# NATIONAL BUREAU OF STANDARDS REPORT

9697

PROJECTS and PUBLICATIONS of the

APPLIED MATHEMATICS DIVISION

A Semi-Annual Report

July through December 1967



U.S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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# NATIONAL BUREAU OF STANDARDS REPORT

**NBS PROJECT** 

**NBS REPORT** 

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9697

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# APPLIED MATHEMATICS DIVISION

July 1 through December 31, 1967

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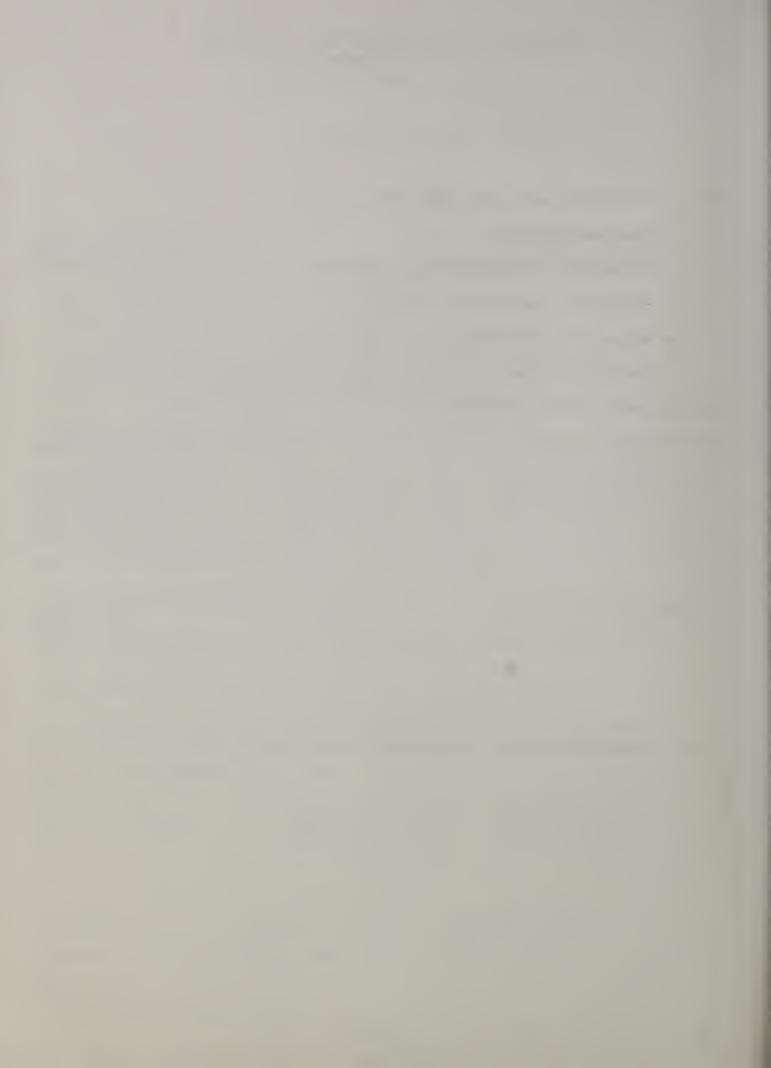
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Formerly Mathematical Physics Section.

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<sup>°</sup>Only unclassified material is included in this report.



# Status of Projects

#### 1. NUMERICAL ANALYSIS

#### RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS

Authorized 8/29/54

Manager: Morris Newman

Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has proved that if

$$A = \begin{pmatrix} -a & b \\ -c & d \end{pmatrix}, B = \begin{pmatrix} -\alpha & -\beta \\ \gamma & \delta \end{pmatrix}$$

are real matrices of determinant 1 and trace  $\geq 2$  such that a, b, c, d,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta \geq 0$ , then A and B generate a free discrete matrix group.

Let G be a finite group of order  $\,n$  with  $\,k$  conjugacy classes and of generator rank  $\,r$  . Then M. Newman has shown that

 $k \ge \log \log n / \log 2$ ,

 $r \leq \Omega(n) \leq \log n / \log 2$ ,

where  $\Omega(n)$  is the total number of primes dividing n .

M. Newman and J.L. Mennicke have shown that  $PSL(3,p^k)$  (p prime and > 3) can be generated by an element of period 2 and an element of period 3.

M. Newman has given a procedure for finding all rational solutions  $\mathbf{x}_{i}$  of the diophantine equation

$$\sin \pi x_1 \sin \pi x_2 \dots \sin \pi x_t = r$$
,

where r is a positive rational. All solutions were determined for t=2.

M. Newman has completed a monograph on matrix representations of groups, which will appear in the AMS series of the NBS. This monograph serves as a complete introduction to the subject and demands only a knowledge of elementary matrix theory from the reader. An appendix on the elements of the theory of algebraic numbers is included.

# Status of Projects

- K. Goldberg has found the area of the minimum circumscribing triangle of a fixed convex quadrilateral. In a related problem, he has found the maximum determinant of those row stochastic matrices of order three which transform a fixed row vector into one with non-negative coordinates.
- S. Haber continued studies of probabilistic methods of approximate evaluation of multiple integrals.
- S. Haber investigated the possibility of numerically inverting Laplace Transforms by use of a real inversion formula due to Boas and Widder. The method was found to be subject to great round-off errors.
- F.W.J. Olver has continued work on the computation of subdominant solutions of second-order linear difference equations. Comparison theorems have been constructed which yield strict and very realistic bounds for the truncation error associated with the new algorithm.

# Publications:

- (1) Semi-groups with zeroids. K. Goldberg. To appear in the Journal of Research NBS.
- (2) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.
- (3) The l.u.b. of a set of determinants. K. Goldberg. To appear in the Journal of Research NBS.
- (4) The 1.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.
- (5) Kleinian groups. L. Greenberg. Accepted for publication in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965).
- (6) On a theorem of Ahlfors. L. Greenberg. American Journal of Mathematics, Vol. 89, No. 1, pp. 56-68 (1967).
- (7) Fundamental Polyhedra for Kleinian groups. L. Greenberg. Annals of Mathematics, Vol. 84, No. 3, pp. 433-441 (1966).
- (8) A theorem on arbitrary functions. S. Haber. Am. Math. Monthly, Vol. 74, No. 8, pp.973-975 (1967).
- (9) A modified Monte-Carlo quadrature II . S. Haber. Mathematics of Computation, Vol. 21, No. 99, pp. 388-397 (1967).
- (10) Midpoint quadrature formulas. S. Haber. Mathematics of Computation, Vol. 21, No. 100, pp. 719-721 (1967).

