

BL5

# NATIONAL BUREAU OF STANDARDS REPORT

9303

PROJECTS and PUBLICATIONS  
of the  
APPLIED MATHEMATICS DIVISION

A Semi-Annual Report  
July through December 1965

For Official Distribution



U.S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS

## THE NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards is a principal focal point in the Federal Government for assuring maximum application of the physical and engineering sciences to the advancement of technology in industry and commerce. Its responsibilities include development and maintenance of the national standards of measurement, and the provisions of means for making measurements consistent with those standards; determination of physical constants and properties of materials; development of methods for testing materials, mechanisms, and structures, and making such tests as may be necessary, particularly for government agencies; cooperation in the establishment of standard practices for incorporation in codes and specifications; advisory service to government agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; assistance to industry, business, and consumers in the development and acceptance of commercial standards and simplified trade practice recommendations; administration of programs in cooperation with United States business groups and standards organizations for the development of international standards of practice; and maintenance of a clearinghouse for the collection and dissemination of scientific, technical, and engineering information. The scope of the Bureau's activities is suggested in the following listing of its four Institutes and their organizational units.

**Institute for Basic Standards.** Applied Mathematics. Electricity. Metrology. Mechanics. Heat. Atomic Physics. Physical Chemistry. Laboratory Astrophysics.\* Radiation Physics. Radio Standards Laboratory.\* Radio Standards Physics; Radio Standards Engineering. Office of Standard Reference Data.

**Institute for Materials Research.** Analytical Chemistry. Polymers. Metallurgy. Inorganic Materials. Reactor Radiations. Cryogenics.\* Materials Evaluation Laboratory. Office of Standard Reference Materials.

**Institute for Applied Technology.** Building Research. Information Technology. Performance Test Development. Electronic Instrumentation. Textile and Apparel Technology Center. Technical Analysis. Office of Weights and Measures. Office of Engineering Standards. Office of Invention and Innovation. Office of Technical Resources. Clearinghouse for Federal Scientific and Technical Information.\*\*

**Central Radio Propagation Laboratory.\*** Ionospheric Telecommunications. Tropospheric Telecommunications. Space Environment Forecasting. Aeronomy.

---

\* Located at Boulder, Colorado 80301.

\*\* Located at 5285 Port Royal Road, Springfield, Virginia 22171.

# NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT

205.0

NBS REPORT

9303

PROJECTS and PUBLICATIONS  
of the  
APPLIED MATHEMATICS DIVISION  
A Semi-Annual Report  
July through December 1965

## IMPORTANT NOTICE

NATIONAL BUREAU OF STANDARDS  
for use within the Government. Before  
and review. For this reason, the pub-  
lished whole or in part, is not authorized  
Bureau of Standards, Washington, D.C.  
the Report has been specifically prepared

Approved for public release by the  
director of the National Institute of  
Standards and Technology (NIST)  
on October 9, 2015

Counting documents intended  
subjected to additional evaluation  
of this Report, either in  
advance of the Director, National  
Government agency for which  
for its own use.



U.S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS

# APPLIED MATHEMATICS DIVISION

July 1 through December 31, 1965

## TECHNICAL ADVISORY PANEL

Francis J. Anscombe, Chairman  
Yale University

Charles R. DePrima, Calif. Institute of Technology	J. P. LaSalle, Brown University
Joaquin B. Diaz, University of Maryland	J. Barkley Rosser, USARC, University of Wisconsin
Ralph E. Gomory, IBM, Yorktown Heights, N. Y.	John Todd, California Institute of Technology
John W. Tukey, Princeton University	

## DIVISION OFFICE

Edward W. Cannon, Ph.D., Chief	
Catherine Hartsfield, Sec'y	Y. S. Sladen, Admin. Officer
Sylvain E. Cappell <sup>ooo</sup>	Hansjörg Oser, Ph.D., Consultant
Robert Hsu, M.S. <sup>o</sup>	Ida Rhodes, M.A., Consultant
Edward Jones <sup>ooo</sup>	Arlene Ruhl, Secretary
Russell A. Kirsch, M.S.	William Watt, M.S.L.

## NUMERICAL ANALYSIS SECTION, Morris Newman, Ph.D., Chief

Richard A. Brualdi, Ph.D.**	Leon Greenberg, Ph.D.*	Joseph Lehner, Ph.D.*
Doris M. Burrell, Secretary	Seymour Haber, Ph.D.	Frank W. J. Olver, D.Sc.
Sharrill Dittman <sup>ooo</sup>	Mary E. Kramer <sup>ooo</sup>	Rosemarie M. Stemmler, Ph.D.**
Karl Goldberg, Ph.D.		

## STATISTICAL ENGINEERING LABORATORY, Joseph M. Cameron, M.S., Chief Joan R. Rosenblatt, Ph.D., Ass't Chief

Raj Chandra Bose, Ph.D.*	Hsien H. Ku, M.S.	Phyllis A. Tapscott, Secretary
Marion T. Carson	Mary G. Natrella, B.A.*	Ruth N. Varner, B.A.*
Veronica Connor, Secretary	Sally T. Peavy, B.S.*	Roy H. Wampler, M.A.
David Hogben, Ph.D.	Janace Speckman, M.A.	Thomas A. Willke, Ph.D.*
Brian L. Joiner, M.S.		

## MATHEMATICAL PHYSICS SECTION, Hansjörg Oser, Ph.D., Acting Chief

Barry Bernstein, Ph.D. <sup>o</sup>	Fritz Oberhettinger, Ph.D.***	Chan Mou Tchen, Ph.D.
James H. Bramble, Ph.D.*	Lawrence E. Payne, Ph.D.*	John P. Vinti, Sc.D.
Doris M. Burrell, Secretary	Walter L. Sadowski, Ph.D.	

## OPERATIONS RESEARCH, Alan J. Goldman, Ph.D., Chief

Jack Edmonds, M.A.	Lambert S. Joel, B.A.	Philip Meyers, M.A.
William G. Hall, B.S.	Kenneth E. Kloss, M.A.	James Root**
Dolores E. Harrison, Sec'y	Joel Levy, M.A.	Christoph J. Witzgall, Ph.D.
John A. Hodgson <sup>ooo</sup>		

\*Part-time  
<sup>o</sup>On leave of absence

\*\*Postdoctoral Resident Research Associate  
<sup>oo</sup>Temporary appointment

\*\*\*Guest worker  
<sup>oo</sup>Student trainee

## N O T I C E

On July 1, 1965 the Computation Laboratory of the Applied Mathematics Division was transferred to the newly created Center for Computer Sciences and Technology in the Institute for Applied Technology of the National Bureau of Standards.

