NATIONAL BUREAU OF STANDARDS REPORT

7110

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1960

For Official Distribution

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to government agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. Research projects are also performed for other government agencies when the work relates to and supplements the basic program of the Bureau or when the Bureau's unique competence is required. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($1.50), available from the Superintendent of Documents, Government Printing Office, Washington 25, D.C.

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NBS PROJECT 11.0

NBS REPORT 7110

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APPLIED MATHEMATICS DIVISION

October 1 through December 31, 1960

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Chan Mou Tchen, Ph.D.
John P. Vinti, Sc.D.

*On leave of absence **Part time °°Guest Worker

°Appointed December 30, 1960

Contents

*Only unclassified projects are included in this report.

Status of Projects

December 31, 1960

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. W. Borsch-Supan has begun programming the numerical computation of lower bounds of the eigenvalues of the prolate spheroidal wave equation using the method developed by N. Bazley and D. W. Fox. The program allows arbitrary parameters of the spheroidal wave equation and arbitrary orders of truncation and intermediate problem. Upper bounds are also computed so as to estimate the eigenvalues to any desired accuracy within single-word precision.

N. Bazley has completed a joint paper with D. W. Fox of the Applied Physics Laboratory, the Johns Hopkins University, entitled "Truncations in the Method of Intermediate Problems for Lower Bounds to Eigenvalues". The paper gives two new procedures for obtaining numerical lower bounds to the eigenvalues of linear operators. The procedures are based on the spectral theory of self-adjoint operators in Hilbert space and apply to several problems in classical and quantum mechanics.

S. Haber obtained an expansion, asymptotic in n, of the error of a numerical quadrature scheme of the form n x Q. Here Q is a fixed numerical quadrature rule, e.g., the trapezoidal rule $\int_a^b f(x)dx \approx \frac{b-a}{2}(f(a) + f(b))$, and n x Q the composite rule in which the interval of integration is divided into n equal subintervals and the rule Q applied in each. He is continuing work on numerical quadrature schemes.

O. Shisha's investigations during this quarter have covered:

(1) A study of approximation to convex (or monotone) functions by means of convex (monotone) polynomials and trigonometric polynomials. The results contain some explicit constructions as well as estimates of accuracy, for approximations of the above type.

(2) An investigation of best Tchebycheff approximation by rational functions, with stress laid on the computational aspect.

(3) Obtaining some results on sequences of analytic functions whose Taylor coefficients are restricted by certain conditions.

(4) Work on a joint paper with Professor J. L. Walsh of Harvard, "The zeros of infrapolynomials with some prescribed coefficients" (a revision of a technical report published in July 1960 and written under Contract AF49 (638)574, 62 p.; for an abstract, see ASTIA Technical Abstract Bulletin, 1 December 1960, p. 72).

(5) A continuation of research on infrapolynomials and their generalizations. Two papers, one on the structure of infrapolynomials with prescribed coefficients and one on the location of the zeros of infrapolynomials, are in manuscript.

P. Davis and O. Shisha are preparing a survey article on the theory of the transfinite diameter.

M. Marcus and M. Newman are preparing a paper in which the permanent function is exhibited as a special case of an inner product on a suitable space of tensors. Using this device several new results on v, k, λ configurations and doubly stochastic matrices have been obtained.

J. Edmonds has defined certain abstract objects called "combinatorial polytopes" which, though only in special cases correspond to geometric objects, are combinatorially the most natural generalization of polyhedra to higher dimensions. The graph-embedding theorem generalizes in n dimensions to one about these objects. The polytopes are valuable in constucting symmetric graphs independent of regular maps. In particular, a k-dimensional quasi-regular polytope can be constructed with an automorphism group isomorphic to any given group with k generators. This group is transitive on the edges and vertices of the polytope, so the graph is symmetric and has this same group of automorphisms. A manuscript entitled "Symmetric graphs and symmetric embeddings" is being prepared.

J. Edmonds has used polytopes and the generalized embedding theorem to prove the following topological result: The Poincaré conjecture is equivalent to the conceptually simpler statement that a simply connected 2-complex is embeddable in E^3 if some neighborhood of its 1-skeleton is.

Publications:

(1) Sequence transformations based on Tchebycheff approximations. J. R. Rice. J. Research NBS, 64B, 227-235 (1960).
(2) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.
(3) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.
(4) Special types of partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.
(5) Bounds for determinants with positive diagonals. E. Haynsworth. Trans. Amer. Math. Soc. 96, 395-399 (1960).
(6) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.
(7) Bound for the P-condition number of matrices with positive roots. E. Haynsworth, P. Davis, and M. Marcus. To appear in the Journal of Research, NBS, Sec. B.
(8) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. J. Research NBS 64B, 211-215 (1960).
(9) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. J. Research NBS 64B, 217-225 (1960).
(10) Best approximations and interpolating functions. J. R. Rice. Submitted to a technical journal.
(11) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.

(12) Another extension of Heinz' inequality. M. Marcus. To appear in the Journal of Research, NBS, Sec. B.
(13) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). To appear in the Canadian Mathematical Bulletin.
(14) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). Submitted to a technical journal.
(15) The permanent function of an inner product. M. Marcus and M. Newman. To appear as a Research Announcement of the Bulletin of the American Mathematical Society.
(16) Comparison theorems for symmetric functions of characteristic roots. M. Marcus. In manuscript.
(17) Chebychev approximations by ab^x+c. J. R. Rice. J. Soc. Ind. Appl. Math. 8, 691-702 (1960).
(18) Some computational problems concerning integral matrices. O. Taussky-Todd. To appear in the Journal of Research, NBS, Sec. B.
(19) Computation problems concerned with the Hilbert matrix. John Todd. To appear in the Journal of Research, NBS, Sec. B.
(20) The permanent function. M. Marcus and F. May (University of British Columbia). To appear in the Canadian Journal of Mathematics. (C)
(21) Symmetric means and matrix inequalities. M. Marcus and P. Bullen (University of British Columbia). To appear in the Proceedings of the American Mathematical Society. (C)
(22) On the completion of a unitary matrix. P. Greiner (Yale University) and M. Marcus. To appear in the Illinois Journal of Mathematics. (C)
(23) On the relation between the permanent and the determinant. M. Marcus and H. Minc (The University of Florida). To appear in the Illinois Journal of Mathematics. (C)
(24) Linear operations on matrices. M. Marcus. In manuscript.

OPERATIONS RESEARCH
Task 1101-12-11115/61-546

Origin: NBS Authorized 12/30/60
Manager: A. J. Goldman

Objective: To conduct research in mathematical and computational techniques relevant to the optimization or improvement of complex systems or activity-patterns.

Background: Since World War II there has been an increasing use of fairly sophisticated scientific and mathematical techniques for the analysis and improvement of complex systems and organizations, for example, in logistics, in the comparison and evaluation of weapons systems without actual physical testing and without study of communications on transportation networks. These areas present novel mathematical and computational problems, and it is extremely desirable to have a research program concerned with the development of new and more powerful theoretical techniques for such problems as distinguished from the solution of immediate problems as they arise one by one. The work is relevant both to the Bureau's practice of supplying

operations research assistance to other Government agencies lacking specialized personnel in that field, and also it is hoped to the activities of NBS itself as a large and complex organization.

