NATIONAL BUREAU OF STANDARDS REPORT

8990

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Semiannual Report

January through July 1965

U.S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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NBS PROJECT	NBS REPORT
205.0	8990

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APPLIED MATHEMATICS DIVISION

January through June 1965

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° On leave of absence
** Postdoctoral Resident Research Associate
°° Temporary Appointment
*** Guest Worker
°°° Student Trainee

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°Only unclassified material is included in this report.

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 20501-12-2050110/55-55

Origin: NBS Authorized 8/29/54
Manager: Morris Newman
Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has determined large classes of pairs of real 2 x 2 unimodular matrices which generate the free product of two cyclic groups, and has applied his results to the classification of all real discrete 2 x 2 representations of the free product of two finite cyclic groups, with J. Lehner.

M. Newman is classifying the normal subgroups of the modular group Γ, by index. For example he has shown that there is no normal subgroup of index $6p$ in Γ, where p is a prime $\equiv -1 \pmod 6$.

L. Greenberg has proved that any finitely generated matrix group always has a subgroup of finite index with no element of trace μ, where μ is any preassigned complex number different from the trace of the identity matrix.

S. Haber derived an error formula for his second modified Monte-Carlo quadrature method, and carried out experimental studies of it. A third modification was proposed, but experiments indicated that it would not be useful.

S. Haber began a study of certain number-theoretical methods of multiple quadrature. He also continued studies related to fix-points of entire functions.

K. Goldberg investigated semi-groups with zeroids. He also continued his investigations of ordered and non-negative vectors.

R.A. Brualdi and M. Newman carried out investigations concerning the permanent of matrices in the convex space Ω_n of all non-negative doubly stochastic matrices of order n. While the permanent is not a convex function on Ω_n, it was shown that for A in Ω_n and $0 \leq \alpha \leq 1$, $\operatorname{per}(\alpha I + (1-\alpha) A) \leq \alpha + (1-\alpha) \operatorname{per}(A)$. They gave a proof of the conjecture of M. Marcus and M. Newman that for A in Ω_n, $\operatorname{per}(I-A) \geq 0$. Here I represents the identity matrix of order n. If we let $p_k(A)$ denote the sum of all the permanental minors of order k and $e_k(A)$ the sum of all the principal permanental minors of order k, then it was shown that

$$p_k(A) \leq \binom{n}{k} \quad \text{and} \quad e_k(A) \leq \binom{n-1}{k} + \binom{n-1}{k-1} \quad \operatorname{per}(A).$$

Other types of results concerning the permanent were obtained. For instance, they define $f(n)$ for n a positive integer to be the smallest order of a 0,1 matrix with permanent equal to k. It was then shown that $\log f(n)$ is asymptotic to $\log \log n$.

F.W.J. Olver is continuing his work in asymptotic expansions under Task 20501-11-2050421/63.

Publications:

(1) Character subgroups of F-groups. M.I. Knopp and M. Newman. J. of Research NBS, 69B, pp. 85-86, 1965.

(2) A theorem on the automorphs of a skew-symmetric matrix. M. Newman. Michigan Mathematical Journal 12, pp. 61-63, 1965.

(3) Bounds for class numbers. M. Newman. American Mathematical Society Proceedings of Symposium for Number Theory, pp. 70-77, 1965.

(4) Real two-dimensional representations of the modular group and related groups. J. Lehner and M. Newman. To appear in Amer. J. Math.

(5) A bounded automorphic form of dimension zero is constant. M.I. Knopp, J. Lehner, and M. Newman. To appear in Duke Mathematical Journal.

(6) Normal subgroups of the modular group which are not congruence subgroups. M. Newman. To appear in Proceedings of the American Mathematical Society.

(7) Congruence subgroups of positive genus of the modular group. M. Knopp and M. Newman. To appear in Illinois Journal of Mathematics.

(8) A functional inequality. S. Haber. To appear in the American Math. Monthly.

(9) A theorem on arbitrary functions. S. Haber. Submitted to a technical journal.

(10) A modified Monte-Carlo quadrature. S. Haber. Submitted to a technical journal.

(11) Hadamard matrices of order cube plus one. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(12) Maximum determinants of certain row stochastic matrices. K. Goldberg. To be submitted to the Journal of Research NBS.

(13) Transformations of ordered vectors. K. Goldberg. To be submitted to the Journal of Research NBS.

(14) Semi-groups with zeroids. K. Goldberg. To be submitted to the Journal of Research NBS.

(15) A note on multipliers of difference sets. R.A. Brualdi. J. of Research NBS, 69B, pp. 87-89, 1965.

(16) Inequalities for permanents and permanental minors. R.A. Brualdi and M. Newman. To appear in the Proceedings of the Cambridge Philosophical Society.

(17) Proof of a permanental inequality. R.A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).

(18) Inequalities for the permanental minors of non-negative matrices. R.A. Brualdi and M. Newman. To appear in the Canadian Journal of Mathematics.

(19) Some theorems on the permanent. R.A. Brualdi and M. Newman. To appear in the Journal of Research NBS.

(20) Convergence and abstract spaces in functional analysis. E. Ordman. Submitted to a technical journal.

(21) Entire solutions of the function equation $\alpha(\beta(z) = \alpha(\gamma(z)) + c$. F. Gross. Submitted to Duke Mathematical Journal.

(22) An analogue of Fermat's last theorem for entire functions. F. Gross. Submitted to American Mathematical Monthly Notes.

(23) Functional equations and fix points. F. Gross. Submitted to the Pacific Journal of Mathematics.

(24) Entire solutions of the functional equation $h(f(z)) = g(z)$. F. Gross. Submitted to the Proceedings of the American Mathematical Society.

(25) A recurrence related to monotone subsequences in permutations. K. Goldberg. To be submitted to the Journal of Research NBS.

ASYMPTOTIC EXPANSIONS

Task 20501-11-2050421/63

Origin: NBS Authorized 9/10/63
Sponsor: U.S. Army Research Office, Durham, N.C.
Manager: F.W.J. Olver
Full task description: July-December 1963 issue, p. 2

Status: CONTINUED. The recently-developed error analysis of phase-integral methods has been applied to problems of wave penetration, including the overdense potential barrier and the approximate harmonic oscillator.

A study of error bounds for asymptotic expansions derived from integral representations has begun.

Publications:

(1) On the asymptotic solutions of second-order differential equations having an irregular singularity of rank one, with an application to Whittaker functions. F.W.J. Olver. To appear in the Journal of the Society for Industrial and Applied Mathematics, Series B. (This paper combines papers (2) and (3) reported Jan.-June, 1964.)

(2) Error bounds for asymptotic solutions of second-order differential equation having an irregular singularity of arbitrary rank. F.W.J. Olver and F. Stenger. To appear in the Journal of the Society for Industrial and Applied Mathematics, Series B.

(3) Error analysis of phase-integral methods I. General theory for simple turning points. F.W.J. Olver. To appear in the Journal of Research of the National Bureau of Standards.

(4) Error analysis of phase-integral methods II. Application to wave-penetration problems. F.W.J. Olver. To appear in the Journal of Research of the National Bureau of Standards.

(5) Error bounds for asymptotic expansions of special functions in the complex plane. F.W.J. Olver. To appear in the Proceedings of a Symposium on Error in Digital Computation, Madison, Wisconsin, April 1965.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

20502-12-2050120/55-0065 **AUTOMATIC CODING**
Origin and Sponsor: NBS
Manager: G. W. Reitwiesner
Full task description: July-September 1954 issue, p. 11
Status: INACTIVE.

20502-40-2050121/57-0216 **MATHEMATICAL TABLES**
Origin and Sponsor: NBS
Manager: I. A. Stegun
Objective: To continue work on long-range mathematical tables projects, update, correct and reissue already published tables.
Status: CONTINUED. Compilation of errata for the previously published tables has continued. New printings have all known errors corrected. Tabulations of functions - by-products of machine computations - are being examined for possible inclusion in further revisions of the various volumes.

20502-12-2050122/63-1999 **CURRENT RESEARCH IN THE COMPUTATION LABORATORY**
Origin and Sponsor: NBS, Section 205.02
Manager: I. A. Stegun
Full task description: July-December 1963 issue, p. 3
Status: CONTINUED. Studies have been carried on in methods of computing mathematical functions particularly slanted toward high speed computations.

3. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS

Task 20503-12-2050131/63-1259

Origin: NBS Authorized 10/1/62
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Status: Continued. R. C. Bose and J. M. Cameron have completed a paper entitled "The bridge tournament problem and calibration designs for comparing pairs of objects". The classical tournament problem calls for arranging v individuals into teams of p players so that a player is teamed the same number of times with each of the other players and also that each player is pitted equally often against each of the other players. The play of the tournament results in the determination of difference in performance of the various pairings of the groups. In the special case when p=2 each team consists of two players and the designs are called bridge tournament designs. In high precision calibration one can measure only the difference between two nominally equal groups so that if v objects are to be intercompared in groups of p objects, then the solutions to the tournament problem provide schedules for the grouping. These designs are useful in weighing and any other measurements where the objects to be measured can be combined into groups without loss of precision or accuracy in the comparisons. This paper presents general methods for constructing of bridge tournament designs, i.e. for the case when p=2, for all $v \leq 50$.

J. M. Cameron's paper on three algorithms for computing the generalized inverse of a matrix has been revised and extended by J. M. Cameron in collaboration with A. J. Goldman (Operations Research Section).

H. H. Ku has completed the revision of his paper, "Notes on the use of propagation of error formulas".

Janace A. Speckman has prepared, for limited distribution on request, a note that provides supplementary documentation of results published in "Estimation for a one-parameter exponential model". This material was deleted from the published paper in the interest of brevity.

Mary G. Natrella has prepared a list of corrigenda for NBS Handbook 91, Experimental Statistics. The table errata have been submitted for publication in Mathematics of Computation.

Brian L. Joiner has initiated an investigation of methods for testing homogeneity of variance, when the individual estimates are based on different numbers of observations.

Publications:

(1) Use of general purpose coding systems for statistical calculations. J. M. Cameron and J. Hilsenrath (NBS Equation of State Section). Proceedings of IBM Scientific Computing Symposium on Statistics, held October 21-23, 1963, IBM Data Processing Div., White Plains, N. Y., 1965, pages 281-299.

(2) A simple method for calculating orthogonal bases for a vector space and its complement. J. M. Cameron. Submitted to a technical journal.

(3) Estimation for a one-parameter exponential model. Janace A. Speckman and Richard G. Cornell (Florida State University). Journal of the American Statistical Association, 60, 1965, pp. 560-572.

(4) Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society of Tool and Manufacturing Engineers.

(5) Three algorithms for computing a generalized inverse. J. M. Cameron and A. J. Goldman (NBS Operations Research Section). To appear in NBS J. Research B. (Math. and Math. Physics).

(6) The bridge tournament problem and calibration designs for comparing pairs of objects. R. C. Bose and J. M. Cameron. To appear in NBS J. Research B. (Math. and Math. Physics).

MEASUREMENT OF RELIABILITY

Task 20503-12-2050130/56-182

Origin: NBS Authorized 3/23/56
Manager: Joan R. Rosenblatt
Full task description: January - March 1956 issue, p. 13

Status: TERMINATED. Studies of the type hitherto reported under this task will in future be reported under Task 20503-12-2050131/63-1259 (see above).

