NATIONAL BUREAU OF STANDARDS REPORT

PROJECTS AND PUBLICATIONS
OF THE
APPLIED MATHEMATICS DIVISION

A SEMI-ANNUAL REPORT

January through June 1971

<NBS>

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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NBS PROJECT **NBS REPORT**

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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION (205.00): Conducts research and provides consulting services to the Bureau and other Federal agencies in various fields of mathematics important in science and engineering, including automatic data processing and operations research, with emphasis on statistical, numerical and combinatorial analysis, and on mathematical physics. Develops tools for mathematical work such as mathematical tables, handbooks, manuals, mathematical models and computational methods, and advises on their use. Provides training in disciplines related to these functions.

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°Only unclassified material is included in this report.

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January 1, 1971 through June 30, 1971

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Walter L. Sadowski, Ph.D.
David J. Sookne, M.A.
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Roy H. Wampler, M.A.
William H. Youden, Ph.D*** [1]

*Part-time
**Postdoctoral Research Associate

***Guest Worker
°On Leave of Absence
°°Located at NBS Boulder Labs.

[1] Deceased, March 1971.

Status of Projects

1. MATHEMATICAL ANALYSIS

RESEARCH IN MATHEMATICAL ANALYSIS

Task 20540-12-2050141/55-57

1550-11

Origin: NBS   Authorized 9/1/54
Manager: H. J. Oser
Full task description: July-September 1954 issue, p. 27

Status: CONTINUED

(a) Consulting and Advisory Services (partly funded under 2050101)

Requests for consultations remained high throughout this reporting period. S.R. Kraft assisted the
Optical Radiation Section in developing various improvements for the computer algorithm used in solving
the integral equation of light interreflection. The original mathematical algorithm was developed
earlier by Kraft. Test runs on the computer are now being carried out. S.R. Kraft and H.J. Oser made
a survey of available methods for modeling flame spread in a building. A short report on these methods
was transmitted to the Technical Analysis Division, which sponsored the activity.

(b) Linear transport equations

R. Kraft obtained generalizations of his method of establishing convergence of semi-discrete approx-
imations of linear transport equations. A paper on this subject is under preparation and further exten-
sions of the ideas to non-linear problems are being studied. Two manuscripts are in the process of being
completed for submission to NBS review.

(c) Continuum Physics

An expository article entitled "Some Elementary Formulas in Matrix Calculus and Their Applications"
was written by J.T. Fong and accepted for publication in the NBS Journal of Research, Section B. The
purpose of the article was to present an elementary introduction to the proper formulation of the chain
rule and the product rule for differentiation in matrix notation as applied to scalar- or matrix-valued
functions of matrix arguments. Using some of the well-known properties of the operator "trace", alter-
native definitions of gradients and simple examples of calculating them are included. Applications of
the so-called "matrix calculus" of differentiable functions in continuum mechanics are illustrated with
two examples.

In collaboration with Dr. John A. Simmons of the Institute for Materials Research, J.T. Fong com-
pleted a manuscript entitled "The Scalar Potential of the BKZ Theory for a Compressible Fluid" in which
a representation theorem for the BKZ scalar potential is proved. An important attribute of the general
BKZ fluid theory is that the "instantaneous response" of such a fluid, defined as the excess stress at
time t due to an instantaneous deformation also at time t, is independent of the prior deformation
history. This fact plus the usual conditions of isotropy and material indifference lead to a represen-
tation of the scalar potential as the sum of three terms, one of which is purely dilatational, another
isochoric, and the third a coupling term. Some implications of the representation theorem are discussed.

Based on an invariant formulation of the isothermal BKZ fluid theory as given recently by Fong and
Simmons, J.T. Fong and L.J. Zapas (Institute for Materials Research) proved that a conformal transforma-
tion of the spatial metric tensor at any past time τ, $-\infty < \tau \le t$, preserves the validity of the prin-
ciple of material indifference as well as the unimodular isotropy group of the BKZ fluid. Efforts to
interpret this extra degree of freedom as a history of environmental degradation are continuing.

FORTRAN LIBRARY IMPROVEMENT AND ALGORITHM DEVELOPMENT
Task 20501-2053587
1550-23

Origin and Sponsor: NBS   Authorized 4/27/70
Manager: H. J. Oser

Status: CONTINUED

D.W. Lozier, L.C. Maximon and W.L. Sadowski developed an approach to the testing of mathematical function algorithms. The approach is based on the choice of exactly machine representable arguments and the use of properly authenticated algorithms to produce reference values to any precision in any number base.

The reference algorithms are subjected to a hierarchy of tests to insure the correctness of the reference values to the desired number of figures. The reference values are stored on tape for use in performance testing.

A fully automated program uses the reference tapes to perform the testing of mathematical routines and supplies the detailed results to the user, including a table of statistics. The program has been used successfully on the NBS UNIVAC 1108 computer to test some of the functions in the FORTRAN library.

It must be stressed that the program applies with appropriate small modifications to other computers and functions other than the elementary functions of the FORTRAN library. At the present time the program is being extended to the IBM 360 series.

The advantage of full automation is the ease and speed with which performance testing can be accomplished.

David J. Sookne developed two FORTRAN algorithms, one for the calculation of Bessel functions $I_n(x)$ and $J_n(x)$, x real, and one for $I_n(z)$ and $J_n(z)$, z complex. For a given order n, argument x, the programs calculate function values of order zero through n for that argument. The algorithms use backward recursion, and the truncation error of the algorithm is strictly bounded so that it is insignificant on the computer being used.

