°, 6500°, 7500°, and 10 000°K, and demonstrations of the degree to which curves reconstituted in

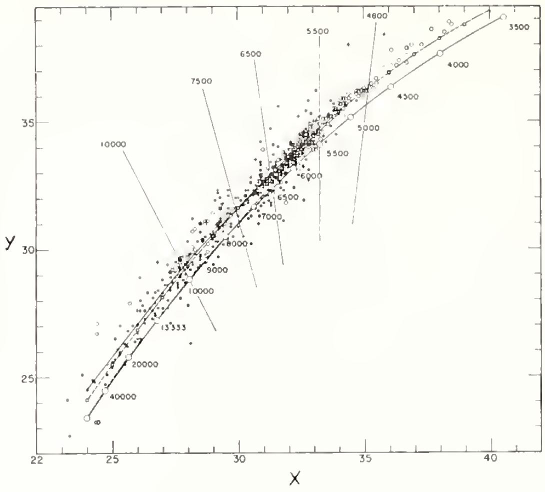

FIG. 2. Chromaticities of daylight compared to the locus of chromaticities implied by the Planck radiation law (open circles connected by solid lines). The temperatures of the complete radiator corresponding to the points at the centers of these open circles are indicated in degrees Kelvin. The straight lines intersecting this locus at 4800°, 5500°, 6500°, 7500°, and 10 000°K correspond to lines of constant correlated color temperature computed by Kelly from the 1960 CIE–UCS diagram. The center of the chromaticity range found for daylight by Nayatani and Wyszecki is shown by solid circles connected with a solid line; that found by Chamberlin, Lawrence, and Belbin, by a dotted line. Chromaticity points computed from the measured spectral distribution curves are indicated by open circles (Condit), by crosses (Henderson and Hodgkiss), and by solid circles (Budde). The locus of chromaticities taken in the present paper to be typical of daylight conforms to the relation: $y=2.870x-3.000x^2-0.275$, and is shown by squares connected by a dotted line.

accord with these scalar multiples may be considered reliable representatives of typical daylight spectral distributions.

CHROMATICITIES OF VARIOUS PHASES OF DAYLIGHT

To derive a curve of typical daylight chromaticities, use has been made of recent visual measurements by Chamberlin, Lawrence, and Belbin[18] on the north sky in southern England, and by Nayatani and Wyszecki[19] on a near-north sky at Ottawa, Canada, and of the chromaticities computed from each of the 622 spectral distributions subjected to characteristic vector analysis. Figure 2 is a central portion of (x,y)-chromaticity diagram of the 1931 CIE coordinate system. It shows as solid circles connected by a solid line the locus derived by Nayatani and Wyszecki, as a dotted line that derived by Chamberlin, Lawrence, and Belbin, as open circles the chromaticity points computed from the 249 spectral distributions obtained by Condit, as vertical crosses the chromaticity points computed from the 274 spectral distributions supplied by Henderson, and as solid circles the chromaticity points computed from the 99 spectral distributions supplied by Budde. In general, the plotted points derived from the spectral distributions agree well with the two lines representing the visual observations, and they cluster closely about the line defined by: $y=2.870x-3.000x^2-0.275$, and shown as squares connected by a solid line on Fig. 2. The Nayatani–Wyszecki line diverges from the Chamberlin–Lawrence–Belbin line for correlated color temperatures greater than 10 000°K. Since there are too few chromaticity points computed from measured spectral distributions to decide between these two lines above 40 000°K, the adopted locus has been drawn about half-way between the two lines. This adopted locus below 40 000°K is drawn closer to the Nayatani–Wyszecki locus because the computed chromaticity points indicate that course. The adopted locus follows the Planckian locus (shown as open circles connected by a solid line on Fig. 2) fairly closely, but stays slightly on the green side. Figure 2 also shows lines of correlated color temperature computed by Kelly[20] from the 1960 CIE–UCS diagram for correlated color temperatures of 4800°, 5500°, 6500°, 7500°, and 10 000°K. Table II gives the chromaticity coordinates at which these lines of correlated color temperature intersect the locus adopted as typical of daylight chromaticities defined by $y=2.870x-3.000x^2-0.275$.

RELATION BETWEEN SCALAR MULTIPLES AND CHROMATICITY COORDINATES

Any spectral distribution curve E_λ reconstituted from a mean and two characteristic vectors derived from a family of measured spectral distributions is computed by Eq. (2) as:

$$E_\lambda=\bar{E}_\lambda+M_1V_{1,\lambda}+M_2V_{2,\lambda}, \qquad (2a)$$

and the tristimulus value X of the irradiance specified by this distribution may be written:

$$X=\sum E_\lambda\bar{x}_\lambda\Delta\lambda=\sum(\bar{E}_\lambda+M_1V_{1,\lambda}+M_2V_{2,\lambda})\bar{x}_\lambda\Delta\lambda,$$

where the weighting function $\bar{x}_\lambda$ is one of the color matching functions of the 1931 CIE standard observer for colorimetry.[1] From the fact that the scalar multiples M_1 and M_2 are constants independent of wavelength for any single reconstituted curve, the expression for

TABLE II. Chromaticity coordinates (x,y) of typical daylight for various correlated color temperatures.

Correlated color temperature (°K)	Chromaticity coordinates x	y
4800	0.3518	0.3634
5500	0.3324	0.3475
6500	0.3127	0.3291
7500	0.2991	0.3150
10 000	0.2787	0.2919

[18] G. J. Chamberlin, A. Lawrence, and A. A. Belbin, Light and Lighting **56**, 73 (1963).

[19] Y. Nayatani and G. Wyszecki, J. Opt. Soc. Am. 53, 626 (1963).

[20] K. L. Kelly, J. Opt. Soc. Am. **53**, 999 (1963).

TABLE III. Scalar multiples of the first two characteristic vectors of the composite data required to reconstitute spectral distribution curves of typical daylight of five correlated color temperatures.

Correlated color temperature (°K)	Scalar multiples of first characteristic vector	Second characteristic vector
4800	−1.140	0.677
5500	−0.784	−0.195
6500	−0.293	−0.689
7500	0.145	−0.752
10 000	1.005	−0.378

X may be written:

$$X = X_0 + M_1 X_1 + M_2 X_2, \tag{3}$$

where X_0 is the X-tristimulus value of the mean distribution, and X_1 and X_2 are the X-tristimulus values of the two characteristic vectors; for example, $X_1 = \sum V_{1,\lambda}\bar{x}_\lambda\Delta\lambda$. By analogous argument for Y and Z we may write:

$$Y = Y_0 + M_1 Y_1 + M_2 Y_2, \tag{3}$$
$$Z = Z_0 + M_1 Z_1 + M_2 Z_2.$$

Note that values of X_0, Y_0, Z_0 are nowhere less than zero, but the tristimulus values of the characteristic vectors may be less than zero because the vectors themselves (see Table I and Fig. 1) are adjusted to zero at 560 nm with both positive and negative values for other parts of the spectrum.

The connection between scalar multiples M_1 and M_2 and chromaticity coordinates x, y follows directly from Eq. 3 and from definitions: $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$. A convenient form of the connection, for $X+Y+Z$ abbreviated as S, is:

$$x = \frac{X_0/S_0 + M_1 X_1/S_0 + M_2 X_2/S_0}{1 + M_1 S_1/S_0 + M_2 S_2/S_0},$$
$$y = \frac{Y_0/S_0 + M_1 Y_1/S_0 + M_2 Y_2/S_0}{1 + M_1 S_1/S_0 + M_2 S_2/S_0}. \tag{4}$$

To find the scalar multiples M_1 and M_2 required to yield a reconstituted spectral distribution of irradiance having any arbitrarily chosen values x and y of chromaticity coordinates, it is necessary to solve for M_1 and M_2 from Eq. 4. A convenient form of solution is:

$$M_1 = \frac{X_0 Y_2 - X_2 Y_0 + (Y_0 S_2 - Y_2 S_0)x + (X_2 S_0 - X_0 S_2)y}{X_2 Y_1 - X_1 Y_2 + (Y_2 S_1 - Y_1 S_2)x + (X_1 S_2 - X_2 S_1)y},$$
$$M_2 = \frac{X_1 Y_0 - X_0 Y_1 + (Y_1 S_0 - Y_0 S_1)x + (X_0 S_1 - X_1 S_0)y}{X_2 Y_1 - X_1 Y_2 + (Y_2 S_1 - Y_1 S_2)x + (X_1 S_2 - X_2 S_1)y}. \tag{5}$$

The tristimulus values of the mean of the composite data and of their first two composite vectors have been calculated and found to be as follows: $X_0 = 102434$, $Y_0 = 106769$, $Z_0 = 123630$; $X_1 = 1866$, $Y_1 = 1914$, $Z_1 = 34810$; $X_2 = 2133$, $Y_2 = 762$, $Z_2 = -2355$. Substitution of these tristimulus values into Eqs. 4 and 5 yielded both the explicit direct connection between the scalar multiples for the composite data and the chromaticity coordinates of the reconstituted spectral distributions of irradiance, and the reverse connection as follows:

$$x = \frac{0.30776 + 0.00561 M_1 + 0.00641 M_2}{1.00000 + 0.11594 M_1 + 0.00162 M_2},$$
$$y = \frac{0.32079 + 0.00575 M_1 + 0.00229 M_2}{1.00000 + 0.11594 M_1 + 0.00162 M_2}, \tag{4a}$$

$$M_1 = \frac{-1.3515 - 1.7703x + 5.9114y}{0.0241 + 0.2562x - 0.7341y},$$
$$M_2 = \frac{0.0300 - 31.4424x + 30.0717y}{0.0241 + 0.2562x - 0.7341y}. \tag{5a}$$

SPECTRAL DISTRIBUTIONS OF DAYLIGHT FOR VARIOUS CORRELATED COLOR TEMPERATURES

To derive a series of related spectral distributions of typical daylight over a considerable range of correlated color temperatures from the composite data, values of scalar multiples M_1 and M_2 were computed by inserting into Eq. (5a) the values of chromaticity coordinates x and y given in Table II for typical daylight chromaticities of correlated color temperatures 4800°, 5500°, 6500°, 7500°, and 10 000°K. Table III shows

TABLE IV. Extension of the mean and the first two characteristic vectors of the composite data given in Table I to the spectral ranges 300 to 330 nm and 700 to 830 nm from Moon's compilation of the spectral absorptance of the earth's atmosphere due to ozone and water vapor.

Wavelength (nm)	Mean	V_1	V_2
300	0.4	0.2	0.0
310	60	45	20
320	296	224	40
330	553	420	85
	(See Table I)		
700	743	−133	96
710	764	−129	85
720	633	−106	70
730	717	−116	76
740	770	−122	80
750	652	−102	67
760	477	−78	52
770	686	−112	74
780	650	−104	68
790	660	−106	70
800	610	−97	64
810	533	−83	55
820	589	−93	61
830	619	−98	65

Table V. Relative spectral irradiance of typical daylight reconstituted from mean and characteristic vectors of the composite data (Tables I and IV) by the scalar multiples of Table III.

Wavelength (nm)	Correlated color temperature (°K) 4800	5500	6500	7500	10 000
300	0.2	0.2	0.3	0.4	0.6
310	23	21	33	52	97
320	68	112	202	298	506
330	132	207	371	550	943
340	163	240	400	573	952
350	190	279	450	627	1011
360	218	307	467	630	977
370	246	344	522	703	1091
380	215	326	500	668	1010
390	267	382	547	700	1006
400	446	610	828	1019	1388
410	516	686	916	1119	1515
420	554	716	935	1128	1503
430	537	679	868	1033	1346
440	704	856	1049	1211	1518
450	827	981	1171	1330	1628
460	864	1004	1178	1323	1594
470	878	999	1149	1272	1503
480	916	1026	1159	1269	1469
490	894	980	1088	1177	1344
500	936	1007	1094	1165	1300
510	949	1008	1078	1137	1246
520	959	1000	1049	1086	1156
530	1011	1042	1077	1105	1153
540	1002	1021	1044	1063	1097
550	1020	1030	1040	1049	1064
560	1000	1000	1000	1000	1000
570	979	973	964	956	943
580	995	977	957	942	914
590	945	914	886	870	848
600	993	944	900	873	835
610	1012	951	896	862	816
620	1014	942	876	836	780
630	983	904	833	787	726
640	1020	923	837	785	716
650	990	889	800	748	683
660	1021	903	802	745	673
670	1075	940	822	755	671
680	1037	900	783	717	638
690	912	797	697	640	567
700	960	829	716	652	573
710	969	849	743	681	602
720	801	702	616	565	500
730	901	793	699	643	572
740	963	850	751	692	617
750	814	719	636	587	524
760	601	528	464	427	379
770	864	759	668	614	545
780	815	718	634	584	520
790	828	729	643	592	527
800	764	674	594	548	488
810	665	587	519	480	429
820	736	650	574	530	472
830	775	683	603	556	496

the values of the scalar multiples so obtained. Reconstituted spectral distributions were then obtained for the wavelength range 330 to 700 nm by applying these scalar multiples to the mean and first two characteristic vectors of the composite data given in Table I in accord with Eq. (2a). To extend these spectral distribution curves to cover the wavelength ranges 300 to 330, and 700 to 830 nm, resort was had to Moon's[12] compilation of data on the spectral absorptance of the earth's atmosphere due to ozone and water vapor. Extensions of the mean and first two characteristic vectors of the composite data in this way are shown in Table IV. Table V shows the reconstituted spectral distributions of typical daylight over the extended range 300 to 830 nm, and Fig. 3 shows a plot of these distributions.

The spectral distributions of typical daylight were also evaluated in an analogous way from the means and first two characteristic vectors derived from the four subsets of data (Condit 249, Henderson 274, from 330 to 700 nm; Henderson 274, from 330 to 780 nm; and Budde 99) as a check on the distributions derived from the composite data comprising all 622 of the measured distributions. The agreement within the spectral range 400 to 700 nm was found to be very satisfactory, but the amount of the ultraviolet component (330 to 390 nm) indicated by the four subsets of data showed significantly different dependences on correlated color temperature. One might be tempted to ascribe these indicated different dependences of ultraviolet content on correlated color temperature to real differences between the atmospheric conditions at Rochester, Ottawa, and Enfield, were it not for the fact that the two Henderson subsets of data (330 to 700 nm and 330 to 780 nm, identical from 330 to 700 nm) also showed this different dependence. It is our view that ultraviolet content is poorly correlated with correlated color temperature, and that the amount of ultraviolet indicated by the composite data should be taken as typical.

In the opinion of the authors, the spectral distributions of irradiance produced by daylight at the earth's surface, shown in Fig. 3, are the most typical that can

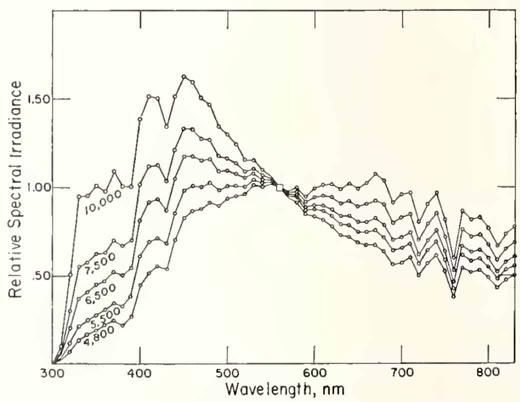

Fig. 3. Spectral distribution of typical daylight for correlated color temperatures: 4800°, 5500°, 6500°, 7500°, and 10 000°K reconstituted from the mean and the first two characteristic vectors of the composite data (622 measured distributions).

TABLE VI. Chromaticity coordinates (x,y) corresponding to the measured spectral distributions shown in Figs. 4–6.

Correlated color temperature (°K) (and Fig. number)	Spectral distribution		Chromaticity coordinates corresponding to the distribution	
			x	y
5500	From Table V		0.3324	0.3475
(Fig. 4)	Condit	51	0.3318	0.3465
	Henderson	107	0.3324	0.3473
	Budde	84	0.3256	0.3334
6500	From Table V		0.3127	0.3291
(Fig. 5)	Condit	120	0.3124	0.3285
	Henderson	67	0.3116	0.3280
	Budde	12	0.3134	0.3295
7500	From Table V		0.2991	0.3150
(Fig. 6)	Condit	241	0.2982	0.3157
	Henderson	247	0.2990	0.3146
	Budde	44	0.2968	0.3100

be derived from the experimental data gathered to date. Although the initial experimental data employed in this analysis refer to different spectral bandwidths of 10 nm and less, the spectral distributions proposed here should be taken as the average of the true values of spectral irradiance over wavelength intervals of 10 nm throughout the spectrum relative to that for the interval 555 to 565 nm with its central wavelength at 560 nm. They are recommended as guides in the development of sources of artificial daylight that might be proposed as standard sources for colorimetry; in particular, the spectral distributions found for correlated color temperatures 5500°, 6500°, and 7500°K are proposed for the role of target curves to CIE Committee E–1.3.1 which has already indicated[6] its intention to develop standard sources of these correlated color temperatures. Should phases of typical daylight of other correlated color temperatures between 4000° and 40 000°K be desired, derivation of other curves of this family can be accomplished by means of Eq. 5a by substitution of other chromaticity coordinates satisfying the relation: $y=2.870x-3.000x^2-0.275$. Reliable curves of relative spectral irradiance of the pinker or greener phases of daylight may also be derived by means of Eq. 5a provided that, for $0.25\leq x\leq 0.38$, the value of the y-coordinate departs from the typical value by not more than 0.008. This range in y-coordinates includes about 90% of the 622 computed chromaticities.

COMPARISONS OF A SAMPLING OF THE MEASURED SPECTRAL DISTRIBUTIONS WITH RECONSTITUTIONS FROM THE MEAN AND FIRST TWO CHARACTERISTIC VECTORS OF THE COMPOSITE DATA

Measured curves were selected from each subset of data by taking those whose corresponding chromaticity points on the 1960 CIE–UCS diagram are closest to the chromaticity points given in Table II for correlated color temperature 5500°, 6500°, and 7500°K. Table VI compares these chromaticity coordinates with those

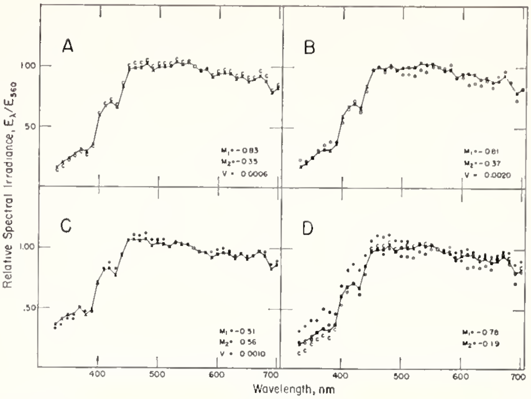

FIG. 4. Comparison of measured spectral distributions of daylight, whose chromaticities are near to that of typical daylight of correlated color temperature 5500°K, with distributions reconstituted from the mean and the first two characteristic vectors derived from the composite data (622 measured distributions). The measured distribution chosen from each subset of data is that whose chromaticity point is nearest on the 1960 CIE–UCS diagram to that for typical daylight of correlated color temperature 5500°K; see Table VI. Quadrants A, B, and C compare measured distributions (A—Condit 51; B—Henderson 107; C—Budde 84) with the reconstitution giving the best least-squares fit. The values of the scalar multiples (M_1 and M_2) so found are indicated together with the variance of the measured distribution from the reconstitution. Quadrant D compares all three of these measured distributions with the reconstituted distribution shown in Fig. 3 for 5500°K.

corresponding to the nine measured curves selected. Figures 4–6 compare the measured relative spectral irradiances with the corresponding curves reconstituted from the mean and first two characteristic vectors by applying the scalar multiples. These scalar multiples give the best least-squares fit and are shown on the plots. Figures 4–6 also compare (lower right quadrant) these measured distributions of spectral irradiance with the reconstituted curves of Fig. 3 and Table V for correlated color temperatures 5500°, 6500°, and 7500°K. The agreement is seen to be satisfactory. There is no indication that spectral distributions reconstituted from the mean and the first two characteristic vectors of the

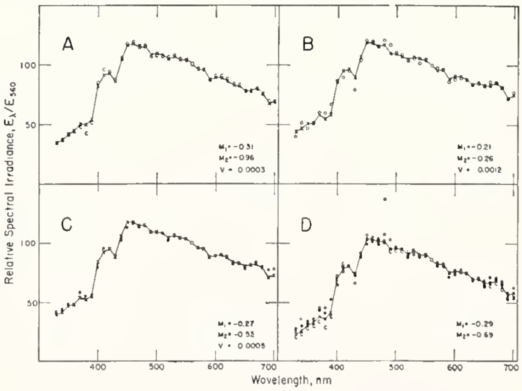

FIG. 5. Same as Fig. 4 except that the correlated color temperature is 6500°K. A—Condit 120; B—Henderson 67; C—Budde 12.

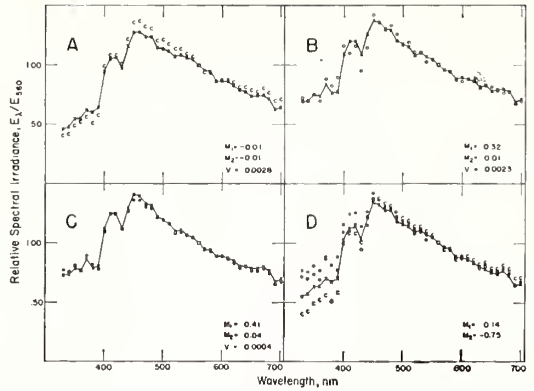

FIG. 6. Same as Fig. 4 except that the correlated color temperature is 7500°K. A—Condit 241; B—Henderson 247; C—Budde 44.

composite data fail importantly to agree with measured spectral distributions corresponding to the same chromaticity. The variance $V(E_\lambda/E_{560})$ for each curve is shown on the plot. The average variance for all nine curves is only 0.0011 which corresponds to a standard deviation of about 3 to 4 on the scale of 100 at the normalizing wavelength 560 nm. These curves show somewhat better than average agreement. The average variance of the 99 Budde curves is 0.0018; for the 249 Condit curves, 0.0020; and for the 274 Henderson curves, 0.0053. Perhaps a more striking indication of the degree to which the individual measured curves agree with the curves reconstituted from the mean and the first two characteristic vectors is to count the number of curves having variances no greater than that ($V=0.0028$) for Condit curve 241 shown on Fig. 6. Of the 99 Budde curves, 89 have variances no greater than 0.0028; of the 249 Condit curves, 222; and of the 274 Henderson curves, 113; so of the 622 curves comprising the composite data, 424 agree with the reconstituted curves more closely than the agreement shown on Fig. 6 for Condit curve 241. The finding that the Henderson measured curves agree less well with their reconstitutions than do the curves measured by Budde and Condit is to be expected from the longer time taken by Henderson to scan the spectrum; but this finding should not be construed to mean that averages of many Henderson curves corresponding to nearly the same chromaticity are in error. The assumption by Henderson and Hodgkiss[14] "that a programme involving many measurements over an extended period would compensate for the random variations during any one run" seems to have been amply justified by the good agreement already noted between curves reconstituted from the Henderson subsets of data and the Condit and Budde subsets.

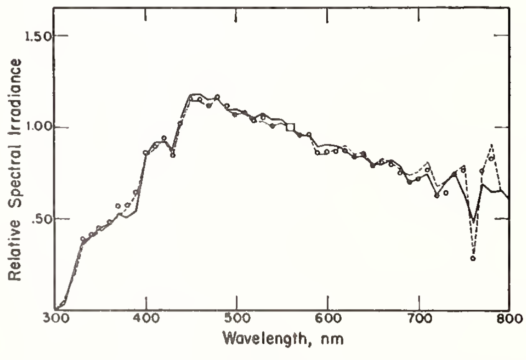

FIG. 7. Comparison of three spectral distributions intended to represent typical daylight at a correlated color temperature of 6500°K. The distribution reconstituted from the mean and first two characteristic vectors of the composite data is shown as a solid line; this distribution is also shown in Fig. 3. The distribution reconstituted from the Henderson data for 330 to 780 nm is shown by open circles, and the distribution derived for the British Standards Institution by taking an average of a selection of measured spectral distributions from the same data is shown by a dotted line.

COMPARISON WITH SPECTRAL DISTRIBUTION CURVE DERIVED FOR THE BRITISH STANDARDS INSTITUTION FOR COLOR MATCHING AND APPRAISAL

A Draft British Standard Specification circulated 13 June 1963 for consideration as a revision of British Standard 950:1941 for artificial daylight for color matching and color appraisal includes a definition of the "spectral power distribution of artificial daylight." This distribution was derived by taking an average of a group of spectral distributions measured by Henderson and Hodgkiss.[14] These measured spectral distributions all had chromaticities in the neighborhood of CIE source C but on the green side of the Planckian locus. The average curve corresponds to the chromaticity coordinates: $x=0.314$, $y=0.329$. Note that these chromaticity coordinates agree well with those derived by us ($x=0.313$, $y=329$; see Table VI) to characterize typical daylight of correlated color temperature 6500°K.

Figure 7 compares our recommendation (solid curve) for this correlated color temperature with the spectral distribution (dotted curve) derived for the British Standards Institution and with the spectral distribution (shown by open circles) derived from the mean and first two characteristic vectors of the Henderson 330–780 nm data. The agreement shown is very encouraging. It demonstrates that, for the spectral range 330 to 700 nm, average daylight agrees well with British daylight of the same correlated color temperature. The discrepancies between 700 and 780 nm may be at least partially ascribed to the smaller bandwidth (1.5 compared to 10 nm) used by Henderson and Hodgkiss, and to the presence of more water vapor in the Enfield atmosphere than in Rochester and Ottawa atmospheres. The good agreement affords a basis for the hope that a single set of related spectral distributions, such as those presented in this paper, may be found suitable for international use.

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(Musterschmidt, Göttingen, Germany, 1966)

This is Judd's last major summary of his views concerning the specification of the appearance of object colors under various conditions of illumination and observation. Although he explained major advances and accomplishments in the field, he regarded the problem as essentially unsolved. His "glimpse of the future" (Sec. 6) indicates that he would have been dissatisfied with the definitions of metric chroma recommended by the CIE in 1976. It is interesting to note that, in this 1965 summary, Judd used his 1935 UCS diagram - not the diagram recommended by the CIE in 1960. The 1976 CIE revision departs still further from his 1935 diagram.

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Color Appearance

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Es wird eine zusammenfassende Übersicht über die Grundlagen gegeben, auf denen eine Voraussage über die Farbwahrnehmungen erstrebt wird. Im einzelnen werden dazu betrachtet: die Spektralwertfunktionen für normales und abweichendes Farbensehen, Simultan- und Sukzessiv-Kontrast, Koeffizientensatz der farbigen Umstimmung (v. Kries), sowie die Trennung in der Wahrnehmung von Körperfarben und Umgebungslicht. Außerdem wird die Voraussage der Farbwahrnehmung bei nicht-ebenen Gegenständen behandelt; ferner werden die Bemühungen um empirische Formeln für Farbton, Helligkeit und Sättigung bei der Körperfarben-Wahrnehmung besprochen, die schon 1940 beschrieben worden sind; die Unzulänglichkeit dieser Formeln wird diskutiert. Zum Schluß wird der sog. „Kräuselungs-Effekt" bei der Helligkeitswahrnehmung vorgestellt, der von Takasaki entdeckt worden ist.

On donne un grand résumé sur les bases sur lesquelles on tente à prédire les perceptions chromatiques. Particulièrement on considère: les fonctions spectrales de la vision colorée normale et abnorme, les contrastes simultané et successif, la loi des coëfficients pour l'adaptation chromatique (v. Kries), aussi bien que la séparation des couleurs-objets de la lumière ambiante. En outre, on parle de la prédiction de la perception de la couleur des objets qui ne sont pas plats, et des efforts à gagner des formules empiriques pour la tonalité, la luminosité et la saturation dans la perception des couleurs-objets, formules que l'on a déjà décrites en 1940; l'insuffisance de ces formules est discutée. On termine par présenter un «effet de crêpage», qui se montre chez la perception lumineuse et qui est découvert par Takasaki.

Summaries of the researches that have formed the basis of predicting color appearance are given as follows: color-matching functions for normal and abnormal vision, simultaneous and successive contrast, v. Kries coefficient law of chromatic adaptation, and perceptual separation of object color and ambient-light color. In addition, prediction of the color appearance of non-flat objects is discussed; and the approach to empirical formulas for hue, lightness and saturation of object-color perceptions developed in 1940 is described and the shortcomings of the formulas are discussed. Finally, the evidence for the "crispening" effect in lightness perception discovered by Takasaki is presented and discussed.

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Mr. Chairman, ladies and gentlemen,

I am mindful of the honor, granted to me, of presenting the opening lecture of this *International Colour Meeting;* and I hope that you will find my remarks on color appearance worthy of your attention.

We possess already some ability to predict color appearance. This ability is based on hundreds of researches in color vision. I expect to review some of these researches, to give the present status of color-appearance prediction as I see it, and to try to look a little into the future. A large fraction of all work on color vision contributes to color-appearance prediction; and I realize that I have not even read many of the European contributions, nor thoroughly assessed those that I have read. So I apologize in advance for any failure on my part to give due credit to the authors of any of these contributions. I will take it as a favor if my attention is called to important work that I have ignored. The opportunity of becoming acquainted with color work being carried on outside of one's own country is one of the most important advantages of an *International Colour Meeting.*

By color appearance I mean the color perceived to belong to the visual object to which attention is directed. The terms used to specify the appearance depend on the mode of appearance, as was first drawn to my attention by the work of David Katz [28]. If the visual object is perceived as not itself emitting light, we may speak of the color as in the object mode. Ostwald called these "bezogene Farben". The terms for this mode are *hue, lightness,* and *chroma* (American term: *saturation).* But if the object is perceived as self-luminous, this may be called the light-source mode. Ostwald's "unbezogene Farben" are colors of this sort and for these colors the terms are *hue, luminosity* (American term: *brightness*), and *saturation.* Lightness refers to the object mode, and varies from a minimum for black to a maximum for white. *Luminosity* refers to the light-source mode, and varies from very dim to dazzling. For example, the color perception of the sun may involve maximum *luminosity* (dazzling); there is no aspect of this color perception that may be correlated with black, white, or any of the intermediate grays.

The color appearance of any object that is perceived as not itself emitting light, is known to depend upon four factors: first, the spectral distribution of the light source; second, the spectral transmittance or reflectance of the object itself; third, the visual response functions (color-matching and other) characterizing the observer; and fourth, upon some as yet incompletely formulated aspect of the scene within which the object is presented.

1. Object in daylight with gray surround

First, let us deal with a particularly simple scene. Let us assume that a flat, opaque object is viewed in daylight against a middle gray surround (say Munsell N 5/) by an observer of normal color vision.

I make this choice because I regard this problem as having been solved to a satisfactory approximation. The *spectral distribution of the source* (daylight) may be taken as that of CIE illuminant C. The *spectral reflectance* (or more precisely the spectral radiance factor for the actual direction of view) can be determined by automatic spectrophotometry. For this simple problem the only aspect of the observer that has to be known is that embodied in his *color-matching functions*, and I could pass over this aspect by saying that by using the color-matching functions of the 1931 CIE standard observer a satisfactory prediction may be obtained of the hue, lightness, and chroma of the color perceived to belong to this flat, opaque object.

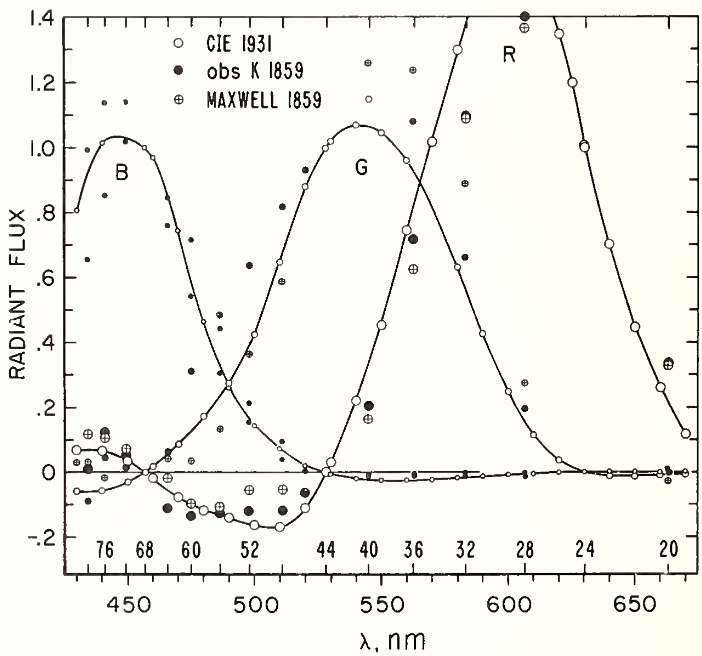

Fig. 1: Radiant flux of the spectrum primaries, 630.2, 528.1, 456.9 nm, required in an additive mixture to match unit radiant flux of the spectrum.
Maxwell's observer K — solid circles
Maxwell (observer J) — crossed circles
1931 CIE standard observer — open circles connected by curves

I think it worthwhile, however, to review briefly the researches that led up to the 1931 CIE standard observer and to indicate what aspect of the visual process is specified by *color-matching functions.* Maxwell [37] made the first determination. Fig. 1 compares Maxwell's determinations for himself and one other observer (K) with the 1931 CIE color-matching functions expressed in terms of Maxwell's choice of primaries. The curves come from the standard observer and the points off the curves are from Maxwell. These curves, R, G, B, indicate the radiant flux of spectrum primaries (630, 528, 457 nm) required in an additive mixture to match unit radiant flux for each part of the spectrum in turn. Note that at the red primary the R-curve is unity; the other two zero. This means simply that one unit of radiant flux at this wavelength color-matches itself, and the same statement holds for the other two primaries, of course, Note also that some of the radiant fluxes indicated by the curves are less than zero. This means simply that the negative amount of the primary, instead of being added to the other two primaries, is added to the part of the spectrum whose color is being determined. For example, at 500 nm, fig. 1 shows that by adding about 0.2 unit of radiant flux of the red primary (630 nm) to one flux unit at 500 nm, there is produced a somewhat desaturated blue-green color that can be matched by about 0.4 unit of the green primary (528 nm) added to about 0.2 unit of the blue primary (457 nm).

The functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ indicated in fig. 1 as R, G, B, respectively, not only show how much of the primaries are required to match each part of the spectrum in turn, they also indicate how much of each primary is required to match any known combination of the various parts of the spectrum. It is on this account that these functions are known as color-matching functions. By using them successful predictions may be made of whether any two lights of known spectral distributions $P_{\lambda,1}$ and $P_{\lambda,2}$, will, or will not, look alike. The *condition for color match* is:

$$\int_0^\infty P_{\lambda,1}\bar{g}(\lambda)\,d\lambda = \int_0^\infty P_{\lambda,2}\bar{g}(\lambda)\,d\lambda$$

$$\int_0^\infty P_{\lambda,1}\bar{r}(\lambda)\,d\lambda = \int_0^\infty P_{\lambda,2}\bar{r}(\lambda)\,d\lambda$$

$$\int_0^\infty P_{\lambda,1}\bar{b}(\lambda)\,d\lambda = \int_0^\infty P_{\lambda,2}\bar{b}(\lambda)\,d\lambda$$

These conditions for color match follow directly from Grassmann's law of additivity well verified for the macular region of the retina. They state merely that if for each spectral component of a compound light the equivalent is given in amounts of the three primaries, the sum of these equivalents will color match the sum of the spectral components making up the compound light. If the conditions for a color match are satisfied because the two lights are physically identical ($P_{\lambda,1} = P_{\lambda,2}$) throughout the visible spectrum, this *spectral* match (as we would say in America) by

OSTWALD's terminology is said to be *isomeric*. In this trivial case, the color-matching functions are not needed. If the two lights have different spectral distributions ($P_{\lambda,1}$ different from $P_{\lambda,2}$ for some wavelengths), the color match is said to be *metameric*; and in this usual case the color-matching functions are crucial.

Now if we think of the visual processes by which a color perception is achieved as consisting of responses of receptors (cones) in the retina leading to signals in the form of nerve impulse being propagated and processed in the *retina*, the *optic nerve*, the *geniculate body*, and the *occipital lobe* of the cortex, we may ask what part of this chain of events is specified by the color-matching functions. If two objects are perceived to have the same colors, it must be that the processes that they cause in the occipital lobe are identical. If they are perceived to be the same when they are viewed side-by-side in the same plane, it must also be that the nerve processes in the geniculate body, and in the retina must also be identical. Furthermore, it must also be that the cone responses elicited by the two objects are identical. Finally if we hold to the currently popular view that the cone responses start with the absorption of radiant energy by photopigments in the cones, we must conclude that *the two objects color match because the light beams received from them cause equal amounts of breakdown of the three cone photopigments.* By this view it follows that *the color-matching functions give information regarding the spectral absorptances of the three photopigments usually assumed to exist in the cones.*

The key to derivation of the spectral absorptances of the three cone photopigments from the color-matching function lies in the choice of primaries. From fig. 1 we saw color-matching functions expressed relative to the spectrum primaries (630, 528, 457 nm) chosen by MAXWELL. From GRASSMANN's law of additivity it can be shown that precisely the same decisions as to color match, or lack of color match, can be obtained from functions R', G', B', as from R, G, B, provided the new functions are weighted sums of the old thus:

$$R' = k_{11}R + k_{12}G + k_{13}B$$
$$G' = k_{21}R + k_{22}G + k_{23}B$$
$$B' = k_{31}R + k_{32}G + k_{33}B$$

where k_{11}, k_{12}, ... k_{33} are constants that may be chosen arbitrarily at any values, positive, negative, or zero provided their determinant differs from zero. It is legitimate therefore to proceed by trial and error to search for constants that will yield response curves, R', G', B', that might correspond to simple spectral absorptance curves; for example, curves nowhere less than zero, and curves showing but one maximum. The choice of primaries suggested in 1807 by THOMAS YOUNG [54] for this purpose is still regarded as closely the most appropriate choice. We take the red and violet primaries close to the extremes of the spectrum, and for the green primary we take the intersection on the Maxwell triangle of the tangents to the spectrum locus at the other two primaries. The König-Dieterici determinations of the color-matching functions were presented [29] with Young's choice of primaries under the name "Elementar-Empfindungs-Curven", and Abney's functions were similarly presented [1] under the name "sensation curves". This choice of primaries was used by Weaver who averaged the König and Abney results to produce the "elementary excitation curves" recommended in 1922 by the *Committee on Colorimetry, Optical Society of America,* under Troland's chairmanship. Fig. 2 compares these color-matching functions with those based on the work of Wright and Guild [10; 52], recommended in 1931 by the CIE, and with those of Stiles and Burch [45] and of Speranskaya [43; 44] for extramacular vision (field subtending 10°) recommended in 1964 by the CIE for large-field colorimetry.

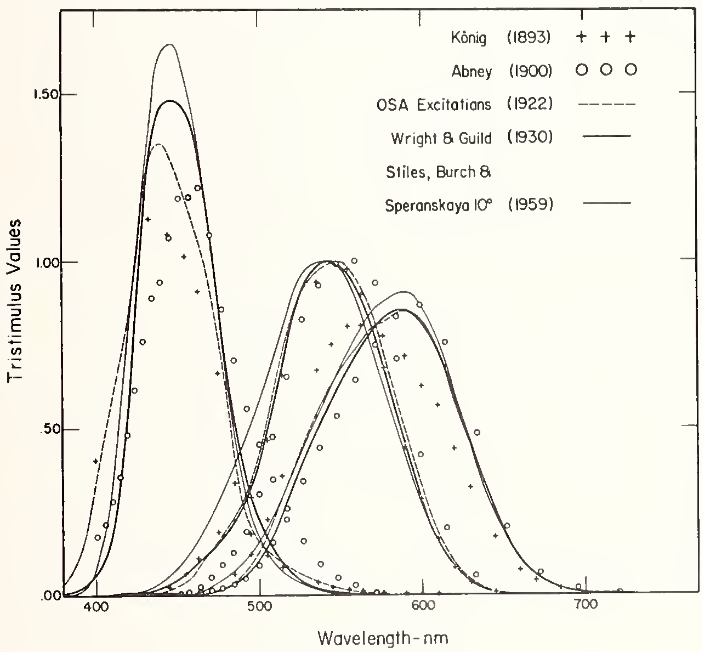

Fig. 2: Color-matching functions expressed relative to the Young primaries (spectrum red, spectrum violet, and green outside spectrum). By the Young theory these functions represent the spectral absorptances of the retinal photopigments multiplied by the spectral transmittances of the ocular media through which light passes to reach the retina.

The color-matching functions in fig. 2 all show the amounts of the Young primaries required to color match unit radiant flux of each part of the spectrum. The scale values for the G'-functions are adjusted to make the maximum be unity; those for the R'- and B'-functions are adjusted to make the areas under the curves equal to that for the G'-function; that is, the so-called equi-energy source (source whose radiant flux per unit wavelength is constant) corresponds for each set of functions to equal amounts of the three primaries. To make the König-Dieterici determinations (shown by crosses) refer to unit amount of spectral radiant flux, it was assumed that König's sunlight was equivalent to sunlight at air mass 2 from the compilation by Parry Moon [38]. Similarly the Abney data (shown by circles) were adjusted by assuming that the positive crater of the carbon arc used by him was equivalent to the blackbody at 3800° K. It will be noted that Weaver's average of these two sets of color-matching functions (shown by the dotted lines) falls fairly satisfactorily between the crosses and circles; so he must have made a fairly similar adjustment. The Guild-Wright determination (shown by heavy solid lines) corroborates the Weaver average suprisingly well considering the rather large differences between the König and Abney data. The Stiles-Burch-Speranskaya determination (shown by light solid lines) departs from these chiefly in ways that correspond to the use of extra-macular retinal regions instead of macular. These color-matching functions should correspond to the product of the spectral absorptance of the three retinal pigments by the spectral transmittance of the ocular media of which the principal absorbers are the lens and macular pigments. It is seen that *the four chief determinations of color-matching functions imply cone pigments of closely the same spectral absorptance.*

Thus ends the historical review of color-matching functions, and the justification of the current view that they serve to evaluate the spectral characteristics of cone photopigments.

To return to the problem of predicting the color appearance of a flat, opaque object in daylight viewed with a middle gray surround by an observer of normal color vision, the next step is the computation of the tristimulus values, X, Y, Z of that object for CIE illuminant C. Many laboratories have automatic digital computers programmed for this purpose.

Finally, the Munsell notation (hue value/chroma) corresponding to X, Y, Z has to be derived by three-dimensional interpolation among the graphs defining the Munsell system (Newhall, Nickerson, Judd [39]). The *National Bureau of Standards* has programmed this interpolation for automatic digital computation (Rheinboldt and Menard [41]).

For these conditions of observation, the Munsell hue correlates quite satisfactorily with the hue of the perceived color; the Munsell value correlates with its lightness; and the Munsell chroma correlates with the chroma of the perceived color.

2. Problem of abnormal vision

Table 1 shows the various types of human color vision. If the observer has one of the dichromatic reduction forms of normal color vision (protanopia, deuteranopia, tritanopia) a fairly satisfactory prediction of hue, lightness, and chroma of the color appearance of the flat object may be computed from the following transformations (Judd [23]) of the 1931 CIE color-matching functions (shown in fig. 3) taken in pairs:

$$R = Y$$
$$G = -0.46\,X + 1.36\,Y + 0.10\,Z$$
$$B = Z$$

The protanopic and deuteranopic Munsell notations of the colors in the Munsell book have already been computed in this way (Judd [24]). Only two hues are perceived, yellow (5Y) and blue (5PB).

If the observer has type 2 monochromatic vision (total color blindness with normal luminosity function), only the Y-function need be used, and the perceived color is that experienced by an observer of normal color vision looking at the corresponding Munsell neutral. The chroma will be zero, and there will be no hue.

If the observer has protanomalous or deuteranomalous vision, the prediction of hue, lightness, and chroma may be made in terms of the opponent-colors theory extended by Jameson and Hurvich [19] with the ad hoc assumption that the predictions for protanomaly are obtained from those for deuteranomly by shifting the response functions uniformly to higher frequencies by an amount corresponding to the change from 560 nm to 540 nm.

Table 1: Types of human color vision

	Responses			Maximum luminosity at	Type designation
	Light - dark	Yellow - blue	Red - green		
Trichromatism	Yes	Yes	Yes	555 nm	Normal
	Yes	Yes	Weak	540 nm	Protanomaly
	Yes	Yes	Weak	560 nm	Deuteranomaly
	Yes	Weak	Yes	560 nm	Tritanomaly
Dichromatism	Yes	Yes	—	540 nm	Protanopia
	Yes	Yes	—	560 nm	Deuteranopia
	Yes	—	Yes	560 nm	Tritanopia
Monochromatism	Yes	—	—	510 nm	Cone blindness
	Yes	—	—	560 nm	—
	Yes	—	—	540 nm	—

The problem of anomalous trichromatism arises because of the presence of at least one photopigment not present in the retina of an observer with normal color vision (De Vries [48]).

The problem of the reduced forms (protanopia, deuteranopia, tritanopia, type 2 monochromatism) arises either because of abnormal distribution of the photopigments in the individual retinal cone, or because of failure of all of the information present in the cone responses to be transmitted to the occipital cortex.

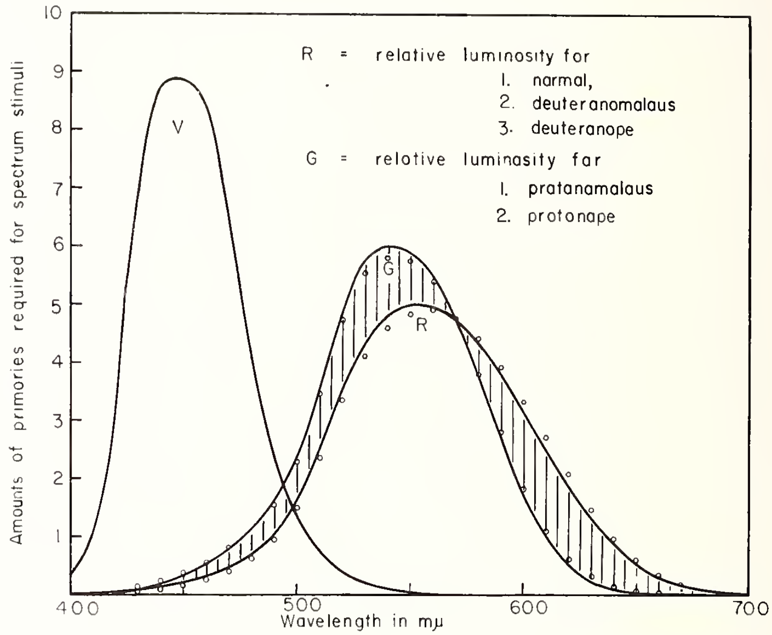

Fig. 3: Color-matching functions for 1931 CIE standard observer expressed relative to primaries at the intersection points of the chromaticity-confusion lines on the MAXWELL triangle for the three types of dichromatic vision: protanopia, deuteranopia, and tritanopia. Curves refer to KÖNIG's theory, small circles, to PITT's measurements (JUDD [23]).

3. Problem of the non-flat object

Fig. 4 shows three diamond-shaped flat objects, one white and two gray, all on a gray surround. The colors perceived to belong to these objects can all be predicted satisfactorily, by the methods already outlined, for an observer of any type of color vision. Fig. 5 shows the same diamond-shaped flat objects arranged to fit together. Some observers see the top diamond as white, and the two lower diamonds as grays of different lightnesses just as in fig. 4. Other observers see this as a picture of a white cube illuminated principally from the top and slightly from one

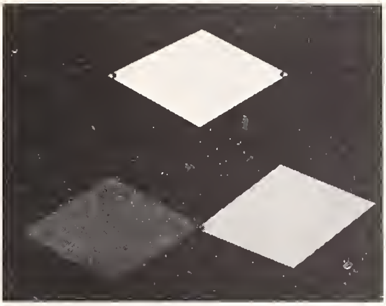

Fig. 4: Three diamond-shaped flat objects usually perceived as white, light gray and middle gray

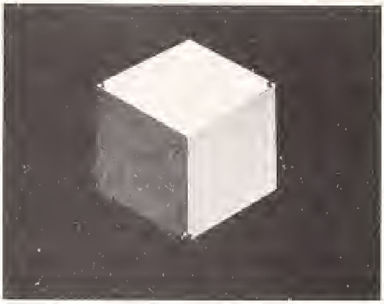

Fig. 5: Three diamond-shaped flat objects arranged so as to be perceptible as a picture of a white cube illuminated from the top and right

side. We have here an indication of the *ambivalence of the observer's response*. When we know all about color appearance we will be able to give two predictions, one successful if the object is perceived as flat; the other, if it is perceived as solid. The difference between these two perceptions must arise at the cortical level.

This is a trivial case of the problem successfully dealt with repeatedly by the artist attempting to portray three-dimensional objects by an array of colors in two dimensions, and obvious rules for the prediction can be formulated for this simple case. If a green color be substituted for the lighter gray in this picture, for example, some observers will see it as a picture of a white cube strongly illuminated by daylight from the top and weakly illuminated by green light from the side.

4. Problem of the non-gray surround

It has been known especially well since Hering's studies [16] that the use of a chromatic surround influences the hue and chroma of the perceived target color. The qualitative influences are conveniently described in terms of the Hering opponent-colors theory. A red surround induces green in the target, a yellow induces blue, and vice versa. If the fixation point is allowed to rove the scene as in the usual way we see things, the eye becomes somewhat adapted to the surround when the fixation point wanders there, and then when the fixation point falls on the target a negative after-image of the surround color is projected onto the target color. This is known as *successive color contrast*.

Even when the fixation point is controlled so that it always falls on the center of the target, there is a qualitatively similar influence of lesser extent. This is known as *simultaneous color contrast*. In ordinary vision the influence of a chromatic surround is no doubt ascribable to some combination of simultaneous and successive contrast.

Simultaneous contrast is an aspect of the processing of the signals from the cones as the signals are transmitted to the cerebral cortex. Probably the processing occurs chiefly in the retina itself. *Successive contrast* is currently supposed by many to correspond to the establishment of a chemical or electrical block in the retinal synapses. It has also been viewed as the result of desensitization of the retinal cones by depletion of the photopigments therein.

The prediction of the influence of a chromatic surround on the color perceived to belong to the target requires a quantitative evaluation of the effects of both simultaneous and successive contrast taken together. A good start in this direction has been made by JAMESON and HURVICH [20] who found the following simple connection between induced red-green, $(r—g)_i$, and yellow-blue, $(y—b)_i$ components and the corresponding components $(r—g)_s$, $(y—b)_s$ of a touching color of the same size:

$$(r—g)_i = —0.56\,(r—g)_s$$

$$(v—b)_i = —0.56\,(y—b)_s$$

For a chromatic surround, it is to be presumed that a similar relation with a constant larger than 0.56 would hold.

5. Problem of the non-daylight source

Suppose now that the flat, opaque object viewed with a middle gray surround is illuminated by a source (say by candle light) far from the chromaticity of daylight. What will the observer with normal color vision see?

If we proceed as for a daylight source, the tristimulus values of the light reflected by the object must be computed relative to the actual source (candle light). These values will differ greatly from those for the same object computed for daylight. This difference is referred to by the *CIE Committee on Color Rendering*, as the *colorimetric shift*. The observer, however, becomes adapted to the candle-lit scene in at least two ways, a rapid way, and a slow way. The rapid way permits him to see a scene much more reddish yellow than the daylight-illuminated scene, and at the same time to see a nonselective object as gray; that is, the observer sees reddish yellow light coming from the object, but by an act of perception, presumably accomplished in the visual cortex of the occipital lobe, the reddish-yellow color is ascribed to the ambient light, rather than to the object. I call this *cortical adaptation*. This phenomenon has been dealt with in the psychological literature under the term *object-color constancy* (by BOCKSCH in 1927 [3], by GELB in 1929 [9], and by JAENSCH in 1921 [17], to mention a few) who differentiated it carefully from simultaneous contrast. *The observer perceives the color of the ambient light as separate from that of the object*, and so discounts the nondaylight color of the ambient light. HELMHOLTZ spoke of discounting the illuminant color *as a judgement*, but extensive later work (KROH in 1921 [32], MARZYNSKI in 1921 [36], HAACK in 1929 [11], JAENSCH and WIEGAND in 1932 [18], KRAVKOV in 1927 [30]) took the view that differentiation of the object color from the color of the ambient light is the *automatic essential characteristics of object-color perception* and is not an intellectual act of judgment. They spoke of any difference between the colors perceived to belong to the patches of light making up a scene and those perceived to belong to the objects depicted by those patches as color transformation. Color transformation acts so as to preserve nearly constant the object-color perception over rather extensive changes in the color of the ambient light.

Any attempt to predict the object-color perception must deal with the question of ambient-light color. On what clues does the observer base his perception of ambient-light color? PIKLER in 1932 [40] made a plausible case for the view that the color of the light scattered within the eyeball is directly sensed and is the color discounted in the formation of object-color perceptions. Others argued for the view that the separation of ambient-light color from object color arises from perception of light scattered by dust particles in the air. A more usual view is that the perception of the ambient-light color arises from the scene, itself, and should be estimated from the colors of any high-lights from glossy objects depicted in the scene, or in the absence of high-lights, as some kind of average of the colors of the light received by the observer's eye from the whole scene. In the absence of definite clues to ambient-light color, the object-color perception may fluctuate widely. So much for assessment of the rapid sort of chromatic adaptation arising from the observer's ability to separate object-color perception from ambient-light perception.

The slow way by which an observer becomes adapted to a candle-lit scene requires about five minutes to reach equilibrium. It is the process responsible for successive contrast, and I call it *retinal chromatic adaptation*. HELMHOLTZ [12] early indicated the connection conceived by him between retinal chromatic adaptation and the three independent mechanisms of YOUNG's theory by saying: "... *the nervous substance in question is less sensitive to new reacting light falling on it than the rest of the retina that was not previously stimulated* ..." Thus an eye which has been acted on by the reddish-yellow light, say, of a candle, is thereafter in a condition in which the blue components of daylight affect it more than the green and red components. HELMHOLTZ continues: "*Accordingly, the effect of fatiguing the retina is not uniformly extended to every kind of stimulation, but chiefly to stimulation similar to the primary stimulation. This fact is explained very simply by* YOUNG's *assumption of three different kinds of sensory nerves for the different colors. For since colored light does not stimulate these three kinds of nerves all to the same extent, different degrees of fatigue must also be the result of different* *degrees of stimulation.*" When the eye has been exposed to reddish-yellow light, then the red-sensitive nerves are strongly stimulated and much fatigued; the green-sensitive nerves moderately stimulated and moderately fatigued; whereas the violet-sensitive nerves will scarcely be fatigued at all. Presentation of daylight to such an eye should thus result in the perception of greenish blue, which agrees with the qualitative facts of successive contrast.

This prediction can in fact be subjected to quantitative test. Suppose one retinal region (say one eye) be maintained in a state of adaptation to daylight, and another retinal region (say the other eye) be pre-exposed to candle light. All stimuli presented to the portion of the retina adapted to candle light will appear more greenish blue than when presented to the daylight-adapted eye, and a measure of this adaptive color shift may be obtained by allowing the observer to alter the light stimulating the daylight-adapted region until he finds a stimulus yielding a color perception equal to that from the reddish-yellow adapted region. This method was used with considerable success by WIRTH [51], by WRIGHT [53], by WALTERS [49], by BREWER [4], by BURNHAM, EVANS, and NEWHALL [5], and others.

HELMHOLTZ' ideas on chromatic adaptation were quantitatively formulated by v. KRIES before 1905 [31]: If R, G, V specify the color of any stimulus for neutral adaptation, then for any chromatic adaptation, the resulting color perception of this stimulus will be similarly specified by R', G', V', where:

$$R' = S_r R, \qquad G' = S_g G, \qquad V' = S_v V,$$

where S_r, S_g, S_v are coefficients specifying the state of chromatic adaptation of the eye following the pre-exposure. This formulation is kown as the v. KRIES *coefficient law*. The studies cited above, and also those by MACADAM [35] and WASSEF [50], indicate that the v. KRIES *coefficient law is a fair first approximation to the experimental facts provided the change in chromatic adaptation is not extreme and provided the luminances of the test stimulus and surround are kept nearly constant.* For such experimental conditions, the primaries used to explain partial color-blindness also yield approximate predictions of hue and chroma of objects viewed under nondaylight sources (HELSON, JUDD and WARREN [14]; HELSON, JUDD and WILSON [15]). A question unresolved by such studies arises from the fact that the optimum choice of primaries (those giving the best fit to the experimental data) varies considerably from one study to another for reasons as yet undetermined.

To apply the v. KRIES coefficient law to prediction of color appearance for nondaylight sources we have only to note that the indicated transformation leaves the primaries unchanged, but only adjusts their relative scale values. If equal amounts of R, G, B indicate an object perceived as white or gray in daylight, then by the v. KRIES coefficient law equal amounts of R', G', B' can be made to indicate white or gray for the perception of the same object by candle light. We have only to adjust the coefficients S_r, S_g, S_b to make the nondaylight source correspond to equal values of R', G', B'. The prediction of hue, lightness, and chroma would proceed as follows:

a) Compute the tristimulus values X, Y, Z of the light coming from the object.

b) Transform these to R, G, B, relative to the primaries used for partial colorblindness.

c) Transform by the v. KRIES coefficient law to R', G', B' to take account of the slow adaptation, which I think is retinal, of the observer to the nondaylight source. The *CIE Committee on Color Rendering* calls the result of this transformation the *adaptive color shift*.

d) By the reverse transformation from R', G', B', obtain X', Y', Z'.

e) Determine the MUNSELL hue, value, and chroma corresponding to X', Y', Z'. The *CIE Committee on Color Rendering* has a name for the difference between the daylight color perception and the nondaylight color perception. They call it the *resultant color shift;* that is, it is the resultant of the colorimetric shift and the adaptive shift.

Fig. 6 shows the resultant color shifts calculated by the v. KRIES coefficient law for eleven objects identified by their MUNSELL notations (HELSON, JUDD, WILSON [15]. The plot is on a portion of the 1931 CIE (x, y)-chromaticity diagram. The circles show the chromaticity points of the 11 objects for CIE illuminant C; the solid dots show the corresponding chromaticity points for illumination by a cool-white fluorescent lamp (4500° K). The difference between circle and dot is the colorimetric shift. The lines from the dots to the arrow heads indicate the adaptive color shift computed by the v. KRIES coefficient law. The vector from circle to arrow head is the resultant of the colorimetric-shift vector and the adaptive-shift vector and is called the resultant color shift. Note that generally the resultant color shift is small compared to the two shifts of which it is the resultant. That is, chromatic adaptation acts largely to counteract the colorimetric shift, and leads to approximate constancy of object-color perception. Note that color constancy for 5P 4/12 is almost perfect; but for others there are deviations from constancy large enough to be of commercial concern. For example, 5Y 8/12 shows a colorimetric shift toward orange, but a larger adaptive shift toward green, so that the resultant is toward green. This accords with the undesirable greenish appearance given to butter by the standard cool-white fluorescent lamp, and indicates why a De Luxe type of lamp may be worth the extra cost.

It is interesting to note that the effects of retinal and cortical adaptation are qualitatively the same; that is, by cortical adaptation we can see nonselectively reflecting objects as gray, by separating the ambient-light color perception from the object-color perception, long before the ambient-

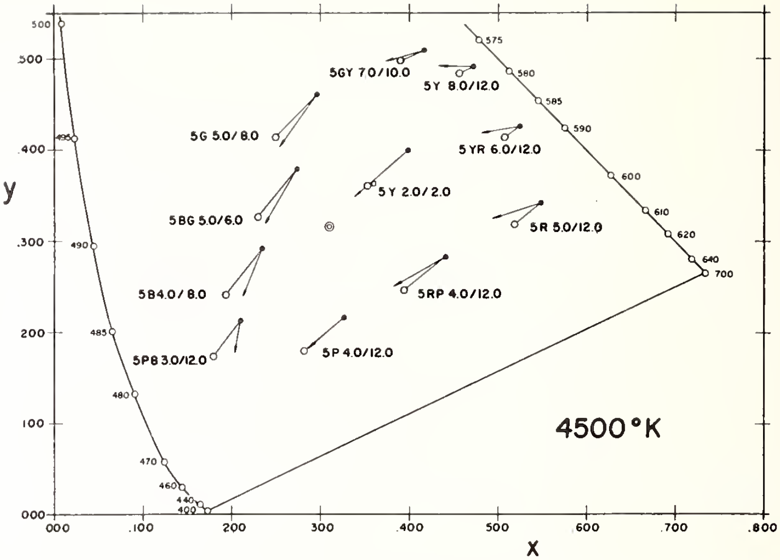

Fig. 6: Chromaticity points for 11 Munsell standards for CIE illuminant C (circles) and for a standard cool-white fluorescent lamp of nominal color temperature 4500°K (dots).
Circles to dots: colorimetric shift
Dots to arrow-heads: adaptive color shift computed from v. Kries coefficient law
Circles to arrow-heads: resultant color shift

light color itself comes by retinal adaptation to appear nearly achromatic. It is too much to hope for that the rapid cortical adaptation would have in detail precisely the same influence as the slower retinal adaptation, and, indeed, careful attention to the appearance of objects in a given scene during retinal adaptation does show significant changes.

One of these changes was noted in work carried out by Helson [13] and Judd [22] and relates particularly to the colors perceived to belong to the darkest parts of the visual field. My American colleagues have been kind enough to refer to this as the Helson-Judd effect. At first these dark parts are seen to correspond to dark gray or black objects, but as retinal adaptation approaches completion these objects are perceived as having hues corresponding to the after-image complementary of the bright parts of the visual field. At the same time the bright parts of the visual field continue to be perceived as of the hue of the ambient light.

In 1938, I made an attempt to write theoretical formulas for the hue, lightness, and chroma of the color perceived to belong to an object in any scene illuminated by any source. The source was to be specified by its distribution; the object, by its tristimulus values for that source; and the surround by some kind of an average of the tristimulus values of the light patches of which it was composed. There are considerable individual variations in the hue, lightness, and chroma of the perceived object color; and I thought this made the theoretical problem simpler because I intended to accept as successful any solution that would give a prediction within this large individual variation. I tried three-components formulations and opponent-colors formulations; but there were always objects for which prediction came nowhere near the observed hues and saturations. This does not mean that such theoretical formulas are not possible. It probably means that I did not have sufficient insight in my selection of variables, nor sufficient skill in applying these theories to succeed. To achieve a theoretical formulation requires that terms for simultaneous contrast, retinal adaptation and cortical adaptation be formulated and combined so as to give a whole series of predictions for the first five minutes that the observer looks at the scene containing the object.

In 1940 I published an empirical formula for hue, lightness, and chroma [22], which fell short of the desired theoretical formula in the following respects:

a) It was intended to apply only to the stable perceptions obtained after five minutes of viewing the scene.

b) It did not evaluate simultaneous contrast, retinal adaptation, and cortical adaptation separately, but lumped them together.

c) It did not in every case yield a prediction within individual-observer variations, though it did not fail spectacularly for any object as my tentative theoretical formulations did.

The basis of this empirical formula is as follows:

a) Lightness L was estimated from the luminous directional reflectance R of the object, and that, R_f, of the surround, by a formula derived from the hyperbolic formula of Adams and Cobb [2].

$$L = \frac{10(R - 0.03)(R_f + 1)}{0.97(R_f + R)}$$

b) Hue was estimated according to direction of the vector on a chromaticity diagram selected for perceptual uniformity of spacing. The vector extends from the achromatic point to the object point.

c) Chroma was estimated by the length of this vector on the same UCS diagram [21].

d) The chromaticity coordinates of the achromatic point depended primarily on those for the surround but were also dependent on the predicted lightness of the color perceived to belong to the object in such a way as to take account of the Helson-Judd effect.

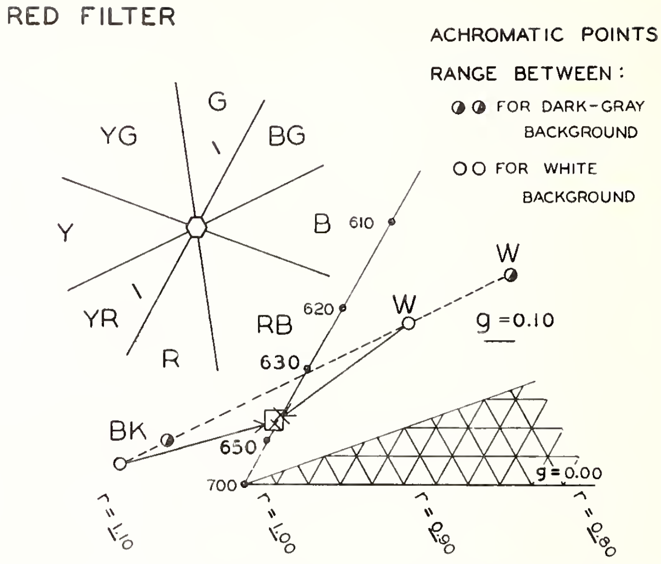

Fig. 7: Achromatic points for object-color perception under red light shown on the Judd 1935 UCS triangle [21]. Chromaticity points for all objects studied fall within square at 645 nm. The hue of the object-color perception is correlated as shown with the direction of the vector from the applicable achromatic point to the object point. Note that for dark objects a blue hue is predicted, but for light objects the predicted hue is yellow-red.

Fig. 7 shows the assignment of hues according to direction on the Judd-UCS triangle [21]. It also shows for red light the locus (dotted line) of achromatic points depending on the predicted lightness. By this red light all objects studied had chromaticity points within the square at wavelength 645 nm. Note that for dark objects the achromatic points fall well outside the gamut of real colors, and permit the prediction of blue-green or blue hues; but for light objects the predicted hue is yellow red. Both of these hue predictions are correct.

Fig. 8 shows by dotted lines the locus of achromatic points for yellow, green, and blue light. All of these loci were determined empirically to achieve the best fit of the observed data.

The outstanding failures of these empirical formulas for hue, lightness, and saturation are as follows:

a) The Bezold-Brücke phenomenon is incorrectly predicted; that is the change of hue caused by a change in luminance is actually zero for unitary yellow, green, and blue; but the indicated change deviates from this pattern.

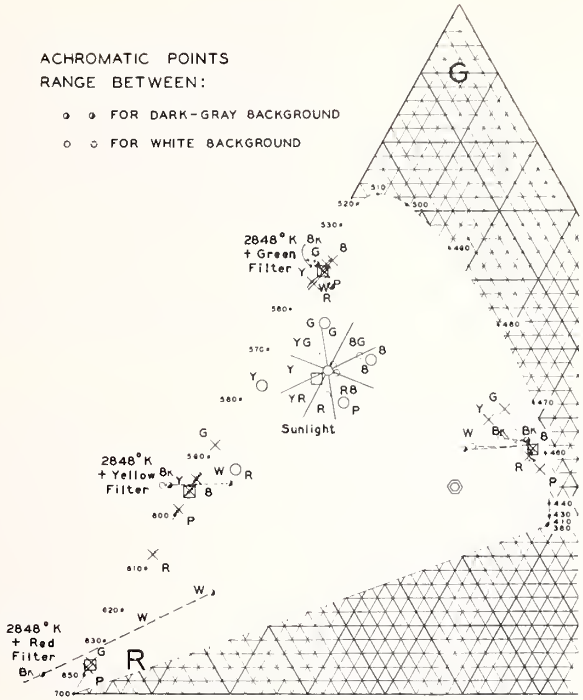

Fig. 8: Chromaticity points on the JUDD 1935 UCS triangle for the 15 objects studied for each of the five light sources used: sunlight, red, yellow, green, and blue. The loci of achromatic points required to take account of the HELSON-JUDD effect are shown as dotted lines.

b) The predicted change in hue of the object caused by changing the light source form daylight to incandescent-lamp light is significantly different from observations by KRUITHOF and BOUMA [33] on the 100 OSTWALD hue-circuit chips.

My present view of these empirical formulas is that *they give fairly good qualitative approximations to the observed facts, but are significantly in error in the quantitative details.* They succeed in predicting correctly the hues observed by LAND [34] from projections of scenes by two primaries (say red light and incandescent-lamp light) [25], even though these hues were obtained by rapid cortical adaptation immediately. LAND showed that chromatic perceptions of object colors could be obtained from two-primary color projections even though the wavelength difference between the primaries amounted to only 20 nm (say by 570 nm for one and 590 nm for the other).

6. A glimpse of the future

One reason why the empirical formulas fall short of the goal may be that *they fail to take account of the extraordinary ability of the human eye to detect small deviations from the surround color.* For example, the formula for chroma is simply the length of the vector from achromatic point to object point. Actually the first small chromaticity difference on a UCS diagram from the achromatic point counts for much more in perception than the last increment to the object point corresponding to a high chroma.

We have had a hint of this ability from SCHÖNFELDER's work [42] on the surround for optimum discrimination. He concluded that the best surround for discriminating two colors is the average of those two colors.

