Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

January through March 1948

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

PREFACE

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from January 1, 1948 to March 31, 1948.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and provide services in various quantitive branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availibility of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J. H. Curtiss, Chief
National Applied Mathematics
Laboratories

Approved:

E. U. Condon, Director
National Bureau of Standards
May 15, 1948

U. S. DEPARTMENT OF COMMERCE
W. Averell Harriman, Secretary

NATIONAL BUREAU OF STANDARDS
E. U. Condon, Director

Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

January through March 1948

CONTENTS

1. ADMINISTRATIVE ACTIVITIES

The first meeting of the Applied Mathematics Executive Council took place on February 2, 1948. The meeting lasted from 9:45 AM until 4:15 PM, with a recess for lunch, and was largely of an informational nature. Reports were presented on the financial and organizational status of the Division.

The second meeting of the Applied Mathematics Executive Council was held on March 22, 1948. The main item of business was the discussion of the computing machine program. A motion was passed authorizing active negotiations with the Eckert-Mauchly Computer Corporation for three "Univacs", to be purchased under a contract providing for check-tests at the Bureau at various stages in the construction of the machines.

The problem-computing laboratory at the unfinished Wind Tunnel at the University of Maryland started operations in January. Several University of Maryland students have been added to the staff of this unit as part-time employees.

The Applied Mathematics Colloquium Series initiated in fiscal year 1947, was resumed with Lecture 6, "Sampling in Carpet Manufacturing", by Mr. O. P. Beckwith of Alexander Smith and Sons Carpet Company, Chairman of Sub-committee B-5, of the A.S.T.M. Committee D-13 on Textile Materials, delivered the evening of 22 January, 1948 at a joint seminar of the NAML and the Graduate School of the U. S. Department of Agriculture. Attendance 30.

Arrangements are in progress for a seventh lecture to be given by Mr. L. H. C. Tippett, Statistician, British Cotton Industry Research Association, on statistical methods of particular value in industrial experimentation, testing, and quality control.

2. STATUS OF PROJECTS AS OF MARCH 31, 1948

Section 1. The Institute of Numerical Analysis

Project: *48R1-1* *Priority:* *3* *Date Auth.* *1/1/48*

Title: Characteristic Roots of Matrices

Origin: NBS

Project Manager: Dr. Olga Taussky Todd

Objective: To find useful limits for the characteristic roots of general matrices with complex elements. The emphasis is upon obtaining well-defined regions in the complex plane inside of which the roots lie.

Background: There are many applications of this research to solving linear systems of equations, to vibration problems, etc.

Comments: There have been numerous previous researches on this problem. The present work is aimed at developing bounds more precise than those already known.

Status: UNDER WAY (NEW)

Publication: "A Recurring Theorem on Determinants" to appear in American Mathematical Monthly.

Project: *48R1-2* *Priority:* *3* *Date Auth.* *1/1/48*

Title: Applications of a. d. c. m. in Algebra and Number Theory

Origin: NBS

Project Managers: Dr. Olga Taussky Todd and Mr. John Todd

Objective: To investigate the possibilities of attacking problems and procuring data in algebra and number theory with automatic digital computing machines.

Background: The main purpose is the general expansion of mathematical knowledge; in particular, knowledge concerning the applicability of automatic computing machines to problems in pure mathematics. In addition to the more obvious examples in elementary number theory where more numerical evidence is needed, there are many parts of algebraic number theory where new theoretical developments may be expected as soon as more numerical data have been obtained.

Status: UNDER WAY (NEW). It is expected that this general program will break down into sub-programs as it develops. Some typical processes in algebra and number theory have been studied and detailed codes and/or flow-diagrams have been prepared. A particular study is being made of the possibility of using automatic digital computing machinery for determining class numbers of algebraic number fields. The results of the investigation to date appear to suggest that the value of the present general-purpose machines may not be so great in combinatorial problems as they appear to be in analytic problems.

Project: 48R1-3 *Priority:* 3 *Date Auth.* 1/1/48

Title: The Numerical Solution of a Class of Integral Equations

Origin: Admiralty Computing Service

Project Manager: Mr. John Todd

Objective: To develop a practical numerical technique for the solution of integral equations of the form

$$y(x) = c(x) - \int_0^x k(x-t)\,y(t)\,dt.$$

Background: The project originally arose in connection with the design of ducts for propulsion purposes.

Status: COMPLETED. A method was developed in collaboration with Dr. A. Erdelyi and Mr. D. H. Sadler to deal with a special problem. The results achieved received some publicity in England (e.g., "Nature" 157 (1946), 571-3) in view of the fact that a task estimated to take 300 man-years was reduced to one of 52 man-hours by the use of this method. The method has now been described in general, and a simple example worked out, where the detail does not obscure the principles.

It is hoped to explore the possibilities of this method, devised for use with desk calculators, on automatic digital computing machines and to compare it with the direct method of approximating to the integral equation by a ("triangular") system of ordinary linear equations.

Publication: Quarterly of Applied Mathematics and Mechanics (Oxford) or NBS Journal of Research.

Project: 48R1-4 *Priority:* 3 *Date Auth.* 1/1/48

Title: Arc Tangent Relations

Origin: Q18, MTAC by J. C. P. Miller

Project Manager: Mr. John Todd

Objective: Preparation of algorithm for reduction of rational and integral arc tangents and its use in providing a definitive table of reductions.

Background: While preparing a table of $\log\Gamma(x+iy)$ (c.f. 46D2-1) Mr. J. C. P. Miller, (Scientific Computing Service Ltd., London) asked when it was possible to express arc tan m/n (m,n integers) as a finite sum of arc tangents of integers so that the addition formula

$$\ln\Gamma(x+iy-1) = \tfrac{1}{2}\ln(x^2+y^2) + i\arctan(y/x) + \ln\Gamma(x+iy)$$

could be used, with the help of tables of logarithms and arc tangents (of integers), to develop the table from pivotal values obtained in a region of good convergence of certain series.

Status: UNDER WAY (NEW). The theoretical work is complete and has been submitted to the American Mathematical Monthly. This includes two short tables of reductions:

1) arc tan n $\quad n \leq 342$
2) arc tan a/b $\quad a^2 + b^2 < 500.$

A larger table of reductions of arc tan a/b ($a < b < 100$) is about 30% complete.

Further theoretical investigations suggested by the study of these tables are in progress - they lead to what are apparently very deep problems in number theory. Some preliminary results, obtained in collaboration with Professor S. Chowla are ready for publication.

Publications: American Mathematical Monthly, M.T.A.C., and B.A.A.S. Series or NBS Applied Mathematics Series.

Project: 48R1-5 *Priority:* 3 *Date Auth.* 1/1/48

Title: Slowly Convergent Series

Origin: NBS

Project Manager: Mr. John Todd

Objective: Exploration of the possibilities of improvement of convergence of series by various transformations using methods of modern analysis.

Background: Isolated attempts, generally from a practical computational aspect, have been made on this problem. The work of J. R. Airey, W. G. Bickley, J. C. P. Miller should be mentioned; in particular, researches by Bickley and Miller commissioned by Admiralty Computing Service in 1944 but not yet completed. A comprehensive theoretical survey is proposed.

Status: NEW

Project: 48R1-6 *Priority:* 3 *Date Auth.* 1/1/48

Title: Hermite Orthogonal Functions

Origin: NBS

Project Manager: Mr. John Todd

Objective: To examine certain conjectures about the behavior of those functions which have been suggested by a study of numerical tables.

Background: The purpose is the general expansion of mathematical knowledge.

Status: NEW

Project: 48E1-1 *Priority:* 3 *Date Auth.* 1/1/48

Title: Numerical Inverting of Matrices of High Order

Origin: NBS

Project Manager: Mr. John Todd

Objective: To evaluate and expound the paper (Bull. Amer. Math. Soc. 53 (1947), 1021-1099) by J. von Neumann and H. H. Goldstine.

Background: This paper of von Neumann and Goldstine is the first in a proposed series devoted to topics in numerical analysis from the point of view of those who have access to an automatic digital computing machine. The importance of the subject treated, and the pioneering nature of the paper, render exploratory treatment and wide dissemination of the results highly desirable.

Status: UNDER WAY (NEW). Two lectures on this have been given: one at the Computation Laboratory (27 Feb. 1948) and the other at NBS (2 March 1948).

Section 2. The Computation Laboratory

Project: 43D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Bessel Functions $I_\nu(x)$; $\pm\nu$ = 1/3, 2/3, 1/4, 3/4.

Origin: NBS

Project Manager: Dr. M. Abramowitz

Objective: To provide tables of $I_\nu(x)$, $\pm\nu$ = 1/3, 2/3, 1/4, 3/4, for x = 0(.001).5(.01)25, 10D or 10S and $e^{-x}I_\nu(x)$ for x ranging from 25 to 30,000 at varying intervals to 10 decimal places. Central differences are also to be tabulated for purposes of interpolation.

Background: The Bessel Functions $J_\nu(x)$ and $I_\nu(x)$ arise in numerous problems in applied mathematics such as hydrodynamics, heat conduction, and elasticity. They are also of importance in the approximation to Bessel functions of large order, and in the approximation of solutions of differential equations of the form $y'' + p(x)y = 0$ in the Stokes transition region where $p(x)$ changes sign. The need for such approximation arises for instance in the theory of wave propagation in stratified media with a constant gradient in the index of refraction; it also arises in many quantum mechanical problems. (See "Quantum Mechanics" by Condon and Morse.) The project was originally proposed by Dr. S. Schelkunoff of the Bell Telephone Laboratories.

Magnitude: Class III.

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Manuscript and introduction completed.

Publication: IN MANUSCRIPT. To be submitted to Columbia University Press.

Project: 43D2-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Intensity Functions

Origin: National Defense Research Committee, Division X

Project Manager: Mr. W. Horenstein

Objective: The tabulation of functions which give the angular distribution of intensity and total light scattered by (1) transparent small spherical particles (such as a fog droplet) and (2) small spherical particles with small absorption coefficient k ($k < 0.01$) as a function of the parameter $\alpha = 2\pi r/\lambda$ when the particle radius r is roughly equal to the wave length λ of the incident light.

Background: Requested specifically by Professor V. K. LaMer of Columbia University. Intensity functions are used, for example,

1. To determine optimum particle size of DDT aeresols.
2. To determine optimum particle size of paint pigments with the view of obtaining maximum covering power of paint.
3. In connection with micro-wave radar studies.

Magnitude: Class III.

Date of Termination: 1/31/48

Comments: Part of the tabular material in the present volume, previously submitted to Professor LaMer, was included in OSRD report 1857 by V. K. LaMer and D. Sinclair. Earlier computations for very opaque particles (large values of k) were carried out for the Naval Research Laboratory.

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.
Publication: To be published in the NBS Applied Mathematics Series.

Project: 43D2-3 *Priority:* 3 *Date Autn.* 7/1/47

Title: Tables of $E_1(z)$, $(z = x + iy)$

Origin: Canadian National Research Council

Project Manager: Mr. A. Hillman

Objective: To prepare tables of the function $E_1(z) = \int_z^\infty (e^{-u})/u \, du$ and related functions for $z = x + iy$.

Table I: $E_1(z) + \log_e z$, $x = 0(.01)1$, $y = 0(.02)1$; 6D

Table II: $E_1(z)$, $x = 0(.02)4$, $y = 0(.02)3(.05)10$; 6D

Table III: $e^x E_1(z)$, $x = 0(.02)4$; $y = 0(.02)3(.05)10$; 6D

Background: The initial motivation for the preparation of a table of exponential integrals for complex arguments was a certain phase of the atomic bomb project which at present is still undisclosed. The table has however found applications in fluid mechanics. (See for instance, article by J. J. Stoker on "Surface Waves in Water of Variable Depth", in the April 1947 issue of the Quarterly of Applied Mathematics.)