As a result of this transfer, a revision of our mailing list has become necessary inasmuch as many subscribers were primarily interested in the activities of the Computation Laboratory, which will no longer be reported in the Projects and Publications report of the Applied Mathematics Division.

Please use the form on the next page to indicate whether or not you wish to continue receiving this report. In the absence of a reply in the affirmative, this report will no longer be sent to you.

NATIONAL BUREAU OF STANDARDS  
Applied Mathematics Division  
Room A-438 - Administration Bldg.  
Washington, D. C. 20234

(2) Fold Back Here

(1) Fold Back Here

We are  
are not interested in receiving further issues of the Applied  
Mathematics Division's Semiannual Projects and Publications reports.

Please print address here  
as it appears on your mail-  
ing label, including the  
key number which is a part  
of your address.



--

C U T  
H E R E

## Contents

Status of Projects <sup>o</sup> as of December 31, 1965. . . . .	1
1. Numerical analysis . . . . .	1
2. Probability and mathematical statistics . . . . .	4
3. Mathematical physics . . . . .	7
4. Operations research . . . . .	10
5. Statistical engineering services . . . . .	14
Lectures and technical meetings . . . . .	15
Publication activities . . . . .	17





# Status of Projects

## 1. NUMERICAL ANALYSIS

### RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 20501-12-2050110/55-55

Origin: NBS

Authorized 8/29/54

Manager: Morris Newman

Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has classified all rational  $2 \times 2$  representations of the modular group, extending the results for real representations previously obtained jointly with J. Lehner.

M. Newman has proved that 0 is always a Weierstrass point of  $\Gamma_0(p^n)$ , where  $p$  is a prime  $\geq 11$  and  $n \geq 2$ . He also showed that if  $m, p$  are integers such that  $p$  is a prime,  $p \mid m$ ,  $p \neq m$  then if 0 is a Weierstrass point of  $\Gamma_0(m)$ , it is also one of  $\Gamma_0(pm)$ .

R.A. Brualdi and M. Newman proved the following result, which partially answers a problem posed by Marshall Hall:

Let  $Q$  be any  $n \times n$  permutation matrix. Let  $f(x)$  be any polynomial with non-negative integral coefficients and with at least 3 non-zero coefficients. Then  $\text{per}(f(Q)) \geq n + 3$ .

L. Greenberg is continuing his studies of Kleinian groups.

J. Lehner is studying automorphic forms on groups whose fundamental domain has infinitely many parabolic cusps.

K. Goldberg has continued his investigation of the l.u.b. of the determinants of certain matrices whose row sums are all equal to 1. Let  $\Sigma_{mn}$  denote the set of all such matrices which are real and  $m$  by  $n$ . Given  $K \in \Sigma_{mn}$  of rank  $m$ , then the l.u.b. of the (absolute values of the) determinants of those  $A \in \Sigma_{mn}$  such that  $KA$  has only non-negative elements, is shown to exist and be positive. Upper and lower bounds are found. Finally it is shown that for such  $A$  with large determinants the elements of  $KA$  which are equal to 0 must form one of a specified number of patterns depending only on  $m$  and  $n$ .

S. Haber continued studies of number-theoretical methods of multiple quadrature. He obtained an improved estimate for the remainder in a quadrature formula of N.M. Korobov. He also found sharp estimates of the irregularity of distribution of two sequences of points, defined by J.G. van der Corput and K.F. Roth, which are of interest for numerical quadrature. The estimates disproved a conjecture of J.H. Halton.

F.W.J. Olver is continuing his work in asymptotic expansions under Task 20501-11-2050421/63.

#### Publications:

- (1) Some theorems on the permanent. R.A. Brualdi and M. Newman. Journal of Research NBS, 69B, pp. 159-163, 1965.
- (2) Inequalities for permanents and permanental minors. R.A. Brualdi and M. Newman. Proceedings of the Cambridge Philosophical Society, Vol. 61, pp. 741-746, 1965.

#### Status of Projects

- (3) Real two-dimensional representations of the modular group and related groups. J. Lehner and M. Newman. American Journal of Mathematics, Vol. 87, No. 4, pp. 945-954, 1965.
- (4) Kernels and the Kronecker product of graphs. R.A. Brualdi. To appear in Proceedings of the American Mathematical Society.
- (5) Inequalities for the permanent minors of non-negative matrices. R.A. Brualdi and M. Newman. To appear in the Canadian Journal of Mathematics.
- (6) Proof of a permanent inequality. R.A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).
- (7) Hadamard matrices of order cube plus one. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
- (8) Maximum determinants of certain row stochastic matrices. K. Goldberg. To be submitted to the Journal of Research NBS.
- (9) Transformations of ordered vectors. K. Goldberg. To be submitted to the Journal of Research NBS.
- (10) Semi-groups with zeroids. K. Goldberg. To be submitted to the Journal of Research NBS.
- (11) A recurrence related to monotone subsequences in permutations. K. Goldberg. To be submitted to the Journal of Research NBS.
- (12) The l.u.b. of a set of determinants. K. Goldberg. To be submitted to the Journal of Research NBS.
- (13) The l.u.b. of a set of determinants of order 3. K. Goldberg. To be submitted to the Journal of Research NBS.
- (14) Upper bounds for the determinant of a row stochastic matrix. K. Goldberg. To be submitted to the Journal of Research NBS.
- (15) Groups preserving ordering in vectors. K. Goldberg. To be submitted to the Journal of Research NBS.
- (16) Pairs of non-singular matrices. K. Goldberg. To be submitted to the Journal of Research NBS.
- (17) On a theorem of Ahlfors. L. Greenberg. To appear in the American Journal of Mathematics.
- (18) Fundamental polyhedra for Kleinian groups. L. Greenberg. To appear in the Annals of Mathematics.
- (19) Kleinian groups. L. Greenberg. To appear in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans (1965) .
- (20) A functional inequality. S. Haber. To appear in the American Mathematical Monthly.
- (21) A theorem on arbitrary functions. S. Haber. Submitted to a technical journal.
- (22) A modified Monte-Carlo quadrature. S. Haber. To appear in Mathematics of Computation.

