PROJECTS and PUBLICATIONS
of the
APPLIED MATHEMATICS DIVISION
A Quarterly Report
January through March 1960

For Official Distribution
THE NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS
APPLIED MATHEMATICS DIVISION

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Status of Projects
March 31, 1960

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth continued work on partitioned matrices and on bounds for the P-condition number of matrices. The paper "A reduction formula for partitioned matrices" was revised and submitted to the NBS Journal of Research. Another paper "Special types of partitioned matrices" was completed, ready for submittal to a technical journal. Two additional papers, "Bounds for the P-condition number of matrices with positive roots" and "Criteria for the reality of matrix eigenvalues" (with M. Drazin) are in manuscript form.

H. F. Weinberger derived the following results on an error bound in the Rayleigh-Ritz approximation of eigenvectors. Let A be a linear hermitian operator with eigenvalues $\lambda_1 \leq \lambda_2 \leq ...$ and corresponding normalized eigenvectors $u_1, u_2, ...$. Upper bounds $k_1 \leq k_2 \leq ... \leq k_M$ for the first $M$ eigenvalues are determined by the Rayleigh-Ritz method. Let $w_1, ..., w_M$ be the corresponding vectors, so that

$$(w_i, w_j) = \delta_{ij},$$

$$(Aw_i, w_j) = k_i \delta_{ij}.$$  

Let lower bounds $\tilde{\lambda}_1 \leq \tilde{\lambda}_2 \leq ... \leq \tilde{\lambda}_N$ for the first $N \geq M$ eigenvalues $\lambda_i$ be known. A bound for the error in norm $(u_p - w_p)$ in the approximation of $w_p$ to the pth eigenvector $u_p$ is found in terms of the $\tilde{\lambda}_i$ and $k_i$ alone. This bound is sharp in the sense that there is an operator A for which equality holds.

N. Bazley completed the investigation of Monte Carlo methods for computing Markov chains by a study of the relationship of the nature of the chain to its eigenvalues with magnitude one. He also worked on the calculation of upper and lower bounds to the eigenvalues of the associated Legendre equation

$$[(x^2-1)y']' + \frac{m^2}{1-x^2} y - \lambda y = 0, \quad y \text{ and } y' \text{ finite at } \pm 1,$$  

for non-integral values of $m$. It is of interest to compare for integral $m$ the results of the approximate methods with the known solutions.
J. R. Rice pursued the following tasks during this quarter:
(a) The manuscript, "Sequence transformations based on Tchebycheff approximations", was completed. This paper deals with convergence acceleration procedures for sequences whose nth term is

\[ \sum_{i=1}^{k} a_i b_i^n + c \quad \text{or} \quad \sum_{i=1}^{k} a_i b_i^n \cos(\theta_i + n\phi_i) + c. \]

It is also shown that the transformations derived are effective for certain other types of sequences.

(b) The IBM 704 program to test a simple method of obtaining rational function approximations was checked out and examples are being run.

(c) Investigations were continued on the feasibility of an "optimized-split" integration method for simultaneous differential equations. The two IBM 704 codes were checked out, which will integrate two particular differential equations, each with different time steps that have been "optimized" so as to use a minimum of integration steps for a given interval. A series of tests were made and the following conclusions were reached:
(i) The method is useful in certain situations; (ii) The usefulness depends critically on the nature of the particular equations being solved; (iii) The optimization procedure should be modified for each particular set of equations if maximum benefit is to be obtained.

(d) Work was continued on the numerical solution of the self-acting gas lubricated journal bearing:

\[ \frac{\partial}{\partial x}[h(x,y)\frac{\partial q}{\partial x}] + \frac{\partial}{\partial y}[h(x,y)\frac{\partial q}{\partial y}] = - \frac{\partial[h(x,y)q]}{\partial x}. \]

\( h(x,y) \) is a given function and \( q(x,y) \) is to be determined. At present the machine solutions are divergent. The reason for this is undetermined although the analysis and programming have been thoroughly checked.

P. J. Walsh (11.2) wrote a code to evaluate methods of numerically satisfying an internal boundary condition involving a normal derivative. This problem comes from the theory of the infinite step bearing. The code is now being checked out.

(e) A long analysis of the problem of approximation by convex and monotonic polynomials was almost completed. Many results were obtained for the general problem of approximating a function in a monotonic norm by a linear combination of functions whose coefficients satisfy a "convex constraint." Algorithms for obtaining both least squares and Tchebycheff approximations by convex and monotonic polynomials are being developed.

