

NATIONAL BUREAU OF STANDARDS REPORT

9896

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION
A Semi-Annual Report
January through June 1968



U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

NATIONAL BUREAU OF STANDARDS

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Reactor Radiation—Linac Radiation—Applied Radiation—Nuclear Radiation.

¹ Headquarters and Laboratories at Gaithersburg, Maryland, unless otherwise noted; mailing address Washington, D. C. 20234.

² Located at Boulder, Colorado 80302.

³ Located at 5285 Port Royal Road, Springfield, Virginia 22151.

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

205.0

NBS REPORT

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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION FUNCTIONS

Conduct research and provide consulting services to the Bureau and other Federal agencies in various fields of mathematics important to physical and engineering sciences, automatic data processing, and operations research with emphasis on statistical, numerical and combinatorial analysis, and mathematical physics. Develop tools for mathematical work such as mathematical tables, handbooks, manuals, mathematical models and computational methods, and advise on their use. Provide training in disciplines related to these functions.

NUMERICAL ANALYSIS SECTION: The advancement of computation and the theory of numerical analysis, particularly in the development of computing algorithms, approximations to functions, and methods to facilitate the use of high speed electronic computers by subject matter specialists. Design of mathematical tables; exploratory calculations on automatic machines. Consulting services and training, and preparation of manuals in these fields. Research in underlying branches of pure and applied mathematics, such as matrix algebra, combinatorial analysis, and number theory.

OPERATIONS RESEARCH SECTION: Development and application of mathematical and computational techniques for the analysis, improvement or optimization of complex systems or activity-patterns. This includes (1) research in specific relevant areas of mathematics, such as linear programming, the theory of linear graphs, and the theory of strategic contests, (2) investigations in the art of constructing useful mathematical models of complex systems, and of obtaining information about the system by applying analytic or simulation methods, and (3) application of these techniques to selected problems, of general methodological significance, arising in the work of the Bureau or of other Government agencies lacking specialized personnel in this field.

STATISTICAL ENGINEERING SECTION: Consulting services in the application of mathematical statistics to physical science experiments and engineering tests, particularly in the design of experiments and in the analysis and interpretation of data. Research on pertinent topics in probability and mathematical statistics. Preparation of reports, manuals, tables, studies of computational methods and other aids to the application of modern statistical methods.

SYSTEMS DYNAMICS SECTION: Research and consulting in applied mathematics basic to physics and engineering, with emphasis on analysis of the dynamic behavior of complex physical systems. This involves, primarily, the development and application of techniques for solving linear and non-linear systems of differential equations and integral equations, or combinations of both. Of concern also is simulation of the behavior of physical systems by means of electronic computers using approximation techniques and semi-analytic methods. Attention is given to problems in plasma dynamics and the behavior of solid matter and multicomponent liquid systems, with emphasis on developing mathematical methods of wide range of applicability beyond the scope of the immediate problem. Investigations are carried out on the special functions encountered in the analysis and algorithms for their evaluation are prepared.

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^oOnly unclassified material is included in this report.

APPLIED MATHEMATICS DIVISION

January 1 through June 30, 1968

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*Part-time

^oOn Leave of Absence

**Postdoctoral Resident Research Associate

^{ooo}Student Trainee

***Guest Worker

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS
Manager: Morris Newman
Full task description: July - September 1954 issue, p. 1

Authorized 8/29/54

Status: CONTINUED. M. Newman and R. Brualdi have shown that the number of formally distinct terms in the permanent of an $n \times n$ circulant is given by

$$\frac{1}{n} \sum_{d|n} \binom{2d-1}{d} \varphi(n/d) ,$$

where φ is the Euler function.

M. Newman and J. Lehner have given a great number of identities for sums of the form $\sum f(k, k')$, where k, k' run over all consecutive pairs of denominators of the Farey fractions of order n . For example,

$$\sum \frac{1}{k + k'} = \frac{6}{\pi^2} (2 \log 2 - 1)n + O(\log n)^2 ,$$

$$\sum \sqrt{kk'} = \frac{32 - 3\pi}{12\pi^2} n^3 + O(n^2 \log n) .$$

M. Newman has proved that for every positive ϵ , the fundamental region of the principal congruence subgroup $\Gamma(n)$ of the modular group Γ consists of arcs of circles of radii $\geq 1/r$, where $r \leq n^{2+\epsilon}$.

M. Newman, I. Stegun, and R. Zucker have prepared FORTRAN versions of the program to solve equations exactly. M. Newman's version will produce the exact solution of any non-singular system of 100 equations or less in which the coefficients are arbitrary integers numerically less than 10^{10} , or the exact solution of any non-singular system of 50 equations or less in which the coefficients are arbitrary integers numerically less than 10^{20} . The running time on the Univac 1108 averages 1 minute for a 40×40 system, but can go as high as 6 minutes for such a system.