In collaboration with F.W.J. Olver, and with suggestions from H.J. Oser, David Sookne performed the necessary error analysis. He then programmed the routines and tested them extensively, reprogramming and retesting until their accuracy was assured.

These algorithms were tested by using the multi-precision arithmetic package of L. Maximon, and the bit-comparison ideas of D. Lozier and W. Sadowski.

The programs are based on a paper by F.W.J. Olver and D.J. Sookne (to be published), and the algorithms and their certifications will also be published.

George W. Reitwiesner studied possible ways to find bounds for polynomial roots by means of Sturm sequences; by use of modular arithmetic an alternate algorithm may be feasible and practicable along the lines suggested by L. Heindel of Bell Telephone Laboratories.

W.H. Elliott prepared high-precision algorithms for the calculation of the arctangent and arcsine functions. These are used in turn to validate the subroutines in the UNIVAC 1108 library.

A talk was presented by D.W. Lozier, L.C. Maximon and W.L. Sadowski at the Approximators Workshop at Argonne National Laboratory on April 14, 1971 entitled "A Bit Comparison Program for Algorithm Testing", and a paper on this title has been submitted to the Communications of the ACM.

The Bessel function algorithms were the subject of a talk by D. Sookne titled "Implementation of Olver's Algorithm for the Bessel Functions $J_n(x)$ and $I_n(x)$", April 13, 1971, at the Approximators Workshop, Argonne, Illinois.

H.J. Oser participated in a panel discussion on "Experimental Error Analysis" at the ACM 10th Annual Technical Symposium at NBS on June 24, 1971.

Publications:

(1) A Bit Comparison Program for Algorithm Testing. D.W. Lozier, L.C. Maximon and W.L. Sadowski. Submitted to Communications of ACM; prepublication as NBS Report No. 10449, June 30, 1971.

(2) Note on Backward Recurrence Algorithms. F.W.J. Olver and D.J. Sookne. Submitted to a technical journal.

MATHEMATICAL METHODS IN PLASMA PHYSICS

Task 20504-12-2050140/59-422

1540-11

Origin: NBS   Authorized   10/31/61
Manager: Walter L. Sadowski
Full task description: October-December 1961 issue, p. 12

Status: COMPLETED

W. L. Sadowski has developed a method of generating stationary solutions of the non-linear Vlasov equa-
tion. Depending on the wave-vector k and the non-linearity parameter, these solutions give Fourier-
Hermite expansion coefficients that increase with the order of the Hermite polynomial. These solutions
cannot be truncated in Hermite space without introducing large truncation errors. Since these stationary
solutions will be generated by the computer in any Fourier-Hermite calculation of the dynamic behaviour
of a non-linear plasma, the possibility of properly truncating the expansion in Hermite space is in
question for those parameters where the coefficients of the stationary solution increase with the order
of the Hermite polynomial. Dynamic calculations in these regions should be verified by other independent
calculations.

A manuscript entitled "On a Source of Numerical Instability in the Solution of the Non-Linear Vlasov
Equation with the Aid of Fourier-Hermite Expansions" is in preparation.

W.L. Sadowski and D.W. Lozier have applied Olver's algorithm to a calculation of a series of integrals
that were intractable by other means. A manuscript is being submitted for review. Future activities
will be reported under project 2050141.

Publications:

(1) Use of Olver's Algorithm to Evaluate Certain Definite Integrals of Plasma Physics Involving
     Chebyshev Polynomials. W.L. Sadowski and D.W. Lozier. Submitted to J. Comp. Physics.

2. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS   Authorized 12/30/60
Manager: A.J. Goldman
Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued heavy. Section staff provided such services in 125 recorded instances, 47 involving assistance to NBS staff. The 125 instances totalled to 711 man-hours. Other agencies assisted included the National Science Foundation, Census Bureau, Office of Economic Opportunity, AID, Federal Communications Commission, Defense Communications Agency, Naval Research Lab, and Office of Naval Research. Requests from industry, universities, professional groups and journals were also met so far as possible.

(2) J. Gilsinn assisted the Metallurgy Division in the analysis of a model for the magnetic moment due to iron impurities in the lattice of a copper-nickel alloy. (Supported under Project 3120401; reported here for convenience.)

(3) W.G. Hall continued assistance to the Army Data Field Systems Command relative to a new tactical artillery fire-control system; this period's efforts focused on compiler redesign. (Reported here for convenience; supported under Project 6505425.)

(4) A.J. Goldman continued as NBS representative to the Mathematical Sciences Division of the National Academy of Sciences. He served on an ad hoc committee to consider J. Res. NBS. He continued as Associate Editor of the Operations Research Society of America's "Transportation Science" journal, and began a term as ORSA's representative to the Conference Board of the Mathematical Sciences. He also served on the selection committee for the Fifth International Symposium on Transportation and Traffic Flow.

Publications

(1) R.D. Elbourne (Div. 650) and Judith F. Gilsinn. Simulation of Air Traffic Control radar beacon assignment plans. NBS Technical Note 568 (3/71).

(2) A.J. Goldman. The adequacy of management science technology for non-military applications in the Federal government. To appear in Proc., Amer. Soc. Public Admin. Workshop on Management Science in the Federal Government (11/69).

(3) A.J. Goldman. Review of "Introduction to Systems Cost-Effectiveness" (K. Seiler III, Wiley-Interscience, 1969). Operations Research 19 (1971), 552-554.