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 20540-12-2050141/55-57

Origin: NBS Authorized 9/1/54
Manager: W.H. Pell
Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. Dr. Bernstein has continued his work on the application of the theory of the elastic fluid (BKZ theory) which he has developed jointly with Dr. E.A. Kearsley and L. Zapas (213.05) to specific problems. In particular, the incompressible elastic fluid theory has been applied to the solution of problems appropriate to experimental situations. Among such have been torsion problems with various strain histories and the problem of biaxial stress relaxation. The responses to a number of homogeneous deformation histories were worked out(this was begun during the last reporting period and noted in July-Dec. 1964 P and P) and written up in a paper entitled "Time Dependent Behavior of an Incompressible Elastic Fluid - Some Homogeneous Deformation Histories.

Investigation was started on finite elasticity theory to find restrictions on the internal energy function due to certain forms of dependence of stress on temperature.

Dr. Bernstein and Dr. C. Hoeve (311.04) have agreed to write a joint article on "Rubberlike Elasticity" for the Encyclopedia of Polymer Science and Technology.

Dr. J.H. Bramble has completed the work reported on in July-Dec. 1964 P and P on the problem of obtaining bounds on the solution of systems of partial differential equations which may be coupled in a non-linear way. A paper has been prepared with the title " A Priori Bounds for Non-linearly Coupled Systems." The general problem of thermoelasticity is treated in detail. Drs. L.E. Payne and J.H. Bramble have submitted for publication a manuscript entitled "A Priori Bounds in the Equations of Classical Incompressible Elasticity" which covers work reported on as in progress during July-Dec. 1964.

Dr. W.H. Pell and A. Kirstein (213.04) have prepared a draft of a paper entitled "Deflection of Centrally Loaded Flat Circular Plates on Equally Spaced Point Supports." This covers their collaborative work having to do with the experimental verification of the theory of point supported elastic plates.

Publications:

(1) Elastic stress-strain relations in perfect elastic fluids. B. Bernstein, E.A. Kearsley, and L.J. Zapas. Submitted to Trans. Soc. Rheology.

(2) Time dependent behavior of an incompressible elastic fluid - Some homogeneous deformation histories. B. Bernstein. Submitted to Acta Mechanica.

(3) A priori bounds in the equations of classical incompressible elasticity. J.H. Bramble and L.E. Payne. To appear in J. of Research NBS, Section B.

(4) A new differential operator of the pure wave type. J.E. Lagnese. Journal of Differential Equations, Vol. 1, No. 2, pp. 171-187, 1965.

PLASMA RESEARCH

Task 20504-12-2050140/59-442

Origin: NBS Authorized 6/30/59
Manager: C.M. Tchen
Full task description: April-June 1959 issue, p. 15

Status: CONTINUED. Dr. C.M. Tchen has continued his research on plasmas in the following areas:

(a) Theory of magnetohydrodynamic turbulence in the solar photosphere.

In the solar photosphere there is found a coupling between the magnetic field and the turbulent velocity field of the plasma, with both spectra developed within approximately the same range of wave numbers. A theory has been developed by Tchen to explain this particular feature.

(b) Diffusion across a magnetic field.

Dr. Tchen has continued his study of the diffusion from the collective motion of plasmas. The basis of this work is quasilinear kinetic theory.

(c) A series of seminar lectures is being presented by Dr. Tchen, at the Institute for Theoretical Physics, University of Marburg, West Germany, for 2 hours per week, from May 6 to November. The seminar presents topics drawn from the results of Dr. Tchen's current work on plasma physics.

Publication:

(1) Stochastic theory of diffusion in a plasma across a magnetic field. C.M. Tchen. Proceedings of the Internal. Symposium on Plasma Diffusion, Feldafing, Germany, pp. 118-123, 1964.

DYNAMICS OF PLASMAS

Task 20504-12-2050417/62-1157

Origin: NBS Authorized 10/3/61
Sponsor: National Aeronautics and Space Administration
Manager: C.M. Tchen
Full task description: October-December 1961 issue, p. 12

Status: CONTINUED. Dr. C.M. Tchen has continued his researchs on the magnetohydrodynamic and kinetic theories of plasmas, with emphasis on the following topics:

(1) Magnetohydrodynamic turbulence.

Clarification on the interaction between turbulence and the magnetic field spectra has been added to a paper by Tchen entitled "Spectra of Stationary and Homogeneous Magnetohydrodynamics Turbulence." This paper will appear in the Physics of Fluids.

(2) Non-linear Landau damping of plasmas.

The linear theory of Landau damping is based on the interaction of wave and particles in a plasma. For treatment of the non-linear problem a method has been devised which is based on the temporal decay of the electrical energy. This method is proved to reproduce the results of the linear theory in a clear and direct way. The non-linear theory involves various correlations and degenerates the fourth order correlations into lower order ones. The results have been presented by Tchen in the Seminar of the Institute for Theoretical Physics, University of Marburg, May-July, 1965.

(3) In collaboration and consultation with the staff of the University of Marburg, Dr. Tchen has initiated the application of the "Diagram Techniques", based on what is known as the "Feynman diagram" in quantum field theory, to the problem of plasma turbulence.

(4) Dr. Tchen, in his capacity as visiting professor, Institute for Theoretical Physics, University of Marburg, Marburg, West Germany, is conducting a seminar based on his current researches. This seminar will run from May to November, two hours per week. Special topics to be covered will be: kinetic equations of plasmas, kinetic theory of plasmas in turbulent motion, non-linear Landau damping, scattering, diffusion across a magnetic field, expansion of a gas into vacuum in the presence of a magnetic field, dynamical problems of plasmas in a gravitational field. The seminar has been well attended by the staff of the five Institutes for Theoretical Physics, and the additional Physikalische Institut (experimental and applied physics).

Publication:

(1) Spectrum of stationary and homogeneous magnetohydrodynamic turbulance. C.M. Tchen. To appear in the Physics of Fluids.

THEORY OF SATELLITE ORBITS

Task 20504-12-2050441/62-1166

Origin: NBS Authorized 1/9/62
Sponsor: National Aeronautics and Space Administration
Manager: J.P. Vinti
Full task description: January-March 1962 issue, p. 12

Status: CONTINUED. Dr. J.P. Vinti has completed the writing of a paper on "Invariant Properties of the Spheroidal Potential for an Oblate Planet." This, the first part of a program for incorporating the third harmonic of the earth's gravitational field into a separable solution for a satellite orbit, is largely concerned with the fundamental physical principles underlying the method. Work is in progress on the preparation of a paper on the second part, containing the detailed solution for the orbit that is required for practical calculations.

Publications:

(1) The spheroidal method in the theory of the orbit of an artificial satellite. J.P. Vinti. To appear in the Proceedings of the Symposium on Celestial Mechanics, held March 1964, at the Mathematisches Forschungsinstitut at Oberwolfach, West Germany.

(2) Effects of a constant force on a Keplerian orbit. J.P. Vinti. To appear in Proceedings of Symposium 25 of the International Astronomical Union, Thessaloniki, Greece, August 15-22, 1964.

OPERATIONS RESEARCH

Task 205-12-2050115/61-546

Origin and Sponsor: NBS Authorized 12/30/60
Manager: Alan J. Goldman
Full task description: October-December 1960 issue, p. 3

Status: CONTINUED. Work during the first half of the reporting period was severely restricted by the priorities of the SST Economic Analysis project (see below). The following activities were carried out by members of the staff:

(1) Special emphasis continued on performing and promoting research to develop theory and computational methods for the optimization and arrangement of discrete systems arising in industry and technology. As before, project funds supported much of the work reported separately below under Combinatorial Mathematics.

(2) P. Meyers continued research on remetrizing a space so as to make the Banach Contraction Theorem applicable. He sharpened, extended and drafted a manuscript on results pertaining to the converse of the contraction principle.

A.J. Goldman and H.D. Mills (IBM Federal Systems Division) analyzed two competitive equilibrium models involving share-of-market, determining the uniqueness and existence of equilibria, as well as the convergence behavior of several plausible iterative schemes.

(3) K. Kloss continued work on various number-theoretic computations. The table of Wilson quotients was extended to 950,000 (19 times the limit of the largest published table); the computations were rearranged so as nearly to double their speed, and the program written so as to detect and compensate for machine errors, thus permitting unattended running during periods when the PILOT computer would otherwise be turned on but idle.

The speed of calculation of Fermat quotients was increased 8-fold. The quotients have been evaluated for all primes $< 31{,}000{,}000$ (15 times the range of any published work), and the related quantities $(a^{p-1}-1)/p \pmod p$ determined for all primes a and p with $3 \leq a \leq 43$ and $p < 5{,}000{,}000$.

The differences between successive primes were examined for numbers $< 128{,}000{,}000$; the first example of a difference > 220 (previously undetected) occurred at 122, 164, 747. At K. Goldberg's suggestion, an investigation was begun of the quotients $W(m)$ arising from the generalized Wilson theorem; results so far include discovery of a new "generalized Wilson prime "($W(5971) \equiv 0 \bmod 5971$), and a proof that, for primes $p > 5$, $W(p^{k+1}) \equiv W(p^k) \bmod p^k$ for all integers $k > 0$. Moreover, $W(3^{k+1}) \equiv W(3^k) \bmod 3^k$ for $k > 1$. It is conjectured that $W(n) \equiv 0 \bmod n$ only if n has no repeated prime factors.

(4) L.S. Joel assumed leadership of the work in modelling some aspects of the textile industry. A number of simulation "runs" were made with the "tufted carpet" and "hosiery" models; the results are being analyzed and documented. K. Kloss prepared a SIMSCRIPT version of the hosiery model. J. Levy continued related studies concerning the important parameters of information (cost, value, accuracy, timeliness) in such an industry, in the context of linear programming and "warehouse problem" models. (Reported here for convenience; supported under Project No. 4270697.)

J. Levy and C. Witzgall continued comparative study of best-path algorithms. A.J. Goldman and C. Witzgall generalized these algorithms to apply to the problem of finding a path which is shortest with respect to one metric, and "not too long" with respect to one or more other metrics; computer implementation is being prepared. P. Meyers and A.J. Goldman used a simple convexity argument to extend previous work by Witzgall to a rather general theorem about transport-cost-minimizing locations of a central facility in a system consisting of radial "nets" together with "beltways"; under mild assumptions each optimal location must lie either on a beltway or at a "hub", not at an intermediate point on a "spoke". (Reported here for convenience; supported by Projects Nos. 4230450 and 4310421.)

J. Levy and A.J. Goldman continued studies of the effects of buffer capacity in certain mail sorting devices. A.J. Goldman completed documenting investigations of mathematical measures of ambiguity for address-coding schemes in mail sorting. P. Meyers continued analysis of a stochastic sorting process. (Reported here for convenience; supported under Project No. 4230450.)

K. Kloss extended his assembly program PEAP for the NBS PILOT computer. The loader was completed. A mechanism was devised for incorporating subroutines from one or more libraries into an assembly. Such subroutines are allowed to share constants and variables with one another and with the main program, and can have an arbitrary number of entry points. Numerous other features were added to the language. A preliminary debugging routine (DRIP) was written; it allows inspection and minor modification of the assembled program (and its variables) using the symbols which were in the symbolic program, even after the assembled program has been loaded and partially executed. (Reported here for convenience; supported under Project No. 4230152.)