K. Goldberg has continued his investigation of the coefficients of the powers of iterates of formal power series. He has related this problem to one involving the powers of linear differential operators.

Status of Projects

S. Haber proved the existence of stochastic quadrature formulas for 5-dimensional cubes, of all degrees of precision. He also constructed some equal-coefficients stochastic formulas of degree 3 for cubes.

S. Haber and C.F. Osgood investigated some problems associated with the number-theoretic methods of numerically evaluating multiple integrals due to N.M. Korobov and E. Hlawka. They showed that certain diophantine estimates used in this theory were best possible; this implied that the best error estimates known in the 2-dimensional case were not obtainable from Korobov's general theory. They also proved various theorems on the existence or non-existence of quadrature formulas of certain types for some classes of periodic functions.

S. Haber began a program of calculations designed to clarify the properties of Korobov's quadrature formulas. The ultimate objective is to find the best formulas of this class, together with error bounds for them. This can be done directly only for formulas using relatively few — up to about 1,000 — points. Programs to do this were written and checked out, and used to calculate some best formulas using 101 or fewer points. Calculations will be continued at the University of Maryland, due to unavailability of time on the NBS computer.

Formulas which are "best" relative to a certain subclass of all Korobov-type formulas were previously calculated by Saltykov, in Russia, Saltykov's calculations were extended, and somewhat improved; and the results compared to the best formulas mentioned above.

F.W.J. Olver has completed an error analysis of the Laplace method for determining asymptotic expansions of definite integrals. Applications have been made to Bessel, gamma and other special functions, and new error bounds have been computed.

Publications:

- (1) Semi-groups with zerooids. K. Goldberg. To appear in the Journal of Research NBS.
- (2) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.
- (3) The l.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.
- (4) On a theorem of Piatetsky-Shapiro and evaluation of multiple integrals. S. Haber and C.F. Osgood. Submitted to a technical journal.
- (5) A combination of classical and Monte Carlo methods for evaluating multiple integrals. S. Haber. To appear in Bull. A.M.S.
- (6) On the sum $\sum \langle \alpha n \rangle^{-t}$. S. Haber and C.F. Osgood. Submitted to a technical journal.
- (7) Maximal normal subgroups of the modular group. M. Newman. To appear in Proc. Amer. Math. Soc.
- (8) Solving equations exactly. M. Newman. J. Res. Nat. Bur. Standards 71B, 171-179(1967).

Status of Projects

- (9) Pairs of matrices generating discrete free groups and free products. M. Newman. Michigan Math. J. 15, 155-160 (1968) .
- (10) A bound for the number of conjugacy classes of a group. M. Newman. J. London Math. Soc. 43, 108-110 (1968).
- (11) Bounds for the number of generators of a finite group. M. Newman. J. Res. Nat. Bur. Standards 71B, 247-248 (1967).
- (12) A diophantine equation. M. Newman. J. London Math. Soc. 43, 105-107 (1968).
- (13) Matrix representations of groups. M. Newman. To appear as AMS-60.
- (14) An extension of Miller's algorithm. F.W.J. Olver. Aplikace Matematiky, vol. 13, pp.174-176 (1968).

2. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Authorized 10/1/62

Status: CONTINUED. Work has begun on preparation of a revised and expanded edition of R. C. Bose, W. H. Clatworthy, and S. S. Shrikhande, Tables of Partially Balanced Designs with Two Associate Classes, Technical Bull. No. 108, North Carolina Agricultural Experiment Station, Raleigh, North Carolina, 1954. The new edition of the tables will be published by the Bureau as a volume in the NBS Applied Mathematics Series.

Professor Clatworthy (State University of New York, Buffalo) will be in charge of the work. The project is a joint effort involving the Bureau, and the University of North Carolina.

Joan R. Rosenblatt has obtained preliminary results in an investigation of the effect on analysis of straight line data of a "cumulative error" component of variance in addition to the "measurement error" variance. The model studied is

$$Y_{i+1} - Y_i = \beta(x_{i+1} - x_i) + e_{i+1} + (d_{i+1} - d_i),$$

$i = 1, \dots, n-1$, where β and the x_i are fixed constants (β unknown) and the e_i and d_i are mutually independent random variables with zero means and variances $(t_i - t_{i-1})\sigma_e^2$, σ_d^2 , respectively.

John Mandel (J. Amer. Statist. Assoc., 1957) defined the two extreme cases, Model A ($\sigma_e^2 = 0$) and Model B ($\sigma_d^2 = 0$) and investigated the properties under Model A(B) of statistical procedures appropriate for B(A). The new results include Mandel's results as extreme cases. Estimates of the variance components σ_e^2 and σ_d^2 are studied.

