

NATIONAL BUREAU OF STANDARDS REPORT

7401

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report
January through March 1962



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1962

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^oOnly unclassified material is included in this report.

Status of Projects

March 31, 1962

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS Task 1101-12-11110/55-55

Origin: NBS

Authorized 8/29/54

Manager: Philip J. Davis

Full Task Description: July-September 1954 issue, p. 1

Status: CONTINUED. O. Shisha has worked in the following areas:

(i) Tchebycheff approximation by rational functions, particularly from the computational standpoint; (ii) infrapolynomials with and without prescribed coefficients and their generalizations, including joint work with Professor J. L. Walsh (Harvard University); (iii) fixed-point theorems.

O. Shisha is preparing a survey volume on the theory of the transfinite diameter.

G. T. Cargo and O. Shisha are working on the following subjects:

(i) inequalities between general means, and (ii) bounds for the zeros of polynomials and power series.

G. Cargo has worked in the following areas: (i) boundary behavior of Blaschke products; (ii) new proofs of various classical boundary behavior theorems by means of Ostrowski's extension of Jensen's theorem; (iii) a connection between interpolation in H^∞ and non-Euclidean geometry; (iv) extensions of the classical theorems of Fatou and F. and M. Riesz; and (v) a connection between Tauberian theorems and normal functions, including an extension of the Hardy-Littlewood Tauberian theorem for curvilinear approach.

S. Haber worked on systematic over-relaxation procedures for the solution of systems of linear equations arising from problems in partial differential equations. He also began a survey of the theory of equidistribution (gleichverteilung) mod one.

F. W. J. Olver has completed the first of two papers on error bounds for Airy function approximations in turning point problems.

M. Rockoff is studying the matrices obtained when one uses unbalanced finite difference approximations to the Laplacian in the discretization of the Dirichlet problem. These matrices are slightly unsymmetric and there are a few off diagonal elements with the sign of the diagonal elements. Although the classical Perron-Frobenius theory for non-negative matrices cannot be applied directly, some results about the existence of positive simple eigenvalues with positive corresponding eigenvectors have been obtained.

Status of Projects

P. J. Davis and Maxine L. Rockoff have undertaken a series of numerical experiments designed to produce positive quadrature formulas for highly multiple integrals.

Publications:

- (1) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the Journal of Mathematical Physics.
- (2) Error bounds for eigenvectors of self-adjoint operators. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Journal of Research NBS, 66B(Mathematics and Mathematical Physics), 1-4, January-March 1962.
- (3) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). To appear in the Pacific Journal of Mathematics.
- (4) Inequalities for the permanent function. M. Marcus and M. Newman. Annals of Mathematics, 75, 47-62, January 1962.
- (5) Modular forms whose coefficients possess multiplicative properties (II). M. Newman. To appear in Annals of Mathematics.
- (6) Tchebycheff approximation by exponentials. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (7) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.
- (8) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (9) Multipliers of difference sets. M. Newman. Submitted to a technical journal.
- (10) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. Submitted to a technical journal.
- (11) Lower bounds to eigenvalues using operator decomposition of the form B^*B . N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.
- (12) Two matrix eigenvalue inequalities. S. Haber. To appear in the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).
- (13) Linear operations on matrices. M. Marcus. In manuscript.
- (14) An extension of Jensen's theorem for derivatives of polynomials and for infrapolynomials. O. Shisha. To appear in the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).
- (15) The zeros of infrapolynomials with prescribed values at given points. O. Shisha and J. L. Walsh (Harvard University). In manuscript.
- (16) Zeros of polynomials and fractional order differences of their coefficients. O. Shisha and G. Cargo. In manuscript.
- (17) Error bounds for first approximations in turning-point problems. F. W. J. Olver. In manuscript.
- (18) The segmental variation of Blaschke products. G. T. Cargo. Submitted to a technical journal.

Status of Projects

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS

Task 1101-12-11411/55-56

Origin: NBS

Authorized 8/13/54

Sponsor: Office of Naval Research

Manager: Morris Newman

Full Task Description: July-September 1954 issue, p.5

Status: CONTINUED. M. Newman is continuing his work on free products of cyclic groups and has found matrix representations for such groups.

A study of the arithmetic minima of quadratic forms and bounds for cofactors has been initiated by M. Newman. It is shown, for example, that if γ_n denotes the Hermite constant, then

$$\gamma_{m+n}^{m+n} \geq \gamma_m^m \gamma_n^n .$$

Some theorems on matrices relevant to the solution of linear systems by iteration have been found by M. Newman. These generalize known results of Seidel and Reich.

The monograph on integral matrices now exists as a first draft, and is being revised by M. Newman.

K. Goldberg and M. Newman considered a finite set S , a subset S_1 , and a function f mapping $(S-S_1) \times S_1$ into S . They proved several results concerning the number of iterations of f necessary to map an element of $S-S_1$ into S_1 when f has certain basic uniqueness properties, and also when S is contained in a group and $f(x,y) = xy^{-1}$.

K. Goldberg continued his study of iterates of formal power series and of the basic equation

$$f(A\{t;z\}) = f(z) \frac{\delta}{\delta z} A\{t;z\} = \frac{\delta}{\delta t} A\{t;z\}.$$

Among the results is the expression $A\{F(x)-z;x\}$ for the formal inverse of the function $F(z) = -\int dz/f(z)$ in the sense that the Taylor series in z of $F^{-1}\{F(x) + z\}$ equals the power series in z of $A\{-z;x\}$.

Publications:

- (1) A note on modular groups. M. Newman. Submitted to a technical journal.
- (2) Congruences for the partition function to composite moduli. M. Newman. Illinois Journal of Mathematics, 6, 59-63, March 1962.
- (3) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (4) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). Submitted to a technical journal.
- (5) Hadamard matrices of order cube plus one. K. Goldberg. In manuscript.
- (6) Some free products of cyclic groups. M. Newman. Submitted to a technical journal.
- (7) Two theorems on matrices. M. Newman. In manuscript.
- (8) Bounds for cofactors and arithmetic minima of quadratic forms. M. Newman. Submitted to a technical journal.

Status of Projects

BOUNDS FOR EIGENVALUES
Task 1101-12-11416/62-1091

Origin: Wright-Patterson AFB

Authorized 10/1/61

Manager: Hansjorg Oser (11.2)

Full Task Description: October-December 1961 issue, p.4

Status: INACTIVE.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter because priority was given to the preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 TABLES OF SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 TABLES OF THE SIEVERT INTEGRAL

HANDBOOK OF MATHEMATICAL FUNCTIONS

Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation

Authorized 12/27/56

Manager: Irene A. Stegun

Full Task Description: October-December 1956 issue, p. 10

Status: CONTINUED. The text, tables, graphs, and examples have been prepared for Chapter 18 (Weierstrass Elliptic Functions). Additional examples are being typed for Chapter 8 (Legendre Functions).

The galley proofs for Chapter 4 (Elementary Transcendental Functions) and Chapter 12 (Struve Functions and Related Functions) were corrected and returned for page proofs. Chapter 25 (Numerical Interpolation, Differentiation and Integration) and Chapter 26 (Probability Functions) are in press. Chapter 16 (Jacobian Elliptic Functions and Theta Functions), Chapter 17 (Elliptic Integrals) and Chapter 29 (Laplace Transforms) are ready for press.

AUTOMATIC CODING

Task 1102-12-11120/55-65

Origin: NBS

Authorized 9/29/54

Manager: J. Wegstein

Full Task Description: July-September issue, p. 11

Status: CONTINUED. Studies on the definition and manipulation of

Status of Projects

artificial mechanical languages continued in connection with participation in the work of the Business Equipment Manufacturers Association (BEMA) subcommittee on standard computer languages.

Work began on efforts to remove ambiguities and obscurities from the ALGOL 60 language. A survey conducted jointly with Regnecentralen, Copenhagen, was in the form of a questionnaire designed by Dr. Peter Naur to determine both the amount of support behind the language and technical opinions on details of the language. Forty-eight replies have been received from eighteen countries and a considerable amount of work is entailed in utilizing this information.

Publications:

- (1) A string language for symbol manipulation based on ALGOL 60. J. H. Wegstein and W. W. Youden. Communications of the ACM, 5, 54-61, January 1962.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS

Task 1103-12-11131/51-2

Origin: NBS

Authorized 7/1/50

Manager: Joan R. Rosenblatt

Full Task Description: July-September 1950 issue, P. 58

Status: CONTINUED. A paper is being prepared for publication to report the results of a study by Churchill Eisenhart, Ann D. Smith, and John Van Dyke on the evaluation of probability points of the coverage of Wilks' unbiased tolerance intervals in random samples of size n from a normal distribution. The table of probability points obtained seemed to contain an anomaly which cast doubt upon the adequacy of the approximation employed when the sample size is small. Further investigation disclosed that the seeming anomaly was actually a characteristic of the distribution. The accuracy of the approximation was investigated by calculating upper and lower bounds for several of the probability points and by comparing some points with values obtained by other methods.

Janace Speckman is making a study of four estimators of the parameter ρ in the model $Y = e^{-\rho t} + \epsilon$. The four methods under consideration are maximum likelihood, least squares, weighted least squares, and partial totals. Monte Carlo results have given estimates of the bias and relative efficiencies when the data come from either a binomial distribution or a distribution which is basically binomial but which has extraneous variation. In addition, tables of estimates have been calculated using the partial totals method for a selected number of equally spaced t values when $t_1=0$.

