NATIONAL BUREAU OF STANDARDS REPORT

7031

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

July through September, 1960

For Official Distribution

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT

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APPLIED MATHEMATICS DIVISION

July 1 through September 30, 1960

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*On leave of absence **Part time °Guest Worker

Contents

*Only unclassified projects are included in this report.

Status of Projects

September 30, 1960

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth worked on inequalities for the elementary symmetric functions of eigenvalues of non-negative Hermitian matrices. These lead to new bounds for the determinants of such matrices. The main results are incorporated in a paper, "Note on a Theorem of Marcus and Lopes", which is in manuscript form.

The paper "Criteria for the Reality of Matrix Eigenvalues" by M. Drazin and E. Haynsworth has been revised.

E. Haynsworth and M. Newman have done some work on addition chains for positive integers. An upper bound has been found for the number of multiplications necessary to generate $2^n - 1$.

M. Marcus and M. Newman are jointly working on the following problems: If A is positive definite with positive entries and $p \geq 1$, does it follow that

$$A_p = (a_{ij}^p)$$

is positive definite, and if so give bounds for the characteristic roots of A_p in terms of those of A. For p a positive integer and for p sufficiently large, integer or otherwise, a satisfactory answer can be given. Another conjecture is that a matrix A satisfying the above conditions can be written as the sum of rank 1 matrices with non-negative entries. If A is simply a matrix with positive entries, then we conjecture that the rank of A is at most the number k of different (in absolute value) terms that appear in the determinant expansion of A. In case k = 1, 2, this is true.

M. Marcus currently has under preparation four papers dealing respectively with convexity properties of elementary symmetric functions, linear operations preserving the classical invariants, transitivity in permutation groups and primitivity indices, and the invariance of singular values under linear operations.

J. Rice has been working a program to solve numerically the non-linear elliptic partial differential equation

$$\frac{\partial(ph^3\frac{\partial P}{\partial x})}{\partial x} + \frac{\partial(ph^3\frac{\partial P}{\partial y})}{\partial y} = -\Lambda\frac{\partial(ph)}{\partial x}$$

which determines the pressure distribution in a gas lubricated bearing.

The final object of this program, once a method of solution was found, was to compute a set of design curves for these bearings. The three physical parameters to be varied are ϵ, the eccentricity (which determines h(x,y) in the equation); Λ and L/D, the length-to-width ratio of the bearing.

A practical method of solution was found, by July, and the computation of the design curves was initiated. At the time this work was interrupted, a complete set of curves had been computed for L/D = 1 and some computations were made for L/D = 2 and 5. The method of solution used is basically an adaptation of the method of overrelaxation. There are two phenomena which appeared that merit mention. In general, one has to underrelax the equations rather than overrelax them for convergence. For linear partial differential equations the optimum relaxation factor is near 1.6. For this equation the optimum relaxation factor decreases as a function of both ϵ and Λ. The range of values is from 0.9 to 0.1. The second phenomena is the dependence of the relaxation factor on the initial estimate of the solution. Indeed there appear to be cases where no relaxation factor leads to convergence--even though the initial estimate is apparently a good one. In these same cases other initial estimates lead to convergence for a large range of relaxation factors. Another method of solution was studied but it is clearly inferior to the overrelaxation method.

S. Haber and P. J. Walsh (11.2) began work on the numerical solution of an eigenvalue problem for a system of three nonlinear ordinary differential equations. The problem arises in a study of the internal structure of stars. A partial program was written and experimental work started.

S. Haber revised a draft of his paper entitled "A Continuity Property of Certain Functions", which is to be submitted to a technical journal.

J. Edmonds has proved the following:

Theorem. For any connected linear graph with an arbitrarily specified cyclic ordering of the edges to each vertex, there exists a topologically unique embedding of the graph in an oriented closed surface so that the clockwise edge orderings around each vertex are as specified and so that the complement of the graph in the surface is a set of discs.

One application of this theorem is to the computation of a set of defining relations for any finite group by suitably embedding its Cayley diagram. The 1-skeleton of a regular map (the combinatorial analog of a regular polyhedron) is symmetric (i.e., its automorphism group is transitive on the vertices and edges). Question: (see Coxeter and Moser, Generators and Relations for Discrete Groups, 1957, p. 116) Is the converse true, i.e., is every symmetric graph the 1-skeleton of some regular map?

J. Edmonds devised an efficient algorithm for finding all regular maps with the complete *n*-graph as 1-skeleton, and found all such maps for $n \leq 11$. There are none for n = 10, thus answering the above question in the negative. Other maps on the list provide counterexamples for a conjecture of Coxeter and Moser (ibid, p. 102) on the non-existence of irreflexible maps of negative characteristic.

A manuscript entitled "Symmetric Embeddings of Symmetric Graphs" is in progress.

Publications:

(1) Tchebycheff approximations by ab^x+c. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.

(2) Split Runge-Kutta for simultaneous equations. J. R. Rice. J. Research NBS 64B, 151-170 (1960).

(3) Sequence transformations based on Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research NBS, Sec. B.

(4) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.

(5) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.

(6) A reduction formula for partitioned matrices. E. Haynsworth. J. Research NBS 64B, 171-174 (1960).

(7) Special types of partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.

(8) Bounds for determinants with positive diagonals. E. Haynsworth. To appear in the Proceedings of the American Mathematical Society.

(9) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.

(10) Bounds for the P-condition number of matrices with positive roots. E. Haynsworth, P. Davis, and M. Marcus. To appear in the Journal of Research, NBS, Sec. B.

(11) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. To appear in the Journal of Research NBS, Sec. B.

(12) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. To appear in the Journal of Research NBS, Sec. B.

(13) Best approximations and interpolating functions. J. R. Rice. Submitted to a technical journal.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman spent the quarter at the University of British Columbia, giving a course entitled "Integral Matrices". Lecture notes have been prepared and are being duplicated. The material is concerned with matrices whose elements are from a principal ideal ring.

K. Goldberg has begun an investigation of problems involving formal power series with positive coefficients. A typical unresolved problem: if f(x) is such a power series, for which polynomials p(x) is f(p(x)) such a power series? He has continued his investigation of the asymptotic properties of Faber polynomials.

J. Gager wrote a program to test a model of baseball proposed by K. Goldberg. The initial tests of the program show good agreement with actual baseball statistics.

K. Kloss wrote a series of number theory codes to test conjectures involving primitive roots, gaps between integers relatively prime to a fixed integer, and the sum of the logarithms of the primes less than X.

L. Clarenbach computed the first 100 roots of $e^z = 1 + z$ with imag $z > 0$, and collaborated with K. Kloss to test the feasability of finding the roots of an analytic function by contour integration.

M. Ellickson wrote a program to evaluate Riesz's sum

$$\sum_{n=1}^{\infty} (-1)^{n-1} x^n/(n-1)!\ \zeta(2n),$$

and found a new real root near x = 1800. The sum is important in the investigation of the Riemann hypothesis.

K. Goldberg has evaluated sums of the type

$$\sum \binom{a_r}{j} p(a_1, \ldots, a_k)$$

taken over all positive a_i such that $a_1 + \ldots + a_k = n$, on the assumption

$$p(a_1)p(a_2, \ldots, a_k) = p(a_1, a_2, \ldots, a_k) + p(a_1 + a_2, a_3, \ldots, a_k)$$

for all positive integers $a_1, a_2, \ldots, a_k$ and all $k \geq 2$.

Publications:

(1) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
(2) Irrational power series. M. Newman. To appear in the Proceedings of the American Mathematical Society.
(3) Periodicity modulo m and divisibility properties of the partition function. M. Newman. To appear in the Transactions of the American Mathematical Society.
(4) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
(5) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
(6) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
(7) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. Submitted to a technical journal.

INFORMATION SELECTION SYSTEMS
Task 1101-12-11412/60-470

Origin: NBS Authorized 9/25/59
Sponsor: National Science Foundation
Managers: K. Goldberg, A. J. Goldman

Status: TERMINATED. The studies have been rounded and drawn to a close due to the cessation of the agreement with the sponsor.

