

NATIONAL BUREAU OF STANDARDS REPORT

7390

PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1961



U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

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^{oo}Temporary appointment

*Part time

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^oOnly unclassified material is included in this report.

Status of Projects

December 31, 1961

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS

Task 1101-12-11110/55-55

Origin: NBS

Authorized 8/29/54

Manager: Philip J. Davis

Full task description: July-September 1954 issue, p. 1

Status: CONTINUED. O. Shisha has worked in the following areas:

(i) Tchebycheff approximation by polynomials and by rational functions, particularly from the computational standpoint; (ii) infrapolynomials, with and without prescribed coefficients and their generalizations. Location of zeros and structure problems; (iii) the location of the critical points of polynomials; and (iv) mechanical translation by the "predictive method."

O. Shisha is preparing a survey volume on the theory of the transfinite diameter.

G. T. Cargo and O. Shisha are working on the following subjects:

(i) inequalities between general means, and (ii) bounds for the zeros of polynomials and analytic functions.

G. T. Cargo has worked in the following areas: (i) boundary behavior of Blaschke products; (ii) new proofs of various classical boundary behavior theorems by means of Ostrowski's extension of Jensen's theorem; (iii) a connection between interpolation in H^∞ and non-Euclidean geometry; and (iv) extensions of the classical theorems of Fatou and F. and M. Riesz.

S. Haber worked on solution of sparse systems of linear equations, with particular reference to systematic over-relaxation procedures and to problems of storage in elimination methods.

F. W. J. Olver has completed his investigation of error bounds for Airy function expansions in turning point problems. The first of two papers, dealing with the first approximation, is now being written.

N. Bazley has completed a joint paper with D. W. Fox (Applied Physics Laboratory, JHU) entitled "Lower bounds to eigenvalues using operator decompositions of the form $B*B$." The method developed in this paper has immediate application in the estimation of frequencies of vibration and buckling loads of elastic plates. The paper has been submitted for publication to a technical journal.

Maxine L. Rockoff has worked on the development of high order finite difference formulas to be used near the boundary of a region in which the

Status of Projects

Dirichlet problem for Poisson's equation is to be solved numerically. As an application of recent work by J. H. Bramble and B. E. Hubbard (University of Maryland), these boundary formulas can be used in conjunction with the usual nine-point approximation to the Laplacian in the interior of the region to obtain a high order overall estimate for the truncation error.

Publications:

- (1) Lower bounds for eigenvalues of Schroedinger's equation. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). *Physical Review*, 124, 483-492, October 1961.
- (2) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). *Canadian Mathematical Bulletin*, 4, 23-27, 1961.
- (3) On the relation between the permanent and the determinant. M. Marcus and H. Minc (The University of Florida). *Illinois Journal of Mathematics*, 5, 376-381, September 1961.
- (4) Best approximations and interpolating functions. J. R. Rice. *Transactions of the American Mathematical Society*, 101, 477-498, December 1961.
- (5) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. *Transactions of the American Mathematical Society*, 99, 298-302, 1961.
- (6) The zeros of infrapolynomials with some prescribed coefficients. O. Shisha and J. L. Walsh (Harvard University). *Journal d'Analyse Mathématique*, 9, 111-160, 1961.
- (7) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the *Journal of Mathematical Physics*.
- (8) Error bounds for eigenvectors of self-adjoint operators. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the *Journal of Research NBS, Section B (Mathematics and Mathematical Physics)*.
- (9) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). To appear in the *Pacific Journal of Mathematics*.
- (10) Inequalities for the permanent function. M. Marcus and M. Newman. To appear in the *Annals of Mathematics*.
- (11) Modular forms whose coefficients possess multiplicative properties. (II). M. Newman. To appear in *Annals of Mathematics*.
- (12) Tchebycheff approximation by exponentials. J. R. Rice. To appear in the *Journal of the Society for Industrial and Applied Mathematics*.
- (13) Criteria for the reality of matrix eigenvalues. M.P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.
- (14) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (15) Multipliers of difference sets. M. Newman. Submitted to a technical journal.
- (16) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. Submitted to a technical journal.
- (17) Lower bounds to eigenvalues using operator decomposition of the form $B*B$. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.

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- (18) Two matrix eigenvalue inequalities. S. Haber. In manuscript.
- (19) Linear operations on matrices. M. Marcus. In manuscript.
- (20) An extension of Jensen's theorem for derivatives of polynomials and for infrapolynomials. O. Shisha. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO
NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS

Authorized 8/13/54

Sponsor: Office of Naval Research

Manager: Morris Newman

Full task description: July-September 1954 issue, p.5

Status: CONTINUED. M. Newman is continuing his work on subgroups of the modular group and has determined the structure of the groups generated by the m th powers of all elements, and their commutator subgroups. A study is underway of free products of cyclic groups and matrix representations for such groups. The subgroups of free products generated by the m th powers of all elements are being studied.

K. Goldberg has constructed incidence algebras of dimension $2n$ and order $p^n + p^{n-1}$ (with p a prime), which are symmetric when $p \equiv 1 \pmod{4}$. The generating group is the set of permutations of the $p^n + p^{n-1}$ cosets of $\Gamma_0(p^n)$ in the modular group Γ . The incidence basis has row sums $1, p^n$ and two each of $(p^\nu - p^{\nu-1})/2$ for $\nu = 1, \dots, n-1$.

K. Goldberg continued his investigation of block designs formed by summing Kronecker products of incidence matrices.

Publications:

- (1) A comment on Ryser's "Normal and intergral implies incidence" theorem. K. Goldberg. American Mathematical Monthly, 68, 770-771, October 1961.
- (2) A note on modular groups. M. Newman. Submitted to a technical journal.
- (3) Congruence properties of the partition function to composite moduli. M. Newman. To appear in the Illinois Journal of Mathematics.
- (4) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (5) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). Submitted to a technical journal.
- (6) Hadamard matrices of order cube plus one. K. Goldberg. In manuscript.
- (7) Some free products. M. Newman. In manuscript.
- (8) Two theorems on matrices. M. Newman. In manuscript.

Status of Projects

BOUNDS FOR EIGENVALUES
Task 1101-12-11416/62-1091

Origin: Wright-Patterson AFB
Manager: N. Bazley

Authorized 10/1/61

Objective: To develop methods which permit rigorous numerical determination of eigenvalues and eigenvectors of linear self-adjoint operators. To apply these methods to unsolved problems in the areas of atomic physics and plate vibrations.

Background: Linear eigenvalue problems play a central role in classical mechanics and quantum mechanics, but have known solutions in special cases only. Therefore, it is important to develop and apply numerical procedures for such problems. It is interesting that in most cases one must employ the spectral theory of linear operators in Hilbert space.

Status: NEW. Preliminary investigations concerning methods for estimating the error in norm between eigenvectors and approximating vectors have been initiated. This includes: (i) methods which involve upper and lower bounds to certain of the eigenvalues together with terms in the approximating vector itself, and (ii) eigenvector estimates which only involve upper and lower bounds to the eigenvalues.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter because priority was given to the preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 TABLES OF SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 TABLES OF THE SIEVERT INTEGRAL

HANDBOOK OF MATHEMATICAL FUNCTIONS

Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56

Manager: Irene A. Stegun

Full Task description: October-December 1956 issue, p. 10

Status: CONTINUED. First drafts of chapter 8 (Legendre functions) and chapter 18 (Weierstrass elliptic functions) are being distributed for comments. The remaining chapters of the Handbook which are in preparation for press are being reviewed for consistency of notation, format, etc.

AUTOMATIC CODING

Task 1102-12-111120/55-65

Origin: NBS

Authorized 9/29/54

Manager: J. Wegstein

Full Task description: July-September 1954 issue, p. 11

Status: CONTINUED. Several ALGOL string programs were written and tested on the 704 Computer including a sort and title-permuting algorithm for use in preparing permuted-title indexes. The syntax for the string language was completely defined using the Backus-normal form.

A series of tutorial lectures on ALGOL 60 was prepared and presented, and is available as a standard repertoire.

Status of Projects

Studies on the definition and manipulation of artificial mechanical languages continued in connection with participation in the work of the Business Equipment Manufacturers Association (BEMA) subcommittee on standard computer languages.

