### NATIONAL BUREAU OF STANDARDS REPORT

7402

PROJECTS and PUBLICATIONS of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

April through June 1962



U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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**NBS PROJECT** 

**NBS REPORT** 

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U. S. DEPARTMENT OF COMMERCE NATIONAL BUREAU OF STANDARDS

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April 1 through June 30, 1962

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June 30, 1962

### 1. NUMERICAL ANALYSIS

# RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54

Manager: Philip J. Davis

Full Task Description: July-September 1954 issue, p. 1

Status: CONTINUED. O. Shisha has worked in the following areas:
(i) The zeros of infrapolynomials with prescribed values at given points (with Professor J. L. Walsh), (ii) Zeros of functions of the form

 $\sum_{k=1}^{n} \frac{c_k}{z^{-z}_k}$  (with Prof. J. L. Walsh), (iii) The structure of infrapolynomials,

- (iv) Tchebycheff approximation by rational functions, particularly from the computational standpoint.
- O. Shisha is preparing a survey volume on the theory of the transfinite diameter.
- G. T. Cargo and O. Shisha have been working on the following topics:
  (i) Inequalities between means, (ii) Zeros of polynomials and power series, and fractional order differences of their coefficients.
- G. T. Cargo worked on ambiguous points of "almost" bounded functions in the open unit disk.
- S. Haber continued work on a survey of the literature of the theory of equidistribution mod one. He showed that if, in a simple Monte Carlo numerical quadrature procedure, the sequence of "random" numbers used is the sequence  $\{(f(n))\}$ , where f is a polynomial of degree two with an irrational coefficient (and "(x)" denotes the fractional part of x), then the usual Monte Carlo error estimate can be established rigorously (if round-off errors are ignored).
- F. W. J. Olver has completed a second paper on error bounds for Airy function expansions in turning-point problems.
- M. Rockoff is comparing iterative methods for the solution of systems of linear equations arising from the use of high order finite-difference approximations to the Laplacian in the discretization of the Dirichlet problem.
- P. J. Davis and Maxine L. Rockoff have undertaken a series of numerical experiments designed to produce positive quadrature formulas for highly multiple integrals.

#### Publications:

- (1) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Journal of Mathematical Physics, 3, 469-471, May-June 1962.
- (2) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). To appear in the Pacific Journal of Mathematics.
- (3) Modular forms whose coefficients possess multiplicative properties (II). M. Newman. Annals of Mathematics, 75, 242-250, March 1962.
- (4) Tchebycheff approximation by exponentials. J. R. Rice. Journal of the Society for Industrial and Applied Mathematics, 10, 149-161, March 1962.
- (5) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Mathematische Zeitschrift, 78, 449-452, March 1962.
- (6) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (7) Multipliers of difference sets. M. Newman. Submitted to a technical journal.
- (8) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. To appear in the Pacific Journal of Mathematics.
- (9) Lower bounds to eigenvalues using operator decomposition of the form B\*B. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in Archive for Rational Mechanics and Analysis.
- (10) Two matrix eigenvalue inequalities. S. Haber. Journal of Research NBS, 66B, 57-58, April-June 1962.
- (11) Linear operations on matrices. M. Marcus. In manuscript.
- (12) An extension of Jensen's theorem for derivatives of polynomials and for infrapolynomials. O. Shisha. Journal of Research NBS, 66B, 53-55, April-June 1962.
- (13) The zeros of infrapolynomials with prescribed values at given points.

  O. Shisha and J. L. Walsh (Harvard University). Submitted to a technical journal.
- (14) Zeros of polynomials and fractional order differences of their coefficients. O. Shisha and G. Cargo. In manuscript.
- (15) Error bounds for first approximations in turning-point problems. F. W. J. Olver. Submitted to a technical journal.
- (16) The segmental variation of Blaschke products. G. T. Cargo. To appear in the Duke Mathematical Journal.

# RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54

Sponsor: Office of Naval Research

Manager: Morris Newman

Full Task Description: July-September 1954 issue, p. 5

Status: CONTINUED. M. Newman is continuing his work on free products of cyclic groups and has found matrix representations for such groups.

A study of the arithmetic minima of quadratic forms and bounds for cofactors has been initiated by M.Newnan. It is shown, for example, that if  $\gamma_n$  denotes the Hermite constant, then

$$\gamma_{m+n}^{m+n} \geq \gamma_m^m \gamma_n^n$$
.

Some theorems on matrices relevant to the solution of linear systems by iteration have been found by M. Newman. These generalize known results of Seidel and Reich.

The monograph on integral matrices now exists as a first draft, and is being revised by M. Newman.

K. Goldberg and M. Newman considered a finite set S, a subset  $S_1$ , and a function f mapping  $(S-S_1) \times S_1$  into S. They proved several results concerning the number of iterations of f necessary to map an element of  $S-S_1$  into  $S_1$  when f has certain basic uniqueness properties, and also when S is contained in a group and  $f(x,y) = xy^{-1}$ .

K. Goldberg continued his investigation of the coefficients in the s-th power of the t-th iterate of a power series  $z + a_1 z^2 + ...$ .

K. Goldberg began an investigation of the transformations of sequences of the type

$$y_n = \sum_{\sum i k_i = n}^{n} c_{1^{k_1}, 2^{k_2}, \dots, n^{k_n}} x_1^{k_1} x_2^{k_2} \dots x_n^{k_n}, \quad n = 1, 2, \dots$$

with particular emphasis on those in which  $c_{1^{k_1},2^{k_2},\ldots n^{k_n}} =$ 

### Publications:

- (1) A note on modular groups. M. Newman. To appear in the Proceedings of the American Mathematical Society.
- (2) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (3) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). To appear in the Proceedings of the American Mathematical Society.

- (4) Hadamard matrices of order cube plus one. K. Goldberg. In manuscript.
- (5) Some free products of cyclic groups. M. Newman. Submitted to a technical journal.
- (6) Two theorems on matrices. M. Newman. To appear in the Journal of Research, NBS, Section B(Mathematics and Mathematical Physics).
- (7) Bounds for cofactors and arithmetic minima of quadratic forms. M. Newman. To appear in the Journal of the London Mathematical Society.

### BOUNDS FOR EIGENVALUES Task 1101-12-11416/62-1091

Origin: Wright-Patterson AFB Authorized 10/1/61

Manager: Hansjorg Oser (11.2)

Full Task Description: October-December 1961 issue, p.4

Status: REACTIVATED. As a first object to study the effectiveness of the method of truncation [N. Bazley and D. W. Fox in Journal of Research NBS, 65B, 105-111, 1961], we have chosen Legendre's differential equation for non-integral coefficients. It turns out in this case that the method yields upper and lower bounds for the eigenvalues with equal ease and preliminary computations give excellent results for the first five eigenvalues. At present, work is being done towards finding satisfactory estimates for eigenvectors. It is doubtful, however, that the method given by Bazley and Fox will give very good bounds for eigenvectors in complicated cases where little is known about the eigenvectors themselves.

### 2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

### MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter because priority was given to the preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 TABLES OF SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 TABLES OF THE SIEVERT INTEGRAL

### HANDBOOK OF MATHEMATICAL FUNCTIONS Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56

Manager: Irene A. Stegun

Full Task Description: October-December 1956 issue, p. 10

Status: CONTINUED. The texts for chapter 8 (Legendre Functions) and chapter 18 (Weierstrass Elliptic Functions) were distributed for comments and are presently being revised. Of the twenty-nine chapters in the Handbook, twenty-three are now in press. Galley proofs for text and table headings and footnotes are being processed for chapters 4, 5, 7, 25 and 26.

### AUTOMATIC CODING Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54

Manager: J. Wegstein

Full Task Description: July-September 1954 issue, p. 11

Status: CONTINUED. Work continued in removing ambiguities and obscurities from the Algol 60 language. J. Wegstein attended a meeting of some of the authors of the Algol 60 report in Rome on April 2 and 3. Extensions to the language were not considered at the meeting. Instead, various proposals for correction and clarification that were submitted by interested parties in response to the Questionnaire in Algol Bulletin no. 14 were used as a guide.

A report that constitutes a supplement to the Algol 60 Report was prepared for eventual publication.

Kenneth Berk is preparing a series of tutorial lectures on the IPL V language and heuristic programming for delivery in August.

In cooperation with Division 12, a preliminary design was prepared for a prototype remote station. The station is to be used by individuals for directly communicating with a computer in general applications such as computing, information retrieval, and machine tutoring.

### 3. PROBABILITY AND MATHEMATICAL STATISTICS

### MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50

Manager: Joan R. Rosenblatt

Full task description: July-September 1950 issue, p. 58

Status: CONTINUED. Janace Speckman has completed a study of four estimators of the parameter  $\rho$  in the model Y =  $e^{-\rho t} + \epsilon$ . The results will be incorporated in a joint paper with R. G. Cornell (Florida State University) which is in preparation.

W. A. Thompson and George Weiss are continuing their study of approximate solutions to the diffusion - growth equation.

W. A. Thompson has completed two related studies entitled "Precision of simultaneous measurement procedures" and "Estimation of dispersion parameters". The first problem arises when data are gathered in such a way that on any given item only one opportunity for measurement occurs, but the item can be observed simultaneously by several instruments. The second study obtains simultaneous confidence intervals for the dispersion parameters of a multivariate normal distribution and provides a table in the bivariate case.

John Van Dyke is studying methods for fitting the straight line  $y=\beta x$  when x is without error and the variance of y is proportional to some power p, p  $\geq$  0, of x, e.g.,  $V(y|x_i) = \sigma^2 x_i^p$ . The "best" (minimum variance unbiased linear) estimator for  $\beta$  is well known to depend on the value of p. Mr. Van Dyke is investigating the properties of estimators which are "best" for integral values of p, if these estimators are applied when p has a fractional value. Let B denote the "best" estimate for p=n (integral), and let  $V_p(B_n)$  denote the variance of B calculated under the assumption that the variance is proportional to  $x^p$ . In particular, the values of p have been found for which

$$Var_{n}(B_{n}) = Var_{n}(B_{n+1}), n$$

A working paper reporting these results is being prepared.

The unsatisfactoriness of confidence intervals based on small samples is being considered by Janace Speckman. The unsatisfactoriness of confidence intervals for the mean of a normal distribution is being measured in two ways: (1) by the probability that a confidence interval will miss including the true mean by an amount greater than or equal to  $\Delta$ , standard deviation known and unknown; and (2) by the distribution of the width of the confidence interval, standard deviation unknown.

H. H. Ku is preparing a paper illustrating the application of the methods treated in "Tests for contingency tables and Markov chains", with emphasis on examples from the physical sciences. The paper will include a table of  $2n\log_e n$  for n=1(1)10,000.

#### Publications:

- (1) On the pedestrian queueing problem. George Weiss. To appear in the Bulletin of the International Statistical Institute.
- (2) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (3) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), and H. H. Ku. To appear in Technometrics.
- (4) Convergence to normality of powers of a normal random variable.

  Norman C. Severo and Lloyd J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (5) Graphs for determining the power of Student's t-test. Mary C. Croarkin. Journal of Research NBS, 66B, 59-70, April-June 1962.
- (6) The relaxation of moments derived from a master equation. Kurt Shuler (Director's Office), George Weiss, Knud Andersen (Director's Office). Journal of Mathematical Physics, 3, 550-556, May-June 1962.

# STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52

Manager: J. M. Cameron

Full Task Description: October-December 1952 issue, p. 60

Status: INACTIVE.

### Publications:

- (1) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (2) A calculus for factorial arrangements. B. Kurkjian (DOFL) and M. Zelen. Annals of Mathematical Statistics, 33, 600-619, June 1962.
- (3) Factorial designs and the direct product. B. Kurjian (DOFL) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.

### STUDY OF NONPARAMETRIC STATISTICAL TECHNIQUES Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55

Manager: Joan R. Rosenblatt

Full task description: October-December 1955 issue, p. 14

Status: CONTINUED. Joan R. Rosenblatt has used an identity connecting certain hypergeometric distributions with distributions of "exceedances" to study the relationships among distribution-free two-sample tests including the Mood-Brown median test and analogous tests using the kth order statistic of two combined samples. The identity is particularly useful for calculating distribution-free confidence limits for the difference between the medians of two continuous random variables. A table was prepared to facilitate the calculation of the confidence limits equivalent to the median test. A further application of the same identity shows that tables of the hypergeometric distribution may be used to calculate tail probabilities for certain two-sample tests based on the number of exceedances.

An interesting property of the sample median has been noted, namely that a median is the limit of the following sequence of "adjusted means": If n observations  $x_1, \ldots, x_n$  have mean  $\overline{x}$ , then the sequence  $m_1, m_2, \ldots$  is obtained from

 $m_1 = \bar{x}$   $m_{k+1} = w_{k1} x_1 + \dots + w_{kn} x_n$ ,  $k=1,2,\dots$   $w_{ki} = |x_i - m_k|^{-1} / \sum_i |x_i - m_{ir}|^{-1}$ .

where

Dr. Rosenblatt is completing a note on this "adjustment" procedure, including also some results for additional types of weights  $\mathbf{w}_{\mathbf{k}i}$ .

W. A. Thompson and T. A. Willke have completed a paper on "A rank sum test for outliers", providing a theoretical study to supplement the paper by W. J. Youden on "The role of laboratories in round robins" in which this rank sum test was proposed. The null hypothesis that ranks are assigned at random to I objects by each of J judges is rejected if the sum of ranks for any object is too large or too small. Dr. Youden gave a table of critical values for the maximum and minimum rank sums for the significance level  $\alpha$  nearest .05. Drs. Thompson and Willke give upper and lower bounds for the  $\alpha$ 's in Youden's table and provide critical values for  $\alpha$  nearest .03 and .01. Asymptotic properties of the test are investigated and power curves are given for several kinds of alternatives.

#### Publication:

(1) The role of laboratories in round robins. W. J. Youden. To appear in Materials Research and Standards.

### MEASUREMENT OF RELIABILITY Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56

Manager: Joan R. Rosenblatt

Full task description: January-March 1956 issue, p. 13

Status: CONTINUED. George Weiss completed a survey of mathematical models in reliability theory and presented the results at an Advanced Seminar on Reliability held in May at the U. S. Army Research Center, Madison, Wisconsin.

Dr. Weiss is also working on a study of systems with spare parts in which it is assumed that the spare parts can fail in storage as well as in use. The results of this study will be reported in a paper submitted to the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

Joan R. Rosenblatt completed the first draft of a paper on "Confidence limits for estimates of the reliability of complex systems". This paper introduces and gives some preliminary evaluation of an approximate nonparametric procedure for calculating confidence limits for a quantity  $R = \Pr\{f(x_1,\ldots,x_k) \geq c\},$  when  $f(x_1,\ldots,x_k)$  and c are given along with a sample of observations of each of the independent random variables  $x_1,\ldots,x_k$  (whose distributions may be unknown). The paper was presented at an Advanced Seminar on the Statistical Theory of Reliability held in May 1962 at the U. S. Army Mathematics Research Center, Madison, Wisconsin. The manuscript is being revised for publication in the Proceedings of that Seminar.

### 4. MATHEMATICAL PHYSICS

# RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54

Manager: W. H. Pell

Full Task Description: July-September 1954 issue, p. 27

Status: CONTINUED. A. Ghaffari is continuing his study of the application of the stroboscopic method to non-linear non-autonomous systems and its extension to higher approximations. Some results concerning the conditions of the existence and stability of periodic oscillations of the Duffing equation have been obtained. These were presented at the meeting of the American Mathematical Society in Atlantic City, New Jersey, April 16-19, 1962.

Further investigations by B. Bernstein on the constitutive equations of viscoelasticity subsumed from those of Green and Rivlin are continuing. The relations that these equations yield for various rubbers, in a state of simple extension, have now been shown to be in complete agreement with the equilibrium data published by various experimental workers. Moreover, tests conducted by the Rheology Section (6.05) indicate strongly that several polymeric materials obey these relations during a simple extension-stress relaxation experiment. An attempt is being made to abstract some of the fundamental ideas from the constitutive equations and to reformulate them in such a way as to describe a fluid. It is felt that the transition from the solid to fluid state, which is gradual for the materials under consideration, will be described by these equations.

J. H. Bramble and L. E. Payne have continued their work on the bounding of solutions of problems in classical linear elasticity. They are currently concerned with the effect of error in the measurement of the elastic moduli on the solution of such problems. A paper concerning their results is in the final stages of preparation.

#### Publications:

- (1) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (2) Pointwise bounds in the first biharmonic boundary value problem.

  J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (3) On Rayleigh's nonlinear vibration equation. A. Ghaffari. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.

# Status of Projects PLASMA RESEARCH Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59

Manager: C. M. Tchen

Full Task Description: April-June 1959 issue, p. 15

Status: CONTINUED. C.M. Tchen is continuing his investigation of the non-linear Fokker-Planck equation for non-Markovian processes which include a memory. The derivation of an equation governing such behavior is based on the kinetic theory, that is, the BBGKY equation; and on the phenomenological theory, that is, the non-linear master equation. The results of both methods are being compared.

#### Publications:

(1) Kinetic equation for plasmas with collective and collisional correlations. C. M. Tchen. Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, Germany, August 28 - September 1, 1961, pp. 825-841.

# THEORY OF SATELLITE ORBITS Task 1104-12-11441/62-1166

Origin: NBS Authorized 1/9/62

Sponsor: National Aeronautics and Space Administration

Manager: J. P. Vinti

Full Task Description: January-March 1962 issue, p. 12

Status: CONTINUED. Through the use of oblate spheroidal coordinates, J. P. Vinti has developed an accurate intermediate orbit for an artificial satellite of an oblate spheroidal planet. This orbit is associated with the Vinti gravitational potential which is closer to the empirically accepted value for the earth than those which have been earlier proposed. The Vinti theory accounts exactly for the effects of the zeroth and second zonal harmonics of the earth's gravitational potential and partly for the effects of the fourth. The next obvious step in the theoretical development is to deduce the effects produced by the residual fourth harmonic, since other investigators using elliptic or quasi-elliptic orbits, have found that such lead to peculiarities in the motion for orbits inclined at approximately 63° to the equator.

To carry out this development the author has re-expressed the integrated kinetic equations of motion, which give the satellite's spheroidal coordinates implicity as functions of time, in terms of a certain set of canonical variables. This set is one of the many sets which would reduce to the fast Delaunay set of the Kepler problem, if one were to neglect the earth's oblateness. The Hamiltonian of this system has been constructed and a canonical transformation, found by use of the von Zeipel-Brouwer method,

has readily yielded the short-periodic effects produced by the residual fourth harmonic. In contradistinction with the case of an elliptical reference orbit, however, it turns out that the von Zeipel-Brouwer method will not yield the secular and long-periodic effects. Fortunately, another method leads to their evaluation, so that the original canonical variables may all be expressed as functions of time. In this first part of the problem no pecularities appear at an orbital inclination of 63°.

The tasks remain of choosing the most convenient mean orbital elements, of inserting the above expressions for the canonical variables into the implicit equations of motion, and of then inverting them to find the spheroidal coordinates as functions of time. If any pecularities are to appear for inclination 63°, they will have to appear in this process of inversion.

### DYNAMICS OF PLASMAS Task 1104-12-11417/62-1157

Origin: NBS Authorized 10/3/61

Sponsor: National Aeronautics and Space Administration

Manager: C. M. Tchen

Full Task Description: October-December 1961 issue, p. 12

Status: CONTINUED. Study of the effects of the collective correlation on plasma oscillations and the Landau damping was continued. The BBGKY equation was taken for the singlet distribution function, and an equation of the Vlasov type was used for the pair correlation function. The Landau damping was found by C. M. Tchen to be decreased by the presence of the correlation, which restrained the extent of the self-consistent field. The equilibrium forms of the singlet distribution function and of the pair correlation function were taken for the unperturbed function. A manuscript is being prepared for publication.

Drs. Tchen and E. Minardi investigated the application of the above theory to other forms of the unperturbed pair correlation function.

## FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55

Sponsor: Office of Naval Research

Manager: F. Oberhettinger.

Full Task Description: July-September 1955 issue, p. 20

Status: INACTIVE.

#### Publication:

(1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

### 5. OPERATIONS RESEARCH

### OPERATIONS RESEARCH Task 1105-12-11115/61-546

Origin and Sponsor: NBS Authorized 12/30/60

Manager: Alan J. Goldman

Full Task Description: October-December 1960 issue, p. 3

Status: CONTINUED. The following investigations in various fields of operations research were carried out by members of the staff:

- (i) Bernice K. Bender continued work on simplification rules for Boolean functions, and continued preparation of a paper concerning her recent investigation of essential-cell content.
- (ii) Mrs. Bender and A.J. Goldman completed a paper containing their previous work on maximal cellular Boolean functions plus some previously unfinished work on Gray codes.
- (iii) C.T. Zahn, Jr. prepared a paper reporting his work on optimal approximation by equivalence relations to binary relations representing two-level and three-level hierarchical structures.

#### Publications:

- (1) Covers and packings in a family of sets. J. Edmonds. To appear in the Bulletin of the American Mathematical Society.
- (2) A property of linear frequency modulation. A. J. Goldman. To appear in the Proceedings of the Institute of Radio Engineers.
- (3) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics.)
- (4) An algorithm for least common multiples. A.J. Goldman. Submitted to a technical journal.

### AIR DEFENSE Task 1105-12-11415/61-544

Origin and Sponsor: U.S. Army Air Defense Command. Authorized 9/30/61

Manager: Lambert S. Joel

Full Task Description: July-September 1961 issue, p. 13

Status: CONTINUED. L. S. Joel and A. J. Goldman continued cooperation with members of Denver Research Institute on model formulation. L. S. Joel and C. T. Zahn, Jr. began coding the first simulation model.

### RADAR STUDY Task 1105-12-11527/60-481

Origin and Sponsor: U.S. Army Signal Air Defense Agency Authorized 9/30/61

Manager: Lambert S. Joel

Full Task Description: July-September 1961 issue, p. 14

Status: COMPLETED. The remaining volume of the final report was finished and sent to the sponsor.

#### Publication:

(1) Black box maximization of circular coverage. C. T. Zahn, Jr. Submitted to the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).

# GRAPH THEORY Task 1105-12-11450/62-1161

Origin: NBS Authorized 1/1/62

Sponsor: Office of Naval Research

Manager: Jack Edmonds

Status: TERMINATED. Work under this task will be continued and reported under Task 1105-12-11455/62-1205.

