Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

A QUARTERLY REPORT

April through June 1950

NATIONAL APPLIED MATHEMATICS LABORATORIES

of the

NATIONAL BUREAU OF STANDARDS

NATIONAL APPLIED MATHEMATICS LABORATORIES

April 1 through June 30, 1950

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Projects and Publications

of the

NATIONAL APPLIED MATHEMATICS LABORATORIES

April through June 1950

Contents

Preface

This is a report on the activities of Division 11 of the National Bureau of Standards for the period from April 1, 1950 through June 30, 1950.

Division 11 is known as the National Applied Mathematics Laboratories. It is the mission of the Laboratories to perform research and to provide services in various quantitative branches of mathematics, placing special emphasis on the development and exploitation of high-speed numerical analysis and modern statistical methodology. The Laboratories maintain an expert computing service of large capacity, and provide consulting services in classical applied mathematics and in mathematical statistics. These services are available primarily to other federal agencies, but under certain circumstances it is possible to perform work for industrial laboratories and universities.

Inquiries concerning the availability of the services of the National Applied Mathematics Laboratories, or concerning further details of any of the projects described in this report, should be addressed to the National Applied Mathematics Laboratories, 415 South Building, National Bureau of Standards, Washington 25, D. C.

E. U. Condon

J. H. Curtiss

Chief

Director
National Bureau of Standards
July 15, 1950

Index of Active Research and Development Projects

NOTE: This index is not intended to cover the numerous special problem solutions, statistical analyses, and other ad hoc services to Government agencies which form an important part of the work of the National Applied Mathematics Laboratories. These services are, however, fully represented in the body of the report.

A. Research: Pure Mathematics

B. Research: Numerical Analysis

C. Research: Applied Mathematics, Physics, Astronomy, and Automatic Translation

D. Mathematical Statistics

*New projects

E. Mathematical Tables

F. Manuals, Bibliographies, Indices and Technical Information

G. Computing Machine Development

*New projects

Announcement

THE NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)

The National Applied Mathematics Laboratories entered into a new era of its history with the completion of the first of the three automatic digital computing machines to be installed in the Laboratories. This is the machine identified with project 11.4/24-49-1 in this series of reports. The final name chosen for the computer was National Bureau of Standards Eastern Automatic Computer, or SEAC for short.

The SEAC, which was built by the NBS Electronic Computers Section under Mr. S. N. Alexander, was sponsored by the Office of the Air Comptroller of the United States Air Forces for use in developing and applying new techniques for high-speed military program planning. The Air Force program, known as Project SCOOP (Scientific Computation of Optimum Programs), represents a pioneer attempt to apply scientific principles to management and administration of large-scale enterprises. The National Bureau of Standards has been collaborating in this program by providing various computing services and carrying on research in mathematical techniques, as well as in the development and construction of computing equipment.*

The new computing machine first began to solve test problems on April 7, 1950. It was formally dedicated on June 20, 1950. The dedication ceremony was attended by a distinguished group of Government officials. The speakers were Dr. E. U. Condon, Lieutenant General E. W. Rawlings, Comptroller, Department of the Air Force, Dr. J.H.Curtiss, chief of the National Applied Mathematics Laboratories, and Mr. S. N. Alexander, chief of the NBS Electronic Computers Section.

The SEAC is a high-speed general purpose digital computer operating in the serial mode and using the acoustic delay line type of internal

*See projects 11.1/1-50-11, 11.2/12-50-2, 11.2/36-49-3, 11.4/24-47-3, 11.4/3-47-4.

memory. It operates at present with a teletype tape input-output mechanism, but soon it will be equipped with multi-channel magnetic tape input-output units. The basic built-in operations are addition, subtraction, multiplication (either with rounding or to full accuracy), division, comparison (i.e. selection of one of two alternative orders depending on the relative size of two numbers), logical transfer (insertion of specified digits of a word into a word held in a designated memory location--partial word transfer), input-output control.

The internal operations are entirely in the binary scale of notation, and if the input and output data for a given problem are to be in the decimal scale, conversion routines must be inserted as part of the program. The instructions for a problem are fed into the machine on the same input mechanism as the data, and are stored with the data in the internal memory for reference as needed. The individual numbers and the instructions are represented in the machine by blocks of binary digits called words. The length of each word is 45 binary digits, of which one is used to represent sign when the word represents a number rather than an instruction. The pulse repetition rate is one microsecond.

The internal memory consists of 64 delay lines, each of which consists primarily of a glass tube filled with mercury, and each of which provides storage for 8 words. Thus the total memory capacity is 512 words, or 23,040 binary digits. The average reference time is 168 microseconds. Provision has been made for eventually expanding the present memory system to eight times its present size. For the immediate future it is planned to supplement it with a 512-word electrostatic memory system based on the so-called Williams-tube principle.

Addition or subtraction time for two 44 digit numbers is 200 microseconds, and multiplication or division takes 2300 microseconds. These rates do not include the time taken by the machine to find the numbers in the memory and return the results to the memory.

There are about 13,000 germanium crystal diodes in the machine and 750 tubes. As was pointed out in a brochure distributed at the dedication program, emphasis in the development was placed on designing circuits especially for computer use rather than adopting the standard procedures of television and radar circuitry. "All computing and switching in the SEAC is performed by germanium crystal diodes rather than by electron tubes. With few exceptions, tubes are used only for power amplification. A standardized tube-and-transformer combination is used throughout the machine to simplify maintenance. The transformers provide a-c coupling between stages and also minimize cross-talk between circuits. These advances in circuit design as well as other unique construction features make it easy to add units to the machine."

The closing words of the address given by the Chief of the National Applied Mathematics Laboratories on the occasion of the dedication of the SEAC were as follows:

"And now it remains to put this remarkable machine to work for the benefit of science and mankind. I hope that I have been able to assure you today that, in the excitement of creating such devices, we have never lost sight of the fact that the human brain still transcends any robot which it has so far created, and that we have made wise and adequate provisions for realizing the full capabilities of this instrument."

Status of Projects

June 30, 1950

I. Institute for Numerical Analysis

(Section 11.1)

1. Research in Numerical Analysis

Key to letter symbols in project numbers:

AE - Algebraic equations
CM - Numerical methods in conformal mapping
ODE - Ordinary differential equations
PD - Partial differential equations
PM - Probabilistic methods in numerical analysis

SOLUTION OF SETS OF SIMULTANEOUS ALGEBRAIC EQUATIONS AND TECHNIQUES FOR THE INVERSION AND ITERATION OF MATRICES
Project 11.1/1-49-AE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The investigation of the distribution of values of a certain class of n-rowed determinants has been completed with the publication (2).

A method for shortening the computation in the unusual case where an exact inverse is required was worked out, and was presented with a numerical example in (3).

Two tests were made on the I.B.M. Card Programmed Calculator of the general iteration scheme described on pp. 1-2 of the Oct-Dec 1949 Projects and Publications. In both tests a problem was chosen deliberately to be difficult and give slow convergence. In one test the usual method for speeding convergence worked well. In the other test an improvement of the method had to be used.

Publications: (1) "Matrix inversion by a Monte Carlo method," by G. E. Forsythe and R. A. Leibler; to appear in July 1950 issue of MTAC. (2) "The extent of n random unit vectors," by G. E. Forsythe and J.W.Tukey; submitted to American Mathematical Monthly. (3) "A method of computing exact inverses of matrices with integer coefficients," by J. B. Rosser; IN MANUSCRIPT at INA.

NUMERICAL METHODS IN CONFORMAL MAPPING
Project 11.1/1-49-CM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The numerical procedure for testing Dr.Ahlfors' method for determining the function U(z) is being set up to calculate U(z) at points on the circular arcs at 3° intervals (see Jan-Mar 1950 status report). The calculation will be done with IBM equipment. The accuracy and rapidity of convergence will be checked against known results.

Publications: (1) "The construction and applications of conformal maps: Proceedings of a symposium", edited by E. F. Beckenbach; to be published by the NBS. The volume will include the following papers written in connection with this project: (i) "A bibliography of numerical methods in conformal mapping," by W. Seidel. (ii) "On conformal mapping of variable regions," by S. E. Warschawski. (iii) "On the convergence of Theodorsen's and Garrick's method of conformal mapping," by A.M.Ostrowski. (iv) "On a discontinuous analogue of Theodorsen's and Garrick's method," by A.M. Ostrowski. (v) "On the Helmholtz problem of conformal representation," by A. Weinstein. (2) "Conformal representation of simply-and multiply-connected regions," by L. Kantorovitch and others; translation from the Russian by W. Seidel; IN MANUSCRIPT at NAML; publication under consideration. (3) "Numerical methods in conformal mapping," by L. Ahlfors; IN MANUSCRIPT, awaiting supplementary numerical work.

STUDIES IN NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS
Project 11.1/1-49-ODE2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Dr. Forsythe wrote a note to make public the idea of random round-off as a mechanism for making round-off errors statistical. In this connection H. A. Rademacher's important article, "On the accumulation of errors in processes of integration," was reproduced in multilith typescript form, since it has been out of print for some time. It originally appeared in the "Proceedings of a (1947) Symposium on Large-Scale Digital Calculating Machinery," Computation Laboratory, Harvard University (Harvard University Press, 1948).

Publications: (1) "Note on the Runge-Kutta method," by W.E.Milne; NBS J. Res. 44, 549-550 (May 1950). (2) "Note on rounding-off errors," by George E. Forsythe. Multilithed typescript available at INA.

STUDIES IN NUMERICAL INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PD1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

Publications: (1) "Numerical methods associated with Laplace's equation," by W. E. Milne; to appear in the Proceedings of the Symposium held at the Harvard Computation Laboratory in September 1949. (2) "Numerical solution of partial differential equations," by E. C. Yowell; to be published in the Proceedings, Scientific Computation Forum, held under the auspices of the I.B.M. Corporation in Endicott, N. Y., November 1949.

SAMPLING TECHNIQUES FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRO-DIFFERENTIAL EQUATIONS
Project 11.1/1-49-PM1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Dr. Kac is testing the method of finding the lowest eigenvalue of Schrödinger's equation by statistical sampling by applying it to the hydrogen atom. This is in preparation for the application of the method to more complicated atomic and molecular systems. (See report for project 11.1/1-50-3, Projects and Publications, July-Sept 1949.) Although simple theoretical calculations indicate violent statistical fluctuations which, a priori, would render the sampling method useless, the data (900 samples) exhibit much smaller fluctuations and lead to remarkably good agreement with the theoretical value. Theoretical studies are under way to explain the enormous discrepancy between the theoretical and the observed fluctuations. Alternative methods of analyzing data are also being studied.

A paper on the mean duration of random walks in n-dimensions is in preparation by Dr. Wasow.

Publications: (1) "Random walks and the eigenvalues of elliptic difference equations," by Wolfgang Wasow; to be published in the NBS Journal of Research. (2) "Sampling methods applied to differential and difference equations, with special reference to equations of the elliptic type," by J.H. Curtiss; to appear in the Proceedings, Scientific Computation Forum, held under the auspices of the I.B.M. Corporation in Endicott, N.Y., November 1949. (3) "The Monte Carlo Method: Proceedings of a symposium," to appear in the NBS Applied Mathematics Series. (4) "A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation," by M. D. Donsker and M. Kac; NBS J. Res. 44, No. 5, 551-557 (May 1950).

DETERMINATION OF EXTREMALS OF FUNCTIONALS
Project 11.1/1-50-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The revision of publication (2) was completed.

A large number of the results given in a paper by Hestenes and Karush (see project 11.1/1-50-3) have been extended to quadratic forms in Hilbert space. This extension gives rise to methods of finding eigenvalues for differential and integral boundary-value problems; preliminary machine computations have been made and the results are encouraging.

Methods have been devised for obtaining eigenvalues of the system $Ax = \lambda Bx$, where B is not the identity matrix. A report on these methods is in preparation.

Publications: (1) "An elementary introduction to the calculus of variations," by M. R. Hestenes; Math. Mag. XXIII, No. 5, 249-267 (May-June 1950); also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A. (2) "Applications of the theory of quadratic forms in Hilbert space to the calculus of variations," by M. R. Hestenes; accepted by the American Journal of Mathematics.

DETERMINATION OF CRITICAL POINTS
Project 11.1/1-50-2

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. The work accomplished in this quarter is reported under project 11.1/1-50-3.

CALCULATION OF EIGENVALUES, EIGENVECTORS, AND EIGENFUNCTIONS OF LINEAR OPERATORS
Project 11.1/1-50-3

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Drs. Feller and Forsythe have completed paper (5), an exposition of a new family of transformations for reducing matrices and obtaining successive eigenvalues and eigenvectors. There is a bibliography and historical summary.

A paper by Drs. Donsker and Kac and further work on sampling methods for obtaining eigenvalues are reported under project 11.1/1-49-PM1.

An 8x8 symmetric matrix was specially constructed by Dr. Rosser so that the eigenvalues would be difficult to compute. Three teams were asked to tackle it using three different methods. Dr. Lanczos used the method described in publication (1) below. Dr. Karush used the gradient method described in publications (3) and (4). Dr. Forsythe was asked to

use classical methods, but found them to be effectively impossible and used common sense together with ad hoc numerical experimentation. All three obtained correct results. Dr. Rosser plans to write a report comparing the solutions.

As part of its computing service, the INA was asked to compute eigenvalues of 8x8 non-symmetric matrices with complex coefficients. The difficulties were so great that the problem has been placed under this project for further study. After trying several methods unsuccessfully, numerical experimentation has yielded a method which it is hoped will be effective. It will be tried on the matrices.

Some preliminary tests have been made on a method for getting eigenvalues by reducing a matrix to triangular form.

Publications: (1) "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators," by C. Lanczos; accepted by the NBS Journal of Research. (2) "Numerical determination of characteristic numbers," by W. E. Milne; accepted by the NBS Journal of Research. (3) "The method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix," by M. R. Hestenes and W. Karush; accepted by the NBS Journal of Research. (4) "An iterative method for finding characteristic vectors of a symmetric matrix," by W. Karush; to be submitted to the Pacific Journal of Mathematics. (5) "New matrix transformations for obtaining characteristic vectors," by W. Feller and G. E. Forsythe; accepted by the Quarterly of Applied Mathematics.

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.1/1-50-4

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Publication (12) below deals with the sequence-to-function transformation R_1:

$$\phi(h) = \frac{2}{\pi} \sum_{n=1}^{\infty} s_n \frac{\sin nh}{n} ,$$

for h 0. As a summability method for sequences $\{s_n\}$ R_1 is not regular, but is related to Lebesgue's regular method. The main result is that R_1 sums the Fourier series of a function at each abscissa of continuity.

A paper by O. Szasz and J. Todd on Cauchy-Riemann sums for infinite intervals is in preparation (see project 11.1/1-49-8, Oct-Dec 1948 issue; also this issue, project 11.2/11-50-4).

Dr. Szasz and Dr. Lukacs are collaborating on a study of some non-negative trigonometric polynomials. Let $0 < b_1 < b_2 < \ldots < b_n$ be n integers and let $g(\theta)$ be the Vandermonde determinant formed from $b_1^2, b_2^2, \ldots b_n^2$ with the first row replaced by $\sin^2 \frac{b_i\theta}{2}$ $(i = 1,\ldots,n)$. The function $g(\theta)$ is then a cosine polynomial. In connection with a problem in probability, the question arose as to when $g(\theta)$ is a non-negative trigonometric polynomial. The following results are typical:

(A) If the b_i are the first n consecutive integers then $g(\theta)$ is non-negative.

(B) If the b_i are the first n consecutive odd integers then $g(\theta)$ is non-negative.