(4) W.A. Horn. Convex homotopy. To appear in Journal of Research NBS 75B (1971).

(5) Roger D. Traub. An ergodic source which is not a function of any finite Markov process. To appear in Journal of Research NBS 75B (1971).

COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS   Authorized 12/30/60
Manager: A.J. Goldman
Full task description: October-December 1964 issue, p.3; April-June 1962, p.15

Status: CONTINUED

(1) J. Gilsinn and C. Witzgall (Boeing Scientific Research Labs) continued documentation on the comparison of shortest-path algorithms.

(2) A.J. Goldman continued studies relating to optimal facility location. He developed an algorithm which either locates a minimax-distance location in a network, or else reduces the problem to one of the network's cyclic components.

(3) W.A. Horn studied a combinatorial problem concerning placement of a minimum number of counters to determine flow volumes in a network.

Publications

(1) J. Edmonds. Submodular functions, matroids, and certain polyhedra. In Proc. 1969 Calgary Conference on Combinatorics (Gordon and Breach, 1970).

(2) A.J. Goldman. Optimal center location in simple networks. Transportation Science $\underline{5}$ (1971), 212-221.

(3) A.J. Goldman. Approximate localization theorems for optimal facility placement. Submitted to a technical journal.

(4) W.A. Horn. Optimal design of sorting networks. Submitted to a technical journal.

(5) W.A. Horn. A simple algorithm for minimizing delay (cost) in a sorting (decision) network. Submitted to a technical journal.

LINEAR AND NON-LINEAR PROGRAMMING
Task 205-12-2050153

Origin and Sponsor: NBS   Authorized 12/30/60
Manager: W.G. Hall
Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) W.G. Hall, R. Jackson and P. Saunders prepared, tested and documented a core-limited code for the revised simplex method. Jackson prepared a simplex quadratic-programming code, and also a Generalized-Lagrange-Multiplier code to handle larger (but separable) quadratic programs. He also adapted an implicit-enumeration code for (0,1) integer programs. L.S. Joel prepared a prototype decomposition-method code. (Largely supported under Project 2053587, funded by the NBS Computer Services Division.)

(2) A.J. Goldman and P.R. Meyers continued study of a "minimax error" approach to imputing values to incompletely determined data.

(3) A.J. Goldman presented a 10-hour introductory course in linear and integer programming for NBS staff. (Supported under Project 6405105; reported here for convenience.)

Publications

(1) W.A. Horn. Optimal container inventory and routing. To appear in Transportation Science.

(2) W. Breen (Northwestern U.) and R. Jackson (then at Southern Methodist U.). An efficient algorithm for solving large-scale portfolio problems. Journal of Financial and Quantitative Analysis VI (1971), 627-637.

OPERATIONS RESEARCH SERVICES FOR TECHNICAL ANALYSIS DIVISION

Task 205-12-2050586

Origin: NBS (Div. 431)   Authorized 7/15/71
Sponsor: NBS, various other agencies
Manager: A.J. Goldman
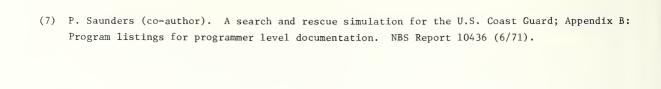Full task description: July-December 1970 issue, p.8

Status: CONTINUED

(1) Project 4314553: J. Gilsinn, A.J. Goldman and R. Jackson continued participation in this study to arrive at modal flow estimates for the Department of Transportation's National Network Simulation. A minimax error approach developed under Project 2050153 was applied to the estimation of pipeline flows, and a model was developed and fitted to aid in the treatment of air cargo. Further implementation of a new traffic assignment algorithm was carried out.

(2) Project 4314542: J. Gilsinn and D. Klavan continued participation in this study, to automate the Interstate Commerce Commission's tracing of shortest paths in the U.S. rail network as an aid in rate evaluation.

(3) Project 4314552: A.J. Goldman assisted in the planning of this study, to evaluate the Shirley Highway Express Busway Demonstration Project.

(4) Project 4310900: A.J. Goldman participated in a number of project and document reviews for the sponsoring division.

(5) Project 4314561: P. Saunders participated in the documentation of this development of a simulation (SARSIM) of the Coast Guard's search and rescue operation.

(6) Project 4314162: P. Saunders and W.A. Horn contributed to several phases of this project to provide planning tools for urban fire departments. A first-cut simulation model for evaluating dispatch policies was developed and exercised on an Alexandria (Va.) scenario. Models were developed to aid in (a) firehouse location and (b) the timing of equipment replacement; some are of the dynamic programming variety, while others use the integer programming capability developed under Projects 2050153 and 2053587.

Publications

(1) J. Gilsinn and D. Klavan (with E. Short and W.A. Steele, Div. 431). A simulation model for estimating airport terminal area throughputs and delays. NBS Report 10592 (5/71); FAA Report FAA-RD71-9.

(2) A.J. Goldman and W.A. Steele (Div. 431), with W.A. Horn. Continued analysis of a capacity concept for runway and final-approach airspace. NBS Report 10589 (4/71).

(3) W.A. Horn. Extension of a capacity concept to dual-use runways and multi-runway configurations. NBS Report 10593 (4/71).

(4) L.S. Joel and W.A. Steele (Div. 431), with J.J. Filliben (205.03) and G.B. Hare (Div. 431). Approaches to evaluating the effects of VFR towers on flow and safety at airports. NBS Report 10562 (4/71).