# Status of Projects

- (11) Maximal normal subgroups of the modular group. M. Newman. To appear in Proc. Amer. Math. Soc.
- (12) Solving equations exactly. M. Newman. To appear in the NBS Journal of Research, Section B.
- (13) The coefficients of the powers of a polynomial. M. Newman. J. Res. Nat. Bur. Standards 71B, 11-12 (1967).
- (14) Note on partitions modulo 5. M. Newman. Math. Comp. 21, 481-482 (1967).
- (15) On Riemann surfaces with maximal automorphism groups. M. Newman and J. Lehner. Glasgow Math. J. 102-112 (1967).
- (16) Doubly stochastic associated matrices. M. Newman and M. Marcus. Duke Math. J. 34, 591-597 (1967).
- (17) Two classical theorems on commuting matrices. M. Newman. J. Res. Nat. Bur. Standards 71B, 69-71 (1967).
- (18) Pairs of matrices generating discrete free groups and free products. M. Newman. To appear in Michigan Math. J.
- (19) A bound for the number of conjugacy classes of a group. M. Newman. To appear in the J. London Math. Soc.
- (20) Bounds for the number of generators of a finite group. M. Newman. To appear in J. Res. Nat. Bur. Standards.
- (21) A diophantine equation. M. Newman. To appear in the J. London Math. Soc.
- (22) Matrix representations of groups. M. Newman. To appear in the AMS series of the NBS.
- (23) Numerical solution of second-order linear difference equations. F.W.J. Olver. J. Res. Nat. Bur. Standards 71B, 111-129 (April-September 1967).
- (24) Bounds for the solutions of second-order linear difference equations. F.W.J. Olver. J. Res. Nat. Bur. Standards 71B (October-December 1967).

# ASYMPTOTIC EXPANSIONS Task 20501-11-2050421/63

Origin: NBS

Authorized 9/10/63

Sponsor: U.S. Army Research Office, Durham, N.C.

Manager: F.W.J. Olver

Full task description: July-December 1963 issue, p. 2

Status: CONCLUDED. The results on error bounds for Laplace's method, the method of steepest descent and the method of stationary phase are being prepared for publication.

# 2. PROBABILITY AND MATHEMATICAL STATISTICS

# RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS

Authorized 10/1/62

Manager: Joan Raup Rosenblatt

Full task description: July - December 1962

Status: CONTINUED. Brian L. Joiner has completed an extensive study of small-sample properties of Cochran-type tests, and related issues.

A Cochran-type test is defined as a test for homogeneity of variance based on a statistic of the form

 $\max_j V_j \neq \sum_{j=1}^k V_j$  where the  $V_j$  are independent estimates of a scale parameter. Two well-known special

cases are the Cochran and Bliss-Cochran-Tukey tests where the  $V_j$  represent respectively the sample variance  $S^2$  and the sample range R. Approximate percentage points have been computed for two new tests of this form in which the  $V_j$  are the standard deviation S and the mean deviation M. Several approximations to the distribution of the sum of independent chi variates are compared and used in the computation of the test based on S. This test was suggested by Churchill Eisenhart.

Some robustness properties of the four tests and Bartlett's test are considered and some numerical results are given. The emphasis throughout is on small sample properties. Equal sized samples of size 3, 5 and 10 in sets of size 3, 5 and 10 are considered. The results are based primarily on a "modified degrees of freedom" approximation due to Le Roux in which it is assumed that the sample variance from a non-normal distribution behaves much as if it were based on a sample of a modified size from the normal distribution. The necessary modification depends on the coefficient of variation of the sample variance. Some results of a limited sampling study are also reported. It appears that there may be appreciable differences in the extent to which the various tests are affected by different kinds of non-normality. There does not appear to be any clear-cut "most robust" test among those considered, but the tests based on the mean deviation and range seem to have a slight overall edge while Bartlett's test appears to be slightly less satisfactory.

Dr. Joiner's study includes a comprehensive survey of the expectation and coefficient of variation of S, M and R in samples from non-normal distributions. New information is given for some (very) discrete distributions and for Tukey random variables defined by the transformation  $X = U^{\lambda}$  -  $(1-U)^{\lambda}$ , where U is uniform.

Some power-type properties of the four Cochran-type tests are also considered under the assumptions that all underlying distributions are normal while (an unknown) one of the variances may be inflated in the ratio  $\theta^2 > 1$ . Cochran's test is shown to be the likelihood ratio test for this class of alternatives. Two functions closely related to the power function are introduced and used in numerical comparisons of the tests. Let P(RC) denote the probability that  $V_i / \Sigma V_j$  is larger than the critical value when the i'th population is indeed the one with the inflated variance. Then it is shown that P(RC) constitutes the bulk of the power function for moderately distant alternatives. The second function is a modified version of the "median significance level." The "median significance level" (MSL) is introduced as an alternative to the power function as a means of assessing the properties of statistical tests. The MSL may be briefly described as the median of the distribution of the observed significance level for a given alternative hypothesis. Elementary examples are given in which the median significance level is compared with several similar criteria.

H. H. Ku has completed an investigation of interaction in multidimensional contingency tables, which has led to a unified treatment from an information-theoretic approach. In 1935, M. S. Bartlett proposed a test for the hypothesis of "no second-order interaction" as related to a 2x2x2 contingency table, based on a suggestion by R. A. Fisher. The problem received renewed interest in recent years, and a number of attempts were made to extend Bartlett's definition from within a wider framework of hypothesis formulation.

Investigation of the problem of second-order interaction from the point of view of information theory (S. Kullback, Information Theory and Statistics, 1959) leads to a unified approach to the analysis of multidimensional contingency tables. Under this approach the principle of minimum discrimination information is proposed and used to generate hypotheses of interest. It is shown that all classical

hypotheses for contingency tables can be generated through the use of this principle when certain marginals are considered fixed.

For each given set of fixed marginals, a unique set of cell probabilities p\* is generated by minimizing

the discrimination information,  $2\Sigma$  p in  $\frac{p}{\pi}$ , for all p's that are consistent with the restraints imposed by the fixed marginals,  $\pi$  being the hypothesis of equal probabilities. The set of p\* corresponds to the cell probabilities representing no interaction, and in the case of no second-order interaction p\* i.jk can be expressed in a logarithmic linear additive form:

$$\ln p_{ijk}^* = \ln \pi_{ijk} + \ln a_{ij} + \ln b_{jk} + \ln c_{ik}$$

where  $a_{ij}$ ,  $b_{jk}$ , and  $c_{ik}$  are functions of cell probabilities of the corresponding fixed two-way marginal tables. The difference between the set of cell probabilities estimated from data and p\* is therefore a measure of interaction.