(C) If the numbers $b_1, \ldots, b_n$ are obtained from the first $(n + 1)$ consecutive integers by omitting the integer $k (1 \leq k \leq n)$ then the trigonometric polynomial $g(\theta)$ is non-negative if and only if $2k \geq n + 1$. A paper is in preparation.

Work is being continued on the Gibbs phenomenon for a class of series-to-function transforms of type

$$\phi(t) = \sum_{n=1}^{\infty} a_n f(nt) ,$$

for $t \sim 0$, see project 11.1/1-50-4, July-Sept 1949 issue.

A paper is being written on identities and inequalities concerning orthogonal polynomials and Bessel functions. This is related to the work done in project 11.1/1-49-SF1, see Apr-Jun 1949 and following issues. New proofs are given, for example, for the Turán inequality for ultraspherical polynomials.

Dr. Dvoretzky is making a certain study of Hausdorff measures. Let $h(x)$ be defined for $0 < x < \infty$ and assume non-negative values. Let S be any linear set of points and $\rho > 0$ arbitrary. Cover S by countably many intervals $I_1, I_2, \ldots$ of lengths $x_1, x_2, \ldots$ all smaller than ρ and denote by $m_\rho (S;h)$ the lower bound of $h(x_1 + h(x_2) + \ldots$ for all such coverings. $m(S;h) = \lim_{\rho \downarrow 0} m_\rho (S;h)$ is called the Hausdorff measure of S with respect to $h(x)$. A study of the relations between measures with respect to different functions is made. A typical result is the following: $0 < m(S;h) < \infty$ implies $0 < m(S;g) < \infty$ and vice versa if and only if $h^*(x)/g^*(x)$ is bounded and bounded away from zero as $x \downarrow 0$; here $h^*(x) = x \inf_{0 < t \leq x} \frac{h(t)}{t}$, and $g^*(x)$ is defined similarly. The results obtained are to be presented at the International Congress of Mathematics. Some of the methods used in the investigation are extensions of those used in A. Dvoretzky, Proc. Camb. Phil. Soc. 44, 13 (1948).

Publications: (1) "On the Gibbs phenomenon for Euler means," by O. Szasz; Acta Litterarum ac. Scient., Szeged. vol. 12, Part B (1950). (2) "Gibbs phenomenon for Hausdorff means," by O. Szasz; accepted for publication in the Transactions of the American Mathematical Society. (3) "On a summation method of O. Perron," by O. Szasz; Mathematische Zeitschrift, 52, 631-636 (1950). (4) "A generalization of S.Bernstein's polynomials to the infinite interval," by O. Szasz; accepted by the NBS Journal of Research. (5) "On some trigonometric transforms," by O.Szasz; for publication in the Pacific Journal of Mathematics. (6) "On positive harmonic functions and ultraspherical polynomials," by W. Seidel and O. Szasz; for publication in the Journal of the London Mathematical Society. (7) "On subharmonic and linear functions of two variables," by E. F. Beckenbach; submitted to Revista, Universidad Nacional de Tucuman (Argentina). (8) "Certain Fourier transforms of distributions," by E. Lukacs and O. Szasz; for publication in the Canadian Journal of Mathematics. (9) "Inequalities concerning ultraspherical polynomials and Bessel functions," by O. Szasz; Proc. Am. Math. Soc. 1, No. 2, 256-267 (April 1950). (10) "On a Tauberian theorem for Abel summability," by O. Szasz; submitted to the Pacific Journal of Mathematics. (11) "On the relative extrema of ultraspherical polynomials," by O. Szasz; for publication in the Bolletino della Unione Matematica Italiana (Firenze). (12) "Tauberian theorems for summability (R_1)," by O. Szasz; IN MANUSCRIPT at INA.

AN ITERATIVE SOLUTION OF FREDHOLM'S INTEGRAL EQUATION
Project 11.1/1-50-8

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

INTEGRAL TRANSFORMS AND NETWORK ANALYSIS
Project 11.1/1-50-9

Origin: NBS — Authorized 3/31/50
Sponsor: Office of Naval Research, USN — Revised 6/30/50
Manager: C. Lanczos

Objective: To replace the traditional expansion theorem of Heaviside, which requires the knowledge of the complex roots of an algebraic equation, by a simple algorithm which obviates the evaluation of the roots.

Background: A Laplace transform is usually inverted by integrating along the imaginary axis and then closing the path of integration through the negative infinite semi-circle. The integration is then reduced to loops around the singularities, which in the case of an electric network are simple poles. To obtain the position of these poles one must get the (generally complex) roots of an algebraic equation, which occasionally may be of rather high degree. This is frequently a cumbersome task. The present method uses conformal mapping by reciprocal radii, transforming the infinite imaginary axis into the unit circle. However, in contrast to the traditional procedure, it is the right half of the complex plane which is mapped inside the unit circle. The Laplace transform is regular in this region and allows expansion into a Taylor series around the center of the circle. Inverting this uniformly convergent series term by term we find that the original function $f(x)$, which has the significance of the transient response (Green's function) of the network, is now expanded into a series of orthogonal Laguerre functions:

$$f(x) = \sum_{k=0}^{\infty} c_k \frac{L_k(2x)}{k!} e^{-x}$$

The coefficients c_k of this expansion are made readily available by a simple division algorithm. The convergence is such that 10 to 12 terms of the series will usually give a good overall representation of the transient response. The "noise" part of the response, which shows up in small humps of the curve and which is usually of minor physical significance, is automatically smoothed out in the representation. This method obviates the solution of an algebraic equation of high degree, and is particularly useful if we want to see how modification of a network parameter influences the transient response. Moreover, this method gives a new solution of the problem of network synthesis, when a network of given order is to be designed which shall approximate a given transient response as closely as possible.

Status: CONTINUED. The principle which led to the solution of the electric network problem in terms of Laguerre polynomials is actually applicable to a larger group of integral transforms. The generating function $g(x)$ of an integral transform may be developed into a suitable system

of orthogonal functions, chosen in such a manner that the coefficients of the expansion shall be determinable in terms of some specific values of the given integral transform f(z).

Apart from solving the inversion problem, the method has further consequences. If the generating function g(x) is replaced by its orthogonal expansion, which is then integrated term by term, an infinite series which generates f(z) everywhere in the right half of the complex plane in terms of equidistant values of f(z) taken along the real axis is obtained. The type of formula thus obtained depends on the nature of the given integral transform. For example the type of integral transform which defines Riemann's zeta-function leads to the Newton-Gregory interpolation formula, while the type of integral transform which defines the Fourier coefficients leads to the Stirling (central difference) type of interpolation. The Laplace transform leads to an interpolation by factorials which remains uniformly convergent even if z recedes to infinity along any ray of the positive half plane.

This method has been applied to obtain expansions of the zeta-function, the logarithm of the gamma-function, and the exponential integral-representative examples of important transcendentals which are definable as integral transforms. The region of convergence of these expansions is the entire right half plane of the complex variable z. They can thus be conceived as interpolations if z is restricted to real positive values, but are extrapolations if z is in the half plane $R(z) > 0$.

Dr. Lanczos presented a talk entitled,"Inversion of the Laplace transform," to the American Mathematical Society Meeting, Berkeley, California, April 27, 28, 1950. A manuscript of this material is in preparation for the Proceedings of the Institute of Radio Engineers.

LANGUAGE TRANSLATION STUDY
Project 11.1/1-50-10

Origin: NBS Authorized 6/1/50
Manager: H. D. Huskey

Objective: To determine the feasibility and practicalness of doing substitution problems on automatic computing machines with a view to ascertaining the optimum type of automatic computer for this purpose. Particular emphasis will be given to the problem of the automatic translation of languages.

Background: Considerable thought has been given to this project by the project manager. Queries to various governmental agencies have confirmed the belief that there is a substantial need for faster and more economical methods of translation than are in use at present. Vast amounts of foreign data, which are not fully utilized by scientists and others due to the fact that no translations are available, have been continually accumulating. If it is shown that translation of this material by machines is practical then a satisfactory solution to the problem will have been found.

Status: NEW. A seminar is being held to consider problems connected with the automatic substitution-translation of languages by a computer such as the National Bureau of Standards Western Automatic Computer. Various members of the University of California at Los Angeles faculty, including several linguistic experts representing various departments, are interested in this project and are attending the seminar as linguistic advisers. It has been decided that for the initial exploratory period

the subject matter is to be restricted to one science and one language, namely, mathematical German.

RESEARCH IN THE MATHEMATICAL THEORY OF PROGRAM PLANNING
Project 11.1/1-50-11

Origin: Office of Air Comptroller, USAF Authorized 6/1/50
Sponsor: " "
Manager: H. D. Huskey

Objective: To pursue a study of the computational methods in linear programing, particularly with respect to the maximization problem, for the purpose of developing new methods, and with the aim of trying out certain of these methods on the National Bureau of Standards Western Automatic Computer.

Background: The sponsor of this project is financing the construction of an automatic computer to be used in the solution of problems in linear programing, and therefore is interested in determining optimum methods for the solution of such problems on a high speed automatic computer.

Comments: It is planned to carry on this project in cooperation with Rand Corporation which is also doing research on this problem.

Status: NEW. A group within the research staff is holding a weekly seminar to familiarize itself with work that has been done in this field. Gertrude Blanch spent one week in Washington, D. C. conferring on the subject with Dr. G. Dantzig of the Office of Air Comptroller. Since her return, Dr. Blanch and H. Luxenberg have presented some material to the seminar. A meeting has been held with Rand Corporation at which time it was decided that members of the staff at Rand would present several lectures, covering work they have done on the subject, at the Institute seminar.

STUDIES IN MATHEMATICAL STATISTICS
Project 11.1/1-50-12

Origin: Office of Naval Research, USN Authorized 6/1/50
Sponsor: " "
Manager: A. Dvoretzky

Objective: To increase knowledge in those fields of mathematical statistics on which certain theories and practices in numerical analysis are based.

Background: The general expansion of scientific knowledge is the principal purpose of this project. There are, moreover, two active projects at the INA which will benefit especially from contributions to mathematical statistics, namely: project 11.1/1-49-PM1, and project 11.1/1-50-11.

Status: NEW. The work on this project was done by Dr. Dvoretzky, in collaboration with Professors Wald and Wolfowitz. The problem worked on during the quarter has equivalent formulations in decision theory and

game theory. In decision theory the problem is to determine sufficient conditions for the replacement of a randomized statistical decision function by an equivalent non-randomized decision function. The replacement is shown to be possible in both non-sequential and sequential decision procedures involving a finite number of atomless distributions and a compact decision space. Counter examples are given for the case of infinitely many distributions.

The dual formulation in terms of a zero-sum two-person game deals with the replacement of mixed strategies by pure strategies.

Publication: "Elimination of randomization in certain statistical decision procedures and zero-sum two-person games," by A. Dvoretzky, A. Wald, and J. Wolfowitz; IN MANUSCRIPT at INA.

COMPUTATION OF THE IMAGINARY ZEROS OF THE RIEMANN ZETA-FUNCTION
Project 11.1/1-50-13

Origin: NBS Authorized 6/1/50
Sponsor: Office of Naval Research, USN
Managers: J. B. Rosser, H. Luxenberg

Objective: To carry out sufficiently extensive computations of the Riemann zeta-function to suggest additional hypotheses concerning the distribution of the imaginary zeros, in the hope that some of these additional hypotheses can either be proved or disproved by mathematical methods and hence that our knowledge of this important function can be increased.

Background: A knowledge of the distribution of the zeros of the Riemann zeta-function is useful in many branches of analytic number theory. Two important hypotheses concerning this distribution have so far defied a proof or disproof. Extensive hand computations have been made, which have failed either to prove or disprove the hypotheses, but which have furnished useful information. If much more extensive computations could be made, it is believed that additional useful information would be forthcoming, including perhaps a decision concerning these hypotheses.

Status: NEW. An improved formula for computation has been worked out and is being checked and coded for the National Bureau of Standards Western Automatic Computer. Certain auxiliary functions which are needed for this formula have been tabulated in the hand-computing room.

2. Mathematical Tables

MATHIEU FUNCTIONS II
Project 11.1/2-45-1

Origin: Applied Mathematics Panel, NDRC
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

SPECIAL TABLE OF BESSEL FUNCTIONS
Project 11.1/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication continued.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

ROCKET NAVIGATION TABLES
Project 11.1/2-48-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. Preparation of the manuscript for publication continued.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLES OF $E_1(z)$, SECOND QUADRANT
Project 11.1/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Table 85% completed.

PUNCHED CARD LIBRARY
Project 11.1/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Comments: A catalog of tables on punched cards which are on file at the Institute may be obtained by addressing the Institute for Numerical Analysis, 405 Hilgard Avenue, Los Angeles 24, California. Within the limits of the program of the computation unit of the Institute, tables will be duplicated upon request, provided the requester furnishes the blank cards. Requests should be addressed directly to the Institute.

Status: CONTINUED. The following tables on punched cards were added to the library:

1. NBS Table of Spherical Bessel Functions, Vol. II. Keypunched but not checked.

2. $(z - 1)\,Z(z)$, $z = x + iy$, $x = 0(.05)2$; 6D
$y = 0(.05)4$; 6D

[$Z(z)$ is the zeta function]. This table came from the Computation Laboratory, University of Toronto, computed on the EDSAC. No checking was done here.

3. Table of integrals relating to modified Bessel-Clifford functions.

$$R(x,a) = \int_0^x e^{y/a} I_0(2\sqrt{y})\,dy, \quad \begin{array}{l} a = 4,8,\ x = 0(.08)1.6(.1)6.2,\ 6D \\ a = 3,6,\ x = 0(.06)1.5(.15)6.45,\ 6D \end{array}$$

$$S(x,a) = \int_1^x e^{y/a} K_0(2\sqrt{y})\,dy, \quad \begin{array}{l} a = 3,\ x = 0(.02)1.5(.1)10(.2)60,\ 8D \text{ or } 8S \\ a = 4,6,8,\ x = 0(.02)1.5(.1)10(.2)36 \\ \qquad (.5)20a,\ 7D \text{ or } 8D. \end{array}$$

$$\exp\left(-\frac{x}{2} - 2\sqrt{x}\right)R(x,a),\ a = 3,4,6,8,\ x = 6(.1)36 \text{ (various)}\ 20a,\ 8D.$$

These functions were computed in connection with 11.1/31-50-6.

4. Tables Relating to Powers of Complex Numbers

Define (1) $U_n(z) = (z^n/n!) = u_n(z) + iv_n(z)$

(2) $T_n(z) = U_n(z)/n = t_n(z) + i\,\tau_n(z)$

(3) $R_i(z) = \sum_{k=0}^{\infty} u_{i+4k}(z)$; $S_i(z) = \sum_{k=0}^{\infty} v_{i+4k}(z)$

$P_i(z) = \sum_{k=0}^{\infty} t_{i+4k}(z)$; $Q_i(z) = \sum_{k=0}^{\infty} \tau_{i+4k}(z)$

$i = 1,2,3,4.$

Available now, for $z = x + iy$:

$x = 0(.1)3.1$; $y = 0(.1)x$; 11D

Status of Projects

$u_n, v_n, t_n \tau_n$, up to point where these functions vanish to 11D

R_i, S_i, P_i, Q_i, i = 1,2,3,4.

3. Development of Automatic Computing Machinery

NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER (SWAC)
(Previously listed as Air Material Command Computing Machine)
Project 11.1/22-49-1

Origin: Office of Air Research, AMC, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The fabrication of all units of the computer has been completed, and all chassis have been individually checked and operated satisfactorily. Checking and testing of the computer as a whole is underway, and simple routines, making use of the entire system, have been run on the machine.

The initial input-output equipment, consisting of a Flexowriter Unit, has undergone the necessary modifications and is operating satisfactorily. The unit consists of two electromatic typewriters, one punched paper tape reader, and one paper tape perforator.

