NATIONAL BUREAU OF STANDARDS REPORT

9407

PROCEDURES FOR PRECISE DETERMINATION
OF THERMAL RADIATION PROPERTIES

PROGRESS REPORT NO. 31

Feb. 1, 1966 - July 31, 1966

RESEARCH AND TECHNOLOGY DIVISION
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

NBS

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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Contract No. DO (33-615)65-1005
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RESEARCH AND TECHNOLOGY DIVISION
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I. SUMMARY

The Photometry and Colorimetry Section of the National Bureau of Standards moved from the Washington, D. C., site to the new facilities at Gaithersburg, Md., during the month of April, 1966. As a result, little experimental work was done during the period March 1 through June 30, since it was necessary to dissassemble and pack all laboratory equipment prior to the move, and then unpack and reassemble the equipment after the move, and then connect all of the required plant facilities.

Advantage was taken of the opportunity to incorporate several desired changes into the equipment.

An error analysis of the integrating sphere reflectometer was made that indicates that the error in absolute reflectance measured with the reflectometer will vary from about +0.7% for a perfectly diffuse specimen to about -0.02% for a perfectly specular specimen. Relative reflectance measurements can be made without any systematic error.

II. LASER SOURCE INTEGRATING SPHERE REFLECTOMETER.

A. Background. The basic design of the laser-source integrating sphere reflectometer has been described in previous reports. It is designed to measure the directional-hemispherical reflectance of specimens at temperatures from room temperature up to 2500°K or above. The reflectometer is capable of measuring either absolute or relative reflectance.

B. Design Modifications. A new radio frequency generator has been procured and installed for use with the integrating sphere reflectometer.

Modifications to the integrating sphere, described in the previous report, have been completed. These modifications should facilitate evacuation of the sphere to a pressure of 10^{-5} torr or less.

A new electronic amplifying and ratioing system has been procured. The system consists of two thermocouple synchronous amplifiers (modified Brower Model 129) and a ratioing multiplexer (Brower Model 150). Tests have shown that the two amplifiers are linear to better than 0.1% over a range of six decades, and that the ratio indicated by the ratioing multiplexer remains constant to 0.1% when both input signals are changed by a ratio of 100 to 1. No crosstalk, either optical or electronic, was observed when the system was used with two lead sulfide detectors viewing the beams from the two ends of the helium-neon laser.

C. Sphere Coatings. Techniques have been developed on another project for applying coatings of sodium chloride that have high reflectance both in the visible and infrared, and are near-perfect diffusers.

D. Analysis of the Integrating Sphere.

1. Basic Theory.

The theory of the integrating sphere is based on two fundamental laws of radiation.

a). The flux received by an elemental area from a point source is inversely proportional to the square of the distance from the source to the receiving area and directly proportional to the cosine of the angle between the normal to the receiving area and the direction of incidence.

b). The flux reflected by a perfect diffuser follows the cosine distribution law, meaning the flux per unit solid angle reflected from a unit surface area in any given direction is proportional to the cosine of the angle between the normal to the surface and the direction of reflection.

When these laws are applied to a sphere having a perfectly diffuse wall of uniform reflectance then it is found that the flux reflected by an area on the sphere irradiates the sphere wall uniformly. Each elemental surface of unit area on the sphere receives the same amount of radiant flux by reflection. As shown in fig. (1), let dA_1 be the reflecting elemental area on the sphere-wall and dA_2 be the receiving area. The two elemental areas are considered to be very small so that they can be treated as plane.

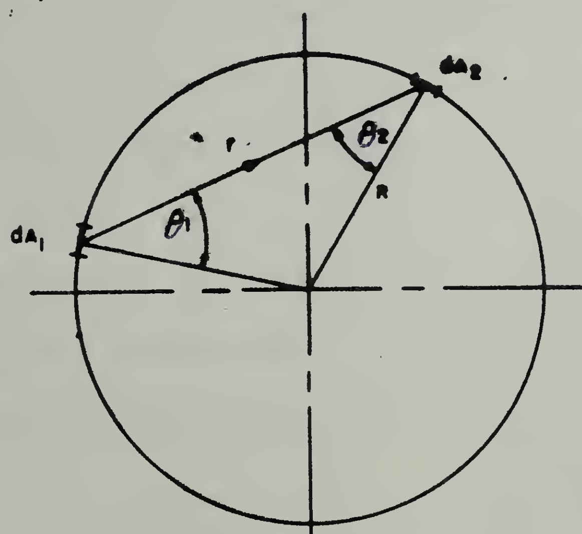

Figure 1.

Then the flux $d^2\Phi$ leaving dA_1 and incident directly on dA_2 is:

$$d^2\Phi = LdA_1 \cos \theta_1 \, d\omega$$

where L is the radiance of dA_1 and $d\omega$ is the elemental solid angle subtended by dA_2 at dA_1.

Since

$$d\omega = \frac{dA_2 \cos \theta}{r^2}$$

$$d^2\Phi = \frac{LdA_1 \, dA_2 \cos \theta_1 \cos \theta_2}{r^2}$$

From fig. (1) we see that

$$\theta_1 = \theta_2 = \theta$$

$$r = (2R \cos \theta)$$

and

$$d^2\Phi = \frac{L dA_1 \, dA_2 \cos^2\theta}{4R^2 \cos^2\theta} = \frac{L dA_1 \, dA_2}{4R^2}$$

For a perfect diffuser the total reflected flux is $dA_1 \pi L$ and therefore the fraction of the flux incident on dA_2 is

$$\frac{d^2\Phi}{\pi L dA_1} = \frac{dA_2}{4R^2\pi} = \frac{dA_2}{A}$$

where $A = 4\pi R^2$ is the total area of the sphere. Therefore, in a sphere whose surface reflects in a perfectly diffuse manner, the irradiance due to reflected flux is equal at all points, regardless of the geometric or areal distribution of the incident flux.

2. Design of the Integrating Sphere.

The sphere which was to be used in the laser integrating sphere reflectometer had to fulfill the following requirements.

a). The reflectometer should yield accurate results regardless of directional distribution of the reflected flux from the sample.

b). It should be possible to measure absolute reflectance, as well as reflectance relative to a reference standard.

c). The specimen heater should not disturb the sphere configuration and should be capable of heating the sample to temperatures from room temperature to 2500°K.

d). The sphere should be capable of operation in vacuum, air, or inert atmosphere.

In order to discuss the sphere it is first necessary to describe its geometry. The sphere is made up of two hemispheres, which are joined by bolted flanges with an O - ring seal. The principal axis of the sphere is defined as the diameter normal to the plane through the joint connecting the hemispheres. The primary plane is defined as the plane containing the principal axis and the center of the entrance port, and the secondary plane of the sphere is the plane containing the principal axis and normal to the primary plane. The entrance port is centered 12° from the principal axis, and by definition is in the primary plane. The detector port is centered 45° from the principal axis and is also located in the primary plane. The field of view of the detector is restricted to a small area of the sphere wall centered around the principal axis in the lower hemisphere. The ports for specimen and comparison standards are placed in secondary plane, 20° on either side of the principal axis. They are thus located symmetrically with respect to the entrance and detector ports. Small shields screen the area viewed by the detector from the specimen and comparison standard ports. See fig. (2).

A sphere designed as described above will meet the requirements established above as indicated in the following paragraphs.

a): Since the field of view of the detector is restircted to a small portion of the sphere wall which is shielded from the sample, no flux reflected from the sample can reach the field of view directly. Thus the reflected flux will be first diffused by the sphere coating before it is incident on the area viewed by the detector. Assuming that the sphere coating reflects uniformly in a perfectly diffuse manner, then the flux received at the field of view is always a constant fraction of the flux reflected by the sample, regardless of its directional distribution. The possibility that flux reflected by the sample is received by the detector directly is eliminated by equipping the detector port with a light trap so that any radiation reaching the detector port from directions outside the field of view is absorbed before reaching the detector.

The field of view of the detector is located symmetrically with respect to sample and comparison standard. When the sphere is used in the comparison mode, the reflected flux from the heated specimen will be compared to that of a water-cooled comparison standard made of the same material and fabricated in the same manner. Therefore, we can assume within reason that the directional distributions of the reflected flux are similar. If we assume furthermore that the directional distribution will not change with increasing surface temperature, then we can even drop the requirement of the sphere-coating to be a perfect diffuser, demanding only the coating to possess a uniform reflectance. We can see this in the following manner. Since sample and standard are located symmetrically with respect to the entrance port and field of view, the reflected fluxes will be distributed symmetrically with respect to the field of view as will be the flux after being reflected by the sphere coating. As a consequence the same fraction of flux leaving the sphere wall will be incident on the field of view in both cases, even if the reflectance function of the coating exhibits a directional variation rather than being a constant.

b): Since it is always desired to obtain an absolute value for reflectance rather than one based on some arbitrary and more or less doubtful standard, it was decided to find a way to use the sphere as an absolute reflectometer. A literature survey revealed that of the many ways possible to achieve this, one method seemed to be especially suitable for the sphere configuration as described above. It has been reported, among others, by McNicholas and can be summarized as follows. Assume a sphere configuration as described above where the comparison standard is now replaced by a spherical cap of the same curvature as the sphere and coated in the same manner as the sphere. If the laser beam is first incident upon the sample shielded from the field of view of the detector, and then upon a portion of the sphere wall not shielded from the view of the detector, then the ratio of the resulting radiance (detector response) when the sample is irradiated, to the radiance (detector response) when the sphere wall is irradiated, is equal to the absolute directional, hemispherical reflectance of the sample.