(5) P. Saunders and E.E. Leyendecker (Div. 431). A search and rescue simulation for the U.S. Coast Guard; Vol. IV: Programmer level documentation for "OPSIM". NBS Report 10433 (6/71).

(6) P. Saunders (co-author). A search and rescue simulation for the U.S. Coast Guard; Appendix A: Flow charts for programmer level documentation. NBS Report 10435 (6/71).

(7) P. Saunders (co-author). A search and rescue simulation for the U.S. Coast Guard; Appendix B: Program listings for programmer level documentation. NBS Report 10436 (6/71).

3. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259
6030-11

Origin: NBS   Authorized 10/1/62
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Status: CONTINUED. Mary G. Natrella has completed the manuscript of a chapter on "Design and
Analysis of Experiments" for inclusion in the third edition of Quality Control Handbook, edited
by J. M. Juran. Based in part on Chapters 11, 12, 13 of Mrs. Natrella's book Experimental
Statistics (NBS Handbook 91), this chapter provides an introduction to the design and analysis
of experiments, including a classification of designs and a summary of important recent developments.
Detailed designs and methods of analysis are given for factorial and fractional factorial designs,
for completely randomized designs, and for the more common types of block designs (e.g., randomized
blocks, balanced incomplete blocks, Latin Squares). Descriptions are given of some special-
purpose designs, such as mixture designs, group screening designs and cross-over designs. Some
simple techniques that are useful in planning and analyzing the results of interlaboratory tests
are included.

Publications:

(1) The percent point function. James J. Filliben. Submitted to a technical journal.

(2) Some properties of the range in samples from Tukey's symmetric lambda distributions.
     Brian L. Joiner and Joan R. Rosenblatt. To appear in J. Amer. Statist. Assoc.

(3) Analysis of information -- An alternative approach to the detection of a correlation
     between the sexes of adjacent sibs in human families. H. H. Ku. Biometrics,
     Vol. 27 (1971), pp. 175-182.

(4) On the analysis of multi-dimensional contingency tables. H. H. Ku, R. Varner and
     S. Kullback (G.W.U.). Journal of the American Statistical Association, Vol. 66 (1971),
     pp. 55-64.

(5) Covariances of two-sample rank sum statistics. Peter V. Tryon. Submitted to a technical
     journal.

DEVELOPMENT OF "OMNITAB"

Task 20503-12-2050131

Origin and Sponsor: NBS   Authorized 11/1/68
Managers: David Hogben, Sally T. Peavy
Full task description: July - December 1968

Status: CONTINUED. David Hogben, Sally T. Peavy and Ruth N. Varner completed a preliminary draft of PART C of the OMNITAB II User's Reference Manual. PART C is an extensive (188 pages) description of all the instructions in the OMNITAB II computing system. Version 5.01, which corrected a number of errors detected in the writing of PART C, was implemented at NBS.

Sally T. Peavy and Ruth N. Varner prepared an overlay and segmentation procedure to enable the implementation of OMNITAB II on computers (such as the CDC 6000 series) in which overlay must be done by reprogramming of the subprograms rather than by system control instructions.

Publications:

(1) Test Problems and Results for OMNITAB II. Ruth N. Varner and Sally T. Peavy. NBS Technical Note 551, October 1970.

(2) Source Listing of OMNITAB II Program. Sally T. Peavy, Ruth N. Varner and David Hogben. NBS Special Publication 339, December 1970.

4. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF

NBS RESEARCH AND TESTING

Task 13911-612050950/51-1
9500-11

Origin: NBS   Authorized 7/1/50
Managers: H. H. Ku, J. R. Rosenblatt
Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau
scientists on the statistical aspects of their investigations. These services vary from short
(one-hour) sessions to extended collaborations involving several man-months; and are concerned
primarily with statistical design of experiments, analysis and interpretation of data, and the use
of computers in statistical analysis of data. Typical examples of the services performed are the
following.

James A. Lechner has assisted the Office of Flammable Fabrics with the precise specification of
a new method for determining the fiber content (wool, cotton, polyester, etc., percent by weight) of
woven fabrics. The new method had been proposed as an alternative to slow "wet chemistry" techniques,
and is based on counting fibers in frayed yarns under a microscope. Dr. Lechner collaborated in
the specification of estimates for the percentage composition and the standard error, taking
account of differences between warp and fill yarns. He also clarified the conditions under which
the method will provide a satisfactory equivalent to the chemical analysis.

Roy H. Wampler has revised his READIT subroutine for inclusion in a machine-independent version of
the computer program MASS developed by R. C. Raybold (Office of Measurement Services) for the
Measurement Analysis Program in mass calibrations. READIT is a Fortran subroutine for reading and
storing free-field numbers (i.e., punched anywhere on a data card). Flexibility is allowed in the
manner in which numbers are punched so that the user is not required to follow fixed formats.

Upon request of Wilfred B. Mann (Radioactivity Section, Center for Radiation Research), H. H. Ku
reviewed various reports of the Interagency Uranium Mining Radiation Review Group, and assisted
Mann in the preparation of a letter to the Surgeon General, commenting on findings relevant to a
reasonable and safe limit exposure to radon daughters for uranium miners.

James J. Filliben assisted Alan Goldman and Lambert Joel (Operations Research Section) in the
analysis of data on accident rates in regard to midair collisions at small airports with and with-
out control towers. The original data were characterized by 1) discreteness and 2) a large
proportion (70-80%) of ties. After some preliminary precautions, the sign test and a modified
sign test (developed to take into account the ties) were applied to the data and the factor
"control tower" was found to be significant. Other recommendations were made regarding 1) the
accuracy of the data and 2) other possibly-relevant factors to be considered in future studies.