An outside case enclosing the entire computer was fabricated and installed by the Universal Sheet Metal Works of Los Angeles. This company also installed the ventilating system for the computer. The necessary room alterations were completed by the University.

The magnetic drum, constructed by the University of California at Berkeley under the direction of Professor Paul Morton, has been delivered. The design and construction of the chassis needed to integrate the drum into the computer system will soon begin.

LOGICAL NOTATION AND BLOCK DIAGRAM SYMBOLISM FOR A.D.C.M.
Project 11.1/22-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Several additions and revisions were made to the preliminary lists of terminology and block diagram symbols to be used in connection with automatic computing machines. Distribution of these lists continued to various groups and individuals interested in the automatic computer field. Comments on the lists are being continually received, and it is planned to reissue the lists, taking into consideration all such comments, as soon as it seems feasible to do so.

Preliminary lists in manuscript form of terminology and block diagram symbols to be used in connection with automatic computing machines are obtainable from the INA.

SEMI-AUTOMATIC INSTRUCTION FOR ELECTRONIC DIGITAL COMPUTERS
Project 11.1/22-50-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

Publications: (1) "Semi-automatic instruction on the Zephyr" by H. D. Huskey; to be published in the Proceedings of the Symposium held at the Harvard Computation Laboratory in September, 1949. (2) "Characteristics of the Institute for Numerical Analysis Computer", by H. D. Huskey; MTAC IV, No. 30, 103-108 (April 1950).

PROGRAMING AND CODING OF PROBLEMS FOR SOLUTION ON THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER
Project 11.1/22-50-2

Origin: NBS Authorized 6/1/50
Sponsor: Office of Air Research, AMC, USAF
Managers: Roselyn Lipkis, H. D. Huskey

Objective: To program and code certain types of mathematical problems, preparing subroutines for such operations as sorting and collating, floating operations, the elementary functions, binary to decimal conversion, etc., and routines for the solution of differential equations, simultaneous linear equations, algebraic equations, etc., for solution on the National Bureau of Standards Western Automatic Computer. The purpose of these general studies is to determine the performance to be expected from this computer on various types of problems, devise test problems to check certain of its design features in actual operation, and also to establish a library of certain basic routines and subroutines of the above-mentioned type which can then be inserted whenever called for in the coding of a problem.

Background: Coding for this computer was carried on during its entire design and construction period at the INA. This study was useful in detecting inadequacies in its original design, and in pointing the way to improvements in the system. During this formative period the original command system underwent various changes. This command system became stabilized during the previous quarter, and the necessary foundation work was completed so that the present project could be inaugurated.

Status: NEW. Diagnostic tests were coded for the arithmetic and memory units. In addition, the following subroutines were completed during the quarter:

1. Log x, with fixed binary points. The locations of the binary points in x and log x are to be specified by the main routine.
2. Binary-coded decimal conversion with fixed binary point whose location is to be specified by the main routine.
3. Floating-point arithmetic operations.
4. e^x, with floating binary point.
5. Binary-coded decimal conversion with floating point.

4. Computing Services

SOLUTION OF A DIFFERENTIAL EQUATION
Project 11.1/31-50-6

Origin: Hughes Aircraft Company
Sponsor: Office of Air Research, AMC, USAF
Manager: Gertrude Blanch

Authorized 11/1/49
Completed 6/30/50

Objective: (a) To obtain the solution and its first two derivatives of the equation

$$-\frac{d^3y}{dx^3} + \frac{2d^2y}{dx^2} - \frac{dy}{dx} + \frac{ay}{x} = \frac{ae^{isx}}{x}$$

for various values of the parameters and subject to certain boundary conditions.

(b) To tabulate three independent solutions of the homogeneous equation and its first two derivatives. (It is known that one solution is a polynomial of degree a.)

(c) To obtain the Wronskian of the three chosen solutions of the homogeneous equation and the six first minors of the Wronskian which involve $y''(x)$, where $y(x)$ is any of the three solutions.

Background: This computation arose in connection with research being performed by the Hughes Aircraft Company under contract with the Air Materiel Command.

Comments: The solutions of the equations involve intimately the "modified" Bessel-Clifford functions of order 0 and 1.

Status: COMPLETED. Tabulation of solutions of the homogeneous equation and of the Wronskian was completed. For the non-homogeneous equation, partial results were submitted for a = 4. In addition, a method for obtaining the solution for small values of x was developed by Dr. Lanczos, by use of Laplace transform theory. This was sufficient for the present needs of the Hughes Aircraft Company.

ANALYSIS OF CIRCULAR SHELL-SUPPORTED FRAMES
Project 11.1/31-50-7

Origin: Lockheed Aircraft Corporation
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. All work which could be done most conveniently by desk machines has been completed, and the results have been recorded systematically. The remaining work will be done by I.B.M. equipment, on a standby-basis. About 20% of the work has been completed.

ANALYSIS OF RAM-JET DATA
Project 11.1/31-50-10

Origin: Marquardt Aircraft Company
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Some work on this project was processed in May, and all requested computations were completed.

FLUTTER ANALYSIS, SWEPT WINGS, II
Project 11.1/31-50-11

Origin: Northrop Aircraft, Inc. Authorized 12/20/49
Sponsor: Office of Air Research, AMC, USAF Completed 6/30/50
Managers: Gertrude Blanch, E. Yowell

Objective: To find the latent roots, whose moduli exceed certain given critical values, for 66 matrices of order 6, composed of complex elements.

Background: This problem arose in connection with a research contract between Northrop Aircraft, Inc., and the Air Materiel Command. The computation is part of the problem of determining critical speeds of aircraft at which flutter phenomena appear.

Status: COMPLETED.

SOLUTION OF A SET OF SIMULTANEOUS DIFFERENTIAL EQUATIONS
Project 11.1/31-50-12

Origin: Hughes Aircraft Company Authorized 12/20/49
Sponsor: Office of Air Research, AMC, USAF Terminated 6/30/50
Manager: Gertrude Blanch

Objective: To obtain the solution of the following set of simultaneous equations:

$$(1)\quad V'' + C_1V' + C_2V = C_3Y' + C_4F(x)W' + C_5G(x)W''$$

$$(2)\quad W''' + A(x)\,W'' + B(x)\,W' = C(x)V$$

$$(3)\quad Z\,Y' = C_6 \sin(\phi - Y) - V_m(x) \sin(W - Y)$$

$$(4)\quad Z' = C_6 \cos(\phi - Y) - V_m(x) \cos(W - Y)$$

for given boundary conditions. The functions A, B, C, F, G and V_m involve exponentials; C_i and ϕ are constants.

Background: The computation arose from a research problem performed by Hughes Aircraft Company for the Air Materiel Command.

Status: TERMINATED.

COMPUTATIONS RELATING TO CRITICAL SPEEDS
Project 11.1/31-50-13

Origin: AiResearch Manufacturing Co. Authorized 3/31/50
Sponsor: Office of Air Research, AMC, USAF Completed 6/30/50
Manager: Gertrude Blanch

Objective: To find the roots τ of the equation

$$0 = \begin{vmatrix} -\alpha_1 - \frac{\tau}{\alpha_1^2} & 0 & -2\alpha_1 + \tau\left(\frac{1}{\alpha_1^2} - \frac{1}{r_1^2}\right) & 0 \\ 0 & -\alpha_2 - \frac{\tau}{\mu\alpha_2^2} & 0 & -2\alpha_2 + \frac{\tau}{\mu}\left(\frac{1}{\alpha_2^2} - \frac{1}{r_2^2}\right) \\ 3\alpha_1 + 2 & 1 & 3\alpha_1 + \frac{\tau}{r_1^2} & 0 \\ 1 & 3\alpha_2 + 2 & 0 & 3\alpha_2 + \frac{\tau}{\mu r_2^2} \end{vmatrix}$$

for various values of the parameters.

Background: This problem is in connection with research performed by AiResearch Mfg. Co. for the Air Materiel Command.

Status: COMPLETED

ROOTS OF FLUTTER MATRICES
Project 11.1/31-50-15

Origin: Douglas Aircraft Company, Inc. Authorized 3/31/50
Sponsor: Office of Air Research, AMC, USAF Completed 6/30/50
Manager: E. Yowell

Objective: To find the eigenvalues of sixteen eighth-order matrices with complex elements.

Background: These solutions are required in certain flutter analyses conducted by the Douglas Aircraft Company in connection with a contract with the USAF.

Status: COMPLETED. The two eigenvalues with largest modulus were found for each matrix. The remaining eigenvalues were not required by Douglas Aircraft. Some experimental computing, investigating methods of attacking this and similar problems, will continue as a research problem.

CALCULATION OF COEFFICIENTS OF POLYNOMIALS
Project 11.1/31-50-16

Origin: Hughes Aircraft Company — Authorized 3/31/50
Sponsor: Office of Air Research, AMC, USAF — Completed 6/30/50
Manager: Gertrude Blanch

Objective: To find the coefficients of a fourteenth-degree polynomial when given the fourteen complex roots. To be performed for a number of such sets of roots.

Status: COMPLETED.

METEOROLOGICAL MEANS
Project 11.1/31-50-17

Origin: Department of Meteorology, U.C.L.A.
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Problem 60% completed.

SLOTTED ANTENNAE
Project 11.1/31-50-18

Origin: North American Aviation, Inc. — Authorized 3/31/50
Sponsor: Office of Air Research, AMC, USAF — Completed 6/30/50
Manager: E. Yowell

Objective: Given a tabulated complex function $F(\theta)$. Required to find

$$S = F(\theta) + m_1 e^{i\psi_1} F(\theta - \tfrac{2\pi}{3}) + m_2 e^{i\psi_2} F(\theta - \tfrac{4\pi}{3})$$

for

$$\begin{matrix}\psi_1 \\ \psi_2\end{matrix} = 0\ (\tfrac{\pi}{4})\ \pi. \qquad \begin{matrix}m_1 \\ m_2\end{matrix} = .25(.25)1$$

Background: The function $F(\theta)$ represents the radiation pattern due to a single slotted cylindrical antenna. The functions S represent the radiation pattern of a cylindrical antenna with three equally spaced slots fed with varying amplitudes and phases. It is sometimes necessary to use a large body of revolution, such as a fuselage, as an antenna. This work is necessary to find a configuration yielding a good radiation pattern.

Status: COMPLETED

COMPUTING SERVICES FOR RESEARCH STAFF OF THE INSTITUTE FOR NUMERICAL ANALYSIS
Project 11.1/32-49-1

Origin: NBS
Sponsor: Office of Naval Research, USN
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Short computations of various types are made for members of research staff as required. Among those of longer range are: (1) Computations for C. Lanczos relating to network analysis and theory of integral equations (see 11.1/1-50-9). (2) Computations for J. B. Rosser. Extension of his work on the approximations to $\pi(x)$ and $\theta(x)$ for x beyond a million, in the range of Lehmer's table of primes. (3) Computations for Drs. Rosser, Hestenes, Karush involving the I.B.M. Card Programmed Calculator in theory of matrices (see 11.1/1-50-3.) (4) Computations by M. Kac, to test a method of finding eigenvalues of differential equations by sampling methods (see 11.1/1-49-PM1.)

REDUCTION OF THEODOLITE DATA
Project 11.1/32-49-2

Origin: Naval Air Missile Test Center (Point Mugu)
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Six flights were calculated during the quarter.

THE DETERMINATION OF THE PERIODS AND AMPLITUDES OF THE LIGHT VARIATIONS OF THE STARS δ SCUTI AND 12 LACERTAE
Project 11.1/32-49-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE.

DETERMINATION OF ORBITS OF COMETS, MINOR PLANETS, AND SATELLITES
Project 11.1/32-49-6

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. About 35% completed.

COMPUTATIONS RELATING TO AIR FLIGHT DESIGN
Project 11.1/32-50-2

Origin: Naval Air Missile Test Center, Point Mugu
Sponsor: Bureau of Aeronautics, USN
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The problem was coded for the I.B.M. Card Programmed Calculator. Necessary"test" calculations to test the coding were completed.

COMPUTATIONS ARISING IN THE THEORY OF HYPERSONIC FLIGHT
Project 11.1/32-50-4

Origin: Naval Ordnance Test Station, Pasadena Annex
Sponsor: Office of Naval Research, USN
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Computations involving various approximation functions were made as requested.

MOMENTS OF ORDER STATISTICS
Project 11.1/32-50-5

Origin: University of Oregon
Sponsor: Office of Naval Research, USN
Manager: E. C. Yowell

Authorized 10/31/49
Completed 6/30/50

Objective: If we define $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ and $F(x) = \int_{-\infty}^{x} f(t)dt$, to obtain:

(a) $u_1(n,j) = \int_{-\infty}^{+\infty} x\, f(x)(F(x))^{j-1}(1 - F(x))^{n-j}\, dx$,

for n=6(1)20 and $\begin{cases} j = 1(1)\frac{n}{2} ; \text{ n even} \\ j = 1(1)\frac{n-1}{2} ; \text{ n odd} , \end{cases}$

(b) $u_2(n,j) = \int_{-\infty}^{+\infty} x^2 f(x)(F(x))^{j-1}(1 - F(x))^{n-j}\, dx$,

for n=6(1)20 and $\begin{cases} j = 1(1)\frac{n}{2} ; \text{ n even} \\ j = 1(1)\frac{n-1}{2} ; \text{ n odd} , \end{cases}$

(c) $u_{1,2}(n,i,j) = \int_{-\infty}^{+\infty}\int_{-\infty}^{y} xy\, f(x)f(y)(F(x))^{i-1}(1-F(y))^{n-j}(F(y)-F(x))^{j-i-1} dx\, dy$

for n=6(1)20, j=2(1)n, i=1(1)j-1.

Values of $u_1, u_2, u_{1,2}$ for all other values $\leq$ n can be easily obtained from these values.

Background: The computations were requested specifically by Professor W. J. Dixon. A paper by H. J. Godwin entitled "Some low moments of order statistics," Ann. Math. Stat. 20, 279 (June 1949), carried out the computation for values of n through 10. Professor Dixon is interested in carrying out this computation for values of n through 20. The mean used by Godwin

is not the same as that used in our present computation. However, there are simple relations between the functions tabulated by Godwin and those tabulated by the Institute for Numerical Analysis.

Comments: A functional check on the accuracy provided by a formula due to H. L. Jones (Ann. Math. Stat. 19, 273) is being used:

$$\sum_{j=1}^{n} E(x_i x_j) = 1 , \quad i = 1,2,\ldots,n$$

where

$$E(x_j) = \frac{n!\, u_1}{(j-1)!\,(n-j)!}$$

$$E(x_j x_j) = \frac{n!\, u_2}{(j-1)!\,(n-j)!}$$

$$E(x_i x_j) = \frac{n!\, u_{1,2}}{(i-1)!\,(j-i-1)!\,(n-j)!}$$

Status: COMPLETED.

RADAR ERROR CALCULATIONS
Project 11.1/32-50-7

Origin: Naval Air Missile Test Center, Point Mugu — Authorized 3/31/50
Sponsor: " " " — Completed 6/30/50
Manager: Gertrude Blanch

Objective: To compute the values of certain combinations of trigonometric functions.

Background: This work is in connection with a theory of tracking errors developed by the sponsor.

Status: COMPLETED.

SUMS OF CROSS PRODUCTS
Project 11.1/32-50-9

Origin: Psychology Department, University of Southern California — Authorized 3/1/50
Completed 6/30/50
Sponsor: Office of Naval Research, USN
Manager: E. Yowell

Objective: Given a set of thirty-four test scores for each 284 subjects. Required to compute the cross products of each score with every other score in the set and sum these cross products over the 284 sets.

Background: These computations are required in connection with research being performed by Psychology Dept., U.S.C., under a contract with the Office of Naval Research.

