

# **NATIONAL BUREAU OF STANDARDS REPORT**

6935

**PROJECTS and PUBLICATIONS**  
of the  
**APPLIED MATHEMATICS DIVISION**  
A Quarterly Report  
April through June 1960

For Official Distribution



**U. S. DEPARTMENT OF COMMERCE**  
**NATIONAL BUREAU OF STANDARDS**

## THE NATIONAL BUREAU OF STANDARDS

### Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to government agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. Research projects are also performed for other government agencies when the work relates to and supplements the basic program of the Bureau or when the Bureau's unique competence is required. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

### Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers. These papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three periodicals available from the Government Printing Office: The Journal of Research, published in four separate sections, presents complete scientific and technical papers; the Technical News Bulletin presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: Monographs, Applied Mathematics Series, Handbooks, Miscellaneous Publications, and Technical Notes.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards (\$1.25) and its Supplement (\$1.50), available from the Superintendent of Documents, Government Printing Office, Washington 25, D.C.

# **NATIONAL BUREAU OF STANDARDS REPORT**

**NBS PROJECT**

**11.0**

**NBS REPORT**

**6935**

**PROJECTS and PUBLICATIONS**  
**of the**  
**APPLIED MATHEMATICS DIVISION**  
**A Quarterly Report**  
**April through June 1960**

## **IMPORTANT NOTICE**

**NATIONAL BUREAU OF STANDARDS**  
Documents intended for use within the Government are not to be distributed outside the Government without the approval of the Director of the National Institute of Standards and Technology (NIST). Such permission is obtained in writing from the Director, NIST, Washington 25, D.C. Such permission is not to be construed as an endorsement of the product or service described herein.

Approved for public release by the  
director of the National Institute of  
Standards and Technology (NIST)  
on October 9, 2015

Progress accounting documents are formally published in the National Bureau of Standards Monograph Series. Reproduction, reprinting, or redistribution, is not authorized unless the National Bureau of Standards, Washington, D.C., is notified and the necessary arrangements are made for the reproduction of copies for its own use.



**U. S. DEPARTMENT OF COMMERCE**  
**NATIONAL BUREAU OF STANDARDS**

# APPLIED MATHEMATICS DIVISION

April 1 through June 30, 1960

## TECHNICAL ADVISORY COMMITTEE

David Blackwell, University of California	Philip M. Morse, Massachusetts
A. S. Householder, Oak Ridge National	Institute of Technology
Laboratory	A. H. Taub, University of Illinois
Mark Kac, Cornell University	J. L. Walsh, Harvard University

## DIVISION OFFICE

Edward W. Cannon, Ph.D., Chief	
Franz L. Alt, Ph.D., Assistant Chief	
Mildred R. Bethany, Secretary	
Myrtle R. Kellington, M.A., Technical Aid	
Luis O. Rodriguez, M.A., Administrative Officer	
Yates S. Sladen, Administrative Assistant	
W. J. Youden, Ph.D., Consultant	Ida Rhodes, M.A., Consultant
Mary B. Sherlin, Secretary	Luba Ross, Clerical Asst
	Irene R. Robinson, M.A.
	Leroy F. Meyers, Ph.D.
	Owen L. McArdle (Sgt.) <sup>o</sup>

## NUMERICAL ANALYSIS SECTION Philip J. Davis, Ph.D., Chief

Norman W. Bazley, Ph.D.**	Alan J. Goldman, Ph.D.	Philip Rabinowitz, Ph.D.
Lois E. Clarenbach	Emilie V. Haynsworth, Ph.D.	John R. Rice, Ph.D.
John R. Edmonds, M.A.	Molly F. Hevenor, B.S., Sec'y	Robert Silverman, Ph.D.
Margaret R. Ellickson	Clifford T. Ireland, B.A.	Hans F. Weinberger, Sc.D.**
Jane C. Gager, B.A.	Kenneth E. Kloss	Charles T. Zahn, Jr.
Karl Goldberg, Ph.D.	Morris Newman, Ph.D.	

## COMPUTATION LABORATORY Don I. Mittleman, Ph.D., Chief

Irene A. Stegun, M.A., Assistant Chief  
Joseph H. Wegstein, M.S., Assistant Chief

Kathleen Amouri	George B. Hatton	Maxine L. Paulsen, B.S.
Robert J. Arms, Ph.D.	Robert J. Herbold, B.A.	Sally T. Peavy, B.S.
Alfred E. Beam, B.A.	Eugene A. Herman, B.S.	B. Stanley Prusch, B.S.
Jeanne M. Beiman, B.S.	Gloria F. Holmes, B.S., Sec'y	George W. Reitwiesner, M.S.
Bernice K. Bender, M.A.	Laurence A. Jackson	Patricia L. Ruttenberg, B.A.**
Wolfgang Borsch-Supan, Ph.D.	Lambert S. Joel, B.A.	Mary W. Shultz
Doris M. Burrell, Sec'y	James C. Lamkin, Jr., B.S.	Elizabeth F. Sutton
Ruth E. Capuano	David S. Liepman	Lois M. Talley
Vernon Dantzler, M.A.	Marion McIlwain, Sec'y	Elmer C. Terry
Charles R. Drew	John P. Menard, B.A.	Ruth N. Varner, B.A.
Mary M. Dunlap, B.S.	Joyce L. Miles	J. D. Waggoner, B.A.
Pearlie M. Fox	Kermit C. Nelson	Philip J. Walsh, B.S.
Gerald M. Galler, B.A.	Peter J. O'Hara, B.S.	Bertha H. Walter
Elizabeth F. Godefroy	Hansjorg Oser, Ph.D.	Guy G. Ziegler, B.S.
William G. Hall, B.S.	Betty J. Pailen	Ruth Zucker, B.A.

## STATISTICAL ENGINEERING LABORATORY Churchill Eisenhart, Ph.D., Chief

Joseph M. Cameron, M.S., Assistant Chief

Marion T. Carson	John N. Mather	Joan R. Rosenblatt, Ph.D.
Mary C. Dannemiller, B.A.	Edmund B. McCue, M.S.	Marilynn Vogt
Lola S. Deming, M.A.	Mary G. Natrella, B.A.	Shirley M. Young, B.A.*
Judith E. Kirsch	Patricia A. Payne, Sec'y	Marvin Zelen, Ph.D.
Hsien H. Ku, M.S.	Charlotte K. Roeca, Sec'y	

## MATHEMATICAL PHYSICS SECTION Edward W. Cannon, Ph.D., Acting Chief

Marian V. Coleman, Sec'y	Lawrence E. Payne, Ph.D.**	John P. Vinti, Sc.D.
Abolghassem Ghaffari, Ph.D.	Chan Mou Tchen, Ph.D.	

\*On leave of absence

\*\*Part time

<sup>o</sup>Guest Worker

## Contents

Status of Projects* as of June 30, 1960.....	1
1. Numerical analysis.....	1
2. Mathematical tables and programming research..	6
3. Probability and mathematical statistics.....	8
4. Mathematical physics.....	11
5. Mathematical and computational services.....	14
6. Statistical and engineering services.....	27
Current applications of automatic computer.....	29
Lectures and technical meetings.....	33
Publication activities.....	35

\*Only unclassified projects are included in this report.





# Status of Projects

June 30, 1960

## 1. NUMERICAL ANALYSIS

### RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS Task 1101-12-11110/55-55

Origin: NBS

Authorized 8/29/54

Manager: P. Davis

Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth has continued work on eigenvalues of partitioned matrices. She has made several additions to the paper "Special types of partitioned matrices" by including new proofs of results by B. Friedman and S. N. Afriat on the characteristic roots of certain special partitioned matrices.

Dr. Haynsworth has collaborated with P. J. Davis and M. Marcus in the completion of a paper on "Bounds for the P-condition number of matrices with positive roots." A relatively simple method is presented for finding an upper and a lower bound for the P-condition number.

H. F. Weinberger studied optimal approximation for functions prescribed at equally spaced points. Explicit upper and lower bounds for the value  $F(u)$  of a linear functional  $F$  applied to a function  $u(x)$  defined on the interval  $0 \leq x \leq 1$  have been found when  $u$  is prescribed at the  $N + 1$  points  $i/N$ ,  $i = 0, \dots, N$ , and a bound for the integral of  $u^{[k]}_2$  is known. These bounds are optimal in the sense that they are attained for functions satisfying the prescribed conditions. Their computation involves the inversion of a matrix of size  $k-1$  rather than  $N$ , which means that  $N$  is permitted to be large. A paper has been prepared.

N. Bazley continued his work on estimating the eigenvalues of the associated Legendre equation for non-integral values of the parameter  $m$ . He has also investigated Kato's method for obtaining bounds on scattering phases.

R. Silverman obtained characterizations of projective planes in terms of Steiner triple systems and these have led to considerations of the structure of triple systems. Also, previous work relating to the structure of metrized Cartesian product spaces was continued. The introduction of a number of statistical concepts has led to several interesting results.

J. R. Rice completed the manuscript on "Interpolating functions and best approximations" (see Jan-Mar 1960 issue, p. 2, paragraph f), and another paper on "Tchebycheff approximation by exponentials" dealing with approximation by a linear combination of exponential functions with polynomial coefficients.

He continued work on a numerical algorithm for the computation of the best Tchebycheff approximation by  $ab^x + c$ . Also, the analysis of approximation by convex and monotonic polynomials was extended to the case

of approximation by polynomials with positive kth derivative; a manuscript is in preparation.

Work by Dr. Rice continued on the numerical solution of the self-acting gas lubricated journal bearing equation

$$\frac{\partial}{\partial x}[h(x,y)\frac{\partial q}{\partial x}] + \frac{\partial}{\partial y}[h(x,y)\frac{\partial q}{\partial y}] = -\Lambda \frac{\partial[h(x,y)\sqrt{q}]}{\partial x}.$$

There is still an undetermined source of divergence in the methods now employed for this equation. Other methods are being evaluated.