(5) Miscellaneous consulting and advisory services were provided for members of four NBS Divisions. Other recipients included the Weapons Systems Evaluation Group, Army Security Agency, Small Business Administration, American Mathematical Society, Washington Operations Research Council, George Washington University and the Civil Service Commission.

Publications:

(1) On measurable sets and functions. A.J. Goldman. Journal of Research NBS, 69B, Nos. 1-2, pp. 99-100, 1965.

(2) A variant of the two-dimensional Riemann integral. A.J. Goldman. To appear in Journal of Research NBS, 69B, 1965.

(3) Realization of semi-multipliers as multipliers. Harriet Fell and A.J. Goldman. To appear in Amer. Math. Monthly (Math. Notes).

(4) Some extensions of Banach's contraction theorem. P. Meyers. To appear in Journal of Research NBS, 69B, 1965.

(5) On Convex Metrics. C. Witzgall. To appear in Journal of Research NBS, 69B, 1965.

(6) Approximating symmetric relations by equivalence relations. C.T. Zahn, Jr. Journal Soc. Ind. Appl. Math. Vol. 12, No. 4, pp. 840-847, 1964.

(7) Barely faithful algebras. Harriet Fell and John Mather. To appear in Amer. Math. Monthly (Math. Notes).

SST ECONOMIC ANALYSIS

Task 20505-12-2050451

Origin: Commerce Dept. (SST Economic Analysis Study) Authorized 8/10/64
Sponsor: Federal Aviation Agency
Managers: A.J. Goldman (205.05), W.G. Hall (205.02)
Full Task Description: July-December 1964 issue, p. 12

Status: CONTINUED. (1) The computer program for simulation of competition over world air routes was applied to the several hundred relevant routes over a wide range of parametric assumptions. The results were appropriately aggregated and reported to the sponsor. (2) Computer programs were prepared to link the simulation model with one developed by a Project contractor; the resulting combination was used for a "market gaming" demonstration involving representatives of 7 major U.S. airlines. (3) The computer program for the previously reported "cost-benefit" model was completed and applied to the analysis of several hundred cases, and the results were reported to the sponsor. An alternative model was similarly developed, implemented and applied, with its results also reported to the sponsor. (4) The model and computer program for assessing balance-of-payment effects of various outcomes and policies were completed, and applied to roughly 150 cases; results were reported to the sponsor.

COMBINATORIAL MATHEMATICS

Task 205-12-2050455/62-1205

Origin: NBS
Sponsor: Army Research Office-Durham
Manager: Jack Edmonds
Full task description: April-June 1962 issue, p. 15

Authorized 5/2/62

Status: Continued.

Edmonds continued work on a theory of matroids and "submodular" set functions. It is of interest in several connections including integer linear programming, n-person games, networks, and projective geometries.

(a) A matroid $M = (E,F)$ is a set E of elements and a family F of so-called independent subsets of E such that (1) subsets of independent sets are independent and (2) for all $A \subset E$, all maximal independent subsets of A have the same cardinality, called the rank $r(A)$ of A. The rank of M is $r(E)$.

(b) A subset E of points in a projective geometry gives rise to a matroid $M = (E,F)$, where $I \in F \iff I$ is not contained in any flat of rank $|I| - 1$ (dimension $|I| - 2$). It is convenient for E (of geometric matroids) to admit duplications of the same geometric point, any two duplications being dependent, and also to admit non-point elements any one of these being dependent. Not all matroids arise from geometries.

(c) A submodular set function f on a set E is one such that

$$f(A \cup B) + f(A \cap B) \leq f(A) + f(B) \quad \text{and}$$

$$f(A) \leq f(A \cup B) \quad \text{for all subsets A and B of E.}$$

(d) A matroid rank function is submodular. Conversely, any integer-valued submodular set function f on a set E yields a matroid $M = (E,F)$, where $I \in F \iff |A| \leq f(A)$ for all non-empty $A \subset I \subset E$.

(e) The rank function r of M of f is given by

$$r(A) = \min (|A_0| + f(A_1) + \ldots + f(A_p)),$$

where $A_0 \cup A_1 \cup \ldots \cup A_p = A$, p not specified. If $f(\emptyset) \geq 0$, then $p = 1$ can be specified.

(f) Let $M_i = (E,F_i)$ be matroids with rank functions r_i. Let k be an integer. $f(A) = -k + \sum r_i(A)$ is submodular and thus yields a certain matroid $M = (E,F)$.

(g) Where the matroids M_i arise from real geometry (in particular, where they are rank 1 matroids), matroid M arises from real geometry.

(h) For any family of matroids $M_i = (E,F_i)$, E can be partitioned into sets E_i such that $E_i \in F_i$ if and only if $|A| \leq \sum r_i(A)$ for all $A \subset E$.

(i) Where $k = 0$ in (f), M is called the straight sum of matroids M_i. It follows from (h) that the straight sum M of matroids M_i is given by: $I \in F \iff I$ is the union of some sets I_i such that $I_i \in F_i$.

(j) If the matroids M_i are each of rank 1, then each is determined by its subset $q_i \subset E$ of elements which are individually independent; in this case, denote by Q the family of q_i's. A straight sum $M = (E,F)$ of rank 1 matroids M_i is called a transversal matroid; it is given by: $I \in F \iff I$ is a transversal (system of distinct representatives) of some subfamily of Q.

(k) Where $k = 1$ in (f), where the M_i's are each of rank 1, and where each member of E is in exactly two members of Q, then the matroid M of (f) is called a graphic matroid. There is an associated graph in which Q is the set of nodes, E is the set of edges, and F is the family of "forests".

(ℓ) Many theorems about general matroids and submodular functions include as special cases interesting theorems about graphs, transversals, geometries, and other structures. For example, consider theorem (h) where each M_i is the same graphic matroid. Two more examples: (e) applied to (j) is the Konig formula for "term rank"; (e) applied to (k) is the Kirchhoff-Whitney formula for "the rank of a graph".

(m) Let f on E be submodular, integer-valued, and $f(A) \geq 0$ for $\phi \neq A \subset E$. Let $M = (E,F)$ be the associated matroid given by (d). Let P be the polyhedron given by the inequalities:

$$0 \leq x_e \leq 1 \text{ for all } e \in E, \quad \text{and}$$

$$\sum_{e \in A} x_e \leq f(A) \text{ for all } \phi \neq A \subset E .$$

The vertices $V(P)$ of P are precisely the zero-one incidence vectors of the members of F. P is called the polyhedron of matroid M.

(n) Where P_1 and P_2 are the polyhedra of any two matroids

$$M_1 = (E,F_1) \text{ and } M_2 = (E,F_2), \quad V(P_1 \cap P_2) = V(P_1) \cap V(P_2) .$$

In general, for three matroid polyhedra in the same variables, the vertices of $P_1 \cap P_2 \cap P_3$ do not have integer values.

Most of the above material is not in manuscript. Other related material appears in publications (2), (3), and (8).

C. Witzgall continued work on some aspects of matching theory. In particular, he obtained results on maximum edge-weight-sum subfraphs with degree-parity constraints as well as degree-bound constraints.

Publications:

(1) Paths, trees and flowers. Jack Edmonds. Canadian Journal of Mathematics 17, pp. 449-467, 1965.

(2) Minimum partition of a matroid into independent subsets. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 67-72, 1965.

(3) Lehman's switching game and a theorem of Tutte and Nash-Williams. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 73-77, 1965.

(4) On the surface duality of linear graphs. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 121-123, 1965.

(5) Maximum matching and a polyhedron with 0,1-vertices. Jack Edmonds. Journal of Research NBS, 69B, pp. 125-130, 1965.

(6) On matching problems. J. Edmonds, A.J. Goldman, C. Witzgall, C.T. Zahn, Jr. Proceedings of Army Research Office Working Group on Computers, ARO-D Report 65-1, pp. 45-50, 1965.

(7) Modification of Edmonds' maximum matching algorithm. C. Witzgall and C.T. Zahn, Jr. Journal of Research NBS, 69B, Nos. 1-2, pp. 91-98, 1965.

(8) Transversals and matroid partition. Jack Edmonds and D.R. Fulkerson. To appear in Journal of Research NBS, 69B, 1965.

6. MATHEMATICAL AND COMPUTATIONAL SERVICES

20502-40-2050647/56-0186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 212.22
Manager: B. S. Prusch
Full task description: July-September 1956 issue, p. 33
Status: CONTINUED. Computations were performed to check 42 laboratory sets of gage blocks as requested by sponsor.

20502-40-2050647/58-0266 DEPOLYMERIZATION PROCESSES
Origin and Sponsor: NBS, Section 311.13
Manager: R. Zucker
Full task description: July-September 1957 issue, p. 36
Status: INACTIVE.

20502-40-2050647/58-0339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 213.05
Manager: H. Oser
Full task description: January-March 1958 issue, p. 38
Status: COMPLETED. No activities can be reported for this period.

20502-40-2050647/60-0486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 221.01
Manager: Ruth Zucker
Full task description: January-March 1960 issue, p. 28
Status: CONTINUED. Production runs were made and results submitted to sponsor.

20502-40-2050647/60-0513 RADIATIVE ENVELOPES OF MODEL STARS
Origin and Sponsor: National Aeronautics and Space Administration
Managers: P. J. Walsh and S. Haber (205.01)
Full task description: July-September 1960 issue, p. 23
Status: INACTIVE.

20502-40-2050647/62-1018 HYDROMAGNETIC PROBLEMS
Origin and Sponsor: Naval Research Laboratory
Manager: Sally Peavy
Full task description: July-December 1964 issue, p. 15
Status: CONTINUED. Old program checked out. New programs are being written in order to reduce execution time.

20502-40-2050647/62-1022 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: April-June 1958 issue, p. 33
Status: CONTINUED. Little work in this period.

20502-40-2050647/62-1027 NEW SYSTEMS
Origin and Sponsor: NBS, Section 205.02
Manager: P. J. Walsh, V. Dantzler, W. Lipton
Full task description: July-September 1961 issue, p. 22
Status: CONTINUED. Version 12 of IBSYS replaced version 10 of IBSYS during the month of April. APARS, describing known errors in version 12, were distributed when the system was placed into operation. The system was modified to include an expanded Input Output Units table for FORTRAN (II) users. The FORTRAN (II) subsystem also produced a memory map at load time. Modifications were placed into IBJØB so that users can reference up to 14 tape units. Additional changes are described in the memorandum announcing version 12. Three members of the staff are giving a series of lectures on IBSYS during the summer months.

IBM has distributed version 13 of IBSYS, and this system is being prepared for use at NBS. It includes an alternate I/O package for FORTRAN IV programmers. This package has its own buffering and data transmission routines and does not use IØCS. Use of the ALTIO package makes an additional 1900 memory cells available for object programs. The FORTRAN IV compiler has its own assembler and no longer uses IBMAP. A phasing technique has been incorporated into the FORTRAN IV compiler which should result in

(NEW SYSTEMS con't)
faster compilations. Three new language features are also available to FORTRAN IV users: 1. Seven Dimensional Arrays, 2. Non-Standard returns from Subroutine Subprograms and 3. Multiple Entry points into Subroutine and Function Subprograms. Additional features will be described when the system is released for use at NBS.