Status: COMPLETED.

CONTOUR INTEGRAL
Project 11.1/32-50-10

Origin: Naval Ordnance Test Station, Pasadena Authorized 6/1/50
Sponsor: " " "
Manager: Gertrude Blanch

Objective: To evaluate the integrals

$$\int_C \left(y - s\frac{dy}{ds}\right) d\zeta$$

$$\int_C \left(x - s\frac{dx}{ds}\right) \frac{d\rho}{\rho}$$

where s is the arc-length $\rho^2 = (x - k_1)^2 + (y - k_2)^2$, $\zeta = \tan^{-1}\frac{y - k_2}{x - k_1}$

and the curves, C, are defined by various functions $y = f_i(x)$.

Background: The shape of the free water surface produced by a flat plate entering water obliquely has been deduced from experimental data obtained at Naval Ordnance Test Station. This curve contains one free parameter. It is planned to predict the value of this free parameter on the basis of the contour integrals proposed in this problem.

Comments: This is part of a general program on water entry problems being carried out at NOTS.

Status: NEW. Problem 50% completed.

ROCKET TRAJECTORIES
Project 11.1/32-50-11

Origin: Naval Air Missile Test Center, Point Mugu Authorized 6/1/50
Sponsor: " " " Completed 6/30/50
Manager: E. Yowell

Objective: To solve numerically two simultaneous second order non-linear ordinary differential equations for various values of the parameters.

Background: These solutions are required in connection with certain investigations of rocket flight being made at the Naval Air Missile Test Center.

Status: COMPLETED (NEW).

"BOILING" COMPUTATIONS
Project 11.1/32-50-12

Origin: Engineering Dept., U.C.L.A. Authorized 6/1/50
Sponsor: Atomic Energy Commission
Manager: Roselyn Lipkis

Objective: To perform certain specified computations involved in data evaluation.

Background: This work arises in connection with research performed by U.C.L.A. Engineering Dept. for the Atomic Energy Commission.

Status: NEW.

COMPUTATIONS OF POTENTIAL FLOW PAST A BODY OF REVOLUTION
Project 11.1/32-50-13

Origin: Naval Ordnance Test Station, Pasadena Annex Authorized 6/1/50
Sponsor: Bureau of Ordnance, USN
Manager: E. Yowell

Objective: To compute the potential flow past the surface of a body of revolution for various angles of attack.

Background: These results are needed by the Naval Ordnance Test Station in connection with research to determine bodies of minimum resistance. The formulae and procedures for making this calculation were specified by the Naval Ordnance Test Station. The theory of the computations is described in two British Air Ministry Reports and Memoranda, No. 2204, "A Family of Streamline Bodies of Revolution suitable for High-Speed or Low-drag Requirements," August 1945; and No. 2071, "A Simplified Theory for Streamline Bodies of Revolution, and its Application to the Development of High-speed Low-drag Shapes," July 1943.

Status: NEW. About 75% completed.

5. Training

Note: In the future training activities will not be reported under separate project headings, but will be noted in the section on "Lectures and Symposia".

CHARACTERISTIC VALUES OF MATHIEU'S DIFFERENTIAL EQUATION
Project 11.1/4-49-6

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for $be_4(s)$ were completed and partially tested.

COURSES ON AUTOMATIC COMPUTING MACHINERY
Project 11.1/4-50-2

Origin: NBS
Sponsor: Various agencies supporting the INA
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. Plans have been made to carry on an intensive course on the National Bureau of Standards Western Automatic Computer at the beginning of the next quarter, in order to assist in orienting the summer mathematical research program at the Institute.

TRAINING PROGRAM TO TEACH METHODS OF PROGRAMING AND CODING FOR THE NATIONAL BUREAU OF STANDARDS WESTERN AUTOMATIC COMPUTER
Project 11.1/4-50-3

Origin: NBS Authorized 6/15/50
Sponsor: Office of Air Research, AMC, USAF
Manager: Roselyn Lipkis

Objective: To train personnel in the procedures to be followed in the programing and coding of problems for the National Bureau of Standards Western Automatic Computer.

Background: This computer is now nearing completion, and therefore it seems advisable to train a small staff in programing and coding procedures. Insofar as possible it is planned to use present employees of the INA, now working in the IBM and hand-computing units, as coders for the automatic computer.

Status: NEW. An out-of-hours course is being held for one hour a week, with approximately twenty-five persons in regular attendance. The logical system of the National Bureau of Standards Western Automatic Computer is being explained in detail, and participants in the course are being assigned mathematical problems to code for solution on this computer.

1. Research

RESEARCH IN CLASSICAL NUMERICAL ANALYSIS
Project 11.2/11-50-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. (1) The development of equally weighted n-point quadrature formulas for integrals of the types

$$\int_0^\infty e^{-x} f(x)dx \text{ and } \int_{-\infty}^\infty e^{-x^2} f(x)dx$$

analogous to the Chebyshev quadrature formulas for finite intervals, was continued. The formulas investigated are of the form

$$(1) \qquad \int_0^\infty e^{-x} f(x)dx = \frac{1}{n} \sum_{i=1}^{n} f(x_i) \text{ , and}$$

$$(2) \qquad \int_{-\infty}^\infty e^{-x^2} f(x)dx = \frac{\sqrt{\pi}}{n} \sum_{i=1}^{n} f(x_i)$$

Given n, and any set of points $x_1, x_2, \ldots, x_n$ (which may be points of the complex x-plane), there exists a largest integer m, $0 \leq m \leq n$, such that the formula gives an exact quadrature if f(x) is any polynomial of degree $\leq m$. This integer m is called the degree of accuracy of the formula; it depends on the choice of the points x_i. In the most interesting cases, the points x_i are the roots of certain special polynomials. Two cases should be distinguished:

Case (a). It is desired that m = n. In this case, for $n \geq 2$ in (1) and $n > 3$ in (2), it is necessary that some of the roots x_i must lie outside of the interval of integration (i.e., the positive real axis in (1) and the real axis in (2)). In this case the coefficients of the polynomials with roots x_i have been determined for all $n \leq 10$.

Case (b). It is desired that all the x_i should lie in the interval of integration. In this case, given n there is an optimum choice of the x_i in the sense of giving the greatest degree of accuracy m. This optimal m has been determined for both (1) and (2) for $n \leq 10$. For each of the values of m so found, the formula of type (1) and of type (2) with least n has been determined explicitly.

(2) Note on powers of quaternions: The following connection exists between integral powers of quaternions, i.e., numbers of the form $(a_0 + ia_1 + ja_2 + ka_3)^n$, and powers of ordinary complex numbers $(a_0 + i\rho)^n$, where $\rho^2 = a_1^2 + a_2^2 + a_3^2$: If $(a_0 + i\rho)^n = \alpha + i\lambda\rho$, then

$$(a_0 + ia_1 + ja_2 + ka_3)^n = \alpha + \lambda (ia_1 + ja_2 + ka_3).$$

In addition to the demonstrations of this relation by H. E. Salzer and Olga Tuassky-Todd, a third elementary demonstration was supplied by Dr. E. W. Cannon. A manuscript giving these proofs and discussing various computational aspects of this relation in detail has been prepared.

(3) Radix table for trigonometric and inverse trigonometric functions: A practical table for 18-place accuracy was calculated and a final version of the accompanying text with illustrations was prepared.

RESEARCH IN MODERN NUMERICAL ANALYSIS: INVESTIGATION OF BERGMAN'S METHOD FOR THE SOLUTION OF THE DIRICHLET PROBLEM FOR CERTAIN MULTIPLY CONNECTED DOMAINS
Project 11.2/11-50-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The integrals

$$D_{m,n} = \iint_B v'_m(z)\ \overline{v'_n(z)}\,dx\,dy,$$

$$v'_{2p}(z) = z^{p-1},\quad v'_{2q+1}(z) = z^{-q-1},\quad p = 1(1)5,\quad q = 0(1)4,\quad z = x + iy$$

have been calculated over the hollow rectangles B for values of the parameters:

$$a = .0625,\ .25,\ .50,\ .75,\ 1;$$

$$b = 0,\ .02,\ .04,\ .06,\ .25,\ .50,\ .75,\ .94,\ .96,\ .98.$$

The elements $B_{m,n}$ of the inverse of the matrix $\|D_{m,n}\|$, $m = 2(1)10$, $n = 2(1)10$, are being computed on the IBM Card Programmed Calculator for use in the formula

$$\tilde{U}^{(10)}(\xi,\eta) = E^{(10)} R\left\{ v_1(\zeta) - \sum_{m=2}^{10}\sum_{n=2}^{10} B_{m,n}\, D_{1,m}\, v_n(\zeta)\right\},$$

where $\zeta = \xi + i\eta$, $R\{w\}$ is the real part of w,

$$E^{(10)} = -2\pi\left[D_{1,1} - \sum_{m=2}^{10}\sum_{n=2}^{10} B_{m,n}\, D_{1,m}\, D_{1,n}\right]^{-1}$$

The expression $\tilde{U}^{(10)}(\xi,\eta)$ is a first approximation to the harmonic function which assumes the value 0 on the outer boundary of B and the value 1 on the inner boundary.

RESEARCH IN MODERN NUMERICAL ANALYSIS: CONDITION OF MATRICES
Project 11.2/11-50-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Mr. Todd continued work on the paper on the condition of the system of linear equations obtained by approximating the biharmonic equation $\nabla^4 w = 0$ by its finite difference analogue.

Publication: "Note on the condition of matrices", by Olga Taussky-Todd, MTAC IV, No. 30, 111-112 (Apr. 1950).

MISCELLANEOUS STUDIES IN PURE MATHEMATICS
Project 11.2/11-50-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Dr. Abramowitz completed a paper entitled "Coulomb wave functions expressed in terms of Bessel-Clifford functions."

Dr. Abramowitz also completed a paper on, and short table of, the function $\int e^{-u^3}\,du$, which is of importance in certain heat flow problems.

Dr. Taussky-Todd continued her survey of the work of A.Scholz and began assembling material for the completion of his article on "Special number fields," as requested by the editors of Enzyklopadie d. math. Wiss.

Dr. Taussky-Todd's first paper on matrix classes and quadratic fields was completed. Further work in this direction is contemplated. She began a study of the properties of the characteristic roots of products of finite matrices, both from the algebraic and arithmetic standpoint.

Mr. Todd completed his paper on certain properties of the Laguerre orthogonal functions. He also completed a paper, written in collaboration with Professor Szasz of the Institute for Numerical Analysis on the convergence, as the sub-interval length tends to zero, of the approximating sums of an infinite integral. (This is a revision and extension of an earlier project 11.1/1-49-8, see Oct-Dec 1948 issue. The paper in question is also referred to in the present issue in project 11.1/1-50-4.)

Publications: (1) "Classes of matrices and quadratic fields", by Olga Taussky-Todd, submitted for publication in the Pacific Journal of Mathematics. (2) "On the relative extrema of the Laguerre orthogonal functions", by John Todd, to appear in Attidella Reale Accademia delle Scienze di Torino.

NUMBER-THEORETICAL TEST PROBLEMS FOR SEAC
Project 11.2/11-50-5

Origin: NBS Authorized 6/1/50
Sponsor: Office of Air Research, AMC, USAF
Manager: J.C.P. Miller

Objective: To study coding and programing problems in connection with various problems in the theory of numbers.

Background: The theory of numbers, in view of its long history and the variety of problems involved offers many opportunities for the study and development of techniques for using the facilities available with electronic computers. Number-theoretical problems are especially useful as test problems for new machines because all answers are exact, so that there is no possibility of not distinguishing between machine failures and rounding errors.

Status: NEW. Problems considered include:

(1) Factorization of numbers. Starting from an original routine prepared by Dr. Alt, which (a) factorizes a number set into the machine and types out all factors in turn, a revised routine has been prepared which as alternatives at will, either (b) lists only prime numbers from a given initial number, or (c) factorizes numbers in any given quadratic sequence $an^2 + bn + c$ (or linear sequences if $a = 0$). Various devices for reducing the time have been incorporated.

(2) The determination of haupt-exponents of 2, that is, to find the least d such that 2^d-1 is a multiple of a given number p (most interesting when p is prime). A simple routine was first prepared to provide a comparison of the performance of the SEAC with the ENIAC and with the EDSAC (Cambridge, England). The ENIAC seems to be two or three times as fast as the SEAC on this problem, and the SEAC seems to be four times as fast as the EDSAC. The original routine has since been modified to produce results more quickly.

(3) Consecutive residues. Dr. Taussky-Todd and Dr. Miller have considered the programing and coding of the following problem for the SEAC: Given a small prime ℓ, to determine all primes p of the form $2k\ell+1$ for which (a) no two ℓ^{th} power residues mod p are consecutive and (b) ℓ is not an ℓ^{th} power residue mod p.

STUDIES IN METHODS OF IMPROVING THE CONVERGENCE OF SERIES
Project 11.2/11-50-6

Origin: NBS Authorized 6/1/50
Sponsor: Office of Air Research, AMC, USAF
Manager: J.C.P.Miller

Objective: To compare and extend methods for obtaining high-accuracy numerical values from series of slow convergence, or of asymptotic and other divergent types.

Background: Much work has been done on many and various methods for obtaining sums of series which are either slowly convergent (e.g., of Dirichlet type, or ultimately approximately geometric with a common ratio approaching r, only a little less than one) or even divergent as with asymptotic series. For example, the Euler transformation is often effective, as is the "Converging Factor" introduced by Airey.

There is need for a monograph which contains accounts of as many of these methods as are useful, accompanied by plentiful numerical examples. Various new developments or extensions of old ones can also be usefully studied.

Status: NEW. A start has been made on the study of existing material, beginning with a partial account prepared by J.C.P. Miller and W. G. Bickley under the auspices of the Admiralty Computing Service in England during 1945-1948. Converging factors have been evaluated for asymptotic series appropriate to the solutions of Weber's equation for parabolic cylinder functions,

$$\frac{d^2y}{dx^2} - (a \pm \tfrac{1}{4} x^2)y = 0.$$

RESEARCH IN LINEAR PROGRAMING
Project 11.2/12-50-1

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Dr. Taussky-Todd participated in the seminar on linear programing which is being held in connection with project 11.1/1-50-11 at the Institute for Numerical Analysis. She reviewed some of the literature relevant to the von Neumann approach.

Efforts at programing the problem of finding "feasible solutions" by means of triangular or almost-triangular coefficient matrices for the IBM Card Programmed Calculator continued and appeared to be successful. The programing of this problem for the SEAC was completed and was tested for one particular case.

For the more general "minimization problem," manual test computations were completed and coding for the SEAC was started.

2. Mathematical Tables

TABLES OF $E_1(z)$, $(z = x + iy)$
Project 11.2/2-43-3

Origin: Canadian National Research Council
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Subtabulation in certain parts of the range is being attempted on the SEAC.

TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Project 11.2/2-46-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations of $\log\Gamma(x+iy)$ for $x = 9(.1)10$ and $y = 0(.1)10$ were completed. Extension of the table to include values for $x = 0(.1)9$ continued.

TABLES OF COULOMB WAVE FUNCTIONS
Project 11.2/2-47-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations were completed for $L = 0(1)5$, 10, 11, 20, 21, $\eta = -5(1)5$, $\rho = 0(.2)5$. Preparation of the final manuscript was underway.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

TABLE OF ANTILOGARITHMS
Project 11.2/2-47-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Checking of the manuscript by differencing continued.

Publication: To be published by the Columbia University Press.

TABLES FOR THE OCCASIONAL COMPUTER
Project 11.2/2-47-4

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1948 issue.

Status of Projects

TABLE OF LAGRANGIAN COEFFICIENTS FOR SEXAGESIMAL INTERPOLATION
Project 11.2/2-48-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Oct-Dec 1949 issue.