Status: NEW. B. K. Bender (11.2) started coding the Boolean simplification method as presented in the NBS report, "Computer simplification of Boolean functions," (with some modifications) for the 704 computer.

Publication:

(1) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. K. Goldberg considered various problems in "invertible" power series:

$$A(z) = z + a_2 z^2 + \ldots, \quad [A(z)]^m = \sum_{n=m}^{\infty} a_n^{(m)} z^n.$$

If $A(B(z)) = z$, E. Jabotinsky has shown that then $n a_n^{(m)} = m b_{-m}^{(-n)}$. This is now generalized to

$$b_{n+m}^{(m)} = \frac{m}{n!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{(n+m+1)(n+m+z)\ldots(2n+m)}{n+m+k} a_{n+k}^{(k)}$$

which implies relations for the positive ($n+m > 0$) and zero ($n+m = 0$) indexed coefficients as well as the known relation ($n+m < 0$). It is also pointed out that Jabotinsky's result implies the inequality $|nb_n|^{-1/n} \geq r^{-1/n} \min_{|z|=r} |A(z)|$ for all r less than the radius of convergence of $z/A(z)$, which in turn gives a lower bound for the radius of convergence of $B(z)$.

Dr. Goldberg investigated the power series $F(z)$, uniquely determined by a fixed invertible $A(z)$, with the property that $F(A(z)) = F(z)A'(z)$. He found simple proofs for the theorems of E. Jabotinsky, M. Kuntz, and J. Hadamard that $A(B(z)) = B(A(z))$ if and only if $F(B(z)) = F(z)B'(z)$, and related results. He showed that there exists an invertible $H(z)$ such that $F(H(z))/H'(z) = z^p + c_A z^{2p-1}$ where $A(z) = z + a_p z^p + \ldots$, $a_p \neq 0$, and c_A is the residue of $1/F(z)$ at $z = 0$.

Dr. Goldberg also showed that if $f(z)$ is any nonlinear power series with only non-negative coefficients then there exists an invertible polynomial

$p(z)$ with some negative coefficients such that $f(p(z))$ has only non-negative coefficients. This disproves a conjecture concerning exp $p(z)$, the simplest counter-example for which is $p(z) = z - Az^2 + z^3 + z^4$ with A less than the positive root of $x^3 + 12x - 6 = 0$ (which is slightly less than 1/2).

M. Newman has completed a manuscript entitled "Congruence properties of the partition function to composite module", in which it is shown that if P(n) denotes the number of unrestricted partitions of n and r is an integer, $0 \leq r \leq 64$, then each of the 65 congruences $P(n) \equiv r \pmod{65}$ has infinitely many solutions n. Similar results are proved for other moduli.

A study of a certain class of modular forms of negative fractional dimension is under way, and recurrence formulae for the coefficients of these forms have been determined.

Publications:

(1) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
(2) Irrational power series. M. Newman. Proc. Amer. Math. Soc. 11, 699-702 (1960).
(3) Periodicity modulo m and divisibility properties of the partition function. M. Newman. To appear in the Transactions of the American Mathematical Society.
(4) Generating functions for formal power series in noncommuting variables. K. Goldberg. Proc. Amer. Math. Soc. 11, 988-991 (1960).
(5) The minima of cyclic sums. K. Goldberg. J. London Math. Soc. 35, 262-264 (1960).
(6) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. To appear in the American Mathematical Monthly.
(7) Some SEAC computations of subsonic flows. P. Davis and P. Rabinowitz. In "Bergman's Linear Integral Operator Method in the Theory of Compressible Fluid Flow," by M. Z. v. Krzywoblocki, pp. 148-172 (Wien, Springer-Verlag, 1960).
(8) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. Submitted to a technical journal.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: P. Davis
Full task description: July-Sept 1955 issue, p. 7

Status: INACTIVE.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter, with priority being given to preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 SIEVERT'S INTEGRAL

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The texts for Chapter 8 (Legendre Functions) and Chapter 18 (Weierstrass Elliptic Functions) were distributed for comments and suggestions. Chapter 4 (Elementary Transcendental Functions) is in press. Drafting of graphs and line cuts has been completed for 14 chapters. Manuscripts of the remaining chapters are undergoing review for updating of references, insertion and checking of cross references, etc.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. A preliminary routine of an automatic operating system for computers of the IBM 7030 or RCA 601 class was prepared. The system would utilize a small satellite computer as a "secretary" for the large computer, and communication between the computers would be by magnetic tapes shared by the computers.

During this quarter, the Tablemaker system was used for a total computer time of 485 minutes. The following table indicates the number of times that the various functions were used.

Function	Number of times used
Curve fit	311
Plot	36
Interpolation	60
Integration	8
Summation	27
Correlation	78

The curve fitting function was expanded to include 12 coefficients, instead of 8.

MATHEMATICAL SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: INACTIVE.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. M. Zelen and M. C. Dannemiller have resumed work on the NBS tables of power points of the analysis-of-variance test. The non-centrality parameter $\emptyset$ of the non-central F distribution is tabulated as a function of the level of significance α, power β, and degrees of freedom ν_1, ν_2 for a variety of these parameters. The main body of this work was done in the Computation Laboratory; see April-June 1951 issue for a full task description. Publication has been held up awaiting the computations at $\nu_1 = \infty$ and $\nu_2 = \infty$. The values of $\emptyset$ when $\nu_1 = \infty$ are now in the process of being computed. This has necessitated the calculation of the .80 and .98 probability points of the χ^2 distribution to 5 decimals. Trial calculations have been started for values of $\emptyset$ at $\nu_2 = \infty$.

J. Van Dyke is investigating numerically the properties of the family of random variables

$$z = Au^{\alpha} - B(1-u)^{\beta} + C,$$

where u is uniformly distributed on (0,1), A and B are positive, and $|\alpha|$, $|\beta|$ are less than unity. This study was suggested by J. W. Tukey of Princeton University, to extend his results for the case A = B, $\alpha = \beta$, C = 0, for which he determined values of A and α leading to "good" approximations (for example) to the normal distribution and to the distribution of Student's t with 1, 2, ..., etc., degrees of freedom.

Publications:

(1) Graphs for bivariate normal probabilities. M. Zelen and N. C. Severo. Ann. Math. Stat. 31, 619-624 (1960).
(2) Selected bibliography of statistical literature, 1930-1957. IV. Markov chains and stochastic processes. Lola S. Deming and D. Gupta (Catholic University). To appear in Journal of Research NBS, Section B, Mathematics and Mathematical Physics.
(3) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research NBS, Section B, Mathematics and Mathematical Physics.
(4) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Biometrika 47, Parts 3 & 4 (1960).
(5) Boscovich and the combination of observations. Churchill Eisenhart. To appear as Chapter 7 in "Boscovich--Essays on His Life and Work", edited by Lancelot Law Whyte.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52
Manager: J. M. Cameron
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. M. Zelen and B. Kurkjian have obtained new results on regression models. These results are contained in "A calculus for factorial experiments" by B. Kurkjian and M. Zelen.