Publications:

- (1) A status report on ALGOL 60. J. Wegstein. Datamation, 7, 24-26, September 1961.
- (2) A string language for symbol manipulation based on ALGOL 60. J. H. Wegstein and W. W. Youden. To appear in Communications of the ACM.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS

Task 1103-12-11131/51-2

Origin: NBS

Authorized 7/1/50

Manager: Churchill Eisenhart

Full task description: July-September 1950 issue, p. 58

Status: CONTINUED. W. A. Thompson, Jr. is studying the precision of simultaneous measurement processes. In the course of this investigation two results in multivariate analysis were obtained: (i) The maximum likelihood equations for estimating the elements of a dispersion matrix which is subject to linear constraints, and (ii) a simultaneous confidence region for the components of an arbitrary dispersion matrix.

A paper has been prepared jointly by Joan R. Rosenblatt and N. C. Matalas (U. S. Geological Survey), reporting their results on the use of regression estimates to augment incomplete sets of data.

Churchill Eisenhart and Anna Glinski completed a study of the reduction in the effective level of confidence that would result if, when in fact σ is known, one were to compute both the normal- and t-based confidence intervals for the population mean, and choose the narrower. Such situations were examined explicitly for nominal confidence levels of 0.90, 0.95, and 0.99; and sample sizes 4, 9, 16, and 25. Values of the corresponding effective levels $\gamma^* = \gamma^*(n)$ were obtained directly by quadrature and by an approximate method based on the bivariate normal $V(h,k)$ function. O. C. Curves were evaluated for the corresponding two-sided tests of significance for $n = 9, 25$, separate consideration being given to (i) the case where the "double-dealer" is reluctant to reject H_0 and (ii) the case where he is looking for an excuse to accept some alternative to H_0 . The results of this study were presented in a joint contributed paper entitled, "On the price of double-dealing in data analysis," at a session of the Section on Physical and Engineering Sciences, American Statistical Association, New York, December 30, 1961.

Anna Glinski and John Van Dyke have nearly completed preparation of some new "Tables for the Fisher-Yates significance test in 2×2 contingency tables." Based on the Lieberman-Owen "Tables of the hypergeometric probability distribution," these new tables extended to $n = 25$ the Finney (Biometrika, vol. 35) and Latscha (Biometrika, vol. 40) tables of one-sided 0.05, 0.025, 0.01, and 0.005 significance levels of the Fisher-Yates test and give the associated 'exact' significance probabilities to four decimal places for $3 \leq n \leq 25$. In the course of this work they detected one obvious typographical error in the Finney table, and 13 errors in the Latscha table. The requisite corrigenda have been submitted for publication in the Table Errata section of "Mathematics of Computation."

Publications:

- (1) On the pedestrian queueing problem. George Weiss. To appear in the Bulletin of the International Statistical Institute.

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- (2) Boscovich and the combination of observations. Churchill Eisenhart. Appeared as Chapter 9 in Roger Joseph Boscovich, Studies of his Life and Work. Edited by Lancelot Law Whyte, Allen and Unwin, Limited, London 1961.
- (3) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (4) Selected bibliography of statistical literature, 1930 to 1957: V. Frequency functions, moments, and graduation. Lola S. Deming. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (5) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), H. H. Ku, and I. J. Good (Admiralty Research Laboratory, England). Submitted to a technical journal.
- (6) Convergence to normality of powers of a normal random variable. Norman C. Severo and Lloyd J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (7) Graphs for determining the power of Student's t-test. Mary C. Croarkin. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN

Task 1103-12-11131/53-1

Origin: NBS

Authorized 10/15/52

Manager: J. M. Cameron

Full task description: October-December 1952 issue, p. 60

Status: INACTIVE.

Publications:

- (1) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (2) A calculus for factorial arrangements. B. Kurkjian (DOFL) and M. Zelen. To appear in the Annals of Mathematical Statistics.
- (3) Factorial designs and the direct product. B. Kurkjian (DOFL) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.

Status of Projects

STUDY OF NONPARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: Joan R. Rosenblatt
Full task description: October-December 1955 issue, p. 14

Status: INACTIVE

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: Joan R. Rosenblatt
Full task description: January-March 1956 issue, p. 13

Status: CONTINUED. Joan R. Rosenblatt and Anna M. Glinski have continued their work on procedures for determining confidence limits for system-performance probabilities which are functions of several distribution functions. Preliminary results on some large-sample approximate procedures have been obtained.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 1104-12-11141/55-57

Origin: NBS

Authorized 9/1/54

Manager: W. H. Pell

Full task description: July-September 1954 issue, p. 27

Status: CONTINUED. B. Bernstein has been studying the formulation of a theory of visco-elasticity by means of subsuming a special case of a much more general theory proposed in a paper by A. E. Green and R. S. Rivlin entitled "The mechanics of nonlinear materials with memory," which appeared in Archive for Rational Mechanics and Analysis, 1, 1-21, 1957. With this formulation Dr. Bernstein has solved the problems of simple tension, simple shear, and torsion of a cylinder.

Dr. Bernstein read a paper entitled "Conditions for second order waves in hypo-elasticity" at the annual meeting of the Society of Rheology at Madison, Wisconsin on November 1, 1961.

A. Ghaffari is investigating the application of the so-called stroboscopic method to the study of some nonautonomous differential equations governing certain problems of nonlinear oscillations. By means of this method the original nonautonomous differential equation is transformed into a system of autonomous differential equations, called the stroboscopic system, which possesses the following main property: the existence of a critical point, stable or unstable, of the stroboscopic system is the criterion for the existence of a periodic solution, stable or unstable, of the original non-autonomous differential equation. Preliminary investigations were conducted concerning the application of this method to the study of the Duffing equation.

L. E. Payne and J. H. Bramble have completed a paper entitled "Error bounds in the pointwise approximation of solutions of elastic plate problems."

J. H. Bramble and B. E. Hubbard (University of Maryland) are preparing a publication concerning the results of a study of bounds for the truncation error in the finite difference analogue of the Dirichlet problem for the Poisson equation.

J. P. Vinti is studying certain elliptic integrals of the third kind in order to develop rapidly converging series for their numerical evaluation.

Publications:

- (1) A priori bounds in the first boundary value problem of elasticity. J. H. Bramble and L. E. Payne. Journal of Research NBS, 65B (Mathematics and Mathematical Physics), 269-276, October-December 1961.
- (2) Some higher order integral identities with application to bounding techniques. J. H. Bramble and B. E. Hubbard. Journal of Research NBS, 65B (Mathematics and Mathematical Physics), 261-268, October-December 1961.

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- (3) Analyticity and probability properties of one-dimensional Brownian motion. A. Ghaffari. Journal of Research NBS, 65B (Mathematics and Mathematical Physics), 251-260, October-December 1961.
- (4) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (5) Pointwise bounds in the first biharmonic boundary value problem. J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (6) On Rayleigh's nonlinear vibration equation. A. Ghaffari. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.

PLASMA RESEARCH

Task 1104-12-11140/59-422

Origin: NBS

Authorized 6/30/59

Manager: C. M. Tchen

Full task description: April-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen has examined the similarity between the method of derivation of the kinetic equation for plasmas on the basis of the BBKGY equation (see C. M. Tchen, Kinetic equation for plasmas with unsteady correlations, Physical Review, 114, 394, 1950) and the method of derivation of the Fokker-Planck equation for Markovian processes on the basis of a master equation, called the Smoluchowski equation (see C. M. Tchen, Enige Wiskundige Betrekkingen Welke een Rol Spelen in Diffusieproblemen, Verhandelingen der koninklijke Nederlandse Akademie van Wetenschappen, Afdeling Natuurkunde, 53, 400, 1944). C. Tchen has proposed a sequence of master equations which include the effects of correlations and, therefore, the memory. On the basis of such a master system, a dynamic equation for non-Markovian stochastic processes was attempted, in the form of a generalized Fokker-Planck equation which could include the memory. An essential problem in the reduction of the sequence of master equations was the degeneration of correlations; however, the technique used for the kinetic theory of plasmas was employed. Besides its applications in random processes with memory, the stochastic method has important applications in the study of the kinetics and transport properties of plasmas.

Publications:

- (1) Kinetic equation for plasmas with collective and collisional correlations. C. M. Tchen. Accepted for publication in the Proceedings of the Fifth International Conference on Ionization Phenomena in Gasses, Munich, Germany, August 28 - September 1, 1961.

Status of Projects

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS

Authorized 12/19/58

Sponsor: Office of Scientific Research, ARDC, USAF

Manager: J. P. Vinti

Full task description: October-December 1958 issue, p. 15

Status: TERMINATED. J. P. Vinti has applied his methods in the theory of orbits when corrections were made for various perturbations. These include forces arising from small terms in the expression for the potential, such as the residual fourth harmonic and odd zonal harmonics.