### Status of Projects

- (23) On the very low temperature behavior of a ferromagnet. S. Haber and S.J. Glass. Submitted to a technical journal.
- (24) On a sequence of points of interest for numerical quadrature. S. Haber. Submitted to a technical journal.
- (25) Real two-dimensional representations of the free product of two finite cyclic groups. J. Lehner and M. Newman. To appear in Proc. Cambridge Philos. Soc.

### ASYMPTOTIC EXPANSIONS

Task 20501-11-2050421/63

Origin: NBS  
Sponsor: U.S. Army Research Office, Durham, N.C.  
Manager: F.W.J. Olver  
Full task description: July-December 1963 issue, p. 2

Authorized 9/10/63

Status: CONTINUED. Work has begun on the error analysis of asymptotic expansions derived from integral representations. Some preliminary results on Laplace's method and the method of steepest descents have been obtained. Applications and extensions are now being studied.

#### Publications:

- (1) Error bounds for asymptotic expansions of special functions in the complex plane. F.W.J. Olver. Error in Digital Computation, Vol. 2, edited by L.B. Rall. John Wiley, New York, 1965.
- (2) On the asymptotic solutions of second-order differential equations having an irregular singularity of rank one, with an application to Whittaker functions. F.W.J. Olver. Journal of the Society for Industrial and Applied Mathematics, Series B, Vol. 2, No. 2, pp. 225-243, 1965.
- (3) Error bounds for asymptotic solutions of second-order differential equations having an irregular singularity of arbitrary rank. F.W.J. Olver and F. Stenger. Journal of the Society for Industrial and Applied Mathematics, Series B, Vol. 2, No. 2, pp. 244-249, 1965.
- (4) Error analysis of phase-integral methods I. General theory for simple turning points. F.W.J. Olver. To appear in the Journal of Research NBS, 69B, Part 4, Oct.-Dec. 1965.
- (5) Error analysis of phase-integral methods II. Application to wave-penetration problems. F.W.J. Olver. To appear in the Journal of Research NBS, 69B, Part 4, Oct.-Dec. 1965.
- (6) Error bounds for asymptotic solutions of differential equations I. The distinct eigenvalue case. F. Stenger (manuscript).
- (7) Error bounds for asymptotic solutions of differential equations II. The general case. F. Stenger (manuscript).

### 3. PROBABILITY AND MATHEMATICAL STATISTICS

#### RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS  
Manager: Joan Raup Rosenblatt  
Full task description: July - December 1962

Authorized 10/1/62

Status: CONTINUED. Brian L. Joiner has expanded his investigation of tests for homogeneity of variance to include a general study of the properties of "Cochran-type" tests, based on a statistic of the form

$$T = \max_i D_i / \sum_i D_i ,$$

where  $D_1, \dots, D_k$  are dispersion measures. For the Cochran test,  $D$  is the sample variance; for the Bliss-Cochran-Tukey test,  $D$  is the sample range. C. Eisenhart has proposed the use of the sample standard deviation for  $D$ , on the grounds that it may provide a more robust test. Approximate percentage points for the Eisenhart test are being computed.

Richard G. Cornell (Florida State University) and Janace A. Speckman have completed a paper entitled "Estimation for a simple exponential model", reporting comparisons of ten methods for estimating  $\rho$  in the statistical model

$$Y = 1 - \exp(-\rho t)$$

where  $Y$  is the expected value of an observed proportion at time (or dosage)  $t$ .

Roy H. Wampler determined, for W. J. Youden, the most "normal-like" triangular distribution. The triangular distribution (symmetrical about zero) that most closely approximates the standardized normal distribution in the sense that the two distributions share the greatest common area has density function

$$t(x) = \alpha - |\alpha^2 x|, \quad -1/\alpha \leq x \leq 1/\alpha, \quad \alpha > 0$$

with  $\alpha = .4425$ . The standard deviation of this triangular distribution is .9226.

Another question posed by W. J. Youden led to an investigation by Brian L. Joiner and Joan R. Rosenblatt of the distribution of observations on a randomly truncated random variable. Let  $x$  and  $y$  denote two observations whose joint distribution is determined in the following way. First, observe  $x$ , which has frequency function  $g(x)$ ,  $-\infty < x < \infty$ . Then, observe  $y$ , which has the conditional distribution  $g(y)/G(x)$ ,  $-\infty < y < x$ , where  $G(x)$  is the cumulative distribution function corresponding to  $g(x)$ . The marginal frequency function for  $y$  is then  $h(y) = -g(y) \ln G(y)$ ,  $-\infty < y < \infty$ . Some properties of  $h(y)$  were determined for various special cases, including uniform and normal distribution for  $g(x)$ .

A library of subroutines for statistical computations for use with the Bureau's OMNITAB program, is being established. The file will include documentation of accuracy (or lack of it), discussion of interpretation of results, and references. Examples of subroutines included are: analysis of residuals, tests for homogeneity of variance, percentage points of various distributions. The basic OMNITAB program (see A general-purpose interpretive program for the calculation of tables of functions and statistical and numerical analysis, by Joseph Hilsenrath, et al., NBS Handbook 101, 1965) provides some basic statistical computations.

Work continued on the development of experiment designs for calibration work, and of related methods of analysis. A weighing design was constructed for the 5, 3, 2, 1, 21, Std 1 series. A note on "least squares in one operation" extended published results to include the case of least squares problems with restraints.



