Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

July through September 1949

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES

July 1 through September 30, 1949

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NATIONAL BUREAU OF STANDARDS
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Projects and Publications
of the
NATIONAL APPLIED MATHEMATICS LABORATORIES

July through September 1949

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from July 1, 1949 through September 30, 1949.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss

Chief

E. U. Condon

Director
National Bureau of Standards
November 15, 1949

Index of Active Research and Development Projects

Note: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, and Astronomy.

D. Mathematical Statistics

E. Mathematical Tables

F. Manuals, Bibliographies, Indices, and Technical Information

G. Computing Machine Development

Status of Projects

September 30, 1949

I. Institute for Numerical Analysis

(Section 11.1)

1. Research in Numerical Analysis

Key to letter symbols in project numbers:

AE - Algebraic equations
AM - Analytical applied mathematics
CM - Numerical methods in conformal mapping
ODE - Ordinary differential equations
PD - Partial differential equations
PM - Probabilistic methods in numerical analysis
SF - Studies of special functions

DETERMINATION OF CHARACTERISTIC VALUES OF MATRICES
Project 11.1/1-49-AE1

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Terminated 9/30/49
Manager: C. Lanczos

Objective: To develop a practical and economical method of evaluating the characteristic values of arbitrary complex matrices.

Background: The calculation of characteristic values of matrices is of importance in many problems in physics, engineering, and applied mathematics. In particular, characteristic values play an important role in flutter and other vibration analysis problems, and it was the inadequacy of methods currently in use for such problems which furnished the motivation for the present project.

Comments: The principle underlying the present investigation consists in iterating a trial vector to set up a system of linear equations having a recurrent matrix, and then designing an algorithm for the solution of the recurrent system. The solution of the circular system yields the coefficients of the characteristic equation. This method is analogous to the method on which project 11.1/1-49-ODE4 is based.

Status: TERMINATED. It was decided that the scope of the objective should be widened, so project 11.1/1-50-3 was set up to replace this project. See current status report for project 11.1/1-50-3.

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Project 11.1/1-49-AE2

Origin: NBS — Authorized 2/15/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Managers: J. B. Rosser, C. Lanczos, Gertrude Blanch, G. E. Forsythe

Objective: To develop practical and economical methods for solving linear and non-linear simultaneous equations in many unknowns, to be used in connection with various types of computing equipment, and to study related problems of matrix inversion and iteration.

Background: Sets of simultaneous algebraic equations and related matrix problems arise repeatedly in all branches of applied mathematics.

Status: CONTINUED. Dr. Lanczos' results are incorporated in his comprehensive work on the eigenvalue problem; see project 11.1/1-50-3. Dr. Blanch studied various techniques for "positioning" the elements of a matrix. She will pursue this matter further during the remainder of the year.

In general, work on this project was subordinated to other interests of the staff. However, at the end of the quarter, a conference on the project was held by Dr. Rosser and greatly increased activity is expected in the future.

SOLUTION OF EQUATIONS IN A SINGLE UNKNOWN
Project 11.1/1-49-AE3

Origin: NBS — Authorized 4/1/49
Sponsor: Office of Naval Research, USN — Revised 9/15/49
Manager: A. M. Ostrowski

Objective: To find new and better ways of dealing with algebraic and transcendental equations in a single unknown.

Background: Such equations arise repeatedly in all branches of applied mathematics.

Status: CONTINUED. Dr. Ostrowski prepared a note giving the results obtained in a study of Vincent's method for separating the real roots of an algebraic equation [Publication (2)]. A summary of the results is included in the previous quarter's status report. The old and rare textbook mentioned was obtained, but did not contribute anything new to the project manager's earlier knowledge of the history of the method.

Dr. Ostrowski continued to work on the improvement of Newton's method for calculating the roots of a polynomial.

Publications: (1) "On two problems in abstract algebra connected with Horner's rule," by A. M. Ostrowski; submitted to the American Mathematical Monthly. (2) "Note on Vincent's theorem," by A. M. Ostrowski; submitted to the Annals of Mathematics.

SOLUTION OF THE TELEGRAPHER'S EQUATION FOR INITIAL CONDITIONS GIVEN ON ONLY ONE CHARACTERISTIC
Project 11.1/1-49-AM2

Origin: NBS
Sponsor: Office of Naval Research, USN
Managers: G. E. Forsythe, Gertrude Blanch

Authorized 11/1/48
Terminated 9/30/49

Objective: (1) To solve the telegrapher's equation

$$\frac{\partial^2 v}{\partial x \partial t} + \frac{1}{4} v = 0$$

for $t > 0$ with the sole boundary condition that v is known for all x when $t = 0$. Here x is essentially terrestrial longitude and t is time. (2) To investigate the solution of a related difference equation. (3) To present the solution in such a form that meteorologists can see its consequences and try it out in forecasting.

Background: This boundary-value problem arises in Rossby's linearized treatment of the equation of non-divergent flow. The harmonic solutions have been discussed many times and it is considered important to see what happens with arbitrary initial conditions. In general, the problem has no unique solution for $-\infty < x < \infty$. The fact that initial conditions are given on only one characteristic makes it a most unusual problem for a hyperbolic partial differential equation. In the present treatment the solution is rendered unique by using the roundness of the earth, i.e., by making the solution periodic in x.

Status: TERMINATED. Although Dr. Forsythe has certain unpublished results on (2) in the Objective, there will be an indefinite delay in preparing these for publication, because of other demands on his time. Hence the project has been terminated for the time being.

Publication: "Solution of the telegrapher's equation with boundary conditions on only one characteristic," by G. E. Forsythe; accepted for publication in the NBS Journal of Research.

THEORETICAL AND PRACTICAL PROBLEMS IN THE APPLICATION OF A.D.C.M. TO THE DETERMINATION OF ORBITS
Project 11.1/1-49-AM3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The previous status report for this project described progress on a manuscript entitled "A Modification and Appraisal of Gibbs' Orbit Method," and a trip to the Yale University Library to consult some recently discovered manuscripts of Willard Gibbs. As a result of the report on these manuscripts a grant of somewhat over $1,000 was made by the National Academy of Sciences Watson Fund to consolidate the new material in the manuscript and complete the work.

The previous report also described progress on a text on rocket navigation. In the present quarter two new chapters were completed; one was completely revised; and several were completely outlined. Numerical investigations and examples make up a sizable portion of the work completed.

Publications: (1) "Introduction to tables for rectilinear and nearly-rectilinear ('nearly parabolic') orbits," to be published in the NBS Applied Mathematics Series; see also project 11.1/2-48-3. (2) "A modification and appraisal of Gibbs' orbit method," by S. Herrick; IN MANUSCRIPT in possession of the author. (3) The text on rocket navigation cited above.

THEORETICAL AMPLITUDE DISTORTION IN A CONDENSER MICROPHONE CIRCUIT
Project 11.1/1-49-AM4

Origin: Department of Physics, UCLA — Authorized 11/1/48
Sponsor: Office of Naval Research, USN — Terminated 9/30/49
Manager: G. E. Forsythe

Objective: (1) To study mathematically the relation between the variable (input) capacitance of the condenser element and the variable (output) voltage across the resistance of an RC condenser microphone circuit. In particular, to find conditions under which the voltage faithfully reproduces the capacitance, as the resistance becomes infinite.
(2) To calculate the amplitudes of the various harmonics in the output voltage, corresponding to a pure sinusoidal input and a finite resistance.

Background: A standard reference on the condenser microphone, E. C. Wente, Physical Review 10, 39-63 (1917) makes some approximations which raised questions about the practical effect of dead capacitance in parallel with a condenser microphone. P. M. Morse, "Vibration and sound," gives a proof which is valid only for infinite resistance. It is believed that no one has presented an exact formula for the amplitudes of the various harmonics for finite R. All these matters are dealt with in the present project which was suggested by Professor R. W. Leonard of the Department of Physics, UCLA.

Comments: Professor Leonard plans an experiment to determine the amplitudes mentioned under (2) above. After his circuit constants are known, a calculation may be carried out to confirm his results. It would involve rapidly convergent series of Bessel functions.

Status: TERMINATED. The project has been inactive for some time, pending an experiment to be performed by Professor Leonard. Since it now appears that there will be an indefinite delay in performing this experiment, the project has been terminated for the present.

NUMERICAL METHODS IN CONFORMAL MAPPING
Project 11.1/1-49-CM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This project was perhaps the most active of the research projects of the Institute during the quarter. The activity was largely analytical and expository, much of it being centered about the preparation of papers for the "Proceedings" of the Symposium held in June (see Publications). However, at the end of the quarter several of the project managers (notably Drs. Warschawski and Ahlfors) had arrived at points in their work at which they felt that extensive numerical experimentation was now justifiable and desirable.

Dr. Beckenbach, who is the editor of the "Proceedings," worked during the quarter almost exclusively on editorial tasks connected with preparing the manuscripts for the printer.

Dr. Seidel, who left the Institute to return to his academic post after the third week of the quarter, wrote up his Bibliography for the Proceedings [Publication (1,i)], and further revised his translation Publication (2).

Dr. Warschawski spent much of the time writing up his paper for the Proceedings [Publication (1,ii)]. This paper has now become a comprehensive review of results dealing with the variation of the mapping function corresponding to deformations in the mapped region. Both "nearly circular" and arbitrary regions are considered. Emphasis is placed on theorems of a quantitative type which give actual estimates for the modulus of the difference between the mapping functions for two regions which are "close" to each other. A considerable amount of foreground research entered into the preparation of the paper.

Dr. Warschawski also began a study of the error term involved in the Bieberbach method of constructing mapping functions by the use of the Rayleigh-Ritz principle. He expects to continue this work at the University of Minnesota; it will be supplemented by numerical experimentation.

Dr. Ostrowski completed two papers giving the results of his study of the convergence features of the method of conformal mapping developed by Theodorsen and Garrick. The contents of these papers had been presented by Dr. Ostrowski at the Symposium on Conformal Mapping held at the Institute for Numerical Analysis in June, and the two papers will appear in the Proceedings [Publications (1,iii) and (1, iv)]. The results contained in the previous quarter's status report are included in the first paper [Publication (1,iii)]. Also included in this paper is the further result that the convergence of the sequence of the derivatives of approximating functions can be established under less restrictive conditions. The second paper [Publication (1, iv)] deals with the matrix analogue of the Theodorsen and Garrick method. The method starts from the approximating trigonometric expression for the singular integral involved in Theodorsen's and Garrick's method, and gives a number of properties of the corresponding matrices. These matrices are singular but present many analogies with orthogonal matrices. The determination of their characteristic numbers enables one to show that the analogue of Warschawski's convergence condition cannot be improved. The numerical procedure involved is appropriate for use with high-speed computing machinery.

Dr. Weinstein, with the help of Mr. R. J. Diamond, one of the summer fellows at the Institute, wrote up the paper which he had presented at the Symposium [Publication (1, v)] during a two-weeks' visit to the Institute.

Dr. Lars Ahlfors was present at the Institute only for the month of August. During his visit he prepared the report referred to below in

Publication (3). This report discusses various ways of constructing the mapping function which maps a polygon on a circle or half plane. The starting point of this report is as follows: Let the polygon lie in the z-plane with vertices at z_k, k = 1,2,...n, and let the angles be denoted by $\alpha_k \pi$. The mapping on the unit circle is then given by

$$z = C \int^{w} \prod_{k=1}^{n} (w - a_k)^{\alpha_k - 1} \, dw,$$

where $a_k = e^{i\omega_k}$ and the constant C are unknown parameters. Three of the ω_k can be chosen arbitrarily, and C determines only the size of the polygon. However, in general one is required to determine the mapping function which lets w = 0 correspond to a prescribed point z_o in the polygon. It is therefore better to make this normalization and treat all differences $\omega_{k+1} - \omega_k$ as unknown. Now

$$\omega_{k+1} - \omega_k = 2 \pi u_k(z_o)$$

where u_k (the harmonic measure) is the harmonic function in the polygon which is equal to one on the side $\overline{z_k z_{k+1}}$ and zero on the rest of the boundary. The most important problem is therefore to compute the harmonic measures of the sides. If this is done for a fixed z_o the inverse function z(w) will be determined explicitly (up to a constant factor) by the Schwarz-Christoffel formula. On the other hand, if two harmonic measures are computed for variable z it is easy to determine the direct mapping function (w(z) will be the intersection of two circular arcs). The report therefore concentrates on finding a numerical method for calculating the harmonic measure of a polygon side,

Two quite different procedures for calculating the harmonic measure are thus proposed. The first is based on polynomials possessing extremal properties on the boundary of the region to be mapped in the unit circle. These polynomials are closely related to the Szegö orthogonal polynomials. An advantage of this method is that it yields simple explicit formulas, but the computation of the formulas is probably prohibitively time-consuming, even for a.d.c.m. Dr. Ahlfors also considers briefly the computing method proposed by Bergman involving areal as opposed to line integrals, but expresses a preference for the line integrals. The second method is a generalization of Schwarz's "alternating method," or method of overlapping regions. The first step is to cover the region to be mapped by a set of overlapping subregions for which the Dirichlet problem is solvable. It is shown that the problem reduces to the solution of an integral equation of the following type:

$$u(x) = \int_a^b K(x,t)u(t)dt + F(x),$$

where F(x) and K(x,t) are explicitly known in a given case (they are obtained from the assumption that the Dirichlet problem is solvable for each of the covering subregions). The report then discusses iteration methods of solving the integral equations. Dr. Ahlfors believes that the second method outlined above is more promising for numerical work, and recommends strongly that it be tested.

Publications: (1) "Proceedings of a symposium on the construction and application of conformal mapping" edited by E. F. Beckenbach; to be published by NBS. The volume will include the following papers written in connection with this project: (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A. M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by Albert Weinstein.

(2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W. Seidel; probably to be published in NBS Applied Mathematics Series.

(3) "Numerical methods in conformal mapping," by Lars Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

MONOGRAPH ON NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE1

Origin: NBS — Authorized 11/26/48
Sponsor: Office of Naval Research, USN — Terminated 9/30/49
Manager: W. E. Milne

Objective: To prepare a systematic exposition of the theory and techniques of integrating differential equations, with special emphasis on ordinary differential equations. The treatise will include chapters on the analysis of remainder terms, the accumulation of errors, relaxation methods, and other topics pertinent to a.d.c.m.