(f) It is well known that the best approximating (in a monotonic norm) nth degree polynomials to a function \( f(x) \) on \([0,1]\) will interpolate the values of \( f(x) \) in at least \( n+1 \) points. This interpolation phenomenon is being studied for a broad class of approximation problems. Theorems of the following types have been established: Theorem 1. Let \( F(A,x) \) be a regular unisolvent function of variable degree and let \( f(x) \) be a given continuous function. If \( F(A^*,x) \) is the best approximation (in the monotonic norm \( \delta \)) to \( f(x) \) and \( F(A^*,x)-f(x) \) has a finite number of zeros, then \( F(A^*,x)-f(x) \) has at least \( k \) simple zeros where \( k \) is the degree of unisolvence of \( F \) at \( A^* \). Theorem 2. Let \( F(A,x),f(x) \) be as given above.
Assume that the monotonic norm \( \delta \) satisfies "condition B" (a new concept in this study). If \( F(A^*, x) \) is the best \( \delta \)-approximation to \( f(x) \), then \( F(A^*, x) - f(x) \) has at least \( k \) strong sign changes where \( k \) is the degree of \( F \) at \( A^* \).

Regularity and condition B are concepts developed in this study. An effort is now being made to establish the necessity of regularity and condition B for Theorems 1 and 2. An example is given of a non-regular unisolvent function of three parameters such that the best least squares approximation to \( (x - \frac{1}{2}) \) is identically zero.

(g) The paper "Tchebycheff approximations by \( ab^x + c \) and \( ab^x \cos(\theta_0 + \theta x) + c \) has been separated into two papers upon the advice of the referee. One of them, entitled "Tchebycheff approximation by \( ab^x + c \)" has been accepted by the Journal of the Society for Industrial and Applied Mathematics. The second paper has not yet been submitted.

Publications:


(6) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Submitted to a technical journal.


(8) Special types of partitioned matrices. E. Haynsworth. In manuscript.


(10) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.

(11) Criteria for the reality of matrix eigenvalues. E. Haynsworth and M. Drazin (RIAS). In manuscript.

(12) Bounds for the P-condition number of matrices with positive roots. E. Haynsworth. In manuscript.

Status of Projects

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has completed a study of periodicity of sequences of integers modulo a non-zero integer, and has obtained many theorems concerned with divisibility properties of partitions. An example is the fact that the unrestricted partition function fills all residue classes infinitely often modulo 2, 5, 13.

Dr. Newman also began work on certain diophantine problems. It was shown, for example, that $1 + (1), 1 + (4) + (2)$ are simultaneously squares if and only if $n = 15$.

K. Goldberg and E. C. Dade have extended their manuscript on the abstract properties of incidence algebras. A paper on the applications to $v, k, \lambda$ designs, through group-generated incidence spaces, is in manuscript.

K. Goldberg has shown that if $\{p_n(t)\}$ is a set of Faber polynomials of a function $f(z)$ with Laurent expansion at $z = 1$ and if there exist constants $a, b$ such that $\sum_{n=1}^{\infty} (p_n(a)-b)$ converges, then $b$ is an integer. If $b$ is positive it is the multiplicity of the root $z = 1$ of $f(z) - f(1)$, and $a = 1/f(1)$. If $b$ is negative then $-b$ is the multiplicity of the pole $z = 1$ of $f(z)$, and $a = 0$. For an example see the task on Information Selection Systems (p. 5), in which $f(z) = ze^{-z}$, $a = e$, $b = 2$.

Publications:

(2) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
Status of Projects

INFORMATION SELECTION SYSTEMS
Task 1101-12-11412/60-470

Origin: NBS
Authorized 9/25/59

Sponsor: National Science Foundation
Managers: K. Goldberg, A. J. Goldman

Status: CONTINUED. K. Goldberg continued his investigations of the problem of classification and of the expected lengths of chains in an infinite flow of data through a computer with a finite memory. He has shown that the expected length of the nth increasing sequence in a random infinite sequence of numbers is \( p_n(e) \), a value very close to 2, where \( \{ p_n(t) \} \) is the set of Faber polynomials of \( ze^{-z} \).

A. J. Goldman began work on coding the boolean simplification program for the IBM 704. Investigation continued on the effects of permuting the individual steps in Phase III of the program. An analysis was started of the maximum number of basic \( k \)-cells in an \( n \)-variable Boolean function having no cell of dimension \( k+1 \).

Publications:
(4) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.
(5) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(6) Optimization models for distribution networks. B. K. Bender (11.2) and A. J. Goldman. In manuscript.

