NATIONAL BUREAU OF STANDARDS REPORT

9507

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Semi-Annual Report

July through December 1966

For Official Distribution

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

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NATIONAL BUREAU OF STANDARDS

INSTITUTE FOR BASIC STANDARDS

APPLIED MATHEMATICS DIVISION

July 1 through December 31, 1966

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Contents

°Only unclassified material is included in this report.

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS
Manager: Morris Newman
Full task description: July - September 1954 issue, p. 1

Authorized 8/29/54

Status: CONTINUED. M. Newman and A.O.L. Atkin have proved that there are only finitely many congruence subgroups of the modular group having just one parabolic class, and have classified all of these. They have also made some progress on the problem of the determination of the congruence subgroups of the modular group of genus zero.

M. Newman has proved that if $p(x)$ is a polynomial with no zeroes inside the unit circle and if r is any positive number, then the nth coefficient of $p(x)^r$ is in absolute value less than cn^{-r-1}, where c is some positive constant independent of n. M. Newman has proved that no three diagonals of the regular n-gon, n odd, intersect in its interior.

M. Newman and J.L. Mennicke have proved that if A is an integral matrix which is not congruent to a scalar matrix modulo m for any $m > 1$ then A is similar by an integral unimodular transformation to a matrix B such that the first row of B is either the first unit vector, its negative, or the second unit vector.

M. Newman, J. Cameron and S. Peavy have put together a matrix inversion routine which includes three absolute error checks and one computational check. The routine is available as a package and should give the prospective user confidence in the results obtained by the machine.

S. Haber continued studies of number-theoretical methods of multiple quadrature, and of Monte Carlo methods. He studied the quadrature fomulas which are obtained by partitioning the interval of integration into subintervals, and taking the midpoints of the subintervals as the abscissas of the formula and the lengths of the subintervals as the corresponding weights. He showed that these formulas all have degree of precision one, and are definite ; and he showed that the "midpoint rule" is the best of them.

S. Haber also continued research on the zeros of certain polynomials in a meromorphic function and its derivatives ; and he attended the American Mathematical Society Summer Institute on Entire Functions.

K. Goldberg has investigated functions of the non-negative integers into themselves which are eventually periodic showing that some of the results of J.D. Becker and E.T. Ordman do not depend on functional relationships.

F.W.J. Olver has begun a further study of Miller's recurrence algorithm for the generation of Bessel and other special functions. By placing the analysis in a more general setting it is proving possible to extend the algorithm by (i) determining automatically the correct starting point (ii) providing a more powerful error analysis (iii) including inhomogeneous terms. Applications are now being tried out.

Publications:

(1) Kernels and the Kronecker product of graphs. R.A. Brualdi. To appear in Proceedings of the American Mathematical Society.

(2) Proof of a permanental inequality. R.A. Brualdi and M. Newman. Quart. J. Math., Oxford, 17, 234-238 (1966).

(3) Hadamard matrices of order cube plus one. K. Goldberg. Proceedings of the American Mathematical Society, Vol. 17, No. 3, pp. 744-746 (June 1966).

(4) Semi-groups with zeroids. K. Goldberg. To appear in the Journal of Research NBS.

(5) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.

(6) The l.u.b. of a set of determinants. K. Goldberg. To appear in the Journal of Research NBS.

(7) The l.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.

(8) Pairs of non-singular matrices. K. Goldberg. Journal of Research NBS, 70B, pp. 155-156, 1966.

(9) Upper bounds for the determinant of a row stochastic matrix. K. Goldberg. Journal of Research NBS, 70B, pp. 157-158, 1966.

(10) Groups preserving ordering in vectors. K. Goldberg. Journal of Research NBS, 70B, pp. 159-160, 1966.

(11) On a theorem of Ahlfors. L. Greenberg. To appear in the Journal of Research NBS.

(12) Kleinian groups. L. Greenberg. Accepted for publication in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965).

(13) Note on normal subgroups of the modular group. L. Greenberg. Accepted for publication in the Proceedings of the American Mathematical Society.

(14) A functional inequality. S. Haber. To appear in the American Mathematical Monthly 73, 1966.

(15) A theorem on arbitrary functions. S. Haber. To appear in the American Mathematical Monthly.

(16) A modified Monte-Carlo quadrature - II. S. Haber. To appear in Mathematics of Computation.

(17) On a sequence of points of interest for numerical quadrature. S. Haber. Journal of Research NBS, 70B, pp. 127-136, 1966.

Publications:

(18) A modified Monte-Carlo quadrature. S. Haber. To appear in Mathematics of Computation 20, 1966.

(19) Note on partitions modulo 5. M. Newman. To appear in Mathematics of Computation.

(20) Doubly stochastic associated matrices. M. Marcus and M. Newman. To appear in the Duke Mathematical Journal.

(21) Classification of normal subgroups of the modular group. M. Newman. To appear in Trans. Amer. Math. Soc.

(22) On punctured Riemann surfaces with maximal automorphism groups. M. Newman and J. Lehner. To appear in Glasgow Math. Journal.

(23) Finding a rank-maximizing matrix block. A.J. Goldman and M. Newman. Journal of Research NBS, 70B, pp. 219-220, 1966.

(24) The coefficients of the powers of a polynomial. M. Newman. To appear in Journal of Research NBS.

ASYMPTOTIC EXPANSIONS

Task 20501-11-2050421/63

Origin: NBS
Sponsor: U.S. Army Research Office, Durham, N.C. Authorized 9/10/63
Manager: F.W.J. Olver
Full task description: July-December 1963 issue, p. 2

Status: CONTINUED. Work has continued on Laplace's method, the method of steepest descent and the method of stationary phase. A better approach to the problem of obtaining error bounds has been discovered and is being exploited.

