

NATIONAL BUREAU OF STANDARDS REPORT

9425

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Semi-Annual Report
January through June 1966

For Official Distribution



U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

205.0

NBS REPORT

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U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through June 30, 1966

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°Only unclassified material is included in this report.



Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS

Authorized 8/29/54

Manager: Morris Newman

Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman and L. Greenberg have been studying the normal subgroups of the modular group Γ . They have shown for example that:

- (1) if there is no normal subgroup of index μ , then there is none of index $p^k \mu$, where p is a prime $> \mu$ and $k \geq 0$,
- (2) there are only finitely many normal subgroups with a given number of parabolic classes, and in particular there are none with p parabolic classes where p is a prime $\equiv -1 \pmod{3}$ and none with $2p$ parabolic classes where p is a prime $\equiv 1 \pmod{12}$,
- (3) with finitely many exceptions the normal subgroups of square-free index are all of genus 1,
- (4) the set of integers μ such that the normal subgroup G is of index μ and Γ/G is solvable has 0 density.

M. Newman and J. Lehner have classified those Riemann surfaces admitting the maximal number of automorphisms, and have found matrix representations over certain algebraic number fields for the associated automorphism groups. These have led to new families of surfaces having the maximal number of automorphisms.

M. Newman and J. Lehner have been considering sums of the type $\sum f(k, k')$, summed over all consecutive pairs of denominators k, k' in the Farey series of order n , and have found asymptotic formulas for this sum for such cases as f a polynomial, and $f = \frac{1}{k k' (k+k')}$. The latter

gives

$$\sum \frac{1}{k k' (k+k')} \sim \frac{12 n \log 2}{\pi^2}.$$

M. Newman and M. Marcus have determined when the associated matrix function $K(A)$ can be doubly stochastic. If K is the r^{th} Kronecker product then $K(A)$ is doubly stochastic if and only if $A = \zeta S$ where ζ is an r^{th} root of unity and S is doubly stochastic; in all other cases A must be a scalar multiple of a permutation matrix.

Status of Projects

M. Newman and J. L. Mennicke have proved the "congruence subgroup property" for all algebraic number fields except the totally imaginary ones.

M. Newman has proved that

$$\lim_{x \rightarrow \infty} \inf \frac{s(x)}{x} \geq \frac{1}{5} + \frac{36}{5.19^4}$$

where $s(x)$ is the number of integers $n \leq x$ such that the number of partitions of n is divisible by 5.

A. J. Goldman and M. Newman have given an algorithm to determine a matrix X such that the matrix

$\begin{bmatrix} A & B \\ C & X \end{bmatrix}$ has largest possible rank, where all matrices are over some given field. If A is $s \times s$,

B $s \times t$, C $t \times s$ and if $\begin{bmatrix} A & B \end{bmatrix}$ has rank β , $\begin{bmatrix} A \\ C \end{bmatrix}$ rank γ then this maximum is just

$\max(s - \beta, s - \gamma)$.

S. Haber continued studies of number-theoretical methods of multiple quadrature, and of the related question of the irregularity of distribution of finite sets of points in the unit square. He also continued experimental studies of his modified Monte-Carlo quadrature methods. He began a study of the zeros of certain polynomials in a meromorphic function and its derivatives.

K. Goldberg has continued his investigation of the maximum determinants of a set of row stochastic matrices. A lower bound was found which is the actual value in all cases computed to date. He began an investigation of those non-negative matrices for which the sum of the squares of the differences of elements appearing in the same row or column is a minimum under certain restrictions.

Publications:

- (1) Kernels and the Kronecker product of graphs. R. A. Brualdi. To appear in Proceedings of the American Mathematical Society.
- (2) Proof of a permanental inequality. R. A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).
- (3) Hadamard matrices of order cube plus one. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
- (4) Semi-groups with zeroids. K. Goldberg. To appear in the Journal of Research NBS.

Status of Projects

- (5) A recurrence related to monotone subsequences in permutations. K. Goldberg. To appear in the Journal of Research NBS.
- (6) The l.u.b. of a set of determinants. K. Goldberg. To appear in the Journal of Research NBS.
- (7) The l.u.b. of a set of determinants of order 3. K. Goldberg. To appear in the Journal of Research NBS.
- (8) Upper bounds for the determinant of a row stochastic matrix. K. Goldberg. To appear in the Journal of Research NBS.
- (9) Groups preserving ordering in vectors. K. Goldberg. To appear in the Journal of Research NBS.
- (10) Pairs of non-singular matrices. K. Goldberg. To appear in the Journal of Research NBS.
- (11) On a theorem of Ahlfors. L. Greenberg. Accepted for publication in the American Journal of Mathematics.
- (12) Kleinian groups. L. Greenberg. Accepted for publication in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965).
- (13) Note on normal subgroups of the modular group. L. Greenberg. Accepted for publication in the Proceedings of the American Mathematical Society.
- (14) A functional inequality. S. Haber. To appear in the American Mathematical Monthly.
- (15) A theorem on arbitrary functions. S. Haber. To appear in the American Mathematical Monthly.
- (16) A modified Monte-Carlo quadrature - II. S. Haber. Accepted for publication in Mathematics of Computation.
- (17) On the very low temperature behavior of a ferromagnet. S. Haber and S. J. Glass. Submitted to a technical journal.
- (18) On a sequence of points of interest for numerical quadrature. S. Haber. To appear in the Journal of Research NBS Section B.
- (19) A modified Monte-Carlo quadrature. S. Haber. To appear in Mathematics of Computation, 1966.
- (20) Note on partitions modulo 5. M. Newman. To appear in Mathematics of Computation.
- (21) Doubly stochastic associated matrices. M. Marcus and M. Newman. To appear in the Duke Math. Journal.