W. A. Thompson and George Weiss are studying approximate solutions of certain classes of nonlinear diffusion equations which arise in genetics and in Skellam's theory of the dispersion of populations.

Publications:

- (1) On the pedestrian queueing problem. George Weiss. To appear in the Bulletin of the International Statistical Institute.
- (2) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (3) Selected bibliography of statistical literature, 1930 to 1957: V. Frequency functions, moments, and graduation. Lola S. Deming. Journal of Research NBS, 66B(Mathematics and Mathematical Physics), 15-28, January-March 1962.
- (4) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), H. H. Ku, and I. J. Good (Admiralty Research Laboratory, England). Submitted to a technical journal.

Statue of Projects

- (5) Convergence to normality of powers of a normal random variable. Norman C. Severo and Lloyd J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (6) Graphs for determining the power of Student's t-test. Mary C. Croarkin. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (7) The relaxation of moments derived from a master equation. Kurt Shuler (Director's Office), George Weiss, Knud Andersen (Director's Office). To appear in the Journal of Mathematical Physics.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN Task 1103-12-11131/53-1

Origin: NBS

Authorized 10/15/52

Manager: J.M. Cameron

Full Task Description: October-December 1952 issue, p. 60

Status: INACTIVE.

Publications:

- (1) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (2) A calculus for factorial arrangements. B. Kurkjian (DOFL) and M. Zelen. To appear in the Annals of Mathematical Statistics.
- (3) Factorial designs and the direct product. B. Kurkjian (DOFL) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.

STUDY OF NONPARAMETRIC STATISTICAL TECHNIQUES Task 1103-12-11131/56-170

Origin: NBS

Authorized 12/15/55

Manager: Joan R. Rosenblatt

Full Task Description: October-December 1955 issue, p. 14

Status: CONTINUED. W. J. Youden has written a paper entitled "The role of laboratories in round robins" in which he proposes a rank sum test for pointing out laboratories with large systematic biases in their measurements. The test may be viewed as supplementary to the Friedman procedure for use of ranks in analysis of variance for a two-way layout. The test criterion is the minimum rank sum. Some properties of this test as an easily used nonparametric method for detecting outliers have been investigated by W. A. Thompson, Jr. and Thomas A. Willke, and a paper entitled "A rank sum test for outliers" is being written. An original table given by Dr. Youden for the test has been expanded and will be included in this paper.

Status of Projects

Publications:

- (1) The role of laboratories in round robins. W. J. Youden. To appear in Materials Research & Standards.

MEASUREMENT OF RELIABILITY

Task 1103-12-11130/56-182

Origin: NBS

Authorized 3/23/56

Manager: Joan R. Rosenblatt

Full Task Description: January-March 1956 issue, p. 13

Status: CONTINUED. George Weiss has completed a survey of mathematical models in reliability theory which will be presented at an advanced seminar on reliability to be held at the U.S. Army Research Center, Madison, Wisconsin. Dr. Weiss is also working on a study of systems with spare parts in which it is assumed that the spare parts can fail in storage as well as in use.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS Task 1104-12-11141/55-57

Origin: NBS

Authorized 9/1/54

Manager: W. H. Pell

Full Task Description: July-September 1954 issue, p. 27

Status: CONTINUED. B. Bernstein is continuing his study of viscoelastic media with the aid of a theory proposed by A. E. Green and Ronald S. Rivlin (Archive for Rational Mechanics and Analysis, 1, 1-21, 1957). The linear theory of viscoelasticity has proven fairly successful for the description of the mechanical behavior of polymers subjected to small strains. Recently, several theories have been suggested which yield qualitatively correct descriptions of the stress-strain relations in visco-elastic materials when the deformations are finite. In general, however, these are too complicated for subjection to quantitative experimental testing. A somewhat simplified theory, subsumed from that of Rivlin and Green, has permitted Dr. Bernstein to solve problems of simple extension, simple shear, and torsion of a cylinder. These solutions are in the process of being tested in the NBS Rheology Section (6.05) by subjecting a material sample to the foregoing deformations in a stress relaxation experiment. Such experiments, if they confirm the theory, will give actual values for the material parameters, some of which are functions rather than constants.

L. E. Payne and J. H. Bramble are continuing their extensive study of the solutions of the Navier equations. The elastic constants which appear in these equations are, in practice, never known precisely, since they are obtained from physical measurements. The question arises as to the error which may arise in the computed values of the displacements and stresses due to error in the determination of these constants, and Drs. Payne and Bramble are studying methods for obtaining bounds for this error. A side result is that their methods can be used to show convergence of the solution of the compressible (i.e., normal) problem to that of the incompressible problem ($\sigma = \frac{1}{2}$) as $\sigma \rightarrow \frac{1}{2}$.

A. Ghaffari is continuing his investigation of nonautonomous nonlinear ordinary differential equations by the use of the "stroboscopic" method of Minorsky. Applying the stroboscopic criterion to Duffing's equation with viscous damping (i) $\ddot{x} + a\dot{x} + x + bx^3 = F \cos \omega t$, A. Ghaffari has obtained the associated stroboscopic system whose critical point, stable or unstable, corresponds to a periodic solution, stable or unstable, of Duffing's equation. Introducing the polar variables r and ψ defined by $r^2 = x^2 + \dot{x}^2 = x^2 + y^2$, $\psi = \arctan (y/x)$, the system equivalent to (i) in phase plane transforms into the system:

$$\begin{cases} dr/dt = -\mu (\alpha r \sin^2 \psi + \beta r^3 \sin \psi \cos^3 \psi - \gamma \sin \psi \cos \omega t) \\ d\psi/dt = -1 - \mu (\alpha \sin \psi \cos \psi + \beta r^2 \cos^4 \psi - \frac{\gamma}{2} \cos \psi \cos \omega t) \end{cases}$$

Status of Projects

If one tries to satisfy the above system by series solutions of the form

$$r(t) = \sum_{n=0}^{\infty} \mu^n r_n(t), \quad \psi(t) = \sum_{n=0}^{\infty} \mu^n \psi_n(t)$$

where μ is a small parameter, it is found that the zeroth and first order terms in these series solutions are, respectively: $r_0(t) = r_0$, $\psi_0(t) = \phi_0 - t$, and

$$\begin{cases} dr_1/dt = -\alpha r_0 \sin^2 \psi_0 - \beta r_0^3 \sin \psi_0 \cos^3 \psi_0 + \gamma \sin \psi_0 \cos \omega t \\ d\psi_1/dt = -\alpha \sin \psi_0 \cos \psi_0 - \beta r_0^2 \cos^4 \psi_0 + \frac{\gamma}{r_0} \cos \psi_0 \cos \omega t \end{cases}$$

where r_0 and ϕ_0 are arbitrary constants. Applying now the stroboscopic transformation, which consists in considering the difference equations obtained by introducing the differences $r_1(2\pi) - r_1(0)$ and $\psi_1(2\pi) - \psi_1(0)$ of the functions $r_1(t)$ and $\psi_1(t)$ during one interval 2π , this gives

$$r_1(2\pi) = -\pi\alpha r_0 + g(\omega) P(\omega, \phi_0), \quad \psi_1(2\pi) = -\frac{3}{4}\pi\beta r_0^2 + \frac{\gamma}{r_0} Q(\omega, \phi_0)$$

with the assumption that $r_1(0) = 0$ and $\psi_1(0) = 0$. The corresponding stroboscopic system is then

$$\begin{cases} dr/d\tau = -\pi\alpha r + g(\omega) P(\omega, \phi) \\ d\phi/d\tau = -\frac{3}{4}\pi\beta r^2 + \frac{g(\omega) Q(\omega, \phi)}{r} \end{cases}$$

where $P(\omega, \phi)$, $Q(\omega, \phi)$ and $g(\omega)$ are known functions, and α , β , and γ are small positive constants. τ is called the stroboscopic time.

Publications:

- (1) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (2) Pointwise bounds in the first biharmonic boundary value problem. J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (3) On Rayleigh's nonlinear vibration equation. A. Ghaffari. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.

PLASMA RESEARCH Task 1104-12-11140/59-422

Origin: NBS

Authorized 6/30/59

Manager: C. M. Tchen

Full Task Description: April-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen is continuing his study of the generalized Fokker-Planck equation for non-Markovian stochastic processes which include a memory. The manuscript to a paper on this subject is being prepared. The subject of most recent study has been the motion of a sphere in a viscous plasma with a magnetic field. The interaction between the velocity and the magnetic field has been investigated.

Status of Projects

Dr. Tchen is currently giving a series of weekly seminars on the dynamics of plasmas at NBS. The topics being covered are: basic processes of collisions and correlations, kinetic equations, field equations, equations of magnetohydrodynamics for the flow of the gas as a whole, equations of magnetohydrodynamics for the separate constituents of the gas mixture, electric conductivity in the presence of a magnetic field, collisionless magnetohydrodynamics, translation and rotation of a sphere in a viscous liquid in the presence of a magnetic field, interaction of the solar plasma with the geomagnetic field, collective correlations of plasmas, dispersion relation, and Landau damping with collective correlations.

Publications:

- (1) Kinetic equation for plasmas with collective and collisional correlations. C. M. Tchen. Accepted for publication in the Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, Germany, August 28 - September 1, 1961.

RESEARCH ON SATELLITE ORBITS Task 1104-12-11440/59-420

Origin: NBS
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full Task Description: October-December 1958 issue, p. 15
Authorized 12/19/58

Status: TERMINATED.

