65

NATIONAL BUREAU OF STANDARDS REPORT

7386a

FIGURES FOR

BLACK BOX MAXIMIZATION

of

CIRCULAR COVERAGE

by

C.T. Zahn, Jr.

Applied Mathematics Division

To

U.S. Army Signal Air Defense Engineering Agency

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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Operations Research Section

Applied Mathematics Division

Final Report (Volume Ia) to the

U.S. Army Signal Air Defense Engineering Agency

(MIPR AB1600-2409-3232)

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ABSTRACT

This volume contains the figures referred to in "Black Box Maximization of Circular Coverage", volume I.

Table of Contents

Figure	Description
1	Coverage by Six Discs
2	Loose Packing of Three Discs
3	Neville's Five Disc Covering
4	Parameters in the Two Disc Case
5	Approximating the Coverage
6	Reflection Short-Cut
7	Points to be Scanned
8	Case (2,9/16)
9	" (2,5/8)
10	" (2,11/16)
11	" (2,3/4)
12	" (2,13/16)
13	" (2,7/8)
14	" (2,15/16)
15	" (3,1/2)
16	" (3,9/16)
17	" (3,5/8)
18	" (3,11/16)
19	" (3,3/4)
20	" (3,13/16)
21	" (3,7/8)
22	" (4,7/16)
23	" (4,1/2)

Table of Contents Continued

Figure	Description
24	Case (4,9/16)
25	" (4,5/8)
26	" (4,11/16)
27	" (5,3/8)
28	" (5,7/16)
29	" (5,1/2)
30	" (5,9/16)
31	" (6,3/8)
32	" (6,7/16)
33	" (6,1/2)
34	" (7,3/8)
35	" (7,7/16)
36	" (8,5/16)
37	" (8,3/8)
38	" (8,7/16)
39	" (9,5/16) Ratio = .8244
40	" (9,5/16) Ratio = .8241
41	" (9,3/8)
42	" (10,5/16)
43	" (10,3/8) Ratio = .9924
44	" (10,3/8) Ratio = .9911
45	" (10,3/8) Ratio = .991
46	" (10,3/8) Ratio = .988

Table of Contents Continued

Figure	Description	
47	Case (10,3/8) Ratio = .990	
48	" (6,1/2)	Different Local Maxima
49	" (6,1/2)	
50	" (6,1/2)	
51	" (6,1/2)	
52	" (6,1/2)	
53	" (10,3/8)	Different Local Maxima
54	" (10,3/8)	
55	" (10,3/8)	
56	" (10,3/8)	
57	" (9,5/16)	Different Local Maxima
58	" (9,5/16)	
59	" (8,3/8)	Different Local Maxima
60	" (8/38)	
61	" (6,5/16)	
62	" (6,3/8)	Different Local Maxima
63	" (6,3/8)	
64	" (6,7/16)	Different Local Maxima
65	" (6,7/16)	
66	" (6,9/16)	Different Local Maxima
67	" (6,9/16)	
68	Computer Configuration for Neville Case.	
69	Neighborhood of a Mesh Point.	

Table of Contents Continued

Figure	Description
70	Maximum Packing
71	Wall Interrupted Coverage by Three Discs.
72	1 Disc Coverage of Wall Region.
73	2 Disc Coverage of Wall Region.
74	3 Disc Coverage of Wall Region.
75	4 Disc Coverage of Wall Region.
76-81	Diagrams for Appendix I.

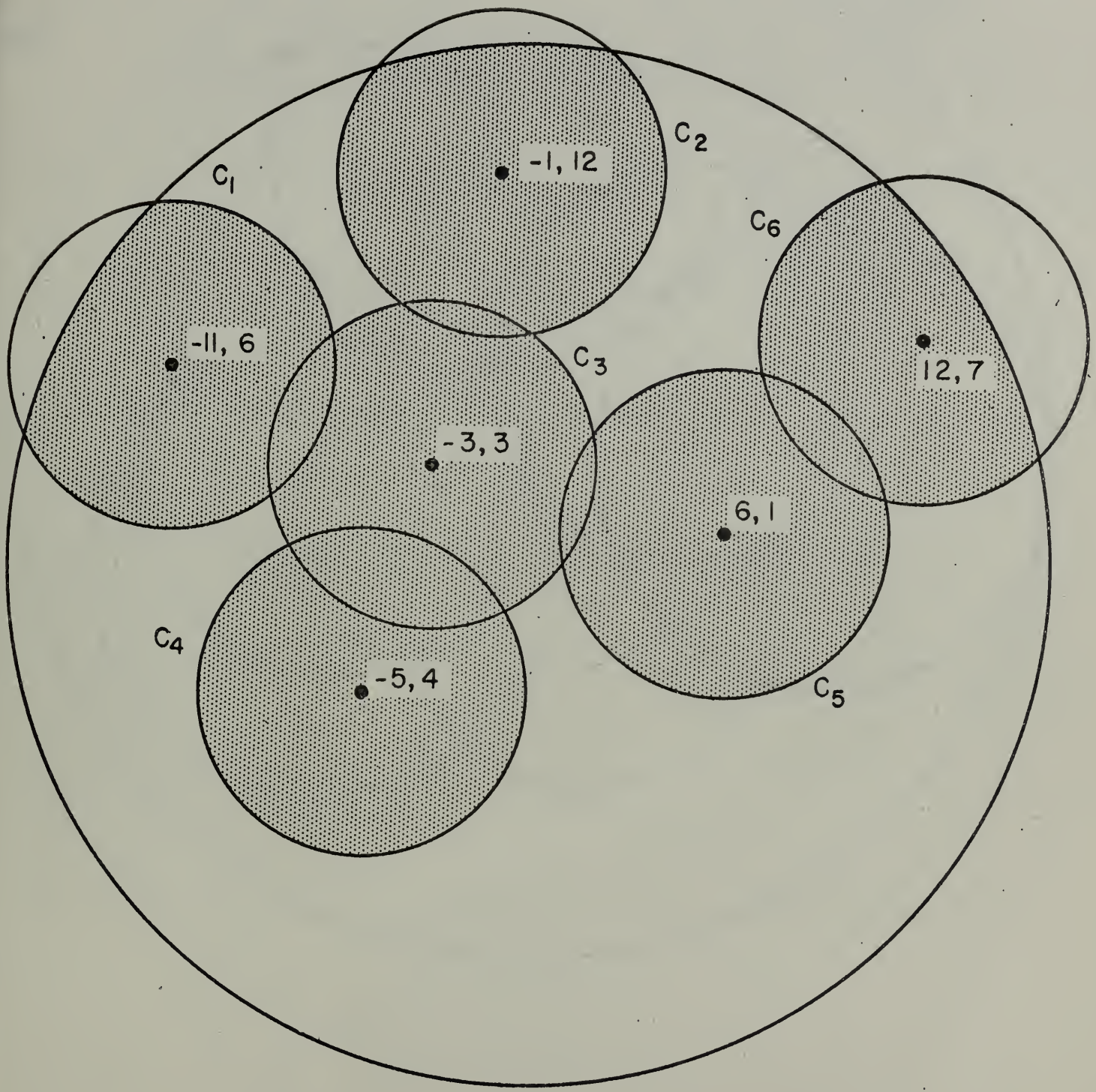

FIG. 1 COVERAGE BY SIX DISCS

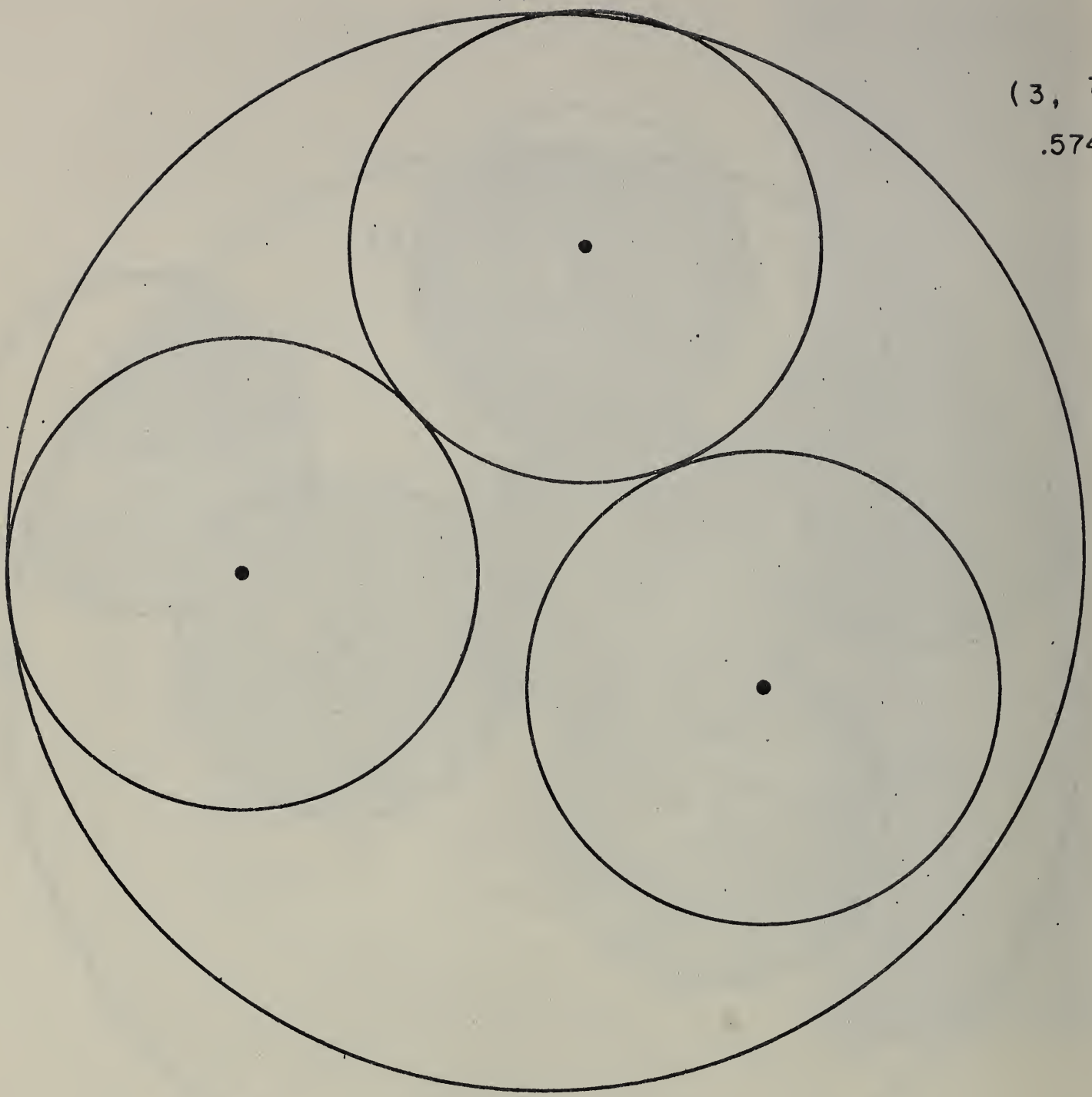

FIG. 2 LOOSE PACKING OF THREE DISCS

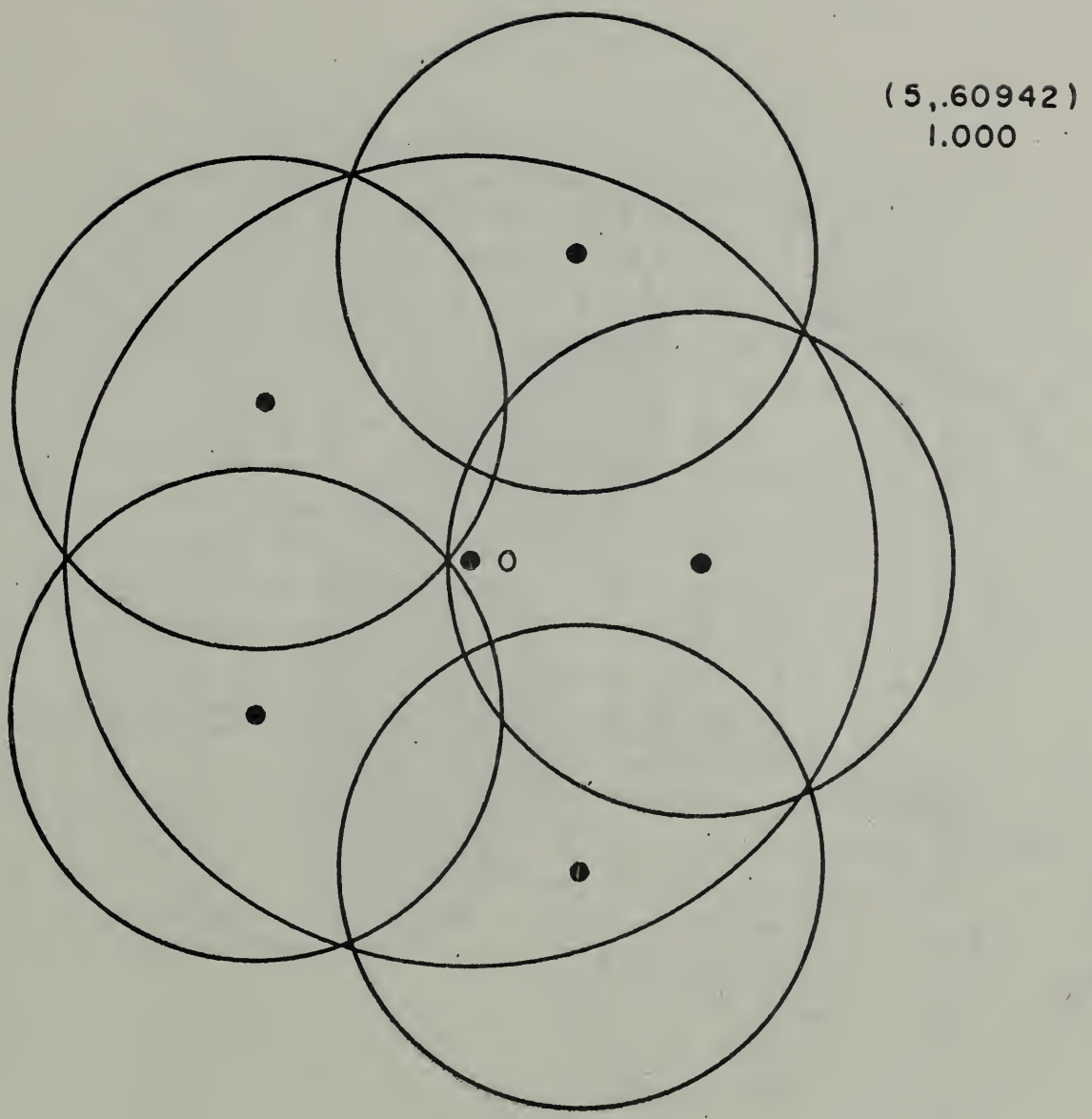

FIG. 3 NEVILLE'S FIVE DISC COVERING

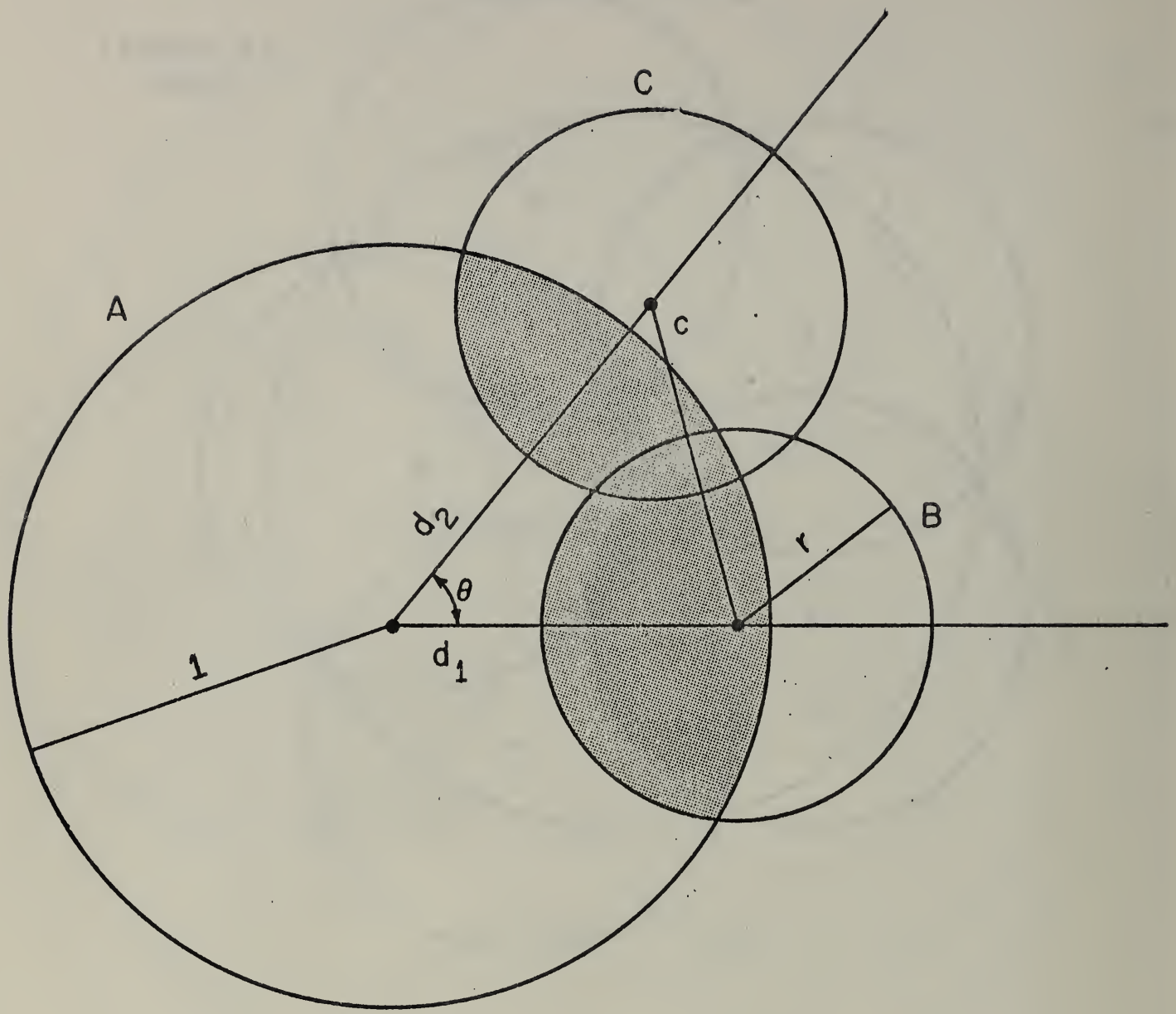

FIG. 4 PARAMETERS IN THE TWO DISC CASE

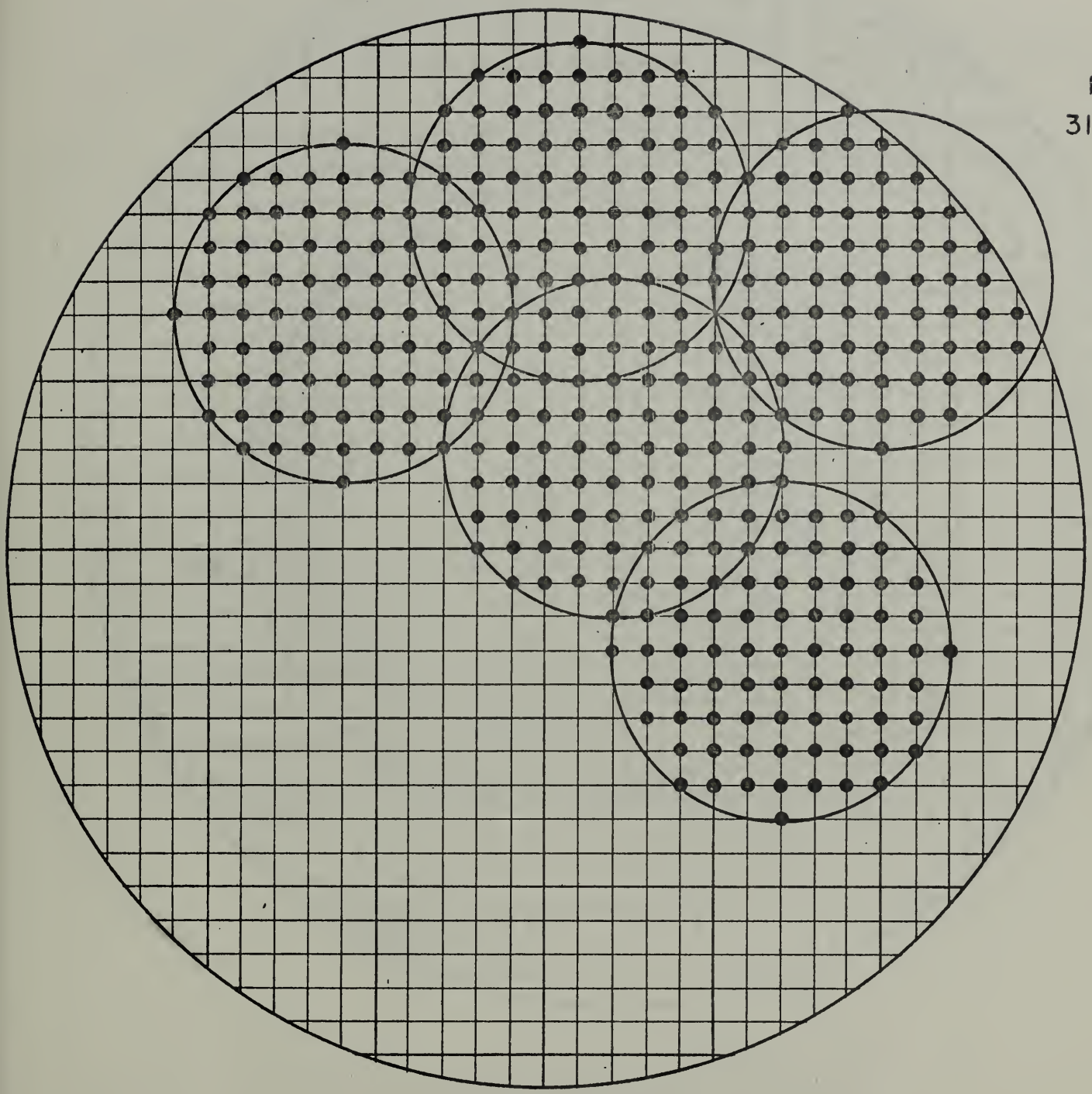

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FIG. 5 APPROXIMATING THE COVERAGE

