SIMPLIFIED ENERGY DESIGN ECONOMICS

PRINCIPLES OF ECONOMICS APPLIED TO ENERGY CONSERVATION AND SOLAR ENERGY INVESTMENTS IN BUILDINGS

ENERGY

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DESIGN
ECONOMICS

PRINCIPLES OF ECONOMICS APPLIED TO ENERGY CONSERVATION AND SOLAR ENERGY INVESTMENTS IN BUILDINGS

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PREFACE

One of the most powerful determinants of building form, as we approach the end of this century, is the cost of energy and energy conservation. Clients expect architects, engineers and all those associated with the building design professions to make economically intelligent and informed energy design decisions on their behalf. The creative challenge posed by this powerful design determinant and the legal consequence of improper energy design decisions are vital presences in today's design practice and compelling reasons for professionals to continue to enhance their design related skills.

This publication is a joint product of the Design and Construction Technology Application Program (DACTAP) and the Building Economics and Regulatory Technology Division, both in the Center for Building Technology. The report is a tool to be used by the design community in making energy decisions. It will aid you in making economic evaluations and logical assessments of costs and benefits inherent in design decisions over time and thus enhance understanding between client and designer. It can serve as a text for classes and self-instruction, as a reference for the drafting table, and as a concise group of problem-solving formats to outline the economic parameters of energy design decisions.

As a handbook it provides the information you need to analyze straightforward economic problems, which comprise perhaps 90 percent of those you will encounter. It will also aid your understanding and facilitate your cooperation with experts retained to conduct more complex economic analysis. This publication will provide you the soundly based self-confidence in economics that you now possess in structural, mechanical and other engineering disciplines related to the design of buildings.

I commend this publication to you for these reasons and sincerely hope that you will use it to expand your analytical skills for your benefit and for that of the design professions.

DACTAP is a new program at the Center for Building Technology, a part of the National Engineering Laboratory in the National Bureau of Standards. DACTAP seeks to understand the technical information needs of designers and builders and to provide useful technical information for application in the design and construction process. The authors, the editor and I will appreciate your comments and reaction to this publication and your suggestions for future work.

Porter Driscoll, AIA
DACTAP Coordinator
Center for Building Technology

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The authors are also indebted to the architects, engineers, urban and regional planners, and others who provided field testing of this report as a teaching manual by attending an extension course, "Design Economics for Energy Conservation in Buildings," presented at the University of California, Berkeley. Steve Petersen, from NBS, joined the authors in teaching that course and thereby contributed to this report.

Special credit is due Vladimir Bazjanac of the University of California, Berkeley, who gave particularly useful insight into making the report suitable for students of architecture and engineering. The NBS Electronic Typesetting staff, directed by Rebecca Morehouse, provided generous assistance in preparing the report for printing. Finally, thanks are due the many persons at NBS who have spent time with the authors discussing energy issues and suggesting significant problems that this report should address.

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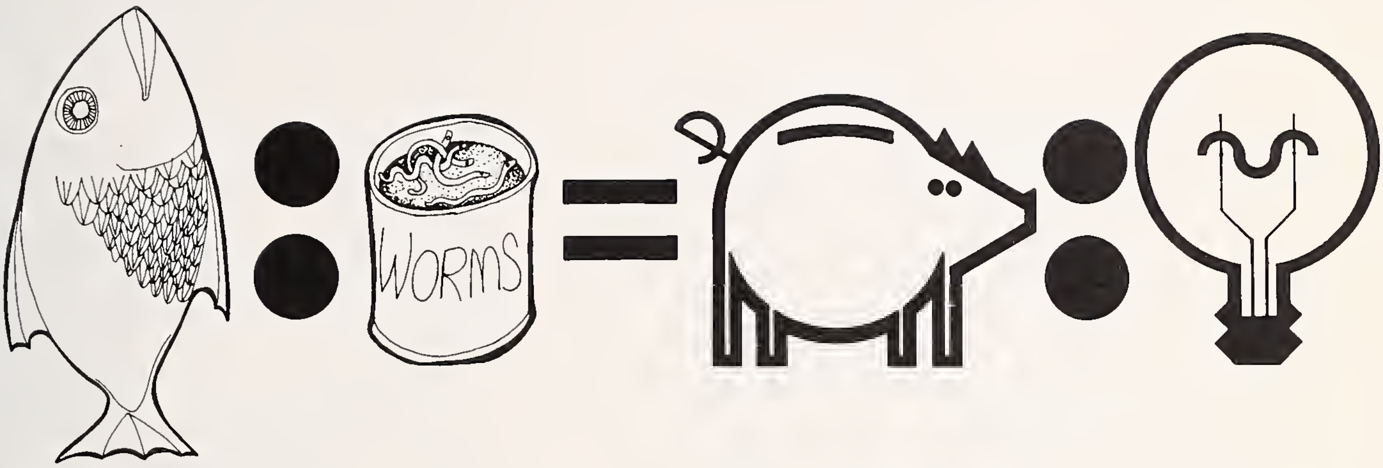


Fish are to bait as savings are to energy conservation costs

INTRODUCTION

Firmness, Commodity, Delight, and Energy

The fuel shortage has added a fourth element to the three the Vitruvian view of architecture saw as essential to all buildings if they were to accommodate the physical and aesthetic needs of humankind. Energy is now, more than at any time in recent history, a crucial element of the art of building.

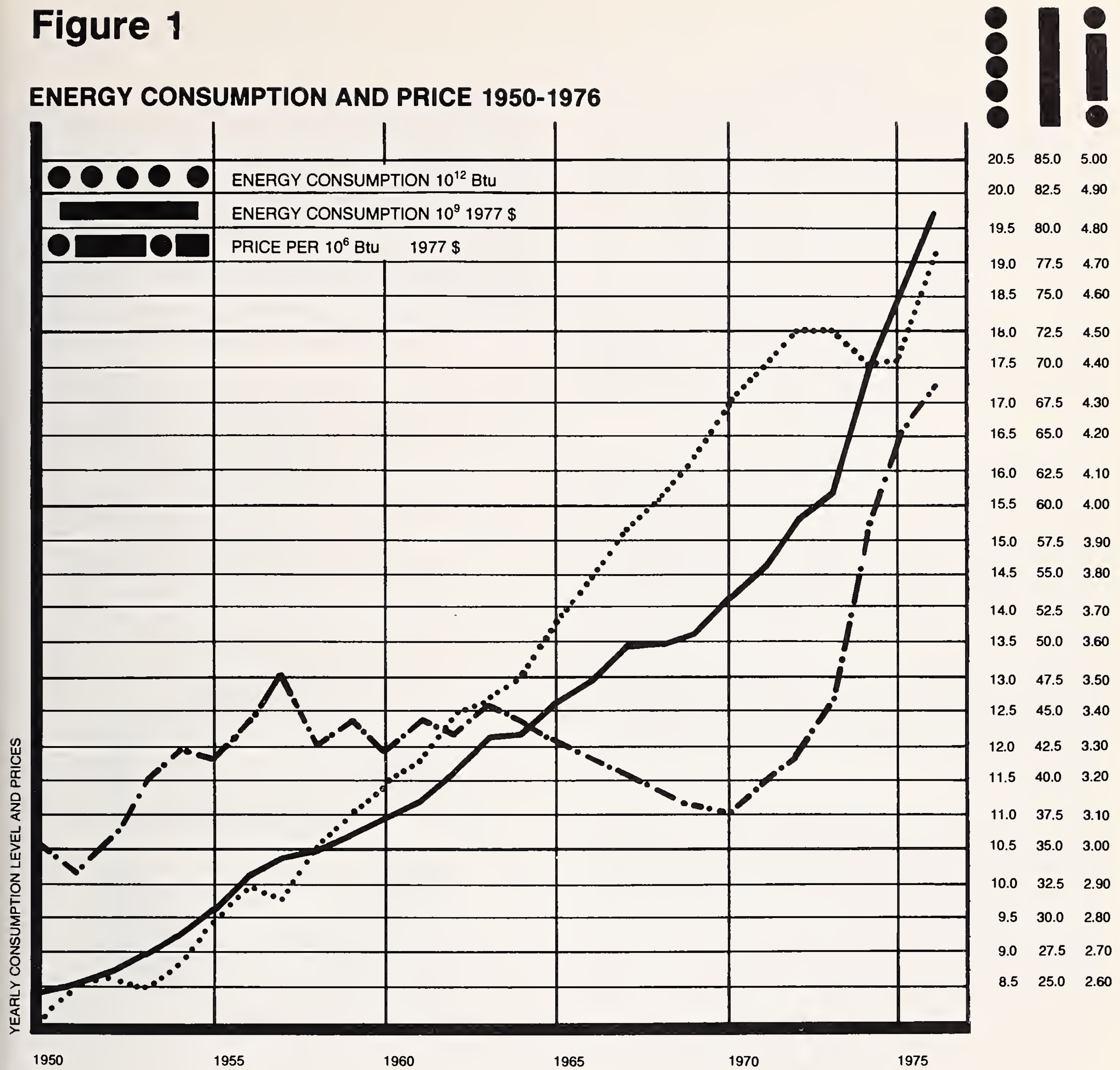
The critical need for energy conserving design stems from the rapid increase in energy costs. A continued scarcity of fuels, increased cost of energy production and delivery, and the mounting demands for energy in an energy-intensive society have combined to render energy a primary economic concern of all those involved in the act of building. The chart (fig. 1)[1] on the opposite page dramatically records energy consumption for building in terms of Btu and dollar expenditures during the past 26 years. During this period physical units of energy consumption increased over 134%. The unit price per 1,000,000 Btu of energy vacillated, declining half the years, but overall increased 44%. The major portion of this increase, 39%, took place during the period 1970-76, and the total dollar expenditure for energy increased by $26 billion. These statistics clearly indicate why energy conservation is crucial to the design community.

Alarming conditions exist in all areas of energy consumption in America as energy prices rise and shortages threaten. Governmental programs, ranging from research and development in renewable energy sources, such as solar energy, to legislation that encourages and sometimes mandates energy conservation, have been instituted to counter these circumstances.[2]

Spurred by the overall rise in building costs, architects, civil and mechanical engineers, builders, their clients, and building owners are rigorously examining all phases of building energy consumption. We find a concentrated examination of the benefits of energy conservation taking place on an unprecedented scale.

Figure 1

ENERGY CONSUMPTION AND PRICE 1950-1976



Purpose of the Book

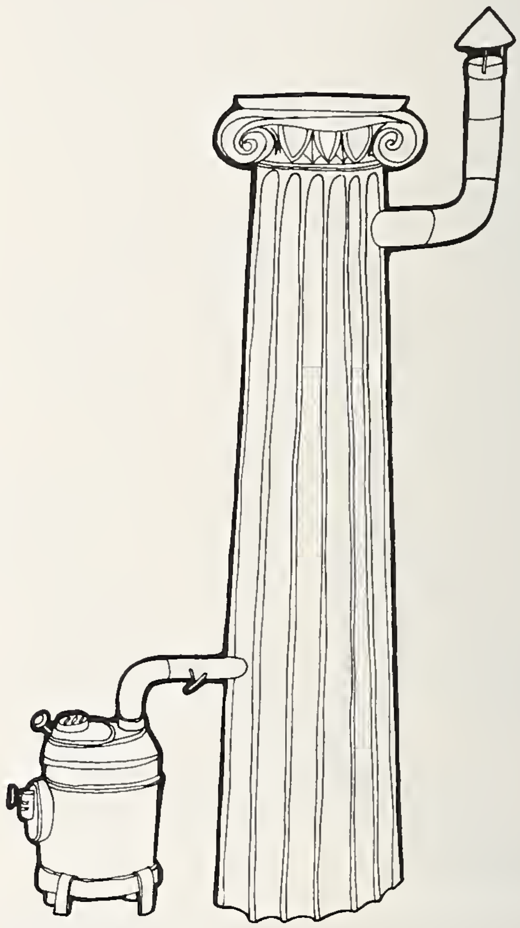
Traditional microeconomic theory+ and engineering economics have long offered useful instruction in the general principles of economic evaluation that could be applied to building. However, there has been little material available to satisfy the specific demands for economic guidelines that present circumstances demand. The purpose of this brief work is to help meet this urgent demand in a simple, comprehensible fashion.

This book is designed as an adaptable instrument for the "design community": architects, engineers, builders, building owners and managers, codes and standards writers, government policy makers, and students of these disciplines.

We recognize that it is probably impossible to provide, in a single brief volume, a guide that will address adequately the range of economic problems faced by each of these members of the design community. Decisions that affect the energy consumption of a building begin with the earliest conceptualization of its basic form and continue throughout the design, financing, construction, and operational phases of the building. The nature of the decisions and the environment in which they must be made vary widely. For example, an architect grappling with the basic configuration of a building on its site, a mechanical engineer selecting an energy control system, and a builder making envelope modifications in a retrofit context must each deal with a different set of problems.

We have adopted a modest goal: to provide a guide to basic economic concepts and tools for solving simple economic problems in energy conservation and for understanding complex problems more clearly. You will find here economic principles and step-by-step examples to aid you in determining the economic efficiency of specific energy investments.

The focus is on economic analysis and choice. The emphasis is on practical method rather than theoretical discussion. Simplified formulas are presented and their applications illustrated. Mathematical derivations and complicated formulas are left to the traditional textbooks in economic theory and engineering economics which you will find referenced in the footnotes. A knowledge of simple mathematics is all that is required to follow the discussion and solve the problems.



+Economic terms are defined in appendix A.

How the Book is Arranged

The concepts of economic efficiency are discussed first, followed by a discussion of the basic principles of economics required to comprehend the economic evaluations of alternative investment decisions. Step-by-step procedures for evaluating the economic desirability of alternative investments are presented to illustrate and reinforce an understanding of the principles.

Five tools of economic analysis are then introduced to equip the reader with the ability to measure economic efficiency in a variety of ways:

1. Life-Cycle Costs (LCC)

2. Net Benefits or Savings (B-C)

3. Savings-to-Investment Ratio (SIR)

4. Internal Rate of Return (IRR)

5. Discounted Payback (DPB)

The advantages and disadvantages of each of these economic tools are discussed and guidelines are given for selecting the appropriate one for dealing with specific types of investment problems.

Discounting, the technique for assessing the time value of money, is then explained and illustrated in a problem analyzing the economics of heat pumps. Following this discussion, the five tools are individually applied with the discounting procedures to solve a problem in solar energy.

A discussion of the general economic factors that affect benefits and costs is presented to aid the reader in selecting the appropriate economic tools to apply to different kinds of economic problems. This is followed by a brief summary.

A series of appendices, including a glossary of economic terms, discount formulas, and tables of compound interest factors, conclude the book. The tables are sufficiently comprehensive to allow problem solving with the economic tools presented here.

CONCEPTS OF ECONOMIC EFFICIENCY

The Objective

Economic efficiency is not the same as engineering efficiency. For example, one furnace may be more "efficient" than another in the sense that it delivers more units of heat for a given quantity of fuel. Yet, it may not be economically efficient if the first cost of the higher-output furnace outweighs its savings in reduced fuel consumption. To achieve economic efficiency in energy conservation, it is necessary to determine the most profitable levels of energy conservation. To determine this optimal level of conservation requires the identification of the most advantageous tradeoff between conserving and supplying energy.

In this section the fundamental principles of economic analysis used to make economically efficient investments in energy conservation are described and illustrated graphically.

Economic analysis is a tool which can be used to determine the tradeoffs necessary (1) to find how much to spend on energy conservation to lower life-time building costs, including investment costs, energy costs, and other recurring and nonrecurring costs; (2) to find the largest possible savings in energy costs for a given conservation budget; or (3) to achieve a targeted reduction in energy costs for the lowest expenditure in conservation.

The second and third applications of economic analysis—obtaining the largest savings in energy costs for a fixed conservation budget and attaining a targeted savings in energy costs for the lowest conservation budget—are more limited in terms of achieving economic efficiency than the first application, which seeks to minimize total building costs or maximize the net benefits from energy conservation.

In the first application, designers or builders may be asked by their clients to include those energy conservation features that will "pay off" in terms of lower life-cycle building costs. In the second application, a building owner may budget a specific sum of money for the purpose of retrofitting a building for energy conservation. In the third application, a designer may be required by State or Federal building standards to reduce the design energy loads of a new building to a specified level.