Publications:

- (1) Formulas for an accurate intermediary orbit of an artificial satellite. J. P. Vinti. *Astronomical Journal*, 66, 514-516, November 1961.
- (2) Intermediary equatorial orbits of an artificial satellite. J. P. Vinti. To appear in the *Journal of Research NBS, Section B (Mathematics and Mathematical Physics)*.

DYNAMICS OF PLASMAS
Task 1104-12-11417/62-1157

Origin: NBS

Authorized 10/3/61

Sponsor: National Aeronautics and Space Administration

Manager: C. M. Tchen

Objective: To develop the theory of the motion of an electrically conducting fluid and to apply the results of this analysis to the study of flows in nozzles and arcs, as well as to the derivation of transport coefficients in plasmas.

Background: In the flow of a conducting fluid, there exist Coulomb forces between the charged particles, in addition to those forces which induced by the moving particles, i.e., the so-called "self-consistent field", representative of the collective phenomena of plasmas. The effects of the collective force on dynamic properties is an important feature of the project. Since oscillations and turbulence exist in such fluids in many of their applications, an investigation of the structure of these phenomena should be of importance in the study of their effects on the flow behavior and transport properties of plasmas. The problems have applications to plasma propulsion and astrophysics.

Status: NEW

Status of Projects

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS

Authorized 9/30/55

Sponsor: Office of Naval Research

Manager: F. Oberhettinger

Full task description: July-September 1955 issue, p. 20

Status: CONTINUED. F. W. J. Olver has completed his review of the entire manuscript.

Publication:

- (1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

5. OPERATIONS RESEARCH

OPERATIONS RESEARCH
Task 1105-12-11115/61-546

Origin: NBS Authorized 12/30/60
Manager: Alan J. Goldman
Full task description: October-December 1960 issue, p.3

Status: CONTINUED. The following investigations in various fields of Operations Research were carried out by members of the staff:

(i) Bernice K. Bender developed the theoretical treatment of essential-cell content of random Boolean functions to a point which adequately explains the results of her previous computer simulation experiments.

(ii) Bernice K. Bender and A. J. Goldman completed their investigation of the occurrence, in a sequence of independent trials with varying success probabilities, of the first run of a consecutive successes immediately preceded by a quota of b or more successes out of c consecutive trials.

Publications:

- (1) On the range of a fleet of aircraft. A. J. Goldman. Journal of Research NBS, 65B (Mathematics and Mathematical Physics), 237-238, October-December 1961.
- (2) A property of linear frequency modulation. A. J. Goldman. Submitted to a technical Journal.
- (3) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. Submitted to a technical journal.
- (4) Minimum covering for a family of sets. J. Edmonds. In manuscript.

AIR DEFENSE
Task 1105-12-11415/61-544

Origin: U. S. Army Defense Command Authorized 9/30/61
Manager: Alan J. Goldman
Full task description: July-September 1961 issue, p. 13

Status: CONTINUED. L. S. Joel and A. J. Goldman continued cooperation with members of Denver Research Institute in the construction of the simulation model. Related mathematical problems were investigated by Bernice K. Bender and A. J. Goldman.

Status of Projects

RADAR STUDY

Task 1105-12-11527/60-481

Origin: U. S. Army Signal Air Defense Agency

Authorized 9/30/61

Manager: Lambert S. Joel

Full task description: July-September 1961 issue, p. 14

Status: CONTINUED. Preparation of the final report to the sponsor continued. One volume of this report describes a detailed study by C. T. Zahn, Jr. of use of "black box maximization" computer methods to determine the placement of n circular discs of radius $r < 1$ which maximizes coverage of a fixed disc of unit radius.

6. MATHEMATICAL AND COMPUTATIONAL SERVICES

1102-40-11645/56-0166 SCF-LCAO SOLUTION OF SOME HYDRIDES

Origin and Sponsor: NBS, Section 5.9

Manager: P. J. Walsh

Full task description: January-March 1956 issue, p. 27

Status: Continued. Most of the integral programs have been checked out. only a few integrals remain to be examined before production runs are made.

1102-40-11645/56-0186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Origin and Sponsor: NBS, Section 2.5

Manager: B. S. Prusch

Full task description: July-September 1956 issue, p. 33

Status: Continued. Computations for 37 sets of gage blocks were completed.

1102-40-11645/57-0236 SELF CONSISTENT FIELD--EIGENVALUES

Origin and Sponsor: NBS, Section 3.6

Manager: P. Walsh

Full task description: April-June 1957 issue, p. 30

Status: Terminated. Production runs are being continued under the direction of the sponsor.

1102-40-11647/58-266 DEPOLYMERIZATION PROCESSES

Origin and Sponsor: NBS, Section 7.6

Manager: Maxine L. Rockoff

Full task description: July-September 1957 issue, p. 36

Status: Reactivated. L. A. Wall (7.6) has requested that an IBM 7090 Fortran code be prepared that will solve a system of equations which describes a chain depolymerization process where initiation for breaking of a particular stable chain Q_n is assumed to be at the end of the chain. The system of equations which describes this phenomenon is:

$$\frac{dQ_n}{d\tau} = - [2+(n-3)\sigma]Q_n + 2(K_0-\sigma/4)Q_{n+2} + 2 \sum_{i=1}^{(N-n-3)/2} K_i Q_{n+2i+2}$$

where $L \leq n \leq N-5$, L and n are even, N odd

$$\frac{dQ_{N-3}}{d\tau} = - \{2+(N-6)\sigma\}Q_{N-3} + 2\{K_0-\sigma/4\}Q_{N-1}$$

Status of Projects

$$\frac{dQ_{N-1}}{d\tau} = - \{ 2 + (N-4)\sigma \} Q_{N-1}$$

$$\text{with } K_i = \sigma \left\{ 1 - \left(1 - \frac{3}{4}\epsilon \right) (1-\epsilon)^i \right\} + \sigma + \epsilon (1-\epsilon)^i$$

where σ and ϵ are functions of the Q_n 's.

This project was formerly entitled "DEPOLYMERIZATION, II" and classified as 3911-61-39952/58-266.

1102-40-11645/58-0339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: H. Oser

Full task description: January-March 1958 issue, p. 38

Status: Continued. Some test runs were made on the computer to study the influence of several parameters upon the solutions. R. S. Marvin (6.05) reported on these computations during a seminar conducted at the University of Kyoto, Japan.

1102-12-11513/59-0348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION

Origin: NBS

Sponsor: Office of Ordnance Research, U. S. Army

Manager: Ida Rhodes (11.0)

Full task description: October-December 1958 issue, p. 26

Status: Continued. A machine code for demonstration of the contemplated translation method on the IBM 704 computer was completed and successfully run on one sentence. The code is now being adopted for the 7090 computer. Compilation of morphological information is continuing for several thousand word stems to be included in a pilot dictionary. Compilation of syntactic information for the same stems ("glossary predictions") has been started.

Publications: (i) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied

Status of Projects

Language Analysis, Teddington, England, September 6-8, 1961.

(ii) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. Mechanical Translation, 6, 33-50, November 1961.

(iii) The hindsight technique in machine translation of natural languages. Ida I. Rhodes and F. L. Alt. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

1102-40-11645/58-0366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U. S. Information Agency, Department of State

Manager: P. J. Walsh

Full task description: April-June 1958 issue, p. 35

Status: Inactive.

1102-40-11645/58-0368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT
SCATTERED BY SPHERICAL PARTICLES

Origin and Sponsor: U. S. Army Signal Research and Development Laboratories.

Manager: H. Oser

Full task description: July-September 1958 issue, p. 32

Status: Continued. Work has continued on preparing the tables of light-scattering functions.

1102-40-11645/59-0394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING

Origin and Sponsor: NBS, Section 4.6

Manager: A. E. Beam

Full task description: October-December 1958 issue, p. 30

Status: Continued. Production runs were continued on the scattering calculation. A new code for computing the photodetachment cross section for H^- was written by P. J. Walsh, and check runs are being made.

1102-40-11645/59-0414 INFINITE SYSTEMS

Origin and Sponsor: NBS, Section 3.0

Manager: Ruth Zucker

Full task description: January-March 1959 issue, p. 28

Status: Inactive.

1102-40-11645/60-0465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS

Origin and Sponsor: NBS, Section 3.2

Managers: P. J. Walsh, J. D. Waggoner

Full task description: October-December 1959 issue, p. 26

Status: Inactive.