Magnitude: Class IV
Date of Termination: 6/30/48

Status: INACTIVE. Computations completed and partly checked.
Publication: To be submitted for publication to the Columbia University Press.

Project: 43D2-4 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Jacobi Elliptic Functions

Origin: NBS

Project Manager: Mr. W. Horenstein

Objective: (a) To prepare tables of the Jacobi elliptic functions: $sn(u,k) = \sin\varphi$, $cn(u,k) = \cos\varphi$, $dn(u,k) = \sqrt{1-k^2\sin^2\varphi}$ where φ is defined by $u = \int_0^\varphi (1-t^2)^{-1/2} (1-k^2t^2)^{-1/2} dt$. These functions are to be tabulated for $k^2 = 0(.01)1$ and for $u = pK$ with $p = 0(.01)1$ and $K = \int_0^{\pi/2} (1-k^2\sin^2\theta)^{-1/2} d\theta$.

(b) To prepare tables of $sn(iu,k') = i\, sn(u,k)/cn(u,k)$, $cn(iu,k') = 1/cn(u,k)$, $dn(iu,k') = dn(u,k)/cn(u,k)$, for same values of u and k as in (a).

Background: Professor Milne-Thompson originally suggested the preparation of a table of Jacobi elliptic functions for complex arguments. Because of the magnitude of this task, it was deemed sufficient to undertake the computation of the functions in question for real and purely imaginary arguments. The known addition formulae would then enable the user to evaluate elliptic functions for complex arguments. The chief applications contemplated by Professor Milne-Thompson were in the field of hydrodynamics.

Subsequently there was extensive correspondence with members of the Mathematical Tables Committee of the BAAS regarding the scope of this table.

The present specifications incorporate the suggestions contained in the correspondence.

Magnitude: Class III

Comments: The proposed tables are to be computed at equidistant intervals of k^2. A similar table is being published (9/30/47) by the Smithsonian Institution in which the tabulation is at equidistant intervals of α = arcsin k.

Status: INACTIVE. Computations 50% completed.

Project: 43D2-5 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Bessel Functions $Y_0(z)$ and $Y_1(z)$

Origin: NBS

Project Manager: Mr. A. Hillman

Objective: To prepare tables of the Bessel functions of the second kind, $Y_0(z)$ and $Y_1(z)$, for $z = \rho e^{i\theta}$, $\rho = 0(.01)10$; $\theta = 0^\circ(5^\circ)90^\circ$, 10D.

Background: Bessel functions of order zero and one arise in the theory of potential, heat conduction, and wave motion, for a domain bounded by a circle or circular cylinder. They occur also in the propagation of electromagnetic waves with a straight wire as a guide, the theory of the skin effect for poorly conducting wires and many other boundary value problems. The tabulation of these functions was originally suggested by Dr. S. Schelkunoff, Bell Telephone Laboratories.

Magnitude: Class III

Date of Termination: 6/30/48

Status: UNDER WAY (CONTINUATION). Final manuscript prepared; checking about 30% completed; introduction in process of preparation.

Publication: To be submitted to the Columbia University Press.

Project: 43D2-6 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Sines and Cosines to Hundredths of a Degree

Origin: NBS

Project Managers: Messrs. M. Abramowitz, J. Laderman, H. E. Salzer

Objective: To prepare a fifteen-place table of sines and cosines at intervals of .01 of a degree. Second central differences are also to be tabulated.

Background: A fifteen-place table of trigonometric functions at an interval of one hundredth of a degree was computed in 1633 by H. Briggs and H. Gellibrand under the title "Trigonometria Brittanica." This table is very scarce. For this reason and in order to meet the frequent demands for a very accurate table of trigonometric functions with decimal subdivision of the degree, the Mathematical Tables Committee of the British Association for the Advancement of Science has suggested the preparation of a fifteen-place table of all the six trigonometric functions at intervals of one thousandth of a degree, A first phase of this program is the preparation of a fifteen-place table of sines and cosines at intervals of one hundredth of a degree.

Magnitude: Class II

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Manuscript and introduction completed.

Publication: IN MANUSCRIPT. To be published in the NBS Applied Mathematics Series.

Project: 45D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Mathieu Functions II

Origin: Applied Mathematics Panel, NDRC

Project Managers: Dr. G. Blanch, Dr. M. Abramowitz

Objective: To prepare a table of the periodic solutions:

$$Se_r(s,t) = \sum_{n=0}^{\infty} De_{2n+p} \cos(2n+p)\,t, \quad (p = 0, 1)$$

$$So_r(s,t) = \sum_{n=1}^{\infty} Do_{2n-p} \sin(2n-p)\,t, \quad (p = 0, 1)$$

for $r = 0(1)15$, $t = 0°(1°)90°$ over the range $s = 0$ to $s = 100$, of the Mathieu differential equation:

$$y'' + (b - s\cos^2 t)\,y = 0.$$

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press 1947). The existing tables by Ince and Goldstein are inadequate. The project was originally proposed by Dr. Philip Morse, now Director of the Brookhaven National Laboratory, and Dr. James Wakelin, formerly of the Office of Naval Research.

Magnitude: Class IV

Date of Termination: 12/31/48

Comments: Related to Project 46D2-2

Status: INACTIVE. Computations about 40% completed.

Publication: To be published by the Columbia University Press.

Project: 46D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Gamma Functions for Complex Arguments

Origin: NBS

Project Manager: Mr. H. E. Salzer

Objective:

a) To prepare a table of $\log_e \Gamma(z)$, $z = x + iy$ for $x = 0(.1)10$, $y = 0(.1)10$, 10D to 12D

b) A table of $\Gamma(z)$ for same arguments as (a).

c) A table of $1/\Gamma(z)$ (near the origin) for $x = 0(.01)p$, $y = 0(.01)q$ where p and q will be at least equal to unity and probably somewhat larger.

Background: Gamma functions for complex arguments occur in numerous physical problems such as the attraction between two particles in a Coulomb field of force. The existing tables are entirely inadequate for the needs of modern nuclear research problems. Originally suggested by Dr. R. D. Evans of the Massachusetts Institute of Technology.

Magnitude: Class IV

Date of Termination: a) 3/30/48, b) 7/30/48, c) 10/30/48

Status: UNDER WAY (CONTINUATION). Computation of $\log\Gamma(x + iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ completed. Computations for $x = 8(.1)9$ in progress.
Publication: To be published by the Columbia University Press.

Project: 46D2-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Mathieu Functions I

Origin: Applied Mathematics Panel, NDRC

Project Manager: Dr. G. Blanch

Objective: An eight-place table of the first 15 odd and 16 even characteristic values b of Mathieu's differential equation

$$y'' + (b - s\cos^2 t)y = 0$$

for s ranging from 0 to 100 at various intervals, and the Fourier coefficients of the solutions corresponding to these characteristic values as well as certain related functions.

Background: Mathieu functions arise in the solution of the wave equation for elliptical domains. Numerous physical applications involving Mathieu functions are described in "Theory and Applications of Mathieu Functions" by N. W. McLachlan (Oxford Press, 1947). The existing tables by Stratton-Morse-Chu-Hutner, Ince and Goldstein are inadequate. The project was proposed by Dr. Philip Morse, now Director of the Brookhaven National Laboratory, and Dr. James Wakelin, formerly of the Office of Naval Research.

Magnitude: Class III

Date of Termination: 12/31/47

Comments: Related to Project 45D2-1

Status: UNDER WAY (CONTINUATION). Manuscript completed, introduction being prepared.
Publication: To be published by the Columbia University Press.

Project: 47D2-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Spheroidal Wave Functions

Origin: NBS

Project Manager: Dr. M. Abramowitz

Objective: a) To prepare tables of the characteristic values for the differential equation of orders $L = 0, 1, 2, \ldots, 10$

$$(1-x^2)w'' - (2m+1)xw' + (b-c^2x^2)w = 0$$

for $m = 0(1)10$ and c^2 ranging from 0 to about 1000. at various intervals.

b) Tables of the solutions of the differential equation corresponding to the characteristic values under (a).

Background: Spheroidal wave functions are the solutions of the wave equation in prolate and oblate spheroidal coordinates. In his introduction to the "Elliptic

Cylinder and Spheroidal Wave Functions," Professor Morse states "Solutions of problems involving the radiation and scattering of waves from strips of material, from wires of finite length and from discs of material, all require the knowledge of the mathematical properties and the numerical values of solutions of the wave equation for these coordinate systems. The solutions are likewise required for the study of the diffraction of waves through slits and circular openings, the absorption of sound by strips or by circular patches of material and the behavior of electrons in diatomic molecules." Originally proposed by Dr. Philip Morse of the Massachusetts Institute of Technology (now Director, Brookhaven National Laboratory.)

Magnitude: Class IV

Status: INACTIVE. Computations completed for:

$m = 1,\ L = 0$ c^2 ranging from 0 to 100 at various intervals

$m = 1,\ L = 1$ c^2 ranging from 0 to 600 at various intervals

$m = 1,\ L = 2,3,4$ c^2 ranging from 0 to 60 at various intervals

Exploratory work has been performed for other values of m and L.

Publication: To be published in the NBS Applied Mathematics Series.

Project: 4702-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables of Coulomb Wave Functions

Origin: NBS

Project Manager: Dr. M. Abramowitz

Objective: Tabulation of the regular solution $F_L(\rho,\eta) = C_L \rho^{L+1} \varphi_L(\rho,\eta)$ and its derivative $F_L'(\rho,\eta) = C_L \rho^L \varphi_L^*(\rho,\eta)$ and of the irregular solution $G_L(\rho,\eta)$ of the differential equation $y'' + \{1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\}y = 0$ where $C_L^2 = \frac{2\pi\eta(1+\eta^2)(4+\eta^2)\ldots(L^2+\eta^2)2^{2L}}{(e^{2\pi\eta}-1)(2L+1)^2[(2L)!]^2}$

Background: This equation arises in the quantum mechanical treatment of two particles moving in a Coulomb field of force; it arises in particular in the problems of proton-proton and proton-neutron interaction. The special case L = 0 occurs in a problem in classical hydrodynamics. Proposed by Professors Philip Morse of MIT, Gregory Breit of Yale University, and Herman Feshbach of MIT.

Magnitude: Class IV

Date of Termination: 12/31/49

Status: UNDER WAY (CONTINUATION). Computations completed:

Values of $\rho\varphi_0$ and φ_0^* for $\eta = 0(.1)10$, $\rho = 0(.05)1.5$.

Values of $\log \rho\varphi_0$ and $\varphi_0^*/\rho\varphi_0$ and their derivatives with respect to η for $\eta = 4$ with $\rho = 1(.05)2(.1)8$ and $\eta = 6,\ 8,\ 10$, with $\rho = 1(.05)2(.1)10$.

Values of $\sigma_0 = \arg\Gamma(1+i\eta)$ and $\Gamma'(-i\eta)/\Gamma(-i\eta)$ for η ranging from 0 to 110 at various intervals.

Computations partially completed:

Values of $\rho\varphi_0$ and φ_0^* and their derivatives with respect to η for $\eta = .5,\ 1.5,\ 2.5$, with $\rho = 1.5(.1)10$ and $\eta = 4$, $\rho = 8(.1)10$. About 40% completed.

Publication: To be published in the NBS Applied Mathematics Series.

Project: 47D2-3 *Priority:* 3 *Date Auth.* 7/1/47

Title: Table of Antilogarithms

Origin: NBS

Project Managers: Dr. M. Abramowitz, Mr. H. E. Salzer

Objective: To prepare a table of 10^x to ten decimal places for x = 0(.00001)1.

