Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

April through June 1951

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

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April 1 through June 30, 1951

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Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

April through June 1951

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Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from April 1, 1951, to June 30, 1951.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

J H Curtiss
Chief

E U Condon

Director
National Bureau of Standards
August 1, 1951.

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

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* New projects

Status of Projects

June 30, 1951

I. Institute for Numerical Analysis

(Section 11.1)

1. Fundamental Research

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Task 1101-11-5100/49-AE2
(formerly 11.1/1-49-AE2)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Meetings were held once a week, in a very informal seminar, to discuss the progress of experimental and theoretical research on solving linear equations at the I.N.A. and sometimes to recommend changes in our computational experiments on I.B.M.. They were attended by Drs. Blanch, Forsythe, Hestenes, John, Lanczos, Motzkin, Stein, and others. The meetings have been discontinued for the summer.

Work continued on I.B.M. equipment with matrices of order six, for two reasons: (1) Much is still to be learned from matrices of order six; (2) Matrices of larger order require reading data in and out of the card-programmed calculator many times, resulting in slower speeds and more operator errors.

To solve a system $Ax = b$, Dr. Stein tried a variation of the optimum $-\alpha$ process (see Oct-Dec 1950 issue). Let α_i^* be the value of α which minimizes $F(x_i - \alpha z_i)$, where

$$F(x) = |Ax - b|^2 \text{ and } z_i = 2A^T(Ax_i - b).$$

Pick a constant β $(0 < \beta < 2)$ and let

$$x_{i+1} = x_i - \beta\alpha_i^* z_i.$$

Dr. Stein tried various values of β and concluded that of these for our matrix the value $\beta = 0.9$ was the best, in that $F(x_i)$ diminished most rapidly in the long run for this β. The decrease of $F(x_i)$ was irregular, suggesting some instability in the transformation from x_i to x_{i+1}. On the average, for one matrix and one start x_0, the method $\beta = 0.9$ resulted in a gain of one decimal of x per 22 iterations at the transformation. The corresponding figure for $\beta = 1.0$ was 180 iterations. The acceleration scheme of Drs. Forsythe and Motzkin (see Oct-Dec 1950 issue) resulted in one decimal gained in x per 11 iterations, but there are two kinds of iterative routines involved. The method $\beta = 0.9$ appears to be a self-accelerating gradient method, requiring only one routine.

Dr. Forsythe is continuing his article on a classification of iterative methods of solving linear equations.

Publications: (1) "The extent of n random unit vectors," by G.E.Forsythe and J. W. Tukey; submitted to a technical journal. (2) "A method of computing exact inverses of matrices with integer coefficients," by J. B. Rosser; accepted by NBS Journal of Research. (3) "Annoted translation of a letter in which Gauss solved linear equations by relaxation," by G. E. Forsythe; submitted to a technical journal. (4) "An extension of Gauss' transformation for improving the condition of systems of linear transformations," by G. E. Forsythe and T. S. Motzkin; submitted to a technical journal.

NUMERICAL METHODS IN CONFORMAL MAPPING
Task 1101-11-5100/49-CM1
(formerly 11.1/1-49-CM1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The calculation undertaken by L. K. Jackson and fully described in the Oct-Dec 1949 issue (see project 11.1/1-49-CM1) has been resumed after a thorough check of equations entering into the calculations. Two iterations have been completed so far. The work is being done on the 604 multiplier. A method suggested by Professor Warschawski for handling the mapping of nearly circular regions, in particular an ellipse with $a/b = 1.2$ is being tried by the Computation Laboratory staff on SEAC.

Publications: (1) "The construction and applications of conformal maps: Proceedings of a symposium," edited by E. F. Beckenbach; to be published in the NBS Applied Mathematics Series. The volume will include the following papers written in connection with this project; (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S, E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A. M. Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A. M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A. Weinstein. (vi) "On subordination in complex theory," by E. F. Beckenbach and E. W. Graham.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Task 1101-11-5100/50-3
(formerly 11.1/1-50-3)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Further studies have been made on the problem of finding eigenvalues and eigenvectors for the system $Ax = \lambda Bx$. The results are being written up. General iterative methods are discussed together with appropriate convergence theorems.

For the case $Ax = \lambda x$, where A is real and symmetric, experiments have been made using the iteration

$$x_{i+1} = x_i - x_i \cdot \xi_i \ , \quad \xi_i = Ax_i - \mu(x_i)x_i \ ,$$

$$\mu . (x) = x^* Ax/x^* . x \ , \quad x_i = \beta / [\mu(\xi_i) - \mu(x_i)] \ ,$$

where $0 < \beta < 1$. The results indicate that for a suitable choice of β (under $\beta = .8$ Or .9) this procedure is significantly better than the power method and the other methods previously devised.

L. J. Paige has given a new and simple proof of necessary and sufficient conditions that two or more matrices can be reduced simultaneously to triangular form. Also, he continued investigations with M.R. Hestenes on iterative methods for obtaining the eigenvalues of the matrix equation $Ax = \lambda Bx$.

T. S. Motzkin and Olga Taussky-Todd have been working on a joint paper relating to pairs of matrices with Frobenius' P-property, a weaker so-called L-property, etc. This work was suggested by M. Kac. It was proved that Hermitian matrices with these properties can be diagonalized simultaneously by similarity transformation. (In this connection, see the work of L. J. Paige in this task.)

Publications: (1) "The separation of close eigenvalues of a real symmetric matrix," by J. B. Rosser, C. Lanczos, M. R. Hestenes, W. Karush; accepted by the NBS Journal of Research. (2) "Alternative derivations of Fox's escalator formulas for latent roots," by G. E. Forsythe; submitted to a technical journal. (3) "Simultaneous reduction of matrices to triangular form," by L. J. Paige, paper presented at the Pullman, Washington meeting of the American Mathematical Society, June 15, 1951; IN MANUSCRIPT. (4) "Determination of the extreme values of the spectrum of a bounded self-adjoint operator," by W. Karush; submitted to a technical journal.

STUDIES IN THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS
Task 1101-11-5100/51-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. 1. W. E. Milne has completed the advanced work on the monograph "Numerical solution of differential equations" (see Jan-Mar 1949 issue). Minor revisions, changes, and additions to the manuscript of the monograph will be made before publication.

2. F. John is continuing his work on the manuscript on parabolic equations. The first part of the manuscript, on applications of difference equations to the solution of differential equations, is almost completed. This part deals with the case of linear parabolic equations in one space dimension on an infinite interval. The second part will deal with the most general nonlinear parabolic equation and bounded intervals. It is shown that, under very general conditions for the linear differential equation, the solution of the difference equation converges to the one of the differential equation for initial data that are bounded and Riemann-integrable. In this connection, the existence of the solution of the differential equation is established for initial and boundary data that are Riemann-integrable and the fundamental solution is constructed.

3. J. W. Green is studying the difference operator in higher dimensional space. In particular he is studying the question of convergence of the solutions of the approximate equations toward the solution of the dif-

ferential equation.

4. G. Blanch has completed four numerical approximations to a parabolic partial differential equation (in two dimensions) having a nonlinear term. This work was done on the card-programmed calculator. One aim of the experiment was to study the effect of replacing the differential operator (which is of order two in this instance) by a difference operator of higher order. Under certain conditions the accuracy of the solution can be improved by using a difference operator of higher order, without unduly complicating the computations. Moreover, it is found that the method used by Hartree and Womersley to improve a solution from computations based on two different mesh intervals, but with the same mesh ratio, is very effective and can be made to give powerful control over the accuracy of the computations. In addition, a study is being made of the optimum mesh ratio (i.e. one that will result in the least amount of work for a given accuracy). Results so far obtained indicate that the largest admissible mesh ratio is not usually the optimum one.

5. The following lectures were presented at the seminar on partial differential equations: (1) "Optimum mesh ratios for parabolic differential equations," by G. Blanch. (2) "The Kolmogorov and Smirnov limit theorems for empirical distribution functions," by H. P. Edmundson. (3) "On equidistant interpolation," by C. Lanczos, (two lectures). (4) "Modes of vibration of a suspension chain," by D. Saxon.

The following papers, closely related to the solution of differential systems, were presented this quarter at meetings: (1) "An invited paper on parabolic equations," by F. John, presented at a seminar of the Mathematics Department, University of California at Los Angeles. (2) "Solution of parabolic equations by finite differences," presented at a meeting of the American Mathematical Society, Palo Alto, California, April 28, 1951. (3) "Integration by weighted sums," by C. Lanczos, presented at a meeting of the American Mathematical Society, Palo Alto, California, April 28, 1951. (4) "On an expansion method for parabolic partial differential equations," by J. W. Green, presented at a meeting of the American Mathematical Society, Pullman, Washington, June 16, 1951.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of a Symposium held at the Harvard Computation Laboratory, September 1949. (2) A paper "On integration of parabolic equations by difference methods. I: The linear equation for the infinite interval," by Fritz John; IN MANUSCRIPT.

PROBABILITY METHODS AND SAMPLING TECHNIQUES
Task 1101-11-5100/51-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue. Dr. W. Wasow is on leave at the Massachusetts Institute of Technology for this semester.

Publications: (1) "Monte Carlo method," proceedings of a symposium held on June 29,30, July 1, 1949, in Los Angeles, California, under the sponsorship of the RAND Corporation and the NBS, with the cooperation of the Oak Ridge National Laboratory. NBS Applied Mathematics Series 12; available from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 30 cents. (2) "A Monte Carlo method for solving a class of integral equations," by R. E. Cutkosky; accepted by the NBS Journal of Research. (3) "On some connections between probability theory and differential and integral equations," by Mark Kac; to be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950. (4) "On the mean duration of random walks," by W. Wasow; NBS J. Res. 46, 462-472 (June 1951), RP2215; available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 10 cents. (5) "Elimination of randomization in certain statistical decision procedures and zero-sum two-person games," by A. Dvoretzky, A. Wald, and J. Wolfowitz; Ann. Math. Stat. 22, 1-21 (Mar. 1951); no reprints. (6) "Uniformly best constant risk and minimax point estimates," by R. T. Peterson, Jr.; accepted by the NBS Journal of Research. (7) "On the duration of random walks," by W. Wasow; submitted to a technical journal. (8) "Random determinants," by R. Fortet; accepted by the NBS Journal of Research. (9) "Some problems on random walk in space," by A. Dvoretzky and Paul Erdös; to be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950. (10) "Various techniques used in connection with random digits," J. von Neümann (summary written by G. E. Forsythe); appeared in "Monte Carlo method" (see publication (1)). (11) "Generation and testing of random digits at the National Bureau of Standards, Los Angeles," by G. E. Forsythe; appeared in "Monte Carlo method" (see publication (1)). (12) "On the duration of random walks," by W. Wasow; submitted to a technical journal. (13) "On the estimation of an eigenvalue by an additive functional of a stochastic process, with special reference to the Kac-Donsker method," by R. Fortet; accepted by NBS Journal of Research. (14) "Additive functionals of a Markoff process," by R. Fortet; submitted to a technical journal. (15) "On some functionals of Laplacian processes," by R. Fortet; accepted by NBS Journal of Research.

VARIATIONAL METHODS
Task 1101-11-5100/51-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The study of variational methods has been applied to simultaneous equations and eigenvalue problems. (See task 1101-11-5100/49-AE2, page 1).

Publications: (1) "A method of gradients for the calculation of roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; NBS J. Res. 47, 45-61 (July 1951), RP2227. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 20 cents. (2) "An iterative method for finding characteristic vectors of a symmetric matrix," by W. Karush; accepted by the Pacific Journal of Mathematics. (3) "Applications of the theory of quadratic forms in Hilbert space to the calculus of variations," by M. R. Hestenes; submitted to a technical journal. (4) "The solution of $Ax = \lambda Bx$," by M. R. Hestenes and W. Karush; accepted by NBS Journal of Research. (5) "On methods for obtaining solutions of fixed end point problems in the calculus of variations," by Marvin Stein; IN MANUSCRIPT.

STUDIES IN APPLIED MATHEMATICS
Task 1101-11-5100/51-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Integration by weighted sums. Numerical integration is based on changing a differential equation into a difference equation and considering the Δ operator as an approximation of the D operator. The generally slow convergence of this process can be speeded up by the solution of an interpolation problem. We obtain f(x) from the given equidistant ordinates by trigonometric interpolation. This gives an analytic expression for f(x) from which the error of the Δ-process can be determined and the proper correction applied. This correction amounts to an additional weighting of the partial sums. Partial differential operators can be similarly treated. Experiments with this method are in progress, which deal particularly with the possibility of applying it to the solution of linear partial differential equations with constant coefficients.

Solution of sets of linear algebraic equations. The previously designed algorithm for solving linear equations and finding the eigenvalues of a matrix (see Apr-Jun 1949 issue, project 11.1/1-49-AE1, p.3) is based on a complete analysis of the matrix A. (See also "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C. Lanczos; published in the NBS J. Res. 45, 255-281 (Oct 1950)). The resultant algorithm is too elaborate, however, if only partial information is required concerning the behavior of the matrix A. In particular, the solution of a set of linear algebraic equations is equivalent to the determination of the eigenvector connected with the eigenvalue zero of a symmetric matrix, while the other eigenvectors are of no interest. Hence it is desirable to find a scheme of greater simplicity which is more specifically adjusted to the problem here involved. The residual vector is purified from the contribution of the large eigenvalues but contains the contribution of the small eigenvalues. A further purification is obtained by adding the first two steps of the initially quoted algorithm. This completes the cycle. If the resultant residual is still too large, the entire cycle can be repeated, replacing b by ρ_m. An arbitrary linear set of equations is thus solvable by a simple and straightforward algorithm with any accuracy desired.

Let G be an arbitrary complex matrix and

$$Gx - g = 0$$

be the given set of equations. We transform this problem into

$$Ax - b = 0$$

with

$$A = \tilde{G}^*G$$

$$b = \tilde{G}^*g$$

(* means transposed, $\sim$ means complex conjugate). The matrix A is Hermitian and positive definite.

Construct a sequence of (n + 1) - dimensional vectors

$$\overline{p}_m = p_m \, , \, (m + 2)^2$$

by the following simple recurrence scheme:

$$p_{m+1} = B\bar{p}_m - p_{m-1}$$

starting with

$$p_o = \frac{4}{\lambda_M} b \quad ,$$

λ_M is the estimated largest eigenvalue of A, and the n-row, (n + 1) - column matrix B is defined by

$$B = 2I - \frac{4}{\lambda_M} A, \quad \frac{4}{\lambda_M} \quad b \ .$$

The approximate solution

$$x_m = \frac{p_m}{(m + 2)^2}$$

gives a residual vector

$$Ax_m - b = \rho_m$$

whose length is bounded by the following inequality:

$$\frac{|\rho_m|}{|x|} \leq \frac{\lambda_M}{(m + 2)^2} \ .$$

<u>Eigenvalue analysis</u>. Let λ_o be an approximate eigenvalue of G. Omit one of the equations of the homogeneous set

$$(G - \lambda_o I)x = 0$$

and replace it thus by an nonhomogeneous set. We now have a problem of the previous type, and the algorithm just described becomes applicable. The resultant approximate solution is near to the (generally complex) eigenvector associated with the eigenvalue

$$\lambda = \lambda_o + \varepsilon \ .$$

We go through the same procedure with the transposed matrix G* and thus obtain the approximate associated vector x*. Now construct the following vectors:

$$b_o = x \ , \quad b_1 = Gb_o, \quad b_2 = Gb_1 \ ,$$

$$b_o^* = x^* \ , \quad b_1^* = G^*b_o^* \ , \quad b_2^* = G^*b_1^* \ ,$$

and obtain the scalars

$$c_o = b_ob_o^* \ , \quad c_1 = b_ob_1^* \ , \quad c_2 = b_1b_1^* \ , \quad c_3 = b_1b_2^* \ .$$

Then in the first approximation

$$\varepsilon = \frac{c_1}{c_o}$$

or in the second approximation

$$\begin{vmatrix} 1 & \varepsilon & \varepsilon^2 \\ c_0 & c_1 & c_2 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0 .$$

This method, which resembles Newton's method of finding the root of an algebraic equation from an approximate value x_0, is particularly valuable where the smallest eigenvalue of a generally complex matrix is desired (we then take $\lambda_0 = 0$) the present method obviates the usual requirement of inverting first the matrix.

The manuscript on equidistant interpolation has been split into two parts, the first dealing with a finite, the second with an infinite interval. The first part, entitled "Practical curve fitting of equidistant data," is complete. A manuscript on the antenna problem is in preparation for the Proceedings of IRE.

Publication: (1) An introduction has been prepared on the application of the tables of Chebyshev polynomials to be included in NBS Applied Mathematics Series 9, "Tables of the Chebyshev pol nomials $S_n(x)$ and $C_n(x)$"; now in press.

MISCELLANEOUS STUDIES IN THEORETICAL PHYSICS
Task 1101-11-5100/51-5

Origin: Office of Naval Research, USN
Sponsor: " "
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. In collaboration with A. S. Cahn, a method was developed for calculating the characteristic frequencies of a suspended inextensible chain vibrating with small amplitude in the plane of the catenary forming the equilibrium configuration. An asymptotic solution was obtained such that the accuracy of the results increases as the mode number increases and/or as the catenary becomes flutter. With the kind assistance of Professors I. Rudnick and R. Leonard of the Department of Physics, University of California at Los Angeles, the characteristic frequencies for the first five modes were measured for various configurations and gave results in good agreement with the theory.

A manuscript entitled "Modes of vibration of a suspended chain," by D. S. Saxon and A. S. Cahn, is now in preparation.

Publications: (1) "The torsion of anisotropic elastic cylinders by forces applied on the lateral surface," by H. Luxenberg; accepted by the NBS Journal of Research. (2) "An analysis of the effect of the discontinuity in a bifurcated circular guide upon plane longitudinal waves," by L. L. Bailin; accepted by the NBS Journal of Research.

A GUIDE TO TABLES ON PUNCHED CARDS
Task 1101-11-5100/51-6

Origin: NBS Authorized 6/27/51
Sponsor: Office of Naval Research, USN
Managers: G. Blanch and E. C. Yowell

Objective: To compile a comprehensive bibliography of tables available on punched cards in the USA and Canada. The classification of tables is to be by subject matter, based on that used by Fletcher, Miller, and Rosenhead in An Index of Mathematical Tables.

