NATIONAL BUREAU OF STANDARDS REPORT

8145

The Use Of Short Codes For The Sorting Of Letter Mail

by

Philip R. Meyers

Technical Report

to

U.S. Post Office Department



U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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The Use Of Short Codes For The Sorting Of Letter Mail

by

Philip R. Meyers

Operations Research Section

Applied Mathematics Division

For

Data Processing Systems Division

Post Office Mechanization Project

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U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS . .

ABSTRACT

Using several alternative assumptions concerning the length of time required to learn to short code mail, we perform computations to determine how many addresses can profitably receive short codes. The relevant parameters are classified into those of little consequence and those of real importance. An analytic expression is derived for the volumes of mail directed to the most frequently occurring addresses. The important parameters are examined to determine how they influence the number of addresses which can receive short codes.

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1. INTRODUCTION

In a recent report [1], a study was made of the relative merits of two stage and one stage coding for a mechanical sorting system. In that report, the use of short codes was considered in determining answers to the major questions. At the time, the efficacy of such codes was left as an open question. We shall now attempt an answer to this question.

By a <u>short code</u> we mean a set of symbols which represent an address on a letter. These symbols are generally chosen for mnemonic convenience in representing a city and state or a street address within some city. They do not follow any straightforward application of some encoding rules. <u>We assume that a</u> coding system which uses short codes as well as encoding rules is harder to learn than one which just uses the latter.

Briefly, we wish to discuss the gains and losses which accrue to coding systems by the introduction of short codes. By their very definition, short codes imply a decrease in coding time. If this were the only consideration, then one would use only short codes and nothing else. However, there are other factors. There is a penalty that the coder pays in the longer training time necessary to learn the more complex coding system. Also, in the language of [1], we might expect <u>orientation time</u>, the average time between the end of the last stroke on one letter and the end of the first stroke on the next one, minus the time for one stroke, to be lengthened in a short code system. Considered in this light, the desirability of short codes and the extent of this desirability become meaningful questions.

In what follows, <u>short codes</u> or sometimes <u>the number of short</u> <u>codes</u> will mean the number of locations to which short codes can be profitably applied, as determined by a criterion which we shall establish later. We assume that this number will be employed in a very gross sense. That is, 200 locations will be short coded rather than 194.

- 1 -

2. ANALYSIS

Consider first the orientation time. If short codes are used, the coder must decide whether a particular letter gets a short code or a regular code, and then proceed to code the letter: it is this decision which increases his orientation time. Notice that every letter has this penalty added to it and not only those that receive short codes. One might say then that a fixed charge T . V is added to a coding system when short codes are introduced, where T is the extra orientation time and V is the volume of mail. It follows that short codes are useful to the extent that the coding time saved exceeds the sum of the extra learning time and the extra orientation time. One might suspect from the above that introducing one short code would be more expensive than none at all because of the extra orientation times added to all of the letters. However, if only one short code is used, the coder has two alternatives (short code or not). We suggest that T would be extremely small in this case and that one short code would be feasible. This reasoning was used in formulating the expressions for extra orientation time.

We have postulated three functions for learning times. They all assume that short codes will be assigned to locations in order of decreasing volume of mail received and that short codes are harder to learn as the number of them increases. Therefore the time required to learn the $n^{\underline{th}}$ short code is a monotonically increasing function of n.

We proceed with a formal development. Let S, represent the volume of mail for one year going to the $j^{\pm h}$ destination from a given city, where a destination is either an address in that city or is another city, and where destinations are ordered

- 2 -

decreasingly by volume. Multiply each S by the number of symbols which would be saved if that destination were to receive a short code rather than a normal one, and label the numbers so obtained S_n , ordered so that $S_i \geq S_{i+1}$ for all i. If now T represents the time required to stroke one symbol then $T_s \cdot S_n$ is the amount of time saved per year by the $n^{\underline{th}}$ most profitable use of short coding. We have assumed here that both <u>distant mail</u> (mail directed out of town) and <u>local mail</u> (mail which is to be delivered within the city under discussion) can receive short codes.

We consider the following forms for the function $R_n \equiv$ the time required to learn the $n\frac{th}{t}$ short code:

$$R_{1,n} = k_{1} \cdot n$$

$$R_{2j,n} = k_{2} \cdot \sum_{j=1}^{n} j$$

$$R_{3,n} = k_{3} \cdot \sum_{j=1}^{n} \varphi_{j}$$

where the k_{j} are suitable positive parameters and φ_{j} is in a technical sense the increment of information related to the $j^{\pm h}$ short code. (For a mathematical development of φ_{j} see Appendix A.) Extra orientation time is also considered as a function of n, and for it we consider the following forms:

With learning time $R_{l,n}$, $T_{l,n}V = \lambda$

With learning time $R_{2,n}$, $T_{2,n}V = \lambda$

With learning time $R_{3,n}$, $T_{3,n}V = \lambda' \cdot \phi_n$

where λ, λ' are suitable positive parameters.

By cost of learning short codes we mean solely the time spent learning these codes and do not mean to include any equipment or instructional costs. These are assumed to have been absorbed into the cost of learning the basic coding technique. We mean to compare this time with the time saved by employing a short code, to determine whether a short code is worth having. To make this comparison we assume that the cost of learning is to be amortized over a period of time and compare this cost with the time saved in coding mail over this period. Returning to our analysis we find that $T_s \cdot S_n - R_n - T_n \cdot V$ is the total value (possibly negative) of introducing the nth short code, where V is the total volume of mail. Thus

(1)
$$T_{s} \cdot S_{n} - R_{n} - T_{n} \cdot V > 0$$

is the condition which must be satisfied for the $n^{\underline{th}}$ short code to be worthwhile. Note that the two penalties which are assumed for short coding systems are different. The extra orientation time for successive new short codes remains constant or decreases. Learning time R_n on the other hand increases with each new short code. Now the coding time saved by each new short code decreases since the volumes to successive locations decrease. Thus the graph of savings per short code is a falling curve^{*}. What we seek is the point of intersection of the savings graph and the sum of the other two graphs^{**}. This intersection represents a situation in which savings just balance costs.