ZEROS AND WEIGHT FACTORS OF THE FIRST SIXTEEN HERMITE POLYNOMIALS
Project 11.2/2-49-1

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Roots for the first 16 polynomials were determined, and computation of the weight factors was started.

RADIX TABLE FOR CALCULATING LOGARITHMS TO MANY PLACES
Project 11.2/2-49-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A preliminary manuscript has been submitted to Dr. Rosser for inspection and comment.

TABLES TO FACILITATE SEQUENTIAL t-TESTS
Project 11.2/2-50-2

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 7 in the NBS Applied Mathematics Series.

TABLE OF CHEBYSHEV POLYNOMIALS
Project 11.2/2-50-3

Origin: NBS
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will be issued as No. 9 in the NBS Applied Mathematics Series.

PROBABILITY TABLES FOR EXTREME VALUES
Project 11.2/2-50-4

Origin: NBS, Section 11.3
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. The inverse table of the range, where the argument is the distribution function, has been completed and checked. All tables are now in form suitable for publication. Computations are underway to prepare interpolation schedules in the few places where they would be useful. The final manuscript with accompanying text was submitted to Dr. E. Gumbel, who originally suggested the compilation of these tables.

BIBLIOGRAPHY OF MATHEMATICAL TABLES AND NUMERICAL ANALYSIS
Project 11.2/2-50-5

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Objective: To prepare, and keep up to date in readily available form, information regarding a) mathematical tables, b) important publications in numerical analysis. The reference file is to indicate not only what publications exist, but also where they are available in this area. Also, c) to maintain a file of errata in mathematical tables, d) to maintain a reprint file on mathematical tables and numerical analysis, e) to maintain a file of reviews, comments, and errata of the publications of the Computation Laboratory.

Status: CONTINUED.

TABLE OF n! and $\Gamma(n + \frac{1}{2})$
Project 11.2/2-50-6

Origin: NBS — Authorized 5/21/50
Sponsor: Office of Air Research, AMC, USAF
Manager: H. E. Salzer

Objective: To prepare for publication an existing manuscript table of n! and $\Gamma(n + \frac{1}{2})$. n! is given to 16 places and $\Gamma(n + \frac{1}{2})$ to 8. The range n = 1(1)1000 is covered.

Background: This is a fundamental table for the use of statisticians, mathematicians, physicists, and engineers, as well as for computers. It will be used, e.g., for the preparation of basic input data for automatic computers.

Status: NEW. Complete manuscript copy, with introduction, was prepared for publication.

Publication: To be submitted for publication in the NBS Applied Mathematics Series.

WAVE FUNCTION FOR LITHIUM
Project 11.2/2-50-7

Origin: NBS — Authorized 6/1/50
Manager: F. L. Alt

Objective: To compute the atomic wave function of the Li^+ 1s2s state.

Background: This project, which was proposed by Professor Gregory Breit of Yale University, is intended as a test problem to demonstrate the feasibility of computing atomic wave functions by means of automatic computing machines. The particular electronic configuration chosen for this problem is interesting because the interpretation of the hyper-fine structure of Li^+ depends on accurate calculations of this configuration. Furthermore, a good deal of manual computation has previously been done on this specific problem, at the suggestion of Professor Breit, by Share, Meyerott, Luke, Geltman, and Clendenin. This should be of help in evaluating the effectiveness of the automatic machine in a project of this nature. At the same time the results obtained by machine computation are expected to be more accurate than those of manual computing.

Comments: Dr. Harold W. Woolley of the NBS Thermodynamics Section (3.2) is assisting in the direction of this project.

Status: NEW.

3. Computing Services

GUST ATTACKS ON DELTA WING
Project 11.2/31-50-1

Origin: Aircraft Laboratory, AMC, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Exploratory work was performed on additional cases prepared by the originator.

HEAT CONDUCTION EQUATION
Project 11.2/33-46-1

Origin: Bureau of Ordnance, USN
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The problem was coded for the SEAC, and one test case was successfully completed.

FOURIER TRANSFORM ADJUSTMENT COMPUTATIONS
Project 11.2/33-49-2

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Computations for various values of the parameters a_i, m_i, were performed when requested.

TABLES OF THERMODYNAMIC PROPERTIES OF GASES
Project 11.2/33-49-5

Origin: NBS, Section 3.2
Sponsor: National Advisory Committee on Aeronautics
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The following tables have been prepared: (a) Ideal gas properties -helium and NO_2, and (b) Free energy -NO_2.

FERMI FUNCTION II
Project 11.2/33-49-10

Origin: NBS, Section 4.4
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. In press.

Publication: The table is being printed by the Government Printing Office and will appear in the NBS Applied Mathematics Series.

EQUILIBRIUM COMPOSITION OF COMBUSTION GASES
Project 11.2/33-49-11

Origin: Lewis Flight Propulsion Laboratory, NACA
Sponsor: " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The partial derivatives of the composition factors with respect to each of the parameters have been computed for all cases for which the composition has previously been determined.

SHOCK WAVE PARAMETERS
Project 11.2/33-49-13

Origin: Bureau of Ordnance, USN
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

BASIC IONOSPHERIC DATA
Project 11.2/33-49-14

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

Publications: Current data are published in the monthly CRPL-F reports of the NBS Central Radio Propagation Laboratory; copy prepared from punched cards will replace hand-made copy as soon as procedures and schedules are worked out. The results of statistical computations will be included in scientific papers where appropriate.

RADIO-TELEGRAPH INTERFERENCE
Project 11.2/33-49-17

Origin: NBS, Section 14.4
Sponsor: "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Observational data were recorded and processed as received.

PERCENTAGE POINTS OF THE ARITHMETIC MEAN IN RANDOM SAMPLES FROM THE SECH AND SECH^2 DISTRIBUTIONS
Project 11.2/33-49-18

Origin, NBS, Section 11.3 — Authorized 5/3/49
Sponsor: " — Completed 6/30/50
Manager: Irene Stegun

Objective: To produce two tables, one relating to the sech distribution, and one to the sech^2 distribution, giving the values of $t_\varepsilon = u_\varepsilon \sqrt{N}$, where u_ε is the solution of

$$\int_{-\infty}^{u_\varepsilon} P_N(u)\,du = 1 - \varepsilon ,$$

for the sech distribution,

$$p_N(u) = \begin{cases} \dfrac{1}{2\cdot(N-1)!}\, u \operatorname{csch} \tfrac{1}{2}\pi u \cdot \prod_{r=1}^{n-1} \left[u^2 + (2r)^2\right], & \text{if } N = 2n \text{ is even} \\ \dfrac{1}{2\cdot(N-1)!} \operatorname{sech} \tfrac{1}{2}\pi u \cdot \prod_{r=0}^{n-1} \left[u^2 + (2r+1)^2\right], & \text{if } N = 2n+1 \text{ is odd;} \end{cases}$$

and for the sech^2 distribution

$$p_N(u) = \frac{2^N}{\pi\sqrt{3}} \int_0^\infty \left(\frac{w}{e^w - e^{-w}}\right)^N \cos \frac{uw}{\sqrt{3}}\, dw.$$

t_ε is to be tabulated to 6D for N = 1(1)15, and for (one-tail) probability levels ε = .001, 005, .010, .025, .050, .100, .200, .250.

Background: The quantity t_ε, called "percentage (or probability) point of the (standardized) mean" for given sample size N and probability ε, is related (by a simple transformation) to the value that will be exceeded with probability ε by the mean of a random sample of N independent observations from a given population. Such percentage points are of fundamental importance in statistical tests of significance and related problems. Approximate values, based on approach to normality, have served in the past when dealing with large samples but have been found inadequate for relatively small samples. Exact percentage points of the mean have previously been computed (project 11.3/1-47-1) for samples from the normal, double-exponential and rectangular distributions. Computations are now needed for samples from the distributions sech

$$p(x)\,dx = \frac{dx}{e^{\frac{1}{2}\pi x} + e^{-\frac{1}{2}\pi x}}\,, \quad -\infty \leq x \leq \infty\,,$$

and sech^2

$$p(x)dx = \frac{\pi}{\sqrt{3}} \frac{dx}{\left(e^{\frac{\pi x}{\sqrt{12}}} + e^{\frac{-\pi x}{\sqrt{12}}}\right)}\,, \quad -\infty \leq x \leq \infty\,,$$

to complete the last two papers mentioned in that project.

Status: COMPLETED. Results were transmitted to the Statistical Engineering Laboratory for use in project 11.3/1-50-1.

RATING OF WATER CURRENT METERS
Project 11.2/33-50-2

Origin: NBS, Section 6.5
Sponsor: "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations were performed as requested.

A PROBLEM IN MOLECULAR STRUCTURE, I
Project 11.2/33-50-3

Origin: Naval Research Laboratory, USN
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Computations for various values of the parameters were being performed as requested.

IONOSPHERIC WINDS
Project 11.2/33-50-7

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Data were being processed as submitted.

CRYSTAL STRUCTURES OF CEMENT COMPOUNDS
Project 11.2/33-50-9

Origin: NBS, Division 9, Portland Cement Research Associate Project
Sponsor: Portland Cement Association
Full project description appears in Oct-Dec 1949 issue.

Status: INACTIVE. Awaiting receipt of specifications of parameters.

WAVE RESISTANCE OF SHIPS, III
Project 11.2/33-50-11

Origin: David Taylor Model Basin, USN
Sponsor: " "
Full project description appears in Oct-Dec 1949 issue.

Status: CONTINUED. Computations were completed for certain specified values of the parameters and the results transmitted to the David Taylor Model Basin.

FREQUENCY DISTRIBUTION IN AN ENSEMBLE
Project 11.2/33-50-12

Origin: NBS, Section 5.2 Authorized 3/31/50
Sponsor: " Completed 6/30/50
Manager: Irene Stegun

Objective: To determine the cumulative frequency distribution, by weight, of glass beads of various sizes in a large ensemble of such beads, on the basis of stratified sampling.

Background: The ensemble of glass beads, which are of near-microscopic size, is to be used in the testing of sieves.

Comments: Requested by Dr.S. Carpenter of Section 5.2.

Status: COMPLETED. Results were transmitted to the originator.

RAY TRACING
Project 11.2/33-50-13

Origin: NBS, Section 1.6
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Additional optical systems were being evaluated as requested, by the method of "skew ray tracing" on the IBM Card Programmed Calculator. Computations by the method of third order aberrations on the same machines are in the final stages of planning. In addition, skew ray tracing was coded for the SEAC, and several test runs

were completed.

WAVE PROPAGATION IN THE IONOSPHERE
Project 11.2/33-50-14

Origin: NBS, Section 14.1
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Exploratory work was being performed.

MOLECULAR STRUCTURE CALCULATIONS, II
Project 11.2/33-50-16

Origin: Naval Research Laboratory, USN
Sponsor: "
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Computations were performed as requested.

COMPUTATION OF KEY FUNCTIONS FOR RADIATION STUDIES
Project 11.2/33-50-17

Origin: NBS, Section 13.0 Authorized 5/21/50
Sponsor: " Terminated 6/30/50
Manager: H. E. Salzer

Objective: To prepare a table of the functions

$$\frac{1}{\frac{n}{x} - \frac{H^{(2)}_{n-\frac{1}{2}}(x)}{H^{(2)}_{n+\frac{1}{2}}(x)}} \text{ and } \frac{1}{x\,H^{(2)}_{n-\frac{1}{2}}(x) - n\,H^{(2)}_{n+\frac{1}{2}}(x)}$$

for $x = 0(.25)10$, $n = 1(1)20$, to about 4S, where $H^{(2)}_{\nu}(x)$ denotes the Hankel function $J_{\nu}(x) - i\,Y_{\nu}(x)$.

Background: These functions arise in a study of radiation from a sphere. Proposed by Mr. P. R. Karr.

Status: TERMINATED (NEW). A preliminary manuscript was computed up to n = 12 and submitted to Mr. Karr.

MOMENTS OF THE DISTRIBUTION OF RANKED EXTREME VALUES
Project 11.2/33-50-18

Origin: N. Y. State Public Service Commission Authorized 5/21/50
Sponsor: National Advisory Committee on Aeronautics
Manager: H. E. Salzer

Objective: To compute the expected value and variance of the r-th ranked value in a random sample of size n from a population having the (cumulative) probability distribution of extreme values, $\exp(-\exp(.-y))$:

$$E(y_r) = C + \sum_{t=0}^{r} (-1)^t \binom{n}{t} \Delta^t \ln(n-t)$$

$$\mu_2(y_r) = \pi^2/6 + \sum_{t=0}^{r} (-1)^t \binom{n}{t} \Delta^t \left[\ln^2(n-t)\right] - \left[E(y_r) - C\right]^2$$

where Δ^t denotes the t-th forward difference, C is Euler's constant, C = .577..., and y_r is the r-th ranked value (by decreasing size, so that $y_{r+1} \leq y_r$, and y_0 is the largest value).

Background: This project complements project 11.2/2-50-4. Whereas the latter is concerned with the probability distribution of extreme values and certain related functions, the present one is concerned with moments of ranked sample values from such a distribution.

The theory of extreme values has applications in such widely diverse problems as estimation of the probability of future flood flows of large magnitude in a given watershed, the distribution of maximum age obtained in a population group over a period of time, maximum pit depths in pipe lines of gun bores or other surfaces subjected to corrosive action, of breaking strength of materials, breakdown limits of dielectrics, of maximum precipitation or other meteorological characteristics, and of maximum gust loads on airplanes.

Comments: This problem was proposed, and the formulas for computation developed by Dr. Bradford F. Kimball.

Status: NEW.

STANDARD LORAN TABLES
Project 11.2/34-50-1: Gulf Coast Chain

Origin: U. S. Navy Hydrographic Office
Sponsor: " "
Full project description appears in July-Sept 1949 issue.

Status: CONTINUED. Calculations were completed and results were submitted to the U. S. Hydrographic Office. Preparation of a final manuscript was underway.

ADAPTATION OF LORAN CALCULATIONS TO CARD PROGRAMMED CALCULATOR AND NBS AUTOMATIC COMPUTER
Project 11.2/34-50-3

Origin: Hydrographic Office, USN — Authorized 6/5/50
Sponsor: " "
Manager: J. H. Levin

Objective: To study calculations of Loran navigation tables from the point of view of adaptation to the IBM Card Programmed Electronic Calculator and to the SEAC.

Background: The present procedure for calculation of Loran tables involves the use of the type 604 IBM Electronic Calculating Punch as well as other standard IBM equipment. It is expected that the number of persons needed, the amount of card handling and the time required for the preparation of Loran tables could be materially reduced by the use of the newly acquired Card Programmed Calculator or the SEAC.

Status: NEW. Investigation of method was in progress.

TRAINER SKY WAVE COMPUTATIONS
Project 11.2/34-50-4: Aleutian Chain
Project 11.2/34-50-6: Atlantic Chain

Origin: Hydrographic Office, USN — Authorized 6/1/50
Sponsor: " " — Completed 6/30/50
Manager: B. S. Prusch

Objective: To prepare a table of sky wave corrections for Loran stations for use with Loran trainer.

Status: COMPLETED (NEW).

STANDARD LORAN TABLES
Project 11.2/34-50-5: Hawaiian Islands Chain

Origin: Hydrographic Office, USN — Authorized 6/1/50
Sponsor: " "
Manager: B. S. Prusch

Objective: Preparation of tables giving coordinates of hyperbolic lines of positions.

Background: Standard Loran Navigation tables are necessary for preparation of charts used by navigators in determining their positions with the aid of certain electronic equipment.

Status: NEW.