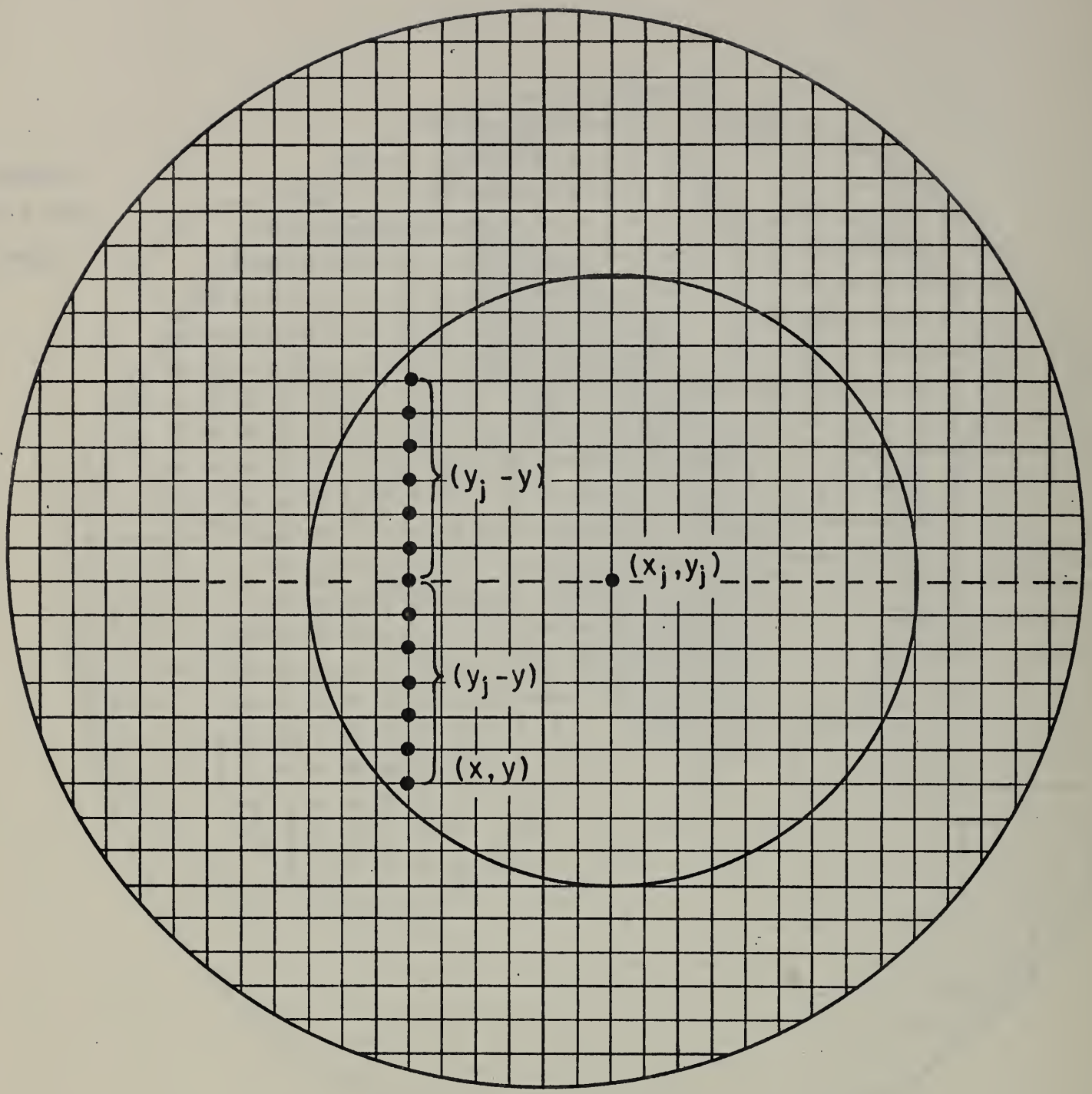


FIG. 6 REFLECTION SHORT - CUT

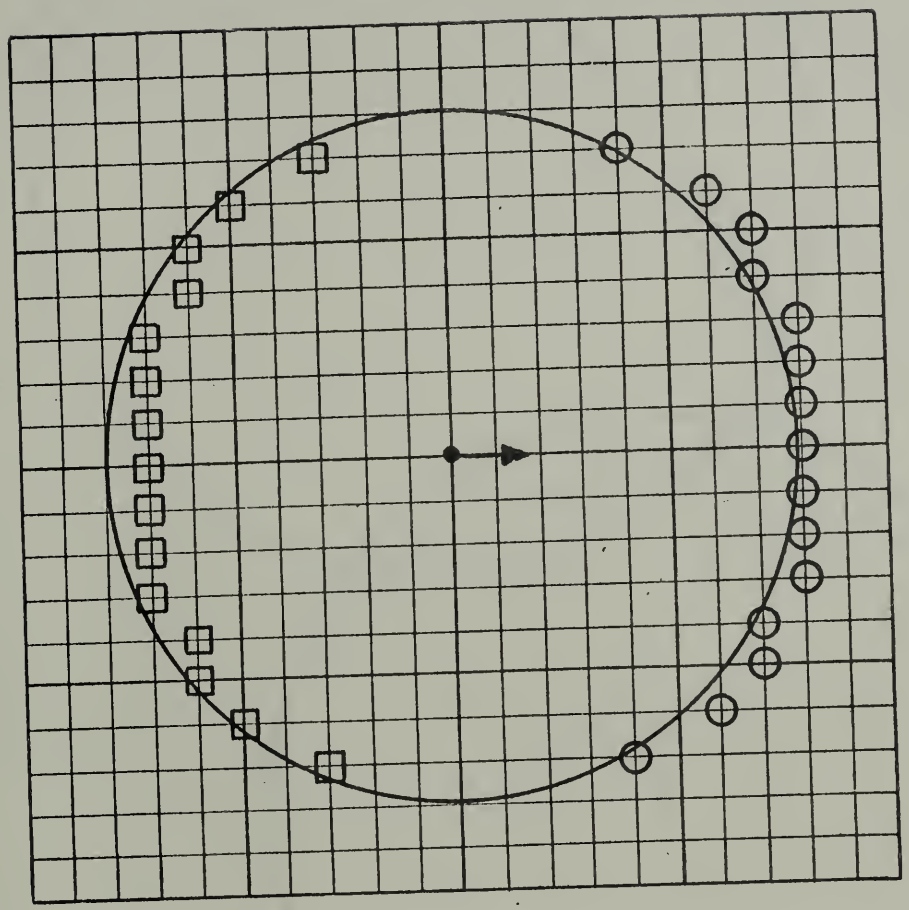


FIG. 7 POINTS TO BE SCANNED

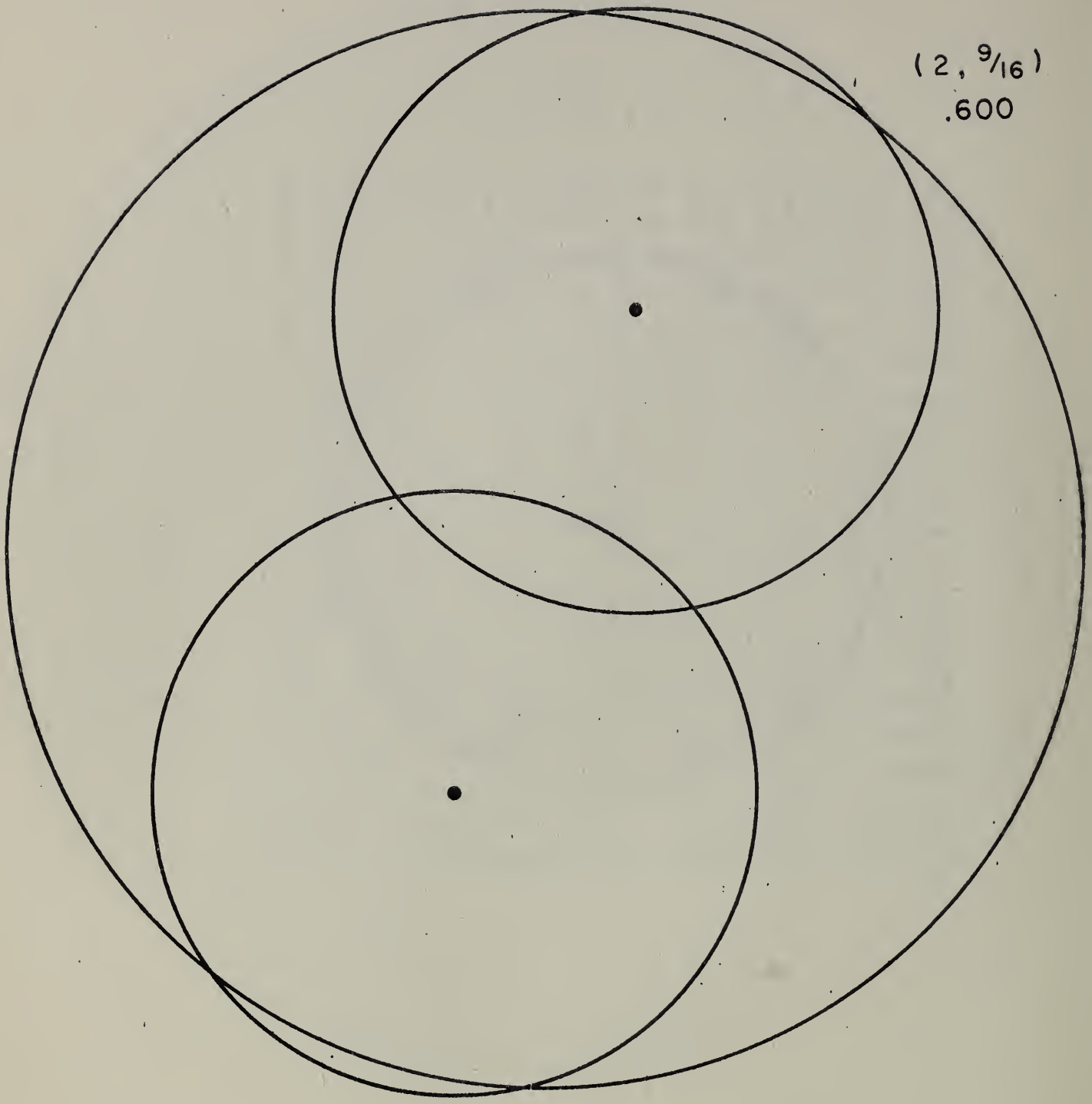


FIG. 8

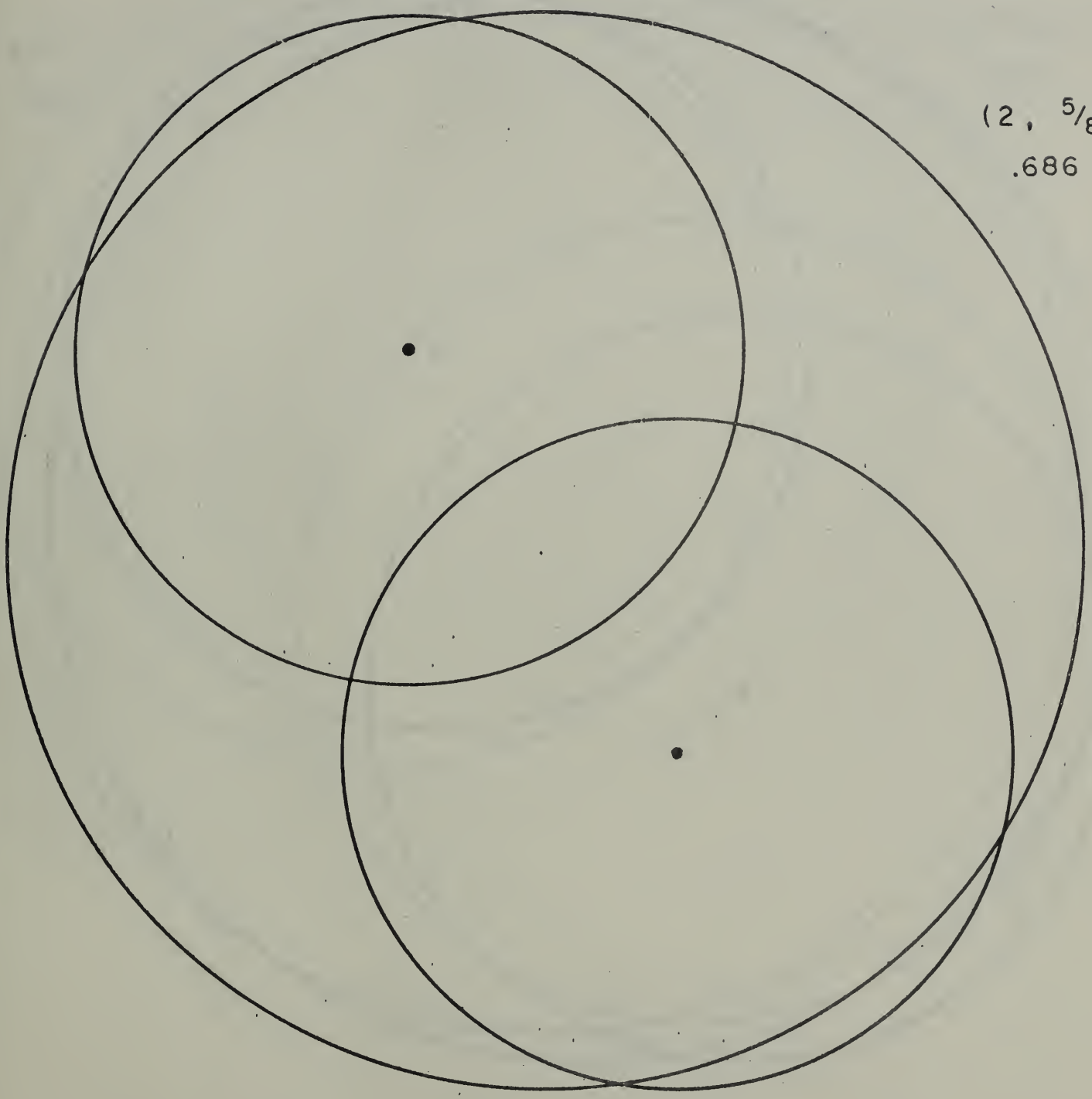


FIG. 9

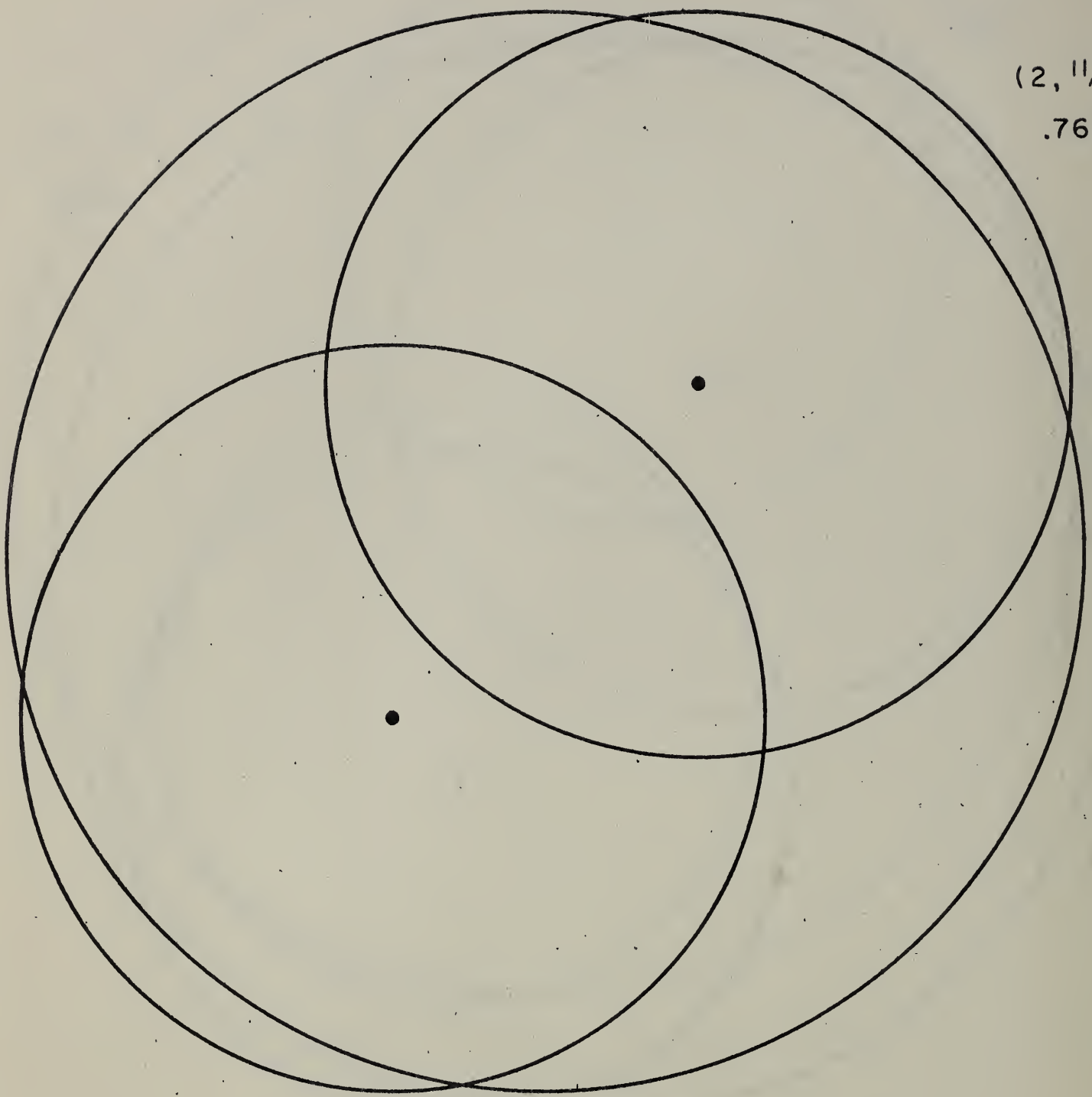


FIG. 10

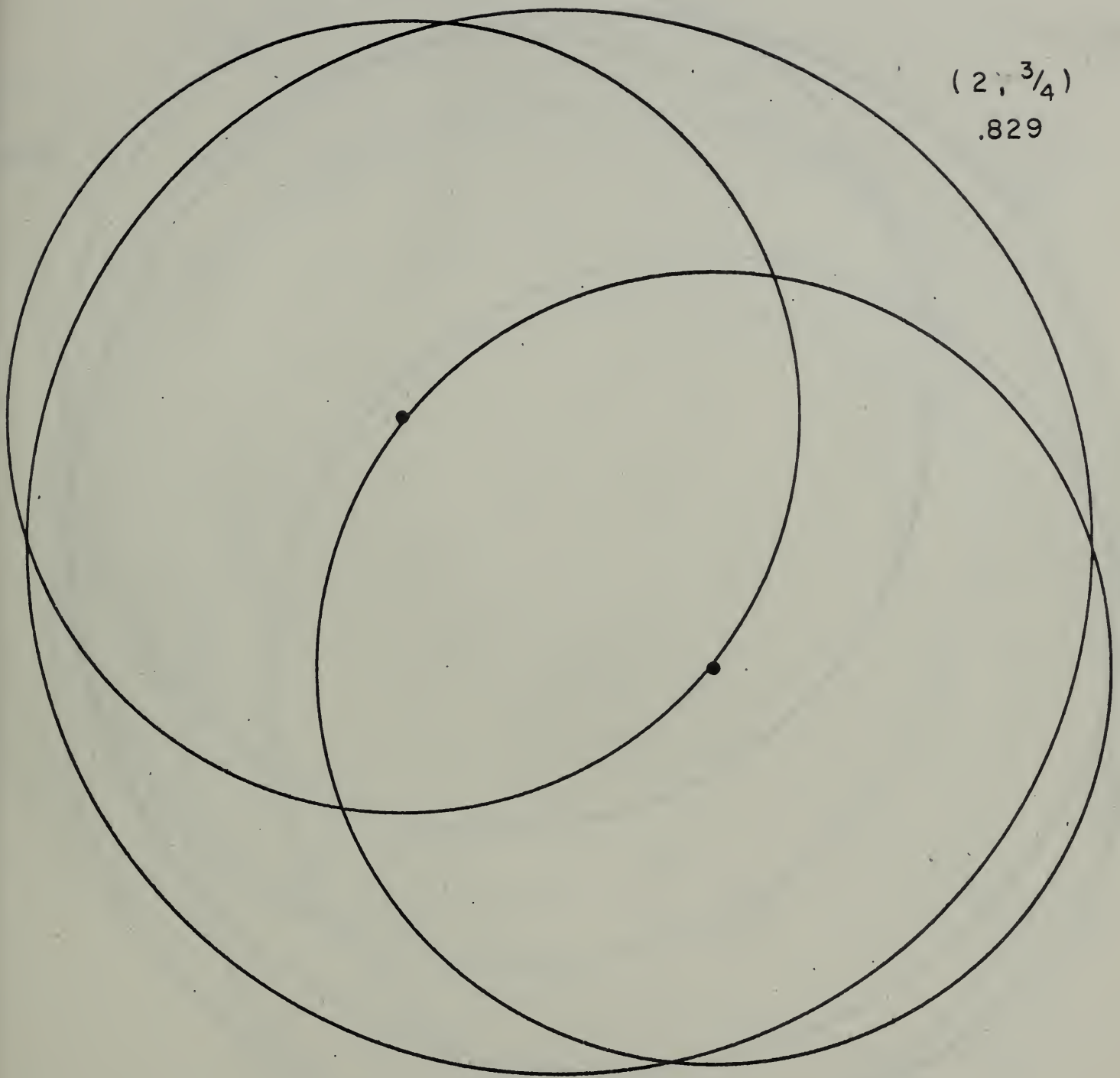


FIG. 11

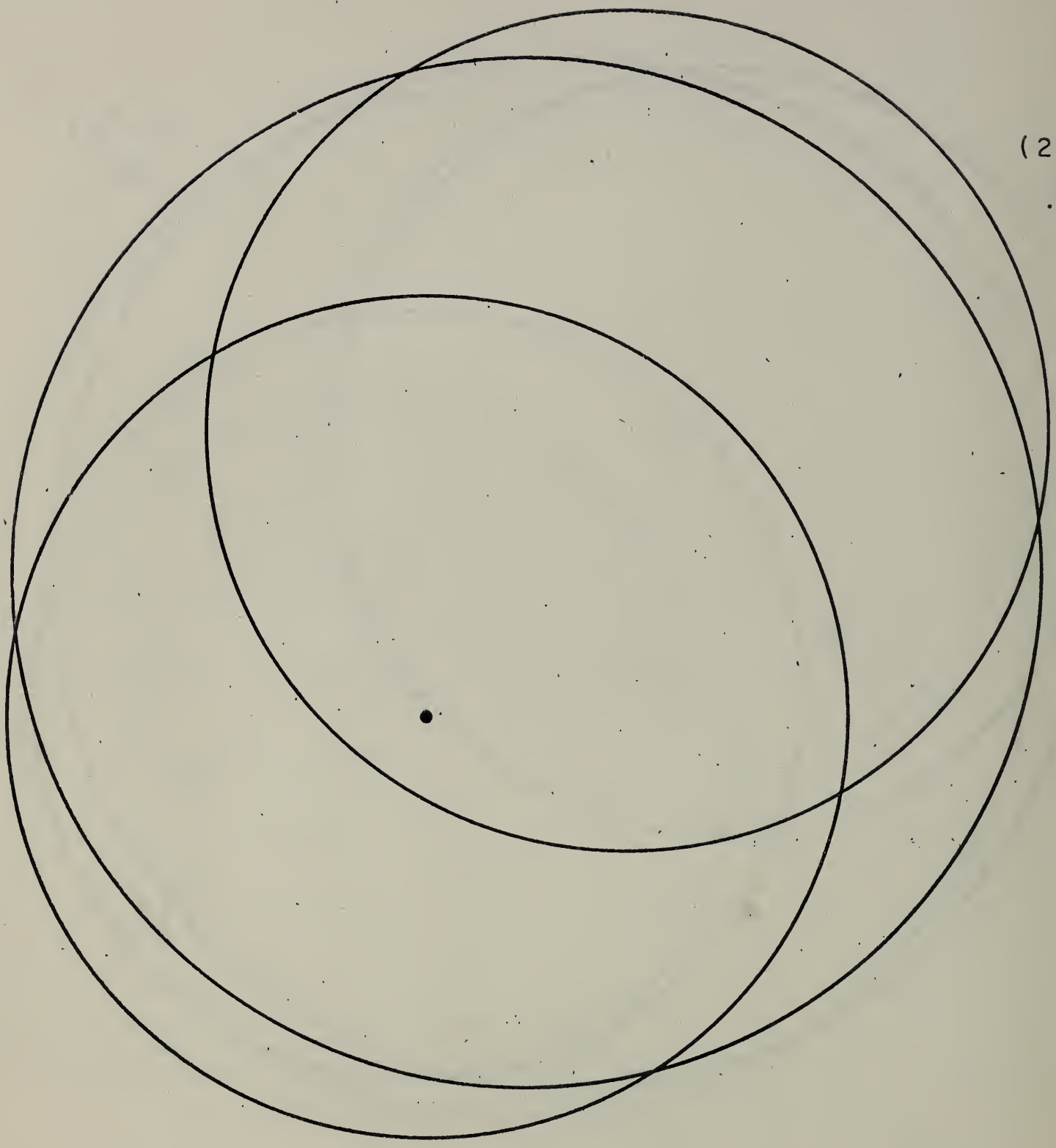


FIG. 12

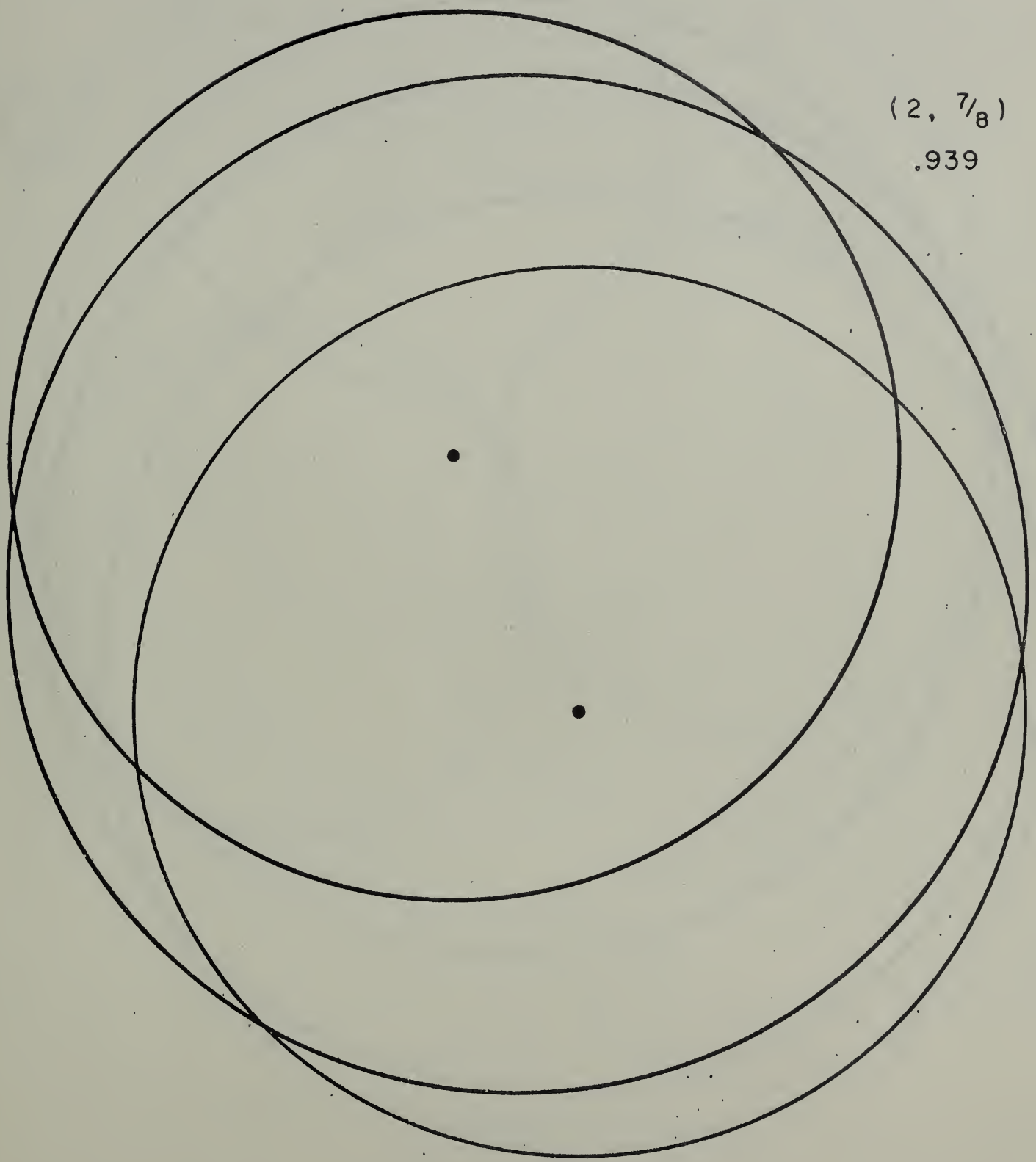


FIG. 13

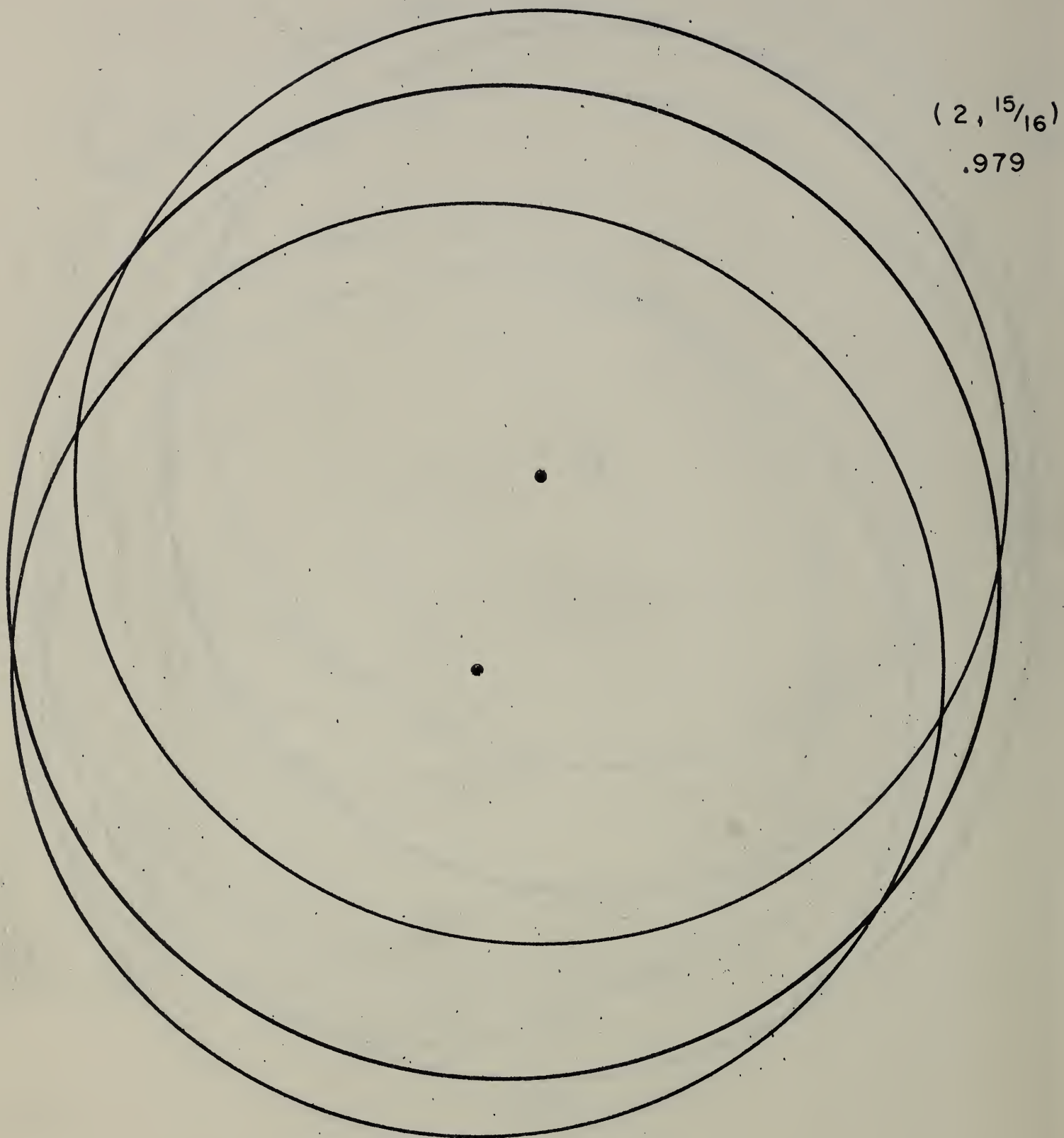


FIG. 14

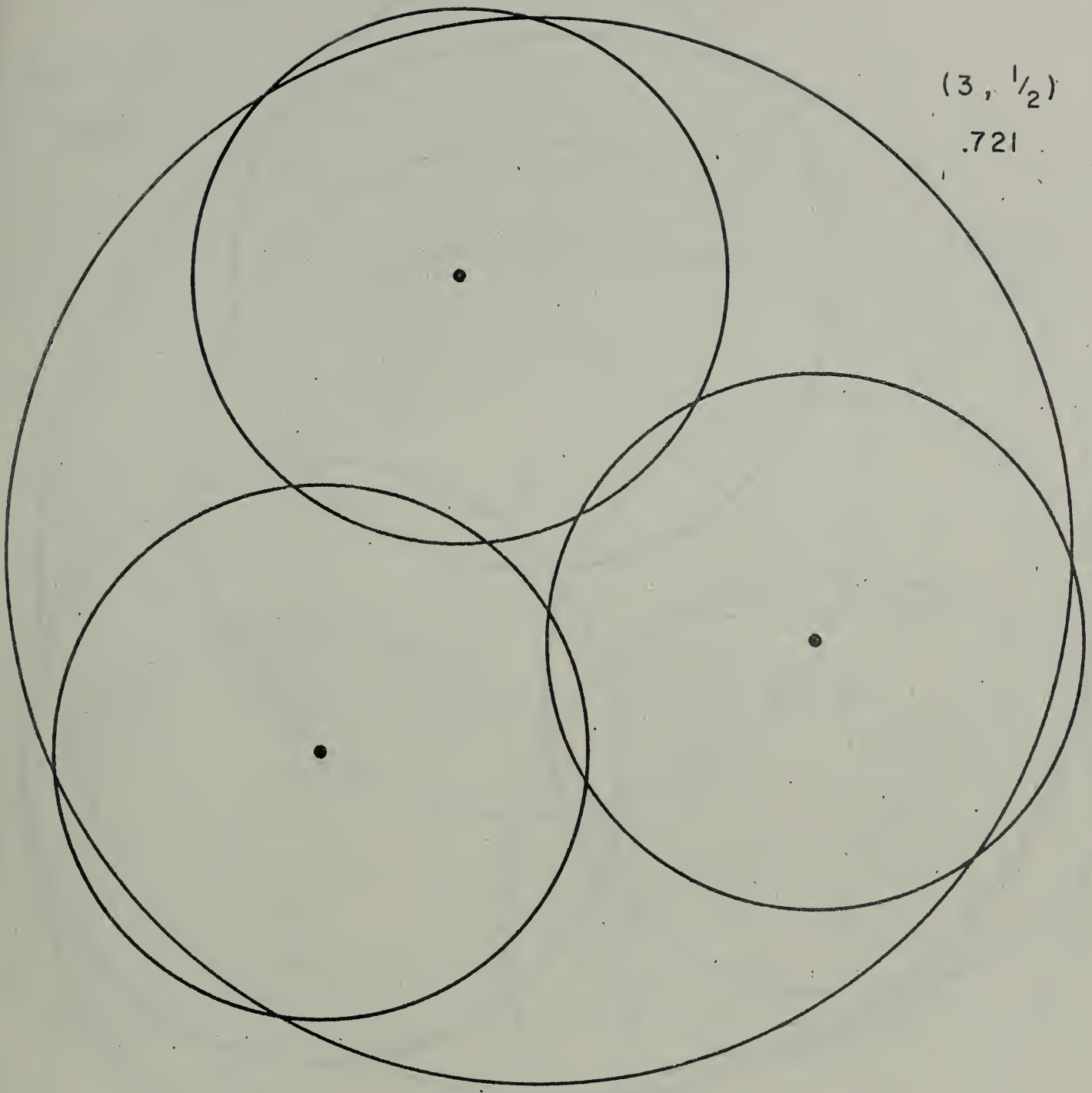
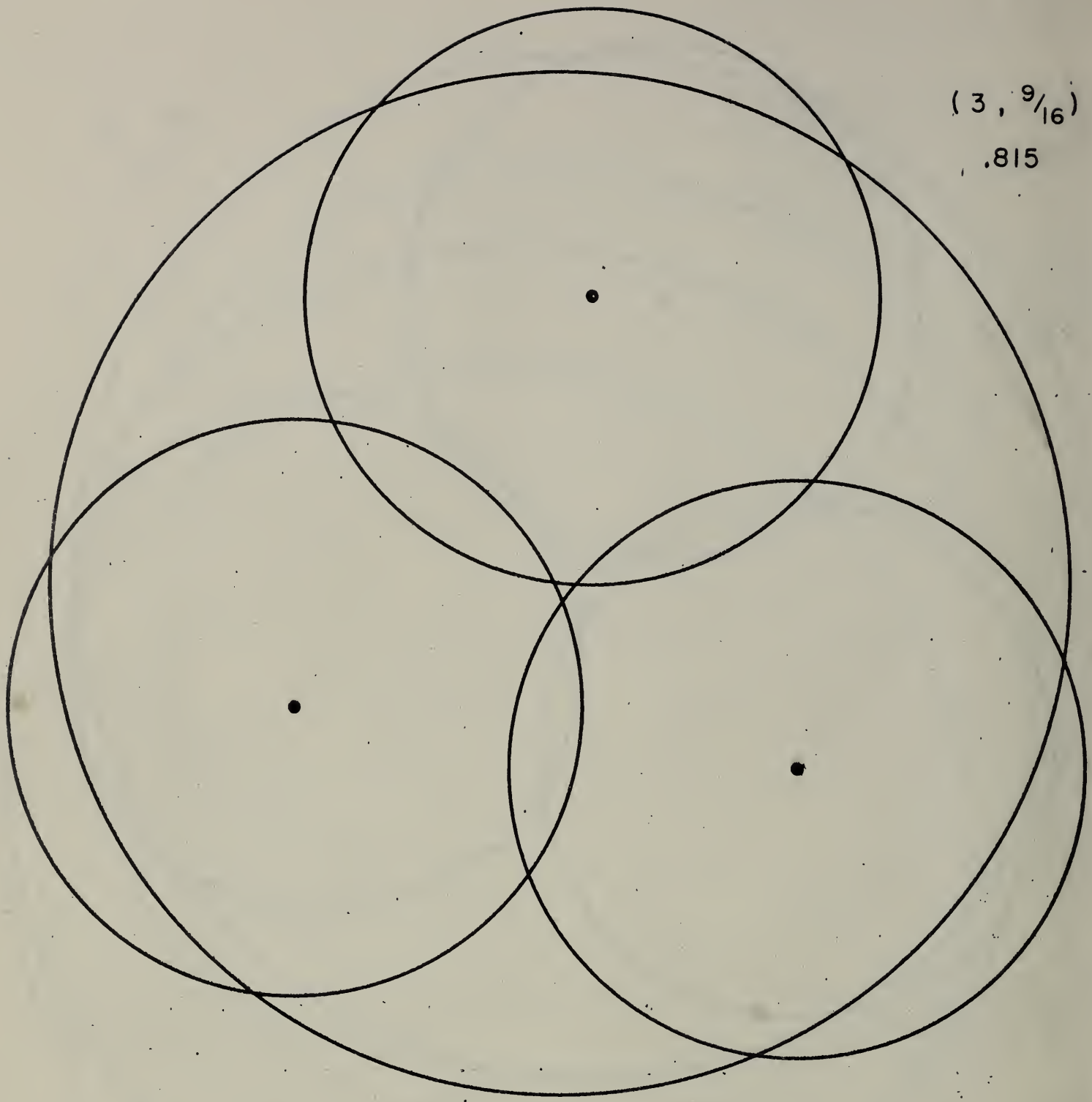


FIG. 15



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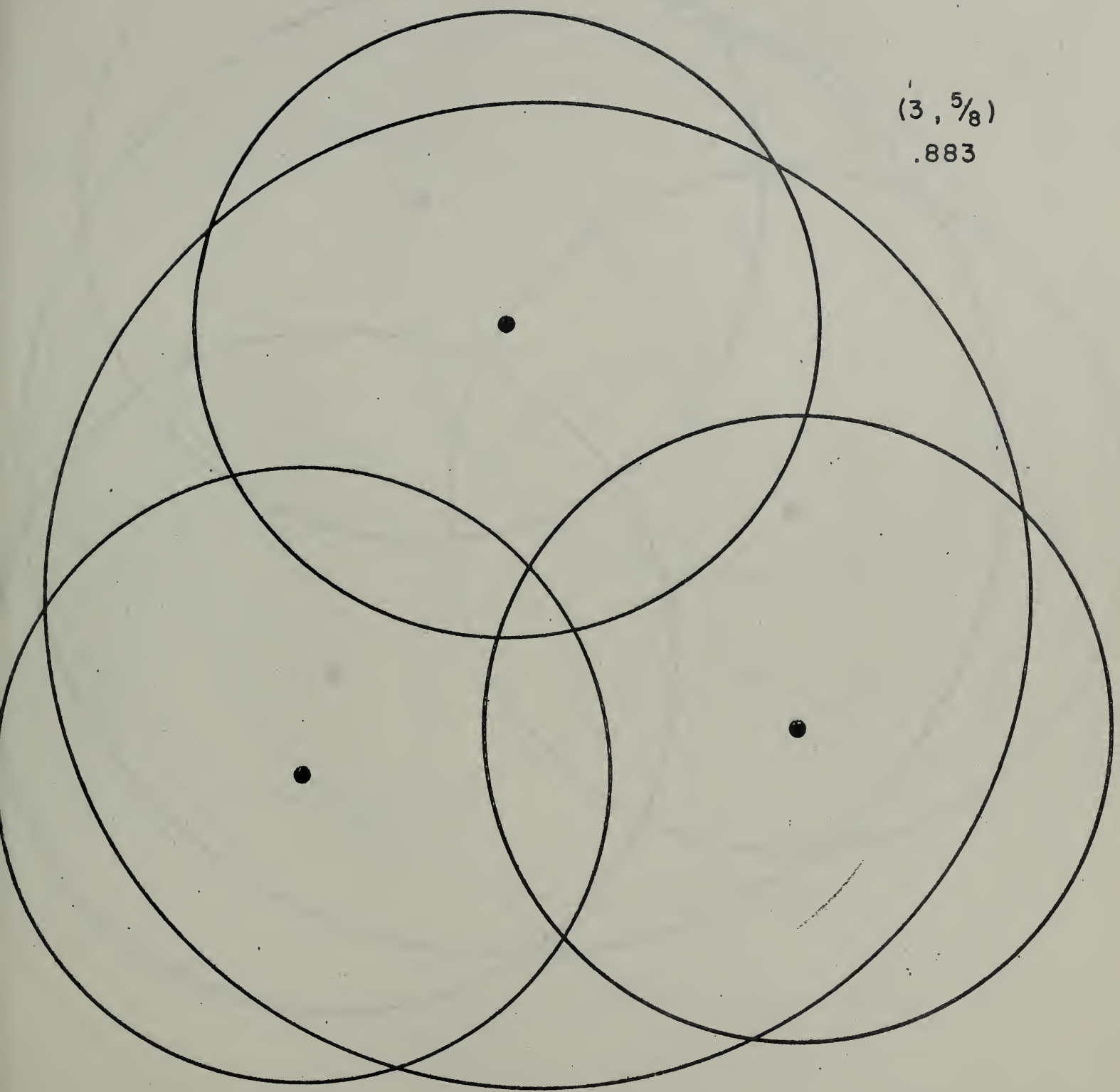


