NATIONAL BUREAU OF STANDARDS REPORT

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SCHEDULING TO MAXIMIZE PASSENGER SATISFACTION

By

Dennis Young

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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ABSTRACT

In this paper, dynamic programming algorithms are developed for scheduling a single vehicle to operate over simple networks. The algorithms are designed to calculate schedules which conform closely to traveler preferences. The traveler preferences are represented by utility functions over the scheduling variables, the departure times and arrival times of trips.

The paper also presents and discusses a set of computer results obtained for the simple case of scheduling a vehicle over a single link (origin to destination) network. The results provide insights into the sensitivity of schedule characteristics to vehicle capabilities and traveler behavior parameters.

Finally, the paper includes a preliminary analysis of the multi-vehicle scheduling problem, and a general discussion of future efforts to be made in this research.

The paper documents research done in the Technical Analysis Division of the National Bureau of Standards for the Northeast Corridor Transportation Project. The research is intended to serve in (1) evaluating alternative transportation systems -- by comparing alternate systems, each in conjunction with its own best schedule (2) developing a computerized scheduling process in an automated system (3) analyzing tradeoffs in the transportation system such as that between speed and capacity of vehicles. The contents of the paper should not be interpreted as

representing policies of the Northeast Corridor Project.

Key Words: utility functions, optimal scheduling, iterative equation, computer algorithm, parametric analysis, transportation

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1. INTRODUCTION

The research described in this paper centers on the issue of efficient use of existing or proposed transportation systems. It focusses on the following question:

Given a transport network, and the number and characteristics of the transportation vehicles, how should the vehicles be scheduled in order to attain a high level of passenger satisfaction?

Of interest, therefore, is that element of passenger satisfaction that is sensitive to schedules--the timing of trips. Thus, the measure of a traveler's satisfaction for a particular trip is the value he has for the pair of numbers (t_d, t_a), where

t_d = time of departure (from traveler's origin)

and

t_a = time of arrival (at traveler's destination).

It is assumed that each prospective passenger is associated with a utility function $U(t_d, t_a)$ which describes his level of satisfaction for any set of values of the variable pair (t_d, t_a). $U(t_d, t_a)$ will, in general, be different for different individuals.

For any set of schedules, and rules for assigning passengers to vehicles, an aggregate utility can be calculated by summing the utilities of the individual passengers. Hence, the problem is to determine schedules (and passenger assignment rules) which maximize the aggregate utility, subject to constraints imposed by the capacities and speeds of the vehicles.

Several remarks are worthy of note, relative to the perspective in which this study has been formulated.

First, the deliberate choice to maximize a measure of "public satisfaction" seems especially appropriate in view of the heavy emphasis accorded to public interests in the Northeast Corridor Transportation Project. This is in contrast to the more usual type of analysis which attempts to minimize costs subject to "level of service" constraints.

Secondly, it should be noted that there are well known theoretical difficulties associated with adding together the utilities of different individuals. This issue is the underlying factor in the discussion of Section 2 concerning normalization of the utility functions. In that discussion, the commensurability of passenger utilities is resolved in terms of the influence that different individuals are allowed to exert on the determination of the schedule.

Third, the formulation here has been greatly influenced by the concept of an adaptive transport system which would be responsive to changes in patterns of demand. In particular, a hypothetical automated system is envisioned in which passengers would "call in" information describing their departure and arrival time preferences, subsequent to which the satisfaction-maximizing schedules would be calculated, and each passenger informed of his assignment to a particular vehicle departure.

Fourth, such an analysis makes possible the evaluation of alternative transportation systems on a consistent basis. That is, if for each alternative system separately the optimal schedule is calculated, then the alternative systems can be compared properly, each in combination with its own particular best schedule.

Finally, the study is formulated so that it may prove well-suited for use with demonstration projects because it involves constructing schedules on the basis of individual preferences represented by data obtainable in interviews. Utilization of individual opinion data to construct schedules, rather than historical "demand over time" data, while involving possible difficulties with regard to veracity, seems essential to satisfying "intrinsic" demand, unmodified by adjustment to existing schedules.

Returning to the technical aspects, the problem posed here is of a sequential nature--e.g., the time at which a vehicle is scheduled to depart a network node affects when it is next available for service. Hence, dynamic programming is a natural analytic tool; this paper is based on a dynamic programming formulation of the problem.

This report is confined mainly to the scheduling of a single vehicle along simple networks. Computer algorithms based on the dynamic programming approach have been developed, and have been utilized in illustrative parametric analyses for the simple case of a vehicle traveling on a single link, origin to destination network. A short list of references to other recent work in passenger scheduling is given at the end of the paper.

2. THE UTILITY FUNCTIONS

As noted in the introduction, an aggregate utility function, of the form

$$U = \sum_i U^{(i)} \tag{2.1}$$

where $U^{(i)}$ is the utility of the i^{th} potential passenger, is to be maximized. $U^{(i)}$ is a function of the two schedule-dependent variables $t_d^{(i)}$, the departure time, and $t_a^{(i)}$, the arrival time, for the i^{th} traveler. This section discusses the proposed form of $U^{(i)}$; the superscripts on $U^{(i)}$, $t_d^{(i)}$, and $t_a^{(i)}$ will be dropped for simplicity so that $U(t_d, t_a)$ will henceforth refer to an individual utility function.

Admittedly it is rather presumptuous to assume that the utility functions are "given", and further that enough information is available with which to determine their functional forms. But the actual proposal of such forms is necessary, in order that the analysis may proceed.

The forms outlined below are proposed on the basis of their simplicity, and correspondence to seemingly reasonable assumptions. Although the specific forms affect the numerical results, they are not essential to the general considerations of this study.

The general form assumed for $U(t_d, t_a)$ involves five parameters, each of which has a <u>behavioral</u> interpretation. It may be possible, therefore, to estimate the distribution of utility functions in the traveling public, <u>via</u> interviews and questionnaires. The first parameter, w, is a weighting coefficient which specifies the relative importances, to the traveler, of his departure time and his arrival time. For some travelers and some types of trips, commuters on their way to work in the morning for example, it is more important to adhere closely to a preferred arrival time than to a departure time. To the contrary, a businessman with an appointment in a distant city the next day may be much more concerned about when his flight departs than when it arrives at his destination. If w is defined such that $0 \leq w \leq 1$, the morning commuter to whom arrival is important has a value of w close to zero, and the businessman has a w value near one. Specifically, the following general form is assumed:

$$U(t_d, t_a) = w\, U_d(t_d) + (1-w)U_a(t_a) \tag{2.2}$$

where $U_d(t_d)$ and $U_a(t_a)$ are component utilities which account for the contributions of departure time and arrival time, respectively.

This seems the simplest way to represent the spectrum of travelers whose attitudes range from completely departure-oriented to completely arrival-oriented.

$U_d(t_d)$ and $U_a(t_a)$ are taken to have identical functional forms; each is to be represented by two parameters. The formulation of $U_d(t_d)$ and $U_a(t_a)$ is based on the following plausible assumptions:

a) A traveler has a certain interval of departure which he considers acceptable; similarly, he has an acceptable interval of arrival time. Hence if t_d^+ and t_d^- are the upper and lower limits respectively, of the departure interval, and t_a^+ and t_a^- are the corresponding arrival time quantities, then

$$U_d(t_d) > 0 \text{ only if } t_d^- \leq t_d \leq t_d^+$$

$$U_a(t_a) > 0 \text{ only if } t_a^- \leq t_a \leq t_a^+$$

and (2.3)

$$U(t_d, t_a) = 0 \text{ if } (t_d < t_d^- \text{ or } t_d > t_d^+ \text{ or } t_a < t_a^- \text{ or } t_a > t_a^+).$$

b) Within the interval he has a most preferred time of departure t_d^*, and a most preferred time of arrival t_a^*. His distribution of departure preferences is such that

$$U_d(t_d) \text{ is non-decreasing for } t_d^- \leq t_d \leq t_d^*$$

$$U_d(t_d) \text{ is non-increasing for } t_d^* \leq t_d \leq t_d^+.$$

The analogous remark pertains to $U_a(t_a)$.

c) For simplification, rather than for reasons critical to methods developed later, it is assumed that preferences are symmetric about the preferred values t_d^* and t_a^*. That is, a one-minute deviation from t_d^* on the "early side," for example, is as serious as a one-minute deviation on the "late side."

Accordingly, the parameters σ_d and σ_a are defined:

σ_d = half-width of the acceptable departure interval.

σ_a = half-width of the acceptable arrival interval.

Thus,

$$t_d^- = t_d^* - \sigma_d, \quad t_d^+ = t_d^* + \sigma_d \tag{2.4a}$$

$$t_a^- = t_a^* - \sigma_a, \quad t_a^+ = t_a^* + \sigma_a. \tag{2.4b}$$

The function $U(t_d, t_a)$ is then fully specified by the five parameters $(w, t_d^*, t_a^*, \sigma_a, \sigma_d)$. See Figure 1 for an illustration of the general form of $U(t_d, t_a)$.

In the following work, the two simplest forms which satisfy the preceding conditions will be used. Both involve a "peak height parameter" ζ unmodified by w, which however is subject to a normalization described below. These forms are the triangular and uniform functions.

The <u>triangular</u> function, shown in Figure 2, is given algebraically by

$$U_d(t_d) = \begin{cases} (\frac{\zeta}{\sigma_d})(t_d - t_d^* + \sigma_d) & \text{if } t_d^* - \sigma_d \leq t_d \leq t_d^* \\ (\frac{\zeta}{\sigma_d})(t_d^* + \sigma_d - t_d) & \text{if } t_d^* \leq t_d \leq t_d^* + \sigma_d \end{cases} \tag{2.5}$$

or,

$$U_d(t_d) = \frac{\zeta}{\sigma_d} (\sigma_d - |t_d - t_d^*|) \text{ if } |t_d - t_d^*| \leq \sigma_d. \tag{2.6a}$$

Similarly,

$$U_a(t_a) = \frac{\zeta}{\sigma_a} (\sigma_a - |t_a - t_a^*|) \text{ if } |t_a - t_a^*| \leq \sigma_a. \tag{2.6b}$$

Hence,

$$U(t_d, t_a) = w\, \zeta[1 - (\frac{1}{\sigma_d})|t_d^* - t_d|] + (1-w)\, \zeta[1 - (\frac{1}{\sigma_a})|t_a^* - t_a|]$$

$$\text{for } |t_d - t_d^*| \leq \sigma_d \ \underline{\text{and}}\ |t_a - t_a^*| \leq \sigma_a, \tag{2.7}$$

$$U(t_d, t_a) = 0 \qquad \text{otherwise.}$$

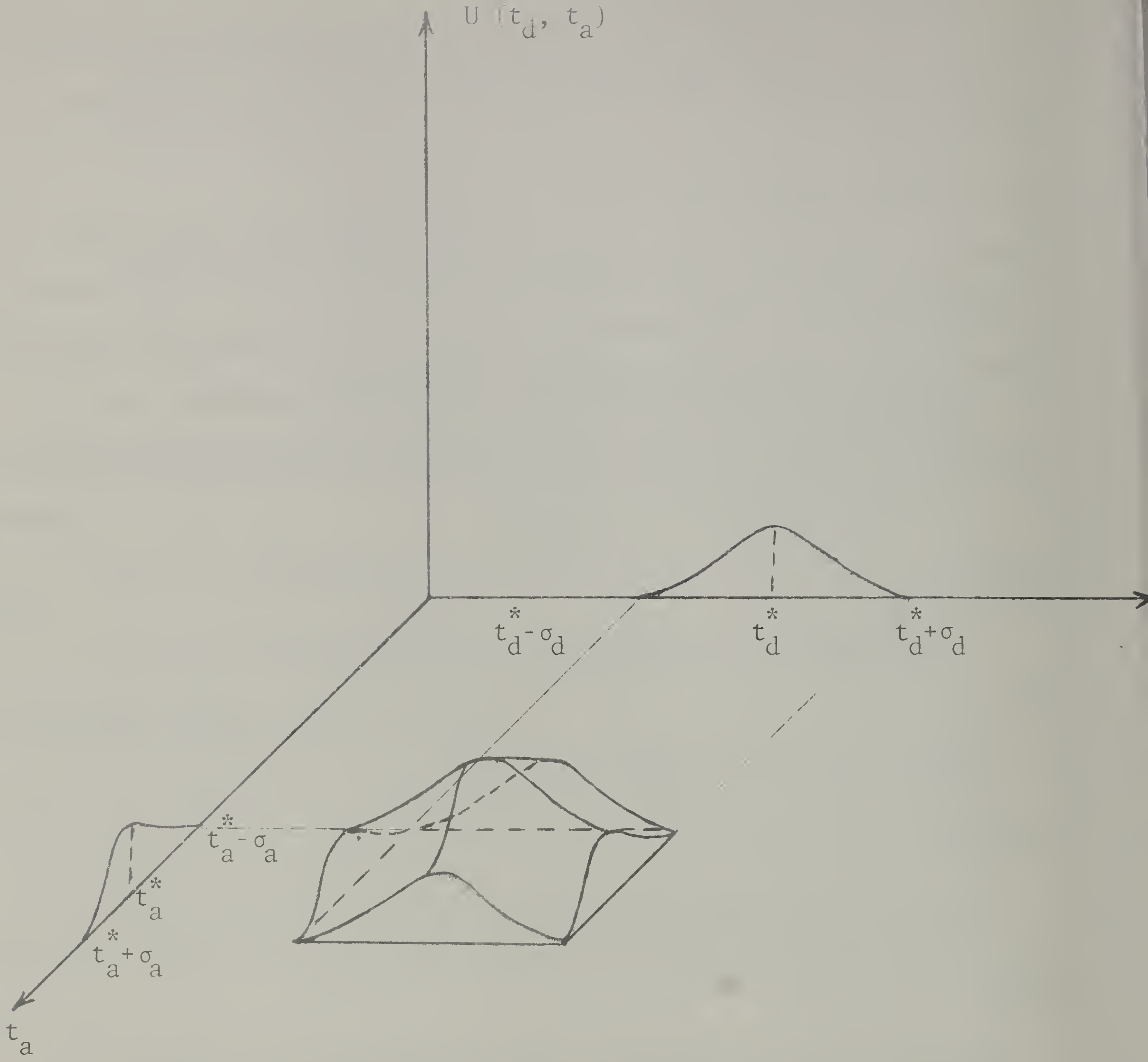

FIGURE 1. INDIVIDUAL UTILITY FUNCTION $U(t_d, t_a)$

In order to investigate the sensitivity of results to the assumption of "peaked" utility functions, such as the triangular one, the uniform function, illustrated in Figure 3, will also be employed:

$$U_d(t_d) = \zeta \text{ (constant) if } |t_d - t_d^*| \leq \sigma_d$$

(2.8)

$$U_a(t_a) = \zeta \text{ (constant) if } |t_a - t_a^*| \leq \sigma_a$$

where ζ is again subject to normalization. Both U_a and U_d are taken equal to the same constant (ζ), within the "acceptable" interval, since it is w that accounts for the relative importances of arrival and departure times.

Thus, for the rectangular function

$$U(t_d,t_a) = w\,\zeta + (1-w)\,\zeta = \zeta$$

$$\text{for } |t_d - t_d^*| \leq \sigma_d \ \underline{\text{and}}\ |t_a - t_a^*| \leq \sigma_a,$$

(2.9)

$$U(t_d,t_a) = 0 \quad \text{otherwise.}$$

Note that adopting the uniform function eliminates the parameter w, and the corresponding degree of freedom (in representing individuals' preferences) that w represents.

The function $U(t_d, t_a)$ must be normalized in some way, to fix the relative intensity of preferences of one individual to another. Unless some normalization is imposed on all functions $U^{(i)}$, certain travelers might exert disproportionate influence over the schedule calculations relative to others. While one can conceive of a normalization based on a deliberate policy of giving more weight to some travelers than others, this would involve political and ethical problems. Thus the "egalitarian" assumption, that each individual should have equal influence, is made

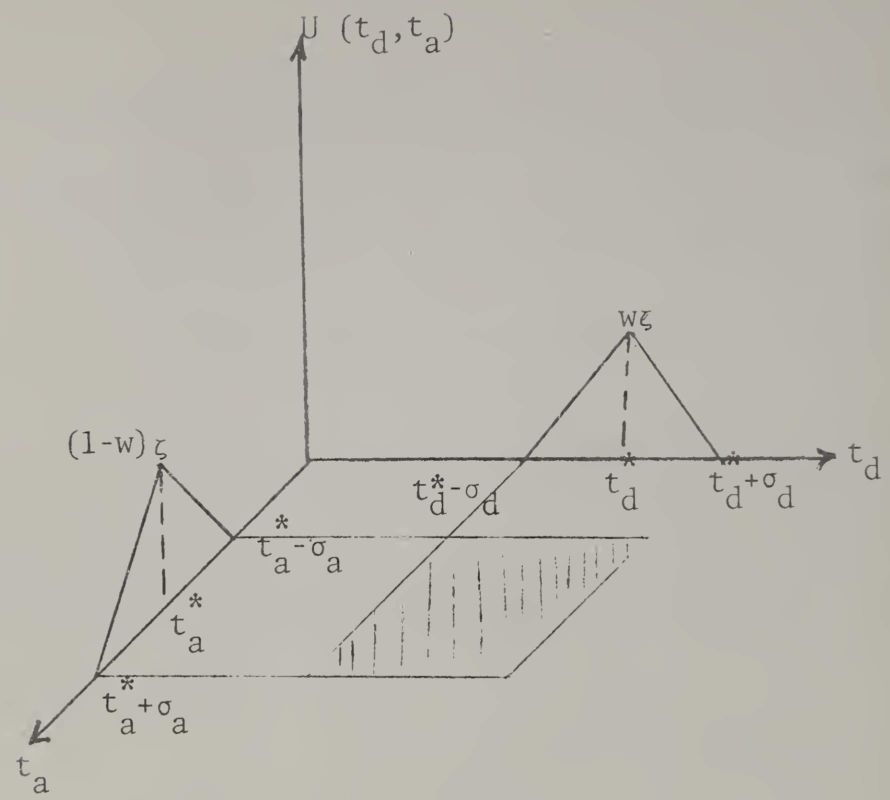

FIGURE 2. TRIANGULAR UTILITY FUNCTION

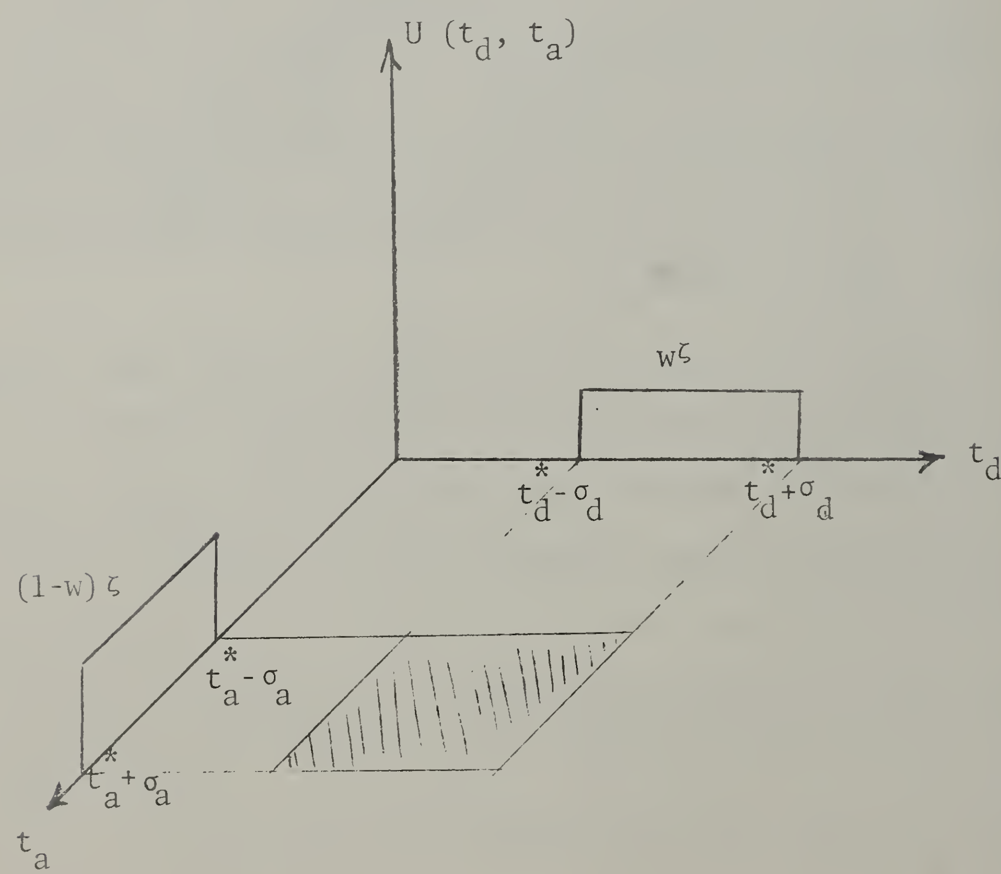

FIGURE 3. UNIFORM UTILITY FUNCTION

here. Two different procedures have been chosen with this purpose in mind: volume normalization and mean-height normalization.

Volume Normalization. The volume under each traveler's $U(t_d, t_a)$ surface is required to be the same. This corresponds intuitively to the idea that each individual is allocated the same "mass" of utility which he can distribute over the (t_d, t_a) plane so that the relative densities at various points reflect his preferences.

For the uniform utility function, the volume is that of a rectangular parallelopiped; normalizing the volume to a value of 1 results in the condition,

$$V = \zeta \cdot 2\,\sigma_a \cdot 2\,\sigma_d = 4\,\sigma_a\,\sigma_d\,\zeta = 1 \tag{2.10a}$$

or

$$\zeta = \frac{1}{4\,\sigma_a\,\sigma_d}\;. \tag{2.10b}$$

In the triangular case, the volume is (by symmetry) given by

$$V = 4\,\zeta \int_{t_d^* - \sigma_d}^{t_d^*} \int_{t_a^* - \sigma_a}^{t_a^*} \Big\{\frac{w\zeta}{\sigma_d}(t_d - t_d^* + \sigma_d)$$

$$+ (1-w)\frac{\zeta}{\sigma_a}(t_a - t_a^* + \sigma_a)\Big\}\,dt_a\,dt_d \tag{2.11a}$$

$$= 4\,\zeta \int_0^{\sigma_d} \int_0^{\sigma_a} \Big\{\frac{w}{\sigma_d}\,x + \frac{(1-w)}{\sigma_a}\,y\Big\}\,dy\,dx = 2\,\zeta\,\sigma_d\,\sigma_a = 1 \tag{2.11b}$$

or

$$\zeta = \frac{1}{2\,\sigma_a\,\sigma_d}\;. \tag{2.11c}$$

Mean-Height Normalization. The mean height of each individual's $U(t_d,t_a)$ surface, over its projection (base) on the (t_d,t_a) plane, is assumed the same. This means that the expected utility for a point (t_d,t_a), chosen at random from an "acceptable rectangle" in the (t_d,t_a) plane, is to be the same for all individuals. Since

mean height = (volume/area of base)

it follows that for the uniform case, the normalization condition is

$$\bar{U} = \text{Vol/Area} = \frac{4\ \sigma_a\ \sigma_d\ \zeta}{4\ \sigma_a\ \sigma_a} = 1 \tag{2.12a}$$

or

$$\zeta = 1. \tag{2.12b}$$

For the triangular case,

$$\bar{U} = \frac{2\ \sigma_d\ \sigma_a\ \zeta}{4\ \sigma_d\ \sigma_a} = 1 \tag{2.13a}$$

or

$$\zeta = 2.$$

Table 1 summarizes the forms of the function $U(t_d,t_a)$ in its various shapes and normalizations.

UTILITY FORMULAS

TABLE 1

Shape	Normalization	$U(t_d,t_a)$ for $\lvert t_d - t_d^* \rvert \le \sigma_d$ and $\lvert t_a - t_a^* \rvert \le \sigma_a$
Triangular	Volume	$\frac{1}{2\,\sigma_a\,\sigma_d}\left\{ w\left(1-\frac{1}{\sigma_d}\lvert t_d^*-t_d\rvert\right) + (1-w)\left(1-\frac{1}{\sigma_a}\lvert t_a^*-t_a\rvert\right)\right\}$
Triangular	Mean-Height	$2w\left(1-\frac{1}{\sigma_d}\lvert t_d^*-t_d\rvert\right)+2(1-w)\left(1-\frac{1}{\sigma_a}\lvert t_a^*-t_a\rvert\right)$
Uniform	Volume	$\frac{1}{4\,\sigma_a\,\sigma_d}$
Uniform	Mean-Height	1

3. DYNAMIC PROGRAMMING

This study is concerned with optimizing the operating policy of a "system" of transport vehicles. The essential properties of such a system can be described by its states, its decision variables, the stage to which it has progressed, and the criterion function to be extremized:

(a) The stage variable is an index of how far the system operation has proceeded; it might in the present context refer to the progress of time, or perhaps to how many potential travelers have either been assigned to vehicle departures or rejected.

(b) The state describes the system's current configuration, which here might be the location and load of every vehicle.

(c) The decision variables describe the possible actions to be taken, causing a transition of the system to a new state at the next stage. In the present case these might be the "hold or dispatch" decisions for all vehicles currently at network nodes.

(d) The criterion function is a sum of values accrued, or cost incurred, at the individual stages; at any stage the value or cost depends on the state of the system and the chosen levels of the decision variables. For this problem, the criterion function is the total utility derived by all of the passengers.

The following notation will be used to denote these concepts:

$(0,T)$ = the time period over which the system is to operate.

K = stage variable - here a discrete time variable assuming values $(0, 1, \ldots, \underline{K})$.

$\underline{\overline{K}}$ = last stage in the process - corresponding to T; therefore

$\underline{\overline{K}}\Delta t = T$ where Δt is the size of each stage increment.

$\underline{x}(K)$ = vector describing the state of the system at stage K.

$\underline{c}(K)$ = vector of decision variables at stage K.

$g(\underline{x}, \underline{c}, \underline{K})$ = state at stage K+1 if decision $\underline{c}$ is made at stage K when the state is $\underline{x}$.

$b(\underline{x}, \underline{c}, K)$ = immediate benefit, if decision vector $\underline{c}$ is chosen at stage K when the state is $\underline{x}$.

$J=\sum_{K=0}^{\overline{K}} b(\underline{x}(K), \underline{c}(K), K)$ = criterion function.

$I(\underline{x}, K)$ = maximum total benefit it is possible to derive in stages K, K+1,...,$\underline{\overline{K}}$, beginning in stage $\underline{x}$ at stage K.

The concept behind $I(\underline{x}, K)$ lies at the heart of the iterative equation ("principle of optimality") that is the basis of dynamic programming:

$$I(\underline{x},K) = \max_{\underline{c}} \{b(\underline{x}, \underline{c}, K) + I\,(g(\underline{x}, \underline{c}, K), K+1)\}, \tag{3.1}$$

where the maximization is over all decisions $\underline{c}$ which are possible when the system is in state $\underline{x}$ at stage K. This equation, which holds for $K<\underline{\overline{K}}$, is supplemented by the "boundary condition,"

$$I(\underline{x}, \underline{\overline{K}}) = \max \{b(\underline{x}, \underline{c}, \underline{\overline{K}})\}. \tag{3.2}$$

In (3.1), $b(\underline{x}, \underline{c}, \underline{K})$ is the immediate gain obtained by choosing $\underline{c}$, while $I\,(g(\underline{x}, \underline{c}, K), K+1)$ is the long term benefit derivable by being in the resulting state $g(\underline{x}, \underline{c}, K)$ at the <u>next</u> stage. Therefore, the equation states that the maximum possible benefit is obtained by executing (at every stage) that set $\underline{c}$ of decisions which maximizes the sum of immediate and long-term benefits.

Using (3.2), one can set $K=\overline{\underline{K}}-1$ in (3.1) and carry out the maximization to determine the function $I(\underline{x}, \overline{\underline{K}}-1)$. Using $I(\underline{x}, \overline{\underline{K}}-1)$, and setting $K=\overline{\underline{K}}-2$, $I(\underline{x}, \overline{\underline{K}}-2)$ can be obtained in the same way. Proceeding from $K=\overline{\underline{K}}$ to $K=0$ in this manner leads ultimately to

$$I(\underline{x}, 0) = \max \{J\} \tag{3.3}$$

where the maximum in (3.3) is over all possible sequences $[\underline{c}(0), \underline{c}(1), \ldots. \underline{c}(\overline{\underline{K}})]$ of decision vectors. For each of the maximizations in this process, the maximizing decision vector $\underline{c}(K)$ is recorded. Once these backwards proceeding calculations reach K=0, the optimal schedule (or "trajectory" or "policy") can be found by proceeding in the "forward" direction, from $K=0$ to $K=\overline{\underline{K}}$, following the set of optimal decisions.

4. SINGLE-LINK ONE-WAY FLOW SCHEDULING FOR ONE VEHICLE

The problem of optimally scheduling a single vehicle, for N potential passengers between a single origin and a single destination, during a specified time period, is considered in this section. See Figure 4.

The time axis over the time period (0, T) is finely divided into equal intervals, whose demarcation points (denoted by 0, 1,..., $\underline{\overline{K}}$) form the "discrete" time axis which will be employed; the stage variable, denoting the "discrete" time, is K.

Let

ϕ = duration of forward trip from origin to destination,

ρ = duration of return trip.

ϕ is assumed to be independent of the vehicle load, and ρ is defined to include minimum required turn-around times at both ends of the route. Corresponding to ϕ and ρ let

F = duration of forward trip measured in discrete time units,

R = duration of return trip measured in discrete time units,

where F and R are integers.

Passengers departing at time t will arrive at time $t+\phi$; also, $\phi+\rho$ is the minimum possible time separation between successive departures of the vehicle from the origin. Equivalently, passengers departing at the stage K corresponding to time t, will arrive at stage K+F, while F+R is the minimum number of stages separating successive departures from the origin.

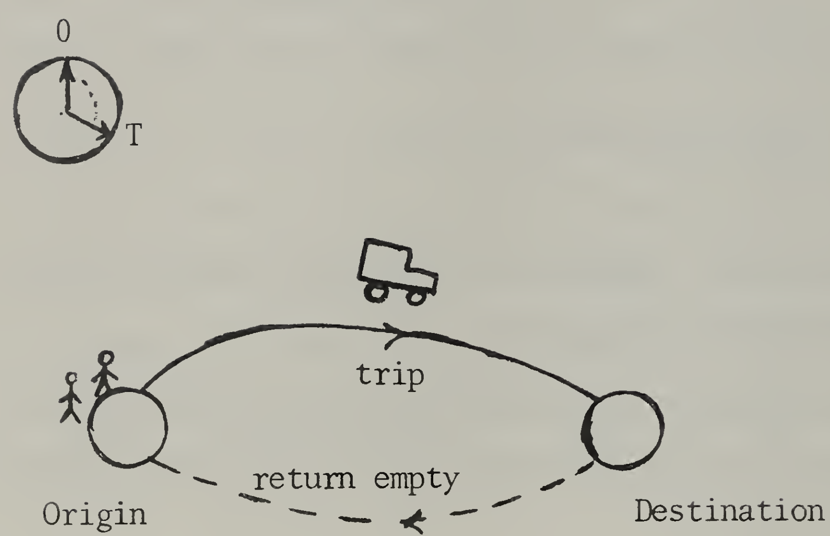

FIGURE 4. SINGLE LINK, ONE VEHICLE, ONE-WAY FLOW PROBLEM

To obtain a dynamic programming formulation, it is necessary to suitably define the state and decision variables, and to specify the transition function $g(\underline{x},\underline{c},K)$ and the immediate-benefit function $b(\underline{x},\underline{c},K)$. There is just one state variable,

x = the stage at which the next departure from the origin is possible. (Thus, $x(K) \geq K$).

Similarly there is a single decision variable c; if $x>K$ the only possible decision is $c=1$, signifying continued movement along the route, but if $x=K$, two possibilities exist: $c=1$ means the vehicle is to be dispatched, and $c=0$ means the vehicle is to be held over at the origin, to (at least) the next stage.

The transition function is given by

$$\begin{aligned} g(x,1,K) &= x \quad \text{if} \quad x>K \\ g(K,1,K) &= K+F+R \\ g(K,0,K) &= K+1. \end{aligned} \tag{4.1}$$

The immediate benefit function is

$$\begin{aligned} b(x,1,K) &= 0 \quad \text{if} \quad x>K \\ b(K,0,K) &= 0 \\ b(K,1,K) &= \textstyle\sum_i U^{(i)}(K,K+F) \end{aligned} \tag{4.2}$$

where the sum $(\sum_i)$ is over all passengers i who at stage K will leave the origin in the vehicle, and $U^{(i)}(K,K+F)$ is the equivalent in discrete time of $U^{(i)}(t,t+\phi)$.

The descriptions of c and b(x,c,K) are incomplete because c should also specify <u>which</u> potential travelers, among those whose intervals of acceptable departure and arrival times include stages K and K+F respectively, are to be dispatched. Making this specification involves two questions:

(a) If the capacity of the vehicle, C, is limiting, which of the eligible travelers should be boarded?

(b) If a traveler is eligible for more than one departure, which one should he take?

Question (b) is eliminated here by assuming that

$$2 \min (\sigma_d, \sigma_a) < \phi+\rho. \tag{4.3}$$

Under this condition, a traveler will be eligible for at most one vehicle departure. For example, if a traveler is eligible for a departure scheduled at t, in the sense that

$$|t-t_d^*| \leq \sigma_d, \quad |t+\phi-t_a^*| \leq \sigma_a,$$

he will no longer be eligible at the earliest possible start of the next round trip.

If there are more than C eligible individuals, the C with the <u>highest</u> values of $U^{(i)}(t,t+\phi)$ are clearly the optimal ones to assign to the vehicle departure at t. Thus, question (a) is also answered.

These assumptions imply that a traveler will be "left behind" if a scheduled departure fails to fall within his acceptable interval or if capacity limitations prevent him from boarding at an acceptable time. It should be noted that condition (4.3), which precludes travelers from being eligible for more than one departure, might be dispensed with, at considerable expense in the dimensionality of the problem; but such a step was believed unwarranted at this stage of the analysis. The aforementioned assumptions will be made in all that follows.