If the complete set of one-way marginals is considered fixed, the set of p\* represents cell probabilities under the independence hypothesis, or the hypothesis of no first-order interaction. If the complete set of two-way marginals is considered fixed, the set of p\* is the cell probabilities representing that of no second-order interaction. In this sense the higher-order no interaction hypothesis can be considered as a hypothesis of "generalized" independence, a concept which unifies the many attempts in the formulation of a second-order interaction hypothesis.

The relationship between minimum discrimination information and maximum entropy is examined and the analogy between the proposed analysis and the analysis of variance using least squares theory is noted. An interpretation of the no interaction hypothesis as equivalent to that "the given marginal tables contain all the information of the full table" reduces the dimension of the table, and hence also the complexity of the analysis. A computer program has been prepared to aid in the calculation of numerical values of  $x^* = np^*$  by a simple convergent iteration process of the given marginal frequencies on  $n \pi_{i,j} k^*$ 

Typical analyses of information tables are given for first-, second-, third-, and mixed-order interactions for four-way contingency tables. Two illustrative examples in the analysis of four-way tables are included.

Churchill Eisenhart (NBS Director's Office) and Harry H. Ku have completed the revision of a paper on the "Expression of uncertainties of final results, and a guide to commonly-used terms and expressions of imprecision, systematic error, and uncertainty associated with a reported value." This paper embodies recommended practices that have been tried out in the Bureau for a number of years.

The OMNITAB instruction STATISTICAL ANALYSIS was revised for the UNIVAC 1108.

Changes in the automatic printout include a revised format to group statistics under appropriate headings such as linear trend statistics and tests for non-randomness, and the addition of several such as the trimmed mean and confidence intervals for the mean and standard deviation. Based on the recent work of E. L. Crow and M. M. Siddiqui, the 5% trimmed mean was changed to the 25% trimmed mean (or, as John Tukey has recently termed it, the midmean).

For the usual straight-line model, in which the independent variable takes on a fixed, known set of values, David Hogben has shown that the sample correlation coefficient is distributed as Q with (n-2) degrees of freedom and non-centrality  $\theta = (\beta/\sigma)\sqrt{\Sigma(x_1-x_1)^2}$ . The Q variate has been defined and studied by Hogben et al. (Ann. Math. Statist. 35, 298-314, 315-318, 1964).

#### Publications:

- (1) Calibration designs based on solutions to the tournament problem. R. C. Bose and J. M. Cameron. To appear in NBS J. Research B. (Math. and Math. Physics).
- (2) The distribution of the sample correlation coefficient with one variate fixed. D. Hogben. To appear in NBS J. Research B. (Math. Sciences).
- (3) The distribution of the sample variance from a two-point binomial population. D. Hogben. Submitted to a technical journal.

- (4) Expression of the uncertainties of final results and a tabular guide to commonly-used terms and expressions. C. Eisenhart (NBS Director's Office) and H. H. Ku. Submitted to a technical journal.
- (5) Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.
- (6) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.

# 3. STATISTICAL ENGINEERING SERVICES

# COLLABORATION ON STATISTICAL ASPECTS OF

#### NBS RESEARCH AND TESTING

Task 13911-61-2050950/51-1

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following.

B. L. Joiner has been involved in a continuing collaboration with the Analytical Chemistry Division and has been engaged in problems in the design of experiments for the evaluation of the inhomogeneity of standard reference materials. As an example, he has extended the work of Anderson and Crump (Technometrics 9(1967)) which contains most of the important results when the cost of a sample can be considered negligible compared to the cost of an analysis and has derived a solution when the cost of a sample is not negligible.

David Hogben has written an appendix, "A statistical consultant's view of OMNITAB," for a revised edition of NBS Handbook 101, OMNITAB, A Computer Program for Statistical and Numerical Analysis. The first of four sections provides additional details on the use of certain OMNITAB commands. The writer's impressions and experiences are expressed in section 2 with the aim of furthering a more effective use of OMNITAB. Section 3 describes how OMNITAB can be used to construct frequency distributions, while illustrating certain powerful features of OMNITAB. The last section proposes and discusses an easy way of using OMNITAB to do the computing for analyzing designed experiments.

Joan R. Rosenblatt has collaborated with the Structures Section of the Building Research Division in development of plans for data analysis, and for sample surveys in additional buildings, for the Survey of Live Loads and Fire Loads in Office Buildings.

Consulting services have been provided for several projects of the Technical Analysis Division: experiments on microfilm reading for the Patent Office; an experiment on comparison of punched card and other systems for transcribing data onto magnetic tape, for the Internal Revenue Service; sample surveys of various card files for the Library of Congress.

# Publications:

- (1) Designs for surveillance of the volt maintained by a small group of saturated standard cells. W. G. Eicke (NBS Electrochemistry Section) and J. M. Cameron. NBS Technical Note 430, U. S. Government Printing Office, October 9, 1967.
- (2) Realistic uncertainties and the mass measurement process: An illustrated review. Paul E. Pontius (NBS Mass and Volume Section) and Joseph M. Cameron. NBS Monograph 103, U. S. Government Printing Office, August 15, 1967.
- (3) A statistical consultant's view of OMNITAB. D. Hogben. To appear as Appendix II in revised edition of NBS Handbook 101, OMNITAB, A Computer Program for Statistical and Numerical Analysis.

#### STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other nongovernmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

The Statistical Engineering Laboratory participated, with the Computer Services Division and the Office of Standard Reference Data, in sponsorship of a three day OMNITAB-OMNITEXT Workshop, August 23-25, 1967. The Workshop was devoted to detailed discussions of a number of user-oriented computer systems developed at the Bureau or having direct application in the computerization of Data Center files and publications. Details of the topics discussed will be found below under "Lectures and Technical Meetings."

Joan R. Rosenblatt served as an advisor to an ad hoc Work Group on Flood Flow Frequency Analysis of the Hydrology Committee, Water Resources Council.

J. M. Cameron submitted a proposal for a study of the ability of calibration laboratories to maintain within laboratory control of their measurement process, and of the extent to which the process is independent of location as well as time, to the National Conference of Standards Laboratories Measurement Agreement Committee at their meeting in Boston on November 3. The proposal called for intensive study of one process to serve as a model for the amount and type of evidence needed to give assurance of consistency, compatability and authenticity of such measurements on a national scale.

