Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

October through December 1950

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES
October 1 through December 31, 1950

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Projects and Publications
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NATIONAL APPLIED MATHEMATICS LABORATORIES

October through December 1950

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from October 1, 1950 to December 31, 1950.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss

Chief

E. U. Condon

Director
National Bureau of Standards
February 1, 1951

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, Astronomy, and Automatic Translation

D. Mathematical Statistics

*New projects

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

*New projects

Status of Projects

December 31, 1950

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-11-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The method of gradients, which has been studied at the Institute for Numerical Analysis principally in connection with eigenvalue problems (see Jun-Sept 1950 issue, task 1101-11-5100/51-3), was applied this quarter to the solution of linear equations.

A linear system, expressible in matrix notation as $A\bar{x} = b$, is solved when one has reduced the error function $F(x) = (Ax - b)^T(Ax - b) = |Ax - b|^2$ to zero. As iterative schemes one may use a class of methods first proposed by Cauchy in 1847: Given an approximate value x_i of x, one first calculates the gradient $z_i = 2A^T(Ax_i - b)$ of the function F at x_i. Then one takes $x_{i+1} = x_i - \alpha_i z_i$ as the next approximant to x, where α_i is a scalar specified in different ways. Most attention has been given by Drs. Motzkin and Forsythe to the "method of optimum α," where α_i is chosen so as to minimize $F(x_i - \alpha z_i)$ over all real α.

Let the distinct eigenvalues of the definite matrix A^TA be numbered so that $0 < \lambda_1 < \ldots < \lambda_n$. By a result of Kantorovich, the norm of the transformation T taking x_i into x_{i+1} is

$$\rho_n = (\lambda_n - \lambda_1)(\lambda_n + \lambda_1)^{-1} < 1,$$

so that the method certainly converges. For large n, ρ_n is likely to be so close to unity that convergence is exceedingly slow, and the utility of the gradient method depends on the possibility of accelerating the convergence. It is conjectured by Drs. Motzkin and Forsythe that, for all general initial vectors x_0, as $i \to \infty$, x_i asymptotically falls into the two-dimensional invariant subspace belonging to the eigenvalues λ_1 and λ_n. If true, this would imply that the endpoints of the vectors x_{2i} asymptotically fall into a line through the solution $x = A^{-1}b$, whence acceleration of the convergence would be easy and efficacious. The conjecture has been proved for n = 3, and numerical experiments suggest its probable validity for all n. The importance of this conjecture lies in the fact that, when convergent, the gradient method with optimal α would behave the same way in many other problems: e.g., minimization of a general functional $F(x)$ (whether x varies over whole space or a

subspace) and determination of eigenvalues.

The gradient method for linear equations is being coded for IBM experiments on the card-programmed calculator. Preliminary runs confirm the conjectured behavior in all respects.

Dr. Forsythe initiated a monograph designed to collect and organize all known methods of solving simultaneous linear equations.

Publications: (1) "The extent of n random unit vectors," by G. E. Forsythe and J. W. Tukey; accepted by the American Mathematical Monthly. (2) "A method of computing exact inverses of matrices with integer coefficients," by J. B. Rosser; accepted by the NBS Journal of Research.

NUMERICAL METHODS IN CONFORMAL MAPPING
Task 1101-11-5100/49-CM1
(formerly 11.1/1-49-CM1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The numerical work necessary to complete this problem is to be scheduled for the card-programed calculator in the next quarter.

Publications: (1) "The construction and applications of conformal maps: Proceedings of a symposium," edited by E. F. Beckenbach; to be published in the NBS Applied Mathematics Series. The volume will include the following papers written in connection with this project; (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A. M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A.Weinstein. (2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W.Seidel; IN MANUSCRIPT at NAML; publication under consideration. (3) "Numerical methods in conformal mapping," by L. Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

CALCULATION OF EIGENVALUES, EIGENVECTORS,AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-11-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In calculating the binding energy of a triton, Harold Gruen and Robert Hayes have been using a modification of a gradient method described in paper (4) and paper (1) on task 1101-11-5100/51-3. The modification consists of successively minimizing the Rayleigh quotient on linear subspaces, so that adjoining subspaces overlap and such that k successive subspaces span the whole space. A report on this method is in preparation.

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C. Lanczos; NBS J. Res. 45, 255-282 (Oct. 1950), RP 2133. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 15 cents. (2) "New matrix transformations for obtaining characteristic vectors," by W. Feller and G. E. Forsythe; accepted by the Quarterly of Applied Mathematics. (3) "The separation of close eigenvalues of a real symmetric matrix," by J. B. Rosser, C. Lanczos, M. R. Hestenes; accepted by the NBS Journal of Research.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-11-5100/51-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. A study of the error in approximating a solution of a quasi-linear parabolic equation by a finite difference scheme was carried out by F. John. For purposes of numerical solution of partial differential equations by finite differences, it is important to obtain a priori estimates for the solution and for the error in the approximation scheme. Such estimates are most easily obtained in the case of parabolic equations. This investigation was stimulated by the work of Lewy, of O'Brien, Hyman, and Kaplan, and of von Neumann on convergence and stability of difference schemes, and also by the related results of W. Wasow and J. H. Curtiss on probability methods applied to the solution of partial differential equations.

The equations considered were of the form

$$u_t = a(x,t,u,u_x)u_{xx} + b(x,t,u,u_x) \quad (a > 0),$$

with initial data $u(x,0) = f(x)$ prescribed for $-\infty < x < +\infty$. The differential equation is replaced by a difference equation using the ordinary forward difference scheme. It is shown that for $\lambda = a(\Delta t/\Delta x^2) < \frac{1}{2}$ the solution of the difference equation approximates that of the differential equation, provided a, b, and f have uniformly continuous and bounded second derivatives. If bounds for the fourth derivatives are available, explicit bounds for the error in the difference scheme can be obtained. In the special case where the differential equation has the simpler form

$$u_t = u_{xx} + b(x,t,u)$$

the convergence of the difference scheme can be proved under the less restrictive assumptions that f and the first derivatives of b are bounded and uniformly continuous. The proof of this theorem involves estimates for the Green's function of the difference equation corresponding to $u_t = u_{xx}$. These estimates also show that the ordinary difference scheme cannot work for $\lambda > \frac{1}{2}$. Work on the extension of these results to the case of more space variables is in progress. Dr. Milne continued the work on his monograph on "Numerical solution of differential equations" (see Jan-March 1949 issue). The study of the two-point boundary value problem for second order differential equations has been completed and written up for the monograph above with the comparison of six different methods. On the subject of partial differential equations, a preliminary chapter has been written for the monograph treating the numerical solution of the simple parabolic equation in two variables,

the simple hyperbolic equation in two variables, and Laplace's equation in two variables. This includes a treatment of relaxation techniques for elliptic differential equations and various suggestions for speeding the convergence. Not yet written up are numerical studies of the use of orthogonal harmonic polynomials in connection with the Dirichlet problem. Further, a study has been made of "angular averages" for approximating the solution of the Dirichlet problem. At the weekly seminar on differential equations, the following lectures were delivered: (1) "A summary of von Neumann's method for integration of the heat equation," by F. Acton. (2) "Probabilistic approach to solutions of equations of the form,

$$\frac{\partial u}{\partial t} + \tfrac{1}{2}\, a(t,x)\, \frac{\partial^2 u}{\partial x^2} + b(x,t)\, \frac{\partial u}{\partial x} = 0,$$

and reduction to normal form, using measure theory," by R. Fortet. (3) "Gradient methods for finding eigenvalues of differential equations," by M. Hestenes. (4) "Basic properties of partial differential equations. Survey of the fundamental properties of parabolic differential equations that are of importance from the viewpoint of numerical solution," by F. John. (5) "Stability and convergence of computing schemes for partial differential equations," by F. John. (6) "The compatibility problem of linear differential equations; use of adjoint system to study the compatibility of overdetermined or underdetermined systems of equations, to establish the existence of a solution satisfying certain boundary conditions, and to accomplish the construction of the Green's function," by C. Lanczos. (7) "Two end-point boundary conditions in ordinary differential equations (discussion of four different numerical methods for obtaining the solution)," by W. E. Milne. (8) "Solution of a differential equation arising in a calculation of the electrical currents produced in the atmosphere by thunderstorms," by D. Saxon. (9) "On the truncation error in the solution of Laplace's equation by finite differences," by W. Wasow.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of the Symposium held at Harvard Computation Laboratory in September 1949. (2) "Numerical solution of partial differential equation," by E. C. Yowell; to be published in the Proceedings of a Scientific Computation Forum, held under the auspices of the International Business Machine Corporation in Endicott, N. Y., December 1949.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-11-5100/51-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Generation of Random Numbers: Dr. G.E.Forsythe, in collaboration with Mr. H. Luxenberg, Mrs. R. Lipkis, and Miss G. Franklin, completed preliminary tests for the random digits generated on the SWAC by method (b) (see July-Sept 1950 issue). The 1,217,370 zeros or ones were summed in groups of 100 consecutive digits, obtaining 12,173 sums S_i, (where $0 \leq S_i \leq 100$). Frequency distributions were obtained for twelve groups of 1000 sums each. The twelve distributions were compared with the corresponding theoretical (binomial) distribution, by a χ^2 test with 23 degrees of freedom. The twelve values of χ^2 ranged

from 9.68 ($P \doteq .99$) to 47.24 ($P << .01$). Only the two extreme values are unexpectedly far from the median. From this experiment one may conclude tentatively that method (b) will generate digits suitable for ordinary statistical sampling but probably not suitable for delicate applications requiring a high degree of independence in the digits.

It is hoped later to use these same digits for the calculation of the least eigenvalue of Schrödinger's equation in one dimension, where the potential $V(x) = x^2$. The results would be comparable with those of Kac and Donsker (see July-Sept 1949 issue, project 11.1/1-50-3).

"Monte Carlo" Techniques: A new sampling method for finding eigenvalues of a Fredholm integral equation with positive definite kernel was proposed by Dr. R. Fortet (see publication (12)). The method is based on a paper by Kac and Sigert (see Journal of Applied Physics, 18, p. 383 (1947)). Dr. Fortet also examined the numerical accuracy of the method of Donsker and Kac for finding eigenvalues of Schrödinger's equation. His conclusions, though somewhat different from those reached by Kac (see publication (5)), confirm the impression that extremely large samples will be needed to obtain reliable results. It was shown by Dr. Wasow that the solution of Laplace's equation by random walks that are based on normal deviates involves a truncation error that is small of exponential order and thus is smaller than the truncation error for any random walk scheme in a lattice.

"Monte Carlo" Experiments: Experiments with Laplace's equation have been planned and coded for SWAC by F. S. Acton and will be run when machine time is available, giving both estimates of the function and a measure of dispersion of these estimates. These experiments are a continuation of those carried out on SEAC by J. Todd.

Theoretical Investigations: In publication (14) Dr. Fortet studies normalized random functions $X_m(t)$ derived from a Poisson process of parameters m. He gives conditions that a given Laplacian (i.e., Gaussian) process $X^*(t)$ may be considered as the limit "in law" of some $X_m(t)$ as $m \to \infty$. (Convergence "in law" is defined in publication (14)). He also studies the asymptotic distribution of functionals of the form

$$L(t,\tau) = \int_t^\tau V\left[t, X_m(t)\right] dt$$

where $V[t,x]$ is a given function. He proves that, under certain conditions, the distribution function of $L(t,t+T)$, if properly normalized, tends toward the Laplace (i.e., normal) distribution, as $T \to \pm\infty$, (for any fixed t). This result is then extended to $\int_t^\tau V[t,X^*(t)]\,dt$.

Another investigation of Dr. Fortet (publication (13)) was concerned with additive functionals $L[t,\tau]$ of general Markoff processes $X(t)$. Let $\psi(t,x;\tau,\xi;v)$ be the conditional characteristic function of $L[t,\tau]$, it being given that $X(t) = x$, $X(\tau) = \xi$, and set

$$\mu(t,x;\tau,e;v) = \int_e \psi(t,x;\tau,x';v)\,d_e F(t,x,\tau,e')$$

where e is any measurable set and $F(t,x,\tau,e')$ is the transition probability of $X(t)$. Then μ satisfies a certain integral equation and, hence, under some assumptions on the regularity and continuity of $X(t)$ and $L[t,\tau]$, a parabolic equation analogous to the classical Kolmogoroff equation.

Continuing his work on the duration of random walks, Dr. Wasow (publication (11)) proved that the distribution function for the duration can be approximated, for a sufficiently concentrated transition probability, by the solution of a boundary value problem for a certain parabolic differential equation.