Background: A list of tables available on punched cards was compiled by W. J. Eckert and published in the October, 1945, issue of Mathematical Tables and Other Aids to Computation. Since then radical changes have been made in IBM equipment, and there has been a great increase in the use of punched card equipment for mathematical computing. As a result, many new and important punched card tables were compiled by laboratories throughout the country, and the editors of Mathematical Tables and Other Aids to Computation felt that an up-to-date bibliography of tables currently available would be of much help to everyone interested in punched card methods for mathematical computations. It was desirable that the compilation be made by those who have intimate knowledge of current and past progress in table making. The managers of this project undertook the task.

Status: NEW. A preliminary draft of available tables has been prepared and circulated for checking among those laboratories who contributed material. The list comprises over 270 entries contributed by about 30 different laboratories. It is hoped that the checked and corrected guide will be available for publication soon in Mathematical Tables and Other Aids to Computation.

STUDIES IN PURE MATHEMATICS
Task 1101-11-5101/50-4
(formerly 11.1/1-50-4)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

Publications: (1) "On some trigonometric transforms," by O. Szász; accepted by the Pacific Journal of Mathematics. (2) "On subharmonic and linear functions of two variables," by E. F. Beckenbach; accepted by Revista, Universidad Nacional de Tucuman (Argentina). (3) "Certain Fourier transforms of distributions," by E. Lukacs and O. Szász; Canadian J. Math. III, no. 2, 140-144 (1951). Reprints available. (4) "On a Tauberian theorem for Abel summability," by O. Szász; Pac. J. of Math. 1, 117-125, (Mar.1951). Reprints available. (5) "Tauberian theorems for summability (R_1)," by O. Szász; accepted by the American Journal of Mathematics. (6) "Relations among certain ranges of vector measures," by A. Dvoretzky; Pac. J. Math. 1, 59-74, (Mar. 1951). Reprints available. (7) "On relative extrema of Bessel functions," by O. Szász; accepted by the Bolletino della Unione Matematica Italiana (Firenze). (8) "On the relative extrema of the Hermite orthogonal functions," by O. Szász; submitted to a technical journal. (9) "On a recursion formula and on some Tauberian theorems," by N.G.de Bruijn and Paul Erdős; accepted by the NBS Journal of Research. (10) "On the Gibbs phenomenon for a class of linear transforms," by O. Szász; accepted by Publications de l'Inst. Math. de l'Acad. Serbe des Sciences, vol. IV. (11) "Recurrent determinants of Legendre and ultraspherical polynomials," by E. F. Beckenbach, W. Seidel, and O. Szász; Duke Math. J. 18, no. 1, 1-10 (March 1951). Reprints available. (12) "Identities and inequalities concerning orthogonal polynomials and Bessel functions," by O. Szász; J. d'Analyse Mathématique (Jerusalem) I, 116-134 (1951). No reprints. (13) "Second order determinants of Legendre polynomials," by G.E.Forsythe; accepted by the Duke Mathematical Journal. (14) "The convergence of Cauchy-Riemann sums to Cauchy-Riemann integrals," by O. Szász and J. Todd; accepted by the NBS Journal of Research.

COMPUTATION OF THE IMAGINARY ZEROS OF THE RIEMANN ZETA FUNCTION
Task 1101-11-5101/50-13
(formerly 11.1/1-50-13)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1950 issue.

2. Applied Research

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Task 1101-21-5102/50-11
(formerly 11.1/1-50-11)

Origin: Office of Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Systematic investigations were pursued in the theory of linear inequalities. Special attention was given to the following four topics: (a) Some new theoretical results on the relaxation method were obtained by T. S. Motzkin and S. Agmon. Experiments by hand computing were performed under the supervision of R. M. Hayes. A. Orden, USAF, made computations on the SEAC based on the method outlined by T. S. Motzkin during lectures presented at the Office of the Air Comptroller, Washington, D. C., during February, 1951. (See Jan-Mar issue). (b) As the relaxation method and other similar methods are based on the iteration of polyhedral (piecewise linear) transformations, studies were made concerning the periodicity behavior of transformations of this kind.
(c) An important line of work with ramified applications in many fields is the study of special systems of linear and nonlinear inequalities pertaining to minimax approximations. A talk on this topic was presented by T.S.Motzkin at the Institute for Numerical Analysis Colloquium Series, May 28, 1951. In particular, a conjecture of E. G. Straus in connection with a problem posed by the RAND Corporation was confirmed. (d) Studies were made in the inter-related field of systems of linear equations. (See task 1101-11-5100/49-AE2,

p. 1).

Consultations were held with G. Dantzig, USAF, F. John, NBS and F. J. Murray, Columbia University. Discussions were also held with several experts regarding the possible development of a special purpose analog machine for solving linear inequalities and programming problems. Participation of the following has been assured in the work of the task during the coming year: L. M. Blumenthal, University of Pennsylvania; I. J. Schoenberg, University of Pennsylvania J. L. Walsh, Harvard University; and S. Agmon, Rice Institute.

A joint NBS-USAF symposium on "Linear inequalities and programming" was held. The program for the symposium is given on page 63 Two talks were delivered by T. S. Motzkin at this symposium. They are entitled "Remarks on the history of linear inequalities" and "New techniques for linear inequalities and optimization." In addition a talk, "Periodicity regions of sequences with a piecewise linear recursion formula," by Dr. Motzkin was presented before the American Mathematical Society, Palo Alto, California, April 28, 1951.

Publication: (1) "Two consequences of the transposition theorem on linear inequalities," by T. S. Motzkin; Econometrica, 19, No. 2, 184-185 (Apr. 1951); also included in this issue is "A note on Motzkin's transposition theorem," by Morton L. Slater; 185-187.

LANGUAGE TRANSLATION STUDY
Task 1101-21-1102/50-10
(formerly 11.1/1-50-10)

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

Publication: (1) "Proposals for the mechanical resolution of German syntax patterns," by Dr. V. Oswald and Mr. S. Fletcher, jr.; Modern Language Forum XXXVI, 1-24 (July 1951).

3. Development

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(Previously listed as Air Materiel Command Computing Machine)
Task 1101-34-5103/49-1
(formerly 11.1/22-49-1)

Origin: Office of Air Research, AMC, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Operation of the computer was continued on a test basis, interspersed with runs of miscellaneous exercises and problems (See task 1101-53-1101/51-38 page 28 and 1101-34-5103/50-2, page 13).

Further work is being done to eliminate noise from the memory circuits. This consists of rearranging certain wiring so as to minimize pickup in the deflection system. The effect of using a.c. heaters was studied by using a d.c. heater supply temporarily. Ripple noise was decreased substantially, partly due to the removal of effects of heater cathode leakage and partly due to the reduction of a.c. fields. The 5JP1 cathode ray tubes were tried in four chassis and were found unsatisfactory due to poor focus and the dependence of focus on intensity. The 5JP1A and 5JP11A cathode ray tubes are about equally satisfactory. The optical tape reader unit was completely checked out and is ready to use with the computer.

Negotiations were begun with local companies concerning the fabrication of the magnetic drum equipment, and the prototyping of the first circuits is under way.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Task 1101-34-5103/49-2
(formerly 11.1/22-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Distribution continued of the preliminary lists of terminology and block diagram symbols to be used in connection with automatic computing machines.

Publication: The preliminary lists mentioned above are obtainable in manuscript form from the Institute for Numerical Analysis.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Task 1101-34-5103/50-1
(formerly 11.1/22-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: H. D. Huskey

Authorized 7/1/49
Terminated 6/30/51

Objective: To establish a system of semi-automatic instruction for all general purpose electronic digital computing machines using as a model the automatic computing machine being constructed under project 11.2/22-49-1.

Background: With the advent of general purpose high-speed digital computing machines the need for a simplified system of coding has become evident. Such a system, by making the machine do much of the tedious and repetitious part of coding, would greatly decrease the work of the coding staff.

Status: TERMINATED.

Publication: (1) "Semi-automatic instruction on the National Bureau of Standards Western Automatic Computer," by H. D. Huskey; to appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory, September 1949.

PROGRAMMING AND CODING OF PROBLEMS FOR SOLUTION ON THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER
Task 1101-34-5103/50-2
(formerly 11.1/22-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. In addition to SWAC test routines, the following problem routines were prepared during the quarter: 1. Linear equations. Solves n simultaneous linear equations, given parameters n,s,t, where 2^{-s} and 2^{-t} are scale factors. Reciprocals of pivotal elements are supplied by operator. 2. Relaxation problem. Given values of a function along the boundary of an 11 x 14 rectangle, solves Laplace equation by relaxation method. 3. Generation of random numbers. Given $c_1 = 5^{13}$, computes $c_i = (c_1)^i \bmod 2^{36}$, adds corresponding digits of 100 values of c_i, finds 1000 such sums, and records frequences of sums s. 4. Tabulation of reciprocals of integers in base 16.

Several new subroutines were also prepared. These include: 1. Double precision. Given numbers a and b, each to 72 binary places in two adjacent addresses, computes a + b, a - b, a x b, or a/b to 72 binary places. 2. Complex numbers. Given complex numbers $x_1 + iy_1$, $x_2 + iy_2$, computes sum, difference, product, or quotient. 3. Complex numbers with square root. 4. Square root. Obtains $\sqrt{n}$, where n has an even number of binary places. 5. Differencing. Computes and types k^{th} differences of numbers on a tape.

4. Mathematical Services

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Task 1101-53-1100/49-1
(formerly 11.1/32-49-1)

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Problems computed on IBM machines were: 1. For G. E. Forsythe: Studies in iterative methods of solving systems of linear equations. 2. For M. Hestenes: (a) Experimental work on iterative methods of solving eigenvalue problems for matrices. (b) Experimental work on iterative methods of solving systems of linear algebraic equations. 3. For P. Wesley: Evaluation of integrals and series used in obtaining numerical results for the problem of electromagnetic radiation from a coaxial type antenna.

Shorter computations on desk calculators were made for various members of the staff. Among such calculations were: 1. For J. B. Rosser: Extension of his work on the approximations of $\pi(x)$ and $\theta(x)$ for x beyond a million in the range of Lehmer's table of primes. Verification of $\pi(x)$ for x = 8,886,111 and $\theta(x)$ for x = 8,040,485 has been established. 2. For W. E. Milne: Computations involved in the study and comparison of various methods of solving differential equations have been completed this quarter.

(see task 1101-11-5100/51-1 page 3). 3. For C. Lanczos: (a) Computations relating to equidistant polynomial interpolations and equidistant curve fitting by a polynomial of minimum order. (b) Computations concerning the increased convergence of integration by weighted sums. (c) Numerical verification of an iterative procedure for the solution of sets of linear algebraic equations, using small matrices (see task 1101-11-5100/51-4, page 6).

In addition, the conformal mapping problem, first undertaken by L. K. Jackson and discontinued, has been resumed, after thorough checking of the equations. (See task 1101-11-5100/49-CM1, page 2). The work is being done on the 604 multiplier, under G. Blanch as task manager.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Task 1101-53-1100/49-4
(formerly 11.1/32-49-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue. (A paper entitled "A least-squares method of determining the periods of variable stars," by E. C. Yowell is in process of preparation.)

NUMERICAL STUDIES OF A NON-LINEAR PARABOLIC DIFFERENTIAL EQUATION
Task 1101-53-1100/51-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. A report on the numerical study is in process of preparation.

MATHIEU FUNCTIONS II
Task 1101-53-1101/45-1
(formerly 11.1/2-45-1)

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Manager: E. C. Yowell
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

SPECIAL TABLE OF BESSEL FUNCTIONS
Task 1101-53-1101/48-2
(formerly 11.1/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Computations were completed. The final manuscript is to be made on a card-controlled typewriter.

TABLES FOR ROCKET AND COMET ORBITS
(formerly listed as ROCKET NAVIGATION TABLES)
Task 1101-53-1101/48-3
(formerly 11.1/2-48-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The checking of the final manuscript has been completed. Pagination of the manuscript is now in progress. Note that the author has changed the title of the book to Tables for Rocket and Comet Orbits.

TABLES OF $E_1(z)$, SECOND QUADRANT
Task 1101-53-1101/49-1
(formerly 11.1/2-49-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Calculations are completed and checked. The final manuscript is to be made on a card-controlled typwriter.

PUNCHED CARD LIBRARY
Task 1101-53-1101/49-2
(formerly 11.1/2-49-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: E. C. Yowell
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. Punching of table of Spherical Bessel Functions was continued. Tables for

$$\nu = -\frac{-27}{2}(1) - 7/2$$

were punched. The following is an addition to the library:

$$x_n + iy_n = (\sigma + i\,\omega)^n$$

where $\sigma = 0(-1) - 10(-2) - 20$

$\omega = 0(1)20(10)400$

$n = 1(1)10;$

Values of x_n and y_n are exact.

REDUCTION OF RAYDIST DATA
Task 1101-53-1101/49-2a
(formerly 11.1/32-49-2)

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Two runs of Raydist data were computed this quarter.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Task 1101-53-1101/49-6
(formerly 11.1/32-49-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: S. Herrick

Authorized 3/21/49
Terminated 6/30/51

Objective: To compute orbit of Halley's comet from 1835 to 1910, and the orbits of the ninth satellite of Jupiter and certain minor planets.

Background: The orbit of Halley's comet from 1835 to 1910 as determined by Cowell and Cromellin in their well-known prediction of the last return, is being checked and experimentally paralleled by a new variation of parameters procedure (Herrick: Publications of the Astronomical Society of the Pacific, October 1948). The calculations are fundamental to a prediction of the next return and to the establishment of the relative merits of the old and new procedures for this prediction. The orbits of the ninth satellite of Jupiter and certain minor planets are to be undertaken in similar experimental calculations.

Status: TERMINATED.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Task 1101-53-1101/50-2
(formerly 11.1/32-50-2)

Origin: Naval Air Missile Test Center, Point Mugu
Sponsor: Bureau of Aeronautics, USN
Manager; R. R. Reynolds
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. The problem has been set up for the differential analyzer at the University of California at Los Angeles, Engineering Department; and work will resume on this task when necessary numerical data are received.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Task 1101-53-1101/50-7
(formerly 11.1/31-50-7)

Origin: Lockheed Aircraft Corporation — Authorized 12/1/49
Sponsor: Office of Air Research, AMC, USAF — Terminated 6/30/51
Manager: E. C. Yowell

Objective: (a) To compute coefficients to be used in the design of circular shell-supported frames. Specifically, to tabulate the solutions of the equation

$$(D^6 + 2D^4 + D^2 - d)\, m = 0 \;;$$

where $D = d/d\theta$ for appropriate boundary conditions and for values of the parameter d = 0, 10, 50, 100, 300, 1000, 2000, 4000, 8000, 25000.
(b) To make certain auxiliary computations for particular cases where the fuselage skin is supported by a finite number of rings.

Background: This project deals with the single problem of circular shell-supported frames subjected to concentrated loadings. The Lockheed Aircraft Corporation developed the mathematical attack of this problem which has been presented in the form of non-dimensional coefficient curves. These curves, while they are developed for only circular frames, may, by means of approximations, be used for nearly any practical frame which has curvature in the region of applied loading. (Ref. "Analysis of Circular Shell-Supported Frames," Lockheed Aircraft Corp. Report No. 42222; "Note on Analyses of Circular Shell-Supported Fuselage Frames," Lockheed Aircraft Corp. Report No. 6821.)

Continuation of the mathematical computations originated by the Lockheed Aircraft Corporation will be accomplished.

Status: TERMINATED.

METEOROLOGICAL MEANS
Task 1101-53-1101/50-17
(formerly 11.1/31-50-17)

Origin: Department of Meteorology, U.C.L.A.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Data were received for the sixth pressure level for February 1949. Computations of fourth, fifth, and sixth levels were 80% completed.

EARTH TIDES
Task 1101-53-1101/51-1

Origin: University of California, Los Angeles, Geophysics Department
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see July-Sept 1950 issue.

EVAPORATION COMPUTATIONS
Task 1101-53-1101/51-3

Origin: Naval Electronics Laboratory
Sponsor: Bureau of Reclamation, Department of the Interior
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Data have been received for the months of September, October, November, and December. Computations have been completed for August, September, and October; computations for November and December have been 10% completed.

BOUNDARY LAYER
Task 1101-53-1101/51-5

Origin: Northrop Aircraft Co., Inc.
Sponsor: Office of Air Research, AMC, USAF
Manager: M. Howard, C. Lanczos

Authorized 9/28/50
Terminated 6/30/51

Objective: To solve, by finite differences, the simultaneous equations

$$\begin{cases} u \dfrac{\partial u}{\partial x} + v \dfrac{\partial u}{\partial y} = UU' + \dfrac{\partial^2 u}{\partial y^2} \ , \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \ . \end{cases}$$

subject to given necessary boundary conditions. U is a given function of x.

Background: This computation arises in connection with a study of the flow in the laminar boundary layer past an airfoil for the case when suction is applied. The method is based upon a translated report on the subject of Laminar Boundary Layers by H. Gortler. The report treats the case where there is no suction ($v = 0$ at $y = 0$). Adaptation of the method for the case where suction is applied ($v = v_0(x)$ at $y = 0$) was made by Northrop Aircraft Co.

Status: TERMINATED. The problem of the boundary layer has been the subject of a great many discussions and elaborate investigations, and there exists extensive literature on the subject. Further substantial advances can be made only by intensive study and by combining analytical research with numerical experimentation. It is the belief of the project manager that purely numerical procedures without proper analytical backing can hardly hope to succeed. Hence this task is terminated herewith, but the project manager expects to continue study on the problem as part of his regular research activity.

ROLLING PULLOUT EQUATIONS OF MOTION
Task 1101-53-1101/51-6

Origin: Cornell Aeronautical Laboratory — Authorized 9/28/50
Sponsor: Office of Air Research, AMC, USAF — Completed 6/30/51
Manager: E. C. Yowell

Objective: To solve nine simultaneous differential equations which describe the motion of an airplane in a rolling pullout.