Publications:

(1) Computer simplification of boolean functions. B. K. Bender (11.2), A. J. Goldman, and R. B. Thomas (12.5). Submitted to a technical journal.
(2) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(3) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(4) Optimization of distribution networks. B. K. Bender (11.2) and A. J. Goldman. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: P. Davis
Full task description: July-Sept 1955 issue, p. 7

Status: INACTIVE.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

MATHEMATICAL TABLES

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter, with priority being given to preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 SIEVERT'S INTEGRAL

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The texts for Chapter 8, Legendre Functions, and Chapter 18, Weierstrass Elliptic Functions, have been completed. Preparation of the associated tables and graphs is well under way. Some revisions are being made in the illustrative examples.

Manuscripts of all remaining chapters are receiving a final review for last minute adjustments and such details as consistency of cross references between chapters, etc.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. Most of the programming on the new automatic operating system for the 704 computer has been completed by G. Galler and G. Ziegler. The system to be called BSBEL will not be placed in operation until the time clock and binary card-to-tape hardware have been installed. A detailed new manual for the system is being written by G. Galler.

A group consisting of representatives of 22 manufacturers and computer-using-laboratories was organized for the purpose of maintaining the ALGOL 60 reference language. The secretary-chairman of this group is temporarily being supported by this automatic coding task.

During this quarter, the Tablemaker system was used for a total computer time of 930 minutes. The following table indicates the number of times that the various functions were used.

Function	Number of times used
Curve fit	194
Plot	180
Interpolation	47
Integration	3

MATHEMATICAL SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A code has been written by J. D. Waggoner to read into core storage the elements of an Hermitian matrix

$$H_{nxn} = (A+iB)_{nxn}, \qquad \text{A and B real,}$$

and to compute and print its eigenvalues and eigenvectors. The input consists of the lower triangular array of the symmetric matrix A and the full, square matrix B. The symmetric matrix

$$C_{2nx2n} = \begin{pmatrix} A & -B' \\ B & A \end{pmatrix}$$

is generated and the roots and vectors of C are computed by an already existing code. The roots of C (appearing in pairs) are equal to the roots of H. Each vector of C has 2n components, the first n of which are the real components of the vector and the second n the imaginary elements of the same vector for a particular eigenvalue of H.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. H. H. Ku continued his study of the application of information theory to the analysis of four-way contingency tables, and presented a seminar on this topic.

C. Eisenhart and Judith E. Kirsch gave analytical and numerical consideration to the following questions: If (1) it is known (e.g., from theoretical considerations) that the dependence of a "response" y on the value of some "independent variable" x is described exactly by a polynomial of the *k*th degree in x, $y = \beta_0 + \beta_1 x + \ldots + \beta_k x^k$; (2) the values of the coefficients of this polynomial are not known; (3) "observed values" $Y_1, Y_2, \ldots, Y_n$ of y corresponding to *n* ($n \geq k$) different values $x_1, x_2, \ldots, x_n$ of x are available; and (4) a polynomial $y = b_0 + b_1 x + \ldots + b_q x$ of degree *q* ($q \leq k$) in x is to be fitted to the observational points (Y_i, x_i) as a basis for estimating the "true" response y corresponding to any chosen value of x -- then does it necessarily follow that for "best results" one should take $q = k$? If it is believed, but not known for certain, *a priori* that $|\beta_k| > 0$, can it happen that although the data indicate (at some reasonable level of significance) that $|\beta_k| > 0$, nevertheless a polynomial of degree *less than k* in x fitted to these data may yield more accurate estimates of y for any chosen x than will a polynomial of degree *k*? Some general formal answers to these questions were obtained and the "situation" illustrated numerically in some detail for a variety of particular cases of linear relations ($k \leq 1$).

Publications:

(1) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.
(2) Selected bibliography of statistical literature, 1930-57. III. Limit theorems. Lola S. Deming. J. Research NBS *64B*, 175-192 (July-Sept 1960).
(3) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(4) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. To appear in Biometrika.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52
Manager: J. M. Cameron
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. M. Zelen and B. Kurkjian have completed their work on a calculus for factorial arrangements. Using this calculus, they were able to obtain new ways of calculating estimates in a general regression model. Their result enables the coefficient matrix of the normal equations to be written as a direct product of matrices of smaller dimension. That is, if the normal equations are (say)

$$Ab = G$$

where A is p x p, b is p x 1, G is p x 1, then A can be written as the direct product (or Kronecker product)

$$A = A_1 \times A_2 \times \ldots \times A_k.$$

Hence it is not necessary to invert the large order matrix A to solve for b, but instead to invert the matrices A_i of smaller dimension. These results are contained in "A calculus for factorial experiments", by B. Kurkjian and M. Zelen, now in manuscript.

Dr. W. J. Youden has devised new designs of the fractional factorial type that permit the identification of two factor interactions by partially confounding these with main effects. This permits the use of highly fractionated designs that approach the classical weighing designs with respect to the small number of observations required while still permitting identification of interactions.

John Mather devised methods for the construction of and enumerated magic rectangles for use as trend elimination designs. These designs provide an order for running the treatment combinations of a two-way classification design so that comparisons among treatment are not upset by any linear trends or drifts in the measurements.

Publications:
(1) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.
(2) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the 4th Berkeley Symposium on Mathematical Statistics and Probability.

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

Publication:

(1) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. Submitted to a technical journal.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. Zelen and M. C. Dannemiller have completed a paper summarizing their work on the robustness of statistical life testing procedures. In connection with this work they investigated two tests proposed in the statistical literature for testing the assumption that failure times come from an exponential distribution. It was found that for small sample sizes the tests are not powerful enough to distinguish between the exponential and the Weibull distributions, unless the shape parameter for the Weibull distribution is large.

Publication:

(1) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. To appear in Technometrics.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. L. E. Payne has obtained pointwise estimates in certain non-well-posed problems for elastic plates. These results are being written up for publication. He has also obtained estimates for the velocity components in the following two-dimensional viscous flow problem: Let a viscous fluid occupy a region $\mathcal{D}$ with boundary C. Assume that inertial effects may be neglected. On a portion Σ of C, the velocity components, the vorticity, and the pressure gradient are measured with known precision. If crude upper bounds for the kinetic energy and for the maximum of the square of the vorticity are known, pointwise bounds for the velocity components at points in $\mathcal{D}$ can be obtained. Furthermore, these bounds may be made close by approximating the data on Σ in the mean square sense. These results are being written up for publication.

A. Ghaffari has been revising the manuscript of the paper on Brownian motion of a free particle. His recent result consists of finding the most general solution of the Chapman-Kolmogoroff functional equation (see Jan-Mar 1960 issue, pp. 14-15) in the interval $(0, \infty)$ in the form

$$(1) \qquad f(x,s;y,t) = \sum_{n=0}^{\infty} \Theta^n(s,t)\psi_n^{(\alpha)}(x)\psi_n^{(\alpha)}(y), \qquad \alpha > -1,$$

where the infinite sequence of functions

$$\psi_n^{(\alpha)}(x) = \frac{\Gamma^{\frac{1}{2}}(n+1)}{\Gamma^{\frac{1}{2}}(n+\alpha+1)} e^{-\frac{x}{2}} x^{\frac{\alpha}{2}} L_n^{(\alpha)}(x)$$

form a complete orthonormal set over $(0, \infty)$, and $L_n^{(\alpha)}(x)$ is the nth Laguerre polynomial defined by

$$L_n^{(\alpha)}(x) = \frac{x^{-\alpha} e^x}{n!} \frac{d^n}{dx^n}\left(x^{n+\alpha} e^{-x}\right),$$

and the factor $\Theta(s,t) = \frac{a(s)}{a(t)} < 1$, $s < t$ is the same as defined before (see Jan-Mar 1960 issue, pp. 14-15). It has been shown that for s and t fixed such that $s < t$ and x,y varying arbitrarily over $(0, \infty)$, the series solution (1) is absolutely and uniformly convergent.