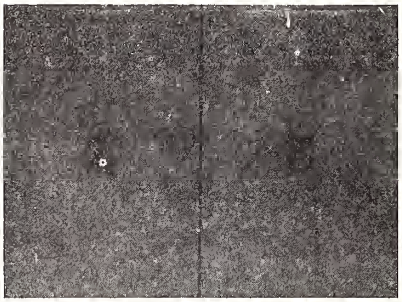

Fig. 9: Identical dark-gray flat objects photographed with identical light-gray surrounds. The perceived lightnesses are likewise identical

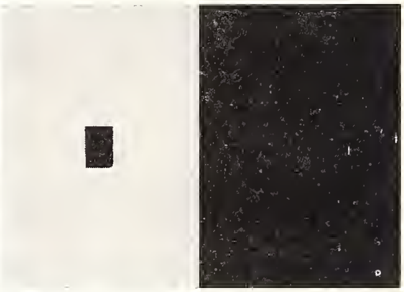

Fig. 10: Identical dark-gray flat objects photographed one with a dark-gray surround, the other with a light-gray surround. The lightnesses of the object-color perceptions are no longer identical

I will conclude this presentation with a brief summary of recent work by TAKASAKI [46] in our laboratory on *the influence of the surround on lightness discrimination.* Fig. 9 shows two identical grays with identical surrounds. The lightness of the grays is observed to be equal. Fig. 10 shows the same grays, one with dark gray surround; the other, light gray. The gray with the dark surround appears lighter. Demonstrations of lightness induction, such as this, have often been presented before; indeed lightness induction was well-known in HERING's time (1878). It was TAKASAKI's purpose, however, to make a thorough *quantitative* study. His method was to choose a dark gray surround and a gray target. Then he asked the observer to choose from a 60-step gray scale mounted on a disk the particular gray to be placed on the lighter gray background to produce

Fig. 11: Observer adjusting position of disk to bring into view a gray object with a light surround such as to appear as light as the fixed gray object on the left with a dark surround

a perception of the same lightness. Fig. 11 shows one of his observers adjusting the position of the disk with her right hand so as to bring this particular gray target into view. Fig. 12 shows the experimental results obtained by TAKASAKI for his own eyes. The abscissa shows the MUNSELL value of the gray target presented with the darker surround. The ordinate shows the MUNSELL value of the gray found to have the same lightness when viewed with the lighter surround. With trivial exceptions *this requires a higher* MUNSELL *value;* so in general, the plotted points representing experimental observations fall above the 45-degree line corresponding to $V_1 = V_2$. In the left half of the figure are presented the experimental points and theoretical curves for background pairs whose value averages to 5/. These are 4/ with 6/, 3/ with 7/, and 1/ with 9/. Note that the lightness induction increases with increase in value difference between the two surrounds.

The right half of the figure corresponds to two background pairs, 1/ and 5/, and 5/ and 9/. Similar results are shown on Fig. 13 for another of the five observers. This observer was chosen because his results correspond to smaller amounts of lightness induction. Note particularly the result for the uppermost condition (1/ and 9/). There is an indication of two maxima of lightness induction.

The *theoretical curves* were derived by setting up two lightness vs. MUNSELL value functions, one for the darker surround, the other for the lighter. The MUNSELL value of the unknown target gray with the lighter surround was read from the latter function to correspond to the lightness of the fixed target gray with the darker surround read from the former. The lower part of Fig. 14 shows four of these lightness functions; one for a surround of MUNSELL value 1/, two for 5/, and one for 9/. To obtain the indicated correspondence between theoretical curves and experimental points it was necessary to derive the series of lightness-value functions from two terms. The first term corresponds to general lightness induction

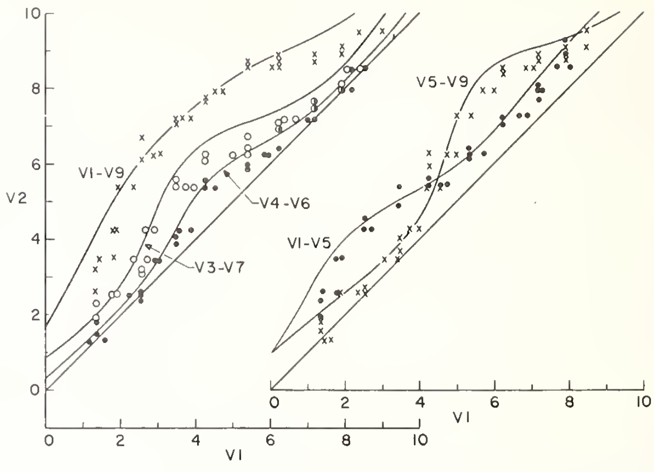

Fig. 12: Munsell value (V2) of object with lighter gray surround appearing equally light as object of Munsell value V1 viewed with darker gray surround. If the surrounds were identical, the observed points would fall on or close to the 45°-line shown on both the left-hand and right-hand plots. For the left-hand plot, the observations by Takasaki for surrounds of Munsell values 4/ and 6/ are shown by dots; for surrounds of 3/ and 7/, by circles; and for surrounds of 1/ and 9/, by crosses. For the right-hand plot, dots correspond to 1/ and 5/; and crosses, to 5/ and 9/.

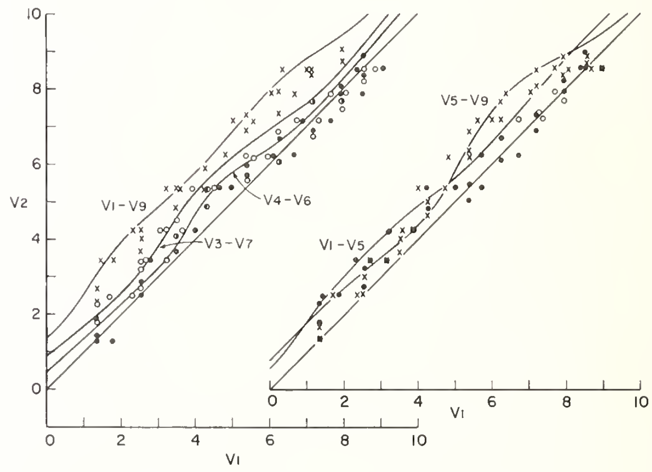

Fig. 13: Same as fig. 12 except that the observations are by another (GTY) of Takasaki's five observers. Note that lightness induction by this observer is considerably less than that shown in fig. 12.

computed by applying the Jameson-Hurvich induction formula to Munsell value. The second term corresponds to what Takasaki calls the *crispening effect*. It acts to increase the slope of the lightness-value function at and near the reflectance of the surround. By using various induction coefficients, coefficients of amount of crispening, and sharpness of crispening, the experimental data for all observers and all the various combinations of surrounds could be accounted for.

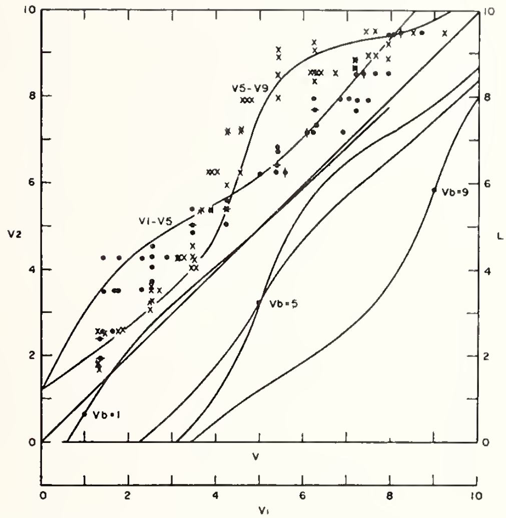

Fig. 14: The left-hand plot shows observations by Takasaki pooled with those by still another observer (GH). Dots correspond to surrounds of Munsell values 1/ and 5/; crosses to 5/ and 9/. The curves on this figure and on figs. 12 and 13 are based on lightness *vs.* value functions chosen for each observer to fit the observations (see text). The right-hand plot shows one such function for a surround of Munsell value 1/; two for 5/; and one for 9/. All of these functions have maximum slope for the Munsell value of the observed object equal to that of the surround. The increase of slope of this function near the Munsell value of the surround is called "crispening" by Takasaki.

These new results of lightness discrimination imply that *the retinal processing of signals from the cones differs considerably in degree from one observer to another*. Discrimination of colors near that of the surround is better for all five observers than that of colors widely different from that of the surround; but for some observers this crispening of discrimination is considerably larger and sharper than for others. My previous view that Munsell value is the best available correlate for lightness of flat objects viewed with a middle gray to white surround is called into question by Takasaki's results, and also by a recent study by Kaneko [27].

Similar nonlinearities in the correlate for chroma were found by the *OSA Committee on Uniform Color Scales*. If general formulas for hue, lightness, and chroma of object-color perceptions cannot soon be derived from theoretical considerations, at least there is promise that considerably improved empirical formulas can be found.

7. Conclusion

If it were possible to predict the hue, lightness, and chroma of the color perceived by any observer to belong to any object in any scene illuminated by any kind of light, then it might be said that the problem of color appearance had been solved. Progress toward this solution has been considerable, but the solution is so far from being obtained that nearly every research paper on color furthers this solution to some degree.

Even if complete theoretical treatments of color matching for both normal and abnormal color vision, successive color contrast, simultaneous color contrast, and separation of object-color perception from ambient-light color perception should be achieved, there would still remain the problem of synthesizing the theories of these various effects to produce a successful prediction of the colors perceived to belong to flat, mat-finish, opaque objects. Except for artists, scarcely anyone has even commenced to grapple with the problems of prediction of the colors perceived to belong to glossy, or translucent, or nonflat objects, though some terminology has been developed (Judd [26]), and Evans [8] has carried out some important exploratory work on conditions for the various modes of appearance.

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Washington 25, D. C., U. S. A.

Eingang des Manuskriptes:
2. Juli 1965

PROGRESS REPORT FOR O.S.A. COMMITTEE ON UNIFORM COLOR SCALES

Proc. Int. Colour Meeting, Lucerne, Switzerland, 1965, pp. 399-407
(Musterschmidt, Göttingen, Germany, 1966)

This, the last "progress" report of the O.S.A. committee written by Judd as its chairman, mentions most of the difficulties that delayed preparation of the set of colors that embody the results of the committee's work. Conclusion 6 on p. 406 is a consequence and quantitative expression of the failure of additivity reported in conclusion 4. The committee's results showed that color space is curved. Much time and committee consideration was devoted to the decision to proceed, nevertheless, by adopting a euclidean color space that does the least possible violence to the observational results. Only by accepting such deviations from the data, to permit formulation of a euclidean color space, could the committee complete its self-imposed task, to make several hundred color cards as nearly equally different as possible. The suggested explanation of low correlation between the observed color differences and the corresponding distances in the adopted color space - as a consequence of near equality (as intended) of the differences between the colors compared - is valid. If they had actually been all exactly equal, and if the color space modeled them exactly (with equal distances) the only sources of variance would be random errors of observation and errors of colorimetry (spectrophotometry and change of the samples between measurement and observation). Such errors, being random and uncorrelated, would result in zero correlation, despite even superb observations and perfect determination of the color space. That higher correlation was obtained (74%) was a consequence of the fact that the observed color differences were not all equal, but significantly different. The correlation could not be much higher because of differences between observers and because the committee was constrained to adopt a euclidean color space, which the committee proved cannot fit the observed results.

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Progress Report for O.S.A. Committee on Uniform Color Scales

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Die 1947 gegründete Kommission führt seit 1955 Untersuchungen mit dem Ziel durch, einige Hundert Farbtäfelchen so auszufärben, daß sie den Pigmentfarbbereich möglichst gleichabständig mit einem Rhomboedergitter durchsetzen. Der erste Versuch (1956) diente der Festlegung von Farbörtern in der Normfarbtafel für Farben empfindungsgemäß gleicher Sättigung (Abstand vom gleichhellen Grau) mit dem Munsell-Helligkeitswert 6/. Im zweiten wurden auf dieser Kurve von Farbörtern 40 Farben mit empfindungsgemäß gleichem Farbtonabstand ermittelt und im dritten die Farbigkeit von fast neutralen Unbunt-Farben der Helligkeitswerte 3/, 6/ und 8/ bestimmt. Die letzte Untersuchung bestand in der Vorbereitung von 43 Farbmustern, die in der Helligkeitsebene 6/ ein Dreieck gleichabständig erfüllen, und in der Überprüfung dieser Gleichabständigkeit durch 70 Beobachter. Jeder Beobachter hatte für 142 Farbunterschiede innerhalb der 43 Proben das Urteil „größer" oder „kleiner" zu fällen. Die Versuchsergebnisse werden mitgeteilt und diskutiert.

Cette commission, qui a été créée en 1947, procède depuis 1955 à des essais ayant pour but, à l'aide de la méthode du réseau rhomboédrique régulier d'établir plusieurs certaines de petits panneaux colorés représentant aussi uniformément que possible la gamme des couleurs. Le premier essai (1956) servit à déterminer le lieu des points de chromaticité dans le diagramme CIE (x, y) pour les couleurs de notations 6/ de Munsell perçues comme ayant la même saturation (distance du gris). Le deuxième essai (1957) a permis de déterminer 40 points de chromaticité de ce lieu répartis uniformement dans la teinte. Le 3ème essai (1961) servit à établir les intervalles de chromie près des tons gris 3/, 6/ et 8/. Le dernier essai permit de préparer 43 panneaux de couleur devant donner une répartition triangulaire uniforme à 6/, ainsi qu'un contrôle de l'uniformité des intervalles par 70 observateurs, dont chacun devait décider entre «plus grand» ou «plus petit» pour 142 différences de paires parmi 43 panneaux de couleur. Les résultats obtenus sont communiqués et expliqués.

This committee, organized in 1947, has been engaged since 1955 in experiments intended to result in the production of several hundreds of painted color chips that would sample the paint gamut as uniformly as possible by the regular rhombohedral lattice-sampling method. The first experiment (1956) determined a locus of chromaticity points on the CIE (x, y)-diagram for colors of Munsell value 6/ perceived as having the same saturation (departure from gray). The second experiment (1957) determined 40 chromaticity points on this locus equally spaced in hue. The third experiment (1961) determined chromaticness spacing near grays of 3/, 6/, and 8/. The latest experiment is a preparation of 43 chips intended to give a uniform triangular sampling at 6/, and a check of the spacing uniformity by 70 observers each making judgments of "larger" or "smaller" for 142 of the pairs of differences among the 43 chips. These experimental results will be presented and discussed.

° Chairman of the *Committee on Uniform Color Scales of the Optical Society of America*

Organization of the Committee

In the report of the *Committee on Colorimetry* of the *Optical Society of America* [12] it is stated: "Color differences consisting solely of chromaticity differences with no luminance differences are rare, and in this case there is no known way of evaluating the noticeability of combined luminance and chromaticity differences. The complete experimental clarification of this problem is one of the major programs yet to be undertaken in the field of colorimetric research. Influences of field size and adaptation to various luminances and chromaticities also await thorough quantitative study."

Two informal conferences on ways to carry out this program were held, one at the February 1947 meeting of the *Optical Society of America* in New York, and the other at the April 1947 meeting of the *Inter-Society Color Council*. Finally, a formal meeting, sponsored by the *National Research Council*, was held in June 1947 at the *National Academy of Sciences* in Washington, D. C. Those present were BALINKIN, BELLAMY, DIMMICK, FOSS, GIBBS (representing the *National Research Council*), GODLOVE, GRANVILLE, JUDD, MACADAM, NICKERSON, and NEWHALL. The usefulness of more comprehensive and accurate information regarding the perceptibility of color differences was discussed at considerable length, and it was brought out that:

1. The *National Bureau of Standards* would use this information to improve the "*NBS unit of color difference*" used in the setting and administration of color tolerances.

2. The MUNSELL *Color Company* would use this information to improve the spacing of the color scales making up the *Munsell Color System.*

3. The *Department of Agriculture* would use the information in its studies of the colorimetry of agricultural products.

4. The *Navy Department* would use information regarding the perceptibility of color differences in solving unnumbered problems arising in the design of fighting equipment.

5. The *plastics industry* would apply this information to the establishment of color tolerances and for evaluating color differences between production batches and the accepted standard.

6. The *photographic industry* would use the information as a tool in its studies and tests of the fidelity of color reproduction of a scene by a given photographic process.

7. Others were interested in this information because it would show under what conditions and to what approximation it is possible to define *an homogeneous isotropic color space* (BALINKIN [2]). Thus, a collection of color chips might be assembled each one of which differed from each of its nearest neighbors by amounts of nearly equal perceptibility (FOSS [4; 5]).

Such a collection would not only cover a given color gamut with the fewest possible number of chips showing color spacing of any given size, but would also be maximally useful in setting color tolerances.

8. Still others were interested in this information because they thought it would aid somewhat in color coordination, or selection of groups of colors to be used together for some specified purpose in which the question of color harmony is important. Possible fields of application lie in interior decoration, exterior painting of prefabricated houses forming a planned community, and in choice of colors for hospital rooms with attendant questions of color therapeutics.

As a result of this meeting, the *Optical Society of America* in December 1947 set up the *O.S.A. Committee on Uniform Color Scales.* This committee met at the October 1948 meeting of the OSA in Detroit to lay out a program of research. In addition to determination of color spacing around some 25 widely distributed points in color space, auxiliary researches were planned on the influence of field size, field luminance, surround luminance, angular separation of field, and chromaticity of the surround. Parts of this research program were parceled out to the various members of the committee. For several years no further meetings were held, but members of the committee went ahead with their tasks individually. Some of these involved a direct experimental attack, others were confined to checks of one or another of the six or seven formulas for color difference already proposed, and still others bore on preparation of color chips to form uniform scales; see bibliographical entries for the years 1949 to 1954 given by Judd [8].

Formulation of the basic task

At the time of the October 1953 meeting of the OSA in Rochester, a committee meeting was called to assess progress made since 1948, and to see whether further coordination of the research efforts was needed. Those who had been concerned with the direct experimental attack held the view that the facts had then been established in their essentials, and said that they did not expect to make any further studies. It is now possible, they said, by interpolation among the 38 colors studied by Brown and MacAdam [3] to assess reliably the perceptibility of any color difference from the tristimulus values of the two colors, and, by appeal to the auxiliary studies (field size, field luminance, color of background, separation of the two fields), to do it for any experimental conditions of interest. Those who had been working with color-difference formulas pointed out correctly that the experimental results are extremely complicated (MacAdam [9]), not suited for convenient engineering applications, and anyway seemed to prove that a truly uniform set of tridimensional color scales in accord with sampling color space by a regular rhombohedral lattice is not possible because color space seems to have a

Gaussian curvature significantly different from zero (Judd [7]). The achievement of most practical value, they said, would be the development of a color-difference formula simple enough to be convenient and accurate enough to represent the essential experimental facts. It developed that no one had been able to form a conception of the nature and importance of the compromise involved in such a formula. Those primarily interested in the production of uniformly spaced color chips (led by Foss) took the position that the time was then ripe to apply everything that has been learned by laboratory experiments to the central problem of producing a set of chips of maximally uniform tridimensional spacing. In this way, they pointed out, the nature of the compromises that have to be made will become evident merely by looking at the chips. We will see, furthermore, they said, whether it is true that interpolations made among the 38 starting colors in the Brown-MacAdam study [3] are reliable. It was decided to embark upon the central project of producing several hundreds of painted color chips that are to sample the paint gamut as uniformly as possible by the regular rhombohedral lattice-sampling method (Wyszecki [14]).

Steps taken to fulfill basic task

The Chairman proposed to select a tentative 500-chip sampling of the color solid by appeal to the Brown-MacAdam spacing and the Munsell spacing (Newhall, Nickerson, Judd [11]), to produce chips of the indicated colors, and to proceed immediately to tests of the uniformity of spacing. The committee, however, voted to carry out some limited checks of the two sets of color-spacing data before producing several hundred color chips. With minor variations, the same methods have been used for four such checks, as follows:

Color Chips. Mat-finish paints of the desired colors were formulated by Davidson. They were applied to hexagonal ceramic tiles of two-inch minimum diameter by Foss. Five or six sets of such chips were provided for study by committee members.

Observing Conditions. Illumination was by natural or artificial daylight of at least 50 foot-candles (538 lx). Each pair of chips generating a color difference to be assessed was viewed with a non-selective surround of luminance factor equal to 30% (*Munsell Neutral* 6/), and some of the pairs were also viewed with other neutral surrounds. This surround also served to separate, by 1/8 to 1/3 inch, the two chips of each pair; and some of the pairs were also viewed in juxtaposition without any separation. Compliance to these conditions was ensured by tile holders made by Bellamy of the Munsell *Color Company.*

Instructions to the Observers. Given two chips on the left forming the left color difference, and two on the right forming the right color difference, the observer was asked to tell whether the right color difference is larger or smaller than the left color difference. Some auxiliary studies also required the observer to estimate by what factor the right difference differs from the left; others required the observer to choose a gray tile appearing equally light as a tile having a chromatic color.

Order of Presentation. Each observer was presented with pairs of colors in the same sequence, but each observer started at a different place in the sequence. The sequence was carefully randomized as to position (above or below) of the tiles forming the differences, position (right or left) of the pair, and introduction of presentations with nonstandard observing conditions (surround differing from MUNSELL 6/, tile pairs viewed in juxtaposition instead of separated by surround).

Reduction of Data. Each pair of differences was scored by subtracting the number of times the right difference was judged smaller from the number of times it was judged larger. By the method of least squares, estimates of the size H_i of the ith color difference were obtained from the model: $S_{ij} = H_i - H_j - E_{ij}$ where S_{ij} is the score obtained by comparing difference i with difference j, and E_{ij} are the errors, the sum of whose squares is to be minimized. A modification of the MORRISSEY-GULLIKSEN method of computation described by JACKSON and FLECKENSTEIN [6] was used.

The four preliminary checks of the BROWN-MACADAM and the MUNSELL color spacings made by these methods are:

(1). Determination of a locus of chromaticity points on the CIE (x, y)-diagram for colors of *Munsell* value 6/ perceived as having the same saturation (departure from gray).

(2). Determination of 40 chromaticity points on this locus equally spaced in hue.

(3). Determination of chromaticness spacing of near grays of Munsell value 3/, 6/, and 8/.

(4). Chromaticness spacing at MUNSELL value 6/.

Table 1 gives the dates, the number of colors, the number of color differences, the number of pairs of differences presented, the number of observers, and the total number of observations for each of these four checks, and indicates who reduced the data.

Fig. 1 shows the constant-saturation locus on the (x, y)-diagram found by the first check. When transformed to the MACADAM (u, v)-diagram recommended as the 1960 CIE-UCS diagram, this locus is seen (Fig. 2) to approximate a circle. The small circles on this locus give the hue spacing found by the second check. Note that this hue spacing does not approximate equal distances on the diagram (maximum distance is 1.8 times minimum) as well as the constant-saturation locus approximates a circle (maximum radius is 1.2 times minimum). The corresponding hue and saturation spacing from the MACADAM [9] ellipses is shown by large circles as interpolated by JUDD and by dots as interpolated by HOWETT.

Table 1: Some details of the four preliminary checks of color spacing

Dates	Purpose	No. of Colors	No. of diffs.	No. of pairs of diffs. shown	No. of observers	No. of observations	Data reduced by:
May, 1955 to June, 1956	Constant saturation locus at value 6/	34	33	384	60	23,040	Nickerson, Judd, and Nimeroff
Sept., 1956 to Oct., 1957	Hue spacing on this locus	40	39	142	102	14,484	Newhall
Dec., 1958 to Sept., 1961	Chromaticness spacing for near grays of Munsell value 3/, 6/, and 8/	21	36	99	97	9,603	Howett
Jan., 1964 to —	Chromaticness spacing for Munsell value 6/	43	147	436	70	9,940	Howett

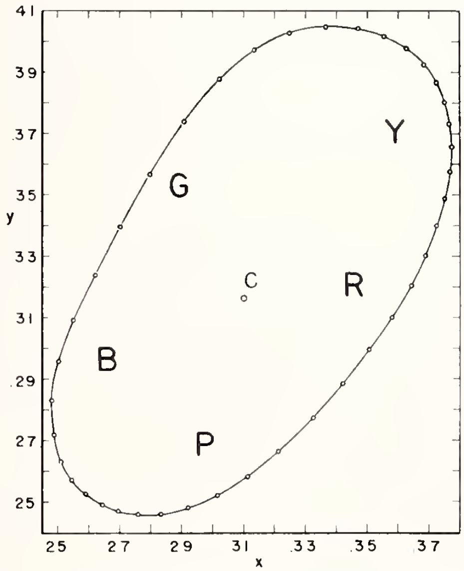

Fig. 1: The closed line (approximately ovoid in shape) indicates the chromaticities on the (x, y)-diagram found by the first preliminary check of the OSA Committee to correspond to equally receptible chromaticity differences from gray (point C). The small circles show the chromaticities on this approximate ovoid found the second check to be uniformly spaced in hue.

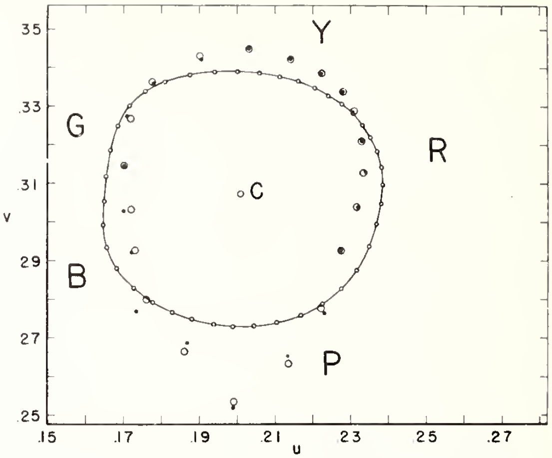

Fig. 2: The solid line and small circles show the data of fig. 1 on the MacAdam (u, v)-diagram recommended in 1960 by the CIE for improved uniformity in chromaticity spacing. The large circles (Judd) and dots (Howett) show evaluations of constant saturations and uniform hue spacing according to the MacAdam ellipses derived by interpolation from the MacAdam [9] graphs of the differential coefficients, g_{11}, g_{12}, and g_{22}.

The third check, chromaticness spacing for near grays of Munsell value 3/, 6/, and 8/, yielded a number of interesting tentative conclusions:

(1). *Hue-scale corroborated.* Except for discrepancies of the order of the spectrophotometric uncertainties, the hue scale found in the second check was corroborated.

(2). *Shape of constant-saturation locus.* Except for a slight flattening on the green side, the loci of constant saturation found at 3/, 6/, and 8/ value have the same shape as in Fig. 1.

(3). *Design data obtained for colors lighter and darker than value 6/.* Davidson had formulated the colors of the tiles to accord approximately with the expansions of the constant-chroma loci of the Munsell renotations with decreasing Munsell value. These expansions were considered to yield approximately uniform spacing for a surround of N 9/. To achieve uniform spacing for a surround of N 6/, it was indicated that the Munsell spacings should be expanded by 22% for value 3/ colors, and expanded by 19% for value 8/ samples. To achieve uniform spacing for gray surrounds of their own value, the Munsell spacings should be expanded by 11% for value 8/ colors, but left unchanged for value 3/ colors.

(4). *Additivity fails by as much as 30%.* Ratio judgments indicated that the sum of the two radial differences from gray in opposite directions is larger by as much as 30% than the difference perceived between the terminal colors.

(5). *Color space negatively curved?* We seem at last to have encountered a hint of the geometrical difficulties discussed at our second meeting (October 1953). The indicated curvature is negative since on the average the perceptual size of the hue differences exceeds that of the saturation differences by a small percentage (3 to 14%).

(6). *Psychometric scale values nonlinearly related to chromaticity-coordinate differences.* MacAdam [10] has shown that the measures H of perceptual size derived in the third check are not proportional to the chromaticity differences, but tend to vary with powers less than one (0.37 to 0.80) of these differences.

The correlation between the scores S_{ij} and the difference $H_{\text{right}} - H_{\text{left}}$ derived from them by least squares was found to be 0.95. We have, however, not been successful in predicting the scores from the chromaticity coordinates obtained spectrophotometrically from the tiles. Regardless of whether these data are evaluated in terms of Munsell spacing, MacAdam's [9] differential coefficient formula, the Adams [1] chromatic-value formula, or the committee's own scales from the first and second checks, no significant correlation has been found. Spectrophotometric errors, and impermanence of the tiles which caused them to drift more or less uniformly toward yellow during the course of the observations, failure of additivity, and negative curvature of color space, could contribute to this poor correlation, but our current view is that this poor correlation is chiefly ascribable to the fact that the color differences between the tiles are with but few exceptions fairly equal. According to the Scheffé S-method [13], only three of the 36 differences depart significantly from the mean difference.

The observations of the fourth check have been completed, and although at present writing (March 1965) the inversion of the matrix of order 148 has not yet been accomplished for the chief determination, the following conclusions from the auxiliary determinations may be stated:

(1). Tiles painted on the basis of information obtained from the first three checks show nonuniformity of spacing by not much more than a factor of 2.

(2). For our conditions of observation, one Munsell value step is perceptually equivalent in size to about 4 Munsell chroma steps.

(3). For a surface of constant lightness in color space the Munsell values of the red, green, and blue colors maximally different from gray are lower than the Munsell value of the gray by as much as 0.8, but for yellow colors lower by not more than 0.1. This result checks closely those previously found by Wyszecki and Sanders [15].

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FUNDAMENTAL STUDIES OF COLOR VISION FROM 1860 to 1960

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Color-vision theories are reviewed in terms of color-matching functions and their transformations. "Any tenable theory of color vision must explain in detail why precisely the conditions for color match expressed by the color-matching functions hold." A review of the determinations of the color-matching functions from Maxwell (1860) to those of Stiles, Burch, and Speranskaya (1959) is followed by a related consideration of partial-color-blindness data and theories, from which chromaticities of the probably "missing" primary sensations in protanopes and deuteranopes are deduced. One indicated conclusion is that there are two classes of deuteranopes: the luminous efficiency function peaks at about 575 nm for one class, whereas it peaks at about 560 nm for the other, which may be compared to 555 nm for persons with normal color vision. Studies of chromatic adaptation are generally in accord with the conclusions derived from partial color blindness, but "progress has been handicapped by lack of crucial evidence that might decide between the alternative theoretical possibilities" that were proposed between 1860 and 1960.

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FUNDAMENTAL STUDIES OF COLOR VISION FROM 1860 TO 1960

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NATIONAL BUREAU OF STANDARDS

What I call fundamental studies of color vision got their start with Maxwell's determination (1860) of the color-matching functions of the normal human eye. Figure 1 (Judd, 1961) compares Maxwell's determinations for himself and one other observer (K) with modern color-matching functions. The open points and the lines drawn through them come from the 1931 CIE standard observer; the points marked otherwise are from Maxwell. Except for the extremes of the spectrum the discrepancies are only slightly more than can be explained by individual differences among observers with normal color vision. These curves, R, G, and B, indicate the radiant flux of spectrum primaries [630, 528, 457 nanometers (nm)] required in an additive mixture to match unit radiant flux for each part of the spectrum in turn. Note that at the red primary the R-curve is unity; the other two, zero. This means simply that one unit of radiant flux at this wavelength color-matches itself, and the same for the other two primaries, of course. Note also that some of the radiant fluxes indicated by the curves are less than zero. To speak of a negative radiant flux makes no sense physically; there is no such thing. But in color matching there is a useful meaning, universally recognized since Maxwell's time, of a negative radiant flux. It means simply that the negative amount of the primary, instead of being added to the other two primaries, is added to the part of the spectrum whose color is being determined. For example, at 500 nm Figure 1 shows that by adding about 0.2 unit of radiant flux of the red spectrum primary (630 nm) to one unit of radiant flux at 500 nm, there is produced a blue-green color that can be matched by about 0.4 unit of the green primary (528 nm) added to about 0.2 unit of the blue primary (457 nm).

The functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ not only show how much of the primaries is required to match each part of the spectrum in turn, but they also indicate how much of the primaries is required to match any known combination of the various parts of the spectrum. On this account these functions are known as color-matching functions. By means of these functions, successful predictions may be made of whether any two lights of known spectral distributions $P_{\lambda,1}$ and $P_{\lambda,2}$ will, or will not, look alike. The conditions for color match are:

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2})\, \bar{r}(\lambda)\, d\lambda = 0,$$

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2})\, \bar{g}(\lambda)\, d\lambda = 0,$$

$$\int_0^\infty (P_{\lambda,1} - P_{\lambda,2})\, \bar{b}(\lambda)\, d\lambda = 0.$$

These conditions for color match follow directly from Grassmann's law (1853) of additivity, well verified (König, 1887; v. Kries, 1905) for the macular regions of the retina. They state merely that if for each spectral component of a compound light the equivalent is given in amounts of the three primaries, the sum of these equivalents will color-match the sum of the spectral components making up the compound light. If the conditions for a color match are satisfied because the two lights are physically identical ($P_{\lambda,1} = P_{\lambda,2}$) throughout the visible spectrum, the match is said to be isomeric (following Ostwald's terminology) or spectral (Nimer-

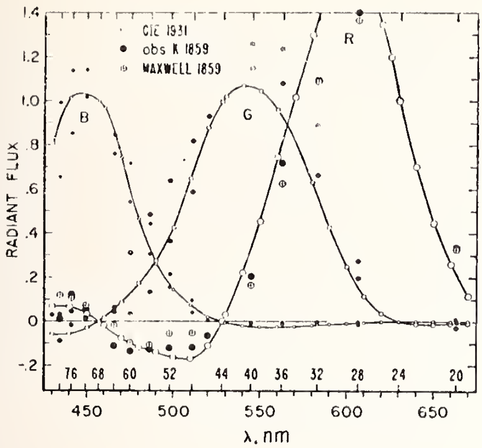

FIG. 1.—Radiant flux of the spectrum primaries, 630.2, 528.1, and 456.9 nm, required in an additive mixture to match unit radiant flux of the spectrum. 1931 CIE standard observer, ○——○; Maxwell's observer K, ●; Maxwell (observer J), ⊗. Note that Maxwell's data expressed in units of radiant flux are in substantial agreement with the 1931 CIE standard observer. The numbers above the base line refer to wavelengths designated on Maxwell's instrumental scale, nonlinear because of prismatic dispersion.

off, 1965). In this trivial case, the color-matching functions are not needed. If the two lights have different spectral distributions, the color match is said to be metameric; and in this usual case the color-matching functions are crucial.

Because of this generality of their application, studies yielding color-matching functions might be said to be fundamental. There are, however, two other reasons for regarding them as fundamental.

Types of Color Vision.—Table 1 summarizes the principal types of human color vision (Judd, 1943).

Normal color vision requires three color-matching functions for its definition, as shown in Figure 1. On this account normal color vision is classed as trichromatic. Trichromatic observers can make light-dark, yellow-blue, and red-green discriminations. There are three types of abnormal trichromatic color vision, known as protanomaly, deuteranomaly, and tritanomaly (rare).

Partially color-blind vision requires but two color-matching functions for its definition, and is known as dichromatic vision. There are three types of dichromatic vision, known as protanopia, deuteranopia, and tritanopia. A dichromat can match any color by a suitable additive combination of but two primary lights; and he is able to make discriminations of but two sorts as indicated in Table 1.

Totally color-blind vision requires but one color-matching function for its definition, and is known as monochromatic vision. A monochromat can match any color by a suitable amount of any light taken as a primary simply by adjusting the primary light to the same brightness, because a monochromat cannot make any discrimination other than light-dark.

Since the color-matching functions provide a basis for classifying human vision into the three types, trichromatic, dichromatic, and monochromatic, this is another sense in which we may regard them as fundamental.

Theories of Color Vision.—Major attention between 1860 and 1960 has been devoted to three categories of color-vision theory: three-components theory, opponent-colors theory, and stage theory. All of these theories, and indeed any tenable theory of color vision, must explain in detail why precisely the conditions for color match expressed by color-matching functions hold. The color-matching

TABLE 1

Types of Human Color Vision

Types	Light-dark	Response Yellow-blue	Red-green	Maximum luminosity at λ (nm)	Type designation
Trichromatism	Yes	Yes	Yes	555	Normal
	Yes	Yes	Weak	540	Protanomaly
	Yes	Yes	Weak	560	Deuteranomaly
	Yes	Weak	Yes	560	Tritanomaly
Dichromatism	Yes	Yes	—	540	Protanopia
	Yes	Yes	—	560	Deuteranopia
	Yes	—	Yes	560	Tritanopia
Monochromatism	Yes	—	—	510	Cone blindness
	Yes	—	—	560	—
	Yes	—	—	540	—

functions thus play the role of quantitative boundary conditions for any theory of color vision. This is the major reason for regarding them as fundamental.

Three-components theory: This theory was briefly stated in 1807 by Thomas Young, and was elaborated by Helmholtz about 50 years later It assumes the existence of three independent response mechanisms in the normal eye: one predominantly sensitive to long-wave light and yielding the response *red;* a second predominantly sensitive to middle-wave light and yielding the response *green;* and a third sensitive to short-wave light and yielding the response *violet.* Young thought that the long-wave and short-wave extremes of the spectrum could excite, respectively, the red and violet responses to the exclusion of the other two. Let us see how color-matching functions yield precisely the response curves implied by this assumption.

The key to this problem is choice of primaries. In Figure 1 are shown color-matching functions expressed relative to the spectrum primaries (630, 528, and 457 nm) chosen by Maxwell. From Grassmann's law of additivity it can be shown that precisely the same decisions as to color-match, or failure to color-match, can be obtained from functions R', G', and B', as from R, G, and B, provided the new functions are weighted sums of the old:

$$R' = K_{11}R + K_{12}G + K_{13}B,$$

$$G' = K_{21}R + K_{22}G + K_{23}B,$$

$$B' = K_{31}R + K_{32}G + K_{33}B,$$

where K_{11}, K_{12}, ..., K_{33} are constants that may be chosen arbitrarily provided the determinant differs from zero. It is legitimate to proceed therefore by trial and error to search for constants that will yield response curves, R', G', and B', in accord with Young's hypothesis; that is, curves with no negative values anywhere, a green curve that is zero for as much as possible of the long-wave and short-wave portions of the spectrum, a red curve that is zero for the short-wave portion, and a violet curve that is zero for the long-wave portion. This choice of constants amounts to choice of red and violet primaries at the extremes of the spectrum.

This possibility was clearly explained by Helmholtz. He states, for example (p. 145): "The choice of the three fundamental colours is somewhat arbitrary... Young may have been guided by the consideration that the terminal colours of the spectrum seem to have special claims by virtue of their positions." The König-Dieterici

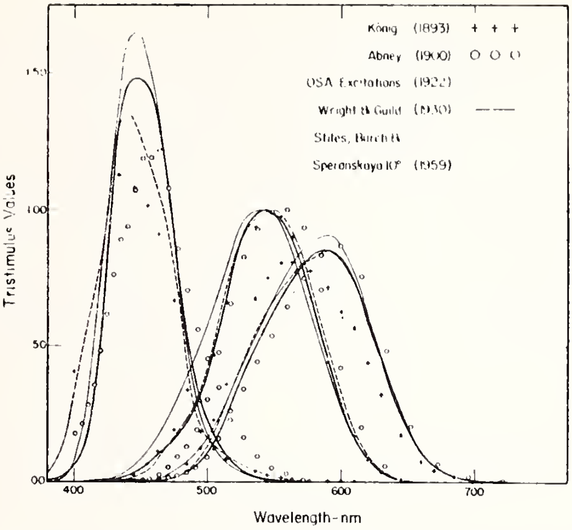

FIG. 2.—Color-matching functions expressed relative to the Young primaries (spectrum red, spectrum violet, and green outside spectrum). By the Young theory these functions represent the spectral absorptances of the retinal photopigments multiplied by the spectral transmittances of the ocular media through which light passes to reach the retina. König (1893), *crosses;* Abney (1900), *circles;* OSA excitations (Troland, 1922), *dashed lines;* CIE standard observer based on Wright and Guild (1930), *heavy lines;* CIE supplementary observer for 10° viewing fields based on Stiles and Burch (1959), and Speranskaya (1959), *light lines.*

determinations of the color-matching functions were presented (1893) with Young's choice of primaries under the name "Elementar-Empfindungs-Curven," and Abney's were similarly presented (1900) under the name "sensation curves." This choice of primaries was used by Weaver, who averaged the König and Abney results to produce the "elementary excitation curves" recommended in 1922 by the Committee on Colorimetry, Optical Society of America, under Troland's chairmanship. Figure 2 compares these color-matching functions with those based on the work of Wright (1929–1930) and Guild (1931) recommended in 1931 by the International Commission on Illumination (CIE), and with those of Stiles and Burch, and Speranskaya (1959) for extramacular vision (field subtending 10°) recommended in 1964 by the CIE for large-field colorimetry.

The color-matching functions in Figure 2 all show the amounts of the Young primaries required to color-match unit radiant flux of each part of the spectrum. The scale values for the G'-functions are adjusted to make the maximum be unity; those for the R'- and B'-functions are adjusted to make the areas under the curves equal to that for the G'-function; that is, the so-called equal-energy source (source whose radiant flux per unit wavelength is constant) corresponds for each set of functions to equal amounts of the three primaries. To make the König-Dieterici determinations (shown by crosses) refer to unit amount of spectral radiant flux, it was assumed that König's sunlight was equivalent to sunlight at air mass 2 from the compilation by Parry Moon (1940). Similarly, the Abney data (shown by circles) were adjusted by assuming that the positive crater of the carbon arc used by him was equivalent to the blackbody at 3,800°K. It will be noted that Weaver's average of these two sets of color-matching functions (shown by dashed lines) falls fairly satisfactorily between the crosses and circles; so he must have made a fairly similar adjustment. The Guild-Wright determination (shown by heavy solid lines) corroborates the Weaver average surprisingly well considering the rather large differences between the König and Abney data. The Stiles-Burch-Speranskaya determination (shown by light solid lines) departs from these chiefly in ways that correspond to the use of extramacular retinal regions instead of macular, and this indicates that the extramacular cones have spectral sensitivities not much different from those of foveal cones.

It will be noted that Young's opinion that the long-wave portion of the spectrum can excite the red response to the exclusion of the other two is not contradicted by any of the color-matching functions. These functions, however, do not permit the view that the short-wave extreme can excite the violet response alone; a small red response (about 1/200 of the maximum) has to be admitted.

There is one final remark on Figure 2. If the additional assumption be made that each type of cone contains its own characteristic photosensitive pigment, then these color-matching functions expressed relative to the Young primaries should correspond to the product of the spectral absorptance of the three retinal pigments by the spectral transmittance of the ocular media (principally lens and macula). When Rushton (1957) announced that by measuring the spectral reflectance of the retina both before and after bleaching by strong light, he had discovered the presence of two bleachable pigments, one with peak absorptance at 590 nm, the other at 540 nm, it was very tempting to point to these two color-matching functions peaking very closely at these two wavelengths, and to say that this is striking corroboration of the Young theory. When the spectral transmittances of the ocular media are divided out of these curves, however, the predicted wavelengths of maximum absorptance are found to be somewhat lower than those indicated by Rushton's very difficult technique.

Partial color blindness: Young's theory has a simple built-in explanation of the three forms of dichromatic vision. These forms (protanopia, deuteranopia, and tritanopia) are reduction forms of normal vision. Any dichromat finds any color match set up by an observer of normal color vision with the same ocular pigmentation to be an acceptable match for him also. The simplest explanation for dichromatic vision by the Young theory is to say that a protanopic observer is an otherwise normal observer who has lost the red response. Similarly, deuteranopia corresponds to loss of the green response, and tritanopia, to loss of the violet. Since in each case the remaining two responses are identical with those of normal vision, the dichromatic observer has no way to detect any error in a normal match. A corollary of this explanation, however, is that an observer who has lost the red response would be blind to the long-wave part of the spectrum where the other two mechanisms have zero sensitivity. Protanopes have reduced sensitivity to long-wave light, but not zero. Similarly, tritanopes have reduced sensitivity to short-wave light, but not zero.

Maxwell was among the first to recognize that dichromatism is a reduced form of normal trichromatism, and he described a method by which to determine precisely the normal color corresponding to the missing primary process. In a letter of June 4, 1855, to G. Wilson, Maxwell wrote: "The mathematical expression of the difference between colour-blind and ordinary vision is that colour to the former is a

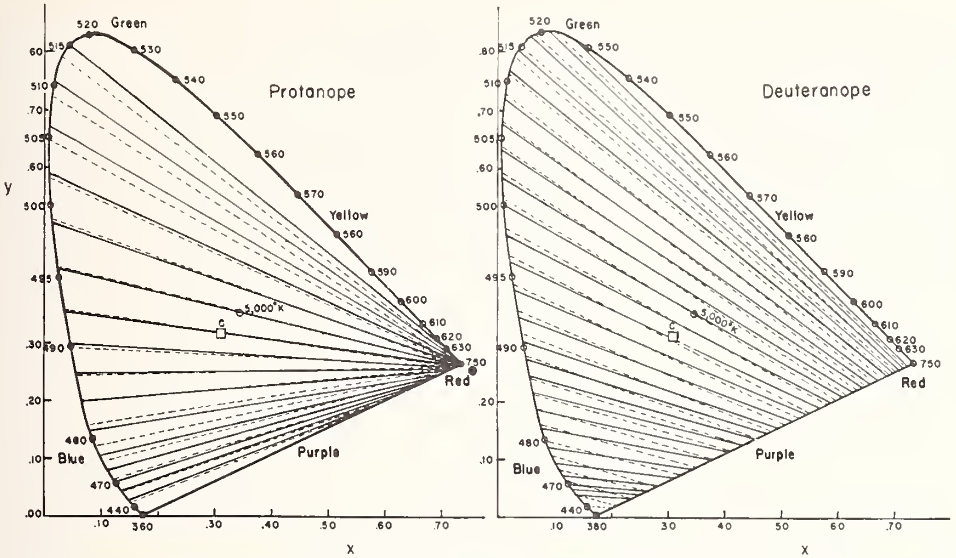

Fig. 3.—Chromaticity confusions of the protanope and deuteranope shown on the (x,y)-chromaticity diagram of the 1931 CIE colorimetric coordinate system from determinations by Pitt (1935). The points along each dotted line correspond to chromaticities found by Pitt to be indistinguishable in dichromatic vision. The solid lines intersect at a single point: protanope at $x = 0.753$, $y = 0.247$; deuteranope at $x = 1.000$, $y = 0.000$.

function of two independent variables, but to an ordinary eye, of three; and that the relation of the two kinds of vision is not arbitrary, but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement, which forms one-third of the apparatus by which we receive sensations of colour. ... If we find two combinations of colours which appear identical to a colour-blind person, and mark their positions on the triangle of colours, then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two. We may in the same way find other lines passing through the series of colours which appear alike to the colour-blind. All these lines either pass through one point or are parallel, according to the standard colours which we have assumed, and the other arbitrary assumptions we may have made. Knowing this law of colour-blind vision, we may predict any number of equations which will be true for eyes having this defect."

Figure 3 shows on the CIE (x,y)-chromaticity diagram by dotted lines the chromaticity confusions found by Pitt (1935) for six protanopes (on the left), and for six deuteranopes (on the right). The protanopic confusion lines are seen to intersect at very nearly one point as stated by Maxwell, and indicate the color for whose perception protanopes have no mechanism. The deuteranopic confusion lines are not as consistent, but the average intersection point is at about $x = 1.10$, $y = -0.10$. The solid lines intersect at $x = 1.00$, $y = 0.00$, which may be taken as an upper y-limit for Pitts' six deuteranopes. By basically the same method, though different in detail, König (1892) also evaluated the intersection points for confusion lines by two protanopes and one deuteranope. Similar evaluations for confusion lines were made by Farnsworth (1955) for one tritanope, and by Thomson and Wright (1953)

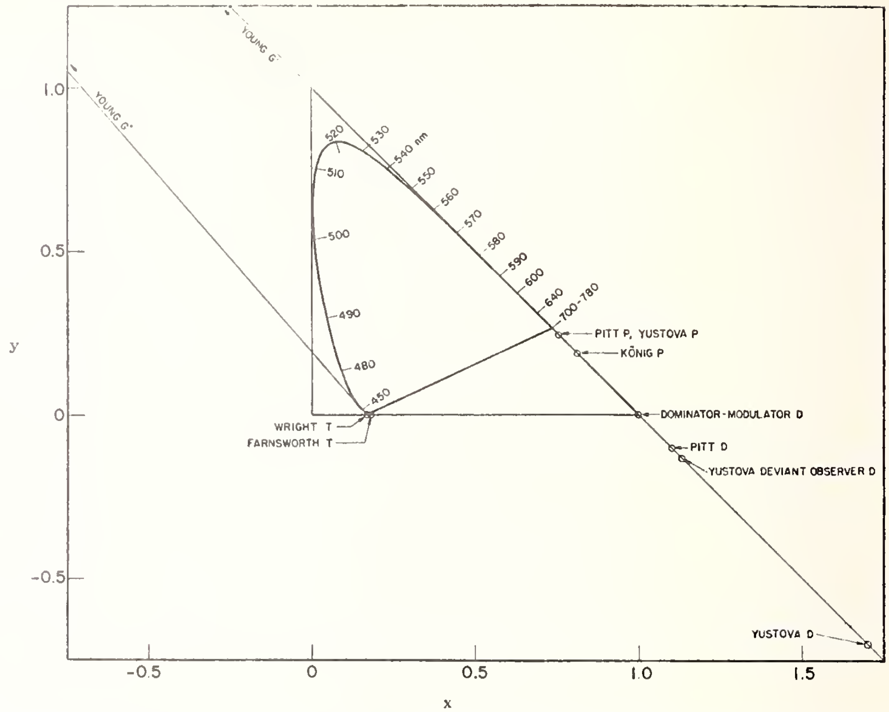

FIG. 4.—Intersection points of the chromaticity-confusion lines for the three types of dichromatic vision: protanopic, *P;* deuteranopic, *D;* and tritanopic, *T,* shown on the (x,y)-chromaticity diagram of the 1931 CIE colorimetric coordinate system. Note that the two determinations (Wright, 1952; Farnsworth, 1955) of the *T*-point fall close to $x = 0.17$, $y = 0.00$, and that the two recent determinations (Pitt, 1935; Yustova, 1953) of the *P*-point fall close to $x = 0.75$, $y = 0.25$. The determinations of the *D*-point (Pitt, 1935; Nuberg and Yustova, 1955), on the other hand, spread over a considerable range. The dominator-modulator theory implies a *D*-point at $x = 1.00$, $y = 0.00$. Young's theory in its original form implies that the deuteranope chromaticity confusion lines are nearly parallel on the (x,y)-chromaticity diagram (see the two straight lines labeled *Young G'*).

for seven tritanopes. Figure 4 shows the locations of the intersection points of the chromaticity-confusion lines for protanopes (*P*), deuteranopes (*D*), and tritanopes (*T*) from the studies mentioned above. König's deuteranope yields an intersection point off the graph.

Yustova (1949, 1953) carried out studies of dichromatic vision, and in 1955 Nuberg and Yustova published an extensive determination of intersection points of chromaticity-confusion lines for four protanopes and 12 deuteranopes. They used a long-overlooked method, also originated by Maxwell (1860), which yields the intersection point from a single pair of colors confused by the dichromat. The method is to obtain the tristimulus values set by the dichromatic observer to match any arbitrarily chosen color and to subtract them from the tristimulus values of the same color set by an observer having normal trichromatic vision. The differences specify the color for the perception of which no mechanism exists in the dichromatic observer (Judd, 1964). The Yustova results are also shown in Figure 4. It will be noted that the point found by Yustova for four protanopes agrees perfectly with that found by Pitt for six protanopes. Only one of the 12 deuteranopes, however, agrees with Pitt's D-point. The other 11 deuteranopes yield intersection points between Pitt's D-point and König's early determination for one deuteranope.

It must be pointed out that the location of the intersection points at various places along the line $z = 0$ in the CIE coordinate system correlates with the wavelength of the maximum of the long-wave response curve for the corresponding dichromat. This long-wave response curve is essentially the dichromatic luminosity function because the contribution of the short-wave response function to luminosity, if not precisely zero, differs negligibly from zero. For the intersection point at $x = 0.75$, $y = 0.25$, as for protanopes, the dichromatic luminosity function peaks at about 540 nm. For $x = 1.00$, $y = 0.00$, as in the dominator-modulator theory (to be discussed presently), the luminosity function is identical with that for normal vision, and peaks at 555 nm. For $x = 1.10$, $y = -0.10$, the peak is at about 560 nm, as for Pitt's six deuteranopes. But for Yustova's 11 deuteranopes yielding $x = 1.7$, $y = -0.7$, the peak is at 573 nm. The luminosity curves found by Hsia and Graham (1957) average to a curve with a peak at about 575 nm in good agreement with the Yustova determination. It should be recalled, also, that Willmer (1949) announced the discovery of two distinct types of deuteranopia, one with normal luminosity function, the other with luminosity function shifted toward the long-wave end of the spectrum compared to normal. If it is true, as announced by Willmer, that there are two types of deuteranope, then perhaps all of the D-points shown on Figure 4 are correct. By this view, all of Pitt's six deuteranopes, one of Hsia and Graham's six deuteranopes, and one of Yustova's 12 deuteranopes should be classed as having essentially normal luminosity functions; the remainder of the deuteranopes studied have luminosity functions shifted from the normal position to the long-wave end of the spectrum.

If the color-matching functions are expressed relative to the P-, D-, and T-points, they may be interpreted as the responses that three different kinds of cones would have to have to explain the three types of dichromatic vision. Figure 5 shows such response curves derived from the CIE 1931 color-matching functions in which the D-point is taken at $x = 1.00$, $y = 0.00$. By this choice, which is barely admissible from Pitt's determination, and inadmissible from Yustova's, deuteranopic luminosity is taken as identical to normal luminosity. The obvious difference between these response curves and those expressed relative to Young's primaries is that the long-wave curve peaks, not at 590 nm, but at 555 nm; and indeed this curve is precisely the normal luminosity function. The other important, though not obvious, difference is that the middle-wave curve does not drop all the way to zero at either end of the spectrum until the other curves do. The short-wave curve remains unchanged. The usual explanation of the differences between these two sets of response curves is that those referred to the Young primaries reflect the spectral absorptances of photosensitive retinal pigments, while those expressed relative to the intersection points of the chromaticity-confusion lines for the three types of dichromatic vision reflect the responses of three types of cones. The fact that the short-wave curves of the two sets are identical is taken to mean that the cones giving violet or blue signals contain only the short-wave-absorbing pigment. The fact that the middle- and long-wave curves expressing cone response are broader than those expressing absorptance of photosensitive pigments may be taken to mean that

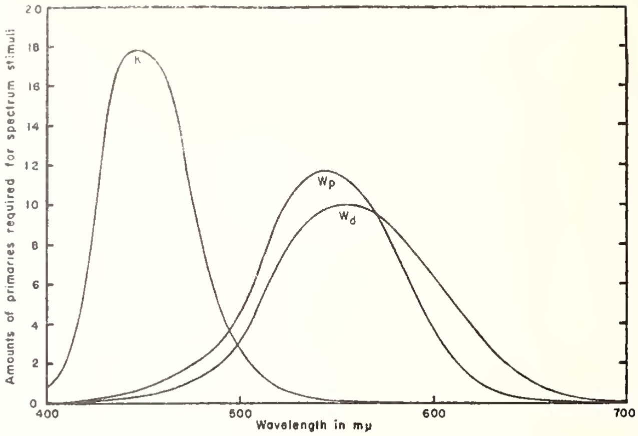

Fig. 5.—Color-matching functions for the 1931 CIE standard observer expressed relative to the primaries implied by the König (1897) tentative form of dominator-modulator theory. The red primary is placed at the protanopic intersection point ($x = 0.753$, $y = 0.247$); the violet primary is placed at the titranopic intersection point ($x = 0.18$, $y = 0.00$); and the green primary is placed at the deuteranopic intersection point ($x = 1.00$, $y = 0.00$) implied by the dominator-modulator theory. By this theory the W_d curve represents for normal color vision both the luminous-efficiency function and the red response function; for deuteranopic vision it is both the luminous efficiency function and the "warm" response function. Protanopic vision does not have this function. Similarly, the W_p curve represents for protanopic vision both the luminous-efficiency function and the "warm" response function. Finally, the K curve represents the response function for the violet modulator in normal vision and for the "cold" modulator for protanopic and deuteranopic vision. Tritanopic vision does not have this function.

the cones giving green signals contain, in addition to middle-wave-absorbing pigment, slight amounts of the other two pigments, and the cones giving red signals contain about equal amounts of middle-wave- and long-wave-absorbing pigments, with perhaps a slight admixture of short-wave-absorbing pigment. This interpretation is based on the idea that the cause of dichromatic vision is a "loss" of one of the three components of normal vision. Another well-explored interpretation of the relation of dichromatic vision to normal trichromatic vision is that proposed by Leber (1869) and Fick (1879). They said that deuteranopia, for example, might be explained by assuming that both the cones signaling red and those signaling green alike contain the same admixture of middle-wave- and long-wave-absorbing pigments. This idea leads to the same cone-response curve as the "loss" hypothesis, but has the advantage that by this view the dichromat could never experience red separate from green, but would always see what unilateral dichromats testify they see: either blue or yellow (Judd, 1948).

The set of response functions in Figure 5 accords with the particular variety of three-components theory tentatively championed by König following his study of tritanopia in 1897. For example, v. Kries in a note specially prepared in 1924 for insertion in the English edition of Helmholtz' *Physiological Optics*, states (p. 412): "König long ago called attention incidentally to the curious fact that the luminosity values of the colours . . . turn out to be very nearly the same function of wave-length for deuteranopes and persons with normal vision. Since in the case of deuteranopes light of long wave-lengths has no effect except on the red component, we must suppose also that this action depends on the wave-length in the same way. But then the further result is that the distribution of luminosity is not affected, or at least almost inappreciably affected, by the addition of the green component by which the deuteranopic visual organ is converted into a normal organ; in other words, that in the case of normal vision also the luminosity goes practically hand in hand with the action of the red component." König was, I believe, the first among three-components theorists to suggest the possibility that the production of the brightness sensation might be entrusted to a single one of the three receptor mechanisms, with the other two mechanisms having the function of modulating this response to produce the chromatic aspects of vision. After Granit's work on retinas of various animals with the microelectrode indicated (1943) that these retinas contained a preponderance of brightness sensors, such theories of color vision came to be known as dominator-modulator theories.

To summarize the 1960 status of three-components theories, there was one for the photosensitive pigments of the retina, and there were several cone-response theories, including dominator-modulator theories, depending on the relationship assumed between deuteranopic and normal vision. All of these theories assume that yellow is produced by the sum of red and green responses and that white is produced by the sum of equal amounts of red, green, and violet responses; but none of them so far discussed here provide any explanation for these assumptions. The Ladd-Franklin theory (1892) is a skillful attempt to formulate a three-components theory overcoming this defect. Its outstanding weaknesses are its failure to account for tritanopia and its implication that the blue process contributes to luminosity for protanopic vision, but subtracts from it for normal and deuteranopic vision.

Opponent-colors theory: This theory was proposed and explained in detail by Ewald Hering in 1878. It is based on an analysis of sensations of color rather than of the stimuli required to evoke them. It assumes that there are six independent unitary colors (red, yellow, green, blue, white, and black), no one of which partakes of any other; that is, for example, yellow is a basic color in its own right, not the product of combining red with green. The Hering theory tacitly assumes that light is absorbed in the receptors by photopigments, that this absorption starts activity in the rest of the visual system, and that this activity is directly responsible for the colors we see. This activity is not found in six separate systems, but in three opposing pairs of processes: black-white, yellow-blue, and red-green. Black and white blend to produce gray, but equal amounts of yellow and blue, and of red and green cancel to zero.

This formulation of human vision supplies a specific model for the colors perceived, which the three-components theory does not. It has been very fruitful in suggesting many researches in such aspects of vision as chromatic adaptation, the influence of the surround on the perceived color, Bezold-Brüche phenomenon, and dependence of color perception on luminance of test field and surround; and it has provided a framework within which the results of such studies can be clearly stated and analyzed.

Just as is true for the various three-components theories, so also do color-matching functions provide quantitative information regarding the dependence of the opponent-color processes on wavelength. It is necessary only to express the color-matching functions relative to the primaries implied by the special assumptions of the particular form of opponent-colors theory under consideration. This quantitative information will be presented under stage theories of vision.

Stage theories of human vision: It has already been pointed out that there are three-components theories for two stages in the visual process: the retinal photopigment stage and the cone-response stage. It was suggested by v. Kries (1905) that the Young-Helmholtz three-components theory may be taken as valid at the receptor level but that the signals from the receptors are so processed that at some later stage the opponent-colors theory of Hering applies. Schrödinger (1925) derived the response curves for the two stages implied by the v. Kries proposal.

A somewhat similar two-stage theory has been used by Hurvich and Jameson (1951 on) in their comprehensive and detailed studies of color vision. The opponent-colors theory has probably received at their hands its most successful quantification and application.

There has been considerable uncertainty as to precisely at what stage of the visual process the signals from the receptors can be said to be organized in opponent colors, whether this be in the retina, the optic nerve, or the occipital lobe of the cortex. Adherents of the opponent-colors theory have been greatly encouraged by the work of Svaetichin and MacNichol (1953–1961) on shallow-water fish with the microelectrode technique. They obtained two types of response curves from the bipolar-cell layer of the fish retina. Both types of curves changed polarity depending on the wavelength of the light introduced into the retina. One corresponded in spectral location and shape to a yellow-blue process of an opponent-colors theory, the other to a red-green process. They suggested that the *Y-B* and *R-G* spectral response curves are expressions of signals delivered by *Y-B* and *R-G* cones, the chromoreceptors which form the basic mechanism for the color vision of this fish. This result for fish vision suggests that the signals in human vision might be organized according to opponent colors in the retinal stage immediately following the receptors themselves.

Müller three-stage theory: This first three-stage theory was developed by G. E. Müller between 1920 and 1930. The three stages are: photopigment stage, cone-response stage, and optic-nerve stage. The photopigment stage follows the three-components theory based on the Young primaries (see Fig. 6, upper-left quadrant). The cone-response stage follows an opponent-colors form with yellowish-red opposing bluish-green, and greenish-yellow opposing reddish-blue; see upper-right quadrant of Figure 6. These response curves (Judd, 1949) derived from the CIE color-matching functions are strikingly similar to those found by Svaetichin and MacNichol from the retinas of shallow-water fish. The optic-nerve stage is the opponent-colors formulation of Hering with red opposing green, and blue opposing yellow (see lower-right quadrant). This is close to the formulation used in the second stage by Schrödinger and by Hurvich and Jameson. The details of Müller's ingenious explanation of all three principal forms of dichromatic vision in terms of these three stages need not be explored here. Some hint of them, however, is shown in the lower-left quadrant. Müller assumed that the photopigment stage contributes directly to luminosity in such proportions as yields closely the protanopic luminosity function (see W_P); then for normal vision the *yR-bG* process makes another contribution of such size as to make up the difference between the normal and protanopic luminosity functions. Protanopia is explained by the loss of the

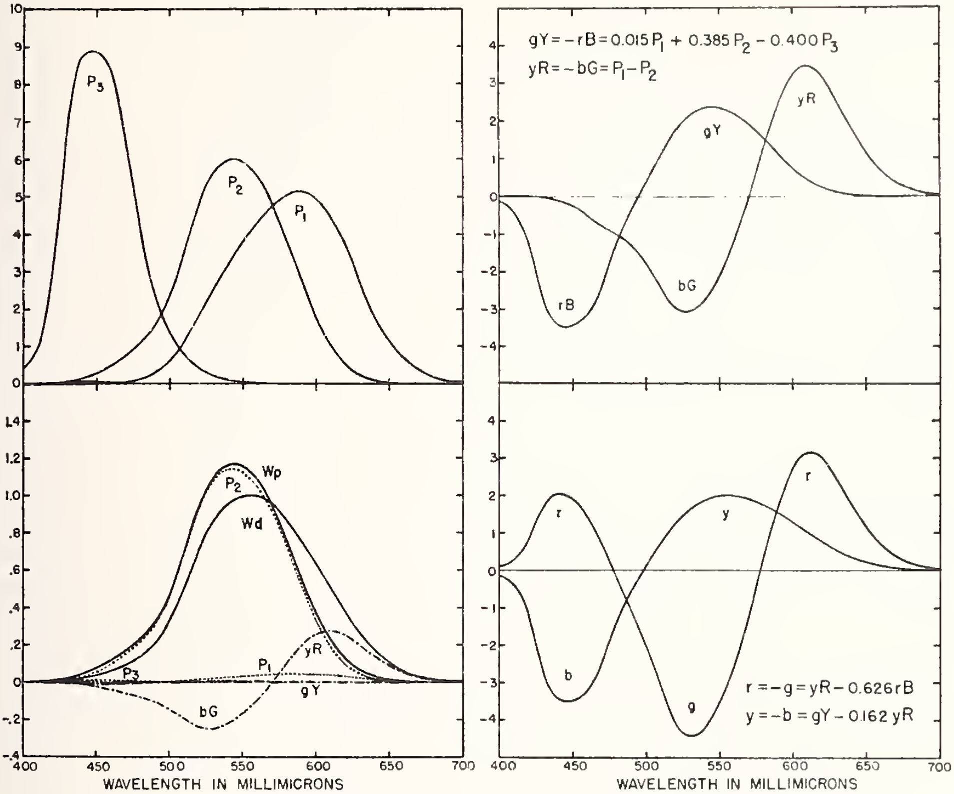

FIG. 6.—Response functions of the Müller stage theory evaluated by means of the 1931 CIE standard observer. *Upper-left* quadrant shows the response functions at the photopigment stage (same as Fig. 2, *heavy lines*). *Upper-right* quadrant shows the opponent-color response functions at the receptor stage. *Lower-right* quadrant shows the opponent-color response functions at the optic-nerve stage. These optic-nerve response functions also correspond to the Hering opponent-colors theory, and to the optic-nerve stage of the Adams theory. The *lower-left* quadrant shows how the Müller theory explains the luminous efficiency W_d function as the sum of contributions from the P_1, P_2, P_3 processes of the photopigment stage and from the yR and gY processes of the receptor stage. By this theory protanopia is a defect of the receptor stage whereby the yR ($= -bG$) process is lost. This loss explains both the inability of the protanopic observer to distinguish red from green, and the shortening of the luminous-efficiency function at the long-wave end whereby the W_d curve is supplanted by the W_p curve. Deuteranopia, on the other hand, is explained as a defect in the optic-nerve stage whereby the r ($= -g$) process is lost. This loss explains the inability of the deuteranopic observer to distinguish red from green, but it implies that the luminous-efficiency function of the deuteranopic observer must be identical to that for the normal observer.

yR-bG process, and it is clear that such a loss, in addition to accounting for inability of the protanope to make these red-green chromatic discriminations, also endows him with a luminosity function peaking correctly at about 540 nm.

Adams three-stage theory: Elliot Q. Adams proposed a three-stage theory of human color vision in 1923, and applied it in 1942 rather successfully to prediction of the perceptual size of color differences. The first and last stages correspond to the Young primaries and to the Hering opponent colors, respectively, just as in the Müller theory. The Adams second stage is a three-components formulation and, surprisingly enough, corresponds precisely to the primaries of the CIE system. Figure 7 shows the CIE color-matching functions in their official form. Adams takes these without change to be the cone-response functions. The central function, which is the normal luminosity function, refers to brightness-perceiving cones, and a portion of their signals is supposed to be transmitted as nonlinear transforms (Munsell value function from Newhall, 1943) of luminance directly to the optic nerve as white signals. The similarly nonlinear signals from the red cones are partially inhibited by another portion of the white signal, and the difference is transmitted to the optic nerve in the form of the Hering red-green response. Similarly, the nonlinear signals from the blue cones are inhibited by the remainder of the signals from the white cones, and the difference is transmitted to the optic nerve in the form of the Hering blue-yellow response. The outstanding weakness of the Adams three-stage theory is that it provides no built-in explanation of two of the three forms of dichromatic vision. It is noteworthy, however, for two reasons. In the first place, the second and third stages are strictly dominator-modulator in type. The Adams theory antedates the coining of the phrase "dominator-modulator" by 20 years. In the second place,it is the first theory of color vision to take explicit account of the possibility that the frequency of firing of the nerve leading from the cones may not be linear with the rate at which radiant flux is absorbed within the cone, but is likely to be a diminishing function of it, such as the logarithm. This ends the summary of the principal theories of human color vision, and the demonstration that color-matching functions are basic to their quantification.

Chromatic Adaptation.—Another method of studying human color vision, less fundamental perhaps, than the information to be derived from setting of color matches, is to submit the eye to pre-exposure of various known kinds, and to study the differences in response so caused. It has been known since studies by v. Kries and König (about 1890) that such pre-exposures of whatever wavelength composition and over a wide range of retinal illuminances [up to 10,000 trolands according to Wright (1936)] have no influence on color matches set up for foveal vision. This "persistence of optical matches," as v. Kries phrased it, means that the color-matching functions whose determination has already been summarized apply to a wide range of chromatic adaptations. All of the theories mentioned above provide for this fact.