The IBM 704 program of P. J. Walsh (11.2) to evaluate methods of numerically satisfying an internal boundary condition involving a normal derivative was partially completed. Several methods are yet to be checked out.

W. Borsch-Supan (11.2) started the investigation of error bounds for the Ritz-Galerkin method applied to inhomogeneous linear ordinary differential equations. For a certain class of second order equations using polynomials as approximating functions, error bounds have been found.

#### Publications:

- (1) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. Rice. J. Research NBS 64B, 91-93 (1960).
- (2) Tchebycheff approximations by  $ab^x+c$ . J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (3) Split Runge-Kutta for simultaneous equations. J. R. Rice. To appear in the Journal of Research NBS, Sec. B.
- (4) Sequence transformations based on Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research NBS, Sec. B.
- (5) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.
- (6) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.
- (7) A reduction formula for partitioned matrices. E. Haynsworth. To appear in the Journal of Research NBS, Sec. B.
- (8) Special types of partitioned matrices. E. Haynsworth. In manuscript.
- (9) Bounds for determinants with positive diagonals. E. Haynsworth. To appear in the Proceedings of the American Mathematical Society.
- (10) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.
- (11) Bounds for the P-condition number of matrices with positive roots. E. Haynsworth, P. Davis, and M. Marcus. In manuscript.
- (12) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. To appear in the Journal of Research NBS, Sec. B.
- (13) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. To appear in the Journal of Research NBS, Sec. B.



RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO  
NUMERICAL ANALYSIS  
Task 1101-12-11411/55-56

Origin: NBS  
Sponsor: Office of Naval Research  
Manager: M. Newman  
Full task description: July-Sept 1954 issue, p. 5

Authorized 8/13/54

Status: CONTINUED. M. Newman derived certain theorems on the structure of modular subgroups, the existence of polynomial bases for classes of automorphic forms, the existence of recurrence formulas of fixed length for the coefficients of modular forms, and the periodicity modulo a given integer of sequences of integers. Tables of coefficients of important modular forms were computed on the 704 (some 100,000 coefficients in all) in order to verify and extend existing tables of D. Ferguson and J. C. P. Miller (Cambridge, England).

K. Goldberg has shown that the word "integral" in H. J. Ryser's well-known "normal and integral implies incidence" theorem can be replaced by the phrase "each non-zero element is at least 1 in absolute value." This has application to the theory of  $v$ ,  $k$ ,  $\lambda$ -designs.

Dr. Goldberg also studied the properties of the coefficients in the power series that is the formal inverse of  $(e^x - 1 - x - \dots - x^n/n!)/(x^n/(n+1)!)$ .

Publications:

- (1) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
- (2) Irrational power series. M. Newman. Submitted to a technical journal.
- (3) Periodicity modulo  $m$  and divisibility properties of the partition function. M. Newman. To appear in the American Journal of Mathematics.
- (4) The incidence equation  $AA^T = aA$ . K. Goldberg. Amer. Math. Month. 67, 367 (1960).
- (5) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
- (6) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
- (7) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

INFORMATION SELECTION SYSTEMS  
Task 1101-12-11412/60-470

Origin: NBS  
Sponsor: National Science Foundation  
Managers: K. Goldberg, A. J. Goldman

Authorized 9/25/59

Status: CONTINUED. K. Goldberg continued his investigation of the expected lengths of chains in a flow of data through a computer with a finite memory. He proved that if  $E_r^{(n)}$  denotes the expected length of the

$r$ th monotone increasing chain in a flow of  $n$  distinct data in random order, then

$$E_r^{(n)} = \sum_{k=r}^n (k!)^{-1} \sum_{j=0}^r (-1)^j \binom{k+1}{j} (r-j)^k,$$

$$E_r = \lim_{n \rightarrow \infty} E_r^{(n)} = \sum_{k=0}^r (-1)^{r-k} ((r-k)!)^{-1} r^k e^{-r},$$

$$|2 - E_r| \sim c(\rho + 1)^{-r} \quad (r \rightarrow \infty)$$

where  $c$  is a positive constant and  $\rho \approx 7.7483$  is the absolute value of the root of  $e^z = 1 + z$  of smallest value.

Work continued on the investigation of the effects of permuting the individual steps in Phase III of the Boolean simplification program. B. K. Bender (11.2) continued coding the program for the IBM 704. A "partial ordering" technique proposed by J. P. Roth appears useful and will ultimately be incorporated in the program.

#### Publications:

- (1) Computer simplification of boolean functions. B. K. Bender (11.2), A. J. Goldman, and R. B. Thomas (12.5). Submitted to a technical journal.
- (2) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.

#### ORTHOGONAL FUNCTIONS IN THE THEORY OF PARTIAL DIFFERENTIAL EQUATIONS Task 1101-12-11413/60-469

Origin: NBS  
Sponsor: Atomic Energy Commission  
Managers: P. Davis, P. Rabinowitz

Authorized 9/25/59

Status: COMPLETED. Investigations that have been carried out by P. J. Davis and P. Rabinowitz include the following:

- (1) The Dirichlet problem was computed for a compressor blade region and the results compared favorably with those achieved by Poritsky using collocation.
- (2) Laplace's equation was solved for a semicircle with mixed boundary conditions. One of the boundary functions used gives the flow through a semicircular channel.
- (3) The biharmonic equation was solved on a square using both Gaussian points and equidistributed points. No significant difference was observed. Furthermore, when the number of equidistributed points was reduced by 80 percent, the results were of the same order as in the original case.

(4) The biharmonic equation was solved for a clamped semicircular plate with uniform loading, concentric loading, circular loading, and linearly increasing loading. 38 functions were used and accuracy better than 1 percent was obtained. The moments  $M_x$ ,  $M_y$ ,  $M$ , and  $M_\theta$  were also computed and accuracy of about 5 percent was achieved.

(5) A subroutine was written to orthonormalize a set of complex vectors with the inner product  $(f,g) = \sum_{i=1}^N w_i f_i \bar{g}_k$ . This subroutine was incorporated in a program to compute the complex orthonormal polynomials over a region. The same program can also be used to obtain a least squares fit of an arbitrary complex function by powers of  $z$  as well as by given special complex functions.

Publications:

- (1) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. Submitted to a technical journal.
- (2) Advances in orthonormalizing computation. P. Davis and P. Rabinowitz. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION

Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health

Authorized 9/30/55

Manager: P. Rabinowitz

Full task description: July-Sept 1955 issue, p. 7

Status: INACTIVE.

## 2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

### Mathematical Tables

The following long-range mathematical table projects are being carried in the Computation Laboratory. Progress continues as dictated by the relative priority in the overall program of the Laboratory and by available funds. All of the table projects were inactive during the past quarter, with priority being given to preparation of the forthcoming "Handbook of Mathematical Functions."

1102-40-11112/47-2 TABLES OF COULOMB WAVE FUNCTIONS

1102-40-11112/51-8 TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS

1102-40-11112/52-37 SPHEROIDAL WAVE FUNCTIONS

1102-40-11112/52-57 SIEVERT'S INTEGRAL

### HANDBOOK OF MATHEMATICAL FUNCTIONS

Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation

Authorized 12/27/56

Manager: I. A. Stegun

Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Chapter 20, Mathieu Functions, has been distributed for comments. Revisions are being incorporated in Chapter 8, Legendre Functions, and in Chapter 18, Weierstrass Elliptic Functions. Chapters 1, 4, 5, 7, and 25 are ready to go to press. All remaining chapters are undergoing final revisions. Drafts of the preface, foreword, and introduction are being reviewed.

### AUTOMATIC CODING

Task 1102-12-11120/55-65

Origin: NBS

Authorized 9/29/54

Manager: J. Wegstein

Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. G. M. Galler and G. Ziegler completed a proposal for expanding the Bell Automatic Operator System to take advantage of the additional equipment being acquired. The new system includes automatic machine time accounting, FORTRAN III, and code checking runs that are passed through the computer entirely via magnetic tape. The



implementation of these changes was begun.

The automatic programming group continued with its plans for developing a data processing compiler. The effort has been broken down into the following areas: (1) Formula translation, string translation; (2) Efficiency, diagnostics, memory allocation; (3) Input-output, editing, tape use; (4) String manipulation; (5) Threaded lists, LISP, ILP5. A manuscript "Proposed String Statements for ALGOL 60" was completed. These proposed statements will permit many useful string manipulations to be expressed in ALGOL 60 notation.

During this quarter, the Tablemaker system was used by 114 people for a total computer time of 994 minutes. The following table indicates the number of times that the various functions were used.

<u>Function</u>	<u>Number of times used</u>
Curve fit	131
Integration	72
Plot	25
Multiple correlation	1
Average and standard deviation	36

#### MATHEMATICAL SUBROUTINES Task 3911-61-39952/56-160

Origin: NBS

Authorized 9/30/55

Managers: Staff

Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. Using the matrix base code described in the Jan-Mar 1960 issue, a subroutine was written to read in matrices  $X_{m \times n}$  and  $S_{m \times r}$  and perform the following calculation:

$$U_{n \times r} = [(X'X)^{-1}X']S$$

Print-out of results for each step above is optional.



### 3. PROBABILITY AND MATHEMATICAL STATISTICS

#### MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS

Task 1103-12-11131/51-2

Origin: NBS

Authorized 7/1/50

Manager: C. Eisenhart

Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. H. H. Ku is studying the application of information theory to the analysis of four-way contingency tables. The minimum discrimination information statistic as defined by S. Kullback is used for the testing of various hypotheses, e.g., independence, homogeneity, symmetry, etc. Adaptation of tests on conditional independence leads also to tests for Markovity of finite chains.