EFFECT OF NUCLEAR RADIATIONS ON HUMAN BEINGS
Project 11.2/35-49-1

Origin: Operational Research Office, U. S. Army (Johns Hopkins University) Authorized 2/1/49
Completed 6/30/50
Sponsor: " " " "
Manager: J. H. Levin

Objective: To systematize data related to the effect of nuclear radiations on the mortality and life expectancy of animals. An auxiliary task is the computation of the ionization produced in different tissues by incident mono-energetic radiations.

Background: The numerical work to be accomplished in this project will contribute toward a research task, being conducted by Johns Hopkins University under contract with the Department of the Army, for the preparation of tables which will show the effect of nuclear radiations on the mortality and life expectancy of human beings. The greater part of the source material was derived from experiments on mammals.

Status: COMPLETED. Results were transmitted to the Operations Research Office.

LINEAR PROGRAMING ON STANDARD PUNCHED CARD MACHINES
(formerly A Problem in Linear Programing)
Project 11.2/36-49-3

Origin: Air Comptroller's Office, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Programs were being calculated as requested by the originators.

4. Training

Note: In the future training activities will not be reported under separate project headings, but will be noted in the section on "Lectures and Symposia".

TRAINING IN NUMERICAL ANALYSIS AND MACHINE COMPUTATION
Project 11.2/4-50-1

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. The training course, "Introduction to Numer-

ical Computation," was continued with a series of ten lectures on Machine Methods of Computation (8 by F. L. Alt, 1 by J. H. Levin, 1 by M. Montalbano). These lectures were attended by 13 staff members. A number of staff members concerned with Linear Programing (projects 11.2/12-50-1 and 11.2/33-49-3) attended conferences on this subject arranged by the Air Comptroller's Office. Test coding for the SEAC was emphasized; the effort spent on it was equivalent to about three full-time mathematicians of this laboratory, plus that of three members of the Machine Development Laboratory. On the IBM Card Programmed Calculator, efforts were concentrated on the wiring of standard plugboards for performing all arithmetic operations with "floating decimal point."

III. Statistical Engineering Laboratory

(Section 11.3)

1. Research in Mathematical Statistics

STATISTICAL PROPERTIES OF SAMPLES OF THREE OBSERVATIONS
Project 11.3/1-49-1

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Preparation of the final manuscript of "Properties of statistics involving the closest pair in a sample of three observations" continued.

It is of interest that the June 1950 issue of the Annals of Mathematical Statistics (pp. 298-301) contains a note by G. R. Seth entitled, "On the distribution of the two closest among a set of three observations". This note presents a general formulation of some of the results contained in Mr. Lieblein's manuscript. It also carries a reference to the effect that Dr. Seth's attention was drawn to this problem while visiting the National Bureau of Standards in 1948 and that his work on it was motivated by the intensive study of statistical properties of the closest pair and other aspects of samples of three observations being conducted by Mr. Lieblein.

ELEMENTARY THEORY OF STOCHASTIC PROCESSES
Project 11.3/1-49-3

Origin: NBS
Manager: Henry B. Mann. (Dr. Mann, who is responsible for technical aspects of this project, undertook the project during a temporary employment by the Bureau from March to June 1949, and is expected to return for brief periods until the work is finished. During his absence Dr. Eisenhart handles inquiries and other matters related to the project.)
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A revised draft of the monograph was received from Professor Mann and was referred to a technical reader for comment.

Also received from Professor Mann, and referred to a technical reader, was a manuscript entitled, "The estimation of parameters in certain stochastic processes," to be submitted to Sankhya. The comments were forwarded to Professor Mann for consideration. This manuscript contains Professor Mann's new results on estimation of parameters of stochastic processes that form a chapter of the aforementioned monograph, which is to be submitted for publication by the National Bureau of Standards.

ESTIMATION OF LOCATION AND SCALE PARAMETERS
Project 11.3/1-50-1

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Considerable work was done on a manuscript on the distribution of the median in random samples of odd size from any parent distribution, which is a revision and extension of the earlier material on "Probability points of the distribution of the median in random samples from any continuous distribution." An abstract of the latter appeared in Ann. Math. Stat. 19, 598-599 (1948). See also Projects and Publications Jan-Mar 1949.

In this connection some research was conducted on the history of the median. For example, some of the original works of Boscovich were studied in which he (i) first proposed and utilized (1757) the principle that, when fitting $Y = a + bX$ to observational or experimental data, the parameters should be determined in accordance with the two conditions that (1) the sum of the positive deviations should be equal to the sum of the negative deviations, and (2), the sum of the absolute values of the deviations should be a minimum; and (ii) first gave in detail (1760), and illustrated the application of, a simple general procedure for obtaining these parameter values. Thus, a "weighted median", i.e., Boscovich's estimate of b, seems to have made its appearance in statistical method quite a few years before Gauss discussed (1816) the median deviation as an estimator of the "probable error" of a distribution of errors, and a century before Fechner discussed (1874) the median itself. A historical note on this work of Boscovich is in preparation.

Dr. Eisenhart spoke on "The distribution of the median in random samples of odd size from any population" at the June 2, 1950 meeting of the Bureau's Statistical Engineering Seminar.

2. Manuals of Statistical Methods

FORMULAS FOR OPERATING CHARACTERISTICS AND SAMPLE SIZES FOR CERTAIN STATISTICAL TESTS
Project 11.3/2-47-2

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STANDARD SAMPLING INSPECTION PROCEDURES
Project 11.3/2-48-1

Origin: Office of Naval Research, and Research and Development Division of the Department of the Army

Authorized 7/15/47
Terminated 6/30/50

Managers: Churchill Eisenhart, J. Lieblein

Objective: To revise, expand, and extend the present Navy Department manual "Standard Sampling Inspection Procedures" (Part D, Chapter 4, of the Administration Manual of the Material Inspection Service, U.S.N.) so as to be suitable for referencing in all types of Government specifications; particularly, in Federal Specifications.

Background: A task group of the Inspection Advisory Council of the War Department undertook during the fiscal year 1947 to effect a revision of the Navy "Procedures" to adapt them to War Department needs. Additional tables recommended by this task group were computed by Section 11.2 in that fiscal year. It was decided that further mathematical work would be needed on the entire task and that the task should be generalized to include all Federal procurement. Theoretical aspects have been assigned to Section 11.3, while responsibility for practical aspects remains in the Department of National Defense.

Status: TERMINATED. A February 1950 draft of MIL-STD-105A, the end product of the work of the Sub-Committee for Revision of JAN Standard 105, on which Dr. Eisenhart served from April-September 1949, is now circulating within the Department of Defense, for comment and approval. If adopted by the Department of Defense, such adoption will mark the end of the first leg of the journey toward the ultimate goal of this project, namely, a Government-wide standard for acceptance sampling inspection by attributes. (For the background of this document, see Projects and Publications July-Sept 1949, 45-46.)

However, several years will be needed to obtain, summarize, evaluate, and act upon the experience gained during the "trial run" of MIL-STD-105A, if adopted and promulgated by the Munitions Board as a standard for the Department of Defense. Also, in view of current rapid developments in the field of acceptance-sampling, it seems almost certain that further revisions will be needed before this standard can be considered reasonably satisfactory for the inspection agencies of the Department of Defense alone. Furthermore, before any final revision of MIL-STD-105A is undertaken, it will be desirable to await results of the research program in the field of statistical aspects of process control and acceptance inspection that has been initiated by the Research and Development Board and is to be carried out under the auspices of the Office of Naval Research, especially since one of the aims of this research program is to reach a practicable solution to the difficult problem of proper balance between the volume and costs of inspection on the one hand, and the volume and value of the product and the consequences of misclassification on the other. Once a standard for acceptance sampling inspection by attributes fully satisfactory to the Department of Defense is achieved, there will then almost certainly be a further delay before Government-wide adoption is feasible, for the following reason: MIL-STD-105A presumes, and its successor also will very likely presume, a full specification of the requirements for an ideal item in each material specification and envisages a statement regarding lot or process

quality (e.g. admissible fraction defective) for each class of defects in the contract (or possibly in the invitation for bids). In view of the present lack of experience in establishment of classifications of defects and of corresponding admissible fractions defective (AQL's, LQS's, or whatever they are called) on the part of many purchasing activities, it is difficult to see how sufficient flexibility of such tolerances can be provided in material specifications to meet the diverse needs of the various purchasing activities of the Government without, in effect, nullifying the sampling requirements of the material specifications involved. Therefore, before MIL-STD-105A or a similar document can be considered suitable for referencing in all types of Government specifications, it seems that it may be necessary to indoctrinate, train, and administer inspectors in the non-military inspection and purchasing activities of the Government in much the same manner as personnel of the various inspection agencies of the Department of Defense are currently trained and administered.

It seems advisable to terminate the present project at this time, rather than hold it in "inactive" status over long periods while awaiting progress in the aforementioned directions in other branches of the Government.

GLOSSARY OF STATISTICAL ENGINEERING TERMINOLOGY
Project 11.3/2-48-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The compilation and revision of definitions continued. Work on such terms as "accuracy", "precision", "reproducibility", "reliability", and so forth, is being closely coordinated with that of Task Group No. 9 on Precision and Accuracy of Committee E-11 of the American Society for Testing Materials. The purpose of the Task Group is to study and designate terms of this character for which it seems necessary, or desirable, to have standard definitions, and to prepare precise definitions of these terms.

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Project 11.3/2-49-1

Origin: NBS
Full project description appears in Jan-Mar 1949 issue.

Status: CONTINUED. The preparation of abstract cards continued.

GUIDE TO TABLES OF NORMAL PROBABILITY INTEGRAL
Project 11.3/2-49-3

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The manuscript was being revised in the light of technical comments received.

EXTREME-VALUE THEORY AND APPLICATIONS
Project 11.3/2-50-1

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Several meetings were held with Dr. Gumbel on technical and editorial matters in connection with the brochure. New applications of the theory were incorporated and preparation of charts was started.

As the probability tables for extreme values neared completion (project 11.2/2-50-4), conferences were held between personnel of the Computation Laboratory and of this section on the final form for publication.

MANUAL ON FITTING STRAIGHT LINES
Project 11.3/2-50-2

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Background research on the subject continued. Dr. Eisenhart spoke on "Assumption economy in the fitting of straight lines" at the April 14, 1950 meeting of the Bureau's Statistical Engineering Seminar.

3. Statistical Services

Note: In the future status reports for the following service activities will not be listed under separate project headings. These activities and similar ones arising subsequently will be summarized under a new comprehensive project for research in applications of statistical method to be initiated on July 1, 1950.

STATISTICAL STUDY OF THE FLOW OF CERTAIN STOCKROOM ITEMS
Project 11.3/31-48-4

Origin: NBS, Section M.4
Full project description appears in Jan-Mar 1949 issue.

Status: INACTIVE.

STATISTICAL ANALYSIS OF THERMOMETRIC MEASUREMENTS
Project 11.3/31-49-4

Origin: NBS, Section 3.1
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The statistical analysis of six sets of readings, covering a period of 6 months, is nearing completion.

Publication: "Comparative tests in a single laboratory" by W. J. Youden; Bul. Am. Soc. Testing Materials, No. 166, 48-51 (May 1950).

WOMEN'S BODY MEASUREMENT STUDY
Project 11.3/31-49-5

Origin: NBS, Section 12.2
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. A careful and detailed study of the body measurements of women considered to be most essential to the proper fit of a pattern or garment and their relationship to age has been made. These measurements are stature, hip girth, and bust girth. This analysis has enabled a tentative system of sizes to be set up by body types within each stature and age group. Means of 49 critical measurements for each of the tentative sizes were computed and from these it is planned, by multiple regression analyses, to determine the increment for each important fitting characteristic for a corresponding increase in stature and girth of hip and bust.

A technical committee met in Washington June 20-21, 1950 to discuss the progress of the analysis and to construct the sizing system which it will recommend to its parent group who requested the project, namely, The Sizing Committee of the Mail Order Association of America.

PRECISION MEASUREMENTS ON STANDARD TEMPERATURE SOURCES
Project 11.3/31-50-2

Origin: NBS, Section 5.0
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

PROBABILITY THEORY OF A CERTAIN DOUBLE BRANCHING SYSTEM
Project 11.3/31-50-3

Origin: NBS, Section 4.5
Full project description appears in July-Sept 1949 issue.

Status: INACTIVE.

GENERAL STATISTICAL SERVICES
Project 11.3/31-50-4

Origin: NBS
Full project description appears in Jan-Mar 1950 issue.

Status: CONTINUED. Numerous advisory services to NBS staff members were performed, for example:

(a) An allocation of specimens from a long panel of metal to a laboratory test and an outdoor test at two locations was worked out. The problem was to determine threshold stress, and gradients in properties across and lengthwise of the panel were expected. The allocation was made in such a way that sets of specimens to be tested at different loads would be as nearly alike as possible.

(b) As a result of previous work on data on high temperature alloys a paper was prepared and a talk given at the 53rd Annual Meeting of ASTM on "The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation."

(c) A large volume of analytical data obtained by spectrochemical methods was processed in the course of a study on the precision of these methods.

(d) An experiment was designed for the determination of the reaction time of fuses. The data from the experiment were subsequently analyzed.

(e) Several conferences were held in connection with problems of stockpiling strategic materials and some preliminary proposals were made for the sampling and testing of these materials.

(f) A numerical procedure that may be carried out with existing tables was developed for evaluating (a) the fraction of within-tolerance items that will be classified as outside tolerance and (b) the fraction of outside-tolerance items that will be classified as within tolerance, when the items are measured by a gage of specified precision and the items of product are normally distributed with known mean and standard deviation.

STATISTICAL SERVICES TO ARMY FIELD FORCES TEST BOARDS
Project 11.3/32-49-1

Origin: Logistics Division, General Staff, U.S.A. Authorized 7/1/48
Manager: W. J. Youden Terminated 6/30/50

Objective: To provide consultation services to the Army Field Forces Test Boards and on other projects of interest to the Logistics Division.

Background: The Test Boards are concerned with planning experimental field tests of Army equipment and weapons and the evaluation of data where the nature of the tests limits the number of the objects tested to small numbers. It is desirable to make use of modern statistical techniques in the conduct of the test programs.

Comments: This is a continuing project. Visits are made to the Boards either on request or to make available the results of studies originating as a consequence of previous visits. On occasion extended visits are made (in one case nearly a month) for the purpose of becoming familiar with the problems of the Test Boards by working alongside of

the officer personnel on current projects.

Status: TERMINATED. A manual on "Statistical methods with special reference to ordnance problems" was prepared and was being edited as the quarter ended. The manual will present techniques for handling data which are expressed qualitatively.

SOLID PROPELLANT DEVELOPMENT TESTS
Project 11.3/32-49-2

Origin: ORDTU, Research and Development Division, Ordnance Dept., U. S. Army

Authorized 7/16/48
Revised 1/7/49
Terminated 6/30/50

Manager: W. J. Youden

Objective: To advise and assist personnel of ORDTU and its contractor on statistical aspects of formulating an experimental program for determining the significant variables affecting the ballistic properties and reproducibility of certain propellants.

Background: By utilization of recent advances in the techniques of statistical inference and the principles and devices of statistical design of experiments, it is expected that economy and increased efficiency can be effected in this research and development program.

Comments: This is the first of a series of projects sponsored by the Research and Development Group, Logistics Division, General Staff, U. S. Army.

Status: TERMINATED. No further requests for assistance were made during this quarter.

PROBABILITY STUDIES OF MISSILE EFFECTIVENESS
Project 11.3/33-49-1

Origin: Bureau of Ordnance, Navy Department, via Electronics Division, NBS

Authorized 6/13/49
Completed 6/30/50

Manager: G. J. Lieberman

Objective: To develop appropriate three-dimensional multi-parameter probability models to depict the performance of certain missiles, and to study the effects of varying the parameters involved in the probability of 'success'

Background: To aid in proper selection of parameter values in missile design.