FIG. 17

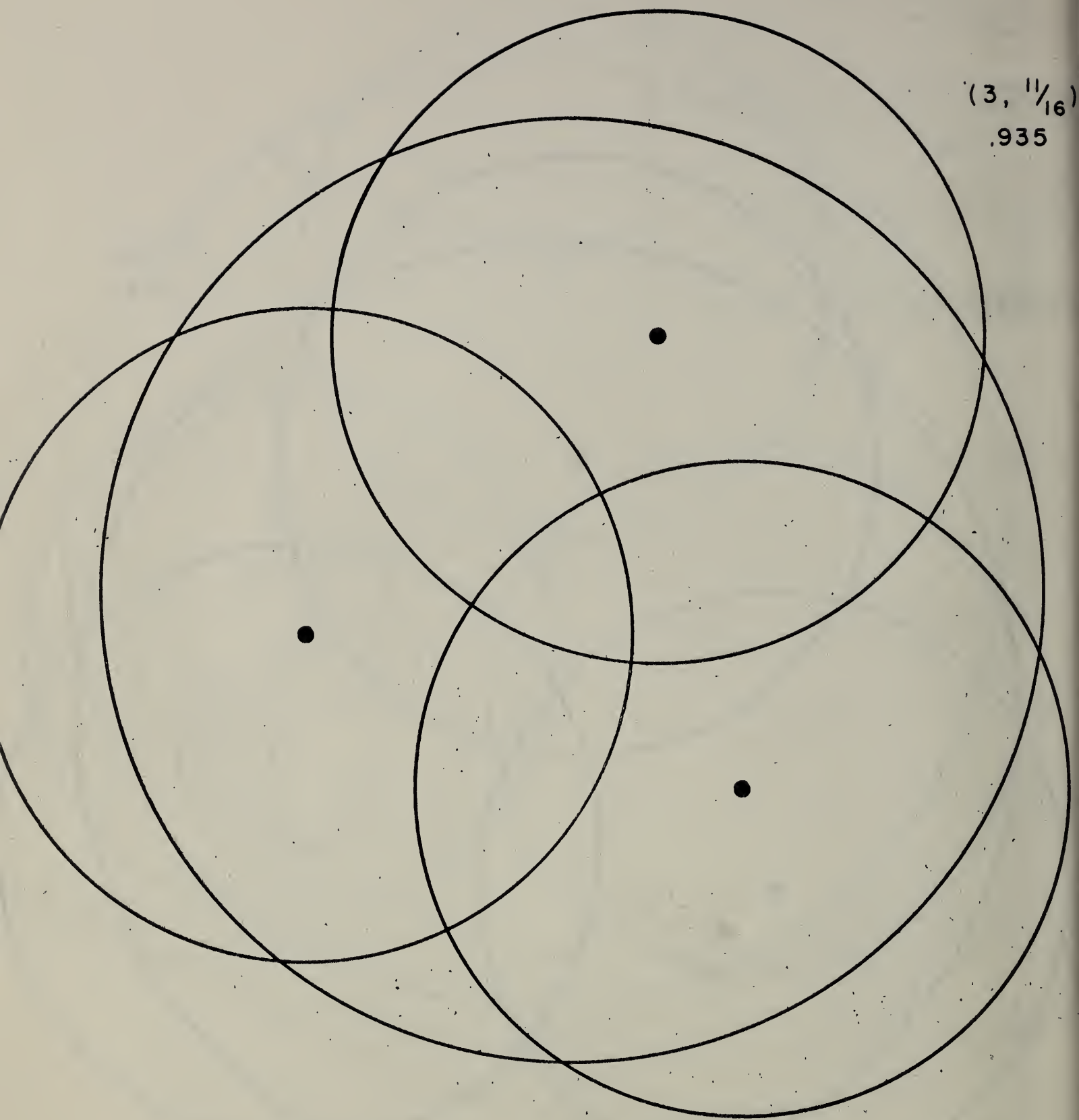


FIG. 18

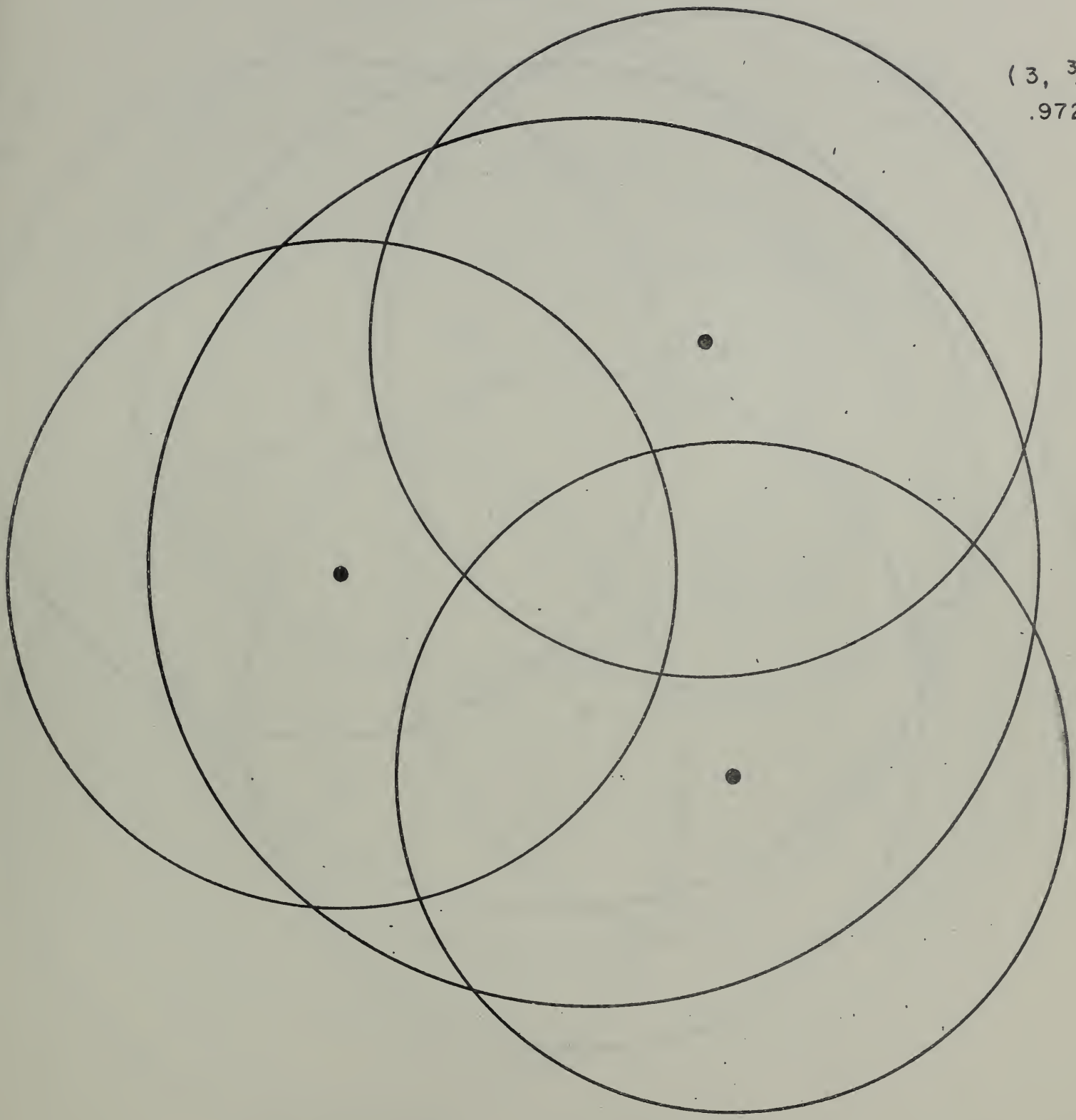


FIG. 19

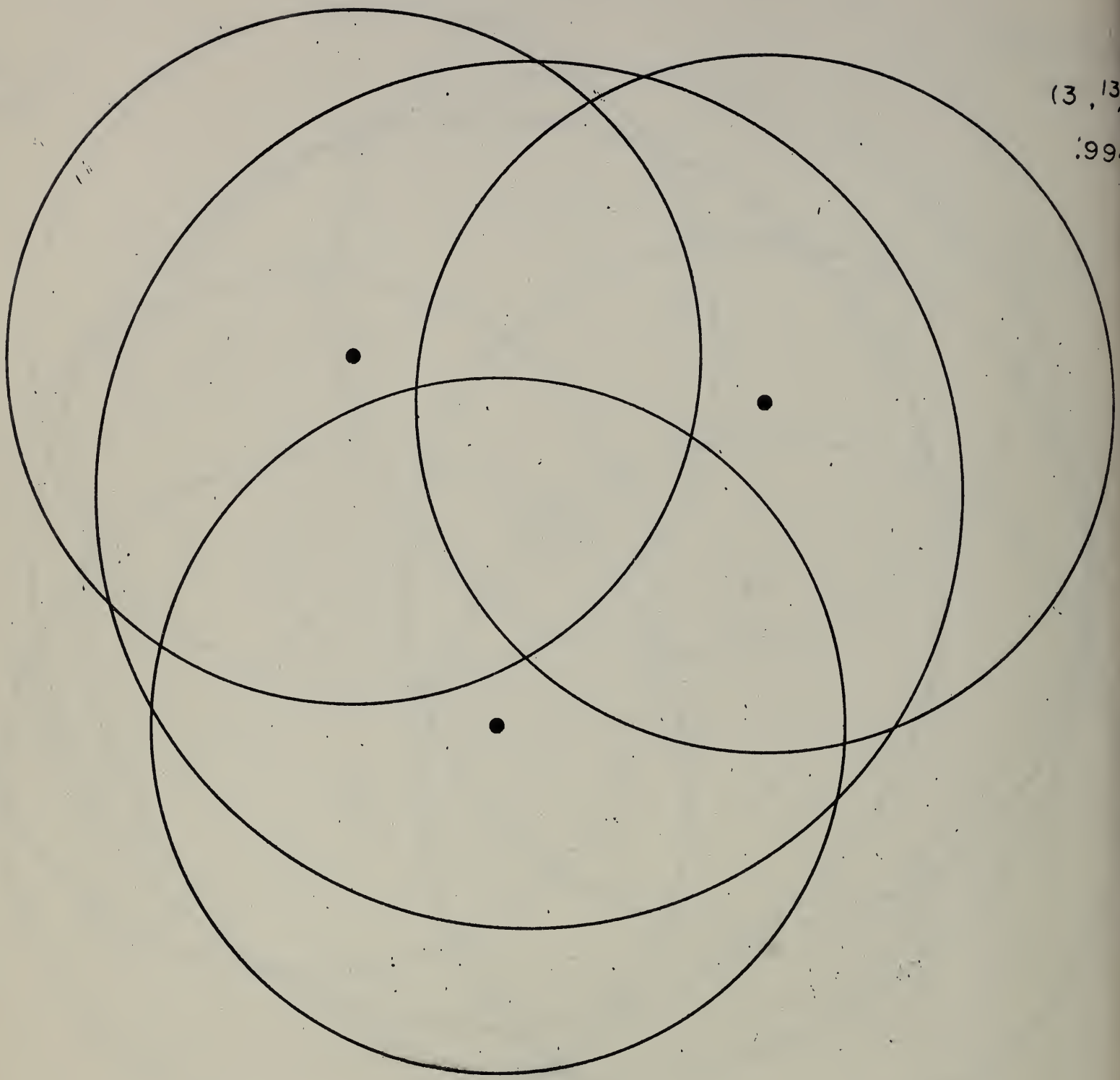


FIG. 20

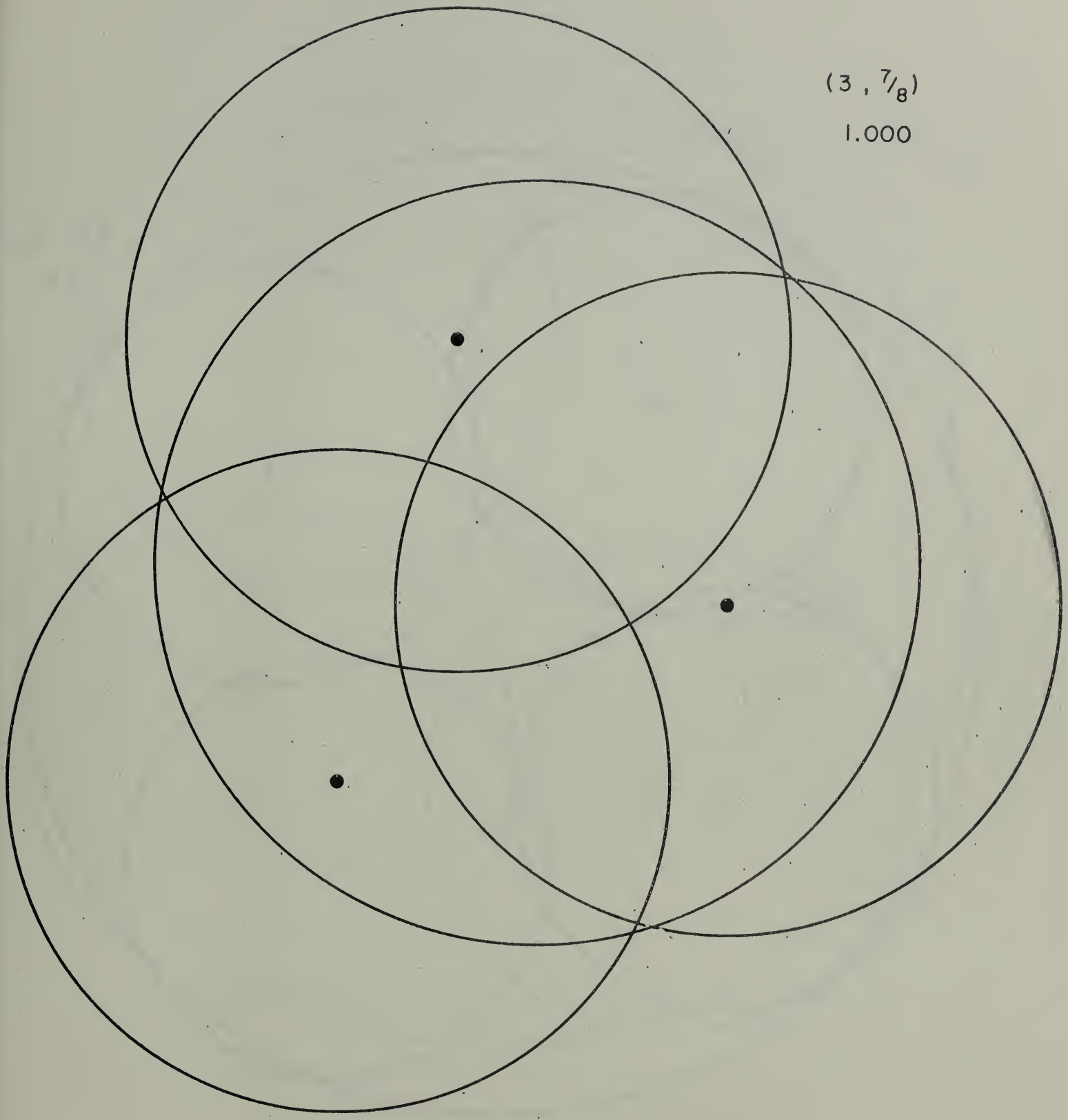
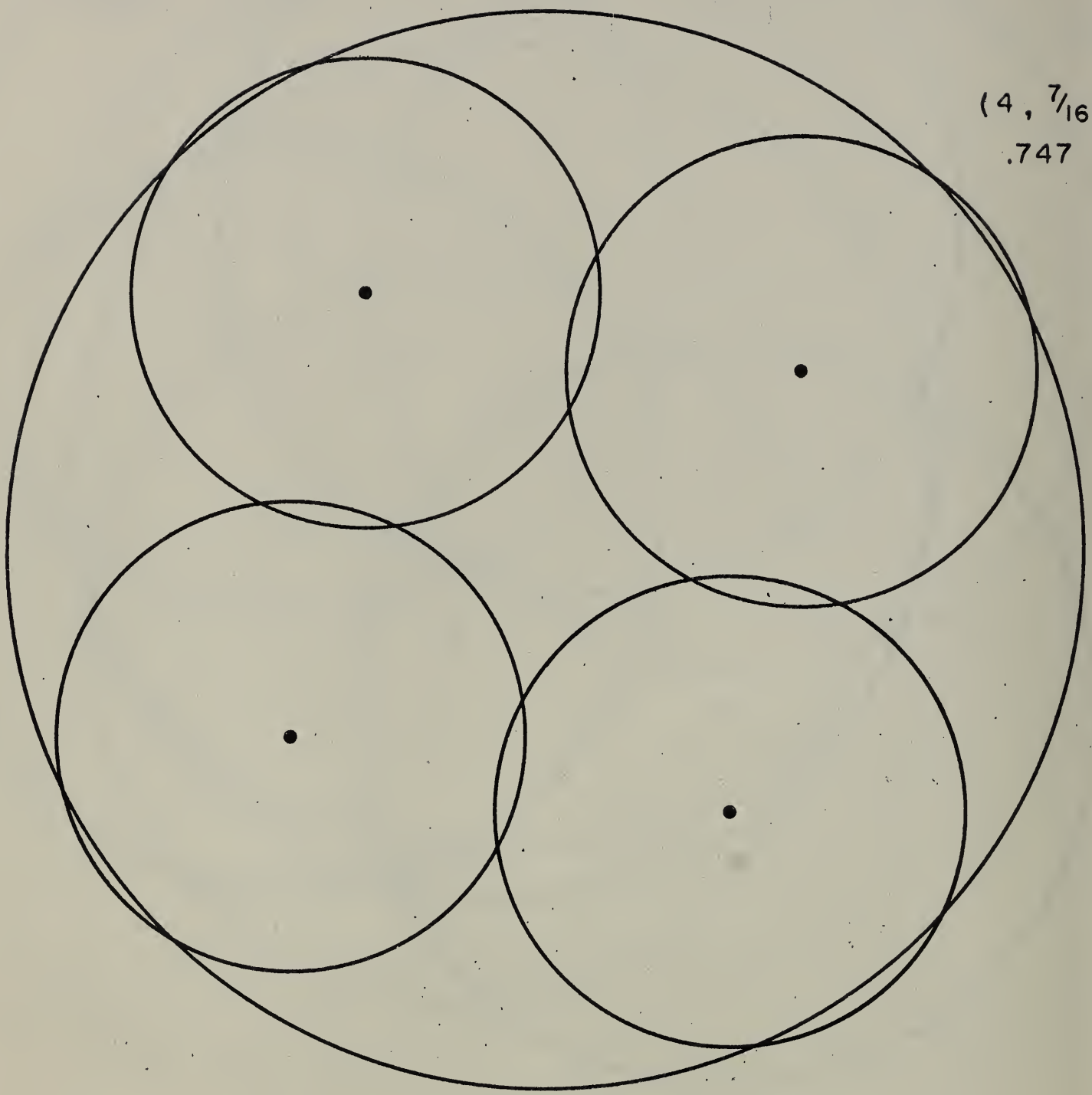


FIG. 21



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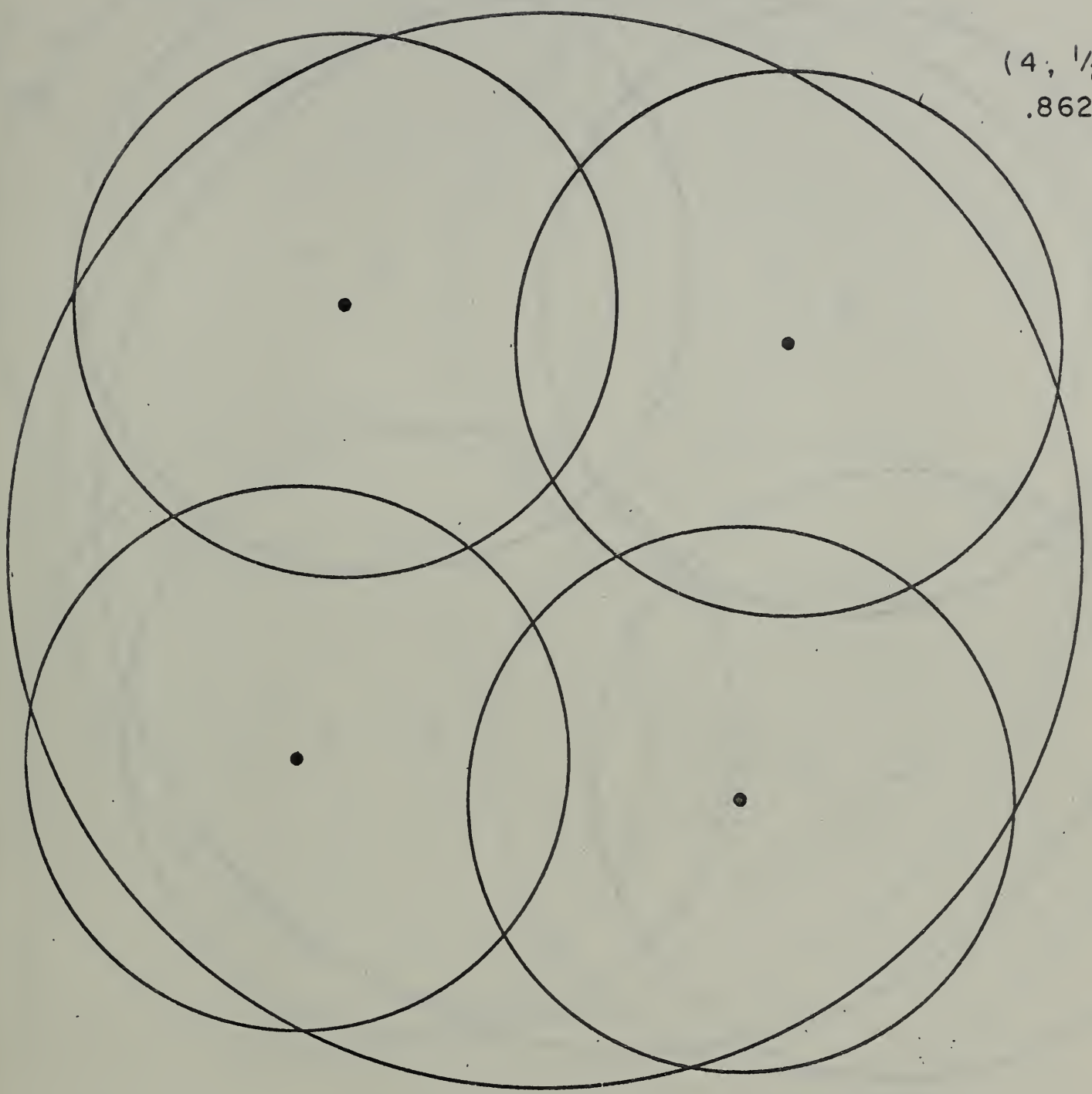

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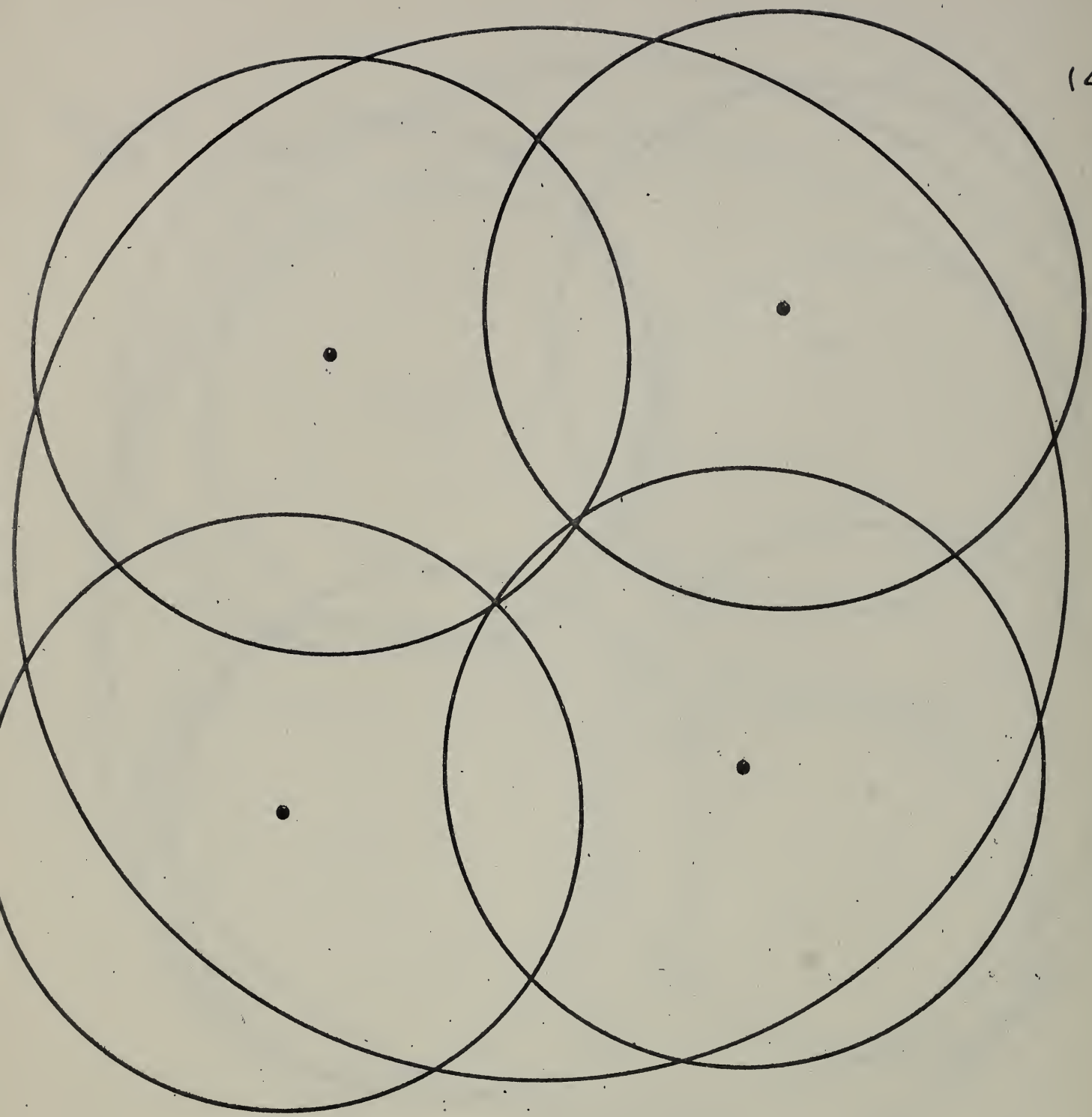

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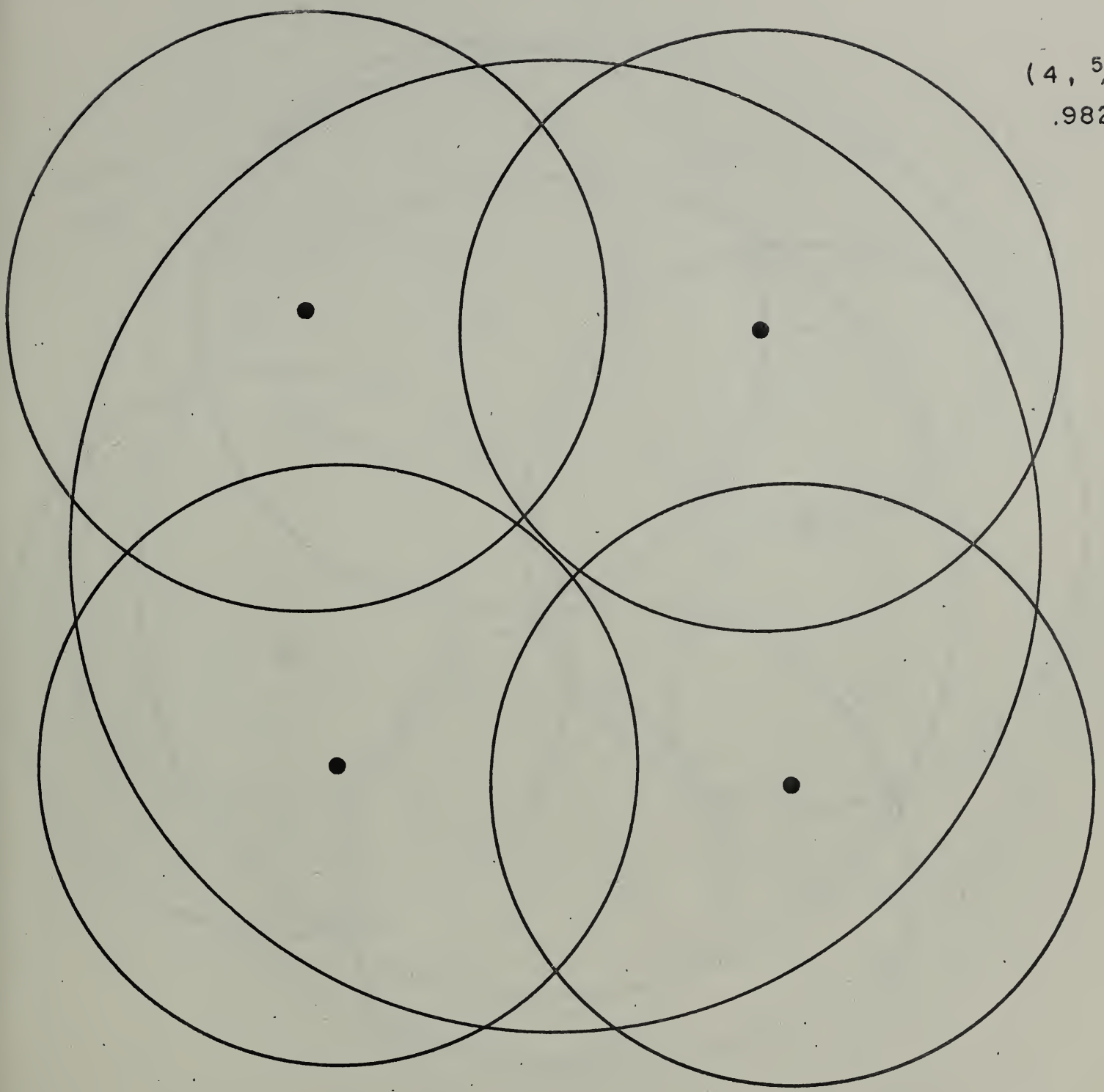

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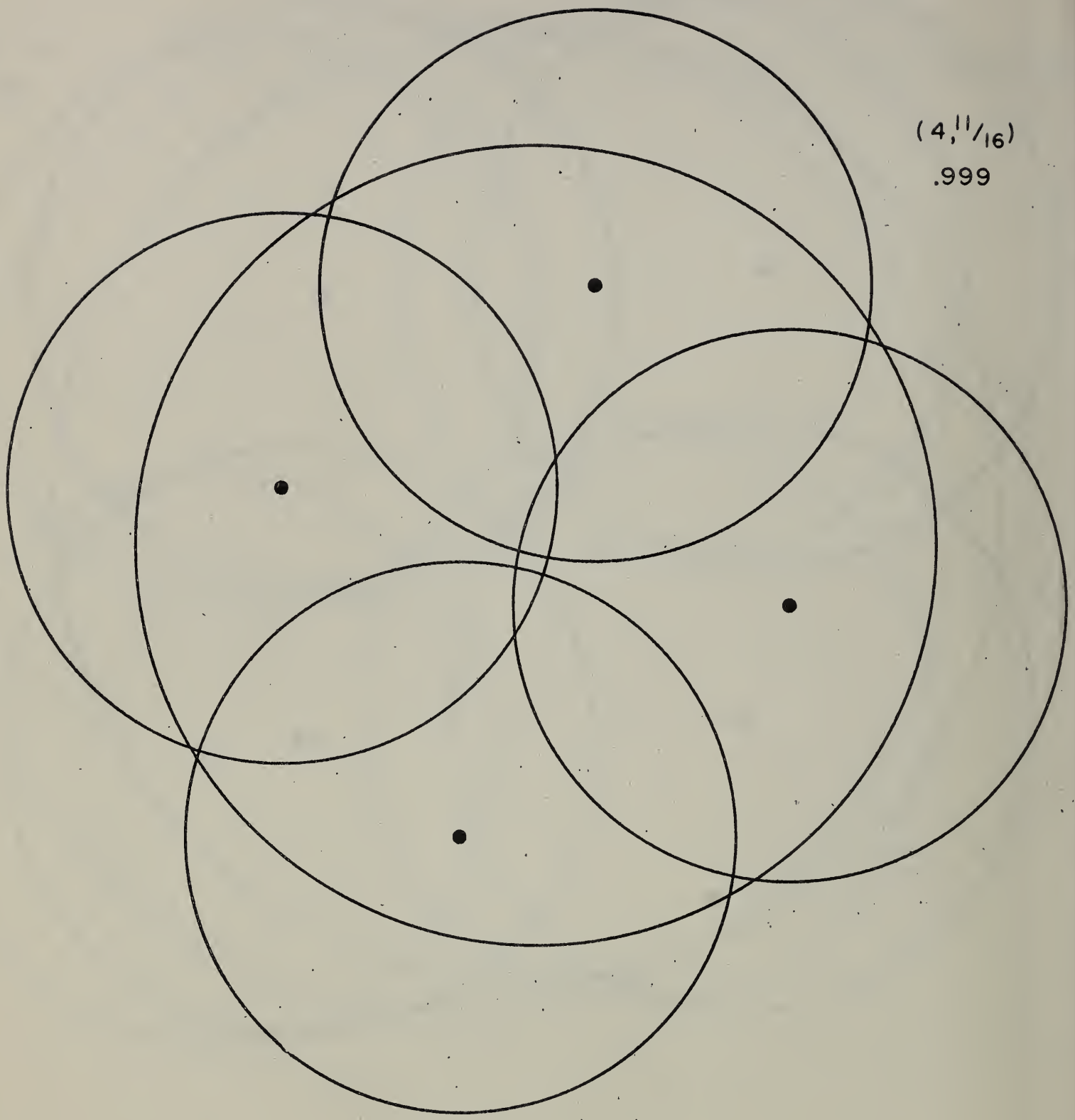

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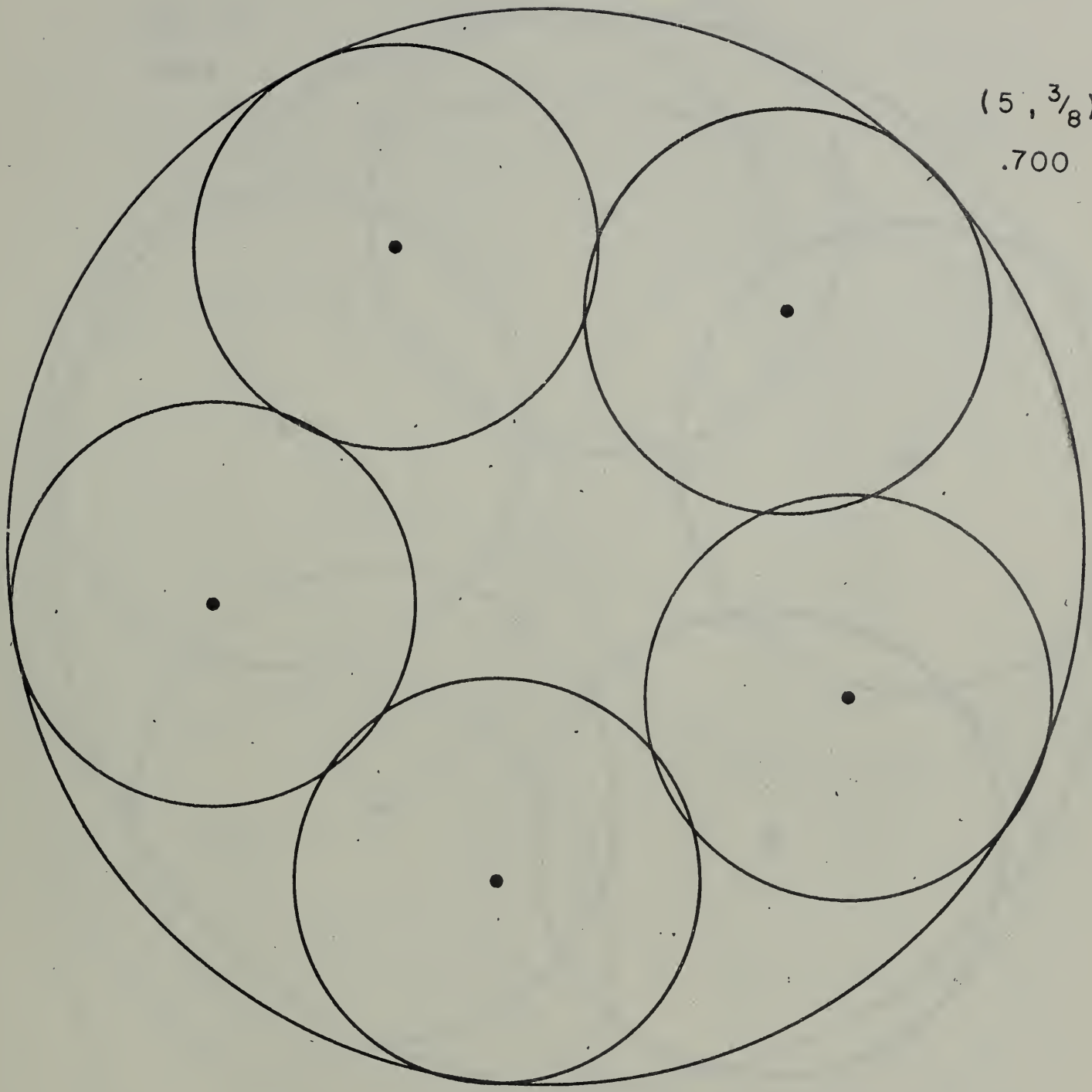

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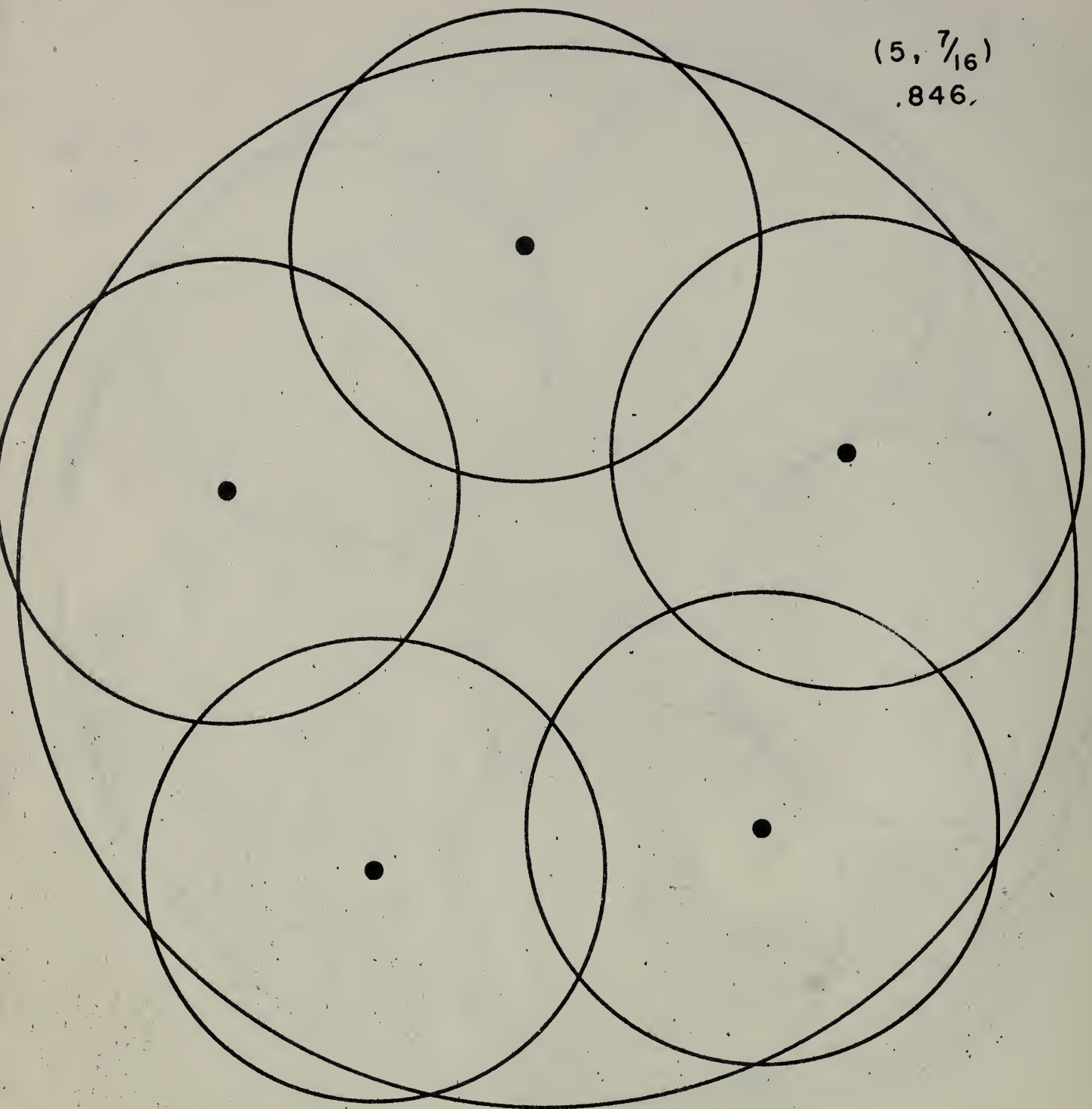

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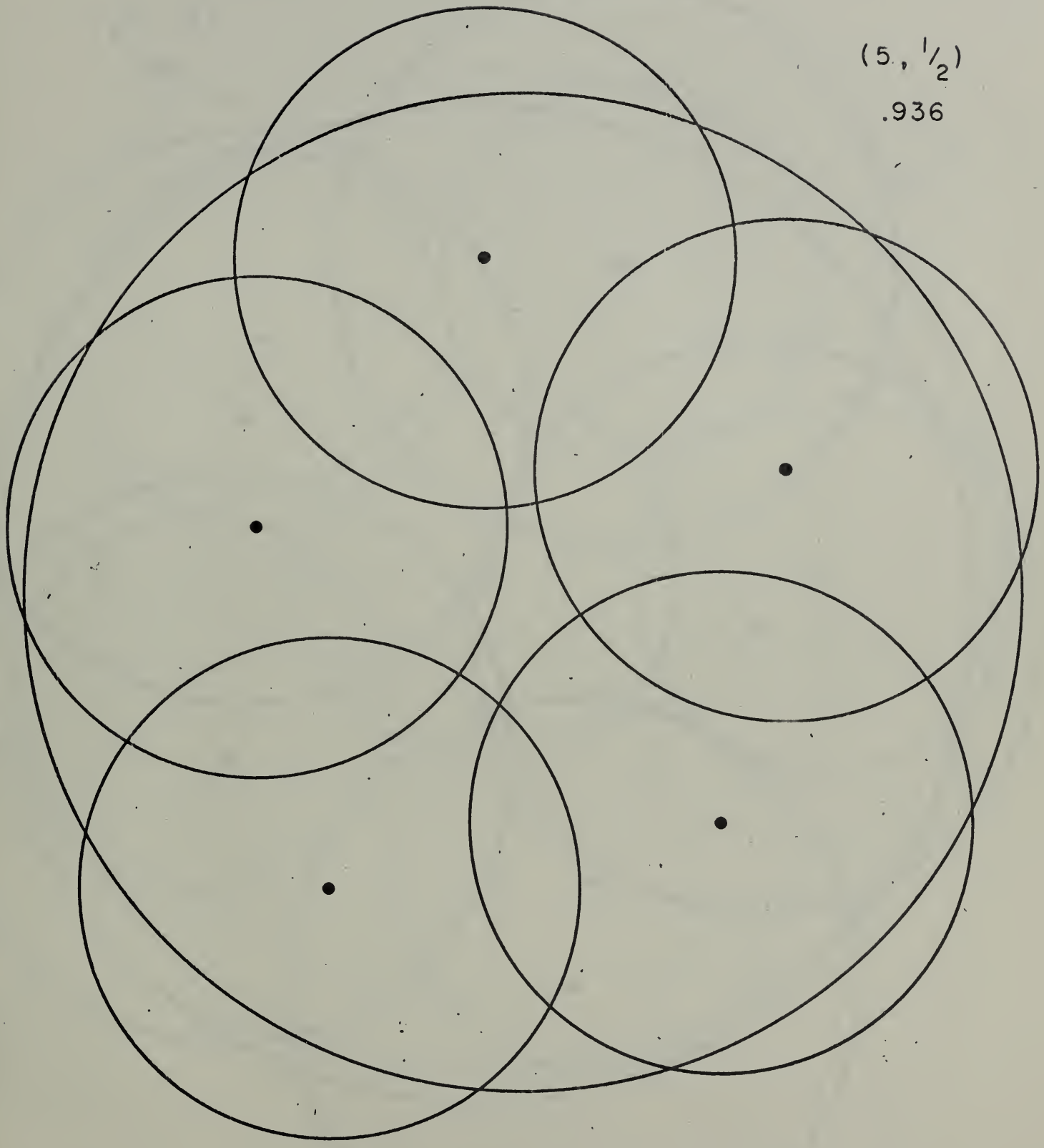

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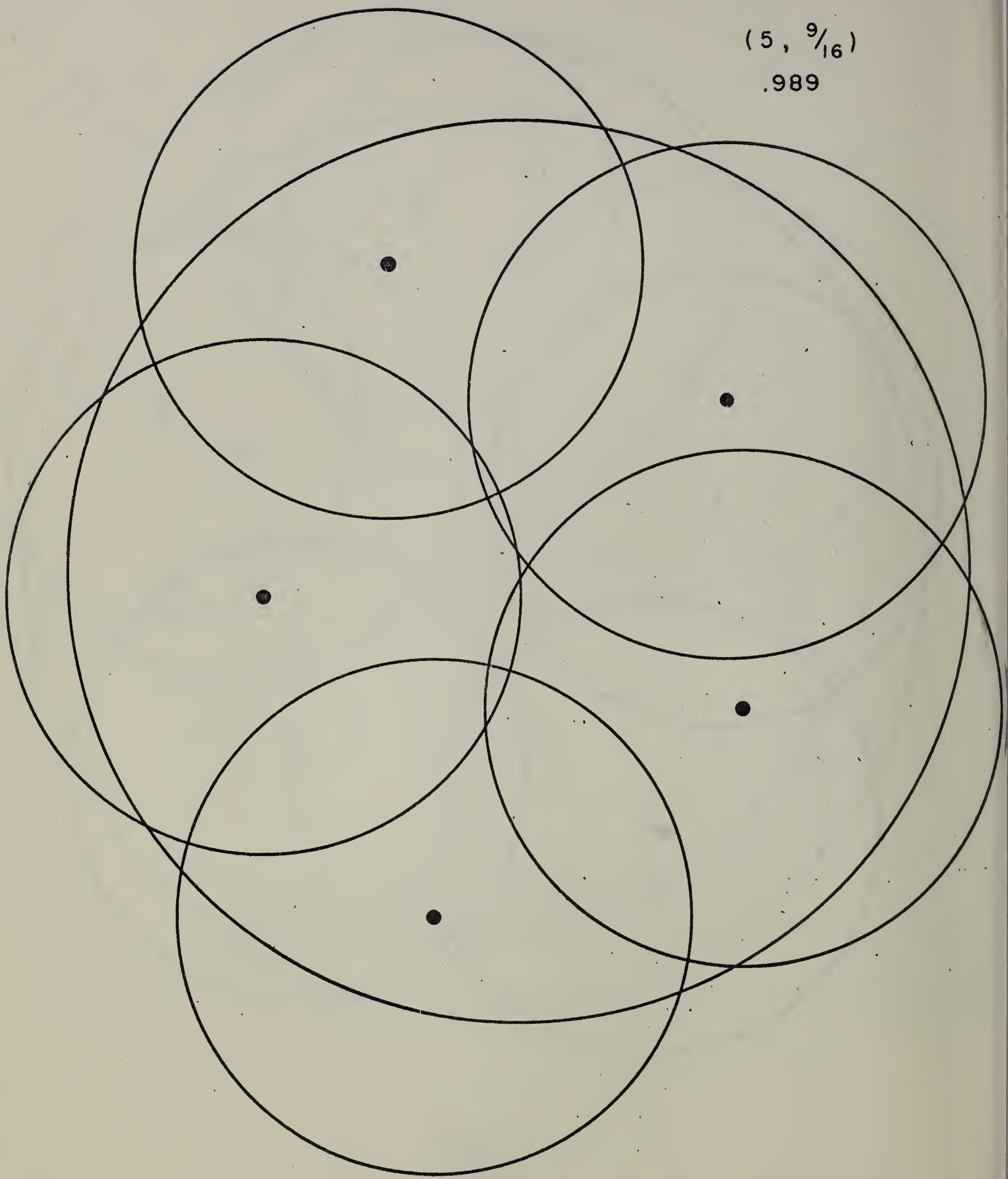