#### Publications:

- (1) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.
- (2) Selected bibliography of statistical literature, 1930-57.  
III. Limit theorems. Lola S. Deming. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
- (3) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
- (4) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Submitted to a technical journal.

#### STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN

Task 1103-12-11131/53-1

Origin: NBS

Authorized 10/15/52

Manager: J. M. Cameron

Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. Revision of the final manuscript for the catalog of fractional factorial designs for the  $2^{m3^n}$  series being prepared under Bureau of Ships sponsorship (see July-Sept 1959 issue, p. 14) continued in the light of comments received from a number of specialists in the theory and practice of experiment design.

M. Zelen and B. Kurkjian (of DOFL) have developed new theory and methods of construction and analysis of asymmetrical factorial experiment designs, providing a general analysis for the use of a certain, wide class of balanced or partially balanced incomplete block designs with asymmetrical factorial designs. The main advantages of these new developments, which are essentially extensions and simplifying

modifications of the earlier research of Zelen are: (i) they apply to any asymmetrical factorial experiment when used in conjunction with a wide class of partially balanced block (PBIB) designs, thereby allowing greater flexibility in making the experiment design fit the physical situation; (ii) there is no restriction that the levels must be a prime (or power of a prime) number; (iii) the analysis is relatively simple no matter how many factors or levels are involved; (iv) the analysis is independent of the concept of confounding in the sense that one can compute the best linear estimates of all main effect and interaction terms without any need to know those terms that are confounded with any of the others; and (v) the analysis is strictly algebraic in nature and avoids difficult combinatorial arguments which would otherwise be required to treat cases involving PBIB designs with more than two associate classes. Preparation of a paper on the research is under way.

J. M. Cameron, by application of modern combinatorial analysis techniques, has achieved a nearly complete enumeration and construction of a new class of balanced incomplete-block "weighing" designs that promise to be of exceptional value in the calibration of groups of standards of nominally equal magnitude.

M. Zelen and B. Kurkjian (DOFL) have made notable progress in developing a special operational calculus for symmetrical and asymmetrical factorial arrangements. The aim of this calculus is to make the requisite but complicated matrix operations easy. A by-product of this investigation is that many more or less standard matrix operations can be carried out using logical rather than arithmetical operations. This should facilitate the analysis of factorial experiments using high speed computers.

#### Publications:

- (1) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.
- (2) Analysis of fractionally replicated  $2^m 3^n$  designs. R. C. Bose and W. S. Connor. Bulletin de l'Institut International de Statistique, XXXVII (31st Session, Brussels, 1958), pp. 141-160 (Published Brussels, 1960).
- (3) Construction of fractional factorial designs of the mixed  $2^m 3^n$  series. W. S. Connor. "Contributions to Probability and Statistics," pp. 168-181 (Stanford University Press, Stanford, California, 1960).

#### STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES

Task 1103-12-11131/56-170

Origin: NBS

Authorized 12/15/55

Manager: J. R. Rosenblatt

Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

## Publications:

- (1) On the power of some rank order two-sample tests. J. R. Rosenblatt. "Contributions to Probability and Statistics," pp. 358-370 (Stanford University Press, Stanford, California, 1960).
- (2) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. Submitted to a technical journal.

## MEASUREMENT OF RELIABILITY

Task 1103-12-11130/56-182

Origin: NBS

Authorized 3/23/56

Manager: M. Zelen, J. R. Rosenblatt

Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. M. Zelen and M. C. Dannemiller have completed a paper summarizing their work on the robustness of statistical life testing procedures. They have found that many of the current life testing procedures are very sensitive to the assumption that failure times follow the exponential distribution. Incident to their studies on robustness, they have developed excellent approximations to the distribution of a sum of Weibull random variables and also to the O. C. curve and average sample number when sequential tests are made on sums of Weibull random variables.

## Publications:

- (1) Analysis of two-factor classifications with respect to life tests. M. Zelen. "Contributions to Probability and Statistics," pp. 508-517 (Stanford University Press, Stanford, California, 1960).
- (2) Statistical models for component aging experiments. J. R. Rosenblatt. IRE National Convention Record, 1960, Vol. 8, Pt. 6, pp. 115-124.
- (3) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. Submitted to a technical journal.

#### 4. MATHEMATICAL PHYSICS

##### RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS

Task 1104-12-11141/55-57

Origin: NBS

Authorized 9/1/54

Manager: E. W. Cannon

Full task description: July-Sept. 1954 issue, p. 27

Status: CONTINUED. L. E. Payne continued work on pointwise bounds in certain non-well-posed problems in the theory of elasticity. A typical problem is the following: An elastic medium occupies a region D and is in static equilibrium under the influence of certain surface forces acting on the boundary C of D. Let a portion C' of the boundary be inaccessible for measurement of boundary data. On C-C' the displacements and surface tractions are measured with known precision. In terms of the measured data on C-C' and a known uniform bound for the displacements inside the medium, pointwise bounds for the stresses and displacements in D are to be determined.

A. Ghaffari has studied the steady state of the probability density function f (see Jan-Mar 1960 issue, pp. 14-15) and has shown that when the time t tends to infinity the limit of the probability  $\psi$ , defined by

$$\psi = \int_{-\infty}^{+\infty} f(x,s,y,t) dy,$$

exists and depends on the initial state defined by (x,s), that is, the probability  $\psi$  is non-oscillatory: The special case  $a(s) = e^{2s}$  leads to the particular solution

$$f(x,s;y,t) = \exp\left(-\frac{x^2+y^2}{2}\right) \sum_{n=0}^{\infty} e^{2n(s-t)} \frac{H_n(x)H_n(y)}{2^n n! \sqrt{\pi}}$$

of the Chapman-Kolmogoroff equation, which satisfies all the prescribed conditions, and the parabolic partial differential equations (see p. 15, Jan-Mar 1960 issue) are reduced to the following simplified forms:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial s} + (1-x^2)f = 0 \quad \text{for } y \text{ and } t \text{ fixed,}$$

$$\frac{\partial^2 f}{\partial y^2} - \frac{\partial f}{\partial t} + (1-y^2)f = 0 \quad \text{for } x \text{ and } s \text{ fixed,}$$

where  $s < t$  as mentioned before.

A. Ghaffari presented a paper on "On partial differential equations of Brownian motion", at the International Conference on Partial Differential Equations and Continuum Mechanics, Mathematics Research Center, U. S. Army, University of Wisconsin, Madison, Wis., June 7-15, 1960.



## Publications:

- (1) The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. J. Research NBS 64C, 115-119 (1960).
- (2) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. J. Fluid Dynamics 7, pt. 4, 529-549 (1960).
- (3) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.
- (4) Upper and lower bounds for the center of flexure. L. E. Payne. J. Research NBS 64B, 105-111 (1960).
- (5) On Stokes flow about a torus. W. H. Pell and L. E. Payne. To appear in Mathematika.

## PLASMA RESEARCH

Task 1104-12-11140/59-422

Origin: NBS

Authorized 6/30/59

Manager: C. M. Tchen

Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen continued his research on plasma statistics and magnetohydrodynamics, at the Max Planck Institute for Physics and Astrophysics, at Munich, Germany, as a Guggenheim Fellow. He investigated the effect of the triple correlation on the plasma shielding. The degenerated triple correlation function (products of singlet distribution functions with double correlation functions) entered in the form of an integrand, in an integral equation governing the double correlation function. This equation was solved, and the role of the shielding was compared with the results of an approximate method given previously (see Tchen, Phys. Rev. 114, 394-411, 1959). Also its connection with the dispersion relation for plasma oscillations was investigated.

Problems of magnetoturbulence have gained some importance in application to astrophysics and thermonuclear reactions. One of the essential features was the transfer of energy between eddies of various sizes. This occurred because of the presence of the nonlinear terms in the dynamic equations. In cooperation with Dr. K. Hain of the Max Planck Institute, a computing scheme was set up to study those phenomena. The equations, however, were degenerated into one dimension, corresponding to the one-dimensional model of Burgers. One of the purposes was to investigate whether such a transfer would agree with the theories of Kolmogoroff and Heisenberg.

## RESEARCH ON SATELLITE ORBITS

Task 1104-12-11440/59-420

Origin: NBS

Authorized 12/19/58

Sponsor: Office of Scientific Research, ARDC, USAF

Manager: J. P. Vinti

Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti has continued work on solving the



kinetic equations of motion for a satellite orbit, expressed in spheroidal coordinates in equations (63) of J. Research NBS 63B, 105-116 (1959). He is also studying the possibility of using a new set of orbital constants, proposed by I. Izsak (private communication). It appears likely that this new set of constants will facilitate further calculations; e.g. they have led to evaluation of the integrals in equations (63) in terms of elliptic integrals and functions. Dr. Vinti is investigating methods for simplifying and solving the resulting equations.

J. P. Vinti presented a paper on the "Theory of the Orbit of an Artificial Satellite, with Use of Spheroidal Coordinates" at the Pittsburgh meeting of the American Astronomical Society, April 19, 1960.

#### FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS

Task 1104-12-11626/56-154

Origin: NBS

Authorized 9/30/55

Sponsor: Office of Naval Research

Manager: F. Oberhettinger

Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

## 5. MATHEMATICAL AND COMPUTATIONAL SERVICES

### 3911-61-39952/54-30 SPECTRUM ANALYSIS

Origin: NBS, Division 4

Manager: W. Bozman (4.1)

Full task description: Jan-Mar 1954 issue, p. 46

Status: Continued. Wavelengths and wavenumbers were computed for 16,000 praseodymium lines and for 2,500 bromine lines. Matrix calculations were made on the energy levels of tantalum (Ta II) and of iron (Fe II). Work on the intensity tables of 70 elements continued. The "Table of Wavenumbers," Vol. I: 2000A to 7000A; Vol. II: 7000A to 1000 microns, has been published.

