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NATIONAL BUREAU OF STANDARDS REPORT

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A DIRECT ELECTRICAL-ANALOG METHOD FOR TWO-DIMENSIONAL TRANSIENT HEAT FLOW ANALYSIS OF FIRE-EXPOSED STRUCTURES

By

D. Gross

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U. S. DEPARTMENT OF COMMERCE
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NBS PROJECT **NBS REPORT**

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ABSTRACT

A continuous-resistance, continuous-capacitance analog model has been constructed and used to simulate a fire-exposed T-beam structure. The model was arranged to permit study of both one and two-dimensional transient heat flow and to allow for Newtonian heat losses to the cool surroundings. Comparison of the results with those of a numerical solution of the problem show agreement within 11%, based on the value of the input signal. Considering the size limitations of the electrical model, edge effects, and the variation in its electrical properties, the agreement is considered reasonable.

1. Introduction

The theory, construction details and performance of an electronic instrument which permits solution of transient heat flow problems by direct electrical analogy has been previously described [1] and the instrument has been used to solve a number of one-dimensional problems. Since the method involves the use of "lumped" resistance and capacitance elements, the extension to two (or three) dimensions could be accomplished by the use of discrete (i.e. lumped) circuit elements arranged in a two- (or three-) dimensional array. Because of simplicity, space considerations and electronic circuitry problems associated with "fast-time" instruments of this type, the use of distributed parameter elements would present distinct advantages. Furthermore, this would eliminate the "lumping" error resulting from the use of a limited number of discrete elements; however, other errors associated with non-uniformity of the conductive medium would generally be introduced.

Conductive-sheet, conductive-solid and conductive-liquid analogs find considerable use in simulating two-and three-dimensional fields governed by Laplace's equation, e. g. steady-state heat flow [2]. For transient heat flow involving potential energy storing elements, however, previous use appears to have been limited primarily to continuous-resistance, lumped-capacitance analogs. Apparently, the first published description of a method utilizing a distributed resistance and capacitance analog was described by Fatt [3].

This report presents data obtained by use of a continuous-resistance, continuous-capacitance analog simulating a fire-exposed T-beam structure and compares the results with corresponding solutions obtained by means of finite difference computations [4]; it is intended to demonstrate the feasibility of using such a model for the approximate solution of one- and two-dimensional transient heat flow problems.

2. Analog Model System

An ideal two-dimensional analog consists of a layer of dielectric material of uniform capacity, to one surface of which is applied a layer of conductive material of uniform resistivity, the other surface being a perfectly conducting layer connected to ground. Ideally both the dielectric and the resistance layer should exhibit the same degree of homogeneity and isotropicity as the thermal body being simulated. Of the more readily available dielectric materials, barium titanate was chosen even though it is known to be nonisotropic to some extent. A 4 by 4 by 1/8 in. barium titanate sheet containing a painted resistance layer on one surface and silver foil on the other surface was specially fabricated by the Centralab Division of Globe Union Mfg. Co. in February 1958. This was the largest size piece which could be conveniently produced at the time; however, the thickness of the dielectric was maintained to a tolerance of ± 0.001 in.

Direct measurements were made of the resistance at several positions across the entire surface and these are listed in Table 1. The non-uniformity of the resistance layer at the measuring point locations was estimated, by means of voltage measurements, to be about ± 10 percent. The time constant (RC) measurements listed in Table 1 were obtained using the circuit shown in Fig. 1. A time constant of 74.7 msec is well suited for measurements by conventional cathode ray oscilloscopes. From these measurements, the capacitance is 0.000464 mfd/cm^2 which corresponds to a dielectric constant, neglecting edge effects, of approximately 1660.

Table 1. Measured Resistance and RC Product

X-direction	R meg	RC sec
1 in.	1.60	.0752
2 in.	1.40	
3 in.	1.40	.0747
4 in.	1.80	.0770
Y-direction		
1 in.		
2 in.	1.60	.0760
3 in.		.0741
4 in.		.0712
Averages	1.56	.0747