Publications:

(1) Randomization and experimentation. W. J. Youden. To appear in the Annals of Mathematical Statistics.
(2) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability.
(3) A calculus for factorial arrangements. B. Kurkjian (Diamond Ordnance Fuze Laboratories) and M. Zelen. Submitted to a technical journal.
(4) Partial confounding in fractional replication. W. J. Youden. Submitted to a technical journal.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: CONTINUED. J. R. Rosenblatt is studying methods for adapting multiple comparison techniques (Scheffé type) for use in the analysis of large scale incomplete paired-comparison experiments.

A. Glinski and J. R. Rosenblatt are investigating some properties of a two-sample procedure proposed by L. Weiss (Naval Research Logistics Quarterly, September 1960) for obtaining a distribution-free confidence interval of fixed length for a quantile of a continuous distribution. One goal of the study is to analyze the effect of various possible ways of choosing certain arbitrary constants, which may be varied at the discretion of the experimenter.

Publication:

(1) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. To appear in the Journal of the American Statistical Association.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: INACTIVE.

Publications:

(1) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. To appear in Technometrics.

(2) On prediction of system behavior. J. R. Rosenblatt. Reprinted in IRE Transactions on Reliability and Quality Control, RQC-9, 23-28 (1960), from "Proceedings of the NYU-Industry Conference on Reliability Theory (June 1958)."

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell .
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. J. H. Bramble and L. E. Payne are studying methods of obtaining bounds for the solutions of problems in the theory of elastic plates. Methods have already been developed for obtaining pointwise bounds in the standard boundary value problems. Dr. Bramble is on the staff of the Institute for Fluid Dynamics and Applied Mathematics of the University of Maryland, and joined the staff of the Mathematical Physics Section in November on a part-time basis.

A paper entitled "A New Decomposition Formula in the Theory of Elasticity" by Drs. Bramble and Payne has been prepared.

A. Ghaffari has previously shown (see January-March 1960 issue, pp. 14-15, and July-September 1960 issue, p. 11) that the most general one-dimensional solutions of the Chapman-Kolmogoroff functional equation on the interval $(0,\infty)$ can be found as series whose terms contain the Laguerre polynomials and which under certain conditions converges absolutely and uniformly over the interval in question. These solutions differ from those given by A. N. Kolmogoroff (Math. Ann. 104, 1931). The steady state of these solutions, which represent the transition probability function in the theory of Brownian motion, has been studied and it has been shown that the limit of the probability as the time tends to infinity depends completely on the initial state, i.e., the probability is non-oscillatory.

An attempt is being made to obtain a derivation of the corresponding second order partial differential equations.

Publications:

(1) The Stokes flow about a spindle. W. H. Pell and L. E. Payne. Quar. Appl. Math. 18, 257-262 (1960).
(2) On some partial differential equations of Brownian motion of a free particle. A. Ghaffari. To appear in the "Proceedings of the International Conference on Partial Differential Equations and Continuum Mechanics," held at the Mathematics Research Center, U. S. Army, Madison, Wis., June 1960.
(3) Pointwise bounds in the Cauchy problem of elastic plates. L. E. Payne. To appear in the Journal of Research NBS, Sec. B.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti is continuing the preparation of a paper in which the problem of finding an accurate intermediary orbit of an artificial satellite is solved. This intermediary orbit corresponds to the gravitational potential proposed by Dr. Vinti (see J. Research, NBS, 63B, 105-116 (1959)), which fits the earth's potential accurately through the second harmonic. The writing is complete through the second order solution for both the secular terms and the periodic terms. Methods are now being devised for improving the accuracy of the secular terms in order to make the solution applicable over a long interval of time.

Publication:
(1) Mean motions in conditionally periodic separable systems. J. P. Vinti. To appear in the Journal of Research, Sec. B, National Bureau of Standards.

PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen is continuing his research on plasma statistics. Currently he is investigating the nonlinear oscillations in plasmas and their Landau damping. A finite number of modes were taken and their interactions analyzed.

Upon the termination of his Guggenheim fellowship at the Max Planck Institute for Physics and Astrophysics in Munich, Dr. Tchen accepted an invitation to work at the Centre d'Etudes Nucleaires, Fontenay-aux-Roses, Seine, France, on problems of the kinetic theory of plasmas. Dr. Tchen returned from a three-months stay at this institution on December 1.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: W. Bozman (13.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The list of intensities of 40,000 lines of 70 elements has been printed on the card-controlled electric typewriter in order of wavelengths, and is now ready for publication. The list arranged by elements is in preparation. Wavenumbers of about 6,000 lines of ytterbium (Yb), 1,700 lines of ruthenium (Ru), and 5,000 lines of titanium (Ti) were calculated. An energy level search code was written by G. C. Racah, and has been used to find 27 levels of the ground configuration (f3), and 40 new even levels (f^2d) of praseodymium (Pr)III.

3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES
Origin: NBS, Section 3.2
Manager: M. L. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Terminated.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. J. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Inactive.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: B. S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants were computed for 41 thermometers. LTS constants were calculated for 22 thermometers.

1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Manager: P. J. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Continued. Production runs were made using the integral programs for dyatomic hydrides. Progress has been made on the integral programs for T_d symmetry. The program for transforming the integrals to a new basis has been written and some successful check-outs were made. A few additional check cases are being considered.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed to check laboratory sets of gage blocks as requested by the sponsor.

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Manager: M. L. Paulsen
Full task description: Jan-Mar 1957 issue, p. 36
Status: Terminated.

1102-40-11645/57-236 SELF CONSISTENT FIELD--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: P. Walsh
Full task description: Apr-June 1957 issue, p. 30
Status: Inactive.

3911-61-39952/56-266 DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Inactive.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Production runs were continued under the direction of the sponsor.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. Work continued on the second network model and the report dealing with it. The case in which travel-distance is euclidean was explored in more detail. There are preliminary indications that many of the previous results can be unified and generalized using a theory of convex functions defined on metric spaces.
Publication: Optimization of distribution models, A. J. Goldman and B. K. Bender; in manuscript.

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.7
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Inactive

1102-40-11645/58-281 PSI EVALUATION
Origin and Sponsor: NBS, Section 4.10
Managers: P. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1957 issue, p. 34
Status: Inactive.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Theoretical work on the electrical network model was continued and incorporated in the mathematical part of a joint publication with R. S. Marvin. No production runs on the computer were made during this period.

1102-12-11513/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Signal Corps and Office of Ordnance Research, U. S. Army
Manager: I. Rhodes (11.0)
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. An NBS report has been prepared on "hindsights", i.e. cases in which a predicted word precedes the predictor. A study was started of semantic ambiguities which can be resolved by means of syntactic information.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: S. Peavy
Full task description: Apr-June 1958 issue, p. 30
Status: Reactivated. The tables of Herschfelder, Curtiss and Bird were not sufficiently accurate for preliminary calculation. It was necessary to recompute tabulated values. In the process of so doing, several unforeseen numerical analysis problems have arisen and are being investigated.