### COMBINATORIAL MATHEMATICS Task 1105-12-11455/62-1205

Origin: NBS Authorized 5/2/62

Sponsor: Army Research Office (Durham)

Manager: Jack Edmonds

Objective: To investigate linear graphs and related combinatorial structures — their extremal properties, their symmetry, and their topology. To formulate computer algorithms for the arrangement of discrete structures of use in technology and operations analysis.

Background: The principal investigator's recent activities in this field have been reported under Operations Research (Project 11115) and Graph Theory (Project 11450).

Status: NEW. For a finite graph G, a matching in G is a subset of its edges such that no two meet the same vertex. A set consisting of 2k+1 vertices in  $G(k \ge 1)$  has weight k and is said to cover an edge e if both endpoints of e are in the set. A set consisting of one vertex has weight 1 and is said to cover an edge e if e meets the vertex. An  $S_0$  cover

of G is a family of odd sets of vertices such that each edge in G is covered by at least one of its members. The Konig theorem for bipartite graphs generalizes to the following Matching-Duality theorem: The maximum cardinality for a matching in a graph G equals the minimum weight-sum for an  $S_0$ - cover of G.

C. T. Zahn, Jr. is studying various refinements of Edmonds matching algorithm and has begun to program it for the 7090.

John Mather is investigating properties of 3-manifolds using Edmonds method for representing them as ordered 2-complexes. He has proved that certain combinatorial operations on the complexes preserve homeomorphism for non-spherical manifolds.

### 6. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-11645/56-0166 SCF-LCAO SOLUTION OF SOME HYDRIDES

Origin and Sponsor: NBS, Section 5.9

Manager: P. J. Walsh

Full Task Description: January-March 1956 issue, p. 27

Status: Continued. Final SCF program for a given molecular symmetry has been checked out with significantly reduced computation time. Preliminary production runs have been completed which test the applicability of the program. Sufficient promise is indicated to warrant the generalization of the integral program which is now in progress.

1102-40-11645/56-0186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Origin and Sponsor: NBS, Section 2.5

Manager: B. S. Prusch

Full Task Description: July-September 1956 issue, p. 33

Status: Continued. Computations for 25 sets of gage blocks were completed.

1102-40-11647/58-0266 DEPOLYMERIZATION PROCESSES

Origin and Sponsor: NBS, Section 7.6

Manager: Maxine L. Rockoff

Full Task Description: July-September 1957 issue, p. 36

Status: Inactive.

1102-40-11645/58-0339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: H. Oser

Full Task Description: January-March 1958 issue, p. 38

Status: Continued. The manuscript of a joint publication with R. S. Marvin

was completed.

1102-12-11513/59-0348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION

Origin: NBS

Sponsor: Office of Ordnance Research, U. S. Army

Manager: Ida Rhodes (11.0)

Full Task Description: October-December 1958 issue, p. 26

Status: Continued. Work proceeded on details of the machine code and on the program for profiling of sentences. The pilot dictionary has progressed to the point where the morphological information is complete for over onethird of the entries.

Publications: (i) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and Ida I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, September 6-8, 1961. (ii) The hindsight technique in machine translation of natural languages. Ida I. Rhodes and F.L. Alt. Journal of Research NBS, 66B, 47-51, Apr.-June, 1962.

1102-40-11645/58-0366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U. S. Information Agency, Department of State

Manager: P. J. Walsh

Full Task Description: April-June 1958 issue, p. 35

Status: Inactive.

1102-40-11645/58-0368 INTENSITY FUNCTIONS OF SCATTERED LIGHT

Origin and Sponsor: U. S. Army Signal Research and Development Laboratories

Manager: H. Oser

Full Task Description: July-September 1958 issue, p. 32

Status: Continued. The Tables of Mie Scattering Functions were proof read and completed to be submitted to the Editorial Committee in the near future.

1102-40-11645/59-0394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING

Origin and Sponsor: NBS, Section 4.6

Manager: A. E. Beam

Full Task Description: October-December 1958 issue, p. 30

Status: Completed.

1102-40-11645/60-0465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS

Origin and Sponsor: NBS, Section 3.2

Managers: P. J. Walsh and J. D. Waggoner

Full Task Description: October-December 1959 issue, p. 26

Status: Terminated. Production runs continued under direction of the

sponsor.

1102-40-11645/60-0466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Full Task Description: October-December 1959 issue, p. 27

Status: Terminated.

1102-40-11645/60-0476 GAS TUBE CHARACTERISTICS, II

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: H. Oser

Full Task Description: October-December 1959 issue, p. 30

Status: Continued. Regular production runs were made by the sponsor

throughout the period.

1102-40-11645/60-0486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS

Origin and Sponsor: NBS, Section 3.0

Manager: Ruth Zucker

Full Task Description: January-March 1960 issue, p. 28

Status: Continued. Code has been rewritten and revised for the 7090.

Additional coding has been included to compute as an option the higher moment integrals  $\int \psi_i \psi_j r^n dr$  where  $\psi_i$  and  $\psi_j$  are members of the same family of wave functions,  $i \neq j$ ,  $0 < n \le 5$ .

The higher moment integrals have application to the interpretation of intensities of infra-red spectra.

Code has been written and checked out. Some production runs have been made.

1102-40-11645/60-0513 RADIATIVE ENVELOPES OF MODEL STARS

Origin and Sponsor: National Aeronautics and Space Administration

Managers: P. J. Walsh and S. Haber (11.1)

Full Task Description: July-September 1960 issue, p. 23

Status: Inactive.

1102-40-11645/61-0516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE

Origin and Sponsor: NBS, Section 4.8

Manager: R. J. Herbold

Full Task Description: July-September 1961 issue, p. 24

Status: Terminated. Results of  $S_n(k^2)$  where n = 0, 1, 2, ..., 9

and  $k^2 = .01, .02, .03, ..., .99$  were computed and transmitted to the sponsor.

3911-61-39952/61-0528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING

Origin and Sponsor: NBS, Section 14.1

Manager: J. D. Waggoner

Full Task Description: July-September 1960 issue, p. 28

Status: Terminated.

1102-40-11645/61-0530 SPECIMEN WAVE LENGTH

Origin and Sponsor: NBS, Section 9.4

Manager: L. Joseph

Full Task Description: July-September 1960 issue, p. 28

Status: Continued. A code was written to determine the parameters using a least squares fit of experimental data. Tests were run and it was found that the solution for the parameters was too sensitive to experimental error.

1102-40-11645/61-0531 HEAT TRANSFER IN CRYSTALS

Origin and Sponsor: NBS, Section 3.1

Manager: H. Oser

Full Task Description: July-September 1960 issue, p. 29

Status: Continued. Some modifications of the program were made, among them an option to generate a large number of different mass distributions in order to study their influence. Results show remarkable agreement with perturbation theory for small amplitudes.

1102-40-11645/61-0538 SPECTRAL REFLECTANCE

Origin and Sponsor: NBS, Section 9.4 Managers: S. Haber (11.1) and P. J. Walsh

Full Task Description: October-December 1960 issue, p. 23

Status: Continued. Further numerical experiments using new experimental programs were performed. It has been concluded that if only two terms corresponding to one family of free electrons and one family of bound electrons are used in the theoretical expression for the reflectivity, the experimental values cannot be fitted. Expressions using three or four terms are being investigated.

1102-40-11645/61-0540 DIFFUSION CALCULATIONS

Origin and Sponsor: Army Chemical Center

Manager: L. Joseph

Full Task Description: January-March 1961 issue, p. 21

Status: Continued. The sponsors have continued coding and running their

own codes.

1102-40-11645/61-0556 TCHEBYCHEFF APPROXIMATION BY RATIONAL FUNCTIONS

Origin and Sponsor: NBS, Section 11.1

Manager: P. J. Walsh

Full Task Description: January-March 1961 issue, p. 22

Status: Continued. Some production runs were made and results are being examined by the sponsor.

1102-40-11645/61-0560 MUSCLE FLEXING

Origin and Sponsor: National Naval Medical Center

Manager: H. Oser

Full Task Description: April-June 1961 issue, p. 22

Status: Inactive.

1102-40-11645/61-0571 NMR SPECTRUM Origin and Sponsor: NBS, Section 15.07

Manager: H. Oser

Full Task Description: April-June 1961 issue, p. 25

Status: Terminated. Production runs are being made directly by the sponsor.

1102-40-11645/62-1009 MONTE CARLO NEUTRON STUDIES

Origin and Sponsor: NBS, Section 4.3

Manager: Sally T. Peavy

Full Task Description: April-June 1961 issue, p. 21

Status: Continued. Two subroutines are ready to be checked out on the 7090.

One is a special randon number routine, and the other is a table look-up and interpolation program.

1105-40-11645/62-1017 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS

Origin: NBS

Sponsor: Post Office Department, Office of Research and Engineering

Managers: Bernice K. Bender and A. J. Goldman

Full Task Description: October-December 1958 issue, p. 22

Status: Terminated. Division 11's work on this task will continue but

will be reported by Division 12.

1102-40-11647/62-1022 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full Task Description: April-June 1958 issue, p. 33

Status: Continued. A small amount of production occurred this quarter.

1102-40-11645/62-1027 NEW SYSTEM

Origin and Sponsor: NBS, Section 11.2

Manager: J. H. Wegstein

Full Task Description: July-September 1961 issue, p. 22

Status: Continued. In December 1962, the Computation Laboratory expects to replace its IBM 1401 computer with an IBM 1410 computer and disk memory which are to be sponsored by the Diamond Ordnance Fuze Laboratory. Plans were started for a monitor system which would use computer, disk, and secretary computer. The disk and tapes are to be used, hopefully, as a stored program system wherein the system retains copies of all programs being checked out and also being used for production computing. The system must do its own cost accounting and the 1410 must also be available for use as an independent computer. Guy Ziegler coded experimental programs for transferring information from 7090 to disk to 1410.

1102-40-11645/62-1030 ELECTROCARDIOGRAPHIC ANALYSIS

Origin: NBS, Section 12.5

Sponsor: Veterans Administration

Manager: R. J. Herbold

Full Task Description: April-June 1959, p. 29

Status: Continued. Production runs were continued. Some histogram plottings of certain peaks and durations of QRS complexes were made. Data tapes were generated for statistical purposes. Some statistical computations were made with these tapes; the data coming from EKG records of known characteristics were grouped accordingly. Covariance and means were computed for each group and were used in likelihood ratio test computations of individual cases.

A short program has been written; it reads and prints digital records which were transformed from analog records by an analog-digital convertor at Mt. Alto Hospital.

A new program, likelihood ratio test with regression, is being written; it will use some data tapes mentioned above but will use additional available data like age or chest measurement.

1102-40-11647/62-1043 MAXIMA AND MINIMA COMPUTATIONS

Origin and Sponsor: NBS, Section 9.0

Manager: Ruth Zucker

Full Task Description: October-December 1961 issue, p. 23

Status: Completed. Additional production runs were made and results

submitted to the sponsor.

1102-40-11647/62-1125 MATRIX COMPUTATIONS

Origin and Sponsor: NBS, Section 9.5

Manager: P. J. Walsh

Full Task Description: October-December 1961 issue, p. 25

Status: Inactive.

1102-40-11647/62-1130 FALLOUT SHELTER COMPUTATIONS

Origin and Sponsor: Office of Civil Defense

Manager: D. I. Mittleman

Full Task Description: October-December 1961 issue, p. 25

Status: Continued. The calculation of protection factors continues and as of the end of this period was 95% completed. The codes required to produce the various summaries have been completed and summaries by standard location, county, county area, state, region, and nation have been compiled on a continuing basis.

