NATIONAL BUREAU OF STANDARDS REPORT
8341

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Semiannual Report

July through December 1963

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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* NBS Group, Joint Institute for Laboratory Astrophysics at the University of Colorado.
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NBS PROJECT
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* Part time — ** Postdoctoral Resident Research Associate — *** Guest worker
° On leave of absence — °° Temporary appointment — °°° Student trainee

Contents

° Only unclassified material is included in this report.

Status of Projects

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: Morris Newman
Full task description: July - September 1954 issue, p. 1

Status: CONTINUED. M. Newman has continued his studies of the modular group Γ. He has proved that if G is a proper subgroup of Γ containing

$$\left\{\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, \Gamma(n)\right\}$$

then $G \subset \Gamma_0(d)$, $d|n$, $d > 1$. This result was used by M. Knopp and M. Newman to prove that if G is a free congruence group of level n, where $(n, 2.3.5.7.13) = 1$, then G is of positive genus.

A study of subgroups of F-groups and characters on such groups has been initiated by M. Knopp, J. Lehner and M. Newman.

Bounds for class numbers of algebraic number fields improving those in the literature have been given by M. Newman. It is shown for example that if p is a prime $\equiv 1 \pmod 4$ then the class number of $R(\sqrt{p})$ is less than $2\sqrt{p}/3$.

Asymptotoc formulas for the number of lattice points in a 3-dimensional sphere have been given by M. Bleicher (U. of Wisconsin) and M. Knopp.

K. Goldberg has considered the problem of determining the sums of distinct permutation matrices which can be made symmetric through left multiplication by a permutation matrix, and for which the multiplier can be chosen to be summetric. The problem has direct application to the construction of Hadamard matrices.

K. Goldberg has continued his investigation of the powers of the iterates of a formal power series.

S. Haber continued his study of a modified Mont-Carlo numerical quadrature method which he had proposed earlier. As applied to integration over the unit n-cube, it consists of subdividing the cube into a large numer of subcubes, choosing a single point at random in each, and averaging the functional values at the points chosen. He showed that the variance of this estimate is always less than that of simple Monte-Carlo quadrature, and that for differentiable functions the standard deviation is asymptotic to $KN^{-\frac{1}{2}-\frac{1}{n}}$, where N is the number of points chosen and K is the L^2 mean of the gradient of the integrand. He also began some numerical experiments with the method.

F. Gross has completed a paper (with Professor E. G. Straus) on exponentials of entire functions. He is now investigating entire solutions to certain functional equations. He is also studying rates of growth of entire functions of one and of several complex variables.

R. Miech is continuing his study of problems in prime number theory.

F. W. J. Olver is continuing his work in aymptotic expansions under Task 1101-11-11421/63

Publications:

(1) On a theorem concerning existence of interpolating functions. R. F. DeMar. Submitted to the Transactions of the American Mathematical Society.
(2) A uniqueness theorem. R. F. DeMar. To be published in Proceedings of the American Mathematical Society.

(3) A note on some quadrature formulas for the interval $(-\infty,\infty)$. S. Haber. To appear in Mathematics of Computation.
(4) Matrix rational completions satisfying generalized incidence equations. E. C. Johnsen. Submitted to a technical journal.
(5) The inverse multiplier for Abelian group difference sets. E. C. Johnsen. To appear in the Canadian Journal of Mathematics.
(6) Weierstrass points of $\Gamma_0(n)$. J. Lehner and M. Newman. To appear in Annals of Mathematics.
(7) Almost primes generated by a polynomial. R. Miech. To appear in Acta Arithmetica.
(8) Symplectic modulary groups. M. Newman and J. R. Smart. To appear in Acta Arithmetica.
(9) A complete description of the normal subgroups of genus one of the modular group. M. Newman. To appear in American Journal of Mathematics.
(10) Normal congurence subgroups of the modular group. M. Newman. American Journal of Mathematics, vol. 85, pp. 419-427 (1963).
(11) Note on the partition function. M. Newman. To appear in American Mathematical Montly.
(12) Free subgrups and normal subgroups of the modular group. M. Newman. To appear in Illinois Journal of Mathematics.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS — Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: Morris Newman
Full task description: July-September 1954 issue, p. 5

Status: INACTIVE. All manuscripts which had not been published when this project was rendered inactive have been transferred to Task 1101-12-11110/55-55.

ASYMPTOTIC EXPANSIONS
Task 1101-11-11421/63

Origin: NBS — Authorized 9/10/63
Sponsor: U. S. Army Research Office, Durham, N.C.
Manager: F. W. J. Olver
Objective: To study the behavior of solutions of differential equations near singular points.

Status: NEW. F.W.J. Olver has been investigating the asymptotic expansions of solutions of ordinary second-order differential equations in the neighborhood of an irregular singularity. In the case of singularities of rank one, new error bounds have been obtained; they are expressed in terms of the variation of the first neglected term of each expansion. Generalizations are now being studied.

Publications:

(1) Error bounds for first approximations in turning-point problems. F. W. J. Olver. Appeared in the Journal of the Society for Industrial and Applied Mathematics, 1963, vol. 11, 748-772.
(2) Error bounds for asymptotic expansions in turning-point problems. F. W. J. Olver. To appear in Journal of the Society for Industrial and Applied Mathematics.
(3) Error analysis of Miller's recurrence algorithm. F. W. J. Olver. Will appear in the issue of Mathematics of Computation of January 1964, vol. 18.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

AUTOMATIC CODING
Task 1102-12-11120/55-0065

Origin: NBS — Authorized 9/29/54
Manager: P. Walsh
Full task description: July - September 1954 issue, p. 11

Status: REACTIVATED. The staff is examining time-sharing systems and the use of computers through remote stations. Several time-sharing systems have been developed throughout the country and steps have been taken to determine their applicability to NBS needs.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11121/57-216

Origin and Sponsor: National Science Foundation — Authorized 12/27/56
Manager: Irene A. Stegun
Full task description: October - December 1956 issue, p. 10

Status: CONTINUED. In press.

CURRENT RESEARCH IN THE COMPUTATION LABORATORY
Task 1102-12-11122/63-1999

Origin and Sponsor: NBS, Section 11.2 — Authorized 8/18/63
Manager: Don I. Mittleman
Objective: To conduct research in various phases of mathematics as they relate to computation.

Background: As a consequence of the service activities of the members of the staff, a number of problems, tangential to meeting the specific needs of a sponsor, arise. These problems have intrinsic value of their own and their solutions contribute to improved understanding of the nature of computational, mathematical techniques as well as improved performance of machine computation.

Status: NEW. Investigations are continuing in the theory of linear ordinary equations in the neighborhood of irregular singular points.

3. PROBABILITY AND MATHEMATICAL STATISTICS

RESEARCH IN PROBABILITY AND MATHEMATICAL STATISTICS
Task 1103-12-11131/63-1259

Origin: NBS Authorized 10/1/62
Manager: Joan Raup Rosenblatt
Full task description: July - December 1962

Status: CONTINUED. J. M. Cameron has written notes entitled "Effect of change of restraint on the inverse of the matrix of normal equations" and "Incorporating restraints in least squares computations", the third and forth of a series of notes on computational problems associated with the solution of the normal equations for estimation in a linear model when the parameters of the model must obey linear restraints.

W. J. Youden worked on a collection of special designs suitable for calibration work. Experiments on electrical circuit analogs of these designs are being performed by Francis Hermach of the Electrical Instruments Section.

A. Bruce Hoadley prepared two related working papers entitled "Comparison of the method of group averages and the method of least squares for fitting a straight line when both variables are subject to error", and "A note on the estimation of error variance in the functional relationship model". A paper summarizing the results of these investigations is being prepared for publication.

H. H. Ku is preparing for publication a collection of propagation-of-error formulas.

Roy H. Wampler is continuing the study initiated by Churchill Eisenhart, with the collaboration of Ann D. Smith and John Van Dyke, on the distribution of tolerance interval coverages in sampling from a normal distribution.

Janace Speckman and Richard G. Cornell (Florida State Univ.) have completed their joint paper on "Estimation for a one-parameter exponential model".

George Weiss has carried further his investigation of mathematical models for pedestrian queueing, and has completed a paper on "The effects of a distribution of gap acceptance functions on pedestrian queues". Two additional papers completed by George Weiss are "The calculation of certain multiple generating functions" and "A simple derivation of the Faxén solution to the Lamm equation". In collaboration with E. W. Montroll (Institute for Defense Analyses), George Weiss initiated computation of tables of the two-dimensional Green's function for the discrete analogue of the Laplace operator

$$J(m,n;\alpha,\beta) = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{\cos mx \cos ny \, dxdy}{(1+\alpha)\,\beta - \alpha \cos x - \cos y}$$

for $m,n = 0(1)5$, $\alpha = 1, 2, 4, 8,$ and $\mu = \beta^{-1} = .01(.01).99$. These tables will be useful in a variety of problems, including random walk problems, solid state problems, and statistical estimation problems in two dimensions.