Publications:

- (1) Intermediary equatorial orbits of an artificial satellite. J. P. Vinti. Journal of Research NBS, 65B (Mathematics and Mathematical Physics), 5-13, January-March 1962.

THEORY OF SATELLITE ORBITS Task 1104-12-11441/62-1166

Origin: NBS
Sponsor: National Aeronautics and Space Administration
Manager: J. P. Vinti
Authorized 1/9/62

Objective: To develop the theory of perturbations produced in the intermediate orbit of a satellite of an oblate planet, where the orbit has been calculated by the theory developed by J. P. Vinti.

Background: Through the use of oblate spheroidal coordinates J. P. Vinti has developed an accurate intermediate orbit for an artificial satellite of an oblate spheroidal planet. This orbit is associated with the Vinti gravitational potential which is closer to the empirically accepted value for the

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earth than those which have been earlier proposed. Vinti finds the Hamilton-Jacobi equation to be separable, and is thus able to reduce the problem to quadratures.

To further improve the accuracy of the orbit, it is necessary to account for various perturbations; it is therefore intended to investigate the following phenomena:

(i) The theory of perturbations produced by the residual fourth harmonic and by the third and fifth zonal harmonics of the planet's gravitational potential.

(ii) The physical interpretation of the theory for the residual fourth harmonic; and explanation of the critical inclination if it is found to occur with the present approach.

(iii) Theory of perturbations produced by the tesseral harmonics which result from ellipticity of the equator and local anomalies, and especially their effects on orbits with a period of approximately one day.

(iv) Derivation of optimum orbits for determining the coefficients in the harmonic development of the gravitational field of the earth, especially when heavy spherical satellites are used. A large ratio of mass to area will reduce the effects of forces such as aerodynamic and electrodynamic drag and those which are not proportional to the mass of the satellite, such as occur due to the bombardment of micrometeorites. This will involve the theoretical calculation of the expected accuracies of the various coefficients, when derived from observations of satellite orbits, as functions of the mass-area ratio of such orbital parameters as apsidal distances and orbital inclinations.

Status: NEW. In the case of drag-free motion of an artificial satellite, neglecting lunar-solar forces and electromagnetic forces, the orbit is given by $\ddot{\mathbf{r}} = -\text{grad } V$. Here V is the gravitational potential of the earth, including the even zonal harmonics, the odd zonal harmonics represent equatorial asymmetry, and the tesseral harmonics represent axial asymmetry. The Vinti potential, which leads to separability with oblate spheroidal coordinates, accounts exactly for the zeroth and second zonal harmonics, and approximately for the fourth. The problem of calculating the corresponding orbit has primarily been solved, even though it is still necessary to account for the residual even harmonics the odd zonal harmonics, and the tesseral harmonics. Preliminary investigations have been instituted concerning the perturbations produced by the residual even harmonics and the odd zonal harmonics. Dr. Vinti is devising appropriate methods for a comprehensive survey of relevant literature concerning perturbation theory.

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DYNAMICS OF PLASMAS
Task 1104-12-11417/62-1157

Origin: NBS Authorized 10/3/61
Sponsor: National Aeronautics and Space Administration
Manager: C. M. Tchen
Full Task Description: October-December 1961 issue, p. 12

Status: CONTINUED. The principal aspects of current investigations under this project are: (i) kinetic theory of a plasma with a self-consistent field, including oscillations; (ii) turbulence in plasmas; and (iii) expansion of a plasma in electric and magnetic fields.

For (i) the singlet distribution function is governed by the BBGKY equation, and the pair distribution function is governed by an equation of the Vlasov type, with self-consistent fields representing the collective force. A dispersion relation for oscillations of small amplitude was found by C. M. Tchen, and the Landau damping with collective correlation was investigated. This theory includes collective correlations and memory effects which are absent in the Landau theory. A manuscript is being prepared.

Drs. C. M. Tchen and E. Minardi investigated the extension of the above collective theory to oscillations of finite amplitude. An iteration method similar to the one used by Montgomery for the Landau damping without collective correlations was used.

A scheme of investigations for items (ii) and (iii) is being prepared.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full Task Description: July-September 1955 issue, p. 20

Status: INACTIVE.

Publication:

- (1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

5. OPERATIONS RESEARCH

OPERATIONS RESEARCH

Task 1105-12-11115/61-546

Origin: NBS

Authorized 12/30/60

Manager: Alan J. Goldman

Full Task Description: October-December 1960 issue, p.3

Status: CONTINUED. Some work begun by J. Edmonds under this task will be continued, and reported under 1105-12-11450. The following investigations in various fields of operations research were carried out by members of the staff:

(i) Bernice K. Bender continued work on simplification rules for Boolean functions, and began preparation of a paper concerning her recent investigation of essential-cell content.

(ii) Mrs. Bender completed study of a simple queueing model arising in optimal buffer-size selection.

(iii) A matrix game is called "completely mixed" if none of its optimal strategies has zero components. A. J. Goldman obtained several theorems which (a) permit identification of a wider class of completely mixed games than was previously possible, and (b) determine how far row and column permutations extend the range of applicability of such identification criteria.

Publications:

- (1) Covers and packings in a family of sets. J. Edmonds. Submitted to a technical journal.
- (2) An algorithm for least common multiples. A. J. Goldman. Submitted to a technical journal.
- (3) Some completely mixed matrix games. A. J. Goldman. In manuscript.
- (4) A property of linear frequency modulation. A. J. Goldman. To appear in the Proceedings of the Institute of Radio Engineers.
- (5) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

AIR DEFENSE

Task 1105-12-11415/61-544

Origin: U.S. Army Air Defense Command

Authorized 9/30/61

Manager: Alan J. Goldman

Full Task Description: July-September 1961 issue, p. 13

Status: CONTINUED. L.S. Joel and A. J. Goldman continued cooperation with members of Denver Research Institute in the construction of the simulation model. Detailed pre-coding analysis was begun.

Status of Projects

RADAR STUDY

Task 1105-12-11527/60-481

Origin: U.S. Army Signal Air Defense Agency

Authorized 9/30/61

Manager: Lambert S. Joel

Full Task Description: July-September 1961 issue, p.14

Status: CONTINUED. One volume of the report was completed and submitted to the sponsor. C. T. Zhan, Jr. continued work on the other volume of this report.

GRAPH THEORY

Task 1105-12-11450/62-1161

Origin: Office of Naval Research

Authorized 1/1/62

Manager: Jack Edmonds

Objective: To investigate linear graphs and related combinatorial structures, especially their extremal properties, symmetry, and topology.

Background: Large-scale problems involving the optimal arrangement of discrete structures arise frequently in operations analysis and in branches of technology such as electric network synthesis. Usually such problems cannot be solved in practice even on a high speed computer by simple enumeration of cases, but require algorithms based on an understanding of the mathematical properties of the structures.

Status: NEW. (i) A totally unimodular matrix is one for which every minor determinant is 0, 1, or -1. Such matrices are of interest in connection with integer linear programming. A special family of them has been described. (ii) With an embedding of a graph G in a closed surface, such that the complement of G in the surface is a family of simply connected regions, there is associated a unique dual graph, obtained by selecting one point in each region to be a vertex and for each edge e in G joining a pair of these vertices by one edge (arc in the surface) crossing e . Theorem: A 1-1 correspondence between the edges of two graphs is a duality with respect to some surface-embedding if and only if for each vertex v of either graph, the edges which meet v correspond to a subgraph which is connected and which has an even number of edges to each of its vertices. If an edge meets v at both ends, its image is counted twice in the corresponding subgraph.

6. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-11645/56-0166 SCF-LCAO SOLUTION OF SOME HYDRIDES

Origin and Sponsor: NBS, Section 5.9

Manager: P. J. Walsh

Full Task Description: January-March 1956 issue, p. 27

Status: Continued. The integral programs have been checked out and some production runs were made. Modifications are being made to allow all the subroutines for a particular type of integral (direct or exchange for diagonal or off-diagonal blocks) to remain in storage while computing the integrals. These changes should significantly decrease the time for computing the integrals in problems which do not require an excessive amount of storage space.

1102-40-11645/56-0186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Origin and Sponsor: NBS, Section 2.5

Manager: B. S. Prusch

Full Task Description: July-September 1956 issue, p. 33

Status: Continued. Computations for 19 sets of gage blocks were completed.

1102-40-11647/58-0266 DEPOLYMERIZATION PROCESSES

Origin and Sponsor: NBS, Section 7.6

Manager: Maxine L. Rockoff

Full Task Description: July-September 1957 issue, p. 36

Status: Continued. A Fortran code using an available differential equation solver with Adams-Moulton integration, variable time step option, and Runge-Kutta restarting procedure has been written and checked out. Fifteen complete solutions have been obtained. Running time averages about 8.5 seconds per time step for the solution of 248 simultaneous equations.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: H. Oser

Full Task Description: January-March 1958 issue, p. 38

Status: Continued. Manuscript writing has nearly been completed. Some machine runs were made to check certain relations which can be theoretically predicted.

1102-12-11513/59-0348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION

Origin: NBS

Sponsor: Office of Ordnance Research, U. S. Army

Manager: Ida Rhodes (11.0)

Full Task Description: October-December 1958 issue, p. 26

Status: Continued. The machine code previously tested on the 704 computer has been converted for the 7090 and successfully tested on that machine. The code now includes the basic logic for syntactic integration, with

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specific syntactic instructions adequate for a few sample sentences; an input, dictionary look-up and morphological analysis routine which are complete but require some revision; and rudimentary routines for profiling (analysis of clauses and phrases) and output. A more comprehensive profiling routine has been programmed in detail but not yet coded. Work on the pilot dictionary, both morphological and syntactic, is continuing.