Helmholtz (1860) early indicated the connection conceived by him between chromatic adaptation and the three independent mechanisms of Young's theory by saying (p. 235): "... the nervous substance in question is less sensitive to new reacting light falling on it than the rest of the retina that was not previously stimulated... Thus [p. 240] an eye which has been acted on by yellow light, say, is thereafter in a condition in which the blue components of white light affect it more than yellow does. Accordingly, the effect of fatiguing the retina is not uniformly extended to every kind of stimulation, but chiefly to stimulation similar to the primary stimulation. This fact is explained very simply by Young's assumption of three different kinds of sensory nerves for the different colours. For since coloured light does not stimulate these three kinds of nerves all to the same extent, different degrees of fatigue must also be the result of different degrees of stimulation. When the eye has been exposed to red, then the red-sensitive nerves are strongly stimulated and much fatigued; whereas the green-sensitive and violet-sensitive nerves are feebly stimulated and not much fatigued. If afterwards white light falls on the eye, the green-sensitive and violet-sensitive nerves will be relatively more af-

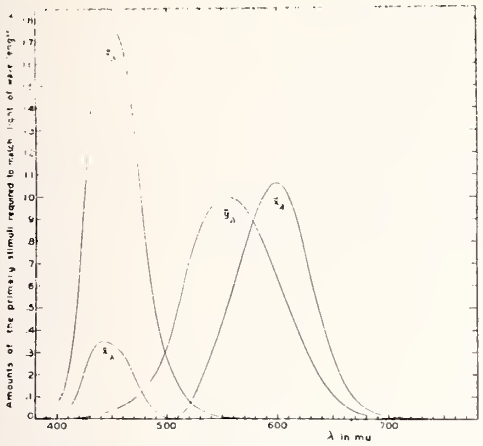

FIG. 7.—Color-matching functions for the 1931 CIE standard observer expressed in the CIE colorimetric coordinate system. The primaries are red ($\bar{x}_\lambda$), green ($\bar{y}_\lambda$), and blue ($\bar{z}_\lambda$), all outside the spectrum. The $\bar{y}_\lambda$-curve is identical to the photopic luminous-efficiency function for normal color vision, and this choice of $\bar{y}_\lambda$-curve ensures that the $\bar{x}_\lambda$- and $\bar{z}_\lambda$-curves are unassociated with luminosity. These color-matching functions apply to the receptor stage of the Adams theory (1923, 1942) which for this stage has the dominator-modulator form.

fected by it than the red-sensitive nerves; and hence the impression of blue-green, which is complementary to red, will predominate in the sensation." This explanation of the qualitative changes caused by local chromatic adaptation is quite successful, and equally good predictions are possible by the opponent-colors theory.

The predictions can, however, be subjected to a quantitative text. Suppose one retinal region (one eye, say) be maintained in a state of adaptation to daylight, and another retinal region (the other eye, say) be pre-exposed to green light. All stimuli presented to the green-adapted portion of the retina will appear more purplish than when presented to the daylight-adapted eye, and a measure of this adaptive color shift may be obtained by allowing the observer to alter the light stimulating the daylight-adapted region until he finds a stimulus yielding a color preception equal to that from the green-adapted region. Helmholtz' ideas on chromatic adaptation were quantitatively formulated by v. Kries, as follows: If R, G, V specify the color of any stimulus for neutral adaptation, then for any chromatic adaptation, the resulting color perception of this stimulus will be similarly specified by R', G', V', where

$$R' = S_r R, \qquad G' = S_g G, \qquad V' = S_v V,$$

where S_r, S_g, S_v are coefficients specifying the state of chromatic adaptation of the eye following the pre-exposure. This formulation is known as the v. Kries coefficient law (v. Kries, 1905; p. 211).

The v. Kries coefficient law will work to some degree or other even if the coordinate system within which R, G, V are evaluated departs rather widely from that based on the true cone responses, but it should work perfectly only for the true cone responses. Studies of chromatic adaptation can therefore be used to check the accuracy of response curves derived by some other means, such as by reference to the protanopic, deuteranopic, and tritanopic intersection points. Studies by Wright (1934), Walters (1942), MacAdam (1956), Brewer (1954), Wassef (1955), and Burnham, Evans, and Newhall (1957) have indicated that the v. Kries coefficient law is at least a good first approximation to the facts, but the primaries yielding the closest approximation to correct predictions do not agree with the dichromatic intersection points. The P- and T-points are found in some studies to be reasonably successful, but the best third primary is extremely variable. For some studies (MacAdam, 1956) it falls within the range of D-points, but for others, far from that range. MacAdam (1958) has suggested a nonlinear extension of the v. Kries law for improved agreement.

A more rewarding line of attack is that by Stiles (1949, 1959) who determined for spectrum colors the least amount required to be added to the center of a large field of another spectrum color to be just detectable. The large field, of course, controls the chromatic adaptation of the eye of the observer, and may render two of the receptor mechanisms relatively insensitive, in which case the thresholds found should be inversely related to the spectral sensitivity of the third mechanism. If the wavelength of the large field is such as to reduce chiefly the sensitivity of but one of the receptor mechanisms, then the threshold for a given test stimulus will still be related chiefly to the sensitivity of the most sensitive of the remaining two. Only if the sensitivities of the two mechanisms are nearly equal does the interpretation of the increment-threshold data depend on an assumption regarding the combination of the responses of two mechanisms to produce an increment threshold.

The result of Stiles' researches by the two-color increment-threshold method is in one sense a very gratifying check on the cone-response curves derivable from the color-matching functions by taking the D-point in agreement with the Yustova values: $x = 1.7, y = -0.7$. One cone mechanism was found to peak at 440 nm, another at 540 nm, and a third at 575 nm. The shape of the "red" cone response curve found by this method does not, however, agree too well with that derived from color-matching functions. Its maximum is considerably more flat. In still another sense the results are puzzling. Not one but three "blue" mechanisms were found with response curves peaking at 440 nm. Furthermore, the response curves for the "green" and "red" cones changed significantly depending on whether the large field provided high or low retinal illuminances. The response for the "green" cone shifted at high illuminances toward somewhat shorter wavelengths, and that for the "red" cone toward longer, so that the maximum was pushed over to 587 nm, that is nearly in agreement with the "red" curve of the photopigment level. Perhaps this means that the photopigment that does not belong in the cone is less stable than the one that does. Evidence both from studies of the v. Kries coefficient law and from the two-color increment-threshold method suggests that chromatic adaptation may be a somewhat more complicated phenomenon than can be adequately described by any simple form of three-components theory.

Summary.—Between 1860 and 1960 several alternative theoretical possibilities were carefully worked out for human color vision but progress has been handicapped by lack of crucial evidence that might decide between these possibilities.

The color-matching functions determined since Maxwell's time (1860) have been reduced to the primaries previously (1807) proposed by Young, and the modern determinations are shown to be in good agreement. The principal theories of color vision are summarized under the headings: three-components (Young, Helmholz, König), opponent-colors (Hering), two-stage theories (v. Kries-Schrödinger and Hurvich-Jameson), and three-stage theories (Müller, Adams). The results of quantifying these theories by appeal to modern determinations of color-matching functions is shown. For example, by the Young theory predictions of the spectral absorptance of three retinal photopigments are derived, and from the Helmholtz-

König theories, the responses of three types of retinal receptors responsible for color vision are derived. The various experimentally determined relations between the three principal forms of partial color-blindness and normal color vision are intercompared and checked against the various theories, and a brief review of the results of studies of chromatic adaptation (v. Kries coefficient law; Stiles two-color increment thresholds) and their theoretical implications is given.

Key words: Chromatic adaptation, color blindness, color-matching functions, color vision, deuteranopia, opponent-colors theory, protanopia, three-components theory of color vision, tritanopia.

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PHYSIOLOGICAL OPTICS AT THE NATIONAL BUREAU OF STANDARDS

Appl. Opt. 6, 13-26 (1967)

The terms "elementary color excitations", "excitation curves", or simply "excitations" used on pp. 17 and 18, in connection with the 1922 report of the Committee on Colorimetry of the Optical Society of America, are equivalent in meaning to the terms "color-matching functions", "color-mixture functions", or "tristimulus weighting functions" which are commonly used in connection with CIE colorimetric data. Similarly, the term "excitation values" has been replaced by "tristimulus values". The account, on p. 18, of why Priest abandoned his efforts to revise the OSA excitation data and supported the British proposal that was based on the determinations by Wright and Guild, which led to adoption of the CIE 1931 standard-observer data for colorimetry, is an interesting and otherwise-unparalleled revelation.

Erratum: P. 22, first column, line 51, change "eight" to "eighty".

Physiological Optics at the National Bureau of Standards

Deane B. Judd

Published work in physiological optics at the National Bureau of Standards is summarized under the headings: evaluation of light-dark patterns, light measurements, color measurement, color differences, and color perception. The bearing of this work on standard methods of the American Society of Testing and Materials, of the Illuminating Engineering Society, and of the American Standards Association, and on recommendations of the International Commission on Illumination is indicated. The purposes and methods of current work are briefly described.

Introduction

In its development of instruments, methods, and standards for measurements made by the human eye, the National Bureau of Standards has obtained information on the response of that physiological mechanism to stimulation by radiant energy. Sometimes this information is specifically sought, but more often it is incidental to another purpose. This paper shows how the need for this information has arisen, recalls some of the most important studies of physiological optics carried out at the National Bureau of Standards, indicates the applications made of their results, and explains briefly the purposes of the current researches. It is convenient to discuss physiological optics under the headings: evaluation of light-dark patterns, light measurement, color measurement, color differences, and color perception.

Evaluation of Light-Dark Patterns

In 1938 Raymond Davis and Milo A. Durand sought a relationship between the legibility of a microcopy of typed material and the resolving power of the microcopying system. By experimenting with various styles of types they found that if the height of the lower case *e* on the film subtended three cycles of the highest spatial frequency resolved, the copy would be legible with difficulty. If the *e* subtended five cycles, the copy was legible without difficulty although serifs and fine details of type were not clearly defined. If the *e* subtended eight or more cycles, the details of the type were clearly defined. Davis included this correlation in the instructions for use of the National Bureau of Standards Microcopy Resolution Test Chart. If one defines a quality index, these instructions may be restated as $m_r e = qR$, where m_r is the resolving power in cycles/mm on the film, e is the height of the lower case *e* on the original in millimeters, q is a quality index having values 3, 5, and greater than 8 as described above, and R is the reduction.[1] McCamy[1] recently succeeded in making a barely legible microcopy with a total reduction of 1200, enough reduction to permit the entire King James version of the Old and New Testaments to be recorded on an area considerably less than a thumbnail. The quality index q of this microcopy was found to be 2.34, in good agreement with Davis' legibility relation. By taking account of the fact that the human observer can average the density along a long band so as to perceive the band even though it may be somewhat broken, McCamy[1] showed that the information capacity of a photographic film in bits/mm² is equal to the square of the resolving power in cycles/mm: $C = m_r^2$. This formula was shown to yield results on four Kodak films in good agreement with information capacity computed by R. Clark Jones. It also indicates the information capacity of the film used (m_r = 1800 cycles/mm) to be 3.24×10^6 bits/mm². McCamy showed on the basis of an analysis of the information characteristics of literal test that the actual information recorded on this film with a reduction of 1200 was 79% of the information capacity computed from this formula.

These studies have served to evaluate the considerable ability of the human visual mechanism to abstract from a pattern afflicted with considerable noise the regularities (lines, letters) therein. These noisy patterns arise from the urge to put the information to be recorded into as small an area of microfilm as possible. A much more widespread use of photography, however, is to produce pictures of as high quality as possible. These pictures are intended to be viewed by the human eye, and the quality of them is appraised by the human eye by little-known processes. The picture containing

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the most information about a given scene is not necessarily judged to be the highest in quality. Judgments of quality are used to set up measures of the speed of photographic materials and to indicate optimum exposures. The National Bureau of Standards has studied methods of estimating exposures for daylight and moonlight photography without the use of an exposure meter. These studies have been carried out in cooperation with ASA Committee PH2.7, and the results have been embodied in the American Standard Photographic Exposure Guide (ASA PH2.7-1966, 7 April 1966). The optimum exposure is found from the speed of the film and the luminance value B_v of the scene which, in turn, is found as the illumination value V_v (derived from latitude, longitude, and apparent solar time) diminished by the scene value X_v which is determined by visual appraisal by the photographer of the scene (distant, nearby, close-up, shaded, unshaded, clear sky, hazy sky, front lighted, side lighted, and so forth). The method works. Is this an important achievement of physiological optics? The author thinks it is. We turn now to visual responses to a simple pattern: two fine dark lines viewed against a light background. The observer must move one line until it is coincident with the other. Washer[2] observed this pattern to determine the effect of magnification on the precision of indoor telescope pointing because it was needed in connection with the design of range finders. He found that the probable error of a single setting for coincidence expressed in seconds of arc (PE_s) was $PE_s = 4.962/M + 0.068$. The constant 4.96 sec is the Vernier acuity of the observer's eye, and this value has corroborated those cited by Walls.

In 1937, Hunter[3] classified glossiness as having five types: (1) specular, (2) sheen, (3) contrast, (4) absence of bloom, and (5) distinctness of reflected image. He showed that suitable goniophotometric correlates of these five types are (1) specular gloss at 60°, (2) specular gloss at grazing incidence, (3) ratio of specular gloss at 45° to nonspecular reflectance factor, (4) ratio of somewhat off-specular reflectance factor to specular gloss, and (5) the same ratio for very slightly off-specular viewing. Hunter and Judd[4] proposed a method for classifying paints according to specular gloss for adoption by ASTM. This proposal was accepted for tentative use, and the current ASTM standard D523-62T is essentially the same. Nimeroff[5] in 1952 showed that the kurtosis index of the goniophotometric curve of polished and sand-blasted glass correlated better with subjective estimates of glossiness than the conventional correlate: 60° specular gloss; and in 1957[6] he showed that judgments of distinctness-of-image glossiness could be predicted better from the ratio of the specular gloss measured with a small aperture to that measured with a large aperture than from any single-parameter gloss method. The two parameter method proposed by Nimeroff has been incorporated into ASTM method D1471-57T.

In 1938, Neeland *et al.*[7] reported on the conspicuousness of airway beacons that are rotated to supply a series of flashes visible to pilots. As a measure of this conspicuousness, they had the observer vary the intensity of a fixed light until it was judged just as conspicuous as the flash of light from the beacon. They showed that the ratio of the fixed-light equivalent intensity I_e of a rotating beacon to its maximum intensity I increases with increasing period of rotation in accord with the Blondel-Rey formula: $I_e/I = t/(a + t)$, where t is duration of an abrupt flash in seconds, and a is a constant equal to 0.2 sec for near-threshold values of I and decreases for flashes considerably above threshold. Neeland *et al.* evaluated I as the maximum intensity of the beacon, and took the duration as the time interval between the rise of intensity to 10% of maximum and the decline to 10% of maximum. They obtained the anomalous result that observation of the beacon at short range sometimes gave lower effective intensities than observation at long range where the maximum intensity was near threshold. The equivalent intensity I_e per unit luminous energy received by the eye from the flash was found to be a maximum for the shortest durations tried (0.001 sec); this suggests that the initial portion of a flash well above threshold desensitizes the retina so the later portions are less effective. In 1957, Douglas[8] showed that an expression of the form originally proposed by Blondel and Rey for the effective intensity of a flashing light, whether or not the flashes are abrupt:

$$I_e = \int_{t_1}^{t_2} I dt/(a + t_2 - t_1)$$

has a maximum value when t_1 and t_2 are taken as the times at which I reaches the absolute threshold value. This definition of the duration of a nonabrupt flash had also been suggested by Blondel and Rey. For some intensity time patterns, an increase in I uniformly without changing the time pattern may result in a decrease in I_e, as found by Neeland *et al.* for rotating beacons, despite the lower values of a corresponding to intensities far above threshold.

In 1957, Projector[9] pointed out that for t less than $a/100$, the Blondel-Rey formula implies, for a given value of effective intensity I_e, a reciprocal relation between I and t in a manner analogous to the equations used for determining photographic exposure. In 1959 Mullis and Projector[10] proposed to define the effective efficiency of a flashing light as the product of a Blondel-Rey Merit Factor by the ratio of the luminous energy in the flash to the electrical energy producing the flash. The Blondel-Rey Merit Factor is defined as five times the ratio of the effective flux in the flash to the effective flux of a flash with the same total luminous energy compressed into an extremely short time interval. By this definition, the effective efficiency is the ratio of the effective flux—computed by the Douglas method with a set equal to 0.2 sec—to the electrical energy producing the flash. Effective intensity by the Douglas method is currently used for the appraisal of all flashing signal lights; see IES *Guide for Calculating the Effective Intensity of Flashing Signal Lights.* Effective efficiency by the Projector method is often used in addition to effective intensity, particularly for appraisal of battery-powered blinking lights.

Light Measurement

Light is the aspect of radiant energy of which a human observer is aware through the stimulation of the retina of the eye. The measurement of light was accomplished by visual photometry at the National Bureau of Standards for many years without determining its precise relation to radiant energy. In 1906, Hyde[11] showed that Talbot's law served to predict within the experimental uncertainties the photometric effect of a rotary sector disk, both for incandescent lamp light and for such light viewed through red, green, and blue filters. Helmholtz stated Talbot's law as follows: "If any part of the retina is excited with intermittent light, recurring periodically and regularly in the same way, and if the period is sufficiently short, a continuous impression will result which is the same as that which would result if the total light received in each period were uniformly distributed throughout the whole period". By applying Talbot's law and the inverse square law, the determination of the intensity of an unknown source by visual comparison to a standard source of the same spectral quality was conveniently possible. Nevertheless, in 1908 Nutting[12] had already raised the question of the relation of light to radiant energy by determining roughly the maximum luminous efficacy of radiant energy. The value obtained (170 lumens/W) was of the right order of magnitude but still rather far from the presently accepted value (680 lumens/W). Nutting used luminous efficiency functions determined by König in Germany in the 1890's for both photopic and scotopic conditions. He pointed out that, because of the Purkinje effect, light cannot be proportional to radiant energy for all ranges, and stated that the logical primary standard of light is one in which the measured quantity is radiant energy measured by some form of radiometer. This suggestion has remained dormant for fifty years, but is now showing some signs of life.

If the test lamp had a spectral distribution different from that of the standard, it usually had a different chromaticity also, and settings for a brightness match not only became much less certain but also strongly dependent on what observer was doing it. The National Bureau of Standards found itself under pressure to supply light standards of a number of chromaticities so that the knotty problem of heterochromatic photometry need not be dealt with by every photometrist. These chromaticities, expressed in terms of color temperature, included 2080°K (carbon filament lamps), 2360°K (acetylene flame, vacuum tungsten lamps), and later 2788°K (gas-filled tungsten filament lamps). The unit of luminous intensity then used (international candle) was embodied in a group of carbon filament lamps, and to calibrate a vacuum tungsten lamp in terms of candles a visual setting for equality of brightness had to be made despite the chromaticity difference. Three techniques were used to improve the reliability of the setting: (1) the step-by-step or cascade method in which the chromaticity difference was broken down into a number of small steps, each one which could be taken with reduced uncertainty; (2) the flicker method in which the lights to be compared were presented alternately at a few cycles per second and the observer adjusted the fields for a minimum of flicker; and (3) the filter method in which the chromaticity difference was reduced nearly to zero by the introduction of an appropriate filter whose luminous transmittance had to be separately determined. Thus we find Priest[13] in 1915 proposing a method of heterochromatic photometry in which the filter is produced by a quartz plate between two polarizers, one fixed and the other adjustable, the luminous transmittance of the filter to be computed by means of a "visibility function based on the average of a large number of determinations of visibility, officially adopted as standard". And in 1916 Middlekauf and Skogland[14] gave the results of an interlaboratory photometric comparison of glass screens and of tungsten lamps involving color difference, showing that each observer maintained a fairly definite relation to the mean of the observers in his laboratory so that significant and large individual observer differences cast doubt on the reliability of the average. Crittenden and Richtmyer[15] measured by equality-of-brightness and flicker photometry the ratio of luminous transmittances of a potassium dichromatic solution and a copper sulfate solution (Y/B ratio) for 115 observers. The ratios ranged from 0.83 to 1.19. They proposed to rely for heterochromatic photometry on the flicker settings of a small number of observers of known Y/B ratio whose settings were to be corrected to give normal values, that is, values characteristic of $Y/B = 1$.

A frontal attack on the basic problem of the relation between light and radiant energy was launched in 1917 by Coblentz and Emerson[16] who determined by flicker photometry the spectral luminous efficiencies of 130 observers, seven of whom were known to be color-blind, and checked the results by equality-of-brightness settings; these agreed for 588 nm but with a wide spread, and gave higher values (by 5% to 500%) in the extremes of the visible spectrum. They proposed a modification of the liquid filter designed by Ives and Kingsbury to duplicate better the average luminous efficiency function found by them, and reported the results of tests of a physical photometer produced by combining this filter with a thermopile. They also gave a formula closely approximating their values of spectral luminous efficiency as a function of wavelength, and in a separate publication[17] used this formula to compute the lumininous efficacy of the blackbody radiator as a function of temperature. The maximum efficacy was found at about 6500°K which gave about 85 lumens/W. They also used this formula in connection with determinations of the maximum luminous efficacy which they found to be about 620 lumens/W.

In 1923, Gibson and Tyndall[18] published the results of what is presently the definitive study of the relation between light and radiant energy. They determined the spectral luminous efficiency functions for fifty-two observers by the cascade method. The spectrum was traversed in twenty-three steps, the field size was 3.5°, and the field luminance varied from 53 millilamberts at 580 nm to about one-twentieth of that luminance at 430 nm and 700 nm. In addition, they made a detailed comparison of the luminous efficiency functions of the eighteen observers studied by them who had previously served as observers for the Coblentz–Emerson study by the flicker method, and they analyzed the differences among the averages of seven separate determinations of spectral luminous efficiency (the two NBS determinations and five others). On the basis of this analysis, values of spectral luminous efficiency were derived to be recommended as representative of normal photopic human vision. They adjusted the recommended values to yield precisely the wavelength center of gravity for the blackbody at 2077°K found by Ives (581.6 nm) from a large number of observers in connection with the development of his physical photometer solution, but warned that the values recommended for the shortwave (violet) extreme are uncertain and must be considered as tentative. In 1924 the Commission Internationale de l'Eclairage (CIE) adopted the Gibson–Tyndall recommended average for tentative use.

In 1924, Tyndall and Gibson[19] showed that this function could be closely approximated by a formula consisting of four exponential terms; and in 1931 Judd[20] extended the recommended values of the function to intervals of 1 nm by osculatory interpolation. In 1931, the CIE made this function one of the three defining the standard observer for colorimetry, and in 1939 the CIE made it the unique fundamental basis for all photometric measurements in the photopic range. It was subsequently adopted by the General Conference on Weights and Measures and has been embodied in the legal standard of light in many countries including the United States.

Luminous flux Φ_v in lumens of a source of spectral concentration of radiant flux $(\Phi_e)_\lambda$ in watts per unit wavelength is

$$\Phi_v = 680 \int_0^\infty (\Phi_e)_\lambda V(\lambda) d\lambda, \qquad (1)$$

where $V(\lambda)$ is the spectral luminous efficiency function recommended in 1923 by Gibson and Tyndall.

In 1938, Gibson *et al.*[21] constructed a combination glass–liquid filter whose spectral transmittances were closely proportional to the standard spectral luminous efficiencies; in 1939 they[22] developed a more reproducible filter satisfying the same condition; and in 1941 Teele[23] constructed a physical photometer by combining this filter with a thermopile and specially designed potentiometer to measure the small values of electromotive force generated by the thermopile. This photometer is in current use at the National Bureau of Standards.

In 1945, Teele[24] constructed a visual photometer for luminescent materials. The luminances of these materials correspond to scotopic and mescopic vision and so cannot be evaluated properly by foveal vision, which, for most observers, is rod-free. The field used is elliptical divided along the major axis which subtends 20°; the minor axis subtends 15°. The scotopic or photopic luminance of the material is taken as the photopic luminance of the standard half-field of color temperature 2360°K found by the observer to appear as bright as the material viewed in the other half-field. Although chromatic differences are so small that they cause little trouble, the between-observer spread was found to vary from about 5% at 10 effective microlamberts to 25% at five thousandths of an effective microlambert.

In 1959, Barbrow[25] showed how to use the standard phototopic luminous efficiency function combined with the maximum value (680 lumens/W) of luminous efficacy to derive, from measurements of color temperature and luminous intensity (lumens per steradian) of an incandescent lamp, the spectral concentration of radiant intensity of the lamp in microwatts per steradian in each band of 10 nm of the spectrum from 400 nm to 700 nm.

Color Measurement

Color is the aspect of light by means of which an observer may distinguish differences between two fields of view of the same size, shape, and structure. Color differences usually correspond to differences in spectral composition of the stimuli. Early in its history the National Bureau of Standards was called upon to recommend methods of measuring and specifying the colors of light sources and those of light reflected by or transmitted through objects by virtue of which the objects themselves may be said to have colors. Thus we find Nutting[26] in 1912 describing "a new precision colorimeter" to measure color in terms of wavelength of dominant or complementary hue, purity, and luminance, and Priest[27] in 1915 proposing to measure the color of cottonseed oils by reference to the same filter (quartz plate between two polarizers) of adjustable spectral transmittance proposed by him for heterochromatic photometry. In 1917, Priest and Peters[28] showed that the saturation of various tints of yellow butter and oleomargarine could be specified by the spectral reflectance ratio for 436 nm and 578 nm.

In 1918, Priest[29] produced a close approximation to average noon sunlight according to Abbot's determinations by combining an incandescent lamp with his quartz rotatory dispersion filter. This artificial sunlight became known as Abbot–Priest average noon sunlight. And in 1923 Lofton[30] specified the color characteristics of white papers by giving the luminous reflectance measured on the Pfund multiple-reflection colorimeter at 625 nm, 550 nm, and 460 nm.

In 1920, Priest[31] presented inductive evidence supporting the conclusion that if any two lights, however different in spectral composition, have equal color they have equal spectral centroids of light; and he also showed[32] that the frequencies of the spectrum colors yielding mixtures that match a standard white are related closely by the rectangular hyperbola: $(580 - f)(f_{comp} - 608) = 220$, where f and f_{comp} are expressed in cycles per trillionth of a second. In the same year Priest *et al.*[33] published a spectrophotometric examination of the Munsell color system by which object colors could be specified, and showed that one of the parameters of the Munsell system—Munsell value—is proportional to the square root of luminous reflectance.

None of the methods of specifying color dealt with in these early papers had general applicability. The Munsell system, for example, did not apply to light sources, and the charts in the Atlas of the Munsell Color system by no means covered the entire gamut of object colors. The quartz rotatory dispersion colorimeter had even greater limitations, and the Nutting colorimeter required standardization of the neutral stimulus used as a reference "white" in the determination of dominant wavelength and purity. The key to a color specification method of general applicability was given in 1922 by the report of the Optical Society of America Committee on Colorimetry written by Troland[34] but based on a preliminary unpublished report by Priest. This key is the set of three tristimulus functions of wavelength defining all color matches set up by normal color vision. The OSA report presented such a set under the name of "elementary color excitations". Troland[34] wrote (p. 579): "One of the main interests of the present Committee is to provide means by which color specifications in terms of different systems can be reduced to a common denominator and, so far as possible, be interconverted. Spectrophotometric data are potentially convertible into the data of any other system whatsoever, but no specifications which are based upon simple color-matching can be reduced to spectrophotometric terms, without additional information. However, a satisfactory common denominator for all systems is apparently provided by the *elementary color excitations*. Values of these excitations can be found which will specify completely the color characteristics of any stimulus, and each member or possible specification in every color system can be reduced to such excitation values, ... In this way the data of separate systems can be definitely intercompared, and can be interconverted in so far as the representations of the several systems overlap; with the obvious restriction that peculiarities of the stimulus—such as spectrophotometric details—which determine no characteristic excitation values, are necessarily lost." The OSA excitations were derived as an average of color-matching data by Abney (1891) in England, and König and Dieterici (1892) in Germany, obtained by matching each spectrum color in turn by appropriate mixtures of three lights chosen arbitrarily as primaries. The color-matching data were reduced to a common set of primaries (spectrum red, spectrum violet, and an extraspectral green, defined on the Maxwell triangle as the intersection of the tangents to the spectrum locus at these two extremes) by application of Grassmann's additivity law which states that equally appearing stimuli added to equally appearing stimuli produce equally appearing sums. The report gives excitation values for the equal energy spectrum adjusted so the areas under the three curves of the products of the excitations by the spectral distribution of Abbot–Priest noon sunlight are equal. The report also gives excitation values for the blackbody as a function of temperature, for the parameters (dominant wavelength and purity) of the Nutting colorimeter, for a selection of colors obtained by the quartz rotary dispersion colorimeter, and for certain of the colors in the Munsell Atlas.

In 1931, Judd[35] extended the OSA excitation curves to intervals of 1 nm by osculatory interpolation.

With great persistence and skill, Priest continued his studies of various methods of color specification. In 1921 he[36] showed by data of four observers of a 3.5° field at moderate brightness with dark surround that the color temperature of the light appearing neither yellowish nor bluish varies from 4600°K to 6500°K with an average of 5300°K. Use of such a stimulus in determinations of dominant wavelength and purity should yield optimum correlation with hue and saturation of the perceived color. In 1922, Priest[37] showed how to measure color temperature by the quartz rotatory dispersion colorimeter; and in 1923 he[38] showed how to measure luminance and spectral centroid of light sources by the same colorimeter. In 1924, he[39] assembled equipment to measure dominant wavelength, purity, and luminance, and embarked on a series of determinations of these quantities by a group of ten observers for spectrally known stimuli as a check on the applicability of the OSA excitations. Priest *et al.*[40] in 1924, Priest[41] in 1926, and Judd[42] in 1926 published formulas for the computation of purity from given tristimulus values. Gibson[43] in 1925 showed that the spectral centroid of the light transmitted by a daylight filter varies linearly with the thickness of the filter for a source of fixed color temperature. If the same filter is used for sources of different color temperature, the spectral centroids of incident and transmitted lights are linearly connected. Peters and Phelps[44] in 1927 found it possible to specify the color of sugar solutions in terms of the number of units of coloring matter in 1 g of saccharine dry substance computed as 206 times the logarithm of the reciprocal of the internal transmittance of the sample at the wavelength (560 nm) of the spectral centroid of the light transmitted by the sample illuminated by Abbot–Priest average noon sunlight. This method is still widely used.

Gibson and Harris[45] in 1927 published a spectrophotometric analysis of the glasses of the Lovibond color system. This system is based on three sets of glass filters (a red, a yellow, and a blue set), and each filter is assigned a number with the intention that, if light is passed through two glasses of the same set in turn, the color produced will equal that of a glass whose number is the sum of those identifying the two being combined, and further with the idea that glasses of equal numbers from each of the three sets will produce a neutral filter (one whose chromaticity is the same as that of the daylight source used in the calibrations). The Lovibond color system has been widely used in commerce for the colors of transparent liquids and solids (paint vehicles, petroleum products, vegetable oils, resins, chemical solutions, and so forth) because of the convenience of the glasses and because the Lovibond notation indicates clearly the approximate color perceived to belong to the specimen. Greens are produced by combinations of yellow and blue Lovibond glasses, purples by red and blue, and orange by red and yellow. The Gibson–Harris spectrophotometric results permitted computation, by means of the OSA excitation curves, of specifications of the color of any combination of Lovibond glasses. The results of computations of this sort were published in 1947 by Haupt and Douglas[46] for about 60 Lovibond red, yellow, and blue glasses, singly and in pairs. Graphs of the computed chromaticity points permitted one to read approximately the Lovibond designation corresponding to any chromaticity within the Lovibond gamut. These graphs also revealed clearly some irregularities in the calibration of the Lovibond glasses in the possession of the National Bureau of Standards. The Gibson–Harris spectrophotometric results, supplemented by additional measurements made in England, also permitted the derivation of the Lovibond red, yellow, and blue units in terms of the spectral internal transmittances of the corresponding glasses. Judd *et al.*[47] carried out this derivation in 1962 and published the chromaticity coordinates of enough one- and two-part combinations of Lovibond glasses illuminated by incandescent lamplight to permit accurate readings of the Lovibond notation for any point on the chromaticity diagram within the Lovibond gamut.

In 1934 Wensel *et al.*[48] established a scale of color temperature by setting up visual color matches between incandescent lamps and the blackbody at three fixed temperatures, the freezing points of platinum (2042°K), rhodium (2233°K), and iridium (2716°K). This scale replaced the Nela scale of color temperature used up to that time and is still being used for calibration of incandescent lamps by the National Bureau of Standards.

Perhaps it is clarifying to point out that the central difficulty grappled with in these early papers on color measurement and color systems is the physiological fact that the human eye does not differentiate between the colors of groups of lights whose spectral distributions are widely different. Pairs of such lights make what is called a metameric match, and the lights are said to be metamers. Observers differ in setting up such matches. A metameric pair for one observer is a bad mismatch for another observer even though both have normal color vision. Varying degrees of lens pigment and macular pigment can cause this. The OSA excitations were an early solution to this problem. In his checks of these excitations by means of measurements of dominant wavelength, purity, and luminance, Priest found as expected that none of the ten observers set metamers in precise accord with the OSA excitations, but the metamers predicted by computation often did not even fall among the experimental matches. Priest had not finished his attempts to revise the OSA excitations to correct these fatal discrepancies when the International Commission on Illumination (CIE) received from the Britsh National Committee a proposal for a revised set of color-matching functions based on determinations of spectral metamers by Wright at the University of London, and by Guild at the National Physical Laboratory. The metamers defined by the British proposal were found to be in considerably improved agreement with Priest's experimental determinations, and Priest decided to support them.

The CIE recommendation of 1931 for a standard observer and coordinate system for colorimetry, and for three standard sources—source A representative of incandescent lamplight, source B representative of noon sunlight, and source C representative of average daylight—has served the immensely important practical purpose of defining standard metamers in satisfactory accord with those set by observers of normal color vision. These recommendations are still in force. The National Bureau of Standards contributed an important feature of the coordinate system, and supplied the liquid filters that, combined with the gas-filled incandescent lamp at a color temperature of 2854°K (source A), were used to produce sources B and C. In 1930, Judd[49] had showed that, if a weighted sum of the color-matching functions is equal to the standard luminous efficiency function, this function could be substituted for any of the original color-matching functions without any change in the metamers specified by them. This idea was incorporated into the 1931 CIE coordinate system for colorimetry. In 1931, Davis and Gibson[50] had developed an extensive series of liquid filters to be used with incandescent lamps for the reproduction of close approximations to sunlight and daylight, and had showed how to use these filters in the determination of color temperature. CIE sources B and C made use of two of these Davis–Gibson filters. Judd[51] published in 1933 a description of the 1931 CIE standard observer and coordinate system for colorimetry, and gave tables facilitating the computation of dominant wavelength and purity from the CIE chromaticity coordinates (x, y).

The 1931 CIE coordinate system for colorimetry has since been applied at the National Bureau of Standards to solve other problems of interconversion of color specifications from one system to another. It was used by Haupt and Douglas[46] and by Judd *et al.*[47] to evaluate the colors of the Lovibond system as already mentioned. And it was used by Gibson and Nickerson[52] in 1940 and Kelly *et al.*[53] to evaluate the colors in the original Atlas of the Munsell System and those in the Munsell Book of Color, respectively. Newhall *et al.*[54] also used it in the Final Report of the OSA Committee on the Spacing of the Munsell Colors as the system within which smoothed and adjusted Munsell notations, known as Munsell renotations, were defined. These Munsell renotations are used by the Munsell Color Company for their current production of color standards.

They have been recognized in ASTM Standard D 1535-58T, and they form one of the alternate methods in American Standard Methods of Measuring and Specifying Color (ASA Z58.7.3-1951). In 1960, Reinboldt and Menard[55] developed a program whereby the Munsell renotations corresponding to any set of tristimulus values in the CIE system can be found by automatic digital computation.

Judd and Kelly[56] in 1939 defined the first ISCC-NBS Method of Designating Colors. This cooperative work of the Inter-Society Color Council and the National Bureau of Standards defined, in terms of the Munsell Book of Color, ranges of colors for each of 319 designations by simple color names. Kelly[57] extended the ISCC-NBS color designations to self-luminous sources viewed with a dark surround. All parts of the CIE (x, y)-chromaticity diagram receive one or another of twenty-two color names (red, reddish orange, orange, yellowish orange, yellow, and so on) except a rather large central area for which no hue name is appropriate. The human observer adapts quickly to lights of these chromaticities viewed with a dark surround and sees them either as without hue or nearly so. The great importance of chromatic adaptation may be seen from the fact that this central area for which no hue name is appropriate covers a majority of the colors in the Munsell book, many of which, viewed with middle gray to white surround, are perceived to have colors far from gray. Kelly and Judd[58] in 1955 published the revised definition of the ISCC-NBS method of color designations in terms of the Munsell renotations, and thus indirectly in terms of the 1931 CIE coordinate system for colorimetry. They also included a dictionary defining about 7,500 color names including 267 groups of synonyms and near synonyms. In 1958, Kelly[59] computed the Munsell renotations for the centroids of the color ranges for the 267 ISCC-NBS designations, and in 1965 the National Bureau of Standards made available charts showing, in the form of glossy paint films on paper, these centroid colors. In the same year, Kelly[60] showed how the centroid charts could be used in three levels for surveys of customer acceptance of color in manufactured goods, and how the ISCC-NBS designations fit into a system of color specification with six levels of precision ranging from thirteen colors used in customer surveys to about five million specifications possible either by the CIE coordinate system or by the Munsell renotation system.

The 1931 CIE standard observer and coordinate system was also used for the statement of the color confusions made by observers with partially color-blind vision, that is, protanopes, deuteranopes, and tritanopes. Judd[61] in 1943 showed how it would be possible for partially color-blind observers to detect camouflaged positions undetectable by observers with normal color vision, and proceeded[62] to derive from the 1931 CIE standard observer a set of color-matching functions which, taking all three together, define normal metamers, but which, taken in pairs, also define the metamers for the three types of partially color-blind vision. These dichromatic metamers were taken from the studies by König in Germany on tritanopia in 1897 and by Pitt in England on protanopia and deuteranopia in 1935. The color-matching functions conform to the three components theory of vision. In 1948, Judd[63] analyzed the published reports of people having one partially color-blind and one color-normal eye, and found that the yellow seen by protanopes and deuteranopes corresponds closely to the hue of the spectrum at 575 nm seen by people with normal color vision, and their blue corresponds to 470 nm. From this finding he developed protanopic and deuteranopic Munsell notations that correlate with what these partially color-blind people see in the same way that conventional Munsell notations correlate with what people with normal color vision see. In 1949, Judd[64] derived from the 1931 CIE standard observer the visual response functions corresponding to the photopigment stage, the receptor stage, and the optic-nerve stage of the Müller stage theory of color vision. The photopigment stage is explained by a three-components theory like that proposed by Young and Helmholtz; the other two stages take formulations of the opponent-colors type, the final stage being precisely the type orginated by Hering. The Müller three-stage theory, quantified in this way, explains metamerism for observers of normal and partially color-blind vision, the final stage explains color perceptions by such observers, and the intermediate stage explains the Priest-Brickwedde data on minimum perceptible purity to be discussed presently in connection with color differences. In 1950, Judd *et al.*[65] showed that the color confusions and spectral luminous efficiencies of an atypical congenital tritanope corresponded in detail with the properties to be expected of a classical tritanope with five or six times normal ocular pigmentation.

One of the chief causes of individual differences in metamers for observers of normal color vision arises from the fact that normal eyes have a more or less dense spot of yellow pigment (probably xanthophyll) on the retina surrounding the fovea and extending from it by 2° to 4°. This pigment is called the macular pigment. Many metamers set up for a large field, for example 10°, fail to match in central vision, one side often appearing red, the other green. This red spot follows the fixation point around on the one side, and a green spot is seen on the other side. This spot, called the Maxwell spot after its discoverer, would seem to be an entoptic projection of the macular pigment, and was so explained by Maxwell. In 1953, Judd[66] showed that a particular form of three components theory, that used to explain partial color-blindness, yields predictions of the colors of the Maxwell spot produced by alternately viewing a uniform field of incandescent lamplight through a dark gray filter and a particular purple filter (Wratten No. 2389) used by Walls for this purpose. The predictions were found to be in nearly perfect agreement with those reported by observers of various types of vision studied by Walls (see Table I).

Table I. Colors of the Maxwell Spot Obtained with Incandescent Lamplight Filtered through a Wratten No. 2389 Purple Filter

Type of color vision	Theory	Experiment (Walls)
Normal	Dark and reddish violet	Red or pink
Protanopia	Dark and blue	Dark or blue
Deuteranopia	Dark	No spot and no color for 16 observers; faintly dark and bluish for 1 observer
Tritanopia	Dark and red	—

Why most deuteranopes would fail to see the Maxwell spot under these conditions is not known. In 1958, Kelly[67] carried out a study of ocular and macular pigmentation by means of a celebrated metameric pair known as the Granville grays. He found that the yellow pigmentation (probably melanin) of the lens was strongly correlated with the age of the observer. Macular pigmentation showed no correlation with age, but some observers' macular pigmentation was found to be greater than that of others by 120 microreciprocal degrees (μrd). A difference of 1 μrd is about the least difference detectable with certainty.

We have seen that the color-matching functions known as the OSA excitation curves were used for about five years before definite evidence was collected that some of the metamers defined by them lay outside the range of metamers set up by observers with normal color vision. The 1931 CIE standard observer first came under fire fifteen years after being recommended. In 1949, Judd[68] corroborated a report by Jacobsen (National Lead Company) that it did not work for large field colorimetry of the rutile and anatase forms of titanium dioxide. Fields of angular subtense considerably greater than 2° (for which the standard observer was derived) are used widely in industry because of the higher precision of color matching achievable thereby. The CIE took up the challenge and developed (1951 to 1959) a supplementary observer for large field colorimetry. In 1964, Nimeroff[69] showed that large field settings on the Donaldson colorimeter by twenty-one observers (eleven from the National Bureau of Standards, ten from the National Research Council of Canada) agreed with the predictions of the proposed CIE supplementary observer for larger field colorimetry in the sense that, for eight of the eleven colors tested, the predicted chromaticity fell among the twenty-one observed chromaticities, three were borderline, and none was outside. Partly because of this support the CIE gave its official stamp of approval for the supplementary observer in that same year. Nimeroff[70] also showed that Grassmann's additivity law, on which the derivation of all color-matching functions must rely, significantly and systematically fails for 10° fields of less than 150 trolands. This failure is like that caused by rod intrusion, but might be owing to interaction between the chromatic response components of the visual system. This result indicates that the 1964 CIE supplementary observer for large field colorimetry should be expected to agree only with experiments such as those carried out at luminances corresponding to considerably more than 170 trolands.

In 1957, Nimeroff[71] showed how to apply the theory of error propagation to chromaticity coordinates computed from spectrophotometric data to predict the chromaticity regions within which observed chromaticity points will fall for a specified fraction of observers provided that the variances of the spectrophotometric data and the between-observer variances of the color-matching functions used for the comparison are both known. Availability of the latter variances permit discrepancies between computed and observed chromaticities to be placed in proper perspective. In 1961, Nimeroff *et al.*[72] computed both the within-observer and between-observer variances of the 1964 CIE supplementary observer from data supplied by Stiles and Burch at the National Physical Laboratory in England and by Speranskaya at the State Optical Institute in Russia. In 1966, Nimeroff[73] showed that the uncertainty ellipses computed from these data agree well with those derived from the settings of color match by eleven actual observers.

The 1931 CIE standard observer has been used widely in recent years for indirect colorimetry by means of automatic spectrophotometers with digital readout, the computations of tristimulus values and chromaticity coordinates being done by high speed digital computers. Photoelectric tristimulus colorimeters, such as that developed at the National Bureau of Standards by Hunter[74] in 1942 and by Nimeroff and Wilson[75] in 1954, calibrated by color standards spectrophotometrically evaluated in the CIE coordinate system, are also used extensively for specimens forming a not too highly metameric match for an available standard. Nimeroff and Yurow[76] in 1965 developed an index for degree of metamerism and showed that it correlates well with the between-observer spread of chromaticity settings on the Donaldson colorimeter.

Color Differences

Interest of the National Bureau of Standards in the perceptual size of color differences became evident at an early date. In 1909, Nutting[77] proposed to form "a natural scale of pure color" by integration of the Steindler data on just noticeable difference in wavelength along the spectrum. In 1920, Karrer and Tyndall[78] determined the relations between visual angle, contrast, and background luminance required for a target to be visible. The data were used in connection with target detection by searchlights. In the same year, Priest *et al.*[33] showed that the value scale embodied in the 1915 Atlas of the Munsell Color System corresponds to Munsell value set proportional to the square root of the luminous directional reflectance factor. They said that, for equal visual spacing, the reflectance factor should form a geometric series in conformity with the Weber–Fechner law, and reported that visual inspection of the Munsell gray scale indicated nonuniformities corresponding to failure to so conform. In 1930, Judd[79] showed that precision in settings of color temperature and the impression of size of the color temperature difference improves with angular subtense of the field and deteriorates with the prominence of any color separating the two fields being compared. Davis[80] in 1931 developed an empirical method for finding the temperature of the blackbody yielding the closest chromaticity match to any light source slightly off the blackbody locus from the chromaticity coordinates of that source. This temperature was, and still is, called the correlated color temperature. In 1932, Judd[81] showed for several types of chromaticity difference (along the spectrum, along lines of constant dominant wavelength, along lines of constant purity, along the blackbody locus, and along the locus of variable Lovibond red combined with 35-yellow) that equal differences in the r coordinate or the g coordinate of the OSA chromaticity diagram, whichever difference is larger, correspond closely to differences found in previously published work to be of the same perceptual size. In 1933, Judd[82] showed that chromaticity differences among lamps of different color temperature perceived to be equal to six observers corresponded closely to equal differences in either the r coordinate of the OSA (r, g)-chromaticity diagram, or in the spectral centroid, or in reciprocal color temperature. In the same year, Priest[83] proposed that the reciprocal of color temperature be used to specify the chromaticities of incandescent lamps and various phases of daylight. The unit was called the microreciprocal degree (μrd or mired) when the quantity $10^6/T_c$ with T_c as color temperature in degrees Kelvin is used. A difference in reciprocal color temperature of 1 μrd is close to the smallest chromaticity difference perceptible with certainty. This scale is still widely used. In the same year, Tyndall[84] evaluated for his own eye the just noticeable difference in dominant wavelength as a function of purity at constant dominant wavelength for various dominant wavelengths; and Judd[85] evaluated for eight observers the intervals in purity perceived as equal for a dominant wavelength of 575 nm. In 1935, Judd[86] developed a projective transformation of the 1931 CIE (x, y)-chromaticity diagram on which equal distances correspond approximately to chromaticness differences perceived as of equal size. Thus, the "natural scale of pure color" proposed by Nutting in 1909 corresponds approximately to steps of equal length along the spectrum locus plotted on this projective transformation, and the Priest–Brickwedde[87] measures of minimum perceptible purity as a function of dominant wavelength correspond to approximately equal radii from the sunlight point, and so on. This transformation was called the "uniform-chromaticity-scale" (UCS) diagram. In 1935, McNicholas[88] used the UCS diagram to estimate the value N_R of Lovibond red that would yield the closest chromaticity match on the 35-yellow, N_R-red locus for an off-locus sample of vegetable oil. In 1936, Judd[89] applied the UCS diagram to the problem of finding the correlated color temperature of any light source whose chromaticity point is slightly off the blackbody locus. The results agreed well with those found by the Davis[80] empirical method already mentioned. In the same year, McNicholas[90] used the UCS diagram to arrange signal colors to be tested on an approximately uniform perceptual scale. In 1939, Breckenridge and Schaub[91] developed a rectangular form of the UCS diagram, known as the RUCS diagram. It has essentially the spacing of the UCS diagram, but the chromaticity point for the equal energy spectrum is placed at the origin of the coordinate system, and the longwave branch of the spectrum locus is set along the vertical. The RUCS diagram has been used to check the spacing of the chromaticity ranges within which signal colors must fall; see the United States Standard for the Colors of Signal Lights, by Breckenridge[92], which appeared in 1964.

In 1960, the CIE recommended the use of a modification of the CIE diagram, with a simplified relation to the CIE (x, y)-chromaticity diagram, proposed by MacAdam in 1937. This modification was recommended for use "whenever a diagram yielding colour spacing perceptually more nearly uniform than the (x, y)-diagram is desired." In 1963, Kelly[93] applied the 1960 CIE–UCS diagram to derivation of the lines of constant correlated color temperature on the (x, y)-diagram; and in 1964 Nimeroff[94] computed the spectral tristimulus values implied by the 1960 CIE–UCS system.

In 1939, Judd[95] used the UCS diagram to derive a measure of the perceptual sizes of color differences in terms of what has now become known as the NBS unit of color difference. Distance on the UCS triangle was weighted by the square root of the reflectance factor to form a measure of the size of the chromaticness component, and the square root of reflectance formed the measure of the size of the lightness component. The size of the color difference is computed as the square root of the sum of the squares of the components weighted so the unit of color difference is about four or five times the size of the just detectable difference. Many commercial products are purchased with color tolerances of about one NBS unit.

In 1950, Judd *et al.*[96] used the NBS unit of color difference to improve the spacing of the Union Colorimeter scale used for lubricating oil and petrolatum. This respaced scale was incorporated into ASTM Method D 1500–58T which is currently in force.

In 1942, Hunter[74] modified the UCS projective transformation to make it agree better with the spacing of the Munsell colors and with spacing data published by Wright in England. Eickhoff and Hunter[97], in the same year, used the revised NBS unit of color difference to express the fading rate of paints exposed outdoors. The colors were measured by photoelectric tristimulus colorimetry.

In 1953, Nimeroff[98] showed that the uncertainties in chromaticity coordinates caused by uncertainty in spectral reflectance or transmittance uniform regardless of wavelength for a representative series of objects corresponded closely to the uncertainties of setting a chromaticity match visually found by MacAdam at Eastman Kodak.

In 1959, Richmond and Harrison[99] obtained estimates of the perceived character and size of about 180 small color differences between specimens of vitreous enamel from each of thirty-four observers. The geometric standard deviations of the size estimates was about 1.8 and the between-observer spread of the character estimates was also similarly large. Estimates for colors perceived as of middle lightness and low saturation were more reliable than those for other colors, but no significant difference in reliability was found between the nineteen observers classed as experienced and the fifteen observers classed as inexperienced.

The current work in physiological optics at the National Bureau of Standards is largely confined to further research on methods of predicting the perceived sizes of color differences from the CIE tristimulus values of the two colors involved. In 1955, a progress report by the Optical Society of America Committee on Uniform Color Scales, written by Judd[100], was published. It indicated that four basic methods of estimating the size of color differences were then in use by industry: one based on projective transformations of the CIE (x, y)-diagram developed at the National Bureau of Standards, a second based on the Munsell spacing, a third based on the Adams chromatic-value diagram, and a fourth based on the MacAdam ellipses. The Committee agreed to embark on the development of a 500-chip uniform tridimensional color scale to be derived by a regular rhombohedral sampling of the paint gamut such as described by Wyszecki[101] in 1954. Each chip whose color fell in the interior of the gamut was to be surrounded by twelve nearest neighbors each differing from the color of the central chip by amounts perceptually as nearly equal as could possibly be achieved. In 1966, Judd[102] published a second progress report detailing the steps taken to achieve this result. The first experiment (1956) determined a locus of chromaticity points on the CIE (x, y)-diagram for colors of Munsell value 6/perceived as having the same saturation (departure from gray). The second experiment (1957) determined forty chromaticity points on this locus equally spaced in hue. The third experiment (1961) determined chromaticness spacing for near grays of 3/, 6/, and 8/. The fourth experiment, still in progress, was based on the production of forty-three chips intended to give a uniform triangular sampling of the paint gamut at 6/. The uniformity of this spacing has been checked by eight observers, each making judgments of "larger" or "smaller" for one hundred and forty-two of the pairs of differences among the forty-three chips. The committee has spent more than a year analyzing these statistical data. These data support the previous finding of MacAdam that perceptual color space has a Gaussian curvature significantly different from zero, and this property prevents any tridimensional sampling from being strictly uniform in spacing. They indicate that the perceptual size of a chromaticity difference is not linearly related to distance on any uniform-chromaticity-scale diagram, but varies with powers less than one (0.37 to 0.80) of these distances. And they support the previous finding of Nickerson that, for equal distances on any chromaticity diagram adjusted for approximately uniform spacing, saturation differences are perceived as smaller than hue differences. Appraisal of these differences by means of any of the four currently used methods does not correlate to any significant degree with the statistical measures of the perceived sizes; but similar measures obtained by fractionating the eighty observers into four groups of twenty observers each show good correlation with each other, and excellent correlation with the results from the entire group of eighty observers. The National Bureau of Standards is currently engaged in studies that are intended to find out what factor is operative in the statistical studies that is not taken into account in any of the current methods of estimating the perceptual size of color differences. Perhaps the presence of two colors defining the second difference in close juxtaposition with those defining the first exerts a significant influence on the perceived size of the first difference and vice versa. Modifications of each of the four currently used methods are being explored in an attempt to find a formula yielding reasonable correlation with these statistical measures of perceived size.

Color Perception

An observer may describe what he sees in his own terms independent of any measures of the stimuli acting on his retina. The terms of color perception are red, yellow, green, blue, and their intermediates for the perceived hues: gray, moderate, and vivid for the perceived saturations; black, dark, medium, light, and white for the lightnesses of colors perceived to belong to non-self-luminous bodies, and dim, bright, and dazzling for the colors perceived to belong to self-luminous bodies or light sources. Interest of the National Bureau of Standards in color perception first became evident in 1907 when Nutting[103] published an expression for the brightness perceived to belong to a spot of light viewed with a dark surround as a function of the luminance of the spot and the absolute threshold of luminance. In 1910, Ives[104] computed how much light from the Welsbach mantel, from the carbon filament lamp, and from the tungsten filament lamp had to be added to mercury arc light to produce the color of daylight, but remarked that the merit of the resulting artificial daylight should be judged by the degree to which the artificial daylight rendered, in accord with natural daylight, the colors of objects to which the eye is particularly sensitive, such as the colors of flesh, lips, and other commonly observed objects. In 1918, Priest[105] announced in discussion of a paper by Troland that the color temperature found by preliminary experiment for a bright spot viewed with dark surround to be perceived without hue is 4000°K; and in 1921 he[106] gave the final values of 4600°K to 6500°K with an average of 5300°K. In 1925, Priest[107] showed that skies perceived as gray may have a higher luminance than objects in the same scene perceived as white. In 1926, he[108] showed that twenty-five observers found either Abbot–Priest average noon sunlight or an equal energy source to be perceived as without hue when viewed alone with a dark surround; but when viewed in juxtaposition neither appeared achromatic or neutral. In the same year, Priest[109] showed that a given element in a scene was perceived as blue, if it was seen as a body of water (Skinner's Lake), but white, if seen (as it really was) as a snow bank.

In 1929, Judd[110] measured the least retinal illumination required in his own eye as a function of wavelength for the production of the "blue arcs of the retina", an entoptic projection of the fibers of the optic nerve, and showed that stimulation of the retinal rods initiated the nerve activity rendering the fibers visible.

In 1936, Judd[111] used the UCS diagram in developing a correlate for paper whiteness. The correlate was the luminous reflectance factor diminished by an appropriate fraction of the distance on the UCS diagram between the chromaticity point for the specimen and the chromaticity point of the standard accepted by the inspector as white. In 1941, Judd[112] proposed a revised formula for whiteness of any object which correlated perceived whiteness with the degree of approach of the object color to the color accepted as white measured in NBS units. With either of these formulas for whiteness it is quite possible to increase the whiteness of a sheet of yellowish paper by dyeing it with blue dye even though the luminous reflectance is thereby considerably decreased. In the same year, Judd[113] objected to the phrases "black sensation" and "white sensation" and proposed definitions of black and white as names for colors perceived to belong to objects themselves perceived as non-self-luminous.

In 1940, Judd[114] developed inductively, from judgments of six observers, empirical formulas for the hue, lightness, and saturation of the color found to belong to any flat object viewed with any surround as functions of the tristimulus values of the flat object and those of the surround. Saturation was expressed as proportional to distance on the UCS diagram between the achromatic point and the object point. Hue depended upon the direction of the line connecting these two points and lightness was expressed as an hyperbolic function, previously proposed by Adams, of the directional reflectance factor of the object and surround. The achromatic point was specified as an empirical function of the surround chromaticity, the object chromaticity and the object lightness. The correlation between formula and the judgments of the six observers was excellent for red surrounds, good for yellow and green, but only fair for blue surrounds. Evidence was found for a significant influence of light scattered by eye media of the observers viewing dark objects with light blue surrounds.

In 1950, Judd *et al.*[115] used the Munsell color system to develop an index of color fidelity that can be used to appraise systems of color television. In 1952, Helson *et al.*[116] showed that the color changes perceived for objects on passing from daylight to incandescent lamp illumination are in approximate conformity to the v. Kries coefficient law applied to the primaries used for the response functions for the three types of dichromatic vision. The v. Kries coefficient law states that, if the colors perceived to belong to objects by an eye adapted to daylight are correlated with the tristimulus values R, G, B, of those objects, then for an eye adapted to a source of nondaylight chromaticity the perceived colors will similarly correlate with tristimulus values R', G', B' found by applying coefficients α, β, γ characterizing the sensitivities of the components of vision as changed by adaptation to the nondaylight source; thus $R' = \alpha R$, $G' = \beta G$, and $B' = \gamma B$. In 1956, Helson *et al.*[117] showed for white, coolwhite, and daylight fluorescent lamps that the v. Kries coefficient law gave predictions within the between-observer spread for 85% of the 390 hue judgments, for 86% of the 390 saturation judgments, and about 97% of the 390 lightness judgments.

In 1954, Wyszecki[118] showed how to determine graphically on the (x, y)-chromaticity diagram the v. Kries transformation based on any choice of fixed points (primaries).

In 1942, Hunter[74] proposed indices of hue, lightness, and saturation of colors perceived to belong to a specimen measured by photoelectric tristimulus colorimetry. These indices apply to specimens illuminated by daylight and viewed with a gray to white surround by an observer with normal color vision. Indices for whiteness and yellowness of near white objects were also proposed.

In 1959, Judd[119] described a number of demonstrations of perceived colors including the blue arcs of the retina, the Maxwell spot, the derivation of the variables' hue, lightness, and saturation (Desert Island experiment), color perceptions of red-green confusers, metamerism, chromatic adaptation, and object-color rendition.

In 1960, Judd[120] analyzed the Land demonstrations of color perceptions arising from two-primary color projections and showed that the formulas published[114] in 1940 predicted the perception of colors of all hues in accord with Land's report for a red primary combined with an incandescent lamp primary.

In 1961 there was published[121] a five-attribute system of describing the nonshape, nontextural, and nontemporal aspects of visual appearance worked out by Judd and Gibson some thirty years previously. The attributes are hue, lightness, saturation, transparency, and glossiness for objects perceived as non-self-luminous and hue, brightness, saturation, and transparency for those perceived as self-luminous.

In 1966, Takasaki[122] studied the influence of a light-dark difference between target and surround on the lightness perception of the target by presenting one gray surround with a fixed gray target side by side with a different gray surround with variable gray targets and asking the observer to select the one making the two targets appear equally light. He found that this influence was particulary marked if one of the targets differed only slightly from the surround, and he referred to this property of the human eye as "crispening". He showed that the Munsell value of the matching gray could be predicted by setting the perceived lightness L_i of the target equal to the following function of the Munsell value V_i of the target, the Munsell value V_{bi} of the surround and the average Munsell value $\bar{V}_b$ of two surrounds,

$$L_i = V_i - C_1 V_{bi} + C_2 \bar{V}_b [(V_i - V_{bi})/C_3] e^{-|V_i - V_{bi}|/C_3}, \quad i = 1,2,$$

and finding the Munsell value V_2 for which $L_1 = L_2$. The constants C_1, C_2, and C_3, which have to be adjusted for each observer, refer to induction, amount of crispening, and reciprocal sharpness of crispening, respectively. Unpublished work shows that formulas of the same type apply to chromaticity variations in the red-green and violet-greenyellow senses at constant luminance. The crispening phenomenon implies that a significant correction to the Munsell value function near the value of the surround should be applied, and it may explain some of the failures of the v. Kries coefficient law to predict the influence of chromatic adaptation on the perceived color.

Current work at the National Bureau of Standards on color perception stems from a study by McNicholas[90] who showed in 1936 that point sources appearing orange or yellow to thirty-eight observers at low intensity appeared shifted in hue toward red or orange, respectively, at ten or fifteen times this low intensity. Similarly, point sources appearing bluish green at low intensities shifted in hue at higher intensities toward green. Both of these shifts are opposite to the hue shifts (Bezold–Brücke phenomenon discovered in the 1870's) found to apply to extended fields of relatively high luminance. The Bezold–Brücke hue shifts correspond to a lower stability of the red-green sense relative to the yellow-blue sense at high luminances. The McNicholas hue shifts correspond to a lower stability of the red-green sense at low values of luminous flux entering the eye (low product of luminance by solid angular size of source). In view of the fact discovered by König in 1894 in Germany and corroborated by Willmer and Wright in 1945 and Hartridge in 1947, both in England, that the normal human eye makes tritanopic mistakes (violet-greenyellow confusion) if the field size is smaller than about 0.1°, the McNicholas hue shifts are difficult to explain. The National Bureau of Standards has undertaken a series of measurements to determine the variation of chromaticity sensibility as a function of field luminance and angular subtense, and to discover to what extent an increase in luminance can compensate for a decrease in solid angular subtense, and vice versa. If it is true that at low luminance and/or angular subtense the normal human eye tends toward tritanopia, these measurements should reveal it, and also show how the transition from trichromatic to dichromatic vision takes place. It is expected that these results will yield a basis not only for explaining the McNicholas hue shifts but also for an improved selection of colors for signalling.

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INTERVAL SCALES, RATIO SCALES, AND ADDITIVE SCALES FOR THE SIZES OF DIFFERENCES PERCEIVED BETWEEN MEMBERS OF A GEODESIC SERIES OF COLORS

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This is one of the most difficult of Judd's papers. The difficulty stems from the subject, not from any lack of clarity of exposition, or ambiguity of terminology.

This very carefully reasoned development will repay attentive reading. It deals with a major complication that jeopardized the work of the OSA Committee on Uniform Color Scales. That difficulty arose from the discovery that the data provided by psychometrics are not linearly related to any quantities that can be determined by physical measurements, even supplemented by the psychophysical data and methods of colorimetry. Judd's discovery, represented by Table II and Fig. 1, made possible the design of a color space that represents adequately the judgments of color differences obtained by the committee. The consequence of this investigation should be remembered: Perceived magnitudes of color differences are not proportional to the corresponding distances in the most appropriate color space, but to those distances raised to some power, about 2/3.

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Interval Scales, Ratio Scales, and Additive Scales for the Sizes of Differences Perceived between Members of a Geodesic Series of Colors

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From larger–smaller judgments of color differences, compared visually two at a time, the perceived sizes may be evaluated on an interval scale. Given numbers B so evaluated, and such that B is linearly connected to some power p, of the physical measure D (such as distance on any chromaticity diagram) of the differences, the additive constant K_{br} such that the numbers $B+K_{br}$ are expressed on a ratio scale may be found from judgments of the ratio of sizes of pairs of differences. To evaluate p, it is sufficient to observe the three differences, 12, 23, and 13 between the pairs of three colors, 1, 2, 3, forming a geodesic series, and chosen so that B_{12} is not much different from B_{23}. The scale formed by the numbers $(B+K_{br})^{1/p}$ is additive if the D scale is additive. If the largest of the color differences judged exceeds the smallest by a factor not greater than 3, a close approximation to the $(B+K_{br})^{1/p}$ scale may be found without evaluating K_{br} by ratio judgments. This approximation is based on the empirical discovery that scales based on the additivity condition: $(B_{12}+K_{bd})^{1/p}+(B_{13}+K_{bd})^{1/p}=(B_{13}+K_{bd})^{1/p}$, though it implies that K_{bd} depends strongly on p, are essentially identical regardless of the choice of p between 1 and $\frac{1}{3}$. It is sufficient therefore, to derive the additive scale by setting $p=1$, and computing K_{bd} as $B_{13}-B_{12}-B_{23}$.

INDEX HEADINGS: Color; Vision; Colorimetry.

A GUIDING principle in the work of the OSA Committee on Uniform Color Scales from its very beginning in 1948 has been that there exists a color space each point of which represents one and only one color, the spacing of the points being uniform in the sense that to a degree of approximation suited to engineering applications the distance between any two points is proportional to the perceived size of the difference between the colors represented by the two points. Equally spaced points along any path in this color space would thus yield a uniform color scale. The committee has been engaged since 1955 in experiments intended to result in the production of several hundreds of painted color chips that would sample this color space by the regular rhombohedral lattice-sampling method.[1] The first experiment (1956) determined a locus of colors of chromaticity points on the CIE (x,y)-diagram for colors of Munsell value 6/ perceived as having the same saturation (departure from gray). The second experiment (1957) determined 40 chromaticity points on this locus equally spaced in hue. The third experiment (1961) determined chromaticness spacing near grays of Munsell value 3/, 6/, and 8/. The latest experiment has determined chromaticness spacing of the entire mat-finish paint gamut at Munsell value 6/. In the course of these studies, the committee evaluated scales of three sorts: interval scales, ratio scales, and additive scales. It is the purpose of this paper to define these three types of scales, to indicate the methods by which the experimental data must be reduced to achieve them, and to restate the guiding principle in more precise terms. In the course of following the above-stated guiding principle, the committee has shown that color space cannot have all the properties included in the statement of this principle.

INTERVAL SCALES AND RATIO SCALES

Let us consider a series ot stimuli, $S_1, S_2, \cdots S_n$, evaluated physically by numbers, $D_1, D_2, \cdots D_n$, and producing responses, $R_1, R_2, \cdots R_n$, evaluated by numbers, $B_1, B_2, \cdots B_n$.

Interval Scale

If for all pairs of samples such that the difference between the members of the pair, S_1 and S_2, is perceived as equal to the difference between some two other samples, S_3 and S_4 (that is, if $R_1-R_2=R_3-R_4$), then the B scale is defined as an interval scale provided that $B_1-B_2=B_3-B_4$, and provided that the converse is true (that is, provided that any two pairs of stimuli, S_5, S_6, and S_7, S_8 for which $B_5-B_6=B_7-B_8$, yield responses such that $R_5-R_6=R_7-R_8$). Stated another way, the B scale is defined as an interval scale if and only if

$$B=m_{br}R-K_{br}. \qquad (1)$$

The subscript br, by which the constants m and K are identified, means that these constants apply to the relation between the B scale and the R scale.

Ratio Scale

If the response R_1, R_2, R_3, and R_4 with R_2 and $R_4 \neq 0$, are such that R_1 is the same multiple (or fraction) of R_2 as R_3 is of R_4 (that is, if $R_1/R_2=R_3/R_4$), then the B scale is defined as a ratio scale provided that $B_1/B_2=B_3/B_4$, and provided that the converse is true. Stated another way: the B scale is defined as a ratio scale if and only if

$$B=m_{br}R. \qquad (2)$$

* Chairman, OSA Committee on Uniform Color Scales.
[1] G. Wyszecki, J. Opt. Soc. Am. 44, 725 (1954); D. B. Judd, J. Opt. Soc. Am. 45, 673 (1955).

Any B scale derived from a model of the form

$$\sum[B_i-B_j-F(f_i-f_j)]^2=\text{a minimum} \qquad (3)$$

by the method of least squares is presumed to approximate an interval scale. Here f_i is the fraction of the total times, S_i is judged to be larger than S_j, and f_j is the fraction of the times S_j is judged to be larger than S_i. The function $F(x)$ may be chosen in many ways including, for example, "normal scores" and also simply $F(x)=x$.[2] When the responses R_i are not very different, as is the case in the committee's experiments, the choice of the function F has very little effect on the B-scale values derived.

If the scores, f_i-f_j, are analyzed by a method, such as the Morrissey–Gulliksen matrix method,[3] in which the average of the responses is assigned arbitrarily a B-scale value of zero (that is: $\sum B_n/n=0$), then this scale obviously could not be a ratio scale. An interval scale for positive quantities like color differences can be converted into a ratio scale by adding a constant K_{br} to each B-scale value. That is, if $B_1, B_2,\cdots B_n$ form an interval scale, then $B_1+K_{br},\cdots, B_n+K_{br}$ form a ratio scale. The value of the additive constant K_{br} may be determined by making ratio judgments. If the ratio judgments indicate that

$$(B_i+K_{br})/(B_j+K_{br})=r_{ij},$$

then

$$K_{br}=(r_{ij}B_j-B_i)/(1-r_{ij}). \qquad (4)$$

In the work of the OSA Committee on Uniform Color Scales, the stimuli S_i have been pairs of simultaneously presented colors of closely constant luminance. They are evaluated physically by distance D_i on a chromaticity diagram between the chromaticity points of the colors forming each pair. The responses R_i are the subjective impressions of the sizes of the differences perceived between the pairs of colors, and the numbers B_i are the interpoint distances derived by the Morrissey–Gulliksen matrix method from larger–smaller judgments. From its derivation, the B scale is presumed to approximate an interval scale; if ratio judgments for a suitable sampling of pairs of these color differences were available, the B scale could be converted to a ratio scale by determining K_{br} in accord with Eq. (4).