ORTHOGONAL FUNCTIONS IN THE THEORY OF
PARTIAL DIFFERENTIAL EQUATIONS
Task 1101-12-11413/60-469

Origin: NBS
Authorized 9/25/59

Sponsor: Atomic Energy Commission
Managers: P. Davis, P. Rabinowitz

Status: CONTINUED. The results of studies during this quarter pursued by P. Rabinowitz include the following:

(a) The Dirichlet problem was solved for various regions including two concentric ellipses, a circle with two holes punched out, and a square with corners removed. The dependence on the boundary functions of the convergence of the method of orthogonal functions for solving the Dirichlet problem was investigated on an ellipse and on a bean-shaped region.
(b) A routine was written for the solution of boundary value problems of the second and third kind for simply-connected planar regions.

(c) A routine was written for the solution of the biharmonic equation \( \Delta^2 u = 0 \) for simply connected planar regions in which the function and its normal derivative are specified on the boundary. This routine was then used to solve such a problem on an ellipse.

(d) The transfinite diameter of two and three collinear segments was studied numerically by means of the orthonormalization routine. It was found that even double precision arithmetic was not good when the powers of \( x \) were orthonormalized. On the other hand, the 3-term recursion formula for orthogonal polynomials gave good results, even in single precision.

P. Davis and P. Rabinowitz have completed a manuscript entitled, "Some geometrical theorems for abscissas and weights of Gauss type." They also have in progress a manuscript entitled, "Advances in orthonormalizing computation," which, among other things, summarizes recent numerical experience.

Publications:
(1) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. In manuscript.
(2) Advances in orthonormalizing computation. P. Davis and P. Rabinowitz. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health
Authorized 9/30/55
Manager: P. Rabinowitz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. Additional runs on the 704 were made during this quarter as requested by the sponsor, still continuing for the slightly modified form of the one-dimensional case.
2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-11112/47-2

Origin: NBS
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-11112/51-8

Origin: Section 11.3, NBS
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-11112/52-7

Origin: NBS
Manager: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-11112/52-37

Origin: NBS
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.
Status of Projects

SIEVERT'S INTEGRAL
Task 1102-40-11112/52-57

Origin: NBS
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46
Status: INACTIVE.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10
Status: CONTINUED. Graphs and tabular material are being prepared for Chapter 8: Legendre Functions, Chapter 18: Weierstrass Elliptic Functions, and Chapter 20: Mathieu Functions. Textual material for these chapters is being revised. All chapters are undergoing review for consistency of notation and format.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11
Status: CONTINUED. In January, J. H. Wegstein attended an international conference in Paris for the purpose of re-writing the international algorithmic language (ALGOL) proposal. The proposal prepared by this conference is called ALGOL 60. If this proposed language gains acceptance, it can be used for communicating scientific computer programs between people on an international level as well as for communicating directly with computers.

A group consisting of A. Beam, G. Galler, S. Peavy, P. Rabinowitz, G. Reitwiesner, G. Urban, J. Wegstein, and W. W. Youden began part-time work on a long-range project aimed at designing a generator for the purpose of generating data processing type computer programs. The group has begun by examining the features which are common to compilers, language translators, special data processing codes, and the intended generator itself. The characteristics sought for the generator are: an efficient object code, independence of any particular computer, notation convenient for humans (hopefully based on the ALGOL language), and capability of bootstrapping itself into existence. The group seeks to define a number of code "packages" which would be coded by hand in the language of any chosen computer (object code). These packages when strung together would constitute the generator.
desired. In addition the generator should be capable of generating an even better version of itself as well as of "bit-chasing" and data processing type codes.


MATHMATICAl SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A General Purpose Orthonormalization Code (BSORTH) along with a complete description for its use was completed by P. J. Walsh and Emilie Haynsworth. This program was contributed to the SHARE organization for distribution to its membership. It can execute any of the following operations:
(a) orthonormalizing a set of vectors with respect to a general inner product;
(b) least squares approximation to a given function or functions by linear combinations of powers (polynomial approximations) or any linear combination of powers, rational functions, transcendental functions and special functions, such as those defined numerically by a set of values;
(c) surface fitting of empirical data in two or more dimensions;
(d) finding the best solution in the least squares sense to a system of m linear equations in n unknowns (n ≤ m);
(e) matrix inversion and solution of linear systems of equations;
(f) expansion of a given function or functions in a series of orthogonal functions, such as a series of Legendre or Chebyshev polynomials.

The polynomial curve fitting code, prepared by P. J. Walsh and described in the July-Sept and Oct-Dec 1959 issues, pp. 11 and 10 respectively, has been checked out and applied to several functions. Successful approximations have been obtained for degrees as high as 20 using 50 and 75 points. Slight modifications will be introduced to prepare the routine as a Fortran function subroutine to be submitted to SHARE. The printout of intermediate calculations (starting at a specified minimum degree) will be made optional. Additional information will be available, such as the Fourier coefficients, residuals, and the sum of squares of residuals.