1102-40-11647/62-1144 THERMAL BOUNDARY LAYERS Origin and Sponsor: University of Maryland

Manager: H. Oser

Full Task Description: October-December 1961 issue, p. 26

Status: Continued. The results for both the forced flow case and the buoyant boundary layer were obtained except for the case Pr = 10.0 where instabilities of the differential equations delayed the computations.

Another problem was submitted by the sponsor. To tabulate a solution  $\phi_3(z)$  of the complex differential equation.

$$i\frac{d^4\phi}{dz^4} + z\frac{d^2\phi}{dz^2} = 0$$

where

$$\phi_4(z) = \int_{-\infty-\infty}^{z} \int_{-\infty}^{z'} t^{1/2} H_{1/3}^{(2)} \left(\frac{2}{3} (it)^{3/2}\right) dt dz'$$

are four independent solutions.  $\phi_4(z)$  can be obtained from the table for  $\phi_3(z)$ .

The problem was analyzed and programmed by L. Joseph and the results were transmitted to the sponsor.

1102-40-11647/62-1155 MORTGAGE LOAN SURVEY

Origin and Sponsor: Federal Home Loan Bank Board

Manager: Ruth Zucker

Full Task Description: January-March 1962 issue, p. 24

Status: Continued. The correlation code has been written and checked out.

Print-out of sample tables have been submitted to the sponsor for approval.

Production runs have not been made because data cards for the survey are not available at present.

1102-40-11647/62-1168 DISPERSION PARAMETERS

Origin and Sponsor: NBS, Section 11.3

Manager: Karen Bedeau

Full Task Description: January-March 1962 issue, p. 25

Status: Completed. Results have been transmitted to the sponsor.

1102-40-11647/62-1171 HOSPITAL PROGRAM PLANNING

Origin and Sponsor: Veterans Administration, Mount Alto Hospital

Manager: Sally T. Peavy

Full Task Description: January-March 1962 issue, p. 26

Status: Continued. Some of the BIMD codes have been converted to NBS system and production runs have been made. Sponsor is in process of analyzing results.

1102-40-11647/62-1174 IMPULSE CALCULATIONS

Origin and Sponsor: NBS, Section 30.0

Manager: A. Beam

Full Task Description: January-March 1962 issue, p. 26

Status: Continued. Coding was completed, checked, and a few runs were made.

1102-40-11647/62-1176 NUMERICAL INTEGRATIONS

Origin and Sponsor: NBS, Section 15.2

Manager: D. Kaplan

Full Task Description: January-March issue, p. 26

Status: Inactive

1102-40-11647/62-1177 ANALYSIS OF VARIANCE

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: Louis Joseph

Full Task Description: January-March 1962 issue, p. 27

Status: Continued. The program was modified to run on the Bell system.

Sample problems were run and the program gave proper answers for a complete factorial design. Wrong answers were obtained for a non-orthogonal design.

Production runs for complete factorials were made and results were given to the sponsor.

1102-40-11647/62-1178 LOGARITHMIC COEFFICIENTS

Origin and Sponsor: NBS, Section 5.3

Manager: R. J. Arms

Full Task Description: January-March 1962 issue, p. 27

Status: Continued: A code has been checked out for performing the basic

calculations. The code is being used presently in test runs for

estimating accuracy and timing.

1102-40-11647/62-1179 CATALOGUE INFORMATION

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: Ruth Varner

Full Task Description: January-March 1962 issue, p. 27

Status: Continued. A 1401 code was written to change the format of some existing data cards. The results were transmitted to the sponsor.

1102-40-11647/62-1182 FOURIER INTEGRAL

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: H. Oser

Objective: To provide a program capable of computing Fourier-transforms of numerically given functions. Special emphasis is to be given to the requirement that meaningful results are produced for higher frequencies.

Background: The Fourier-transforms

$$A(w) = \int_{0}^{T} f(t) \cos wt dt$$

$$B(w) = \int_{0}^{T} f(t) \sin wt dt$$

for pulses 
$$F(t) = \begin{cases} f(t) & t < T \\ 0 & t > T \end{cases}$$

are computed with increasing difficulty as the frequency w increases. An ordinary trapezoidal or Simpson rule fails very quickly as the oscillations increase. Two methods were tried and both found to be so successful that they were incorporated into the final program.

(a) Piecewise linear interpolation on f(t). If f(t) is approximated by a piecewise linear function and if the arguments are assumed to be equidistant, the following set of formulas results:

$$A(w) = \sum_{i=1}^{n-1} \frac{\Delta_i f(t_i)}{w^2 \Delta x} \Delta_i \cos wt_i + \left[ f(t_n) \sin wt_n - f(t_1) \sin wt_1 \right] / w$$

$$B(w) = \sum_{i=1}^{n-1} \frac{\Delta_i f(t_i)}{w^2 \Delta x} \Delta_i \sin wt_i - \left[f(t_n) \sin wt_n - f(t_1) \cos wt_1\right]/w$$

where  $\triangle_{i}$  is the forward-difference operator:

$$\Delta_{i}^{f(t_{i})} = f(t_{i+1}) - f(t_{i}), \text{ etc.}$$

(b) Quadratic interpolation on f(t) (Filon's method). The formulas resulting from putting a quadratic parabola through three successive points and integrating the expressions have the advantage that they do not require more points than Simpson's rule for the integration of f(t) alone. The formulas can be found in L.N.G. Filon: "On a quadrature formula for trigonometric integrals." Proc. Roy. Soc. Edinburgh 49, (1929), 38 ff.

It should be noted here that both methods suffer from a heavy figure loss when the frequencies become of the order of several hundred radians.

We have therefore decided to compute those parts involving trigonometric functions in double precision. A. Beam wrote a Fortran program for both methods (a) and (b) and several test functions were used to find excellent agreement between the two methods, the choice of which depends naturally on the nature and the precision of the given function f(t).

The problem was submitted by G. Ausman (DOFL).

Status: New.

1102-40-11647/62-1188 STEPWISE REGRESSION

Origin and Sponsor: Ft. Belvoir, U. S. Army

Manager: P. J. Walsh

Objective: For m independent variables and one dependent variable to calculate simple correlation coefficients for all combinations of the variables two at a time. To select the significant ones among the independent variables, and calculate the regression coefficients for the significant variables. To calculate a prediction table for all observations based upon the regression coefficients.

Background: To determine the functional relationship between a dependent variable, radiance, and a number of related and unrelated independent variables that establish the observed radiances of objects in a thermal background scene.

The problem was communicated by R. E. Deighton (Ft. Belvoir).

Status: New. A SHARE library code was used to perform the above calculation.

Production runs are now being made.

1102-40-11647/62-1189 SEQUENTIAL METHODS TABLES

Origin and Sponsor: Quartermaster Research and Engineering Field Evaluation Agency, U. S. Army

Manager: R. J. Arms

Objective: To generate tables for sequential analysis beyond the present published ones.

Background: Scope of tables is that recommended in a report by B. K. Ghosh and Harold Freeman of Groton Associates, Inc. for Headquarters Quartermaster Research and Engineering Command. The report is entitled "Introduction to sequential experimentation." These tables will serve to implement the theory in the design of experiments and will be of aid in analysis of results.

Status: New. Some tables have been completed. Further programming is in progress.

1102-40-11647/62-1190 INTERFEROMETER CALCULATIONS

Origin and Sponsor: NBS, Section 2.5

Manager: L. Joseph

Objective: For a set of equally spaced numbers  $\beta_i$ , to solve the system of equations:

(1) 
$$\sin (\theta_i - \alpha_i) = \frac{e}{a} \sin \alpha_i = n \sin (\theta_i - \beta_i)$$

for  $\theta_i$  and  $\alpha_i$ , where e, a, and n are given parameters.

Background: To calculate tables for use of an interferometer, it was necessary to solve the above system of equations and then to compute the quantities:

$$\begin{split} \delta_{a,i} &= -\text{nu}(\cos \beta_i - \cos \beta_{i-1}) + \sqrt{a^2 - e^2 \sin^2 \alpha_{i-1}} - \sqrt{a^2 - e^2 \sin^2 \alpha_i} \\ &+ \text{ne} (\cos \beta_i - \cos \beta_{i-1}) + \text{na} [\cos (\theta_i - \beta_i) - \cos (\theta_{i-1} - \beta_{i-1})] \end{split}$$

$$\delta_{u,i} = -nu(\cos \beta_i - \cos \beta_{i-1})$$

$$\delta_{v,i} = -nv(\sin \beta_i - \sin \beta_{i-1}).$$

The problem was transmitted by J. B. Saunders (2.5).

Status: Completed (New). The variable  $\theta_i$  was eliminated from (1) to obtain:

(2) 
$$\alpha_i + \arcsin(\frac{e}{a}\sin\alpha_i) - \arcsin(\frac{e}{na}\sin\alpha_i) - \beta_i = 0$$
.

A Fortran program was written to solve (2) using the Newton-Raphson method, and then to compute the  $\theta_i$  and the  $\delta$ 's. To avoid loss of significant figures double precision was used in the calculations. Also, formulas for the difference of two sines and the difference of two cosines were used.

The production runs were made and the results were given to the sponsor.

1102-40-11647/62-1191 CAB TAPE CONVERSION

Origin and Sponsor: Civil Aeronautics Board

Manager: G. G. Ziegler

Objective: To edit and convert those data files created via the Domestic Airline Traffic Survey from the format used on the IBM 7090 into a format which could be used on the IBM 1401; and to prepare a table of inter-city mileages for those city pairs involved in this survey.

Background: Since the inception of the Domestic Airline Traffic Survey, eleven reports have been produced using the NBS computer services. The generation of these reports resulted in the creation of data files applicable for continued processing on the IBM 7090. In March of this year, the Civil Aeronautics Board, desiring to transfer this problem to their own computer installation, requested that these files be converted into a format that could be used on the IBM 1401. It was further requested that a table of inter-city mileages be provided that could be used in the preparation of a master file of all city pairs involved in a given report.

The problem was submitted by J. FitzGibbon(CAB).

Status: New. A total of sixty-nine data tapes were processed representing approximately 4.6 million items of information.