W. H. Pell attended the 1960 Summer Seminar in Applied Mathematics on "Modern Physical Theories and Associated Developments" at the University of Colorado, July 24 to August 19.

Publications:

(1) On the Stokes flow about a torus. W. H. Pell and L. E. Payne. Mathematika 7, 78-92 (1960).

(2) The Stokes flow about a spindle. To appear in the Quarterly of Applied Mathematics.

(3) On some partial differential equations of Brownian motion of a free particle. A. Ghaffari. To appear in the Proceedings of the International Conference on Partial Differential Equations and Continuum Mechanics held at the Mathematics Research Center, U.S. Army, Madison, Wis., June 1960.

PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen is investigating the dynamics of a system of particles with short and long range forces. Usually the Boltzmann equation is used for particles interacting with a short range force, while the Fokker-Planck equation is used for particles with a long range force of interaction. Both equations can be derived separately from the integration of the Liouville equation. Two different basic hypotheses concerning the weakening of correlations, however, were found to be necessary for the closure of the sequence of equations. This difficulty of two opposing hypotheses becomes serious when one treats the kinetics of a system of particles in which both the short and long range forces coexist, for example, in electrons and ions with hard cores, partially ionized gases, and plasma mixed with hard magnetic cores as in interstellar clouds. Tchen has attempted to bring to the hypotheses a common basis, and to suggest a method for the derivation of a kinetic equation applicable for both the cases of short and long range forces. This investigation has been made while Tchen was working as a Guggenheim Fellow at the Max Planck Institute for Physics and Astrophysics, Munich, Germany, and, since September 1960, at the Theoretical Plasma Section of the Centre d'Etudes Nucleaires, Fontenay-aux-Roses, Seine, France.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti has prepared a paper on "Mean Motions in Conditionally Periodic Separable Systems." In this paper he shows that in any conditionally periodic nonsingular Staeckel system the mean frequency n_k of any coordinate q_k is equal to the corresponding fundamental frequency $\nu_k \equiv \partial\alpha_1/\partial J_k$, where α_1 is the energy and J_k is the corresponding action variable. It therefore applies to all non-polar orbits of a satellite moving in a gravitational field which leads to separability, and can thus be used to find the mean motions in the corresponding intermediary orbit.

Dr. Vinti is also preparing a paper, from material largely worked out, giving the solution of the problem of satellite motion for the potential proposed in J. Research Nat. Bureau of Standards 63B, 105-116 (1959). With use of certain orbital elements directly related to initial conditions, the solution gives both secular terms and periodic terms correct through the second order in a small parameter k proportional to J_2. Here J_2 is a measure of the planetary oblateness, being the coefficient of the second harmonic in the spherical harmonic expansion of the potential.

Publication:

(1) Theory of the orbit of an artificial satellite with use of spheroidal coordinates. J. P. Vinti. Abstract: Astronomical J. 65, 353 (1960).

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS — Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Manager: W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Computation of wavelengths and wavenumbers continued for praseodymium and bromine (Br). A list of the mean values of Br lines was prepared for use in computing energy levels. The list of 70,000 punched cards for the intensity tables was finished, and the data are being prepared for publication on the card-controlled typewriter. Matrices to determine the separation of nearest neighboring eigenvalues were computed for tantalum (Ta), ruthenium (Ru), and thorium (Th).

3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES
Origin: NBS, Section 3.2
Manager: M. L. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. J. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Production runs were made as requested by the sponsor. The "improved" codes were adjusted to overcome several errors noted in the runs.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: B. S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants were computed for 51 thermometers under test. LTS constants were calculated for 22 thermometers.

1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: P. J. Walsh
Full task description: Jan-Mar issue, p. 27
Status: Continued. Control routines have been written that select integral routines, previously completed, in the order required for subsequent operations in obtaining self-consistent field SCF solutions. A complete SCF problem for diatomic hydrides was checked out satisfactorily. Considerable progress has been made on a routine that will transform the integrals to a new basis.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed for the checking of 17 laboratory sets of gage blocks.

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Manager: M. L. Paulsen
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-11645/57-236 SELF CONSISTENT FIELD--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: P. Walsh
Full task description: Apr-June 1957 issue, p. 30
Status: Inactive.

3911-61-39952/56-266 DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Inactive.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Several Fourier summations were calculated for the lysozyme chloride and arginine crystals by means of the MIT program. Codes were written to revise and translate data to the format required by the Fourier program.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. The results for the first network model (uniform spacing of subareas) have been written up and submitted to the sponsor. For the second model, the optimal locations for any subset of the sorting centers have been found in terms of the locations of the remaining center. A power-series solution has been found for the optimal partition; the first few terms have simple forms and give an adequate approximation.

C. T. Ireland (summer student employee) began the analysis of a version in which "long-haul" and "short-haul" transportation are distinguished by a threshold distance. He also studied the results of permitting straight-line transportation, instead of permitting only horizontal and vertical routes.

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Inactive.

1102-40-11645/58-281 PSI EVALUATION
Origin and Sponsor: NBS, Section 4.10
Managers: P. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1957 issue, p. 34
Status: Inactive.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. A joint paper with R. S. Marvin is in preparation. An electrical network is described which serves as a model to represent the viscoelastic properties of rubber-like materials. The network equations are derived and the relations to the mechanical quantities are shown. The mathematical problems in connection with several inverse Laplace transformations are presented and the transformations are carried out numerically.

1102-12-11513/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes (11.0)
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. Work is proceeding on machine programming for syntactic predictions, on planning for clause and phrase recognition, and on compilation of inflectional paradigmatic forms. Machine programming has been started for the generation of morphological information from stems and endings. An NBS report on clauses and phrases has been prepared.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: S. Peavy
Full task description: Apr-June 1958 issue, p. 30
Status: Reactivated. A few small programs are being written in conjunction with the classical case.

3711-60-0009/58-360 DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: J. P. Menard
Full task description: Apr-June 1958 issue, p. 32
Status: Terminated. Productions runs will be continued under the direction of the sponsor and the machine time will be reported in the section of this report entitled "Current Applications of Automatic Computer".

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. Production runs have been submitted to the sponsor.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Manager: P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Inactive.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Continued. Production runs were continued under the direction of the sponsor.

1102-40-11645/59-389 FREQUENCY ALLOCATION

Origin and Sponsor: Civil Aeronautics Administration

Manager: L. S. Joel

Full task description: Oct-Dec 1958 issue, p. 29

Status: Continued. On the computational side, bookkeeping programs for the system have been written and are being checked. A complete list of the currently assigned and projected stations has been received from the sponsor and is being transcribed onto IBM cards for use as input to the "one interchange" code. A flow chart has been written for extending the program to interchange "chains" of length 3.

The theoretical work is based on the observation that an efficient method for finding the optimal pattern of allocations would solve a substantial generalization of a certain problem in the theory of linear graphs. J. Edmonds is seeking such a method (based on the algorithm of C. Berge for the original problem, that of finding a "maximal isolated set" of vertices). One by-product of the research has been the following generalization of a result by B. K. Bender and A. J. Goldman: A connected regular graph with only even circuits has exactly two maximal isolated sets of vertices. This has led to a study of hamiltonian circuits in such graphs, and also to a simple proof of Peterson's theorem that every regular graph of even degree contains a set of circuits passing through each vertex exactly once.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II

Origin and Sponsor: NBS, Section 4.6

Manager: A. E. Beam

Full task description: Oct-Dec 1958 issue, p. 30

Status: Continued. A code has been written which computes the photodetachment cross section of H^-, using the Pekeris bound function.