The analog model was arranged so that probes could be positioned on its conductive surface through vertically aligned holes in two sheets of clear plastic placed as shown in Fig. 2. Although a measuring grid of this type was selected for convenience, the system is basically continuous and does not restrict measurements to any finite number of space points. Because of symmetry, one-half of the T-beam may be represented by the inverted L-section of Figure 3. A series of 19 "fixed" probes were placed at selected points and were spring-loaded to ensure good contact with the surface. The tips of the 0.075 in. dia. probes were beveled to provide a contact area of approximately 0.03 in. dia. Each of the probes was connected to an individual discharge circuit and, through a manual selector switch, to the high impedance measuring circuit. One additional "movable" probe was used to make measurements at any other grid point desired. The line representing the fire-exposed surface was outlined on the resistive paint surface using highly conductive silver paint and then covered with a sheet of aluminum foil formed to the same outline. The lower plastic sheet was fastened down firmly assuring good contact at the aluminum foil-silver paint interface. A lead connected to the aluminum foil electrode was fed a voltage-time signal derived from a photoformer-type of signal generator which simulated the shape of the prescribed temperature-time curve. Another lead from the silver foil on the lower surface of the model was connected to the instrument ground.

3. Two-dimensional T-beam Problem

3.1 Thermal

The problem chosen for study was one for which numerical solutions by means of a digital computer were available [4]. The lower surfaces (ceiling) of a long horizontal slab and beam structure (T-beam) are subjected to the standard fire exposure defined for fire endurance tests [5] while the upper surface (floor) loses heat by radiation and Newtonian cooling to an ambient maintained at the initial temperature of the structure. Although consideration was given in ref. 4 to a wide range of physical and thermal parameters, this study was concerned only with the following:

Beam to slab ratio:	4.2	
Slab thickness:	4 in.	(10.16 cm)
Beam width:	8 in.	(20.32 cm)
Concrete density:	140 lb/ft^3 ($2.24\ g/cm^3$)	
Specific heat:	0.577 Btu/lb F (2.41 watt sec/g C)	
Thermal conductivity:	0.905 Btu/hr ft F (0.0156 watt/cm C)	
Thermal diffusivity:	0.0112 ft^2/hr (0.00289 cm^2/sec)	

The loss of heat from the unexposed upper surface by radiation and convection was assumed to take place according to the relation

$$q = 0.275\, t^{4/3} + 0.173 \left[\left(\frac{t + t_o}{100} \right)^4 - \left(\frac{t_o}{100} \right)^4 \right]$$

where q = rate of heat loss, Btu/hr ft^2

t = temperature rise above initial, deg F

t_o = initial temperature, 68 F

3.2 Electrical

Analogous thermal and electrical elements for both the one-dimensional and two-dimensional cases are listed in Table 2.

Table 2. Analogous Thermal and Electrical Elements.

Element	One-Dimensional Case		Two-Dimensional Case (Conductive Sheet Analog)	
	Symbol	Unit	Symbol	Unit
Thermal Resistance	$\frac{x_t}{k}$	$\frac{cm^2 C}{watt}$	$\frac{1}{k}$	$\frac{cm C}{watt}$
Electrical Resistance	R	megohm	R	megohm/ square
Thermal Capacitance	$\rho s x_t$	$\frac{watt\text{-}sec}{cm^2 C}$	$\rho s A_t$	$\frac{watt\text{-}sec}{cm\ C}$
Electrical Capacitance	C	μfd	cA_e	μfd

In addition to the capacity, resistivity and time-scaling factors in the one-dimensional case, a linear scale factor $\lambda = L_t/L_e$ is introduced in the two-dimensional case to account for the use of a small-scale electrical model. Based on the dimensions and the measured electrical values of the analog model, the values of the scaling factors (in c.g.s. units) were as follows:

$$m = \frac{1}{kR} = \frac{1}{(.0156)(1.56)} = 41.1$$

$$n = \frac{\rho s}{c}\left(\frac{L_t}{L_e}\right)^2 = \frac{(2.24)(2.41)}{(.000464)}(5)^2 = 2.91 \times 10^5$$

$$\varphi = 1$$

$$\lambda = \frac{L_t}{L_e} = \frac{4}{0.8} = 5$$

For a three-hour fire exposure duration, each time division on the read-out oscilloscope was chosen to represent 4 minutes of thermal time and the oscillator was therefore set for a frequency of

$$f = \frac{m\ n\ \phi}{t_t} = \frac{(41.1)(2.91 \times 10^5)(1)}{4 \times 60} = 49.8 \text{ kc.}$$

