NATIONAL BUREAU OF STANDARDS REPORT

7196

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1961

For Official Distribution

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT 11.0

NBS REPORT 7196

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A Quarterly Report

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U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION

January 1 through March 31, 1961

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*On leave of absence **Part time °°Guest Worker

Contents

*Only unclassified projects are included in this report.

Status of Projects

March 31, 1961

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. W. Borsch-Supan has performed commputations using the eigenvalue estimation method of N. Bazley and D. Fox on the IBM 704. A code has been written to compute lower and upper bounds for the eigenvalues of the spheroidal wave equations. First results show that the method of N. Bazley and D. Fox gives very satisfactory bounds in that case.

A method of estimating the errors of a given set of approximations to the zeros of a polynomial has been found by W. Borsch-Supan. With this method rounding errors and errors in the coefficients of the polynomial may be taken into account.

N. Bazley has completed a joint paper with D. Fox which gives new numerical estimates to the eigenvalues of the helium atom, an anharmonic oscillator, and a radial Schroedinger equation. Several recent developments in the theory of intermediate problems are also presented. The paper has been submitted for publication to a technical journal.

O. Shisha has worked in the following areas:

(1) Approximation by analytic functions whose Taylor coefficients are (geometrically) restricted, e.g., lie in a given sector.

(2) Best Tchebycheff approximation by rational functions.

(3) Infrapolynomials and their generalizations; problems of location of zeros and structure.

(4) Approximation to convex or monotone functions by means of convex or monotone polynomials and trigonometric polynomials.

(5) Mechanical translation (with Mrs. Ida Rhodes).

P. Davis and O. Shisha are preparing a survey on the theory of the transfinite diameter.

M. Marcus completed a recent manuscript entitled "The sum of the elements of the powers of a matrix" (with K. Goldberg and M. Newman). At present research is being done on inequalities for symmetric functions of positive numbers. Also M. Marcus is assisting M. Newman and F. May in the preparation of notes on integral matrices.

S. Haber obtained a posteriori bounds on the total error in certain methods of numerical solution of ordinary differential equations. He is continuing work on the propagation of error in such methods.

Status of Projects

Publications:

(1) On a theorem of I. Schur concerning matrix transformations. M. Marcus and F. May. Arch. Math. 11, 401-404 (1960).
(2) On the unitary completion of a matrix. M. Marcus and P. Greiner. Illinois J. Math. 5, 152-158 (1961).
(3) The permanent function as an inner product. M. Marcus and M. Newman. Research Announcement, Bull. Amer. Math. Soc. 67, 223-224 (1961).
(4) Bound for the P-condition number of matrices with positive roots. M. Marcus, P. J. Davis, and E. V. Haynsworth. J. Research NBS 65B (Math. and Math. Phys.), 13-14 (1961).
(5) A structure theorem for infrapolynomials with prescribed coefficients. O. Shisha. Abstract: Notices, Amer. Math. Soc. 8, 66 (Feb. 1961).
(6) On approximation by analytic functions whose Taylor coefficients lie in a sector. O. Shisha. Abstract submitted for publication.
(7) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.
(8) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Transactions of the American Mathematical Society.
(9) Special types of partitioned matrices. E. Haynsworth. J. Research NBS 65B (Math. and Math. Phys.), 7-12 (1961).
(10) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.
(11) Best approximations and interpolating functions. J. R. Rice. To appear in the Transactions of the American Mathematical Society.
(12) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical.
(13) Another extension of Heinz' inequality. M. Marcus. To appear in the Journal of Research NBS, Sec. B (Math. and Math. Phys.).
(14) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). To appear in the Canadian Mathematical Bulletin.
(15) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). Submitted to a technical journal.
(16) Comparison theorems for symmetric functions of characteristic roots. M. Marcus. To appear in the Journal of Research NBS, Sec. B (Math. and Math. Phys.).
(17) Some computational problems involving integral matrices. O. Taussky-Todd. J. Research NBS 65B (Math. and Math. Phys.), 15-17 (1961).
(18) Computation problems concerned with the Hilbert matrix. John Todd. J. Research NBS 65B (Math. and Math. Phys.), 19-22 (1961).
(19) Symmetric means and matrix inequalities. M. Marcus and P. Bullen (University of British Columbia). To appear in the Proceedings of the American Mathematical Society.
(20) On the relation between the permanent and the determinant. M. Marcus and H. Minc (The University of Florida). To appear in the Illinois Journal of Mathematics.
(21) Linear operations on matrices. M. Marcus. In manuscript.
(22) Lower bounds for eigenvalues of Schroedinger's equation. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.

Status of Projects

(23) Truncations in the method of intermediate problems for lower bounds to eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Physics).
(24) Optimal approximation for functions prescribed at equally spaced points. H. F. Weinberger. To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Phys.).

OPERATIONS RESEARCH
Task 1101-12-11115/61-546

Origin: NBS Authorized 12/30/60
Manager: A. J. Goldman
Full Task Description: Oct-Dec 1960 issue, p. 3

Status: CONTINUED. Work proceeded on the IBM 704 Boolean simplification program, and on preparation of two papers on related theoretic questions.

J. Edmonds has found an algorithm for zero-one linear programming --that is, where the constraint matrix and solution vector are to consist of zeros and ones. It is applicable to several well-known combinatorial topics, e.g. the problem of Quine, block designs, error-correcting codes, and graph coloring. Abstract follows:

> A cover of a finite class of sets is a subclass whose union contains all the elements of these sets. For the problem of finding a cover with fewest sets, the algorithm takes the form given by the following theorem: If a cover C of a class S ($C \subset S$, $\cup\{C\} = \cup\{S\}$) is not minimum then there exists a tree whose vertices are alternately distinct sets of C and S - C and whose edges are distinct elements contained in the sets which they join, such that each S - C vertex meets exactly two edges of the tree, and such that interchanging the roles of the vertex-sets yields a smaller cover.

Publications:

(1) Partial algorithms for minimal covers. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(2) Some results on Boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(3) On the range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.

Status of Projects

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. K. Goldberg continued his investigation of combinatorial inequalities, with an emphasis on equalities between positive forms which imply certain inequalities. For example, if p_r denotes the normalized rth elementary symmetric function of $x_1, \ldots, x_n$ and $p_r(i,j)$ the same function for the same set excluding x_i and x_j, then the equality

$$n^2(n-1)^2(p_r^2 - p_{r+1}p_{r-1}) = (n-r-1)(r-1)\sum_{i<j}\left\{p_{r-1}^2(i,j) - p_r(i,j)p_{r-2}(i,j)\right\}(x_i - x_j)^2 + (n-1)\sum_{i<j} p_{r-1}^2(i,j)\,(x_i - x_j)^2$$

proves the inequality $p_r^2 \geq p_{r+1}p_{r-1}$ by induction for all real $x_1, \ldots, x_n$.

