NATIONAL BUREAU OF STANDARDS REPORT

8168

Review of Elementary Theory of the Photometry of Projection Apparatus

by

C. A. Douglas

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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Photometry and Colorimetry Section
Metrology Division

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Definition of Symbols

L	Luminance of source S
d	Distance between the projector and the point of observation or measurement
d_c	Critical distance
E	Illuminance at the point of observation or measurement
F	Focal length of the optic of the projector
I	Intensity of the projector
I'	Apparent intensity of the projector
I_s	Intensity of the source, S
I_i	Intensity of the image S' of the source
M	Magnification of optical system
R	Radius of the objective (lens or reflector)
r	Radius of the source S
r'	Radius of image S' of source S
S	Light source (assumed to be of uniform luminance)
S'	Source image formed by the projector
t	Transmittance of the optical system, if lens type; reflectance of the optic, if reflector type
u	Distance between the source and the objective
v	Distance between the image and the objective
y	Radius of flashed zone on the objective
θ	The beam spread produced by a spread lens
$\bar{y}$	Dimension of asymmetric source in x-y plane
$\bar{Y}$	Dimension of asymmetric optic in x-y plane
$\bar{z}$	Dimension of asymmetric source in x-z plane
$\bar{Z}$	Dimension of asymmetric optic in x-z plane

Review of Elementary Theory of the Photometry of Projection Apparatus

C. A. Douglas

ABSTRACT

In this report equations based upon simple geometric relations are developed for the illuminance produced by a projector at a distance from the projector. When the beam is radially symmetrical but not collimated and the image, virtual or real, subtends a smaller angle at the point of observation than does the objective of the projector, illuminance varies inversely as the square of the distance to the image. If the angle subtended by the image is larger than that subtended by the objective, the illuminance varies inversely as the square of the distance to the objective. The distance at which the two angles are equal is defined as the critical distance. Equations relating critical distance to the radius of the source, the radius of the objective, and the magnification of the system are developed. Approximations for use when the beam of the projector is asymmetric are developed. Very good agreement was found between the computed variation of illuminance with distance and the measured variation of illuminance with distance for a projector forming a virtual image 150 feet behind the objective.

1. INTRODUCTION

The intensity, I, of a projector is obtained by measuring the illuminance, E, on a surface a distance, d, from the projector and computing I from the relation

$$I = E\ d^2 \qquad (1)$$

The use of this relation without qualification implies that the value of I is independent of the distance at which the illuminance, E, is measured, or for which E is computed.

If measurements could be made at increasing distances, in a perfectly transmitting atmosphere (or, practically, if corrections are made for atmospheric losses), of the illuminance produced by a projector emitting a collimated beam, the value of the product $E\ d^2$ increases and approaches a limiting value.

The relation

$$I = \lim_{d \longrightarrow \infty} E\ d^2 \qquad (2)$$

may be considered as the defintion of I.

In practice it is usually found for these projectors that beyond a certain distance, the critical distance, there is no measurable change in the intensity computed by means of equation 1.*

The concept of intensity of a projector is valid, and useful, only when this critical distance is exceeded, for only then can illuminance at one distance be computed from measurements of illuminance at another distance.

*Such terms as "minimum inverse-square distance" and "limiting distance for the application of the inverse-square law" are frequently used for this distance. The term "critical distance" is used throughout this report as it is a brief, convenient term.

In the photometry of projectors, the photometric distance is usually made greater than this critical distance whenever it is feasible to do so. Hence, determination of the critical distance is important. The relation between critical distance for a collimated-beam projector and the dimensions of the projector and the source have been extensively treated[1-5]. For a parabolic reflector with a spherical source at the focus, the critical distance along the axis of the reflector is

$$d_c = [RF/r][1 + (R/2F)^2] \qquad (3a)$$

and for a disk source

$$d_c = [RF/r][1 + (R/2F)^2]/[1 - (R/2F)^2] \qquad (3b)$$

where

- d_c is the critical distance,
- R is the distance from the axis to the edge of the reflector,
- r is the radius of the source, and
- F is the focal length of the reflector.

Similar equations may be written for a spherical lens[4].

The relation given in the IES Lighting Handbook (Third Edition, page 4-35)[5] may be written as

$$d_c = Rd_e/r \qquad (3c)$$

where d_c and R are as before, and d_e is the distance from the focal point to the edge of the reflector.

For each of these conditions, d_c is given approximately by

$$d_c = RF/r. \qquad (4)$$

The error in computing d_c from this relation is insignificant in most practical applications, particularly when r is small in comparison to R, and F is two or more times R.