Publications:

(1) Error bounds of asymptotic solutions of differential equations I. The distinct eigenvalue case. F. Stenger. J. 70B3-180, 167-186 (July-September 1966).

(2) Error bounds for asymptotic solutions of differential equations II. The general case. F. Stenger. J. 70B3-181, 187-210 (July-September 1966).

2. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS Authorized 10/1/62
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Status: CONTINUED. Solomon Kullback (George Washington University) and H. H. Ku are investigating the problem of interaction in multidimensional contingency tables, from the point of view of information theory (Kullback, Information Theory and Statistics, 1959). The hypothesis of no rth-order interaction is defined in the sense of an hypothesis of "generalized" independence of classifications with fixed rth-order marginal restraints. For a three-way table, with given cell probabilities π_{ijk}, it is shown that the minimum discrimination information for a contingency table with marginals $p_{ij.}$, $p_{.jk}$, and $p_{i.k}$ is given by the set of cell probabilities $p^*_{ijk} = a_{ij}\, b_{jk}\, c_{ik}\, \pi_{ijk}$ where a_{ij}, b_{jk}, and c_{ik} are functions of the given marginal probabilities, that is, $\ln \frac{p^*_{ijk}}{\pi_{ijk}} = \ln a_{ij} + \ln b_{jk} + \ln c_{ik}$, representing no second-order interaction. The minimum discrimination information statistic, asymptotically distributed as χ^2 with appropriate degrees of freedom, is $2 \sum_{ijk} f_{ijk} \ln f_{ijk} - 2 \sum_{ijk} f^*_{ijk} \ln f^*_{ijk} \geq 0$ where f_{ijk} are the observed cell frequencies and f^*_{ijk} are the "no interaction" cell frequencies uniquely determined by a simple convergent iteration process of the marginals on π_{ijk}. For lower order marginal restraints, the usual independence hypotheses are generated. It is shown that set p^*_{ijk} satisfies definitions of no second order interaction in a 2x2x2 table given by Bartlett (J. Roy. Statist. Soc. Suppl. 2, 248-252, 1935) and no interaction in a rxsxt table by Roy and Kastenbaum (Ann. Math. Statist. 27, 749-757, 1956), and is also related to that given by Good (Ann. Math. Statist. 34, 911-934, 1963).

J. M. Cameron continued his work on development of experiment designs especially suited to calibration work. An important class of designs involves comparing members of a reference set of objects with members of a set of unknown objects. The sum of known values of the reference objects is usually taken as the restraint.

One method for constructing a calibration design for $2v$ objects involves the use of the incidence matrix of a symmetric balanced incomplete block design, i.e., BIB for which $v=b$, $r=k$, $\lambda = r(r-1)/(v-1)$. The resulting design turns out to be relatively simple to analyze.

Other results on calibration designs for two equal groups have been obtained, including construction and analysis of designs which provide for estimating a left-right "position effect" in measurements of differences between objects.

Brian L. Joiner continued his investigation of the properties of the sample range in samples from Tukey's family of random variable Z defined by

$$Z = U^{\lambda} - (1-U)^{\lambda} , \quad U \text{ uniform on } (0,1) .$$

It is observed that Plackett's (Biometrika 34, 120) upper bound on the expected value of the range is attained for samples of n when $\lambda = n-1$. The expected value and coefficient of variation of the range have been computed for various n and for values of λ chosen to cover a wide range of values of $\beta_2 = \mu_4/\mu_2^2$. The results complement those surveyed by D. R. Cox (Biometrika 41, 469).

Roy H. Wampler has been investigating the problem of rounding error in least squares fits of polynomials in an effort to exhibit quantitatively the effects of various factors that determine accuracy.

A number of test problems have been run on different computers using various algorithms. The results obtained show that orthonormalizing codes provide much better accuracy than do codes where elimination methods are used for matrix inversion. The test problems varied the range of X (the independent variable) as well as the degree and coefficients of the polynomial.

It was found that it is possible to get evidence from the computer regarding the severity of the rounding error by refitting to the predicted values of Y after carrying out the usual fit of a kth degree polynomial to n observed Y's. Let $\hat{Y}_i$ denote the predicted values from the first fit, and $\hat{\hat{Y}}_i$ the predicted values from the refit. Let (C_{ij}) denote the inverse of the matrix $X'X$. Then

$$S_j = \frac{\sqrt{\sum_i (\hat{Y}_i - \tilde{Y}_i)^2}\ \sqrt{C_{jj}}}{\sqrt{n - k - 1}}$$

is a measure of the rounding error in each coefficient, $\hat{\beta}_j$. In all cases tested, whenever the elements of the matrix $X'X$ contain no more digits than the computer's capacity, then the rounding error in the computer's $\hat{\beta}_j$ was never more than 5.2 times S_j.

R. L. Chamberlain prepared a summary of the results of published studies of the behavior of two pseudo-random number generators, the mixed congruential and the multiplicative congruential. The mixed congruential method defines a sequence of integers $\{X_i\}$ defined by

$$X_{i+1} = A X_i + C \pmod{P},$$

and the special case $C=0$ is the multiplicative congruential method.

Ten tests (or types of tests) for randomness are described and the published test results are listed for each set of parameter values (X_0, A, C, P) that has been proposed.

Publications:

(1) Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society of Tool and Manufacturing Engineers.

(2) Notes on the use of propagation of error formulas. H. H. Ku. NBS J. Research C. (Engineering and Instrumentation), Vol. 70C, 1966, pages 263-273.

(3) Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.

(4) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.

(5) Useful alternatives to Chauvenet's rule for rejection of measurement data. T. A. Willke. Submitted to a technical journal.

(6) A note on contaminated samples of size three. T. A. Willke. NBS J. Research B. (Math. and Math. Physics), Vol. 70B, 1966, pages 149-151.

3. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING

Task 13911-61-1390951/51-1

Origin: NBS Authorized 7/1/50
Managers: J. M. Cameron, H. H. Ku
Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following.

David Hogben assisted S. B. Garfinkel (NBS Radiation Physics Division) in adaptation of his use of a triangular P.B.I.B. design in preparing point-source radioactive standards. Modifications were needed to take into account radioactive decay in the source. The appropriateness of an additive model was investigated and a method of analysis was developed using observations adjusted for the decay constant of the source, with weights.

The section has continued its collaboration with a number of the Bureau's calibration laboratories on statistical and computational aspects of their work. Mass calibration, voltage calibration, and polygon calibration are among the programs in which there is continuing collaboration.

Brian L. Joiner, in collaboration with Donald McSparron and Robert C. Raybold (NBS Metrology Division), is working on the analysis of data from past records of the H.C.P. (horizontal candle power) and Lumen standards, and on experiment designs, to develop the statistical procedures for evaluation of the precision of the calibration processes associated with these standards.

Janace A. Speckman has been working with Donald McSparron (NBS Metrology Division) to develop nonlinear least squares procedures involved in the calibration of a thermal pile for the use of laser beam in an experiment in radiometry.

J. M. Cameron and H. H. Ku assisted Seymour Edelman (NBS Mechanics Division) in the planning of a round-robin interlaboratory experiment on the testing of accelerometers.

Joan R. Rosenblatt worked with Herbert Dixon (NBS Analytical Chemistry Division) on least squares procedures that might be proposed as alternatives to existing procedures (approximate calculations or an optical fitting technique) for fitting a hyperbola to a cooling curve that arises in connection with a method for determining the fraction of impurities in high-purity substances.

A continuing program of collaboration with experimental scientists in activation analysis and Mossbauer spectroscopy has been begun by David Hogben.

During the fall semester, three members of the section taught in-hours courses in the NBS Graduate School program: Statistics of Measurement (Mary G. Natrella) and Computer Programming Laboratory (Sally T. Peavy and Ruth Varner).

In collaboration with Philip J. Walsh (NBS Computer Services Division), Sally T. Peavy completed preparation of a manual on deck preparation for IBSYS jobs to be run on the Bureau's computer.

Publications:

(1) Statistical evaluation of uncertainties associated with the reported values. H. H. Ku. Appendix in Analytical Procedures for Isotopic Analysis, edited by W. R. Shields, NBS Technical Note 277, July 25, 1966, pages 77-99.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies — Authorized 3/31/58
Manager: J. M. Cameron
Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

David Hogben visited the White Sands Missile Range to present lectures on the use of the OMNITAB program and as followup provided advisory services on the solution of problems using OMNITAB.

J. M. Cameron participated in the presentation of a course on Computer Science, for undergraduate students from three Maryland colleges. This course is part of a collaborative program on training in the computer sciences, supported in part by the National Science Foundation.

The Bureau, jointly with the Harry Diamond Laboratories (Department of the Army), served as host organization for the Twelfth Conference on Design of Experiments in Army Research, Development and Testing, in October 1966. The conference was held at the NBS Gaithersburg site.

Numerous inquiries were received and handled in two categories: use of the OMNITAB program, and statistical tables.

STANDARD PROGRAMMING AIDS FOR STATISTICAL COMPUTATIONS

Project 4512115

Origin: NBS — Authorized 1/21/66
Manager: J. M. Cameron
Full task description: January - June 1966 issue, p. 9

Status: CONTINUED. Sally T. Peavy has prepared, for distribution to NBS computer users, a description of procedures for use of the BMD package of statistical programs. Programs are on magnetic tape and the user can select the necessary program by specifying the name of the desired program.

Programs for inverting a matrix or solving a set of linear equations with a computed check on the accuracy of the inverted matrix have been made available for use on several computers that are used by the NBS staff. These programs were prepared by Sally T. Peavy and Morris Newman from a SHARE library subroutine. An internal check on machine computation is incorporated as well as the error bounds which depend on the following:

Let A be the input matrix to be inverted and X the result of the inversion. Define $Y = I-AX$. In order to guarantee that X be a good approximation to A^{-1}, it is necessary to have $ERR = N(X)N(Y)/[1-N(Y)]$ small, where $N(\ )$ is a norm. The matrix inversion package computes ERR for three different norms.

The main part of a program called OMNISTAT has been written by Alfred Beam (University of Maryland). It is based on the principles of the OMNITAB program, modified and extended to provide (i) increased size and flexible dimensions for the worksheet, (ii) addition of tape commands for extended storage capability and (iii) convenient addition and deletion of subroutines. This program will be available for the IBM 7090/94 computer in both single and double precision.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 20540-12-2050141/55-57

Origin: NBS — Authorized 9/1/54

Manager: H. Oser

Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. In cooperation with the Metallurgy Division (312.00) of NBS two papers were presented on the whisker growth problem: (i) Drs. Simmon, (312.05), Corriell (312.03), and Oser (205.00) attended the International Conference on Whisker Growth in Boston, Mass. and presented a paper "On the solution of the Stefan problem on whisker growth" to be published in a special issue of the J. Phys. Chem. Solids.

(ii) H. Oser presented a paper on the mathematical aspects of the same Stefan problem during the International Congress of Mathematicians, August 1966, in Moscow, Russia.

Dr. W.L. Sadowski continued his work on the numerical solution of the nonlinear Vlasov equation. The eigenfunction expansion was extended to include travelling waves in addition to standing waves. This extension makes it possible to calculate the nonlinear conductivity of a plasma. An experimental study of truncation errors in the double expansion was undertaken. An implicit method was developed in conjunction with the explicit method to obtain better numerical stability. Scaling of the coefficient matrix was introduced to examine in detail the reconstitution of the solution in terms of eigenfunctions. A code was developed to study propagation of energy through the modes caused by nonlinear mode-mode coupling. This particular study may throw some light on the behavior of a turbulent plasma. A part of the analysis and coding on the above was done by Zella Ruthberg. A manuscript is in preparation.