Background: The literature of numerical methods in differential equations is at present scattered, incomplete, and often aimed at very special cases. A convenient, up-to-date treatise is very much needed. (See also project 11.1/1-49- DE2.)

Comments: Project 11.1/1-49-ODE2 is intended to furnish some of the groundwork for this project.

Status: TERMINATED. The project was terminated when the project manager returned to his regular academic post at Oregon State College. Work is continuing there on the project. It is expected that Dr. Milne will return to the staff of the Institute some time during the fiscal year 1951, at which time the project will be completed.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Dr. Milne left the I.N.A. for his academic post at the beginning of the quarter, but a considerable amount of work continued to be done on this project, both at the I.N.A. and by Dr. Milne at home.

In particular, Publication (3) was prepared by Dr. Ostrowski and Mr. Jackson from correspondence with Professor Bieberbach. The correspondence was initiated by Dr. Ostrowski when he acted as a reader for Publication (2) and noticed that no proof was available for a pertinent formula in Bieberbach's "Theorie der Differentialgleichungen." Bieberbach's reply provided an improvement of the formula but contained minor algebraic errors, which were corrected by Dr. Ostrowski and Mr. Jackson. A principal result is that if $y(x)$ denotes the solution of

$$\frac{dy}{dx} = f(x,y),$$

on $|x - x_0| \leq a$, and $y_1(x)$ denotes the approximation to $y(x)$ obtained by the Runge-Kutta rule (starting with correct initial values), then

$$\left|y(x) - y_1(x)\right| \leq K\left|x - x_0\right|^5,$$

where K is a constant explicitly given in terms of bounds of $f(x,y)$ and its derivatives so chosen as to be governed simultaneously by the magnitude of a.

Dr. Forsythe studied the results of Drs. Huskey and Hartree (NBS J. Res. 42, 57 to 62 (1949) on numerical integration of the system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = - x$$

by the Heun method. Comparison of the numerical solutions with the true sine-cosine solution of the system revealed surprising deviations of the round-off errors from those expected. In the article cited, Hartree gave one criterion for determining certain values of t where qualitatively bad round-off errors might be expected, but most of the anomalies were still left unexplained. In the present investigation the Hartree criterion has been amplified so that it provides an estimate of the round-off error expected to occur in the vicinity of each point t. This estimate actually explains most of the anomalies found in the Huskey data and gives a method of compensating for these errors to a considerable degree.

Huskey's round-off errors behaved mysteriously essentially because the mathematical functions involved have subtle number-theoretical properties. It is felt that too much ad hoc analysis would be required to compensate every integration for its round-off errors. Therefore, in numerical integration high-speed automatic digital computing machines, it is proposed that one force the round-off to obey known statistical distribution laws by the following device: Whenever a number is to be curtailed and rounded, let the rounding be "up" or "down" according to some unbiased random procedure. Experiments on IBM machinery are testing this method on the above system.

Publications: (1) "A note on the numerical integration of differential equations," by W. E. Milne; to be published in the NBS Journal of Research. (2) "Note on the Runge-Kutta method," by W. E. Milne; IN MANUSCRIPT in possession of the author. (3) "On the remainder of the Runge-Kutta formula in the theory of ordinary differential equations," by L. Bieberbach; edited and revised by A. M. Ostrowski and L. K. Jackson of the Institute for Numerical Analysis; submitted to the Quarterly of Applied Mathematics.

Note: A paper by Dr. Milne entitled "Numerical determination of characteristic numbers," previously listed under this project, now appears in the publications for project 11.1/1-50-3.

APPROXIMATE DETERMINATION OF THE CHARACTERISTIC FUNCTIONS AND CHARACTERISTIC VALUES OF ARBITRARY LINEAR DIFFERENTIAL OPERATORS
Project 11.1/1-49-ODE4

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: C. Lanczos
Authorized 2/15/49
Terminated 9/30/49

Objective: To develop a practical and economical method of obtaining the characteristic values and functions of arbitrary linear differential operators.

Background: In many problems of physics and engineering, the determination of the characteristic solutions of a linear differential equation is of great importance. The frequently employed Rayleigh-Ritz method is applicable only to self-adjoint systems, it requires integrations, and is basically restricted to the characteristic value which is least in absolute value. The present study is aimed at developing a method which will not have these limitations.

Comments: The principle underlying the present investigation consists in constructing a trial function which together with its iterations up to a certain order satisfies the given boundary conditions, and then determining the characteristic equation by a method entirely analogous to that used in project 11.1/1-49-AE1.

Status: TERMINATED. It was decided that the scope of the objective should be widened, so project 11.1/1-50-3 was set up to replace this project. See current status report for project 11.1/1-50-3.

STUDIES IN NUMERICAL INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PD1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Although the project manager returned to his academic post, some numerical work continued on the project at the Institute and the project manager returned to the staff as a consultant for a short period in September. The study mentioned in the previous status report was completed and was incorporated into a paper presented at the Symposium held at the Harvard Computation Laboratory in September.

Publications: "Numerical methods associated with Laplace's equation," by W. E. Milne; IN MANUSCRIPT in possession of the author; to appear in the Proceedings of the above Symposium.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Experimental work on the integration of Laplace's equation was performed by Mr. Ernest Elyash, a summer fellow of the Institute, under the direction of William Feller. The joint RAND-INA Seminar on Stochastic Processes continued to meet regularly every Thursday throughout the quarter under the direction of William Feller, who spoke at a number of the meetings. Other papers were given by Mr. H. Kahn of RAND, and Mr. Kac of the Institute.

Mr. Feller is preparing an exposition on the connection between stochastic processes and differential equations, based on his lectures to the summer fellows (see project 11.1/4-49-7) and to the joint RAND-INA seminar.

Publication: "Proceedings of a Symposium on the Monte Carlo Method," a volume presenting the papers given at the Symposium on June 29, 30, and July 1, 1949, is in preparation.

GENERATION AND TESTING OF RANDOM DIGITS
Project 11.1/1-49-PM2

Origin: NBS — Authorized 4/1/49
Sponsor: Office of Naval Research, USN — Completed 9/30/49
Managers: G. E. Forsythe, R. P. Peterson, Jr., E. Yowell, Jr.

Objective: (1) To devise machine methods of generating random digits by arithmetical processes. (2) To test whether the methods give random digits suitable for use in various Monte Carlo sampling processes.

Background: Random digits (or random-sampling digits) are needed for many purposes in "experimental mathematics," applied statistics, and numerical analysis. (See for example project 11.1/1-49-PM1.) Tippett and Kendall-Smith in England have published tables of random-sampling digits obtained by laborious hand work with tables and machines of the roulette-wheel type. The RAND Corporation has generated random digits from the thermionic noise. It is inherent in the nature of a.d.c.m., however, that neither precomputed digits now produced by a non-reproducible process are very well suited for calculations. It therefore seems essential to have techniques for internal generation of random digits (1) which will be repeatable, and (2) which will give digits which behave like the products of a truly random process.

Comments: Some of the Atomic Energy Commission Laboratories have experimented with the arithmetic generation of sampling digits. However, the reports at the Symposium on the Monte Carlo Method held at the INA on June 29, 30, July 1, 1949, showed a need for more basic investigations.

Status: COMPLETED.

Publication: The results will appear in the "Proceedings of a Symposium on the Monte Carlo Method," to be published by the NBS.

DETERMINANTS OF ORTHOGONAL POLYNOMIALS
Project 11.1/1-49-SF1

Origin: NBS Authorized 11/29/48
Sponsor: Office of Naval Research, USN Completed 9/30/49
Managers: E. F. Beckenbach, W. Seidel, G.E.Forsythe

Objective: To derive identities, inequalities, and convexity properties of minors of recurrent determinants whose elements are orthogonal polynomials, e.g.,

$$\Delta(n, h, k; x) = \begin{vmatrix} P_n(x) & P_{n+h}(x) \\ P_{n+k}(x) & P_{n+h+k}(x) \end{vmatrix},$$

where $P_n(x)$ is the Legendre polynomial of degree n; and to develop applications of these results, especially to the problem of moments.

Background: This is primarily a long-range project aimed at extending general mathematical knowledge. However, orthogonal polynomials are of great importance in applied mathematics, and any further information concerning them has an appreciable probability of being immediately useful.

Status: COMPLETED. In a forthcoming paper [Publication (2)], E. F. Beckenbach, W. Seidel and O. Szasz have studied recurrent determinants of ultraspherical polynomials and have shown, among other things, that the quadratic forms

$$\sum_{\mu,\nu=0}^{n} \left[P_{\mu+\nu}^{(\lambda)}(x)/P_{\mu+\nu}^{(\lambda)}(1) \right] u_\mu \bar{u}_\nu,$$

where $P_n^{(\lambda)}(x)$ is the ultraspherical polynomial of degree n and order λ, are positive definite for $\lambda > 0$, $n = 0,1,2,\ldots$, and $x > 1$. In the present paper it is shown that the Hermitian forms

$$\sum_{\mu,\nu=0}^{n} \left[P_{|\mu-\nu|}^{(\lambda)}(x)/P_{|\mu-\nu|}^{(\lambda)}(1) \right] u_\mu \bar{u}_\nu$$

are positive definite for $\lambda > 0$, $n = 0,1,2,\ldots$ and $-1 < x < 1$, a result conjectured by G. Szegö. (This is the problem discussed in the second paragraph of the previous status report for this project.) This result follows from the fact that the harmonic function,

$$\tfrac{1}{2} + R\left\{ \sum_{n=1}^{\infty} \left[P_n^{(\lambda)}(x)/P_n^{(\lambda)}(1) \right] z^n \right\}$$

is positive in the unit circle $|z| < 1$ for $-1 \leq x < 1$ and $\lambda > 0$. For the special case $\lambda = 1$ this result was proved earlier by G. Szegö.

Publications: (1) "Second order determinants of Legendre polynomials," by G. E. Forsythe; to be submitted to the Duke Mathematical Journal. (2) "Recurrent determinants of orthogonal polynomials, Part I: Legendre and ultraspherical polynomials," by E. F. Beckenbach,

W. Seidel, and O. Szasz; accepted by the Duke Mathematical Journal, subject to revision and condensation.

DETERMINATION OF EXTREMALS OF FUNCTIONALS
Project 11.1/1-50-1

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes

Objective: To study numerical and analytical methods of determining the maxima and minima of functionals; in particular, integrals of the types customarily studied in the calculus of variation.

Background: A substantial part of modern physical theory can be stated in terms of variational principles. In view of this fact, the calculus of variation is playing a continually expanding role in the solution of problems in applied physics. The existing literature on calculus of variation is extensive, but the numerical aspects have not kept pace with the theoretical ones.

Status: NEW. Publication (1) was completed, and a certain amount of time was spent in revising Publication (2).

Publications: (1) "Some elementary problems in the calculus of variations," by M. R. Hestenes; submitted to the Mathematics Magazine, also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A. (2) "Quadratic forms in Hilbert space, with applications in the calculus of variation," by M. R. Hestenes; accepted, subject to revision, by the American Journal of Mathematics.

DETERMINATION OF CRITICAL POINTS
Project 11.1/1-50-2

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Manager: M. R. Hestenes

Objective: To study numerical methods of determining critical points of functions of more than one variable, and in particular, to develop practical methods for locating maxima and minima of functions of many variables.

Background: Critical point determinations arise in the theory of aircraft design, the theory of games, and in many other physical and economic applications.

Status: NEW. Four closely related methods of locating maxima and minima of functions were studied. Each one is a generalization or modification of the Newton-Raphson iterative method of finding roots of equations. In the first method, first and second order derivatives are used to find the roots of the first derivatives, exactly as in the Newton-Raphson method. The second method is really only a geometric interpretation of the first and third methods. In this method level surfaces of the function in the neighborhood of a critical point

are approximated by ellipsoids. The centers of these ellipsoids are found, and a process is set up whereby the successive centers converge toward the critical point in question. In the third method, the function is interpolated by successive polynomials of the second degree, and the respective minima or maxima of these polynomials are then determined. The last method is the so-called method of steepest ascent. In this method the first problem is that of selecting a norm whereby to give a meaning to the term "steepest ascent." It is found that when a suitable norm has been chosen, the method reduces simply to the first one mentioned above. The first three methods are about equally satisfactory as regards convergence; choice will depend on convenience in specific cases. Extensions to the calculus of variations have been considered. Progress on this aspect will be reported on in later status reports for project 11.1/1-50-1. Random sampling methods have been investigated in a preliminary way, but without satisfactory results as yet.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Project 11.1/1-50-3

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Managers: C. Lanczos, W. Feller, G. E. Forsythe, M. Kac

Objective: To develop practical and economical methods of solving eigenvalue problems, to be used in connection with various types of computing equipment.

Background: The determination of the eigenvalues (or characteristic values), eigenvectors, and eigenfunction expansions of matrices, differential equations, integral equations, etc. is of fundamental importance in many problems of physics and engineering. The interest of the Institute for Numerical Analysis in such problems originated in the fact that at the time when work first began on the predecessor of this project (see Comments below), current methods of determining the eigenvalues of matrices were inadequate for certain matrices which arise in vibration analysis problems.

Comments: This project represents a continuation and amplification of projects 11.1/1-49-AE1 and 11.1/1-49-ODE4.

Status: NEW. (I) The manuscript of Dr. Lanczos referred to in the previous quarter's status report for project 11.1/1-49-AE1 was incorporated into the more general treatment represented below in Publication (1). This treatment also includes the material on integral equations and differential operators described in earlier status reports for project 11.1/1-49-ODE4. As certain mathematical details of the Lanczos method have already been given in these earlier status reports, no attempt will be made here to describe the method mathematically as it finally emerges in Publication (1). The advantages claimed for the method are as follows: (a) The iterations are used in the most economical fashion, obtaining an arbitrary number of eigenvalues and eigensolutions by one single set of iterations, without reducing the order of the matrix. (b) The rapid accumulation of fatal rounding errors, common to all iteration processes if applied to matrices of high dispersion (large "spread" of the eigenvalues), is effectively counteracted by the method of "minimized iterations." (c) The method is directly translatable into analytical terms by replacing summation by integration. We then get a rapidly convergent analytical iteration process by which the eigenvalues and eigensolutions of linear differential and integral equations may be obtained.