The basic dynamic programming equation (3.1) reads,

$$I(x,K) = 0+I(x,K+1) \quad \text{if} \quad x>K, \tag{4.4}$$

$$I(K,K) = \max \begin{cases} 0+I(K+1,\ K+1) \\ \sum_i U^{(i)}(K,K+F) + I(K+F+R,K+1). \end{cases} \tag{4.5}$$

The equations may be simplified by invoking (4.4) to show that

$$I(x,K) = I(x,x) \quad \text{for} \quad x>K. \tag{4.6}$$

Hence states for which $x \neq K$ (where the vehicle is not at the origin node) can be ignored and the iterative equation is reducible to (4.5); defining $I(K,K) = I(K)$ and using (4.6), (4.5) becomes

$$I(K) = \max \begin{cases} 0+I(K+1) \\ \sum_i U^{(i)}(K,K+F) + I(K+F+R). \end{cases} \tag{4.7}$$

With the assumption that the vehicle is ready to depart from the origin at $K=0$, the desired maximum $I(\underline{x},0)$ is just $I(0)$ as obtained using (4.7). Because this recursion equation involves stages F+R units apart, the "set" of boundary conditions,

$$I(K) = 0 \quad \text{for} \quad \overline{K} \leq K \leq \overline{K} + F+R \tag{4.8}$$

will be used.

The other significant quantities are simplified in notation as follows:

$$c(x,K) \longrightarrow c(K)$$

$$g(x,c,K) \longrightarrow g(c,K)$$

$$b(x,c,K) \longrightarrow b(c,K)$$

A block diagram for the scheduling calculation is given in Figure 5. A sample problem is worked out in detail, below.

Example 4.1. Suppose that five people, each with a triangular utility pattern, wish to go from origin to destination during the time interval $t=0$ to $t=T=10$. Their utilities are characterized by the following table:

Traveler	w	t^*_d	σ_d	t^*_a	σ_a
1	0.7	1.5	1.0	3.0	1.5
2	0.2	3.0	1.0	5.0	1.0
3	0.5	2.0	1.0	4.5	0.5
4	0.4	5.0	0.5	7.0	0.5
5	0.1	9.0	1.0	11.5	0.25

Let the vehicle capacity $C=2$, the one way trip time $\phi=2$, and assume negligible turnaround time so that $\phi=\rho=2$. Further, let the time axis be divided into intervals of $\Delta t=0.5$; hence $\underline{\bar{K}}=20$, and $F=R=4$. Finally, assume volume normalization for the utility functions.

Figure 6 shows the distribution of utility over the (t_d, t_a) plane. The line $t_a=t_d+\phi$ is the travel line, on which any feasible departure-arrival pair must lie. The bounded regions are the "acceptable rectangles"

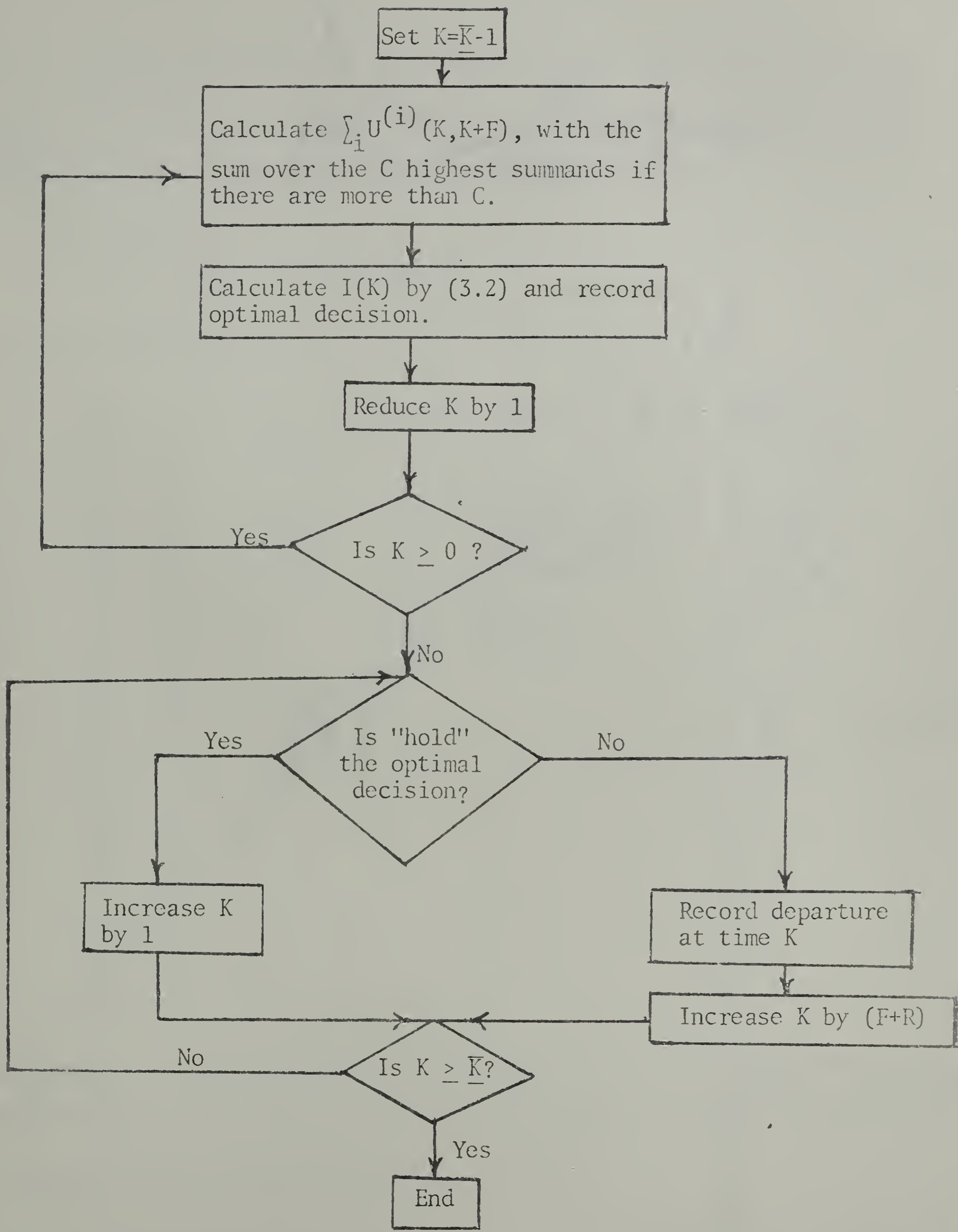

FIGURE 5. SCHEDULE CALCULATION

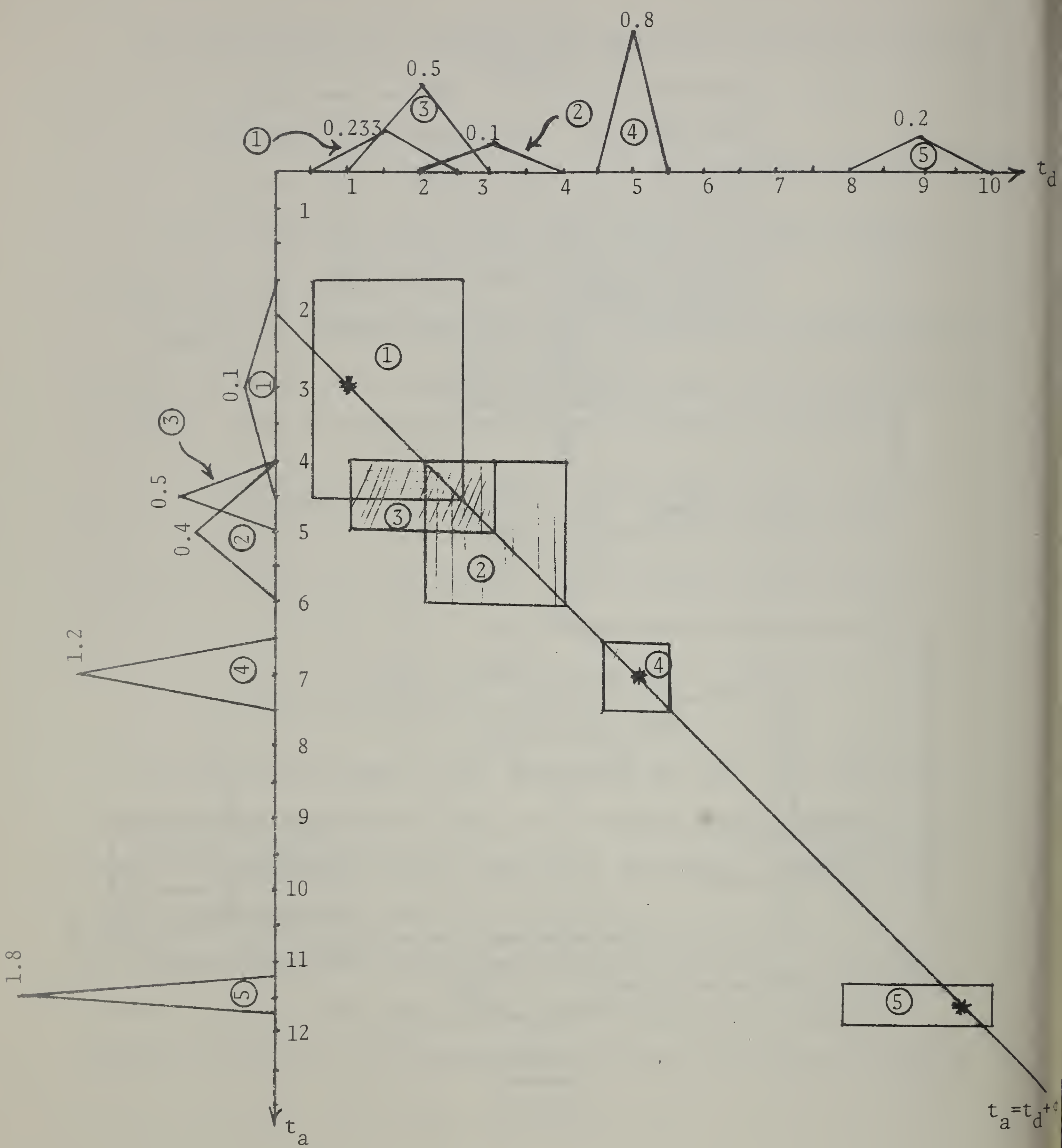

FIGURE 6. DISTRIBUTION OF UTILITY IN THE (t_d, t_a) PLANE

of (t_d, t_a) pairs for each of the travelers. The value of $U^{(i)}$, for any point in the i^{th} traveler's rectangle, is given by the sum of the heights of that traveler's triangles drawn along the axes.

The values of

$$U(K) = b(1,K) = \sum_i U^{(i)}(K,K+F) \tag{4.9}$$

are calculated as follows:

K	(t_d, t_a)	Traveler Utility 1	2	3	4	5	Total Utility U(K)
0	0, 2.0	-	-	-	-	-	0
1	0.5, 2.5	0.067	-	-	-	-	0.067
2	1.0, 3.0	0.217	-	-	-	-	0.217
3	1.5, 3.5	0.300	-	-	-	-	0.300
4	2.0, 4.0	0.150	0	0.5	-	-	0.65
5	2.5, 4.5	0	0.25	0.75	-	-	1.0
6	3.0, 5.0	-	0.50	0	-	-	0.5
7	3.5, 5.5	-	0.25	-	-	-	0.25
8	4.0, 6.0	-	0	-	-	-	0
9	4.5, 6.5	-	-	-	0	-	0
10	5.0, 7.0	-	-	-	2.0	-	2.0
11	5.5, 7.5	-	-	-	0	-	0
12	6.0, 8.0	-	-	-	-	-	0
13	6.5, 8.5	-	-	-	-	-	0
14	7.0, 9.0	-	-	-	-	-	0
15	7.5, 9.5	-	-	-	-	-	0
16	8.0, 10.0	-	-	-	-	-	0
17	8.5, 10.5	-	-	-	-	-	0
18	9.0, 11.0	-	-	-	-	-	0
19	9.5, 11.5	-	-	-	-	1.9	1.9
20	10.0, 12.0	-	-	-	-	-	0

The entries (-) above indicate traveler ineligibility at the particular stage K. Note that at the edge of the acceptable rectangles, travelers are eligible but may have zero utility.

The algorithm proceeds as follows. At any stage, if either value of decision variable c yields the same value of I(K), c is taken to be 0 (no dispatch):

K	I(K+1)	U(K) + I(K+8)	I(K)	Optimal c
19	0	1.9 + 0	1.9	1
18	1.9	0 + 0	1.9	0
17	1.9	0 + 0	1.9	0
16	1.9	0 + 0	1.9	0
15	1.9	0 + 0	1.9	0
14	1.9	0 + 0	1.9	0
13	1.9	0 + 0	1.9	0
12	1.9	0 + 0	1.9	0
11	1.9	0 + 1.9	1.9	0
10	1.9	2.0 + 1.9	3.9	1
9	3.9	0 + 1.9	3.9	0
8	3.9	0 + 1.9	3.9	0
7	3.9	0.25 + 1.9	3.9	0
6	3.9	0.5 + 1.9	3.9	0
5	3.9	1.0 + 1.9	3.9	0
4	3.9	0.65 + 1.9	3.9	0
3	3.9	0.3 + 1.9	3.9	0
2	3.9	0.217 + 3.9	4.117	1
1	4.117	0.067 + 3.9	4.117	0
0	4.117	0 + 3.9	4.117	0

Thus I(0) = 4.117 and, following the optimal decisions from K=0 on, the schedule is:

Departure No.	K	t_d	t_a	Passengers
1	2	1	3	1
2	10	5	7	4
3	19	9.5	11.5	5

Passengers 2 and 3 are unaccommodated.

The departures are indicated with *'s in Figure 6. By coincidence, the same schedule results in this example, if mean height normalization is used. This is not generally the case, however. A full discussion of the effects of alternate normalizations is deferred until Section 8.

5. SINGLE-LINK TWO-WAY FLOW SCHEDULING FOR ONE VEHICLE

In this section, the simplicity of a single link (with end nodes 1 and 2) and one vehicle of capacity C, is retained. Rather than having one node exclusively a traffic origin and the other a destination, however, it is assumed that in the time interval t=0 to t=T, there are

N_{12} potential passengers from 1 to 2,

N_{21} potential passengers from 2 to 1.

Let the trip durations be denoted by

λ_{12} = length of trip from 1 to 2,

λ_{21} = length of trip from 2 to 1,

with L_{12} and L_{21} the corresponding trip durations measured in discrete time units. The turn-around times are treated as negligible, but could easily be incorporated.

Recall that in the last section, a formulation was found for which it was necessary to consider only those states in which the vehicle was at a node, available for dispatch. Such a formulation is chosen here at the outset.

The condition

$$2 \min (\sigma_d, \sigma_a) < \lambda_{12} + \lambda_{21} \tag{5.1}$$

analogous to (4.2), is assumed to hold. There is a single state variable,

x = node at which vehicle is currently located (1 or 2),

and a single decision variable c, with values 0 and 1 signifying "hold vehicle" and "dispatch vehicle," respectively. The iterative equations, analogous to (4.7), are

$$I(1,K) = \max \left\{ \begin{matrix} 0 + I(1,K+1) & \text{"hold"} \\ U_1\ (K) + I(2,K+L_{12}) & \text{"dispatch"} \end{matrix} \right\} \quad (5.2)$$

$$I(2,K) = \max \left\{ \begin{matrix} 0 + I(2,K+1) & \text{"hold"} \\ U_2\ (K) + I(1,K+L_{21}). & \text{"dispatch"} \end{matrix} \right\} \quad (5.3)$$

Here $U_1\ (K) = \sum_i U_1^{(i)}\ (K,K+L_{12})$, where the sum is over all passengers at node 1 who are eligible to leave for node 2 at stage K; if more than C are eligible, the C with highest values of $U_1^{(i)}\ (K,K+L_{12})$ are included in the sum. $U_2\ (K)$ is defined similarly. The "boundary conditions" are

$$I(1,K) = 0 \quad \text{if} \quad \underline{\overline{K}} \leq K \leq \underline{\overline{K}} + L_{21} \quad (5.4a)$$

$$I(2,K) = 0 \quad \text{if} \quad \underline{\overline{K}} \leq K \leq \underline{\overline{K}} + L_{12}. \quad (5.4b)$$

Figure 7, the analog to Figure 5 of Section 4, shows the overall process. The notation

$$x' = \left\{ \begin{matrix} 1 & \text{if } x = 2 \\ 2 & \text{if } x = 1 \end{matrix} \right.$$

x_o = node at which vehicle resides at K = 0,

is used in the figure.

The following example illustrates the use of the algorithm. Mean-height normalization on triangular utilities is employed.

Example 5.1 Assume the following parameter values:

$T = \underline{\overline{K}} = 20\ (\Delta t=1)$

$\lambda_{12} = \lambda_{21} = L_{12} = L_{21} = L = 3$

$N_{12} = N_{21} = 3$

$C = 2$

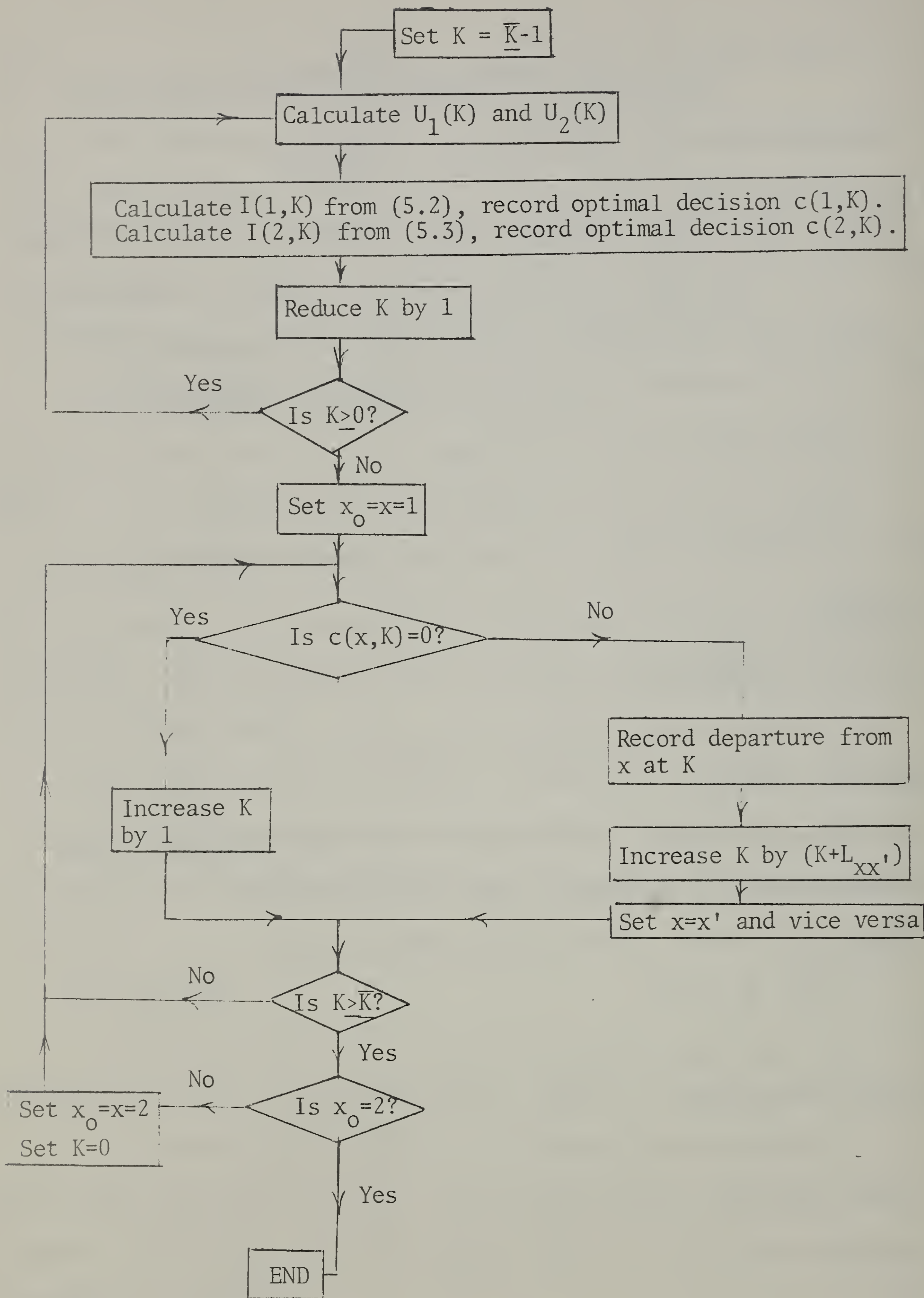

FIGURE 7. SCHEDULING CALCULATION, TWO-WAY FLOW

Node	Traveler	w	t_d^*	σ_d	t_a^*	σ_a
1	1a	0.5	5	1	8	2
	1b	0.2	11	3	15	2
	1c	0.9	17	2	19	1
2	2a	0.01	3	2	6	0.5
	2b	0.4	10	1	12	3
	2c	0.7	12	2	16	1

Figure 8 illustrates the utility distributions for travelers from nodes 1 and 2, respectively.

Utilities

K	(t_d, t_a)	Node 1			$U_1(K)$	Node 2			$U_2(K)$
		1a	1b	1c		2a	2b	2c	
0	0, 3	–	–	–	0	–	–	–	0
1	1, 4	–	–	–	0	–	–	–	0
2	2, 5	–	–	–	0	–	–	–	0
3	3, 6	–	–	–	0	2.0	–	–	2.0
4	4, 7	0.5	–	–	0.5	–	–	–	0
5	5, 8	2.0	–	–	2.0	–	–	–	0
6	6, 9	0.5	–	–	0.5	–	–	–	0
7	7, 10	–	–	–	0	–	–	–	0
8	8, 11	–	–	–	0	–	–	–	0
9	9, 12	–	–	–	0	–	1.2	–	1.2
10	10, 13	–	0.267	–	0.267	–	1.6	–	1.6
11	11, 14	–	1.2	–	1.2	–	0.4	–	0.4
12	12, 15	–	1.867	–	1.867	–	–	1.4	1.4
13	13, 16	–	0.933	–	0.933	–	–	1.3	1.3
14	14, 17	–	0	–	0	–	–	0	0
15	15, 18	–	–	0	0	–	–	–	0
16	16, 19	–	–	1.1	1.1	–	–	–	0
17	17, 20	–	–	1.8	1.8	–	–	–	0
18	18, 21	–	–	–	0	–	–	–	0
19	19, 22	–	–	–	0	–	–	–	0
20	20, 23	–	–	–	0	–	–	–	0

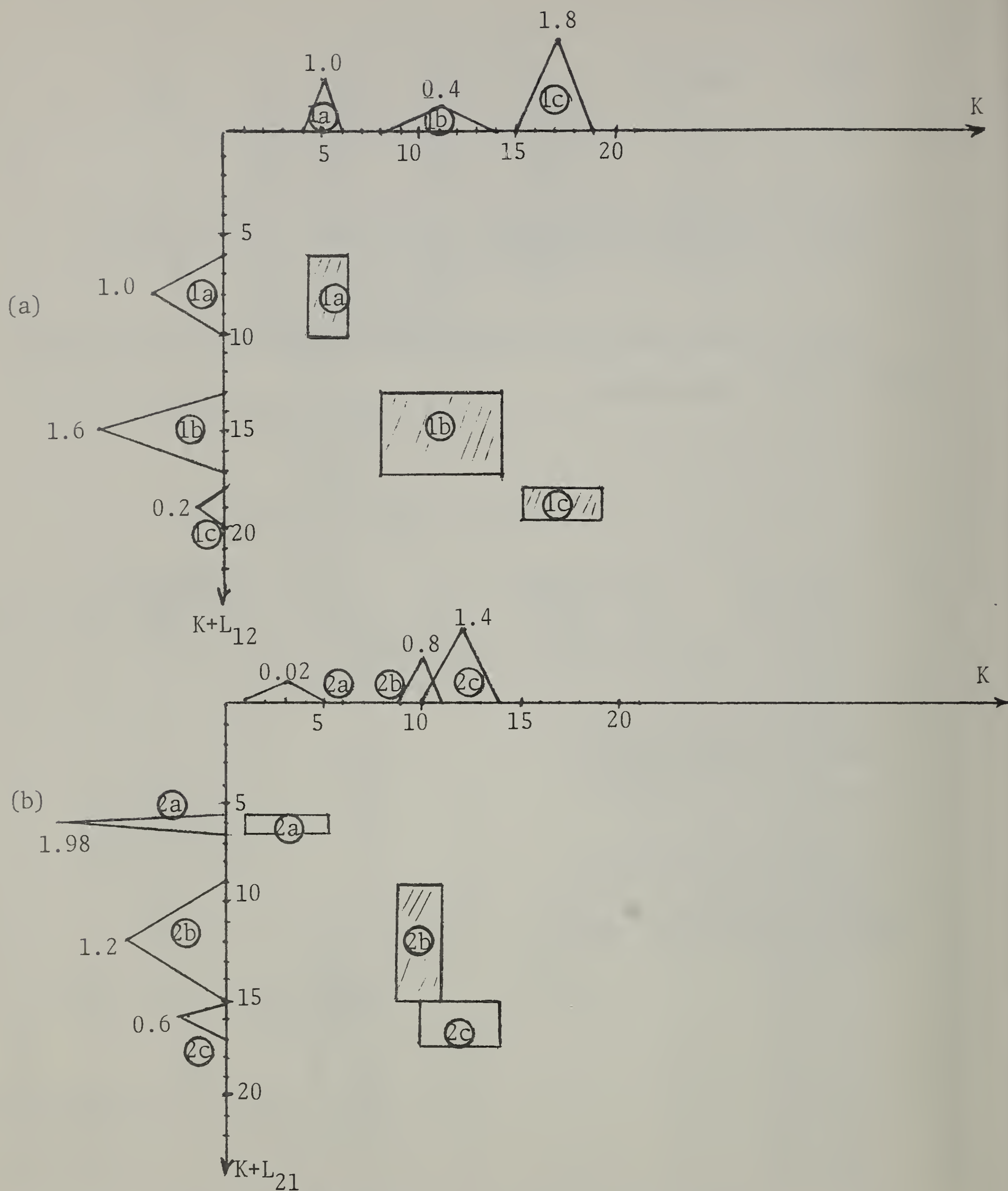

Figure 8. UTILITY DISTRIBUTIONS: (a) NODE 1, (b) NODE 2

Iterating with equations (5.2) and (5.3) yields the results below. Again, c=0 is chosen if both values of c yield equal benefit.

K	x	I(x,K+1)	$U_x(K)+I(x',K+3)$	I(x,K)	Optimal c
19	1	0	0 + 0	0	0
	2	0	0 + 0	0	0
18	1	0	0 + 0	0	0
	2	0	0 + 0	0	0
17	1	0	1.8 + 0	1.8	1
	2	0	0 + 0	0	0
16	1	1.8	1.1 + 0	1.8	0
	2	0	0 + 0	0	0
15	1	1.8	0 + 0	1.8	0
	2	0	0 + 0	0	0
14	1	1.8	0 + 0	1.8	0
	2	0	0 + 1.8	1.8	1
13	1	1.8	.933 + 0	1.8	0
	2	1.8	1.3 + 1.8	3.1	1
12	1	1.8	1.867 + 0	1.867	1
	2	3.1	1.4 + 1.8	3.2	1
11	1	1.867	1.2 + 1.8	3.0	1
	2	3.2	0.4 + 1.8	3.2	0
10	1	3.0	0.267 + 3.1	3.367	1
	2	3.2	1.6 + 1.8	3.4	1
9	1	3.367	0 + 3.2	3.367	0
	2	3.4	1.2 + 1.867	3.4	0
8	1	3.367	0 + 3.2	3.367	0
	2	3.4	0 + 3.0	3.4	0

K	x	I(x,K+1)	$U_x(K)+I(x',K+3)$	I(x,K)	Optimal c
7	1	3.367	0 + 3.4	3.4	1
	2	3.4	0 + 3.367	3.4	0
6	1	3.4	0.5 + 3.4	3.9	1
	2	3.4	0 + 3.367	3.4	0
5	1	3.9	2.0 + 3.4	5.4	1
	2	3.4	0 + 3.367	3.4	0
4	1	5.4	0.5 + 3.4	5.4	0
	2	3.4	0 + 3.4	3.4	0
3	1	5.4	0 + 3.4	5.4	0
	2	3.4	2.0 + 3.9	5.9	1
2	1	5.4	0 + 3.4	5.4	0
	2	5.9	0 + 5.4	5.9	0
1	1	5.4	0 + 3.4	5.4	0
	2	5.9	0 + 5.4	5.9	0
0	1	5.4	0 + 5.9	5.9	1
	2	5.9	0 + 5.4	5.9	0

Coincidentally, I(1,0) = I(2,0) = 5.9; the benefit incurred by using an optimal schedule is indifferent to the node at which the vehicle resides at K=0. However, it should be noted that if the initial node is node 1 then the optimal decision is to depart (with the vehicle empty) for node 2 immediately. Figure 9 shows the trajectories found by applying the optimal decisions.

The passengers are accommodated by the schedules as follows:

i) Vehicle Initially at Node 1.

Depart Station	Time	Passengers
1	0	None
2	3	2a
1	6	1a
2	10	2b
1	17	1c

ii) Vehicle Initially at Node 2

Depart Station	Time	Passengers
2	3	2a
1	6	1a
2	10	1b
1	17	1c

Passengers 1b and 2c are unaccommodated.

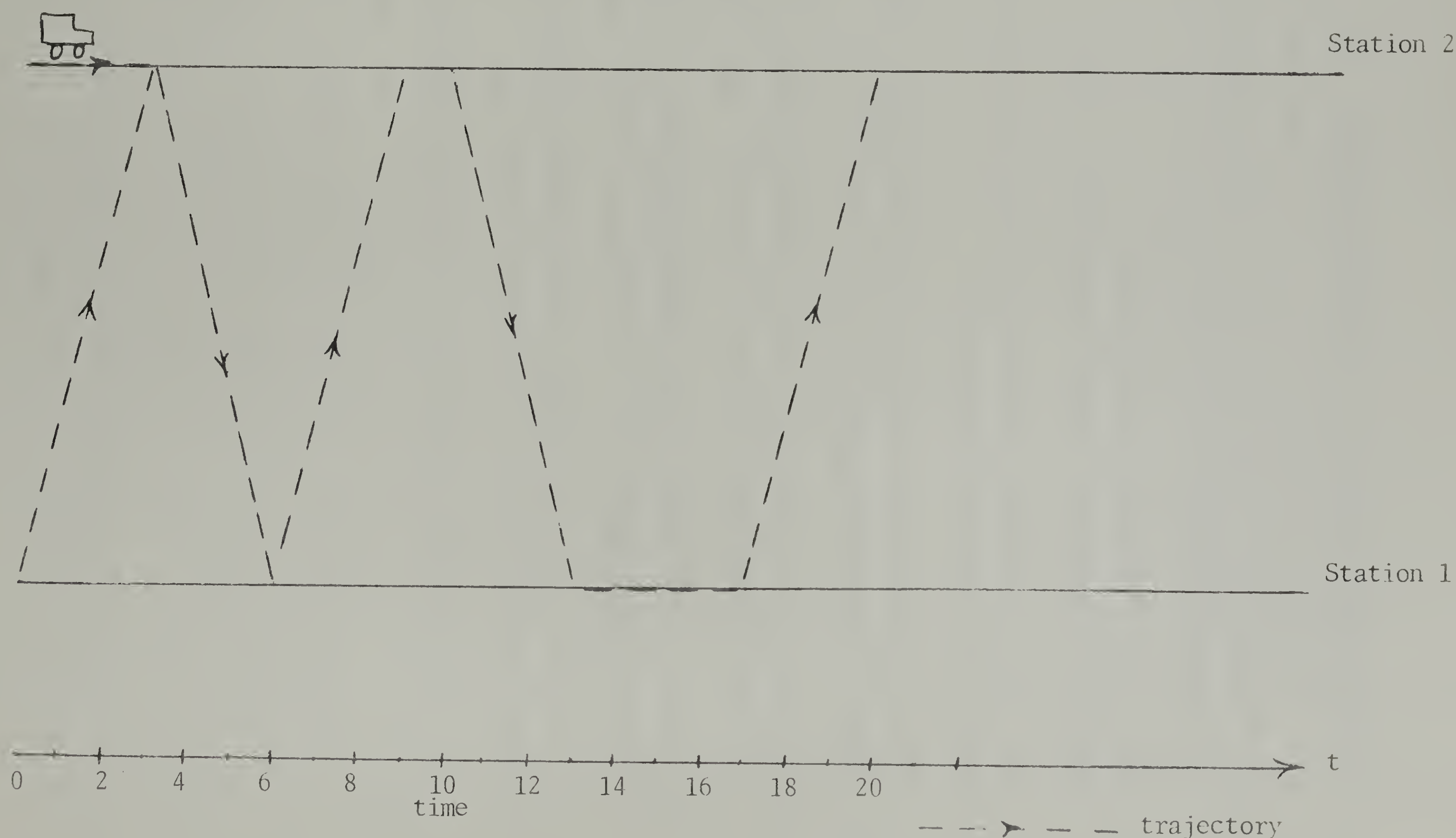

FIGURE 9. VEHICLE TRAJECTORIES FOR EXAMPLE 5.1

6. SCHEDULING ONE VEHICLE OVER AN EXTENDED NETWORK

The algorithms described heretofore, for optimal scheduling of a single vehicle along one link to match (one-way or two-way) passenger preferences, can be extended to a class of more complex networks for which few new ideas or computational requirements are necessary.

The basic premises for such generalization are these:

(a) That the analog of equation (4.3) hold, i.e. that any traveler is eligible for just one vehicle departure.

(b) That the routing sequence in which the vehicle visits nodes of the network, is specified a priori, and is cyclic in nature. Thus no attempt is made here to determine optimal routing of the vehicle.

(c) That the "hold or dispatch" decisions are required only at a limited number of the nodes, called dispatching points. All other nodes, to be called stopping points, are characterized by fixed stopping times for the vehicle. Thus travel time along the route, between any two successive dispatching points, is a constant of the model.

(d) That passengers travel within or between consecutive dispatching points only; thus, the longest trips are those from one dispatching point to the next along the route. In order to be more general a "through passenger" traveling past one or more dispatching stations might be regarded as being discharged and replaced by a fictitious new passenger, at each dispatch point in his journey. Such an assumption involves certain difficulties, however. In particular, schedules may result which fail to accommodate all the fictional passengers along the segments of the total trip. Hence, the "real" passenger would be accommodated for only a portion of his full journey.

The generalization of the earlier algorithms involves some preliminary steps with regard to the traveler utilities and capacity limitations. First, the utility functions $U^{(i)}(t_d, t_a)$ for persons wishing to arrive or depart at intermediate stopping points, must be "referred" to dispatching point coordinates. Secondly, utilities must be aggregated in such manner as to make full use of capacity. These two matters are discussed below.