Publications: (1) "Random walks and eigenvalues of elliptic difference equations," by Wolfgang Wasow; to be published in the NBS

Journal of Research. (2) "Sampling methods applied to differential and difference equations, with special reference to equations of the elliptic type," by J. H. Curtiss; Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., November 1949, 87-109; reprints available. (3) "The Monte Carlo Method: Proceedings of a symposium held in Los Angeles, June 1949," to appear as NBS Applied Mathematics Series 12; in press. (4) "A Monte Carlo method for solving a class of integral equations," by R. E.Cutkosky; IN MANUSCRIPT. (5) "On some connections between probability theory and differential and integral equations," by Mark Kac; to be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950. (6) "On the mean duration of random walks," by W. Wasow; accepted by the NBS Journal of Research. (7) "Elimination of randomization in certain statistical decision procedures and zero-sum two-person games," by A. Dvoretzky, A. Wald, and J. Wolfowitz; accepted by the Annals of Mathematical Statistics. (8) "Some problems on random walk in space," by A. Dvoretzky and Paul Erdös; to be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950. (9) John von Neumann, "Various techniques used in connection with random digits," (summary written by G. E. Forsythe); to appear in "The Monte Carlo method: Proceedings of a symposium held June 29, 30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office. (10) "Generation and testing of random digits at the National Bureau of Standards, Los Angeles," by G. E. Forsythe; to appear in"The Monte Carlo method: Proceedings of a symposium held June 29, 30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office. (11) "On the duration of random walks," by W. Wasow; accepted by the NBS Journal of Research. (12) "On the estimation of an eigenvalue by an additive functional of a stochastic process, with special reference to the Kac-Donsker method," by R. Fortet;IN MANUSCRIPT. (13) "Additive functionals of a Markoff process," by R. Fortet; IN MANUSCRIPT. (14) "On some functionals of Laplacian processes," by R. Fortet; submitted to a technical journal.

VARIATIONAL METHODS
Task 1101-11-5100/51-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. For application of the gradient method to the solution of linear equations, see task 1101-11-5100/49-AE2. Dr. Marvin Stein has given a convergence proof for a generalization of Newton's method applied to systems of ordinary differential equations. In addition he has given a convergence proof of a gradient method for solving such a system under stronger hypotheses. These methods are concerned primarily with 2-point boundary value problems.

Publications: (1) "A method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; accepted by the NBS Journal of Research. (2) "An iterative method for finding the characteristic vectors of a symmetric matrix," by W. Karush; submitted to a technical journal. (3) "Applications of the theory of quadratic forms in Hilbert space to the calculus of variations," by M. R. Hestenes, IN MANUSCRIPT.

(4) "The solutions of $Ax = \lambda Bx$," by M. R. Hestenes and W. Karush; submitted to a technical journal. (5) "On methods for obtaining solutions of fixed end point problems in the calculus of variations," by Marvin Stein; doctorate thesis accepted by the University of California at Los Angeles, IN MANUSCRIPT.

STUDIES IN APPLIED MATHEMATICS
Task 1101-11-5100/51-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Stirling's approximation of the Gamma function is one of the most interesting approximations of a highly transcendental function by elementary transcendentals. The Stirling expansion has an asymptotic character, i. e., the series does not converge for any value of n, but the properly truncated series approximates the correct value of the function with great accuracy if n is sufficiently large. The Stirling expansion has thus the character of an expansion about a point (in this case the point $n = \infty$).

The present investigation which makes use of a modified Stirling formula of high precision applies the principles of expansions in the large to the problem of approximating the Gamma function over extended portions of the complex domain. The following integral representation, which follows directly from Euler's definition of the Gamma function

$$\Gamma(z + 1) = (\gamma z + 1)^{z+1} \int_0^1 (-x^\gamma \log x)^z dx$$

is well adapted for approximations, because the function $f(x) = -x^\gamma \log x$ in the range between 0 and 1 follows very closely the course of a parabola, provided that γ is properly chosen. Replacing $f(x)$ by the osculating parabola at the point of maximum, the following approximation is obtained:

$$\Gamma(z + 1) = \sqrt{2\pi}\,(z + \mu)^{z+\frac{1}{2}}\, e^{-(z+\mu)}(1 + \varepsilon)$$

where $\mu = \frac{1}{\gamma} - \frac{1}{2}$. The above reduces to the first term of Stirling's formula, if we set $\mu = \varepsilon = 0$. If we choose $\mu = 0.26$, we find

$$|\varepsilon| < 0.02 \quad \text{for } R(z) \geq 0 .$$

Hence an elementary approximation of the Gamma function is obtained which gives $z! = \Gamma(z + 1)$ with a relative error not exceeding .02 at any point of the complex half-plane.

Instead of neglecting the remainder we can expand it into an infinite series. Applying the principles of orthogonal function systems, we obtain an expansion which is no longer asymptotic, but uniformly convergent for all values of z in the entire positive half-plane, assuming μ is a positive constant. The form of this expansion is as follows:

$$\Gamma(z + 1) = \sqrt{2\pi}\,(z + \mu)^{z+\frac{1}{2}}\, e^{-(z+\mu)} K(z) \qquad (1)$$

where

$$K(z) = a_0 + \frac{a_1 z}{z+1} + a_2 \frac{z(z-1)}{(z+1)(z+2)} + \cdots \qquad (2)$$

The coefficients a_k are obtainable by recursion. The strong convergence obtainable by these expansions is demonstrated by the following two, three, and four-term approximations, The error estimates given hold again for the entire positive half-plane of the complex variable z:

$$\mu = 1.5,\ K_1(z) = 1 + \frac{0.4596}{z+1} + \varepsilon\ , \quad |\varepsilon| < 7.10^{-4}$$

$$\mu = 2.5,\ K_2(z) = 1 + \frac{2.189618}{z+1} - \frac{0.231727}{z+2} + \varepsilon,\ |\varepsilon| < 5.10^{-5}$$

$$\mu = 3.5,\ K_3(z) = 1 + \frac{2.1897711}{z+1} - \frac{0.2329512}{z+2} + \frac{0.0015300}{z+3} + \varepsilon,\ |\varepsilon| < 1.10^{-6}$$

Note that μ changes with the number of terms taken in the approximation for optimum convergence. Analytically interesting is the fact that the expansion (2) places the poles of the Gamma function at the proper points z = -1, -2, -3,.... Hence we may raise the question whether the validity of the representation (1) may not be extendable to the negative half-plane. This is actually the case. The expansion (2) remains convergent in the entire region $R(z) > -\mu$, except at the poles z = -1, -2,... . We thus have an expansion which represents the Gamma function not only in the entire positive half-plane, but even in arbitrarily large portions of the negative half-plane, provided that μ is chosen as a sufficiently large positive constant.

Publications: (1) A manuscript on the antenna problem is in preparation for the Proceedings of the IRE. (2) An introduction has been prepared on the applications of the tables of Chebyshev polynomials to be included in NBS Applied Mathematics Series 9, "Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$"; now in press. (See task 1102-21-1104/50-3a, page 31.)

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-11-5100/51-5

Origin: Office of Naval Research, USN
Sponsor: " "
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Work which is underway on several problems in theoretical physics is described in the following paragraphs.

An attempt is being made by H. Gruen to calculate the binding energy of the tritium nucleus, using as interaction potential for this three-body system the well-known potential determined from a study of the low energy neutron-proton and proton-proton interactions. A Rayleigh-Ritz treatment reduces the Schrödinger equation to a matrix eigenvalue problem, and, to date, lowest eigenvalues and eigenvectors have been obtained by the "gradient method" for various matrices including an 18 x 18 [see task 1101-11-5100/51-3, papers (1) and (4)]. This last yields a binding energy of about 65% of the experimentally known value. It is hoped that careful choice of higher order matrices will significantly improve the convergence and provide an answer, once

and for all, to the question of whether the system is adequately described by the assumed interaction.

Calculations of the electromagnetic radiation pattern from coaxial structures are being made by J. P. Wesley. Using the exact solutions to two simpler problems, an approximate solution has been obtained for the radiation from a coaxial wave guide with the center conductor extending a finite distance past the mouth of the guide. The work remaining to be done is almost entirely computational in nature.

An investigation in the field of magneto-hydrodynamics is being initiated by R. K. Golden. This rather new field has possible applications to many important problems in physics ranging from the origin of cosmic rays to the magnetic properties of the earth.

Other problems being actively considered include a calculation of the electric currents produced in the atmosphere by thunderstorms, the interaction of electrons with crystal lattices in simple one-dimensional crystals, and the scattering of protons by nuclei.

STUDIES IN PURE MATHEMATICS
Task 1101-11-5101/50-4
(formerly 11.1/1-50-4)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In publication (14) Dr. Szász defines a given function $\phi(t)$ and associates with a series $\sum_1^\infty a_n$ a linear transform

$$\Phi(h) = \sum_1^\infty a_n \phi(nh).$$

The transform is said to present a Gibbs phenomenon, if

$$G = \frac{2}{\pi} \lim_{\substack{h\to 0 \\ t\to 0}} \sup \sum_{n=1}^\infty \frac{\phi(nh)}{n} \sin nt > 1.$$

The main result of the paper is that for regular transforms the relation

$$G = \max_{0<\gamma<\infty} \frac{2}{\pi} \int_0^\infty \phi(t) \frac{\sin \gamma t}{t}\, dt$$

is valid. The following examples are discussed:

1) The Riesz means,
2) $\phi(t) = \exp(-t^\lambda)$,
3) $\phi(t) = \frac{2}{\pi} \mathrm{Si}(t)$,
4) $\phi(t) = \frac{te^{-t}}{1-e^{-t}}$.

Publications: (1) "Gibbs phenomenon for Hausdorff means," by O. Szász; Transactions of the American Mathematical Society 69, n. 3, 440-456; reprints available. (2) "A generalization of S. Bernstein's polynomials to the infinite interval," by O. Szász; NBS J. Res. 45, 239-245 (Sept. 1950), RP 2131. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents. (3) "On some trigonometric transforms," by O. Szász; accepted by the Pacific Journal of Mathematics. (4) "On positive harmonic functions and

ultraspherical polynomials," by W. Seidel and O. Szász; accepted by the Journal of the London Mathematical Society. (5) "On subharmonic and linear functions of two variables," by E. F. Beckenbach; accepted by Revista, Universidad Nacional de Tucuman (Argentina). (6) "Certain Fourier transforms of distributions," by E. Lukacs and O. Szász; accepted by the Canadian Journal of Mathematics. (7) "On a Tauberian theorem for Abel summability," by O. Szász; accepted by the Pacific Journal of Mathematics. (8) "On the relative extrema of ultraspherical polynomials," by O. Szász; Bolletino della Unione Matematica Italiana Ser. III, Ann. V, n.2, 125-127 (Jun. 1950); reprints available. (9) "Tauberian theorems for summability, (R_1)," by O. Szász; accepted by the American Journal of Mathematics. (10) "Relations among certain ranges of vector measures," by A. Dvoretzky, A. Wald, and J. Wolfowitz; accepted by the Pacific Journal of Mathematics. (11) "On relative extrema of Bessel Functions," by O. Szász; accepted by the Bolletino della Unione Matematica Italiana (Firenze) Florence, Italy. (12) "On the relative extrema of the Hermite orthogonal functions," by O. Szasz; accepted by the Bolletino della Unione Matematica Italiana (Firenze) Florence, Italy. (13) "On a recursion formula and on some Tauberian theorems," by Paul Erdös; accepted by the NBS Journal of Research. (14) "On the Gibbs phenomenon for a class of linear transforms," by O. Szász; accepted for publication in the 1950 Yearbook of the University of Belgrade, Yugoslavia.

COMPUTATION OF THE IMAGINARY ZEROS OF THE RIEMANN ZETA-FUNCTION
Task 1101-11-5101/50-13
(formerly 11.1/1-50-13)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1950 issue.

2. Applied Research

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-21-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF
Sponsor: " "
Manager: T. S. Motzkin
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A new method has been evolved by T. S. Motzkin for maximization and for the solution of systems of linear inequalities. This method, called the double description method, consists essentially in determining for any k first inequalities the vertices of the (bounded or unbounded) convex polyhedron defined by them. Preliminary small-scale tests on hand computing and I.B.M. machines have been carried

out and seem to compare very favorably with the classical method of elimination. The adaptability of the method for high-speed computing machines is being investigated. Both exact and probabilistic estimates of the number of operations needed have been proposed, partly in discussions with F. John. Theoretical and experimental comparison is also contemplated with the simplex method. The double description method seems to have natural advantages in case of a variable extremal form. On the other hand, the simplex method seems always advisable for amelioration of crude results; however, the final computation should be performed by one of the iterative methods for systems of linear equations. Concerning the latter, see task 1101-11-5100/49-AE2 for a method by G. E. Forsythe and T. S. Motzkin.