Note: When the measurement of illuminance is made at distances less than a critical distance, the value of the product Ed^2 will be less than I. This value of the product is frequently defined as the apparent intensity. In this report it will be so defined, and it will be represented by the symbol I'_x where the subscript is the distance for which the value of the apparent intensity, I', was computed. Note that I'_x is usually applicable in the determination of the illuminance on a surface only when the distance from the source to the illuminated surface is very nearly equal to the distance x.

Although many projectors are intended for use as collimated-beam projectors, with the source at the focus, there are many types of projectors where this is not the case. For example, in the lens cells of the Fresnel-Lens Optical Landing System, the source is between the focus and the lens so that the lens forms a virtual image of the source 150 feet behind the lens. Spread lenses used in conjunction with collimated beams form either real images in front of the spread lens or virtual images behind the spread lens. Hence, an analysis of the factors affecting critical distances and the relation between illuminance and distance of non-collimated beam projectors is of value. The purpose of this report is such an analysis.

Throughout this report rigor and detailed accuracy of analysis have been subordinated in order to obtain simple, easily interpreted relations free from the complicating, and often confusing, effects of correction terms. The relations developed are, therefore, approximations only. They are, however, sufficiently accurate to be used in a qualitative analysis of the performance of a projector system. The performance of the usual types of projector systems is significantly affected by such factors as errors in the shape of the optics and by the variations in luminance across the surface of the source. Such factors vary from unit to unit and can not be readily treated in a theoretical analysis. Quantitative analyses of projector systems can, and should, be made by direct measurement on a photometric range.

2. CASE I. PROJECTORS HAVING THE SOURCE BETWEEN THE FOCUS AND THE OBJECTIVE

2.1 Illuminance Produced by a Positive Lens

Consider the optical system shown in Figure 1.

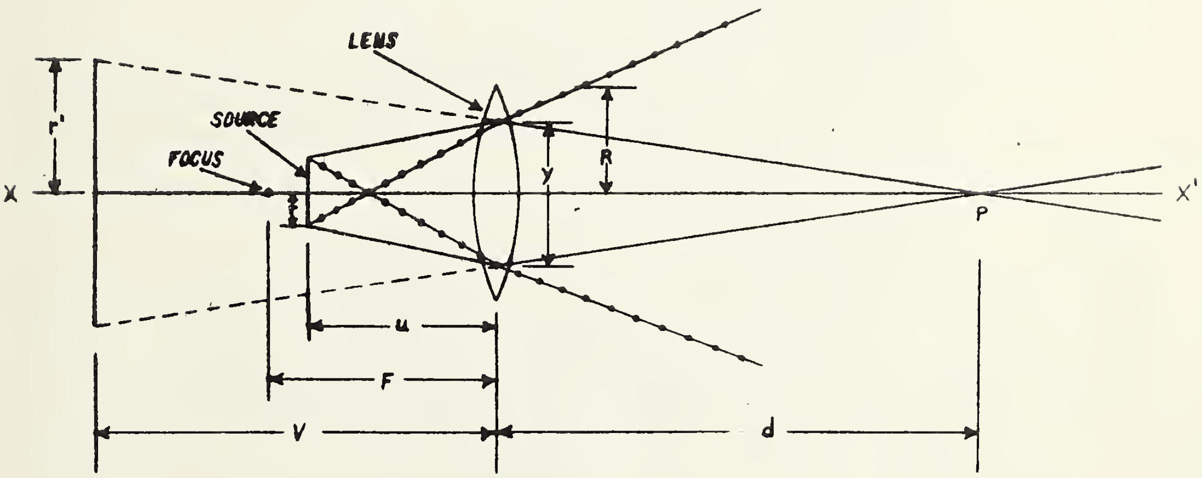

Figure 1. Optical system with source between focus and positive objective (lens). (not to scale)

The system is symmetric about the line xx' and consists of a positive lens of focal length F and radius R. (Throughout this analysis F is assumed to be large in comparison to R.)

A source, S, of radius r and of uniform luminance is placed at a distance u, less than F, from the lens. The system will form a virtual image, S', of the source at a distance v from the lens. Assume the observer's eye, or the photometer, is at the point P, a distance d from the lens. When the virtual image, S', is viewed through the lens from the point P and d is less than a critical distance d_c, to be determined later, a zone on the lens of radius y will be flashed, that is, appear to be the source of light.

The illuminance at P from this zone will be

$$E = \pi y^2 Lt/d^2 \qquad (d \leq d_c) \qquad (5)$$

where L is the luminance of the source S, and

t is the transmittance of the lens.

