NATIONAL BUREAU OF STANDARDS REPORT

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Progress Report
on
Viscoelastic Behavior
of Dental Amalgam

U.S. DEPARTMENT OF COMMERCE

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* NBS Group, Joint Institute for Laboratory Astrophysics at the University of Colorado.
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Progress Report on Viscoelastic Behavior of Dental Amalgam

By

P. L. Oglesby*
G. Dickson*
R. D. Davenport*
M. S. Rodriguez**
W. T. Sweeney***

* Dental Research Section, National Bureau of Standards, Washington, D. C. 20234.
** Guest Worker, National Bureau of Standards. Presently teaching dental materials at Loyola University Dental School, New Orleans, Louisana.
*** Chief, Dental Research Section, National Bureau of Standards, Washington, D. C. 20234.

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Viscoelastic Behavior of Dental Amalgam

Abstract

Measurements made on dental amalgam in tension indicate that amalgam exhibits three types of viscoelastic phenomena: (1) instantaneous elastic strain, (2) retarded elastic strain (transient creep) and (3) viscous strain (steady state creep). The combination of elastic plus retarded strain can be represented by an equation of the form $\epsilon = A\sigma + B^2\sigma^2$ where A and B are functions of time but not of the stress, σ. The viscous strain rate can be represented by an equation of the form $\dot{\epsilon}_V = K\sigma^m$ where K and m are constants of the material. By applying a nonlinear generalization of the Boltzmann superposition principle to a general equation describing the creep behavior of amalgam, the results of creep tests can be directly related to the results of stress-strain tests.

1. Introduction

There is no information in the literature on the viscoelastic behavior of dental amalgam except the limited amount that could be obtained from stress-strain relationships [1,2]. The stress-strain data indicate that amalgam exhibits a nonlinear relation between stress and strain over the entire range of stress investigated; this has been found to be true in both tension and compression. It has been further noted that the shape of the stress-strain curves vary with the strain rate indicating that the strain developed is not only dependent on the applied stress but also upon time of application of the stress. The results of tensile stress-strain investigation by Rodriguez [1] indicated that dental amalgam might be a viscoelastic material. It was, therefore, the objective of this study (1) to make an exploratory investigation of the types of viscoelastic phenomena exhibited by dental amalgam in tension (2) to describe the viscoelastic phenomena exhibited by amalgam in terms of available viscoelastic theory and (3) to make available a practical example and method of application of viscoelastic theory to a dental material.

2. Theory

In general a strain-hardened material will exhibit at least one or more of three types of viscoelastic phenomena; (1) elastic behavior (2) viscous behavior (steady-state creep) and (3) retarded elastic behavior (transient creep).

The elastic behavior is a linear function of the applied stress and completely recoverable; that is to say the elastic strain developed in a material is directly proportional to the applied stress and disappears instantaneously upon removal of the stress. The instantaneous elastic response of a material is commonly represented by a spring as shown in Figure 1-A.

$$\epsilon_e = J_o\sigma \qquad (1)$$

where

ϵ_e = the elastic strain developed in the material

σ = the applied stress

J_o = the proportionality constant called the elastic compliance which is the reciprocal of the elastic modulus.

In the case of viscous behavior (steady-state creep), the strain developed in the material is a function of both the applied stress and the time of application of the stress, but the viscous strain is nonrecoverable upon removal of the stress, that is to say it is permanent. The viscous strain is a linear function of time, and the viscous strain rate for any given stress value is a constant; on the other hand the viscous strain may or may not be a linear function of the applied stress. The viscous response of a material can be represented by the behavior of a dashpot as shown in Figure 1-B.

The viscous strain rate $\dot{\epsilon}_V$ for a material exhibiting a linear relation to the applied stress is as follows:

$$\dot{\epsilon}_V = \frac{1}{\eta}(\sigma - \sigma_o) \qquad (2)$$

where

$\dot{\epsilon}_V$ is the viscous strain rate

η is the coefficient of viscosity of the material

σ is the applied stress

σ_o is the applied stress level at which the material first begins to exhibit viscous flow and is called the yield stress.

A material exhibiting viscous flow in accordance with the above relation is said to exhibit ideal plastic behavior (Bingham behavior) while a material which obeys the above equation in its viscous flow but has a value of σ_o which is zero, is said to be a material which exhibits Newtonian flow. Materials demonstrating viscous flow which is a nonlinear function of the applied stress; usually obey one of the following two equations in their viscous response

$$\dot{\epsilon}_V = K (\sigma - \sigma_o)^m \quad \text{or} \quad \dot{\epsilon}_V = A \sinh B\sigma \qquad (3), (4)$$

where K and m, and A and B are constants at any given temperatures. A material exhibiting nonlinear viscous response described by the first equation is said to be pseudo plastic [3] in its viscous behavior in either case whether σ_o is or is not zero. On the other hand a material demonstrating nonlinear viscous response according to the second equation does not have a yield stress σ_o and is said to exhibit viscous flow (plastic flow) in accordance with the Kauzmann rate theory [4]. This type of viscous flow occurs in metals at temperatures near their melting points [5,6,7,8].

The retarded elastic response of a material may be represented by a single model or by either a finite or an infinite series of models called Voigt elements. A Voigt element is shown in Figure 1-C.

Retarded elastic behavior (transient creep) which can be described by means of a single Voigt element is represented by the following equation:

$$\epsilon_R = J \left(1-e^{-t/\tau}\right) \sigma \qquad (5)$$

where

ϵ_R is the retarded elastic strain developed in the material

σ is the applied stress

J is the retarded elastic compliance

τ is the retardation time of the material.

Retarded elastic behavior corresponding to a finite series of Voigt elements is represented by the following equation:

$$\epsilon_R = \sigma \sum_{i=1}^{n} J_i\left(1-e^{-t/\tau_i}\right) \qquad (6)$$

where

ϵ_R is the retarded elastic strain developed in the material

σ is the applied stress

J_i is the retarded elastic compliance of each Voigt element

τ_i is the retardation time of each Voigt element

Retarded elastic response represented by an infinite number of Voigt elements is described by the following equation:

$$\epsilon_R = \sigma \int_0^{\infty} J(\tau) \left(1-e^{-t/\tau}\right) d\,\tau \qquad (7)$$

In all of the above equations for the representation of retarded elastic strain, the relation between the retarded strain and the applied stress is assumed to be linear; however, in many materials this relation has been found to be nonlinear. The mathematical description of the nonlinear case has been treated in various publications [9, 10]. For example, if the retarded elastic strain (transient creep) is related to the applied stress by a second degree equation in which the time response can be represented by an infinite number of Voigt elements, such a retarded elastic response can be described by the following equation:

$$\epsilon_R = \sigma \int_0^\infty J(\tau) \left(1-e^{-t/\tau}\right) d\tau + \sigma^2 \int_0^\infty B(\gamma) \left(1-e^{-t/\gamma}\right) d\gamma \qquad (8)$$

where

ϵ_R is the retarded elastic strain

σ is the applied stress

$J(\tau)$ is the continuous distribution of retarded elastic compliance as a function of the variable retardation time τ of the continuous distribution of Voigt responses in the linear stress response element of the material

$B(\gamma)$ is the continuous distribution of retarded elastic compliances as a function of the variable retardation time γ of the continuous distribution of Voigt elements in the nonlinear second degree stress response of the material.

In a material which exhibits all three types of behavior (elastic, viscous, and retarded elastic), and in which the three types of behavior are additive (obey a superposition relation), the creep strain developed in the material at any time t under an applied stress σ may be related to the strains due to the individual behaviors by the following equations:

$$\epsilon = \epsilon_o + \epsilon_R + \epsilon_v \qquad (9)$$

where

ϵ is the creep strain developed in the material at any time t under an applied constant stress

ϵ_o is the instantaneous elastic strain developed in the material

ϵ_R is the retarded elastic strain developed in the material

ϵ_v is the viscous strain developed in the material.

These individual responses as was noted earlier may be linearly or nonlinearly related to the stress. For example, in a material which exhibits all three phenomena linearly related to the applied stress then using the linear equation of each response given earlier, the above equation takes the form:

$$\epsilon_R = J_o \sigma + \sigma \int_0^\infty J(\tau) \left(1-e^{-t/\tau}\right) d\tau + \frac{\sigma t}{\eta} \qquad (10)$$

Next, consider an example of a material which exhibits all three types of behavior, but in which the viscous strain developed is nonlinearly related to the applied stress as follows:

$$\epsilon_v = K (\sigma - \sigma_o)^m t \qquad (11)$$

while the retarded elastic strain is nonlinearly related to the applied stress by an equation of second degree as given earlier and the instantaneous is assumed to be linear; thus, the above superposition equation for creep strain takes the following form:

$$\epsilon = J_o \sigma + \sigma \int_0^\infty J(\tau) \left(1-e^{-t/\tau}\right) d\tau + \sigma^2 \left[\int_0^\infty B(\gamma) \left(1-e^{-t/\gamma}\right) d\gamma \right]^2 + K(\sigma - \sigma_o)^m t \qquad (12)$$

Where the creep strain is linearly related to the applied stress, the strain can be written as a product of a general creep compliance, which is a function of time only. and the applied stress as given by the following equation:

$$\epsilon = J(t)\sigma \tag{13}$$

where

ϵ is the creep strain developed

σ is the applied stress

$J(t)$ is the general creep compliance

For example, in the linear creep behavior as seen in equation (10), the general creep compliance J(t) is of the form:

$$J(t) = J_o + \int_0^\infty J(\tau)\ \left(1-e^{-t/\tau}\right)\ d\ \tau + \frac{t}{\eta} \tag{14}$$

In the case that creep strain is linearly related to the applied stress, the stress-strain behavior for a material may be related to the creep behavior by means of the linear superposition principle developed by Boltzmann [11] as shown below:

$$E\ (\sigma_t) = \int_0^{\sigma_t} J(T-\theta)\ d\ \sigma\ (\theta) \tag{15}$$

or

$$E\ (T) = \int_0^T J(T-\theta)\ \frac{d\sigma(\theta)}{dt}\ d\ \theta \tag{16}$$

where

σ is the applied stress related to the experimental time θ in a prescribed experimental functional relation.

$E(T)$ is the strain of a stress-strain relation measured at a time T subsequent to the variable time θ.