(II) In the course of studying and testing the Lanczos method referred to above, Dr. Milne developed a procedure for calculating eigen-

values, based essentially on the Lanczos method, but modified in such a way as to provide a simple numerical routine for the computation. In Publication (2), this procedure is set forth, together with a number of numerical illustrations. The exposition is limited to second order differential equations. The essential steps in Dr. Milne's procedure are these:

$$\text{Let } L(u) = \sum_{0}^{2} P_k(x)\, D^{2-k}\, u$$

be the differential operator in question. The problem is to find the characteristic values of for which the differential system

$$L(u) + \lambda^2 u = 0 ,$$

$$u + g\, Du = 0 \qquad \text{at } x = a ,$$

$$u + G\, Du = 0 \qquad \text{at } x = b ,$$

possesses non-zero solutions in $a \leq x \leq b$. A fictitious partial differential system is now introduced:

$$D_t^2\, V = L(V)$$

$$V + g\, Dx\, V = 0 \quad \text{at } x = a$$

$$V + G\, D_x\, V = 0 \quad \text{at } x = b$$

$$V = F(x) \quad \text{when } t = 0$$

$$D_t\, V = 0 \quad \text{when } t = 0$$

A solution in the form

$$(*) \qquad V = \sum c_k u_k(x) \cos \lambda_k t$$

is assumed. A network is set up in the (x,t) space and the partial differential system is replaced by an approximately equivalent difference equation. The values of the function V at the vertices of the network may be considered as coefficients of a set of homogeneous simultaneous linear equations with a non-trivial solution, and by substitution of (*) into the equation a corresponding set of equations involving the eigenvalues λ_k is derived. From these it is evident that the eigenvalues of the difference equations are proportional to the roots of an equation of the type

$$\sum_{k=0}^{n} A_k \cos k\mu = 0.$$

(III) Dr. W. Feller and Dr. G. E. Forsythe collaborated on a study of the transformation of a given square matrix A into another matrix B such that the characteristic values and vectors of A can be determined from those of B. The initial ideas were mainly those of Dr. Feller. As of the end of the quarter, a joint paper was being prepared [Publication (3) below]. Technical details of this work are as follows:

Two well-known but previously unrelated numerical procedures for obtaining the eigenvalues and eigenvectors of an n-th order square matrix A both begin with an iteration to get one characteristic value λ_1 and its associated vectors, followed by a transformation of A to a new matrix B from which the characteristic value λ_1 is absent. After obtaining by iteration one characteristic value of B and associated vectors, one can reconstruct another characteristic value and vectors of A. One can then start over with B. (Lanczos' procedure referred to in (I) and (II) above is of another type.) One procedure, "deflation," is symmetry-preserving (meaning that if A is symmetric, then so is B), but not order-reducing (order-reducing means that B is of order n-1). The other procedure, the Frazer-Duncan-Collar reduction method, is order-reducing, but

not symmetry-preserving.

In the present investigvation a two dimensional set S of matrix transformations of A to B is found, each member of which permits one to proceed from B's characteristic values and vectors to those of A. The deflation procedure and the Frazer-Duncan-Collar reduction procedure both appear as natural special members of the set S. A certain one-dimensional subset of S is symmetry-preserving, while another one-dimensional subset is order-reducing. Two special transformations are both order-reducing and symmetry-preserving and are therefore recommended for practical work.

The theorems apply to both defective and non-defective matrices.

(IV) Dr. M. D. Donsker and Dr. M. Kac developed a method for finding the lowest eigenvalue and corresponding eigenfunctions of Schrödinger's equation by the so-called Monte Carlo technique. Their work is contained in the paper cited in Publication (4). Their method is based on the following considerations:

Let $X_1, X_2, X_3, \ldots$ be independent identically distributed random variables each having mean 0 and standard deviation 1 and let $S_k = X_1 + X_2 + \ldots + X_k$. Under certain general assumptions on $V(x)$, the most severe of which is that $V(x)$ be non-negative, it can be shown that the limiting distribution function $\sigma(\alpha,t)$ of the random variable

$$S = \frac{1}{n} \sum_{k<nt} V \frac{S_k}{\sqrt{n}}$$

is such that

$$(*) \qquad \int_0^\infty \int_0^\infty e^{-\alpha - st} \, d_\alpha \sigma(\alpha,t) dt = \int_{-\infty}^{\infty} \psi(x) dx$$

where $\psi(x)$ is the fundamental solution of the differential equation

$$\frac{1}{2} \frac{d^2\psi}{dx^2} - (s + V(x))\psi = 0$$

subject to the conditions

$$\psi(x) \to 0 \qquad x \to \pm \infty$$

$$\left|\psi'(x)\right| < M \qquad x \neq 0$$

$$\psi'(+0) - \psi'(-0) = 2.$$

The fundamental solution $\psi(x)$ of the aforementioned differential equation is expressible in terms of the normalized eigenfunction $\{\psi_j(x)\}$ and eigenvalues λ_j of the one dimensional Schrödinger eigenvalue problem

$$\frac{1}{2} \frac{d^2\psi}{dx^2} - V(x)\psi(x) = -\lambda\psi \qquad \lambda > 0$$

as

$$(**) \qquad \psi(x) = \sum_j \frac{\psi_j(0)\,\psi_j(x)}{s + \lambda_j} .$$

Substitution of (**) into (*) and subsequent inversion with respect to s permits the lowest eigenvalue to be expressed as

$$\lambda_1 = \lim_{t\to\infty} -\frac{1}{t} \log \int_0^\infty e^{-\alpha} \, d_\alpha \sigma(\alpha,t).$$

The distribution function $\sigma(\alpha,t)$ is then calculated by a sampling process in which the value of S above is repeatedly determined for a suitably large value of n. Generalizations to multidimensional Schrödinger equations are considered. The theory for the second eigenvalue is also stated.

In a numerical example in which λ_1 was actually 0.71, with n = 200 the Monte Carlo approximation was 0.74.

(V) Dr. Kac considered the following classical eigenvalue problem: Let B be a simply connected two-dimensional region of area $|B|$ bounded by the curve C. Within B let $u = u(x,y)$ satisfy the equation

$$\Delta u + \lambda u = 0, \text{ with } u = 0 \text{ on } C.$$

Then according to a well-known result of Weyl, the number of eigenvalues less than a given λ is asymptotically equal to $|B| \lambda / 4\pi$ as $\lambda \to \infty$. There is a corresponding result due to Carleman for the squares of the eigenfunctions. Dr. Kac found new short proofs of these results based on the theory of diffusion.

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C.Lanczos; to be submitted to the NBS Journal of Research. (2) "Numerical determination of characteristic numbers," by W. E. Milne; to be published in the NBS Journal of Research. (3) A paper embodying results of W. Feller and G. E. Forsythe to be submitted to the Quarterly of Applied Mathematics. (4) "A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation," by M. D. Donsker and M. Kac; to be published in the NBS Journal of Research.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.1/1-50-4

Origin: NBS Authorized 7/1/49
Sponsor: Office of Naval Research, USN
Managers: Various members of the staff

Objective: To conduct various studies in those fields of pure mathematics on which the theory and practice of numerical analysis are based; also to carry out certain investigations in pure mathematics which require large amounts of computing and so may result incidentally in a contribution to the science of computing.

Background: The principal scientific purpose of this project is simply the general expansion of mathematical knowledge. It is important in conducting research work at the scientific level of that of the Institute to insure freedom of inquiry to the staff, and in particular, to avoid setting up barriers to research at the apparent (and probably quite ephemeral) boundaries of a specific program. However, there are certain secondary aims of this project which make it contribute tangibly to the program of the Institute. Two of these may be inferred directly from the statement of the Objective. A third is that it is to the advantage of the Institute to have mathematical members of its research staff retain their proficiency in the techniques and logic of pure mathematics, as effective research in numerical analysis and applied mathematics nearly always makes use of the methods of pure mathematics.

Status: NEW. The work on this project during the quarter was done mainly by Dr. Otto Szasz. He wrote two papers, Publications (1) and (2) below, on the Gibbs phenomenon as it appears in applying certain summation methods to trigonometric series, and a third one, Publication (3),

on the Perron method of summability.

A principal result of the first paper is that if $s_n(t)$ denotes the Euler mean of the series $\sum_1^\infty \sin nt/n$, with parameter r lying in the interval

$$0 < r \leq 1, \text{ then } \lim s_n(t_n) = \int_0^{r\tau} \frac{\sin y}{y}\, dy$$

as $nt \to \tau$, $nt_n^2 \to 0$.

The second paper contains a discussion of the Gibbs phenomenon for Hausdorff means. Hausdorff means of a sequence $\{s_n\}$ are defined by

$$h_n = \sum_0^n \binom{n}{\nu} s_\nu \int_0^1 u^\nu (1-u)^{n-\nu}\, d\psi(u),$$

where $\psi(u)$ is of bounded variation, $\psi(1) = 1$, $\psi(+0) = \psi(0) = 0$.

The main problem here is to give conditions on the function $\psi(u)$ which are necessary and sufficient in order that the h_n means of the series

$\sum_1^\infty \frac{\sin nt}{n} = \frac{1}{2}(\pi - t)$ present a Gibbs phenomenon. When applied to Hölder means of order $p > 0$, in which case

$$\psi(u) = \frac{1}{\Gamma(p)} \int_0^u (\log \tfrac{1}{x})^{p-1}\, dx,$$

the existence of a constant p_0 is proved, so that the Gibbs phenomenon occurs for $p < p_0$, but not for $p \geq p_0$. It is found that $p = 0.5826\ldots$. In contrast to the equivalent Cesàro means Gronwall found the critical value $0.4395\ldots$.

The third paper contains a comparison of the Perron and Cesàro methods of summability. The Perron transform of a series $\sum_0^\infty a_n$ is

$$\phi_\chi(x) = a_0 + \sum_1^\infty a_n \frac{x}{x+\chi} \frac{x+1}{x+\chi+1} \cdots \frac{x+n-1}{x+\chi+n-1} \ldots$$

The series is summable to s in the Perron sense if $\lim_{x\to\infty} \phi_\chi(x) = s$. The main result of this paper is that this summability method is more powerful than Cesàro summability of order χ. The essential tool is an integral representation of $\phi_\chi(x)$.

Publications: (1) "On the Gibbs phenomenon for Euler means," by O. Szasz; accepted for publication in Acta Litterarum ac. Scient., Szeged. (2) "The Gibbs phenomenon for Hausdorff means," by O. Szasz; IN MANUSCRIPT. (3) "On a summation method of O. Perron," by O. Szasz; to be submitted for publication in Mathematische Zeitschrift.

2. Mathematical Tables

MATHIEU FUNCTIONS II
Project 11.1/ 2-45-1

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The coefficients De_k and Do_k which were transmitted from the Computation Laboratory have been checked in connection with a problem for the Air Materiel Command. The computations of the functions which had been started will be continued.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

DECOMPOSITIONS OF ARC TANGENTS
Project 11.1/2-48-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The manuscript is being typed, and the final proofs are being subjected to tests under the supervision of J. Todd.

Publications: The table will be submitted for publication in the NBS Applied Mathematics Series; the table will include the decompositions for integral arguments $n \leq 2{,}100$. "A problem on arc tangent relations," by J. Todd has been accepted by the American Mathematical Monthly.

SPECIAL TABLE OF BESSEL FUNCTIONS
Project 11.1/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Although this project was reported completed, work is continuing on the preparation of the manuscript for publication.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

ROCKET NAVIGATION TABLES
Project 11.1/2-48-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Although this project was reported completed, work is continuing on the preparation of the manuscript for publication.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLES OF $E_1(z)$, SECOND QUADRANT
Project 11.1/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The auxiliary functions $U_n(z)$ and $T_n(z)$ were completed for $n \leq 17$.

PUNCHED CARD LIBRARY
Project 11.1/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles, 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following tables on punched cards were added to the library:

1. $\sin x$, $\cos x$, Δ, δ^2; $x = [0(.0001)1; 11D]$ with maximum error of 1 unit in last place. These were computed in connection with the study of round-off error by G. E. Forsythe.
2. Table of Bessel-Clifford Functions. The functions
$$x^{-\frac{n}{2}} J_n(2\sqrt{x}),\ x^{-\frac{n}{2}} I_n(2\sqrt{x}),\ x^{-\frac{n}{2}} K_n(2\sqrt{x}),\ x^{-\frac{n}{2}} I_n(x)$$
are now available on punched cards for x ranging between 0 and 410 at various intervals, for n = 0 and 1. For $x > 6.2$, the functions
$$e^{2\sqrt{x}}\, x^{-\frac{1}{2}n} K_n(2\sqrt{x}) \text{ and } e^{-2\sqrt{x}}\, x^{-\frac{1}{2}n} I_n(2\sqrt{x}) \text{ are given.}$$

3. $E_2(x) = \int_1^\infty (e^{-xu}/u^2)du$, $x = [0(.001)2; 7D]$

$x = [2(.001)10; 5S \text{ or } 6S]$, $x = [10(.001)16; 13D]$.

4. The NBS "Table of Spherical Bessel Functions," Vol. 1, has been punched, but not checked. North American Aviation Corp. is punching the NBS "Table of Circular and Hyperbolic Tangents and Cotangents." The Consolidated Vultee Aircraft Corp. has undertaken the punching of the NBS "Tables of Natural Logarithms," Vol. III and IV.

3. Development of Automatic Computing Machinery

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.1/22-49-1

Origin: Air Materiel Command, USAF
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The arithmetic unit contract has been given to the Cole Instrument Company of Los Angeles. They have produced prototypes of both the M and A Registers, and have returned them to be checked. The M Register was checked and found satisfactory; production on the M Register is now under way at the Cole Instrument Company. Checking on the A Register will soon be completed. It will then be returned to the Cole Instrument Company and production on it will begin at that time. Delivery of the completed chassis for the arithmetic unit is expected by not later than November 15.