The real efficiency of any possible solution of large systems of inequalities hinges on probabilistic problems some of which have been developed by T. S. Motzkin and J. H. Curtiss. A first step to the solution of such problems has been attempted by R. Fortet in publication (1). The author considers determinants of the form

$$D_n = \left|\left| x_{ij} + X_{ij} \right|\right| ,$$

where the x_{ij}'s are given numbers and the X_{ij}'s are random numbers; he gives the values of $E(D_n)$ and $E(D_n^2)$ under some assumptions. Let $P_1,\ldots,P_n$, Q be n + 1 fixed points in a Euclidean space E_n, and $M_1,\ldots,M_n$ be n independent random points in E_n, P_i being the "mean position" of M_i; these M_i's determine a random plane π ; if $n \to +\infty$ and if the distance h from Q to the plane $(P_1,\ldots,P_n)$ remains constant and > 0, the probability that Q is above π tends toward 1. The preceding results on D_n are applied to the study of the distance from Q to π.

In publication (2) T. S. Motzkin presents a simple method of proving, by means of his transposition theorem, Farkas' combination lemma and the minimax principle used in linear programing (related to the one of von Neumann). Studies in the general theory of linear inequalities and convex bodies also have an application to the solution of partial differential equations by random walk procedures (see task 1101-11-5100/51-2 for a description of the work by W. R. Wasow).

The activities outlined above were centered around a seminar on linear programing which convened fifteen times during the quarter and included the following lectures: (1) "Introduction," by H. D. Huskey. (2) "Geometric considerations," by T. S. Motzkin. (3) "Probability problems," by J. H. Curtiss. (4) "The connection between linear programing and game theory," by G. Brown (RAND Corp.). (5) "Algebraic criteria," by T. S. Motzkin. (6) "Methods of solving systems of linear equations," by G. E. Forsythe. (7) "Linear programing activities in Washington, D. C.," by G. B. Dantzig (USAF). (8) "Duality in convex geometry," by T. S. Motzkin. (9) "Dantzig's simplex method of solution of the Koopmans-Hitchcock transporation problem," by H. Luxenberg. (10) "Minima under general inequality restrictions," by F. John. (11) "The connection between random determinants and a probability problem arising in linear programing," by R. Fortet. (12) "The double description method of maximization,: by T. S. Motzkin. (13) "On a gradient method for solving linear equations," by G. E. Forsythe.

Conferences with G. B. Dantzig (USAF), J. Todd (NBS), C. Tompkins (ONR), A. W. Tucker (Princeton University), and J.von Neumann (Institute for Advanced Study) served to guide the above activities.

Preparations are underway for a symposium in Maximization and Convex Geometry to be held in June 1951.

Publications: (1) "Random determinants," by R. Fortet, IN MANUSCRIPT. (2) "Two consequences from the transposition theorem on linear

inequalities," by T. S. Motzkin; accepted by Econometrica. (3) A Bibliography of the linear programing literature available at the INA prepared under the direction of H. D. Huskey and available in duplimat form at the INA. (4) A first draft of a general bibliography has been prepared under the direction of T. S. Motzkin and is now in manuscript form.

LANGUAGE TRANSLATION STUDY
Task 1101-21-1102/50-10
(formerly 11.1/1-50-10)

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The pre/publication report of the results of the work of Dr. Oswald and Mr. Fletcher on the automatic translation of mathematical German by a computer such as the SWAC was presented by Dr. Oswald to the seminar which is being conducted on the subject at the Institute for Numerical Analysis. As a result of this presentation the report was revised by the authors and is now being prepared for publication.

3. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(Previously listed as Air Materiel Command Computing Machine)
Task 1101-34-5103/49-1
(formerly 11.1/22-49-1)

Origin: Office of Air Research, AMC, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Operation of the computer continued on a test basis, interspersed with miscellaneous mathematical exercises and problems.

Coupling time constants were found to be inadequate in the addition circuits. This caused failures on products of certain numbers due to carry configurations. Accordingly, these time constants were suitably increased. Conversion of the CRT amplifiers to 2C51 type tubes continued and has been approximately 80 percent completed. The motor generator set, acquired from the University of California at Los Angeles (no charge basis), broke down and had to be returned to Westinghouse Corporation for repair. The difficulty of obtaining replacement parts for this equipment has delayed its completion; however, Westinghouse has promised to have the necessary repairs completed early next quarter. The motor generator set will make possible isolation from line voltage variations, which should result in more reliable operation. The magnetic tape unit has not yet arrived from Raytheon Corporation, but promise has been made of delivery early next quarter. Preparations are underway to integrate this tape unit into the computer system. Similarly,

work proceeded on the design and construction of the chassis needed to integrate the magnetic drum in the SWAC computer system.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-34-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Distribution continued of the preliminary lists of terminology and block diagram symbols to be used in connection with automatic computing machines. Considerable comment has been received on these lists, particularly from England, where the lists were reissued by the Electronic Glossary and Symbols Panel of the Central Radio Bureau to persons whom the committee thought likely to be interested in the subject. Efforts are being made to coordinate the work being done on this project with that of groups in other organizations interested in the subject.

Publications: The preliminary lists mentioned above are obtainable in manuscript form from the Institute for Numerical Analysis.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Task 1101-34-5103/50-1
(formerly 11.1/22-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

Publications: (1) "Semi-automatic instruction on the Zephyr," by H. D. Huskey; to appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory, September 1949.

PROGRAMING AND CODING OF PROBLEMS FOR SOLUTION ON THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER
Task 1101-34-5103/50-2
(formerly 11.1/22-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. In addition to many test routines used for checking out the computer, the following problem routines were prepared during the quarter:

1. Solution of the set of simultaneous differential equations

$$\dot{x} = yz$$
$$\dot{y} = -xz$$
$$\dot{z} = -k^2xy$$

with initial conditions $x = 0$, $y = z = 1$, $\dot{x} = 1$, $\dot{y} = \dot{z} = 0$, at $t = 0$. (The solutions of these equations are the Jacobi elliptic functions with modulus k.)

2. Computation of $N_2(2m)$, the number of solutions of the equation

$$2m = p_1 + p_2,$$

where p_1 and p_2 are prime, $p_1 \geqq p_2$, and m takes on various integral values up to 100,000. (The values obtained will be used to check a conjecture of Hardy and Littlewood concerning the asymptotic representation for N_2.)

3. Computation of the least eigenvalue of the Schrödinger equation for the harmonic oscillator

$$\tfrac{1}{2}\psi'' + (\lambda - x^2)\psi = 0,$$

subject to certain conditions on ψ. (A Monte Carlo method is used.)

4. A problem for the Office of the Air Comptroller.

5. Solution by the Monte Carlo method of the Laplace equation

$$\Delta u = 0$$

in a circle, with $u = xy$ on the boundary.

4. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-53-1100/49-1
(formerly 11.1/32-49-1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Among the problems involving the use of IBM equipment were:

1. For H. Gruen: Computation of elements of a 22 x 22 matrix as described below (task 1101-11-5100/50-3).
2. For T. S. Motzkin: A test of his method for solving a problem in linear programing (task 1101-21-5102/50-11).

Shorter computations on desk calculators were made for various members of the staff. Among such calculations are:

1. For J. B. Rosser: Extension of his work on the approximations of $\pi(x)$ and $\theta(x)$ for x beyond a million in the range of Lehmer's table of primes. Verification of $\pi(x)$ for $x = 4{,}876{,}800$ and $\theta(x)$ for $x = 2{,}676{,}445$ has been established.

2. For H. Gruen: Computation of elements of a matrix. [This work is needed in connection with a problem in finding the lowest eigenvalue and corresponding eigenvector of a matrix which arises from the Rayleigh-Ritz variational principle applied to the Schrödinger equation for a nuclear three-body problem with tensor forces (task 1101-11-5100/50-3).]

3. For T. S. Motzkin: Computations relating to his work on linear programing (task 1101-21-5102/50-11).

4. For W. E. Milne: Computations involved in the study and comparison of various methods of solving differential equations (task 1101-11-5100/51-1).

5. For D. Saxon: Computations pertaining to the evaluation of a Fourier Transform (task 1101-11-5100/51-5).

6. For C. Lanczos: (a) Computations relating to a precision approximation of the Gamma function in the entire right half plane. (b) Computations concerning the radiation of a cylindrical antenna. (task 1101-11-5100/51-4).

7. For G. E. Forsythe: Statistical test of randomness of digits generated on SWAC (task 1101-11-5100/51-2).

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI and 12 LACERTAE

Task 1101-53-1100/49-4
(formerly 11.1/32-49-4)

Origin: [illegible]
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 195[illegible] issue.

Status: CONTINUED. One possible way in which the period of a variable star can be determined is to represent the light curve of the star by a Fourier series and to determine the fundamental period of the Fourier series. As a test of the practicability of this method, a set of Fath's observations of 12 Lacertae was analyzed. Fath has shown that there are at least three incommensurate periods of variation in the light curve of this star. The first test, using a Fourier series in the primary period alone, showed a very slow rate of convergence. The second test, using a Fourier series in both the primary and secondary periods, converged in six iterations.

In the case of the successful test, the approximating series used was

$$m = m_0 + a_1 \sin\omega_1 t + a_2 \cos\omega_1 t + a_3 \sin\omega_2 t + a_4 \cos\omega_2 t.$$

This equation was linearized by expanding in a Taylor's series around a point in the seven-dimensional parameter space and neglecting all terms of second or higher order. The result of this expansion gave a linear equation in the seven first order corrections to be applied to the assumed parameters in order to obtain the solution. This observational equation was then evaluated for each of 132 observations and the set of 132 observational equations in seven unknowns solved by the standard least squares techniques. Since the observational equations used were linear approximations to the true non-linear equations, the solution yields a better approximation to the parameters rather than the best approximation. By successive iterations, however, the least squares solution of the non-linear equation can be approached.

This process becomes practicable when rapid computing equip-

ment is available. The tests in question were made on an IBM Card Programmed Calculator. Each iteration took about six hours, and another three hours were spent in setting up the problem on the machine. A total of one week's work gave a seven parameter fit to 132 observations with a precision of one part in five million for the periods and better than one part in ten thousand in the amplitudes.

The above abstract was presented at the American Astronomical Society Meeting at Haverford, Pa., December 27-30, 1950.

Publication : A paper entitled "A least-squares method of determining the periods of variable stars," by E. C. Yowell is in process of preparation.

MATHIEU FUNCTIONS II
Task 1101-53-1101/45-1
(formerly 11.1/2-45-1)

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Some further computations relating to $Se_0(s,x)$ were made during this quarter.

SPECIAL TABLE OF BESSEL FUNCTIONS
Task 1101-53-1101/48-2
(formerly 11.1/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Computations were completed. Final manuscript is to be made on a card-controlled typewriter.

ROCKET NAVIGATION TABLES
Task 1101-53-1101/48-3
(formerly 11.1/2-48-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Computations were completed. Final manuscript is to be made on a card-controlled typewriter.

TABLES OF $E_1(z)$, SECOND QUADRANT
Task 1101-53-1101/49-1
(formerly 11.1/2-49-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Calculations are completed and checked. The final manuscript is to be made on a card-controlled typewriter.

PUNCHED CARD LIBRARY
Task 1101-53-1101/49-2
(formerly 11.1/2-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. Checking was started on the NBS Table of Spherical Bessel Functions (see Apr-Jun 1950 issue). Tables for $\nu = 1/2(1)13/2$ have been differenced and partially checked. No new tables have been added to the library this quarter.

REDUCTION OF RAYDIST DATA
Task 1101-53-1101/49-2a
(formerly 11.1/32-49-2)

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Two runs of Raydist data were calculated during this quarter.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Task 1101-53-1101/49-6
(formerly 11.1/32-49-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Task 1101-53-1101/50-2
(formerly 11.1/32-50-2)

Origin: Naval Air Missile Test Center, Point Mugu
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Task 1101-53-1101/50-7
(formerly 11.1/31-50-7)

Origin: Lockheed Aircraft Corporation
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE.

ANALYSIS OF RAM-JET DATA
Task 1101-53-1101/50-10
(formerly 11.1/31-50-10)

Origin: Marquardt Aircraft Company
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE.

"BOILING" COMPUTATIONS
Task 1101-53-1101/50-12
(formerly 11.1/32-50-12)

Origin: Engineering Dept., U.C.L.A.
Sponsor: Atomic Energy Commission
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Data reduction has continued throughout the quarter.

METEOROLOGICAL MEANS
Task 1101-53-1101/50-17
(formerly 11.1/31-50-17)

Origin: Department of Meteorology, U.C.L.A.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Data gathered for January 1949, at all pressure levels, has been completed. Data for February 1949, at one pressure level, is 80 per cent completed.

EARTH TIDES
Task 1101-53-1101/51-1

Origin: University of California, Los Angeles, Geophysics Department
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE.