K. Goldberg continued to accumulate data for a test of his baseball model based on the 1959 American League season. Preliminary results indicate excellent agreement.

K. Goldberg continued his investigation of conditions for commutativity of formal power series under substitution.

M. Newman has completed a study of a certain subgroup of the modular group, which implies arithmetic results about the number of representations of an integer n by the form

$$\sum_{i=1}^{r} x_i^2 + q\sum_{i=1}^{s} y_i^2, \qquad q \text{ an odd prime.}$$

In addition, the structure of the subgroup of the modular group generated by the pth powers of the elements has been determined.

Publications:

(1) Congruence properties of the parition function to composite moduli. M. Newman. To appear in the Illinois Journal of Mathematics.
(2) Subgroups of the modular group and sums of squares. M. Newman. Amer. J. Math. 82, 761-778 (1960).
(3) Periodicity modulo m and divisibility properties of the partition function. M. Newman. Trans. Amer. Math. Soc. 97, 225-236 (1960).
(4) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. To appear in the Journal of Mathematical Analysis and Applications.
(5) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. To appear in the American Mathematical Monthly.
(6) A Stirling number proof of Von Staudt's theorem. K. Goldberg. Submitted to a technical journal.

Status of Projects

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: P. Davis
Full task description: July-Sept 1955 issue, p. 7

Status: TERMINATED. The code for the computations with ordinary differential equations has been turned over to the sponsor, who is acquiring his own computing machine.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter, with priority being given to preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 SIEVERT'S INTEGRAL

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Chapters 12 (Struve Functions and Related Functions), 25 (Numerical Interpolation, Differentiation, and Integration), and 26 (Probability Functions) are in press. Review of remaining chapters continues for consistency of notation, updating of references, cross-references, etc. Preparation of graphs and line cuts is also being continued.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. A manual "BS BEL, An Input-Output and Monitor System for the IBM 704" was completed by G. M. Galler and G. G. Ziegler.

The development of a string manipulation language has continued along two lines. Several special purpose languages such as LISP, IPL, and COMIT have been examined for basic ideas and common features. Various data processing problems such as file maintenance, report generating, natural language translation, and program compilation have been examined for basic and often recurring operations which should be included in a string manipulation language. The results of these investigations are leading to the design of a string manipulation language.

Status of Projects

R. Herbold has modified the plotting subroutine in the Tablemaker system to give the user the option of setting the printer to 8 lines per inch or 40 lines per inch for plotting data. Linear interpolation was also added as an option to permit smoother curve plotting. During this quarter, the Tablemaker system was used for a total computer time of 522 minutes. The following table indicates the number of times that the various functions were used.

Function	Number of times used
Curve fit	218
Plot	110
Finite Difference	15
Standard deviation, mean, sum	136

MATHEMATICAL SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: INACTIVE.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. Using the normal approximation to the distribution of $\bar{x} + ts$ and the NBS "Tables of the Bivariate Normal Distribution and Related Functions" (Applied Mathematics Series 50), C. Eisenhart and A. Glinski investigated for sample sizes n = 4, 9, 16, and 25 and nominal confidence level $1-\alpha = 0.95$, the magnitude of the reduction in the level of confidence that would result from always selecting the narrower of the two confidence intervals $\bar{x} + K_{\frac{1}{2}\alpha}\sigma$, $\bar{x} + t_{\frac{1}{2}\alpha}s$ when σ is known.

In connection with the NBS Theory of Errors Colloquium:

(1) A. Glinski evaluated the ratio of the mean deviation δ to the standard deviation σ for a variety of unimodal symmetrical distributions ranging in kurtosis from the rectangular distribution ($\beta_2 = 1.8$) to Student's distribution for 3 degrees of freedom ($\beta_2 = \infty$), obtaining the following results:

β_2	Distribution	$\delta/\sigma\sqrt{\frac{1}{2}\pi}$
1.8	Rectangular	1.085
2.4	Pearson II	1.032
3.0	Normal	1.000
4.2	Pearson VII	0.965
	Sech^2	.958
5.0	Pearson VII	.952
	Sech	.930
6.0	Pearson VII	0.940
	Double Exponential	.886
9.0	Pearson VII	.914
∞	Student (4d.f.)	.886
	Student (3d.f.)	.798

(2) For the same distributions, C. Eisenhart prepared a table of standardized deviates corresponding to central areas of .50, .60, .80, .90, .95, .98, .99, and .999, showing great sensitivity of all but the 95% points to the kurtosis of the underlying distribution; the 95% points being amazingly stable for $\beta_2 \geq 3$.

β_2	1.8	2.4	3.0	4.2	5.0	6.0	9.0	∞
$\left\|\frac{x-\mu}{\sigma}\right\|$	1.645	1.844	1.960	1.995	1.999	1.998	1.991	1.963
				2.020	2.061	2.118		1.837

(3) C. Eisenhart, A. D. Smith, and J. Van Dyke began numerical evaluation of probability points of coverage distributions corresponding to Wilks' unbiased tolerance intervals in small samples from a normal distribution.

Status of Projects

Work on the calculation of tables of the non-central F and χ^2 distributions is continuing. The non-central parameter φ for $\nu_1 = \infty$ has been calculated for all contemplated percentage points. Trial calculations have been made and a code written by M. C. Dannemiller for the calculation with respect to the non-central χ^2 distribution. The code is currently being checked.

J. Van Dyke is continuing the numerical investigation of the distributions of the family of random variables

$$z = Au^{\alpha} - B(1-u)^{\beta} + C$$

where u is uniformly distributed on (0, 1). The case $\alpha = 0.1$, $\beta = 0.2$ has been completed and the computation scheme has been written up in preparation for computer programming.

W. J. Youden developed a simple statistic particularly suited for limited amounts of data. The statistic consists of an interval formed by adding k times the sample range to the maximum value and subtracting k times the sample range from the maximum value in the sample. Confidence limits for n = 2, 3, 4, and 5 for several values of k were established largely by sampling normal deviates.

Publications:

(1) Index to the distributions of mathematical statistics. Frank A. Haight. J. Research NBS 65B, 23-60 (1961).

(2) Selected bibliography of statistical literature, 1930-1957. IV. Markov chains and stochastic processes. Lola S. Deming and D. Gupta (Catholic University). J. Research NBS 65B, 61-93 (1961).

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52
Manager: J. M. Cameron
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. W. J. Youden continued the study of a special class of weighing designs particularly appropriate for the determination of physical constants. These designs also find application in the evaluation of routine testing procedures.

M. Zelen and B. Kurkjian (DOFL) are continuing work on a calculus for factorial designs and are attempting to extend it to include fractional factorial designs. A paper has been written for presentation at the 1961 meeting of the International Statistical Institute summarizing their work on the use of the direct product in factorial designs.