BAIT PRO PISCIS

Figure 2a

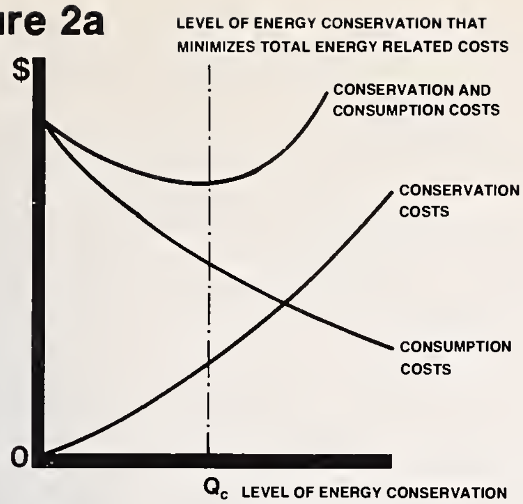


Figure 2b

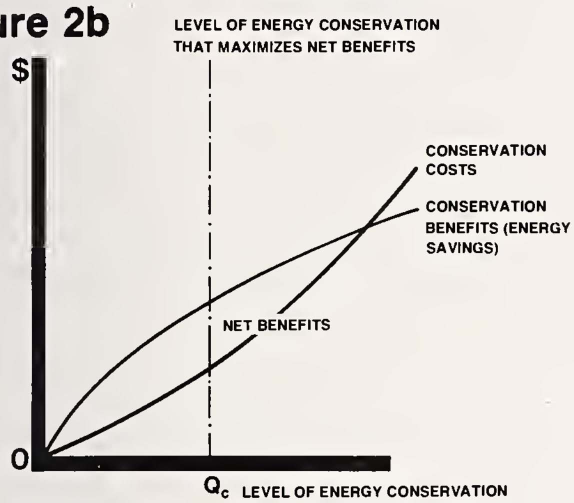
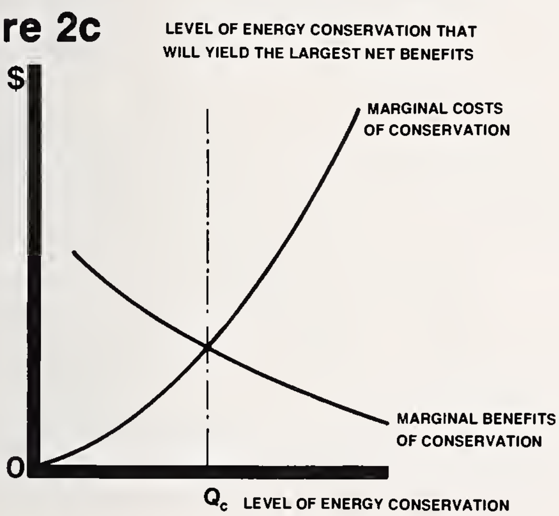


Figure 2c

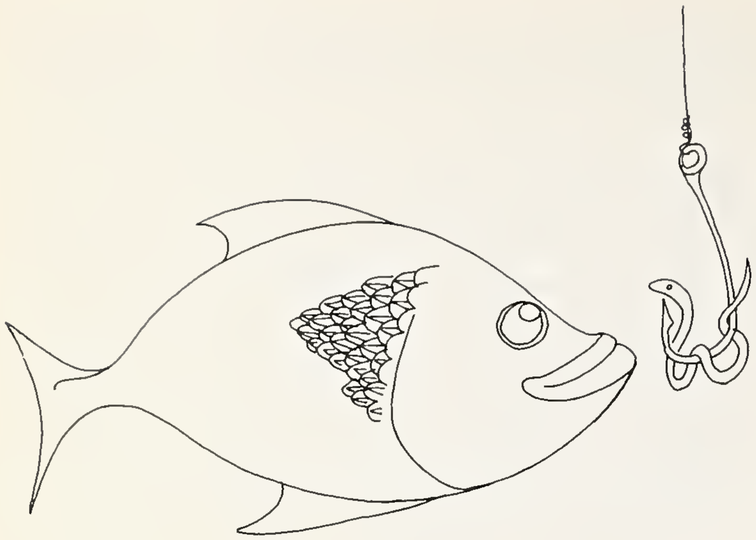


Graphical Illustrations

In figures 2a, 2b, and 2c the physical quantity of inputs used to conserve energy is measured on the horizontal axis and dollar costs are measured on the vertical axis. In 2a the upward sloping line from left to right indicates the rising total dollar costs of conservation as the physical quantity of inputs to conserve energy is increased, and the downward sloping line from left to right indicates the declining total cost of consumption as conservation is increased. Initially the rise in conservation costs is shown to be more than offset by the fall in energy consumption costs, but eventually, as more conservation is undertaken, the rise in conservation costs can be seen to become greater than the fall in energy consumption costs. This is reflected in the combined cost curve (the upper U-shaped curve), which falls to a minimum point and then rises. The most economically efficient level of energy conservation is that for which the combined cost curve is at a minimum, as indicated by "Q_c."

Another way of describing this concept is in terms of maximizing the net benefits from energy conservation as shown in 2b and 2c. Using this approach, the reductions in energy costs are the benefits, and the objective is to find the level of inputs to conserve energy for which the difference between the costs and the benefits of conservation is greatest.

Figure 2b indicates that the total costs of conservation tend to rise slowly at first but then begin to rise sharply as more and more inputs of conservation are acquired and difficulties begin to be experienced in carrying out the conservation. The result is that the total cost curve typically bends upward. Total energy savings (benefits), on the other hand, tend to rise at a decreasing rate as more and more inputs of conservation are added to a building. As long as the benefits curve lies above the costs curve, the energy conservation is profitable. The conservation level at which the curves are most distant, with the benefits above costs, is most profitable. The level at which the curves intersect indicates a breakeven investment, i.e., benefits are fully offset by costs. The cost curve rising above the benefits curve indicates that the energy conservation investment loses money.



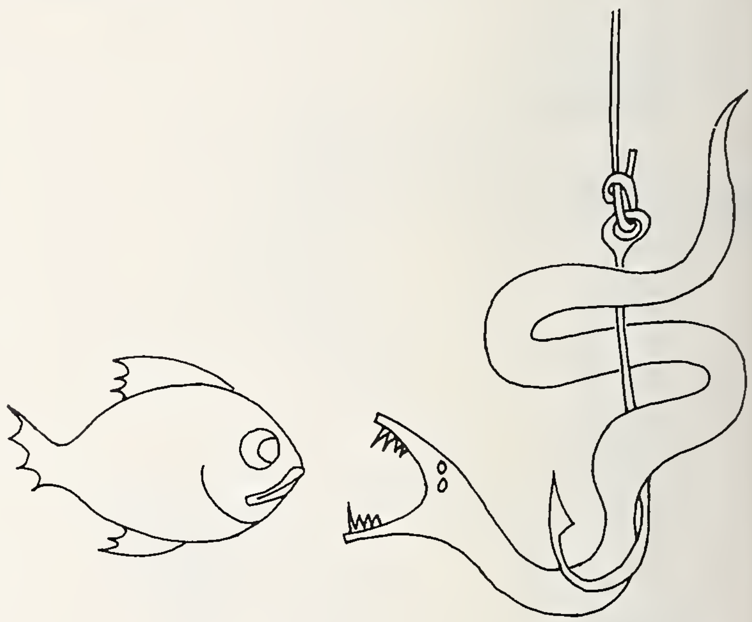
ECONOMIC RETURN ON INVESTMENT

Figure 2c shows how "marginal analysis" can be used to find the level of conservation which will yield the largest net benefits. Figure 2c depicts the changes in the total benefits and costs curves of 2b as the inputs of energy conservation are increased. The level of energy conservation where these marginal costs and benefits curves intersect is the most profitable level of energy conservation as indicated in figure 2b. This is the level at which the costs of adding one more unit of conservation are just equal to the corresponding additional benefits in terms of energy savings; that is, the level at which "marginal costs" and "marginal benefits" are equal.

For lower levels of conservation, the additional benefits from increasing conservation by another unit are greater than the additional costs, and it pays to invest more. For higher levels of conservation, the costs of additional conservation exceed its benefits and the level of total net benefits begins to fall. The most economically efficient level of conservation is indicated in each of figures 2a, 2b, and 2c as "Q_c."

Because savings and costs of alternative conservation techniques tend to differ, it will usually be necessary to make tradeoffs among techniques, investing more in some and less in others. For example, if another dollar invested in insulation offers a larger return than a dollar invested in solar energy, it will pay to increase the amount of insulation before investing in solar. Because the energy savings from most conservation techniques will tend to decrease as the investment in the technique is increased, it will usually pay at some point to shift further investment to some other technique.

The most profitable level of each technique in combination occurs when an additional dollar spent on one of the techniques will yield the same dollar return as that spent on each of the other techniques. In economic terms, this is the combination of techniques where the ratio of marginal savings to marginal costs is equal for all of the techniques. If the ratio is higher for one technique than another, it will pay to shift resources from the technique with the lower ratio to the one with the higher ratio in order to increase total net savings. Figures 2a, 2b, and 2c are constructed under the assumption that the most profitable combination of techniques is being considered for each level of energy conservation.



UNECONOMIC RETURN ON INVESTMENT

MEASURING BENEFITS AND COSTS

Kinds of Benefits and Costs

To determine the economic attractiveness of an investment, it is necessary to measure the benefits and costs associated with it. In broad terms, benefits from a conservation investment include both the monetary value of the resulting energy savings and the nonmonetary value of other beneficial effects of the investment. "Nonmonetary benefit" is defined as a benefit to which it is difficult to assign a dollar value. It does not mean that the benefit has no value.

There may be monetary and nonmonetary benefits that are enjoyed directly by the person or organization who invests in energy conservation as well as monetary and nonmonetary benefits that extend beyond the investor. An example of a monetary benefit that accrues directly to the investor in conservation is the dollar savings in fuel costs. An example of a more difficult to quantify benefit to the investor is the improvement in occupant comfort that may result when conservation tightens the building envelope and reduces drafts. Other possible examples of nonmonetary benefits to the investor from conservation are the achievement of a degree of energy independence, the prestige that may accompany the use of a new technology such as solar energy, and the respect that may be accorded private actions taken in the public interest.

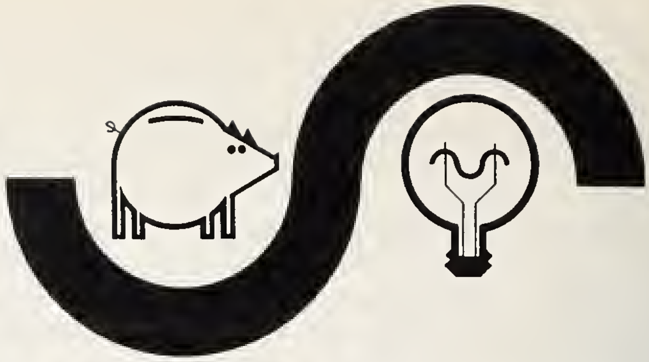
An example of a monetary benefit that may accrue beyond the investor, in this case to the nation as a whole, is the improvement in the U.S. balance of payments from reduced imports of oil. Societal benefits that are more difficult to quantify are the impacts of conservation on U.S. strategic vulnerability and on environmental air quality.

There may also be monetary and nonmonetary benefits that extend beyond the investor but not to the entire nation. For example, the planting of trees adjacent to a building for landscaping purposes may improve the environmental quality of the community, in addition to reducing the investor's air conditioning costs by shading the structure. Public recognition of "spillover" effects of private investor decisions may be reflected in political and legal requirements, such as a requirement for planting around a building as a condition for approval of building permits.

Although most costs of conservation tend to be measurable in dollars and incurred directly by the investor, there may also be costs that are nonmonetary and extending beyond the investor. For example, some conservation investments might be felt to have a negative impact on building aesthetics that will adversely affect the owner, the occupants, and the community.

While both monetary and nonmonetary benefits and costs are important, whether accruing only to the investor or to society, the focus of this report is on monetary benefits and costs to the investor. Monetary values are emphasized simply because the state of the art of measuring benefits and costs makes it difficult to assign values to nonmonetary benefits.[3] The focus on monetary values, however, is not to suggest that nonmonetary benefits should be given any less consideration in decisionmaking. Quantifying as many benefits and costs as possible in dollar terms reduces the guesswork and provides a single measure against which the more subjective elements may be compared.

The focus on benefits and costs to the investor, rather than to the nation, reflects the assumption that most investors in the private sector make decisions primarily on the basis of direct monetary benefits to them, although they are often influenced by community and national concerns. As a result, the five economic evaluation methods, presented here as tools to aid the design community in determining economically efficient investments in energy conservation, describe dollar costs and benefits solely to the investor.



Five Economic Tools

The five economic tools described here are Life-Cycle Costs (LCC), Net Benefits or Savings (B-C), Savings-to-Investment Ratio (SIR), Internal Rate of Return (IRR) and Discounted Payback (DPB).[4]

The first four are comprehensive analytical tools that can be used to evaluate investments in energy conservation. They consider both first costs and future costs and savings. Because they all look at the significant costs and benefits over the life of an investment, they are often referred to collectively as life-cycle techniques. The fifth tool, the discounted payback method (DPB), does not fully use the life-cycle approach. It nevertheless may be quite useful to designers under certain circumstances, as, for example, when the client requires rapid recovery of investment funds or when the durability of investment assets is highly uncertain. Each of the five tools considers the timing of cash flows and associated cost of money.

Life-Cycle Costs (LCC): Life-cycle costing sums the energy costs of the building together with the net costs of purchase and installation (less any salvage value), maintenance, repair, replacement, and all other costs attributed to the conservation investment. This includes the cost of money over the life of the investment. The investment that has the lowest total life-cycle cost while meeting the investor's objective and constraints is the preferred investment.

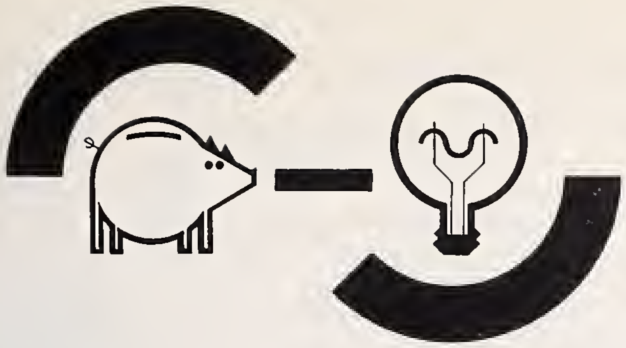
All cash amounts are generally converted to either present value or annual value dollars. Present Value is defined as the equivalent value of past and future dollars corresponding to today's values. Annual Value means that all past, present, and future costs are converted to an equivalent constant amount recurring annually over the evaluation period. The conversion process for both present value and annual value dollars is called discounting.

Following is a general formula for finding the total life-cycle costs of an energy conservation investment:

Life-Cycle Costs
Purchase and Installation Costs
Salvage Value
Maintenance and Repair Costs
Replacement Costs
Energy Costs

$$LCC = P - S + M + R + E,$$

where all costs are in life-cycle present value or annual value dollars and adjusted for taxes and incentives. An example using this tool appears on page 22.



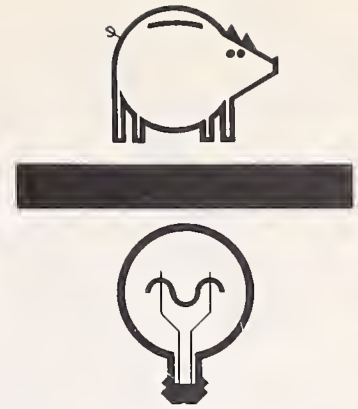
Net Benefits or Savings (B-C): This tool finds the difference between the life-time dollar energy savings and life-time dollar costs of a conservation investment. Net benefits or savings may be expressed in either present value or annual value dollars. This tool applies to the same types of investments as the life-cycle cost (LCC) tool, but is formulated somewhat differently as is shown below:+

Net Benefits or Savings
Reduction in Energy Costs
Differential Purchase and Installation Costs
Differential Salvage Value
Differential Maintenance and Repair Costs
Differential Replacement Costs

$$(B\text{-}C) = E^* - (P^* - S^* + M^* + R^*),$$

where all costs and benefits are in present value or annual value dollars and adjusted for taxes and incentives. An example using this tool appears on page 23.

+The values of "E," "P," "S," "M," and "R" in this and subsequent equations, where accompanied by an asterisk (*), represent the difference between the present value or annual value costs for an energy conserving investment and its alternative. While the previous LCC formula must be applied to each of two investments being compared, the (B-C) formula is applied directly to the difference between two alternative investments. The LCC formula corresponds to figure 2a, while the (B-C) formula corresponds to figure 2b.



Savings-to-Investment Ratio (SIR): Like the two preceding tools, the SIR is based on discounted cash flows. However, savings and investment costs are expressed as a ratio rather than a dollar amount. For positive net savings, the ratio must be greater than one. The higher the ratio, the more dollar savings realized per dollar of investment.

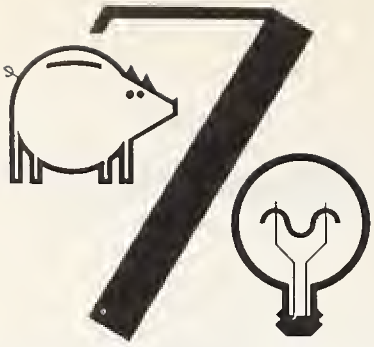
Following is a general formula for computing the savings-to-investment ratio:+

Savings-to-Investment Ratio
Reduction in Energy Costs
Differential Maintenance and Repair Costs
Differential Purchase and Installation Costs
Differential Salvage Value
Differential Replacement Costs

$$SIR = (E^* - M^*) \div (P^* - S^* + R^*),$$

where all costs are in present value or annual value dollars and adjusted for taxes and incentives. An example using this tool is given on page 24.

+The ratio is sensitive to whether cost elements are subtracted from savings in the numerator or added to costs in the denominator. For ratios greater than one, adding costs to the denominator will reduce the ratio more than subtracting the same costs from the numerator. Placement of salvage values, maintenance costs, and replacement costs in the numerator or denominator of the ratio sometimes varies in application, depending on the costs for which the investor is trying to otain the highest return. The formulation shown above, however, is widely used.



Internal Rate of Return (IRR): This tool finds the rate of return on an investment. This is the interest rate, stated as a percent, for which life-time dollar savings are just equal to life-time dollar costs. The calculated IRR is compared to the investor's minimum acceptable rate of return to determine if the investment is desirable.

The IRR is generally calculated by a structured process of trial and error.+ Selected compound rates of interest are used to discount the cash flows until a rate is found for which the net value of the investment is zero or close to zero.

Following is a general formula for the internal rate of return:

Internal Rate of Return (IRR = i)
Reduction in Energy Costs
Discount Factor for Interest Rate i (UPW*,i,N)
Differential Purchase and Installation Costs
Differential Salvage Value
Discount Factor for Interest Rate i (SPW,i,j)
Differential Maintenance and Repair Costs
Discount Factor for Interest Rate i (UPW,i,N)
Differential Replacement Costs
Discount Factor for Interest Rate i (SPW,i,j)

Find i such that $(\overline{E} \cdot a) - [\overline{P} - (\overline{S} \cdot b) + (\overline{M} \cdot c) + (\overline{R} \cdot b)] = 0$

The bar over the symbols, e.g., "$\overline{E}$," indicates that the cost differences have not yet been converted to present or annual values. The terms a, b, and c refer to discounting factors that are explained on pages 16–20 and listed in appendices C and D. A sample problem is solved by this tool on page 25.

+IRR programs available for some programmable desk calculators may be helpful in solving for IRR.

Discounted Payback (DPB): This tool measures the elapsed time between the point of initial investment and the point at which accumulated savings, net of other accumulated costs, are sufficient to offset the initial investment cost. Costs and savings are adjusted to account for the changing value of money over time. If a time adjustment is omitted, the tool is termed "simple payback."

For investors who seek a rapid turnover of investment funds, the investment increases in desirability as the payback period decreases. However, a shorter payback time does not necessarily indicate the most economically efficient investment. An investment with a longer payback period may prove more profitable than an investment with a shorter payback period if it continues to yield savings for a longer period of time.