ASYMPTOTIC EXPANSIONS
Task 20501-11-2050421/63

Origin: NBS
Sponsor: U. S. Army Research Office, Durham, N. C.
Manager: F. W. J. Olver
Full task description: July-December 1963 issue, p. 2

Authorized 9/10/63

Status: CONTINUED. The preliminary error analyses of Laplace's method and the method of steepest descent have been improved upon, and some applications made. A similar investigation of the method of stationary phase is now being carried out.

Publications:

- (1) Error bounds of asymptotic solutions of differential equations I. The distinct eigenvalue case. F. Stenger. To appear in the Journal of Research NBS.
- (2) Error bounds for asymptotic solutions of differential equations II. The general case. F. Stenger. To appear in the Journal of Research NBS.

2. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS

Authorized 10/1/62

Manager: Joan Raup Rosenblatt

Full task description: July - December 1962

Status: CONTINUED. T. A. Willke has completed his study of contaminated samples of size three. Estimation of the mean and standard deviation using the closest two of three observations in a sample from a normal population with contamination by slippage of the mean is investigated by a sampling study. Lieblein's results (NBS J. Res., Vol. 48, 1955, pages 255-268), which indicated that the use of these statistics is not advisable for non-contaminated samples, are borne out by this study for contaminated samples as well.

David Hogben has investigated problems that arise in fitting a straight line with repeat measurements and a "between" component of variance. Suppose observations can be represented by $y_{ij} = \alpha + \beta x_i + \epsilon_i + \eta_{ij}$, where ϵ_i represents non-linearity and the η_{ij} , associated with variation of n_i observations at x_i , are assumed to behave as $NID(0, \sigma_\eta^2)$. Least squares estimators for the unknown parameters α and β are given when (i) ϵ_i are assumed to be $NID(0, \sigma_\epsilon^2)$ and (ii) the ϵ_i represent systematic errors.

In case (i) it is noted that although the ϵ_i and η_{ij} are independent, the random errors $(\epsilon_i + \eta_{ij})$ are not independent. The "best" estimator for β is obtained by weighting the means of the observations at each x_i by $w_i = n_i/(n_i + \theta)$, where $\theta = \sigma_\eta^2/\sigma_\epsilon^2$. Since θ is usually unknown, two other estimators are discussed, where the weights are 1 and n_i (the case usually considered) respectively. Numerical results of a study of the efficiency of these two estimators are discussed for a wide variety of situations. It is seen that the estimator obtained using equally weighted means is more efficient, unless σ_η^2 is substantially greater than σ_ϵ^2 . In case (ii) $w_i = n_i$ yields the "best" estimator.

Brian L. Joiner has investigated properties of the standard deviation, mean deviation and range for small samples from some non-normal distributions. For example, Tukey random variables, defined by the transformation

$$Z = U^\lambda - (1-U)^\lambda$$

where U is uniformly distributed on $(0,1)$, are such that properties of the sample range are easily obtainable. The results are being examined in connection with a study of the general problem of robustness of scale parameter estimators.

Brian L. Joiner has investigated the problem of the interpretation of the results of independent tests of significance of the same null hypothesis. An expository note has been written giving 4 tables useful in the interpretation of multiple significance levels.

Brian L. Joiner has prepared an annotated list of references on probability plotting in the hope that it will serve as a general introduction to the subject for those who are interested primarily in applications.

H. H. Ku worked out an illustration of the analysis of multi-way contingency tables by the information approach, using data that were presented and analyzed in a different way by Greenberg and White (J. Amer. Statis. Assoc., Vol. 60, 1965, pp. 1035-1045). The analysis of information tables was given for testing hypotheses of independence, homogeneity, conditional independence and homogeneity, and Markovity.

Publications:

- (1) Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society of Tool and Manufacturing Engineers.
- (2) The bridge tournament problem and calibration designs for comparing pairs of objects. R. C. Bose and J. M. Cameron. NBS J. Research B. (Math. and Math. Physics), Vol. 69B, 1965, pages 323-332.
- (3) Notes on the use of propagation of error formulas. H. H. Ku. To appear in NBS J. Research C. (Engineering and Instrumentation).
- (4) Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.
- (5) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.
- (6) Useful alternatives to Chauvenet's rule for rejection of measurement data. T. A. Willke. Submitted to a technical journal.
- (7) A note on contaminated samples of size three. T. A. Willke. To appear in NBS J. Research B. (Math. and Math. Physics).

3. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING

Task 13911-61-1390951/51-1

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku

Full task description: July - September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data.

Typical examples of the services performed are the following. (a) The Statistical Engineering Laboratory is intimately involved in the planning and execution of the transfer from old to new Bureau headquarters of the national electrical standard used to maintain the unit of electromotive force (emf) or voltage (the "standard volt"). This standard consists of 44 cells, housed under controlled conditions and a number of secondary or travelling groups to aid in the transfer. The experiment design for the transfer calls for a number of steps, at each of which one or more subgroups of the cells are transported and intercomparisons among the cells are performed and analyzed. J. M. Cameron devised experiment designs for the intercomparisons of groups of cells and with Janace A. Speckman, developed computer programs for analysis of the data.

(b) H. H. Ku continued work on the statistical evaluation of uncertainties of reported determinations of isotopic abundance ratios and atomic weights of selected elements by mass spectrometric methods. He prepared an Appendix to "Analytical Procedures for Isotopic Analysis," edited by W. R. Shields (NBS Analytical Mass Spectrometry Section), discussing the application of general principles of experiment design and data analysis in this context and providing a detailed example illustrating the various steps leading from original data to the reported uncertainties for the isotopic ratio of bromine.

(c) The section has continued its collaboration with a number of the Bureau's calibration laboratories, on statistical and computational aspects of their work.

(d) In connection with the Bureau's National Standard Reference Data System program, Joan R. Rosenblatt participated in a Workshop on Mathematical and Statistical Analysis of Experimental Data and Techniques for Information Storage and Retrieval, held at the Thermophysical Properties Research Center, Purdue University.

(e) During the spring semester, two members of the section taught in-hours courses: Fortran Programming (Ruth Varner) and Nonparametric Statistical Techniques (Joan R. Rosenblatt).

Publications:

- (1) Statistical evaluation of uncertainties associated with the reported values. H. H. Ku. To appear as Appendix in Analytical Procedures for Isotopic Analysis, NBS Technical Note 277, edited by W. R. Shields.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January - March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

Senior personnel from precision measurement and calibration laboratories attended a three-day seminar on Precision and Accuracy in Measurement and Calibration conducted by the Statistical Engineering Laboratory. The seminar was held twice, for two groups of about twenty participants each. The following lectures were presented.

The National Measurement System. R. D. Huntoon
(Director, Institute for Basic Standards)

Measurement as a Production Process. P. E. Pontius
(Chief, Mass and Volume Section) and J. M. Cameron

Realistic Evaluation of Precision and Accuracy.
Churchill Eisenhart (Director's Office)

Design of Experiments. W. J. Youden
(George Washington University)

Propagation of Error in a Chain of Laboratories.
J. R. Rosenblatt

Calibration Curves. H. H. Ku

Case Histories. J. M. Cameron

The Emerging Role of Computers in the Laboratory.
J. Hilsenrath (Chief, Equation of State Section)

Reporting Final Results. C. Eisenhart

In addition to the lectures, there were discussion sessions for follow-up of questions raised at the lectures and for exchange of experience among the participants. Other notable activities included (a) presentation by J. M. Cameron and David Hogben of a series of lectures on the Bureau's OMNITAB program for staff of the Calibration Laboratory at White Sands Missile Range; (b) statistical computations for the Subcommittee on Administrative Procedures of the Senate Judiciary Committee; (c) advisory service to the Bureau of Labor Statistics in obtaining multiple regression computation procedures with a satisfactory freedom from round-off error.

STANDARD PROGRAMMING AIDS FOR STATISTICAL COMPUTATIONS

Project 4512115

Origin: NBS
Manager: J. M. Cameron

Authorized 1/21/66

Objective: To develop programming aids for statistical calculations.

Background. Computer utilization for statistical calculations in the analysis of experimental data is of special importance because the applicability of statistical methods crosses subject matter boundaries. However, potential users find programming to be a troublesome and often insurmountable barrier to computer usage even though subroutines for the basic statistical calculations are available. Almost always the scientist has problems related to data input to the computer and has special data handling needs (transformations, temperature corrections, tests for aberrant results, etc.) prior to the statistical analysis. Usually he has special output needs in addition to that of the basic analysis (values for environmental factors, summary statements, special tables, etc.). It is these special needs that require the major programming effort and cause the bottleneck when treated one at a time in the fashion of work orders in a job shop.

There is need for an easier method for communication with a machine in the language of the experimenter. NBS has made some progress through the development of a general-purpose coding system, called OMNITAB, which permits communication with the machine in ordinary English sentences. A similar program for statistics would save much time and make machines available to a large group of scientists who now cannot afford to use them. The goal of this project is to develop such a statistical program which can be called on with a word and sentence structure as simple and as near ordinary spoken language as feasible.

Status: NEW. This project is a part of the program of the new NBS Center for Computer Science and Technology.

The first task has been initiated, to collect and make operational on the NBS machine existing routines, especially general purpose packages such as BMD, and to provide documentation of their properties and to evaluate their performance.