Brian L. Joiner is studying the comparison of the Normal and Weibull distributions, according to the criterion

$$D(p) = \min_{\mu,\sigma} \max_{x} \left| N(x;\mu,\sigma) - W(x;p) \right|$$

where

$$W(x;p) = \begin{cases} 0 & x < 0 \\ 1 - \exp\{-x^p\} & x \geq 0 \end{cases}, \quad p > 0,$$

$$N(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{ -\tfrac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2 \right\} dt,$$

$$-\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$

$D(p)$ has a minumum near $p = 3.06232$. For fixed p, the minimizing mean $\mu(p)$ and standard deviation $\sigma(p)$ of the normal distribution are compared with the mean, median, standard deviation, and other characteristics of the Weibull distribution $W(x;p)$.

Brian L. Joiner has taken up the investigation of the random variable defined by $z = Ax^{\alpha} - Bx^{\beta} + C$, where x has uniform distribution on the unit interval. Computations are being made to permit comparison of the percentage points and moments of the distribution of z with those of tabulated distributions.

Publications:

(1) Realistic evaluation of the precision and accuracy of instrument calibration systems. Churchill Eisenhart. Proceedings National Conference of Standards Laboratories, held at NBS Boulder Laboratories, Aug. 8-10, 1962, NBS Misc. Publication 248, pages 63-89, August 1963.

(2) A note on contingency tables involving zero frequencies and the $2\hat{I}$ test. H. H. Ku. Technometrics 5, 398-400, August 1963.

(3) Experimental Statistics. May G. Natrella. NBS Handbook 91, 1963.

(4) On an extreme rank sum test for outliers. W. A. Thompson, Jr., and T. A. Willke. To appear in Biometrika, December 1963.

(5) An analysis of pedestrian queueing. George Weiss. Journal of Research NBS-B. (Mathematics and Mathematical Physics), 67B, 229-243, Oct.-Dec., 1963.

(6) A note on a generalized elliptic integral. George H. Weiss. To appear in Journal of Research NBS-B. (Mathematics and Mathematical Physics).

(7) The calculation of certain multiple generating functions. George H. Weiss. To appear in Journal of Research NBS-B. (Mathematics and Mathematical Physics).

(8) The effects of a distribution of gap acceptance functions on pedestrian queues. George Weiss. To appear in Journal of Research NBS-B. (Mathematics and Mathematical Physics).

(9) A simple derivation of the Faxén solution to the Lamm equation. George Weiss. To appear in Journal of Math. Physics.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: Joan Raup Rosenblatt
Full task description: January-March 1956 issue, p. 13

Status: CONTINUED. Albert Romano completed a working paper summarizing the results of his investigation of models for the statistical analysis of transistor aging experiments.

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. Dr. Barry Bernstein is continuing his work with E. A. Kearsley and L. Zapas, of Section 6.05, which consists of a study aimed at a description of the mechanical response of polymers. Experimental work in 6.05 has been continuously conducted over the last year in which materials were subjected to simple extension histories suggested by the formula proposed by Bernstein, Kearsley, and Zapas. (See (1) below.) Recent experiments have tended to confirm the appropriateness of this formula, and some schemes have been considered for computing the response of these materials. It is expected that computational work will begin soon.

Preliminary theoretical work on the thermodynamics of viscoelastic materials has shown promise of leading to the formulation of a proper thermodynamical theory.

Dr. L. E. Payne and J. H. Bramble are continuing their intensive study of the problem of determining error bounds of a priori type for the solutions of boundary value problems in classical elasticity. Their current investigation is concerned with obtaining pointwise bounds for elastic plates with mixed boundary conditions. A man uscript is in preparation.

The study of Drs. J. P. Vinti and R. M. Langer on the measurement of drag on a manned satellite of the earth ahs been concluded.

J. E. Lagnese has been investigating the existence of second order, linear partial differential operators of hyperbolic type which satisfy Huygens' principle. In the self-adjoint case, a large class of such operators has been found; this class includes, as a special case, the set of all previously known Huygens' operators. Several important properties of these operators have been found, in particular, properties relating to their fundamental solutions.

For operators with constant principal part and coefficients depending on only one of the independent variables, the use of Fourier transforms enables one to reduce the question of Huygens' principle to a consideration of special properties of solutions of second order ordinary differential equations containing a parameter.

A manuscript dealing with a portion of these results has been prepared.

The system of differential equations

$$\frac{dx}{dt} = x(x + y + 1) \qquad \frac{dy}{dt} = y(ax + by + c) \tag{1}$$

is encountered in the thory of oscillations, astrophysics, and chemical kinetics, and has been studied by N. N. Serebriakova [Pryklad. Math. Mech., vol. 27, no. 1, 1963]. Dr. A. Ghaffari has obtained new results concerning the behavior of solutions of the system at infinity, and showed that if the parameters a, $b \neq 1$, then the equator is an integral curve of (1). Using Bendixson's criterion, Dr. Ghaffari has demonstrated the non-existence of limit cycles of (1).

The discussion of the second-order equation

$$\frac{d^2x}{dt^2} + f(x) = 0, \qquad x(0) = c_1, \qquad \frac{dx(0)}{dt} = c_2, \tag{2}$$

in the phase plane is classic, see J. J. Stoker, Non-linear Vibrations, 1950. Application of

standard methods leads to the associated equation

$$\left(\frac{dx}{dt}\right)^2 + F(x) = C\,, \qquad x(0) = c_1 \tag{3}$$

where $F'(x) = 2f(x)$, $C = c_2^2 + F(c_1)$. Hochstadt [Am. Math. Monthly 70, 1086-87 (1963)] has pointed out by considering the case $f(x) = x$ that it can happen that (2) may have unique solutions, whereas (3) does not. He obtained a criterion which will insure that a solution of (3) will also satisfy (2). Dr. Ghaffari has discussed the case $f(x) = 2x^2(1 - x)$, and showed that for suitable choice of C, a solution may be obtained which satisfies Hochstadt's criterion.

Publications:

(1) A study of stress relaxation with finite strain. B. Bernstein, E. A. Kearsley, and L. J. Zapas. Transactions of the Society of Rheology VII. 391-410 (1963).

(2) Pointwise bounds in the first biharmonic boundary value problem. L. E. Payne and J. H. Bramble. To appear in the Journal of Mathematics and Physics.

(3) Error bounds in the pointwise approximation of solutions of elastic plate problems. J. H. Bramble and L. E. Payne. Jounal of Research NBS 67B, 157-168 (1963).

(4) Drag compensation and measurement with manned satellites: feasibility study. R. M. Langer and J. P. Vinti. Journal of Research NBS 67C, 247-249 (1963).

(5) On Rayleigh's non-linear vibration equation. A. Ghaffari. To appear in the Proceedings of the International Symposium on Non-linear Vibrations, sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 16-18, 1961.

THEORY OF SATELLITE ORBITS
Task 1104-12-11441/62-1166

Origin: NBS — Authorized 1/9/62
Sponsor: National Aeronautics and Space Administration
Manager: J. P. Vinti
Full task description: January-March 1962 issue, p. 12

Status: CONTINUED. The analysis of the drag-free motion of an artificial satellite has been continued by J. P. Vinti. The perturbations of the standard Vinti orbit by the third harmonic have now been derived, along with those produced by the residual fourth harmonic. These derivations, utilizing an adaptation of the von Zeipel method, hold good whenever the inclination I is sufficiently far removed from the critical values 63.4° and 116.6°.

Work is now in progress on the case where I is close to either of these critical values, with two purposes. The first is to incorporate the solution for the critical regions into the general algorithm for calculating an orbit by the spheroidal method, in such a way that the critical solution will merge into the previous solution at the boundaries $63.4^\circ \pm \epsilon$ and $116.6^\circ \pm \epsilon$, where ϵ is still to be specified. The second is to attempt to make the critical inclinations physically reasonable.

Publications:

(1) The spheroidal method for satellite orbits. J. P. Vinti. Proceedings of the First International Symposium on the Use of Artificial Satellites for Geodesy, North Holland Publishing Co. Amsterdam. pp. 12-16 (1963).

(2) Zonal harmonic perturbations of an accurate reference orbit of an artificial satellite. J. P. Vinti. J. Research NBS. 67B, No. 4, pp. 191-222 (1963).

(3) Drag compensation and measurement with manned satellites: feasibility study. R. M. Langer and J. P. Vinti. J. Research NBS 67C, No. 3, pp. 247-249 (1963).

PLASMA RESEARCH
Tsk 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: April-June 1959 issue, p. 15

Status: CONTINUED. Research by Dr. C. M. Tchen has been directed toward the following problems:
(1) Statistical theory of hydromagnetic turbulence. An analytic formulation of the mechanism of the non-linear transfer of energy between the eddies and the spectral distribution of the kinetic and magnetic energies in the wave-number space has been made. It is assumed that the conducting fluid is incompressible, and the Heisenberg hypothesis for the non-conducting fluid was generalized to include the effect of the magnetic field. A report on this is in preparation.

An attempt has been made to construct a molecular theory for turbulence in a non-conducting field. This has been helpful in achieving a better understanding of the non-linear transfer mechanism, but needs more elaboration.

(2) Interaction of the solar wind with the geomagnetic field. A derivation has been made of the shape of the boundary between the dipole magnetic field of the earth and the uniform solar plasma. The disturbance in the geometrical configuration of the field lines by the plasma stream was analyzed by an approximate theory. A manuscript if in preparation. Analysis of the non-stationary problem representing the commencement of the solar wind has been attempted, using a one-dimensional model and the method of characteristics.