The IBM QUIKTRAN time sharing system was made available at NBS. IBM 1050 equipment communicates with QUIKTRAN via a data phone. QUIKTRAN operates on an IBM 7044 located in New York City.

20501-12-2050514/62-1091 LOWER BOUNDS FOR EIGENVALUES
Origin: Wright-Patterson AFB
Manager: H. Oser
Full task description: October-December 1961 issue, p. 4
Status: CONTINUED. A manuscript is in preparation which describes the results of the computation for perturbed Lagrange, Hermite, and Legendre operators.

20502-40-2050647/62-1130 FALLOUT SHELTER COMPUTATIONS
Origin and Sponsor: Office of Civil Defense
Manager: Maxine Paulsen
Full task description: October-December 1961 issue, p. 25
Status: CONTINUED. Processing second generation data through P.C.U. 101.

20502-40-2050647/62-1203 CYLINDRICAL SHOCK WAVE
Origin and Sponsor: NBS, Section 221.04
Managers: Sally Peavy and S. Haber
Full task description: April-June 1962 issue, p. 30
Status: COMPLETED. Problem turned over to sponsor for further results.

20502-40-2050647/63-1240 SECRET SERVICE FORGERY PROJECT
Origin and Sponsor: Treasury Department, U.S. Secret Service
Manager: M. Paulsen
Full task description: July-December 1962 issue, p. 33
Status: INACTIVE.

20502-40-2050647/64-1450 GLASS BEAD DATA
Origin and Sponsor: NBS, Section 421.07
Manager: R. Zucker
Full task description: See January-March 1961 issue, p. 22, PARTICLE SIZE CALCULATIONS
Status: COMPLETED.

20502-40-2050647/64-1479 NUCLEAR QUADRUPOLE
Origin and Sponsor: NBS, Section 222.04
Manager: P. J. Walsh
Full task description: January-June 1964 issue, p. 21
Status: INACTIVE.

20502-40-2050647/64-1488 INTERPLANETARY CALCULATIONS
Origin and Sponsor: NASA
Manager: R. J. Arms
Full task description: January-June 1964 issue, p. 22
Status: CONTINUED. New programs included point plotting onto micro film and two-dimensional interpolation routines.

20502-40-2050647/64-1569 NERVE FIBERS
Origin and Sponsor: U. S. Naval Medical Research Institute
Manager: R. J. Arms
Full task description: July-December 1964 issue, p. 17
Status: CONTINUED. The steady-state problem is essentially completed. Analysis of the transient problem has begun.

20500-12-2050404/65-1456 RESEARCH ON BIOLOGICAL PATTERN DATA PROCESSING Authorized 1-21-64
Origin: NBS
Sponsor: National Institutes of Health
Manager: Russell A. Kirsch
Full task description: January-June 1964 issue, p. 19.

Status: CONTINUED. It is convenient to divide the work done during the present period into four categories; the analysis of images, the synthesis of images, the linguistic research, and the development of supporting research tools.

In analysis of images, Dr. George A. Moore in "Design for a Preferred Language for the Command of Automatic Analysis of Micrographs" describes several programs that have been useful in the analysis of micrographs primarily for applications in metallurgy.

Two programs for the IBM 7094 do smoothing and obtain transformed images by a modified wave propagation type of technique. In synthesis there was a study of generative devices for specifying the structure of pictures primarily of a line drawing nature. One program for the production of such line drawing images was experimented with on the IBM 7094. Another program obtains differentiated or derivative images from scanned photographs.

In research of a linguistic nature, the main effort was to develop a grammar for a part of the professional subdialect used by neuropathologists in describing photographs of tissue, "PLACEBO IV, Rules, Concordance, Sample Computer Generation" by W. C. Watt, NBS Tech. Note 255. A subsequent version, PLACEBO V, is currently being worked on. Two studies on micrograrmmars were made, "Prerequisites to the Utility of Micrograrmmars" by W. C. Watt, NBS Tech. Note 258, and the other is a study of the general properties and desiderata of micrograrmmars as tools for use in interrogation systems.

Some preparatory work was done in arranging materials for informant work with neuropathologists to attempt to elicit from them descriptions of photographs of neuropathological interest.

In supporting research, two programs were written to produce concordances for large grammars. Three versions of parsers were investigated.

Finally, a NBS seminar was conducted on an occasional basis with invited speakers from both within and outside the government speaking on the subject of language and picture processing.

20500-12-2050406/65 RESEARCH ON A PICTURE LANGUAGE MACHINE Authorized 5-1-61
Origin: NBS
Sponsor: National Science Foundation
Manager: Russell A. Kirsch
Full task description: July-December 1963 issue, p. 17.

Status: INACTIVE.

Publications:

The Analysis, Synthesis, and Description of Biological Images. L. E. Lipkin, W. C. Watt, and R. A. Kirsch. To appear in the Annals of the New York Academy of Sciences.

20502-40-2050630/65-1632 RATIO VARIABLES
Origin and Sponsor: Office of the Comptroller of the Currency
Manager: Ruth Zucker
Objective: To edit and prepare a set of tables relating to banks in 40 cities.
Background: Approximately 1512 banks each consisting of 35 variables were involved in the survey. The data information was about loans, assets, deposits, etc. Tables were desired of 16 ratios to test the consistency of the data of each bank; also other tables based on financial size of banks and population size of cities were cross tabulated for certain ratios desired. The problem was transmitted by Franklin R. Edwards.
Status: NEW. Completed. Tables were submitted to the sponsor.

20502-40-2050630/65-1634 TABLES
Origin and Sponsor: NBS, Section 311.01
Manager: J. D. Waggoner
Objective: To determine the parameters ℓ and t of the distribution function $W(r) = Ar^{\ell}\exp(-\alpha r^{t})$ (1)

from the following reduced moments of W(r):

$$\mu(p,s) = \frac{< r^{p}>}{< r^{p/s}>^{s}} - 1 \qquad (2)$$

$$< r^{p}> = A \int_{0}^{\infty} r^{p+\ell}\exp(-\alpha r^{t})\, dr \qquad (3)$$

$$< r^{s}> = A \int_{0}^{\infty} r^{s+\ell}\exp(-\alpha r^{t})\, dr \qquad (4)$$

Therefore,

$$\mu(p,s) = \frac{\Gamma\left(\frac{p+\ell+1}{t}\right)\left[\Gamma\left(\frac{\ell+1}{t}\right)\right]^{s-1}}{\left[\Gamma\left(\frac{p/s+\ell+1}{t}\right)\right]^{s}} - 1 \qquad (5)$$

Background: $\mu(p,s)$ are given independently from Monte-Carlo computations of the moments of non-self intersecting random walks on a given lattice. Values for $< r^{p}>$, $< r^{s}>$ and $\mu(p,s)$ are given.
Status: NEW. Completed. Program was written to find ℓ and t from eq. 5 which agree best with the given values of $\mu(p,s)$. The parameters ℓ and t should be the same for all the reduced moments.

20502-40-2050630/65-1645 SCALE VALUES
Origin and Sponsor: G. L. Howett, Section 212.11, Photometry and Colorimetry.
Manager: Ruth Zucker
Objective: To evaluate experimental data known as "paired comparisons". Sets of color differences (each difference consisting of 2 color samples) were compared one against the other with respect to some specified quality. The responses were either +1 or -1. The objective of the analysis was to derive a scale value for each color difference used in the experiment, the scale value indicating the size of the color difference.
Background: T_i and T_j are the scale values of color difference i and j, respectively, and if S_{ij} is the score resulting from a comparison of difference i and j, we want the scale values to be such as to minimize the sum of the squares $\sum_{i,j} \left(T_i - T_j - S_{ij}\right)^2$

The determination of scale values minimizing the above has been formulated in matrix terms. (See H. Scheffé, The Analysis of Variance, Wiley, 1959.)
The basic objective of the study is to produce a set of color samples such that the difference between any two neighboring samples in the array is constant as perceived by the average color-normal human observer.
Status: NEW. Code was written to perform matrix calculations. The scale values for all the color differences used in the experiment were tabulated. Other statistical measures permitting various significance tests were also computed. Numerous production runs were made and submitted to the sponsor.

20502-40-2050630/65-1659 PROJECT SUMMARY REPORT
Origin and Sponsor: NBS, Division 123
Manager: Irene A. Stegun
Objective: To adapt the procedure for obtaining the Project Cost Report prepared at the Boulder Laboratories, to the general NBS project accounts.
Background: The system of reporting project costs was devised within the Ionosphere Research and Propagation Division to provide management, in an easy and economical way, with current fiscal information.
Status: CONTINUED. Maintenance of master files as a backup measure and processing reports by institutes were introduced into the procedure. The automation of the treatment of purchase orders and requisitions was also effected. Reports have been produced biweekly starting with pay period 3.

20502-40-2050630/65-1665 ELASTIC RING PROBLEM
Origin and Sponsor: NBS, Section 213.04, R. A. Mitchell
Manager: Philip J. Walsh
Objective: To investigate the relationships between the shape of elastic proving rings and their corresponding deflection characteristics, load capacity, and weight. The class of rings under study has consisted of elliptical rings with sinusoidal variation of cross section dimensions around the ring.

A strain energy analysis, involving the assumption that ring cross sections remain plane during deflection, resulted in the following equations for deflection and load capacity.

$$\delta = \frac{P}{E}\int \frac{1}{bt}\left\{\left[N' + (1-\nu^2)\frac{M'}{\rho}\right]N' + \frac{12}{5}(1+\nu)V' - (1-\nu^2)\left(\frac{M'}{e}+N'\right)\left(\frac{r\cos\theta}{\rho}\right)\right\} ds$$

$$P_o = \frac{\sigma}{\left(\frac{1}{bt}\right)_o + \text{abs}\left\{\left[\frac{t-2e}{bte(2\rho-t)}\right]_o \left[\left(\frac{\int\frac{r\cos\theta}{btep}ds - \int\frac{\sin\theta dr}{bt\rho} - \int\frac{r\cos\theta d\theta}{btp}}{\int\frac{1}{btep}ds}\right) - C_1\right]\right\}}$$

$$P_{\pi/2} = \frac{\sigma\left[bte\left(\frac{2\rho-t}{t-2e}\right)\right]_{\pi/2}}{\left[\frac{\int\frac{r\cos\theta}{bte\rho}ds - \int\frac{\sin\theta dr}{bt\rho} - \int\frac{r\cos\theta d\theta}{bt\rho}}{\int\frac{1}{bte\rho}ds}\right]}$$

In the equations M', N', V', b, t, e, ρ and r represent loading and geometry functions that vary around the ring.
Background: The corresponding strain energy analysis for the special case of a circular ring of uniform cross section has been used in the design of proving rings for many years. The present analysis extends the method to a more general class of elliptical rings whose cross section varies sinusoidally around the ring.
Status: NEW. A program was written to evaluate the formulae given above. Test cases were run and agreed with hand-calculated experiments. The formulae depend basically upon seven parameters. Approximately 500 cases were computed during production runs.