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. Production runs made by the sponsor. Because of lack of personnel time, there has been delay in the development of two new codes.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Manager: P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. New cases are being set up for production runs. In some instances the code did not converge to the maximum radiation between lobes, and changes have been introduced into the original code. These modifications are being checked.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Inactive.

1102-40-11645/59-389 FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Adminstration
Manager: L. S. Joel
Full task description: Oct-Dec 1958 issue, p. 29
Status: Continued. J. Edmonds is seeking practical extensions of recent applications of the classical alternating path method to certain graph extremal problems in an attempt to assign frequencies to stations in the best manner. One promising algorithm, for selecting from a given graph a large number of vertices no pair of which are joined by an edge, is to find in the graph, from which some vertices have been tentatively selected, a bipartile subgraph joined to the complement only by edges going to unselected vertices and such that the vertices of one part are selected and the larger number of vertices of the other part is unselected. One improves the selection by exchanging parts. The pilot (or sample) program has been used for limited production due to the exigency of FAA requirements. A flow chart for a full-scale system of programs has been initiated. Meetings with the FAA were started for the purpose of detailing the system requirements.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II
Origin and Sponsor: NBS, Section 4.6
Manager: A. E. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. Several check runs were made using the code for photodetachment cross section of H^-.

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. Tape testing codes have been developed. Smoothing techniques, suggested by the sponsor, have been code checked. Statistical analysis of the ventricular gradient awaits further tape testing.

1102-40-11645/59-445 OIL SUPPLY
Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Inactive.

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. M. Beiman, W. G. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. All programs have been modified to do more extensive checking to detect machine errors. These programs have been checked out and results are being transmitted to the sponsor.

The sorting phase of the program is now being done on the IBM 7090.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Reactivated, to compute values of additional functions.

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1959 issue, p. 26
Status: Continued. The code to diagonalize matrices was used to produce eigenvalues and eigenvectors for a variety of matrices.

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. Further testing on the adjustment of parameters is in progress.

1102-40-11645/60-467 TRANSISTOR SIMULATION
Origin and Sponsor: NBS, Section 12.1
Manager: G. W. Reitwiesner
Full task description: Oct-Dec 1959 issue, p. 27
Status: Inactive.

1102-40-11645/60-475 IONOSPHERIC SOUNDINGS
Origin and Sponsor: NBS, Section 82.40
Manager: M. L. Paulsen
Full task description: Oct-Dec 1959 issue, p. 29
Status: Continued. The usual monthly virtual height input data cards (approx. 20,000) were received and computations completed yielding the desired information. This was returned to the Boulder Laboratories as input for other programs. The Boulder Laboratories plan to assemble the Fortran Program, written for the IBM 704, on their new CDC 1604.

1102-40-11645/60-476 GAS TUBE CHARACTERISTICS, II
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: H. Oser, W. Borsch-Supan
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. The program has been written and checked out. The first production runs have been made.

1102-12-11122/60-479 PROCESSING OF DIAGRAMS
Origin and Sponsor: NBS, Section 11.0
Managers: F. L. Alt (11.0), S. T. Peavy, R. J. Herbold
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Moments were computed for a number of geometrical figures, in order to test the sensitivity of the numerical computation to random errors.

1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: R. Zucker
Full task description: Jan-Mar 1960 issue, p. 28
Status: Continued. Production runs have been made for 23 cases.

1102-40-11645/60-489 INVERSION OF LINE PROBE DATA
Origin and Sponsor: NBS, Section 3.1
Manager: R. Herbold
Full task description: Jan-Mar 1960 issue, p. 29
Status: Terminated. Production runs will be continued under the direction of the sponsor and the machine time will be reported in the section of this report entitled "Current Applications of Automatic Computer."

1102-40-11645/60-501 KANSAS RIVER SYSTEM
Origin and Sponsor: Corps of Engineers, U. S. Army, Office of District Engineers, Kansas City District
Manager: S. Peavy
Full task description: Apr-June 1960 issue, p. 24
Status: Inactive.

1102-40-11645/60-504 ELECTROSTATIC-FOCUSING PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. Beam
Full task description: Jan-Mar 1960 issue, p. 30
Status: Completed. No runs were made during this quarter.

1102-40-11645/60-506 COMMODITY PRICE INDICES
Origin and Sponsor: U. S. World Bank, Statistics Division
Manager: J. C. Lamkin, Jr.
Objective: To derive price indices, relative to a base time period, which will depict changes in the economic importance to various geographic areas of commodities produced in those areas and traded in world markets.
Background: A method is needed which portrays more graphically the economic situation of world areas whose well-being depends largely upon the sale of a few raw materials to more industrialized regions.
The request was transmitted by Chas. G. Goor (WB).
Status: New. Programming of the problem was almost completed.

3911-61-39952/60-507 TRANSISTOR AGING BEHAVIOR STUDIES
Origin and Sponsor: NBS, Section 1.6
Manager: R. Varner
Full task description: Apr-June 1960 issue, p. 24
Status: Terminated. Continued as task 3911-61-39952/61-528.

3911-61-39952/60-508 MODEL ADSORPTION ISOTHERMS
Origin and Sponsor: NBS, Section 5.2
Manager: J. P. Menard
Full task description: Apr-June 1960 issue, p. 25
Status: Continued. The existing code has been modified slightly to allow greater flexibility in the integration routine. The range of parameters has been extended and the output has been revised. Further production runs will be made using this code.
Attention is invited to a correction in the first two lines of the status in the July-Sept 1960 issue. They should read: "For the case $c > 6.75$, the transformation $\zeta = x/\sqrt{2\cdot c}$, $d\zeta = dx/\sqrt{2\cdot c}$ has been introduced."

1102-40-11645/60-510 H_2^+ BOMBARDMENT
Origin and Sponsor: Naval Research Laboratory
Manager: W. Borsch-Supan
Full task description: Apr-June 1960 issue, p. 26
Status: Continued. A report is in preparation.

1102-40-11645/60-513 RADIATIVE ENVELOPES OF MODEL STARS
Origin and Sponsor: National Aeronautics and Space Administration
Managers: S. Haber (11.1) and P. J. Walsh
Full task description: Jul-Sept 1960 issue, p. 23
Status: Continued. The program was checked out and various parameters were determined after numerical experiments. Several solutions were obtained and transmitted to the sponsor.

1102-40-11645/61-516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE
Origin and Sponsor: NBS, Section 4.8
Manager: R. J. Herbold
Full task description: Jul-Sept 1960 issue, p. 24
Status: Continued. The 704 program is being revised to compute $S(k^2)$ and $pe(\rho,h)$ in double precision mode.

1102-40-11645/60-519 SCALAR WAVE SCATTERING
Origin and Sponsor: NBS, Section 6.0
Manager: R. Zucker
Full task description: Jul-Sept 1960 issue, p. 24
Status: Completed. Results have been transmitted to the sponsor.