The control unit chassis are nearing completion, and the racks to hold these chassis are in the process of being manufactured. Completion of the racks and partial checking out of the control unit are anticipated by November 1.

A complete cathode ray tube memory unit has been built and operated successfully in storing 512 points on the face of the tube. Although this unit operated entirely satisfactorily it was found that the gating circuitry could be simplified. Hence, a second model was constructed making use of simplified gating circuitry. This second memory tube unit is now being tested in the cathode ray tube test unit.

The machine will be put into operation initially with only 256 spots on the face of the cathode ray tubes, - that is, 256 words in the high speed memory. However, tests with the present memory units indicate that there would be no great difficulty in storing 1024 words in the memory with, possibly, a rule about reference to nearby points on the memory. (Experiments by Division 13 of NBS and by other groups have indicated that such a rule can not be avoided). Coding personnel definitely prefer the larger memory even at the price of such a rule. The Zephyr is being constructed in such a way that the memory tube capacity can be increased to 1024 at any future date without greatly altering the rest of the machine. From the above it will be seen that some sacrifices are being made in order to complete the machine as quickly as possible.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Project 11.1/22-49-2

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Comments on the existing terminology and notation have been received from various interested persons in the field of automatic computing machines. When deemed worthwhile by the committee these comments and suggestions have been incorporated in the practices of the coding group at the Institute, and in the reports issued on the automatic computer being designed and constructed under project 11.1/22-49-1. It is hoped that in this manner a system of terminology, notation, and symbolism, which will be widely acceptable, can be established. The system is still expanding, and is continuously being subjected to practical test by the coding group. It is expected that the period of test will extend into the early stages of operation of the machine being constructed in project 11.1/22-49-1.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Project 11.1/22-50-1

Origin: NBS Authorized 7/1/49
Sponsor: Air Materiel Command, USAF
Manager: H. D. Huskey

Objective: To establish a system of semi-automatic instruction for all general purpose electronic digital computing machines using as a model the automatic computing machine being constructed under project 11.2/22-49-1.

Background: With the advent of general purpose high-speed digital computing machines the need for a simplified system of coding has become evident. Such a system, by making the machine do much of the tedious and repetitious part of coding, would greatly decrease the work of the coding staff.

Status: NEW. Dr. Huskey presented his preliminary results in a talk entitled "Semi-automatic instruction on the Zephyr" at a Symposium on Large-Scale Calculating Machinery which was held at Harvard University in September 1949. The subject has also been discussed with Mr. E.F.Moore, who has submitted a paper on the subject to the Mathematical Tables and Other Aids to Computation.

4. Computing Services

SEPARATION OF EXPONENTIALS
Project 11.1/31-48-2

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

TEST OF AN ADAPTATION OF THE MONTE CARLO METHOD
Project 11.1/31-49-10

Origin: The Rand Corporation — Authorized 11/16/48
Sponsor: Air Materiel Command, USAF — Completed 8/31/49
Managers: L. L. Bailin, Roselyn A. Siegel

Objective: By use of an adaptation of the so-called "Monte Carlo" method, to obtain a statistical solution of the neutron transport equation.

$$(1)\quad \gamma \frac{\partial \psi}{\partial x} + \psi = \frac{\gamma}{2} \int_{-1}^{+1} \psi(x, \gamma')\, d\gamma',$$

and compare the solution with one obtained by analytic methods.

Background: One considerable disadvantage in using the Monte Carlo method to solve an equation such as (1), where the independent variable is greatly attenuated for certain physically interesting situations, is that often an impossibly large number of "particles" must be calculated in order to obtain a reasonably accurate statistical approximation to the answer. This project is to test a conjecture that this difficulty can be removed by substituting $\psi = \phi\, e^{-\alpha x}$ and solving the resulting equation for ϕ. This corresponds to a physical situation where particles may multiply as well as be absorbed.

Status: COMPLETED.

Publication: Mr. Herman Kahn of the Rand Corporation expects to publish his results in Nucleonics.

RADIATION CHARACTERISTICS OF A TURNSTILE ANTENNA SHIELDED BY A SECTION OF A METALLIC TUBE CLOSED AT ONE END
Project 11.1/31-49-12

Origin: North American Aviation Corporation — Authorized 3/4/49
Sponsor: Air Materiel Command, USAF — Completed 9/30/49
Manager: L. L. Bailin

Objective: To determine by theoretical methods the radiation characteristics of the antenna, i.e., the far field radiation pattern and polarization, the behavior of the propagating modes in the cylindrical shielding can, the impedence seen by the antenna, the field and power gain functions, subject to the following assumptions:

(i) The antenna consists of perpendicularly crossed wires, of equal length, excited by a sinusoidal current distribution which differs 90° in time phase between members of the cross.

(ii) The shield consists of a cylindrical can open at one end. The plane of the antenna is perpendicular to the axis of the cylinder; the center of the crossed wires is on the axis.

(iii) The shield is assumed to be a perfect conductor; the effect of the antenna supports is neglected.

Background: This problem was proposed by the Aerophysics Laboratory of the North American Aviation Corporation as a step in obtaining a satisfactory basis for efficient design of antennas of this type.

Comments: Professors A. Baños, Jr., and D. S. Saxon of the University of California at Los Angeles are acting as consultants on this problem.

Status: COMPLETED. The results were transmitted to the North American Aviation Corporation.

Publication: Paper by Mr. L. L. Bailin covering the methods of radiation characteristics will be submitted for publication in the NBS Journal of Research.

OPTIMUM STAR CONFIGURATIONS
Project 11.1/31-49-14

Origin: Hughes Aircraft Company, Inc. — Authorized 5/25/49
Sponsor: Air Materiel Command, USAF — Completed 8/8/49
Manager: E. Yowell

Objective: To find the parameters of certain types of star-shaped figures which maximize the area between the star and an enclosing circle.

Background: This problem arises in the study of methods of packing solid rocket fuel in cylindrical rockets.

Status: COMPLETED. The results were forwarded to the Hughes Aircraft Corporation. For certain sets of parameters no solution exists and mathematical criteria were derived for the determination of this point.

COMPUTATIONS RELATING TO THEODOLITE SYSTEMS
Project 11.1/31-50-1

Origin: Boeing Aircraft Corporation — Authorized 8/1/49
Sponsor: Air Materiel Command, USAF — Completed 8/8/49
Manager: E. Yowell

Objective: To compute certain quantities by formulae furnished by the Boeing Aircraft Corporation for parameters corresponding to various positions of a sighted object.

Background: The problem arose in research work of the Boeing Aircraft Corporation under contract to the Air Materiel Command.

Status: COMPLETED.

TABLE RELATING TO A LINEAR ANTENNA ARRAY
Project 11.1/31-50-2

Origin: Hughes Aircraft Corporation — Authorized 8/1/49
Sponsor: Air Materiel Command, USAF — Completed 9/10/49
Manager: E. Yowell

Objective: To compute

$$f(n,\psi) = 10 \log_{10} \left| \frac{1 + e^{-x}}{Z + e^{-x}} \right|^2$$

for $x = n \pi \sqrt{3}/360$; $Z = \exp(i\psi)$;

$n = 0(1)135$; $\psi = 0(1^{\circ})180^{\circ}$.

Background: The problem arose in connection with a study of relative currents, performed by Hughes Aircraft Corporation as part of a contract with the Air Materiel Command.

Status: COMPLETED.

STUDIES OF A DIFFERENTIAL EQUATION
Project 11.1/31-50-3

Origin: Hughes Aircraft Corporation — Authorized 8/1/49
Sponsor: Air Materiel Command, USAF — Completed 9/30/49
Managers: Gertrude Blanch, E. Yowell

Objective: To find the sinusoidal solutions of the equation

$$y'' - y' + \frac{ay}{x} = \eta(x) ,$$

where

$$\eta(x) = \left[\sin\left(sx + \lambda \frac{\pi}{2}\right) \right] / x^{c}, \lambda = 0, 1; c = 0,1; s = 1(1)8; \text{ and } a = 4.$$

To tabulate the function up to a point where the asymptotic expansion gives approximately two significant figures.

Background: These solutions are desired in connection with research being performed by the Hughes Aircraft Corporation under a contract with the Air Material Command.

Status: COMPLETED.

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Project 11.1/32-49-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Manager: Gertrude Blanch

Authorized 9/2/48
Revised 11/15/49

Objective: To assist the research staff of the Institute for Numerical Analysis by performing small computational tasks.

Background: A function of the Computation Unit of the Institute for Numerical Analysis is to provide facilities for the numerical experimentation needed in the research work of the Institute. Furthermore, the publications of the research staff frequently contain small mathematical tables and numerical demonstrations of method, and it is the duty of the Computation Unit to furnish this material.

Status: CONTINUED. Among the more extensive computations during this quarter were:

1. Application of the method of C. Lanczos for the determination of eigenvalues and eigenvectors. (See 11.1/1-50-3, section I).
2. Studies in the problem of finding maxima and minima on surfaces, for M. R. Hestenes. (See 11.1/1-50-1 and 11.1/1-50-2).
3. Studies in generating random digits, for G. Forsythe. (See 11.1/1-49-PM2).
4. The determination of "Gibbs' constant" for Hölder means, for O. Szasz. (See 11.1/1-50-4).
5. Approximation studies for A. M. Ostrowski. (See 11.1/1-49-AE3).
6. Application of the Monte Carlo method to the numerical solution of the Dirichlet problem, for harmonic functions in a rectangle; for W. Feller: From any point in the boundary of a region a random walk is made along a square grid. The walk is stopped when a boundary is reached, and the value of the function at the boundary point is computed. The average value of the boundary function for a large number of walks gives the value of the function at the starting point.
7. Obtaining the limit A^n, as n approaches infinity, for a stochastic matrix A by calculating the projective operator of the space, complementary to that spanned by the vectors of the matrix (A - I). The problem was performed by H. Weinberger, under the direction of W. Feller.
8. Computations relating to the zeros of the L-series, for J. Barkley Rosser.
9. Problems in number theory, performed by Robert Douthitt at the request of D. H. Lehmer on IBM equipment. Mr. Douthitt punched and accumulated the quotients of the first 10,000 integers. He likewise punched and accumulated the values of $\frac{3n^2}{\pi^2}$ for the same range. A prediction of D. H. Lehmer was not borne out by the experiment.
10. Study of round-off error, for G. Forsythe and E. Yowell. Stepwise integrations, of considerable length, of the sine and cosine functions were performed using a certain method of random round-off.

11. Computation of lowest eigenvalue of Schrödinger's equation by a Monte Carlo method, for M. Kac, M. Donsker and E. Yowell. See project 11.1/1-50-3, section (IV).

REDUCTION OF THEODOLITE DATA
Project 11.1/32-49-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Project 11.1/32-49-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Project 11.1/32-49-6

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. About 35% completed. Work was continued by Dr. S. Herrick and his staff at the University of California at Los Angeles, using Institute for Numerical Analysis I.B.M. equipment, when machine time was available.

TEST OF A VARIATIONAL PROCEDURE FOR DETERMINING THE LOWEST ENERGY VALUE OF A SIMPLE QUANTUM MODEL
Project 11.1/32-50-1

Origin: NBS Authorized 8/1/49
Sponsor: Office of Naval Research, USN
Manager: H. Gruen

Objective: To test the feasibility of a variational method for computing quantum states by applying the method to the following problem in nuclear physics.

The operator

$$H = \begin{cases} -\left[\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_0 \right], & |x| \leq a , \\ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} , & |x| \geq a , \end{cases}$$

is the Hamiltonian for a particle of a mass m in a potential well of a depth V_0 and a width 2a. h is Planck's constant divided by 2π.

A trial wave function is chosen as

$$\Phi(x) = \begin{cases} (x^2 - a^2)^9 , & |x| \leq a , \\ 0 , & |x| \geq a . \end{cases}$$

We define

$$F_n(A) = A + \frac{\beta_n(A)}{\beta_{n-1}(A)},$$

where n = 1,3,5,..., and

$$\beta_n(A) = \int_{-a}^{+a} \Phi(x) \, (H - A)^n \, \Phi(x) dx \Big/ \int_{-a}^{+a} \left[\Phi(x)\right]^2 dx.$$

The minimum values of F(A) for A ranging from -500 to +500 for odd values of $n \leq 9$ will be computed and the results compared with the theoretical conjecture (see Background) that an optimum n_0 exists where $\min F_{n_0}(A)$ is the best approximation to E_0 obtainable with the given trial wave function.

Background: The variational method together with the conjectured results was suggested by Dr. Edward Gerjuoy of the University of Southern California.

Status: NEW

CHARACTERISTIC VALUES OF MATHIEU'S DIFFERENTIAL EQUATION
Project 11.1/4-49-6

Origin: NBS
Sponsor: Air Materiel Command, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Characteristic values were being computed for s beyond the range of the published tables, as outlined in previous status reports. At the end of the quarter, $be_4(s)$ was available at an interpolable interval in $1/\sqrt{s}$, from 0 to .1 (i.e. for s from 100 to ∞).

SUMMER FELLOWSHIP EDUCATIONAL PROGRAM
Project 11.1/4-50-1

Origin: NBS — Authorized 7/1/49
Sponsor: Office of Naval Research, USN — Completed 8/26/49
Managers: E. C. Yowell, H. D. Huskey, W. Feller, A. M. Ostrowski

Objective: To acquaint a selected group of graduate students with some of the trends and techniques of modern numerical analysis.

Background: The Summer Fellowship program was suggested by Professors Morgan Ward and H. F. Bohnenblust of the California Institute of Technology.

Status: COMPLETED. The following courses were given:

(1) "Techniques of present-day computing". Application of electronic punched card equipment to solution of mathematical problems. Dr. E. C. Yowell and associates. (2) "Theory of automatic digital computing machinery". Logical aspects of the design of present and proposed systems; programming and problem preparation. Dr. H. D. Huskey and associates. (3) "Seminar in probability methods." Introduction to the partial differential equations which arise in the theory of stochastic processes, and study of sampling methods for solving these equations. Dr. Feller and members of the staff. (4) "Seminar in the inversion of matrices of high order." Chiefly a study of the paper, "Numerical inverting of matrices of high order," by J. von Neumann and H. H. Goldstine, Bul. Amer. Math. Soc., 53, 1021 to 1099, (1947). Dr. A. M. Ostrowski.

