

NATIONAL BUREAU OF STANDARDS REPORT

9616

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Semi-Annual Report
January through June 1967



U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through June 30, 1967

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^{ooo} Student Trainee

*** Guest Worker

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*Only unclassified material is included in this report.

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS

Manager: Morris Newman

Authorized 8/29/54

Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has proved that if p is a prime > 3 and n is any integer > 5 such that $(n,6)=1$ and $n = p$ or n divides $\frac{p+1}{2}$, then $LF(2,p)$ may be generated by elements A, B such that A is of period 2, B is of period 3, and AB is of period n . A consequence of this is that there exist infinitely many maximal normal subgroups of the modular group Γ which are not congruence groups.

M. Newman has prepared an expository article on the classical theorems on arbitrary sets of commuting matrices, giving proofs by the elements of the theory of group representations.

M. Newman has made a study of a congruential method for finding the exact solution of a system of linear equations over the rationals, and has prepared an article describing the method, the detailed program, and the numerical results obtained using the Q32 computer at SDC in Santa Monica, California via a remote console. The results were uniformly excellent and the method will be incorporated into Omnitab.

M. Newman has proved that if

$$s(x) = \lim_{x \rightarrow \infty} \inf \frac{1}{x} \sum_{\substack{p(n) \equiv 0 \pmod{5} \\ n \leq x}} 1, \quad ,$$

where $p(n)$ is the unrestricted partition function, then $s(x) > \frac{1}{5}$. That is, $p(n)$ is divisible by 5 more than it should be "on the average".

S. Haber continued studies of probabilistic and number-theoretical methods for approximate evaluation of multiple integrals. He developed some features of a general class of quadrature formulas which combine classical and Monte-Carlo methods to gain some of the advantages of both. These "randomized quadrature formulas" include his two previously discussed Modified Monte Carlo methods. A new formula of this type was found, which has degree-of-precision 2, and for which the quadrature error goes to zero as $N^{-(1/2+3/s)}$ (where N is the number of points at which the integrand is evaluated, and s is the number of variables of integration).

K. Goldberg has found solutions A to the matrix equation $AA^T + A^T A = aI + bJ$ with the condition $(A + A^T)J = cJ$, and has found the complete solution in certain cases.

Status of Projects

K. Goldberg has investigated the sum $\sum_{k=1}^m (-1)^{k-1} \binom{m}{k} \binom{n}{k} k! m^{-k} n^{-k}$ and the conjecture that it is greater than $1 - (1 - m^{-1})^{-m}$ for $mn > 1$.

K. Goldberg has investigated the distribution of values of the sum $\sum_{j=1}^{n-k} s_j s_{j+k}$ over all sequences s_1, s_2, \dots, s_{n-1} .

F.W.J. Olver has developed a new algorithm for the computation of subdominant solutions of second-order linear difference equations. Applications have been made to Bessel, Anger-Weber, and Struve functions, and to Clenshaw's method for the solution of ordinary differential equations in Chebyshev series. Compared with the well-known Miller algorithm, the new method has the advantages of determining automatically the correct starting point, including inhomogeneous terms, and being susceptible to powerful error analyses.

Publications:

- (1) Kernels and the Kronecker product of graphs. R.A. Brualdi. To appear in Proceedings of the American Mathematical Society.
- (2) Semi-groups with zeroids. K. Goldberg. To appear in the Journal of Research NBS.
- (3) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.
- (4) The l.u.b. of a set of determinants. K. Goldberg. To appear in the Journal of Research NBS.
- (5) The l.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.
- (6) Kleinian groups. L. Greenberg. Accepted for publication in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965).
- (7) A theorem on arbitrary functions. S. Haber. To appear in the American Mathematical Monthly.
- (8) A modified Monte-Carlo quadrature - II. S. Haber. To appear in Mathematics of Computation.
- (9) A functional inequality. S. Haber. Appeared in the American Mathematical Monthly, Vol. 73, No. 10 pp-1103-1104 (1966).
- (10) On a theorem of Ahlfors. L. Greenberg. Appeared in American Journal of Mathematics (Jan. 1967).
- (11) Fundamental Polyhedra for Kleinian groups. L. Greenberg. Appeared in Annals of Mathematics. (Nov. 1966).

Status of Projects

- (12) Note on normal subgroups of the modular group. L. Greenberg. Proceedings of the American Mathematical Society, Vol. 17, No. 5, pp. 1195-1198 (Oct. 1966).
- (13) Classification of normal subgroups of the modular group. M. Newman. Trans. Amer. Math. Soc. 126, pp. 267-277 (1967).
- (14) Maximal normal subgroups of the modular group. M. Newman. Submitted to a technical journal.
- (15) Solving equations exactly. M. Newman. In manuscript.
- (16) The numerical solution of second-order linear difference equations. F.W.J. Olver. To appear in the Journal of Research NBS.
- (17) The diophantine equation $P(x,y) = (xy + d)z$. C. Osgood. To appear in the Journal of Research NBS.
- (18) A method in diophantine approximation (II). C. Osgood. To appear in Acta Arithmetica.