Comments: Technical liaison with the Electronics Division (Div. 13) will be provided by Dr. H. K. Skramstad, Chief of the Missile Dynamics Section (13.9).

Status: COMPLETED. The results were incorporated in two technical reports to be transmitted to the Bureau of Ordnance.

4. Training

Note: In the future training activities will not be reported under separate project headings, but will be noted in the section on "Lectures and Symposia".

STATISTICAL DESIGN OF EXPERIMENTS
Project 11.3/4-50-2

Origin: NBS, Educational Committee — Authorized 3/1/50
Managers: J. Mandel, W. J. Youden — Completed 6/30/50

Objective: To offer a three-term in-hours course in the principles and devices of the statistical design of scientific experiments and engineering tests, and associated techniques of statistical inference.

Background: A repetition, at the request of the NBS Educational Committee, on a three-term basis with additional material, of a two-term in-hours course on the same subject offered last year (see Jan-Mar 1949 issue of this publication.)

Comments: This course is listed, with the above title and designated as A116.1-A116.3, on page 8 of the Announcement of Courses for 1949-1950 of the Graduate School of the National Bureau of Standards.
The first term and a half of the course was taught entirely by Mr. John Mandel, Section 7.5, NBS; and the remainder is being taught by Dr. W. J. Youden, Section 11.3.

Status: COMPLETED. The second half of the course closed on June 6 with 14 students completing the course. Topics covered included balanced incomplete blocks, confounding, and fractional replication.

ACCURACY AND PRECISION
Project 11.3/4-50-3

Origin: NBS, Personnel Division — Authorized 3/1/50
Manager: Churchill Eisenhart — Completed 6/30/50

Objective: To present a series of three lectures on (1) the operational meaning of accuracy and precision and (2) the practical evaluation and representation of the accuracy and precision of a process of measurement; and to prepare a set of lecture notes on the material presented.

Background: A repetition, with some modifications, of material on the same subject presented last year (see project 11.3/4-49-3) a full description of which appears in the Jan-Mar 1949 issue of this publication. The lecture notes are to form a portion of "The Handbook for Professional Employees" being prepared by the Personnel Division of the National Bureau of Standards.

These lectures are to be offered as part of the general series entitled "Introduction to Research", one phase of the Junior Professional Training Program of the Bureau.

Status: COMPLETED. The three lectures were presented last quarter as previously reported. Preparation for distribution of notes on these lectures continued. Notice will be given when the notes become available.

CURVE FITTING
Project 11.3/4-50-4

Origin: NBS, Personnel Division — Authorized 3/1/50
Manager: Churchill Eisenhart — Completed 6/30/50

Objective: To present a series of three lectures on (1) the fitting of linear and curvilinear functions to quantitative data by the method of least squares and otherwise, (2) statistical techniques for judging the goodness of fit, and (3) evaluation and representation of the precision with which the parameters involved are estimated; and to prepare a set of lectures on the material presented.

Background: A repetition, with some modifications, of material on the same subject presented last year (see project 11.3/4-49-6) a full description of which appears in the Jan-Mar 1949 issue of this publication.
These lectures are to be offered as part of the general series entitled "Introduction to Research", one phase of the Junior Professional Training Program of the Bureau. The lecture notes are to form a portion of "The Handbook for Professional Employees" being prepared by the Personnel Division of the National Bureau of Standards.

Comments: The write-up of much of the substance of these lectures, together with other related material, in the form of a "Manual on fitting of straight lines", is the aim of project 11.3/2-50-2.

Status: COMPLETED. The three lectures were presented on April 7, 14, 21, with about 70 persons in attendance on each occasion.

IV. Machine Development Laboratory

(Section 11.4)

1. Design and Construction of Automatic Digital Computing Machines

Note: The machine design and construction projects are being performed in cooperation with the Electronics Division of the Bureau.

THE BUREAU OF THE CENSUS COMPUTING MACHINE
Project 11.4/21-47-1

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The new status of the Eckert-Mauchly Computer Corporation--as a subsidiary of the Remington-Rand Corporation--was the source of considerable uncertainty concerning UNIVAC production. There were extended discussions with Remington-Rand, as principal stockholder in E-MCC, on the Government UNIVAC contracts. By the end of the quarter it appeared that E-MCC would continue to manufacture the UNIVACs for which it had contracted with the Bureau.

THE NAVY COMPUTING MACHINE
Project 11.4/22-47-2

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The contractor continued work on mercury-line storage and tape-handling units, as authorized by the Bureau. Substantial progress was made on two computers under another Government contract, certain components of which are in the nature of prototypes for the Bureau-sponsored machine.

AIR MATERIEL COMMAND COMPUTING MACHINE
Project 11.4/23-49-1

Origin: Air Materiel Command, USAF
Sponsor: Office of Air Research, AMC, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. In accordance with bid procedure, a contract was executed with the General Electric Company for the construction of this computer. Work has begun on the machine by the contractor at its Electronic Park Laboratories in Syracuse, New York.

THE AIR COMPTROLLER'S COMPUTING MACHINE
Project 11.4/24-47-3

Origin: Office of the Air Comptroller, USAF
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)
Project 11.4/24-49-1

Note: This computer has been previously referred to as the NBS Interim Computer and as the NBS Automatic Computer I.

Origin: NBS
Sponsor: Air Comptroller's Office, USAF
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer was placed in operation during the quarter. The dedication date was June 20; however, the computer had been used for the solution of miscellaneous mathematical exercises and problems since April 7. Complete details are given in the announcement on page 1.

A 512-word Williams tube memory and faster input-output equipment will be added without interference with use of the computer for computation.

ARMY MAP SERVICE COMPUTING MACHINE
Project 11.4/25-49-1

Origin: Army Map Service, U.S.A.
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. This computer is being constructed by the Eckert-Mauchly Computer Corporation under a contract in the form of a supplement to the Census UNIVAC contract. The status is the same as that given for project 11.4/21-47-1.

DEPARTMENT OF THE ARMY COMPUTER DESIGN
Project 11.4/26-49-1

Origin: Department of the Army — Authorized 12/15/48
Sponsor: " " — Terminated 6/30/50
Manager: E. W. Cannon

Objective: To design a large-scale electronic digital computer which will be suited to specified needs of the Department of the Army.

Background: It was considered by the group of the Department of the Army dealing with the Bureau that an electronic computer of the EDVAC type would serve their needs along the lines of high-speed computing. Because of Dr. Lubkin's comprehensive knowledge of design features of the EDVAC type computer, from both the electronics and the logical systems viewpoint, the requested design was considered to be a feasible undertaking for the Bureau.

Comments: This project concerns the design of a system and the determination of circuit constants for a large-scale electronic computer of the EDVAC type. The work under this project and under project 11.4/24-47-3 is being conducted by the Bureau as far as possible. Division 11 is responsible for the mathematical sufficiency of the design, and Division 13 for its engineering soundness.

Status: TERMINATED. The work on this project had become limited to work common to this project and to project 11.4/24-49-1 (NBS Eastern Automatic Computer). With the placing of the SEAC in operation this project was terminated.

2. Programing Studies

PROGRAMING OF PROBLEMS FOR SOLUTION ON AUTOMATIC DIGITAL COMPUTING MACHINES
Project 11.4/3-47-4

Origin: Bureau of the Census, Department of the Navy, Department of the Air Force, and Department of the Army.
Sponsors: " " " " "
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. Work under this project during the quarter under review was devoted entirely to preparation of problems for solution on the NBS Eastern Automatic Computer (SEAC).

I. Programing of the following problems and routines was completed, and they were run successfully on the SEAC.
   1. Optical ray tracing. See project 11.2/33-50-13.
   2. Diagnostic test routine.
   3. Code for checking subroutines with fixed binary point.
   4. Demonstration problem calculating Barlow's Tables.
   5. Linear programing for the Air Force. (This work is continuing). See project 11.2/12-50-1.

6. A heat flow problem. See project 11.2/33-46-1.

II. Programing of the following problems and routines was completed, and they are being checked.
1. A problem of x-ray penetration.
2. A routine for solving 80 simultaneous linear equations in 80 unknowns.
3. A routine for inverting a matrix of order 80.
4. A routine for checking subroutines with a floating binary point.
5. A modified diagnostic test routine.
6. A routine for checking the 512 word mercury memory.

III. Analysis and preliminary coding have been completed for the following problems.
1. Calculation of Loran tables. See project 11.2/34-50-3.
2. Fourier synthesis of crystal structure. See project 11.2/33-50-9.
3. Transportation problem. See project 11.2/36-49-1, Jan-Mar 1949 issue.
4. The evaluation of elliptic functions. See project 11.2/2-43-4, Apr-Jun 1949 issue.

CODING RELATED TO THE UNIVAC SYSTEM
Project 11.4/3-49-2

Origin: The Bureau of the Census
Sponsor: " "
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see Jan-Mar 1950 issue.

CODING RELATED TO THE RAYTHEON COMPUTER
Project 11.4/3-49-2

Origin: Mathematics Branch, Office of Naval Research
Sponsor: Office of Naval Research, USN
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE. For status to date see July-Sept 1949 issue.

3. Technical Reports on Computing Machinery

THE MTAC SECTION
Project 11.4/4-47-1

Origin: Committee on High-Speed Computing of the National Research Council.
Full project description appears in Apr-Jun 1949 issue.

Status: CONTINUED. The material for the Automatic Computing Machinery

Section of the October 1950 issue of Mathematical Tables and Other Aids to Computation was assembled and edited. In addition to this material, the galley and page proofs for the April 1950 issue and the galley proofs for the July 1950 issue of the journal were corrected and returned to the editor.

BIBLIOGRAPHY ON HIGH-SPEED AUTOMATIC COMPUTING MACHINERY
Project 11.4/42-49-2

Origin: NBS
Full project description appears in Apr-Jun 1949 issue.

Status: INACTIVE For status to date see July-Sept 1949 issue.

Lectures and Symposia

Applied Mathematics Division Technical Meetings

KOONS, Florence (Bureau of Census). Statistical applications of automatic digital computing machines. April 10, 1950.

ALT, F. L. Equilibrium composition of combustion gases. April 24, 1950.

MILLER, J.C.P. (Scientific Computing Service, London). The printing of mathematical tables. May 9, 1950.

YOUDEN, W. J. Combination of probabilities. May 22, 1950.

HESTENES, M. R. (NBS and the University of California at Los Angeles). An iterative method of finding characteristic values of matrices. May 25, 1950.

Numerical Analysis Colloquium Series
(Los Angeles, California)

KAPLANSKY, I. (University of Chicago). Polynomial identities in rings. April 3, 1950.

SZASZ, Otto (NBS and University of Cincinnati). Tauberian theorems. April 17, 1950.

PINNEY, E. (University of California, Berkeley). Elastic wave problems and their computation: (1) Elastic vibrations. May 8,1950. (2) The fundamental wave types. May 9, 1950.

PRAGER, W. (Brown University). The extremum principles of the mathematical theory of elasticity and their application to the solution of boundary value problems: Part I, May 22, 1950; Part II, May 23, 1950.

JOHN, F. (New York University). Surface waves in liquids of constant depth. June 1, 1950.

DVORETZKY, A. (NBS and Hebrew University, Jerusalem). On Brownian paths. June 5, 1950.

KAC, M. (NBS and Cornell University). Brownian motion and the Dirichlet problem. June 19, 1950.

FEKETE, M. (Hebrew University, Jerusalem). Sequences of factors of invariance in the theory of Fourier series. June 26, 1950.

Statistical Engineering Seminars

EISENHART, C. On assumption economy in the fitting of straight lines. April 14, 1950.

LIEBERMAN, G.J. On the evaluation of the probability-volume of an irregular three dimensional region by Monte Carlo methods. May 12, 1950.

WILLIAMS, J.F. (Bureau of Customs). On improvement of test measurements through use of statistical methods in the U. S. Customs Service. May 19, 1950.

EISENHART, C. Distribution of the median in random samples of odd size from any population. June 2, 1950.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J.M. The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation. Presented at the 53rd Annual Meeting of the American Society for Testing Materials, Atlantic City, N. J., June 28, 1950.

DVORETZKY, A. Eliminations of mixed strategies. Presented at the Rand Corporation, Santa Monica, California, June 2, 1950.

FORSYTHE, G.E. Digital computation. Presented at the Geophysics Seminar at the University of California at Los Angeles, California, April 21, 1950.

HANDY, B.F.,Jr. The IBM Card-Programmed Calculator and some of its applications. Presented to the Mathematics Club of Howard University, Washington, D. C., April 13, 1950.

HESTENES, M.R. Mathematical applications to automatic digital computers. Presented to the Mathematics Department of Santa Barbara College, May 11, 1950.

HUSKEY, H.D. (1) Automatic digital computing machines and their applications to library work. Presented at a meeting of the Special Librarian's Association, Southern California Chapter, held at the NBS Institute for Numerical Analysis, April 22, 1950. (2) Applications of the National Bureau of Standards Western Automatic Computer. Presented at the Summer and Pacific General Meeting of the American Institute of Electrical Engineers, Pasadena, California, June 13, 1950. (3) Computing machines and their applications. Presented at a meeting of the American Society for Engineering Education, at the University of Washington, Seattle, June 20, 1950. (4) The National Bureau of Standards Western Automatic Computer. Presented at the Naval Air Missile Test Center, Point Mugu, California, June 29, 1950.

JACKSON, L.K. A generalization of subharmonic functions. Presented at the American Mathematical Society meeting, Berkeley, California, April 28, 1950.

KARUSH, W. An iterative method for finding characteristic vectors of a symmetric matrix. Presented at the American Mathematical Society meeting, Berkeley, California, April 28, 1950.

LACEY, E., D.F. RUTLAND, H.D. HUSKEY, H.T. LARSON. Design features of the National Bureau of Standards Western Automatic Computer. Presented by E. Lacey at the Summer and Pacific General Meeting of the American Institute of Electrical Engineers, Pasadena, California, June 13, 1950.

LANCZOS, C. (1) A new application of the Laplace transform. Presented at the Naval Ordnance Test Station, Pasadena, California, April 7, 1950. (2) Inversion of the Laplace transform. Presented at the American Mathematical Society Meeting, Berkeley, California, April 28, 1950. (3) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. Presented at the Mathematical Symposium, Oklahoma A and M University, Stillwater, Oklahoma, June 28, 1950.

MARDEN, E.C. Tracing of skew rays through an optical lens system, as programed for the SEAC. Presented at a demonstration of the SEAC to the NBS staff, June 23, 1950.

MILLER, J.C.P. Recurring decimals. Presented to the Mathematics Club, University of Maryland, May 23, 1950.

PEED, Patricia K. The technical secretary. Presented at Santa Monica City College, Santa Monica, California, April 19, 1950.

RHODES, I. Heat flow problem. $\theta_t = \theta_{xx} + e^{-\frac{i}{\theta}}$ as programed for the SEAC. Presented at a demonstration of the SEAC to the NBS staff, June 23, 1950.

ROSSER, J.B. Real zeros of real Dirichlet L-series. Presented at the American Mathematical Society meeting, Berkeley, California, April 29, 1950.

SZASZ, O. Gibbs phenomenon for Hausdorff means. Presented at the American Mathematical Society meeting, Berkeley, California, April 28, 1950.

TUASSKY-TODD, O. Symmetrization of systems of linear equations. Presented at the Washington meeting of the American Mathematical Society, April 28-29, 1950.