The evaluation of this cross section is essentially an integration of the product of the bound state function for H^- times the electron dipole operator $r_1 + r_2$ times the final continuum function representing an ejected electron and ground state H atom. Three of the six coordinates involved may be ignored, so there remains a three-fold numerical integration. We use the Pekeris bound function for H^-, which is a 203-parameter function in terms of the perimetric coordinates u, v, and w, rather than the original coordinates r_1, r_2, and r_{12}. This function is computed once at the 15^3 mesh points required for the Gaussian quadrature method, and then stored. The final state function is taken to be either a plane wave or the variational electron-hydrogen P-wave scattering function which has been previously computed. The evaluation of this function in terms of the u, v, and w variables is relatively fast and is evaluated for each desired energy of electron ejection, to be used with the stored bound function in the triple quadrature.

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. (1) Successful experiments have been performed in screening readings with respect to their ventricular gradient. The ventricular gradient is by definition the time average,

$$VG = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x(t)dt,$$

the time interval being one beat cycle.
(2) A 60-cycle correction procedure has been checked out.
(3) On checkout is a code for computing an "average" EKG reading.

1102-40-11645/59-445 OIL SUPPLY
Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Inactive.

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. M. Beiman, W. G. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. A completed survey was turned over to the CAB for a final checkout. All program phases are in the process of being rewritten so that more checks on all tape operations may be made to assure that no information is lost due to machine tape failures.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Reactivated, to compute values of additional functions.

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1959 issue, p. 26
Status: Inactive.

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. Work on the van der Waal's area continued in cooperation with Professor Nesbet, now in Paris, in the form of integral tests for that region. It was found that the number of quadratures in the numerical integration had to be increased to obtain the desired accuracy. Testing was slowed considerably because of the need to communicate with Professor Nesbet on program details at various stages along the way. What is believed to be the final parameter adjustment was worked out just as the period came to a close,and awaits testing.

1102-40-11645/60-467 TRANSISTOR SIMULATION
Origin and Sponsor: NBS, Section 12.1
Manager: G. W. Reitwiesner
Full task description: Oct-Dec 1959 issue, p. 27
Status: Inactive.

1102-40-11645/60-475 IONOSPHERIC SOUNDINGS
Origin and Sponsor: NBS, Section 82.40
Manager: M. L. Paulsen
Full task description: Oct-Dec 1959 issue, p. 29
Status: Continued. Processing for the Boulder Laboratories of virtual height data cards in large quantity (several thousands) continued--covering computation of hourly electron density profiles, tabulation of the results, and determination of additional related parameters. These results are used as input data for other programs at the Boulder Laboratories.

1102-40-11645/60-476 GAS TUBE CHARACTERISTICS, II
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Managers: H. Oser, W. Borsch-Supan
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Beyond the task described in the Oct-Dec 1959 issue, p. 30, an investigation is required into the nonstationary behavior of charge density, current and electrical field. A breakdown of the gas tube discharge induced by changes in external voltage or electron production of the cathode has been observed experimentally. The mechanism of this breakdown is to be investigated by means of the following system of one ordinary and two partial differential equations:

$$(1)\qquad \frac{\partial q_-}{\partial t} = \alpha(E)\cdot v_-(E)\cdot q_- - \frac{\partial}{\partial x}(v_-(E)\cdot q_-)$$

$$(2)\qquad \frac{\partial q_+}{\partial t} = \alpha(E)\cdot v_-(E)\cdot q_- + \frac{\partial}{\partial x}(v_+(E)\cdot q_+)$$

$$(3)\qquad \frac{\partial E}{\partial x} = \epsilon_o^{-1}(q_- - q_+)$$

where q_+ are the charge densities, v_+ and v_- the velocities of ions and electrons respectively with the latter given as functions of the electrical field E, and $\alpha(E)$ the number of ionizations per unit of path length.

On the boundary x = d (anode), the ion current $v + q_+$ vanishes; on the boundary x = 0 (cathode) the electron current $v - q_-$ is prescribed as a function of the ion current at the cathode and the mean electron current inside the tube. A boundary condition for the field E is provided by an ordinary first order differential equation with respect to time, describing the voltage in an external electrical curcuit. Initial values for q_-, q_+ and the external voltage are given.

For reasons of stability, the integration of the above system by difference methods requires very short time steps due to the electron velocity v_-, which is large compared with v_+. For computations covering larger time intervals, the electron density q_- is therefore regarded as being quasi-stationary, and $\partial q_-/\partial t$ is replaced by 0 on the left-hand side of equation (1).

While in the general case the system may be solved by straightforward explicit difference methods, in the quasi-stationary case the system of two ordinary equations resulting from (1) and (3) has to be treated iteratively.

The equations have been translated into finite difference equations. The stability of these equations has been studied and the program is being written.

1102-12-11122/60-479 PROCESSING OF DIAGRAMS
Origin and Sponsor: NBS, Section 11.0
Managers: F. L. Alt (11.0), S. T. Peavy, R. J. Herbold
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Checking of the computer code was completed. For a set of diagrams--photographs of the 26 roman capital letters in one particular type font--moments up to the sixth order were computed in a test run.

1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: R. Zucker
Full task description: Jan-Mar 1960 issue, p. 28
Status: Continued. The code is in production. In one of the production runs, vibrational transition probabilities for the second positive ($C^3\pi_u$ - $B^3\pi_g$) band system of nitrogen agreed quite favorably with previous published results.

1102-40-11645/60-489 INVERSION OF LINE PROBE DATA
Origin and Sponsor: NBS, Section 3.1
Manager: R. Herbold
Full task description: Jan-Mar 1960 issue, p. 29
Status: Continued. Production runs have been made. The procedure has been improved to permit variable data smoothing.

1102-40-11645/60-501 KANSAS RIVER SYSTEM

Origin and Sponsor: Corps of Engineers, U. S. Army, Office of District Engineers, Kansas City District

Manager: S. Peavy

Full task description: Apr-June 1960 issue, p. 24

Status: Continued. The program has been revised by the sponsor to allow for additional variations in the natural phenomena of the river system under study. The new program was submitted for reassembly and has been checked out.

1102-40-11645/60-504 ELECTROSTATIC-FOCUSING PROBLEM

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: A. Beam

Full task description: Jan-Mar 1960 issue, p. 30

Status: Continued. Several production runs were made.

3911-61-39952/60-507 TRANSISTOR AGING BEHAVIOR STUDIES

Origin and Sponsor: NBS, Section 1.6

Manager: R. Varner

Full task description: Apr-June 1960 issue, p. 24

Status: Continued. The transistor aging studies are being continued in NBS Section 14.1 and the computation task now appears as task 3911-61-39952/61-528 on page 34.

3911-61-39952/60-508 MODEL ADSORPTION ISOTHERMS

Origin and Sponsor: NBS, Section 5.2

Manager: J. P. Menard

Full task description: Apr-June 1960 issue, p. 25

Status: Continued. For the case $c > 6.75$, the following transformation $\xi = x/\sqrt{2}\ {}^2$, $d\xi = dx/\sqrt{2}\ {}^2$ has been introduced. The model adsorption isotherm and its first moment have been rewritten, each as a sum of two integrals as follows:

$$\Theta = \frac{1}{\sqrt{\pi}} \left\{ \int_{-\infty}^{\xi(\theta_1)} e^{-\xi^2} \theta(\xi) d\xi + \int_{\xi(\theta_2)}^{\infty} e^{-\xi^2} \theta(\xi) d\xi \right\}$$

$$X = \frac{\sigma}{\sqrt{\pi/2}} \left\{ \int_{-\infty}^{\xi(\theta_1)} e^{-\xi^2} \theta(\xi)\xi d\xi + \int_{\xi(\theta_2)}^{\infty} e^{-\xi^2} \theta(\xi)\xi d\xi \right\}$$

where

$$\xi(\theta) = \frac{RT}{\sqrt{2}\sigma} \left\{ \frac{\theta}{1-\theta} - c\theta + n \frac{\theta}{1-\theta} - \ln y \right\}$$

and

$$\int_0^{\theta_1} g(\theta)d\theta + (\theta_2 - \theta_1) g(\theta_1) = \int_0^{\theta_2} g(\theta)d\theta$$

where

$$g(\theta) = \frac{\theta}{1-\theta} e^{\frac{\theta}{1-\theta} - c\theta}, \quad 0 < \theta_1 < \theta_2 < 1.$$

A separate code has been written to determine the values of θ_1 and θ_2 for a given value of c.