ASYMPTOTIC EXPANSIONS

Task 20501-11-2050421/63

Origin: NBS

Sponsor: U.S. Army Research Office, Durham, N.C.

Authorized 9/10/63

Manager: F.W.J. Olver

Full Task description: July-December 1963 issue, p. 2

Status: CONTINUED. Little work has been carried out on this project during the present period.

2. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Authorized 10/1/62

Status: CONTINUED. R. C. Bose and J. M. Cameron have prepared a paper on "Calibration designs based on solutions to the tournament problem." In high precision calibrations one measures differences between nominally equal objects or group of objects and establishes a value for the individuals with reference to one or more standards. The solutions of the classical tournament problem, which calls for arranging v individuals into teams of p players so that a player is teamed the same number of times with each of the other players and also that each player is pitted equally often against each of the other players, provide balanced designs for scheduling the measurements. These designs are useful in weighing and other measurements when the objects to be measured can be combined into groups without loss of precision or accuracy in the comparisons.

This paper presents solutions to the tournament problem for all $v \leq 13$ and for $p \leq \frac{v}{2}$.

(Solutions for $v \leq 50$ and $p=2$ were published previously.) The statistical analysis, a worked example, and computational procedures are given.

R. L. Chamberlain is continuing his review of pseudo-random number generators and tests for randomness of sequences of random numbers. A computer program is being prepared to carry out the various tests.

Roy H. Wampler is continuing the evaluation of computer programs for multiple regression and polynomial regression. Results on a set of test problems have been obtained for over a dozen different programs.

Brian L. Joiner is conducting a study of Cochran-type tests for testing homogeneity of variance. A Cochran-type test is based on a statistic of the form

$$Y = \max_j \frac{V_j}{\sum_{j=1}^k V_j},$$

where the V_j are k independent estimates of a scale parameter. Two well-known special cases are Cochran's test based on sample variances, and the Bliss-Cochran-Tukey test based on sample ranges. These two tests are being compared with each other and with two new ones: a test proposed by C. Eisenhart based on sample standard deviations, and a test based on sample mean deviations.

David Hogben prepared an algorithm for computing points for plotting the boundary of a joint confidence region for the parameters of a straight line.

Publications:

- (1) Calibration designs based on solutions to the tournament problem. R. C. Bose and J. M. Cameron. To appear in NBS J. Research B. (Math. and Math. Physics).
- (2) Chapter 2--Statistical concepts in metrology. H. H. Ku. Handbook of Industrial Metrology, sponsored by the American Society of Tool and Manufacturing Engineers, Prentice-Hall, Inc., 1967, pages 20-54.
- (3) Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.
- (4) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.

3. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS

RESEARCH AND TESTING

Task 13911-61-1390951/51-1

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following.

J. M. Cameron collaborated with W. G. Eicke, NBS Electrochemistry Section, in the preparation of a technical note that describes a procedure for maintaining surveillance over a small group of saturated standard cells. The measurement process is briefly discussed and the principle of left-right balance as a means of eliminating certain systematic errors is developed. Specific designs and their analysis for intercomparing 3, 4, 5 and 6 cells in a single temperature controlled environment are given. Procedures for setting up control charts on the appropriate parameters are given, and a technique is described for detecting certain types of systematic errors.

Roy H. Wampler is developing a computer program for performing the computations for voltage cell calibrations, and producing a fully-detailed calibration report, with additional output of data for use in a surveillance program.

Joan R. Rosenblatt collaborated with the Sound Section on the statistical analysis required for a detailed report giving the scientific basis for the International Standards Organization ISO Recommendation R389, "Standard Reference Zero for the Calibration of Pure-Tone Audiometers," November 1964. The uncertainties of the sound pressure levels corresponding to threshold involved the uncertainties of "transfer factors" that were used to adjust five different types of equipment to a common basis. The measurement errors in threshold determinations and in the transfer factor determinations were evaluated, to show their individual and combined effects on the results after the weighted average was readjusted to obtain levels for the five different equipments.

Paul E. Pontius, NBS Mass and Volume Section, and J. M. Cameron have prepared for publication an illustrated review of the concepts and operations involved in measuring the mass of an object. The importance of viewing measurement as a production process is emphasized and control-chart methods for evaluating process parameters are presented. The use of one of the Laboratory's standards as an additional unknown in routine calibrations provides an accuracy check and, as time goes on, the basis for precision and accuracy statements.

David Hogben is collaborating with members of the NBS Activation Analysis and Radiochemical Analysis Sections on a variety of problems. Examples are: (1) the use of propagation of error formulas and linear calibration curves in an investigation to establish some systematic error in oxygen activation analysis with ^{14}N ; (2) the use of analysis of variance, sampling techniques and computing methods for testing homogeneity of Kale leaf, White Oak leaf and Beef liver samples with respect to amount of various trace elements as determined by activation analysis. The samples are to serve as biological Standard Reference Materials. (3) the use of a PBIB design to determine the effect of plate thickness, window width and composition on area of Mossbauer resonant absorption for $\text{SnO}_2\text{-Al}_2\text{O}_3$ mixtures and use of the results to design the collection of data for constructing calibration curves.