The specifics of "referring" a potential traveler to dispatching points is as follows. Let A and D be consecutive dispatching points, between which lie the i^{th} traveler's origin B and destination C (in that order); perhaps B=A and/or C=D. A time t_d of departure from B implies that the vehicle departed from A at time $t_d - \lambda_{AB}$. (λ_{AB} is the travel time from A to B along the route, including stopping times at stopping nodes up to and including B.) A time t_a of arrival at C implies that the vehicle will reach D at time $t_a + \lambda_{CD}$. Thus the traveler can be replaced by a fictitious one with origin A and destination D, whose utility function $V^{(i)}$ is

$$V^{(i)}(t_d, t_a) = U^{(i)}(t_d + \lambda_{AB}, t_a - \lambda_{CD}). \tag{6.1}$$

Once all travelers have been so referred, the utilities must be aggregated at each dispatch node in a manner analogous to that in the earlier algorithms:

The state variable will be

x = dispatching point at which the vehicle is currently located.

Also let

D_x = dispatching point next after x in the routing sequence.

L_x = length of trip from x to D_x, measured in <u>discrete</u> time units.

Then the aggregated utility is

$$V_x(K) = \sum_i V^{(i)}(K,K+L_x),$$

where the sum is over the (fictitious) set of travelers eligible to leave node x at stage K.

The utility aggregation procedure used previously must be modified to ensure best use of vehicle capacity. The original procedure was to count the C highest utility travelers at each node. Here, however, this is inadequate because travelers, in general, ride only a portion of the route segment between dispatching points. Hence, if a passenger travels only part of the route between two dispatching points, he leaves an empty seat which might well be utilized by someone else, during the remaining part of the route.

The situation can be illustrated by considering the route segment A-B-C-D where A and D are the consecutive dispatching points. Suppose all travelers riding between A and D in the A→D direction have been referred, and the aggregated utility $V_A(K)$ is to be calculated for use in the dynamic programming routine for "hold or dispatch" decisions from node A. $V_A(K)$ should be the maximum sum of utilities resulting from an optimal assignment of passengers to the vehicle along the route. Calculation of $V_A(K)$ is complicated by the fact that certain travelers are in competition for seats while others are not. For example, travelers from A to B and A to C are in competition, but travelers from A to B and B to D are not. Hence passenger trip segments must be "meshed" together, to derive a passenger assignment list which maximizes the sum of utilities of passengers assigned to the vehicle.

The problem of optimally assigning passengers to the vehicle along the route between dispatching nodes is analogous to the problem of finding a "minimal-cost flow" between a source node and a sink node in a graph. A graph is comprised of a set of nodes, and a set of edges which connect the nodes. In the present context, the nodes are the stations (A, B, C, and D). The source node (A) is the dispatching node from which the trip emanates; the sink node (D) is the dispatching station at which the trip terminates. The edges are the travelers, each of whom can be thought of as a connecting edge between his origin node and his destination node. Each edge (traveler) is associated with a cost and a "flow capacity." The cost is equal to the negative of the utility that the traveler holds for his trip. A flow capacity value of one is assigned to each edge. Thus a unit flow from source to sink is equivalent to a set of "connecting" travelers whose trip segments combine to make up a total trip between the successive dispatching nodes (A and D). Zero cost edges are included in the graph, between each pair of consecutive nodes, to represent potentially empty seats on each of the trip segments. Figure 10 illustrates the graph which represents travelers along the A-B-C-D route.

Using the graph representation, the problem of optimally assigning passengers to the vehicle is equivalent to the problem of finding that flow from source to sink, of magnitude C (the vehicle capacity), which has minimum cost (maximum utility). The minimal-cost flow problem is readily solved using established algorithms.[1] An illustrative solution, for a vehicle capacity C=2, is shown in Figure 10. The aggregate utility $V_A(K)$ is equal to the negative sum of costs on the edges which are included in the minimal-cost flow solution.

Finally, the basic iterative equation is

$$I(x,K) = \max \begin{cases} 0 + I(x,K+1) & \text{("hold")} \\ V_x(K) + I(D_x,K+L_x). & \text{("dispatch")} \end{cases} \qquad (6.2)$$

Examples of networks which are of the type discussed in this section, are illustrated in Figure 11. A variety of other configurations can easily be imagined. Thus the methods of this section make it possible to schedule a single vehicle on a fairly general array of networks.

The section concludes with a brief example to illustrate some of the ideas put forth here.

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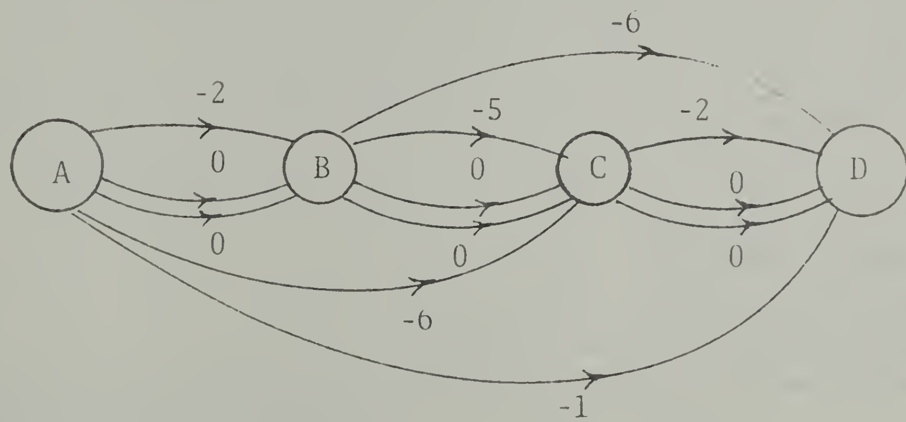
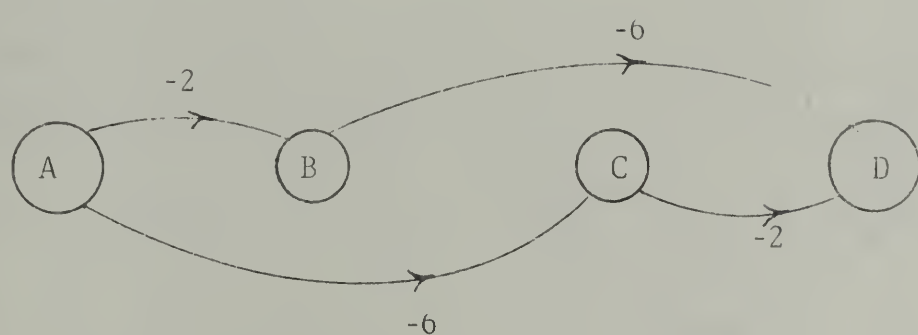
[1] See for example Busacker, R. G. and Saaty, T. L., "Finite Graphs and Networks," McGraw Hill, 1965, pp. 225-260.

Passenger	Origin	Destination	Utility
1	A	D	1
2	A	C	6
3	A	B	2
4	B	D	6
5	B	C	5
6	C	D	2

Graph:



Minimal-Cost Flow: C=2

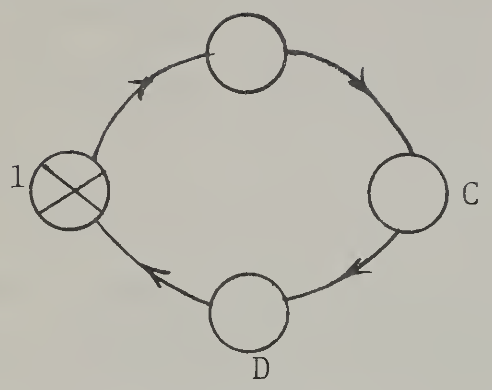


Passengers: Seat #1: Passengers 3 and 4
Seat #2: Passengers 2 and 6

Total Utility: $V_A(K) = 16$.

FIGURE 10. GRAPH REPRESENTATION OF PASSENGER ASSIGNMENT PROBLEM

(a) Loop



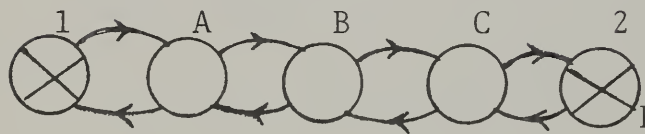
Dispatching Points: node 1
Routing: 1-B-C-D-1

$x = 1$

$D_x = 1$

$L_x = L_{1B}+L_{BC}+L_{CD}+L_{D1}$

(b) Line of stations

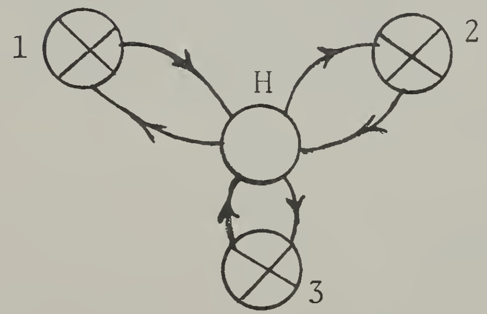


Dispatching Points: nodes 1, 2
Routing: 1-A-B-C-2-C-B-A-1

$$D_x = \begin{cases} 2 \text{ if } x=1 \\ 1 \text{ if } x=2 \end{cases}$$

$$L_x = \begin{cases} L_{1A}+L_{AB}+L_{BC}+L_{C2} \text{ if } x=1 \\ L_{2C}+L_{CB}+L_{BA}+L_{A1} \text{ if } x=2 \end{cases}$$

(c) Hub



Dispatching Points: Nodes 1, 2, 3

Routing: 1-H-2-H-3-H-1

$$D_x = \begin{cases} 2 \text{ if } x = 1 \\ 3 \text{ if } x = 2 \\ 1 \text{ if } x = 3 \end{cases}$$

$$L_x = \begin{cases} L_{1H}+L_{H2} \text{ if } x = 1 \\ L_{2H}+L_{H3} \text{ if } x = 2 \\ L_{3H}+L_{H1} \text{ if } x = 3 \end{cases}$$

FIGURE 11. EXAMPLES OF EXTENDED NETWORKS

Example 6.1

Given the network

with routing 1-A-2-A-1, and dispatching points 1 and 2, and the following set of travelers wishing to depart over the interval t=0 to T=20:

Traveler	Origin	Destination	w	t^*_d	σ_d	t^*_a	σ_a
1	1	2	0.5	5	1	8	2
2	1	A	0.2	11	3	13	2
3	A	2	0.9	18	2	19	1
4	2	1	0.01	3	2	6	0.5
5	2	A	0.4	10	1	11	3
6	A	1	0.7	14	2	16	1

Assume $\overline{K}$=T=20 (Δt=1), and

$L_{1A}=L_{A1}=1$,

$L_{2A}=L_{A2}=2$,

$C=2$.

Travelers 2, 3, 5, and 6 must be referred to the dispatching stations. Hence, the following fictious set of travelers is to be considered:

Traveler	Origin	Destination	w	t^*_d	σ_d	t^*_a	σ_a
1	1	2	0.5	5	1	8	2
2'	1	2	0.2	11	3	15	2
3'	1	2	0.9	17	2	19	1
4	2	1	0.01	3	2	6	0.5
5'	2	1	0.4	10	1	12	3
6'	2	1	0.7	12	2	16	1

From this point, the two-way algorithm as discussed in Section 5 can be employed directly to schedule between nodes 1 and 2. It will be noted that the 'new' set of traveler parameters above corresponds exactly to those in Example 5.1. The solution is the one calculated there, but

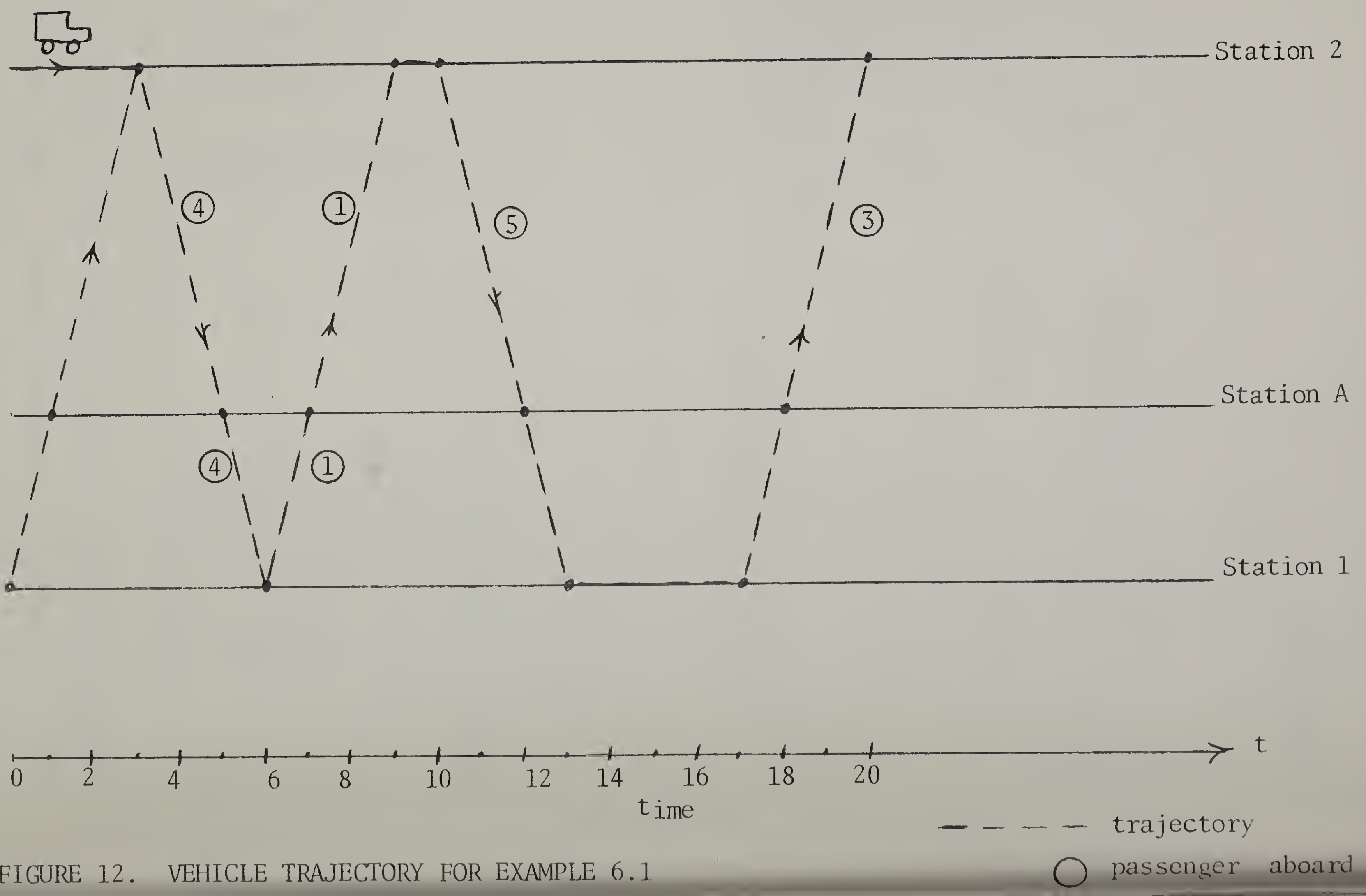


FIGURE 12. VEHICLE TRAJECTORY FOR EXAMPLE 6.1

illustrated in the present context in Figure 12. Note that the algorithm does not allow for an initial departure from an intermediate stopping point such as node A.

7. MULTI-VEHICLE SCHEDULING

The bulk of this paper is devoted to scheduling of a single vehicle--a starting point for the study of more general scheduling problems. The purpose of the present section is to describe some possible approaches to scheduling a fleet of several vehicles, with (possibly) different speeds and capacities. To date this research effort has not included much work in the multi-vehicle area, so that what follows is more indicative of likely future work than of accomplished analysis.

The natural inclination, in proceeding to the case of several vehicles, is to extend the one-vehicle formulation. Thus, let

s_j = dispatching station (node) from which the next departure of vehicle j can occur,

K_j = next stage at which vehicle j can depart from s_j,

V = total number of vehicles.

Then the state of the system is described by the vector

$$\underline{x} = (\underline{s}, \underline{K})$$

where

$$\underline{s} = (s_1, s_2, ---, s_V),$$
$$\underline{K} = (K_1, K_2, ---, K_V).$$

It will be shown shortly that the size of the state space in the dynamic programming formulation will offer great difficulty in computational requirements. To alleviate the problem somewhat, the following modification is made in the definition of $\underline{x}$:

Let $\underline{x} = (\underline{s}, \underline{T})$

where

$$\underline{T} = (T_1, T_2, ---, T_V)$$

and

$T_j = K_j - K$ = number of stages until the j^{th} vehicle could be dispatched from s_j.

Using T_j rather than K_j results in a considerable reduction, since K_j has $\overline{\underline{K}}+1$ possible values, whereas T_j has at most only $(L_{max}+1)$ possible values, where L_{max} is the largest number of stages separating any two consecutive (dispatching) nodes. It should be noted that this definition of state could have been used in the single vehicle algorithms, but to no particular advantage.

The <u>decision</u> vector $\underline{c}$ is given by

$$\underline{c} = (c_1, c_2, ---, c_V)$$

where

$c_j = \Phi$ (null decision for j^{th} vehicle)

$c_j = 0$ if "hold" is ordered for j^{th} vehicle

$c_j = 1$ if "dispatch" is ordered for j^{th} vehicle.

Clearly,

$$c_j(K) = \Phi \text{ if } T_j \neq 0,$$

$$c_j(K) = 0 \text{ or } 1 \text{ if } T_j = 0.$$

Using the notation

L_j = number of stages required, by vehicle j, to go from its most recent dispatching node to the next one,

s_j' = dispatching node next after the current one, for the j^{th} vehicle,

the transition vector $\underline{g}(\underline{x}, \underline{c}, K)$ has components $g_j(x_j, c_j, K)$ given by

$$g_j(x_j, \Phi, K) = (s_j, T_j - 1),$$
$$g_j(x_j, 0, K) = (s_j, 0),$$
$$g_j(x_j, 1, K) = (s_j', L_j).$$

The "immediate benefit" function $\underline{b}(\underline{x}, \underline{c}, K)$ would be defined in terms of a set of aggregate utility functions, classified by station (node) and by vehicle speeds and capacities. Obvious complications arise regarding the assignment of particular travelers to particular vehicles. No definitive treatment has been devised up to now to deal with these complexities.

Turning to the basic computational problem, consider the amount of computation and computer storage required by the basic dynamic programming equation (3.1), repeated here for convenience:

$$I(\underline{x}, K) = \max_{\underline{c}} \{b(\underline{x},\underline{c},K) + I(g(\underline{x},\underline{c},K), K+1)\}. \tag{3.1}$$

The amount of computation depends primarily on the number of terms to be calculated and compared in finding the maximum. If there are $V(\underline{x})$ vehicles at dispatching points when the system is in state $\underline{x}$, then the number of different decision vectors in (3.1) is $2^{V(\underline{x})}$. While $V(\underline{x})$ could be as great as V, in reality it may be only a small fraction of V for most states.

As is generally the case for dynamic programming calculations, computer storage requirements are particularly heavy. The results of the maximization (3.1) for <u>each</u> $\underline{x}$ need to be recorded, and to obtain them, it is necessary to store all $I(g(\underline{x},\underline{c},K), K+1)$ at the succeeding stage, K+1. Thus the dominant storage requirement is twice the number of different states $\underline{x}$, in the state space. Suppose for simplicity that the number of dispatching nodes S is the same for each of the V vehicles. Then there are S possible values for each s_j, and hence S^V possible choices for the "next dispatching node" vector $\underline{s}$. (Recall $\underline{x} = (\underline{s}, \underline{T})$.) Further, T_j takes on one of (L_j+1) values, 0 through L_j inclusive. Suppose any two successive dispatching nodes are at least L* stages apart; then a <u>lower bound</u> on the number of states is

$$S^V(L^*+1)^V.$$

If for example S=10, L*=2, and V=5, a fairly modest network, the storage requirement exceeds 2×10^7!

While this general mode of analysis still seems well worth pursuing, the need to explore a variety of other tools is clear. There are a number of techniques within the dynamic programming context which may prove useful in dealing with the difficulties of multi-vehicle scheduling. These include the use of Lagrange multipliers to reduce the dimensionality of the problem, and the application of polynomial approximations to reduce computer memory requirements for storing functions (e.g. $I(\underline{x}, t)$).[2]

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[2]See Bellman and Dreyfus, "Applied Dynamic Programming," Princeton University Press, 1962.

A rather different approach, in which attaining the true optimum is sacrificed for computational simplicity, is that of sequential optimization. In the present context, this would involve using the single-vehicle algorithm to schedule vehicles one at a time. The procedure would be to schedule a vehicle, "remove" the accommodated passengers and revise the aggregate utility functions accordingly, schedule the next vehicle, and so on.

The quality of the solutions obtained by this method will of course depend on the order in which the vehicles, characterized by speed, capacity, and routing sequence, are scheduled. It should be interesting to analyze the comparative merits of different ordering rules. The performance of the methods as a whole can at least be indicated by comparing results with those from a full multi-vehicle dynamic programming formulation, on systems simple enough for the latter to be computationally feasible.

Finally, it may be worthwhile to seek an iterative procedure for successively improving an initial "guess" at a good schedule. This starting schedule might be found by sequential optimization or by some other means. The determination of a suitable "improvement routine" is yet to be studied.

8. NUMERICAL RESULTS AND SENSITIVITY ANALYSES

The algorithm for scheduling a single vehicle over a single link with one-way demand has been implemented in a FORTRAN computer program. The details of the computer code are presented in Appendix A. This section presents a discussion of the results of its application to investigate the sensitivity of optimal schedule characteristics to changes in the important problem parameters.

The single link, one-way, one vehicle case, although extremely simple, provides a format in which to study some of the fundamental aspects of a transportation system, such as capacity limitations, variable vehicle speed, and passenger preference characteristics. Accordingly, it is hoped that results obtained here have some general significance.

Several parameters are to be studied. The (one-way) trip time, ϕ, is of prime importance as a measure of vehicle speed. Capacity, C, is a second quantity basic to studying system operation. The trade-off between speed and capacity is a subject of obvious importance and will be investigated here.

The properties of the traveler utility functions are also to be studied for their influence on the optimal schedules. The effects of the half-width parameters (σ) and the weighting parameter (w), as well as the functional form and normalization criterion, are of interest here.

Finally, the question of variability of results with changes in the precision of calculation (size of the stage increments in the dynamic programming calculation), and with different sets of input travelers, will be addressed.

Design and execution of the experiments by which to perform the analyses involved some preliminary preparation; additional assumptions had to be made and data had to be generated from which meaningful results, based on estimates of "realistic" traveler behavior, could be obtained.

The following assumptions were made with regard to traveler behavior:

(a) Each traveler is either totally departure-oriented or totally arrival-oriented, e.g., $w^{(i)}=0$ or 1. For each traveler, $w^{(i)}$ is generated from a binomial distribution with parameter p, where

$$p = \text{Probability } \{w^{(i)}=0\} = 1 - \text{Probability } \{w^{(i)}=1\}.$$

(b) The half-width parameters for each traveler's utility function are selected such that if a traveler is departure-oriented, his acceptable arrival interval is sufficiently wide so that, over the full range of trip times considered in this experiment, if a vehicle departed within his acceptable departure interval it would arrive within the acceptable arrival interval. Analogously, for arrival-oriented travelers, if a vehicle arrives within the acceptable arrival interval, it must necessarily have departed within the acceptable departure interval. Assumption (b) is implemented as follows:

(i) Each traveler is assigned a value $w^{(i)}=0$ or 1.

(ii) If $w^{(i)}=1$, $\sigma_d^{(i)}$ is generated as a random number between 0 and some specified maximum value σ_{MAX}. If $w^{(i)}=0$, $\sigma_a^{(i)}$ is so generated instead. It is assumed that the same value of σ_{MAX} is applicable for departures and arrivals.

(iii) Assuming $w^{(i)}=1$, assumption (b) translates into the inequalities:

$$t_d^{*(i)}-\sigma_{MAX}+\phi_f \geq t_a^{*(i)}-\sigma_a^{(i)}, \tag{8.1}$$

$$t_d^{*(i)}+\sigma_{MAX}+\phi_s \leq t_a^{*(i)}+\sigma_a^{(i)}, \tag{8.2}$$

where

ϕ_f = trip time for <u>fastest</u> trip considered,

ϕ_s = trip time for <u>slowest</u> trip considered.

Inequality (8.1) requires that if a vehicle departs at the earliest possible time within the widest possible interval of departure, on the vehicle of highest speed, then arrival will not occur earlier than the earliest acceptable arrival time. Inequality (8.2) requires that if departure occurs as late as possible, within the widest possible departure interval, on the slowest vehicle considered, then arrival will still occur before the end of the acceptable arrival interval.

Corresponding to (8.1) and (8.2), for the <u>arrival-oriented</u> traveler, are the inequalities

$$t_a^{*(i)}+\sigma_{MAX}-\phi_f \leq t_d^{*(i)}+\sigma_d^{(i)}, \tag{8.3}$$

$$t_a^{*(i)}-\sigma_{MAX}-\phi_s \geq t_d^{*(i)}-\sigma_d^{(i)}. \tag{8.4}$$

Following assumption (4.3), the following additional inequality is required to ensure that no individual is included on more than one vehicle departure:

$$\sigma_{MAX} \leq \phi_f. \tag{8.5}$$

Hence, in generating a set of traveler parameters, (8.1) through (8.5) was assumed to hold. The following steps were actually taken to generate the data:

(a) Table 2, taken from the Port of New York Authority report, "New York's Domestic Air Passenger Market," 1965, gives the percent of total passengers leaving in any hour, between 7 a.m. and 12 midnight, from New York airports on holidays, and the same data for weekends. For the purpose of this study, time $t=0$ was taken as 7 a.m. and $t=T=17$ was taken as 12 midnight; further, each percentage point was regarded as a single traveler, yielding a total of 100 travelers desiring to leave over the interval $(0, T)$. The distribution for holidays was used as the basis for the input data throughout most of the analysis; the weekend distribution was used for a comparison of results, given at the end of this section.

(b) A range of trip times (ϕ_f, ϕ_s) was selected, over which sensitivity to the parameter ϕ would be tested. ϕ_s, the trip time for the slowest vehicle to be considered, was set at 3 hours; ϕ_f, the fastest considered trip time, was set at 1 hour. The nominal trip time in this 3:1 range of trip duration is

$$\phi_n = \frac{\phi_s + \phi_f}{2} = 2 \text{ hours.} \tag{8.6}$$

(c) The parameters $t_d^{*(i)}$ and $t_a^{*(i)}$, for each traveler i, were generated as follows: Within each hourly interval, the travelers desiring to depart in that hour, as given by Table 2, were assigned desired departure times $t_d^{*(i)}$ randomly over that hour, using the random number generating

Table 2. PERCENTAGE BREAKDOWN OF FLIGHT DEPARTURE TIMES FOR PASSENGERS DEPARTING NEW YORK

Departure Time	t	Holidays	Weekends
7 - 8 a.m.	0-1	3	4
8 - 9 a.m.	1-2	6	5
9 - 10 a.m.	2-3	16	9
10 - 11 a.m.	3-4	3	5
11 - 12 noon	4-5	8	5
12 - 1 p.m.	5-6	6	6
1 - 2 p.m.	6-7	2	6
2 - 3 p.m.	7-8	3	5
3 - 4 p.m.	8-9	9	8
4 - 5 p.m.	9-10	11	9
5 - 6 p.m.	10-11	3	7
6 - 7 p.m.	11-12	3	11
7 - 8 p.m.	12-13	10	8
8 - 9 p.m.	13-14	1	3
9 - 10 p.m.	14-15	12	5
10 - 11 p.m.	15-16	0	1
11 - 12 p.m.	16-17	4	3
Total		100	100

Source: "New York's Domestic Air Passenger Market", 1965, Port of New York Authority, Table 30, page 80.

function on the CDC 3100 computer. Subsequently, each traveler was assigned the desired arrival time

$$t_a^{*(i)} = t_d^{*(i)} + \phi_n = t_d^{*(i)} + 2.0. \tag{8.7}$$

Of course, since $w^{(i)}=0$ or 1 and assumption (b) holds, concerning the half-width parameters, only one of the numbers $t_d^{*(i)}$, $t_a^{*(i)}$ has significance for any particular traveler. Hence (8.7) does not imply any (strict) functional relationship between travel time and the way an individual specifies his desired departure and arrival times.

(d) Next, the parameter p was chosen, subsequent to which each traveler was assigned a value of $w^{(i)}$ according to the binomial distribution with parameter p.

(e) The value of σ_{MAX} was specified in accordance with (8.5). Then, for each traveler, the half-width parameter of the stressed interval ($\sigma_d^{(i)}$ if $w^{(i)}=1$, and $\sigma_a^{(i)}$ if $w^{(i)}=0$) was picked as a random number between 0 and σ_{MAX}. The half width parameter of the unstressed interval was then set at

$$\sigma_a^{(i)} = \phi_n = 2 \text{ if } w^{(i)} = 1, \tag{8.8}$$

$$\sigma_d^{(i)} = \phi_n = 2 \text{ if } w^{(i)} = 0.$$

Note that (8.8) in conjunction with (8.7) and (8.5) satisfies the requirements (8.1) and (8.2) if $w^{(i)}=1$, and (8.3) and (8.4) if $w^{(i)}=0$. The effects of changing the half-width parameters were studied by varying σ_{MAX}.

A computer print-out of the set of traveler parameters, using p=0.5 and the holiday travelers of Table 2, is shown in Figure 13. This set was used through the bulk of the experiments.

KFIN = 340 NO. OF PASSENGERS = 100 T = 17.00000

TRIP TIME = 2.00000 SIGMAX = 1.00000 P = .50000

I	W(I)	SIGD(I)	SIGA(I)	TDSTAR(I)	TASTAR(I)
1	1.00000	.36170	2.00000	.93887	2.93887
2	0	2.00000	.15614	.36170	2.36170
3	1.00000	.57502	2.00000	.21727	2.21727
4	1.00000	.69967	2.00000	1.15614	3.15614
5	0	2.00000	.46292	1.50857	3.50857
6	1.00000	.05102	2.00000	1.57502	3.57502
7	0	2.00000	.52444	1.88565	3.88565
8	1.00000	.55754	2.00000	1.69967	3.69967
9	1.00000	.66091	2.00000	1.43567	3.43567
10	1.00000	.54114	2.00000	2.46292	4.46292
11	1.00000	.33416	2.00000	2.88047	4.88047
12	1.00000	.87520	2.00000	2.05102	4.05102
13	0	2.00000	.23583	2.35613	4.35613
14	0	2.00000	.47590	2.52444	4.52444
15	0	2.00000	.08471	2.57248	4.57248
16	1.00000	.89551	2.00000	2.55754	4.55754
17	1.00000	.83336	2.00000	2.67726	4.67726
18	0	2.00000	.17636	2.66091	4.66091
19	0	2.00000	.35687	2.62023	4.62023
20	1.00000	.22636	2.00000	2.54114	4.54114
21	0	2.00000	.61257	2.65075	4.65075
22	1.00000	.20907	2.00000	2.33416	4.33416
23	0	2.00000	.26864	2.76700	4.76700
24	1.00000	.78287	2.00000	2.87520	4.87520
25	1.00000	.04121	2.00000	2.40188	4.40188
26	0	2.00000	.51130	3.23583	5.23583
27	1.00000	.16293	2.00000	3.46780	5.46780
28	0	2.00000	.36102	3.47590	5.47590
29	0	2.00000	.90591	4.48049	6.48049
30	0	2.00000	.93169	4.08471	6.08471
31	0	2.00000	.57937	4.55930	6.55930
32	0	2.00000	.91353	4.89551	6.89551
33	0	2.00000	.09853	4.94278	6.94278
34	0	2.00000	.25082	4.83336	6.83336
35	0	2.00000	.00410	4.18534	6.18534
36	1.00000	.57961	2.00000	4.17636	6.17636
37	0	2.00000	.73737	5.04287	7.04287
38	0	2.00000	.74376	5.35887	7.35887
39	1.00000	.99819	2.00000	5.83375	7.83375
40	1.00000	.54309	2.00000	5.22636	7.22636
41	1.00000	.19281	2.00000	5.31698	7.31698
42	0	2.00000	.34212	5.61257	7.61257
43	1.00000	.20926	2.00000	6.58562	8.58562
44	0	2.00000	.76942	6.20907	8.20907
45	0	2.00000	.57199	7.14618	9.14618
46	0	2.00000	.46629	7.26864	9.26864
47	0	2.00000	.17309	7.57429	9.57429
48	1.00000	.69416	2.00000	8.78287	10.78287
49	1.00000	.23095	2.00000	8.84825	10.84825
50	1.00000	.58435	2.00000	8.04121	10.04121
51	0	2.00000	.42630	8.14819	10.14819
52	0	2.00000	.92007	8.51130	10.51130
53	1.00000	.87505	2.00000	8.90533	10.90533
54	0	2.00000	.56184	8.16293	10.16293
55	0	2.00000	.23461	8.37108	10.37108
56	0	2.00000	.98516	8.36102	10.36102
57	0	2.00000	.80870	9.13525	11.13525
58	0	2.00000	.86622	9.90591	11.90591
59	0	2.00000	.75333	9.24740	11.24740

(Continued)

60	0	2.00000	.26776	9.93160	11.93160
61	0	2.00000	.89727	9.43670	11.43670
62	0	2.00000	.68571	9.57937	11.57937
63	1.00000	.23485	2.00000	9.44739	11.44739
64	0	2.00000	.08681	9.91353	11.91353
65	1.00000	.80738	2.00000	9.17680	11.17680
66	1.00000	.02773	2.00000	9.09853	11.09853
67	0	2.00000	.48787	9.30604	11.30604
68	0	2.00000	.06274	10.25082	12.25082
69	1.00000	.08139	2.00000	10.36810	12.36810
70	0	2.00000	.85826	10.00410	12.00410
71	0	2.00000	.21185	11.51682	13.51682
72	0	2.00000	.80089	11.57961	13.57961
73	0	2.00000	.61496	11.46985	13.46985
74	1.00000	.34486	2.00000	12.73737	14.73737
75	0	2.00000	.75572	12.18202	14.18202
76	0	2.00000	.96021	12.74376	14.74376
77	0	2.00000	.09594	12.97593	14.97593
78	1.00000	.26454	2.00000	12.99819	14.99819
79	0	2.00000	.87896	12.78028	14.78028
80	1.00000	.76686	2.00000	12.54309	14.54309
81	1.00000	.96973	2.00000	12.88951	14.88951
82	0	2.00000	.64211	12.19281	14.19281
83	0	2.00000	.06181	12.10673	14.10673
84	1.00000	.04062	2.00000	13.34212	15.34212
85	0	2.00000	.49270	14.76207	16.76207
86	1.00000	.67790	2.00000	14.25926	16.25926
87	1.00000	.94014	2.00000	14.38221	16.38221
88	1.00000	.79059	2.00000	14.76940	16.76940
89	1.00000	.43430	2.00000	14.16474	16.16474
90	0	2.00000	.02690	14.57199	16.57199
91	1.00000	.41780	2.00000	14.48381	16.48381
92	0	2.00000	.44944	14.46629	16.46629
93	0	2.00000	.99233	14.28133	16.28133
94	0	2.00000	.50832	14.17309	16.17309
95	1.00000	.02783	2.00000	14.62956	16.62956
96	0	2.00000	.79996	14.69416	16.69416
97	0	2.00000	.48728	16.76188	18.76188
98	1.00000	.26039	2.00000	16.23095	18.23095
99	1.00000	.09379	2.00000	16.86065	18.86065
100	1.00000	.66667	2.00000	16.58435	18.58435

FIGURE 13. TRAVELER UTILITY PARAMETERS

One interesting statistic by which to characterize the traveler set is the "maximum benefit it is possible to derive if every traveler were accommodated at his most desired time of departure (if $w^{(i)}=1$) or arrival ($w^{(i)}=0$.)" For the set of travelers of Figure 13, this quantity, denoted

MPB = maximum possible benefit,

is calculated to be:

	Triangular Utility		Rectangular Utility	
	Volume	Mean-Height	Volume	Mean-Height
MPB:	177.6	200	88.8	100

A flow diagram of the program which was designed to facilitate study of parametric variations, is shown in Figure 14. The program exhibits five distinct branches, each of which carries out variation of a particular parameter. The mode of variation for any particular run is "read in" by giving the appropriate "M" parameter the value 1:

M1 = 1 → variation of trip time, ϕ.