A similar approach has been worked out for investigating the process parameters associated with mass spectrometer measurements of very small particles in a number of laboratories. The experimental design gives estimates of components of variance and provides procedures for establishing that the measurement processes are in a state of control. Designs for surveillance of groups of standards have been sent to a number of laboratories upon request, and these designs serve as the basis for establishing within laboratory control of measurement processes.

#### 4. MATHEMATICAL PHYSICS

#### RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 20540-12-2050141/55-57

Origin: NES Authorized 9/1/54

Manager: H. J. Oser

Full task description: July-September 1954 issue, p. 27

Dr. J. T. Fong joined the then Mathematical Physics Section on September 1, 1967 following a year of postdoctoral research associateship with the Engineering Mechanics Section (213.04). Progress in the following three projects initiated prior to September 1 continued:

- (a) In collaboration with Dr. R. deWit of the Lattice Defects and Microstructures Section (312.03) J. T. Fong investigated the geometric foundation of several continuum theories of crystal imperfections with particular interest in the non-Riemannian geometry of Kondo's theory (1952) and the distant parallelism geometry of Kroner's theory (1955). Attempts in unifying the geometric framework and in extending the static theories to a dynamic one are being made.
- (b) A joint study of the effect of the viscoelastic properties of a string on its dynamic response to a transverse impact was completed by J. T. Fong in association with Dr. J. C. Smith of the Polymer Interface Section (311.07). A manuscript entitled "On the coupling of longitudinal and transverse waves in a linear three-element viscoelastic string subjected to transverse impact" is prepared and will be submitted to a technical journal in January 1968.
- (c) A numerical experiment on the solution of an initial and boundary value problem consisting of a quasi-linear hyperbolic system of partial differential equations in two independent varibles with a moving boundary of a <u>priori</u> unknown speed is being planned. The problem resulted from a physical experiment on the generation of strong shocks in a thin metal tube subjected to complex dynamic loadings which J. T. Fong initiated at the Engineering Mechanics Section in cooperation with Dr. L. R. Hettche (213.04). As a first step in this research project, a weekly workshop meeting on the question of stability and convergence of the related finite-difference scheme was conducted by J. T. Fong with the participation of members of the Mathematical Physics Section.

A manuscript entitled "On the generation of two new types of strong discontinuities in a multi-stress problem of rate-independent dynamic plasticity" was completed by J. T. Fong and has been submitted to a technical journal.

#### PLASMA RESEARCH

Task 20504-12-2050140/59-442

Origin: NBS Authorized 10/31/61

Manager: H. J. Oser

Full task description: October-December 1961 issue, p. 12

Status: CONTINUED. Dr. W. L. Sadowski and Mrs. Z. Ruthberg continued to work on the two-stream instability in plasmas. Expansions were obtained for a model in which the two electrons in streams are at a different temperature than the rest of the plasma.

Work was also continued on the evaluation of truncation errors in the double expansion of the solution of the Vlasov equation. Numerical experiments have been carried out to estimate the effect of truncation on the accuracy of the solution. An operational approach for evaluating work is in progress to derive the full set of equations that govern truncation error propagation.

#### Publications

(1) On Some Aspects of the Eigenfunction Expansion of the Solution of the Non-Linear Vlasov Equation. W. L. Sadowski. NASA SP-153, April 1967. (A conference report on a symposium held at Langley Field, Virginia: Computer Simulation of Plasma and Many Body Problems).

#### PLASMA RESEARCH

#### Task 20504-12-2050417/62-1157

Origin: NBS Authorized 6/30/59

Sponsor: NASA

Manager: C. M. Tchen

Full task description: April-June 1959 issue, p. 15

Status: TERMINATED. Dr. Tchen completed investigations on plasma turbulence in a magnetic field. Cases of collisional dissipation and collisionless dissipation were studied. Turbulent diffusion was also derived from the theory. The problem is relevant to experiments in plasma fusion and to plasmas in upper atmosphere. Nonlinear problems of plasma oscillations have attracted great interest in recent years, but the theoretical effort has been exclusively on quasilinear approximations or weak turbulence. Tchen's emphasis was on strong turbulence, combining the techniques of plasma physics and hydrodynamic turbulence.

In order to gain an insight into the eddy diffusivity caused by the transfer of momentum across the turbulent spectrum into smaller scales, a cascade approximation was initiated by Tchen. The method lead to some new results of spectral functions, and at the same time enabled the clarification and comparison between the theories of turbulence by Heisenberg, Kolmogoroff, Chukoff and Kraichnan.

The diffusion of plasma in a magnetic field is important in plasma experiments, in the upper atmosphere (e.g. Barium cloud experiments), and in the Earth's magnetosphere. However, the most important diffusion is not the molecular diffusion, but is the turbulent diffusion. A theory was attempted by Tchen, and was based on the generalized Langevin equation for the motion of particles, and on the hydrodynamic theory for the electrostatic correlation. The diffusion coefficient was found to be inversely proportional to the first power of the external magnetic field, and proportional to the root mean square of the turbulent kinetic energy. The latter result cannot be obtained from a theory of weak turbulence, but is a consequence of strong turbulence. The result appears to agree with the experimental findings at Columbia University.

The problems are now being continued by Tchen as a professor at the City University of New York.

#### Publications

- (1) Turbuleuce in a Rarefied Plasma, Proceedings on Advanced Problems in Fluid Mechanics, Poland, Fluid Mechanics Transactions 3, 689 (1967).
- (2) Turbulence by Electrostatic Fluctuations, "Lectures in Theoretical Physics", C. M. Tchen, Vol.9c (Kinetic Theory), 265, (1967). Edited by W. E. Brittin, Gordon and Breach, Science Publishers, Inc., New York.
- (3) Spectrum of Turbulence in a Plasma with a Strong Magnetic Field, accepted for publication at Proc. Summer Institute on Noulinear Problems in Plasmas, Paris.
- (4) Fundamentals of Turbulence, C. M. Tchen (coauthor with J. Menkes), IDA Report P. 283 (1967).
- (5) Turbulence in Stratified Atmosphere, C. M. Tchen, G. E. Report R67SD45 (1967).