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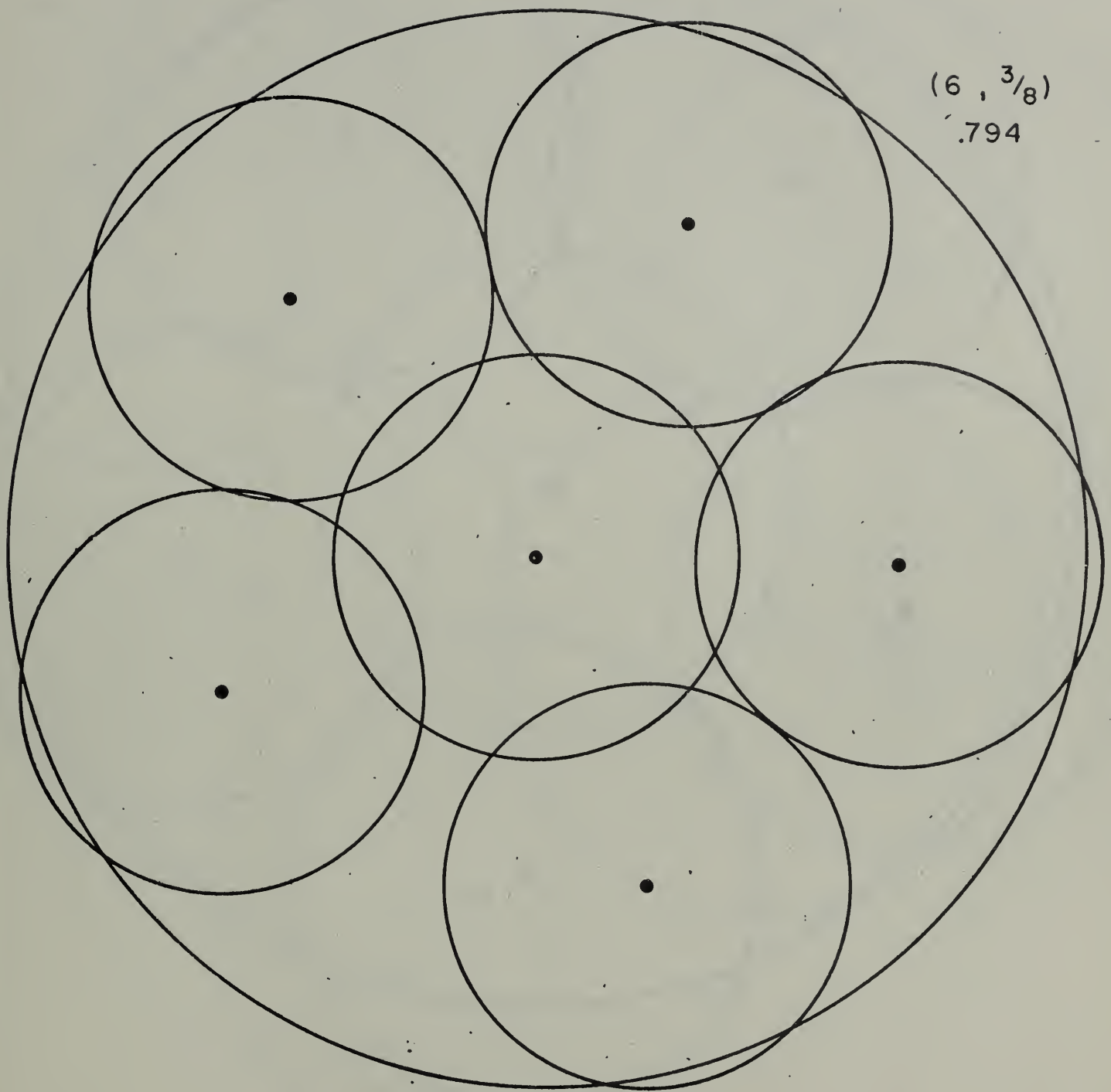

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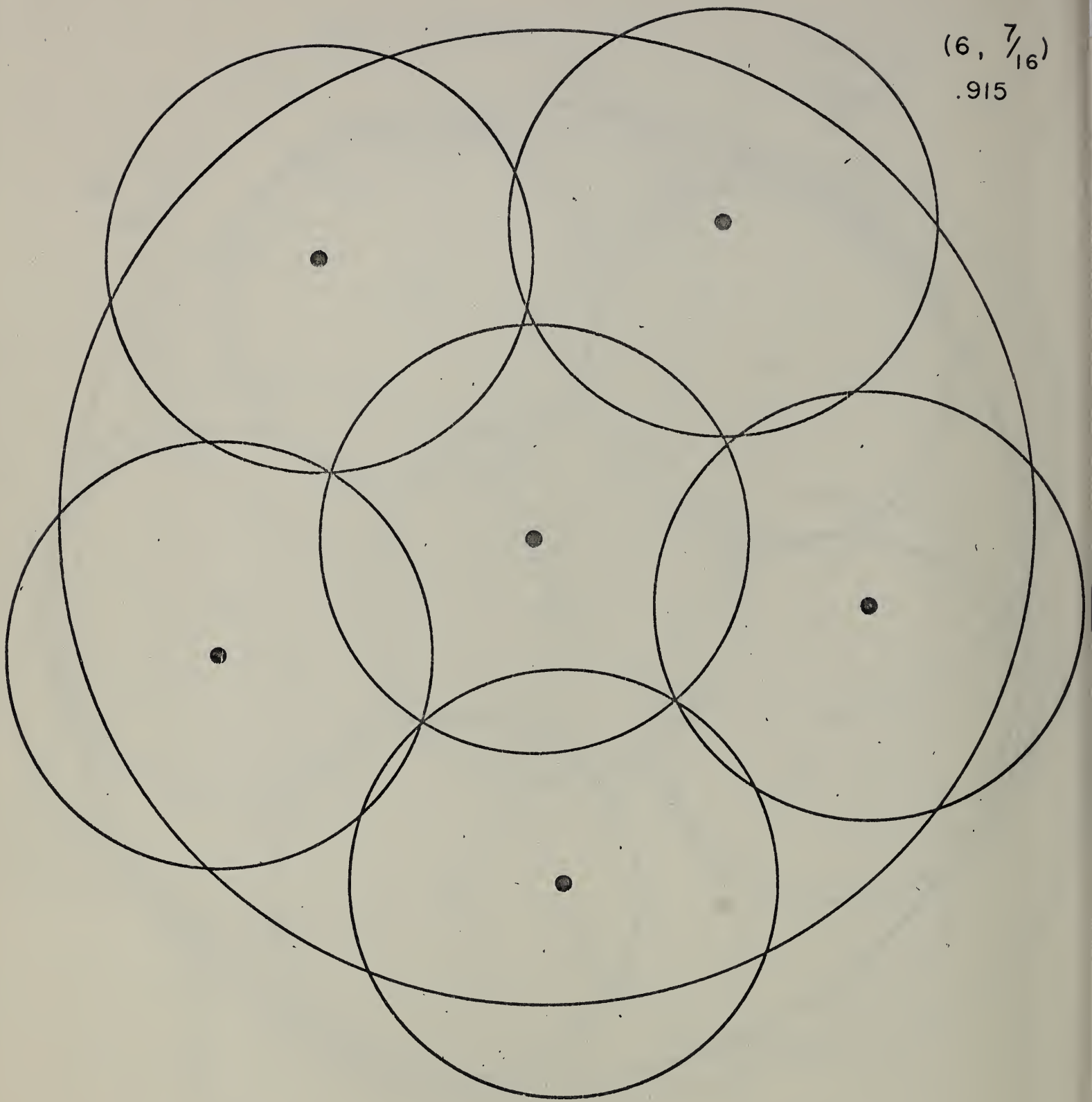

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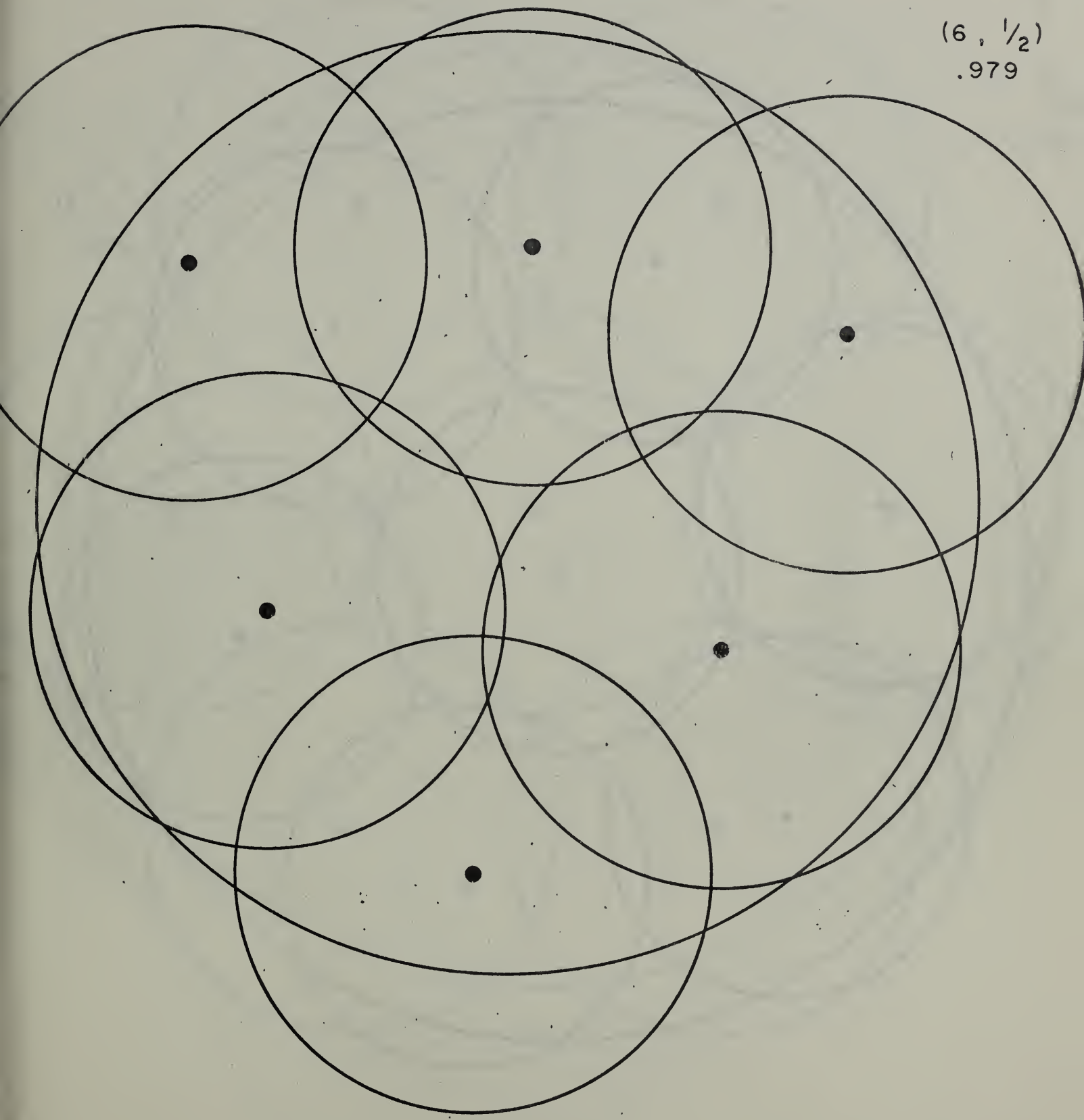

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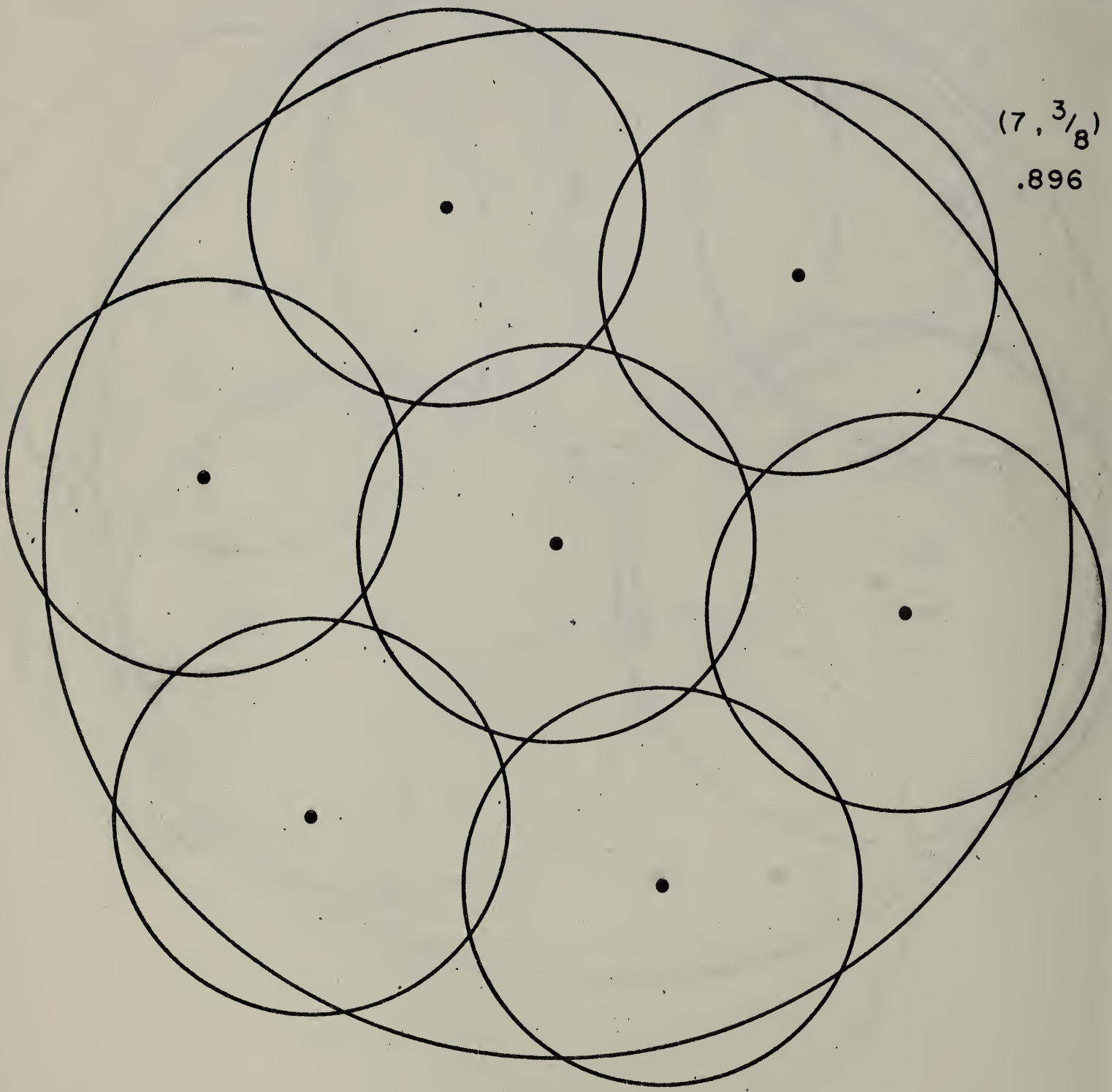

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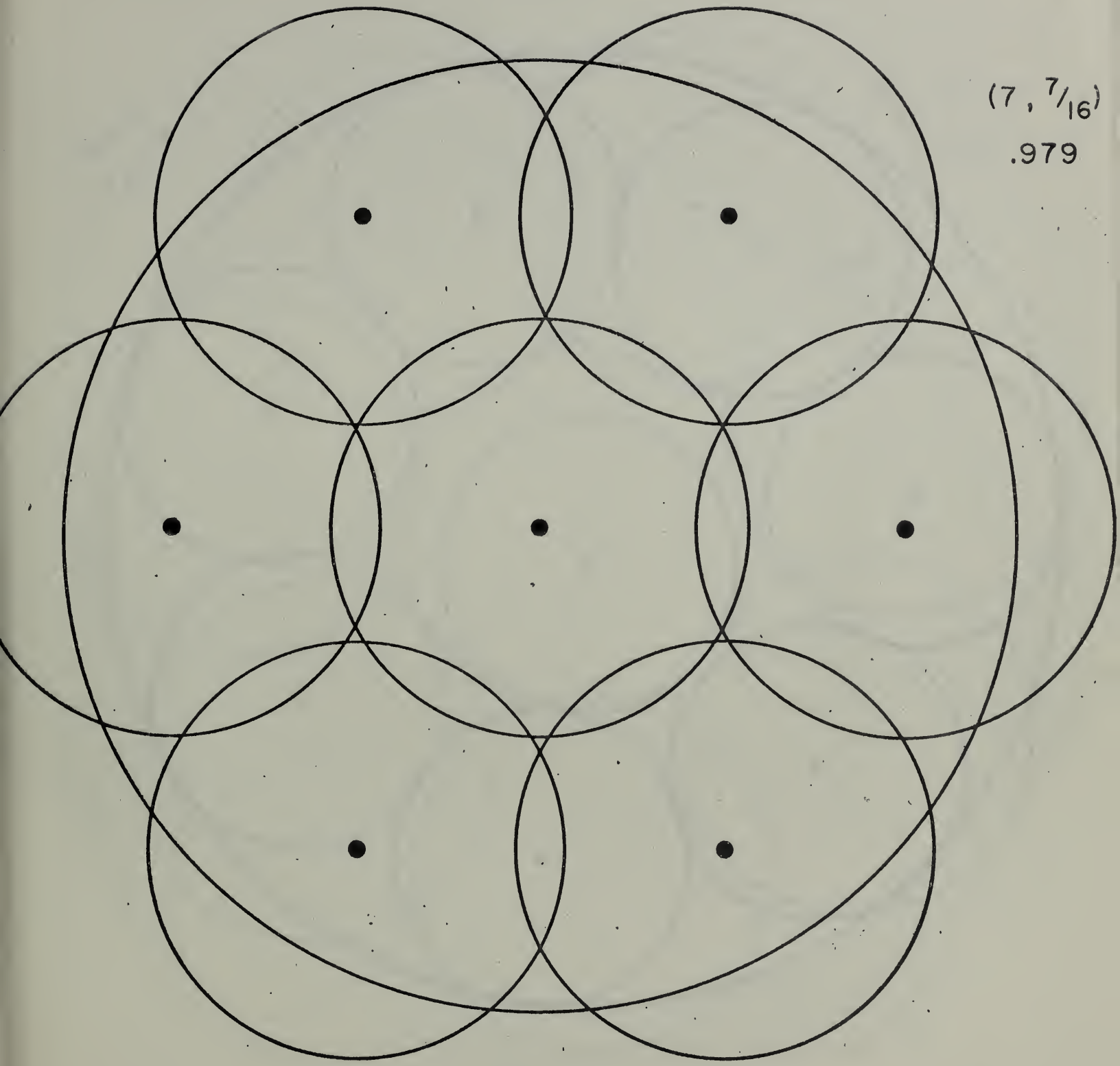

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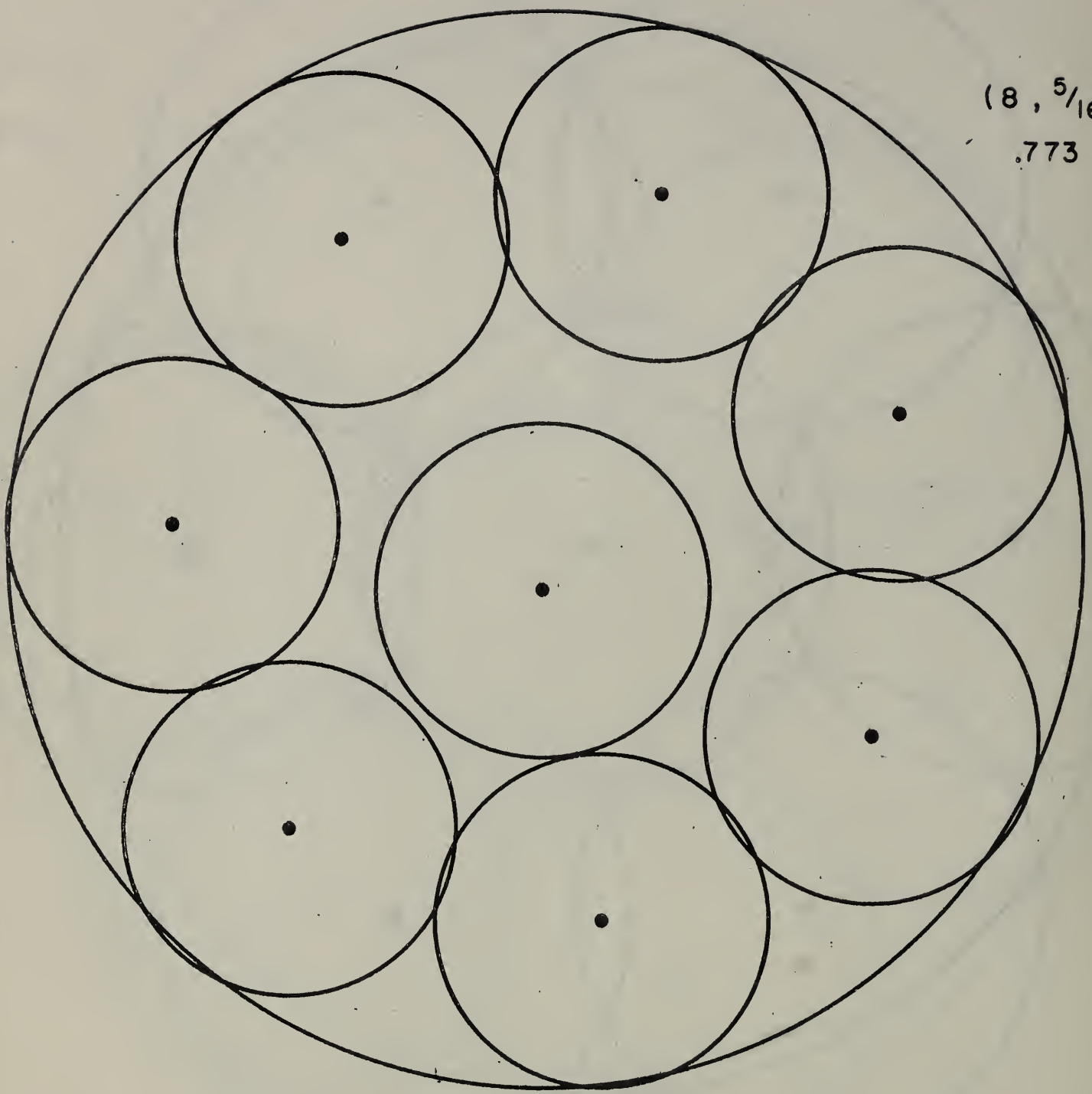

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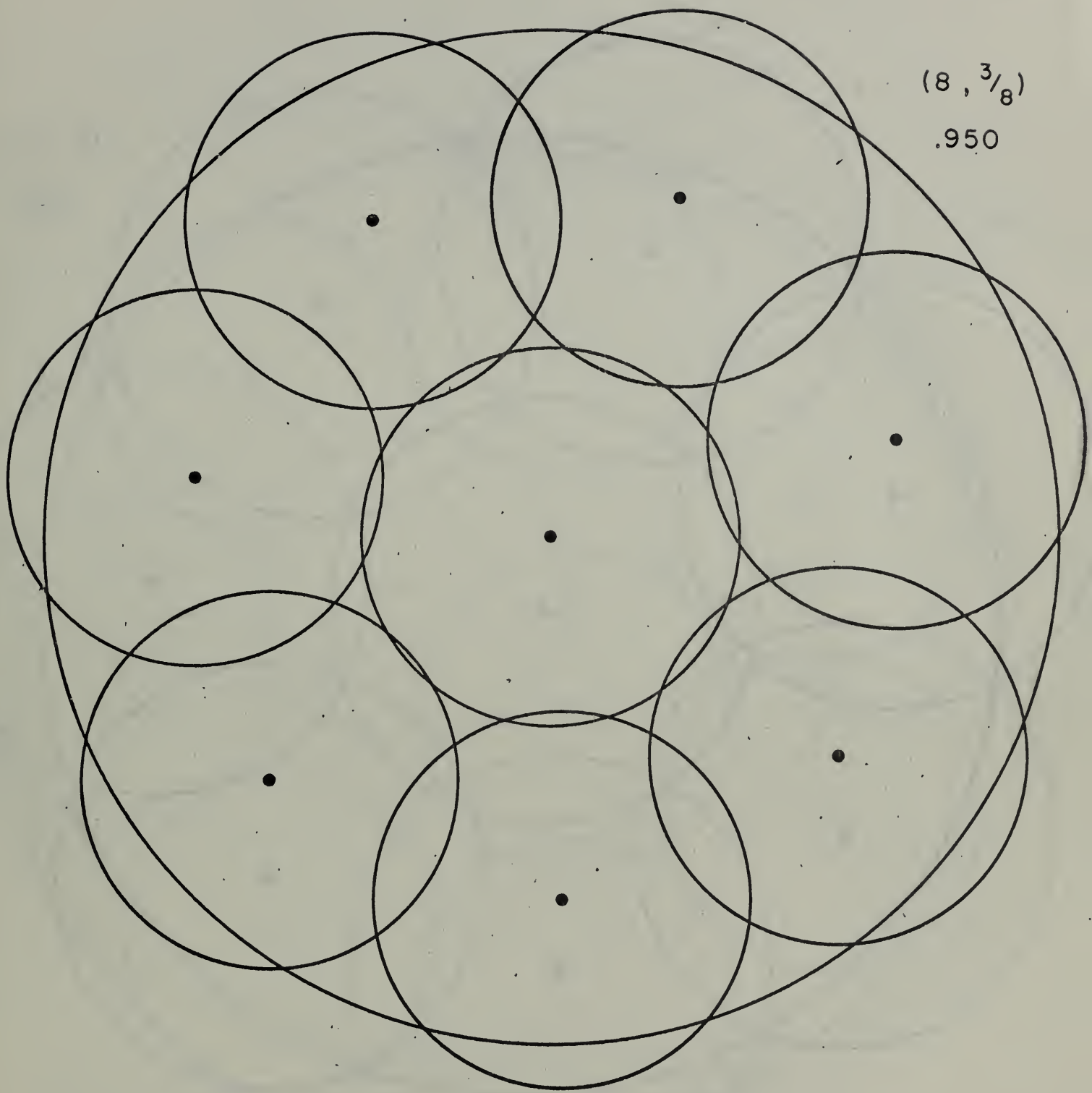

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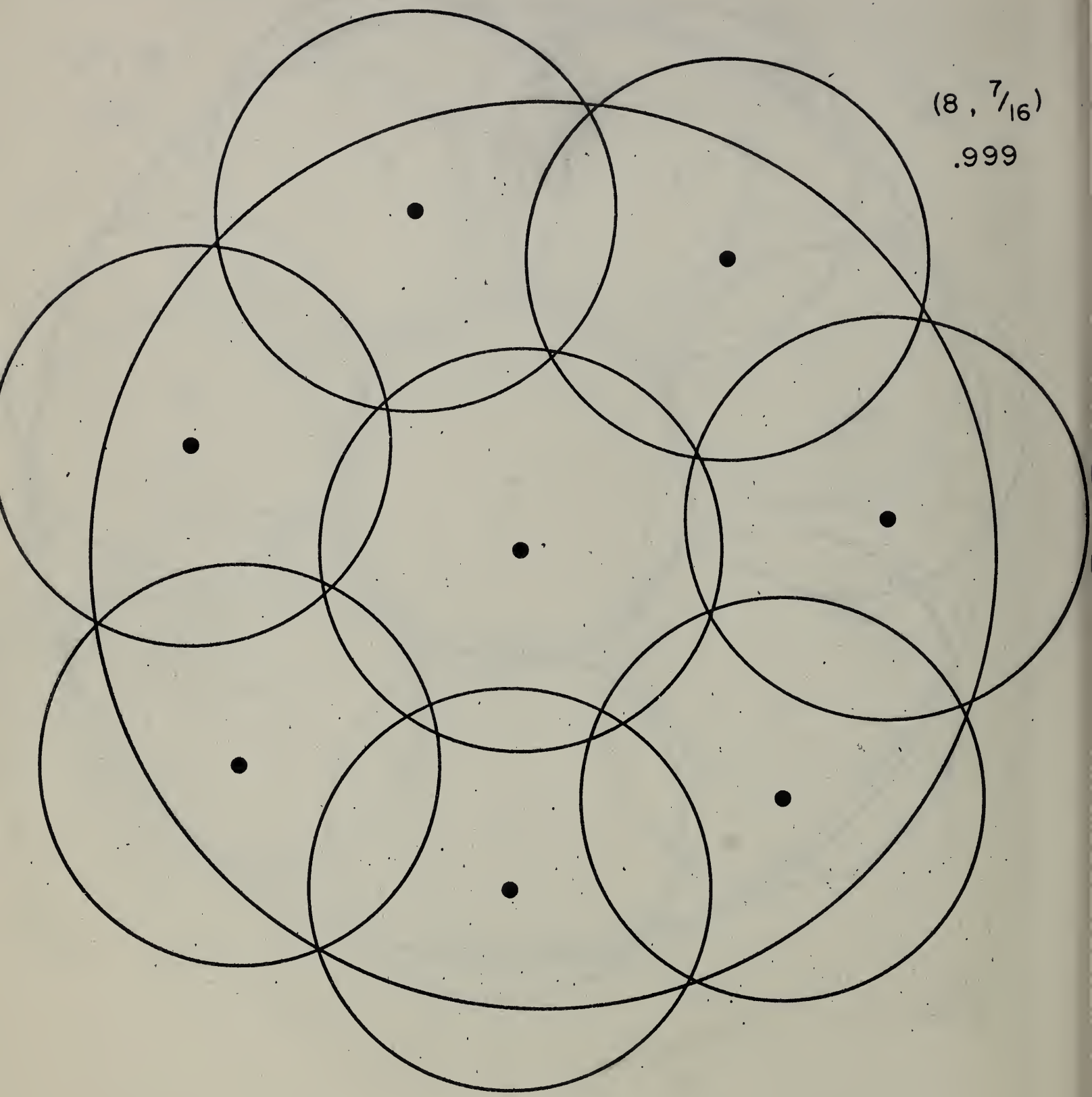

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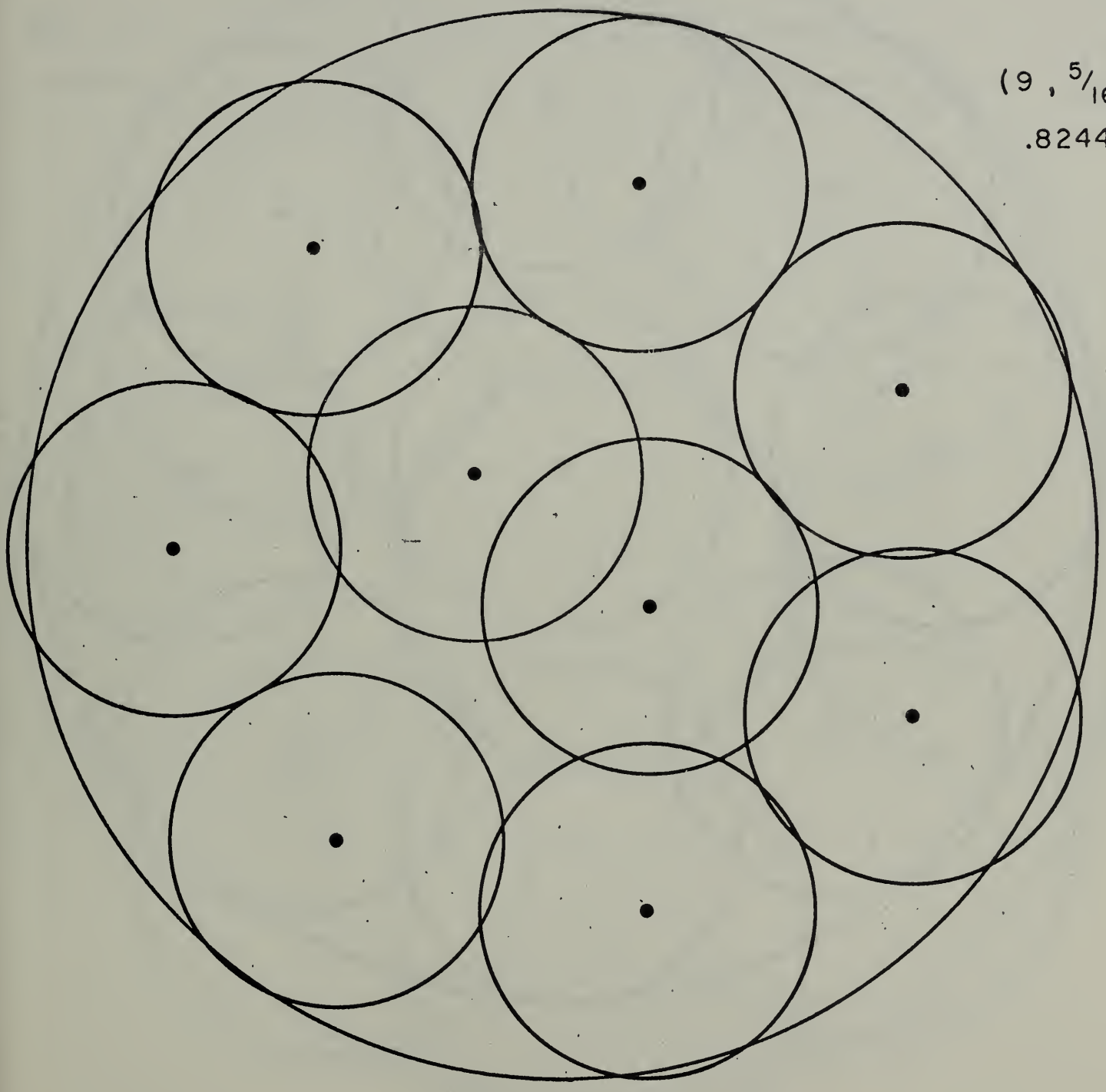

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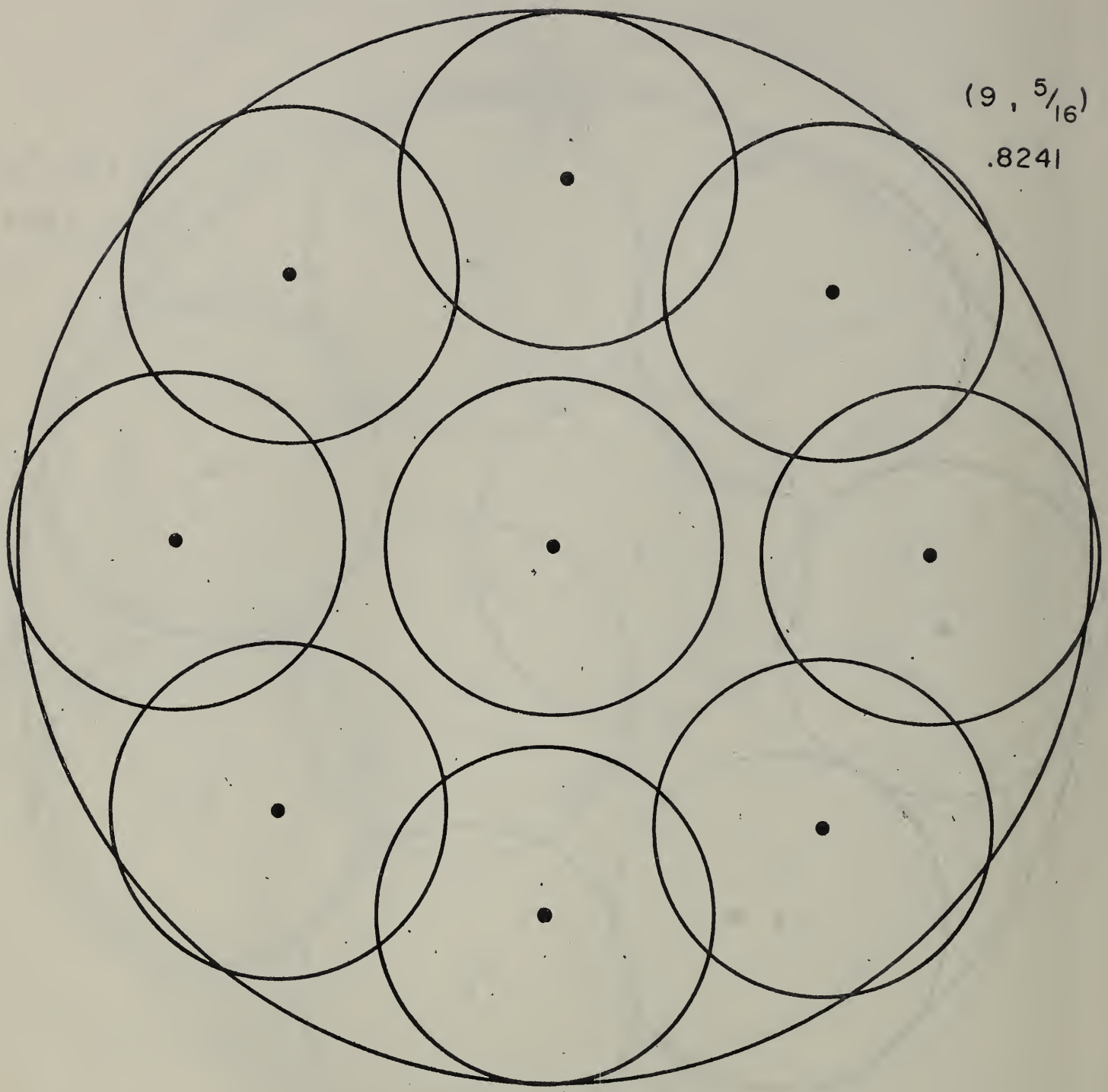

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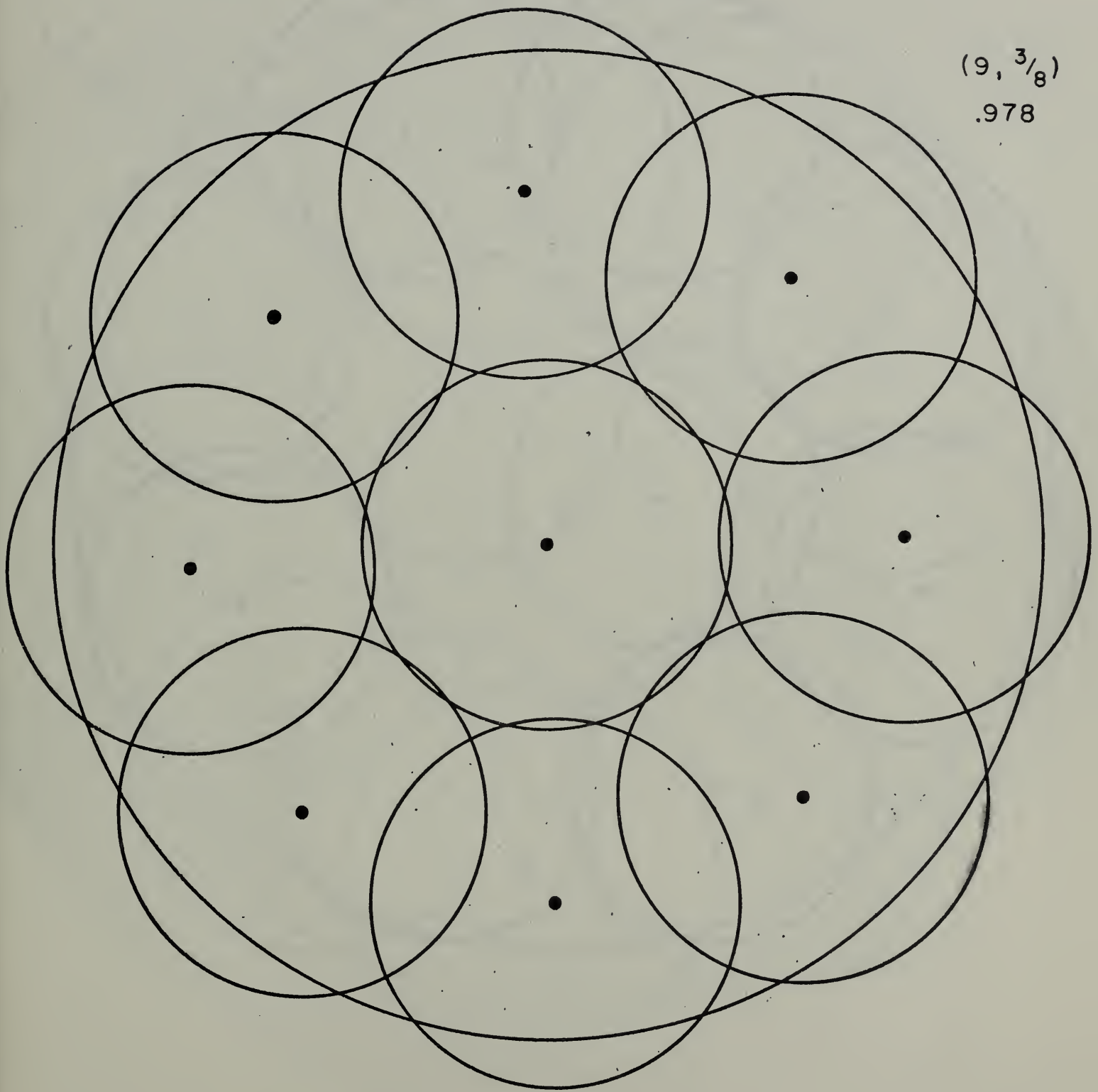