The following formula shows how to find the payback period, Y:

Summation from years 1 to Y
Reduction in Energy Costs in year j
Differential Maintenance and Repair Costs in year j
Differential Replacement Costs in year j
Differential Salvage Value in year j
Differential Purchase and Installation Costs

Find Y such that $\sum_{j=1}^{Y} (E^*_j - M^*_j - R^*_j + S^*_j) = P^*$

where all costs are in present value or annual value dollars and adjusted for taxes and incentives. A sample problem is solved by this tool on page 26.

Advantages, Disadvantages, Recommended Applications

COMMON ECONOMIC QUESTIONS AND RECOMMENDED TOOLS FOR SOLUTION

	LCC	B-C	SIR	IRR	DPB
HOW CAN SAVINGS BE COMPARED TO COSTS?	●	●	●	●	●
HOW LARGE AN INVESTMENT TO MAKE?	●	●			
HOW TO FIND THE LEVEL OF MAXIMIZED DOLLAR BENEFITS?	●	●			
HOW MUCH OVERALL COSTS WILL BE LOWERED BY INCREASED CONSERVATION?	●	●			
HOW CAN PROJECTS DIRECTLY COMPETING FOR THE SAME PURPOSE (e.g., R-10 INSULATION VERSUS R-19 INSULATION) BE COMPARED?	●	●			
HOW CAN DIFFERENT PURPOSE PROJECTS COMPETING FOR THE SAME BUDGET (e.g., INCREASING THE THERMAL MASS OF THE BUILDING WALLS VERSUS THE USE OF EXTERIOR OVERHANGS TO SHADE THE WINDOWS) BE COMPARED?			●	●	
HOW TO FIND THE RATE OF RETURN ON INVESTMENT?				●	
HOW SOON WILL ENERGY INVESTMENTS BE PAID OFF BY SAVINGS?					●

● HOW can savings be compared to costs?

The five tools described are not equally appropriate for evaluating all types of energy conservation investment decisions. However, if a simple "accept-reject" investment decision is all that is desired, then any of the tools described can be used as described below:

LCC The total life-cycle costs must be lower with the investment than without it.

B-C The net dollar benefits must be positive.

SIR The ratio of dollar benefits to dollar costs must be greater than one.

IRR The internal rate of return must be greater than that minimally acceptable to the investor.

DPB The payback period must be shorter than the expected life of the investment and must meet the investor's timing requirements for recovery of investment funds.

However, to maximize net savings from conservation investments, choices among competing investments should be based on more sophisticated evaluation criteria than those just stated, and the choice of tool becomes important. Each tool can be used to attain specific objectives and each has unique advantages, disadvantages, and recommended applications.

● HOW large an investment to make?

● HOW to find the level of maximized dollar benefits?

● HOW much overall costs will be lowered by increased conservation?

● HOW can projects directly competing for the same purpose (e.g., R-10 insulation versus R-19 insulation) be compared?

LCC and B-C are the tools recommended to find the economically efficient size of a conservation investment. Although the SIR and IRR techniqes may also be used to determine the most advantageous size of a project if applied correctly to increments in the investment, LCC and B-C are less apt to be misapplied.+

If the life-cycle costs fall with added investment, it is profitable to increase the investment. Or if net savings increase, it is profitable to increase the investment.

By measuring the change in costs and the amount of savings from energy conservation investments, the LCC and B-C tools also provide an answer to the fourth question, "How much will overall costs be lowered?" Additionally, either tool will provide an answer to the fifth question, "How can directly competing projects for the same purpose be compared?" This question is answered by determining the project that minimizes life-cycle costs or maximizes net savings. If, for example, total costs are lower or net savings are higher with R-19 insulation in the attic than with R-10 insulation, it pays to use R-19 insulation, other things equal. These investments are often termed "mutually exclusive" because undertaking one generally precludes undertaking the other.

Despite the fact that the LCC and B-C tools are effective in addressing the above five questions, they are not always effective for solving other kinds of investment problems, such as comparing projects that have different purposes but that compete for a limited budget.

Although the life-cycle measures of LCC and B-C indicate whether total building costs are higher or lower with or without the conservation investment, they do not distinguish between large and small investments that result in the same net dollar energy savings. That is, they do not provide an indication of the return on the investment dollar.

+Because the SIR and IRR measures may begin to fall before the economically efficient size of an investment is reached, they can be used to size an investment efficiently only if used to measure the economic efficiency of each *increment* of the investment rather than of the total. The LCC and B-C measures, on the other hand, can be applied to the total investments for purpose of sizing.

LCC and B-C both have essentially the same advantages and disadvantages. The two are generally interchangeable.

● HOW can different purpose projects competing for the same budget (e.g., increasing the thermal mass of the building walls versus the use of exterior overhangs to shade the windows) be compared?

SIR or IRR are recommended for determining the ranking to be given to competing investments which are not direct substitutes for one another, where it is assumed that the optimal size of each project has already been determined.+ These investments are often termed "nonmutually exclusive" because, aside from budget limitations, undertaking one investment does not necessarily preclude also undertaking the other. SIR and IRR are recommended because both reflect the return on investment dollar and can be used to rank investment projects to determine the combination that will result in the largest total return for a given budget.

LCC and B-C are not recommended for ranking competing nonmutually exclusive investments because investments selected on the basis of their LCC or B-C ranking may not yield the highest total net benefits for a limited budget. For example, suppose the investor has $12,000 to be spent on an energy conservation retrofit package and the following four projects have been identified as possible candidates:

(1) replacement of all windows at a cost of $2,000 and an expected life-cycle savings of $6,000,

(2) a new energy control system with a cost of $12,000 and an expected life-cycle savings of $20,000,

(3) adding thermal insulation to the walls at a cost of $4,000 and an expected savings of $9,000, and

+For maximum economic efficiency, the sizing of individual projects and the allocation of limited funds among competing projects cannot be independent decisions. A condition for achieving the economically efficient combination of projects is that to the extent possible the last dollar spent on each project yields the same benefits as that spent on all the other competing projcets. However, in practice, potential projects are often compared based on predetermined sizes for each.

(4) an improved maintenance routine that is expected to cost $6,000 and save $12,000.

The following table shows the comparative ranking of the four projects, first according to the B-C tool, and second according to the SIR tool (the LCC rankings would be the same as the B-C, and IRR rankings would be the same as the SIR).

Project

Investment Alternatives	Investment Cost	Expected Savings	Net-Benefits	B-C Ranking	SIR	SIR Ranking
	($)	($)	($)			
1	2,000	6,000	4,000	(4)	3	(1)
2	12,000	20,000	8,000	(1)	1 2/3	(4)
3	4,000	9,000	5,000	(3)	2 1/4	(2)
4	6,000	12,000	6,000	(2)	2	(3)

Ranking by the B-C tool would suggest the selection of project 2 for a cost of $12,000 and a net savings of $8,000, while ranking by the SIR tool would indicate the selection of projects 1, 3, and 4 for a total cost also of $12,000 and a total net savings of $15,000. Selecting the projects according to their SIR (or IRR) results in the realization of an extra $7,000 in net savings from the fixed budget of $12,000 over the net savings from the projects selected by their B-C ranking.

Of course for a simple example like this one, the combination of projects with the highest total net benefits can be found simply by trying different combinations of projects costing $12,000 and adding their net benefits. When there are many projects under consideration, however, the use of the SIR or IRR ranking to guide project selection can be an easier, more direct approach.

● HOW to find the rate of return on investment?

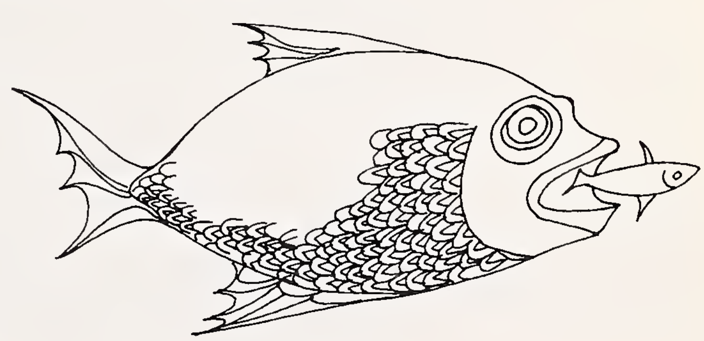
The IRR tool is the only one of the five tools that measures economic efficiency in terms of the rate of return on investment. It is also unique among the tools discussed in that it does not require the specification of a discount rate. However, it does require an estimation of the minimum acceptable rate of return against which the calculated rate of return is compared to determine the desirability of an investment. And, estimating the minimum acceptable rate of return is similar to estimating the discount rate. This tool, like the SIR, has the advantage of indicating the relative economic efficiencies of alternative investments and can be used to rank competing projects in descending order of their IRR's.

The IRR has several disadvantages. It is subject to misinterpretation in sizing projects because as an investment is expanded, the rate of overall return may fall while the rate on additional investment may continue to exceed the investor's minimum attractive rate of return. It may prove cumbersome to calculate and can, under certain conditions, result in indeterminant or multiple solutions.

● HOW soon will energy investments be paid off by savings?

Because it provides a measure of the time period necessary to recover funds, the DPB tool is recommended when a fast turnaround on investment funds is required. This is often a critical factor to speculative investors. DPB is also recommended when the principal assets have highly uncertain life expectancies and the economic viability of the investment hinges on a minimum life.

The feature of rapid recovery of funds, however, may be overemphasized with the result that less efficient, short-term ventures are favored over more efficient, long-term investments. The principal disadvantage of DPB is that, even based on discounted benefits and costs, it does not provide a full measure of an investment's profitability because it does not include benefits and costs that occur after the payback date is reached. This problem can be averted by supplementing DPB with one of the four comprehensive life-cycle evaluation tools.



DISCOUNTING

The Method

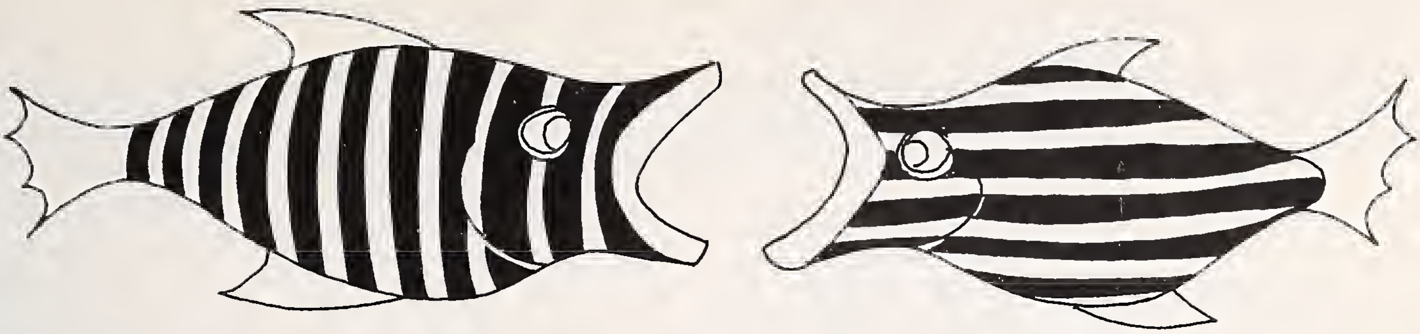
Energy conservation costs often consist primarily of first costs, i.e., costs incurred during the time of initial design and construction or retrofitting. The benefits, on the other hand, typically accrue over the life span of the project in the form of yearly energy savings. To compare benefits and costs that accrue at different points in time, it is necessary to put all cash flows on a time equivalent basis. The method for accomplishing this is called discounting.

The value of money is time dependent for two reasons: first, inflation erodes the buying power of the dollar, and second, money can be invested over time to yield a return over and above inflation. While it is possible that deflation might also occur, inflation is stressed because it expresses the more common condition in recent times. For these reasons, a given dollar amount today will be worth more than that same dollar amount in a year's time.

For example, if there were no inflation and investors could at best earn 10% interest per annum in a risk free savings account, they would find a given dollar amount this year equivalent in value to that amount plus 10% a year hence. They could be expected to be indifferent between $100 now and $110 a year from now unless they had better investment opportunities available. A 10% rate of interest would indicate the investor's time preference for money. If there were additionally a 5% rate of inflation, investors would require $115 a year from now in order to be indifferent between that future amount and $100 today. The higher the time preference, the stronger the desire for money now rather than in the future and the higher the rate of interest required to increase future cash flows sufficiently to make them equal to a given value today. The rate of interest at which an investor feels adequately compensated for trading money now for money in the future is the appropriate rate to use for converting present sums to future equivalent sums and future sums to present equivalent sums, i.e., the rate for discounting cash flows for that particular investor. This rate is often called the discount rate.

For a simplified treatment of the basic procedures of cash flow analysis, all costs incurred during the planning, design, and construction phases are here termed "first costs" and treated as though they are all incurred at the beginning of the investor's time horizon. In practice, of course, the budgeting, time scheduling, and financing of costs during planning, design and construction can be extremely complex and crucial to a project's success.[5]

To evaluate the economic efficiency of an energy conservation investment correctly, it is necessary to estimate the values of the various expenditures and savings that accrue over time and convert them all to values in a common base year. Usually, all past and future values are converted to equivalent present values, or all past, present, and future values are converted to equivalent annual values.



Sample Discounting Problem

The remainder of this section illustrates how to derive an investment's total life-cycle costs by putting all cash flows on a time equivalent basis, i.e., by discounting. The discounting procedure is illustrated in a sample problem of purchasing, installing, maintaining, and operating a heat pump. This type of cost analysis would be required, for example, if the life-cycle costs of a heat pump were to be compared to those of an alternative heating/cooling system to determine the most cost-effective system.

The life-cycle costs in the sample problem are shown for the heat pump alone, and not for alternative heating/cooling systems. Although the example is intended only to clarify the discounting process, and not to compare alternative heating/cooling systems, realistic estimates are used for the heat pump costs.

The life-cycle cost calculations are shown alternatively for two reference times. The first is the present, called "present value." The second is a yearly time scale, called "annual value," whereby all costs are expressed as though they occur in uniform yearly amounts over the study period. These two reference points are the most common in economic evaluations of investments.

The future is a third reference point in time that can be used in discounting. Appendices B and C, respectively, show discount formulas and factors for discounting cash flows to present, annual, and future values.

The life-cycle costs of an investment of more than 1 year's duration will necessarily be lower in absolute dollars when expressed as an *annual* value than when expressed as a present value; nevertheless, they are equivalent values in time and will both give the same relative ranking of conservation investment priorities. This equivalence is demonstrated in the following heat pump illustration.

Problem Assumptions: A residential heat pump, not including the duct system, costs $1,500 to purchase and install. The heat pump has a useful life of 15 years and incurs annual maintenance costs of $50 a year over its useful life. A compressor replacement is required in the eighth year at a cost of $400.

The yearly electricity cost for using the heat pump is $425 based on the price of electricity at the beginning of the investor's time horizon. Electricity prices are projected to escalate at a rate of 7% compounded annually.

The discount rate is 10% including inflation. No salvage value is expected from the heat pump at the end of 15 years.

It should be noted that to focus on the discounting procedures, we accept these assumptions as given. In practice, there may be uncertainty as to what assumptions reasonably describe an investment. The problem of uncertainty is discussed later in this handbook.

Problem Solution: The total costs of the heat pump system include costs of purchase, installation, maintenance, replacement, and electricity for operation. Using the present as the base time reference point, we must convert the above listed costs to their present values before adding them. If we assume that the purchase and installation cost is incurred at the base reference point, the present, the $1,500 is already in present value terms.

Figure 3 diagrams the conversion of the other cash flows to present values. The first task is to convert the stream of yearly maintenance costs to its present value. The maintenance costs, as shown in the cash flow diagram of figure 3, are $50 per year, measured in dollars of the years in which they occur.+

We follow the practice here of compounding interest at the end of each year, and the costs and benefits in the future are always considered to occur at the end of the year in which they occur. Alternatively, cash flows can be assumed to occur at the beginning or the middle of each year, as well as continuously throughout the year. The present refers to the beginning of year one.

The discounting operation for calculating the present value of maintenance costs is to multiply the yearly maintenance costs times the uniform present worth (UPW) factor. The UPW is a multiplicative factor taken from the Table of Discount Factors in appendix C, table C-6.++

Discount factors allow one to calculate present values for both uniformly recurring values and one-time future values, as well as recurring values that increase over time. In the case of maintenance costs, the costs are uniform and recurring and, therefore, the UPW factor is the appropriate discount factor to use.

+For purpose of this illustration, maintenance costs are assumed to remain the same in current dollars as might be the case if they were fixed by a long-term contractual agreement.

++Discount formulas, described in appendix B, can also be used to obtain present or annual values. Multiplicative discount factors, described in appendix C, are easier to use, however, and we therefore emphasize their use in this paper.

Table C-6 is searched for the column that provides the discount factors for a 10% discount rate. We find a factor of 7.606 for 15 years (N=15) in the 10% column. Multiplying this factor times $50 gives a present value of maintenance equal to $380. Note that the $380 present value of maintenance costs is much less than the sum of $50 for 15 years (i.e., $750). This illustrates the importance of discounting to compensate for time differences of cash flows.

The second task is to convert the one-time future cost of the compressor replacement, $400, to its present value. The operation for calculating the present value of compressor replacement is to multiply the future value of the compressor replacement times the single present worth factor (SPW). Finding the SPW table (C-2) in appendix C, we move down the first column to N=8 and across to the column with 10% discount factors to find a value of .467. Multiplying this factor times $400 gives a present value cost of the compressor replacement of $187, as shown in figure 3. Again note that discounting makes a significant difference in the measure of costs. Failing to discount the $400 would result in an overestimate of cost of $213 ($400–$187).