* For $T_{3,n}$ which decreases this would not generally be true. ** For $T_{3,n}$ some empirical tests have shown this intersection to be unique, except at n=10. Since this is less than the lowest number of short codes which are feasible under any circumstances, we feel safe in saying the intersection is unique.

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3. THE COMPUTATION

A program based on (1) was written for the IBM 7090 computer. Lists of the largest local and distant destinations for Baltimore mail were compiled from [2] and [3]. The entries of these lists were grouped in blocks of 10 and an average volume was computed for each block. The two new lists were then combined, in proportions corresponding to one and two stage mail, into lists decreasingly ordered by volume and the above mentioned average volumes were denoted $S_n^{'}$. These were then used to compute φ_n and $R_{3,n}$. The comparison expressed by (1) was made until the quantity became negative at which point the number of blocks was recorded and the procedure started again on a new set of data.

The values used for N_L , the number of strokes saved by short codes, were 6 and 9 for street addresses and 4 for cities and states. We used three values for stroke time $T_s:0.2$ sec., 0.25 sec. and 0.33 sec.

The estimates for learning time and extra orientation time were less well known than the other parameters. Therefore we chose much wider ranges for these parameters. We chose values between 8 minutes and 95 minutes for $R_{1.200}$.

The sequence φ_j , Fig l, is decreasing and hence $R_{3,n}$ is an increasing sequence with decreasing slope, i.e. $R_{3,n}$ is convex upward. $R_{1,n}$ is a straight line and $R_{2,n}$ is convex downward. Fig. 5 shows all three sequences where $R_{1,n}$ and $R_{2,n}$ have been normalized so that

 $R_{1,200} = R_{2,200} (R_{1,200} = R_{1,n} \text{ with } n = 200)$ and $R_{1,n}$ and $R_{3,n}$ have normalized so that

 $\sum_{n=1}^{200} R_{1,n} = \sum_{n=1}^{200} R_{3,n}$.

- 5 -



We might also have normalized so that

200
$$\sum_{n=1}^{200} R_{1,n} = \sum_{n=1}^{200} R_{2,n} = \sum_{n=1}^{200} R_{3,n}$$

or

$$R_{1,200} = R_{2,200} = R_{3,200}$$

but chose not to. (See Appendix B for an analysis of these situations.)

For extra orientation times we chose values from 0.0 seconds, corresponding to no extra orientation, up to 0.4 seconds, corresponding to λ =0.002 seconds.

In [2] we found that .509 of the total outgoing mail remains within Baltimore. Using this to get proportions for one and two stage coding we found that S_n' approximately satisfies

2a)
$$\log S_n = -0.88 \log n - 1.27$$

for two stage coding and

2b)
$$\log S_n = -0.90 \log n - 1.07$$

for one stage. (See Appendix C for the details of the computation and the range of validity.)

It should be noted that S_n has no relation to $R_{1,n}$ or $R_{2,n}$ but that it is intimately connected with $R_{3,n}$. If the ability to learn something is dependent on the frequency with which it appears then $R_{3,n}$ has a validity which the others lack.

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4. DISCUSSION OF RESULTS

For moderate values of n, (in this case $n \leq 250$) the functions $R_{1,n}$ and $R_{2,n}$ have similar values. (See Appendix B.) Further, within the ranges under consideration for k_1 and k_2 we find that 250 is close to an upper bound for the number of short codes. See Tables 1-6. Thus there are few differences in the number of short codes as determined by $R_{1,n}$ and $R_{2,n}$. For this reason we shall discuss $R_{1,n}$, the simpler of the two, with the understanding that $R_{2,n}$ is also under consideration.

The maximum number of short codes with $R_{l,n}$ is 290 (260 for $R_{2,n}$), and hence 2 character short codes are feasible in all cases. For $R_{3,n}$ this maximum is 410, Tables 7-9. This is well below the 676 alphabetic combinations possible but it is large ehough so that one might consider longer short codes. We say this because with 410 short codes many more of the combinations would be used and the probability of costly miscodings, as opposed to invalid codings (which a machine would reject), is high. We shall demonstrate below that a small change in N_L will not seriously affect our results. Thus we continue on the assumption that only 2 character codes are being used.

The values chosen for N_L bound those under consideration for a regular code, [5]. Even with these values, short codes do not change greatly. The maximum change is 60 short codes and this change represents only .25 of the number of short codes considered there, See Tables 1-9. On this basis we assert that N_L is of minor importance in determining short codes.

In the light of the last remark, we assume that the functional form demonstrated for S_n holds for S_n as well. This is not accurate in that each S_n is multiplied by its appropriate N_L . Fig 8 shows log S_n vs log n.