But $$y/d = r'/(v + d)$$

and $$r'/r = v/u = M$$

where

M is the magnification of the system.

Therefore

$$y = rvd/u(v + d),$$

or $$y = rMd/(v + d),$$

and $$E = \pi r^2 Lt/u^2(1 + d/v)^2, \qquad (d \leq d_c) \qquad (6a)$$

or $$E = \pi r^2 M^2 Lt/(v + d)^2, \qquad (6b)$$

and $$E = \pi (r')^2 Lt/(v + d)^2 . \qquad (6c)$$

Equations 6 are of the form

Illuminance = Constant / (Distance)^2

where the distances are measured, not from the lens, but from the position of the virtual image. Thus the condition of equation 1 is

met, and the numerator of the right-hand side is, by definition, an intensity.

From the definition of luminance

$$I_s = \pi r^2 L \tag{7}$$

where

I_s is the intensity of the source itself.

Hence $$E = I_s M^2 t/(v + d)^2. \qquad (d \leq d_c) \tag{8}$$

Similarly the intensity of the image, I_i, may be defined as

$$I_i = \pi (r')^2 Lt.$$

Then $$I_i = I_s M^2 t = E(v + d)^2. \tag{9}$$

If the radius of the virtual image, r', is less than the radius of the lens, the flashed zone will never fill the lens however large, d, is made. For this condition, equations 6, 8, and 9 are valid for all distances. However, if r' is greater than R and the distance, d, is equal to or greater than a critical distance, d_c, the entire lens will be filled and the illuminance at a point on the axis will be given by

$$E = \pi R^2 Lt/d^2. \qquad (d \geq d_c) \tag{10}$$

Equation 10 may be written as

$$E = I/d^2, \qquad (d \geq d_c) \tag{11}$$

where $$I = \pi R^2 Lt.$$

I is a constant and hence is, by definition, the intensity of the projector, applicable to all distances equal to, or greater than, the critical distance.

Note that when d is equal to d_c, equations 8 and 11 are both valid. Hence at this distance

$$I = I_i d^2/(v + d)^2. \qquad (d = d_c) \qquad (12)$$

When the distance, u, between the source and the lens becomes very nearly equal to the focal length of the lens, F, the distance to the virtual image, v, the magnification, M, and the radius of the image, r', become very large, becoming infinite when u is equal to F. From equation 6a

$$\lim_{u \to F} E = \pi r^2 Lt/F^2, \qquad (d < d_c) \qquad (13a)$$

or, combining with equation 7, we have

$$\lim_{u \to F} E = I_s t/F^2. \qquad (13b)$$

Thus, for a projector producing a collimated beam, the illuminance produced on a surface is constant, independent of distance, for distances less than the critical distance. (Note that this condition, in effect, means that the distance is measured from the image and is infinite, as is the magnification. However, the ratio of distance to magnification is equal to the focal length of the objective.)

2.2 Determination of the Critical Distance

Note from equation 10 that for distances greater than the critical distance, the axial illuminance varies inversely as the square of the distance and is independent of the position of the source with respect to the focus. However, the critical distance, d_c, is not independent of the position of the source with respect to the focus. It may be determined as follows.

If there is a critical distance, at this distance

$$y = R. \qquad (d = d_c)$$

Therefore,

$$R = rvd_c/u(v + d_c) = vd_c/u(1 + d_c v)$$

or

$$R = rMd_c/(v + d_c).$$

Hence

$$d_c = Ru/(r - uR/v), \tag{14a}$$

and

$$d_c = Rv/(rM - R). \tag{14b}$$

Note that as u approaches F, v becomes infinite, and equation 14a becomes the familiar relation used for collimated-beam projectors,

$$\lim_{u \to F} d_c = RF/r. \tag{4}$$

From equations 14a and 14b, note that d_c becomes infinite when

$$r = Ru/v = R/M, \tag{15}$$

for then the radius, r', of the virtual source is just equal to the radius of the lens, R, and the lens will appear flashed only when viewed from an infinite distance. If r is less than R/M, the value obtained for d_c is negative indicating that r' is less than R and that the virtual source will not completely fill the lens at any viewing distance. Hence, as stated earlier, under this condition equation 9 is valid for all distances.

3. CASE II. PROJECTORS HAVING THE FOCUS BETWEEN THE SOURCE AND THE LENS

3.1 Illuminance Produced by a Positive Lens

If the distance from the source to the lens, u, is greater than the focal length of the lens, F, the system will form a real image of the source a distance v from the lens, as shown in Figure 2.