All codes have been written and production runs are in progress. It is assumed that further modifications may have to be made in order to cover the complete range of all the parameters and still maintain significance. Attempts to decrease the machine time necessary for each case are under consideration.

1102-40-11645/60-510 H_2^+ BOMBARDMENT

Origin and Sponsor: Naval Research Laboratory

Manager: W. Borsch-Supan

Full task description: Apr-June 1960 issue, p. 26

Status: Continued. The integrals have been evaluated for several sets of parameters. Some minor changes of the code have been made. A table of the function $\emptyset(\lambda)$ (see Apr-June 1960 issue, p. 26) and a description of some analysis regarding the computation of $\emptyset(\lambda)$ is in preparation as a report.

1102-40-11645/60-513 RADIATIVE ENVELOPES OF MODEL STARS

Origin and Sponsor: National Aeronautics and Space Administration

Managers: S. Haber and P. J. Walsh

Objective: To solve the boundary value problem:

$$\frac{dq}{dx} = \frac{px^2\beta}{t}$$

$$\frac{dt}{dx} = -c\,\frac{p^2\beta^2}{x^2 t^{8.5}}\left(1 + F\,\frac{t^{4.5}}{p^{\beta}}\right)$$

$$\frac{dp}{dx} = \frac{-pq^{\beta}}{x^2 t}$$

$$\beta = 1 - B\,\frac{t^4}{p} \leq 0\cdot$$

$x_f \leq x \leq 1$; x_f determined by $\frac{pdt}{t\,dp}(x_f) = 0.4000$;

$q(1) = 1,\ \beta(1) = \beta_o,\ t(1) = \left[\frac{c}{10^{6.1502}\lambda}\right]^{1/4},$

$w(x_f) = K(v(x_f))$ where K is a tabulated function, $v = \frac{\beta q}{6.25xt}$, $w = \frac{\beta^2 x^2 p}{2.5t^2}$.

F and B are given constants.

The eigenvalue c and the solution are to be determined for various values of β_o and λ.

Background: The problem arises in the study of the evolution and structure of stars. The equations describe the envelope of a model star consisting of a core in which energy transfer is by convection and an envelope in which energy transfer is by radiation.

The problem was transmitted by S. S. Huang (NASA).

Status: New. A program for numerical solution of the problem has been written, and code checking and numerical experimentation are in progress.

1102-40-11645/61-516 RADIATION FIELD FROM A CIRCULAR DISK SOURCE

Origin and Sponsor: NBS, Section 4.8

Manager: R. J. Herbold

Objective: To evaluate $S_n(k^2)$ and $p_\ell(\rho,h)$ in the expressions

$$S_n(k^2) = \int_0^\pi \frac{d\theta}{(1-k^2\cos\theta)^{n+\frac{1}{2}}}$$

$$p_\ell(\rho,h) = \int_s P_\ell(\cos\theta)dr(\theta,\rho,h).$$

Background: In shielding calculations, photometry and related studies, a class of problems occurs involving the response of a radiation detector to uniform finite plane sources radiating according to some angular distribution law $g(\theta)$, where θ is the direction from the source-plane normal. If $g(\theta)$ is expressed as a series of coefficients g_ℓ of its Legendre expansion, the flux integral at the detector reduces to the sum

$$\sum_{\ell=0}^{\infty} (\ell+\tfrac{1}{2})g_\ell\, p_\ell$$

which converges rapidly in many cases of interest.

The procedure for computing $S_n(k^2)$ and $p_\ell(\rho,h)$ has been planned in detail by the sponsor. The problem was communicated by J. H. Hubbell (4.8).

Status: New. A 704 program has been written and checked. $S_n(k^2)$ for $n = 0(1)9$, $k^2 = 0(.01)99$; and $p_\ell(\rho,h)$ for $L = 1(1)9$ have been evaluated and results have been transmitted to the sponsor.

1102-40-11645/60-519 SCALAR WAVE SCATTERING

Origin and Sponsor: NBS, Section 6.0

Manager: R. Zucker

Objective: To evaluate

$$2 + h_m = 2(1-\beta_m) \qquad (m = 0)$$

$$h_m = -\beta_m \cdot \frac{4ik}{ik+\ _m} \cdot \frac{L_+^{(o)}(\eta_o)}{L_+^{(o)}(\eta_m)} \cdot \frac{1}{J_o(j_m)} \qquad (m \geq 1)$$

where β_m are the solutions of the system of equations

$$\frac{1}{\left\{L_+^{(o)}(ik)\right\}^2} \frac{\beta_m}{1-\beta_m} = 1 - \frac{2ik}{1-\beta_m} \sum_{r=1}^{n} \frac{\beta_r}{ik+\eta_r} \qquad (m = 0)$$

$$\frac{1-\beta_o}{ik+\eta_m} = \frac{L_m}{2\eta_m} \beta_m + \sum_{\substack{r=1 \\ r\neq m}}^{n} \frac{\beta_r}{\eta_m+\eta_r} \qquad (m = 1, 2, \ldots, n)$$

$$\eta_m = \sqrt{\left(\frac{j_m}{a}\right)^2 - k^2} \qquad \alpha = k\cdot a$$

and $$L_m = \frac{(2\eta_m)^2}{\left\{(ik+\eta_m)L_+^{(o)}(\eta_m)\right\}^2} + 1 .$$

$$\ell n L_+^{(o)}(\eta_m) = -i(j_m^2-\alpha^2)^{\frac{1}{2}} \frac{1}{\pi} \int_o^{\alpha} \frac{\tan^{-1}\left\{-J_1(u)/N_1(u)\right\}}{(j_m^2-u^2)(\alpha^2-u^2)^{\frac{1}{2}}} u\, du$$

$$+ (j_m^2-\alpha^2)^{\frac{1}{2}} \frac{1}{\pi} \int_o^{\alpha} \frac{\ell n[\pi J_1(u)\left\{(J_1(u))^2+(N_1(u))^2\right\}^{\frac{1}{2}}]}{(j_m^2 - u^2)(\alpha^2 - u^2)^{\frac{1}{2}}} u\, du$$

$$- (j_m^2- \alpha^2)^{\frac{1}{2}} \frac{1}{\pi} \int_o^{\infty} \frac{\ell n\left\{1/[2I_1(u)K_1(u)]\right\}}{(j_m^2 + u^2)(\alpha^2 + u^2)^{\frac{1}{2}}} u\, du$$

where J_1, N_1, I_1 and K_1 are Bessel functions and $J_1(j_m) = 0$.

Background: Although an exact solution of the problem of the scattering by a dielectic cylindrical rod of finite length is not known, this problem is important in light scattering experiments and in the design of antennae and microphones. The solution is well known when the length of the rod is infinite, but theoretical knowledge of the scattering due to finite obstacles in 3 dimensions is essentially limited to those of ellipsoidal shape. In order to make some progress, the first simplification as given above is to make the rod semi-infinite in length. (See the "Scattering of a Scalar Wave by a Semi-infinite Rod of Circular Cross Section," D. S. Jones, Phil. Trans. Roy. Soc. Series A, No. 934, Vol. 247, pp. 499-528, 1955.) The problem was submitted by Dr. Matsui (6.1).

Status: New. To avoid the singularity, a transformation of variables was made in the first two integrals of $\ell n L_+^{(o)}(\eta_m)$. The code has been written and checked out, and production runs were make for $\alpha = 0(.1)3$, $m = 0, 1, \ldots, 6$.

1102-40-11645/61-520 HEAT EXPANSION

Origin and Sponsor: NBS, Section 2.4

Manager: H. Oser

Objective: To determine the best fit in the least square sense to a given set of heat expansion coefficients and temperatures.