M2 = 1 → variation of capacity, C.

M3 = 1 → variation of σ_{MAX}.

M4 = 1 → variation of binomial parameter, p.

If none of the M's are specified to be 1, the program executes a single schedule calculation, with all the parameter values read in, and no parametric variation performed. The subroutines ADUTIL, SCHED1, and ASSIGN, alluded to in Figure 14, are described in Appendix A.

An example of the output from a single schedule calculation is exhibited in Figure 15, for the case of ϕ=3.0, C=10, triangular, volume-normalized utilities, and $\underline{K}$=340. ($\underline{K}$=340 corresponds to stage increment size Δt=3 minutes.)

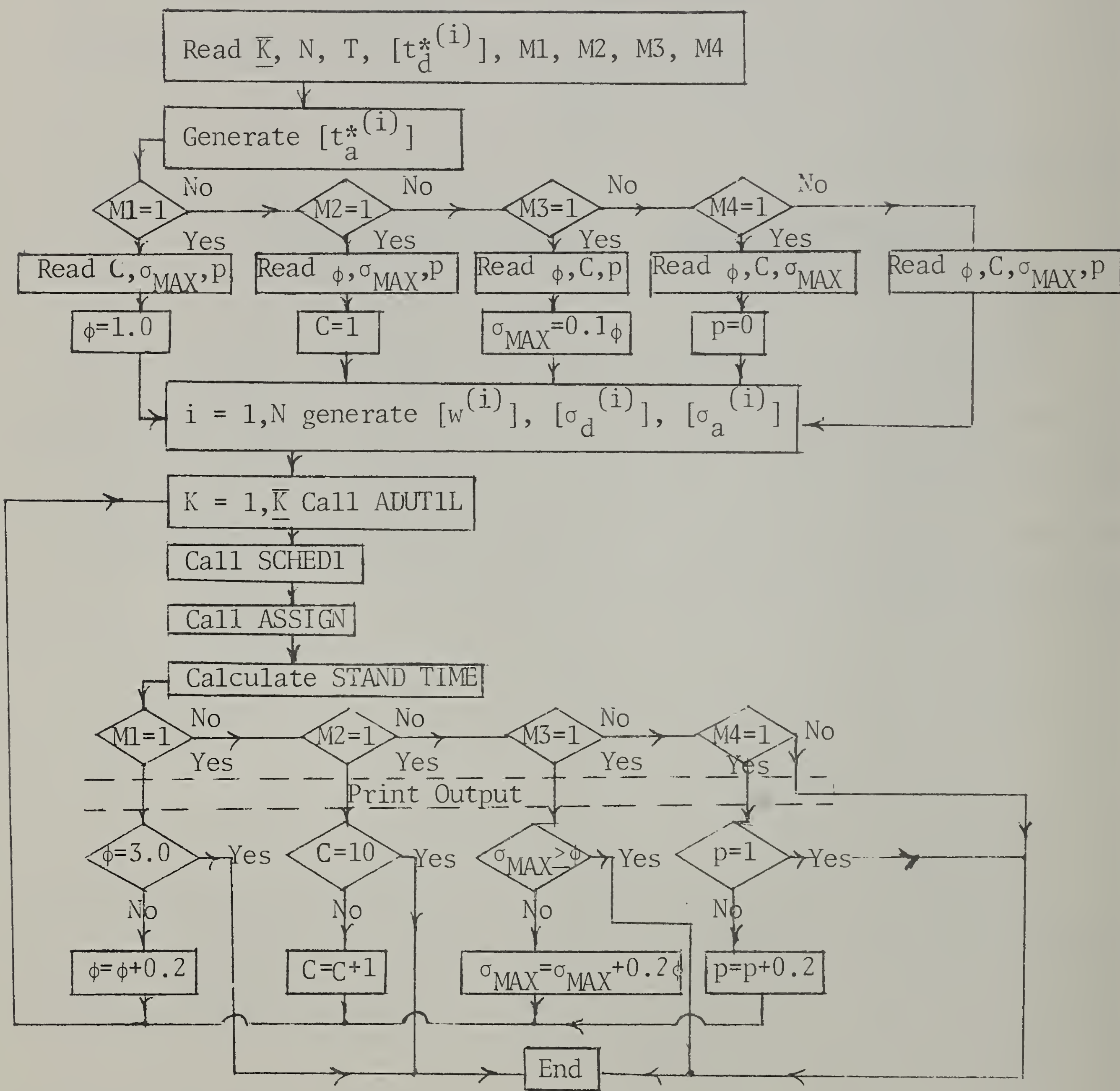


FIGURE 14. SENSITIVITY TEST PROGRAM

STANDING TIME = 4.85000

CAPACITY = 10 IMAX(0) = 21.85204

DEPARTURE NO.	NUMBER OF PASSENGERS BOARDED	DEPARTURE TIME
1	10	2.40000
2	9	9.10000
3	2	16.85000

PASSENGER NO.	ASSIGNED TO DEPARTURE NO.
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	1
11	0
12	1
13	0
14	0
15	0
16	1
17	1
18	0
19	0
20	1
21	0
22	1
23	0
24	1
25	1
26	1
27	0
28	1
29	0
30	0
31	0
32	0
33	0
34	0
35	0
36	0
37	0
38	0
39	0
40	0
41	0
42	0
43	0
44	0
45	0
46	0
47	0
48	2
49	0
50	0
51	0
52	0
53	2
54	0
55	0
56	[illegible]

(Continued)

57	0
58	2
59	0
60	2
61	2
62	2
63	0
64	0
65	2
66	2
67	0
68	0
69	0
70	2
71	0
72	0
73	0
74	0
75	0
76	0
77	0
78	0
79	0
80	0
81	0
82	0
83	0
84	0
85	0
86	0
87	0
88	0
89	0
90	0
91	0
92	0
93	0
94	0
95	0
96	0
97	0
98	0
99	3
100	3

FIGURE 15: OUTPUT SCHEDULE AND PASSENGER ASSIGNMENTS

The "STANDING TIME" print-out gives the amount of time, during the optimal schedule, that the vehicle is not in motion.

In the remainder of this section, results of the computer runs will be illustrated and discussed. Unless otherwise noted, the following inputs were used:

$\underline{K} = 340$; hence $\Delta t = T/\underline{K} = .05$ hrs. = 3 minutes.

Traveler set, generated from Table 1, holiday column.

N = 100 travelers.

Triangular utility functions.

$p = 0.5$, $\sigma_{MAX} = 1.0$.

Variation of I(0) with trip time. Figures 16 and 17 display the behavior of I(0), the benefit attained by the optimal schedule, as a function of trip time, ϕ. Figure 16 shows the volume-normalized case, and Figure 17 the mean-height normalized case. The right-hand ordinate gives I(0) as a percent of the MPB.

These graphs display several interesting properties. I(0) is generally decreasing with increased trip time, as should be expected. A faster vehicle is able to make more trips per time period than a slower one, or at least able to make the same number of trips at more opportune times. However, it will be noted, in Figures 16 and 17, that I(0) is not always invariably decreasing with increasing ϕ, a fact that is somewhat disturbing. For example, in Figure 16, a trip time of 2.25 incurs more benefit than a trip time of 2.0, at capacities greater than 6.

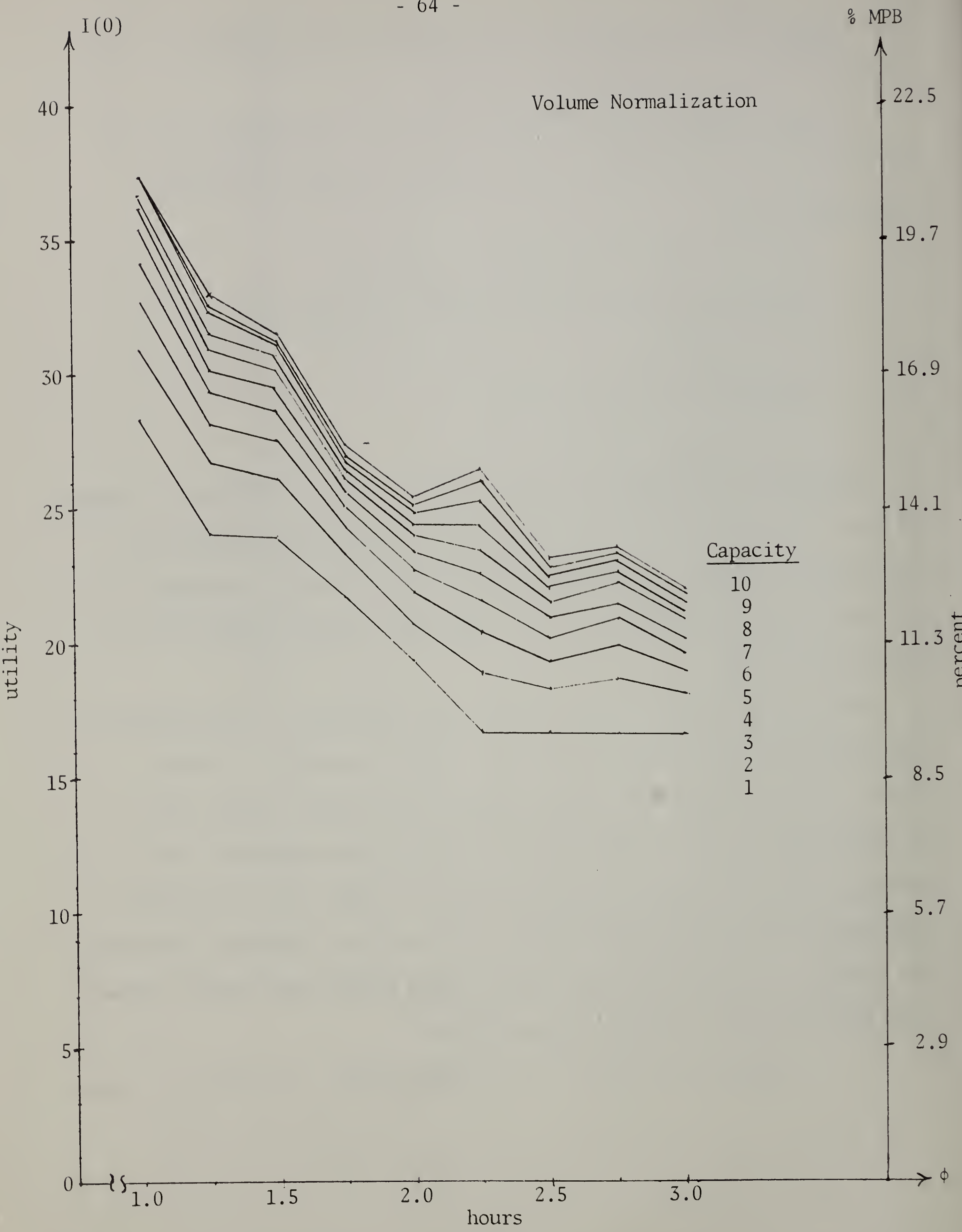


FIGURE 16. DERIVED BENEFIT AS A FUNCTION OF TRIP DURATION

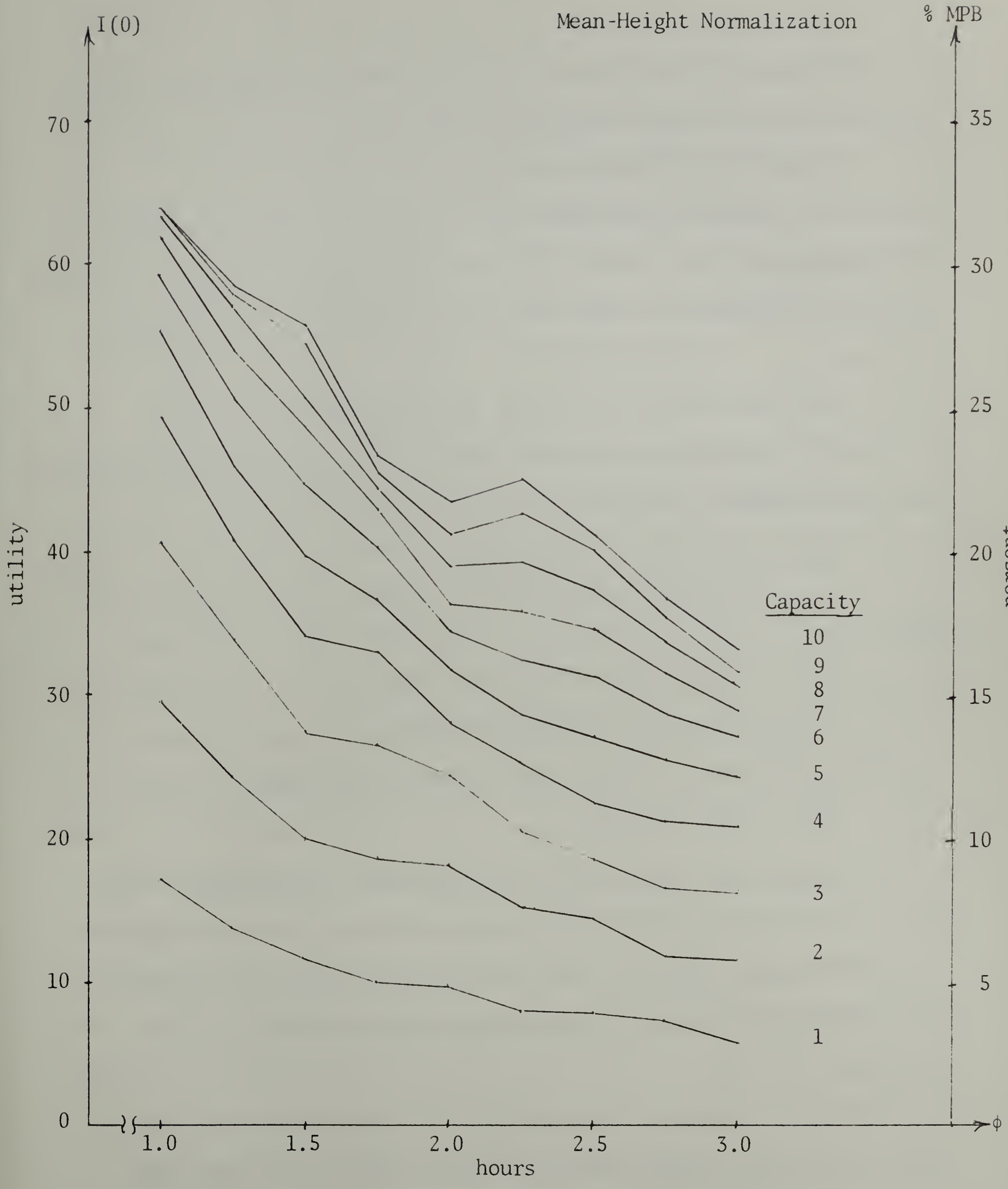


FIGURE 17. DERIVED BENEFIT AS A FUNCTION OF TRIP DURATION

The reason for this anomalous behavior is that the immediate benefit function (for the "depart" decision at stage K),

$$b(1,K) = \sum_i U^{(i)}(K,K+F) = U(K),$$

changes with changes in the value of F. In certain instances, changes in U(K) <u>compensate</u> for increases in trip time, and lead to increases in benefit. The source of variation in U(K) lies in the structure of the individual utilities:

$$U^{(i)}(K,K+F) = w^{(i)}U_d^{(i)}(K) + (1-w^{(i)})U_a^{(i)}(K+F). \qquad (8.9)$$

Since it has been assumed here that all travelers are either completely departure-oriented or completely arrival-oriented,

$$U(K) = \sum_{i\varepsilon D} U_d^{(i)}(K) + \sum_{i\varepsilon A} U_a^{(i)}(K+F) \qquad (8.10)$$

where

D = set of departure-oriented travelers

A = set of arrival-oriented travelers.

Hence, U(K) can be divided into two parts such that

$$U(K) = U_a(K) + U_d(K+F). \qquad (8.11)$$

Now suppose that <u>all</u> travelers were departure-oriented (p=0). Then $U(K) = U_d(K)$ would be unaffected by changes in F. On the other hand, if all travelers were oriented towards arrival (p=1), then $U(K) = U_a(K+F)$ would respond to a change in F by <u>shifting</u> along the time axis, while <u>maintaining</u> its shape.

However, if p=0 or 1, the shape of U(K) is not maintained when F is varied; hence it is <u>a priori</u> possible that increases in benefit, <u>via</u>

modification of U(K), can result from increases in trip duration, in a traveler population of <u>mixed</u> orientation. If p=0, U(K) is unchanged with changing F, and I(0) should monotonically decrease with increasing F; if p=1, U(K) shifts with changing F, leading to shifting schedules, but not to non-monotonic behavior of I(0). These contentions are born out in Figures 18 and 19, which shows the curves of I(0) versus trip time, corresponding to the parameter values p=0 and p=1, for volume and mean-height normalizations respectively. The curves are indeed monotonic.

The details of the process by which I(0) deviates from monotonicity are illustrated by comparing the results of the schedule calculations, as trip time is increased from 2 to 2.25, in the volume-normalized case, capacity C=10, p=0.5. Figure 16 shows that the decrease in trip time results in a decrease in I(0). Table 3 summarizes what happens.

Departures 1 and 2 takes place at the same times in the optimal schedules for either value of trip time. Thus t_{d_1} = 2.40, and t_{d_2} = 9.10 remain the best times to depart, despite the added flexibility of a faster vehicle; but marginal changes in U(K) preclude the vehicle from incurring as much benefit at the faster speed (ϕ=2.0) as at the slower one (ϕ=2.25). Departure 3 is more complicated; decreasing the trip time has two effects here: it affects U(K) so as to make the old departure time t_{d_3} = 14.30 less advantageous at ϕ=2.0 than it was at ϕ=2.25. However, the smaller trip time allows the vehicle to leave at an <u>earlier</u>, more beneficial departure time, t_{d_3} = 13.35.

The lists of passengers on each trip is informative. Consider departure 1. Passengers 15, 18, and 23 are all arrival-oriented travelers who are eligible if ϕ=2.25, but become ineligible if trip time is decreased to ϕ=2.0.

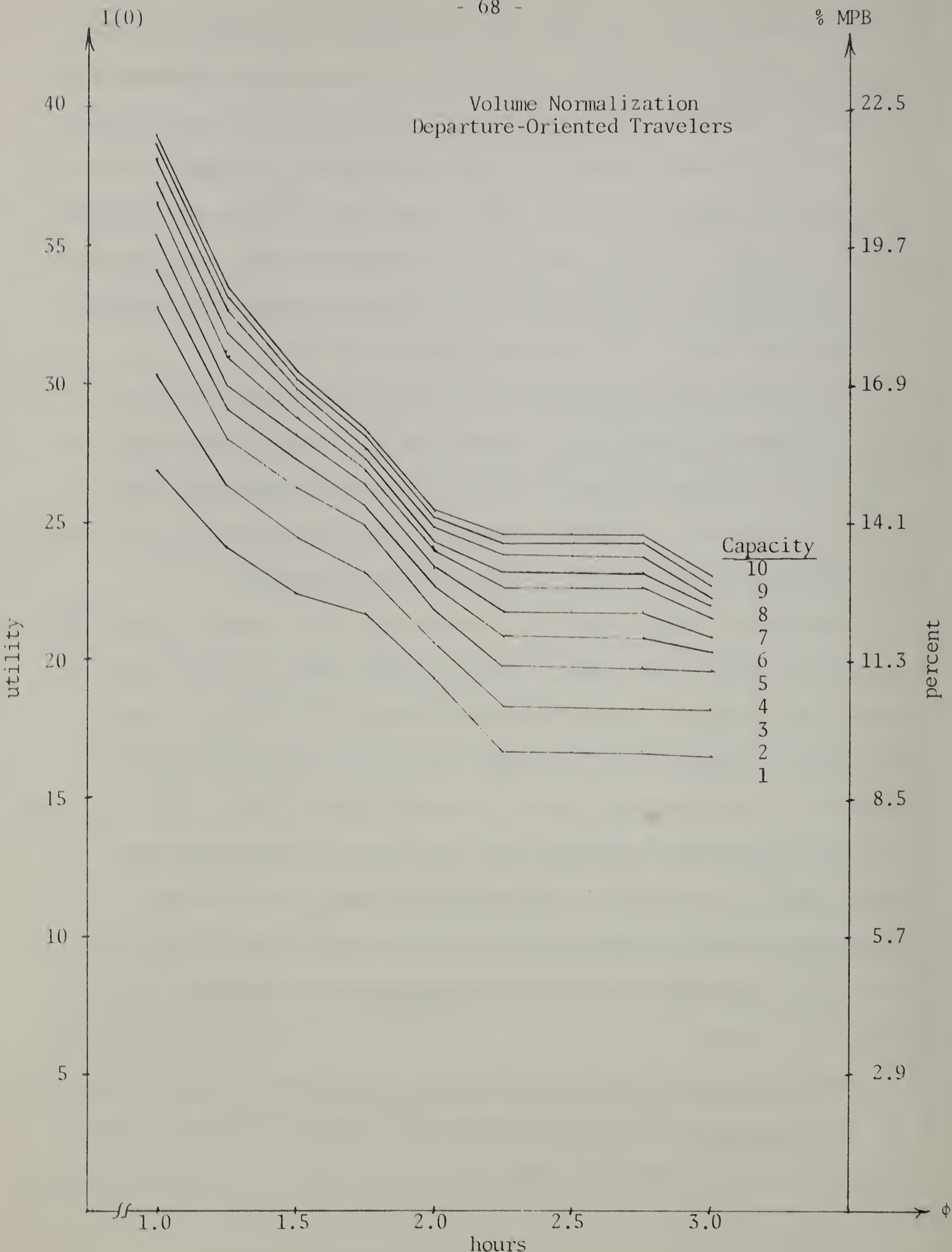


FIGURE 18. BENEFIT-TRIP TIME CURVES, UNIFORMLY DEPARTURE ORIENTED TRAVELERS, VOLUME NORMALIZED

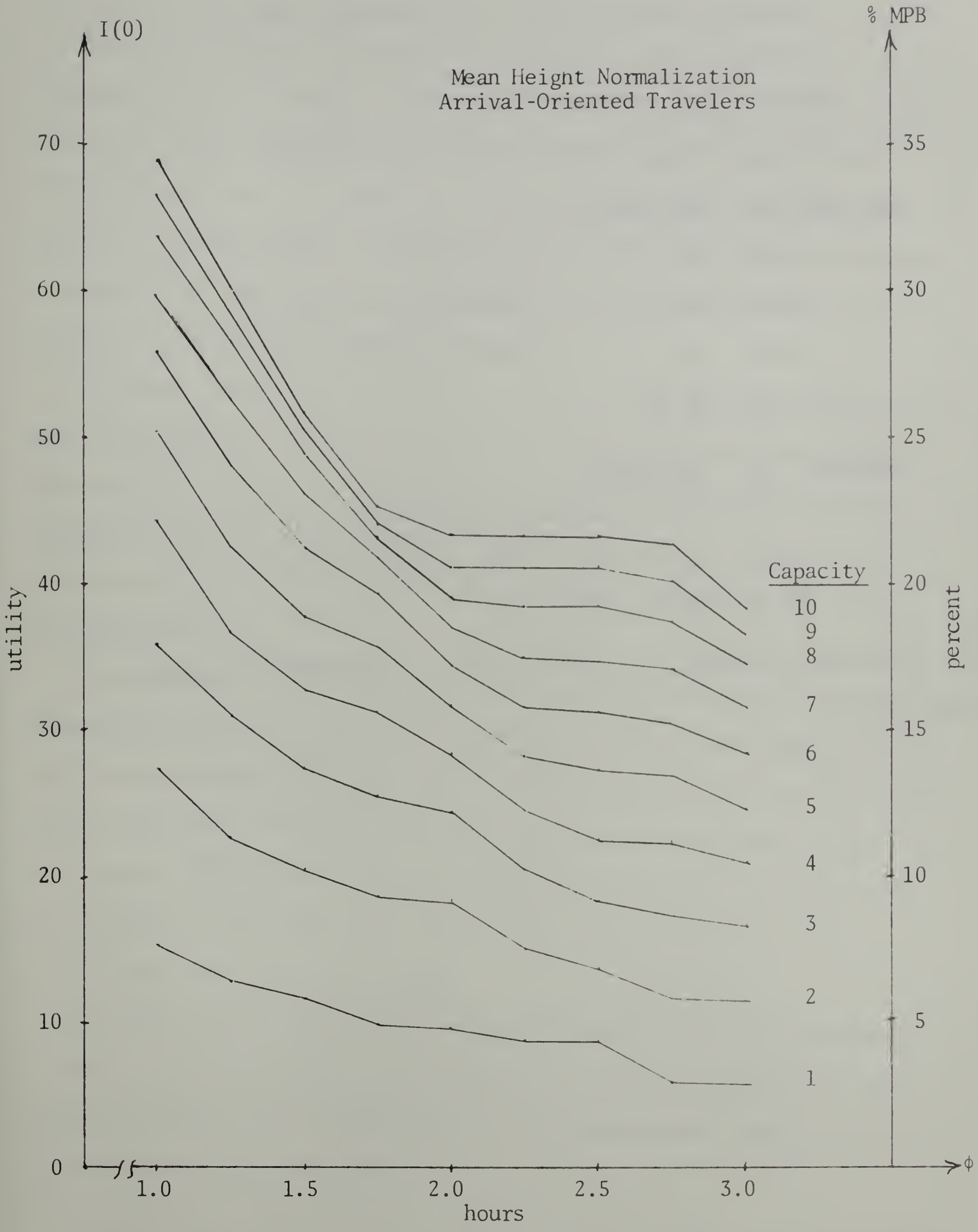


FIGURE 19. BENEFIT-TRIP TIME CURVES, UNIFORMLY ARRIVAL-ORIENTED TRAVELERS, MEAN-HEIGHT NORMALIZED

TABLE 3. SCHEDULING CHANGES AS TRIP TIME DECREASES FROM $\phi=2.25$ to $\phi=2.0$

Volume - normalization, Capacity = 10, p=0.5.

I. $\phi = 2.25$ $I(0) = 26.484$

Departure	t_d	Passengers
1	2.40	10, 14, 15, 18, 19, 20, 21, 22, 23, 25
2	9.10	48, 53, 57, 58, 59, 61, 62, 65, 66, 67
3	14.30	85, 86, 87, 88, 89, 90, 91, 92, 93, 96

II. $\phi = 2.0$ $I(0) = 25.407$

Departure	t_d	Passengers
1	2.40	10, 13, 14, 16, 17, 19, 20, 21, 22, 25
2	9.10	48, 52, 53, 57, 59, 61, 62, 65, 66, 67
3	13.35	76, 79, 81, 84, 93

Departure	Passengers at $\phi=2.25$	replaced by	Passengers at $\phi=2.0$
1	15, 18, 23	$\longrightarrow$	13, 16, 17
2	58	$\longrightarrow$	52
3	85 thru 92, 96	$\longrightarrow$	76, 79, 81, 84

Passengers 16 and 17 are departure-oriented, and are therefore eligible regardless of trip duration. However, they are "marginal" travelers whose utilities are the smallest of all those included on the trip when $\phi=2.0$. (This is verified by the fact that passenger 17 is not included on the departure when capacity is reduced to 9, and both 16 and 17 are excluded if capacity is lowered to 8.) Passenger 13 is arrival-oriented; he is eligible if $\phi=2.0$ but ineligible if $\phi=2.25$.

All other passengers on departure 1, are included for both values of ϕ. When the transition from $\phi=2.25$ to $\phi=2.0$ is made, the following occurs: Passengers 15, 18, and 23 are dropped, and replaced by the low priority travelers 16 and 17, plus the newly eligible passenger 13. The result is a net loss in utility derived from this passenger exchange on the first scheduled departure.

Consider departure 2. Passenger 58, included on the slower trip, is exchanged for passenger 52 on the faster trip. Both 52 and 58 are arrival-oriented but eligible for <u>both</u> trips. The utilities of these two travelers are such that they exchange positions of priority as ϕ changes from 2.25 to 2.0. Further, passenger 58's utility for the slower trip is greater than 52's utility for the faster one; hence a net loss in utility is again realized from the passenger exchange.

Of course, those other arrival-oriented travelers who are accommodated at either value of ϕ, such as passenger 14, 19, and 21, also exhibit net changes in utility; the overall effect is the pathological behavior of I(0) currently under discussion.

A second interesting property of the variation of I(0) with trip time can be identified clearly in Figure 18 and 19. The curves of these graphs exhibit "plateau" regions in which decreases in ϕ yield zero incremental benefit. This effect is quite separate from the previous discussion on monotonicity since the traveler populations under present discussion are uniformly departure-oriented (p=0) or uniformly arrival-oriented (p=1). The plateau merely indicates that, in certain ranges of vehicle speed capability, there is just one best schedule, the benefit of which cannot be exceeded unless a certain "threshold" trip time is achieved. This phenomenon could have interesting policy connotations with regard to decisions on vehicle capability. Although somewhat hidden in the case where $p \neq 0$ or 1, the "plateau" effect is still noticeable; see Figures 16 and 17, where p=0.5.