(3) Interaction of a plasma source with a uniform magnetic field. This is a variation of problem (2). A steady source of plasma is placed in a uniform magnetic field, and the theoretical shape of the boundary between the plasma and the magnetic field is derived. The problem has application to the streaming of the solar wind in a galactic magnetic field. A paper on this has been completed.

DYNAMICS OF PLASMAS
Task 1104-12-11417/62-1157

Origin: NBS
Manager: C. M. Tchen
Full task description: April-June 1959 issue, p. 15

Status: CONTINUED. Work is in progress on the following problems in the kinetics of plasmas:
(1) Kinetic equation for rapidly varying processes in plasmas. The theory of such an equation is based on the BBGKY hierarchy of equations, The usual closure procedure leads to a system of two equations determining the singlet distribution function and the correlation function. The time variation of the correlation function is to play an important role in the theory presently being developed. This gives the effect of a memory in the evolution of the singlet distribution function and it modifies the dielectric property of the medium.

(2) Diffusion of the correlation function in a plasma. With an initial correlation function given, one may ask how it evolves in time and in space before it reaches its asymptotic equilibrium value. This question has been studied, and it is found that the time evolution of the correlation function can be interpreted by a modified diffusion process.

(3) Plasma oscillations with correlations. Here the dispersion relation, the modes of oscillations and the Landau damping have been studied by including the effects of the self-consistent field in the correlation function.

(4) Nonlinear damping and excitation in a plasma. At the present stage of study of this problem the nonlinear Landau damping from the Vlasov equation has been computed. The data are being analyzed by W. L. Sadowski and C. M. Tchen.

Preliminary reports for the problems (1) - (3) have been prepared, and the results are being compared with the results obtained by Dr. Tchen using other methods of attack. In the case of problem (2), it is planned to increase the amplitude and to consider the excitation problem. Dr. K. Hain, on leave from the Institute for Plasma Physics, Munich, Germany, has recently joined Section 11.04 of NBS and will work on this problem.

HYPERVELOCITY IMPACT
Task 1104-12-11418/63-1373

Origin and Sponsor: Applied Physics Laboratory
Johns Hopkins University

Authorized 4/22/63

Manager: Barry Bernstein

Status: CONTINUED. An attempt to calculate the effect of impact of two metal cylinders has been made. The analysis is based on the mathematical model (due to R. L. Bjork, Proc. Xth International Astron. Congress, London, 1959. Springer-Verlag, Wien, pp. 505-514 (1960)) of an inviscid gas with the introduction of an artificial viscosity (J. von Neumann and R. D. Richtmyer, J. App. Phys. 21, 232-237 (1960)) to eliminate the necessity of considering possible discontinuous solutions. It was found that an IBM-STRETCH computer program existed at Los Alamos which could be used for this problem. The problem was run at Los Alamos and the results reported to the sponsor. The results were analyzed and the resolution was found to be somewhat too coarse to give the desired information.

OPERATIONS RESEARCH
Task 1105-12-11115/61-546

Origin and Sponsor: NBS — Authorized 12/30/60
Manager: Alan J. Goldman
Full task description: October-December 1960 issue, p. 3

Status: CONTINUED. The following investigations in various fields of operations research were carried out by members of the staff:

(1) C. Witzgall and A. J. Goldman continued investigating mathematical models of distribution networks. Illustrative results based on Chicago residential density data were obtained. Asymptotic analysis of related functions defined by integrals is in progress.

(2) W. Sillars, J. Levy and A. J. Goldman continued investigations of the effects of buffer capacity in a simple flow system. P. Meyers is analyzing a (previously simulated) stochastic sorting process, and carried out numerical studies on the usefulness of exceptions to the rules in encoding information. D. Kleinman continued work on the computer simulation of a class of mail sorting devices.

(3) L. S. Joel, K. Kloss and J. Levy (primarily) are participating in a Commerce Department analysis of some aspects of the textile industry. (Reported here for convenience; supported under Project No. 30409). A. J. Goldman continued assisting the Interagency Committee on Oceanography in connection with an operations research study.

(4) H. Fell and J. Mather proved that "barely faithful algebras" (in which each element is a right and left zero-divisor, but no non-zero element is a right or left annihilator) exist in all dimensions ≥ 4, but not in lower dimensions. H. Fell and A. J. Goldman improved the existing procedure for imbedding a given algebra A as a left ideal in an algebra B so that every left semi-multiplier of A (linear transformation T obeying $T(xy)=(Tx)y$ is realized as left multiplier of B. A. J. Goldman and D. Kleinman obtained a counter-example to a standing conjecture related to the maximum number of iterations for the simplex method of linear programming. D. Kleinman worked out the relationship between optimal solutions of transportation problems and those of imbedded assignment problems.

(5) P. Meyers is investigating variants of the Banach Contraction Theorem, an important tool in establishing convergence of iterative processes. The emphasis is on the possibility of remetrizing to turn a non-contraction into a contraction.

(6) A. J. Goldman completed collaboration with M. Zelen (now at NIH, National Cancer Institute) on a paper dealing with least squares estimation. C. T. Zahn, Jr. revised his paper on relational approximations to incorporate additional results.

Publications:

(1) Realization of semi-multipliers as multipliers. Harriet Fell and A. J. Goldman. To appear in Am. Math. Monthly. (Math. Notes).

(2) Barely faithful algebras. Harriet Fell and John Mather. Submitted to a technical journal.

(3) Examples relating to the simplex method. A. J. Goldman and Daniel Kleinman. To appear in Operations Research, Vol. 12 (1964).

(4) Approximating symmetric relations by equivalence relations. C. T. Zahn, Jr. Submitted to a technical journal.

COMMERCIAL REFILE PROBLEM DCA
Task 1105-12-11465/63-1494

Origin and Sponsor: Defense Communication Agency — Authorized 6/19/63
Manager: Lambert S. Joel
Full task description: June 19, 1963

Objective: To analyze various instrumentalities of the Defense Communications System and if possible, to determine optimal structure and operational procedures according to appropriately developed cost/effectiveness/ and feasibility criteria.

Background: The DCS processes a large number of messages to, from, and within the military establishment. It is quite desirable to minimize annual costs while maintaining adequate quality and accuracy of service.

Status: NEW. Two problems were investigated. One is that of finding the best set of "relay points" from the military network to commercial communications facilities; a disrete linear programming formulation was obtained, and computational procedures are being developed. The second problem is the rational derivation of "value numbers" for the links of a communication net; a formulation based in part on L. Shapley's analysis of n-person games has been developed and is being explored.

COMBINATORIAL MATHEMATICS
Task 1105-12-11455/62-1205

Origin: NBS — Authorized 5/2/62
Sponsor: Army Research Office-Durham
Manager: Jack Edmonds
Full task description: April-June 1962 issue, p. 15

Status: CONTINUED. C. Witzgall and C. T. Zahn, Jr. prepared their paper on the "predecessor" algorithm for maximum matching in graphs. Zahn, examining in depth the generalized matching theory, (1) analyzed limitations of the predecessor method and (2) simplified the treatment by Edmonds. Witzgall proved the existence of certain alternating paths in certain matched graphs. A. J. Goldman prepared a paper, listed below, on some other results in this area.

J. Mather wrote the first draft of a long paper on the immersion of complexes in Euclidean space. Mather and Zahn constructed counterexamples to a conjecture on minimum boolean formulas.

J. Edmonds, investigating with Witzgall various topics on graphs and combinatorics, obtained many negative results and a few positive ones including (1) minimum normal formulas for symmetric boolean functions, (2) conditions for the existence of k-edge connected graphs with prescribed degrees, (3) an efficient algorithm for deciding isomorphism of trees and of series-parallel networks, (4) with C. Y. Chao (U. of Pennsylvania), a construction for graphs with prescribed automorphisms, (5) a simplification in inter-programming theory, (6) with A. J. Goldman, the convex hull of the (0,1)-vectors corresponding to the trees in a graph.

Publications:

(1) Paths, trees, and flowers. Jack Edmonds. To appear in the Canadian Journal of Mathematics.

(2) On the surface duality of graphs. Jack Edmonds. Submitted for publication.

(3) Optimal matchings and degree-constrained subgraphs. A. J. Goldman. To appear in Journal of Research NBS, Section B (Math. and Mathematical Physics) 68B, (1964).

1102-40-11645/56-0166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9 (now Division 15)
Manager: P. J. Walsh
Full task description: January-March 1956 issue, p. 27
Status: CONTINUED. Calculations are run on the computer by the sponsor. Future activities will be reported in the section "Current Applications of Automatic Computer".

1102-40-11645/56-0186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-September 1956 issue, p. 33
Status: CONTINUED. Computations were performed to check 41 laboratory sets of gage blocks as requested.

1102-40-11647/58-0266 DEPOLYMERIZATION PROCESSES
Origin and Sponsor: NBS, Section 7.6
Manager: Maxine L. Rockoff
Full task description: July-September 1957 issue, p. 36
Status: INACTIVE

1102-40-11645/58-0339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: January-March 1958 issue, p. 38
Status: CONTINUED. Several computer runs were made on polymer fractions and blends with the aim to find ways of computing the creep function for polymer blends. The most promising way seems to be linear superposition of the stress relaxation functions and determining the creep function from

$$t = \int_0^t G(t - \tau)\, J\,(\tau)\, d\tau .$$

A program has been written at Bell Telephone Laboratories which determines J when G is numerically given. Test runs are under way which will determine the accuracy of this method.