Background: The approximating function is given by Grüneisen's formula:

$$\beta(T) = \frac{Qc_v}{(Q - kE)^2}$$

where c_v and E are given by P. Debye as

$$c_v = 3R\left[12\left(\frac{T}{\theta}\right)^3 \int_0^{\theta/T} \frac{y^3 dy}{e^y - 1} - 3\frac{\theta/T}{e^{\theta/T} - 1}\right]$$

$$E = \int_0^T c_v \, dT$$

θ, Q and k are the constants to be determined for a given set of data.

The problem is nonlinear in nature and an iterative scheme has to be used for the solution. Assuming θ to be known for the moment the problem of minimizing the expression

$$S(\theta) = \sum_{i=1}^{n} [(Q-kE(T_i))^2 \beta(T_i) - Q\, c_v(T_i)]^2$$

leads to a cubic equation in Q. The constant k is then explicitly given as a function of Q. The iteration is essentially Muller's method, i.e., fitting a second order parabola through three points of the function S(θ), whose minimum is used as a new approximation for θ.

The solution of this problem is required for the interpretation of length measurements presently being obtained in the Metrology Division of NBS. The problem was submitted by R. K. Kirby (2.4).

Status: New. A code has been written by E. A. Herman and is being checked out.

1100-40-11460/61-522 SPECIAL CONSULTING SERVICES

Origin and Sponsor: National Aeronautics and Space Administration

Manager: E. W. Cannon (11.0)

Full task description: Apr-June 1960 issue, p. 26

Status: Continued. The Committee members visited computational facilities of the National Aeronautics and Space Administration and discussed organization and operational procedures with personnel encountered. Both "in-house" and contract facilities were included in the visits. Preparation of the Committee report was begun.

1102-40-11645/61-527 NUCLEAR MAGNETIC RESONANCE

Origin and Sponsor: NBS, Section 15.7

Manager: H. Oser

Objective: To compute a table of the shielding function of the benzene ring, i.e., its magnetic field distribution.

Background: C. E. Johnson and F. A. Bovey (J. Chem. Phys. 29 (1958), 1012) used the free electron model of Pauling to calculate the magnetic field around a benzene ring, which is rotating around all axes in an external magnetic field. Recent measurements of the magnetic field distribution of benzene rings, made in the Chemistry Division of the NBS, go beyond the range of these tables and therefore the need for an extension has arisen.

The problem was submitted by E. Lustig (15.7).

Status: New. A FORTRAN code has been written and is being checked out.

3911-61-39952/61-528 ANALYSIS OF EXPERIMENTAL DATA ON TRANSISTOR AGING

Origin and Sponsor: NBS, Section 14.1

Manager: R. Varner

Objective: To obtain bounds on the range of variability of the characteristics of transistors caused by within-day and also between-day transistor variability, ambient temperature variability, and test set variability. To determine the distribution of parameter values so as to decide what, if any, transformations will be necessary before conducting statistical analysis. To test for any significant effects of soldering the transistors to the test plug. To detect any other possible sources of trouble before subjecting the transistors to controlled aging conditions.

Background: The above information is required for use in the planning of a study of transistor aging behavior in which approximately 550 type 2N396 transistors will be used. The study will incorporate aging treatments that are various combinations of ambient temperature, collector power dissipation and collector voltage.

This problem is a continuation of task 3911-61-39952/60-507 which originated in Section 1.6. With project and sponsor transferred to Section 14.1, the task is continued under this new number for accounting purposes.

The problem was transmitted by G. T. Conrad, Jr.

Status: Continued. A code has been written to tabulate and plot the distribution of transistor readings.

1102-40-11645/61-530 SPECIMEN WAVELENGTH
Origin and Sponsor: NBS, Section 9.4
Manager: J. P. Menard
Objective: To solve the equation

$$E_{sc\lambda o} = 8\beta_o \Big\{ 2(E_{s\lambda}-1)[(1-\beta_o^2)(e^{\sigma_o D} - e^{-\sigma_o D}) + 4\beta_o]$$

$$+(1+\beta_o)^3 e^{2\sigma_o D} - (1-\beta_o)^3 e^{-2\sigma_o D} + 2\beta_o(1-\beta_o^2)$$

$$/[n_\lambda(n_\lambda+1)^2][(1+\beta_o)^4 e^{2\sigma_o D} + (1-\beta_o)^4 e^{-2\sigma_o D} - 2(1-\beta_o^2)^2]\Big\}$$

for
1) n_λ - Spectral index of refraction of the coating matrix
2) $E_{s\lambda}$ - emittance of the substrate into the coating
3) σ_o - extinction coefficient
4) β_o - effective emittance

as a function of wavelength. The normal spectral emittance of the composite specimen, $E_{sc\lambda o}$, and the thickness of the coating, D, are known. The extinction coefficient, σ_o, and the effective emittance, β_o, are further defined in terms of the absorption coefficient of the coating α_λ, and the scattering coefficient of the coating, S_λ as follows:

$$\sigma_o = \sqrt{\alpha_\lambda(\alpha_\lambda + 2\ S_\lambda)}$$

$$\beta_o = \sqrt{\alpha_\lambda\ /\ (\alpha_\lambda + 2S_\lambda)}$$

Background: The above equation arises in the study of the radiant flux emitted by a composite specimen of a partially transparent, light-scattering coating applied to a completely opaque substrate. If we consider the plus direction to be from the substrate to the coating-air interface, and the minus direction to be from the air to the coating-substrate interface, we can speak of the components of the radiant flux as the following:

(1) energy originating in the substrate and transmitted in the plus direction by the coating

(2) energy originating in the coating and emitted in the plus direction

(3) energy originating in the coating, emitted in the minus direction, reflected by the substrate and transmitted by the coating in the plus direction

Components due to internal reflection at the coating-air interface are to be neglected in the preliminary analysis.

The present methods of obtaining a composite specimen of a specified spectral emittance and coating thickness are very slow and tedious. It is hoped that by specifying values of $E_{sc\lambda o}$ and D in above equation and solving for n_λ, $E_{s\lambda}$, σ_o, and β_o, it will be possible to use these results in predicting the composition of the desired specimen and thus reduce the number of trials necessary in obtaining a composite specimen of specified spectral emittance and coating thickness.

The problem was transmitted by J. Richmond (9.4).

Status: New.

1102-40-11645/61-531 HEAT TRANSFER IN CRYSTALS

Origin and Sponsor: NBS, Section 3.1

Manager: H. Oser

Objective: To solve a system of ordinary differential equations

(1) $$M\ddot{x} + Kx = 0,$$

where M is a diagonal matrix and K is cyclic and symmetric. The initial conditions are the same for all problems:

$$\text{a)}\quad x(0) = 0$$

$$\text{b)}\quad \dot{x}(0) = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

To be computed are the kinetic and potential energies for various numbers of particles as functions of time. Results for periodic mass distributions are to be compared with randomly distributed masses. Rather large numbers of mass particles are anticipated (of order 100 and more).

Background: Assuming a spatial period in the crystal leads to a cyclic model, mathematically equivalent to $x_i = x_{i+n}$. The propagation of a disturbance as given above is described by the differential equation (1) whose solution can be written:

(2)
$$x(t_o + \Delta t) = [\cos(B\cdot\Delta t)]\cdot x(t_o) + [B^{-1} \sin B\cdot\Delta t]\cdot \dot{x}(t_o)$$
$$\dot{x}(t_o + \Delta t) = -[B \sin(B\cdot\Delta t)]\cdot x(t_o) + [\cos(B\cdot\Delta t)]\cdot \dot{x}(t_o)$$

where $B = \sqrt{M^{-1}K}$. We note here that in spite of K being singular no inverses or roots of K appear in the final result (2), because only even powers of B remain in the expressions for x(t) and $\dot{x}(t)$.