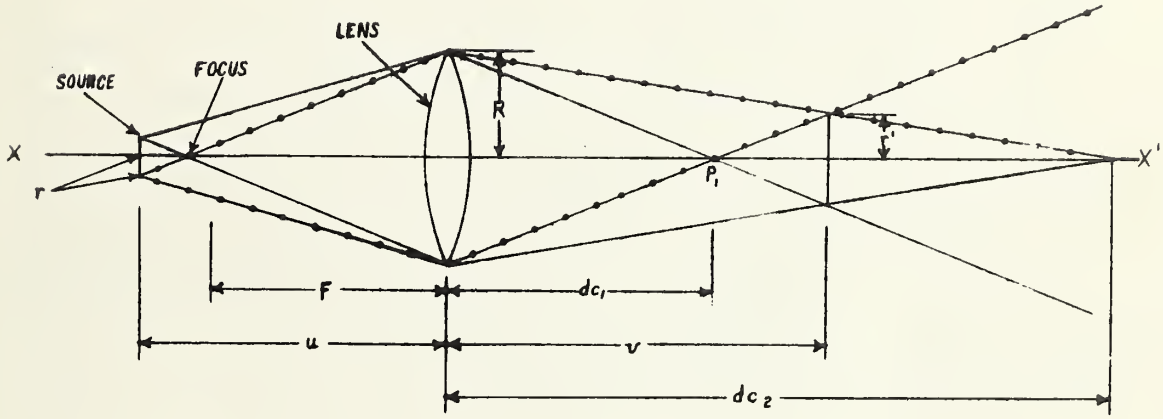

Figure 2. Optical system with focus between source and positive objective (lens).

Consider first the case in which the radius of the image, r', is less than the radius of the lens. Since light rays pass from every point on the lens to every point on the image, it is evident from Figure 2, on which the limiting rays from the edge of the lens are shown, that rays from the edge of the lens will cross the axis only in the region

between the points P_1 and P_2. In this region the entire lens will appear to be flashed. The illuminance at a point on the axis is, as for the flashed lens of Case I,

$$E = \pi R^2 Lt/d^2 . \qquad (d_{c1} \leq d \leq d_{c2}) \qquad (10)$$

The inverse square law holds with distance measured from the lens to the point of observation, or measurement, and as before

$$I = \pi R^2 Lt. \qquad (d_{c1} \leq d \leq d_{c2}) \qquad (11)$$

If the lens is viewed from a point P_3 on the axis between the point P_1 and the lens, the entire lens will not be flashed for rays from the outer part of the lens to the image cross the axis at, or beyond, the point P_1. Instead, a zone of radius y will appear flashed. As is evident from the limiting rays shown on Figure 3,

$$y = r'd/(v - d), \qquad (16a)$$

or

$$y = rMd/(v - d) = rd/u(1 - d/v). \qquad (16b)$$

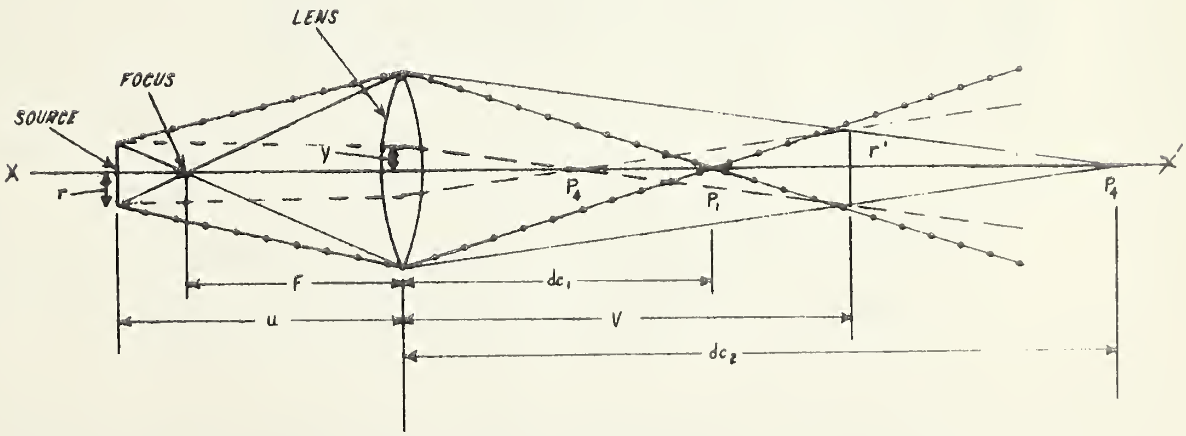

Figure 3. Flashed zone of an optical system forming real image; point of observation between objective and image.