Publications:

(1) Fractional factorial designs for experiments with factors at two and three levels. W. S. Connor and Shirley Young. To appear as NBS Applied Mathematics Series 58.

Status of Projects

(2) Partial confounding in fractional replication. W. J. Youden. Submitted to a technical journal.
(3) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.
(4) A calculus for factorial arrangements. Badrig Kurkjian (DOFL) and M. Zelen. Submitted to a technical journal.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. In 1954, W. S. Connor proposed a "method of line segments" for obtaining simultaneous confidence limits for a straight line $y = a + bx$ when observations have been taken at two values of x. J. R. Rosenblatt has noticed that this can be extended to "method of parabolic segments" for obtaining simultaneous confidence limits for a parabola $y = a + bx + cx^2$ when observations have been taken at three values of x. The attractive simplicity of Connor's method is lost in the generalization. A study of Connor's method is being prepared for publication.

Publication:
(1) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. To appear in the Journal of the American Statistical Association.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS
Managers: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. C. Dannemiller and M. Zelen have written an expository paper summarizing their recent work on robustness of life tests at the invitation of the editor of Missile Design and Development.

Publications:
(1) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and Mary C. Dannemiller. Technometrics 3, 29-49 (1961).
(2) Robustness of life testing procedures based on the exponential distribution. Mary C. Dannemiller and M. Zelen. Missile Design and Development pp. 35-36, 99 (Mar. 1961).

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. A. Ghaffari has found earlier (see July-Sept 1960 issue, p. 11) the most general solution on the interval $(0,\infty)$ of the Chapman-Kolmogoroff functional equation for the transition probability $f(x,s;y,t)$ of a one-dimensional Brownian motion, where x refers to the state of a particle at instant s, and y to the state at instant t. The corresponding second-order partial differential equation satisfied by f is

$$\alpha(x,s)\,\frac{\partial^2 f}{\partial x^2} + \beta(x,s)\,\frac{\partial f}{\partial x} + 2\,\frac{a(s)}{a'(s)}\,\frac{\partial f}{\partial s} = 0$$

where α and β are known rational functions of x and $\Theta(s,t) = a(s)/a(t)$.

Analyticity and probability properties of the solutions for the interval $(-\infty,\infty)$ as well as $(0,\infty)$ (see Jan-Mar 1960 and succeeding issues) and the asymptotic behavior of the probability functions have been investigated previously. The study has now been concluded.

L. E. Payne and J. H. Bramble continued their work on pointwise bounds in elastic plate problems. They completed a manuscript on "A Priori Bounds in the First Boundary Value Problem in Elasticity". In this paper they obtain upper and lower bounds for the strain energy and pointwise bounds for the displacements and stresses in an elastic medium if the displacements are prescribed on the boundary. These bounds are of a priori type; they involve undifferentiated data and are expressed in terms of integrals on which the Rayleigh-Ritz method may be employed to make the error small.

Publications:

(1) On some partial differential equations of Brownian motion of a free particle. A. Ghaffari. To appear in the "Proceedings of the International Conference on Partial Differential Equations and Continuum Mechanics," held at the Mathematics Research Center, U. S. Army, Madison, Wis., June 1960.
(2) Pointwise bounds in the Cauchy problem of elastic plates. L. E. Payne. To appear in the Journal of Research NBS, Sec. B.

Status of Projects

PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen continued an investigation of the structure of turbulence in a plasma from two points of view: (1) the Vlasov equation with the self-consistent electric field is integrated to obtain the equations of moments, which are reduced to a non-linear equation of velocity fluctuations; (2) the magnetohydrodynamic equations are reduced to two equations of fluctuations in the velocity and magnetic field. The two cases correspond, respectively, to electrohydrodynamic turbulence and magnetohydrodynamic turbulence. The fluctuations may be subjected to a Fourier analysis. Kolmogoroff and Heisenberg's hypothesis is used to describe the mechanism of transfer of energy between the various harmonics. The spectral functions are obtained for the turbulent kinetic and magnetic energies.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti has prepared a manuscript for publication on "Theory of an Accurate Intermediary Orbit for Satellite Astronomy". This gives an orbit which corresponds to the gravitational potential proposed by J. P. Vinti in the Journal of Research, NBS, 63B, 105-116 (1959). The solution presented in the present paper gives the results in terms of certain uniformizing variables the periodic parts of which are correct through the second order in the oblateness parameter k (proportional to J_2, which is a measure of the planetary oblateness, and is the coefficient of the second harmonic in the spherical harmonic expansion of the potential) and the secular parts of which are exact for the intermediary orbit. These exact results for the secular terms are expressed without the use of elliptic integrals of the third kind and involve only certain rapidly converging series.

Publication:

(1) Mean motion in conditionally periodic separable systems. J. Vinti. To appear in the Journal of Research of the National Bureau of Standards, Sec. B.

Status of Projects

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. The manuscript was completed and the Introduction was prepared.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The intensity table arranged in order of wavelength is ready for publication; the list by elements is nearly ready for printing of the master sheets on the card-controlled typewriter. Wavelengths and wavenumbers were calculated for approximately 5,000 praseodymium lines, and least squares calculations for constants involving wavelength were determined for approximately 10,000 lines. Calculations for energy levels of the third spectrum of praseodymium (Pr III) are nearly completed; 7 new levels were found for the F_2^3 configuration, 15 for the F^2d, 15 for the $4F^26p$, and 5 for the $4F^26s$ configurations. Theoretical matrix calculations were made for both Pr III and Yb III.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. J. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Production runs were made under the sponsor's direction.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: B. S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants were calculated for 58 thermometers under test, and LTS constants for 21 thermometers.

1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: P. J. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Programming of the direct integrals for T_d symmetry has been completed. The exchange integrals are being programmed.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed to check 20 laboratory sets of gage blocks.

Status of Projects

1102-40-11645/57-236 SELF CONSISTENT FIELD--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: P. Walsh
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. Some cases were run using the SCF program. Further checks are being made on the program for transforming integrals to a new basis.

3911-61-39952/56-266 DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Inactive.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Inactive.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. Work continued on analytical problems in the optimization of distribution networks. Literature pertaining to some experimental automated sorting centers was examined, and visits to these installations were made.

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Inactive.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Theoretical studies have been concluded on questions concerning inverse Laplace transformations by means of asymptotic series. A paper on these investigations is being prepared.