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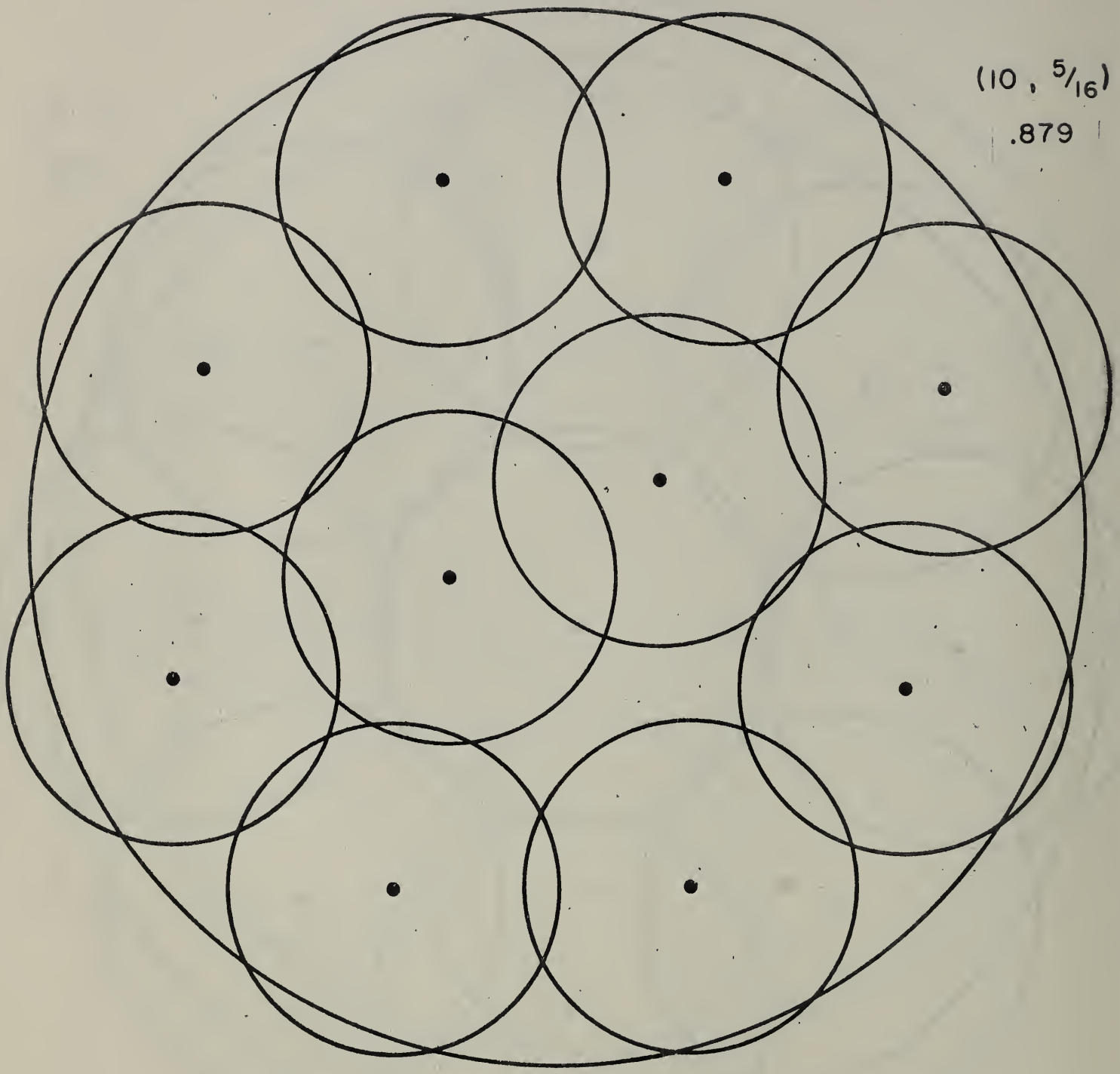