Background: Under combat conditions it is often important for a plane to execute a pullout with a roll. It is known that this results in extraordinary stresses. The Cornell Aeronautical Laboratory is making a study of this problem for the Air Materiel Command.

Status: COMPLETED. Results were transmitted to contractor. Task 1101-53-1101/51-34 (see p. 26) is a study of simplified versions of the rolling pullout equations.

RAYDIST DATA ANALYSIS
Task 1101-53-1101/51-10

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Oct-Dec 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

EQUATIONS OF PRESSURE SYSTEMS
Task 1101-53-1101/51-15

Origin: Naval Ordnance Test Station (Inyokern) Authorized 1/15/51
Sponsor: Bureau of Ordnance, USN Terminated 6/30/51
Manager: F. S. Acton

Objective: To solve a set of four simultaneous equations consisting of three ordinary nonlinear differential equations and one algebraic equation.

Background: This problem arose in connection with research being performed at the Naval Ordnance Test Station.

Status: TERMINATED.

RANGE ERROR COMPUTATION
Task 1101-53-1101/51-16

Origin: Naval Air Missile Test Center, (Point Mugu) Authorized 2/15/51
Sponsor: Bureau of Ordnance, USN Completed 6/30/51
Manager: E. C. Yowell

Objective: Given the coordinates of a transmitter and of three receivers, to calculate the following quantities for each of 111 specified points. Let ℓ_0, m_0, and n_0 be the direction cosines from the transmitter to a point in space, and ℓ_i, m_i, and n_i (where $i = 1,2,3$) be the direction cosines from the receivers to the point. If the matrix (a_{ij}) is the inverse of the matrix

$$\begin{matrix} \ell_0 + \ell_1 & m_0 + m_1 & n_0 + n_1 \\ \ell_0 + \ell_2 & m_0 + m_2 & n_0 + n_2 \\ \ell_0 + \ell_3 & m_0 + m_3 & n_0 + n_3 \end{matrix} ,$$

the quantities desired are

$$\frac{\Delta x}{\Delta p} = (a_{11}^2 + a_{12}^2 + a_{13}^2)^{\frac{1}{2}}$$

$$\frac{\Delta y}{\Delta p} = (a_{21}^2 + a_{22}^2 + a_{23}^2)^{\frac{1}{2}}$$

$$\frac{\Delta z}{\Delta p} = (a_{31}^2 + a_{32}^2 + a_{33}^2)^{\frac{1}{2}} .$$

The computation is to be made for each of 4 sets of three receivers.

Status: COMPLETED. The results were transmitted to the contractor.

CALCULATION OF A MEASURE OF ASSOCIATION
Task 1101-53-1101/51-17

Origin: University of Southern California, Psychology Department
Sponsor: Office of Naval Research, USN
Manager: F. S. Acton
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. After the correlations were computed a factor analysis for 12 to 16 factors was requested for the 54 x 54 correlation matrix. Test computations on a 10 x 10 matrix were completed to check the method. The work on the 54 x 54 matrix is 30% completed.

COMPUTATIONS IN CONNECTION WITH PROGRAM ANALYSIS
Task 1101-53-1101/51-18

Origin: Operations Analysis Office
Sponsor: Office of Air Comptroller, USAF
Full project description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

STATISTICAL SMOOTHING
Task 1101-53-1101/51-19

Origin: Stanford Research Institute, Stanford University
Sponsor: Office of Research Operations, U. S. Army
Manager: M. Howard
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Eighty-nine cases have been completed.

ROCKET GRAIN BURNING
Task 1101-53-1101/51-21

Origin: Naval Ordnance Test Station (Inyokern), USN
Sponsor: Bureau of Ordnance, USN
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Preliminary studies of possible steady-state solutions have been made and reported to the sponsor. A conference was held with sponsor, and the planning of the computations is now in progress.

EQUATIONS OF COMBUSTION
Task 1101-53-1101/51-22

Origin: Naval Ordnance Test Station (Inyokern), USN
Sponsor: Bureau of Ordnance, USN
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. The differential system was solved for three sets of parameters and is now being solved for a last set. This work is being performed on the differntial analyzer at the University of California at Los Angeles, Engineering Department.

COMPUTATIONS RELATING TO HARBOR DEVELOPMENT STUDY
Task 1101-53-1101/51-23

Origin: Hydraulic Structures Laboratory, California Institute of Technology
Sponsor: Bureau of Yards and Docks, USN
Manager: E. C. Yowell

Authorized 2/28/51
Completed 6/30/51

Objective: To evaluate the expressions

$$T(s,u) = \frac{2\pi}{c} \sum_{m} \frac{1}{N_m} \sin^2 \gamma_m \left[Se(s,u)\right]^2 \qquad (1)$$

$$I(s,u,\phi) = \frac{4\pi}{c} \sum_{m,n} \frac{1}{N_m N_n} \sin \gamma_m \sin_n \cos(\gamma_n - \gamma_m)\, V_{m,n}\,, \qquad (2)$$

where

$$V_{m,n} = Se_m(s,u)\, Se_n(s,u)\, Se_m(s,\phi)\, Se_n(s,\phi).$$

$Se_m(s,x)$ are the even periodic solutions (period π or 2π) of Mathieu's differential equation $y'' + (b - s\cos^2 x)y = 0$ normalized so that $Se_m(s,0) = 1$.

N_n and N_m are normalization factors. The symbols γ_m and γ_n are angles which can be expressed in terms of the tabulated "joining factors" $f_{e,m}(s)$. (See NBS "Tables relating to Mathieu Functions," now in press, Columbia University Press, New York, N. Y.) In the above

$$c = \sqrt{s} = \frac{\pi\nu}{L}.$$

The expressions are to be evaluated for:

$$c = \frac{\pi}{2},\ \pi,\ 2\pi,\ 3\pi$$

$$u = 0^\circ(15^\circ)90^\circ$$

$$\phi = 0^\circ(5^\circ)180^\circ.$$

Background: If waves of certain height pass through a gap in a breakwater, a disturbance is set up on the sheltered side of the breakwater. In equations (1) and (2), T represents the total energy passing through the gap and I presents the intensity of wave energy at different angles ϕ

behind the breakwater. The angle u is the angle of approach of the waves, D is the gap width, and L the wave length of the approaching waves.

Comments: The derivation of equations (1) and (2) is given in a paper, "The diffraction of waves by ribbons and slits," by Philip M. Morse and Pearl J. Rubenstein, Phys. Rev. 54, p. 895-898, (Dec. 1, 1938).

Status: COMPLETED. The results were transmitted to the contractor.

COMPUTATION IN CONNECTION WITH A STUDY OF POLARIZATION OF LIGHT
Task 1101-53-1101/51-25

Origin: Department of Meteorology, U.C.L.A.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Work requested by the sponsor has been fully completed. Other supplementary computations may be called for later.

During the past quarter the sponsor widened the field of investigation to include two additional ψ-functions for the same parameters τ and μ. In addition a set of tables relating to molecular scattering, defined by equations (183) to (186), Chapter X of Radiative Transfer, by S. Chandrasekhar (1950), Oxford, Clarendon Press, has been completed. The computed results are being studied by the sponsor. The final report will be written after the sponsor decides whether any further computations are desired.

SOLUTION OF SETS OF ALGEBRAIC EQUATIONS
Task 1101-53-1101/51-26

Origin: North American Aviation — Authorized 3/30/51
Sponsor: Atomic Energy Commission — Completed 6/30/51
Managers: G. Blanch, M. Howard

Objective: To compute the coefficients and solve seven sets of six simultaneous linear algebraic equations in six unknowns.

Background: These equations arose in the course of research being performed by North American Aviation for the Atomic Energy Commission.

Status: COMPLETED. The results have been transmitted to the contractor.

CONVERSION OF HEXIDECIMAL NUMBERS
Task 1101-53-1101/51-28

Origin: Naval Air Missile Test Center (Point Mugu) — Authorized 3/30/51
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Jan-Mar 1951 issue.

Status: INACTIVE.

AIRPLANE WINDSHIELD DEICING AND DEFOGGING
Task 1101-53-1101/51-29

Origin: North American Aviation, Inc.
Sponsor: Office Air Research, AMC. USAF
Manager: D. S. Saxon

Authorized 6/22/51
Terminated 6/30/51

Objective: To calculate the temperature distribution in a slab as a function of time when the temperature of the surrounding medium changes linearly in time. In particular, this was to be applied to the temperature distribution through the windshield of an airplane during a dive. A wide range of the parameters involved was requested.

Background: Basic theoretical work was completed and supplied by North American in Technical Report AFTR No. 6118. This work was checked and was found to be correct.

Comments: Analysis of the problem indicated that the theoretical formulation could only be regarded as a semi-quantitative description of the actual physical situation of the diving airplane. Hence it was recommended that calculations be made for just a limited number of parameter values including those for a specific case of interest.

Status: TERMINATED (NEW). This task was terminated without any computational work being performed. The contractor decided to do the computations on his own analog equipment.

COMPUTATIONS IN CONNECTION WITH LATTICE ARRANGEMENTS
Task 1101-53-1101/51-30

Origin: North American Aviation, Inc.
Sponsor: Atomic Energy Commission
Manager; E. C. Yowell

Authorized 6/22/51
Completed 6/30/51

Objective To substitute certain specified parameters in various given algebraic equations.

Background: Equations were developed by the client in the course of research for the Atomic Energy Commission.

Status: COMPLETED. The results were transmitted to the contractor.

TRAINING PROGRAM FOR AIR FORCE CADETS
Task 1101-53-1101/51-31

Origin: Office of Air Research, AMC, USAF and
National Bureau of Standards
Sponsor: Office of Air Research, AMC, USAF
Manager: Captain G. R. Johnston

Authorized 3/15/51
Completed 6/30/51

Objective: To train USAF airmen in computing methods, the logical design of high-speed digital computers, and in coding and programing techniques for the SWAC, IBM, and hand machines.

Background: This training program was put into effect in order that airmen could be used as trainees to alleviate the shortage of trained personnel in the computation field.

Comments: Eight airmen were selected by the OAR for this program. The program began on March 19 and is to continue for 89 days.

Status: COMPLETED.

TABLES OF THE BIVARIATE NORMAL DISTRIBUTION FUNCTION
Task 1101-53-1101/51-32

Origin: Division 13, NBS Authorized 5/31/51
Sponsor: Office of Chief of Ordnance, U. S. Army
Manager: G. Blanch

Objective: To compile a table of the bivariate normal distribution function, defined by

$$L(h,k,r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^\infty \int_k^\infty \exp\left[\frac{-(x^2 + y^2 - 2rxy)}{2(1 - r^2)}\right] dx\, dy \quad .$$

Background: The function L(h,k,r) has been tabulated and published in Karl Pearson's Tables for Statisticians and Biometricians, part II, (1931) p. 78-137, Biometric Laboratory, University College, London, for the following range of parameters:

$$\pm\, r = 0(.05)1 \; ; \;\; h,\, k = 0(.1)\, 2.6 \; .$$

This table has been extended by Dr. Evelyn Fix to cover a range of h and k up to 4, for the same range of r. This extension is unpublished, but available in typewritten form at the University of California Statistical Laboratory, Berkeley. The major portion of the compilation will comprise these two basic tables. They will be unified and perhaps augmented by suitable auxiliary tables to increase their usefulness. The tables will be submitted for publication in the NBS Applied Mathematics Series. Also the values will be keypunched, so as to be useful in conjunction with IBM computations.

Status: NEW. A conference was held with E. Fix, J. Neyman, and D. H. Lehmer to discuss the handling of the tables. Dr. Fix left her tables with the INA to be rounded to six decimals on IBM equipment. The keypunching of the entries in Pearson's tables is in progress.

PRESSURE FIELDS OF POTENTIAL FLOW PAST A BODY OF REVOLUTION
Task 1101-53-1101/51-33

Origin: Naval Ordnance Test Station (Pasadena) Authorized 6/22/51
Sponsor: Bureau of Ordnance, USN
Manager: R. R. Reynolds

Objective: To compute the potential, velocity, and pressure on the surface of a body of revolution.

Background: The Naval Ordnance Test Station needs these results in a research project to determine bodies of minimum resistance.

Comments: In Task 1101-53-1101/50-13 (see July-Sept 1950 issue, p.30) an approximate procedure was used to determine a body for which a given velocity distribution was valid. Some of the results of that task are now being utilized to obtain the pressure distribution.

Status: NEW.

SIMPLIFIED ROLLING PULLOUT EQUATIONS
Task 1101-53-1101/51-34

Origin: Cornell Aeronautical Laboratory Authorized 6/22/51
Sponsor: Office of Air Research, AMC, USAF
Manager: E. C. Yowell

Objective: To solve nine simultaneous differential equations which describe in a simplified way the motion of an airplane in a rolling pullout.

Background: This task is related to task 1101-53-1101/51-6. Under combat conditions it is often important for a plane to execute a pullout with a roll. It is known that this results in extraordinary stresses. The Cornell Aeronautical Laboratory is making a study of this problem for the Office of Air Research. This problem is to test whether a simplified model is accurate enough to be used for prediction purposes rather than an exact model (see task 1101-53-1101/51-6, p.19).

Status: NEW. Integrations are 20% completed.

FREQUENCY RESPONSE STUDY
Task 1101-53-1101/51-35

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 6/22/51
Sponsor: Bureau of Aeronautics, USN
Manager: E. C. Yowell

Objective: 1. To prepare a table of

$$(\sigma + i\,\omega)^n = x_n + iy_n \quad \text{for the range}$$

$$\sigma = 0(-1)-10(-2)-20; \ \omega = 0(1)20(10)400 \ ; \ n = 0(1)10 \ .$$

2. For certain given values of k_{ij}, to form the sums

$$K_i = \sum_{j=1}^{7} k_{ij} x_j$$

$$K_i' = \sum_{j=1}^{7} k_{ij}' y_j$$

3. To solve the set of equations

$$K_1 a + K_2 b = K_3$$

$$K_1' a + K_2' b = K_3'$$

Parts 2 and 3 are computed for the same range of σ and ω as part 1.

Background: The problem arises in connection with the work of the originator.

Status: NEW. The table of $(\sigma + i\omega)^n$ is completed. The total computations are 60% completed.

LOW MOMENTS OF ORDER STATISTICS
Task 1101-53-1101/51-36

Origin: University of Oregon — Authorized 6/22/51
Sponsor: Office of Naval Research, USN
Manager: L. H. Miller

Objective: To find the first, second, and cross moments of the order statistics, associated with the normal distribution, for samples of size 6 to 20 inclusive.

Background: This is a continuation of project 11.1/32-50-5, whose full project description appears in the Oct-Dec 1949 issue, p. 23.

Status: NEW. A new method for checking the consistency of computed results indicates no significant errors in the first or second moments. It shows that, for samples of size greater than 10, specific cross moments are incorrect. The method has been used to smooth the values where errors seem to exist. The smoothed data are being subjected to additional checks to determine whether they are satisfactory or whether major recomputation is necessary.

THREE NON-LINEAR DIFFERENTIAL EQUATIONS
Task 1101-53-1101/51-37

Origin: Hughes Aircraft Company — Authorized 6/22/51
Sponsor: Office of Air Research
Manager: R. R. Reynolds

Objective: To solve the system

$$\frac{R^2}{\lambda}\ddot{X} + U_X\dot{X} + V_X X = W_X, \quad \frac{R^2}{\lambda}\ddot{Y} + U_Y\dot{Y} + V_Y Y = W_Y, \quad \frac{R^2}{\lambda}\ddot{Z} + U_Z\dot{Z} + V_Z Z = W_Z$$

where

$$U_X = (\eta - Y)\dot{Y} + (\zeta - Z)\dot{Z}, \quad U_Y = (\zeta = Z)\dot{Z} + (\xi - X)\dot{X}, \quad U_Z = (\xi - X)\dot{X} + (\eta - Y)\dot{Y},$$

$$V_X = (\dot{\eta} - \dot{Y})\dot{Y} + (\dot{\zeta} - \dot{Z})\dot{Z}, \quad V_Y = (\dot{\zeta} - \dot{Z})\dot{Z} + (\dot{\xi} - \dot{X})\dot{X}, \quad V_Z = (\dot{\xi} - \dot{X})\dot{X} + (\dot{\eta} - \dot{Y})\dot{Y},$$

$$W_X = U_X\dot{\xi} + V_X\,\xi\,, \quad W_Y = U_Y\dot{\eta} + V_Y\,\eta\,, \quad W_Z = U_Z\dot{\zeta} + V_Z\,\zeta\,,$$

$$\xi = b\,\omega\,(t_o - t) + a\sin\omega\,(t_o - t), \ \eta = \sqrt{2}\,a\cos\omega\,(t_o - t)\ ,$$

$$\zeta = b\,\omega\,(t_o - t) - a\sin\omega\,(t_o - t)\ ,$$

where a, b, ω, λ, and t_o are certain given constants and subject to given initial conditions.

Comments: This is similar to task 1101-53-1101/51-27, (see Jan-Mar 1951 issue, p. 26) where ξ , η , ζ were simpler functions.

Status: NEW.

PROBABILITY DISTRIBUTION OF KOLMOGOROV STATISTIC
Task 1101-53-1101/51-38

Origin: University of Washington — Authorized 6/22/51
Sponsor: Office of Air Research, USAF
Manager; Roselyn Lipkis

Objective: To compute

$$P(n,c) = \frac{n!e^n}{n^n} R_{o,n}(c)\ ,$$

where

$$R_{o,o}(c) = 1$$

$$R_{i,o}(c) = 0,\ i \neq 0$$

$$R_{i,k}(c) = 0,\ |i| \geq c$$

$$R_{i,k+1}(c) = e^{-1} \sum_{s=0}^{2c-1} R_{i+1-s,k}\ \frac{1}{s!}\ ,\ |i| \leq c - 1$$

for n = 1(1)100 and c = 1(1)15.

Background: From this table, non-parametric confidence intervals for cumulative distribution may be established for sample size n, by inverse interpolation on c/n for various probabilities P(n,c). The computation was proposed by Professor Z. W. Birnbaum of the University of Washington, who derived the estimate of the truncation error in the recursion formula above.