$\frac{d\sigma(\theta)}{d\theta}\ d\ \theta$ is the increment of stress from time θ to time $(\theta + d\ \theta)$.

For example, applying the Boltzmann superposition principle to the example of creep given in equation (10) gives the following relation between creep behavior and the strain measured in a stress-strain test [12]:

$$E\ (T) = J_o\sigma + \int_0^\infty\int_0^\infty J(\tau)\ \left(1-e^{-\frac{T-\theta}{\gamma}}\right)\ d\ \tau\ \frac{d\sigma(\theta)}{d\ \theta}\ d\ \theta + \int_0^T \frac{\sigma(\theta)}{\eta}\ d\ \theta \tag{17}$$

An excellent discussion of the application of the Boltzmann superposition principle as applied to linear viscoelastic phenomena is given by Leaderman [13]. However, if a material exhibits a nonlinear relation in its retarded strain (transient creep) and the applied stress, as discussed earlier, the linear Boltzmann superposition principle cannot be applied in its present form to relate strain of a stress-strain test on the material to the creep strain. A generalization of Boltzmann's superposition principle developed by Nakada [10] may be used to relate nonlinear retarded elastic creep behavior to the stress-strain behavior of the material as given:

$$E_R\ (\sigma) = \int_0^{\sigma(T)} \Psi(T-\theta)\ d\ \sigma\ (\theta) + \int_0^{\sigma(T)}\int_0^{\sigma(T)} \Phi\ (T-\theta)\ \Phi\ (T-\phi)\ d\ \sigma\ (\theta)\ d\ \sigma\ (\phi)$$

$$+ \int_0^{\sigma(T)}\int_0^{\sigma(T)}\int_0^{\sigma(T)} E(T-\theta)\ E(T-\phi)\ E(T-\alpha)\ d\ \sigma\ (\theta)\ d\ \sigma\ (\phi)\ d\ \sigma\ (\alpha)\quad + ----- \tag{18}$$

Now for example, consider the application of the nonlinear superposition principle of Nakada as applied to the case of retarded strain (transient creep) related to the applied stress by a second degree equation as shown earlier:

$$\epsilon_R = \sigma \int_0^\infty J(\tau)\left(1-e^{-t/\tau}\right) d\tau + \sigma^2 \left[\int_0^\infty B(\gamma)\left(1-e^{-t/\gamma}\right) d\gamma\right]^2 \quad (19)$$

now

$$\psi(t) = \int_0^\infty J(\tau)\left(1-e^{-t/\tau}\right) d\tau$$

and

$$\Phi(t) = \int_0^\infty B(\gamma)\left(1-e^{-t/\gamma}\right) d\gamma$$

where $\psi(t)$ and $\Phi(t)$ are the respective first degree and second degree retardation functions as given in the nonlinear superposition principle. Therefore, applying the nonlinear superposition principle to the above retarded creep example one has:

$$E_R\left(\sigma(T)\right) = \int_0^{\sigma(T)} \int_0^\infty J(T)\left(1-e^{-\frac{T-\theta}{\tau}}\right) d\tau\, d\sigma(\theta)$$

$$+ \int_0^{\sigma(T)} \int_0^{\sigma(T)} \int_0^\infty \int_0^\infty B(\gamma)^2 \left(1-e^{-\frac{T-\theta}{\gamma}}\right)\left(1-e^{-\frac{T-\phi}{\gamma}}\right) d\gamma\, d\gamma\, d\sigma(\theta)\, d\sigma(\phi) \quad (20)$$

Considering an example of a material which exhibits all three phenomena but in which the creep strain is a nonlinear function of the applied stress according to equation (12), then applying the nonlinear superposition principle, the stress-strain behavior of the material may be related to the creep behavior by the following equation:

$$E = \sigma J_o + \int_\infty^T \int_0^\infty J(\tau)\left(1-e^{-\frac{[T-t]}{\gamma}}\right) d\tau\, d\sigma(t)$$

$$+ \int_{-\infty}^T \int_{-\infty}^T \int_0^\infty B(\gamma)\left(1-e^{-\frac{[T-\theta]}{\gamma}}\right)\left(1-e^{-\frac{[T-\phi]}{\gamma}}\right) d\gamma\, d\sigma(\theta)\, d\sigma(\phi)$$

$$+ \text{-------} \qquad + \int_0^T K\, \sigma^m(t)\, dt \quad (21)$$

3. Materials

The specimens were prepared using a commercial alloy for dental amalgam certified to comply with American Dental Association Specification No. 1. This alloy (composition approximately Ag 70%, Sn 26%, Cu 3.5%, Zn 0.5%) was mixed with mercury and condensed into a mold as described by Rodriguez and Dickson [1] to produce a specimen with dimensions as shown in Figure 2. Specimens were aged for at least one week to obtain essentially full mechanical strength [14].

4. Procedure

The dumbbell-shaped specimen was placed in the grips and Tuckerman optical strain gages were mounted on opposite sides of the specimen as shown in Figure 3. To obtain the creep curve, readings were taken on the strain gages, a weight was suspended from the lower grip and a second strain gage reading was taken immediately. Strain readings were then made at 15 second intervals for 4 minutes and at increasingly longer intervals until the strain rate became constant (usually after approximately 1.5 hours). At the end of this period, the load was removed, a strain reading was taken immediately and the recovery curve was followed by reading first at 15 second intervals and then at longer intervals until the strain became constant.

Strain readings obtained on the two sides of the specimen were averaged and strain was plotted against time to obtain the loaded creep and unloaded recovery curves. Readings on the strain gages were normally made to the nearest 2×10^{-5} inch. Since a gage length of 0.25 inch was used this is equivalent to a strain of 8×10^{-5}. Thus in the results given below differences in strain of 1×10^{-4} are approximately equal to the minimum reading difference.

Loads placed on the specimen (with a nominal 0.01 in^2 cross sectional area) varied from 5 to 40 lbs. giving stresses from approximately 500 to 4,000 psi. Most specimens were used for several runs, first at high and then at lower stresses. The first loaded creep run was considered a strain hardening treatment and data obtained on these runs were not used in the calculation of results other than for viscous strain rate. All runs were made at 23 ± 1°C.

5. Results and Discussion

The creep curves (both loaded and recovery curves) of strain hardened dental amalgam as shown for a number of different stress levels in Figure 4 indicate that at room temperature amalgam exhibits three different types of viscoelastic phenomena: (1) instantaneous elastic strain (2) retarded elastic strain (transient creep) and (3) viscous strain (steady-state creep).

The viscous strain rate was determined from the loaded portion of the creep curve by taking the slope of the straight line portion of the curve, and was also determined from the recovery portion of the creep curve by dividing the value of the recovery strain (the permanent strain in the specimen) by the total time the load was on the specimen. The viscous strain rates for any given creep curve as calculated from the loaded and recovery portions of the curve were found to agree fairly well as shown in Table 1. The log of the viscous strain rate was found to be a linear function of the log of the applied stress, as shown in Figure 5; thus the viscous strain rate could be related to the applied stress by the following equation:

$$\dot{\epsilon}_v = K \sigma^m \qquad (22)$$

where

$\dot{\epsilon}_v$ is the viscous strain rate

σ is the applied stress

K and m are constant of the material

The value of m for amalgam is the value of the slope of the curve in Figure 5. While the value of K is the antilog of the viscous strain rate value at a value of applied stress σ of 1 psi. The values of K and m for the dental amalgam used in this investigation were found to be K equals 2.85 and 4.98×10^{-19} and m equals 3.99 and 3.92 from loaded and unloaded data respectively.

The strain developed in amalgam due to the other two phenomena (1) instantaneous elastic strain and (2) retarded strain can be determined from the strain recovery since these two types of strain are recoverable while the viscous strain is not. Thus, at any given load the strain values taken from the creep curve after the sample has been unloaded (that is in the recovery portion of the creep curve) are subtracted from the strain value on the creep curve at the instant just before unloading of the specimen. This difference is plotted against recovery time $t_1 = T_1 - T_u$ where T_u is the time at which the specimen was unloaded and T_1 is the time of the strain value on the recovery portion of the curve. These difference values, ϵ', are seen plotted against the recovery time for each load or stress in Figure 6. These plots are a measure of the combination of the elastic and retarded elastic strain behavior of dental amalgam as a function of time for various stress levels. In theory the same plot may be obtained from the loaded portion of the creep curve by taking values off the loaded creep curve and subtracting the viscous strain accumulated in the specimen at that time. The accumulated viscous strain at any time may be calculated by multiplying the viscous strain rate by the time corresponding to that value on the creep curve. Thus the difference between the creep curve value on the loaded protion and the viscous strain value at a corresponding time is a measure of the combination of the instantaneous and retarded elastic strain. However, a small error in the viscous strain rate causes a large error in the difference value. Therefore, the plot of the combination of elastic and retarded elastic strain versus time as obtained from the loaded creep curve is subject to large possible error; for this reason the combination elastic behavior is obtained from the recovery portion of the creep curves.