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						TABLI	E 2				
Tl,	n ^(sec)					$T_s = $	25				
	0.00	7	10	13	18	26	7	10	13	18	26
	0.04	6	9	12	16	21	7	10	12	16	22
	0.10	6	8	10	13	16	6	8	10	13	16
k	0.20	5	6	7	8	10	5	6	8	9	10
	0•30	3	5	5	6	6	4	5	5	6	7
*	0.40	3	3	3	3	4	4	4	5	5	5
R _{l,} ı	n(min)	95	48	32	16	8	95	48	32	16	8
^R 1,1	n ^(min)	95	48	32	16	8	95	48	32	16	8
^T l,1	(sec)	1					·····				
	Q . Q	0	9	11	16	23	7	9	12	17	24
	0.04	5	8	10	14	18	7	8	10	15	20
	0.10	5	7	8	11	13	6	8	8	11	14
	0.20	4	5	5	6	8	5	6	7	8	8
	0•30	3	4	4	λ.	4	4	5	5	6	7
	0.40	3	3	3	3	3	4	4	5	5	5

TABLE 1

Т	=	•	2
0			

T _{l,n} (sec)			l Stag	ge N _L =	6	l Stage N _L =9				
0.00	6	8	10	15	21	6	5 9	12	16	23
0,04	6	8	9	13	17	6	5 8	10	14	18
0.10	5	7	8	9	11	5	7	8	11	14
0.20	4	5	6	7	7	Ц	- 5	6	7	8
0.30	4	24	5	5	5	4	- 4	5	5	5
0.40	3	4	4	4	5	3	3 4	4	4	5
										en ober desensed over a son wa
R _{l,n} (min)	95	48	32.	16	8	. 9	95 48	32	16	8

2 Stage N_L=6

2 Stage $N_{L}=9$

T_{l.n}(sec)

11 و ا										
0.00	5	8	10	14	21	6	9	11	16	23
0.04	5	7	9	12	16	6	8	10	14	18
0.10	4	6	7	9	11	5	7	8	11	13
0.20	3	4	4	5	6	3	5	6	7	8
0.30	3	3	3	3	4	3	3	3	24	5
0.40	3	3	3	3	3	3	3	3	3	3

Tables 1-9 Short Codes x 10⁻¹.

All Have Same Format.

T _s	=.	33

T_{l,n}(sec)

0.00	8	12	15	21	29	8	12	15	21	29
0.04	8	11	14	18	24	8	11	14	19	25
0.10	7	10	12	16	19	7	10	12	16	20
0.20	6	8	9	12	13	6	8	10	12	14
0.30	5	6	7	8	9	5	6	8	9	10
0.40	4	5	5	6	6	5	5	6	6	7
R _{l,n} (min)	95	48	32	16	8	95	48	32	16	8
-										

Τl,	n ^(sec)						
	0.00	7	11	13	19	26	8
	0.04	7	10	12	16	21	7
	0.10	6	8	10	13	17	7
	0.20	5	6	8	9	11	6
	0.30	4	5	5	6	7	5
	0.40	3	4	4	4	5	5

8 8

					TABLE	4				
					$T_s = .2$	2				
T _{l,n} (sec)					5					
0.00	9	12	14	18	22	10	11	14	18	22
0.04	9	11	13	16	19	9	11	13	16	19
0.10	8	9	11	13	15	8	10	10	12	15
0.20	6	7	7	8	8	6	7	8	8	9
0.30	4	4	5	5	5	5	5	5	5	5
0.40	3	3	3	3	3	4	4	4	4	4
	1									
R _{l,n} (min)	95	48	32	16	8	95	48	32	16	8

T_{l.n}(sec)

⊥,n` `					•					
0.00	9	12	13	16	21	9	11	13	17	21
0.04	8	. 10	12	14	17	8	10	12	15	18
0.10	7	8	9	11	12	7	8	10	11	13
0.20	5	5	6	6	6	6	7	7	7	8
0.30	3	4	4	<u>)</u>	4	5	5	5	5	5
0.40	3	3	3	3	3	4	4	4	4	4

					TABLE 5	5				
T _{l,n} (sec)					T_=-2	5				
0.*00	10	13	15	19	24	10	13	15	19	24
0.04	10	12	14	17	21	10	12	14	17	21
0.10	8	11	12	15	16	9	11	12	14 14	17
0.20	7	8	9	10	11	7	8	9	10	11
0.30	5	6	6	6	7	6	6	6	7	8
0.40	3	4	4	4	4	5	5	5	5	5
R (min)	95	<u>)</u> 18	20	16	8		1.8	20	16	8
"l,n"	<u> </u>	+0		10		2)	40	<u> </u>		
T _{l,n} (sec)										
0.00	9	12	14	17	22	10	12	14	18	23
0.04	9	11	12	15	19	9	11	13	16	20
0.10	8	9	10	12	14	8	10	11	13	15
0.20	6	6	7	8	9	7	8	8	8	9
0•30	4	4	4	5	5	6	6	6	7	7
0.40	3	3	3	3	3	5	5	5	5	5

TABLE 6

T_=	•	33
-----	---	----

T_,n(sec)					G					
0.00	11	15	16	21	26	11	14	17	21	26
0.04	11	14	15	19	23	11	14	16	19	24
0.10	10	12	14	16	20	10	12	14	17	20
0.20	8	10	11	13	15	9	10	11	13	14
0.30	7	8	8	9	10	7	8	9	10	10
0.40	6	6	6	7	7	6	6	7	7	8
R _{l,n} (min)	95	48	32	16	8	95	48	32	16	8
T _{l,n} (sec)					1					
0 \$00	10	13	15	19	24	11	13	16	20	25
0.04	10	12	14	17	21	10	13	15	18	22
0.10	9	11	12	15	17	9	11	13	15	18
0.20	7	9	10	11	12	8	9	10	11	12
0.30	6	6	6	7	8	7	8	8	8	8
0.40	4	4	5	5	5	6	6	7	7	7