Background: The function 10^x is of basic importance. The best existing table is that of J. Dodson, Antilogarithmic Canon, London, 1742, which is out of print. The proposed publication, which was suggested by Mr. H. E. Salzer of the Computation Laboratory, will be an improvement over Dodson's table from the standpoint of accuracy and format.

Magnitude: Class III

Date of Termination: 12/31/47

Status: INACTIVE. Final manuscript prepared and 25% checked by differencing.

Publication: IN MANUSCRIPT. To be published by Columbia University Press.

Project: 47D2-4 *Priority:* 3 *Date Auth.* 7/1/47

Title: Tables for the Occasional Computer.

Origin: NBS

Project Manager: Entire technical staff.

Objective: To prepare an improved and amplified version of the Jahnke-Emde tables.

Background: The preparation of an improved version of the Jahnke-Emde tables had been originally suggested by Professor E. P. Wigner of Princeton University who submitted suggestions for the contents of the contemplated volume. This matter was discussed with Professor Tukey of Princeton, Professor Barkley J. Rosser of Cornell University and others.

Magnitude: Class IV.

Status: INACTIVE.

Project: 48D2-1 *Priority:* 3 *Date Auth.* 12/26/47

Title: Tables for X-Ray Diffraction Analysis

Origin: Section 9.7 NBS

Project Manager: Dr. M. Abramowitz

Objective: To prepare tables of $\frac{\lambda}{2}\csc\theta$ for $\theta = 0^\circ(.01)90^\circ$ for various values of λ.

Background: These tables are based on new experimental values of the wave lengths. They will supersede previously published tables using incorrect wave lengths.

Specifically requested by Mr. H. E. Swanson.

Magnitude: Class II

Date of Termination: 6/30/48

Status: UNDER WAY (CONTINUATION). Computation for two values of λ completed.

Publication: To be published in the NBS Applied Mathematics Series.

Project: 46S2-1 *Priority:* 2B *Date Auth.* 7/1/47

Title: Heat Conduction Equation

Origin: Bureau of Ordnance, Navy Department

Project Manager: Mrs. I. Rhodes

Objective: To obtain numerical solutions of the non-linear partial differential equation $\frac{\partial\theta}{\partial t} = k\frac{\partial^2\theta}{\partial x^2} + e^{-1/\theta}$ (k = constant) satisfying the initial and boundary conditions: $\theta(x,0) = \theta_0$; $\theta(0,t) = \theta_1$, for various values of θ_0 and θ_1.

Background: The problem had its origin in the investigation of the flow of heat from a hot thermostatic bath into an explosive substance in the form of a sphere immersed in the bath. For practical applications it is permissible to consider the explosive as a plane slab of infinite extent.

Magnitude: Class IV

Date of Termination: 12/31/47

Status: INACTIVE. Computations completed for:

$\theta_0 = .0600$, $\theta_1 = .1000$. $\theta_0 = .0600$, $\theta_1 = .141003$; $\theta_0 = .0600$, $\theta_1 = .2390$.

Preliminary computations carried out for other values of θ_0 and θ_1.

Publication: Report to be submitted to Bureau of Ordnance for photo-offset reproduction.

Project. 47S2-1 *Priority:* 2C *Date Auth.* 7/1/47

Title: Shock Wave Computations

Origin: Bureau of Ordnanc , Navy Department

Project Manager: Mr. William Horenstein

Objective: The determination of a number of shock wave parameters corresponding to $\gamma = c_p/c_v = 1.1$ and $5/3$ and the representation of the results in graphical form.

Background: These computations were originally requested by the Explosive Section of the Bureau of Ordnance, and are now sponsored by Dr. Raymond Seeger of the Naval Ordnance Laboratory. Earlier computations of a similar nature were used in the preparation of Explosives Research Report No. 13, "Regular Reflection of Shocks in Ideal Gases," and Explosives Research Report No. 14, "Interaction of Shock Waves in Water-Like Substances," as well as in checking experiments on three-shock solutions in air which were carried on at the Princeton Station of Division 2, NDRC.

Magnitude: Class IV

Date of Termination: 12/31/47

Status: UNDER WAY (CONTINUATION). Work now in progress for $\gamma = 5/3$ and $\zeta = 0, .1, .3, .5, .7, .9, 1.0$. Computation for $\zeta = .1, .3$ are completed. Computations for $\zeta = .5$ are in progress.

Publication: Manuscript to be submitted to Bureau of Ordnance, Navy Department.

Project: 47S2-2 *Priority:* 2C *Date Auth.* 7/1/47

Title: Computations for Meteorological Project, N. Y. U.

Origin: Office of Naval Research

Project Manager: Mr. J. Laderman

Objective: To perform computations required in objective analyses of meteorological elements. These computations include the determination of least squares solutions for divergence, wind velocity, barometric pressure, etc., based on data collected from weather stations throughout the eastern part of the U. S. Also the determination of large scale eddy stresses, stream lines, etc.

Background: The results obtained in addition to being of current interest in general circulation investigations are expected to establish the feasibility of constructing a machine to carry out these calculations. These computations were requested by Dr. H. Panofsky of New York University and Dr. J. Von Neumann of the Institute for Advanced Study.

Magnitude: Class III

Date of Termination: 12/31/48

Status: UNDER WAY (CONTINUATION). New problems are being processed as prior ones are completed.

Publication: Manuscript submitted to Dr. H. Panofsky of Meteorological Project, New York University.

Project: 48S2-5 *Priority:* 3 *Date Auth.* 7/1/47

Title: Computation of Lattice Sums

Origin: Section 7.7 NBS

Project Manager: Mr. J. Laderman

Objective: To compute the sum of the quantities

$$\frac{W}{K} = \frac{3(a^2 - b^2 + c^2)(a^2 + b^2 - c^2)(-a^2 + b^2 + c^2) + 8a^2 b^2 c^2}{8(a^2 b^2 c^2)^{5/2}}$$

over the face centered cubic and hexagonal lattices, where a^2, b^2, c^2, are given functions of n_1, n_2, n_3, and m_1, m_2, m_3, with n_i and m_j taking on all possible integral values satisfying certain inequalities. A total of about 21000 terms (values of $\frac{W}{K}$) have to be computed and summed.

Background: The purpose of this project is to obtain the difference in the lattice sums of the third order Van der Waals interaction for the two closest packed lattices, the hexagonal and the face centered cubic. The problem was proposed by B. M. Axilrod; approved by Dr. Teller, University of Chicago, and Dr. J. Weyl, Office of Naval Research.

Magnitude: Class III

Date of Termination: 1/31/48

Status: UNDER WAY (CONTINUATION). Computations about 60% completed.

Project 48S2-7 *Priority: 2B* *Date Auth.* 8/28/47

Title: Problem in the Theory of Atomic Spectra II

Origin: Section 4.1, NBS

Project Manager: Mr. A. Hillman

Objective: Tabulation of the radial integrals F^k and G^k for specified values of n, L, n', L', where

$$F^k(n,L,n',L') = \int_0^\infty r_1^k R_1^2(n,L)\, dr_1 \int_{r_1}^\infty \frac{R_2^2(n',L')}{r_2^{k+1}}\, dr_2$$

$$+ \int_0^\infty \frac{R_1^2(n,L)}{r_1^{k+1}}\, dr_1 \int_0^{r_1} r_2^k\, R_2^2(n',L')\, dr_2$$

$$G^k(n,L,n',L') = 2\int_0^\infty r_1^k R_1(n,L)\, R_1(n',L')\, dr_1 \int_{r_1}^\infty \frac{R_2(n,L)\, R_2(n',L')}{r_2^{k+1}} dr_2$$

where

$$R_1(n,L) = \sqrt{\frac{(n-L-1)!}{n^2[(n+L)!]^3}}\, e^{-r_1/n} \left(\frac{2r_1}{n}\right)^{L+1} L_{n+L}^{2L+1}\left(\frac{2r_1}{n}\right) \text{ and}$$

L_{n+L}^{2L+1} is the Associated Laguerre Polynomial.

Background: The integrals arise in the theory of atomic spectra. Proposed by Dr. George Shortley.

Magnitude: Class IV

Status: INACTIVE. Negotiations are under way with the proposer to reduce the scope of the project.

Project: 48S2-8 *Priority: 2B* *Date Auth.* 7/18/47

Title: Computation of the function E(u,q)

Origin: Naval Research Laboratory

Project Manager: Mr. H. E. Salzer

Objective: Computation of the function

$$E(u,q) = \left| \int_{t_0}^{t_1} R^{\frac{1}{2}} I\, e^{-135.33 j R [1+\cos(t-u)]} dt \right|^2$$

where

$$R = \csc^2 t [2\sqrt{1+a \sin t \cos(t-q)} - a \sin t \cos q - 2 \cos t]$$

$$a = 2\cos q - \sin q; \quad \tan t_0 = -a/2; \quad t_1 = t_0 + 90^\circ; \quad j = \sqrt{-1}$$

$$I = e^{-2.613(t-t_0-45^\circ)^2}$$

$E(u,q)$ is to be tabulated for $q = 35^\circ(3^\circ)80^\circ$ and for a 4° range of u centered near the root of $\frac{d}{dt}\{R[1+\cos(t-u)]\} = 0$ for $t = t_0 + 45^\circ$ and for 20 values of u and 16 values of q.

Background: The integral arose in the course of an antenna theory investigation.

Magnitude: Class IV.

Date of Termination: 6/30/48

Status: INACTIVE. Negotiations have been initiated to reconsider the scope of the project.

Publication: Manuscript to be submitted to Naval Research Laboratory.

Project: 48S2-9 *Priority:* 3 *Date Auth.* 8/6/47

Title: Guided Missile Computations

Origin: Section 13.7 NBS

Project Manager: Dr. G. Blanch

Objective: Tabulation and graphs of Z, $\frac{dZ}{du}$, $\frac{d^2Z}{du^2}$, where

$$\frac{d^3Z}{du^3} - \frac{1}{f}\frac{d^2Z}{du^2} + \frac{1}{u}\frac{dZ}{du} - \frac{Z}{u^2} = 0$$

for f = 1, 2, 4, ∞, and various initial conditions.

Background: The differential equation arose in the theory of the trajectory of a homing guided missile. Requested by Dr. H. K. Skramstad of the National Bureau of Standards.

Magnitude: Class III

Date of Termination: 2/28/48

Status: COMPLETED.

Publication: Manuscript submitted to Section 13.7 NBS.

Project: 48S2-10 *Priority:* 2B *Date Auth.* 8/5/47

Title: Subsonic Compressible Flow Calculations

Origin: Office of Naval Research

Project Manager: Mr. W. Horenstein

Objective: To find particular solutions of the differential equation:

$$\frac{1-M^2}{\rho^2}\frac{\partial^2\psi}{\partial\theta^2}+\frac{w}{\rho}\frac{\partial}{\partial w}\left(\frac{w}{\rho}\frac{\partial\psi}{\partial w}\right)=0,$$

for the stream function, ψ, and the analogous differential equation for the velocity potential, φ, suitable for the calculation of vortex patterns, where w and θ are hodograph coordinates, $M=\frac{w}{a}$ = local Mach number, a = acoustic velocity, and ρ = density of the fluid.

Background: The differential equation arose in connection with a problem of turbine design under study at the General Electric Company, Schenectady, N. Y. The mathematical formulation of the problem is due to Professor Bergman and Mr. Hans Kraft of the Turbine Generator Engineering Division of the GE Company. The equations are of general interest in that their solution will furnish a basis for a direct numerical construction of any practically occurring subsonic compressible flow whose description in the hodograph plane is one-valued.