FIG. 42

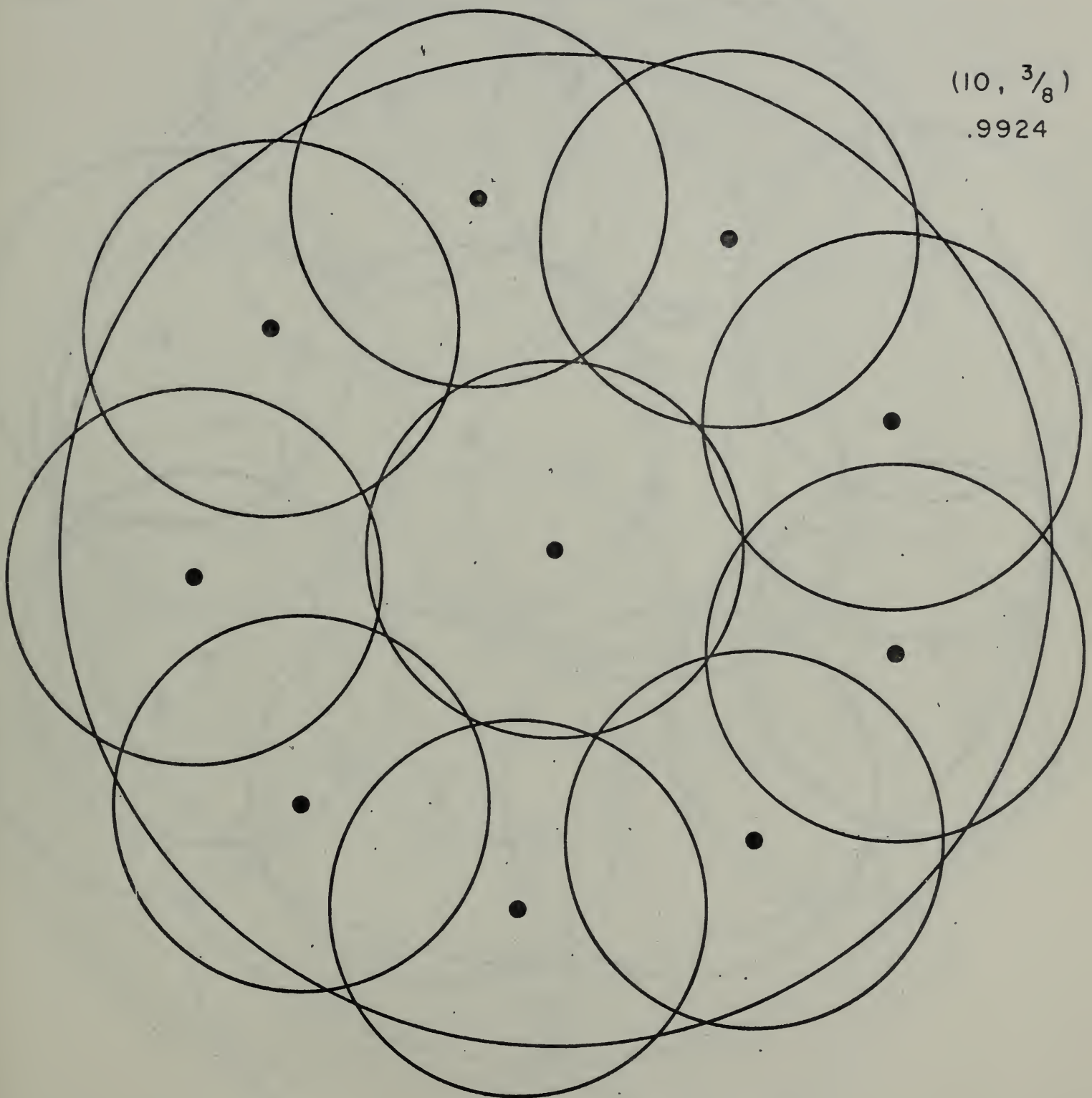

FIG. 43

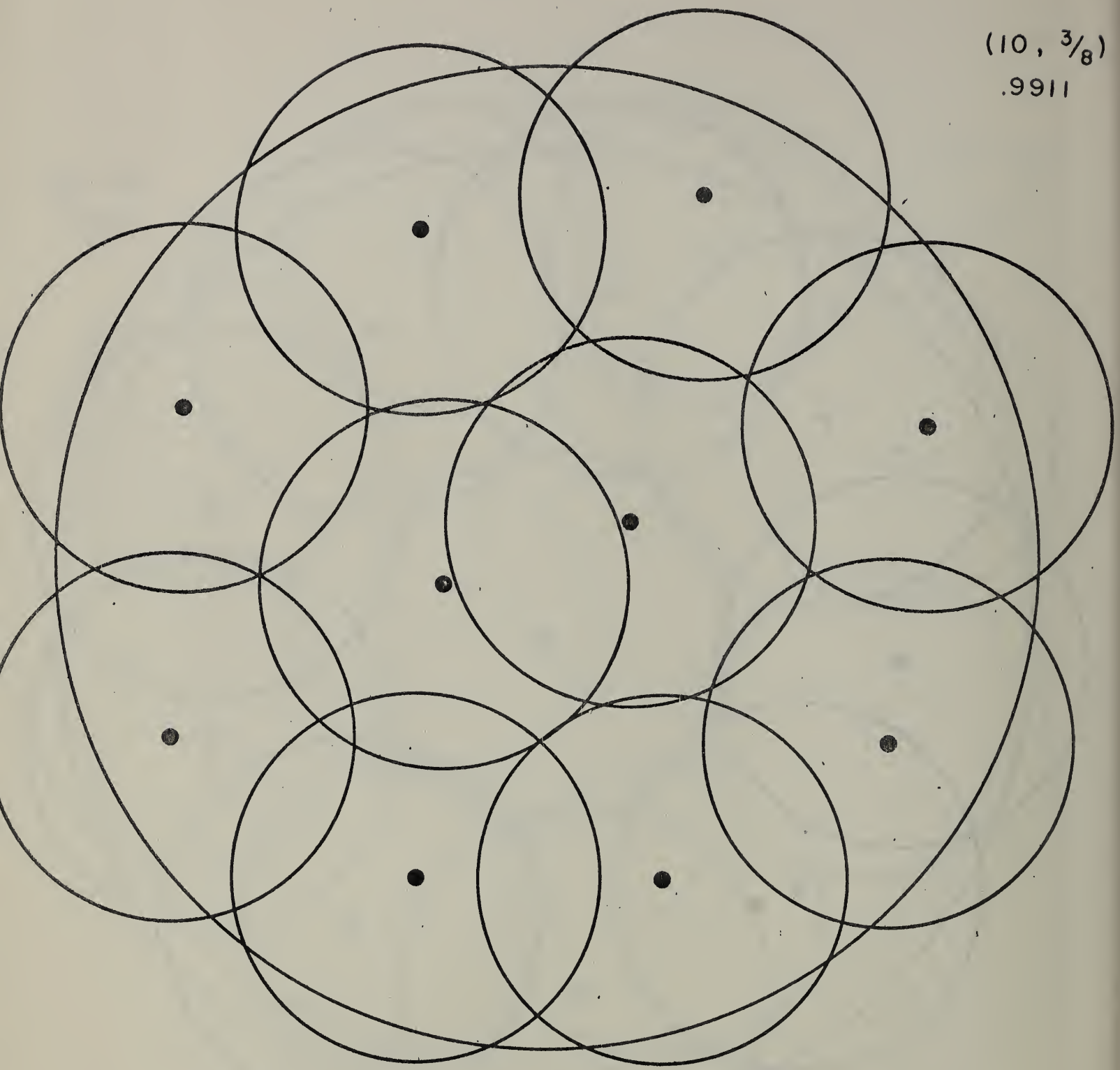

FIG. 44

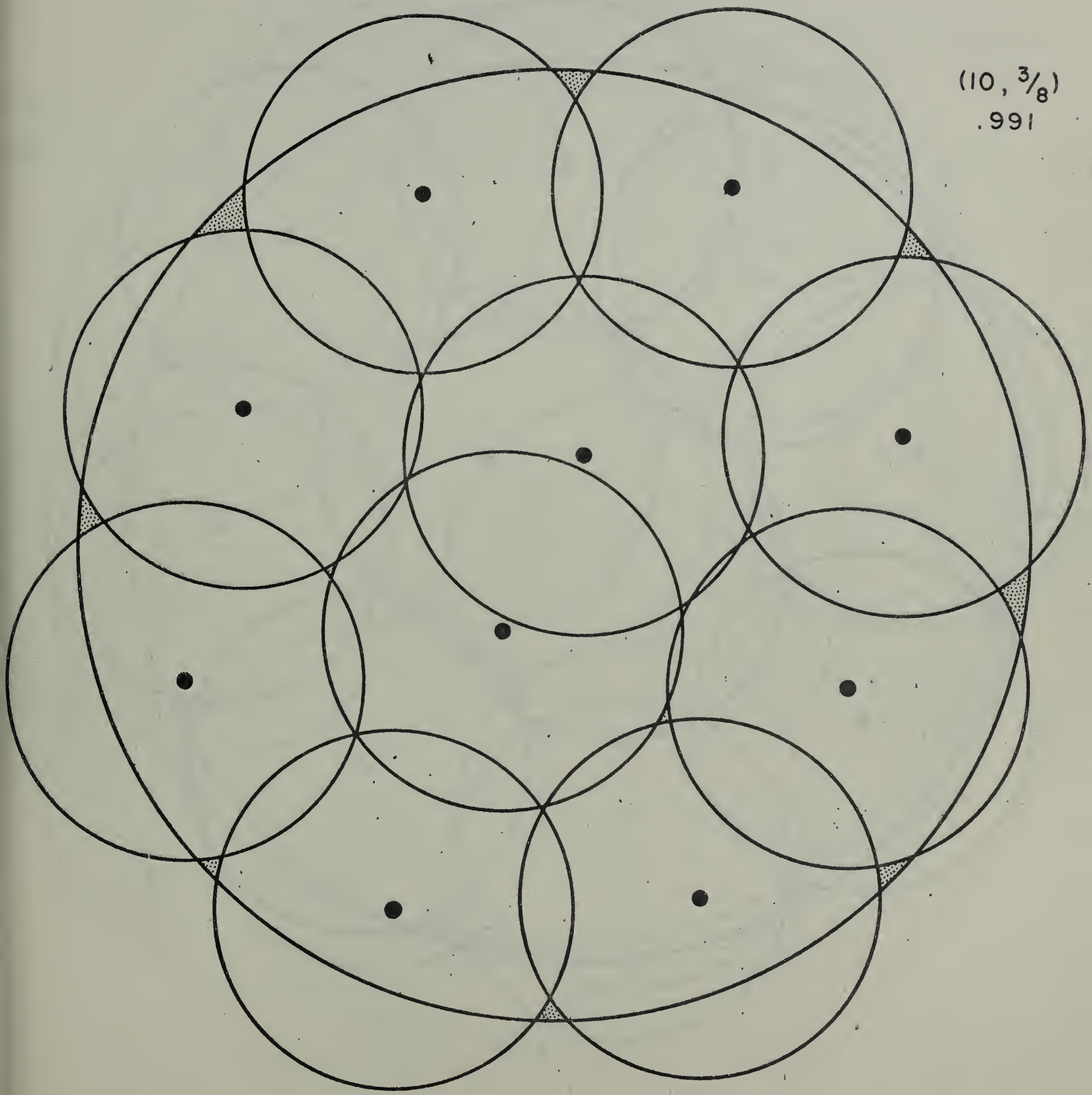

FIG. 45

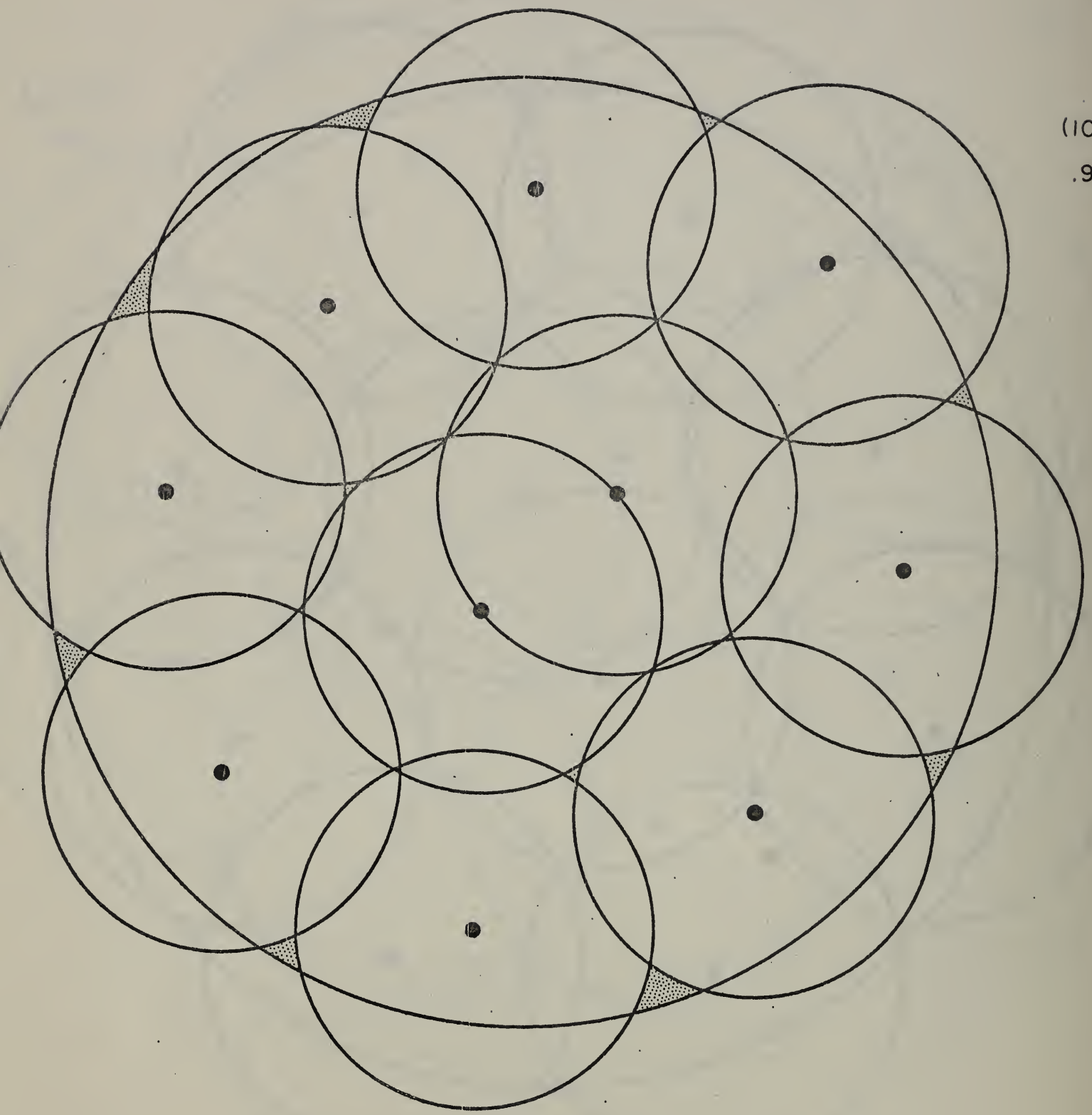

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FIG. 46

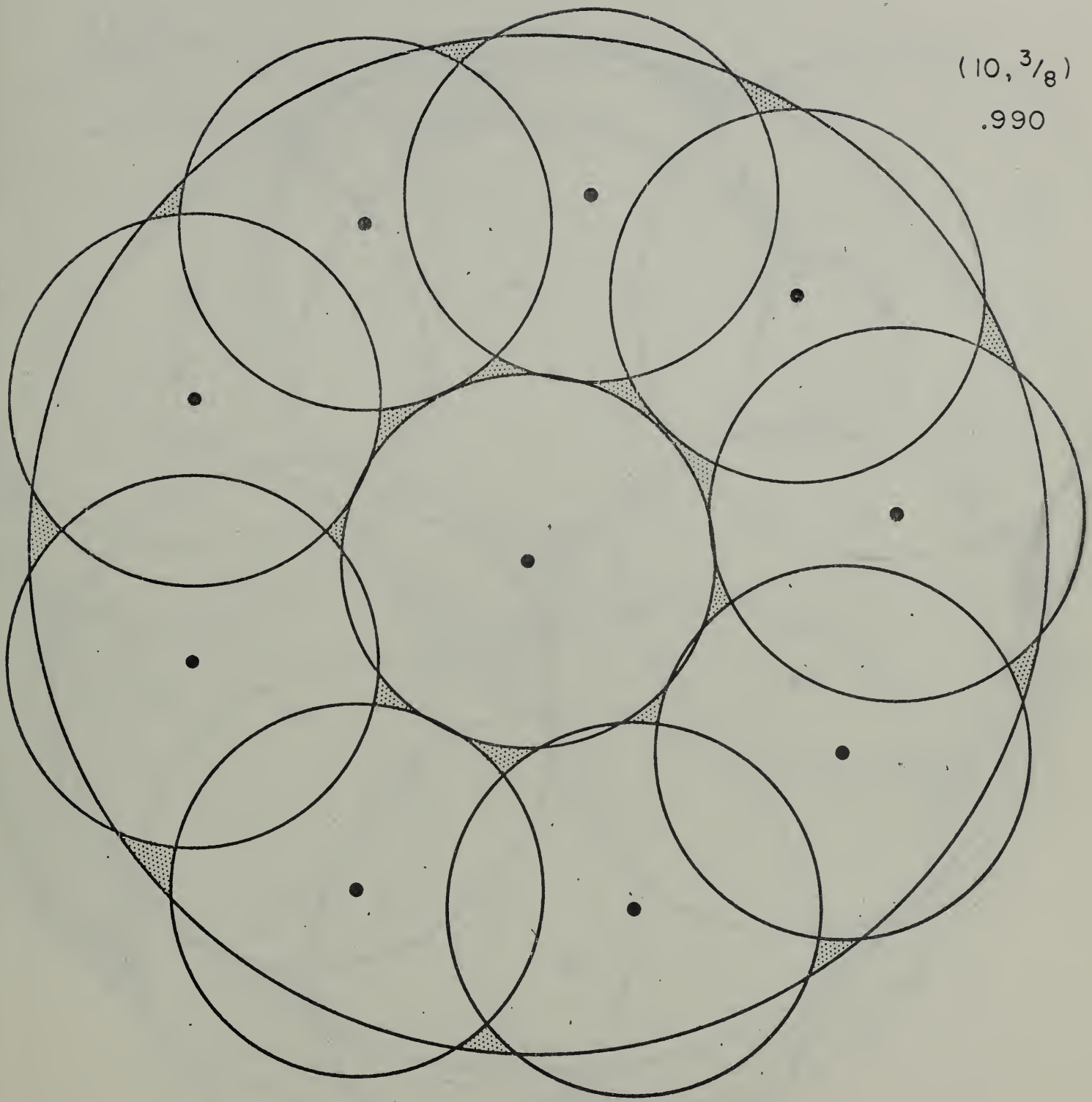


FIG. 47

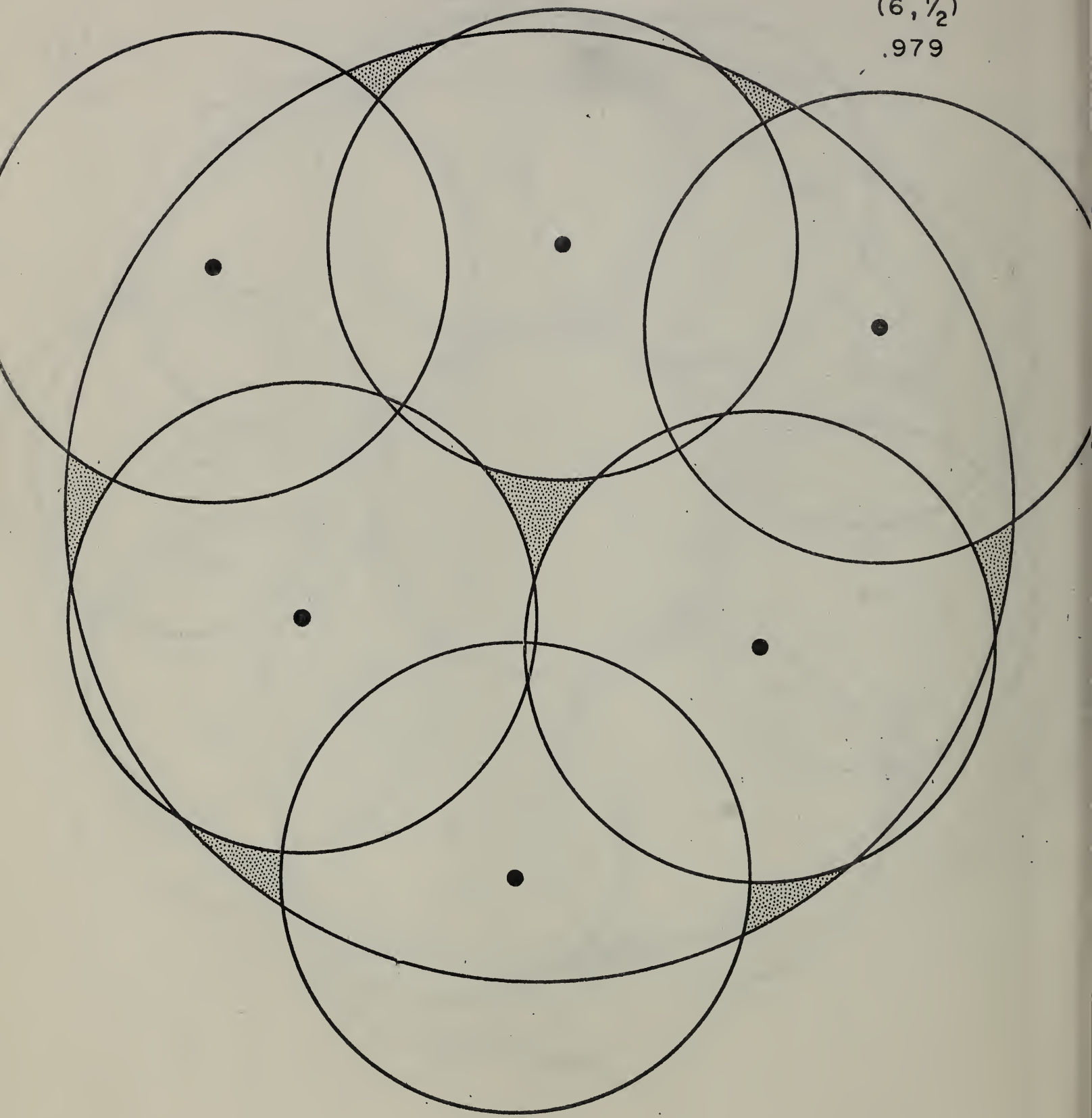


FIG. 48

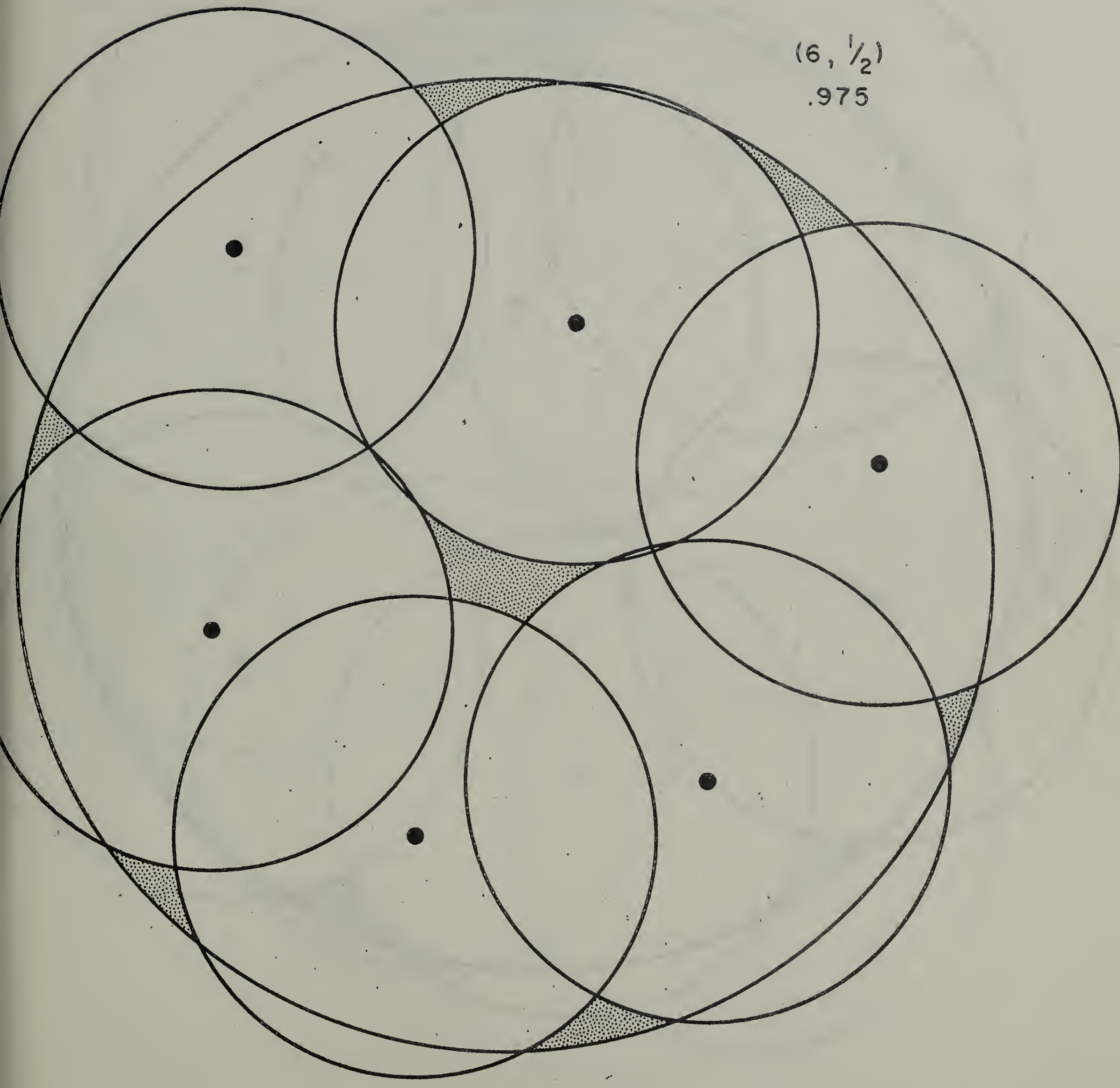


FIG. 49

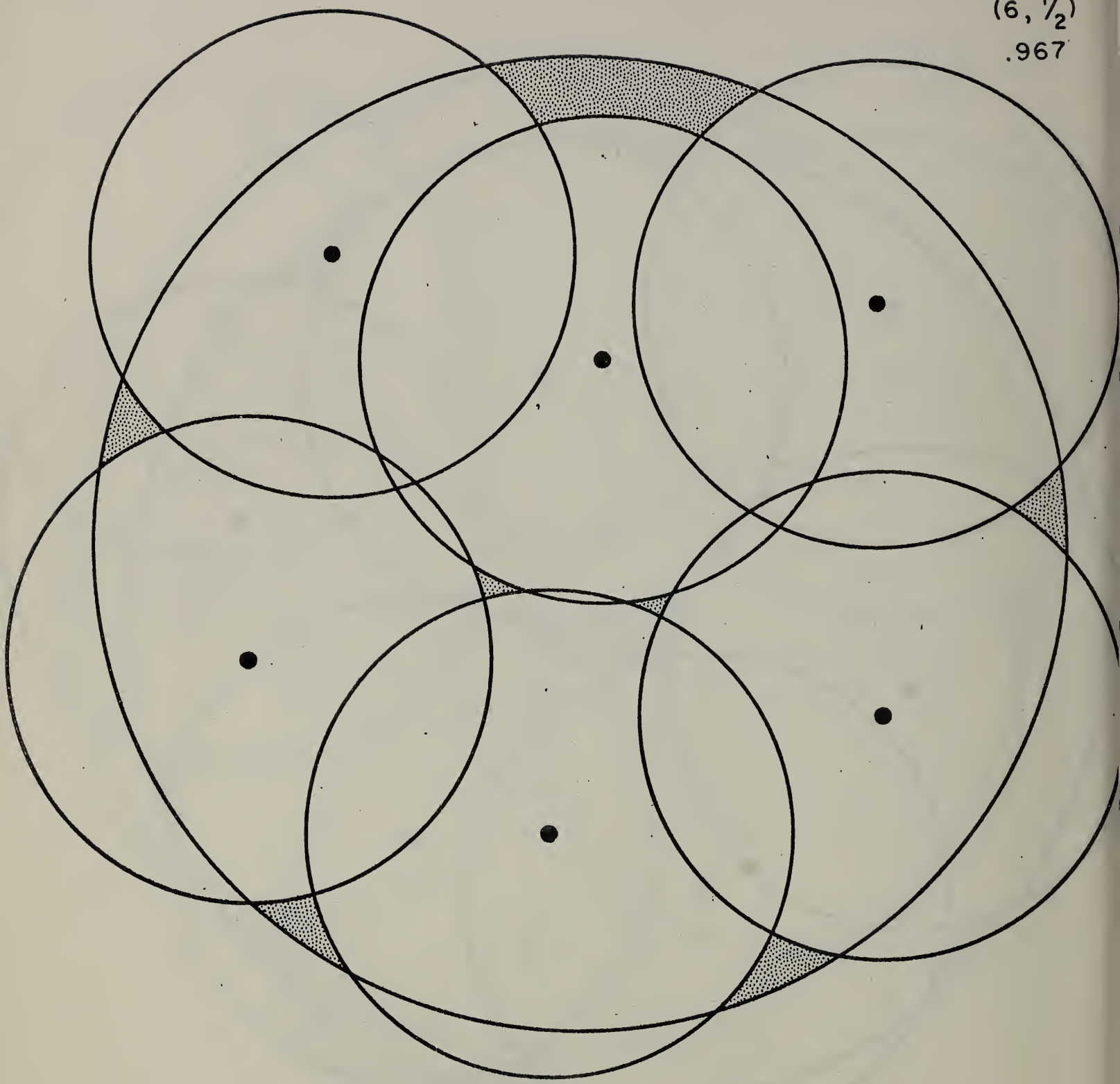


FIG.50

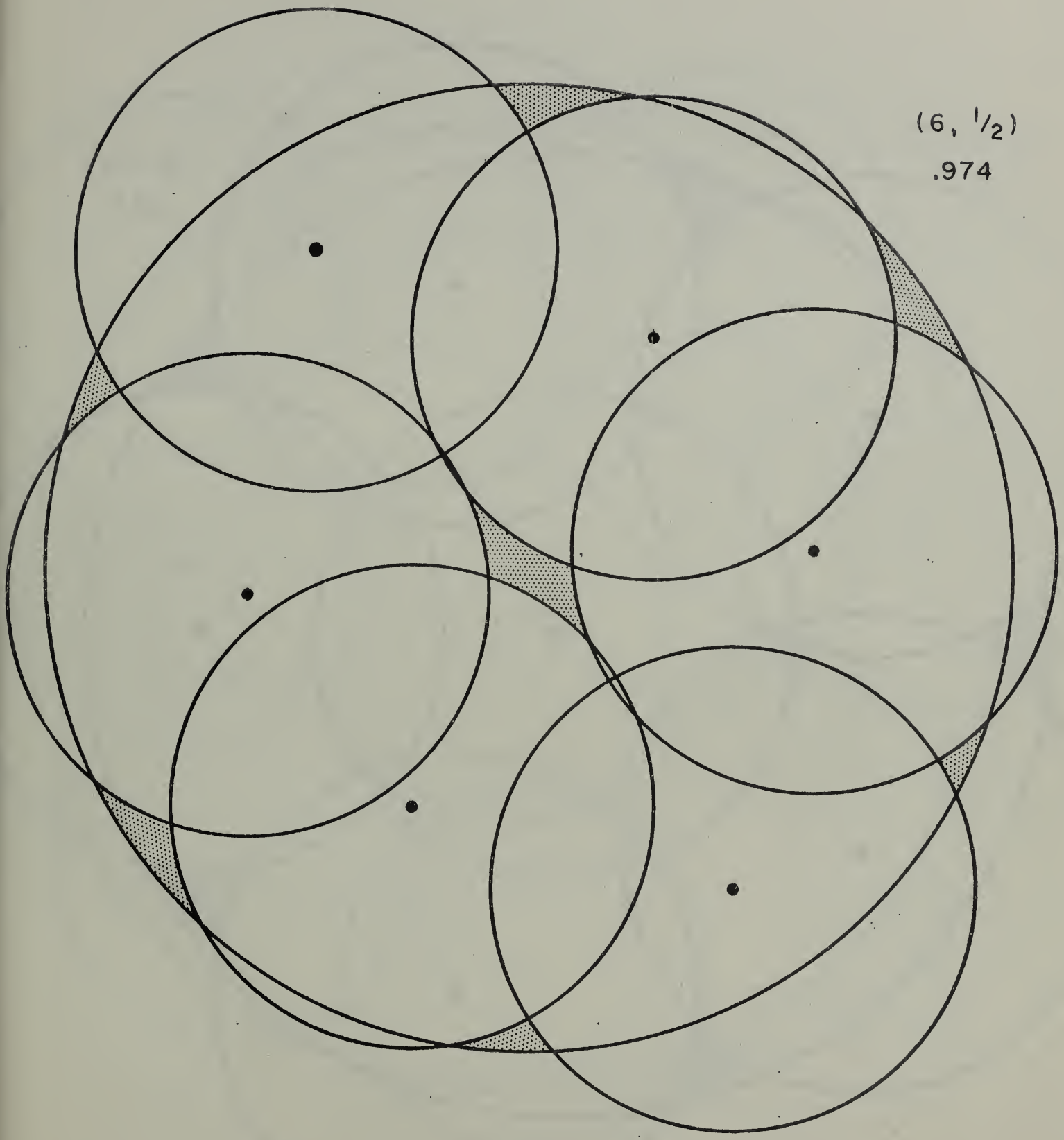


FIG. 51

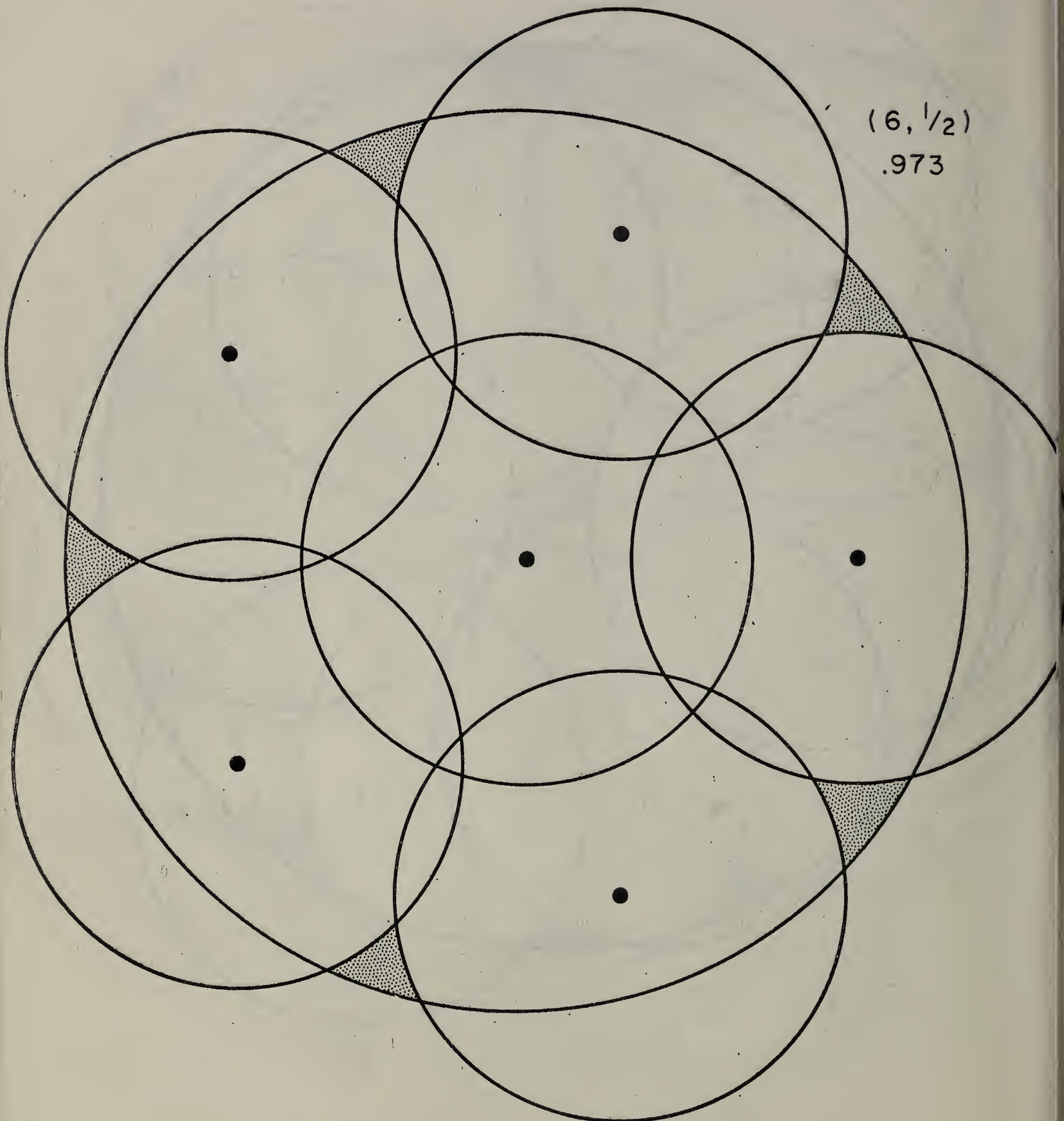


FIG. 52

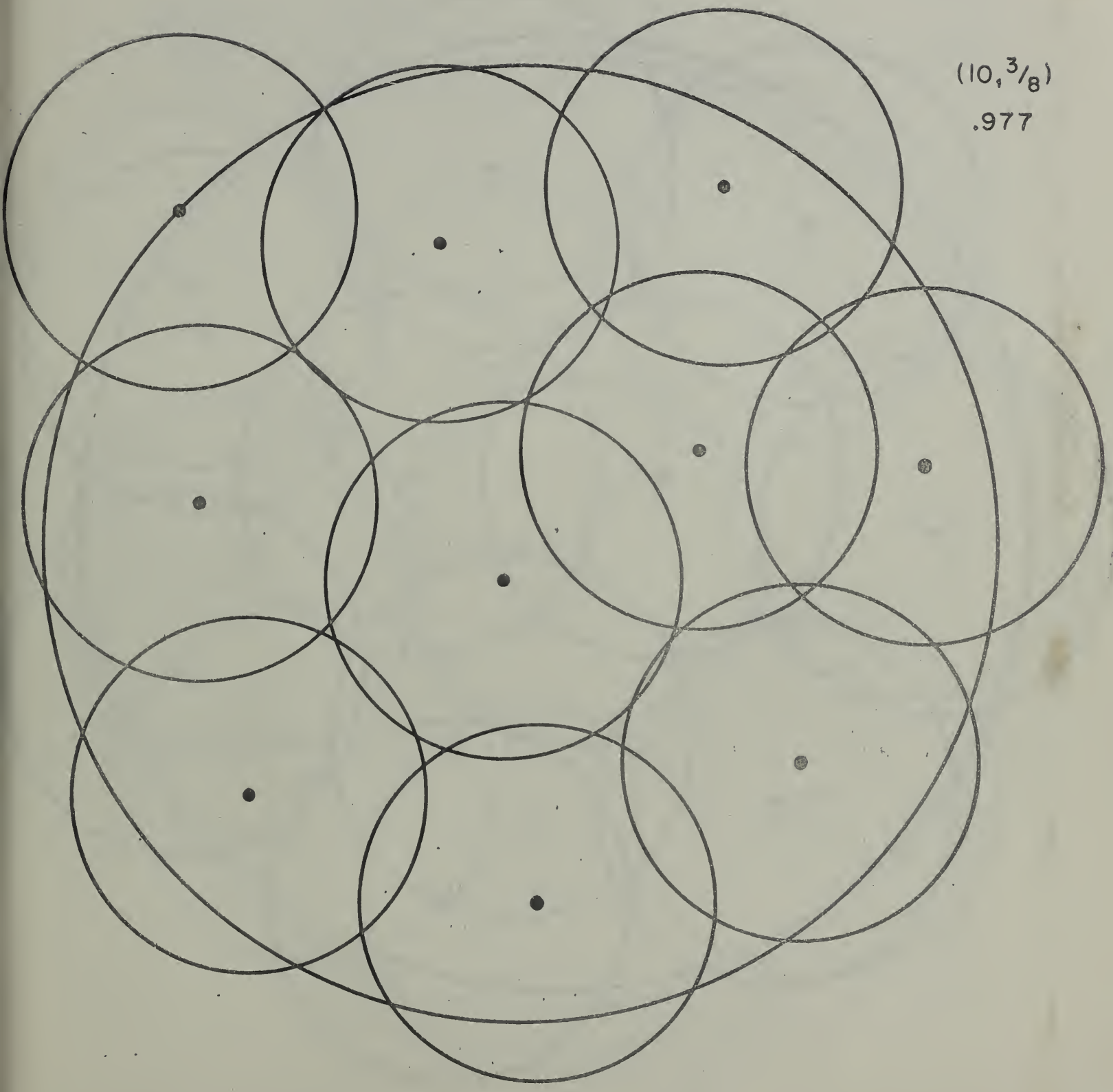


FIG. 53

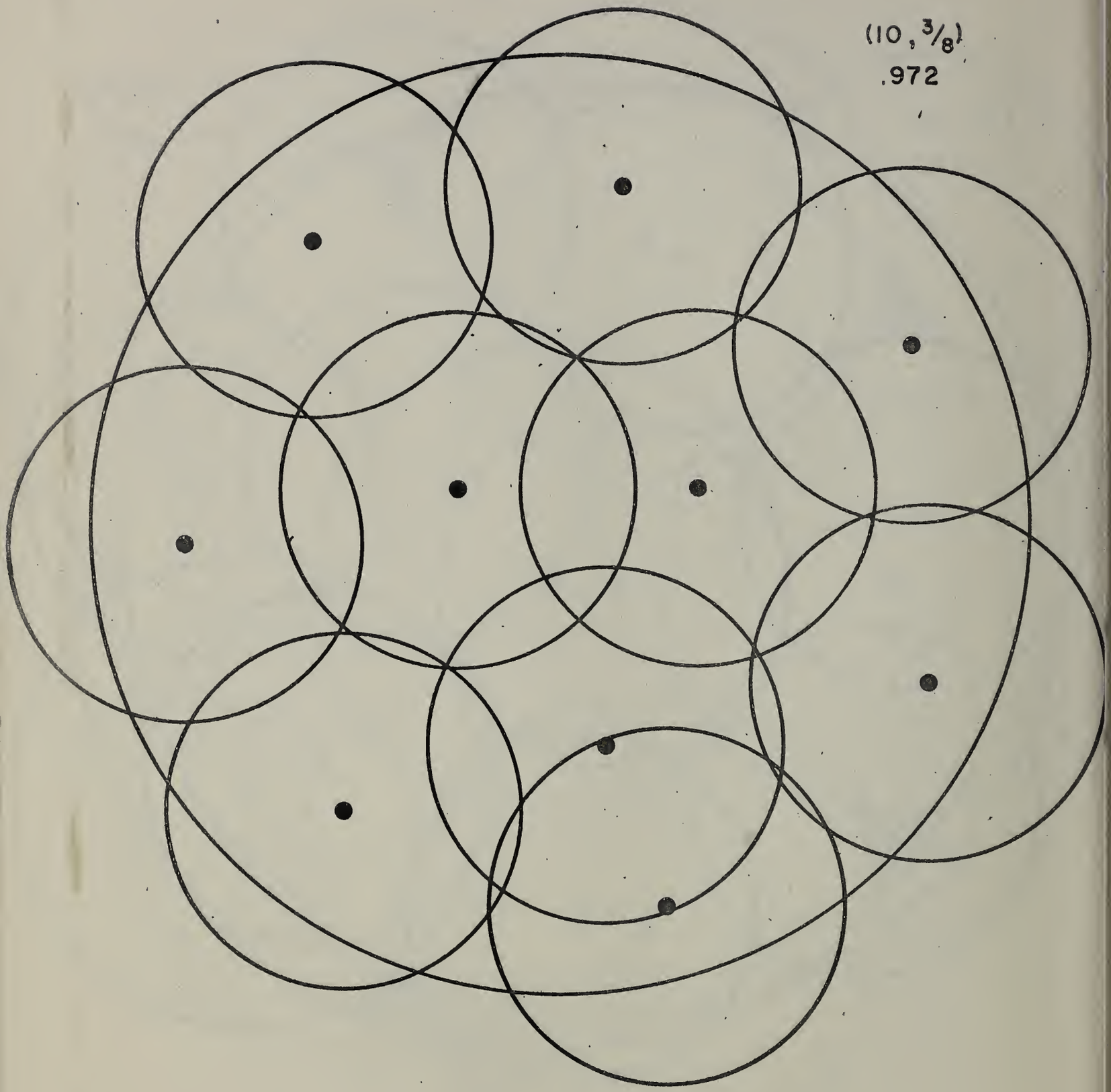


FIG. 54

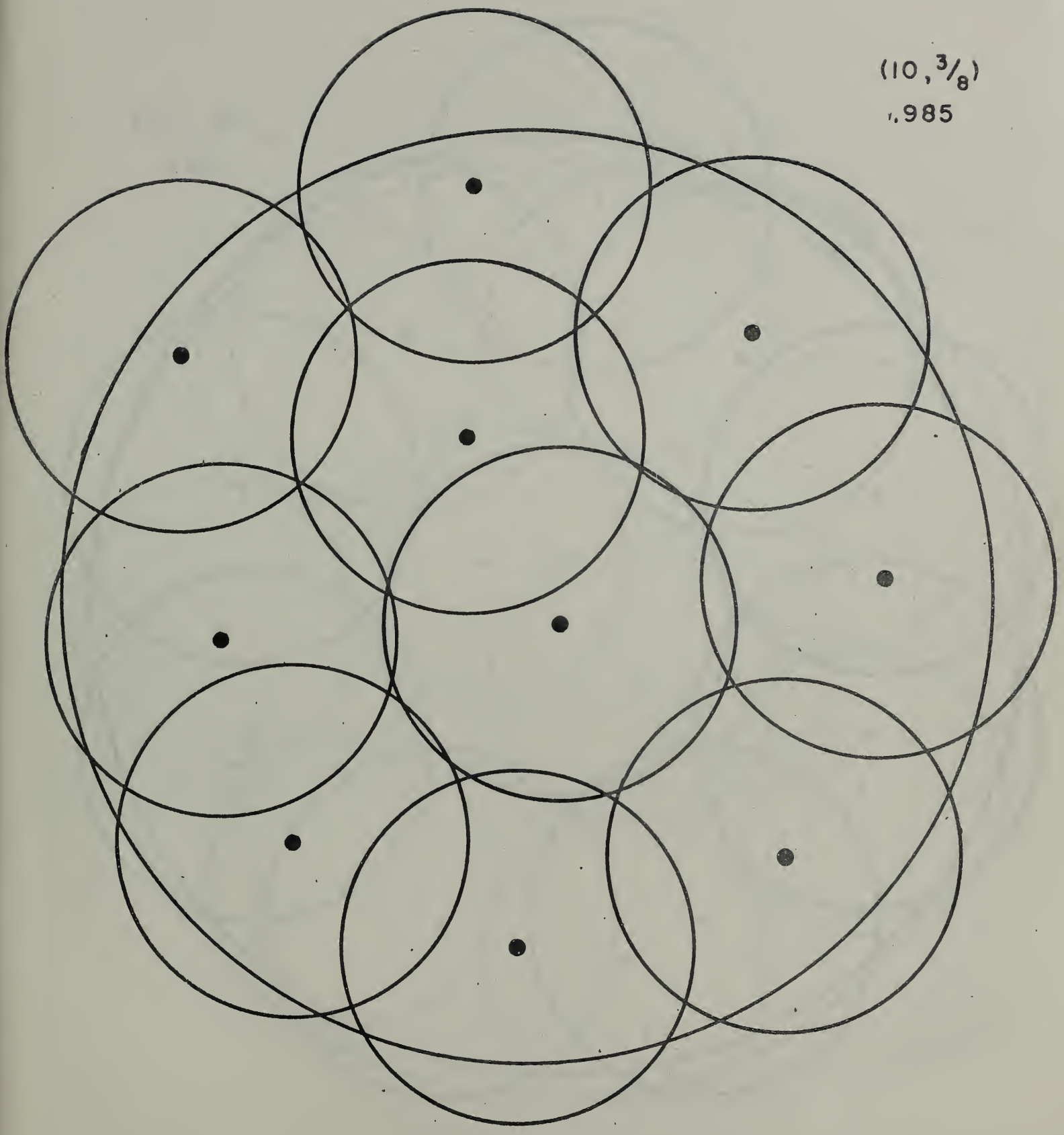


FIG. 55

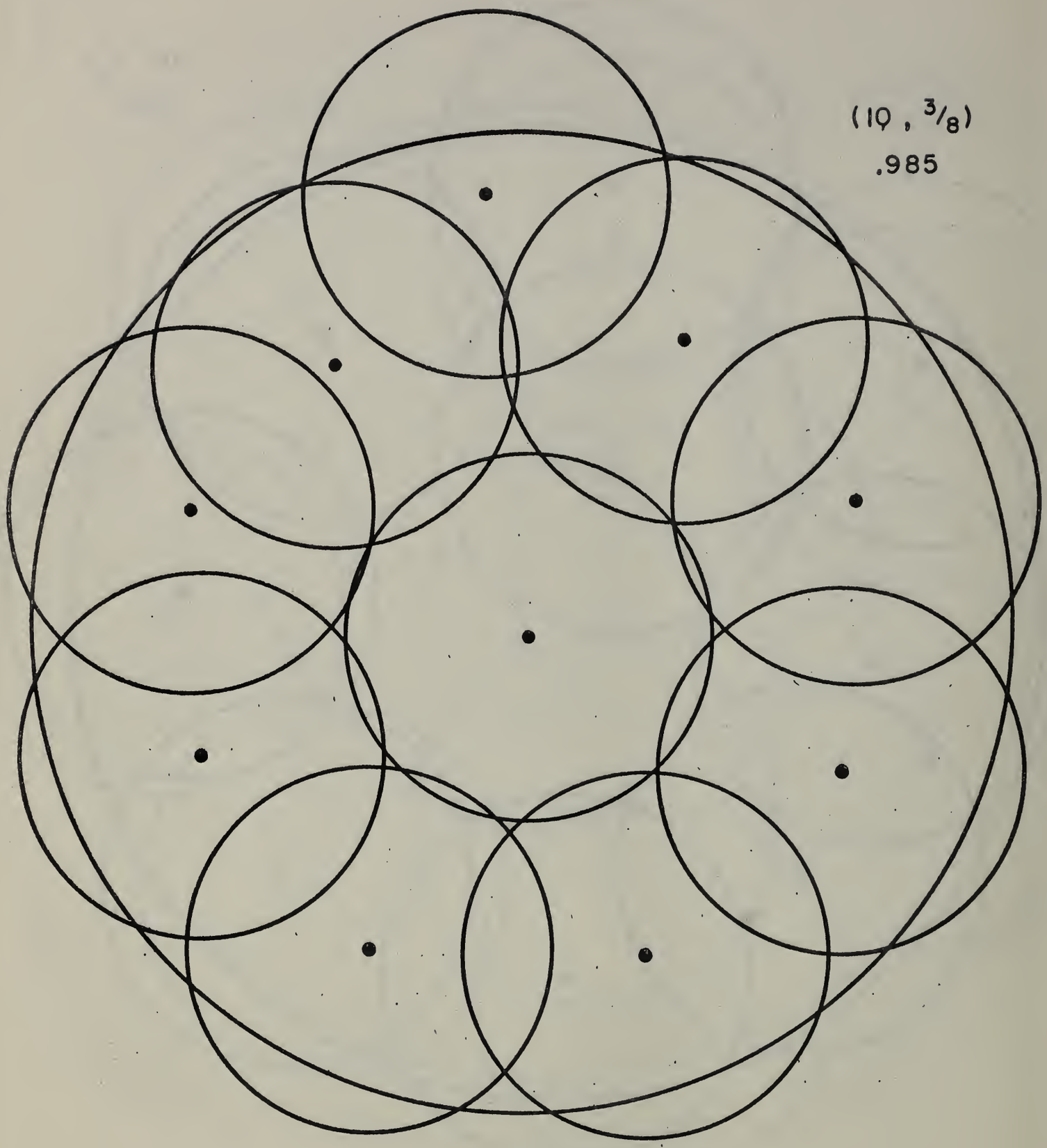


FIG. 56

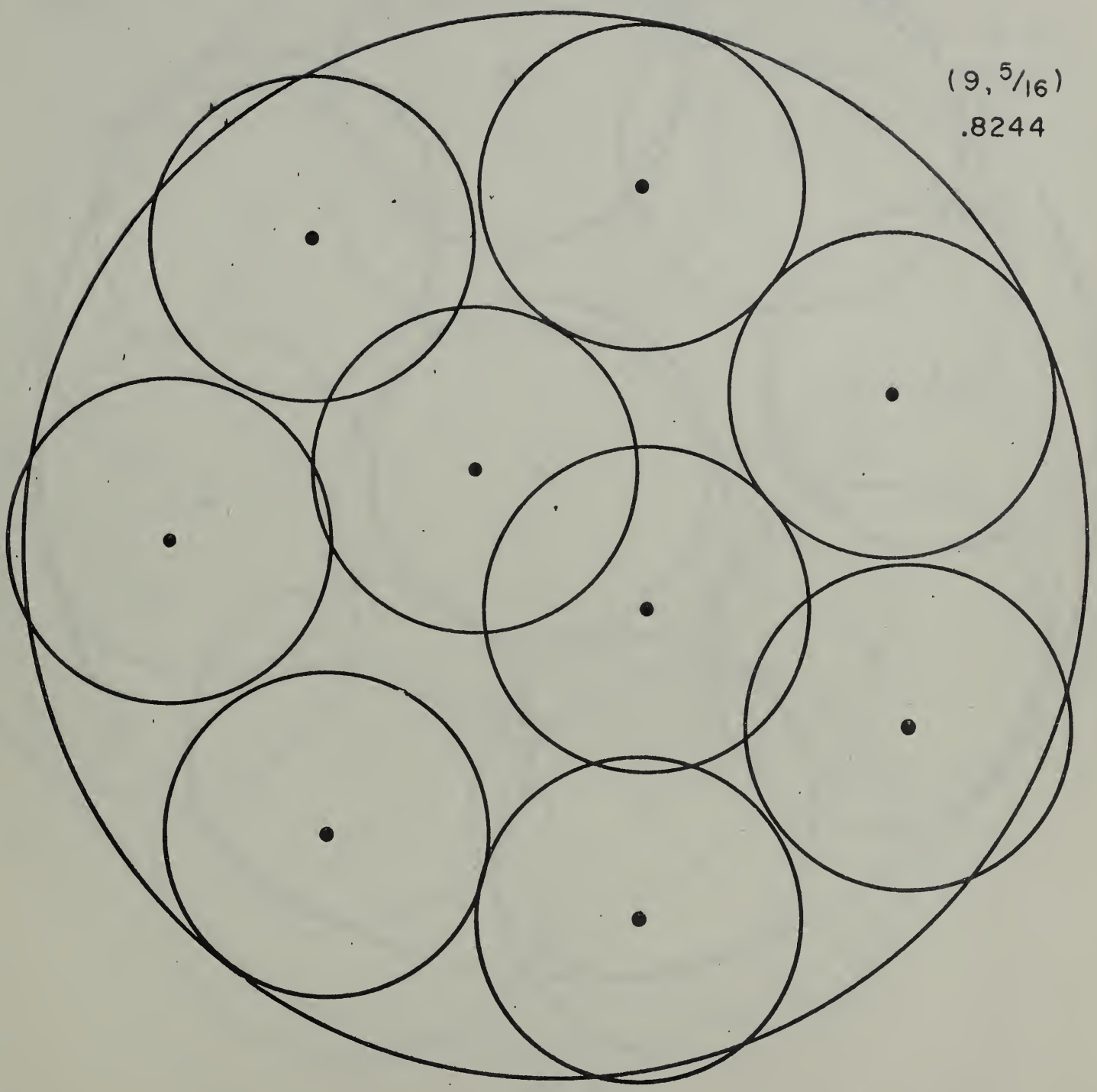


FIG. 57