A trip was made to NASA Langley field in June by Dr. Sadowski and Dr. Oser upon invitation of NASA to consult on some problems of numerical stability in the solution of the nonlinear Vlasov equation. A talk was presented at that time by Dr. Sadowski entitled "Solution of the Nonlinear Vlasov Equation by Finite Difference Methods" . A manuscript is in preparation.

A one-week seminar was organized at the Bureau in July to discuss some aspects of numerical experimentation in plasma physics and on the quasilinear theory based on the Bogolyubov-Krylov asymptotic methods.

Dr. Sadowski was invited to participate in the Physics of Reentry Conference sponsored by ARPA-IDA in November.

PLASMA RESEARCH

Task 20504-12-2050140/59-442

Task 20504-12-2050417/62-1157

Origin: NBS — Authorized 10/3/61
Sponsors: NASA and NBS — and 6/30/59
Manager: C.M. Tchen

Full task description: October-December 1961 issue, p. 12
April-June 1959 issue, p. 15

Status: CONTINUED. The research on plasma turbulence covered the spectral analysis of plasma turbulence. The spectrum of electrostatic potential was found theoretically to follow the

k^{-5} law, in agreement with the NASA Lewis Center experimental results [Dr. J. Reece Roth, NASA Lewis Research Center]. This spectrum was first found on the basis of the Fourier decomposition of the Navier Stokes equation in a manuscript "Spectrum of Turbulence in a Plasma with a Strong Magnetic Field", to be published in the Proceedings of the Summer Institute on Nonlinear Problems in Plasmas, Paris.

In the absence of the magnetic field, the coupling between the turbulent velocity and the electrostatic field gives rise to a diffusion by electrostatic fluctuations. This investigation gave a spectrum k^{-5} in electrostatic potential and a turbulent velocity spectrum k^{-3}. These results were reported in a manuscript entitled "Turbulence by Electrostatic Fluctuations", by C.M. Tchen, accepted for publication at the Proceedings of the Summer Institute on Basic Kinetic Problems, University of Colorado, Boulder, Colorado.

The k^{-5} law of spectrum was verified by Dr. Roth experimentally to be independent of the intensity of the magnetic fields applied. The effects of the gradients will be the next topic of investigation, as Dr. Roth's experiments can cover positions where such effects are important.

The Facultyof Sciences, University of Paris, organized the International Summer Institute on Nonlinear Phenomena in Plasmas, September 5-23, 1966. Upon the invitation of the University and at its expense , C.M. Tchen attended the Institute and presented 3 lecture series. There were 17 invited lecturers, of which 5 came from the United States, a number of observer-lecturers and graduate students. C.M. Tchen was an invited lecturer, and also served as a panelist on the session "Turbulence and Mode Coupling Effects in Plasmas".

The Summer Institute covered the following areas: Quasilinear theory of plasma oscillations, mode-mode interactions, strong turbulence, scattering of electromagnetic waves, mathematical methods and models, plasmas in solids and weakly ionized gases, plasmas in astrophysics. The most significant developments in nonlinear behavior of plasmas were contributed in the sessions on strong turbulence and plasmas in solids. In the session on strong turbulence, the spectral distributions of turbulence were obtained theoretically for the cases with or without an external magnetic field, and the new experimental results of turbulent spectra from the Zeta machine by Dr. Robinson, U.K. Atomic Energy Research Establishment, Culham, showed a long portion of the turbulent spectrum in agreement with Dr. Tchen's theory. The spectrum covered the longest range of frequencies ever known among the various plasma laboratories in the United States. This agreement confirmed the sound foundation of the theory, enabling the extension of the theory to more complicated cases, and helped to advance and accelerate the work.

In addition, C.M. Tchen visited the Centre d'Etudes Nucleaires, Fontenay-aux-Roses, and presented a lecture. The Centre d'Etudes Nucleaires has a strong experimental group working on controlled thermonuclear fusion experiments, and a supporting theoretical group working on plasma instability, scattering radiation and turbulence in plasmas.

5. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS — Authorized 12/30/60
Manager: Alan J. Goldman
Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued to increase. Section staff provided such services in 115 recorded instances; of these 63 involved assistance to NBS staff (covering 11 different NBS divisions or other units). The 115 instances totalled to 892 man-hours. Other agencies assisted included Bureau of the Budget, Bureau of Public Roads, National Science Foundation, Post Office Department, National Institutes of Health, Atomic Energy Commission, Rural Electrification, Coast Guard, Office of Naval Research, and Army Corps of Engineers. Requests from universities, industry, professional groups and journals were also met.

(2) W. Hall continued participation in the design of a data store for inter-urban transport analyses. (Reported here for convenience; supported under Project 4314690.) He and A.J. Goldman continued service on the Technical Advisory Panel of a major Bureau of Public Roads project.

(3) L.S. Joel and A.J. Goldman continued participation in an analysis aimed at developing methods to estimate proper protection levels for various elements in a communications network. A technical report describing early phases of the work was submitted to the sponsor. (Reported here for convenience; supported under project 4556455.)

(4) C. Witzgall, with C. Mestenyi of U. of Maryland, prepared a manuscript on "Stable Evaluation of Polynomials".

(5) P.R. Meyers prepared two manuscripts documenting results obtained under a previous project on "Convergence of Iterative Methods".