During the spring semester, Sally T. Peavy taught a course on FORTRAN programming and David Hogben taught a course, "Introduction to mathematical and statistical analysis of laboratory data," organized around the Bureau's OMNITAB program. Sally T. Peavy and Ruth N. Varner taught a summer course on the EXEC. II operating system for the UNIVAC 1108.

Publications:

- (1) Designs for surveillance of the volt maintained by a small group of saturated standard cells. W. G. Eicke (NBS Electrochemistry Section) and J. M. Cameron. To appear as NBS Technical Note 430, U. S. Government Printing Office.
- (2) Realistic uncertainties and the mass measurement process: An illustrated review. Paul E. Pontius (NBS Mass and Volume Section) and Joseph M. Cameron. To appear as NBS Monograph 103, U. S. Government Printing Office.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

A four-day Seminar on Precision and Accuracy in Measurement and Calibration was presented to metrologists from industrial and governmental laboratories. The Seminar dealt with the concepts of a National Measurement System as presented by Robert D. Huntoon, Director of Institute for Basic Standards, and of the need for viewing measurement as a production process. These concepts were developed in presentations by Churchill Eisenhart, NBS Senior Research Fellow and in talks by Paul E. Pontius of the Mass and Volume Section and J. M. Cameron of the Statistical Engineering Laboratory.

Lectures were presented on Propagation of error in a chain of laboratories (by Joan R. Rosenblatt), Calibration curves (by David Hogben), Accuracy ratios (by H. H. Ku), Design of experiments and interlaboratory comparisons (by J. M. Cameron) and Control chart techniques (by B. L. Joiner).

The participation of personnel from Bureau laboratories apart from the Statistical Engineering Laboratory added considerably to the value of the program. Woodward G. Eicke of the Electrochemistry section presented some new methods for intercomparing groups of voltage standards and maintaining surveillance over the measurement process involved. Robert C. Raybold of the Mass and Volume section presented an account of measurement problems in photometry and steps in the evaluation of process parameters.

The problem sessions were highlighted by the presence of NBS measurement specialists from seven different calibration laboratories.

The seminar, which last year involved 38 participants, was attended by a total of 53 people representing 39 industrial and university laboratories and included 12 from Army, Navy, Air Force or NASA Laboratories.

A National Bureau of Standards - Bureau of the Census Joint Seminar on Errors of Measurement, bringing together senior mathematical statisticians from the two agencies, was held on June 27, 1967 at the National Bureau of Standards.

The morning session, on applications of statistics in the evaluation of precision and accuracy of physical science measurements, was chaired by Churchill Eisenhart, Senior Research Fellow, Office of the Director, NBS.

The afternoon session, on measurement-error research at the Bureau of the Census, was chaired by Morris H. Hansen, Assistant Director for Research and Development, Bureau of the Census.

There were frequent consultations with other agencies concerning the evaluation of computer programs. Test problems and other evaluation services were done for the Bureau of Labor Statistics, the Social Security Administration, the National Institutes of Health, and the Atomic Energy Commission.

Staff members served as consultants to (1) the Water Resources Council, with an interagency working group on methods for flood frequency analysis, (2) the National Center for Health Statistics, reviewing plans for a loudness-balance experiment to be used for calibration of earphones.

STANDARD PROGRAMMING AIDS FOR STATISTICAL COMPUTATIONS

Project 4512115

Origin: NBS

Authorized 1/21/66

Manager: J. M. Cameron

Full task description: January - June 1966 issue, p. 9

Status: CONTINUED. Work was begun on specifications for statistical programs which would contain an essentially complete analysis, be based on the "best" computational algorithm and be properly documented. Programs for descriptive statistics, fitting of low-order polynomials, and least squares analysis are being used as test cases and the results are being incorporated in OMNITAB. The documentation phase is proving the biggest obstacle because of the variety of statistical models that may underlie a given array of data.

4. MATHEMATICAL PHYSICS
RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 20540-12-2050141/55-57

Origin: NBS
Manager: H. Oser

Authorized 9/1/54

Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. Dr. W.L. Sadowski and Mrs. Z. Ruthberg have completed their work on the numerical solution of the non-linear Vlasov equation. Both the non-linear Landau damping and dispersion relations were obtained. Some results seem to be applicable to the solar corona. Work was continued on the two-stream instability in plasmas. Preliminary results were obtained and look promising.

Discussions were held with Drs. Wilson, Konradi and co-workers pertaining to a cooperative study of charged particle trapping by the magnetic field in the aurora. The work is in the very early stages but will be continued.

Dr. Sadowski was invited for a work visit by Drs. Firor and Nakagawa of NCAR, Boulder, Colorado. The visit was very fruitful.

Talks were presented by Dr. Sadowski on his work to NCAR personnel in April and at NASA, Greenbelt, in May.

Mrs. Ruthberg and Dr. Sadowski were invited to attend a conference on numerical models in plasma physics in April in Williamsburg, Va.