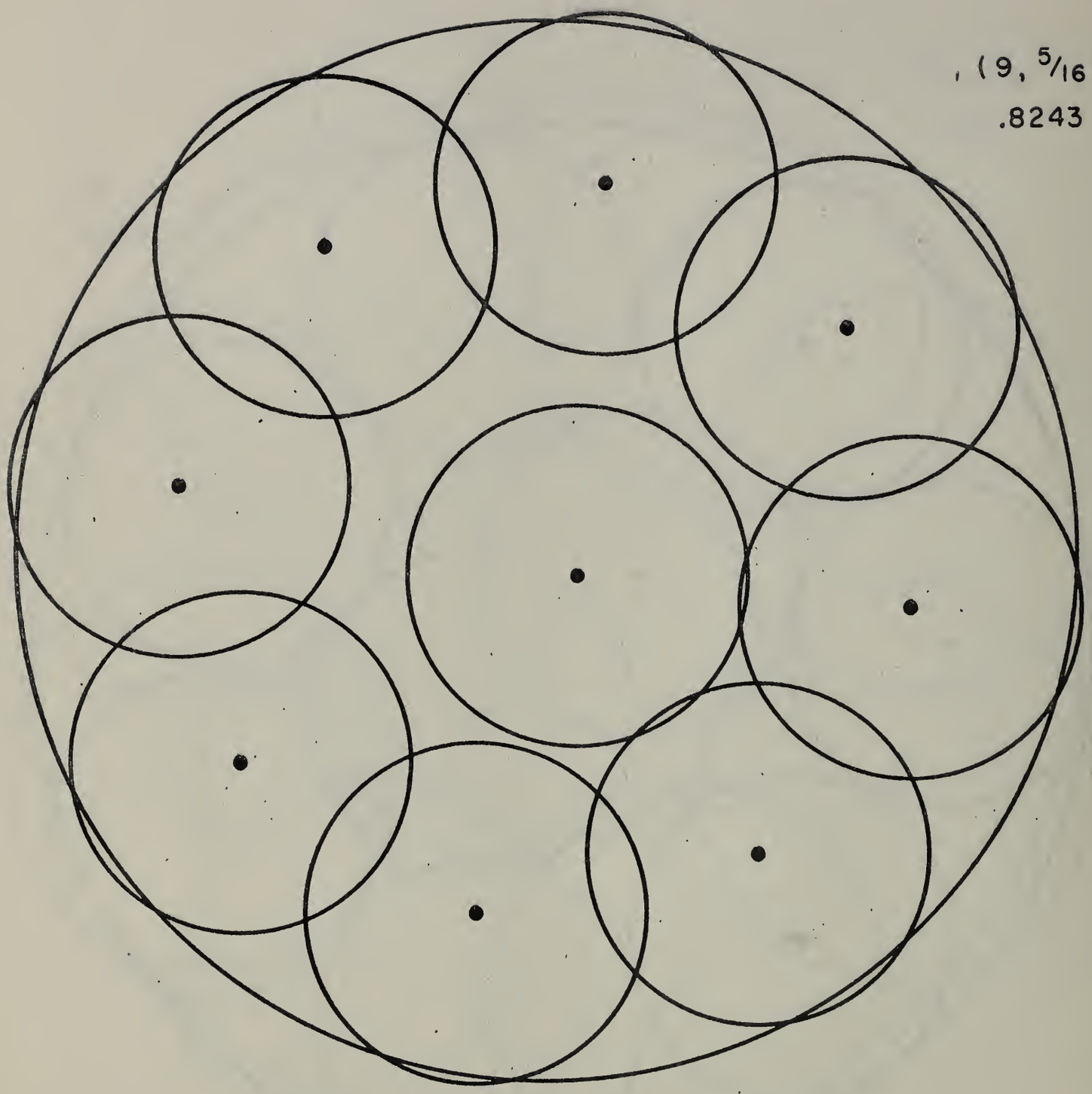


FIG. 58

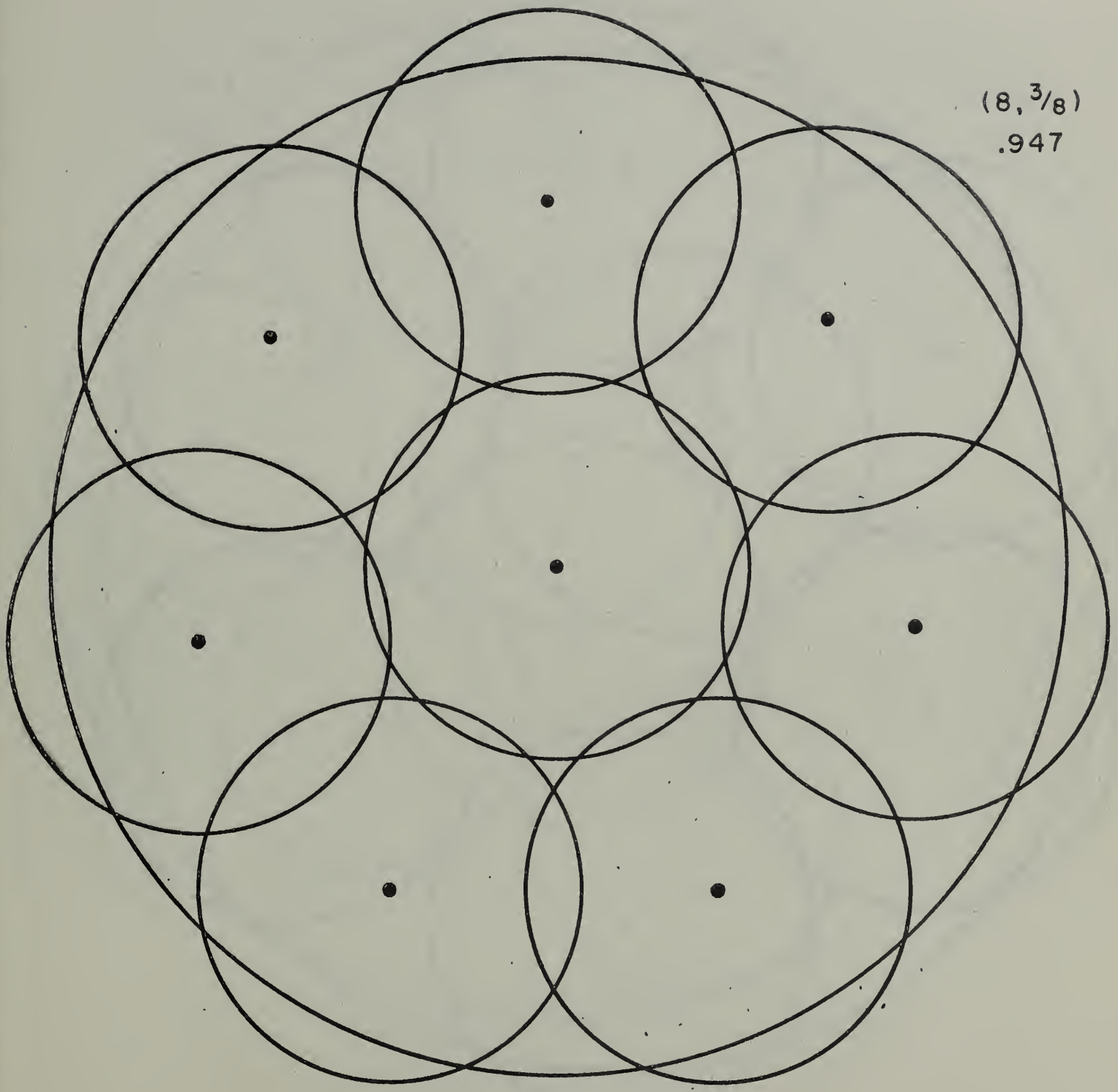


FIG. 59

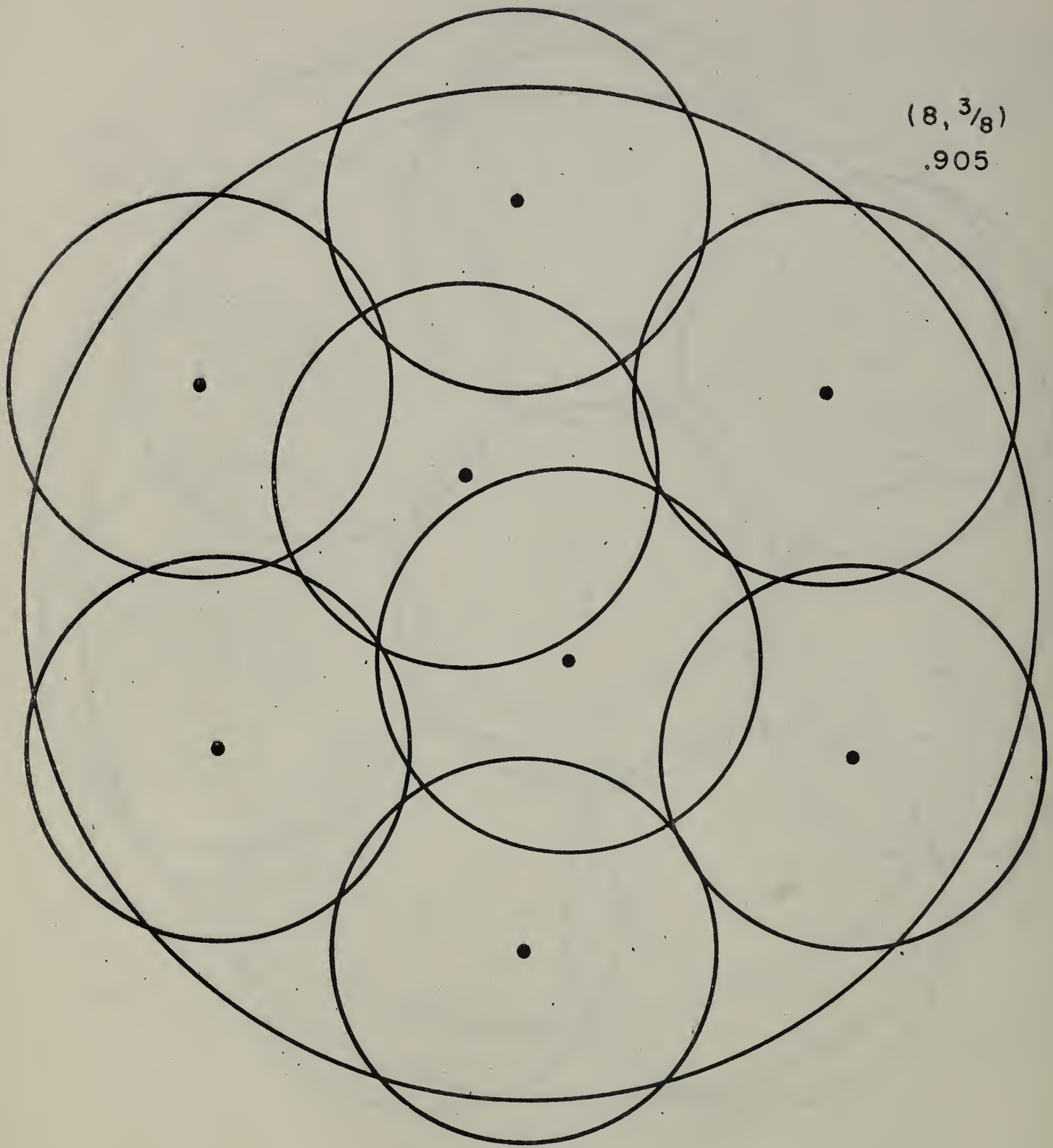


FIG. 60

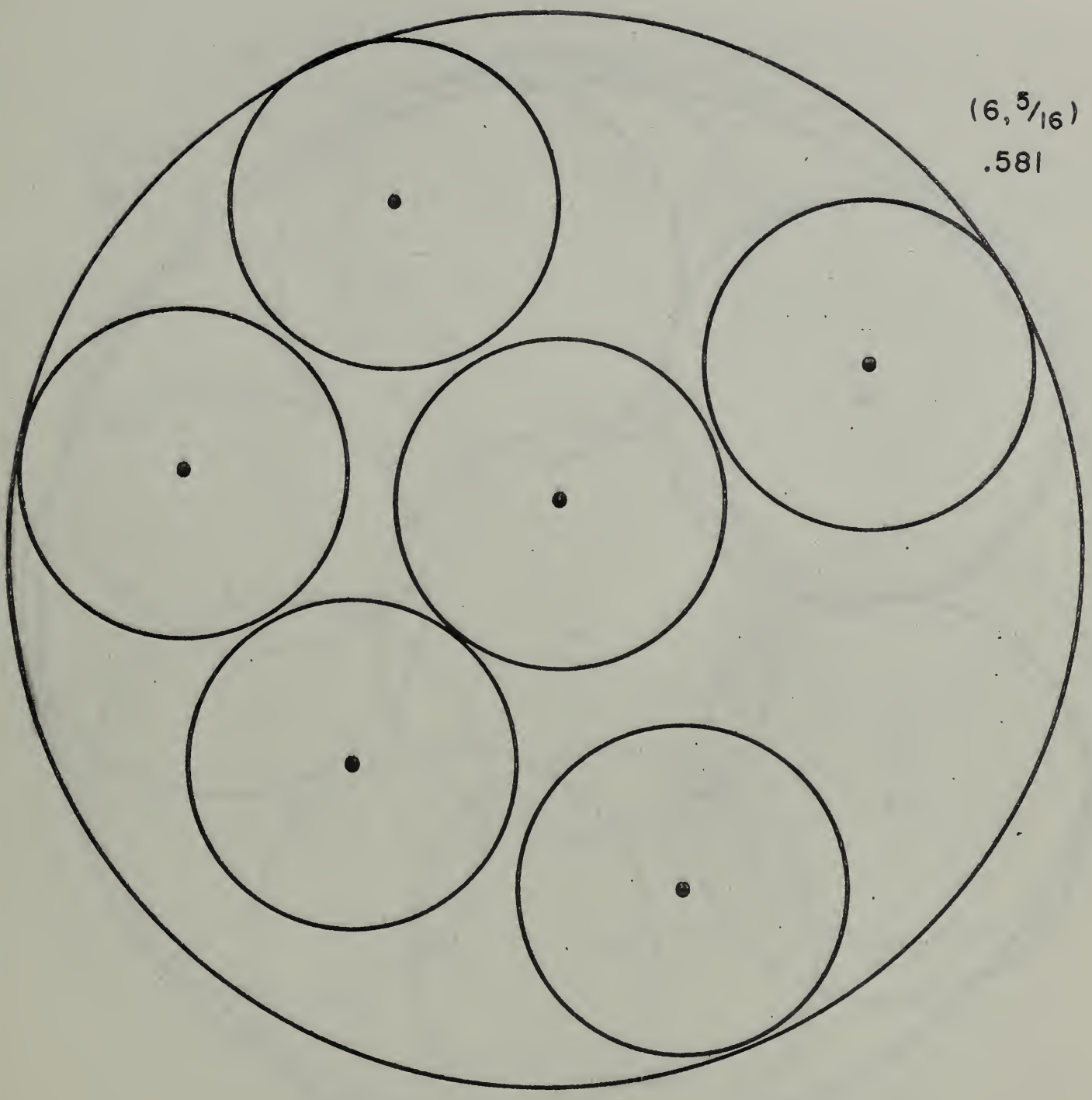


FIG. 61

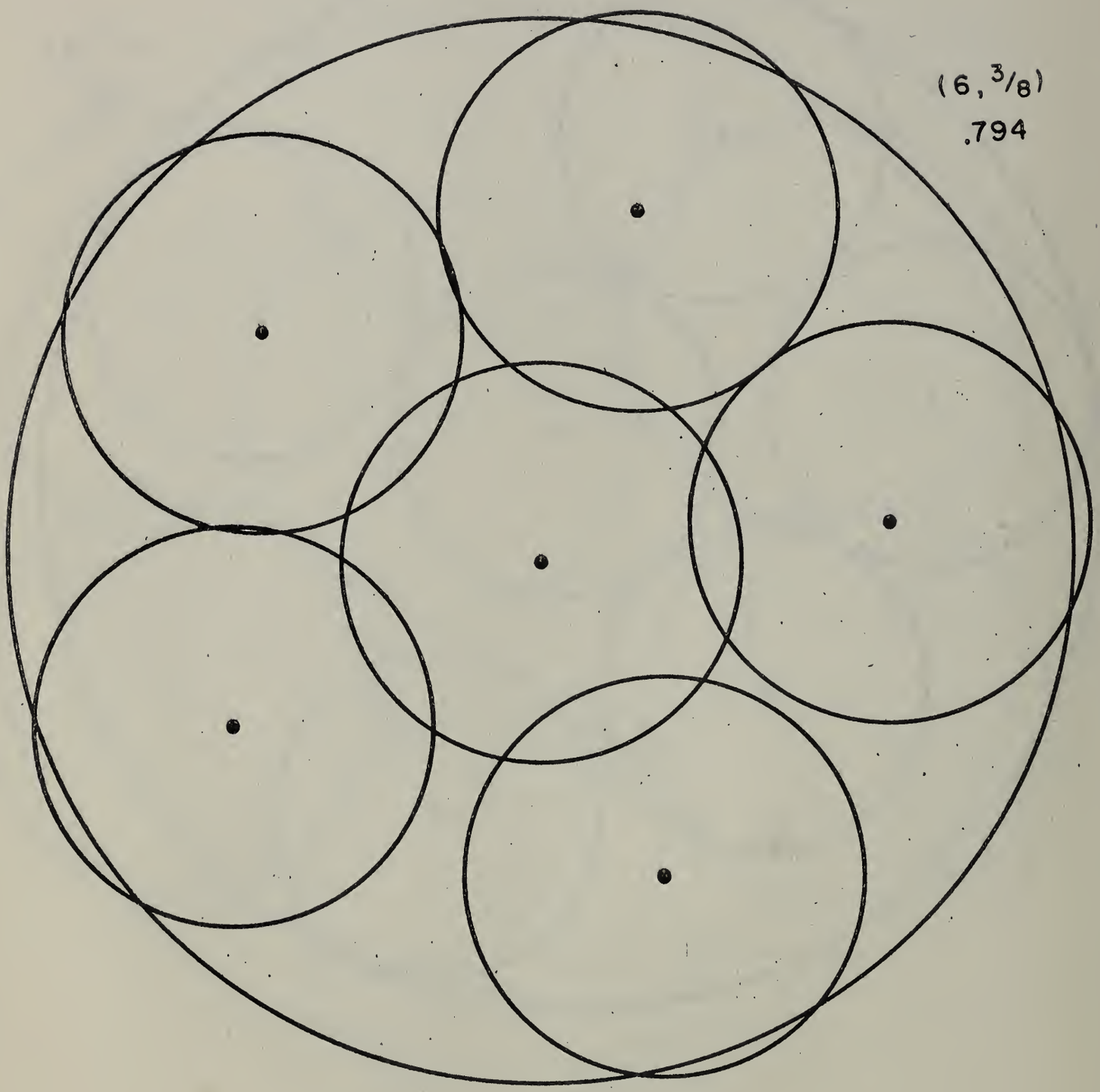
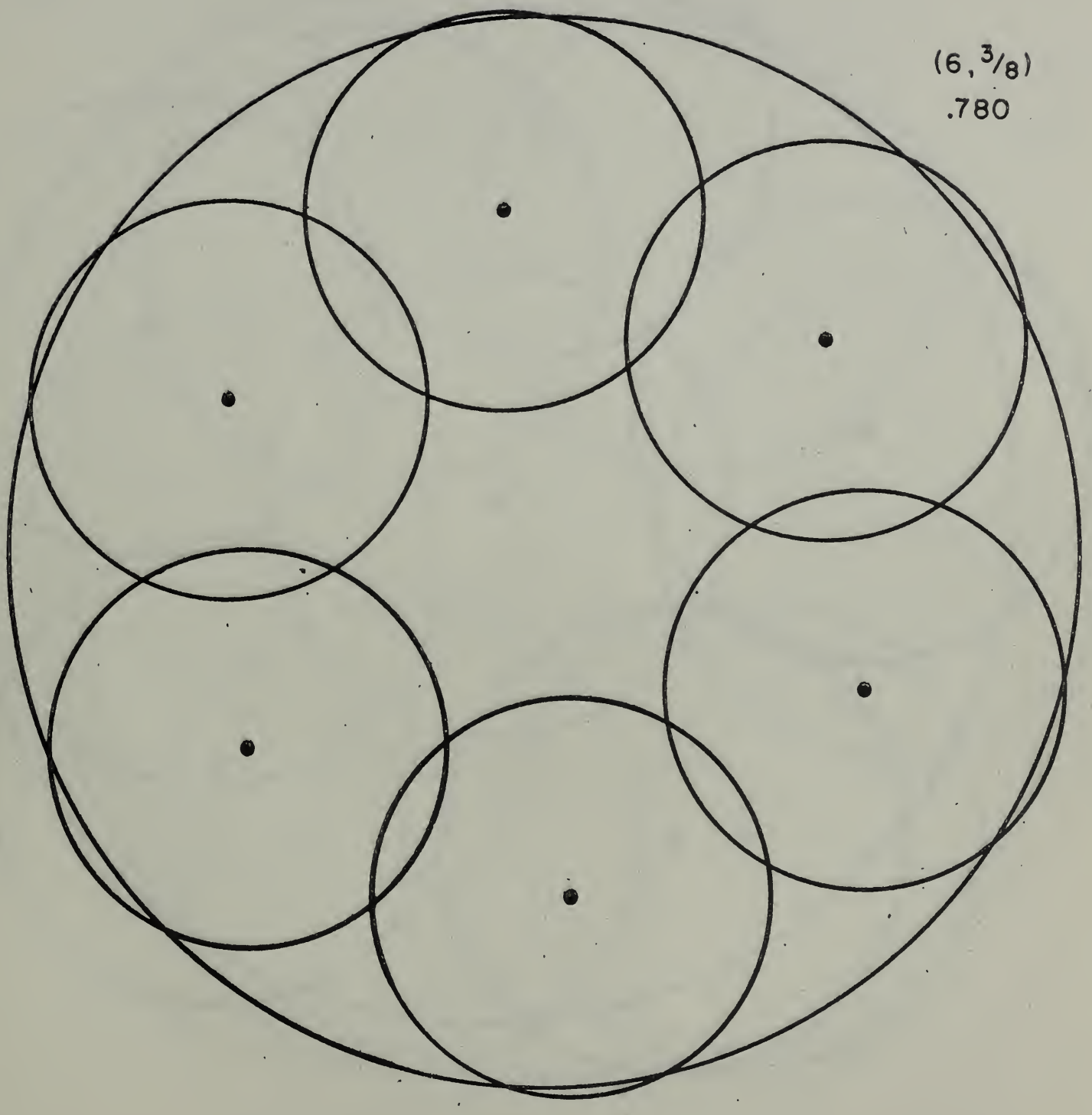


FIG. 62



FIG, 63

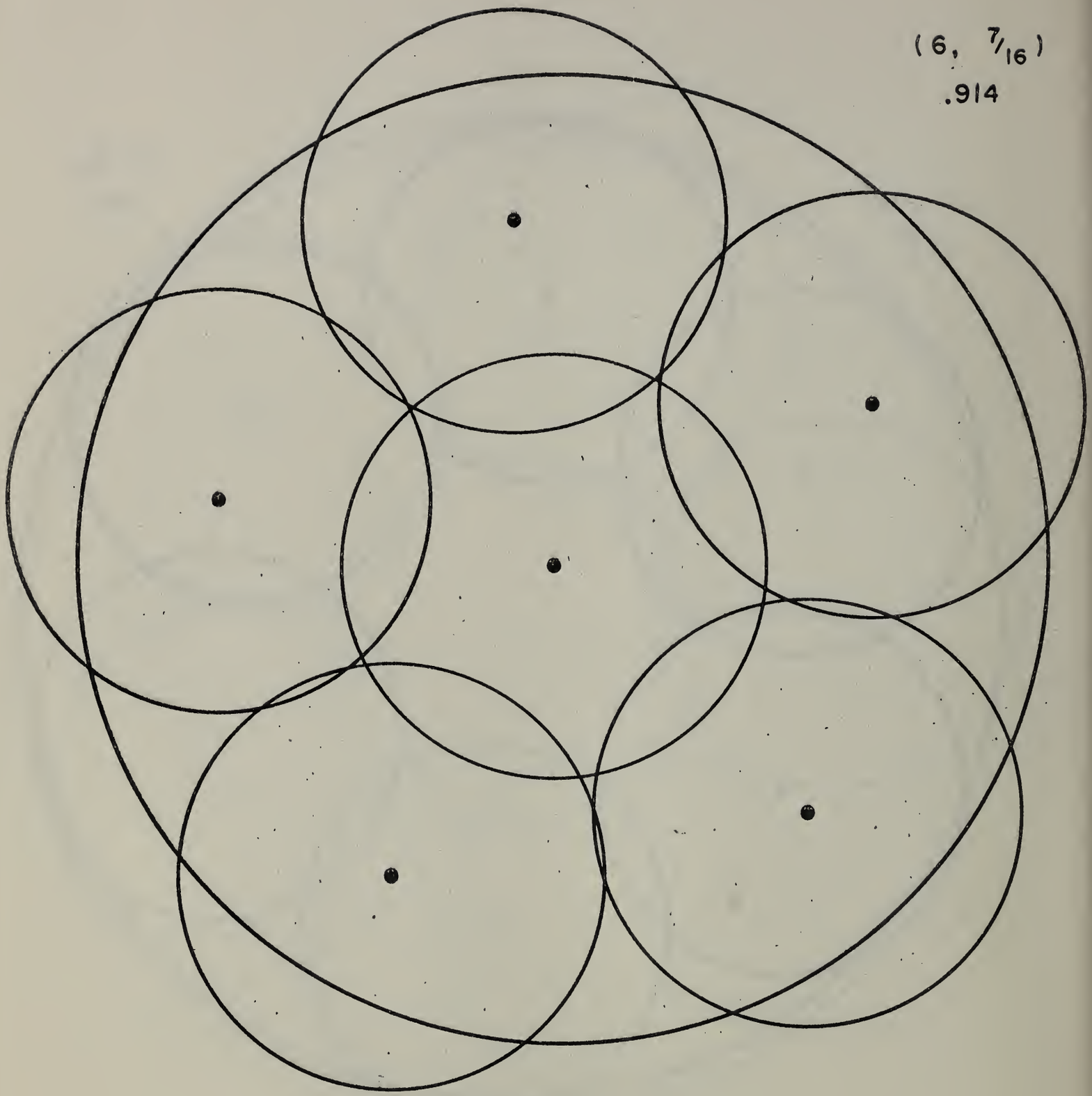


FIG. 64

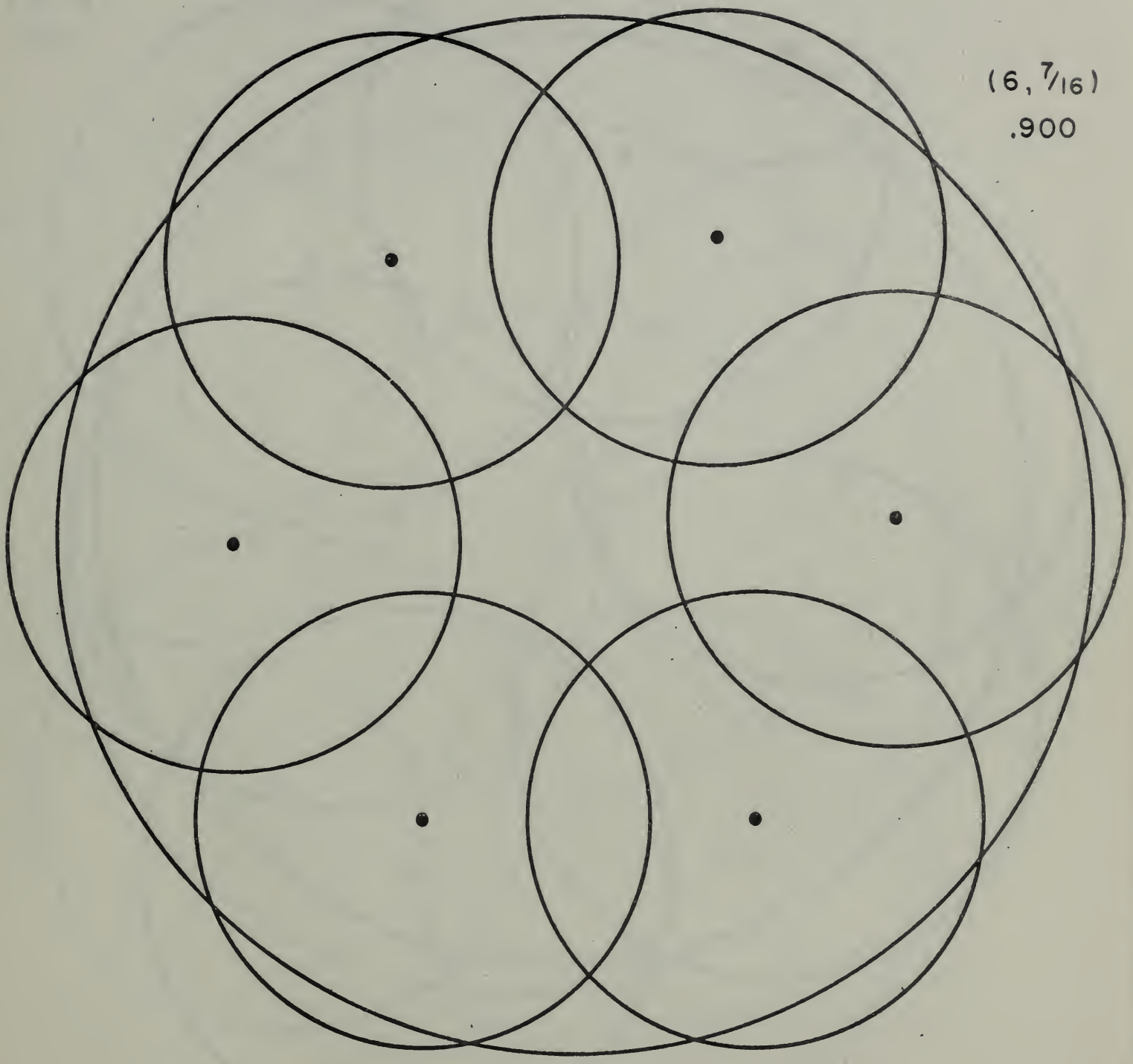


FIG. 65

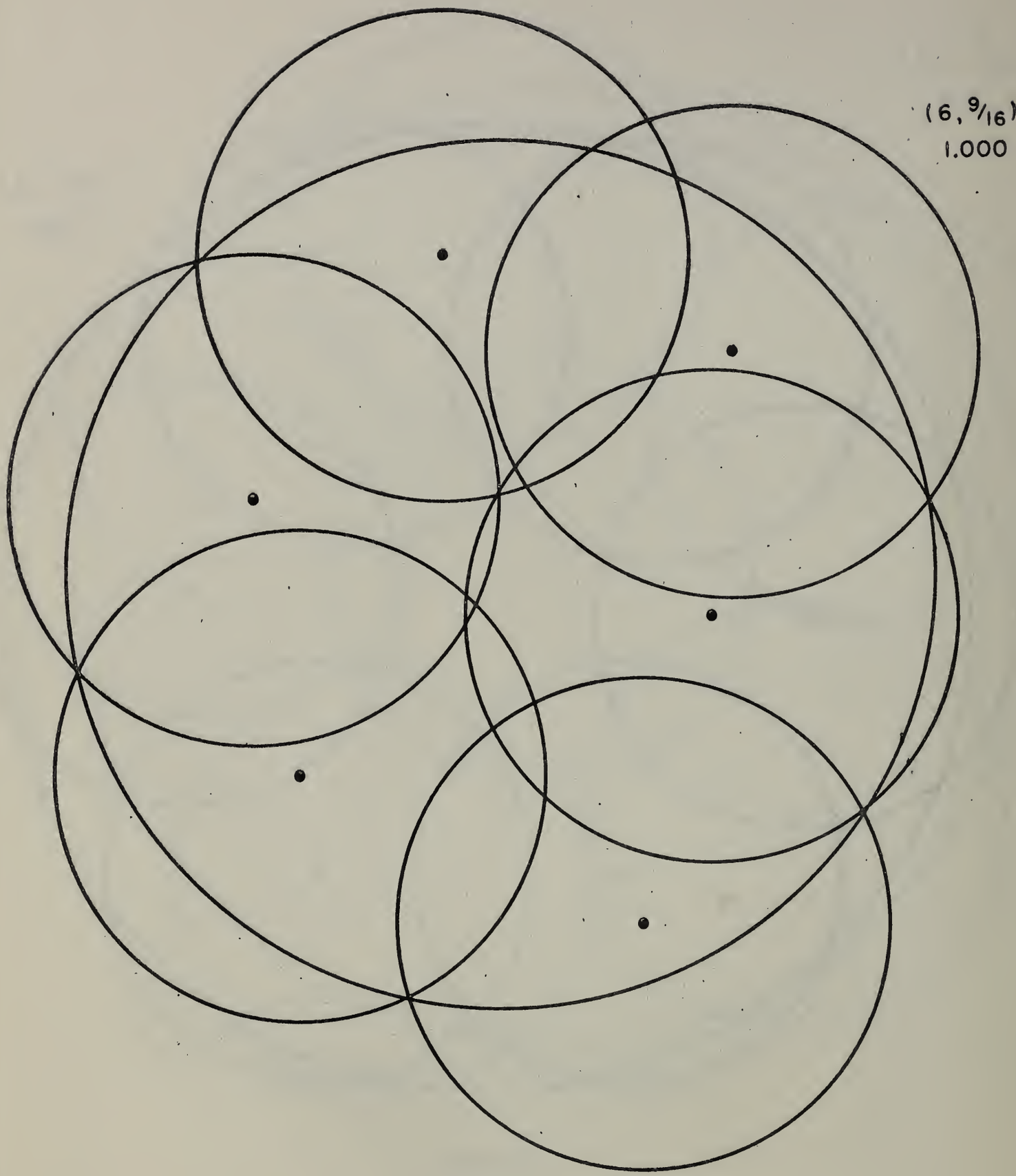


FIG. 66

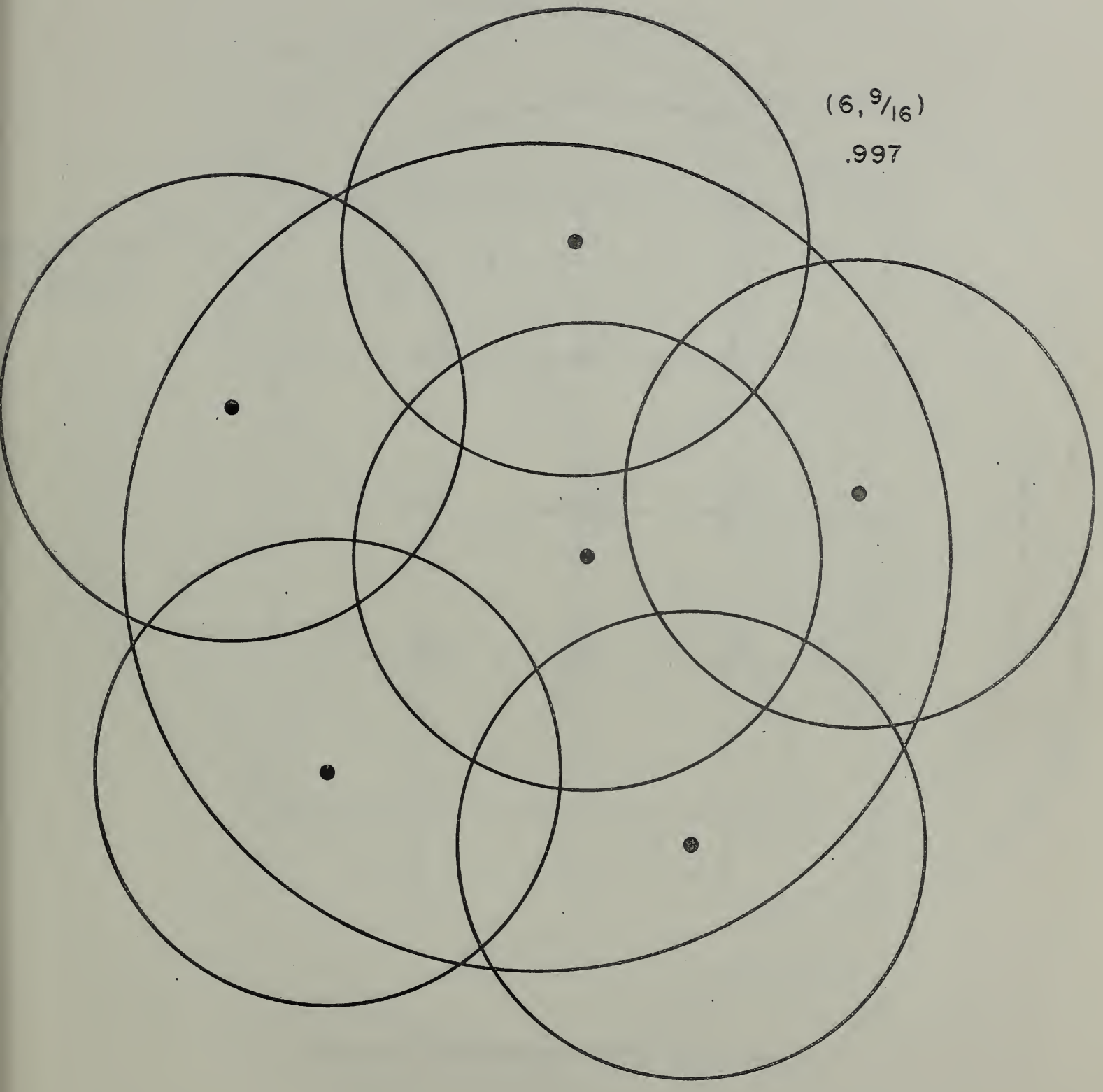


FIG. 67

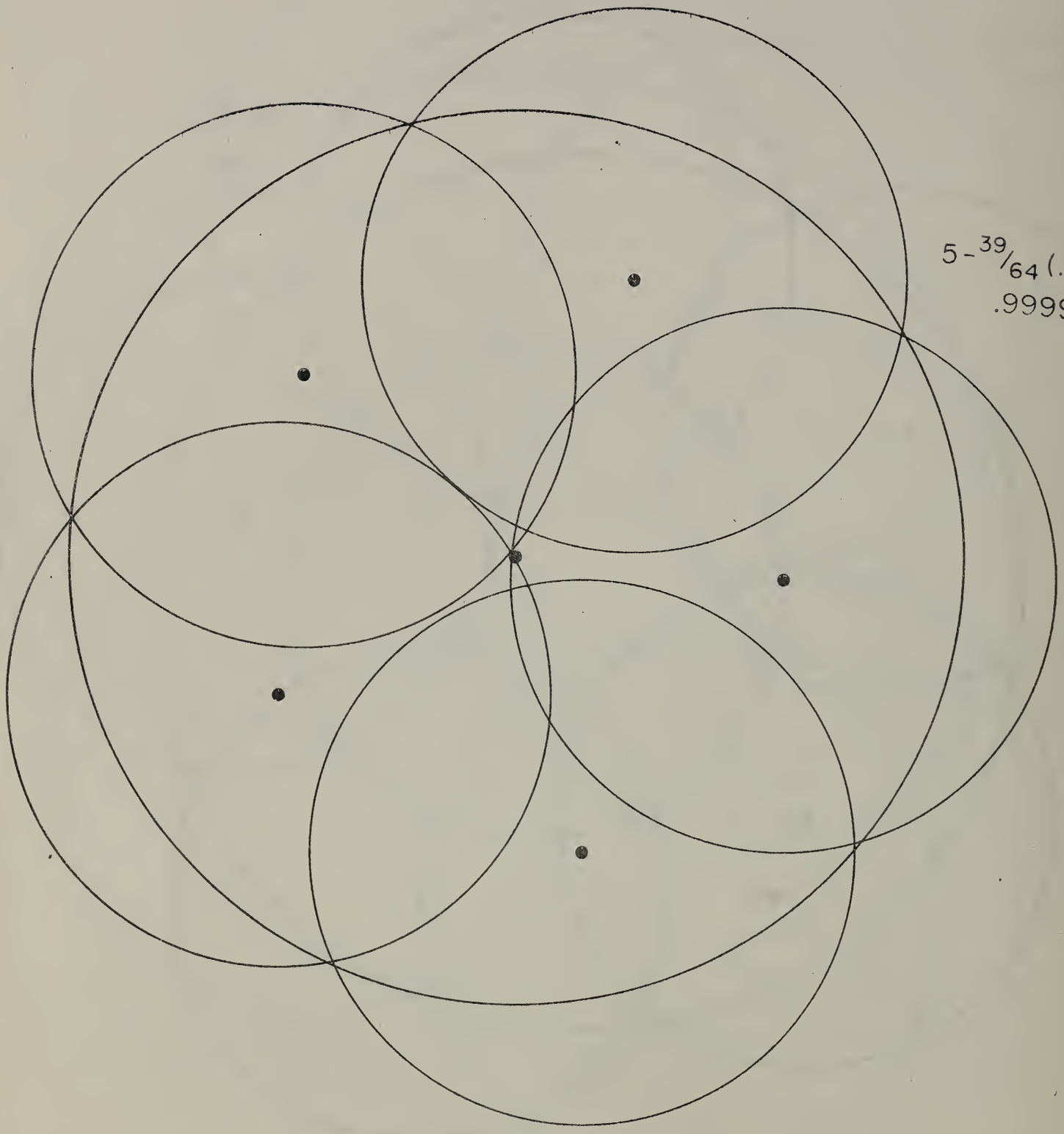


FIG. 68 COMPUTER CONFIGURATION FOR NEVILLE CASE

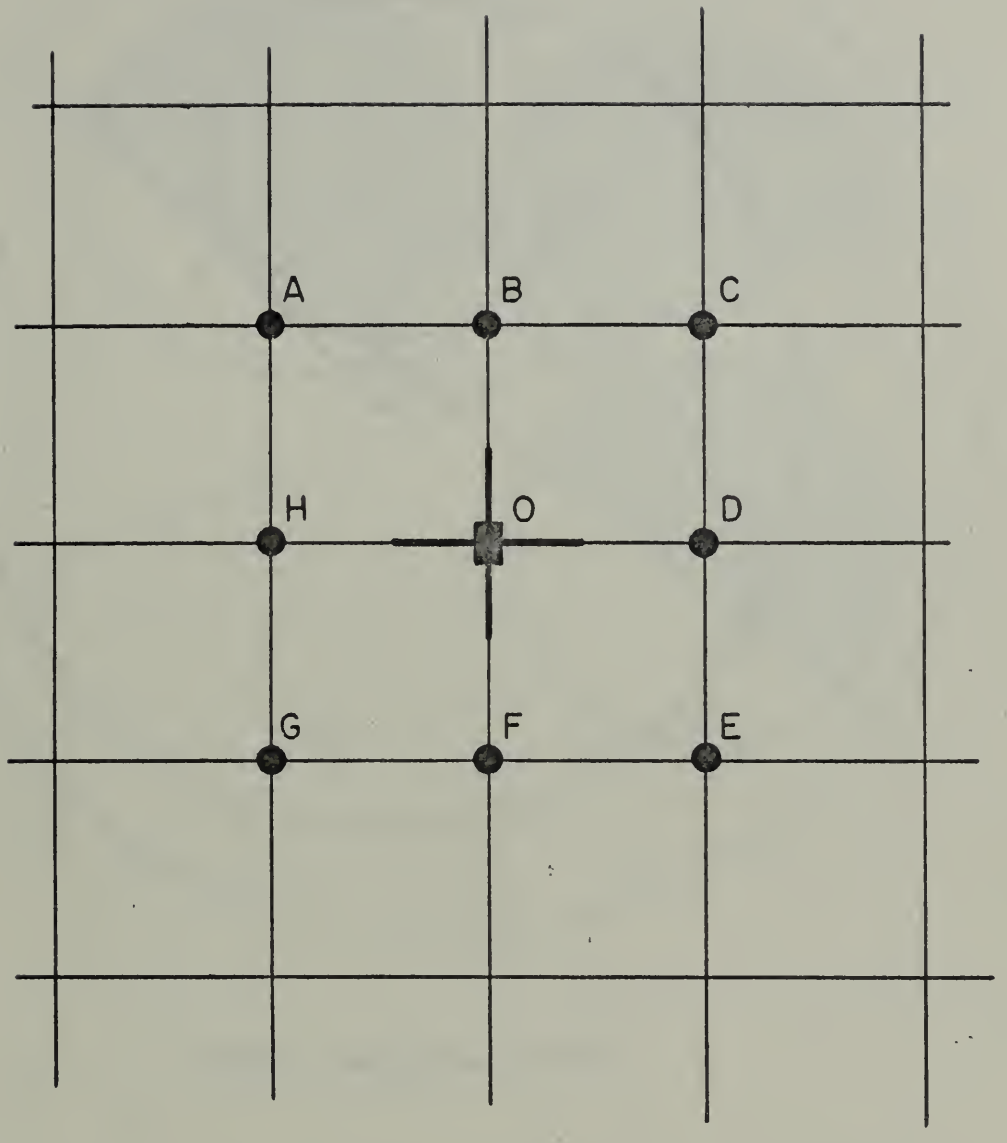


FIG. 69 NEIGHBORHOOD OF A MESH POINT

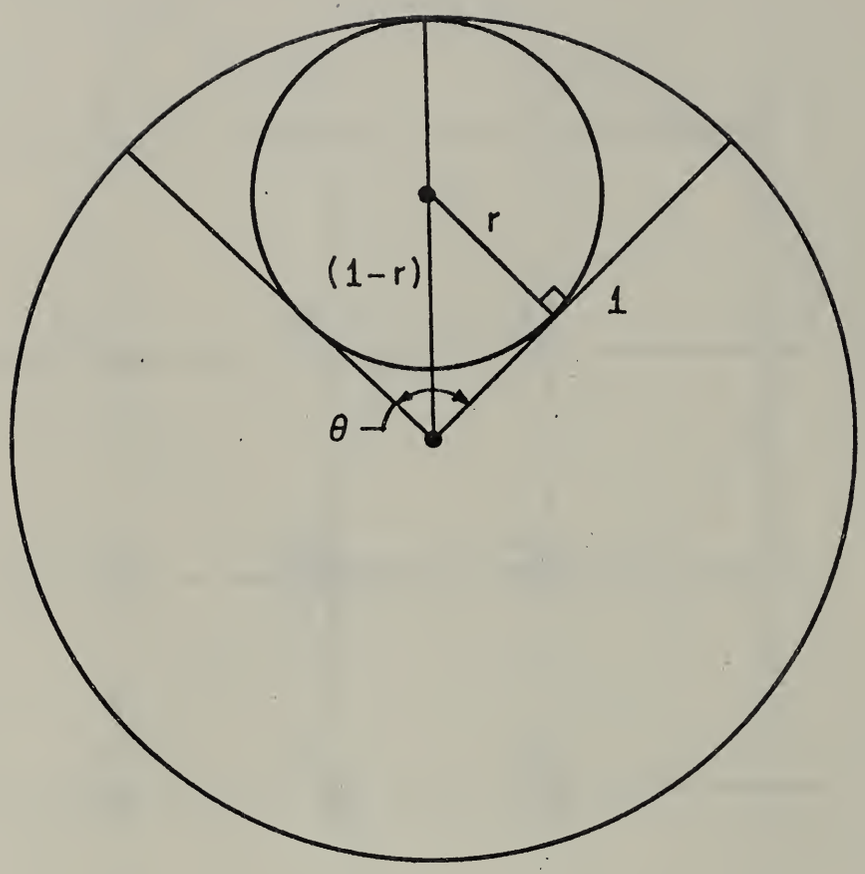


FIG. 70 MAXIMUM PACKING

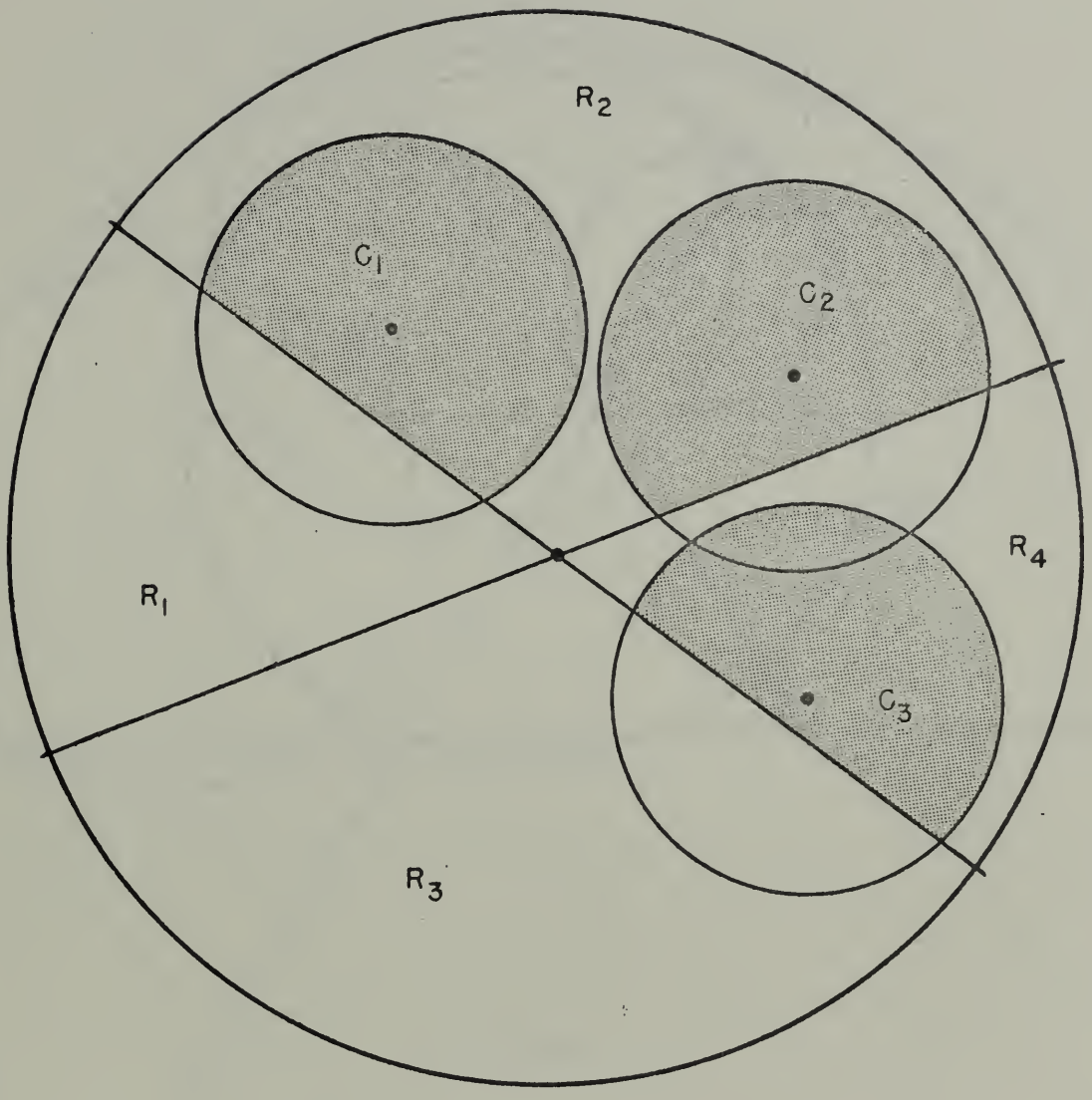


FIG. 71 WALL INTERRUPTED COVERAGE BY THREE DISCS.