Publications: (i) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and Ida I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, September 6-8, 1961.
(ii) The hindsight technique in machine translation of natural languages. Ida I. Rhodes and F. L. Alt. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

1102-40-11645/58-0366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U. S. Information Agency, Department of State

Manager: P. J. Walsh

Full Task Description: April-June 1958 issue, p. 35

1102-40-11645/58-0368 INTENSITY FUNCTIONS OF SCATTERED LIGHT

Origin and Sponsor: U. S. Army Signal Research and Development Laboratories

Manager: H. Oser

Full Task Description: July-September 1958 issue, p. 32

Status: Continued. Tables were printed out on a card operated typewriter and proof reading has begun.

11-2-40-11645/59-0394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING

Origin and Sponsor: NBS, Section 4.6

Manager: A. E. Beam

Full Task Description: October-December 1958 issue, p. 30

Status: Continued. The code for computing the photodetachment cross section for H^- was checked and production runs were made. Production runs were essentially completed on the scattering calculation.

1102-40-11645/59-0414 INFINITE SYSTEMS

Origin and Sponsor: NBS, Section 3.0

Manager: Ruth Zucker

Full Task Description: January-March 1959 issue, p. 28

Status: Completed.

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1102-40-11645/60-0465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS

Origin and Sponsor: NBS, Section 3.2

Managers: P. J. Walsh and J. D. Waggner

Full Task Description: October-December 1959 issue, p. 26

Status: Inactive.

1102-40-11645/60-0466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Full Task Description: October-December 1959 issue, p. 27

Status: Inactive.

1102-40-11645/60-0476 GAS TUBE CHARACTERISTICS, II

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: H. Oser

Full Task Description: October-December 1959 issue, p. 30

Status: Continued. Production has continued under direction of the sponsor.

1102-40-11645/60-0486 MORSE WAV. FUNCTIONS AND FRANCK-CONDON FACTORS

Origin and Sponsor: NBS, Section 3.0

Manager: Ruth Zuckner

Full Task Description: January-March 1960 issue, p. 28

Status: Inactive.

1102-40-11645/60-0513 RADIATIVE ENVELOPES OF MODEL STARS

Origin and Sponsors: National Aeronautics and Space Administration

Managers: P. J. Walsh and S. Haber (11.1)

Full Task Description: July-September 1960 issue, p. 23

Status: Continued. Production runs were made for a variety of initial conditions using the modified code. These changes in formulae were suggested during the last quarter.

1102-40-11645/61-0516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE

Origin and Sponsor: NBS, Section 4.8

Manager: R. J. Herbold

Full Task Description: July-September 1961 issue, p. 24

Status: Continued. A program to compute $S_n(k^2)$ where

$$S_n(k^2) = \sum_{i=0}^{\infty} \frac{\pi n!}{2^{6i}} \frac{(2n+4i)!}{(2n)! (n+2i)! (i!)^2}$$

was written in triple precision mode using A. Beam's Multiple Precision Package, BS MPP.

Status of Projects

Results of $S_n(k^2)$ where
 $n=0,1,2, \dots, 9$ and
 $k^2 = .01, .02, .03, \dots, .99$

were computed and transmitted to the sponsor.

3911-61-39952/61-0528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING

Origin and Sponsor: NBS, Section 14.1

Manager: J. D. Waggoner

Full Task Description: July-September 1960 issue, p. 28

Status: Continued. Programs have been turned over to the sponsor.

1102-40-11645/61-0530 SPECIMEN WAVELENGTH

Origin and Sponsor: NBS, Section 9.4

Manager: L. Joseph

Full Task Description: July-September 1960 issue, p. 28

Status: Continued. A new formula was derived by the sponsor. A code is now being written in order to test the practicality of determining the parameters in the formula by means of least squares fitting of experimental data.

1102-40-11645/61-0531 HEAT TRANSFER IN CRYSTALS

Origin and Sponsor: NBS, Section 3.1

Manager: H. Oser

Full Task Description: July-September 1960 issue, p. 29

Status: Continued. A number of changes were made in the code to compute typical heat transfer parameters in certain lattices. Production was continued.

1102-40-11645/61-0532 VIBRATIONAL ENERGY LEVELS FOR IONIC MOLECULES

Origin and Sponsor: Georgetown University

Manager: P. J. Walsh

Full Task Description: October-December 1960 issue, p. 21

Status: Terminated.

1102-40-11645/61-0538 SPECTRAL REFLECTANCE

Origin and Sponsor: NBS, Section 9.4

Manager: S. Haber (11.1) and P. J. Walsh

Full Task Description: October-December 1960 issue, p. 23

Status: Continued. Numerical experimentation was continued and new testing programs were written.

Status of Projects

1102-40-11645/61-0540 DIFFUSION CALCULATIONS

Origin and Sponsor: Army Chemical Center

Manager: L. Joseph

Full Task Description: January-March 1961 issue, p. 21

Status: Continued. Two programs were modified and reassembled for the 7090.
No production runs have been made with these reassembled codes.

1102-40-11645/61-0542 STUDENT LOAN DATA

Origin and Sponsor: Department of Health, Education, and Welfare

Manager: Ruth Zucker

Full Task Description: October-December 1960 issue, p. 24

Status: Completed.

1102-40-11645/61-0556 TCHEBYCHEFF APPROXIMATION BY RATIONAL FUNCTIONS

Origin and Sponsor: NBS, Section 11.1

Manager: P. J. Walsh

Full Task Description: January-March 1961 issue, p. 22

Status: Continued. The code has been completely checked out and the sponsor is considering cases for production runs.

1102-40-11645/61-0559 THERMOCOUPLE CALIBRATION

Origin and Sponsor: NBS, Section 3.1

Manager: Karen A. Bedeau

Full Task Description: January-March 1961 issue, p. 23

Status: Completed. The code has been written and checked out. Production runs are under the direction of the sponsor.

1102-40-645/61-0560 MUSCLE FLEXING

Origin and Sponsor: National Naval Medical Center

Manager: H. Oser

Full Task Description: April-June 1961 issue, p. 22

Status: Inactive.

1102-40-11645/61-0562 CUBIC LATTICES

Origin and Sponsor: NBS, Section 7.06

Manager: L. Joseph

Full Task Description: April-June 1961 issue, p. 22

Status: Completed.

1102-40-11645/ 61-0571 NMR SPECTRUM

Origin and Sponsor: NBS, Section 15.07

Manager: H. Oser

Full Task Description: April-June 1961 issue, p. 25

Status: Continued. Production runs were continued under the direct supervision of the sponsor.

Status of Projects

1102-40-11645/62-1009 MONTE CARLO NEUTRON STUDIES

Origin and Sponsor: NBS, Section 4.3

Manager: Sally T. Peavy

Full Task Description: April-June 1961 issue, p. 21

Status: Continued. Preliminary conversions from IBM 704 codes to IBM 7090 codes have been completed.

1105-40-11645/62-1017 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS

Origin: NBS

Sponsor: Post Office Department, Office of Research and Engineering

Managers: Bernice K. Bender and A. J. Goldman

Full Task Description: October-December 1958 issue, p. 22

Status: Continued. Bernice K. Bender continued participation in the evaluation of simulation models for automatic mail-sorting. Mrs. Bender and A. J. Goldman derived formulae for calculating the overflow rate in a parcel sorting device in terms of buffer capacity and number of operator lanes. C. T. Zahn began preparation of a set of tables based on these formulae. W. Sillars is applying the calculus of variations to some aspects of the network problem.

1102-40-11647/62-1022 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full Task Description: April-June 1958 issue, p. 33

Status: Continued. Project has been continually in production with results being forwarded to the sponsor.

1102-40-11645/62-1027 NEW SYSTEM

Origin and Sponsor: NBS, Section 11.2

Manager: J. H. Wegstein

Full Task Description: July-September 1961 issue, p. 22

Status: Continued. Two minor improvements were made to the monitor system. The system was made to compute the number of output records via the system for each run, and project numbers were changed to five digits within the system. Work was begun to develop a model which can describe the computer and peripheral equipment and their performance. A complete timing of all card, tape, and computer activity for an entire day was made. This data remains to be analysed.

Status of Projects

1102-40-11645/62-1030 ELECTROCARDIOGRAPHIC ANALYSIS

Origin: NBS, Section 12.5

Sponsor: Veterans Administration

Manager: R. J. Herbold

Full Task Description: April-June 1959, p.29

Status: Continued. Conversions of IBM-704 programs to IBM-7090 programs are now complete; the majority of the old programs were recoded to take care of recent changes. There are now six major subroutines and one main routine. The functions of these six subroutines are: (i) To determine beginnings and endings of P-wave, QRS-complex and T-wave of digitalized electrocardiographic analog records of single heart beats; (ii) To compute time integrals of QRS-complex and T-wave; (iii) To compute eigenvalues, eigenvectors, polar vectors and matrix angles of P-wave, QRS-complex and T-wave; (iv) To find instantaneous vectors of QRS-complex and QRS-T segment; (v) To find maximal and scalar vectors of P-wave, QRS-complex and T-wave. (vi) To analyze QRS-complex, i.e., to determine beginnings and endings of Q, R, and S waves. Production runs by all six major subroutines were made and results were transmitted to the sponsor. Approximately one thousand selected electrocardiographic records were analyzed.