#### 5. OPERATIONS RESEARCH

#### CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Alan J. Goldman

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

- (1) Demand for miscellaneous consulting and advisory services continued heavy; responsiveness was somewhat limited by loss of two senior staff members. Section staff provided such services in 61 recorded instances; of these 33 involved assistance to NBS staff, covering 8 NBS divisions or other units. The 61 instances totalled to 440 recorded man-hours. Other agencies assisted included National Institutes of Health, Bureau of Employees' Compensation, Atomic Energy Commission, Bureau of Public Roads, Agency for International Development, Defense Supply Agency, and the Department of Transportation. Requests from universities, industry, professional groups and journals were also met.
- (2) W. Hall continued assistance to the Army Data Field System Command in a major evaluation and planning effort. (Reported here for convenience; supported under Project 4566427.) He also collaborated in constructing a simulation model for a highway maintenance depot.
- (3) J. Gilsinn, W. Horn and A.J. Goldman are assisting in a study to determine locations and sizings for inland marine cargo consolidation centers. (Reported here for convenience; supported under Project 4314422.) Horn completed a manuscript extending work of Browder on the existence of a fixed point for a map with domain and range in a Banach space. Goldman chaired the "Applications to Transportation" session of the Association for Computing Machinery's 1967 National Meeting.
- (4) I.S. Joel and A.J. Goldman continued participation in an analysis aimed at developing methods to estimate proper protection levels for various elements in a communications network. (Reported here for convenience; supported under Project 4556455.) They also began assisting the Department of Housing and Urban Development in connection with systems analyses of urban transport. (Reported here for convenience; supported under Project 2050625.)

# Publications

- (1) A.J. Goldman. Operations research research and government O.R. NBS Misc. Publ. 294 (12/67),

  Operations Research; Proceedings of a conference for Washington area government agencies,

  pp. 13-18.
- (2) W.A. Horn. Some fixed point theorems for compact maps and flows in Banach spaces. Submitted to a technical journal.
- (3) P.R. Meyers. A converse to the Banach contraction theorem. Journal of Research NBS, 71B (1967) pp. 73-76.
- (4) P.R. Meyers. Contractive semigroups and uniform asymptotic stability. Submitted to a technical journal.

# COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS Manager: Jack Edmonds Authorized 12/30/60

Full task description: October-December 1964 issue, p.3; April-June 1962 issue, p.15

Status: CONTINUED.

(1) Jack Edmonds presented a series of lectures on "Combinatorial methods" at the Amer. Math. Soc. Summer Seminar on Mathematics of the Decision Sciences, Stanford University, in July.

- (2) Presented "Matroids and the greedy algorithm" at the Mathematical Programming Symposium, Princeton University, in August.
  - (3) Further developed his methods of "alternation and shortest bidirected paths".
- (4) Found a "labeling method" for network flows such that the number of labelings before reaching maximum flow is bounded by a cubic polynomial in the number of nodes, regardless of the arc-capacities. This is in contrast with the labeling method of Ford and Fulkerson, which depends for termination on the magnitudes and relative commensurability of the arc-capacities.

# Publications

- (1) Jack Edmonds. Optimum Branchings. To appear in Journal of Research NBS, 71B (1967).
- (2) Jack Edmonds. Systems of distinct representatives and linear algebra. To appear in Journal of Research NBS, 71B (1967).

#### LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS Manager: Jack Edmonds

Authorized 12/30/60

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

- (1) Scott Lockhart, Ellis Johnson (visiting from Yale), and Jack Edmonds completed computer implementation of their algorithm for "The matching problem -- a solvable case of integer linear programming". A statistical analysis of observed running times went very well. A presentation was made at the Mathematical Programming Symposium, Princeton, in August. The Fortran program (5000 cards) is available and a full report is near completion.
- (2) Sharon Rose prepared certain related programs including one for the Chinese postman's problem.
- (3) A.J. Goldman and P.R. Meyers are examining minimax error selection of a discrete bivariate distribution with given marginals. L.S. Joel and J. Levy examined questions pertaining to symmetric assignment matrices, and to the convex hulls of matrices representing certain classes of permutations. W.A. Horn continued work on a manuscript on convex minimization one variable at a time.

# Publications

(1) Toward an algebraic characterization of convex polyhedral cones. R.J.B. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. Submitted to a technical journal.

# MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2058456

Origin: Technical Analysis Division, NBS Authorized 3/1/66

Sponsor: Northeast Corridor Transportation Project, Dept. of Transportation

Manager: A.J. Goldman

Full task description: January-June 1966 issue, p. 14

Status: CONTINUED (from 2050456).

(1) J. Gilsinn, A.J. Goldman, W.A. Horn and J. Levy prepared roughly 25 memoranda on various phases of the Northeast Corridor Transportation Project, including reviews of several contractors' reports. W. Hall continued participation in the freight simulation effort.

- J. Gilsinn prepared a program for efficient modification of the network "inputs" to the passenger simulation. She designed and began implementing a method for representing capacity constraints in the simulation.
- P. Saunders converted a Johns Hopkins University code for univariate dynamic programming, and two codes for solving capital allocation problems, to operational status on the NBS computer. She refined and began experiments with scheduling algorithms developed by D. Young (Technical Analysis Division). She and W.A. Horn are beginning refinement and conversion of scheduling and fleet estimation methods developed at MIT.
- (2) J. Levy continued the development of methods for evaluating feedback vs. non-feedback methods of regulating flow in a transport network. W. Horn completed a manuscript on optimal allocation of service times among N incoming streams sharing a single server. He prepared a working paper on a concept for automated transportation with non-scheduled departures. He continued work on choosing a minimum-cost planar network joining N given points. A.J. Goldman prepared a report proving the uniqueness of trip distributions by a gravity model.

#### Publications

- (1) A.J. Goldman, P.R. Meyers, J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.).

  Analysis of a market split model. Northeast Corridor Transportation Project Tech. Paper No. 8.

  To appear in Journal of Research NBS, 72B (1968).
- (2) A.J. Goldman and G.L. Nemhauser (Johns Hopkins U.). A transport improvement problem transformable to a best-path problem. Northeast Corridor Transportation Project Tech. Paper No. 9.

  To appear in Transportation Science.

#### MATHEMATICAL ANALYSES OF TOPICS IN ARMS CONTROL

Task 205-12-2050458

Origin and Sponsor: Arms Control and Disarmament Agency Manager: L.S. Joel

Authorized 3/1/67

#### Status: TERMINATED

- (1) Documentation of previously reported work was undertaken.
- (2) S. Rose (Technical Analysis Division) adapted the "matching problem" computer program prepared under Project 2050152 to solve the Inspector's Tour (=Chinese Postman's) Problem.
- (3) L.S. Joel, M.H. Pearl and R.D. Traub studied combinatorial probability problems relating to the likelihood that a large "random" game with more than 2 players has an equilibrium point in pure strategies.

# Publication

(1) K. Goldberg (205.01), A.J. Goldman and M. Newman (205.01). The probability of an equilibrium point. To appear in Journal of Research NBS, 68B (1968).

#### RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING

20500-12-2050404/651456 Authorized 1-21-64

Origin: NBS

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: January-June 1964 issue, p. 19

Status: Continued. A large part of the activity on this project during the present period was intimately connected with the use of large remote time sharing computers. The Q32 computer in California continues as the main workhorse for the project. Somewhat less use is made of the MAC computer in Massachusetts. The M44 computer in New York is no longer being used. Three new machines have, however, been used fairly extensively. The first is the PDP-6 at Stanford University. The use of this machine from our consoles was at the request of NIH to experiment with programs written at Stanford for the generation of chemical structures. This remote experimentation was proved feasible and the continued experiments are now being done by two groups at NIH. Another NIH group was helped by us in using the PDP-6 at Maynard, Massachusetts and the Q32 computer in California. They are continuing this work. A final new computer was used at the request of the Harry Diamond Laboratory to experiment with some information retrieval applications on an SDS-940 computer in California. In these last three cases we have enabled other agencies to use remote computers.

The most directly productive work this period involved the further development of the Q32 computer system for manipulation of biological images and their descriptions.

A program was written (by Becker) to do a syntactic analysis of linear strings which were 1-dimensional counterparts of pictures with respect to certain important properties.

Many new programs were written in assembly language (by Mrs. Rhodes) and in the higher level Lisp language (by Kirsch) on Q32 for manipulating biological images. These included tape control, counting, image printing, density histogram computing, area measurement, multiple image Boolean transformation, local Boolean transformation, and various thresholding programs. With this set of programs all of which are available via the Lisp language, powerful image processing programs can easily be written on-line on the computer. Further development of additional programs is continuing.

We have also combined the image processing capability with a previously reported capability for manipulating list structures which describe these images. This enables us to handle images and their descriptions interchangeably in the machine.

Because of the usefulness of Lisp for these experiments we have partially supported the construction (by D. Orser) of a Lisp interpreter on the 1108 computer.

A powerful rotating mirror microscope scanner that was built commercially on contract to the National Cancer Institute is to be connected to Dr. Lipkin's PDP8-Linc computer. P. Stein has commenced the design of the connection hardware for the scanner.

Our educational efforts this period were of two types. In the informal type several workers have spent periods of two weeks or more working with us. They have been from the University of Pennsylvania, the Cancer Institute, NIH-DCRT and (for lesser times) from other universities and government agencies. The more formal type of education has been via a weekly seminar presented (by Kirsch) on the Theory of Computers and Computation. This seminar is covering the theory of automata, neuron nets, Turing machines, recursive function theory and algebraic linguistics. An overly enthusiastic response from between 25 and 50 people represents the interest in the subject of several organizations (in order of participation) the NBS Computer Center, NIH, the rest of NBS, NASA and a few other university and industrial groups.

Finally during this period, Kirsch participated in various committees and consulting efforts. These included the Pharmacology-Toxicology Information program at NIH, an Advisory Committee to the National Cancer Institute, the Association for Computing Machinery Awards Committee, and miscellaneous consulting for CIA, Harry Diamond Laboratories and NIH.

#### MATHEMATICAL TABLES

20500-40-2050121/57-216 Origin and Sponsor: NBS Manager: I. A. Stegun

Full task description: July-December 1964 issue, p.4

Status: CONTINUED. Assistance to NBS divisions, other government agencies, industry, and universities in the evaluation of computing techniques, identification of functions and their tabulation, etc. continued at the rate of 12 per month.

A correction list is continually being maintained for AMS 55- the Handbook of Mathematical Functions as well as other volumes in the AMS series

A sixth printing of the Handbook was released in November and a second edition with additions and corrections of CUP 13- Tables relating to Mathieu Functions - was released in August.

Work continued on the examination of computing algorithms for special functions and the effectiveness or pitfalls of available programs.

# Lectures and Technical Meetings

# OMNITAB-OMNITEXT Workshop

sponsored by
Computer Services Division
Statistical Engineering Laboratory
Office of Standard Reference Data
August 23-25, 1967

# Session 1. OMNITAB Implementations

WALSH, P. J. (Computer Services Division) Introduction.

GILBERT, W. J. (Computer Services Division) Features and status of the new FORTRAN version of OMNITAB.

CHAMBERIAIN, R. L. (NBS and Iowa State University) Implementation of OMNITAB on the IBM 360/50 at Iowa State University.

BAUM, L. (University of Pennsylvania) Implementation of Pentab on the IBM 7040 at the University of Pennsylvania.

DAVIS, F. (Thermophysical Properties Research Center, Purdue University) An improved spline fitting program.

# Session 2. Numerical and Statistical Methods

(Statistical Engineering Laboratory) Hogben on OMNITAB.

REILLY, M. L. (Heat Division) Numerical interpolation, differentiation, and integration.

CAMERON, J. M. (Statistical Engineering Laboratory) Statistics and matrix operations on OMNITAB and OMNISTAT.

HILSENRATH, J. (Office of Standard Reference Data) PRECISE: A multiple-precision version of OMNITAB.

#### Session 3. Computer-assisted Text Preparation

WAIBEL, K. (Office of Standard Reference Data) Computer-assisted text preparation on the IBM 7094.

BERNS, G. M. (Washington Scientific Center, IBM) FORMAT: A text processing program for the IBM 360.

VAN HORN, J. (V.I.P. Systems, Inc.) On-line text editing.

HOGBEN, David

DUNCAN, B. C. (Chemistry Division) The general purpose document image code.

# Session 4. Text Editing and Typesetting

BULL, G. (Hatfield College of Tech., England, and Dartmouth College) The Dartmouth College EDITOR.

THOMPSON, R. C. (Office of Standard Reference Data) Prospects for simplified systems for computerized type setting.

MESSINA, C. G. (Office of Standard Reference Data) OMNITEXT: A series of interlocking utility programs for text and data manipulation.

# Session 5. Concluding Session

WELLS, M. B.

(The Los Alamos Scientific Laboratory) MADCAP Project: Motivation, Implementation and Rewards.

A Round Table on "On-line command languages." Walter J. Gilbert, NBS: Gordon Bull, Dartmouth; Mike Keegan, Sandia Corp.; R. L. Chamberlain. NBS: Lew Baum, U. of Pennsylvania.

# Applied Mathematics Division Lectures

GOLDMAN, Dr. A. J. Konigsberg's Bridges and Adelaide's Buses. Scientific Staff Meeting, September 22, 1967.