To simulate convective and radiative heat losses, it is common to use non-linear resistive elements [1,6]. To maintain the continuous, geometrical nature of a two-dimensional analog would require the use of a distributed voltage-sensitive resistive film. In the absence of such a material, it becomes necessary to employ discrete elements, so that a combined geometric and network analog results. The size of the connector contact is important since, if too large, there is the tendency to form a short circuit between adjacent contacts, whereas if too small, the flow lines tend to crowd [7]. Because of a lack of non-linear elements of the desired properties and since simple resistors may approximate non-linear resistors over a limited voltage range, conventional resistors were used. The contact size was approximately 0.03 in. dia. and eight heat loss resistors were arranged 0.4 in. apart as shown in Fig. 3. The values of the individual resistors were determined by linear approximations to the electrical equivalent of the heat loss curve over the approximate temperature ranges (derived from the numerical solution). The appropriate current-voltage curve was obtained by use of the relation

$$\frac{q}{i} = 10^6 \frac{\Delta t}{m\ \Delta E}$$

where $\frac{\Delta t}{\Delta E} = 144$ F/volt, as measured.

These resistor values check very closely (approximately 6%) the values obtained by employing the "effective" surface heat transfer coefficient of Fig. 4 (Ref. 4), and using the relation $R_f = \frac{1}{mh_{eff}}$.

4. Discussion of Results

Temperature rise data on the two-dimensional analog model of the reinforced concrete T-beam problem were obtained at several locations as indicated in Fig. 3. For direct comparison with the numerical results given in ref. [4], all data were taken at a thermal time of 2 hours after application of the standard temperature-time fire exposure along the surface designated F. Section A-A is sufficiently removed from two-dimensional effects and hence the data in Fig. 4 represents the one-dimensional temperature gradient for a horizontal slab whose lower surface is fire-exposed and which loses heat from its upper ("unexposed") surface by Newtonian cooling. The analog results are within 7 %, based on the value of the input signal, of the corresponding results obtained by numerical solution of the finite difference equations. In Figures 5 and 6 are shown the temperature gradients along Sections B-B and C-C, where two-dimensional heat flow patterns are evident. Here, the analog and numerical values do not differ by more than 11%.

Higher analog values along part of the B-B axis of symmetry may be due to difficulties associated with measurements along an edge surface. Although this problem also exists along the other edge, its effect is partially masked by the influence of the heat loss resistors at the "unexposed" surface. Some of the factors involved in edge measurements include:

(1) the conventional edge effect for a parallel plate capacitor which results in increase in capacitance close to the edge [8].

(2) the finite dimension of the probe tip and its proximity to the edge,

and (3) the imperfect registration of the grounding plate, which is actually recessed approximately 0.03 in. from the edge.

To avoid the edge effect at the axis of symmetry, it would be preferable to utilize the complete T-section. This would also provide twice the measuring capabilities (and averaging advantages), but would probably require a larger area model for reasonable accuracy.

In most experimental work, the difference or error between the measured and true potentials is expressed as a percent of the true potential. However, in analog-system measurements, this expression is not suitable since the true potential may be very small or even zero. A more feasible method for gaging the performance of an analog system is to express the difference between the measured and true potentials as a percent of the largest potential difference existing in the field, generally the voltage difference between boundary electrodes [2]. This method has been employed here to compare the analog and the finite difference results, neither of which can be considered true values.

It should be borne in mind that solutions of transient heat flow problems by finite difference computations are only approximations and involve errors which depend upon the space and time grids chosen and the associated problems of instability and convergence. It is generally difficult to estimate the magnitude of such errors. Because of a lack of knowledge of the accuracy of the numerical computations, analog measurements may be as much a check on finite difference calculations as the reverse.

In fire endurance tests, the performance of the test construction is measured by the time period during which it withstands the standard fire exposure without failure according to prescribed physical and thermal criteria. For wall or floor constructions, thermal failure generally involves a temperature elevation on the "unexposed" surface of 250 degrees F. Structural failure of stressed structural steel members usually occurs at a temperature of about 1000 F and therefore particular attention was paid to the lower portion of the beam where reinforcing members would be most subject to high temperature. Studies of this kind are useful both for estimating relative fire endurance times and as an aid in selecting appropriate steel and concrete combinations for particular applications.