The third task is to convert the yearly electricity cost of heating and cooling to its present value. The electricity costs for a year, evaluated at the base year or reference point, is shown in figure 3 to be $425. A price escalation rate for electricity of 7% per annum is assumed. This is reflected yearly in the cash flow diagram in terms above the line. For example, $425 $(1.07)^5$ is the predicted cost of energy in the fifth year after energy inflation has been

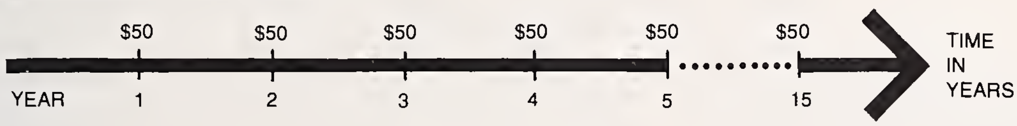
Figure 3

DETERMINATION OF PRESENT VALUE, LIFE-CYCLE COSTS OF A HEAT PUMP FOR HEATING AND COOLING

TASK DESCRIPTION [a]	CASH FLOW DIAGRAM	DISCOUNTING OPERATION [b]

FIND P, GIVEN A

Find the present value (P_m) of the $50 annual maintenance costs (A_m) over 15 years.



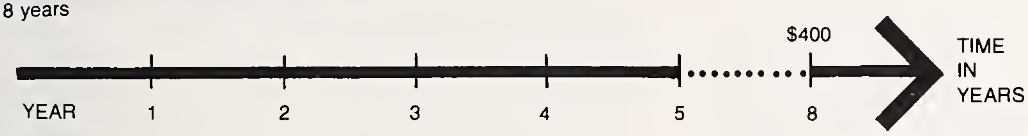
$P_m = (A_m)$ (UPW)

$P_m =$ ($50) (UPW, 10%, 15 yrs.)

$P_m =$ ($50) (7.606) = $380.

FIND P, GIVEN F

Find the present value (P_c) of the $400 future cost of replacing compressor (F_c) at the end of 8 years



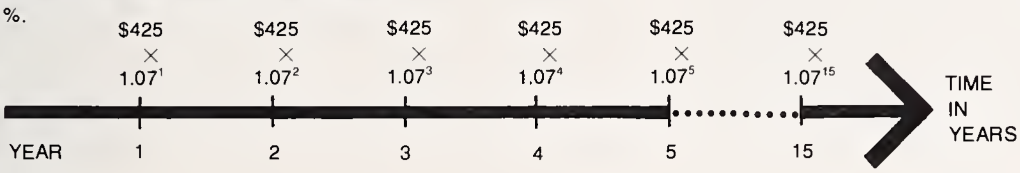
$P_c = (F_c)$ (SPW)

$P_c = (F_c)$ (SPW, 10%, 8 yrs.)

$P_c =$ ($400) (.467) = $187.

FIND P, GIVEN A WITH ESCALATION

Find the present value (P_e) of the annual electricity costs (A_e) over 15 years, beginning with an annual cost of $425, and with electricity cost escalation of 7%.



$P_e = (A_e)$ (UPW*)

$P_e =$ ($425) (UPW*, 10%, 15 yrs., 7% escalation)

$P_e =$ ($425) (12.109) = $5,146.

FIND THE TOTAL PRESENT VALUE OF THE HEAT PUMP

PRESENT VALUE = Installation Cost + P_m + P_c + P_e

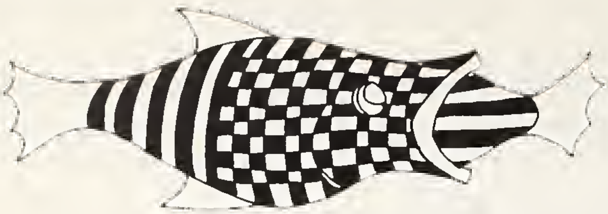
PRESENT VALUE = $1,500 + $380 + $187 + $5,146

PRESENT VALUE = **$7,213**

a P = present value
A = annual value
F = future value

b UPW = uniform present worth factor
SPW = present worth factor
UPW* = uniform present worth factor with energy escalation

Subscript
m = maintenance costs
c = compressor cost
e = electricity costs



added. Appendix D, table D-2, provides a table of modified uniform present value factors (UPW*) for a discount rate of 10% and energy escalation rates from 1% to 10%. The modified uniform present value discount formula that includes energy price escalation is given in appendix B. The asterisk following UPW (i.e., UPW*) means that a term for energy price escalation is included.

The discounting operation (shown in fig. 3) for finding the present value of electricity costs is to multiply the base year electricity cost times the appropriate UPW* factor in appendix D. Locating year 15 in the first column of appendix D-2 and looking across the table for the UPW* factor under the escalation rate of 7%, the value 12.109 is found. Multiplying this factor by $425 gives a present value of electricity costs of $5,146. Note once again that failing to discount would overestimate costs by $1,229 ($6,375–$5,146). Discounting with a UPW factor that does not incorporate energy price escalation would underestimate costs by $1,913 ($5,146–$3,233).

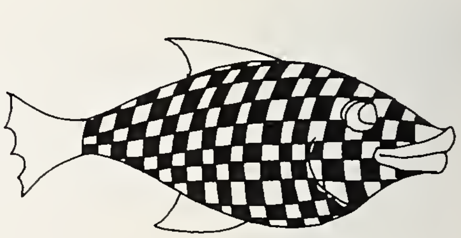
The final operation described in figure 3 is to sum the purchase and installation costs and the present values of maintenance, compressor replacement, and electricity costs. Total life-cycle costs of the heat pump in present value terms is found to be $7,213. This is the cost figure that a designer would need to compare the cost effectiveness of this heat pump with alternative heating/cooling systems.

Figure 3 provides a model for the designer who must calculate present values from a number of benefit and cost streams. It can be seen that many different kinds of benefits and costs occurring in future years can be handled either with the SPW, the UPW factor, or the UPW* factor with price escalation.[6]

Only one discounting operation is required for converting the present value costs of the heat pump to annual value terms. The total present value amount is multiplied by the uniform capital recovery factor (UCR) for 10% and 15 years. The UCR factor, found in table C-4, is .131. Multiplying this factor by the total present value of $7,213 gives the cost of the heat pump as $945 in annual value terms.

Note that annual values are not the same as average yearly values. For example the installation cost of $1,500, divided by 15 years, or $100 per year, will not be the annual value of installation costs. Because average yearly values do not include discounting, they give erroneous estimates of benefits and costs. The two figures, $945 in annual value terms over 15 years and $7,213 in present value terms, are time equivalent values, made consistent through the application of discounting.

Using the discounting procedures described above, together with appendices B, C, and D, the building decision maker can formulate and solve many conservation investment problems.



5 ECONOMIC TOOLS

A PROBLEM IN FIVE SOLUTIONS

To illustrate the five tools described earlier, a solar energy system with a conventional backup system is evaluated for cost effectiveness against a conventional energy system used alone.

Problem Assumptions

For the purpose of this illustration all costs are evaluated with a discount rate of 10%, a fuel price escalation rate of 5%, and a time horizon of 20 years.

The combined solar/conventional energy system is assumed to cost $20,000 which is $8,000 more than the conventional system alone.[7]

Major components of the solar/conventional system are anticipated to last 20 years and be without value after that time.

The conventional system if used alone is estimated to require a major replacement at the end of the 15th year, calculated to cost $6,000, and to have a salvage value of $4,000 at the end of 20 years, the end of the time horizon. Maintenance costs for purposes of this illustration are assumed to be no higher for the combined solar/conventional system than for the conventional system alone.

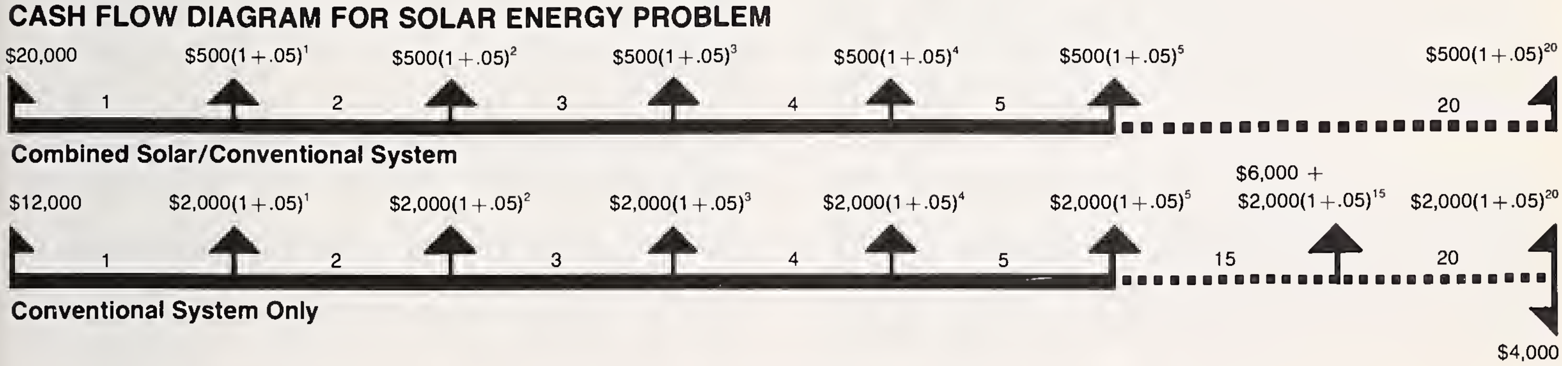
The combined system is assumed to result in substantial fuel savings, reducing base year energy costs from $2,000 to $500. A fuel price rise of 5% per year and a discount rate of 10% are assumed.

Figure 4 shows two cash flow diagrams for this problem, the upper part for the combined system, the lower part for the conventional system. Upward pointing arrows indicate expenditures; downward pointing arrows, receipts.

The values for both systems are purely hypothetical and for the purpose of illustration only.

Figure 4

CASH FLOW DIAGRAM FOR SOLAR ENERGY PROBLEM



A

Table A: Life-Cycle Costing

		TYPE OF ENERGY SYSTEM	COMBINED SOLAR/ CONVENTIONAL	CONVENTIONAL ONLY
1		PERIOD OF ANALYSIS	20 Yrs.	20 Yrs.
2	P	INITIAL INVESTMENT COSTS	$20,000	$12,000
3	(S)[a]	SALVAGE	0	$4,000 20th Yr.
4	S	PRESENT VALUE OF SALVAGE	0	$596
5	(M)	MAINTENANCE + REPAIR COSTS	0	0
6	M	PRESENT VALUE OF MAINTENANCE + REPAIR	0	0
7	(R)	REPLACEMENT COSTS	0	$6,000 15th Yr.
8	R	PRESENT VALUE OF REPLACEMENT COSTS	0	$1,434
9	(E)	BASE-YEAR ENERGY COSTS	$500	$2,000
10	E	PRESENT VALUE OF ENERGY COSTS	$6,359	$25,436
11	LCC	PRESENT VALUE OF TOTAL COSTS	$26,359	$38,274

LCC = P - S + M + R + E

For COMBINED SOLAR/CONVENTIONAL LCC = 20,000 - 0 + 0 + 0 + 6,359 = $26,359

For CONVENTIONAL ONLY LCC = 12,000 - 596 + 0 + 1,434 + 25,436 = $38,274

[a]Present values are used in the LCC formula. Symbols for numbers expressed in other terms are enclosed in parentheses to indicate they cannot be inserted directly into the formula.

LCC Problem Solution: The relevant information from the problem assumptions are entered in table A in lines 1, 2, 3, 5, 7, and 9. Lines 4, 8, and 10 are filled in after the following calculations:

Line 4 lists the present value of the salvage value of the conventional system as $596. This figure was obtained by multiplying the $4,000 of estimated salvage value in the 20th year times the SPW factor in appendix C-2 for a 10% discount rate, i.e., [($4,000)(.149)=$596].

Line 8 gives the present value of replacement costs as $1,434 for the conventional system. This figure was obtained by multiplying the $6,000 in replacement costs times the single present worth factor (SPW) in appendix C-2 for 10% and the 15th year, i.e., [($6,000)(.239)=$1,434].

Line 10 gives the present value of fuel costs for the combined system calculated from the base-year cost of $500. With an expected fuel price rise of 5% per year and a discount rate of 10%, the present value of energy costs for the combined system over a 20-year time horizon amounts to $6,359. This figure was derived using the UPW* factor from appendix D, table D-2, for 20 years at 5% price escalation and a 10% discount rate, i.e., [($500)(12.718)=$6,359].

The present value of fuel costs for the conventional system used alone, also given in line 10, is $25,436. This figure is calculated by multiplying the base-year fuel cost of $2,000 times the same UPW* factor for 20 years and 5% price escalation that was used to calculate present value fuel costs for the combined system, i.e., [($2,000)(12.718)=$25,436].

Line 11 gives the total life-cycle cost for the solar/conventional system as $26,359. This is the sum of lines 2, 6, 8, and 10 and the subtraction of line 4. The total life-cycle cost for the conventional system is $38,274, obtained in the same way.

A comparison of the total costs of the two systems over the 20-year time horizon indicates that the cost of the combined solar/conventional system is $11,915 less than the costs of the conventional system alone and is therefore the preferred investment.

B

Table B: Net Benefits (Savings)

1		PERIOD OF ANALYSIS	20 Yrs.
2	$\bar{E}$	BASE-YEAR SOLAR ENERGY SAVINGS	$1,500
3	E*	PRESENT VALUE OF ENERGY SAVINGS	$19,077
4	P*	DIFFERENTIAL SOLAR INVESTMENT COSTS	$8,000
5	S*	PRESENT VALUE OF DIFFERENTIAL SALVAGE VALUE	-$596
6	M*	PRESENT VALUE OF DIFFERENTIAL MAINTENANCE COSTS	0
7	R*	PRESENT VALUE OF DIFFERENTIAL REPLACEMENT COSTS	-$1,434
8	B-C*	NET PRESENT VALUE SAVINGS	$11,915

$(B\text{-}C^*) = E^* - (P^* - S^* + M^* + R^*) =$

$\$19{,}077 - [\$8{,}000 - (-\$596) + 0 + (-\$1{,}434)] = \$11{,}915$

*(Indicates the differences between the present value costs for the combined solar/conventional system and for the conventional system alone)

$\bar{E}$ (Indicates the differences between costs prior to discounting)

B-C Problem Solution: The above net-benefits solution is based on the same costs and assumptions used in the preceding example. However, most of the values in table B differ from those in table A because they are shown for differences in the two heating systems. Using differences requires applying the formula only once for comparing the economic efficiency of the two heating systems.

The combined solar/conventional system is assumed to cost $8,000 more than the conventional system (Line 4), to save $1,500 in energy costs in the base year (Line 2), and to last 20 years without a major replacement cost.

Line 3 gives the present value of energy savings over the 20-year period of analysis as $19,077. This figure was derived using the UPW* factor for 20 years at 10% discount and a yearly 5% escalation rate, i.e., [(12.718)($1,500)=$19,077].

Line 7 shows that the combined system requires $1,434 less in replacement costs in present value terms than the conventional system used alone.

Line 5 shows that the combined system results in less salvage value than the conventional system alone due to the replacement for the conventional system late in the study period which has remaining life at the end of the study period. The net effect of the lower replacement cost and lower salvage value is a cost advantage for the combined solar/conventional system.

The total net savings are found by subtracting the differential costs of the system from the savings. Table B shows the present value of net savings to be $11,915, the same results obtained by finding the difference in the present value of total life-cycle costs of the two alternatives in line 11 of table A.

C

Table C: Savings-to-Investment Ratio

1	E	BASE-YEAR SOLAR ENERGY SAVINGS	$1,500
2	E*	PRESENT VALUE OF ENERGY SAVINGS	$19,077
3	P*	DIFFERENTIAL SOLAR INVESTMENT COSTS	$8,000
4	S*	PRESENT VALUE OF DIFFERENTIAL SALVAGE VALUE	-$596
5	M*	PRESENT VALUE OF DIFFERENTIAL MAINTENANCE COSTS	0
6	R*	PRESENT VALUE OF DIFFERENTIAL REPLACEMENT COSTS	-$1,434
7	E* - M*	SIR NUMERATOR	$19,077
8	P* - S* + R*	SIR DENOMINATOR	$7,162
9	SIR RATIO		2.66

$$SIR = (E^* - M^*) \div (P^* - S^* + R^*) =$$

$$(\$19{,}077 - 0) \div [\$8{,}000 - (-\$596) + (-\$1{,}434)] = 2.66$$

SIR Problem: This Savings-to-Investment ratio solution uses the same hypothetical cost data presented in the preceding two illustrations.

Line 2 shows the present dollar value of energy savings over the 20-year period of analysis (this was calculated in the previous example) as $19,077. Since there are no differential maintenance costs, the numerator of the SIR is $19,077.

Lines 3, 4, and 6 combine to form the denominator of the SIR. The lower replacement costs and the lower salvage value together offset somewhat the higher solar investment costs, and the denominator of the SIR is $7,162.

The Savings-to-Investment Ratio of 2.66 indicates an average gross return per investment dollar of $2.66.

D

Table D: Internal Rate of Return

TRIAL INTEREST RATES			20%	25%
1	EI*	PRESENT VALUE OF ENERGY SAVINGS	$9,773	$7,634
2	P*	DIFFERENTIAL SOLAR INVESTMENT COSTS	$8,000	$8,000
3	SI*	PRESENT VALUE OF DIFFERENTIAL SALVAGE VALUE	-$104	-$48
4	MI*	PRESENT VALUE OF DIFFERENTIAL MAINTENANCE COSTS	0	0
5	RI*	PRESENT VALUE OF DIFFERENTIAL REPLACEMENT COSTS	-$390	-$210
6	BI-CI	NET PRESENT VALUE BASED ON THE TRIAL INTEREST RATES (1) - [(2) - (3) + (4) + (5)]	$2,059	-$204

$(\bar{E} \cdot a) - [\bar{P} - (\bar{S} \cdot b) + (\bar{M} \cdot c) + (\bar{R} \cdot b)] = 0$

For i = 20%

($1,500) (6.515) - [($8,000) - (-$4,000) (.026) + (0) + (-$6,000) (.065)] =

($9,773) - [($8,000) - (-$104) + 0 + (-$390)] = $2,059

For i = 25%

($1,500) (5.089) - [($8,000) - (-$4,000) (.012) + (0) + (-$6,000) (.035)] =

($7,634) - [($8,000) - (-$48) + 0 + (-$210)] = -$204

Therefore, 20% <IRR< 25% By interpolation, $\text{IRR} = .20 + \left[.05\left(\frac{\$2{,}059}{\$2{,}059 + \$204}\right)100\right] = 24.5\%$

(Note: "$\bar{E}$," "$\bar{P}$," "$\bar{S}$," "$\bar{M}$," and "$\bar{R}$," are symbols indicating the differences between the energy conserving investment and its alternative that have not yet been converted to present values. "Ei*," "P*," "Si*," "Mi*," and "Ri*," are symbols indicating the present values of the amounts based on a discount rate i, where i is first assumed equal to 20% and then to 25%.)