Yale University, New Haven, Conn. Computation of Optimal Policies in JOHNSON, Ellis L. Infinite Horizon Dynamic Programming. September 7, 1967.

Yale University, New Haven, Conn. Computer Implementation of Network JOHNSON, Ellis L. Algorithms. September 14, 1967.

(Mechanics Division, Rheology Section, National Bureau of Standards, KEARSLEY, Dr. E. A. Washington, D. C.) Multiplication of Tensors of Rank 2. August 14, 1967.

(Department of Mathematics, University of Göttingen, Göttingen, MENNICKE, Prof. J. L. Germany) Some Results on Matrix Groups. September 27, 1967.

RABINOWITZ, Prof. P. (Department of Mathematics, Weizmann Institute, Rehovoth, Israel) Error Estimates in Numerical Integration Using Chebyschev Polynomials. August 30, 1967.

OBERHETTINGER, Prof. F. (Department of Mathematics, Oregon State College, Corvallis, Oregon) On The Zeros of Functions Generated by a Finite Fourier Transform. August 23, 1967.

# NBS In-hours Courses Given by Staff Members

VARNER, Ruth N. (with P. J. Walsh, Computer Services Division) FORTRAN Programming for Beginners. Fall semester.

# Applied Mathematics Division Seminars

Computer and Computation Theory, December 1967-May 1968. KIRSCH, R. A.

NEWMAN, Dr. M. Group Representations. September 1967-May 1968.

# Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J. M. Establishment and Maintenance of a Local Standard. Presented at the Instrument Society of America, Chicago, Illinois, September 13-14,1967.

> Generation and Testing of Pseudo-Random Numbers. Presented at the Washington Chapter, Association for Computing Machinery, Washington, D. C., December 6, 1967.

EDMONDS, J. Matroids and Extremal Combinatorics. Presented at the International Symposium on Mathematical Programming, August 14.18, 1967, Princeton University.

GOLDMAN, A. J. The Probability of an Equibrium Point. November 1-3, 1967. Presented at the 32nd National Meeting ORSA, Chicago, Illinois.

JOHN, E. E., EDMONDS, J. and LOCKHART S.

The Degree-Constrained Subgraph Problem. Presented at the International Symposium by E. L. Johnson at the International Symposium on Mathematical Programming, August 14-18, 1967, Princeton University.

KU, H. H.

Uncertainty Statements for Calibrations. Presented at the National Conference of Standards Laboratories Workshop, Chicago, Illinois, September 14, 1967.

HOGBEN, David

The Distribution of the Sample Correlation Coefficient with One Variable Fixed. Presented at the Institute of Mathematical Statistics' Annual Meeting, Washington, D. C., December 30, 1967.

JOINER, B. L.

The Median Significance Level and Other Small Sample Measures of Test Efficacy. Presented at the Institute of Mathematical Statistics' Annual Meeting, Washington, D. C., December 28, 1967.

KIRSCH, R. A.

Remote Communication - Computer Networks in Biological Communication. Presented at the U. S. Veterans Administration Hospital, Martinsburg, West Virginia, November 15, 1967.

Experiences and Extrapolations from the Use of Large Time-Sharing Systems Over Long Distances. Presented at the Seminar in Computer Science. The Johns Hopkins University, Baltimore, Maryland, December 4, 1967.

NEWMAN, M.

The Structure of Normal Subgroups of the Modular Group. Presented at a conference on "Discontinuous Groups and Automorphic Functions", University of Wisconsin, Madison, Wisconsin, July 10-14, 1967.

Linear Fractional Groups and Maximal Normal Subgroups of the Modular Group. Presented to the Department of Mathematics, University of Notre Dame, Notre Dame, Indiana, December 1, 1967.

OLVER, F. W. J.

The Numerical Solution of Second-Order Linear Difference Equations. Presented at the Computing Laboratory, University of Oxford, England, September 8, 1967.

An Extension of Miller's Algorithm. Presented to the Conference on "Basic Problems of Numerical Analysis" organized by the Czechoslovakia Academy of Sciences, Liblice, Czechoslovakia, September 11-15, 1967.

OSER, H. J.

On the Solution of the Stefan Problem for Whisker Growth. Presented at the Illinois Institute of Technology, Chicago, Illinois, July 21, 1967.

Functional and Numerical Analysis of Monotone Operators. Presented at the Mathematics Colloquim, Florida State University, Tallahassee, Florida, September 29, 1967.

The Stefan Problem of One-Dimensional Crystal Growth. Presented at the Physics Colloquim, Florida State University, Tallahassee, Florida, September 29, 1967.

TCHEN, C. M.

A Cascade Theory of Plasma Turbulence. Presented at the Applied Physics Laboratory, the Johns Hopkins University, Silver Spring, Maryland, July 29, 1967.

Derivation of Turbulent and Density Spectra. Derivation of Anamolous Diffusion. Presented at the International Conference on Ionization Phenomena in Gases, Vienna, Austria, August 27-September 2, 1967.

Derivation of Turbulent Spectrum and Eddy Viscosity. Presented at the Symposium on Advanced Problems and Methods in Fluid Mechanics, Tarda, Poland, September 18-23, 1967.

Problems in Turbulence. Presented at the Space Sciences Laboratory, General Electric Company, Valley Forge, Pennsylvania, August 15, 1967.

# TCHEN, C. M

Plasma Turbulence. Presented at the Institute for Plasma Physics, Garching, Munchen, Germany, September 5, 1967.

Atmospheric Turbulence and Diffusion. Presented at the Institute for Extraterrestrial Physics, Max Planck Institute for Physics and Astro-Physics, Munchen, Germany, September 6, 1967.

Hydrodynamic Turbulence. Presented at the APS Fluid Mechanics Division Annual Meetings, Bethelehem, Pennsylvania, November 20, 1967.

Cascade Theory of Turbulence. Presented at the Graduate Center for Aero-Sciences, Cornell University, Ithaca, New York, November 29, 1967.

#### PUBLICATIONS ACTIVITIES

#### 1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

#### 1.1 Mathematical Tables

Handbook of Mathematical Functions, AMS 55, Sixth Printing (with corrections), November 1967.

Tables Relating to Mathieu Functions, AMS 59, (Second Edition with Additions), August 1967.

#### 1.3 Technical Papers

Optimum Branchings. Jack Edmonds. J. of Research NBS, 71B (1967)

Systems of direct representatives and linear algebra. Jack Edmonds. J. of Research NBS 71B (1967)

Designs for surveillance of the volt maintained by a small group of saturated standard cells. W. G. Eicke (NBS Electrochemistry Section) and J. M. Cameron. NBS Technical Note 430, U. S. Government Printing Office, October 9, 1967.

