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COMPLEX DIELECTRIC CONSTANT AND DISSIPATION FACTOR OF FOLIAGE

by
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Report For

U. S. Naval Ordnance Laboratory

Inter-Agency Project Order No. P.O. 7-0011

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DIELECTRIC CONSTANT AND LOSS FACTOR OF FOLIAGE

Introduction:

This report presents data on the dielectric properties of living foliage, plant material, and clay soil. This data is of potential value to investigators concerned with the effects of plant material and earth on electromagnetic radiation at radio and microwave frequencies. The report includes a description of the measurement theory and equations, experimental apparatus and techniques, a calibration check using water as a reference material, a discussion of sources of error and experimental data on a variety of specimens at 23°C and over a frequency range of 10^5 Hz to 4.2×10^9 Hz. Estimates of precision, calculated from experimental data, are presented at representative frequencies.

Theory of Measurement:

The measurement of dielectric constant of a material at frequencies up to about 10^{10} Hz (cycles per second) is conveniently related to the measurement of the admittance of a coaxial cylindrical transmission line with a specimen of the material occupying some of the space between the coaxial conductors. The characteristic admittance of a section of the line filled with a material of relative complex dielectric constant $\kappa^* = \kappa' - j\kappa''$ and relative complex permeability μ^* is given by,

$$Y_{\kappa*} = 2\pi\left(\frac{\kappa^* \epsilon_o}{\mu^* \mu_o}\right)^{1/2} (\ln B/A)^{-1}, \tag{1}$$

and the propagation constant $\gamma_{\kappa*}$ of this section is given by,

$$\gamma_{\kappa*} = j\,\omega\,(\kappa^* \epsilon_o\, \mu^* \mu_o)^{1/2}, \tag{2}$$

where ϵ_o and μ_o are the permittivity and permeability of vacuum, A and B are the radii of the inner and outer conductors of the line, $j = \sqrt{-1}$, and ω is the angular frequency of the electromagnetic radiation which is assumed to be low enough to propagate in the TEM mode only.

The admittance Y_M at point M in the line (see figure 1) can be expressed in terms of the admittance Y_L at point L and the characteristic impedance and propagation constant of the line by the relationship [1][1]:

$$Y_M = Y_{\kappa*} \frac{Y_L + Y_{\kappa*} \tanh \gamma_{\kappa*}\ell}{Y_{\kappa*} + Y_L \tanh \gamma_{\kappa*}\ell} \tag{3}$$

where ℓ is the length of line between M and L.

Referring to figure 1 we assume a uniform line open-circuited on the right and with a sample of thickness ℓ_s inserted in the line. The fringing field at the end of the line must be taken into account either by assuming a load capacitance at the line end or by assuming that the open is located a short distance (determined by measurement) beyond the line end.

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[1] Numbers in brackets refer to references listed at the end of this report.

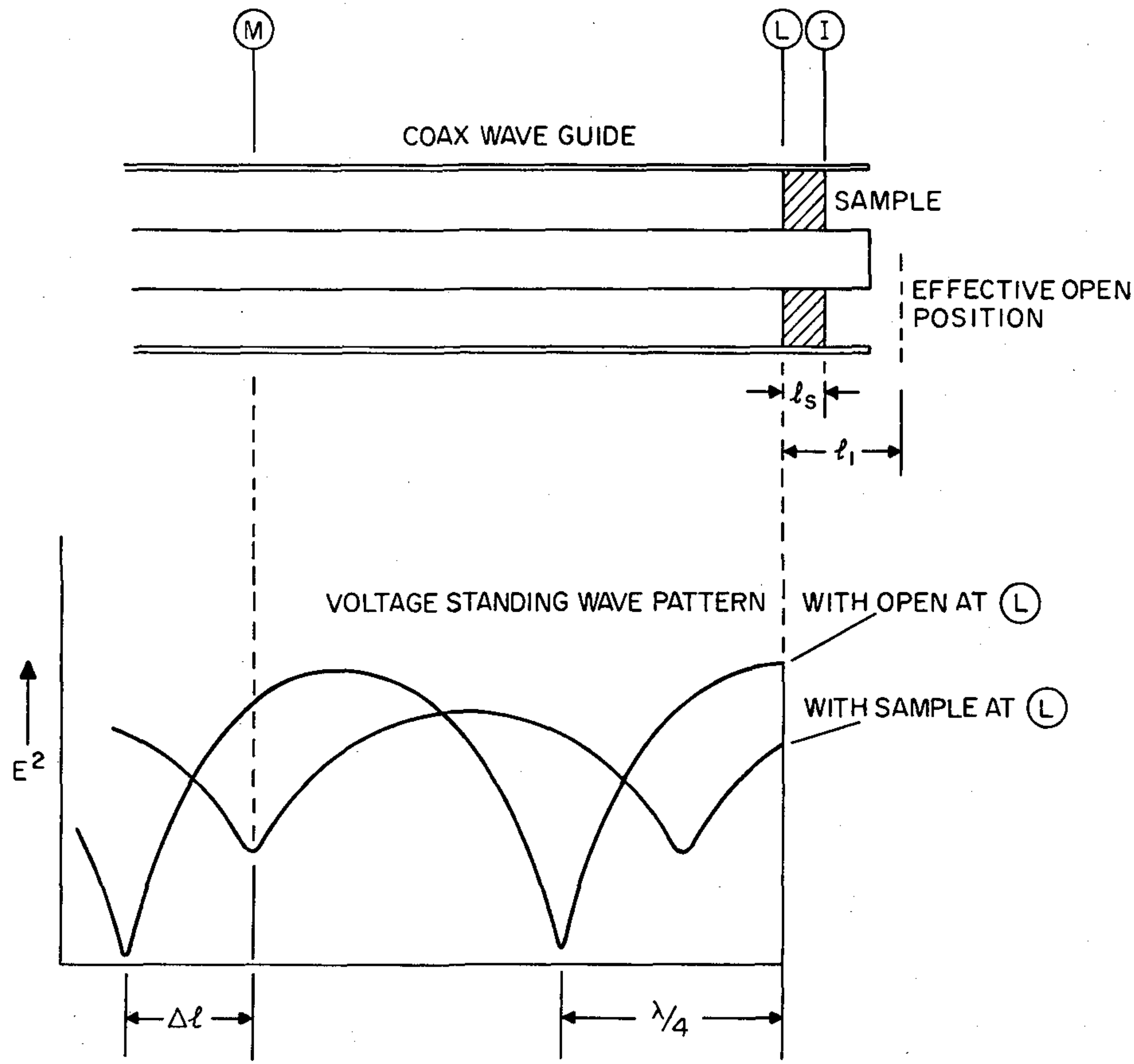

Figure 1. A schematic diagram of the coaxial wave guide used for the leaf measurements and the corresponding standing wave patterns assumed in the derivation of the working equations.

Using this latter approach we define ℓ_1 to be the length from this effective open position to the front (left hand) face of the sample.

Using equation (3) for Y_1, with a load admittance of zero, and a characteristic admittance Y_0 and transmission coefficient γ_0 for air (same as equations (1) and (2) with $\kappa^* = \mu^* = 1$) we get

$$Y_1 = Y_0 \tanh \gamma_0 (\ell_1 - \ell_s) \tag{4}$$

If we now transform this admittance Y_1 through the sample, again using equation (3), we get

$$Y_L = Y_{\kappa^*} \frac{Y_0 \tanh \gamma_0 (\ell_1 - \ell_s) + Y_{\kappa^*} \tanh \gamma_{\kappa^*} \ell_s}{Y_{\kappa^*} + Y_0 \tanh \gamma_0 (\ell_1 - \ell_s) \tanh \gamma_{\kappa^*} \ell_s} \tag{5}$$

Substituting $Y_{\kappa^*} = Y_0 \left(\frac{\kappa^*}{\mu^*}\right)^{1/2}$ and $\gamma_{\kappa^*} = \gamma_0 \ (\kappa^* \mu^*)^{1/2}$ and simplifying,

$$Y_L = Y_0 \frac{\tanh \gamma_0 (\ell_1 - \ell_s) + \left(\frac{\kappa^*}{\mu^*}\right)^{1/2} \tanh \gamma_0 \ (\kappa^* \mu^*)^{1/2} \ \ell_s}{1 + \left(\frac{\mu^*}{\kappa^*}\right)^{1/2} \tanh \gamma_0 (\ell_1 - \ell_s) \tanh \gamma_0 (\kappa^* \mu^*)^{1/2} \ell_s} \tag{6}$$

This equation becomes much more tractable if the arguments of the tanh can be kept small. Writing $\gamma_0 = j \frac{2\pi}{\lambda}$ where λ is the radiation wave length in air, consider the size of the arguments $j \frac{2\pi}{\lambda} (\ell_1 - \ell_s)$ and $\frac{2\pi \ell_s}{\lambda} (\kappa^* \mu^*)^{1/2}$ in equation (6). The smallest wave length we will use in the measurements reported here is 7.1 cm (corresponding to the maximum frequency of 4.2 GHz). The value of ℓ_s is kept below 0.04 cm and ℓ_1 is measured to be about 0.3 cm (measurement described on page 9). Hence the maximum values of the above arguments are

$$\left.\frac{2\pi(\ell_1-\ell_s)}{\lambda}\right|\max \cong \frac{2\pi x0.3}{7.1} \cong 0.26$$

and for $\kappa^* \sim 50\ (1-0.3\ j)$ (gotten from the measured high frequency data) and $\mu^* \sim 1$ (for non-magnetic materials)

$$\frac{2\pi\ell_s}{\lambda}(\kappa^*\mu^*)^{1/2} \max \cong \frac{2\pi x0.04x7}{7.1} \cong 0.25$$

It is true that κ^* becomes larger at lower frequencies but λ becomes large much faster than $\sqrt{\kappa^*}$ and this latter argument becomes smaller than 0.25. Letting u* represent the complex quantity (x+jy), we can expand the tanh u* as follows:

$$\tanh u^* = u\left(1-\frac{u^{*2}}{3}+\ .\ .\ .\right)=x\left(1-\frac{x^2}{3}+y^2+\ldots\right)+jy\left(1-x^2+\frac{y^2}{3}+\ldots\right)$$

We see that the largest error due to dropping all terms but the first in the tanh expansions in equation (6) is of the order of $(0.26)^2$ compared to 1, or about 7%. This error is much less than the accuracy required of these measurements and we can approximate equation (6) by,

$$Y_L = Y_o \frac{\gamma_o(\ell_1-\ell_s)+\kappa^*\gamma_o\ \ell_s}{1+\gamma_o^2\ \mu^*\ell_s(\ell_1-\ell_s)} \tag{7}$$

By the arguments above we can also neglect the second term in the denominator (remembering $\mu^* \cong 1$ for the non-magnetic samples measured in this study), and introducing $\kappa^* = \kappa'-j\kappa''$ we find,

$$Y_L = Y_o\ \gamma_o\ \left[(\kappa'-1)\ell_s+\ell_1-j\ \kappa''\ \ell_s\right] \tag{8}$$

It may be comforting to the reader who is more familiar with lumped circuit measurements to show a correspondence with our linearized transmission line equation. One can replace in equation (8) $Y_o = \sqrt{\frac{C_o}{L_o}}$ and $\gamma_o = j\,\omega\sqrt{L_o\,C_o}$ where L_o and C_o are the capacitance and inductance per unit length in air. This gives,

$$Y_L = j\,\omega\,C_o\,[(\kappa'-1)\ell_s+\ell_1 - j\,\kappa''\,\ell_s] \tag{9}$$

Thus the admittance of the part of the line occupied by the sample is just the capacitive susceptance of the empty line enhanced by the relative complex dielectric constant κ^*. This simplification results from being able to use very thin specimens - in this case single thicknesses of leaves - and has been previously described under the name "thin sample method"[2].

We now go to the other part of the problem - getting Y_L in terms of measured quantities. We will consider the low frequency bridge measurements and the high frequency slotted line measurements separately.