FOURTH ORDER EQUATIONS
Task 1101-53-1101/51-2

Origin: Naval Ordnance Test Station (Pasadena) Authorized 9/28/50
Sponsor: Bureau of Ordnance, USN Completed 12/31/50
Managers: G. Blanch, E. C. Yowell

Objective: To compute

$$\alpha_1 = 2\zeta_1\omega + \frac{\alpha_2 - (\omega^2 + \frac{1}{\omega^2})}{2\zeta_1\omega}$$

$$\alpha_3 = \frac{2\zeta_1}{\omega} + \frac{\alpha_2 - (\omega^2 + \frac{1}{\omega^2})}{\frac{2\zeta_1}{\omega}}$$

for $\alpha_2 = 3(1)6(2)20(5)30(10)50(50)100$,

$\zeta_1 = .02(.02).10(.10).80(.20)1.00(.50)3.00(1.00)6.00(2.00)10.00,$

$\omega = 1.0(.1)\sqrt{\alpha_2}.$

Background: These tables are useful in solving rapidly fourth degree algebraic equations of certain types.

Status: COMPLETED. Results have been transmitted to the Naval Ordnance Test Station.

EVAPORATION COMPUTATIONS
Task 1101-53-1101/51-3

Origin: Naval Electronics Laboratory
Sponsor: Bureau of Reclamation, Department of the Interior
Full project description appears in the July-Sept 1950 issue.

Status: CONTINUED. Computation of data received for the months of April, May, and June, 1950, have been completed.

BOUNDARY LAYER
Task 1101-53-1101/51-5

Origin: Northrop Aircraft Co., Inc.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in the July-Sept 1950 issue.

Status: CONTINUED. Computations have been halted pending further investigation of the theory by C. Lanczos.

ROLLING PULLOUT EQUATIONS OF MOTION
Task 1101-53-1101/51-6

Origin: Cornell Aeronautical Laboratory Authorized 9/28/50
Sponsor: Office of Air Research, AMC, USAF
Managers: G. Blanch, F. S. Acton

Objective: To solve nine simultaneous differential equations which describe the motion of an airplane in a rolling pullout.

Background: Under combat conditions it is often important for a plane to execute a pullout with a roll. It is known that this results in extraordinary stresses. The Cornell Aeronautical Laboratory is making a study of this problem for the Air Materiel Command.

Status: CONTINUED. Test computations on desk calculators are about 75 per cent completed. The programing of the calculations for the I.B.M. Card Programmed Calculator are in progress.

COMPUTATIONS IN CONNECTION WITH ONE-SIDED CONFIDENCE INTERVALS
Task 1101-53-1101/51-7

Origin: University of Washington Authorized 12/1/50
Sponsor: Office of Air Research, USAF
Manager: E. C. Yowell

Objective: To compute the value of ε defined by

$$P(n,\varepsilon) = 1(1-\varepsilon)^n - \varepsilon \sum_{j=1}^{[n(1-\varepsilon)]} \binom{n}{j} \left(1 - \varepsilon - \frac{j}{n}\right)^{n-j} \left(\varepsilon + \frac{j}{n}\right)^{j-1}$$

where $[x]$ = integral part of x such that $P(n,\varepsilon) = 1 - \alpha$ to four decimals for α = .10, .05, .01, .001

n = 5, 8, 10, 20, 40, 50.

Background: This computation arose in connection with a contract between ONR and the University of Washington to investigate distribution of the Kolmogoroff statistic.

Comments: This task is sponsored by the OAR in answer to a request from ONR because of interest of the Operations Analysis Group of USAF in this problem. The computations were specifically requested by Professor Z. W. Birnbaum of the University of Washington.

Status: NEW. All computations have been completed. Final checks are in progress.

LEAST SQUARES DETERMINATION OF CONSTANTS IN A CERTAIN DIFFERENTIAL EQUATION
Task 1101-53-1101/51-8

Origin: Ames Aeronautical Laboratory Authorized 12/1/50
Sponsor: National Advisory Committee for Aeronautics
Manager: E. C. Yowell

Objective: To determine the constants b, k, C_o, and C_1 in the equation

$$\frac{d^2 q}{dt^2} + b\frac{dq}{dt} + kq = C_o \delta(t) + C_1 \frac{d\delta}{dt}$$

from experimentally determined functions $\delta(t)$ and $q(t)$.

Comments: The methods for computing this problem were specified by Ames Aeronautical Laboratory.

Status: NEW. The computing was completed. The final check is being made and a report is being written to submit to the contractor.

ROOTS OF POLYNOMIALS
Task 1101-53-1101/51-9

Origin: Ames Aeronautical Laboratories Authorized 12/1/50
Sponsor: National Advisory Committee for Aeronautics
Manager: E. C. Yowell

Objective: To compute the roots of 487 algebraic polynomials of fourth and fifth degree.

Status: NEW. The computing was completed. The final checks are being made and a report is being written to submit to the contractor.

RAYDIST DATA ANALYSIS
Task 1101-53-1101/51-10

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 12/1/50
Sponsor: Bureau of Aeronautics, USN
Manager: E. C. Yowell

Objective: To calculate the coordinates of an object being followed by a Raydist tracking system.

Status: NEW.

CALCULATION OF POLYNOMIAL COEFFICIENTS
Task 1101-53-1101/51-11

Origin: Hughes Aircraft Company Authorized 11/30/50
Sponsor: Office of Air Research, USAF Completed 12/31/50
Manager: M. Howard

Objective: To find the coefficients of a twentieth-degree polynomial when given the twenty complex roots. To be performed for four sets of roots.

Status: NEW, COMPLETED.

VIBRATION DENSITY ANALYSIS
Task 1101-53-1101/51-12

Origin: Office of Air Research, AMC, USAF Authorized 12/1/50
Sponsor: " " Completed 12/31/50
Manager: G. Blanch

Objective: To fit regression lines to data comprising 28 separate charts and to calculate the corresponding σ_{y_s}.

Background: These data were collected on various types of aircraft in various flight conditions. For the 28 charts, the regression

lines and the standard deviations were calculated.

Status: NEW, COMPLETED.

II. Computation Laboratory

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-21-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Mr. Salzer worked on the following two topics:

(1) Formulas for calculating the error function of a complex variable. A note on this topic (cf. Jan-Mar. 1950 issue, p. 34) was revised, following suggestions of a referee.

(2) Sets of formulas for finding the argument for which a function has a given derivative have been determined. One set is given in terms of the actual tabular values and another in terms of the differences. A note containing these results has been submitted to a technical journal.

Dr. L. Fox (Exchange Consultant) worked on three topics during his stay at the Computation Laboratory:

(1) Relaxation and step-by-step methods for the solution of (ordinary) differential equations.

(2) The use of large intervals in numerical integration of differential equations.

(3) Tabulation of the exponential integral for large values of the argument.

Notes covering the work will be submitted to technical journals.

Publications: (1) "Formulas for calculating the error function of a complex variable," by H. E. Salzer; submitted to a technical journal. (2) "Formulas for finding the argument for which a function has a given derivative," by H. E. Salzer; submitted to a technical journal.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-21-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE.

RESEARCH IN MODERN NUMERICAL ANALYSIS: CONDITION OF MATRICES
Task 1102-21-1104/50-3
(formerly 11.2/11-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-21-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Dr. O. Taussky-Todd has completed the manuscript of a second paper on "Classes of matrices and quadratic fields."

Work on properties of characteristic roots of polynomials in finite matrices, studied from the arithmetic standpoint, was continued. A report on this was submitted to the American Mathematical Society Annual Meeting, Dec 27-30, 1950, by Dr. O. T-Todd and Mr. J. Todd.

Dr. Milton Abramowitz completed a table of $\int_0^{\phi} (\sin x)^{1/3}\, dx$ for $\phi = 0(1^{\circ})90^{\circ}$ to 8D and a table of $\frac{4}{3} (\sin \phi)^{-4/3} \int_0^{\phi} (\sin x)^{1/3}\, dx$ for $\phi = 0(\frac{1}{2}^{\circ})\ 180^{\circ}$ to 8S. These functions occur in a problem in heat transfer. The tables have been submitted for publication.

Dr. Abramowitz has begun a study of a problem in heat convection in laminar flow. This requires the solution to the differential equation

$$y'' + (1/x)\, y' + (1 - x^2/\beta^2)y = 0$$

subject to $y(0) = 1$, $y(\beta) = 0$. Available accounts of the problem are not complete.

Publications: (1) "Classes of matrices and quadratic fields," by O. Taussky-Todd; accepted for publication in the Pacific Journal of Mathematics. (2) "Tables of the functions $\int_0^{\phi} \sin^{1/3} x\, dx$ and $\frac{4}{3} (\sin \phi)^{-4/3} \int_0^{\phi} \sin^{1/3} x\, dx$," by M. Abramowitz; submitted to a technical journal.

NUMBER-THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-21-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. (1) The table of factors of $n^2 + 1$, n = 1(1)25000 has been completed, under the direction of Dr. John W.Wrench.

(2) Consecutive residues. This problem (see Apr-Jun 1950 issue, p. 31) which is connected with the so-called Legendre-Sophie St. Germain criterion has been completed as far as possible with the present tables of primitive roots (see (3) below). The results are now under investigation.

(3) A routine for the determination of least primitive roots of primes beyond the range of Cunningham's table has been prepared and tested.

STUDIES IN METHODS OF IMPROVING THE CONVERGENCE OF SERIES
Task 1102-21-1104/50-6
(formerly 11.2/11-50-6)

Origin: NBS — Authorized 6/1/50
Sponsor: Office of Air Research, AMC, USAF — Terminated 12/31/50
Manager: J.C.P.Miller

Objective: To compare and extend methods for obtaining high-accuracy numerical values from series of slow convergence, or of asymptotic and other divergent types.

Background: Much work has been done on many and various methods for obtaining sums of series which are either slowly convergent (e.g., of Dirichlet type, or ultimately approximately geometric with a common ratio approaching r, only a little less than one) or even divergent as with asymptotic series. For example, the Euler transformation is often effective, as is the "Converging Factor" introduced by Airey.

There is need for a monograph which contains accounts of as many of these methods as are useful, accompanied by plentiful numerical examples. Various new developments or extensions of old ones can also be usefully studied.

Status: TERMINATED, due to the return of Dr. J.C.P.Miller to England.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-21-1104/51-6

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. A two-dimensional case has been investigated in detail, and the coding for a three-dimensional case has been checked but not yet run (see task 1101-11-5100/51-2). The random numbers which

have been used were obtained by the following process, suggested by Dr. O. Taussky-Todd. The residues (mod 2^{42}) of an odd power of 5 have period 2^{40}; these residues (divided by 2^{42}) were used as random numbers. The reduction (mod 2^{42}) can be accomplished by a single operation--low multiplication--on the SEAC.

Estimates of the solution at various points in the square (at the center and near the boundaries) were obtained and compared with the theoretical results. The actual and theoretical variances at representative points were obtained and compared. Control charts were made. It appeared that the above method for the generation of random numbers was satisfactory for the present purpose. It is proposed to submit these random numbers to the usual randomness tests.

NON-NEGATIVE TRIGONOMETRIC POLYNOMIALS
Task 1102-21-1104/51-7

Origin: NBS Authorized 12/26/50
Manager: H. E. Salzer

Objective: To find the root of the algebraic equation

$$Q_s(z,n) \equiv \int_{-z}^{1} (1-t^2)^{n-1}\, t^{2(s-1)} (t^2-z^2)\,dt = 0 \qquad (1)$$

located in the interval (0,1) for the values s = 1,2 and n = 2,3,4,5,6,7, 8,9,10,15,20 as well as the asymptotic value $\rho_s = \lim_{n=\infty} \sqrt{n+s-\frac{1}{2}}\; z_s(n)$ where $z_s(n)$ is the solution of equation (1). It is known that ρ_s satisfies the transcendental equation

$$(2s-1)\Gamma(s-\tfrac{1}{2})\left[\tfrac{1}{2} - \frac{\rho^2}{2s-1}\right] - 2\int_0^{\rho} (\rho^2 x^{2s-2} - x^{2s}) e^{-x^2} dx = 0. \qquad (2)$$

It is also known that exactly one root $z_s(n)$ of (1) exists in (0,1), and that

$$0 < z_s(n) < \sqrt{\frac{s-\frac{1}{2}}{n+s-\frac{1}{2}}}.$$

The values of ρ_s are only needed for s = 1,2.

Background: In a joint paper E. Lukacs and O. Szász study certain trigonometric polynomials (task 1103-11-1107/51-2, item no. 3). It appears that the non-negativity of some of these trigonometric polynomials depends on the solutions of the above equations. It is therefore of interest to include a brief table of the roots of these equations in the paper which is currently being prepared.

Status: NEW. Computations were completed for s = 1.

ANALYSIS OF SAMPLING PLANS
Task 1102-21-1104/51-8

Origin: NBS Authorized 12/26/50
Managers: J. H. Levin and J. M. Cameron

Objective: To evaluate the operating characteristics of certain acceptance sampling plans by generating random samples from a normal probability distribution and applying the acceptance criteria of the plans to a large number of such samples.