Magnitude: Class II

Status: UNDER WAY (CONTINUATION). An attempt is being made to develop a numerical solution of the problem.

Project: 48S2-12 *Priority:* 2C *Date Auth.* 9/19/47

Title: Loran Log Project

Origin: Section 14.1, NBS

Project Managers: Mr. J. Laderman, Mrs. I. Rhodes

Objective: To analyze observations on the "Musk-Calf" low frequency Loran system. The observations are to be recorded on punched cards, gross errors are to be detected and various statistical analyses are thereafter to be performed.

Background: The analysis was undertaken primarily to ascertain the cause of the discrepancies in the determination of locations by means of the low frequency Loran system of navigation. The results will also be of value in studies of the ionosphere and on anomalous propagation phenomena.

Magnitude: Class III

Date of Termination: 12/31/48

Status: UNDER WAY (CONTINUATION). Computations required for determining standard deviations and correlations are being performed.

Project: *48S2-13* *Priority: 2B* *Date Auth.* *11/13/47*

Title: Electron Ejection Problem

Origin: Clinton National Laboratories, Atomic Energy Commission

Project Manager: Mr. A. Hillman

Objective: Tables of the internal conversion coefficient $\beta = \frac{2\pi\alpha kI}{(I+1)(2I+1)}S$ where $\alpha = 1/137.03$ = fine structure, $I = 1(1)5$, and S is a complicated expression involving Gamma functions for complex arguments and hypergeometric functions for complex values of the parameters and the argument. All parameters and arguments are functions of the atomic number Z in the range from 20 to 90 and the energy of radiation k ranging from 0 to 5.

Background: The above calculations arise in the problem of electron ejection from atomic shells by nuclear gamma rays, and are expected to contribute considerably to the understanding of nuclear structure with particular emphasis on the determination of nuclear energy levels.

Magnitude: Class IV

Date of Termination: 12/31/48

Comments: Exact calculations have been done heretofore for one nucleus only (Z = 84) whereas present day experiments require a knowledge of internal conversion coefficients throughout the periodic tables.

Status: INACTIVE. Computation postponed pending final decision by Oak Ridge National Laboratory.

Project: *48S2-14* *Priority: 2A* *Date Auth.* *12/9/47*

Title: Loran Stations

Origin: U. S. Navy Hydrographic Office

Project Manager: Dr. M. Abramowitz

Objective: Preparation of tables giving coordinates of hyperbolic lines of positions.

Background: Loran Navigation Tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Magnitude: Class III

Date of Termination: 2/1/48

Status: UNDER WAY (CONTINUATION). Computations completed, manuscript being prepared.

Publication: Tables to be published by U. S. Navy Hydrographic Office.

Project: *48S2-15* *Priority: 2B* *Date Auth.* *2/19/48*

Title: Air Force Test Problem

Origin: Air Comptroller, USAF

Project Manager: Mr. J. Laderman

Objective: To find the diet to be formed from 77 different foods with known nutritive values and known cost so that specified quantities of nine nutrients will be obtained at minimum cost.

Background: This is an experimental problem in simultaneous linear algebraic relationships, designed to study certain extremal properties of such relationships. The setting up of a computational procedure for this type of problem is the main objective rather than the solution of this specific diet problem. The computational procedure for this problem is to follow the lines of a procedure prepared by Dr. George B. Dantzig of the Air Comptroller's office after consultation with Professor John von Neumann, Dr. Olga Taussky Todd of Section 11.1, and others.

Magnitude: Class II

Date of Termination: April 10, 1948

Comments: The project is a part of the mathematical work supporting Project 48D4-3.

Status: UNDER WAY (NEW). Computations are 75% completed.

Project: 48S2-16 *Priority:* 2C *Date Auth.* 2/20/48

Title: Friction Coefficients

Origin: Stevens Institute of Technology

Project Manager: Miss Irene Stegun

Objective: To check the table of Schoenheer Friction Coefficients for Reynolds Numbers 1×10^5 to 1×10^{10}.

Background: This work is part of a research project being carried out by the Hydromechanics Sub-committee of the Research and Technical Committee, Society of Naval Architects and Marine Engineers to establish a uniform practice in the shipbuilding and ship design industry for the calculation of the skin friction of ships and other hydrodynamic bodies and the expansion of towing tank model test data to full scale. The research activities of the Society are being sponsored in part by the Office of Naval Research, Navy Department, under Research and Development Contract N6-ONR-420.

Magnitude: Class II

Date of Termination: March 31, 1948

Status: COMPLETED.

Publication: Manuscript submitted to Experimental Towing Tank, Stevens Institute of Technology, Hoboken, N. J.

Project: 48S2-17 *Priority:* 2B *Date Auth.* 2/18/48

Title: Particle Distribution Problem

Origin: Oak Ridge National Laboratory

Project Manager: Mr. A. Hillman

Objective: To determine the frequency distribution of the distances traveled by particles (e. g., neutrons) passing through matter. The method consists in actually tracing the life history of many particles, determining distances between collisions and types of collisions by experimental sampling methods. Each particle is traced until it either disappears or its energy falls below a prescribed value.

Background: This problem is an example of the so-called "Monte Carlo" method which is now under consideration by nuclear scientists as a useful tool for solving a variety of diffusion problems. Such problems can generally also be formulated in terms of integro-differential equations, but these equations are sometimes intractible from the point of view of obtaining numerical results.

Magnitude: Class III

Date of Termination: July 1948

Comments: A basic reference for such problems is an abstract by Ulam and von Neumann in the Bulletin of the American Mathematical Society, vol. 53, (1947), p. 1120.

Status: UNDER WAY (NEW). A procedure for carrying out the computations on IBM machines has now been developed.

Project: 48S2-18 *Priority: 2C* *Date Auth. 2/20/48*

Title: Shock Wave Problem

Origin: ONR contract with IAS

Project Manager: Miss Irene Stegun

Objective: To make hydrodynamical calculations, involving shocks, with difference equations in order to test whether w (shock width) can be chosen large enough to produce desired results without introducing serious instability.

Background: Problem was proposed by Dr. von Neumann.

Magnitude: Class II

Date of Termination: 3/23/48

Status: UNDER WAY (NEW). Computations completed for various values of the nominal shock width. Further computations suspended pending conference with Dr. R. Richtmeyer of Institute of Advanced Study.

Publication: Manuscript to be submitted to Dr. R. Richtmeyer of Institute for Advanced Study.

Project: 4852-19 *Priority:* 2B *Date Auth.* 3/8/48

Title: Klein-Nishina Formula

Origin: Section 4.5, NBS

Project Manager: Miss Irene Stegun

Objective: To evaluate R = N/D, where

$$N = [1-2n(n+1)]\log\frac{m}{n} + n(n+2)\frac{m-n}{m} + n(m-n) + \frac{m^2-n^2}{2m^2}$$

and

$$D = [1-2n(n+1)]\log\frac{n+2}{n} + 4n + 2\left[\frac{1+n}{(2+n)^2}\right]$$

for a number of values of n and m.

Background: This was a pilot study undertaken in connection with an extensive calculation of x-ray scattering being initiated in Section 4.5. Proposed by Dr. U. Fano.

Magnitude: Class I

Date of Termination: 3/8/48

Status: COMPLETED.

Publication: MANUSCRIPT submitted to Dr. U. Fano, NBS.

Section 3. The Statistical Engineering Laboratory

Project: 47R3-1 *Priority: 3* *Date Auth. 7/1/47*

Title: The Arithmetic Mean and the Median as Estimators of Location Parameters of Probability Distributions.

Origin: Section 11.3, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: To evaluate percentiles and other features of the distributions of the arithmetic mean, the median, and other estimators of parameters of location, in random samples from normal (Gaussian), Cauchy, Laplace (double-exponential), rectangular, sech, $sech^2$ (derivative of the "logistic"), and "contaminated normal", populations.

Background: This project stems from a study, undertaken in connection with another Project of the S series, of various procedures that have been advocated, over the years, for rejecting, or giving less weight to, anomalous or extreme observations. The motivating question was as follows: If the practice of reporting the medians of sets of measurements, instead of their arithmetic means, is adopted as a way of reducing the effect of occasional anomalous observations (due, perhaps, to faulty measurement, but possibly to more chance fluctuations) on the reported "averages", then what losses, if any, in accuracy and precision are to be expected when the measuring process is actually in control,

Previous studies of the relative merits of the arithmetic mean and the median have, almost without exception, concentrated on comparing the moments of their distributions in large samples, since in large samples the distributions of both the mean and the median tend to normality. Comparison of the arithmetic mean and the median in small samples (e.g., of 3, 5 or 7 observations) has been virtually neglected. The present approach via percentiles promises to give valuable new information.

Magnitude: Class II

Status: INACTIVE. About 70% completed.

Project: 47R3-2 *Priority: 3* *Date Auth. 7/1/47*

Title: The Mean Deviation, Standard Deviation, and Range as Estimators of Scale Parameters (Measures of Dispersion) of Probability Distributions.

Origin: Section 11.3, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: To evaluate percentiles and other features of the distributions of these estimators in small random samples from normal (Gaussian) and various non-normal (see Project 47R3-1) populations.

Background: Previous studies of the relative merits of the mean deviation, standard deviation, and range as estimators of scale parameters of probability distributions have, in the main, concentrated on (a) evaluation of adjustment factors for rendering them unbiased estimators of, say, the standard deviation of the population; and (b), comparison of their "efficiencies" (as measured by the ratios of their sampling variances when so adjusted). Since their distributions

in small samples are non-normal and generally differ in form, comparisons of their "efficiencies" in small samples may not truly represent their relative merits with regard to accuracy and precision in such cases. The approach via percentiles and other features (e.g., the probability of underestimating the true value of the relevant scale parameter) is expected to yield important new information.

Magnitude: Class II

Status: INACTIVE. About 10% completed.

Project: 47R3-3 *Priority:* 3 *Date Auth.* 7/1/47

Title: Statistical Tests of Significance for 2 x 2 Tables When the Number of Observations is Small.

Origin: Program Committee, Institute of Mathematical Statistics.

Project Manager: Dr. Churchill Eisenhart

Objective: To compare Fisher's "exact" test, Barnard's "C.S.M." test, and certain other statistical tests for data arranged in 2 x 2 tables with respect to (a) scope, i.e., conditions for which the respective tests are valid, and (b) operating characteristics (i.e., bias, power, etc.) under the conditions for which they are jointly valid.

Background: The project was undertaken in connection with an invited address given at the Symposium on 2 x 2 Tables sponsored by the Institute of Mathematical Statistics at the New Haven, Connecticut meeting on September 2, 1947.

Magnitude: Class II

Date of Termination: June 30, 1948

Status: INACTIVE. About 60% completed.

Tables and graphs have been prepared showing the operating characteristics of the "exact" and "C.S.M." tests when used as tests of the equality of the parameters of two binomial distributions from which samples of sizes n_1 and n_2, respectively, are drawn independently at random. The ".05" level of significance (in one case, an .0007 level) was adopted, and the tables and graphs were constructed for the cases of $n_1 = n_2 = 3$; $n_1 = 4$, $n_2 = 7$; and $n_1 = n_2 = 7$. A three-dimensional cardboard model of the "power surface" was constructed showing for the case of $n_1 = 4$, $n_2 = 7$, the greater power of the C.S.M. test relative to the exact test when both are used as tests of the equality of the parameters of two binomial distributions - the type of problem for which the C.S.M. test was expressly developed.

Publication: DITTOED copies of a synopsis of Dr. Eisenhart's New Haven address, including an annotated bibliography and the tables and graphs mentioned above can be obtained from the Statistical Engineering Laboratory while the supply lasts.