The illuminance at P_3 will then be

$$E = \pi y^2 Lt/d^2,$$

or

$$E = \pi r^2 M^2 Lt/(v - d)^2. \qquad (d \leq d_{c1}) \qquad (17)$$

Similarly, for the point P_4 (see Figure 4) where d is equal to or greater than d_{c2}

$$y = rMd/(d - v), \qquad (18)$$

and

$$E = \pi r^2 M^2 Lt/(d - v)^2. \qquad (d \geq d_{c2}) \qquad (19)$$

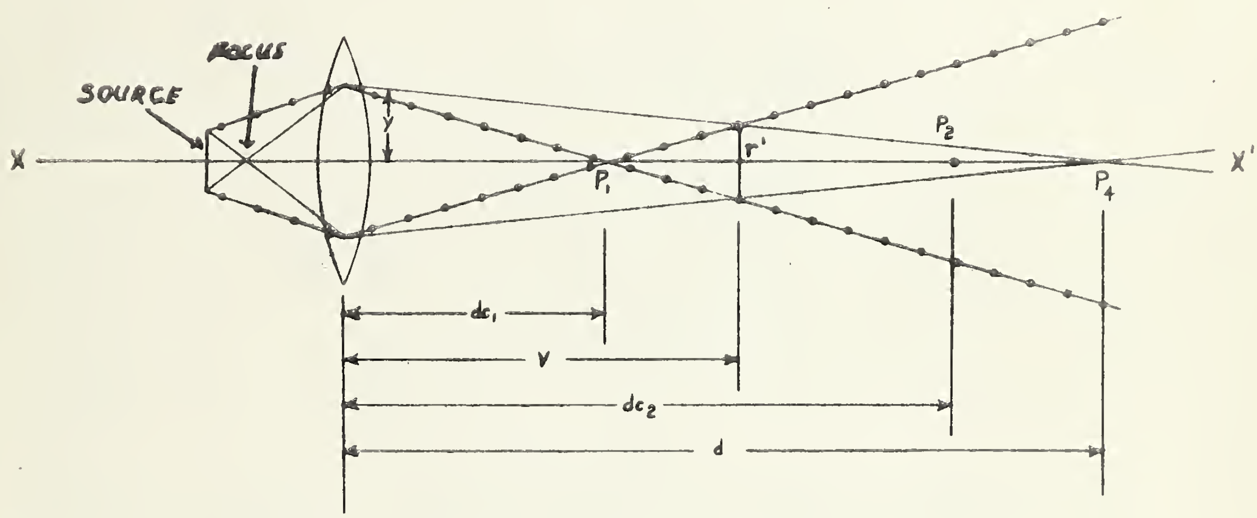

Figure 4. Flashed zone of an optical system forming real image; image between point of observation and objective.

Equations 17 and 19 are of the form of equations 6. Note that $(d - v)$ and $(v - d)$ are both the distance between the point of observation and the image formed by the lens.

Hence, as in equations 8 and 12,

$$E = I_s M^2 t/(v - d)^2, \qquad (20)$$

and

$$E = I_i/(v - d)^2. \qquad (21)$$

3.2 Determination of the Critical Distances for Lenses Forming Real Images

If r' is less than R, the critical distances, d_{c1} and d_{c2}, may be found by substituting R for y, and d_{c1} for d in equation 16, and d_{c2} for d in equation 18, obtaining

$$d_{c1} = Rv/(R + Mr) \qquad (22a)$$

or

$$d_{c1} = Ruv/(Ru + rv), \qquad (22b)$$

and

$$d_{c2} = Rv/(R - Mr) \qquad (23a)$$

or

$$d_{c2} = Ruv/(Ru - rv). \qquad (23b)$$

Since

$$r' = Mr,$$

when r' is equal to or greater than R, d_{c2} becomes infinite, and the lens is flashed for all distances greater than d_{c1}. For this condition equation 10 is applicable for all distances greater than d_{c1}.

When the source is placed at the focus,

$$u = F,$$

M and v become infinite, and equation 22 reduces to equation 4,

$$d_{c1} = RF/r, \tag{4}$$

and d_{c2} becomes infinite.

3.3 Measurement of Beam Angles

When the optical system forms an image, virtual or real, of the source instead of producing a collimated beam, and the point of observation or measurement is at a distance such that the objective of the system is not completely flashed, angles of observation as well as illuminance are determined using the distance between the image and the point of observation as a base, not the distance between the objective and the point of observation. That is, the vertex of the angle between the line of sight and the axis of the light is located at the image of the source, not at the objective of the projector. However, in practice, for example when a light is mounted on a goniometer, it is usually more convenient to measure angles whose vertices are at the projector. The relation between the two angles is easily found by the use of Figure 5.