20502-40-2050630/65-1675 EIGENVALUES OF MATRICES
Origin and Sponsor: NBS, Section 221.03
Manager: Walter Lipton
Objective: To find the eigenvalues and eigenvectors of several 16x16 matrices. The actual solution was done using a SHARE program, and programs were written here to set up the input and output.
Background: These matrices were generated as a result of experiments in low temperature physics.
Status. NEW. Completed.

20502-40-2050630/65 MICROFILM BLEMISH ANALYSIS
Origin and Sponsor: NBS, Section 221.13
Manager: Walter Lipton
Objective: To determine the causes of blemishes on microfilm by statistical analyses correlating such items as storage conditions, processor, and frequency of use with the frequency and type of blemish.
Background: Blemishes on microfilm were classified into types and inspectors were trained to record their occurrence on data cards. About 7500 rolls of film from 40 users were checked and the data given on cards to NBS.
Status: NEW. Several types of classifications have been completed and analyzed, and the results used to decide on more meaningful methods of classification, which are now being programmed.

20502-40-2050630/65-1689 DATA REDUCTIONS
Origin and Sponsor: NBS, Section 212.13
Manager: Sally T. Peavy
Objective: The program is designed to measure the granularity of photographic materials.
Background: When photographic images are viewed under sufficient magnification, they are seen to be inhomogeneous arrangements of silver grains in gelatin. This impression or sensation of nonuniformity in the image produced on the consciousness of the observer is termed graininess. The term granularity is used to designate the objective aspect of these inhomogeneities.
Status: NEW. Code has been completed, checked and results handed to sponsor for further checking.

20502-40-2050630/65-1704 and 1708 LEAST SQUARES
Origin and Sponsor: NBS, Section 223.21
Manager: Bertha Walter
Objective: To fit curves to a series of experimental points.
Background: Recently experiments have been carried out to determine precisely the desorption order for chemisorbed nitrogen on tungsten. A large number of data points were taken and fitted to the best straight line on a log-log plot.
Status: NEW. Completed.

20502-40-2050630/65 BILLING AUTOMATION
Origin and Sponsor: NBS, Section 121.01
Managers: Ruther Zucker and Irene A. Stegun
Objective: To write FORTRAN codes for the preparation of sequential billing reports.
Background: These reports are presently being prepared using EAM equipment. In order to create more current reports and have codes which might be used on other equipment, the present codes are being prepared in the FORTRAN language.
Status: NEW. Continued. Codes for editing, match-merging and preparation of reports and master files have been written and are in the process of being checked. Parallel runs are being compared for consistency.

7. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING

Task 13911-31-1390951/51-1

Origin: NBS Authorized 7/1/50
Managers: J. M. Cameron, H. H. Ku
Full task description: July-September 1950 issue, p. 60

Status: CONTINUED. J. M. Cameron developed a new design for use in the calibration of mass standard of the 5,3,2,1,1,1 series. A collection of designs is being prepared in collaboration with R. C. Raybold of the mass and volume section. These designs differ from those currently in the literature in that they involve the calibration of an additional weight of known value to serve as an accuracy check.

J. R. Rosenblatt has continued her assistance to the Technical Analysis Division and has collaborated on a variety of problems involving such diverse items as a study of ocean freight costs and the use of discriminant functions in problems related to the Northeast Corridor Transportation project.

Mary G. Natrella presented an in-hours course on "Statistics of Measurement for Scientists and Engineers." This course provides an introduction to the use of NBS Handbook 91, Experimental Statistics.

J. M. Cameron collaborated with J. Hilsenrath (Equation of State Section) in teaching a course entitled: Introduction to Mathematical and Statistical Analysis of Laboratory Data. This course presents an outline of the use of the Bureau's general purpose computing program OMNITAB for the statistical and numerical analysis of experimental data.

Publications:

(1) Evaluation of exact solutions to the Lamm equation. I. Billick (Macromolecules Synthesis and Structure Section) and G. H. Weiss. Submitted to a technical journal.

(2) The evolution or designed experiments. W. J. Youden. Proceedings, IBM Scientific Computing Symposium on Statistics, held October 21-23, 1963, IBM Data Processing Div., White Plains, N. Y., 1965, pages 59-67.

(3) Uncertainties associated with proving ring calibration. T. E. Hockersmith (Mechanics Division) and H. H. Ku. To appear in the Transactions of the Instrument Society of America.

(4) Mortality patterns in eight strains of flour beetles. W. J. Youden, D. B. Mertz and T. Park (Univ. of Chicago). Submitted to a technical journal.

(5) Evaluation of analytical data. W. J. Youden. To appear in Encyclopedia of Industrial Analysis.

(6) Uncertainties associated with proving ring calibration error. T. E. Hockersmith (Mechanics Division) and H. H. Ku. Instrument Society of America Journal, 12, 1965, pp. 73-77.

STATISTICAL SERVICES

Task 20503-40-2050132/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: January-March 1958 issue, p. 45

Status: CONTINUED. This is a continuing project which involves providing, upon request, statistical services to other governmental agencies, universities, industrial organizations, and other non-governmental agencies. Approximately 30 such requests are handled per month ranging from short conferences to collaboration involving several days work.

Over 25 inquiries concerning mathematical tables primarily for use in statistics were handled and as a result the section has expanded its efforts in the area of tables. Perhaps a fourth of the consulting problems relate to computational methods or similar aspects of computer useage.

The section has participated in the analysis of data from the mass measurement systems of White Sands Missile Range and of Huntsville and has provided special weighing designs for the calibration of 10,000-50,000Kg weights.

Under this project the section does work for the American Standards Association, ASTM, ASTME, and other technical societies. Members of the section have served, for example, as Chairman of the Section on Physical and Engineering Sciences of the ASA, as Associate Editor for the Society for Industrial and Applied Mathematics and as Program Secretary for the Institute of Mathematical Statistics.

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

THIS IS A RECORD OF THE USE OF THE IBM 7094 FOR THE PERIOD OF JANUARY 1, THROUGH JUNE 30, 1965

TASK NUMBER	TITLE	AS	CC	PR	TOTAL
NBS SERVICES		(M I	N U	T E	S)
51-0002 20503	STATISTICAL ENGINEERING	55	1	230	286
63-0003 20503	CLASS+++	0	0	40	40
54-0030 22201	SPECTRUM ANALYSIS++	7	7	639	653
54-0031 22201	SPECTRUM ANALYSIS++	0	0	52	52
54-0032 22201	SPECTRUM ANALYSIS++	26	0	27	53
54-0033 22207	SPECTRUM ANALYSIS++	136	100	773	1009
54-0034 22207	SPECTRUM ANALYSIS++	10	1	231	242
55-0065 20502	AUTOMATIC CODING	3	2	0	5
55-0082 22101	THERMOMETER CALIBRATION+	0	0	257	257
56-0131 21212	CALCULATIONS IN OPTICS+	0	0	1	1
57-0219 22102	THERMAL PROPERTIES+	28	23	91	142
57-0250 21211	SPECTROPHOTOMETRIC DATA+	8	0	15	23
58-0256 42106	COMPOSITE WALL STUDIES++	223	16	83	322
58-0272 22104	EQUATION OF STATE++	49	2	274	325
58-0314 22104	APPROXIMATIONS FOR GAS MIXTURES	11	0	4	15
60-0489 22101	INVERSION OF LINE PROBE DATA+	26	4	76	106
61-0523 23101	NEUTRON CROSS SECTION STUDIES++	160	69	42	271
61-0559 22101	THERMOCOUPLE CALIBRATION+	1	36	46	83
62-1000 42305	POST OFFICE OPERATIONS STUDY++	59	22	36	117
62-1003 22341	MOLECULAR SPECTROSCOPY+	0	0	9	9
62-1005 23104	RADIATION INTERACTION++	255	201	1303	1759
62-1006 23104	RADIATION INTERACTION++	372	509	644	1525
62-1011 22205	DISPERSION INTEGRALS++	40	1	18	59
62-1013 31100	STATISTICAL METHODS++	0	1	1	2
62-1015 22311	THERMAL FUNCTIONS++	0	0	1	1
62-1019 12500	NBS PERSONNEL REPORT++	3	68	281	352
62-1027 20502	NEW SYSTEM	17	2	120	139
62-1029 31306	D-SPACING CALCULATIONS+	23	1	42	66
62-1033 31306	CRYSTAL STRUCTURE CALIBRATION++	35	0	580	615
62-1034 22201	PHOTOIONIZATION CROSS SECTION++	146	4	449	599
62-1035 31101	CREEP DATA ANALYSIS++	103	53	73	229
62-1036 31105	FILM THICKNESS++	18	2	4	24
62-1038 31111	STANDARDIZATION ANALYSES++	15	13	2	30
62-1052 21212	BLACK BOX COMPUTER SERVICE+	8	0	2	10
62-1055 31204	ELLIPSOIDAL COMPUTATION++	5	3	5	13

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER	TITLE	AS	CC	PR	TOTAL
NBS SERVICES		(M I N U T E S)			
62-1064 21221	GAGE BLOCK STUDIES++	0	0	19	19
62-1066 21102	STANDARD CELLS++	6	0	21	27
62-1080 31302	BLACK BOX COMPUTER SERVICE+	0	0	31	31
62-1081 31301	BLACK BOX COMPUTER SERVICE+	10	1	15	26
62-1089 31305	ELASTIC CONSTANTS++	0	0	15	15
62-1107 21305	OSCILLATING SPHERE++	0	0	9	9
62-1163 42501	TRANSISTOR AGING BEHAVIOR++	64	125	0	189
62-1165 22341	NMR SPECTRA ANALYSES+	0	0	4	4
62-1181 42304	NTDC++	26	91	0	117
62-1185 42103	HEAT TRANSFER CALCULATIONS+	249	142	190	581
62-1195 31102	LIGHT SCATTERING++	6	5	2	13
62-1203 22104	CYLINDRICAL SHOCK WAVE	5	0	16	21
62-1212 42108	COLOR DIFFERENCES	0	0	3	3
63-1222 31101	DILATOMETRIC DATA CALCULATIONS+	9	28	19	56
63-1231 22207	BLACK BOX COMPUTER SERVICE+	0	0	20	20
63-1234 42103	VAPOR TRANSMISSION++	43	4	3	50
63-1237 22101	PYROMETRY++	0	0	24	24
63-1263 22351	LINEAR CLASSICAL SYSTEM++	13	0	1	14
63-1276 42502	INSTRUMENTATION++	17	0	0	17
63-1277 21102	BLACK BOX COMPUTER SERVICE++	0	0	1	1
63-1285 20505	RTS FUNDS++	1	3	0	4
63-1287 22104	DATA ANALYSES OF GASES++	0	0	42	42
63-1289 22105	IONIZED GASES++	287	30	29	346
63-1290 22100	MOLECULAR ENERGY LEVELS++	0	0	30	30
63-1291 31101	JOB CALCULATIONS++	43	27	55	125
63-1302 31103	COMPUTER CALCULATIONS++	2	0	3	5
63-1309 23101	LINEAR REGRESSION ANALYSIS++	47	2	49	98
63-1315 22103	VIRIAL COEFFICIENTS++	76	16	66	158
63-1318 42103	THERMISTOR PROGRAM++	5	1	0	6
63-1323 22100	PLASMA TRANSPORT++	3	0	9	12
63-1325 23101	THERMOFLUX++	191	19	245	455
63-1333 21212	BLACK BOX COMPUTER SERVICE+	3	2	6	11
63-1340 22103	FUNCTION OF TEMPERATURE++	0	0	95	95
63-1342 21301	OMNITAB+	4	6	24	34
63-1351 21102	TEST DATA++	14	62	24	100
63-1375 22104	THERMAL PROPERTIES+	187	11	46	244
63-1378 42305	DCA++	67	92	184	343
63-1388 22102	COMBUSTION CALORIMETRY++	0	0	29	29
63-1399 22102	HEAT MEASUREMENT++	0	1	10	11
64-1400 22202	STATISTICS++	122	1	325	448
64-1401 42107	LONG TIME CEMENT STUDY 1++	4	0	183	187