Dr. Sadowski presented a paper entitled "On Some Aspects of Eigenfunction Expansion of the Solution of the Nonlinear Vlasov Equations"

PLASMA RESEARCH

Task 20504-12-2050140/59-442
Task 20504-12-2050417/62-1157

Origin: NBS
Sponsors: NASA and NBS
Manager: C.M. Tchen

Authorized 10/3/61
and 6/30/59

Full task description: October-December 1961 issue, p. 12
April-June 1959 issue, p. 15

Status: CONTINUED. Dr. C.M. Tchen developed a cascade theory of turbulence, in which the big eddies contribute in the evolution of energy, and the small eddies contribute transport properties. The hierarchy of dynamical equations governing the big eddies is solved, using a quasinormal approximation, and the dynamical equation governing the small eddies is solved by means of a quasilinear approximation. It should be noted that the dynamical equations determining the ensemble of big and small eddies can neither be solved by the quasinormal approximation, nor by the quasilinear approximation.

Status of Projects

In the framework of hydrodynamic turbulence, the theory derives a formula for the eddy viscosity. Interestingly enough the formula yields the spectral law $k^{-3/2}$, in agreement with theory proposed by Kraichnan, if a certain elementary approximation is made. However, with a more complex approximation, the formula of eddy viscosity derived by Tchen degenerates to the empirical formula of Heisenberg, and therefore leads finally to the spectral law $k^{-5/3}$, in agreement with the Kolmogoroff-Heisenberg theories. The cascade theory is developed for the following problems:

- (a) plasma turbulence with external magnetic field,
- (b) plasma turbulence in the presence of an external magnetic field, and
- (c) magnetohydrodynamic turbulence.

Three manuscripts giving the results of the above 3 problems were completed. (Publication No. 4, 5 and 6)

The main results are:

- (1) The density spectrum follows a k^{-5} law, in agreement with several experiments (Princeton, NASA Cleveland and NASA Langley).
- (2) The Bohm diffusion is derived for a turbulent plasma.
- (3) The electrostatic spectrum follows a spectral law k^{-3}
- (4) The spectra for mhd turbulence are derived under various circumstances, e.g. equipartition or non-equipartition between turbulent and magnetic energies.

Publications:

- (1) Turbulence in a Rarefied Plasma, Proceedings on Advanced Problems in Fluid Mechanics, Poland, Fluid Mechanics Transactions, 3, 689-692 (1967).
- (2) Spectrum of Turbulence in a Plasma with a Strong Magnetic Field, accepted for publication at the Proceedings of the Summer Institute on Nonlinear Problems in Plasmas, Paris, France.
- (3) Turbulence by Electrostatic Fluctuations, to be published at the Proceedings of the Summer Institute on Basic Kinetic Problems, University of Colorado, Boulder, Colorado.
- (4) Cascade Theory of Plasma Turbulence, Proceedings of the Reentry Plasma Turbulence Meeting at the Institute for Defense Analyses, Nov. 29-30, 1966, Washington, D.C., p. 4.1 to 4.35.
- (5) Spectral Distributions of Turbulence in a Plasma with Collisional and Collisionless Dissipations, will be published as an NBS Report, and is prepared for publication in an outside technical journal.
- (6) Cascade Process in Magnetohydrodynamic Turbulence will be published as an NBS Report, and is prepared for publication in a technical journal.

5. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS
Manager: Alan J. Goldman

Authorized 12/30/60

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued heavy. Section staff provided such services in 128 recorded instances; of these 79 involved assistance to NBS staff, covering 11 NBS divisions or other units. The 128 instances totalled to 862 man-hours. Other agencies assisted included Environmental Science Services Administration, Bureau of Public Roads, Highway Research Board, Post Office Department, Atomic Energy Commission, Civil Aeronautics Board, Securities Exchange Commission, Public Health Service, Customs Bureau, Economic Development Agency and NASA. Requests from universities, industry, professional groups and journals were also met.

(2) W. Hall continued participation in the design of a data store for inter-urban transport analyses. He delivered a number of lectures on current data storage and retrieval systems, and is assisting the Army Data Field System Command in a major evaluation effort. (Reported here for convenience; supported under Projects 4314690, 4566427, 4314433 respectively.)

(3) L.S. Joel and A.J. Goldman continued participation in an analysis aimed at developing methods to estimate proper protection levels for various elements in a communication network. (Reported here for convenience; supported under Project 4556455.) Goldman served as a judge in the first essay contest conducted by the Operations Research Society's Cost-Effectiveness Section.

(4) W.A. Horn investigated extensions of Browder's work on the existence of a fixed point of a continuous map f with domain and range in a Banach space. Affirmative results were obtained for the case in which (a) f carries a convex set into itself, or a suitable set of iterates carry a convex set into itself or a subset of itself, and also (b) f is compact, or completely continuous, or the space is uniformly convex and f is nonexpansive. A manuscript is in progress. He also derived a refinement, of Koopman's formula for the distribution of first-detection time in a random search, which is symmetric in the speeds of searcher and evader.

(5) K. Kloss continued work relating to the NBS PILOT computer: He extended the capabilities of the compiler (COMPILOT), in particular to include an improved formatted output. He continued the work (reported in the previous issue, pp. 12-13) on bases for spaces of isotropic tensors; the rank 8 calculations were completed, and those for rank 10 begun. Kloss used COMPILOT to investigate several number-theoretic conjectures, including some concerned with representing primes in the form $x^2 + 11y^2$ and some concerning weighted sums of Legendre symbols. The conjecture, that the smallest Fibonacci number divisible by a given prime is not divisible by the square of the prime, was verified for primes $< 50,000$.