For a bridge measurement such as we have used from 10^5 Hz to 250 MHz we actually measure the capacitance and conductance of the coax line at point M. We can calculate the capacitance and conductance at point L by using an inverted form of equation (3),

$$Y_L = Y_o\,\frac{Y_M - Y_o\tanh\gamma_o\,\ell}{Y_o - Y_M\tanh\gamma_o\,\ell}\ . \tag{10}$$

For a lossless line $\tanh \gamma_0 \ell = j \tan \beta\ell$ where $\beta = 2\pi/\lambda$. Substituting $G+j\omega C = Y$ for both Y_L and Y_M and equating real and imaginary parts we find

$$G_L = G_M \frac{1 + \tan^2 \beta\ell}{1+2 \frac{\omega C_M}{Y_o} \tan \beta\ell + \frac{G_M^2+(\omega C_M)^2}{Y_o^2} \tan^2 \beta\ell} \tag{11}$$

and

$$C_L = C_M \frac{1-\tan^2\beta\ell + \frac{\omega^2 C_M^2 - Y_o^2 + G_M^2}{Y_o \omega C_M} \tan \beta\ell}{1+2 \frac{\omega C_M}{Y_o} \tan \beta\ell + \frac{G_M^2+(\omega C_M)^2}{Y_o^2} \tan^2 \beta\ell} \tag{12}$$

For the slotted line, if we know the position of a minimum with an open circuit at L then we can calculate the admittance Y_L of any load at L from the shift in the minimum and the voltage standing wave ratio due to the load. See figure 1. Again we can use equation (10) to get Y_L in terms of Y_M. Since Y_M is real (probe at a minimum) and equal to G_M, and ℓ is some odd multiple of quarter wave lengths less the shift $\Delta\ell$, and remembering that $\tan\left(\frac{\pi}{2} - x\right) = \cot x$ equation (10) reduces in this case to

$$Y_L = Y_o \frac{\frac{G_M}{Y_o} - j \cot \beta \Delta \ell}{1-j \frac{G_M}{Y_o} \cot \beta \Delta \ell} \tag{13}$$

The ratio $\frac{G_M}{Y_o}$ is the measured voltage standing wave ratio, r. Multiplying through by $\tan \beta\Delta\ell$ and separating real and imaginary parts we get for the conductance and capacitance at L,

$$G_L = Y_o \frac{r(1+\tan^2 \beta\Delta\ell)}{r^2+\tan^2 \beta\Delta\ell} \tag{14}$$

$$C_L = \frac{Y_o}{\omega} \frac{(r^2-1)\tan \beta\Delta\ell}{r^2+\tan^2 \beta\Delta\ell} \tag{15}$$

Before writing the final working equations we shall introduce one further complication into equation (9). Because a leaf has such a high dielectric constant (similar to that of water) the effect of an air gap between the sample and holder is severe. That is, the sample can be considered a large admittance in series with a comparable size gap admittance and the series combination is a considerably smaller admittance. To reduce the air gap errors it was decided to measure the leaves immersed in water, thereby increasing the gap admittance by a factor of the order of 100 and also preventing the drying of the leaves during measurement. Considering the sample to be made up of leaf of thickness ℓ_ℓ and relative permittivity κ_ℓ^* and water of thickness $(\ell_s-\ell_\ell)$ and dielectric constant κ_w^*, we can follow the procedure used in obtaining equation (9) using one additional transformation to account for the water and we obtain an expression analogous to equation (9) as follows:

$$Y_L = j\, Y_o\, \beta\, [\kappa_\ell'\, \ell_\ell + \kappa_w'\, (\ell_s-\ell_\ell) + (\ell_1-\ell_s)] \tag{16}$$

$$+ Y_o\, \beta\, [\ell_\ell \kappa_\ell'' + (\ell_s-\ell_\ell)\, \kappa_w'']$$

Again letting $Y_L = G_L + j\,\omega\, C_L$, separating real and imaginary parts, and solving for κ'_ℓ and κ''_ℓ, the real and imaginary parts of the relative permittivity of the leaf sample we have,

$$\kappa'_\ell = \frac{\omega\, C_L}{Y_o \beta \ell_\ell} - \kappa'_w \left(\frac{\ell_s}{\ell_\ell} - 1 \right) - \frac{(\ell_1 - \ell_s)}{\ell_\ell} \tag{17}$$

and

$$\kappa''_\ell = \frac{G_L}{Y_o \beta \ell_\ell} - \kappa''_w \left(\frac{\ell_s}{\ell_\ell} - 1 \right) \tag{18}$$

The κ'_w and κ''_w are gotten from tables of dielectric constants [8], ℓ_s is the constructed length of the coax chamber containing the water and leaf, G_L and C_L are the values of conductance and capacitance given by equations (11) and (12) at bridge frequencies and by (14) and (15) at slotted line frequencies, Y_o is taken as the nominal value for the GR line used $(50\ \Omega)^{-1}$, $\beta = \omega/c$ where ω is 2π x frequency determined either by measuring the wave length in the slotted line or measuring frequency directly with a counter and the leaf thickness ℓ_ℓ is the average of several measurements with an ordinary mechanical micrometer, and c is the velocity of light.

In equations (13) through (15), $\Delta\ell$ is the difference between the measured minimum for the loaded line and the minimum for the line with an open at point L of figure 1. This latter was measured by putting a copper shorting disk at the end of the line and measuring two adjacent minima. The midpoint of these less the distance between L and the shorting disk was taken to be the minimum with an open at L. At lower frequencies where only one minimum could be measured, the minimum with the open at L was taken to be $\lambda/4$ where λ was computed from the counter-measured frequency and the known velocity of light. The distance between this calculated minimum for the open at L and the actual measured minimum with the unloaded line equals ℓ_1 in figure 1.

Measurement Procedure:

The measurements at 100, 200 and 400 kHz were made with a Cole-Gross type transformer bridge described previously [3]. From 2 MHz to 250 MHz a Boonton RX meter type 250A [4], fitted with a type N coax adapter terminal was used, and from 450 MHz to 4.2 GHz a Hewlett Packard model 805A slotted line was used with the HP model 612A and 616B signal generators and HP model 415B standing wave indicator. The sample holder consists of about 10 cm of General Radio type 874L10 air line [5] cut off at one end so that the inner and outer conductors are flush and connected to a type N male connector which permits rapid connecting and disconnecting with both the slotted line and the RX meter. The sample chamber in the cut off end of the air line was constructed as follows. A small wafer of polystyrene foam was fitted snugly into the line and the surface was coated with wax to prevent any water from getting inside the line. A piece of shim containing a rectangular step ~ 0.04 cm deep was then used to remove the excess wax and create a sharp-cornered water-tight pocket in the end of the line. A polystyrene cap was fitted

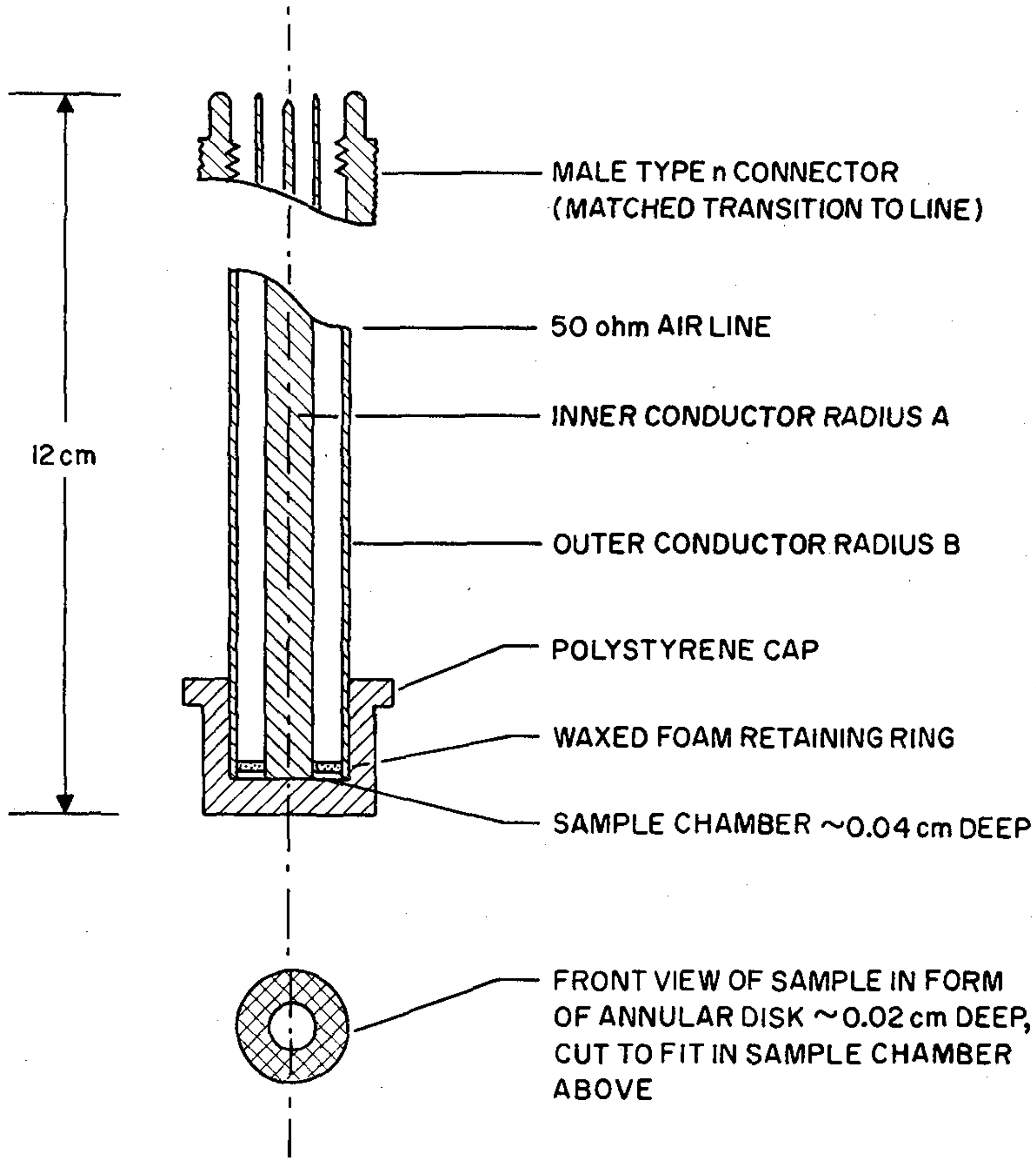

Figure 2a. A diagram of the coaxial line, sample holder and a sample disk used for dielectric measurements on foliage.

over the end of the line and served as the outer surface and seal for the sample pocket. This cap was secured by a locking nut on the line. The holder is illustrated in figure 2a and b.

The leaf samples were cut to shape to fit snugly in the sample holder by means of a specially made "cookie cutter". Immediately after cutting, the sample was weighed and measured for thickness and then immersed in a dish of distilled water. With the sample holder also immersed and visible bubbles removed, the leaf wafer was placed in the sample pocket and the cap secured in place. The transparent cap permits inspection of the outer surface of the leaf for trapped air bubbles and if any were observed the load procedure was repeated. The unit was then taken to the various measurement systems for electrical measurements. The measurements generally took less than one half hour and the first measurement was always repeated at the end of a run to see what electrical changes if any had taken place. Also the sample was weighed after measurement to see what weight changes, if any, had occurred. For the measurement of tree branches the same type of wafer specimens were prepared and with clay the sample chamber was merely packed with the clay samples.

Densities were determined by taking the ratio of measured weight to measured volume (all specimens had the same area as determined by the sample holder and cookie-cutter dimensions). Moisture content was determined by weighing the leaf before measurement and after completely drying it at 110°C. The weight loss divided by the initial weight was taken to be the

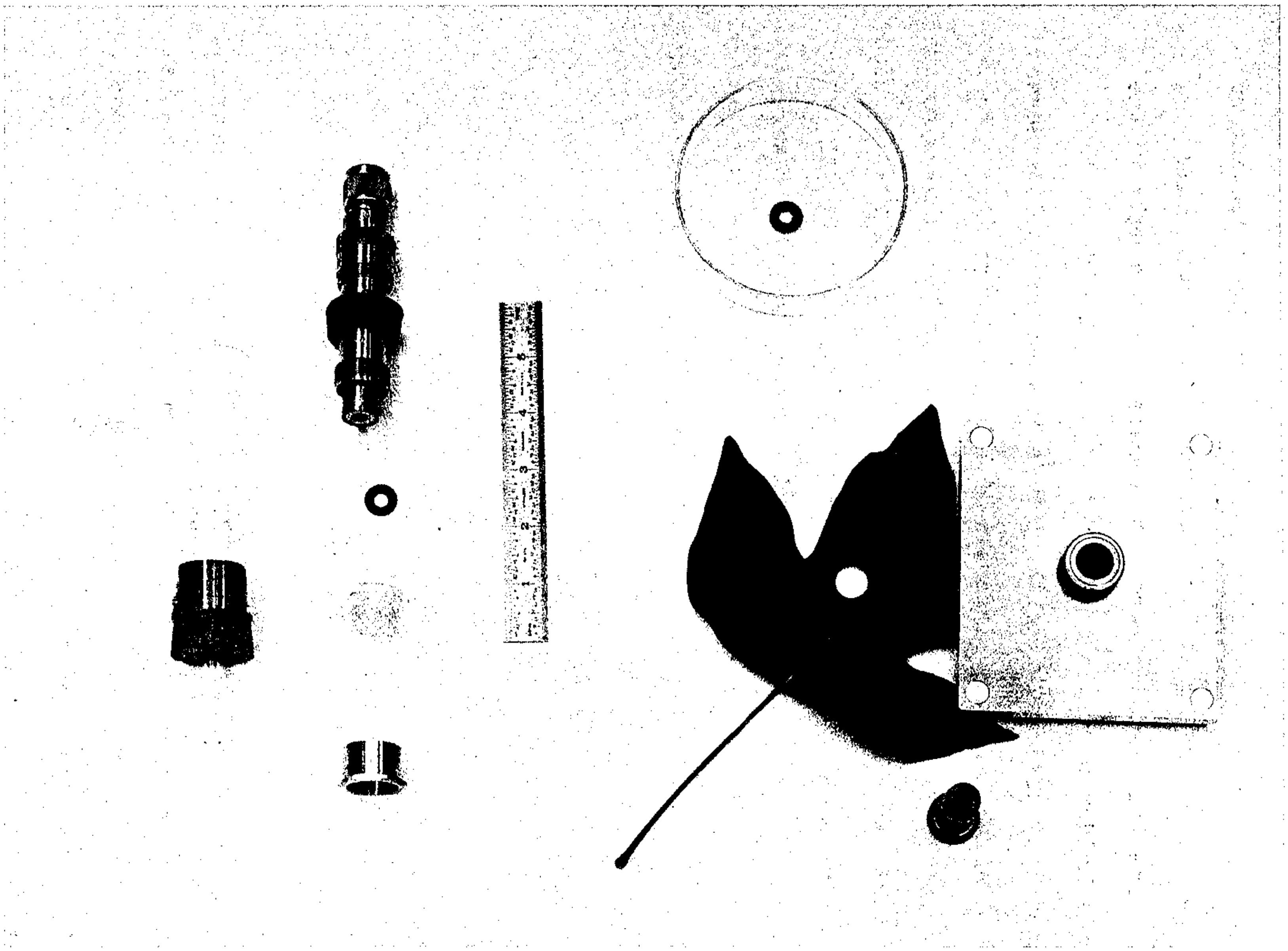

Figure 2b. Photograph of sample "cookie cutter", sample holder, sample, end cap and retainer, and thermostating water jacket.

moisture content of the leaf during measurement and this figure was expressed as a percent of the total weight.