IRR Problem Solution: The above Internal Rate of Return solution also uses the hypothetical cost data employed in the previous three examples.

Visual inspection is used to identify trial interest rates that might balance benefits and costs in terms of present value dollars. The columns indicate the selected trial interest rates of 20 and 25%.

Line 1 shows the present value of energy savings calculated at 20 and 25%. The amount of $9,773 for the present value of energy savings is found by multiplying the UPW* factor for 20 years at 20% by the base-year energy savings of $1,500. The amount of $7,634 is found by multiplying the UPW* factor for 20 years at 25% by the base year energy savings.

Line 2 lists the solar/conventional differential investment cost. This cost which occurs initially is already in present value dollars.

Line 3 gives the present value of differential salvage. Recall that salvage is assumed to be less for the combined solar/conventional system than for the conventional system alone; therefore, the value is negative. The present value amount of -$104 is found by multiplying the SPW factor for 20 years and 20% by the estimated $4,000 in salvage foregone. The present value amount of -$48 is found by multiplying the SPW factor for 20 years at 25% by the amount of salvage foregone.

Line 4 shows that no difference in maintenance costs is assumed for the two alternatives.

Line 5 gives the present value of differential replacement costs as a negative amount because the combined solar/conventional system is assumed to require less replacement than the conventional system alone. The present value amount of -$390 is found by multiplying the SPW factor for 15 years and 20% by the estimated $6,000 replacement costs saved in the 15th year. The present value amount of -$210 is found in a similar way by using the SPW factor for 15 years and 25%.

Line 6 gives the net present value of the combined components of costs evaluated first at 20% and then at 25%. It indicates a saving of $2,059 for the 20% trial interest rate and a loss of $204 for the 25% trial interest rate. Thus the internal rate of return on the investment lies between 20% and 25%. By interpolation it can be found that 24.5% is the internal rate of return on investment.

E

Table E: Discounted Payback

Y	$\sum_{j=1}^{Y} E^*_j$	$\sum_{j=1}^{Y} M^*_j$	$\sum_{j=1}^{Y} R^*_j$	$\sum_{j=1}^{Y} S^*_j$	P^*	$\sum_{j=1}^{Y}$ B-C
YEARS INTO THE INVESTMENT	PRESENT VALUE OF CUMULATIVE ENERGY SAVINGS	PRESENT VALUE OF CUMULATIVE DIFFERENTIAL MAINTENANCE COSTS	PRESENT VALUE OF CUMULATIVE DIFFERENTIAL REPLACEMENT COSTS	PRESENT VALUE OF DIFFERENTIAL SALVAGE VALUE	DIFFERENTIAL SOLAR INVESTMENT COSTS	PRESENT VALUE OF NET SAVINGS
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	$1,433	0	0	0	$8,000	-$6,567
2	2,801	0	0	0		-5,199
3	4,103	0	0	0		-3,897
4	5,349	0	0	0		-2,651
5	6,537	0	0	0		-1,463
6	7,673	0	0	0		-327
7	8,756	0	0	0		+756

Find Y such that $\sum_{j=1}^{Y} (E^*_j - M^*_j - R^*_j + S^*_j) = P^*$

For $Y = 6$, $(\$7,673 - 0 - 0 + 0) < \$8,000$

For $Y = 7$, $(\$8,756 - 0 - 0 + 0) > \$8,000$

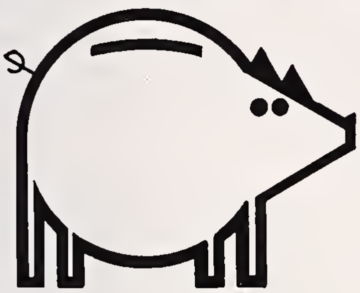
Therefore $6 < Y < 7$

DPB Problem Solution: The above discounted payback solution is calculated for the same hypothetical investment problem used to illustrate the four previous economic tools.

Cumulative discounted savings and recurring costs are compared each year with the initial cost until the present value of net savings becomes positive. Given energy price escalation at a rate of 5% and discounting at 10%, we find that energy savings offset investment costs in the seventh year.

Column 2 lists the present value of cumulative energy savings. Figures in Column 2 are found by multiplying the base-year energy savings of $1,500 by the UPW* factor from table D-2 for a 10% discount rate, a 5% escalation rate, and the stated number of years into the investment as indicated by Column 1. For the first year, $0.955 \times \$1,500 = \$1,433$, and for the seventh year $5.837 \times \$1,500 = \$8,756$. The seventh year is the first year the cash flow becomes positive. This indicates that the time of payback of the initial investment of $8,000 occurs in the seventh year, before year end.

FACTORS AFFECTING BENEFITS AND COSTS



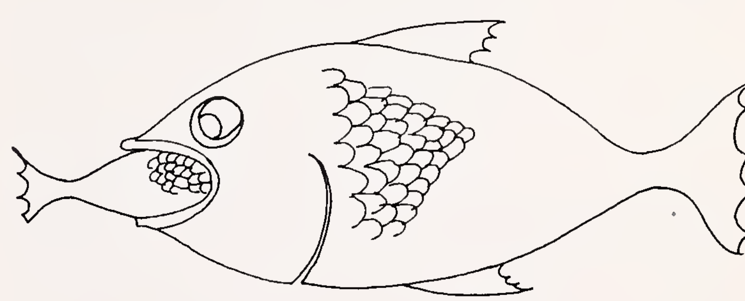
To this point we have discussed economic efficiency, described discounting, and demonstrated the application of five economic tools as well as the advantages and disadvantages of their use. The purpose of this section is to clarify elements that affect the use of these tools to measure benefits and costs.

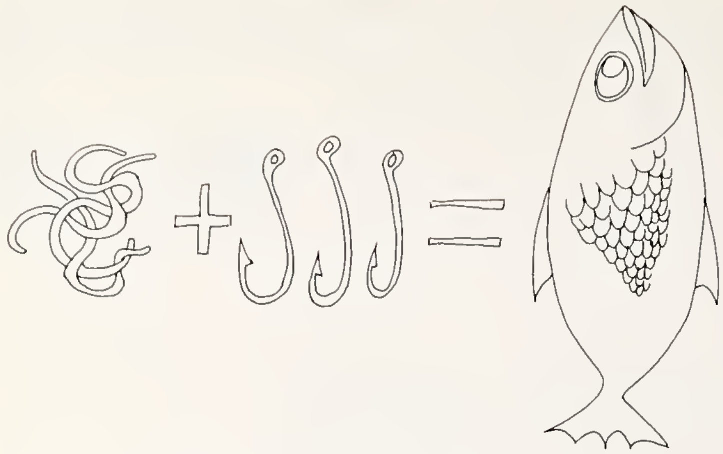
The elements to be discussed are discount rates, incentives, inflation, salvage values, taxes, time horizons and uncertainty. They are arranged in alphabetical order. Their relative importance depends on the characteristics of a particular investment.

Discount Rate

The discount rate is a rate of interest used to convert benefits and costs occurring at different times to a common time. The discount rate is selected to reflect the investor's time preference for money. Once a discount rate has been chosen it can be used either in discount formulas or to select discount factors to carry out various discounting procedures.

The selection of a discount rate may be guided by the level of return on alternative investment opportunities, on the cost of borrowing money, or, in the case of public organizations, on legislative or mandated requirements. The United States Office of Management and Budget (OMB) currently requires Federal agencies to use a discount rate of 10% in excess of the rate of inflation for evaluating most government investments.[8]





If an investor is unsure of the potential return on an alternative investment, the cost of borrowing can be used as the discount rate. However, the earning rate available on alternative investments should take precedence over the borrowing rate as an indicator of the appropriate discount rate and should be taken into account regardless of whether the money is borrowed or not. In selecting the appropriate discount rate, the client might be asked for the after-tax rate of return on other investments, in the case of a homeowner, for example, the rate of interest received on savings accounts. These may vary considerably by firm and industry. If the evaluation is being made for a government client, it is important to determine if there are legislative or mandated requirements for the discount rate.

Clients may request that high risk projects be evaluated with higher discount rates than those with low risk.[9] Risk can also be treated in other ways, such as basing benefit and cost estimates on probabilities of occurrence, incorporating contingency estimates of cash flow into the calculations, or by employing sensitivity analysis to assess the impact of different time horizons or of different amounts of energy savings on the profitability of the investment. (Probability and sensitivity analyses are discussed in more detail later in this section.)

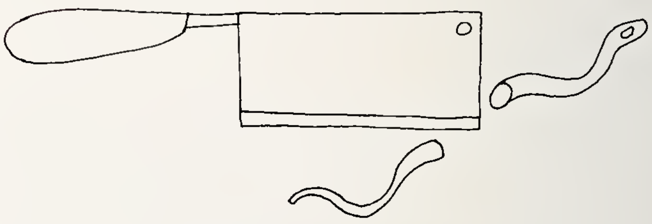
Discount rates may be expressed in either "nominal" or "real" terms. Nominal rates include the effects of inflation and the real earning power of money invested over time. "Market rates" are nominal rates because they reflect both inflation and real earning power. A real rate reflects only the real earning power of money, and therefore is lower than a nominal rate, given the same conditions.

Ensuring that dollar estimates of benefits and costs and discount values are compatible with one another by either including inflation or excluding inflation from all values is a basic principle to be followed in discounting. A real discount rate is appropriate if inflation is removed from the cash flows prior to discounting. A nominal rate is appropriate if cash flows are inflated.

For example, the fuel escalation rate must be consistent with the discount rate in terms of either including or excluding inflation. If a nominal rate of discount is used, then the projected rate of total change in energy prices must be used. This was the approach used in the problem in the earlier section on discounting. On the other hand, if a real rate of discount is used, the differential rate of energy price change—that is, the projected escalation rate of energy prices minus the average escalation rate of prices in general—is appropriate.

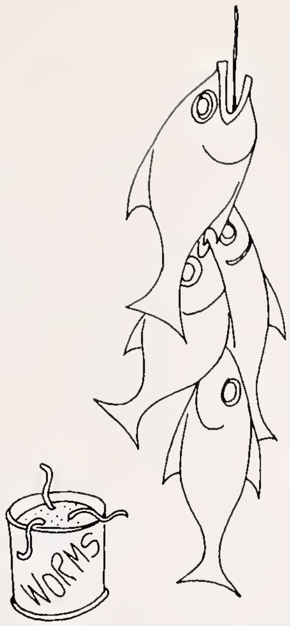
The discount rate is one of the most dramatic factors affecting the net benefits of conservation investments. The rate selected may make a project seem either economic or uneconomic. For example, a project which has positive net savings when evaluated using a 6% discount rate might yield negative net savings when evaluated at 7%.

As the discount rate is increased, the present value of any given future stream of costs or benefits becomes smaller. High discount rates tend to favor projects with quick payoffs over projects with deferred benefits.



COST ADJUSTMENT

INTERNAL RATE OF RETURN



Incentives

An incentive is a positive inducement to encourage a particular type of behavior or action. Incentives are considered in economic evaluations of conservation and solar energy investments because they can affect the economic viability of an investment and its optimal size. Their cash values should be discounted to present values just as are other cost savings. Following are examples of incentives for energy conservation and solar energy provided by the government.

Grants: Cash subsidies in specified amounts are sometimes made to purchasers of energy conservation equipment. The National State/Federal Combined Program for Providing Subsidies to Residential Users of Solar Hot Water Heaters is an example. The cost of a solar hot water heater to the recipient of a grant is the life-cycle cost of the system minus the government grant.

Taxes: Taxes may be used as a means of providing several types of incentives. Income tax credits for conservation and solar expenditures provide a subsidy by allowing specific deductions from the investor's taxes owed. Property tax exemptions for energy conservation and solar energy capital equipment eliminate or reduce property taxes that would otherwise add to annual costs. Liberal depreciation allowances for energy conservation and solar energy investments reduce taxable income. The imposition of higher taxes on nonrenewable energy sources raises their prices and encourages investments in conservation and renewable energy. The elimination of tax deductibility for business fuel expenses would further encourage energy conservation and solar energy investments in commercial buildings.

Government Cost Sharing: Another form of governmental incentive is provided when one unit of government bears a specified percentage of the costs of an investment made either by another unit of government or by an investor in the private sector. The National Energy Act, for example, provides Federal cost sharing for energy conservation in schools, hospitals, local government, and public care buildings.

Loan Interest Subsidies and Guarantees: Loan subsidies provide for loans at rates below the market rate and reduce borrowing costs to make energy conservation and solar energy more economical. Loan guarantees may induce lending institutions to provide loans on more liberal terms or to make loans when they otherwise would not. Conservation designs that are uneconomic without subsidies may in fact be cost effective if subsidies are included in the economic evaluation. Federal and State energy offices and associations such as the National Conference of State Legislatures are potential sources of information on available subsidies for energy conservation.

Inflation

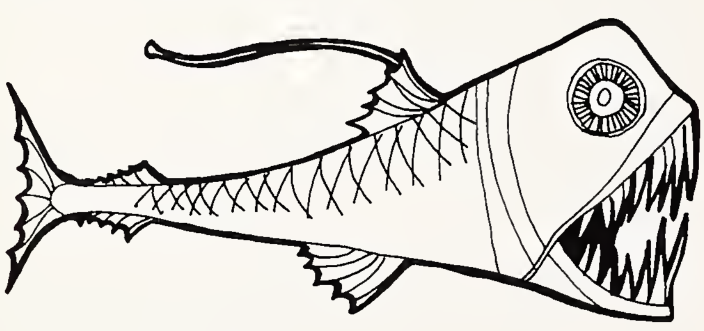
Inflation is a rise in the general price level. Although all prices cannot be expected to rise or fall together and in the same amount, average price increases in specific and general categories of goods and services can be measured.[10] In making economic evaluations of energy conservation and solar energy investments, it is important that price inflation as indicated by average price increases in the economy be eliminated from estimates of benefits and costs.

As was shown in figure 1, fuel prices have increased dramatically in recent years—so rapidly that they should be given special attention in evaluating investments to conserve nonrenewable energy. Since benefits from conservation vary directly with fuel prices, assumptions regarding the change in fuel prices over time have a major impact on the predicted benefits of a conservation project. Projected energy prices are usually based on contractually stated prices, extrapolated trends from historical prices, or government/industrial predictions of future prices. The Department of Energy's price projections, for example, provide estimates of prices of energy for residential and commercial users in 1978 constant dollars for the period of 1977 to 1990.[11] These government price projections are updated periodically.

Other prices affecting the benefits and costs of conservation investment over time are those related to operation, maintenance, and replacement. The possibility of inflation affecting these prices, as well as energy prices, should be considered.

There are several basic methods of handling inflation reflected in future prices. One is to eliminate it from inflated cash flows by applying a price deflator index.[12] Future prices may be expressed in constant dollars in some base year, as in the case of the Department of Energy's projections described above. The prices in constant dollars may rise, but they will reflect real price changes (i.e., increases over and above the average inflation rate for all goods and services) rather than changes due to inflation. The constant dollar prices must then be discounted with a real discount rate to arrive at present or annual values.

As indicated earlier, a second way of handling inflation is to discount cash flows that contain inflation with a nominal discount rate that reflects both real changes in value and the expected inflation rate.

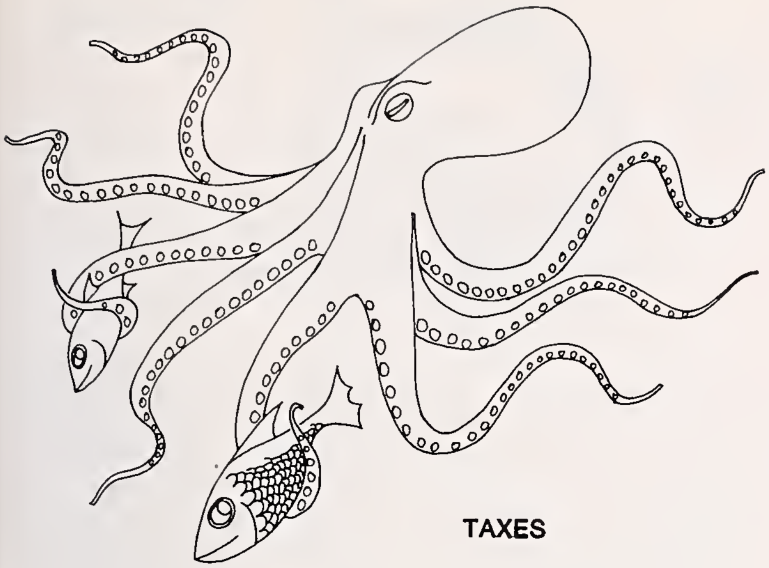


The impact of inflation on the economic viability of conservation and solar investments depends on which prices are inflated most, as well as on institutional arrangements such as taxes. For example, the higher the escalation rate of energy relative to other prices, the more economical investments to conserve nonrenewable energy will be. On the other hand, for commercial properties depreciation write-offs for tax purposes will become less significant with higher rates of general inflation because the depreciation is based on the original "book value" of the investment.

Salvage Value

For the purpose of evaluating the economic feasibility of an investment, its salvage value is defined broadly here to encompass (1) its residual value, net of the cost of disposal, whenever it is removed or replaced during the study period; (2) the value remaining at the end of the study period; or (3) the value recovered through resale at the end of the study period. If an existing investment is being compared with a new alternative, the current salvage or resale value of the existing investment is used to compare against the first cost of the new alternative. The present value of salvage value can generally be expected to decrease, other things equal, as the discount rate rises, the equipment deteriorates, and the time horizon lengthens.

A measure of salvage value is the amount that could be added to the selling price of the building due to the energy conservation or solar investment. It might be assumed that, with perfect information, a building buyer would be willing to pay an additional amount for an energy conserving or solar-equipped building equal to the capitalized value of fuel savings, net of costs, over the remaining life of the conserving investment. If the investor's time horizon is the same as the useful life of the investment, there will be no salvage value; if the time horizon is shorter, there may be salvage. Even if potential energy savings remain, there will be no salvage value to the current owner unless these savings can be expected to be reflected in the resale value of the building or unless the energy conservation devices can be removed and sold or used in another application. These possibilities are very uncertain. Thus estimating salvage value with any reliability can be very difficult.