1102-40-11647/62-1193 SOLUTION TO SECOND ORDER PARTIAL DIFFERENTIAL ELLIPTIC EQUATION

Origin and Sponsor: NBS, Section 3.08 Managers: P. J. Walsh and R. J. Arms

Objective: To solve

$$\nabla^2 \varphi = -ks + e^{\varphi}$$
 with  $\nabla \varphi \cdot n = 0$  on surface,

and

$$\nabla^2 g + \nabla g \nabla \varphi = -\frac{\partial \varphi}{\partial x}$$

subject to the boundary conditions

$$g = 0$$
 on  $x = 0$  and  $x = 1$  planes

$$7 \text{ g} \cdot \text{n} = 0$$
 on  $y = 0$  and  $y = 1$  planes

$$7 g \cdot n = 0$$
 on  $z = 0$  and  $z = 1$  planes

To calculate

$$\int_{0}^{1} \int_{0}^{1} \int_{0}^{1} e^{i\hat{Q}} dx dy dz$$

$$\int_{0}^{1} \left\{ \int_{0}^{1} \int_{0}^{1} e^{i\boldsymbol{\sigma}} \left[ 1 + \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \right] d\mathbf{y} dz \right\} d\mathbf{x}$$

 $0 \le x$ , y,  $z \le 1$ . k will assume many values.

$$S = \begin{cases} 1 & x, y, z \le h \alpha \\ 0 & x, y, z > h \alpha \end{cases}, \text{ where } \alpha \text{ is an input parameter.}$$

Background: The effective diffusion constant D' for the self-diffusion of ions in a periodic three-dimensional electrostatic potential  $\Psi$  is given by

$$D' = D_0 \quad \left\{ 1 + \frac{\iiint e^{tO} \left( \frac{\partial g}{\partial x} \right) dV}{\iiint e^{tO} dV} \right\} ,$$

where  $D_0$  is the hydrodynamic diffusion constant and  $\phi = -\epsilon \psi$ kT. The integrals are evaluated over one periodic interval or cube. The function g satisfies the differential equation

$$\nabla(e^{\varphi} \nabla g) = -\frac{\partial e^{\varphi}}{\partial x}$$

For a certain model of a polyelectrolyte solution, the potential is given by the solution of the following Poisson-Boltzmann equation

$$\nabla^2 \varphi = -kS(x, y, z) + e^{\varphi},$$

where S(x, y, z) is a step function and K a parameter which depends on the charge on the polyion.

The problem was submitted by S. Coriell, Section (3.8).

Status: New. The problem has been coded and is now being checked using several sets of parameters.

1102-40-11647/62-1196 HEAT OF ADSORPTION

Origin and Sponsor: NBS, Section 15.2

Manager: Ruth Varner

Objective: To solve for  $dN_3/dN_1$  and  $dN_2/dN_1$ , given the values of X, Y, and  $\theta$ at designated values of  $\theta$ , the pairs of linear equations

$$\alpha_1 \left( dN_3 / dN_1 \right) + \beta_1 \left( dN_2 / dN_1 \right) + \gamma_1 = 0$$

$$\alpha_2 (dN_3/dN_1) + \beta_2 (dN_2/dN_1) + \gamma_2 = 0$$

where  $\alpha_1$ ,  $\beta_1$ ,  $\gamma_1$ ,  $\alpha_2$ ,  $\beta_2$ , and  $\gamma_2$  are evaluated as functions of X, Y, and  $\theta$ .

Background: Calculations, according to the Pace lattice liquid theory, of the population ratio data necessary for calculations of isosteric heats of adsorption for argon on carbon black and on boron nitride were needed. The problem was submitted by C. Prenzlow (15.2).

New. The 7090 code necessary for the calculation has been written Status: and production runs have been made. The results have been transmitted to the sponsor.

1102-40-11647/62-1201 UHF-TV

Origin and Sponsor: Federal Communications Commission

Manager: W. Hall

Objective: To obtain information on coverage of signals in big-city areas, particularly information concerning the availability of television service as furnished by UHF-and VHF-TV stations on the Empire State Building.

Background: Data have been collected on measurements and observations for 5000 typical receiver locations in the metropolitan area of New York City. Analysis of the transmission systems with respect to propagation of signals and operation of equipment must be made on submitted data and, where practical, certain relationships resolved that may be defined after preliminary information has been developed.

The problem was submitted by G. Waldo, (FCC).

Status: New.

1102-40-11647/62-1202 ANALYSIS OF ION REACTION DATA

Origin and Sponsor: NBS, Section 13.6

Manager: Maxine Rockoff

Objective: To develop a code which will permit interpretation of experimental data on ion reaction rates in terms of a realistic model. The reaction data is complicated by the effects of diffusion and certain unavoidable instrumental difficulties. The code performs the following integration:

$$I(t_{i}) = N \int_{t_{i}}^{t_{i}} \left\{ \sqrt{\frac{D}{t_{\pi}}} \left[ \exp \frac{-(d + va_{1} - vt)^{2}}{4Dt} - \exp \frac{-(d + vw_{1} - vt)^{2}}{4Dt} \right] + \frac{1}{2} v \left[ \operatorname{erf} \frac{d + vw_{1} - vt}{\sqrt{4Dt}} - \operatorname{erf} \frac{d + va_{1} - vt}{\sqrt{4Dt}} \right] \right\} e^{-\beta t} dt$$

The parameters N, D and  $\frac{d}{v}$  are varied to make the computed integral fit experimental values.

Background: Interpretation of data on ion reactions in gases is difficult because of the fact that many phenomena are generally occurring at the same time. The number of experimental parameters that can be changed is large and the effect on the observations is quite involved. Many errors of interpretation are traceable to assuming that the physical measurements can be interpreted in terms of simple models.

The problem was submitted by E. C. Beaty (13.6).

Status: New. A Fortran code has been written and checked out for analysis of data taken without ion reaction. It has been found that effects of diffusion are not as well understood as previously thought. It is not possible to interpret the data in terms of simple diffusion. An effort is being made to correct this difficulty before proceeding to the analysis of the more complicated data.

1102-40-11647/62-1203 CYLINDRICAL SHOCK WAVE

Origin and Sponsor: NBS, Section 3.7

Managers: Sally Peavy and S. Haber

Objective: Study the flow behind an expanding cylindrical shock wave by method of characteristics, from the region of very strong shock wave (similarity solution of Taylor and Lin) to the region of weak shock (Oshima's quasi-similar solution).

Background: This problem is associated with experiments involving the explosion of a metallic wire by means of a sudden discharge of electrical energy through the wire.

The problem was submitted by D. H. Tsai (3.7).

Status: New. A program is in process of being written.

1102-40-11647/62-1204 LAPLACE AND FOURIER TRANSFORMS

Origin and Sponsor: NBS, Section 11.2

Manager: Maxine Rockoff

Objective: To study Laplace and Fourier transforms for the solution of

the following equations:

$$\frac{\partial Z_1(z,t)}{\partial t} = -D_1 \alpha_m^2 Z_1 + D_1 \frac{\partial^2 Z_1}{\partial z^2} + v_1 \frac{\partial Z_1}{\partial z} - \alpha_1 Z_1 + \alpha_2 Z_2$$

$$\frac{\partial Z_2(z,t)}{\partial t} = -D_2 \alpha_m^2 Z_2 + D_2 \frac{\partial^2 Z_2}{\partial z^2} + v_2 \frac{\partial Z_2}{\partial z} + \alpha_1 Z_1 - \alpha_2 Z_2$$

Background: These equations arise in the study of ion diffusion and reactions in gases, see task 1102-40-11647/62-1202.

Status: New. Completed. When  $\alpha_1 = \alpha_2 = 0$  (no ion reaction) the solution is given by

$$Z(z,t) = \frac{\exp(-D \ O_{m}^{2} \ t)}{\sqrt{4 \pi D t}} \int_{0}^{\infty} f(y) \left[ \exp\left(-\frac{(z-y+vt)^{2}}{4Dt}\right) - \exp\left(-\frac{(z+y+vt)^{2}}{4Dt} + \frac{vy}{D}\right) \right] dy$$

where f(y) = Z(y,0).

1102-40-11647/62-1206 CALCULATION OF AVERAGE THEORETICAL DEPRECIATION RESERVES

Origin and Sponsor: Department of the Treasury

Manager: R. Herbold

Objective: To compute tables of P<sub>Lj</sub> (N,i);

$$P_{\ell,j}(N,i) = \frac{\sum_{k=1}^{\ell} A_{j}(k,N)(1+i)}{\sum_{k=1}^{N-k} (1+i)}$$

$$A_{j}(k,N) = \frac{1}{2} [R_{j}(k,N) + R_{j}(k-1,N)]$$

$$R_1(k,N) = k/N$$

$$R_2(k,N) = 1 - (1 - 2/N)^k$$

$$R_3(k,N) = k[2N - (k-1)]/N(N+1)$$

$$R_4(k,N) = 1 - (1 - 1.5/N)^k$$

where  $\ell$ , j, N, i are inputs.

Background: The problem arises from plans for a new tax depreciation policy. In an effort to predict the effects of alternate proposals the several tables are to be computed.

The problem was submitted by M. David (Dept. of the Treasury). Status: New. The program to compute and print  $P_{\ell,j}(N,i)$  was written and checked out. A production run was made to get tables of  $P_{\ell,j}(N,i)$  for requested  $\ell$ , j, N, and i. The results were transmitted to the sponsor.

1102-40-11647/62-1209 MIXED INITIATION OF DEPOLYMERIZATION

Origin and Sponsor: NBS, Section 7.6

Manager: Maxine Rockoff

Objective: To solve the following system:

$$\frac{d Q_{N}}{d \tau} = -\left[\frac{2 + \mu(N-3)}{1 + \mu} + (N-1) \sigma\right] Q_{N}$$

$$\frac{d Q_{N-1}}{d \tau} = -\left[\frac{2 + \mu(N-4)}{1 + \mu} + (N-2) \sigma\right] Q_{N-1} + \left[\sigma + \epsilon \left(\frac{2}{1 + \mu} + \sigma\right)\right] Q_{N}$$

$$\frac{d Q_{n}}{d \tau} = -\left[\frac{2 + \mu(n-3)}{1 + \mu} + (n-1) \sigma\right] Q_{n} + \left[\sigma + \epsilon \left(\frac{2}{1 + \mu} + \sigma\right)\right] Q_{n+1}$$

$$+ \sum_{i=1}^{N-n-1} K_{i} Q_{n+1+i} \quad (L \le n \le N-2)$$

where N, L,  $\sigma'$ , Z',  $\mu$ , and the  $\mathbf{Q_i'}$ s at  $\tau$  = 0 are input parameters and

$$\sigma = \frac{\sigma' \left[ (2-3\mu) \sum_{i=L}^{N} Q_i + \mu_{i=L}^{N} iQ_i \right]^{\frac{2}{2}}}{\left[ 2 + (N-3) \mu \right]^{\frac{1}{2}}}$$

$$Z = \frac{Z' \left\{ \left[ 2(2-3\mu) - (1+\mu) \sigma' \right] + \left[ 2\mu + (1+\mu) \sigma' \right] N \right\} \left\{ (2-3\mu) \sum_{i=L}^{N} Q_i + \mu_{i=L}^{N} iQ_i \right\}^{\frac{1}{2}}}{\left[ 2 + (N-3) \mu \right]^{\frac{1}{2}} \left\{ \left[ 2(2-3\mu) - (1+\mu) \sigma \right] \sum_{i=L}^{N} Q_i + \left[ 2\mu + (1+\mu) \sigma \right] \sum_{i=L}^{N} iQ_i \right\}}$$

$$\mathcal{E} = \frac{1}{Z+1}$$

$$\mathtt{K}_{\mathtt{i}} = (\frac{2 \ \mu}{1 + \mu} + \sigma) \ [1 - (1 - \epsilon)^{\mathtt{i}}] + (\frac{2}{1 + \mu} + \sigma) \ \epsilon (1 - \epsilon)^{\mathtt{i}} + \sigma$$

Background: The equations describe a chain depolymerization process in which initiation for breaking of a particular stable chain can occur either at the end of the chain or randomly.

The problem was communicated by L. A. Wall (7.6)

Status: New. A four term recursion formula for the computation of the derivatives has been found and FAP coded. A Fortran main program has been coded using an available differential equation solver with Adams-Moulton integration, variable time-step option and Runge-Kutta restarting procedure. These codes are being checked out.