The demise of the PILOT halted these and related efforts; their continuation on the computational facilities now available appears either impractical or awkward.

(6) K. Kloss participated in the following projects with members of Division 455's Advanced Programming Techniques group (supported under Project 4556152; reported here for convenience):

(a) Initiated and assisted in the implementation of the LISP 1.5 programming language on the new NBS computer (with D. Orser, R. Freemore, and P. Mason).

(b) Designed a programming language NEMO ("Non-Existent-Machine-Oriented") aimed at allowing machine-level programming while still remaining highly hardware-independent.

(c) Investigated some novel uses of UNIVAC 1106 FORTRAN V, specifically ways of making object code modifications and of achieving a recursive subroutine calling capability.

(7) Kloss investigated and compared the suitability of various commercial computers for research in picture processing, and began conversion of the PAX II picture-processing language to the UNIVAC 1106. (Supported under Project 2050404; reported here for convenience.)

Publications

- (1) A.J. Goldman. Operations research research. To appear in Proceedings of April 1966 Operations Research Conference for Non-Defense Washington Area Federal Agencies.
- (2) W.A. Horn. A modification of the random search formula. Submitted to a technical journal.
- (3) P.R. Meyers. A converse to the Banach contraction theorem. To appear in Journal of Research NBS, 71B (1967).
- (4) P.R. Meyers. Contractive semigroups and uniform asymptotic stability. Submitted to a technical Journal.
- (5) C. Mesztenyi (U. of Maryland) and C. Witzgall. Stable evaluation of polynomials. Journal of Research NBS, 71B (1967), pp. 11-17.

COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full task description: October-December 1964 issue, p.3; April-June 1962 issue, p.15

Status: CONTINUED.

(1) Jack Edmonds, in collaboration with D.R. Fulkerson of the RAND Corporation prepared a paper called "Bottleneck extrema", on a theory of solution of the fundamental class of optimization problems of the form: $\min_x (\max f(x))$, where $x \in R$, where R is a member of a prescribed family of subsets of a set E^R , and where f is a prescribed real-valued function on E .

He prepared a paper, called "An introduction to matching", on his blossom method for solving the class of integer-linear programs called matching problems. The paper was presented in a series of lectures at the University of Michigan.

He continued collaboration with Professor Ellis Johnson, of the Industrial Engineering Department of Yale University, on development and implementation of the blossom method.

He drafted a paper, called "Bidirected paths", to be presented in a series of lectures at the American Math Society Summer Seminar on the Decision Sciences at Stanford University in July.

He drafted a paper, called "Matroids and the greedy algorithm", to be presented at the Mathematical Programming Symposium at Princeton University in August.

He lectured at NIH (Division 431 auspices) on the assignment problem and on the critical path method. He prepared notes on linear programming for limited distribution at NBS.

Publications

- (1) Jack Edmonds. SDR's and linear algebra. Submitted to a technical journal.
- (2) Jack Edmonds. Optimal branchings. To be submitted to a technical journal.
- (3) P.R. Meyers. Minimum number of subsets to distinguish individual elements. Journal of Research NBS, 71B (1967), pp. 21-22.

LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) C. Witzgall documented his work with R. Wets (Boeing Scientific Research Laboratories) on the face-structure of polyhedral sets defined by linear inequalities.

(2) W. Horn studied "one-variable-at-a-time" methods of convex minimization.

Publications:

- (1) Algorithms for frames and lineality spaces of cones. R.J.B. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. Journal of Research NBS, 71B (1967), pp. 1-7.
- (2) Toward an algebraic characterization of convex polyhedral cones. R.J.B. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. To appear in Numerische Mathematik.

MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2050456

Origin: Technical Analysis Division, NBS

Authorized 3/1/66

Sponsor: Northeast Corridor Transportation Project, Dept. of Transportation

Managers: A.J. Goldman, P.R. Meyers

Full task description: January-June 1966 issue, p. 14

Status: CONTINUED.

(1) J. Gilsinn, A.J. Goldman, W. Horn and P.R. Meyers prepared roughly 25 memoranda on various phases of the Northeast Corridor, including reviews of several contractors' reports. Gilsinn and Meyers continued participation in conceptual and implementation phases of the Project.

J. Gilsinn designed and conducted initial experiments with a program to trace out multimodal trips. She and K. Kloss further detailed the design of a program for efficient modification of the network "inputs" to the simulation. W. Hall is participating in the planning of the freight simulation model.

(2) J. Levy continued the development of methods for evaluating feedback vs. non-feedback methods of regulating flow in a transport network. W. Horn continued work on the problem of choosing a planar network joining N given points, so as to minimize the sum of construction costs and travel-related costs. He also considered the optimal placement of cross-links between parallel main lines. Horn extended previous results (Berkeley, unpublished) on optimal allocation of service times among N incoming streams sharing a single server.

(3) K. Kloss designed and partially implemented a language to permit quick encoding of the shortest-path algorithms collected and flow-charted by C. Witzgall. (The current phase of this effort has been interrupted by the demise of the NBS PILOT computer.)