Status of Projects

1102-40-11645/60-0466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Full task description: October-December 1959 issue, p. 27

Status: Inactive.

1102-40-11645/60-0476 GAS TUBE CHARACTERISTICS, II

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: H. Oser

Full task description: October-December 1959 issue, p. 30

Status: Continued. Production runs were made by the sponsor.

1102-12-11122/60-0479 PROCESSING OF DIAGRAMS

Origin and Sponsor: NBS, Section 11.0

Managers: F. L. Alt (11.0), Sally T. Peavy, R. J. Herbold

Full task description: October-December 1959 issue, p. 30

Status: Completed. The results obtained have been submitted for publication in a technical journal.

1102-40-11645/60-0486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS

Origin and Sponsor: NBS, Section 3.0

Manager: Ruth Zucker

Full task description: January-March 1960 issue, p. 28

Status: Continued. Additional results of continued production runs were forwarded to the sponsor.

1102-40-11645/60-0506 COMMODITY PRICE INDICES

Origin and Sponsor: U. S. World Bank, Statistics Division

Manager: Jeanne M. Beiman

Full task description: October-December 1960 issue, p. 19

Status: Terminated.

1102-40-11645/60-0513 RADIATIVE ENVELOPES OF MODEL STARS

Origin and Sponsors: National Aeronautics and Space Administration

Managers: P. J. Walsh and S. Haber (11.1)

Full task description: July-September 1960 issue, P. 23

Status: Continued. The sponsor suggested a few modifications and some runs were made using the new formulae. The solutions were submitted to the sponsor for further examination.

Status of Projects

1102-40-11645/61-0516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE

Origin and Sponsor: NBS, Section 4.8

Manager: R. J. Herbold

Full task description: July-September 1961 issue, p. 24

Status: Reactivated. The IBM 704 program for computing $h^n q_n(\rho, h)$ was converted to an IBM 7090 program. Values for $h^n q_n(\rho, h)$ were then computed when $h = .01, .02, \text{ and } .05$.

3911-61-39952/61-0528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING

Origin and Sponsor: NBS, Section 14.1

Manager: J. D. Waggoner

Full task description: July-September 1960 issue, p. 28

Status: Continued. The IBM 704 programs were converted to the IBM 7090 programs and they are now being checked.

1102-40-11645/61-0530 SPECIMEN WAVELENGTH

Origin and Sponsor: NBS, Section 9.4

Manager: L. Joseph

Full task description: July-September 1960 issue, p. 28

Status: Inactive.

1102-40-11645/61-0531 HEAT TRANSFER IN CRYSTALS

Origin and Sponsor: NBS, Section 3.1

Manager: H. Oser

Full task description: July-September 1960 issue, p. 29

Status: Continued. The machine program was expanded to include various initial conditions of the mass-system. Control runs were made which proved to be very satisfactory.

1102-40-11645/61-0532 VIBRATIONAL ENERGY LEVELS FOR IONIC MOLECULES

Origin and Sponsor: Georgetown University

Manager: P. J. Walsh

Full task description: October-December 1960 issue, p. 21

Status: Continued. Production runs were made to compute the vibrational energy levels for the hydrogen molecule.

1102-40-11645/61-0538 SPECTRAL REFLECTANCE

Origin and Sponsor: NBS, Section 9.4

Manager: S. Haber (11.1)

Full task description: October-December 1960 issue, p. 23

Status: Continued. Programs embodying a new numerical approach to the problem have been written and checked ; experimental calculations are in progress.

Status of Projects

1102-40-11645/0540 DIFFUSION CALCULATIONS

Origin and Sponsor: Army Chemical Center

Manager: L. Joseph

Full task description: January-March 1961 issue, p. 21

Status: Continued. The sponsors wrote and ran their own codes during this quarter.

1102-40-11645/61-0542 STUDENT LOAN DATA

Origin and Sponsor: Department of Health, Education, and Welfare

Manager: Ruth Zucker

Full task description: October-December 1960 issue, p. 24

Status: Continued. An additional 20,000 cards were collected for the student loan survey. Approximately 50,000 cards are now included in the new 140 cross-tabulation tables with their corresponding negative and nonresponse tables. The tables were computed and submitted to the sponsor.

1102-40-11645/61-0556 TCHEBYCHEFF APPROXIMATION BY RATIONAL FUNCTIONS

Origin and Sponsor: NBS, Section 11.1

Manager: P. J. Walsh

Full task description: January-March 1961 issue, p. 22

Status: Continued. Several test cases have been made and are being evaluated by the sponsor.

1102-40-11645/61-0559 THERMOCOUPLE CALIBRATION

Origin and Sponsor: NBS, Section 3.1

Manager: Karen A. Bedeau

Full task description: January-March 1961 issue, p. 23

Status: Continued. The original program for phases II and III has been converted to run on the 7090 computer, and it is in production under the direction of the sponsor. A new code has also been written for phases II and III. This code uses a least squares polynomial fit to determine a quadratic or cubic equation that best approximates input data for temperature and incremental changes in EMF. The equation for these incremental EMF's is then added to the standard curve equation to give EMF as a function of temperature for the given thermocouple. This code is being checked out.

1102-40-11645/61-0560 MUSCLE FLEXING

Origin and Sponsor: National Naval Medical Center

Manager: H. Oser

Full task description: April-June 1961 issue, p. 22

Status: Continued. A different model of a muscle, consisting of two elastic elements of different properties was suggested by the sponsor. The code is in the process of being checked out.

Status of Projects

1102-40-11645/61-0562 CUBIC LATTICES

Origin and Sponsor: NBS, Section 7.06

Manager: L. Joseph

Full task description: April-June 1961 issue, p. 22

Status: Continued. A code was written for a random walk in a tetrahedral lattice. A production run was made on the 7090. Results were given to the sponsor.

1102-40-11645/61-0571 NMR SPECTRUM

Origin and Sponsor: NBS, Section 15.07

Manager: H. Oser

Full task description: April-June 1961 issue, p. 25

Status: Continued. A number of 6-spin spectra were computed with the program from Mellon Institute. For checking purposes, P. J. Walsh has written a code which generates the submatrices occurring when symmetrized spin functions are used. Eigenvalues, eigenvectors, intensities and intensity ratios are compared with the results of the programs not using symmetrized spin functions. The code is expected to facilitate finding the input parameters for the other programs.

1102-40-11645/62-1009 MONTE CARLO NEUTRON STUDIES

Origin and Sponsor: NBS, Section 4.3

Manager: Sally T. Peavy

Full task description: April-June 1961 issue p. 21

Status: Inactive.

1102-40-11645/62-1016 POST OFFICE OPERATION RESEARCH

Origin and Sponsor: NBS, Section 12.5

Manager: A. E. Beam

Full task description: July-September 1961 issue, p. 21

Status: Completed. Production runs were made and turned over to the sponsor.

1105-40-11645/62-1017 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS

Origin: NBS

Sponsor: Post Office Department, Office of Research and Engineering

Managers: Bernice K. Bender , A. J. Goldman

Full task description: October-December 1958 issue, p.22

Status: Continued. Bernice K. Bender continued participation in the evaluation of simulation models for automatic mail-sorting. Work was resumed on the network model.

Status of Projects

1102-40-11647/62-1022 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full task description: April-June 1958 issue, p. 33

Status: Reactivated. Production runs were subjected to minor program changes. This task was formerly classified as 1102-40-11645/58-361.

1102-40-11645/62-1025 OPERATING SYSTEMS

Origin and Sponsor: NBS, Section 11.2

Manager: G. Ziegler

Full task description: July-September 1961 issue, p. 22

Status: Terminated.

1102-40-11645/62-1027 NEW SYSTEM

Origin and Sponsor: July-September 1961 issue, p. 22

Manager: J. H. Wegstein

Full task description: July-September 1961 issue, p. 22

Status: Continued. Modifications of the Bell Monitor System were completed and this system was placed in operation when the IBM 7090 and 1401 computers were installed in November. Minor debugging and updating was continued during the rest of this quarter.

1102-40-11645/62-1030 ELECTROCARDIOGRAPHIC ANALYSIS

Origin: NBS, Section 12.5

Sponsor: Veterans Administration

Manager: R. J. Herbold

Full task description: April-June 1959 issue, p. 29

Status: Continued. Conversion of the IBM 704 programs to IBM 7090 programs has been temporarily suspended.