The basic possibility of obtaining the absolute value of reflectance, regardless of the directional distribution of the reflected flux, by this method can be proven as follows. Let Φ_0 be flux initially entering the sphere, A_V the area of the field of view of the director, A the total sphere area, $\rho_S(\theta,\varphi;\ 2\pi)$ the directional, hemispherical reflectance of the sample, ρ_w, the reflectance of the sphere wall and $L'(\theta',\varphi')$ the radiance of the sample in the direction (θ',φ'), due to reflected flux.

The flux Φ_0 is incident upon the sample from the direction (θ,φ). The flux intercepted by a small area dA of the sphere in the direction (θ',φ') as seen from the sample is

$$L'\ (\theta',\ \varphi')\ A_i\ \cos\ \theta'\ d\omega',$$

where $d\omega'$ is the elemental solid angle subtended by dA at the sample and A_i is the irradiated portion of the sample area.

The flux leaving dA is:

$$\rho_w\ L'(\theta',\varphi')\ A_i\ \cos\ \theta'\ d\omega'.$$

Since we assume the sphere coating to be a perfectly diffuse reflector, therefore the fraction of this flux reaching the area A_V is equal to the configuration factor $F_{dA \rightarrow A_V}$ which in the case of the sphere has been shown to be $\frac{A_V}{A}$.

Thus the irradiance of A_V due to the flux incident on dA is

$$\frac{1}{A_V} \rho_w L'(\theta',\varphi') A_i \cos\theta' \, d\omega' \frac{A_V}{A}$$

$$= \frac{1}{A} \rho_w L'(\theta', \varphi') A_i \cos\theta' \, d\omega'.$$

As shown by Taylor and many others the irradiance is increased by the factor $\frac{1}{1 - \rho_w}$ due to multiple reflections within the sphere. Therefore, the irradiance of A_V resulting from reflected flux incident on dA is

$$\frac{\rho_w}{A\,(1 - \rho_w)} \; L'(\theta', \varphi') A_i \cos\theta' \, d\omega'.$$

The total irradiance due to all the reflected flux is

$$\frac{\rho_w}{A\,(1 - \rho_w)} \; A_i \int_0^{2\pi} L'(\theta',\varphi') \cos\theta' \, d\omega'$$

where the integration is performed over the solid angle of 2π. This gives rise to a radiance leaving A_V of

$$\frac{1}{\pi} \; \frac{\rho_w}{A\,(1 - \rho_w)} \; A_i \int_0^{2\pi} L'(\theta',\varphi') \cos\theta' \, d\omega'.$$

Since $A_i \int_0^{2\pi} L'(\theta',\varphi') \cos\theta' \, d\omega$ is equal to the total reflected flux Φ' from the sample, we can also write the radiance of A_V to be

$$\frac{1}{\pi} \; \frac{\rho_w}{A(1 - \rho_w)} \; \Phi'$$

To effectively evaluate the initial flux Φ_o we direct the beam onto the sphere wall at a point which is not shielded from A_V. The flux leaving the sphere wall is $\rho_w\Phi_o$. The fraction of this flux received by A_V is again determined by the configuration factor from the sphere area irradiated to A_V or $\frac{A_V}{A}$. The irradiance of A_V is then

$$\frac{1}{A_V}\,\rho_w\Phi_o\,\frac{A_V}{A} = \frac{\rho_w\Phi_o}{A}.$$

Again the irradiance is increased by multiple reflections within the sphere by the factor $\frac{1}{1-\rho_w}$. Therefore the total irradiance of A_V is:

$$\frac{\rho_w}{A(1-\rho_w)}\,\Phi_o.$$

This results in a radiance leaving A_V of

$$\frac{1}{\pi}\,\frac{\rho_w}{A(1-\rho_w)}\,\Phi_o.$$

The ratio of the radiances is

$$\frac{\frac{1}{\pi}\,\frac{\rho_w}{A(1-\rho_w)}\,\Phi'}{\frac{1}{\pi}\,\frac{\rho_w}{A(1-\rho_w)}\,\Phi_o} = \frac{\Phi'}{\Phi_o} = \rho_S(\theta, \varphi; 2\pi).$$

Since $\frac{\Phi'}{\Phi_o}$ is by definition the directional, hemispherical reflectance of the sample.

It might be useful to restate the assumptions made or implied in the derivation. First it is required that the sphere coating represents a perfectly diffuse reflector of uniform reflectance. Secondly the sum of the areas of all openings is negligible compared with the total sphere area and finally the presence of the shield does not change measurably the irradiance of the sphere. A more detailed investigation of these points is presented in the next section.

When the reflectance of the heated specimen is measured relative to a cool standard, then the spherical cap is exchanged for a sample-holder with a shield attached to it. Thus it is possible to use the sphere in the absolute as well as the relative mode.

c). The method chosen for heating the sample was induction heating. The heating unit, which is described in more detail in another section, is mounted outside the sphere and does not disturb the sphere configuration in any way.

d). All ports can be closed by quartz windows with O-ring seals. Also, the heating unit has been built vacuum tight. Slots arranged concentrically around the sample allow evacuation of the sphere. Inert gas can be introduced through openings in the flange around the entrance port, and removed through the evacuation slots by means of a valve located in the bottom of the heating unit.

3. Error Analysis of the Integrating Sphere.

Many authors have dealt with the errors involved in the use of an integrating sphere for reflectance measurements. However, none of the published equations can be applied directly to this sphere. The purpose of this section is therefore to evaluate the systematic errors which arise from this specific sphere geometry, rather than to develop general integrating sphere theory.

The assumptions under which the following expressions will be derived are that the sphere coating is a perfectly diffuse reflector of uniform reflectance and that all flux leaving through the openings of the sphere is lost. In addition, we assume that the detector signal is strictly proportional to the radiance of the field of view A_V. This means that the detector-amplification-recorder system has to be linear over the dynamic range used.

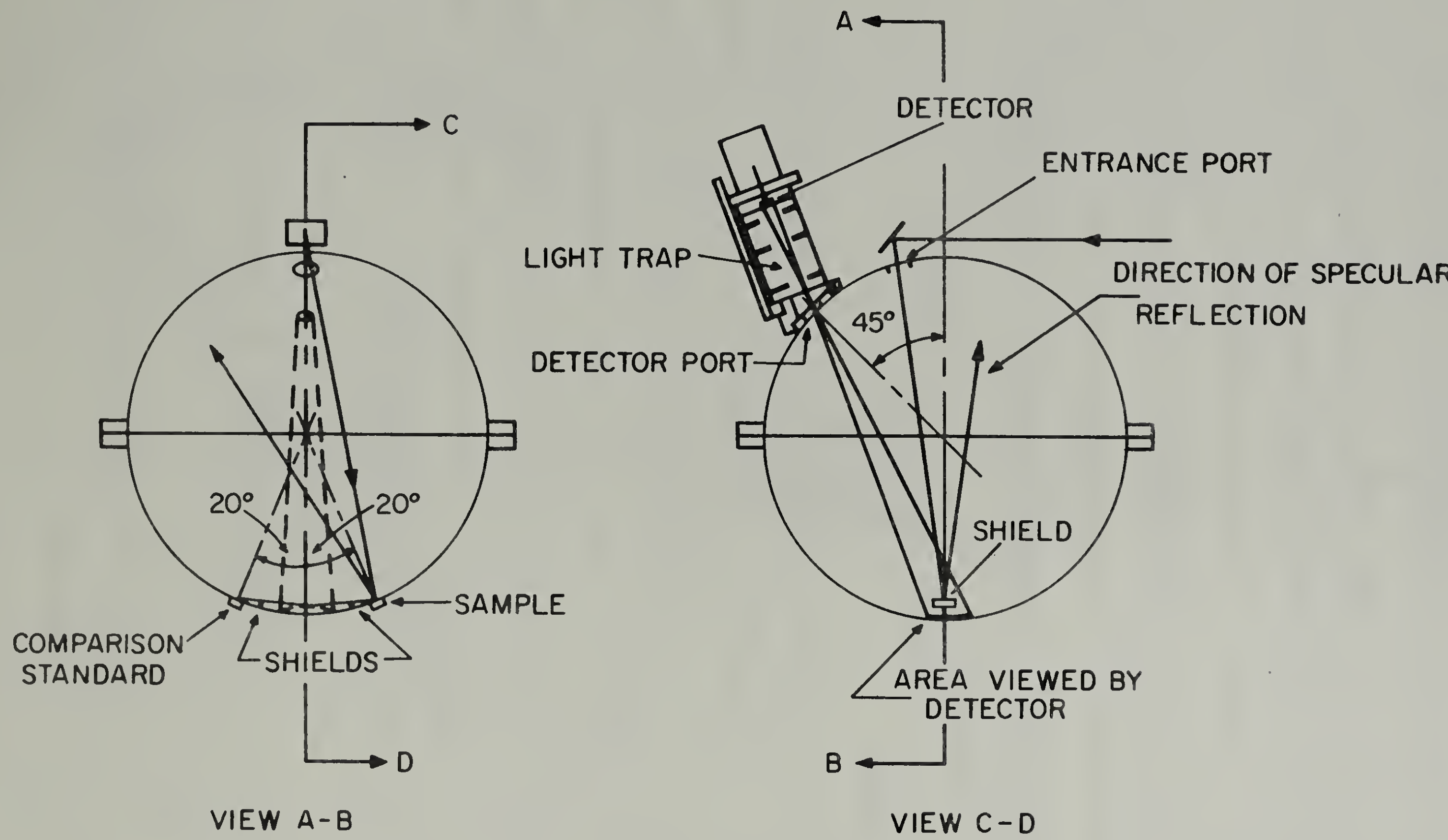
C
D
20°
20°
SAMPLE
COMPARISON
STANDARD
SHIELDS
VIEW A-B
A
B
DETECTOR
ENTRANCE PORT
LIGHT TRAP
DIRECTION OF SPECULAR
REFLECTION
45°
DETECTOR PORT
SHIELD
AREA VIEWED BY
DETECTOR
VIEW C-D

Figure 2

We will consider two cases. First, assume the sample to be a perfectly diffuse reflector. In this case, the flux intercepted by the shield after its initial reflection from the sample will be a maximum. In the second case, the sample is treated as a perfect mirror. As can be seen from Fig. (2), in this case no flux reflected once only by the specimen will be incident on the shield. Since the directional distribution of a real sample is somewhere between that of a perfect diffuser and that of a perfect mirror we will be able to place limits on the magnitude of the error caused by the real sample. (Excluded from samples are retroreflectors, but this is not a severe limitation, since almost all technical materials have a distribution which agrees with the statement above.)