The three-dimensional model showing the power surfaces of the exact and the C.S.M. tests for the case discussed above is available for examination in the Statistical Engineering Laboratory.

Project: 4703-1 *Priority:* 3 *Date Auth.* 7/1/47

Title: Power Function of Analysis-of-variance Tests, Requirements for New Tables of.

Origin: Section 11.3, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To formulate requirements for a set of new tables of the integral and percentage points of Tang's distribution

$$p(u) = \sum_{i=0}^{\infty} \frac{\lambda^i e^{-\lambda}}{i!\, B\left(\frac{f_1+2i}{2}, \frac{f_2}{2}\right)} \left(\frac{u}{1+u}\right)^{\frac{f_1+2(i-1)}{2}} \left(\frac{1}{1+u}\right)^{\frac{f_2+2}{2}}, \quad 0 \leq u \leq \infty,$$

the integral of which furnishes the power function (or, operating characteristics) of analysis-of-variance procedures for making decisions with regard to the presence or absence of fixed (constant) relations of specified form among the means of sub-sets of a statistical population.

Background: During the fall of 1946, Professors Jerzy Neyman (University of California, Berkeley) and Abraham Wald (Columbia University, New York City) discussed with Dr. A. N. Lowan, Chief, The Computation Laboratory (Section 11.2), the possibility of having The Computation Laboratory prepare a set of new tables of the integral and/or percentage points of "the power function of the analysis-of-variance tests". By joint letter dated 31 October 1946, they submitted a specific request to Dr. Lowan. This letter was subsequently referred to the Statistical Engineering Laboratory (Section 11.3) for consideration in the light of the broad, over-all tabulation needs of mathematical statistics.

Magnitude: Class II

Date of Termination: May 1948

Comments: The above estimated date of termination refers to probable date of completion of a final formulation of the requirements of the proposed tables, not to the probable date of completion of the computation of the tables, if authorized as a project of the Computation Laboratory.

Copies of the recommendation outlined below, when completed, will be furnished on request, and comments and suggestions regarding order of preparation, urgencies, etc., of the tables envisaged will be welcomed.

Status: UNDER WAY (CONTINUATION). About 98% completed.

A detailed recommendation, being prepared in the light of comments and advice received from W. G. Cochran, R. A. Fisher, H. Hotelling, J. Laderman, A. N. Lowan, W. G. Madow, J. Neyman, E. S. Pearson, P. C. Tang, J. W. Tukey, A. Wald, S. S. Wilks and others, to be submitted for consideration as a computation project, was nearing completion as the quarter ended. Three volumes of tables

are being proposed: (1) A volume containing tables of (a) "percentage points" of the non-central t-distribution (to which Tang's distribution above reduces when $f_1 = 1$) for a selection of values of λ and f_2; and (b), "power levels" of λ for one-sided and two-sided t-tests of linear hypotheses regarding means, for a selection of "significance levels" (α), "powers" ($1-\beta$), and "degrees of freedom" (f_2). (2) A volume containing tables of (a) percentage points of the non-central chi-square distribution (to which Tang's distribution reduces when $f_2 = \infty$) for a selection of values of λ and f_1; and (b), power levels of λ for chi-square tests of linear hypotheses regarding means, for a selection of significance levels, powers, and degrees of freedom (f_1). (3) A comprehensive volume of tables of (a) percentage points of Tang's distribution above for a selection of values of λ, f_1, and f_2; and (b), power levels of λ for the analysis-of-variance (F or z) tests of linear hypotheses regarding means, for a selection of significance levels, powers, and combinations of degrees of freedom (f_1 and f_2).

Project: 4703-2 *Priority:* 3 *Date Auth.* 7/1/47

Title: Formulas for Operating Characteristics and Sample Sizes for Certain Statistical Tests.

Origin: Section 11.3, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To provide a useful collection of formulas for the operating characteristics and the number of observations needed for certain single-sample one-sided tests of statistical hypotheses, with instructions for their application.

Background: Procedures are given in the statistical literature (e.g., in textbooks, journal articles) for determining operating characteristics (discriminating power) and the number of observations needed (cost) for certain single-sample one-sided tests of statistical hypotheses, but they generally require the use of specialized probability tables. It does not appear to have been generally recognized that many problems of these types can be satisfactorily handled by means of relatively simple approximate formulas requiring for their use only certain readily accessible and easily remembered normal-probability deviates.

Magnitude: Class II

Date of Termination: June 1948

Status: UNDER WAY (CONTINUATION). About 85% completed.

Approximate and exact formulas for acceptance numbers, operating characteristics, and the number of observations needed for the common single-sample one-sided and two-sided tests of the following were assembled or developed by Mr. Uttam Chand, guest worker from India, during the summer of 1947: (1) the mean of a normal population, when σ is (is not) specified, (2) the difference of the means of two

normal populations, when σ is (is not) specified, (3) the variance of a normal population, (4) the ratio of the variances of two normal populations (5) the fraction of a normal population exceeding a specified value, (6) the correlation coefficient of a bi-variate normal population, (7) the parameter of binomial and Poisson-exponential populations, and (8) the difference of the parameters of binomial and Poisson-exponential populations. These formulas are incorporated by Mr. Chand in a preliminary draft of a paper to be submitted for publication in an appropriate statistical journal and as a manual of the Applied Mathematics Series of the NBS, after preparation of a set of examples illustrating applications and the accuracies of formulas is completed. Work on determining the accuracy of the formulas in cases (1), (2), (5), and (7) is virtually completed and is in progress for the other cases.

Publication: The manuscript of the above mentioned draft is available for consultation in the Statistical Engineering Laboratory.

A "Preliminary Note on Accuracy of Certain Formulas for Sample Size and Acceptance Number in the case of Single-Sample Acceptance-Sampling by Attributes" was prepared by J. M. Cameron and C. Eisenhart, and circulated to various interested persons. Dittoed copies may be obtained from the Section Office.

Project: 4803-1 *Priority:* 3 *Date Auth.* 7/15/47

Title: Standard Sampling-Inspection Procedures

Origin: Office of Naval Research, Research and Development Division, War Department.

Project Managers: Dr. Churchill Eisenhart and Mr. Julius Lieblein

Objective: To revise, expand, and extend the present Navy Department manual "Standard Sampling Inspection Procedures" (Part D, Chapter 4, of the Administration Manual of the Material Inspection Service, U.S.N.) so as to be suitable for referencing in all types of Government specifications; particularly, in Federal Specifications.

Background: A task group of the Inspection Advisory Council of the War Department undertook during the fiscal year 1947 to effect a revision of the Navy "Procedures" to adapt them to War Department needs. Additional tables recommended by this task group were computed by Section 11.2 in that fiscal year. It was decided that further mathematical work would be needed on the entire task and that the task should be generalized to include all Federal procurement. Theoretical aspects have been assigned to Section 11.3, while responsibility for practical aspects remains in the Department of National Defense.

Magnitude: Class III

Date of Termination: June 30, 1949

Status: INACTIVE. About 5% completed.

Project: 48D3-2 *Priority:* 3 *Date Auth.* 1/23/48

Title: Introduction to the NBS Handbook of Physical Measurements

Origin: Office of the Director, NBS

Project Managers: Dr. J. H. Curtiss and Dr. Churchill Eisenhart

Objective: To provide a summary introduction to modern concepts and techniques of mathematical and statistical analysis that pertain to the design of experiments, and the reduction, analysis interpretation of experimental data.

Background: During the past two-and-a-half decades the classical Theory of Errors has been supplanted by a more general, more rigorous, and in some cases simpler, Theory of Statistical Inference; the tools of numerical mathematics (e.g. computing machines, mathematical tables) have been improved in scope, efficiency, and reliability; the techniques of statistical analysis that pertain to the reduction, analysis, and interpretation of experimental data have been sharpened and expanded considerably; and a vigorous science of the statistical design of experiments has come into being and become a widely recognized discipline.

Magnitude: Class II

Date of Termination: December 1948

Status: NEW

Project: 48D3-3 *Priority:* 3 *Date Auth.* 1/23/48

Title: Glossary of Statistical Engineering Terminology

Origin: Section 11.0, NBS

Project Manager: Miss Celia S. Martin

Objective: To prepare a glossary of the statistical terminology associated with acceptance sampling and process control, statistical analysis and interpretation of experimental and test data, and statistical design of experiments and tests.

Background: The application of statistical concepts and techniques to acceptance sampling and process control has given rise to new terms, and many everyday terms are used with very specific connotations. The relatively new art of the statistical design of experiments and tests also has a special vocabulary. Finally, the concepts, principles, and techniques of statistical inference as applied to the analysis and interpretation of experimental and test data have been revised and expanded considerably during the past two decades with consequent changes in the meanings of terms, and the introduction of new terms.

It is highly desirable, therefore, that a glossary of statistical engineering terminology be prepared, to eliminate some of the present confusion in this field, and to facilitate wider understanding of the subject.

Magnitude: Class II

Date of Termination: 30 June 1948

Comments: Related to, providing vocabulary for, but having a somewhat broader base than Project 48D3-2.

Status: UNDER WAY (NEW). About 10% completed.

Preparation of a list of terms that require definition, i.e. terms for which exact meanings would be helpful, is nearing completion. Some correspondence was exchanged with the Committee on Standards of the American Society for Quality Control which is engaged in a somewhat similar undertaking with respect to quality control terms and symbols.

Project: 47S3-2 *Priority:* 2B *Date Auth.* 7/1/47

Title: Pre-ignition Rating of Spark Plugs

Origin: Division 3, NBS

Project Manager: Dr. J. H. Curtiss

Objective: To determine statistical tolerance limits for preignition ratings, in 17.6 test engine, of automotive spark plugs of different manufacturers.

Background: The NBS has been requested, by the Electrical Supplies Committee of the Federal Specifications Board, to formulate a classification of automotive spark plugs into groups on the basis of pre-ignition rating in the 17.6 test engine, as a possible substitute for the classification given in para. B-2 of Federal Specification W-P-506.

Magnitude: Class I

Date of Termination: September 1948

Comments: Factors needing study include: (a) possible trends in the successive rating of individual spark plugs; (b) errors inherent in the testing method, e.g. degree of reproducibility of test results in repeated tests on a given machine; and (c) mutual consistency of results of tests conducted by different laboratories. These are to be investigated by means of statistically designed experiments.

Status: UNDER WAY (CONTINUATION). About 40% completed.

A "Proposed Revision of Federal Specification W-P-506 for Plugs; Spark" was prepared, in collaboration with Dr. F. G. Brickwedde, 3.0, and Mr. G. F. Blackburn, 3.6.

Project: 47S3-3 *Priority:* 2C *Date Auth.* 7/1/47

Title: Statistical Studies of Clinical Thermometer Testing

Origin: Section 3.1, NBS

Project Manager: Mr. J. M. Cameron

Objective: To determine whether, and in what form, acceptance sampling is feasible in inspection and testing of clinical thermometers.

Background: Various government agencies (e.g., Veterans Administration) that purchase clinical thermometers under contracts referencing Federal Specification GG-T-311; Thermometers, Clinical use the Bureau as the inspection and testing agency for these thermometers. Although the aforementioned specification in-

cludes a provision (Par. F-3) for acceptance or rejection of an entire delivery on the basis of the characteristics and performance of a sample of thermometers from the delivery, many contracts reference the paragraph (F-2) calling for complete inspection and testing of 100% of the thermometers of a delivery. At times backlogs of serious proportions have resulted from this practice. The present project is intended to explore ways and means of reducing the volume of inspection and testing without significant loss of protection to the purchasers.