Status of Projects

1102-12-11513/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes (11.0)
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. An NBS Report entitled "Handling of Adapted and Compound Words in the National Bureau of Standards' Scheme of Mechanical Translation" has been prepared. It deals with such problems as word stems not found in the dictionary, including words of non-Russian origin, and composite stems. Work continued on the other aspects of the project.
Publications: (1) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, Sept. 6-8, 1961.
(2) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: S. Peavy
Full task description: Apr-June 1958 issue, p. 30
Status: Reactivated. A small program and subroutine have been written and are in the process of being checked out. The program involves calculating b*, g*, for a certain g_m and $\rho(r)$.

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. New programs for data generation have been checked out.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Manager: P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. Production runs were made under the direction of the sponsor.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Inactive.

Status of Projects

1102-40-11645/59-389 FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Administration
Manager: L. S. Joel
Full task description: Oct-Dec 1958 issue, p. 29
Status: Continued. The revised computer program is being debugged. The interim program was used to solve several problems.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II
Origin and Sponsor: NBS, Section 4.6
Manager: A. E. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. The variational calculation of electron-hydrogen atom scattering has been extended to include p and d configuration in the trial function. Coding for this is essentially completed. A code is being prepared to calculate the energy eigenvalues of H^-, making use of part of the scattering trial function. As the lowest eigenvalue of H^- is well known, this will serve as a check on various subroutines used in the scattering calculation as well as allowing us to search for the existence of any higher bound eigenvalues.

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. Production has been continued. New test codes are in progress. A wave recognition procedure, utilizing smoothing transformations has been successful.

1102-40-11645/59-445 OIL SUPPLY
Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Inactive.

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. M. Beiman, W. G. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. Production runs of the four quarters for 1960 data have been made and the results submitted to the sponsor.

Status of Projects

1102-40-11645/60-462 CORRELATION OF FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Completed. Results transmitted to the sponsor.

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1959 issue, p. 26
Status: Inactive.

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. An attempt is being made to increase the accuracy of the integral evaluation by introducing more points in the Gaussian quadrature evaluation. Some runs were made with this modification and the results have been submitted to the sponsor for analysis.

1102-40-11645/60-467 TRANSISTOR SIMULATION
Origin and Sponsor: NBS, Section 12.1
Manager: G. W. Reitwiesner
Full task description: Oct-Dec 1959 issue, p. 27
Status: Inactive.

1102-40-11645/60-475 IONOSPHERIC SOUNDINGS
Origin and Sponsor: NBS, Section 82.40
Manager: M. L. Paulsen
Full task description: Oct-Dec 1959 issue, p. 29
Status: Completed. The Boulder Laboratories have written their own code to replace the IBM 704 Fortran Program and other calculations performed by their IBM 650. This new code was prepared especially for their CDC 1604 and is not an assembly of the Fortran code as was indicated in the last report.

1102-40-11645/60-476 GAS TUBE CHARACTERISTICS, II
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: H. Oser, W. Borsch-Supan
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Additional production runs have been completed, showing the dependence upon different parameters. The truncation errors have been reduced by formal solution of the corresponding finite and infinitesimal problem with constant field. A publication is in preparation.

Status of Projects

1102-12-11122/60-479 PROCESSING OF DIAGRAMS
Origin and Sponsor: NBS, Section 11.0
Managers: F. L. Alt (11.0), S. T. Peavy, R. J. Herbold
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. The results so far obtained on the computer have been analyzed. It appears that the"truncation errors" caused by using a finite grid, as well as the errors due to noise, were small enough to be tolerated in all the cases studied. They do not interfere, for example, with discrimination of patterns as similar as "O" and "Q". In fact, moments of order up to 4 or 5 seem to be sufficient for such purposes.

1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: R. Zucker
Full task description: Jan-Mar 1960 issue, p. 28
Status: Continued. Additional results of continued production runs were forwarded to the sponsor.

1102-40-11645/60-501 KANSAS RIVER SYSTEM
Origin and Sponsor: Corps of Engineers, U. S. Army, Office of District Engineers, Kansas City District
Manager: S. Peavy
Full task description: Apr-June 1960 issue, p. 24
Status: Inactive.

1102-40-11645/60-504 ELECTROSTATIC-FOCUSING PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. Beam
Full task description: Jan-Mar 1960 issue, p. 30
Status: Reactivated. Several production runs were made for the sponsor.

1102-40-11645/60-506 COMMODITY PRICE INDICES
Origin and Sponsor: U. S. World Bank, Statistics Division
Manager: J. C. Lamkin, Jr.
Full task description: Oct-Dec 1960 issue, p. 19
Status: Continued. Programming of price index computations is complete. Production runs will be made when data is received from the sponsor.

3911-61-39952/60-508 MODEL ADSORPTION ISOTHERMS
Origin and Sponsor: NBS, Section 5.2
Manager: J. P. Menard
Full task description: Apr-June 1960 issue, p. 25
Status: Inactive.

Status of Projects

1102-40-11645/60-510 H_2^+ BOMBARDMENT
Origin and Sponsor: Naval Research Laboratory
Manager: W. Borsch-Supan
Full task description: Apr-June 1960 issue, p. 26
Status: Completed. A report on the results of this project has been prepared, and publication is planned.

1102-40-11645/60-513 RADIATIVE ENVELOPES OF MODEL STARS
Origin and Sponsor: National Aeronautics and Space Administration
Managers: S. Haber (11.1) and P. J. Walsh
Full task description: Jul-Sept 1960 issue, p. 23
Status: Continued. Improvements were made in the program and further solutions were obtained.

1102-40-11645/61-516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE
Origin and Sponsor: NBS, Section 4.8
Manager: R. J. Herbold
Full task description: Jul-Sept 1960 issue, p. 24
Status: Continued. A new program to compute p (ρ,h) in double precision mode is checked out and results have been transmitted to the sponsor.

3911-61-39952/61-528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING
Origin and Sponsor: NBS, Section 14.1
Manager: R. Varner
Full task description: Jul-Sept 1960 issue, p. 28
Status: Completed. Several minor modifications were made in the existing programs. Production runs are being made under the direction of the sponsor.

1102-40-11645/61-530 SPECIMEN WAVELENGTH
Origin and Sponsor: NBS, Section 9.4
Manager: L. Joseph
Full task description: Jul-Sept 1960 issue, p. 28
Status: Inactive.

1102-40-11645/61-531 HEAT TRANSFER IN CRYSTALS
Origin and Sponsor: NBS, Section 3.1
Manager: H. Oser
Full task description: Jul-Sept 1960 issue, p. 29
Status: Continued. During this period code checking has been completed and trial production runs were made. The trial runs with maximum time interval Δt show that optimal programming, for the subroutine to multiply two matrices, is necessary in order to obtain reasonable computing times. With smaller time steps the method is satisfactory with respect to speed and accuracy. The stability against propagation of round-off errors is quite good for obtaining at least five-place accuracy for a signal which propagates completely through a system of 200 mass-points.