Perfectly Diffuse Sample.

Let Φ_o be the flux initially incident on the sample. Then $\rho_S \Phi_o$ is reflected diffusely. Of this flux, the fraction $F_{A_i \to A_{sh}}$ is intercepted by the shield where $F_{A_i \to A_{sh}}$ is the configuration factor from the irradiated area on the sample to the shield. Fig. (3) shows the geometry in its correct proportions. Since the illuminated spot is only about 1/8" in diameter, we can use the small area approximation for the configuration factor, namely

$$F_{\Delta A_i \to A_{sh}} = \frac{1}{\Delta A_i} \int_{A_{sh}} \int_{\Delta A_i} \frac{\cos\theta_1 \cos\theta_2 \, dA_{sh}}{\pi r^2} \approx \int_{A_{sh}} \frac{\cos\theta_1 \cos\theta_2 \, dA_{sh}}{\pi r^2}$$

This configuration factor is given in [1] and was calculated to 0.017. At this point we will adopt an abbreviation for the configuration factors involved. The letter A for area will be omitted and the areas we are dealing with are designated by their subscripts.

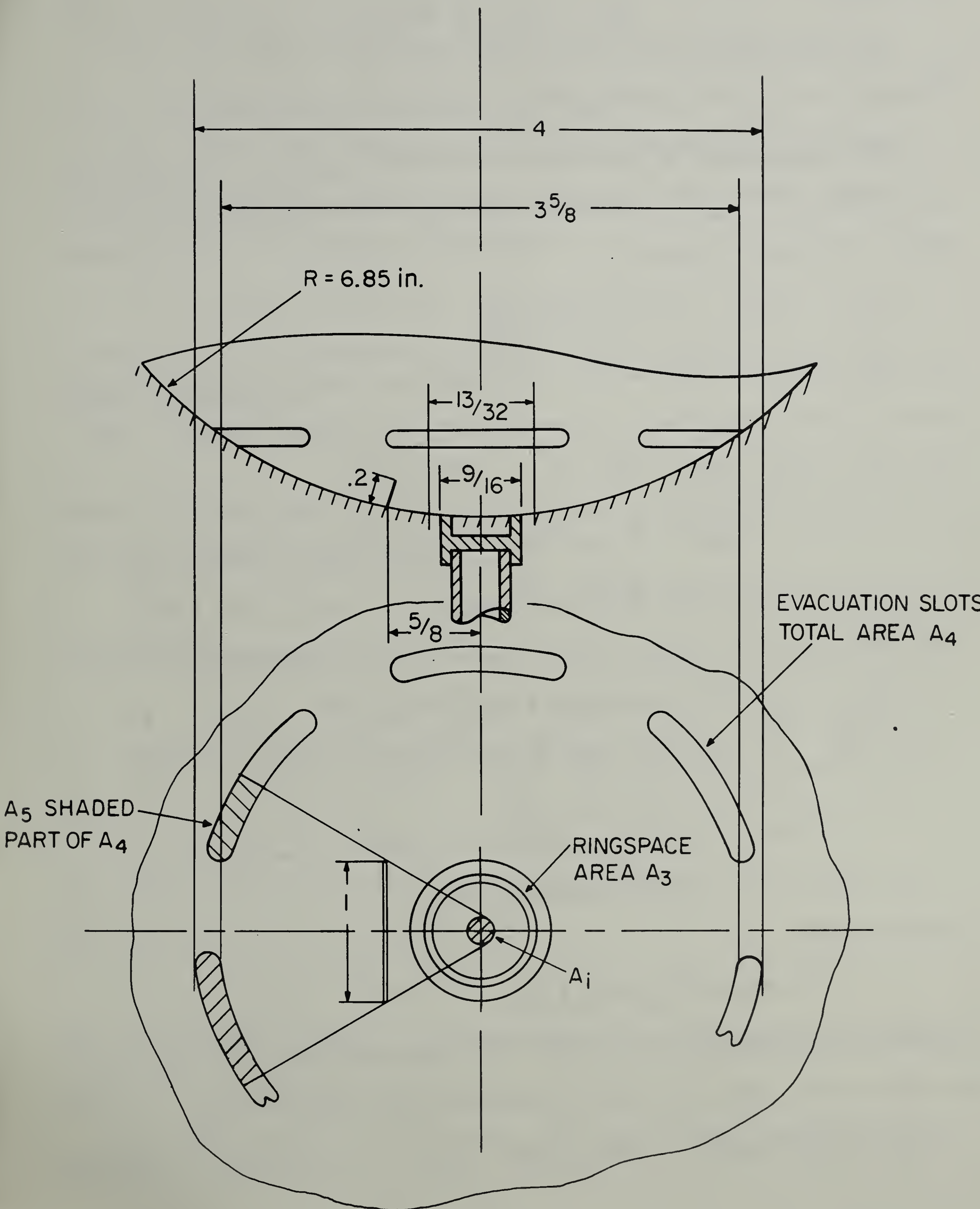
4
$3^5/_8$
R = 6.85 in.
13/32
.2
9/16
5/8
EVACUATION SLOTS
TOTAL AREA A_4
A_5 SHADED
PART OF A_4
RINGSPACE
AREA A_3
A_i

Figure 3

For example:

$$F_{A_1 \to A_{sh}} = F_{i,sh}$$

and

$$F_{A_{sh} \to A_1 + A_2 + A_3 + A_4} = F_{sh,1234}$$

The flux incident on the shield is therefore $\rho_s \Phi_o F_{i,sh}$. Of this flux the fraction $(1 - \rho_{sh})$ is absorbed, where ρ_{sh} is the reflectance of the shield. Leaving the shield is

$$\rho_{sh} \rho_s \Phi_o F_{i,sh}.$$

From figs. 2 and 3 we can see that the shield can view all openings of the sphere with the exception of the shaded part of A_4 which we name A_5. The flux lost out the openings is

$$\rho_{sh} \rho_s \Phi_o F_{i,sh} (F_{sh,1} + F_{sh,2} + F_{sh,3} + F_{sh,6})$$

$$= \rho_{sh} \rho_s \Phi_o F_{i,sh} F_{sh,1236}$$

where the subscript 1 stands for the area of the entrance port

2 for the area of the detector port

3 for the area of the ring space around the sample

6 for $A_6 = A_4 - A_5$

with A_4 being the area of the evacuation slots and

A_5 the shaded part of A_4.

Another portion

$$\rho_s \Phi_o F_{i,sh} \rho_{sh} F_{sh,s}$$

of the flux is received by the sample. The remaining flux is incident on the sphere wall. The flux which is incident on the sphere wall is

$$\rho_{sh} \rho_s \Phi_o F_{i,sh} - \rho_{sh} \rho_s \Phi_o F_{i,sh} (F_{sh,1236} + F_{sh,s}) = \rho_{sh} \rho_s \Phi_o F_{i,sh} (1 - F_{sh,1236s})$$

Where the subscript s symbolizes the sample area.

After these preliminary remarks we can proceed to investigate the diffuse case. The flux Φ_o is incident on the sample and $\rho_s\Phi_o$ is diffusely reflected. Of the once reflected flux $\rho_s\Phi_o$ the fraction $F_{i,126}$ is lost through the openings A_1, A_2, and A_6 (A_3 cannot be "seen" by the sample). Incident on the wall directly is therefore:

$$\rho_s\Phi_o - \rho_s\Phi_o F_{i,126} - \rho_s\Phi_o F_{i,sh} = \rho_s\Phi_o(1 - F_{i,126} - F_{i,sh})$$

In addition to the flux which is incident on the wall directly we have another contribution

$$\rho_{sh}\rho_s\Phi_o F_{i,sh}(1 - F_{sh,1236s})$$

which, as we have seen, is the portion of the flux reflected by the shield and then received by the sphere wall.