Modifications of the OMNITAB program are being prepared that will make it better suited to the needs of statistical analysis--first of all, to provide for larger data arrays.

Preparation of "standard subroutines" has begun with work on development of a matrix inversion subroutine that incorporates computation of an error bound for the inverse matrix.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 20540-12-2050141/55-57

Origin: NBS

Authorized 9/1/54

Manager: H. Oser

Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. Drs. Simmons and Corriell of the Metallurgy Division and H. Oser completed their joint efforts on the one-dimensional whisker-growth equation. A full numerical solution for wide ranges of the parameters is now on hand. Both the short-time and long-time asymptotic results have been numerically confirmed. The physical significance of the results lies in the fact that they open the way to determining surface diffusion constants for crystals and for deriving meaningful expressions for the mean stay time of adatoms on such crystals.

The physical results were presented at the "International Conference on Crystal Growth" in Boston, Mass., June 20-25, 1966 and will be published in the "Journal of Chem. and Physics of Solids".

The theoretical part of the investigations is to be presented at the International Congress of Mathematicians, to be held at Moscow, Russia in August, 1966.

Dr. W. Sadowski continued his work on the numerical solution of the Vlasov equation. He is able now to follow several plasma oscillations and observe Landau damping for different amounts of nonlinearity in the system. A new approach was developed to solving the non-linear Vlasov equation in higher dimensions. The technique consists of an expansion in terms of eigenfunctions of the linearized equation. The new method extends the range of the finite-difference method towards longer wave lengths, but overlaps sufficiently with the earlier method to ascertain full agreement between the two.

PLASMA RESEARCH

Task 20504-12-2050140/59-442

Task 20540-12-2050417/62-1157

Origin: NBSA

Authorized 10/3/61
and 6/30/59

Sponsors: NASA and NBS

Manager: C.M. Tchen

Full task description: October-December 1961 issue, p. 12
April-June 1959 issue, p. 15

Status: CONTINUED. The research in theoretical plasma dynamics is funded on NBS project No. 20504-12-2050140/59-442 and NASA Contract R-127/09-022-024, in the form of a joint program. The progress in the 6 months consists of 2 parts

(i) Turbulence in a Rarefied Plasma, and

(ii) Spectrum of Turbulence in a Plasma with a Strong Magnetic Field.

Status of Projects

The research on "Turbulence in a rarefied plasma" emphasized the kinetic equation governing a turbulent plasma.

The derivation of the kinetic equation is based on the nonlinear Vlasov equation and the turbulent dielectric behavior of the turbulent medium. The method was equivalent to the test particle method, but extended to take account of the turbulent medium. A manuscript is in preparation.

The results of the research on the "Spectrum of Turbulence in a Plasma with a Strong Magnetic Field" were reported at the NASA Contractors Meeting, April 1966, and are as follows.

The equilibrium spectral distributions of the density and electrostatic fluctuations in a low β plasma (magnetic pressure \gg particle pressure) were investigated in the inertial and dissipative ranges. In the inertial range the electrostatic potential spectrum follows the k^{-5} law. Two methods were used: (i) The wave-mixing method determines the wave-wave interaction in the cascade process; it is a generalization of the Heisenberg theory of hydrodynamic turbulence, and gives a foundation of the concept of "eddy diffusivity". The full universal spectrum, including the inertial and dissipative ranges, was found as the solution of a nonlinear integral equation. (ii) The similitude method determines the inertial range of the spectrum, and is a generalization of the Kolmogoroff theory in hydrodynamics. An expression for the turbulent diffusion was found and depends on the turbulent density. The relation between the turbulent and the Bohm diffusions was obtained. A manuscript is in preparation.

The following talks were presented:

- 1) Nonlinear Oscillation of Collision Free Plasma, Drexel Institute of Technology, Philadelphia, Pa., April 20, 1966.
- 2) Kinetic Theory of the Turbulent Damping of Plasma, City College, New York, N.Y., April 22, 1966.
- 3) NASA Contractors Meeting, Washington, D.C., April 26, 1966.

Publication:

- (1) Turbulence in a rarefied plasma. C.M. Tchen. Accepted for publication in the Proceedings of the Polish Symposium on Advanced Problems in Fluid Mechanics, to appear in the fall of 1966.

5. OPERATIONS RESEARCH

CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS
Manager: Alan J. Goldman
Full task description: October-December 1960 issue, p.3

Authorized 12/30/60

Status: CONTINUED.

(1) Demand for miscellaneous consulting and advisory services continued heavy. Section staff provided such services in 74 recorded instances; of these 37 involved assistance to NBS staff (covering 9 different NBS divisions or other units). The 74 instances totalled to 357 man-hours. (That these data are substantial underestimates is indicated by the fact that totals for the second half of the period, after a reminder of the importance of reporting such services, exceed those for the first half by over 75%.) Other agencies assisted included the Bureau of the Budget, Interstate Commerce Commission, National Science Foundation, Bureau of Public Roads, National Institutes of Health, Federal Aviation Agency, and Office of Civil Defense. Requests from universities, industry, professional groups and journals were also met.