1102-40-11647/62-1212 COLOR DIFFERENCES

Origin and Sponsor: NBS, Section 10.9

Manager: J. D. Waggoner

Objective: The present investigation, initiated in 1956, is a joint effort of the Metallic Building Materials Section, National Bureau of Standards, and the Porcelain Institute, working through its Research Associateship at the Bureau. The goals are two-fold: (1) to evaluate the weather resistance of new types of porcelain enamel that have been introduced during the past 15 years, and (2) to develop reliable tests for predicting weather resistance.

The present IBM 704 computer program is being changed to allow use of the new 7090 computer.

The program is being arranged so that the computer will calculate:

- (1) Color stability index (Ics) and gloss index (Ig) averages for each exposure site.
- (2) Ics and Ig grand averages for any selected number of sites.
- (3) The standard deviation of these averages.
- (4) Group enamels by types and by acid spot test ratings, then calculate Ics and Ig grand averages.
- (5) The standard deviation of each grand average calculated in (4) above.

Background: Increased use of porcelain enamel for architectural panels has caused manufacturers to take a greater interest in the causes and effects of weathering. The ultimate objective of the investigation is to develop an accelerated test to determine weather resistance. By the use of the accelerated test, fabricators will be able to determine if enamels will be good or poor when exposed to weathering. Information obtained from weathered specimens will give a basis for developing this test.

The problem was submitted by R. S. Thompsen (10.9)

Status: New. The code has been written for the 7090 and test runs have been made. Results have been submitted to the sponsor for evaluation.

### 7. STATISTICAL ENGINEERING SERVICES

### COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50

Managers: W. J. Youden, J. M. Cameron

Full task description: July-September 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

- (i) Voltage cell calibrations. A compilation of incomplete block designs appropriate for voltage cell calibrations (or any other calibration process where only differences are measured) was prepared by J. M. Cameron for C. A. Law of the Electrochemistry Section.
- (ii) Weight calibrations. Churchill Eisenhart and J. M. Cameron continued work with Paul E. Pontius and L. B. Macurdy of the Mass and Scales Section in developing efficient and convenient methods for the analysis of the precision of weighings, and for the detection of possible drifts, changes in precision, etc.
- (iii) Angular standards. A program for the machine computation of the analysis of data from the calibration of polygons used for angular standards was prepared and used in the analysis of data from studies of the effect of environmental factors on the precision and accuracy of the calibrations. This work was done by J. M. Cameron for D. B. Spangenberg and C. E. Haven of the Engineering Metrology Section.
- (iv) Color scales. Joan R. Rosenblatt conferred with G. L. Howett of the Photometry and Colorimetry Section on the design of two experiments. One of these, continuing earlier work, is part of the experimental program of the Committee on Uniform Color Scales of the Optical Society of America. A large-scale incomplete paired-comparison experiment is being designed for the evaluation of perceptual distances between adjacent color samples in an array of supposedly equidistant samples. The second experiment is a colormatching experiment which will provide information about the loci of "constant hue" perceptions that are obtained when chroma (intensity) varies while value (lightness) is held constant. Observers will be asked to match color specimens with a fixed array of "standards". An analysis was made to determine some of the possible effects of various proposed schemes for requiring observers to subdivide the interval between adjacent standards.
- J. R. Rosenblatt presented an in-hours course on "Nonparametric statistical techniques."

#### Publications:

(1) Distribution of total service time for a fixed observation interval. W. S. Connor and Norman C. Severo. Journal of the American Statistical Association, <u>57</u>, 376-386, June 1962.

- (2) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. Journal of the Optical Society of America, 52, 685-691, June 1962.
- (3) On the realistic measurement of precision and accuracy. Churchill Eisenhart. To appear in the Proceedings of the 8th National Aero-Space Instrumentation Symposium.

## STATISTICAL SERVICES Task 1103-40-11625/58-348

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January-March 1958 issue, p. 45

Status: CONTINUED. Miscellaneous services were performed for the following agencies: (i) Veterans Administration Hospital, Perry Point, Md. (ii) Agricultural Marketing Services, U. S. Department of Agriculture.

This is a record of the use of the IBM 7090 for the period of April 1 through June 30, 1962.

TASK NUMBER TITLE				CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NBS SERVI	CES		( M	I N	U T	E S )
51-0002 54-0030 54-0031 54-0032 54-0033 54-0055 55-0082 56-0131 56-0166 56-0171 57-0219 57-0236 57-0250 57-0252 58-0256	11.3 13.1 13.1 13.1 13.1 11.1 3.1 2.2 15.0 3.8 3.2 3.8 2.1 4.4	STATISTICAL ENGINEERING SPECTRUM ANALYSIS++ SPECTRUM ANALYSIS++ SPECTRUM ANALYSIS++ SPECTRUM ANALYSIS++ SPECTRUM ANALYSIS++ RESEARCH IN NUMERICAL ANALYSIS THERMOMETER CALIBRATION+ CALCULATIONS IN OPTICS+ SCF-LCAO SOLUTION OF HYDRIDES+ TRANSPORT THEORY INTEGRALS++ THERMAL PROPERTIES+ SCF EIGENVALUES+ SPECTROPHOTOMETRIC DATA+ NEUTRAL MESON EXPERIMENTS++ COMPOSITE WALL STUDIES++	12 109 5 0 21 0 16 0 1 4 5 0 13 177 70	3 50 150 4 246 0 38 0 1 434 5 49 2 8 151 63	36 142 34 23 643 16 0 90 0 10 0 60 31 5 231 146	51 301 189 27 910 16 54 90 2 448 10 114 33 26 559 279
58-0272 58-0314 59-0394 59-0403 59-0421 59-0433 59-0440 60-0474 60-0493 61-0523 61-0530 61-0531 61-0556 61-0559 61-0571 61-0824 61-0825 61-0826	3.7 3.7 13.6 2.1 12.5 2.1 87.1 2.5 3.8 4.7 9.4 3.1 9.4 11.1 3.1 15.4 11.2 11.2	EQUATION OF STATE++  APPROXIMATIONS FOR GAS MIXTURES+  SCATTERING BY HYDROGEN ATOMS  COMPUTATION OF COLOR FADINGS+  TRAFFIC SIMULATION++  COLOR OF SIGNALS++  NUMERICAL MAPPING++  GAGE BLOCK STABILITY+  POISSON DISTRIBUTION FUNCTION++  NEUTRON CROSS SECTION STUDIES++  SPECIMEN WAVELENGTHS  HEAT TRANSFER IN CRYSTALS  SPECTRAL REFLECTANCE DATA  CHEBYSHEV APPROXIMATIONS  THERMOCOUPLE CALIBRATION  NMR SPECTRUM  SYSTEMS CONVERSION+++  TRAINING+++  TRAINING+++	214 114 7 0 4 20 6 0 49 12 3 8 14 0 0	324 426 144 4 6 3 65 0 257 37 10 18 0 7	206 1367 15 7 0 12 21 3 279 12 0 189 25 14 7 46 59 0	744 1907 166 11 10 35 92 3 585 61 13 215 39 21 17 46 80 16 24

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TASK NUMBER NBS SERVICES:		TITLE	ASSENBLY			TOTAL TIME ON COMPUTER
			( M	I N	UTI	E S )
61-0856	11.2	IBM 7090 FORTRAN INSTRUCTION+++	0	3	0	3
61-0995	11.2	ERROR DETECTION+++	1	0	267	268
62-1000	12.5	POST OFFICE OPERATIONS STUDY++ MOLECULAR SPECTROSCOPY+	142	171	6	319
62-1003 62-1005	15•4 4•3	RADIATION INTERACTION++	9 75	305 33	59 9	373 117
62-1006	4.3	RADIATION INTERACTION++	10	0	0	10
62-1007	4 • 3	RADIATION SHIELDING++	0	28	3	31
62-1008	4.3	GAMMA RAY PENETRATION++	49	372	6	427
62-1009	4.3	MONTE CARLO NEUTRON STUDIES	1	0	0	1
62-1011	13.5	DISPERSION INTEGRALS++	7	Ö	1	8
62-1013	7.0	STATISTICAL METHODS++	4	1	0	5
62-1015	15.1	THERMAL FUNCTIONS++	43	5	0	48
62-1019	41.0	NBS_PERSONNEL_REPORT++	1	9	56	66
62-1020	3 • 3	EIGENVALUES+	27	131	4	162
62-1028	11.2	GENERAL SUBROUTINES+++	1	0	12	13
62-1029 62-1033	9•7 9•7	D-SPACING CALCULATIONS+ CRYSTAL STRUCTURE CALIBRATION++	0	0 3	17 70	17 73
62-1034	30.0	PHOTOIONIZATION CROSS SECTION++	52	4	26	82
62-1035	7.7	CREEP DATA ANALYSIS++	30	29	7	66
62-1036	7.7	FILM THICKNESS++	0	3	40	43
62-1038	7.5	STANDARDIZATION ANALYSES++	12	104	1	117
62-1043	9.0	MAXIMA AND MINIMA COMPUTATIONS	0	0	2	2
62-1047	9 • 0	BLACK BOX COMPUTER SERVICE+	1	0	4	5
62-1052	2.0	BLACK BOX COMPUTER SERVICE+	0	0	6	6
62-1055	8 • 4	ELLIPSOIDAL COMPUTATION++	0	8	0	8
62-1064	2 • 4	GAGE BLOCK STUDIES++	0	0	8	8
62-1066	1.2	STANDARD CELLS++	0	0	13	13
62-1080 62-1081	9•2 9•1	BLACK BOX COMPUTER SERVICE+ BLACK BOX COMPUTER SERVICE+	1	2 0	20	23
62-1085	11.2	MATHEMATICAL SUBROUTINES	0	13	1	1 14
62-1089	9.6	ELASTIC CONSTANTS++	8	0	21	29
62-1101	9.6	BLACK BOX COMPUTER SERVICE+	0	0	8	8
62-1102	6.8	BLACK BOX COMPUTER SERVICE+	Ō	Ō	6	6
62-1106	3.0	GAS PROPERTIES++	0	0	4	4
62-1149	5.3	LEAST SQUARES COMPUTATIONS	0	0	7	7
62-1162	10.7	CEMENT AGING STUDIES++	22	190	950	1162
62-1163	14.1	TRANSISTOR AGING BEHAVIOR++	197	490	9	696
62-1164	3.2	THERMODYNAMIC PROPERTIES++	0	0	2	2
62-1165	15.2	NMR SPECTRA ANALYSES+	0	0	37	37
62 <b>-</b> 1168 62 <b>-</b> 1170	11.3 7.7	DISPERSION PARAMETERS HIGH PURITY POLYMERS++	20 1	5 0	0 2	2 <b>5</b> 3
62-1176	30.0	IMPULSE CALCULATIONS	1	44	8	53
62-1178	5.3	LOGARITHMIC COEFFICIENTS	8	70	5	83
	7 4 5		-			9 9