Status of Projects

1102-40-11645/61-532 CALCULATION OF VIBRATIONAL ENERGY LEVELS FOR IONIC MOLECULES

Origin and Sponsor: Georgetown University
Manager: P. J. Walsh
Full task description: Oct-Dec 1960 issue, p. 21
Status: Continued. Code checking on the problem described in the previous report (Oct-Dec 1960) has been completed and some production runs were made. These results were submitted to the sponsor for analysis.

1102-40-11645/61-536 SECULAR EQUATIONS

Origin and Sponsor: NBS, Section 13.2
Manager: Ruth Zucker
Full task description: Oct-Dec 1960 issue, p. 22
Status: Continued. Production runs were continued as requested by the sponsor.

1102-40-11645/61-537 MASS ACTION LAW

Origin and Sponsor: NBS, Section 5.2
Manager: H. Oser, J. Lamkin
Full task description: Oct-Dec 1960 issue, p. 23
Status: Continued. Program completed and being operated by sponsor.

1102-40-11645/61-538 SPECTRAL REFLECTANCE

Origin and Sponsor: NBS, Section 9.4
Manager: W. Borsch-Supan
Full task description: Oct-Dec 1960 issue, p. 23
Status: Continued. Programs are running which allow tabulation of the reflectance as a function of the wave length as well as a least square fit by changing all of the parameters. The results indicate that a reasonable approximation of the measurements by the given functions over the whole range of measured wave length is not possible. For shorter intervals of the wave length, however, approximations are quite good.

1102-40-11645/61-540 DIFFUSION CALCULATIONS

Origin and Sponsor: Army Chemical Center
Manager: L. Joseph
Objective: To calculate the areas in the x,y - plane bounded by certain constant density curves.
Background: The density distributions treated describe the diffusion through the atmosphere of a substance which deteriorates with time. The cases treated include the point source, the normal line source and the finite line source. The equations used are modifications of equations given by Sutton.

The problem was transmitted by C. G. Whitacre, (ACC).

Status: New. Fortran programs have been written for the normal line source and the finite line source. Preliminary production runs have been made and given to the sponsor for further study.

Status of Projects

1102-40-11645/61-542 STUDENT LOAN DATA
Origin and Sponsor: Department of Health, Education, and Welfare
Manager: R. Zucker
Full task description: Oct-Dec 1960 issue, p. 24
Status: Continued. Approximately 140 cross-tabulation tables, with their corresponding negative and non-response tables, were computed and submitted to the sponsor.

1102-40-11645/61-551 PARTICLE SIZE CALCULATIONS
Origin and Sponsor: NBS, Section 10.7
Manager: R. Zucker
Objective: To calculate the number, volume, and weight distributions, as well as surface area and number per unit weight, of glass beads sampled from two populations.
Background: The calculations arise in the study of experimental and computational problems entailed in the preparation of a particle size standard of known size distribution. Since some of the beads contained air bubbles, their weight and sedimentation rate did not correspond to their diameter.

The problem was transmitted by C. M. Hunt (10.7).

Status: New. The code has been written and checked out. The data have been processed in groups of 100 and 1,000 to determine the variation from sample to sample for two populations. These results have been transmitted to the sponsor.

1102-40-11645/61-556 TCHEBYCHEFF APPROXIMATION BY RATIONAL FUNCTIONS
Origin and Sponsor: NBS, Section 11.1
Manager: P. J. Walsh
Objective: Given a real function f(x), its (finite, real) domain S, and non-negative integers m and n; let

$$(1) \qquad J = \inf \max_{x \in S} \left| f(x) - \frac{\sum_{\nu=0}^{n} a_\nu x^\nu}{\sum_{\nu=0}^{m} b_\nu x^\nu} \right|$$

where the inf is taken over all possible choices of real numbers $a_0, \ldots, a_n, b_0, \ldots, b_m$ subject to

$$(2) \qquad \sum_{\nu=0}^{m} b_\nu x^\nu > 0 \quad \text{throughout } S.$$

Status of Projects

To prepare a code:

a) To compute J.
b) Given a positive δ, to determine a_ν and b_ν (subject to (2)), such that

$$\max_{x \in S} \left| f(x) - \frac{\sum_{\nu=0}^{n} a_\nu x^\nu}{\sum_{\nu=0}^{m} b_\nu x^\nu} \right|$$ will differ from J by less than δ.

Background: This problem of approximation arises in many physical and engineering problems. Experience shows that in many cases one gets a much closer approximation to a given function by using rational functions than by using polynomials having the same number of parameters (i.e., coefficients). Hence it is desirable to calculate Tchebycheff approximations to a given function by rational functions.

The problem was transmitted by O. Shisha (11.1).

Status: New. The problem has been programmed and code checking is in progress.

1102-40-11645/61-557 STRUCTURE DETERMINATION

Origin and Sponsor: NBS, Section 13.5

Manager: K. Bedeau

Objective: To compute $\sum_{k=1}^{x} S_k$ for various values of α, β and sets of x triples $(h_{1,k}, h_{2,k}, h_{3,k})$ where S_k is of the form:

$$\sum_i \cos 2\pi[(a_i + b_i\alpha)h_{1,k} + (c_i + d_i\alpha)h_{2,k} + (e_i + f_i\beta)h_{3,k}]^2$$
$$+ \sum_i \sin 2\pi[(a_i + b_i\alpha)h_{1,k} + (c_i + d_i\alpha)h_{2,k} + (e_i + f_i\beta)h_{3,k}]^2$$

Background: The above summations arise in the structure determination of solid β-oxygen.

The problem was transmitted by E. M. Horl (13.5).

Status: New. A code has been written and checked out. Several production runs have been made and the results turned over to the sponsor.

1102-40-11645/61-559 THERMOCOUPLE CALIBRATION

Origin and Sponsor: NBS, Section 3.1

Manager: K. Bedeau

Objective: To write a program for processing raw data taken directly from measuring instruments and to derive temperature-EMF tables for thermocouples. The program will consist of three independent phases:

(1) To determine a set of from three to nine EMF-temperature relationships for the thermocouple.
(2) To fit the results of phase 1 to a set of polynomials for EMF in terms of temperature from 0°C to 1450°C.
(3) To generate a table for each thermocouple.

Status of Projects

Background: Potentiometer and resistance readings are at present processed by hand to calibrate thermocouples. A considerable saving in time, an increase in accuracy, and a decrease in the temperature intervals at which the EMF is tabulated will be made possible by the program.

The problem was transmitted by J. P. Evans (3.1).

Status: New. A program is being written for phase 1.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. The following examples are representative of the statistical assistance and advice that members of this section provided during the quarter to Bureau personnel of other divisions:

(1) Pyrometer calibrations. H. H. Ku analyzed data on pyrometer calibrations for H. J. Kostkowsky of the Temperature Physics Section. The between-day and between-observer components of variance were estimated in connection with the study of precision of the calibration process.