The combination elastic strain (instantaneous and retarded elastic strain) values are seen in Table 2 tabulated against time for various applied stress levels. The combination strain becomes asymptotic with time in accordance with theory as seen in Figure 6. The combination elastic strain values were plotted as a function of the various stress levels for corresponding time; as shown in Figure 7. The combination elastic strain is seen to be a nonlinear function of the applied stress. When the combination strain values were divided by their corresponding stresses and then plotted against the corresponding stress for a fixed time a linear plot was obtained for each fixed time as illustrated in Figure 8; this result indicated that the combination elastic behavior of amalgam as a function of applied stress under the test conditions could be represented by an equation of the form:

$$\epsilon' = A(t)\,\sigma + B^2(t)\,\sigma^2 \tag{23}$$

where

ϵ' is the combination of elastic and retarded elastic strain

σ is the applied stress

$A(t)$ and $B(t)$ are constants for any given time value and are functions of time but not of stress. The value of $A(t)$ for any time value is the intercept at $\sigma = 0$ of the plot for that time value as shown in Figure 8 while $B^2(t)$ is the slope of the straight line for that time value. It is also noted in Figure 8 that as a function of stress the combination strain divided by the stress is a straight line for all values of t. This indicated that over all ranges of t the combination elastic strain obeys the same functional relation to the stress. Thus from Figure 6 it was concluded that the retarded elastic strain as a function of time could be described by means of conventional viscoelastic theory assuming either a finite or infinite number of Voigt elements while the combination elastic strain being a nonlinear function of stress indicated the need for the use of nonlinear viscoelastic theory as developed first by Leaderman[9] and later more generally by Nakada [10]. Therefore, the values for $A(t)$ and $B^2(t)$ were determined by fitting curves to the data by the method of least squares and were tabulated as a function of time as shown in Table 3. The $A(t)$ values were plotted as a function of the corresponding t values as shown in Figure 9. The $A(t)$ values are seen to approach an asymptote as $t \longrightarrow \infty$, thus when the values of the asymptote minus the $A(t)$ values are plotted as a function of the corresponding time values, Figure 10, the curve is seen to fall off in a complicated exponential behavior which could be represented by viscoelastic theory assuming an infinite number of Voigt elements or a continuous spectrum. Thus, it has been assumed that $A(t)$ could be represented by the following equation from linear viscoelastic theory since $A(t)$ is the linear term in stress:

$$A(t) = J_o + \int_0^\infty J(\tau)\left(1-e^{-t/\tau}\right) d\tau \tag{24}$$

$$A(t) = J_o + \int_0^\infty J(\tau)\, d\tau - \int_0^\infty J(\tau)\, e^{-t/\tau} d\tau \tag{25}$$

$$A(t) = A_{as} - \int_0^\infty J(\tau)\, e^{-t/\tau} d\tau \tag{26}$$

$$A_{as} - A(t) = \int_0^\infty J(\tau)\, e^{-t/\tau} d\tau \tag{27}$$

where

$A(t)$ is the creep compliance term which is linear in stress

J_o is instantaneous elastic compliance

$J(\tau)$ is the retarded elastic compliance spectrum as a function of the retardation time τ

A_{as} is the asymptote value of A(t) as t becomes very large.

Thus, equation (26) indeed does describe the asymptotic behavior of A(t) as a function of time as seen in Figure 9 while equation (27) could be used to describe the complicated exponential-like behavior seen in Figure 10. The linear creep compliance term A(t) is plotted as a function of log t to obtain a sigmoidal curve as shown in Figure 11. The first plateau of the sigmoidal plot at very small values of t should correspond to J_o (the instantaneous elastic compliance)[15], however, the curve does not extend to short enough time values to determine the value of the plateau which would be J_o. Therefore, it is concluded that instantaneous elastic behavior of amalgam cannot be separated from the combination elastic behavior and thus is indeterminate from the data obtained in this investigation. The plot in Figure 11 indicates the need for improved experimentation on amalgam in which strain measurements can be made at very short times after loading or removal of the load from the specimen.

Next the nonlinear creep compliance term $B^2(t)$ was plotted against t as shown in Figure 12. The value of $B^2(t)$ is seen to approach an asymptote with increasing time. When the asymptote value of B(t) minus B(t) is plotted against time it decreases in a complicated exponential-like form as shown in Figure 13. Thus using the nonlinear theory of Nakada [10] it is concluded that the experimental B(t) for amalgam could be described by the following equation:

$$B(t) = \sqrt{B^2(t)} = \int_0^\infty B(\gamma)\left(1-e^{-t/\gamma}\right) d\gamma \qquad (28)$$

which would indeed describe the behavior of the curves seen in Figure 12 and Figure 13. It is therefore concluded that the combination elastic behavior of dental amalgam in creep under the test conditions used can be described by means of the following equation from viscoelastic theory [10,15]:

$$\epsilon' = A(t)\,\sigma + B^2(t)\,\sigma^2 \qquad (29)$$

$$\epsilon' = J_o\,\sigma + \sigma\int_0^\infty J(\tau)\left(1-e^{-t/\tau}\right) d\tau + \left[\int_0^\infty B(\gamma)\left(1-e^{-t/\gamma}\right) d\gamma\right]^2 \sigma^2 \qquad (30)$$

and since the loaded portion of the creep curve for amalgam is also composed of viscous strain ϵ_v as well as combination elastic strain ϵ' then the strain on the loaded portion is composed of the sum of the two as follows:

$$\epsilon = \epsilon' + \epsilon_v \qquad (31)$$

Thus

$$\epsilon = J_o\,\sigma + \sigma\int_0^\infty J(\tau)\left(1-e^{-t/\tau}\right) d\tau + \left[\int_0^\infty B(\gamma)\left(1-e^{-t/\gamma}\right) d\gamma\right] \sigma^2 + K\,\sigma^m t \qquad (32)$$

Therefore applying both linear and nonlinear viscoelastic theory to the experimental behavior of amalgam under the test conditions used, a general equation describing the creep behavior of amalgam as given by equation (32) above is obtained.

Applying the nonlinear generalization of the Boltzmann superposition principle as developed by Nakada [10], to the above creep equation for amalgam, the stress-strain curves for various stress rate conditions were calculated for amalgam from the creep data and compared to the experimental stress-strain curves obtained under those conditions. It is seen in Figure 14 that good agreement is obtained between the calculated and experimental stress strain curves. Thus, it is concluded that in the case of dental amalgam the results of creep tests can be directly related to those of stress-strain tests by use of viscoelastic theory [10,15].

6. Conclusion

Dental amalgam exhibits three types of viscoelastic phenomena: (1) instantaneous elastic strain (2) retarded elastic strain (transient creep) and (3) viscous strain (steady state creep).

The instantaneous elastic strain is assumed to be proportional to the applied stress but the methods used in this study did not provide an independent determination of instantaneous strain.

The combination of elastic plus retarded strain is a nonlinear function of stress and can be represented by an equation of the following form:

$\epsilon' = A\sigma + B^2 \sigma^2$ where A and B are functions of time but not of stress

The viscous strain rate is also a nonlinear function of stress and can be represented by an equation of the form:

$\dot{\epsilon}_V = K\sigma^m$ where K and m are constants of the material

A general equation describing the creep behavior of amalgam was obtained by the application of both linear and nonlinear viscoelastic theory to the experimental behavior of amalgam under the test conditions used. By applying the nonlinear generalization of the Boltzmann superposition principle to this equation the results of creep tests can be directly related to results of stress-strain tests of amalgam.

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TABLE 1

VISCOUS STRAIN RATES

Specimen - Run	Stress PSI	Strain per Minute Loaded* $X 10^{-5}$	Strain per Minute Unloaded** $X 10^{-5}$
41-40-1	3807	5.6531	6.3511***
41-40-2	3807	5.9759	5.8037
39-40-1	3731	4.9184	5.7449***
39-40-2	3731	5.2273	5.0822
40-40-1	3731	7.4476	8.0372***
43-35-1	3458	3.1200	3.7086***
43-35-2	3458	3.2339	3.2762
43-35-3	3458	3.4171	3.4389
44-30-1	2959	1.9183	1.9778***
44-30-2	2959	1.6821	1.6769
44-30-3	2959	1.7387	1.7185
41-30-3	2855	2.8429	2.9555
44-25-4	2465	0.9067	0.8867
35-25-1	2375	1.0521	1.3312***
35-25-2	2375	0.9762	1.0621
35-25-3	2375	1.0288	1.0803
38-25-1	2366	1.0909	1.3967***
38-25-2	2366	0.7832	0.8211
38-25-3	2366	0.7846	0.8061
35-20-4	1900	0.3843	------
35-20-5	1900	0.4192	0.4348
38-20-4	1892	0.3487	0.3581
35-15-6	1425	0.1109	0.1133
38-15-5	1419	0.0761	0.0768
35-10-7	950	0.0306	0.0338
38-10-6	946	0.0232	0.0248
38-10-7	946	0.0170	0.0178
35-05-8	475	0.0083	0.0066

* From slope of straight portion of loaded creep curve

** From strain remaining in specimen after unloading and recovery

*** These values include effects of strain hardening and were not used in calculating the relation between stress and viscous strain rate.

TABLE 2

RECOVERY OF ELASTIC (INSTANTANEOUS PLUS RETARDED) STRAIN

Time (Min.)	Strain* at: 475 psi	947 psi	1422 psi	1896 psi	2384 psi	2933 psi	3458 psi	3761 psi
	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$	$X\ 10^{-4}$
0.25	0.745	1.300	2.113	3.062	3.729	5.281	6.301	7.085
0.50	0.945	1.301	2.163	3.212	4.186	5.831	7.101	7.964
1.00	0.845	1.301	2.297	3.495	4.660	6.431	7.601	8.530
2.00	1.045	1.303	2.597	3.663	5.030	6.740	8.268	9.370
3.00	1.045	1.336	2.763	3.763	5.254	7.073	8.846	10.021
5.00	1.046	1.450	2.914	3.963	5.563	7.506	9.468	10.727
10.00	1.047	1.675	3.065	4.471	5.997	8.239	10.224	11.991
20.00	1.147	1.968	3.199	4.662	6.468	9.106	11.201	13.244
30.00	1.286	1.971	3.421	4.962	6.700	9.307	11.668	13.827
40.00	1.366	1.985	3.654	5.228	7.055	9.656	11.969	14.343
50.00	1.445	2.000	3.787	5.295	7.265	9.857	12.169	14.704
60.00	1.445	2.014	3.830	5.362	7.340	10.140	12.602	14.963
70.00	1.445	2.029	3.864	5.396	7.415	10.413	12.735	15.250
80.00	1.445	2.043	3.982	5.429	7.498	10.428	12.891	15.400
90.00	1.445	2.058	4.110	5.472	7.582	10.476	13.035	15.542

* Each value is an average of 2 to 7 determinations of the difference between strain recorded at the time indicated and strain recorded immediately prior to removal of the tensile stress.

TABLE 3

VALUES OF A(t) AND $B^2(t)$ IN EQUATION

$$\epsilon' = A(t)\,\sigma + B^2(t)\,\sigma^2$$

Time Min.	A(t) $\times 10^{-7}$	$B^2(t)$ $\times 10^{-11}$
0.25	0.922	2.707
0.50	1.122	2.873
1.00	1.303	2.828
2.00	1.309	3.367
3.00	1.280	3.904
5.00	1.322	4.284
10.00	1.363	4.986
20.00	1.383	5.804
30.00	1.409	6.110
40.00	1.544	6.082
50.00	1.589	6.190
60.00	1.581	6.454
70.00	1.570	6.678
80.00	1.588	6.728

ϵ' is combination of instantaneous and retarded elastic strain

σ is stress

A(t) and $B^2(t)$ are constants for any time and are functions of time but not of stress.