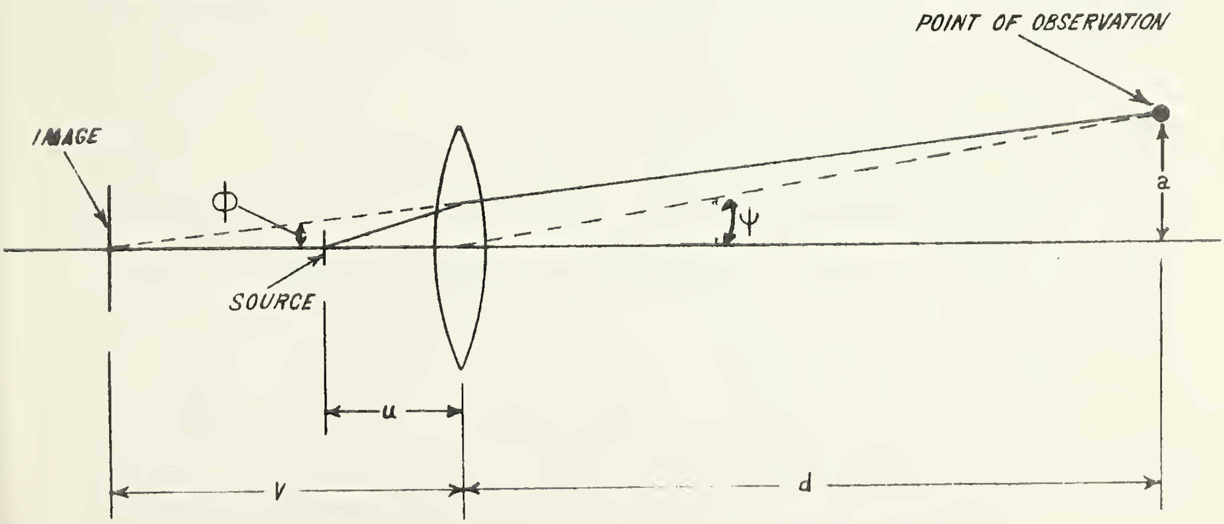

Figure 5. Correction of angles of optical system forming virtual image.

Since

$$\tan \psi = a/d$$

and

$$\tan \phi = a/(d + v),$$

for small angles

$$\psi \doteq a/d,$$

and

$$\phi \doteq a/(d + v).$$

Hence

$$\phi \doteq [d/(d + v)]\psi, \tag{24a}$$

or

$$\phi \doteq [d/(d + Mu)]\psi. \tag{24b}$$

3.4 Asymmetric Beams

Throughout the preceding discussion it has been assumed that the optical system is symmetric about the optical axis of the system. Thus, a disk or spherical source and an optical system composed of spherical elements have been assumed. In such systems the beam pattern is the same in every plane containing the axis of the optical system.

However, lights having unequal vertical and horizontal beam spreads are often required. These unequal beam spreads are obtained by using sources in which the width (the dimension in the z direction) and the height (the dimension in the y direction) are not equal and/or by adding cylindrical elements, with the axis of the cylinders parallel to the y or z axis. When cylindrical elements are used, the image of the source formed by a narrow section in the

y direction through the center of the lens will not be the same distance from the lens as the image formed by a narrow section in the z direction. That is, the projection of the image of the source on the x-y plane will not be in the same position as the projection of the image on the x-z plane. Under these conditions the equations of the type given below will give approximate values. The applicable equations of Section 2.2 are applied to determine if the viewing, or test, distance exceeds the critical distance using the dimensions of the projection on the x-y and the x-z planes, in turn, to determine d_{cy} and d_{cz}, respectively.

Thus, if the projection system forms a vertical image,

$$d_{cy} = \bar{Y}u_y/(\bar{y} - u_y\bar{Y}/v_y), \tag{14c}$$

or

$$d_{cy} = \bar{Y}v_y\ (\bar{y}M_y - \bar{Y}) \tag{14d}$$

where

$\bar{Y}$ is the dimension of the lens in the x-y plane,

$\bar{y}$ is the dimension of the source in this plane, and

the subscript y refers to quantities measured in the x-y plane.

Similar equations can be written for d_{cz} where $\bar{Z}$, $\bar{z}$, and the subscript z replace $\bar{Y}$, $\bar{y}$, and the subscript y.