H. H. Ku and S. Kullback (George Washington University) have applied the procedure of minimum discrimination information estimation to five examples of contingency table analysis given by Joseph Berason in his paper in Biometrics, Vol. 24, No. 1, 1968. The results are compared with those given by Grizzle using maximum likelihood and those given by Berkson using minimum logit χ^2 .

Publications:

- (1) Calibration designs based on solutions to the tournament problem. R. C. Bose and J. M. Cameron. NBS J. Research B. (Math. and Math. Physics), 71B (1967) 149-160.
- (2) The distribution of the sample correlation coefficient with one variable fixed. D. Hogben. NBS J. Research B. (Math. Sciences), 72B (1968) 33-36.
- (3) The distribution of the sample variance from a two-point binomial population. D. Hogben. Submitted to a technical journal.
- (4) For samples of size 3 from a normal distribution the range and the mean deviation have the same distribution except for a scale factor. B. L. Joiner and J. R. Rosenblatt. Submitted to a technical journal.
- (5) A guide to commonly used terms and expressions of imprecision, systematic error, and uncertainty associated with a reported value. H. H. Ku. To appear in Measurement and Data.
- (6) Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.

- (7) The problem of interaction in multidimensional contingency tables, an information theoretic approach. H. H. Ku. To appear in NBS J. Research B. (Math. Sciences).
- (8) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Biometrics, 23 (1967) 717-737.

3. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF

NBS RESEARCH AND TESTING

Task 13911-61-2050950/51-1

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following.

A series of 15 seminars emphasizing the application of statistical techniques in the design, analysis and interpretation of scientific data was conducted. As a result of collaboration with members of the Analytical Chemistry Division, the value of such a series was recognized by B. L. Joiner, who presented many of the lectures and arranged for talks by other members of the Bureau staff. Details of the topics covered will be found below under "Lectures and Technical Meetings." Average attendance during the series was about 50, with participation from many Bureau divisions.

Two members of the staff were invited to present lectures as part of Seminars sponsored by Bureau calibration laboratories. H. H. Ku talked on "Statistics in Counting" in the Precision Measurement Seminar on Radiation Quantities. J. M. Cameron participated in the Precision Measurement Seminar on Length Measurements.

Janace A. Speckman collaborated with C. S. McCamy and S. R. Wiley (Image Optics and Photography Section) in preparation of a paper reporting results of a survey of blemishes on processed microfilm. Descriptions of blemishes had been obtained by microscopic inspection of 7411 rolls of film and were analyzed in conjunction with data on the processing and storage histories of the film.

Publications:

- (1) A statistical consultant's view of OMNITAB. D. Hogben. Appendix II in revised edition of NBS Handbook 101, OMNITAB, A Computer Program For Statistical and Numerical Analysis, U. S. Government Printing Office, January 1968.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies
Manager: J. M. Cameron
Full task description: January - March 1958 issue, p. 45

Authorized 3/31/58

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

A four-day Seminar on Precision and Accuracy in Measurement and Calibration was presented to metrologists from 23 industrial and 10 governmental laboratories. The Seminar dealt with the concepts of a National Measurement System as presented by Dr. Shirleigh Silverman, Associate Director for Academic Liaison, and of the need for viewing measurement as a production process. These concepts were developed in presentations by Churchill Eisenhart, NBS Senior Research Fellow and in talks by Paul E. Pontius of the Mass and Volume Section and J. M. Cameron of the Statistical Engineering Laboratory.

Lectures were presented on Propagation of error in a chain of laboratories (by Joan R. Rosenblatt), Calibration curves (by David Hogben), Accuracy ratios (by H. H. Ku), Design of experiments and interlaboratory comparisons (by J. M. Cameron) and Control chart techniques (by B. L. Joiner).

The participation of personnel from Bureau laboratories apart from the Statistical Engineering Laboratory added considerably to the value of the program. Woodward G. Eicke of the Electrochemistry section presented some new methods for intercomparing groups of voltage standards and maintaining surveillance over the measurement process involved. Robert C. Raybold of the Mass and Volume section presented an account of measurement problems in photometry and steps in the evaluation of process parameters. Dr. Henry J. Kostkowski, of the Radiation Thermometry Laboratory presented material on the accuracy of temperature measurements, and Thomas L. Zapf of the Radio Standards Laboratory in Boulder discussed problems in radio frequency measurements.

J. M. Cameron participated in a joint Army-Navy-Air Force task group on electrical measurements held at Newark AFS, Ohio. As a result of the meeting collaboration between Norman Belecki of Newark AFS, Robert McCallum, Sandia Corporation and W. G. Eicke of the Electrochemistry section of NBS was begun to extend the ideas of the PILOT program (interlaboratory collaboration to establish performance characteristics of measurement processes) in mass voltage measurements.