TAXES

Taxes

Taxes may have an impact on the economic viability of conservation investments in two ways. One way is as a mechanism for providing direct financial subsidies; the other way is through regular tax laws, such as existing property tax laws, sales taxes, and income tax laws.

Property Taxes: Because many energy conservation investments are capital intensive, they tend to increase the cost of a building and raise the value of the property tax. This effect reduces the net savings from capital-intensive conservation investments.

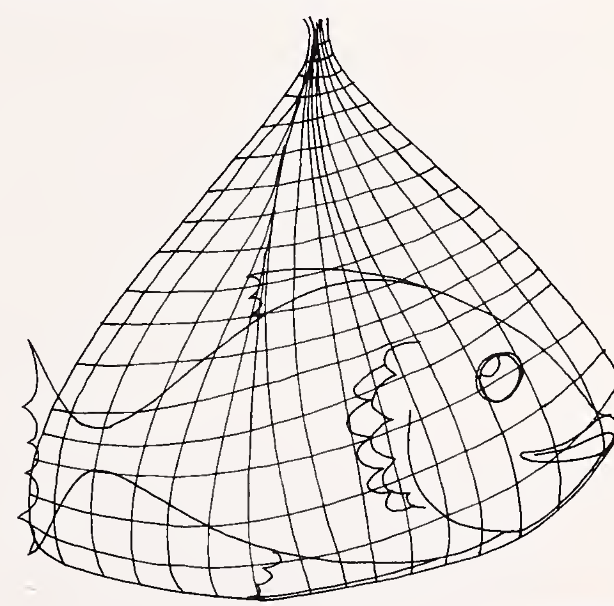
Income Taxes: The deductions from taxable income of interest on loans and depreciation on capital investments have positive effects on conservation and solar investments. The deduction from taxable income of fuel expenses as a business expense, on the other hand, has a negative effect on conservation and solar investments because it effectively reduces the cost of fuel to business. Conserving fuel may result in after-tax dollar savings for businesses that are less than the before-tax value of the fuel saved. For homeowners, dollars saved from fuel conservation are savings which are not taxed.

Time Horizon

The time horizon is simply the period of economic analysis, measured usually in years. For a conservation investment, it is the length of time over which costs and benefits from conservation are calculated. The time horizon is the "life cycle" in life-cycle cost analysis.

The selection of a time horizon or life cycle is based on a concept of investment life or on the personal time perspective of the investor. There are often no fixed time horizons for investment projects. The life of the building or the particular conservation investment may be used as the time horizon, as well as the investor's planned time of occupancy of the building.

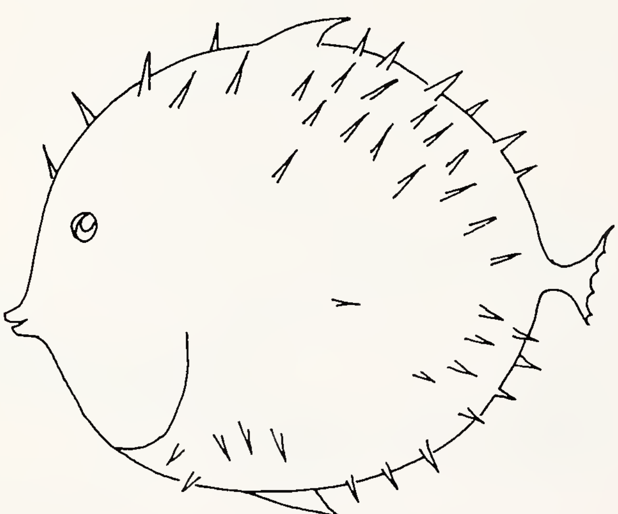
Two concepts of investment life may be considered in an evaluation. One is "useful life," the other, "economic life." Useful life is the period over which the investment has value in conserving energy. Economic life is the time period over which the investment is the least-cost means of providing a specified type of conservation. The economic life, in most instances, will be shorter than the useful life. In theory, evaluations of investments should be based on their economic life. In practice, the economic life is probably more difficult to predict than useful life, and the concept of useful life more often enters analyses.



NET BENEFITS

The personal time perspective of the building owner will depend on the objective and circumstances of the owner. A speculative builder planning for immediate sale, for example, may view the relevant time horizon as that period of ownership from the acquisition of property to the first sale of the building. Thus, although the useful life of a solar domestic hot water heating system might be 20 years, the speculative builder might project his economic time horizon for only 1 year. If the speculator does not anticipate gain through a higher selling price of the building due to the potential savings to the buyer, then the solar investment with its high first cost is unlikely to be economic in the speculator's evaluation. Such conditions might also be anticipated by the owner of a residence who expects to move before the end of the payback period for conservation investments.

If the client that requests an economic evaluation of energy conservation investments does not specify a period of analysis, the designer or analyst must select one. Federal, State, and local guidelines may provide an answer for public buildings. Mortgage lending periods for buildings, normally ranging from 20 to 30 years, may provide an index of building life. The time horizon for government buildings is usually longer than private buildings because they tend to have one owner, are built to rigid specifications, and are generally well maintained. Information from research reports and literature from manufacturers of energy conservation materials, as well as warranties and guarantees may also provide guidelines.



INFLATION

SALVAGE VALUE

As public awareness of the cost of energy and potential savings from conservation increases and as conservation becomes increasingly capitalized in the resale prices of buildings, time horizons for conservation investments in the private sector are likely to increase. From the perspective of national economic efficiency, a time horizon based on a concept of life rather than brief, speculative periods is more appropriate. As buyers become aware of the potential benefits from energy conservation design, the responsiveness of the speculative market will predictably improve and considerations of the long-term net benefits from conservation will begin to take precedence.

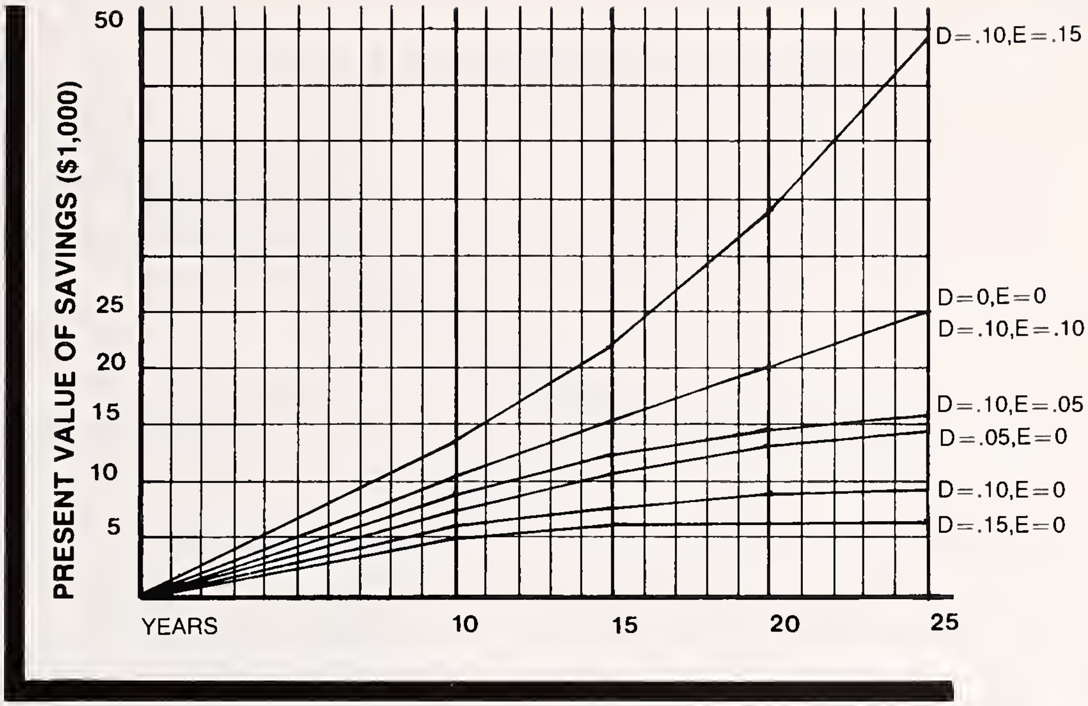
The selection of a time horizon can affect the value of the net benefits from conservation and thereby can affect investment decisions. The impact of varying time horizons depends in part upon three related factors: the discount rate, the rate of fuel price escalation, and salvage value.

Uncertainty

Evaluations of benefits and costs from energy conservation design are only as good as the values in formulating the analysis. Some of the life-cycle costs and most of the life-cycle benefits from conservation design accrue in the future. The design community will therefore experience uncertainty as to the correct values to use in predicting future benefits and costs.

Two analytical techniques that can be used to help make decisions about conservation investments whose economic payoffs are uncertain are *sensitivity analysis* and *probability analysis.*[13]

Figure 5



SENSITIVITY OF FUEL SAVINGS TO TIME HORIZONS, DISCOUNT RATES AND ENERGY ESCALATION RATES.

D = Discount Rate E = Energy Escalation

Sensitivity Analysis: This technique involves a test of the responsiveness of life-cycle costs, net benefits, or other economic measures to alternative values of key factors about which there is uncertainty. It shows decision makers how the economic viability of a conservation project changes as, for example, fuel price escalation, discount rates, time horizons and other critical factors vary.

To illustrate, figure 5 shows the sensitivity of fuel savings by a solar heating system to three critical factors: time horizons (0 to 25 years), discount rates (D equals 0, 5, 10 and 15%), and energy escalation rates (E equals 0, 5, 10, and 15%). The present value of savings in each case is based on a base-year fuel savings of $1,000.

Note that, other things equal, savings increase over time, but more slowly with higher discount rates and more quickly with higher escalation rates. The impact of fuel price escalation is most apparent when comparing the top curve of the graph (D=10, E=.15) with one close to the bottom (D=.10, E=0). The present value of savings at the end of 25 years is about $50,000 for a fuel price escalation rate of 15%, and only about $8,000 for an escalation rate of 0%, other things equal. Whereas the Btu savings and initial prices are the same, the present value of the Btu savings varies widely depending on the selection of the escalation rate of fuel prices and the discount rate.

Although impact scenarios such as these illustrated in figure 5 do not show the analyst what parametric values to choose, they do show decision makers the impact of alternative assumptions. Knowing the consequences of error may help analysts make better decisions about conservation investments with uncertain outcomes.

Table F

EXPECTED VALUE OF COMPRESSOR REPLACEMENT

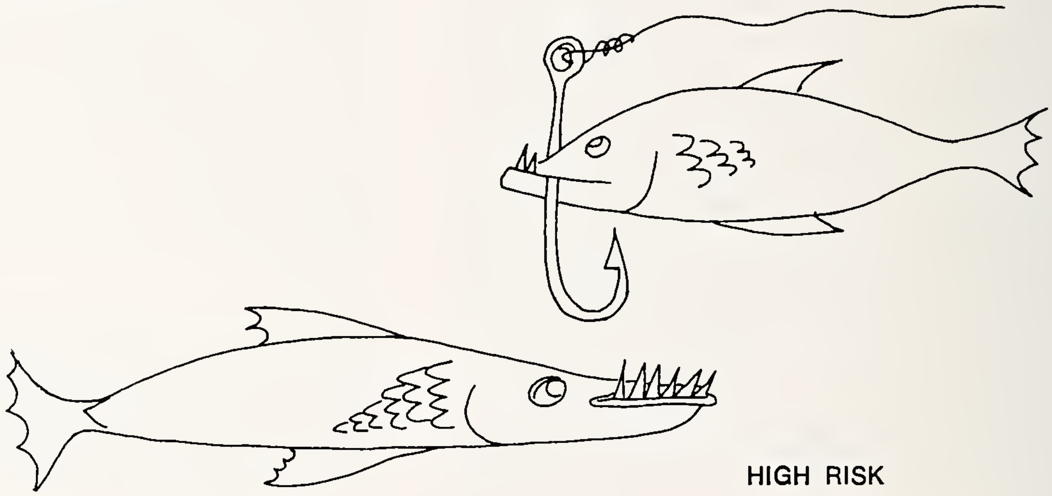
Year of Replacement	Probability	Cost ($)	SPW 10% Discount Rate	Expected Present Value Cost ($)
6	.1	400	.565	23
7	.2	400	.513	41
8	.6	400	.467	112
9	.1	400	.424	17
			Expected value of compressor replacement:	$193

Expected Value of Cost = Cost × Probability × SPW

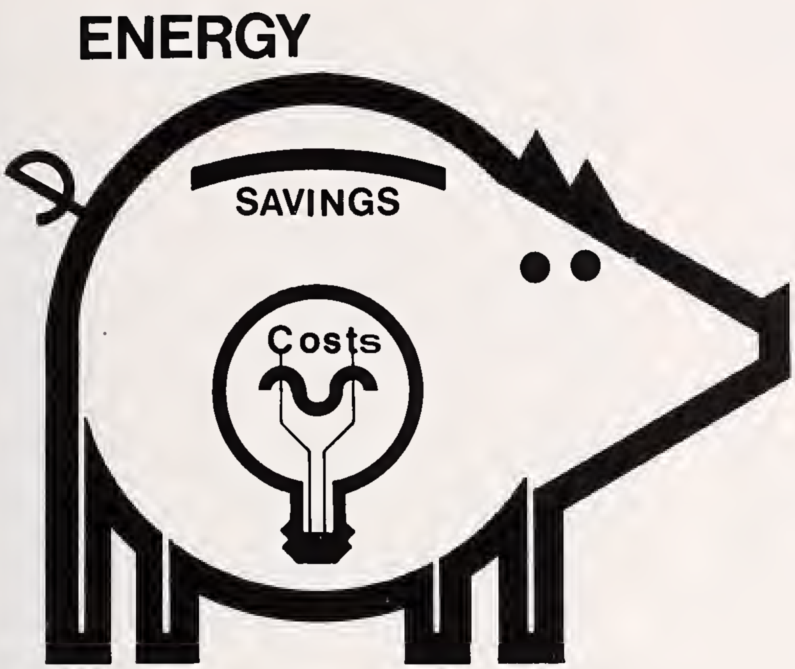
Probability Analysis: This technique is sometimes called "expected value analysis." It is used to evaluate the benefits and costs of an event whose expected chance of occurrence can be predicted.[14] Often historical data are available for generating probability data for existing technologies. But for many new conservation technologies, probability data may not be available. Computer simulation is sometimes used to provide probability data on innovative technologies.

An example of using probability analysis is shown in table F. The heat pump case described earlier in figure 3 as an illustration of discounting is used once again.

The heat pump case treated the replacement of a compressor. The expected value of the compressor replacement, as measured in present value dollars using probability analysis, is shown in table F to be $193. Note that this differs from the $187 estimated in figure 3. While it is unlikely that the exact cost of replacing the compressor will be predicted using a probabilistic approach, generally, over a large number of applications, the difference between the actual cost and the predicted cost will be less than in the case when a single point estimate is employed.



HIGH RISK



SUMMARY

To the average member of the design community, the problem of carrying out an economic analysis of a particular type of investment problem often appears to be a difficult, even an incomprehensible task. Most of the available books on economic analysis assume that the reader has previously acquired a knowledge of fundamental principles, and thus they proceed to the more complex aspects of analysis.

It is true that some economic problems are exceedingly difficult, but it is also true that many of the problems that frequently occur are surprisingly simple in their solution. This modest handbook was compiled with this fact in mind. With knowledge of the various economic terms and techniques explained here, the reader should be more able to understand economic analysis and to carry out simple economic evaluations.

We have tried to explain economic principles without the use of advanced mathematics or the derivation of formulas. Complex economic problems are not treated, although now the reader will have a better comprehension of them. We have furnished a kit of tools for design professionals to help them solve simple economic problems and better appreciate those of greater complexity.

We do not believe that a little knowledge is a dangerous thing, but that it is rather the best means of whetting the appetite for more. We hope that this introductory handbook will encourage our readers to plan their energy investments with as much care and understanding as Vitruvius counseled in designing buildings for firmness, commodity, and delight.

Footnotes

(1) Chart figures for *Energy Consumption 10^{12} Btu* were obtained from *Buildings' Energy Facts and Trends* (Draft in Preparation), Oak Ridge National Laboratory, Oak Ridge, Tennessee, November, 1977, pp. 7-10. Data for 1976 were obtained from Mr. Charles Reading, U.S. Department of Interior.

Figures charted for *Energy Consumption 10^9 in 1977 Dollars* were obtained by multiplying the annual energy consumption by the appropriate fuel price. Prices for natural gas, electricity distributed, and fuel oil were taken from *Buildings' Energy Facts and Trends* (Draft). Coal prices were calculated by extrapolation using the Anthracite Coal (stove size) Historical Price Index taken from the *Historical Statistics of the United States,* Department of Commerce, September, 1975.

Chart figures for *Price Per 10^6 Btu in 1977 Dollars* reflect a weighted average of the four energy sources (gas, electricity, petroleum, and coal) converted from current year dollars to 1977 dollars by the Implicit Price Deflator for Personal Consumption Expenditures. The deflator index was taken from the *Survey of Current Business,* U.S. Department of Commerce, July, 1977, Table 8-8.

(2) For example, see State of California, Energy Resources Conservation and Development Commission, Conservation Division, *Energy Conservation Design Manual for New Non-Residential Buildings,* October, 1977, a document that describes the requirements for energy conservation that must be met prior to the application for a nonresidential building permit in California.

(3) Often, however, dollar values are assigned to effects which are difficult to measure. See, for example, the measurement of the cost of airport noise pollution on an urban environment and of congestion costs on wilderness recreation in *Theory and Measurement* of *Economic Externalities,* ed. Steven A. Y. Lin (New York; Academic Press, 1976).

(4) Our emphasis is on a simplified treatment of these methods. For a more in-depth description of these methods, see Eugene L. Grant and W. Grant Ireson, *Principles of Engineering Economy,* 5th ed. (New York; The Ronald Press Co., 1970); E. J. Mishan, *Cost-Benefit Analysis: An Introduction (New York; Praeger, 1971); Ajit K. Dasgupta and D. W. Pearce, Cost-Benefit Analysis: Theory and Practice* (New York; Harper and Row, 1972); and Peter G. Sassone and William A. Scheffer, *Cost-Benefit Analysis: A Handbook* (New York; Academic Press, 1978).