(6) K. Kloss carried out various projects centering around the NBS PILOT computer. Specifically he: (a) Designed and implemented a compiler, COMPILOT, patterned after BASIC and FORTRAN. The system accepts algebraic expressions of considerable generality, including implicit function calls, arbitrarily complex and numerous subscripts, and intermixed integer and real-valued variables. Array dimensions are allowed to vary during execution. Input/output is format-free and particularly simple to use, though sufficiently versatile to generate attractive report style printout.

The system includes a subroutine library. Programs, data, and output are communicated via a teletype. Because of this, the system can be (and has been) utilized from a remote terminal equipped with a dataphone.

(b) At the request of Dr. W. Sadowski (205.04), adapted a program, to solve certain partial differential equations of interest in plasma physics, for PILOT. The original program consisted of 800 FORTRAN II statements, many involving array references. The program was hand-translated into only 1300 PILOT machine language instructions. It has been debugged, but no production runs will be made until a subroutine package is prepared to provide magnetic tape output.

In the course of the transliteration, some novel machine methods were developed for efficiently referencing elements of a matrix and for handling recursively-called subroutines.

(c) At the request of Dr. Kearsley (213.05), began an investigation of isotropic tensors. The basic problem is to determine a basis for the space of isotropic tensors of given rank. For each rank, it is possible to display a collection of tensors which span the space and a set of homogeneous linear equations which includes all the dependence relations which hold for the collection.

The problem is solved by reducing the given set of relations to an independent set. If there remain r relations and if the spanning set of tensors contains s members, then there must be $s-r$ independent tensors; the relations can be used to display this basis and to represent the r dependent tensors in terms of the basis.

For ranks greater than 7, the numbers r,s are too large to make the manual calculation practical. The work is therefore being mechanized.

All the coefficients in the original set of dependence relations are ± 1. It is conjectured that the reduction to a set of independent relations can be done in such a way that no coefficients other than ± 1 arise. The original equations are represented by a coefficient matrix; the goal is to triangularize this matrix.

A special "programming language" was designed and implemented to perform various simple operations on such a matrix. For the rank 7 case, this language was used to check the conjecture mentioned above, and suggested a heuristic algorithm for automating the reduction.

At present a program exists which, for rank 8, generates the spanning set of tensors, the dependence relations, and the coefficient matrix. The reduction algorithm is being encoded.

(d) Continued the investigation of the congruence $a^{p-1} \equiv 1 \pmod{p^2}$ (a,p positive prime numbers). No new solutions were found, and the range of the calculations is currently as follows:

$$\begin{aligned} a &= 2, & p &\leq 206{,}000{,}000 \\ a &= 29, & p &\leq 102{,}445{,}000 \\ a &= 41, & p &\leq 104{,}519{,}000 \\ a &= 79, & p &\leq 21{,}248{,}000 \\ a &= 83, & p &\leq 22{,}215{,}000 \\ a &= 89, & p &\leq 36{,}000{,}000 \\ a &= 97, & p &\leq 41{,}087{,}000 \end{aligned}$$

Because of the historical interest in the case $a = 2$, the calculations are being repeated to insure accuracy. (The program, however, includes the error check that $a^{p-1} \equiv 1 \pmod{p}$ must hold.) The recomputation has been done for $p \leq 142{,}000{,}000$.

Publications:

(1) A.J. Goldman and M. Newman (205.01). Finding a Rank-Maximizing Matrix Block. Journal of Research NBS, 70B (1966), 219-220.

(2) A.J. Goldman. Operations Research Research. To appear in Proceedings of Operations Research Conference for Non-Defense Washington Area Federal Agencies (4/20/66).

(3) A.J. Goldman, A. Hobbs (Division 423) and L. Joel. The Concept of Attack Worth of Telecommunications Network Elements. NBS Report 9368, 7/66.

(4) P.R. Meyers. A Converse to the Contraction Theorem. To appear in Journal of Research NBS, 71B (1967).

(5) C. Witzgall and C. Mestenyi (U. of Maryland). Stable Evaluation of Polynomials. To appear in Journal of Research NBS, 71B (1967).

COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS — Authorized 12/30/60
Manager: Jack Edmonds
Full task description: October-December 1964 issue, p.3 April-June 1962 issue, p.15

Status: CONTINUED.

(1) Edmonds presented the paper "Optimum Branchings" at the International Seminar on Graph Theory and its Applications, sponsored by the International Computation Center, Rome, July, 1966.

He completed a paper which relates the theory of "systems of distinct representives" to the linear algebra of indeterminates.

He continued work on matroids and on combinatorial algorithms.

(2) P.R. Meyers solved the following problem: In a set S with n elements, what is the minimum cardinality of a family F of subsets such that each element of S is the intersection of some sub-family of F ?

Publications:

(1) Jack Edmonds. Optimum Branchings. To appear in "Theorie des Graphes, Actes des Journees Internationales de l' I.C.C., Rome 1966". Dunod, 1967.

(2) Jack Edmonds. SDR's and Linear Algebra. Submitted to the American Mathematical Monthly.

(3) A.J. Goldman. Realizing the Distance Matrix of a Graph. Journal of Research NBS, 70B (1966), 153-154.

(4) P.R. Meyers. Minimum Number of Subsets to Distinguish Individual Elements. To appear in Journal of Research NBS, 71B (1967).

(5) John Mather. Invariance of the homology of a lattice. To appear in the Proceedings of the American Mathematical Society. Vol 17, No 5. Oct. 1966. pp. 1220.

LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS Authorized 12/30/60
Manager: C. Witzgall
Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) In collaboration with R. Wets (Boeing Scientific Research Laboratories), C. Witzgall studied the face-structure of polyhedral sets defined by linear inequalities.

(2) J. Root continued to investigate applications of statistical decision theory and modern control theory to quality control problems of machine control and material identification. A manuscript is in progress.

Publication:

(1) R. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. Algorithms for Frames and Lineality Spaces of Cones. To appear in Journal of Research NBS, 71B (1967).

MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2050456

Origin: Technical Analysis Division, NBS Authorized 3/1/66
Sponsor: Office of High Speed Ground Transportation, Dept. of Commerce
Managers: A.J. Goldman, P.R. Meyers

Full Task description: July-December 1966 issue, p.14

Status: CONTINUED.

(1) A.J. Goldman and P.R. Meyers prepared roughly 25 memoranda on various phases of the Northeast Corridor Transportation Project, including reviews of several contractors' reports. J. Gilsinn and Meyers continued participation in conceptual and implementation phases of the Project simulation. Gilsinn and K.E. Kloss are considering the problem of efficient modification of the network "inputs" to the simulation.

(2) J. Levy continued the development of methods for evaluating feedback vs. non-feedback methods for regulating flow in a transport network. A.J. Goldman, together with G. Nemhauser of Johns Hopkins U., generalized previous results on the reducibility of certain transport investment problems to best-path problems. Goldman and Meyers, together with J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.) completed an analysis of certain market split models. In a purchased report draft, W. Horn explored the problem of choosing a planar network joining N given points, so as to minimize the sum of construction costs and travel-related costs.

(3) C. Witzgall, assisted by J. Gilsinn, revised his survey of labelling algorithms for determining shortest paths. He and K.E. Kloss continued plans for a macro language to facilitate network manipulations.

Publications:

(1) J. Gilsinn, R. Crow and R. Lerman (Office of High Speed Ground Transportation), L. McKay and S. Rose (Division 431). Method for the Initial Superdistrict Projections of Northeast Corridor Population, Employment and Income. Northeast Corridor Transportation Project Tech. Paper No. 6. 8/66.

(2) A.J. Goldman. A Transport Improvement Problem Transformable to a Best-Path Problem. NBS Report 9326, 1966.

(3) A.J. Goldman, P.R. Meyers, J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.). Analysis of a Market Split Model. NBS Report 9384, 1966.

RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING

20500-2050404/65-1456 Authorized 1-21-64
Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch
Full task description: January-June 1964 issue, p. 19

Status: CONTINUED. R. Kirsch and K. Kloss made changes in the PAX picture processing language which is being maintained at NBS on the MIT-MAC computer in Massachusetts. The language is now being used as a research tool by two groups at MIT, by one at the University of Pennsylvania, and by one at the University of Maryland as a result of our efforts.

Several experiments on remote communication between NBS, a remote computer, and a third party were performed. Kirsch interchanged both picture data and programs on-line with NIH, MIT, the University of Pennsylvania and the University of Maryland.

Mrs. Rhodes wrote programs for computing gradients of images in PAX. With these programs Kirsch was able to produce 3-dimensional reconstructions of a nerve cell from optical serial section data prepared by Dr. Lipkin at NIH.

Experiments were performed by Kirsch on modifications to an automatic syntax analyzer written at SDC in LISP 1.5 on the Q-32 computer. Mrs. Rhodes is continuing this work.

Joseph Becker wrote a report on the syntax of picture representation.

Dr. Rosenfeld wrote two reports on texture analysis and on grammars for pictures.

Kirsch and Dr. Lipkin are working on a model for the structure and morphogenesis of C.N.S. tissue.

Mrs. Rhodes made several studies of programming capabilities of different computers for NIH. Kirsch served as a consultant to NIH on the pharmacology and toxicology research program and the digital computer research and technology program.

RESEARCH ON A PICTURE LANGUAGE MACHINE

20500-12-2050406/65 Authorized 5-1-61
Origin: NBS
Sponsor: National Science Foundation
Manager: Russell A. Kirsch
Full task description: July-December 1963 issue, p. 17

Status: CONTINUED. Several searches were made through the physics literature by Kirsch with Dr. Alt using the remote MAC computer facilities. Considerable use was made of the computer plus a two remote party hookup both for instructional purposes and to try out novel file interrogation methods.

MATHEMATICAL TABLES

20500-40-2050121/57-216
Origin and Sponsor: NBS
Manager: I. A. Stegun
Full task description: July-December 1964 issue, p. 4

Status: CONTINUED. The fifth printing (with corrections) of AMS 55 - the Handbook of Mathematical Functions appeared during the period. The correction file, updated reference file and "improved methods" file for the volume are continually being maintained to facilitate the preparation of a second edition and to aid in our consulting and advisory service.

A second edition of CUP13 - Tables Relating to Mathieu Functions has been approved for publication in the AMS series.

Preliminary explorations are underway on the possibility of either including a section or supplement to AMS 55 on tested algorithms for the computation of special functions.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

BRUTER, Claude — (Mathematics Department, University of Waterloo, Ontario, Canada) A Small Survey on Mathematical Programming with Integer Variables. December 29, 1966.

FEIX, M. — (NASA Langley Research Center, Hampton, Va.) Philosophy of the Numerical Experiments. July 25, 1966.

Water Bay Model Application to the Theory of Inhomogeneous Plasmas. Minimum Energy Principle. Case of Gravitational Forces. July 27, 1966.

Numerical Analytical Properties of the Vlasov Equation. Fourier-Hermite Expansion Techniques. July 26, 1966.

Krylov-Bogolyubov Method for the Wave Equation. Case of the Cold Plasma. Physical Picture of the Coupling between Longitudinal and Transverse Wave. July 28, 1966.

PETERSON, Elmor — (Department of Mathematics, West Virginia University, Morgantown, West Virginia.) Geometric Programming. November 29, 1966.

REINER, Irving — (Department of Mathematics, University of Illinois, Urbana, Illinois) Tensor Products of Matrices over a Field of Characteristic p. July 25, 1966.

NBS In-hours Courses Given by Staff Members

NATRELLA, Mary G. — Statistics of Measurement. Fall Semester

PEAVY, Sally T. and VARNER, Ruth N. — Computer Programming Laboratory. Fall semester.