The leaf specimens were collected locally from living plants and measurements were made shortly after removal from the plants. Specimens A (Bamboo) included leaves newly formed this spring and leaves which had been formed last year and were in varying stages of yellowing. Specimens B (Tulip Tree) were all new leaves collected during the first month after their formation. The twig specimens were taken from growing tulip tree branches and sliced into appropriate sized samples.

<u>Checking Measurement System for Accuracy and Precision</u>:

The system was first checked by measuring the empty holder with the sample chamber containing air and with the polystyrene end cap in place. The measured capacitance of the chamber plus fringing capacitance is given in table 1. Then distilled water was placed in the chamber and a full range of measurements were made. In this case we have a water sample completely displacing air so that we understand the quantities in equations (17) and (18) to represent the following; κ'_ℓ, κ''_ℓ are for water, κ'_w and κ''_w are for air ℓ_ℓ is the thickness of water, ℓ_s = thickness of sample = ℓ_ℓ. The resulting measured values of κ' and κ'' are given in table 1. Values for water calculated from the literature for a temperature of 23°C are given in table 1 for comparison. It is apparent that the system gives a fairly

Table I Check of System Using Air and Water as References

Frequency (GHz)	Empty Holder Capacitance (pf) [a]	Water-Filled Holder Capacitance (pf) [a]	Water (Calc) [b] κ'	Water (Calc) [b] κ''	Water (literature) [c] κ'	Water (literature) [c] κ''
0.002	0.42	2.56	79.6	7.44	79.	
0.005	0.42	2.60	79.9	2.06	79.	
0.01	0.41	2.60	79.8	1.49	79.	0.36
0.02	0.40	2.59	79.8	0.60	79.	
0.03	0.39	2.59	80.1	0.21	79.	0.120
0.05	0.40	2.58	79.6	0.78	79.	0.200
0.07	0.44	2.61	78.8	1.30	79.	0.281
0.1	0.43	2.56	77.4	1.09	79.	0.401
0.2	0.44	2.56	77.2	2.46	79.	0.802
0.25	0.44	2.60	78.5	2.48	79.	1.003
0.45	0.43	2.54	76.8	2.80	79.	1.80
0.70	0.46	2.57	77.2	4.09	78.9	2.80
1.0	0.49	2.57	76.1	5.14	78.8	4.00
1.2	0.39	2.44	73.0	5.67	78.7	4.80
.1	0.39	2.56	78.9	6.28	78.0	8.32
.4	0.56	2.81	81.7	14.2	76.6	13.2
.2	0.34	2.26	70.3	15.4	75.3	16.0

[a] Numbers calculated from data by equations 11, 12, 14, 15.

[b] Numbers calculated by equations 17, 18.

[c] Values calculated from ref [8] for T ~ 23°C. These values are approximate because of sensitivity of data to temperature and need to interpolate from data reported at 20 and 25° C.

constant value for the empty holder capacitance and values for the dielectric constant of distilled water to within 10% accuracy. Thus we may conclude that as far as the equipment is concerned, we anticipate accuracies of the order of 10%. (We are here neglecting the sizeable differences between the measured and literature values for κ''.) This figure is probably optimistic at the highest frequencies where the error is greatest and pessimistic in the more accurate low frequency range.

A check of the precision of the leaf measurements was made as follows. Several leaves were collected from various parts of plant types A (Bamboo) and B (Tulip Tree). Samples were cut and measured at one bridge frequency (200 MHz) and one slotted line frequency (1.0 GHz). The sample was removed from the holder, replaced and again measured. Each sample was measured three separate times in this way and six separate samples were measured from specimen type A and five from specimen type B. The results are shown in table 2, together with the measured values of thickness, weight, density and moisture content. The scatter was analyzed as follows. For each of the three characteristics, κ', κ'', and D, and for each type of specimen and frequency, an analysis of variance was made to ascertain the components of variance for

Table II - a   Specimen A   Frequency 200 MHz

Leaf Number	Thick-ness (cm)	Weight (gms)	Density (gms/cm³)	Moisture Content %	κ'	Average κ' for each leaf	Std. dev. for each leaf	κ''	Ave. κ'' for each leaf	Std. dev. for each leaf	D $= \kappa''/\kappa'$	Ave. D for each leaf	Std. dev. for each leaf
A-1	0.0106	0.0111	0.804	66.	66.4 59.1 61.2	62.2	3.76	66.7 67.1 65.7	66.5	0.72	1.00 1.14 1.07	1.07	0.0700
A-2	0.0102	0.0105	0.795	64.	66.9 66.9 66.2	66.7	0.40	58.8 59.3 58.8	59.0	0.29	0.879 0.886 0.889	0.885	0.0051
A-3	0.0099	0.0103	0.798	63.	61.6 59.9 66.2	62.6	3.26	56.2 53.2 53.8	54.4	1.59	0.912 0.887 0.811	0.870	0.0526
A-4	0.0102	0.0105	0.795	64.	61.6 64.6 60.8	62.3	2.00	59.6 57.3 58.0	58.3	1.18	0.967 0.888 0.955	0.937	0.0426
A-5	0.0096	0.0100	0.800	62.	56.5 57.3 56.5	56.8	0.46	45.0 45.4 44.8	45.1	0.30	0.804 0.792 0.793	0.796	0.0066
A-6	0.0102	0.0104	0.789	63.	67.3 66.4 67.1	66.9	0.47	65.1 63.1 63.1	63.9	1.15	0.973 0.950 0.939	0.954	0.0173
Average value					62.9			57.8			0.919		
Std. dev. of combined random errors					4.13 (6.56%)			7.61 (13.2%)			0.098 (10.7%)		
Std. dev. of instrumental errors only					2.21 (3.51%)			0.99 (1.71%)			0.040 (4.35%)		
Std. dev. of errors due to leaf variability					3.49 (5.55%)			7.55 (13.06%)			0.090 (9.79%)		

Table II - b   Specimen A   Frequency 1 GHz

Leaf Number	Thick-ness (cm)	Weight (gms)	Density (gms/cm³)	Moisture Content %	κ′	Average κ′ for each leaf	Std. dev. for each leaf	κ″	Ave. κ″ for each leaf	Std. dev. for each leaf	D = κ″/κ′	Ave. D for each leaf	Std. dev. for each leaf
A-1	0.0106	0.0111	0.804	66.	39.8	38.2	2.25	14.6	14.4	0.252	0.367	0.377	0.0173
					35.6			14.1			0.397		
					39.1			14.4			0.367		
A-2	0.0102	0.0105	0.795	64.	43.7	42.7	1.79	13.6	13.5	0.264	0.311	0.317	0.0079
					43.7			13.7			0.314		
					40.6			13.2			0.326		
A-3	0.0099	0.0103	0.798	63.	41.9	42.4	1.66	11.7	11.6	0.153	0.280	0.273	0.0098
					41.1			11.4			0.278		
					44.3			11.6			0.262		
A-4	0.0102	0.0105	0.795	64.	42.2	43.4	1.12	12.7	13.3	0.520	0.301	0.306	0.0050
					44.4			13.6			0.307		
					43.7			13.6			0.311		
A-5	0.0096	0.0100	0.800	62.	40.9	39.8	1.96	9.4	9.33	0.115	0.231	0.236	0.0087
					40.9			9.4			0.231		
					37.5			9.2			0.246		
A-6	0.0102	0.0104	0.789	63.	39.9	42.2	2.73	13.6	13.8	0.208	0.342	0.328	0.0164
					41.4			13.7			0.332		
					45.2			14.0			0.310		

	κ′	κ″	D
Average value	41.4	12.6	0.306
Std. dev. of combined random errors	2.59 (6.26%)	1.89 (15.0%)	0.049 (16.0%)
Std. dev. of instrumental errors only	1.87 (4.52%)	0.28 (2.22%)	0.012 (3.92%)
Std. dev. of errors due to leaf variability	1.67 (4.03%)	1.86 (14.8%)	0.048 (15.7%)

Table II - d Specimen B Frequency 1 GHz

Leaf Number	Thick-ness (cm)	Weight (gms)	Density (gms/cm³)	Moisture Content %	κ'	Average κ' for each leaf	Std. dev. for each leaf	κ''	Ave. κ'' for each leaf	Std. dev. for each leaf	D $= \kappa''/\kappa'$	Ave. D for each leaf	Std. dev. for each leaf
B-1	0.0208	0.0210	0.776	77.	45.4	46.2	0.737	8.55	8.47	0.124	0.188	0.183	0.00503
					46.5			8.54			0.184		
					46.8			8.33			0.178		
B-2	0.0221	0.0230	0.801	76.	43.2	43.3	0.503	8.31	8.44	0.135	0.192	0.195	0.00264
					42.8			8.44			0.197		
					43.8			8.58			0.196		
B-3	0.0221	0.0250	0.871	78.	41.8	42.2	0.346	9.45	9.42	0.023	0.226	0.223	0.00231
					42.4			9.41			0.222		
					42.4			9.41			0.222		
B-4	0.0236	0.0251	0.818	78.	42.6	43.9	1.212	9.97	9.91	0.095	0.234	0.226	0.00723
					44.1			9.80			0.222		
					45.0			9.96			0.221		
B-5	0.0262	0.0286	0.842	80.	42.2	44.2	2.107	9.96	10.05	0.081	0.236	0.228	0.00907
					44.0			10.10			0.229		
					46.4			10.10			0.218		
B-6	0.0208	0.0219	0.813	77.	41.0			8.75			0.213		
Average value					44.0			9.26			0.211		
Std. dev. of combined random errors					1.76 (4.00%)			0.77 (8.32%)			0.021 (9.95%)		
Std. dev. of instrumental errors only					1.17 (2.66%)			0.10 (1.08%)			0.0059 (2.80%)		
Std. dev. of errors due to leaf variability					1.32 (3.00%)			0.77 (8.32%)			0.020 (9.48%)		

Table II - C Specimen B Frequency 200 MHz

Leaf Number	Thick-ness (cm)	Weight (gms)	Density (gms/cm^3)	Moisture Content %	κ'	Average κ' for each leaf	Std. dev. for each leaf	κ''	Ave. κ'' for each leaf	Std. dev. for each leaf	D $= \kappa''/\kappa'$	Ave. D for each leaf	Std. dev. for each leaf
B-1	0.0208	0.0210	0.776	77.	50.4 50.4 53.8	51.5	1.96	39.2 38.5 37.6	38.4	0.80	0.778 0.764 0.698	0.747	0.0427
B-2	0.0221	0.0230	0.801	76.	49.4 49.6 52.1	50.4	1.50	35.7 37.9 37.3	37.0	1.14	0.722 0.764 0.717	0.734	0.0258
B-3	0.0221	0.0250	0.871	78.	46.3 48.9 48.8	48.0	1.47	43.6 43.9 42.9	43.5	0.51	0.942 0.989 0.880	0.907	0.05467
B-4	0.0236	0.0251	0.818	78.	51.4 52.1 53.1	52.2	0.85	45.0 45.0 43.9	44.6	0.64	0.875 0.863 0.827	0.855	0.0250
B-5	0.0262	0.0286	0.842	80.	48.4 52.7 55.1	52.2	3.39	46.4 44.9 44.9	45.4	0.87	0.959 0.852 0.814	0.875	0.0751
B-6	0.0208	0.0219	0.813	77.	46.9	46.9	-	38.7	38.7	-	0.826	0.826	-
Average value					50.8			41.8			0.823		
Std. dev. of combined random errors					2.40 (4.72%)			3.83 (9.16%)			0.095 (11.54%)		
Std. dev. of instrumental errors only					2.03 (4.00%)			0.26 (0.62%)			0.048 (5.83%)		
Std. dev. of errors due to leaf variability					1.29 (2.54%)			3.82 (9.14%)			0.082 (9.96%)		

instrumental error and for variability between leaves [6]. From these components, standard deviations,as well as coefficients of variation (expressed in percent) were derived. Tables II include a summary of the statistical analysis. As anticipated the scatter due to the leaf alone was generally greater than scatter due to instrumental errors alone, a large part of the leaf scatter being due most likely to errors in thickness measurements rather than to variations in the biological structure of the leaf. A reasonable estimate of the uncertainty in the thickness is 0.001 cm. This amounts to about 5% for specimens B and 10% for specimens A. These estimates can account for most of the observed standard deviations of the measured quantities. At any rate there is no indication that uncertainties exceed the 20% value imposed on the measurements in the original measurement program request.