Publications:

- (1) A.J. Goldman, P.R. Meyers, J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.). Analysis of a market split model. Northeast Corridor Transportation Project Tech. Paper No. 8; also submitted to a technical journal.
- (2) A.J. Goldman and G.L. Nemhauser (Johns Hopkins U.). A transport improvement problem transformable to a best-path problem. Northeast Corridor Transportation Project Tech. Paper No. 9; also submitted to a technical journal.

MATHEMATICAL ANALYSES OF TOPICS IN ARMS CONTROL

Task 205-12-2050458

Origin and Sponsor: Arms Control and Disarmament Agency
Manager: L.S. Joel

Authorized 3/1/67

Objectives: To investigate the potential of various disciplines of mathematical operations research for developing models and methods applicable to arms control analyses. To investigate methods for bringing existing models closer to operational usefulness.

Background: Although the potential of the mathematical theory of games to assist in understanding arms control problems has received some attention, other possibly useful disciplines --- network theory, control theory, etc. --- have been relatively unexplored in this connection. Further study of the applicability of various mathematical disciplines can be expected to yield new insights.

Status: NEW

(1) The "Inspector's Tour Problem", of finding a shortest tour through a network in which each link is traversed (inspected) at least once, coincides with the "Chinese Postman's Problem" for which a solution method can be obtained based on J. Edmonds' matching-theory algorithms. Computational implementation of this method is in progress.

(2) A formula was derived for the probability that a "random" 2-person game (not in general zero-sum) has an equilibrium point in pure strategies. The limiting probability, for large games, is the surprisingly large $1-1/e=0.632^+$, in sharp contrast with the limit of zero for games of direct conflict.

(3) A conceptual study was begun of "games of imperfect execution", in which the strategy actually executed by a "player" (e.g., a team or alliance) can differ from the one formally chosen (e.g., by the team captain).

(4) Some relevant mathematically oriented behavioral-science literature was reviewed.

RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING

Authorized 1-21-64

20500-12-2050404/65-1456

Origin: NBS

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: January-June 1964 issue, p. 19

Status: CONTINUED. Mrs. J. Prewitt who is a guest worker from the University of Pennsylvania did some experiments with the PAX language to devise algorithms for identifying chromosomes and blood cells. Some of this work involved her own data and some involved data obtained by linking to Dr. Tretiak's data at M. I. T.. Kirsch used these data and some of our own on nerve cells for experimenting with algorithms for boundary detection and other homogeneous Boolean transformations on PAX images. Several primitive functions to be used by higher level languages (like LISP) have been written by Kirsch in LISP on the Q32 computer.

During this period experiments were performed on a third large time sharing system, the M44, located at IBM Research in New York. The M44, the MAC system in Massachusetts and the Q32 system in California were all used fairly heavily from the same consoles at NBS and a study was made of the different features of these systems. The comparison was useful because of the wide disparities in hardware configurations, system sophistication, user activity, and quality of maintenance among the three systems.

Because of the interest in the use of pattern recognition languages a meeting was held, at NBS, of users of PAX who discussed criteria for the design of higher level picture processing languages. It appeared as a conclusion that until more research is done in two dimensional syntax such languages as LISP will prove adequate for many of the pattern recognition problems. As a consequence of this conclusion work is being started on building some PAX operations into LISP 1.5 on the Q32 within the LAP assembly language.

J. Becker wrote a program in TINT that arranges standard geometric objects in two dimensions according to a standard topology that represents the arrangement of nerve cell parts. The program arranges the objects so as to preserve the inviolability of membrane boundaries.

Kirsch and Mrs. Rhodes wrote several programs for manipulation of files which are used for describing the structural organization of tissue. These tree search programs used in conjunction with the LISPEDIT language on Q32 make it very easy to build large structures for describing tissue organization. Kirsch and Dr. Lipkin have continued to use these tools. Much of the recent application has been at the level of cellular organization rather than at the level of gross morphology as previously.

RESEARCH ON A PICTURE LANGUAGE MACHINE

Authorized 5-1-61

20500-12-2050406/65

Origin: NBS

Sponsor: National Science Foundation

Manager: Russell A. Kirsch

Full task description: July-December 1963 issue, p. 17

Status: CONTINUED. The work that was done using the remote file system on PROJECT MAC was summarized in a paper by Alt and Kirsch. This terminates the work on this project.

MATHEMATICAL TABLES

20500-40-2050121/57-216

Origin and Sponsor: NBS

Manager: I. A. Stegun

Full task description: July-December 1964 issue, p. 4

Status: CONTINUED. Assistance to NBS divisions, other government agencies, industry and universities in the evaluation of computing techniques, identification of functions and their tabulation, etc. was provided in 67 recorded instances.

A correction list has been submitted for the sixth printing of AMS 55 - the Handbook of Mathematical Functions. Work continued on the updated reference and "improved methods" files.

To facilitate access to material contained in the Handbook experiments were performed in mathematical information retrieval, utilizing remote time-sharing computer systems and standard programming languages.