Status: NEW. The table has been completed for c = 1(1)13, and it may be sufficient. Values for c = 14 and 15 will be computed later if necessary. These computations were done on the SWAC.

REDUCTION OF HYDROGRAPHIC DATA
Task 1101-53-1101/51-39

Origin: Scripps Oceanographic Institute — Authorized 6/22/51
Sponsor: University of California
Manager: E. C. Yowell

Objective: Given sets of observations of salinity S, temperature T, and pressure P, of samples of sea-water:

1. Compute Cl, the chlorinity, by the formula

$$Cl = \frac{S-0.030}{1.8050}$$

2. Compute

$$\sigma_{S,0,0} = -0.069 + 1.4708\, Cl - 0.001570\, Cl^2 + .0000398\, Cl^3 .$$

3. Compute the auxiliary quantities A_T, B_T, Σ_T where

$$A_T = 10^{-3}\, T(4.7867 - 0.098185T + .0010843T^2)$$

$$B_T = 10^{-6}\, T(18.030 - 0.8164T + 0.01667T^2)$$

$$\Sigma_T = -\frac{(T - 3.98)^2\, (T + 283)}{503.570\, (T + 67.26)}$$

4. Compute $\sigma_{S,T,0}$ by the formula

$$\sigma_{S,T,0} = \Sigma_T + (\sigma_{S,0,0} + 0.1324)\, [1 - A_T + B_T(\sigma_{S,0,0} - 0.1324)] .$$

5. Compute $\alpha_{S,T,P}$ by the formula

$$\alpha_{S,T,P} = \alpha_{S,T,0} - 10^{-9}\, P\, \alpha_{S,T,0} \left\{ \frac{4886}{1 + 0.0000183P} - [227 + 28.33T - 0.551T^2 + 0.004T^3] \right.$$

$$+ 10^{-4} P \left[105.5 + 9.50T - 0.158T^2\right] - 10^{-8} (1.5P^2T)$$

$$- \frac{\sigma_{S,0,0} - 28}{10} \left[147.3 - 2.72T + 0.04T^2 - 10^{-4}P(32.4-0.87T + 0.002T^2)\right]$$

$$+ \left(\frac{\sigma_{S,0,0} - 28}{10}\right)^2 \left[4.5 + 0.1T - 10^{-4}P(1.8-0.06T)\right] \Big\}$$

The quantities σ and α are related by the equations $\alpha = \frac{1}{\rho} \sigma = 10^3(\rho -1)$.

6. Compute the anomaly in specific volume

$$\delta = \alpha_{S,T,P} - \alpha_{35,0,P}$$

7. Compute the anomaly in geopotential distance between isobaric surfaces P_1 and P_2

$$\Delta D = \int_{P_1}^{P_2} \delta \, DP \ .$$

Background: The data consists of measurements of temperature T (in degrees centigrade), and salinity S(in parts per mille), of a sample of water taken at a depth corresponding to a sea-pressure P(in decibars). The density ρ, or the specific volume is a function of all three parameters. It is desired to determine the specific volume for all points of observation by the method given by V. Bjerknes and J. W. Sandstrom, Dynamic Meteorology and Hydrography, part 1, Carnegie Institute of Washington, Pub. No. 88, 1910.

Comments: These data are collected by Scripps Oceanographic Institute in connection with a study of Pacific currents.

Status: NEW.

AIRCRAFT STABILITY EQUATIONS
Task 1101-53-1101/51-41

Origin: Naval Air Missile Test Center (Point Mugu) Authorized 6/29/51
Sponsor: Bureau of Aeronautics, USN Completed 6/30/51
Manager: R. R. Reynolds

Objective: The equations

$$m(\dot{v}-wp + ur) = mg \sin \int_0^t pdt + Y_\beta \frac{v}{u} \ \ Y_r r + D \frac{v}{u}$$

$$m(\dot{w}-uq + vp) = mg \cos \int_0^t pdt - L_\alpha \frac{w}{u} - L_q q \ \ D \frac{w}{u}$$

$$I_{xx}\dot{p} + (I_{zz}-I_{yy})rq = L_r r + L_\beta \frac{v}{u} \ \ L_{\delta_\alpha} \delta_\alpha$$

$$I_{yy}\dot{q} + (I_{xx}-I_{zz})rp = M_q q + M_{\dot{\alpha}} \frac{\dot{w}}{u} + M_\alpha \frac{w}{u} + M_{\delta_\varepsilon} \delta_\varepsilon$$

$$I_{zz}\dot{r} + (I_{yy}-I_{xx})pq = N_r r + N_p p + N_\beta \frac{v}{u} + N_{\delta_\alpha} \delta_\alpha$$

are to be solved for the variables v(t), p(t), q(t), r(t), subject to the initial conditions $v = w = p = q = r = 0$ at $t = 0$. The other quantities appearing are constants, except $M_{\delta_\varepsilon}\delta_\varepsilon = F(t)$.

Background: This problem arose in the investigation of three-dimensional aircraft stability at the Naval Air Missile Test Center.

Status: COMPLETED. (NEW). This problem was submitted for solution on the differential analyzer at the University of California at Los Angeles.

EQUATIONS OF FLIGHT
Task 1101-53-1101/51-42

Origin: Naval Air Missile Test Center (Point Mugu) — Authorized 6/29/51
Sponsor: Bureau of Aeronautics, USN — Completed 6/30/51
Manager: R. R. Reynolds

Objective: To solve the system

$$\ddot{x} = \frac{g}{w} (T - .0682 C_D \rho V^2) \cos \theta$$

$$\ddot{y} = \frac{g}{w} (T - .0682 C_D \rho V^2) \sin \theta - g$$

$$\tan \theta = \frac{\dot{y}}{\dot{x}}, \quad V = (\dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$$

for x(y), $M(t) = V/a$, x(t) with 11 sets of initial conditions for x, y, $\dot{x}$, $\dot{y}$, θ. The functions w(t), $C_D(M)$, $\rho(y)$, a(y), $g(x^2 + y^2)$ are given graphically.

Background: Solutions are required for theoretical investigations being carried on by the contractor.

Comments: This problem is a continuation of task 1101-53-1101/51-13 (see Jan-Mar 1951 issue, p. 19).

Status: COMPLETED (NEW). This problem was submitted for solution on the differential analyzer at the University of California at Los Angeles. Results were transmitted to the contractor.

II. Computation Laboratory

(Section 11.2)

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Task 1102-21-1104/50-1
(formerly 11.2/11-50-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. H. E. Salzer studied Lucas' method for the determination of real and complex zeros of polynomials. A preliminary report on this has been prepared. He also planned a table to assist in the numerical determination of the Laplace transform of a function given at equal intervals.

J. Todd indicated how the Laguerre approximate quadrature could be used conveniently to tabulate the functions $E_1(z)$ when $|z|$ is large. (See task 1102-21-1104/43-3, p. 34)(Also for a comparison see "Table-making for large arguments. The exponential integral," by L. Fox and J.C.P. Miller which will appear in Mathematical Tables and Other Aids to Computation V (July 1951)).

Publications: (1) "Formulas for calculating the error function of a complex variable," by H. E. Salzer; MTAC V, 67-70 (Apr. 1951); reprints available. (2) "Formulas for finding the argument for which a function has a given derivative," by H. E. Salzer; submitted to a technical journal.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Task 1102-21-1104/50-2
(formerly 11.2/11-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The inversion of fifty 10 x 10 matrices is in process.

RESEARCH IN MODERN NUMERICAL ANALYSIS: CONDITION OF MATRICES
Task 1102-21-1104/50-3
(formerly 11.2/11-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Task 1102-21-1104/50-4
(formerly 11.2/11-50-4)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. O. Taussky-Todd continued work on the characteristic roots of sums of matrices and carried out research on a problem of M. Kac. A manuscript on characteristic roots of quaternion matrices has also been completed.

Publications: (1) "Classes of matrices and quadratic fields," by O. Taussky-Todd; published in the Pac. J. of Math. 1, 127-132 (Mar. 1951); reprints available. (2) "Classes of matrices and quadratic fields, II," by O. Taussky-Todd; accepted by the Journal of the London Mathematical Society. (3) "Arnold Scholz," by O. Taussky-Todd; accepted for publication in the Crelle Journal fur Reine und Angewandte Mathematik. (4) "Tables of the functions $\int_0^\varphi \sin^{1/3}x\,dx$ and $(4/3)\sin^{-4/3}\varphi \int_0^\varphi \sin^{1/3}x\,dx$," by M. Abramowitz; accepted by the NBS Journal of Research.

NUMBER-THEORETICAL TEST PROBLEMS FOR SEAC
Task 1102-21-1104/50-5a
(formerly 11.2/11-50-5)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. A new routine for the table of least primitive roots was written which incorporates the following improvements: (a) squares are not used as trial divisors, (b) an indication is included as to whether or not 10 is a primitive root and (c) neater printings in groups of five are included. The table was started again and has been used as a test of the electrostatic memory.

(2) The table of Wilson quotients was completed extending through all primes less than 10,000.

Publication: (1) "The third Wilson prime and an extended table of Wilson quotients," by K. Goldberg; submitted to a technical journal.

SOLUTION OF LAPLACE EQUATION BY MONTE CARLO METHOD
Task 1102-21-1104/51-6

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. A detailed program of tests for the random numbers in use has been worked out in consultation with the Statistical Engineering Laboratory. A report on the results obtained will be issued as a working paper. In addition, further experiments in three and four dimensions have been carried out. An experiment in 16 dimensions is in progress. Results will be described in forthcoming working papers.

ANALYSIS OF SAMPLING PLANS
Task 1102-21-1104/51-8

Origin: NBS
Full project description appears in Oct-Dec 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

HEAT CONVECTION IN LAMINAR FLOW THROUGH A TUBE
Task 1102-21-1104/51-9

Origin: NBS
Full project description appears in Jan-Mar 1951 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

2. Applied Research: Tables and Experimental Computations

TABLES OF $E_1(z)$, $(z = x + iy)$
Task 1102-21-1104/43-3
(formerly 11.2/2-43-3)

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The subtabulation was completed on the SEAC. Preparation of the manuscript is in progress. (See task 1102-21-1104/50-1, p. 32).

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Task 1102-21-1104/46-1
(formerly 11.2/2-46-1)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Values of the real and imaginary parts of the $\log_e \Gamma(z)$, where $z = x + iy$ for $x = 0(.1)10$ and $y = 0(.1)10$, have been checked. A useful by-product is a skeleton table of $\Gamma(z)$ itself.

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-21-1104/47-2
(formerly 11.2/2-47-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The final manuscript has been prepared for Volume II.

Publication: (1) "Tables of Coulomb Wave Functions," vol. I, NBS Applied Mathematics Series 17; now in press.

TABLE OF ANTILOGARITHMS
Task 1102-21-1104/47-3
(formerly 11.2/2-47-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of the manuscript by differencing was completed. This table will be published by the Columbia University Press.

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Task 1102-21-1104/48-2
(formerly 11.2/2-48-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for the six point formulas have been checked.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES

Task 1102-21-1104/49-2
(formerly 11.2/2-49-2)

Origin: NBS
Manager: H. E. Salzer

Authorized 3/14/49
Completed 6/30/51

Objective: To tabulate representations of numbers approximately between $1 + 10^{-20}$ and 10 in the form $\pi_i p_i$ where p_i are prime numbers, e_i are positive or negative integers, and the product is taken over the first seven primes (i = 1,2,...,7).

Background: For many years radix tables have been devised; they were tables of $\log(1 \pm 10^{-m})$, where n and m are integral, to aid in the calculation of logarithms to more than 20 places. Uhler recently produced such a table for calculating logarithms to about 130 places.

The present table in conjunction with values of the logarithms of seven basic primes will facilitate the computation of logarithms and antilogarithms to about 330 significant figures. The logarithms of the primes from 2 to 17 are known with the required accuracy, except for log 13 which will be calculated as part of this project.

This project was proposed by Dr. J. Barkley Rosser of Cornell University, who evolved the method of the present computation.

Status: COMPLETED.

FERMI FUNCTION, II

Task 1102-21-1104/49-10
(formerly 11.2/33-49-10)

Origin: NBS, Section 4.4
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as "Table for the analysis of β -spectra," NBS Applied Mathematics Series 13.

TABLES TO FACILITATE SEQUENTIAL t-TESTS

Task 1102-21-1104/50-2a
(formerly 11.2/2-50-2)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: Irene Stegun

Authorized 9/1/49
Completed 6/30/51

Objective: To publish a table of the solution $z(L,n,\delta)$ of the equation:

$$L = \log F\left[\frac{n}{2}, \frac{1}{2}; \frac{\delta^2 z}{2}\right] - \frac{n\delta^2}{2}$$

for n = 1(1)200; δ = .1(.1)1(.2)2, 2.5; ± L = 2(1)7, ln 19, ln 99.

Background: These tables were originally computed by the Mathematical Tables Project for the Statistical Research Group at Columbia University during the war for use in connection with sequential testing. They had not been published before.

Status: COMPLETED.

Publication: (1) "Tables to facilitate sequential t-tests," NBS Applied Mathematics Series 7; available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 45 cents.

TABLE OF CHEBYSHEV POLYNOMIALS
Task 1102-21-1104/50-3a
(formerly 11.2/2-50-3)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press. The checking of the galley proofs is proceeding.

Publication: The table is being printed by the Government Printing Office and will be issued as NBS Applied Mathematics Series 9.

PROBABILITY TABLES FOR EXTREME VALUES
Task 1102-21-1104/50-4a
(formerly 11.2/2-50-4)

Origin: NBS, Section 11.3
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The final manuscript was completed.

Publication: (1) "Probability tables for analysis of extreme-value data"; to appear in the NBS Applied Mathematics Series.

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Task 1102-21-1104/50-5
(formerly 11.2/2-50-5)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Objective: To prepare, and keep up to date in readily available form, information regarding a) mathematical tables, b) important publications in numerical analysis. The reference file is to indicate not only what publications exist, but also where they are available in this area. Also, c) to maintain a file of errata in mathematical tables, d) to maintain a reprint file on mathematical tables and numerical analysis, e) to maintain a file of reviews, comments, and errata of the publications of the Computation Laboratory.

Status: CONTINUED

TABLE OF n! and $\Gamma(n + \frac{1}{2})$
Task 1102-21-1104/50-6a
(formerly 11.2/2-50-6)

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Manager: H. E. Salzer

Authorized 5/21/50
Completed 6/30/51

Objective: To prepare for publication an existing manuscript table of n! and $\Gamma(n + \frac{1}{2})$.n! is given to 16 places and $\Gamma(n + \frac{1}{2})$ to 8. The range n = 1(1)1000 is covered.

Background: This is a fundamental table for the use of statisticians, mathematicians, physicists, and engineers, as well as for computers. It will be used, e.g., for the preparation of basic input data for automatic computers.

Status: COMPLETED.

Publication: (1) "Tables of n! and $\Gamma(n + \frac{1}{2})$ for the first thousand values of n," by H. E. Salzer; NBS Applied Mathematics Series 16; available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 15 cents.

WAVE FUNCTION FOR LITHIUM
Task 1102-21-1104/50-7
(formerly 11.2/2-50-7)

Origin: NBS
Full project description appears in Apr-Jun 1950 issue.

Status: INACTIVE. For status to date see Oct-Dec 1950 issue.

CRYSTAL STRUCTURE PROBLEM FOR POINT ATOMS
Task 1102-21-1104/51-2

Origin: Naval Research Laboratory, USN
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The computations were performed as requested.

ANALYSIS OF CRYSTAL STRUCTURE
Task 1102-21-1104/51-3

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Complete sets of interplanar spacings have been calculated for powder patterns of about 20 substances. This is being done for a project of Division 9, Section 7 of the NBS on identification of crystalline materials for powder patterns. Structure factors have been calculated for calcium chloride hexahydrate, and about 20 short sets have been calculated for models of the magnesium hydroxide structure.

COLLECTED SHORT MATHEMATICAL TABLES OF THE COMPUTATION LABORATORY
Task 1102-21-1104/51-4

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Volume I is in preparation.

TABLE OF THE BESSEL FUNCTION $Y_n(x)$
Task 1102-21-1104/51-10

Origin: NBS
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. The punched cards are now being checked.

RESISTIVITY OF A NOTCHED STRIP
Task 1102-21-1104/51-12

Origin: NBS
Manager: H. E. Salzer

Authorized 5/31/51
Completed 6/30/51

Objective: To calculate x and y satisfying the relation

$$Z = x + jy = \frac{c}{\pi} \cosh^{-1} \left[\frac{2e^{\pi v}\cos\pi u - (a + 1)}{a - 1} + j \frac{2e^{\pi v}\sin\pi u}{a - 1} \right]$$

$$- \frac{g}{\pi} \cosh^{-1} \left[\frac{-2ae^{-\pi v}\cos\pi u + (a + 1)}{a - 1} + j \frac{2ae^{-\pi v}\sin\pi u}{a - 1} \right] + j (g - c)$$

where $j = \sqrt{-1}$ and $a = c^2/g^2$ for the following values of v,u,g, and c:

$g = 1; \quad c = 1.5, 2.0, 2.5; \quad u = 0(0.25)1.0;$

$v = -1, -0.5, 0(0.25)1.0, 1.5, 2.0.$

Background: This problem arose in connection with measurements of the dielectric constant of heterogeneous dielectrics, in which certain unexpected results were obtained. The results of the present computation may be applicable to the resistivity of a sheet in which holes have been punched. This task was proposed by Dr. Harvey Curtis, NBS 30.0.

Status: COMPLETED (NEW). The results were turned over to Dr. Curtis.

3. Mathematical Services

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Task 1102-53-1106/49-2
(formerly 11.2/33-49-2)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were performed as requested.

LINEAR PROGRAMMING ON STANDARD PUNCHED CARD MACHINES
Task 1102-53-1106/49-3
(formerly 11.2/36-49-3)

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Calculations on programming models were carried out as requested. Twelve models were completed; six more are in progress.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/49-13
(formerly 11.2/33-49-13)

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were performed as requested.