Anomalous values of κ' and κ'' were measured above 1 GHz. For a GR line higher order modes are expected above frequencies of $9.5 / \sqrt{\kappa}$ GHz [7]. In the leaf filled chamber this ratio falls between 1 and 2 GHz and could account for excessive scatter occasionally observed in the measurements above 1 GHz.

The Effect of Non-Ideal Contact Between Sample and Holder:

With a sample of high admittance the errors introduced by a low admittance contact between sample and holder can be severe. Let us assume that the sample admittance is adequately represented by a parallel conductance G_ℓ and capacitance C_ℓ giving a sample admittance $Y_\ell = G_\ell + j\omega C_\ell$. Assume that this sample admittance is coupled to the line by a gap admittance $Y_s = G_s + j\omega C_s$. Under these conditions the apparent admittance one measures for the series combination of sample plus gap is $Y_M = Y_\ell Y_s/(Y_\ell + Y_s)$ which by the usual manipulation reduces to

$$Y_M = \frac{G_\ell G_s(G_\ell+G_s)+\omega^2(C_s^2 G_\ell + C_\ell^2 G_s)+j\omega\left[(C_\ell+C_s)\omega^2 C_\ell C_s + G_s^2 C_\ell + G_\ell^2 C_s\right]}{(G_\ell+G_s)^2 + \omega^2 (C_\ell+C_s)^2} \tag{19}$$

Normalizing this equation in terms of $\kappa'' = G/\omega C_0$ and $\kappa' = C/C_0$ where C_0 is an effective cell constant we get for the measured κ'_M and κ''_M in terms of the actual sample κ'_ℓ and κ''_ℓ and the effective gap κ'_s and κ''_s the following equations:

$$\kappa''_M = \frac{\kappa''_s\kappa''_\ell(\kappa''_s+\kappa''_\ell)+\kappa'^2_s\kappa''_\ell+\kappa'^2_\ell\kappa''_s}{(\kappa''_s+\kappa''_\ell)^2+(\kappa'_s+\kappa'_\ell)^2} \tag{20}$$

$$\kappa'_M = \frac{\kappa'_s\kappa'_\ell(\kappa'_s+\kappa'_\ell)+\kappa''^2_s\kappa'_\ell+\kappa''^2_\ell\kappa'_s}{(\kappa''_s+\kappa''_\ell)^2+(\kappa'_s+\kappa'_\ell)^2} \tag{21}$$

TABLE III

Comparison of In-Air and In-Water Measurements

Freq (MHz)	Sample B-6 Measured in Water κ'_ℓ	κ''_ℓ	Sample B-6 Measured in Air κ'_M(exp)	κ''_M(exp)	Calculated from (20) and (21) κ'_M(calc)	κ''_M(calc)
10	156	491	132	222	132	222
50	59	120	56	72	67	80
100	54	66	45	42	53	48
250	47	34	40	26	43	26
450	46	15	37	11	41	12
1000	41	9	35	6	37	7
2100	36	7.6	34	4	33	6

We have already mentioned that the measurements were made with the sample in water in order to lower the gap admittance and reduce gap errors. In order to quantify this statement, measurements were made on the same leaf (Tulip Tree, B-6,table 2) in water and in air. Assuming that κ_{ℓ}' and κ_{ℓ}'' are the values measured in water (i. e. the correct leaf values) we applied equations (20) and (21) to calculate the values we would get when the sample was measured in air assuming a fixed κ_{s}' and κ_{s}''. Using values of $\kappa_{s}' = 340$ and $\kappa_{s}'' = 3600/f$ (where f is the frequency in MHz) we then calculated the values in table 3.

The agreement in table 3 is very good considering the fact that it requires (1) that the contact of leaf and holder is the same for both in-water and in-air measurement and (2) that the conductivity and capacitance of the gap does not change with frequency. Thus the basic picture of the gap being an effective G-C pair in series with the leaf allows us to make the in-air and in-water data self-consistent, and we assume this model is correct.

Now let us examine the in-water measurements. Here we have a gap filled with water of a dielectric constant of 80, and we thus expect that κ_{s}' with water is roughly 80 times what we found for κ_{s}' with air or 27,000. Using a dc ohm meter we also observed that the dc conductivity of the holder containing leaf in water was roughly 5 times that of the holder containing leaf in air.

With this observation we assumed that κ_s'' with water was 5x36 f or 180 f (f = frequency in MHz). We then used equations (20) and (21) to find what values of κ_ℓ' and κ_ℓ'' would give the observed κ_M' and κ_M'' for the in-water measurements. The largest error was found to be about 10% at 10^5 Hz with specimen B-6 and the error became less than 2% at and above 10 MHz.

These calculations (1) are consistent with our assumption that the in-water measurements are much more accurate than the in-air measurements (2) show that the in-air values of κ_ℓ' and κ_ℓ'' can be reduced to give the in-water values of κ_ℓ' and κ_ℓ'' within 20% and (3) that the water-gap errors for the data reported here are probably less that 20% at 10^5 Hz and negligible above 10^7 Hz.

Experimental Results:

The specimens used for the data presented in this report in addition to those in table 2 are listed in table 4 together with a brief description of the sample and some values for thickness, weight, density, moisture content and conditions of measurement. Specimens A (Bamboo) were collected from plants growing outside in Bethesda, Maryland on 5-2-67. Specimens B and C were gathered from several different Tulip Trees around Gaithersburg, Maryland from 4-26-67 (B-80) to 5-8-67 (B-77) shortly after the new leaves were formed. Specimens D (clay soil) were gathered outside the NBS laboratories at

TABLE IV

Description of Samples Measured

Number	Type	Thickness (cm)	Weight (gms)	Density (gms/cm^3)	Moisture Content (Weight %)	Temp. of Measurement °C	Measured in
A-63	Bamboo - young leaf	0.0102	0.103	0.78	63	22.7	Water
A-52	Bamboo - older leaf, dull green	0.0102	0.0106	0.80	52	23.0	Water
A-38a	Bamboo - older leaf, yellowish	0.0132	0.0114	0.97	38	23.0	Water
A-38b	Bamboo - older leaf, yellowish	0.0114	0.0161	0.94	38	23.0	Water
A-0	Bamboo - old leaf, woody, brown	0.0107	0.0068	0.49	0	23.0	Water
B-80	Tulip Tree - youngest leaf	0.0229	0.0237	0.80		24.0	Water
B-77	Tulip Tree - young leaf	0.0208	0.0219	0.81	77	24.0	Air and Water
B-68	Tulip Tree- young leaf	0.0157	0.0182	0.89	68	23.0	Air
B-49	Tulip Tree - young leaf	0.0114	0.0130	0.88	49	23.0	Air
B-0	Tulip Tree - young leaf	0.0152	0.0060	0.30	0	23.0	Air
C-58	Tulip Tree Branch - slice along branch, outer layer	0.0381	0.0402	0.81	58	24.1	Water
C-30	Tulip Tree Branch - slice perpendicular to branch, woody material	0.0356	0.0166	0.36	30	23.5	Water
C-42	Tulip Tree Branch - slice along branch, outer layer	0.0356	0.0332	0.72	42	23.0	Water
C-5	Tulip Tree Branch - slice along branch, outer layer	0.0356	0.0113	0.24	5	23.0	Water
C-0	Tulip Tree Branch - slice along branch, outer layer	0.0356	0.0116	0.25	0	23.0	Air

CONTINUED

Description of Samples Measured

Number	Type	Thickness (cm)	Weight (gms)	Density (gms/cm^3)	Moisture Content (Weight %)	Temp. of Measurement °C	Measured in
D-34	Clay soil - soggy	0.0406			34	23.2	Air
D-22	Clay soil - wet	0.0406			22	23.3	Air
D-14	Clay soil - moist	0.0406			14	23.3	Air
D-0	Clay soil - dry	0.0406	0.0592	1.12	0	23.0	Air
E-1	Philodendron (varigated)	0.0251	0.0260			23.6	Water
E-2	Philodendron (green)	0.0305	0.0326			23.5	Water
F-1	Passiflora (young leaf)	0.0127	0.0130			23.6	Water
F-1	Passiflora (old leaf)	0.0203	0.0219			23.6	Air and Water
G-1	Dandelion (old leaf)	0.0305	0.0306	0.78		23.4	Water
H-1	Buckhorn Plantain (old leaf)	0.0343	0.0358	0.82		23.5	Water
I-1	Dogwood Blossom	0.0216	0.0168	1.03		23.5	Water
J-1	Sugar Maple (young leaf)	0.0114	0.0136	0.92		23.6	Water

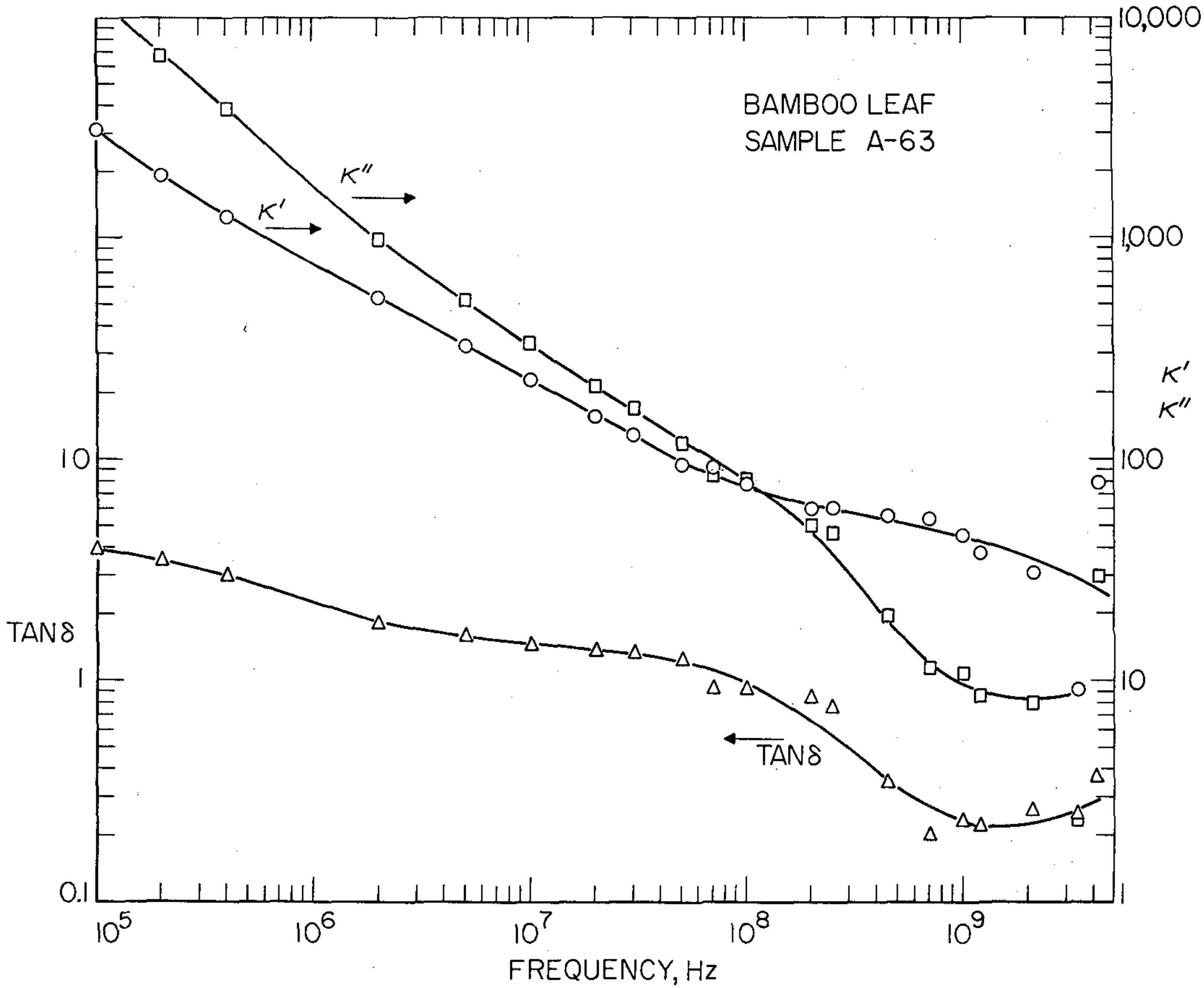

Figure 3. The real and imaginary parts of the relative dielectric permittivity and the loss factor of a sample (A-63) of bamboo leaf as a function of frequency at 23°C.