### 3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES

Origin: NBS, Section 3.2

Manager: M. L. Paulsen

Full task description: Jan-Mar 1954 issue, p. 48

Status: Inactive.

### 3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS

Origin: NBS, Division 9

Managers: P. J. O'Hara, S. Block (9.7)

Full task description: Jan-Mar 1955 issue, p. 18

Status: Continued. Routine crystallographic calculations were carried out for the several crystals currently being studied.

### 3911-61-39952/55-82 THERMOMETER CALIBRATIONS

Origin: NBS, Section 3.1

Manager: B. S. Prusch

Full task description: Jan-Mar 1955 issue, p. 20

Status: Continued. ITS constants were calculated for 50 thermometers under test. Thermometer codes are being updated to more efficiently use the larger 704 memory.

### 1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES

Origin and Sponsor: NBS, Section 5.9

Managers: E. V. Haynsworth (11.1), P. J. Walsh

Full task description: Jan-Mar 1956 issue, p. 27

Status: Continued. Code checking has been completed on programs which compute and sort direct and exchange integrals as input data for the A-matrix program. A code to generate and sort the VST integrals has also been written and checked out. These programs are now being linked to the A-matrix and SCF programs.

## 1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS

Origin and Sponsor: NBS, Section 2.5

Manager: B. S. Prusch

Full task description: July-Sept 1956 issue, p. 33

Status: Continued. Computations were performed to check 24 laboratory sets of gage blocks.

## 1102-40-11645/57-219 THERMAL PROPERTIES

Origin and Sponsor: NBS, Section 3.2

Manager: R. N. Varner

Full task description: Oct-Dec 1956 issue, p. 30

Status: Terminated. Production runs have been transferred to the direction of the sponsor. Hereafter, production time on the 704 will be reported in the section of this Report, "Current Applications of Automatic Computer."

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING  
MACHINERY TO PAYROLL OPERATIONS

Origin: NBS, Section 40.0

Managers: M. L. Paulsen

Full task description: Jan-Mar 1957 issue, p. 36

Status: Inactive.

## 1102-40-11645/57-236 SELF CONSISTENT FIELDS--EIGENVALUES

Origin and Sponsor: NBS, Section 3.6

Manager: E. V. Haynsworth

Full task description: Apr-June 1957 issue, p. 30

Status: Continued. Further calculations involving the linear symmetric  $N_3$  molecule were performed.

## 3911-61-39952/58-266 DEPOLYMERIZATION, II

Origin: NBS, Section 7.6

Manager: L. S. Joel

Full task description: July-Sept 1957 issue, p. 36

Status: Inactive.

## 1102-40-11645/58-269 MOLECULAR STRUCTURE, IV

Origin and Sponsor: Naval Research Laboratory, USN

Manager: P. J. O'Hara

Full task description: July-Sept 1957 issue, p. 38

Status: Continued. A three-dimensional Fourier Summation program written by William Sly and David Shoemaker of Massachusetts Institute of Technology is being tested. Input data for about 8,000 reflections has been set up and several small trial runs were completed.

## 1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS

Origin: NBSSponsor: Post Office Department, Office of Research and EngineeringManagers: B. K. Bender, A. J. GoldmanFull task description: Oct-Dec 1958 issue, p. 22

Status: Continued. Investigation of mathematical models of distribution networks continued. In the current model, the service area is a square uniform sink-source of mail, partitioned into sorting regions by lines parallel to the sides. Optimal locations have been derived for any subset of the sorting centers, assuming given (not necessarily optimal) positions for the others. For a fixed degree of decentralization, a satisfactory truncated power-series approximation was found for the partition which minimizes total transportation costs. B. K. Bender has determined the optimal way of merging mail (so as to avoid excessive loading costs) along a one-dimensional array of pickup points each generating the same amount of mail.

Publication: Optimization models for distribution networks. B. K. Bender and A. J. Goldman. In manuscript.

## 1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES

Origin and Sponsor: NBS, Section 3.2Manager: J. P. MenardFull task description: Oct-Dec 1957 issue, p. 32Status: Inactive. For status to date, see Oct-Dec 1959 issue, p. 19.

## 1102-40-11645/58-281 PSI EVALUATION

Origin and Sponsor: NBS, Section 4.10Managers: P. Walsh, J. D. WaggonerFull task description: Oct-Dec 1957 issue, p. 34

Status: Reactivated. Changes were required in the original problem to allow for greater flexibility in parameter input data. In addition, the following tables were desired:

Table 1.  $A_{ik} = \cos(\gamma_{ik} - \alpha)$

$$B_{ik} = \cos(\gamma_{ik} + \alpha)$$

where

$$\gamma_{ik} = \cos^{-1}(\cos\phi\cos\beta_i - \sin\phi\sin\beta_i \cos\delta_k)$$

Table 2.

$$C_{ikl} = \cos^{-1}(E_{ik}) - \cos^{-1}(F_{ikl})$$

$$D_{ikl} = \cos^{-1}(E_{ik}) + \cos^{-1}(F_{ikl})$$

where

$$E_{ik} = \frac{\cos\phi - \cos\beta_i \cos\gamma_{ik}}{\sin\beta_i \sin\gamma_{ik}}; \quad F_{ikl} = \frac{\cos\alpha - \cos\theta_l \cos\gamma_{ik}}{\sin\theta_l \sin\gamma_{ik}}$$

The original code was rewritten in FORTRAN and added to the FORTRAN code for computing Tables 1 and 2. Results for several sets of parameters have been submitted to the sponsor.



## 1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor: NBS, Section 3.4

Manager: H. Oser

Full task description: Jan-Mar 1958 issue, p. 38

Status: Continued. Production runs were made under the direction of the sponsor.

## 1102-12-11513/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION

Origin: NBS

Sponsor: Office of Ordnance Research, U. S. Army

Manager: I. Rhodes (11.0)

Full task description: Oct-Dec 1958 issue, p. 26

Status: Continued. Programming for the 704 computer is proceeding on the numerous subroutines and branches of the code corresponding to different syntactic predictions. A first report on "profilng", i.e., the delimitation of clauses and phrases within a sentence, has been written. Compilation of the inflectional forms of several thousand Russian words has been started, in preparation for that part of the machine code which generates predictions and morphological information from stems and inflectional endings.

Publications: (1) The outlook for machine translation. F. L. Alt.

"Proceedings, Western Joint Computer Conference, San Francisco, 1960," pp. 203-208. (2) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

## 1102-40-11645/58-358 REDUCED CROSS-SECTIONS

Origin and Sponsor: NBS, Section 3.2

Manager: R. J. Arms

Full task description: Apr-June 1959 issue, p. 30

Status: Completed.

## 3711-60-0009/58-360 DIFFUSION COEFFICIENTS

Origin: NBS, Section 5.2

Manager: J. P. Menard

Full task description: Apr-June 1958 issue, p. 32

Status: Continued. A code for the function  $D_t(c)$  (see Jan-Mar 1960 issue, p. 21) has been written and checked out on the IBM 704. Production runs will be made under the sponsor's direction.

## 1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION

Origin and Sponsor: Naval Research Laboratory

Manager: R. J. Arms

Full task description: Apr-June 1958 issue, p. 33

Status: Continued. Production runs, as requested, have been submitted to the sponsor.



## 1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS

Origin and Sponsor: U. S. Information Agency, Department of State

Manager: P. J. Walsh

Full task description: Apr-June 1958 issue, p. 35

Status: Continued. Table 4 (see Apr-June 1958 issue, p. 35) was completed for  $\lambda = 10.0$ . The antenna analysis code produced information for approximately 75 antennae.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT  
SCATTERED BY SPHERICAL PARTICLES

Origin and Sponsor: U. S. Army Signal Research and Development  
Laboratories, Atmospheric Physics Branch,  
Belmar, N. J.

Manager: H. Oser

Full task description: July-Sept 1958 issue, p. 32

Status: Continued. Production runs were made under the direction of the sponsor.

## 1102-40-11645/59-377 LOGICAL DIAGRAM REDUCTION

Origin and Sponsor: NBS, Section 12.3

Managers: W. G. Hall

Full task description: Apr-June 1959 issue, p. 25

Status: Terminated. Production runs on the 704 will continue under the direct supervision of the sponsor and machine time will be reported in the section of this Report entitled, "Current Applications of Automatic Computer."

## 1102-40-11645/59-388 HEAT PUMP CALCULATIONS

Origin and Sponsor: NBS, Section 10.3

Manager: R. Zucker

Full task description: Jan-Mar 1959 issue, p. 26

Status: Terminated. Production runs were continued using the heating and cooling data. Future runs on the 704 will be made directly by the sponsor and machine time will be reported in the section of this Report on "Current Applications of Automatic Computer."

## 1102-40-11645/59-389 FREQUENCY ALLOCATION

Origin and Sponsor: Civil Aeronautics Administration

Manager: L. S. Joel

Full task description: Oct-Dec 1958 issue, p. 29

Status: Continued. G. Galler has extended the code to permit sequences of interchanges. Exploratory studies of the mathematical analysis of the station network continued.

1102-40-11645/59-394    VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING  
                             BY HYDROGEN ATOMS, II

Origin and Sponsor:    NBS, Section 4.6

Manager:    A. E. Beam

Full task description:    Oct-Dec 1958 issue, p. 30

Status:    Continued.    Production runs were continued under the direction of the sponsor. The new code for computing the photodetachment cross section for  $H^-$  was completed and checked, and runs were made under direction of the sponsor.