The feasibility of using a continuous-resistance, continuous-capacitance analog of the type described for two-dimensional transient heat flow analysis has been demonstrated. Considering the size limitations of the electrical model and the variations in its electrical properties, the comparison with the numerical solutions, at least for the locations and time period measured, were considered reasonable. The exploratory nature of the analog model described in this report leave a number of questions regarding experimental technique, measurement error and overall accuracy as yet unevaluated. In particular, the magnitude of possible errors introduced by edge effects, by contact resistance and by capacitive coupling between probes and lead wires should be evaluated. For situations requiring increased precision, a more thorough exploration should be made of the dielectric properties of barium titanate (or alternate dielectrics), especially with regard to piezoelectric, temperature, hysteresis nonisotropic and dielectric loss effects.

5. Acknowledgments

The probes and mounting assembly for the analog model were made by William H. Bailey. Two Antioch College cooperative students, Glenn Crout and Peter Kleban, worked out experimental details and made some of the early measurements.

6. References

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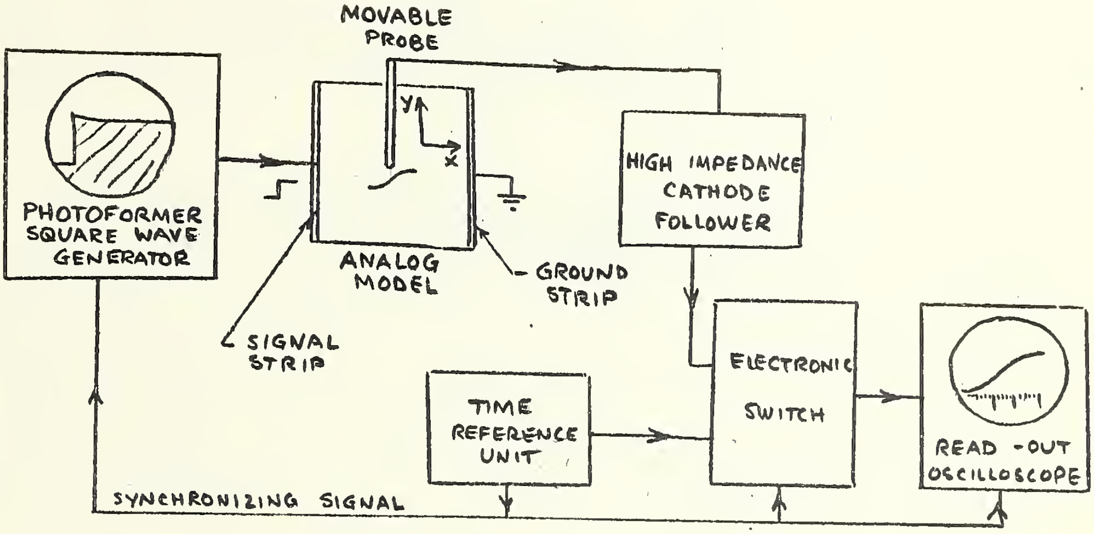

Figure 1. Schematic diagram for measurement of the time constant of a continuous-resistance, continuous-capacitance analog.