The summer fellows were: Robert J. Diamond, California Institute of Technology; Robert C. Douthitt, University of California, Berkeley; Ernest Elyash, Cornell University; Harold Gruen, University of California, Los Angeles; James G. C. Templeton, Princeton University; and Hans Weinberger, Carnegie Institute of Technology.

1. Mathematical Tables

TABLES OF $E_1(z)$, $(z = x + iy)$
Project 11.2/2-43-3

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations completed; preparation of final manuscript for publication was continued.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Project 11.2/2-46-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

MATHIEU FUNCTIONS, I
Project 11.2/2-46-2

Origin: Applied Mathematics Panel, NDRC — Authorized 7/1/47
Manager: Gertrude Blanch — Completed 9/15/49

Objective: To compute an eight-place table of the first 15 odd and 16 even characteristic values b of Mathieu's differential equation

$$y'' + (b - s\cos^2 t)y = 0$$

for s ranging from 0 to 100 at various intervals, and the Fourier coefficients of the solutions corresponding to these characteristic values, as well as certain related functions.

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu functions," by N. W. McLachlan (Oxford Press, 1947). The existing tables by Stratton-Morse-Chu-Hutner, Ince and Goldstein are inadequate. The project was proposed by Dr. Philip Morse and Dr. James Wakelin.

Comments: Related to project 11.1/2-45-1 (formerly 11.2/2-45-1, prior to April 1949).

Status: COMPLETED.

Publication: Manuscript transmitted to Columbia University Press for publication.

TABLES OF COULOMB WAVE FUNCTIONS
Project 11.2/2-47-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations completed; checking was continued.

TABLE OF ANTILOGARITHMS
Project 11.2/2-47-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations completed; checking was continued.

TABLES FOR THE OCCASIONAL COMPUTER
Project 11.2/2-47-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Project 11.2/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN HERMITE POLYNOMIALS
Project 11.2/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Project 11.2/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

TABLE OF POWERS OF COMPLEX NUMBERS
Project 11.2/2-50-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF Authorized 9/1/49
Manager: Irene Stegun

Objective: To publish a table of the exact values of the first 25 powers of $z = x + iy$ for $x, y = 0(1)10$.

Background: The manuscript of such a table, which is basic for a great many computations, had been prepared several years ago by the Mathematical Tables Project, under Mr. Herbert Salzer. At the present time it is intended to check and publish this manuscript.

Status: NEW. Checking of galley proofs was in progress.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 8 in the NBS Applied Mathematics Series.

TABLES TO FACILITATE SEQUENTIAL TESTS
Project 11.2/2-50-2

Origin: NBS Authorized 9/1/49
Sponsor: Office of Air Research, AMC, USAF
Manager: Irene Stegun

Objective: To publish a table of the solution $z(L,n,\delta)$ of the equation:

$$L = \log F\left[\frac{n}{2}, \frac{1}{2}; \frac{\delta^2 z}{2}\right] - \frac{n\delta^2}{2}$$

for $n = 1(1)200$; $\delta = .1(.1)1(.2)2, 2.5$; $\pm L = 2(1)7$, ln 19, ln 99.

Background: These tables were originally computed by the Mathematical Tables Project for the Statistical Research Group at Columbia University during the war for use in connection with sequential testing. They had not been published before.

Status: NEW. Checking of galley proofs was in progress.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 7 in the NBS Applied Mathematics Series.

TABLE OF CHEBYSHEV POLYNOMIALS
Project 11.2/2-50-3

Origin: NBS Authorized 9/1/49
Sponsor: Office of Air Research, AMC, USAF
Manager: Irene Stegun

Objective: To publish a table of the Chebyshev polynomials $S_n(x)$, $C_n(x)$ for $n = 2(1)12$ and $x = 0(.001)2$.

Background: The manuscript of this table was prepared some time ago by the Mathematical Tables Project but publication was postponed. These polynomials are frequently used as approximations to more complicated functions. They are especially convenient for use in connection with high-speed automatic computing machines.

Status: NEW. Checking of galley proofs was in progress.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 9 in the NBS Applied Mathematics Series.

2. Computing Services

RANGE LAG OF BOMBS
Project 11.2/31-49-2

Origin: Air Materiel Command, USAF — Authorized 6/25/49
Sponsor: Office of Air Research, AMC, USAF — Completed 8/1/49
Manager: F. L. Alt

Objective: To approximate the range lag of bombs as a simple analytic function of altitude and air-speed at time of release, and of ballistic coefficient.

Background: The range lag, i.e., the difference between the actual range (horizontal travel) of a bomb and that range which would occur in a vacuum, is fitted by means of least squares by an expression of form $My + Ny^{3/2}$, where y is the altitude of release and where M and N are coefficients which in turn are to be fitted by simple functions of air-speed and ballistic coefficient. The reports are desired by contractors of the Air Materiel Command for the design of certain compensating equipment.

Status: COMPLETED. Manuscript transmitted to the Air Materiel Command.

GUST ATTACKS ON DELTA WING
Project 11.2/31-50-1

Origin: Aircraft Laboratory, AMC, USAF — Authorized 9/15/49
Sponsor: Office of Air Research, AMC, USAF — Estimated Completion 12/49
Manager: Irene Stegun

Objective: To evaluate by numerical computation the basic unsteady variations in lift caused by a sharp-edged vertical wind gust attacking a portion of a delta wing in supersonic flight.

Status: NEW. Preliminary computations were started.

HEAT CONDUCTION EQUATION
Project 11.2/33-46-1

Origin: Bureau of Ordnance, Department of the Navy
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Project 11.2/33-49-2

Origin: Naval Research Laboratory
Sponsor: Naval Research Laboratory
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameter a_1, m_1, were performed when requested.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Project 11.2/33-49-5

Origin: Section 3.2, NBS
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Tables of zero pressure properties were completed for atomic oxygen, atomic hydrogen, and atomic nitrogen. Tables of free energy properties were completed for N_2, O_2, normal H_2, atomic oxygen, atomic nitrogen and atomic hydrogen. A table of vapor pressure of oxygen was also completed. Prior to this quarter, tables of zero pressure properties had been completed for N_2, O_2, A, air, and normal H_2; and computations for real gas properties had been completed for dry air, normal H_2, and moist air.

ADMINISTRATIVE RECORDS ON PUNCHED CARDS
Project 11.2/33-49-8

Origin: Division M, NBS
Sponsor: Division M, NBS
Manager: A. H. Rosenthal

Authorized 12/15/48
Terminated 9/15/49

Objective: To establish and maintain punched-card records of certain administrative items, such as stock records, travel data, etc. To prepare up-to-date printed listings of the records as required.

Background: Such records were formerly kept by manual methods. The punched card method will make it possible to compile necessary periodic reports in much less time and will make record data more easily and quickly available for use in statistical studies.

Status: TERMINATED. The project was transferred to the NBS Budget and Management Division (M).

FERMI FUNCTION II
Project 11.2/33-49-10

Origin: Section 4.4, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations completed and checked. Preparation of auxiliary tables and of an introduction to accompany publication of the table was started.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Project 11.2/33-49-11

Origin: Lewis Flight Propulsion Laboratory, NACA
Financial Sponsor: Lewis Flight Propulsion Laboratory, NACA
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were started.

SHOCK WAVE PARAMETERS
Project 11.2/33-49-13

Origin: Bureau of Ordnance, Department of the Navy
Sponsor: Bureau of Ordnance, Department of the Navy
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were completed, and preparation of the manuscript for publication of the tables of $I_n(x,c_v)$ was under way. An investigation of the interpolability of these tables was initiated.

BASIC IONOSPHERIC DATA
Project 11.2/33-49-14

Origin: Section 14.1, NBS
Sponsor: Section 14.1, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Recording and processing of the observation data was continued.

Publications: Current data are published in the monthly CRPL-F reports of the NBS Central Radio Propagation Laboratory; copy prepared from punched cards will replace hand-made copy as soon as procedures and schedules are worked out. The results of statistical computations will be included in scientific papers where appropriate.

SEQUENTIAL SAMPLING PLANS
Project 11.2/33-49-15

Origin: Hq. Quartermaster Corps Inspection Service, Department of the Army — Authorized 4/14/49, Completed 9/30/49
Sponsor: National Applied Mathematics Laboratories
Manager: Irene Stegun

Objective: To design truncated sequential sampling plans which have OC curves matching those of a given set of single sampling plans at two points.

Background: This set of sequential sampling plans is to be used in the revision of JAN Standard 105.

Comments: Supervision is carried out in close contact with Mr. J. Cameron of the NBS Statistical Engineering Laboratory.

Status: COMPLETED. Manuscript transmitted to the originator.

STRAIN ROSETTE MEASUREMENTS
Project 11.2/33-49-16

Origin: David Taylor Model Basin, USN — Authorized 4/14/49
Sponsor: David Taylor Model Basin, USN — Completed 9/15/49
Manager: Irene Stegun

Objective: To evaluate a series of strain rosette measurements taken at the David Taylor Model Basin.

Background: "Strain rosette measurements" consist in determining strains in three coplanar directions, two being perpendicular to each other and the third at a 45° angle. From these measurements it is desired to determine the magnitude and directions of the principal strains.

Status: COMPLETED. Manuscript transmitted to the David Taylor Model Basin.

RADIO-TELEGRAPH INTERFERENCE
Project 11.2/33-49-17

Origin: Section 14.4, NBS
Sponsor: Section 14.4, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Recording of data was continued.

PERCENTAGE POINTS OF THE ARITHMETIC MEAN IN RANDOM SAMPLES FROM THE SECH AND SECH^2 DISTRIBUTIONS
Project 11.2/33-49-18

Origin: Section 11.3, NBS
Sponsor: Section 11.3, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations about 75% completed.

COMPUTATION OF NORMAL MODES OF BOX BEAM
Project 11.2/33-49-20

Origin: Section 6.4, NBS
Sponsor: Section 6.4, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. 75% completed.

TABLES OF CABLE FUNCTIONS
Project 11.2/33-49-21

Origin: David Taylor Model Basin, USN — Authorized 6/25/49
Sponsor: David Taylor Model Basin, USN — Completed 8/49
Manager: S. Prusch

Objective: To evaluate the functions

$$\log_e \tau = \int_0^{\phi} \frac{P}{Q}\, d\phi\,, \qquad \sigma = \int_0^{\phi} \frac{T}{Q}\, d\phi$$

$$\eta = \int_0^{\phi} \frac{\tau \sin\phi}{Q}\, d\phi\,, \qquad \xi = \int_0^{\phi} \frac{\tau \cos\phi}{Q}\, d\phi$$

where $P = f \frac{\cos\phi}{|\cos\phi|} + w \sin\phi$, $Q = -|\sin\phi| \sin\phi + w \cos\phi$,

and $w = \sec\phi_c - \cos\phi_c$, for $f = .01(.01).03$, $\phi_c = 0(5^\circ)90^\circ$,

$\phi = [\phi_c(1^\circ)\phi_c + 180^\circ]$.

Background: These functions facilitate the determination of the shape of and tensions in a flexible cable that is held in a uniform stream. The effects of the weight of the cable and frictional drag are included. The types of configurations that may be treated by means of these functions are most general.

Status: COMPLETED. Manuscript transmitted to the David Taylor Model Basin.

WAVE RESISTANCE OF SHIPS
Project 11.2/33-50-1

Origin: David Taylor Model Basin, USN — Authorized 9/1/49
Sponsor: David Taylor Model Basin, USN
Manager: J. H. Levin

Objective: To calculate functions $M_n(\gamma) = \int_0' \xi^n x_i \sin\gamma\xi \, d\xi$, $M_n'(\gamma) = \int_0' \xi^n \cos\gamma\xi \, d\xi$ for $n = 0(1)5(2)11$, and for approximately 150 values of γ in the range $1 \leq \gamma \leq 100$; to calculate also specified products of the type M_iM_k, $M_i'M_k'$. The above integrals are evaluated by the use of the recursion formulae:

$$M_n(\gamma) = -\frac{\cos\gamma}{\gamma} + \frac{n \sin\gamma}{\gamma^2} - \frac{n(n-1)}{\gamma^2} M_{n-2}$$

$$M_n'(\gamma) = \frac{\sin\gamma}{\gamma} + \frac{n \cos\gamma}{\gamma^2} - \frac{n(n-1)}{\gamma^2} M'_{n-2}$$

Background: For simple forms the wave resistance of a ship is given to a first approximation by Michell's integral:

(1) $$R = \frac{8\rho g}{\pi} \frac{B^2H^2}{L} \int_{\gamma_0}^{\infty} \left\{ I^{+2}(\gamma) + \mathfrak{I}^{+2}(\gamma) \right\} \frac{(\gamma/\gamma_0)^2}{\sqrt{(\gamma/\gamma_0)^2 - 1}} d$$

(2) $$I^+(\gamma) = \int_0' \sum n \, a_n \xi^{n-1} \cos\gamma\xi \, d\xi \int_0' (1 - \zeta^m) e^{-\theta\zeta} \, d\zeta$$

(3) $$I^+(\gamma) = \int_0' \sum n \, a_n \xi^{n-1} \sin\gamma\xi \, d\xi \int_0' (1 - \zeta^m) e^{-\theta\zeta} \, d\zeta$$

where n, m are integers, the quantity preceding the integral sign in (1) is a dimensional factor, and $\theta = \frac{\gamma^2 (\text{constant})}{\gamma_0}$. This approximation can be used as a basis for more complicated cases. An equivalent expression (Havelock's integral) is:

$$R = \frac{8\rho g}{\pi}\frac{B^2H^2}{L}\int_0^{\pi/2}\left\{I+2\,(\gamma_0 \sec\theta) + \mathcal{J}^{+2}(\gamma_0 \sec\theta)\right\}\sec^3\theta\, d\theta$$

where $\gamma_0 \sec\theta$ is substituted for γ. The expressions $M_n(\gamma)$ and $M_n'(\gamma)$ are necessary for the evaluation of either Michell's integral or Havelock's integral.