1102-12-11403/59-0348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: U. S. Army Signal Corps and U. S. Army Research Office
Manager: I. Rhodes (11.0)
Full task description: October-December 1958 issue, p. 26
Status: CONTINUED. All three parts of our scheme have been encoded and checked out on the 7094. This is a necessarily limited routine, since we have been experimenting all along with only two very difficult Russian sentences. We are now engaged in applying the mechanical scheme to numerous other sentences, based upon our small internally stored sample glossary. If these prove successful, we shall attempt a public demonstration of the efficacy of our method.

1102-40-11645/60-0486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: Ruth Zucker
Full task description: January-March 1960 issue, p. 28
Status: CONTINUED. Production runs were made and results submitted to Sponsor.

1102-40-11645/60-0513 RADIATIVE ENVELOPES OF MODEL STARS
Origin and Sponsor: National Aeronautics and Space Administration
Managers: P. J. Walsh and S. Haber (11.1)
Full task description: July-September 1960 issue, p. 23
Status: INACTIVE

1102-40-11645/61-0532 CALCULATION OF VIBRATIONAL ENERGY LEVELS FOR IONIC MOLECULES
Origin and Sponsor: Georgetown University
Managers: H. Oser and P. Walsh
Full task description: October-December 1960 issue, p. 21
Status: COMPLETED. All production runs were carried out and results transmitted to the sponsors.

1102-40-11645/61-0538 SPECTRAL REFLECTANCE
Origin and Sponsor: NBS, Section 9.4
Managers: S. Haber (11.1) and P. J. Walsh
Full task description: October-December 1960 issue, p. 23
Status: CONTINUED. New data on the reflectivity of Rhodium and on its optical constants were obtained from outside sources. As a check on the accuracy of the new and old data, it was decided to see whether they satisfied the Kramers-Kronig relations. Routines necessary for the calculation of the phase from the reflectivity were written andchecked out, and applied to the data. The results were inconclusive and indicated a need for more data on the reflectivity of Rhodium in the ultra-violet.

1102-40-11647/62-1022 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: April-June 1958 issue, p. 33
Status: CONTINUED: A small amount of calculation has been completed. It is expected that the project will continue on a limited basis.

1102-40-11645/62-1027 NEW SYSTEM
Origin and Sponsor: NBS, Section 11.2
Manager: R. J. Herbold
Full task description: July-September 1961 issue, p. 22
Status: CONTINUED. The IBM system (IBSYS) has been in a limited use and it is hoped that it will be used more in the future. A special program is being coded to make the OMNITAB system more flexible under IBSYS. The target date for completion of the insertion of OMNITAB into IBSYS system is January 1, 1964.

1102-40-11645/62-1030 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Section 12.5
Manager: Robert Herbold
Sponsor: Veterans Administration
Full task description: April-June 1959, p. 29
Status: CONTINUED. Production runs are being conducted by the sponsor. Future activities will be reported in the section "Current Applications of Automatic Computer."

1101-12-11416/62-1091 BOUNDS FOR EIGENVALUES
Origin: Wright Patterson AFB
Manager: H. Oser
Full task description: October-December 1961 issue, p. 4
Status: CONTINUED. Activities on this project are reported under tasks 0532 and 1390.

1102-40-11647/62-1130 FALLOUT SHELTER COMPUTATIONS
Origin and Sponsor: Office of Civil Defense
Manager: D. I. Mittleman
Full task description: October-December 1961 issue, p. 25
Status: CONTINUED. Since the completion of the first year's work, the problem has been relatively dormant. We are presently in an updating stage which will include building constructions completed more recently as well as corrections to material that is already in the permanent file. A new modification of the calculation has been the lowering of the minimum protection factor to 10 from the previously used figure of 20.

1102-40-11647/62-1155 MORTGAGE LOAN SURVEY
Origin and Sponsor: Federal Home Loan Bank Board
Manger: Ruth Zucker
Full task description: January-March 1962 issue, p. 24
Status: CONTINUED. Production runs continued under the sponsor's direction.

1102-40-11647/62-1178 LOGARITHMIC COEFFICIENTS
Origin and Sponsor: NBS, Section 5.3
Manager: R. J. Arms
Full task description: January - March 1962 issue, p. 27
Status: INACTIVE.

1102-40-11647/62-1179 CATALOGUE INFORMATION
Origin and Sponsor: HDL
Manager: Ruth Varner
Full task description: January-March 1962 issue, p. 27
Status: CONTINUED. A new program was requested by the sponsor. The purpose of the new program is to search a magnetic tape with bibliographic entries such as subject codes and other pertinent information in order that a list of documents may be submitted to a requester who has specified his interest in particular subjects. The program will be written for the 1410.

1102-40-11647/62-1189 SEQUENTIAL METHODS TABLES
Origin and Sponsor: Quartermaster Research and Engineering Field Evaluation Agency, U. S. Army
Manager: R. J. Arms
Full task description: April-June 1962 issue, p. 26
Status: INACTIVE. For lack of funds this project has been inactive this period.

1102-40-11647/62-1193 SOLUTION TO SECOND ORDER PARTIAL DIFFERENTIAL ELLIPTIC EQUATION
Origin and Sponsor: NBS, Section 3.08
Manager: P. J. Wash
Full task description: April-June 1962 issue, p. 28
Status: INACTIVE.

1102-40-11647/62-1196 HEAT OF ADSORPTION
Origin and Sponsor: NBS, Section 15.2
Manager: Ruth Varner
Full task description: April-June 1962 issue, p. 29
Status: INACTIVE

1102-40-11647/62-1203 CYLINDRICAL SHOCK WAVE
Origin and Sponsor: NBS, Section 3.7
Managers: Sally Peavy and S. Haber
Full task description: April-June 1962 issue, p. 30
Status: CONTINUED. Several computations were performed in order to check the mesh interval and resulting computer time. Formulation of inner boundary condition will have to be checked by sponsor.

1102-40-11647/62-1212 COLOR DIFFERENCES
Origin and Sponsor: NBS, Section 10.9
Manager: J. D. Waggoner
Full task description: April-June 1962 issue, p. 33
Status: CONTINUED. Several runs were made and results were submitted to the sponsor.

1102-40-11647/63-1219 SHOCK TUBE DATA
Origin and Sponsor: Diamond Ordnance Fuze Laboratories
Manager: L. Joseph
Full task description: July-December 1962 issue, p. 32
Status: TERMINATED

1102-40-11647/63-1223 CARDIOVASCULAR DYNAMICS
Origin and Sponsor: Univ. of Pennsylvania, Bockus Research Institute
Manager: M. Rockoff
Full task description: July-December 1962 issue, p. 32
Status: TERMINATED

1102-40-11647/63-1240 SECRET SERVICE FORGERY PROJECT
Origin and Sponsor: Treasury Department, U. S. Secret Service
Manager: M. Paulsen
Full task description: July-December 1962 issue, p. 33
Status: INACTIVE.

1102-40-11647/63-1258 SPECTRAL ANALYSIS
Origin and Sponsor: NBS, Section 15.03
Manager: H. Oser
Full task description: July-December 1962 issue, p. 33
Status: COMPLETED. Results defy a satisfactory interpretation at this time. Further theoretical work will be necessary before resumption of this task.

1102-40-11647/63-1335 MATRIX OF POLYNOMIALS
Origin and Sponsor: NBS, Section 13.1
Manager: Ruth Zucker
Full task description: January-June issue 1963, p. 25
Status: COMPLETED. Results were transmitted to the sponsor.

1102-40-11647/63-1341 LINE WIDTHS
Origin and Sponsor: NBS, Section 13.2
Manager: Maxine Paulsen
Full task description: January-June 1963 issue, p. 26
Status: COMPLETED

1102-40-11647/63-1352 NEAR NATIONAL EMERGENCY ALARM REPEATER
Origin and Sponsor: OCD
Manager: Louis Joseph
Full task description: January-June 1963 issue, p. 26
Status: TERMINATED.

1102-40-11647/63-1355 STUDY OF ELECTRONIC ENERGY BANDS IN THE RUTILE CRYSTAL
Origin and Sponsor: NBS, Section 13.4
Managers: P. Walsh and A. Gregg
Full task description: January-June 1963 iss, p. 26
Status: CONTINUED. The codes for generating the various energy matrices were checked and production runs were made. The results have been submitted to the sponsor.

1102-40-11647/63-1364 BaF ELEMENT RESPONSE TIME
Origin and Sponsor: NBS, Section 14.4
Manager: Maxine Paulsen
Full task description: January-June 1963 issue, p. 27
Status: COMPLETED.

1102-40-11647/63-1368 HEART DISEASE CONTROL
Origin and Sponsor: Public Health Service
Manager: Sally Peavy
Fuss task description: January-June 1963 issue, p. 27
Status: CONTINUED. Program is written and in process of being checked out.