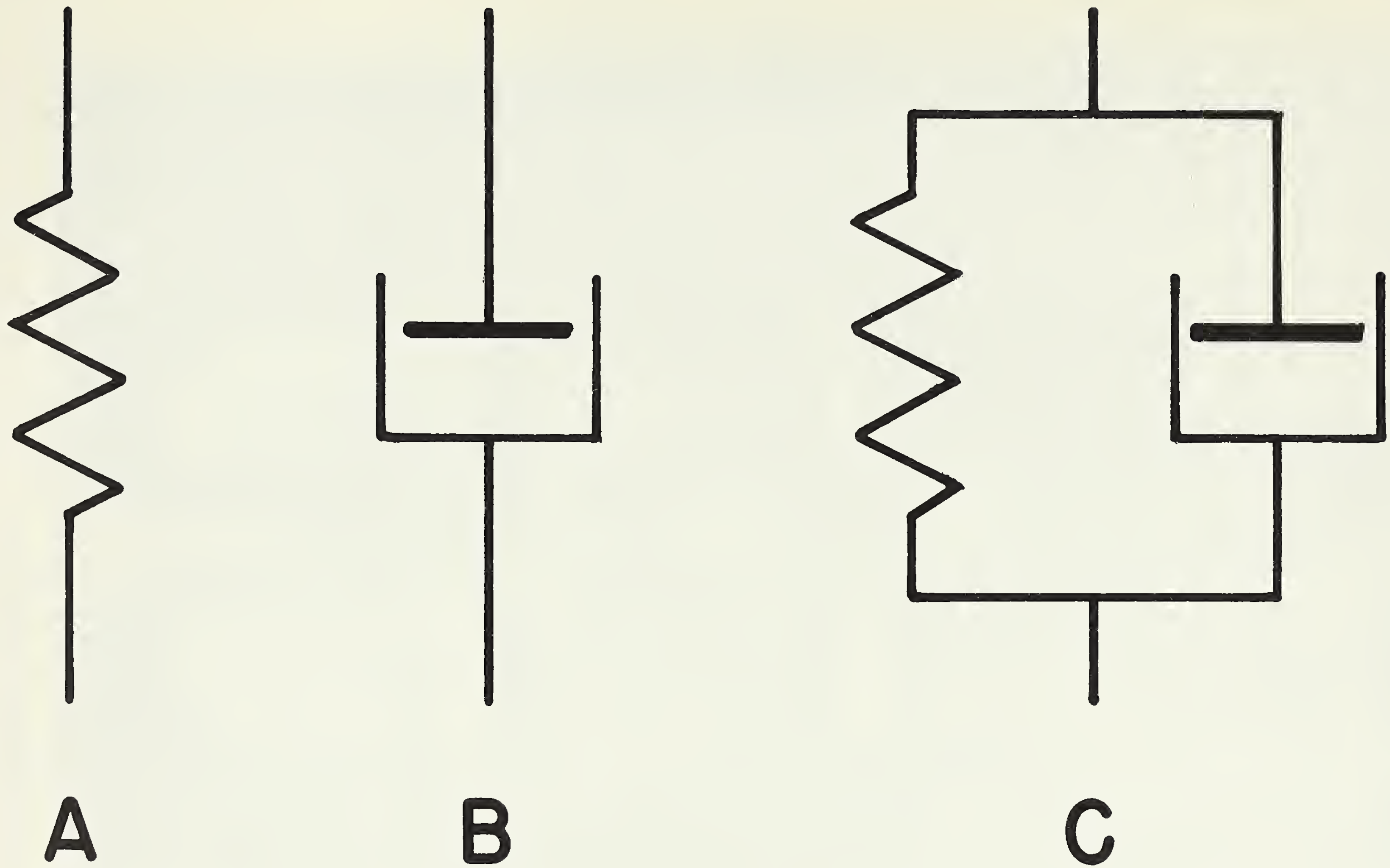

Figure 1. Models representing types of viscoelastic behavior: A-spring-elastic behavior, B- dashpot - viscous behavior, C-Voigt element - retarded elastic behavior.

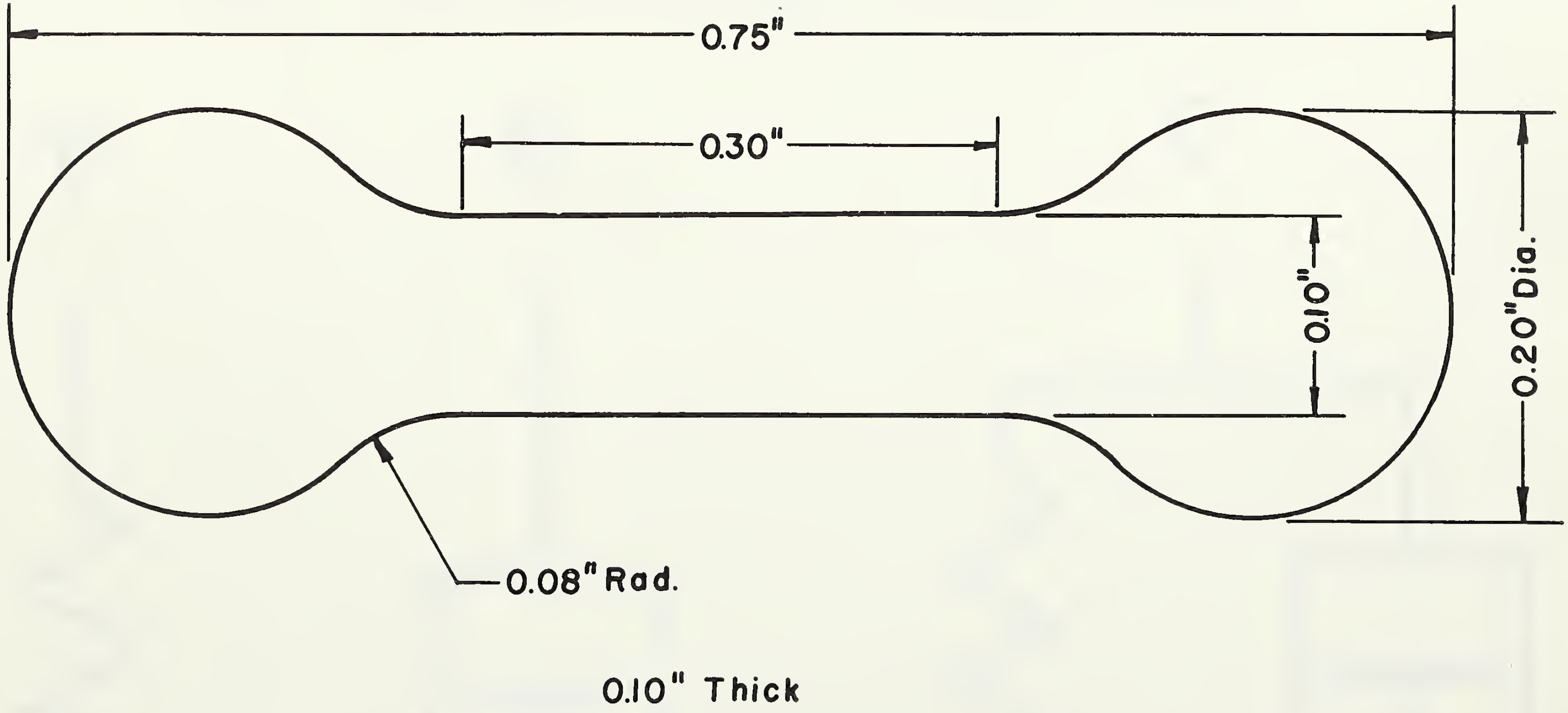

Figure 2. Dimensions of the dumbbell-shaped tensile specimen of amalgam.

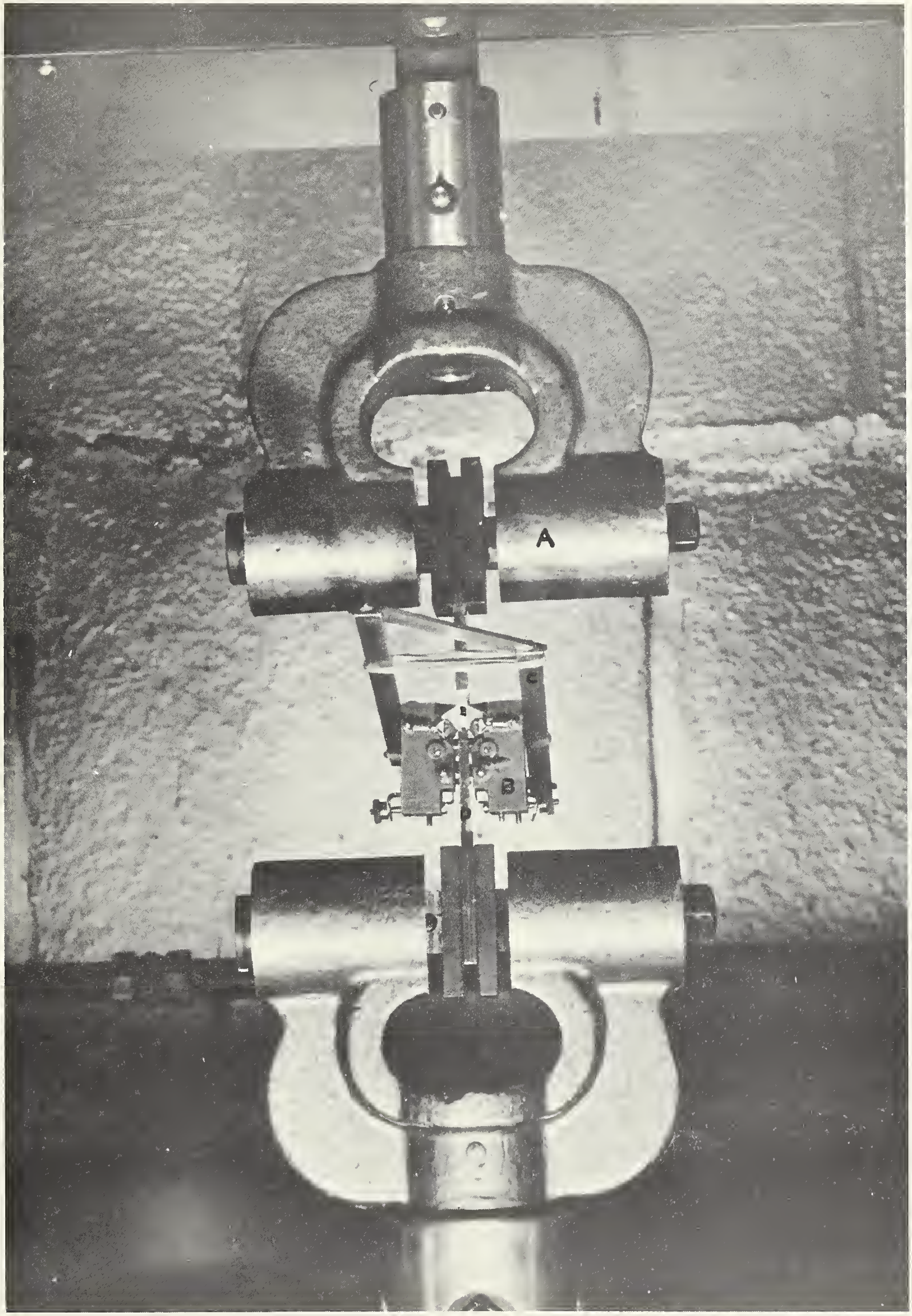

Figure 3. Tensile specimen in position for load application with optical strain gages mounted on opposite sides of the specimen.