All the flux incident on the sphere wall after one reflection by the specimen is therefore:

$$\rho_s\Phi_o(1 - F_{i,126} - F_{i,sh}) + \rho_{sh}\rho_s\Phi_o F_{i,sh}(1 - F_{sh,\ 1236a})$$

$$= \rho_s\Phi_o[1 - F_{i,126} - F_{i,sh} + \rho_{sh}F_{i,sh}(1 - F_{sh,1236s})]$$

$$= \rho_s\Phi_o\{1 - F_{i,126} - F_{i,sh}\ [1 - \rho_{sh}(1 - F_{sh,1236s})]\}$$

Up to this point no flux was incident on the detectors field of view A_V. Only flux reflected by the sphere wall is able to reach A_V. This flux is

$$\rho_w\rho_s\Phi_o\{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\}.$$

The flux striking A_V after Φ_o has undergone one reflection from the sphere wall is

$$\Phi_{v,1} = \frac{A_V}{A}\rho_w\rho_s\Phi_o\{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\}$$

To evaluate the magnitude of the losses the configuration factors have been calculated:

$$F_{i,sh} = 0.017$$

$$F_{i,126} = 0.0051$$

$$F_{sh,s} = 0.021$$

$$F_{sh,1} = 0.00069$$

$$F_{sh,2} = 0.00026$$

$$F_{sh,3} = 0.032$$

$$F_{sh,6} = 0.019$$

$$F_{sh,1236s} = 0.073$$

From this we see that 1.7% of the flux reflected by the sample undergoes interreflection with the shield. To find out what fraction of flux contained in the sphere is received by the shield after the flux has been uniformly distributed over the entire sphere surface we divide the sphere into hemispheres by a plane containing the shield. Let A_{h1} be the area of the hemisphere facing the front side of the shield and A_{h2} the area of the hemisphere facing the back side of the shield. All the flux leaving A_{sh1}, the front side of the shield, has to strike A_{h1} or

$$F_{sh1,h1} = 1$$

From the law of reciprocity we know that

$$A_{sh1}F_{sh1,h1} = A_{h1}F_{h1,sh1}$$

or

$$F_{h1,sh1} = \frac{A_{sh1}}{A_{h1}} F_{sh1,h1} = \frac{A_{sh1}}{A_{h1}}$$

For the same reason $F_{h2,sh2} = \dfrac{A_{sh2}}{A_{h2}}$.

Now if Φ is the total flux leaving the sphere wall then approximately $\Phi/2$ is contained in each hemisphere. The flux received by A_{sh1} is

$$\frac{\Phi}{2}\frac{A_{sh1}}{A_{h1}} = \frac{\Phi}{2}\frac{A_{sh}}{A/2} = \Phi\frac{A_{sh}}{A}$$

where $A_{sh}/A = 0.00031$. This means that only 0.03% of the flux contained in the sphere after it has been uniformly distributed is incident on the front side of the shield, where almost all shield losses are caused since A_{sh1} views all openings except a part of A_4 (See fig. 3). Therefore it is justified to use the following approximation. The exact losses due to the shield are taken into account only before the flux has been diffused by the sphere wall. After the flux has been distributed over the entire sphere surface we treat the shield as having a surface area of $2A_{sh}$ with a reflectance of ρ_{sh}.

The flux contained in the sphere after one wall reflection was:

$$\Phi_1 = \rho_w\rho_s\Phi_o\{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\} + \rho_s^2\Phi_oF_{i,sh}\rho_{sh}F_{sh,s}$$

$$= \rho_w\rho_s\Phi_o\{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})] + \rho_s\frac{\rho_{sh}}{\rho_w}F_{i,sh}F_{sh,s}\}$$

Now we define an average sphere reflectance $\bar{\rho}_w$ such that

$$\bar{\rho}_w(A + 2A_{sh}) = \rho_{sh}2A_{sh} + \rho_w[A - (A_1 + A_2 + A_3 + A_4 + A_s)] + \rho_sA_s$$

or

$$\bar{\rho}_w = \frac{1}{A + 2A_{sh}}\{2A_{sh}\rho_{sh} + \rho_w[A - (A_1 + A_2 + A_3 + A_4 + A_s)] + \rho_sA_s\}$$

then if Φ_n is the flux contained in the sphere after n reflections the flux after n + 1 reflections is

$$\Phi_{n+1} = \bar{\rho}_w\Phi_n$$

After any reflection n the fraction $\frac{A_v}{A}$ of the flux Φ_n is incident on A_v. Therefore the flux received by A_v is:

$$\Phi_v = \sum_{n=1}^{\infty} \Phi_{v,n}$$

$$= \Phi_{v,1} + \Phi_{v,2} + \ldots$$

$$= \Phi_{v,1} + \frac{A_v}{A} \sum_{n=2}^{\infty} \Phi_n$$

$$= \Phi_{v,1} + \frac{A_v}{A} [\Phi_1 \bar{\rho}_w + \Phi_1 \overline{\rho_w^2} + \Phi_1 \overline{\rho_w^3} + \ldots]$$

$$= \Phi_{v,1} + \frac{A_v}{A} \Phi_1 \bar{\rho}_w \frac{1}{1 - \bar{\rho}_w}$$

Therefore

$$\Phi_v = \frac{A_v}{A} \rho_w \rho_s \Phi_o \{1 - F_{i,126} - F_{i,sh} [1 - \rho_{sh}(1 - F_{sh,1236s})]\}$$

$$+ \frac{A_v}{A} \rho_w \rho_s \Phi_o \{1 - F_{i,126} - F_{i,sh} [1 - \rho_{sh}(1 - F_{sh,1236s})] + \rho_s \frac{\rho_{sh}}{\rho_w} F_{i,sh} F_{sh,s}\} \frac{\overline{\rho_w}}{1 - \overline{\rho_w}}$$

$$= \frac{A_v}{A} \rho_w \rho_s \Phi_o \{1 - F_{i,126} - F_{i,sh} [1 - \rho_{sh}(1 - F_{sh,1236s})]\}.$$

$$\{1 + \frac{\overline{\rho_w}}{1 - \overline{\rho_w}} (1 + \frac{\rho_s \rho_{sh} F_{i,sh} F_{sh,s}}{\rho_w \{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\}})\}.$$

When we check the order of magnitude of the term

$$\frac{\rho_s \rho_{sh} F_{i,sh} F_{sh,s}}{\rho_w \{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\}}$$

then we see that it is much smaller than 1 and therefore can be neglected. (The maximum value this term can obtain is about 0.0004). Neglecting this term we can further simplify the expression to:

$$\Phi_v = \frac{A_v}{A} \rho_w \rho_s \Phi_o \{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\} \frac{1}{1 - \overline{\rho_w}}$$

Since we assume that the signal from the detector-amplifier-recorder system is proportional to the radiance of A_v, S_1, the signal when the beam is first incident on the sample, becomes;

$$S_1 = K \frac{1}{\pi A} \rho_w \rho_s \Phi_o \{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]\} \frac{1}{1 - \bar{\rho}_w}$$

where K is a proportionality constant.

To measure Φ_o the beam is directed onto an area A_w [1/] of the sphere surface with an unobstructed view of A_v. Reflected off the sphere wall is $\rho_w \Phi_o$ of which the fraction $\frac{A_v}{A}$ is received by A_v.

Thus

$$\Phi_{v,1} = \frac{A_v}{A} \rho_w \Phi_o$$

Of the flux $\rho_w \Phi_o$ the fraction $F_{w,124}$ is lost through the openings A_1, A_2, and A_4. (The opening A_3 and the sample area are screened off by the shield.) Another fraction $F_{w,sh}$ is incident on the backside of the shield A_{sh2} of which $\rho_{sh} F_{w,sh}$ is reflected. The flux reflected off the shield is therefore

$$\rho_{sh} \rho_w F_{w,sh2} \Phi_o$$

A portion of this flux is in turn lost through that part of A_4 which can be viewed by A_{sh2}. This flux is therefore

$$\rho_{sh} \rho_w \Phi_o F_{w,sh2} F_{sh2,4}$$

The flux incident on the sphere wall directly is

$$\rho_w \Phi_o - \rho_w \Phi_o F_{w,124} - \rho_w \Phi_o F_{w,sh2} = \rho_w \Phi_o (1 - F_{w,124sh2})$$

of which the fraction $\frac{A_v}{A}$ is again incident on A_v. In addition to this there is a small amount of flux

$$\rho_{sh} \rho_w \Phi_o F_{w,sh2} F_{sh2,v}$$

1/ A_w is located at the position of the comparison standard, figure 2.

which is reflected off the shield and then received by A_v. With this

$$\Phi_{v,2} = \frac{A_v}{A} \rho_w^2 \Phi_o(1 - F_{w,124sh2}) + \rho_w \rho_{sh} \Phi_o F_{w,sh2} F_{sh2,v}$$

$$= \frac{A_v}{A} \rho_w^2 \Phi_o(1 - F_{w,124sh2} + \frac{A}{A_v} \frac{\rho_{sh}}{\rho_w} F_{w,sh2} F_{sh2,v})$$

The flux remaining after two wall reflections is

$$\Phi_2 = \rho_w^2 \Phi_o(1 - F_{w,124sh2}) + \rho_w \rho_{sh} \Phi_o F_{w,sh}(1 - F_{sh2,4})$$

$$= \rho_w^2 \Phi_o[1 - F_{w,124sh2} + \frac{\rho_{sh}}{\rho_w} F_{w,sh}(1 - F_{sh2,4})].$$

From this point on we assume that the flux is uniformly distributed over the whole surface of the sphere and treat the following reflections in the same manner as before. Therefore the sum of all the flux incident on A_v after the second wall reflection is

$$\sum_{n=3}^{\infty} \Phi_{v,n} = \frac{A_v}{A} \Phi_2 \frac{\overline{\rho_w}}{1 - \overline{\rho_w}}$$

and therefore Φ_v becomes

$$\Phi_v = \frac{A_v}{A} \rho_w \Phi_o + \frac{A_v}{A} \rho_w^2 \Phi_o(1 - F_{w,124sh2} + \frac{A}{A_v} \frac{\rho_{sh}}{\rho_w} F_{w,sh2} F_{sh2,v})$$

$$+ \frac{A_v}{A} \rho_w^2 \Phi_o[1 - F_{w,124sh2} + \frac{\rho_{sh}}{\rho_w} F_{w,sh}(1 - F_{sh2,4})] \frac{\overline{\rho_w}}{1 - \overline{\rho_w}}$$

$$= \frac{A_v}{A} \rho_w \Phi_o\{1 + \rho_w[(1 - F_{w,124sh2}) \frac{1}{1 - \overline{\rho_w}} + \frac{\rho_{sh}}{\rho_w} F_{w,sh2,v} + \frac{\overline{\rho_w}}{1-\overline{\rho_w}} (1-F_{sh2,4})]\}$$

Hence S_2, the signal when the beam is first incident on the sphere wall, becomes:

$$S_2 = K\frac{1}{\pi A} \rho_w \Phi_o\{1 + \rho_w[(1 - F_{w,124sh2}) \frac{1}{1 - \overline{\rho_w}} + \frac{\rho_{sh}}{\rho_w} F_{w,sh2}(\frac{A}{A_v} F_{sh2,v}$$

$$+ \frac{\overline{\rho_w}}{1 - \overline{\rho_w}} (1 - F_{sh2,4}))]\}$$

The ratio of these two signals $\frac{S_1}{S_2}$ is therefore

$$\frac{S_1}{S_2} = \rho_s \frac{1 - F_{i,126} - F_{i,sh}[1 - \rho_{sh}(1 - F_{sh,1236s})]}{(1 - \overline{\rho_w})\{1 + \rho_w[(1 - F_{w,124sh2}) \frac{1}{1-\overline{\rho_w}} + \frac{\rho_{sh}}{\rho_w} F_{w,sh2}(\frac{A}{A_v} F_{sh2,v} + \frac{\overline{\rho_w}}{1-\overline{\rho_w}} (1-F_{sh2,4}))]\}}$$

or

$$\rho_s = \frac{S_1}{S_2} C$$

where C is a correction factor.