The solution is obtained by repeated application of the transformation matrix

$$T(\Delta t) = \begin{bmatrix} [\cos(B\cdot\Delta t)], & [B^{-1}\cdot \sin(B\ \Delta t)] \\ -[B\cdot \sin B\ \Delta t], & [\cos(B\cdot\Delta t)] \end{bmatrix}$$

on the vector $y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$. The problem was transmitted by R. Rubin (3.1).

Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Transistor aging experiments. J. R. Rosenblatt and H. H. Ku are collaborating in the preparation of an experiment design for a large-scale experiment on the aging of transistors being conducted by G. Conrad of the Engineering Electronics Section (14.01). A complex physical relation exists between the three factors under the experimenters' control and the chief factor under study.

(2) Color mixture data. J. R. Rosenblatt and M. C. Dannemiller prepared an example showing how variances and covariances for color mixture functions can be used in interpreting the results of color-matching experiments, for inclusion in their joint paper with I. Nimeroff.

(3) Calibration of volumetric apparatus. W. J. Youden has studied the measurement process for calibrating volumetric glassware to ascertain whether sampling methods could be used in place of complete inspection. This work was done in collaboration with C. T. Collett of the Volumetry and Densimetry Section (2.07).

(4) Rejection of observations. Monte Carlo simulations of a method for the rejection of outlying observations are being carried out on the Bureau's computer to determine the operating characteristics of the method. A machine program is being prepared by M. C. Dannemiller for John Mandel, Organic and Fibrous Materials Division.

Publications:

(1) Variability of color-mixture data. I. Nimeroff (NBS Photometry and Colorimetry Section), J. Rosenblatt and M. C. Dannemiller. In manuscript.
(2) Some observations on evaluating accuracy. W. J. Youden. To appear in Proceedings of the American Society for Testing Materials.
(3) The sample, the procedure, and the laboratory. W. J. Youden. Submitted to a technical journal.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Preparation of the text for final publication is nearing completion.

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the Veterans' Administration Hospital, Perry Point, Md., for whom a computer program for performing discriminant analysis on data from the VA's cooperative studies on chemotherapy in psychiatry was completed.

Current Applications of Automatic Computer

The record of the use of the IBM 704 for the period July 1 through September 30 is as follows:

Task No.		Title	Assembly	Checking	Production
			(MINUTES)		
NBS:					
11110/55-55	11.1	Research in numerical analysis	542	241	1536
11411/55-56	11.1	Research in mathematical topics applicable to numerical analysis		4	182
11413/60-469	11.1	Orthogonal functions in the theory of partial differential equations	24	25	723
11120/55-65	11.2	Automatic coding	44	120	226
39952/56-160	11.2	Mathematical subroutines	256	232	144
39951/51-1	11.3	Statistical engineering	23	27	223
39952/54-30	4.1	Spectrum analysis	188	123	393
39952/55-68	9.7	Crystal structure calculations	31	36	858
39952/55-82	3.1	Thermometer calibrations			259
39952/56-131	2.2	Calculations in optics*	17	9	91
11645/56-166	5.9	SCF-LCAO solution of some hydrides*	44	256	112
11645/56-171	3.2	Collision integrals used in transport theory**	48	78	28
11645/57-216	11.2	Handbook of Mathematical Functions	5	20	
11645/57-219	3.2	Thermal properties*	180	144	292
11645/57-246	4.8	Radiation diffusion**	516	56	402
11645/57-249	9.4	Color differences*			14
39952/57-250	2.3	Automatic reduction in spectrophotometric data*	33	55	64
11645/57-251	1.6	Current noise and fixed resistors*	2		
11645/57-252	4.11	Detecting efficiency in a neutral meson experiment**	73	100	363
39952/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*			48
11645/58-255	4.8	Chi functions**	337	81	843
11645/58-256	10.6	Composite walls**	204	68	232
11645/58-260	12.5	Prototype accounting**	59	327	87
11645/58-267	2.1	Munsell color system conversion*			76

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/58-271	6.3	Simultaneous equations for potential flow**	16		23
/58-272	3.2	Thermodynamic properties of real gases			9
/58-274	9.7	Calculations for d-spacings II*	18	4	37
/58-275	7.8	Crystallography**			164
/58-294	4.8	Nuclear scattering of photons*			9
/58-306	2.1	Interpolation of color mixture functions*			53
/58-308	3.4	Oscillating sphere*			9
/58-314	3.2	Approximations for gas mixtures*	194	75	357
/58-333	9.0	Calcium hydroxide*	404	158	873
/58-339	3.4	Viscoelasticity properties of materials		23	5
/58-357	3.3	Eigenvalues**	6		22
/58-358	3.2	Reduced cross sections	10	4	160
/58-360	5.2	Diffusion coefficients*	3	31	61
/59-387	30.4	Nuclear reactor design**			103
/59-388	10.3	Heat pump calculations*			35
/59-390	12.5	Electrocardiogram**	29		
/59-394	4.6	Slow electron scattering by hydrogen atoms	212	252	138
/59-395	7.7	Adsorption study**	108	42	57
/59-403	2.1	Computation of color fadings*			34
/59-409	12.5	Bank Board**	61	111	811
/59-417	2.4	Spectrum analysis of ruthenium**			108
/59-418	4.8	P-Wave equation*	61	29	40
/59-428	12.5	Radio intensities**	151	68	135
/59-440	82.10	Mapping**	75	328	306
/59-446	85.10	Ionospheric data**	28	66	168
/60-457	12.5	Public Housing problem**	30	36	399
/60-466	3.2	Electronic properties of simple molecular systems		14	42
/60-474	2.5	Gage block stability*			23
/60-475	82.40	Ionospheric soundings	17	64	642
11122/60-479	11.0	Processing of diagrams	13	29	5
11645/60-486	3.6	Morse wave functions	5		762
/60-487	5.9	Parabolic curve fitting*			2
/60-489	3.1	Inversion of line probe data	19	.47	39
/60-493	3.7	Poisson distribution function**	231	239	521
/60-494	82.0	Atmospheric transmission**	89	131	79
/60-495	6.4	Engineering mechanics**	9	5	
/60-508	5.2	Model adsorption isotherms	20	181	224
/60-514	3.9	Flame spectra**			42
/60-515	13.5	Convolution integral**	81	82	

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/61-516	4.8	Radiation field from a circular disk source	13	9	
/61-519	6.0	Scalar wave scattering	33	56	29
/61-520	2.4	Heat expansion	22	42	
/61-523	4.0	Neutron cross section computations**	48	21	
/61-525	3.8	Curve fitting of wave functions**	32		21
/61-526	3.0	Crystal field calculations**	5		
/61-528	14.1	Transistor aging	4		
		Totals (NBS Services)...	4,673	4,149	13,793
OUTSIDE:					
11645/53-45	SC	Air defense tactics°		66	126
/58-269	NRL	Molecular structure, IV	451	174	204
/58-270	PO	Post Office problem	3	84	196
/58-276	NOL	General kinetics, I**			6,704
/58-278	NOL	Polaris**	56	333	62
/58-325	VA	Covariance analysis	4	22	2
/58-335	DOFL	Roots of Bessel functions**		2	14
/58-348	OOR	Russian-to-English machine translation	12	23	9
/58-361	NRL	Spectrum of dipole radiation	48	26	455
/58-368	SC	Intensity functions of light scattered by spherical particles	5	46	
/59-371	NRL	ASWAP°	30	144	195
/59-373	DOFL	Rhinitis**	94	4	395
/59-389	CAA	Frequency allocation	16		
/59-407	DOFL	Fourier coefficients*	198	53	147
/59-408	NASA	NASA**	187	52	849
/59-411	HEW	Fitting of exponential curves**	445	172	173
/59-415	DOFL	Complex Legendre functions*	17		35
/59-416	DOFL	Analysis of power supply experiments**	70	52	72
/59-419	DOFL	Neutrons°	31		8
/59-423	WB	Weather Bureau**			54,056
/59-425	CU	Molecular orbitals*			608
/59-434	CIW	Petrological computations*	6		2
/59-435	VA	Electrocardiographic analysis	6	27	284
/59-441	GK	Systems engineering**		183	1,779
/59-445	NPSA	Oil supply	4		
/59-447	BPRO	Public Roads study**	115	28	5,225
/60-450	ACC	Chemical warfare°	100	14	885
/60-453	AMS	Data conversion*			318