TASK NUMBER		TITLE	ASSEMB <b>LY</b>	CODE CHEÇKING	PRODUCTION	TOTAL TIME ON COMPUTER
CEOTYME CON	•		( M	I N	U T E	s )
62-1185 62-1190 62-1192 62-1193 62-1194 62-1195 62-1196 62-1187 62-1198 62-1199 62-1202 62-1209 62-1210 62-1211 62-1211 62-1212 62-1214 62-1217 62-2003	12.4 10.3 2.5 15.5 3.8 15.1 7.2 15.2 2.4 5.6 7.8 13.6 7.6 2.5 10.9 9.6 10.1 11.2 11.2	NTDC++ HEAT TRANSFER CALCULATIONS INTERFEROMETER CALCULATIONS LIQUID STRUCTURE ELLIPTIC DIFFERENTIAL EQUATIONS MEMBRANE TRANSPORT++ LIGHT SCATTERING++ EQUATIONS IN XY THETA FRUSTRATED REFLECTIONS++ BLACK BOX COMPUTER SERVICE+ COMPLEXITY CONSTANTS++ ANALYSIS OF ION DATA MIXED INITIATION THREAD FORK STRESS++ TECHNICAL INFO RETRIEVAL++ COLOR DIFFERENCES DECAY CURVE FUNCTION SHOESTRING DATA++ TRAINING+++ RESEARCH+++	0 70 10 0 1 28 0 8 0 0 8 2 0 8 8 0	12 269 4 7 8 17 34 0 0 48 42 12 6 38 1 17 3 2	0 917 2 2 0 1 1 3 8 1 0 0 0 0 0 0 0 0	12 1256 16 6 8 37 18 45 8 1 48 50 14 6 46 9 17 5 2
		Totals (NBS Services)	1783	5097	6391	13271
NON-NBS SER	RVICES					
57-0216 58-0269 58-0348 59-0407 59-0409 59-0425 59-0434 60-0457 60-0458 60-0476 60-0486 60-0492 60-0506 61-0532 61-0550 61-0569	NSF NRL OOR DOFL FSLIC CU GC PHA CAB DOFL U ONT IMF WBANK GU ACC GWU AGO	HANDBOOK OF MATHEMATICAL TABLES MOLECULAR STRUCTURE IV+ MACHINE TRANSLATION OF RUSSIAN FOURIER COEFFICIENTS+ BANK BOARD REPORTS++ MOLECULAR ORBITALS PETROLOGICAL COMPUTATIONS+ PUBLIC HOUSING PROBLEM++ AIRLINE TRAFFIC SURVEY+ GAS TUBE CHARACTERISTIC II MORSE WAVE FUNCTION++ MONETARY RESEARCH REPORTS++ VIBRATIONAL ENERGY LEVELS DIFFUSION CALCULATIONS LOGISTICS RESEARCH++ HUMAN FACTORS RESEARCH++	2 6 5 1 72 0 6 11 4 5 3 97 0 2 62 83 27	0 13 29 0 775 0 5 92 6 1 123 47 0 52 69 82	4 15 0 1489 18 27 85 1486 572 44 89 13 0 187 396 74	6 34 34 1 2336 18 38 188 1496 578 170 233 13 2 301 548 183

TASK NUMBER NON-NBS SERVICES		TITLE	ASSEMBLY TIME	CODE	PRODUCTION TIME	ON COMPUTER
			( M	I N	UT	ES)
61-0829	BPR	HIGHWAY TRAFFIC STUDIES++	0	44	578	622
61-0830	BPR	HIGHWAY TRAFFIC STUDIES++	0	0	112	112
61-0849	BPR	HIGHWAY TRAFFIC STUDIES++	52	64	121	237
61-0865	BPR	HIGHWAY TRAFFIC STUDIES++	5	16	505	526
61-0902	BPR	HIGHWAY TRAFFIC STUDIES++	19	18	71	108
61-0903	BPR	HIGHWAY TRAFFIC STUDIES++	2	0	376	378
61-0945	WB	FORECASTING++	0	0	516	516
62-1014	NIH	METABOLIC DISEASES++	91	39	720	850
62-1017	PO	POST OFFICE STUDIES	21	0	3	24
62-1018	NRL	HYDROMAGNETIC PROBLEMS+	27	105	274	406
62-1021	DCH	HIGHWAY STUDIES++	238	648	3545	4431
62-1022	NRL	SPECTRUM OF DIPOLE RADIATION	0	9	8	17
62-1023	NSF	IMAGE PROCESSING++	8	27	- 40	75
62-1030	VA	ELECTROCARDIOGRAPHIC ANALYSIS	203	147	478	828
62-1032	QM	SUPPLY PROGRAMMING PROBLEMS++	11	14	12	37
62-1044	FCC	RADIO INTENSITIES++	13	0	33	46
62-1046	BPR	TRAFFIC PREDICTION++	22	106	200	328
62-1056	DOFL	PD ENGINEERING++++	16	3	113	132
62-1071	DOFL	RHINITIS STUDIES++	0	0	2	2
62-1073	DOFL	COMPLEX LEGENDRE FUNCTIONS++	149	84	65	298
62-1076	NAS	EVALUATION OF APPLICATIONS+	6	0	80	86
62-1096	DOFL	VULNERABILITY STUDY++++	12	103	67	182
62-1110	ICC	ICC SYSTEMS STUDY++	72	141	2.8	241
62-1113	DOFL	TRANSPORT ANALYSES++++	0	47	123	170
62-1115	DOFL	BLACK BOX COMPUTER SERVICE+	2	0	0	2
62-1121	CARIN	CARNEGIE INSTITUTE STUDIES	115	0	34	149
62-1130	COENG	FALLOUT SHELTER COMPUTATIONS	58	654	1335	2047
62-1134	HARVU	STATISTICAL DECISION THEORY++	0	10	0.	10
62-1140	VA	VA MEDICAL++	0	0	230	230
62-1141	COENG	FALLOUT SHELTER COMPUTATIONS	27	129	6951	7107
62-1143	NIH	MOLECULAR INTERACTION++	0	0	1	1
62-1144	UOFMD	THERMAL BOUNDARY LAYERS	51	79	78	208
62-1146	BPR	GRAVITY MODEL STUDIES++	2	33	523	558
62-1147	MPSA	JET FUEL PROCESSING++	5	0 5	0	<b>5</b> 5
62 <b>-</b> 1151 62 <b>-</b> 1154	DARCO AGO	STATISTICAL DECISION THEORY++ MILITARY RESEARCH DATA++	0	0	36	36
62-1155	FSLIC	MORTGAGE LOAN SURVEY	0	73	8	81
62-1158	GC	MINERALOGY STUDIES++	20	35	47	102
62-1169	U ONT	ATOMIC COLLISIONS++	61	34	108	203
62-1171	VA	HOSPITAL PROGRAM PLANNING	20	34	126	180
J- 11,1	* * 1	THE THE THOUGHT I EMITTED	- 0	•		-00

TASK NUMBER TITLE NON-NBS SERVICES				ASSENBLY TIME	CHECKING	PRODUCTION TIME	TOTAL TIME
				( M	N J	UT	ES)
62-1172	PEACE	PEACE CORPS EVAL	UATIONS++	508	49	892	1449
62-1173	CAB	AIRLINES MERGER		12	23	58	93
62-1175	DOFL	ION DISTRIBUTION	S	17	11	0	28
62-1177	DOFL	ANALYSIS OF VARI	–	0	0	4	4
62-1179	DOFL	CATALOG INFORMAT	ION	0	10	0	10
62-1182	DOFL	FOURIER INTEGRAL		0	4	3	7
62-1183	DOFL	BLACK BOX COMPUT	ER SERVICE+	1	0	11	12
62-1186	MELPR	EIGENVALUES		0	0	4	4
62-1188	FTBEL	STEPWISE REGRESS		5	1	7	13
62-1189	QM	TABLES FOR SEQUE	NTIAL METHODS	67	132	41	240
62-1191	CAB	TAPE CONVERSIONS	ICCLI	53	118	490	661
62-1197	UMD	HIGH ENERGY PHYS	105++	5	1	435	441
62-1200 62-1201	VA FCC	STATISTICS++	コノシバコ	0 35/m 10	0	5	5
62-1201	TREAS	UHF TV TAX DEPRECIATION			100	2.8	138
62-1208	DOFL	TAPE LISTING++	REVISION	_	3	4	12
02-1206	DOIL	TAPL LISTINGTT		0	12	43	55
	Totals	(NON-NBS Services	also also also	2397	4459	24071	30927
			DIES+4				
	TOTAL T	IME FOR THE QUARTER	(MINUTES)	4180	9556	30462	44198
	TOTAL T	IME FOR THE QUARTER	(HOURS)	70	159	508	737

<sup>+</sup> Problem programmed in the Computation Laboratory, production runs continued under direction of sponsor.

<sup>++</sup> Problem programmed by the sponsor and run under his direction.

<sup>+++</sup> Functions pertain to the internal operations of the Computation Laboratory.

<sup>++++</sup> Classified task.

### Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

### Applied Mathematics Division Lectures

- WALZ, A. (Technische Hochschule Karlsruhe, Germany). The influence of Mach number and heat transfer on laminar separation at a wavy wall. April 25, 1962.
- MORDELL, L. J. (University of Arizona and St. John's College, Cambridge).

  On some recent work in number theory. May 31, 1962.
- GILLIS, J. (Institute for Space Studies). A problem in hydrodynamic stability. June 4, 1962.
- SCHATTAN, R. A non-approximation theorem. June 12, 1962.

### Seminar in Mathematics

A series of weekly seminars led by Dr. Barry Bernstein with the cooperation of Dr. Oved Shisha, was conducted to cover the Princeton lecture notes, "Advanced Calculus" by H. K. Nickerson, D. C. Spencer, and N. E. Steenrod (D. Van Nostrand Co., 1959). April-June 1962.

### Mathematical Statistics Seminar

- EISENHART, C. The astonishing effectiveness of the asterisk treatment or the unwisdom of treating the 'worst'. April 17, 1962.
- WILLKE, T. A. A rank sum test for outliers. May 4, 1962.
- THOMPSON, W. A. Precision of simultaneous measurement procedures.
  May 22, 1962.
- SIDDIQUI, M. M. (NBS Boulder Laboratories). (i) Autocorrelation analysis of a continuous sample. June 14, 1962; (ii) Regression analysis of a continuous sample. June 19, 1962.

### Lectures and Technical Meetings

# Presented by Members of the Staff at Meetings of Outside Organizations

- ALT, F. L. (i) Digital pattern recognition by moments, (ii) Mathematical theory of management problems, (iii) Machine translation of natural languages—a survey. April 12-13, 1962. These talks were presented at Rutgers State University, New Brunswick, New Jersey and Douglas College, Newark, New Jersey. (iv) Machine translation of natural languages. Presented at the National Convention of the Institute of Radio Engineers, Washington, D. C., June 26, 1962.
- EDMONDS, J. Covers and packings in a family of sets. Presented at the seminar in Operations Research, Massachusetts Institute of Technology, Cambridge, April 11, 1962.
- EISENHART, C. and CROARKIN, M. C. The unwisdom of treating the 'worst'.