Joan Rosenblatt has assisted R. K. Kirby (Crystallography Section, Inorganic Materials Division)
to calculate least squares cubic spline approximations with fixed knots. The approximations are
needed to smooth data on thermal expansion, for use in characterizing a quartz Standard Reference
Material. The methods are applicable for similar thermal expansion data and are based on a
representation that leads to convenient computations using the FIT command in OMNITAB. Additional
applications were made for smoothing data on thermal expansion of copper, iron, and cobalt, in
preparation of a Standard Reference Data compilation.

Peter V. Tryon and Brian L. Joiner have continued their collaboration with a number of people in the Electromagnetics Division concerning the evaluation and improvement of their measurement systems. In depth analyses have revealed a variety of places where statistical concepts can be brought to bear resulting in considerable improvement in the accuracy of the result as well as a better understanding of errors involved in the measurement. Frequently there is also a cost reduction due to increased efficiency of the measuring process.

Brian L. Joiner has developed an experiment design for calibration of three sets of 48 thermal voltage converters (TVC), one at each of three voltage levels. At each voltage level, a check standard is included, and the calibration uses two working standards. Each TVC is calibrated at ten frequencies. The experiment design provides for detection of "outlying" observations and of linear drift of the measuring device. A computer program was written to generate random permutations subject to various restrictions and to print out the complete schedule for making 1300 individual measurements for each set of TVC's. In addition to providing information about the measurement process as a by-product, the experiment design allows a saving of about 25% (two man-weeks) over the procedure followed hitherto.

Peter V. Tryon and Brian L. Joiner have continued their collaboration with personnel in the Cryogenic Flow Facility concerning the evaluation of the Facility and of selected flow meters undergoing test. The accuracy statement resulting from this collaboration has been selected by the Institute Office for circulation to all sections as an example of the type of statement desired for all measurement services.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346
5040-11

Origin: Various Agencies   Authorized 3/31/58
Sponsor: NBS
Manager: J. R. Rosenblatt
Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

David Hogben and Sally T. Peavy presented lectures on OMNITAB II at a two-week Seminar and Workshop on On-Line Computing, organized by the Intergovernmental Bureau for Informatics, June 7-18, in Rome. Under the general direction of Joseph Hilsenrath (NBS Office of Standard Reference Data), the workshop was arranged with the cooperation of: IFIP Administrative Data Processing Group (headquarters Amsterdam); Language and Systems Development, Inc. (Silver Spring, Md.); Teletype Corp. (Skoki, Ill.); Centro di Calcolo Interfacolta, Universita di Roma; Honeywell Information Systems Italia; Instituto Nazionale dell'Informazione (Rome); Olivetti Corp. (Rome); SIP Societa Italiana per l'Esercizio Telefonico.

OMNITAB II was implemented on a UNIVAC 1108 under Exec 8, for operation in conversational mode from remote consoles. Participants in the workshop represented computer installations in Italy, Greece, France, Israel, and Czechoslovakia.

With the help of the workshop participants and the staff of the University of Rome computing center, an Italian version (limited) of OMNITAB II was successfully implemented on the University of Rome's computer (UNIVAC 1108, Exec 8).

Brian L. Joiner and Peter V. Tryon are consulting with Del Caldwell of the U. S. Navy Metrology Engineering Center, Pomona, California, on several electromagnetic measurement problems. Areas investigated thus far include attenuation, power and voltage.

Members of the staff served in various editorial and committee assignments, including the following. H. H. Ku is an associate editor of the AMERICAN STATISTICIAN. Joan R. Rosenblatt is a member of the editorial board of the SIAM JOURNAL ON APPLIED MATHEMATICS (Soc. Indust. and Applied Math.). Roy H. Wampler is a member of the IMS Mathematical Tables Committee. Brian L. Joiner is a member of the IMS Annals Index Committee. David Hogben has served as 1971 Program Chairman for the ASA Section on Physical and Engineering Sciences.

Publications:

(1) Randomization and the draft lottery. Joan R. Rosenblatt and James J. Filliben. Science, Vol. 171 (1971), pp. 306-308.

5. OTHER ACTIVITIES

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

1540-11

Origin: NBS   Authorized 8/29/54
Manager: Morris Newman
Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has determined the Smith normal form of various (v, k, λ) configurations. Since isomorphic designs necessarily have the same Smith normal form, these results are of use in showing certain designs to be non-isomorphic.

M. Newman has completed a book entitled "Integral Matrices" . The book provides a survey of the theory of matrices with elements from a principal ideal ring, and contains a complete account of classical matrix theory over a field.

M. Newman and R. Merris have begun the study of an explicit isomorphism from the algebra of $n \times n$ matrices onto the ideal in the group algebra corresponding to an irreducible character χ of degree n . Many classical inequalities for matrix functions have been generalized and extended via this concept.

M. Newman has completed a set of programs to do matrix operations and polynomial operations modulo p . In cooperation with Dr. K. Uchida of Tohoku University, it has been shown that the class number of the pth cyclotomic field is bigger than 1 for all primes $p \geq 23$, thereby settling an old conjecture.

R. Merris proved some inequalities for the matrix functions

$$e_r(A) = \sum_{g \in G} \chi(g) E_r(A_{1g(1)}, \ldots, A_{ng(n)})$$

where G is a subgroup of S_n and χ is an irreducible character on G .