The magnitude of the corection factor, in percent of the correct value, is given by

$$\frac{\rho_{s,\ correct} - \rho_{s,\ measured}}{\rho_{s,\ correct}} 100 = \frac{\frac{S_1}{S_2} C - \frac{S_1}{S_2}}{\frac{S_1}{S_2} C} 100 = \frac{C - 1}{C} 100$$

For a particular geometry C is a function of ρ_w, ρ_s, and ρ_{sh}.

$$C = C\ (\text{geometry}, \rho_w, \rho_s, \rho_{sh})$$

In this case the shield is coated with the same material as the surface of the sphere and therefore

$$\rho_{sh} = \rho_w$$

All the configuration factors were calculated.

$$F_{i,126} = 0.0051$$

$$F_{i,sh} = 0.017$$

$$F_{sh,1236s} = 0.00575$$

$$F_{w,124sh2} = 0.0072$$

$$F_{w,sh2} = 0.0017$$

$$F_{sh2,v} = 0.0672$$

$$F_{sh2,4} = 0.0079$$

In addition we have

$$\overline{\rho_w} = 0.9939\rho_w + 0.00033\rho_s$$

and

$$\frac{A}{A_v} = 155.35$$

Using these values C reduces to

$$C = \frac{1.00075 - 0.00033\rho_s + 0.00017\rho_w)}{0.9779 + 0.0169\rho_w}$$

The table below shows the calculated correction factor C and its percentage of the correct value as a function of wall and sample reflectance.

Table I

$\rho_s = 0.1$

ρ_w	0.4	0.6	0.8	0.9	1.0
C	1.016	1.013	1.009	1.008	1.006
$\frac{C-1}{C}$ 100	1.59	1.25	0.92	0.75	0.57

$\rho_s = 0.95$

ρ_w	0.4	0.6	0.8	0.9	1.0
C	1.016	1.012	1.009	1.007	1.0055
$\frac{C-1}{C}$ 100	1.57	1.19	0.89	0.71	0.55

Perfectly Specular Sample.

Again let Φ_o be the flux which is initially incident on the sample. Then $\rho_s\Phi_o$ is the flux which is reflected by the sample in a specular manner and strikes the sphere wall. (From fig. 2 it can be seen that no flux will be lost through any of the openings and none will be received by the shield.) Leaving the sphere wall then is

$$\rho_W \rho_S \Phi_o$$

of which the fraction $\frac{A_v}{A}$ is incident on the detectors field of view. Therefore,

$$\Phi_{v,1} = \frac{A_v}{A} \rho_W \rho_S \Phi_o$$

A fraction of the flux leaving the sphere wall is incident on the sample and specularly reflected towards the entrance port. If A_{i2} is the area where the specularly reflected beam struck the sphere wall initially, then

$$\rho_W \rho_S \Phi_o F_{i2,s}$$

is the flux which is in turn specularly reflected towards A_1. This flux appears to originate at the image of A_{i2} which we call $A_{\overline{i2}}$ as seen in the mirrorlike sample. The fraction of the flux leaving A_{i1} which is lost through A_1 due to specular reflection by the sample is therefore determined by the configuration factor from the image of A_{i1} to A_1 or $F_{\overline{i1},1}$. See Fig. 4.

Since the areas involved are very small compared with the distance, it can be assumed that the configuration factor is essentially the same for any point of $A_{\overline{i1}}$ and A_1 and therefore

$$F_{\overline{i1},1} = \frac{\cos \theta_{\overline{i1}} \cos \theta_1}{\pi B^2} A_1$$

where $\theta_{\overline{i1}}$ is the angle between the normal to $A_{\overline{i1}}$ and the line of sight

θ_1 the analogous angle for A_1

and B the distance separating A_1 and $A_{\overline{i1}}$.

From fig. 4 it can be seen that

$$\cos \theta_{\overline{i1}} = \cos \theta_1 = \cos \theta$$

and

$$B = 2(2R \cos \theta).$$

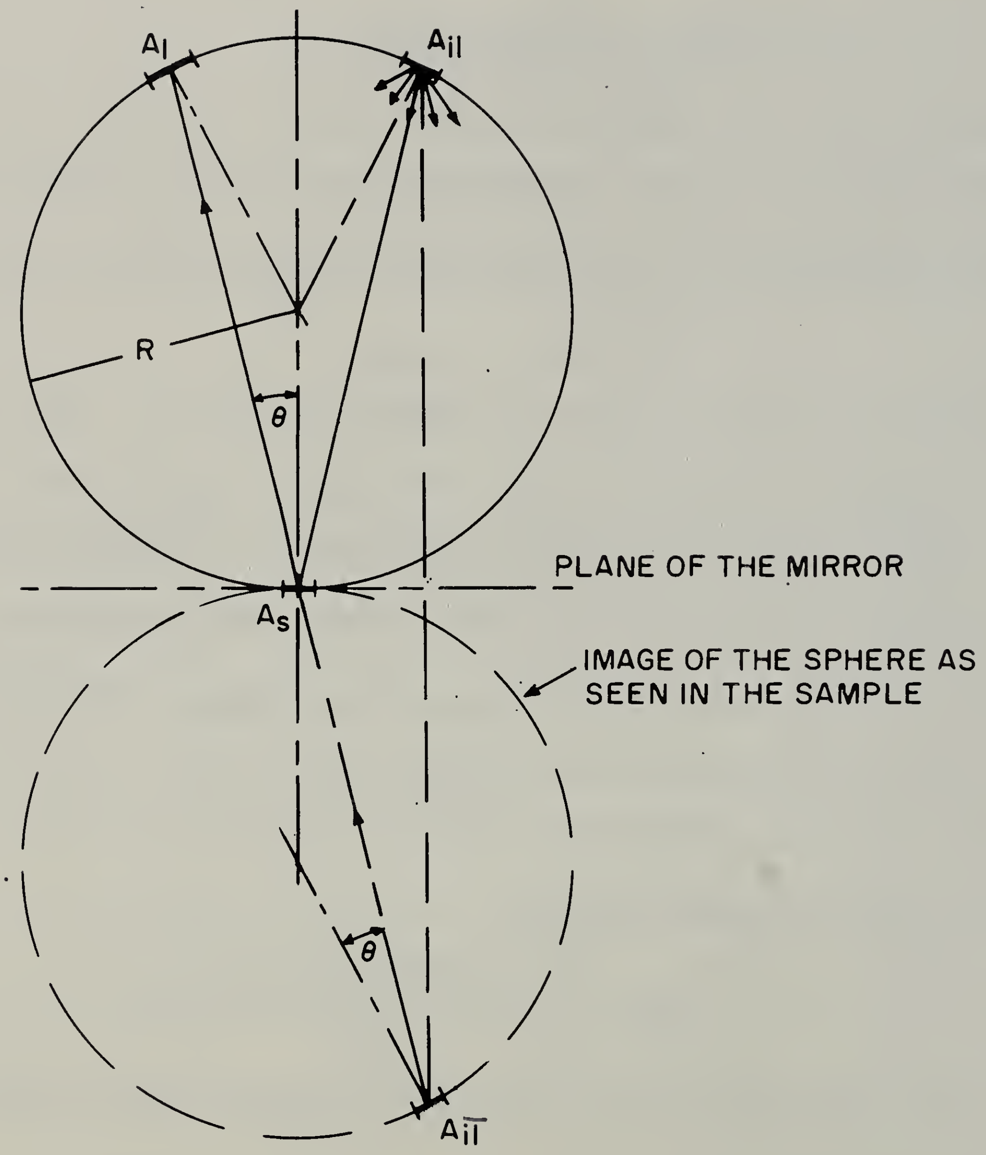
A_I
A_{iI}
R
θ
PLANE OF THE MIRROR
A_s
IMAGE OF THE SPHERE AS SEEN IN THE SAMPLE
θ
$A_{i\bar{I}}$