If the observation distance is less than the critical distance in both the x-y and the x-z planes, that is,

$$d \leq d_{cy}$$

and

$$d \leq d_{cz},$$

equation 6 may be written as

$$E = \pi \bar{y}\bar{z} L t / u_y u_z (1 + d/v_y)(1 + d/v_z) \tag{6d}$$

or

$$E = \pi \bar{y}\bar{z} M_y M_z L t / (v_y + d)(v_z + d). \tag{6e}$$

Equation 8 becomes

$$E = I_s M_y M_z t / (v_y + d)(v_z + d), \tag{8b}$$

and equation 9 becomes

$$I_i = E(v_y + d)(v_z + d). \tag{9b}$$

Equation 9b may be considered as the definition of the intensity of the system for these conditions.

When $\quad d \leq d_{cy}$

and $\quad d \geq d_{cz}$,

combining equations 6 and 10, we have

$$E = \pi y \bar{z} M_y L t / (v_y + d)(d). \tag{25}$$

From equations 9 and 11 we have

$$I_i = E(v_y + d)(d). \tag{26}$$

Similar equations can, of course, be written interchanging the y's and the z's.

Similar expressions can, of course, be developed for systems which form real images in either or both planes by using the terms (v - d) or (d - v) in place of (v + d) as applicable.

3.5 Numerical Example

The principles developed in Section 3.4 were applied to a cell of the Fresnel-Lens Optical Landing System. The cell has a lenticular lens with vertical cylindrical elements forming a real image of the

source ahead of the unit to produce a wide horizontal spread, with the following dimensions (approximate):

$\bar{Z}$ = 0.165 inch

(2$\bar{Z}$ is the width of one cylindrical element)

$\bar{z}$ = 0.375 inch

v_z = 0.5 inch (computed from design beam spread of 40°)

u_z = 24 inches.

From these dimensions it is apparent that the first critical distance in the x-z (horizontal) plane, d_{cz1}, is very small. (This is usually the case in the plane in which there is an appreciable spread).

From equation 22b,

$$d_{cz1} = 0.165 \times 24 \times 0.5/(0.165 \times 24 + 0.375 \times 0.5)$$

$$d_{cz1} = 0.48 \text{ inch} = 0.04 \text{ foot.}$$

The second critical distance in the horizontal plane is also very small. From equation 23b,

$$d_{cz2} = 0.52 \text{ inch.}$$

In the x - y (vertical) plane, the optical system forms a virtual image of the source 150 feet behind the lens. Pertinent dimensions are as follows:

$\bar{Y}$ = 4 3/4 inches = 0.39 foot

$\bar{y}$ = 1/32 inch = 0.00285 foot

v_y = 150 feet

u_y = 2 feet

M_y = 75

$r'_y = M_y\bar{y}$ = 0.00285 x 75 = 0.213 foot

Since r'_y is less than Y, the lens will never be completely flashed in the y direction.

The relation between illuminance produced by the system and the "intensity" is

$$I_i = E\ (v_y + d)(d - v_z) \qquad (27)$$

or

$$I_i = E\ (150 + d)(d - 0.04)$$

where d is in feet.

Or, since v_z is very small compared to typical values of d,

$$I_i = E\ (150 + d)(d). \qquad (28)$$

Measurements were made at three distances of the illuminance produced by a Fresnel-lens Optical Landing System cell and the intensity was computed by equation 28 with the following results:[6/]

Photometric Distance (feet)	Peak Intensity, I_i (kilocandelas)
50	32.0
100	31.3
920	34.8

The consistency of the values of I_i is surprisingly good.

3.6 PRACTICAL CONSIDERATIONS

3.6.1 Simplified Equations

The equations which have been developed, although they describe the performance of a projector, are in forms which often are too complex to be used to best advantage by the photometrist, who, given the task of determining the intensity distribution of a projector, desires only sufficient information to permit him to choose a photometric distance greater than the critical distance. Often he is given very little information concerning the design parameters of the projector. The relations given below contain only parameters which are easily measured.

Consider first a projector which forms a virtual image of the source. Equation 14a may be written as

$$R/d_c = (r - uR/v)/u. \qquad (14a)$$

Then

$$R/d_c \leq r/u, \qquad (29)$$

or

$$d_c \geq Ru/r. \qquad (30)$$

Hence, if the photometric or observation distance is equal to or greater than Ru/r, illuminance varies inversely as the square of the distance from the objective.

Note that relation 29 becomes equation 4 when the source is at the focus. Note also that R/d_c is one-half the angle subtended by the objective of the projector at the critical distance and that r/u is one-half the angle subtended at the projector by the source.