The indices n of the functions $M_n(\gamma)$, $M_n'(\gamma)$ depend upon exponents of terms in polynomials used for approximating ship forms. With the chosen indices (exponents) the whole range of normal ship forms can be covered.

Besides, the same functions $M_n(\gamma)$, $M_n'(\gamma)$ admit of solving resistance problems of totally submerged bodies near the surface, gliding vessels, and problems of behavior of a ship in a seaway.

Thus these functions are of basic importance in Theoretical Naval Architecture.

Status: NEW. Computations were started.

RATING OF WATER CURRENT METERS
Project 11.2/33-50-2

Origin: Division 6.5, NBS — Authorized 9/15/49
Sponsor: Division 6.5, NBS
Manager: J. H. Levin

Objective: To represent the relationship between velocity of current and meter reading by a fitted curve, which is to be used in calibrating the meter.

Background: In rating current meters, a series of towing tests is made in still water. The meter is towed through the water at a known velocity and the revolutions per second of the meter recorded. The curve representing the velocity is called the rating curve. For purposes of calibration the rating curve may be approximated by two intersecting straight lines. The intersection occurs at about the same meter reading for each type of meter. These lines are used in the calibration of the meters. The equations of the lines are obtained from the observed data on punched card machines. Also obtained are the deviations of the observed current velocities from the current velocities calculated from the lines.

Status: NEW. Computations were performed as requested.

A PROBLEM IN MOLECULAR STRUCTURE
Project 11.2/33-50-3

Origin: Naval Research Laboratory, USN — Authorized 9/1/49
Sponsor: Naval Research Laboratory, USN
Manager: Irene Stegun

Objective: To tabulate values of the function

$$f = e^{\frac{-h}{4ah+1}(R-L)^2} e^{\frac{q^2}{2}} \sqrt{|q|} \left[I_{-\frac{1}{4}}\left(\frac{q^2}{2}\right) + I_{\frac{1}{4}}\left(\frac{q^2}{2}\right) \right], \text{ for } q > 0,$$

$$f = e^{\frac{-h}{4ah+1}(R-L)^2} e^{\frac{q^2}{2}} \sqrt{|q|} \left[I_{-\frac{1}{4}}\left(\frac{q^2}{2}\right) - I_{\frac{1}{4}}\left(\frac{q^2}{2}\right) \right], \text{ for } q < 0$$

where

$$q = \frac{L\,h\,(1-4a\beta) - R\,(h+\beta)}{(4\,ah+1)^{\frac{1}{2}} \left[h(4ah+1) - 4\,a\,(h+\beta)\right]^{\frac{1}{2}}},$$

a and L are assigned constants, and R ranges from 3.9 to 4.6, β from 1 to 7, and h from 20 to 100.

Background: This function occurs in investigations of molecular structure. It represents the probability distribution of the distance R of two atoms in the internal vibratory motion of the molecule, where L is the equilibrium distance of the atoms at rest and h, β are damping factors relating to the vibratory motion.

Comments: This problem was formulated by Dr. J. Karle and Mr. H. Hauptman of the Naval Research Laboratory.

Status: NEW

STANDARD LORAN TABLES
Project 11.2/34-50-1: Gulf Coast Chain

Origin: U. S. Navy Hydrographic Office — Authorized 9/1/49
Sponsor: U. S. Navy Hydrographic Office
Manager: J. H. Levin

Objective: Preparation of tables giving coordinates of hyperbolic lines of positions.

Background: Standard Loran Navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: NEW. Preliminary computations started.

EFFECT OF NUCLEAR RADIATIONS ON HUMAN BEINGS
Project 11.2/35-49-1

Origin: Operational Research Office, U.S.Army (John Hopkins University)
Sponsor: Operational Research Office, U.S.Army (John Hopkins University)
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations of attenuation of radiation for additional energies were started.

TABLES FOR COMBAT CREW PLANNING
Project 11.2/36-49-2

Origin: Air Comptroller's Office, USAF
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations had been completed and checked on punched cards. Preparation of final manuscript continued.

Publication: Tables to be included by the Air Comptroller's Office in the next edition of the Air Force Technical Manual on Combat Crew Planning.

A PROBLEM IN LINEAR PROGRAMING
Project 11.2/36-49-3

Origin: Air Comptroller's Office, USAF
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status. CONTINUED. Several satisfactory procedures for completely solving this type of problem on IBM machines have been devised and rested. Present efforts are being directed to the attainment of greater speed of solution and greater generality in the type of problem handled.

MAGNETIC MOMENT OF ELECTRONS
Project 11.2/37-49-1

Origin: Atomic Energy Commission — Authorized 6/30/49
Sponsor: Atomic Energy Commission — Completed 9/15/49
Manager: M. Oliphant

Objective: To evaluate analytically a certain four-fold integral of a rational function of four variables.

Background: The integral represents effects of fourth-order electromagnetic interactions on the magnetic moment of electrons.

Comments: The problem was proposed by Drs. R. Karplus and M. Kroll of the Institute for Advanced Study, Princeton.

Status: Terminated - at the request of the originators, who are reconsidering the scope of the problem.

3. Training

TRAINING OF PERSONNEL IN THE OPERATION OF NEW IBM MACHINES
Project 11.2/4-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED.

1. Research in Mathematical Statistics

THE MEAN DEVIATION, STANDARD DEVIATION, AND RANGE AS ESTIMATORS OF SCALE PARAMETERS (MEASURES OF DISPERSION) OF PROBABILITY DISTRIBUTIONS
Project 11.3/1-47-2

Origin: NBS
Full project description appears in Oct-Dec 1948 issue.

Status: INACTIVE.

Publications: (1) Probability center lines for standard deviation and range charts. Churchill Eisenhart. Industrial Quality Control VI, No. 1, 24-26 (July 1949). (2) The relative frequencies with which certain estimators of a normal population tend to underestimate its value. Churchill Eisenhart and Celia S. Martin. To be submitted to the Journal of the American Statistical Association. An abstract appears in Annals of Mathematical Statistics XIX, No. 1, 600 (Dec.1948).

STATISTICAL TESTS OF SIGNIFICANCE FOR 2 x 2 TABLES WHEN THE NUMBER OF OBSERVATIONS IS SMALL
Project 11.3/1-47-3

Origin: NBS
Full project description appears in Oct-Dec 1948 issue.

Status: INACTIVE.

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Project 11.3/1-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. About 90% completed. Preparation of Publication (1), giving a complete discussion of the theoretical and numerical results developed under this project, approached completion.

Publications: (1) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. To be submitted to the NBS Journal of Research. (2) The fallacy of the best two out of three. For this summary of the results of the study and their practical implications see Tech. News Bull. 33, No. 8,(1 July 1949).

STATISTICAL PROCEDURES FOR INTERPOLATED MEDIANS
Project 11.3/1-49-2

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Project 11.3/1-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Manager: Henry B. Mann. (Dr. Mann,who is responsible for technical aspects of this project, undertook the project during a temporary employment by the Bureau from March to June 1949, and is expected to return for brief periods until the work is finished. During his absence Dr. Eisenhart handles inquiries and other matters related to the project.)

Status: CONTINUED. A first draft of a monograph on elementary theory of stochastic processes had been prepared by Dr. Mann, assisted by Dr. Miriam L. Yevick. It consists of six chapters: 1) Stochastic processes with a continuous parameter, 2) The Markoff process with continuous increment, 3) The Fourier analysis of stochastic processes, 4) Stationary Markoff processes, 5) Markoff processes with discontinuous increment, and 6) Differential processes modified by mechanical devices.

During July, a limited number of copies of the draft was circulated to certain selected individuals for their comments. It is anticipated that Dr. Mann will return in December 1949 to revise the document in the light of comments received and to consider in detail the inclusion of references, illustrative examples, and instructional aids. Dr. Mann plans to use the monograph in a course on stochastic processes that he will give next spring as a Visiting Professor, at the University of California, Berkeley. It is anticipated that a final draft of the monograph will then be prepared.

2. Manuals of Statistical Methods

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS
Project 11.3/2-47-2

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STANDARD SAMPLING-INSPECTION PROCEDURES
Project 11.3/2-48-1

Origin: Office of Naval Research, and Research and Development Division of the Department of the Army
Full project description appears in Apr-Jun 1949 issue.

Comments: In order to foster a wider appreciation and understanding of some of the standard sampling-inspection systems now in use in the Department of Defense and elsewhere, a one-term, in-hours course in the basic statistical principles, devices, and techniques involved was offered at the Bureau (project 11.3/4-49-7) during the period Mar-Jun 1949. See Projects and Publications Apr-Jun 1949.

Status: CONTINUED. Work on this project from April 1949 to date took the form of active participation by Dr. Eisenhart in the work of the Sub-Committee for Revision of JAN Standard 105, which is a sub-committee of the Sampling Inspection Committee of the Munitions Board Material Inspection Agency. The full membership of this Sub-Committee consists of Mr. David H. Schwartz (Quartermaster Corps Inspection Service), chairman, Churchill Eisenhart (National Bureau of Standards), Lee Goulden (Inspection Administration, Navy Department), O. C. Griffith (Air Materiel Command), Frank E. Grubbs (Ballistics Research Laboratories, Aberdeen Proving Ground), Mary J. Natrella (Bureau of Ships, Navy Department), W. R. Pabst, (Bureau of Ordnance, Navy Department), and E. W. Rote (Air Materiel Command). Each member serves solely as an expert in his field, not as an official representative of his agency.

The present JAN Standard 105 is simply the Navy's "Appendix X" (General Specifications for Inspection of Material, Appendix X, Standard Sampling Inspection Tables for Inspection by Attributes, issued by the Navy Department, April 1946) with a new cover, converting it into "JAN-STD 105, Sampling Inspection Tables for Inspection by Attributes", and bearing such changes in the general instructions as were necessary to convert it to a joint Army-Navy-Air Force document pending the promulgation of a revised standard. Various changes in the present JAN Standard 105 have seemed necessary, or at least desirable, in the light of the needs of the various inspection agencies of the Department of Defense, and their experiences with "Appendix X", the present standard, and related documents. Among such changes are:

1) Replacement of the AQL ranges used as column headings of the tables by single AQL values. The demand for this change was especially strong from inspection personnel at the operating level.

2) Provision of tables (i.e., acceptance-sampling plans) for larger (i.e. "looser") AQL values. The largest AQL accommodated by"Appendix X" is 6.4%, whereas in the Quartermaster Corps, Signal Corps, and elsewhere, inspection by attributes may be called for in cases where AQL's of 15% or 20% are characteristic of the product concerned.

3) Inclusion of tables for inspection on a defects-per-unit basis as well as for inspection on a percent defective (i.e., percent-having-at-least-one-defect) basis. Experience of the Quartermaster Corps, Signal Corps, Army Ordnance Inspection services, and others, with defects-per-unit inspection has seemed to indicate that the indoctrination and training of inspectors at the operating level would be simplified by merging percent-defective and defects-per-unit inspection procedures in a single standard.

4) Replacement of Acceptable-Quality Level (AQL) by Lot-Quality Standard (LQS). The concept of an Acceptable-Quality Level as originally introduced referred to "the maximum percent defective which can be considered satisfactory as a process average", and was used in this sense in standard inspection procedures issued during World War II by the Army Ordnance De-

partment, Quartermaster Corps, and Signal Corps. It thus represented an engineering standard of permissible non-perfection adopted in recognition of the fact that to demand perfection in purchased materials was unrealistic, or at least, impractical and uneconomic; and was related to long-run experience, that is, to the process average. The Navy's "Appendix X", on the other hand, introduced the same term and symbol (AQL) as the "percentage of defective items in an inspection lot such that the sampling plan will result in the acceptance of 95% of submitted inspection lots containing that percentage of submitted material". The AQL so defined was thus a property of the acceptance-sampling procedure concerned, and related to the acceptance of lots by the procedure. Much confusion has resulted from the two-fold difference between these two definitions of the same term and symbol. Administrative and operating-level inspection personnel seem, in the main, to find the engineering-standard approach more natural, and easier to comprehend and employ. Legal counsel advise that the contractual relationship between the purchaser and the supplier generally relates to the purchase of lots, and that the quality requirement must therefore be expressed in terms of lots not process averages. Accordingly, an engineering standard representing the contracted-for quality of lots seems to be needed.

5) Provision of an orderly reduction of the Producer's Risk with increasing lot size. In view of the somewhat different practical circumstances attending the disposition of a large rejected lot and a small rejected lot, the risk of erroneously rejecting a lot of contracted-for quality or better should, it seems, be less for large than for small lots. Also, with lot-by-lot acceptance sampling it is generally not possible, to provide the same protection to the purchaser against acceptance of small lots of very poor quality as against large lots of such quality. Some, oftem considerable, improvement in this situation, from the purchaser's viewpoint, can be achieved by increasing slightly for small lots the producer's risk of having lots of contracted-for quality erroneously rejected by the acceptance-sampling procedure.

As the quarter drew to a close, a final draft of the Sub-Committee's proposed revision of JAN Standard 105 neared completion. Thus the end of this the first leg of the journey toward the ultimate goal - a Government-wide standard for acceptance-sampling by attributes - seems to be in sight.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Project 11.3/2-48-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Project 11.3/2-49-1

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Project 11.3/2-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

3. Statistical Services

STATISTICAL STUDIES OF CLINICAL THERMOMETER TESTING
Project 11.3/31-47-3

Origin: NBS, Section 3.1 Authorized 7/1/47
Manager: J. M. Cameron Completed 9/30/49

Objective: To determine whether, and in what form, acceptance sampling is feasible in inspection and testing of clinical thermometers.