Mathematical Statistics Seminar

BIRNBAUM, Allan — (Courant Institute of Mathematical Sciences, New York University, New York, New York.) Optimal Robustness: A general approach, with applications to linear estimation of location. November 10, 1966.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J. M. — Design of experiments. Presented at the University of Maryland, Department of Mathematics, December 14, 1966.

EDMONDS, J. — Optimum Arborescences. Presented at the International Seminar on Graph Theory and its Applications, Rome, Italy, sponsored by International Computation Center. July 6, 1966.

GOLDMAN, A. J. — A Transport Improvement Problem Transformable to a Best-Path Problem. Presented at the National Meeting of the Operations Research Society of America, Durham, North Carolina. October 17, 1966. (With G. Nemhauser, Johns Hopkins University.)

Mathematical Investigations for the Northeast Corridon Transportation Project. Presented at the NECTP Review Conference, Cacapon Lodge, Berkely Spring, West Virginia, November 17, 1966.

GOLDMAN, A. J. — Asymptotic Analysis of a First-Moment Integral and its Derivatives. Presented at the Mathematical Association of America Meeting, Reiss Science Building, Georgetown University, Washington, D. C. December 10, 1966.

HALL, W. — Application of FFS to Northeast Corridor Files. Presented to various government agencies. Sponsored by TAD (431), September 28, 1966.

Application of FFS to Northeast Corridor Files. Presented to various government agencies. Sponsored by TAD (431), November 8, 1966.

HOGBEN, D. — OMNITAB Program. Presented at the Harry Diamond Laboratory, Washington, D.C. July 12, 1966.

Fitting a Straight Line with Repeat Measurements and a Between Component. Presented before the Biometric Society, Los Angeles, California, August 15, 1966.

Fitting a Straight Line with Repeat Measurements and a Between Component. Presented before the Department of Mathematics, University of Maryland, October 26, 1966.

KIRSCH, R. A. — Pattern Recognition in Radiology. Presented at the University of Chicago, Chicago, Illinois. October 21, 1966.

Picture Syntax. Presented at the IEEE Meeting, Las Croabas, Puerto Rico, October 26, 1966.

MEYERS. P. R. — Minimum Number of Subsets to Distinguish Individual Elements. Presented at the Mathematical Association of America, Reiss Science Building, Georgetown University, Washington, D. C. December 10, 1966.

NEWMAN, M. — Five lectures on "Group Representations". Presented to the Physics Faculty, University of Nijmegen, Holland, October 20-October 26, 1966.

OLVER, F. W. J. — The Liouville-Green (or WKB) Approximation. Presented to the Departments of Mathematics and Physics, University of Toronto, Toronto, Ontario, Canada, July 22, 1966.

Error Analysis of Asymptotic Expansions and Asymptotic Solutions of Second-Order Differential Equations Near an Irregular Singularity. Presented at the Canadian Mathematical Summer Institute, University of Alberta, Edmonton, Alberta, Canada, July 25-29, 1966.

Error Bounds for Asymptotic Expansions of Special Functions in the Complex Plane. Presented at the National Physical Laboratory, Glazebrooke Hall, Teddington, England, September 1-7, 1966.

Error Analysis of Asymptotic Expansions. Presented at the University Mathematical Laboratory, University of Cambridge, England, September 8, 1966.

Error Bounds forWatson's Lemma. Presented at the International Congress of Mathematicians, Moscow University, U.S.S.R., August 16-26, 1966.

OSER, H. — On the Solution of a Stefan Problem. Presented at the International Congress of Mathematicians, Moscow University, U.S.S.R., August 16-26, 1966.

ROSENBLATT, J. R. — Distribution-free Two Sample Tests. Presented at the University of Maryland, Department of Mathematics, November 9, 1966.

SADOWSKI, W. — Solution of the Non-linear Vlasov Equation by Finite Diference Methods. Presented at NASA Langley Research Center, Hampton, Virginia, July 8, 1966.

TCHEN, C. M. — Spectrum of Turbulence in a Plasma with a Strong Magnetic Field. Presented at NASA Langley Research Center, Hampton, Virginia, July 13, 1966.

Nonlinear Scattering of a Laser Beam from a Plasma. Presented at NASA Langley Research Center, Hampton, Virginia, July 14, 1966.

1) Spectrum of Plasma Turbulence
2) Nonlinear Incoherent Scattering from a Plasma
3) Turbulence in a Rarefied Plasma
Presented at NBS Laboratories and ITSA of ESSA, Boulder, Colo., August 1-12,1966.

TCHEN, C. M.

1) Quasilinear Theory of Incoherent Scattering
2) Plasma Turbulence
Presented at the Summer Institute on Basic Kinetic Theories, University of Colorado, Boulder, Colorado, August 9-11, 1966.

1) Plasma Turbulence in a Magnetic Field
2) Turbulence in a Rarefied Plasma
3) Nonlinear Scattering of a Laser Beam from a Plasma.
Presented at the International Summer Institute on Nonlinear Phenomena in Plasmas, University of Paris, Orsay, France, September 5-23, 1966.

Plasma Turbulence. Presented at the Centre d''Etudes Nucleaires, Fontenay-aux-Roses, Seine-et-Oise, France, September 23, 1966.

Spectrum of Turbulence in a Plasma with a Strong Magnetic Field. Presented at Lewis Research Center, NASA, Cleveland, Ohio, October 31, 1966.

Turbulence in Plasmas. Presented at the International Symposium on Plasma Turbulence and Shock Waves, Case Institute of Technology, Cleveland, Ohio, November 1, 1966.

Plasma Turbulence in a Magnetic Field. Presented at the Annual Meeting of the Plasma Physics Division, A.P.S., Boston, Massachusetts, November 2-5, 1966.