					TAB	LE 7				
T _{l,n} (sec)					Ts	=.2				
0.00	4	6	9	16	25	4	8	11	18	29
0.04	4	6	9	16	25	4	8	11	18	28
0.10	4	6	9	15	24	4	8	10	18	28
0.20	3	6	8	14	22	4	7	10	17	26
0+30	3	5	7	13	20	3	7	9	16	26
0.40	3	5	6	12	19	3	6	8	15	24
R _{l,n} (min)	95	48	32	16	8	95	48	32	16	8
T _{l,n} (sec)										
0.00	5	7	9	16	28	5	8	11	19	31
0.04	5 5	7 7	9 9	16 16	28 27	5	8 8	11 11	19 19	31 30
0.04 0.10	5 5 5	7 7 7	9 9 9	16 16 16	28 27 26	5 5 5	8 8 8	11 11 10	19 19 18	31 30 29
0.04 0.10 0.20	5 5 5 5	7 7 7 6	9 9 9 8	16 16 16 15	28 27 26 25	5 5 5 5	8 8 8 7	11 11 10 10	19 19 18 17	31 30 29 29
0.00 0.04 0.10 0.20 0.30	5 5 5 5 4	7 7 6 6	9 9 9 8 8	16 16 16 15 14	28 27 26 25 24	5 5 5 5 5 4	8 8 8 7 7	11 11 10 10 10	19 19 18 17 16	31 30 29 29 29

TABLE	8

Т	=	25
S		

T _{l,n} (sec)		2			5						
0 *00	4	8	11	18	29	5	9	13	21	33	
0.04	4	7	11	18	29	5	9	13	21	33	
0.10	4	7	10	18	28	5	9	13	21	33	
0.20	4	7	10	17	27	5	8	12	20	31	
0.30	4	7	10	16	25	5	8	12	20	30	
0.40	4	6	9	14	23	4	8	11	18	28	
										1	
R _{l,n} (min)	95	48	32	16	8	95	48	32	16	8	
T _{l,n} (sec)											
0,00	5	8	11	20	32	. 5	10	13	22	36	
0.04	5	8	11	20	31	5	10	13	22	36	
0.10	5	8	11	19	30	5	10	13	22	36	
0.20	5	8	11	18	29	5	9	12	20	34	
0.,30	5	8	10	17	28	5	7	12	20	33	

					TABLE	9				
T _{l,n} (sec)					T _s =.3	33				
0.00	5	10	14	22	35	7	12	10	26	40
0.04	5	10	13	22	35	7	12	16	26	39
0.10	5	10	13	21	35	6	12	16	25	39
0.20	5	9	12	20	33	6	11	15	25	38
0•30	5	9	12	20	32	6	11	15	24	38
0.40	5	9	12	19	31	6	10	15	23	35
R (min)	05	1.8	20	16	8	05	<u>),8</u>	20	16	8
"l,n("""")				10	0	2)	40	<u>ے</u> ح		
T _{l,n} (sec)										
0.00	6	10	14	25	39	7	12	16	27	41
0.04	6	10	14	24	39	7	12	16	27	41
0.10	6	10	14	24	37	7	12	16	26	41
0.20	6	10	13	22	37	6	12	16	25	41
0.30	6	9	13	22	36	6	11	15	25	40

6 10

15

23 39

6 9 13 20 34

0.40

We shall say that a parameter is <u>important</u> if short codes can double as the parameter takes on its allowed values. With this criterion, T_s is not an important parameter, see Figure 2. This seemingly arbitrary distinction does in fact effectively divide the parameters into classes whose influence differs by as much as an order of magnitude.

In examining Figure 2 one should not be misled by the near linearity of some of the graphs. This is due to the existence of only three points on the graphs. One can show (see Appendix D) that linear variation of short codes with T_s would imply a form for S_n different from the observed one.

If we accept T_s and N_L as being relatively unimportant^(*) then we can simplify the analysis for $R_{l,n}$. If

$$\log S_n = a \log n + b$$
 then $S_n = Kn^a$ for some K.

We seek n such that

(3)
$$Kn^{a} = k_{1}n+\lambda$$

or

(4)
$$K_n^{a-1} = k_1 + \frac{\lambda}{n}$$
.

Now a \approx - 1.0 (see eqs(2a) and (2b)) and we have

$${\rm Kn}^{-2} \approx {\rm k_l} + \frac{\lambda}{n}$$
 .

(*) We need really only ignore N_{T} to be able to proceed.





FIGURE 2.

Thus changes in $T_{l,n}$ are divided by n and as n increases $\Delta T_{l,n}$ becomes less important. On the other hand, the influence of $R_{l,n}$ is not mitigated through division by n, but persists via k_l . Thus as n increases and $\Delta S_n = K[n^{-2}-(n+1)^{-2}]$ decreases, the influence of changes in K also increases. This is readily observed in Tables 1-3.

It is easiest to understand how $T_{l,n}$ influences short codes if we examine it in conjunction with S_n . Recall that $T_{l,n}$ is constant. Thus if we consider the intersection of its graph with that of S_n we see that $T_{l,n}$, if positive rather than zero,

determines an upper bound $S_n = \frac{T_{l,n} \cdot V}{T_s}$ for the number of

short codes. If $T_{l,n}$ is large then this upper bound is small and the difference between successive values of $S_n (\approx K [n^{-2} - (n+1)^{-2}])$ is relatively large. If we now add $R_{l,n}$ to $T_{l,n}$ then the following statements are true. First, since there are only a few codes the difference in $R_{l,n}$ due to changes in k_l is not very large. Second, since the difference between successive values of S_n is large the number of short codes does not change very much with the assumed form of the learning time function.

If $T_{l,n}$ is small then the upper bound determined by $T_{l,n}$ is large and thus the difference between successive values S_n is small. In this case $\Delta R_{l,n}$ does build up and the number of short codes varies significantly with $R_{l,n}$.