1102-40-11645/59-407    FOURIER COEFFICIENTS

Origin and Sponsor:    Diamond Ordnance Fuze Laboratories, Department of  
                             the Army

Manager:    R. Zucker

Full task description:    Jan-Mar 1959 issue, p. 27

Status:    Terminated.    Production runs will be continued under the direction of the sponsor and the machine time will be reported in the section of this Report entitled "Current Applications of Automatic Computer."

1102-40-11645/59-414    INFINITE SYSTEMS

Origin and Sponsor:    NBS, Division 3

Manager:    R. Zucker

Full task description:    Jan-Mar 1959 issue, p. 28

Status:    Completed.    Results have been transmitted to the sponsor.

1102-40-11645/59-435    ELECTROCARDIOGRAPHIC ANALYSIS

Origin:    NBS, Division 12.5

Sponsor:    Veterans Administration

Manager:    R. J. Arms

Full task description:    Apr-June 1959 issue, p. 29

Status:    Continued.    Wave detection problems have been considered. The present study is one of finding transformations suitable for smoothing the data. In particular, in view of machine print-outs, it appears advisable to use Fourier techniques for subtracting 60-cycle waves. For further smoothing, local polynomial fitting may be sufficient. Experiments with polynomials are in progress.

1102-40-11645/59-444    HOSPITAL SUBSISTENCE ITEMS SUPPLY

Origin and Sponsor:    Veterans Administration

Manager:    L. S. Joel

Full task description:    Apr-June 1959 issue, p. 29

Status:    Completed.    The problem considered was that of optimally selecting the locations and employment of intermediate storage facilities (warehouses) in the delivery of a single frozen-food commodity from a single supplier to a group of Veterans Administration hospitals. The mathematical features of the problem have been analyzed, the features rendering it more difficult than standard transportation problems have been identified,

and related computation techniques have been cited. Although progress in mathematical knowledge, computation techniques and digital computer rapidity during the past three years have moved the frontiers of the "state of the art" enormously closer to the point where the VA problems can be coped with, nevertheless that point has not yet been reached. In any case, it is suggested that the problem is inherently so big and so complicated as to require a larger faster computer than the IBM 650 now available to the VA. (The assigned scope of this study included neither the development of new mathematical techniques nor the detailed adaptation of existing ones for the purposes of the problem.)

1102-40-11645/59-445 OIL SUPPLY

Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy

Manager: L. S. Joel

Full task description: Apr-June 1959 issue, p. 30

Status: Continued. (1) Program errors prevented carrying out comparative tests with the MIT code. No symbolic listing is known to exist at present. In consequence, it is planned to write a revised input program for NYTRI at NBS. (2) Specific allocation problems were run as they were received from the sponsor.

1102-40-11645/60-452 CONSULTING SERVICES FOR PACIFIC MISSILE RANGE

Origin and Sponsor: NBS, Section 12.3

Manager: G. W. Reitwiesner

Full task description: July-Sept 1959 issue, p. 29

Status: Completed. A consolidated team composed of personnel of the U. S. Navy Bureau of Weapons, the Pacific Missile Range, and NBS Divisions 11 and 12 evaluated the PMR real-time computation system proposals. A harmonious recommendation was reached by 21 April 1960.

1102-40-11645/60-453 DATA CONVERSION

Origin and Sponsor: Army Map Service

Manager: J. M. Beiman

Full task description: July-Sept 1959 issue, p. 30

Status: Terminated. Production runs are continuing under direction of the sponsor, and machine time on the 704 will be reported in this Report in the section on "Current Applications of Automatic Computer."

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY

Origin and Sponsor: Civil Aeronautics Board

Managers: J. M. Beiman, W. G. Hall

Full task description: July-Sept 1959 issue, p. 31

Status: Continued. Production runs were made and results have been submitted to the sponsor.



## 1102-40-11645/60-459 TRANSCENDENTAL EQUATIONS

Origin and Sponsor: NBS, Section 9.1

Manager: R. Zucker

Full task description: July-Sept 1959 issue, p. 31

Status: Completed. The requested calculations were carried out and the results have been transmitted to the sponsor.

1102-40-11645/60-460 DESIGN EQUATIONS FOR MAGNETRON INJECTION  
ELECTRON GUNS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of  
the Army

Manager: A. E. Beam

Full task description: July-Sept 1959 issue, p. 32

Status: Completed. The equations were solved as requested and the results were transmitted to the sponsor.

## 1102-40-11645/60-462 CORRELATION OF FUNCTIONS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of  
the Army

Manager: G. W. Reitwiesner

Full task description: July-Sept 1959 issue, p. 33

Status: Completed. All the functions have been computed as requested.

## 1102-40-11645/60-464 CORRELATION MATRIX FOR PHS DATA

Origin and Sponsor: Public Health Service

Manager: J. M. Beiman

Full task description: July-Sept 1959 issue, p. 34

Status: Terminated. Production runs are to be continued under the direction of the sponsor, and the machine time will be reported henceforth in the Section, "Current Applications of Automatic Computer."

## 1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS

Origin and Sponsor: NBS, Section 3.2

Managers: P. J. Walsh, J. D. Waggoner

Full task description: Oct-Dec 1959 issue, p. 26

Status: Continued. Molecular orbital calculations have been performed for the assumed ground state configurations of  $\text{He}_2$ . Various choices were made for the screening constants and internuclear separations. In some cases the molecular integrals were printed as well as the Hartree-Fock eigenvalues, overlap matrices, etc.

## 1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Full task description: Oct-Dec 1959 issue, p. 27

Status: Continued. The Nesbet Molecular Program for IBM 704 became operative at NBS during this quarter. Various test cases were run to insure working operation to the degree it has been achieved elsewhere (MIT and Institute for Molecular Research, Paris). Production was begun on the interaction of two neon atoms. A curve for the repulsive region  $3.0 \leq r \leq 6.0$  au. has been completed. Work continues on the van der Waal's region where, because of the large value of internuclear distance ( $R > 6$  au.), more testing of the program to insure accuracy of the molecular integrals is in progress.

## 1102-40-11645/60-467 TRANSISTOR SIMULATION

Origin and Sponsor: NBS, Section 12.1

Manager: G. W. Reitwiesner

Full task description: Oct-Dec 1959 issue, p. 27

Status: Inactive, pending reconsideration of the terms of the problem.

## 1102-40-11645/60-471 INTERLABORATORY STANDARDIZATION OF TESTING PROCEDURES

Origin and Sponsor: NBS, Section 7.3

Manager: A. E. Beam

Full task description: Oct-Dec 1959 issue, p. 28

Status: Terminated. Production runs were made under the direction of the sponsor. Hereafter, this task will be reflected in this Report as machine time on the 704 in the section on "Current Applications of Automatic Computer."

## 1102-40-11645/60-475 IONOSPHERIC SOUNDINGS

Origin and Sponsor: NBS, Section 82.40

Manager: M. L. Paulsen

Full task description: Oct-Dec 1959 issue, p. 29

Status: Continued. Many thousands of true height (h'f) data cards have been received, transcribed to tape, then processed using the 704 code. The results have been sent to Boulder on magnetic tape.

## 1102-40-11645/60-476 GAS TUBE CHARACTERISTICS, II

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: H. Oser

Full task description: Oct-Dec 1959 issue, p. 30

Status: Continued. Some improvements in the code have been made to extend the limit of convergence of the method. Production runs were continued under the direction of the sponsor.

In order to include space-charge effects, a system of three partial differential equations has to be solved. A study has been completed on the stability of the finite difference method to be used.



## 1102-12-11122/60-479 PROCESSING OF DIAGRAMS

Origin and Sponsor: NBS, Section 11.0

Managers: F. L. Alt (11.0), S. T. Peavy, R. J. Herbold

Full task description: Oct-Dec 1959 issue, p. 30

Status: Continued. A 704 tape was prepared on SEAC with letter diagrams. One of the codes was checked out, and checking of the analysis code was started.

1102-40-11645/60-480 LARGE SIGNAL CALCULATIONS FOR A VOLTAGE TUNEABLE  
MAGNETRON

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: A. E. Beam

Full task description: Oct-Dec 1959 issue, p. 30

Status: Completed. All desired production runs were made.

## 1102-40-11645/60-484 POLYMER CRYSTALLIZATION

Origin and Sponsor: NBS, Section 7.6

Manager: H. Oser

Full task description: Jan-Mar 1960 issue, p. 28

Status: Terminated. Production runs were made directly by the sponsor, and time on the 704 is recorded now and henceforth in this Report in the section on "Applications of Automatic Digital Computer."

## 1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS

Origin and Sponsor: NBS, Section 3.0

Manager: R. Zucker

Full task description: Jan-Mar 1960 issue, p. 28

Status: Continued. Additional coding to evaluate the R-Centroid

$$\frac{\int \psi_v, \psi_v, r dr}{\int \psi_v, \psi_v, dr}$$

was incorporated into the code.

## 1102-40-11645/60-489 INVERSION OF LINE PROBE DATA

Origin and Sponsor: NBS, Section 3.1

Manager: R. Herbold

Full task description: Jan-Mar 1960 issue, p. 29

Status: Inactive.

## 1102-40-11645/60-500 DENTAL RESEARCH COLORIMETRY

Origin and Sponsor: NBS, Section 7.8

Manager: J. P. Menard

Full task description: Jan-Mar 1960 issue, p. 30

Status: Completed. The data was processed using the new code, and the results were transmitted to the sponsor.

## 1102-40-11645/60-501 KANSAS RIVER SYSTEM

Origin and Sponsor: Corps of Engineers, U. S. Army, Office of District Engineers, Kansa City District.