The introduction of a 32K core memory has made the formation of a matrix base code feasible. A code has been built by P. J. Walsh and J. D. Waggoner, which contains the more frequently used matrix operations listed below. Some of these subroutines were obtained from the SHARE library; others were written by members of the staff. For short matrix problems or for problems involving small matrices (which frequently arise), only a small
SAP code needs to be assembled and added to the binary base code.

1. \( A(n \times m) + B(n \times m) = C(n \times m) \)

2. \( A(n \times m) \cdot B(m \times r) = C(n \times r) \)

3. \( A(n \times m) \cdot B^T(m \times r) = C(n \times r) \) \quad \{ \text{Performs matrix multiplication without transposing } B. \}

4. \( B^T(n \times m) \cdot A(m \times r) = C(n \times r) \)

5. \( A(n \times n) \cdot B(n \times r) = C(n \times r) \) \quad \{ \text{Where } A \text{ is a diagonal matrix stored as a vector with components } a_{ii}, i=1,2,...,n. \}

6. \( B(m \times n) \cdot A(n \times n) = C(m \times n) \)

7. \( A^T(n \times n) = B(n \times n) \)

8. \( A^{-1}(n \times n) = B(n \times n) \)

9. Replace the \((i,j)\)-element of \(A(m \times n)\) by the contents of the accumulator.

10. Replace the diagonal elements of \(A(n \times n)\) by the components of vector \(b\), with option of replacing all off-diagonal elements of \(A\) by zero or leaving them unaltered.

11. Select the diagonal elements of \(A(n \times n)\) to form vector \(b\).

Some experiments are being performed in curve fitting using the orthonormalization subroutine. Two problems have been presented which read as follows:

1. Given \(X_i\) and the corresponding functional values \(Y_i\), \(i=0,1,2,...,n\), find the best fitting polynomial in the least squares sense subject to the following constraints: (a) that the polynomial pass through \((X_0,Y_0)\); and (b) that the derivative at \(X_0\) is \(K\).

2. The second problem is the same as 1. but subject to the further restriction that \(Y(0) = 0\).

In both problems, polynomials of degrees 2 through 5 have been requested. For case 1, the function \((Y-Z)\) will be fitted by linear combinations of \((X-X_0)^2, (X-X_0)^3, \ldots\), where \(Z=a+bX_0\), the straight line passing through \(X_n\) and possessing slope \(K\). For case 2, the function \((Y-Z)\) will be fitted by linear combinations of \(X(X-X_0)^2, X(X-X_0)^3, \ldots\), where \(Z=aX_0+bX_0^2\), the parabola passing through \(X_0\) and having derivative \(K\).

A Fortran program was written by G. W. Reitwiesner to mechanize a part of the preparation of the Computation Laboratory monthly personnel time reports.
3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. M. Zelen and N. C. Severo have extended their work on normal approximations to the chi-square distribution to include approximations for small degrees of freedom.

C. Eisenhart and K. N. Dietmeier studied the extent to which the level of confidence would be lowered if, when \( \sigma \) is known, one always chose the narrower of the normal and the studentized confidence intervals for the population mean.

Publications:
(6) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS
Manager: J. M. Cameron
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. The catalog of fractional factorial designs for the \( 2^m3^n \) series prepared under Bureau of Ships sponsorship (see July-Sept 1959 issue, p. 14) is now being revised for publication in the light of comments received from a number of specialists in the theory and practice of experiment design.
Publications:


STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

Publications:


(2) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. Submitted to a technical journal.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt completed a paper on "Statistical Models for Component Aging Experiments." A general framework is proposed for probabilistic models representing the results of experiments in which (say) electronic components are exposed to controlled "aging" conditions and measured periodically until "failure".

M. Zelen has found the exact distribution of the sum of

$$Y = \sum_{1}^{n} x_1$$

of random variables from a Weibull population having the cumulative distribution

$$F(x) = 1 - e^{-x^p/\theta}.$$
namely,

$$\Pr(Y \leq y) = \left(\frac{py^p}{\theta}\right) \sum_{k=0}^{\infty} (-1)^k \left(\frac{y^p}{\theta}\right)^k \frac{E[\sum W_i]^k}{\Gamma(pk+pn+1)}$$

where the $(W_i)$ are independent random variables each equal to the $p$th power of a gamma variable with parameter $p$.

M. Zelen and M. C. Dannemiller are preparing a paper summarizing their work on the robustness of statistical life testing procedures. They have found that many of the current life testing procedures are very sensitive to the assumption that failure times follow the exponential distribution. Incident to their studies on robustness, they have developed excellent approximations (i) to the distribution of a sum of Weibull random variables, and (ii) to the O.C. curve and average sample number when sequential tests are made on sums of Weibull random variables. These methods are based on a general life probability density function

$$p(x) = \frac{e^{-x} x^a}{\Gamma(a+1)} \sum_{s=0}^{k} a_s L_s^a(x)$$

where $L_s^a(x)$ are Laguerre polynomials.