## 7. STATISTICAL ENGINEERING SERVICES

### COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING

Task 13911-61-1390951/51-1

Origin: NBS

Authorized 7/1/50

Managers: J. M. Cameron, H. H. Ku

Full task description: July-September 1950 issue, p. 60

Status: CONTINUED. This is a continuing project involving cooperation with other Bureau scientists on the statistical aspects of their investigations. These services vary from short (one-hour) sessions to extended collaborations involving several man-months; and are concerned primarily with statistical design of experiments, analysis and interpretation of data, and the use of computers in statistical analysis of data. Typical examples of the services performed are the following. (a) The section has continued its collaboration with a number of the Bureau's calibration laboratories. Especially noteworthy is the section's work on the statistical and computational aspects of the Pilot Program of the Mass and Volume section. The procedures of the Pilot Program are designed to assist other laboratories in being self-sufficient once they have a value for a starting standard. (b) As part of the effort of providing improved designs for calibration, a new design for the 5, 3, 2, 1, 21, Std 1 series was created. New designs for intercomparisons of two groups of 10 standard volt cells were prepared and used on some groups involved in the move to Gaithersburg. (c) Work has begun on methods for the evaluation of the flatness of surface plates. (d) Analysis of data on the atomic weight of chromium and the isotopic ratios of magnesium were analyzed for the Analytical Mass Spectrometry section. (e) Data on the strength of glass has been analyzed to determine which of a number of possible probability distributions provide the best representation of the data. (f) The section has continued its collaboration with the Office of Product Standards particularly with regard to revision of lumber and apparel sizing standards. (g) Dr. Rosenblatt has continued her work for the Technical Analysis Division.

Statistical computations were carried out for other Bureau scientists, largely through the use of OMNITAB. A library of OMNITAB statistical routines (e.g. on the analysis of residuals) has been started in order to facilitate the use of the more sophisticated statistical techniques.

#### Publications:

- (1) Uncertainties associated with proving ring calibration. T. E. Hockersmith (Mechanics Division) and H. H. Ku. Proceedings, Instrument Society of America Annual Conference, Vol. 19, Part 1, 1954, pages 73-77.
- (2) Mortality patterns in eight strains of flour beetles. W. J. Youden, D. B. Mertz and T. Park (Univ. of Chicago). Biometrics, Vol. 21, 1965, pages 99-114.
- (3) Evaluation of analytical data. W. J. Youden. To appear in Encyclopedia of Industrial Analysis.

## STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January-March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

The most notable of these services were: (a) participation in the 5th Interservice Measurement Audit at Sandia Corporation, Albuquerque, November 16-18 and The Fifth Annual Army Audit at White Sands. (b) inquiries concerning mathematical tables primarily for use in statistics were handled and as a result the sections expanded efforts in the area of tables. Perhaps a fourth of the consulting problems relate to computational methods or similar aspects of computer usage.

Under this project the section does work for the National Conference of Standards Laboratories, American Standards Association, ASTM, ASTM E, and other technical societies. Members of the section have served, for example, on committees of the American Statistical Association, as Associate Editor for the Society for Industrial and Applied Mathematics and as Program Secretary for the Institute of Mathematical Statistics.

### Publications:

- (1) Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society of Tool and Manufacturing Engineers.
- (2) The bridge tournament problem and calibration designs for comparing pairs of objects. R. C. Bose and J. M. Cameron. To appear in NBS J. Research B. (Math. and Math. Physics).
- (3) Notes on the use of propagation of error formulas. H. H. Ku. Submitted to a technical journal.
- (4) Estimation for a simple exponential model. Richard G. Cornell (Florida State University) and Janace A. Speckman. Submitted to a technical journal.
- (5) Useful alternatives to Chauvenet's rule for rejection of measurement data. T. A. Willke. Submitted to a technical journal.

4. MATHEMATICAL PHYSICS  
RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS  
Task 20540-12-2050141/55-57

Origin: NBS  
Manager: H. Oser  
Full task description: July-September 1954 issue, p. 27

Authorized 9/1/54

Status: CONTINUED. The level of activity was greatly reduced with Dr. Bramble's departure from NBS and Dr. Bernstein's being on leave of absence through most of the reporting period.

Work was consultative in nature. Assistance was given to the Engineering Mechanics Section of NBS on the computation of special functions; also to the Metal Physics Section on the integration of nonlinear integral equations describing the growth of one-dimensional metal whiskers in a vapor atmosphere; and to the Office of Business Economics of the Commerce Department on the exact solution of a nonlinear partial differential equation which is to describe certain aspects of the Nation's economy (together with Dr. A. Goldman of the Operations Research Section) .

Publications:

- (1) Elastic stress-strain relations in perfect elastic fluids. B. Bernstein, E.A. Kearsley, and L.J. Zapas. Transactions of the Society of Theology, Vol. 9, Part 1, pp. 27-39, (1965) .
- (2) Time dependent behavior of an incompressible elastic fluid - Some homogeneous deformation histories. B. Bernstein. To appear in Acta Mechanica.
- (3) A priori bounds in the equations of classical incompressible elasticity. J.H. Bramble and L.E. Payne. To appear in Journal of Research NBS, Section B.
- (4) The fundamental solution and Huygens' principle for decomposable differential operators. J.E. Lagnese. Archive for Rational Mechanics and Analysis, Vol. 19, No. 4, pp. 299-307, 1965 .

PLASMA RESEARCH  
Task 20504-12-2050140/59-442

Origin: NBS  
Manager: C.M. Tchen  
Full task description: April-June 1959 issue, p. 15

Authorized 6/30/59

Status: CONTINUED. Dr. Tchen has investigated the foundation of the quasilinear method for solving the nonlinear Vlasov equation. This is done as a generalization of the customary technique of many time scales.

DYNAMICS OF PLASMA  
Task 20540-12-2050417/62-1157

Origin: NBS  
Sponsor: National Aeronautics and Space Administration  
Manager: C.M. Tchen  
Full task description: October-December 1961 issue, p. 12

Authorized 10/3/61

Status: CONTINUED. Dr. C.M. Tchen has continued his research on the kinetic and magnetohydrodynamic theories of ionized gases, with emphasis on the following topics:

(1) Magnetohydrodynamic turbulence.

A paper on the spectrum of stationary and homogeneous magnetohydrodynamic turbulence has been published in *Physics of Fluids*. The theory covers the strong coupling between the magnetic field and turbulence. The investigation of the case of a weak coupling has been continued. A weak coupling occurs when the two spectra show a distinct shift against each other.

(2) Plasma turbulence.

The problem of turbulence in a rarefied plasma has recently called the attention of many investigators, because not only has it important applications to provide a heating mechanism of plasmas in thermonuclear experiments, and to account for a diffusion or dissipation mechanism in collision free shock waves in interplanetary space, but also it is an important problem in nonlinear mathematics and physics. Tchen has completed a manuscript on "Turbulence in a Rarefied Plasma", which he presented on invitation at the Symposium for Advanced Problems in Fluid Mechanics, September 1965, Jurata, Poland. The paper is expected to appear in the Proceedings of the Symposium.