Background: Various Government agencies (e.g., Veterans Administration) that purchase clinical thermometers under contracts referencing Federal Specification GG-T-311; Thermometers, Clinical, use the Bureau as the inspection and testing agency for these thermometers. Although the aforementioned specification includes a provision (Par. F-3) for acceptance or rejection of an entire delivery on the basis of the characteristics and performance of a sample of thermometers from the delivery, many contracts reference the paragraph (F-2) calling for complete inspection and testing of 100% of the thermometers of a delivery. At times backlogs of serious proportions have resulted from this practice. The present project is intended to explore ways and means of reducing the volume of inspection and testing without significant loss of protection to the purchasers.

Comments: Work on an additional report, "Analysis of the reproducibility of the tests for accuracy and consistency of readings," initiated under this project, was discontinued and will be renewed under another project, project 11.3/31-49-1.

Status: COMPLETED. Two cooperative experiments carried out at the Johns Hopkins Hospital to determine (1) the reproducibility of clinical temperature readings under customary hospital conditions and procedures and (2) the correlation of the NBS mechanical hard-shaker tests with manual shaking results of doctors and nurses, were analyzed and evaluated and the results reported by letter to Dr. Rider of the Johns Hopkins Hospital. Other work bearing on this project was transferred to project 11.3/31-49-1 under which more suitable data recently became available.

WOOL CONTENT OF BLANKETS
Project 11.3/31-47-6

Origin: NBS and Division of Statistical Standards, Bureau of the Budget. Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Work on this project from March to September 1949, took the form of active participation by Dr. Eisenhart in the work of a Task Group on Revision of Army Material and Test Method Specifications for Textiles, set up under Sub-Committee B-5 on Presentation and Interpretation of Data, of Committee D-13 on Textiles, of the American Society for Testing Materials, in response to a request from the Sub-Committee on Fibers and Fabrics of the National Research Council's Advisory Board on Quartermaster Research and Development.

The membership of the Task Group consists of Mr. O. P. Beckwith (Alexander Smith and Sons Carpet Company), chairman, Stanley Backer (Research and Development Branch, Quartermaster Corps), Churchill Eizenhart (National Bureau of Standards), John C. Hintermaier (Cluett Peabody and Company), Arthur Lipman (Philadelphia Quartermaster Depot), David H.Schwartz (Quartermaster Corps Inspection Service), and Gerald Winston (Research and Development Branch, Quartermaster Corps). Each member serves solely as an expert in his field, not as an official representative of his organization.

During recent months there had been numerous instances of disagreement over the interpretation of physical- and chemical-test requirements of Quartermaster Corps textile-materials and textile-test-methods specifications. Study of the situation revealed that there was a difference between the intent of the Research and Development specification writer and the interpretation of the Inspection Services and the contractors. The Task Group was charged with the clarification of such specifications so as to leave no room for misinterpretation, and to eliminate any inconsistencies between test-methods specifications and material specifications. In this connection, the following ambiguities or omissions were found to be characteristic of existing specifications of the types under consideration: 1) ambiguity on whether the physical and chemical requirements applied to the average of all the units in a lot or to each unit individually, 2) omission of information on the quantity of cloth constituting an individual "unit" of production for test purposes, 3) ambiguity regarding the number of measurements of a given property or characteristic to be made on each unit, 4) omission of information on the consistency of repeated determinations of a particular property made in accordance with a prescribed method under controlled conditions by a single operator-equipment combination and by different operator-equipment combinations, and ambiguity in the relation of such information, in conjunction with the variability of the material itself, to the required number of determinations per unit, and 5) omission of an engineering standard of permissible nonperfection, representing contracted-for lot quality, in recognition of the fact that to demand perfection (i.e. no defective units, no defects per unit) is unrealistic, or at least impractical and uneconomic.

A proposed revision of the Quartermaster Corps textile-test-method specification and a model material specification embodying the above considerations were prepared by the Task Group and submitted late in September 1949 to the Sub-Committee on Fibers and Fabrics for its consideration.

EFFECT OF GASOLINE AND OIL ADDITIVES ON CARBON AND GUM FORMATION
Project 11.3/31-47-8

Origin: NBS, Division 3 — Authorized 7/1/47
Managers: J. M. Cameron, W. J. Youden — Completed 9/30/49

Objective: To advise and assist personnel of Section 3.5, NBS, with the statistical aspects of the planning and conduct of experiments to determine the effect of gasoline and oil additives on carbon and gum formation of engines.

Background: By utilization of recent advances in the techniques of statistical inference and the principles of experimental design, it is expected that economy and increased efficiency will be effected in this research and testing program. The experiment involves the testing of over 20 additives in combination with a "control" gasoline and oil mixture on 80 similar engines, the problem being to design the most efficient experiment to determine the performance of the various additives.

Status: COMPLETED. An interoffice memorandum summarizing the statistical aspects of the tests was sent to Division 3.

STATISTICAL STUDY OF THE FLOW OF CERTAIN STOCKROOM ITEMS
Project 11.3/31-48-4

Origin: NBS, Section M.4
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STATISTICAL ANALYSIS OF THERMOMETRIC MEASUREMENTS
Project 11.3/31-49-4

Origin: NBS, Section 3.1
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Analysis of the data from all 6 of the tests carried out by the Temperature Measurements Section (3.1) was completed, and the results were presented informally to that section.

WOMEN'S BODY MEASUREMENT STUDY
Project 11.3/31-49-5

Origin: NBS, Section 12.2
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Certain age groupings were decided upon after a study of the frequency distributions by single year of age and for seven control dimensions (stature, hip girth, bust girth, waist girth, abdominal extension girth, abdominal extension arc, neck to bust length). Stature-

hip girth bivariate frequency distributions were then obtained for each of the age groupings. From these, recommendations were made for four stature groups (short-short, short, regular, tall).

During the quarter several conferences of the Washington area group were held and one conference was held in New York City with representatives of the Mail Order Association of America, to whom recommendations were presented and plans developed for continuation of the project.

EXPERIMENTAL ARRANGEMENTS FOR MEASUREMENT OF RADIOACTIVITY
Project 11.3/31-50-1

Origin: NBS, Section 4.4 — Authorized 8/1/49
Manager: W. J. Youden — Completed 9/30/49

Objective: To review the statistical aspects of the problem of comparing standards of radioactivity.

Background: Mr. H. H. Seliger pointed out that extreme care was necessary to avoid drifts and changing conditions when using Geiger counters for comparing the radioactivity of different sources. Customary practice achieves this by making every second observation a determination with the primary standard.

Status: COMPLETED. Experimental trials showed that a Latin Square design accomplished the same purpose of controlling the effects of changing conditions with a marked reduction in the number of determinations on the standard.

PRECISION MEASUREMENTS ON STANDARD TEMPERATURE SOURCES
Project 11.3/31-50-2

Origin: NBS, Section 5.0 — Authorized 8/1/49
Manager: W. J. Youden

Objective: To ascertain the precision of measurements directed to the determination of the reproducibility of temperatures set by the freezing point of chemical substances.

Background: A previous study by F. W. Schwab and Edward Wichers (NBS RP 1647) on the freezing temperature of benzoic acid is being extended to other chemicals. The reproducibility of these temperature sources is apparently close to the limits detectable by the most refined methods of temperature measurement. It is necessary to devise methods to determine whether the small differences found between repeated measurements may be ascribed to the temperature source or the method of measuring the temperature.

Status: NEW. A schedule for the intercomparison of sources and thermometers was developed which increased the precision of the comparison of sources and thermometers. Consideration is being given to means of determining whether the source of the temperature or the method of measuring the temperature is the limiting factor in the observed result.

PROBABILITY THEORY OF A CERTAIN DOUBLE BRANCHING SYSTEM
Project 11.3/31-50-3

Origin: NBS, Section 4.5 Authorized 9/1/49
Manager: J. Lieblein

Objective: To study the probability distribution of two types of particles in a certain multiplicative system, with particular reference to developing statistical tests of the "crossover" probability based on observed frequencies.

Background: Each of an initial set of similar particles (type A) splits into two, and each of the two new particles has probability P (the "crossover" probability of the system) of being a new type, B, and probability (1-P) of remaining of type A. If the new particle is of type A, it, in turn, can produce both types A and B, according to the same probability law, but if of type B it can produce only type B. The successive generations of A and B particles thus produced constitute an important type of multiplicative (branching) process in two variables which does not appear to have been specifically treated heretofore. This problem has been communicated by Dr. Fano in connection with biophysical investigations.

Status: NEW. About 60% completed. Explicit values for several of the moments of the distribution of both types of particles in the nth generation have been evaluated and other features of the distribution were in process of investigation. Everett and Ulam have developed a theory which implicitly contains these results, but which, because of its completely general formulation, is in terms not practicable to apply in the present problem. It therefore was deemed advisable to attack the problem directly by adapting and developing some of the theory outlined in a paper by T. E. Harris dealing with branching systems of only one type of particle, thereby leading more directly and efficiently to the desired results.

STATISTICAL SERVICES TO ARMY FIELD FORCES TEST BOARDS
Project 11.3/32-49-1

Origin: Logistics Division, General Staff, U.S.A Authorized 7/1/48
Sponsor: Research and Development Group, General Staff, U. S. Army
Manager: W. J. Youden

Objective: To provide consultation services to the Army Field Forces Test Boards and on other projects of interest to the Logistics Division.

Background: The Test Boards are concerned with planning experimental field tests of Army equipment and weapons and the evaluation of data where the nature of the tests limits the number of the objects tested to small numbers. It is desirable to make use of modern statistical techniques in the conduct of the test programs.

Comments: This is a continuing project. Visits are made to the Boards either on request or to make available the results of studies originating as a consequence of previous visits. On occasion extended visits are made (in one case nearly a month) for the purpose of becoming

familiar with the problems of the Test Boards by working alongside of the officer personnel on current projects.

Status: CONTINUED. (1) Criteria and brief tables were provided for comparing the aiming errors of one system with those of another system, and the aiming error of a given system with a prescribed performance when only a few measurements are in hand. (2) In the event that one of the aiming errors appears unusually large and is therefore possibly a gross error it was pointed out the median error has certain advantages or alternatively that a statistical criterion is available to assist in determining whether the measurement in question may be considered a gross error. (3) These methods were extended to errors measured in space as well as in two dimensions. (4) The investigation of aiming errors over a range of conditions when only very limited numbers of firings are made presents a particularly difficult problem. A technique was proposed for this situation.

SOLID PROPELLANT DEVELOPMENT TESTS
Project 11.3/32-49-2

Origin: ORDTU, Research and Development Division, Ordnance Dept., U. S. Army
Sponsor: Research and Development Group, General Staff, U. S. Army
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE

PROBABILITY STUDIES OF MISSILE EFFECTIVENESS
Project 11.3/33-49-1

Origin: Bureau of Ordnance, Navy Department, via Electronics Division, NBS
Sponsor: Bureau of Ordnance, Department of the Navy
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. With the aid of Professor A. H. Bowker, acting chairman of the Department of Mathematical Statistics, Stanford University, who served as a special consultant for three weeks, the basic assumptions and probability models were set up. The various data necessary for the solution of the problem were listed, and action was taken on their procurement.

TECHNICAL SYMBOLISM AND TERMINOLOGY
Project 11.3/4-50-1

Origin: NBS Educational Committee Authorized 9/1/49
Manager: Lola S. Deming

Objective: To offer an in-hours "editorial course" designed especially for typists and clerks who are responsible for transcribing, typing, proofing, and editing scientific material. Topics covered to include: Greek alphabet (recognition, pronunciation, writing, and typing);

graphs, charts, and tables (scales, legends, general format, etc.); exponents, subscripts, radicals, other mathematical and chemical symbols; vocabulary and spelling of common scientific terms and proper names; rounding (rules and practices); significant figures.

Background: Because the preparation of a large proportion of manuscripts at the Bureau involves transcribing, typing, and editing of mathematical and scientific material, a course of this nature seemed useful. It was offered for the first time last year and was received with sufficient enthusiasm to cause the Educational Committee to ask to have it repeated this year and extended to two terms to include more material with several sessions conducted by a chemist to cover chemical terminology and vocabulary.

Comments: This course is listed, with the above title, and designated F111.1, on page 17 of the Announcement of Courses for 1949-1950 of the Graduate School of the National Bureau of Standards.

Status: NEW. In the absence of appropriate published text material, dittoed notes were made for distribution to the class and to serve later as the backbone of a manual for the guidance of NBS clerks and typists in preparing, proofing, and editing technical manuscripts.

IV. Machine Development Laboratory

(Section 11.4)

1. Design and Construction of Automatic Digital Computing Machines

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Project 11.4/21-47-1

Origin: The Bureau of the Census
Sponsor: The Bureau of the Census
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Two check points of the UNIVAC contract were successfully passed by the contractor, one relating to engineering design (the BINAC check point) and the other concerned with auxiliary equipment (the UNISERVO check point). UNIVAC design modifications, recommended by the contractor to expedite construction, were found to decrease the UNIVAC operational speed by 15 to 20 per cent, but were considered acceptable.

THE NAVY COMPUTING MACHINE
Project 11.4/22-47-2

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The logical design of this computer is essentially complete. Instruction codes have been made available by the Raytheon Company for analysis by the programing group of this laboratory. Mercury-line storage and tape-handling units were placed under construction during the quarter, and in addition fairly extensive prototyping of computer components was under way.

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.4/23-49-1

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computer proposals by the Engineering Research Associates were proposals of machines whose cost would exceed the funds available for the project. General Electric cost estimates were lower but were estimates made by design engineers and had not been submitted to the General Electric Production Department. Plans for advertising for bids for this computer had been made by the end of the quarter.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Project 11.4/24-47-3

Origin: Office of the Air Comptroller, USAF
Sponsor: " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation. The contract is in the form of a supplement to the Census UNIVAC contract. The status is the same as given for project 11.4/21-47-1.

THE NBS INTERIM COMPUTER
Project 11.4/24-49-1

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The logical design has been frozen. The memory unit has been fully engineered and sufficient prototypes of important components have been built and tested to provide the contractor with the data necessary for him to proceed with construction. Construction of components has been under way by the Electronic Computer Section of the Bureau. It is expected that the computer will be assembled and under test early in 1950.