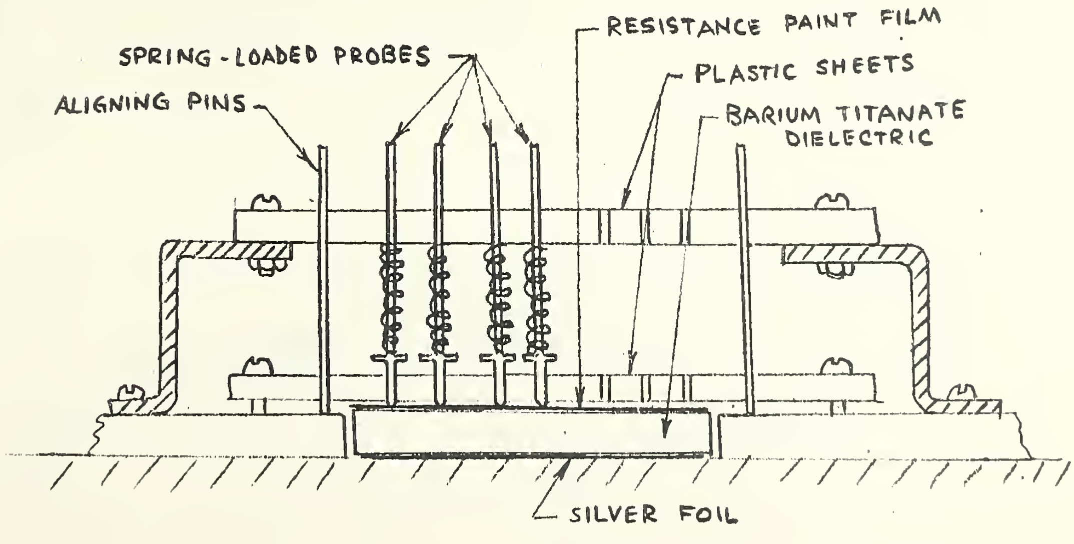

Figure 2. Mounting details of analog model

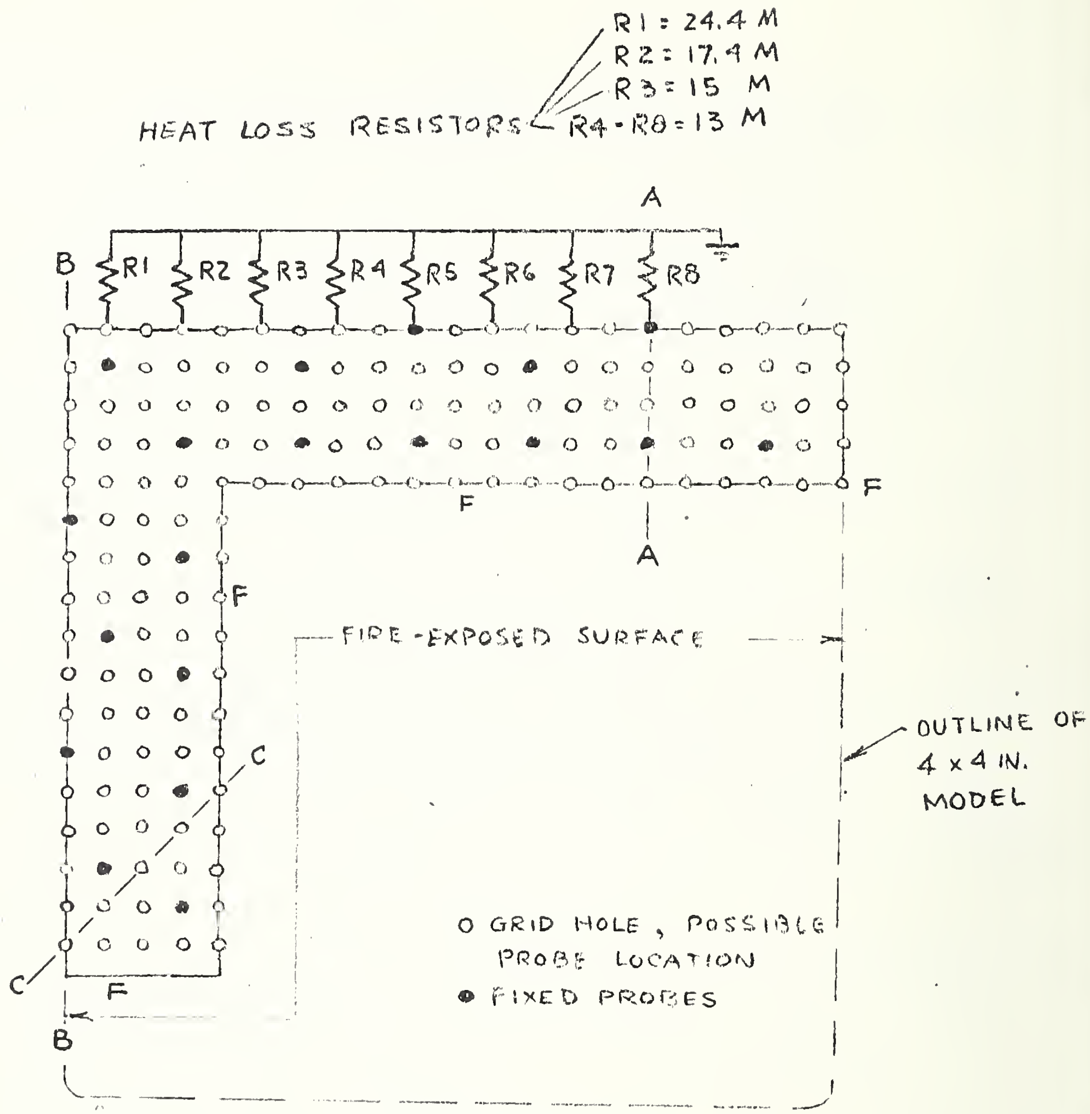

Figure 3. Grid pattern for analog model of fire-exposed T- beam

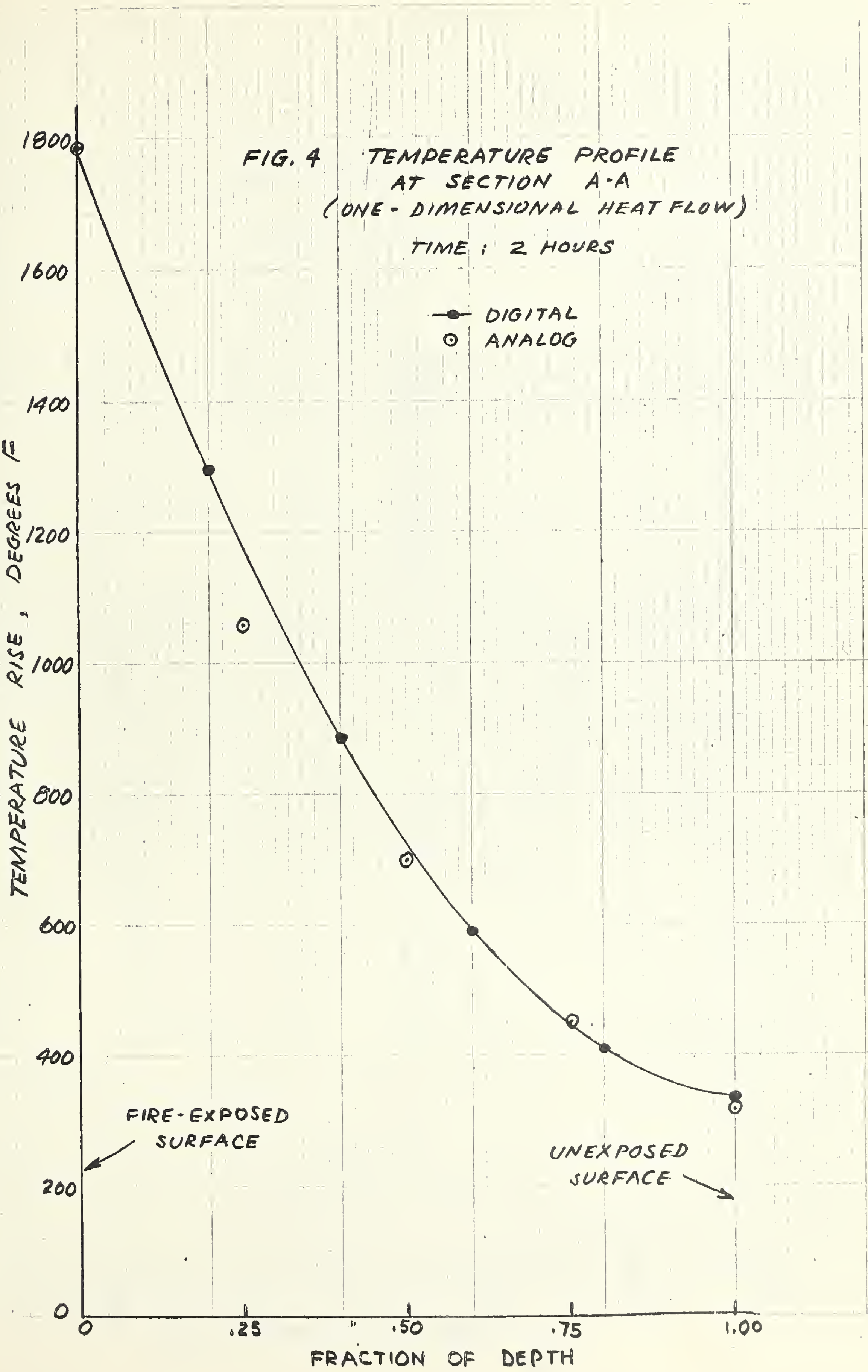
FIG. 4 TEMPERATURE PROFILE
AT SECTION A-A
(ONE-DIMENSIONAL HEAT FLOW)
TIME : 2 HOURS
DIGITAL
ANALOG
TEMPERATURE RISE, DEGREES F
1800
1600
1400
1200
1000
800
600
400
200
0
FIRE-EXPOSED SURFACE
UNEXPOSED SURFACE
0
.25
.50
.75
1.00
FRACTION OF DEPTH