YOUDEN, W. J. (1) Biometric clinic: a discussion of practical problems submitted by pharmacologists. Presented at a meeting of the Biometric Society, held jointly with the American Society for Pharmacology and Experimental Therapeutics, Atlantic City, N. J., April 19, 1950. (2) Improving the precision of engineering measurements. Presented to the staff of the Naval Experiment Station, Annapolis, Md., April 24, 1950. (3) The examination of experimental data. Presented to the members of the IN-Training Course in

Statistics at the U. S. Naval Engineering Experiment Station, Annapolis, Md., April 27, 1950. (4) Proving and improving U.S.P. chemical assays. Presented at the U.S. Pharmacopeial Convention and Conferences on Current Problems of U.S.P. Revision, Washington, D. C., May 8,1950. (5) Designs in chemistry. Presented at the 28th Annual Meeting of the Virginia Academy of Science, Roanoke, Va., May 13, 1950. (6) "Statistical design of experiments, and sampling for testing". Presented at a staff meeting of the NBS Electrodeposition Section, May 24, 1950.

WASOW, W. (1) On random walks and eigenvalues of elliptic difference equations. Presented at the American Mathematical Society meeting in Berkeley, California, April 28, 1950. (2) Mathematical problem occurring in the theory of hydrodynamic stability. Presented at the Peripatetic Seminar at the University of Southern California, June 5, 1950. (3) Mean duration of random walks in n dimensions. Presented at Stanford University, Palo Alto, California, June 8, 1950.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) A new formula for inverse interpolation. H. E. Salzer. NBS Mathematical Table MT30. (Originally published in the Bulletin of the American Mathematical Society, August 1944.) Reissued by the National Bureau of Standards March 1950. Available from Superintendent of Documents, Government Printing Office, Washington 25, D. C., 5 cts.

(2) On the computation of Mathieu functions. G. Blanch. NBS Mathematical Table MT37. (Originally published in the Journal of Mathematics and Physics, February 1946.) Reissued by the National Bureau of Standards April 1950. Available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 5 cts.

1.3 Technical Papers

(1) A sampling method for determining the lowest eigenvalue and the principal eigenfunction of Schrödinger's equation. M.D.Donsker and M. Kac. NBS J. Res. 44, No. 5, 551-557 (May 1950). Available as RP2102 from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 10 cts.

(2) An elementary introduction to the calculus of variations. M.R. Hestenes. Math. Mag. XXIII, No. 5, 249-267 (May-June 1950). Reprints available. Also to be published as a chapter in a survey of mathematics edited by Professor Glenn James of U.C.L.A.

(3) Characteristics of the Institute for Numerical Analysis Computer. H. D. Huskey. MTAC IV, No. 30, 103-108 (Apr. 1950). Reprints available.

(4) Note on the Runge-Kutta method. W.E. Milne. NBS J. Res. 44, 549-550 (May 1950). Available as RP2101 from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., 5 cts.

(5) Inequalities concerning ultraspherical polynomials and Bessel functions. O. Szasz. Proc. Am. Math. Soc. 1, No. 2, 256-267 (Apr. 1950). Reprints available.

(6) On a summation method of O. Perron. O. Szasz. Mathematische Zeitschrift 52, 631-636 (1950). Reprints available.

(7) On the Gibbs phenomenon for Euler means. O. Szasz. Acta Litterarum ac Scient., Szeged, 12, Part B (1950).

(8) Notes on numerical analysis 2: Note on the condition of matrices. O. Taussky-Todd. MTAC IV, No. 30, 111-112 (Apr. 1950). Reprints available.

(9) Comparative tests in a single laboratory. W. J. Youden. Bul. Am. Soc. Testing Materials No. 166, 48-51 (May 1950). Reprints available.

1.4 Reviews and Notes

(1) Errata in "Tables relating to Hankel integrals of order zero," by L. Schwarz, published in Luftfahrtforschung, 20, No. 12, 341-372 (1943), and translated by J. Lotsof, Cornell Aeronautics Laboratory, May 1946. A. H. Rosenthal, MTAC IV, No. 30, 100 (Apr. 1950).

(2) Latin and Youden squares. TNB 34, No. 5, 69-71 (May 1950). Mimeographed copies available.

1.5 Miscellaneous Publications

How statistics improves physical, chemical, and engineering measurements. W. J. Youden. Revised text of a lecture, presented earlier by Dr. Youden, has been issued in mimeographed form by (and copies are available from) the Committee on Experimental Design, Agricultural Research Administration, U. S. Department of Agriculture.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION JUNE 30, 1950.

2.1 Mathematical Tables

(1) Tables of the exponential function e^x. NBS Mathematical Table MT2. Third edition in press, Government Printing Office.

(2) Table of coefficients for obtaining the first derivative without differences. H. E. Salzer. NBS Applied Mathematics Series 2. Reprinting in process, Government Printing Office.

(3) Tables to facilitate sequential t-tests. NBS Applied Mathematics Series 7. In press, Government Printing Office.

(4) Tables of the Chebyshev polynomials $S_n(x)$ and $C_n(x)$. NBS Applied Mathematics Series 9. In press, Government Printing Office.

(5) Tables for conversion of X-ray diffraction angles to interplanar spacing. H. Swanson. In press, Government Printing Office.

(6) Table of Arctangents of rational numbers. J. Todd. In press, Government Printing Office.

(7) Tables of Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments. In press, Columbia University Press.

(8) Tables relating to the Mathieu functions. In press, Columbia University Press.

(9) Table for the analysis of β spectra. To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

2.3 Technical Papers

(1) Tables of integrals of Struve functions. M. Abramowitz. Accepted for publication in the Journal of Mathematics and Physics.

(2) Machine methods for finding the characteristic roots of a matrix. F.L.Alt. To appear in the Proceedings, Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., November 1949.

(3) An analysis of the effect of the discontinuity in a bifurcated circular guide upon plane longitudinal waves. L. L. Bailin. Accepted for publication in the NBS Journal of Research.

(4) On subharmonic, harmonic and linear functions of two variables. E. F. Beckenbach. To appear in Revista, Universidad Nacional de Tucuman (Argentina).

(5) On subordination in complex theory. E. F. Beckenbach and E. W. Graham. Submitted to the Bulletin of the American Mathematical Society; also to appear in "The construction and applications of conformal maps: Proceedings of a symposium." to be published by the National Bureau of Standards.

(6) Recurrent determinants of orthogonal polynomials. Part I: Legendre and ultraspherical polynomials. E. F. Beckenbach, W. Seidel and O. Szasz. Submitted to the Duke Mathematical Journal.

(7) A method for reducing the amount of inspection. J.M.Cameron and W. J.Youden. Submitted to Industrial Quality Control.

(8) The selection of a limited number from many possible conditioning treatments for alloys to achieve best coverage and statistical evaluation. J.M.Cameron and W.J.Youden. To appear in the Bulletin of the American Society for Testing Materials.

(9) A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space. R.H.Cameron. Submitted to the Duke Mathematical Journal.

(10) Forced oscillations in non-linear systems. M.L.Cartwright. Accepted for publication in the NBS Journal of Research.

(11) The application of statistical procedures to the preparation of industrial specifications and acceptance procedures. J.H.Curtiss. To appear in the Proceedings of the International Statistical Conferences.

(12) Sampling methods applied to differential and difference equations with special reference to equations of the elliptic type. J.H.Curtiss. To appear in the Proceedings, Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, New York, November 1949.

(13) Generation and testing of random digits at the National Bureau of Standards, Los Angeles. G.E.Forsythe. To appear in "The Monte Carlo method: Proceedings of a symposium held June 29,30, July 1, 1949, in Los Angeles, California," now in press, Government Printing Office.

(14) Matrix inversion by a Monte Carlo method. G.E.Forsythe and R. A. Leibler (Naval Communications Station, Washington, D. C.). Accepted for publication in the July 1950 issue of Mathematical Tables and Other Aids to Computation.

(15) New matrix transformations for obtaining characteristic vectors. W.Feller and G.E.Forsythe. Accepted for publication in the Quarterly of Applied Mathematics.

(16) Second order determinants of Legendre polynomials. G.E.Forsythe. Submitted to the Duke Mathematical Journal.

(17) The extent of n random unit vectors. G.E.Forsythe and J.W.Tukey (Princeton University). Submitted to the American Mathematical Monthly.

(18) Applications of the theory of quadratic forms in Hilbert space to the calculus of variations. M.R.Hestenes. Accepted for publication in the American Journal of Mathematics.

(19) The method of gradients for the calculation of the characteristic roots and vectors of a real symmetric matrix. M.R.Hestenes and W.Karush. Accepted for publication in the NBS Journal of Research.

(20) Semi-automatic instruction on the Zephyr. H.D.Huskey. To appear in the Proceedings of a Symposium on large-scale digital calculating machinery, held at the Harvard Computation Laboratory in September 1949.

(21) Systems of extremals for the simplest isoperimetric problem. M. Karlin. Submitted to the Bulletin of the American Mathematical Society.

(22) An iterative method for finding characteristic vectors of a symmetric matrix. W.Karush. Submitted to the Pacific Journal of Mathematics.

(23) An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. C. Lanczos. Accepted for publication in the NBS Journal of Research.

(24) Certain Fourier transforms of distributions. E.Lukacs and O.Szasz. Accepted for publication in the Canadian Journal of Mathematics.

(25) Numerical determination of characteristic numbers. W.E.Milne. Accepted for publication in the NBS Journal of Research.

(26) Generalization of a theorem of Osgood to the case of continuous approximation. A.M.Ostrowski. Submitted to the Bulletin of the American Mathematical Society.

(27) Note on an infinite integral. A.M.Ostrowski. Submitted to the Duke Mathematical Journal.

(28) Note on Vincent's theorem. A.M.Ostrowski. Accepted for publication in Annals of Mathematics.

(29) On two problems in abstract algebra connected with Horner's rule. A.M.Ostrowski. Submitted to the American Mathematical Monthly.

(30) On a discontinuous analogue of Theodorsen's and Garrick's method. A.M.Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(31) On the convergence of Theodorsen's and Garrick's method of conformal mapping. A.M.Ostrowski. To be included in "The construction and applications of conformal maps: Proceedings of a sym-

posium," to be published by the National Bureau of Standards.

(32) Transformations to speed the convergence of series. J.B.Rosser. Accepted for publication in the NBS Journal of Research.

(33) Real roots of real Dirichlet L-series. J.B.Rosser. Accepted for publication in the NBS Journal of Research.

(34) Coefficients for polar complex interpolation. H.E.Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(35) Formulas for numerical differentiation in the complex plane. H.E.Salzer. Accepted for publication in the Philosophical Magazine.

(36) Formulas for numerical integration of first and second order differential equations in the complex plane. H.E.Salzer. Accepted for publication in the Journal of Mathematics and Physics.

(37) Checking and interpolation of functions tabulated at certain irregular logarithmic intervals. H.E.Salzer. Accepted for publication in the NBS Journal of Research.

(38) A bibliography of numerical methods in conformal mapping. W. Seidel. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(39) On positive harmonic functions and ultraspherical polynomials. W. Seidel and O. Szasz. For publication in the Journal of the London Mathematical Society.

(40) A generalization of S. Bernstein's polynomials to the infinite interval. O. Szasz. Accepted for publication in the NBS Journal of Research.

(41) Gibbs' phenomenon for Hausdorff means. O. Szasz. Submitted to the Transactions of the American Mathematical Society.

(42) On a Tauberian theorem for Abel summability. O. Szasz. Submitted to the Pacific Journal of Mathematics.

(43) On some trigonometric transforms. O. Szasz. Submitted to the Pacific Journal of Mathematics.

(44) On the relative extrema of ultraspherical polynomials. O. Szasz. For publication in Bollettino della Unione Matematica Italiana (Firenze).

(45) Classes of matrices and quadratic fields. O. Taussky-Todd. Submitted to the Pacific Journal of Mathematics.

(46) On the relative extrema of the Laguerre orthogonal functions. John Todd. For publication in Attidella Reale Accademia delle Scienze di Torino.

(47) On conformal mapping of variable regions. S.E.Warschawski. To be included in "The construction and applications of conformal maps: Proceedings of a symposium," to be published by the National Bureau of Standards.

(48) Random walks and the eigenvalues of elliptic difference equations. W. Wasow. Accepted for publication in the NBS Journal of Research.

(49) A note on the four by four Latin squares. W. J. Youden. Accepted by Biometrics.

(50) Use of statistics to determine the precision of test methods. W. J. Youden and J.M. Cameron. To be published by the American Society for Testing Materials.

(51) The incorporation of subroutines into a complete problem on the National Bureau of Standards Automatic Computer I. The staff of the Machine Development Laboratory. To appear in Mathematical Tables and Other Aids to Computation.

(52) Numerical solution of partial differential equations. E.C.Yowell. To appear in the Proceedings, Scientific Computation Forum, held by the International Business Machines Corporation, Endicott, N. Y., November 1949.

2.4 Reviews and Notes

(1) Review of "Proceedings of the Berkeley symposium on mathematical statistics and probability." C. Eisenhart. For publication in Psychometrika.

(2) A review of E.C.Berkeley's "Giant brains or machines that think." G.E.Forsythe. To appear in the Journal of Chemical Education.

(3) Note on M.T.E. 117. A Webb and Airey-Adams-Bateman-Olsson error. J.C.P. Miller. Submitted to Mathematical Tables and Other Aids to Computation.

2.5 Miscellaneous Publications

(1) Some problems for numerical analysis. (Four papers presented at the Symposia on Numerical Analysis and Automatic Calculating Machinery, held at the NBS Institute for Numerical Analysis, Los Angeles, California, June 1948). To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(2) The Monte Carlo method; Proceedings of a symposium held on June 29, 30, July 1, 1949, in Los Angeles, California, under the sponsorship of the Rand Corporation, and the NBS, with the cooperation of the Oak Ridge National Laboratory. To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(3) The construction and applications of conformal maps: Proceedings of a symposium held at the NBS Institute for Numerical Analysis, Los Angeles, Calif., June 1949. To appear in the NBS Applied Mathematics Series. In press, Government Printing Office.

(4) The role of a statistical consultant in a research organization. C. Eisenhart. To appear in the Proceedings of the International Statistical Conferences.

(5) High-speed computing and accounting. H.D.Huskey and V.R.Huskey. To appear in the August 1950 issue of the Journal of Accounting.

EXPLANATION OF PROJECT DESCRIPTIONS

The project descriptions appearing in this report are reproduced from the Project Forms used in the project control system of the National Applied Mathematics Laboratories. With a view toward making this report more useful, an explanation of certain terms used in the Project Forms is given here.

Date of Authorization. This is the date on which work on the project was authorized by the Chief of the National Applied Mathematics Laboratories.

Status. Here is given the narrative of the progress to date on the project. Certain descriptive terms are used to indicate at a glance the nature of the activity on the project during the period to which the entry applies. These terms, with their explanations, are as follows:

"NEW" means that the Laboratories made a commitment within the quarter covered by the report to work on the project.

"CONTINUED" means that the work was initiated prior to the quarter covered by the report, and was in progress during the quarter.

"INACTIVE" means that the Laboratories made a commitment prior to the quarter covered by the report to work on the project, but no work of any consequence was performed on the project during the quarter.

"COMPLETED" means that all the technical work, including the preparation of manuscripts of the final reports (if any) has been completed. In the case of tables for which the galley proof or page proof is to undergo extensive mathematical checks, the designation "Completed" is employed only after these checks have been performed.

"TERMINATED" means that, although all aspects of the objective were not achieved, it was necessary to terminate the project due to circumstances beyond the control of the Laboratories.

Publication. This entry, when it appears, gives information as to the availability, or expected availability, of the results of the project. "In Manuscript" means that the results have been written up and are available for reference at the Laboratories, and furthermore are in a form suitable for photo-offset or other means of reproduction. In the case of "Completed" projects for which manuscripts of reports are in the process of publication, further periodic entries are not made under *Status* or *Publication* to record the successive steps of the publication procedure, such as the reading of galley proofs, etc.