Figure 4

With this

$$F_{\overline{i1},1} = \frac{\cos^2\theta}{\pi\ 16R^2\ \cos^2\theta} A_1 = \frac{A_1}{4(4\pi R^2)} = \frac{A_1}{4A}$$

or the fraction of flux which is leaving A_{i1} and is specularly reflected out the entrance port is equal to the area of the entrance port divided by four times the total sphere area. However one restriction has to be observed. If, as in figure 5, the entire entrance hole can be seen from $A_{\overline{i1}}$ through the mirror then

$$F_{\overline{i1},1} = \frac{A_1}{4A} \text{ as indicated.}$$

If on the other hand the sample is so small that it restricts the sight from $A_{\overline{i1}}$ to A_1, as shown in fig. 6, then the configuration factor to use is not $\frac{A_1}{4A}$ but $F_{\overline{i1},s}$ which is equal to $F_{i1,s} = \frac{A_s}{A}$. The condition for $F = \frac{A_1}{4A}$, where F is now used for the fraction specularly reflected out A_1 is

$$\frac{A_1}{4A} < \frac{A_s}{A} \quad \text{or} \quad A_1 < 4A_s$$

and

$$F = \frac{A_s}{A} \quad \text{if} \quad A_1 \geq 4A_s$$

If $A_1 < 4A_s$ then $\rho_w\rho_s^2\Phi_o \frac{A_1}{4A}$ is lost through A_1: Since

$$\rho_w\rho_s\Phi_o F_{i1,s} = \rho_w\rho_s^2\Phi_o \frac{A_s}{A}$$

was reflected specularly towards the entrance port, therefore

$$\rho_w\rho_s^2\Phi_o \left(\frac{A_s}{A} - \frac{A_1}{4A}\right) = \rho_w\rho_s^2\Phi_o \frac{1}{A} (A_s - \frac{1}{4} A_1)$$

is incident on the sphere wall around the entrance port.

If $A_1 \geq 4A_s$ then all of the specularly reflected flux is lost through A_1. In this case the flux lost is

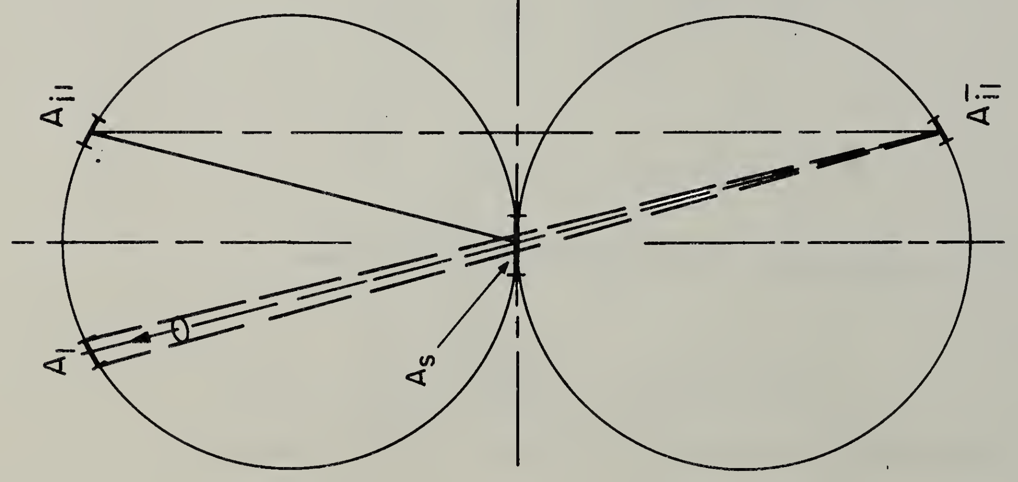

Figure 5

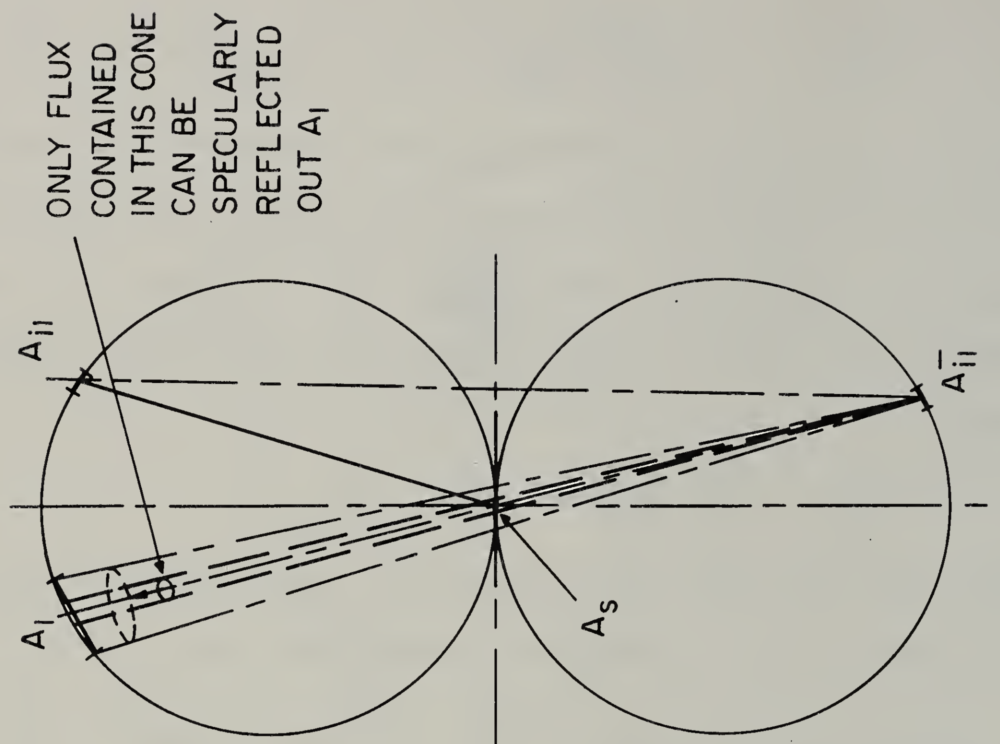

Figure 6

$$\rho_s \rho_w \Phi_o \frac{A_s}{A} \ . \quad \text{(This includes the flux absorbed at the sample.)}$$

For our sphere $A_1 = 0.994 \text{ in}^2$ and

$A_s = 0.196 \text{ in}^2$

therefore $A_1 > 4A_s$ and the flux lost is $\rho_w \rho_s \Phi_o \frac{A_s}{A}$.

Once the flux is uniformly distributed over the entire surface area of the sphere then we need additional information to calculate the hole loss caused by specular reflection at the sample. Now it is necessary to know the area from which diffusely reflected flux is able to reach A_1 via the specular sample. Let this area be A_{i2}. Now, if Φ is the flux uniformly distributed over the sphere wall then $\frac{A_{i2}}{A} \Phi$ is the flux leaving the area A_{i2}. Of this flux the fraction $\frac{A_s}{A}$ is received by the specular sample and reflected towards the entrance hole. This reflected flux strikes the sphere wall within an area centered around the entrance port. If A_{i3} is this irradiated area, then the fraction lost out the entrance port is $\frac{A_1}{A_{i3}}$. Hence the additional loss through the detector port caused by the specular sample becomes

$$\frac{A_1}{A_{i3}} \ \frac{A_{i2} A_s}{A^2} \ \Phi$$

In the same way, there exists another portion of flux which is specularly reflected into the detector port and absorbed by the light trap without reaching the detector. This flux can be expressed as

$$\frac{A_2}{A_{i5}} \ \frac{A_{i4} \ A_s}{A^2} \ \Phi$$

where A_{i5} and A_{i4} are the corresponding areas to A_{i3} and A_{i2}. The areas A_{i2} and A_{i3} (or A_{i5} and A_{i4}) **can** be found graphically as shown in Fig. 7. The method corresponds to finding the area on the sphere which is irradiated by the mirror A_s if the entrance port is replaced by a diffusely emitting source of equal area and then in turn finding the irradiated area around the entrance port due to the diffusely reflecting area A_{i2} or A_{i4} respectively.

Now assume that after n reflections the flux is uniformly distributed over the sphere wall. Let this flux be Φ_n. Then the problem is to calculate Φ_{n+1}.

Of the flux Φ_n the fraction $\frac{1}{A}(A_1 + A_2 + A_3 + A_4)$ is lost through the openings directly. An additional fraction

$$\frac{A_s}{A^2}\left(A_1 \frac{A_{i2}}{A_{i3}} + A_2 \frac{A_{i4}}{A_{i5}}\right)$$

is lost due to specular reflections of the sample. (No flux can be specularly reflected out A_3 since A_3 is the same plane as A_s and no flux can be specularly reflected out A_4 since A_4 is arranged symmetrically around the normal to A_s. Therefore for any part of A_4 the corresponding area from which the flux would have to originate is again a part of the opening A_4 with an effective $\rho = 0$.) (See fig. 3) With this the total hole loss becomes

$$\frac{1}{A}\left[A_1 + A_2 + A_3 + A_4 + \frac{A_s}{A}\left(A_1 \frac{A_{i2}}{A_{i3}} + A_2 \frac{A_{i4}}{A_{i5}}\right)\right]\Phi_n$$

The portion $\frac{A_s}{A}\Phi_n$ is incident on the sample which absorbs

$$\frac{A_s}{A}(1 - \rho_s)\Phi_n$$

Another part, $\frac{2A_{sh}}{A}$, is received by the shield which absorbs the flux

$$\frac{2A_{sh}}{A}(1 - \rho_{sh})\Phi_n$$

The remaining flux

$$\Phi_n - \left[\frac{1}{A}(A_1 + A_2 + A_3 + A_4) + \frac{A_s}{A} + \frac{2A_{sh}}{A}\right]\Phi_n$$