Joan R. Rosenblatt assisted the staff of a Presidential Task Force on Communications by using the OMNITAB program to prepare a variety of tables and least squares computations.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS

AND RELATED FIELDS

Task 20540-12-2050141/55-57

Origin: NBS
Manager: H. J. Oser

Authorized 9/1/54

Full task description: July-September 1954 issue, p. 27

In collaboration with Dr. R. deWit of the Lattice Defects and Microstructure Section (312.03), J. T. Fong continued to investigate the geometric foundation of several continuum theories of dislocations such as those of Kondo (1952), Bilby, Boullough and Smith (1955), Kroner (1955-60) and deWit (1967), with the goal of unifying them by means of three basic notions in Ricci calculus, namely, the definition of a covariant basis, the notion of a point transformation in a geometric manifold, and the associated concept of the dragging along a coordinate system. The first notion enables one to deduce the basic geometric equations in each theory by appealing to the theory of a totally integrable system of first order partial differential equations. Based on the remaining two notions, the concept of a Lie derivative representing the time rate of change of an object field with respect to a contravariant velocity vector field for infinitesimal deformations is shown to be the natural tool for the study of motions in a continuum of distributed dislocations.

A talk entitled "On the application of dislocation theory to engineering problems" was given by Dr. Fong, February 19, 1968 at the Westinghouse Research Laboratories Lecture Series in Mechanics of Materials, Westinghouse Electric Corporation, Churchill Borough, Pittsburg, Pa.

Dr. George B. DeLancey joined the then Mathematics Physics Section under a Postdoctoral Research Associateship on November 22, 1967. Dr. DeLancey has focused his primary attention on the numerical solution of coupled heat and mass transfer problems with nonlinear reaction kinetics. The flux expressions are based on the postulates of irreversible thermodynamics.

A scheme for integrating the diffusional contribution to the transfer process has been developed which permits the use of a maximum time step in each of the coupled equations. Each time step depends upon the associated eigenvalue in the transport coefficient matrix. Work is continuing on (i) an appropriate mesh configuration in spherical coordinates to account for the singular coefficient in the transport equations at the origin: (ii) accurate approximations for the reaction velocities.

The theoretical assumptions associated with existing and new techniques for measuring cross diffusion coefficients in both isothermal and nonisothermal systems are being investigated.

A paper entitled "An Analysis of Nonisothermal Multicomponent Mass Transfer" was presented by Dr. DeLancey at the meeting of the American Institute of Chemical Engineers in Tampa, Florida. Open seminars based on this presentation were conducted within the Systems Dynamics Section.

Publications

- (1) DeLancey, G. B., and S. H. Chiang, "The Dufour Effect in Liquid Systems", to appear in A. I. Ch. E. Journal.
- (2) DeLancey, G. B., and S. H. Chiang, "Nonisothermal Multicomponent Diffusion; Gas Absorption," to be submitted to A. I. Ch. E. Journal.
- (3) DeLancey, G. B. and S. H. Chiang, "Coupled Diffusion and Heat Transfer with Linear Kinetics," to be submitted to Chem. Eng. Sci.
- (4) A manuscript entitled "On the coupling of longitudinal and transverse waves in a linear three-element viscoelastic string subjected to transverse impact," by Dr. J. T. Fong with co-author J. C. Smith of the Polymer Interface Section (311.07) has been accepted for publication by the J. of Research NBS (B: Mathematics and Mathematical Physics).

PLASMA RESEARCH

Task 20504-12-2050140/59-442

Origin: NBS
Manager: H. J. Oser

Authorized 10/31/61

Full task description: October-December 1961 issue, p.12

Status: CONTINUED. Dr. W.L. Sadowski and Mrs. Z.G. Ruthberg examined the problem of truncation in Fourier-Hermite space of the solution of the non-linear Vlasov equation. The approach led to a combinatorial study of a two-term non-linear differential operator. A manuscript on the general properties of such an operator is in preparation. This combinatorial problem has also been studied on the computer with the aid of the LISP language in cooperation with Mrs. E. Fong (System Research and Development Division). Specific application of the combinatorial technique to the Vlasov operator has been studied. The problem is at the point where FORTRAN programming is feasible and will be carried out.

A study of the application of rational function approximations to the eigenfunction has been initiated. This work is in progress.

A study of the properties of the Hermite expansion for the two-stream problem has been completed. Temperature renormalization has been examined and found not to have significant effect on numerical stability for the values of the non-linearity parameters studied so far. It will, however, be significant in the future for higher values of non-linearity. A manuscript on the work is in preparation.

5. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS

Manager: Alen J. Goldman

Authorized 12/30/60

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued heavy. Section staff provided such services in 80 recorded instances; of these 47 involved assistance to NBS staff, covering 7 NBS divisions or other subunits. The 80 instances totalled to 635 recorded man-hours. Other agencies assisted included National Institutes of Health, Atomic Energy Commission, Bureau of Public Roads, U.S. Information Agency, Agency for International Development, and the Bureau of Prisons. Requests from universities, industries, professional groups and journals were also met.

(2) J. Gilsinn, W. Horn and A.J. Goldman continued assistance in a study to determine locations and sizings for inland marine cargo consolidation centers. Goldman and L.S. Joel continued assistance to the Department of Housing and Urban Development in connection with systems analyses of urban transport. (Reported here for convenience; supported under Projects 4314422 and 2050625 respectively.)

(3) W.G. Hall continued assistance in developing a simulation model for a highway maintenance depot. He also continued helping the Army Data Field Systems Command in planning, design and numerical analysis aspects of a proposed new tactical artillery-fire control system. He initiated assistance to the National Highway Safety Bureau in planning and evaluating the data-systems elements of its activities. (Reported here for convenience; supported under Projects 4565425 and 2050625 respectively.)

(4) P.R. Meyers continued the study of "contractification" of self-mappings of metric spaces by remetrization, obtaining conditions for the simultaneous contractification of a finite family of commuting maps. A.J. Goldman investigated what probability distributions, of inputs per period to a "unitization" facility, would yield a time-invariant distribution for the fractional residues per period.

Publications

- (1) K. Goldberg (205.01), A.J. Goldman, and M. Newman (205.01). The probability of an equilibrium point. Journal of Research NBS, 72B (1968), pp. 93-101.
- (2) A.J. Goldman. Fractional container-loads and topological groups. To appear in Operations Research.
- (3) W.A. Horn. Some fixed point theorems for compact maps and flows in Banach spaces. Submitted to a technical journal.
- (4) P.R. Meyers. Contractive semigroups and uniform asymptotic stability. Submitted to a technical journal.

COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full task description: October-December 1964 issue, p. 3; April-June 1962 issue, p.15

Status: CONTINUED.

J. Edmonds wrote a draft of "A labeling method for maximum network flows which is bounded by a polynomial in the number of nodes" and together with R. Karp of IEM, developed other improvements in network flow methods. He rewrote "Matroids and the greedy algorithm" for publication in the proceedings of the recent Mathematical Programming Symposium. He lectured on "Topics in combinatorics and optimization" at the 3rd Waterloo Combinatorics Conference. He continued various studies on matroids, lattices, polyhedra, subisomorphisms and combinatorial algorithms.

Publications

- (1) J. Edmonds, and D.R. Fulkerson (RAND Corporation). Bottleneck extrema. To appear in J. Combinatorial Theory.
- (2) J. Edmonds. Systems of distinct representatives and linear algebra. Journal of Research NBS, 71B (1967), pp. 241-245.
- (3) J. Edmonds. Optimum branchings. Journal of Research NBS, 71B (1967), pp. 233-240.
- (4) J. Edmonds. Matroid Partition. In Mathematics of the Decision Sciences, Amer. Math. Soc. Lectures in App. Math. vol. 11, pp. 335-345. (Paper (3), above, also appears in this volume, pp. 346-363.)

LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full task description: October-December 1960 issue, p. 3

Status: CONTINUED.

A.J. Goldman and P.R. Meyers continued studies on the minimax error selection of incompletely specified discrete probability distributions, with "error" given by the L_1 norm. For univariate distributions, two manuscripts were prepared dealing respectively with (a) componentwise bounds and (b) rank ordering as the "incomplete specification". As a by-product, Goldman solved the problem of choosing a non-negative matrix (x_{ij}) , with prescribed row and column sums, to minimize $\max_{i,j} x_{ij}$.

Publications

- (1) A.J. Goldman. The minimax transportation problem. To appear in Transportation Science (Letters to Editor).

MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2058456

Origin: Technical Analysis Division, NBS

Authorized 3/1/66

Sponsor: Northeast Corridor Transportation Project, Dept. of Transportation

Manager: A.J. Goldman

Full task description: January-June 1966 issue, p. 14

Status: CONTINUED (from 2050456)

(1) J. Gilsinn, A.J. Goldman and W.A. Horn prepared roughly 25 memoranda on various phases of the Northeast Corridor Transportation Project, including reviews of several contractors' reports.

(2) P. Saunders is carrying out conversion and adaptation of MIT scheduling research for application to high-speed trains in the Corridor, including (a) extension of the methods to cover 2 classes of service (local and express) and (b) integration of the algorithm with the demand forecasting models used elsewhere in the Project.

(3) C. Witzgall (with J. Gilsinn) completed a classification survey of labelling methods for shortest-path problems, including flow-charts for over 20 variants. J. Gilsinn is carrying out computational experiments aimed at comparing the performances of several of these algorithms on several types of networks.