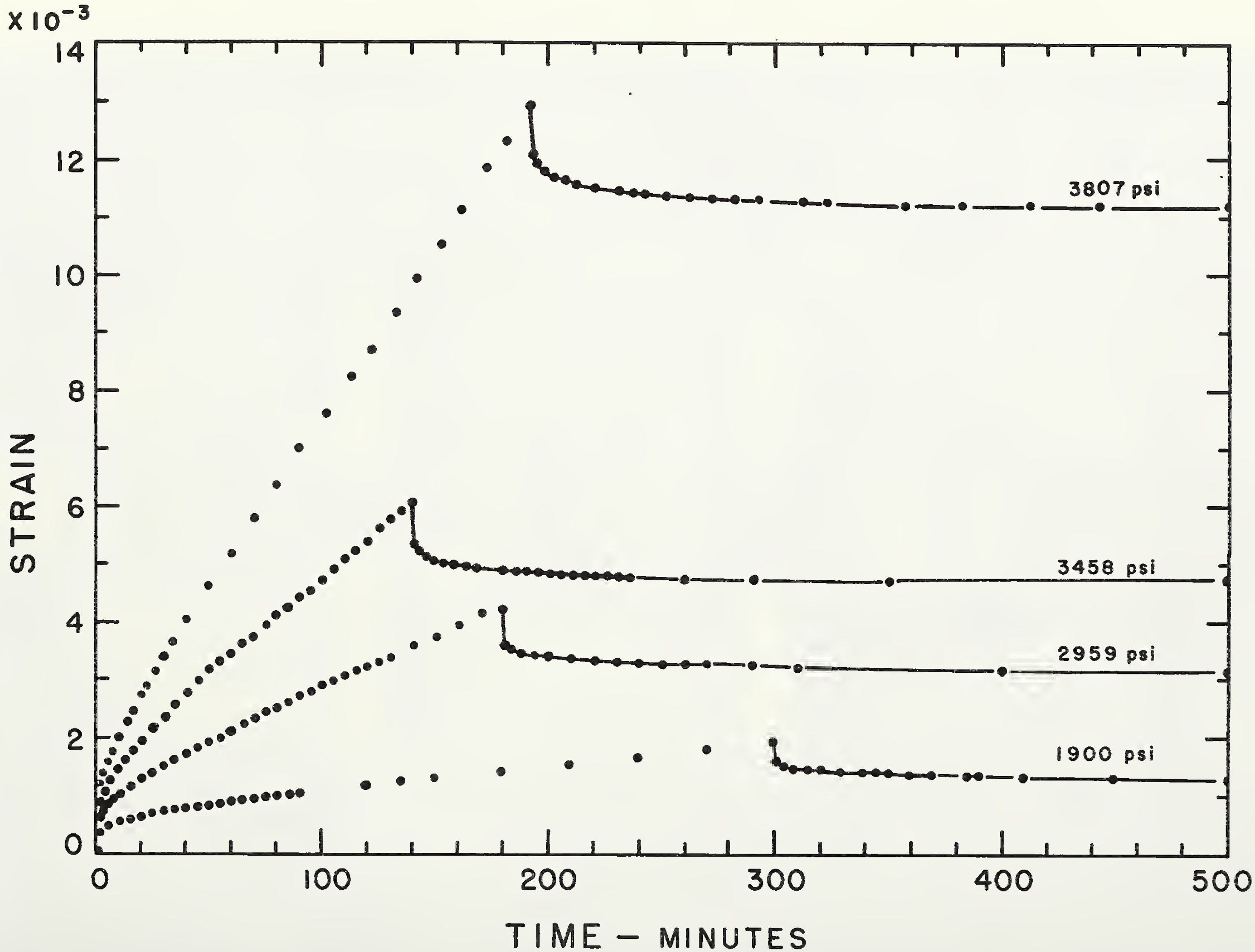

Figure 4. Creep and recovery of amalgam loaded in tension.

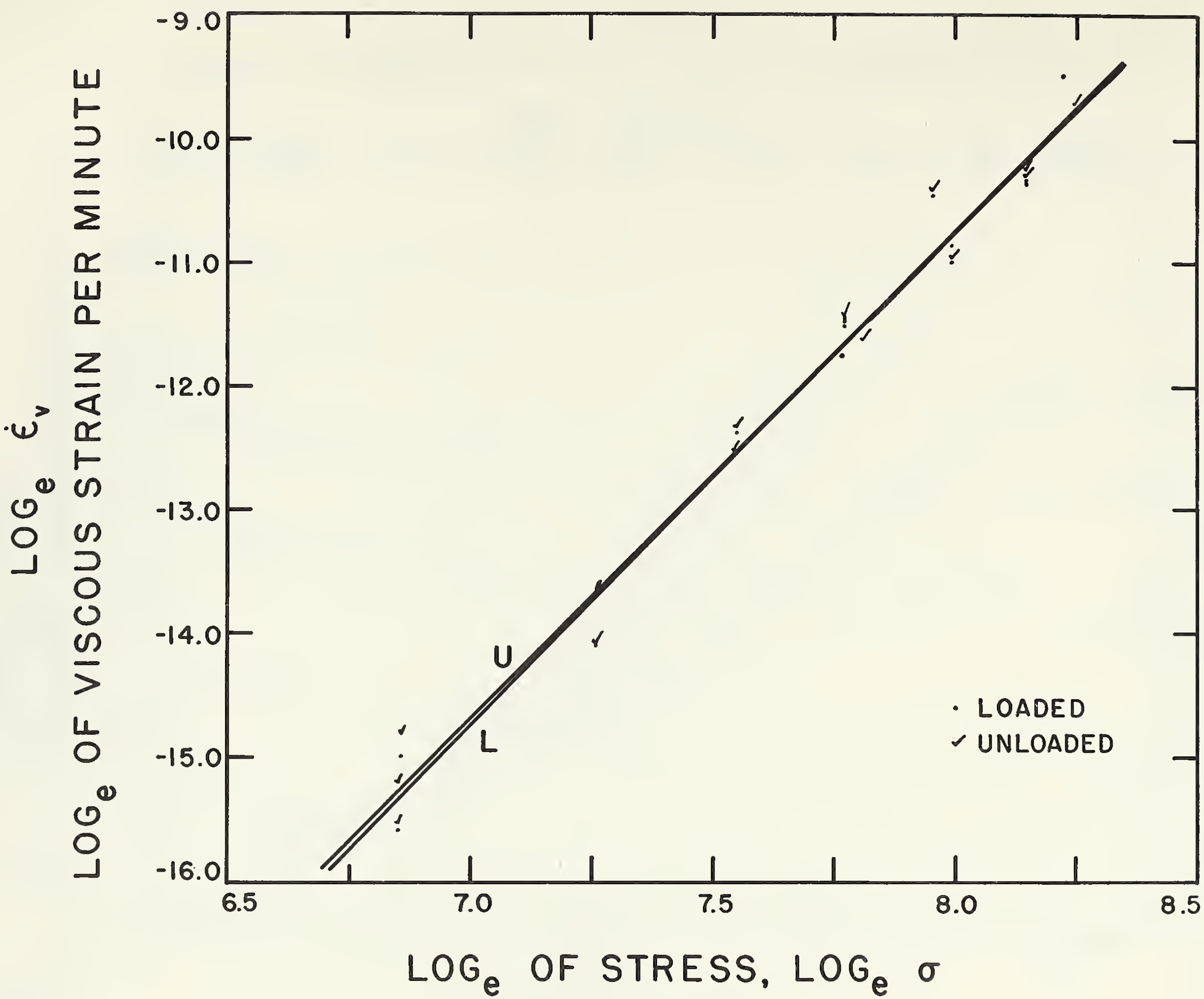

Figure 5. Relationships between viscous strain rate and stress. Straight lines are least-squares fits of equation $\log \dot{\epsilon}_v = \log K + m \log \sigma$ to the data. Loaded values are from straight portion of creep curve; unloaded values are from strain remaining in specimen after unloading and recovery.