(2) Ship steel. H. H. Ku collaborated with C. Staugaitis of the Mechanical Metallurgy Section on the statistical aspects of his final report on the notch-toughness of ship steel.

(3) Modulus of rupture of glass specimens. M. C. Dannemiller and J. M. Cameron completed the study of the modulus of rupture of glass specimens. A memorandum describing the results and the statistical techniques which were employed was sent to M. J. Kerper of the Glass Section.

(4) Distribution of standardized gaps. M. C. Dannemiller collaborated with J. Mandel of the Organic and Fibrous Materials Division in an empirical sampling study to determine the distribution of standardized gaps arising from a normal population.

Publications:

(1) Propagation of error in a chain of standards. A. G. McNish (Metrology Division) and J. M. Cameron. IRE Trans. Instrumentation I-9, 101-104 (1960).
(2) Stress-rupture tests at 1350F on Type 304 stainless steel. W. D. Jenkins and W. A. Willard (Metallurgy Division), and W. J. Youden. Materials Res. Standards 1, 104-108 (1961).
(3) How to evaluate accuracy. W. J. Youden. To appear in Materials Research and Standards.
(4) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability.
(5) Systematic errors in physical constants. W. J. Youden. Submitted to a technical journal.
(6) Variability of color-mixture data. I. Nimeroff (Photometry and Colorimetry), J. R. Rosenblatt, and M. C. Dannemiller. In manuscript.

Status of Projects

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Page proofs of the Tables Section have been corrected. Text for two other sections are in the hands of the contractor to the Office of Ordnance Research, Department of the Army. The remaining two sections are in the final stages of preparation for publication.

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. A discriminant analysis was performed on data from the Veteran's Administration Cooperative Chemotherapy Study V.

Current Applications of Automatic Computer

The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
NBS:					
11110/55-55	11.1	Research in numerical analysis	33	4	272
11411/55-56	11.1	Research in mathematical topics applicable to numerical analysis	9	24	25
11120/55-65	11.2	Automatic coding	6	11	
39952/56-160	11.2	Mathematical subroutines	236	122	474
39951/51-1	11.3	Statistical engineering	91	30	382
39952/54-30	13.1	Spectrum analysis	180	94	1627
39952/55-68	9.7	Crystal structure calculations	28	4	414
39952/55-82	3.1	Thermometer calibrations	8		225
39952/56-131	2.2	Calculations in optics*	12	3	59
11645/56-166	5.9	SCF-LCAO solution of some hydrides*			57
11645/56-171	3.2	Collision integrals used in transport theory**	95	139	77
11645/57-219	3.2	Thermal properties*	123	84	312
11645/57-223	3.2	Self-consistent fields		4	45
11645/57-246	4.8	Radiation diffusion**	1154	501	1456
39952/57-250	2.3	Automatic reduction in spectrophotometric data*			83
11645/57-251	1.6	Current noise and fixed resistors*	14		2
11645/57-252	4.11	Detecting efficiency in a neutral meson experiment**	35	14	74
39952/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*			31
11645/58-255	4.8	Chi functions**	923	259	999
11645/58-256	10.6	Composite walls**	78	72	73
11645/58-260	12.5	Prototype accounting**	29	120	152
11645/58-271	6.3	Simultaneous equations for potential flow**	5		4
11645/58-272	3.2	Thermodynamic properties of real gases	9	6	52
11645/58-274	9.7	Calculations for d-spacings II*			56
11645/58-275	7.8	Crystallography**			22
11645/58-294	4.8	Nuclear scattering of photons*	5		59

Current Applications of Automatic Computer

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/58-308	3.4	Oscillating sphere*			11
11645/58-314	3.7	Approximations for gas mixtures*	212	274	415
/58-333,	9.0	Calcium hydroxide*	63	5	93
/58-357	3.3	Eigenvalues**	60	24	313
/58-358	3.2	Reduced cross sections	127	42	220
/59-387	30.4	Nuclear reactor design**			78
/59-388	10.3	Heat pump calculations*			1
/59-394	4.6	Slow electron scattering by hydrogen atoms	120	108	21
/59-395	7.7	Adsorption study**	52	60	41
/59-403	2.1	Computation of color fadings*			3
/59-409	12.5	Bank Board**	152	752	1503
/59-414	3.0	Infinite systems	9	55	
/59-417	2.4	Spectrum analysis of ruthenium**			118
/59-418	4.8	P-Wave equation*	174	97	171
/59-421	12.5	Traffic simulation**	18	92	
/59-428	12.5	Radio intensities**	199	33	583
/59-433	2.3	Color of signals**			31
/59-440	82.10	Mapping**	104	617	1402
/60-449	2.1	Spectral line colorimetry	13		2
/60-457	12.5	Public Housing problem**	33	166	1281
/60-466	3.2	Electronic properties of simple molecular systems		158	136
/60-474	2.5	Gage block stability*	12		19
/60-484	7.6	Polymer crystallization			10
/60-486	3.6	Morse wave function			148
/60-489	3.1	Inversion of line probe data			3
/60-493	3.7	Poisson distribution function**	343	133	189
/60-508	5.2	Model adsorption isotherms	7	25	
/60-514	3.9	Flame spectra**			30
/60-515	13.5	Convolution integral**		39	
/61-516	4.8	Radiation field from a circular disk source	55	131	34
/61-520	2.4	Heat expansion			14
/61-523	4.0	Neutron cross section computations**	163	125	483
/61-525	3.8	Curve fitting of wave functions**			8
/61-526	3.0	Crystal field calculations	30		
/61-528	14.1	Transistor aging	50	54	296
/61-530	9.4	Specimen wave lengths	29	4	
/61-531	3.1	Heat transfer in crystals	104	59	132
/61-536	3.2	Secular equations			17
/61-537	15.2	Mass action law	9	34	58
/61-538	9.2	Curve fitting for spectra reflectance data	63	69	9