(4) J. Levy and M.H. Pearl continued the development of methods for evaluating feedback vs. non-feedback methods for regulating flow at a merge point in a transport network. Several working papers on this subject have been prepared.

(5) A.J. Goldman (with R. Ku of the NBS Technical Analysis Division) performed logical analyses and numerical experiments with variants of several demand forecasting models prepared by contractors. Considerable progress was achieved in obtaining improved model versions. Computational simplifications for other models were determined. An analysis was made of a "consumers' surplus" approach to evaluating user benefits from transport improvements. L.S. Joel is serving as technical monitor on another demand modelling contract.

Publications

- (1) A.J. Goldman, P.R. Meyers, J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.). Analysis of a market split model. *Journal of Research NBS*, 72B (1968), pp. 43-60.
- (2) A.J. Goldman and G.L. Nemhauser (Johns Hopkins U.). A transportation problem transformable to a best-path problem. *Transportation Science* 1 (1967), pp. 295-307.
- (3) A.J. Goldman. Evaluating user benefits from transport improvements. NBS Report 9809 (April, 1968).
- (4) W.A. Horn. Allocating service periods to minimize average delay. *Journal of Research NBS*, 72B (1968).
- (5) W.A. Horn. Optimal networks connecting N points in the plane. To appear in Proceedings of Fourth International Symposium on Traffic Flow (Karlsruhe, Germany; June, 1968).
- (6) C. Witzgall. On labelling algorithms for determining shortest paths in networks. NBS Report 9840 (May, 1968).

RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING

Task 20500-12-2050404/651456

Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch
Full task description: January-June 1964 issue, p. 19

Authorized 1/21/64

Status : CONTINUED. During this period the Q32 computer at Systems Development Corporation has become our main research tool giving us the experience to help in the planning and design of other systems. Thus the planning for the image processing facility at the Division of Computer Research and Technology at NIH has been largely influenced by the kind of image processing that we have been doing on the Q32 in LISP. The LISP implementation on the Q32 does not provide any special image processing facilities but rather general-purpose list structure manipulation capabilities. Consequently we added to this LISP system several functions using the assembler LAF to provide basic image processing capabilities at an efficiency greater than that provided by the LISP language. All of these functions are embedded in LISP and hence usable as ordinary LISP functions. The basic programs written include:

- a. A program for computing arbitrary Boolean functions of two binary image planes.
- b. A program for computing an arbitrary Boolean function of every point in a single binary plane expressed as a function of arbitrary neighbors of that point.
- c. A function for counting neighborhood points of each point in a plane and thresholding the resultant counts.
- d. A program for shifting a plane by a specified x and y.
- e. A program for isolating an arbitrary set of connected points in a plane, hence for selecting out continuous connected objects or blobs.

Using the basic functions, high level functions were written in LISP that enabled us to get density histograms of discrete objects in pictures, programs that enabled us to experiment with various types of binary threshold operations on the multiple level pictures, and programs to select arbitrary sections of images for printing display on the Teletype printer. These programs were used in conjunction with inputs on magnetic tape that contained complete scans for a piece of cortex tissue.

Since there was no LISP available locally at NBS a small interpreter was written for the 1108 Univac to enable LISP programs to be run. This interpreter is presently not complete, although most programs not involving numerical computation can be run.

Plans are in the final stages for the connection of a galvanometer scanner for the Linc-PDP-8 computer in Dr. Lipkin's laboratory. Preliminary investigation of the scanner and suitability of connecting it to the machine has been completed as well as has some design on the actual interconnection hardware. During the coming fiscal year this will provide a major tool in scanning images directly through the microscope with programmable light sources as well as microscope and stage control. Since the images to be produced will be available in magnetic tape form, all the image processing capabilities developed on the Q32 computer can be devoted to the processing of these microscope images.

Hardware was designed and constructed for the connection of an NIH microspectrophotometer with controlled monochromator light source for connection to a Linc-PDP-8 computer. When this connection is complete the microspectrophotometer will be under the complete control of the computer with direct readings obtainable through analog to digital conversion hardware in the computer.

MATHEMATICAL TABLES

Task 20500-40-2050121/57-216

Origin and Sponsor: NBS

Manager: I. A. Stegun

Full task description: July-December 1964 issue, p.4

Status: CONTINUED. A correction list and updating bibliographic material are continually being maintained for all published volumes. Errata are corrected in subsequent printings. A correction list was submitted for a seventh printing of AMS 55- the Handbook of Mathematical Functions.

Assistance in the evaluation or modification of computing techniques, identification of functions and their tabulation continued at the rate of about 10 consultations per month to NBS divisions, other government agencies, industry and universities.