Gaithersburg, Maryland on 5-11-67 and were either dried or moistened with distilled water to prepare the desired samples. Specimens E and F were of house grown plants and the remaining specimens were obtained outside around Gaithersburg, Maryland around the end of April.

The dielectric data for Specimens A, B and C are shown as a function of frequency in figures 3, 4 and 5. In figure 4 a second set of data is shown (dashed lines) for a leaf collected 2 weeks earlier than B-77, to show the sort of variation one might expect between leaves of different ages. The significant features of the results are the broad maxima in tan δ at about 10^7 Hz, the high frequency level of the κ' curve a little below the value for water, and the increases in tan δ at 10^5 and 4×10^9 Hz indicating additional maxima beyond both the highest and lowest frequencies reported here. Figure 5 for branch material shows similar results but with a somewhat lower tan δ maximum, a longer plateau in κ', and a straighter κ'' curve. These features will be interpreted in the next section.

Figures 6-13 show the effect on κ' and κ'' of differing moisture contents. Figures 6-9 are for leaves and in general, as one would expect, both κ' and κ'' tend to be higher as the moisture content is higher. For bamboo there are in addition to moisture content differences, structural differences. The

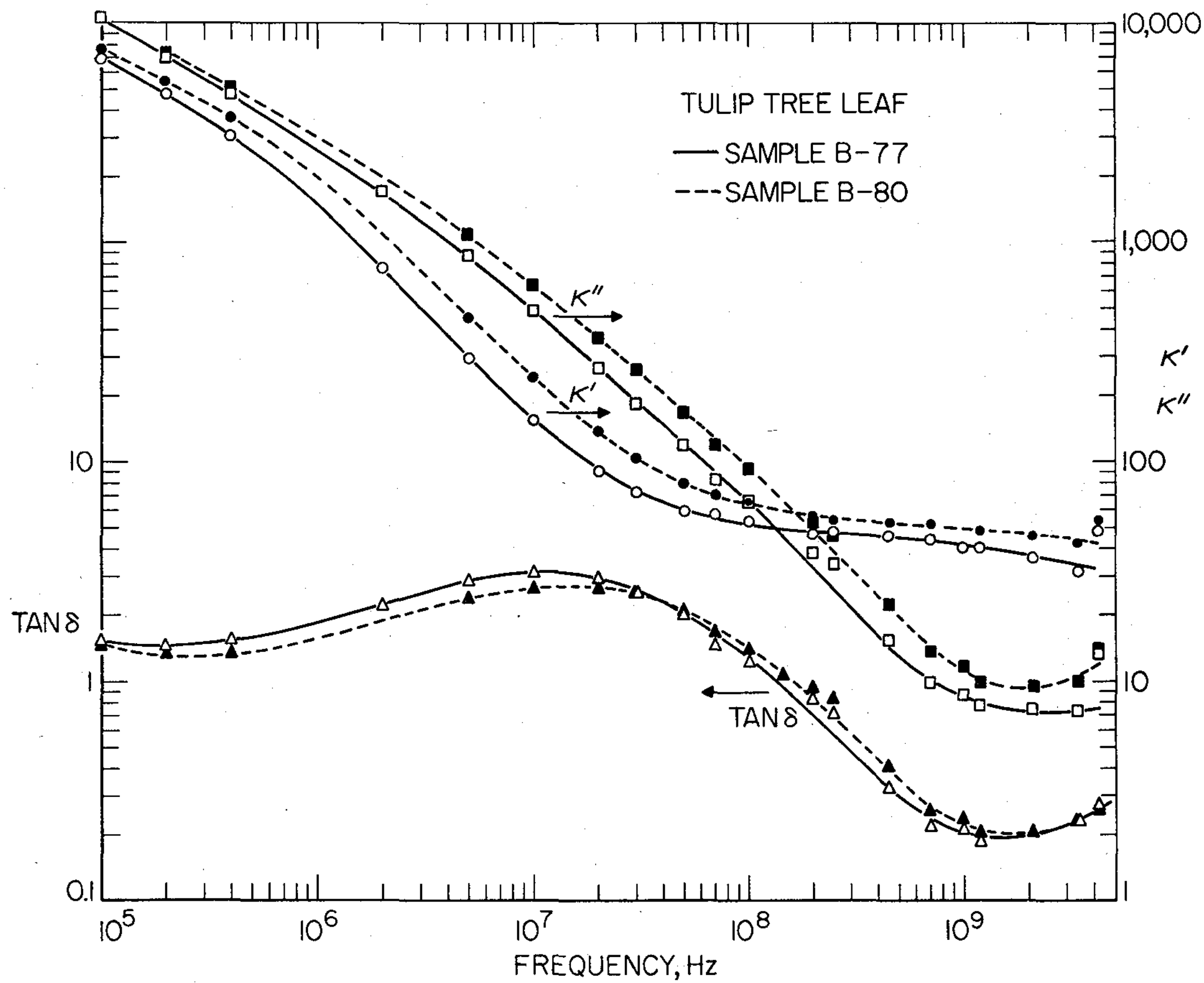

Figure 4. The real and imaginary parts of the relative dielectric permittivity and the loss factor of two samples (B-77 and B-80) of tulip tree leaves as a function of frequency at 23°C.

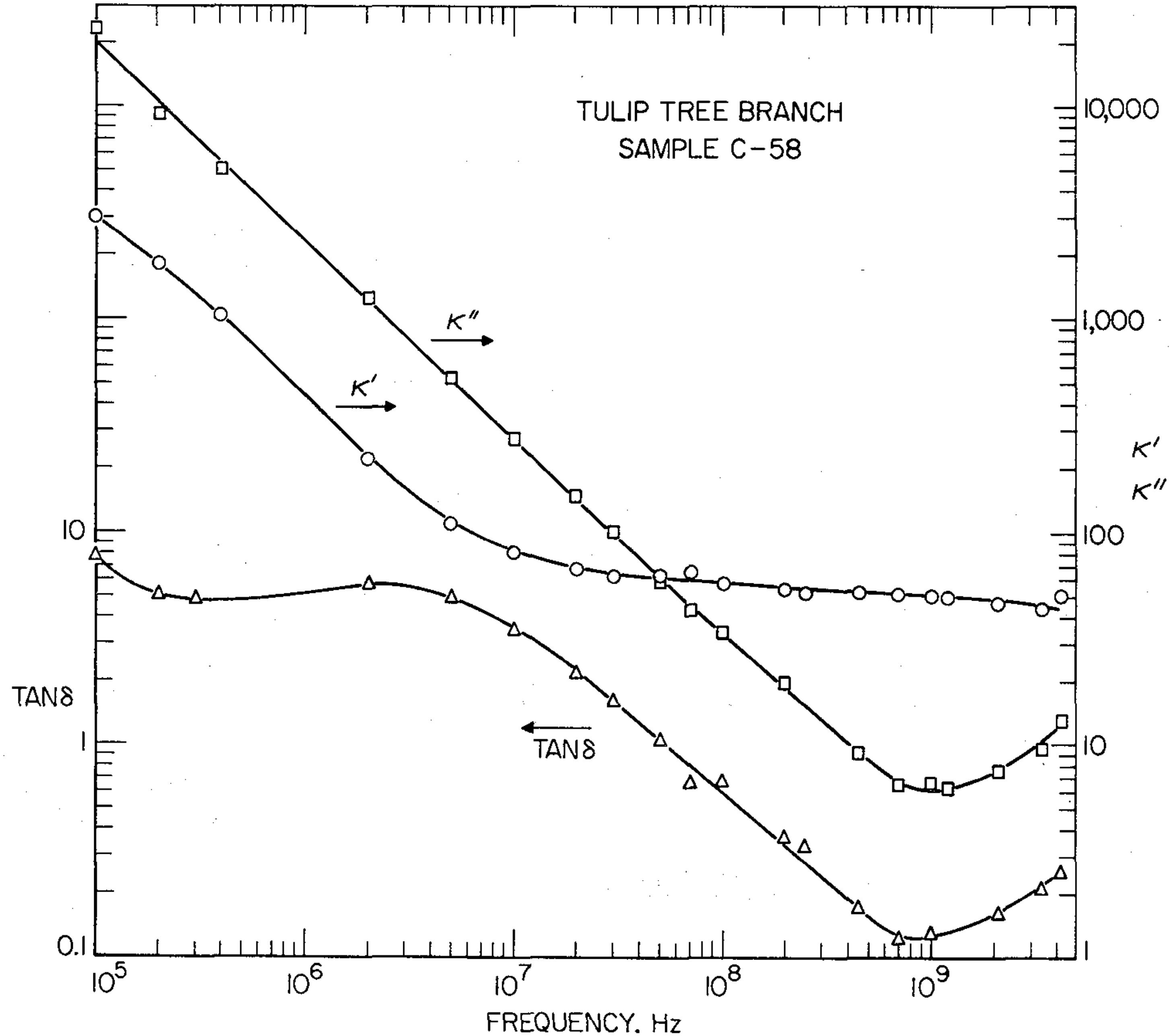

Figure 5. The real and imaginary parts of the relative dielectric permittivity and the loss factor of a sample (C-58) of tulip tree branch as a function of frequency at 23°C.

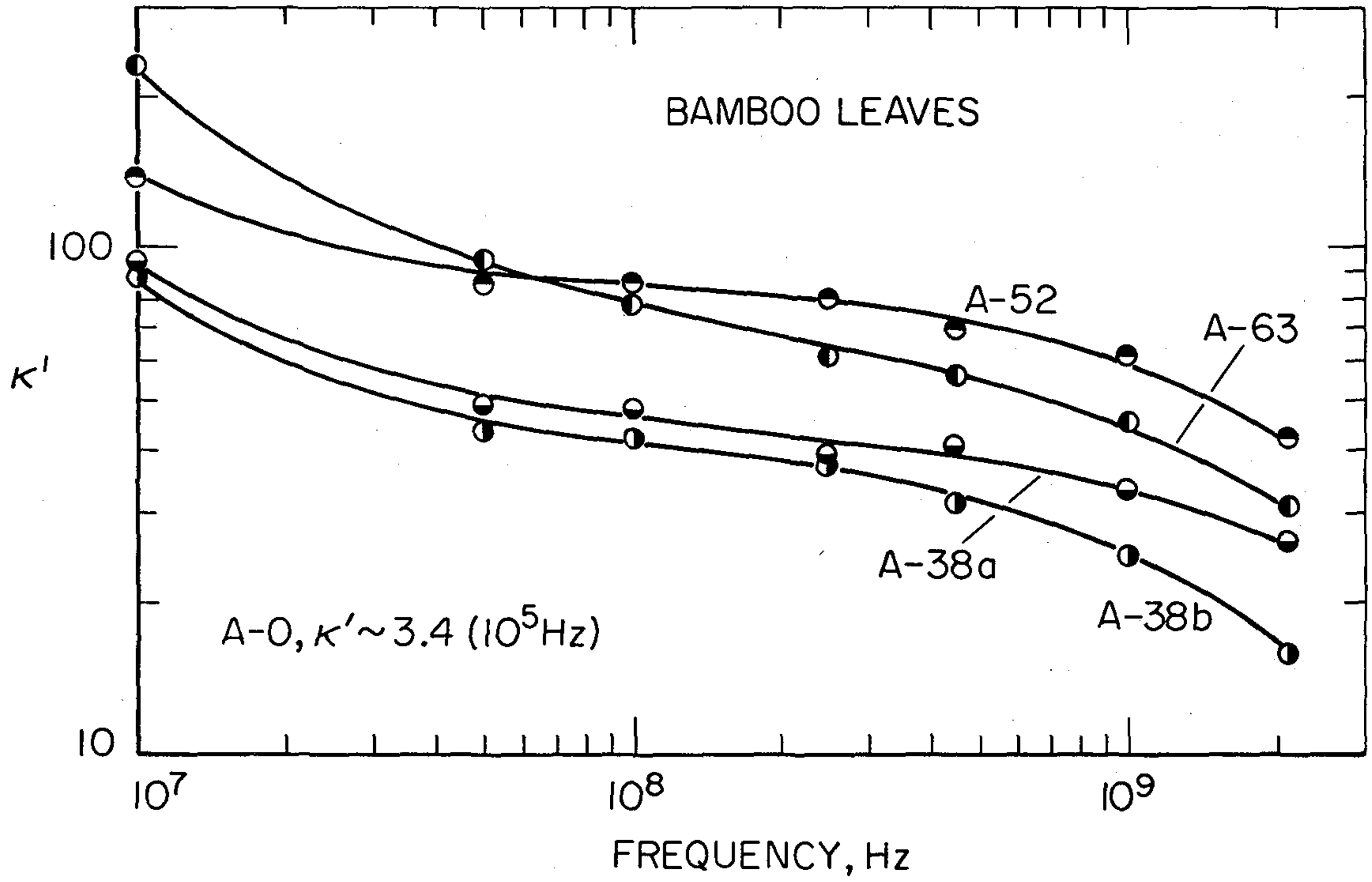

Figure 6. The relative dielectric permittivity of four samples (A-52, A-63, A-38a and A-38b) of bamboo leaves as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