(5) For a discussion of the budgeting of probable costs, scheduling of disbursements and control of costs during programming, master planning, preliminary drawings, working drawings, specifications and construction; see *Creative Control of Building Costs,* ed. Wilhan-Dudley Hunt, Jr., AIA (New York; McGraw Hill Book Co., 1967).

(6) To treat future energy costs that are expected to escalate at changing rates over time, several additional steps, beyond those illustrated here, are required.
The procedure and worksheets for calculating the present value of energy savings when multiple escalation rates are used are provided in Rosalie T. Ruegg's, et al., *Life-Cycle Costing,* National Bureau of Standards Science Series 113, September 1978, pp. 18-21. UPW* factors incorporating multiple energy price escalation rates as projected by the Energy Information Administration of the U.S. Department of Energy are provided in Rosalie T. Ruegg and John S. McConnaughey's *Manual for the Federal Energy Management Program,* National Bureau of Standards Special Report (In Press), 1979, Appendix B.

(7) Here the problem is presented as one of determining whether it will pay to have a solar energy system of a specified design, size, and cost. In practice a more common problem will be to determine the optimal design and size for a system and the cost effectiveness of that system. For a more in-depth treatment of the economics of solar energy systems, including optimization analysis, see Rosalie T. Ruegg and G. Thomas Sav, *Microeconomics of Solar Energy,* National Bureau of Standards Special Report, 1980, in press.

(8) OMB Circular A-94. Other rates are required for different types of investment. For example, for decisions on leasing or purchasing real property, OMB requires a discount rate of 7%.

(9) For a discussion of investment decisions under high risk, see Henry A. Latane, *Criteria for Choice Among Risky Ventures;* John W. Pratt, *Risk Aversion In The Small and In The Large;* Jack Hirshleifer, *Risk, The Discount Rate, and Investment Decisions;* ed. Stephen H. Archer and Clarke A. D'Ambrosio, *The Theory of Business Finance,* 2nd ed. (New York; MacMillan Publishing Co., Inc., 1976).

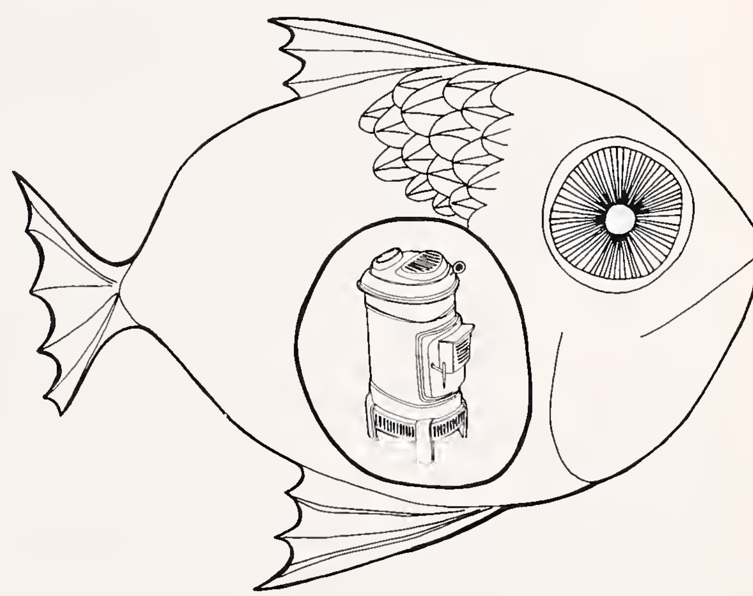
(10) For a description of price indexes and how to use them, see U.S. Department of Labor, Bureau of Labor Statistics, *Monthly Labor Review,* any issue.

(11) Department of Energy; *Historic and Forecasted Energy Prices by U.S. Department of Energy; Region and Fuel Types for Three Microeconomic Scenarios and One Imported Oil Price Escalation Scenario,* DOE/ETA-0102/27, December, 1978, Energy Information Administration Clearinghouse, 1726 M Street, N.W., Room 210, Washington, D.C. 20461.

(12) For past and present price indices, see U.S. Department of Labor, *Monthly Labor Review,* Current Issue. For a description of how to construct a constant dollar price deflator based on past inflation rates for use in adjusting future current dollar payments, see U.S. Office of Management and Budget Circular No. A-104, June 14, 1972.

(13) For a general text on decisionmaking in the design process under conditions of uncertainty, see Myron Tribus, *Rational Descriptions, Decisions and Designs* (New York; Pergamon Press, 1969). For an overview of alternative approaches to dealing with uncertainty and criteria for decisionmaking under uncertainty, see Ajit K. Dasgupta and D. W. Pearce, *Cost-Benefit Analysis: Theory and Practice* (New York; Harper and Row, 1972, pp. 174-178).

(14) Probabilities may be "objective" based on frequency of occurrences information or "subjective," based on intuition. For a more in-depth discussion of probability analysis see *ibid.,* pp. 174-198.



APPENDICES

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APPENDIX A

GLOSSARY OF ECONOMIC TERMS

Annual Value: Benefits or costs occurring in uniform amounts annually, or the uniform annual equivalents of past, present or future benefits.

Benefit-Cost Analysis: A means of evaluating alternative projects or investments by comparing the discounted value of total expected benefits with the discounted value of total expected costs for each alternative.

Benefit-Cost Ratio: Benefits expressed as a ratio to costs, where both are discounted to a present or annual value; the ratio must be greater than one for an investment to be economically efficient.

Constant Dollars: Values expressed in terms of the general purchasing power of the dollar in the base year. Constant dollars do not reflect price inflation.

Current Dollars: Values expressed in terms of actual prices of each year. Current dollars reflect price inflation.

Discount Rate: The rate of interest reflecting the time value of money that is used to convert benefits and costs occurring at different times to equivalent values at a common time.

Discounted Payback Period: The time required for the cumulative net benefits derived from an investment to pay back the investment cost, considering the time value of money.

Discounting: A technique for converting cash flows that occur over time to equivalent amounts at a common point in time.

Economic Efficiency: Maximizing net benefits or minimizing costs for a given level of benefits.

Economic Life: The period of time over which an investment is considered to be the least-cost alternative for meeting a particular objective.

Future Value (Worth): The value of a present dollar amount at some point in the future, considering the time value of money.

Incentive: A positive inducement to encourage a particular type of behavior or action.

Inflation: A rise in the general price level resulting from a decline in the purchasing power of the dollar.

Internal Rate of Return: The interest rate for which the total discounted benefits from an investment equal its total discounted costs.

Grants: Cash payments for the purpose of encouraging a particular practice or the use of a good or service by reducing its net cost to the owner or user.

Investment Cost: The sum of the planning, design, and construction costs necessary to provide a finished project ready for use.

Life Cycle: The period of time between the starting point and cutoff date of analysis, over which the costs and benefits of a certain alternative are incurred.

Life-Cycle Cost: The total of all relevant costs associated with an activity or project during the time it is analyzed. For buildings, life-cycle costs include all costs of owning, operating, and maintaining a building over its period of analysis, including its energy costs.

Marginal Analysis: Evaluating incremental changes in costs and benefits resulting from incremental changes in an investment.

Monetary Benefits: Benefits assigned a dollar value.

Net Benefits: The difference between benefits and costs, evaluated in present or annual value dollars.

Nonmonetary Benefits: Benefits to which it is difficult to assign dollar values.

Operation and Maintenance Costs: The costs associated with the normal operation and maintenance of a system, often accumulated on a recurring basis.

Payback: (See Discounted and Undiscounted Payback Period)

Present Value (Worth): Past and future cash flows expressed in time-equivalent amounts as of the present time, adjusted for inflation and the time value of money.

Probability Analysis: A technique, also called expected value analysis, used to evaluate the dollar value of an event whose expected chance of occurrence can be predicted.

Salvage Value: The net sum to be realized from disposal of an asset, net of disposal costs, at the time of its replacement, resale, or at the end of the study period.

Sensitivity Analysis: Testing the outcome of an evaluation by altering the values of key factors about which there is uncertainty.

Time Horizon (Study Period): A period of economic analysis over which time the costs and benefits of an investment are calculated.

Undiscounted Payback Period (Simple Payback): The length of time necessary for the cumulative benefits or savings resulting from an investment to recover the original cost of the investment, not considering the time value of money.

Useful Life: The period over which an investment is considered to have value.

APPENDIX B

DISCOUNT FORMULAS

FORMULA NAME		
Single Compound Amount Formula (SCA)	To find F when P is known	$F=P\ (1+i)^N$
Single Present Worth Formula (SPW)	To find P when F is known	$P=F\ \frac{1}{(1+i)^N}$
Uniform Sinking Fund Formula (USF)	To find A when F is known	$A=F\ \frac{i}{(1+i)^N-1}$
Uniform Capital Recovery Formula (UCR)	To find A when P is known	$A=P\ \frac{i\,(1+i)^N}{(1+i)^N-1}$
Uniform Compound Amount Formula (UCA)	To find F when A is known	$F=A\ \frac{(1+i)^N-1}{i}$
Uniform Present Worth Formula (UPW)	To find P when A is known	$P=A\ \frac{(1+i)^N-1}{i\,(1+i)^N}$
Uniform Present Worth Formula Modified (UPW*)	To find P when A is escalating at rate e	$P = A\ \frac{(1+e)}{(i-e)} \left[1- \left(\frac{1+e}{1+i}\right)^N\right]$

Where:

P = a present sum of money.

F = a future sum of money, equivalent to P at the end of N periods of time at an interest or discount rate of i.

i = an interest or discount rate.

N = number of interest or discounting periods.

A = an end-of-period payment (or receipt) in a uniform series of payments (or receipts) over N periods at i interest or discount rate.

e = rate of escalation of A in each of N periods.

C-1
SCA

APPENDIX C

SINGLE COMPOUND AMOUNT FORMULA $F = P\,(1+i)^N$

A present sum of money (P) times SCA factor for appropriate discount rate (i) gives (F) future value of (P). N=Number of discount periods

DISCOUNT FACTORS

	DISCOUNT RATES						
N	6%	8%	10%	12%	15%	20%	25%
1	1.060	1.080	1.100	1.120	1.150	1.200	1.250
2	1.124	1.166	1.210	1.254	1.322	1.440	1.563
3	1.191	1.260	1.331	1.405	1.521	1.728	1.953
4	1.263	1.360	1.464	1.574	1.749	2.074	2.441
5	1.338	1.469	1.611	1.762	2.011	2.488	3.052
6	1.419	1.587	1.772	1.974	2.313	2.986	3.815
7	1.504	1.714	1.949	2.211	2.660	3.583	4.768
8	1.594	1.851	2.144	2.476	3.059	4.300	5.960
9	1.689	1.999	2.358	2.773	3.518	5.160	7.451
10	1.791	2.159	2.594	3.106	4.046	6.192	9.313
11	1.898	2.332	2.853	3.479	4.652	7.430	11.642
12	2.012	2.518	3.138	3.896	5.350	8.916	14.552
13	2.133	2.720	3.452	4.633	6.153	10.699	18.190
14	2.261	2.937	3.797	4.887	7.076	12.839	22.737
15	2.397	3.172	4.177	5.474	8.137	15.407	28.422
16	2.540	3.426	4.595	6.130	9.358	18.488	35.527
17	2.693	3.700	5.054	6.866	10.761	22.186	44.409
18	2.854	3.996	5.560	7.690	12.375	26.623	55.511
19	3.026	4.316	6.116	8.613	14.232	31.948	69.389
20	3.207	4.661	6.727	9.646	16.367	38.338	86.736
21	3.400	5.034	7.400	10.804	18.822	46.005	108.420
22	3.604	5.437	8.140	12.100	21.645	55.206	135.525
23	3.820	5.871	8.954	12.552	24.891	66.247	169.407
24	4.049	6.341	9.850	15.179	28.625	79.497	211.758
25	4.292	6.848	10.835	17.000	32.919	95.396	264.698
26	4.549	7.396	11.918	19.040	37.857	114.475	330.872
27	4.822	7.988	13.110	21.325	45.535	137.371	413.590
28	5.112	8.627	14.421	23.884	50.066	164.845	516.988
29	5.418	9.317	15.863	26.750	57.576	197.814	646.235
30	5.743	10.063	17.449	29.960	66.212	237.376	807.794
31	6.088	10.868	19.194	35.555	76.144	284.852	1009.742
32	6.453	11.737	21.114	37.582	87.565	341.822	1262.177
33	6.841	12.676	23.225	42.092	100.700	410.186	1577.722
34	7.251	13.690	25.548	47.143	115.805	492.224	1972.152
35	7.686	14.785	28.102	52.800	133.176	590.668	2465.190
40	10.286	21.725	45.259	93.051	267.864	1469.772	7523.164
45	13.765	31.920	72.890	163.988	538.769	3657.262	22958.874
50	18.420	46.902	117.391	289.002	1083.657	9100.438	70064.923

SINGLE PRESENT WORTH FORMULA $P = F \frac{1}{(1+i)^N}$

A future sum of money (F) times SPW factor for appropriate discount rate (i) gives (P) present value of (F).

DISCOUNT FACTORS

	DISCOUNT RATES						
N	6%	8%	10%	12%	15%	20%	25%
1	0.943	0.926	0.909	0.893	0.870	0.833	0.800
2	.890	.857	.826	.797	.756	.694	0.640
3	.840	.794	.751	.712	.658	.579	0.512
4	.792	.735	.683	.636	.572	.482	0.410
5	.747	.681	.621	.567	.497	.402	0.328
6	.705	.630	.565	.507	.432	.335	0.262
7	.665	.584	.513	.452	.376	.279	0.210
8	.627	.540	.467	.404	.327	.233	0.168
9	.592	.500	.424	.361	.284	.194	0.134
10	.558	.463	.386	.322	.247	.162	0.107
11	.527	.429	.351	.288	.215	.135	0.086
12	.497	.397	.319	.257	.187	.112	0.069
13	.469	.368	.290	.229	.163	.094	0.055
14	.442	.341	.263	.205	.141	.078	0.044
15	.417	.315	.239	.183	.123	.065	0.035
16	.394	.292	.218	.163	.107	.054	0.028
17	.371	.270	.198	.146	.093	.045	0.022
18	.350	.250	.180	.130	.081	.038	0.018
19	.331	.232	.164	.116	.070	.031	0.014
20	.312	.215	.149	.104	.061	.026	0.012
21	.294	.199	.135	.093	.053	.022	0.009
22	.278	.184	.123	.083	.046	.018	0.007
23	.262	.170	.112	.074	.040	.015	0.006
24	.247	.158	.102	.066	.035	.013	0.005
25	.233	.146	.092	.059	.030	.010	0.004
26	.220	.135	.084	.053	.026	.009	0.003
27	.207	.125	.076	.047	.023	.007	0.002
28	.196	.116	.069	.042	.020	.006	0.002
29	.185	.107	.063	.037	.017	.005	0.002
30	.174	.099	.057	.033	.015	.004	0.001
31	.164	.092	.052	.030	.013	.004	0.001
32	.155	.085	.047	.027	.011	.003	0.001
33	.146	.079	.043	.024	.010	.002	0.001
34	.138	.073	.039	.021	.009	.002	0.000
35	.130	.068	.036	.019	.008	.002	0.000
40	.097	.046	.022	.011	.004	.001	0.000
45	.073	.031	.014	.006	.002	.000	0.000
50	.054	.021	.009	.004	.001	.000	0.000

C-3 USF

UNIFORM SINKING FUND FORMULA $A = F \frac{i}{(1+i)^N - 1}$

A future sum of money (F) times USF factor for appropriate discount rate (i) gives (A) an end-of-period payment equivalent over (N) discount periods to (F).

DISCOUNT FACTORS

	DISCOUNT RATES						
N	6%	8%	10%	12%	15%	20%	25%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.485	0.481	0.476	0.472	.465	0.455	0.444
3	.314	.308	.302	.296	.288	.275	0.262
4	.229	.222	.215	.209	.200	.186	0.173
5	.177	.170	.164	.157	.148	.134	0.122
6	.143	.136	.130	.123	.114	.101	0.089
7	.119	.112	.105	.099	.090	.077	0.066
8	.101	.094	.087	.081	.073	.061	0.050
9	.087	.080	.074	.068	.060	.048	0.039
10	.076	.069	.063	.057	.049	.039	0.030
11	.067	.060	.054	.048	.041	.031	0.023
12	.059	.053	.047	.041	.034	.025	0.018
13	.053	.047	.041	.036	.029	.021	0.015
14	.048	.041	.036	.031	.025	.017	0.012
15	.043	.037	.031	.027	.021	.014	0.009
16	.039	.033	.028	.023	.018	.011	0.007
17	.035	.030	.025	.020	.015	.009	0.006
18	.032	.027	.022	.018	.013	.008	0.005
19	.030	.024	.020	.016	.011	.006	0.004
20	.027	.022	.017	.014	.010	.005	0.003
21	.025	.020	.016	.012	.008	.004	0.002
22	.023	.018	.014	.011	.007	.004	0.002
23	.021	.016	.013	.010	.006	.003	0.001
24	.020	.015	.011	.008	.005	.003	0.001
25	.018	.014	.010	.008	.005	.002	0.001
26	.017	.013	.009	.007	.004	.002	0.001
27	.016	.011	.008	.006	.004	.001	0.001
28	.015	.010	.007	.005	.003	.001	0.000
29	.014	.010	.007	.005	.003	.001	0.000
30	.013	.009	.006	.004	.002	.001	0.000
31	.012	.008	.006	.004	.002	.001	0.000
32	.011	.007	.005	.003	.002	.001	0.000
33	.010	.007	.005	.003	.002	.000	0.000
34	.010	.006	.004	.003	.001	.000	0.000
35	.009	.006	.004	.002	.001	.000	0.000
40	.006	.004	.002	.001	.001	.000	0.000
45	.005	.003	.001	.001	.000	.000	0.000
50	.003	.002	.001	.000	.000	.000	0.000

UNIFORM CAPITAL RECOVERY FORMULA $A = P \frac{i(1+i)^N}{(1+i)^N - 1}$

A present sum of money (P) times UCR factor for appropriate discount rate (i) gives (A) an end-of-period payment equivalent over (N) discount periods to (P).