1102-40-11647/63-1377 WISKER GROWTH IN A VAPOR ATMOSPHERE
Origin and Sponsor: NBS, Section 8.5
Managers: H. Oser and J. A. Simmons (8.5)
Full task description: January-June 1963 issue, p. 28
Status: INACTIVE

1102-40-11647/64-1410 INTEGRO-DIFFERENTIAL EQUATIONS Authorized 8/12/63
Origin and Sponsor: Institute for Defense Analyses
Manager: R. J. Arms
Objective: The integro-differential equations were reduced to a system of partial differential equations.

$$1)\quad s_{\ell\ell} = e_r / s_r$$

$$e_t = s_t\, e_r / s_r - rf(r)/s s_r$$

Where f(r) is a known function; r, ℓ, and t are independent variables; and s, e are unknown functions. Boundary conditions are given on the surfaces $\ell=0$, $t=0$, $r=0$. The solution s,e is wanted in a part of the positive octant.
Status: NEW. The sytem 1) was replaced by a finite difference system. Results have been submitted to the sponsor.

1102-40-11647/64-1433 NMR SPECTRA Authorized 8/14/63
Origin and Sponsor: Food and Drug Administration
Manager: L. Joseph
Objective: To convert a FORTRAN program for the calculation of NMR (nuclear magnetic resonance) spectra so that it can be run on the Bell Monitor System on the IBM 7094.
Background: FREQINT IV is a program from the MELLON INSTITUTE for the calculation of NMR spectra expected for a given set of chemical shifts and coupling constants. It is written in the FORTRAN language to be run on the IBM 7094. In order for it to run on the Bell Monitor System certain minor changes were required.
In addition, an extra option was put in to enable part of the printout to be suppressed.
Status: NEW. The program was modified as indicated above. Code check and production runs have been made. Further runs will be made by the sponsor. Submitted by Dr. E. Lustig.

1102-40-11647-1415 SINGLE CRYSTAL DATA Authorized: 8/13/63
Origin and Sponsor: NBS, Section 5.6
Manager: Don I. Mittleman
Objective: To prepare a volume using the Mergenthaler linofilm equipment at the Government Printing Office and to organize magnetic tapes for information storage and retrieval of these data.
Background: The past techniques for the preparation of this volume are unsatisfactory from the standpoint that the volume appears too long after it has been prepared. Using modern computer techniques and Mergenthaler equipment available at the Government Printing Office, it is hoped to produce this volume on a more timely basis and at lower cost and to make revision and updating a bi-product of the operation. It is also hoped that having the information available on magnetic tape will facilitate research work in this field.
Status: NEW. The work was started and as of the present is dormant.

1102-40-11647/64-1381 EXPANSION OF POLYELECTROLYTES Authorized 12/14/63
Origin and Sponsor: NBS, Section 3.8
Manager: J. D. Waggoner
Objective: To determine theoretically the average size and shape of an electrically charged polymer molecule (polyelectrolyte) in solution. In particular, to compute by approximation, a Gaussian distribution, i.e. $m(r) = e^{-Br^2}$, where B is a parameter which depends on the total number of chain elements in the polymer.
Background: Various considerations have been made in connection whith a polyelectrolyte and what happens if one has charge of the same sign distributed along the chain, if it is clear that the charge will, by virtue of the Coulomb repulsion, tend to expand the polyion and change its average shape. On the basis of theoretical considerations, it is required that the density of polyion matter is proportional to the product of the initial (uncharged) distribution and the Boltzmann factor, $\exp[p\varphi/kt]$. Here p is the fraction of the chain elements carrying charge and $\varphi(r)$ is the total electrostatic potential. Based on the assumption that the counter-ions are distributed according to the Boltzmann distribution, the solution of the Poisson-Boltzmann equation will yield the potential and the polyion shape.
Status: NEW. Completed. A FORTRAN code was written to solve the Poisson-Boltzmann equation. Several runs were made and the results were submitted to the sponsor along with the code. Future activities will be reported under "Current Applications of Automatic Computer".

1102-40-11647/64-1438 MATRIX CALCULATION Authorized 9/19/63
Origin and Sponsor: NBS, Section 8.5
Manager: J. D. Waggoner
Objective: To perform computations in connection with the calculation of the diffusion coefficient of tracer ions in a NaCl type ionic crystal.
Background: Assuming the diffusion mechanism to consist of tightly bound anion-cation vacancy pairs, it is desired to study the evidence that this mechanism is the dominant one under certain experimental conditions. The calculation involves finding the probabilities of correlated exchanges of a tracer with a vacancy pair. By comparing the results of the calculation with experiment, it should be possible to determine the ratio of jump frequencies, and the anion and cation vacancies.
Status: NEW. Completed. A FORTRAN code was written to perform these computations. Several runs were made and the results were submitted to the sponsor.

1100-12-11404/63-1456 RESEARCH ON A PICTURE LANGUAGE MACHINE Authorized 5/1/61
Origin: NBS
Sponsor: National Science Foundation
Manager: Russell A. Kirsch
Objective: To perform investigations on the development of an automatic programming system for processing information retrieval prescriptions which are addressed to collections of documents which consist of interrelated pictures and natural language text.
Background: In order to mechanize information retrieval systems which use text and pictures, it is necessary to provide a formal (i.e., mechanical) set of procedures for processing the text and pictures. Since such information is ordinarily meant only for human consumption, it is necessary that these procedures accomplish the equivalent of human visual pattern recognition and linguistic syntactical analysis. The development of a general-purpose research tool like a Picture Language Machine will provide the capability for exploring more complex problems in pattern recognition and linguistic analysis than could be done without this tool. Furthermore, of the many research efforts in pattern recognition or in linguistic analysis, none attempts to tie together in this way these two kinds of information processes to attack problems which human beings as processors of information have the natural equipment for solving.
Status: A prototype Picture Language Machine was partly programmed and partly hand simulated. This prototype was able to accept English sentences which described pictures containing geometrical shapes and by mechanical operation upon the sentences and the pictures determine which sentences were correct descriptions of which pictures.
To accomplish this simulation, a general purpose syntactical analyzer for phrase structure grammars was written for the 7094 computer. A grammar for the relevant fragment of English was also written. This grammar was the basis for several others written that included larger fragments of English. A set of programs in the STRIP language for the SEAC computer was constructed for automatic analysis of the simple pictures. Finally, an algorithm was written and programmed for translating from the syntactically analyzed sentences to a logical calculus representation.
A small study was also made of the use of grammars in inductive inference. A pictorial grammar was written for the geometric arrays of symbols used in various textbooks for denoting Boolean algebraic expressions. A comparative analysis was completed for the list processing languages COMIT, FLPL, IPL-V, and LISP.

7. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING

Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: J. M. Cameron, H. H. Ku
Full task description: July-September 1950 issue, p. 60

Status: CONTINUED. During this period members of the section provided statistical assistance to a number of Bureau personnel. The following are representative examples:
Hsien H. Ku continued his collaboration with T. Hockersmith of the Engineering Mechanics Section on the statistical aspects or proving ring calibration particularly the estimation of the uncertainty to be attached to such calibrations.
Brian L. Joiner has been working with M. G. Kerper and T. Scuderi of the Glass Section on problems related to the use of the Weibull distribution in studies of the strength of glass.
W. J. Youden collaborated with W. B. Mann and B. Garfinkel of the Radiation Physics Division on problems of experimental design and uncertainty statements relating to radiation standards.
G. Weiss has collaborated with H. Andersen, I. Oppenheim, and K. Shuler (Director's Office) in the preparation of a paper on "Exact conditions for the preservation of a canonical distribution in Markovian relaxation processes".
Mary G. Natrella is teaching an in-hours course on Statistics of Measurement. For scientists and engineers, this course provides an introduction to the use of NBS Handbook 91, Experimental Statistics. Fourteen people, from NBS and other government agencies in the Washington area, are registered for the first semester.
J. M. Cameron collaborated with J. Hilsenrath (Equation of State Section) in teaching a course entitled: Introduction to Mathematical and Statistical Analysis of Laboratory Data. This course presents an outline of the use of the Bureau's general purpose computing program OMNITAB for the statistical and numerical analysis of experimental data.

Publications:

(1) Mathematics and experimental science. W. J. Youden. The Science Teacher 30, 23-26, September 1963.

(2) Measurement agreement comparisons. W. J. Youden. Proceedings National Conference of Standards Laboratories, held at NBS-Boulder, Aug. 1962. NBS Misc. Pub. 248, pp. 147-151, August 1963.

(3) Editorial: Parable of the Fisherman. W. J. Youden. The Physics Teacher 1, 120-121, Sept. 1963.

(4) Families of distributions for hourly median power and instantaneous power of received radio signals. M. M. Siddiqui (NBS Boulder Laboratories) and George Weiss. J. Res. NBS--D. (Radio Propagation), 67D, 753-762, Nov.-Dec. 1963.

(5) Statistics of irreversible termination in homogeneous anionic polymerization. Fred Gornick (NBS Macromolecules Synthesis and Structure Section), Bernard D. Coleman (Mellon Institute), and George Weiss. To appear in J. Chemical Physics.

(6) Exact conditions for the preservation of a canonical distribution in a Markovian relaxation process. H. Andersen, I. Oppenheim, K. Shuler (Director's Office), and G. Weiss. To appear in J. Math. Physics.

(7) Exact Faxen solution for centrifugation when sedimentation depends linearly on concentration. George Weiss and Irwin H. Billick (Macromolecules Synthesis and Structure Section). Submitted to a technical journal.

(8) Mathematical models for personnel promotion. E. L. Crow (Boulder Laboratories) and George Weiss. Submitted to a technical journal.

(9) Simplified statistical quantity control. W. J. Youden. Report of the 48th National Conference on Weights and Measures, NBS Misc. Pub. No. 254, 1963, pp. 97-102.