ARMY MAP SERVICE COMPUTING MACHINE
Project 11.4/25-49-1

Origin: Army Map Service, U.S.A.
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Army Map Service Computer construction contract, with the Eckert-Mauchly Computer Corporation, is in the form of a supplement to the Census UNIVAC contract, providing for delivery of a duplicate of the Census UNIVAC system. The status is the same as given for project 11.4/21-47-1.

DEPARTMENT OF THE ARMY COMPUTER DESIGN
Project 11.4/26-49-1

Origin: Department of the Army
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The memory units are under construction by the contractor. Little direct work on the machine was done during the quarter under review; however, much of the work on the NBS Interim Computer was applicable to this project (see project 11.4/24-49-1)

2. Programing Studies

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Project 11.4/3-47-4

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Emphasis was placed upon investigation of the utility of the UNIVAC as a sorting device. A working group, consisting of representatives of the Bureau, the Bureau of the Census, the USAF and the Navy, was organized to study techniques of electronic sorting and to evaluate existing and proposed devices for high-speed sorting.

CODING ON THE E.R.A. COMPUTER
Project 11.4/3-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Programing analyses were performed in connection with the evaluation of the E.R.A. proposals for an Air Materiel Command computer (see project 11.4/23-49-1).

CODING RELATED TO THE UNIVAC SYSTEM
Project 11.4/3-49-2

Origin: The Bureau of the Census
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. An analysis of the latest instruction code submitted by the Eckert-Mauchly Computer Corporation for the UNIVAC was made in order to determine the effect of the suggested changes on the programing of problems for the computer and the relative efficiency of the new instructions.

CODING RELATED TO THE RAYTHEON COMPUTER
Project 11.4/3-49-3

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Raytheon Manufacturing Company revised the codes and design of its machine. Some programing was done for the purpose of comparing the speed of its performance with that of the UNIVAC.

PERFORMANCE TEST FOR THE BINAC
Project 11.4/3-49-4

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The test routines for the BINAC which had been prepared at the Bureau were rejected by the Eckert-Mauchly Corporation as too time-consuming, and they submitted two of their own tests instead. These tests were analyzed and that one chosen which more closely tested the performance specifications desired by the Bureau. The test was applied, and with the assistance of the Census Bureau the results were checked and evaluated.

3. Technical Reports on Computing Machinery

THE MTAC SECTION
Project 11.4/4-47-1

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for the Automatic Computing Machinery section of the January 1950 issue of Mathematical Tables and Other Aids to Computation was assembled and edited. A technical paper entitled "Magnetic Drum Storage for Digital Information Processing Systems," by Arnold A. Cohen, describes a machine constructed by Engineering Research Associates, Inc. In a discussion article, "Solution of Differential Equations by Recurrence Relations," by John Todd, the author shows, by consideration of a very simple example, that recursive methods for the solution of ordinary differential equations, while very attractive from the point of view of mechanization, must be used with care. The example is discussed in some detail to exemplify a few of the concepts used in modern numerical analysis, such as truncation error, rounding-off error, strict estimates, probabilistic estimates, and stability. Attention is drawn to work of L. Collatz, who discussed similar problems for partial differential equations, and to recent work of L. Fox and E. T. Goodwin, who have developed an iterative

process for the use of recursion formulae and applied it successfully in several cases.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Project 11.4/42-49-2

Origin: Section 11.4, NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. About 85% completed.

Numerical Analysis Colloquium Series
(Los Angeles, California)

HARTREE, D. R. (The Cavendish Laboratory, Cambridge, England). (1) "Forced oscillation of non-linear systems." July 6, 1949. (2) "Some electron space-charge calculations." July 7, 1949. (3) "A method for the numerical integration of first-order differential equations." July 8, 1949.

LANCZOS, C. "The calculation of eigenvalues." July 25, 1949.

HELMER, O. (Rand Corporation). "Methods in the theory of games." August 1, 1949.

ISAACS, R. (Rand Corporation). "Monodiffric functions." August 8, 1949.

DONSKER, M. "Calculation of eigenvalues of the Schrödinger equation by Monte Carlo methods." August 15, 1949.

GRUEN, H. (U.C.L.A. and National Bureau of Standards.) "Properties of light nuclei." August 22, 1949.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

(1) Papers presented at the meeting of the American Mathematical Society held at the University of Colorado, August 30-September 2, 1949:

BECKENBACH, E. F. "A class of mean value functions."

OSTROWSKI, A. M. "On Theordorsen's and Garrick's method of conformal mapping."

ROSSER, J. B. "Real roots of Dirichlet L-series."

SZASZ, O. "On the Gibbs' phenomenon for Euler means."

TODD, J. "The condition of certain matrices." Presented by title.

(2) Papers presented at A Symposium on Large-Scale Digital Computing Machinery held at Harvard University under the sponsorship of the Navy Department Bureau of Ordnance and Harvard University, September 13-16, 1949:

HUSKEY, H. D. "Semi-automatic instruction on the Zephyr."

LANCZOS, C. "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators."

MILNE, W. E. "Numerical methods associated with Laplace's equation."

(3) Invited addresses presented at meetings of the Peripatetic Society.

HUSKEY, H. D. "The National Bureau of Standards electronic digital computer program." At the University of Southern California, August 22, 1949.

KAC, M. "Zeros of random functions." At the University of California at Los Angeles, July 25, 1949.

WARSCHAWSKI, S. E. "On the behavior of the mapping function in conformal mapping." At the University of Southern California, August 22, 1949.

(4) Invited talks given at the Rand Corporation, Santa Monica, California.

HESTENES, M. R. "Problem of Bolza with applications to paths of least time." September 13, 1949.

KAC, M. "Deviations between theoretical and empirical distributions." August 2, 1949.

(5) Other invited talks:

MILNE, W. E. "A new method of numerical solution of differential equations." Presented at the National Bureau of Standards, Washington, D. C., September 9, 1949.

OSTROWSKI, A. M. "On the continuity of roots of algebraic equations." Presented to the Mathematics Department of the University of Maryland, September 22, 1949.

Publication Activities

1. Publications which appeared during the quarter

1.1 Mathematical Tables

(None)

1.2 Manuals, Bibliographies and Indices

(None)

1.3 Technical Papers

(1) Formulas for the percentage points of the distribution of the arithmetic mean in random samples from certain symmetrical universes. Uttam Chand. NBS J. Res. 43, No. 1, 79-80 (July 1949). Available as RP2007 from the Superintendent of Documents, Government Printing Office, Washington 25, D.C. at 10 cents.

(2) Probability center lines for standard deviation and range charts. C. Eisenhart. Industrial Quality Control, VI, No. 1, 24-26 (July 1949). Reprints available.

2. Manuscripts in the Process of Publication September 30, 1949

2.1 Mathematical Tables

(1) Tables of the binomial probability distribution. NBS Applied Mathematics Series 6. Being printed by the Government Printing Office.

(2) Tables to facilitate sequential tests. NBS Applied Mathematics Series 7. Being printed by the Government Printing Office.

(3) Tables of powers of complex numbers. NBS Applied Mathematics Series 8. Being printed by the Government Printing Office. Exact values of z^n for $z = x + iy$, $x, y = 0(1)10$, $n = 0(1)25$; exact values of x^n for $x = 0(1)10$, $n = 0(1)25$.

(4) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. Being printed by the Government Printing Office.

(5) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. Being printed by Columbia University Press.

(6) Tables relating to the Mathieu functions. Being printed by the Columbia University Press.

(7) Table of coefficients in numerical integration formulae. NBS Mathematical Table 22. To be reissued. Being printed by the Government Printing Office. (Originally published in J. Math. Phys. XXII, No. 2, 49-50, June 1943.)

(8) Table of Fourier coefficients. NBS Mathematical Table 23. To be reissued. Being printed by the Government Printing Office. (Originally published in J. Math. Phys. XXII, No. 3, 136-147, Sept. 1943.)

2.3 Technical Papers

(1) Asymptotic expansions of spheroidal wave functions. M.Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) Tables of integrals of Struve functions. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(3) A class of mean value functions. E. F. Beckenbach. Submitted to the American Mathematical Monthly.

(4) Metric differential geometry. E. F. Beckenbach. Submitted to the Mathematics Magazine.

(5) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. Submitted to the Bulletin of the American Mathematical Society; also to appear in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(6) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Submitted to the Duke Mathematical Journal.

(7) Table of modified Bernoulli polynomials. G. Blanch and R. Siegel. Accepted for publication in the NBS Journal of Research.

(8) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Submitted to the Duke Mathematical Journal.

(9) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(10) Acceptance sampling by variables, with special reference to the case in which quality is measured by average or dispersion. J. H. Curtiss. To appear in a special supplement to the Journal of the American Statistical Association.

(11) A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation. M. D. Donsker and M. Kac. Accepted for publication in the NBS Journal of Research.

(12) Exact particle trajectories for nonviscous flow in a plane with a constant coriolis parameter. G. E. Forsythe. Accepted for publication in the Journal of Meteorology. (Originally submitted under the title, Some exact solutions of the equations of non-viscous flow in a plane.)

(13) Solution of the telegrapher's equation with boundary conditions on only one characteristic. G. E. Forsythe. Accepted for publication in the NBS Journal of Research.

(14) Numerical integration for linear sums of exponential functions. R. E. Greenwood. Submitted to the Quarterly of Applied Mathematics.

(15) 1949 Ephemeris of Jupiter's ninth satellite. S. Herrick. To be published as a leaflet by the Astronomical Society of the Pacific.

(16) Some elementary problems in the calculus of variations: M. R. Hestenes. Submitted to the Mathematics Magazine; also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A.

(17) Quadratic forms in Hilbert space, with applications in the calculus of variations. M. R. Hestenes. Accepted for publication in the American Journal of Mathematics.

(18) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Submitted to the Bulletin of the American Mathematical Society.

(19) The remainder in linear methods of approximation. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(20) A note on the numerical integration of differential equations. W. E. Milne. Accepted for publication in the NBS Journal of Research.

(21) Numerical determination of characteristic numbers. W.E.Milne. Accepted for publication in the NBS Journal of Research.

(22) Generalization of a theorem of Osgood to the case of continuous approximation. A. M. Ostrowski. Submitted to the Bulletin of the American Mathematical Society.

(23) On some generalizations of the Cauchy-Frullani integral. A. M. Ostrowski. Accepted for publication in the Proceedings of the National Academy of Sciences.

(24) Note on an infinite integral. A. M. Ostrowski. Submitted to the Duke Mathematical Journal.

(25) Note on Vincent's theorem. A. M. Ostrowski. Submitted to the Annals of Mathematics.

(26) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to the American Mathematical Monthly.

(27) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(28) On the convergence of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(29) Formulas for complex Cartesian interpolation of higher degree. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(30) Polynomials for best approximation over semi-infinite and infinite intervals. H. E. Salzer. Accepted for publication in Mathematics Magazine.

(31) Coefficients for polar complex interpolation. H. E. Salzer. Accepted for publication in Journal of Mathematics and Physics

(32) Tables of zeros and weight factors of the first 15 Laguerre polynomials. H. E. Salzer and Ruth Zucker. Accepted for publication in the Bulletin of the American Mathematical Society.

(33) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Philosophical Magazine.

(34) Formulas for numerical integration of first and second order differential equations in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(35) A bibliography on numerical methods in conformal mapping, W. Seidel. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(36) Inequalities concerning ultraspherical polynomials and Bessel functions. O. Szasz. Submitted to the Bulletin of the American Mathematical Society.

(37) Summation of slowly convergent series with positive terms. O. Szasz. Submitted to the Journal of Mathematics and Physics.

(38) On the Gibbs phenomenon for Euler means. O. Szasz. Submitted by request for publication in the 12th Anniversary volume of the Acta Scientiarum Mathematicarum (Hungary).

(39) A remark concerning the characteristic roots of the finite segments of the Hilbert matrix. O. Taussky-Todd. Accepted for publication in the Oxford Quarterly Journal.

(40) A recurring theorem on determinants. O. Taussky-Todd. Accepted for publication in the American Mathematical Monthly.

(41) A problem on arc tangent relations. J. Todd. Accepted for publication in the American Mathematical Monthly.

(42) The condition of certain matrices. John Todd. Accepted for publication in the Quarterly Journal of Mechanics and Applied Mathematics.

(43) The condition of a certain matrix. John Todd. Accepted for publication in the Proceedings of the Cambridge Philosophical Society.

(44) On conformal mapping of variable regions. S. E. Warschawski. To be included in the "Proceedings of a symposium on the construction and applications of conformal mapping," to be published by the National Bureau of Standards.

(45) Statistics in analytical chemistry. W. J. Youden. Accepted for publication in Transactions of the New York Academy of Sciences.

2.4 Reviews and Notes

Symposia on conformal mapping and Monte Carlo method. J. H. Curtiss. To appear in the October 7th issue of Science.

At the request of the following journals, staff members prepared abstract reviews of technical books and articles, as follows:

Mathematical Reviews
E. F. Beckenbach - 2

U. S. Quarterly Book List
E. W. Cannon - 2

2.5 Miscellaneous Publications

(1) Some recent trends in applied mathematics. J. H. Curtiss. Accepted for publication in the American Scientist.

(2) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

(3) Operational aspects of instrument design. Churchill Eisenhart. To appear in the October 7th issue of Science.

EXPLANATION OF PROJECT DESCRIPTIONS

The project descriptions appearing in this report are reproduced from the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With a view toward making this report more useful, an explanation of certain terms used in the Project Forms is given here.

Date of Authorization. This is the date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Status. Here is given the narrative of the progress to date on the project. Certain descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to which the entry applies. These terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the quarter covered by the report to work on the project.

"CONTINUED" means that the work was initiated prior to the quarter covered by the report, and was in progress during the quarter.

"INACTIVE" means that the Laboratories made a commitment prior to the quarter covered by the report to work on the project, but no work of any consequence was performed on the project during the quarter.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof is to undergo extensive mathematical checks, the designation "Completed" is employed only after these checks have been performed.

"TERMINATED" means that, although all aspects of the objective were not achieved, it was necessary to terminate the project due to circumstances beyond the control of the Laboratories.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "In Manuscript" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "Completed" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under *Status* or *Publication* to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.