GEODESIC SERIES OF COLORS

Colors $C_1, C_2,\cdots C_n$ are said to form a geodesic series of colors if all of the intermediate colors, $C_2, C_3,\cdots C_{n-1}$, lie on the geodesic in color space connecting the extreme colors, C_1 and C_n. Color space is any three-dimensional space each point of which represents one and only one color, and in which each color is represented by one and only one point. Given two separate points in color space representing two separate colors, there exist infinitely many paths starting with the one point and terminating in the other. For any of these infinitely many paths it is conceptually possible for an observer to determine the number of thresholds of color difference required to traverse this path from one end to the other. That path yielding the smallest number of thresholds is the geodesic path. Examples of color spaces are those formed by plotting the tristimulus values, X, Y, Z, of the colors along orthogonal axes, by plotting the chromaticity coordinates x, y, and the luminance Y greater than zero along orthogonal axes, or by plotting Munsell hue H, value V, and chroma C in cylindrical coordinates, that is, by plotting H according to angle, C as the distance from the black–white axis, and V as distance parallel to this axis.

ADDITIVE SCALES

If the stimuli S_{gh} are differences among a geodesic series of colors, and if color 2 is an intermediate along the geodesic between any two colors 1 and 3, then the physical measures D_{gh} of the stimuli S_{gh} form an additive scale if and only if the sum of D_{12} and D_{23} equals D_{13}. Most physical measures are expressed on essentially additive scales; in particular, distance D on any chromaticity diagram computed from the usual formula

$$D=[(x_2-x_1)^2+(y_2-y_1)^2]^{\frac{1}{2}}$$

corresponds to the length of the straight line on the chromaticity diagram between the points (x_1,y_1) and (x_2,y_2); it would be nonadditive only for such geodesic series of colors as depart significantly from this straight line. Distance measured along the geodesic between any two points on the chromaticity diagram conforms to the additivity condition except for such geodesics as leave the plane of constant luminance. That is, if the geodesic between points 1 and 3 corresponding to colors of the same luminance has such a course in color space that the intermediate color represented by point 2 has closely the same luminance as colors 1 and 3, then distance measured along the geodesic in the chromaticity diagram is additive. Only if color 2 has a significantly different luminance than that of colors 1 and 3 would this distance scale be nonadditive. It has been presumed in the work of the committee that restriction of the difference between the terminal colors of the same luminance to amounts less than about 24 NBS units (4 Munsell chroma steps) is sufficient to make distance on any chromaticity diagram additive.

Similarly, the numbers B_i that evaluate the responses R_i to the stimuli S_i formed from a geodesic series of colors are on an additive scale if and only if the sum of B_{12} and B_{23} equals B_{13}, where colors 1, 2, and 3 are

[2] H. A. David, *The Method of Paired Comparisons* (Hafner Publishing Co., New York, 1963), Sec. 1.3 and Ch. 4, especially p. 55.

[3] J. E. Jackson and M. Fleckenstein, Biometrics 13, 51 (1957).

as defined above. The condition for an additive scale is

$$B_{12}+B_{23}=B_{13}. \tag{5}$$

If the geodesic series does not depart significantly from a straight line on the chromaticity diagram, then the D scale is known to be an additive scale and if the B scale is linearly connected to the D scale, then the numbers, B_1, $B_2, \cdots B_n$, may be converted into an additive scale by adding a constant K_{bd} to each number, that is, B_1+K_{bd}, $B_2+K_{bd}, \cdots B_n+K_{bd}$ evaluate the responses R_1, $R_2, \cdots R_n$ on an additive scale. The required constant K_{bd} may be computed[4] as

$$K_{bd}=B_{13}-B_{12}-B_{23}. \tag{6}$$

Proof: If the B-scale is linearly connected to the D scale it must be that $B=m_{bd}D-K_{bd}$, or $D=(B+K_{bd})/m_{bd}$. If the D scale is additive, then $D_{12}+D_{23}=D_{13}$, or

$$(B_{12}+K_{bd})/m_{bd}+(B_{23}+K_{bd})/m_{bd}=(B_{13}+K_{bd})/m_{bd},$$

whence it is seen that the $(B+K_{bd})$ scale is additive, and it is also seen that

$$B+K_{bd}=m_{bd}D.$$

That is, the D scale and the $(B+K_{bd})$ scale differ only by some constant factor m_{bd}, relating the sizes of the units of the two scales. The subscript *bd* by which the constants m and K are identified means that these constants apply to the relation between the B and D scales.

IMPLICATIONS OF AN ADDITIVE $(B+K_{bd})$ SCALE FOR THE RESPONSES R

If by observing several geodesic series of colors it is established within experimental uncertainty that the $(B+K_{bd})$ scale is linearly connected to an additive D scale, what does this imply about the connection between the responses R_n and the physical measures D_n? If the numbers B_i have been derived by applying the criterion of Eq. (3), it is presumed that the relation between B_i and R_i is that given by Eq. (1). From the additivity of the $(B+K_{bd})$ scale we have

$$B_{12}+K_{bd}+B_{23}+K_{bd}=B_{13}+K_{bd},$$

and from Eq. (1) may be written

$$m_{br}R_{12}-K_{br}+K_{bd}+m_{br}R_{23}-K_{br}+K_{bd}=m_{br}R_{13}-K_{br}+K_{bd},$$

whence it is seen that only if $K_{br}=K_{bd}$, does R satisfy the requirement of additivity: $R_{12}+R_{23}=R_{13}$. If the B and D scales are linearly connected (that is, if $B=m_{bd}D-K_{bd}$), we have already seen that

$$B+K_{bd}=m_{bd}D.$$

That is, the $(B+K_{bd})$ scale differs from the D scale only by a constant of proportionality. If the responses R are themselves proportional to the measures D of the stimuli, this would satisfy the condition: $K_{br}=K_{bd}$.

IMPLICATIONS OF THE HYPOTHESIS THAT THE $(B+K_{br})$ SCALE IS PROPORTIONAL TO SOME POWER p, LESS THAN ONE, OF D

We have seen that the assumption that B is linearly connected to D implies that the $(B+K_{bd})$ scale differs from the D scale only by a constant of proportionality, where K_{bd} is given by Eq. (6). MacAdam[5] has proposed the hypothesis that the psychometric scale values $B+K_{br}$, where K_{br} is determined by ratio judgments in accord with Eq. (4), are really proportional to chromaticity differences D raised to a power p, thus

$$B+K_{br}=m_{bd}D^p. \tag{7}$$

He has found evidence from the work of the committee on three seven-color clusters each centering on a gray, one for each of Munsell values 3/, 6/, and 8/, that the hypothesis is valid, and that $0.37<p<0.80$. From some very preliminary experiments, Judd and Howett[6] reported ratio judgments from two geodesic series of approximately equally spaced colors indicating that the double difference was perceived as only 1.5 to 1.7 times as large as the single difference. The implication of this preliminary result is shown as follows:

p	Single difference $(B_{12}+K_{br})$	Implied perceived size of the double difference $(B_{13}+K_{br})/(B_{12}+K_{br})=2$
1	1	2.00
0.50	1	1.41
0.333···	1	1.26

The values of $(B_{13}+K_{br})/(B_{12}+K_{br})$ found by Howett and Judd (1.5 to 1.7) are seen to correspond to values of p between 1 and 0.50. From the law that the perceived size of the double difference D_{13} is equal to 2^p times the perceived size of the single differences $R_{12}=R_{23}=1$, of which the double difference is composed, that is,

$$(B_{13}+K_{br})/(B_{12}+K_{br})=2^p,$$

we may solve explicitly for p

$$p=\log\left[\frac{B_{13}+K_{br}}{B_{12}+K_{br}}\right]\Big/[\log 2=0.301]. \tag{8}$$

Or, to generalize, if we have n colors along a geodesic producing $n-1$ equal differences, that is, differences

[4] W. S. Torgerson, *Theory and Methods of Scaling* (John Wiley & Sons, New York, 1958), p. 271.

[5] D. L. MacAdam, J. Opt. Soc. Am. **53**, 754 (1963).

[6] G. L. Howett, *Report of Eighth Meeting of OSA Committee on Uniform Color Scales*, 20 November 1964, p. 4, unpublished.

TABLE I. Demonstration of the degree to which $[(B_{13}+K_{bd})/(B_{12}+K_{bd})]^{1/p}$ approaches 2 for K_{bd} approximated from Eq. (11), B_{12} taken equal to $B_{23}=-0.5$, and $B_{13}=0.7$.

p	K_{bd} approximated from Eq. (11)	$B_{12}+K_{bd}$	$B_{13}+K_{bd}$	$\frac{B_{13}+K_{bd}}{B_{12}+K_{bd}}$ implied ratio of perceived sizes	$(B_{12}+K_{bd})^{1/p}$	$(B_{13}+K_{bd})^{1/p}$	$\left(\frac{B_{13}+K_{bd}}{B_{12}+K_{bd}}\right)^{1/p}$ implied ratio D_{13}/D_{12} of the physical measures
1	1.700	1.200	2.400	2.00	1.200	2.400	2.000
$\frac{3}{4}$	2.266	1.766	2.966	1.68	2.134	4.261	1.997
$\frac{2}{3}$	2.548	2.048	3.248	1.59	2.931	5.853	1.996
$\frac{1}{2}$	3.397	2.897	4.097	1.41	8.393	16.785	2.000
$\frac{1}{3}$	5.094	4.594	5.794	1.26	6.956	194.507	2.006
$\frac{1}{4}$	6.791	6.291	7.491	1.19	1566.30	3148.89	2.010
$\frac{1}{8}$	13.579	13.079	14.279	1.09	856241589.	1728153633.	2.018

such that

$$R_{12}=R_{23}=R_{34}\cdots=R_{n-1,n}$$

$$\frac{R_{1n}}{R_{12}}=\frac{B_{1n}+K_{br}}{B_{12}+K_{br}}=(n-1)^p, \quad (9)$$

then

$$p=\log\left[\frac{B_{1n}+K_{br}}{B_{12}+K_{br}}\right]\Big/\log(n-1).$$

If we have a D scale such that also $D_{12}=D_{23}=\cdots D_{n-1,n}$, then the law may be stated as MacAdam did:

$$B+K_{br}=m_{bd}D^p \quad \text{or} \quad R_{1n}/R_{12}=(D_{1n}/D_{12})^p.$$

Since these judgments involved three colors from Eq. (8) ($n=3$), the values of p implied by the ratios 1.5 and 1.7 may be computed as follows:

$\frac{B_{13}+K_{br}}{B_{12}+K_{br}}$	$\log_{10}$	$p=\log\left[\frac{B_{13}+K_{br}}{B_{12}+K_{br}}\right]\Big/0.301$
1.5	0.176	0.58
1.6	0.204	0.68
1.7	0.230	0.76

Since the implied values of p (0.58 to 0.76) found by Howett and Judd fall within the range of 0.37 to 0.80 found by MacAdam, the Howett–Judd preliminary determination corroborates the MacAdam values found from the three seven-color clusters in which K_{br} was evaluated from ratio judgments by many observers.

DETERMINATION OF K_{bd} FROM OBSERVATIONS OF 43 VALUE 6/ COLORS

For the linear hypothesis, $(B+K_{bd})=m_{bd}D$, the value of K_{bd} for the interpoint distances among the 43 value 6/ colors used in the latest experiment of the committee, was intended to be found from observations of three double differences among triads of colors along three separate near-geodesic series. The following table identifies the three triads of differences, gives the corresponding values of B, and the values of K_{bd} computed from Eq. (6):

Geodesic series	B_{12}	B_{23}	B_{13}	K_{bd}
2–6–12	−0.335	−0.767	0.615	1.717
37–38–39	−0.581	−0.605	0.614	1.800
41–42–43	−0.130	−0.741	0.695	1.566
			Average	1.694

If the linear hypothesis is assumed, we would therefore expect to compare the numbers $B+K_{bd}=B+1.694$ with the corresponding physical measures D of the color differences to determine the applicability of each particular D scale [distance on (x,y) diagram, distance on (u,v) diagram, distance on Munsell hue–chroma diagram, and so on] for the prediction of the perceived size of the corresponding differences. The three series were chosen by taking the smallest pairs of single differences along a near-geodesic. It is to be expected that B_{12} and B_{23} should be less than average for such a choice, and B_{13} greater than average; and it may be noted from the above table that the single differences have B values less than zero, and the double differences greater than zero, as expected.

For the MacAdam nonlinear hypothesis given in Eq. (7), $(B+K_{bd})=m_{bd}D^p$, we would expect to compare the values of $(B+K_{bd})^{1/p}$ with the corresponding physical measures D of the color differences, because by this hypothesis $(B+K_{bd})^{1/p}$ is proportional to D. Since the D scale is assumed to be additive, K_{bd} must be adjusted so that $(B+K_{bd})^{1/p}$ is additive; that is, the condition to be satisfied by K_{bd} is that

$$(B_{12}+K_{bd})^{1/p}+(B_{23}+K_{bd})^{1/p}=(B_{13}+K_{bd})^{1/p}.$$

For $p=\frac{1}{2}$, an explicit solution for K_{bd} can be found readily:

$$K_{bd}=B_{13}-B_{12}-B_{23}+[2(B_{13}-B_{12})(B_{13}-B_{23})]^{\frac{1}{2}}. \quad (10)$$

By comparing Eq. (6) with Eq. (10), we see immediately that K_{bd} depends upon the value of p which, as indicated in Eq. (8), can be evaluated by ratio judgments. Note that the first three terms of Eq. (10) are identical to Eq. (6), and further note that the last term in Eq. (10) can never be zero unless one of the single differ-

TABLE II. Dependence of the values of $2[(B+K_{bd})/K_{bd}]^{1/p}$ on the value of p over the range $p=1$ to $p=\frac{1}{3}$.

		$2[(B_{ij}+K_{bd})/K_{bd}]^{1/p}$				
		$p=1$	$p=\frac{3}{4}$	$p=\frac{2}{3}$	$p=\frac{1}{2}$	$p=\frac{1}{3}$
i, j	B_{ij}	$K_{bd}=2.000$	$K_{bd}=2.943$	$K_{bd}=3.414$	$K_{bd}=4.828$	$K_{bd}=7.657$
1, 2	−1	1.00	1.18	1.19	1.26	1.31
1, 3	0	2.00	2.00	2.00	2.00	2.00
1, 4	1	3.00	2.95	2.94	2.91	2.89
1, 5	2	4.00	4.00	4.00	4.00	4.00
2, 3	−1	1.00	1.18	1.19	1.26	1.31
2, 4	0	2.00	2.00	2.00	2.00	2.00
2, 5	1	3.00	2.95	2.94	2.91	2.89
3, 4	−1	1.00	1.18	1.19	1.26	1.31
3, 5	0	2.00	2.00	2.00	2.00	2.00
4, 5	−1	1.00	1.18	1.19	1.26	1.31
Average	0	2.00	2.06	2.06	2.09	2.10

ences is judged to be of the same size as the double difference of which it is a part. Explicit solutions for K_{bd} for values of p different from 1 and $\frac{1}{2}$ have not been found, though K_{bd} for any actual values of p, B_{12}, B_{23}, and B_{13}, can readily be found by trial and err or.

From a considerable number of such trial-and-error solutions, we discovered that K_{bd} is closely a linear function of $1/p$. This discovery led to the following approximate formula for K_{bd}:

$$K_{bd} \doteq B_{13}-B_{12}-B_{23}-2^{\frac{1}{2}}(1-1/p) \times[(B_{13}-B_{12})(B_{13}-B_{23})]^{\frac{1}{2}}. \quad (11)$$

Table I illustrates how closely the approximate values of K_{bd} found from Eq. (11) conform to the additivity condition for the particular choice of values $B_{12}=B_{23}=-0.5$, $B_{13}=0.7$, which were chosen as representative of the experimental values already given for the three near-geodesic series of colors studied by the committee. This choice of values on the B scale corresponds to three equally spaced colors on a geodesic, and the ratio $[(B_{13}+K_{bd})/(B_{12}+K_{bd})]^{1/p}$ on a truly additive scale yielded by precise values of K_{bd} would be precisely 2. Note from the last column of Table I that the approximate values of K_{bd} found from Eq. (11) yield values of this ratio that depart from 2 by no more than 0.2 of 1% for p ranging between 1 and $\frac{1}{8}$. Note also that this ratio for $p=1$ given in column 5 gives the ratio of the perceived sizes closely equal to 2^p in accord with Eq. (8); that is, whatever value may be found for K_{br} from Eq. (4), there exists a value of p for which Eq. (8) is satisfied by setting $K_{br}=K_{bd}$.

A number of other choices of B_{12}, B_{23}, and B_{13} for which $B_{12}=B_{23}$ were tried with precisely identical results, and this suggested that the degree of approximation to which Eq. (11) gives K_{bd} is independent of the choice of B_{12} and B_{13}. That this is indeed true for $B_{12}=B_{23}$ may be seen by writing out the expression for the ratio $(B_{13}+K_{bd})/(B_{12}+K_{bd})$ obtained by substituting the value of K_{bd} from Eq. (11), thus:

$$(B_{13}+K_{bd})/(B_{12}+K_{bd}) =[2-(1-1/p)2^{\frac{1}{2}}]/[1-(1-1/p)2^{\frac{1}{2}}], \quad B_{12}=B_{23}. \quad (12)$$

In this expression the term $(B_{13}-B_{12})$ cancels out, yielding the implied ratio of perceived sizes as simply a function of p. The degree to which Eq. (11) gives correct values of K_{bd} thus depends on the degree to which the right-hand expression in Eq. (12) approximates 2^p. Note that for $p=1$, this expression is precisely 2^1; for $p=\frac{1}{2}$, it is precisely $2^{\frac{1}{2}}$; and for $p=0$, it is precisely 2^0.

To explore to what extent the spacing of the $(B+K_{bd})^{1/p}$ scale is dependent on p, take values of B applying for $p=1$ to all differences among five equally spaced colors along a geodesic, and compute values on the $(B+K_{bd})^{1/p}$ scales for other values of p with K_{bd} computed from Eq. (11). Table II shows the 10 values of B_{ij}. Note that they have been chosen so that $\Sigma B_{ij}=0$ in accord with the convention, already mentioned in connection with Eq. (3), of the Morrissey–Gulliksen matrix method. The size of the unit of the $(B+K_{bd})$ scale increases rapidly with $1/p$, the factor of proportionality being approximately $K_{bd}^{1/p}$. The scale formed by $2[(B+K_{bd})/K_{bd}]^{1/p}$ is relatively independent of p both as to spacing and as to size of unit. Table II shows this scale for $p=1$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$, that is, for the range of values of p indicated by MacAdam's studies.[5] Note that for B_{ij} equal to 0 and 2, the value of $2[(B+K_{bd})/K_{bd}]^{1/p}$ is 2.00 and 4.00, respectively, independent of the value of p; but there is a slight dependence (less than 0.3 of a unit) for $B_{ij}=-1$ and $+1$. Figure 1 extends this comparison of the scales yielded by $2[(B+K_{bd})/K_{bd}]^{1/p}$ for various values of p in the range $\frac{1}{3}$ to 1 over the range 0 to 7 on the scale for $p=1$. Note that for the range 1.5 to 4.5, it makes little difference which of these scales is chosen, the maximum discrepancy being less than 0.3 unit, or less than 10% of the total range of 1.5 to 4.5. Uncertainties

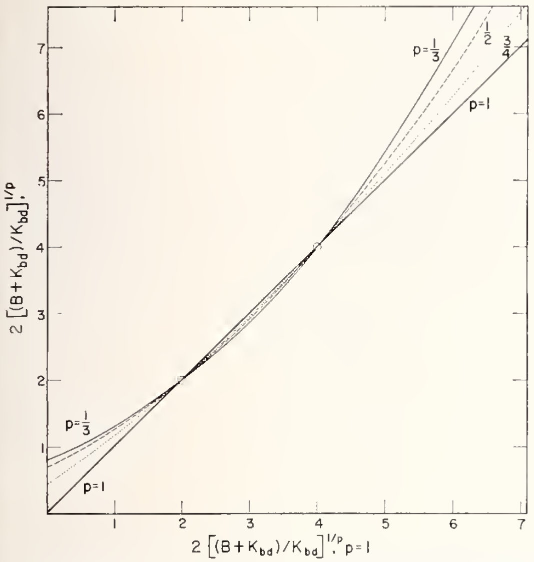

FIG. 1. Scales of $2[(B+K_{bd})/K_{bd}]^{1/p}$ for $p=\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 plotted against the scale for $p=1$. Note that within scale values varying by a factor of 3 (1.5 to 4.5) it makes little difference which of these scales is chosen.

in the values of B_{ij} determined experimentally are somewhat greater than 10% of the range from hi hest to lowest values of B_{ij}.

REDUCTION OF INTERVAL SCALES OF COLOR DIFFERENCE TO RATIO SCALES AND TO ADDITIVE SCALES

We are now in position to indicate the procedures by which interval scales of color difference can be reduced to ratio scales or to additive scales. Given a set of values of B found experimentally on a B scale by the Morrissey–Gulliksen method such that B is linearly connected to some power p of the physical measure D of the stimuli. Required first to derive a ratio scale from the B scale. Required second to derive an additive scale from the B scale.

The direct way to derive a ratio scale is to obtain judgements of the ratio of sizes of pairs of color differences thus:

$$(B_i+K_{br})/(B_j+K_{br})=r_{ij}$$

and to compute K_{br} from Eq. (4), thus:

$$K_{br}=(B_i-r_{ij}B_j)/(r_{ij}-1).$$

The $(B+K_{br})$ scale is the desired ratio scale, and the linear connection of B to some power of D is given by Eq. (7):

$$B+K_{br}=m_{bd}D^p.$$

To find the value of p implied by K_{br} it is sufficient to observe differences 12, 23, and 13 among colors 1, 2, and 3, forming a geodesic or near-geodesic series, and chosen so that $B_{12}=B_{23}$, and to find p from Eq. (8), thus:

$$p=(1/0.301)\log[(B_{13}+K_{br})/(B_{12}+K_{br})].$$

The required additive scale is then given by the basic assumption stated in Eq. (7): $B+K_{br}=m_{bd}D^p$, which implies that $(B+K_{br})^{1/p}$ is proportional to D, the additive physical measure. The $(B+K_{br})^{1/p}$ scale is the desired additive scale.

In the studies so far carried out by the OSA Committee on Uniform Color Scales, the stimuli S_i (color differences) have always been chosen so as to produce responses R_i as nearly identical as possible by using all information available to the Committee either from previously published studies of color spacing or from its own previous work. If the stimuli are chosen so that the ratio of the maximum response R_{max} to the minimum response R_{min} is less than 3, then it is possible to derive an additive scale from the observed values of B_{ij} without making any ratio judgments at all provided it be assumed, as indicated by MacAdam's work,[5] that the value of p lies somewhere between $\frac{1}{3}$ and 1. The method is to observe differences 12, 23, and 13 among colors 1, 2, and 3, forming a geodesic or near-geodesic series, and chosen so that B_{12} is approximately the same as B_{23}. The value of K_{bd} is then found for $p=1$ from Eq. (11) as

$$K_{bd}=B_{13}-B_{12}-B_{23},$$

in which case the desired additive scale is the $(B+K_{bd})$ scale; that is, a correctly spaced D scale is proportional to the $(B+K_{bd})$ scale well within the experimental uncertainties of determining the values of B if K_{bd} is found for $p=1$; see Table II and Fig. 1. No determination of p is needed.

If a ratio scale is desired, K_{br} must be determined by ratio judgments in accord with Eq. (4), and the scale is the $(B+K_{br})$ scale. This is proportional to the $(B+K_{bd})^p$ scale, where p is found from Eq. (8) and K_{bd} is found for $p=1$ from Eq. (11). The ratio scale depends sharply on the value of p; the additive scale much less sharply. If $\frac{1}{3}\leq p\leq 1$, and $R_{max}\leq 3\,R_{min}$, the dependence on p is not significant.

RESTATEMENT OF GUIDING PRINCIPLE

If the MacAdam finding, corroborated by the preliminary determinations of Howett and Judd, that ratio scales are significantly nonadditive (p significantly less than one in the model: $B+K_{br}=m_{bd}D^p$), is accepted, we must also recognize that there cannot be a color space uniform in the sense that the distance between any two points is proportional to the perceived size of the difference between the colors represented by the two points. Consider n equally spaced colors lying along a geodesic, generating $n-1$ color differences of one unit, $n-2$ differences of two units, . . . and one difference of $n-1$ units. All the one-unit differences must be separated in ratio space by the same unit distance, all the two-unit differences must be separated by 2^p, all of the three-unit differences by 3^p, and so on. If, as seems likely, p is about $\frac{2}{3}$, the geodesic would be represented in ratio space by a line far from straight, and these theoretical departures of the geodesic from a straight line are far from being within experimental error. For example, the one-unit differences could be represented by one-unit separations; but the two-unit differences would have to be represented by separations of only 1.59 instead of 2; and the three-unit differences by separations of 2.08 instead of 3.

There remains, however, the possibility of developing a color space that is uniform in the sense that the perceived sizes of the differences between pairs of colors identified by points equally separated in color space will, to a degree of approximation suited to engineering applications, be equal. In such a color space, distance D must be adjusted as closely as possible to be proportional to additive color scales of the form $B+K_{bd}$, where K_{bd} is found from Eq. (6). If it is desired to predict from this color space the ratio judgment for two color differences with different separations D_1 and D_2 in uniform color space, the prediction would be given by D_1^p/D_2^p, where p is evaluated as in Eq. (8).

A FLATTERY INDEX FOR ARTIFICIAL ILLUMINANTS

Illum. Eng. 62, 593-598 (1967)

Judd's statement of the justification and need for this index, which might otherwise seem frivolous, is worthy of serious consideration. His activity in this project was typical of much of his work - reaction to clear needs of industry and the public, which led him into many unconventional applications of colorimetry - and resulted in such novel contributions as this.

In a note pencilled on his reprint of this article, Judd concluded that Eq. (46) can be used, with the full differences between exact and preferred reproductions, as listed in Table II, if the requirement that the flattery index of the reference illuminant be about 90 is abandoned. Judd calculated that it would be about 45, instead.

Consideration of this or a closely related index, which may be renamed "preference index", is currently (1978) on the working program of Technical Committee 3.2 of the CIE.

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TRANSACTION IES

A Flattery Index for Artificial Illuminants

By Deane B. Judd

ARTIFICIAL illuminants are used for a large variety of purposes. Prominent among these are (1) to permit light-dark discrimination of objects (reading, performance of office work, assemblage of non-color-coded parts in manufacture), (2) critical examination of colored objects (as in color matching, inspection of goods being considered for purchase in a salesroom, application of makeup in the home or in a public rest room, or diagnosis of disease by a physician), and (3) appreciative viewing of colored objects (foliage in the home or garden, foods in the dining room or restaurant, or human complexion in the living room, office. or cocktail lounge).

The accepted way to appraise artificial illuminants for the first purpose (light-dark discrimination of objects) is to measure lumen output of the light source, or illuminance of the working plane for a lighting installation.

The recommended way, rapidly becoming accepted, to appraise artificial illuminants for the second purpose (critical examination of colored objects) is by general and special indices of color rendering.[1] These indices of color rendering permit evaluation of the degree to which the artificial illuminant imparts to objects their "true" colors. For illuminants of high color temperature, people regard the color rendered by daylight as the true color of an object; and for illuminants of low-color temperature, people regard the color rendered by incandescent lamp light as the true color. For critical examination of colored objects, the user of the artificial illuminant must choose one giving a sufficient approximation to the truth.

No way has yet been developed to appraise artificial illuminants to be used for appreciative viewing of colored objects. General lighting for the home, office, factory, restaurant, reception room, or ballroom, is not intended for critical appraisal of colored objects. If a lighting installation for these purposes flatters the people viewed there, makes everybody appear to glow with health, it will be preferred to one that is mercilessly revealing of the true state of health. Similarly, a lighting installation for the dining room or restaurant should be such as to make food appear as appetizing as possible. When the food has been placed on the table, the time has already passed for critical examination of the food colors to reveal inferior quality in grocery-store products, or minor errors in preparation. We like to maintain an optimistic viewpoint, even though this involves an element of pretense. We use cosmetics for this purpose, and nobody worries about the element of concealment and subterfuge involved. A lighting installation that promotes an optimistic viewpoint by flattery likewise performs a valuable service, so the purpose of this paper is to devise a way to evaluate the degree to which an artificial illuminant succeeds in flattering people and objects viewed under it.

A paper presented before the National Technical Conference of the Illuminating Engineering Society, September 10-14, 1967, Montreal, Canada. AUTHOR: National Bureau of Standards, Washington, D. C.

Basis of the Flattery Index

The Subcommittee on Color Rendering of the IES Light Source Committee, on 29 August 1966, appointed a group to report on Guide Lines for Color Rendition Calculations. At a meeting of this group held at the Shoreham Hotel, Washington, D. C., on 12 October 1966 (Members: C. W. Jerome, Chairman, I. Meister, G. Pracejus, L. Thorington; Guests: C. L. Crouch, D. B. Judd, D. Nickerson, F. Studer) it was pointed out by Pracejus that the color-rendering index of a light source may correlate poorly with public preference of the source for general lighting purposes. He put in a plea for an "Application Index" based on preference studies that might supplement the color-rendering index. I remarked that the color-rendering index penalizes *any* departure from the true colors of objects produced by the light sources being appraised. If a light source of low-color-rendering index was preferred for general lighting to one of higher-color-rendering index, it must be true that some of the distortions were such as to flatter the object, and that these flattering distortions were preferred by the observers to the true colors. To test whether this hypothesis is true, I offered to draw up a flattery index based on the work of Sanders[2] and Newhall[3] on colors preferred or remembered for various natural objects (complexions, foods, foliage). If the values of flattery index of light sources were to be found to correlate well with the results of preference studies of those light sources, the hypothesis would be established. Such a flattery index, like the color-rendering index, would be based solely on the spectral distribution of the source to be tested. It would have an advantage over an "Application Index" based on preference studies of the light sources themselves because values of it could be calculated by an automatic computer for a light source of any known spectral distribution without the need for any lengthy preference study.

The basis of the flattery index is similar to the color-rendering index except that the target colors will not be the true colors computed for the standard reference source, but instead will be the preferred colors of the test samples viewed under the standard reference source. Note that a flattery index based on this principle might have a higher value for a source different from the reference source than for the reference source itself. The lamp manufacturer thus may be able to develop an artificial source superior in flattery to daylight or to any of the conventional incandescent sources, and therefore capable of achieving greater public acceptance for appreciative viewing of selected colored objects.

The IES Group on Guide Lines for Color Rendition Calculations agreed that such a flattery index would be worth developing, and they individually expressed interest in trying it out.

Tentative Definition of Flattery Index

The general color-rendering index R_a, recommended by the CIE,[1] is defined by the formula:

$$R_a = 100 - 4.6\,\overline{\Delta E_a} \qquad (1)$$

where $\overline{\Delta E_a}$ is the arithmetical mean of the eight values $\Delta E_{a,i}$ of chromaticity difference for the eight test samples, to be calculated as follows:

$$\Delta E_{a,i} = 800\{[(u_{o,i} - u_o) - (u_{K,i} - u_K)]^2 + [(v_{o,i} - v_o) - (v_{K,i} - v_K)]^2\}^{1/2} \qquad (2)$$

where:

- $u_{K,i}, v_{K,i}$ are the UCS-coordinates of any test samples (index i) under the lamp to be tested (index K).
- $u_{o,i}, v_{o,i}$ are the UCS-coordinates of any test samples under the reference illuminant (index o).
- u_K, v_K are the UCS-coordinates of the lamp to be tested (index K).
- u_o, v_o are the UCS-coordinates of the reference illuminant (index o).

The UCS chromaticity coordinates[4] are related to chromaticity coordinates (x,y) and tristimulus values (X,Y,Z) in the standard coordinate system for colorimetry[5] as follows:

$$\begin{aligned} u &= 4x/(-2x + 12y + 3) \\ &= 4X/(X + 15Y + 3Z) \\ v &= 6y/(-2x + 12y + 3) \\ &= 6Y/(X + 15Y + 3Z) \end{aligned} \qquad (3)$$

It will be noted that if the test lamp K has the same spectral distribution as the reference illuminant o, then it will have the same chromaticity coordinates ($u_K = u_o$, $v_K = v_o$) and will yield the same chromaticity coordinates for each of the test samples i ($u_{K,i} = u_{o,i}$, $v_{K,i} = v_{o,i}$). Insertion of these values into Equation (2) shows that $\Delta E_{a,i} = 0$ for each test sample; so the average of the chromaticity differences for all eight test samples must likewise be zero ($\overline{\Delta E_a} = 0$). Substitution of this value into Equation (1) shows $R_a = 100$; that is, the general color-rendering index R_a is defined so that the reference illuminant is characterized by $R_a = 100$, and no other source can have a value of color-rendering index higher than 100. In other words, a value of 100 means perfectly true color rendering, and no source can render colors more than 100 per cent true.

It is proposed to define a flattery index R_f such that the reference illuminant is assigned a value of about 90. We can think of a hypothetical source that would render the test samples precisely as observers prefer to see them. Such a hypothetical source should be assigned the value of 100, but it is by no means certain, or even likely, that there exists a spectral distribution which would render all of the test colors precisely as observers prefer to see them. Observers' preferences are likely to be self-contradictory. For example, it is likely that observers would prefer to see the colors of nearly all of the test samples rendered so as to have somewhat higher saturations than those yielded by the reference illuminant. but if the test samples are large in number, not highly selective, and well-distributed in hue, this is not possible.

To assure that the flattery index R_f for the reference illuminant will have the value of 90, and that a test source yielding precisely the colors preferred for each of n test samples will yield a value of 100, the definition of R_f may be written:

$$R_f = 100 - 10(\overline{\Delta E_{f,K}})/(\overline{\Delta E_{f,o}}) \qquad (4)$$

where $\overline{\Delta E_f}$ is the weighted arithmetical mean of the n values $\Delta E_{f,i}$ of the chromaticity difference for the n test samples to be calculated as follows:

$$\Delta E_{f,i} = 800\{[(u_{o,i} + \Delta u_{f,i} - u_o) - (u_{K,i} - u_K)]^2 + [(v_{o,i} + \Delta v_{f,i} - v_o) - (v_{K,i} - v_K)]^2\}^{1/2} \quad (5)$$

where: $\Delta u_{f,i}$, $\Delta v_{f,i}$ are the chromaticity-coordinate increments that have to be added to the UCS-coordinates ($u_{o,i}$ $v_{o,i}$) of any test samples (index i) for the reference illuminant (index o) to produce the UCS-coordinates of the preferred color of the test sample; and the other symbols are as in Equation (2).

The chromaticity difference $\Delta E_{f,i}$ between the color rendered by the reference illuminant for a test sample and the preferred color for that sample is:

$$\Delta E_{f,i} = 800\ [(\Delta u_{f,i})^2 + (\Delta v_{f,i})^2]^{1/2} \quad (6)$$

and Equation (5) yields this expected result if u_K and $u_{K,i}$ are set, respectively, equal to u_o and $u_{o,i}$ and v_K and $v_{K,i}$ are set equal to v_o and $v_{o,i}$, respectively.

It is proposed to use 10 test samples in the computation of flattery index: samples 1 through 8 used in the general color-rendering index supplemented by samples 13 and 14 used for special color rendering indices. The reasons for this choice are to be explained in connection with evaluation of the preferred colors.

Evaluation of Preferred Colors

As stated by Buck and Froelich,[6] "there is one surface, the average human complexion, which presents itself under nearly every lighting installation and which consciously or unconsciously often becomes the criterion by which the job is evaluated." According to Sanders[2] the preferred color of the human complexion differs importantly from the average actual color by being redder and more saturated. Both the (x,y) and (u,v) chromaticity coordinates of the preferred color of the human complexion found by Sanders are given in Table I for CIE source C. Also shown in Table I are the coordinates computed from the curve reported by Buck and Froelich[6] for the average of 78 Caucasians. They also give the average spectral reflectance curves of eight women with and without cosmetics, and Table I also shows the chromaticity coordinates computed from these curves. The values of $\Delta u_{f,i}$ and $\Delta v_{f,i}$, where i refers to a representative of complexion color, indicated by combining Sanders' preferred color with the Buck and Froelich actual average color are +0.028 and +0.018. The values inferred from the Buck-Froelich measurements of eight women with and without cosmetics are +0.013 and +0.004. I have taken the average ($\Delta u_{f,13} = 0.020$, $\Delta v_{f,13} = 0.011$) to apply to sample 13 (Munsell notation 5YR 8/4) to define the preferred color of complexions and propose to give it 35 per cent of the total weight.

Next in importance are food colors. Buck and Froelich[6] state: "In lighting for homes, restaurants, stores, etc., the appearance of merchandise, of food, and of appointments may be equally as important as that of people." Sanders[2] determined preferred colors for tea, butter, and potato chips, but found that of these three only butter showed a significant discrepancy beween actual and preferred color. The chromaticity coordinates shown in Table I for butter indicate values of $\Delta u_{f,i}$ and $\Delta v_{f,i}$ equal to −0.007 and −0.010; that is, Sanders found that the preferred color for butter is less saturated than the ac-

Table I—Chromaticity coordinates (x, y) and (u, v) of actual colors of some natural objects compared to those of the preferred or remembered colors

Natural Object	Chromaticity Coordinates				Differences (Preferred or remembered minus actual)		
	x	y	u	v	Δu	Δv	$[(\Delta u)^2/(\Delta v)^2]^{12}+$
Average Caucasian complexion[6]	0.377	0.342	0.237	0.323	+ 0.028	+ 0.018	0.033
Preferred[2]	.441	.379	.265	.341			
Complexion (average of 8 women)							
No cosmetics[6]	.373	.341	.236	.322	+ 0.013	+ 0.004	.014
With cosmetics[6]	.395	.345	.249	.326			
Butter							
Actual[2]	.403	.415	.225	.347	− 0.007	− 0.001	.013
Preferred[2]	.375	.386	.218	.336			
Foliage[7]							
Actual	.325	.369	.192	.327	− 0.037	− 0.006	.037
Remembered	.266	.368	.155	.321			
Green grass							
Actual[7]	.346	.415	.190	.342			
Remembered[7]	.248	.415	.132	.333	− 0.058	− 0.009	.058
Remembered[3]	.305	.438	.160	.344	− 0'030	+ 0.002	.030

tual color but of closely the same hue. I propose to take sample 2 (Munsell notation 5Y 6/4) as representative of the color of butter, but for reasons to be explained later take $\Delta u_{f,2} = \Delta v_{f,2} = 0.000$, and give it 15 per cent of the total weight.

The only other natural object whose preferred color seems to differ from the actual is green foliage or grass. Newhall, Burnham and Clark[3] and Bartleson[7] indicate that the colors of foliage and green grass are remembered as considerably less yellowish and somewhat more saturated than they really are. Table I shows that if we take the remembered color as identical with the preferred color the values of $\Delta u_{f,i}$ and $\Delta v_{f,i}$ for foliage would be -0.037 and -0.006, and for green grass they would be -0.058 and -0.009, according to Bartleson, and -0.030 and $+0.002$ according to Newhall, Burnham and Clark. There is excellent agreement as to direction of the difference between the remembered and the actual colors of foliage and green grass, but it is hard to believe that these huge shifts would be preferred for chlorophyll-colored foods (lettuce, spinach, green peas). With rather less support than was found for the preferred colors of complexions and butter we take somewhat arbitrarily, $\Delta u_{f,14} = -0.020$, and $\Delta v_{f,14} = 0.000$, and apply these values to sample 14 (Munsell notation: 5GY 4/4) with 15 per cent of the total weight.

The remainder of the weight (35 per cent) is parceled out among the seven samples (Nos. 1 and 3 through 8) by assigning five per cent of the total to each. The preferred colors, except for samples 2 and 3, are based on the reports by Newhall, Burnham and Clark[3] and by Bartleson,[7] that memory colors are more saturated than original colors. Newhall et al state (p. 56) that "Significantly more purity . . . . (was) required to complete the color matches by memory than was necessary for the simultaneous matches." Bartleson states (p. 77), "There is evidence of increased saturation in the memory colors." Although no published proof has been found that the preferred colors are likewise more saturated than the originals, this seems to be a reasonable presumption. It is consistent with the proposal by Pracejus at the meeting on 12 October 1966 to evaluate the merit of a light source by the area on the CIE-UCS diagram enclosed by test colors Nos. 1 to 8 rather than by the average color distortion. Rather arbitrarily, therefore, we have introduced centrifugal shifts (maximum absolute value of $\Delta u_{f,i}$ or $\Delta v_{f,i}$ equal to 0.01). Such a shift for sample 1 cancels the centripetal shift indicated by the difference between preferred and actual butter colors, and justifies the values $\Delta u_{f,2} = \Delta v_{f,2} = 0.000$. For sample 3 (Munsell notation: 5GY 6/8), close to some chlorophyll-colored foods, the preferred color is taken not only as more saturated, but also somewhat less yellowish; so we have somewhat arbitrarily taken $\Delta u_{f,3} = -0.010$, and $\Delta v_{,f3} = +0.004$. Table II lists the test samples by number, i, gives the Munsell notations of them, the values of $\Delta u_{f,i}$, $\Delta v_{f,i}$, and $[(\Delta u_{f,i})^2 + (\Delta v_{f,i})^2]^{1/2}$. Fig. 1 shows on the 1960 CIE-UCS diagram, the chromaticity points (base of arrows) for the 10 test samples for CIE source D_{6500} and the adopted chromaticity points (heads of arrows) for the corresponding preferred colors.

The adopted values of $\Delta u_{f,i}$ and $\Delta v_{f,i}$ refer to natural overcast sky light and to artificial illuminants (such as CIE source C) intended to approximate it. To the extent that these adopted values are supported by experiment, they may also be taken for light sources of correlated color temperature greater than 3500°K. Some adjustment of them would probably be required for sources of lower correlated color temperature (such as incandescent lamp light and warm-white fluorescent light), but these adjustments should probably be postponed until the extent of correlation between the present tentative definition

Table II—Identification of the 10 test samples used in the definition of flattery index by Munsell notations, preliminary choice of chromaticity differences (Δu, Δv) and distance on (u, v)-diagram between their preferred and actual colors for D_{6500} as the light source, and weight (percentage of the total) used in taking the average to obtain $\overline{\Delta E}_{f \cdot K}$ for insertion in Eq. 4 defining a preliminary form for R_f', of flattery index (see Eq. 4a)

Test Sample	Munsell Notation	Chromaticity Differences Δu	Δv	$[\Delta u^2 + \Delta v^2]^{1/2}$	Weight Per Cent,
1	7.5R 6/4	+ 0.010	+ 0.004	0.011	5
2	5Y 6/4	0.000	0.000	0.000	15
3	5GY 6/8	− 0.010	+ 0.004	0.011	5
4	2.5G 6/6	− 0.010	+ 0.005	0.011	5
5	10BG 6/4	− 0.010	− 0.002	0.010	5
6	5PB 6/8	− 0.006	− 0.010	0.012	5
7	2.5P 6/8	+ 0.004	− 0.010	0.011	5
8	10P 6/8	+ 0.010	− 0.005	0.011	5
13	5YR 8/4	+ 0.020	+ 0.011	0.023	35
14	5GY 4/4	− 0.020	0.000	0.020	15
			Weighted Average	0.01490	

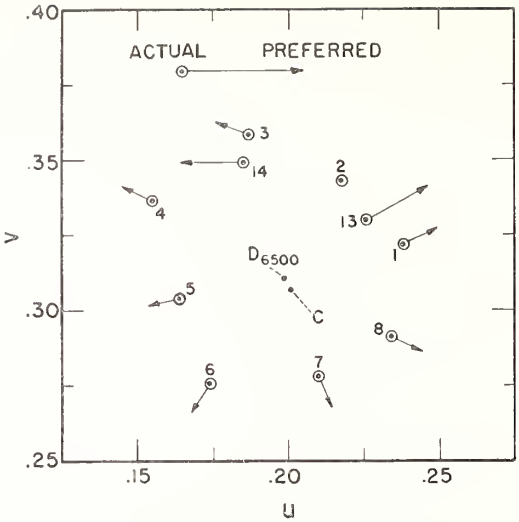

Figure 1. Chromaticities for the ten test samples for reference illuminant D_{6500} shown on the 1960 CIE-UCS diagram (bases of arrows). The chromaticities of these true colors of the test samples for the reference illuminant are compared with those tentatively adopted in Table II as corresponding to the preferred colors of those samples (heads of arrows).

of flattery index and preference studies for light sources above 3500°K has been determined.

Scaling of the Flattery Index

A preliminary definition of flattery index R'_f was based on the chromaticity differences between preferred and actual colors given in Table II from the summary of preferred and remembered colors given in Table I. It will be noted from Table II that the weighted average departure of the preferred colors from the actual colors of the 10 test samples is 0.01490. But, multiplied by 800 this is the value of $\overline{\Delta E_{f,o}}$ appearing in Equation (4) which may now be written simply as:

$$R'_f = 100 - 0.839\ \overline{\Delta E_{f,K}} \tag{4a}$$

If the reference illuminant is taken as the test source, $\overline{\Delta E_{f,K}}$ would have the value $800 \times 0.01490 = 11.92$, and the value of R'_f becomes $100 - 0.839 \times 11.92 = 100 - 10 = 90$, as intended.

Jerome and Nickerson have criticized this preliminary definition (R'_f) of flattery index on the ground that the scale is compressed compared to that of the general color-rendering index R_a by about a factor of five, thus hindering comparisons between the color-rendering index of a light source and its flattery index. This scale compression can be seen by comparing the constant (4.6) in Equation (1) with that (0.839) in Equation (4a). What we need, they said, is a flattery index that evaluates the departures of the preferred colors in exactly the same way that the general color-rendering index evaluates the departures from the actual colors rendered by the standard. Then the values of the two indices (flattery and color-rendering) will be on the same scale so that direct comparison becomes possible. It is known, they said, that increasing the color-rendering of a standard cool white fluorescent lamp, for example, by preparing a deluxe version of it, also makes objects look better; and so the flattery index should be scaled to rise with the color-rendering index by a comparable amount.

To meet the criticism of Jerome and Nickerson, which seems to have considerable merit, requires that the flattery index R_f be defined with precisely the same constant (4.6) used for color-rendering index; that is, the formula has to be:

$$R_f = 100 - 4.6\ \overline{\Delta E_{f,K}} \tag{4b}$$

If the flattery index of the standard is to be kept near 90, the values of the chromaticity differences between preferred and actual colors must be decreased by a factor of five. Table III shows these

Table III—Same as Table II except that the chromaticity differences applicable to the redefinition of flattery index R_f as in Eq. 4b are substituted for those applicable to the preliminary definition R_f'

Test Sample	Munsell Notation	Chromaticity Differences			Weight
		$u_{f,i}$	$v_{f,i}$	Vector Lengths	
1	3.6R 6/4	0.0020	0.0008	0.0022	5
2	5Y 6/4	.0000	.0000	.0000	15
3	5GY 6/8	− .0020	.0008	.0022	5
4	2.5G 6/6	− .0020	.0010	.0022	5
5	10BG 6/4	− .0020	− .0004	.0020	5
6	5PB 6/8	− .0012	− .0020	.0023	5
7	2.5P 6/8	.0008	− .0020	.0022	5
8	10P 6/8	.0020	− .0010	.0022	5
13	5YR 8/4	.0040	.0022	.0046	35
14	5GY 4/4	− .0040	.0000	.0040	15

Table IV—Color-rendering indices (R_a) and flattery indices (R_f) for some artificial light sources of interest

Identification of source			Standard*	R_a	R_f
Super Examolite, Nickerson No. 26			R7500	90.6	86.9
Super DeLuxe Cool-White Nickerson No' 50			P4400	86.0	82.9
Fluorescent White	"	" 74	P3600	63.0	62.4
Standard Cool White	"	" 81	P4500	69.9	66.9
DeLuxe Cool White	"	" 86	P4200	85.2	82.6
Soft White	"	" 152	P3800	72.9	73.0
Color-Improved Mercury (Pracejus, 4/21/67)			P4100	49.9	48.1
DeLuxe-White Mercury (Pracejus, 4/21/67)			P3600	47.2	51.1
Multi-Vapor (Pracejus, 4/21/67)			P4800	67.3	70.2

* Correlated color temperature preceded by *R* for reconstituted daylight, or by *P* for Planckian.

revised values. The average vector length found with the same weights is, of course, one-fifth that shown in Table II: $0.01490/5 = 0.00298$. For the standard source the value of flattery index by this revised definition would be: $R_f = 100 - 4.6 \times 800 \times 0.00298 = 89$, which is, as intended, near to 90.

Table IV shows color-rendering indices (R_a) and flattery indices (R_f) for some artificial light sources of interest.

Summary

A flattery index for light sources and lighting installations intended for the appreciative viewing of objects (complexions, foods, foliage) has been developed as a tentative measure of the degree to which the lighting installation produces the preferred colors of objects. It is modeled after the general color-rendering index; it uses 10 of the 14 test samples selected for testing color rendition; it uses the same method to determine the reference or standard illuminant; and it uses precisely the same scale. It gives complexion color about one-third of the total weight, food colors about one-third of the total weight, and the remaining weight is distributed equally among six test samples not representing complexion or foods. The differences between the preferred and actual colors on which this redefinition of flattery index is based are, however, only one-fifth of those indicated by the literature on preferred and remembered colors. These arbitrary choices of weights and preferred colors are, of course, subject to change. The selection of test samples is, itself, subject to change. Perhaps it would work just as well to use the same eight test samples by which the general color-rendering index is defined.

It is not expected that this redefinition of flattery index has yielded the most sound or most useful measure, and this redefinition is not recommended, in its present form, for immediate practical use. It is offered as a suggestion for consideration and possible study by those concerned with appraisal of the performance of light sources.

If the idea of a flattery index is found to be attractive to the present Subcommittee on Color Rendering, they might undertake to revise, adjust, and validate the present redefinition. In this way a form of flattery index, worthy of adoption for practical use, might be developed after several years of active work.

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INTRODUCTION TO *THEORY OF COLOURS* BY JOHANN WOLFGANG VON GOETHE

(MIT Press, Cambridge, Mass., 1970), pp. v-xvi

As Robert W. Burnham wrote in his review of this book [J. Opt. Soc. Am. 60, 988 (1970)] Judd's Introduction "is a splendid contribution." Burnham quoted Judd's comment that "Instead of attacking the physicists of his own day, who deserved it, for their neglect of the subjective aspect of color, he attacked their predecessor, Newton, who did not." In developing this thought, Burnham quoted from a puzzling passage in Judd's Introduction. It is the quotation from Newton's *Opticks* that appears at the top of page vii. Following that quotation, Judd wrote, "Newton's view that color is a sensation is also widely held today." But, immediately after that, Newton continued, "colors in the object are nothing but a disposition to reflect this or that sort of rays more copiously than the rest; in the rays they [colors] are nothing but their [the rays] dispositions to propagate this or that motion into the sensorium, and in the sensorium they [colors] are sensations of those motions under the forms of colors." Evidently, Newton was as confused as was Goethe as to the nature of colors. When the ambiguities of Newton's pronouns are resolved as in [ ] above, we see that Newton thought of colors equivocally as properties of 1. objects, 2. rays (i.e., light), and 3. a class of sensations, and that he was finally reduced to using a circular definition of the relevant class of sensations. In his attack on Newton, Goethe was certainly mistaken and intemperate, and as Judd wrote, "Goethe's explanation of color makes no physical sense at all", but neither Newton nor any more recent writer has formulated completely consistent ideas about color, either. As Judd concluded, "Perhaps after 160 years, Goethe's mystical theory may come to be recognized as foreshadowing, however dimly, the next important advance in the theory of color."

INTRODUCTION.

The enigma of color has attracted the interest and attention of many of the most gifted intellects of all time. Aristotle, Grimaldi, Newton, Goethe, Hegel, Schopenhauer, Young, Maxwell, Helmholtz, Hering, and Schrödinger all have been intrigued by color and have contributed to our knowledge of it.

Aristotle based his view of color on the observation that sunlight on passage through, or reflection from, an object is always reduced in intensity, or darkened. Since by this operation colors may be produced, he viewed color as a phenomenon arising out of the transition from brightness to darkness, which in a sense it is; or, stated less clearly as it usually is, Aristotle viewed color as a mixture, or blend, or commingling, or superposition, or juxtaposition of black and white. An essential part of this view, widely held up to Newton's time (1642 to 1727), is that all true and pure light, such as light from the sun, has no color, and color must be some sort of constituent or material permeating opaque and transparent objects and media, capable of altering or degrading the pure light incident upon them. Some doubts as to the correctness of Aristotle's view began to arise early in the seventeenth century because of the discovery of what we now name interference colors—colors of thin films, such as soap bubbles—which change markedly with angle of view. These films seem to have every kind of color in them at the same time and to contaminate the incident sunlight in different ways depending on thickness of the film and direction of passage of sunlight through it.

The discovery in 1665 by Newton that light from the sun could be bent to varying degrees by a prism so as to produce a spectrum of colors ranging from red (rays least bent), through orange, yellow, green, and blue, to violet (rays most bent) provided the basis for rejecting Aristotle's view that color comes from objects and permitted substitution for it of the view that color is a property of light. This view has been supported by the great advance in our understanding of the various physical phenomena (interference, scattering, and diffraction as well as refraction and absorption) by which color may appear. This view is, indeed, a view widely held today. It states that wavelength composition of a light beam serves to define its color, and it suggests that physics holds the key to the enigma of color.

We must pause here to make clear that this extreme view that color is a property of light, and of light alone, though it arose among Newton's followers and was based on his discovery, was not shared by Newton himself. He states in a passage, lately much quoted, from his *Opticks:* "And whenever I seem to speak of radiation or rays, coloured or imbued with colour, I should like it always so expressed that it does not sound philosophical or special, but intelligible to the general public; since those ideas are accepted which people, watching experiments of this kind, can themselves comprehend. Indeed, rays, properly expressed, are not coloured. There is nothing else in them but a certain power or disposition which so conditions them that they produce in us the sensation of this or that colour." Newton's view that color is a sensation is also widely held today, but, though Grimaldi, the great Italian pioneer in optics had already expressed a closely similar view in the very year (1665) of Newton's discovery, Newton's followers quickly forgot it.

It must not be supposed, either, that Newton is responsible for the great oversimplification that wavelength of light determines color. Newton's view of light was that it consists of corpuscles or small particles flying through space away from every source of light. The wave theory was not substantiated until many years after Newton's death in 1727 by Thomas Young (1773–1829) who successfully maintained from his experiments on interference that "radiant light consists of undulations of the luminiferous ether.' Independently of Young, the French physicist, Fresnel (1788-1827) disproved Newton's corpuscular theory of light by experiments supporting the view that light is due to wave motion, and that these waves are perpendicular, or transverse, to the direction of propagation of the waves. The wave theory, once established, explained in a simple and brilliant way the colors produced by scattering, diffraction, interference (for example, in thin films), polarization, and refraction. It became easy to ignore Newton's view of color as a sensation, usually, but not always, originating out of radiation, and to say simply that waves of length 400 to 450 nm (billionths of a meter) are violet; 450 to 480, blue; 480 to 560, green; 560 to 590, yellow; 590 to 620, orange; and 620 to 800 nm, red.

To Johann Wolfgang von Goethe (1749–1832), student of the arts, theatrical director, and widely acclaimed author of the master works *Iphigenia at Taurus, Egmont,* and *Faust,* this simple theory of color was the result of mistaking an incidental result for an elemental principle. His study of color phenomena, which extended over many years, had led him to an explanation of color more akin to that of Aristotle than to the new physics that he did not understand. In a period of his life described by one literary critic as "a long interval, marked by nothing of distinguished note" he wrote out a clear and systematic description of all of his extended observations of color phenomena interspersed with the arguments supporting his explanation of them. Instead of attacking the physicists of his own day, who deserved it, for their neglect of the subjective aspect of color, he attacked their predecessor, Newton, who did not. All of physics, he implies, had got off to a misguided start because of its reliance on Newton. He says (Paragraph 726): "A great mathematician was possessed with an entirely false notion on the physical origin of colours; yet, owing to his great authority as a geometer, the mistakes which he committed as an experimentalist long became sanctioned in the eyes of a world ever fettered in prejudices." Again (Paragraph 725): "The theory of colours . . . has suffered much, and its progress has been incalculably retarded by having been mixed up with optics generally, a science which cannot dispense with mathematics; whereas the theory of colours, in strictness, may be investigated quite independently of optics."

Of his own theory Goethe was supremely confident. He writes in his Introduction (page lviii): "From the philosopher, we believe we merit thanks for having traced the phenomena of colours to their first sources, to the circumstances under which they simply appear and are, and beyond which no further explanation respecting them is possible." He believed it his duty to set everybody straight, to expose the fallacies of the Newtonian theory by a detailed application of his own theory to all known color phenomena, and he hoped that the medical practitioner, the investigator of nature, the chemist, the practical dyer, and the artist would applaud his achievement, make use of it, and push its development forward. Up to the present time, this hope has gone largely unfulfilled.

The science of optics took little notice either of Goethe's attacks, or of his account of subjective color phenomena, or of the old-fashioned, nonmathematical explanation of color contained in his *Theory of Colours* published in 1810. The physicists of Goethe's time were of the opinion, of course, that Goethe was the one who had mistaken an incidental result for an elemental principle, not they, and they largely ignored him. Nevertheless, this book caused a great stir among the many readers of Goethe's masterpieces of fiction. According to Helmholtz (*Physiological Optics,* Chapter 19, The Simple Colors): "The great sensation produced in Germany by Goethe's *Farbenlehre* was partly due to the fact that most people, not being accustomed to the accuracy of scientific investigation, are naturally more disposed to follow a clear artistic presentation of the subject than mathematical and physical abstractions. Moreover, Hegel's natural philosophy used Goethe's theory of colour for its purposes. Like Goethe, Hegel wanted to see in natural phenomena the direct expression of certain ideas or of certain steps of logical thought." A similar long-lasting popular interest in England led Charles Lock Eastlake to prepare, after a lapse of 30 years and with sympathetic and scholarly notes, his accurate and free-flowing translation into English, here reissued.

The general public nowadays has a considerable understanding of, and appreciation for, the accomplishments of physical science. In view of the fact the Goethe's explanation of color makes no physical sense at all, one might wonder why it is considered appropriate to reissue this English translation of the *Theory of Colours.* From what standpoints might an intelligent and well-informed nonspecialist approach this 160-year-old book expounding a largely repudiated theory of color?

1. Goethe's *Theory of Colours* can be read, first of all, for the beauty and sweep of his conjectures regarding the connection between color and philosophical ideas, and for the flavor of life in Europe just after our revolutionary war. This book does not have to be studied to be enjoyed. Goethe's subjective, rather mystical, theory of colors permits him to speak most persuasively regarding color harmony and aesthetics; it seems to make green the symbol of both heaven and hope as opposed to red, the symbol

of earthly power. If these conjectures evoke in some readers a responsive thrill, then they must to a degree correspond to an artistic truth. Others will regard them as pure fantasy, charmingly stated, but valuable chiefly as an indication of early nineteenth century beliefs and modes of thought.

2. Goethe's *Theory of Colours* may be read as a guide to the study of color phenomena. In this book a master of prose describes the production of color by all means available to a household in eighteenth century Weimar, and of course, easily available here and now. He tells what equipment (vessels, diaphragms, lenses, prisms, and so on) is required to produce the color, he tells what to do with the equipment, and he tells what you ought to see. Goethe had a passion for careful observation and accurate reporting that may come as a surprise from a theatrical director and famous author of fiction. Most of Goethe's explanations of color have been thoroughly demolished, but no criticism has been leveled at his reports of the facts to be observed; nor should any be. This book can lead the reader through a demonstration course not only in subjectively produced colors (after images, light and dark adaptation, irradiation, colored shadows, and pressure phosphenes), but also in physical phenomena detectable qualitatively by observation of color (absorption, scattering, refraction, diffraction, polarization, and interference).

3. Finally, Goethe's *Theory of Colours* can serve to prepare the reader for unprejudiced consideration of new solutions to the enigma of colors. Goethe's own solutions are interspersed with his directions for experiencing the colors. Some of the explanations are correct, but most of them spring from Goethe's own version of Aristotle's view of color, largely repudiated and so far unproductive. Nevertheless Goethe was a master salesman of his own ideas. A reader who attempts to follow the logic of Goethe's explanations and who attempts to compare them with the currently accepted views might, even with the advantage of 1970 sophistication, become convinced that Goethe's theory, or at least a part of it, has been dismissed too quickly.

For example, Goethe does not deny that light from a slit allowed to pass through a prism permits an observer to see the succession of colors reported by Newton: violet, blue, green, yellow, orange, red (Paragraph 214), but he maintains that, far from being the fundamental phenomenon by which to explain color, it is an unimportant, incidental result of a truly basic fact. This fact (Paragraph 198) is that circumscribed objects must be *displaced* by refraction in order to exhibit an appearance of color. The displacement, not the refraction, in Goethe's view, is the pertinent circumstance. If the edge of a white figure is displaced over a dark boundary by viewing through a prism, he says (Paragraph 204), a narrow blue edge appears next to this boundary, and a broader blue-red border appears next to the blue edge; but if the edge of a black figure is displaced in the same way over the light boundary, a narrow yellow edge appears next to the boundary and a broader yellow-red border appears next to the yellow edge. Both types of border colors thus tend toward red, which is, for Goethe, the most powerful and intense of all colors. A white area circumscribed by a dark surround, viewed through a prism, may thus give rise to the series yellow-red, yellow, white, blue, blue-red; but if the white area be narrowed, the yellow and blue colors can be made to overlap and produce green. The series thus becomes yellow-red, yellow, green, blue, blue-red, in close agreement with what was reported by Newton. But refraction through a prism is not necessary for these effects, says Goethe (Paragraph 239): "... thus the colours produced by refraction may be fitly explained by the doctrine of the semi-transparent mediums. For where dark passes over light, as the border of the semi-transparent accessory image advances, a yellow appears; and, on the other hand, where a light outline passes over the dark background blue appears." Furthermore, Goethe says (Paragraph 247): "Having now sufficiently investigated the exhibition of colour in this phenomenon, we repeat that we cannot admit it to be an elementary phenomenon.

On the contrary, we have traced it to an antecedent and a simpler one; we have derived it, in connexion with the theory of secondary images, from the primordial phenomenon of light and darkness, as affected or acted upon by semi-transparent mediums."

Note how closely Goethe adheres to Aristotle's view that color arises from the transition of brightness to darkness. Note also how utterly Goethe ignores wavelength. He never mentions the word.

The advantage of trying to follow Goethe's explanations of color phenomena is that, by the time you have succeeded in doing so, your thoughts have become so divorced from the wavelength explanation of color, that you can begin to think about color theory relatively unhampered by prejudice, either ancient or modern. Remember that if an observer is in a room illuminated entirely by light from the long-wave extreme of the spectrum, he ought, by the wavelength explanation of color, to see nothing but objects having colors intermediate to red and black. The observable fact is, however, that he sees, in addition to these, pink, yellowish red, purplish gray, and deep greenish blue objects. Again, if the room is illuminated entirely by two kinds of spectrum light, middle-wave and long-wave, he ought, by the wavelength explanation of color, to see nothing but black, red, green, and mixtures of them including various kinds of yellow. Actually he sees objects having colors of all hues including blue. The blue is said by adherents of the wavelength theory of colors to be caused by chromatic contrast, a subjective phenomenon; and this is another way of admitting that wavelength is not all there is to color. Goethe knew all about chromatic contrast (Paragraphs 47 to 50). Whenever we try to predict what colors will be perceived to belong to objects under non-daylight, we are likely to find that the wavelength explanation of color falls down badly. It may be significant that these scenes must involve, as an essential element, images of "circumscribed objects" insisted upon by Goethe (Paragraph 191). Perhaps, after 160 years, Goethe's mystical theory may come to be recognized as foreshadowing, however dimly, the next important advance in the theory of color.

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In order to define a color space, the discrepancy from 2π of the ratio of the circumference of a constant-chroma circle to its radius, C, should be proportional to the area of the circle, i.e., to C^2. The "fan-crinkled" surface discussed on pages 47 and 48 has infinite curvature at C = 0, even though a limited sector of it can be flattened, as mentioned in the discussion.

Errata: P. 39, second column, 13th line from bottom, omit space between ΔE. In Eq.(1), place Δ (Greek cap delta) ahead of E (no space between). P. 43, third column, 7th line from bottom correct $V_x - V_y$ and $V_z - V_y$. P. 44, first column, line 27, insert "of" before Eq.(6); 3rd line from bottom, change "plain" to "plane". P. 46, second column, line 15, remove "-" in front of C_2; third column, line 1, change "Radium" to "radius". P. 48, Eq.(15), in second line, change = to - (minus) and remove bar over C^2. Between second and third line, remove bar over $4\Delta V^2$. Third line, move $4\Delta V2]^{1/2}$ to end of second line. P. 50, Fig. 9 caption, last line, delete "g" before "gray" and "?" after it; delete "i" after "nor". P. 51, second column, line 20, insert comma after "amount". P. 52, first column, line 10, place overbars on V and C, as they appear in Eq.(17); line 19, delete "s" after f_h; between line 21 and 22, remove "s,"; third line from bottom, change Eq.(18) to Eq.(17).

Ideal color space

by Deane B

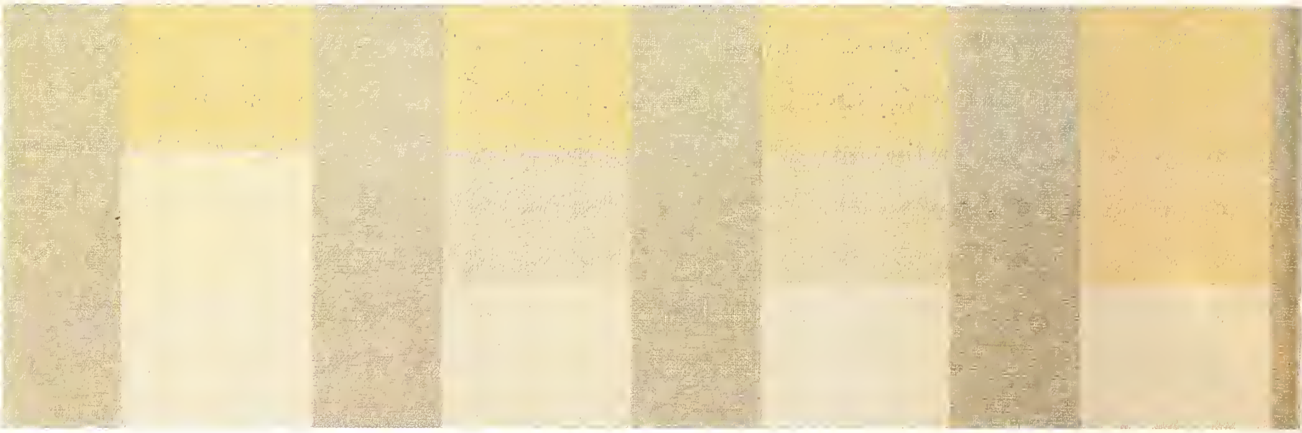


Figure 1.

In the purchase of raw material (mineral products, agricultural products), of finished articles (wearing apparel, automobiles, refrigerators, and so on), but particularly of components to go into a finished article (textiles and thread for wearing apparel, plastics and upholstery for an automobile, enamel for the body of a refrigerator, plastic door for the freezing compartment) it is often necessary to specify ahead of time the color that the product or component must have in order to serve its purpose. The simplest way to do this, still very often used, is to supply with the order a sample, swatch, or chip of the desired color so that the vendor may avoid delivering goods that have to be returned.

Now, strictly speaking, no batch of raw material, no group of finished articles or components, is really uniform in color. It is just as impossible to have two articles, or swatches of precisely the same color, as it is to have two Johansson gauge blocks of precisely the same length. The color differences may be too small for an inspector to detect, but they are there. Quite apart from this theoretical color range, not precisely zero, each raw material, and each finished component, has a color range depending on the method of producing it: Color differences between different samples of a batch of raw material, or between different manufactured components, are often large enough to permit easy detection by the inspector. He comes to recognize, more or less accurately, the color range appropriate to a particular raw material or finished component, and to allow for it. Nevertheless it is easily possible for the producer's inspector to judge that a departure from the color standard is within the color range appropriate to the manufacturing process, and yet to have the inspector for the purchaser reject the article as off-color. This can, and does, occur when both inspectors are conscientiously trying to carry out their duties to the best of their abilities, and with perfect objectivity and good faith. It is obvious that the problem of stating ahead of time the desired color really involves a statement of the range of acceptable colors. The maximum difference between the color standard and an acceptable color is called the color tolerance.