Several new programs were written in order to: (i) compute calibration constants, (ii) print scalar leads, (iii) print spatial magnitude and direction (iv) plot spatial direction for QRS-complex, (v) call for any of the six major subroutine and store results on tapes, and (vi) read in results stored on tapes and do some histogram plottings. Programs to compute statistical results of data stored on tapes are currently being written by B. McNeill of Mt. Alto Hospital. Further production runs await the outcome of sponsor's analysis of results already transmitted to him. Some digitalized electrocardiographic records are being deleted due to various reasons, e.g. some of the original analog records failed to get digitalized properly. Some refinements have been made; the main program will now test and reject certain records.

1102-40-11647/62-1043 MAXIMA AND MINIMA COMPUTATIONS

Origin and Sponsor: NBS, Section 9.0

Manager: Ruth Zucker

Full Task Description: October-December 1961 issue, p. 23

Status: Continued. Production runs were made and results submitted to the sponsor.

1102-40-11647/62-1125 MATRIX COMPUTATIONS

Origin and Sponsor: NBS, Section 9.5

Manager: P. J. Walsh

Full Task Description: October-December 1961 issue, p. 25

Status: Inactive.

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1102-40-11647/62-1130 FALLOUT SHELTER COMPUTATIONS

Origin and Sponsor: Office of Civil Defense

Manager: D. I. Mittleman

Full Task Description: October-December 1961 issue, p. 25

Status: Continued. All programming needed to produce the Protection Factor calculations have been completed and production has begun. The coding to produce the necessary summaries continues.

1102-40-11647/62-1144 THERMAL BOUNDARY LAYERS

Origin and Sponsor: University of Maryland

Manager: H. Oser

Full Task Description: October-December 1961 issue, p. 26

Status: Continued. The code has been checked out for the case of forced flow. Preliminary results were given to the sponsor to determine accuracy requirements and stepsizes for future runs.

1102-40-11647/62-1155 MORTGAGE LOAN SURVEY

Origin and Sponsor: Federal Home Loan Bank Board

Manager: Ruth Zucker

Objective: To edit the data and prepare a series of tables relating to conventional mortgage loans on one family nonfarm homes. Two fields of data with other control fields are to be cross tabulated for the number of borrowers, amount of the mortgage, types of lending institutions, etc. Weighted averages and standard error of averages are to be obtained for the various tabulations. The subpopulations of the survey universe for the United States are: (i) eighteen selected SMSA's (Standard Metropolitan Statistical Areas); each of these 18 SMSA's are also subdivided; (ii) the remaining SMSA's; and (iii) the non-SMSA's.

Background: The need for reliable information on trends in home mortgage interest rates is widely felt throughout the mortgage financing field. Focus of attention will be on the cost of home financing to the borrower. The study will be confined to information on loans for the purchase of single family homes in order to increase the homogeneity of elements that influence the structure of interest rates and related terms. The survey will be conducted by questionnaires to a selected sample of voluntarily participating institutions.

The problem was transmitted by Mr. Torrence(Federal Home Loan Bank Board).

Status: New. An editing code has been written and checked out. The correlation code has been written and is in the process of being checked.

Status of Projects

1102-40-11647/62-1168 DISPERSION PARAMETERS

Origin and Sponsor: NBS, Section 11.3

Manager: Karen Bedeau

Objective: A program is to be written to compute values of ℓ and u which may be used in connection with the theorem below. This involves solving the following equations given values of α , β , and n for ℓ and u respectively:

$$(i) \quad 1-\alpha = P\{2\ell \leq \chi_{2n}^2\} - \frac{\sqrt{\pi}}{2^{(n-1)/2} \Gamma(n/2)} \ell^{(n-1)/2} e^{-\ell/2} P\{\ell \leq \chi_{n+1}^2\}$$

$$(ii) \quad 1-\beta = P\{2\ell \leq \chi_{2n}^2 \leq 2u\} - \frac{\sqrt{\pi}}{2^{(n-1)/2} \Gamma(n/2)} \{ \ell^{(n-1)/2} e^{-\ell/2} + u^{(n-1)/2} e^{-u/2} \} \\ \cdot P\{\ell \leq \chi_{n+1}^2 \leq u\}$$

Background: If a statistical sample of size $n+1$ is collected from a p -variate normal distribution having dispersion matrix (σ_{jj}) and if $(a_{jj})/n$ denotes the usual unbiased estimate of σ_{jj} , we are then concerned with the following statistical theorem:

The probability is at least $1-\alpha$ that the following system of relations holds:

$$u^{-1} a_{jj} \leq \sigma_{jj} \leq \ell^{-1} a_{jj}$$

$$j = 1 \dots p$$

$$| \sigma_{jj'} - \frac{1}{2}(u^{-1} + \ell^{-1}) a_{jj'} | \leq \frac{1}{2}(\ell^{-1} - u^{-1}) (a_{jj} a_{jj'})^{\frac{1}{2}}$$

$$j \neq j'$$

This problem was transmitted by W. A. Thompson, Jr. (11.3)

Status: NEW. A code has been written and is being checked out.

Status of Projects

1102-40-11647/62-1171 HOSPITAL PROGRAM PLANNING

Origin and Sponsor: Veterans Administration, Mount Alto Hospital

Manager: Sally T. Peavy

Objective: To analyze inter-hospital differences of effective treatment of patients.

Background: It was considered advisable to provide additional information that would be relevant for program planning of hospitals.

This problem was transmitted by J. Stumpf (Mount Alto Hospital).

Status: NEW. Preliminary conversion of the BIMD codes to the NBS system has been completed.

1102-40-11647/62-1174 IMPULSE CALCULATIONS

Origin and Sponsor: NBS, Section 30.0

Manager: A. Beam

Objective: To evaluate complex gamma functions and a generalized hypergeometric function ${}_3F_2$ with complex coefficients. These in turn will be used to tabulate the probabilities of excitation and dissociation of a Morse oscillator involved in a head-on collision with an atom.

Background: The expressions for the probability are derived from the Impulse Approximation. This quantum mechanical approximation assumes that during the "time of collision" the intramolecular potential may be treated as a perturbation. Thus the calculations are designed for the case of strong non-adiabatic collisions in the thermal dissociation of diatomic molecules.

This problem was transmitted by F. Mies (30.0).

Status: NEW. Most of the codes for the problem were written and assembled.

1102-40-11647/62-1176 NUMERICAL INTEGRATIONS

Origin and Sponsor: NBS, Section 15.2

Manager: D. Kaplan

Objective: To evaluate the following integrals:

$$\int_0^{\sqrt{3/2\pi}} e^A dy, \int_0^{\sqrt{3/2\pi}} A e^A dy \quad \text{where}$$

$$A = \frac{C}{T} \{ 2 q^{-3} m(y) - q^{-6} l(y) \}$$

$$m(y) = (1 + 4y + y^2)(1-y)^{-5} - 1$$

$$l(y) = (1 + y) (1 + 24y + 76y^2 + 24y^3 + y^4)(1-y)^{-11}$$

Input parameters are symbolized by C, T, and q.

Status of Projects

Background: Computations of Devonshire coefficients and their subsequent use in cumbersome numerical integrations is necessary if one wishes to apply "liquid" theories to multilayer adsorption problems. These computed coefficients, and parameters evaluated from them, are now being used to determine heats of absorption of argon on various uniform surfaces so that data observed in these laboratories may be evaluated.

This problem was transmitted by C. F. Prenzlöw (15.3)

Status: NEW. Simpson's rule was used taking 1500 iterations between the two limits of integration.

1102-40-11647/62-1177 ANALYSIS OF VARIANTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: Louis Joseph

Objective: To modify an analysis of variance or covariance program so that it will run on the Bell system.

Background: Share Distribution Number 1212 contains a FORTRAN programmed code: Analysis of Variance or Covariance by John R. Howell. This project consists of modifying the code so that it will run on the Bell system.

This problem was transmitted by A. B. Parks (Diamond Ordnance Fuze Laboratories).

Status: NEW.

1102-40-11647/62-1178 LOGARITHMIC COEFFICIENTS

Origin and Sponsor: NBS, Section 5.3

Manager: R. J. Arms

Objective: To integrate the equation:

$$y'' = z \sinh (zy) / x^4$$

Initial conditions for small x's are determined from an experimentally verified approximation formula. The solution is then used in evaluation of logarithmic activity coefficients.

Background: This equation represents the solution of Poisson-Boltzman equation for the potential of ions, and was previously studied numerically; however, only in a very limited sense.

This problem was transmitted by R. A. Robinson (5.3).

Status: NEW.

1102-40-11647/62-1179 CATALOGUE INFORMATION

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: Karen A. Bedeau

Objective: The ultimate objective is to transfer these cards to a tape which can from time to time be updated and from which the information can be retrieved in various types of print outs, e.g., shelf number, subject, and author. The first code which must be written, however, is one to convert all of the 10,000 items to the same format. At the present there are

Status of Projects

two different formats. Next, all cards of each item which contain titles (some 35,000 with some 4000 distinct titles) are to be changed to more efficient titles once this is accomplished, the entire set of 120,000 cards must be sorted into items.

Background: Approximately 10,000 technical documents (items) have been processed at DOFL over the past two years. The information on each of these documents is recorded on about 12 IBM cards.

This problem was transmitted by B. Altmann (Diamond Ordnance Fuze Laboratories).

Status: NEW.

7. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3911-61-39951/51-1

Origin: NBS

Authorized 7/1/50

Managers: W. J. Youden, J. M. Cameron

Full Task Description: July-September 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(i) Interlaboratory tests. W. J. Youden has developed a test for the detection of laboratories with pronounced systematic errors. His procedure is based on the sum of ranks obtained by laboratories participating in a round robin involving M materials and has the advantages of being a nonparametric test and of being easily understood and applied.

(ii) Weight calibrations. Churchill Eisenhart and J. M. Cameron have been cooperating with Paul E. Pontius and L. B. Macurdy of the Mass and Scales Section in developing efficient and convenient methods for the analysis of the precision of weighings, and for the detection of possible drifts, changes in precision, etc.

Joan R. Rosenblatt is presenting an in-hours course on "Nonparametric statistical techniques." The course is intended to cover a large number of statistical tests with emphasis on the related confidence interval procedures.

Publications:

- (1) Distribution of total service time for a fixed observation interval. W. S. Connor and Norman C. Severo. To appear in the Journal of the American Statistical Association.
- (2) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. To appear in the Journal of the Optical Society of America.
- (3) Accuracy of analytical procedures. W. J. Youden. Journal of the Association of Official Agricultural Chemists, 45, 169-173, February 1962.
- (4) Experimentation and measurement. W. J. Youden. Appeared as a paperback edition in the Vistas of Science book series, National Science Teachers Association, 1962.
- (5) Statistical problems arising in the establishment of physical standards. W. J. Youden. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume III, 321-335, 1961.
- (6) Interpreting preliminary measurements. W. J. Youden. Materials Research & Standards, 1, 987 and 991, December 1961.

Status of Projects

- (7) How to evaluate accuracy of analytical procedures. W. J. Youden. Fourth Annual NPI Conference on Chemical Control Problems, sponsored by the National Plant Food Institute, 36-47, March 1962.
- (8) Systematic errors in physical constants. W. J. Youden. *Technometrics*, 4, 111-123, February 1962.

STATISTICAL SERVICES
Task 1103-40-11625/58-348

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full Task Description: January-March 1958 issue, p. 45

Status: INACTIVE.

Current Applications of Automatic Computer

This is a record of the use of the IBM 7090 for the period of January 1 through March 31, 1962.

<u>TASK NUMBER</u>	<u>TITLE</u>	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER					
		(M	I	N	U	T	E	S)
NBS SERVICES:										
51-0002	11.3 STATISTICAL ENGINEERING	25		28			61			114
54-0030	13.1 SPECTRUM ANALYSIS++	155		48			139			342
54-0031	13.1 SPECTRUM ANALYSIS++	4		1			37			42
54-0032	13.1 SPECTRUM ANALYSIS++	66		57			6			129
54-0033	13.1 SPECTRUM ANALYSIS++	125		803			29			957
54-0034	13.1 SPECTRUM ANALYSIS++	0		0			13			13
55-0055	11.1 RESEARCH IN NUMERICAL ANALYSIS	48		60			0			108
55-0082	3.1 THERMOMETER CALIBRATION+	0		0			51			51
56-0131	2.2 CALCULATIONS IN OPTICS+	4		1			0			5
56-0166	15.0 SCF-LCAO SOLUTION OF HYDRIDES+	10		175			17			202
56-0171	3.8 TRANSPORT THEORY INTEGRALS++	171		142			3			316
57-0219	3.2 THERMAL PROPERTIES+	40		24			14			78
57-0236	3.8 SCF EIGENVALUES+	0		18			28			46
57-0250	2.1 SPECTROPHOTOMETRIC DATA+	50		11			0			61
57-0252	4.4 NEUTRAL MESON EXPERIMENTS++	565		61			111			737
58-0256	10.6 COMPOSITE WALL STUDIES++	269		105			130			504
58-0266	7.6 DEPOLYMERIZATION PROCESSES	36		42			513			591
58-0272	3.7 EQUATION OF STATE++	44		60			1			105
58-0314	3.7 APPROXIMATIONS FOR GAS MIXTURES+	274		581			127			982
59-0394	13.6 SCATTERING BY HYDROGEN ATOMS	18		77			419			514
59-0433	2.1 COLOR OF SIGNALS++	21		7			21			49
59-0440	87.1 NUMERICAL MAPPING++	9		24			0			33
60-0449	2.1 SPECTRAL LINE COLORIMETRY	1		0			0			1
60-0474	2.5 GAGE BLOCK STABILITY+	0		0			2			2
60-0489	3.1 INVERSION OF LINE PROBE DATA+	0		42			8			50
60-0493	3.8 POISSON DISTRIBUTION FUNCTION++	191		222			288			701
61-0516	4.8 RADIATION FIELD FROM A DISK	13		41			0			54
61-0523	4.7 NEUTRON CROSS SECTION STUDIES++	19		11			90			120
61-0530	9.4 SPECIMEN WAVELENGTHS	5		0			0			5
61-0531	3.1 HEAT TRANSFER IN CRYSTALS	119		120			91			330
61-0538	9.4 SPECTRAL REFLECTANCE DATA	7		4			8			19
61-0546	11.5 OPTIMIZATION TECHNIQUES	6		14			0			20
61-0556	11.1 CHEBYSHEV APPROXIMATIONS	7		35			2			44
61-0559	3.1 THERMOCOUPLE CALIBRATION	31		30			9			70
61-0571	15.4 NMR SPECTRUM	0		22			38			60

Current Applications of Automatic Computer

TASK NUMBER		TITLE	ASSEMBLY TIME	CHECKING CODE	PRODUCTION TIME	TOTAL TIME ON COMPUTER
			(M I N U S)			
61-0824	11.2	SYSTEMS CONVERSION+++	16	2	15	33
61-0826	11.2	TRAINING+++	65	73	26	164
61-0856	11.2	IBM 7090 FORTRAN INSTRUCTION+++	13	1	0	14
61-0993	11.2	TAPE CHECK+++	0	0	700	700
61-0995	11.2	ERROR DETECTION+++	0	121	71	192
62-1000	12.5	POST OFFICE OPERATIONS STUDY++	117	89	17	223
62-1003	15.4	MOLECULAR SPECTROSCOPY+	55	155	7	217
62-1005	4.3	RADIATION INTERACTION++	7	0	0	7
62-1008	4.3	GAMMA RAY PENETRATION++	0	123	2	125
62-1009	4.3	MONTE CARLO NEUTRON STUDIES	2	0	0	2
62-1011	13.5	DISPERSION INTEGRALS++	1	0	2	3
62-1015	15.1	THERMAL FUNCTIONS++	41	10	16	67
62-1019	41.0	NBS PERSONNEL REPORT++	3	115	39	157
62-1028	11.2	GENERAL SUBROUTINES+++	21	33	16	70
62-1029	9.7	D-SPACING CALCULATIONS+	0	0	28	28
62-1033	9.7	CRYSTAL STRUCTURE CALIBRATION++	44	24	217	285
62-1034	30.0	PHOTOIONIZATION CROSS SECTION++	26	12	5	43
62-1035	7.7	CREEP DATA ANALYSIS++	46	36	0	82
62-1036	7.7	FILM THICKNESS++	0	0	23	23
62-1038	7.5	STANDARDIZATION ANALYSES++	44	67	4	115
62-1043	9.0	MAXIMA AND MINIMA COMPUTATIONS	2	0	5	7
62-1047	9.0	BLACK BOX COMPUTER SERVICE+	4	1	0	5
62-1051	1.0	BLACK BOX COMPUTER SERVICE+	4	0	1	5
62-1055	8.4	ELLIPSOIDAL COMPUTATION++	0	9	0	9
62-1060	15.0	BLACK BOX COMPUTER SERVICE+	0	0	1	1
62-1064	2.4	GAGE BLOCK STUDIES++	0	3	20	23
62-1066	1.2	STANDARD CELLS++	0	0	8	8
62-1080	9.2	BLACK BOX COMPUTER SERVICE+	3	22	10	35
62-1081	9.1	BLACK BOX COMPUTER SERVICE+	24	10	37	71
62-1085	11.2	MATHEMATICAL SUBROUTINES	1	2	0	3
62-1089	9.6	ELASTIC CONSTANTS++	16	9	10	35
62-1106	3.0	GAS PROPERTIES++	0	8	0	8
62-1118	13.4	HEAT TRANSFER STUDIES++	0	0	22	22
62-1129	3.3	DIAGONALIZATION OF MATRICES+	1	3	4	8
62-1133	5.0	CHEMISTRY CALCULATIONS++	6	0	10	16
62-1139	9.2	BLACK BOX COMPUTER SERVICE+	0	0	2	2
62-1149	5.3	LEAST SQUARES COMPUTATIONS	0	5	5	10
62-1150	7.8	STRESS-STRAIN RELATIONSHIPS	4	0	0	4
62-1160	82.2	PREDICTION STUDIES++	36	534	82	652
62-1162	10.7	CEMENT AGING STUDIES++	15	18	18	51
62-1163	14.1	TRANSISTOR AGING BEHAVIOR++	30	88	0	118
62-1164	3.2	THERMODYNAMIC PROPERTIES++	0	0	2	2