The objective of a projector forming a real image of the source will appear flashed at all distances equal to or greater than the image distance if (and only if) the radius of the image is equal to or greater than the radius of the objective. Hence,

$$r' = rv/u \geq R$$

or

$$v \geq Ru/r. \qquad (31)$$

(The distance v can usually be determined readily from direct observation of the position of the image.)

Hence, when a real image is formed at a distance equal to or greater than Ru/r, illuminance varies inversely as the square of the distance from the objective for all distances greater than v. It is apparent from simple geometric relations (see Figure 2) that, if the condition of relation 31 is met, the objective will be flashed for distances in the range $v/2$ to v inclusive.

3.6.2 Application to Spread Lenses

Projectors often contain lens elements to increase the beam spread of the light from the projector. These lens elements form images, real or virtual, which are too small to flash the element. Hence, distance should be measured from the position of the image. (Note these elements usually are asymmetric and the image distances in the y and z planes are different. For simplicity, only one plane is treated here.) The geometric relations of these spread elements are illustrated in Figure 6.

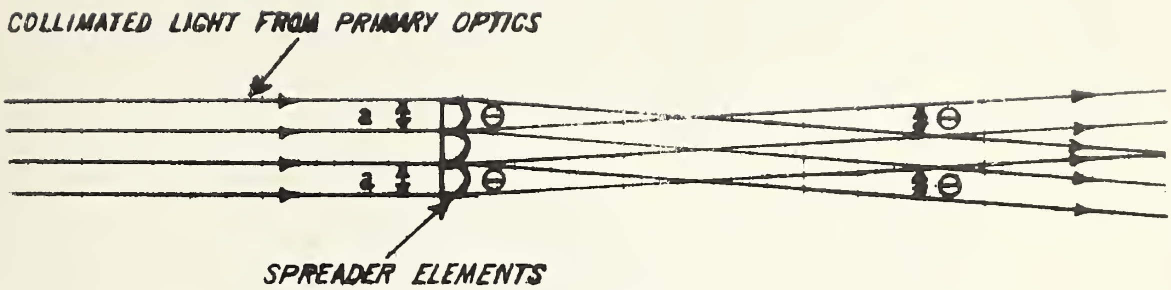

Figure 6. Geometric relations for a spreader element.

The angle θ, in degrees, is commonly called the beam spread of the lens. The relation between image distance and beam spread is

$$v = (a/2) \operatorname{ctn} \theta/2 \ . \tag{32}$$

The following approximation is sufficiently accurate for most purposes,

$$v \approx 60a/\theta \tag{33}$$

when θ is in degrees.

With the usual projectors v is so small in comparison to d that no significant error results when the term $(v + d)$ is replaced by d. For example, consider a 5° spread lens with cylindrical elements one inch wide. From equation 33, v is found to be 12 inches. As this is a rather extreme example, v will usually be considerably less than this. Note that when a spread lens is used, each spreader element acts as a separate projector, and that usually these elements

will not be completely flashed at practical observation distances. Under these conditions, distance should properly be measured from the images produced by these spreader elements. The criteria determining the photometric distance are then:

1. The photometric distance, d, should be large in comparison to the image distance v.
2. The angular subtense of the light unit from the point of measurement should be small in comparison to the beam spread of the light.

3.6.3 Incandescent Lamps as Sources

The filament of an incandescent lamp is not a source of uniform luminance. Hence, the overall dimensions of the filament should not be considered as the size of source in determining the critical distance. Instead, the size of the smallest separable element of the filament should be used[5/]. The smallest separable element will usually be the filament wire itself but at times it will be the helix. The dimensions of both the filament wire and the helix must be considered, as the separate turns of the filament wire may be discernible from the central part of the reflector while only the helix may be discernible from the outer part of the reflector[7/].

3.6.4 Application to Objectives of Small f - Number.

The development of the equations of this report is based on the assumption that the radius of the objective is small compared to its focal length. In practice, the radius of the objective is often as large or larger than the focal length.

Frequently, when applied to projection systems of the latter type, these equations yield results sufficiently accurate for engineering purposes if u and v are measured to the most distant part of the objective instead of along the axis of the projection system[5,7].

4. CONCLUSION

A review of the elementary theory of the photometry of projectors has been made. Use is made of simple geometric relations to develop equations relating distance with the illuminance produced by projectors. Particular attention has been given to projectors producing non-collimated beams and to the effect of spreader elements in the optical system. Equations have been developed relating critical distance to the radius of the source, the radius of the objective, and the magnification of the system. Equations applicable to asymmetric beams have been developed. The agreement between values computed from these relations and direct measurements is very good.

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