1102-40-11645/61-520 HEAT EXPANSION
Origin and Sponsor: NBS, Section 2.4
Manager: H. Oser
Full task description: Jul-Sept 1960 issue, p. 26
Status: Completed. The original code was rewritten and extended by K. A. Bedeau and has been checked out completely. Production runs were made for 29 elements and have been turned over to the sponsor. Future production runs will be carried out under the direct supervision of the sponsor.

1100-40-11460/61-522 SPECIAL CONSULTING SERVICES
Origin and Sponsor: National Aeronautics and Space Administration
Manager: E. W. Cannon (11.0)
Full task description: Apr-June 1960 issue, p. 26
Status: Completed. A report has been transmitted to the sponsor.

1102-40-11645/61-527 NUCLEAR MAGNETIC RESONANCE
Origin and Sponsor: NBS, Section 15.7
Manager: H. Oser
Full task description: Jul-Sept 1960 issue, p. 27
Status: Completed. The table of the shielding function for the benzene ring has been computed and graphs of the isoshielding lines have been drawn by the hand computer group.

3911-61-39952/61-528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING
Origin and Sponsor: NBS, Section 14.1
Manager: R. Varner
Full task description: Jul-Sept 1960 issue, p. 27
Status: Continued. Several minor modifications have been made in the existing codes. Production runs are being made by the sponsor.

1102-40-11645/61-530 SPECIMEN WAVELENGTH
Origin and Sponsor: NBS, Section 9.4
Manager: J. P. Menard
Full task description: Jul-Sept 1960 issue, p. 28
Status: Continued. A Fortran program has been compiled so that trial runs can be made to determine whether the present method of solution will converge to the desired solution. It is anticipated that more sophisticated methods may be necessary to obtain convergence.

1102-40-11645/61-531 HEAT TRANSFER IN CRYSTALS
Origin and Sponsor: NBS, Section 3.1
Manager: H. Oser
Full task description: Jul-Sept 1960 issue, p. 29
Status: Continued. A master code and five subroutines have been written in Fortran by J. D. Waggoner. These subroutines were necessary to take advantage of the special character of the matrices, which are very sparsely populated in general. At the end of the quarter, the code was almost ready for production.

1102-40-11645/61-532 CALCULATION OF VIBRATIONAL ENERGY LEVELS FOR IONIC MOLECULES
Origin and Sponsor: Georgetown University
Manager: P. J. Walsh
Objective: To find the vibrational energies and related vibrational quantum numbers. This data will be analyzed in an attempt to obtain a functional formulation for the vibrational energy levels of ionic molecules.
Background: Two potentials for ionic molecules are being tested. One potential consists of the Coulomb attraction and a nuclear repulsion term ($B\, e^{-r/\rho}$) only. The second potential adds a repulsive polarization term to the first potential. A number of "ionic molecules (e.g., L_iI, NaCL, I_nF) will be tested. The problem was transmitted by E. Finn (GU).
Status: New. The code was written and is being checked out.

1102-40-11645/61-534 CURVED JET COMPUTATIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: W. Borsch-Supan and R. Varner
Objective: To solve the differential equations:

$$f_1''' + 2F_o f_1' + 6F_o'' f_1 = 4\eta G_o' \qquad \text{where } f_1(0) = f_1'(0) = f_1'(T) = 0$$

$$g_1' = -3(\cosh\eta)^{-2} f_1' \qquad \text{where } g_1(T) = 0$$

$$f_2''' + 2F_o f_2'' - 4F_o' f_2' + 10F_o'' f_2 = 4[3\eta(\cosh\eta)^{-2} f_1' + g_1] + 2(f_1')^2 - 6f_1 f_1''$$

$$\text{where } f_2(0) = f_2''(0) = f_2''(T) = 0$$

$$y_8' = -2(\cosh\eta)^{-2} f_2' - (f'_1)^2 \qquad \text{where } y_8(0) = 0$$

To compute:

$$N_2 = -2y_8(T)$$

$$g_2 = \frac{3}{4}\left\{y_8 - N(G_o - \tfrac{1}{2})\right\}$$

where:

η is used as the independent variable; prime means differentiation with respect to η;

$$F_o = t = \tanh \eta;$$

$$G_o = \tfrac{1}{2} - \frac{3}{4}t(1 - \frac{1}{3}t^2);$$

T is used in place of infinity; y_8 is an auxiliary variable used for convenience.

Background: DOFL is conducting a research and development effort on Fluid Stream Amplifiers, devices which operate by means of boundary layer and fluid stream interaction effects. Part of this effort is an investigation of the behavior of fluid jets. A series of differential equations to describe velocity and pressure distributions in a curved turbulent jet separating regions of different pressures has been written. These equations are submitted to the NBS Computation Laboratory for numerical analysis and solution on the IBM 704 computer.

Status: Completed (New). A Fortran 704 program has been written and checked out. Requested runs were made and results were transmitted to the sponsor.

1102-40-11645/61-536 SECULAR EQUATIONS

Origin and Sponsor: NBS, Section 13.2

Manager: R. Zucker

Objective: To analyze infra-red spectrum of the asymmetric-top molecule D_2O (heavy water) by assigning the observed lines to transitions between energy levels, and relating those energy levels to more basic parameters, the molecular constants.

Background: In an asymmetric-top molecule, with three different moments of inertia, the energy levels must be obtained as eigenvalues of matrix equations of high order. The molecular constants used include coefficients of terms of power higher than quadratic in the quantum numbers J and K, representing the distortion of the molecule due to centrifugal stretching. These higher power terms appear in the diagonal and first- and second- off diagonal matrix elements. By comparing the eigenvalues, computed with different sets of molecular constants, with the observed energy levels, the best set of constants can be obtained. The eigenvectors corresponding to the optimum set are used in further calculations of the intensity of the spectroscopic transitions. The code may be used to facilitate the calculation of energy levels and derivation of centrifugal stretching constants of other asymmetric-top molecules.

The problem was transmitted by W. S. Benedict (13.2).

Status: New. The code has been written and checked out. Production runs were made with 5 sets of constants for matrices of order 2 to 17.

1102-40-11645/61-537 MASS ACTION LAW

Origin and Sponsor: NBS, Section 5.2

Manager: H. Oser

Objective: To compute tables of the chemical mass action law for a two-component mixture:

$$k = (c-x)^n \cdot \left(c-\frac{n}{m}x\right)^m$$

for various inputs as follows:

a) Given n,m and a given number of pairs (x,c), to compute k.
b) Given quadruples (k,c,m,n), to determine x from these given values.

Background: The problem arises in the study of chemical reactions of associated colloid components. Knowledge of the computed constant k leads to a determination of the free energy of the system.

The problem was submitted by Stanley Wasik (5.2).

Status: New. Rather high values of m and n occur, which requires a fairly wide range of exponents. A multiple-precision arithmetic was therefore required. The code was written by J. Lamkin.