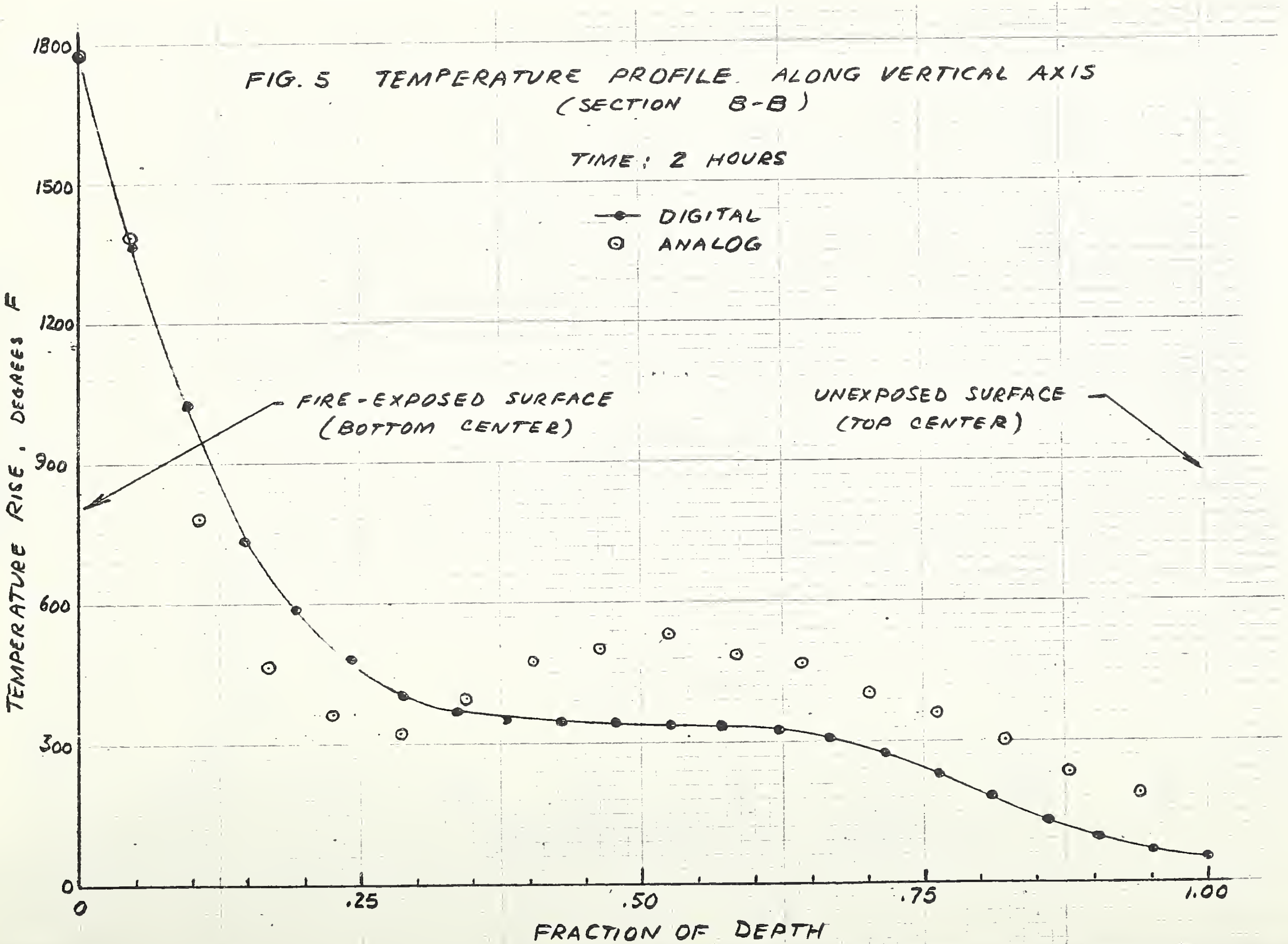

FIG. 5 TEMPERATURE PROFILE ALONG VERTICAL AXIS (SECTION B-B)