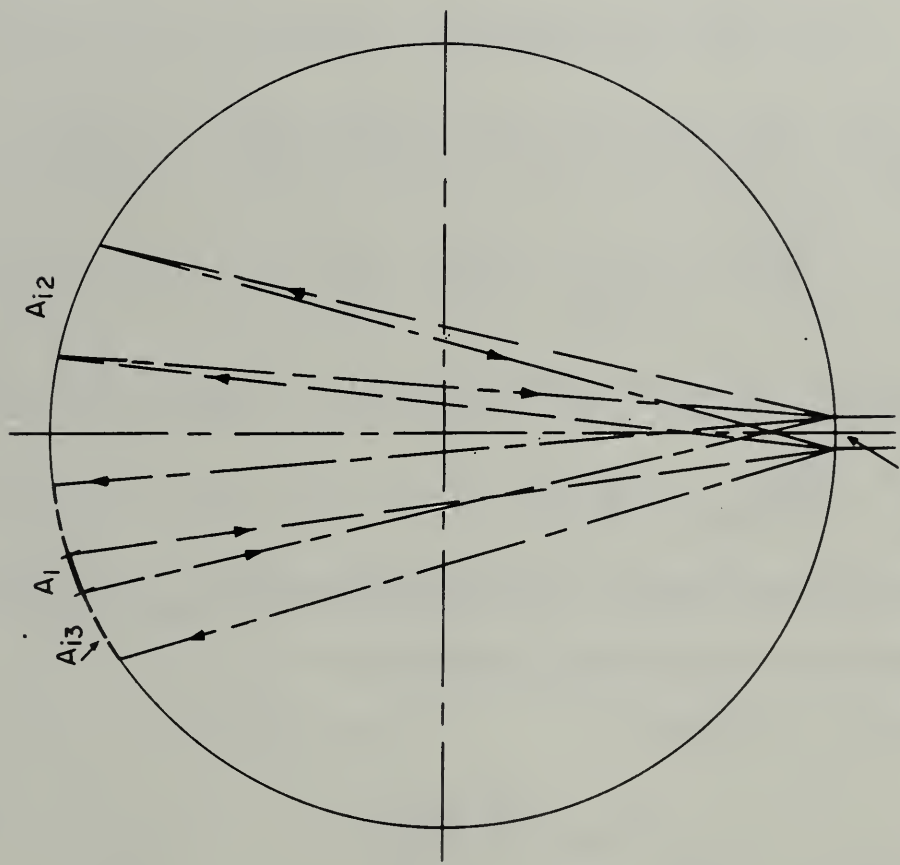

Figure 7

is incident on the sphere wall which absorbs

$$\Phi_n\{1 - \frac{1}{A}[A_1 + A_2 + A_3 + A_4 + A_s + 2A_{sh}]\}(1 - \rho_w)$$

Therefore the flux contained in the sphere after n + 1 wall reflections becomes:

$$\Phi_{n+1} = \Phi_n - \frac{1}{A}\left[A_1 + A_2 + A_3 + A_4 + \frac{A_s}{A}\left(A_1 \frac{A_{12}}{A_{13}} + A_2 \frac{A_{14}}{A_{15}}\right)\right]\Phi_n$$

$$- \frac{A_s}{A}(1 - \rho_s)\Phi_n - \frac{2A_{sh}}{A}(1 - \rho_{sh})\Phi_n - \frac{1}{A}\{A - [A_1 + A_2$$

$$+ A_3 + A_4 + A_s + 2A_{sh}]\}(1 - \rho_w)\Phi_n$$

$$\Phi_{n+1} = \Phi_n\{1 - \frac{1}{A}\left[A_1 + A_2 + A_3 + A_4 + \frac{A_s}{A}\left(A_1 \frac{A_{12}}{A_{13}} + A_2 \frac{A_{14}}{A_{15}}\right)\right] - \frac{A_s}{A}(1 - \rho_s)$$

$$- \frac{2A_{sh}}{A}(1 - \rho_{sh}) - \frac{1}{A}\{A - [A_1 + A_2 + A_3 + A_4 + A_s + 2A_{sh}]\}(1-\rho_w)\}$$

$$\Phi_{n+1} = \Phi_n\{\rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s + 2A_{sh})] - \frac{A_s}{A^2}\left(A_1 \frac{A_{12}}{A_{13}} + A_2 \frac{A_{14}}{A_{15}}\right)$$

$$+ \frac{2A_{sh}}{A}\rho_{sh} + \frac{A_s}{A}\rho_s\}.$$

Now we define an effective sphere reflectance such that $\rho_{eff} = \dfrac{\Phi_{n+1}}{\Phi_n}$ where ρ_{eff} can be calculated from the above equation.

We had that $\Phi_{v,1}$ is given by

$$\Phi_{v,1} = \frac{A_v}{A}\rho_w \rho_s \Phi_o$$

The flux left after one wall reflection is $\frac{A_v}{A}\rho_w \rho_s \Phi_o$. Of that, the fraction

$$\frac{1}{A}(A_1 + A_2 + A_3 + A_4)$$

is lost out openings directly. As discussed earlier an additional hole loss occurs by specular reflection, which is given by

$$\rho_w \rho_s \Phi_o \frac{A_s}{A}$$

Another fraction $F_{i2,sh}$ is incident on the shield which absorbs $F_{i2,sh}(1 - \rho_{sh})$ of it. The remaining flux is received by the sphere wall which absorbs the fraction $(1 - \rho_w)$ of it. Received by the wall is

$$\rho_w \rho_s \Phi_o - \frac{1}{A}(A_1 + A_2 + A_3 + A_4)\rho_w \rho_s \Phi_o - \frac{A_s}{A} \cdot \rho_w \rho_s \Phi_o - F_{i2,sh}\, \rho_w \rho_s \Phi_o$$

$$= \rho_w \rho_s \Phi_o [1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4) - \frac{A_s}{A} - F_{i2,sh}].$$

Therefore $\Phi_{v,2}$ is given by

$$\Phi_{v,2} = \frac{A_v}{A} \rho_w^2 \rho_s \Phi_o [1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}]$$

The remaining flux after two reflections at the sphere wall is

$$\Phi_2 = \rho_w \rho_s \Phi_o - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s)\rho_w \rho_s \Phi_o - F_{i2,sh}\, \rho_w \rho_s \Phi_o (1 - \rho_{sh})$$

$$- \rho_w \rho_s \Phi_o [1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}](1 - \rho_w)$$

$$\Phi_2 = \rho_w^2 \rho_s \Phi_o \{1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}\,(1 - \frac{\rho_{sh}}{\rho_w})\}.$$

As in the previous cases, we assume from now on that the flux is evenly distributed over the surface of the sphere and that after any reflection n the fraction $\frac{A_v}{A}\Phi_n$ is incident on A_v.

Therefore

$$\sum_{n=3}^{\infty} \Phi_{v,n} = \frac{A_v}{A}(\Phi_2 \rho_{eff} + \Phi_2 \rho_{eff}^2 + \ldots) = \frac{A_v}{A}\Phi_2 \frac{\rho_{eff}}{1 - \rho_{eff}}$$

and $\sum_{n=1}^{\infty} \Phi_{v,n}$ becomes

$$\frac{A_v}{A}\rho_w\rho_s\Phi_o + \frac{A_v}{A}\rho_w^2\rho_s\Phi_o[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}] + \frac{A_v}{A}\rho_w^2\rho_s\Phi_o\{1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}\left(1 - \frac{\rho_{sh}}{\rho_w}\right)\}\frac{\rho_{eff}}{1 - \rho_{eff}}$$

$$= \frac{A_v}{A}\rho_w\rho_s\Phi_o\{1 + \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}] + \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) - F_{i2,sh}(1 - \frac{\rho_{sh}}{\rho_w})]\frac{\rho_{eff}}{1 - \rho_{eff}}\}$$

$$= \frac{A_v}{A}\rho_w\rho_s\Phi_o\{1 + \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s)]\frac{1}{1 - \rho_{eff}} - \rho_w F_{i2,sh}[1 + (1 - \frac{\rho_{sh}}{\rho_w})\frac{\rho_{eff}}{1 - \rho_{eff}}]\}$$

In this case

$$\rho_{sh} = \rho_w$$

and the signal from the detector becomes

$$S_1 = K\frac{1}{\pi A}\rho_w\rho_s\Phi_o\{1 + \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s)]\frac{1}{1 - \rho_{eff}} - \rho_w F_{i2,sh}\}$$

When the beam is directed onto the sphere wall then $\Phi_{v,1}$ and $\Phi_{v,2}$ are exactly the same as the diffuse case since no flux is incident on the sample before the flux has been reflected twice by the sphere wall. In fact, we can use the same equation for Φ_v if we replace the average wall reflectance $\rho_{\overline{w}}$ as used in the diffuse case by the effective sphere reflectance as defined in the specular case. Therefore S_2, the signal when the beam is first incident on the sphere wall, is given by

$$S_2 = K\frac{1}{\pi A}\rho_w\Phi_o\{1 + \rho_w[(1 - F_{w,124sh2})\frac{1}{1 - \rho_{eff}} + F_{w,sh2}(\frac{A}{A_v}F_{sh2,v} + \frac{\rho_{eff}}{1 - \rho_{eff}}(1 - F_{sh2,4}))]\}$$

and the S_1/S_2 ratio of the two signal becomes

$$\frac{S_1}{S_2} = \rho_s \frac{\{1 + \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s)]\frac{1}{1-\rho_{eff}} - \rho_w F_{i2,sh}\}}{\{1 + \rho_w[1 - F_{w,124sh2})\frac{1}{1-\rho_{eff}} + F_{w,sh2}\left(\frac{A}{A_v}F_{sh2,v} + \frac{\rho_{eff}}{1-\rho_{eff}}(1-F_{sh2,4})\right)]\}}$$

or

$$\rho_s = \frac{S_1}{S_2} C,$$

where the correction factor C is defined by the equation above.