(2) L.S. Joel continued leadership in the work in modelling some aspects of the textile industry. A semi-technical report on the "tufted carpet" simulation was issued. (Reported here for convenience; supported under project 4314690.)

(3) W. Hall continued participation in the design of a data store for inter-urban transport analyses. (Reported here for convenience; supported under project 4314423.) He and A.J. Goldman are serving on the Technical Advisory Panel of a major Bureau of Public Roads project.

(4) J. Levy continued studies of the effects of buffer capacity in certain mail sorting devices. P. Meyers revised and extended his study of the use of "short codes" in encoding letter addresses. (Reported here for convenience; supported under project 4230450.)

(5) L.S. Joel and A.J. Goldman continued participation in an analysis aimed at developing methods to estimate proper protection levels for various elements of a communications system. K. Kloss continued participation in the development of a general purpose program for simulating a telephone network. (Reported here for convenience; supported under projects 4235455 and 4235454 respectively.)

(6) K. Kloss organized and, with W. Allman (Div. 431), taught an NBS course in simulation programming languages. The languages presented were Militrans, Simscript, GPSS-III, and DYNAMO. The course objective was to enable the students to choose a suitable programming language for a given simulation task. (Reported here for convenience; supported under project 1250959.)

(7) K. Kloss proved the following theorem: For positive relatively prime integers m, n define

$$F(m, n) = (m^{\phi(n)} - 1)/n,$$

where $\phi(n)$ is the Euler phi function. Then if p is an odd prime not dividing m , and k is a positive integer,

$$F(m, p^k) \equiv F(m, p^{k+1}) \pmod{p^k}.$$

This is very similar to a congruence reported earlier for Wilson quotients.

(8) K. Kloss carried out several number-theoretic calculations on the NBS PILOT computer. These included: Discovery of all prime factors $< 1,000,000,000$ in the first 1000 Fibonacci and Lucas numbers; testing the first 20 roots of the Riemann zeta function for linear independence (with Dr. R. Stemmle, 205.01); and tabulating the positive numbers $N < 120,000$ for which $x^2 - Ny^2 = -1$ admits no integer solutions.

(9) K. Kloss wrote various pieces of "system programming" for the PILOT computer. Included were: A debugging routine to print selected portions of memory during a running program, in a variety of formats, and showing the symbolic names of locations; a magnetic tape monitor allowing one to manipulate and edit a tape library; and some extensions of the PEAP assembler, including the ability to change input devices in the midst of an assembly. (Reported here for convenience; partially supported under project 4236555.)

Publications:

- (1) A.J. Goldman and M. Newman (205.01). Finding a Rank-Maximizing Matrix Block. To appear in Journal of Research NBS, 70B, 1966.
- (2) A.J. Goldman. Operations Research Research. To appear in Proceedings of Operations Research Conference for Non-Defense Washington Area Government Agencies (4/20/66).
- (3) Some number-theoretic computations. K.E. Kloss. Journal of Research NBS, 69B, 335-336, 1965.

COMBINATORIAL METHODS

Task 205-12-2050152

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full task description: October-December 1964 issue, p.3 April-June 1962 issue, p.15

Status: CONTINUED.

(1) J. Edmonds spent the first four months of this period at the University of Waterloo, teaching "Extremal Combinatorics" and working with several people there.

(2) Edmonds showed that a matroid M_1 is homomorphic to a matroid M_2 on the same set of elements, if and only if there is a matroid M which reduces to M_1 and contracts to M_2 .

(3) An "arborescence" (or "connected branching") T is a tree whose edges are directed so that each is directed toward a different node. Exactly one node of T , called the root, has no edge of T directed toward it. Where G is a directed graph with a real numerical weight for each edge, Edmonds found a good algorithm for finding in any given G a spanning arborescence with prescribed root (if there is one) whose edges have maximum total weight. For any directed graph, he characterized the polyhedron whose vertices correspond to the spanning arborescences of the graph. An application is minimum-cost relaying of a message to all points, when costs of direct point-to-point communication are known.

(4) A report which catalogs labeling algorithms for determining shortest paths in networks has been drafted by C. Witzgall. It is intended as a preparation for large scale numerical testing of these algorithms, based on a macro language for handling network algorithms. Ideally such a language requires modifications and extensions of macro-assembling software. Work in this direction has been started by K. Kloss (Supported in part under project 2050456.)

Publications:

- (1) New Methods and Problems in Combinatorial Optimization. A.J. Goldman. Washington Operations Research Council Newsletter, Vol. 5, No. 8 (April 1966), 4-8.
- (2) Realizing the Distance Matrix of a Graph. A.J. Goldman. To appear in Journal of Research NBS, 70B, 1966.

LINEAR AND NON-LINEAR PROGRAMMING

Task 205-12-2050153

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: C. Witzgall

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) In collaboration with R. Wets (Boeing Scientific Research Laboratories), C. Witzgall completed examining the problems of finding vertices and edges of the convex hull of a finite set of points.