1102-40-11647/62-1043 MAXIMA AND MINIMA COMPUTATIONS

Origin and Sponsor: NBS, Section 9.0

Manager: Ruth Zucker

Objective: To compute $F(x)$ and $G(x)$, and to determine the number of maxima for $F(x)$.

$$F(x) = \frac{s^2 x}{1+s^2 x^2} + \frac{3(1-s)}{R^2} \left[\frac{2bsx + (a-1)(1-s^2 x^2)}{x(1+s^2 x^2)^2} \right]$$

Status of Projects

$$G(x) = \frac{s}{1+s^2 x^2} + \frac{3(1-s)}{R^2} \left[\frac{b(1-s^2 x^2) - 2sx(a-1)}{x(1+s^2 x^2)^2} \right]$$

$$a + ib = L \coth L$$

$$\text{where } L = R \sqrt{\frac{x}{1+x^2} [x(1-s) + i(1+sx^2)]}$$

for the parameters $0 < s < 1$, $0 < R < 100$ and $0 < x < \infty$.

$$s = \frac{t_1}{t} = \frac{C_L}{C_H + C_L} \quad , \quad R = \frac{Y_0}{\sqrt{Kt_1}} \quad 0 < s < 1$$

In these equations C_L is the specific heat of lattice, C_H is the specific heat of paramagnetic materials, Y_0 is the radius of the specimen, K is the heat conductivity, t is the relaxation time, w is the measured frequency, and $x = wt$. The expressions $F(x)/s$ and $G(x)/s$ correspond to the imaginary and real part of susceptibilities.

Background: The above equations arise in the study of paramagnetic relaxation at liquid helium temperature. The dispersion found was explained to be the effects of size and thermal conductivity of the paramagnetic specimen. This effect has been formulated by J. Eisenstein, Physical Review, vol. 84, p. 548, 1951.

The problem was transmitted by P. H. Fang (9.0).

Status: New. Tables using a rough grid in R and S to obtain an overall picture of the function were submitted to the sponsor.

1102-40-11645/62-1074 CONTOUR PLOTTING

Origin and Sponsor: Diamond Ordnance Fuze Laboratories

Manager: R. J. Herbold

Full task description: July-September 1961 issue, p. 23

Status: Terminated.

Status of Projects

1102-40-11647/62-1125 MATRIX COMPUTATIONS

Origin and Sponsor: NBS, Section 9.5

Manager: P. J. Walsh

Objective: To compute the matrix $L = RS^T(SS^T)^{-1}$ when the matrices S and R are given. Certain rows of matrix L are to be selected in order to form a matrix T which will contain all ones in the last row. The row sums of $(TT^T)^{-1}$ are to be computed also.

To compute the matrices M^{-1} , MA, KA, and KM^{-1} when the matrices M, A, and K are given. Certain rows of KM^{-1} are to be selected in order to form a matrix T which will contain all ones in the last row. The row sums of $(TT^T)^{-1}$ are to be computed also.

Background: These calculations are aimed at solving problems in such fields as analytical chemistry, in which a multiplicity of analytical results, each depending on several constituents, must be interpreted simultaneously. The specific test problem is a determination of the four-phase compatibility tetrahedron in a region of the system $CaO-SiO_2-Al_2O_3-Fe_2O_3$ from X-ray and chemical analyses of a series of synthetic mixtures.

This problem was submitted by F. Ordway (9.5).

Status: New. The codes were written and checked. The results of some production runs were submitted to the sponsor.

1102-40-11647/62-1130 FALLOUT SHELTER COMPUTATIONS

Origin and Sponsor: Office of Civil Defense

Manager: D. I. Mittleman

Objective: (i) Prepare codes which will produce protection factors and capacities for shelter areas for radioactive fallout.

(ii) Use these codes on data submitted on building characteristics.

(iii) To prepare codes to compute summaries of the calculated protection factors and shelter capacities.

(iv) To produce for distribution the summaries of objective three.

Background: The President of the United States has stated that shelter protection against radioactive fallout will be provided for the United States. The responsibility for this task rests with the Department of Defense, Office of Civil Defense. Phase 1 is an inventory of the present potential shelter areas available to the people of the United States. The technical data needed to accomplish this survey was indicated by the Radiation Physics Division of the National Bureau of Standards; the data itself is being collected under the direction of the Corps of Engineers, U. S. Army, and is being prepared by the Department of Commerce, Bureau of the Census for transmittal to the NBS Applied Mathematics Division for processing.

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Due to the vastness of this problem, task numbers 1141, 1142, and 1156 are also fallout shelter numbers.

This problem was transmitted by P. S. Visher (Department of Defense).

Status: New

1102-40-11647/62-1144 THERMAL BOUNDARY LAYERS

Origin and Sponsor: University of Maryland

Manager: H. Oser

Objective: To solve a number of systems of nonlinear ordinary differential equations which occur in the study of thermal boundary layers under free and forced flow conditions, for a selected set of Prandtl numbers.

Background: Based on the assumption that similar solutions exist, Prandtl's boundary layer equations can be transformed into an infinite system of ordinary differential equations.

(i) For the case of forced flow along a flat plate under the influence of a constant free-stream velocity V_∞ and a given temperature difference $T_w - T_a$ between the plate and the surrounding medium, the first six equations are as follows:

zeroth approximation:

$$f''_0 + f_0 f''_0 = 0 \qquad f_0(0) = f'_0(0) = 0, \quad T_0(0) = 1$$

$$T''_0 + \text{Pr} f_0 T'_0 = 0 \qquad f'_0(\infty) = 2 \qquad T_0(\infty) = 0$$

first approximation:

$$f'''_1 + f''_1 f_0 - 2f'_0 f'_1 + 3f''_0 f_1 + 8T_0 = 0 \qquad f_1(0) = f'_1(0) = f'_1(\infty) = 0$$

$$T''_1 + \text{Pr}(f_0 T'_1 - 2f'_0 T_1 + 3f_1 T'_0) = 0 \qquad T_1(0) = T_1(\infty) = 0 \quad (i = 1, 2)$$

second approximation:

$$f'''_2 + f_0 f''_2 - 4f'_0 f'_2 + 5f''_0 f_2 + 3f_1 f''_1 - 2(f'_1)^2 + 8T_0 = 0$$

$$T''_2 + \text{Pr}(f_0 T'_2 - 4f'_0 T_2 + 3f_1 T'_1 - 2f'_1 T_1 + 5f_2 T'_0) = 0$$

(ii) In the case of a vertical hot plate in an otherwise resting fluid the driving forces are merely given by the local density gradients due to

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temperature differences. Gravitational forces are balanced by buoyancy forces. The first six differential equations in this case are as follows:

zeroth approximation:

$$g_0''' + 3g_0 g_0'' - 2(g_0')^2 + T_0 = 0$$

$$g_0(0) = g_0'(0) = 0, T_0(0) = 1$$

$$T_0'' + 3Pr g_0 T_0' = 0$$

$$g_0'(\infty) = 0, T_0(\infty) = 0$$

first approximation:

$$g_1''' + 3g_0 g_1'' - 2g_0' g_1' + g_0'' g_1 + T_1 = 0$$

$$g_1(0) = g_1'(0) = T_1(0) = 0$$

$$T_1'' + Pr(g_1 T_1' + 2g_0' T_1 + 3g_0 T_1') = 0$$

$$g_1'(\infty) = T_1(\infty) = 0 (i = 1, 2)$$

second approximation:

$$g_2''' + 3g_0 g_2'' - g_0'' g_2 + g_1 g_1'' + T_2 = 0$$

$$T_2'' + Pr(g_1 T_1' + 2g_1' T_1 - g_2 T_1' + 3g_1 T_2' + 4g_0' T_2) = 0$$

Only the first two equations in each set are nonlinear. The higher approximations are solutions of systems of two linear equations. It is planned to solve the nonlinear equations by an iterative method similar to one used by H. Weyl. The subsequent equations will be solved by Runge-Kutta integration.

This problem was transmitted by A. A. Szewczyk (University of Maryland).

Status: New

7. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3911-61-39951/51-1

Origin: NBS

Authorized 7/1/50

Managers: W. J. Youden, J. M. Cameron

Full task description: July-September 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(i) Calibration of gage blocks. A study of precision in the calibration of gage blocks by an intercomparison scheme with two sets of master blocks was made by H. H. Ku for John S. Beers of the Length Section from data involving three sets of blocks. A comparison of the precision by this scheme with the precision of calibration by the interferometer was made and the results have been written up in draft form.