It is interesting to determine the point at which $T_{l,n}$ ceases to be an important factor. For this we use the measure of importance introduced above. We shall say that $T_{l,n}$ is <u>not</u> the overriding factor when for fixed $T_{l,n}$ short codes can double as k_l varies. Of course this value of $T_{l,n}$ (the one at which it ceases to be overriding) depends on S_n as well as $R_{l,n}$. In

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FIG. 3 B= R In C= T_S

FIG. 40.	FIG.4b.
B= T _{In}	B = T 3 n
C = T _S	C = T _S



Table 13 we list the approximate values of $T_{l,n}$ at which this doubling occurs. To a certain extent $R_{l,n}$ exhibits the same properties as $T_{l,n}$. Thus if learning time is sufficiently large the maximum number of short codes is low although not as low as with the largest value of $T_{l,n}$. Also, as learning time decreases, the importance of $T_{l,n}$ increases. If we again apply the criterion that the number of short codes double, in this case as $T_{l,n}$ varies for fixed $R_{l,n}$, then Table 14 shows the values at which $R_{l,n}$ is no longer the overriding factor.

If we now consider the information theoretic approach (involving R_{3,n} and T_{3,n}), we find that T_{3,n} has only a negligible influence on short codes. At n = j, R_{3,n} = k₃ $\Sigma \quad \phi_i$ and T_{3,j} = $\lambda_3 \phi_j$. Thus orientation time adds an amount proportional to the last and hence smallest (see Appendix A) of the terms ϕ_j whose sum is R_{3,j}. It is clear therefore that T_{3,j} is not influential.

As defined $R_{3,n}$ is proportional to the amount of information in a system with n short codes. As mentioned earlier $R_{3,n}$'s dependence on S'_n gives it a validity the others lack.

In Figure 5 we have plotted the three learning functions. We see that $R_{3,n}$ drops off sharply from the other functions and thus it is not surprising that $R_{3,n}$ allows a much broader range for short codes. If it proves an adequate representation of the true shape of the learning curve it will allow for very sensitive testing when better parameter values are established.

The total net saving (the difference between coding time saved and learning costs) associated with a coding system which uses short codes is certainly less than the coding time saved. This last is concentrated in the first few short codes used. If we use the approximation $S_n = \frac{K}{n^2}$ then 90% of the coding time saved by using 400 short codes n^2 is saved by using 50 short codes

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,



TABLE 13

^T l,n	(0.0	sec	0.04	sec	0.10	sec "	0.20	sec	. C	• 30	sec
l Stage N _I	_=6	16	min	16	min	8	min					
NI	=9	32	min	16	min	16	min	18	min			
2 Stage N $_{\rm I}$	_=6	32	min	16	min	16	min	8	min	-		
NI	_=9	16	min	16	min	16	min	32	min			
l Stage N _I	_=6	16	TT	32	tt	16	TT	8	TT		16	min
N	=9	16	11	16	11	16	TT	8	11			
2 Stage N _L	_=6	16	TT	32	11	16	TT .	8	rr			
N	_=9	16	11	16	TT	8	tT					
l Stage N _L	=6	16	11	16	11	16	**	16	11			
N _T	=9	16	11	16	11	16	11	16	11		8	11
2 Stage N _L	=6	16	ff	16	tt	16	tt	8	11			
\mathbb{N}_{L}	=9	16	tt	16	TT	8	**					

TABLE 14	
----------	--

R _{l,n}	(8 min	16 n	nin	3 2 r	nin	48 r	nin	95 m	iin
l Stage N_=	6	0.l sec	0.1	sec	0.1	sec	•04	sec	•04	sec
N _L =	9	0.l "	0.1	tt	0.1	**	•04	ŤŤ	•04	TT
2 Stage N _L =	6	0.2 "	0.1	ŦŤ	0.1	ff	0.1	ŤŤ		
N_{L} =	9	0.2 "	0.2	TT	0.2	11	0.1	TT	•04	TT
l Stage N_=	6	0.2 "	0.3	ŧT	0.2	ŤŤ	0.2	TT	0.1	TT
N _L =	9	0.2 "	0.1	TT	0.1	TT	0.1	TT		
2 Stage N _L =	6	0.2 "	0.2	ff	0.1	TŤ	0.1	TT	0.0	TT
N _L =	9	0.l "	0.1	ŤŤ	0.04	ŤŤ	0.1	ŤŤ		
l Stage N _L =	6	0.2 "	0.2	**	0.1	tt	0.1	11	0.04	TT
N _L =	9	0.2 "	0.2	TT	0.1	TT	0.1	ŤŤ		
2 Stage N_=	6	0.2 "	0.2	ŤŤ	0.2	ŤŤ	0.1	ŤŤ	0.1	TT
NL=	9	0.1 "	0.1	ff	0.04	ŤŤ	0.04	**		

and 95% of this time is saved by using 90 short codes. However an examination of the actual values S_n shows that this is not a very accurate estimate. The approximation $S_n = K/n^2$ is heavily weighted by the first term and thus for our use here K/n^2 is not a sufficiently accurate choice for S_n .

We cannot use the analytic expression for S_n but we can examine the data and determine the short codes which yield 90% of the gross and the net savings, Table 15. We first notice that the same number of short codes yield 90% of the gross saving in each of the four cases, one stage, two stage, $N_L = 6$ and $N_L = 9$. That is, 290 short codes yields 90% of the saving of 400 short codes in each of these cases. Similarly 160 short codes gives 90% of the saving of 200 short codes. These are gross savings.