Lectures and Technical Meetings

Applied Mathematics Division Lectures

- KALMAN, Professor Gabor (Department of Mathematics, Brandeis University, Waltham, Mass.)
Relativistic Many-Particle Systems. March 20, 1967.
- KLOSS, K. E. (NBS Applied Mathematics Division) The NBS "PILOT" Computer: An
Introduction. NBS Administration Building. January 4, 1967.
- MYCIELSKI, Prof. Jan (Case Institute of Technology, Cleveland, Ohio) Games of Pursuit and
Evasion. May 9, 1967.
- TUTTE, Prof. W. (University of Waterloo, Waterloo, Ontario) Enumeration of Planar
Graphs. April 5, 1967.
- WAMPLER, Roy H. (Metrology Division Seminar) Pitfalls in Least Squares Calculations.
January 26, 1967.

NBS Courses Conducted By Staff Members

- EDMONDS, Jack Games and Linear Relations. February 1967 - May 1967.

NBS In-hours Courses Given by Staff Members

- HOGBEN, David Introduction to Mathematical and Statistical Analysis of Laboratory Data.
Spring Semester.
- PEAVY, Sally T. FORTRAN Programming. Spring Semester.
- PEAVY, Sally T. and
VARNER, Ruth N., et al. EXEC. II Operating System for the UNIVAC 1108. Summer course.

Mathematical Statistics Seminar

- COHEN, J. W. (Delft Technical University, Delft, Holland.) Recent Results in
Queueing Theory. June 8, 1967.
- LEWIS, P. A. W. (Thomas J. Watson Research Center, IBM.) Statistical Engineering Seminar:
A Computer Program for the Statistical Analysis of Series of Events.
March 28, 1967.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

- CAMERON, J. M. Calibration Designs. Presented at the Johns Hopkins University, Depart-
ment of Biostatistics, January 25, 1967.
- EDMONDS, Jack Network Analysis. Presented at the National Institutes of Health,
Bethesda, Maryland, June 14, 1967.
- EDMONDS, Jack Matching Theory. Presented at the University of Michigan, Ann Arbor,
Michigan, June 5-9, 1967.
- GOLDMAN, A. J. Two Mathematical Models of Market Split. Presented at the Washington
Operations Research Council, National 4-H Center, Chevy Chase, Maryland,
February 20, 1967.
- GOLDMAN, A. J. and
MEYERS, P. R. Analysis of a Market Split Model. Presented at the Society for Indus-
trial and Applied Mathematics, Shoreham Hotel, Washington, D.C., June 14,
1967. (With J. M. McLynn and R. H. Watkins, Davidson, Talbird & McLynn,
Inc.)

- HALL, W. G. Introduction to Linear Programming and Game Theory, III. Presented at the NBS "Introduction to Computer Sciences" Course for local students, April 21, 1967.
- The Formatted File System. Presented at the Space Data Control Branch, NASA Goddard Space Flight Center, Maryland, June 20, 22, 27, 29, 1967.
- HORN, W. A. Allocations of Servicing Periods which Minimize Average Delay for N Time-Shared Traffic Streams. Presented at the Society for Industrial and Applied Mathematics, Shoreham Hotel, Washington, D.C., June 13, 1967.
- JOEL, L. S. Introduction to Linear Programming. Presented at the Civil Service Commission, Course for local students, March 14, 1967.
- Introduction to Linear Programming and Game Theory, I, II. Presented at the NBS "Introduction to Computer Sciences" Course for local students, April 7, 14, 1967.
- KLOSS, K. E. "Comparative Aspects of Simulation Languages". Presented at the Institute of Management Sciences, Civil Service Auditorium, February 2, 1967.

Papers and Invited Talks Presented
by Members of the Staff at Meetings
of Outside Organizations

- KU, H. H. Interaction in Multi-dimensional Contingency Tables. Presented at the Institute of Mathematical Statistics, Columbus, Ohio, March 23-25, 1967.
- MEYERS, P. R. Contractive Semigroups and Uniform Asymptotic Stability. Presented at the Society for Industrial and Applied Mathematics, Shoreham Hotel, Washington, D.C., June 14, 1967.
- OSER, H. J. Numerical Solution of Ordinary First-Order Differential Equations. Presented at the Division of Computer Research and Technology Seminar, National Institutes of Health, Bethesda, Maryland, May 12, 1967.
- The Problem of One-Dimensional Crystal Growth. Presented at the Sigma Xi Society, Washington, D.C., May 15, 1967.
- Stability Problems in the Numerical Solution of Ordinary Differential Equations. Presented at the Division of Computer Research and Technology Seminar, National Institutes of Health, Bethesda, Maryland, May 26, 1967.
- ROSENBLATT, J. R. Prediction of the Reliability of Complex Systems. Presented at Pennsylvania State University, Department of Industrial Engineering, University Park, Pennsylvania, April 13, 1967.
- Distribution-Free Two-Sample Tests. Presented at Pennsylvania State University, Department of Mathematics, University Park, Pennsylvania, April 14, 1967.
- Applications of Statistics in Colorimetry. Presented at Madison College, Harrisonburg, Virginia, April 20, 1967.
- Prediction of the Reliability of Complex Systems. Presented at Bridgewater College, Bridgewater, Virginia, April 20, 1967.
- SADOWSKI, W. L. The Numerical Solution of Plasma Problems. Presented at the NASA Laboratory, Langley Field, Va., April 19-21, 1967.
- Numerical Solution of the Nonlinear Vlasov Equation. Presented at NASA, Bldg. 1, Greenbelt, Md., May 15, 1967.
- TCHEN, C. M. Plasma Turbulence. Presented at NBS Thermodynamic Colloquium, January 16, 1967.
- Turbulence by Electrostatic Fluctuations. Presented at Princeton University, Princeton, N.J., March 15, 1967.