Variation of I(0) with Capacity. Figures 20 and 21 display I(0) versus C, for the volume-normalized and mean-height normalized cases, respectively. In both cases smooth asymptotic increasing behavior is observed. A striking difference in the range of variation between the two graphs is observed, however. A large variation in I(0) over the capacity range C=1 to C=10, is observed in the mean-height case, Figure 21 ; however, a small variation over that range occurs in the volume case, Figure 20. This behavior is attributable to the ranges of variation in the "heights" of the individual utility functions under each mode of normalization. Under volume normalization, the maximum height of any utility surface varies inversely

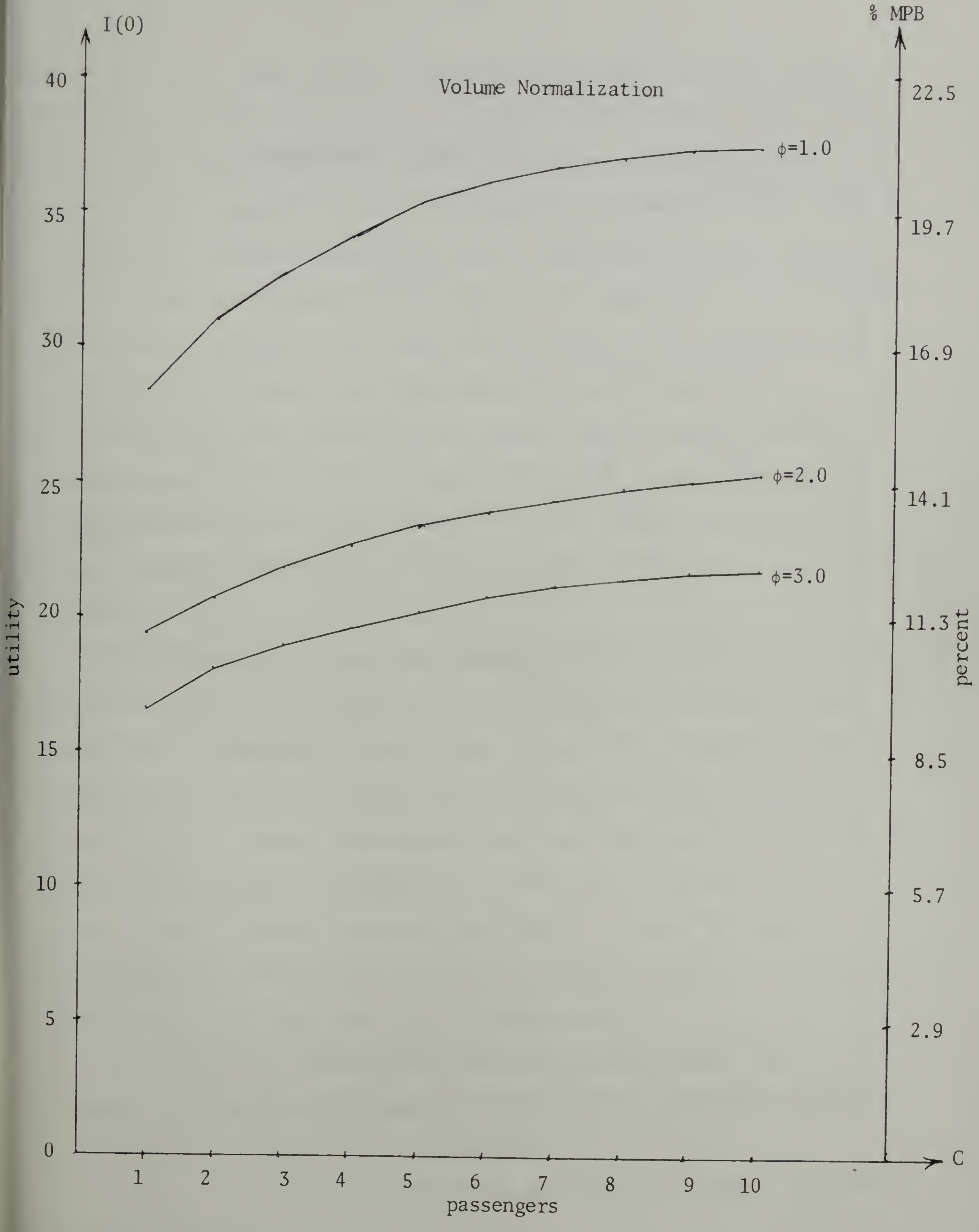


FIGURE 20. BENEFIT AS A FUNCTION OF CAPACITY

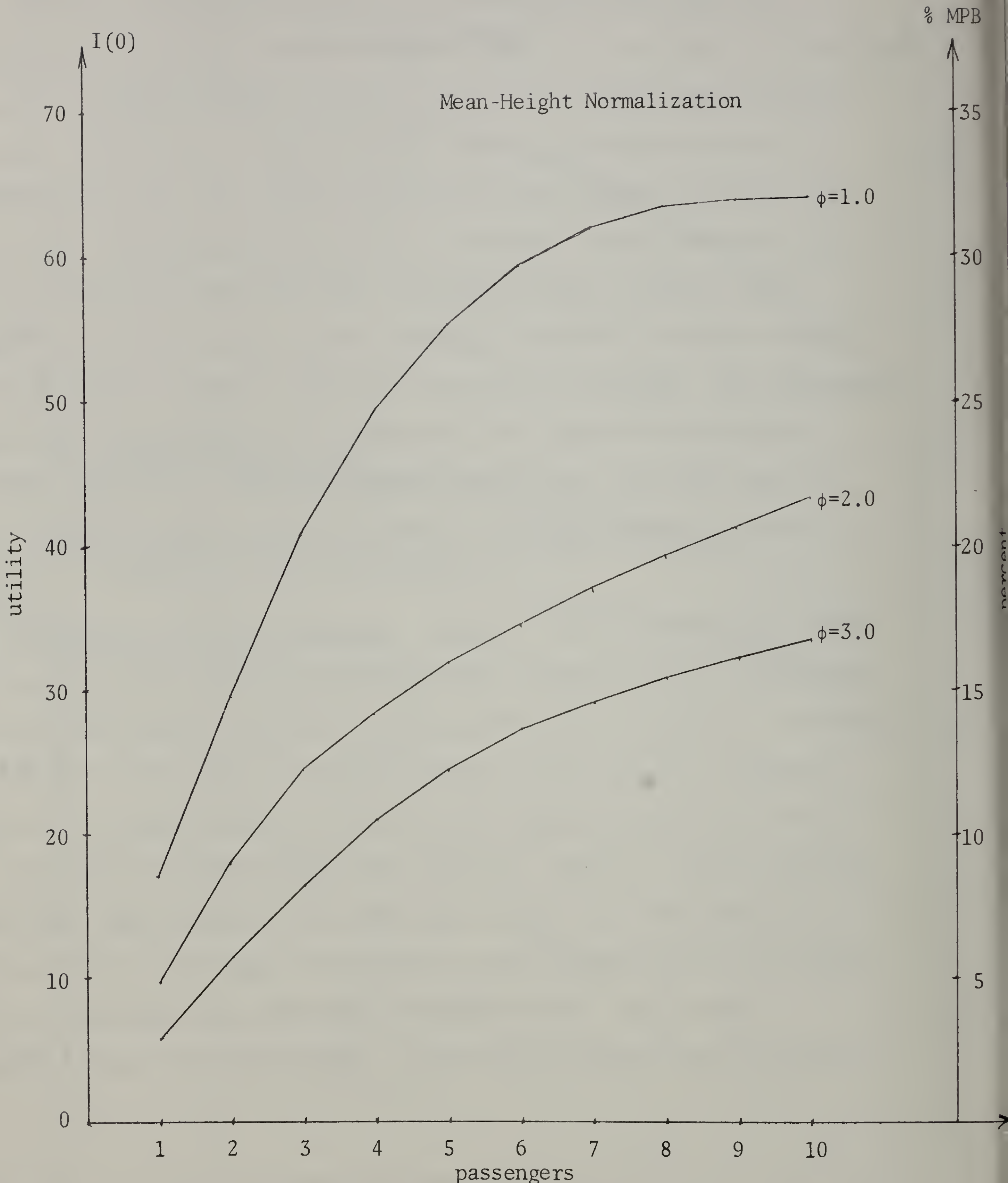


FIGURE 21. BENEFIT AS A FUNCTION OF CAPACITY

with the area of the acceptable interval; hence travelers exhibit utilities varying from those that are low in height and spread over a large area, to those that are very tall and confined to a narrow acceptable interval. As a result, the aggregate utility is composed of components of widely varying magnitude. According to the algorithm developed here, the major component utilities are accommodated first. Hence at low capacities it is probable that the major portion of the "available" utility will be collected, and that additional capacity will accommodate only minor components.

Under mean-height normalization, however, one can expect that the component utilities for all travelers eligible for a given departure are commensurate with one another. Hence it can be expected that vehicles of larger capacity will achieve substantially more benefit than ones of lesser capacity.

Trade-off of Capacity and Trip Duration. Figures 22 and 23 show the contours of constant I(0) in the plane of ϕ and C, for the volume-normalized and mean-height normalized representations, respectively. Figure 22 displays some deviate points which result from the non-monotonicity in Figure 16. Similar points occur under mean-height normalization, but not at the values of I(0) plotted in Figure 23. A comparison of these two graphs reveals that the contour lines are considerably "steeper" in Figure 22, indicating that capacity has less import relative to speed under volume normalization than under mean-height normalization.

However, the same general trend appears in both figures: the steepness of the curves decreases as I(0) decreases. This means that in

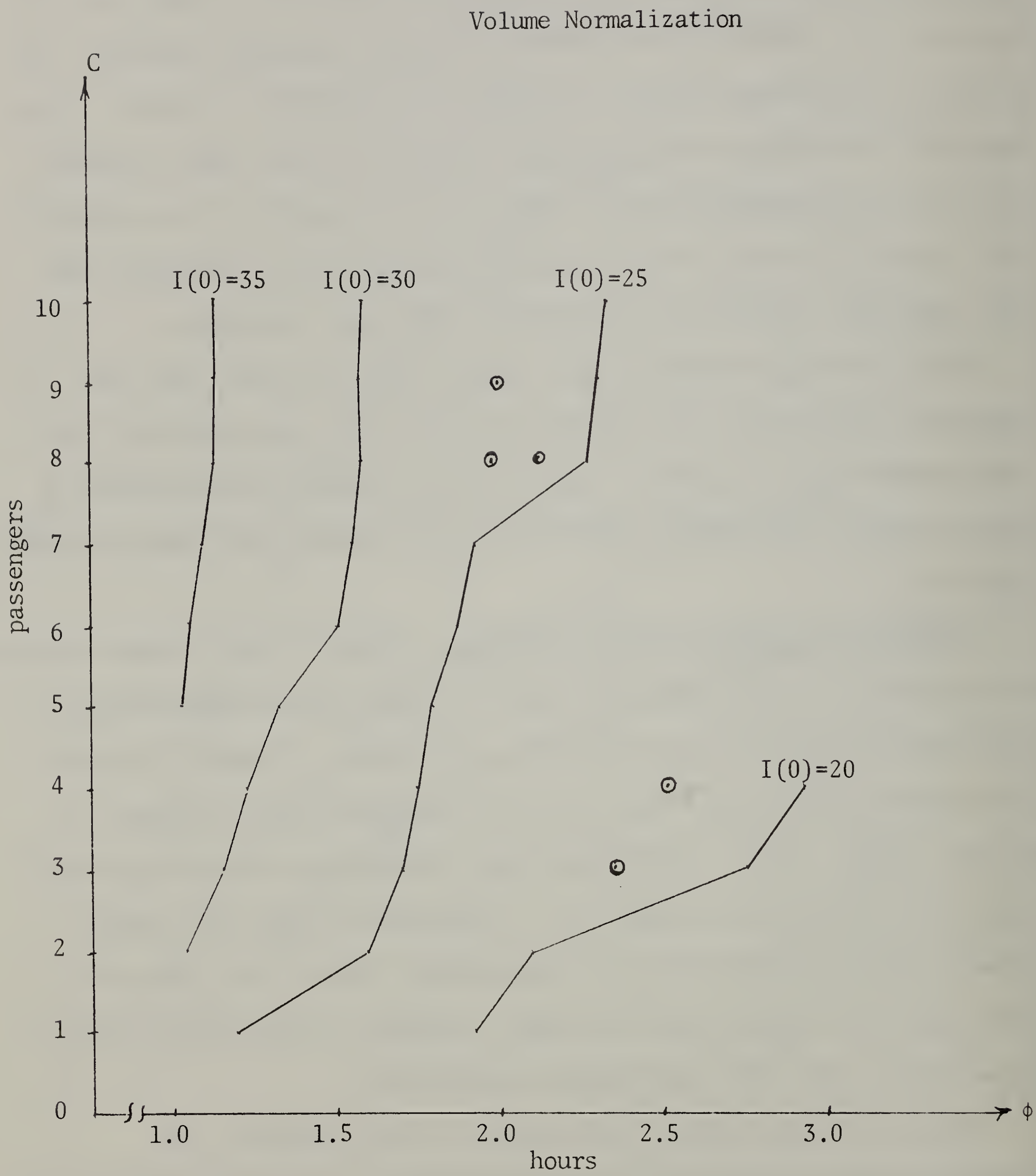


FIGURE 22. CURVES OF CONSTANT BENEFIT

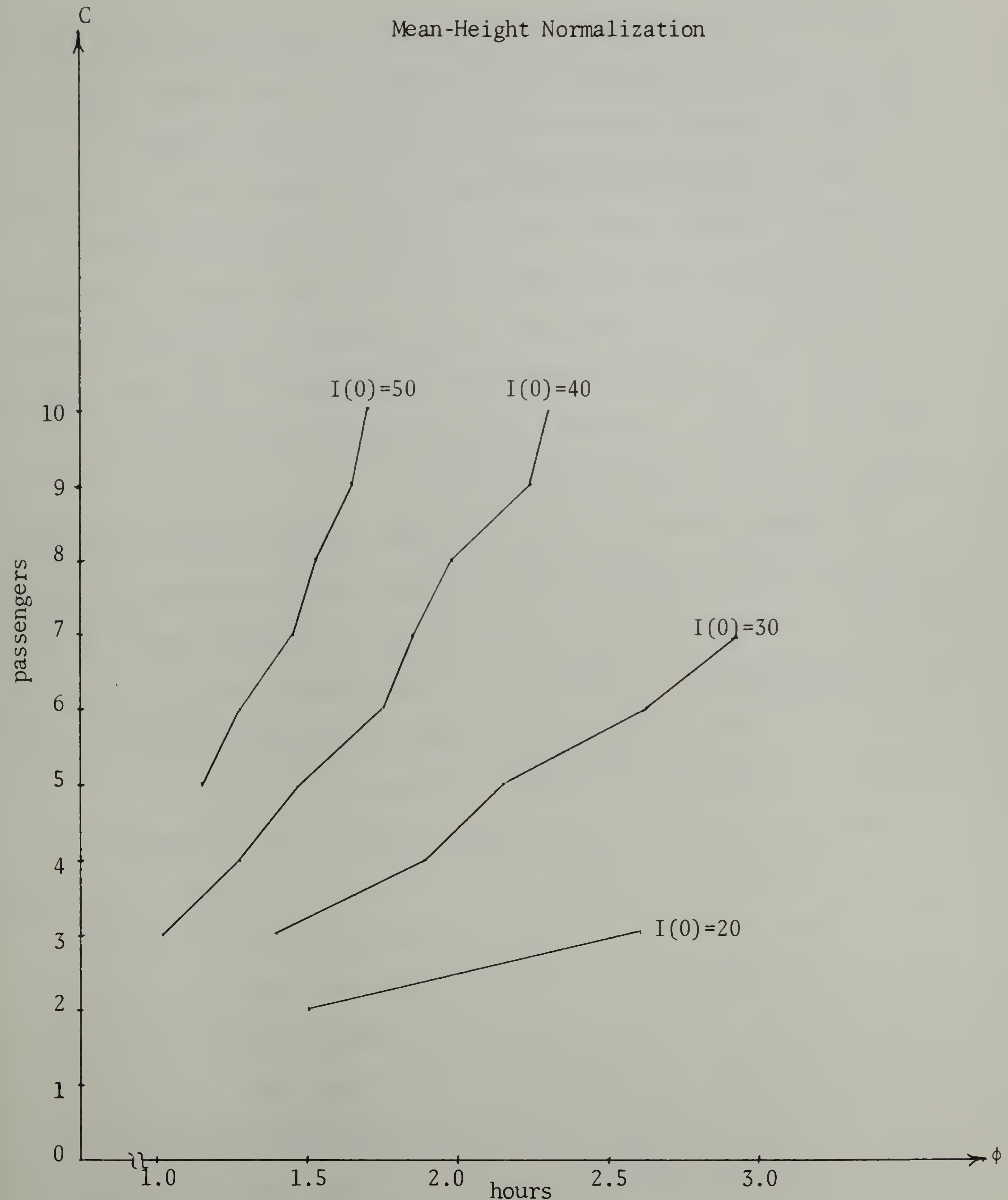


FIGURE 23. CURVES OF CONSTANT BENEFIT

a "high benefit" system more is to be gained by increasing vehicle speed, than by increasing capacity. To the contrary, in "low-benefit" systems, increases in capacity are more valuable than increases in speed.

Figures 24 and 25 exhibit the contours of constant I(0) for uniformly oriented populations; Figure 24 corresponds to the departure-oriented, volume normalized population (p=0) of Figure 18, and Figure 25 corresponds to the arrival-oriented (p=1), mean height normalized population of Figure 19. These figures are not plagued with deviate points of the kind shown in Figure 22, because non-monotonic variations of I(0) with ϕ do not occur for uniformly-oriented populations.

The contour I(0)=21.73 of Figure 24 does, however, exhibit some rather strange behavior as a result of the "plateau" effect discussed previously. The range of points from ϕ=2.25 through ϕ=2.75, at capacity value C=5, all lie on the same contour, I(0)=21.73. The same behavior occurs at other values of I(0) (not plotted) in both Figure 24 and Figure 25, and is merely another manifestation of the plateaus of Figures 18 and 19.

Variation of Schedules with ϕ and C. Figures 26 and 27 exhibit sets of optimal schedules, for vehicle capacity C=5, for the volume and mean-height normalizations, respectively. Each line segment represents a scheduled departure in the optimal schedule for the particular value of ϕ shown at the extreme right. The digit printed to the immediate right of each line segment is the number of passengers included on that trip.

In Figure 26, the volume-normalized case, one characteristic of the schedules--their stability--is particularly striking. Over broad ranges

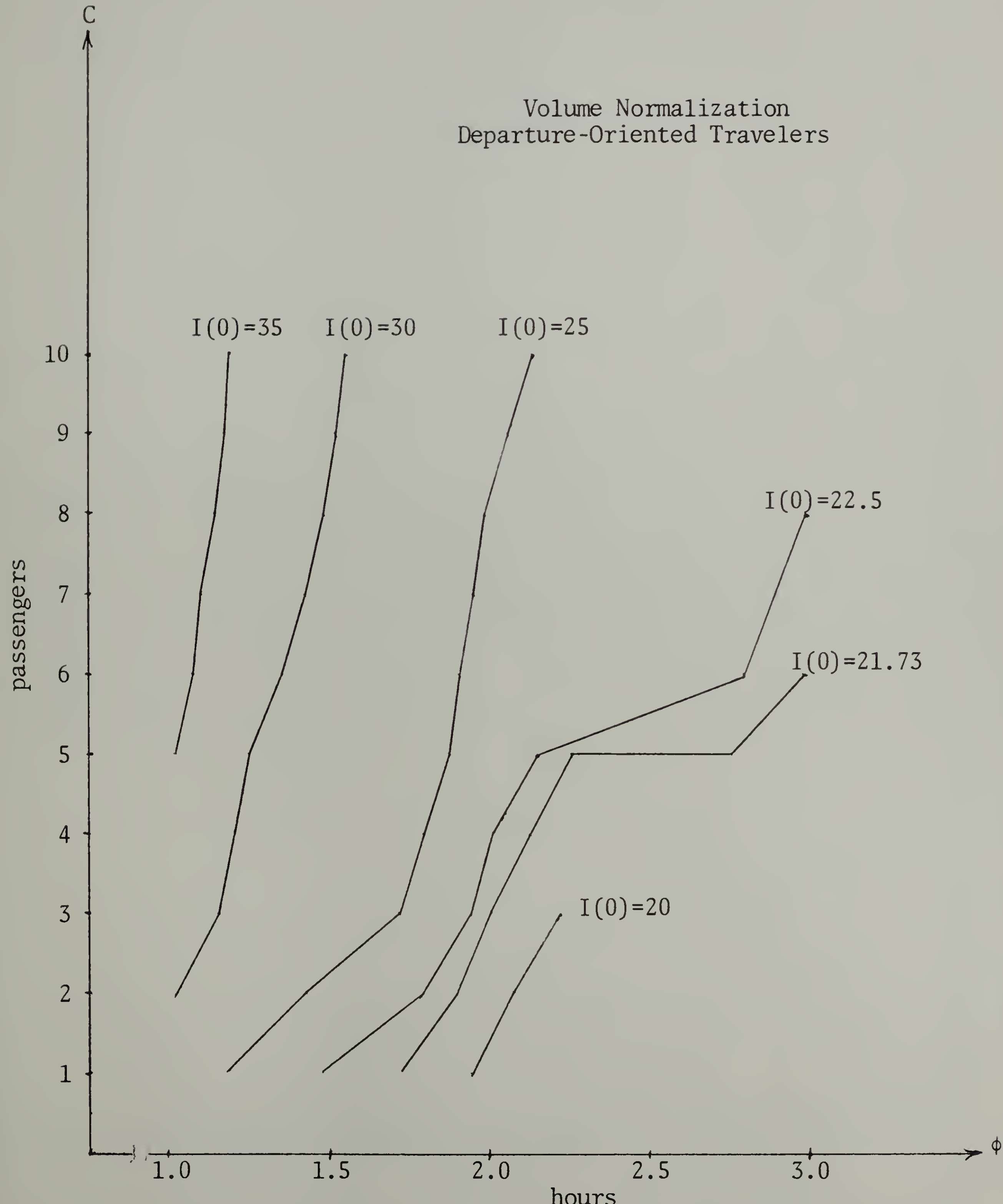


FIGURE 24. CURVES OF CONSTANT BENEFIT, UNIFORM TRAVELER ORIENTATION

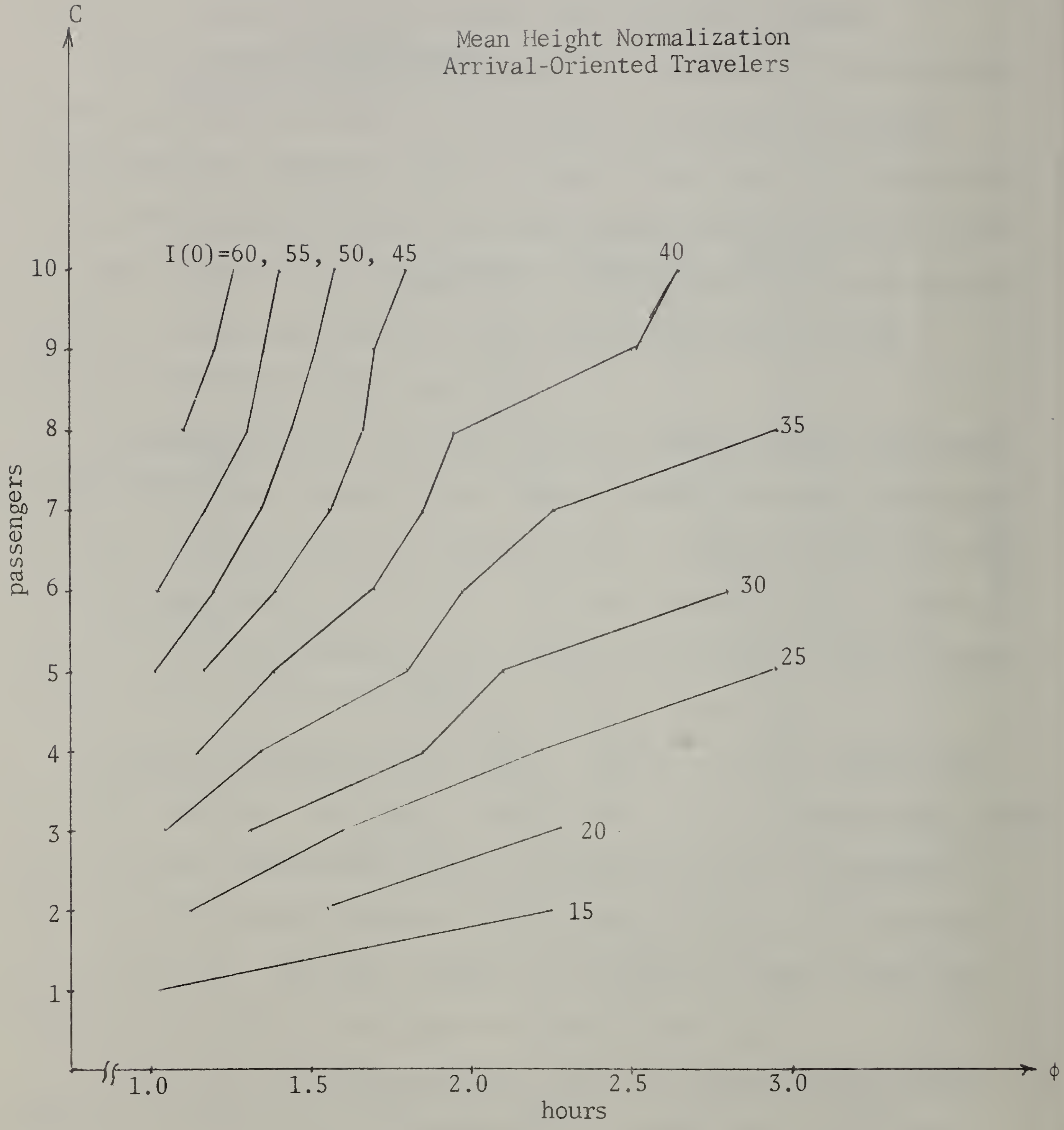


FIGURE 25. CURVES OF CONSTANT BENEFIT, UNIFORM TRAVELER ORIENTATION

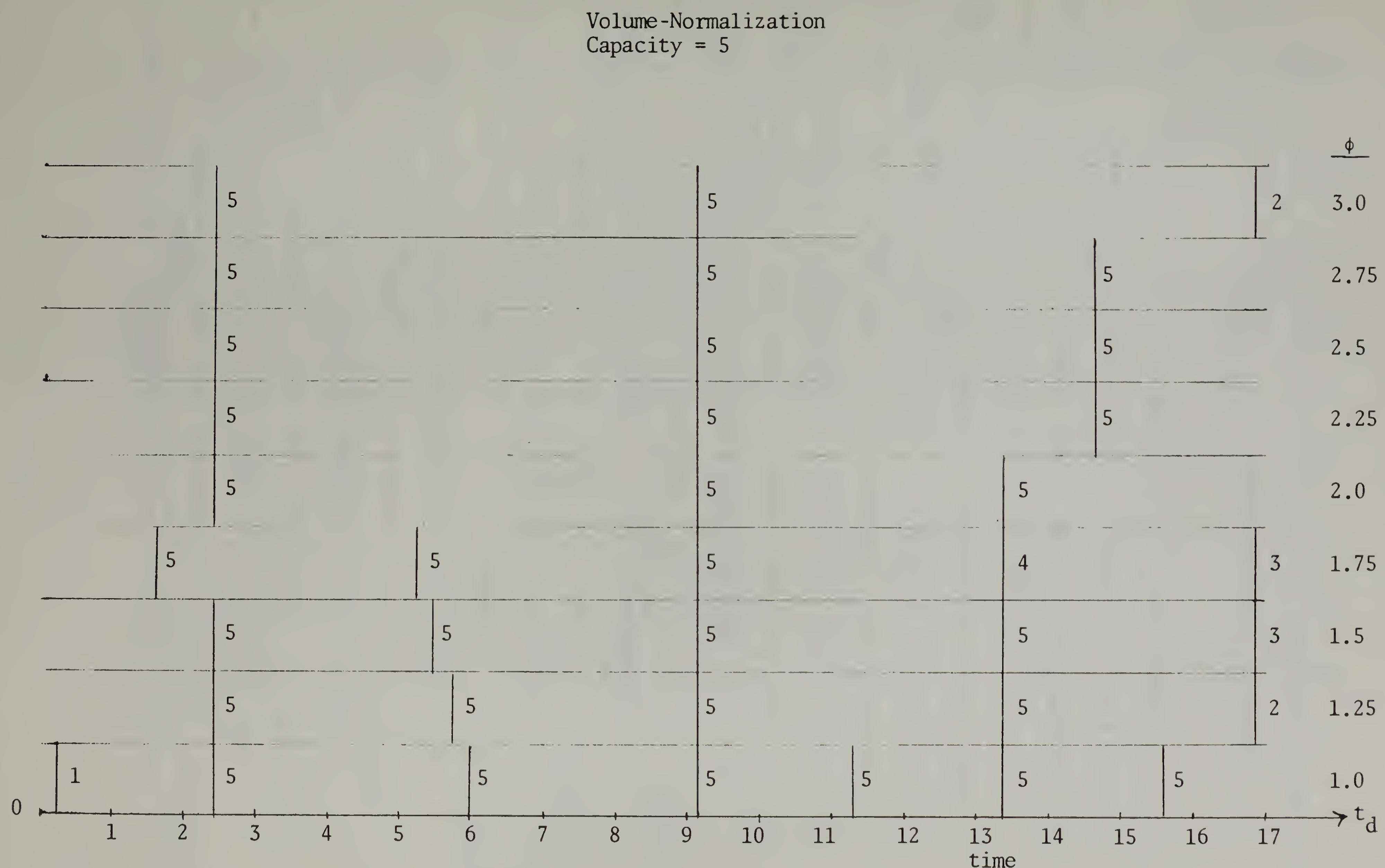


FIGURE 26. OPTIMAL SCHEDULES AS TRIP LENGTH INCREASES

Mean-Height Normalization
Capacity = 5

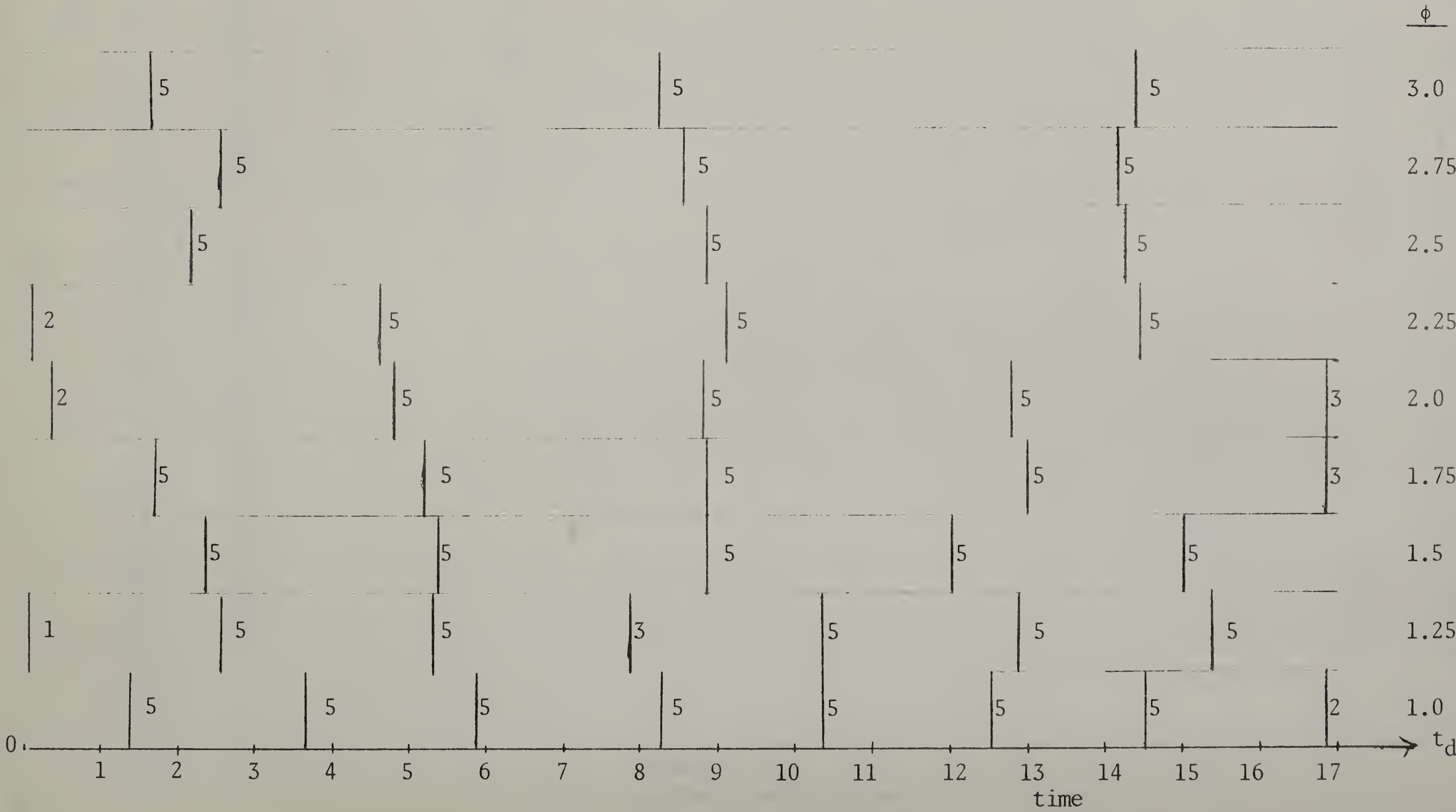


FIGURE 27. OPTIMAL SCHEDULES AS TRIP LENGTH INCREASES

of ϕ, the schedules remain relatively fixed; further, particular departures, such as the ones at t_d=2.40 and t_d=9.10 remain unchanged throughout even larger ranges.

This behavior is explained by the fact that, under volume-normalization, the benefit derived by a schedule is made up substantially of a few major components. The trips in a schedule tend to center around accommodation of a few high utility (narrow-acceptance interval) travelers. Hence the schedule remains unaffected by changes in ϕ unless such changes affect accommodation of these major components. For example, the departure at t_d=2.40 is tailored to the preference of traveler 25, while departure at t_d=9.10 accommodates the preferences of traveler 66. Both 25 and 66 are departure-oriented, with very narrow acceptance intervals centered at the above mentioned departures. On the other hand, passenger 33 is also a high utility (narrow interval) traveler, but is arrival-oriented. Traveler 33 provides an example of how a change in ϕ affects accommodation of a major utility component, which in turn affects the schedules. Traveler 33 supplies the principal utility contribution toward the departure at t_d=5.95, in the case where ϕ=1.0 in Figure 26. As ϕ is incremented in steps of 0.25, up to ϕ=1.75, the corresponding departure shifts to the left on the departure time axis, in steps of 0.25, so that traveler 33's preferred arrival time is adhered to. Another situation in which a change in ϕ is likely to affect the schedule is when such change permits accommodation of either an extra, or a different, more "beneficial", principal component utility. (A more beneficial principal component is one which is located at a preferred time at which the <u>aggregate</u> utility is greater.) Examples of such situations occur at the transitions from $\phi=2.25$ to $\phi=2.0$, and from $\phi=2.0$ to $\phi=1.75$. In the former instance, passenger 95 is replaced by the more "beneficial" passenger 84. In the latter instance, major utility component traveler 99 is added to the schedule.

In contrast to the volume case, the mean-height normalized case of Figure 27 exhibits considerably less regularity. The large discrepancy in the magnitudes of the individual utility components which comprise the benefit derived by the schedules under volume normalization, does not exist here. Hence the stability phenomenon does not occur.

Figures 28 and 29 exhibit the sets of optimal schedules obtained by varying the capacity from 1 to 10, at $\phi=2.0$, in the volume and mean-height normalizations, respectively. Notice that under volume normalization, the schedule remains completely unchanged throughout the entire range! On the contrary, there is considerable fluctuation in Figure 29. This is just a further illustration of the contrasting stability effects of the alternate normalization procedures.

It is interesting that, under mean height normalization at least, similar changes occur in the schedules from increasing capacity <u>or</u> from decreasing speed. This can be seen by comparing Figures 27 and 29. Increased capacity tends to favor schedules with fewer trips, at times preferable to many travelers. Decreases in vehicle speed (increases in ϕ), although degrading with respect to overall benefit attained, tend to have the same effect on the schedules simply because the vehicle becomes less capable of making many trips in the specified time period.