Background: For certain acceptance sampling plans it is not possible to express explicitly the probability of acceptance for material of a given quality. In such cases an approximation to the desired probability can be obtained by sampling methods, which has meant forming random samples from a table of random numbers and applying the acceptance criteria of the plan and noting the proportion of samples passing. If the random numbers can be generated in the SEAC such a sampling plan can be evaluated in a short time.

Status: NEW. A method of generating random normal variates internally on SEAC was developed; it uses the random numbers generated by the process described in the status of the immediately preceding task. Preliminary tests of randomness were performed on these normal variates, and no departure from normality was detected. The operating characteristic curves of two acceptance sampling plans (which cannot be evaluated explicitly) were determined by generating samples from a normal population on the machine and counting the percentage of such samples passing the acceptance criteria of the plans.

2. Applied Research: Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-21-1104/43-3
(formerly 11.2/2-43-3)

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Subtabulation in certain parts of the range will be carried out on the SEAC. The completion of this phase of the project has been delayed due to lack of machine time.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-21-1104/46-1
(formerly 11.2/2-46-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1950 issue.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-21-1104/47-2
(formerly 11.2/2-47-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The manuscript of Vol. I has been completed, computations for Vol. II were completed and checked, and the manuscript is in preparation.

Publication: Volume I is being printed by the Government Printing Office and will be issued as a number in the NBS Applied Mathematics Series.

TABLE OF ANTILOGARITHMS
Task 1102-21-1104/47-3
(formerly 11.2/2-47-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of the manuscript by differencing continued.

TABLES FOR THE OCCASIONAL COMPUTER
Task 1102-21-1104/47-4
(formerly 11.2/2-47-4)

Origin: NBS — Authorized 7/1/47
Manager: Entire technical staff — Terminated 12/31/50

Objective: To prepare an improved and amplified version of the Jahnke-Emde tables.

Background: The preparation of an improved version of the Jahnke-Emde tables had been originally suggested by Professor E. P. Wigner of Princeton University who submitted suggestions for the contents of

the contemplated volume. This matter was discussed with Professor Tukey of Princeton, Professor Barkley J. Rosser of Cornell University, and others.

. Status: TERMINATED, in favor of task 1102-21-1104/51-4, "Collected Short Mathematical Tables of the Computation Laboratory."

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-21-1104/48-2
(formerly 11.2/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN HERMITE POLYNOMIALS
Task 1102-21-1104/49-1
(formerly 11.2/2-49-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were completed and checked. Preparation of the accompanying manuscript is in progress.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Task 1102-21-1104/49-2
(formerly 11.2/2-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date, see July-Sept 1950 issue.

FERMI FUNCTION, II
Task 1102-21-1104/49-10
(formerly 11.2/33-49-10)

Origin: NBS, Section 4.4
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as "Table for the analysis of β-spectra," NBS Applied Mathematics Series 13.

TABLES TO FACILITATE SEQUENTIAL t-TESTS
Task 1102-21-1104/50-2a
(formerly 11.2/2-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as "Tables to facilitate sequential t-tests," NBS Applied Mathematics Series 7.

TABLE OF CHEBYSHEV POLYNOMIALS
Task 1102-21-1104/50-3a
(formerly 11.2/2-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as NBS Applied Mathematics Series 9.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-21-1104/50-4a
(formerly 11.2/2-50-4)

Origin: NBS, Section 11.3
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The final manuscript was completed.

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-21-1104/50-5
(formerly 11.2/2-50-5)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. As part of this task a list has been prepared showing the location in the Washington libraries of those tables described in the Fletcher, Miller, and Rosenhead Index. This list is being reproduced.

TABLE OF n! and $\Gamma(n+\frac{1}{2})$
Task 1102-21-1104/50-6a
(formerly 11.2/2-50-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. In press.

Publication: "Tables of n! and $\Gamma(n+\frac{1}{2})$ for the first thousand values of n," by H. E. Salzer; accepted for publication in the NBS Applied Mathematics Series.

WAVE FUNCTION FOR LITHIUM
Task 1102-21-1104/50-7
(formerly 11.2/2-50-7)

Origin: NBS
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Program for SEAC is underway.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-21-1104/51-4

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Preparation of the manuscript is in progress.

RELATIVE ABUNDANCE OF THE ELEMENTS
Task 1102-21-1104/51-1

Origin: Applied Physics Laboratory, Johns Hopkins University
Authorized 9/28/50
Completed 12/31/50
Manager: J. H. Levin

Objective: To solve the following system of differential equations:

$$\frac{d\xi_n}{d\tau} = -\left(1 + \frac{3}{2}\right)\xi_n - P_1 \xi_n \xi_1 - P_2 \xi_n \xi_2 - P_3 \xi_n \xi_3 - P_4 \xi_n \xi_4$$

$$\frac{d\xi_i}{d\tau} = \xi_n - P_1 \xi_n \xi_1 - \frac{3}{2\tau}\xi_1$$

$$\frac{d\xi_i}{d\tau} = P_i \xi_n \xi_{i-1} - P_i \xi_n \xi_1 - \frac{3}{2\tau}\xi_i, \quad (i = 2,3,\ldots,).$$

In the above equations τ represents time, ξ_n is the relative abundance of neutrons, and ξ_i is the relative abundance of the element having atomic number i. P_i is essentially the effective neutron capture volume swept out per second by nuclei of element having atomic number i.

Background: The above equations arise out of a formulation of the theory of the "neutron capture process" for the formation of the elements, suggested by R. A. Alpher, H. Herman, and G. Gamow. This theory, in explaining the relative abundance of the elements, takes into account formation of elements by neutron capture, radioactive decay of neutrons, and dilution of matter resulting from expansion of the universe.

Comments: References: (1) A neutron-capture theory of the formation and relative abundance of the elements, R. A. Alpher, The Physical Review, vol. 74 (Dec.1, 1948). (2) On the relative abundance of the elements, R. A. Alpher and H. Herman, The Physical Review, vol. 74, (Dec. 15, 1948). (3) Remarks on the evolution of the expanding universe, R. A. Alpher and H. Herman, The Physical Review, vol. 75 (Apr. 1, 1949).

Status: COMPLETED. Results were turned over to the originators.

CRYSTAL STRUCTURE PROBLEM FOR POINT ATOMS
Task 1102-21-1104/51-2

Origin: Naval Research Laboratory, USN
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE.

ANALYSIS OF CRYSTAL STRUCTURE
Task 1102-21-1104/51-3

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. A number of revisions are being incorporated in the program for SEAC.

X-RAY PENETRATION PROBLEM
Task 1102-21-1104/51-5

Origin: NBS
Manager: O. Steiner

Authorized 9/28/50
Completed 12/31/50

Objective: To solve on the SEAC the following integral equations for $a_{\ell n}(\lambda)$:

$$\mu(\lambda)\, a_{\ell n}(\lambda) = \int_{\lambda_0}^{\lambda} k(\lambda,\lambda')\, P_\ell(1-\lambda+\lambda')\, a_{\ell n}(\lambda')\, d\lambda'$$

$$-\alpha \sum_{n'=0}^{n-1}\left[\frac{\ell+1}{2\ell+1}\, a_{\ell+1,n'}(\lambda') + \frac{\ell}{2\ell+1}\, a_{\ell-1,n'}(\lambda')\right]$$

$$+ \lambda_0 k(\lambda_0,\lambda) P_\ell(1-\lambda+\lambda_0)\, \frac{\alpha}{\mu_0}\left[1-\frac{\alpha}{\mu_0}\right]^n,$$

where $k(\lambda',\lambda)$ is the Klein-Nishina cross-section,

P_ℓ is the ℓ^{th} Legendre polynomial,

μ is total cross section (varying with material),

$\alpha = \mu$(minimum), and

$a_{\ell n}(\lambda)$ are used to solve the transport equation governing X-ray penetration.

Background: The transport equation governing X-ray penetration of materials has been reduced to the above chain of Volterra integral equations. It is desired to investigate the possibility of using the SEAC in the solution of such chains of integral equations. If this problem can be easily handled on the SEAC, then a program will be considered for determining X-ray penetrations for a wide variety of cases.

Comments: This project was proposed by Dr. L. Spencer, NBS 4.8.

Status: COMPLETED. A number of test cases were successfully computed on the SEAC, and the results were turned over to Dr. L. Spencer, NBS 4.8, who will account for them in a forthcoming publication.

Status of Projects

3. Mathematical Services

HEAT CONDUCTION EQUATION
Task 1102-53-1106/46-1
(formerly 11.2/33-46-1)

Origin: Bureau of Ordnance, Department of the Navy — Authorized 7/1/47
Manager: Ida Rhodes — Completed 12/31/50

Objective: To obtain numerical solutions of the non-linear partial differential equation

$$\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2} + e^{-1/\theta} \quad (k = \text{constant})$$

satisfying the initial and boundary conditions; $\theta(x,0) = \theta_0$ and $\theta(0,t) = \theta_1$, for various values of θ_0 and θ_1.

Background: The problem had its origin in the investigation of the flow of heat from a hot thermostatic bath into an explosive substance in the form of a sphere immersed in the bath. For practical applications it is permissible to consider the explosive as a plane slab of infinite extent.

Status: TERMINATED. With the successful computation of test cases on the SEAC, it is considered that the primary purpose of this task, which was to determine the feasibility of a numerical approach to this problem, has been accomplished.

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Task 1102-53-1106/49-2
(formerly 11.2/33-49-2)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameters a_i, m_i were performed when requested.

LINEAR PROGRAMING ON STANDARD PUNCHED CARD MACHINES
Task 1102-53-1106/49-3
(formerly 11.2/36-49-3)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Programs were being calculated as requested by the originators.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/49-13
(formerly 11.2/33-49-13)

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

GUST ATTACKS ON DELTA WING
Task 1102-53-1106/50-1
(formerly 11.2/31-50-1)

Origin: Aircraft Laboratory, AMC, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations on the cases requested so far are 90% completed.

STANDARD LORAN TABLES
Task 1102-53-1106/50-1a (formerly 11.2/34-50-1):Gulf Coast Chain
Task 1102-53-1106/50-5 (formerly 11.2/34-50-5): Hawaiian Islands Chain
Task 1102-53-1106/51-5: Marshall Islands Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Calculations were completed, and results were submitted to the U. S. Hydrographic Office. Preparation of a final manuscript was underway.

RESEARCH IN LINEAR PROGRAMING
Task 1102-53-1106/50-2
(formerly 11.2/12-50-1)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Research on numerical methods and efforts at coding such methods for SEAC were continued. In particular, a code for the "transportation problem" was completed and two small test cases were successfully run on the machine. These cases refer to relatively small matrices containing no more than 200 elements. In order to handle larger cases, it would be necessary either to increase the internal memory of the machine or to change the code so as to make use of the external memory.

Publication: "Bounds of characteristic roots of matrices, II," by O. Taussky-Todd, accepted by the NBS Journal of Research.

A PROBLEM IN MOLECULAR STRUCTURE, I
Task 1102-53-1106/50-3
(formerly 11.2/33-50-3)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations for various values of the parameters were performed as requested.

ADAPTATION OF LORAN CALCULATIONS TO CARD PROGRAMMED CALCULATOR AND NBS AUTOMATIC COMPUTER
Task 1102-53-1106/50-3a
(formerly 11.2/34-50-3)

Origin: Hydrographic Office, USN
Sponsor: " "
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A program for the IBM Card Programmed Calculator was completed and is going to be used by the Hydrographic Office. Programing for the SEAC is in progress.

WAVE RESISTANCE OF SHIPS, III
Task 1102-53-1106/50-11
(formerly 11.2/33-50-11)

Origin: David Taylor Model Basin, USN
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Computation of certain auxiliary functions is in progress.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-53-1106/50-16
(formerly 11.2/33-50-16)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations were performed as requested.

MOMENTS OF THE DISTRIBUTION OF RANKED EXTREME VALUES
Task 1102-53-1106/50-18
(formerly 11.2/33-50-18)

Origin: N. Y. State Public Service Commission
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1950 issue.

Status: INACTIVE.

ANALYSIS OF UNDERWATER SOUND MEASUREMENTS
Task 1102-53-1106/51-6

Origin: Underwater Sound Laboratory, USN
Sponsor: " "
Full project description appears in the July-Sept 1950 issue.

Status: CONTINUED. Computations are in progress.

PROGRAM COMPUTATION ON THE SEAC
Task 1102-53-1106/51-7

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in the July-Sept 1950 issue.

Status: CONTINUED. Three additional programs were completed, and a fourth is in progress. Revisions in the code, taking advantage of new machine features such as increased memory and use of magnetic wire and tape for input, output, and storage, are being incorporated.

FLOW IN SUPERSONIC NOZZLES
Task 1102-53-1106/51-13

Origin: Naval Ordnance Laboratory Authorized 12/15/50
Sponsor: " " "
Manager: M. M. Andrew

Objective: To calculate the variation of flow in a test section of a supersonic nozzle for a given range of Mach numbers.