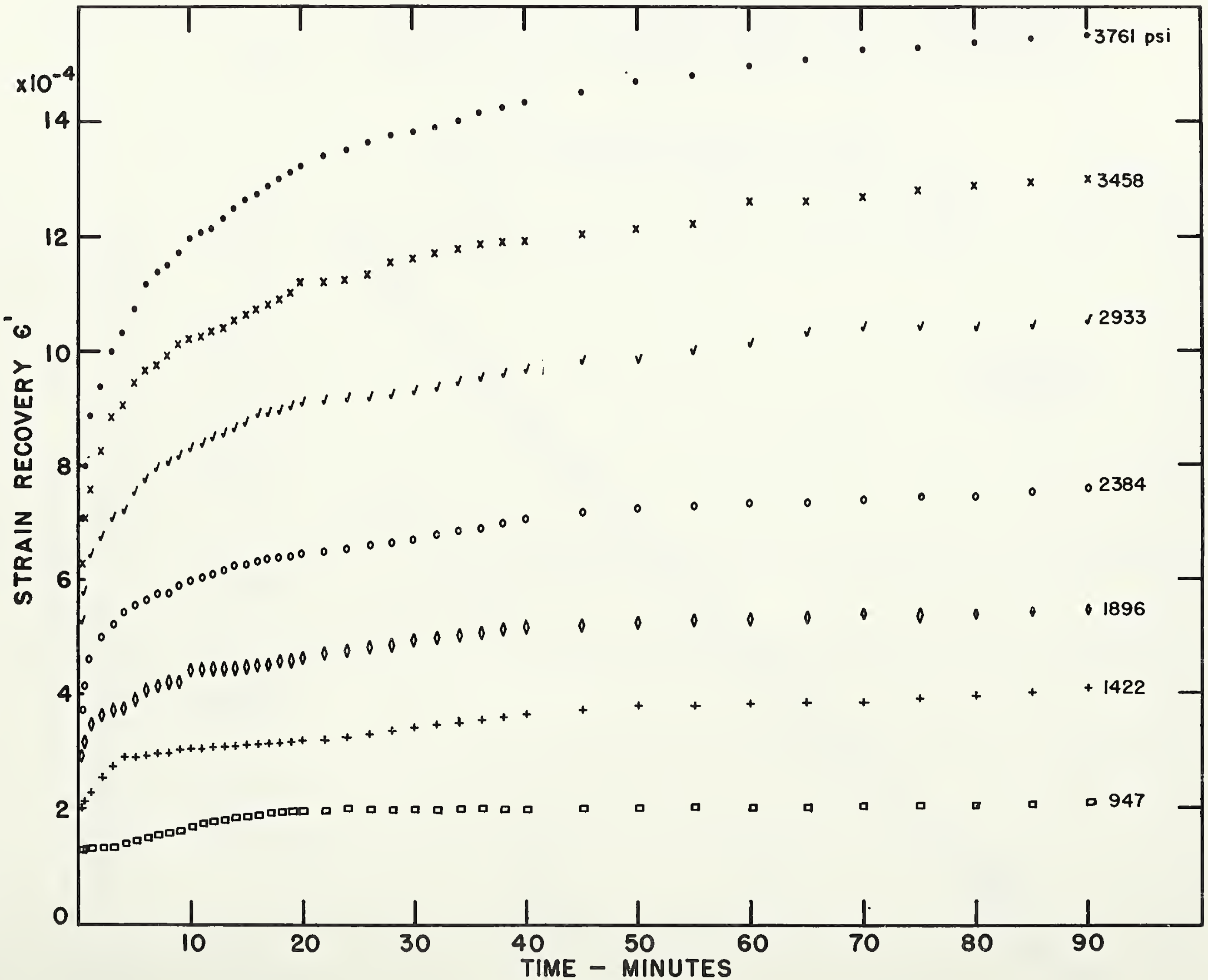

Figure 6. Strain recovery after release of tensile stress. Each curve is an average of 2 to 7 runs.

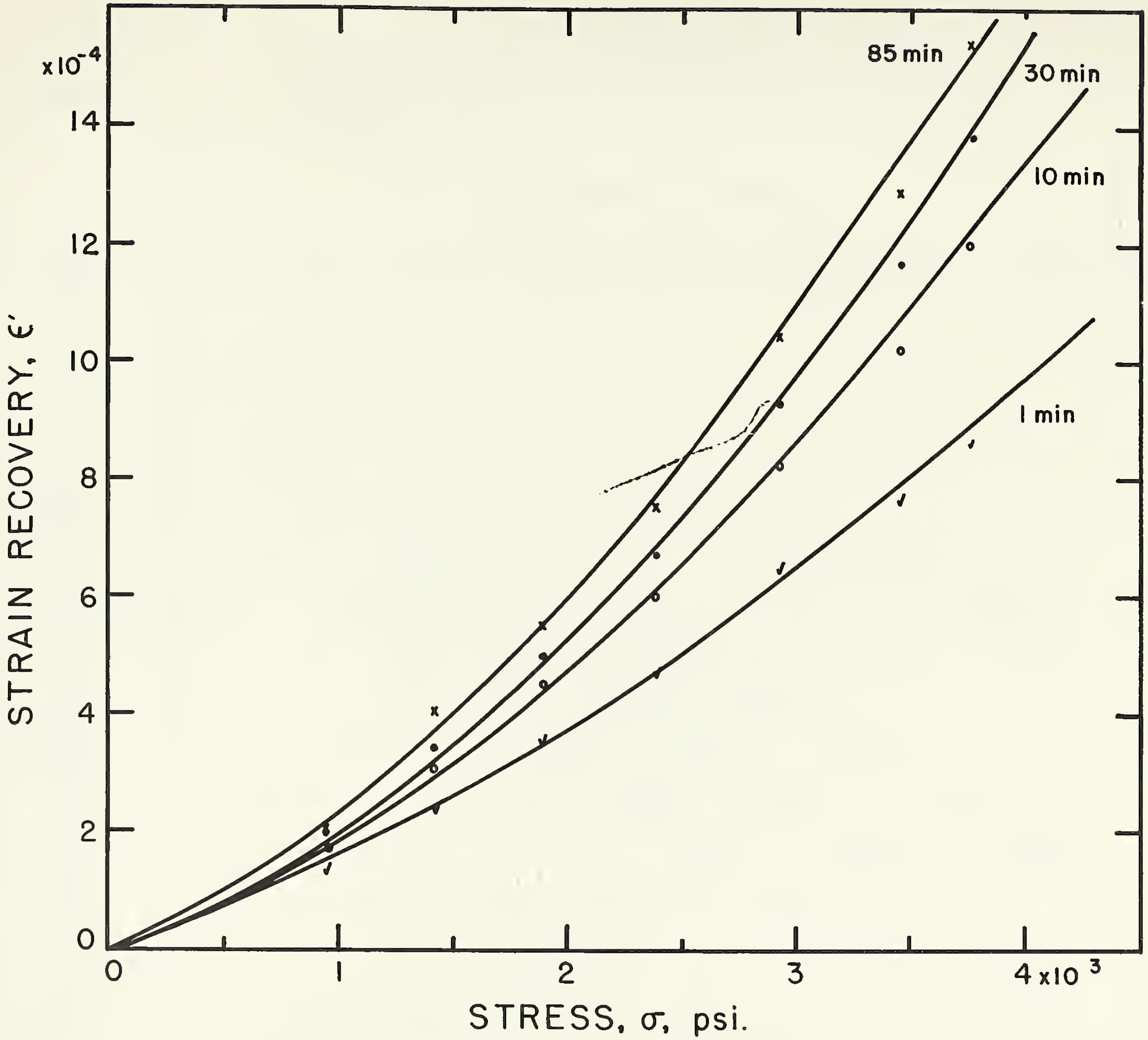

Figure 7. Relationships between recovery strain, stress and time. Plotted points are averages of 2 to 7 determinations. Curves are least-squares fits of the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$ to the data.

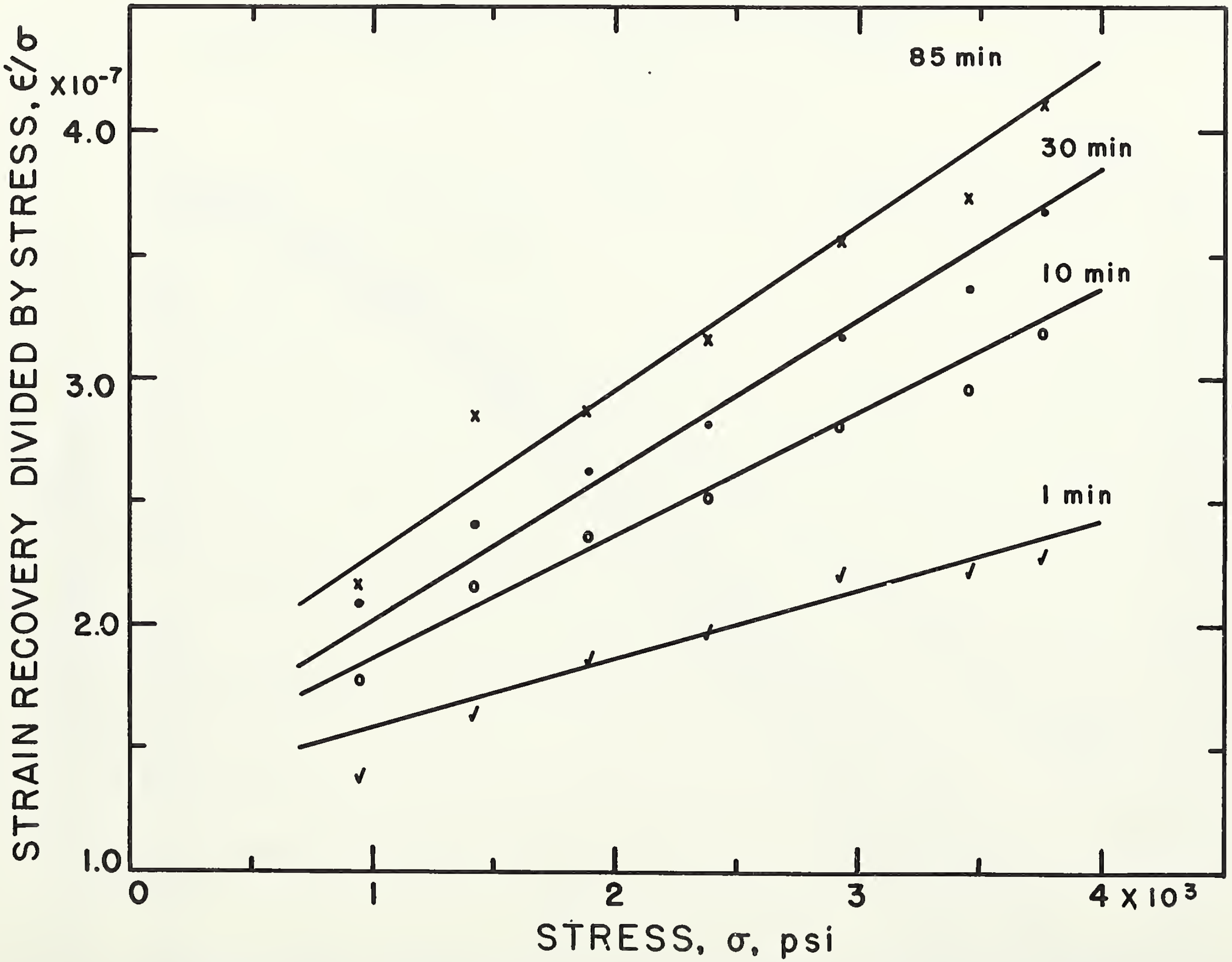

Figure 8. Relationships between recovery strain, stress and time. Plotted points are averages of 2 to 7 determinations. Straight lines represent the constants obtained by least-squares fit of the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$ to the data.