(10) Sampling and statistical design. W. J. Youden. To appear in Proceedings, Symposium on Environmental Measurements, U. S.Public Health Service, 1963.

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies — Authorized 3/31/58
Manager: J. M. Cameron
Full task description: January-March 1958 issue, p. 45

Status: CONTINUED. Senior personnel from precision measurement and calibration laboratories attended a one-week seminar on Precision and Accuracy in Measurement and Calibration conducted by the Statistical Engineering Laboratory. In addition to lecture and discussion sessions, the fourteen conferees who participated were given the opportunity to visit the NBS laboratories most closely related to their fields.
The session chairman, Churchill Eisenhart, speaking on "Calibration as a Process" and "Expression of Uncertainties," dealt with measurement of the precision and assessment of the accuracy of a measurement process. W. J. Youden, speaking on "Estimation of Precision" and "Establishing Statistical Control," discussed experimental designs for calibration, how to detect possible systematic error in comparisons, and how to run and interpret interlab studies. H. H. Ku spoke on "Propagation of Systematic and Random Error." J. M. Cameron, after presenting the general theory and techniques of "Least Squares," demonstrated applications of least squares, as well as applications of quality control of a measurement process, in a "Case History -- Mass Calibration."

This is a record of the use of the IBM 7094 for the period of July 1 through December 31, 1963.

TASK NUMBER		TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NBS SERVICES			(M I N U T E S)			
51-0002	11.3	STATISTICAL ENGINEERING	86	70	144	300
63-0003	11.3	CLASS+++	0	0	22	22
54-0030	13.1	SPECTRUM ANALYSIS++	79	146	318	543
54-0031	13.1	SPECTRUM ANALYSIS++	0	0	23	23
54-0032	13.1	SPECTRUM ANALYSIS++	0	0	33	33
54-0033	13.1	SPECTRUM ANALYSIS++	25	19	2448	2492
54-0034	13.1	SPECTRUM ANALYSIS++	42	0	147	189
55-0055	11.1	RESEARCH IN NUMERICAL ANALYSIS	26	249	34	309
55-0056	11.1	RESEARCH IN MATHEMATICAL TOPICS	4	31	31	66
55-0065	11.2	AUTOMATIC CODING	0	10	28	38
55-0082	3.1	THERMOMETER CALIBRATION+	2	3	122	127
56-0166	15.0	SCF-LCAO SOLUTION OF HYDRIDES+	20	80	67	167
57-0219	3.2	THERMAL PROPERTIES+	20	73	168	261
57-0236	3.8	SCF EIGENVALUES+	2	0	60	62
57-0250	2.1	SPECTROPHOTOMETRIC DATA+	8	6	25	39
57-0252	4.4	NEUTRAL MESON EXPERIMENTS++	88	442	235	765
58-0256	10.6	COMPOSITE WALL STUDIES++	81	142	152	375
58-0272	3.7	EQUATION OF STATE++	0	3	0	3
58-0314	3.7	APPROXIMATIONS FOR GAS MIXTURES	137	90	23	250
58-0339	6.5	VISCOELASTICITY PROPERTIES	5	5	19	29
59-0433	2.1	COLOR OF SIGNALS++	0	0	1	1
60-0489	3.1	INVERSION OF LINE PROBE DATA+	50	93	6	149
61-0523	4.7	NEUTRON CROSS SECTION STUDIES++	0	9	0	9
61-0538	9.4	SPECTRAL REFLECTANCE DATA	14	0	27	41
61-0559	3.1	THERMOCOUPLE CALIBRATION+	3	15	29	47
61-0562	7.6	CUBIC LATTICES+	8	10	97	115
61-0995	11.2	ERROR DETECTION+++	0	0	16	16
62-1000	12.5	POST OFFICE OPERATIONS STUDY++	79	440	47	566
62-1003	15.4	MOLECULAR SPECTROSCOPY+	1	18	56	75
62-1005	4.3	RADIATION INTERACTION++	67	87	211	365
62-1006	4.3	RADIATION INTERACTION++	375	391	56	822
62-1007	4.3	RADIATION SHIELDING++	61	60	40	161
62-1011	13.5	DISPERSION INTEGRALS++	0	6	0	6
62-1013	7.0	STATISTICAL METHODS++	0	19	11	30
62-1015	15.1	THERMAL FUNCTIONS++	0	0	1	1

TASK NUMBER / NBS SERVICES	TITLE	ASSEMBLY TIME (MINUTES)	CODE CHECKING (MINUTES)	PRODUCTION TIME (MINUTES)	TOTAL TIME ON COMPUTER (MINUTES)
62-1019 41.0	NBS PERSONNEL REPORT++	6	41	149	196
62-1027 11.2	NEW SYSTEM	119	120	109	348
62-1029 9.7	D-SPACING CALCULATIONS+	0	0	14	14
62-1033 9.7	CRYSTAL STRUCTURE CALIBRATION++	37	20	541	598
62-1034 30.0	PHOTOIONIZATION CROSS SECTION++	14	2	20	36
62-1035 7.7	CREEP DATA ANALYSIS++	1	2	24	27
62-1036 7.7	FILM THICKNESS++	6	15	72	93
62-1038 7.5	STANDARDIZATION ANALYSES++	32	14	0	46
62-1052 2.0	BLACK BOX COMPUTER SERVICE+	1	0	8	9
62-1055 8.4	ELLIPSOIDAL COMPUTATION++	0	2	6	8
62-1064 2.4	GAGE BLOCK STUDIES++	0	0	16	16
62-1066 1.2	STANDARD CELLS++	0	0	15	15
62-1080 9.2	BLACK BOX COMPUTER SERVICE+	1	0	128	129
62-1081 9.1	BLACK BOX COMPUTER SERVICE+	21	4	19	44
62-1089 9.6	ELASTIC CONSTANTS++	10	2	16	28
62-1102 6.8	BLACK BOX COMPUTER SERVICE+	12	2	22	36
62-1107 6.5	OSCILLATING SPHERE++	15	12	13	40
62-1125 9.5	MATRIX COMPUTATIONS	0	42	17	59
62-1157 11.4	PLASMA RESEARCH++	102	22	5	129
62-1163 14.1	TRANSISTOR AGING BEHAVIOR++	28	114	34	176
62-1170 7.7	HIGH PURITY POLYMERS++	0	2	5	7
62-1181 12.4	NTDC++	12	29	0	41
62-1185 10.3	HEAT TRANSFER CALCULATIONS+	85	55	105	245
62-1187 2.4	FRUSTRATED REFLECTIONS++	0	0	40	40
62-1195 7.2	LIGHT SCATTERING++	0	7	0	7
62-1196 15.2	EQUATIONS IN XY THETA	2	6	13	21
62-1203 3.7	CYLINDRICAL SHOCK WAVE	4	9	4	17
62-1211 12.5	TECHNICAL INFO RETRIEVAL++	1	1	0	2
62-1212 10.9	COLOR DIFFERENCES	1	8	7	16
63-1226 12.0	DTS - KWIC++	37	313	198	548
63-1231 13.0	BLACK BOX COMPUTER SERVICE+	1	0	11	12
63-1237 3.1	PYROMETRY++	0	9	0	9
63-1241 12.5	IICASP++	0	25	22	47
63-1250 12.0	KWIC++	9	36	30	75
63-1252 11.5	ARMY ORDNANCE++	0	7	0	7
63-1257 7.8	CALC OF CALCIUM PHOSPHATE++	0	0	1	1
63-1259 11.3	RESEARCH IN PROBABILITY++	42	47	64	153
63-1263 15.5	LINEAR CLASSICAL SYSTEM++	24	0	78	102
63-1276 14.2	INSTRUMENTATION++	52	56	14	122
63-1281 2.4	CORRECTION-SMEARING	9	0	0	9
63-1285 11.5	RTS FUNDS++	2	1	1	4