In the course of examination of computing algorithms for special functions, R. Zucker discovered various pitfalls and inefficiencies in the 1108 FORTRAN library subroutines. For example, there was no extension of the range of argument in the double precision subroutine for the hyperbolic sine and cosine. The tangent subroutine involved both the sine and cosine subroutines - in each of which the angle is reduced to the first octant and then a seventh or eighth degree polynomial evaluated.

Lectures and Technical Meetings

Seminar on Statistics for Chemists and Physicists

- JOINER, B. L. The utilization of basic statistical methods in the interpretation of data. January 17.
- JOINER, B. L. The use of OMNITAB for statistical calculations. January 24.
- KU, H. H. (and Churchill Eisenhart, Office of the Director). Reporting results and the expression of uncertainties. January 31.
- EISENHART, C. (Office of the Director). Realistic evaluation of precision and accuracy. February 8.
- JOINER, B. L. Basic considerations in the planning of experiments. February 21.
- JOINER, B. L. The use of control charts in checking measurement stability. February 28.
- JOINER, B. L. Functional relationships fitted to data by the method of least squares. March 6.
- JOINER, B. L. Basic concepts in the "analysis of variance." March 13.
- KU, H. H. Propagation of error. March 20.
- JOINER, B. L. Basic distributions: Gaussian, Student's t, F, etc. March 27.
- MANDEL, J. (Office of the Director, Institute for Materials Research). The analysis of data in the evaluation of materials and instruments. April 3.
- MANDEL, J. (Office of the Director, IMR). The analysis of data in the evaluation of materials and instruments, continued. April 10.
- JOINER, B. L. Some problems arising in testing for homogeneity of materials. May 1.
- ROSENBLATT, J. R. Some techniques that do not depend on the assumption of normality. May 8.
- MANDEL, J. (Office of the Director, IMR). The conduct and analysis of interlaboratory studies and the role of standard reference materials. May 15.

Mathematics Division Expository Seminar

- NEWMAN, M. How to approximate numbers. February 28.
- OLVER, F. W. J. Numerical use of recurrence relations. March 27.
- KU, H. H. The analysis of categorical data in cross tabulations. May 1.
- KIRSCH, R. A. Theoretical limitations of computers and some practical consequences. May 22.
- GOLDMAN, A. J. Analysis of a model for market split. June 26.

Applied Mathematics Division Lectures

- SHISHA, Dr. Oved (Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio). Bounds for ratios and differences of means. June 10.
- LOUBALL, P. (University of California at Berkeley). A mathematical model for trip distribution. February 8.

NBS In-hours Courses Given by Staff Members

HALL, W. Analysis I. A beginning course in analytic geometry and calculus.
HOGBEN, D. Introduction to mathematical and statistical analysis of laboratory data.
PEAVY, Sally T. FORTRAN programming for beginners.
VARNER, Ruth N. New topics in FORTRAN.

Papers and Invited Talks

Presented by Members of the Staff

at Meetings of Outside Organizations

DELANCEY, G. B. An analysis of non-isothermal multicomponent mass transfer. American Institute of Chemical Engineers, Tampa, Fla., May 22.
EDMONDS, J. A labeling method for maximum network flows which is bounded by a polynomial in the number of nodes. Mathematics Department, Princeton University, Princeton, N. J., February 20.
Topics in combinatorics and optimization. Third Waterloo Conference, Ontario, Canada, May 19-31.
FONG, J. T. On the application of dislocation theory to engineering problems. Westinghouse Electric Corporation Research and Development Center, Pittsburgh, Pa., February 19.
GOLDMAN, A. J. The probability of an equilibrium point. Operations Research Society of America. Los Angeles, Calif., May 1-3.
Konigsberg's bridges and the traveling dynamiter. Joint Board on Science Education's Conference on Mathematics Teaching. Naval Ship Research and Development Center, Carderock, Md., January 20.
HOGBEN, D. OMNITAB for UNIVAC 1108. Social Security Administration, Baltimore, Md., February 29.
KU, H. H. Contingency tables with fixed marginals. University of Maryland, College Park, Md., March 27.
OLVER, F. W. J. The numerical solution of difference equations. Department of Mathematics, Geneva College, Beaver Falls, Pa., April 1.
ROSENBLATT, J. R. Analysis of straight line data: The effect of a cumulative error component. 4th Monmouth Conference on Statistics and Quality Assurance, West Long Branch, N. J., April 20.

Systems Dynamics Section Seminar

LIPPS, F. B. (Geophysical Fluid Dynamics Laboratory, Environmental Science Services Administration.)
Numerical experiments in thermal convection. April 3.
RIMON, Y. (Naval Ship Research and Development Center.)
Solution of the Navier-Stokes equation for incompressible flow over a disc or sphere. May 15.
LUGT, H. J. (Naval Ship Research and Development Center.)
The final period of decay of arbitrary localized disturbances in a viscous fluid. June 19.