DISCOUNT FACTORS

	DISCOUNT RATES						
N	6%	8%	10%	12%	15%	20%	25%
1	1.060	1.080	1.100	1.120	1.150	1.200	1.250
2	0.545	0.561	0.576	0.592	.615	0.655	0.694
3	.374	.388	.402	.416	.438	.475	0.512
4	.289	.302	.315	.329	.350	.386	0.423
5	.237	.250	.264	.277	.298	.334	0.372
6	.203	.216	.230	.243	.264	.301	0.339
7	.179	.192	.205	.219	.240	.277	0.316
8	.161	.174	.187	.201	.223	.261	0.300
9	.147	.160	.174	.188	.210	.248	0.289
10	.136	.149	.163	.177	.199	.239	0.280
11	.127	.140	.154	.168	.191	.231	0.273
12	.119	.133	.147	.161	.184	.225	0.268
13	.113	.127	.141	.156	.179	.221	0.265
14	.108	.121	.136	.151	.175	.217	0.262
15	.103	.117	.131	.147	.171	.214	0.259
16	.099	.113	.128	.143	.168	.211	0.257
17	.095	.110	.125	.140	.165	.209	0.256
18	.092	.107	.122	.138	.163	.208	0.255
19	.090	.104	.120	.136	.161	.206	0.254
20	.087	.102	.117	.134	.160	.205	0.253
21	.085	.100	.116	.132	.158	.204	0.252
22	.083	.098	.114	.131	.157	.204	0.252
23	.081	.096	.113	.130	.156	.203	0.251
24	.080	.095	.111	.128	.155	.203	0.251
25	.078	.094	.110	.128	.155	.202	0.251
26	.077	.013	.109	.127	.154	.202	0.251
27	.076	.091	.108	.126	.154	.201	0.251
28	.075	.090	.107	.125	.153	.201	0.250
29	.074	.090	.107	.125	.153	.201	0.250
30	.073	.089	.106	.124	.152	.201	0.250
31	.072	.088	.106	.124	.152	.201	0.250
32	.071	.087	.105	.123	.152	.201	0.250
33	.070	.087	.104	.123	.152	.200	0.250
34	.070	.086	.104	.123	.151	.200	0.250
35	.069	.086	.104	.122	.151	.200	0.250
40	.066	.084	.102	.121	.151	.200	0.250
45	.065	.083	.101	.121	.150	.200	0.250
50	.063	.082	.101	.120	.150	.200	0.250

C-5
UCA

UNIFORM COMPOUND AMOUNT FORMULA $F = A \dfrac{(1+i)^N - 1}{i}$

An end-of-period payment (A) times UCA factor for appropriate discount rate (i) gives (F) future value of (A) over (N) discount periods.

DISCOUNT FACTORS

N	DISCOUNT RATES 6%	8%	10%	12%	15%	20%	25%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.060	2.080	2.100	2.120	2.150	2.200	2.250
3	3.184	3.246	3.310	3.374	3.472	3.640	3.813
4	4.375	4.506	4.641	4.779	4.993	5.368	5.766
5	5.637	5.867	6.105	6.353	6.742	7.442	8.207
6	6.975	7.336	7.716	8.115	8.754	9.930	11.259
7	8.394	8.923	9.487	10.089	11.067	12.916	15.073
8	9.897	10.637	11.436	12.300	13.727	16.499	19.842
9	11.491	12.488	13.579	14.776	16.786	20.799	25.802
10	13.181	14.487	15.937	17.549	20.304	25.959	33.253
11	14.972	16.645	18.531	20.655	24.349	32.150	42.566
12	16.870	18.977	21.384	24.133	29.002	39.581	54.208
13	18.882	21.495	24.523	28.029	34.352	48.497	68.760
14	21.015	24.215	27.975	32.393	40.505	59.196	86.949
15	23.276	27.152	31.772	37.280	47.580	72.035	109.687
16	25.673	30.324	35.950	42.753	55.717	87.442	138.109
17	28.213	33.750	40.545	48.884	65.075	105.931	173.636
18	30.906	37.450	45.599	55.750	75.836	128.117	218.045
19	33.760	41.446	51.159	63.440	88.212	154.740	273.556
20	36.786	45.762	57.275	72.052	102.444	186.688	342.945
21	39.993	50.423	64.002	81.699	118.810	225.026	429.681
22	43.392	55.457	71.403	92.503	137.632	271.031	538.101
23	46.996	60.893	79.543	104.603	159.276	326.237	673.626
24	50.816	66.765	88.497	118.155	184.168	392.484	843.033
25	54.865	73.106	98.347	113.334	212.793	471.981	1054.791
26	59.156	79.954	109.182	150.334	245.712	567.377	1319.489
27	63.706	87.351	121.100	169.374	283.569	681.853	1650.361
28	68.528	95.339	134.210	190.699	327.104	819.223	2063.952
29	73.640	103.966	148.631	214.583	377.170	964.068	2580.939
30	79.058	113.283	164.494	241.333	434.745	1181.882	3227.174
31	84.802	123.346	181.943	271.293	500.957	1419.258	4034.968
32	90.890	134.214	201.138	304.848	577.100	1704.109	5044.710
33	97.343	145.951	222.252	342.429	664.666	2045.931	6306.887
34	104.184	158.627	245.447	384.521	765.365	2456.118	7884.609
35	111.435	172.317	271.024	431.663	881.170	2948.341	9856.761
40	154.762	259.057	442.593	767.091	1779.090	7343.858	30088.655
45	212.744	386.506	718.905	1358.230	3585.128	18281.310	91831.496
50	290.336	573.770	1163.909	2400.018	7217.716	45497.191	280255.693

UNIFORM PRESENT WORTH FORMULA $P = A \frac{(1+i)^N-1}{i(1+i)^N}$

An end-of-period payment (A) times UPW factor for appropriate interest rate (i) gives (P) a present sum of money equivalent to (A) over (N) discount periods.

DISCOUNT FACTORS

	DISCOUNT RATES						
N	6%	8%	10%	12%	15%	20%	25%
1	0.943	0.926	0.909	0.893	0.870	0.833	0.800
2	1.833	1.783	1.736	1.690	1.626	1.528	1.440
3	2.673	2.577	2.487	2.402	2.283	2.106	1.952
4	3.465	3.312	3.170	3.037	2.855	2.589	2.362
5	4.212	3.993	3.791	3.605	3.352	2.991	2.689
6	4.917	4.623	4.355	4.111	3.784	3.326	2.951
7	5.582	5.206	4.868	4.564	4.160	3.605	3.161
8	6.210	5.747	5.335	4.968	4.487	3.837	3.329
9	6.802	6.247	5.759	5.328	4.772	4.031	3.463
10	7.360	6.710	6.145	5.650	5.019	4.192	3.571
11	7.887	7.139	6.495	5.938	5.234	4.327	3.656
12	8.834	7.536	6.814	6.194	5.421	4.439	3.725
13	8.853	7.904	7.103	6.424	5.583	4.533	3.780
14	9.295	8.244	7.367	6.628	5.724	4.611	3.824
15	9.712	8.559	7.606	6.811	5.847	4.675	3.859
16	10.106	8.851	7.824	6.974	5.954	4.730	3.887
17	10.477	9.122	8.022	7.120	6.047	4.775	3.910
18	10.828	9.372	8.201	7.250	6.128	4.812	3.928
19	11.158	9.604	8.365	7.366	6.198	4.843	3.942
20	11.470	9.818	8.514	7.469	6.259	4.870	3.954
21	11.764	10.017	8.649	7.562	6.312	4.891	3.963
22	12.042	10.201	8.772	7.645	6.359	4.909	3.970
23	12.303	10.371	8.883	7.718	6.399	4.925	3.976
24	12.550	10.529	8.985	7.784	6.434	4.937	3.981
25	12.783	10.675	9.077	7.843	6.464	4.948	3.985
26	13.003	10.810	9.161	7.896	6.491	4.956	3.988
27	13.211	10.935	9.237	7.943	6.514	4.964	3.990
28	13.406	11.051	9.307	7.984	6.534	4.970	3.992
29	13.591	11.158	9.370	8.022	6.551	4.975	3.994
30	13.765	11.258	9.427	8.055	6.566	4.979	3.995
31	13.929	11.350	9.479	8.085	6.579	4.982	3.996
32	14.084	11.435	9.526	8.112	6.591	4.985	3.997
33	14.230	11.514	9.569	8.135	6.600	4.988	3.997
34	14.368	11.587	9.609	8.157	6.609	4.990	3.998
35	14.498	11.655	9.644	8.176	6.617	4.992	3.998
40	15.046	11.925	9.779	8.244	6.642	4.997	3.999
45	15.456	12.108	9.863	8.283	6.654	4.999	4.000
50	15.762	12.233	9.915	8.304	6.661	7.999	4.000

D-1
UPW*

APPENDIX D

MODIFIED UNIFORM PRESENT WORTH FORMULA

$$P = A \frac{(1+e)}{(i-e)} \left[1 - \left(\frac{1+e}{1+i}\right)^N\right]$$

An end-of-base-year payment (A) times UPW* factor for appropriate discount (i) and price escalation rate (e) gives (P), a present sum of money equivalent to (A) escalating at rate e over (N) discount periods.

8% Discount Rate

U P W FACTORS MODIFIED FOR ENERGY PRICE ESCALATION

	RATE OF ENERGY PRICE ESCALATION									
N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.935	.944	.954	.963	.972	.981	.991	1.000	1.009	1.019
2	1.810	1.836	1.863	1.890	1.917	1.945	1.972	2.000	2.028	2.056
3	2.628	2.679	2.731	2.783	2.836	2.890	2.945	3.000	3.056	3.112
4	3.393	3.474	3.558	3.643	3.730	3.818	3.908	4.000	4.093	4.189
5	4.108	4.226	4.347	4.471	4.598	4.729	4.863	5.000	5.141	5.285
6	4.777	4.936	5.099	5.268	5.443	5.623	5.809	6.000	6.197	6.401
7	5.402	5.606	5.817	6.036	6.264	6.500	6.745	7.000	7.264	7.538
8	5.987	6.239	6.501	6.776	7.062	7.361	7.674	8.000	8.341	8.696
9	6.534	6.837	7.154	7.488	7.838	8.207	8.593	9.000	9.427	9.876
10	7.046	7.401	7.777	8.173	8.593	9.036	9.505	10.000	10.524	11.077
11	7.525	7.935	8.370	8.834	9.326	9.850	10.407	11.000	11.630	12.301
12	7.972	8.438	8.936	9.469	10.039	10.649	11.302	12.000	12.747	13.547
13	8.391	8.914	9.476	10.082	10.733	11.434	12.188	13.000	13.875	14.817
14	8.782	9.363	9.991	10.671	11.407	12.203	13.066	14.000	15.012	16.110
15	9.148	9.787	10.483	11.239	12.062	12.905	13.936	15.000	16.161	17.426
16	9.490	10.188	10.951	11.786	12.699	13.700	14.797	16.000	17.320	18.768
17	9.810	10.567	11.398	12.312	13.319	14.428	15.651	17.000	18.489	20.134
18	10.110	10.924	11.824	12.819	13.921	15.142	16.497	18.000	19.670	21.525
19	10.390	11.262	12.230	13.307	14.507	15.843	17.335	19.000	20.861	22.942
20	10.651	11.580	12.618	13.777	15.076	16.532	18.165	20.000	22.063	24.386
21	10.896	11.881	12.987	14.230	15.629	17.207	18.988	21.000	23.277	25.856
22	11.125	12.166	13.340	14.666	16.167	17.870	19.802	22.000	24.502	27.353
23	11.339	12.434	13.676	15.086	16.690	18.520	20.610	23.000	25.738	28.878
24	11.539	12.688	13.996	15.490	17.199	19.159	21.410	24.000	26.985	30.431
25	11.727	12.928	14.302	15.879	17.694	19.785	22.202	25.000	28.244	32.013

MODIFIED UNIFORM PRESENT WORTH FORMULA

$$P = A \frac{(1+e)}{(i-e)} \left[1 - \left(\frac{1+e}{1+i}\right)^N\right]$$

An end-of-base-year payment (A) times UPW* factor for appropriate discount (i) and price escalation rate (e) gives (P), a present sum of money equivalent to (A) escalating at rate e over (N) discount periods.

10% Discount Rate
U P W FACTORS MODIFIED FOR ENERGY PRICE ESCALATION

	RATE OF ENERGY PRICE ESCALATION									
N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.918	0.927	0.936	0.945	0.955	0.964	0.973	0.982	0.991	1.000
2	1.761	1.787	1.813	1.839	1.867	1.892	1.919	1.946	1.973	2.000
3	2.535	2.584	2.634	2.684	2.735	2.787	2.839	2.892	2.946	3.000
4	3.246	3.324	3.403	3.483	3.566	3.649	3.734	3.821	3.910	4.000
5	3.899	4.009	4.123	4.239	4.358	4.480	4.605	4.734	4.865	5.000
6	4.498	4.645	4.797	4.953	5.115	5.281	5.453	5.630	5.812	6.000
7	5.048	5.234	5.428	5.628	5.837	6.053	6.277	6.509	6.750	7.000
8	5.553	5.781	6.019	6.267	6.526	6.796	7.078	7.372	7.680	8.000
9	6.017	6.288	6.572	6.871	7.184	7.512	7.858	8.220	8.601	9.000
10	6.443	6.758	7.090	7.441	7.812	8.203	8.616	9.053	9.513	10.000
11	6.834	7.194	7.575	7.981	8.411	8.868	9.354	9.870	10.418	11.000
12	7.193	7.598	8.030	8.491	8.983	9.510	10.072	10.672	11.314	12.000
13	7.523	7.972	8.455	8.973	9.530	10.127	10.770	11.460	12.202	13.000
14	7.825	8.320	8.853	9.429	10.051	10.723	11.449	12.233	13.082	14.000
15	8.102	8.642	9.226	9.860	10.549	11.296	12.109	12.993	13.954	15.000
16	8.357	8.941	9.576	10.268	11.268	11.849	12.752	13.738	14.818	16.000
17	8.592	9.218	9.903	10.653	11.477	12.382	13.377	14.470	15.674	17.000
18	8.807	9.475	10.209	11.018	11.910	12.896	13.984	15.189	16.523	18.000
19	9.004	9.713	10.496	11.362	12.323	13.390	14.576	15.895	17.363	19.000
20	9.186	9.934	10.764	10.688	12.718	13.867	15.151	16.588	18.196	20.000
21	9.351	10.139	11.015	11.996	13.094	14.326	15.711	17.268	19.022	21.000
22	9.504	10.329	11.251	12.287	13.454	14.769	16.255	17.936	19.840	22.000
23	9.645	10.505	11.471	12.562	13.796	15.196	16.784	18.591	20.650	23.000
24	9.774	10.668	11.678	12.822	14.124	15.607	17.299	19.235	21.454	24.000
25	9.892	10.819	11.871	13.068	14.437	16.003	17.800	19.867	22.250	25.000

D-3
UPW*

MODIFIED UNIFORM PRESENT WORTH FORMULA

$$P = A\,\frac{(1+e)}{(i-e)}\left[1-\left(\frac{1+e}{1+i}\right)^N\right]$$

An end-of-base-year payment (A) times UPW* factor for appropriate discount rate (i) and price escalation rate (e) gives (P), a present sum of money equivalent to (A) escalating at rate e over (N) discount periods.

12% Discount Rate

U P W FACTORS MODIFIED FOR ENERGY PRICE ESCALATION

RATE OF ENERGY PRICE ESCALATION

N	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.902	.911	.920	.929	.938	.946	.955	.964	.973	.982
2	1.715	1.740	1.765	1.791	1.816	1.842	1.868	1.894	1.920	1.947
3	2.448	2.495	2.543	2.591	2.640	2.690	2.740	2.791	2.842	2.894
4	3.110	3.183	3.258	3.335	3.413	3.492	3.573	3.655	3.739	3.825
5	3.706	3.810	3.916	4.025	4.137	4.252	4.369	4.489	4.612	4.738
6	4.244	4.380	4.521	4.666	4.816	4.970	5.129	5.293	5.462	5.636
7	4.729	4.900	5.078	5.262	5.452	5.650	5.856	6.068	6.289	6.517
8	5.166	5.373	5.589	5.814	6.049	6.294	6.550	6.816	7.094	7.383
9	5.561	5.804	6.060	6.328	6.609	6.903	7.213	7.537	7.877	8.234
10	5.916	6.197	6.492	6.804	7.133	7.480	7.846	8.232	8.639	9.069
11	6.237	6.554	6.890	7.247	7.625	8.026	8.451	8.902	9.381	9.889
12	6.526	6.880	7.256	7.658	8.086	8.542	9.029	9.549	10.103	10.694
13	6.787	7.176	7.593	8.039	8.518	9.031	9.581	10.172	10.805	11.486
14	7.022	7.446	7.902	8.394	8.923	9.494	10.109	10.773	11.489	12.263
15	7.234	7.692	8.187	8.723	9.303	9.931	10.613	11.352	12.155	13.026
16	7.426	7.916	8.449	9.028	9.659	10.346	11.095	11.911	12.802	13.775
17	7.598	8.120	8.689	9.312	9.993	10.738	11.555	12.450	13.433	14.512
18	7.754	8.306	8.911	9.575	10.306	11.109	11.994	12.970	14.046	15.234
19	7.894	8.475	9.114	9.820	10.599	11.461	12.414	13.471	14.643	15.945
20	8.020	8.629	9.302	10.047	10.874	11.793	12.815	13.954	15.224	16.642
21	8.134	8.769	9.474	10.258	11.132	12.108	13.199	14.420	15.789	17.337
22	8.237	8.897	9.632	10.454	11.374	12.405	13.565	14.869	16.340	18.000
23	8.330	9.013	9.778	10.636	11.600	12.687	13.914	15.302	16.875	18.660
24	8.414	9.119	9.912	10.805	11.813	12.954	14.249	15.720	17.396	19.309
25	8.489	9.216	10.035	10.961	12.012	13.207	14.568	16.123	17.904	19.947

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