For all three learning times 90% of the net saving attainable by using n short codes is attained by between 50 and 60 per cent of n short codes. We have examined two cases for $R_{1,n}$ and $R_{2,n}$ namely with no orientation time and with moderate orientation time. For $R_{3,n}$ this was not necessary since the orientation time is negligible.

Finally, two facts should be emphasized. First all of the results are only approximations to the results that a strict application of (1) would give. This is so because the data were handled in blocks of ten and therefore the number of short codes is only correct to the 10's place. That is, when an answer of 70 codes appears, we can only be sure that at least 60 and fewer than 70 codes is the number of codes one should use. From the kind of data available, it seems clear that any answer which had more accuracy would be formally better but of no additional practical value. We have employed this device so as to cut down on computer time and for the above reason we feel that there is no undue loss of accuracy.

TABLE 15

90% Of The Gross And Net Savings For Using

	100 Short Codes	200 Short Codes	300 Short Codes	400 Short Codes
.9X Gross				
Saving is				
Attained by	80 s.c.	160 s.c.	220 s.c.	290 s.c.
.9X Net				
Saving				
R _{l,n} , T=0.0	60 s.c.	100 s.c.	170 s.c.	
R _{l,n} , T=0.1	60 s.c.	100 s.c.		
R _{2,n} , T=0.0	60 s.c.	120 s.c.		
R _{2,n} ,T=0.1	60 s.c.	110 s.c.		
R _{3,n}	50 s.c.	110 s.c.	160 s.c.	200 s.c.

•

Second, the different learning times and orientation times give very different results. These functions were chosen on the basis of our feelings and discussions with other who have some competence in the field of training people. They are not a definitive set of choices but rather likely candidates. One needs to determine which (if any) of the models is appropriate before using the results.

A final admonition involves our choice of n = 200 as the point of normalization. We chose n = 200 not because we had any special information about it, but rather because it was about one half as many codes as we intuitively expected as an upper limit. To this extent we have prejudged our results. This makes no difference for the linear case ($R_{1,n}$ and $T_{1,n}$), but for the non-linear ones it does make a difference.

APPENDIX A

The Increment of Information

Let k be the number of short codes assigned. If we now consider all locations other than those labelled S_j , j = 1, 2, ..., kas receiving the same code, then we can say that the information associated with such a coding system is

$$I_{k} = -\sum_{j=1}^{k} s_{j}' \log_{2} s_{j}' - (1 - \sum_{j=1}^{k} s_{j}') \log_{2} (1 - \sum_{j=1}^{k} s_{j}')$$

See for instance [4].

We now consider short coding one additional location and form

$$\begin{split} \varphi_{k+1} &= I_{k+1} - I_{k} \\ &= -\sum_{j=1}^{k+1} s_{j}^{'} \log_{2} s_{j}^{'} - (1 - \sum_{j=1}^{k+1} s_{j}^{'}) \log_{2} (1 - \sum_{j=1}^{k+1} \log_{2} s_{j}^{'}) - \\ &\{ -\sum_{j=1}^{k} s_{j}^{'} \log_{2} s_{j}^{'} - (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} (1 - \sum_{j=1}^{k} \log_{2} s_{j}^{'}) \} \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{\substack{i=1\\k+1\\j=1}}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{\substack{j=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) - \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{\substack{i=1\\k+1\\j=1}}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{\substack{j=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) - \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{\substack{i=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{\substack{j=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) - \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{\substack{i=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{\substack{j=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{\substack{i=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{\substack{i=1\\k+1\\l-\sum}}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + (1 - \sum_{j=1}^{k} s_{j}^{'}) \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_{j}^{'} \right) + \\ &= -s_{k+1}^{'} \log_{2} \left(\sum_{j=1}^{k} s_$$

Thus ϕ_k is the increment of information related to the $k^{\underline{th}}$ short code.

APPENDIX B

The Normalization of R_{1,n}, R_{2,n} and R_{3,n}

The normalization $R_{1,200} = R_{2,200}$ implies that 200 $k_1 = k_2 \sum_{j=1}^{200} j = k_2 \frac{200(201)}{2}$

or

$$k_1 = \frac{201}{2} k_2$$
.

If we normalize so that

$$\sum_{j=1}^{200} R_{1,j} = \sum_{j=1}^{200} R_{2,j} \text{ then }$$

we have

$$k_{1} \frac{(200)_{201}}{2} = k_{2}' \frac{\sum_{j=1}^{200} j(j+1)}{2} = k_{2}' \frac{200(201)(202)}{2\times3}$$

or

$$k_1 = \frac{202}{3} k_2'$$
.

Thus

$$k'_2 = \frac{3}{2} \frac{201}{202} k_2 \approx \frac{3}{2} k_2$$
.

With this normalization we find that

$$R_{1,n} = R_{2,n} \text{ at } j \approx 133, \text{ since if}$$

$$k_{1}j = k_{2}' \frac{j(j+1)}{2} \text{ then}$$

$$\frac{202}{3} k_{2}'j = k_{2}' \frac{j(j+1)}{2} \text{ and}$$

$$j+1 = \frac{404}{3} \text{ or } j \approx 133 \text{ .}$$

Thus $R_{l,n} < R_{2,n}$ for n > 133. It follows that the number of short codes one should use with $R_{2,n}$ under this normalization is bounded above by the number with $R_{l,n}$ whenever n > 133. See Figure 6a.

If we had normalized so that

$$R_{1,200} = R_{3,200}$$

then $R_{3,j} > R_{1,j}$ for j < 200 and the number of short codes as determined by $R_{3,j}$ (for zero extra orientation time) is less than the number as determined by $R_{1,j}$. More important,

$$R_{3,50} > .54 R_{1,200}$$
 and $R_{3,100} > .75 R_{1,200}$

so that short codes with $R_{3,n}$ are considerably less than those with $R_{1,n}$ for n < < 200. For this reason we did not choose the above mentioned normalization. See Figure 6b.