(ii) Calibration of glass beads. A preliminary study has been made for C. M. Hunt of the Inorganic Building Materials Section by H. H. Ku on the size and volume distribution of glass beads to be used for standard samples. Distributions obtained by microscopic count and Coulter counter were compared and the limits of uncertainty of the percentage of beads below a certain diameter were estimated.

(iii) Spectrographic measurements. A statistical evaluation of the precision of measurements made by an X-ray spectrograph was carried out by J. M. Cameron for B. Leonard Bean of the Inorganic Building Materials Section.

(iv) Sampling. Members of the Section collaborated with Robert Good of the Planning Staff on the use of sampling methods in two studies in the field of management and administration.

(v) Dental materials. W. J. Youden collaborated with personnel of the Dental Research Section on studies of the physical properties of dental materials

Publications:

- (1) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. Journal of Research NBS, 65A (Physics and Chemistry), Pp. 475-483, November-December, 1961.
- (2) Experimental design and ASTM committees. W. J. Youden. Materials Research & Standards, 1, 862-867, November 1961.
- (3) What is the best value? W. J. Youden. Journal of the Washington Academy of Sciences, 51, 95-97, October 1961.
- (4) Distribution of total service time for a fixed observation interval. W. S. Connor and Norman C. Severo. To appear in the Journal of the American Statistical Association.
- (5) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. To appear in the Journal of the Optical Society of America.

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- (6) Accuracy of analytical procedures. W. J. Youden. To appear in the Journal of the Association of Official Agricultural Chemists.
- (7) Experimentation and measurement. W. J. Youden. To appear as a paper-back edition in the Vistas of Science book series, National Science Teachers Association.
- (8) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, 1960.
- (9) The interpretation of preliminary measurement. W. J. Youden. To appear in Materials Research & Standards.

STATISTICAL SERVICES

Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies

Authorized 3/31/58

Manager: J. M. Cameron

Full task description: January-March 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(i) Association of Official Agricultural Chemists: W. J. Youden prepared a report for their Committee on Statistics concerning the evaluation of interlaboratory tests.

(ii) Sandia Corporation, Albuquerque: W. J. Youden served as consultant to their Standards Laboratory for the period November 14-16 on problems of precision and accuracy arising in their calibration work.

Current Applications of Automatic Computer

This is a record of the use of the IBM 704 and IBM 7090 for the period
October 1 through December 31.

<u>Task No.</u>		<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
<u>NBS SERVICES:</u>			(M I N U T E S)		
51-0002	11.3	Statistical engineering	15	24	71
54-0030	13.1	Spectrum analysis**	196	80	493
54-0031	13.1	Spectrum analysis**	2	3	11
54-0032	13.1	Spectrum analysis**	0	29	0
54-0033	13.1	Spectrum analysis**	162	209	137
54-0034	13.1	Spectrum analysis**	0	6	10
55-0055	11.1	Research in numerical analysis	0	3	39
55-0056	11.1	Research in mathematical topics	0	0	82
55-0065	11.2	Automatic coding	5	43	5
55-0082	3.1	Thermometer calibration*	0	67	146
56-0131	2.2	Calculations in optics*	12	19	31
56-0166	15.0	SCF-LCAO solution of hydrides*	0	74	60
56-0171	3.8	Transport theory integrals**	79	301	634
57-0219	3.2	Thermal properties*	37	42	2
57-0236	3.8	SCF eigenvalues	0	0	29
57-0250	2.1	Spectrophotometric data*	4	14	50
57-0252	4.4	Neutral meson experiments**	252	67	43
58-0256	10.6	Composite wall studies**	155	111	141
58-0314	3.7	Approximations for gas mixtures*	446	498	547
58-0339	6.5	Viscoelasticity properties	0	7	0
59-0372	11.3	Statistical analysis*	0	0	7
59-0394	13.6	Scattering by hydrogen atoms	11	213	465
59-0403	2.1	Computation of color fadings*	0	31	10
59-0421	12.5	Traffic simulation**	0	57	0
59-0433	2.1	Color of signals**	3	4	59
59-0440	82.1	Mapping**	45	411	1100
60-0474	2.5	Gage block stability*	3	0	14
60-0489	3.1	Inversion of line probe data*	0	0	38
60-0493	3.8	Poisson distribution function**	175	71	281
61-0516	4.8	Radiation field from a disk	3	1	5
61-0523	4.7	Neutron cross section studies**	115	39	102
61-0526	3.0	Crystal field calculations**	10	0	0
61-0528	14.1	Transistor aging study	139	83	116
61-0531	3.1	Heat transfer in crystals	6	17	0
61-0538	9.4	Spectral reflectance data	26	3	2

Current Applications of Automatic Computer

<u>Task No.</u>		<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
<u>NBS SERVICES:</u>			(M I N U T E S)		
61-0546	11.5	Optimization techniques	5	0	145
61-0556	11.2	Chebyshev approximations	21	33	4
61-0559	3.1	Thermocouple calibration	30	29	10
61-0562	7.6	Cubic lattices	11	10	56
61-0571	15.4	NMR spectrum	10	19	16
61-0824	11.2	Systems conversion***	102	116	17
61-0825	11.2	Training***	22	54	0
61-0826	11.2	Training***	22	39	16
61-0993	11.2	Tape check***	0	0	1095
61-0995	11.2	User error ***	0	4	11
62-1000	12.5	Post office operations study**	42	52	16
62-1003	15.4	Molecular spectroscopy*	1	51	11
62-1005	4.3	Radiation interaction**	2	1	133
62-1006	4.3	Radiation interaction**	31	21	0
62-1008	4.3	Gamma ray penetration**	58	432	0
62-1011	13.5	Dispersion integrals**	3	0	26
62-1012	12.5	Quantitative model research**	22	38	0
62-1013	7.0	Statistical methods**	0	2	0
62-1015	5.9	Thermal functions**	0	13	0
62-1019	41.0	NBS personnel report**	14	187	62
62-1028	11.2	General subroutines***	21	16	7
62-1029	9.7	D-Spacing calculations*	0	8	9
62-1033	9.7	Crystal structure calibration**	31	10	72
62-1034	30.0	Photoionization cross section **	32	0	99
62-1035	7.7	Creep data analysis**	30	34	5
62-1036	7.7	Film thickness**	0	2	8
62-1038	7.5	Interlaboratory standardization**	17	8	0
62-1043	9.0	Maxima and minima computations	12	0	5
62-1051	1.0	Black box computer service*	0	2	0
62-1055	8.4	Ellipsoidal computation**	0	9	0
62-1057	10.0	Black box computer service*	8	0	0
62-1063	4.0	Black box computer service*	1	0	1
62-1064	2.4	Gage block studies**	0	0	38
62-1066	1.2	Standard cells**	0	0	46
62-1068	5.2	Spectrochemical analyses*	0	0	6
62-1080	9.2	Black box computer service*	0	1	15
62-1081	9.1	Black box computer service*	2	0	10
62-1084	3.3	Black box computer service*	3	0	3
62-1107	6.5	Oscillating sphere**	0	0	11
62-1112	12.5	Directory data**	11	0	55
62-1122	3.8	Curve fitting for wave functions **	22	6	0
62-1123	10.3	Black box computer service*	0	0	54
62-1125	9.5	Matrix computations	0	1	1
62-1127	9.1	Thoria investigations**	3	2	0
62-1128	15.2	Least squares computations*	0	0	1