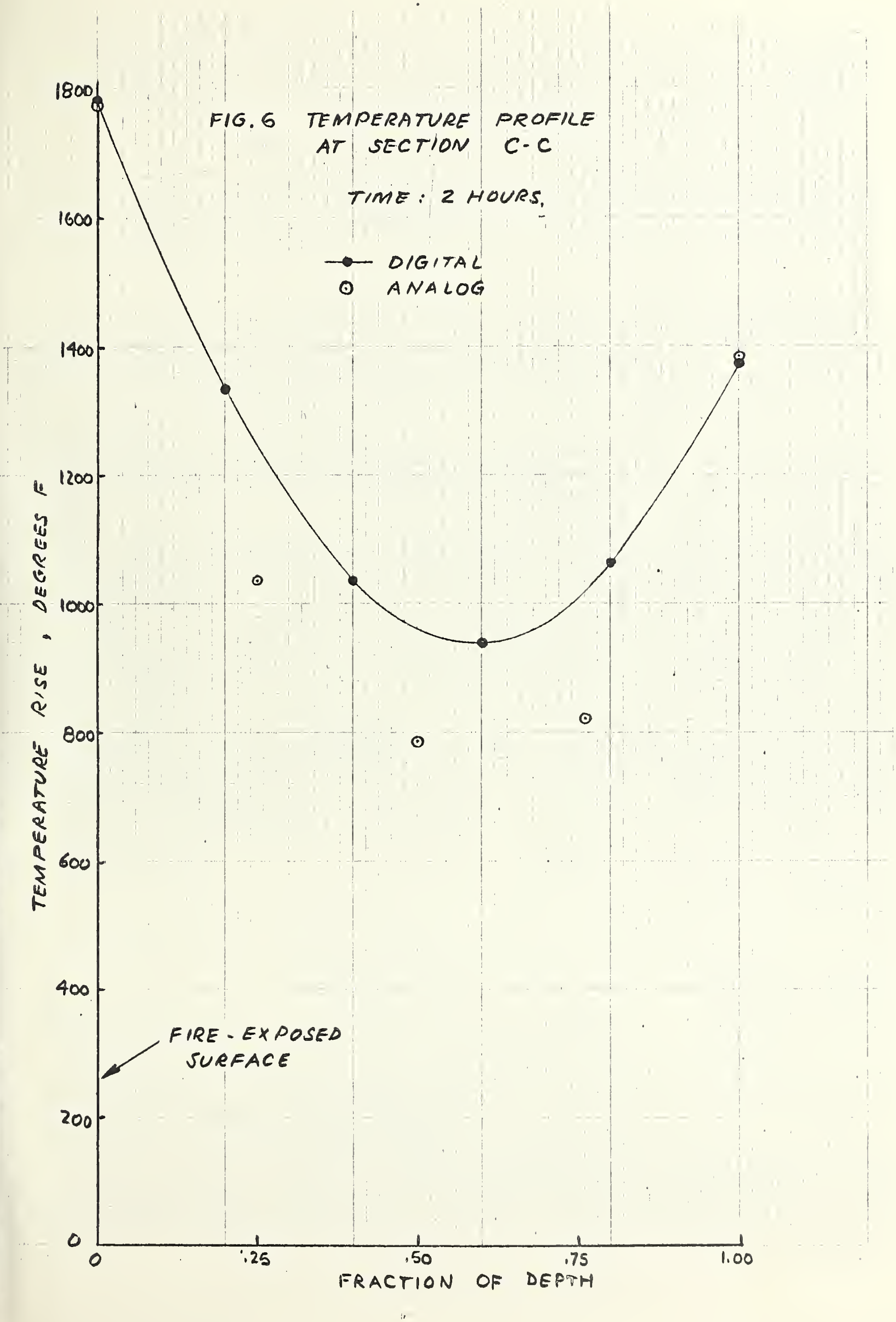
FIG. 6 TEMPERATURE PROFILE AT SECTION C-C
TIME: 2 HOURS.
DIGITAL
ANALOG
TEMPERATURE RISE, DEGREES F
1800
1600
1400
1200
1000
800
600
400
200
0
0
.25
.50
.75
1.00
FRACTION OF DEPTH
FIRE-EXPOSED SURFACE

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