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APPENDIX C

The Functional Form of S

Tables 10-12 are the values for S_n' and S_n for one and two stage coding with $N_L=6,9$ symbols. Figure 7 is a plot of log S_n' vs log n. A least squares straight line was fitted to this data in the standard manner. Thus if log $S_n' = y$ and log n = x, the line y = ax+b is determined by the equations

$$b \cdot N + a \cdot \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i$$

$$b \cdot \sum_{i=1}^{N} x_{i} + a \cdot \sum_{i=1}^{N} x_{i}^{2} = \sum_{i=1}^{N} x_{i} y_{i}$$

where N is the number of data points and x_{i} and y_{i} are the respective values.

For one stage we have

40 $b_1 + 47.92 a_1 = -92.90$ 47.92 $b_1 + 63.0216 a_1 = -116.2328$

and for two stage

40 $b_2 + 47.92 a_2 = -85.76$ 47.92 $b_2 + 63.0216 a_2 = -107.7713$.

From these equations we have $a_1 = -0.8962$, $b_1 = -1.0703$ and $a_2 = -0.8798$, $b_2 = -1.2685$.

	TABLE 10a						
s _n F	'OR ONE STAG	E		s, F	OR TWO STAGE		
	BALTIMORE				BALTIMORE		
l.	7•77 %	21.	0.44	l.	5.14 %	21.	0.39
2.	3.01	22.	0.43	2.	2.90	22.	0•37
3•	2•38	23.	0.41	3•	1.99	23.	0.35
4.	2.19	24.	0•39	4.	1.57	24.	0•33
5•	1.86	25.	0.38	5.	1.23	25.	0.32
6.	1.64	26.	0•35	6.	1.09	26.	0+31
7•	1.38	27.	0•34	7•	0•93	27.	0.30
8.	1.12	28.	0+33	8.	0.91	28.	0.29
9•	0•99	29.	0.32	9•	0.78	29.	0.28
10.	0.85	30.	0.32	10.	0.74	30.	0.27
11.	0.76	31.	0.31	11.	0.67	31.	0.26
12.	0.70	32.	0.28	12.	0.66	32.	0.25
13.	0.69	33•	0.28	13.	0.59	33•	0.24
14.	0.63	34.	0.28	14.	0.56	34•	0.23
15.	0•59	35•	0.26	15.	0.52	35•	0.22
16.	0.59	36.	0.25	16.	0.50	36.	0.22
17.	0.52	37•	0.24	17.	0.46	37•	0.21
18.	0.51	38.	0.24	18.	0.•45	38.	0.21
19.	0.49	39•	0.23	19.	0.42	39•	0.20
20.	0.47	40.	0.23	20.	0.42	40.	0.19

TABLE 11a

Log S_n' FOR ONE STAGE

COM	IPUTED	OBS	ERVED	COM	COMPUTED OB		0BS	BSERVED	
l.	-1.07	l.	-1.12	21.	-2.25		21.	-2.22	
2.	-1.34	2.	-1.31	22.	-2.27		22.	-2.27	
3.	-1.50	3•	-1.53	23.	-2.29		23.	-2.29	
4.	-1.61	¥.	-1.62	24.	-2.31		24.	-2.29	
5.	-1.70	5•	-1.73	25.	-2.32		25.	-2.31	
6.	-1.77	6.	-1.78	26.	-2.34		26.	-2.34	
7.	-1.83	7.	-1.81	27.	-2.35		27.	-2,35	
8.	-1.88	8.	-1.85	28.	-2.37		28.	-2.37	
9•	-1.92	9•	-1.94	29.	-2.38		29.	-2.38	
10.	-1.97	10.	-1.96	30.	-2.40		30.	-2.39	
11.	-2.00	11.	-2.01	31.	-2.41		31.	-2.41	
12.	-2.04	12.	-2.01	32.	-2.42		32.	-2.42	
13.	-2.07	13.	-2.05	33•	-2.43		3 <u>3</u> •	-2.45	
14.	-2.10	14.	-2.07	34•	-2.44		34.	-2.48	
15.	-2.13	15.	-2.11	35•	-2.45		35•	-2.49	
16.	-2.15	16.	-2.12	36.	-2.47		36.	-2.49	
17.	-2.17	17.	-2.14	37•	-2.48		37•	-2.50	
18.	-2.20	18.	-2.17	38.	-2.49		38.	-2.51	
19.	-2.22	19.	-2.20	39•	-2.50		39•	-2.53	
20.	-2.24	20.	-2.20	40.	-2.50		40.	-2.54	

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FIGURE 7.

TABLE 11b

Log S_n

FOR TWO STAGE

COM	PUTED	OBS	ERVED	COM	PUTED	OBS	SERVED
l.	-1.29	l.	-1.27	21.	-2.41	21.	-2.43
2.	- 1.55	2.	- 1•53	22.	-2.43	22.	-2.45
3•	-1.70	3.	-1.69	23.	-2.45	23.	-2.47
4.	-1.80	4.	-1.80	24.	-2.49	24.	-2.48
5.	-1.91	5.	-1.88	25.	-2.50	25.	-2.50
6.	-1.97	6.	-1.95	26.	-2.51	26.	-2.52
7.	-2.04	7•	-2.02	27.	-2.52	27•	-2.53
8.	-2.05	8.	-2.06	28,	-2.54	28.	-2.54
9.	-2.11	9.	-2.10	29.	-2.55	29.	-2.55
10.	-2.13	10.	-2.15	30.	-2.57	30.	-2.57
11.	-2.17	11.	-2.18	31.	-2.58	31.	-2.58
12.	-2.18	12.	-2:22	32.	-2.60	32.	-2.60
13.	-2.22	13.	-2.25	33•	-2.62	33•	-2.61
14.	-2.25	14.	-2.28	34.	-2.64	34.	-2.61
15.	-2.28	15.	-2.31	35•	-2.66	35•	-2.62
16.	-2.30	16.	-2.32	36.	-2.66	36.	-2.64
17.	-2.34	17.	-2.35	37•	-2.68	37•	-2.65
18.	-2.35	18.	-2.38	38.	-2.68	38.	-2.66
19.	-2.38	19.	-2.39	39•	-2.70	39•	-2.67
20.	-2.39	20.	-2.41	40.	-2.71	40.	-2.68