Current Applications of Automatic Computer

<u>Task No.</u>		<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
<u>NBS SERVICES:</u>			(M I N U T E S)		
62-1129	3.3	Diagonalization of matrices*	2	1	2
62-1133	5.0	Chemistry calculations**	19	4	7
62-1135	9.5	Angle computations**	4	0	3
62-1137	9.0	Black box computer service*	2	3	0
62-1138	12.0	Tape conversion**	0	17	4
62-1139	9.2	Black box computer service*	0	1	2
62-1148	83.6	Tropospheric analyses**	<u>3</u>	<u>3</u>	<u>0</u>
		Totals (NBS Services)	2520	3756	6812
<u>NON-NBS SERVICES:</u>					
58-0269	NRL	Molecular structure IV*	26	68	266
58-0319	FTMON	Auto tag studies*	0	42	76
58-0348	OOR	Machine translation of Russian	4	36	0
58-0368	SC	Light scattering by particles	0	11	1032
59-0407	DOFL	Fourier coefficients*	33	50	0
59-0409	FSLIC	Bank board reports**	8	48	562
59-0425	CU	Molecular orbitals	0	0	185
59-0434	GC	Petrological computations*	63	133	103
59-0441	USRED	Systems engineering**	0	0	558
60-0457	PHA	Public housing problem**	0	63	216
60-0458	CAB	Airline traffic survey*	18	231	41
60-0476	DOFL	Gas tube characteristic II	0	24	103
60-0486	UONT	Morse wave function**	0	0	41
60-0492	IMF	Monetary research reports**	149	25	91
61-0513	NASA	Orbiting studies	32	3	13
61-0532	GU	Vibrational energy levels	0	18	142
61-0540	ACC	Diffusion calculations	57	19	219
61-0542	HEW	Student loan survey	0	7	683
61-0545	WH	Nuclear reactor design**	0	0	1063
61-0560	NMC	Muscle flexing	6	5	0
61-0569	AGO	Human factors research**	212	94	9
61-0829	BPR	Highway traffic studies**	0	0	243
61-0849	BPR	Highway traffic studies**	102	118	259
61-0865	BPR	Highway traffic studies**	0	0	383
61-0902	BPR	Highway traffic studies**	0	0	748
61-0903	BPR	Highway traffic studies**	0	0	700
61-0945	WB	Forecasting**	0	51	935
62-1002	UOC	Self consistent fields*	0	25	0
62-1014	NIH	Metabolic diseases**	214	229	126
62-1016	PO	Post office operation research	0	0	159
62-1018	NRL	Hydromagnetic problems*	72	128	10
62-1021	DCH	Highway studies**	8	20	314

Current Applications of Automatic Computer

<u>Task No.</u>	<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
<u>NON-NBS SERVICES:</u>		(M I N U T E S)		
62-1022	NRL Spectrum of dipole radiation	17	37	63
62-1023	NSF Image processing**	0	123	95
62-1030	VA Electrocardiographic analysis	386	218	61
62-1031	PHS Heart disease control**	0	8	8
62-1032	QM Supply programming problems**	18	14	175
62-1040	DOFL Black box computer service*	6	14	11
62-1041	DOFL Black box computer service*	0	0	5
62-1044	FCC Radio intensities**	0	0	66
62-1046	BPR Traffic prediction**	15	10	67
62-1056	DOFL PD engineering ^o	29	8	62
62-1067	NIH Cancer studies**	349	424	2300
62-1071	DOFL Rhinitis studies**	7	6	6
62-1073	DOFL Complex legendre functions**	190	155	21
62-1076	NAS Evaluation of applications*	137	2	30
62-1096	DOFL Vulnerability study ^o	27	12	0
62-1099	COENG Nuclear reactor studies**	0	0	50
62-1110	ICC ICC systems study**	17	17	0
62-1111	HEW Radiological health studies**	3	43	0
62-1113	DOFL Transport analyses ^o	7	100	0
62-1115	DOFL Black box computer service*	11	8	2
62-1116	DOFL Black box computer service*	0	0	2
62-1119	BPR Highway traffic studies**	0	0	122
62-1134	HARVU Statistical decision theory**	4	65	0
62-1140	VA VA medical**	0	0	13
62-1141	CENG Fallout shelter computations	0	5	2
62-1143	NIH Molecular interaction**	6	0	3
62-1145	AGRIC Farm economics**	0	0	57
62-1147	MPSA Jet fuel processing**	<u>4</u>	<u>24</u>	<u>4</u>
Totals (Non-NBS Services)		2237	2741	12505
Total time for the quarter (MINUTES)		4757	6497	19317
Total time for the quarter (HOURS)		79	108	322

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

*** Functions pertaining to internal operations of the Computation Laboratory.

^o Classified task.

Note: The IBM 704 computer ceased operations on October 23, 1961; the IBM 7090 computer began operations on November 6, 1961.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

EDMONDS, J. and NEWMAN, M. Combinatorial Analysis. October 18 - 19, 1961.

GOLDMAN, A. J. 'Operations research' research. October 27, 1961.

HOPKINS, H. G. (British Armament and Research Establishment). Axially symmetric plasticity theory. November 7, 1961.

SETH, B. R. (Indian Institute of Technology, Kharagpur, India). Boundary layer thickness. December 19, 1961

Applied Mathematics Division Seminar

CARGO, G. T. Analytic functions in the open unit disk. This was a series of nine lectures concerning the development of the convergence continuation theorems of Stieltjes, Vitali, and Blaschke. The theory of Blaschke products and the rudiments of Nevanlinna's theory of functions of bounded characteristic were developed and discussed. October 10 - December 19, 1961.

Papers and Invited Talks

Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. Pattern recognition by moments. Presented before the American Mathematical Society, 583rd Meeting, Cambridge, Massachusetts, October 28, 1961.

BRAMBLE, J. H. An approximation method in elliptic boundary value problems. Presented as a RIAS lecture at the University of Maryland, College Park, Maryland. December 19, 1961.

Lectures and Technical Meetings

- EDMONDS, J. Graphs and extrema. Presented before the Mathematics Department, Princeton, New Jersey, November 20, 1961.
- EISENHART, C. and GLINSKI, Anna M. On the price of double-dealing in data analysis. Presented before the Section on Physical and Engineering Sciences, American Statistical Association, New York, December 30, 1961.
- GOLDMAN, A. J. On the range of a fleet of aircraft. Presented at the meeting of the Operations Research Society of America, San Francisco, California, November 8 - 10, 1961.
- NEWMAN, M. Subgroups of the modular group. Presented at the Mathematics Colloquium, University of Illinois, Urbana, Illinois, November 9, 1961.
- ROSENBLATT, Joan R. Nonparametric and shortcut statistics. Presented at the Seminar on Statistical Methods for Federal Executives, U. S. Department of Agriculture Graduate School, Washington, D. C., November 16, 1961.
- SHISHA, O. Convexity. Presented before the Mathematics Club, Washington and Lee High School, Arlington, Virginia. December 11, 1961.
- WEGSTEIN, J. H. (i) A tutorial on ALGOL 60, Part I. Presented before the National Security Agency, Fort Meade, Maryland, December 1, 1961. (ii) The use and translation of artificial languages. Presented before the American Association for the Advancement of Science, Denver, Colorado. December 7, 1961.
- YOU DEN, W. J. (i) Modern statistical quality control concepts for materials testing. Presented to the National Lime Association, Washington, D. C. October 5, 1961. (ii) How to evaluate accuracy of analytical procedures. Presented at the meeting of the National Plant Food Institute, Washington, D. C., October 25, 1961, and at the meeting of the Association of Official Agricultural Chemists, October 29, 1961. (iii) Statistical applications in the chemical industries. Presented at the Eastman Kodak Company, Rochester, New York, December 5, 1961. (iv) Dice, data, and deductions. Presented before the Rochester Society for Quality Control, Rochester, New York, December 5, 1961.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Technical Notes, Manuals and Bibliographies

- (1) Advances in orthonormalizing computation. P. J. Davis and P. Rabinowitz. Appeared as a chapter in Advances in Computers. Volume II. Edited by Franz L. Alt. Academic Press, New York and London, October 1961.
- (2) Boscovich and the combination of observations. Churchill Eisenhart. Appeared as Chapter 9 in Roger Joseph Boscovich, Studies of his Life and Work. Edited by Lancelot Law Whyte, Allen and Unwin, Ltd., London 1961.

1.3 Technical Papers

The following papers appeared in the Journal of Research NBS, 65B (Mathematics and Mathematical Physics), October-December 1961:

- (1) On the evaluation of the function $\phi(\lambda) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} e^{u \ln u + \lambda u} du$ for real values of λ . W. Börsch-Supan. Pp. 245-250.
- (2) Some higher order integral identities with application to bounding techniques. J. H. Bramble and B. E. Hubbard (University of Maryland). Pp. 261-268.
- (3) A priori bounds in the first boundary value problem in elasticity. J. H. Bramble and L. E. Payne. Pp. 269-276.
- (4) Analyticity and probability properties of one-dimensional Brownian motion. A. Ghaffari. Pp. 251-260.
- (5) On the range of a fleet of aircraft. A. J. Goldman. Pp. 237-238.

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- (6) Lower bounds for eigenvalues of Schroedinger's equation. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Physical Review, 124, 483-492, October 1961.
- (7) A comment on Ryser's "Normal and integral implies incidence" theorem. K. Goldberg. American Mathematical Monthly, 68, 770-771, October 1961.