Turbulence in Ionosphere. Presented at the Arecibo Ionospheric Synposium, San Juan, Puerto Rico, November 8, 1966.

Turbulence in Plasmas. Presented at Temple University, Philadelphia, Penna., November 14, 1966.

1) Turbulence in Fully and Weakly Ionized Plasmas
2) Spectrum and Diffusion in a Turbulence Plasma with a Magnetic Field.
Presented at the Symposium on Plasma Turbulence and Wakes, Institute for Defense Analysis, Arlington, Virginia, November 29-30, 1966.

Plasma Turbulence in a Wake. Presented at the General Electric Company, Valley Forge, Penna., December 8-9, 1966.

Publication Activities

1.0 PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.1 Mathematical Tables

Handbook of Mathematical Functions, AMS 55, Fifth Printing (with corrections),(August 1966).

1.3 Technical Papers

Proof of a permanental inequality. R. A. Brualdi and M. Newman. Quart. J. Math., Oxford, 17, 234-238 (1966).

Hadamard Matrices of Order Cube Plus One. K. Goldberg. Proceedings of the American Mathematical Society, Vol. 17, No. 3, pp. 744-746 (June 1966).

Pairs of Non-Singular Matrices. K. Goldberg. J. of Research NBS, 70B, pp. 155-156,(1966).

Upper Bounds for the Determinant of a Row Stochastic Matrix. K. Goldberg. J. of Research, NBS, 70B, pp. 157-158,(1966).

Groups Preserving Ordering in Vectors. K. Goldberg. J. of Research NBS, 70B, pp. 159-160,(1966).

Realizing the Distance Matrix of a Graph. A. J. Goldman. J. of Research NBS, 70B, 153-154,(1966).

Finding a Rank-Maximizing Matrix Block. A. J. Goldman and M. Newman (205.01). J. of Research NBS, 70B, 219-220,(1966).

On a Sequence of Points of Interest for Numerical Quadrature. S. Haber. J. of Research, NBS, 70B, pp. 127-136,(1966).

Notes on the Use of Propagation of Error Formulas. H. H. Ku. (Engineering and Instrumentation), J. of Research, NBS, 70C, 263-273,(1966).

Statistical Evaluation of Uncertainties Associated with the Reported Values. H. H. Ku. Appendix in Analytical Procedures forIsotopic Analysis, NBS Technical Note 277. Edited by W. R. Shields, July 25, 1966 .

Invariance of the Homology of a Lattice. John Mather. Proceedings of the American Mathematical Society, 17, No. 5. p. 1220, (October 1966).

Error Bounds of Asymptotic Solutions of Differential Equations I. The Distinct Eigenvalue Case. F. Stenger. J. of Research NBS. 70B, pp. 167-186(July-September 1966).

Error Bounds for Asymptotic Solutions of Differential Equations II. The General Case. F. Stenger. J. of Research NBS, pp. 187-210 (July-September 1966).

Effects of a Constant Force on a Keplerian Orbit. J. P. Vinti. Proceedings of Symposium 25 of the International Astronomical Union held August 15-22, 1966 in Thessaloniki, Greece. pp. 355-362, No. 46 (1966)

A Note on Contaminated Samples of Size Three. T. A. Willke. J. of Research NBS, 70B, pp. 149-151 (1966).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.1 Tables Relating to Mathieu Functions (Second Edition of CUP 13). To appear in the AMS Series.

2.3 Technical Papers

Optimum Branchings. Jack Edmonds. To appear in "Theorie des Graphes", Actes des Journees Internationales de l' I.C.C., Rome (1966).

SDR's and Linear Algebra. Jack Edmonds. To appear in the American Mathematical Monthly, 1967.

Operations Research Research and Government O. R.. A. J. Goldman. To appear in Proceedings of Operations Research Conference for Non-Defense Washington Area Federal Agencies.

Note on Normal Subgroups of the Modular Group. L. Greenberg. Accepted for publication in the Proceedings of the American Mathematical Society.

A Functional Inequality. S. Haber. To appear in the American Mathematical Monthly 73, 1966.

A Theorem on Arbitrary Functions. S. Haber. To appear in the American Mathematical Monthly.

A Modified Monte-Carlo Quadrature - II. S. Haber. To appear in Mathematics of Computation.

A Modified Monte-Carlo Quadrature. S. Haber. To appear in Mathematics of Computation 20, 1966.

Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society to Tool and Manufacturing Engineers.

Analysis of Information -- An Alternative Approach to the Detection of a Correlation Between the Sexes of Adjacent Sibs in Human Families. H. H. Ku. Submitted to a technical journal.

Note on Partitions Modulo 5. M. Newman. To appear in Mathematics of Computation.

Classification of Normal Subgroups of the Modular Group. M. Newman. To appear in Trans. American Math. Soc.

The Coefficients of the Powers of a Polynomial. M. Newman. To appear in Journal of Research NBS.

Turbulence in a Rarefied Plasma. C. M. Tchen. To appear in the Proceedings on Advanced Problems in Fluid Mechanics, Poland.

Spectrum of Turbulence in a Plasma with a Strong Magnetic Field. C. M. Tchen. Accepted for publication in the Proceedings of the Summer Institute on Nonlinear Problems in Plasmas. Paris, France.

Turbulence by Electrostatic Fluctuations. C. M. Tchen. Accepted for publication in Proceedings of the Summer Institute on Basic Kinetic Problems, University of Colorado, Boulder, Colorado.

Algorithms forFrames and Lineality. J. B. Wets and C. Witzgall. To appear in Journal of Research NBS.

Stable Evaluation of Polynomials. C. Witzgall and C. Mestenyi (U. of Maryland). To appear in Journal of Research NBS, 71B, 1967.


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