(3) Solution of the Nonlinear Vlasov Equation.

The nonlinear plasma oscillations are studied by means of the Vlasov equation. The computations were carried out by Dr. W. Sadowski on an electronic computer at the Faculté des Sciences, University of Paris, Orsay, France. Some of the physical features are being analyzed by Tchen on the basis of a quasilinear theory of plasma oscillations. The numerical data covering plasma oscillations exhibiting strong nonlinear behavior have still to be checked as to the effects of the numerical errors and numerical instabilities.

(4) In the capacity of a visiting professor, Institute for Theoretical Physics, University of Marburg, West Germany, Tchen completed a seminar on his current research in plasma physics: plasma turbulence, kinetic equation for turbulent plasma, scattering, anomalous diffusion, quasilinear theory of plasmas. It ran from May to November 1965, for 2 hours per week. While in Europe Tchen attended the International Symposium on Ionization Phenomena in Gases, August 1965, Belgrade, Yugoslavia. He also visited and presented talks at the following institutions: (i) Faculté des Sciences, University of Paris, Orsay, France; (ii) Centre d'Études Nucléaires Fontenay aux Roses, Seine et Oise, France; (iii) Centre d'Études Nucléaires, Saclay, Seine et Oise, France; (iv) Institute for Plasma Physics, Jutphaas, Netherlands.

Publications:

- (1) Spectrum of stationary homogeneous magnetohydrodynamic turbulence. C.M. Tchen. *Physics of Fluids*, Vol. 8, No. 9, pp. 1659-1667, September, 1965.



## Status of Projects

- (2) Turbulence in a rarefied plasma. C.M. Tchen. Accepted for publication at the Proceedings of the Symposium for Advanced Problems in Fluid Mechanics, September 1965, Jurata, Poland.

## THEORY OF SATELLITE ORBITS

Task 20504-12-2050441/62-1166

Origin: NBS  
Sponsor: National Aeronautics and Space Administration  
Manager: J.P. Vinti  
Full task description: January-March 1962 issue, p. 12

Authorized 1/6/62

Status: COMPLETED. The proposed program for incorporating the third harmonic of the earth's gravitational field into a separable solution for a satellite orbit was successfully completed. The fundamental physical principles are outlined in the paper "Invariant properties of the spheroidal potential of an oblate planet" while the detailed solution for the orbit is given in the paper "Inclusion of the third zonal harmonic in an accurate reference orbit of an artificial satellite." Both papers have been accepted for publication in the NBS Journal of Research, Section B (January-March 1966) .

Dr. Vinti is now with the Mathematics Department of the State University of North Carolina, at Raleigh, N.C.

## 5. OPERATIONS RESEARCH

### CONSULTATION IN MATHEMATICAL OPERATIONS RESEARCH

Task 205-12-2050151

Origin and Sponsor: NBS  
Manager: Alan J. Goldman  
Full task description: October-December 1960 issue, p.3

Authorized 12/30/60

Status: CONTINUED. This is a continuation of part of Task 2050-12-2050115/61-546.

(1) Miscellaneous consulting and advisory services were provided by Section staff in 67 recorded instances (many involving multiple conferences); of these 30 involved assistance to NBS staff (covering 6 different NBS divisions or other units). The 67 instances totalled to 460 man-hours. Other agencies assisted included the Office of Business Economics, Bureau of Public Roads, National Science Foundation, Veterans Administration, Office of the Secretary of the Army (Civil Defense) and Bureau of the Budget. Requests from universities, industry and professional groups and journals were also met.

(2) L.S. Joel continued leadership of the work in modelling some aspects of the textile industry. Reports are in preparation describing the "tufted carpet" and "hosiery" simulation models and the results obtained with them. (Reported here for convenience; supported under Project 4314690.)

(3) J. Levy and A.J. Goldman continued studies of the effects of buffer capacity in certain mail sorting devices. P. Meyers completed documenting his investigation of a stochastic sorting process, and his study of alternative methods of encoding letter addresses. He and A.J. Goldman helped complete and document an evaluation of a proposed sorting sub-system. (Reported here for convenience; supported under Project 4230450.)

(4) P. Meyers and A.J. Goldman (with J. McLynn and R. Watkins of Davidson, Talbird and McLynn) studied nonlinear generalizations of certain demand split models. They reviewed various aspects of a transport simulation model, and helped evaluate the outputs of contracts let by NBS. K. Kloss studied the usefulness of various simulation languages for the tasks at hand, while W. Hall is participating in the design of a data store for inter-urban transport analyses. (Reported here for convenience; supported under Project 4310421.)

(5) K. Kloss participated in the design and construction of a general purpose program for simulating a telephone network. The program differs from earlier efforts at NBS and elsewhere in the level of detail considered and in its versatility. All the delays actually encountered by a telephone call are simulated - e.g., the time required to dial the number, the time required by the various pieces of hardware in the switching machinery, the time required for a person to respond to a ringing phone. The program allows for alternate routings, for calls with various priorities and the ability to pre-empt calls of lower priority, for the "crank-back" and "spill forward" routine doctrines, and so on. (Reported here for convenience; supported under project 4235454.)

(6) K. Kloss designed and implemented a simple algebraic programming language for the NBS PILOT computer. The language allows the user to perform an arbitrary sequence of simple arithmetic operations on variables designated symbolically. A sequence of calculations can be carried out for each value of a given variable in a specified range. The full accuracy of the computer is made available to the user, and operations combining real (i.e. floating point) numbers and integers are allowed. The language permits easy specification of highly legible, neatly formatted output. The system is largely self-teaching; typing an illegal statement immediately yields a very specific error comment which usually enables the user to retype the line correctly.

(7) K. Kloss made some improvements in the PEAP assembly program for PILOT. These include: allowing input, as well as a limited editing ability, from a teletype; providing a much faster card reading routine; permitting symbolic specification of the unit and format in an input-output instruction; and laying most of the groundwork for a sophisticated macro-instruction capability.

(8) At the request of R.A. Kirsch (205.00), K. Kloss revised the PAX picture-processing language to run in the MIT MAC time sharing system. This made it possible to try out very quickly schemes for recognizing certain characteristics of pictorial data. It was found, moreover, that most runs on MAC took only half the computer time required by the NBS IBM 7094; this was attributed to the savings offered by MAC's disk storage versus the NBS magnetic tapes. A version of PAX was also produced which allows one to easily turn picture-processing programs into FORTRAN callable subroutines. The PAX language was extended to permit easy transmission of pictorial data to and from a FORTRAN program. (Reported here for convenience; supported under project 2050404.)