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER	NBS SERVICES	TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
			(M I N U T E S)			
63-1287	3.7	DATA ANALYSES OF GASES++	7	37	62	106
63-1289	3.8	IONIZED GASES++	137	9	0	146
63-1290	3.0	MOLECULAR ENERGY LEVELS++	8	8	16	32
63-1291	7.1	JOB CALCULATIONS++	0	39	7	46
63-1300	4.1	MAXIMUM SIGMA++	3	58	8	69
63-1302	7.3	COMPUTER CALCULATIONS++	0	0	8	8
63-1308	2.5	BUTTRESS THREADS++	4	47	14	65
63-1309	4.2	LINEAR REGRESSION ANALYSIS++	0	0	12	12
63-1315	3.3	VIRIAL COEFFICIENTS++	18	9	8	35
63-1318	10.3	THERMISTOR PROGRAM++	17	6	32	55
63-1320	9.7	CRYSTAL STRUCTURE	0	5	2	7
63-1323	3.0	PLASMA TRANSPORT++	0	0	53	53
63-1325	4.7	THERMOFLUX++	46	74	100	220
63-1326	3.2	HEAT MEASUREMENT++	0	10	26	36
63-1332	3.1	TEMPERATURE PHYSICS++	0	2	4	6
63-1333	2.2	BLACK BOX COMPUTER SERVICE+	0	0	1	1
63-1337	12.0	RICASIP PT1++	0	1	2	3
63-1338	15.4	SECTION COMPUTATIONS++	0	0	7	7
63-1340	3.3	FUNCTION OF TEMPERATURE++	1	0	10	11
63-1341	13.2	LINE WIDTH	7	14	3	24
63-1342	6.1	OMNITAB+	1	0	1	2
63-1343	3.1	OMNITAB+	0	3	11	14
63-1351	1.2	TEST DATA++	0	0	8	3
63-1355	13.4	RUTILE BAND STRUCTURE	3	36	3	42
63-1359	13.5	OMNITAB+	0	21	2	23
63-1375	3.7	THERMAL PROPERTIES+	60	78	54	192
63-1378	12.5	DCA++	23	231	133	392
63-1381	3.8	POLY-ELECTROLYTES	33	90	20	143
63-1382	2.3	SIGMA COMPUTATIONS	0	0	5	5
63-1388	3.2	COMBUSTION CALORIMETRY++	0	2	0	2
63-1389	6.4	PROVING RINGS++	0	24	12	36
63-1390	30.0	FOKKER-PLANCK	34	36	259	329
64-1395	10.9	EMITTANCE DATA	0	5	2	7
64-1396	12.0	SYNCRETIC CODE++	0	18	3	21
64-1398	03.2	PILOT PROJECT LPHC++	2	2	0	4
64-1400	13.2	STATISTICS++	60	0	143	203
64-1401	10.7	LONG TIME CEMENT STUDY 1++	60	2	82	144
64-1402	10.7	LONG TIME CEMENT STUDY 2++	34	2	162	198
64-1405	6.8	TEMPERATURE SENSING++	0	0	14	14
64-1406	6.8	HYPERSONIC COMBUSTION++	68	87	4	159
64-1407	5.2	SPECTROANALYSIS++	2	0	3	5

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER	NBS SERVICES	TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
			(M I N U T E S)			
64-1408	10.1	ELASTIC SOLIDS	9	5	0	14
64-1412	4.2	REF++	38	105	84	227
64-1416	9.3	OMNITAB+	0	0	2	2
64-1417	13.7	DISPERSION INTEGRATION++	0	0	1	1
64-1418	2.6	STATISTICAL COMPUTATION++	1	0	68	69
64-1419	2.6	STATISTICAL COMPUTATION++	0	0	62	62
64-1420	3.2	OMNITAB+	0	0	2	2
64-1421	12.5	HAYSTAQ++	15	61	0	76
64-1423	3.7	COORDINATE ANALYSIS++	204	345	35	584
64-1427	8.5	ELECTRON DIFFRACTION	0	2	2	4
64-1428	9.5	NUMERICAL INTEGRATION++	3	4	7	14
64-1431	3.7	RESEARCH++	29	14	8	51
64-1437	7.08	AMALGAM STRAIN-TIME DATA++	0	3	64	67
64-1438	8.0	MATRIX OPERATIONS	2	1	0	3
64-1440	10.9	OMNITAB+	2	0	4	6
64-1443	4.10	MAGNET TEST PROGRAM++	9	1	11	21
64-1444	9.9	SILICA X-RAY PATTERNS++	0	0	11	11
64-1445	30.0	TEXTILE INDUSTRY STUDY++	1	0	41	42
64-1448	15.0	BLACK BOX COMPUTER SERVICE+	1	2	6	9
64-1449	15.5	OXYGEN BANDS++	0	0	7	7
64-1450	10.7	GLASS BEAD DATA	1	1	2	4
64-1453	3.1	RES THERMOMETER CALC++	0	3	0	3
64-1454	7.4	ACTIVE ENERGIES++	23	6	7	36
64-1455	30.35	REACTOR DESIGN++	0	0	2	2
64-1456	11.0	INFORMATION RETRIEVAL++	8	10	17	35
64-1459	4.0	GIANT RESIDENCE ANALYSIS++	2	1	13	16
64-1460	13.5	FIELD EMISSION++	22	6	0	28
64-1462	4.23	POSITRON PRODUCTION++	0	20	2	22
64-1463	13.0	TRANSITION PROBABILITIES	0	0	4	4
64-1464	10.7	OMNITAB+	0	0	1	1
64-1465	4.33	CROSS SECTION APPROX++	14	0	3	17
64-1468	5.3	LEAST SQUARES	0	1	0	1
64-1473	3.7	POLAR GASES++	1	31	2	34
64-1474	15.0	ATOM CORRELATION++	5	0	4	9
63-3003	11.2	MACHINE TIME ONLY+++	3	16	0	19
63-3005	11.2	FREE MACHINE TIME+++	18	10	1	29
63-3008	11.2	SECRETARYS MACHINE TIME+++	3	5	171	179
64-3011	11.2	ERROR-USER+++	0	0	40	40
		TOTALS (NBS SERVICES).	3011	5151	8503	16665

CURRENT APPLICATIONS OF AUTOMATIC COMPUTER

TASK NUMBER		TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NON-NBS SERVICES			(M	I N U	T E	S)
58-0348	OOR	MACHINE TRANSLATION OF RUSSIAN	21	46	1	68
58-0366	USIA	RADIATION PATTERNS OF ANTENNAS	0	0	2	2
59-0407	HDL	FOURIER COEFFICIENTS+	1	1	6	8
59-0409	FSLIC	BANK BOARD REPORTS++	0	13	300	313
59-0425	CU	MOLECULAR ORBITALS+	33	158	93	284
59-0434	GC	PETROLOGICAL COMPUTATIONS+	45	64	78	187
59-0441	USRED	SYSTEMS ENGINEERING++	7	97	46	150
60-0457	PHA	PUBLIC HOUSING PROBLEM++	8	49	229	286
60-0476	HDL	GAS TUBE CHARACTERISTIC II	0	0	735	735
60-0486	U ONT	MORSE WAVE FUNCTION++	4	0	59	63
60-0492	IMF	MONETARY RESEARCH REPORTS++	23	286	127	436
60-0506	WBANK	WORLD BANK REPORTS++	44	8	102	154
61-0513	NASA	ORBITING STUDIES	0	13	5	18
61-0532	GU	VIBRATIONAL ENERGY LEVELS+	1	73	157	231
61-0540	ACC	DIFFUSION CALCULATIONS+	40	37	268	345
61-0569	AGO	HUMAN FACTORS RESEARCH++	125	129	60	314
61-0830	BPR	HIGHWAY TRAFFIC STUDIES++	22	8	638	668
61-0902	BPR	HIGHWAY TRAFFIC STUDIES++	0	0	340	340
61-0903	BPR	HIGHWAY TRAFFIC STUDIES++	48	7	1315	1370
61-0945	WB	FORECASTING++	0	0	136	136
62-1004	BUSHP	RHOMBIC ANTENNAS+	1	2	34	37
62-1014	NIH	METABOLIC DISEASES++	257	448	1354	2059
62-1018	NRL	HYDROMAGNETIC PROBLEMS+	55	99	86	240
62-1021	DCH	HIGHWAY STUDIES++	84	342	2567	2993
62-1023	NSF	IMAGE PROCESSING++	0	74	38	112
62-1030	VA	ELECTROCARDIOGRAPHIC ANALYSIS	763	1211	1310	3284
62-1044	FCC	RADIO INTENSITIES++	18	16	49	83
62-1046	BPR	TRAFFIC PREDICTION++	363	44	1737	2144
62-1056	HDL	PD ENGINEERING++++	10	18	225	253
62-1071	HDL	RHINITIS STUDIES++	17	0	0	17
62-1076	NAS	EVALUATION OF APPLICATIONS+	0	23	1	24
62-1096	HDL	VULNERABILITY STUDY++++	0	1	1	2
62-1110	ICC	ICC SYSTEMS STUDY++	33	38	355	426
62-1113	HDL	TRANSPORT ANALYSES++++	39	110	267	416
62-1114	HDL	RADIATION EFFECTS++	13	3	0	16
62-1119	BPR	HIGHWAY TRAFFIC STUDIES++	0	0	21	21
62-1121	CARIN	CARNEGIE INSTITUTE STUDIES++	127	14	11	152
62-1130	COENG	FALLOUT SHELTER COMPUTATIONS	92	276	312	680
62-1131	USIA	CANTILEVER RETAINING WALL++	2	0	30	32
62-1140	VA	VA MEDICAL++	53	5	192	250
62-1158	GC	MINERALOGY STUDIES++	1	46	258	305