Current Applications of Automatic Computer

TASK NUMBER	TITLE	ASSEMBLY TIME	CODE CHECKING	PRODUCTION TIME	TOTAL TIME ON COMPUTER
NBS SERVICES:		(M I N U T E S)			
62-1165 15.2	NMR SPECTRA ANALYSES+	5	0	25	30
62-1168 11.3	DISPERSION PARAMETERS	15	6	0	21
62-1170 7.7	HIGH PURITY POLYMERS++	1	0	0	1
62-1174 30.0	IMPULSE CALCULATIONS	15	6	0	21
62-1176 15.2	NUMERICAL INTEGRATIONS	4	2	2	8
Totals (NBS Services)		3020	4453	3708	11181
NON-NBS SERVICES:					
57-0216 NSF	HANDBOOK OF MATHEMATICAL TABLES	3	4	5	12
58-0269 NRL	MOLECULAR STRUCTURE IV+	228	220	398	846
58-0348 OOR	MACHINE TRANSLATION OF RUSSIAN	8	12	0	20
59-0407 DOFL	FOURIER COEFFICIENTS+	68	27	53	148
59-0409 FSLIC	BANK BOARD REPORTS++	94	1182	588	1864
59-0425 CU	MOLECULAR ORBITALS	0	3	87	90
59-0434 GC	PETROLOGICAL COMPUTATIONS+	74	71	36	181
59-0441 USRED	SYSTEMS ENGINEERING++	3	18	0	21
60-0457 PHA	PUBLIC HOUSING PROBLEM++	5	34	10	49
60-0458 CAB	AIRLINE TRAFFIC SURVEY+	10	865	360	1235
60-0476 DOFL	GAS TUBE CHARACTERISTIC II	5	0	313	318
60-0492 IMF	MONETARY RESEARCH REPORTS++	100	1	73	174
60-0506 WBANK	WORLD BANK REPORTS++	1	3	3	7
61-0513 NASA	ORBITING STUDIES	0	0	5	5
61-0540 ACC	DIFFUSION CALCULATIONS	10	10	31	51
61-0569 AGO	HUMAN FACTORS RESEARCH++	120	87	62	269
61-0829 BPR	HIGHWAY TRAFFIC STUDIES++	0	289	106	395
61-0830 BPR	HIGHWAY TRAFFIC STUDIES++	0	35	22	57
61-0849 BPR	HIGHWAY TRAFFIC STUDIES++	8	16	0	24
61-0865 BPR	HIGHWAY TRAFFIC STUDIES++	0	12	228	240
61-0878 BPR	HIGHWAY TRAFFIC STUDIES++	0	67	0	67
61-0902 BPR	HIGHWAY TRAFFIC STUDIES++	8	36	0	44
61-0903 BPR	HIGHWAY TRAFFIC STUDIES++	5	199	67	271
61-0945 WB	FORECASTING++	0	72	1991	2063
62-1004 BUSHP	RHOMBIC ANTENNAS+	10	1	3	14
62-1014 NIH	METABOLIC DISEASES++	98	38	257	393
62-1018 NRL	HYDROMAGNETIC PROBLEMS+	147	367	213	727
62-1021 DCH	HIGHWAY STUDIES++	270	2137	681	3088
62-1022 NRL	SPECTRUM OF DIPOLE RADIATION	0	35	9	44
62-1023 NSF	IMAGE PROCESSING++	32	117	60	209
62-1030 VA	ELECTROCARDIOGRAPHIC ANALYSIS	236	356	111	703
62-1031 PHS	HEART DISEASE CONTROL++	3	91	18	112
62-1032 QM	SUPPLY PROGRAMMING PROBLEMS++	53	47	9	109
62-1040 DOFL	BLACK BOX COMPUTER SERVICE+	1	7	11	19

Current Applications of Automatic Computer

TASK NUMBER	TITLE	ASSEMBLY TIME	CHECKING CODE	PRODUCTION TIME	TOTAL TIME ON COMPUTER			
		(M	I	N	U	T	E	S)
62-1041	DOFL BLACK BOX COMPUTER SERVICE+	0	0			4		4
62-1044	FCC RADIO INTENSITIES++	24		27		31		82
62-1046	BPR TRAFFIC PREDICTION++	6		104		25		135
62-1056	DOFL PD ENGINEERING++++	0		0		68		68
62-1067	NIH CANCER STUDIES++	0		0		41		41
62-1073	DOFL COMPLEX LEGENDRE FUNCTIONS++	61		277		92		430
62-1076	NAS EVALUATION OF APPLICATIONS+	52		2		13		67
62-1096	DOFL VULNERABILITY STUDY++++	26		63		140		229
62-1110	ICC ICC SYSTEMS STUDY++	28		210		6		244
62-1113	DOFL TRANSPORT ANALYSES++++	19		133		187		339
62-1115	DOFL BLACK BOX COMPUTER SERVICE+	3		0		5		8
62-1119	BPR HIGHWAY TRAFFIC STUDIES++	0		6		36		42
62-1121	CARIN CARNEGIE INSTITUTE STUDIES	49		0		0		49
62-1130	COENG FALLOUT SHELTER COMPUTATIONS	97		471		87		655
62-1134	HARVU STATISTICAL DECISION THEORY++	0		107		0		107
62-1140	VA VA MEDICAL++	0		2		10		12
62-1141	COENG FALLOUT SHELTER COMPUTATIONS	0		137		516		653
62-1143	NIH MOLECULAR INTERACTION++	4		0		2		6
62-1144	UOFMD THERMAL BOUNDARY LAYERS	48		15		0		63
62-1145	AGRIC FARM ECONOMICS++	0		0		71		71
62-1147	MPSA JET FUEL PROCESSING++	24		427		2		453
62-1151	DARCO STATISTICAL DECISION THEORY++	0		21		0		21
62-1154	AGO MILITARY RESEARCH DATA++	0		0		4		4
62-1155	FSLIC MORTGAGE LOAN SURVEY	7		56		0		63
62-1158	GC MINERALOGY STUDIES++	24		90		55		169
62-1159	GC CRYSTALLOGRAPHY	0		13		0		13
62-1167	NASA ORBIT EQUATIONS	12		19		12		43
62-1169	U ONT ATOMIC COLLISIONS++	26		35		27		88
62-1171	VA HOSPITAL PROGRAM PLANNING	10		2		0		12
62-1172	PEACE PEACE CORPS EVALUATIONS++	284		42		108		434
62-1173	CAB AIRLINES MERGER	25		438		71		534
62-1177	DOFL ANALYSIS OF VARIANTS	13		4		0		17
62-1183	DOFL BLACK BOX COMPUTER SERVICE+	2		0		0		2
Totals (NON-NBS Services)		2444	9160	7423	19027			
TOTAL TIME FOR THE QUARTER (MINUTES)		5464	13613	11131	30208			
TOTAL TIME FOR THE QUARTER (HOURS)		91	227	186	503			

+ PROBLEM PROGRAMMED IN THE COMPUTATION LABORATORY; PRODUCTION RUNS CONTINUED BY SPONSOR.

++ PROBLEM PROGRAMMED BY THE SPONSOR AND RUN UNDER HIS DIRECTION.

+++ FUNCTIONS PERTAIN TO THE INTERNAL OPERATIONS OF THE COMPUTATION LABORATORY.

++++ CLASSIFIED TASK.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

BATEMAN, P. (University of Illinois and the University of Pennsylvania).
Distribution of prime numbers. March 13, 1962.

Dynamics of Plasmas Seminar

- TCHEN, C. M. (i) Magnetohydrodynamic equations in the absence of particle collision. January 29, 1962; (ii) Motion of a sphere in a conducting fluid with a magnetic field. February 6, 1962; (iii) Interaction of a plasma with a magnetic field. February 13, 1962; (iv) Magnetosonic waves. February 20, 1962; (v) Expansion of an interstellar gas cloud in a magnetic field. February 27, 1962; (vi) Collective correlation of plasmas. March 6, 1962; (viii) Plasma oscillations with collective correlations. March 13, 1962; (xi) Dispersion relation for plasma oscillations with collective correlations. March 20, 1962; (xii) Landau damping. March 27, 1962.
- HAIN, K. (Max Planck Institute for Physics and Astrophysics, Munich, Germany; and Institute for Plasma Physics, Garching, Germany). (vii) Recent developments in numerical solutions of plasma and magnetohydrodynamic problems. March 12, 1962.
- MESTEL, L. (University of Cambridge, England; and the Institute for Advanced Study, Princeton, New Jersey). (ix) On equatorial acceleration in a magnetic star. March 14, 1962; (x) How to short-circuit a cosmical battery. March 14, 1962

Mathematical Statistics Seminar

WILKINSON, G. (Division of Mathematical Statistics, CSIRO, Adelaide, Australia). A quadratic form inversion method for obtaining least squares solutions. March 20, 1962.

Lectures and Technical Meetings

Papers and Invited Talks

Presented by Members of the Staff
at Meetings of Outside Organizations

ALT, F. L. (i) Machine translation of natural languages. January 25, 1962, and (ii) Management of a mathematical research and service facility. January 26, 1962. Both talks were presented before the Research Society of America, Wilmington, Delaware. (iii) A mathematical theory of management problems. February 15, 1962, and (iv) Digital pattern recognition by use of moments. February 16, 1962. Both talks were presented at the Virginia Polytechnic Institute, Blacksburg, Virginia. (v) Pattern recognition by use of moments. Presented at the International Business Machines General Production Laboratory, Education Department, Endicott, New York, February 26, 1962, and at the University of Delaware, Newark, Delaware, March 1, 1962. (vi) Management of a mathematical research and service facility, and (vii) Mathematical theory of management problems. Both talks were presented at the University of Delaware, Newark, Delaware, March 2, 1962. (viii) Management of a mathematical research and service facility, and (ix) Digital pattern recognition by use of moments. Both presented at the Nova Scotia Technical College, Halifax, Canada, March 15, 1962. (x) Recent progress in machine translation of natural languages. Presented to the Nova Scotia Institute of Science, Halifax, Canada, March 16, 1962. (xi) Digital pattern recognition, and (xii) Recent progress in machine translation of natural languages. Both presented at the Stevens Institute of Technology, Mathematics Department and Computer Center, Hoboken, New Jersey, March 20, 1962. (xiii) The mathematical theory of management problems. Presented at the IBM Systems Research Institute, New York, March 28, 1962. (xiv) Machine translation of natural languages. Presented at the IBM Watson Laboratory, Columbia University, New York, March 29, 1962.