GUST ATTACKS ON DELTA WING
Task 1102-53-1106/50-1
(formerly 11.2/31-50-1)

Origin: Aircraft Laboratory, AMC, USAF — Authorized 9/15/49
Sponsor: Office of Air Research, AMC, USAF — Completed 6/30/51
Manager: Irene Stegun

Objective: To evaluate by numerical computation the basic unteady variations in lift caused by a sharp-edged vertical wind gust ttacking a portion of a delta wing in supersonic flight.

Status: COMPLETED. The results were turned over to the originators.

STANDARD LORAN TABLES
Task 1102-53-1106/50-1a (formerly 11.2/34-50-1): Gulf Coast Chain
Task 1102-53-1106/50-5 (formerly 11.2/34-50-5): Hawaiian Islands Chain
Task 1102-53-1106/51-5: Marshall Islands Chain
Task 1102-53-1106/51-18: Marianna Islands Chain
Task 1102-53-1106/51-30: North Atlantic Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The calculations were completed for the Marianna Islands Chain, and the manuscript was submitted to the U. S. Hydrographic Office. Calculations are in progress for the North Atlantic Chain. Work has been completed for the Gulf Coast Chain, Hawaiian Islands Chain, and Marshall Islands chain.

RESEARCH IN LINEAR PROGRAMMING
Task 1102-53-1106/50-2
(formerly 11.2/12-50-1)

Origin: Air Comptroller's Office USAF
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The study of sampling methods for inverting matrices and determining eigenvalues was continued. Experiments in the inversion of a 16x16 Leontieff matrix are being carried out on the SEAC. Assistance is being given to the Air Comptroller's Office in the preparation of a report on the Symposium on Linear Inequalities and Programming jointly sponsored by the Department of the Air Force and the National Bureau of Standards.

Publication: (1) A revised version has been prepared of the bibliography for the paper, "Bounds for characteristic roots of matrices, II," by O. Taussky-Todd; the paper was published in the NBS J. Res. 46, 124-125 (Feb. 1951), RP2184. (Available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 5 ¢).

A PROBLEM IN MOLECULAR STRUCTURE, I
Task 1102-53-1106/50-3
(formerly 11.2/33-50-3)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations for various values of the parameters were performed as requested.

ADAPTATION OF LORAN CALCULATIONS TO CARD PROGRAMMED CALCULATOR AND NBS AUTOMATIC COMPUTER
Task 1102-53-1106/50-3a
(formerly 11.2/34-50-3)

Origin: Hydrographic Office, USN
Sponsor: " "
Full project description appears in Apr-Jun 1950 issue.

Status: CONTINUED. Programming is now in progress.

WAVE RESISTANCE OF SHIPS, III
Task 1102-53-1106/50-11
(formerly 11.2/33-50-11)

Origin: David Taylor Model Basin, USN
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Computations were performed as requested.

MOLECULAR STRUCTURE CALCULATIONS, II
Task 1102-53-1106/50-16
(formerly 11.2/33-50-16)

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations were performed as requested.

ANALYSIS OF UNDERWATER SOUND MEASUREMENTS
Task 1102-53-1106/51-6

Origin: Underwater Sound Laboratory, USN
Sponsor: " "
Manager: B. Dove

Authorized 9/28/50
Completed 6/30/51

Objective: To make a time series study of radar and periscope bearing errors.

Background: This data was obtained from bearing measurements on a moving target by radar-type devices and periscopes. A time series study is necessary to establish standards on the expected accuracy of bearing measurements.

Status: COMPLETED.

PROGRAM COMPUTATION ON THE SEAC
Task 1102-53-1106/51-7

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Computations on triangular programming models were carried out as requested, using the new system of codes which takes advantage of the magnetic tape mechanisms. Five models were computed on a monthly basis yielding results which are more accurate than and three times as detailed as the previous quarterly calculations. Further improvements in the codes were made which included (1) provision in the computation for losses in storage, (2) additional flexibility in the application of a coefficient to an activity level, (3) reduction of magnetic tape movement, (4) checking designed for the magnetic tapes, and (5) provisions for resuming computation after a break. Coding for the recently developed punched card reader was accomplished also. All data for future models will be read from punched cards rather than from teletype tape.

FLOW IN SUPERSONIC NOZZLES
Task 1102-53-1106/51-13

Origin: Naval Ordnance Laboratory
Sponsor: "
Manager: M. M. Andrew
Full project description appears in Oct-Dec 1950 issue.

Status: CONTINUED. Code checking was completed on the SEAC, and test runs were made. Further analysis of the problem seems necessary.

CORPORATE INCOME TAX
Task 1102-53-1106/51-14

Origin: U. S. Treasury Department
Sponsor: "
Full project description appears in Oct-Dec 1950 issue.

Status: CONTINUED. Work on this task has been resumed, and computations were performed as requested.

INTERNAL CONVERSION COEFFICIENTS FOR L-SHELL
Task 1102-53-1106/51-19

Origin: Atomic Energy Commission, Oak Ridge National Laboratories
Sponsor: " " "
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Programming of the problem is largely completed. The over-all code involves the use of various specialized subroutines the coding of which is partially completed.

X-RAY PENETRATION
Task 1102-53-1106/51-20

Origin: Atomic Energy Commission, New York Office
Sponsor: " "
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. It was decided to solve this problem for three kinds of sources, namely, plane monodirectional, point isotropic, and angular. Because of storage limitations of the SEAC, it is necessary to divide the computation into two parts, using high and low source energies separately.

As some cases can be combined, only four different codes are needed. Three of these are completed and the fourth is in the process of being checked. As for the actual computation, the high energy part is nearly completed for plane and point isotropic sources and just started for angular sources. The low energy part has to wait for the last code.

LIQUID-VAPOR TRANSITION
Task 1102-53-1106/51-22

Origin: Naval Medical Research Institute
Sponsor: "
Full project description appears in Jan-Mar 1951 issue.

Status: CONTINUED. Integral equations were solved for three values of the parameters. The IBM procedure was prepared, and calculations for eight values of the parameters are in process.

SPECTRUM STUDY
Task 1102-53-1106/51-28

Origin: Weather Bureau — Authorized 3/30/51
Sponsor: "
Manager: R. Brooks

Objective: To obtain a large number of serial products and perform auto-correlations on weather data.

Background: Meteorologists have long felt that the atmosphere showed signs of "pulsating." The present project has been originated by the U. S. Weather Bureau to determine possible evidence of periodicity in atmospheric pressure data.

Status: NEW. The necessary procedures were prepared, and the autocorrelation calculations are in process.

SYNCHROTRON-OSCILLATION RESONANCE
Task 1102-53-1106/51-29

Origin: Office of Naval Research, USN   Authorized 6/28/51
Sponsor: " "
Manager: Ida Rhodes

Objective: To integrate the equation

$$\theta'' + \frac{4\pi}{\sqrt{3}} (1 + \alpha N)(\tfrac{1}{2} - \sin \theta) = - \beta \sin 2\pi N$$

for α = .000711, .0001778,

β = .00474, .0237, .0474, .0711, .0948

over the range N = -100(1/64)100 to obtain the asymptotic behavior of the solution.

Background: This problem is connected with the design of a proton synchrotron. It is desired to determine the effect of the resonance between the magnet-power-supply-ripple harmonics and the nonlinear, time-varying phase oscillations of the protons. This resonance may seriously attenuate the proton beam or may turn out to be benign. A lengthy integration of the differential equation of this oscillation is necessary for which the SEAC is appropriate. The problem was proposed by Dr. N. Blackman, ONR.

Status: NEW.

INTEGRALS OF LAGUERRE POLYNOMIALS
Task 1102-53-1106/51-31

Origin: Director of Intelligence, Air Targets Division, USAF   Authorized 6/28/51
Sponsor: " "
Manager: J. A. Hershberger

Objective: To tabulate the Laguerre polynomials $L_n(x)$ and their integrals for n = 2(1)6 and x = 0(.001)5 and make additional calculations based on the results.

Background: These polynomials are used in the analysis of experimental data and the synthesis of empirical formulas. In particular, they are employed in considerations involving area problems.

Status: COMPLETED (NEW). Results were turned over to the originators.

WAVE RESISTANCE, IV
Task 1102-53-1106/51-33

Origin: David Taylor Model Basin, USN   Authorized 6/28/51
Sponsor: " "
Manager: Irene A. Stegun

Objective: To evaluate the integral

$$R = 2C_0 \int_{w_0}^{\infty} \frac{w \cosh^2 \left[w(1-f/h)\right] \left\{ M_1 \left[\frac{ac}{hF_h} \sqrt{w \tanh w} \right] \right\}^2}{\cosh^2 w \sqrt{w^2 - \frac{w \tanh w}{F_h^2}}} dw$$

for various values of F_h, f/h, a/h. Here

$$M_1(j) = \int_0^1 \xi \sin j\xi \, d\xi ,$$

w_0 is a root of the equation

$$w_0 = \frac{1}{F_h^2} \tanh w_0 ,$$

and C_0 is a constant.

Background: This integral arises in the computation of the wave resistance of a spheroid on shallow water.

Status: NEW. The original request was completed. Additional parameters have been added.

SHOCK WAVE PARAMETERS
Task 1102-53-1106/51-38

Origin: Bureau of Ordnance, USN   Authorized 6/28/51
Sponsor: " "
Manager: Irene A. Stegun

Objective: Determinatior of detonation pressure, temperature, and velocity for various explosives, making use of tables prepared in task 1102-53-1106/49-13.

Background: See task 1102-53-1106/49-13, p. 40.

Status: NEW. Exploratory computations are underway.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Task 0302-51-2606/49-5
(formerly 11.2/33-49-5)

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were performed as requested.

BASIC IONOSPHERIC DATA
Task 1401-34-1412/49-14
(formerly 11.2/33-49-14)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were performed as requested.

RADIO-TELEGRAPH INTERFERENCE
Task 1404-34-1423/49-17
(formerly 11.2/33-49-17)

Origin: NBS, Section 14.4
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were performed as requested.

RATING OF WATER CURRENT METERS
Task 0605-41-0621/50-2
(formerly 11.2/33-50-2)

Origin: NBS, Section 6.5
Sponsor: "
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

IONOSPHERIC WINDS
Task 1401-11-1401/50-7
(formerly 11.2/33-50-7)

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Data were processed as requested.

RAY TRACING
Task 0202-21-2308/50-13
(formerly 11.2/33-50-13)

Origin: NBS, Section 2.2
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations were performed as requested.

NUMERICAL SOLUTION OF A SET OF DIFFERENTIAL EQUATIONS CHARACTERIZING A DEPOLYMERIZING SYSTEM
Task 0700-12-0700/51-2

Origin: NBS, 7.0
Sponsor: "
Full project description appears in July-Sept 1950 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

DETERMINATION OF THE CHARACTERISTICS OF A DAMPER
Task 0604-31-3518/51-6

Origin: NBS, 6.4
Sponsor: "
Manager: Ida Rhodes

Authorized 6/28/51
Completed 6/30/51

Objective: To vary the five parameters controlling the theoretical effectiveness of the damping system over a wide range and, from the results, to determine the optimum values of the parameters.

Background: Preliminary work had indicated the feasibility of adding a spring-supported mass to the 160 cps, 10 g Ramberg vacuum tube accelerometer in order to damp out the natural frequency vibration of the tube. This mechanical system was investigated theoretically to determine, for design purposes, what the relative size of the added spring-mass system should be and what vibration characteristics it should have for optimum performance.

Status: COMPLETED (NEW). Results were turned over to the originators.

COMPUTATION OF SINGLE AND DOUBLE WALL SOUND TRANSMISSION INTEGRALS
Task 0601-31-3527/51-7

Origin: NBS   Authorized 6/25/51
Sponsor: "
Manager: N. Levine

Objective: To compute the integral

$$\tau = 2 \int_0^1 \frac{v\,dv}{A^2}$$

where

$$A^2 = 1 + 4\left[R(R+1)+p^2v^2\right] + 4\sin^2 bv\left\{\left[R(R+1)+p^2v^2\right]^2 - p^2v^2\right\}$$

$$- 4pv \sin 2bv\left[R(R+1)+p^2v^2\right],$$

R is a damping parameter, b is a parameter which depends on the ratio of an airspace thickness to the wave length of sound, and p is a parameter depending on the mass reactance of the wall and the stiffness reactance resulting from the existence of flexural waves.

Background: This integral represents the ratio of transmitted to incident energy in the case of a random sound field striking a wall consisting of two partitions separated by an airspace. The variable v is the cosine of the angle of incidence.

Comments: This problem was formulated by the Sound Section, National Bureau of Standards. The occurrence of flexural waves in the wall is taken into account.

Status: NEW. The preliminary coding was finished and the code checking was started on the SEAC.

TABLES OF POWER POINTS OF ANALYSIS-OF-VARIANCE TESTS
Task 1304-34-6351/51-8

Origin: Section 11.3, NBS   Authorized 3/26/51
Sponsor: "
Manager: M. Abramowitz

Objective: To prepare tables of the power points of the analysis-of-variance tests of linear hypothesis regarding means of normal populations; specifically, to prepare tables of the inverse function $\phi(f_1, f_2, \alpha, \beta)$ defined by

$$\sum_{m=} \frac{\lambda^m e^{-\lambda}}{m!} \cdot \frac{B_x(\frac{1}{2} f_1 + m, \frac{1}{2} f_2)}{B(\frac{1}{2} f_1 + m, \frac{1}{2} f_2)} = \beta$$

where

$$\lambda = \lambda(f_1, f_2, \alpha, \beta) = \tfrac{1}{2}(f_1 + 1)\, \phi^2(f_1, f_2, \alpha, \beta).$$

The equation

$$x = x(f_1, f_2, \alpha)$$

is the upper α-probability level of the incomplete beta-function ratio defined by

$$\frac{1}{B(\frac{1}{2}f_1, \frac{1}{2}f_2)} \int_0^{x(f_1,f_2,\alpha)} t^{\frac{1}{2}f_1-1}(1-t)^{\frac{1}{2}f_2-1}dt = \frac{B_x(\frac{1}{2}f_1, \frac{1}{2}f_2)}{B(\frac{1}{2}f_1, \frac{1}{2}f_2)} = 1 - \alpha;$$

to 3D for

$$f_1 = 1(1)10, 12, 15, 20, 24, 30, 40, 60, 120, \infty$$

$$f_2 = 1(1)10, 12, 15, 20, 24, 30, 40, 60, 80, 120, 240, \infty$$

and all combinations of $\underline{\alpha}$ and $\underline{\beta}$ derivable from

$$\alpha = \underline{0.001}, 0.005, \underline{0.01}, \underline{0.02} (0.01) \underline{0.05}, 0.10, \underline{0.20}, 0.50$$

$$\beta = 0.001, \underline{0.01}, \underline{0.05}, \underline{0.10}, 0.20, 0.30, \underline{0.50}, 0.80, \underline{0.90}$$

that satisfy the condition

$$\beta < 1 - \alpha.$$

(The underscoring above indicates the most urgent tables; see Comments).

Background: These tables are needed to make possible rational and efficient planning of experiments that aim to determine whether there exist any nonzero differences among a set of true (i.e., 'population') means. If $x_1, x_2, \ldots, x_k$ are independently and normally distributed about $\xi_1, \xi_2, \ldots, \xi_{k2}$, with variances $\sigma^2/w_1, \sigma^2/w_2, \ldots, \sigma^2/w_k$, respectively, and s^2 is an estimator of σ^2 distributed as $\chi^2\sigma^2/f_2$ independently of the x's, then the probability is β that the analysis-of-variance test of the hypothesis H_0 that $\xi_1 = \xi_2 = \ldots = \xi_k$, when conducted at the α-level of significance, will fail to detect the falsity of H_0 when the differences among the ξ's are such that

$$\sum_{i=1}^{k} \frac{w_i(\xi_1 - \xi_0)^2}{\sigma^2} = (f_1+1)\, \phi^2 (f_1, f_2, \alpha, \beta) ,$$

with

$$\xi_0 = \sum_{i=1}^{k} w_i \xi_i / \sum_{i=1}^{k} w_i$$

and

$$f_1 = k - 1 .$$

In other words, when the probability of rejecting H_0 when it is true (an "error of the 1st kind") is set at $\underline{\alpha}$; the probability of accepting H_0 when it is false (an "error of the 2nd kind") is β for all alternatives to H_0 corresponding to $\phi(f_1, f_2, \alpha, \beta,)$. Thus, the test has 'power' $\underline{1 - \beta}$, i.e., probability $\underline{1 - \beta}$ of detecting the falsity of H_0, relative to the alternatives to H_0 corresponding to $\phi(f_1, f_2, \alpha, \beta)$, which is, therefore, a $(1 - \beta)$-power point of the test.

The preparation of such tables was originally urged by Professor Jerzy Neyman, University of California, and Professor Abraham Wald, Columbia University, in a joint proposal submitted to The Computation Laboratory in the fall of 1946.

Comments: These tables constitute an extension of those given by Emma Lehmer in the December 1944 issue of the Annals of Mathematical Statistics, her "β" corresponding to our "$1-\beta$". In our notation, Mrs. Lehmer's four tables give the values of $\phi(f_1, f_2, \alpha, \beta)$ to 3D, or 2D, for the four (α, β)-combinations derivable from

$$\alpha = 0.01,\ 0.05$$

$$\beta = 0.20,\ 0.30\ ,$$

for f_1 as specified above, and $f_2 = 2(2)20$ and thence as specified above.

The present project calls for the preparation of a total of 84 Lehmer-type individual tables, and covers a somewhat broader area in the (α, β)-plane than the 125 tables originally proposed. Of these 84 tables, the 15 corresponding to underscored values of α and β are urgently needed. The remaining 69 are less urgent but their preparation is essential to a reasonably complete tabulation of "the power points of analysis-of-variance tests of linear hypotheses regarding means of normal populations."

The present formulation of these tables is the end product of Project 47D2-1 (See Jan-Mar 1948 issue, p. 23) and completes that project.

Status: NEW. The IBM procedure is being prepared.