Background: The contour of a supersonic nozzle can be determined, neglecting boundary layer effects, such that the flow in a test section of nozzle will be uniform for one assumed Mach number. If the ratio of the nozzle diameter at the throat to the nozzle diameter at the test section is varied, the Mach number of the flow will change and the flow will become non-uniform. The problem is to calculate the amount of this non-uniformity in the flow. Analytically, the problem is one in potential theory where it is necessary to integrate the following system of hyperbolic equations over the area of a meridional nozzle section:

$$\frac{\partial \theta}{\partial \phi} = \frac{1}{M\left(1+\frac{M^2}{3}\right)^{3/2}} \frac{\partial M}{\partial \psi}$$

$$\frac{\partial M}{\partial \phi} = \frac{M}{(M^2-1)\left(1+\frac{M^2}{5}\right)^{3/2}} \frac{\partial \theta}{\partial \psi}$$

where ϕ are the potential lines of the flow
ψ are the stream lines of the flow
M is the Mach number
θ is the angle of the stream line with respect to the axis of symmetry of the nozzle.

Status: NEW. Programing for SEAC is completed and coding is 30% completed.

CORPORATE INCOME TAX
Task 1102-53-1106/51-14

Origin: U. S. Treasury Department — Authorized 12/1/50
Sponsor: " " — Completed 12/31/50
Manager: M. Abramowitz

Objective: To compute on punched cards the Federal corporation income tax for a sample of corporations under various hypothetical tax rates.

Background: These computations are needed in the evaluation of various proposals for revising the corporate income tax structure.

Status: COMPLETED. (NEW). Results were transmitted to the originator.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-51-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Tables of the first four virial coefficients for nitric oxide were completed. Tables of the temperature derivatives are under way.

BASIC IONOSPHERIC DATA
Task 1401-34-1412/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

RADIO-TELEGRAPH INTERFERENCE
Task 1404-34-1423/49-17
(formerly 11.2/33-49-17)

Origin: NBS, Section 14.4
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

RATING OF WATER CURRENT METERS
Task 0605-41-0621/50-2
(formerly 11.2/33-50-2)

Origin: NBS, Section 6.5
Sponsor: "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were performed as requested.

IONOSPHERIC WINDS
Task 1401-11-1401/50-7
(formerly 11.2/33-50-7)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Data were processed as requested.

CRYSTAL STRUCTURES OF CEMENT COMPOUNDS
Task 9099-00-9009/50-9
(formerly 11.2/33-50-9)

Origin: NBS, Division 9, Portland Cement Research Associate Project
Sponsor: Portland Cement Association
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. Specification for parameters is pending.

RAY TRACING
Task 0202-21-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 1.6
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Skew rays have been traced and third order aberrations computed for optical systems consisting of spherical lens surfaces, as requested. A method of tracing rays with non-spherical surfaces has been devised and tried out in a number of cases.

WAVE PROPAGATION IN THE IONOSPHERE
Task 1401-11-1400/50-14
(formerly 11.2/33-50-14)

Origin: NBS, Section 14.1 — Authorized 3/31/50
Sponsor: " — Completed 12/31/50
Managers: M. Abramowitz, Gertrude Blanch

Objective: To solve a pair of simultaneous ordinary differential equations of order 4 with two-point boundary conditions, describing the propagation of electromagnetic waves of vertical incidence in a magnetic field of general intensity and direction.

Background: The equations arise in the magneto-ionic theory of wave propagation in the ionosphere.

Comments: Specifically requested by Drs. A. E. McNish and J. Feinstein of the Ionospheric Research Laboratory. The computations will be performed largely by the Institute for Numerical Analysis.

Status: COMPLETED. Results were turned over to the originators.

ELECTRON TRAJECTORIES
Task 1202-34-6354/51-1

Origin: NBS, Section 12.2 — Authorized 9/28/50
Manager: M. M. Andrew — Completed 12/31/50

Objective: To calculate solution on the SEAC of the non-linear differential equation

$$\frac{d^2\psi}{d\theta^2} = \eta_0 \Omega_0 \cos\psi \sin(\theta + \delta)$$

for $\psi(0) = 0$ and $\frac{d\psi(0)}{d\theta} = \Omega_0$. The exit velocity Ω_f of the electron from the electromagnetic field is to be calculated for $\eta_0 = .025(.05)0.475$, $\Omega_0 = .025(.025)0.35$ and $\delta = 0\ (\frac{\pi}{4})\ \frac{7\pi}{4}$. Ω_f is defined as $\frac{d\psi(\theta_f)}{d\theta}$ with $\psi(\theta_f) = \pi$. A total of 1120 Ω_f are to be computed, which will require about $5\frac{1}{2}$ hours time on the SEAC.

Background: This differential equation, which represents a one-dimensional electron trajectory in an electromagnetic field, arose from a study of Dr. R. T. Young of the NBS Tube Laboratory, in considering the efficiency of operation of tubes of the klystron type.

Status: COMPLETED. Results were turned over to the originators.

NUMERICAL SOLUTION OF A SET OF DIFFERENTIAL EQUATIONS CHARACTERIZING A DEPOLYMERIZING SYSTEM
Task 0700-12-0700/51-2

Origin: NBS, 7.0
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE.

THREE COIL SYSTEM FOR PRODUCING A UNIFORM MAGNETIC FIELD
Task 0303-11-2608/51-3

Origin: NBS, Division 3 Authorized 10/14/50
Sponsor: "
Manager: Irene Stegun

Objective: To solve the following equations

$$C_3 = (C_3)_a + (C_3)_b - (C_3)_c = 0$$
$$C_5 = (C_5)_a + (C_5)_b - (C_5)_c = 0 ,$$

where

$$C_3 = -\left(\frac{I}{\ell}\right)\left[\beta^3 (\beta^2 + \ell^2)^{-3/2} - \alpha^3(\alpha^2 + \ell^2)^{-3/2}\right]$$

$$C_5 = -\left(\frac{I}{12\ell^3}\right)\left[\beta^3(2\beta^4 + 7\beta^2\ell^2 + 20\ell^4)(\beta^2 + \ell^2)^{-7/2} - \alpha^3(2\alpha^4 + 7\alpha^2\ell^2 + 20\ell^4)(\alpha^2 + \ell^2)^{-7/2}\right],$$

for

(a) $\ell = \frac{1}{2} B, \quad \alpha = a - \frac{1}{2} D_0 , \quad \beta = a ,$

(b) $\ell = x_0 + \frac{1}{2}B , \quad \alpha = a - \frac{1}{2} D_0 , \quad \beta = a + \frac{1}{2}D_0 ,$

(c) $\ell = x_0 - \frac{1}{2}B , \quad \alpha = a - \frac{1}{2} D_0 , \quad \beta = a + \frac{1}{2}D_0 ,$

for the unknowns... $\frac{B}{a}$, $\frac{D_0}{a}$ and $\frac{x_0}{a}$, for 10 values of the parameter $\frac{BD_0}{a^2}$ ranging between .2 and .35.

Background: A set of modified Helmholtz coils for producing magnetic fields up to 450 gauss, with extreme flatness of field, is being designed and constructed in section 3.3 for use in superconductivity investigations. These computations are for obtaining design data.

Status: NEW.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS

Task 1103-11-1107/47-2
(formerly 11.3/2-47-2)

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The final manuscript was received from Professor Chand. A summary table of the formulas treated, together with formal specifications of the situations to which they are respectively applicable, is being prepared, after which the manuscript is to be submitted for publication by the Bureau.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY

Task 1103-11-1107/48-3
(formerly 11.3/2-48-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date, see Apr-Jun 1950 issue.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE

Task 1103-11-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Work on the preparation of abstract cards continued. The present bibliography covers essentially the material reviewed in Mathematical Reviews plus such supplementary material gathered from various sources (such as obituaries, lists of publications, etc.) as becomes available from time to time. It was decided to extend the scope and the usefulness of the guide by including gradually the literature covered by the Zentralblatt. It is planned to photostat the relevant abstracts and thus to prepare abstract cards for the literature of the decade preceding the foundation of Mathematical Reviews. As it is highly desirable to have a subject index to the bibliography, the problems of indexing the abstract files were discussed. Several members

of the staff of SEL attended conferences organized by the Bureau's Office of Basic Instrumentation, which has similar problems of indexing.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Task 1103-11-1107/49-3
(formerly 11.3/1-49-3)

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The revision of the manuscript of Professor H. B. Mann's monograph on the theory of stochastic processes is almost completed. The monograph was surveyed with great care, and a number of desirable additions and modifications of proofs and of some details was noted. The next step will be a conference with Professor Mann on these matters, which is to be held in January 1951.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Task 1103-11-1107/49-3a
(formerly 11.3/2-49-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The manuscript has been revised in the light of comments by technical readers. The references to available tables have been considerably expanded to include additional citations given by Fletcher, Miller, and Rosenhead, "Index of Mathematical Tables," and to include more recent text books in the field.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Task 1103-11-1107/50-1
(formerly 11.3/1-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE.

EXTREME-VALUE THEORY AND APPLICATIONS
Task 1103-11-1107/50-1a
(formerly 11.3/2-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Three of the four lectures have been typed on ditto stencils and a major portion of the charts have been reworked and traced by a draftsman in form suitable for preliminary reproduction.

When this stage is completed the dittoed manuscript will be ready for final review by the author, Dr. Gumbel, prior to its submission for publication by the Bureau.

As the work progressed, further correspondence with the author was required, chiefly in connection with additional technical changes pertaining to the charts for the monograph and the manuscript for the probability tables for extreme values. The latter publication, being prepared under NAML task 1102-21-1104/50-4a, is awaiting the retyping of certain portions to incorporate the suggested changes.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-11-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS
Manager: J. H. Curtiss
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Member of the Statistical Engineering Laboratory Staff conferred with Dr. C. P. Winsor of Johns Hopkins University and jointly prepared a list of mathematical models of commonly encountered situations in which the fitting of straight lines to the data may be appropriate. A selected bibliography of reference material was prepared.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-11-1107/51-1

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The tables have been checked and are awaiting final preparation for publication.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-11-1107/51-2

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. 1. A preliminary draft of the manuscript of a joint paper by E. Lukacs and O. Szász entitled "Some non-negative trigonometric polynomials connected with a problem in probability" was completed and sent to Professor O. Szász. A few points require clarification before the paper can be submitted for publication.

2. The Hungarian journal, Matematikai Lapok, is publishing a series of articles entitled "Thirty years of mathematics in the Soviet Union," written by Alfred Renyi. The title of the first paper is "Foundations of probability theory"; that of the second is "New lines of research in probability theory." Two more parts are to follow. These articles contain interesting material but are practically inaccessible to the American public since they are written in Hungarian. It is intended to

prepare a translation of the papers dealing with probability and statistics and to issue them either as working papers or as reports.

3. Mr. Lieblein obtained the theoretical comparison between the operating characteristic curves (cumulative distributions) for the distributions

$$f_1(t) = \frac{m}{\sqrt{\pi}} e^{-\frac{1}{4}m^2t^2} , \qquad 0 \leq t < \infty , \qquad (1)$$

where $t = |x_1 - x_2| / m$, and $m = E|x_1 - x_2| = 1.1284$ is the mean range for samples of 2 from a unit normal distribution; and

$$f_2(t') = \frac{3\sqrt{2}}{\pi} m' \int_{\sqrt{3/2}\, m't'}^{\infty} e^{-\frac{1}{2}\left[x^2 + \frac{1}{2}(m')^2(t')^2\right]} dx, \quad 0 \leq t' < \infty, \qquad (2)$$

where $t' = (x' - x'')/m'$, and $m' = E(x' - x'') = .4535$ is the mean of the difference between the two closest values in a sample of 3 from the normal. This comparison is needed in an inquiry by Dr. Youden into possible methods for discriminating between a set of data obtained from groups of triplicate measurements by discarding the outlying one from each and a set consisting of actual duplicates. Work sheets have been laid out for making the numerical comparisons with sufficient accuracy.

4. Dr. Eisenhart formulated, in outline, a mathematical theory of direct-measurement processes in which the strong law of large numbers plays a central role, and which, in addition to representing sequences of independent measurements, makes provision for intra-class correlation and/or serial correlation of the measurements as well. This very general mathematical formulation of direct-measurement processes was developed in preparation for his talk, "On the meaning and representation of accuracy and precision," at the (Bureau's) Atomic and Radiation Physics Staff Meeting, 18 December 1950; and for his invited address on "The specification of precision" at a joint meeting of the American Society for Quality Control, the American Statistical Association, the Biometrics Society, and the Institute of Mathematical Statistics, Chicago, Illinois, 27 December 1950.