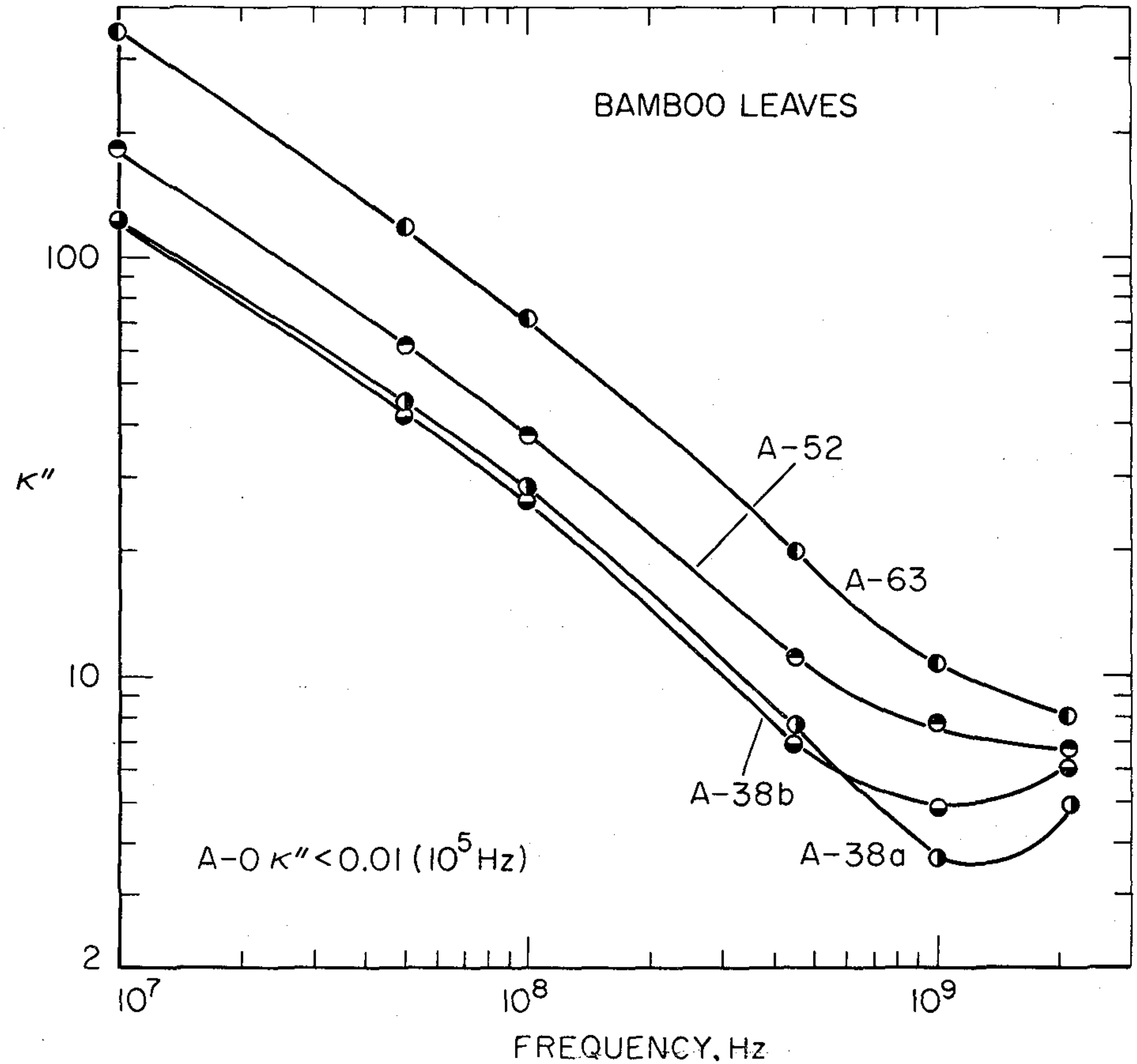

Figure 7. The loss index of four samples (A-52, A-63, A-38a and A-38b) of bamboo leaves as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

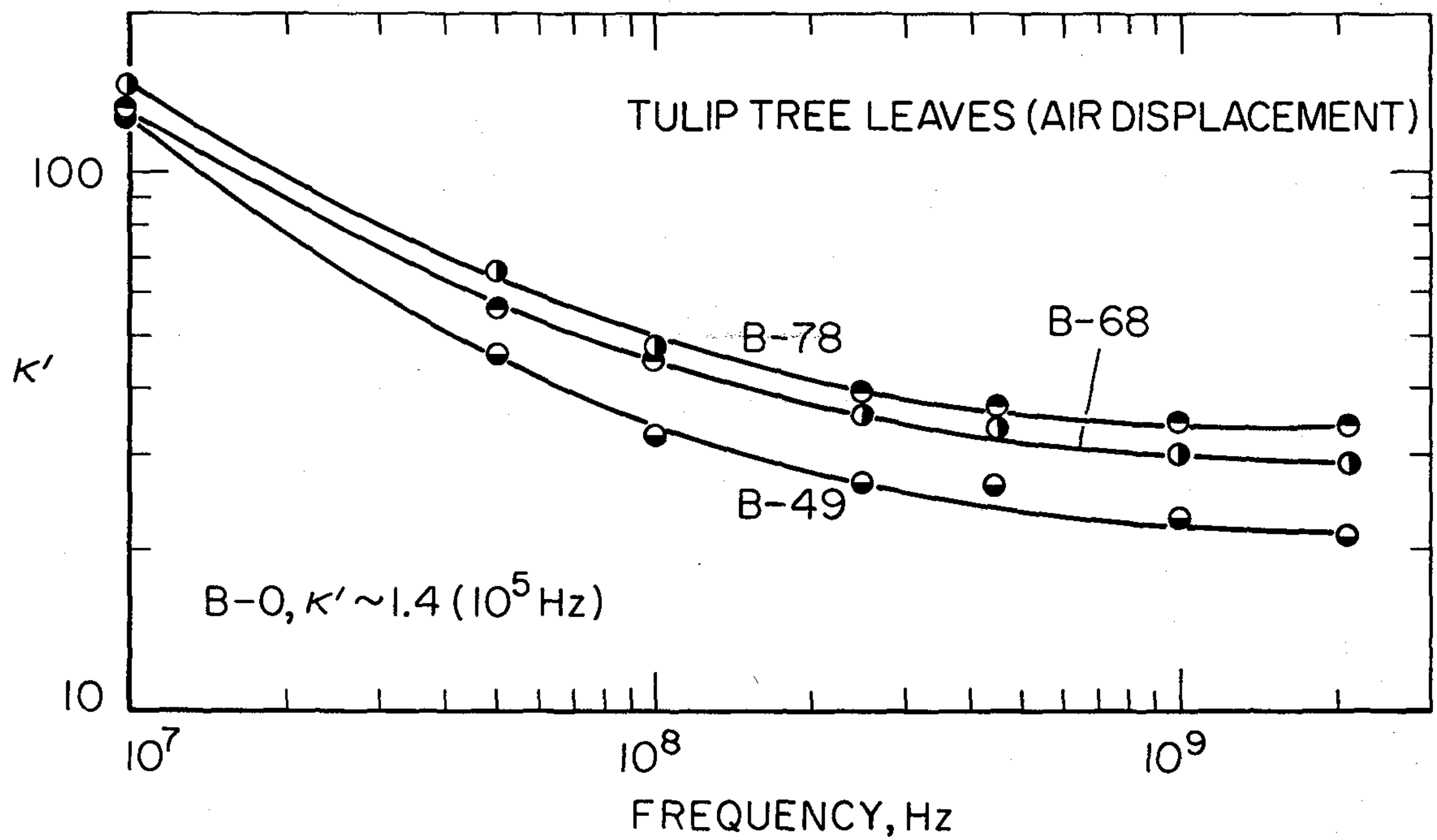

Figure 8. The relative dielectric permittivity of three samples (B-78, B-68, B-49) of tulip tree leaves as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

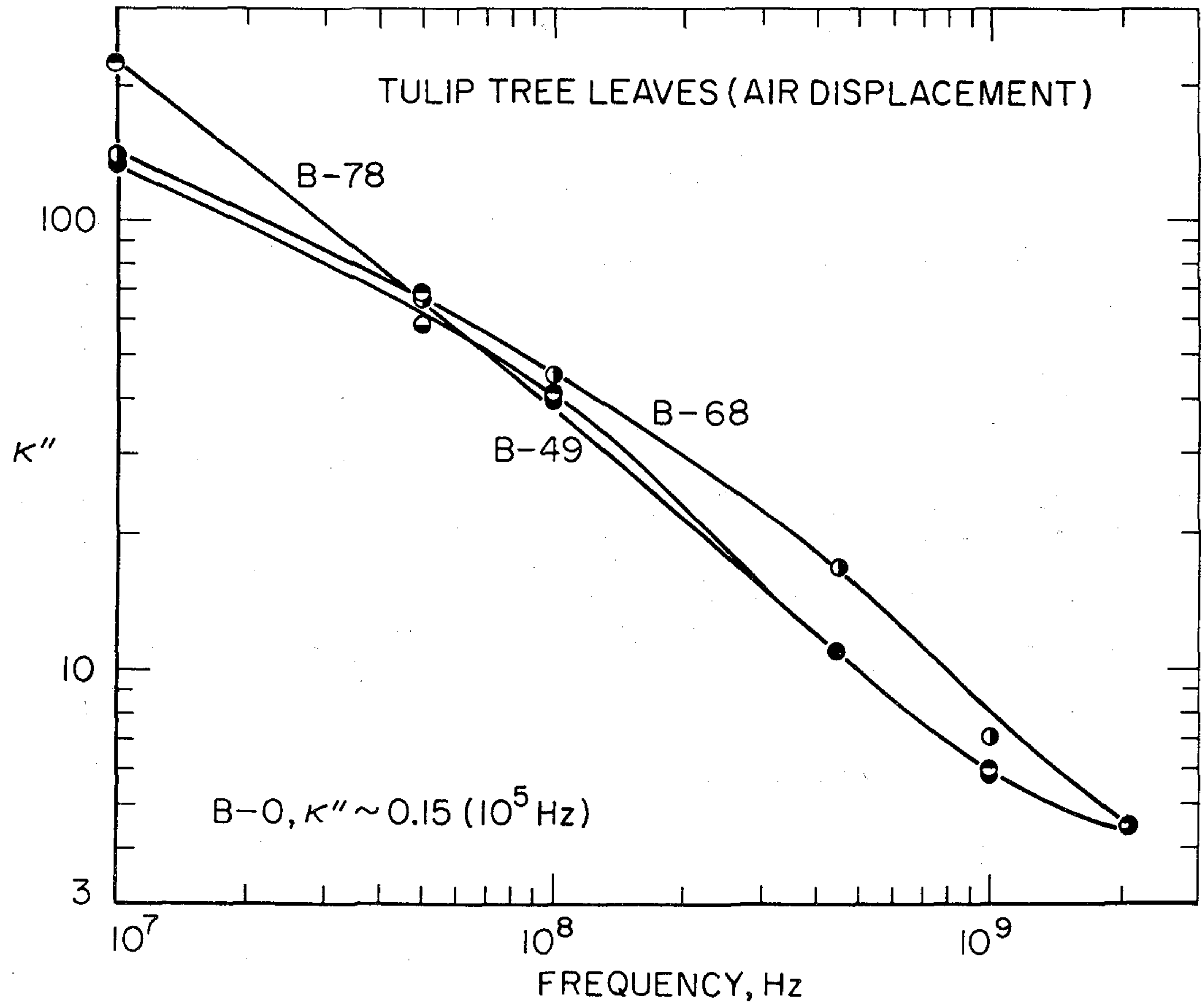

Figure 9. The loss index of three samples (B-78, B-68, B-49) of tulip tree leaves as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