Karl Goldberg continued work on a forthcoming booklet of tables of elementary combinatorial numbers and relations among them. He began work on a second booklet on functions of partitions including symmetric functions and coefficients of formal power series.

S. Haber, together with L. Flatto of Yeshiva University, showed the existence of formulas for numerical integration of functions of n variables that integrate exactly all polynomials of degree 3 or lower and use only n+1 points. This settled, in the negative, a conjecture of Hammer and Stroud.

S. Haber found an improvement of an error estimate, due to S.L. Sobolev, for integration of periodic functions by the trapezoid rule.

F.W.J. Olver has developed new asymptotic approximations for the Whittaker functions for large values of the parameter m . These approximations are in terms of elementary functions (for positive m) and Airy functions(for pure imaginary m) and both hold uniformly for the interval $(0, \infty)$ of the independent variable.

F.W.J. Olver and David J. Sookne have prepared a comprehensive algorithm for computing Bessel functions $J_n(x)$ and $I_n(x)$ of real arguments and integer orders based on recurrence procedures. A feature of the algorithm is rigorous control of truncation and rounding errors. D. Sookne is extending this work to complex arguments.

M. Sheingorn found new sufficiency conditions for weighted polynomial approximation. He also clarified parts of DeBrange's paper on the Ramanujan hypothesis.

Publications:

(1) A class of representations of the full linear group. R. Merris and S. Pierce. J. Algebra, Vol. 17 (1971) 346-351.

(2) An improvement of the Fischer inequality. R. Merris. J. of Research 75B (1971) 76-79.

(3) Orthogonal groups of positive definite multilinear functionals. S. Pierce. Pacific Jour. of Math., Vol. 33, No. 1 (1970) 183-189 .

(4) 2-generator groups and parabolic class numbers. M. Newman. To appear in the Proc. of the Amer. Math. Soc.

(5) Invariant factors of combinatorial matrices. M. Newman. To appear in Israel J. of Math.

(6) Bounded matrix groups. M. Newman and S. Pierce. To appear in the Journal of Linear Algebra.

(7) On the Smith normal form. M. Newman. To appear in the J. of Research,NBS.

(8) Units in cyclotomic number fields. M. Newman. To appear in J. Reine Angew Math.

(9) A property of the triangle groups. J. Lehner. To appear in the J. of Research, NBS.

(10) A dominance theorem for partitioned hermitian matrices. R. Merris. Submitted to a technical journal.

(11) Elementary divisors of higher degree associated transformations. R. Merris and S. Pierce. Submitted to a technical journal.

(12) Conjecture and theorem on permanents. R. Merris. Submitted to a technical journal.

(13) The Bell numbers and r-fold transitivity. R. Merris and S. Pierce. To appear in the J. Comb. Theory(A).

(14) Isomorphic symmetry classes of tensors. R. Merris and W. Watkins. Submitted to a technical journal.

(15) Group algebra ideals and symmetry classes of tensors. R. Merris and W. Watkins. Submitted to a technical journal.

(16) Monotonicity of positive semidefinite hermitian matrices. R. Merris and S. Pierce. Submitted to a technical journal.

(17) Inequalities for matrix functions. R. Merris. To appear in Journal of Algebra.

(18) An inequality for positive semidefinite hermitian matrices. R. Merris. Submitted to a technical journal.

(19) Two characterizations of $(A_q(U))^*$. M. Sheingorn. Submitted to a technical journal.

(20) Trace function I . R. Merris. Submitted to a technical journal.

(21) Multiplicative maps of matrix semigroups over Dedekind rings. S. Pierce. Submitted to a technical journal.

(22) The error in numerical integration of analytic functions. S. Haber. To appear in the Quarterly of Applied Mathematics.

(23) On certain optimal quadrature formulas. S. Haber. To appear in Journal of Research, NBS.

(24) The existence of simple quadratures. S. Haber. Submitted to a technical journal.

(25) A quadrature formula of degree three. S. Haber and L. Flatto. Submitted to a technical journal.

BIOMEDICAL IMAGE AND LANGUAGE PATTERN PROCESSING
Task 20500-2050404
1550-22

Origin: NBS   Authorized: 1/21/64

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: January-June issue 1964, p. 19

Status: CONTINUED. The set of programs available for image processing in the LISP language continues to provide an almost complete set of image processing and list processing tools. At the LISP level, a complete morphological analyzer enables us to decompose scanned images into many different partitionings of the image field corresponding to covers of a morphological decomposition tree. This program has been used to produce resynthesis of several scans of nerve cells. Analyses of structural parts of the images continue to be obtained along with high quality reproductions on an ink-jet-printer. The high quality of the scanned images produced encouraged us to produce 3 dimensional serial section reconstructions from thick (4×10^{-5} meter) tissue sections. Among equipment developments there was further construction of the optical bench scanning system; connection of a line printer to the PDP-8; completion of construction of scan axis drivers; and installation of a TV monitor for remote microscope viewing.

SCANNING MICROSCOPE PROCESSING
Task 20500-2050408
1550-22

Origin: NBS

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: July-December issue 1968, p. 14

Status: CONTINUED. Progress during this period included items mentioned under 2050404 as well as the following. Help continued to be given in the development of a cell population dynamics model for solid tumor growth. Scans continued to be made and analyzed for white blood cells to study some nuclear fine structure problems.