Magnitude: Class II

Date of Termination: June 1948

Status: UNDER WAY (CONTINUATION). About 90% completed.

The results of this study are now being written up in a series of reports. The first of these, "An experiment in Acceptance Sampling of Clinical Thermometers", was being typed as the quarter ended, for limited circulation to secure comments and suggestions for revision, if any. A second, entitled "An Analysis of Recent Inspection Records" is virtually ready for typing for similar circulation. Three additional reports, for which material has been (or, is being) assembled, are projected: "Analysis of the Reproducibility of the Tests for Accuracy and Consistency of Readings", "Considerations Appropriate to the Revision of Clinical Thermometer Specifications", and "Proposed Revision of Present Clinical Thermometer Specifications".

Project: 4783-5 *Priority:* 2C *Date Auth.* 7/1/47

Title: Physical Constants of Leather

Origin: Section 7.4, NBS

Project Managers: Dr. Churchill Eisenhart and Miss Celia S. Martin

Objective: To advise and assist personnel of Section 7.4 in evaluating the accuracy and precision of methods of measuring the physical constants of leather.

Background: The development and application of statistical methods to industrial process control within the last quarter century has indicated that measurement and testing procedures can fruitfully be regarded as production processes, the "product" being measurements, and that the validity of conclusions based on measurements obtained by a given procedure may be open to question unless the procedure is shown to be in a "state of statistical control". When regarded from this viewpoint, evaluation of the accuracy and precision of a measurement or testing procedure involves (a) showing that it is in a "state of statistical control" (i.e., among other things, that successive measurements are independent, and that the process is free from trends and non-random shifts after adjustment for known or likely sources of bias); and (b) evaluating the parameters of the procedure.

It is expected that projects of this nature will increase the value, for purposes of scientific inference, of measurements taken at the Bureau, and increase the usefulness of test results as a basis for decisions.

Magnitude: Class II

Date of Termination: Latter half of Fiscal 1948

Status: INACTIVE. About 80% completed.

Project: 47S3-6 *Priority:* 3 *Date Auth.* 7/1/47

Title: Wool Content of Blankets

Origin: Section 7.5, NBS; Division of Statistical Standards, Bureau of the Budget.

Project Manager: Dr. Churchill Eisenhart

Objective: To develop a procedure for sampling a lot of part-wool blankets and for taking one or more specimens from each of the sample blankets in order to determine the wool content and weight of the blankets with reasonable assurance.

Background: The present Federal Specifications for blankets gives no procedure, and the A.S.T.M. Standard, a very inadequate procedure, for sampling a lot of blankets. Neither specification gives any instructions for taking one or more specimens from each of the sample blankets for tests and analysis. The scanty instructions given in general specifications on the number of specimens to be subjected to any particular test apply only to the verification of the precision of the test procedures and leave variability of product out of consideration.

The omission of a proper sampling plan for blankets is serious as the specifications requires a minimum wool content and a minimum weight without specifying whether these minima apply to the lot, sample, blankets or specimen. Therefore, the manufacturer will generally either have to furnish a large excess of wool content and weight over the requirement or else take a big chance that his blankets will be rejected. The purchaser has the same risk of obtaining a large quantity of material that is deficient in wool content and weight by accepting material on the basis of results obtained on a piece or pieces that are not representative of the material.

It is the aim of this project to determine the variation in wool content and weight from point to point within a blanket and from blanket to blanket within a lot. This will furnish a basis for a rational sampling procedure as well as for giving an operational meaning to the terms minimum "percent wool" and "weight" as commonly applied to blankets.

Magnitude: Class II

Date of Termination: December 1948

Comments: This project stems from a project completed in the fiscal year 1947 dealing with the components of the variance of a wool content determination based on a small piece taken at random from a part-wool blanket.

Status: UNDER WAY (CONTINUATION). About 10% completed.

A conference was held at A.S.T.M. Headquarters in Philadelphia on 8 March 1948, attended by representatives of the National Bureau of Standards (Dr. Eisenhart, Section 11.3 and Mr. R. F. Tener, Section 7.5), Bureau of the Budget (Dr. W. Edw. Deming), the A.S.T.M., and blanket manufacturers. As a result of this conference, a Task Group on Sampling Blankets for Wool Fibre Content and Weight has been set up under Sub-committee B-5 on Sampling, Presentation and Interpretation of Data, of Committee D-13 on Textile Materials, of the A.S.T.M. The membership of the Task Group includes representatives of the Government (Messrs. Deming, Eisenhart, and Tener), of the A.S.T.M., and of blanket manufacturers. Dr. Eisenhart has been appointed Chairman of the Task Group.

Project: 47S3-7 *Priority:* 2C *Date Auth.* 7/1/47

Title: Flammability of Textiles

Origin: Section 10.2 (formerly of Section III-6), NBS

Project Manager: Mr. J. M. Cameron

Objective: To evaluate the sampling and testing clauses in "Flammability of Textiles - Recommended Commercial Standard TS 4350" and in "Flameproofing of Textiles" (NBS Circular C 455); to develop alternative and additional procedures as necessary.

Background: Proposed evaluation originally requested by Mr. S. H. Ingberg (of III-6), recently retired, whose work is being carried on by Dr. Marjorie W. Sandholzer of Section 10.2 (formerly of III-6).

Magnitude: Class I

Date of Termination: September 1948

Status: INACTIVE. About 1% completed.

Special test equipment was sent to the participating laboratories during February 1948. It is estimated that 3 months will be required for the testing and collection of the data in a form suitable for analysis.

Project: 47S3-8 *Priority:* 2C *Date Auth.* 7/1/47

Title: Effect of Gasoline and Oil Additives on Carbon and Gum Formation

Origin: Division 3

Project Manager: Mr. J. M. Cameron

Objective: To advise and assist personnel of Section 3.5, NBS, with the statistical aspects of the planning and conduct of experiments to determine the effect of gasoline and oil additives on carbon and gum formation of engines.

Background: By utilization of recent advances in the techniques of statistical inference and the principles of experimental design, it is expected that economy and increased efficiency will be effected in this research and testing program. The experiment involves the testing of over 20 additives in combination with a "control" gasoline and oil mixture on 80 similar engines, the problem being to design the most efficient experiment to determine the performance of the various additives.

Magnitude: Class I

Date of Termination: June 1948

Status: UNDER WAY (CONTINUATION). About 40% completed.

Data from test runs using only a base fuel were analyzed to determine whether the engines used in the experiments were properly synchronized. Data from a first run to determine the effect of an additive on carbon and gum formation were analyzed also and indicated the variability of the test results to be such that

only large reductions in the amount of gum and carbon deposited due to the effect of the additive were likely to be detected in tests of feasible size. As a result of this latter analysis it was decided that a different base fuel should be used in future tests.

Project: 4783-8 *Priority:* 3 *Date Auth.* 7/1/47

Title: Teen-age girls' body-measurement study

Origin: Section 12.2, NBS

Project Managers: Dr. Churchill Eisenhart and Mrs. Lola S. Deming

Objective: Reduction and analysis of certain body-measurement data for "teen-age" girls in order to establish standard "teen-age" size designations for wearing apparel. The analysis proposed will be based chiefly on two bivariate frequency diagrams: hip girth versus stature, and hip girth versus maximum chest girth. A set of representative areas will then be chosen, with attention to practicability and statistical efficiency in regard to "coverage" from which garments, patterns, and forms can be sized to guarantee an accurate fit for a large proportion of the teen-age population.

Background: Various consumer-, distributor-, and producer- groups have for some time indicated displeasure with the current diverse sizing systems for wearing apparel, especially for teen-age girls. It is proposed, therefore, to develop a sensible sizing system for this group by analysis and study of actual data on body measurements - the system to depend on (i.e. be expressed in terms of) a small number of basic body measurements, selected with due regard to practicability and statistical efficiency. For this study a "teen-age girl" is defined as (1) one who is not less than 12 years and not more than 17 years of age, and (2) one who has a "bust development" (i.e. difference between chest girth at arm scye and maximum chest girth) of one centimeter or more.

Some years ago 37 body measurements were taken on approximately 70,000 school girls between the ages of 4 and 17 years by carefully trained anthropometrists, for the Textiles and Clothing Division, Bureau of Home Economics, USDA. The cards on which these data were punched were loaned to NBS; the present study will be based on data from those cards pertaining to teen-age girls as defined above.

Magnitude: Class II(3); Class II(2)

Date of Termination: June 1, 1948 (very approximate since actual work on this project must be arrested from time to time to allow for meetings with interested outside organizations).

Comments: In addition to developing a sizing system that insures accurate fitting of a large proportion of teen-age girls, it is hoped to develop also auxiliary sizing areas for "slims" and "stouts" in order to cover all girls except the very small group admittedly requiring individually made-to-measure clothing.

Status: UNDER WAY (CONTINUATION). About 50% completed.

Thirty-seven body measurements punched on two cards for each of 14,811 teen-age girls have been sorted out from approximately 65,000 girls of broader age

groups. A bivariate frequency distribution of hip girth and stature by single centimeters has been prepared and studied. Seven "regular-size" areas and seventeen auxiliary size areas, each 4cm., have been constructed. For each of these twenty-four size areas, frequency distributions of seven basic measurements and a table of means for each of the thirty-seven measurements have been prepared.

Project: 4853-1 *Priority:* 2B *Date Auth.* 12/15/47

Title: Statistical Theory of Diffraction Gratings

Origin: Section 4.0, NBS

Project Manager: Dr. J. H. Curtiss

Objective: To study the distribution of the intensity of spectral lines obtained from a diffraction grating when the spacings between the lines on the grating are subject to random errors.

Background: The problem of ruling diffraction gratings has been attacked in the past with a considerable amount of engineering ingenuity, but successful solutions have been rare. To assist in determining a program for the Bureau in diffraction gratings, Division 4 has undertaken to review and develop the theory of such gratings. The present project is a part of the Division 4 study. Proposed by Dr. R. D. Huntoon.

Magnitude: Class II

Date of Termination: 2/28/48

Comments: The mathematical problem is closely related to the problem of "random flights" first proposed by Karl Pearson and later worked on by Lord Rayleigh.

Status: UNDER WAY (CONTINUATION).

The mean value of the intensity has now been computed for (1) the case in which the position of the n-th line is given by $d_n = nd + \varepsilon_n$, and (2) the case in which the position of the n-th line is given by $d_n = d_{n-1} + \varepsilon_n$, where d is the nominal distance between lines and $\varepsilon_1, \varepsilon_2, \ldots$ are identically distributed independent random variables. The dispersion and asymptotic distribution of the intensity have also been calculated in Case (1). These results for Case (1) were known heretofore, but with a restriction to a Gaussian distribution for ε_n.

Project: 4853-2 *Priority:* 3 *Date Auth.* 2/26/48

Title: Sampling of Baled Wools by Core Boring

Origin: Task Group on Sampling of Packaged Wools by Core Boring, Committee D-13 on Textiles, American Society for Testing Materials.

Project Manager: Dr. Churchill Eisenhart

Objective: To advise and assist the originating task group, on statistical aspects of the development of a limited number of alternative sampling plans for each of several levels of precision requirements for "shrinkage" determinations, for a number of combinations of within-bale and among-bale components of variance, for various lot sizes.

Background: The accurate and precise determination of the "shrinkage" of baled wools (i.e. the weight of the baled wool minus the net weight of the clean wool fiber) is of interest not only to the producers and marketers of wool, and to manufacturers of wool products, but also to the Bureau of Customs of the Department of Treasury, and to the Wool Division of the Production and Marketing Administration of the Department of Agriculture. These two Government agencies have led the way in the development of equipment and techniques for withdrawing samples of wool (cores) from a bale to furnish a basis for the determination of the shrinkage of the wool in the baled samples, and thence of the lot. On the other hand, the problem of the total number of cores needed, and their optimal allocations within and among bales, which is largely a statistical problem, has not been given adequate attention. Accordingly the aforementioned task group has requested the advice and assistance of Dr. W. Edwards Deming, Advisor in Sampling, Bureau of the Budget, and of personnel of the Statistical Engineering Laboratory, National Bureau of Standards in connection with this problem.