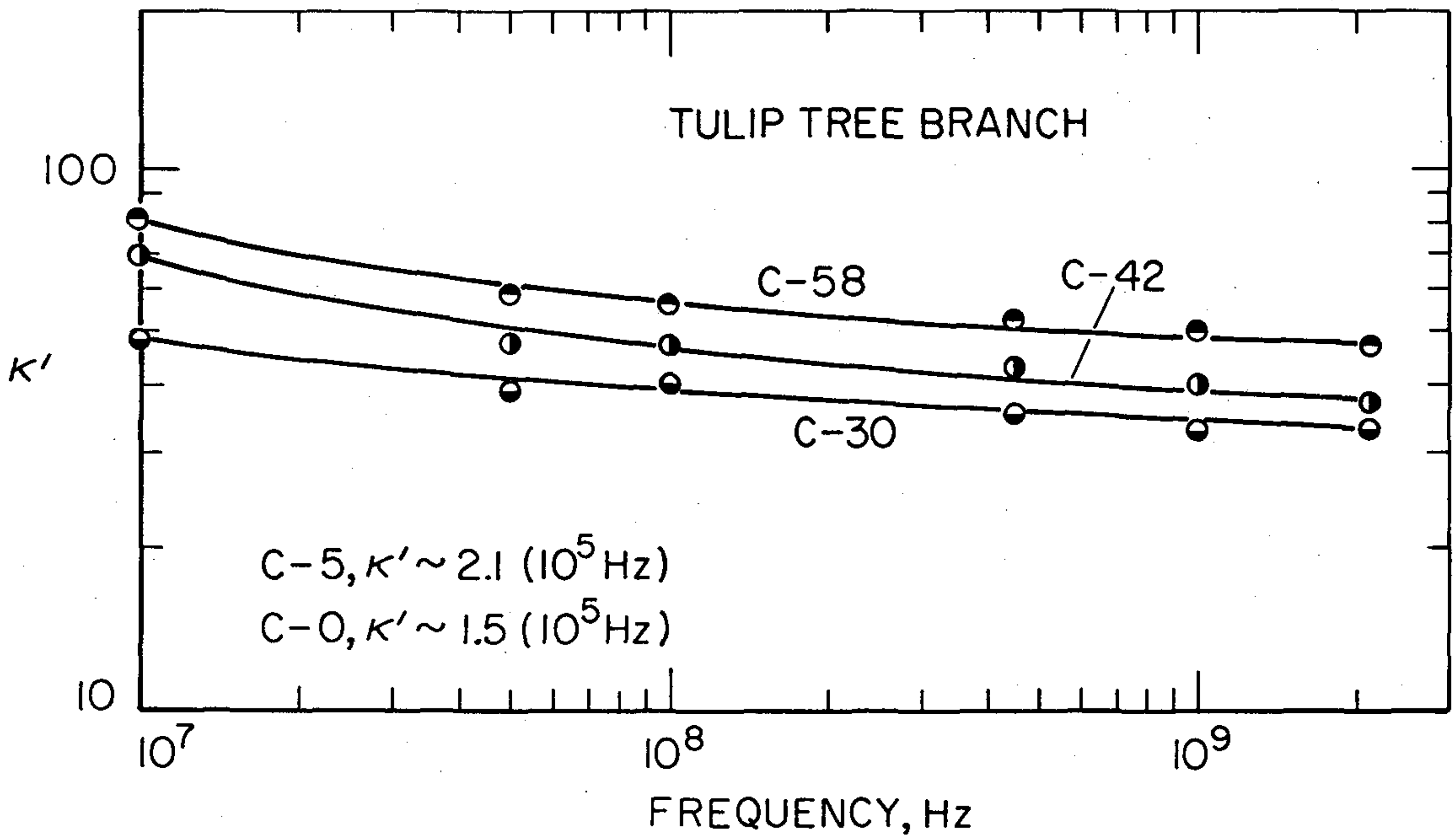

Figure 10. The relative dielectric permittivity of three samples (C-58, G-42, C-30) of tulip tree branch as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The values at the lower left are for a dry sample and one containing 5% water.

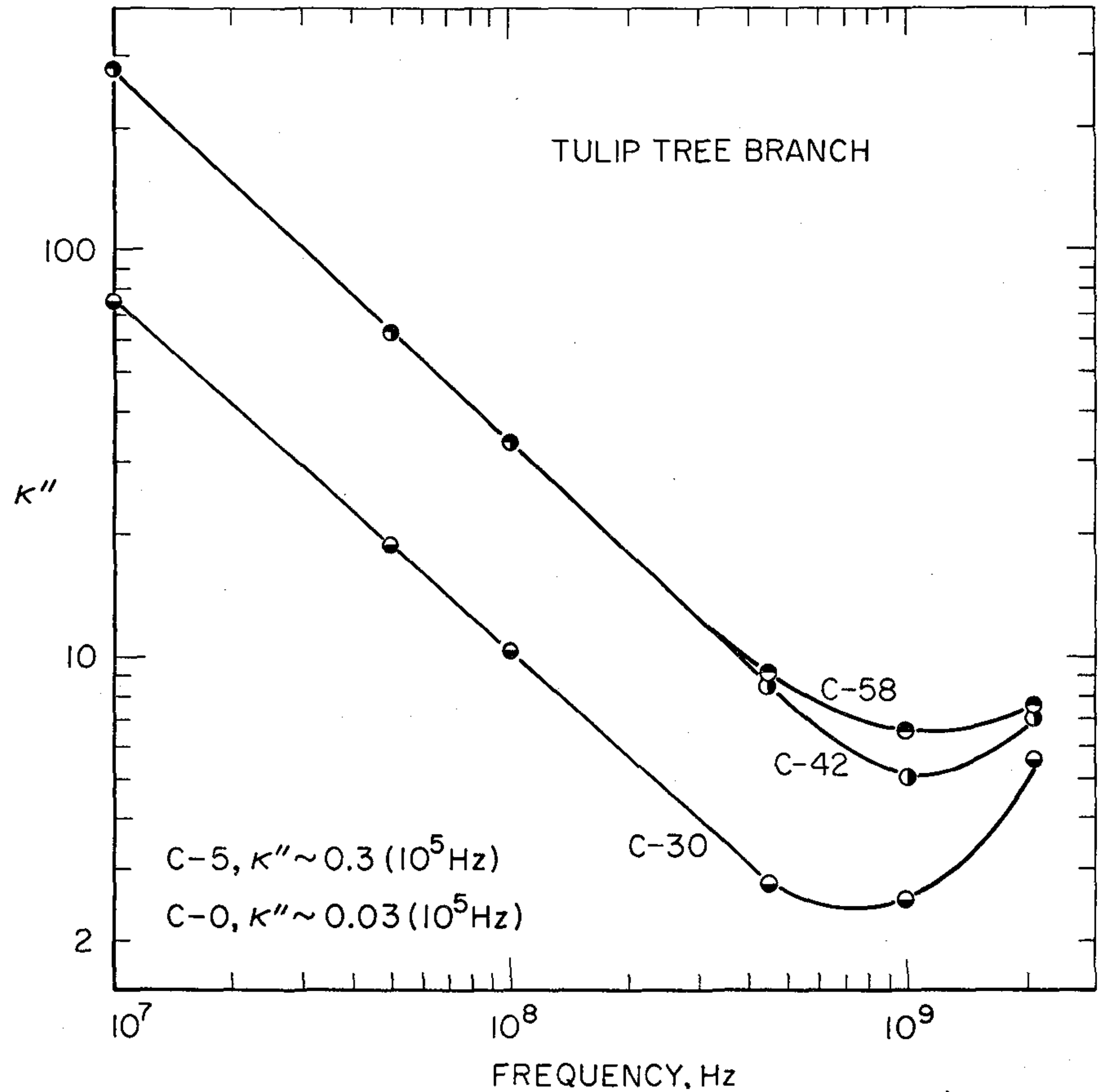

Figure 11. The loss index of three samples (C-58, G-42, C-30) of tulip tree branch as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The values at the lower left are for a dry sample and one containing 5% water.

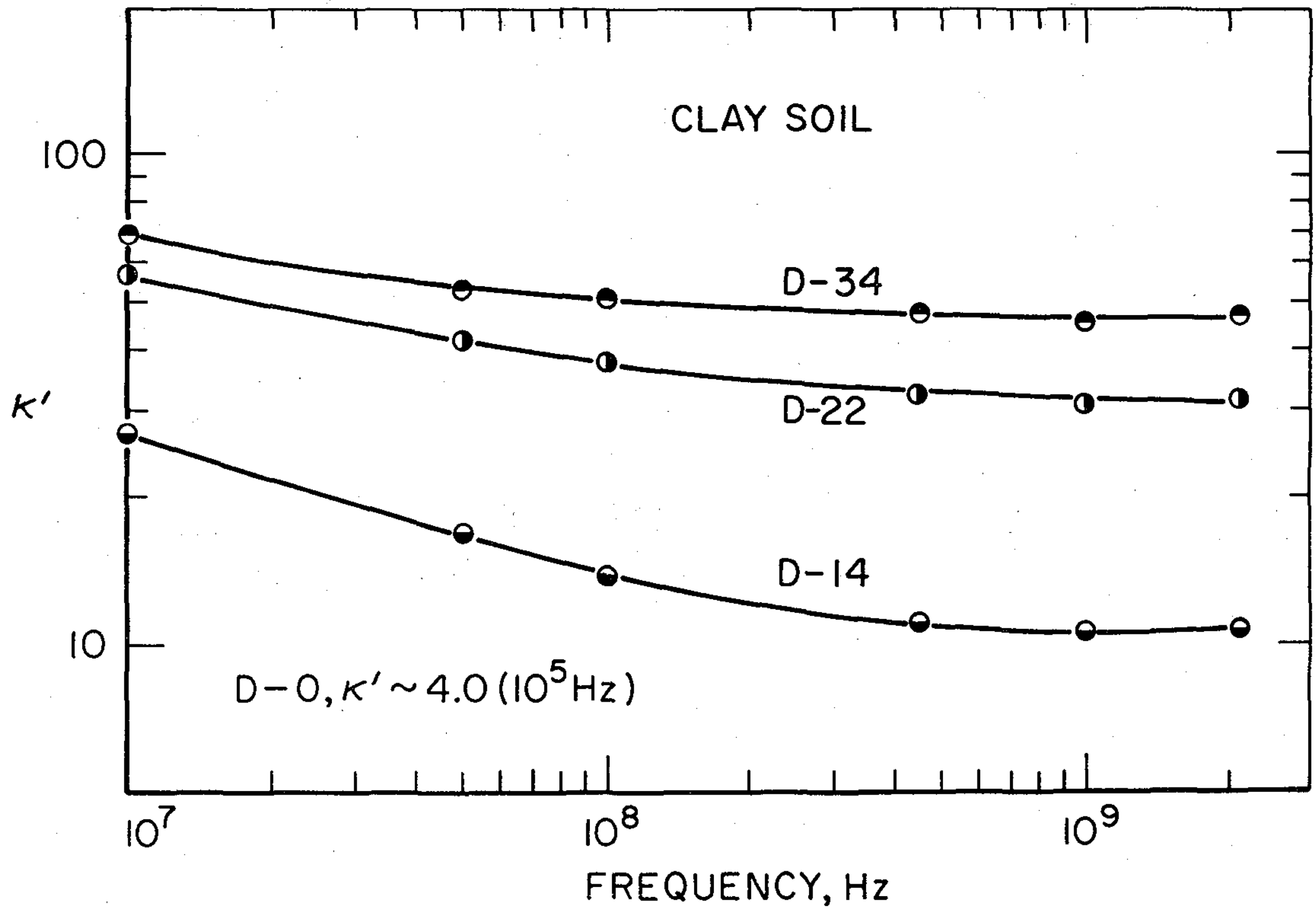

Figure 12. The relative dielectric permittivity of three samples (D-34,D-22, D-14) of clay soil as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

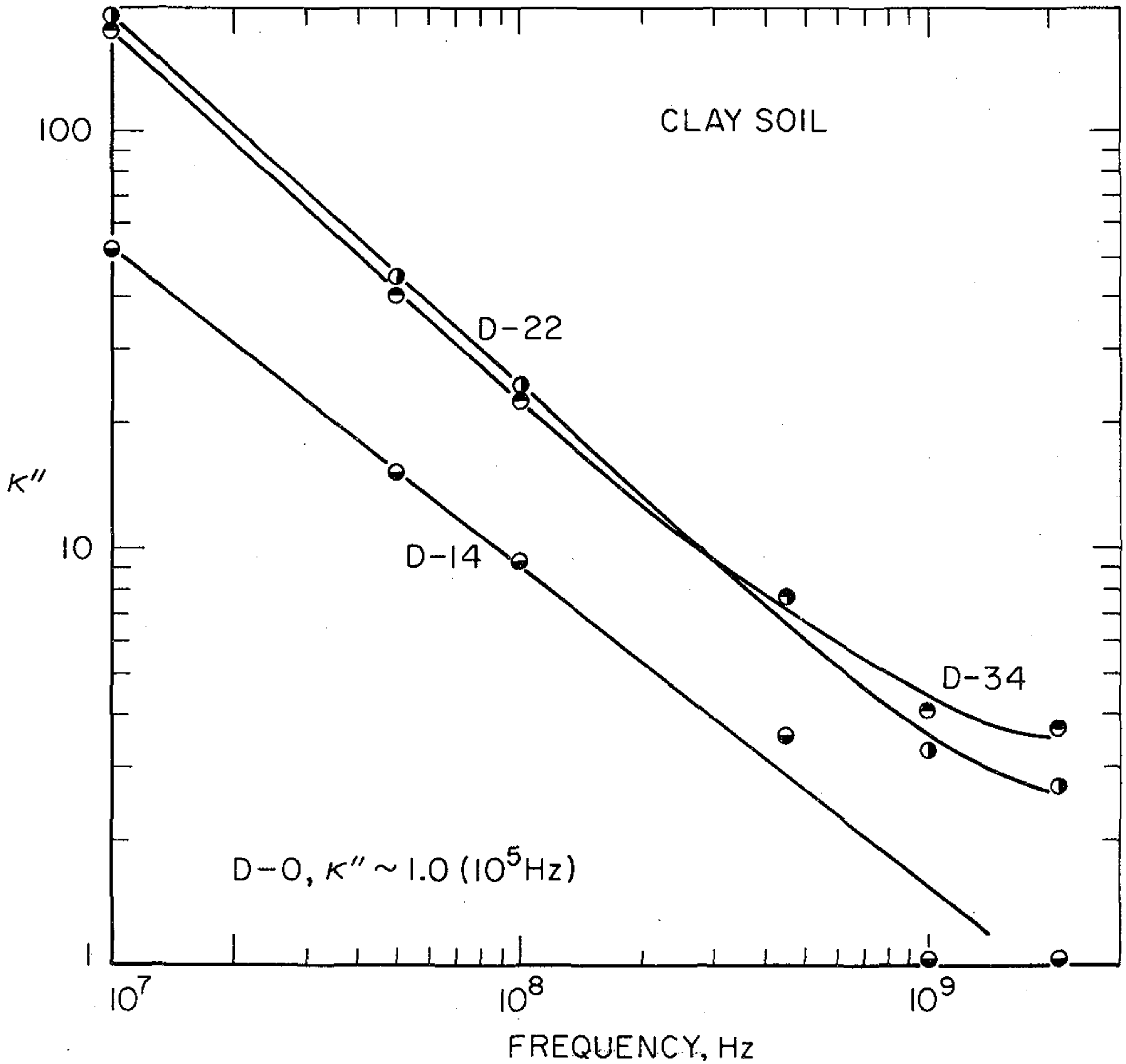

Figure 13. The loss index of three samples (D-34, D-22, D-14) of clay soil as a function of frequency at 23°C. The sample number denotes the moisture content of the sample. The value at the lower left is for a dry sample.