PUBLICATIONS ACTIVITIES

1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.3 Technical Papers

Calibration designs based on solutions to the tournament problem. R. C. Bose and J. M. Cameron. *J. of Research NBS*, 71B, pp. 149-160.

Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. *Biometrics*, 23, pp. 717-737, 1967.

Matroid Partition. J. Edmonds. In *Mathematics of the Decision Sciences*, Amer. Math. Soc. Lectures in Appl. Math. vol. 11, pp. 335-345 (1968).

The probability of an equilibrium point. K. Goldberg (205.01), A. J. Goldman, and M. Newman (205.01). *J. of Research NBS*, 72B, pp. 93-101 (1968).

Analysis of market split model. A. J. Goldman, P. R. Meyers, J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.). *J. of Research NBS*, 72B, pp. 43-66 (1968).

A transportation problem transformable to a best-path problem. A. J. Goldman and G. L. Nemhauser (John Hopkins U.). *Transportation Science* 1, pp. 295-307 (1968).

The distribution of the sample correlation coefficient with one variable fixed. D. Hogben. *J. of Research NBS*, 72B, pp. 33-36 (1968).

A statistical consultant's view of OMNITAB. D. Hogben. Appendix II in revised edition of *NBS Handbook 101, OMNITAB, A Computer Program for Statistical and Numerical Analysis*, U. S. Government Printing Office, January 1968.

Solving equations exactly. M. Newman. *J. of Research NBS*, 71B, pp. 171-179 (1967).

Pairs of matrices generating discrete free groups and free products. M. Newman. *Michigan Math. J.* 15, 155-160 (1968).

A bound for the number of conjugacy classes of a group. M. Newman. *J. London Math. Soc.* 43, 108-110 (1968).

Bounds for the number of generators of a finite group. M. Newman. *J. of Research NBS*, 71B, 247-248 (1967).

A diophantine equation. M. Newman. *J. London Math. Soc.* 43, 105-107 (1968).

An extension of Miller's algorithm. F. W. J. Olver. *Aplikace Matematiky*, vol. 13, pp. 174-176 (1968).

2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Monograph

Matrix representations of groups. M. Newman. To appear as AMS 60.

2.3 Technical Papers

Citation searching and bibliographic coupling with remote on-line computer access. F. L. Alt and R. A. Kirsch. To appear in the *J. of Research NBS*.

Bottleneck extrema. J. Edmonds, and D. R. Fulkerson (RAND Corporation). To appear in *J. Combinatorial Theory*.

Minimax error selection of a discrete univariate distribution with prescribed componentwise bounds. A. J. Goldman and P. R. Meyers. To appear in the *J. of Research NBS*.

On a theorem of Piatetsky-Shapiro and evaluation of multiple integrals. S. Haber and C. F. Osgood. Submitted to a technical journal.

A combination of classical and Monte Carlo methods for evaluating multiple integrals. S. Haber. To appear in Bull. A.M.S.

On the sum $\sum \langle \alpha_n \rangle^{-t}$. S. Haber and C. F. Osgood. Submitted to technical journal.

The distribution of the sample variance from a two-point binomial population. D. Hogben. Submitted to a technical journal.

Allocating service periods to minimize average delay. W. A. Horn. To appear in J. of Research NBS, 72B, (1968).

Optimal networks connecting N points in the plane. W. A. Horn. To appear in Proceedings of Fourth International Symposium on Traffic Flow (Karlsruhe, Germany; June, 1968)

Some fixed point theorems for compact maps and flows in Banach spaces. W. A. Horn. Submitted to a technical journal.

For sample of size 3 from a normal distribution the range and the mean deviation have the same distribution except for a scale factor. B. L. Joiner and J. R. Rosenblatt. Submitted to a technical journal.

A guide to commonly used terms and expressions of imprecision, systematic error, and uncertainty associated with a report value. H. H. Ku. To appear in Measurement and Data.

Analysis of information -- An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.

The problem of interaction in multidimensional contingency tables, an information theoretic approach. H. H. Ku. To appear in J. of Research NBS.

Contractive semigroups and uniform asymptotic stability. P. R. Meyers. Submitted to a technical journal.

Error bounds for the Laplace approximation for definite integrals. F. W. J. Olver.

2.4 NOTES

The minimax transportation problem. A. J. Goldman. To appear in Transportation Science (Letters to Editor).

Fractional container-loads and topological groups. A. J. Goldman. To appear in Operations Research.

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, chemistry, and engineering. Comprehensive scientific papers give complete details of the work, including laboratory data, experimental procedures, and theoretical and mathematical analyses. Illustrated with photographs, drawings, and charts.

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Technical Notes. This series consists of communications and reports (covering both other agency and NBS-sponsored work) of limited or transitory interest.

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