Current Applications of Automatic Computer

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/61-546	11.1	Operations research in optimization techniques	21	27	22
/61-549	4.2	Least squares computations			7
/61-551	10.7	Particle size calculations	7	12	24
/61-553	12.1	Card-to-tape conversion			23
/61-557	13.5	Structure determination	38	12	9
		Miscellaneous			15
		Totals (NBS Services).	5,340	4,698	15,005
OUTSIDE:					
11645/57-200	DTMB	(DTMB)**			449
/58-269	NRL	Molecular structure, IV			109
/58-270	PO	Post Office problem	8	62	32
/58-276	NOL	General kinetics, I**			7658
/58-325	VA	Covariance analysis	14	8	28
/58-335	DOFL	Roots of Bessel functions**			84
/58-348	OOR	Russian-to-English machine translation	18	20	
/58-361	NRL	Spectrum of dipole radiation	90	33	57
/58-366	USIA	Radiation patterns of antennas	11	26	80
/58-368	SC	Intensity functions of light scattered by spherical particles	3	23	13
/59-371	NRL	ASWAP°	1628	3	2347
/59-373	DOFL	Rhinitis**	23		36
/59-389	CAA	Frequency allocation	4	9	34
/59-407	DOFL	Fourier coefficients*	142	58	36
/59-408	NASA	NASA**	3		23
/59-411	HEW	Fitting of exponential curves**	237	202	782
/59-415	DOFL	Complex Legendre functions*	209		16
/59-416	DOFL	Analysis of power supply experiments**	31		70
/59-419	DOFL	Neutrons°	6	62	892
/59-423	WB	Weather Bureau**			1251
/59-425	CU	Molecular orbitals*			45
/59-434	CIW	Petrological computations*	10		2
/59-435	VA	Electrocardiographic analysis	114	358	1116
/59-437	BPR	Highway study	41	167	1271
/59-441	GK	Systems engineering**	77	107	555
/59-447	BPRO	Public Roads study**	302	657	7530
/60-454	GE	G.E.**	39	79	297
/60-458	CAB	Domestic airline traffic survey	205	624	3809
/60-462	DOFL	Correlation of functions	153		271

Current Applications of Automatic Computer

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/60-476	DOFL	Gas tube characteristic II	21	20	552
/60-481	SC	Radar study°	21		19
/60-492	IMF	Monetary research reports**	181	12	468
/60-496	BPA	Short circuit program**			16
/61-499	DOFL	Equilibrium calibration	25	81	86
/60-502	USA	Quartermaster mathematics programming**	213	21	465
/60-504	DOFL	Electrostatic focusing			50
/60-506	WB	World Bank**	4	24	2
/60-512	DOFL	Data transformation**	28	9	61
/60-517	DOFL	Shock tube**	139	41	489
/61-518	DOFL	Reliability of binary counter**			144
/61-532	GU	Vibrational energy levels for ionic molecules**	4	52	115
/61-533	U MD	Datatrol computations**	9	392	70
/61-540	EDARS	Edgewood Arsenal°	51	40	170
/61-542	HEW	Student loan data		99	1140
/61-543	DOFL	Fourier analysis°	16	20	5
/61-545	WEST	Nuclear reactor design**			6788
/61-550	GWU	Logistics research**	14	6	
/61-552	DOFL	Transistor data**	34	14	8
/61-561	OSR	Zone refinements for quartz**	3		
		Totals (Outside)	4,131	3,349	39,541
		Total time for the quarter (MINUTES).....	9,471	8,047	54,546
		Total time for the quarter (HOURS)	158	134	909

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

° Classified task.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

AGMON, S. (New York University) Growth properties of the resolvent of a class of operators with applications to non-self-adjoint differential boundary problems. March 23.

MORDELL, L. J. (St. Johns College, Cambridge University and University of Notre Dame) Dedekind sums. February 17.

Theory of Errors Colloquium

EISENHART, C. This series, open to members of the Bureau staff, aims to provide a thorough consideration of selected topics in the Theory of Errors that are especially pertinent to the calibration and basic measurements programs of the Bureau. The following subjects have been treated this quarter: (1) The postulate of direct measurement and the strong law of large numbers. January 12; (2) Some mathematical models of measurement processes. January 26; (3) The measurement of unprecision. February 23; (4) The probable error--never was what it used to be. March 9; (5) More on intervals of the form $(\bar{x}-kS, \bar{x}+kS)$. March 23.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. Recent results in machine translation. Presented at the Massachusetts Institute of Technology, Cambridge, Mass., March 17.

BAZLEY, N. (1) Lower bounds for eigenvalues. Presented at the Rensselaer Polytechnical Institute, Troy, New York, January 9, 1961. (2) Lower bounds for eigenvalues of the Schrödinger equation. Presented at the meeting of the American Physical Society, New York City, February 2.

BENDER, B. K., and GOLDMAN, A. J. A simple signal identification model. Presented at the meeting of the American Mathematical Society, Washington, D. C., January 23-26.

Lectures and Technical Meetings

BORSCH-SUPAN, W. Errors in the computation of polynomial roots. Presented at the Oak Ridge National Laboratory, March 21.

CAMERON, J. M. Propagation of error in a chain of standards. Presented at the American Institute of Electrical Engineers, Washington, D. C., February 21.

DAVIS, P. J. Weak theory convergence and its application to approximation. Presented at the Department of Mathematics, Georgetown University, Washington, D. C., January 6.

EDMONDS, J. R. Symmetric embeddings of complete graphs. Presented at the meeting of the American Mathematical Society, Washington, D. C., January 23-26.

EISENHART, C. Precision and accuracy--experiment design aspects. Presented before the American Society for Quality Control, Metropolitan Section, Seton Hall University, South Orange, New Jersey, February 9. To fit or not to fit; that is the question... (1) Presented at the Statistics Seminar, Yale University, March 20. (2) Presented at the Statistics Colloquim, Harvard University, March 22.

HABER, S. An asymptotic expansion of the error of numerical quadrature schemes. Presented at the meeting of the American Mathematical Society, Washington, D. C., January 23-26.

MARCUS, M. Convex functions and matrix inequalities. Presented at the Graduate Mathematics and Applied Science Colloquium, Adelphi College, Garden City, L. Is., March 16.

RHODES, I. (1) Mechanical translation. Presented at the Moore School of Engineering, University of Pennsylvania, Philadelphia, Pa., February 1. (2) Machine translation by means of electronic computers. Presented at the Mathematics In-Service Institute, District of Columbia Teachers College, Washington, D. C., February 25. (3) Machine translation. Presented before the Rochester Section, Optical Society of America, Rochester, N. Y., March 21.

ROSENBLATT, J. R. Statistical problems in colorimetry and paired comparison experiments. Presented before the Department of Statistics, Virginia Polytechnic Institute, Blacksburg, Va., February 23.

SHISHA, O. A structure theorem for infrapolynomials with prescribed coefficients. Presented at the meeting of the American Mathematical Society, Washington, D. C., January 23-26.