TASK NUMBER	TITLE	AS	CC	PR	TOTAL
NBS SERVICES		(M I N U T E S)			
64-1406 21306	HYPERSONIC COMBUSTION++	0	1	0	1
64-1407 31002	SPECTROANALYSIS++	0	0	4	4
64-1408 42101	ELASTIC SOLIDS	3	0	2	5
64-1416 31303	OMNITAB+	1	0	53	54
64-1418 21231	STATISTICAL COMPUTATION++	1	0	108	109
64-1419 21231	STATISTICAL COMPUTATION++	62	6	16	84
64-1420 22102	OMNITAB+	3	3	52	58
64-1423 22104	COORDINATE ANALYSIS++	27	43	8	78
64-1437 31105	AMALGAM STRAIN-TIME DATA++	1	0	21	22
64-1438 31200	MATRIX OPERATIONS	1	0	70	71
64-1440 42108	OMNITAB+	60	0	28	88
64-1445 42706	TEXTILE INDUSTRY STUDY++	6	0	189	195
64-1448 22300	BLACK BOX COMPUTER SERVICE+	0	3	19	22
64-1453 22101	RES THERMOMETER CALC++	0	9	1	10
64-1456 20500	INFORMATION RETRIEVAL++	26	655	342	1023
64-1462 23123	POSITRON PRODUCTION++	1	33	0	34
64-1463 22200	TRANSITION PROBABILITIES	0	0	27	27
64-1470 42305	PICNIC PROJECT++	3	45	25	73
64-1473 22104	POLAR GASES++	51	5	87	143
64-1474 22300	ATOM CORRELATION++	25	0	942	967
64-1476 22102	THERMOVELOCITY++	48	29	79	156
64-1478 23121	LEAST SQUARES++	0	0	9	9
64-1483 31306	POWDER PATTERNS++	0	0	19	19
64-1484 21303	OMNITAB+	0	0	16	16
64-1486 22101	OMNITAB+	0	0	12	12
64-1487 21301	VIBRATION CALIBRATION++	4	0	74	78
64-1492 23101	ELECTROMAG CROSS SECT++	17	358	139	514
64-1493 21304	PROVING RINGS++	0	0	116	116
64-1495 20100	FLEX TO LINOFILM	10	2	1	13
64-1496 21221	EXP FOR INVAR TAPE++	0	0	1	1
64-1500 12504	FORTRAN CLASS	16	6	2	24
64-1502 31207	POT POLYELECTROLYTE++	5	0	0	5
64-1503 21301	OMNITAB+	9	30	133	172
64-1523 21211	FORTRAN CLASS++	2	15	0	17
64-1537 42501	FIST++	17	0	0	17
64-1540 42305	DESCRIPTORS++	1	44	2	47
64-1547 31001	MOSSBAUER++	18	0	84	102
64-1552 21105	DIPOLE++	44	2	145	191
64-1559 42305	IPRS++	53	145	99	297
65-1563 21221	THERMAL EXPANSION++	0	43	1	44
65-1573 20505	FAA++	1821	1499	3260	6580

TASK NUMBER	TITLE	AS	CC	PR	TOTAL
NBS SERVICES		(M I N U T E S)			
65-1574 21102	OMNITAB+	0	0	2	2
65-1575 22101	HIGH TEMP ENTH++	0	0	2	2
65-1581 21323	TRIAL DATA++	7	0	4	11
65-1583 22105	TIME INTERVALS++	16	2	0	18
65-1584 23122	THERMOFLUX++	26	9	60	95
65-1586 21321	OMNITAB++	56	5	15	76
65-1589 31306	CRYSTAL DATA++	5	0	667	672
65-1594 22104	SHOCK WAVE++	299	167	180	646
65-1601 22105	SOUND PROPAGATION++	0	0	22	22
65-1605 21222	OPTICAL COMPUTATION++	18	75	25	118
65-1615 22205	ANGFOL++	0	5	17	22
65-1622 40000	RESEARCH++	1	0	17	18
65-1624 20505	FISH-ORI++	59	14	118	191
65-1625 42103	PROPERTIES OF AIR++	134	39	5	178
65-1627 22300	CLASSICAL SYSTEMS++	58	7	37	102
65-1628 31103	HEAT CAPACITY++	16	0	0	16
65-1629 40000	MANAGEMENT OBJECTIVES++	1	0	1	2
65-1630 42600	NE CORRIDOR++	420	38	41	499
65-1634 31101	GAMMA FUNCTIONS	1	0	0	1
65-1635 31307	INTEGRAL EVALUATION	10	1	0	11
65-1637 31302	STANDARD GLASSES++	0	0	12	12
65-1638 31100	OPTICAL SCAN++	0	7	0	7
65-1639 43205	D.C.A.++	0	184	33	217
65-1640 23111	INVERSION MATRICES	5	0	0	5
65-1642 42305	CHINESE PROBLEM++	0	0	1	1
65-1643 42305	MITRE++	0	3	0	3
65-1645 21211	SCALE VALUES	25	6	11	42
65-1646 21107	STANDARD CAPACITORS++	17	7	3	27
65-1647 23101	FRENCH LANGUAGE++	9	0	203	212
65-1648 20502	PAYROLL+	109	41	71	221
65-1649 22105	GASES AND HIGH TEMP++	26	0	0	26
65-1654 21304	DIFFERENTIAL TRANSFORMERS++	0	0	3	3
65-1658 31306	SINGLE CRYSTAL DETERMIN++	61	43	47	151
65-1659 12300	KEMPER REPORT+	71	21	682	774
65-1660 31400	EQUIPOISE 3A++	30	9	80	119
65-1663 21211	DATA REDUCTION	1	2	0	3
65-1664 21211	EMISSIVITY DATA++	0	5	0	5
65-1665 21304	DIMENSIONAL PARAMETERS	7	7	5	19
65-1666 22103	ANALYSIS EXPER DATA++	0	0	23	23
65-1667 22104	INTEGRAL EQUATION++	219	34	94	347
65-1668 30000	FLUORITE LATTICE	44	4	12	60

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER	TITLE	AS	CC	PR	TOTAL
NBS SERVICES		(M I	N U	T E	S)
65-1671 30011	TESTING ASPHALT++	8	0	6	14
65-1674 42108	REFLECTANCE++	7	0	3	10
65-1675 22103	EIGENVALUES OF MATRICES	6	17	19	42
65-1676 21213	MICROFILM DATA	13	25	77	115
65-1683 22103	RESISTANCE THERMOMETRY++	0	0	7	7
65-1684 23133	LINEAR ACCEL RESEARCH++	30	0	9	39
65-1685 21102	REFERENCE DATA++	6	0	13	19
65-1688 31104	MONTE CARLO++	24	7	10	41
65-1689 21213	DATA REDUCTIONS	14	8	0	22
65-1690 40000	CUC++	42	0	431	473
65-1692 21104	LOW TEMP BEHAVIOR++	5	0	54	59
65-1697 22102	OMNITAB+	2	0	20	22
65-1699 25100	MICROWAVE SPECTRAL TABLES++	11	0	102	113
65-1700 41001	JANOF++	4	5	0	9
65-1701 31301	LAOCOON++	0	0	47	47
65-1704 22321	LEAST SQUARES	0	0	2	2
65-1705 20502	SEQUENCE BILLING	85	12	40	137
65-1706 42304	CIRCULAR A-55++	0	0	31	31
65-1708 22321	LEAST SQUARES	0	0	1	1
65-1711 42300	AF DEMONSTRATION PROJECT++	4	8	0	12
65-1712 21211	VAPOR LAMP PROGRAM++	2	0	2	4
65-1715 21305	OMNITAB+	7	0	0	7
65-1719 43100	OMNITAB+	57	0	14	71
63-1999 20502	RESEARCH	10	0	0	10
63-3003 20502	MACHINE TIME ONLY+++	8	4	8	20
63-3005 20502	FREE MACHINE TIME+++	366	52	62	480
63-3007 20502	COMPILER EVALUATION	9	4	0	13
63-3008 20502	SECRETARYS MACHINE TIME+++	89	25	23	137
63-3009 20502	COST ACCOUNTING	42	8	9	59
63-3010 20502	NEW SYSTEMS+++	28	11	149	188
64-3011 20502	ERROR-USER+++	0	0	248	248
65-3012 20502	TAPE TEST+++	0	0	37	37
	TOTALS (NBS SERVICES)	8312	5707	18401	32420

TASK NUMBER		TITLE	AS	CC	PR	TOTAL
NON-NBS SERVICES			(M I N U T E S)			
58-0366	67	RADIATION PATTERNS OF ANTENNAS	0	0	4	4
59-0407	21	FOURIER COEFFICIENTS+	38	1	0	39
59-0434	90	PETROLOGICAL COMPUTATIONS+	20	5	38	63
59-0441	21	SYSTEMS ENGINEERING++	204	3005	555	3764
60-0457	86	PUBLIC HOUSING PROBLEM++	5	64	240	309
60-0476	21	GAS TUBE CHARACTERISTIC II	9	22	1150	1181
60-0492	90	MONETARY RESEARCH REPORTS++	190	81	304	575
60-0506	80	WORLD BANK REPORTS++	343	0	529	872
61-0540	21	DIFFUSION CALCULATIONS+	10	193	86	289
61-0569	21	HUMAN FACTORS RESEARCH++	102	21	97	220
61-0830	90	HIGHWAY TRAFFIC STUDIES++	35	0	327	362
61-0903	90	HIGHWAY TRAFFIC STUDIES++	45	10	1093	1148
61-0945	13	FORECASTING++	0	0	742	742
62-1004	17	RHOMBIC ANTENNAS+	0	0	21	21
62-1014	75	METABOLIC DISEASES++	234	294	2476	3004
62-1018	17	HYDROMAGNETIC PROBLEMS+	228	170	28	426
62-1021	99	HIGHWAY STUDIES++	80	477	1509	2066
62-1030	36	ELECTROCARDIOGRAPHIC ANALYSIS	1120	2689	1558	5367
62-1044	27	RADIO INTENSITIES++	63	0	32	95
62-1046	90	TRAFFIC PREDICTION++	589	90	1946	2625
62-1056	21	PD ENGINEERING++++	32	0	213	245
62-1071	21	RHINITIS STUDIES++	0	0	1	1
62-1076	90	EVALUATION OF APPLICATIONS+	28	6	63	97
62-1113	21	TRANSPORT ANALYSES++++	47	26	1586	1659
62-1114	21	RADIATION EFFECTS++	139	4	62	205
62-1119	90	HIGHWAY TRAFFIC STUDIES++	1	1	328	330
62-1121	90	CARNEGIE INSTITUTE STUDIES++	62	0	36	98
62-1130	43	FALLOUT SHELTER COMPUTATIONS	8	9	240	257
62-1140	36	VA MEDICAL++	4	0	222	226
62-1158	90	MINERALOGY STUDIES++	0	47	89	136
62-1169	90	ATOMIC COLLISIONS++	0	3	62	65
62-1171	36	HOSPITAL PROGRAM PLANNING+	139	166	1301	1606
62-1179	21	CATALOG INFORMATION+	9	2	116	127
62-1216	90	ARIZONA++	3	0	553	556
63-1236	13	DATATROL++	12	1	240	253
63-1239	75	PUBLIC HEALTH SERVICE++	25	10	98	133
63-1246	75	SCREENING EVALUATION+	34	17	83	134
63-1249	90	ISOTOPE TRACER ANALYSIS++	4	47	54	105
63-1262	17	NUCLEONICS++	64	362	0	426
63-1264	17	NUCLEONICS++	289	285	678	1252
63-1271	13	ECONOMICS STUDY++	21	1	223	245