TCHEN, C. M.

Cascade Process in Magnetohydrodynamic Turbulence. Presented at the University of Connecticut (Department of Aerospace Engineering), March 16, 1967.

Spectral Distributions of Turbulence in a Plasma with Collisional and Collisionless Dissipations. Presented at the Theoretical Division of NASA Goddard Space Center, Greenbelt, Md., May 18, 1967.

Hydrodynamics Turbulence. Presented at the Theoretical Division of NASA Goddard Space Center, Greenbelt, Md., June 13, 1967.

Spectra of Turbulence in a Collisionless Plasma. Presented at the International Symposium on Diffusion and Fluctuations, Princeton University, Princeton, N.J., June 27, 1967.

Publication Activities

1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.3 Technical Papers

Analysis of a market split model. A. J. Goldman, P. R. Meyers, with J. M. McLynn and R. H. Watkins (Davidson, Talbird & McLynn). Technical Paper No. 8, U.S. Department of Transportation, (April 1967).

A transport improvement problem transformable to a best-path problem. A. J. Goldman and G. L. Nemhauser (Johns Hopkins U.). Technical Paper No. 9, U.S. Department of Transportation (April 1967).

On a theorem of Ahlfors. L. Greenberg. American Journal of Mathematics. (Jan. 1967).

Fundamental Polyhedra for Kleinian Groups. L. Greenberg. Annals of Mathematics. (Nov. 1966).

Note on normal subgroups of the modular group. L. Greenberg. Proceedings of the American Mathematical Society, Vol. 17, No. 5, pp. 1195-1198 (Oct. 1966).

A functional inequality. S. Haber. American Mathematical Monthly, Vol. 73, No. 10, pp. 1103-1104 (1966).

Chapter 2--Statistical concepts in metrology. H. H. Ku. Handbook of Industrial Metrology, sponsored by the American Society of Tool and Manufacturing Engineers, Prentice-Hall, Inc., pp. 20-54 (1967).

Minimum number of subsets to distinguish individual elements. P. R. Meyers. J. of Research NBS, 71B, pp. 21-22 (1967).

Stable evaluation of polynomials. C. Mesztenyi (U. of Maryland) and C. Witzgall. J. of Research NBS, 71B, pp. 11-17 (1967).

Classification of normal subgroups of the modular group. M. Newman. Trans. Amer. Math Soc. 126, pp. 267-277 (1967).

The coefficients of the powers of a polynomial. M. Newman. J. of Research NBS, 71B1, pp. 9-10 (1967).

On the solution to the Stefan problem for whisker growth. J. A. Simmons, H. Oser, and S. R. Coriell. Crystal Growth (Supplement to the Journal of Physics and Chemistry of Solids) Proceedings of an International Conference on Crystal Growth, Boston, June 1966, Pergamon Press, New York, 1967.

Turbulence in a Rarefied Plasma. C. M. Tchen. Proceedings on Advanced Problems in Fluid Mechanics, Poland, Fluid Mechanics Transactions, 3, pp. 689-692 (1967).

Cascade Theory of Plasma Turbulence. C. M. Tchen. Proceedings of the Reentry Plasma Turbulence Meeting at the Institute for Defense Analyses, Washington, D.C., pp. 4.1 to 4.35, Nov. 29-30, 1966.

Algorithms for frames and lineality spaces of cones. R. J. B. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. J. of Research NBS, 71B, pp. 1-7 (1967).

2.0 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.1 Mathematical Tables

Tables Relating to Mathieu Functions (Second Edition of CUP 13). To appear as AMS 59.

2.3 Technical Papers

A modification of the random search formula. W. A. Horn. Submitted to a technical journal.

A converse to Banach's contraction theorem. P. R. Meyers. To appear in J. of Research NBS, 71B (1967).

Contractive semigroups and uniform asymptotic stability. P. R. Meyers. Submitted to a technical journal.

Maximal normal subgroups of the modular group. M. Newman. Submitted to a technical journal.

Two classical theorems on commuting matrices. M. Newman. To appear in J. of Research NBS.

The numerical solution of second-order difference equations. Frank W. J. Olver. To appear in J. of Research NBS.

A method in diophantine approximation (II). C. Osgood. To appear in Acta Arithmetica.

Realistic uncertainties and the mass measurement process: An illustrated review. Paul E. Pontius (NBS Mass and Volume Section) and Joseph M. Cameron. To appear as NBS Monograph 103, U. S. Government Printing Office.

Toward an algebraic characterization of convex polyhedral cones. R. J. B. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. To appear in Numerische Mathematik.