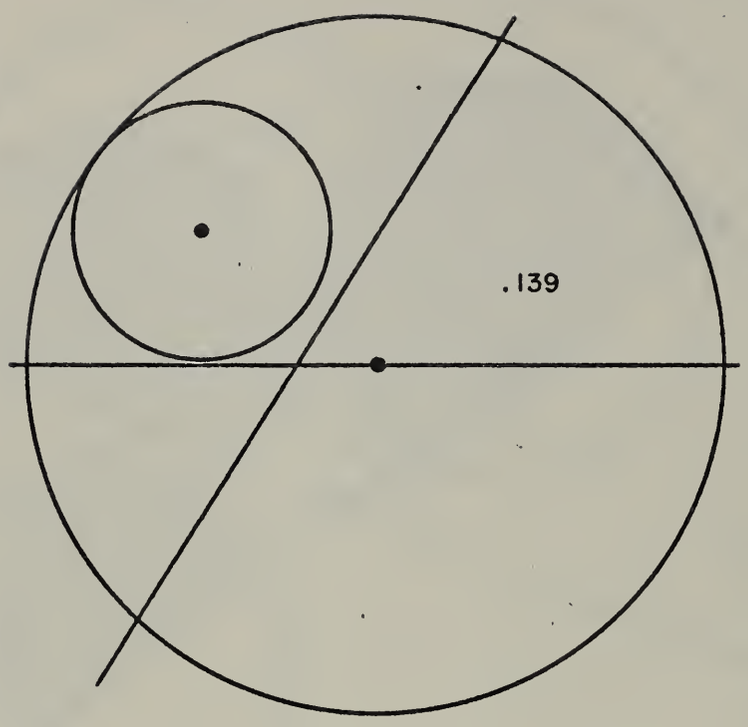


FIG. 72

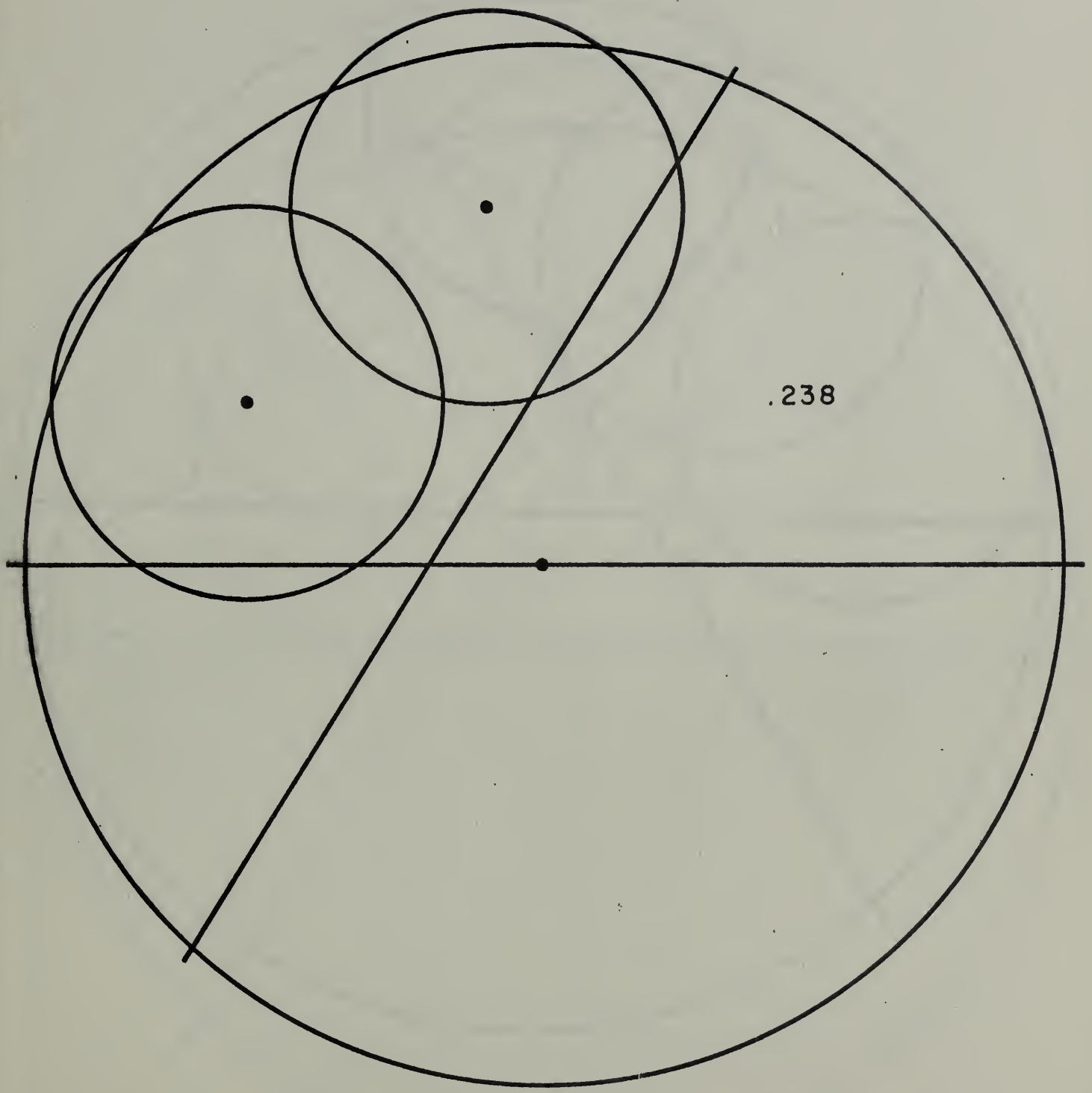


FIG. 73

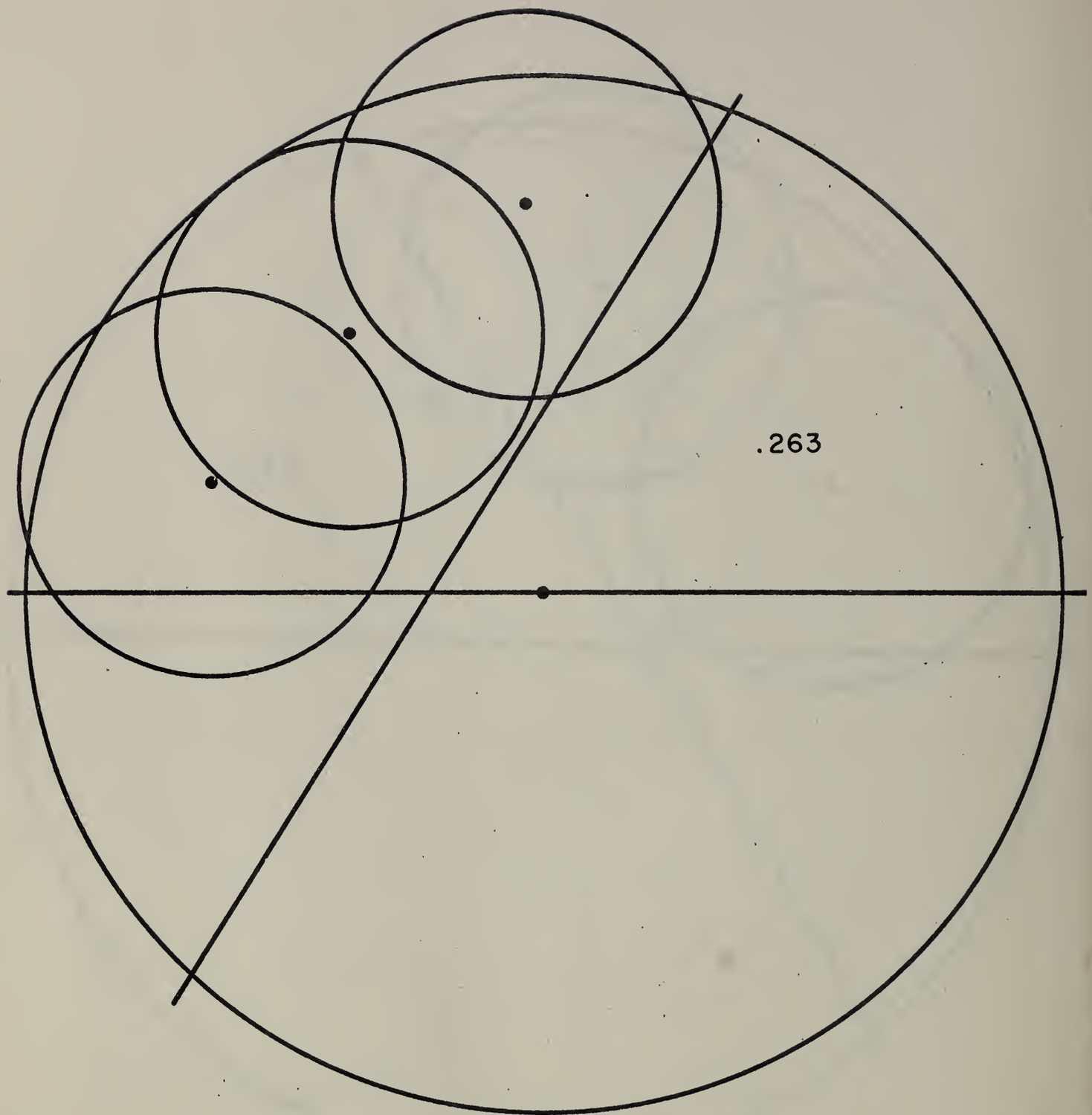


FIG. 74

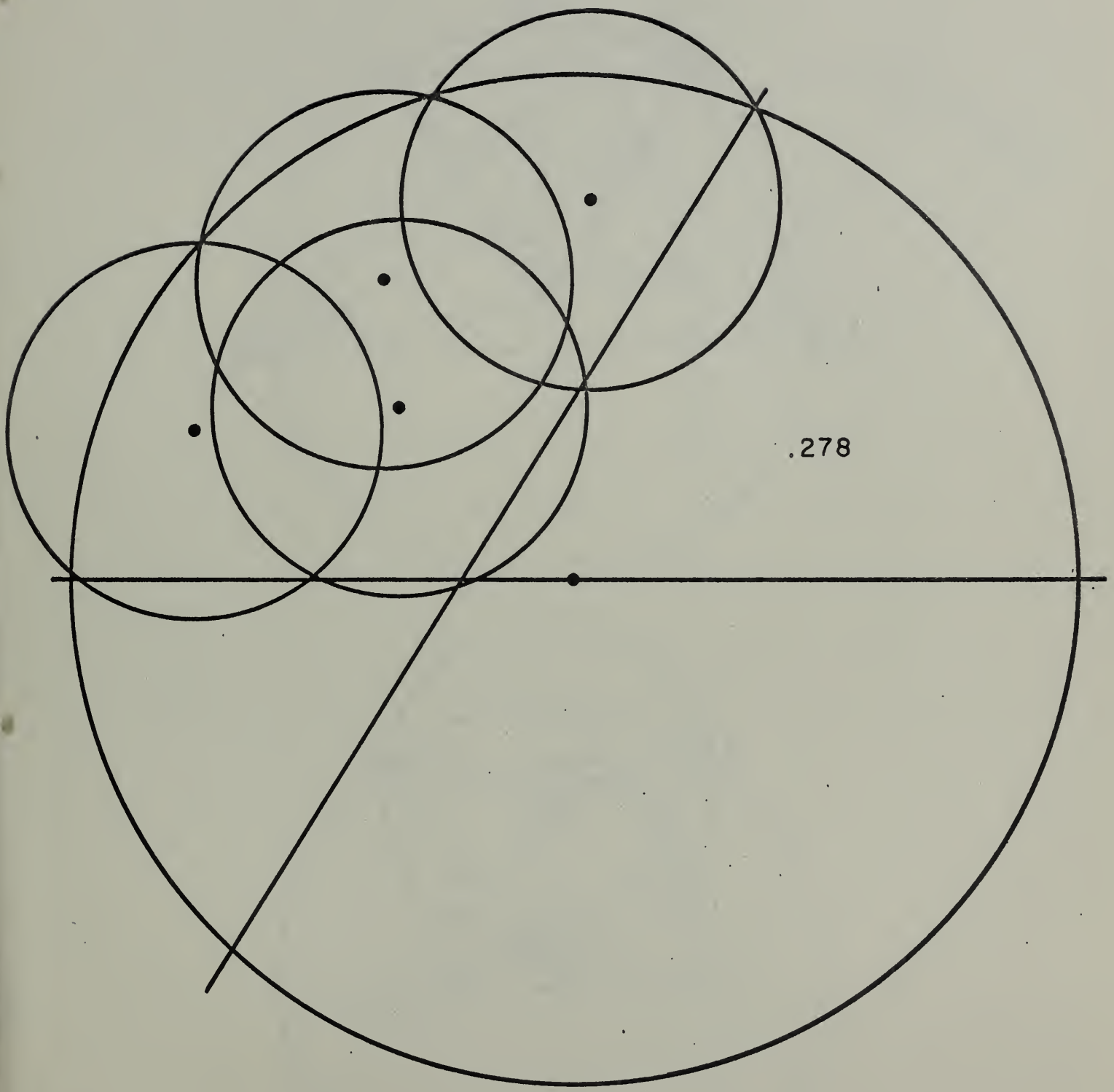
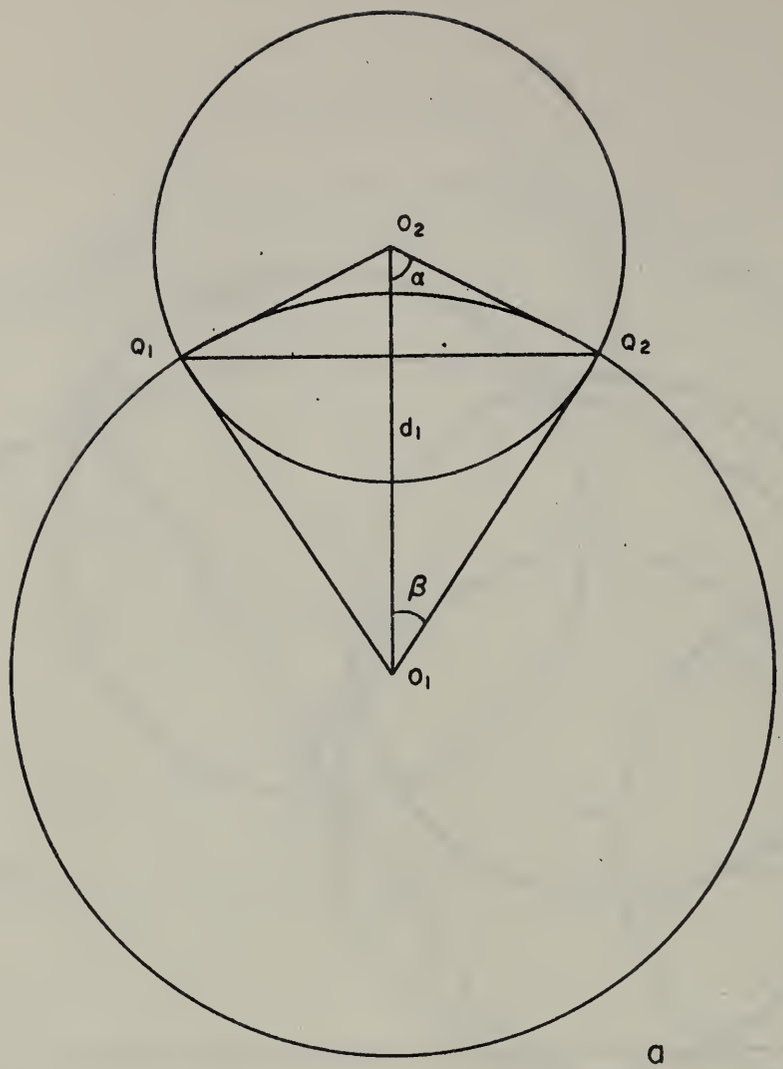


FIG. 75



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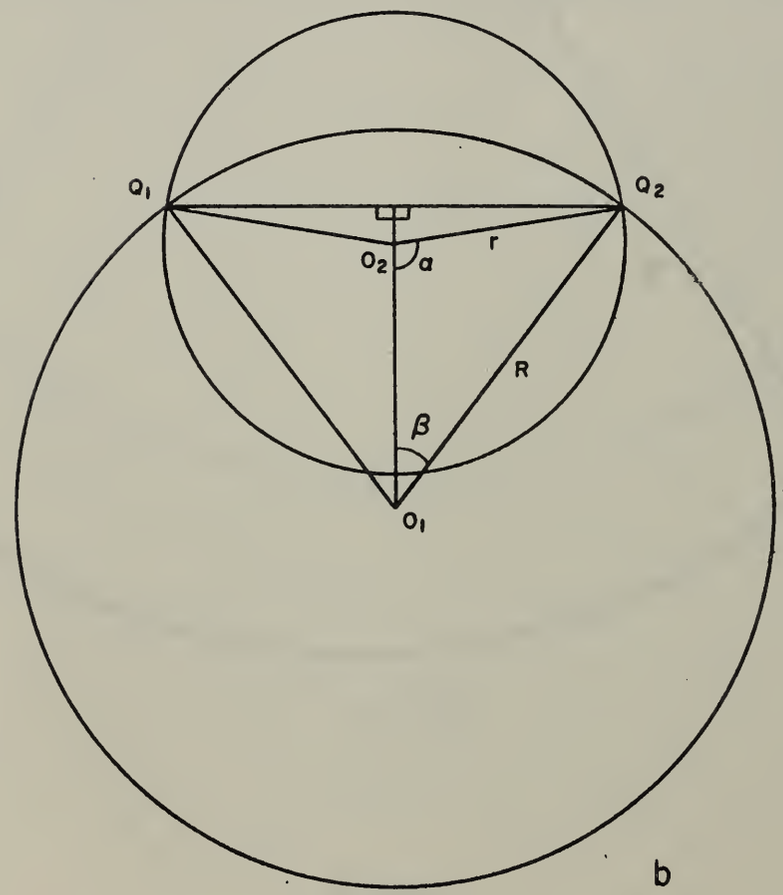
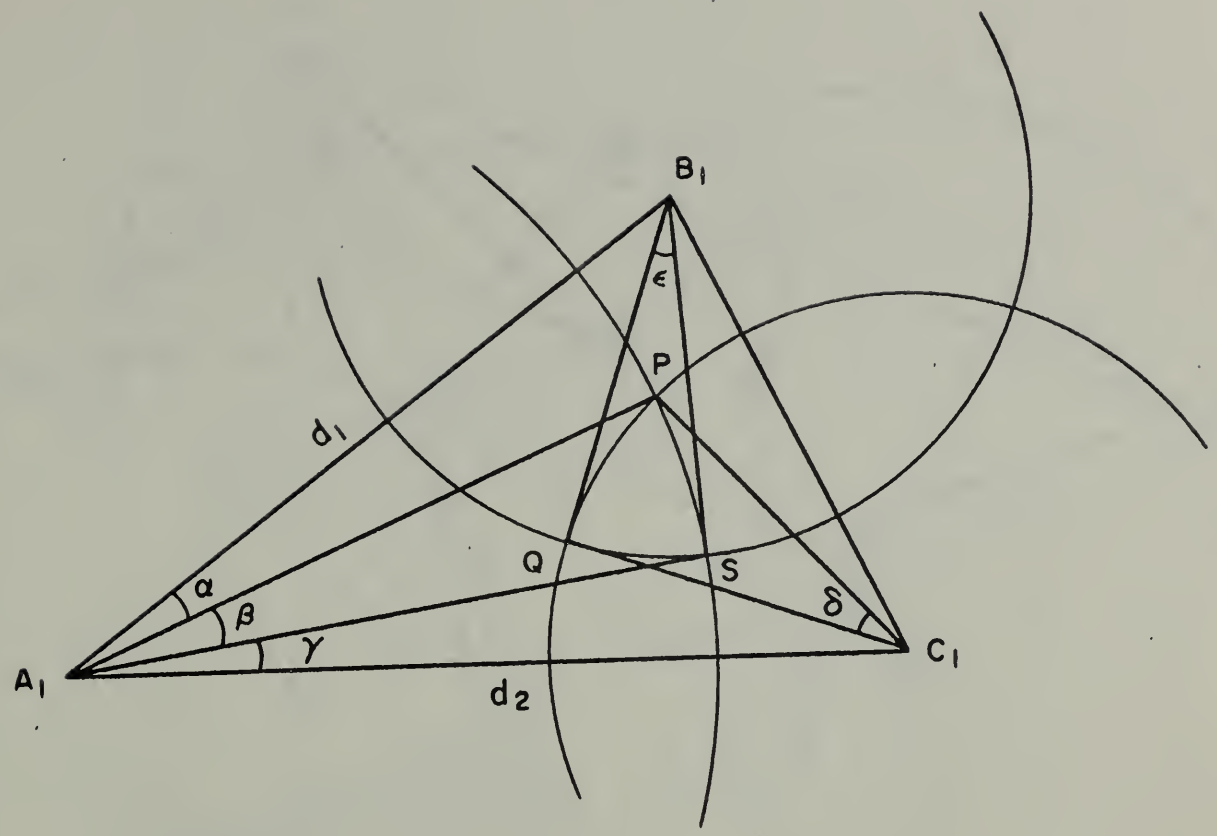
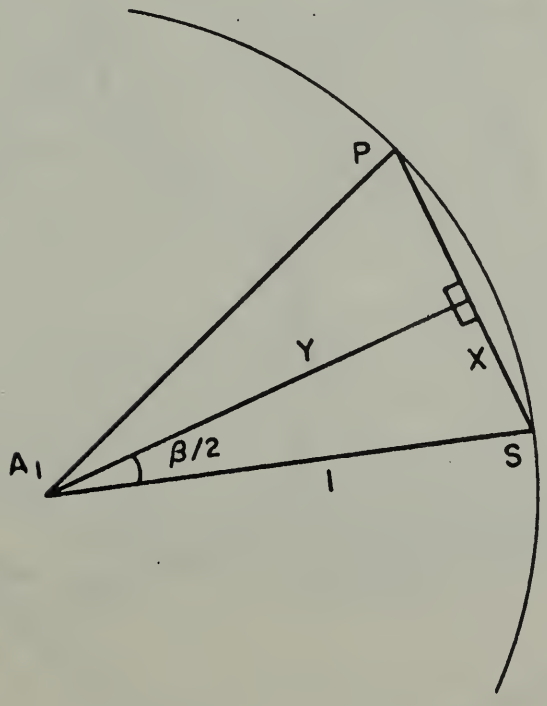


b

FIG. 76

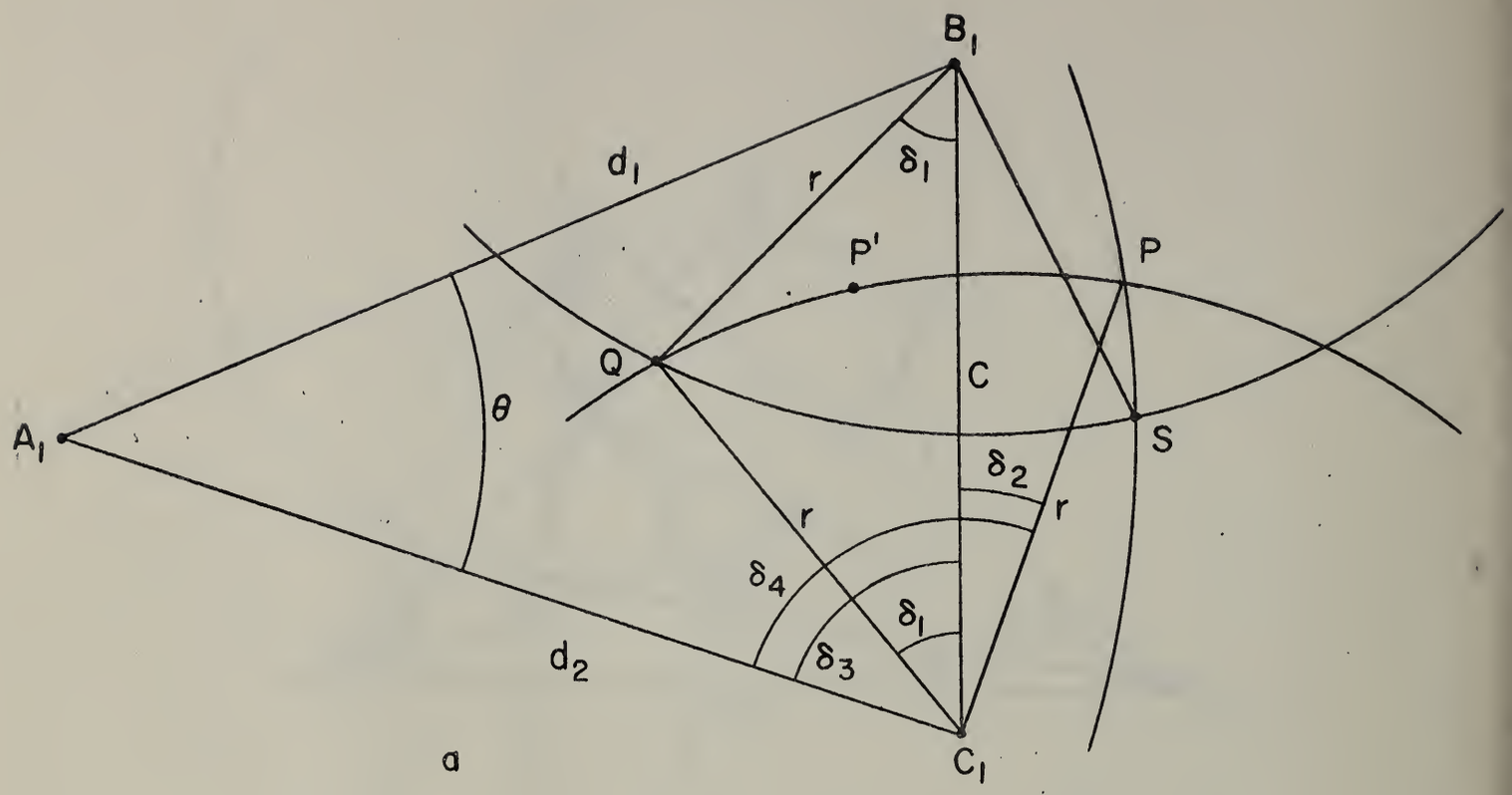


a



b

FIG. 77



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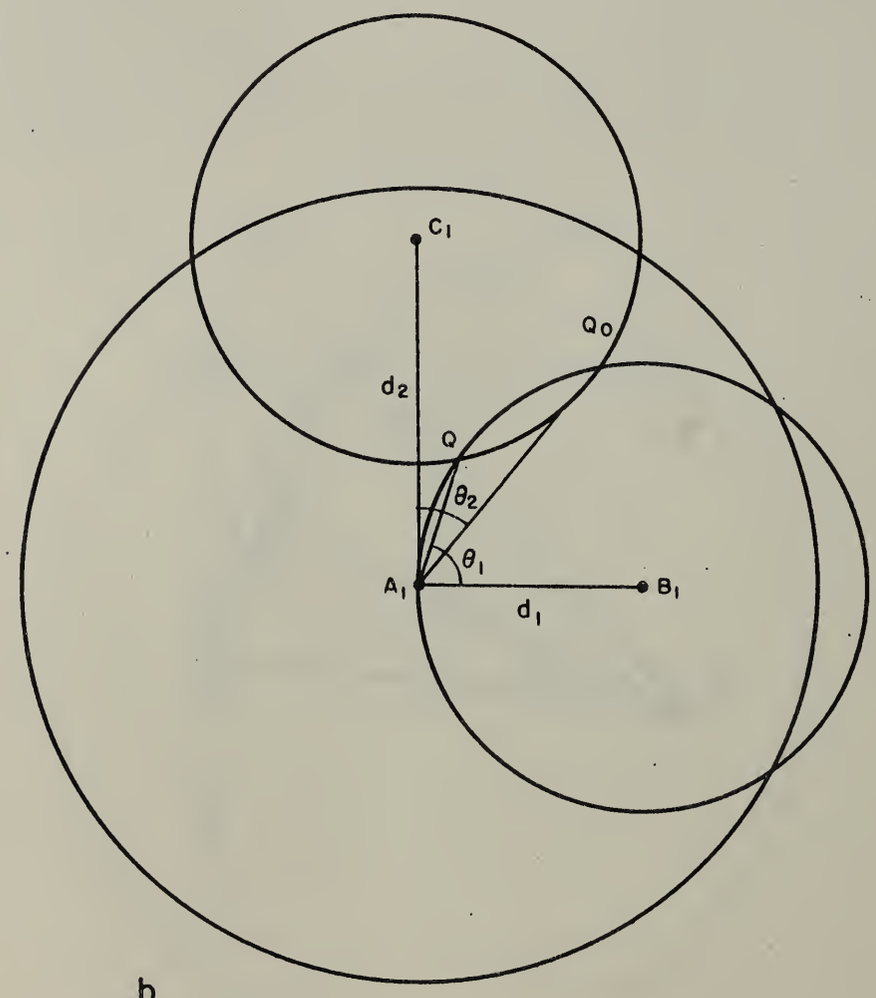

b

FIG. 78

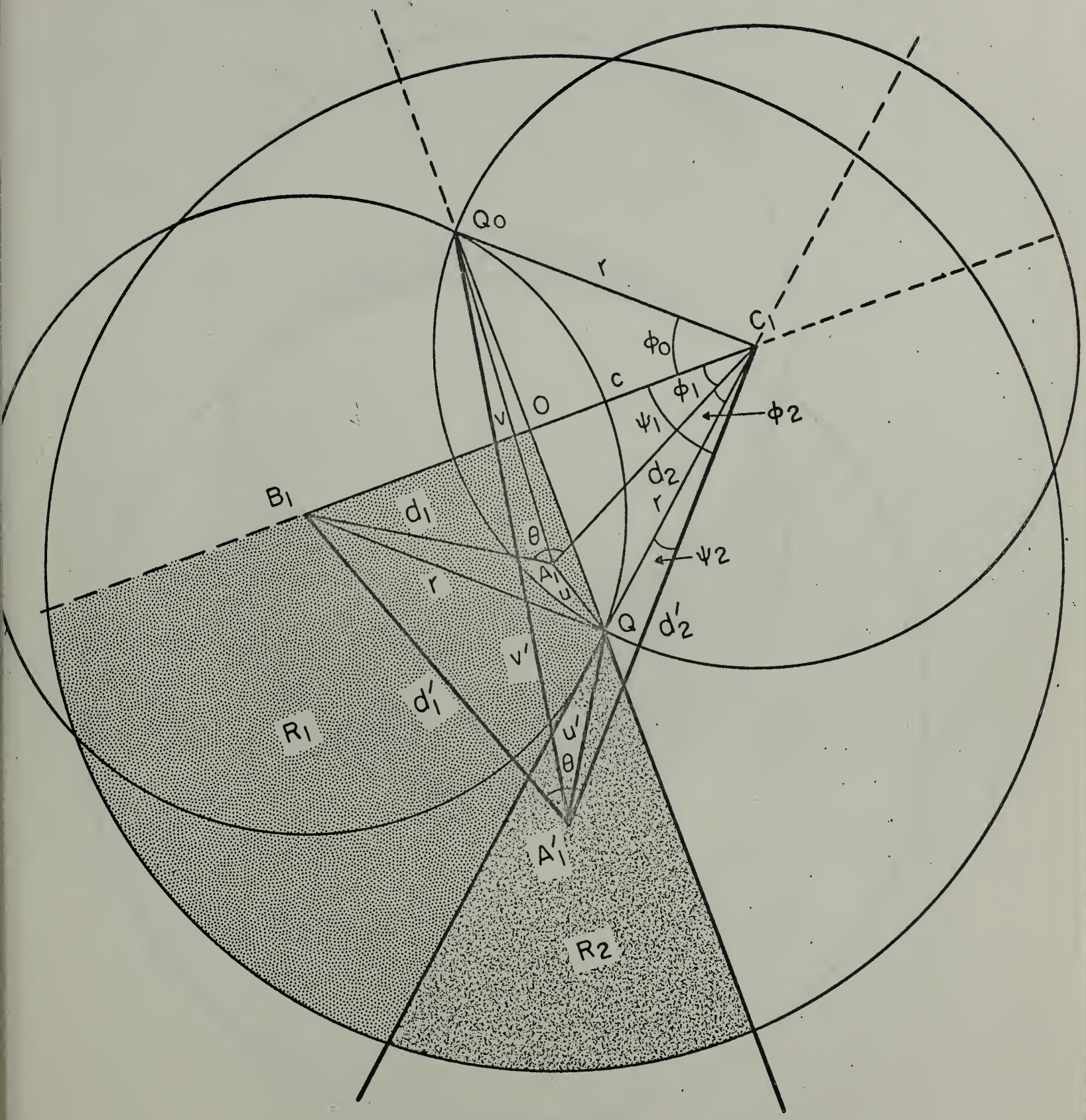

FIG. 79

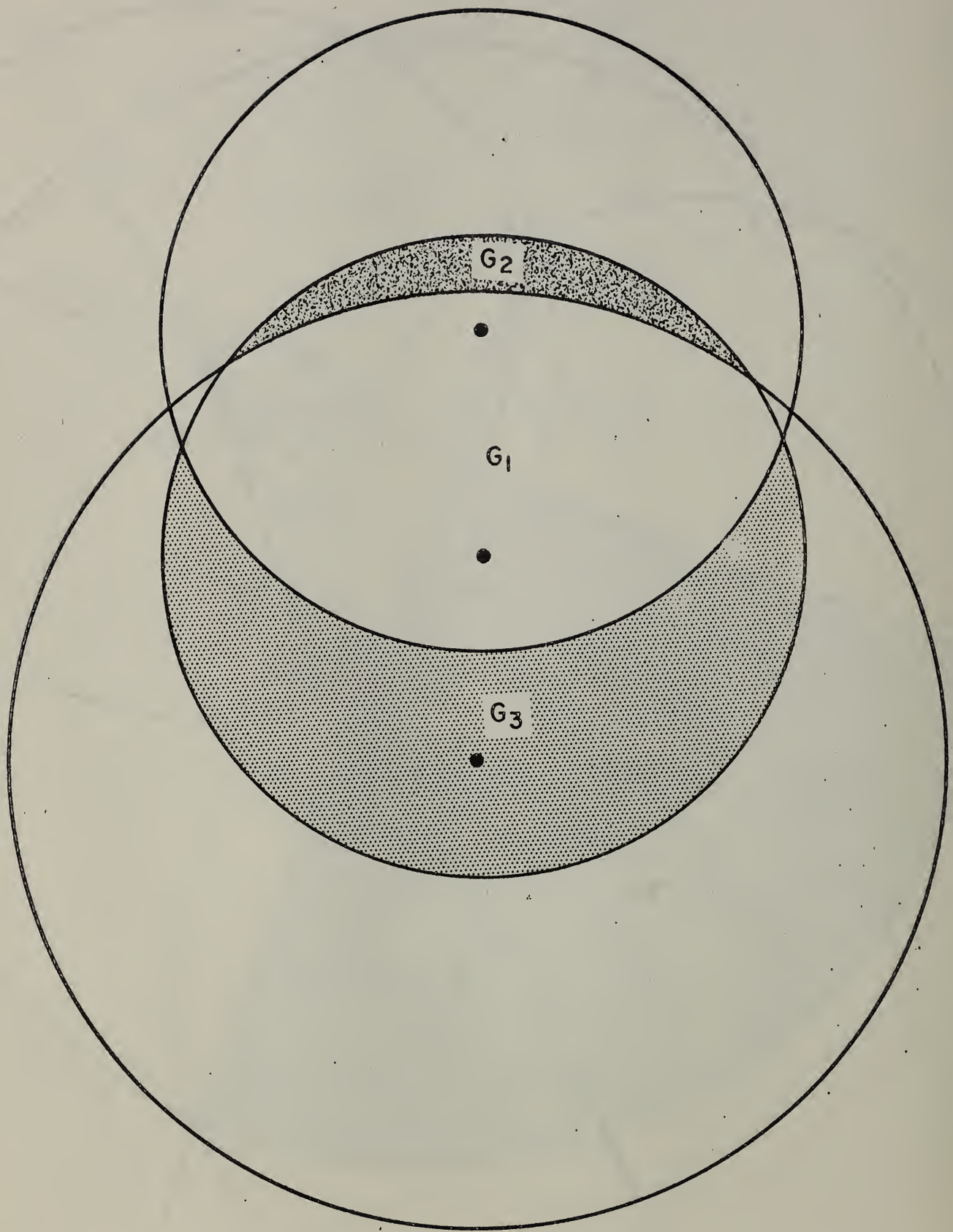

FIG. 80

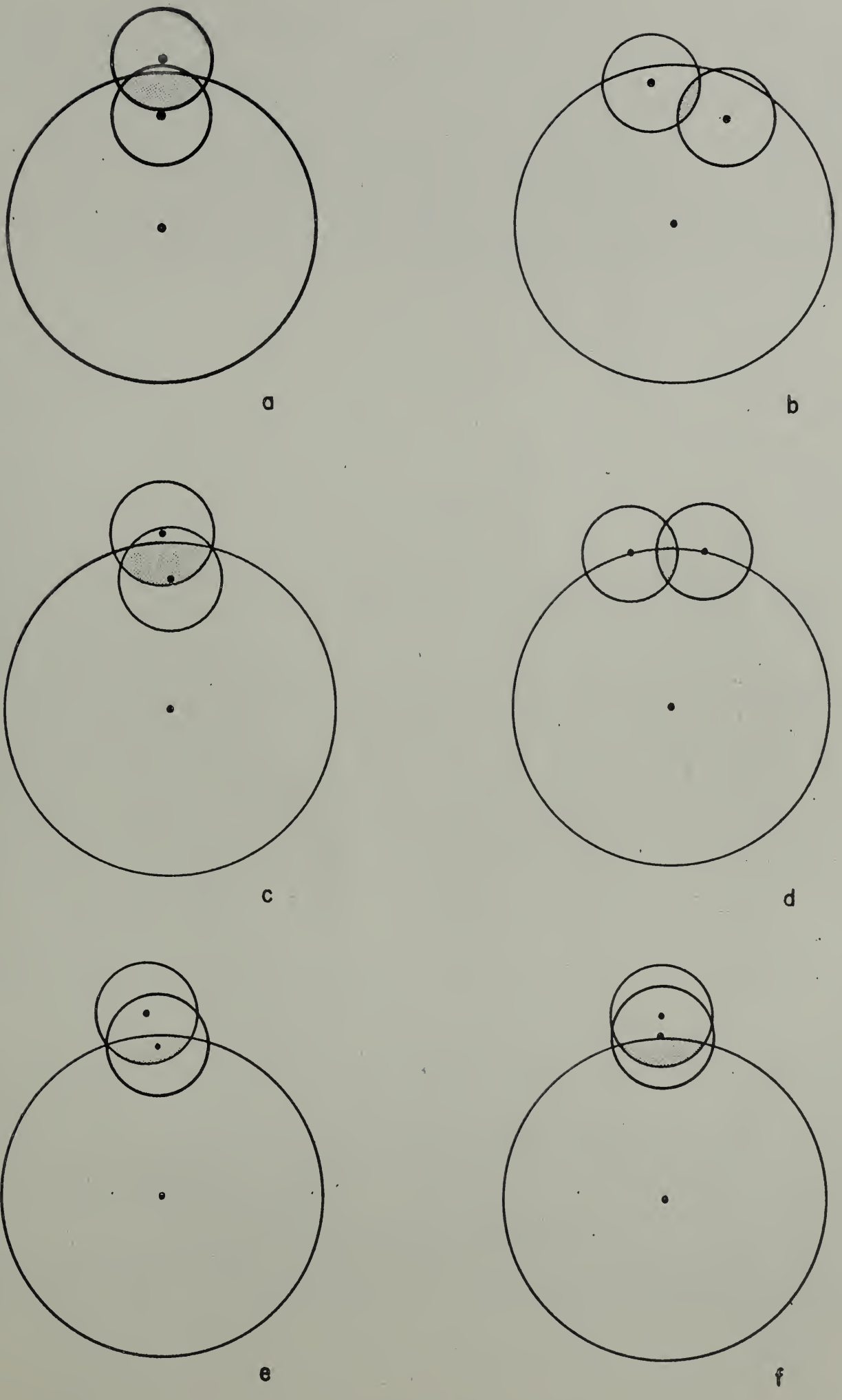

FIG. 81

USCOMM-NBS-DC

U. S. DEPARTMENT OF COMMERCE
Luther H. Hodges, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D. C.

Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics. High Voltage.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research. Crystal Chemistry.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Polymers. Macromolecules: Synthesis and Structure. Polymer Chemistry. Polymer Physics. Polymer Characterization. Polymer Evaluation and Testing. Applied Polymer Standards and Research. Dental Research.

Metallurgy. Engineering Metallurgy. Microscopy and Diffraction. Metal Reactions. Metal Physics. Electrolysis and Metal Deposition.

Inorganic Solids. Engineering Ceramics. Glass. Solid State Chemistry. Crystal Growth. Physical Properties. Crystallography.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials. Metallic Building Materials.

Applied Mathematics Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Computer Technology. Measurements Automation. Engineering Applications. Systems Analysis.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics.

Instrumentation. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Physical Chemistry. Thermochemistry. Surface Chemistry. Organic Chemistry. Molecular Spectroscopy. Molecular Kinetics. Mass Spectrometry.

Office of Weights and Measures.

BOULDER, COLO.

Cryogenic Engineering Laboratory. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Cryogenic Technical Services.

CENTRAL RADIO PROPAGATION LABORATORY

Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services. Vertical Soundings Research.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Systems. Applied Electromagnetic Theory. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

RADIO STANDARDS LABORATORY

Radio Physics. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time-Interval Standards. Millimeter-Wave Research.

Circuit Standards. High Frequency Electrical Standards. Microwave Circuit Standards. Electronic Calibration Center.

NBS