TABLE 12a

		$N_{L} = 6$	S _n F	OR ONE STA	GE	N _T = 9	
1.	.3108	21.	.0204	l.	•3108	- 21.	.0252
2.	•1314	22.	.0196	2.	•1971	22.	•0236
3.	.1204	23.	.0188	3.	.1204	23.	.0222
4.	.0952	24.	.0184	4.	•0952	24.	•0208
5.	•0744	25.	.0172	5.	•0744	25.	.0204
6.	•0656	26.	.0172	6.	.0656	26.	.0196
7.	•0554	27.	.0164	7.	.0630	27.	.0192
8.	•0448	28.	.0152	8.	.0554	28.	.0188
9.	•0420	29.	.0148	9•	.0531	29.	•0174
10.	•0396	30.	.0140	10.	•0459	30.	.0172
11.	•0354	31.	.0136	11.	•0448	31.	.0164
12.	•0340	32.	.0132	12.	•0396	32.	.0162
13.	•0306	33•	.0132	13.	•0394	33.	•0156
14.	•0304	34.	•0128	14.	.0351	34.	.0152
15.	•0276	35•	•0124	15.	.0340	35•	.0148
16.	.0264	36.	.0116	16.	•0306	36.	.0140
17.	.0252	37•	•0116	17.	•0304	37.	.0132
18.	•0236	38.	•0108	18.	.0282	38.	.0132
19.	•0234	39•	.0164	19.	•0276	39.	.0128
20.	.0208	40.	•0104	20.	.0258	40.	.0124

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			S	FOR I	WO	STAGE		
		$N_{L} = 6$	n				$N_{L} = 9$	
l.	.2056	21.	.0184		l.	.2610	21.	•0236
2.	.1740	22.	•0180		2.	•2056	22.	•0234
3•	•0796	23.	.0168		3•	•0837	23.	•0216
4.	.0628	24.	•0168		4.	•0796	24.	.0208
5.	•0558	25.	.0156		5•	•0666	25.	.0198
6.	.0492	26.	•0156		6.	.0628	26.	•0184
7.	•0444	27.	•O144		7.	•0594	27.	.0180
8.	•0436	28.	.0140		8.	•0504	28.	.0171
9•	•0396	29.	.0132		9.	•0492	29.	•0168
10.	•0364	30.	.0132		10.	.0450	30.	•0160
11.	•0336	31.	•0124		11.	•0436	31.	.0156
12.	•0312	32.	.0120	4	12.	•0405	32.	•0152
13.	•0300	33.	•0116	:	13.	•0378	33•	.0140
14.	•0270	34.	.0114	:	14.	•0364	34•	.0140
15.	•0268	35•	•0108	:	15•	•0333	35•	•0132
16.	•0252	36.	•0108	:	16.	•0312	36.	.0132
17.	•0236	37•	•0104	:	17.	•0288	37•	•0128
18.	•0222	38.	•0100	:	18.	.0270	38.	.0124
19.	•0208	39•	.0096		19.	.0268	39•	•0118
20.	.0162	40.	.0092	:	20.	.0252	40.	.0116

The forty data points represent the four hundred locations which get the highest volumes of mail. We make no assertion about the validity of this linear representation beyond this point. Within this range Tables lla & llb give the computed values y'_i and if we compute

$$\epsilon_{\text{RMS}} = \begin{bmatrix} \sum_{i=1}^{N} (y_i - y'_i)^2 \end{bmatrix}^{\frac{1}{R}} \quad \text{we find}$$

 $\epsilon_{\rm RMS,1} = 0.02$; $\epsilon_{\rm RMS,2} = 0.02$ to the nearest hundredth. We feel that this is sufficiently close to a straight line for our analysis.





APPENDIX D

The Functional Form Of S_n Under The Assumption That n Varies Linearly With T_s .

If learning time is of the form $R_{l,n}$ and orientation $T_{l,n}$ then the assumption that short codes vary proportionately with $T_s \left(\frac{\partial n}{\partial T_s} = c\right)$ leads to

$$T_s \cdot S_n - (k_1 \cdot n + \lambda) = 0$$
 and if we apply $\frac{\partial}{\partial T_s}$

$$S_n^{+T}s \cdot \frac{dS_n}{dn} \frac{\partial n}{\partial T}s \cdot \frac{\partial n}{\partial T}s = 0$$
.

Substituting $\frac{\partial n}{\partial T_s} = c$, we have $cT_s \frac{dS_n}{dn} + S_n = ck_1$ $dS_n + (S_n - ck_1)/cT_s dn = 0$ $cT_s dS_n/(S_n - ck_1) + dn = 0$

where the variables are now separated. This leads to the solution

$$S_n = \alpha \exp(-n/cT_s) + ck_l$$
 where $\alpha = \exp(c_l/cT_s)$

and c_1 is a constant of integration. Thus we are led to an exponential function rather than a power function for S_n .

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