Publications: (1) "Maximum likelihood estimate of position derived from measurements performed by hyperbolic instruments," by E. Lukacs; accepted by NBS Journal of Research. (2) "On the stochastic independence of symmetric and homogeneous linear and quadratic statistics," by E. Lukacs; submitted to a technical journal for publication.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3000-21-0002/51-1

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this project fell into two main categories:

A. The Design of Experiments. Typical examples were: (1) An allocation of specimens from seven strips of titanium alloy to seven test loads at three test locations was constructed so that the comparison of threshold stress values at a location could be unaffected by possible difference in properties of the strips. (2) An experiment involving electrodeposited metals was designed so that the reproducibility of a salt spray test could be determined for each of four testing laboratories.

B. The Development or Selection of the Appropriate Statistical Methods for Analysis and Interpretation of Data. Examples were: (1) The Gumbel extreme value theory was recommended as the appropriate technique for estimating the maximum wind expected for a given long time period. This maximum expected wind was to be used for determining load factors in building construction codes. (2) Two problems involving the use of regression analysis were investigated, and the need for a weighted analysis was demonstrated in both cases. (3) Various authors were assisted in the evaluation and representation of the accuracy and precision of their results, and, at the invitation of the Division of Atomic and Radiation Physics, Dr. Eisenhart spoke "On the meaning and representation of accuracy and precision" before their Seminar of December 18, 1950.

RESEARCH ON APPLICATION OF THEORY OF EXTREME VALUES TO GUST LOAD PROBLEMS
Task 1103-21-5106/51-1

Origin: NACA, Dynamic Loads Division
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Attempts were made during the quarter to bring the available basic data within known types of probability models, mainly the class of compound Poisson distributions studied by Neyman, Feller, and others. The data did not satisfy any of the tests of agreement with the simple or compound Poisson distributions that could be brought to bear on them; and it appeared likely that the observed airplane accelerations due to gusts were not governed by random factors alone, but were to a considerable extent influenced by operating practice. To settle this question definitely it would be necessary to have certain specialized data more amenable to deeper analysis, together with fuller indication of the kinds of conditions under which the airplanes were flown.

Other questions to be studied further include the adaptation of extreme-value theories to data giving the maximum gust loads encountered in flights.

IV. Machine Development Laboratory

(Section 11.4)

in cooperation with

Electronic Computer Section

(Section 12.3)

1. Development: Design and Construction of Automatic Digital Computing Machines

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This requirement will be fulfilled with a UNIVAC which is being constructed by the Eckert-Mauchly Computer Corporation under contract CST 10279. During the quarter negotiations were conducted to revise the specifications to provide a machine with additional self-checking features. In addition arrangements were completed whereby the contractor will provide operation and maintenance services during the first year of the machine's operation. It is expected that the first UNIVAC will be completed during the first quarter of 1951.

THE NAVY COMPUTING MACHINE
Task 1104-34-5107/47-2
(formerly 11.4/22-47-2)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Raytheon Manufacturing Company, contractor, made substantial progress on the two computers which it is constructing under another Government contract and which will precede in delivery the computer being constructed under contract with the Bureau. The contractor estimated that on its first computer the engineering design was 100% completed and the actual construction 50% completed.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/ 24-47-3)

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation, under a supplement to the Census UNIVAC contract. The design status is the same as that of task 1104-34-5107/47-1. Delivery of the computer is expected during the second quarter of 1951.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Note: This computer has been previously referred to as the NBS Interim Computer and as the NBS Automatic Computer I.

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During this period, the SEAC was operated on a 24-hours-a-day, seven-days-a-week basis. Of the 168 hours of working time each week, 16 hours were reserved for preventive maintenance operations on the machine, 76 hours were allocated for engineering development and testing of new computer equipment, and 76 hours were reserved for the solution of problems. Of this latter 76 hours, only that portion of time in which problem-solutions or coding-checks were turned out correctly by the machine is considered "productive" time; the remainder of the 76 hours is considered "down" time for the SEAC, even where the machine was in good operating order but idle.

During the quarter, a magnetic wire input mechanism was installed and used on a limited basis. In addition, substantial progress was made on the installation of the electrostatic memory. It is anticipated that both the magnetic input equipment and the electrostatic memory will be in routine operation during the first quarter of 1951.

The following tabulation shows the percentage of problem-solution time which was productive during October, November, and December, 1950.

Week of:

October					November				December			
1st	8th	15th	22nd	29th	5th	12th	19th	26th	3rd	10th	17th	24th
53%	20%	75%	88%	72.5%	53%	76%	73%	63%	84%	67%	81%	92%
Average for October: 61.7%					Average for November: 66%				Average for December: 81%			

See task 1104-53-1108/47-4 for information on specific problems for which the SEAC has been used.

AIR MATERIEL COMMAND COMPUTING MACHINE
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Work continued on this computer by the General Electric Company, at its Electronic Park Laboratories, in Syracuse, New York.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, U.S.A.
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The design status is the same as that of task 1104-34-5107/47-1. Delivery of this computer, dependent upon the Eckert-Mauchly production facilities, is expected to follow closely that of the Air Comptroller's UNIVAC.

2. General Services

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Task 1104-53-1108/47-4
5108/
(formerly 11.4/3-47-4)

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A large variety of problems have been coded and run successfully on the SEAC. A tabular summary of the problems worked on during the past six months follows:

Problems Coded and Run on SEAC
July 1 - December 31, 1950

Name	Time Spent on Coding Analysis, Consultation, Preparation of Tape, Checking, etc.	Machine Time (1)
Jacobi Elliptic Functions (see Jul-Dec 1947 issue, project 43D2)	½ hours	10½ hours
Number Theoretical Test Problems (see task 1102-21-1104/50-5a)	177 "	200½ "
Optical Ray Tracing (see task 0202-21-2308/50-13)	24 "	1½ "
Analysis of Crystal Structure (see task 1102-21-1104/51-3)	202½ "	½ "
Gamma Ray Penetration (see task 1102-21-1104/51-5)	54 "	5 "
Relative Abundance of the Elements (see task 1102-21-1104/51-1)	120 "	48½ "
Linear Programing - OAC (see task 1102-53-1106/50-2)	2387½ "	525½ "
Determination of Optimum Values for Subsampling Design - BuCensus (see task 1102-53-1106/51-1)	383 "	68 "
Electron Trajectories (see task 1202-34-6354/51-1)	80 "	40 "
LaPlace Equation by Monte Carlo Method (see task 1102-21-1104/51-6)	156½ "	26½ "
Four classified tasks	1891 (3) "	211 "
Circuit Design(C. Page)	-- (2)	72 "
Van der Pol's Equations	15 "	17½ "
Banana River Task	4 "	8½ "
Social Security Agency (see task 1104-53-5108/51-1)	--	19 "
Analysis of Sampling Plans (see task 1102-21-1104/51-8)	101½ "	9½ "
$E_1(z)$ (see task 1102-21-1104/43-3)	-- (2)	9 "
Non-Linear Spring (C. Page)	-- (2)	7 "
Totals	5596½ hours	1280 hours

(1) Machine time = Good operation time + code checking + downtime.

(2) Coded by personnel outside the SEAC unit.

(3) The estimate for one of these projects includes the time spent through 10/31/50 only.

CODING RELATED TO THE UNIVAC SYSTEM
Task 1104-53-5108/49-2
(formerly 11.4/3-49-2)

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

CODING RELATED TO THE RAYTHEON COMPUTER
Task 1104-53-5108/49-3
(formerly 11.4/3-49-3)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS OF THE SOCIAL SECURITY AGENCY
Task 1104-53-5108/51-1

Origin: Social Security Agency Authorized 12/31/50
Sponsor: " "
Manager: Ida Rhodes

Objective: The purpose of the survey will be to investigate the applicability of automatic digital electronic computing to the record keeping and statistical operation of the Division of Accounting Operations of the Social Security Agency.

Background: The Social Security Agency has requested this survey to be performed by staff members of the Machine Development Laboratory of the National Bureau of Standards, with the cooperation of procedural consultants of the Division of Accounting Operations. Equipment requirements are to be evaluated by the Electronics Division of the Bureau. In connection with the survey, representative problems of the Division are to be tried out on existing computing machines. At the conclusion of the survey, a report of findings and recommendations are to be submitted to the Division.

Status: NEW. During the quarter, instruction in the principles of electronic computation has been given to members of the staff of the Division of Accounting Operations. A program of instructions specifically applicable to Social Security Agency problems has been run on the SEAC.

THE MTAC SECTION
Task 1104-51-1109/47-1
(formerly 11.4/4-47-1)

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for inclusion in the April 1951 issue of Mathematical Tables and Other Aids to Computation was assembled, edited, and forwarded to Prof. D. H. Lehmer, editor. In addition, galley and page proofs for the January issue were edited.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Task 1104-51-1109/49-2
(formerly 11.4/42-49-2)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

Lectures and Symposia

Applied Mathematics Division Technical Meeting

FOX, L. (National Physical Laboratory, Teddington, England). On the work of the Mathematics Division of the National Physical Laboratory. October 18, 1950.

Statistical Engineering Seminars

SICHEL, H. S. (CSIR, Union of South Africa). (1) Statistical evaluation of a mine and grade control of the ore. December 13, 1950. (2) Absence, accident, and error proneness. December 15, 1950.

Numerical Analysis Colloquium Series
(Los Angeles, California)

KNOPP, K. (University of Tubingen, Germany). Analytic continuation by the summability methods of Euler and Borel. October 9, 1950.

LEWY, H. (University of California, Berkeley). (1) On solutions of difference equations and of differential equations. October 31, 1950. (2) On a method of solving linear equations if the solution is integral. November 2, 1950.

FORTET, R. Additive functionals of a Markoff process. November 13,1950.

WASOW, W. On the duration of random walks. December 4, 1950.

POLYA, G. (Stanford University). Two problems of approximation. December 18, 1950.

Papers and Invited Talks Presented by Members of the Staff at Outside Meetings

CAMERON, J. M. Use of components of variance in preparing schedules for sampling of baled wools. Presented at a meeting of the American Statistical Association, Chicago, Illinois, December 27, 1950.

CURTISS, J. H. Sampling methods in the solution of partial differential equations. Presented at the Peripatetic Seminar of the California Institute of Technology, the University of Southern California, and the University of California at Los Angeles, November 6, 1950.

EISENHART, C. (1) Some notes on statistical prediction. Presented to the Washington Section, American Society for Quality Control, October 24, 1950. (2) On the meaning and representation of precision and accuracy. Presented at a meeting of the NBS Atomic and Radiation Physics Division, December 18, 1950. (3) The specification of precision of measurements. Presented to the Biometrics Section of the American Society for Quality Control and the Institute of Mathematical Statistics, Chicago, Ill., December 27, 1950.

FORTET, R. (1) Random function derived from a Poisson process. Presented at the Statistical Laboratory, University of California, Berkeley, November 7, 1950. (2,3) Additive functionals of a Markoff process. Presented to the Mathematics Department, Stanford University, November 9, 1950; also presented at Cornell University, November 29, 1950. (4) Random function derived from a Poisson process. Presented to the Mathematics Department, Cornell University, November 30, 1950. (5) Low and large numbers and the ergodic theory. Presented at the Peripatetic Seminar in Mathematics, University of Southern California, December 4, 1950. (6) Study of parabolic equations with the aid of a relative Markoff process. Presented to the Department of Mathematics, University of California, Berkeley, December 7, 1950.

FOX, L. (National Physical Laboratory, Teddington, England) and YOUDEN, W. J. On the work of the Mathematics Division of the National Physical Laboratory. Presented at NBS Scientific Staff Meeting, November 10, 1950.

HUSKEY, H. D. (1) Automatic computing machines. Presented to the Glendale-Burbank Engineers' Club, Glendale, California, October 10, 1950. (2) The future of large-scale automatic digital computing machines and computation laboratories. Presented at Wright-Patterson Air Force Base, Dayton, Ohio, December 7, 1950. (3) Tube experience in the SWAC. Presented at the IRE/AIEE/RDB Joint Conference on Electron Tubes for Computers, Atlantic City, N. J., December 12, 1950.

LIPKIS, R. Coding for the SWAC. Presented at the U. S. Naval Air Missile Test Center, Point Mugu, California, October 19, 1950.

LUXENBERG, H. Programing for the SWAC. Presented at the U. S. Naval Air Missile Test Center, Point Mugu, California, October 12, 1950.

RHODES, I. Coding of the SEAC. Presented at a technical seminar of the Applied Mathematics Division, U. S. Naval Ordnance Laboratory, White Oak, Maryland, October 20, 1950.

SAXON, D. S. On properties of solids. Presented at the Physics Seminar, University of California at Los Angeles, October 30, 1950.

SALZER, H. E. (1) Formulas for finding the argument for which a function has a given slope. Presented by title at the meeting of the American Mathematical Society, Evanston, Illinois, November 24, 25, 1950. (2) Equally weighted quadrature formulas over semi-infinite and infinite intervals. Presented by title at the Annual Meeting of the American Mathematical Society, Gainesville, Florida, December 27-29, 1950.

TAUSSKY, O. Classes of matrices. Presented to the Columbian Mathematics Club, George Washington University, Washington, D. C., December 4, 1950.