Manager: S. Peavy

Objective: To prepare a program to simulate the operation of a system of reservoirs through an entire flood cycle, including emptying and flood storage, taking into account the operational criteria essentially as set forth in the reservoir regulation manuals.

Background: The problem arises in connection with the study of flood control. The program was originally written for the 650 but, because the running time was excessive, programming for the 704--using FORTRAN--was decided upon. The new program has been submitted to the NBS for checking and running.

Mr. Otto Steiner, of the Kansas City District, transmitted the request to the NBS.

Status: New. The program has been checked out and a test run has been made so that the sponsor may compare the results with those obtained on the 650.

## 1102-40-11645/60-504 ELECTROSTATIC-FOCUSING PROBLEM

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: A. Beam

Full task description: Jan-Mar 1960 issue, p. 30

Status: Continued. The code was completed and checked. Several production runs were made.

## 1102-40-11645/60-505 EARTH TEMPERATURE PROBLEM

Origin and Sponsor: Geophysical Laboratory, Carnegie Institution of Washington

Manager: W. W. Youden (12.3)

Objective: To calculate a table describing the thermal history of the earth due to radioactive heating, based on a simple earth model. The calculations involve the solutions of complex trigonometric equations for 2688 sets of parameters.

Background: The calculations are required in the study of radioactive heating of the earth. Account is taken of the radial position, thermal diffusivity, and the thickness of the radioactive shell, as well as the finite half-life of the important radionuclides.

The problem was transmitted by S. Clark (Geophysical Laboratory, CIW).

Status: Completed (New). The results were transmitted to the sponsor.

## 3911-61-39952/60-507 TRANSISTOR AGING BEHAVIOR STUDIES

Origin and Sponsor: NBS, Section 1.6

Manager: R. Varner

Objective: To obtain bounds on the range of variability of the characteristics of transistors caused by within-day and also between-day transistor variability, ambient temperature variability, and test set

variability; to determine the distribution of parameter values so as to decide what, if any, transformations will be necessary before conducting statistical analysis; to test for any significant effects of soldering the transistors to the test plug; to detect any other possible sources of trouble before subjecting the transistors to controlled aging conditions.

Background: The above information is required for use in the planning of a study of transistor aging behavior in which approximately 550 type 2N396 transistors will be used. The study will incorporate aging treatments that are various combinations of ambient temperature, collector power dissipation and collector voltage.

The problem was transmitted by G. T. Conrad, Jr.

Status: New.

3911-61-39952/60-508 MODEL ADSORPTION ISOTHERMS

Origin and Sponsor: NBS, Section 5.2

Manager: H. Oser

Objective: To compute the model adsorption isotherm

$$\Theta = \int_{-\infty}^{+\infty} f(x) \Theta(x) dx$$

and its first moment

$$X = \int_{-\infty}^{+\infty} x f(x) \Theta(x) dx$$

for various temperatures, pressures and values of a parameter  $c$ .

Background: The gas adsorption isotherm equation can be written in the form

$$(1) \quad y \exp \frac{x}{RT} = \frac{\Theta}{1-\Theta} \exp \left( \frac{\Theta}{1-\Theta} - c\Theta \right)$$

where  $x$  is the difference between the heat of adsorption at a particular point and the average heat of adsorption over the total surface.  $\Theta$  is the fractional volume of adsorption  $V/V_m$  and  $R, T$  and  $c$  are constants. The heat of adsorption is assumed to have a Gaussian distribution

$$f(x) = e^{-x^2/2}$$

For  $c > 27/4$  the isotherm equation (1) shows two extrema in the interval  $\Theta(0,1)$  which are physically insignificant. A modification of equation (1) is therefore necessary to satisfy the 2nd law of thermodynamics.

The problem was submitted by Messrs. M. B. Wallenstein (30.30), F. G. Carpenter (5.2) and V. R. Deitz (5.2).

Status: New. A program has been written to compute the integrals desired and has been checked out. The cases where  $c > 27/4$  are dealt with in a special code, which is in preparation.

1102-40-11645/60-510  $H_2^+$  BOMBARDMENT

Origin and Sponsor: Naval Research Laboratory

Manager: W. Borsch-Supan

Objective: To compute the double integrals

$$y_1 = \frac{k_1}{\Delta_1} \int_0^{x_{UL}} \int_{-\epsilon}^{\infty} e^{-(w^2/2\Delta_1^2)} (ax)^{-1} \phi(\lambda) e^{-\alpha x} dw dx$$

$$y_j = \frac{k_j}{\Delta_j} \int_0^{x_{UL}} \int_{-\epsilon}^{\infty} e^{-(w^2/2\Delta_j^2)} (ax)^{-1} \phi(\lambda) (1 - e^{-\alpha x}) dw dx, \quad j = 2, 3$$

for certain values of the parameters  $\epsilon$  and  $x_{UL}$ , where

$$\lambda = \frac{\epsilon + w}{ax} - \ln x - b,$$

$$\phi(\lambda) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \exp(u \ln u + \lambda u) du,$$

$k_1, k_2, k_3, \Delta_1, \Delta_2, \Delta_3, a, b, c, \alpha$  are given constants, and  $\sigma$  is an arbitrary positive number.

Background: The value  $y_1 + y_2 + y_3$  gives the total yield of gamma-rays from a target of thickness  $x_{UL}$  bombarded by an  $H_2^+$  beam of a mean energy exceeding the  $p, \gamma$ -resonance energy by  $\epsilon$ . The energy distribution of the incoming beam is assumed to be Gaussian with standard deviation  $\Delta_1$ . During the penetration of the target, the beam is subject to energy loss by ionization and to dissociation. The results are to be compared with measurements.

The problem was transmitted by R. O. Bondelid (NRL).

Status: Some analysis has been done regarding the replacing of infinitesimal processes by finite ones. A table of  $\phi(\lambda)$  with 3 to 4 significant figures and asymptotic formulas for this function including error bounds have been obtained. The code for the numerical evaluation of the double integrals has been written and is being checked.

1100-40-11460/61-522 SPECIAL CONSULTING SERVICES

Origin and Sponsor: National Aeronautics and Space Administration

Manager: E. W. Cannon (11.0)

Objective: To render assistance to NASA in the review of management and technical operations of computational facilities.

Background: The diversity of the applications of modern electronic computers and the variety of computing systems in use require intensive planning to achieve effective operation of an extensive computational facility. By request of the National Aeronautics and Space Administration an advisory committee has been formed to provide assistance in reviewing and planning the organization and operation of NASA computational facilities. NBS is providing the chairman for this Committee.

Status: New.



## 6. STATISTICAL ENGINEERING SERVICES

### COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING Task 3911-61-39951/51-1

Origin: NBS

Authorized 7/1/50

Managers: W. J. Youden, J. Cameron

Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Precision of gage block calibrations. Statistical analyses were made by H. H. Ku on one year's data on gage block calibrations, for John S. Beers of the Length Section, to determine the precision of the measurement process and to ascertain whether any evidence of non-random effects was present.

(2) Color scales. Methods were formulated for the analysis of data obtained in a paired comparison experiment on the magnitudes of color-differences. This work was done by J. R. Rosenblatt for G. L. Howett of the Colorimetry Section in connection with his work for the Committee on Uniform Color Scales of the Optical Society of America.

(3) Transistor aging experiments. Some rough analysis and scrutiny of preliminary measurements on 500 transistors were done by J. R. Rosenblatt and H. H. Ku in preparation for determining the experiment design for a large-scale aging experiment being conducted by G. Conrad of the Engineering Electronics Section.

(4) Precision on the calibration of U-234 standards. Statistical analyses on data of mass-spectrometer measurements of U-233 concentrations were carried out by H. H. Ku in connection with the work of W. R. Shields, 4.03, on the calibration of eight uranium standards.

(5) Data analysis. The analysis of data from tests for homogeneity of standard reference samples of metal alloys using a trend elimination design was prepared by J. M. Cameron and coded for the Bureau's electronic computer by R. Varner and R. J. Herbold. This code is now being used by the Spectrochemistry Section of the Bureau.

#### Publications:

- (1) Some canons of sound experimentation. Churchill Eisenhart. Bulletin de l'Institut International de Statistique, XXXVII (31st Session. Brussels, 1958), pp. 339-350 (Published Brussels, 1960).
- (2) Statistical aspects of the cement testing program. W. J. Youden. Proc. ASTM 59, 1120-1128 (1959).
- (3) Variability of color-mixture data. I. Nimeroff (NBS Photometry and Colorimetry Section), J. Rosenblatt and M. C. Dannemiller. In manuscript.
- (4) How to evaluate accuracy. W. J. Youden. Submitted to a technical journal.



## Status of Projects

- (5) Multivariable investigations. W. J. Youden. SAE Journal 68, No. 6, 65-67 (June 1960).
- (6) The sample, the procedure, and the laboratory. W. J. Youden. Submitted to a technical journal.
- (7) Statistics--engineering viewpoint. W. J. Youden. To appear in Journal of Engineering Education.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC  
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51  
Manager: W. J. Youden  
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS  
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54  
Manager: C. Eisenhart  
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. Work on preparing the text for final publication is nearing completion.

STATISTICAL SERVICES  
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58  
Manager: J. M. Cameron  
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) U. S. Geological Survey: Investigations were continued by J. R. Rosenblatt on several mathematical models associated with measurement processes in hydrology.

(2) Veterans' Administration Hospital, Perry Point, Md.: A computer program is being prepared for performing discriminant analysis on data from the VA's cooperative studies on chemotherapy in psychiatry.