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER		TITLE	AS	CC	PR	TOTAL
NON-NBS SERVICES			(M I N U T E S)			
63-1272	90	ROADS STUDY++	0	0	166	166
63-1293	13	BODDY CALCULATION++	57	9	41	107
63-1299	21	1410 PROGRAM++	26	0	11	37
63-1305	21	ARMY++	0	0	35	35
63-1307	21	MISCELLANEOUS PROGRAMMING++	0	1	0	1
63-1314	90	FLORIDA HIGHWAYS++	2	7	1119	1128
63-1317	72	SORTING AND TABULATING	134	81	261	476
63-1336	17	ARC++	3	0	1505	1508
63-1350	21	ME DATA++	8	177	37	222
63-1360	26	FEDERAL POWER COMMISSION++	7	0	9	16
63-1365	21	1410++	38	30	67	135
63-1371	20	ALTERNATE TAX PLANS++	39	0	1648	1687
63-1386	90	PUBLIC ROADS++	1	0	0	1
63-1391	75	BIOMEDICAL STA PROG++	12	0	157	169
64-1394	21	ARMY COST MODEL (RAND)++	0	0	390	390
64-1403	21	WORLD TEMPERATURE DIST++	44	0	64	108
64-1404	21	BUDGET PROG.++	2	0	28	30
64-1414	21	AD 70 PROGRAM++	134	13	84	231
64-1429	75	RESEARCH MISC++	79	27	391	497
64-1433	75	NMR SPECTRA	0	3	26	29
64-1435	13	CAPITOL COEFFICIENTS++	6	0	17	23
64-1436	21	DIPOLE MOMENT COMP++	0	0	2	2
64-1439	21	SHOCK PRESSURES++	0	0	7	7
64-1447	75	SOCIAL SECURITY RES++	1	0	85	86
64-1457	17	SOLAR RADIATION DATA RED++	78	4	13	95
64-1467	17	THEORET NUCLEAR PHYSICS++	146	170	736	1052
64-1482	90	BIOPHYSICS++	40	0	119	159
64-1488	80	INTERPLANETARY CALC	218	184	131	533
64-1498	20	REGRESSION EQUATION++	0	2	0	2
64-1504	55	1970 PROJECTIONS++	0	1	21	22
64-1516	21	ECM STUDY++	377	91	273	741
64-1526	21	BATTERY PROGRAM++	26	4	105	135
64-1551	21	AD CONVERSION++	10	23	0	33
64-1554	21	PREDICT PROGRAM++	137	33	4906	5076
65-1562	17	DIPOLE CALC++	0	0	2	2
65-1564	90	RADC CONTRACT++	113	64	805	982
65-1569	17	EXCAVATION	14	50	27	91
65-1570	13	PIPE STRESS++	42	0	0	42
65-1572	21	LANCE++	0	0	5	5
65-1576	43	RAIL COAST PROGRAMS++	0	0	2	2
65-1577	21	EIGENVALUES++	385	129	24	538

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER		TITLE	AS	CC	PR	TOTAL
NON-NBS SERVICES			(M I N U T E S)			
65-1590	13	BALANCE OF PAYMENTS++	1	0	5	6
65-1595	21	ANALOG TO DIGITAL TAPE++	0	0	3	3
65-1597	21	FACTORIAL ANOV++	285	41	61	387
65-1604	13	OBE++	0	158	1	159
65-1607	90	NSF++	0	0	3	3
65-1609	36	MARTINSBURG++	0	0	94	94
65-1612	21	CORG AMMUNITION STUDY++	51	542	245	838
65-1614	75	BEDS++	69	871	5	945
65-1617	21	TIME ANALYSIS++	5	0	0	5
65-1618	14	CRYSTALLOGRAPHY++	61	1	516	578
65-1620	90	ECONOMETRIC MODEL++	52	5	123	180
65-1621	13	MARINE DATA++	306	0	550	856
65-1623	90	UNIV PA TRAFFIC STUDIES++	1	3	143	147
65-1626	75	PHYSIO ANALYSIS++	7	0	70	77
65-1631	21	ANALOG RESEARCH++	27	15	22	64
65-1632	13	LOAN/INTEREST RATES	11	2	10	23
65-1633	13	R/D++	110	0	198	308
65-1636	90	PA++	19	0	840	859
65-1644	90	HTR-HAWAII++	12	6	490	508
65-1650	21	RETAIL SALES TAX++	26	14	111	151
65-1651	21	RATIOS++	3	3	4	10
65-1652	21	GUIDANCE PROBLEM++	11	1	1	13
65-1653	21	HIGH ALT FUZE COST++	84	30	258	372
65-1656	90	SE WISC REGIONAL PLANNING++	1	0	196	197
65-1657	90	TRANSPORTATION STUDY++	32	193	84	309
65-1661	90	ECONOMIC PERFORMANCE++	3	3	31	37
65-1662	17	BETA PARAMETERS++	31	29	449	509
65-1670	21	NEW STATION OVERLAYS	5	0	14	19
65-1673	21	VARIABLE LENS++	5	1	0	6
65-1677	90	RESOURCES++	51	0	70	121
65-1678	17	BUDOCKS TECH PROB++	0	0	38	38
65-1679	21	OSCILLATOR++	9	0	30	39
65-1680	21	RESEARCH++	0	0	33	33
65-1682	21	REPORT GENERATOR	39	5	94	138
65-1686	75	DELAWARE ESTUARY STUDY++	40	0	6	46
65-1687	21	CURVE FITTING++	46	49	17	112
65-1696	18	SAP PILOT STUDY	33	0	24	57
65-1702	21	LEVER ESCAPEMENT STUDY++	62	0	12	74
65-1709	13	SST++	489	55	134	678
65-1710	21	PERSHING FUZE++	0	0	123	123
65-1718	21	ENCOUNTER ANALYSIS++	1	9	3	13

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER		TITLE	AS	CC	PR	TOTAL
NON-NBS SERVICES			(M I N U T E S)			
65-1720	21	GREEN++	8	2	0	10
65-1723	30	BOB-ICC++	2	C	0	2
		TOTALS (NON-NBS SERVICES)	8413	11250	37413	57076
		TOTALS (NBS AND NON-NBS)	16725	16957	55814	89496

\+ PROBLEM PROGRAMMED IN THE COMPUTATION LABORATORY, PRODUCTION RUNS CONTINUED UNDER DIRECTION OF SPONSOR.

++ PROBLEM PROGRAMMED BY THE SPONSOR AND RUN UNDER HIS DIRECTION.

+++ FUNCTIONS PERTAIN TO THE INTERNAL OPERATIONS OF THE COMPUTATION LABORATORY.

++++ CLASSIFIED TASK.

AS ASSEMBLY TIME.

CC CODE CHECKING TIME.

PR PRODUCTION TIME.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publications Activities.

Applied Mathematics Division Lectures

KNOPP, M. (University of Wisconsin, Madison, Wisconsin) Series of five lectures on Automorphic Functions. June 7 through June 11, 1965.

MORDELL, L. J. (University of Illinois, Urbana, Illinois) The Diophantine Equation $y^2=ax^3+bx^2+cx+d$. April 21, 1965.

POWELL, M. J. D. (Atomic Energy Research Establishment, Hartwell, Didcot, Berks, England) On Calculating Unconstrained Minima of Differentiable Functions of Several Variables. May 20, 1965.

RABINOWITZ, P. (Brown University, Providence, Rhode Island) Numerical Experiments in the Solution of Laplace's Equation by Particular Functions. February 16, 1965.

NBS Courses Conducted by Staff Members

KLOSS, K. Programming for the NBS "Pilot" Computer. February 9 - May 25, 1965.

LEVY, J. Markov Chains and Applications in Dynamic Programming. September 1964 - January 1965.

NBS In-hours Courses Given by Staff Members

PEAVY, S. T. Fortran Programming.

WALSH, P. J. Symbolic Programming for the IBM 7090/7094.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

BERNSTEIN, B. Thermodynamics of elastic fluids. Presented at Indiana University, Bloomington, Indiana, January 7, 1965.

Finite visco-elastic strain -- A realistic theory. Presented at Purdue University, Lafayette, Indiana, January 8, 1965.

Thermodynamics of elastic fluids. Presented at Cornell University, Ithaca, New York, February 24, 1965.

Thermodynamics of elastic fluids. Presented at West Virginia University, Morgantown, West Virginia, April 26, 1965.

BRUALDI, R. A. Inequalities for the permanent. Presented to the Department of Mathematics, University of Michigan, Ann Arbor, Michigan, February 10, 1965.

CAMERON, J. M. Calibration designs. Rutgers--The State University, New Brunswick, N. J., April 9, 1965. (American Statistical Association--Biometric Society--Institute of Mathematical Statistics Program of Visiting Lecturers in Statistics).