EISENHART, C. On the measurement of precision and accuracy. Presented at the joint meeting of the Philadelphia Section of the American Society for Quality Control, and the Philadelphia Council of the American Society for Testing and Materials, Philadelphia, Pennsylvania, March 15, 1962.

GOLDMAN, A. J. Implementing an operations research program. A panel discussion concerning Lectures on Operations Research and Management of Federal Programs, Held at the Brookings Institute, Washington, D. C., March 15, 1962.

RHODES, Ida I. Machine translation by means of electronic computers. Presented in cooperation with the Westinghouse Science Talent Search, Washington, D.C., March 1, 1962; and also at the U.S. Naval Ordnance Laboratories, Mathematics Department, White Oak, Maryland, March 5, 1962.

Lectures and Technical Meetings

ROSENBLATT, Joan R. (i) Confidence limits for reliability estimates. Presented before the Professional Group on Reliability and Quality Control of the Institute of Radio Engineers, Washington Chapter, Washington, D.C., February 15, 1962. (ii) Careers in science. Presented to the Joint Board on Science Education at the Conference for Women in Science, Washington, D.C., March 31, 1962.

WEGSTEIN, J. H. Problems in programming. Presented before the New York State Institute of Electronic Data Processing. Albany, New York, February 26, 1962.

WILLKE, T. A. Statistics and probability as tools of science. Presented at the Maryland Academy of Sciences, Easton, Maryland, March 28, 1962.

YOUDEM, W. J. (i) Opportunities in science. Presented at the Woodrow Wilson High School, Washington, D.C., January 8, 1962. (ii) The sample, the procedure, and the laboratory. Presented before the Binghamton Section of the American Chemical Society, Binghamton, New York, February 6, 1962. (iii) Measurement--basis of all science. This lecture was presented before local sections of the American Chemical Society at (1) Corning, New York, February 5, 1962; (2) Schenectady, New York, February 7, 1962, (3) Syracuse, New York, February 8, 1962; and (4) Canton, New York, February 9, 1962. (iv) Experimentation and measurement. Presented at the School Contacts Committee of the Joint Board on Science Education, National Academy of Sciences, Washington, D.C., March 12, 1962. (v) Introduction to statistics. Presented at the Hewlett-Packard Company, Palo Alto, California, March 16, 1962.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Technical Notes, Manuals and Bibliographies

- (1) Selected bibliography of statistical literature, 1930 to 1957: V. Frequency functions, moments, and graduation. Lola S. Deming. Appeared in the Journal of Research NBS, 66B, 15-28, January-March 1962.
- (2) Experimentation and measurement. W. J. Youden. Appeared as a paperback edition in the Vistas of Science book series. Produced by the National Science Teachers Association in cooperation with the National Bureau of Standards. Published by Scholastic Magazines, Inc., New York, January 1962.

1.3 Technical Papers

The following papers appeared in the Journal of Research NBS, 66B (Mathematics and Mathematical Physics), January-March 1962:

- (1) Error bounds for eigenvectors of self-adjoint operators. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Pp. 1-4
- (2) Intermediary equatorial orbits of an artificial satellite. J. P. Vinti. Pp. 5-13.

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- (3) Inequalities for the permanent function. M. Marcus and M. Newman. Appeared in Annals of Mathematics, 75, 47-62, January 1962.
- (4) A string language for symbol manipulation based on ALGOL 60. J. H. Wegstein and W. W. Youden. Communications of the ACM, 5, 54-61, January 1962.
- (5) Accuracy of analytical procedures. W. J. Youden. Journal of the Association of Official Agricultural Chemists, 45, 169-173, February 1962.
- (6) How to evaluate accuracy of analytical procedures. W. J. Youden. Fourth Annual NPMI Conference on Chemical Control Problems, sponsored by the National Plant Food Institute, pp. 36-47, March 1962

Publication Activities

- (7) Congruences for the partition function to composite moduli. M. Newman. Illinois Journal of Mathematics, 6, 59-63, March 1962.
- (8) Interpreting preliminary measurements. W. J. Youden. Materials Research & Standards, 1, 987 and 991, December 1961.
- (9) Statistical problems arising in the establishment of physical standards. W. J. Youden. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume III, 321-335, 1961.
- (10) Systematic errors in physical constants. W. J. Youden. Technometrics, 4, 111-123, February 1962.

2 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.1 Mathematical Tables

- (1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

2.2 Technical Notes, Manuals and Bibliographies

- (1) Handbook of mathematical functions. To appear in the NBS Applied Mathematics Series.
- (2) Experimental statistics. Mary G. Natrella. To be published as ORDP 20-110, 111, 112, 113, 114 by the Army Research Office, Durham, Duke Station, Durham, North Carolina.

2.3 Technical Papers

- (1) Digital pattern recognition by moments. F. L. Alt. To appear in the Journal of the Association for Computing Machinery.
- (2) Fifteen years ACM. F. L. Alt. Submitted to a technical journal.
- (3) Safety levels in military inventory management. F. L. Alt. Submitted to a technical journal.
- (4) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, September 6-8, 1961.
- (5) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the Journal of Mathematical Physics.

Publication Activities

- (6) Lower bounds to eigenvalues using operator decompositions of the form B^*B . N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.
- (7) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (8) Pointwise bounds in the first biharmonic boundary value problem. J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (9) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960
- (10) The segmental variation of Blaschke products. G. T. Cargo. Submitted to a technical journal.
- (11) Distribution of total service time for a fixed observation interval. W. S. Connor and N. C. Severo. To appear in the Journal of the American Statistical Association.
- (12) Graphs for determining the power of Student's t-test. Mary C. Croarkin. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (13) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.
- (14) Covers and packings in a family of sets. J. Edmonds. Submitted to a technical journal.
- (15) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (16) On Rayleigh's nonlinear vibration equation. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.
- (17) An algorithm for least common multiples. A. J. Goldman. Submitted to a technical journal.
- (18) A property of linear frequency modulation. A. J. Goldman. To appear in the Proceedings of the Institute of Radio Engineers.
- (19) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

Publication Activities

- (20) Two matrix eigenvalue inequalities. S. Haber. To appear in the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).
- (21) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), H. H. Ku, and I. J. Good (Admiralty Research Laboratory England). Submitted to a technical journal.
- (22) A calculus for factorial arrangements. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. To appear in the Annals of Mathematical Statistics.
- (23) Factorial designs and the direct product. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.
- (24) On the determination of the eigenvalues and eigenvectors of certain matrices. A. N. Lowan. Submitted to a technical journal.
- (25) Stability criteria for problems involving cylindrical and spherical symmetry. A. N. Lowan. Submitted to a technical journal.
- (26) Stability criteria for the Peaceman-Rachford difference scheme. A. N. Lowan. Submitted to a technical journal.
- (27) Stability criteria for various difference schemes associated with the problem of the vibrating bar. A. N. Lowan and R. J. Arms. Submitted to a technical journal.
- (28) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (29) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (University of Florida). To appear in the Pacific Journal of Mathematics.
- (30) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. Submitted to a technical journal.
- (31) A note on modular groups. M. Newman. Submitted to a technical Journal.
- (32) Bounds for cofactors and arithmetic minima of quadratic forms. M. Newman. Submitted to a technical journal.
- (33) Modular forms whose coefficients possess multiplicative properties (II). M. Newman. To appear in Annals of Mathematics.

Publication Activities

- (34) Multipliers of difference sets. M. Newman. Submitted to a technical journal.
- (35) Some free products of cyclic groups. M. Newman. Submitted to a technical journal.
- (36) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). Submitted to a technical journal.
- (37) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (38) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. To appear in the Journal of the Optical Society.
- (39) The hindsight technique in machine translation of natural languages. Ida Rhodes and F. L. Alt. To appear in the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).
- (40) Tchebycheff approximation by exponentials. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (41) Convergence to normality of powers of a normal random variable. N. C. Severo and L. J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (42) An extension of Jensen's theorem for the derivative of a polynomial and for infrapolynomials. O. Shisha. To appear in the Journal of Research NBS, Section B(Mathematics and Mathematical Physics).
- (43) The relaxation of moments derived from a master equation. K. Shuler (Director's Office), G. Weiss, K. Andersen (Director's Office). To appear in the Journal of Mathematical Physics.
- (44) Kinetic equation for plasmas with collective and collisional correlations, C. M. Tchen. Accepted for publication in the Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, Germany, August 28-September 1, 1961.
- (45) On the pedestrian queueing problem. G. H. Weiss. To appear in the Bulletin of the International Statistical Institute.
- (46) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (47) The role of laboratories in round robins. W. J. Youden. To appear in Materials Research & Standards.

U. S. DEPARTMENT OF COMMERCE

Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS

A. V. Astin, *Director*



THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D. C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Polymers. Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Microscopy and Diffraction. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics.

Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Molecular Kinetics. Mass Spectrometry.

Office of Weights and Measures.

BOULDER, COLO.

Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. Microwave Circuit Standards. Electronic Calibration Center.