(9) C. Witzgall, with C. Mesztenyi (U. of Maryland), investigated how to choose "reference abscissae" to improve the accuracy of polynomial evaluation by Newton's formula.

(10) A.J. Goldman is participating in a analysis aimed at developing methods to guide proper protection levels for various elements of the Defense Communications System. (Reported here for convenience; supported under Project 4235455.)

Publications:

- (1) Realization of semi-multipliers as multipliers. Harriet Fell and A.J. Goldman. Amer. Math. Monthly (Math. Notes), 72, pp. 639-641, 1965.
- (2) Barely faithful algebras. Harriet Fell and John Mather. Amer. Math. Monthly (Math. Notes), 72, pp. 1001-1003, 1965.
- (3) A variant of the two-dimensional Riemann integral. A.J. Goldman. Journal of Research NBS, 69B, 185-188, 1965.
- (4) Some number-theoretic computations. K.E. Kloss. To appear in J. Research NBS, 69B, 1965.

COMBINATORIAL METHODS

Task 205-12-205-152

Origin and Sponsor: NBS

Authorized 12/30/60

Manager: Jack Edmonds

Full Task Description: October-December 1964 issue, p.3      April-June 1962 issue, p.15

Status: CONTINUED. This task was formerly part of 205-12-2050115/61-546, and related work was reported under 205-12-2050455/62-105.

(1) Jack Edmonds continued research in the field of matroids and "submodular" set functions along the lines reported for the last period, and began investigating the representation theory of matroids.

(2) C. Witzgall began a study of the comparative merits of labeling algorithms for determining shortest paths in networks. He continued a general examination, begun in connection with the coding of Edmonds' matching algorithm, of ways of representing networks in a computer. This work tends in the direction of developing a macro language for handling network algorithms. The shortest-path algorithms of Moore and Dantzig have been coded, the latter with the assistance of J. Hodgson. The Moore algorithm was specialized to deal with large acyclic networks. In collaboration with A.J. Goldman shortest path algorithms have been adapted to the more general problems of constrained shortest paths, and most profitable routing before maintenance. One version of an algorithm for most profitable routing before maintenance has been coded. (Supported in part by Project 4314421.)

Publications:

- (1) Transversals and matroid partition. Jack Edmonds and D.R. Fulkerson. Journal of Research NBS, 69B, pp.147-154, 1965.

SST ECONOMIC ANALYSIS

Task 20505-12-2050451

Origin: Commerce Dept. (SST Economic Analysis Study)

Authorized 8/10/64

Sponsor: Federal Aviation Agency

Managers: A.J. Goldman, W.G. Hall

Full Task Description: July-December 1964 issue, p.12

Status: COMPLETED. All results have been transmitted to the sponsor.

## COMBINATORIAL MATHEMATICS

Task 205-12-2050455/62-1205

Origin: NBS

Sponsor: Army Research Office-Durham

Manager: Jack Edmonds

Full task description: April-June 1962 issue, p. 15

Authorized 5/2/62

Status: COMPLETED. Current and future work in this field will be reported under Project 2050-12-2050152 (Combinatorial Methods).

## LINEAR AND NON-LINEAR PROGRAMMING

Task 2050-12-2050153

Origin and Sponsor: NBS

Manager: C. Witzgall

Full task description: October-December 1960 issue, p.3

Authorized 12/30/60

Status: CONTINUED. This is a continuation of part of Task 2050-12-2050115/61-546.

(1) C. Witzgall examined the problem of finding the vertices and edges of the convex hull of a finite set of points. Algorithms based on the simplex method were developed, coded, and proved quite successful. The sign-patterns of matrices representing polyhedra, and their relationship to geometric properties, are being studied.

(2) Witzgall also studied differentiability properties of convex functions.

(3) A.J. Goldman and P.R. Meyers extended previous results on the transport-cost-minimizing locations of a central facility in a network of roads.

(4) J.G. Root investigated applications of statistical decision theory and modern control theory to quality control problems of machine control and material classification. The quality control problem has been formulated as a modern control problem; optimal control and classification strategies have been found for certain fairly general cases.

Publications:

(1) On Convex Metrics. C. Witzgall. Journal of Research NBS, 69B, pp.175-177, 1965.

## CONVERGENCE OF ITERATIVE METHODS

Task 2050-12-2050154

Origin and Sponsor: NBS

Manager: Philip R. Meyers

Full Task Description: October-December 1960 issue, p.3

Authorized 12/30/60

Status: CONTINUED. This is a continuation of part of Task 2050-12-2050115/61-546.

(1) Results on when a "local contraction" on a complete metric space could be made into a contraction by a suitable (complete) remetrization were further clarified, extended to periodic behavior, and documented. These theorems facilitate establishing the rapid convergence of certain iterative methods.

(2) The hypotheses of the "converse to the Contraction Principle" were significantly weakened. The result now reads: If  $X$  is a (completely) metrizable space, and  $f: X \rightarrow X$  is a continuous map with fixed point  $x_0$  which has both the global property that  $f^n(x) \rightarrow x_0$  for all  $x$  in  $X$ , and the local property that  $f^n(U) \rightarrow \{x_0\}$  for some open neighborhood  $U$  of  $x_0$ , then  $f$  is a contraction for some (complete) metric of  $X$ . Corollary: If some iterate of  $f: X \rightarrow X$  is a contraction for some (complete) metric on  $X$ , then so is  $f$  itself.

(3) The last corollary can be interpreted as referring to a "discrete flow" on  $X$  . The corresponding problem for continuous flows, and its relation with the concept of asymptotic stability, are under investigation.

Publications:

- (1) Some Extensions of Banach's Contraction Theorem. Philip R. Meyers. Journal of Research NBS, 69B, pp. 179-184, 1965.