Publication Activities

- (8) Radiation field from a circular disk source. J. H. Hubbell, R. L. Bach, and R. J. Herbold. Journal of Research NBS, 65C (Engineering and Instrumentation), 249-264, October-December 1961.
- (9) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). Canadian Mathematical Bulletin, 4, 23-27, 1961.
- (10) On the relation between the permanent and the determinant. M. Marcus and H. Minc (University of Florida). Illinois Journal of Mathematics, 5, 376-381, September 1961.
- (11) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and M. C. Dannemiller. Journal of Research NBS, 65A (Physics and Chemistry), 475-483, November-December 1961.
- (12) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. Mechanical Translation, 6, 33-50, November 1961.
- (13) The National Bureau of Standards method of syntactic integration. Ida Rhodes. Proceedings of the National Symposium on Machine Translation, University of California, Los Angeles, February 2-5, 1960. Edited by H. P. Edmundson. Pp. 39-44. Published by Prentice-Hall, 1961.
- (14) Best approximations and interpolating functions. J. R. Rice. Transactions of the American Mathematical Society, 101, 477-498, December 1961.
- (15) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Transactions of the American Mathematical Society, 99, 298-302, 1961.
- (16) The zeros of infrapolynomials with some prescribed coefficients. O. Shisha and J. L. Walsh (Harvard University). Journal d'Analyse Mathématique, 9, 111-160, 1961.
- (17) Formulae for an accurate intermediary orbit of an artificial satellite. J. P. Vinti. Astronomical Journal, 66, 514-516, November 1961.
- (18) A status report on ALGOL 60. J. H. Wegstein. Datamation, 7, 24-26, September 1961.
- (19) Experimental design and ASTM committees. W. J. Youden. Materials Research & Standards, 1, 862-867, November 1961.
- (20) What is the best value? W. J. Youden. Journal of the Washington Academy of Sciences, 51, 95-97, October 1961.

Publication Activities

2 MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.1 Mathematical Tables

- (1) Tables of Fourier transforms of absolutely continuous distribution functions. Fritz Oberhettinger. To appear in the NBS Applied Mathematics Series.

2.2 Technical Notes, Manuals and Bibliographies

- (1) Handbook of mathematical functions. To appear in the NBS Applied Mathematics Series.
- (2) Experimentation and measurement. W. J. Youden. To appear as a paperback edition in the Vistas of Science book series, National Science Teachers Association.
- (3) Experimental statistics. Mary G. Natrella. To be published as ORDP 20-110, 111, 112, 113, 114 by the Army Research Office, Durham, Duke Station, Durham, North Carolina.
- (4) Selected bibliography of statistical literature, 1930 to 1957: V. Frequency functions, moments, and graduation. Lola S. Deming. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).

2.3 Technical Papers

- (1) Digital pattern recognition by moments. F. L. Alt. Submitted to a technical journal.
- (2) **Safety levels in military inventory management. F.L. Alt. Submitted to a technical journal.**
- (3) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, September 6 - 8, 1961.
- (4) A procedure for estimating eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the Journal of Mathematical Physics.
- (5) Lower bounds to eigenvalues using operator decompositions of the form B^*B . N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.

Publication Activities

- (6) Error bounds for eigenvectors of self-adjoint operators. N. W. Bazley and D. W. Fox. (Applied Physics Laboratory, JHU). To appear in Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (7) Conditions for second order waves in hypo-elasticity. B. Bernstein. To appear in the Transactions of the Society of Rheology.
- (8) Pointwise bounds in the first biharmonic boundary value problem. J. H. Bramble and L. E. Payne. Submitted to a technical journal.
- (9) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960
- (10) Distribution of total service time for a fixed observation interval. W. S. Connor and N. C. Severo. To appear in the Journal of the American Statistical Association.
- (11) Graphs for determining the power of Student's t-test. Mary C. Croarkin. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (12) Criteria for the reality of matrix eigenvalues. M.P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.
- (13) Roger Joseph Boscovich and the combination of observations. Churchill Eisenhart. To appear in Actes du Symposium International Roger Boscovich 1961.
- (14) Precision and accuracy--experiment design aspects. Churchill Eisenhart. To appear in Conference on Applications of Statistical Methods in the Chemistry Industry.
- (15) On Rayleigh's nonlinear vibration equation. To appear in the Proceedings of the International Symposium on Nonlinear Vibrations. Sponsored by the Academy of Sciences of the Ukrainian SSR, Kiev, USSR, September 12-18, 1961.
- (16) A property of linear frequency modulation. A. J. Goldman. Submitted to a technical journal.
- (17) The first run preceded by a quota. A. J. Goldman and Bernice K. Bender. Submitted to a technical journal.
- (18) Tests for contingency tables and Markov chains. S. Kullback (George Washington University), M. Kupperman (George Washington University), H.H. Ku, and I.J. Good (Admiralty Research Laboratory England). Submitted to a technical journal.

Publication Activities

- (19) A calculus for factorial arrangements. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. To appear in the Annals of Mathematical Statistics.
- (20) Factorial designs and the direct product. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. To appear in the Bulletin of the International Statistical Institute.
- (21) On a direct method in the calculus of variations. A. N. Lowan. Submitted to a technical journal.
- (22) On the determination of the eigenvalues and eigenvectors of certain matrices. A. N. Lowan. Submitted to a technical journal.
- (23) Stability criteria for problems involving cylindrical and spherical symmetry. A. N. Lowan. Submitted to a technical journal.
- (24) Stability criteria for the Peaceman-Rachford difference scheme. A. N. Lowan. Submitted to a technical journal.
- (25) Stability criteria for various difference schemes associated with the problem of the vibrating bar. A. N. Lowan and R. J. Arms. Submitted to a technical journal.
- (26) On the maximum number of zeros in the powers of an indecomposable matrix. M. Marcus and F. May. Submitted to a technical journal.
- (27) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (University of Florida). To appear in the Pacific Journal of Mathematics.
- (28) Inequalities for the permanent function. M. Marcus and M. Newman. To appear in Annals of Mathematics.
- (29) The sum of the elements of the powers of a matrix. M. Marcus and M. Newman. Submitted to a technical journal.
- (30) A note on modular groups. M. Newman. Submitted to a technical journal.
- (31) Congruence properties of the partition function to composite moduli. M. Newman. To appear in the Illinois Journal of Mathematics.
- (32) Modular forms whose coefficients possess multiplicative properties (II). M. Newman. To appear in Annals of Mathematics.
- (33) Multipliers of difference sets. M. Newman. Submitted to a technical journal.

Publication Activities

- (34) Note on a subgroup of the modular group. M. Newman and J. R. Smart (New York University). Submitted to a technical journal.
- (35) The structure of some subgroups of the modular group. M. Newman. To appear in the Illinois Journal of Mathematics.
- (36) Variability of spectral tristimulus values. I. Nimeroff (Photometry and Colorimetry Section), Joan R. Rosenblatt, and Mary C. Dannemiller. To appear in the Journal of the Optical Society.
- (37) The hindsight technique in machine translation of natural languages. Ida Rhodes and F. L. Alt. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (38) Tchebycheff approximation by exponentials. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (39) Convergence to normality of powers of a normal random variable. N. C. Severo and L. J. Montzingo, Jr. To appear in the Bulletin of the International Statistical Institute.
- (40) Kinetic equation for plasmas with collective and collisional correlations, C. M. Tchen. Accepted for publication in the Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, Munich, Germany, August 28-September 1, 1961.
- (41) Intermediary equatorial orbits of an artificial satellite. J. P. Vinti. To appear in the Journal of Research NBS, Section B (Mathematics and Mathematical Physics).
- (42) A string language for symbol manipulation based on ALGOL 60. J. H. Wegstein and W. W. Youden. To appear in Communications of the ACM.
- (43) On the pedestrian queueing problem. G. H. Weiss. To appear in the Bulletin of the International Statistical Institute.
- (44) Accuracy of analytical procedures. W. J. Youden. To appear in the Journal of the Association of Official Agricultural Chemists.
- (45) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
- (46) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, 1960.
- (47) The interpretation of preliminary measurements. W. J. Youden. To appear in Materials Research & Standards.

U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*



THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics. Electrolysis and Metal Deposition.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Crystal Growth. Physical Properties. Constitution and Microstructure.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics.

Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Molecular Kinetics. Mass Spectrometry.

Office of Weights and Measures.

BOULDER, COLO.

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Interval Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