# Current Applications of Automatic Computer

The record of the use of the IBM 704 for the period April 1 through June 30 is as follows:

<u>Task No.</u>	<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
(M I N U T E S)				
<u>NBS:</u>				
11110/55-55	11.1 Research in numerical analysis	422	264	312
11411/55-56	11.1 Research in mathematical topics applicable to numerical analysis	29	45	60
11413/60-469	11.1 Orthogonal functions in the theory of partial differential equations	112	68	981
11120/55-65	11.2 Automatic coding	245	71	758
39951/56-160	11.2 Mathematical subroutines	12	41	142
39951/51-1	11.3 Statistical engineering	87	9	200
39952/54-30	4.1 Spectrum analysis	48	93	420
39952/55-68	9.7 Crystal structure calculations	17		536
39952/55-82	3.1 Thermometer calibrations			138
39952/56-131	2.2 Calculations in optics*	20	16	11
11645/56-166	5.9 SCF-LCAO solution of some hydrides*	194	225	95
11645/56-171	3.2 Collision integrals used in transport theory**	28	247	24
11645/57-219	3.2 Thermal properties*	25	15	90
39952/57-236	3.2 Self-consistent fields—eigenvalues			18
11645/57-246	4.8 Radiation diffusion**	85	184	1117
39952/57-247	6.1 Mechanical impedance*			5
11645/57-249	9.4 Color differences*			139
39952/57-250	2.3 Automatic reduction in spectrophotometric data*	28	23	89
11645/57-251	1.6 Current noise and fixed resistors*	15		19
11645/57-252	4.11 Detecting efficiency in a neutral meson experiment**	326	484	1436
39952/58-254	2.3 Reproduction of color- and spectral-energy distribution of daylight*	1	14	5
11645/58-255	4.8 Chi functions**	190	170	1160
11645/58-256	10.6 Composite walls**	73	11	118
11645/58-260	12.5 Prototype accounting**	23	47	129

<u>Task No.</u>		<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
(M I N U T E S)					
11645/58-270	12.5	Mathematical problems related to postal operations	14	113	122
/58-271	6.3	Simultaneous equations for potential flow**	10		3
/58-272	3.2	Thermodynamic properties of real gases		15	65
/58-274	9.7	Calculations for d-spacings II*			40
/58-275	7.8	Crystallography**			113
/58-281	4.10	Psi evaluation*	16	7	18
/58-294	4.8	Nuclear scattering of photons*			39
/58-306	2.1	Interpolation of color mixture functions*			16
/58-308	3.4	Oscillating sphere*	7	19	33
/58-314	3.2	Approximations for gas mixtures*	366	240	469
/58-333	9.0	Calcium hydroxide*	228	26	437
/58-339	3.4	Viscoelasticity properties of materials	5	37	55
/58-357	3.3	Eigenvalues**	2		48
/58-360	5.2	Diffusion coefficients*	2	46	
/59-377	12.3	Logical diagram reduction			441
/59-383	10.5	Design flow capacity curves*	5	2	
/59-387	30.4	Nuclear reactor design**			454
/59-388	10.3	Heat pump calculations			8
/59-394	4.6	Slow electron scattering by hydrogen atoms	29	411	684
/59-395	7.7	Adsorption study**	13	1	10
/59-403	2.1	Computation of color fadings*			23
/59-409	12.5	Bank Board**	3	137	1322
/59-417	2.4	Spectrum analysis of ruthenium**			141
/59-418	4.8	P-Wave equation*	143	92	107
/59-428	12.5	Radio intensities**	173	147	89
/59-429	6.6	Watch jewel performance*		5	
/59-440	82.10	Mapping**	64	494	1246
/59-446	85.10	Ionospheric data**		38	230
/60-457	12.5	Public Housing problem**	19	87	386
/60-466	3.2	Electronic properties of simple molecular systems		122	803
/60-467	12.1	Transistor simulation		7	
/60-474	2.5	Gage block stability*			19
/60-475	82.40	Ionospheric soundings			359
11122/60-479	11.0	Processing of diagrams	41	173	
11645/60-484	7.6	Polymer crystallization	21	20	107
/60-486	3.6	Morse wave functions	5	129	388
/60-487	5.9	Parabolic curve fitting*			14
/60-489	3.1	Inversion of line probe data	22	12	18

<u>Task No.</u>	<u>Title</u>	<u>Assembly</u>	<u>Checking</u>	<u>Production</u>
(M I N U T E S)				
11645/60-493 3.7	Poisson distribution function**	387	715	58
/60-494 82.0	Atmospheric transmission**	151	67	42
/60-495 6.4	Engineering mechanics**	18	10	
/60-500 7.8	Dental research colorimetry	17	6	27
/60-507 1.6	Transistor aging behavior studies	32	17	49
/60-508 5.2	Model adsorption isotherms	8	50	17
/60-509 10.0	Cement testing**	5		
/60-514 3.9	Flame spectra**			31
/60-515 13.5	Convolution integral**	8		
Totals (NBS Services)...		3,794	5,272	16,533

OUTSIDE:

11645/53-45 SC	Air defense tactics°		15	13
/58-269 NRL	Molecular structure, IV	351	92	483
/58-276 NOL	General kinetics, I**			5,264
/58-278 NOL	Polaris**	90	436	126
/58-319 HPBA	Auto tag°		7	411
/58-325 VA	Covariance analysis	48	38	25
/58-335 DOFL	Roots of Bessel functions**			106
/58-340 DOFL	M5-17 Fuze Data°	28	26	213
/58-348 OOR	Russian-to-English machine translation	21	72	
/58-361 NRL	Spectrum of dipole radiation	4	2	396
/58-366 USIA	Radiation patterns of antennas		12	120
/58-368 SC	Intensity functions of light scattered by spherical particles	55	72	13
/59-371 NRL	ASWAP°	263	371	232
/59-373 DOFL	Rhinitis**	66	9	
/59-407 DOFL	Fourier coefficients*	118	88	38
/59-408 NASA	NASA**	3,041	1,062	14,077
/59-411 HEW	Fitting of exponential curves**	317	5	821
/59-415 DOFL	Complex Legendre functions*	27	6	5
/59-416 DOFL	Analysis of power supply experiments**		2	10
/59-419 DOFL	Neutrons°	36	15	5,716
/59-423 WB	Weather Bureau**			20,415
/59-425 CU	Molecular orbitals*			285
/59-434 CIW	Petrological computations*	50	4	87
/59-435 VA	Electrocardiographic analysis	5	79	199
/59-441 GK	Systems engineering**		1,845	1,643
/59-445 NPSA	Oil supply			436
/59-447 BPRO	Public Roads study**	30	18	3,308



<u>Task No.</u>	<u>Title</u>	<u>Assembly    Checking    Production</u>		
		(M I N U T E S)		
11645/60-450 ACC	Chemical warfare <sup>o</sup>	8	97	176
/60-454 GE	G.E.**			1,290
/60-456 DOFL	Firing circuit equations*			101
/60-458 CAB	Domestic airline traffic survey	78	106	3,248
/60-460 DOFL	Design equations for magnetron injection electron guns			22
/60-462 DOFL	Correlation of functions	7	11	
/60-464 PH	Correlation matrix for PHS matrix*	10	6	8
/60-465 CU	Calculations in molecular quantum mechanics			35
/60-472 DOFL	Probabilistic encounter analysis*	9		37
/60-476 DOFL	Gas tube characteristic II		83	234
/60-480 DOFL	Large signal calculations for a voltage-tuneable magnetron			5
/60-482 UMD	Fluid dynamics**			2
/60-485 DOFL	Constant pressure data**	186	29	570
/60-492 IMF	Monetary research reports**			53
/60-496 BPA	Short circuit program**	183	85	56
/60-501 CEng	Kansas River Reservoir system**	32	13	8
/60-502 USA	Quartermaster mathematics programming**	15	8	581
/60-504 DOFL	Electrostatic focusing	5	103	952
/60-505 GPH	Earth temperature	17	8	13
/60-506 WB	World Bank**	34		41
/60-510 NRL	H <sub>2</sub> <sup>+</sup> -bombardment	24	63	37
/60-511 DCCE	DCCE problem**			243
/60-512 DOFL	Data transformation	42	13	
	Totals (Outside) .....	<u>5,200</u>	<u>4,901</u>	<u>62,154</u>
Total time for the quarter (MINUTES).....		8,994	10,173	78,687
Total time for the quarter (HOURS) .....		150	170	1,311

\* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

\*\* Problem programmed by sponsor and run under his direction.

<sup>o</sup> Classified task.

# Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

## Applied Mathematics Division Lectures

MORDELL, L. J. (University of Cambridge and University of Colorado)  
Reflections of a mathematician. May 31.  
On a Pellian equation conjecture. June 1.  
Recent work in number theory. June 2.  
Rational points on cubic curves and surfaces. June 3.

## Mathematical Statistics Seminar

JACKSON, Julius L. (NBS Free Radicals Research) The crystallization of polymers and the parking problem. April 20.

## Statistical Engineering Laboratory Reliability Seminars

BARLOW, Richard E. (Sylvania Electronic Defense Laboratory, Mountain View, California) Applications of semi-Markov processes to counter and reliability problems. June 27.

## Papers and Invited Talks

### Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. The outlook for machine translation. Presented at the Western Joint Computer Conference, San Francisco, California, May 4.

BAZLEY, N. W. (1) Approximation of eigenvalues for differential operators. Presented at Aeronautical Research Laboratories, Wright-Patterson Air Force Base, Ohio, April 27. (2) Error bounds in Rayleigh-Ritz eigenvector approximation. Presented at the International Conference on Partial Differential Equations and Continuum Mechanics, held at the Mathematics Research Center, U. S. Army, Madison, Wis., June 7-15.

DAVIS, P. J. Some geometrical theorems for Gaussian weights and abscissas. Presented at a meeting of the Mathematical Association of America, held at the University of Virginia, Charlottesville, Va., May 7.