20500-12-2050404/65-1456      RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING      Authorized 1-21-64  
Origin:    NBS  
Sponsor:   National Institutes of Health  
Manager:   Russell A. Kirsch  
Full task description:    January-June 1964 issue, p. 19

Status: CONTINUED. During this period there was intensive development of programs with the use of the MAC time-sharing system located in Massachusetts from interrogation consoles located at the Bureau. Kenneth Kloss made extensive modifications to his STANDPAX homogeneous picture processor and inserted this into the time-sharing system. Several pictures of optical serial sections from photomicrographs which had been recorded locally on punch cards were sent to MIT and processed on the time-sharing system using STANDPAX. The experiments consisted of automatic differentiation of cell boundaries in photographs and the assignment of primitive syntactic labels to pictorial components. Several language processing programs were written in the MAC system primarily in the COMIT II programming language concurrent with the writing of the language while changes were being made in Chicago on the MIT machine involving COMIT II. These changes were used here at the Bureau by R. A. Kirsch in writing a syntax directed translator from the PLACEBO IV grammar to produce partially formalized tree diagrams from sentences randomly specified by the grammar. The experience gained in using such a large time-sharing system as MAC was useful both to the project and to others both of the sponsoring agency and NBS who plan to use a time-sharing system in the future. The system was used from about a half-dozen different locations and extensive interchange of program information was made among these locations and with other groups in Chicago, Wisconsin, and Massachusetts. Over a wide range of different kinds of problems, a ratio of about 30 to 1 was observed for console interaction time compared to computing time.

Some attention was given to psycholinguistic problems to be encountered by men communicating with computers in a language which to most intents is English. In particular, the habitability hypothesis of W. Watt has been greatly refined.

20500-12-2050406/65      RESEARCH ON A PICTURE LANGUAGE MACHINE      Authorized 5-1-61  
Origin:    NBS  
Sponsor:   National Science Foundation  
Manager:   Russell A. Kirsch  
Full task description:    July-December 1963 issue, p. 17

Status: REACTIVATED. Activity was resumed on this project in the area of research on discourse analysis. W. Watt made a thorough canvass of various fields related to discourse analysis problems, particularly ancient and modern Rhetoric, stylistics, and applied Discourse Analysis. There were some preliminary insights developed into intersentential structural relations of a syntactic nature.

## Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publications Activities.

### Applied Mathematics Division Lectures

- COTSAFTIS, M. (Centre d' Études Nucléaires, Fontenay aux Roses (Seine-et Oise), Paris, France) Stability of Conservative autonomous Linear Systems. November 24, 1965.
- PORITSKY, H. (Schenectady, New York) Applications of Schwartz-Christoffel Integrals in Field Problems. July 22, 1965.

### NBS Courses Conducted by Staff Members

- KLOSS, K. Programming for the NBS "PILOT" Computer. Fall Semester.

### NBS In-hours Courses Given by Staff Members

- PEAVY, S. T. Introduction to Digital Computers. September 1965-January 1966.

### Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

- CAMERON, J. M. Measurement processes: statistical and computational requirements. Presented at Instrument Society of America, Los Angeles, California, October 7, 1965.
- CAMERON, J. M. Statistics in the calibration laboratory. Presented at Stevens Institute of Technology, Hoboken, New Jersey, October 21, 1965.
- EDMONDS, J. R. Series of lectures on Graphs, August 16-20, 1965 (part of a short course given at UCLA), University of California, Los Angeles.
- EDMONDS, J. R. Matroids and combinatorial extrema. Presented before the Mathematics Department of the City University of New York, November 1, and 8, 1965, New York, New York.
- GOLDMAN, A. J. Game theory. Presented before the Bureau of Public Roads Advanced Math. Group, October 8, 1965, GSA Auditorium, Washington, D. C.
- GOLDMAN, A. J. A competitive equilibrium model featuring share-of-market. Presented before the Operations Research Seminar of John Hopkins University, Baltimore, Maryland, November 9, 1965.
- GOLDMAN, A. J. with MEYERS, P.R. A domination theorem for optimal locations. Presented before the Operations Research Society of America, Houston, Texas, November 4-5, 1965.
- KIRSCH, R. A. Discussion on Project INTREX planning. Woods Hole, Mass., August 2-6, 1965.
- KIRSCH, R. A. Session Chairman on Graphical information processing, California Polytechnic Institute, San Dimas, California, August 9-12, 1965.
- KIRSCH, R. A. Syntactical theory of programming and graphic displays. National Security Agency, Ft. Meade, Maryland, October 13, 1965.
- KIRSCH, R. A. Multiple access non-computational uses of computers, National Institute of Neurological Diseases and Blindness, National Institutes of Health, Bethesda, Maryland, November 19, 1965.
- NEWMAN, M. Some results on permanents. Presented to the Department of Mathematics, University of California, Santa Barbara, California, July 15 and 21, 1965.
- NEWMAN, M. Modular functions and additive number theory. Presented at the Conference on Number Theory, held from September 6-12, 1965 in the Math. Forschungsinstitut, Oberwolfach, Germany.

OLVER, F. W. J. The Liouville-Green (or WKB) approximation. Presented to the Department of Mathematics, Brown University, Providence, Rhode Island, October 7, 1965.

OLVER, F. W. J. The Liouville-Green (or WKB) approximation. Presented at the Rensselaer Polytechnic Institute, Troy, New York, October 11, 1965.

OLVER, F. W. J. The Liouville-Green (or WKB) approximation. Presented to the Department of Applied Mathematics and Computer Science, University of Virginia, Charlottesville, Virginia, October 14, 1965.

OLVER, F. W. J. The Liouville-Green (or WKB) approximation. Presented to the Department of Mathematics, University of Delaware, Newark, Delaware, November 18, 1965.

OSER, H. Remote processing. Presented before the Seventh Annual Highway Engineering Exchange Program, Madison, Wisconsin, September 15, 1965.

OSER, H. New developments in data transmission. Presented before the Annual Meeting of the American Association of State Highway Officials Subcommittees on Communications and Electronics, New York, New York, October 6, 1965.

ROSENBLATT, J. R. Confidence limits for the reliability of complex systems. Presented at Stevens Institute of Technology, Hoboken, New Jersey, October 20, 1965.

TCHEN, C. M. Magnetohydrodynamic turbulence. Presented at the Laboratory for Plasma Physics, Faculty of Sciences, University of Paris, Paris, France, September 21, 1965.

TCHEN, C. M. Statistical theory of magnetohydrodynamic turbulence. Presented to the Faculty of Sciences, University of Paris, Paris, France, October 7, 1965 (Orsay).