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/60-454	GE	G.E.**	17		541
/60-458	CAB	Domestic airline traffic survey	209	904	3,362
/60-462	DOFL	Correlation of functions	45	19	
/60-470	NSF	Information selection system**	5	16	
/60-476	DOFL	Gas tube characteristic II		6	405
/60-481	SC	Radar study°	244	192	4
/60-492	IMF	Monetary research reports**	49		30
/60-496	BPA	Short circuit program**	52	14	68
/60-501	CEng	Kansas River Reservoir system**	44	27	
/60-502	USA	Quartermaster mathematics programming**	13	14	257
/60-504	DOFL	Electrostatic focusing			423
/60-506	WB	World Bank**	8		29
/60-510	NRL	H_2^+-bombardment			106
/60-512	DOFL	Data transformation **	55	21	2
/60-517	DOFL	Shock tube**	39	15	111
/61-518	DOFL	Reliability of binary counter**	18	137	18
/61-529	RCA	RCA AFC**			54
		Miscellaneous			15
		Totals (Outside)	2,696	2,870	78,228

	Assembly	Checking	Production
Total time for the quarter (MINUTES).....	7,369	7,019	92,021
Total time for the quarter (HOURS)	123	117	1,534

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.
** Problem programmed by sponsor and run under his direction.
° Classified task.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Lectures

MORDELL, L. J. (University of Cambridge and University of Colorado) The sum $\sum [x_n/(x_{n+1}+x_{n+2})]$ and some others. September 19.

Mathematical Statistics Seminars

ZELEN, M. A survey of methods for generating random deviates. July 6. A calculus for factorial arrangements. August 5.

KU, H. H. An application of information theory to the analyses of contingency tables. August 19.

Statistical Engineering Laboratory Reliability Seminars

DANNEMILLER, Mary C. The robustness of statistical life testing procedures, II. July 22.

MORRISON, Donald F. (National Institutes of Health) Statistical properties of systems with spare components. September 30.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

DANNEMILLER, M. C. and ZELEN, M. The robustness of life testing procedures derived from the exponential distribution. Presented at the Annual Meeting of the American Statistical Association, Stanford University, Stanford, Calif., August 25.

EISENHART, C. Accuracy and precision. Presented at the Gordon Research Conferences, Section on Statistics in Chemistry and Chemical Engineering, New Hampton, N. H., July 12.

HAYNSWORTH, E. Boolean algebra. Presented at a National Science Foundation Institute, Stetson University, Deland, Fla., July 7.

RABINOWITZ, P. Use of orthogonal functions in solving linear elliptic differential equations. Presented at the 15th National Conference of the Association for Computing Machinery, Milwaukee, Wis., August 25.

RICE, J. R. (1) Interpolating functions and best approximations on [0,1]. (2) Interpolating functions and best Tchebycheff approximations. (3) Linear interpolating functions and best L_p approximations. Presented at the meeting of the American Mathematical Society, East Lansing, Mich., August 29.

YOUDEN, W. J. (1) How to evaluate accuracy. Presented at the Symposium on Quality of Observations, ASTM Annual Meeting, Atlantic City, N. J., July 1. (2) The enduring values. Presented before the Spectrochemical Society of Pittsburgh, Pa., September 21.

ZELEN, M. (1) A calculus for factorial arrangements. (2) (With Kurkjian, B.) A general theory of confounding for mixed factorial experiments. (3) (With Severo, N. C.) Normal approximation to the chi-square and non-central F probability functions. All presented at the Institute of Mathematical Statistics Annual Meetings, Stanford University, Calif., August 23-26.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.3 Technical Papers

The following papers appeared in J. Research NBS 64B, July-Sept 1960 (Mathematics and Mathematical Physics):

(1) Split Runge-Kutta method for simultaneous equations. J. R. Rice. Pp. 151-170.

(2) A reduction formula for partitioned matrices. E. Haynsworth. Pp. 171-174.

(3) Selected bibliography of statistical literature, 1930-1957. III: Limit theorems. Pp. 175-192.

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(4) Mechanized conversion of colorimetric data to Munsell renotations. W. C. Rheinboldt and J. P. Menard. J. Opt. Soc. Amer. 50, 802-807 (1960).

(5) On Stokes flow about a torus. W. H. Pell and L. E. Payne. Mathematika 7, 78-92 (1960).

1.4 Reviews and Notes

(1) Theory of the orbit of an artificial satellite with use of spheroidal coordinates. J. P. Vinti. Abstract only: Astronomical J. 65, 353 (1960).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Technical Notes, Manuals, and Bibliographies

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

2.3 Technical Papers

(1) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. To appear in the Journal of Research, NBS, Sec. B.

(2) Computer simplification of Boolean functions. B. K. Bender, A. J. Goldman, and R. B. Thomas (Data Processing Systems). Submitted to a technical journal.

(3) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960.

(4) Bounds for the P-condition number of matrices with positive roots. P. J. Davis, E. Haynsworth and M. Marcus. To appear in the Journal of Research, NBS, Sec. B.

(5) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. Submitted to a technical journal.

(6) A comment on Ryser's "Normal and Integral Implies Incidence" theorem. K. Goldberg. Submitted to a technical journal.

(7) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(8) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(9) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.

(10) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.

(11) Bounds for determinants with positive diagonals. E. V. Haynsworth. To appear in the Proceedings of the American Mathematical Society.

(12) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.

(13) Special types of partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.

(14) Irrational power series. M. Newman. To appear in the Proceedings of the American Mathematical Society.

(15) Periodicity modulo m and divisibility properties of the partition function. M. Newman. To appear in the Transactions of the American Mathematical Society.

(16) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.

(17) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.

(18) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

(19) Best approximations and interpolating functions. J. R. Rice. Submitted to a technical journal.

(20) Sequence transformations based on Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B.

(21) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.

(22) Tchebycheff approximations by ab^x+c. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.

(23) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.

(24) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. To appear in Biometrika.

(25) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. To appear in the Journal of Research, NBS, Sec. B.

(26) Some observations on evaluating accuracy. W. J. Youden. To appear in Proceedings of the American Society for Testing Materials.

(27) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(28) Statistical problems arising in the establishment of physical standards. W. J. Youden. To appear in the Proceedings of the 4th Berkeley Symposium on Mathematical Statistics and Probability, 1960.

(29) The sample, the procedure, and the laboratory. W. J. Youden. Submitted to a technical journal.

(30) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. Submitted to a technical journal.

(31) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. To appear in Technometrics.

(32) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.

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U.S. DEPARTMENT OF COMMERCE
Frederick H. Mueller, *Secretary*

NATIONAL BUREAU OF STANDARDS
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THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

ELECTRICITY. Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics.

METROLOGY. Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

HEAT. Temperature Physics. Heat Measurements. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research. Equation of State. Statistical Physics. Molecular Spectroscopy.

RADIATION PHYSICS. X-Ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

CHEMISTRY. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

MECHANICS. Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Combustion Controls.

ORGANIC AND FIBROUS MATERIALS. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

METALLURGY. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

MINERAL PRODUCTS. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

BUILDING RESEARCH. Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

APPLIED MATHEMATICS. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

DATA PROCESSING SYSTEMS. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Applications Engineering.

ATOMIC PHYSICS. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics.

INSTRUMENTATION. Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Office of Weights and Measures.

BOULDER, COLO.

CRYOGENIC ENGINEERING. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

IONOSPHERE RESEARCH AND PROPAGATION. Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services.

RADIO PROPAGATION ENGINEERING. Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

RADIO STANDARDS. High frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

RADIO SYSTEMS. High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Space Telecommunications.

UPPER ATMOSPHERE AND SPACE PHYSICS. Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.

NBS