- GHAFFARI, A. On some partial differential equations of Brownian motion of a free radical. Presented at the International Conference on Partial Differential Equations and Continuum Mechanics held at the Mathematics Research Center, U. S. Army, Madison, Wis., June 7-15.
- HAYNSWORTH, E. (1) Regions containing the characteristic roots of a matrix. Presented at a meeting of the American Mathematical Society, New York City, April 14-16. (2) Bounds for determinants, and (3) Criteria for the reality of matrix eigenvalues, both presented at Mathematics Colloquia, University of North Carolina, Chapel Hill, N. C., April 11 and May 12 resp.
- RABINOWITZ, P. Use of orthogonal functions in numerical analysis. Presented at a Mathematics Colloquium, Oregon State College, Corvallis, Oregon, May 10.
- RICE, J. R. Approximation by monotonic and convex polynomials. Presented at a meeting of the American Mathematical Society, New York City, April 14-16.
- ROSENBLATT, J. R. Non-parametric tests. Presented before the Washington Section, American Society for Quality Control (1960 Course on Quality Control), Washington, D. C., April 12.
- VINTI, J. P. Theory of the orbit of an artificial satellite, with use of spheroidal coordinates. Presented at a meeting of the American Astronomical Society, Pittsburgh, Pa., Apr. 18-21.
- WEGSTEIN, J. Common languages for the uncommon folks. Presented before the Washington Chapter, Association for Computing Machinery, Washington, D. C., April 21.
- YODEN, W. J. (1) Everyday applications of statistics. Presented at a meeting of the Montgomery County Mathematics Teachers Association, North Bethesda Junior High School, April 6. (2) Constrained randomization. Presented at the University of Paris, Paris, France, May 19. (3) The enduring values. Presented at a meeting of the Gordon Conference on Information Processing for Critical Tables of Scientific Data, New Hampton, New Hampshire, June 23.
- ZELEN, M. The robustness of statistical life testing procedures. Presented at the Westinghouse Research Laboratories, Pittsburgh, Pa., June 23.

# Publication Activities

## 1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

### 1.3 Technical Papers

The following papers appeared in J. Research NBS 64B, Apr-June 1960 (Mathematics and Mathematical Physics):

- (1) Non-self-adjoint boundary value problems in ordinary differential equations. W. Greub and W. C. Rheinboldt. Pp. 83-90.
- (2) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. R. Rice. Pp. 91-93.
- (3) Upper and lower bounds for the center of flexure. L. E. Payne. Pp. 105-111.

\* \* \* \* \*

The following paper appeared in J. Research NBS 64C, Apr-June 1960 (Engineering and Instrumentation):

The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. Pp. 115-119.

\* \* \* \* \*

- (1) The outlook for machine translation. F. L. Alt. "Proceedings, Western Joint Computer Conference, San Francisco, 1960," Pp. 203-208.
- (2) Analysis of fractionally replicated  $2^m 3^n$  designs. R. C. Bose and W. S. Connor. Bulletin de l'Institut International de Statistique XXXVII (31st Session, Brussels, 1958), pp. 141-160 (Published Brussels, 1960).
- (3) Construction of fractional factorial designs of the mixed  $2^m 3^n$  series. W. S. Connor. "Contributions to Probability and Statistics," pp. 168-181 (Stanford University Press, Stanford, California, 1960).
- (4) Some canons of sound experimentation. Churchill Eisenhart. Bulletin de l'Institut International de Statistique XXXVII (31st Session, Brussels, 1958), pp. 339-350 (Published Brussels, 1960).



- (5) The incidence equation  $AA^T = aA$ . K. Goldberg. Amer. Math. Month. 67, 367 (1960).
- (6) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. J. Fluid Dynamics 7, pt. 4, 529-549 (1960).
- (7) On the power of some rank order two-sample tests. J. R. Rosenblatt. "Contributions to Probability and Statistics," pp. 358-370 (Stanford University Press, Stanford, California, 1960).
- (8) Statistical models for component aging experiments. J. R. Rosenblatt. IRE National Convention Record, 1960, Vol. 8, Pt. 6, pp. 115-124.
- (9) Multivariable investigations. W. J. Youden. SAE Journal 68, No. 6, 65-67 (June 1960).
- (10) Statistical aspects of the cement testing program. W. J. Youden. Proc. ASTM, 59, 1120-1128 (1959).
- (11) Analysis of two-factor classifications with respect to life tests. Marvin Zelen. "Contributions to Probability and Statistics," pp. 508-517 (Stanford University Press, Stanford, California, 1960).

## 2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

### 2.2 Technical Notes, Manuals, and Bibliographies

- (1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.
- (2) Selected bibliography of statistical literature, 1930-1957. III. Limit theorems. L. S. Deming. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
- (3) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

### 2.3 Technical Papers

- (1) Reliability of Monte Carlo methods in computing finite Markov chains. N. Bazley and P. J. Davis. To appear in the Journal of Research, NBS, Sec. B.
- (2) Computer simplification of Boolean functions. B. K. Bender, A. J. Goldman, and R. B. Thomas (Data Processing Systems). Submitted to a technical journal.

- (3) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960.
- (4) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. Submitted to a technical journal.
- (5) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
- (6) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
- (7) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
- (8) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.
- (9) A reduction formula for partitioned matrices. E. V. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.
- (10) Bounds for determinants with positive diagonals. E. V. Haynsworth. To appear in the Proceedings of the American Mathematical Society.
- (11) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.
- (12) Irrational power series. M. Newman. Submitted to a technical journal.
- (13) Periodicity modulo  $m$  and divisibility properties of the partition function. M. Newman. To appear in the Transactions of the American Mathematical Society.
- (14) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
- (15) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.
- (16) On Stokes flow about a torus. W. H. Pell and L. E. Payne. To appear in Mathematika.
- (17) Mechanized conversion of colorimetric data to Munsell renotations. W. C. Rheinboldt and J. P. Menard. To appear in the Journal of the Optical Society of America.
- (18) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

- (19) Sequence transformations based on Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B.
- (20) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.
- (21) Split Runge-Kutta for simultaneous equations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B.
- (22) Tchebycheff approximations by  $ab^x + c$ . J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
- (23) Tchebycheff approximations by functions unisolvant of variable degree. J. R. Rice. To appear in the Proceedings of the American Mathematical Society.
- (24) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Submitted to a technical journal.
- (25) Error bounds in the Rayleigh-Ritz approximations of eigenvectors. H. F. Weinberger. To appear in the Journal of Research, NBS, Sec. B.
- (26) How to evaluate accuracy. W. J. Youden. Submitted to a technical journal.
- (27) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.
- (28) Statistics--engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.
- (29) The sample, the procedure, and the laboratory. W. J. Youden. Submitted to a technical journal.
- (30) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. Submitted to a technical journal.
- (31) The robustness of life testing procedures derived from the exponential distribution. M. Zelen and M. C. Dannemiller. Submitted to a technical journal.
- (32) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.



**U.S. DEPARTMENT OF COMMERCE**

**Frederick H. Mueller, *Secretary***

**NATIONAL BUREAU OF STANDARDS**

**A. V. Astin, *Director***



**THE NATIONAL BUREAU OF STANDARDS**

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colo., is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

**WASHINGTON, D.C.**

**ELECTRICITY.** Resistance and Reactance. Electrochemistry. Electrical Instruments. Magnetic Measurements. Dielectrics.

**METROLOGY.** Photometry and Colorimetry. Refractometry. Photographic Research. Length. Engineering Metrology. Mass and Scale. Volumetry and Densimetry.

**HEAT.** Temperature Physics. Heat Measurements. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research. Equation of State. Statistical Physics. Molecular Spectroscopy.

**RADIATION PHYSICS.** X-Ray. Radioactivity. Radiation Theory. High Energy Radiation. Radiological Equipment. Nucleonic Instrumentation. Neutron Physics.

**CHEMISTRY.** Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

**MECHANICS.** Sound. Pressure and Vacuum. Fluid Mechanics. Engineering Mechanics. Combustion Controls.

**ORGANIC AND FIBROUS MATERIALS.** Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

**METALLURGY.** Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

**MINERAL PRODUCTS.** Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

**BUILDING RESEARCH.** Structural Engineering. Fire Research. Mechanical Systems. Organic Building Materials. Codes and Safety Standards. Heat Transfer. Inorganic Building Materials.

**APPLIED MATHEMATICS.** Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

**DATA PROCESSING SYSTEMS.** Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Applications Engineering.

**ATOMIC PHYSICS.** Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics.

**INSTRUMENTATION.** Engineering Electronics. Electron Devices. Electronic Instrumentation. Mechanical Instruments. Basic Instrumentation.

Office of Weights and Measures.

**BOULDER, COLO.**

**CRYOGENIC ENGINEERING.** Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

**IONOSPHERE RESEARCH AND PROPAGATION.** Low Frequency and Very Low Frequency Research. Ionosphere Research. Prediction Services. Sun-Earth Relationships. Field Engineering. Radio Warning Services.

**RADIO PROPAGATION ENGINEERING.** Data Reduction Instrumentation. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation-Terrain Effects. Radio-Meteorology. Lower Atmosphere Physics.

**RADIO STANDARDS.** High frequency Electrical Standards. Radio Broadcast Service. Radio and Microwave Materials. Atomic Frequency and Time Standards. Electronic Calibration Center. Millimeter-Wave Research. Microwave Circuit Standards.

**RADIO SYSTEMS.** High Frequency and Very High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Space Telecommunications.

**UPPER ATMOSPHERE AND SPACE PHYSICS.** Upper Atmosphere and Plasma Physics. Ionosphere and Exosphere Scatter. Airglow and Aurora. Ionospheric Radio Astronomy.