In addition to the terms already used previously we need ρ_{eff} which was given by

$$\rho_{eff} = \rho_w[1 - \frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s + 2A_{sh})] + \frac{2A_{sh}}{A}\rho_{sh} + \frac{A_s}{A}\rho_s$$

$$- \frac{A_s}{A^2}\left(A_1 \frac{A_{i2}}{A_{i3}} + \frac{A_{i4}}{A_{i5}}\right)$$

$$= 0.9939\rho_w + 0.00033\rho_s - 3.628(10^{-7})$$

where ρ_{sh} was set equal to ρ_w.

From this we see that ρ_{eff} cannot be distinguished from $\overline{\rho_w}$ and

$$\rho_{eff} = \overline{\rho_w} = 0.9939\rho_w + 0.00033\rho_s$$

then

$$\frac{1}{A}(A_1 + A_2 + A_3 + A_4 + A_s) = 0.0061$$

$$F_{i2,sh} = 5.62\ (10^{-5})$$

With this the correction factor becomes

$$C = \frac{1 - 0.00033\rho_s}{1 + 0.0166\rho_w + 0.00033\rho_s - 0.01592\rho_w^2}$$

The table below gives the correction factor C and its percentage for a perfectly specular sample as a function of wall and sample reflectance.

Table II.

$\rho_s = 0.1$

ρ_w	0.4	0.6	0.8	0.9	1.0
C	0.9959	0.9957	0.9968	0.9979	0.9992
$\frac{C-1}{C}100$	-0.41	-0.43	-0.32	-0.21	-0.08

$\rho_s = 0.95$

ρ_w	0.4	0.6	0.8	0.9	1.0
C	0.9955	0.9952	0.9965	0.9974	0.9987
$\frac{C-1}{C}100$	-0.47	-0.48	-0.37	-0.26	-0.13

The relatively large correction factors for a perfectly diffuse reflector originate mainly from the fact that part of the flux reflected at polar angles around 75° is intercepted by the shield and part of the flux reflected at polar angles around 85° is trapped by the evacuation slots. Most real materials, even if they are considered to be good diffusers, reflect considerably less at these large polar angles than does a perfect diffuser. This means that for any real material the correction factor is much smaller than that calculated on the basis of a perfect diffuser. Especially for metals which reflect predominantly in the specular direction even when roughened, the correction factor will be much closer to that for a perfect mirror than that for a perfect diffuser.

Since it would be very difficult to calculate the correction factor for every real sample, the ratio $\frac{S_1}{S_2}$ was set equal to the directional hemispherical reflectance. In this case the correction factors from tables 1 and 2 or the curves of fig. 8 show the maximum error for a perfect diffuser. The actual error for any real sample will be much smaller than those calculated on the basis of the two extreme flux distributions.

III. SHALLOW CAVITY PROCEDURE FOR MEASUREMENT OF TOTAL NORMAL EMITTANCE OF NONMETALS AT TEMPERATURES ABOVE 1800°K.

The equipment for measurements by the shallow cavity procedure has been moved into the new laboratories at Gaithersburg, Maryland. The radio frequency generator used with this equipment is shared with another project, and will not be available for use with the shallow cavity equipment for several weeks.

IV. REFERENCES

[1] Hamilton, D.C. and Morgan, W.R. NACA TN 2836.

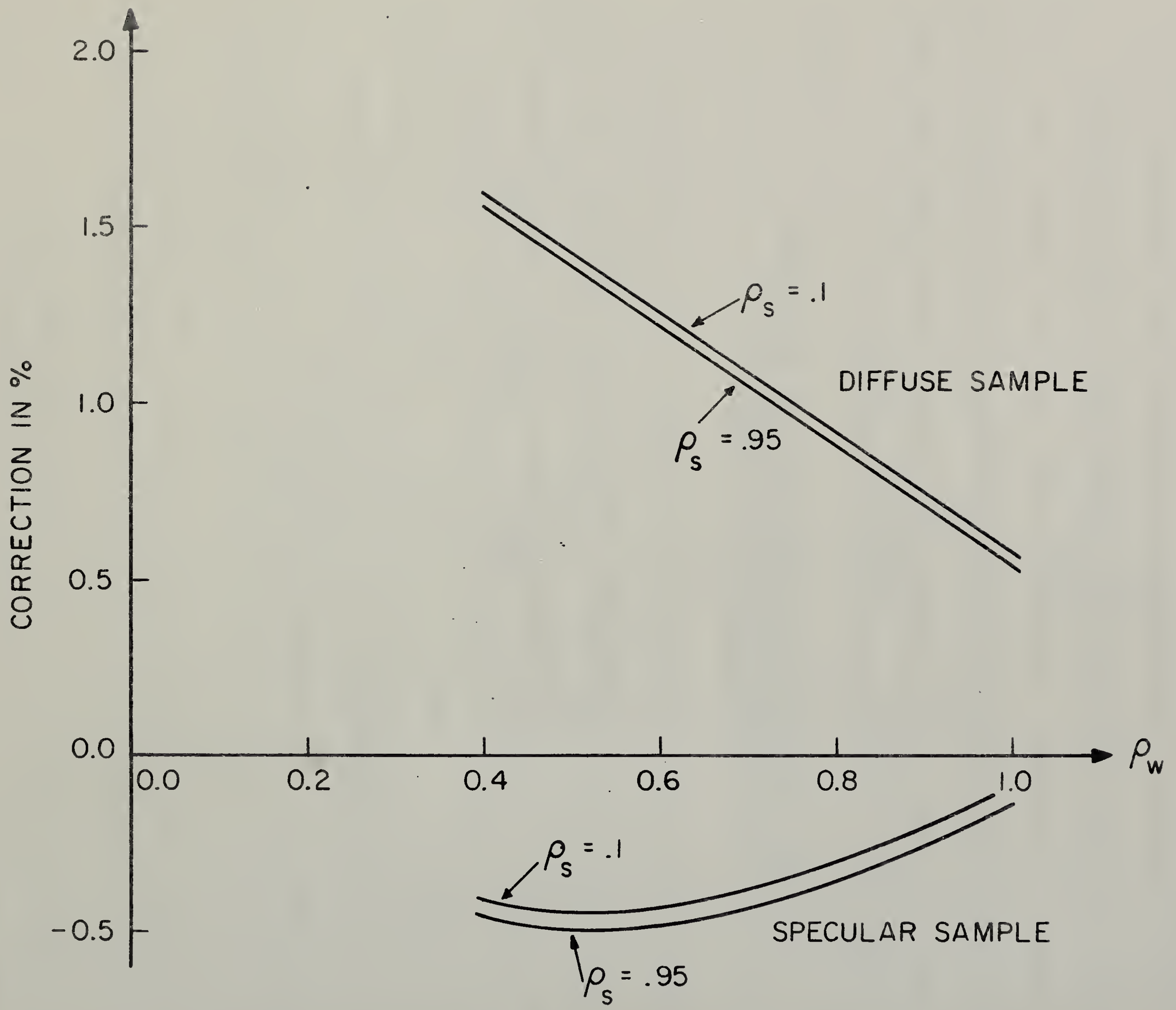
2.0
1.5
1.0
0.5
0.0
-0.5
CORRECTION IN %
0.0
0.2
0.4
0.6
0.8
1.0
ρ_w
$\rho_s = .1$
$\rho_s = .95$
DIFFUSE SAMPLE
$\rho_s = .1$
$\rho_s = .95$
SPECULAR SAMPLE

Figure 8

List of Symbols

Areas:

A	= Total surface area of the sphere $A = 4R^2\pi$
A_1	= Area of the entrance port
A_2	= Area of the detector port
A_3	= Area of the ring space around the sample
A_4	= Area of evacuation slots
A_5	= Part of A_4 screened off from the irradiated area of the sample
A_6	= $A_4 - A_5$
A_{sh}	= Area of one side of the shield
A_{sh1}	= Area of the shield facing the sample
A_{sh2}	= Area of the shield not facing the sample
A_{h1}	= Area of the hemisphere facing A_{sh1}
A_{h2}	= Area of the hemisphere facing A_{sh2}
A_v	= Area of the field of view of the detector
A_s	= Area of the sample
A_i	= Part of A_s initially irradiated by Φ_0
A_{i1}	= Area of the irradiated spot on the sphere due to the perfect specular reflection of Φ_0 at the sample
$A_{\overline{i1}}$	= Image of A_{i1} as seen in a perfect sample
A_{i2}	= Part of the sphere wall from which flux is able to reach A_1 due to perfect specular reflection at the sample
A_{i3}	= Part of the sphere area centered around the entrance port which is irradiated by flux leaving A_{i2} due to perfect specular reflection at the sample
A_{i4}	= Area analogous to A_{i2} in connection with A_2
A_{i5}	= Area analogous to A_{i3} in connection with A_2

Fluxes:

Φ	= Flux [Watt]
Φ_0	= Flux initially entering the sphere
Φ'	= Flux reflected by the sample regardless of direction
Φ_v	= Sum of flux incident on the field of view of the detector after Φ_0 has undergone infinite many reflections
$\Phi_{v,n}$	= Flux incident on A_v after Φ_0 has undergone n reflections at the sphere wall
Φ_n	= Flux contained in the sphere after Φ_0 has undergone n wall reflections

Reflectances:

ρ_s	= Directional, hemispherical reflectance of the sample correctly expressed as $\rho(12°; 2\pi)$
ρ_w	= Reflectance of the sphere wall
ρ_{sh}	= Reflectance of the shield
$\bar{\rho}_w$	= Average sphere wall reflectance as defined on page 16
ρ_{eff}	= Effective reflectance of the sphere wall as defined on page 29

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