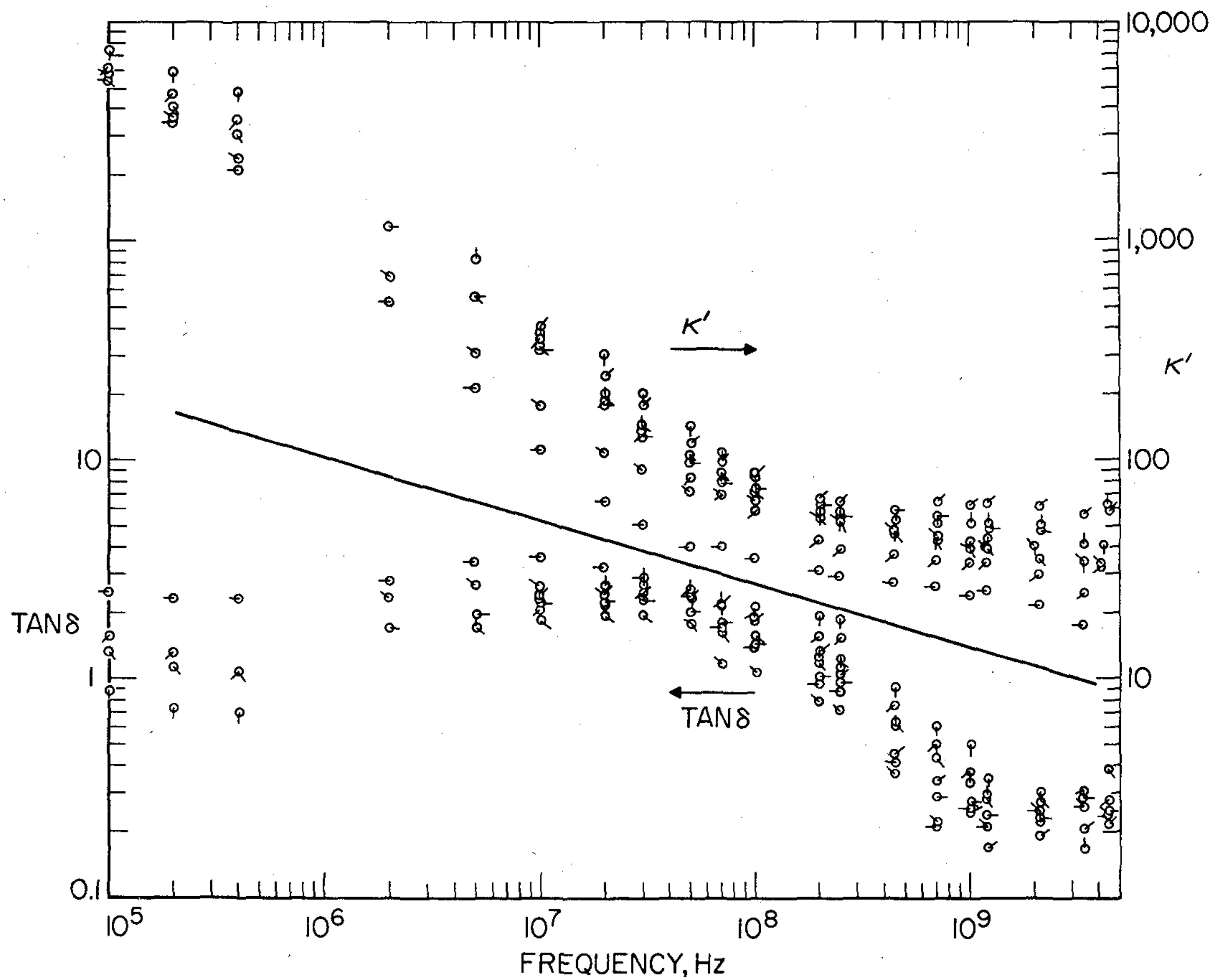

Figure 14. The relative dielectric permittivity and loss factor of leaf samples of eight different plants (E-1 philodendron (variegated), E-2 philodendron (green) F-1 passiflora (young leaf), F-2 passiflora (old leaf) G-1 dandelion H-1 plantain, I-1 dogwood blossom J-1 maple) as a function of frequency around 23°C. Samples are characterized more fully in Table IV.

leaves with less moisture were older leaves and contained more cellulose (larger dry weight) than the leaves with higher moisture content. For the tulip tree samples the same leaves were measured as they became drier and one notices in figures 8 and 9 that the differences for different moisture contents was not so great as in figures 6 and 7. This is to be expected because as a fresh leaf dries it also shrinks and tends to keep a fairly constant κ' and κ''. The older bamboo leaves were drier without having shrunken. The results for tulip tree branch material are shown in figures 10 and 11.

The results for clay soil (figures 12 and 13) do not greatly differ from the results for plant material, except that there is a more striking dependence on moisture content. The κ' curves for soil are flatter than those for leaves and more like the curves for branch material.

The final figure 14 shows results for a wide variety of leaves and indicates the overall similarity in the dielectric properties of foliage. There were no outstanding departures from the general dielectric behavior. The most unusual sample studied - the dogwood blossom - gave the greatest deviation from the average density and dielectric constant values for the leaves.

The dry specimen measurements presented special problems because the κ^* values were too low for the limited sensitivity of the equipment used. In each case the dry samples were measured with the low frequency Cole-Gross-type bridge and these results were included in captions on the graphs. The dry-sample values of κ' can be interpreted as maximum values over the entire frequency range, and κ'' values can reasonably be expected to be at least a factor of 10 below the κ' values.

Interpretation of the Data:

Most of the observed features of the dielectric data on leaves can be explained by a rather reasonable model for the leaf which includes separate cells containing water with large concentrations of ionic impurities. κ' within a cell would be constant and close to that of water (~ 80) and κ'' would be a straight line of slope -1 on the log κ'' vs log frequency plot. The sap within the leaf cells would also reflect the high frequency drop in κ' and increase in κ'' due to the dipolar relaxation of the water molecules. If one includes the effect of the cell walls which partially block the flow of ionic charge then one expects interfacial polarization and a Debye-like relaxation region. At high frequencies the behavior would be similar to that described above, but at lower frequencies κ' values would increase, the slope of κ'' would decrease and tan δ would exhibit a maximum. This essentially describes the results observed for leaves except at the very low frequencies. Here the measurement procedure enters the picture and in the holder used in these measurements we must include the effect of the gap between holder and specimen as discussed previously. The slight upturn in tan δ at the lowest frequencies measured is to be regarded as an effect of the measurements and not a property of the leaf itself. Of course for any usual calculations the interface between a leaf and its surroundings must be taken into consideration and will usually be responsible for a low frequency relaxation.

The above model predicts that the position of the tan δ maximum is sensitive to the average size of the leaf cells. The further the ions must travel to attain equilibrium the longer the time and hence the lower the frequency of the tan δ maximum. Thus the younger leaf in figure 4 could have a higher frequency tan δ maximum because of smaller cells, and the considerably lower frequency of tan δ maximum in figure 5 would imply considerably larger water-containing chambers. The failure of figure 3 to show a distinct maximum in tan δ can be interpreted as due to an unusually wide spread in cell-sizes which broadens the relaxation down into the region where the gap effects enter. The effects of cell structure are much reduced in the case of branch material and clay. These results exhibit the flat κ' and near -45 degree κ'' and tan δ curves expected for a conduction mechanism without interfacial blocking effects.

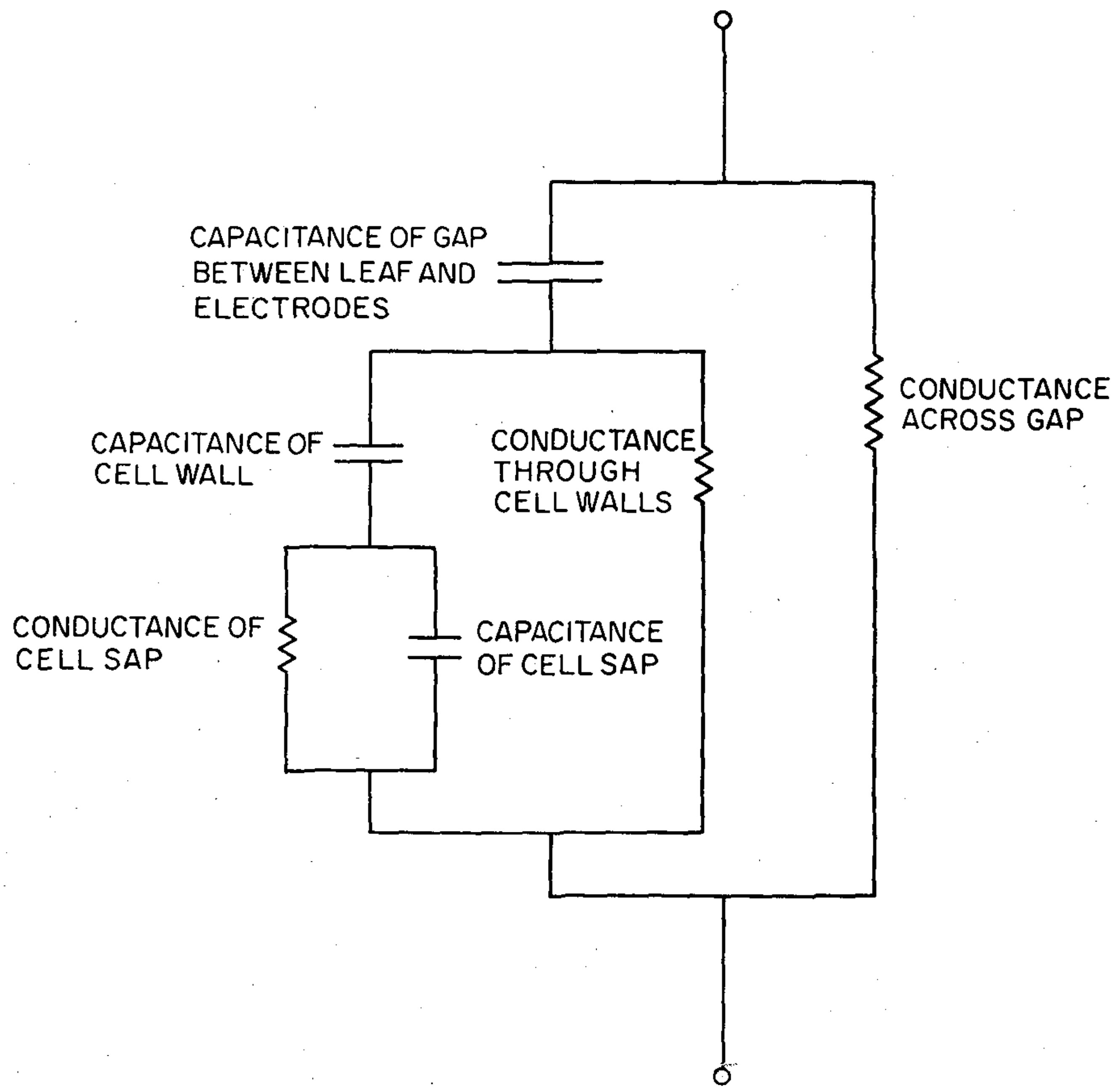

Figure 15. A proposed circuit equivalent of a leaf sample in a coaxial holder which qualitatively accounts for the observed dielectric behavior.

In accord with the above considerations, a general equivalent circuit for a leaf is shown in figure 15. It should be mentioned that the single components shown are average values and (due to differences in cell structure and orientation throughout a leaf) we can expect a distribution of values for some components in this circuit, leading to relaxation broadening.

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