Volume Normalization
$\phi=2.0$

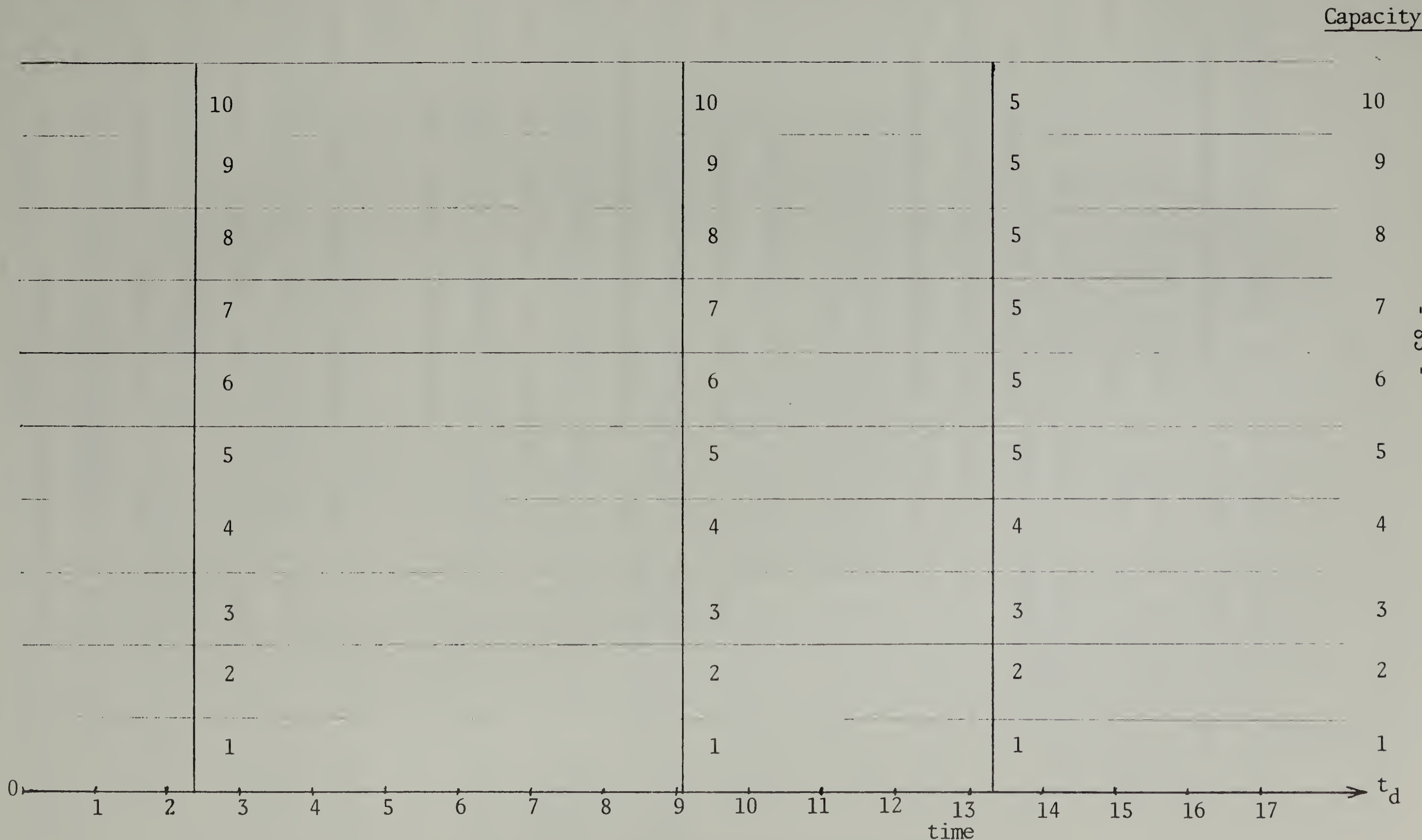
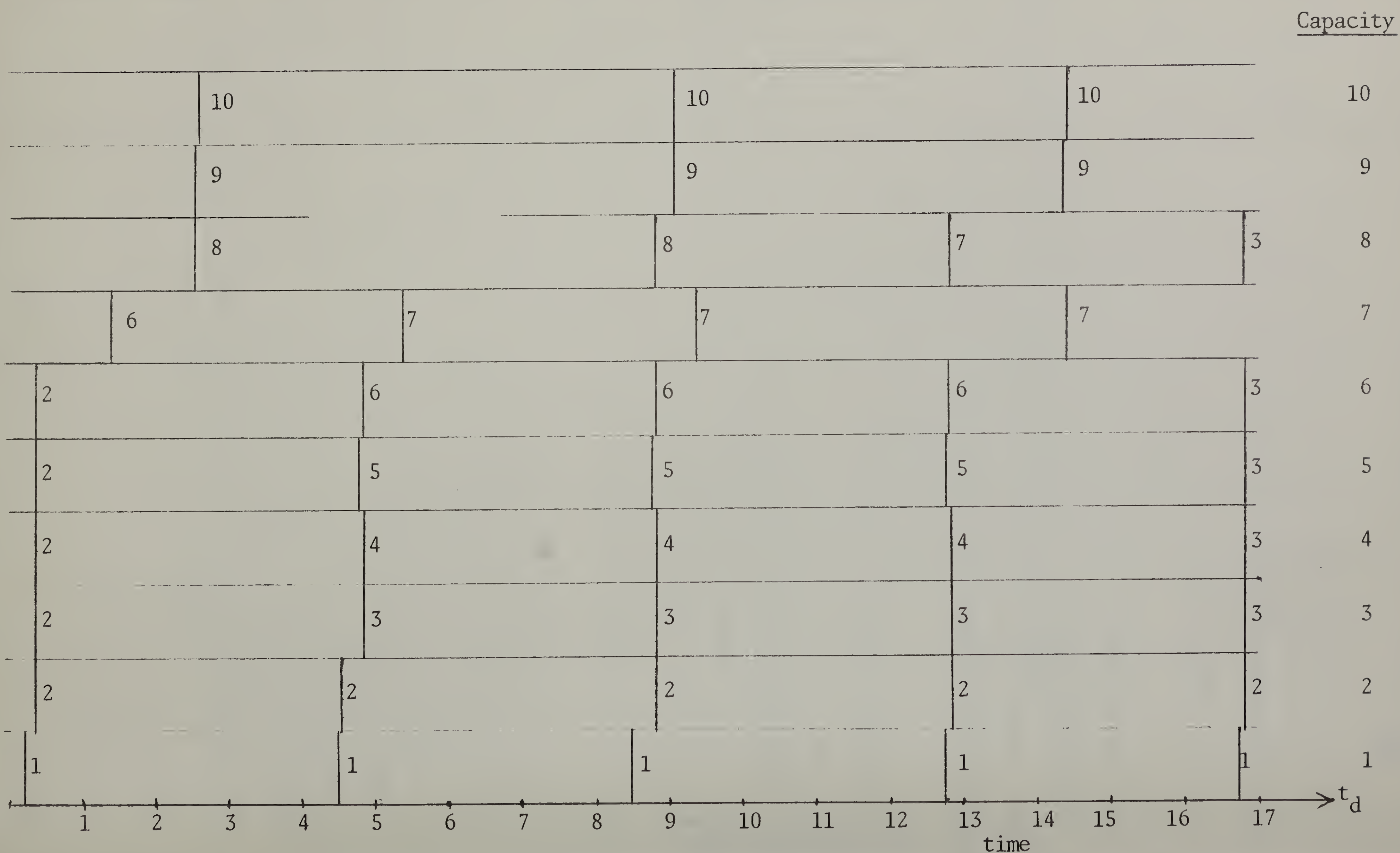


FIGURE 28. OPTIMAL SCHEDULES AS CAPACITY INCREASES

Mean-Height Normalization
ϕ=2.0



IGURE 29. OPTIMAL SCHEDULES AS CAPACITY INCREASES

Variation with σ_{MAX}. Figure 30 exhibits the variation of I(0) with parameter σ_{MAX}, under the alternate normalizations. Recall that σ_{MAX} is the upper limit of the uniform distribution from which the value of $\sigma_d^{(i)}$ is picked if $w^{(i)}=1$, or $\sigma_a^{(i)}$ is picked if $w^{(i)}=0$. From the mean-height curve the absolute value of I(0) may be read off the left-hand ordinate, and I(0) as a percent of MPB may be read off the right-hand ordinate. For volume normalization there are two curves because the value of the MPB varies with σ_{MAX}. The solid curve shows the behavior of the absolute value of I(0) (read from the left-hand ordinate) while the dotted curve exhibits I(0) as a percent of MPB (read from the right hand ordinate).

The solid volume normalization curve in Figure 30 reflects the fact that under volume-normalization, the benefit is principally derived from relatively few "tall and narrow" component utility surfaces. Thus, as σ_{MAX} is increased, the heights of the principal utility components decrease, and so does the overall (absolute) benefit. However, the dotted curve and the mean-height curve demonstrate that relatively greater benefit can be achieved by increasing σ_{MAX}, since widening of the intervals over which travelers are willing to travel, enables the accommodation of a larger number of passengers.

Figures 31 and 32 show the effects of changes in σ_{MAX} on the schedules, under both normalizations. Before analyzing these figures it is important to note the manner in which the traveler half-width parameters were modified during the computer runs (see Figure 14).

For each traveler (i) the pertinent half-width parameter ($\sigma_d^{(i)}$ if $w^{(i)}=1$, $\sigma_a^{(i)}$ if $w^{(i)}=0$) was initially generated as a random number between zero and $\sigma_{MAX}=0.2$. Following each complete scheduling calculation,

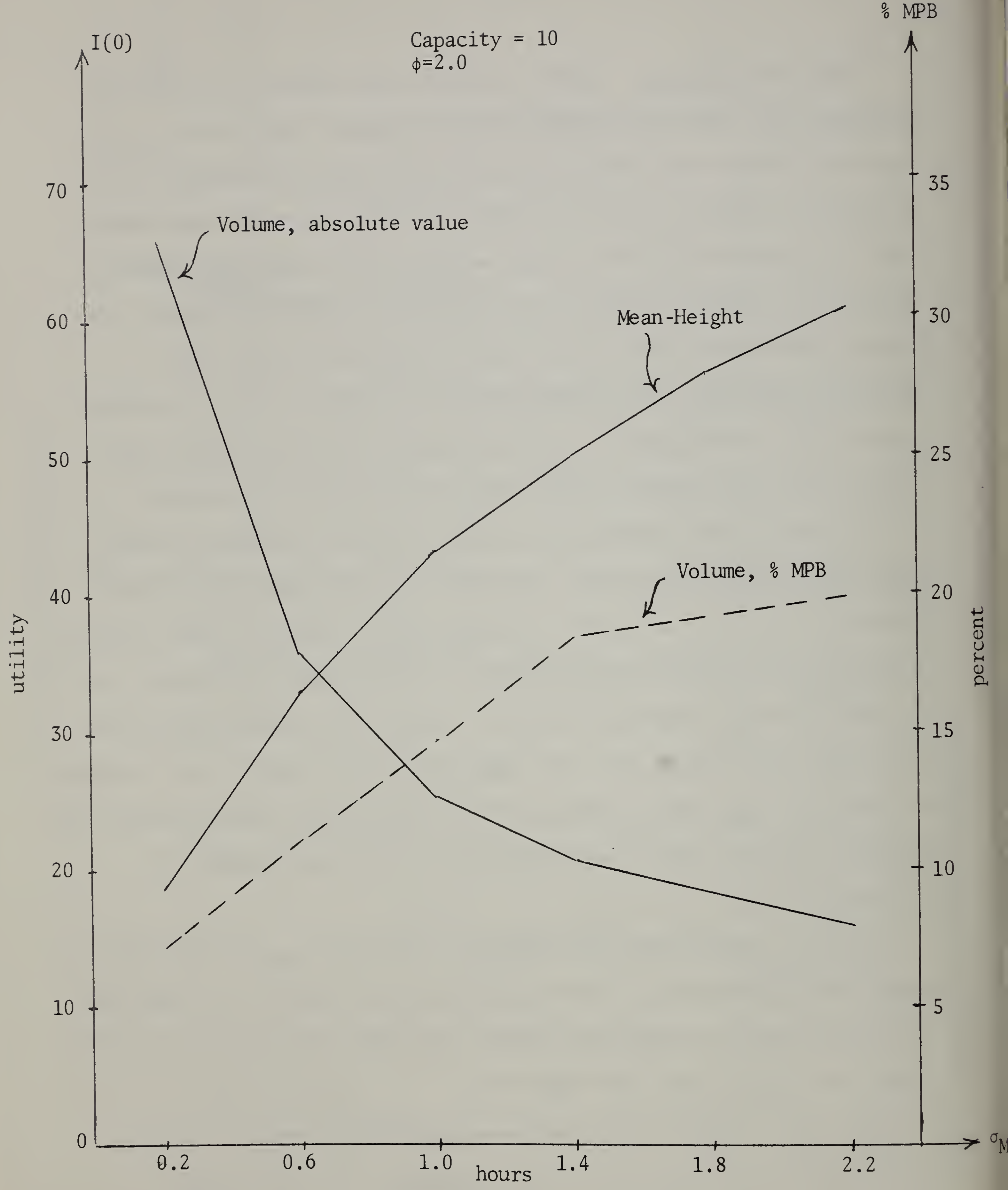


FIGURE 30. VARIATION OF BENEFIT WITH INTERVAL HALF-WIDTHS

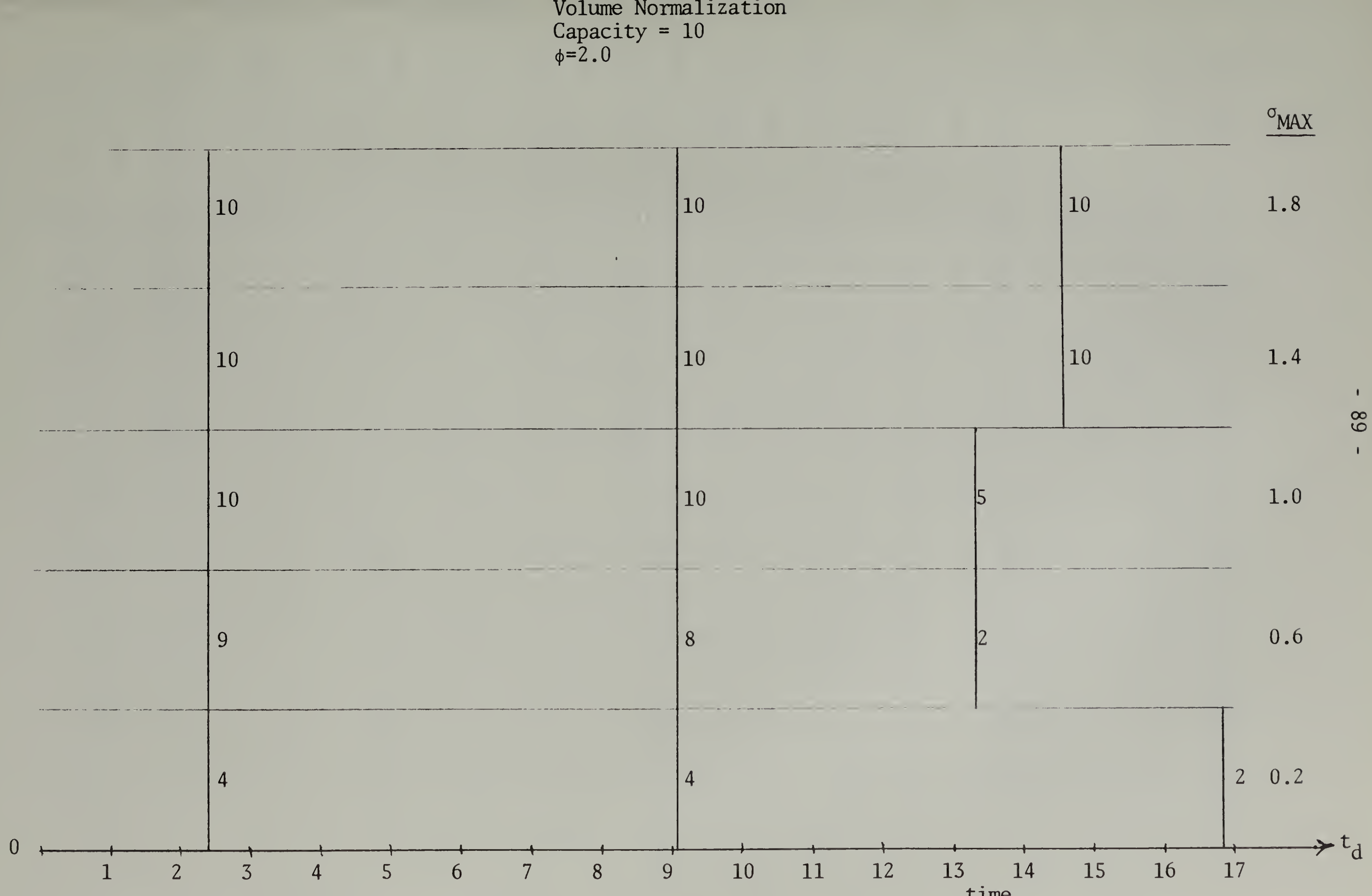


FIGURE 31. OPTIMAL SCHEDULES AS σ_{MAX} INCREASES

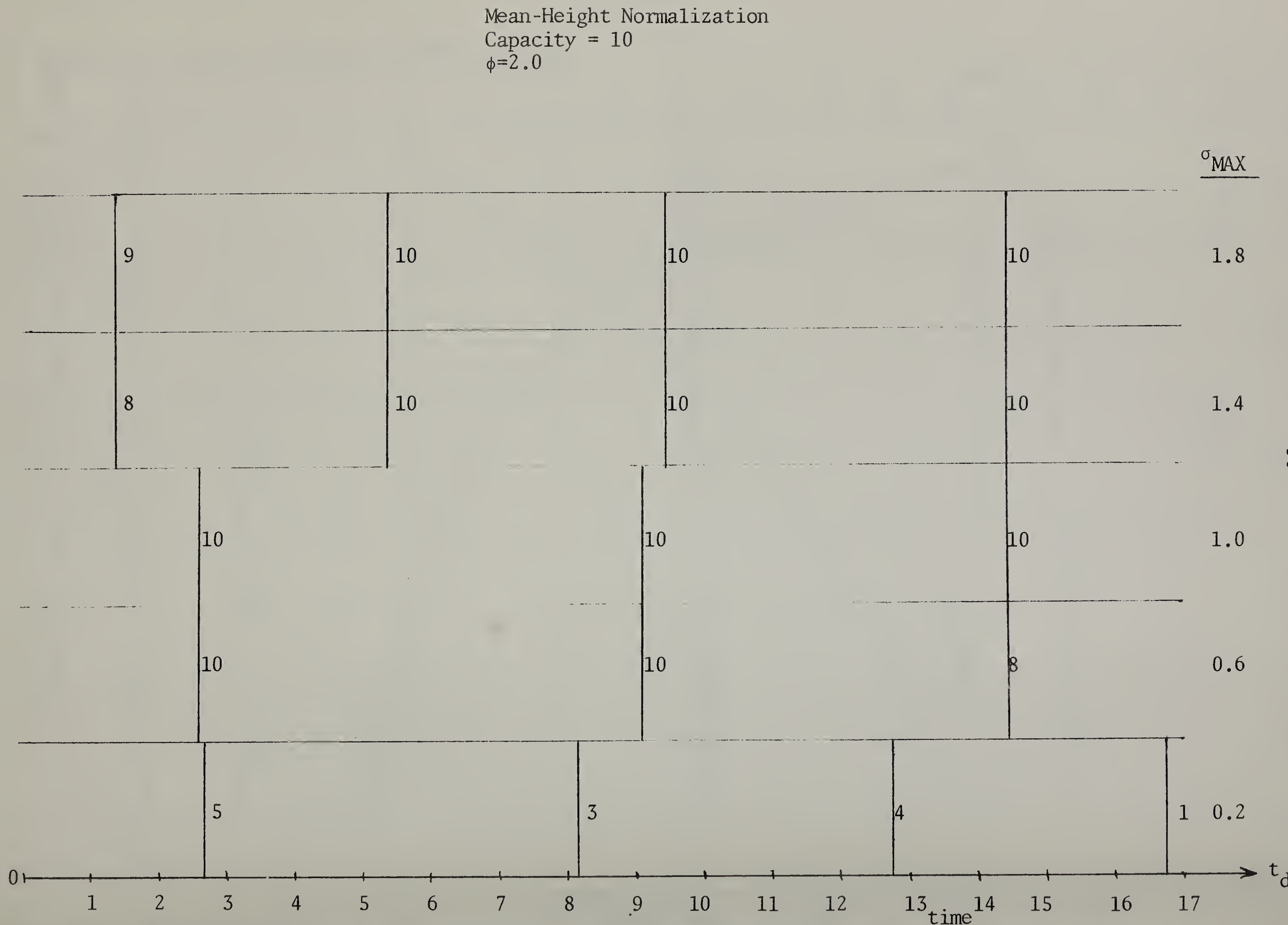


FIGURE 32. OPTIMAL SCHEDULES AS σ_{MAX} INCREASES

σ_{MAX} was incremented by 0.4; accordingly, the appropriate half-width parameters were multiplied by the expansion factor,

$$e = \frac{\text{new value of } \sigma_{MAX}}{\text{previous value of } \sigma_{MAX}} . \qquad (8.12)$$

Hence, the $\sigma^{(i)}$ values for the travelers become distributed between zero and the new value of σ_{MAX}, but the travelers maintain their relative preferences with respect to one another.

In Figure 31, the volume normalized case, except for the third departure, the schedule is unaffected by varying σ_{MAX}. The shift in departure 3 points up the way in which the variation in the $\sigma^{(i)}$ can affect the schedules. It has been noted before that, under volume normalization, the schedule tends to center about a few major utility components. In the third departures of Figure 31, there are several such components; which is "chosen" depends on the value of σ_{MAX} largely as a result of the degree of precision with which the time axis is divided into stages. With $\underline{\overline{K}}$=340 in the current runs (Δt=3 minutes), the departure decision can be considered only at each 3 minute demarcation point. At small values of $\sigma_{MAX}/\Delta t$ it is entirely possible that some very tall, narrow utility components fall entirely or almost entirely between successive grid points on the time axis, and are, therefore, overlooked in the calculation..

This is the case with travelers 95, 84, and 99. With σ_{MAX}=0.2, departure 3 falls at the preferred time for traveler 99, although the maximum utilities of travelers 84 and 95 each exceed 99's utility. However, the utilities of 84 and 95 fall between grid points. When σ_{MAX} is increased to 0.6, traveler 84 replaces 99 as the principal component in the third departure, which is now rescheduled for 84's preference.

Although traveler 95's utility exceeds 84's, it still lies substantially between grid points until σ_{MAX} is augmented to 1.4, at which point the schedule switches to 95's departure preference.

In Figure 32, the mean-height case, variation of σ_{MAX} causes somewhat more shifting than in Figure 31. The primary reason for the modifications here is the increased benefit that results from accommodating the greater number of travelers that become eligible for departures at various points, as a result of increased σ_{MAX}.

Variation of I(0) with p. Figure 33 exhibits the random-like variations of I(0) (expressed in percent of MPB) as the binomial parameter p, which determines the mixture of departure and arrival orientations in the traveler population, is varied from 0 to 1. However, the results of Figure 33 may not entirely be the result of changes in p since it will be recalled from Figure 14 that the generation of the $\sigma_d^{(i)}$ and $\sigma_a^{(i)}$ parameters is performed subsequent to the generation of the $w^{(i)}$ parameters. Thus, after each increment in the value of p in the computer runs, new values of $w^{(i)}$ and $\sigma^{(i)}$ were generated for each traveler. The fact that a different set of $\sigma^{(i)}$, generated from a common distribution, was used at each value of p may have affected the results despite the fact that benefit is plotted as a percent of MPB in the figure, to "normalize" the benefits attained at different values of p.

The random behavior of the curves in Figure 33 is reasonable, however; there is no a priori reason to expect a functional relationship between p and I(0). The greater range of fluctuation under volume normalization is attributable to the fact that the changes which occur in the magnitude

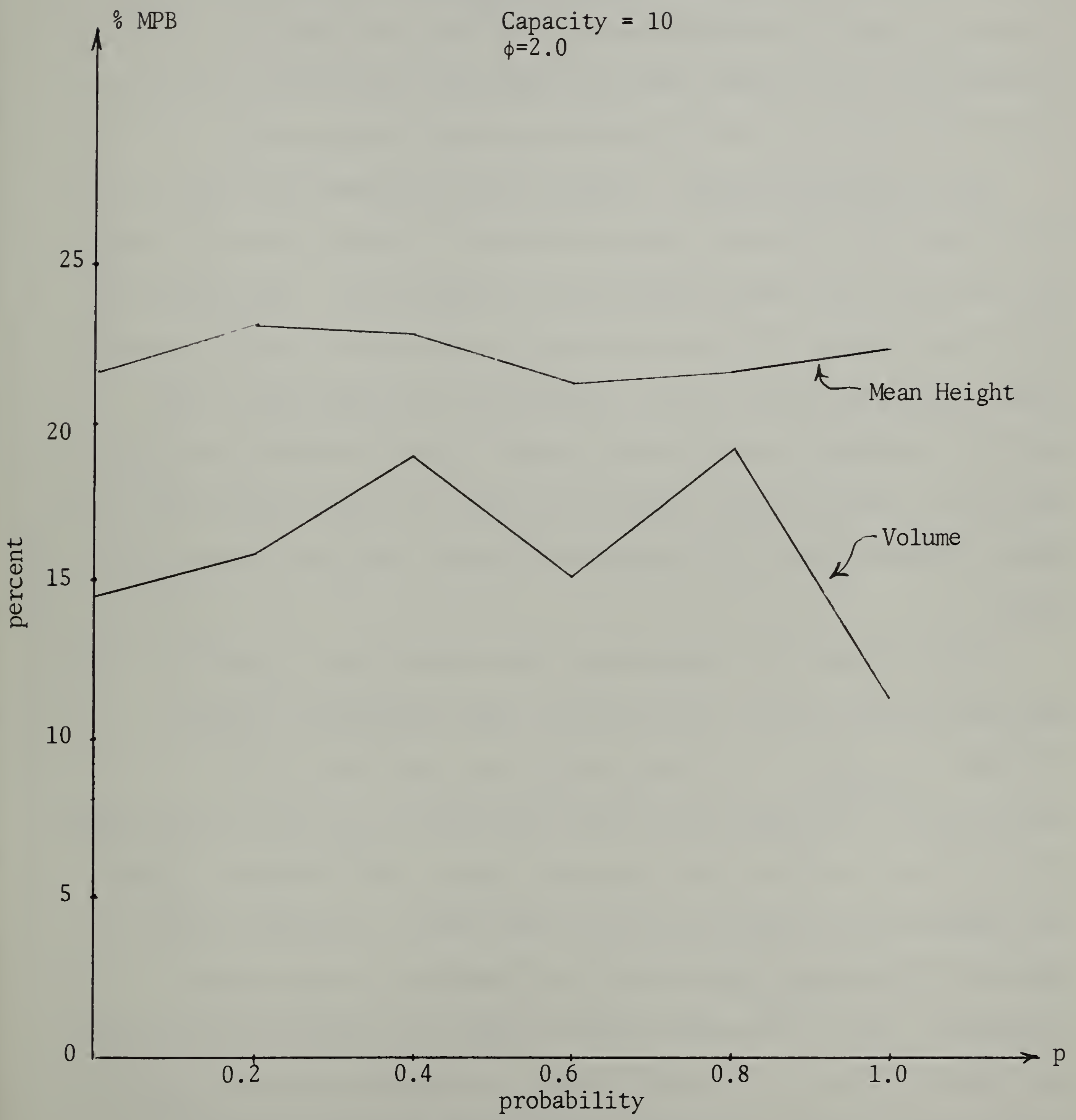


FIGURE 33. VARIATION OF BENEFIT WITH BINOMIAL PARAMETER p

and distribution of the principal utility components as a result of changing p (via the parameter generating mechanism of Figure 14) can be expected to be more radical than the changes that take place in the overall distribution of utility under mean-height normalization.

Use of Uniform Utilities. Figures 34 and 35 compare the curve of I(0) versus trip duration ϕ, using uniform and triangular utilities, under the volume and mean-height normalizations, respectively. The ordinate in each figure measures I(0) as a percentage of the maximum possible benefit (MPB). This facilitates a better comparison of the magnitudes of the benefits resulting from each utility scheme since the MPB for triangular utilities is twice that for uniform utilities; the latter results from having normalized both the triangular and uniform functions to the same values.

The two figures show contrasting effects that result, under the different normalizations, from switching from triangular to uniform utilities. In Figure 35, the mean-height case, the substitution of uniform utility functions has a "smoothing" effect; the curve is transformed into one that is monotonic and reasonably smooth. The phenomenon whereby the immediate benefit function b(1,K) varies with ϕ, discussed earlier, is apparently mitigated by eliminating the "peakedness" of the utility components. This is reasonable since a change in ϕ, under uniform utility, will have no effect on a traveler's value for departure at some stage K unless the variation is sufficient to cause that traveler to become

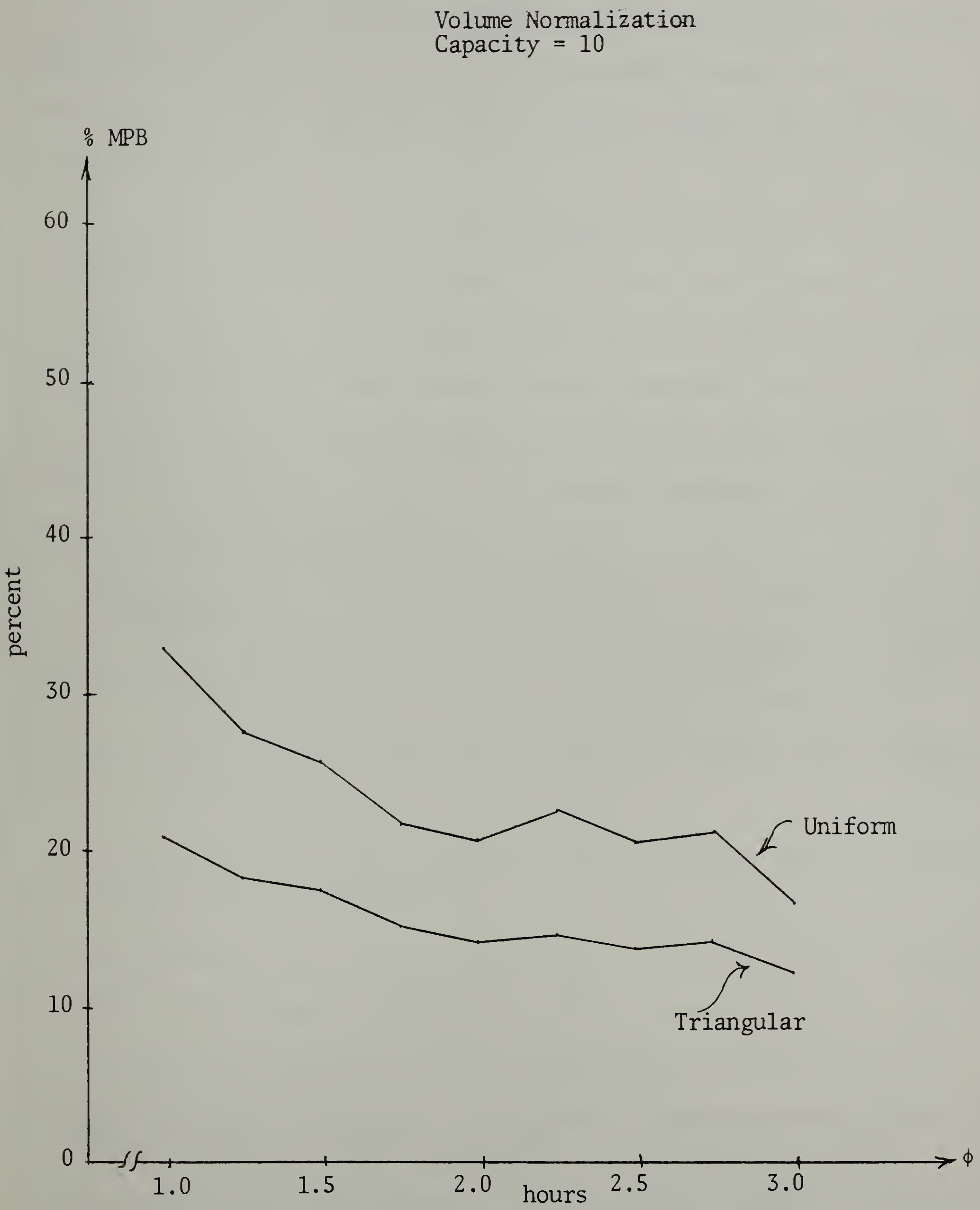


FIGURE 34. BENEFIT VARIATION WITH ϕ, UNIFORM AND TRIANGULAR UTILITIES

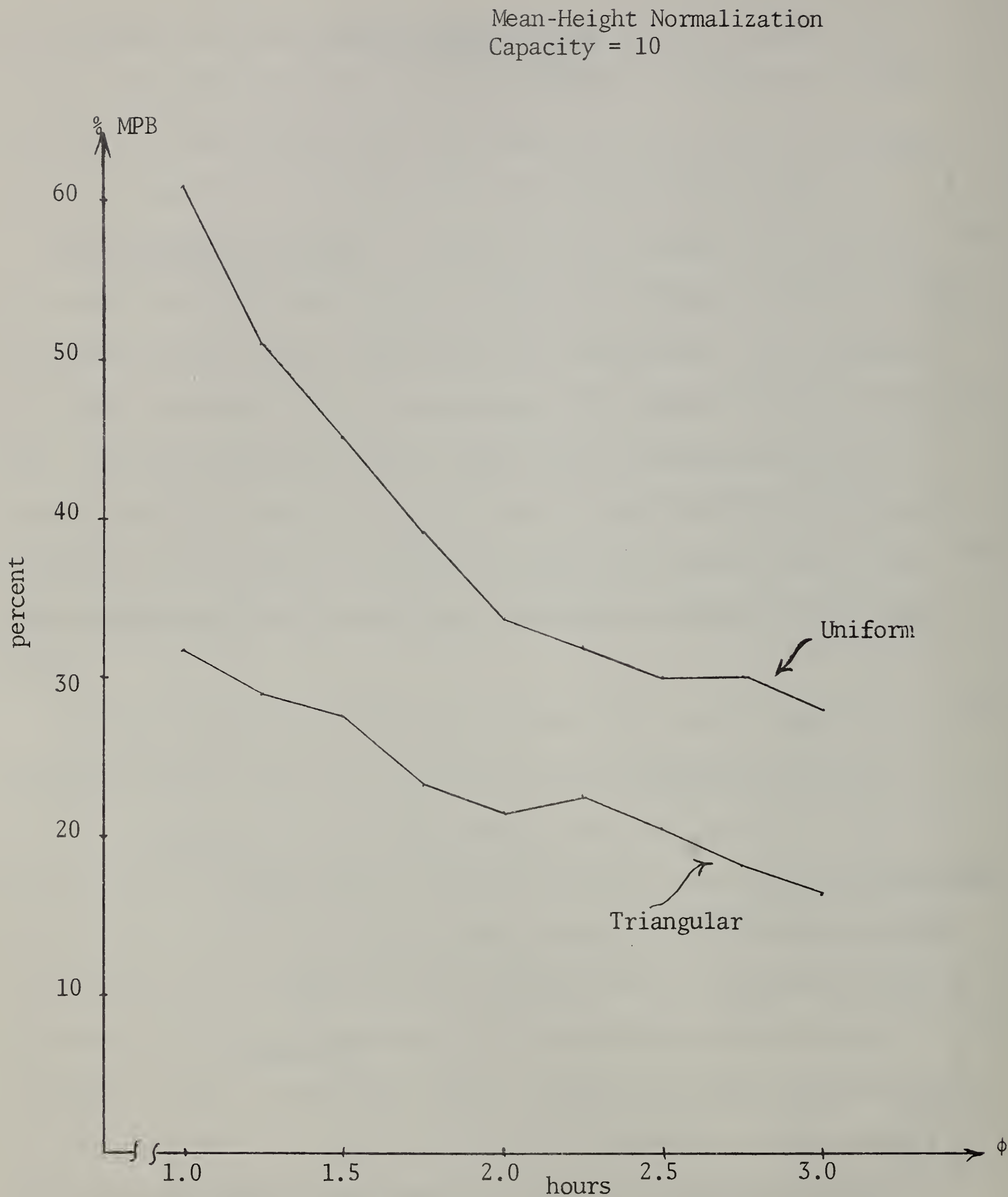


FIGURE 35. BENEFIT VARIATION WITH ϕ, UNIFORM AND TRIANGULAR UTILITIES

ineligible at that stage (or to become eligible if he were not). Further, under mean-height uniform utility, if a passenger becomes ineligible at some K, he is likely to be replaced by another traveler, of equal utility, who becomes eligible.