(2) C. Witzgall proved the existence of "orthogonal solutions" of pairs of complementary polar conical sets. This generalizes the duality theorem of linear programming, which (in one formulation) makes the corresponding assertion for pairs of complementary orthogonal linear manifolds.

(3) J.G. Root continued to investigate applications of statistical decision theory and modern control theory to the quality control problems of machine control and material identification. Also studied was the problem of identifying the structure of a stochastic control system.

Publication:

- (1) R. Wets (Boeing Scientific Research Laboratories) and C. Witzgall. Algorithms for Frames and Lineality Spaces of Cones. Submitted to a technical journal.

CONVERGENCE OF ITERATIVE METHODS

Task 205-12-2050154

Origin and Sponsor: NBS
Manager: Philip R. Meyers

Authorized 12/30/60

Full task description: October-December 1960 issue, p.3

Status: CONTINUED.

(1) The previously reported "converse to the Banach Contraction Principle" was applied to several questions concerning continuous flows. For example, if $\{T_t: t \geq 0\}$ is a 1-parameter semigroup of continuous operators on a space, it was shown that either every one or none of $\{T_t: t > 0\}$ is a contraction on some (complete) metric d_t . This has the following implication in stability theory: A dynamical system $[x(t), x \in X, t \geq 0]$ is asymptotically stable if and only if each operator $T_t: X \rightarrow X$ ($t > 0$) defined by $T_t(x(0)) = x(t)$, when restricted to a suitable subspace X_0 of X , is a contraction on some complete metric.

(2) Some examples were collected of earlier results in the literature which are "explained" by the results of the project. Completing documentation is now seen as the only remaining task; two manuscripts are in progress.

Publication:

- (1) P.R. Meyers. On the Converse to the Contraction Mapping Principle. University of Maryland doctoral thesis, 1966.

MATHEMATICAL METHODS FOR HIGH SPEED GROUND TRANSPORTATION STUDY

Task 205-12-2050456

Origin: Technical Analysis Division, NBS
Sponsor: Office of High Speed Ground Transportation, Dept. of Commerce
Managers: A.J. Goldman, P.R. Meyers
Objectives: (1) To develop mathematical models and methods useful in analysis and synthesis of

Authorized 3/1/66

possible advanced inter-urban transport systems. (2) To provide mathematical assistance to OHSGT's Northeast Corridor Transportation Project.

Background: The Northeast Corridor Project involves an ambitious attempt to use comprehensive and systematic decision methods as aids in selecting among alternative means of meeting the Corridor's growing inter-urban transport needs. This attempt both requires and stimulates new developments and applications of mathematical operations research.

Status: NEW

- (1) A.J. Goldman and P.R. Meyers prepared roughly 25 memoranda on various phases of the Northeast Corridor Project.

(2) J. Levy is developing methods for evaluating feedback vs. non-feedback methods for regulating flow in a transport network. A.J. Goldman studied the reducibility of certain transport investment problems to shortest-path problems. Goldman and Meyers continued work on the optimal location in a network of a central facility such as a transport terminal. With J. McLynn and R. Watkins (Davidson, Talbird and McLynn, Inc.), they continued a study of demand split models in which the elasticities of the demand fractions for the various transport alternatives, with respect to changes in the parameters characterizing these alternatives, are separable functions of the demand fractions.

(3) C. Witzgall undertook a computationally oriented survey of labeling algorithms for determining shortest paths. He and K.E. Kloss initiated plans for a macro language for network manipulations. (This was supported in part under Project 2050152.) Relative to the Project simulation, Kloss reviewed various available simulation languages, while L.S. Joel is examining alternative methods of random number generation.

(4) The above includes work done prior to 3/31/66 on an Interdivision Work Order basis.

RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING

20500-2050404/65-1456

Authorized 1-21-64

Origin: NBS

Sponsor: National Institutes of Health

Manager: Russell A. Kirsch

Full task description: January-June 1964 issue, p. 19

Status: CONTINUED. Extensive experiments were performed with a large time sharing system in California. Several computations connected with mathematical functions were performed by I.A. Stegun using the time sharing system's elaborate debugging capabilities for on-line programming.

Several programs written in on-line LISP are intended to result in a facility for dealing with natural language and pictures and structural descriptions of both of them.

Programs were written to produce high quality digital picture representations of scanned photomicrographs using the sixteen level Stromberg-Carlson 4020 cathode ray tube printer.

A serious linguistic study of the structural description on a class of pictorial images was made by W. C. Watt which resulted in NBS Report 9050. He exhibited a phrase structure grammar for a class of pictures (cattlebrands) and a partial analysis of the corresponding natural language descriptions.

Dr. Rosenfeld did research on the analysis and synthesis of textures in pictures. It is hoped that these procedures will enable us to specify textures as primitives along with shape in higher level structural descriptions of biological images.

RESEARCH ON A PICTURE LANGUAGE MACHINE

20500-12-2050406/65

Authorized 5-1-61

Origin: NBS

Sponsor: National Science Foundation

Manager: Russell A. Kirsch

Full task description: July-December 1963 issue, p. 17

Status: CONTINUED. During this period a study was made of methods for the syntactical description of graphic images. Some results were obtained on a structural approach to picture processing and the relationships between picture description, picture perception and picture synthesis by a machine.