1102-40-11645/61-538 SPECTRAL REFLECTANCE

Origin and Sponsor: NBS, Section 9.4

Manager: W. Borsch-Supan

Objective: To establish a program for the IBM 704 which fits a certain analytic expression to a set of given data (R_i, λ_i), $i = 1, 2, \ldots N$. The analytic expression for R as a function of λ is given by the following formulas using complex quantities:

$$R = \left|(N-1)/(N+1)\right|^2,$$

$$N^2 = 1 - \frac{G}{w(w+iH)} + \sum_{j=1}^{J} \frac{G_j}{w_j^2 - w^2 + iwH_j}$$

$$w = 2\pi c/\lambda$$

where J and c are given constants. The square root $N = \sqrt{N^2}$ is to be taken so that R is smaller than 1. The parameters G, H, G_j, H_j, w_j are to be chosen in such a way that a best fit--e.g., in the sense of least square sum--is obtained.

Background: R is the reflectance of a certain metal, λ the wavelength of the incoming light. The formulas given above are based on the classical electron theory on reflection of light. The goal is to describe the reflectance of any metal by a few data, namely the parameters G, H, G_j, H_j, w_j.

The request was transmitted by W. N. Harrison (9.4).

Status: New. In order to get good initial approximations for an iterative approach to the problem, the family of functions $R(\lambda)$ has been split into a few linear one-parameter families, which are going to be tabulated first. Coding for this part has started.

1102-40-11645/61-542 STUDENT LOAN DATA

Origin and Sponsor: Department of Health, Education and Welfare

Manager: R. Zucker

Objective: To edit the data and prepare a series of tables relating to the National Defense Student Loan. As many as three fields of data with other control fields are to be cross tabulated for the number of borrowers, amount of money borrowed by states, types of institutions, etc. Percentages were also obtained for the various tabulations. Separate tabulations are to be made for negative and non-responses.

Background: The National Defense Student Loan Program has the dual aim of helping to provide higher educational opportunities for able students in need of financial aid and of recruiting teachers for the public schools. Since the survey consists of a sample of about 32,000 questionnaires with 43 fields of data for rapid processing, electronic equipment was needed.

The problem was transmitted by R. Hall (HEW).

Status: New. Editing, generalized cross-tabulation and specified column print routines were written and checked out. Sample tables were submitted to the sponsor for approval.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Color difference measurements. J. R. Rosenblatt developed techniques for describing between-observer and within-observer variability in a large-scale incomplete paired comparison experiment. A study is being made of multiple comparison procedures appropriate for such experiments. This work is being done for G. L. Howett of the Photometry and Colorimetry Section.

(2) Voltage standards. Schedules for the intercomparison of small groups of standard voltage cells were worked out for C. A. Law of the Electrochemistry Section.

(3) Electric meters. A draft of a report by M. G. Natrella and A. Glinski on the factors to be considered in effectively applying statistical sampling techniques to the testing of electric meters was prepared for F. K. Harris of the Electrical Instruments Section.

(4) Thermometric standards. Data on temperature measurements of naphthalene cells were analyzed by H. Ku for D. Enagonio of the Pure Substances Section.

(5) Modulus of rupture measurements. A study was undertaken for H. J. Kerper of the Glass Section to find a probability distribution which will adequately describe the distribution of the modulus of rupture of glass specimens. The four distributions which were under consideration are: the normal, the Weibull, the log-normal, and the extreme value. The allied problem of finding the zero strength of glass, i.e., the pressure below which glass specimens will not fracture, was also studied.

Publications:

(1) Variability of color-mixture data. I. Nimeroff (Photometry and Colorimetry), J. R. Rosenblatt, and M. C. Dannemiller. In manuscript.
(2) How to evaluate accuracy. W. J. Youden. To appear in Materials Research and Standards (formerly ASTM Bulletin).
(3) The sample, the procedure, and the laboratory. W. J. Youden. Anal. Chem. 32, 23A-37A (1960).
(4) Precision and accuracy--experiment design aspects. Churchill Eisenhart. To appear in "Proceedings of a Symposium on Statistical Methods in the Chemical Industry," held by the ASQC, New Hampton, N.H., 1960.
(5) Propagation of error in a chain of standards. A. G. McNish (Metrology) and J. M. Cameron. To appear in IRE Transactions on Instrumentation.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager : C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Preparation of the text in final form for publication is near completion.

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: INACTIVE.

Current Applications of Automatic Computer

The record of the use of the IBM 704 for the period October 1 through December 31 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
NBS:					
11110/55-55	11.1	Research in numerical analysis			40
11411/55-56	11.1	Research in mathematical topics applicable to numerical analysis			25
11120/55-65	11.2	Automatic coding	37	83	217
39952/56-160	11.2	Mathematical subroutines	223	115	363
39951/51-1	11.3	Statistical engineering	25	44	106
39952/54-30	13.1	Spectrum analysis	237	307	1420
39952/55-68	9.7	Crystal structure calculations	2		477
39952/55-82	3.1	Thermometer calibrations	16		142
39952/56-131	2.2	Calculations in optics*	1	8	73
11645/56-166	5.9	SCF-LCAO solution of some hydrides*	5	226	73
11645/56-171	3.2	Collision integrals used in transport theory**	156	237	296
11645/57-219	3.2	Thermal properties*	14		64
11645/57-223	3.2	Self-consistent fields			37
11645/57-246	4.8	Radiation diffusion**	672	69	113
11645/57-249	9.4	Color differences*			7
39952/57-250	2.3	Automatic reduction in spectrophotometric data*	10		123
11645/57-251	1.6	Current noise and fixed resistors*			8
11645/57-252	4.11	Detecting efficiency in a neutral meson experiment**	53		85
39952/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*			34
11645/58-255	4.8	Chi functions**	516	185	262
11645/58-256	10.6	Composite walls**	148	107	158
11645/58-260	12.5	Prototype accounting**	9	185	175
11645/58-271	6.3	Simultaneous equations for potential flow**			21
11645/58-274	9.7	Calculations for d-spacings II*			76
11645/58-275	7.8	Crystallography**			21
11645/58-308	3.4	Oscillating sphere*			15

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/58-314	3.7	Approximations for gas mixtures*	290	251	840
/58-333	9.0	Calcium hydroxide*	53	3	221
/58-339	3.4	Viscoelasticity properties of materials			5
/58-357	3.3	Eigenvalues**			109
/58-358	3.2	Reduced cross sections	15	20	314
/59-388	10.3	Heat pump calculations*		4	4
/59-394	4.6	Slow electron scattering by hydrogen atoms	78	170	88
/59-395	7.7	Adsorption study**	2	69	13
/59-403	2.1	Computation of color fadings*			6
/59-409	12.5	Bank Board**	41	125	782
/59-417	2.4	Spectrum analysis of ruthenium**			165
/59-418	4.8	P-Wave equation*	101	102	20
/59-428	12.5	Radio intensities**	100	86	749
/59-440	82.10	Mapping**	129	546	681
/59-446	85.10	Ionospheric data**	56	65	504
/60-449	2.1	Spectral line colorimetry			5
/60-457	12.5	Public Housing problem**			666
/60-466	3.2	Electronic properties of simple molecular systems		32	67
/60-474	2.5	Gage block stability*			24
/60-475	82.40	Ionospheric soundings			312
11122/60-479	11.0	Processing of diagrams	20	38	23
11645/60-486	3.6	Morse wave functions		7	21
/60-489	3.1	Inversion of line probe data			31
/60-493	3.7	Poisson distribution function**	298	422	235
/60-509	10.0	Cement testing**			11
/60-514	3.9	Flame spectra**			97
/60-515	13.5	Convolution integral**			7
/61-516	4.8	Radiation field from a circular disk source	55	5	
/61-519	6.0	Scalar wave scattering	4	26	29
/61-520	2.4	Heat expansion	62	24	57
/61-521	3.5	Matrix manipulation	5		4
/61-523	4.0	Neutron cross section computations**	142	176	
/61-525	3.8	Curve fitting of wave functions**			15
/61-527	5.7	Nuclear magnetic resonance	40		37
/61-528	14.1	Transistor aging	104	38	93
/61-531	3.1	Heat transfer in crystals	153	17	2
/61-536	3.2	Secular equations	8	17	61