However, in Figure 34, the volume-normalized case, the behavior of I(0) versus ϕ with uniform utilities is very much like the behavior with triangular functions. Here, although they are non-peaked, the rectangular utility components still have widely varying magnitudes. Hence, the modification in b(1, K) which results from varying ϕ, via the change in eligibility of the individual travelers, remains substantially as it was under the triangular formulation.

Figures 36 and 37 show comparisons of the optimal schedules obtained with the two utility shapes, for the alternate normalizations. In each figure the line segments below the axes represent the schedules under uniform utility, while the segments above the axes represent the triangular utility schedules.

It will be observed that, under both normalizations, there is considerable similarity between the schedules obtained using rectangular and triangular formulations. However, the nature of the deviations between the uniform and triangular schedules, is different under different normalizations. In Figure 36 (volume normalization), the scheduled departures for the uniform and triangular cases frequently coincide, but when they do not coincide they are usually substantially apart. For example, at ϕ=1.0, the third, fourth, fifth, sixth, and seventh departures

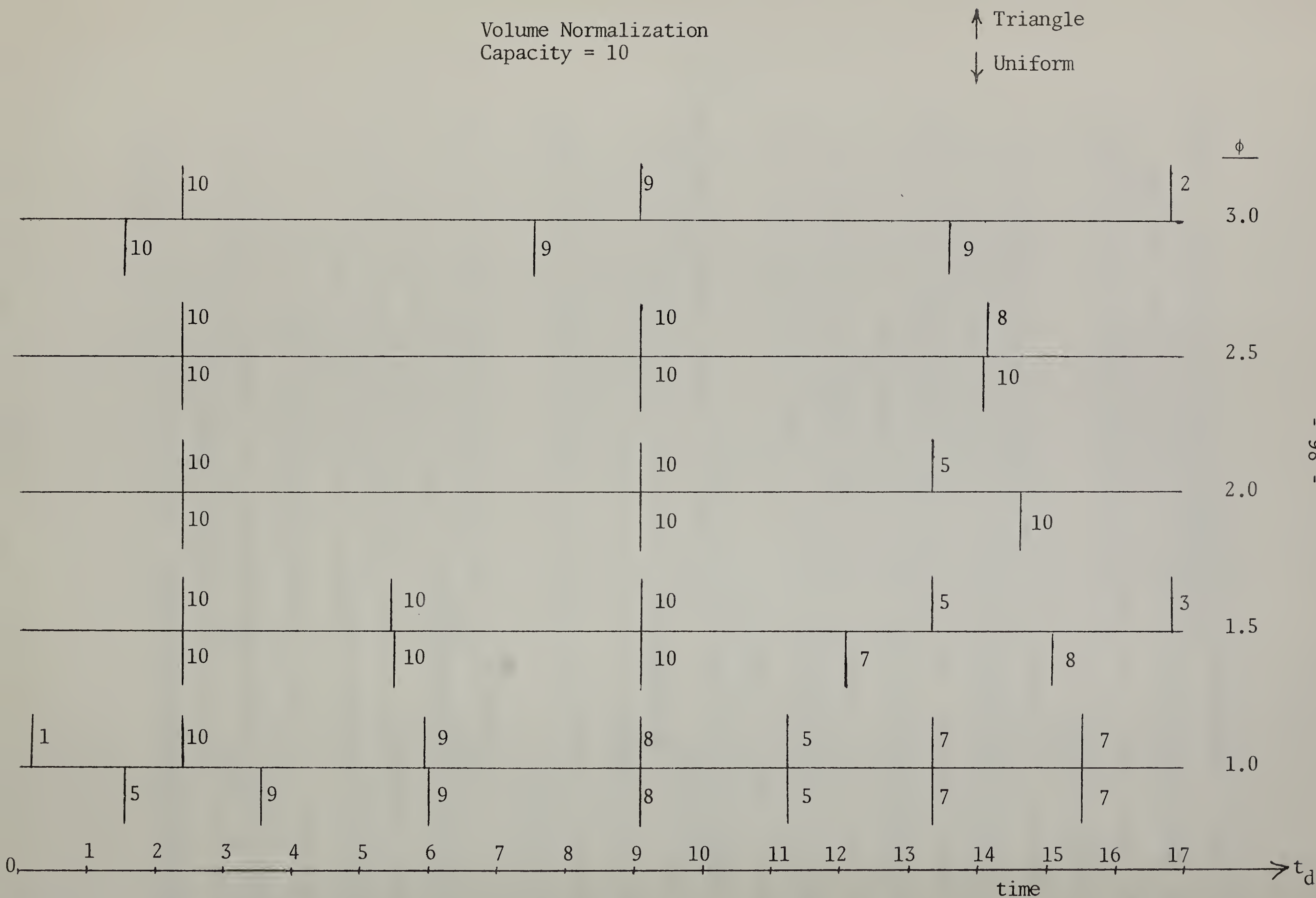


FIGURE 36. COMPARISON OF SCHEDULES WITH UNIFORM AND TRIANGULAR UTILITY

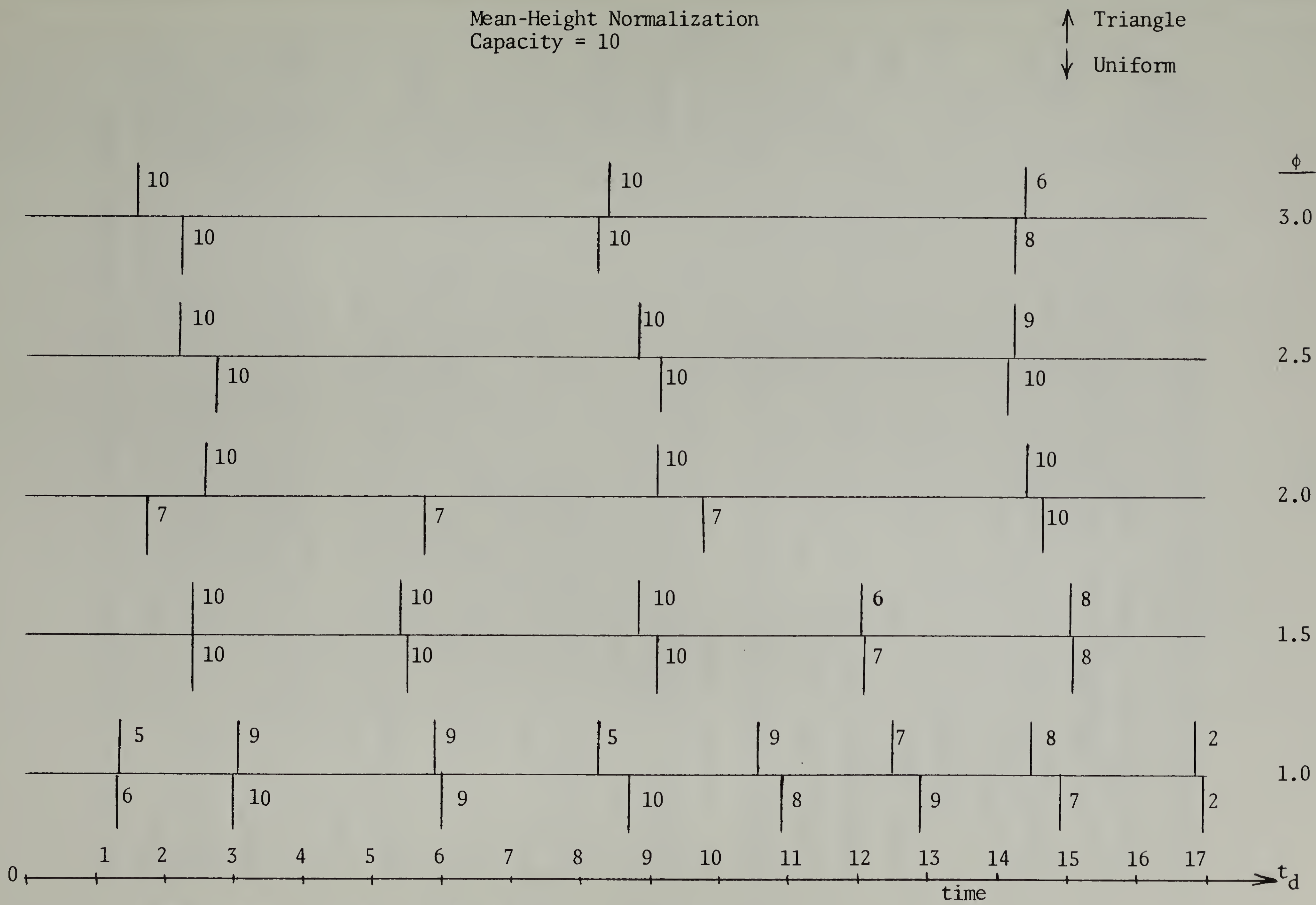


FIGURE 37. COMPARISON OF SCHEDULES WITH UNIFORM AND TRIANGULAR UTILITY

in the schedules are coincident, or nearly so. However, the first and second departures in either schedule occur at very different times. On the other hand, under mean-height normalization in Figure 37, the departures scheduled under the triangular and uniform formulations are almost never coincident, but almost always in fairly close proximity to each other.

This behavior is in keeping with the observations that have been made before regarding the effects of particular normalizing procedures. Under volume normalization, for both the triangular and rectangular forms, the schedules tend to center around principal component utilities. Hence departures under either formulation are frequently coincident about the same travelers. However, the use of rectangular utilities does cause some difference. It sometimes happens that "tall, narrow" utility components are located substantially between two stage grid points on the time axis. Under triangular utility, the magnitude of these utilities at the grid points are much smaller than the peak values. However, with uniform utilities this is not the case. Hence, under the uniform formulation, departures in the schedule might center around travelers whose utilities become considerably reduced at the stage interval points, when the triangular form is used.

Cases in point (at $\phi=1$, Figure 36) are travelers 6 and 15 about whom departures 1 and 2 center under uniform utility, but whose preferred departure times lie midway between successive stage points. (Refer to Figure 13.) From evidence found thus far, it is reasonable to conjecture that, under volume-normalization, generally closer agreement between uniform and triangular schedules would obtain if the time axis were more finely divided.

Under mean-height normalization, the effect of peakedness in the utility functions is more noticeable. Changing from triangular to uniform utilities significantly modifies the localized variations in the aggregate utility, although the general shape is preserved. Thus, although there is normally a one to one correspondence between departures in the schedules under the two utility forms, there is always some difference in schedule times. Sometimes, however, as when $\phi=2$ in Figure 37, the deviation between the triangular and uniform aggregate utility functions is sufficient to affect the schedule more radically.

Lastly, it will be noted that generally more passengers are accommodated in the schedules derived under rectangular utility functions. This follows from the fact that these schedules are less geared to the particular preference times of individual travelers, but more to departure times for which a substantial number of travelers have positive value.

Effect of Stage Increment Size. Figure 38 shows the variation of I(0) with ϕ, under volume-normalization, for four different stage increment sizes. The solid curves ($\underline{K}$=85, 170 and 340) result from using stage increment sizes of Δt=0.2, 0.1, and 0.05 hours, respectively. Note that all grid points on the time axis corresponding to $\underline{K}$=85 are included in the set of grid points corresponding to $\underline{K}$=170 and $\underline{K}$=340. Similarly, all grid points corresponding to $\underline{K}$=170 are included in the set of grid points for $\underline{K}$=340. The dashed curve ($\underline{K}$=425, Δt=0.04 hrs.) is the only case in Figure 38 which has grid points not in common with the others.

It is obvious from Figure 38 that the stage increment size has significant impact. For example, as illustrated by the solid curves, a decrease in precision ($\underline{K}$) exaggerates the non-monotonic behavior. The reason for this is rooted in the mechanisms by which a change in ϕ affects the overall derived benefit of an optimal schedule. Recall that a decrease in ϕ has two, sometimes conflicting effects. First, it modifies the immediate benefit function b(1, K) in a way that sometimes reduces the overall derivable benefit. Second, it increases the vehicle flexibility so that trips can be made at more opportune times. Now, decreasing $\underline{K}$ reduces the number of grid points on the time axis at which departure decisions may be considered, thus restricting the flexibility with which the vehicle can be scheduled. Hence, reducing the precision in this way hampers the mechanism by which benefit-reducing changes in b(1, K) can be compensated for.

A case in point is the transition from ϕ=2.25 to ϕ=2.0. The schedules are shown in Table 4, for two values of $\underline{K}$. The differences occur only in the third departure. When ϕ=2.25, the calculation using $\underline{K}$=340 schedules departure 3 at t_d=14.65; since the latter is not a grid point with $\underline{K}$=170, an inferior departure, at t_d=14.30, is scheduled in this case. This accounts for the small difference in the magnitude of I(0) at ϕ=2.25, in the curves of Figure 38. When ϕ is reduced to 2.0, the schedule calculated

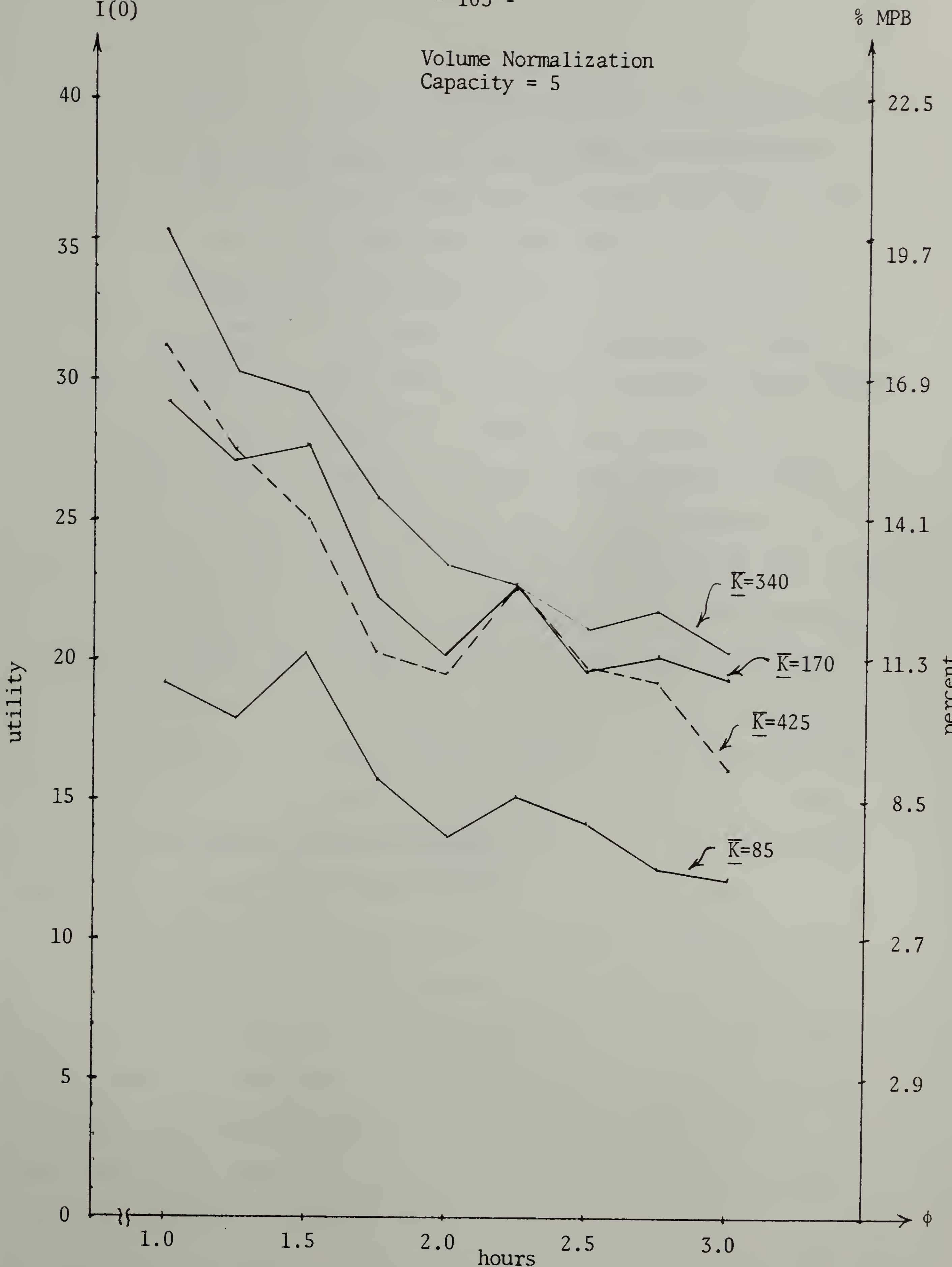


FIGURE 38. EFFECT OF STAGE INCREMENT SIZE ON BENEFIT-TRIP TIME CURVES

TABLE 4

COMPARISON OF SCHEDULES USING TWO DIFFERENT VALUES OF $\overline{K}$

	$\overline{K} = 170$		$\overline{K} = 340$	
ϕ	t_d	Passengers	t_d	Passengers
2.25	2.40	18, 19, 22, 23, 25	2.40	18, 19, 22, 23, 25
	9.10	59, 61, 65, 66, 67	9.10	59, 61, 65, 66, 67
	14.30	86, 89, 90, 91, 92	14.65	85, 88, 91, 95, 96
2.0	2.40	10, 13, 20, 22, 25	2.40	10, 13, 20, 22, 25
	9.10	57, 59, 65, 66, 67	9.10	57, 59, 65, 66, 67
	16.90	97, 99, 100	13.35	76, 79, 81, 84, 93

Volume-normalized Triangular Utilities

Vehicle Capacity C=5

with $\underline{K}$=340 yields a new (superior) departure time of 13.35, yielding an over all increase in benefit. But t_d=13.35 is not a valid grid point when $\underline{K}$=170; the best time for the third departure here, t_d=16.90, leads to an overall benefit <u>less</u> than that incurred when ϕ=2.25. The major source of discrepancy in the benefits at ϕ=2.0, is the fact that the major utility traveler 84 has a preferred departure of t_d=13.34, with half-width parameter σ_d=0.04. Hence, his utility is completely confined within the grid points of the $\underline{K}$=170 calculation.

The dashed curve of Figure 38 points up the fact that not only is the number of grid points (i.e. the size of the stage increments) important but also the <u>location</u> of these points along the time axis. The value $\underline{K}$=425 indicates greater precision than $\underline{K}$=340. Yet schedules computed using grid points at every 0.04 hour interval attain less benefit than those computed on the basis of 0.05 hr. intervals. The reason for this is that the preferences of high utility travelers, in the population used here happen to fall more closely to grid points for $\underline{K}$=340 than to those of $\underline{K}$=425, and since the utility components can be very "tall and narrow" under volume normalization, the differences can be significant.

Figure 39 shows the comparison of benefit curves, under mean-height normalization. Here, the stage increment size has much less import. Although the same mechanisms are at work here as ϕ is varied, the effects are substantially eliminated because the aggregated utility is much more "regular". In particular, there are no tall, narrow peaks (which sometimes fall between grid points under volume-normalization) to make the size of the stage increment crucial.

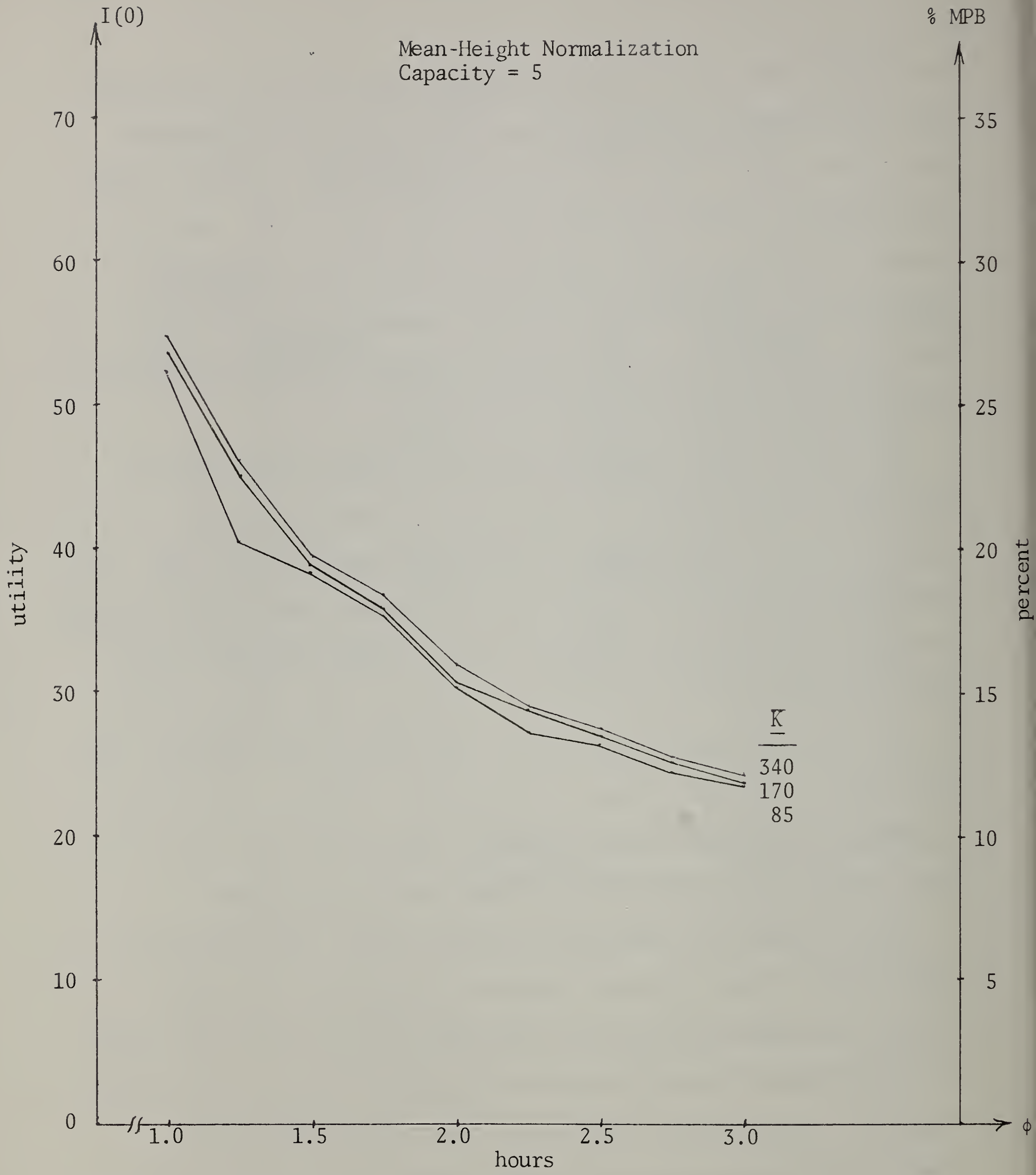


FIGURE 39. EFFECT OF STAGE INCREMENT SIZE ON BENEFIT-TRIP TIME CURVES

Table 5 compares the schedules generated with the different increment sizes. The columns for volume normalization display substantial differences, particularly where departures in a schedule corresponding to higher precision ($\underline{K}$) fall between the grid points of a lower precision schedule. However, in the column for mean-height normalization, the schedules are mostly very similar. The case of ϕ=2.0 is an exception in which the more precise calculation finds a five trip schedule of greater benefit than the four trips scheduled with $\underline{K}$=170.

In summary, the extreme effects of volume normalization make the precision of calculation crucial; in contrast, schedules based on mean-height normalization are less sensitive to the size of the stage increment.

Use of Alternate Input Data. As mentioned earlier, the results here have been based on the distribution of "holiday" travelers shown in the first column of Table 2. For purposes of comparison, several computations were performed using a new passenger distribution, derived from the "weekend" travelers in column two in that table. The new set of travelers were assigned, by the program illustrated in Figure 14, a set of $\sigma^{(i)}$ and $w^{(i)}$ parameters identical to those of the original travelers, using the values σ_{MAX}=1.0 and p=0.5. The sets of parameters for the new (weekend) and old (holiday) travelers differ only with respect to the distributions of $[t_d^{*(i)}]$ and $[t_a^{*(i)}]$ along the time axis.

TABLE 5

COMPARISON OF SCHEDULES

	VOLUME			MEAN-HEIGHT		
		t_d			t_d	
ϕ	$\bar{K}$=85	$\bar{K}$=170	$\bar{K}$=340	$\bar{K}$=85	$\bar{K}$=170	$\bar{K}$=340
1	.20	0.20	0.20	1.40	1.40	1.35
	2.40	2.40	2.40	3.60	3.60	3.65
	4.40	4.40	5.95	5.80	5.80	5.85
	6.40	6.60	9.10	8.20	8.20	8.25
	8.40	9.10	11.25	10.40	10.30	10.35
	10.40	11.10	13.35	12.40	12.50	12.50
	12.60	13.10	15.55	14.40	14.50	14.50
	14.60	16.90	-	16.40	16.90	16.85
	16.80					
2	2.40	2.40	2.40	0.40	1.30	0.35
	8.40	9.10	9.10	4.80	5.30	4.80
	13.00	16.90	13.35	8.80	9.30	8.80
				12.80	14.20	12.80
				16.80	-	16.85
3	2.40	2.40	2.40	1.60	1.60	1.65
	10.40	9.10	9.10	8.20	8.30	8.25
	16.80	16.90	16.85	14.40	14.40	14.40

t_d = departure time from origin

Figure 40 shows the curves of I(0) versus trip time ϕ. The solid curves represent the results from the new (weekend) data source, while the dashed curves are the results with the old (holiday) data. It is apparent that for the new distribution of travelers, decreases in vehicle speed are generally more serious. The fact that the old travelers were distributed somewhat in widely separated "clumps" (see Table 2) permitted the vehicle, even at the slower speeds, to accommodate large groups of travelers. The new travelers are more evenly distributed, however; the longer trip durations therefore present a greater disadvantage here.

Although the specific traveler distributions have an important effect on the curves of Figure 40, it is also apparent that the general properties by which these curves have been previously characterized under either normalization, are retained. The same comment applies to Figure 41, which displays the variation of benefits with vehicle capacity. In this graph, the fact that benefits under the old traveler distribution exceed those for the new distribution increasingly at the higher values of capacity, again is directly related to the greater uniformity of the weekend traveler distribution.

<u>Concluding Comment</u>. This section has presented some of the highlights of a substantial number of computer computations, intended to study the parametric variations of the one-way, single-link system. Valuable information has been obtained to facilitate choice of the structure of the utility functions to be used in future work. For instance, it has

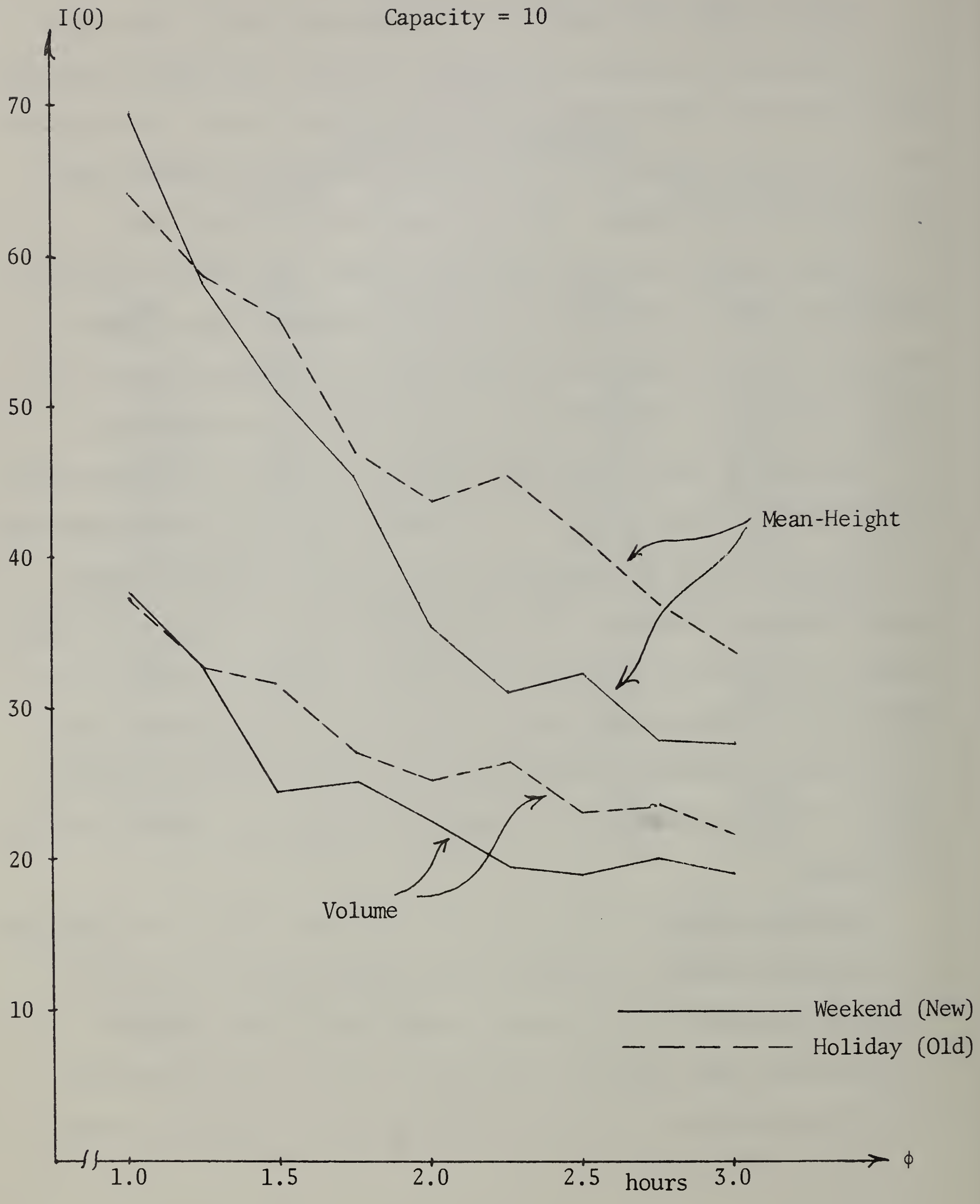


FIGURE 40. BENEFIT-TRIP TIME CURVES USING ALTERNATE TRAVELER SETS

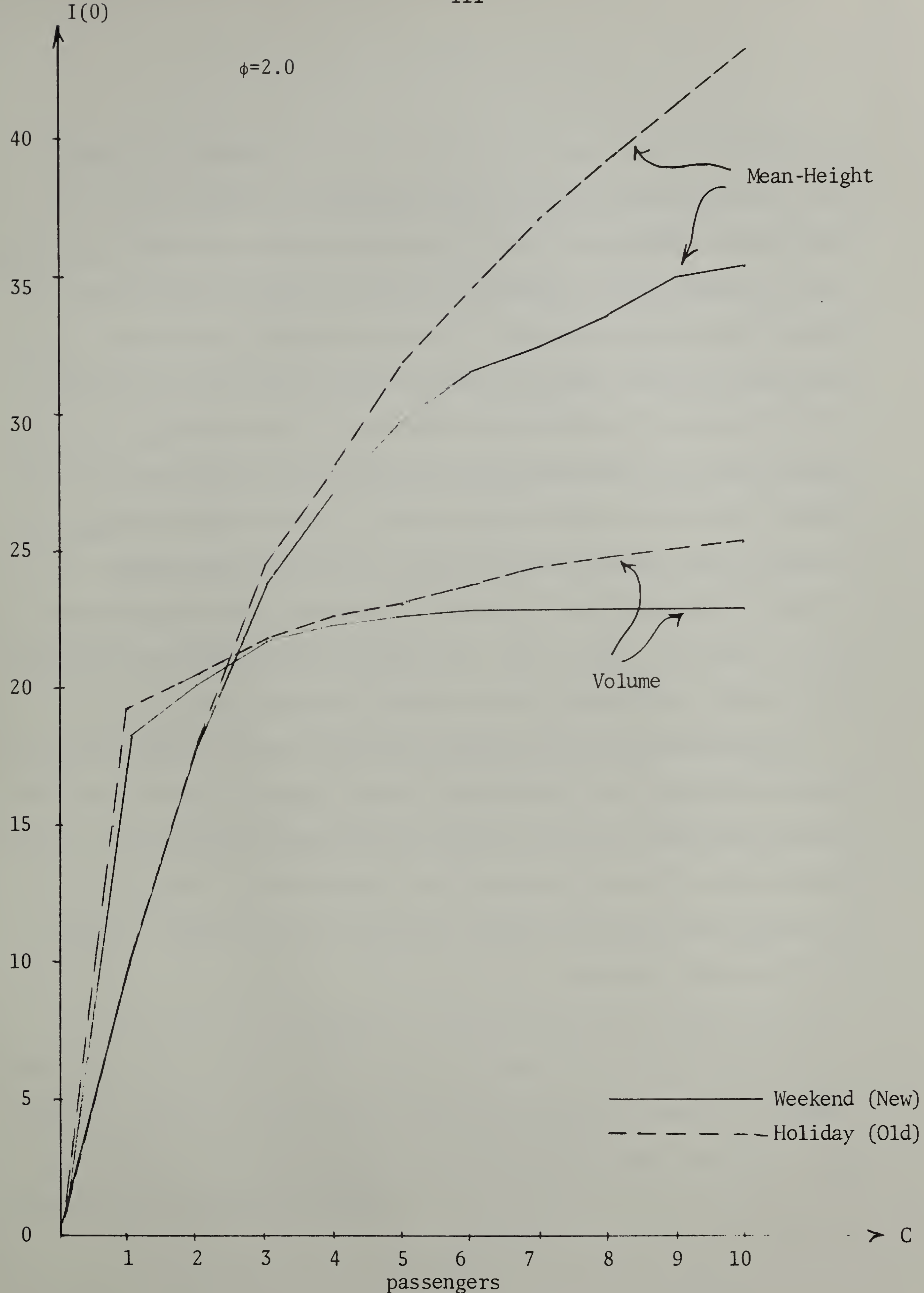


FIGURE 41. BENEFIT-CAPACITY CURVES USING ALTERNATE TRAVELER SETS

been learned that the question of normalization is critical. On the basis of the results obtained, volume-normalization appears to be unacceptable. The normalization procedure was intended to lend equal weight to each traveler, in the schedule calculation. Volume-normalization seems to have done the opposite. Also, volume normalization seems to have under-represented the trade-off value of capacity for speed by promoting a large discrepancy in the relative importances of the individual travelers. Another feature of volume normalization, which does not reflect on its validity but tends to make it unattractive, is that it tends to exaggerate the computational difficulties associated with choosing the stage increment. Hence of the two alternative procedures, mean height normalization is superior. But the fact that results are so sensitive to normalization warrants further scrutiny of the utility representation.