  Presented before the Statistics Section of the Virginia Academy of Science, Norfolk, May 11, 1962.
- EISENHART, C. On the measurement of realized precision and accuracy. Presented at the ISA Eighth National Aero-Space Instrumentation Symposium, Washington, D. C., May 23, 1962.
- GHAFFARI, A. Application of the stroboscopic method to a nonlinear equation of nonautonomous character. Presented before the American Mathematical Society, Atlantic City, New Jersey, April 18, 1962.
- RHODES, Ida. Machine translation by means of electronic computers. Presented at the U. S. Army Research Office, Durham, North Carolina, May 25, 1962.
- ROCKOFF, Maxine L. and ROSENBLATT, Joan R. A panel discussion on Careers in Science at a special program for high school girls in the greater Washington area, sponsored by the Committee on Women in Science Joint Board on Science Education, Washington Academy of Sciences. Held at Georgetown University, April 7, 1962.
- ROSENBLATT, Joan R. Confidence limits for estimates of the reliability of complex systems. Presented at the U. S. Army Mathematics Research Center, University of Wisconsin, Madison, April 30, 1962.
- TCHEN, C. M. (i) Collective correlation of plasmas. Presented at the National Conference on Electronics, Prague, Czechoslovakia, April 25, 1962. (ii) Kinetic equation and plasma oscillations. Presented at the Institute for Plasma Physics, Garching, Germany, May 2, 1962. (iii) Plasma oscillations with correlations. Presented at the Max Planck Institute for Physics and Astrophysics, Munich, Germany, May 3, 1962. (iv) Some problems of plasma dynamics. Presented at

### Lectures and Technical Meetings

- the U. K. Atomic Energy Research Establishment, Culham, England, May 9, 1962. (v) Some problems of magnetohydrodynamics. Presented before the Physics Department, Delhi University, Delhi, India, May 11, 1962. (vi) Kinetic Equations with Correlations. Presented at the National Physical Laboratory, New Delhi, India, May 14, 1962. (vii) Some mathematical problems connected with magnetohydrodynamics. Presented at the National Aeronautical Laboratory, Bangalore, India, May 16, 1962. (viii) Some mathematical problems connected with controlled fusion processes. Presented at the National Tsin Hwa University, Taipei, Taiwan, June 26, 1962.
- THOMPSON, W. A. and WILLKE, T. A. A rank sum test for outliers. Presented before the Virginia Academy of Science, Norfolk, May 12, 1962.
- VINTI, J. P. The spheroidal method for satellite orbits. Presented at COSPAR-IAG Symposium on the Use of Artificial Satellites for Geodesy, Washington, D. C., April 26, 1962.
- WEGSTEIN, J. H. (i) Algol 60 programming I, and (ii) Algol 60 programming II. Presented before the Illinois University Computing Center, Carbondale, May 10-11, 1962. (iii) Algorithmic languages. Presented before the Student Chapter of the Association for Computing Machinery, Carbondale, Illinois, May 10, 1962. (iv) Programming languages. Presented at the National Convention of the Institute of Radio Engineers, Washington, D. C., June 26, 1962.
- WEISS, G. H. A survey of some mathematical models in the theory of reliability. Presented at the U. S. Army Mathematics Research Center, University of Wisconsin, Madison, May 9, 1962.
- YOUDEN, W. J. (i) Statistical methods in biological and chemical experimentation, March and April 1962. This was the general title for a series of seminars given at the various installations in Sydney, Brisbane, Canberra, Melbourne, Adelaide, and Perth of the Commonwealth Scientific and Industrial Research Organization (CSIRO), Australia. (ii) Systematic errors in physical constants, March 27, 1962; and (iii) Teaching of statistics in the university, March 28, 1962. Both talks were given at the University of New South Wales, Sidney. (iv) Sophisticated chemistry, March 29, 1962. Presented before the Statistical Society of New South Wales and the Royal Australian Chemical Institute, Sydney. (v) Constrained randomization, April 12, 1962. Presented before the Canberra Statistical Society, Canberra. (vi) A study of ranking scores. Presented before the Scuola di Stude Superiori Sugli Idrocarburi, Milan, Italy, May 15, 1962.

### 1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

### 1.3 Technical Papers

- (1) Fifteen years ACM. F. L. Alt. Communications of the ACM, 5, 300-307, June 1962.
- (2) Digital pattern recognition by moments. F. L. Alt. Journal of the Association for Computing Machinery, 9, 240-258, April 1962.
- (3) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Journal of Mathematical Physics, 3, 469-471, May-June 1962.
- (4) The reflection of logistics in electronic computer design. E. W. Cannon. Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 7, 365-371, December 1960.
- (5) Distribution of total service time for a fixed observation interval. W. S. Connor and N. C. Severo. Journal of the American Statistical Association, 57, 376-386, June 1962.
- (6) Graphs for determining the power of Student's t-test. Mary C. Croarkin. Journal of Research NBS, 66B, 59-70, April-June 1962.
- (7) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Mathematische Zeitschrift, 78, 449-452, March 1962.
- (8) Two matrix eigenvalue inequalities. S. Haber. Journal of Research NBS, 66B, 57-58, April-June 1962.
- (9) A calculus for factorial arrangements. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. Annals of Mathematical Statistics, 33, 600-619, June 1962.
- (10) Modular forms whose coefficients possess multiplicative properties (II). M. Newman. Annals of Mathematics, 75, 242-250, March 1962.
- (11) Tchebyshev approximations by exponentials. J. R. Rice. Journal of the Society for Industrial and Applied Mathematics, 10, 149-161, March 1962.

- (12) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. Journal of the Optical Society of America, 52, 685-691, June 1962.
- (13) The hindsight technique in machine translation of natural languages. Ida Rhodes and F. L. Alt. Journal of Research NBS, 66B, 47-51, April-June 1962.
- (14) An extension of Jensen's theorem for the derivative of a polynomial and for infrapolynomials. O. Shisha. Journal of Research NBS, 66B, 53-55, April-June 1962.
- (15) Kinetic equation for plasmas with collective and collisional correlations. C. M. Tchen. Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, Germany, August28-September 1, 1961, pp. 826-841.
- (16) The relaxation of moments derived from a master equation. K. Shuler (Director's Office), G. Weiss, K. Andersen (Director's Office). Journal of Mathematical Physics, 3, 550-556, May-June 1962.

#### 2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

#### 2.1 Mathematical Tables

(1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

### 2.2 Technical Notes, Manuals, and Bibliographies

- (1) Handbook of mathematical functions. To appear in the NBS Applied Mathematics Series.
- (2) Experimental statistics. Mary G. Natrella. To be published as ORDP 20-110, 111, 112, 113, 114 by the Army Research Office, Durham, Duke Station, Durham, North Carolina.
- (3) Selected bibliography of statistical literature, 1930 to 1957: VI. Theory of estimation and testing of hypotheses, sampling distributions, and theory of sample surveys. Lola S. Deming. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

### 2.3 Technical Papers

(1) Safety levels in military inventory management. F. L. Alt. To appear in Operations Research.

- (2) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and Ida Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, September 6-8, 1961.
- (3) Lower bounds to eigenvalues using operator decompositions of the form B\*B. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in Archive for Rational Mechanics and Analysis.
- (4) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (5) Pointwise bounds in the first biharmonic boundary value problem.

  J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (6) The segmental variation of Blaschke products. G. T. Cargo. To appear in the Duke Mathematical Journal.
- (7) Normal functions, Montel's property, and interpolation in H. G. T. Cargo. Submitted to a technical journal.
- (8) A connection between Tauberian theorems and normal functions. G. T. Cargo. To appear in the Bulletin of the American Mathematical Society.
- (9) Covers and packings in a family of sets. J. Edmonds. To appear in the Bulletin of the American Mathematical Society.
- (10) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (11) On the realistic measurement of precision and accuracy. Churchill Eisenhart. To appear in Proceedings of the Eighth National Aero-Space Instrumentation Symposium.
- (12) Ferroelectric switching and the Sievert integral. P. H. Fang (Physical Properties Section) and Irene A. Stegun. Submitted to a technical journal.
- (13) On Rayleigh's nonlinear vibration equation. A. Ghaffari. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.
- (14) An algorithm for least common multiples. A. J. Goldman. Submitted to a technical journal.

- (15) A property of linear frequency modulation. A. J. Goldman. To appear in the Proceedings of the Institute of Radio Engineers.
- (16) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (17) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), and H. H. Ku. To appear in Technometrics.
- (18) Factorial designs and the direct product. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.
- (19) On the determination of the eigenvalues and eigenvectors of certain matrices. A. N. Lowan. Submitted to a technical journal.
- (20) Stability criteria for problems involving cylindrical and spherical symmetry. A. N. Lowan. Submitted to a technical journal.
- (21) Stability criteria for the Peaceman-Rachford difference scheme.

  A. N. Lowan. Submitted to a technical journal.
- (22) Stability criteria for various difference schemes associated with the problem of the vibrating bar. A. N. Lowan and R. J. Arms. Submitted to a technical journal.
- (23) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (24) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (University of Florida). To appear in the Pacific Journal of Mathematics.
- (25) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. To appear in the Pacific Journal of Mathematics.
- (26) A note on modular groups. M. Newman. To appear in the Proceedings of the American Mathematical Society.
- (27) Bounds for cofactors and arithmetic minima of quadratic forms.

  M. Newman. To appear in the Journal of the London Mathematical Society.
- (28) Multipliers of difference sets. M. Newman. Submitted to a technical journal.

- (29) Some free products of cyclic groups. M. Newman. Submitted to a technical journal.
- (30) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). To appear in the Proceedings of the American Mathematical Society.
- (31) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (32) Two theorems on matrices. M. Newman. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (33) Modulary groups of t X t matrices. M. Newman and J. R. Smart (New York University). Submitted to a technical journal.
- (34) Error bounds for first approximations in turning-point problems. Frank W. J. Olver. Submitted to a technical journal.
- (35) Convergence to normality of powers of a normal random variable.

  N. C. Severo and L. J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (36) Collective correlation of plasma. C. M. Tchen. To appear in the Proceedings of the Second National Conference on Electronics, Prague, Czechoslovakia, April 25, 1962.
- (37) The spheroidal method for satellite orbits. J. P. Vinti. To appear in the Proceedings of the International Symposium on the Use of Artificial Satellites for Geodesy, Washington, D. C., April 26, 1962.
- (38) The zeros of infrapolynomials with prescribed values at given points. J. L. Walsh (Harvard University) and O. Shisha. Submitted to a technical journal.
- (39) On the pedestrian queueing problem. George Weiss. To appear in the Bulletin of the International Statistical Institute.
- (40) The reliability of a system in which spare parts deteriorate in storage. George Weiss. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (41) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (42) The role of laboratories in round robins. W. J. Youden. To appear in Materials Research & Standards.

### U. S. DEPARTMENT OF COMMERCE Luther H. Hodges, Secretary

### NATIONAL BUREAU OF STANDARDS

A. V. Astin. Director



### THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

### WASHINGTON, D.C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics. Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics, Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

**Polymers**, Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Microscopy and Diffraction. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems, Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics, Spectroscopy, Infrared Spectroscopy, Far Ultraviolet Physics, Solid State Physics, Electron Physics, Atomic Physics, Plasma Spectroscopy.

Instrumentation, Engineering Electronics, Electron Devices, Electronic Instrumentation, Mechanical Instruments, Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Elementary Processes. Mass Spectrometry. Photochemistry and Radiation Chemistry.

Office of Weights and Measures.

### BOULDER, COLO.

Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

#### CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. lonosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Frequency Utilization. Modulation Research. Antenna Research. Radiodetermination.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. High Latitude lonosphere Physics. lonosphere and Exosphere Scatter. Airglow and Aurora. lonospheric Radio Astronomy.

### RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Radio Plasma. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. High Frequency Calibration Services. Iligh Frequency Impedance Standards. Microwave Calibration Services. Microwave Circuit Standards. Low Frequency Calibration Services.



T St. Marches .