Publications:


(2) **Are life testing procedures robust?** M. Zelen and M. C. Dannemiller. Appeared in "Proceedings of the Sixth National Symposium on Reliability and Quality Control" (1960), pp. 185-189.

(3) **Statistical models for component aging experiments.** Joan R. Rosenblatt. To appear in the "Institute of Radio Engineers National Convention Record."
The most general solution of one-dimensional Brownian motion, which is governed by Chapman-Kolmogoroff functional equation

\[ f(x,s;y,t) = \int_V f(x,s;z,u)f(z,u;y,t)\,dz, \quad s < u < t, \]

was given by A. Ghaaffari (Bull. Amer. Math. Soc. 57, 1951) in the form

\[ f(x,s;y,t) = \sum_{n=0}^{\infty} \theta^n(s,t)\varphi_n(x)\varphi_n(y). \]

\( V \) denotes the interval \((-\infty, \infty)\), and the function \( f \), being Lebesgue-measurable in \( V \), is the transition probability that the Brownian particle (assumed spherical) passes from state \( x \) at the instant \( s \) to any one of the states of a set \( V \) at the instant \( t \). The infinite sequence of parabolic cylinder functions

\[ \varphi_n(x) = \frac{e^{-x^2/2} H_n(x)}{\sqrt{2^n n! \sqrt{\pi}}} \]

form a complete orthonormal set over \((-\infty, \infty)\), and

\[ \theta(s,t) = \frac{a(s)}{a(t)} \]

where \( a(s) \) is a positive increasing function \( \neq 0 \), and \( H_n(x) \) is the \( n \)th Hermite polynomial. It was proved that, for \( s \) and \( t \) fixed such that \( s < t \) and \( x,y \) varying arbitrarily over \((-\infty, \infty)\), the series solution (2) is absolutely and uniformly convergent. It was shown also that the series solution (2) is doubly square integrable with respect to Lebesgue measure. A. Kolmogoroff obtained (Math. Ann. 104, 1931) a class of particular solutions of equation (1) and showed that his solutions verify two linear partial differential equations of the second order and parabolic type.

Now A. Gnaaffari has shown that the series solution (2), which is different from those given by A. Kolmogoroff, satisfies, for \( y \) and \( t \) fixed,
Status of Projects

the parabolic linear partial differential equation

\[
\frac{\partial^2 f}{\partial x^2} + 2 \frac{a(s)}{a'(s)} \frac{\partial f}{\partial s} + (1-x^2)f = 0
\]

and, for \( x \) and \( s \) fixed, its formal adjoint. A paper is in preparation.

L. E. Payne has been working on a method for obtaining pointwise bounds in the Cauchy problem for the biharmonic equation. Such estimates are needed in the treatment of certain elastic plate problems.

Publications:


PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen investigated the mechanism of shielding in the three-body plasma statistics. The system of B-B-G-K-Y equations (Bogoliubov, Born, Green, Kirkwood, and Yvon) was made closed by degenerating the triple correlation function, and reduced to a system of two equations relating the singlet distribution functions and the double correlation functions. The shielding, which modifies the static potential, was represented by such degenerated correlation functions (products of singlet distribution functions with double correlation functions). They caused some mathematical difficulties, as they entered in the form of an integral equation with different indices. Thus, in fact, one had to solve a system of integral equations.

The solution of the system was attempted by the following three methods: (a) Use of singular integral equations, (b) use of the dielectric property of the screening cloud, (c) use of immobilized screening cloud.

Method (a) was very much simplified when one assumed that the time scale of the correlation function was very small as compared with that
of the distribution function. Method (b) considered the screening cloud as a macroscopic medium having some dielectric property, determined by the Poisson equation. Method (c) considered the screening cloud as a medium with an average immobilized geometry. Method (c) had also served as a basis for the derivation of the kinetic equation by Tchen (see Phys. Rev. 114, 394-411, 1959).