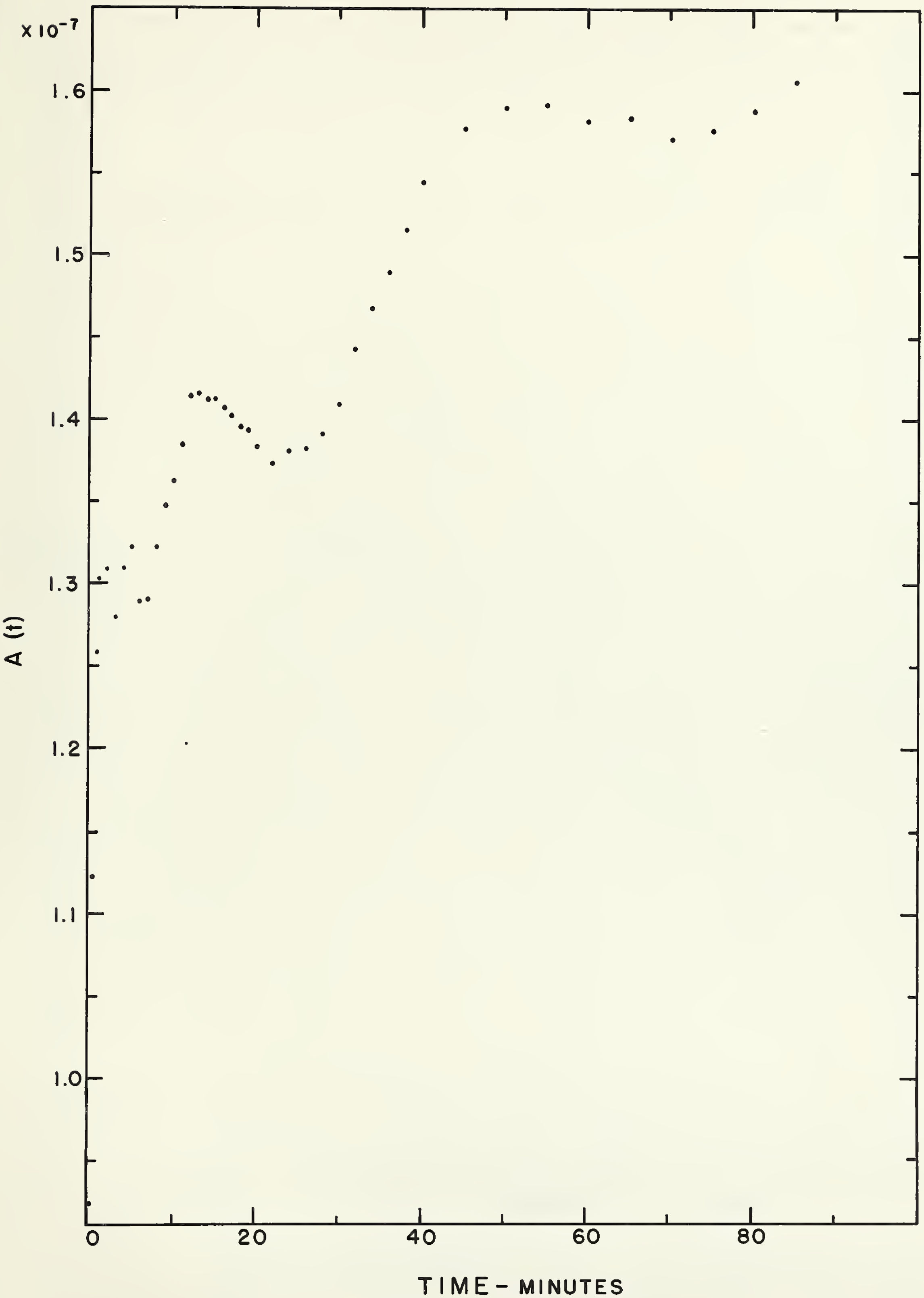

Figure 9. Variation with time of the linear creep compliance term, A(t), in the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$.

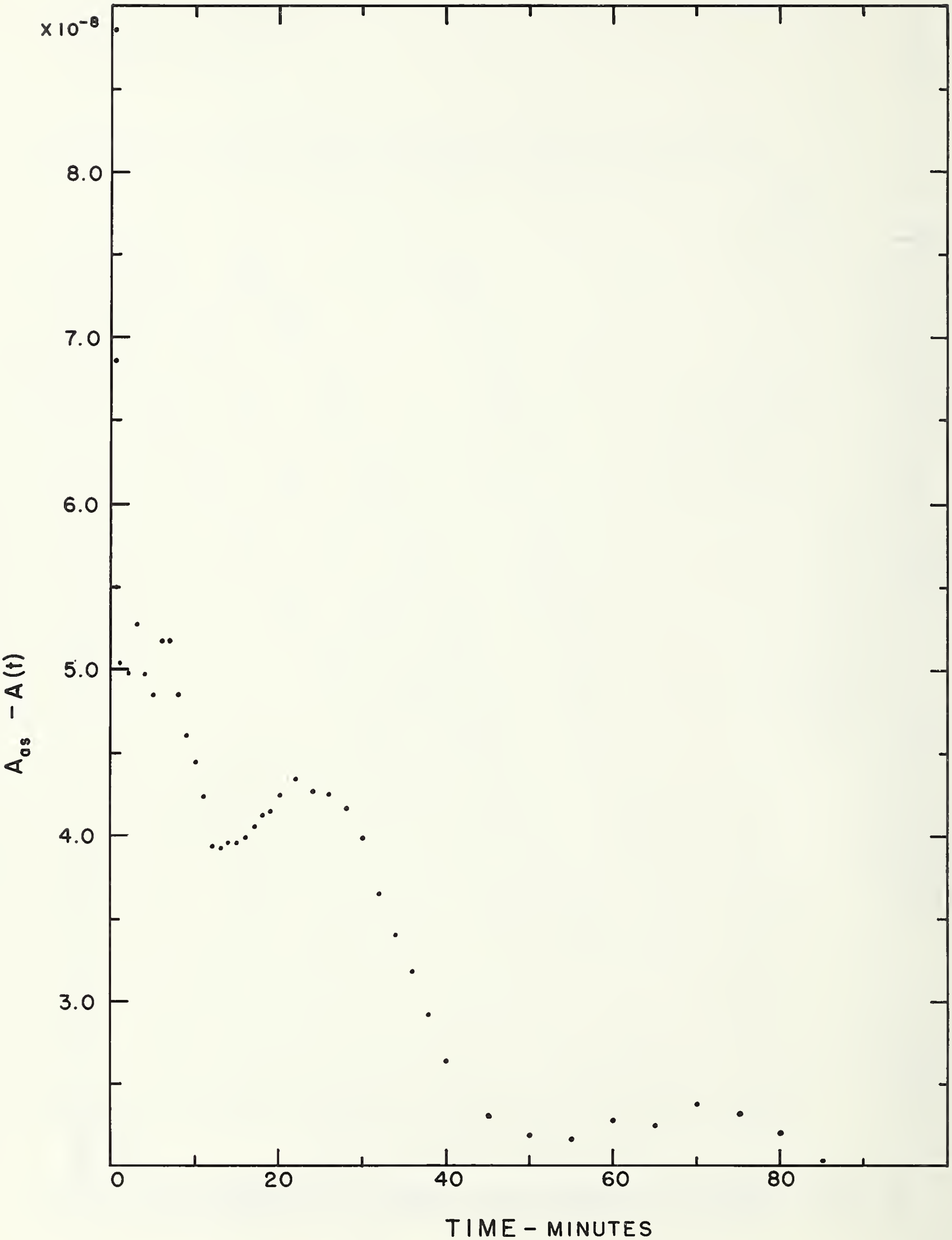

Figure 10. Variation with time of $(A_{as}-A(t))$ where A(t) is the linear creep compliance term in the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$ and A_{as} is the asymptote value of A(t) as t becomes large.

Figure 11. Variation with log of time of the linear creep compliance term, A(t), in the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$.