A set of experiments were performed with a remote time sharing system to search the physics literature for various kinds of citation references. Several searches of different kinds on the MIT file of 25,000 articles were made for the National Standard Reference Data Center.

A study by Mr. Watt which resulted in the NBS report "Acceptance, Acceptability, Grammaticality, Sentencehood" was made to investigate the distinction among different notions of grammaticalness in formal grammars.

MATHEMATICAL TABLES

20500-40-2050121/57-216

Origin and Sponsor: NBS

Manager: I. A. Stegun

Full task description: July-December 1964 issue, p. 4.

Status: CONTINUED. A correction file, updated reference file and "improved methods" file are being maintained for AMS 55-the Handbook of Mathematical Functions. The fourth printing (with corrections) appeared during the period and corrections were submitted for the fifth printing.

Since the volume covers almost the entire field of special functions, the updated file will not only facilitate the preparation of a second edition of AMS 55 but also the second editions of more extensive tables of the relative specialized functions in the AMS series. An immediate purpose is being served by the updated reference file in providing a more update "Index of Mathematical Tables" for our advisory and consultative service.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publications Activities.

Applied Mathematics Division Lectures

- BLEISTEIN, DR. NORMAN (Courant Institute of Mathematical Sciences, New York Univ., New York, N.Y.)
Uniform expansion of integrals with stationary points near algebraic singularities. April 22, 1966.
- CHRISTIAN, PROF. ULRICH H. (The Johns Hopkins University, Baltimore, Md.)
On modular groups. January 7, 1966.
- DANZER, DR. LUDWIG (University of Göttingen, Germany)
On the discriminant of plane point sets of given diameter. March 14, 1966.
- McLANE, DR. C. KEITH (Environmental Science Services Administration, Boulder, Colorado)
Density Correlation and Diffusion in a Plasma. May 5, 1966.
- MORDELL, PROF. LOUIS J. (Department of Mathematics, University of Illinois, Urbana, Ill.)
Number theory. March 1, 1966 - May 31, 1966.

NBS Courses Conducted by Staff Members

- KLOSS, K. E. Simulation programming languages. Fall Semester.

NBS In-hours Courses Given by Staff Members

- ROSENBLATT, JOAN R. Non-parametric Statistical Techniques. February 1966 - May 1966.
- VARNER, RUTH N. Fortran Programming. February 1966 - May 1966.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

- CAMERON, J. M. Computer Programs for Analysis of the Calibration Data.
Presented at the National Conference of Standards Laboratories,
National Bureau of Standards, Gaithersburg, Maryland, May 10, 1966.
- GOLDMAN, A. J. Operations Research Research. Presented before the Washington
Area Government Agencies Operations Research Conference,
Smithsonian Institution. April 20, 1966.
- KIRSCH, R. A. Language Problems in Artificial Intelligence. Computational
Sciences Seminar, Univ. of Chicago, Chicago, Ill.,
April 5, 1966.
- KIRSCH, R. A. Correspondence between One-Dimensional and Two-Dimensional
Languages in Pattern Recognition. Computer Sciences Colloquium,
Univ. of Wisconsin, Madison, Wisconsin, April 6, 1966.
- KIRSCH, R. A. Articular Structure of Biological Images. Biophysics Department
Colloquium, University of Minnesota, Minneapolis, Minnesota,
April 11, 1966.

KU, H. H.	Accuracy and Precision. Presented at George Washington University, Wash., D.C., June 7, 1966.
ROSENBLATT, J. R.	1. Evaluation of Uncertainties in Experimental Results. 2. Statistical Tools for Data Analysis. Presented at Purdue University, West Lafayette, Indiana, March 9 & 10, 1966.
ROSENBLATT, J. R.	Distribution-free Two-sample Tests. Presented at the University of Pennsylvania, Philadelphia, Pennsylvania, April 4, 1966.
ROSENBLATT, J. R.	Applications of Statistics in Colorimetry. Presented at the University of Pennsylvania, Philadelphia, Pennsylvania, April 4, 1966 and at the College of Notre Dame of Maryland, Baltimore, Maryland, April 5, 1966.
SADOWSKI, W. L.	Nonlinear oscillations in a collisionless plasma. Presented at the Institutes for Environmental Research, Washington, D.C., March 10, 1966.
SADOWSKI, W. L.	Some problems associated with the numerical solution of the one-dimensional Vlasov equation. Presented at NASA Langley Research Center, Hampton, Virginia, June 9, 1966.
TCHEN, C. M.	Nonlinear oscillations of a collision free plasma. Presented at Drexel Institute of Technology, Philadelphia, Pa., April 20, 1966.
TCHEN, C. M.	Kinetic theory of the turbulent damping of plasma. Presented at City College, New York, N.Y., April 22, 1966.
TCHEN, C. M.	Turbulence in a rarefied plasma. Presented at the NASA Contractors Meeting, Washington, D.C., April 26, 1966.
WITZGALL, C. J.	On sign-patterns in matrices representing polyhedra. Presented before the Math. Dept., University of California, San Diego, Calif., January 6, 1966.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THIS PERIOD.