Operations research research and government O.R., A. J. Goldman, NBS Misc. Publ. 294, Washington area government agencies, pp. 13-18.

A theorem on arbitrary functions. S. Haber. Am. Math. Monthly, Vol. 74, No. 8, pp. 973-975 (1967).

A modified Monte-Carlo quadrature II. S. Haber. Mathematics of Computation, Vol. 21, No. 99, pp. 388-397 (1967).

Midpoint quadrature formulas. S. Haber. Mathematics of Computation, Vol. 21, No. 100, pp. 719-721 (1967).

A converse to the Banach contraction theorem. P. R. Meyers. J. of Research NBS,  $\overline{71B}$ , pp. 37-76 (1967).

Note on partitions modulo 5. M. Newman. Mathematics of Computation, Vol. 21, No. 99, 481-482 (1967).

On Riemann surfaces with maximal automorphism groups. M. Newman and J. Lehner. Glasgow Math. J. 102-112 (1967).

Doubly stochastic associated matrices. M. Newman and M. Marcus. Duke Math. J. 34, 591-597 (1967).

Two classical theorems on commuting matrices. M. Newman. J. of Research NBS, 71B, 69-71 (1967).

Numerical solution of second-order linear difference equations. F. W. J. Olver. J. of Research NBS, 71B, 111-129 (April-September 1967).

Realistic uncertainties and the mass measurement process: An illustrated review. Paul E. Pontius (NBS Mass and Volume Section) and Joseph M. Cameron. NBS Monograph 103, U. S. Government Printing Office, August 15, 1967.

Turbulence by electrostatic fluctuations, "Lectures in Theorectical Physics", C. M. Tchen Vol. 9c(Kinetic Theory), 265, (1967). Edited by W. E. Brittin, Gordon and Breach, Science Publishers, Inc., New York.

Fundamentals of turbulence. C. M. Tchen. (Coauthor with J. Menkes), IDA Report P283 (1967).

Turbulence in stratified atmosphere. C. M. Tchen. G. E. Report R67SD45(1967).

#### 2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

#### 2.3 Technical Papers

Calibration designs based on solutions to the tournament problem. R. C. Bose and J. M. Cameron. To appear in J. of Research NBS (Math. Sciences).

Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.

The distribution of the sample correlation coefficient with one variate fixed. D. Hogben. To appear in J. of Research NBS (Math. Sciences).

The distribution of the sample variance from a two-point binomial population. D. Hogben. Submitted to a technical journal.

Some fixed point theorems for compact maps and flows in Banach spaces. W. A. Horn. Submitted to a technical journal.

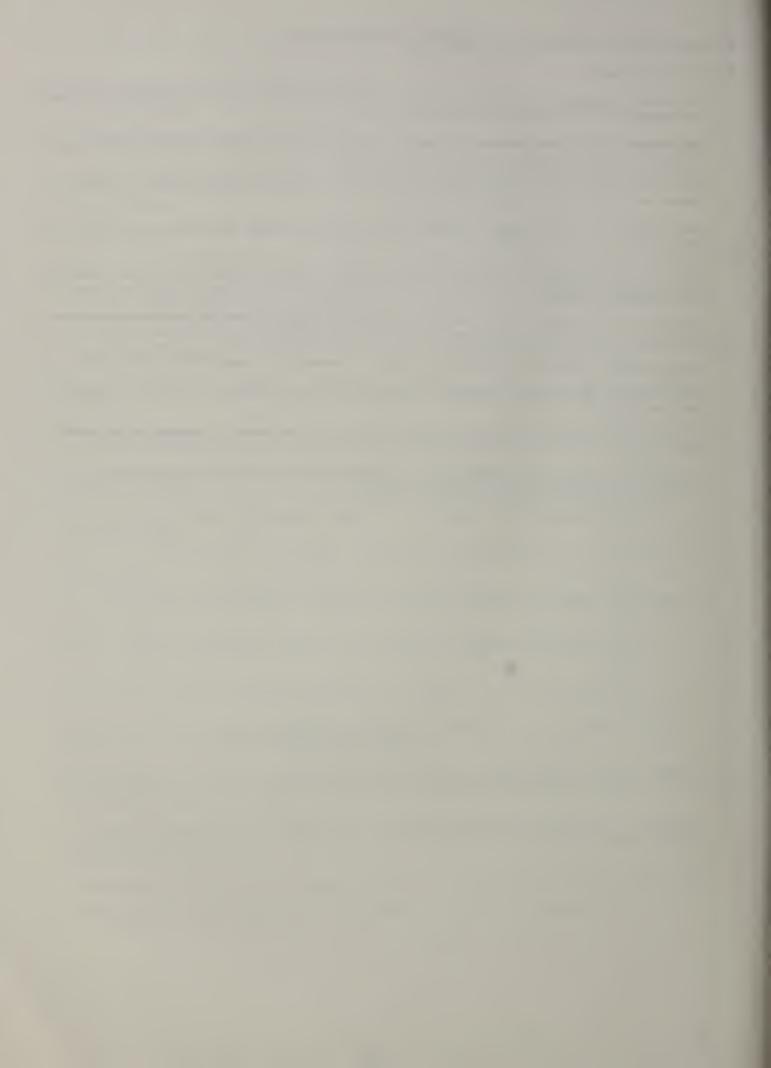
Analysis of information -- An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.

Maximal normal subgroups of the modular group. M. Newman. To appear in Proc. Amer. Math. Soc.

Pairs of matrices generating discrete free groups and free products. M. Newman. To appear in Michigan Math. J.

A bound for the number of conjugacy classes of a group. M. Newman. To appear in the J. London Math. Soc.

Bounds for the solutions of second-order linear difference equations. F. W. J. Olver. J. of Research NBS, 71B, (October-December 1967).



# NBS TECHNICAL PUBLICATIONS

#### **PERIODICALS**

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, chemistry, and engineering. Comprehensive scientific papers give complete details of the work, including laboratory data, experimental procedures, and theoretical and mathematical analyses. Illustrated with photographs, drawings, and charts.

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The best single source of information concerning the Bureau's research, developmental, cooperative and publication activities, this monthly publication is designed for the industry-oriented individual whose daily work involves intimate contact with science and technology—for engineers, chemists, physicists, research managers, product-development managers, and company executives. Annual subscription: Domestic, \$1.50; foreign, \$2.25\*.

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Technical Notes. This series consists of communications and reports (covering both other agency and NBS-sponsored work) of limited or transitory interest.

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