Between September 1959 and February 1960, Dr. Tchen conducted his research at the Theoretical Physics Division of the U. K. Atomic Energy Research Establishment at Harwell, England, as a Guggenheim Fellow. His emphasis was on plasma dynamics and applications to thermonuclear fusion. Since March 1960, he has continued his research at the Max-Planck Institute for Physics and Astrophysics, at Munich, Germany. His emphasis has been on fundamental plasma problems and applications of magnetohydrodynamics to astrophysics.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti continued work on the problem of solving the kinetic equations of motion for a satellite orbit, expressed in spheroidal coordinates. The kinetic equations are the equations (63) given in his paper, J. Research NBS 63B, 105-116 (1959), expressing the spheroidal coordinates \( \rho, \eta, \phi \) implicitly as functions of time. The development of the problem has lead to a solution, in which both secular terms and periodic terms are correct through order \( k^2 \). There remain many algebraic calculations to be done, however, in order to obtain the final formulas necessary to calculate an orbit. The polar coordinates will then follow directly from the spheroidal coordinates. It is of some interest to note that the secular terms calculated in Vinti's manner from the kinetic equations agree with the secular terms as calculated from the action variables \( J_i \), with use of the theorem

\[
\nu_i = \frac{\partial \alpha_i}{\partial J_i}
\]


FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.
5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The line list for 12,000 thorium lines has been prepared for publication. About 15,000 lines of praseodymium have now been measured, and their wave lengths and wave numbers calculated. The main table of intensities of 70 elements has been punched on cards, ready for listing and sorting into tables arranged by element and by wave length.

The interferometry codes have been completed and have been used for computation of observed patterns.

A code has been written to add every spectrum line of a list to every energy level of a second list, and then to select the sums that repeat, within a tolerance, a given number of times.

3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES
Origin: NBS, Section 3.2
Manager: M. L. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. Although no additional codes have been written, two codes written earlier, one for interpolation-editing and one that permits 3-, 4-, and 5-point interpolation in a table of many arguments (see Jan-Mar 1959 issue, p. 16) have been used by various members of the Thermodynamics Section (3.2) staff in other aspects of the project.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. J. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Least squares refinements were carried out for several crystals during the past quarter. Only in the case of the potassium cobalti tungstate crystal was the approximate structure sufficiently well determined to allow the least squares program to produce an improved structure.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: B. S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for 42 thermometers.
1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. V. Haynsworth (11.1), P. J. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Reactivated. Programs are being prepared to compute integrals and to sort them in a specific arrangement required by the A-matrix subroutine. Another program is planned which will generate a matrix (similar in form to the A-matrix) and select certain eigenvectors, computed by the SCF subroutine, for pre- and post-multiplication on sections of the matrix generated.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed for 25 laboratory sets of gage blocks.

1102-40-11645/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: R. N. Varner
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. Production runs were made to determine the thermodynamic properties of a number of elements and their compounds. The results are included in an NBS Report prepared in the Thermodynamics Section (3.2), "Preliminary Report on the Thermodynamic Properties of Selected Light Element Compounds."

1102-40-11645/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Completed. Computations have been carried out as requested, and the results have been transmitted to the sponsor.

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. L. Paulsen, P. L. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.
1102-40-11645/57-236  SELF CONSISTENT FIELDS--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. V. Haynsworth (11.1)
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. Calculations involving linear symmetric $N_3$ molecule were performed.

3911-61-39952/58-266  DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. Work was resumed on the code for the solution of the differential equations with terminal initiation for large sized ($\approx 1000$) polymer chains. Code checking is almost completed.

1102-40-11645/58-269  MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. The atomic coordinates and isotropic temperature factors of the N-benzyl dihydronics-tinamide crystal were refined using the least squares program of Dr. W. Busing (Oak Ridge National Laboratory). After detection and correction of a few errors in the observed data, it was possible to obtain a satisfactory molecular structure for the crystal.

1102-40-11645/58-270  MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman (11.1)
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. Investigation of mathematical models of distribution networks continued. The optimal value of the "decentralization parameter" $N$ has been determined for the model described in the last report (Oct-Dec 1959, p. 19), assuming a simple formula for the cost of the routing-sorting operations. In particular, a highly centralized system proves optimal if efficiencies of scale can reduce operating cost by as much as 50 percent. A more general model was examined in which the service area is partitioned into (possibly unequal) regions by lines parallel to the sides; the optimal locations of the sorting installations for any such partition have been derived, and some progress has been made toward determining the optimal partition (for fixed $N$).
1102-40-11645/58-272  THERMODYNAMIC PROPERTIES OF REAL GASES

Origin and Sponsor:  NBS, Section 3.2
Manager:  J. P. Menard
Full task description:  Oct-Dec 1957 issue, p. 32
Status:  Inactive. For status to date, see Oct-Dec 1959 issue, p. 19.

1102-40-11645/58-304  TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES

Origin and Sponsor:  NBS, Section 3.2
Manager:  P. J. Walsh, J. D. Waggoner
Full task description:  Oct-Dec 1957 issue, p. 40
Status:  Terminated. Some code checking was carried out; then the code was given to the sponsor who will directly supervise the production runs. Production time on the 704 will be reported hereafter in the section of this Report, "Current Applications of Automatic Computer."