Lectures and Technical Meetings

YOUDEN, W. J. (1) Systematic errors in physical constants. Presented at the meeting of the Advanced Seminars on Experimental Designs, held by the U. S. Army at the University of Wisconsin, Madison, Wis., January 10. (2) Elementary statistical design. Presented before the Society of Plastic Engineers, Washington, D. C., January 25. (3) Elementary statistical design. Presented before the American Society for Quality Control, Seton Hall University, South Orange, N. J., February 4. (4) Systematic errors in physical constants. Presented before the Standards Division of the American Ordnance Association, Albuquerque, N. Mex., February 15. (5) Better answers with less work. Presented at the Joint Meeting of the Buffalo-Niagara Sections of the American Statistical Association and the American Society for Quality Control, Tonawanda, N. Y., February 27. (6) Applications of statistics in experiments at NBS. Presented at The Johns Hopkins University, School of Hygiene and Public Health, Baltimore, Md., March 10. (7) Test programs for the statistical evaluation of dental materials. Presented before the Dental Materials Group, International Association for Dental Research, Boston, Mass., March 23-26.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

The following papers appeared in J. Research NBS 65B (Math. and Math. Phys.), Jan-Mar 1961:

(1) Bound for the P-condition number of matrices with positive roots. P. J. Davis, E. Haynsworth and M. Marcus. Pp. 13-14.

(2) Selected bibliography of statistical literature, 1930-1957: IV. Markov chains and stochastic processes. L. S. Deming and D. Gupta (Catholic University). Pp. 61-93.

(3) Special types of partitioned matrices. E. Haynsworth. Pp. 7-12.

(4) Index to the distributions of mathematical statistics. Frank A. Haight. Pp. 23-60.

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(5) Robustness of life testing procedures based on the exponential distribution. M. C. Dannemiller and M. Zelen. Missile Design and Development, pp. 35-36, 99; March 1961.

(6) Stress rupture tests at 1350F on Type 304 stainless steel. W. D. Jenkins and W. A. Willard (Metallurgy Division), and W. J. Youden. Materials Res. Standards 1, 104-108 (1961).

(7) Propagation of error in a chain of standards. A. G. McNish (Metrology Division) and J. M. Cameron. IRE Trans. Instrumentation I-9, 101-104 (1960).

(8) Periodicity modulo m and divisibility properties of the partition function. M. Newman. Trans. Amer. Math. Soc. 97, 225-236 (1960).

(9) Subgroups of the modular group and sums of squares. M. Newman. Amer. J. Math. 82, 761-778 (1960).

(10) The permanent function as an inner product. M. Marcus and M. Newman. Research Announcement, Bull. Amer. Math. Soc. 67, 223-224 (1961).

Publication Activities

(11) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. Technometrics 3, 29-49 (1961).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Technical Notes, Manuals, and Bibliographies

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Fractional factorial designs for experiments with factors at two and three levels. To appear as Applied Mathematics Series 58.

2.3 Technical Papers

(1) Recognition of clauses and phrases in machine translation of languages. F. L. Alt and I. Rhodes. To appear in the Proceedings of the International Conference on Machine Translation of Languages and Applied Language Analysis, Teddington, England, Sept. 6-8, 1961.

(2) Automatic screening of normal and abnormal electrocardiograms by means of a digital electronic computer. R. J. Arms with H. V. Pipinger and F. W. Stallman (Veterans Administration, Mt. Alto, and Georgetown University School of Medicine).

(3) Lower bounds for eigenvalues of Schroedinger's equation. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). Submitted to a technical journal.

(4) Truncations in the method of intermediate problems for lower bounds to eigenvalues. N. W. Bazley and D. W. Fox (Applied Physics Laboratory, JHU). To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Phys.).

(5) A new decomposition formula in the theory of elasticity. J. H. Bramble and L. E. Payne. To appear in the Journal of Research, NBS, Sec. B.

(6) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington Univeristy, Washington, D. C., 1960.

(7) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. To appear in the Journal of Mathematical Analysis and Applications.

(8) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.

Publication Activities

(9) Combination of observations. C. Eisenhart. To appear as Chapter 7 in "Boscovich--Essays on His Life and Work", ed. by Launcelot Law Whyte.

(10) Precision and accuracy--experiment design aspects. C. Eisenhart. To appear in "Proceedings of a Symposium on Statistical Methods in the Chemistry Industry", to be held by the ASQC, Spring, 1961.

(11) On some partial differential equations of Brownian motion of a free particle. A. Ghaffari. To appear in the "Proceedings of the International Conference on Partial Differential Equations and Continuum Mechanics", held at Madison, Wis., 1960.

(12) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. To appear in the American Mathematical Monthly.

(13) A Stirling number proof of Von Staudt's theorem. K. Goldberg. Submitted to a technical journal.

(14) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.

(15) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.

(16) Another extension of Heinz's inequality. M. Marcus. To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Phys.).

(17) Comparison theorems for symmetric functions of characteristic roots. M. Marcus. To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Phys.).

(18) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). To appear in the Canadian Mathematical Bulletin.

(19) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). Submitted to a technical journal.

(20) Congruence properties of the partition function to composite moduli. M. Newman. To appear in the Illinois Journal of Mathematics.

(21) Pointwise bounds in the Cauchy problem of elastic plates. L. E. Payne. To appear in the Journal of Research, NBS, Sec. B.

(22) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

(23) Best approximations and interpolating functions. J. R. Rice. To appear in the Transactions of the American Mathematical Society.

Publication Activities

(24) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.

(25) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Transactions of the American Mathematical Society.

(26) Mean motion in conditionally periodic separable systems. J. Vinti. To appear in the Journal of Research, NBS, Sec. B.

(27) Optimal approximation for functions prescribed at equally spaced points. H. F. Weinberger. To appear in the Journal of Research, NBS, Sec. B (Math. and Math. Phys.).

(28) Partial confounding in fractional replication. W. J. Youden. Submitted to a technical journal.

(29) How to evaluate accuracy. W. J. Youden. To appear in Materials Research and Standards (formerly ASTM Bulletin).

(30) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(31) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, 1960.

(32) Systematic errors in physical constants. W. J. Youden. Submitted to a technical journal.

(33) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. To appear in the Journal of the American Statistical Association.

(34) A calculus for factorial arrangements. M. Zelen and B. Kurkjian (Diamond Ordnance Fuze Laboratories). Submitted to a technical journal.

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Electricity. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics.

Metrology. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

Heat. Temperature Physics. Heat Measurements. Cryogenic Physics. Equation of State. Statistical Physics.

Radiation Physics. X-ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

Analytical and Inorganic Chemistry. Pure Substances. Spectrochemistry. Solution Chemistry. Standard Reference Materials. Applied Analytical Research.

Mechanics. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Rheology. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics. Electrolysis and Metal Deposition.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Crystal Growth. Physical Properties. Constitution and Microstructure.

Building Research. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics. Operations Research.

Data Processing Systems. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Applications Engineering.

Atomic Physics. Spectroscopy. Infrared Spectroscopy. Solid State Physics. Electron Physics. Atomic Physics.

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Ionosphere Research and Propagation. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services.

Radio Propagation Engineering. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Interval Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

Radio Systems. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems.

Upper Atmosphere and Space Physics. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.


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