PRECISE DETERMINATION OF THE PARAMETER OF DISPERSION EQUATION FOR SEVERAL TYPES OF OPTICAL GLASS
Task 0202-12-0202/51-10

Origin: NBS, 1.6 — Authorized 6/25/51
Sponsor: " — Completed 6/30/51
Manager: I. A. Stegun

Objective: (1) Evaluation of the constants a,b,c and ℓ of the equation

$$n^2 = a + b\lambda^2 + \frac{c}{\lambda^2 - \ell^2}$$

by the method of least squares from given values of n and λ.

(2) Computation of n and the residuals, (n_s-n formula), for the specified values of n and λ.

(3) Computation of n by the formula for various values of the wavelength λ.

Background: The results will enable one to express more accurately the refractivities of glass for the use in optical design throughout a longer interval of the spectrum than is now used in optical glass catalogues.

Status: COMPLETED (NEW). Results were turned over to the originator.

CONDUCTION OF HEAT INWARD FROM A PLANE SURFACE EXPOSED TO HIGH INTENSITY RADIATION
Task 1002-11-4720/51-14

Origin: NBS 10.0 Authorized 6/28/51
Sponsor: "
Manager: M. Abramowitz

Objective: To solve the integral equation

$$\theta = \frac{\sqrt{\alpha}}{k\sqrt{\pi}} \qquad \exp\left(\frac{-x^2}{4\alpha(t-\tau)}\right) (t-\tau)^{-\frac{1}{2}} q(\theta)d\tau$$

where $q(\theta)$ is a given function of θ, for various values of the parameters.

Background: This problem arises in the study of temperature distributions within a solid material bounded by a plane surface which is exposed to high intensity radiation. The problem was proposed by Dr. W. F. Roeser, 10.0.

Status: NEW. Exploratory computations are in progress.

III. Statistical Engineering Laboratory

(Section 11.3)

1. Fundamental Research in Mathematical Statistics

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Task 1103-11-1107/48-3
(formerly 11.3/2-48-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date, see Apr-Jun 1950 issue.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-11-1107/49-1a
(formerly 11.3/2-49-1)

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Work on the preparation of abstract cards was continued, and the backlog of unprocessed abstracts was reduced. Conferences were held with Mr. Hilsenrath regarding the TIP-bibliography and with Mr. Terry regarding the cards of the McBee Company. It appears desirable to have some mechanical means of finding all the cards which pertain to a desired subject-matter class. Members of the staff discussed various possible methods of organizing the files. The abstracts drawn from one volume of Mathematical Reviews are currently being classified. It is expected that this will yield information about multiple classifications. This information has to be obtained before the final decision is made on the manner of recording the subject classification.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Task 1103-11-1107/49-3
(formerly 11.3/1-49-3)

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The typing of the final draft of the manuscript of Professor Mann's monograph on the theory of stochastic processes was completed. A copy was sent to Professor Mann for checking prior to the manuscript's submission to the Editorial Committee. Simultaneously, a copy is being read at the Bureau. Arrangements have been made for the preparation of an index.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Task 1103-11-1107/49-3a
(formerly 11.3/2-49-3)

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The manuscript "A guide to the tables of the normal probability integral," was accepted for publication as a volume in the NBS Applied Mathematics Series.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Task 1103-11-1107/50-1
(formerly 11.3/1-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. To aid in the write-up of results previously obtained in the present study, the systematic recording of known properties of, and historical notes on, the sech, sech^2, Laplace (double exponential), Cauchy, and rectangular distributions was begun. Asymptotic distributions of the mean, median, mean deviation, and standard deviation in samples from the generalized double-exponential,

$$f(x) = \frac{p}{2\delta\Gamma(\frac{1}{p})} e^{-\left|\frac{x-\lambda}{\delta}\right|^p}, \quad (p > 0),$$

were studied also, since for $1 \leq p \leq 2$ this distribution seems to "interpolate" between the Laplace and the normal distribution.

EXTREME-VALUE THEORY AND APPLICATIONS
Task 1103-11-1107/50-1a
(formerly 11.3/2-50-1)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The dittoed draft of the monograph by E. J. Gumbel entitled "Lectures on the theory of extreme values and their practical applications" was reviewed by several readers. The manuscript is being revised to incorporate the minor changes suggested.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-11-1107/50-2
(formerly 11.3/2-50-2)

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The first draft of a write-up, which is to be incorporated in the manual on "Analysis of straight line data," was completed for the case of regression where one variable is accurately known. Other chapters to be included in the manual were outlined in detail.

TABLE TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-11-1107/51-1

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The preparation of the introduction to the table was begun.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-11-1107/51-2

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. (1) E. Lukacs and O. Szász completed the final draft of a joint paper entitled "Some non-negative trigonometric polynomials connected with a problem in probability."

(2) Some progress was made on translation of the second paper by A. Renyi published in the Hungarian Journal "Matematikai Lapok." This journal is publishing a series of articles "Thirty Years of mathematics in the Societ Union" (in Hungarian). The second paper discusses new lines of research in probability theory.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3000-21-0002/51-1

Origin: NBS
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. Activity under this project fell into two main categories:

A. The Design of Experiments: Typical examples were: (1) The construction of a design for the location and evaluation of possible sources of constant error and for the estimation of the precision of calibration of a 24-sided polygon; (2) The design of an experiment to determine the magnitude of variability among small units from a large aggregate of cement stored in a silo with a view to developing optimum sampling schemes.

B. The Development or Selection of the Appropriate Methods for Analysis and Interpretation of Data. For example, the methods of probit analysis were used to estimate the level of noise considered objectionable by 50% of television viewers from data giving the percentage voting "objectionable" at different levels of noise. A standard error for the estimate can be computed when this method is used.

A manual on sampling inspection by variables, modeled on MIL-STD-105A (by attributes), was prepared and submitted to the Ordnance Development Division.

RESEARCH ON APPLICATION OF THEORY OF EXTREME VALUES TO GUST LOAD PROBLEMS
Task 1103-21-5106/51-1

Origin: NACA, Dynamic Loads Division
Full project description appears in July-Sept 1950 issue.

Status: CONTINUED. The study of statistical properties of estimators of extreme-value parameters was continued. Special emphasis was placed on estimators, alternative to those of Gumbel and Kimball, which, for large samples, achieve computational simplicity without too great a sacrifice of accuracy and precision.

A first draft of a report to the National Advisory Committee for Aeronautics embodying the results of the above work was prepared. It was found that, when the parameter $\beta = 1/\alpha$ is known, one, two, or three order statistics can be determined from a sample in such a way that their mean, when corrected for bias, will estimate the parameter u with considerably greater efficiency than is obtainable using certain methods suggested by Gumbel and by Kimball. Also, the efficiency of a Gumbel estimator of β, which requires calculation of the sample standard deviation, can probably be improved considerably by use of certain quasi-ranges involving two or four of the sample values, regardless of whether u is known or not.

In addition, a progress report summarizing work under the project for the period July 1, 1950, to March 31, 1951, together with a proposal for further research were prepared and submitted to the NACA.

Publications: (1) "Maximum likelihood estimates of position derived from measurements performed by hyperbolic instruments," by E. Lukacs; accepted by the NBS Journal of Research. (2) "On the stochastic independence of symmetric and homogeneous linear and quadratic statistics," by E. Lukacs; submitted to a technical journal for publication.

IV. Machine Development Laboratory

(Section 11.4)

in cooperation with

Electronic Computer Section

(Section 12.3)

1. Development: Design and Construction of Automatic Digital Computing Machines

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Task 1104-34-5107/47-1
(formerly 11.4/21-47-1)

Origin: The Bureau of the Census
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. During the past quarter, the first UNIVAC system has been in partial operation for the Bureau of the Census. A formal dedication was held on June 14 at which time the installation was officially transferred to the custody of the Bureau of the Census. The NBS continues to represent this Bureau through the contract for maintenance service by Eckert-Mauchly (subsidiary Remington Rand, Inc.) for the period that the UNIVAC is to remain in the manufacturer's plant in Philadelphia. The machine is now yielding useful output on small area Census tabulations. The full output expected from the UNIVAC system has however not yet been achieved because of intermittent operation of the card-to-tape converter. The contractor is continuing to make alterations to improve the converter and thus far has not submitted the second converter for approval.

THE NAVY COMPUTING MACHINE
Task 1104-34-5107/47-2
(formerly 11.4/22-47-2)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Development work at the Raytheon Manufacturing Company on the computer has continued. Detailed planning for the ONR-NBS machine will follow the completion of the Special Devices Center machine. A firm delivery date cannot yet be set.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Task 1104-34-5107/47-3
(formerly 11.4/24-47-3)

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The UNIVAC System No. 2 is well along in assembly and test. Some procurement difficulties have delayed the installation of the remaining memory elements. Sufficient testing has been done to indicate that the machine should be ready for trial run as soon as the missing components are available. However, it does not appear likely that the delivery date of July 15, 1951, can be met. It is possible that late August will see this equipment ready for acceptance tests.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Task 1104-34-5107/49-1
(formerly 11.4/24-49-1)

Note: This computer has been previously referred to as the NBS Interim Computer and as the NBS Automatic Computer I.

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Williams' tube memory has been operated successfully with 256 spots for several eight-hour sessions on the supersonic nozzle problem. Engineering tests show that the system can be adjusted so that the maximum machine-call rate of one word every 48 microseconds does not interfere with stored information. The Williams' memory has also been used successfully with 512 spots on a problem in dynamics (involving two degrees of freedom) which concerns the construction of sensitive apparatus related to airplane safety. It is now being tried out (with 512 spots) in conjunction with the acoustic memory on a problem requiring the full capacity of both memories. Additional difficulties were encountered with the magnetic input, but routine operation was achieved during the quarter. A punched-card-to-magnetic-tape converter has been built and is being tested on the SCOOP problem. It is not yet in routine operation. The Automonitor has been completed and installed, and a corresponding change has been made in the control console panel. This permits the machine to print out all the steps of a computation <u>automatically</u> as an aid to checking the correctness of new codes.

Routine solution of problems has continued yielding good operation for 71% of the scheduled problem time (see task 1104-53-1108/47-4, p. 60).

WRIGHT DEVELOPMENT CENTER COMPUTING MACHINE
(formerly AIR MATERIEL COMMAND COMPUTING MACHINE)
Task 1104-34-5107/49-1a
(formerly 11.4/23-49-1)

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The Arithmetic Unit has been completed and has undergone static operating tests by the contractor, the General Electric Co. The remaining switching and control circuitry is about 80% completed. Most of the rack wiring has been completed, and the assembly into cabinets is underway. The machine is designed to operate at 156 kilocycles per second for the main repetition rate. The Memory Unit consisting of magnetic drums has been designed and is being machined. Memory circuits are in the final stages of development. The chances of completion on schedule appear fairly good.

ARMY MAP SERVICE COMPUTING MACHINE
Task 1104-34-5107/49-1b
(formerly 11.4/25-49-1)

Origin: Army Map Service, U.S.A.
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The final assembly of UNIVAC No. 3 is coming along quite satisfactorily, and the machine should be ready for some form of testing in late fall.

2. General Services

PROGRAMMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Task 1104-53-1108/47-4
5108
(formerly 11.4/3-47-4)

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A large variety of problems have been coded and run successfully on the SEAC. A tabular summary of the problems worked on during the past three months follows:

Problems Coded and Run on SEAC-April 1-June 30, 1951

Name	Time Spent on Coding Analyses, Consultation, Preparation of Tape, Checking, etc.	Machine Time 1/
Two classified tasks	702	45
Linear Programming-OAC (see task 1102-53-1106/50-2)	1581	292
Linear Inequalities-OAC	186	37
Aircraft Deployment and Calculation-OAC	179	7
Transportation Problem-OAC	16	5
Inversion of Matrices by Monte Carlo Method-OAC	108	6
Minimization Task-OAC	82	-
Computations in Fluid Dynamics-NOL	189	7
X-Ray Penetration Problem	315	53
Sound Transmission Problem	280	3
Internal Coefficients for L-Shell	487	14
Number Theoretical Test Problem (see task 1102-21-1104/50-5a)	26	185
Integration of a System of Differential Equations (Westinghouse)	-	8
Analysis of Crystal Structure (Ordway)(see task 1102-21-1104/51-3)	31	34
Tables of $E_1(z)$(See task 1102-21-1104/43-3)	-	35
Loran task	28	-
Particle Trajectory Problem	42	3
Solution of Laplace Equations by Monte Carlo Method	212	23
Coulomb Wave Function-OAR	10	-
Dirichlet Problem (Solution of Integral Equation by Iteration)	39	4
Long Range Proving Ground Task	2/	18
Social Security Agency Task (see task 1104-53-5108/51-1)	2/	1
Dynamics Problem	2/	1
Meteorology Problem	2/	1
ONR Problem	2/	2
Spring Noise Problem	2/	31
	4513 hours	815 hours

1/ Machine time = good operation time + code checking + down time.
2/ Coded by personnel outside the SEAC unit.

CODING RELATED TO THE UNIVAC SYSTEM
Task 1104-53-5108/49-2
(formerly 11.4/3-49-2)

Origin: The Bureau of the Census
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1951 issue.

CODING RELATED TO THE RAYTHEON COMPUTER
Task 1104-53-5108/49-3
(formerly 11.4/3-49-3)

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

INVESTIGATION OF THE APPLICABILITY OF AUTOMATIC DIGITAL ELECTRONIC COMPUTING TO PROBLEMS OF THE SOCIAL SECURITY AGENCY
Task 1104-53-5108/51-1

Origin: Social Security Agency
Sponsor: "
Full project description appears in Oct-Dec 1950 issue.

Status: CONTINUED. A complete survey of the work in the Statistical Branch of the Program Analysis Division was made; in addition, a class was held of ten representatives of all the branches of this division for the purpose of coding instruction. Subroutines were coded for a typical statistical analysis problem which is to be run on the SEAC and UNIVAC; these subroutines will be run on the SEAC as soon as pertinent IBM cards are available. A survey was made of the work of the Division of Claims which supervises the adjudication and disbursement of payments to the almost 4 million beneficiaries through the six area offices in the United States.

THE MTAC SECTION
Task 1104-51-1109/47-1
(formerly 11.4/4-47-1)

Origin: Committee on High-Speed Computing of the National Research Council
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for inclusion in the October 1951 issue of Mathematical Tables and Other Aids to Computation was assembled, edited, and forwarded to the editor. In addition the galley and page proofs for the July 1951 issue were edited.

Symposium on Linear Inequalities and Programming

Sponsored by Department of the Air Force and
National Bureau of Standards, Washington, D.C.,
June 14, 15, 16, 1951.

Thursday, June 14 - 9.30 A.M. Linear Inequalities. G. B. DANTZIG, Chairman,
- Welcoming talk. E. U. CONDON.
- Theorems of alternatives for pairs of matrices. A. W. TUCKER
- Remarks on the history of work on linear inequalities. T.S.MOTZKIN
- Optimization in unitary spaces. E. W. BARANKIN
- Polyhedral cones. A. CHARNES
- Geometrical significance of the simplex method. J.P. MAYBERRY
- Discussion.

Thursday, June 14 - 2.00 P.M. Applications. T. C. KOOPMANS, Chairman.
- Choice models and non-choice models. W. W. LEONTIEF
- Research program of Project SCOOP. M. K. WOOD.

Friday, June 15 - 9.30 A.M. Computational Theory and Techniques. J.TODD, Chairman.
- Welcoming talk. GEN. F. J. DAU
- New techniques for linear inequalities and optimization. T.S.MOTZKIN
- Methods of solving linear equations. G. FORSYTHE
- Gradient methods in Lagrangian problems and their game theoretical interpretation. L. HURWICZ
- Properties of Leontief matrices. Y. K. WONG
- Discussion.

Friday, June 15 - 2.00 P.M. Problems of Formulation. P.A.SAMUELSON,Chairman.
- Efficiency prices for decentralized decisions. T. C. KOOPMANS and G. DEBREU.
- Suitability scales for allocation problems. J. L. HOLLEY.
- Personnel classification. D. F. VOTAW, JR. and A. ORDEN.
- Design of an optimal battery of tests. M. A. WOODBURY.

Saturday, June 16 - 9.30 A. M. Non-linear and Linear Problems. E.D.SCHELL, Chairman.
- Theory of the transportation problem. M. M. FLOOD
- Convex programming. D. W. BLACKETT
- Least ballast shipping required to meet a specified shipping program. I. HELLER
- Problem of contract awards. L. GOLDSTEIN
- A gasoline blending problem. W. COOPER
- Discussion

Saturday, June 16 - 2.00 P.M. Computational Theory and Techniques. F.L. ALT, Chairman.
- Matrix inversion. H. H. GOLDSTINE
- A unified technique for matrix inversion, linear inequalities, games, optimization. A. ORDEN
- A short proof of the dual theorem. G. B. DANTZIG
- The projective method and computational experience. C. B. TOMPKINS
- Discussion.

Applied Mathematics Seminars

DIAZ, J. P. (University of Maryland). Upper and lower numerical bounds for quadratic integrals. April 13, 1951.

TRENT, H. M. (Naval Research Laboratory, USN). The impact of analog methods on modern research. April 27, 1951.

ORDEN, A. (Air Comptroller's Office, USAF). Duality in linear programming.

Numerical Analysis Colloquium Series
(Los Angeles, California)

VAN DANTZIG, D.(University of California, Berkeley and Mathematisch Centrum, Amsterdam, Holland). Some recent results in non-parametric methods. April 2, 1951.

LANCZOS, C. Studies in equidistant interpolation. April 30, 1951.

SHERMAN, S. (Lockheed Aircraft Corporation). Games of attrition and games of reconnaissance. May 14, 1951.

MOTZKIN, T. S. (NBS and UCLA). Minimax approximation. May 28, 1951.

GERJUOY, E. (University of Southern California). (1) The conservation laws in classical and quantum mechanics. June 4, 1951. (2) The quantization of classical non-realistic systems. June 11, 1951.

MILNE, W. E. (NBS and Oregon State College). Difference operators associated with the Laplacian differential operator. June 18,1951.

Papers and Invited Talks Presented by Members of the Staff at Outside Organizations

ALT, F. L. High speed computing machines and their application to physics. Presented at the weekly seminar of the John Hopkins University Applied Physics Laboratory, Silver Spring, Maryland, April 20, 1951; also presented to the Physics Department Seminar, Howard University, Washington, D. C., May 28, 1951.