TASK NUMBER		TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NON-NBS SERVICES			(M I N U T E S)			
62-1169	U ONT	ATOMIC COLLISIONS++	19	5	93	117
62-1171	VA	HOSPITAL PROGRAM PLANNING+	283	165	147	595
62-1172	PEACE	PEACE CORPS EVALUATIONS++	40	16	51	107
62-1179	HDL	CATALOG INFORMATION+	0	6	19	25
63-1218	HDL	INTEGRAL EVALUATION	0	3	4	7
63-1221	BPR	RHODE ISLAND++	0	0	56	56
63-1232	NORAD	MISSILES++	0	25	9	34
63-1236	COMM	DATATROL++	187	27	292	506
63-1239	PHS	PUBLIC HEALTH SERVICE++	0	0	46	46
63-1243	WSMAS	HIGHWAY STUDIES++	1	0	15	16
63-1246	PHS	SCREENING EVALUATION+	11	5	2	18
63-1249	RC	ISOTOPE TRACER ANALYSIS++	0	0	36	36
63-1253	GU	BLACK BOX COMPUTER SERVICE++	3	57	54	114
63-1254	DEFCO	HIGH FREQUENCY PROPAGATION++	137	27	352	516
63-1256	NCTA	TRANSIT STUDY++	0	8	78	86
63-1264	NRL	NUCLEONICS++	144	4	25	173
63-1271	COMM	ECONOMICS STUDY++	8	15	102	125
63-1272	BPR	ROADS STUDY++	17	0	449	466
63-1280	UARIZ	NIH++	12	49	3	64
63-1284	HDL	STATISTICAL ANALYSIS++	0	1	1	2
63-1293	COMM	BODDY CALCULATION++	18	9	0	27
63-1299	HDL	1410 PROGRAM++	15	1	9	25
63-1301	HDL	SERGEANT SPARE PARTS++	23	0	3	26
63-1305	DSA	ARMY++	45	14	4018	4077
63-1307	HDL	MISCELLANEOUS PROGRAMMING++	28	6	23	57
63-1310	HDL	SHOCK WAVE TEST++	58	14	28	100
63-1313	IDA	OMNITAB+	10	1	7	18
63-1317	AID	SORTING AND TABULATING	24	16	124	164
63-1324	HEW	GENERAL KINETICS++	51	635	3	689
63-1330	BROOK	INVESTMENT++	0	0	8	8
63-1336	NAVWE	ARC++	60	2	3286	3348
63-1345	HDL	ROCKET TRAJECTORIES++	32	60	17	109
63-1350	HDL	ME DATA++	0	0	22	22
63-1352	OCDM	NEAR	7	8	569	584
63-1356	NIH	COMPUTER CONSULTING	16	201	51	268
63-1357	BPR	HIGHWAY STUDIES++	0	0	16	16
63-1358	PHS	TRAINING GRANTS	16	35	352	403
63-1360	FPC	FEDERAL POWER COMMISSION++	0	0	190	190
63-1361	HDL	ANTENNA CALCULATION++	10	0	9	19
63-1362	VA	RESEARCH++	0	0	32	32
63-1365	HDL	1410++	8	1	0	9

TASK NUMBER		TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NON-NBS SERVICES			(M I N U T E S)			
63-1367	BPR	PUBLIC ROADS++	0	0	89	89
63-1368	PHS	HEART DISEASE	60	29	4	93
63-1371	TREAS	ALTERNATE TAX PLANS++	11	0	1015	1026
63-1374	OCDM	FEDERAL RESEARCH++	0	52	93	145
63-1386	BPR	PUBLIC ROADS++	0	0	57	57
63-1391	HEW	BIOMEDICAL STA PROG++	11	0	90	101
63-1393	NASA	COMPUTER SYSTEMS	40	9	41	90
64-1394	DSA	ARMY COST MODEL (RAND)++	45	0	2181	2226
64-1409	AU	SOUND FIELDS	0	23	8	31
64-1410	IDA	INTEGRAL EQUATION	26	3	121	150
64-1411	HDL	AUTOCORRELATION++	0	34	4	38
64-1413	HDL	TRANSFER EQUATION++	0	0	8	8
64-1414	HDL	AD 70 PROGRAM++	81	37	146	264
64-1426	DCH	DC HIGHWAY++	10	52	39	101
64-1429	HDL	RESEARCH MISC++	27	7	42	76
64-1430	HDL	LANCE ECM STUDY++	13	0	15	28
64-1432	BRINS	BROOKINGS++	0	0	63	63
64-1433	HEW	NMR SPECTRA	4	1	5	10
64-1434	AMERD	AMERAD++	28	1	0	29
64-1435	OBE	CAPITOL COEFFICIENTS++	0	0	19	19
64-1436	HDL	DIPOLE MOMENT COMP++	0	0	3	3
64-1441	BROOK	STAT STUDY OF DIVIDENDS++	0	0	10	10
64-1446	HDL	TIME VARYING INDUCTANCE++	1	0	0	1
64-1451	DEF	PROGRAM 2++	0	0	502	502
64-1452	HDL	BESSEL FUNCTIONS++	2	13	3	18
64-1457	NRL	SOLAR RADIATION DATA RED++	3	0	9	12
64-1458	HDL	ANALIGHT++	3	0	0	3
64-1461	COMM	BP ANALYSIS++	0	0	2	2
64-1467	NRL	THEORET NUCLEAR PHYSICS++	69	26	0	95
64-1469	DEFSU	LEAST SQUARES	0	0	101	101
		TOTALS (NON-NBS SERVICES)	4067	5462	28816	38345
		TOTALS (NBS AND NON-NBS)	7078	10613	37319	55010

\+ PROBLEM PROGRAMMED IN THE COMPUTATION LABORATORY, PRODUCTION RUNS CONTINUED UNDER DIRECTION OF SPONSOR.

++ PROBLEM PROGRAMMED BY THE SPONSOR AND RUN UNDER HIS DIRECTION.

+++ FUNCTIONS PERTAIN TO THE INTERNAL OPERATIONS OF THE COMPUTATION LABORATORY.

++++ CLASSIFIED TASK.

AS ASSEMBLY TIME.

CC CODE CHECKING TIME.

PR PRODUCTION TIME.

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

BETCHOV, R. (Aerospace Corporation, Los Angeles, California) Statistical theory of turbulance. December 3, 1963.

CLENSHAW, C. (National Physical Laboratory, Great Britain) Slowly-convergent Chebyshev series expansions. October 17, 1963.

HOH, F.C. (Princeton University, Princeton, New Jersey) Electron resonance in plasma column. October 7, 1963.

LEMKE, C.E. (Rensselaer Polytechnic Institute, Troy, New York) Bimatrix games and quadratic-like programs. December 13, 1963.

NBS Scientific Staff Meeting

CANNON, E.W. Highlights of Activity in the Applied Mathematics Division, October 25, 1963

1. M. Newman. Recent research on the modular group. Numerical Analysis Section.
2. J. M. Cameron. Curve fitting. Statistical Engineering Laboratory.
3. A. J. Goldman. Overflows and firehouses. Operations Research Section.

Mass and Scale Section Staff Meeting

CAMERON, J.M. Statistics of mass calibration. October 7, 1963.

Lectures and Technical Meetings

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CAMERON, J.M. — The use of general purpose coding systems for statistical calculations. IBM Scientific Computing Symposium on Statistics, Yorktown Heights, N.Y., Oct. 23, 1963.

The method of least squares. Monmouth College, West Long Branch, N.J., October 30, 1963. (Institute of Mathematical Statistics Visiting Lecturer Program.)

EDMONDS, J. — The Existence of connected graphs with prescribed degrees. Presented before the Mathematical Association of America. December 14, 1963.

GHAFFARI, A. — On a new approximation method for nonlinear nonautonomous differential equations. Sponsored by Mathematics Research Laboratory; Aerospace Research Laboratories; and the United States Air Force, Wright-Patterson Air Force Base, Ohio. August 1, 1963.

GOLDMAN, A.J. — Optimal matchings and degree-constrained subgraphs. Presented before the Mathematical Association of America. December 14, 1963.

Contributions of mathematical concepts to management. Seminar at Industrial College of the Armed Forces, Fort McNair. September 3, 1963.

KEARSLEY, E.A. & BERNSTEIN, B. — A theory of compressible isotropic elastic media for which pressure may effect shear. Fourth Int. Congress Rheology, Providence, Rhode Island, August 26, 1963.

KIRSCH, R. A. — Participation in "Project MAC". Summer study sponsored by Advanced Research Projects Agency at the Massachusetts Institute of Technology, Cambridge, Mass. July 8, July 16, July 25, August 5, 1963.

Participation in "Notation Workshop" sponsored by National Bureau of Standards, at Warrenton, Virginia. September 6, 1963.

"Our resources in information processing - where are the gaps?" Panel presented by the Association for Computing Machinery, University of Maryland.

"Automata theoretic methods in pattern recognition." AEC Computer Information Meeting, University of Illinois, Urbana, Illinois. November 1, 1963.

KLOSS, K. — Some recent number-theoretic research using a high-speed calculator. Presented at the PiMuEpsilon Fraternity in conjunction with the Math. Association of America Meeting at Boulder, Colorado, August 27, 1963.

LAGNESE, J.E. — Self-adjoint differential operators of the pure wave type. Presented at the U. S. Naval Ordnance Laboratory. White Oak, Silver Spring, Maryland, Sept. 30, 1963.

MEYERS, P. — A remark on the contraction theorem. Presented before the Mathematical Association of America. December 14, 1963.

NEWMAN, M. — Bounds for class numbers. Presented at the American Math. Society Meeting at Pasadena, California, November 21, 1963.

OLVER, J.W.J. — The Liouville-Green (or WKB) approximation. Presented at the University of Maryland, November 20, 1963.

Error bounds for asymptotic expansions. Presented at the University of Maryland, November 27, 1963.

PEAVY, S.T. Ways to communicate with electronic computers. Presented at the American Association of University Women Board Room, Washington, D.C. Oct. 2 and 9, 1963. Sponsor: U. S. Civil Service Commission.

ROSENBLATT, R.R. Activities of the committee on women in scienc of the Joint Board on Science Education. December 20, 1963.

Confidence limits for the reliability of a complex system. Pittsburgh Chapter, American Statistical Association, November 12, 1963.

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2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

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