The first step toward specification of color tolerance by the purchaser is to give the supplier two swatches instead of one. The first may be or standard now thought of a the central color of an acc range, in which case the other represents one of the colors boundary of this range. The will be told that any article parts in color from the s swatch by not more than the t swatch will be accepted, and others will be rejected. Altern both swatches may represent ary colors; one on the one sid center of the acceptable colo the other on the opposite si this choice of swatches, the will be told that the acceptab is that "between" the colors two swatches. If the color of a ed item is "between" the colo two swatches, known as lim ards, the item will be accepted

These two-swatch methods fying a color range help the i considerably more than t swatch method of specifying the central color of the range derstandings as to acceptab range are less frequent when swatch method is used. But it ous that the guidance given inspector by two swatches is complete. Exactly what is meant by one color being "between" two other colors? Figure 1 shows four triads of colors, each triad making a horizontal row. The extremes of each triad are intended to be identical; the left extremes are gray; the right, grayish yellow. These two colors might be limit colors for a rather large acceptable range. The central colors are all different. Which, if any, would you say are between the limit colors? One inspector might consider the central color in the first row to be between the two limit colors; another might say, no, it is lighter than either of them; so it should be rejected.

Similarly, the method of one swatch to define the central color, and another to give a color on the boundary of the acceptable range, provides the inspector with very incomplete guidance. If the central sample in the first row of Fig. 1 be taken as the standard for the central color, and if the grayish yellow sample just to the right of it be taken to define a color on the boundary of the acceptable range, this boundary sample shows how far the color of a finished article may depart from the standard in the direction of yellow; but it does not show how much lighter a color may be and still be acceptable, or how much darker, or grayer, or more saturated, or redder, or greener it may be. Even if light, dark, red, green, gray, and yellow limits were to be given (and such limits would certainly help the inspector to make objective and less equivocal judgments of acceptable or nonacceptable), still the inspector would not be able unequivocally to judge whether a finished article whose color departed from standard by being, for example,both darker and greener than the standard fell, or did not fall, within the acceptable range. The fact is that complete guidance by color swatch requires an infinite number of swatches, one for each of the infinitely many directions of color difference from the standard color. Needless to say, complete guidance by color swatch is never provided. The largest number that has come to my attention is twelve, intended to define the color tolerance in each of twelve equally spaced directions.

Now the terms (range, between, boundary, direction) that we have used to discuss the color-tolerance problem express geometrical ideas; and, indeed, there is no other way to discuss color tolerance with any degree of success. Color is a tridimensional quantity. This we see from the fact that an inspector with normal color vision can make color distinctions of only three kinds: light-dark, yellow-blue, and red-green. All other color distinctions are combinations of these three. In this paper we will state the properties of the ideal geometrical model for colors, will describe the properties of some models that have already been constructed so as to see to what extent these properties approach the ideal, and finally we will inquire whether it is possible to construct a geometrical model really possessing these ideal properties.

IDEAL COLOR SPACE

In any geometrical model of color, each color is represented by a point. Since color is tridimensional, the array of points must also be tridimensional; so it is quite natural and useful to speak of the array as a color space. Each point has a particular location in this color space, and the relation of one color to another may be described in terms of the direction and length of the straight line in the model connecting the points corresponding to the two colors.

Ideal color space is a tridimensional array of points, each representing a color, so located that the length of the straight line (straight in the Euclidian sense) connecting any two points is proportional to the perceived size of the difference between the colors represented by the points. An acceptable color range defined by two swatches, one for the central color, the other defining the size of the color tolerance, becomes in ideal color space simply the group of points lying within a sphere whose center is the point corresponding to the color of the first swatch, and whose radius is the distance between the points representing the colors of the two swatches. It will be noted that the point representing the tolerance swatch lies on this spherical boundary; for any other direction of difference from the central color there exists a color point on the spherical boundary defining a tolerance color whose perceived difference from the central color is the same as that to the color of the tolerance swatch. Figure 2 shows a cross-section of this spherical boundary passed through the point representing the central color.

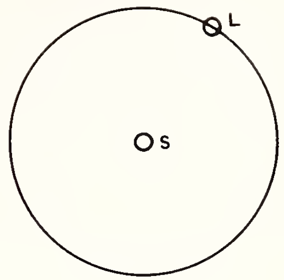


Figure 2: An acceptable color range defined by two-swatches, S for central color, L the size of the color tolerance, becoming in ideal color space the group of points lying within a sphere whose center is the point corresponding to the color of the first swatch, and whose radius is the distance between the points representing the colors of the two swatches. The point for the color standard is at the center of the circular area; that for the tolerance swatch somewhere on the edges of this area. Colors represented by points within the circular area of any such cross-section being acceptable; those colors by the points outside; unacceptable.

What might be meant by one color being between two other colors can also easily be made clear in terms of ideal color space. It might mean that the point for the one color must lie somewhere on the straight line connecting the points corresponding to the two other colors, or that the point lies somewhere in the sphere one of whose diameters is this straight line, or something intermediate to these two interpretations. Figure 3 indicates these possibilities by cross-sections of three possible boundaries: the sphere, the straight line, and an intermediate.

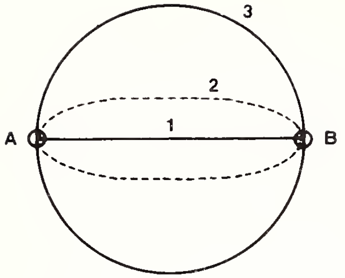


Figure 3: Three possible interpretations of the requirement that to be acceptable a color shall be 'between' two limit colors, defined in terms of ideal color space, indicating possibilities by cross-sections of three possible boundaries: the sphere, straight line, and an intermediate, points A and B representing the two limit colors. The most liberal interpretation 3 of the requirement would accept any color whose point fell within the circular area. The strictest would require the point 1 to fall on the connecting straight line between A and B. 2, the elliptical area, indicates an interpretation of intermediate strictness.

To permit the boundary to be spherical yields the largest number of acceptable colors, but to require the point representing the "between" color to lie strictly on the diameter of this sphere means that the probability of producing such a color approaches zero. This latter interpretation makes the requirement that the acceptable color lie between two other colors an unreasonable requirement. Nevertheless, a close approach to it, in the form of a very eccentric ellipsoid of revolution about this line (see boundary 2 in Figure 3) becomes appropriate whenever the color range of the manufactured component is extended along one direction in ideal color space much more than in any other. For example, trademark colors produced by a transparent or translucent layer of ink laid down on paper by a printing process vary chiefly in the direction corresponding to thickness variation of the ink film. By supplying a swatch or chip of the palest color acceptable, and another of the deepest color, the purchaser can give the printer excellent guidance for the degree to which ink-layer thickness must be controlled. Whether the proportion of colorants in the ink is correct within acceptable limits, is less well controlled; but this may be a less frequent source of misunderstanding between printer and company owning the trademark. More explicit guidance is sometimes given by supplying the printer with six or eight color chips corresponding to six or eight film thicknesses, the two extremes being unacceptable, the remainder acceptable. With this sort of guidance, the meaning of the finished product (trade-mark) having a color "between" a pale limit and a deep limit becomes clear. It means between them on the particular color locus indicated approximately by the six or eight color chips. This locus, in general, is different from the locus of colors defined by a straight line in ideal color space; but the smaller the color tolerance, the more nearly must the two loci coincide.

Straight lines in ideal color space have been given a special name; they are called color geodesics. A geodesic between two points is the path of minimum distance between those points. In any color space not possessing the ideal property that distance between two points is proportional to the perceived size of the difference between the colors represented by those points, the color geodesics will not be precisely straight lines, but will have to be somewhat curved.

If the location of points in ideal color space be defined by the variables, p,q,r, to be plotted along mutually perpendicular axes, then the basic property of ideal color space is that the perceived size, Δ E, of the difference between the colors corresponding to points p_1, q_1, r_1 and p_2, q_2, r_2 would be simply that of Euclidean geometry: 1.

$$E=k[(p_1-p_2)^2+(q_1-q_2)^2+(r_1-r_2)^2]^{1/2},$$

where k is a constant of proportionality connecting the units of the p-, q-, and r-scales with the unit in which the perceived size, ΔE, of the color difference is expressed. The variables, p,q,r, have so far not been discovered, and perhaps the reason is that they do not exist. The discovery of such variables or even a reasonable approximation thereto, would be of such great practical assistance in the specification of color and color tolerances, that many attempts have been made to develop them. We proceed now to a brief consideration of some color spaces that have been developed.

COLOR SPACE BASED ON COLORANT PROPORTIONS

The colors of raw materials are determined primarily by the amount and identities of the mineral, plant and animal pigments naturally occurring in them. Those of manufactured components depend either on such pigments or on synthetic pigments and dyes developed for this purpose by chemists. Each pigment has the property of absorbing more or less strongly for each part of the spectrum the radiant energy incident on it. This property is the chief determinant of the color change caused by introduction of the pigment into a manufactured component. It is possible to specify the color of a pigmented body of known lightdiffusing substrate (paper, textiles, white plastic, white ceramic, or white paint) or of known non-lightdiffusing substrate (clear plastic, glass, or gelatine) in terms of the identities and amounts of the pigments incorporated into it. If these be limited to three pigments or dyes, the amounts of the three colorants can serve to define a color space. Within the gamut of colors producible by various amounts of the three colorants each color corresponds to a point defined by the amounts, and each point so defined corresponds to one, and only one, color. An acceptable range of colors may be specified in terms of the ranges in these amounts; such specifications are quite practical, and much used. Examples are the specification of the color of an element of a color photograph in terms of the amounts of magenta, yellow, and cyan dyes present there, or specification of color in terms of the number of units of Lovibond red, yellow, and blue glasses required to produce a match for the color to be specified. This type of color space, however, cannot be used for all color-tolerance specificatio[ns] because many commercially important colors fall outside of the gamuts of these systems. Furthermore, the perceived size of the color difference caused by adding a fixed amount of any one of the colorants diminishes rapidly with the amount of that colorant already present. The first unit added may produce a color difference from the white, or perfectly clear, color of the substrate that will be perceived as large; but adding one unit when 100 units are already present usually produces a color change scarcely perceptible. Just noticeable differences among near-white colors are represented in colorant-proportion color spaces by relatively small distances compared to those among colors (black, red, green, blue) quite different from white. Color spaces based on colorant proportions thus differ drastically from ideal color space in which the distance between two color points must be proportional to the perceived size of the difference between the two colors.

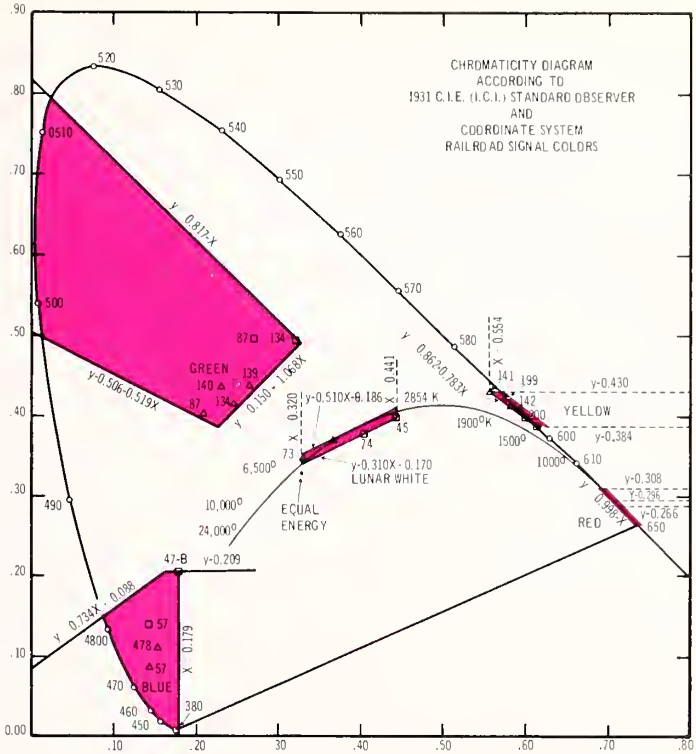


Figure 4 shows, on the (x,y) diagram, the chromaticity ranges adopted by the CIE for red, yellow, green, blue, and white signal colors. Note that on this diagram the acceptable chromaticity range for green signals extends over a much larger area than those for signals for other colors. On the chromaticity diagram implied by ideal color space these areas would be at least approximately of the same size.

CIE-XYZ COLOR SPACE

If more than three colorants, say four, are independently varied with a given substrate, the spectral character of the reflected or transmitted radiant energy can be made to vary with four degrees of freedom; but since the normal human eye can respond in only three independent ways, not all of the different spectral distributions so produced will correspond to different colors. Pairs of different spectral distributions will be obtained whose difference cannot be detected by the eye of the inspector, who sees them as having identical colors. Such pairs of spectral distributions are said to be metameric pairs, and to produce a metameric color match. To produce a color space applicable to all colors, we must build a system that gives the same evaluation to the members of all metameric pairs in spite of their differences of spectral distribution. The colorimetric coordinate system recommended by the International Commission on Illumination (CIE) in 1931 has this property. Since the spectral distribution E_λ of the incident flux is modified by the spectral radiance factor B (λ) of the color chip, then in the CIE system the color is specified by the tristimulus values, X,Y,Z, defined as:

$$
\begin{aligned}
X &= \int_0^\infty E_\lambda \beta(\lambda)\bar{x}(\lambda)d\lambda, \\
Y &= \int_0^\infty E_\lambda \beta(\lambda)\bar{y}(\lambda)d\lambda, \qquad (2)\\
Z &= \int_0^\infty E_\lambda \beta(\lambda)\bar{z}(\lambda)d\lambda
\end{aligned}
$$

where x(λ), y(λ), z(λ) are weighting functions roughly describably as expressing the red-, green-, or blue sensitivities, respectively, of the normal human eye. The tristimulus values, X,Y,Z, of a color correspond to the amounts of certain specified red, green, and blue lights that must be added together to produce a metameric match for that color. The condition for color match of two colors is that simultaneously: $X_1 = X_2$, $Y_1 = Y_2$, $Z_1 = Z_2$. By inspection of Eq 2 it may be seen that if B (λ) is the same function of wavelength for two swatches or samples, then this condition for color match is automatically fulfilled. It may also be noted from Eq 2, that this condition may be fulfilled even though the spectral radiance factors B (λ) are different for the two swatches. These cases are the matameric color matches.

By plotting on mutually perpendicular axes the tristimulus values, X,Y,Z, a color space may be formed. Each point in this space corresponds to one, and only one, color; and each color is represented by one, and only one, point in the space. Any desired color range may be specified in terms of the tristimulus values, X,Y,Z. For example, the requirement that the X value of an acceptable color fall between a maximum value, X_x, and a minimum value, X_n; that its Y value fall between Yx and Y_n; and its Z value, between Z_x and Z_n, specifies a rectangular parellelopiped in this color space. If X_x=X_n = Y_x-Y_n = Z_xx-Z_n, the acceptable range corresponds to a cube in XYZ-space. Automatic color sorters,

Figure 6

based on photoelectric tristimulus colorimetry, have been developed that will throw into one bin any ceramic tile, or plastic component, whose color corresponds to a point within such a rectangular parallelopiped, and will throw into a reject bin, any tile or component whose color corresponds to a point outside this range. This machine inspection does away with the need for a human inspector, and makes practicable the inspection of every component delivered. With human inspection, it is customary to inspect only a small percentage of the delivered components; and the details of the sampling procedure, to avoid misunderstandings, have to be agreed upon beforehand.

As in the colorant-proportion color spaces, an increment of one unit of X, Y, or Z corresponds to a perceived size of color change that diminishes with increase of X, Y, or Z. In CIE-XYZ space, just noticeable differences among near-black colors (X, Y, and Z approaching zero) are represented by relatively small distances compared to just noticeable among colors (white, yellow, red, green) quite different from black. CIE-XYZ color space thus differs drastically from ideal color space. The practical disadvantage is simply that for each color, a separate determination has to be made of how big to make the acceptable range. To a purchaser using the two-swatch method of indicating an acceptable range, this disadvantage may not appear very serious. He is used to making a separate decision for each color when he chooses the tolerance swatch. If, however, an ideal color space could be constructed, the purchaser would have only to decide whether to apply a strict (say, one just noticeable difference), a moderate (say 4 jnd's), or a loose tolerance (say 40 jnd's). Regardless of the desired color, all of the remaining tolerance information could be derived from the ideal space.

CIE-XYZ color space has the advantage that it provides for location of the color point corresponding to any sample, swatch, or chip, by means of spectrophotometry of that sample. The reduction of the spectrophotometric data is accomplished in accord with Eq 2. Other color spaces, including those based on colorant proportions, differ from CIE-XYZ space only in the distribution of the points within the space; and the three variables defining such spaces are determinable functions of the tristimulus values X,Y,Z. It follows that the variables p,q,r, of Eq 1, defining ideal color space, are also, if they exist at all functions of X, Y, and Z; that is:

3. $p = f_p(X,Y,Z)$, $q = f_q(X,Y,Z)$,
$r = f_r(X,Y,Z)$.

CIE-Yxy SPACE

Perhaps the currently most used color space is that in which the Y tristimulus value is taken as one variable and the other two variables, x,y, are the following functions of the tristimulus values, X,Y,Z:

4. $x = X/((X+Y+Z)$,
$y = Y/(X+Y+Z)$

The variables, x,y, are known as chromaticity coordinates of the color, X,Y,Z. A third chromaticity coordinate, z, defined as $z = Z/(X + Y + Z)$, is not necessary because $z = 1 - x - y$. For self-luminous areas the Y tristimulus value corresponds to the luminance of the area. For transparent specimens (non-lightdiffusing glass, plastic, or gelatine) the Y value corresponds to luminous transmittance. For opaque and translucent specimens (paper, textiles, paint, and pigmented plastics and ceramics) the Y value corresponds to luminance factor, The array of points specified by the triplet of variables, Y,x,y, does not quite qualify as a color space. Not every color has one, and only one, point of this array corresponding to it. The point $X = Y = Z = 0$ maps not into a single point in Yxy space, but into the entire plane $Y = 0$. Note from Eq 4 that for this color x and y may take on any value whatsoever. If the Y tristimulus value refers to the luminance of a self-luminous area, this failure to qualify as a color space is purely nominal. Note that the colors of all self-luminous areas are indeed mapped by one, and only one, point in Yxy space because $Y = 0$ does not correspond to a self-luminous area. But if Y corresponds to the luminous transmittance of a non-lightdiffusing object, or to the luminance factor of a light-diffusing object, this failure, though it applies only to ideal black, signals a great expansion of chromaticity scales for near blacks. CIE-Yxy space is used quite satisfactorily to indicate the rather large color ranges acceptable for colored traffic signals (railway, highway, airway, marine). None of these ranges refers to colors at all close to $X = Y = Z = 0$. Fig. 4 shows, on the (x,y)-diagram, the chromaticity ranges adopted by the CIE for red, yellow, green, blue, and white signal colors. Note that on this diagram the acceptable chromaticity range for green signals extends over a much larger area than those for signals of other colors. On the chromaticity diagram implied by ideal color space these areas would be at least approximately of the same size. Variations in the sizes of these areas on the (x,y)-diagram indicate roughly how much (x,y)-spacing departs from the ideal. CIE–Yxy space thus departs from ideal color space not only by having very open chromaticity scales for Y approaching zero, but also by having rather open chromaticity scales for green colors.

Figure 8

CIE-U*V*W* SPACE*

Improvements in chromaticity spacing were achieved by Judd in 1935, MacAdam in 1937, Breckenridge and Schaub in 1939, and Hunter in 1941. These improvements take the form of transformations of the (x,y)-diagram. In 1960 the CIE recommended the transformation proposed by MacAdam in 1937. Instead of the chromaticity coordinates x and y, MacAdam proposed the use of the chromaticity diagram formed by plotting along perpendicular axes the chromaticity coordinates, u,v, defined as follows:

5. $$u = 4x/(-2x + 12y + 3) = 4X/(X + 15Y + 3Z),$$
$$v = 6y/(-2x + 12y + 3) = 6Y/(X + 15Y + 3Z).$$

On the (u,v)-diagram the acceptable chromaticities shown on Fig. 4 correspond to areas much more nearly of the same size for the five signal colors.

An improvement in lightness spacing was achieved by plotting $Y^{1/2}$, the square-root of the luminance factor, instead of Y, itself. This function of Y was used in the original form of the Munsell color system. A further improvement was achieved by using the Munsell value function, V, related to luminance factor relative to magnesium oxide, Y/Y_{mgo}, as follows:

$$6.\quad 100Y/Y_{MgO} = 1.2219V - 0.23111V^2 + 0.23951V^3 - 0.021009V^4 + 0.0008404V^5.$$

This awkward, but more accurate, definition of a uniform lightness scale is used in the current Munsell color system, as to be described presently.

Finally the inordinately wide chromaticity spacing in Yxy-space for near blacks has been avoided by plotting instead of x and y, the differences, $x\text{-}x_o$, and $y\text{-}y_o$, both multiplied by some function of Y. Scofield in 1943 used $Y^{1/2}$ as this function. The chromaticity coordinates, x_o, y_o, refer to an achromatic (white or gray) color.

It is not necessary to go into detail regarding the various combinations of these three kinds of improvements that have been proposed. In 1963 Wyszecki proposed a color space embodying all of these improvements having any practical value, and in 1964 the CIE recommended this color space for use whenever a better approximation to uniform spacing than that yielded by the CIE-XYZ color space is desired. The 1964 CIE-U*V*W* color space is formed by plotting on mutually perpendicular axes the variables U*,V*,W*, related to the tristimulus values, X,Y,Z, of the color as follows:

$$7.\quad W^* = 25Y^{1/3} - 17,\ 1<Y<100,$$

$$U^* = 13\,W^*(u-u_o),$$

$$V^* = 13\,W^*(v-v_o).$$

where u and v are chromaticity coordinates recommended in 1960 by the CIE (see Eq 5), and u_o, v_o are the chromaticity coordinates of an achromatic (white or gray) color. The variables, U*,V*,W*, may be taken as approximations of the variables, p,q,r, defining ideal color space, and the estimate of the perceived size of the difference between any two colors is computed according to Eq 1.

One trivial defect of U*V*W*-space is obvious from inspection of the definition of W* in Eq 7. W* is defined only for luminance factors, Y, greater than 1% of that of the perfect diffuser; so U*V*W*-space is not defined for near blacks. This trivial defect could be avoided by defining W* as the Munsell value function of Y as in Eq 6. The actual definition of W* is a close approximation to the Munsell value function multiplied by 10 for Y greater than 1%, and it was adopted, in spite of the lower limit being greater than zero, because of its superior convenience. Note, for example, that it can be solved explicitly for Y, thus:

$$8.\quad Y = \left[(W^* + 17)/25\right]^3$$

The scales of U*,V*,W have been adjusted so that if the constant, k, of Eq 1 be set equal to unity, the estimates, ΔE, of perceived size of color difference are given in units about equal to 4 just noticeable differences. A color difference of the size of one such unit has been found by the National Bureau of Standards to be the largest difference tolerable for many commercial products, and units of about this size have come to be known as NBS units of color difference. Wyszecki has checked the uniformity of color spacing of CIE-U*V*W*-space for several groups of randomly and rather widely spaced colors, and has found that the estimates ΔE of the sizes of the color differences derived from CIE-U*V*W*-space correlated well with estimates of the sizes made by actual observers having normal color vision. The degree of correlation that he found was that only 10% of the estimates made by actual observers differed from those computed from CIE-U*V*W* space by more than 25%. Since estimates of the sizes of color differences made by randomly selected observers often vary by factors as large as 2, this degree of correlation is quite sufficient to make CIE=U*V*W* space of great practical value in specifying color tolerances and testing delivered goods for compliance with those tolerances. The chief obstacle to universal acceptance of CIE-U*V*W* space for color-tolerance specification is the fact that many industrial firms use other color spaces (Adams chromatic-value space, Munsell color space) that they regard as even closer approximations to ideal space, or use another method (MacAdam ellipses) that they have found to correlate with estimates of the sizes of color difference by actual inspectors more closely than any color space yet developed.

ADAMS CHROMATIC-VALUE SPACE

In 1923 Adams formulated a noteworthy theory of color vision, and in 1942 he developed a color space that would conform to ideal color space if this theory were correct. By this theory the initial response to the color stimulus, X,Y,Z, is one of diminishing returns like the Munsell value function, and the lightness response V_y according to the Adams theory is simply the Munsell value function of Y/Y_{MgO} as given in Eq 6. The initial responses to the other two tristimulus values, X and Z, are denoted V_x and V_z, respectively, and are defined as the Munsell value functions of X/X_{MgO} and Z/Z_{MgO}, respectively. They may be evaluated from Eq 6 by substituting X/X_{MgO} and Z/Z_{MgO}, respectively, for Y/Y_{MgO}. By the Adams theory the response V_y is transmitted directly to the brain as the lightness response, but the other two initial responses, V_x and V_z, do not go directly to the brain but are compared by nerve connections in the retina to the lightness response, V_y, so that the responses transmitted to the brain are $V_x\text{-}V_y$ and $V_z\text{-}V_y$, respectively. The $V_x\text{-}V_y$ response corresponds to red if V_x is greater than V_y; and to green, if it is less. For achromatic (white or gray) colors, that is, colors of the same chromaticity as magnesium oxide (M_gO), $V_{x}\text{-}V_y$ is zero. Similarly $V_{z}\text{-}V_y$ is a blue-yellow response; the excess of V_z over V_y measures the blueness of the response; and that of V_y over V_z, the yellowness. The lightness response, V_y, the red-green response, $V_x\text{-}V_y$, and the blue-yellow response, $V_z\text{-}V_y$, letermine the perceived color, and if suitably scaled and plotted along nutually perpendicular axes these hree variables should produce an ideal olor space. The most widely used caling factors yield the following variables: $0.23V_y$, $V_x\text{-}V_y$, and $0.4(V_z\text{-}V_y)$. Substitution of these variables for p, q, and r in Eq 1 yields the usual Adams chromatic-value formula or estimating the perceived size ΔE of he difference between any two colors pecified by their tristimulus values, X_1,Y_1,Z_1 and X_2,Y_2,Z_2.

$$\Delta E = \{ (0.23\Delta V_y)^2 + [\Delta(V_x - V_y)]^2 + [0.4\Delta(V_z - V_y)]^2 \}^{1/2}$$

where ΔV_y stands for $(V_y)_1 - (V_y)_2$
$\Delta(V_x - V_y)$ stands for $(V_x - V_y)_1 - (V_x - V_y)_2$, and
$\Delta(V_z - V_y)$ stands for $(V_z - V_y)_1 - (V_z - V_y)_2$.

Eq 9 has been considerably used in ndustry, and in the more convenient, hough perhaps less accurate, form in which the cube-root function is taken or the V-function instead Eq 6, it is till under study by Reilly and Glasser who are exploring the improvements n spacing that can be achieved by pplying it to other primary colors han those of the CIE-XYZ system. The conformity of the Adams chromatic-value color space to ideal color pace is remarkably good, and goes far oward extablishing as essentially orrect the theoretical ideas on which it is based. Except for rather wide pacing of extremely saturated colors f purplish blue hue, the Adams hromatic-value space seems to conform to ideal color space even more losely than does CIE-U*V*W* space.

MUNSELL COLOR SPACE

The variables are Munsell hue, Munsell value, and Munsell chroma. The definition of Munsell value in rms of Y/Y_{MgO} is given by Eq 6. In Munsell space, Munsell value is plotted long a vertical scale; colors whose Munsell values are the same correspond to points in a horizontal plain, nd this convention is usual also for CIE-Yxy space, CIE U*V*W* space,

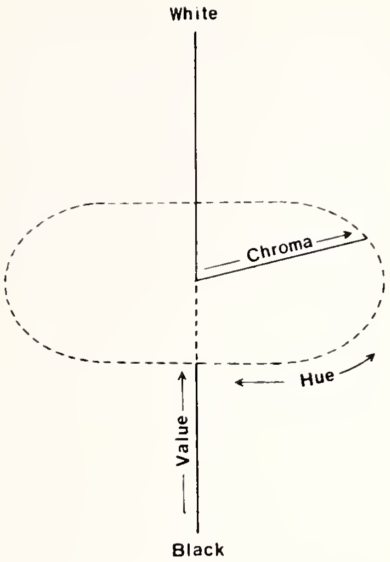


Figure 5: The dimensions of Munsell Color space. The location of points is given in Munsell space by polar coordinates, the angle being specified by Munsell hue, the radius by Munsell chroma. The variables, hue, value, and chroma, serve to locate one, and only one, point; so the Munsell variables define a color space.

and Adams chromatic-value space. The location of points in any one of these planes, instead of being given by rectangular coordinates is given in Munsell space by polar coordinates, the angle being specified by Munsell hue, the radius by Munsell chroma. The Munsell variables, hue, value, and chroma, serve to locate one, and only one, point in space for each color; and each color corresponds to one, and only one, point; so the Munsell variables define a color space. Fig. 5 indicates the dimensions of Munsell color space.

In Munsell color space, colors having constant hue and chroma correspond to a vertical line, and spacing along such lines is in accord with the Munsell-value function determined experimentally to yield uniform spacing; that is, if the value difference between colors A and B is the same as that between colors C and D, all corresponding to points on this vertical line, then these colors have been found experimentally to be such that the perceived difference between colors A and B is closely the same as that perceived between colors C and D. This property of Munsell color space thus conforms to ideal color space.

In Munsell color space, the locus of colors of constant value and constant chroma is a circle whose center is the point representing the achromatic, or neutral, or gray color of that value. Since the spacing along the scales of Munsell chroma was determined experimentally to be perceptually uniform, this property of Munsell color space also conforms closely to ideal color space. Munsell chroma of a color means the departure of the color from the gray of the same value. The locus of colors of constant Munsell chroma regardless of value is a cylinder in Munsell color space centered on the neutral axis, plotted vertical, on which points representing the various grays between black and white are located.

Finally, the colors of constant Munsell hue and constant Munsell value are represented by points lying along a horizontal straight line intersecting the neutral axis in Munsell color space. The selection of colors to be represented along such a line was made experimentally so as to yield color perceptions of constant hue. Such lines in ideal color space would correspond to geodesics between the point representing a chromatic color and that representing the nearest gray. Perhaps colors represented by points along such geodesics do yield color perceptions of constant hue. Such lines in ideal color space would correspond to geodesics between the point representing a chromatic color and that representing the nearest gray. Perhaps colors represented by points along such geodesics do yield color perceptions of constant hue. Schrodinger in 1920 in his important treatment of higher color metrology made this assumption. The direction of such a line is identified in Munsell color space by the Munsell hue notation, and along any locus of constant value and constant chroma the colors have been chosen experimentally so as to be perceptually equally spaced for equal increments in Munsell hue notation. In this respect also Munsell color space corresponds closely to ideal color space.

Munsell color space is defined by giving in CIE-Yxy space the loci of constant Munsell hue and constant Munsell chroma for a number of Munsell values ranging between zero for black and 10 for white as follows: 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Like other color spaces intended to approximate ideal color space, Munsell color space is thus defined by variables that are functions of the tristimulus values, X,Y,Z, even though there are no explicit formulas for these functions. The estimate of the perceived size of the difference between two colors defined by their Munsell notations: $H_1\ V_1/C_1$, and $H_2\ V_2/C_2$, according to distance in Munsell color space has been derived by Godlove from Eq 1:

10. $$\Delta E = \left[2C_1C_2(1 - \cos 3.6\Delta H) + (\Delta C)^2 + (4\Delta V)^2\right]^{1/2}.$$

where ΔH, ΔC, ΔV are abbreviations for H_1-H_2, C_1-C_2, and V_1-V_2, respectively, and 3.6 ΔH gives the angular separation in degrees between the directions from the neutral axis defined by the two Munsell hues, H_1 and H_2. The unit of ΔE in Eq 10 is one Munsell chroma step, or about 6 NBS units.

Munsell color space is largely responsible for the development of the geometrical concept of ideal color space. Munsell color space conforms within experimental error to the basic idea that the perceived size of the difference between two colors is proportional to the distance between the points representing them in Munsell space for three kinds of loci: the vertical lines of constant Munsell chroma and hue, the horizontal lines of constant Munsell value and hue, and the circles of constant Munsell value and chroma. Whether the same proportionality holds for loci other than these three kinds will be discussed after consideration of the MacAdam ellipses.

Munsell color space is exceptionally convenient for color and color-tolerance specification. For swatches viewed against a gray background Munsell hue corresponds closely to the hue of the color perception; Munsell value, to its lightness; and Munsell chroma, to its saturation. The Munsell Book of Color is widely distributed, and copies have been continously available by purchase since 1929. This book shows color chips identified by Munsell hue, value and chroma (H V/C); so the purchaser may see by direct inspection the size of a departure from the standard that corresponds to any color tolerance set in Munsell terms.

IS IT POSSIBLE TO CONSTRUCT AN IDEAL COLOR SPACE?

Three of the color spaces just discussed (CIE-U*V*W*, Adams chromatic value, and Munsell) approach to a considerable degree ideal color space. CIE-U*V*W* space was empirically developed so as to have a maximally simple connection to CIE-XYZ space. Adams chromatic-value space is based on the Adams theory of color vision; only the Munsell value function and scaling factors were experimentally determined. Munsell color space is based on color scales of three kinds, hue-scales, value-scales, and chroma-scales, each adjusted experimentally to be perceptually uniform. These scales are, however, combined in accord with the theoretical concept of ideal color space to yield the formula (Eq 10) for perceived size of color difference; so Munsell color space has a partly theoretical and partly experimental basis. In all of these spaces equally luminous colors (colors for which Y is constant) are represented by points in a horizontal plane.

In 1942 MacAdam reported the results of an experimental study of the sensitivity of one observer (P. G. Nutting) to chromaticity variations among equi-luminous colors specified by points in the (x,y)-plane. The only theoretical assumption that he made was that chromaticities perceived as equally different from a standard chromaticity should correspond to points on an ellipse in the (x,y)-plane centered on the point representing the standard chromaticity. Such ellipses were derived to represent his data for 25 standard chromaticities. Each one of these ellipses could be rectified (transformed into the circle demanded by ideal color space) by a change in coordinate system. It is necessary only to take one axis of the new coordinate system parallel to the major axis of the ellipse and to take the other axis parallel to the minor axis, and then adjust the scales along these axes to transform the ellipse into a circle. In this way 25 local maps of the two-dimensional diagram representing equi-luminous colors could be derived such that each local map conforms to a plane in ideal color space. The trouble was that these local maps did not fit together to form a large, all-inclusive plane in ideal color space. Much as local maps of the earth's surface can be shown with sufficient accuracy on plane surfaces, but the map of the whole earth requires a surface of constant positive curvature, so MacAdam found that to map the entire gamut of chromaticities with uniform spacing required a surface far from flat. It had a dome (positive curvature) in the middle, representing near-grays, and ruffles (negative curvature) around the edges.

This finding that equiluminous colors could not be represented with uniform perceptual spacing by points on a plane surface considerably disturbed those who had been thinking about color and color tolerances in terms of ideal color space. If you cannot make a uniform map of equiluminous colors on a plane surface, then how can you make a uniform map of all colors in accord with ideal color space? Geometers deal with curved space. They say that curved three-dimensional space can be embedded in uncurved space of dimension higher than three just as curved surfaces may be embedded in uncurved three dimensional space; but it is hard to understand what bearing this would have on color tolerances. It all seem to add up to the conclusion that strictly ideal color space cannot b developed.

Adherents of ideal color space tend ed to ascribe to experimental erro MacAdam's finding that a curve surface is required for the uniforn mapping of chromaticities, and to point out that the data came from one observer only. Maybe data by another observer would indicate a different surface. It was noted by Farnsworth that by changing Nutting's data only slightly beyond the experimental uncertainties, they could be fitted onto a plane. MacAdam did not see why his experimental solution for chromaticity tolerances should be forced to fit any preconceived theoretical idea. To get a purely experimental estimate ΔE of the perceived size of the chromaticity difference between two colors of the same luminance, he wrote the general formula for an ellipse in the (x,y)-chromaticity plane:

$$\Delta E = \left[g_{11}(\Delta x)^2 + 2g_{12}(\Delta x)(\Delta y) + g_{22}(\Delta y)^2\right]^{1/2} \qquad 11.$$

where the metric coefficients, g_{11}, g_{12}, g_{22}, were found by interpolation on the (x,y)-chromaticity diagram among the 25 sets of values derived from Nutting's data. Two color technologists, Davidson and Ingle, particularly concerned with setting industrial color tolerances, independently checked results computed from Eq 11 against their data on acceptable color ranges arrived at by consensus of inspectors, and found better correlation than could be obtained by any of the then existing approximations to ideal color space. Since that time a considerable segment of industry in America has used the MacAdam ellipses to set and administer color tolerances.

The improved correlation obtainable by departing from the properties of ideal color space seemed to indicate that this model departs significantly from the facts of perception of the sizes of color differences. If color space really is curved, this would be one reason why ideal color space cannot exist. Although sensitivity ellipses derived by the MacAdam method do indeed vary considerably from one observer to another, those who, like myself, have been used to dealing with color differences in terms of ideal color space, have begun to think of other kinds of experimental evidence that this kind of color space does not exist.

MUNSELL HUE AND CHROMA STEPS COMPARED

For the assessment of the perceptual size of color differences involving both a hue and a chroma component, it is necessary to know the relative importance of the Munsell hue and the Munsell chroma steps.

If Munsell color space is an ideal color space, the perceptual importance of the Munsell hue step relative to that of the Munsell chroma step would be that indicated by Eq 10. If we consider two colors of the same value and chroma ($\Delta V = 0, \Delta C = 0, C_1 - C_2 = C$) that differ by one Munsell hue step ($H = 1$), Eq 10 shows that the perceptual importance of the color difference is:

$$\Delta E_{\Delta H=1} = C\left[2(1 - \cos 3.6°)\right]^{1/2}$$

which indicates correctly that the perceptual importance of one Munsell hue step is proportional to the chroma C of the two colors. One Munsell hue step between two colors of rather high chroma, say /10, is perceptually bigger in ideal color space by a factor of 10 than that between two near-grays of chroma equal to /1. If now we consider two colors of the same hue ($\Delta H = 0$) and the same value ($\Delta V = 0$) that differ by one chroma step ($\Delta C = 1$), Eq 10 shows that the perceptual importance of the color difference is:

$$\Delta E_{\Delta C=1} = 1,$$

and this corresponds to the statement that the unit of color difference used in Eq 10 is one chroma step. The perceptual importance of one Munsell hue step relative to that of one Munsell chroma step is thus:

$$\Delta E_{\Delta H=1}/\Delta E_{\Delta C=1} \qquad 12.$$

$$= C\left[2(1 - \cos 3.6°)\right]^{1/2}$$

$$= C\left[2(1 - 0.998027)\right]^{1/2}$$

$$= C(0.003946)^{1/2} = 0.0628_2 C$$

If the whole 100 Munsell hue steps of the hue circuit at chroma C were stepped off, one at a time, the total color difference would have the size of 6.282C. This total corresponds in ideal color space to the length of the perimeter of a regular polygon of 100 sides inscribed in a circle of Radium C. If the hue circuit were stepped off by hue intervals approaching zero, the total difference would correspond in ideal color space to the circumference of the circle, itself, whose length is $2\pi C = 6.28_3 C$.

As far as I know, the very first formula for the perceptual size of color differences ever published is the 1936 Nickerson index of fading in which the relative importance of Munsell hue, value, and chroma steps was evaluated experimentally for colors viewed with gray surround whose hues differed by less than 10 steps. The Nickerson index, I, of fading is:

$$I = (C/5)(2\Delta H) + 6\Delta V + 3\Delta C, \quad \Delta H < 10. \qquad 13.$$

By this formula, the contribution of one Munsell hue step would be: $I_{\Delta H=1} = 2C/5$; and the contribution of one Munsell chroma step would be: $I_{C=1} = 3$. The perceptual importance of one Munsell hue step relative to that of one Munsell chroma step determined experimentally by Nickerson in 1936 for hue intervals of less than 20 steps is thus: 14.

$$I_{\Delta H=1}/I_{\Delta C=1} = 2C/15 = 0.1333\,C$$

and this differs from the maximum ratio (0.0628 C) permitted by ideal color space by slightly more than a factor of 2. Some adherents of ideal color space rejected this violation and ascribed it to experimental error. They welcomed the Godlove formula (Eq 10) because it is in strict accord with ideal color space. Maybe this was a backward step.

If we ask of the MacAdam surface, on which equally important chromaticity differences correspond to equal distances, what would be the perceptual importance of one Munsell hue step relative to that of one Munsell chroma step, we get the answer that it depends on the chroma. For colors of very small departures from gray (very small Munsell chromas) the ratio should be 0.0628 C, because a strictly local map accords with ideal color space. For somewhat higher chromas, those corresponding to the end of the dome-shaped portion of the MacAdam surface, the ratio should be less than

0.0628 C; and for high chromas extending into the ruffled portions of the MacAdam surface the ratio might be greater than 0.0628 C. The Nickerson index of fading, as we have seen, implies for colors differing in hue by less than 10 Munsell hue steps a ratio of 0.1333 C. Fig. 6 shows two clusters, each of 7 colors intended to be equiluminous, that is, of the same Munsell value, presented on a gray surround of somewhat lower Munsell value. This surround color is chosen because the MacAdam ellipses were derived for this condition. The central colors of the two clusters are intended to be identical grays; the six peripheral colors are intended to differ from the central gray by amounts that are perceptually equal (constant Munsell chroma), and the hues of the six peripheral colors are intended to be equally different (different by one-sixth of the hue circuit, or by $\Delta H = 100/6 = 16\ 2/3$. The first cluster shows small departures from gray (Munsell chroma about 2) the second, large departures (Munsell chroma about /8). If the reproduction of the colors in Fig. 6 accords with the above-stated intentions, and if you perceive hue differences relative to chroma differences in accord with ideal color space (that is, in accord with Eq 1 or Eq 10), you should see the radial color differences in both clusters as precisely the same size as the peripheral differences. Note the peripheral colors of each cluster correspond to the apices of a regular hexagon in ideal color space composed of six equilateral triangles. Note also that Eq 10 for $C_1, = C_2, = C\ \Delta C = 0$, $\Delta V = 0$, $\Delta H = 100/6$, indicates:

$$\Delta E_{\Delta H = 100/6} = C\left[2(1 - \cos 60°)\right]^{1/2} = C$$

If your appraisal accords with the Nickerson index of fading, you should see in both clusters the peripheral (Munsell hue) differences considerably larger than the radial (Munsell chroma) differences. If your appraisal accords with the MacAdam surface, you should see the radial differences as larger than the peripheral in the first cluster, but not in the second. Many people will say that it makes no sense to ask them to compare the perceptual size of a hue difference with that of a saturation difference. They say that the two differences are quite different in nature, which is true, and that the task is impossible. Others will say that the comparison is possible, but can be made only with very low precision, and this is also true. Nevertheless color inspectors faced with the definition of an acceptable range of colors by the two-swatch method, one swatch for the central color, the other for the tolerance, have to make these judgments repeatedly as a part of their everyday task, and the judgments have to be in accord with those made by the ultimate consumer a large fraction of the time or the employer will find somebody else to do the inspecting.

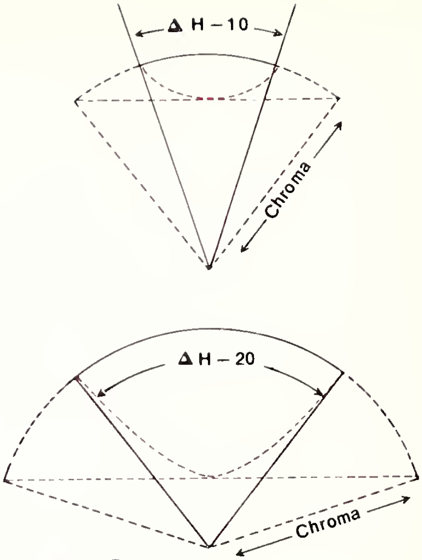


Figure 7:

.... Color geodesics (dotted lines) implied by weighting Munsell hue steps twice as much, relative to Munsell chroma steps, as that implied by the Euclidean geometry of ideal color space, shown on the Munsell hue-chroma diagram, and derived from the fan-crinkled surface.

———— Half extended fan.

– – – – Fully extended fan.

Note that for a hue interval of 20 Munsell steps the length of the geodesic is not much less than 2C; even for the 10-step hue interval the geodesic departs considerably from the chord corresponding to Euclidian geometry [Godlove formula (5)].

The Nickerson index of fading does not imply that the chromaticity surface has to be curved to make distance between two points proportional to the perceived size of the difference between the equiluminous colors represented by the points. It suggests for each segment of 10 Munsell hue steps (one-tenth of the hue circuit), the kind of a surface produced by a half-extended fan that, when fully extended, covers about one-fifth of the circle. The number of ribs in the fan may be increased indefinitely without altering the property that peripheral distances along the surface are about double those on a plane surface for the same angular interval. Such a surface might be called a half-extended fan-shaped surface, or a fan-crinkled surface. It is composed of a large number of triangles, the plane of each triangle making a dihedral angle of about 60° with its neighbor. This surface is not curved because it may be made into a plane surface simply by extending the fan. All points of a fan-crinkled surface may be made to come as close as desired to the central plane simply by increasing the number of ribs in the fan.

Increasing the hue range of a fan-crinkled surface even moderately beyond the 10 hue steps for which the Nickerson index of fading was experimentally verified produces some rather surprising results. For example, if the hue range of the fan be increased to 25 Munsell hue steps (one quarter of the hue circuit), extending the fan-crinkled surface to form a plane surface yields a semi-circle, see Fig. 7. Straight lines on this extended surface should conform to geodesics for colors of constant Munsell value. The straight line bridging the interval of 25 Munsell hue steps is simply the diameter of this semi-circle. The increase in the hue range of the Nickerson index of fading from 10 to 25 thus implies that the shortest path from one hue, say 5R, to another differing by 25 Munsell hue steps, say 10Y, passes through gray and that the total length of the geodesic is 2C. Increasing the hue range of the fan-crinkled surface to 50 Munsell hue steps makes the extended plane surface into a circle. The shortest distance on this circle bridging a hue interval of 50 is likewise 2C. It is, o

:ourse, not legitimate to jump directly rom one end of the fan to the other; o the indicated geodesic would pass rom, say 5R, to N (gray) with a ength of C, and then back up the ame line, which now refers to the other end of the extended fan, to the pposite hue, 5BG, with a length of C, naking a total of 2C. This result agrees vith ideal color space, and can be btained from Eq 10; but the implica-ion that gray is both precisely etween 5R and 10Y and precisely etween 5R and 5BG is rather hard to ake. Furthermore, by this model, gray es precisely between any two hues iffering by 25 or more and 50 or less lunsell hue steps. It would seem that he super-importance of a Munsell hue :ep found to apply to small hue inter-als progressively diminishes as the hue iterval becomes larger. A formula xtending the Nickerson index of fad-ig to hue intervals of any size by pplying this idea can easily be written y adjustment of Eq 10. We need only iultiply the first term by a function f ΔH varying smoothly from a num-ical value of 4 for ΔH approaching ero, to a value of unity for ΔH ap-roaching 50. One such function is $[4/3 - \cos 3.6 \Delta H]_2$, and by applying this inction the revision of Eq 10 ecomes:

$$I = \left[32C_1 C_2 (1 - \cos 3.6\Delta H)/(3 = \cos 3.6\Delta H)^2 + \Delta C^2 + 4\Delta V^2\right]^{1/2} \qquad 15.$$

here does not seem to be a geometri-l model agreeing with Eq 15; at least have not been able to think of one. any case, if we believe the Nicker-n index of fading, it seems that we ust believe that the best approxima-on to ideal color space that can be veloped for colors viewed with a ay surround will fail to satisfy Eq 1 factors greater than two.

IMINISHING RETURNS IN OLOR-DIFFERENCE PERCEPTION

Another kind of experimental evi-ence that ideal color space does not xist might be called the law of dimin-hing returns in color-difference erception. It has recently been inde-endently pointed out by two mem-ers, MacAdam and Helm of the OSA Committee on Uniform Color Scales, that if three colors, A, B, C, are equally spaced along a geodesic, the perceived size of the difference between colors A and C is importantly less than twice the perceived size of that either between colors A and B or between colors B and C. The double difference is perceived as somewhere between 1.4 and 1.8 times the size of either of the two single differences. Perhaps the just-discussed diminishing of the super-importance of the Munsell hue step relative to a fixed chroma interval as the hue interval is increased is a special case of the law of diminishing returns in color-difference perception.

Now it is easy to arrange three points in space, one for each of the three colors, A, B, C, so that the distance AC is 1.6 times distance AB or BC. This arrangement corresponds to the apices of an isosceles triangle whose base is 1.6 times as long as either of the two equal sides. We cannot pretend, however, that the space in which this arrangement of points is located corresponds to ideal color space. In ideal color space color geodesics are straight lines; this arrangement of points implies that the color geodesic passing through A, B, and C is a broken line. If we believe the law of diminishing returns, and if we wish to save something of the concept of ideal color space, we might drop the requirement that in ideal color space the distance between two points shall be proportional to the perceived size of the difference between the colors represented by the points, and be satisfied with the less stringent requirement that color differences perceived as equal shall correspond to equal distances in ideal color space. Even the strictly local maps of chromaticity formed by rectifying the MacAdam ellipses correspond to ideal color space only in this less stringent sense, if the law of diminishing returns in color-difference perception be accepted. MacAdam has revised Eq 11 to read:

$$\Delta E = \left[g_{11}(\Delta x)^2 + 2g_{12}(\Delta x)(\Delta y) + g_{22}(\Delta y)^2\right]^{p/2} \qquad 16.$$

where p is a number somewhere between 0.3 and 0.8. Any value of p less than unity corresponds to some degree of diminishing returns in color-difference perception.

INFLUENCE OF SURROUND COLOR

The final kind of experimental evidence that ideal color space does not exist has to do with the influence on perception of size of color difference exerted by the colors surrounding the two whose difference is being appraised. The only way an inspector can avoid having a surround when he compares the sample of delivered goods with the swatch defining the desired color is to have either or both the sample and standard swatch so large that together they fill his entire visual field. Although I have seen standard textile swatches as large as one meter square so as to permit convenient fulfillment of this condition, it is by far more usual that practical considerations force the inspector to view the sample and standard colors against some other color serving as background.

It is well-known that the perception of a target color, though primarily determined by the tristimulus values of the target color, itself, also depends importantly on the tristimulus values of the surround color. Perception of a target color is thus a function of at least six variables. Fig. 8 shows strips of four slightly different yellowish green colors, one half against a yellowish green background, the other half against a violet background. You will note that the perceptions of these yellowish green colors have been shifted toward higher saturations by the violet surround, and toward gray by the yellowish green surround. Thus it is impossible to predict simply from the tristimulus values of the light reflected from the targets themselves what the perceptions will be. A specification of the kind of light coming to the eye of the observer from the surround is also needed. Any color space, like the Munsell space, in which location of the color point is intended to correlate with the perception of the target color, and which is based only on the variables defining the target color, cannot possibly hold for all surround colors. Strictly speaking, it can hold for but one surround color; and practically speaking it can hold approximately only for a limited range of surround colors. Munsell color space correlates with color perception to a good approximation for any of a series of surround colors ranging from middle gray to white. For any of these surround colors, Munsell hue corresponds fairly well with the hue of the perception; Munsell value, with its lightness; and Munsell chroma, with its saturation.

How then can we hope to have a color space, like the CIE-U*V*W*, the Adams chromatic value, or the Munsell color space, based only on the tristimulus values of the target colors, conform to the characteristic of ideal color space that equally different colors are represented by equally distant points? Do these spaces also refer only to a limited range of surround colors? There is a basis for hoping that the perceived size of the difference between two colors is relatively unaffected by the surround color. It has been noted from Fig. 8 that the influence of a chromatic surround is to shift the perception toward the complementary color. If the perceptions of both of two nearly identical colors being compared are shifted by nearly identical amounts, the perception of the size of the difference between the two colors might be left nearly unaltered by a change in surround color. If you pay attention to the size of the difference perceived by you between any two contiguous members of the series of four yellowish green strips shown on Fig. 8, you can see to what extent color-difference perception by you remains unaffected by a change in surround color.

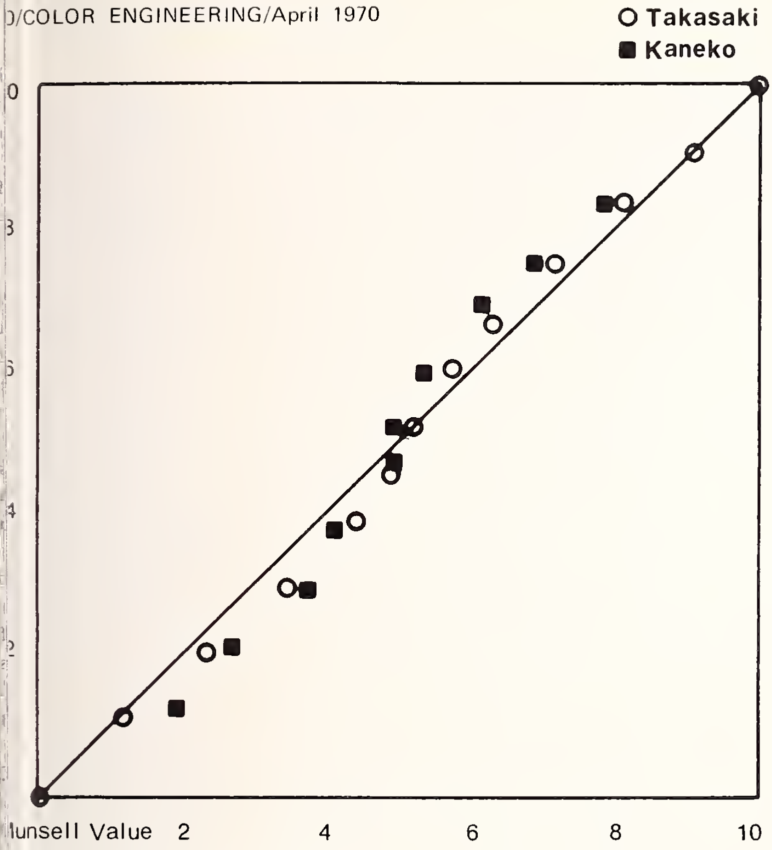
It is, however, recognized by inspectors that there are indeed some very unfavorable choices of surround color. If the standard is a near-white and the tolerance swatch is a slightly different near-white, use of a black surround can make the perceptions of these two colors identical; that is, they both are perceived to be identical near-white colors. One way to think of this phenomenon is that the black surround induces so much white into both the standard and the tolerance color that the induced white drowns out the small difference between them. Similarly, two slightly different near-blacks can be made to appear identical by using a white surround. It is common practice to select a surround color not much lighter, nor much darker, than the two colors to be compared. The determination of the Munsell notation of any unknown color by comparison with the color chips of the Munsell book can be carried out by means of a surround of middle gray (N 5/); but this determination can be made more precise by using a light-gray mask (say N 9/) for near whites, and a dark gray (say N 1/) for near-blacks.

In 1933 Schönfelder carried out an experimental study of the influence of the surround color on precision of color matching, and he announced what may be called Schönfelder's law that the most favorable surround color is the average of those being compared. This law implies that a large chromatic difference between surround color and the average of the two colors being compared, as well as a large lightness difference, can interfere with color-difference perception. By this law the perceived sizes of the differences between the yellowish green strips shown in Fig. 8 should be smaller for the violet surround than for the yellowish green surround which should be the optimum choice.

If we believe Schönfelder's law, and there seems to be no contrary evidence, we must believe that a color space giving a good approximation to ideal color space for colors viewed with a white surround, must give a much poorer (by at least a factor of two) approximation for colors viewed with a black surround. In spite of the fact that color-difference perception is less dependent on surround color than is color perception, itself, a marked change in surround color requires a redefinition of the approximation to ideal color space.

Recently Jameson and Hurvich established experimentally that to a first approximation the change of color perception of any color induced by a shift of surround color is proportional to the size of that shift. Thus, to a first approximation all of the yello ish green colors of the strips shown Fig. 8 should be shifted uniformly b substituting the yellowish green f the violet surround, leaving the diff ences between the colors of contig ous strips unchanged. This fi approximation to the facts does n conform to Schönfelder's law. St more recently, Takasaki confirmed t validity of this first approximatio but also found that the change i duced in the perceptions of colors ve close either to that of the first su round, or to that of the second, is s nificantly higher (by at least a fact of two) than the change induced in t perceptions of colors considerably d ferent from either surround col Takasaki called this phenomenon t crispening effect. The crispening eff is in agreement with Schönfelder's l that the surround color most facili ting the discrimination of two colors the average of those two colors. Fig shows lightness determined by Tal saki for six observers as a function Munsell value for a middle gray (N surround. The crispening effect corr ponds to an increase of slope at Mu sell value equal to 5/. Kaneko's s more recent redetermination of lig ness scales, also shown on Fig. reveals a similar crispening effect.

If construction of a color space planned with the intention of appro mating ideal color space for color c ferences judged against a surround fixed color, it is obvious that to co form to Takasaki crispening, or Schönfelder's law, the color of the s round must be a singular point arou which the color spacing must be larged. Furthermore, if construction another color space is envisaged conform to color-difference perc tion for colors viewed with a fixed s round of some different color, a si lar singular point must be allowed f but it will appear at a different lo tion in the space. In a surface corr ponding to equiluminous colors t singular point would appear at apex of a dome, thus requiring t surface to have positive curvat around the singular point. Perhaps central dome in the MacAdam surf may be ascribed to Takasaki crisp



gure 9: *Influence of the surround color fixed at middle gray (Munsell N5/0) on e lightness scale. Kaneko found his lightness scale by choice of grays to produce a ale perceived as having equal lightness steps. Takasaki derived his lightness scale om determinations of the gray viewed with one gray surround that appeared ually light as another gray viewed on a different gray surround. Note that the htness scale defined by the Munsell value function does not apply to a surround lor and at middle g gray? nor i to a surround of any fixed gray.*

g. MacAdam used a gray surround hose luminance was one-half that of e target colors. Since Kaneko and akasaki both found experimental evi-ence of lightness crispening, the ales in all directions, not just those constant lightness, from the point presenting the surround color would ave to be opened up. This must be hat is meant by curvature of color ace being different from zero (posi-ve) in the neighborhood of a point. ispening thus also implies that it is ipossible to develop ideal color space the colors to be discriminated are ewed with a surround of fixed color.

If we are going to keep alive the ope of constructing an ideal color pace, we must not require that the olors to be distinguished be judged ith a fixed surround color. If, how-ver, every pair of colors were to be judged either with a surround color equal to the average of the colors forming the pair to conform to Schönfelder's law, or if the two colors had such an angular extent that together they filled the visual field, then color space would not have to have any singular points, and perhaps an ideal color space could still be constructed.

ANGULAR SEPARATION OF TARGET COLORS

In his comparison of a sample of delivered goods with the swatch defining the center of the acceptable color range, the inspector will, if it is at all physically possible, place the sample and swatch side by side so that none of the surround color intrudes between them. He does this because it facilitates his discrimination of the color difference between them. Sometimes the shape of sample and swatch prevent them from being placed together so as to produce a sharp dividing line. One example would be textile swatches with frayed edges; another would be metal plaques covered with vitreous enamel that piles up at the edges to form a cylindrical surface known as the "bead" so that when the two plaques are touching a dark line is seen between them. In such cases the comparison might be made by means of an optical device, such as a bi-prism, which prevents the inspector from seeing, and being bothered by, the dividing line or area having a color different from either sample or swatch. It is more common, however, to make a mask providing a favorable surround color with two holes in it, one through which to view the sample, the other for the swatch, so as to hide the dividing area between sample and swatch by extending the surround color to cover it.

Traub and Balinkin have studied the interference of an angular separation of the two colors being compared on the perception of the color difference between them. They found a marked interference in the perception of lightness difference, but the perception of chromatic differences was scarcely, if at all, interfered with. This experimental finding seems to be easy to accommodate in the construction of color spaces intended to approximate the ideal. To accommodate the use of a greater angular separation it seems only to be necessary to decrease the spacing of the surfaces representing colors of constant lightness. The chromaticness spacing may be left unchanged.

The three spaces already presented as approximating to a considerable degree the properties of ideal color space permit such an adjustment to be made simply. The adjustment is simple because each of these three spaces (CIE-U*V*W*, Adams chromatic-value, Munsell) represent equiluminous colors (colors of constant Y tristimulus value, or constant Munsell value) by points in a horizontal plane. Such colors are perceived as having very closely, but not quite, constant lightness. Wyszecki and Sanders have shown that of the colors of constant Y tristimulus value those departing considerably from gray are perceived as lighter than near-grays if a gray surround is used. To accommodate the Balinkin-Traub finding with maximum convenience might require the construction of a color space in which the colors of constant lightness, rather than the colors of constant Y tristimulus value, are represented by points in a horizontal plane.

REDEFINITION OF THE CONCEPT OF IDEAL COLOR SPACE

We have seen that the simple concept of ideal color space, with which we started, cannot be made to conform to the experimental facts. Maybe there is still something useful left in the concept. Perhaps the experimental facts so far established will still permit a less stringent, more complicated concept to be formulated that is worthy of attention.

The law of diminishing returns in color-difference perception requires that we give up the requirement that distance in ideal color space be proportional to perceived color difference, but we could still hope to develop a space in which equal distances would correspond to equal perceived sizes of color difference.

The influence of surround color on color-difference perception requires that we give up the thought that ideal color space can be defined regardless of the surround color simply as a function of the tristimulus values of the light reflected into the inspector's eye from the two specimens whose color difference is being appraised. We may, however, still hope to develop an ideal color space for some one specified set of viewing conditions. Schönfelder's law and the Takasaki crispening effect alike indicate that this some one set of viewing conditions cannot be that of a fixed surround color, because such a choice introduces a singular point in the neighborhood of which the color space is positively curved. We could still hope to develop an ideal color space applying to color-difference perceptions made each time either with a surround chosen in accord with Schönfelder's law, or with such large samples that together they fill enough of the inspector's visual field that it makes no difference what surround color fills the rest of it.

One possible redefinition of ideal color space is thus that it consists of a method of locating points in tridimensional space, each point representing one, and only one color, and each color corresponding to one, and only one, point so that all pairs of points separated by any fixed distance correspond to pairs of colors perceived to differ by the same amount provided that the appraisal of the perceived size be carried out with optimal surround colors chosen in accord with Schönfelder's law that the surround color be the average of the two colors being compared.

This redefinition is possible in the sense that existing experimental data have not proved it impossible. The MacAdam data, obtained with a gray surround of luminance equal to one-half that of the two target colors, and reduced on the assumption that the locus of equi-luminous colors perceived to differ equally from any one central color is an ellipse in the (x,y)-plane, do not bear on this redefinition of ideal color space because they were not obtained with surround colors chosen in accord with Schönfelder's law. Similarly the data leading to the Nickerson index of fading fail to bear on this question because of the gray surrounds used. The Nickerson data show that to predict the perceived size of color differences for gray surrounds of Munsell value intermediate to those of the two colors being compared we cannot use Eq 1 or Eq 10, derived from it, but must use some such formula as Eq 15. It is possible that the spacing of color points by existing Munsell color space is not significantly different from that of ideal color space based on Schönfelder's law. Nobody has disproved this.

COLOR SPACES BASED ON GRAY SURROUNDS

The great interest in color spac based on observations of colors wit gray surrounds is that in such spaces movement of the color point awa from the gray axis correlates with co or perceptions of increasing saturatio and tangential movements in the spac (movements orthogonal to centripet movements) correspond to perceptio of differing hues. The gray surroun provides a valuable reference point. I such spaces, so to speak, the directio indicated by the mile-posts are reli ble, but the numbers of miles have t be computed by a formula, like Eq 1 that cannot conform simply to the di tances between the color points, an are on this account somewh confusing.

Ideal color space, on the oth hand, based on Schönfelder's law pe mits the numbers of miles on the sig posts to be given reliably, but th direction toward gray is somewh ambiguous. Helson and Michels ha shown that for nearly every chrom ticity there exists a surround col such as to cause the target color to perceived as without hue; that is, gr can be nearly anywhere. Each su round color selected in accord wi Schönfelder's law yields a differe location for gray, which thus may b not anywhere at all, but anywhe within a large central range. As C Foss, to whom I am indebted for th concept of ideal color space redefine has often remarked, you can get lost a color space based on surround colo sliding around to conform to the av age of the colors being compare there is no unique place for gra which is the necessary anchor poi for the perception of hue a saturation.

It seems likely that the perceiv sizes of color differences apprais with any fixed gray surround could predicted from ideal Schönfelder col space. It would be necessary only write the formula for perceived size color difference from Euclidian geo etry with three correction terms. Fi raise the Euclidian distance to t power, p, as in Eq 16; this corrects t perceived size of the color differen in accord with the law of diminishing returns. Second, multiply by a term, f_s, for lightness and chromaticness crispening evaluated by Takasaki. At its June 1967 meeting in Washington, CIE Committee E-1.3.1, Colorimetry, suggested for study an expression for f_s:

$$f_s = \frac{15 + \left[\bar{C}^2 + 16(\bar{V}-V_s)^2\right]^{1/2}}{5 + \left[\bar{C}^2 + 16(\bar{V}-V_s)^2\right]^{1/2}} \qquad 17.$$

where V and C are the average value and chroma for the two samples being compared, and V_s is the Munsell value of the gray surround. Third, multiply the tangential component by a term, f_h, for the super-importance of hue differences relative to chroma (radial) differences. CIE Committee E-1.3.1 has suggested for study an expression for f_hs:

$$f_h = \left[\frac{4}{3 - \cos(3.6\Delta H)}\right]^2. \qquad 18.$$

where $_{s,}$H is the hue difference in Munsell steps between the two colors being compared.

Existing Munsell color space refers to gray surrounds, but not to a fixed gray, or else the Munsell gray scale would show a local expansion near that fixed gray such as shown on Fig. near N 5/. Munsell color space thus must be thought of as referring to appraisal of color difference with a gray surround of Munsell value intermediate to those of the two colors being compared. The same remark applies to the CIE-U*V*W* space and to Adams chromatic-value space because both of these spaces are based on the Munsell value function. For this surround condition (gray of sliding lightness) the formula to predict perceived size of color differences would also be based on Euclidian geometry with three correction terms, but the term, f_s, taking crispening into account would refer only to chromaticness crispening as in the expression obtainable from Eq 18 by setting V = V_s, thus:

$$f_{s,\bar{V}=V_s} = (15 + \bar{C})/(5 + \bar{C}). \qquad 19.$$

SUMMARY

The concept of color space has been traced from its origin in the practical problems of color tolerances. It has been redefined to take account of the law of diminishing returns on color-difference perception and Schönfelder's law that the optimum surround color is the average of the two colors being compared. The relation between redefined color space and the formulas for the perceived size of differences between colors viewed against any fixed gray surround has been indicated.

AUTHOR's NOTE: Portions of this article appeared in three parts in the Sandoz Palette (Sandoz, Ltd., Basle, Switzerland) Nos. 29, 1968; 30, 1969 and 31, 1969, and appears here with permission.

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CIE 1960 UCS DIAGRAM AND THE MÜLLER THEORY OF COLOR VISION
(with G. T. Yonemura)

Proc. Int. Color Association, Stockholm, Sweden, 1969, pp. 266-274
(Musterschmidt, Göttingen, Germany, 1970)

Although the defining equations (2) of the CIE 1960 UCS diagram are used, the proposed formula (6) for color difference does not correspond to circles on the 1960 UCS diagram. The formula has been tested, with success as shown by Fig. 2, with wavelength-discrimination data for normal, protanopic, and tritanopic observers. Constant-hue loci predicted by the formula are shown in the article entitled "Perceptually uniform spacing of equiluminous colors and the loci of constant hue", which is included in this book. Equally perceptible chromaticness differences, in addition to wavelength discrimination, predicted with the formula, are compared with observational results in J. Opt. Soc. Am. 60, 1407-1409 (1970). Whether the formula overcomes the deficiencies of the CIE 1960 UCS diagram for representing, for normal observers, all just-noticeable color differences with equal-size circles, remains to be investigated and reported.

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Deane B. Judd* and **Gary T. Yonemura***, WASHINGTON D. C.: C 2

CIE 1960 UCS Diagram and the Müller Theory of Color Vision

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The shape of the CIE 1960 UCS diagram can be closely duplicated by a chromaticity diagram generated by the Müller second stage. The angles subtended at the convergence points of the chrolaticity-confusion lines for propanopia and tritanopia by any two chromaticity points are measures of the size of the difference perceived between the two corresponding chromaticities by protanopic and tritanopic vision, respectively. By using the square-root of the sum of the squares of these angles, the chromatic sensibility to wavelength change in the spectrum is also accounted for quantitatively for normal vision including the secondary maximum of sensibioity in the neighborhood of 420 nm.