CAMERON, J. M. Discussion: Symposium on Bulk Sampling. Presented at 54th Annual Meeting of the American Society for Testing Materials, Atlantic City, N. J., June 18, 1951.

CURTISS, J. H. Monte Carlo methods and the calculation of eigenvalues. Presented at the meeting of the American Physical Society, Washington, D. C., April 27, 1951.

EISENHART, C. (1) The postulate of direct measurement. Presented to the Physics Colloquium, Wesleyan University, Middletown, Conn., April 23, 1951. (2) On gage precision and product variability--statistical aspects. Presented to the Southern Connecticut Section of the American Society for Quality Control, Bridgeport, Conn., May 9, 1951.

GREEN, J. W. On an expansion method for parabolic partial differential equations. Presented to the American Mathematical Society Meeting, Pullman, Wash., June 16, 1951.

LANCZOS, C. Investigation of the radiation of a cylindrical antenna. Presented to the Southwestern IRE Conference, Dallas, Tex., April 20, 1951.

LIEBLEIN, J. On certain estimators for the parameters of the distribution of the largest value. Presented by title at the 6th West Coast meeting of the Institute of Mathematical Statistics, Santa Monica, Calif., June 16, 1951.

LUKACS, E. Certain stochastic processes and the estimation of their parameters. Lecture presented to technical personnel of the U.S. Naval Ordnance Test Station, Pasadena Annex, Calif., April 2, 1951.

LUXENBERG, H. Automatic computing machines. Presented to the California Alpha Chapter of Pi Mu Epsilon, April 5, 1951.

MOTZKIN, T. S. Periodicity regions of sequences with a piecewise linear recursion formula. Presented to the American Mathematical Society, Palo Alto, Calif., April 28, 1951.

PAIGE, L. J. Simultaneous reduction of matrices to triangular form. Presented to the American Mathematical Society, Pullman, Wash., June 16, 1951.

YOUDEN, W. J. (1) Technique for locating the source of variability in a process. Presented before American Chemical Society Annual Meeting, Boston, Mass., April 4, 1951. (2) Interpretation of chemical data. Presented before 623rd Meeting of the Chemical Society of Washington, April 12, 1951. (3) Single-linked designs. Presented at the Virginia Academy of Sciences, Lynchburg, Va., May 11, 1951. (4) Interpretation of chemical data. Presented at 5th National Convention, American Society for Quality Control, Cleveland, O., May 23, 1951.

<u>Papers presented at the meeting of the American Mathematical Society, Stanford University, Stanford, California, April 28, 1951</u>

FORSYTHE, G. E. Generation and testing of 1,217,370 "random" digits on the SWAC.

FORSYTHE, G. E., and MOTZKIN, T. S. Acceleration of the optimum gradient method.

JOHN, F. Solution of parabolic equations by finite differences.

LANCZOS, C. Integration by weighted sums.

STEIN, M. L. Solutions of fixed end point problems in the calculus of variation.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. Available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 45 cents.

(2) Table of Arctangents of rational numbers. J. Todd. NBS Applied Mathematics Series 11. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $1.50.

(3) Tables of n! and $\Gamma(n+\frac{1}{2})$ for the first thousand values of n. H. E. Salzer. NBS Applied Mathematics Series 16. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 15 cents.

(4) Tables relating to the Mathieu functions. Available from Columbia University Press, New York 27, N. Y., $8.00.

1.3 Technical Papers

(1) Recurrent determinants of Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Duke Math. J. 18, No.1, 1-10 (March 1951). Reprints available.

(2) The use of components of variance in preparing schedules for the sampling of baled wools. J.M. Cameron. Biometrics 7, No. 1, 83-96 (March 1951). Reprints available.

(3) The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation. J. M. Cameron and W. J. Youden. A.S.T.M. Proceedings 50, 951-964 (1950). Reprints available.

(4) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R. H. Cameron. Duke Mathematical Journal 18, No. 1, 111-130 (March 1951). Reprints available from the author, University of Minnesota, Minneapolis 14, Minn.

(5) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J.H.Curtiss. Proceedings of the International Statistical Conferences, Washington, D. C., September 6-18, 1947; Vol. III, 25th Session of the International Statistical Institute, Part A, 351-357. Reprints available.

(6) Elimination of randomization in certain statistical decision procedures and zero-sum two-person games. A. Dvoretzky, A. Wald, and J. Wolfowitz. Ann. Math. Stat. 22, 1-21 (Mar. 1951). No reprints.

(7) Relations among certain ranges of vector measures. A. Dvoretzky. Pac. J. Math. 1, 59-74 (Mar. 1951). Reprints available.

(8) Generation and testing of random digits at the National Bureau of Standards, Los Angeles. G. E. Forsythe. Included in "Monte Carlo method": "Proceedings of a symposium held June 29, 30, July 1, 1949, in Los Angeles, California," NBS Applied Mathematics Series 12, available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 30 cents.

(9) Relaxation and step-by-step methods. L. Fox. MTAC V, 92-95 (Apr. 1951).

(10) A method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix. M. R. Hestenes and W. Karush. NBS J. Res. 47, 45-61 (July 1951), RP2227. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 20 cents.

(11) Certain Fourier transforms of distributions. E. Lukacs and O. Szász. Canadian J. Math. III, No. 2, 140-144 (1951). Reprints available.

(12) Two consequences of the transposition theorem on linear inequalities. T. S. Motzkin. Econometrica 19, No. 2, 184-185 (Apr. 1951).

(13) Proposals for the mechanical resolution of German syntax patterns. V. Oswald and S. L. Fletcher, jr. Modern Language Forum XXXVI, 1-24 (July 1951).

(14) Formulas for calculating the error function of a complex variabl H. E. Salzer. MTAC V, 67-70 (Apr. 1951). Reprints available.

(15) Identities and inequalities concerning orthogonal polynomials and Bessel functions. O. Szász. J. D'Analyse Mathématique (Jerusalem) I, 116-134 (1951). No reprints available.

(16) On a Tauberian theorem for Abel summability. O. Szasz. Pac. J. Math. 1, 117-125 (Mar. 1951). Reprints available.

(17) Classes of matrices and quadratic fields. (I). O. Taussky-Todd. Pac. J. Math. 1, 127-132 (Mar. 1951). Reprints available.

(18) On the mean duration of random walks. W. Wasow. NBS J. Res. 46, 462-472 (June 1951), RP2215. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D.C., 10 cents.

1.5 Miscellaneous Publications

(1) Problems for numerical analysis of the future. (Four papers presented at the Symposia on Numerical Analysis and Automatic Calculating Machinery, held at the NBS Institute for Numerical Analysis, Los Angeles, California, June 1948). Applied Mathematics Series 15. Available from Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 20 cents.

(2) Monte Carlo method; Proceedings of a symposium held on June 29, 30, July 1, 1949, in Los Angeles, Calif., under the sponsorship of the RAND Corporation, and the NBS, with the cooperation of the Oak Ridge National Laboratory. NBS Applied Mathematics Series 12

Available from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 30 cents.

(3) The Institute for Numerical Analysis of the National Bureau of Standards. J. H. Curtiss. Office of Naval Research Monthly Research Report, 8-17 (May 1951). Reprints available. (The article includes a bibliography.) Also an abbreviated version appeared in Amer. Math. Mo. 58, 372-379 (June-July 1951).

(4) Using mathematics for design. (How industry can make better use of mathematics.) An interview with J. H. Curtiss. Product Engineering 22, 156-159 (June 1951). No reprints.

(5) The role of a statistical consultant in a research organization. C. Eisenhart. Proceedings of the International Statistical Conferences, Washington 25, D. C., September 6-18, 1947, Vol.III, 25-th Session of the International Statistical Institute, Part A, 308-313. Reprints available.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1951.

2.1 Mathematical Tables

(1) Tables of the exponential function e^x. Formerly NBS Mathematical Table MT2. Third edition to be issued as NBS Applied Mathematics Series 14. In press, Government Printing Office.

(2) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(3) Table for the analysis of β spectra. NBS Applied Mathematics Series 13. In press, Government Printing Office.

(4) Tables of Coulomb wave functions, Vol. I. NBS Applied Mathematics Series 17. In press, Government Printing Office.

(5) Tables for rocket and comet orbits. S. Herrick. NBS Applied Mathematics Series. In press, Government Printing Office.

(6) Probability tables for analysis of extreme-value data. To appear in the NBS Applied Mathematics Series.

(7) Table of the integral $\int_0^x e^{-u^3} du$. M. Abramowitz. Accepted by the Journal of Mathematics and Physics.

(8) Tables of the functions $\sin^{1/3} x \, dx$ and $(4/3) \sin^{-4/3} \phi \quad \sin^{1/3} x dx$. M. Abramowitz. Accepted by the NBS Journal of Research.

(9) The third Wilson prime and an extended table of Wilson quotients. K. Goldberg. Submitted to a technical journal.

(10) Table of the zeros and weight factors of the first twenty Hermite polynomials. H. E. Salzer, R. Zucker and R. Capuano. Accepted for publication in the NBS Journal of Research.

2.2 Manuals, Bibliographies, Indices

(1) A guide to the tables of the normal probability integral. To appear in the NBS Applied Mathematics Series.

2.3 Technical Papers

(1) An analysis of the effect of the discontinuity in a bifurcated circular guide upon plane longitudinal waves. L. L. Bailin. Accepted for publication in the NBS Journal of Research.

(2) On subharmonic, harmonic, and linear functions of two variables. E. F. Beckenbach. To appear in Revista, Universidad Nacional de Tucuman (Argentina).

(3) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. To appear in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(4) Tables for constructing and computing the operating characteristics of single sampling plans. J. M. Cameron. Submitted to a technical journal.

(5) On the derivation and accuracy of certain formulas for sample sizes and operating characteristics of non-sequential sampling procedures. U. Chand. Accepted for publication in the NBS Journal of Research.

(6) A Monte Carlo method for solving a class of integral equations. R. E. Cutkosky. Accepted for publication in the NBS Journal of Research.

(7) Some problems on random walk in space. A. Dvoretzky and P. Erdös. To be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950.

(8) On a recursion formula and on some Tauberian theorems. N. G. de Bruijn and P. Erdős. Accepted by the NBS Journal of Research.

(9) Second order determinants of Legendre polynomials. G. E. Forsythe. Accepted by the Duke Mathematical Journal.

(10) Alternative derivations of Fox's escalator formulas for latent roots. G. E. Forsythe. Submitted to a technical journal.

(11) The extent of n random unit vectors. G. E. Forsythe and J. W. Tukey (Princeton University). Submitted to a technical journal.

(12) An extension of Gauss' transformation for improving the condition of systems of linear transformations. G. E. Forsythe and T. S. Motzkin. Submitted to a technical journal.

(13) On some functionals of Laplacian processes. R. Fortet. Accepted for publication in the NBS Journal of Research.

(14) On the estimation of an eigenvalue by an additive functional of a stochastic process with special reference to the Kac-Donsker method. R. Fortet. Accepted for publication in the NBS Journal of Research.

(15) Random determinants. R. Fortet. Accepted for publication in the NBS Journal of Research.

(16) Additive functionals of a Markoff process. R. Fortet. Submitted to a technical journal.

(17) Practical solution of linear equations and inversion of matrices. L. Fox. Accepted for publication in the NBS Journal of Research.

(18) Applications of the theory of quadratic forms in Hilbert space to the calculus of variations. M. R. Hestenes. Submitted to a technical journal.

(19) The solution of $Ax = \lambda Bx$. M. R. Hestenes and W. Karush. Accepted by the NBS Journal of Research.

(20) Semi-automatic instruction on the National Bureau of Standards Western Automatic Computer. H. D. Huskey. To appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory, September 1949.

(21) On some connections between probability theory and differential and integral equations. M. Kac. To be published in the Berkeley Second Symposium on Mathematical Statistics and Probability, held by the University of California, 1950.

(22) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Accepted by the Bulletin of the American Mathematical Society.

(23) Determination of the extreme values of the spectrum of a bounded self-adjoint operator. W. Karush. Submitted to a technical journal.

(24) An iterative method for finding characteristic vectors of a symmetric matrix. W. Karush. Accepted by the Pacific Journal of Mathematics.

(25) Properties of statistics involving the closest pair in a sample of three observations. J. Lieblein. Accepted by the NBS Journal of Research.

(26) A method of summing infinite series. S. Lubkin. Accepted by the NBS Journal of Research.

(27) Maximum likelihood estimates of position derived from measurements performed by hyperbolic instruments. E. Lukacs. Accepted by the NBS Journal of Research.

(28) On the stochastic independence of symmetric and homogeneous linear and quadratic statistics. E. Lukacs. Submitted to a technical journal.

(29) The torsion of anisotropic elastic cylinders by forces applied on the lateral surface. H. Luxenberg. Accepted for publication in the NBS Journal of Research.

(30) Numerical methods associated with Laplace's equation. W. E. Milne. To appear in the Proceedings of a Symposium held at Harvard Computation Laboratory, September 1949.

(31) Note on an infinite integral. A. M. Ostrowski. Accepted for publication in the Duke Mathematical Journal.

(32) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. Submitted to a technical journal.

(33) On a discontinuous analogue of Theodorsen's and Garrick's method. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(34) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A. M. Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(35) Uniformly best constant risk and minimax point estimates. R. T. Peterson, jr. Accepted for publication by the NBS Journal of Research.

(36) A method of computing exact inverses of matrices with integer coefficients. J. B. Rosser. Accepted by the NBS Journal of Research.

(37) The separation of close eigenvalues of a real symmetric matrix. J. B. Rosser, C. Lanczos, M. R. Hestenes, and W. Karush. Accepted by the NBS Journal of Research.

(38) Formulas for numerical differentiation in the complex plane. H. E. Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(39) An elementary note on powers of quaternions. H. E. Salzer. Submitted to a technical journal.

(40) Formulas for finding the argument for which a function has a given derivative. H. E. Salzer. Submitted to a technical journal.

(41) A bibliography of numerical methods in conformal mapping. W. Seidel. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(42) The hypergeometric and Legendre functions with applications to integral equations of potential theory. C. Snow. Revised version approved for publication in the NBS Applied Mathematics Series. (Formerly NBS MT15, out of print).

(43) Tauberian theorems for summability (R_1). O. Szász. Accepted for publication by the American Journal of Mathematics.

(44) On some trigonometric transforms. O. Szász. Accepted by the Pacific Journal of Mathematics.

(45) On the relative extrema of the Hermite orthogonal functions. O. Szász. Submitted to a technical journal.

(46) On the relative extrema of Bessel functions. O. Szász. Accepted for publication by the Bolletino della Unione Matematica Italiana (Firenze).

(47) On the Gibbs phenomenon for a class of linear transforms. O. Szász. Accepted by "Publications de l'Inst. Math. de l'Acad. Serbe des Sciences," vol. IV.

(48) The convergence of Cauchy-Riemann sums to Cauchy-Riemann integrals. O. Szász and J. Todd. Accepted by the NBS Journal of Research.

(49) Classes of matrices and quadratic fields, II. O. Taussky-Todd. Accepted for publication in the Journal of the London Mathematical Society.

(50) On conformal mapping of variable regions. S.E.Warschawski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published in the National Bureau of Standards Applied Mathematics Series.

(51) On the duration of random walks. W. Wasow. Submitted to a technical journal.

(52) The interpretation of chemical data. W. J. Youden. To appear in conference papers of the 5th National Convention of the American Society for Quality Control.

(53) Technique for locating sources of variability in a process. W. J. Youden. Submitted to a technical journal.

2.5 Miscellaneous Publications

(1) The construction and applications of conformal maps: Proceedings of a symposium held at the NBS Institute for Numerical Analysis, Los Angeles, California, June 1949. To appear in the NBS Applied Mathematics Series. In press, U. S. Government Printing Office.

(2) Annotated translation of a letter in which Gauss solved linear equations by relaxation. G. E. Forsythe. Submitted to a technical journal for publication.

(3) High-speed computing and accounting. H. D. Huskey and V. R. Huskey. To appear in the Journal of Accountancy.

Appendix

Numerical Integration of the Rolling Pullout Equations for an Airplane (Task 1101-53-1101/51-6)

During the course of the research program at Cornell Aeronautical Laboratories, it was desired to compute the theoretical dynamical loading on the tail of an airplane as that plane was performing a specified maneuver-- a rolling pullout. The theoretical problem was formulated as a set of eight simultaneous first order nonlinear differential equations and was submitted to the Institute for Numerical Analysis for solution.

The basic coordinate system is a rectangular three dimensional system fixed in the airplane. In level flight the positions of the axes and their directions as seen by the pilot are as follows: the z axis is along the direction of gravity, with positive end pointing downwards; the x-axis is in the direction of flight, with positive end pointing forward; the y axis lies perpendicular to the line of flight and in the horizontal plane, with positive end pointing along the right wing.

The variables in the equations are:

a) The three components of linear velocity, u, v, and w, in the positive x, y, and z directions, respectively. The forward velocity u is the perturbation to the initial constant velocity U_1 and is a small term compared to U_1.

b) The three angular velocities p, q, and r, about the x, y, and z axes respectively.

c) The angles θ and ϕ which define the orientation of the co-ordinate system with respect to a system fixed on the surface of the earth. The angle θ is the angle measured in a vertical plane between the x axis and the intersection of that vertical plane with a horizontal plane passing through the origin. It is positive if the positive end of the x axis lies above the horizontal plane. The angle ϕ is measured in a plane perpendicular to the x axis between the y axis and the intersection of that plane and the horizontal plane. It is positive if the positive end of the y axis is below the horizontal plane.