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/61-537	15.2	Mass action law	15	6	
		Miscellaneous		4	8
		Totals (NBS Services)...	3,895	3,819	10,737
OUTSIDE:					
11645/58-269	NRL	Molecular structure, IV	454	796	899
/58-270	PO	Post Office problem	20	59	142
/58-276	NOL	General kinetics, I**			9777
/58-341	NRL	Cosmic rays	28		13
/58-348	OOR	Russian-to-English machine translation	49	81	57
/58-361	NRL	Spectrum of dipole radiation	58	44	2869
/58-366	USIA	Radiation patterns of antennas	3	26	75
/58-368	SC	Intensity functions of light scattered by spherical particles	15	4	262
/59-371	NRL	ASWAP°		49	10
/59-373	DOFL	Rhinitis**	25		67
/59-389	CAA	Frequency allocation		54	71
/59-407	DOFL	Fourier coefficients*			9
/59-408	NASA	NASA**	20		70
/59-411	HEW	Fitting of exponential curves**	327	92	798
/59-415	DOFL	Complex Legendre functions*	120		9
/59-416	DOFL	Analysis of power supply experiments**	89		34
/59-419	DOFL	Neutrons°	13		19
/59-423	WB	Weather Bureau**		26	6166
/59-425	CU	Molecular orbitals*			108
/59-434	CIW	Petrological computations*	92	48	40
/59-435	VA	Electrocardiographic analysis	68	304	916
/59-437	BPR	Highway study	94	85	415
/59-441	GK	Systems engineering**		122	886
/59-445	NPSA	Oil supply	95	323	157
/59-447	BPRO	Public Roads study**	162	608	12726
/60-450	ACC	Chemical warfare°	7	11	
/60-454	GE	G.E.**			494
/60-458	CAB	Domestic airline traffic survey	159	465	2593
/60-462	DOFL	Correlation of functions	61		
/60-465	CU	Calculations in molecular quantum mechanics			3
/60-470	NSF	Information selection system**		14	
/60-476	DOFL	Gas tube characteristic II	80	58	477

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/60-481	SC	Radar study°	97	109	211
/60-492	IMF	Monetary research reports**	407	35	23
/60-496	BPA	Short circuit program**			26
/60-502	USA	Quartermaster mathematics programming**	38	45	340
/60-506	WB	World Bank**			49
/60-510	NRL	H_2^+-bombardment			25
/60-512	DOFL	Data transformation**	39	27	52
/60-517	DOFL	Shock tube**	132	101	476
/61-518	DOFL	Reliability of binary counter**		49	
/61-532	GU	Vibrational energy levels for ionic molecules**	7	30	
/61-533	U MD	Datatrol computations**	30	17	
/61-540	EDARS	Edgewood Arsenal°	23	21	
/61-542	HEW	Student loan data	12	125	752
/61-543	DOFL	Fourier analysis°	5	3	
/61-545	WEST	Nuclear reactor design**			941
/61-547	U MD	U MD**			38
		Totals (Outside)	2,829	3,831	43,095
		Total time for the quarter (MINUTES).....	6,724	7,650	53,832
		Total time for the quarter (HOURS)	122	129	897

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

° Classified task.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

POLLACK, H. O. (Bell Telephone Laboratories, Murray Hill, N. J.) Distribution of L^2 norm for square integrable entire functions of exponential type. Energy relations for band-limited functions. October 20.

ADKINS, John E. (University of Nottingham, England) Exact solutions and perturbation methods in finite elasticity. December 6.

OBERHETTINGER, Fritz (Oregon State College) Chebychef approximation and Zolotareff rational functions. Part I, December 7. Part II, December 9.

Mathematical Statistics Seminar

DAVID, Herbert A. (Virginia Polytechnic Institute) The method of paired comparisons. November 21.

Theory of Errors Colloquium

EISENHART, C. Background and Evaluation of the Method of Least Squares: (1) Adjustment of observations, the principle of the arithmetic mean, and least-residuals methods. October 6; (2) Least residuals, Legendre's principle, and non-probabilistic least squares, October 27; (3) Laws of errors and Gauss' first "proof" of the method of least squares, November 10.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. (1) Problems for superspeed computers. Presented at the First Annual One-Day Technical Symposium of the Washington, D. C. Chapter of the Association for Computing Machinery, October 20. (2) Machine translation of natural languages. Presented at the Institute of Mathematical Sciences, New York University, New York City, December 1.

ARMS, R. J. Digital computation with electrocardiographic readings. Presented at a Conference on Electrical Techniques in Medicine and Biology, Washington, D. C., November 1.

BAZLEY, N. The method of truncation in intermediate problems for lower bounds to eigenvalues. Presented at the U. S. Naval Ordnance Laboratory, Silver Spring, Md., November 21.

BENDER, B. K., and GOLDMAN, A. J. Maximal cellular boolean functions. Presented (by title only) at the meeting of the American Mathematical Society, Worcester, Mass., October 22.

CANNON, E. W. The National Bureau of Standards Computation Laboratory. Presented at the First Annual One-Day Technical Symposium of the Washington, D. C. Chapter of the Association for Computing Machinery, October 20.

EDMONDS, J. R. A combinatorial representation for polyhedral surfaces. Presented at the meeting of the American Mathematical Society, Worcester, Mass., October 22.

EISENHART, C. (1) To fit or not to fit, that is the question... Presented before the Pittsburgh Chapter of the American Statistical Association, Pittsburgh, Penn., October 11. (2) Probability models for measurement with a linear scale. Presented at a meeting of the Mathematical Association of America, held at the University of Maryland, College Park, Md., December 3.

GOLDMAN, A. J. Optimization models of distribution networks. Presented at a meeting of the Operations Research Society of America, Detroit, Mich., October 11.

WEGSTEIN, J. H. (1) The ALGOL language. Presented at the Naval Ordnance Laboratory, Silver Spring, Md., October 17. (2) ALGOL, compiler and problem-oriented language. Presented before the General Dynamics Technical Computer Panel on Analog and Digital Computing, New York City, October 19.