EDMONDS, J. R.	The Chinese postman's problem. Presented before the Operations Research Society of America, Boston, Mass., May 6, 1965. Also at the NATO-ONR-UCAL Advanced Study Institute on Integer Programming and Network Flows, Tahoe City, California, June 30, 1965.
	Concave set functions, matroids and certain polyhedra. Presented before the NATO-ONR-UCAL Advanced Study Institute on Integer Programming and Network Flows, Tahoe City, California, June 23, 1965.
GOLDMAN, A. J.	Non-cooperative games involving share-of-market, (with H. D. Mills). Presented at the Princeton Conference on Game Theory, Princeton, New Jersey, April 5, 1965. Also at Weapon Systems Evaluation Group Symposium, Arlington, Virginia, June 16, 1965.
	Most profitable routing before maintenance, (with C. J. Witzgall). Presented before the Operations Research Society of America, Boston, Mass., May 7, 1965.
	Government Applications of Operations Research. Presented before the CSC Executive Seminar in Operations Research, Washington, D C., March 23, 1965.
KIRSCH, R. A. and WATT, W. C.	Natural and Artificial Language and Intelligence. Presented at the Center for the Information Sciences, Lehigh University, Bethlehem, Pennsylvania, March 18, 1965.
KIRSCH, R. A.	Pattern Recognition Machines. Panel participation at the IFIP Congress 65, New York City, May 25, 1965.
OLVER, F. W. J.	Error bounds for asymptotic expansions of special functions in the complex plane. Presented at the Symposium on April 26, 1965, at the U. S. Army, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin. (The lecture was delivered by Dr. H. Oser due to unavoidable absence of F. W. J. Olver).
	Evaluation of special functions. Presented at the International Federation for Information Processing Congress, New York, N. Y., May 28, 1965.
ROSENBLATT, J R.	Confidence limits for the reliability of complex systems. Department of Industrial Engineering and Administration, Cornell University, Ithaca, N. Y., April 13, 1965.
	Statistics and the Rainbow. Hollins College, Virginia, April 26, 1965. (American Statistical Association--Biometric Society--Institute of Mathematical Statistics Program of Visiting Lectures in Statistics).
	Predicting the reliability of complex systems. Hollins College, Virginia, April 27, 1965, (Program of Visiting Lecturers in Statistics).
	Distribution-free two-sample tests. Graduate Colloquium, Department of Statistics, Virginia Polytechnic Institute, Blacksburg, Virginia, April 27,1965.
TCHEN, C. M.	Magnetohydrodynamic turbulence. Presented at the NASA Contractor's Conference on Fluid Physics, Washington, D. C , March 16, 1965.
	Dr. Tchen was invited and participated in the NASA Symposium on Collision Free Shock Waves, NASA Ames Research Center, California, March 1-3, 1965.
	Dr. Tchen was invited for discussions and presented a talk on "Turbulence in a Reacting Gas" at the Institute for Defense Analysis Conference, Washington, D. C., April 30, 1965.

TCHEN, C. M.	Dr. Tchen presented a series of seminar lectures on "Special Topics of Plasma Physics", at the Institute for Theoretical Physics, University of Marburg, Marburg, West Germany, where he is invited as a visiting professor from May 6 to November 6, 1965.
VINTI, J. P.	Translational and rotational invariance of the spheroidal potential. Presented at the Boeing Scientific Research Laboratories, Seattle, Washington, April 12, 1965.
WALSH, P. J.	Components and capabilities of computers. Sponsored by U. S. Civil Service Commission, Washington, D. C., April 8, 1965.
WILLKE, T. A.	Rank statistics and multivariate distributions. Methodology Section, Washington Statistical Society, Washington, D. C., March 11, 1965.
WITZGALL, C. J.	Integer quadratic programming. Presented at NATO-ONR-UCAL Advanced Study Institute on Integer Programming and Network Flows, Tahoe City, California, July 1, 1965.
YOUDEN, W. J.	Quality control in textiles. American Association of Textile Chemists and Colorists, New York, N. Y., January 14, 1965.
	Statistical applications in the physical sciences. Department of Statistics, Harvard University, Cambridge, Mass., March 10, 1965.
	Measurement. Delaware Teachers of Science, Wilmington, Delaware, March 20, 1965.
	The evolution of designed experiments. Food and Drug Administration, Washington, D. C., March 23, 1965.
	Seminar on statistical problems of the Stormfury projects. U. S. Weather Bureau, University of Minnesota, College of Medical Sciences, Minneapolis, Minn., April 6, 1965.
	Discussion of papers on the analysis of experiments. American Society for Quality Control, West Long Branch, N. J., April 17, 1965.
	Statistical design of experiments. Seminar on Statistical Methods for Federal Executives, U. S. Department of Agriculture Graduate School, Baltimore, Maryland, April 19, 1965.
	A statistical technique for analytical chemists. Virginia Academy of Science, Richmond, Virginia, May 7, 1965.
	Evolution of experimental design. Department of Statistics, University of Chicago, Chicago, Illinois, June 1, 1965.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THIS PERIOD

1.3 Technical Papers

Localized-induction concept on a curved vortex and motion of an elliptic vortex ring. R. J. Arms and Francis R. Hama. The Physics of Fluids 8, No. 4, 553-559, April 1965.

A note on multipliers of difference sets. R. A. Brualdi, J. of Research, 69B, pp. 87-89, 1965.

Use of general purpose coding systems for statistical calculations. J. M. Cameron and J. Hilsenrath (NBS Equation of State Section). Proceedings of IBM Scientific Computing Symposium on Statistics, held October 21-23, 1963, IBM Data Processing Division, White Plains, N. Y., 1965, pp. 281-299.

Paths, trees and flowers. Jack Edmonds. Canadian Journal of Mathematics 17, pp. 449-467, 1965.

Minimum partition of a matroid into independent subsets. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 67-72, 1965.

Lehman's switching game and a theorem of Tutte and Nash-Williams. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 73-77, 1965.

On the surface duality of linear graphs. Jack Edmonds. Journal of Research NBS, 69B, Nos. 1-2, pp. 121-123, 1965.

Maximum matching and a polyhedron with 0,1-vertices. Jack Edmonds. Journal of Research NBS, 69B, pp. 125-130, 1965.

On matching problems. J. Edmonds, A J. Goldman, C. Witzgall, C. T. Zahn, Jr. Proceedings of Army Research Office Working Group on Computers, ARO-D Report 65-1, pp. 45-50, 1965.

On measurable sets and functions. A J. Goldman. Journal of Research NBS, 69B, Nos. 1-2, pp. 99-100, 1965.

Uncertainties associated with proving ring calibration error. T. E. Hockersmith (Mechanics Division) and H. H. Ku. Instruments Society of America Journal, 12, 1965, pp. 73-77.

Character subgroups of F-groups. M I. Knopp and M. Newman. Journal of Research NBS, 69B, pp. 85-86, 1965.

A new differential operator of the pure wave type. J. E. Lagnese. Journal of Differential Equations, Vol. 1, No. 2, pp. 171-187, 1965.

A theorem on the automorphs of a skew-symmetric matrix. M. Newman. Michigan Mathematical Journal 12, pp. 61-63, 1965.

Bounds for class numbers. M. Newman. American Mathematical Society Proceedings of Symposium for Number Theory, pp. 70-77, 1965.

Relaxation of a Lorentz Gas with a Repulsive r^{-s} Force Law. Hansjörg Oser, Kurt E. Shuler (Director's Office) and G. H. Weiss. J. Chem. Phys., 41, (1964) 2661-2666.

Some Remarks on Certain Generalized Dedekind Sums. H. Rademacher. Acta Arithmetica, 9, Section 1, pp. 97-105 (1964).

Estimation for a one-parameter exponential model. Janace A. Speckman and Richard G. Cornell (Florida State University). Journal of the American Statistical Association, 60, (1965), pp. 560-572.

Stochastic theory of diffusion in a plasma across a magnetic field. C. M Tchen. Proc. Internal. Symposium on Plasma Diffusion, Feldafing Germany, pp. 118-123, (1964).

Modification of Edmonds' maximum matching algorithm. C. Witzgall and C. T. Zahn, Jr. Journal of Research, NBS, 69B, Nos. 1-2, pp. 91-98, (1965).

The evolution or designed experiments. W. J Youden. Proceedings, IBM Scientific Computing Symposium on Statistics, held October 21-23, 1963, IBM Data Processing Division, White Plains, New York, (1965), pp. 59-67.

Approximating symmetric relations by equivalence relations. C T. Zahn, Jr. Journal of Soc. Indust. Appl. Math., Vol 12, No. 4 (1964.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.3 Elastic stress-strain relations in perfect elastic fluids. B. Bernstein, E. Kearsley and L. Zapas. Submitted to Transaction of the Society of Rheology.

Some theorems on the permanent. R. A. Brualdi and M. Newman. To appear in the J. of Research, Section B, NBS.

Inequalities for permanents and permanental minors. R. A Brualdi and M. Newman. To appear in the Proceedings of the Cambridge Philosophical Society.

Proof of a permanental inequality. R. A. Brualdi and M. Newman. To appear in the Quarterly Journal of Mathematics (Oxford).

Inequalities for the permanental minors of non-negative matrices. R. A Brualdi and M. Newman. To appear in the Canadian Journal of Mathematics.

Kernels and the Kronecker product of graphs. R. A. Brualdi. To appear in Proceedings of the American Mathematical Society.

Scattering properties of concentric soot-water spheres for visible and infrared light. R W. Fenn (U. S Army Electronics Labs) and H. Oser. To appear in Journal of Applied Optics.

Hadamard matrices of order cube plus one. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

A variant of the two-dimensional Riemann integral. A. J. Goldman. To appear in the Journal of Research, NBS, 69B, (1965).

Equivalence of certain inequalities complementing those of Cauchy-Schwarz and Hölder. A. J. Goldman, J. B Diaz and F. T. Metcaff. To appear in Journal of Research, NBS, 69B.

An analogue of Fermat's last theorem for entire functions. Fred Gross. To appear in American Math. Monthly Notes.

Entire solutions of the function equation $a(\varphi(z)) = a(\gamma(z)) + c$. F. Gross. Submitted to the Duke Mathematical Journal.

A functional inequality. S. Haber. Submitted to Proceedings of Am. Math. Soc.

A theorem on arbitrary functions. S. Haber. Submitted to a technical journal.

appear in Duke Mathematical Journal.

Chapter IC - Statistical Concepts of a Measurement Process, and Chapter ID - Statistical Analysis of Measurement Data. H. H. Ku. To appear in Industrial Metrology, American Society of Tool and Manufacturing Engineers.

The fundamental solution and Huygens' Principle for decomposable differential operators. J. E. Lagnese. To appear in Archive for Rational Mechanics and Analysis.

Real two-dimensional representations of the modular group and related groups. J. Lehner and M. Newman. To appear in Amer. J. Math.

Some extensions of Banach's contraction theorem. P. Meyers. To appear in J. of Research, NBS, 69B, (1965).

Error bounds for asymptotic expansions of special functions in the complex plane. F. W. J. Olver. To appear in the Proceedings of a Symposium on Error in Digital Computation, Madison, Wisconsin, April 1965.

On the asymptotic solutions of second-order differential equations having an irregular singularity of rank one, with an application to Whittaker functions. F W J Olver. To appear in the Journal of the Society for Industrial and Applied Mathematics, Series B. (This paper combines papers (2) and (3) reported Jan-June 1964).

Error bounds for asymptotic solutions of second-order differential equation having an irregular singularity of arbitrary rank. F. W J. Olver and F. Stenger. To appear in the J. of the Society for Industrial and Applied Mathematics, Series B.

Convergence and abstract spaces in functional analysis. E. Ordman. Submitted to a technical journal.

Spectrum of stationary and homogeneous magnetohydrodynamic turbulence. C. M. Tchen. To appear in the Physics of Fluids.

Effects of a constant force on a Keplerian orbit. J P. Vinti. To appear in Proceedings of Symposium 25 of the International Astronomical Union, Thessaloniki, Greece, August 15-22 (1964).

A prerequisite to the utility of microgrammers. W. C. Watt. To appear as Technical Note 258.

PLACEBO IV, Rules, Concordance, Sample Computer Generation. W. C. Watt. To appear as NBS Technical Note 255.

On convex metrics. C. Witzgall. To appear in J. of Research, NBS, 69B, (1965).

Morality patterns in eight strains of flour beetles. W. J. Youden, D. B. Mertz and T Park (Univ. of Chicago). Submitted to a technical journal.

Evaluation of analytical data. W. J Youden. To appear in Encyclopedia of Industrial Analysis.


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