1.1 Mathematical Tables

Handbook of Mathematical Functions, AMS 55, Fourth Printing (with corrections), December 1965.

1.3 Technical Papers

The bridge tournament problem and calibration designs for comparing pairs of objects. R. C. Bose and J. M. Cameron. Journal of Research NBS, 69B, pp. 323-332, 1965.

Inequalities for the permanental minors of non-negative matrices. R. A. Brualdi and M. Newman. Canadian Journal of Mathematics, Vol. 18, pp. 608-615, 1966.

Scattering properties of concentric soot-water spheres for visible and infrared light. R. W. Fenn and H. Oser. Journal of Applied Optics, Vol. 4, pp. 1504-1509, November, 1965. (Journal listing previously incorrect.)

New methods and problems in combinatorial optimization. A. J. Goldman. Washington Operations Research Council Newsletter, Vol. 5, No. 8, pp. 4-8, April 1966.

The analysis, synthesis, and description of biological images. L. E. Lipkin, W. C. Watt, and R. A. Kirsch. Annals of the New York Academy of Sciences, Vol. 128, Art. 3, pp. 984-1012, Jan. 1966.

Some number-theoretic computations. K. E. Kloss. Journal of Research NBS, 69B, pp. 335-336, 1965.

Real two-dimensional representations of the free product of two finite cyclic groups. J. Lehner and M. Newman. Proceedings of the Cambridge Phil. Soc., Vol. 62, pp. 135-141, 1966.

Error analysis of phase-integral methods I. General theory for simple turning points. F.W.J. Olver. Journal of Research NBS, 69B, pp. 271-290, 1965.

Error analysis of phase - integral methods II. Application to Wave-Penetration problems. F.W.J. Olver. Journal of Research NBS, 69B, pp. 291-300, 1965.

Invariant properties of the spheroidal potential of an oblate planet. J. P. Vinti. Journal of Research NBS, 70B, pp. 1-16, 1966.

Inclusion of the third zonal harmonic in an accurate reference orbit of an artificial satellite. J. P. Vinti. Journal of Research NBS, 70B, pp. 17-46, 1966.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.3 Technical Papers

Kernels and the Kronecker product of graphs. R. A. Brualdi. To appear in Proceedings of the American Mathematical Society.

Proof of a permanental inequality. R. A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).

Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.

Upper bounds for the determinant of a row stochastic matrix. K. Goldberg. To appear in the Journal of Research NBS.

Groups preserving ordering in vectors. K. Goldberg. To appear in the Journal of Research NBS.

Pairs of non-singular matrices. K. Goldberg. To appear in the Journal of Research NBS.

Realizing the Distance Matrix of a Graph. A. J. Goldman. To appear in Journal of Research NBS, 70B, 1966.

Finding a Rank-Maximizing Matrix Block. A. J. Goldman and M. Newman (205.01). To appear in Journal of Research NBS, 70B, 1966.

On a theorem of Ahlfors. L. Greenberg. Accepted for publication in the American Journal of Mathematics.

Kleinian groups. L. Greenberg. Accepted for publication in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965).

Note on normal subgroups of the modular group. L. Greenberg. Accepted for publication in the Proceedings of the American Mathematical Society.

Fundamental polyhedra for Kleinian groups. L. Greenberg. To appear in the Annals of Mathematics.

A modified Monte-Carlo quadrature - II. S. Haber. Accepted for publication in Mathematics of Computation.

On the very low temperature behavior of a ferromagnet. S. Haber and S. J. Glass. Submitted to a technical journal.

On a sequence of points of interest for numerical quadrature. S. Haber. To appear in the Journal of Research NBS, Section B.

Notes on the use of propagation of error formulas. H. H. Ku. To appear in the Journal of Research NBS, Section C. (Engineering and Instrumentation).

Analysis of information--An alternative approach to the detection of a correlation between the sexes of adjacent sibs in human families. H. H. Ku. Submitted to a technical journal.

Statistical evaluation of uncertainties associated with the reported values. H. H. Ku. To appear as Appendix in Analytical Procedures for Isotopic Analysis, NBS Technical Note 277, edited by W. R. Shields (Analytical Mass Spectrometry Section).

Note on partitions modulo 5. M. Newman. To appear in Mathematics of Computation.

Error bounds for asymptotic solutions of differential equations I. The distinct eigenvalues case. F. Stenger (manuscript).

Error bounds for asymptotic solutions of differential equations II. The general case. F. Stenger (manuscript).

Turbulence in a rarefied plasma. C. M. Tchen. Accepted for publication in the Proceedings of the Polish Symposium on Advanced Problems in Fluid Mechanics, to appear in the fall of 1966.

Useful alternatives to Chauvenet's rule for rejection of measurement data. T. A. Willke. Submitted to a technical journal.

A note on contaminated samples of size three. T. A. Willke. To appear in the Journal of Research NBS, Section B. (Math. and Math. Physics).