YOUDEN, W. J. (1) Statistics for skeptics. Presented at a meeting of the American Chemical Society, Charlotte, N. C., October 3. (2) The enduring values. Presented before the Cleveland Chapter of the American Society for Quality Control, October 7; also presented at the Sixth Conference on the Design of Experiments in Army Research, Development, and Testing, held at Aberdeen Proving Ground, Md., October 20. (3) Experimental design. Presented before the Washington, D. C. Section, American Society for Quality Control, November 1. (4) What happens when different laboratories evaluate the same item? Presented before the Binghamton Section of the American Society for Quality Control, November 14.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

The following papers appeared in J. Research NBS 64B, Oct-Dec 1960 (Mathematics and Mathematical Physics):

(1) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. Pp. 211-215.

(2) Sequence transformations based on Tchebycheff approximations. J. R. Rice. Pp. 227-235.

(3) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. Pp. 217-225.

* * * * *

(4) Generating functions for formal power series in noncommuting variables. K. Goldberg. Proc. Amer. Math. Soc. 11, 988-991 (1960).

(5) The minima of cyclic sums. K. Goldberg. J. London Math. Soc. 35, 262-264 (1960).

(6) Bounds for determinants with positive diagonals. E. V. Haynsworth. Trans. Amer. Math. Soc. 96, 395-399 (1960).

(7) Irrational power series. M. Newman. Proc. Amer. Math. Soc. 11, 699-702 (1960).

(8) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. Quart. Appl. Math. 18, 257-262 (1960).

(9) On prediction of system behavior. J. R. Rosenblatt. Reprinted in IRE Transactions on Reliability and Quality Control, RQC-9, 23-28 (1960), from "Proceedings of the NYU-Industry Conference on Reliability Theory (June 1958)".

(10) Chebyshev approximation by $ab^x + c$. J. R. Rice. J. Soc. Ind. Appl. Math. 8, 691-702 (1960).

(11) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Biometrika 47, Pts. 3 & 4, 411-416 (1960).

(12) The sample, the procedure, and the laboratory. W. J. Youden. Anal. Chem. 32, 23A-37A (1960).

(13) Graphs for bivariate normal probabilities. M. Zelen and N. C. Severo. Ann. Math. Stat. 31, 619-624 (1960).

1.5 Miscellaneous

(1) Some SEAC computations of subsonic flows. P. Davis and P. Rabinowitz. In "Bergman's Linear Integral Operator Method in the Theory of Compressible Fluid Flow," by M. Z. v. Krzywoblocki, pp. 148-172 (Wien, Springer-Verlag, 1960).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Technical Notes, Manuals, and Bibliographies

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Sec. B.

(3) Fractional factorial designs for experiments with factors at two and three levels. To appear as Applied Mathematics Series 58.

(4) Selected bibliography of statistical literature, 1930-1957: IV. Markov chains and stochastic processes. L. S. Deming and D. Gupta (Catholic University). To appear in the Journal of Research, NBS, Sec. B.

2.3 Technical Papers

(1) Automatic screening of normal and abnormal electrocardiograms by means of a digital electronic computer. R. J. Arms with H. V. Pipinger and F. W. Stallman (Veterans Administration, Mt. Alto, and Georgetown University School of Medicine).

(2) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960.

(3) Bounds for the P-condition number of matrices with positive roots. P. J. Davis, E. Haynsworth and M. Marcus. To appear in the Journal of Research, NBS, Sec. B.

(4) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. Submitted to a tehcnical journal.

(5) Criteria for the reality of matrix eigenvalues. M. P. Drazin (RIAS) and E. V. Haynsworth. Submitted to a technical journal.

(6) Boscovich and the combination of observations. C. Eisenhart. To appear as Chapter 7 in "Boscovich--Essays on His Life and Work", ed. by Lancelot Law Whyte.

(7) Combination of observations. C. Eisenhart. To appear in the "Boscovich Memorial Volume," ed. L. L. Whyte.

(8) Precision and accuracy--experiment design aspects. C. Eisenhart. To appear in "Proceedings of a Symposium on Statistical Methods in the Chemistry Industry," held by the ASQC, New Hampton, N. H., 1960.

(9) On some partial differential equations of Brownian motion of a free particle. A. Ghaffari. To appear in the "Proceedings of the International Conference on Partial Differential Equations and Continuum Mechanics," held at Madison, Wis., 1960.

(10) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. To appear in the American Mathematical Monthly.

(11) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.

(12) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.

(13) Special types of partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.

(14) Another extension of Heinz's inequality. M. Marcus. To appear in the Journal of Research, NBS, Sec. B.

(15) A note on normal matrices. M. Marcus and N. Khan (Muslim University, India). To appear in the Canadian Mathematical Bulletin.

(16) The invariance of symmetric functions of singular values. M. Marcus and H. Minc (The University of Florida). Submitted to a technical journal.

(17) Propagation of error in a chain of standards. A. G. McNish (Metrology Division) and J. M. Cameron. To appear in IRE Transactions on Instrumentation.

(18) Periodicity modulo m and divisibility properties of the partition function. M. Newman. To appear in the Transactions of the American Mathematical Society.

(19) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.

(20) Pointwise bounds in the Cauchy problem of elastic plates. L. E. Payne. To appear in the Journal of Research, NBS, Sec. B.

(21) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

(22) Best approximations and interpolating functions. J. R. Rice. Submitted to a technical journal.

(23) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.

(24) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.

(25) Some computational problems concerning integral matrices. O. Taussky-Todd. To appear in the Journal of Research, NBS, Sec. B.

(26) Computational problems concerned with the Hilbert matrix. John Todd. To appear in the Journal of Research, NBS, Sec. B.

(27) Mean motion in conditionally periodic separable systems. J. Vinti. To appear in the Journal of Research, NBS, Sec. B.

(28) Partial confounding in fractional replication. W. J. Youden. Submitted to a technical journal.

(29) How to evaluate accuracy. W. J. Youden. To appear in Materials Research and Standards (formerly ASTM Bulletin).

(30) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(31) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, 1960.

(32) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. To appear in the Journal of the American Statistical Association.

(33) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. To appear in Technometrics.

(34) A calculus for factorial arrangements. M. Zelen and B. Kurkjian (Diamond Ordnance Fuze Laboratories). Submitted to a technical journal.

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WASHINGTON, D.C.

ELECTRICITY. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics.

METROLOGY. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

HEAT. Temperature Physics. Heat Measurements. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research. Equation of State. Statistical Physics. Molecular Spectroscopy.

RADIATION PHYSICS. X-Ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

CHEMISTRY. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

MECHANICS. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Combustion Controls.

ORGANIC AND FIBROUS MATERIALS. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

METALLURGY. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

MINERAL PRODUCTS. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

BUILDING RESEARCH. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

APPLIED MATHEMATICS. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

DATA PROCESSING SYSTEMS. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Applications Engineering.

ATOMIC PHYSICS. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics.

INSTRUMENTATION. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

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CRYOGENIC ENGINEERING. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

IONOSPHERE RESEARCH AND PROPAGATION. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services.

RADIO PROPAGATION ENGINEERING. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

RADIO STANDARDS. High frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

RADIO SYSTEMS. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Space Telecommunications.

UPPER ATMOSPHERE AND SPACE PHYSICS. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.


NBS