Insights have been obtained here, which may be useful toward policy decisions with regard to schedules and vehicle characteristics. The effects of traveler arrival-departure orientations, and the observed characteristics of the I(0) contours are examples of this. Finally, computational experience has been obtained which will be helpful toward extension of this research.

9. SUMMARY AND OUTLOOK

This report covers an exploratory phase of a research effort in passenger scheduling. Results obtained here will be useful primarily for the insights they provide toward future development of the analysis. The work has had several interesting aspects. Effort has been directed toward formulation of traveler utilities as functions of the scheduling variables. Analysis of scheduling one vehicle has been substantially advanced. Numerical results have been obtained for the single link, one-way problem, which point out the effects of various representations of traveler utility, and the ramifications of changes in system characteristics such as speed and capacity. Finally, preliminary methodology has been put forth for the analysis of multi-vehicle systems.

Avenues for future research lead in several directions. First, future development of the concept of traveler utility is necessary; additional research into the form that the utility functions should take, and into the question of relative magnitudes of utility among individuals, is needed to enhance the usefulness of the utility concept. In particular, an "economic" interpretation of utility, such as a person's "willingness to pay" for a trip, might provide insights into the utility questions.

A second area for development is the incorporation of costs into the scheduling model. The present formulation takes no account of operating expenses. Further, a model which combines an economic concept of traveler utility with an accounting of costs would be useful toward economic evaluation of alternative systems.

Extension of the analysis to deal with a fleet of several vehicles is of obvious practical importance. In addition, the question of routing is highly significant. Future effort will be directed toward the development of algorithms to schedule vehicles optimally over the dimensions of time <u>and</u> space.

Finally, all the analysis here has been deterministic in nature. Incorporation of uncertainties in variables such as trip times and passenger preference parameters would be a highly valuable asset. Although computationally difficult, an effort in this direction should eventually be made.

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APPENDIX A: THE COMPUTER PROGRAM

This appendix contains the documentation of the FORTRAN computer program, for single-vehicle one-way demand problems, that was operated to produce the results presented in Section 8.

The bulk of the program consists of three subroutines:

ADUTIL (for utility aggregation)
SCHED1 (for schedule calculation)
ASSIGN (for assigning passengers to departures).

These will be discussed, in turn, below. The "main frame" program coordinates these subroutines; it reads the input data, calls the subroutines in order, and prints the desired information. See Figure 42.

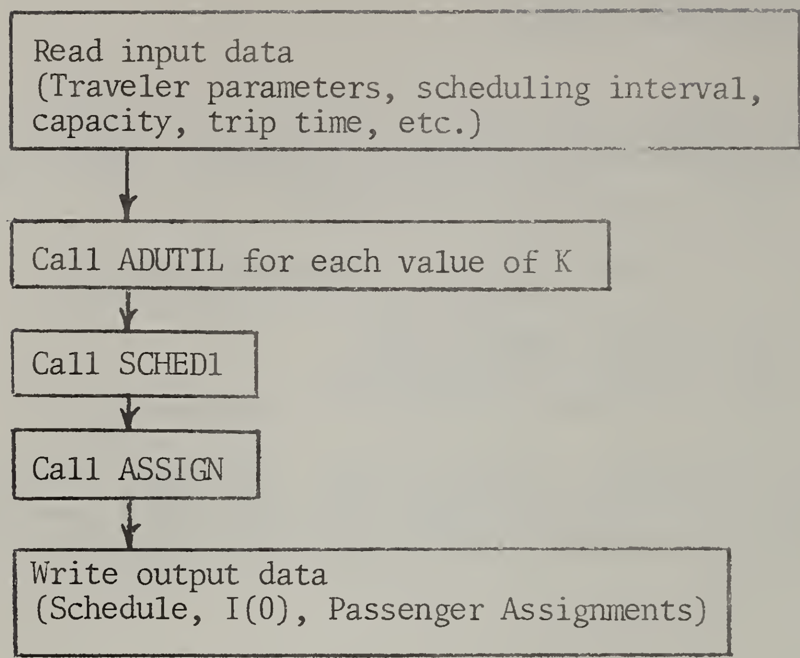


FIGURE 42. THE MAIN FRAME

A listing for ADUTIL, for the triangular, volume normalized case is given in Figure 43. The following is a list of variables and their definitions:

K = stage variable
T = upper limit of scheduling interval
KFIN = $\overline{K}$, last stage of process
N = total number of travelers
ICAP = vehicle capacity, C
TRTIM = ϕ, one way travel time
w(I) = $w^{(i)}$
SIGD(I) = $\sigma_d^{(i)}$
SIGA(I) = $\sigma_a^{(i)}$
TDSTAR(I) = $t_d^{*(i)}$
TASTAR(I) = $t_a^{*(i)}$
V(I) = value of utility for i^{th} traveler, at current K.
J = index tallying eligible travelers
UTIL = $\sum_i U^{(i)}(K,K+F)$, aggregate utility
R=R1/R2 = expresses K/KFIN as real rather than integer; R*T converts K to real time.

Q = an array of eligible traveler utilities; the first C elements in Q comprise UT1L in the case of capacity limitations.

TEM = an intermediate variable in the loop that determines the smallest value in the array Q.

The flow diagram of ADUTIL is shown in Figure 44. ADUTIL is called to calculate aggregate utility (UTIL) at a stage K. During the operation, travelers are called and processed one at a time. The process, for each traveler, begins by determining whether, at the specified K, a departure falls within his acceptable intervals of departure and arrival time. If not, the value of traveler utility V(I) is set to zero, and the next traveler is called. If departure at K is acceptable, V(I) is calculated according to the appropriate utility function (and normalization criterion). Then the index J, which counts the number of eligible travelers, is increased by one. A determination is then made to see if J has exceeded the vehicle capacity. If J has not exceeded capacity, the traveler utility, V(I), is assigned the next place in the array Q; Q fills from Q(1) through Q(ICAP) during the time that J is less than capacity. Once V(I) is assigned to an element of Q, the aggregate utility UTIL is increased by V(I), and the next traveler is called.

Suppose, however, that for the current traveler J exceeds capacity. In this case, V(I) is assigned to the last element in Q, Q(ICAP+1). Now Q consists of C+1 components, only C of which can make up UTIL, because of the capacity restriction. Following the rule that the travelers will be boarded, highest utilities first, and thus a vehicle leaving at stage K will accrue the sum of the C greatest component utilities, the array Q is now scanned to find the smallest component. That component is placed in the Q(ICAP+1) position. The value of UTIL is then modified by adding V(I) and subtracting Q(ICAP+1). It is possible, of course, that V(I) remains in the Q(ICAP+1) position during the scan, in which case UTIL is left unchanged.

The routine ends after the last traveler (I=N) is processed, leaving UT1L at its final value.

The schedule calculation subroutine, SCHED1, applies the dynamic programming algorithm to determine the optimal schedule. A listing is shown in Figure 45. Variable definitions not previously given are:

```
                                    3200  FORTRAN    (2,1)              /  /
      SUBROUTINE ADUTIL(K,T,KFIN,N,ICAP,TRTIM,W,SIGD,TDSTAR,SIGA,TASTAR,
     1                 V,J,UTIL)
C     SUBROUTINE TO AGGREGATE UTILITY AT STAGE K
      DIMENSION W(200),SIGD(200),TDSTAR(200),SIGA(200),
     1          TASTAR(200),V(200),Q(200)
      I = 1
      J = 0
      UTIL = 0.
      R1 = K
      R2 = KFIN
      R  = R1/R2
   10 IF((TDSTAR(I)-SIGD(I)).GT.T*R) 20,16
   16 IF((TDSTAR(I)+SIGD(I)).LT.T*R) 20,17
   17 IF((TASTAR(I)-SIGA(I)).GT.(T*R+TRTIM)) 20,18
   18 IF((TASTAR(I)+SIGA(I)).LT.(T*R+TRTIM)) 20,91
C      10 THRU 18 TAKE CARE OF PERSONS WITH ZERO UTILITY WITHIN
C     CURRENT INTERVAL
   91 V(I)=  W(I)*(2.-(2./SIGD(I))*ABS(TDSTAR(I)-T*R))
     1       + (1.-W(I))*(2.-(2./SIGA(I))*ABS(TASTAR(I)-(T*R+TRTIM)))
      V(I) = V(I)/(4.*SIGA(I)*SIGD(I))
C     ABOVE STEP USED IN NORMALIZED VOLUME CASE
C     J IS AN INDEX TALLYING TRAVELERS WITH NONZERO UTILITY
      J = J+1
      IF(J.LE.ICAP) 25,35
   35 Q(ICAP+1) = V(I)
      DO 68 NN=1,ICAP
      IF(Q(NN).LT.Q(NN+1)) 62,68
   62 TEM = Q(NN)
      Q(NN) = Q(NN+1)
      Q(NN+1) = TEM
   68 CONTINUE
      UTIL = UTIL+V(I)-Q(ICAP+1)
      GO TO 30
   20 V(I) = 0.
      GO TO 30
   25 UTIL = UTIL + V(I)
      Q(J) = V(I)
   30 IF(I-N) 32,40,32
   32 I = I+1
      GO TO 10
   40 RETURN
      END

        3200 FORTRAN DIAGNOSTIC RESULTS - FOR   ADUTIL

ERRORS
```

FIGURE 43. SUBROUTINE ADUTIL (FORTRAN)

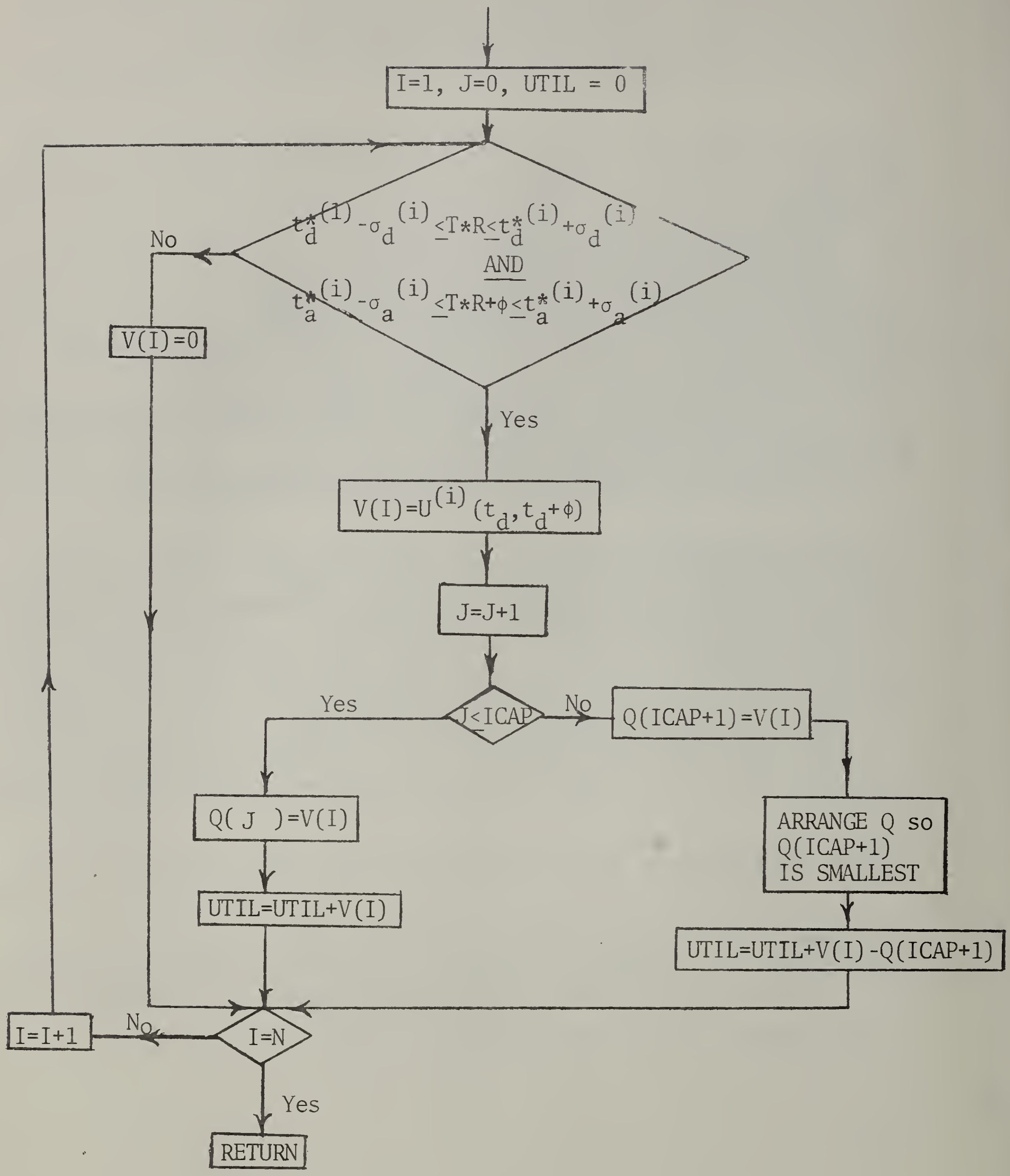


FIGURE 44. SUBROUTINE ADUTIL

```
      SUBROUTINE SCHED1(HEADWAY)
C      N PERSONS,1 WAY, SINGLE VEHICLE AND LINK
      DIMENSION IMAX(500),C(500),TIMDEP(300),INTDEP(300),TEMP(2),
     1          UTIL(500)
      REAL IMAX
      INTEGER C
      COMMON IMAX,C,KFIN,TRTIM,LASDEP,TIMDEP,T,UTIL,INTDEP
C     IMAX(K) IS THE MAX BENEFIT FUNCTION, C(K) IS THE DECISION VECTOR
      H1 = KFIN
      H2 = H1*HEADWAY/T
      L = H2+0.5
C      L IS THE HEADWAY (OR SINGLE VEH. RND. TRIP TIME) EXPRESSED IN
C      DISCRETE UNITS, ROUNDED TO AN INTEGER
      KK = KFIN+1
C      THE DEVICE KK=K+1 IS USED TO ENABLE STORAGE AT K=0 IN ARRAYS
C        IMAX,C,ETC.
      IMAX(KFIN+1) = 0.
      C(KFIN+1) = 0
  111 KK= KK - 1
      JC = 1
  121 IF(JC.EQ.1) 131,135
  131 B = 0.
      KKNEXT = KK+1
      GO TO 141
  135 IF(KK.EQ.1) 136,137
  136 B = 0.
      GO TO 138
  137 B = UTIL(KK-1)
  138 KKNEXT = KK+L
  141 IF(KKNEXT.GT.KFIN+1) 143,145
  143 KKNEXT = KFIN+1
  145 TEMP(JC) = B+IMAX(KKNEXT)
C     B IS THE IMMEDIATE BENEFIT,TEMP(1) IS THE MAX BENEFIT FCN AT K
C       IF C(K)=0,TEMP(2) FOR C(K)=1
      IF (JC-2) 147,151,147
  147 JC = 2
      GO TO 121
  151 IF(TEMP(1)-TEMP(2)) 157,155,155
  155 IMAX(KK) = TEMP(1)
      C(KK) = 0
      GO TO 159
  157 IMAX(KK) = TEMP(2)
      C(KK)=1
  159 IF(KK.NE.1) 111,161
C     WHEN KK = 1 AT 159 ALGO. PROCEEDS TO TRAJECTORY CALC. BELOW
  161 NUMDEP = 0
  163 IF(C(KK)-1)171,165,171
  165 NUMDEP = NUMDEP + 1
      INTDEP(NUMDEP) = KK-1
      KK = KK+L
      IF(KK.LE.(KFIN+1)) 163,175
  171 KK = KK+1
      IF(KK.LE.(KFIN+1)) 163,175
  175 LASDEP = NUMDEP
      DO 179 NUMDEP=1,LASDEP
  179 TIMDEP(NUMDEP) = ACTIME(INTDEP(NUMDEP),KFIN,T)
C      LASDEP IS THE TOTAL NUMBER OF DEPARTURES,TIMDEP(NUMDEP) GIVES
C      TIME OF EACH DEPARTURE,INTDEP(NUMDEP) THE DISCRETE INTERVAL OF
C      EACH DEPARTURE
      RETURN
```

FIGURE 45. SUBROUTINE SCHED1 (FORTRAN)

HEADWAY = minimum time between successive departures. (HEADWAY = $\phi+\rho$; see Section 3)

L = HEADWAY expressed as an integer number of stage units

KK=K+1; in order to store values at K=0, this index is used as a substitute stage variable

IMAX(KK) = maximum benefit function

C(KK) = decision variable

H1,H2 = variables intermediate to the conversion of HEADWAY to an integer

JC = intermediate value of C(KK)

TIMDEP = the array of departure times in the optimal schedule

TEMP = a two component array, the elements of which represent tentative values of IMAX(KK)

LASDEP = the total number of departures in the optimal schedule

B = the immediate benefit b(1,K)

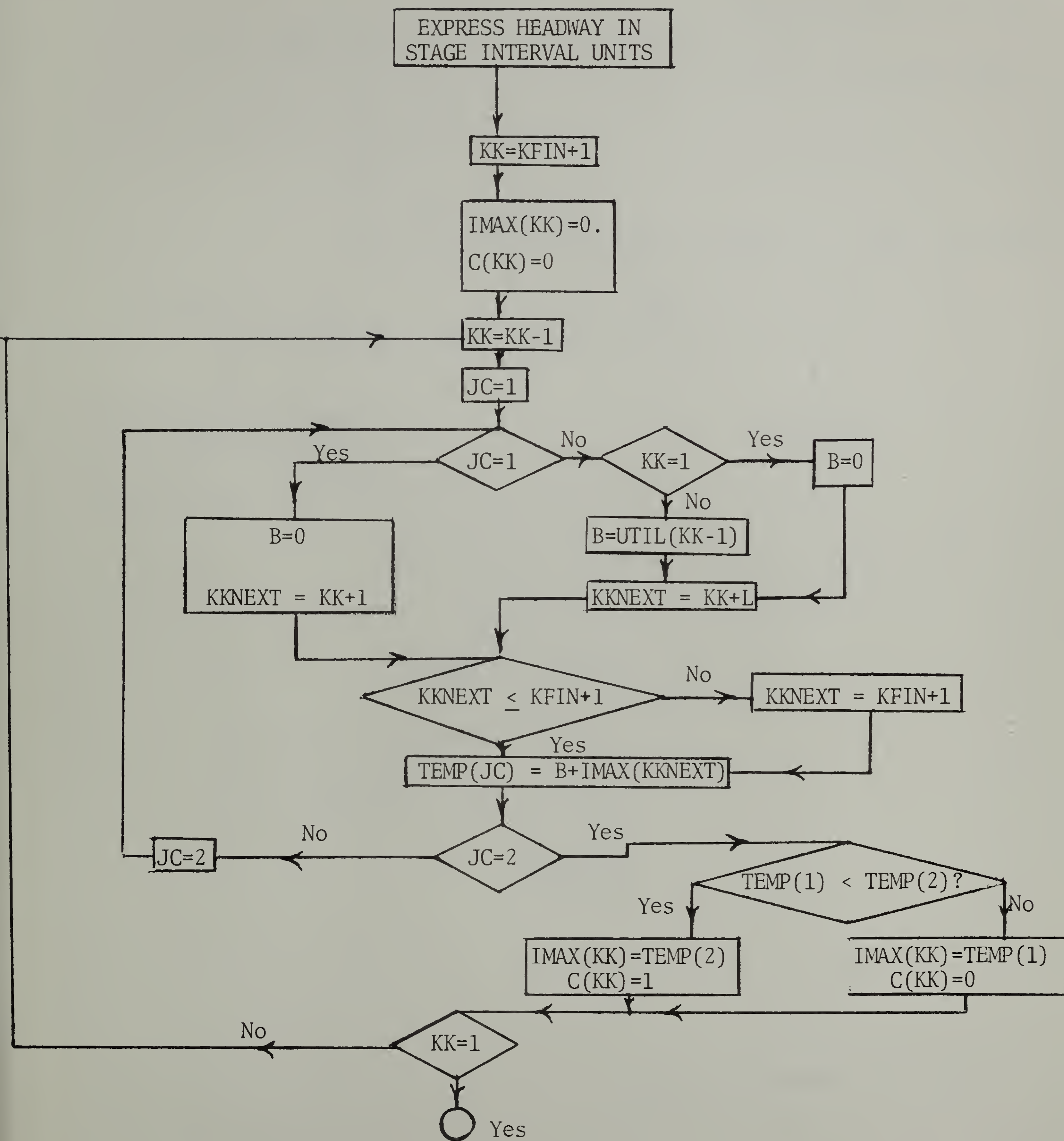
$$\text{KKNEXT} = \begin{cases} \text{KK+1} & \text{if } c(\text{KK}) = 0 \\ \text{KK+L} & \text{if } c(\text{KK}) = 1 \end{cases}$$

NUMDEP = index which tallies the number of departures, as they are determined

ACTIME(K,KFIN,T) = a function subprogram, listed in Figure 47, which converts the stage value to actual time units.

A flow diagram for SCHED1 is shown in Figure 46. The first step is a conversion of HEADWAY into stage interval units; the converted value is denoted by L. Stage index KK is initialized at KFIN+1, and IMAX(KFIN+1) and C(KFIN+1) are also initialized, to 0.

The program proceeds backwards towards KK=1. At each stage the benefit is calculated for each possible decision, subsequent to which IMAX(KK) is set equal to the maximum benefit, and C(KK) to the


EXPRESS HEADWAY IN STAGE INTERVAL UNITS
KK=KFIN+1
IMAX(KK)=0.
C(KK)=0
KK=KK-1
JC=1
JC=1
Yes
No
KK=1
Yes
B=0
No
B=0
KKNEXT = KK+1
B=UTIL(KK-1)
KKNEXT = KK+L
KKNEXT ≤ KFIN+1
No
KKNEXT = KFIN+1
Yes
TEMP(JC) = B+IMAX(KKNEXT)
JC=2
No
JC=2
Yes
TEMP(1) < TEMP(2)?
Yes
No
IMAX(KK)=TEMP(2)
C(KK)=1
IMAX(KK)=TEMP(1)
C(KK)=0
KK=1
No
Yes

Continued

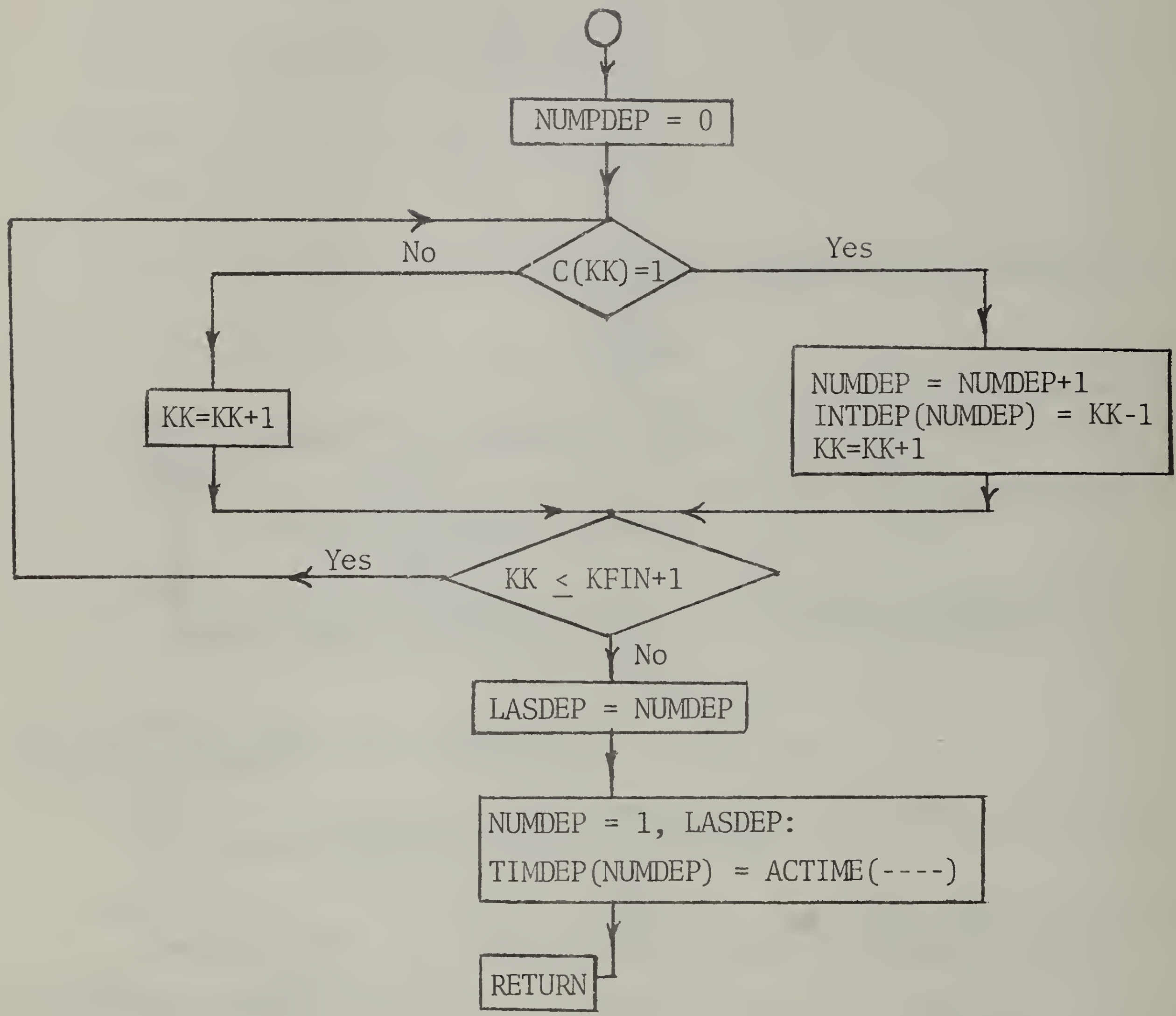


FIGURE 46. SUBROUTINE SCHED1

```
                                    3200  FORTRAN     (2,1)
      FUNCTION ACTIME(K,KFIN,T)
C     ACTIME CONVERTS INTERVAL INDEX TO ACTUAL TIME
      R1 = K
      R2 = KFIN
      ACTIME = T*R1/R2
      RETURN
      END

       3200 FORTRAN DIAGNOSTIC RESULTS - FOR   ACTIME

NO ERRORS
```

FIGURE 47. FUNCTION SUBPROGRAM ACTIME

3200 FORTRAN (2.1) /

```
      SUBROUTINE ASSIGN(ICAP)
      DIMENSION MPASS(200),NUMPAS(300),WAIT(100),V(200),
     1          INTDEP(300),W(200),SIGD(200),TDSTAR(200),SIGA(200),
     2          TASTAR(200),UTIL(500),C(?00),IMAX(500),TIMDEP(300)
      COMMON IMAX,C,KFIN,TRTIM,LASDEP,TIMDEP,T,UTIL,INTDEP,
     1         N,W,SIGD,TDSTAR,SIGA,TASTAR,NUMPAS,MPASS
      NUMDEP = 1
      DO 303 I=1,N
  303 MPASS(I)=0
  301 CALL ADUTIL(INTDEP(NUMDEP),T,KFIN,N ,CAP,TRTIM,W,SIGD,TDSTAR,SI
     1     TASTAR,V,J,UTEMP)
      JPOS = 0
      JFULL = J-ICAP
      I=1
      NPAS=0
  304 IF(V(I).GT.0.)305,310
  305 IF(JFULL.GT.0) 325,309
  309 MPASS(I)=NUMDEP
  313 NPAS = NPAS+1
      GO TO 310
  310 IF(I.EQ.N)    317,315
  315 I = I+1
      GO TO 304
  317 NUMPAS (NUMDEP) = NPAS
      IF(NUMDEP.EQ.LASDEP) 375,319
  319 NUMDEP = NUMDEP + 1
      GO TO 301
  325 JPOS = JPOS + 1
      IF(JPOS.GT.JFULL) 329,327
  327 WAIT(JPOS)=V(I)
      GO TO 315
  329 WAIT(JFULL+1) = V(I)
      DO 333 MM=1,JFULL
      IF(WAIT(MM).GT.WAIT(MM+1)) 331,333
  331 X=WAIT(MM)
      WAIT(MM)=WAIT(MM+1)
      WAIT(MM+1)=X
  333 CONTINUE
      DO 337 II=1,I
      IF(WAIT(JFULL+1).EQ.V(II))335,337
  335 IF(MPASS(II).EQ.0) 341,337
  337 CONTINUE
  341 MPASS(II)=NUMDEP
      GO TO 313
  375 RETURN
      END
```

3200 FORTRAN DIAGNOSTIC RESULTS - FOR ASSIGN

NO ERRORS

FIGURE 48. SUBROUTINE ASSIGN (FORTRAN)

corresponding decision value (0 or 1). (At KK=1 the immediate benefit B is (arbitrarily) set to zero, since no value is stored for UTIL(0).)

The completed maximization process yields IMAX(1), the maximized total utility. Next, calculations proceed from KK=1 toward KK=KFIN+1, to recover the optimal schedule by following the optimal decisions. The array INTDEP stores the stage values at which departures take place. These are converted to an array TIMDEP of successive departure times by function subroutine ACTIME.

The listing of subroutine ASSIGN is shown in Figure 48. The variables which have not been defined yet, are:

MPASS(I) = departure number to which passenger I is assigned; a value of zero indicates no accommodation.

JPOS = index tallying the number of eligible passengers encountered thus far.

JFULL = the number of eligible passengers in excess of capacity

NPAS = index tallying number of passengers boarded on the NUMDEPth departure

WAIT = array, which in case of capacity limitations, stores prospective passenger utilities

MM = index used in ordering elements of WAIT

II = index used in procedure which picks out passengers of highest utility in WAIT.

The flow diagram for ASSIGN is given in Figure 49. This routine is entered for each of the successive departures in the optimal schedule (NUMDEP=1 through LASDEP). The stage INTDEP(NUMDEP) corresponding to the current departure is found; using this stage, ADUTIL is called to produce J, the number of passengers eligible for departure, from which is calculated JFULL, the excess of demand (J) over capacity (ICAP). Each passenger (I=1,...,N) is treated in turn; V(I) is calculated, and if it is positive, that (I-th) passenger is assigned to the current departure if JFULL≤0, and put on a waiting list if JFULL>0. In the latter case, the array WAIT is first filled, from WAIT(1) through WAIT(JFULL). As each subsequent traveler with V(I) > 0 is identified, his V(I) is entered as a WAIT(JFULL+1) and the elements of WAIT are permuted so that the largest becomes WAIT(JFULL+1); the traveler corresponding to the maximum is then assigned to the departure. When the process is complete (I=N) all but the JFULL eligible travelers left on the waiting list, will have been assigned to the departure.

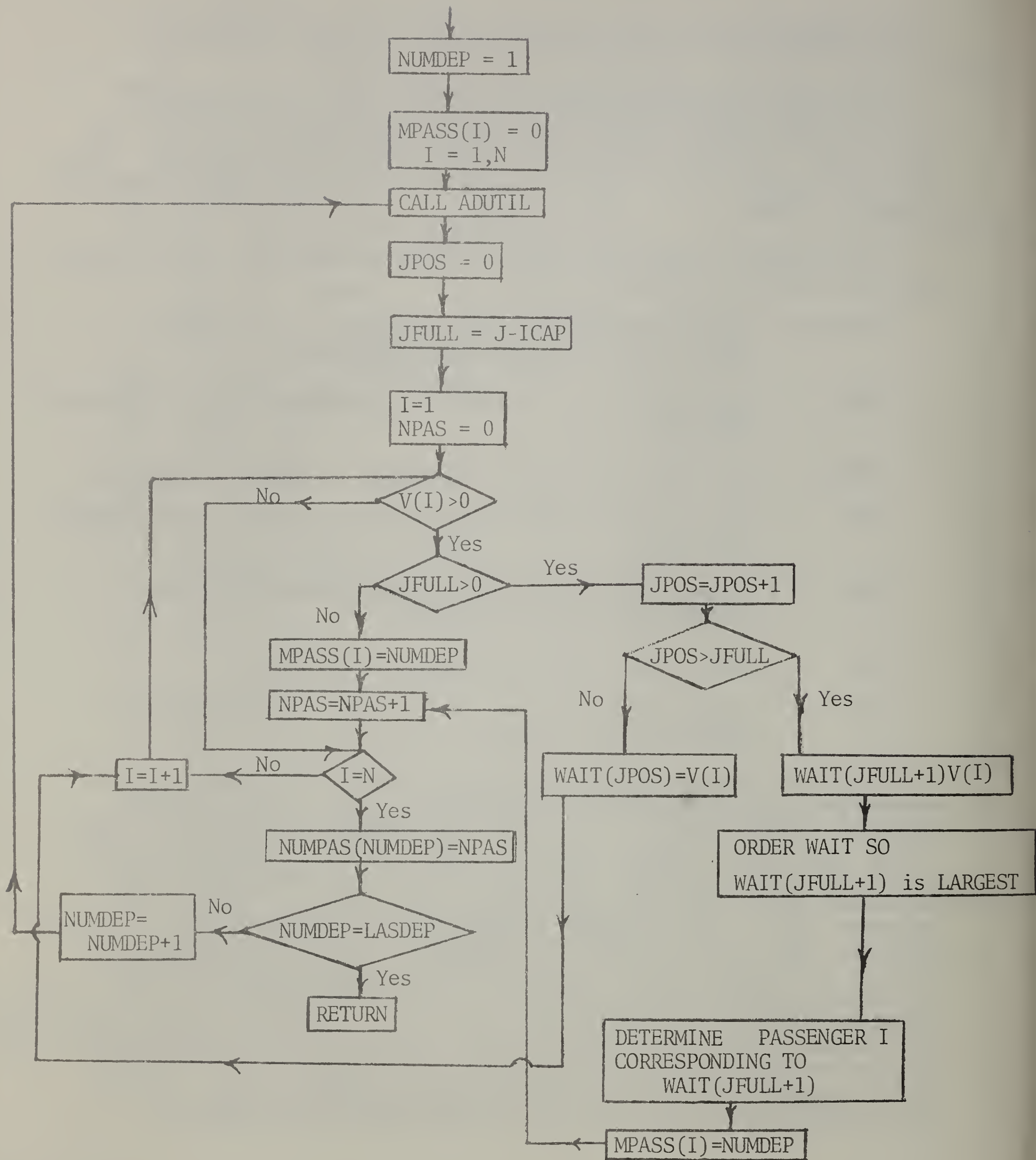


FIGURE 49. SUBROUTINE ASSIGN

The running time of one full schedule calculation, involving the three subroutines, including print-out time but excluding compilation, using $\underline{K}=340$, $N=100$, on the CDC 3100 machine is approximately 25 seconds.