1102-40-11645/58-307  STUDY OF SURFACE TENSION

Origin and Sponsor:  NBS, Section 9.2
Manager:  R. J. Arms
Full task description:  Oct-Dec 1957 issue, p. 43
Status:  Completed.

1102-40-11645/58-316  INTERSECTION CAPACITY STUDY

Origin and Sponsor:  Bureau of Public Roads
Managers:  S. T. Peavy, J. M. Cameron
Full task description:  Jan-Mar 1958 issue, p. 33
Status:  Terminated, at the request of the sponsor.

1102-40-11645/58-339  COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS

Origin and Sponsor:  NBS, Section 3.4
Manager:  H. Oser
Full task description:  Jan-Mar 1958 issue, p. 38
Status:  Continued. The theoretical investigations have been completed during this period. Expansions have been found to cover the whole t-axis for stress relaxation and creep. A Fortran program has been written for a small region from $10^{-3}$ to 1 sec which had not been covered before. We distinguish now four time regions ($t$ in sec):

$$0 \leq t \leq 10^{-8},$$
$$10^{-8} \leq t \leq 10^{-4},$$
$$10^{-4} \leq t \leq 10^{2},$$
$$10^{2} \leq t \leq \infty.$$
For each of these regions expansions have been found which are numerically practical and allow one to compute stress relaxation and creep to any desired accuracy. A paper on the subject is being prepared by H. Oser together with R. S. Marvin (3.4).

1102-40-11645/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes (11.0)
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. Programming for the 704 computer is proceeding principally on that portion of the translation scheme which compares each incoming source word with the "predictions" arising from earlier words in the same sentence or from the rules of grammar. The program will consist of about 16 subroutines—one for each kind of prediction—and a main routine which accomplishes the necessary switching and bookkeeping. Each subroutine requires numerous branches corresponding to the different ways in which predictions can be satisfied. To date the main routine has been written and tested; subroutines are being coded one branch at a time, as required by sample texts. An outline has been made for the routines needed to set up predictions. Preliminary studies are under way for the treatment of clauses and phrases.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: R. J. Arms
Full task description: Apr–June 1959 issue, p. 30

3711-60-0009/58-360 DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: J. P. Menard
Full task description: Apr–June 1958 issue, p. 32
Status: Reactivated. A code is now being written for the IBM 704, which will compute

\[ D_t(c) = \frac{-1}{2t} \frac{\partial}{\partial x} \int_{c_1}^{c} (x-x)dc \]

where

\[ \bar{x} = \frac{1}{c} \int_{c_1}^{c_m} x(c)dc. \]

The fringe numbers \( x_i \) and the concentrations \( c_i, i=1,2,\ldots,m \), are input data.
1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION  
Origin and Sponsor: Naval Research Laboratory  
Manager: R. J. Arms  
Full task description: Apr-June 1958 issue, p. 33  
Status: Continued. Production runs were continued, and results were forwarded to the sponsor.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS  
Origin and Sponsor: U. S. Information Agency, Department of State  
Manager: P. J. Walsh  
Full task description: Apr-June 1958 issue, p. 35  
Status: Continued. Calculations on Table (d), (see Apr-June 1958 issue, p. 35), were carried out for parameter L/λ = 6.6(1.1)8.0. The code is being set up to calculate Table (d) for 4λ = 8.1(1.1)10.0. The antenna code was used to produce results for three different antennae, and results have been submitted to the sponsor.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES  
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.  
Manager: H. Oser  
Full task description: July-Sept 1958 issue, p. 32  
Status: Continued. Production runs for scattering diagrams were made during the whole period, and the results were turned over to the sponsor.

1102-40-11645/59-377 LOGICAL DIAGRAM REDUCTION  
Origin and Sponsor: NBS, Section 12.3  
Managers: W. G. Hall, C. Coleman (4.1)  
Full task description: Apr-June 1959 issue, p. 25  
Status: Continued. Minor revisions were made in the code to make running automatic.

1102-40-11645/59-388 HEAT PUMP CALCULATIONS  
Origin and Sponsor: NBS, Section 10.3  
Manager: R. Zucker  
Full task description: Jan-Mar 1959 issue, p. 26  
Status: Continued. Production runs were continued using the heating and cooling data.

1102-40-11645/59-389 FREQUENCY ALLOCATION  
Origin and Sponsor: Civil Aeronautics Administration  
Manager: L. S. Joel  
Full task description: Oct-Dec 1958 issue, p. 29  
Status: Continued. A demonstration run of the "one interchange" code was
made for the sponsor. The code will be used in production as soon as a complete input data file is available. The program is to be extended to compute chains of single interchanges.

Study of the mathematical analysis of the station network was continued.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II
Origin and Sponsor: NBS, Section 4.6
Manager: A. E. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. Production runs were continued under the direction of the sponsor. A new code is being written to compute the photodetachment cross section for H\(^{-}\), making use of the symmetric p-wave results.