TAUSSKY, O. and TODD, J. The characteristic roots of products of matrices with rational integral elements. Presented by title at the Annual Meeting of the American Mathematical Society, Gainesville, Florida, December 27-29, 1950.

YOUDEN, W. J. (1) Improving the precision of measurements. Presented at a Conference on Scientific Method in Industrial Production, held by the Royal Statistical Society, Sheffield, England. (2) The precision of comparative measurements, (3) Best two out of three, (4) Measurement schedules and precision, presented at three Colloquia at the National Physical Laboratory, Teddington, England. (5) Applications of statistical methods in the Physical Sciences. Presented at a seminar in Biometry, Oxford University, England, October 27, 1950. (6) Statistics and planning of tests at elevated temperatures. Presented at a meeting of the Society for Experimental Stress Analysis, New York, New York, November 27, 1950. (7) What is an average brood? Presented to Fish and Wild Life Service, Department of Interior Seminar, Washington, D. C., December 5, 1950. (8) Statistical evaluation of data. Presented to meeting of the American Chemical Society, Philadelphia, Pennsylvania, December 13, 1950. (9) Revolution in methods of experimentation. Presented at a Joint Meeting of the American Statistical Association and the Institute of Mathematical Statistics, Chicago, Illinois, December 28, 1950.

YOWELL, E. C. A least-squares method of determining the periods of variable stars. Presented at the Astronomical Society Meeting, Haverford, Pa., December 27, 1950.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables for conversion of X-ray diffraction angles to interplanar spacing. H. Swanson. NBS Applied Mathematics Series 10. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., $1.75 (buckram).

1.3 Technical Papers

(1) Forced oscillations in non-linear systems. M. L. Cartwright. NBS J. Res. 45, 514-518 (Dec. 1950), RP 2166. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(2) Sampling methods applied to differential and difference equations with special reference to equations of the elliptic type. J. H. Curtiss. Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, New York, November 1949, pp. 87-109. Reprints available.

(3) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. C. Lanczos. NBS J. Res. 45, 255-282 (Oct. 1950), RP 2133. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 15 cents.

(4) Note on Vincent's theorem. A. M. Ostrowski. Ann. Math. 52, 702-707 (Nov. 1950). Reprints available.

(5) Real roots of real Dirichlet L-series. J. B. Rosser. NBS J. Res. 45, 505-514 (Dec. 1950), RP 2165. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(6) A generalization of S. Bernstein's polynomials to the infinite interval. O. Szász. NBS J. Res. 45, 239-245 (Sept. 1950), RP 2131. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents.

(7) Gibbs' phenomenon for Hausdorff means. O. Szász. Transactions of the American Mathematical Society 69, n.3, 440-456 (Nov.1950). Reprints available.

(8) On the relative extrema of ultraspherical polynomials. O.Szász. Bollettino della Unione Matematica Italiana Ser. III, Ann. V, n. 2, 125-127 (Jun. 1950). Reprints available.

(9) On the relative extrema of the Laguerre orthogonal functions. John Todd. Bolletino della Unione Matematica Italiana Ser. III, Ann. V, n. 2, 122-125 (1950). Reprints available.

(10) A note on the four by four Latin squares. W. J. Youden. Biometrics 6, 289-290 (Sept. 1950). Reprints available.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1950.

2.1 Mathematical Tables

(1) Tables of the exponential function e^x. Formerly NBS Mathematical Table MT2. Third edition to be issued as NBS Applied Mathematics Series 14. In press, Government Printing Office.

(2) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. In press, Government Printing Office.

(3) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(4) Table of Arctangents of rational numbers. J. Todd. NBS Applied Mathematics Series 11. In press, Government Printing Office.

(5) Table for the analysis of β spectra. NBS Applied Mathematics Series 13. In press, Government Printing Office.

(6) Tables of n! and $\Gamma(n+\frac{1}{2})$ for the first thousand values of n. H. E. Salzer. Accepted for publication in the NBS Applied Mathematics Series.

(7) Tables of Coulomb wave functions, Vol. I. Accepted for publication in the NBS Applied Mathematics Series.

(8) The hypergeometric and Legendre functions with applications to integral equations of potential theory. C. Snow. Revised version approved for publication in the NBS Applied Mathematics Series. (Formerly NBS MT15, out of print).

(9) Tables relating to the Mathieu functions. In press, Columbia University Press.

(10) Table of the integral $\int_0^x e^{-u^3} du$. M. Abramowitz. Accepted by the Journal of Mathematics and Physics.

(11) Tables of the functions $\int_0^\phi \sin^{1/3}x\, dx$ and $(4/3)\sin^{-4/3}\phi \int_0^\phi \sin^{1/3}x dx$ M. Abramowitz. Submitted to a technical journal.

2.3 Technical Papers

(1) Coulomb wave functions expressed in terms of Bessel-Clifford functions and Bessel functions. M. Abramowitz. Accepted by the Journal of Mathematics and Physics.

(2) Machine methods for finding the characteristic roots of a matrix. F. L. Alt. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., December 1949.

(3) An analysis of the effect of the discontinuity in a bifurcated circular guide upon plane longitudinal waves. L. L. Bailin. Accepted for publication in the NBS Journal of Research.

(4) On subharmonic, harmonic, and linear functions of two variables. E. F. Beckenbach. To appear in Revista, Universidad Nacional de Tucuman (Argentina).

(5) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. To appear in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(6) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel, and O. Szász. Accepted by the Duke Mathematical Journal.

(7) A problem in precision cam design. J. Blum. Accepted by the NBS Journal of Research.

(8) The use of components of variance in preparing schedules for sampling of baled wools. J. M. Cameron. Submitted to a technical journal.

(9) The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation. J. M. Cameron and W. J. Youden. Accepted for publication in the A.S.T.M. Proceedings.

(10) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Accepted by the Duke Mathematical Journal.

(11) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(12) Some problems on random walk in space. A. Dvoretzky and P. Erdös. To be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950.

(13) Elimination of randomization in certain statistical decision procedures and zero-sum two-person games. A. Dvoretzky, A. Wald, and J. Wolfowitz. Submitted to a technical journal.

(14) Relations among certain ranges of vector measures. A. Dvoretzky, A. Wald, and J. Wolfowitz. Accepted for publication by the Pacific Journal of Mathematics.

(15) On a recursion formula and on some Tauberian theorems. P. Erdös. Accepted by the NBS Journal of Research.

(16) New matrix transformations for obtaining characteristic vectors. W. Feller and G. E. Forsythe. Accepted for publication in the Quarterly of Applied Mathematics.

(17) Generation and testing of random digits at the National Bureau of Standards, Los Angeles. G. E. Forsythe. To appear in "The Monte Carlo method: Proceedings of a symposium held June 29,30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office.

(18) Second order determinants of Legendre polynomials. G.E.Forsythe. Accepted by the Duke Mathematical Journal.

(19) The extent of n random unit vectors. G.E. Forsythe and J. W. Tukey (Princeton University). Accepted by the American Mathematical Monthly.

(20) On some functionals of Laplacian processes. R. Fortet. Submitted to a technical journal.

(21) Quadratic forms in Hilbert space, with applications in the calculus of variations. M. Hestenes. Accepted for publication in the American Journal of Mathematics.

(22) A method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix. M.R. Hestenes and W. Karush. Accepted for publication in the NBS Journal of Research.

(23) The solution of $Ax = \lambda Bx$. M. R. Hestenes and W. Karush. Submitted to a technical journal.

(24) Semi-automatic instruction on the Zephyr. H. D. Huskey. To appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory, September 1949.

(25) On some connections between probability theory and differential and integral equations. M. Kac. To be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950.

(26) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Accepted by the Bulletin of the American Mathematical Society.

(27) Determination of the extreme values of the spectrum of a bounded self-adjoint operator. W. Karush. Submitted to a technical journal.

(28) An iterative method for finding characteristic vectors of a symmetric matrix. W. Karush. Submitted to a technical journal.

(29) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. Accepted by the NBS Journal of Research.

(30) A method of summing infinite series. S. Lubkin. Accepted by the NBS Journal of Research.

(31) Maximum likelihood estimates of position derived from measurements performed by hyperbolic instruments. E. Lukacs. Accepted by the NBS Journal of Research.

(32) Certain Fourier transforms of distributions. E. Lukacs and O. Szász. Accepted by the Canadian Journal of Mathematics.

(33) On the stochastic independence of symmetric and homogeneous linear and quadratic statistics. E. Lukacs. Submitted to a technical journal.

(34) Numerical methods associated with Laplace's equation. W. E. Milne. To appear in the Proceedings of a Symposium held at Harvard Computation Laboratory, September 1949.

(35) Two consequences from the transposition theorem on linear inequalities. T. S. Motzkin. Accepted for publication in Econometrica.

(36) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Accepted by the Bulletin of the American Mathematical Society.

(37) Note on an infinite integral. A. M. Ostrowski. Accepted for publication in the Duke Mathematical Journal.

(38) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(39) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(40) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(41) Transformations to speed the convergence of series. J.B.Rosser Accepted for publication in the NBS Journal of Research.

(42) A method of computing exact inverses of matrices with integer coefficients. J. B. Rosser. Accepted by the NBS Journal of Research.

(43) The separation of close eigenvalues of a real symmetric matrix. J. B. Rosser, C. Lanczos, M. R. Hestenes, W. Karush. Accepted by the NBS Journal of Research.

(44) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(45) Formulas for numerical integration of first and second order differential equations in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(46) Checking and interpolation of functions tabulated at certain irregular logarithmic intervals. H. E. Salzer. Accepted for publication in the NBS Journal of Research.

(47) An elementary note on powers of quaternions. H. E. Salzer. Submitted to a technical journal.

(48) Radix table for obtaining trigonometric and inverse trigonometric functions to many places. H. E. Salzer. Submitted to a technical journal.

(49) Formulas for calculating the error function of a complex variable. H. E. Salzer. Submitted to a technical journal.

(50) Formulas for finding the argument for which a function has a given derivative. H.E. Salzer. Submitted to a technical journal.

(51) A bibliography of numerical methods in conformal mapping. W. Seidel. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards Applied Mathematics Series.

(52) On positive harmonic functions and ultraspherical polynomials. W. Seidel and O. Szász.. Accepted for publication in the Journal of the London Mathematical Society.

(53) On a Tauberian theorem for Abel summability. O. Szász. Accepted for publication by the Pacific Journal of Mathematics.

(54) Tauberian theorems for summability (R_1). O. Szász. Accepted for publication by the American Journal of Mathematics.

(55) On some trigonometric transforms. O. Szász. Accepted by the Pacific Journal of Mathematics.

(56) On the relative extrema of the Hermite orthogonal functions. O. Szász. Submitted to a technical journal.

(57) On the relative extrema of Bessel functions. O. Szász. Accepted for publication by the Bolletino della Unione Matematica Italiana (Firenze).

(58) Identities and inequalities concerning orthogonal polynomials and Bessel functions. O. Szász. Submitted to a technical journal.

(59) On the Gibbs phenomenon for a class of linear transforms. O. Szász. Accepted for publication in the 1950 Yearbook of the University of Belgrade, Yugoslavia.

(60) The convergence of Cauchy-Riemann sums to Cauchy-Riemann integrals. O. Szász and J. Todd. Accepted by the NBS Journal of Research.

(61) Classes of matrices and quadratic fields. O. Taussky-Todd. Accepted for publication in the Pacific Journal of Mathematics.

(62) Bounds for characteristic roots of matrices, II. O.Taussky-Todd. Accepted by the NBS Journal of Research.

(63) On conformal mapping of variable regions. S. E. Warschawski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(64) Random walks and the eigenvalues of elliptic difference equations. W. Wasow. Accepted for publication in the NBS Journal of Research.

(65) On the mean duration of random walks. W. Wasow. Accepted by the NBS Journal of Research.

(66) Numerical solution of partial differential equations. E. C. Yowell. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., December 1949.

(67) A Monte Carlo method for solving Laplace's equation. E. C. Yowell. To appear in the Proceedings of a Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., December 1949.

2.5 Miscellaneous Publications

(1) Problems for numerical analysis of the future. (Four papers presented at the Symposia on Numerical Analysis and Automatic Calculating Machinery, held at the NBS Institute for Numerical Analysis, Los Angeles, California, June 1948). To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(2) The Monte Carlo method; Proceedings of a symposium held on June 29, 30, July 1, 1949, in Los Angeles, California, under the sponsorship of the Rand Corporation, and the NBS, with the cooperation of the Oak Ridge National Laboratory. NBS Applied Mathematics Series 12. In press, Government Printing Office.

(3) The construction and applications of conformal maps: Proceedings of a symposium held at the NBS Institute for Numerical Analysis, Los Angeles, California., June 1949. To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(4) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

(5) High-speed computing and accounting. H. D. Huskey and V. R. Huskey. To appear in the Journal of Accountancy.