CHEMICAL BIOLOGICAL INFORMATION PROCESSING
Task 20500-2050410
1550-22

Origin: NBS

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: July-December issue 1968, p. 14

Status: CONTINUED. Advisory work continued on Chemical Biological Information Handling techniques. Advisory capacity continued to consist of evaluation of submitted research proposals from various corporations and institutions as well as periodic site visits to review project and research proposals in connection with the development of a Chemical Biological Information Processing System. Studies were continuing of proposals for the development of computer graphics and data management systems for use in pharmacology.

MATHEMATICAL AIDS

Task 20500-40-2050121/57-216

Origin and Sponsor: NBS
Manager: I.A. Stegun
Full task description: July-December 1964 issue, p.4.

Status: CONTINUED. Checking continued of questionable formulas and values in AMS 55 - the Handbook of Mathematical Functions. The errata list is updated in preparation for further corrected printing of the volume.

Consultations continued at the rate of about 11 per month with staff of NBS, other government agencies, industry and universities. Topics covered for the most part were tables and programs for special functions with emphasis on computing techniques and pitfalls.

Comments and suggestions relative to the library of FORTRAN programs for special functions are being carefully considered. The error function subroutine has been successfully tested and is in use on major computers in the U. S. There has been an implementation in ALGOL and successful testing on a foreign computer. The paper on the exponential integral is being revised to include a further evaluation of alternative methods of computation.

Publications:

  Handbook of Mathematical Functions, AMS 55, Ninth Printing (with corrections), November 1970.

Lectures and Technical Meetings

EDMONDS, J.
Optimum Branching Systems. Also Chairman, 2 Sessions on Networks and Graphs. 39th National Meeting, Operations Research Society of America, Dallas, Texas. May 6-7.

FILLIBEN, J. J.
Simple and Robust Linear Estimation of the Location Parameter of a Symmetric Distribution. Department of Statistics, Princeton University. March 15.

Linear Estimation of the Location Parameter of a Symmetric Distribution. Institute of Mathematical Statistics, American Statistical Association (Section on Physical and Engineering Sciences), and Biometric Society (East North American Region), University Park, Penn., April 21.

GOLDMAN, A. J.
Optimal Location of Transport-Related Facilities. (Sponsored by Office of Naval Research and the George Washington University), Rosslyn, Va. March 17.

Chairman, Session on Systems Analysis in Government. 39th National Meeting, Operations Research Society of America, Dallas, Texas. May 7.

Some Topics in Optimal Facility Location. Cornell University (Operations Research Seminar). May 11.

JOINER, B. L.
Errors and Statistics. Laser Power and Energy Measurements Seminar, NBS Boulder Laboratories, January 14.

OMNITAB — A Fantastic Way to Compute, Statistics Department, Pennsylvania State University., February 19.

LOZIER, D.
MAXIMON, L.
SADOWSKI, W.
A Bit Comparison Program for Algorithm Testing. Argonne National Laboratory, Argonne, Illinois. April 14.

MERRIS, R.L.
A Characterization of n-fold Transitivity, University of Miami, Department of Mathematics, Coral Gables, Florida. April 26.

The Induced Inner Product in Tensor Spaces, Florida State University, Department of Mathematics, Tallahassee, Florida. April 30.

Some Inequalities for Matrix Eigenvalues. Aerospace Research Laboratories, Wright-Patterson AFB, Ohio. May 20.

NEWMAN, M.
Geometry of Numbers. University of Wisconsin, Department of Mathematics, Madison Wisconsin. May 20.

OSER, H.J.
Experimental Error Analysis. ACM 10th Annual Technical Symposium, NBS. June 24.

SOOKNE, D.J.
OLVER, F.W.J.
Implementation of Olver's Algorithm for the Bessel Functions $J_n(x)$ and $I_n(x)$. Argonne National Laboratory, Argonne, Illinois. April 13.

TRYON, P.V.
A Class of Non-Parametric Tests for Homogeneity Against Ordered Alternatives. Institute of Mathematical Statistics, University Park, Penna., April 21.

Applied Mathematics Division Expository Seminar Series

SIMMONS, John A.
(Metal Physics Section, Metallurgy Division, Institute for Materials Research). Physical Principles and Mathematical Techniques of Field Theories. January 20.

DEWIT, Roland
(Lattice Defects and Microstructures Section, Metallurgy Division, Institute for Materials Research). Generalized Functions and Fourier Transforms in Classical Field Theories. February 16.

KEARSLEY, Elliot A.
(Rheology Section, Mechanics Division, Institute for Basic Standards). Non-Linear Field Theory: An Example to Illustrate the Connection Between Constraints and Measurement of Physical Properties. March 17.

CASELLA, Russell C.
(Reactor Radiation Division, Institute for Materials Research). Harmonic Oscillator in Quantum Mechanics – Part I. April 21. Part II. May 19.

KAHN, Arnold H.
(Solid State Physics Section, Inorganic Materials Division, Institute for Materials Research). Coherent States of an Oscillator and an Electron in a Magnetic Field. June 16.

Applied Mathematics Division Lectures

NIKOLAI, Paul J.
(Applied Mathematics Research Laboratory – Aerospace Research Laboratories AFSC). (Wright-Patterson AFB, Ohio).

Calculation of the Permanent of an Incidence Matrix. April 5.

ATKIN, Oliver
(University of Arizona, Department of Mathematics, Tucson, Arizona).

$K_2(\phi)$ and Gauss' First Proof . April 12.