Magnitude: Class I

Date of Termination: 6/30/48

Status: UNDER WAY (NEW). About 5% completed.

The objectives and general plan of procedure of the Task Group of which Dr. Eisenhart has been made a member, were defined at its 20 February meeting. It was agreed that the role of the Statistical Engineering Laboratory in this undertaking would be as stated above under "Objective". Sets of data received from various members of the Task Group were analyzed and used to prepare a tentative formulation of a set of charts showing the relation between precision of estimate and variance components and the number of cores per bale, and the number of bales per lot, for various lot sizes.

Project: 48E3-1 *Priority: 2C* *Date Auth. 9/1/47*

Title: Techniques of Statistical Inference

Origin: Educational Committee, NBS

Project Manager: Dr. Churchill Eisenhart

Objective: To present an in-hours graduate-level course, with calculus a pre-requisite, in modern mathematical statistics and applications, and to prepare a set of official lecture notes for this course.

Magnitude: Class III

Date of Termination: June 1948

Comments: The first set of 20 lectures, constituting in-hours course A2.1, were delivered during the period September-December 1947. A list of the topics covered, together with a statement of the lecture notes, tables, etc., for which copies are available, will be found in the Report for the preceding quarter, covering the period October-December 1947.

Status: UNDER WAY (CONTINUATION). About 35% completed.

The second set of 20 lectures (of a total of 60) have been delivered.

A total of 7 persons continued in attendance during this second term of the course: 5 from NBS, 1 from the Naval Medical Research Center, and 1 from the Geophysical Laboratory of the Carnegie Institution of Washington.

The topics of the lectures of this second term, in-hours course A2.2, were: 1. Some inventory problems. 2. Statistical confidence limits for the number of occurrences in future samples of specified size, based on an observed sample of known size and composition, I: Poisson-exponential sampling. 3. Idem, II: binomial sampling. 4. Some unusual estimation problems: (1) estimating the unknown size of a sample, (2) estimating the size of a population. 5. Removing the limitations of simple sampling, I: Poisson binomial sampling. 6. Idem, II: Lexis sampling. 7. Detailed discussion of final examination of course A2.1 (Cameron). 8. Removing the limitations of single sampling, III: Coolidge sampling. 9. Problem of k binomial samples: specification of the null hypothesis, and various admissible classes of alternatives. 10. Dispersion indices, I: testing homogeneity of binomial samples theory. 11. and 12. Proposed Civil Service Commission Definition for Statistics Series P-190-0. 13. Dispersion indices, II; testing homogeneity of binomial samples, applications (Cameron). 14. Dispersion indices, III: adequacy of χ^2 approximation. 15. and 16. Dispersion indices, IV and V: approximate power functions. 17. and 18. Methods of treating zero-value observations in using the sign test. 19. and 20. An application of the method of maximum likelihood.

While most of these lectures were recorded by Sound Scriber, preparation of the official lecture notes has lagged seriously. About 5 of these lectures have been written up in part, but only 1 (Lecture 1) has been dittoed in full to date.

Publication: DITTOED copies are available of the official lecture notes for Lecture 1 above, and of a "Table for Predicting Outcome of Second Sample on Basis of a First Sample - Poisson Case", prepared for use in connection with Lecture 2 above.

Section 4. The Machine Development Laboratory

Project: 47D4-1 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Bureau of the Census Computing Machine

Origin: The Bureau of the Census

Project Manager: Dr. E. W. Cannon

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for the preparation of census reports.

Background: As a result of wartime work on electronic computing machines and related developments, it has appeared for some time that a revolution is imminent in methods of compilation and tabulation of statistical data. The construction and successful opera ion of an electronic digital computing machine, the ENIAC, and other developments in the electronic computing machine field clearly pointed out the possibility of constructing electronic digital equipment to carry out the types of manipulation of data involved in both the regular census compilations and in the newer sampling techniques. It is expected that the proposed electronic equipment will perform at increases in speed over existing equipment of a factor of 10 to 20 in some operations, and up to 100 or 200 or even more for other operations.

Magnitude: Class III

Date of Termination: December, 1949

Comments: This project is related to Project 47D4-2, in that contractors for each project were made aware of the performance specifications for both projects, and were informed that a single model might finally be selected for both projects. Project 47D4-2 involves mathematical work related to the present project as well as to other projects.

Status: UNDER WAY (CONTINUATION). Due to the change in performance specifications of the magnetic tape handling device for use with the Eckert-Mauchly machine and since this seemed to be the weakest link in the development, it has been decided as a preliminary step to enter into a development-type contract with Eckert-Mauchly Computer Corporation for the completion of a prototype of this tape handling device. The contract negotiations are now underway. Design specifications have been submitted by the Eckert-Mauchly Computer Corporation. These were sent to the National Research Council Committee on High Speed Calculating Machines and the committee has prepared a preliminary report. This preliminary report was presented to the Executive Council of the National Applied Mathematics Laboratories at a meeting on March 22, 1948. At this meeting the Executive Council approved contracting with the Eckert-Mauchly Computer Corporation for construction of a census machine. The responsibility for this contracting and the essential engineering monitoring of the contract will be assumed by Division 13.

Project: 47D4-2 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Navy Computing Machine

Origin: Mathematics Branch, Office of Naval Research

Project Manager: Dr. E. W. Cannon

Objective: To design and construct an automatic-sequenced electronic digital computing machine suitable for general mathematical computation.

Background. The project was undertaken to meet the need, recognized by the Mathematics Section of ONR, for faster and more efficient computing machinery than that now existing. Included among the problems at which the machine is aimed are the following: (a) Problems involving the systematic handling of large linear arrays (e.g., determination of the characteristic roots of matrices arising in vibration theory and quantum mechanics; solutions of systems of linear equations such as those which arise in vibration problems, metallurgical problems, weather problems, multivariate statistical analysis); (b) problems involving the solution of linear and non-linear partial differential equations, such as those which arise in the study of supersonic phenomena, turbulent flow, flow of viscous fluids, weather problems, servo-mechanisms, non-linear electrical oscillations and so on.

Magnitude: Class III

Date of Termination: December, 1949

Comments: See Comments for 47D4-1

Status: UNDER WAY (CONTINUATION). Design specifications completed by the Raytheon Corporation were considered by the NRC committee on High Speed Calculating Machines, and its preliminary report was given to the Applied Mathematics Executive Council Meeting on March 22, 1948. The cost estimates for machines submitted by the Raytheon Corporation were outside the NBS computing machine budget, so it was decided that Raytheon machines could not be included in the NBS program for the time being. A study program has been submitted by Engineering Research Associates of St. Paul, Minnesota, in which a proposal is made to consider the Raytheon and Eckert-Mauchly designs and produce an integrated design.

Project: 47D4-3 *Priority:* 2B *Date Auth.* 7/1/47

Title: The Air Comptroller's Computing Machine

Origin: Office of the Air Comptroller, Air Force

Project Manager: Dr. E. W. Cannon

Objective: To develop specifications for, and to construct an automatic-sequenced electronic digital computing machine suitable for use by the U. S. Air Force in program planning and control.

Background: The Air Comptroller's Office requires a high-speed and flexible computing machine to calculate detailed programs consistent with general policy decisions, and to facilitate rapid recomputation of programs to meet budgetary and other limitations. The problems involved are of wide applicability, and a part of the present project consists in formulating them mathematically. It is expected that the primary computation problem to be solved by the machine will consist of finding rapidly the solutions of large systems of simultaneous equations containing up to 1000 unknowns. The computer must be able to store and classify large quantities of data, and to refer rapidly for needed items to huge tables of

organization, equipment, supply and other similar data. These tables will contain millions of items. It is required, in addition, that the printing devices associated with the computer will be capable of extremely high-speed printing of the complete details of the Air Force's programs that have been computed.

Magnitude: Class III

Date of Termination: June, 1949

Comments: This project and Projects 47D4-1, 47D4-2, and 47D4-4 are interrelated Project 47D4-4 serves to coordinate the mathematical direction of the three computing machine projects.

Status: UNDER WAY (CONTINUATION). Dr. Olga Taussky Todd has been considering the mathematical phases of the project and in addition, Dr. J. von Neumann has rendered assistance. Mrs. Todd has been studying the connection of the mathematical problem with published work on convexity and inequalities, in particular the publications of Dines and McCoy. Following a suggestion of von Neumann she has also been studying fixed point theorems and Lebesgue's Pflastersatz from the point of view of constructibility. This part of the investigation is in itself of great importance for a very wide class of problems. A simple form of the problem in terms of nutrition has been programmed on the UNIVAC. A more complicated nutrition problem is being worked out by the Computation Laboratory; see project 48S2-15. The applicability of analogue computers and rotating-drum electronic digital computing machines is being studied.

Project: 47D4-4 *Priority:* 2C *Date Auth.* 7/1/47

Title: Programming of Problems for Solution on Automatic Digital Computing Machines

Origin: Bureau of the Census, Navy Department and the Air Force

Project Manager: Mrs. Ida Rhodes

Objective: To program certain general types of mathematical and statistical routines such as sorting, collating, the solution of linear systems, square rooting, etc., which frequently recur in the solutions of larger problems proposed by the Bureau of the Census, the Navy and the U. S. Air Force for solution on automatic digital computing machines. Thereby to detect deficiencies in, and effect improvements in, the design of proposed machines; also to establish a library of routines for the above mentioned types of problems and thus eliminate the necessity for the programmer to repeat the construction of a program whenever he is confronted with certain problems.

Background: The project was primarily undertaken to insure proper coordination and mathematical direction of Projects 47D4-1 and 47D4-2. A secondary justification lies in the fact that when automatic computing machinery becomes generally available, it will be necessary to have collections of programs for the routine mathematical operations, so that problem preparation can be expedited as much as possible.

Magnitude: Class III

Date of Termination: June 30, 1948

Comments: This project serves as the foundation of the mathematical directions of Projects 47D4-1 and 47D4-2. The performance of proposed automatically-sequenced electronic digital computing machines is carefully analyzed. The project is expected to serve as groundwork for the preparation of manuals of operation for the automatic computing machines constructed under the supervision of the Bureau.

Status: UNDER WAY (CONTINUATION). Instruction codes have been prepared for both elementary and somewhat complicated problems on both the UNIVAC and the Raytheon machines. A study of coding on the Reeves EDVAC was started. The solving of one hundred simultaneous linear equations on the UNIVAC and on a magnetic drum machine proposed by Engineering Research Associates was considered. A preliminary study indicated that the UNIVAC was 200 to 300 times faster than the magnetic drum machine, but since this result was not based on systematic programming specifically aimed at improving the accessibility of numbers stored on a rotated drum, ERA was asked to consider the problem further. Dr. Tompkins of ERA was asked to consult with Dr. Dantzig concerning the Air Force Procurement problem, and evaluate the performance of the ERA machine on this problem preliminary to further analysis by the Bureau.

Project: 47S4-1 *Priority:* 2C *Date Auth.* 7/1/47

Title: The MTAC Section

Origin: Committee on High-Speed Computing of the National Research Council

Project Manager: Miss Edith Norris

Objective: To assemble and edit material for a Section entitled "Automatic Computing Machinery" in the quarterly Mathematical Tables and Other Aids to Computation (MTAC), published by the National Research Council.