The eight differential equations are given below, together with one algebraic equation which defines the component of acceleration $a_z(t)$ normal to the flight path.

$$B\dot{q} = M_{\delta_e} \delta_e(t) + (C-A)pr + E(p^2+r^2) - M_q \left(q + \frac{M_{\dot{w}}\dot{w}}{M_q}\right) + M_{\Delta\alpha}\Delta\alpha \qquad (1)$$

where $$M_{\delta_e} = C_{m_{\delta_e}} \Omega S C_H$$

$$M_q = \left.\frac{dC_L}{d\alpha}\right)_{o_{HT}} \frac{AR_{HT} + 2}{AR_{HT}\sqrt{1-M^2} + 2} \quad \frac{\ell_t^2 S_{HT}\Omega}{U_1 + u}$$

$$\frac{M_{\dot{w}}\dot{w}}{M_q} = (\bar{F}+\bar{G}M+\bar{H}M^2) \quad \frac{(U_1+u)\dot{w} - w\dot{u}}{(U_1+u)^2}$$

$$M_{\Delta\alpha} = \left.\frac{dC_L}{d\alpha}\right)_o \frac{AR + 2}{AR\sqrt{1-M^2} + 2} \left\{ -\bar{B}+\bar{C} \left.\frac{dC_L}{d\alpha}\right)_o \frac{AR + 2}{AR\sqrt{1-M^2}+2} (2\alpha_{A_1}+\Delta\alpha)\right\} \Omega S C_H$$

$$a_z = -g\cos\Delta\alpha - Z_c\cos\Delta\alpha - Z_s\sin\Delta\alpha + Z_{\delta_e}\delta_e(t) \qquad (2)$$

where $$Z_c = \left.\frac{dC_L}{d\alpha}\right)_o \left(\frac{AR + 2}{AR\sqrt{1-M^2}+2}\right) \frac{\Delta\alpha\,\Omega S}{m}$$

$$Z_s = \left\{ a+k\left[\left.\frac{dC_L}{d\alpha}\right)_o \left(\frac{AR + 2}{AR\sqrt{1-M^2}+2}\right) (\alpha_{A_1} + \Delta\alpha)\right]^{2.25} + f\beta^2 \right\} \frac{\Omega S}{m}$$

$$Z_{\delta_e} = C_{m_{\delta_e}} \frac{\Omega S}{m} \frac{C_H}{\ell_t}$$

$$\dot{w} = a_z + g\cos\theta\cos\phi - vp + (U_1 + u)q \qquad (3)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \qquad (4)$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \qquad (5)$$

$$\dot{u} = g(\sin\Delta\alpha - \sin\theta) + X_s\sin\Delta\alpha + vr - X_c\cos\Delta\alpha - wq \qquad (6)$$

where

$$X_s = \left.\frac{dC_L}{d\alpha}\right)_o \frac{AR + 2}{AR\sqrt{1-M^2} + 2} \Delta\alpha \frac{\Omega S}{m}$$

$$X_c = \left\{ a+k\left[\left.\frac{dC_L}{d\alpha}\right)_o \left(\frac{AR + 2}{AR\sqrt{1+M^2} + 2}\right) (\alpha_{A_1} + \Delta\alpha)\right]^{2.25} + f\beta^2 \right\} \frac{\Omega S}{m} - C_{D_1} \frac{\rho}{2} U_1^2 \frac{S}{m}$$

$$\dot{v} = g\cos\theta\sin\phi + Y_\beta\beta - Y_{pz_t} p z_t + wp + Y_{pb} pb - r(U_1+u) \qquad (7)$$

where

$$Y_\beta = \left[\frac{dCy}{d\beta}\Big)_{W+F} - \frac{dCL}{d\alpha}\Big)_{o_{VT}} \frac{AR_{VT} + 2}{AR_{VT}\sqrt{1-M^2} + 2} \frac{S_{VT}}{S}\right] \frac{\Omega S}{m}$$

$$Y_{p_{z_t}} = \frac{dCL}{d\alpha}\Big)_{o_{VT}} \frac{AR_{VT} + 2}{AR_{VT}\sqrt{1-M^2} + 2} \frac{\Omega S_{VT}}{m(U_1+u)}$$

$$Y_{pb} = \left[\bar{x} \frac{F}{G\sqrt{1-M^2}+H} + \frac{1}{AR}\frac{dCL}{d\alpha}\Big)_o \frac{(AR + 2)}{AR\sqrt{1 - M^2} + 2}(\alpha_{A_1} + \Delta\alpha)\right] \frac{\Omega S}{2m(U_1+u)}$$

$$A\dot{p} = L_1\left(C_{1_{p_o}} \frac{pb}{2(U_1+u)} + C_{1_{\beta_{o_{W+F}}}}\beta + C_{1_{\delta_a}} \delta_a(t)\right) + L_\beta\left(\frac{r\ell_t}{U_1+u} - \beta\right) \quad (8)$$

$$+ L_p p + L_r r + E(\dot{r} + pq) - (C-B)qr$$

where

$$L_1 = \frac{F}{G\sqrt{1-M^2}+H} \Omega Sb$$

$$L_\beta = \frac{AR_{VT} + 2}{AR_{VT}\sqrt{1-M^2} + 2} \frac{dCL}{d\alpha}\Big)_{o_{VT}} \Omega S_{VT} z_t$$

$$L_p = C_{1_{p_{o_{HT}}}} \frac{AR_{HT} + 2}{AR_{HT}\sqrt{1-M^2}+2} \frac{b_{HT}^2}{b^2} \frac{S_{HT}}{S} \frac{b^2}{2(U_1 + u)} \Omega S$$

$$L_r = \left\{K(\Delta\alpha + \alpha_{A_1}) + \Delta C_{1_{r_o}} \frac{J}{\dot{K}\sqrt{1-M^2} + L}\right\} \frac{b^2 \Omega S}{2(U_1+u)}$$

$$C\dot{r} = (N_{p_1}p + N_N N(t) + N_r r + N_{p_2}p + N_\beta \beta) - E(rq-\dot{p}) - (B-A)pq \quad (9)$$

where

$$N_{p_1} = N_1 \frac{K_2 b}{2(U_1+u)}$$

$$N_1 = \frac{\bar{P}\,(\alpha_{A_1} + \Delta\alpha)\,\Omega Sb}{(\bar{R}\sqrt{1-M^2} + \bar{S})(\bar{Q}\sqrt{1-M^2} + \bar{T})}$$

$$N_N = N_1 \frac{dCL}{d\alpha}\Big)_o \frac{AR + 2}{AR\sqrt{1-M^2} + 2}$$

$$N_r = \left\{2\frac{dCL}{d\alpha}\Big)_{o_{VT}} \frac{AR_{VT} + 2}{AR_{VT}\sqrt{1-M^2}+2} \frac{S_{VT}\ell_t^2}{Sb^2} + K_3(\Delta\alpha + \alpha_{A_1})^2 + \Delta C_{n_r}\right\} \frac{b^2\Omega S}{2(U_1+u)}$$

$$N_{p_2} = \frac{\Delta C_{n_p}}{2} \frac{b^2 \Omega S}{U_1+u}$$

$$N_\beta = \left\{ \left(\frac{dCL}{d}\right)_{o_{VT}} \frac{AR_{VT} + 2}{AR_{VT}\sqrt{1-M^2}+2} \frac{S_{VT}\ell_t}{Sb} + C_{n_{\beta_{W+F}}} \right\} \Omega Sb$$

In these equations, the following quantities are functions of other variables as follows:

$$M = \frac{1}{S} \left\{ (U_1 + u)^2 + v^2 + w^2 \right\}^{\frac{1}{2}}$$

$$\Omega = \frac{\rho}{2} \left\{ (U_1 + u)^2 + v^2 + w^2 \right\}$$

$$\Delta\alpha = \tan^{-1} \frac{w}{U_1+u}$$

$$\beta = \tan^{-1} \frac{v}{U_1+u}$$

The following symbols stand for empirical polynomials in M.

(1) The symbols a and k are cubics in M.

(2) The symbols $\overline{B}$ and $\overline{C}$ are quadratics in M.

All other symbols are constants or known functions of t.

The boundary conditions are: at $t = 0$, $u = v = w = p = q = r = \theta = \phi = 0$. Taken together with the specified functions $\delta_e(t)$, $\delta_a(t)$, $N(t)$, this requires that all first derivatives are also zero. The known functions are given by the formulae

$\delta_e(t)$	$= -.08726646t$	for	$0 \leq t \leq 0.4$
	$= -.03490658$	for	$0.4 \leq t \leq 3.0$
$\delta_a(t)$	$= 0$	for	$0 \leq t \leq 0.8$
	$= 1.745(t - 0.8)$	for	$0.8 \leq t \leq 1.0$
	$= 0.3490$	for	$1.0 \leq t \leq 3.0$
$N(t)$	$= 0$	for	$0 \leq t \leq 0.8$
	$= .02596(t - 0.8)$	for	$0.8 \leq t \leq 1.0$
	$= .005191$	for	$1.0 \leq t \leq 3.0$

Some exploratory computations were made on desk calculators using a simplified system of equations. The simplifications consisted of replacing M and Ω by linear functions of u, replacing the arctangents by the first two terms of the power series expansions, and discarding terms in the differential equations which appeared to contribut less than 5% to the maximum value of the derivative. This approximate system was integrated at an interval of 0.1 seconds in the region $0 \leq t \leq 0.8$ seconds, and at an interval of 0.01 seconds in the region $0.8 \leq t \leq 1.2$ seconds. These preliminary calculations showed that the choice of the order of evaluation of the differential equations was reasonable, that several variables, especially p and v, were closely interdependent, and that the equations themselves seemed to contain no major errors.

A preliminary investigation of the nature of the solution in the neighborhood of the origin was also made. In this region, the variables v,p,r, and ϕ will remain at their initial zero value due to the nature of the driving functions $\delta_a(t)$ and N(t). Since θ enters only through its cosine and u enters only in the combination $U_1 + u$ where U_1 is large, θ and u can be set equal to zero in the neighborhood of the origin. If in addition M and Ω are assumed constant and $\tan^{-1}\frac{w}{U_1}$ is replaced by $\frac{w}{U_1}$, a set of two differential equations with constant coefficients and one algebraic equation is obtained. These equations are equation (1) for $\dot{q}$, equation (3) for w, and equation (2) for $a_z(t)$. If q and w are eliminated from this set, an equation is obtained which can be written symbolically, namely,

$$(aD^2 + bD + c)\, a_z(t) = (eD^2 + fD + g)\, \delta_e(t)$$

If $\delta_e(t)$ is considered a known function and is any combination of powers, exponentials and trigonometric functions, then $a_z(t)$ will have the exact form of $\delta_e(t)$. However, a multiple of the solution of the equation

$$(aD^2 + bD + c)\,\zeta = 0$$

must be added to the general solution for $a_z(t)$ in order to satisfy the boundary conditions. In the particular case solved here, ζ is of the form

$e^{-1.80t}(\alpha_1 \sin 3.18t + \alpha_2 \cos 3.18t)$. Obviously, ζ becomes small as t increases. Consequently the effect of multiples of ζ introduced in any manner will vanish as time increases.

If $a_z(t)$ is considered known and of the form specified above, then the solution for $\delta_e(t)$ will be of the exact form of $a_z(t)$. A multiple of the solution of the equation

$$(eD^2 + fD + g)\eta = 0$$

must be added to the general solution for $\delta_e(t)$ to satisfy the boundary conditions. In the particular case solved here, η is of the form $\beta_1 e^{17.94t} + \beta_2 e^{-19.78t}$. If β_1 is not zero, η increases exponentially. To avoid this, the boundary conditions must be chosen with great care so $\beta_1 = 0$. However, in numerical integration, roundoff error may introduce a $\beta_1 \neq 0$ at any step. If this is done, the solution obtained is soon erroneous by the exponential factor $\beta_1 e^{17.94t}$.

This analysis is valid only in that region where the approximation of the complete equations by the simplified set with constant coefficients can be made. Outside this region, a more rigorous study of the propagation of roundoff error is necessary.

The integration of the complete equations was carried out on the IBM Card Programmed Calculator (CPC). The general plan used was to store the eight values of the functions and the eight values of the derivatives in the storage unit. The interval of integration h and the function of the variables M, $\Delta\alpha$, and β were stored in four of the accumulating counters, leaving three counters free in which to carry out the evaluation of the differential equations.

To follow the procedure for solution, assume the memory is filled with functions and derivatives evaluated at time t_n. The problem is to find the values at time $t_n + h$. Let x_n be the value of any variable at time t_n, and x_{n+1} be the value of any variable at time $t_n + h$. Then the process used was to extrapolate all variables using the formula

$$x_{n+1} = x_n + h\dot{x}_n.$$

Each equation was evaluated using these extrapolated values of the functions and each yielded a value of a derivative $\dot{x}_{n+1}$ at time $t_n + h$. As soon as a derivative was computed, it was used in the trapezoidal rule

$$x_{n+1} = x_n + \frac{h}{2} (\dot{x}_n + \dot{x}_{n+1})$$

to integrate the variable in question. This integrated value was used instead of the extrapolated value in all subsequent calculations and replaced its corresponding value x_n in the memory. After all variables were integrated, the derivatives were recomputed using the integrated values of all the functions. These new values of the derivatives then replaced the corresponding values of $\dot{x}_n$ in the memory system. All functions and all derivatives were punched out at the end of each integration step to insure a permanent record of the time history of each function and each derivative.

The variables u,v,w,p, and r were carried to six decimal places, θ and ∅ were carried to seven, and q was carried to eight. The derivative of each variable was carried to one less decimal place than its corresponding function. All operations were performed with an eight digit plugboard arrangement of the CPC.

This general computation scheme was altered slightly in practice. The problem was divided into two parts at t = 0.8 seconds. Throughout the first part the variables v,p,r, and ∅ were zero, as demonstrated previously. The four remaining differential equations were evaluated in the order $\dot{q}$,$\dot{w}$, $\dot{\theta}$,$\dot{u}$. This cycle was repeated three times at each time point. As each equation was evaluated, the best available value of each variable was used. This was a value extrapolated by the formula

$$x_{n+1} = x_n + h\dot{x}_n$$

in the early stages of the computation, or a value integrated by the trapezoidal rule during the later stages of the computation. The adopted value of u_{n+1} was integrated on the second cycle, while the adopted values of

q_{n+1}, w_{n+1}, and θ_{n+1} were integrated on the third cycle. θ and u were integrated by the trapezoidal rule, while q and w were integrated by the formula

$$x_{n+1} = x_n + \frac{h}{12} (5\dot{x}_{n+1} + 8\dot{x}_n - \dot{x}_{n-1}).$$

As each adopted value was integrated, it replaced the value of that function at the preceding time point in the memory of the CPC. Similarly, the values of the derivatives computed on the third cycle replaced their corresponding previous values in the CPC memory.

This integrating procedure was performed on the CPC at a speed of 10 time steps per hour. The interval used in the integration was h = 0.01 seconds, giving a running time of about 8 hours for this section of the problem.

In the region $0.8 \leq t \leq 3.0$, all eight differential equations must be simultaneously integrated. These were evaluated in the order $\dot{q},\dot{w},\dot{q},\dot{\theta},\dot{\phi},\dot{u},\dot{v},\dot{p},\dot{r},\dot{v},\dot{p}$, and $\dot{r}$. As before, the best available value of each variable was used at all times. The adopted values of the variables q,v, p, and r were integrated the second time their derivatives were computed, while the adopted values of w,θ,ϕ, and u were integrated the first time their derivatives were computed. As each variable was integrated, its new value x_{n+1} replaced its old value x_n in the CPC memory. At the end of this integrating cycle, all the differential equations were again evaluated in the order $\dot{q},\dot{w},\dot{\theta},\dot{\phi},\dot{u},\dot{v},\dot{p}$, and $\dot{r}$. These values of the derivatives replaced the old values in the memory of the CPC.

This cycle of operations was performed on the CPC at a rate of 5 steps per hour. The interval used in this region was h = 0.005 seconds, giving a running time of about 88 hours for this section of the problem.

The following checks were applied. One step of the integration was evaluated by a desk computer. These results were checked against the CPC answers, and agreement was demanded for every approximation to a derivative or to a function, and for every term in each approximation. When this agreement had been obtained, the CPC control deck was considered checked. The results of the integration were differenced to test for smoothness. A few random CPC errors were picked up by this check, and some work had to be rerun to correct these errors. When all observable errors had been removed, the functions and derivatives throughout the range of the integration had fourth differences less than 100 in units of the last place carried.

Some numerical estimates were made of the size of certain error terms. The first of these was the estimate of the truncation error, which was made from a consideration of Laplace's formula written in terms of backward differences,

$$\int_{x_{n-1}}^{x_n} f(t)dt = h \left\{ f(x_n) - \left[1/2 \nabla_n + 1/12 \nabla_n^2 + 1/24 \nabla_n^3 + \ldots \right] f(x_n) \right\} .$$

The trapezoidal rule is equivalent to the first two terms of the series. Since the third differences of the derivatives were small compared to the second differences, the truncation error was approximated by the first dropped term, namely, $(-h/12)\nabla_n^2 f(x)$. Hence the sum of the absolute values of the second difference of the derivative was formed and was multiplied by $\frac{h}{12}$. This provided an estimate of the truncation error in each variable.

A second source of error was the use of an incorrect approximation to the derivative in the integration formula. The derivative of each variable was computed at least twice and the next to the last approximation was integrated. The difference between the last two approximations to any derivative was taken as an approximation to the error in the value of the derivative. Hence the sum of the absolute value of this difference was formed and was multiplied by $\frac{h}{2}$, the weight factor of the derivative in the integration formula.

Third, the roundoff error was estimated as a half unit per step in the last place carried. Because of the difference of only one decimal place in function and derivative, a roundoff error in the derivative will not directly affect the last place carried in the function, as long as the interval of integration is less than 0.1. As this condition was fulfilled, the roundoff error of the derivative could be neglected.

Finally, the total error was estimated by summing these three components. It is obvious that this estimate in no way sets a limit on the error, but rather provides an order of magnitude estimate that should not be too erroneous for well behaved functions. In this problem, the permissible error in any variable was set at 3% of its maximum variation. The greatest error, according to the estimate made by the system outlined above, was 75% of the permissible error.

F. Acton and R. Reynolds aided in the analysis. The desk calculations were computed under the supervision of G. Blanch; the IBM coding and supervision of the IBM computations were done by E. C. Yowell. The following persons participated in the hand and IBM computing: F. Gordon, R. Horgan, G. Kimble, W. Paine, A. Rosenthal, and L. Wilson. This report was prepared by E. C. Yowell.

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