Die Form der CIE-UCS-Farbtafel 1960 kommt einer Farbtafelform sehr nahe, die der zweiten Stufe der Müller-Theorie des Farbensehens entspricht. Die Winkel an den Konvergenzpunkten der Verwechslungs-Geraden für Protanopen und Tritanopen sind ein Maß für die von diesen Dichromaten-Typen wahrgenommenen Farbart-Unterschiede. Nimmt man die Quadratwurzel aus der Summe der Quadrate dieser Winkel, so kann man die Farbart-Unterschiedsempfindlichkeit bei Wellenlängen-Änderungen im Spektrum damit auch für farbennormalsichtige Beobachter quantitativ richtig beschreiben, einschließlich des sekundären Maximums bei 420 nm.

La façon du diagramme de chromaticité CIE-UCS 1960 est très semblable à celle de la deuxième phase suivant la théorie de Müller de la vision colorée. Les angles aux points de convergence des droites de confusion des protanopes et des tritanopes peuvent être pris comme mesure des différences de chromaticité perçues par ces sortes de dichromats. En extrayant la racine carrée de la somme des carrés de ces angles on peut reproduire correctement la courbe de sensibilité aux chromaticités spectrales tout aussi bien pour les personnes de vision normale, le maximum secondaire près de 420nm inclus.

1. Introduction

In a paper on *Recent Developments of Thomas Young's Color Theory*, delivered in 1886 before the *British Association in Birmingham*, A. KÖNIG [1] stated that *"it ought not to be difficult in the construction of a chromaticity diagram so to modify the adopted arbitrary assumptions that the separation of two points on it would give a measure for the difference in sensation between the colors corresponding to them, ...*" In 1932 one of us [2] drew attention to the fact that the (r, g)-chromaticity diagram based on the so-called *"OSA excitation curves"* yields some approach to uniform chromaticity scales, and in 1935 a MAXWELL triangle yielding approximately uniform chromaticity scales (known as the UCS diagram) was

* *Colorimetry and Spectrophotometry Section, National Bureau of Standards, Washington, D. C.*

defined [3] by reference to the colorimetric coordinate system recommended by the *International Commission on Illumination (CIE)* [4].

In 1937 MacAdam [5] showed that essentially the same chromaticity spacing could be achieved by plotting the v-chromaticity coordinate against the u-coordinate in a rectangular coordinate system based on the simple transformation of tristimulus values, X, Y, Z in the 1931 CIE colorimetric coordinate system:

$$\begin{aligned} U &= (2/3)\,X \\ V &= Y \\ W &= -0.5\,X + 1.5\,Y + 0.5\,Z \end{aligned} \qquad (1)$$

The V-function, like the Y-function, is equal to be standard spectral luminous efficiency function. The (u, v)-chromaticity coordinates are related to the 1931 CIE tristimulus values, X, Y, Z and to the chromaticity coordinates, x, y in this simple way:

$$\begin{aligned} u &= 4\,X/(X + 15\,Y + 3\,Z) = 4\,x/(-2\,x + 12\,y + 3) \\ v &= 6\,Y/(X + 15\,Y + 3\,Z) = 6\,y/(-2\,x + 12\,y + 3) \end{aligned} \qquad (2)$$

In 1939 Breckenridge and Schaub [6] developed a variation of the UCS diagram in which the equi-energy point was made the origin of a rectangular coordinate system (known as the RUCS diagram), and applied it to the specification of the colors of signal lights (railway, highway, marine, and airplane traffic) [7]. In 1942 Scofield, Judd, and Hunter [8] published the definition of the "alpha-beta" diagram that first, like the RUCS diagram, placed achromatic colors at the origin, second could be approximated closely by simple functions of the readings, A, G, B, obtained from a photoelectric tristimulus colorimeter through amber, green and blue tristimulus filters, respectively, and finally gave a chromaticity spacing more in accord with Wright's study [9] of the RUCS diagram than any previously formulated diagram.

In 1945 the O.S.A. *Committee on Colorimetry* [10] made use of a method devised by MacAdam [11] to derive a rectangular-coordinate UCS diagram having the same spacing as the Judd UCS diagram, and this diagram is included in the final report of the committee [12; p. 301].

In 1944 Farnsworth [13] described a rectilinear uniform chromaticity scale diagram designed so as to make the Munsell value 5, chroma 10 locus [14] approximate a circle. In 1949 Neugebauer [15] made use of the MacAdam 1937 (u, v)-diagram in a form representing achromatic object colors at the origin by plotting $v - v_0$ against $u - u_0$, where u_0, v_0 are the chromaticity coordinates of the illuminant. This form of UCS diagram has been known as the MacAdam-Neugebauer diagram. In 1959 Sugiyama and Fukuda [16] defined still another UCS diagram by adjusting the spacing to accord maximally with that of the Munsell Color System [14] and with the MacAdam ellipses [17].

In 1960, the CIE recommended the 1937 MacAdam (u, v)-diagram [18] for use whenever a projective transformation of the CIE (x, y)-diagram is desired to give chromaticity spacing perceptually more nearly unifom than the (x, y)-diagram itself. In the third edition of the *International Lighting Vocabulary,* the diagram defined by Eq. (2) is called the *CIE 1960 UCS diagram.*

König's idea of a uniform-chromaticity-scale diagram has thus received widespread attention and has come to fruition in the form of an international recommendation after 75 years. It should not be supposed, however, that the separation of two points on the recommended UCS diagram is an exact measure of the perceived size of the difference between the chromaticities represented by the points; this separation is an approximate measure. The degree of approximation, however, has been shown by Wyszecki and Wright [19] to be sufficient for many practical purposes, and this degree is probably close to the highest achievable by projective transformation of the 1931 CIE (x, y)-diagram. Less restricted types of transformation should permit improved agreement with the experimental facts; and plane diagrams for this purpose derived by nonlinear transformations have achieved some success; see, for example, those derived by SINDEN [20], ADAMS [21], MOON and SPENCER [22], FARNSWORTH [23], GLASSER et al. [24], NICKERSON, JUDD, and WYSZECKI [25], and SAUNDERSON and MILNER [26]. The MUNSELL Renotation System [14] itself is equivalent to a series of diagrams derived by nonlinear transformation of the (x, y)-diagram, one for each level of MUNSELL value, and if the Euclidian distance element derived by GODLOVE [27] is used, these are UCS diagrams plottable on planes.

Further refinements of these empirical models not subject to the restriction of being plottable on a plane have been derived; see, for example, NICKERSON [28], MACADAM [29], FRIELE [30], and CHICKERING [31].

In 1949 one of us [32] derived response functions for the three stages of the MÜLLER theory of color vision [33; 34] and showed how the chromatic response functions of the second stage could be combined with the standard spectral luminous-efficiency function, $\bar{y}(\lambda)$, to account for the minimum perceptible luminance purity determined by PRIEST and BRICKWEDDE [35] and by PURDY [36]. The two chromatic response functions of the second stage correspond to a yellowish red (yR) process whose negative is bluish green (bG) and a greenish yellow (gY) process whose negative is reddish blue (rB). The failure of the (yR-bG)-process accounts in the MÜLLER theory for protanopia; that of the (gY-rB)-process, for tritanopia. These response functions are defined in terms of the tristimulus values, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$, of the spectrum in the 1931 CIE system [4] by the equations:

$$\begin{aligned} yR(\lambda) = -bG(\lambda) &= 5.741\,\bar{x}(\lambda) - 4.601\,\bar{y}(\lambda) - 1.140\,\bar{z}(\lambda) \\ gY(\lambda) = -rB(\lambda) &= -0.932\,\bar{x}(\lambda) + 2.751\,\bar{y}(\lambda) - 1.819\,\bar{z}(\lambda) \end{aligned} \tag{3}$$

The colorimetric coordinate system proposed for the second stage of the MÜLLER theory [32], and formed of these two chromatic response functions combined with the standard spectral luminous-efficiency function $\bar{y}(\lambda)$ yields a chromaticity diagram with scales departing maximally from perceptual uniformity because the diagram is degenerate.

This paper will show how to fit the $yR(\lambda)$ and $gY(\lambda)$ functions into a coordinate system yielding a UCS diagram, and will present a new distance element for the perceptibility of chromaticity differences based on the MÜLLER theory of color vision.

2. Protanopic and Tritanopic Convergence Points as Primaries

The UCS diagram based on protanopic and tritanopic convergence points as primaries is that of the colorimetric coordinate system formed by the yR and gY functions (Eq. 3) of the MÜLLER second stage combined

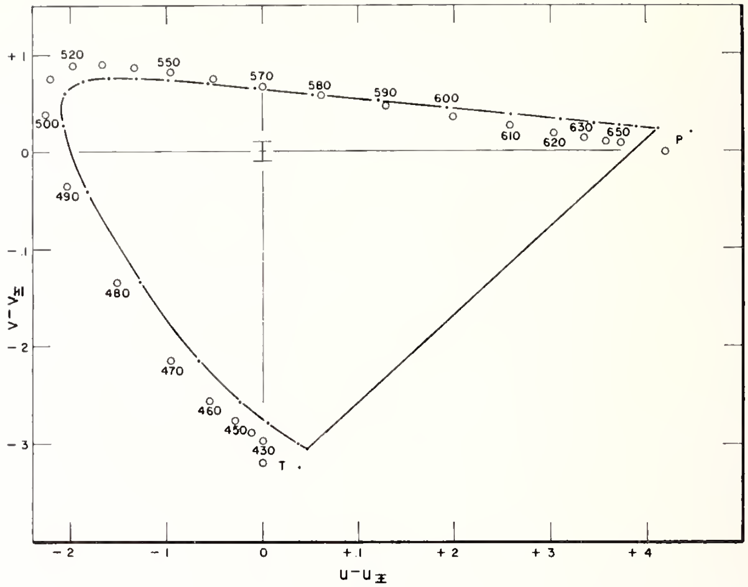

Fig. 1. The chromaticity diagram derived in accord with the second stage of the Müller theory [$0.42\,yR/S$ and $-0.32\,rB/S$ from Eq. (4) plotted in rectangular coordinates] compared with the CIE 1960 UCS diagram in the Neugebauer [15] form.

with the direct contribution $L(P_i)$ of the primary processes (P_1, P_2, P_3) of the first stage to luminosity. $L(P_i)$ thus for the Müller theory corresponds to protanopic luminosity:

$$\begin{aligned} yR &= 5.741\,X - 4.601\,Y - 1.140\,Z \\ rB &= 0.932\,X - 2.751\,Y + 1.819\,Z \\ 11.7\,L(P_i) &= -5.417\,X + 15.982\,Y + 1.135\,Z \end{aligned} \quad (4a)$$

$$\begin{aligned} X &= 0.2391\,yR - 0.0967\,rB + 0.0855\,[11.7\,L(P_i)] \\ Y &= 0.0810\,yR - 0.0024\,rB + 0.0855\,[11.7\,L(P_i)] \\ Z &= 0.4966\,rB + 0.0855\,[11.7\,L(P_i)] \end{aligned} \quad (4b)$$

Fig. 1 shows as large circles the chromaticity points of the spectrum colors on the chromaticity diagram formed by plotting $0.42\,yR/S$ as abscissa with $-0.32\,rB/S$ as ordinate, where $S = yR + rB + 11.7\,L(P_i)$. The solid dots refer to the spectrum locus of the Neugebauer [15] form of the CIE 1960 UCS diagram (Eq. 2) obtained by plotting $u - u_0$ as abscissa and $v - v_0$ as ordinate, where u_0, v_0 are the chromaticity coordinates (0.2105, 0.3158) of the equi-energy point.

It will be noted that the degree of difference in spacing of the two diagrams is not large enough to have any practical significance. The diagrams have the equi-energy point in common. The diagram yielded by Eq. 4a in accord with the second stage of the MÜLLER theory has the protanopic convergence point (0.42, 0.00), and the tritanopic convergence point (0.00, −0.32); the corresponding points are shown as solid dots for the CIE 1960 UCS diagram. It is concluded that the CIE UCS diagram, developed empirically, yields spacing in accord with the second stage of the MÜLLER theory of color vision. The accommodation of the CIE 1960 UCS diagram within the second stage suggests that for the MÜLLER theory the processes determining discrimination of chromatic differences occur in the retina; the third stage must be thought of as the process by which the discriminations made in the retina are recoded for transmission along the optic nerve with no loss of information.

3. Proposed Measure of Perceived Size of Chromatic Differences for Dichromats

If the length of the line connecting two points on a uniform chromaticity diagram is an approximate measure of the size of the chromatic difference perceived for an observer with normal color vision between the chromaticities represented by the two points, what measure is appropriate for dichromatic vision? To a dichromat, the chromaticities represented by any point on any straight line passing through the dichromatic convergence point are all identical. The component of the vector on the UCS diagram along such lines contributes nothing to dichromatic discrimination; only the component orthogonal to the direction of the chromaticity-confusion line counts. It is natural, therefore, to take the angular separation of the confusion lines containing the chromaticity points as this measure; that is, if we have two chromaticities specified by chromaticity coordinates u_1, v_1 and u_2, v_2 on the UCS diagram, the perceived size ΔC_d of the chromaticity difference for a dichromat whose convergence point is at u_d, v_d is:

$$\Delta C_d = K_d \left\{ \tan^{-1}\left[\frac{v_1 - v_d}{u_1 - u_d}\right] - \tan^{-1}\left[\frac{v_2 - v_d}{u_2 - u_d}\right]\right\} \tag{5}$$

The small open circles connected by dot-dash lines on Fig. 2 show the wavelength difference just detectable in protanopic vision computed by this measure for $K_d = 0.81$ with the angles expressed in degrees, and for $u_d = 0.658$, $v_d = 0.334$. This choice of $K_d = 0.81$ implies that a change in the direction of the chromaticity confusion line by 1.23° is required for a protanope to detect the chromaticity difference. Note that the wavelength-sensibility curve predicted by this measure for protanopic vision shows a pronounced minimum of about 1.2 nm at about 481 nm and rises to 10 nm

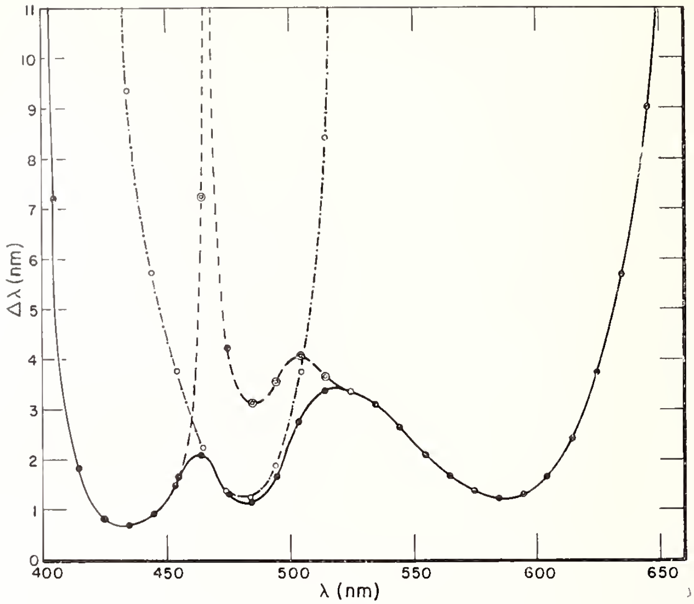

Fig. 2. Wavelength difference just detectable by chromaticity discrimination according to the Müller theory for protanopic (circles connected by dot-dash line), tritanopic (double circles connected by dashed line), and normal (solid circles connected by solid line) vision as a function of wavelength.

both at 434 and 517 nm in good agreement with the experimental determination reported by Pitt [37].

The double circles connected by dashed lines on Fig. 2 show analogous predictions of the wavelength difference just detectable in tritanopic vision computed from Eq. (5) also for $K_d = 0.81$. The tritanopic convergence point has been taken as required by the Müller theory at $x = 0.164$, $y = -0.004$ by setting $u_d = 0.249$, $v_d = -0.009$. Note that this predicted curve of tritanopic sensibility to wavelength difference among spectrum colors has three minima (430, 487, and 585 nm), one sharp maximum (467 nm) and one relative maximum (504 nm) as well as two indefinitely high maxima at the spectrum extremes. We might say that the computations indicate "*good discrimination in the yellow region of the spectrum, discontinuity in the blue-green wavelengths where wavelength discrimination virtually disappears, and, most striking of all, very keen wavelength discrimination in the far violet.*" These are, however, not our words, but those of Wright [38] describing wavelength discrimination found experimentally by him for four tritanopes.

4. Proposed General Measure of Perceived Size of Chromatic Differences

In the Müller second stage, normal vision may be regarded as the result of combining the protanopic discriminative ability with that of the tritanope. The Müller second stage thus suggests the following general measure of perceived size of chromatic differences:

$$\Delta C = [(K_p \Delta \Theta_p)^2 + (K_t \Delta \Theta_t)^2]^{1/2} \qquad (6)$$

where $K_p \Delta \Theta_p$ is Eq. (5) applied to protanopic vision, and $K_t \Delta \Theta_t$ is Eq. (5) applied to tritanopic vision. The assumption on which this proposal is based is that whatever the protanope can discriminate, the normal observer, endowed with both protanopic and tritanopic discriminative ability, can see better. The solid circles connected by a solid curve on Fig. 2 correspond to a combination of the two dotted curves in accord with Eq. (6). It will be noted that, by this view, tritanopic vision accounts for nearly all of the ability of the normal observer to discriminate wavelength by chromaticity difference in the spectrum up to 430 nm at the short-wave end and from 520 nm on in the long-wave end; only between 430 and 520 nm does protanopic discriminative ability contribute appreciably. This prediction accords well with experiments reported by Bedford and Wyszecki [39].

The argument by which Eq. (6) is proposed may seem somewhat circular. The CIE 1960 UCS diagram is used to justify the measure for dichromatic discriminations given by Eq. (5); then a new measure, Eq. (6), based on Eq. (5) is proposed that implies that distance on the UCS diagram does not really accord precisely with the perceived size of chromaticity difference after all. The justification of this circular argument lies in whether a more successful model of chromatic discrimination is thereby discovered. The model need not necessarily be that suggested by the Müller second stage; it might be that suggested by the Müller third stage, a combination of deuteranopia and tetartanopia; it might be a combination of all three established forms of dichromatic vision as in the three-components theory; or it might be based on the view, suggested by Walraven [40], that normal color vision be considered a combination of tritanopia with deuteranopia.

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CHOOSING PLEASANT COLOR COMBINATIONS

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This distillation of the most significant conclusions of the Helson-Lansford report, with examples of estimation of proposed color combinations, does not present any terminological difficulties.

Choosing Pleasant Color Combinations

Deane B. Judd

This article, prepared by Dr. Judd, is a comprehensive interpretation, and summary of the highlights of the IERI study by Dr. H. Helson and Dr. T. Lansford on the effect of background colors on the pleasantness of object colors, under different light sources. The monumental IERI report by Helson and Lansford, titled "The Role of Spectral Energy of Source and Background Color in the Pleasantness of Object Colors," was published in July 1970 by the Optical Society of America, in their publication, Applied Optics.

Dr. Judd's excellent summary provides guidelines for the use of the Helson-Lansford data on pleasantness of object-background color combinations.

Modern man has the capability of controlling the visual qualities of the spaces in which he works, and of those in which he relaxes. Much effort has been expended to determine how much light is required for various tasks, and how it should be distributed.[1] The importance of choosing colors for the bounding surfaces (ceiling, walls, floor, draperies, desk tops, furniture) that will please the prospective occupants is recognized, but because the pleasantness of color combinations is such a complicated business, no very specific advice has been available.

Pleasantness of color combinations is not only complicated, but it is also controversial. There are those who contend that choice of colors should not be left to an engineer guided by rules established by practice. Only artists, they say, have the ability to visualize from the design of the space how various color combinations will work out, and so make a successful choice. But not every artist can become a successful interior designer. Granted that he may visualize the appearance of a space in advance of its decoration and so judge whether he would like it, this is no guarantee that the client, or anyone else, will find it pleasing. An interior designer must have had experience in pleasing people as well as the ability to visualize color combinations in various contexts. No one can predict whether a stranger will like a decor; but perhaps the probability of acceptance by randomly selected strangers can be predicted to a useful degree.

In spite of the fairly widespread view that pleasantness of color combinations is too individual a matter and too complicated to be a suitable subject for scientific study, the Illuminating Engineering Research Institute some years ago decided to risk a few thousands of dollars for such a study. In particular, it seemed clear that the influence of changing from incandescent-lamp light to fluorescent lighting of one sort or another on selection of colors for living spaces should be studied by the illuminating engineering fraternity, rather than by architects or interior designers. The result of this investment is the Helson-Lansford report on the Role of Spectral Energy of Source and Background Color in the Pleasantness of Object Colors.[2] The data therein presented do indeed indicate the complicated nature of the subject, and some years of analysis were required to discover and present the very significant trends established by them. It is the purpose of the present paper to draw attention to this report by giving a brief summary of it, and to explore to what extent practical information on choice of colors may be found in it.

Author: National Bureau of Standards, Washington, D.C.

PHILOSOPHY OF THE STUDY

The most direct way to find out whether an object of one color viewed against the background of another pleases someone is to ask him. Ten observers, five men and five women, students with varied ages, backgrounds, outlooks and interests at the University of Texas, were asked to rate each object-background color combination for pleasantness on a scale of nine categories ranging from one for very, very unpleasant, through five for neutral or indifferent, to nine for very, very pleasant.

OBJECT COLORS VIEWED

Colors of 20 hues uniformly distributed throughout the hue circle were shown. For each hue there were grayish colors, one light, one dark, and one intermediate; there were also vivid colors for each hue, and light, medium and dark versions of colors departing moderately from gray. These 125 object colors, in the form of two-inch (5-cm) square* color chips, were shown on each of 25 background colors, including five red colors, five yellow, four green, four blue, three neutrals (black, gray, white), and one orange, one green-yellow, one purple-blue, and one red-purple. The number of object-background combinations studied was thus 25 by 125, equal to 3125 combinations. Each of these object and background colors is specified in the report by its Munsell notation (Hue Value/Chroma). The sampling of the color solid was thus both extensive and systematic.

ILLUMINANTS USED

The color chips were viewed, about 12 at a time, against the background color in a light-tight booth illuminated only by light from one or another of five sources: incandescent lamp, standard warm white fluorescent lamp, deluxe cool white fluorescent lamp, improved (deluxe equivalent) daylight fluorescent lamp, and Macbeth daylight lamp (incandescent lamp with Corning Daylite filter). Each of the 3125 object-background color combinations was rated under each of these five light sources by each of the five men and five women observers, making 31,250 ratings to comprise the main study, each rating being the average of five individual ratings. Tables AI to AV of the report give the 15,625 ratings resulting from averaging those of the two sexes.

*The Helson-Lansford paper gives the size of the color chips as one by one-and-a-half inches, but a later communication from Dr. Helson corrects this to two inches square.

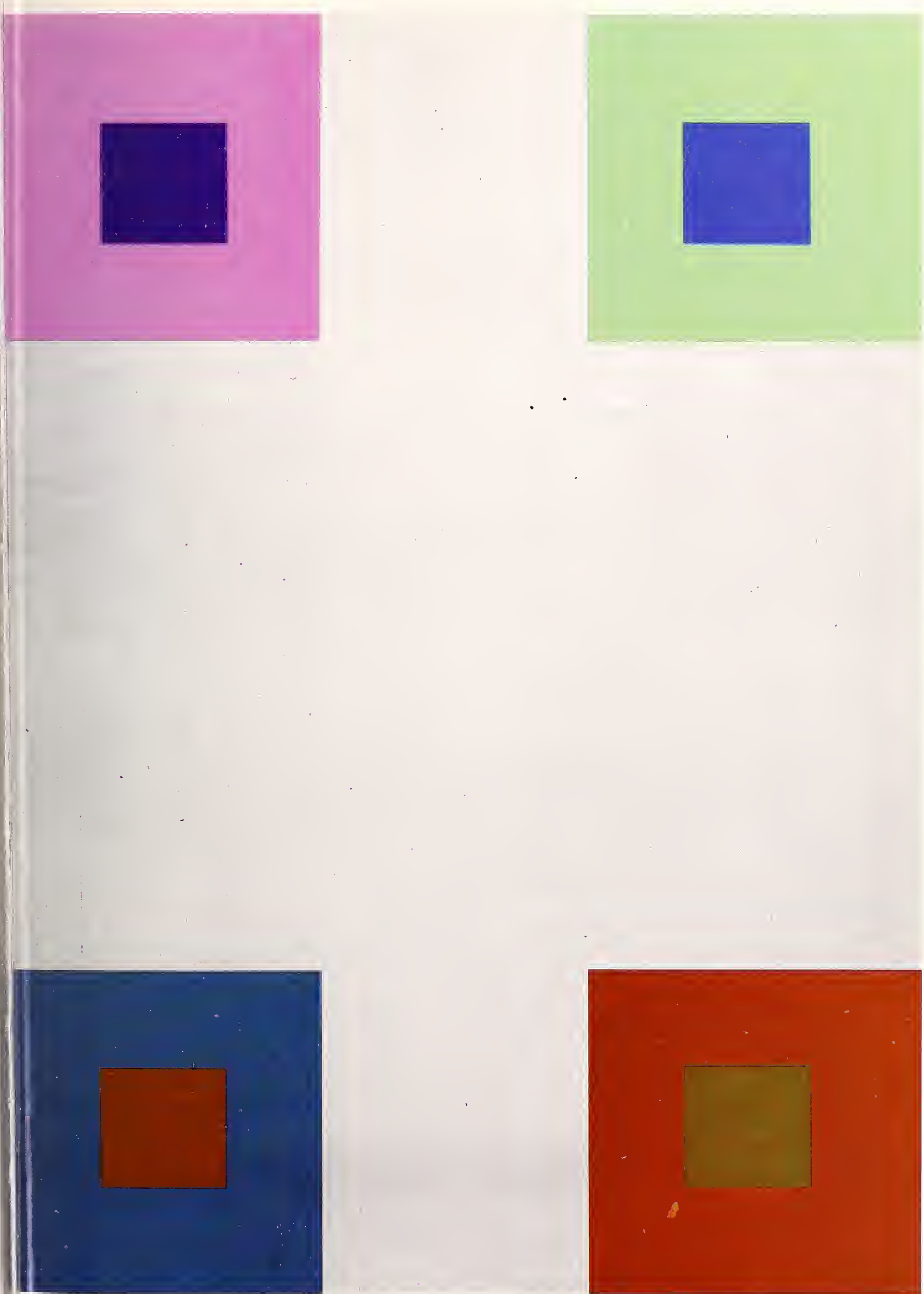
AUXILIARY STUDIES

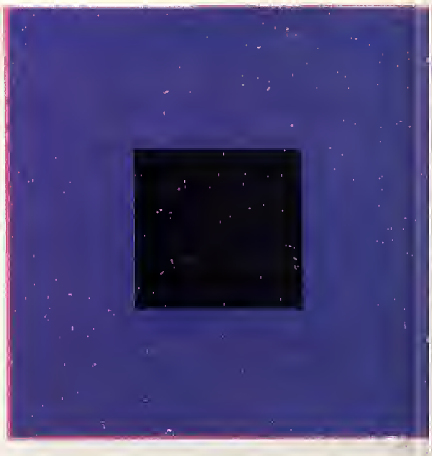
In addition to the massive main study, two important auxiliary studies are included in the Helson-Lansford report. Complexions and foods were rated for pleasantness against a neutral gray background illuminated by each of the five sources in turn; see Table VI giving 60 pleasantness ratings. The 25 background colors, viewed so as to take up so large a proportion of the visual field of each observer that contrast effects had no influence on the ratings, were also rated for pleasantness; Table XI gives 125 ratings.

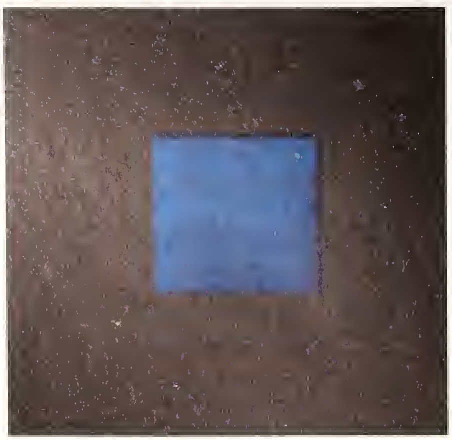
ANALYSIS OF RESULTS

The primary data of the Helson-Lansford report thus consist of 15,810 pleasantness ratings. To assist in discovering what these data indicate, various averages of the pleasantness ratings were taken, and various statistical analyses were carried out. To give an idea of the complexity of the subject and the thorough way it had been studied, the averages are identified, the analyses are indicated, and the tables giving the results are listed. This list may serve as introduction to the study of the Helson-Lansford report.

Description	Number of Entries	Location in the Report
Averages of ratings for 4 to 7 colors of the same Munsell hue		
For each of 20 hues, 25 background colors and 5 illuminants	2500	Tables AI to AV
Averages of ratings for 10 to 14 colors of the same Munsell hue family		
For each of 10 hue families, 25 background colors, and 5 illuminants	1250	Tables AI to AV
Averages of ratings for 3 neutrals		
For each of 25 background colors and 5 illuminants	125	Tables AI to AV
Averages of ratings under 5 illuminants		
For each of 125 object colors on each of 25 background colors:	3125	Table BI
For each of 125 object colors, the ratings for the best background colors and for the worst, arranged in rank order of pleasantness:	1361	Table XIV
Averages of ratings for 4 to 7 colors of the same Munsell hue averaged for 5 illuminants		
For each of 20 hues on		







each of 25 background colors	500	Table BI
Averages of ratings of 10 to 14 colors of the same Munsell hue family averaged for 5 illuminants		
For each of 10 hue families on each of 25 background colors	250	Table BI
Averages of ratings for 3 neutrals		
For each of 25 background colors	25	Table BI
Averages of ratings on 25 background colors		
For each of 125 object colors and each of 5 illuminants	625	Tables AI to AV
Averages of mean ratings on 25 background colors		
For each of 20 hues, 10 hue families, and 1 neutral under each of 5 illuminants	155	Tables AI to AV
Averages of ratings on 25 background colors averaged for 5 illuminants		
For each of 125 object colors:	125	Table BI
Arranged in rank order of pleasantness:	125	Table IX
Averages of ratings of 125 object colors taken from Tables AI-AV		
For 25 background colors arranged for each of 5 illuminants in rank order of pleasantness	125	Table VII
Averages of ratings of 125 object colors taken from Table VII and averaged for 5 illuminants		
For 25 background colors arranged in rank order of pleasantness	25	Table VIII
Averages of ratings of 3 to 7 colors of the same family averaged for 5 illuminants taken from Table BI and averaged for 25 background colors		
For 21 families (20 hues plus neutral):	21	Table BI
Arranged in rank order of pleasantness:	21	Table X
Averages of ratings of 10 to 14 colors of the same Munsell hue family averaged for 25 background colors and 5 illuminants		
For 10 hue families	10	Table BI
Average ratings of object colors of low, medium, and high chroma		
For each of 5 illuminants:	15	Table XII
Average ratings for 5 illuminants:	5	Table XII
Averages for low, medium, and high chroma:	3	Table XII
Average ratings of object colors of low, medium, and high Munsell value		
For each of 5 illuminants:	15	Table XIII
Average ratings for 5 illuminants:	5	Table XIII
Averages for low, medium, and high value:	3	Table XIII
Average ratings of 25 background colors viewed under 5 illuminants so as to subtend a large solid angle		
For each of 25 background colors averaged over 5 illuminants:	25	Table XI
For each of 5 illuminants averaged over 25 background colors:	5	Table XI
Average of all:	1	Table XI
Average ratings of foods and complexions		
For each of 5 illuminants separately for men and women:	10	Table VI
For each of 5 illuminants:	5	Table I
Averages of ratings of 125 object colors on 25 background colors from Table BI		
For 5 illuminants	5	Table I
Total number of entries giving pleasantness-rating averages	10,465	

Note that, because of the large number of parameters studied (background color, hue, value and chroma of object color, and spectral energy of illuminant), it has required nearly two-thirds as many entries for the averages taken to assist in the interpretation of the pleasantness ratings as there were ratings in the report of the main study; compare 10,465 with 15,810. This is quite apart from tables presenting the results of various statistical analyses, which we now proceed to list as a further inventory of the numerical information contained in the Helson-Lansford report.

Description	Number of Entries	Location in the Report
Levels of significance of the experimental variables (sex of observer, illuminant, and background color) and their interactions by color families		
For 21 families (20 hues plus neutral)	67	Table III

Choosing Pleasant Color Combinations . . .

Description	Number of Entries	Location in the Report
Sex differences in effects of sources of illumination on pleasantness ratings		
For 15 hues	73	Table IV
Differential effects of background colors on pleasantness ratings of 8 hues by men and women	30	Table V
Frequency distributions of value contrasts of the best and poorest object-background color combinations		
For 9 value contrasts (0 to 8 Munsell value steps)	18	Table XV
Frequencies in per cents of object-background value contrasts of the 5 best and 5 poorest color combinations for each of 125 object colors		
For low (1 to 3), medium (4 to 6), and high (7 to 9) Munsell values of object color for each of low, medium, and high Munsell values of background color	18	Table XVI
Frequency distributions of chroma contrasts in the best and poorest object-background color combinations		
For 8 chroma contrasts (0 to 14 Munsell chroma steps)	16	Table XVII
Frequencies in per cents of object-background chroma contrasts of the 5 best and 5 poorest color combinations for each of the 122 object colors		
For low (0 to 2), medium (4 to 6), and high (10 to 14) Munsell chromas of object color for each of low, medium, and high Munsell chromas of background color	18	Table XVIII
Frequencies of positive, negative, and zero Munsell value and chroma contrasts for the best and poorest object-background color combinations together with results of a chi-square significance test	21	Table XIX
Frequencies of differences in hue between object and background colors in the best and poorest color combinations		
For 10 sizes of hue differences (from 0 for combinations of the same hue in intervals of 5 Munsell hue steps each to 10 for combinations in which the hue of the object color is complementary to that of the background color) plus neutrals	22	Table XX

The Tables within the Helson-Lansford report contain the answers to many questions frequently raised, in choosing pleasant color combinations. Some of these are:

Q Does choice of illuminant have an influence on pleasantness of color combinations?

A (Table I of the report). Yes, a significant, though small, influence; see Fig. 3 of the report. Choice of background color is much more important.

Q Does pleasantness of color combinations depend significantly on sex of observer, choice of illuminant, choice of background color, and interactions of these variables for each of the 21 color families studied?

A (Table III of the report). Choice of illuminant and background color exerts a strong influence for each color family. Sex of observer by itself is significant for only one hue, but in conjunction with choice of background color and choice of illuminant the influence is significant for eight or more of the 21 families.

Q How can sex of observer be a matter generally of indifference for all color families except one, but important in conjunction with choice of illuminant?

A Table IV Women rate all illuminants higher than men do for object colors of "warm" hues (red to yellow); men rate them higher for "cool" object colors (hues from blue to red-purple).

Q How about the influence of choice of background color for women and for men?

A (Table V). Women rate 23 or more of the 25 background colors higher for object colors of "warm" hues (yellow reds); men prefer "cool" object colors on more than 17 of the 25 background colors.

Q Are there background colors that are especially good (or poor) with one choice of illuminant, and not in another?

A (Table VII). The rule is that if a color makes a good background under one illuminant, it does well under all; but there are important exceptions.

Q What makes a color good (or poor) as a background?

A (Table VIII). To be good a background color must be either light or dark; being intermediate in lightness makes it poor.

Q What makes a color pleasant (or unpleasant) as an object?

A (Table IX). Consistently pleasant object colors have hues in the green to blue to purple range, or they differ greatly from gray (by at least eight Munsell chroma steps). Consistently unpleasant colors for objects have hues between yellow and yellowish-green, or they depart from gray by a slight amount (not zero).

Q Are there preferred hues for object colors?

A (Table X). Yes; the hue range green to blue to purple is preferred for object colors. The hue range yellow to green-yellow is rated only slightly above 5.0 (indifferent).

Q Are saturated colors preferred over grayish colors?

A (Table XII). Yes; the preference is slight, but almost perfectly consistent for the five illuminants studied.

Q How does lightness of an object color affect its pleasantness?

A (Table XIII). No consistent effect. Light, medium, or dark colors may be good, bad or indifferent depending on other factors.

Q Can an object color be changed from pleasant to unpleasant merely by changing the background color?

A (Table XIV). Yes, 23 out of 125 object colors studied were so changed. In addition, about 50 others were depressed by 2.0 or more pleasantness categories.

Q Does the light-dark contrast between object color and background color influence pleasantness of the combination, and if so in what way?

A (Tables XV, XVI, and XIX). To be pleasant an object color must stand out from the background color by being definitely either lighter or darker. This is the most consistent and striking single requirement for pleasantness discovered in the Helson-Lansford report.

Q Is it more pleasant to combine a vivid color with a grayish color than to combine two colors equally different from gray?

A (Tables XVII, XVIII, and XIX). Vivid colors combined with grayish colors tend to favor pleasantness. Combining equally vivid colors produces about the same frequency of poor as good combinations.

Q What about the "law" of the hues of harmonizing colors that identifies as pleasant, combinations of colors of the same hue, or of analogous hues, or of strongly contrasting hues, but states that color combinations involving hue differences of intermediate size (about *one-fourth* of the hue circle) should be avoided because they are ambiguous (neither the same, nor analogous, nor strongly contrasting)?

A (Table XX). There is very little reliance to be placed in this "law." Combinations involving colors of the same hue or of near-complementary hues occur more frequently among the best combinations than among the poorest; and those showing hue differences of one-twentieth of the hue circle are more often among the poorest than the best. The important finding is that good and poor combinations are found for all sizes of hue difference.

PRINCIPAL CONCLUSIONS

This summary of the Helson-Lansford report is intended to indicate the method, the parameters, and the extent of the study, and to show the kind of questions for which quantitative answers are supplied. Despite its comprehensiveness, the report does not give final scientific answers to all questions of the pleasantness of object-background color combinations, and the authors fully recognize this. More observers and more colors will eventually have to be used; but the Helson-Lansford report is a magnificent first step. The new information supplied by it should be immensely valuable to everyone who must choose color combinations—artists, industrial designers, architects, interior designers, landscape architects, and fashion designers, as well as illuminating engineers.

The question now to be explored is how the Helson-Lansford report may be used in choice of colors for the bounding surfaces (ceiling, walls, floor, draperies, desk tops, furniture) of living and work spaces.

CHOICE OF COLORS FOR LIVING AND WORK SPACES

There are two distinctly different ways of using the Helson-Lansford report. The first way is to study the generalizations derived from analysis of the quantitative data, obtain an adequate comprehension of their limitations, then apply these "rules of color harmony" to choice of colors for the particular living or work space being designed. This is the method suggested by the American Standard Practice for Office Lighting[1] which states (4.4.1.d): "The person planning the colors for the interior should have a knowledge of color harmonies and an understanding of the importance of maintaining the recommended reflectances." The contribution of the Helson-Lansford report, as should be evident even from the over-simplified summary just presented, is that it has for the first time made quantitatively precise some rules of color harmony, and has largely rewritten others. The color combinations shown at the tops of the illustration pages, together with their pleasantness ratings taken from Table XIV, are among the best discovered by the Helson-Lansford study. Those at the bottoms of the pages are among the worst. If these examples of pleasant and unpleasant combinations of colors seem right or reasonable to you, you might very well make successful choices by applying the Helson-Lansford generalizations alone.

If, on the other hand, the highly-rated combinations seem to you not much better, or even poorer, than the combinations given the low ratings, and you cannot see any sense to these ratings, then your appreciation of the pleasantness of color combinations must be quite different from those of the ten observers taken to represent typical aesthetic judgments. Your chance of choosing color combinations with a high probability of pleasing a random selection of strangers is likely to be rather low if based simply upon your personal judgment or your understanding of color harmony, and recourse to the second method is likely to be required for success. Perhaps the choices of the most distinguished artist, or the most experienced interior designer, could also be improved in customer acceptance if they, too, had the patience to apply the second method.

The second method is based on the property of pleasantness ratings demonstrated in the Helson-Lansford report that small changes in the object color or the background color, or both, produce only small changes in pleasantness ratings; see Figs. 6 to 15 of the report for a demonstration of this law for changes in object-color hue by ten Munsell hue steps. This law means that pleasantness of color combinations based on judgments of ten observers, far from being capricious, is dependent in an orderly and continuous way on the two colors involved. Thus, each of the 15,625 ratings listed in the Helson-Lansford Tables AI to AV refers not only to the object-background color combination actually viewed under that illuminant, but also with little loss of significance to combinations involving colors not much different from the object color actually present viewed against a background color also not much different from that actually used. Now since the Helson-Lansford report gives, for each of five illuminants, pleasantness ratings of 125 object colors well distributed in color space for each of 25 background colors, likewise well distributed, it follows from this law of continuity that reliable estimates of pleasantness ratings may be obtained for any color combination by interpolation among the ratings given in Tables AI to AV. The second way of using the Helson-Lansford report consists, therefore, of determining by interpolation the pleasantness ratings of all combinations of colors suggested for a given design. The ratings, thus determined, will confirm a happy choice, or warn against a poor one. Helson and Lansford, in their discussion of the basic data of Tables AI to AV, state, "These tables can be used in a practical way by looking up the rating in the table most closely approximating the source, object color, and background color used in the viewing situation. It is also possible to interpolate between values given in the five tables."

Let us see what is involved in this process of interpolating for the pleasantness rating of 5BG 3/6 viewed against 5Y 8/2 in deluxe cool white fluorescent light. This might be a ceramic-vase color viewed against a wall color. Table AIV gives first the following ratings for nearby colors viewed on the same background, and second the ratings of the same object color viewed on backgrounds not differing much from 5Y 8/2:

Same background color		Same object color	
10G 2/2 on 5Y 8/2	6.8	5BG 3/6 on 5R 8/2	7.8
10G 5/6 on 5Y 8/2	7.8	5BG 3/6 on N 10/	7.0
10BG 3/6 on 5Y 8/2	7.7	5BG 3/6 on 5G 8/2	6.7
5B 4/8 on 5Y 8/2	7.7	Average	7.0
Average	7.5		

The actual observed value of pleasantness rating for 5BG 3/6 on 5Y 8/2 is 7.3 for cool white fluorescent light.

Try the same process for 10 R 5/10 on 5G 8/2 for warm white fluorescent light. This might be the combination of an office-floor color with a wall color. Reference to Table AIII shows:

Same background color		Same object color	
5R 5/8 on 5G 8/2	6.5	10R 5/10 on 5Y 8/2	6.1
5YR 5/8 on 5G 8/2	6.0	10R 5/10 on 2.5PB 8/2	6.2
Average	6.2	Average	6.2

The rating actually observed for 10R 5/10 on 5G 8/2 is 6.6.

Finally try interpolating for the pleasantness ratings of both 5B 8/4 on 5GY 7/10 and the reverse, 5GY 7/10 on 5B 8/4, for incandescent-lamp light. Table AII shows:

Same background color		Same object color	
10BG 8/2 on 5GY 7/10	5.8	5B 8/4 on 5Y 8/2	5.4
10B 8/4 on 5GY 7/10	5.8	5B 8/4 on 5Y 9/12	5.5
		5B 8/4 on 5G 8/2	4.8
Average	5.8	Average	5.2

The rating actually observed for 5B 8/4 on 5GY 7/10 is 5.5. One of the colors (blue) might be considered for a schoolroom wall, the other for the desk tops. For the reverse combination Table AII shows:

Same background color	Same object color	
No information; 5B 8/4 was not used as a background color.	5GY 7/10 on 5G 8/2	5.9
	5GY 7/10 on 5G 5/4	5.9
	5GY 7/10 on 2.5PB 8/2	4.7
	5GY 7/10 on 2.5PB 5/4	5.5
	Average	5.5

Tables AI to AV show many object-background color combinations whose pleasantness rating changes importantly on interchange, but for this combination the rating seems to be little changed.

In selection of nearby colors for object and background from Tables AI to AV to arrive at an interpolated pleasantness rating, large hue differences may be tolerated, moderate saturation differences, but only small lightness changes unless balanced by about equal lightness differences in the opposite direction; see questions answered by Tables XX, XVII, and XV, respectively. Whether it is worthwhile in taking the averages to weight the ratings by proximity of the colors chosen from Tables AI to AV to those of the tentative pair to be evaluated in pleasantness is doubtful.

Determination of pleasantness ratings by interpolation among the ratings listed in Tables AI to AV can serve as a guide for choices of color combinations for any purpose. This method involves the following steps:

1. Determine the approximate Munsell notations of the two colors, object and background, making up the combination. This determination may easily be done with sufficient accuracy by comparing them to the chips in the Munsell Book of Color *under the kind of light to be used to view the combination.* For this purpose Munsell hue correct within five Munsell steps, Munsell chroma within two Munsell steps, and Munsell value within one-half Munsell step is quite accurate enough.
2. Determine by interpolation from the appropriate one of Tables AI to AV the pleasantness rating.
3. Repeat the process for other tentative choices of colors.
4. In making the final choice, bear in mind that differences in pleasantness rating of less than one category step determined by this method are of doubtful significance; that is, a color combination found by the interpolation method to have a pleasantness rating of 6.0 (mildly pleasant) should not necessarily be rejected in favor of one having a rating of 6.5 (halfway between pleasant and mildly pleasant).

Uncertainty in validity of these ratings arises from several sources:

1. The uncertainty of the original ratings in Tables AI to AV, themselves, each such rating being the average of ten, one by each of ten observers.
2. The uncertainty of deriving by interpolation the rating for a color combination not actually observed.
3. The possible failure of the ten observers to make esthetic judgments in accord with those who are to live or work in the space being designed.
4. The error caused by taking the color combination out of one context (flat object of angular subtense about 7° viewed against a considerably larger surround produced by a

flat surface in the same plane, and hence equally illuminated) into another (a room in which the two surfaces whose colors are being selected may have about equal angular subtenses, may be seen as merely contiguous, and may not be in the same plane, and hence may not be equally illuminated*).

Pleasantness ratings derived by this method for any one of the five illuminants actually studied do not suffer any loss of validity at all from the fact that color appearance of a given Munsell chip (identified by Munsell hue, value, and chroma) varies considerably depending on which of these five illuminants is used. Since the actual object and surround surfaces are specified by Munsell hue, value, and chroma determined by visual comparison under the one of the five illuminants to be used, the detailed data in Tables AI to AV are precisely applicable. For example, to take an extreme case, there exist specimens so metameric in daylight to chips of green color shown in the Munsell book that they look brown in incandescent-lamp light. Such a specimen would color-match quite a different Munsell chip depending on the illuminant used, and the Munsell notation of that different chip is precisely appropriate for the one of Tables AI to AV referring to that illuminant. This quite different color (say brown instead of green) has already been studied by the ten observers taken to represent typical esthetic judgments.

Pleasantness ratings derived by this interpolation method for illuminants intermediate in spectral distribution to any two of the illuminants for which detailed results are available in Tables AI to AV suffer little loss of validity because of the additional interpolation between the two illuminants. For illuminants having spectral distributions not intermediate to any pair of the five studied, but only slightly different from one of them, the method can still have some validity; but if the spectral distribution is quite different from any of the five studied (such as that of a green fluorescent lamp), no meaningful extrapolation of pleasantness rating can be made.

*If the room is designed to provide considerably more light on one surface (say the floor) than on a contiguous surface (say the lower wall), the Munsell notation of the first color should be found by illuminating the sample (for the floor) correspondingly more than the samples of the Munsell book.

Finally, there is a discrepancy between the light leaving the light source (incandescent-lamp light, cool white fluorescent, and so forth) and that incident on the objects and backgrounds studied. This discrepancy arises from multiple reflections within the light-tight booth used for the study. For example, the objects studied with a vivid yellow background were illuminated by somewhat more yellowish light than the same objects with the same light source against a less yellowish background (black, gray, or blue). Much the same discrepancy between nominal and actual illuminant exists in the living space being designed; so the pleasantness ratings may be taken to apply without change.

CHOICE OF COLORS FOR CEILINGS, WALLS, FLOORS AND DESK TOPS

The guidelines just given for using the Helson-Lansford report to check the pleasantness of tentative choices of color combinations assume that the colors tentatively chosen are, except for pleasantness, otherwise appropriate to their intended use. For example, Table XIV of the report shows vivid red (5R 4/14) on near-black backgrounds (2.5PB 2/2, 5G 2/2, N 1/) to rate between pleasant (7.0) and very pleasant (8.0), but neither of these colors is appropriate for the ceiling, wall, or floor of a work space. Surfaces of these colors reflect too small a fraction of the incident light to be seriously considered; they waste too much light. Similarly, a black object on a pale yellow background rates as pleasant (6.8), but to fill a large fraction of the observer's visual field with each of the

Table I

Room Surface	Office Lighting		School Lighting	
	Reflectance per cent	Munsell value	Reflectance per cent	Munsell value
Ceilings	80–92	9.0–9.6	70–90	8.5–9.5
Walls	40–60	7.0–8.0	40–60	7.0–8.0
Floors	21–39	5.1–6.8	30–50	6.0–7.5
Desk tops	26–44	5.6–7.1	35–50	6.4–7.5

members of this combination would produce fatigue for an observer subjected to it for extended periods of time. The pupil of an eye adapted to black must contract whenever the pale yellow is fixated, and the muscles controlling the pupil size tire. So you soon get too much of these colors, juxtaposed in large areas; the combination is too contrasty to live with.

The American Standard Practice for Office Lighting and Guide for School Lighting, already referred to,[1] give requirements for reflectances of ceilings, walls, floors, and desk tops for economical lighting and avoidance of visual loss. The chief requirements on reflectance, and their equivalents in Munsell value are shown in the accompanying Table I.

These quite necessary requirements cramp the style of the designer choosing colors for pleasantness alone; they rule out most of the very pleasant object-background color combinations. Recall that the answer to the question: "What makes a color good (or poor) as a background?" supplied by Table VIII of the report is: "To be good, a background color must be either light or dark; being intermediate in lightness makes it poor." The colors recommended for ceilings qualify for good background colors by being quite light (Munsell value 8.5 or higher); but walls, floors, and desk tops can be only moderately light (Munsell value not more than 8.0). The pleasantness lesson to be learned is to stay near the light boundary for walls, floors, and desk tops so as to enhance the pleasantness of dark objects viewed against them as backgrounds.

Recall also that the answer to the question: "Does the light-dark contrast between object color and background color influence pleasantness of the combination?" supplied by Tables XV, XVI, and XIX is: "To be pleasant an object color must stand out from the background color by being definitely either lighter or darker." Now if the wall, floor, and desk-top colors are chosen as light as possible to provide good backgrounds against which to see objects, then the wall-floor, the wall-desk top, and the floor-desk top combinations will have minimal, rather than maximal, light-dark contrast, and cannot be highly pleasant. The designer cannot have it both ways at the same time; he must make a compromise. These considerations give some hint as to why it is often argued that choice of colors cannot usefully be reduced to a formula that, if followed, will guarantee success.

The data in the Helson-Lansford report can, however, give a type of assistance not previously available to the designer in his choice of colors. The following procedure is suggested:

1. Make tentative choices of colors for ceiling, walls, floor, and desk tops that conform to the reflectance requirements designed to avoid visual loss and waste of light. Suggestions for these choices may be obtained from Table BI of the report by noting the pleasantness ratings of combinations of colors satisfying, or nearly satisfying, these requirements expressed in terms of Munsell value.

2. Revise these tentative choices slightly to make them comply precisely with the reflectance requirements. Determine reflectance of the surfaces for the actual illuminant planned for the living space being designed.

3. Estimate the pleasantness ratings for each pair of the revised selections by interpolation in the appropriate one of Tables AI to AV, as already explained in detail.

The merit of these ratings is that they express the judgments of ten observers carrying out the first comprehensive study of the pleasantness of color combinations. A combination with a high rating has a higher probability of pleasing a random selection of observers drawn from the same population group than a combination with a low rating. Some designers will avoid using this information on the ground that the esthetic judgments of ten students of the University of Texas are not pertinent, however varied their interests; others, on the ground that the influence on pleasantness of taking a color combination out of context is unknown, or even unknowable. Still other designers will welcome this information because it seems an unnecessary risk to use a color combination that ten observers have already found to be unpleasant when choice of a combination rated pleasant by them can readily be made.

References

1. See, for example, American Standard Practice for Office Lighting, A132.1-1966, sponsored by the Illuminating Engineering Society, and American Standard Guide for School Lighting, A23.1-1962, sponsored jointly by the American Institute of Architects, the Illuminating Engineering Society, and the National Council on Schoolhouse Construction.

2. Helson, H. and Lansford, T., "The Role of Spectral Energy of Source and Background Color in the Pleasantness of Object Colors," *Applied Optics*, Vol. 9, July 1970, p. 1513.

PREDICTION OF TARGET VISIBILITY FROM THE COLORS OF TARGET AND SURROUND
(with A. A. Eastman)

Illum. Eng. 66, 256-266 (1971)

No problems of terminology are presented by this clear account of a straightforward study, except possibly the word "target" that appears in the title. It is used in the general sense that has become customary in the U.S. illuminating engineering literature. It refers to any and all visual objects and configurations that observers might have to detect, recognize, or manipulate. Almost all of the available literature has been concerned with light - dark pattern contrasts. This is one of the first substantial efforts to evaluate the contribution to visibility of chromatic components of contrast.

Errata: P. 258, Table II, the second and third sets of column headings should be omitted; p. 264, second column, first line of discussion, "Deane" is misspelled.

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IES TRANS-ACTION

Prediction of Target Visibility From the Colors of Target and Surround

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Arthur A. Eastman

THE TARGET visibilities of each of about 300 combinations of target and surround colors have been measured by one of us by means of a recently developed contrast-threshold visibility meter.[1] The targets and surrounds were Munsell papers, and the targets were of such size and distance from the observer (AAE) that they subtended 10 minutes of arc. The experimental details and the results for about half of these target-surround combinations have been published,[2] and it was pointed out also in discussion of this paper[3] that a close correspondence exists between these measures of visibility and computations based on a modification of the 1964 CIE uniform color space[4] even though the colors of target and surround were simply approximated from published values[5] obtained by spectrophotometric measurements of other samples of the same nominal Munsell notation.

The purpose of the present paper is to derive a method of predicting target visibility along these lines that will be based upon all 263 target-surround combinations measured[2] for light-colored surrounds (Munsell value 8/).

Theory of the Visibility Meter

If Y_t, Y_s and Y_w refer to luminance of target, surround, and the white reflected by the beam splitter then the luminances of target and surround viewed through the beam splitter are:

$$Y_{tv} = TY_t + RY_w$$

$$Y_{sv} = TY_s + RY_w$$

whence we find:

$$(Y_t - Y_s)/(Y_{tv} - Y_{sv}) = 1/T \qquad (1)$$

that is, if the beam splitter be adjusted so as to yield the threshold condition, then the numerical value of $1/T$ gives the luminance difference between target and surround expressed in multiples of the threshold. The same relation also holds for differences in the other tristimulus values, X and Z.

Table I gives the chromaticity coordinates, x, y, and luminous reflectance factor, Y, of the 45 Munsell samples used as targets computed from the spectral distribution of the illuminant used, that produced by 40-watt standard cool white fluorescent lamps.

Table II gives the 263 values of $1/T$, each found as a mean of 20 settings of the threshold for all target-surround combinations studied. Note that there were six colors, all of Munsell value 8/, used as surrounds for each of about 44 target colors: N 8/, R 8/4, Y 8/12, G 8/6, B 8/4 and P 8/4. In addition six darker colors, R 7/8, R 6/10, R 5/12, G 7/6, G 6/6, G 5/8, were used in an auxiliary study to test directly the influence on visibility of interchanging target and surround colors. The interpretation of these auxiliary results is reserved for presentation in another paper.

A paper presented at the Annual Conference of the Illuminating Engineering Society, September 14-17, 1970, Vancouver, B.C. AUTHORS: Office of Colorimetry, Institute of Applied Technology, National Bureau of Standards, Washington, D.C., and Applied Research Laboratory, Large Lamp Dept., General Electric Co., Nela Park, Cleveland, Ohio, respectively.

Table I. Chromaticity coordinates, *x*, *y*, and luminous reflectance factor, *Y*, in per cent for 45 Munsell samples illuminated by light from a 40-watt fluorescent lamp computed from the measured spectral reflectances of the Munsell samples and the measured spectral distribution of the lamp.

Munsell notation H V/C	Chromaticity coordinates *x*	*y*	Reflectance *Y*	Munsell notation H V/C	Chromaticity coordinates *x*	*y*	Reflectance *Y*
R 5/12	0.5319	0.3647	19.28	B 5/6	0.2871	0.3438	17.16
R 6/10	.4910	.3786	29.92	B 6/6	.2989	.3477	27.61
R 7/8	.4579	.3843	44.42	B 7/6	.3125	.3636	39.22
R 8/4	.4100	.3883	59.32	B 8/4	.3359	.3737	55.48
YR 5/8	0.5085	0.4316	21.17	PB 5/10	0.2630	0.2677	16.78
YR 6/12	.5212	.4431	34.56	PB 6/8	.3005	.3189	28.84
YR 7/10	.4979	.4347	45.04	PB 7/6	.3258	.3392	41.29
YR 8/4	.4263	.4095	60.62	PB 8/2	.3624	.3752	58.02
Y 5/6	0.4582	0.4716	20.85	P 5/10	0.3289	0.2738	17.17
Y 6/8	.4654	.4863	32.11	P 6/8	.3466	.3110	27.07
Y 7/10	.4679	.4941	48.84	P 7/8	.3552	.3288	40.34
Y 8/12	.4747	.4970	65.60	P 8/4	.3642	.3585	57.61
GY 5/6	0.4108	0.5006	21.26	RP 5/10	0.4360	0.3209	17.66
GY 6/8	.4162	.5220	32.73	RP 6/10	.4348	.3294	27.78
GY 7/10	.4242	.5245	45.40	RP 7/8	.4223	.3513	42.48
GY 8/8	.4167	.4898	59.88	RP 8/6	.4050	.3628	56.63
G 5/8	0.3209	0.4727	18.10				
G 6/6	.3421	.4437	28.48	MgO	0.3757	0.3869	100.00
G 7/6	.3525	.4360	41.02				
G 8/6	.3558	.4288	56.45				
				N 5/	0.3738	0.3864	19.75
BG 5/6	0.3031	0.3993	17.33	N 6/	.3746	.3852	29.46
BG 6/6	.3120	.4023	27.24	N 7/	.3741	.3853	42.31
BG 7/4	.3397	.4001	40.70	N 7.5/	.3751	.3865	50.60
BG 8/2	.3599	.3950	58.00	N 8/	.3744	.3870	58.09

Extension of CIE Uniform Color Space to Small Fields

It has been pointed out[3] that to adapt 1964 CIE uniform color space to predictions of the visibility of targets subtending angles considerably smaller than one degree the formula for the perceived size of the color difference should be changed from that for targets of large angular subtense:

$$\Delta E_{60\ \min} = [(\Delta U^*)^2 + (\Delta V^*)^2 + (\Delta W^*)^2]^{1/2} \quad (2)$$

to some such form as:

$$\Delta E_{10\ \min} = [(\Delta U^*/20)^2 + (\Delta W^*/2)^2]^{1/2} \quad (3)$$

Reduction of the target subtense from 60 to 10 minutes of arc produces what is known as small-field tritanopia[6] and justifies the omission of ΔV^*, the measure of difference in the violet-greenyellow sense. It also reduces markedly the visibility of the red-green component of the color difference, measured by ΔU^*, and that of the light-dark component, measured by ΔW^*, somewhat, and so leads to a formula for perceptual size of color difference between the surround color and the target color such as Eq. 3, derived from preliminary analysis of the data of Table II.

The variables, U^*, V^*, W^*, plotted along mutually perpendicular axes to produce 1964 CIE uniform color space, are nonlinear transformations of the chromaticity coordinates, x, y, and luminous reflectance, Y, in per cent according to the 1931 CIE standard observer and coordinate system, thus:

$$W^* = 25Y^{1/3} - 17, \text{ for } 1 \leq Y \leq 100;$$

$$U^* = 13W^*(u - u_o), \quad u = 4x/(12y - 2x + 3)$$

$$V^* = 13W^*(v - v_o), \quad v = 6y/(12y - 2x + 3) \quad (4)$$

where u_o, v_o, refer to the achromatic or neutral reference color. Eqs. 2 and 3 give estimates of the perceived size of the differences between any two colors, U^*_1, V^*_1, W^*_1, and U^*_2, V^*_2, W^*_2, where $\Delta U^*_1 = U^*_1 - U^*_2$ with similar definitions for ΔV^* and ΔW^*. The units of ΔE are known as 1964 CIE units of color difference. One such unit is approximately equal to a four-fold multiple of the difference threshold.

Table II. Visibility of 263 target-surround combinations determined experimentally as values of 1/T, and the predictions of ΔE of these visibilities as computed from Eqs. 5, 9 and 10 by means of the constants given by the next to the last line of Table III. An asterisk (*) indicates that the target is lighter than the background.

Target color	$1/T$	ΔE	Target color	$1/T$	ΔE	Target color	$1/T$	ΔE
Target color	$1/T$	ΔE	Target color	$1/T$	ΔE	Target color	$1/T$	ΔE
Target color	$1/T$	ΔE	Target color	$1/T$	ΔE	Target color	$1/T$	ΔE
			A. Surround = R 8/4					
R 7/8	13.4	2.27	YR 8/4	2.46	0.31*	Y 8/12	6.52	1.17*
R 6/10	21.7	5.21	YR 7/10	12.5	2.09	Y 7/10	11.6	1.96
R 5/12	28.3	7.85	YR 6/12	18.2	4.08	Y 6/8	21.9	5.45
			YR 5/8	27.3	7.55	Y 5/6	28.3	8.16
GY 8/8	8.35	1.05*	G 8/6	12.0	2.04	BG 8/2	9.97	1.39
GY 7/10	15.8	3.43	G 7/6	19.6	4.67	BG 7/4	19.9	4.69
GY 6/8	23.5	5.96	G 6/6	25.7	7.31	BG 6/6	26.0	7.74
GY 5/6	31.1	8.50	G 5/8	32.0	9.87	BG 5/6	31.1	9.98
B 8/4	13.6	2.06	PB 8/2	8.05	1.11	P 8/4	7.22	0.97
B 7/6	21.9	5.18	PB 7/6	17.6	4.40	P 7/8	17.5	4.11
B 6/6	28.0	7.61	PB 6/8	25.4	7.18	P 6/8	25.1	7.00
B 5/6	33.4	10.00	PB 5/10	33.0	10.08	P 5/10	30.3	9.45
RP 8/6	6.09	0.48	N 8/	7.51	0.96			
RP 7/8	14.1	2.92	N 7/	16.3	3.74			
RP 6/10	23.5	6.06	N 6/	24.6	6.43			
RP 5/10	29.8	8.73	N 5/	29.8	8.74			
			B. Surround = Y 8/12					
R 8/4	7.54	1.15	YR 8/4	7.18	0.86			
R 7/8	15.8	3.38	YR 7/10	14.4	3.20	Y 7/10	14.1	3.00
R 6/10	23.8	6.23	YR 6/12	19.6	5.12	Y 6/8	24.0	6.43
R 5/12	30.7	8.78	YR 5/8	28.7	8.48	Y 5/6	32.0	9.05
GY 8/8	10.6	1.82	G 8/6	14.0	2.92	BG 8/2	12.4	2.28
GY 7/10	17.5	4.41	G 7/6	22.6	5.64	BG 7/4	22.6	5.66
GY 6/8	24.8	6.91	G 6/6	28.3	8.22	BG 6/6	29.1	8.64
GY 5/6	31.1	9.39	G 5/8	34.5	10.71	BG 5/6	35.7	10.82
B 8/4	14.7	2.99	PB 8/2	11.2	2.05	P 8/4	10.4	1.98
B 7/6	23.0	6.14	PB 7/6	19.8	5.39	P 7/8	19.4	5.12
B 6/6	30.7	8.52	PB 6/8	29.5	8.10	P 6/8	26.0	7.93
B 5/6	37.7	10.84	PB 5/10	35.1	10.91	P 5/10	32.5	10.30
RP 8/6	10.5	1.61	N 8/	11.4	1.91			
RP 7/8	17.5	4.01	N 7.5/	15.2	3.20			
RP 6/10	22.4	7.04	N 7/	20.9	4.75			
RP 5/10	30.7	9.61	N 6/	28.0	7.38			
			N 5/	34.0	9.62			
			C. Surround = G 8/6					
R 8/4	8.44	2.02*	YR 8/4	8.68	2.28*	Y 8/12	12.1	2.90*
R 7/8	10.45	1.85	YR 7/10	9.24	1.78	Y 7/10	6.76	0.83
R 6/10	15.9	3.85	YR 6/12	13.0	2.81	Y 6/8	15.2	3.74
R 5/12	22.1	6.21	YR 5/8	21.9	5.82	Y 5/6	21.5	6.30
GY 6/8	5.10	1.09*				BG 8/2	2.76	0.65*
GY 7/10	9.53	1.59	G 7/6	12.8	2.79	BG 7/4	13.4	2.82
GY 6/8	16.8	4.09	G 6/6	19.6	5.38	BG 6/6	20.3	5.79
GY 5/6	23.0	6.58	G 5/8	24.3	7.85	BG 5/6	26.6	7.97
B 8/4	4.32	0.23	PB 8/2	3.89	0.93*	P 8/4	5.01	1.10*
B 7/6	14.2	3.30	PB 7/6	13.0	2.62	P 7/8	12.5	2.52
B 6/6	20.3	5.69	PB 6/8	21.3	5.29	P 6/8	18.6	5.21
B 5/6	25.1	8.00	PB 5/10	27.3	8.08	P 5/10	24.3	7.53

Table II—*continued*

RP 8/6	7.53	1.92*	N 8/	4.02	1.07*			
RP 7/8	11.4	2.06	N 7.5/	5.85	0.77			
RP 6/10	18.4	4.55	N 7/	11.3	2.08			
RP 5/10	24.6	6.94	N 6/	18.8	4.63			
			N 5/	24.3	6.84			
			D. Surround = B 8/4					
R 8/4	9.73	2.01*	YR 8/4	8.98	2.29*	Y 8/12	12.2	2.97*
R 7/8	9.51	1.58	YR 7/10	8.31	1.52	Y 7/10	5.49	0.59
R 6/10	15.9	3.65	YR 6/12	12.2	2.59	Y 6/8	15.0	3.59
R 5/12	21.3	6.07	YR 5/8	19.9	5.69	Y 5/6	22.1	6.19
GY 8/8	5.95	1.19*	G 8/6	2.65	0.23*	BG 8/2	2.98	0.71*
GY 7/10	7.96	1.45	G 7/6	11.9	2.68	BG 7/4	12.2	2.70
GY 6/8	16.0	3.98	G 6/6	19.1	5.28	BG 6/6	20.1	5.69
GY 5/6	22.8	6.48	G 5/8	24.6	7.78	BG 5/6	25.1	7.89
			PB 8/2	4.29	0.94*	P 8/4	5.18	1.07
B 7/8	13.3	3.19	PB 7/6	11.3	2.49	P 7/8	10.78	2.34
B 6/6	19.9	5.59	PB 6/8	19.4	5.19	P 6/8	17.4	5.09
B 5/6	25.7	7.92	PB 5/10	25.1	8.00	P 5/10	23.5	7.44
RP 8/6	7.44	1.84*	N 8/	4.21	1.09*			
RP 7/8	9.42	1.80	N 7.5/	5.19	0.53			
RP 6/10	16.5	4.38	N 7/	9.79	1.91			
RP 5/10	23.3	6.83	N 6/	17.1	4.51			
			N 5/	21.9	6.74			
			E. Surround = P 8/4					
R 8/4	3.29	0.95*	YR 8/4	3.50	1.23*	Y 8/12	7.79	1.99*
R 7/8	11.6	1.63	YR 7/10	9.62	1.47	Y 7/10	9.32	1.04
R 6/10	17.6	4.36	YR 6/12	14.3	3.24	Y 6/8	18.4	4.50
R 5/12	24.6	6.92	YR 5/8	23.3	6.59	Y 5/6	23.8	7.15
GY 8/8	4.16	0.52*	G 8/6	7.79	1.07	BG 8/2	5.58	0.48*
GY 7/10	12.3	2.44	G 7/6	15.6	3.67	BG 7/4	15.7	3.69
GY 6/8	18.9	4.96	G 6/6	21.7	6.28	BG 6/6	22.6	6.70
GY 5/6	24.8	7.47	G 5/8	27.0	8.79	BG 5/6	28.0	8.91
B 8/4	8.65	1.07	PB 8/2	4.12	0.20*			
B 7/6	17.4	4.18	PB 7/6	14.2	3.43	P 7/8	12.7	3.19
B 6/6	23.8	6.58	PB 6/8	21.5	6.17	P 6/8	19.8	6.02
B 5/6	29.1	8.93	PB 5/10	27.0	9.01	P 5/10	25.7	8.41
RP 8/6	4.52	0.81	N 8/	3.42	0.17*			
RP 7/8	11.2	2.18	N 7.5/	7.88	1.22			
RP 6/10	18.4	5.17	N 7/	13.5	2.79			
RP 5/10	24.0	7.74	N 6/	19.2	5.45			
			N 5/	26.0	7.72			
			F. Surround = N 8/					
R 8/4	4.20	0.94*	YR 8/4	3.99	1.21*	Y 8/12	8.37	1.91*
R 7/8	11.6	1.80	YR 7/10	9.82	1.65	Y 7/10	8.87	1.15
R 6/10	18.3	4.49	YR 6/12	14.4	3.37	Y 6/6	18.4	4.59
R 5/12	24.0	7.03	YR 5/8	23.0	6.69	Y 5/6	25.7	7.24
GY 8/8	3.42	0.36*	G 8/6	7.88	1.07	BG 8/2	5.08	0.43
GY 7/10	12.3	2.51	G 7/6	15.9	3.75	BG 7/4	15.5	3.77
GY 6/8	19.2	5.04	G 6/6	21.9	6.36	BG 6/6	22.6	6.78
GY 5/6	26.6	7.56	G 5/8	27.3	8.88	BG 5/6	27.0	9.00
B 8/4	8.13	1.10	PB 8/2	3.46	0.15	P 8/4	3.36	0.17
B 7/6	16.6	4.26	PB 7/6	14.5	3.51	P 7/8	14.7	3.29
B 6/6	22.1	6.67	PB 6/8	21.7	6.25	P 6/8	22.1	6.12
B 5/6	29.1	9.02	PB 5/10	28.3	9.10	P 5/10	28.7	8.50
RP 8/6	5.00	0.90	N 7.5/	8.31	1.31			
RP 7/8	11.4	2.33	N 7/	14.8	2.88			
RP 6/10	18.9	5.28	N 6/	22.4	5.54			
RP 5/10	25.1	7.84	N 5/	27.6	7.81			

Spectral Distribution of Retinal Image of Target and Surround

It was also noted[3] that surrounds reflecting more short-wave flux than the target caused, almost without exception, the targets to be harder to detect at an angular subtense of 10 minutes than surrounds reflecting less short-wave flux than the target. It was suggested that this regular departure of the experimental results from the model represented by Eq. 3 might be ascribable to selective scattering within the eyeball[7, 8] of short-wave flux from the surround in excess of that scattered from the target. It is evident also that chromatic aberration of the lens system of the human eye also may misdirect short-wave flux from the surround so that it falls on the retinal area corresponding to the image of the target formed by flux of whatever wavelength is accurately focused. For targets of large angular subtense the spectral distribution of target and surround may be so close to those of the light leaving the target and surround that Eq. 2 could be expected to apply. For targets of angular subtense as small as 10 minutes, however, Eq. 3 evaluated in accord with the external flux could not be expected to give correct predictions if the fluxes forming the retinal images differ significantly from external fluxes.