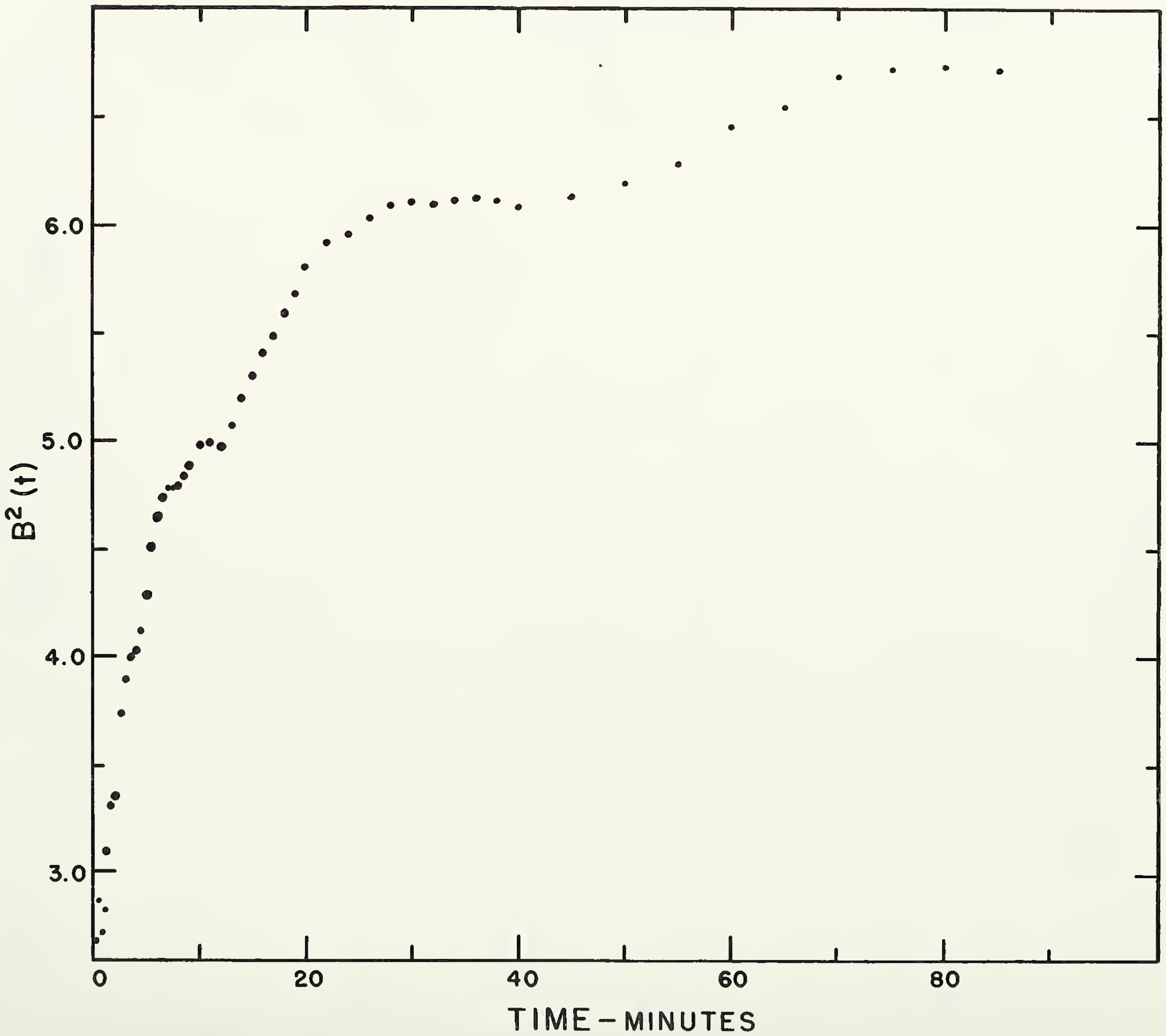

Figure 12. Variation with time of nonlinear creep compliance term, $B^2(t)$, in the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$.

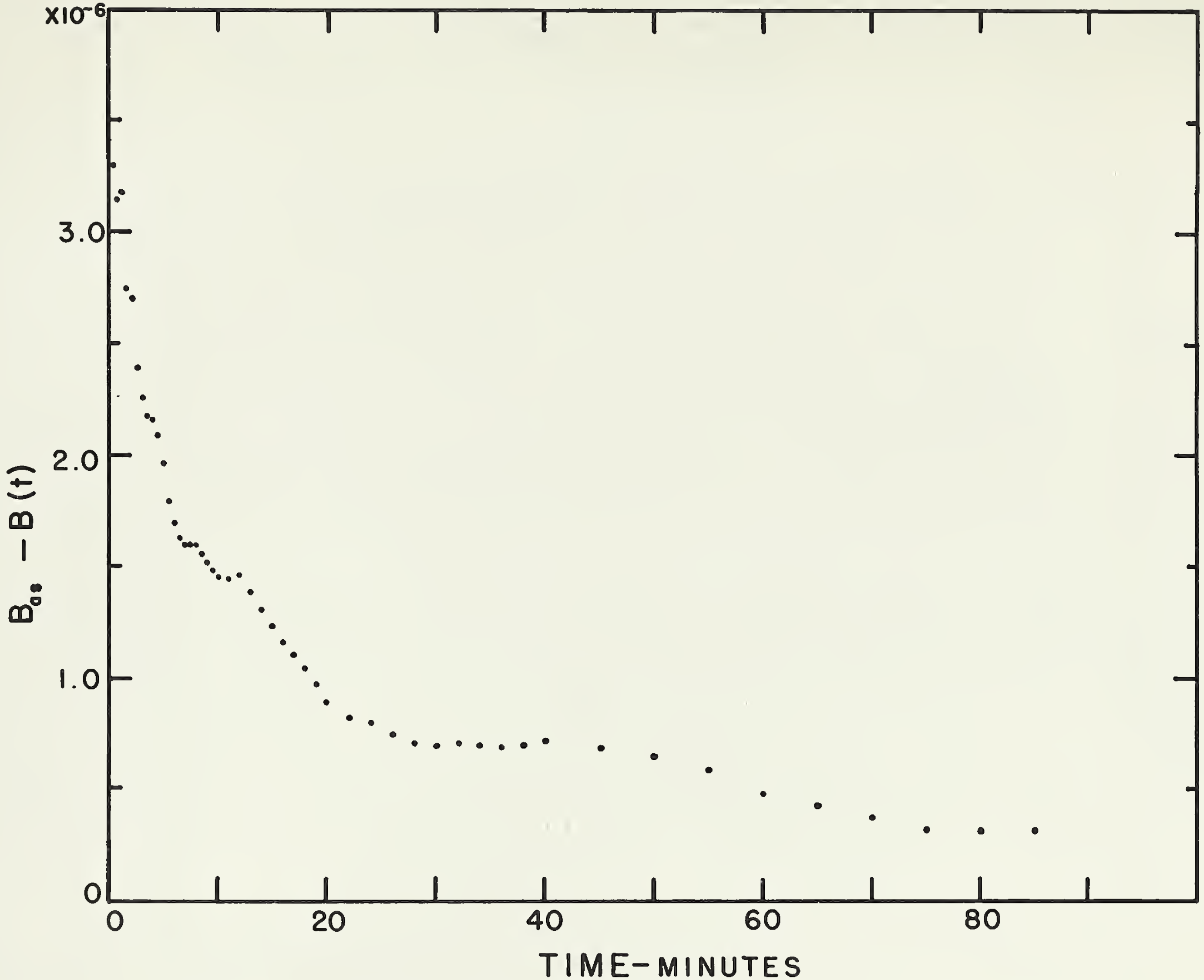

Figure 13. Variation with time of $\left(B_{as}-B(t)\right)$ where $B^2(t)$ is the nonlinear creep compliance term in the equation $\epsilon' = A(t)\sigma + B^2(t)\sigma^2$ and B_{as} is the asymptote value of $B(t)$ as t becomes large.

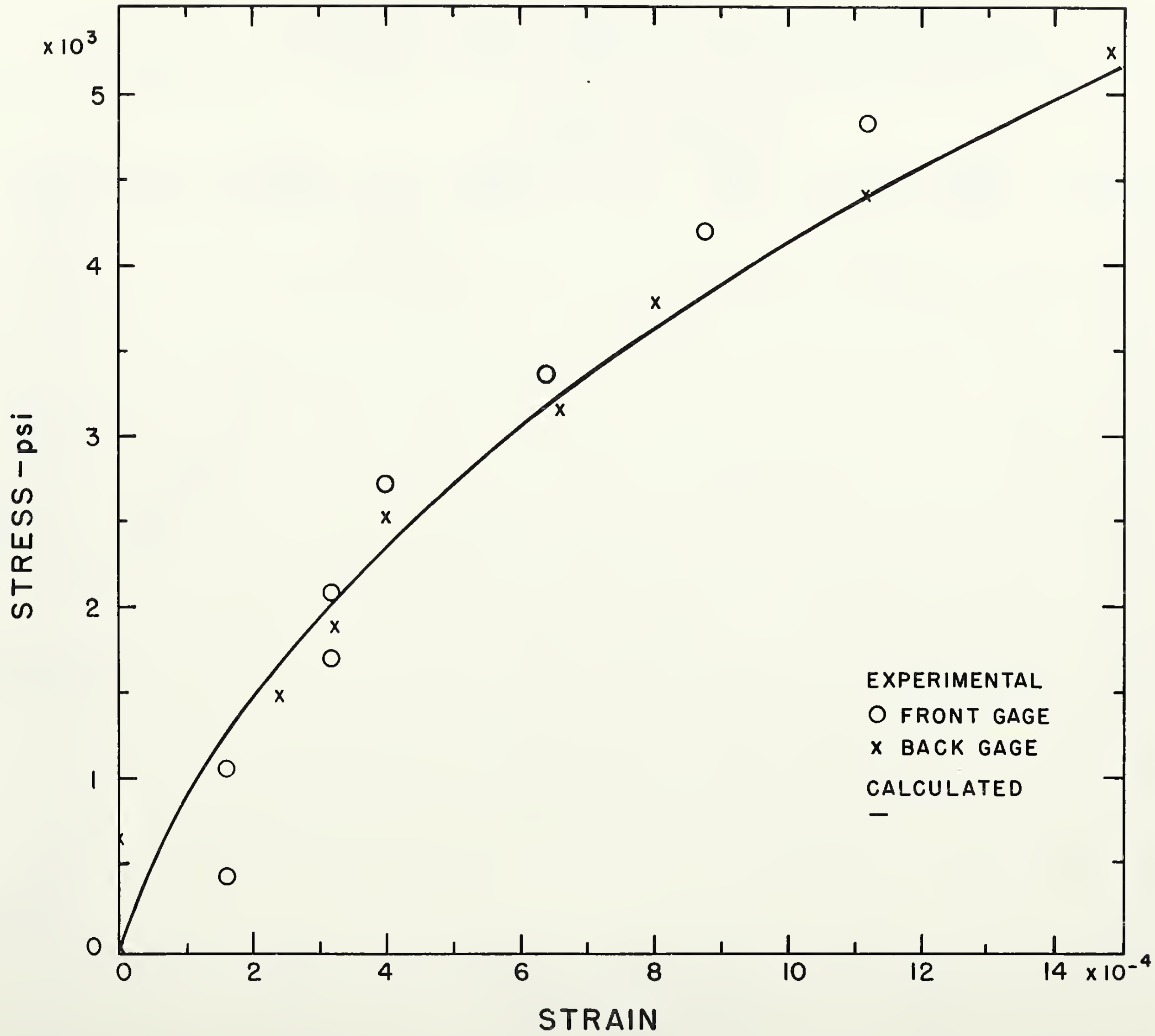

Figure 14. Comparison of calculated and experimental stress-strain curves.