TCHEN, C. M. Kinetic theory of turbulence in a rarefied plasma. Presented to the Division of Applied Physics, Centre d'Études Nucléaires, Saclay, Gif-sur-Yvette, Paris, France, October 8, 1965.

TCHEN, C. M. Turbulence in plasmas. Presented at the Institute for Plasma Physics, Jutphaas, Netherlands, October 12, 1965.

WATT, W. C. Microgrammars. Harvard University, Computation Laboratory, Cambridge, Massachusetts, December 10, 1965.

WITZGALL, C. J. On most profitable routing before maintenance. Presented before the Mathematics Seminar of Boeing Scientific Research Laboratories, December 6, 1965, Seattle, Washington.



# Publication Activities

## 1. PUBLICATIONS THAT APPEARED DURING THIS PERIOD

### 1.3 Technical Papers

Elastic stress-strain relations in perfect elastic fluids. B. Bernstein, E. A. Kearsley, and L. J. Zapas. Transactions of the Society of Rheology, Vol. 9, Part 1, pp. 27-39, 1965.

Some theorems on the permanent. R. A. Brualdi and M. Newman. Journal of Research NBS, 69B, pp. 159-163, 1965.

Inequalities for permanents and permanental minors. R. A. Brualdi and M. Newman. Proceedings of the Cambridge Philosophical Society, Vol. 61, pp. 741-746, 1965.

Transversals and matroid partition. Jack Edmonds and D. R. Fulkerson. Journal of Research NBS, 69B, pp. 147-153, 1965.

Scattering properties of concentric soot-water spheres for visible and infrared light. R. W. Fenn and H. Oser. U. S. Army Electronics Laboratories, Vol. 4, pp. 1504-1509, November 1965.

A variant of the two-dimensional Riemann integral. A. J. Goldman. Journal of Research NBS, 69B, pp. 185-188, 1965.

The fundamental solution and Huygens' principle for decomposable differential operators. J. E. Lagnese. Archive for Rational Mechanics and Analysis, Vol. 19, No. 4, pp. 299-307, 1965.

Real two-dimensional representations of the modular group and related groups. J. Lehner and M. Newman. American Journal of Mathematics, Vol. 87, No. 4, pp. 945-954, 1965.

Some extensions of Banach's contraction theorem. Philip R. Meyers. Journal of Research, NBS, 69B, pp. 179-184, 1965.

Error bounds for asymptotic expansions of special functions in the complex plane. F. W. J. Olver. Proceedings of a Symposium on Error in Digital Computation, Madison, Wisconsin, Vol. 2, pp. 55-75, 1965.

On the asymptotic solutions of second-order differential equations having an irregular singularity of rank one, with an application to Whittaker functions. F. W. J. Olver. Journal of the Society for Industrial and Applied Mathematics, Series B, Vol. 2, No. 2, pp. 225-243, 1965.

Error analysis of phase-integral methods I. General theory for simple turning points. F. W. J. Olver. J. of Research, NBS, 69B, Part 4, Oct-Dec. 1965.

Error analysis of phase-integral methods II. Application to wave-penetration problems. F. W. J. Olver. J. of Research, NBS, 69B, Part 4, Oct-Dec. 1965.

Error bounds for asymptotic solutions of second-order differential equations having an irregular singularity of arbitrary rank. F. W. J. Olver and F. Stenger. Journal of the Society for Industrial and Applied Mathematics, Series B, Vol. 2, No. 2, pp. 244-249, 1965.

Spectrum of stationary homogeneous magnetohydrodynamic turbulence. C. M. Tchen. The Physics of Fluids, Vol. 8, No. 9, pp. 1659-1667, September 1965.

On convex metrics. C. J. Witzgall. Journal of Research NBS, 69B, pp. 175-177, 1965.

Mortality patterns in eight strains of flour beetles. W. J. Youden, D. B. Mertz and T. Park (Univ. of Chicago). Biometrics, Vol. 21, pp. 99-114, 1965.

## 2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

- 2.3 Time dependent behavior of an incompressible elastic fluid - Some homogeneous deformation histories. B. Bernstein. To appear in Acta Mechanica.

The bridge tournament problem and calibration designs for comparing pairs of objects. R. C. Bose and J. M. Cameron. To appear in NBS J Research B (Math. and Math. Physics).

Kernels and the Kronecker product of graphs. R. A. Brualdi. To appear in Proceedings of the American Mathematical Society.

Inequalities for the permanental minors of non-negative matrices. R. A. Brualdi and M. Newman. To appear in the Canadian Journal of Mathematics.

Proof of a permanental inequality. R. A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).

On a theorem of Ahlfors. L. Greenberg. To appear in the American Journal of Mathematics.

Fundamental polyhedra for Kleinian groups. L. Greenberg. To appear in the Annals of Mathematics.

Kleinian groups. L. Greenberg. To appear in the Proceedings of the Conference on quasi-conformal mapping, discontinuous groups and moduli, New Orleans.

A theorem on arbitrary functions. S. Haber. Submitted to a technical journal.

A modified Monte-Carlo quadrature. S. Haber. To appear in Mathematics of Computation.

Some number-theoretic computations. K. E. Kloss. Journal of Research, NBS, 69B, No. 4, October-December 1965.

The analysis, synthesis, and description of biological images. L. E. Lipkin, W. C. Watt, and R. A. Kirsch. To appear in the Annals of the New York Academy of Sciences.

Turbulence in a rarefied plasma. C. M. Tchen. Accepted for publication at the Proceedings of the Symposium for Advanced Problems in Fluid Mechanics, (Jurata, Poland).

Invariant properties of the spheroidal potential of an oblate planet. J. P. Vinti. Accepted for publication in the NBS Journal of Research, Section B (Jan-March 1966).

Inclusion of the third zonal harmonic in an accurate reference orbit of an artificial satellite. J. P. Vinti. Accepted for publication in the NBS Journal of Research, Section B (Jan-March 1966).

Useful alternatives to Chauvenet's rule for rejection of measurement data. T. A. Willke. Submitted to a technical journal.

Evaluation of analytical data. W. J. Youden. To appear in Encyclopedia of Industrial Analysis.

## 2.4 Reviews and Notes

Materials for PLACEBO V. W. Watt. NBS Technical Note 251, 1965.