Given a light source of spectral irradiance E_λ, illuminating a target of spectral reflectance factor $R_t(\lambda)$ viewed against an identically illuminated surround of spectral reflectance factor $R_s(\lambda)$ by a perfectly focused eye, free of spherical and chromatic aberration, whose media have a spectral absorptance $\alpha(\lambda)$ but are perfectly non-light-scattering. The retinal images of target and surround would have the following spectral distributions:

Target	Surround
$E_\lambda R_t(\lambda)[1-\alpha(\lambda)]$	$E_\lambda R_s(\lambda)[1-\alpha(\lambda)]$

If, however, an eye having the chromatic aberration characteristics of real human eyes be substituted, then light of some wavelength λ, is diverted because of chromatic aberration, or intraocular scattering, away from the image of the target to be distributed over the image of the surround, and if the same fraction $S(\lambda)$ is diverted onto the image of the target that without scattering or chromatic aberration would have contributed to the retinal image of the surround, then the spectral distribution of the target image becomes:

$$E_{\lambda,ti} = [1-S(\lambda)]E_\lambda R_t(\lambda)[1-\alpha(\lambda)] + S(\lambda)E_\lambda R_s(\lambda)[1-\alpha(\lambda)] = E_\lambda[1-\alpha(\lambda)]\{[1-S(\lambda)]R_t(\lambda) + S(\lambda)R_s(\lambda)\} \quad (5)$$

If the target is small compared to the surround (such as a 10-minute target on a 120-minute surround), the spectral distribution of the surround image will not differ significantly from that, $E_\lambda R_s(\lambda)[1-\alpha(\lambda)]$, for the aberration-free and scatter-free eye. By this model the difference between the spectral distributions $E_{\lambda,ti}$ of the target image and that $E_{\lambda,si}$ of the surround image is:

$$E_{\lambda,ti} - E_{\lambda,si} = E_\lambda[1-\alpha(\lambda)]\{[1-S(\lambda)]R_t(\lambda) - [1-S(\lambda)]R_s(\lambda)\} = E_\lambda[1-\alpha(\lambda)][1-S(\lambda)][R_t(\lambda)-R_s(\lambda)] \quad (6)$$

Note that, by this model, the correct prediction for the case of $R_t(\lambda)$ identical to $R_s(\lambda)$ is given; that is, the target would then be undetectable because the target image and the surround image are likewise identical. Note also that this model predicts the same size of spectral-distribution differences between target and surround if they are interchanged. In spite of this spectral distribution symmetry of the model, Eq. 3 for perceived size of color differences yields lower predicted sizes for red targets on green surrounds, than for green on red, as it must do to agree with experiment.

The suggestion that scattering of light within the eye of the observer might account for the difficulty of detecting targets with short-wave surrounds has been studied by setting: $S(\lambda) = 0.0256/\lambda^4$, in accord with Rayleigh scattering inversely proportional to the fourth power of the wavelength in micrometers; see Fig. 1. Note that the constant 0.0256 has been chosen as 0.40^4; so no greater degree of light scattering is admissible by this suggestion, for this would make $S(0.40)$ greater than one, the maximum fraction possible.

The suggestion that chromatic aberration of the lens system of the human eye is responsible for the difficulty has been studied by defining $S(\lambda)$ in accord with the following argument. If, because of defocussing, the flux directed by a focused eye into a circle of radius r_o on the retina is spread out uniformly over a greater circular area of radius r, then the fraction of the flux falling within the circle of radius r and failing to contribute to the

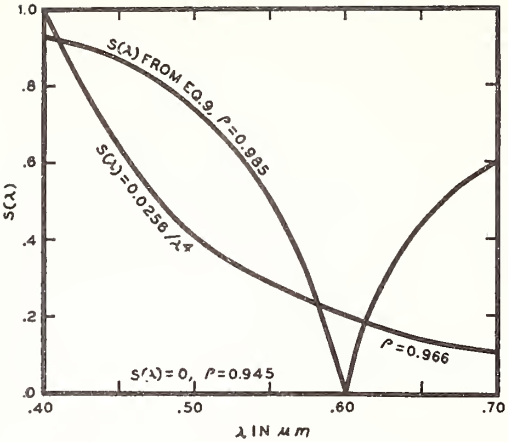

Figure 1. Three choices of the wavelength function $S(\lambda)$ indicating what fraction of the flux reflected from the target (surround) is misguided by the lens system and optical media of the eye so as to fall on the image of the surround (target). Note that for the assumption of a perfect eye, $S(\lambda) = 0$, the correlation coefficient between experimentally determined visibility $1/T$ and the perceptual size of the color difference between target and surround ΔE computed from Eq. 10 for $k_u = 0.05$, $k_v = 0.00$, $k_w = 0.05$, is 0.945; from the Rayleigh-scattering model, $S(\lambda) = 0.0256/\lambda^4$, is 0.966; and from the chromatic-aberration model, $S(\lambda)$ from Eq. 9, and k_u in Eq. 10 equal to 0.04, is 0.985.

target image may be taken as an evaluation of the function $S(\underline{\lambda})$ and is:

$$S(\underline{\lambda}) = (r^2 - r_o^2)/r^2 \qquad (7)$$

Of course, chromatic aberration produces a graded, rather than uniform halo; but this simplified first approximation introduces sufficiently well the indicated wavelength dependence of $S(\lambda)$. From the data of Wald and Griffin,[9] Ivanoff,[10] and Bedford and Wyszecki,[11] it may be shown that the refractive power of the eye is closely equal to $\underline{a} + \underline{b}/\lambda^2$, where $\underline{a}$ and $\underline{b}$ are constants independent of wavelength λ. If the wavelength for which the eye is in focus is λ_o, and if the radius of a circular image of 10-minute angular subtense is increased from r_o to $\underline{r}$, where the difference in radius $r - r_o$, is proportional to the absolute value of the difference in refractive power, then:

$$r - r_o = Kb \,|\, (1/\lambda^2) - (1/\lambda_o^2) \,| \qquad (8)$$

where Kb is a constant of proportionality to be adjusted to fit the experimental data of Table II.

By combining Eqs. 7 and 8 we can derive an expression for $S(\lambda)$ explicitly in terms of wavelength in micrometers and two constants, λ_o, the wavelength for which the observer's eye is in focus, and Kb/r_o, both of which may be adjusted to fit the experimental data:

$$S(\lambda) = \frac{[1 + (Kb/r_o)\,|\,(1/\lambda^2) - (1/\lambda_o^2)\,|\,]^2 - 1}{[1 + (Kb/r_o)\,|\,(1/\lambda^2) - (1/\lambda_o^2)\,|\,]} \qquad (9)$$

Incidentally, in the combined constant, Kb/r_o, only the constant, K is to be adjusted to fit the data of Table II. The constant $\underline{b}$ is determined by the change in refractive power of the eye with wavelength, and the constant r_o for a target subtending 10 minutes of arc is about 25 micrometers. Fig. 1 shows $S(\lambda)$ computed by Eq. 9 for $\lambda_o = 0.60$ μm, and for $Kb/r_o = 0.80$.

Optimization of Constants in the Chromatic-aberration Model

The method of adjustment was by trial and error. For each tentative choice of the constants, λ_o and Kb/r_o, of Eq. 9, and k_u and k_v of the general form of Eq. 2:

$$\Delta E = [(k_u \Delta U^*)^2 + (k_v \Delta V^*)^2 + (k_w \Delta W^*)^2]^{1/2} \qquad (10)$$

the coefficient of correlation ρ was obtained by digital computer between the values of ΔE found from Eq. 10 for $k_w = 0.50$, and the 263 experimental values of visibility given in Table II as $1/T$ for surrounds of Munsell value 8/. Table III shows the values of product-moment correlation coefficient ρ for the most significant of the 40 determinations made. It also shows values of the associated coefficient of alienation[12], $(1 - \rho^2)^{1/2}$, and coefficient of forecasting efficiency[13], $1 - (1 - \rho^2)^{1/2}$. The coefficient of alienation gives the fraction of the variation in visibility, $1/T$, not accounted for by the model, while the coefficient of forecasting efficiency gives the fraction that is.

From the last entry in Table III, it will be noted that by counting only the light-dark components of the color differences between target and surround already more than 50 per cent of the variations in observed visibility are accounted for. From the first entry in Table III, it will be noted that Eq. 3 applied to the external spectral distributions accounts for more than 67 per cent of the variations. The improvement from 50 to 67 per cent is the result of taking into account the red-green as well as the light-dark components. The application of Eq. 3 to the spectral distributions estimated for the retinal image by the Rayleigh-scattering model brings a considerable further improvement by accounting for more than 74 per cent of this variation. The chromatic-aberration model, however, makes possible an even larger improvement by accounting for more than 82 per cent of the variations. To do this the wavelength

λ_o of perfect focus was placed between 0.60 and 0.64 micrometers, the constant Kb/r_o was set between 0.60 and 0.85, and the constants of Eq. 10, k_u and k_v, were placed between 0.035 and 0.050 and 0.000 and 0.010, respectively. Table II, third column, shows values of ΔE computed from Eqs. 5, 9 and 10 by setting $\lambda_o = 0.60$, $Kb/r_o = 0.80$, $k_u = 0.040$, and $k_v = 0.000$.

It will be noted from Table II that, as expected, the color differences expressed in multiples of the threshold as $1/T$ are about four times as large as the same differences ΔE expressed in 1964 units of color difference by computation from the spectral reflectance factors of the target and surround and the spectral distribution of the illuminant (standard cool white fluorescent). The standard error of estimating values of $1/T$ by this method may be computed from the standard deviation of the 263 measures of $1/T$ (about 9.0) by multiplying by the coefficient of alienation given as 0.175 in Table III, and is $\pm$ 1.6. A plot of the values of $1/T$ against those of ΔE in Table II bears this out. The number of target-surround combinations departing from the average curve by more than 1.6 multiples of the threshold is about one-half the number that departs by less than this. See Fig. 2.

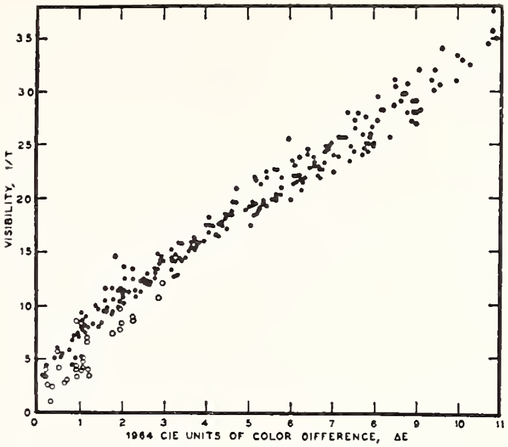

Figure 2. Plot of Visibility ($1/T$) of 266 target-surround combinations vs 1964 Units of Color Difference (ΔE) from Table II. The open circles are for targets lighter than their backgrounds.

Summary and Conclusions

1. Visibility determinations by the contrast-threshold visibility meter of 263 combinations of target and surround colors have been made by one of us (AAE) for angular subtense of target set at 10 minutes of arc; see Table II. The target color in each of these combinations was either definitely darker than the surround color, or of about the same lightness.

2. These experimental data have been analyzed relative to several theoretical models with the following results: .

A. About two-thirds of the variations in visibility found experimentally are explained by computing from a modification of 1964 CIE uniform color space, the size of the color difference between target and surround. The modification takes the form of neglecting entirely variations in V^* (violet to greenyellow), counting variations in U^* (red to green) at only one-twentyfifth their large-field values, and counting variations in W^* (light to dark) at only one-half; see Eq. 3, Table III, and Fig. 1.

B. About three-quarters of the variations are explained by applying the same modification of 1964 color space, not to the external distributions, but to the spectral distributions of the corresponding retinal images computed on the assumption that these images are degraded by Rayleigh scattering by the media of the one eye used to obtain data; see Eqs. 3, 6, Table III, Fig. 1.

C. More than four-fifths of the variations are explained by computing the spectral distributions of the retinal target image on the assumption that these images are degraded by chromatic aberration; see Eqs. 5, 9 and 10, Table III, and Fig. 1.

3. The following conclusions have been drawn:

A. The contrast-threshold visibility meter[1] is useful in fundamental psychophysical studies of the factors determining target detection as well as to evaluation of visibility in practical cases.

B. For targets subtending 10 minutes at the eye of the observer, and not appreciably lighter than the surround color, visibility depends upon the degree to which the retinal image of the target is degraded by the optical imperfections of the human eye. It seems to be sufficient for our one observer (AAE) to take account simply of the degradations produced by chromatic aberration alone on the assumption that the wavelength of perfect focus is about 0.60 μm.

C. It seems probable to us that the visibility ($1/T$) of any circular target, not appreciably lighter than the surround, subtending 10 minutes of arc, and viewed by the same observer (AAE) on any uniform surround, whatever the colors of target and surround may be, can be reliably predicted from Eqs. 5, 9 and 10 for $\lambda_o = 0.60$ μm, $Kb/r_o = 0.80$, $k_u = 0.04$, $k_v = 0.00$ and $k_w = 0.50$.

D. Our success with the one single observer strongly suggests that modifications of 1964 CIE uniform color space combined with the chromatic-aberration model can be applied successfully to

Table III. Coefficients of correlation, ρ, alienation, $(1-\rho^2)^{1/2}$, and forecasting efficiency, $1-(1-\rho^2)^{1/2}$, for various models for the fraction $S(\lambda)$ of light of wavelength λ diverted from the image of the target to be distributed over the image of the surrounds of Munsell value 8/. The total number of target-surround combinations is 263. Lightness constant, k_w, set at 0.50.

Model				Coefficient of correlation	Coefficient of alienation	Coefficient of forecasting efficiency
$S(\lambda) = 0$				0.945	0.327	0.673
$S(\lambda) = 0.0256/\lambda^4$ (Rayleigh scattering)				0.966	0.259	0.741
Chromatic-aberration models						
λ_o	Kb/r_o	k_u	k_v			
0.58	0.60	0.050	0.000	0.973	0.231	0.769
.58	.80	.050	.000	.976	.218	.782
.58	.90	.050	.000	.977	.213	.787
.58	1.00	.050	.000	.977	.213	.787
0.62	0.90	0.050	0.000	0.980	0.199	0.801
.64	.80	.050	.000	.982	.189	.811
.62	.80	.050	.000	.982	.189	.811
.61	.80	.050	.000	.983	.184	.816
0.60	0.60	0.050	0.000	0.9832	0.183	0.817
.60	.80	.060	.000	.9833	.182	.818
.59	.90	.050	.000	.9836	.180	.820
.62	.60	.050	.000	.9839	.179	.821
0.64	0.60	0.050	0.000	0.9840	0.178	0.822
.60	.85	.050	.000	.9842	.177	.823
.60	.75	.050	.000	.9842	.177	.823
.60	.80	.050	.000	.9843	.176	.824
0.60	0.80	0.035	0.000	0.98447	0.176	0.824
.60	.80	.040	.010	.98453	.175	.825
.60	.80	.040	.005	.984540	.175	.825
.60	.80	.040	.000	.984542	.175	.825
—	0.00	0.000	0.000	0.870	0.493	0.507

predict visibility determinations of 10-minute targets made by most other observers provided suitable adjustment be made of the constants, such as wavelength for perfect focus, known so far for the one observer (AAE) only.

E. Finally, we think that modifications of 1964 CIE uniform color space applied to the spectral distributions of retinal images of target and surround computed from the chromatic-aberration model can be successfully applied to targets of any angular subtense whatever by adjustment of the constant Kb/r_o of Eq. 9, and constants, k_u, k_v, k_w, of Eq. 10. These constants vary from $kb/r_o \rightarrow \infty$, $k_u = k_v = O$, $k_w \rightarrow O$ for angular subtense of target approaching zero, to $Kb/r_o = O$, $k_u = k_v = k_w = 1$ for targets subtending 60 minutes in such a way that $k_v \leq k_u \leq k_w$. The precise dependence of these constants on angular subtense of targets remains to be determined experimentally for a sufficiently large group of observers.

Acknowledgment

We take pleasure in acknowledging with thanks the assistance of Dr. Gary T. Yonemura, Office of Colorimetry, Institute of Applied Technology, National Bureau of Standards, who wrote out and tested the program for evaluating by digital computer the spectral distributions of the retinal images of the targets from Eqs. 6 and 9 for various choices of λ_o and Kb/r_o, for evaluating the corresponding color differences ΔE between retinal images of target and surround by Eq. 10 for various choices of k_u and k_v, and for calculating the correlation coefficients between ΔE, so found, and the experimentally determined visibilities. It should be obvious that without such a computer program this study would have been quite unfeasible.

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DISCUSSION

H. Richard Blackwell:* It is generally recognized that implementation of the method described in RQQ Report No. 4 for evaluating the degree of veiling reflections produced by different lighting installations will ultimately require predetermination of task contrast from physical data on the task and the installation. This procedure has been shown[1,2] to be equivalent to predetermining the relative level of task visibility for achromatic tasks, provided certain restrictions with respect to lighting directionality and task specularity are observed. However, many tasks have chromatic as well as achromatic contrast components. In such cases, predetermination will involve additional complications. Task properties will have to be assessed as a function of wavelength, and the spectral characteristics of direct and reflected light rays reaching the task must be known. Finally, it will be necessary to have a calculational method for combining achromatic and chromatic components of contrast.

Judd and Eastman have made the first major contribution to the solution of this complex problem. They have succeeded in developing a computational method for combining the effects of achromatic and chromatic contrast components to predict task visibility. The method requires evaluation of a constant representing the chromatic aberration of the individual eye involved, and this constant may well vary significantly among different members of the population. Population data will be required before serious practical use of the method can be considered.

The authors restrict their analysis to visual tasks "not appreciably lighter than the surround color," implying difficulty in applying it to tasks appreciably lighter than the surround. The discusser suggests that this difficulty may not be fundamental but related rather to the method used for collecting the experimental data with the Eastman Contrast Threshold Visibility Meter (CTM). As indicated elsewhere,[3] Eastman has now developed a procedure in which the luminance veil is matched to the task luminance in the manner advocated for use of the Visual Task Evaluator.[4] However, this procedure was not used in obtaining the data and an artifact may have been introduced. This possibility can be investigated most simply by repeating some measurements for tasks darker and lighter than their surround with the new procedure.

The discusser notes what seems a discrepancy between the conclusion reached in the present paper and that originally reached by Eastman.[5] Judd and Eastman seem to conclude that chromatic contrasts always enhance the effects of existing luminance contrasts, whereas Eastman suggested that chromatic contrasts sometimes reduce the effects of existing luminance contrasts. Which is it?

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Dean B. Judd and Arthur A. Eastman:* We are gratified to note Dr. Blackwell's appraisal of our work as "the first major contribution to solution" of the problem of "combining achromatic and chromatic components of contrast." We agree that population data on chromatic aberration of the eye will be required before serious practical use of the method can be considered.

Like Dr. Blackwell, we are reluctant to conclude from observations by a single observer that 10-minute targets lighter than the surround are inherently harder to detect than those darker than the surround. We have no explanation for this experimental indication and are searching for an artifact that might explain it away.

Following Dr. Blackwell's suggestion, one of us (AAE) has determined the visibilities of ten target-surround combinations of neutral gray by means of Contrast Threshold Meter No. 2, and has obtained repeat determinations by a second observer (JFM). The targets and surround were illuminated by standard cool white fluorescent lamps so that the surround (N 7/) had a luminance $Y_s = 100$ fL.

Each visibility determination was obtained as the average of 25 settings taken five in succession for five series. Table 4 gives the values, so found, of transmittance T of the beam splitter and of the reflectance R of the instrument (product of reflectance of beam splitter and that of the auxiliary mirror) by which the veiling luminance required to produce the threshold condition was produced from Munsell neutral 7/ used for the surround. It also gives in footlamberts the surround luminance Y_{sv} for the thershold condition computed as $100(R + T)$. The threshold contrast defined as $(Y_{tv} - Y_{sv})/Y_{sv}$ has been computed in per cent as $100[T/(R+T)][(R_t/R_s)-1]$. Table 4 also gives these results. It is evident that these repeat determinations support the previous experimental indication. Targets lighter than the surround seem to be about 20 per cent harder to detect than those darker than the surround. We have not yet found any artifact that could explain away this indication.

Dr. Blackwell further remarks, "Judd and Eastman seem to conclude that chromatic contrasts always enhance the effects of existing luminance contrasts whereas Eastman suggested that chromatic contrasts sometimes reduce the effects of existing luminance contrasts. Which is it?"

Our model does indeed imply that at the retinal level chromatic contrasts always enhance the effects of existing

*Authors.

Table 4—Contrast thresholds corresponding to visibility determinations of ten target-surround combinations; five with targets darker than the surround, and five lighter. Surround was Munsell N 7/, R_s = 42.60. Illumination was by standard cool-white fluorescent lamps so as to make the surround luminance 100 fL.

Targets: Munsell Notation	Targets: Reflectance in % R_t	Wedge Set for Threshold: Reflectance, R: AAE	Reflectance, R: JFM	Transmittance, T: AAE	Transmittance, T: JFM	Surround Luminance = 100(R + T), fL: AAE	JFM	Contrast in Percent = [T/(T + R)] $\left[\frac{R_t}{R_s} - 1\right]$: AAE	JFM
N 2/	3.2	0.3710	0.3665	0.0155	0.0184	38.65	38.49	−3.7	−4.4
N 3/	6.3	.3690	.3648	.0170	.0194	38.60	38.42	−3.8	−4.3
N 4/	12.3	.3655	.3600	.0190	.0223	38.45	38.23	−3.5	−4.1
N 5/	19.9	.3540	.3460	.0253	.0290	37.93	37.50	−3.6	−4.1
N 6/	29.7	.3170	.3180	.0434	.0430	36.04	36.10	−3.6	−3.6
				Average for targets darker than surround				−3.64	−4.12
N 7.5/	50.9	0.2600	0.2320	0.0755	0.0920	33.55	32.40	4.4	5.5
N 8/	58.4	.3215	.3055	.0412	.0493	36.27	35.48	4.2	5.2
N 8.5/	68.0	.3513	.3133	.0266	.0300	37.79	34.33	4.2	5.2
N 9/	78.4	.3640	.3622	.0200	.0210	38.40	38.32	4.4	4.6
N 9.5/	88.0	.3706	.3667	.0158	.0183	38.64	38.50	4.4	5.1
				Average for targets lighter than surround				4.32	5.14

luminance contrasts. The model also implies that because of chromatic aberration the spectral distribution of the retinal image of the target differs importantly from that of the light leaving the target. It is an experimental fact that sometimes substitution of a target-surround combination, involving both a chromatic and a light-dark component, for a combination of the same light-dark contrast, without any chromatic component, sometimes makes the target less, rather than more, visible. In this sense addition of an external chromatic component may sometimes reduce rather than increase visibility. Our model predicts this.

Take the case of a spectrally nonselective gray (Munsell notation: N 6.9/) of luminous reflectance factor 42.3 per cent viewed on a light nonselective gray surround (N 7.9/) of luminous reflectance factor equal to 57.4 per cent. The luminance contrast for this target-surround combination is: $C = (57.4 - 42.3)/57.4 = 15.1/57.4 = 0.26$ or 26 per cent. Compare this to a second target-surround combination composed of the same gray (N 6.9/) viewed on a light blue surround (B 7.9/4) of the same luminous reflectance factor, 57.4 per cent, as the light gray surround of the first combination; see the accompanying Fig. A for the three spectral reflectance curves involved. By substituting the second combination for the first we have in effect added a chromatic component to a luminance contrast of 26 per cent. Application of our model shows, however, that a lower rather than a higher visibility is predicted. The details of the computation are shown in Table 5, accompanying this discussion.

The first column gives the spectral reflectance factors $R(\lambda)$ of the Munsell sample 5B 8/4 used in our study. The second shows luminous efficiencies $\bar{y}(\lambda)$ at the corresponding wavelengths. The third shows the products $R(\lambda)\bar{y}(\lambda)$. The ratio of the sums of these columns:

$$\Sigma R(\lambda)\bar{y}(\lambda)/\Sigma\bar{y}(\lambda) = 6.126/10.672 = 0.574$$

shows that for an equal-energy source this light blue surround (5B 7.9/4) has the same luminous reflectance factor

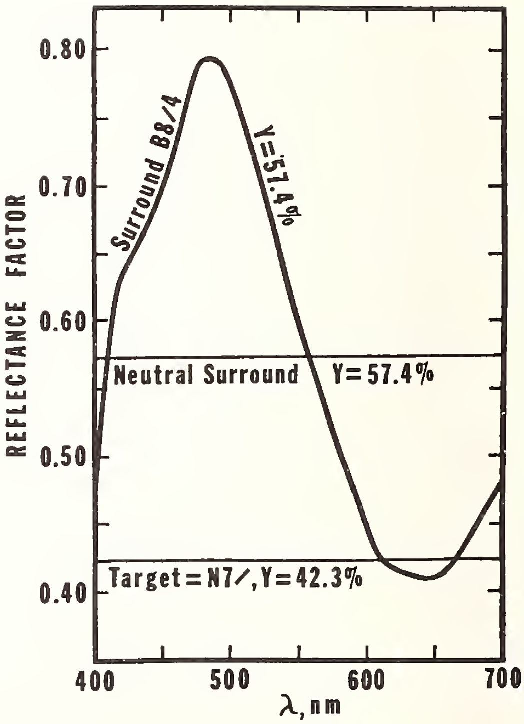

Figure A.

Table 5—Computation of light-dark contrast at the retinal level for two target-surround combinations: N 6.9/ on N 7.9/, and N 6.9/ on B 7.9/4

Wave Length λ	Reflectance of B 7.9/4 R_s	Luminous Efficiency $\bar{y}(\lambda)$	Product $R_s\bar{y}(\lambda)$	For λ_o = 600 nm, $Kb/r_o = 0.8$ $1 - S(\lambda)$	$(R_t - R_s)[1 - S(\lambda)]$ N 6.9/ on N 7.9/	$(R_t - R_s)[1 - S(\lambda)]$ N 6.9/ on B 7.9/4	$\bar{y}(\lambda)(R_t - R_s)[1 - S(\lambda)]$ N 6.9/ on N 7.9/	$\bar{y}(\lambda)(R_t - R_s)[1 - S(\lambda)]$ N 6.9/ on B 7.9/4
430	0.655	0.0116	0.0076	0.104	−0.016	−0.024	−0.0002	−0.0003
440	.677	.023	.0156	.118	− .018	− .030	− .0004	− .0007
450	.702	.058	.0267	.134	− .020	− .037	− .0008	− .0014
460	.736	.060	.0442	.152	− .023	− .048	− .0014	− .0029
470	.774	.091	.0704	.174	− .026	− .061	− .0024	− .0056
480	.794	.139	.1104	.198	− .030	− .073	− .0042	− .0101
490	.795	.208	.1654	.225	− .034	− .084	− .0071	− .0175
500	.775	.323	.2503	.256	− .039	− .090	− .0126	− .0291
510	.740	.503	.3722	.291	− .044	− .092	− .0221	− .0463
520	.703	.710	.4991	.332	− .050	− .093	− .0355	− .0660
530	.667	.862	.5750	.378	− .057	− .092	− .0491	− .0793
540	.630	.954	.6010	.432	− .065	− .089	− .0620	− .0849
550	.597	.995	.5940	.494	− .075	− .086	− .0746	− .0856
560	.564	.996	.5612	.566	− .085	− .080	− .0846	− .0796
570	.535	.952	.5093	.650	− .098	− .073	− .0933	− .0695
580	.505	.870	.4394	.749	− .113	− .061	− .0983	− .0531
590	.473	.757	.3581	.864	− .130	− .043	− .0984	− .0326
600	.443	.631	.2795	1.000	− .151	− .020	− .0953	− .0126
610	.428	.503	.2123	.869	− .131	− .004	− .0659	− .0020
620	.417	.381	.1589	.768	− .116	+ .005	− .0442	+ .0019
630	.413	.265	.1094	.687	− .104	+ .007	− .0276	+ .0019
640	.410	.175	.0718	.620	− .094	+ .008	− .0164	+ .0014
650	.410	.107	.0439	.566	− .085	+ .007	− .0091	+ .0007
660	.415	.061	.0253	.521	− .079	+ .004	− .0048	+ .0002
670	.430	.032	.0138	.482	− .073	− .003	− .0023	− .0001
680	.440	.017	.0076	.449	− .068	− .012	− .0012	− .0002
690	.468	.0082	.0038	.420	− .063	− .019	− .0005	− .0002
	Total	10.672	6.126				−0.9143	−0.6735
	Ratio		0.574				−0.0857	−0.0631

(57.4 per cent) as the light gray surround (N 7.9/) assumed for the first combination.

The fourth column shows values of $1 \cdot S(\lambda)$ with $S(\lambda)$ computed from Eq. 9 and plotted in Fig. 1. The fifth and sixth columns evaluate $[R_t(\lambda) - R_s(\lambda)][1 - S(\lambda)]$, column 5 for the gray target (N 6.9/) on the light gray surround (N 7.9/), column 6 for the same gray target on the light blue surround (B 7.9/4). Note that entries in column 5 are simply values of $(0.574 - 0.423)\ [1 - S(\lambda)] = 0.151\ [1 - S(\lambda)]$ because both target and surround are taken as spectrally nonselective grays of reflectance factors 42.3 and 57.4 per cent, respectively. Columns 5 and 6 show the differences in spectral distribution of the fluxes forming the retinal images of target and surround according to Eq. 6 provided the surrounds and target are irradiated by an equal-energy source ($E\lambda = 1$) and viewed by an absorption-free eye so that $a(\lambda) = 0$. Columns 7 and 8 are found by multiplying by the spectral luminous efficiencies $\bar{y}(\lambda)$. They represent the corresponding differences in luminous terms. Most of the entries in columns 5 through 8 are less than zero because the target (N 6.9/) is darker than either of the two surrounds (N 7.9/ and B 7.9/4). The sums of columns 7 and 8 indicate that although, externally evaluated, the gray target contrasts by 26 per cent from either the light gray (N 7.9/) or in the light blue (B 7.9/4) surround, according to the chromatic-aberration model proposed by us the gray-on-gray light-dark contrast is $(57.4 - 48.8)/57.4 = 8.6/57.4 = 0.15$ or 15 per cent. The light-dark contrast of the gray-on-blue combination is, however, reduced even more to $(57.4 - 51.1)/57.4 = 6.3/57.4 = 0.11$ or 11 per cent. Our model thus predicts for this example that substitution of a light blue surround for a light gray surround of the same luminous reflectance factor produces at the retinal level a reduced light-dark contrast, and as a matter of fact reduces the visibility of the target because the chromatic (red-green) component of the color difference between target and surround for either of these combinations is negligible at the retinal level.

A glance at Fig. A will make clear what is going on here. Note that in the neighborhood of 600 nm the difference in spectral reflectance factor between the gray target and the light blue surround is considerably less than that for the two grays. Note also that by our model this spectral region is the only spectral region in reasonably sharp focus.

COLOR IN VISUAL SIGNALLING

Paper in Color Vision, Symposium at 1971 Spring Meeting, Committee on Vision, Nat. Res. Council, pp. 65-82 (Nat. Acad. Sci., Washington, D.C., 1973)

Figure 7, and the formula on p. 71, are the culmination of Judd's work on target and signal detection. This review of that work should not present any terminological problems to readers who have read this far in this collection.

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NATIONAL BUREAU OF STANDARDS

Color in Visual Signaling

Visual signaling refers to detection from a distance and identification by eye of a light or an object whose presence has a previously established meaning. The presence of an airplane flying at night is signaled by its running lights, and there are visual signals to control traffic on railroads, on highways, and at sea. These are examples of *self-luminous* visual signals.

The presence of an obstruction to aviation may be signaled by day by the alternate stripes of aviation orange and white painted on the obstruction, and ground-to-air signals may be set up for daytime use by displaying horizontal panels painted with fluorescent paint. These are examples of visual signaling by means of objects made visible by daylight illumination. Such signals are *non-self-luminous.*

The eye does not care whether the signal is self-luminous or non-self-luminous, but self-luminous signals usually have a dark, and non-self-luminous signals, a bright, surround. This difference is important.

FACTORS INFLUENCING SIGNAL DETECTION AND IDENTIFICATION

Whether a message is delivered by a signaling device depends on a number of variables, some of which are given below:

1. Distance between observer and the light or object forming the signal
2. Size of the light or object
3. Rate at which luminous energy enters the eye of the observer from the signal area. This rate depends on distance and size of the signaling device and on its luminance. If the signal area is brighter than the surround, the pertinent flux is that from the signaling device itself. If it is darker, the pertinent flux is what would enter the eye if the signaling object were not there; that is, the luminous flux that drives the visual mechanism comes from the bright surround rather than from the object. A black object silhouetted against the horizon sky may make a very conspicuous visual signal.
4. Fog, smoke, or haze of the atmosphere between the signaling device and observer
5. Chromaticity difference between the signal and its surround
6. Texture of the surround, whether nearly uniform, cut up by sharp shadows, or punctuated by street lights
7. Visual system of the observer, whether normally acute or amblyopic, or trichromatic, dichromatic (partially color-blind), or monochromatic (totally color-blind).

Other obvious factors are whether the observer has uncorrected refractive errors, has his eyes pointed approximately in the right direction, or, indeed, has them open at all.

ROLE OF THE CHROMATIC ASPECT OF THE VISUAL SIGNAL

The role of the chromatic aspect of the visual symbol is twofold. First, it has some bearing on the visibility of the signal. Second, it is often entrusted with the responsibility of identifying the signal, either in conjunction with a spatial or temporal pattern, or by itself. This second part is probably the more important and depends importantly on the number of colors in the signal system, in addition to factors 1 through 7, above.

Although the effect of the chromaticity of the signal on its visibility is of minor importance, I will discuss this effect first because research carried out by the Office of Colorimetry in its last few years of existence at the National Bureau of Standards has gone a long way toward determining what this effect is. Furthermore, this research also suggests ways of determining whether a visual signal of a given luminance and size will be successfully identified at a given distance by an observer equipped with a given visual system. However, these suggestions require validation.

EASTMAN VISIBILITY DATA FOR 10-MINUTE TARGETS ON LIGHT SURROUNDS

Eastman (1968b) determined the visibilities of about 300 combinations of target and surround colors by means of his newly developed contrast–threshold visibility meter (1968a). The targets subtended 10 min of arc, and the immediate surrounds subtended 120 min of arc and were either lighter than the targets or of about the same lightness. The surround luminance was about 40 foot-lamberts (FL); the target luminances ranged between about 8 and 42 FL. Both target and surround colors ranged through all hues and had Munsell chromas close to the maximum for that Munsell value and hue.

The Eastman visibility meter evaluates the visibility of a target–surround combination by permitting the observer to reduce the contrast of the scene until the target is just visible. The contrast reduction is achieved by having the observer look at the scene through a beam-splitting wedge of adjustable transmittance and reflectance. Light diffused from a white standard illuminated by the same light as that used for the scene is added by reflection from the beam splitter at the same time as the light coming from the scene is reduced by transmission through the beam splitter. Since the transmittance, T, and reflectance, R, of the beam-splitting wedge add up to approximately the same sum regardless of the portion of the wedge chosen by the observer, the luminance of the scene displayed by the visibility meter is kept approximately constant.

If Y_t, Y_s and Y_w refer to luminance of target, surround, and the white reflected by the beam splitter, then the luminances of target and surround viewed through the beam splitter are:

$$Y_{tv} = TY_t + RY_w,$$
$$Y_{sv} = TY_s + RY_w.$$

From that we find:

$$\frac{(Y_t - Y_s)}{Y_{tv} - Y_{sv}} = \frac{1}{T}, \tag{1}$$

that is, the actual luminance difference between target and surround, expressed as a multiple of the threshold, is equal to the reciprocal of the transmittance of the beam splitter adjusted to the threshold condition.

ANALYSIS OF THE EASTMAN DATA

This analysis led to a method that made it possible to predict the experimentally determined sizes of color differences between target and surround expressed as a multiple of threshold (Judd and Eastman, 1971). The basis of the method is to modify the color space recommended in 1964 by the CIE for 60-min fields to apply to 10-min fields. The recommendation was made pending development of an improved coordinate system for use "whenever a three-dimensional spacing perceptually more nearly uniform that that provided by the XYZ-system is desired." This space is formed by plotting variables, U^*, V^*, W^* (defined in terms of the tristimulus values, X, Y, Z, of the color) along mutually perpendicular axes:

$$W^* = 25Y^{1/3} - 17, \text{ for } 1 \leqslant Y \leqslant 100;$$

$$U^* = 13W^*(u - u_o), \ u = \frac{4X}{(X + 15Y + 3Z)}$$

$$V^* = 13W^*(v - v_o), \ v = \frac{6Y}{(X + 15Y + 3Z)},$$

where u_o, v_o, refer to the achromatic or neutral reference color. The formula recommended for estimating the perceptual size of a color difference, ΔE, is:

$$\Delta E_{60\,\text{min}} = [(\Delta U^*)^2 + (\Delta V^*)^2 + (\Delta W^*)^2]^{1/2}, \tag{2}$$

where $\Delta U^* = U_1^* - U_2^*$. The subscripts 1 and 2 refer to the two colors defining the color difference, and similar definitions are used for ΔV^* and ΔW^*. Note that this formula is a simple square root of the sum of the squares.

Figure 1 shows a cross section of the boundary surface of this color space. This boundary surface is the locus of points representing the colors of the spectrum; the large circles show this spectral locus and are located by plotting $v - v_0$ against $u - u_0$ for the spectral colors. A very similar locus is shown by the small dots, which were computed for the spectral colors according to the second stage of the Müller theory of vision. The close agreement between these two loci indicates (Judd and Yonemura, 1970) that CIE 1964 Uniform Chromaticity Scale (UCS) color space has a theoretical as well as an empirical basis and accords with the Müller stage theory.

The CIE 1964 UCS space permits easy computation of predictions

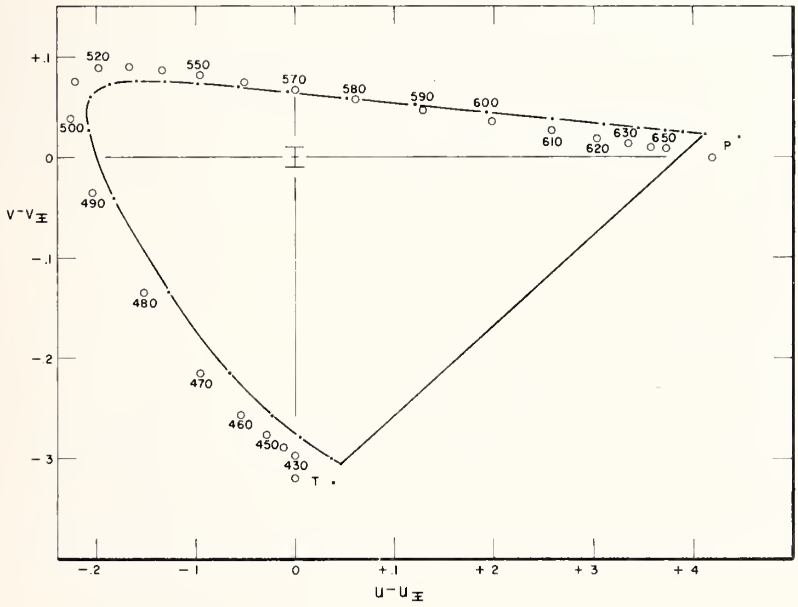

FIGURE 1 Horizontal cross section of CIE 1964 $U^*V^*W^*$ color space (large circles) compared with the spectra locus of a chromaticity diagram based upon the second stage of the Müller theory. The CIE 1964 $U^*V^*W^*$ color space and the Müller second stage differ by amounts that are insignificant in practice.

of perceived size for monochromatic (totally color-blind) and dichromatic (protanopic, deuteranopic, and tritanopic) vision as well as for normal trichromatic vision. The first term of the difference formula, $(\Delta U^*)^2$, refers to the red–green component of the color difference; the second, $(\Delta V^*)^2$, to the violet–green yellow component; and the third, $(\Delta W^*)^2$, to the light–dark component. For an observer who confuses red with green (protanope or deuteranope), the formula for perceived size of color difference would be essentially changed only by deletion of the first term, and the corresponding color space would collapse to a vertical plane intersecting the v–v_o axis. For an observer who confuses violet with green yellow (tritanope), the second term should be deleted, and the resulting color space would likewise collapse to a vertical plane, this time intersecting the cross section along the u–u_o axis. For an observer who can make no chromatic distinctions at all (totally color-blind), both the first and second terms would be deleted, and the color space would collapse to a vertical line intersecting the cross section at the point, u_o, v_o, corresponding to an achromatic color.

The predictions based in this way on Formula 2 are only approximate, and by using the angle version (Judd and Yonemura, 1970) of the formula, considerably better predictions for dichromatic vision may be obtained. For predicting the detection of visual signals by observers with normal vision, however, the simple $U^* V^* W^*$-system is amply successful.

SMALL-FIELD AND LOW-LUMINANCE TRITANOPIA

When luminous flux entering the pupil of the eye from the signal area is insufficient to drive the visual mechanism, the violet–green yellow component of color differences is lost first. The flux may be insufficient because the luminance of the signal area is low or because the angular extent of the signal area is small (Hartridge, 1947). It is plausible that luminous flux entering the eye should be a precise measure of the loss of violet–green yellow discrimination relative to red–green, and it was pointed out by Farnsworth (1955) that this is approximately true. Constant luminous flux entering the eye from the target is equivalent to constancy of the product $Y \times a^2$, Y for luminance, a for angular subtense of signal. Recent determinations (Yonemura and Kasuya, 1969) are, however, fitted better by the function $Y \times a^{3/2}$; so perhaps luminous flux entering the eye from the target is only an approximate determinant of the ratio of violet–green yellow to red–green discrimination. At any rate, over a considerable range in angular subtense (between about 1 and 60 min) too small a signal to detect can be compensated by an increase in its luminance.

To make the CIE 1964 space apply to dim targets of small angular subtense as well as to bright targets subtending more than 60 min of arc, we must generalize the color-difference formula by introducing the factors k_u, k_v, and k_w, which indicate fractions of normal discriminations in the red–green, violet–green yellow, and light–dark dimensions, respectively. Thus:

$$\Delta E = [(k_u \Delta U^*)^2 + (k_v \Delta V^*)^2 + (k_w \Delta W^*)^2]^{1/2} . \qquad (3)$$

When this formula is applied to Eastman's data, which were obtained using a stimulus of 10 min of arc, setting $k_v = 0$ resulted in the best predictions. This result confirms that Eastman's visibility settings were made by tritanopic vision. Introduction of even a slight positive value for this constant (for example, $k_v = .005$) resulted in a poorer correlation between predicted and observed values.

DEGRADATION OF RETINAL IMAGE BY VISUAL DEFECTS

Although the correlation coefficient obtainable by assuming a perfect image was quite respectable ($r = .945$), the deviations between experiment and prediction showed almost perfect regularity. These deviations suggested that the light forming the retinal image of the target was being contaminated by short-wave flux misdirected from the surround. Without any such contamination the expressions for the spectral distributions of the retinal images of target and surround are obviously:

$$\text{Target image} = E_\lambda R_t(\lambda)\,[1 - \alpha(\lambda)],$$

$$\text{Surround image} = E_\lambda R_s(\lambda)\,[1 - \alpha(\lambda)],$$

where E_λ is the spectral distribution of light incident on target and surround, R_t and R_s are spectral reflectance factors of target and surround, respectively, and $\alpha(\lambda)$ is the spectral absorptance of the ocular media.

If, however, we have an eye that, through chromatic aberration or intraocular scattering, diverts to the surround a fraction, $S(\lambda)$, of the light that should have gone to the target image, and diverts to the target the same fraction $S(\lambda)$ of the light that should have gone to the surround, the expression for the spectral distribution of the target becomes:

$$E_{\lambda-ti} = [1-S(\lambda)]\, E_\lambda R_t(\lambda)\, [1-\alpha(\lambda)] + S(\lambda)\, E_\lambda R_s(\lambda)\, [1-\alpha(\lambda)]. \quad (4)$$

For the retinal image of the surround, however, the expression given above for $S(\lambda) = 0$ is not significantly in error because the area of the 120-min surround is much greater than that of the 10-min target.

The first assignment of values for $S(\lambda)$ was made on the assumption that the misdirection of flux within the eye was caused by Rayleigh scattering of the ocular media. Some improvement in correlation coefficient resulted (.966 up from .945), but the residuals were still highly suggestive of more misdirection of short-wave flux than could result from Rayleigh scattering, even assuming that 100 percent of the flux at 400 nm was so misdirected.

The second assignment of values of $S(\lambda)$ was made on the basis that the misdirection of flux within the eye was caused by chromatic aberration of the lens of the eye. It was assumed that the eye was in perfect focus for one wavelength only and that at other wavelengths misdirection of flux from the surround to target image would occur. Figure 2 shows the three assignments of values for $S(\lambda)$. $S(\lambda) = 0$ corresponds to an eye that misdirects no flux at all, and this gave a correlation coefficient of .945. $S(\lambda) = \frac{.0256}{\lambda^4}$ corresponds to an eye afflicted with maximum Rayleigh scattering, and this gave a correlation coefficient of .966. $S(\lambda)$ set equal to flux misdirected by chromatic aberration for perfect focus of flux of 600 nm gave a correlation coefficient of .985. The results of this study are summarized in Table 1. The light–dark component of the color difference between target and surround accounts for about half of the experimentally found variations in visibility. The light–

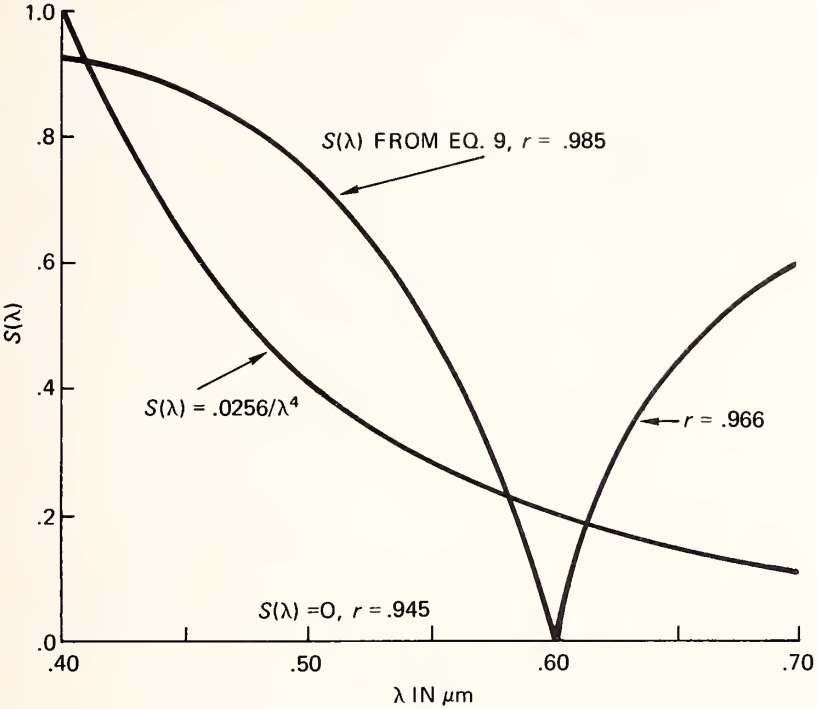

FIGURE 2 Variation of the fraction $S(\lambda)$ with wavelength: (a) for the hypothetical, scatter-free, aberration-free lens system; (b) for a lens system afflicted with maximum Rayleigh scattering, $S(\lambda) = \frac{.0256}{\lambda 4}$; and (c) for a scatter-free lens system afflicted by chromatic aberration except for perfect focus of flux of 600 nm.

dark and red–green components, taken together, account for about two thirds of the variations. Adding the assumption of Rayleigh scattering can raise this fraction from two thirds to three fourths, but other evidence indicates that scattering by the ocular media is nearly the same for all wavelengths. Substitution of an account of the influence of chromatic aberration can raise this fraction to four fifths, which corresponds to a correlation coefficient of .985. Figure 3 shows a plot of the experimental value of $1/T$ against the predictions on this latter basis. The degree of agreement implied by a correlation coefficient of .985 is obvious, but this plot can also show that the 1964 CIE unit of color difference used for the cal-

TABLE 1 Total Correlations between Observed and Predicted Visibility Using Various Assumptions

Predictions Obtained from the Following Assumptions:	Fractions of Normal Discrimination k_u	k_v	k_w	$S(\lambda)$	Correlation Coefficient	Fraction of Visibility Variation Explained
Only the light-dark component of the external difference	0	0	.50	0	.870	.507
The light-dark and red-green components of the external difference taken together	.04	0	.50	0	.945	.673
Maximum contamination of the retinal image of the target by Rayleigh scattering	.04	0	.50	$\frac{.0256}{\lambda^4}$	.966	.741
Contamination of the retinal image of the target by chromatic aberration model lens system in perfect focus at 600 nm	.04	0	.50	[a]	.985	.825

[a] From chromatic-aberration model; see Figure 2.

culated color differences is close to four times the threshold for 10-min targets, just as it is for fields of 60-min subtense.

IMPLICATIONS FOR COLOR IN VISUAL SIGNALING FROM EASTMAN'S DATA

These data imply that the chromatic aspect of a non-self-luminous signal has only minor bearing on whether the signal will be detected for visual signals subtending 10 min of arc. By reducing the angular subtense from 60 to 10 min, Eastman's red-green discrimination was reduced relative to light-dark discrimination by more than a factor of 10. (Compare k_u = .04 with k_w = .50.) Violet-green yellow discrim-

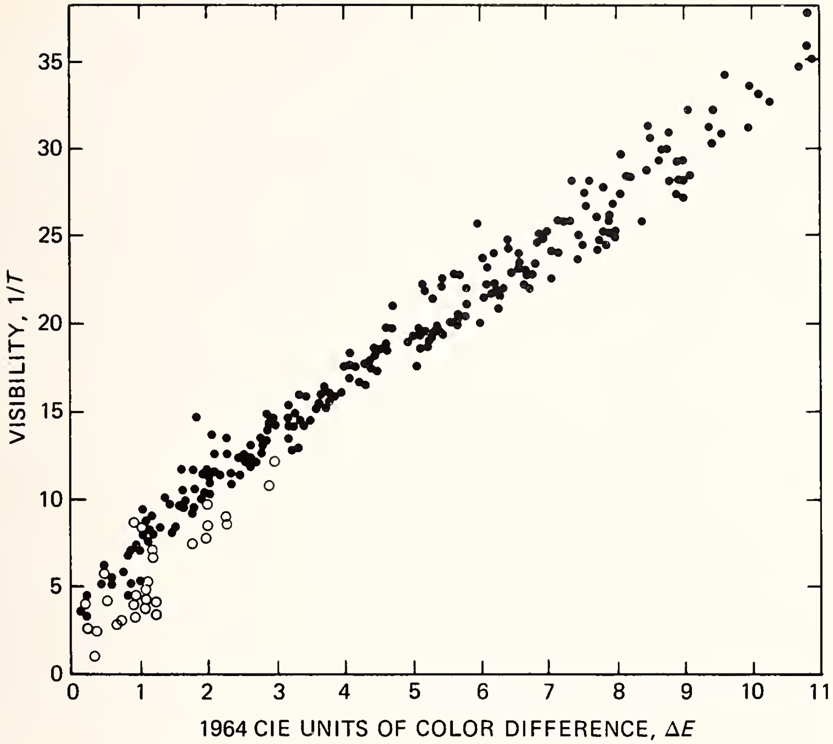

FIGURE 3 Relation between the visibility, $1/T$, obtained experimentally in multiples of the threshold on the Eastman contrast-threshold visibility meter, and the size of the color difference between target and surround, computed from the spectral reflectance factors and spectral distribution of illuminant (cool-white fluorescent), and expressed in 1964 CIE units. For the angular subtense of 10 min of arc applying to the experimentally determined visibilities, just as for angular subtenses greater than 60 min, the 1964 CIE unit of color difference corresponds approximately to four times the threshold.

ination has no bearing at all on Eastman's detection of a 10-min signal of luminance 50 FL.

The conclusion is based on a wide sampling of target colors displayed on light and not very chromatic surrounds of many hues but only one observer and only one angular subtense (10 min). A summary follows of work with a small number of target colors (9) viewed on each of only 2 surround colors (neutral gray and foilage green) but with 29 observers and for 5 angular subtenses of targets ranging from 2 to 30 min of arc (Judd and Yonemura, 1969).

TARGET CONSPICUITY AND ITS DEPENDENCE ON COLOR AND ANGULAR SUBTENSE

In this experiment the observer was shown target–surround combinations in which the light–dark difference was negligible and either the red–green difference or the violet–green yellow difference was negligible. He then rated them for conspicuity by comparing them with a series of six known light–dark differences, which were produced by gray targets of the same size as the chromatic targets displayed against a lighter gray surround. Surround luminance was close to 5 FL. The conspicuity ratings were obtained by each of 29 observers, most of whom made a repeat set of ratings about one week later. These observations serve to evaluate k_u and k_v relative to k_w. The dependence of k_w on angular subtense at constant luminance of surround was taken from Blackwell's (1946) work.

Figure 4 shows the individual conspicuity ratings of the 29 observers for each of five angular subtenses for the most conspicuous target–surround combination (vivid red on foliage green) and the least conspicuous (gray on foliage green). There are considerable individual differences in these ratings; moreover, the average ratings for the two combinations differ by highly significant amounts. The red–green difference declines in conspicuity with decline in angular subtense down to about 8 min of arc at the same rate as the light–dark differences with which it was compared, and so it is represented here as a horizontal line. The violet–green yellow difference, on the other hand, decreases more rapidly and has become negligible at a subtense of about 7 min.

Figure 5 shows the average conspicuity ratings for all nine target–surround combinations. The combinations involving a substantial red–green component of difference (solid lines) decline in conspicuity with decline in angular subtense more slowly than the violet–green yellow combinations (dotted lines).

Figure 6 shows the relation between the conspicuity equivalent value of W^*_e for 29-min subtense, and E, computed from the formula giving estimates of color-difference size for fields of 60-min subtense. The best-fitting straight line through the origin for the points 1, 2, 7, and 8, corresponding to red–green differences, has a higher slope than the line (dot-dash) fitting the points 4, 5, 6, and 9, corre-

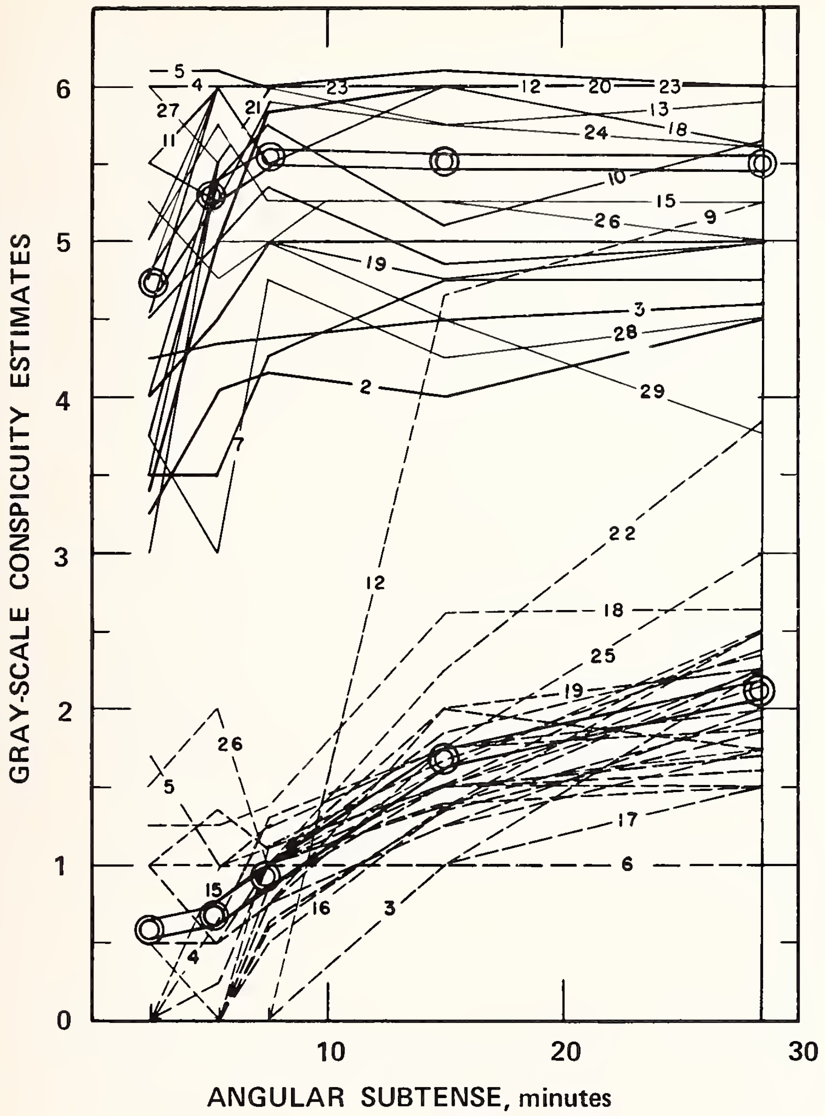

FIGURE 4 Conspicuity estimates of all 29 observers for two target–surround color combinations: red on foliage green, and gray on foliage green. The estimates are given by identifying the gray target of equivalent conspicuity on a gray surround of approximately Munsell N 6/0. Target 1 was intended to match the surround; target 2 was darker by about one half Munsell value step; target 3, by about one Munsell value step; and so on up to gray target 6, which was darker than the surround by about 2.5 Munsell value steps. The conspicuity estimates made by each observer for the five angular subtenses are identified by connecting the corresponding points by light solid lines for the red-on-foliage-green combination; those for gray on foliage green, by dotted lines. The average estimates for all 29 observers are indicated by double circles connected by heavy double lines.

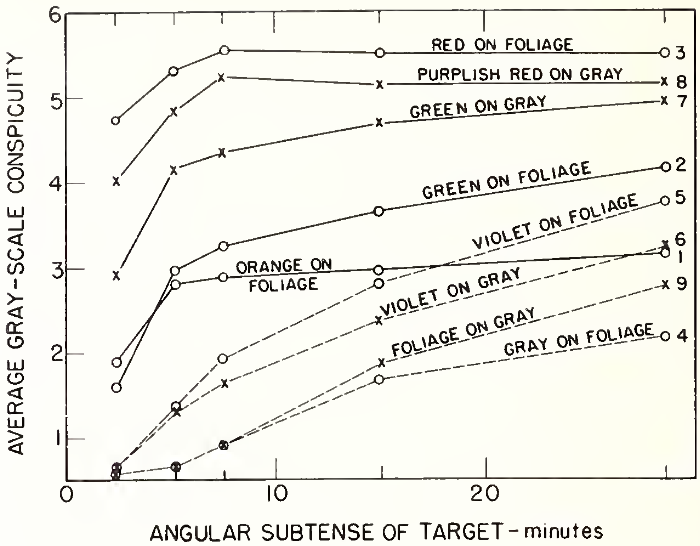

FIGURE 5 Average conspicuity estimates for all 29 observers for all nine target-surround color combinations. The conspicuity estimates for red targets remain essentially constant down to an angular subtense of 8 min of arc; those involving only violet-green yellow differences (dotted lines) decline rapidly in conspicuity within this range of angular subtense; and the two green targets differing in the red-green dimension from the surround color decline slowly.

sponding to violet-green yellow differences. This indicates that discrimination of violet-green yellow differences has suffered more than has red-green discrimination by changing the field size from 60 to 29 min at 5 FL.

Figure 7 shows the dependence of the constants k_u, k_v, k_w on angular subtense for luminance of surround equal to 5 FL. This dependence was derived from the conspicuity comparison with light-dark differences by 29 observers. The variation of k_w, fraction of 60-min light-dark discrimination, was taken from Blackwell's (1946) results. Note that at 5 FL the average observer seems to lose ability to make red-green discriminations at an angular subtense of about 2 min of arc and that he loses his violet-green yellow discrimination at about 7 min of arc.

The uncertainty of the values of k_u and k_v shown in Figure 7 can be estimated from the internal consistency of the conspicuity estimates at about 20 percent of the values shown. It is of some interest to see

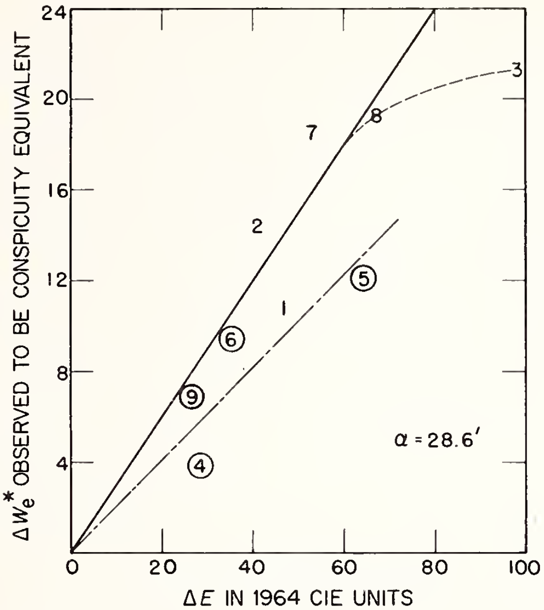

FIGURE 6 The two relations (solid line for red–green differences and dashed line for violet–green yellow differences) between the target–surround color difference and the difference in lightness index W^* (for the gray-scale target–surround difference found experimentally to have equivalent conspicuity). The constant of proportionality is higher for the former than for the latter.

how the values of $k_u = .04$, $k_v = 0$ for $k_w = .50$ found for 40 FL by one observer, Eastman, compares with these values obtained by averaging conspicuity estimates made by 29 observers at 5 FL. According to Figure 7, k_w reaches the value .50 at about 13 min of arc. For this angular subtense both k_u and k_v are considerably higher than the values found to apply to Eastman's settings of threshold; compare .15 with .04 for k_u, and .03 with 0 for k_v. Judging from the individual variations shown in Figure 4, Eastman's values of k_u and k_v relative to k_w are somewhat too low to fall within range found by the observers, even allowing for an uncertainty estimate of 20 percent based on internal consistency. This discrepancy suggests that the actual uncertainty of the values of k_u and k_v relative to k_w shown in Figure 7 is somewhat greater than 20 percent of the values.

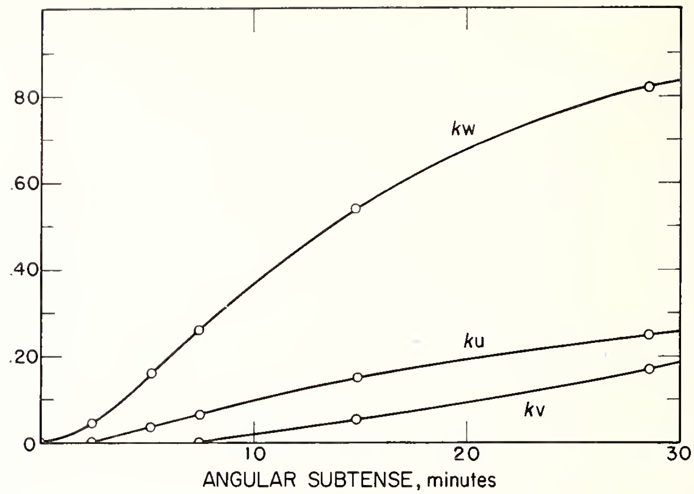

FIGURE 7 The variation with angular subtense in the constants k_u, k_v, and k_w of Figure 4 required to fit the experimental results of Figure 6. The variation of k_w was taken from the 1946 data of Blackwell, that of k_u and k_v, from the estimates relative to gray-scale differences. The ability of the observers to detect red–green differences went to zero for circular fields subtending about 2 min of arc. For fields smaller than this at surround luminances less than 5 FL, the observers behaved as if totally color-blind. Also the ability of the observers to detect violet–green yellow differences went to zero at about 8 min of arc. For fields of angular subtense between 2 and 8 min, the observers behaved like tritanopes. For fields subtending more than 15 min, the observers distinguished all three kinds of color differences: violet–green yellow, red–green, and light–dark.

USE OF k_u AND k_v TO ASSESS VISIBILITY OF NON-SELF-LUMINOUS SIGNALS

The values of k_u and k_v shown in Figure 7 can be used to estimate the degree to which the chromatic aspect of the signal bears on its visibility. For a sufficiently low value of $Y \times a^2$, the chromatic aspect has no bearing whatsoever on the visibility of the signal nor on its identification ($k_u = k_v = 0$). By increasing the size of the signal or its luminance or by decreasing the distance between the signaling device and the observer, non-self-luminous signals that differ in the red–green dimension from the surround color may be made easier to detect and perhaps identifiable even though violet–green yellow discrimination is ineffective (k_u up to .10, $k_v = 0$). This greater ease of detection is, however, marginal at best. At still closer range, chromatic differences of any sort between signal and surround may serve to increase visibility of the non-self-luminous signal and to contribute to its identification. A signal system intended for short-range use may have many more colors than one intended for long-range use, which may be feasible for only two colors, red and green. These qualitative conclusions are by no means unknown to signal practice, but the values given in Figure 7 permit estimates of at least semiquantitative validity to be made.

The bearing of the chromatic aspect of a self-luminous signal on its visibility against a completely dark surround may be estimated by assuming a nominally achromatic color for the surround about equal to that of a Planckian radiator at 15,000 K (Helson and Michels, 1948).

APPLICATION TO IDENTIFICATION OF SELF-LUMINOUS SIGNALS

The size of the chromatic departure of any signal colors from any other signal color in the system should correlate with the degree of success experienced by observers attempting to discriminate them. This size may be estimated in 1964 CIE units of color difference by computations involving values of k_u and k_v for the particular luminance, size, and distance of interest. For a given design of signaling device, the maximum signaling distance at which identification is possible could be estimated by computation not only for observers having normal trichromatic vision but also for tritanopes and for red–green confusers. All of these statements assume clear air and a uniform, dark surround. Problems of fog, smoke, haze, or grossly nonuniform surrounds might be solved more easily by use of k_u and k_v, but require information not discussed here for complete solution. Since the surround is much darker than the signaling area, no account need be taken of flux misdirected by defects in the lens system of the eye. The worst that can happen from chromatic aberration is that a purple signal will be seen as red with a blue halo.

SUMMARY

Two experiments have been described, one by Eastman on visibility of target–surround color combinations, the other by Judd and Yonemura on conspicuity of red–green and violet–green yellow differences compared to light–dark, and suggestions have been made as to the application of these results in determining the bearing of chromatic differences on the visibility and identification of signals.