WIRZ, H.J.
(Institute fur Überschalltechnik, Technical University, Berlin, Germany) Implicit Finite-Difference Schemes for Integrating the Time-Dependent Navier – Stokes Equations. May 21.

NBS Course

GOLDMAN, A.J.
Introduction to Linear and Integer Programming. May 24-28.

Statistics Seminar (Boulder)

LAWTON, William H.
(Eastman Kodak Co.) Nonlinear Regression. May 24.

Self-Modeling Nonlinear Regression. May 25.

In-hours Courses, NBS Graduate School

NATRELLA, Mary G.
Statistics of Measurement. Fall Semester.

PEAVY, Sally T.
FORTRAN Programming for Beginners. Fall Semester.

ROSENBLATT, Joan R.
Nonparametric Statistical Techniques. Spring Semester.

Publication Activities

1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.3 Technical Papers

Submodular Functions, Matroids,and Certain Polyhedra. J. Edmonds. Proc. 1969 Conference on Combinatorics, Gordon and Breach, 1970.

Optimal Center Location in Simple Networks. A.J. Goldman. Transportation Science 5, 212-221, (1970).

Analysis of Information -- An Alternative Approach to the Detection of a Correlation Between the Sexes of Adjacent Sibs in Human Families. H.H. Ku. Biometrics, Vol. 27(1971), pp.175-182.

On the Analysis of Multi-Dimensional Contingency Tables. H.H. Ku, R.Varner and S. Kullback (G.W.U) Journal of the American Statistical Association, Vol. 66 (1971),pp. 55-64.

An Improvement of the Fischer Inequality. R. Merris. J. of Research 75B (1971) 76-79.

A Class of Representations of the Full Linear Group. R. Merris and S. Pierce. J. Algebra. Vol.17 (1971) 346-351.

Orthogonal Groups of Positive Definite Mulilinear Functions. S. Pierce. Pacific Journal of Math. Vol. 33, No. 1 (1970) 183-189.

Randomization and the Draft Lottery. Joan R. Rosenblatt and James J. Filliben. Science, Vol. 171 (1971), pp. 306-308.

1.4 Notes and Reviews

Simulation of Air Traffic Control Radar Beacon Assignment Plans. R.D. Elbourne (Division 650) and Judith F. Gilsinn. NBS Technical Note 568(3/71).

Review of "Introduction to Systems Cost-Effectiveness" (K. Seiler III, Wiley-Interscience,1969). A.J. Goldman, Operation Research 19, 552-554(1971).

Test Problems and Results for OMNITAB II. Ruth N. Varner and Sally T. Peavy. NBS Technical Notes 551, October 1970.

1.5 Books

Source Listing of OMNITAB II Program. Sally T. Peavy, Ruth N. Varner, and David Hogben. NBS Special Publication 339, December 1970.

2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.3 Technical Papers

Approximate Localization Theorems for Optimal Facility Placement. A. J. Goldman. Submitted to a technical journal.

The Error in Numerical Integration of Analytic Functions. S. Haber. To appear in the Quarterly of Applied Mathematics.

On Certain Optimal Quadrature Formulas. S. Haber. To appear in Journal of Research, NBS.

The Existence of Simple Quadratures. S. Haber. Submitted to a technical journal.

A Quadrature Formula of Degree Three. S. Haber and L. Flatto. Submitted to a technical journal.

Optimal Design of Sorting Networks. W.A. Horn. Submitted to a technical journal.

A Simple Algorithm for Minimizing Delay (Cost) in a Sorting (Decision) Network. W.A. Horn. Submitted to a technical journal.

Convex Homotopy . W.A. Horn. To appear in J. of Research NBS 75B(1971).

Optimal Container Inventory and Routing. W. A. Horn. To appear in Transportation Science.

Some Properties of the Range in Sample from Tukey's Symmetric Lambda Distributions. Brian L. Joiner and Joan R. Rosenblatt. To appear in J. Amer. Statistics Assoc.

A Property of the Triangle Groups. J.Lehner. To appear in the J. of Research, NBS.

A Bit Comparison Program for Algorithm Testing. D.W Lozier, L.C. Maximon, and W.L. Sadowski. Submitted to Communications of ACM; prepublication as NBS Report No. 10449, June 30, 1971.

Trace Functions I. R. Merris. Submitted to a technical journal.

A Dominance Theorem for Partitioned Hermitian Matrices. R. Merris. Submitted to a technical journal.

The Bell Numbers and r-Fold Transitivity. R. Merris and S. Pierce . To appear in the J. Comb. Theory(A).

Monotonicity of Positive Semidefinite Hermitian Matrices. R. Merris and S. Pierce. Submitted to a technical journal.

Group Algebra Ideals and Symmetry Classes of Tensors. R. Merris and W. Watkins. Submitted to a technical journal.

On the Smith Normal Form. M. Newman. To appear in the J. of Research, NBS.

2-Generator Groups and Parabolic Class Numbers. M. Newman. To appear in the Proc. of the Amer. Math. Soc.

Invariant Factors of Combinatorial Matrices. M. Newman. To appear in the Israel J. of Math.

Bounded Matrix Groups. M. Newman and S. Pierce. To appear in the Journal of Linear Algebra.

An Ergodic Source which is not a Function of any Finite Markov Process. Roger D. Traub. To appear in the J. of Research NBS 75B(1971).

Covariances of Two-Sample Rank Sum Statistics. Peter V. Tryon. Submitted to a technical journal.