Background: This Section is to serve as a news letter, a medium for exchange of opinions, and a vehicle for the publication of shorter technical papers, in the field of automatic high-speed calculating machinery. The need for such a service has been pointed out repeatedly by groups interested in such machinery. The decision to sponsor this task and to assign the basic editorial work to the NAML was made at a joint meeting of the Committees on High-Speed Computing and on Mathematical Tables and Other Aids to Computation in New York in April, 1947.

Magnitude: Class II

Status: UNDER WAY (CONTINUATION). Page proof for the April, 1948, issue of MTAC has been edited and sent to the editors. In addition, material for the July, 1948, issue has been compiled and sent to the editors.

3. PUBLICATION ACTIVITIES

3.1 Publications which appeared during the quarter

3.1.1 Mathematical Tables

(1) Tables of the Bessel Functions $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$. Number 1 of the Applied Mathematics Series of the National Bureau of Standards.

(2) Coefficients for expressing the first twenty-four powers in terms of Legendre polynomials. H. E. Salzer. Mathematical Tables and Other Aids to Computation. **3**. No. 21, 16 (January 1948).

3.1.2 Manuals, Bibliographies, and Indices

(None)

3.1.3 Technical Papers

(1) The square root method for solving simultaneous linear equations. Jack Laderman. Mathematical Tables and Other Aids to Computation. 3, No. 21, 13 (January 1948).

(2) RP1847. A statistical analysis of some mechanical properties of manila rope. Sanford B. Newman and J. H. Curtiss. Journal of Research of the National Bureau of Standards, **39**, 551 (1947).

3.1.4 Reviews and Notes

(1) An Index of Mathematical Tables, A. Fletcher, J. C. P. Miller, and L. Rosenhead. Review by Churchill Eisenhart. Journal of American Statistical Association, **42**, 468 (1947).

(2) Errata in a Russian paper by H. S. Koshliakov. H. E. Salzer. Mathematical Tables and Other Aids to Computation, **3**, No. 21, 41 (January 1948).

3.1.5 Miscellaneous Publications

(1) A Federal program in applied mathematics. J. H. Curtiss. Science, **107**, 257 (1948).

(2)* Selected Techniques of Statistical Analysis, for Scientific and Industrial Research and Production and Management Engineering, by Statistical Research Group, Columbia University (Edited by Churchill Eisenhart, Millard W. Hastay, W. Allen Wallis). McGraw-Hill Book Company, Inc., November 28, 1947; pp. xiv plus 473. (During various periods in 1946 and 1947, Dr. Eisenhart, a former member of the Statistical Research Group, Columbia University, worked (out-of-hours)

*This item was inadvertently omitted from the preceding Report which covered the period July-December 1947.

on the editing of this volume, which constitutes a portion of the Summary Technical Report of the Applied Mathematics Panel, National Research and Development. He was directly responsible for the texts of five of its seventeen chapters. In addition, with the assistance of Miss Celia S. Martin of the Statistical Engineering Laboratory, he prepared the 43-page Index to this volume. Seven of the basic mathematical tables in the volume were checked, and corrected where necessary, by Mr. J. Laderman assisted by other members of the staff of the Computation Laboratory, under the direction of Arnold N. Lowan.)

3.2 Manuscripts in the Process of Publication as of March 31, 1948.

3.2.1 Mathematical Tables

(1) Table of coefficients for obtaining the first derivative without differences. H. E. Salzer. Number 2 of the Applied Mathematics Series of the National Bureau of Standards.

(2) Tables of the confluent hypergeometric function $F(\frac{1}{2}n, \frac{1}{2}; x)$ and related functions. Number 3 of the Applied Mathematics Series of the National Bureau of Standards.

(3) Table of coefficients for interpolating in functions of two variables. To appear in the Journal of Mathematics and Physics.

(4) MT 14. Table of probability functions, Vol. II. Second edition. Being printed by the Government Printing Office.

(5) Bessel functions of fractional orders, Vol. I. Columbia University Press.

(6) Coefficients for facilitating trigonometric interpolation. H. E. Salzer. Submitted to the Journal of Mathematics and Physics.

(7) Coefficients for expressing the first 30 powers in terms of the Hermite Polynomials. H. E. Salzer. Submitted to Mathematical Tables and Other Aids to Computation.

(8) Coefficients for complex quartic, quintic, and sextic interpolation within a square grid. H. E. Salzer. Accepted for publication in Journal of Mathematics and Physics.

3.2.2 Manuals, Bibliographies, and Indices

(Note: A comprehensive bibliography of the Division has been authorized and a first draft has been prepared. The project has been inactive during the period under review.)

(1) A Guide to Tables of the Normal Probability Integral by Celia S. Martin. Being circulated for comment with a view to publication as a Miscellaneous Publication of the National Bureau of Standards.

3.2.3 Technical Papers

(1) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J. H. Curtiss. To appear in the Proceedings of the Inte national Statistical Institute.

(2) Electronic digital computing in England. Harry D. Huskey. Submitted to Mathematical Tables and Other Aids to Computation.

(3) On the reality of zeros of Bessel Functions. A. Hillman. Submitted to the Bulletin of the American Mathematical Society.

(4) A problem of J. C. P. Miller on arc tangent relations. John Todd. Submitted to the American Mathematical Monthly.

(5) Bounds for characteristic roots of matrices. Olga Taussky Todd. Submitted to the Duke Mathematical Journal.

(6) Bessel Functions expansions for Coulomb Wave Functions. M. Abramowitz. Submitted to the Journal of Mathematics and Physics.

(7) The Density of Reducible Integers. S. D. Chowla and John Todd. Submitted to the Canadian Journal of Mathematics.

(8) On the Inverse Functions of $\frac{\tanh w}{w} = z$ and $\frac{\coth w}{w} = z$. A P. Hillman and H. E. Salzer. Submitted to the Journal of Mathematics and Physics.

3.2.4 Reviews and Notes

(1) Re iew of a paper by T. Szele, for Mathematical Reviews. Olga Taussky Todd.

(2) In roduction to Mathematical Statistics, by Paul G. Hoel. Review by Churchill Eisenhart. To appear in Science.

(3) Theoretical and Numerical Treatment of Diffraction by a Circular Aperature. (Ph.D. T esis), Groningen, Batavia, Holland On Spheroidal Wave Functions of Order Zero, Journal of Mathematics and Physics, 24. 79 (1947); both by C. J. Bouwkamp. Peviews by G. Blanch for Mathematical Tables and Other Aids to Computation, scheduled for April, 1948 issue.

3.2. Miscellaneous Publications

(1) Statistical positions in the Federal Service, Unsigned editorial, prepared by Churchill Eisenhart in collaboration with Benjamin J. Tepping, Bureau of the Census, for the April 1948 issue of the American Statistician.

(2) The role of a statistical consultant in a research organization. Churchill Eisenhart. Abridged version of an address of the same title presented to the International Statistical Institute in September 1947, prepared for publication in the April 1948 issue of the American Statistician.

(3) Some Trends in Applied Mathematics. J. H. Curtiss. Preliminary draft submitted to the editors of the American Scientist for comment.

APPENDIX

Explanation of Project Descriptions

The project descriptions appearing in Sect. 2 of this report are reproductions of the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With the view of making this report more useful, an explanation of certain of the symbols and standard terms used in the Project Forms will now be given.

Project Number. Each project of the Laboratories is identified by a four-location symbol called the Project Number, which appears in the upper left hand corner of the Forms. The first location in the symbol designates the fiscal year in which a Project Form for the project was first prepared; e.g., 45 for 1945, 47 for 1947, etc. (For projects under way as of July 1, 1947, the fiscal year designated is that in which the Form would have been prepared under the present rules.) The letter in the second location denotes the class of project: R stands for research, D for developmental (usually of aids for work in mathematics), S for service, E for educational. The third symbol location denotes the Section of Division 11, to which primary responsibility for the project has been assigned, and the last symbol is a serial number within the section.

Priority. Priority rankings are assigned to each project as a guide for the staff and in recognition of the interests of clients of the Laboratories, The system of rankings is as follows:

Priority 1. This category consists of those projects, the early completion of which is essential to the success of current or impending operations of another division of the Bureau of Standards, or another Government agency, or an important industrial or academic laboratory.

Priority 1A. This priority is assigned only to those projects whose results are immediately needed for purposes of national security.

Priority 1B. This priority is assigned to those projects whose results are, for economic and/or administrative reasons, urgently needed by clients of the National Applied Mathematics Laboratories.

Priority 1C. This priority is assigned to those projects which meet the requirements of the Priority 1 category, but which are not so critically related to the success of current or impending operations of other laboratories.

Priority 2. This category consists of projects of obvious importance, the completion of which will increase the efficiency of and promote economy in the National Applied Mathematics Laboratories, other divisions of the Bureau, other Government agencies, and industrial or academic laboratories.

Priority 2A. This priority is assigned to special and presumably nonrecurrent projects which, if brought to an early and successful conclusion, will almost surely contribute materially to the effectiveness, efficiency and economy of current operations.

Priority 2B. This priority is assigned to special and presumably nonrecurrent projects which will probably contribute to the efficiency and economy of current operations; or which will increase the usefulness of a forthcoming publication of the Laboratories which otherwise has been assigned to Priority 1 or 2, or which is otherwise ready for the printer.

Priority 2C. This priority is assigned to those projects of a routine or recurrent nature, the results of which are integrated with the operations of other laboratories, but which do not satisfy the requirements of Priority 1.

Priority 3. This category consists of projects which have no urgent application to any particular activity of the National Applied Mathematics Laboratories or the clients of the Laboratories, but which are worth while prosecuting, provided that they do not delay problems of higher priority. The base-load projects of the National Applied Mathematics Laboratories, such as the preparation and distribution of major mathematical tables not urgently needed for special work of other laboratories, are assigned to this category.

Date of Authorization. The date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Title. Self-explanatory

Origin. "

Background. "

Magnitude. This is an estimate of the size of a task. At the outset of a project such an estimate will often be only an educated guess, so this entry is subject to change as the work progresses. Four classes are used to designate magnitude: Class I means 5 man-days or less, Class II means more than 5 but not more than 100 man-days, Class III means more than 100 but not more than 1000 man-days, and Class IV means more than 1000 man-days.

Work performed outside the Laboratories on contracts (other than contracts for personal services) is not included in the calculation of magnitude. When more than one Section of the Laboratories is involved in a project, separate entries are made for each Section. The section numbers are then placed in parentheses after the magnitude designations.

Date of Termination. This is the date on which it is estimated that work will terminate. In cases where commitments have been made to outside organizations, the agreed upon completion date is used here. In the case of projects upon which no commitments have been made to outside organizations, this entry is subject to modification as the work progresses, and in certain cases involving low-priority R and D projects, no date of termination is given at all.

Comments. Related projects are mentioned here, together with other relevant information.

Status. Here is given the narrative of the progress to date on the project. In making the entries, certain standard descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to

which the entry applies. These standard terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, but no work of any consequence has been performed as of the date of the report.

"UNDER WAY (NEW)" means that the Laboratories made a commitment within the 3 months preceding the date of the report to work on the project, and that work was in progress during that period.

"UNDER WAY (CONTINUATION)" means that the work was initiated more than 3 months preceding the date of the report and was in progress during the 3 months preceding the date of the report.

"INACTIVE" means that the Laboratories made a commitment more than 3 months preceding the date of the report, to work on the project, but no work of any consequence was performed on the project during the last 3 months.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof are to undergo extensive mathematical checks, the designation "COMPLETED" is employed only after these checks have been performed.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "IN MANUSCRIPT" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "COMPLETED" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under Status or Publication to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.