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PERCEPTUALLY UNIFORM SPACING OF EQUILUMINOUS COLORS AND THE LOCI OF CONSTANT HUE

Proc. [Symposium on] Color Metrics, Driebergen, Holland, September 1971, pp. 147-159 (AIC/Holland, c/o Institute for Perception TNO, Soesterberg, 1972)

This represents the culmination of Judd's studies of the connections between colorimetry and color perception. He brings together, as has no other author, the most reliable data on constant-hue loci, and all of the reasonably successful methods of approximating them by computations based on the parameters of colorimetry.

PERCEPTUALLY UNIFORM SPACING OF EQUILUMINOUS COLORS AND THE LOCI OF CONSTANT HUE

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INTRODUCTION

Schrödinger (1920) suggested that the most direct transition from any highly saturated color to neutral is that which passes through colors seen to have the same hue. He said (MacAdam, 1970) "It seems to me probable that the most rapid transition from any highly saturated hue to huelessness takes place without any change of hue, because such a change, as a superfluous, newly added distinguishing characteristic between neighboring colors of the series, would be liable to prolong the series."

According to Schrödinger's formulation, the shortest paths to a hueless color, that is, the color geodesics to neutral, are as shown in Fig. 1 for equiluminous colors. This formulation is based on the assumption that the red, green, and blue primaries proposed by König and Dieterici (1892) are the true fundamental colors.

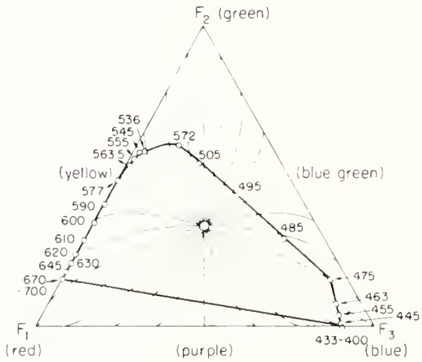

Fig. 1. Theoretical constant-hue loci (Schrödinger, 1920).

* Because of illness of dr. Judd, this paper was read by dr. I. Nimeroff.

DETERMINATIONS OF CONSTANT-HUE LOCI

In planning the Munsell color system, A. H. Munsell (Tyler and Hardy, 1940) originally intended to assign Munsell hue notations in accord with additive mixtures on a Maxwell disk. The rule was to be that when a chromatic color is mixed additively with a neutral (white, gray, or black), the hue of the mixture would be taken to be the same as that of the chromatic color. By this rule the intention was to make constant Munsell hue mean the same as constant dominant wavelength.

Observation of colors formulated to have constant dominant wavelength showed, however, that this definition of constant hue was usually unacceptable. Even the first Atlas of the Munsell Color System issued in 1915 (Gibson and Nickerson, 1940) deviated from this definition, and the color standards issued by the Munsell Color Company were progressively revised to deviate more and more from constant dominant wavelength to produce the Munsell Book of Color (Glenn and Killian, 1940; Kelly, Gibson and Nickerson, 1943). The colors of this book were then subjected to a systematic scrutiny (Newhall, 1940; Newhall, Nickerson, and Judd, 1943) that resulted in the definition of the Munsell renotations. The degree to which the present Munsell renotation hue departs from constant dominant wavelength may be seen from Fig. 2

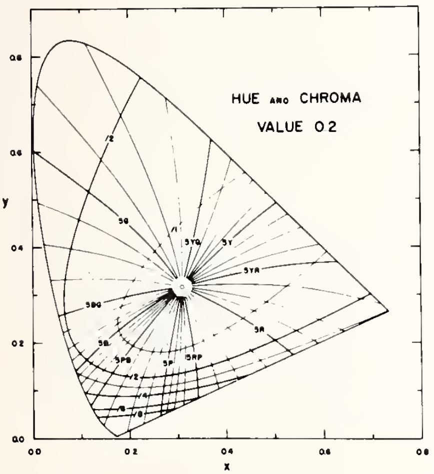

Fig. 2. Loci of constant Munsell hue and chroma.

taken from a paper by Judd and Wyszecki (1956). It will be noted that, with one exception, the lines radiating out from the center of the chromaticity diagram are somewhat curved. These lines, based on extensive observations by more than 40 observers, define constant Munsell renotation hue; straight lines from the center correspond to constant dominant wavelength. These data constitute the first results on loci of constant hue to be considered. Note that for blue the locus is curved.

A second body of data is that by MacAdam (1950) who reported for two observers the loci of constant hue determined with various surrounding colors. These loci indicated that the hueless point is controlled largely by the color of the surround, but otherwise agreed, in general, with the curvatures of constant-hue lines in the Munsell renotation system.

A third body of data is that by Wilson and Brocklebank (1955) who matched various rotary mixtures on the Maxwell disk of each of 120 relatively pure colors, forming a hue scale, with a white by having the observer point out the one of the 120 pure colors yielding a hue match. Fig. 3 shows some of their

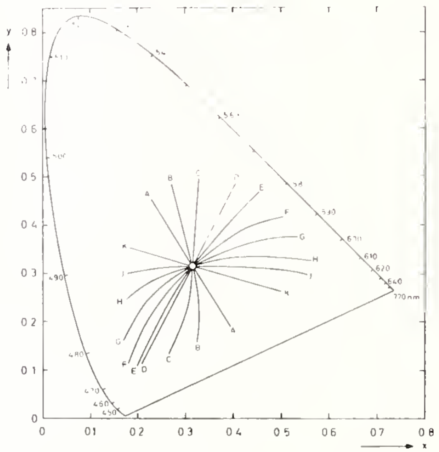

Fig. 3. Experimental constant-hue loci (Wilson and Brocklebank, 1955).

results. They agree with the Munsell and MacAdam data in general. Note, however, that the line of constant hue from the violet end of the spectrum is straight. Schrödinger made a check of this point, and remarked that the theoretical result based upon the König blue primary at 460 nm was wrong. He suggested that the choice of primary should have been violet, instead of blue, to accord with König's later studies of observers with tritanopic vision (MacAdam, 1970, p. 180). So much for data on constant-hue loci by experiment.

DETERMINATION OF COLOR GEODESICS

The task of determining the geodesic between two colors by direct experimentation is so formidable that to my knowledge no report of the results of such an experiment has ever been published. What must be done is to find a series of colors corresponding to each of a reasonable sampling of the paths in color space leading from the one color to the other, determine the number of perceptually equal steps in each path, and then pick out the path having the least number of steps. Even if we pick two colors of the same luminance and investigate only paths involving constant luminance, the task is still discouragingly formidable. Suppose, however, you have a plane diagram whose points represent equi-luminous colors and such that you believe it has uniform chromatic spacing; then it is easy to find the shortest path implied by that belief. Simply draw a straight line, and there you have the implied geodesic.

In the Munsell color system, equiluminous colors are represented by planes whose points are identified by a polar-coordinate system with chroma corresponding to the radius vector, and hue, to angle. The spacing in such planes is intended to be uniform, and is as uniform as experiments carried out between 1915 and 1929 could make it. As we have seen, however, the colors to be represented along any radius vector were chosen to have constant hue. It was always supposed, though rarely stated in those days, that constancy of hue implied shortest path to neutral. Nickerson (1936) proposed an index of the perceptual size of a color difference. This index was called the Index of Fading. This index had the form of a sum of suitably weighted hue, chroma; and value components of the color difference: and this form of index clearly implies that, if the two colors have the same Munsell hue and value ($\Delta H = \Delta V = 0$), then constancy of hue does indeed imply the shortest path to neutral because the formula for the Index of Fading evaluates the difference as proportional solely to the chroma difference, that is to the length of a straight line, some part of the radius vector. The assumption implied by the Munsell color system is thus equivalent to Schrödinger's assumption stated backward; that is, instead of saying, as Schrödinger did, that we have an evaluation of the geodesics to neutral, and we assume that they correspond to constant hue, the Munsell system says we have determined the loci of constant hue, and we assume that they are the shortest paths to neutral.

MacAdam (1942) reported on an extensive series of determinations of the sensitivity of one observer (P. G. Nutting, Jr.) to chromaticity change at constant luminance. The standard deviation of 50 settings for a color match was used as the inverse measure of sensitivity, and the result of traversing each of 25 chromaticities in several directions (4 to 9) yielded what have become known as the "MacAdam Ellipses". MacAdam was impressed at the time by the fact that the spacing implied by these ellipses could only be represented on a plane by making adjustments somewhat larger than the estimated uncertainties. So MacAdam avoided deriving any such plane diagram for the time being. Moon and Spencer (1943) and Fry (1945), however, did not hesitate to do so; nor did Farnsworth (1957). The plane diagram derived by Fry to convert the MacAdam ellipses into equal-sized circles accorded with the modulation theory proposed by Fry. This theory is based on the opponent-color pairs, red-green and yellow-blue, as in the Hering theory, and it also accommodated the facts of both protanopia and deuteranopia. Like Schrödinger, Fry then "assumed that the lines of constant hue are straight lines radiating from white" on his

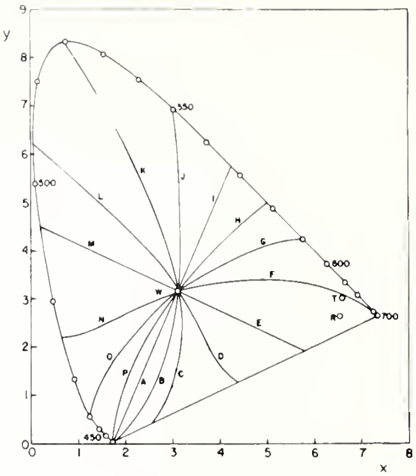

Fig. 4. Constant-hue loci derived (Fry, 1945) from MacAdam (1942) ellipses.

(u'',v'')-diagram. By reverse transformation to the (x,y)-chromaticity diagram he obtained the constant-hue loci shown in Fig. 4. Note that there are two directions in which constant-hue accords with constant dominant wavelength —one in the violet-greenyellow direction agreeing with tritanopic confusions with neutral, the other in the red-green direction agreeing with deuteranopic confusions with neutral.

After a delay of 27 years, MacAdam (1969) derived his own nonlinear transform of the (x,y)-diagram adjusted to reduce as closely as possible the MacAdam ellipses to equal-size circles. This transform was produced by plotting in rectangular coordinates two variables, XI and ETA. XI and ETA are ten-term polynomials in auxiliary variables, a and b, which are, in turn, projective transforms of the chromaticity coordinates, x,y. The 26 constants involved in the double transformation were optimized by automatic digital computer. Fig. 5 shows the curved lines in the (x,y)-chromaticity diagram that correspond

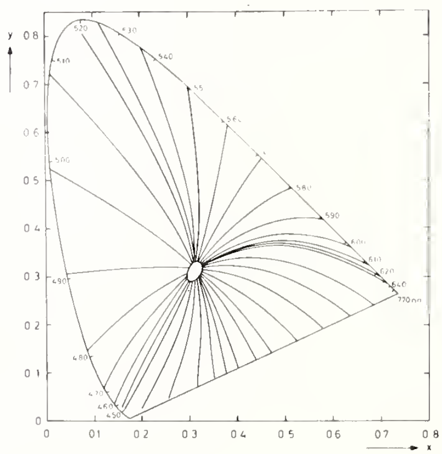

Fig. 5. Constant-hue loci derived (MacAdam, 1969) from the MacAdam (1942) ellipses.

to straight lines drawn on the (XI-ETA)-chromaticity diagram intended to have uniform spacing. In accord with the usual assumption MacAdam has labeled these curved lines "constant-hue loci". Note that this strictly empirical finding from the MacAdam ellipses indicates that the line of constant dominant wavelength to the violet extreme of the spectrum is very closely the shortest path: but for blue and for reddish violet the shortest paths are curved if plotted on a mixture diagram.

Still another determination of the geodesics implied by the MacAdam ellipses was recently reported by Muth and Persels (1971). He started with the formula for the perceived size of color differences derived from the MacAdam ellipses by Friele (1965, 1966) and refined by MacAdam (1966) and Chickering (1967). By computer he determinded the tri-dimensional geodesics required to pass from various of the spectrum colors to a white of unit luminance. The relative luminance of the spectrum color was adjusted until the geodesic approached white with a zero slope. The vertical projections of these tridimensional geodesics onto the (x,y)-chromaticity diagram are shown by courtesy of Persels in Fig. 6.

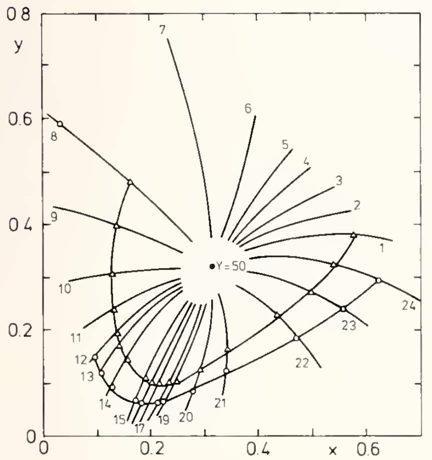

Fig. 6. Vertical projections of Persel's three-dimensional constant-hue loci derived from the Friele (1965, 1966) and MacAdam (1966) formulas with constants optimized by Chickering (1967). (from Muth and Persel, 1971).

This is the last derivation of the geodesics implied by the MacAdam ellipses to be considered. The shapes of these geodesics plotted on the (x,y)-chromaticity diagram agree well.

A method independent of the MacAdam ellipses has been used by Yonemura (1970) to derive geodesics to neutral. This method is based on the opponent-colors theory as exemplified by the CIE 1960 UCS diagram originally derived by MacAdam (1937) some years before he developed the MacAdam ellipses. This diagram has been shown by Judd and Yonemura (1970) to accord with the second stage of the Müller theory. It is a projective transformation of the (x,y)-

chromaticity diagram. Yonemura made use of measures of red-green and violet-greenyellow activity that took the form of angles subtended by the chromaticity-difference vector at the tritanopic convergence point, and at the deuteranopic convergence point, respectively. The argument is that normal vision may be viewed as a combination of tritanopic and deuteranopic vision as originally suggested by Walraven and Bouman (1965). Since the chromaticities along any line passing through the tritanopic convergence point cannot be distinguished by the tritanope, the angle measure of red-green distinction made by him is mandatory. By this argument the diagram produced by plotting in rectangular coordinates the angles subtended at the deuteranopic convergence point against those at the tritanopic convergence point is a uniformly spaced chromaticity diagram. It is, of course, a nonlinear transform of the (x,y)-chromaticity diagram. If the spacing of this nonlinear transform is accepted as a useful approximation to uniform spacing, then the series of equiluminous colors defined by any straight line on this diagram is a useful approximation Neutral colors are represented at the origin of the coordinate system formed to the geodesic between the colors represented by the extremes of the line. by plotting θ_{vgy} against θ_{rg}. Any straight line from this origin in a direction between red and greenyellow not only corresponds to a geodesic but also to colors for which the ratio of red to greenyellow is constant. But this is the condition for constancy of hue. So by this theoretical construct, no separate assumption that a color geodesic corresponds to a locus of constant hue is required; the construct already implies this correspondence. Fig. 7 is taken from Yonemura (1970) and compares the geodesics to neutral derived from his angle version of the CIE 1960 UCS diagram with some of the loci of constant hue according to the Munsell renotations. Note that the agreement is good even in the violet

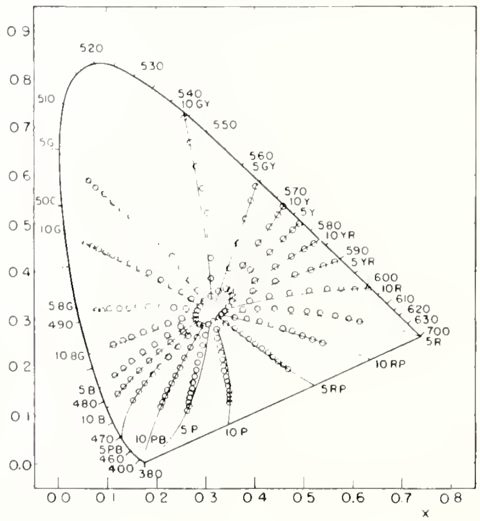

Fig. 7. Constant-hue loci (Yonemura, 1970) compared to Munsell-renotation-hue loci.

to reddish violet hue region where the theoretical prediction accords better with the MacAdam and the Wilson-Brocklebank data than with the Munsell renotations. Note that the constant-hue loci are straight for only four directions, those toward the tritanopic convergence point and its complementary, and those toward the deuteranopic and its complementary.

The second theoretical construct to be considered is essentially that of the stage theory of color vision proposed by Adams in 1923, and elaborated by him in 1941. The measure of red-green activity in the third stage is taken as $V_x - V_y$, and that of yellow-blue activity as $V_y - V_z$, where V_t stands for the Munsell value function of tristimulus value T. Adams recognized that this theoretical construct failed to agree with the Munsell constant-hue loci in the blue to violet region because the predicted locus for blue is straight rather than curved, but he suggested that further experiments might support the theory rather than the Munsell renotations.

Glasser, Reilly, et al (1958) have for some years been exploring the adaptability of the Adams chromatic-value diagram to predictions of the perceived sizes of color differences. They used the cube-root function instead of the Munsell-value function, and they did not restrict it to the *XYZ* primaries of the 1931 CIE coordinate system as Adams did, but chose three other primaries RGV adjusted to improve the predictions of perceived size of color differences. In a discussion of these choices with Reilly in June 1970, it became clear that choice of *R*, *G*, and *V* to correspond to the convergence points of the three types of dichromatic vision, as suggested by Schrödinger in 1920, would make the Adams chromatic-value diagram yield much improved predictions of the constant-hue loci. I have worked out the constant-hue loci implied by it. The hue index used was $(R^{1/3} - G^{1/3})/(G^{1/3} - V^{1/3})$. Note that if $R = G$, this index remains constant at zero regardless of the values of *R*, *G*, and *V*, and if $G = V$, it is constant at infinity. This accounts for the straight lines in the directions

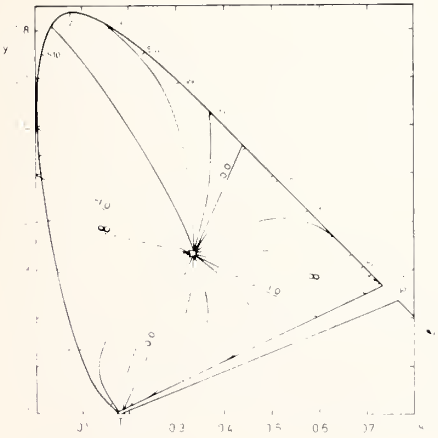

Fig. 8. Cube-root constant-hue loci. Straight constant-hue loci pass through dichromatic convergence points P, D and T.

of the tritanopic and protanopic convergence points and their complementaries. There is also a straight line in the direction of the deuteranopic convergence point and the opposite direction toward its complementary. All points on this line correspond to $R = V$, and for such points the hue index becomes: $(1 - G^{1/3})/(G^{1/3} - 1) = -1$, regardless of the value of $G^{1/3}$. Fig. 8 shows the lines of constant hue implied by this variation of the Adams chromatic-value diagram. The main features of the experimental loci are duplicated, but the predicted curvatures are somewhat greater than those observed. This discrepancy indicates that a power intermediate to 1/3 and 1 might be found that would make predictions in accord with the experimental facts.

CONCLUSIONS

The assumption, made by Schrödinger (1920) that geodesics to neutral are also loci of constant hue, seems to have been amply supported by experiment. Two theoretical constructs yielding fairly successful predictions of constant-hue loci, yield this assumption as a part of their prediction.

The success of the Yonemura (1970) angle version of the CIE 1960 ICS diagram supports the suggestion that normal vision may usefully be described as a combination of tritanopic with deuteranopic discriminations as suggested by Walraven and Bouman (1965).

The limited degree of success of the Adams chromatic-value diagram, applied to the R, G, and V primaries explaining the three types of dichromatic vision as "loss" variations of normal vision still gives significant support to the view that the deviations of the loci of constant hue from constant dominant wavelength arise simply from a nonlinear connection between stimulus and response; that is, a response compression.

By this theoretical view, any direction from the neutral point yielding a straigth line on the mixture diagram as a locus of constant hue must be toward the point representing a primary color process.

There are thus many more or less equally successful ways to explain color geodesics and the loci of constant hue. Psychophysics has thus indicated a number of explanations, but it could easily take many years of physiological study to decide which, if any, of these explanations is correct.

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DISCUSSION*

Guth: In the preliminary theory which I outlined in my paper, curvilinear hue lines are caused by the fact that the Bvs$Y$ system inhibits the Rvs$G$ system less and less as a stimulus is desaturated. How do the theories which you discussed cause the curvilinear loci?

Wyszecki: At the end of dr. Judd's paper you will find an answer, in that he clearly thinks in terms of a non-linear relation between stimulus and response.

Guth: That would mean that the slope of the response vs. intensity function is greater for $B-Y$ than for $R-G$?

Wyszecki: I would say: that follows naturally.

Guth: It can be seen, then that for both the Bezold-Brücke effect and the curvilinear hue lines (the Abney effect) there are two equally-tenable explanations. One hypothesizes that there are different intensity-response functions, whereas the other uses a between mechanism inhibitory effect.

Nimeroff: It would take further investigation to determine which of these two hypotheses best fit the available data. We may not be able to resolve this problem even with additional information.

I would have expected that under your hypothesis the constant lines from a suitable white point would be straight lines.

Wyszecki: 1. A recent experimental determination of constant hue lines by Robertson (Stockholm meeting) may be worth mentioning.

2. It is of interest to note that many models of color vision (line elements, etc.) predict remarkably well the observed hue lines despite the fact that these models are based on fundamentally different principles of construction.

Judd: My omission of a reference to the work of Robertson was inadvertent, and I am glad to follow Dr. Wyszecki's suggestion that it be included. (Robertson, 1969).

Richter: I would like to add that in my own opponent colour model I can compute the hue lines of the Munsell system in good agreement. I get two straight lines in the yellow region for 10 Y and in the purple blue. This description is better than by the many other theories. which get straight hue lines in the chromaticity diagram for 3 up to 6 directions. Also my model is able to describe the change of hue with luminance factor in good agreement with the Munsell hue curves for this case (Richter, 1970).

Boynton: Recent work by Savoie and Cornsweet, (1971) reported in Sarasota has shown that the wavelengths usually thought to yield invariant hue with changes in intensity, do not actually do so. The older work by Purdy was based on only two levels (100 and 1000 trolands). Between these levels, Savoie and Cornsweet found hue changes.

* The discussion was supplemented wich dr. Judd's written comments.

Vos: It should be added that Savoie and Cornsweet worked with 5 msec flashes, that is in a quite different situation from that of Purdy, e.g.

Nimeroff: It may be that luminance is the reason that the several investigations arrive at somewhat different sets of constant-hue lines.

Hard: Could constant hue-determination result in different experimental data if we look for a constant ratio between e.g. red and yellow in a simultaneous comparison situation of two samples, or if we try, by absolute judgment of the ratio, to look at the two samples each at a time without comparison, like in the NCS-experiments?

Nimeroff: If the observers can more readily make the constant-hue judgment, when the latter question is put to them than when the former question is put to them, then better, more consistent results may be obtained.

Wright: I find it of great interest that although Dr. Judd's paper deals with uniform spacing of equi-luminous colours, yet he has illustrated his argument with the non-uniform (x,y) -chromaticity chart and not with the so-called uniform (u,v) chart. I personally applaud this as the (u,v) chart seems to me an unsatisfactory chart to illustrate general chromaticity relations. The feature which makes the (u,v) chart unsatisfactory is the compression of the important yellow-green, yellow, orang area of the chart. This is brought out by the distribution of the samples on the (x,y) and (u,v) charts prepared by one of our students, Miss P. Dunster, which I will put on display.

McLaren: The Adams-Nickerson UCS diagram has been found to be free from the compressions which spoil both the (x,y) and (u,v) diagram; in addition Munsell samples of constant hue plot as straight lines.

Judd: Like Dr. Wright I have seen demonstrations that the (u,v)-chromaticity diagram is unduly compressed along the white to yellow locus; but there are conditions for which the (u,v) spacing is correct. The Priest-Brickwedde data (1926, 1938) on minimum perceptible colorimetric purity corroborate it perfectly, and Dr. Wright's own data (1943) indicate that the spacing from white to yellow for his conditions of observation should be even more compressed than is shown on the (u,v)-diagram. Use of high luminance and large angular subtense of target, and surrounds of the same high luminance open up more white-yellow discriminations than they do red-green discriminations for the same range of colours. I think that the (u,v)-spacing is much more nearly correct than the (x,y)-spacing for average viewing conditions.

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REVIEW: SOURCES OF COLOR SCIENCE, BY DAVID L. MACADAM, EDITOR

(MIT Press, Cambridge, Mass., 1970).
Supplement to ISCC News Letter no. 222 (Jan.-Feb. 1973)

This unabridged version of the review that was published in the ISCC News Letter no. 217 (March-April 1972) was found among Dr. Judd's unpublished papers by Dorothy Nickerson. It contains many otherwise unpublished reflections by Judd on great contributions to our knowledge of color.

After "sets" in the first line of the second column on p. 1, the words "in motion" seem to be needed to complete the thought. In the second column of p. 2, the statement that Young substituted green and violet for yellow and blue in 1845 is incorrect. To correct this mistaken impression, which is given by the book, insert at the end of the first paragraph on p. 51 of the book:

> "(from Phil. Trans. 1802, p. 375) In consequence of Dr. Wollaston's correction of the description of the prismatic spectrum . . . it becomes necessary to modify the supposition that I advanced in the last Bakerian lecture . . . substituting red, green, and violet for red, yellow and blue."

Near the top of the second column of p. 3, there is a stronger indictment of Goethe than Judd included in his "Introduction" to Goethe's _Theory of Colours_: ". . . he seems never to have tried to verify Newton's experimental results but contented himself with deriding them and with abusing Newton himself."

The ambiguity of the word "color" is illustrated in the same column, where Judd wrote "the surface being illuminated by two different colors", which is in apparent conflict with his seemingly approving statement, in the "Introduction" to Goethe's _Theory of Colours_, that "color is a sensation". Actually, "color" is properly used in all three senses indicated by Newton in the statement quoted in the introductory paragraph that precedes Judd's "Introduction" to Goethe's _Theory of Colours_ (in this volume). Judd himself uses the phrases "true color of the object" and "color of the background" in the same passage on p. 3 of this review.

The difficulties Judd reported in following Schrodinger's papers may be attributed, to some extent, to typographical errors in the book. On p. 161, in the second unnumbered equation and in Eq. (4), the integral signs should be replaced by bold-face capital S, standing for summation. In Eq.(15'), plus signs should be inserted between the first and second and third terms under the square-root sign. The page reference in the sixth line from the bottom of p. 171 should be 180. In Eq.(28), the symbol in the numerator of the first term should be h_z, not hz. In Eq.(2) on p. 188, r_1 should be x_1 and dx_1 should be dx_i. In the eighth line from the bottom of p. 190, the reference should be to fig. 13 (not 17). In the next line, the reference to "dash-line curve" should be to "solid dots".

Sources of Color Science

by David L. MacAdam

(The MIT Press, Cambridge Massachusetts, and London, England, pp. x, 282, 1970)

Reviewed by Deane B. Judd

When I was asked in October 1970 to review this book, I was glad to agree to undertake it, but because of illness I was prevented from fulfilling this commitment. When Dr. Burnham renewed his request more than a year later, I was doubly glad, first because I have recovered enough to complete this assignment, and second because I welcome the chance to support Dr. MacAdam in his unique tutorial project. I think he has done color science a great service by selecting excerpts from the writings of those who have shown the greatest insight into the mysteries of color, editing them by incorporation of modern(1970) terminology, and arranging for publication of this compilation in a most legible and attractive format by the MIT Press. Dr. MacAdam has done his best to resolve the contradictions among these basic views, and has left them in a form that encourages the reader to attempt his own resolution of them.

The excerpts were taken from 26 different publications by 15 authors. The importance attributed by Dr. MacAdam to each author may be judged by the number of pages devoted to excerpts from his writings. Schrödinger was granted 60 pages, followed by Guild (47), v. Kries (26), Newton (24), Polyak (23), Maxwell (22) and v. Helmholtz (17). The remaining eight authors in chronological order are Plato, Aristotle, Palmer, Young, Grassman, F. E. Ives, L. F. Richardson, and Le Gros Clark.

In my ignorance of the old literature I was prepared to quibble about devoting any space to writers, except Isaac Newton, before 1800. From our comparatively advanced state of present-day knowledge about color, much of what these writers had to say we now recognize as wrong, but it is probably good for us to learn that some of our fundamental color concepts are far from modern. For example, Aristotle said that it is only in light that the color of a thing is seen, that color sets in movement, not the sense organ, but what is transparent, e.g. the air, extending continuously from object to eye, and that in turn sets the sense organ causing color to appear at the external boundary of the transparent medium. He also knew that color signals go from sense organ to the brain and said that the eye is an offshoot from the brain. But Aristotle sought the explanation of chromatic color as a mystical juxtaposition, or perhaps a superposition, or perhaps an interpenetration, of black and white, that produces agreeable colors if the ratio of black to white can be expressed by simple-integer ratios, like 3 to 2 or 3 to 4.

Palmer (1777) stated that the surface of the retina is compounded by particles of three different kinds, one kind moved by yellow rays of light, another by blue, and the third by red. Complete uniform motion of these particles produces the sensation of white; the absolute want of motion, the sensation of darkness. But he entertained the paradoxical thought that colored objects absorb the rays analogous to the colors of which they seem painted and are perceived only by the reflection of the other rays.

Grassmann (1853), formulator of the laws generally recognized as the basis of modern colorimetry, cannot be faulted on mathematical theory, but he was sufficiently unfamiliar with the facts of color mixture that he supposed homogeneous red light to form the transition color between the violet and red of the ordinary (impure) spectrum. This supposition led him to state wrongly that to every color belongs another homogeneous color, which, when mixed with it, gives colorless light.

In the preface Dr. MacAdam emphasizes that he is not a linguist and has not examined the original Greek texts. The selections have been ruthlessly pruned, he says, and all selections, whether based on published translations, literal translations especially prepared for this book, or works originally published in English, have been freely edited, by substitution of modern (1970) terminology and by elimination of circumlocutions made unnecessary by such terminology. The advantages of this plan are particularly evident in the more extensively excerpted and more modern works to which we now turn attention.

Often the most confusing bar to comprehension of these works is uncertainty about what the author intended his words to mean. No such uncertainty handicaps these edited excerpts. They have a clarity achievable only by a master scholar of color science such as Dr. MacAdam, and this is far beyond what any purely linguistic scholar could hope to do.

These excerpts from the writings of the foremost thinkers about color also pack a punch greater than the writing of any one color scientist because of the greater diversity of the views and concepts developed by any one author, and because the concepts are defended, not by someone-else's rehash of the arguments, but by the most pithy statements that their originators have managed to put together. The clash of these views is most dramatic, and the reader is forced to some hard thinking to decide whether the different views are compatible, and, if not, to decide which view is correct. This compilation is thus in a sense an intellectual mystery. By juxtaposition of conflicting views the reader is almost forced to decide for himself where the complicated truth lies. The likelihood that no two readers will come precisely to the same decision is merely an evidence that our modern views of color measurement are by no means cut and dried, though their underlying unity becomes ever clearer with additional study of their sources. The greater the knowledge of the reader, the more valuable he is likely to find this book; but even the beginning student, by skipping some of the passages from Schrödinger and Guild, will find that this compilation provides a quick and very readable introduction to the outstanding physiological problems of color science.

The enormous literature on the psychology of color, and the dependence of the color perceived to belong to a central field on the characteristics of the surrounding field have been purposely omitted as outside the competence of the editor.

The 24 pages excerpted from the writings of Newton include the complete descriptions of the experiments by means of which Newton concluded:

(1) The light of the sun consists of rays differently refrangible.

(2) The heterogeneous rays of compound light may be separated from one another by means of a prism.

(3) All homogeneous light has its proper color corresponding to its degree of refrangibility; that color cannot be changed by reflections and refractions.

(4) Colors may be produced by composition which shall be like the colors of homogeneous light as to the appearance of color, white or gray may be produced by too much composition, and colors (purples) may also be produced by composition which are not like any of the colors of homogeneous light.

(5) The whiteness of the sun's light is compounded of all the homogeneous colors mixed in a due proportion.

(6) In a mixture of homogeneous colors, the quantity and quality of each being given, the color of the compound may be found by the center of gravity principle.

It is obvious from the description of his experiments that Newton referred to the appearance of colors only when viewed with dark surround, but he does not state this necessary condition. His law of color mixtures is correct, but Newton supposed that to make it work the homogeneous colors had to be represented by points placed around the arc of a circle with a space on that arc, to represent the purple colors. Nearly 200 years later, Maxwell carried out the experiments required to corroborate Newton's laws of color mixture, but these same experiments showed that the locus of points representing homogeneous colors is much more nearly along two straight lines intersecting at the point representing the green color corresponding to 510 nm.

The single page devoted to Young includes almost all that this famous scientist ever wrote about color. We see that he first (1802) adopted the then current view that red, yellow, and blue are the principal colors; but later (1845) adopted the choice, red, green, and violet, still held by most modern adherents of the three-components theory. Palmer, however, enunciated the three-components theory in 1777, thus antedating Young's statement by 25 years.

Maxwell wrote as clearly about color and color measurement as anyone, before or since. He was the first to state the connection between partial color blindness (dichromatic vision), and normal (trichromatic) vision. He said that the mathematical expression of the difference between color-blind observers and ordinary vision is that color to the former is a function of two independent variables, but to an ordinary eye, color is a function of three independent variables, and that the relation of the two kinds of vision is not arbitrary but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement that forms one third of the apparatus by which we receive the sensations of color. Maxwell's determination of the tristimulus values of the spectrum agree remarkably well with the 1931 CIE standard observer. He was the first to show how to construct a chromaticity diagram, still often called the Maxwell color triangle, and his description of the spectrum locus, though he overemphasizes its resemblance to two straight lines intersecting in the green, is not seriously wrong. He said that the chart of the spectrum may be described as consisting of two straight lines meeting in a point representing the color of the spectrum (about 510 nm). Proceeding from this green toward the red end of the spectrum we find the different colors lying *almost* exactly in a straight line. On the blue side of primary green the color equations are seldom so accurate. The colors, however, lie in a line which is *nearly* straight. Instead of the red, green, and violet mentioned by Young as the principal colors, Maxwell speaks of red, green, and *blue;* but he also states that primary blue is a sensation differing little from that excited by the parts of the spectrum near 434 nm. The difference is thus a matter of name only, because most modern authorities use the hue name, violet, for this part of the spectrum.

Forty years later (1900), F. E. Ives made use of a simplified version of the tristimulus values of the spectrum determined by Maxwell to develop the theory of the trichromatic process of color photography. This simplified version made the spectrum locus plot precisely along two straight lines intersecting at the point corresponding to 527 nm. By adjusting the spectral sensitivities of the photographic plates by dyes and by auxiliary filters to conform to these simplified tristimulus-value curves, and by projecting the positives so procured by means of the purest possible red, green, and blue lights, Ives was able to produce a much more accurate reproduction to the eye of the colors of the objects photographed than he had been able to do by trial and error, and was convinced that this success would never have been accomplished without the aid of this theory.

In the brief excerpt given from the first edition (1866) of the Helmholtz Physiological Optics, there is a clarifying discussion of Aristotle's view that the various colors are produced by a mixture of black and white, and of Goethe's then recent attempt to push ahead in researches based on this idea. Of course, Goethe could not believe that the purest of all colors, white, could be analyzed into homogeneous components, as shown by Newton, and he was so sure of this that he seems never to have tried to verify Newton's experimental results, but contented himself with deriding them and with abusing Newton himself. Even Maxwell's early writings (1856) spoke of white as the purest of all colors, but by 1857 he reversed himself, and thereafter spoke of the spectrum colors as maximally pure. Helmholtz recognized the fundamental importance of Newton's experiments and pointed out that Huygens' hypothesis that light consists of undulations of an elastic medium is not inconsistent with them. One must only substitute different frequencies of vibration for Newton's different degrees of refrangibility. Huygens' hypothesis had the advantage of explaining the colors of thin plates by interference. Helmholtz also discussed the puzzling effects of color perception based on learning by experience. He pointed out that anybody who has had experience with the effect of mixing colored light may occasionally fancy that he really does see the simple colors (e.g. red and green) in a compound color (yellow). The most curious illusion is when two simple colors are seen in the same place at the same time, the surface being illuminated simultaneously by two different colors, one of which predominates at certain places and the other at other places. In a case of this kind we often imagine we see the two colors simultaneously at the same place, one through the other as it were. The effect is very much as if objects were seen through a colored veil or mirrored in a colored surface. We have learned by experience, even under such circumstances, to form a correct judgment of the true color of the object, and this distinction between the color of the background and that of the light that is irregularly distributed over it is taken into consideration in all similar cases. This is an example of the psychological color phenomena that are only briefly touched upon by the MacAdam excerpts.

Helmholtz discussed clearly the dissimilar results of light mixture and pigment mixture, as Maxwell had done before him, and he points out that the quality of every color impression depends on three variables, luminosity, hue, and saturation, as Grassmann had done previously. He made a rather trivial improvement in Newton's circular locus of the purest colors by introducing a chord of the circle to represent the mixtures of homogeneous red and violet light that form purple colors, but from Young's brief statement of the three-components hypothesis, he sketched the distribution of the three fundamental sensations, red, green, and violet, throughout the spectrum in truly remarkable agreement with the results of measurements Maxwell had not yet undertaken. It was Helmholtz' view that the essential thing in Young's hypothesis is the idea that all color sensations are composed of three processes in the nervous substance that are perfectly independent of one another.

In the excerpts from the writings of Johannes v. Kries (1878-1905) the method used by him to study adaptation phenomena is described in these terms. A part of the retina whose border passes through the center of the retina is exposed for some time to what is called the adapting light. At the end of the adapting time, on the same spot, the reacting light falls; on the neighboring spot, not exposed previously, the comparison light falls. The comparison light and the reacting light meet, therefore, in the stared-at point. One of the two is varied in successive experiments until they appear equal at the beginning of the comparison. In this way there is found a light required to produce in a rested portion of the retina, the same color as the reacting light for the part of the retina previously exposed to the adapting light.

There is an assumption involved in this procedure. It is assumed that the sensitivity of the rested portion of the retina is not changed by exposure of the contiguous portion to the adapting light. If the comparison light is weak or absent, the after-image of the adapting light is seen to have a halo that extends into the area intended to be in a rested, or dark-adapted, state. Von Kries regarded the halo as due to a spread of the stimulation and not from a changed excitability. Now, since the study of adaptation has to do with the determination of excitabilities, he maintains that it is permissible to regard the immediate neighborhood of the after-image as unchanged if rather strong reacting and comparison lights are always used. This precondition of the v. Kries experiments should not be disregarded.

The great importance of adaptation phenomena is that they permit experimental variations of visual sensations and the determination of whether such variations are attributable to three individual components, as demanded by the Young-Helmholtz theory, and what those components are. A basic assumption is that the more the excitability of each component is reduced, the greater activation that component receives from the adapting light.

From his adaptation experiments v. Kries first established the persistence of optical matches; that is, if two objectively different lights are found to match (form a metameric pair) after adaptation to any one adapting light, then they will also be found to match after adaptation to any other light. This is a prerequisite for modern colorimetry. Von Kries gives the argument by which he concludes from the persistence of optical matches that there are only three components in play. Note that if experimental conditions are such that the degree of freedom introduced by rod vision is added to the three degrees of freedom available from cone vision, optical matches no longer persist for all adaptations.

Also described are the experiments in adaptation that enabled v. Kries to conclude that the primary nervous processes of the Young-Helmholtz theory correspond to a red, slightly more saturated and less yellowish than any spectrum red, to a green of a hue close to that of the spectrum at 517 nm but much more saturated, and to a blue or violet color somewhat more saturated than the spectrum color whose wavelength remained open to question. These are the colors that, if they could be produced, would be found to be invariant in quality regardless of adaptation, and v. Kries said they seem to be quite different from the primary colors computed from dichromatic color systems (by Maxwell's method).

Finally there is excerpted a statement of the famous v. Kries coefficient law by means of which, given the color points invariable to chromatic adaptation, and given the matching color (optimally near neutral) for any one combination of adapting and reacting lights, the matching color for any other reacting light may be computed for that same adapting light. The coefficients give the fractions of the rested-eye excitabilities of the three components, and serve to characterize the adaptive state produced by that adapting light. Like the law of the persistence of optical matches, the coefficient law breaks down under experimental conditions permitting intrusion of rod vision.

For this reviewer the English translation of the monumental Outline of a Theory of Color Measurement for Daylight Vision by Erwin Schrödinger, edited and brought up to date by insertion of modern colorimetric terminology is Dr. MacAdam's most valuable achievement in this compilation. I spent many weeks in 1928 studying this 1920 work with very limited comprehension of the original German and my first reading of the MacAdam version was like a revelation. With the exception of a few passages in Part II, Advanced Color Measurement, all statements appeared to me marvelously clarified. It is true that these exceptions are more or less basic to the whole treatment. These passages refer to equations 2 to 6 dealing with the differential equation of the area element that, in the sense of Riemann geometry, is perpendicular to the direction of the vector corresponding to constant chromaticity. Equations 23a to 26 were intended to prove that in the space generated by the square-root transform of tristimulus values, these vectors are plane logarithmic spirals. The difficult passages also include the entire section 8 on the course of the geodesic curves in tristimulus-value space. I suspect that the mathematical treatment is sound, but is too brief to be followed by anyone with only the limited mathematical insights possessed by the reviewer. Many readers, like myself, may have to take these derivations on faith. The MacAdam version has made clear to me for the first time what Schrödinger was trying to do.

Schrödinger treats as elementary color measurement the derivation and use of what we now know as color-matching functions, first determined by Maxwell. This kind of color measurement is elementary because it includes only information derivable from settings of equality between two fields. The treatment is based upon the König and Dieterici determination of color-matching functions and is elegant both from the algebraic and geometric standpoints. He uses two types of equations, as many British writers do: ordinary equations and color equations. Color equations take the form of: $F = a_1F_1 + a_2F_2 + a_3F_3$, in which F stands for any color, F_1, F_2 and F_3 stand for three primary or calibration colors, and a_1, a_2, a_3, for their amounts or tristimulus values. Note that in a color equation the equal sign does not mean numerical equality but rather "color matches," the plus signs do not mean scalar addition of two numbers but rather "addition of color components in a colorimeter," and the expression a_1F_1 does not mean the product of two scalars but rather should be read "a_1 units of primary color F_1." The transformation of tristimulus values (a_1, a_2, a_3) of a color expressed in terms of one set of primaries to those expressed in terms of another set is given in determinant form with the scales of both sets of primaries defined in terms of the color of the undispersed light (sunlight) whose normal spectrum was used by König and Dieterici to obtain the color-matching functions.

The manifold of colors, or color space, is a tridimensional system of affine structure based only on the equality judgment. Transformation to another set of calibration colors is linear and homogeneous. Only such properties of shapes in color space that remain unchanged by such transformation are affine and significant for this color space. By such transformations straight lines are converted into straight lines, plane surfaces into plane surfaces. Angles have no meaning because they may, by affine transformation, be completely deformed; nor do ratios of lengths.

In affine color space every color is represented by a vector extending from the origin $X_1 = X_2 = X_3 = 0$ to a length that for that one direction is proportional to the radiance of the stimulus; but the ratio of lengths of two vectors having different directions has no meaning. Therefore we cannot conclude from such ratios what the luminance ratios are. The colors corresponding to vectors of different directions have different chromaticities, but from the ratio of angles no conclusions as to the ratio of chromaticity differences can be drawn, unless the three directions are coplanar, in which case the larger angle from some central direction (say, the white direction) must correspond to a greater chromaticity (or saturation) difference. For departures from the white direction not all coplanar, we do not know whether a larger angle from the white direction means a greater or a less saturation of the corresponding color. The colors of the spectrum are represented by vectors forming a cone that encloses the bag or bundle of vectors for colors producible by all possible combinations of lights from the spectrum. The vectors corresponding to colors produced by combining extreme spectrum red with extreme spectrum violet lie in one plane and complete the boundary of the vector bundle corresponding to all real colors.

To transform color specifications relative to a set of real calibration colors to a form corresponding to the Young-Helmholtz theory requires the use of unreal calibration colors; that is, colors outside the gamut of real colors. For such calibration colors none of the tristimulus values of the spectrum will be less than zero, so they may be interpreted as excitabilities of the three components of color vision postulated by that theory.

In Part II, Advanced Color Measurement, methods of determining brightness, hue, and saturation are suggested together with a geometry appropriate to them. It is assumed that a measure of the dissimilarity of two colors can be set up in this geometry in a manner originated by Helmholtz to accord with the Fechner law such that for all just distinguishable color pairs this measure has a constant value. The Helmholtz line element:

$$ds^2 = \frac{1}{3}\left[\frac{dx_1^2}{x_1^2} + \frac{dx_2^2}{x_2^2} + \frac{dx_3^2}{x_3^2}\right] \quad (1)$$

is rejected because it implies a luminosity function equal to the cube root of the product, $x_1x_2x_3$, which is grossly different from the experimental facts. Schrödinger proposed a modified line element as probably approximating the experimental facts most closely:

$$ds^2 = \frac{1}{ax_1+bx_2+cx_3}\left[\frac{adx_1^2}{x_1} + \frac{bdx_2^2}{x_2} + \frac{cdx_3^2}{x_3}\right] \quad (2)$$

where x_1, x_2, x_3 are tristimulus values of the color, and a, b, c, are the luminosity coefficients.

This differential formula suffices to express the dissimilarity of two colors that differ by not much more than the threshold, but this is not sufficient to deal with brightness measurement in general that requires an assessment of the brightness of two colors of quite dissimilar chromaticities. The further assumption is therefore made that the dissimilarity of any colors is judged according to the magnitude of $\int ds$ for the shortest connecting path (geodesic path) between the two color points in the manifold. The shortest connecting path has to be followed, not the straight line in the vector space, but that curve for which that integral assumes the smallest value.

If we have one particular color A required to find the color B of not too different chromaticity that is seen as equally bright, the radiance of the stimulus producing this color is adjusted until the dissimilarity of the two colors is a minimum. In the affine color space discussed under elementary color measurement, we might think of locating this second color B by choosing the length of the vector so that the perpendicular to that vector passes through the end of the vector representing the color A. Then we can ask which length of the vector having the first direction (that of the vector representing color A) has the same brightness as the color represented by the second vector, and by the same method this length is found to be less than that of the vector originally chosen. By endowing this affine geometry with metrical properties that seem intuitively reasonable we have managed to achieve a definition of brightness such that if color A has the same brightness as color B, and color C has the same brightness as color B and the same chromaticity as color A, nevertheless color C is producible by a radiance less than that required to produce A, and is therefore of brightness less than A. This contradictory property of brightness equalities proves that the affine color space built up simply from judgments of the equality of two fields is not appropriate to represent the results of judgments of settings for minimum dissimilarity.

For colors of not too different chromaticities, colors of constant luminance are seen as of constant brightness. If the tristimulus values of a color x_1, x_2, x_3 are referred to calibration colors of the same luminance, then the luminance of the color is simply $x_1 + x_2 + x_3$, and in x_i space constant luminance corresponds to a plane. By introducing the square-root transform, thus:

$$Xl_1 = x_1^{\frac{1}{2}},\ Xl_2 = x_2^{\frac{1}{2}},\ \text{and}\ Xl_3 = x_3^{\frac{1}{2}} \quad (3)$$

we find that this plane in x_i-space transforms into a sphere centered on the origin ($Xl_1 = Xl_2 = Xl_3 = 0$) provided the values of Xl_i are plotted orthogonally. The line element (Eq. 2) based on the Fechner law becomes:

$$ds^2 = 4(dXl_1^2 + dXl_2^2 + dXl_3^2)/(Xl_1^2 + Xl_2^2 + Xl_3^2) \quad (4)$$

and if we wish to set up separate measures of luminance and chromaticity difference, the variables are the length of the radius vector, r, and the angle ϕ subtended at the origin $x_1 = x_2 = x_3 = 0$. Expressed in these variables the line element becomes:

$$ds^2 = 4(dr^2\, r^2 + d\phi^2) = 4\left[(d\ln r)^2 + d\phi^2\right] \quad (5)$$

By this line element, the lines of constant chromaticity are geodesics, and the geodesics between colors of the same brightness are great circles on the sphere centered on the origin, or ellipses in x_i-space. The brightness difference between two colors of the same chromaticity but having vectors of length r and r' is measured by ln(r/r'). Thus, the geodesics expressed by the variables, ln r and ϕ are straight lines. It is assumed that the geodesics between the neutral color $x_1 = x_2 = x_3$, and all other colors of the same brightness correspond to loci of constant hue. The measure of saturation is simply the angle ϕ in XI_i-space. In x_i-space this measure becomes:

$$2 \arccos \frac{a(x_1)^{\frac{1}{2}} + b(x_2)^{\frac{1}{2}} + c(x_3)^{\frac{1}{2}}}{(a + b + c)(ax_1 + bx_2 + cx_3)^{\frac{1}{2}}}$$

which is, of course, equal to zero for the neutral condition: $x_1 = x_2 = x_3$, and with a, b, c, being the luminosity coefficients as before.

The line element proposed by Schrödinger based on the Fechner law was subjected by him to a few checks with experiment. He showed that the just noticeable wavelength difference found by Uhthoff among equally bright spectrum colors from 490 to 640 nm agreed in a general way with the implications of the proposed line element, though the agreement was no better than that achieved by the otherwise unacceptable line element proposed by Helmholtz 50 years previously. He noted that the lines of equal-brightness geodesics deviated from the lines of constant dominant wavelength in almost precisely the way opposite to that found by Abney for the lines of constant hue on dilution with white light. Schrödinger's own repetition of Abney's experiment seemed, however, to support the line element and contradict Abney's experimental result.

Finally Schrödinger pointed out that the line element proposed is such as to imply that luminance variations at constant chromaticity correspond to geodesics, and so implies that such changes correspond to constant hue. He admitted that this implication of the line element is contradicted by the well-established Bezold-Brücke phenomenon, but pointed out that Brücke ascribed the hue change to the fact that the primary least represented in the color being judged drops near or below the threshold, and so becomes ineffective. This corresponds to a failure of the Fechner law for stimulation approaching zero, and we should expect a line element based precisely on the Fechner law as an approximation not to predict a phenomenon corresponding to a failure of that law. This line element thus has had only limited support from experiment, and Schrödinger did not indeed claim very much for it. Schrödinger was, however, the first to inquire carefully into the kind of geometry appropriate to represent color phenomena. His method of approach has great and permanent value and should not be considered as marred by the indifferent success of the first line element proposed by him.

The excerpts of the writings of J. Guild consist of reprints of two papers, Some Problems of Visual Perception, and Interpretation of Quantitative Data in Visual Perception, both presented at the 1932 *Discussion on Vision* organized in London by the Physical Society. The trichromatic theory, whatever form it takes, is an attempt to explain one fundamental empirical fact. This fact cannot be stated by saying, as many carelessly do, that any stimulus, whatever its spectral distribution, can be matched, as regards the color it evokes, by a mixture of three other stimuli. This statement is simply wrong. No matter what three stimuli are chosen it is easy to find other stimuli not matchable by a mixture of these three. Even the statement by v. Kries who says: "the resultant of all the various light stimuli, so far as sensations are concerned, can be completely represented as a function of three variables," will not serve. The principle here stated is the basis of modern colorimetric practice, but like that practice it applies only to a restricted range of experimental conditions, avoidance of rod intrusion, for example, in the sense that within this restricted range the most precise measurements fail to disprove it. For Guild the basic empirical fact is that to set a color match between two fields the observer need never operate more than three independent controls. This statement is true regardless of the adaptation of the eye to darkness or to excessively bright fields, and for Guild any explanation of it is a trichromatic theory.

A reception system for radiation consists of (1) a receptor, (2) a coupling, (3) an indicator, and (4) a cognizer. It frequently happens in experiments with radiation that simultaneously several reception systems are exposed to the same radiation. The individual reception systems may have receptors of different types and indicators of different modalities, but they usually have one cognizer in common. If any two or more of the reception systems contribute to the value of the same one (and only one) of the response parameters, or if they contribute to various parameters in the same proportion, it is evident that these systems are simply acting as a single composite reception system. Such reception systems may be said to form a cumulative group. The fundamental fact that in visual experiments three independently operated controls are in general necessary and always sufficient to effect a complete visual match between two samples of radiation means simply that there are three, and only three, independent cumulative groups of reception systems in simultaneous operation.

For the human visual system the rods and cones are the receptors, the nerve or nerve chain leading from the receptors to the visual center constitute the coupling, and the central connections form the location of the indicator. The indicator supplies an effective presentation of the stimulus to the cognizer, the psychological element of the system. The sensation is the reaction of the cognizer to the effective presentation. The relative sensitivity of the reception system depends entirely on the properties of the receptors.

Normal color discrimination requires an appreciable area; so it must be described in terms of the operation of cumulative groups of reception systems. Schrödinger never considers this basic aspect of vision.

There are three and only three independent systems in operation simultaneously in the normal human visual system. Whatever may be the actual number of elementary systems, the allocation of such types of receptor as may exist to central connections of such modalities as may exist must be of such character that the systems form three cumulative groups. The resultant peripheral properties (spectral sensitivity) and the resultant central modality of each group must differ from those of the other two groups.

Vision is characterized by an essential triplicity of function involving at least a threefold differentiation of the retinal receptors, and at least a threefold differentiation in central behavior. This is all that should be meant by the name trichromatic theory. There are four possibilities:

(1) Only three dissimilar types of receptor and only three dissimilar central modalities, all receptors of one type being coupled to central connections of one modality. This possibility seems to be what Young and Helmholtz had in mind, and it can apply only to experimental conditions ruling out rod vision.

(2) Only three dissimilar types of receptor, but any greater number of dissimilar modalities in the central connections. This possibility would indeed account for the existence of three, and only three, independent cumulative groups of reception systems in simultaneous operation, but do we not know that there are at least four dissimilar types of receptors — three types of cones plus one type of rod.

(3) Only three dissimilar modalities in the various central connections, but any greater number of dissimilar types of receptor. To the reviewer it seems that this possibility is the correct one, with the greater number of receptor types set at four.

(4) Any number, not less than three, of receptor types, and central connections of any number of different modalities, not less than three. The essential triplicity of visual function may be achieved without any special distribution of properties of the individual receptors alone or of the central connections alone, although it does not rule out such special distributions. The variations of the spectral properties of the individual receptors, for example, might be variation from three mean types, but it might also be a normal variation from one mean type; and the same thing for variations in central-connection modalities. Three independent reception systems might be obtained provided that the allocation of receptor types and central modalities were not also a random selection.

Guild's definition of trichromatic theory permits any of these possibilities to form a trichromatic theory, and since one of them must be valid, he has defined trichromatic theory in a way that prevents it from ever being proved wrong. By this definition, most forms of Hering opponent-colors theory are trichromatic theories.

The "persistence of optical equations" so carefully studied by v. Kries is not, by this view, a necessary consequence of the trichromatic theory; nor does it follow from this theory that selective adaptation should necessarily be explicable as a mere change of the relative sensitivities of the three reception systems, without alteration of their individual spectral sensitivities as implied by the v. Kries coefficient law.

In his paper, Interpretation of Quantitative Data in Visual Problems, Guild adopts the narrow view of measurement that its basis must be analogous in principle to the process of measuring a distance by finding the number of measuring rods, each of equal length, which have to be laid end to end to occupy the distance in question. The criterion of equality of magnitude of two quantities of the same kind, simultaneously observed under identical conditions, is the fundamental criterion in the construction of all scales of measurement. In establishing photometric relations, the most precise work requires that an appreciable area of the retina shall be used, and that the property of the viewed surface by virtue of which it is visible shall be uniformly distributed over its area. This property is the *rate of emission of radiant energy per unit area of surface (projected normally to the line of sight) per unit solid angle in the direction of the line of sight.* The modern name for this property is *radiance.*

The complete specification of the stimulus is given by the distribution of its radiance throughout the spectrum. In photometry the criterion of equality in magnitude is that two juxtaposed fields appear equally bright. Guild says that a photometrist can make valid judgments of brightness equality even though the chromaticities of the two fields are quite different, and that the uncertainty of the settings becomes smaller as the chromaticity difference is reduced. The radiances E_A and E_B of the two half-fields are under the control of the photometrist, and when he has adjusted them to appear equally bright, the result of the experiment is simply that under the actual conditions prevailing as regards the size, shape, and position with respect to the observer of the half-fields, and at the actual time when the observation was made, the radiance E_A of field A evoked the same sensation of brightness in this observer as the radiance of the field B. In symbolic form this result may be written: $E_A K_A = E_B K_B$, where K is a quantity proportional to luminous efficacy. If similar experiments comparing fields C, D, and so on with field A are carried out, we are immediately tempted to write:

$$E_B K_B = E_C K_C = E_D K_D \text{ and so on}$$

and to regard these as equal quantities of the entity that has been measured in the photometric process since each has been separately found as having the same brightness as $E_A K_A$, but this is not correct, says Guild. It assumes that the quantity $E_A K_A$ can be regarded as a standard of constant magnitude, but the only thing known to be constant in all of the settings is the radiance E_A. Nothing in the nature of the photometric process either proves or disproves the constancy of K_A under the conditions of the various comparisons, which, though identical in all other aspects, necessarily differ with respect to time.

In order to obtain from the photometric process something that is really measurable we must write the observation equations in the form:

$$E_A = E_B(K_B/K_A),\ E_A = E_C(K_C/K_A)$$

and so on. The quantities (K_B/K_A), (K_C/K_A), and so on are simply dimensionless ratios. From this series of equal quantities a scale of measurement may perhaps be built up that gives a quantitative measure of magnitude for the entity, brightness, of which these quantities are samples.

The quantitative measure is called luminance; its dimensions are the same as those of radiance. In the case of luminance the photometric process provides a practical means of obtaining a series of equal quantities. To construct a quantitative scale from this series, we must assume them to be additive without alteration of their individual significance. That is to say, the ratio K_B/K_A associated with radiance E_B must be assumed to remain constant for all values of the total luminance and for all qualities of the total radiance that may result from adding $E_B K_B/K_A$ to other members of the series $E_C K_C/K_A$, $E_D K_D/K_A$, and so on, and a similar constancy must be assumed for K_C/K_A, K_D/K_A, and so on. From this assumption it follows that the luminance associated with a radiance of any one given quality (spectral distribution) is proportional to the radiance. If we select the luminance associated with E_B as a standard unit, the luminance of a surface emitting nE_B will numerically equal *n* units of luminance.

The fact that this luminance scale involves the quite arbitrary assumption of the constancy of the various ratios K_B/K_A, K_C/K_A, and so on, does not imply that we could have obtained a scale with other properties by making some other assumption. No scale with other properties would be a quantitative measure of brightness. Insofar as there can be a measure of brightness, that measure must be luminance, and we have to find by suitable tests within what range of stimulus values the luminance scale has any practical significance. The series of comparisons having the most direct significance is that in which the radiances E_B, E_C, E_D, and so on, are concentrated in narrow wave bands throughout the visible spectrum. In this case the ratios K_B/K_A, K_C/K_A, and so on are known as values of spectral luminous efficiency. It will be seen that the assumption of constancy of the various ratios under all conditions is equivalent to the assumption of a constant shape for the curve of spectral luminous efficiency. Only within the luminance range in which this curve is in fact constant, to within the errors of experiment, is it true that luminance has any meaning as a quantitative measure of brightness. Intrusion of rod vision destroys this meaning; so the applicable luminance ranges depend upon the size and location of the retinal region involved. At the upper end of the luminance scale, the shape of the spectral luminous-efficiency curve also becomes seriously disturbed. For these conditions also use of photometric quantities is illegitimate.

Guild likewise sternly insists that photometry has meaning only with regard to luminance of an extended area. The idea that a particular luminous efficacy may be regarded as associated with radiant energy of a particular quality without reference to its spatial distribution is rather prevalent. This idea underlies the technical photometrist's definitions of luminous flux, luminous intensity, and allied entities. We are not entitled to form any such general conceptions. Guild's views in this respect have found few adherents, but nobody seems to have pointed out in what respects they are wrong. The established techniques of photometry are, in general, convenient and appear to be useful. Can it be that this usefulness is illusory?

In his analysis of color measurement, Guild covers much the same ground as Schrödinger in his elementary color measurement. Guild, however, insists that the tristimulus values by which a color are specified must be expressed in luminance terms to provide a true measurement on quantitative scales. The implication is that color matches hold only for experimental conditions yielding a substantially unchanged shape of the curve of spectral luminous-efficiency, but it would seem that only the shapes of the individual color-matching functions are significant. Note that Schrödinger treats color measurement as elementary, and does not deal with brightness measures until he comes to advanced color measurement. The Schrödinger hue and saturation measures are regarded by Guild as no measures at all.

L. F. Richardson in his paper, Measurability of Sensations of Hue, Brightness, or Saturation, mentions three ways to measure these sensations as distinct from stimuli: (E) by counting small equal-appearing intervals, (J) by counting just-perceptible intervals, and (R) by directly estimating the ratio of unequal intervals, both much larger than the least perceptible. As individuals differ in color vision and in aesthetic opinions, the standardizing institutions, he says, are led to measure stimuli, not sensations — to do physics, as being easier than psychology. Guild in discussion said that sensation does indeed have a quantitative aspect, but we cannot measure the magnitude of a sensation. We can make subjective estimates of this magnitude, but these estimates do not constitute measurement. Richardson in reply said that it is a mistake to attempt, as Guild does, to forbid the study of the functional relationship between magnitudes of sensations and those of stimuli. He does so for the reason that they are philosophically noncomparable phenomena. Nevertheless some of these functional relationships have been discovered and many others will no doubt be discovered later.

In his paper, Retinal Structure and Color Vision, Polyak gives a clear description of the anatomy of the central fovea including macular pigment, cones of the rod-free area, rods and cones in the surrounding regions, but particularly the various types of nerve cells of the retina, and how they are interlocked to form a complex retinal tissue. He describes bipolar cells, ganglion cells, and interrelations of the retinal neurons, and he presents an analysis of the synaptical relationships of retinal neurons. He points out that the mop bipolar cells are capable of being stimulated both by the rods and by the cones, either at the same time or alternately. The midget ganglion cells in the rod-bearing regions may also serve as a common rod and cone pathway. Only the midget bipolar cells (besides the cones) preserve the character of a strictly cone mechanism. Since in the rod-free foveal center there is complete hue distinguishability, we may conclude that cones by themselves may act as color receptors. Again, since there is no concrete evidence of any further differentiation of the foveal cones into several varieties, the central cones must be declared pretty much alike in their function, and all central cones seem to be able to react in exactly the same way to every hue to which the eye responds at all. If there is a triplex factor basically responsible for selective spectral responsiveness on the cone level this factor is not of the kind demonstrable by ordinary microscopical methods.

If there is no differentiation of the cones into three types, they would then merely furnish a dynamical "material" for other structures of the visual system to work with. In such a case the bipolars and ganglion cells must in some way be the carriers of the process by which the global cone excitation is transformed and directed into one or the other channel according to the spectral position of the stimulus, its magnitude, and other qualities. Note, however, that this suggestion is contrary to Guild's contention that the receptors, and receptors alone, determine the spectral sensitivity of each reception system. We can certainly agree with Polyak's later statement that "what arrive in the center are the impulses that originate in the cones but that are in many ways modified by the intervening neurons." The modification cannot, however, be such as to produce two different impulses in the center for two stimuli yielding identical cone impulses.

The final paper of the MacAdam collection, Laminar Pattern of the Lateral Geniculate Nucleus Considered in Relation to Color Vision, by Wilfried E. Le Gros Clark provides evidence of a segregation of red, green and violet reporting fibers in the visual systems of men and macaques. The evidence is provided by an analysis of the main primary optic center, the lateral geniculate nucleus through which retinal impulses are relayed to the cerebral cortex. The geniculate nucleus is made up of six well-defined cell laminae which are quite separate in the area for central vision, but are partly fused in the area for peripheral vision. If one eye is removed, three layers undergo rapid degeneration, the first, fourth, and sixth of the geniculate nucleus on the side opposite to the eye, and the second, third and fifth of the nucleus on the same side. Thus each nucleus consists fundamentally of two sets of three layers, each set related to one eye.

If a very small and localized lesion in the central part of the retina is made experimentally, a circumscribed patch of transneuronal-cell atrophy appears in all three of the corresponding laminae. It has been inferred from this observation that from each local spot in the central area of the retina three types of fiber pass back in the optic tract, one to each lamina. If the laminae are labeled a (for outermost), b and c (for innermost), layer a is composed of much fewer and larger cells. At a fixation point the a fibers are relatively very few or may be absent altogether, but the relative number of a fibers increases progressively towards the periphery of the retina.

It has been suggested that if the a fibers correspond to the violet factor of trichomasy, and the b and c fibers to red and green, this would explain the smaller extent of the red- and green-perceiving retinal fields. The paucity (or possible absence) of the a fibers from the foveal center accords well with the evidence that here the retina is relatively insensitive to violet. More direct evidence is supplied by experiments with monkeys kept for several weeks in light from which the short-wave end of the spectrum is completely excluded. The cells in the central retinal zone of cell lamina a may undergo marked atrophic changes, though those of cell laminae b and c also show some degree of cell atrophy.

Furthermore two cases of diabetic amblyopia reported by Rönne showed central scotomas for red and green, and in both cases there was a corresponding degeneration in the central retinal areas of layers b and c, while the whole of layer a appeared to be intact. All this evidence is presumptive, according to Le Gros Clark, and needs further corroboration before the existence of a three-fiber connection between retina and geniculate, one fiber for red, another for green, and a third for violet, can be regarded as proved.

APPENDIX 1: PUBLICATIONS OF DEANE B. JUDD

(A) indicates Abstract

Standardization of telephone rates (with C. A. Wright). Paper at 25th Convention U.S. Independent Telephone Assn., Chicago, Nov. 15-18, 1921, 31 p. (Engineering Experiment Sta., Ohio State Univ., Columbus, 1921).

Standardization of telephone rates (with C. A. Wright). Bull. 27, Eng. Exper. Sta., Ohio State Univ., Columbus, 74 p. (1923).

Physical characteristics of complementary pigments. M.A. thesis, Ohio State Univ., Columbus, 28 p. (1923).

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The probabilities imposed by the scoring system of modern lawn tennis. Unpub. ms., 17 p. (1924).

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The computation of colorimetric purity. J. Opt. Soc. Am. Rev. Sci. Instr. 13, 133-154 (1926).

A quantitative investigation of the Purkinje afterimage. Ph.D. thesis, Cornell Univ., Ithaca, N.Y., 89 p. (1926); Am. J. Psychol. 38, 507-533 (1927); J. Opt. Soc. Am. Rev. Sci. Instr. 14, 143A (1927).

Apparatus for the control of stationary stimuli. Am. J. Psychol. 38, 107-112 (1927); J. Opt. Soc. Am. Rev. Sci. Instr. 14, 470A (1927).

Sensibility to wavelength difference and the precision of measurement of dominant wavelength for yellow colors of high saturation (with I. G. Priest). J. Opt. Soc. Am. Rev. Sci. Instr. 14, 470A (1927).

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Least retinal illumination by spectral light required to evoke the "blue arcs of the retina". Program, Michelson Meeting Opt. Soc. Am., Nov. 1-3, 27A (1928); J. Opt. Soc. Am. 18, 172A (1929); J. Res. Nat. Bur. Stand. (U.S.), 2, 441-451 (1929) RP43.

Calibration of sixty-five 35-Yellow Lovibond glasses (with I. G. Priest, K. S. Gibson, and G. K. Walker). J. Res. Nat. Bur. Stand. (U.S.), 2, 793-819 (1929) RP58.

Review: Colour and Colour Theories, by Christine Ladd-Franklin. J. Opt. Soc. Am. Rev. Sci. Instr. 19, 103-105 (1929).

Comment on a paper by G. W. Thompson, "The true tinting strength of white pigments". Proc. Am. Soc. Test. Mater. 29, Part II, 8-11 (1929).

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Reduction of data on mixture of color stimuli. J. Res. Nat. Bur. Stand. (U.S.), 4, 515-548 (1930) RP163.

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16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This book is a collection of fifty-seven papers written by Deane B. Judd, a staff members of the National Bureau of Standards from 1926 to 1969, and an internationally recognized authority on color. The contents of this collection include some of the major contributions of Dr. Judd to such areas as the measurement and specification of color, spectrophotometry, color appearance and spacing, and color vision. Each paper is preceded by an introduction which provides general commentary on the article and explains the terminology used. Some introductions also direct the reader to related articles in the collection and point out significant developments, such as international agreements, which were based on Judd's work. A list of the more than 200 articles written by Dr. Judd is included in an appendix.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)*

Color; color spacing; color vision; colorimetry; Deane Judd; selected writings; spectrophotometry.

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