1102-40-11645/59-407 FOURIER COEFFICIENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 27
Status: Inactive.

1102-40-11645/59-414 INFINITE SYSTEMS
Origin and Sponsor: NBS, Division 3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 28

1102-40-11645/59-415 COMPLEX LEGENDRE FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 29
Status: Terminated. Additional production runs were made on the 704. Future production runs will be made under the direction of the sponsor, and machine time will be reported in the section of this report entitled, "Current Applications of Automatic Computer."

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. Exploratory experiments in the evaluation of electrocardiographic readings are being conducted. Various computational procedures are being applied to medical data to see if the results can be
used to reveal a meaningful medical interpretation. One experiment using Fourier analysis and the least squares method was carried out, but the results were unsuccessful. A revised procedure is being devised. An averaging routine has been checked out.

1102-40-11645/59-444  HOSPITAL SUBSISTENCE ITEMS SUPPLY

Origin and Sponsor: Veterans Administration
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. A paper by G. Dantzig and P. Wolfe of the RAND Corporation, "Decomposition Principles for Linear Programming," describes a method that may be used for solution of multi-stage transportation problems. Preliminary study indicates that it may have application to the supplier-depot-consumer structure of VA's subsistence system.

1102-40-11645/59-445  OIL SUPPLY

Origin and Sponsor: Military Petroleum Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Continued. The Bureau has obtained a binary deck of the MIT transportation program and plans to test it in solving a 270x250 petroleum allocation schedule. Since the MIT program is not at present in SHARE, the significant operating differences between it and NYTRl (the SHARE identification for the program customarily used to solve transportation problems) are noted:

1) In data preparation, NYTRl requires all \( c_{ij} \) to be listed explicitly, whereas the MIT program permits the use of lexicographic indices for listing only the nonzero elements of sparse matrices. The program generates the computational \( c_{ij} \) matrix. (DTMB has written a set of data preparation programs for use with NYTRl, but they apparently contain residual errors after debugging. These codes were produced ad hoc for a specific problem and were "discarded" after use; so they were not edited for submission to SHARE.)

2) Intermediate feasible solutions may be saved and printed out with NYTRl, but not with the MIT code. Thus with NYTRl it is sometimes possible to make changes in the \( c_{ij} \) and to start with a "good" feasible solution obtained from a previous computation. This can save many iterations.

3) To solve disparate problems consecutively with NYTRl, the program must be read into the computer for each run. The MIT program permits the use of a consolidated data deck to solve several problems consecutively.

4) NYTRl can print alternate optima if they exist; the MIT code cannot.

5) Both programs use fixed point arithmetic. To use the MIT code, \( a_i, b_j, c_{ij} \) must be less than 131,071 \((2^{17}-1)\); for NYTRl they may be as large as \( 10^{10}-1 \).
Further evaluation of these two programs is planned: a set of problems will be run with both programs to test under what conditions NYTRl is faster than the MIT code. Whenever a symbolic listing of the MIT program can be obtained, it will be studied to determine if it can be modified easily to obtain the output advantages of NYTRl while retaining the convenience of the MIT data input.

In addition, a new method for solving the transportation problem, not based on the simplex method, that has been proposed by E. M. Beale, is being studied for computer application.

1102-40-11645/60-452 CONSULTING SERVICES FOR PACIFIC MISSILE RANGE
Origin and Sponsor: NBS, Section 12.3
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 29
Status: Continued. For further background, a visit was made by D. Mittleman and G. Reitwiesner to the Ballistic Research Laboratories at Aberdeen Proving Ground, Md.

1102-40-11645/60-453 DATA CONVERSION
Origin and Sponsor: Army Map Service
Manager: J. M. Beiman
Full task description: July-Sept 1959 issue, p. 30
Status: Continued. Production runs were continued as requested by the sponsor.

1102-40-11645/60-456 FIRING CIRCUIT EQUATIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 30
Status: Completed. The results were transmitted to the sponsor (see July-Sept 1959 issue, p. 30).

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. M. Beiman, W. G. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. Production runs are being made to check and process submitted data. A program is being written to edit the data processed by the above runs.

1102-40-11645/60-459 TRANSCENDENTAL EQUATIONS
Origin and Sponsor: NBS, Section 9.1
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. Production runs were made for 29 values of P for values of u=0(.05).5, and the results were submitted to the sponsor.
1102-40-11645/60-460 DESIGN EQUATIONS FOR MAGNETRON INJECTION ELECTRON GUNS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. E. Beam
Full task description: July-Sept 1959 issue, p. 32
Status: Inactive. For status to date, see Oct-Dec 1959 issue, p. 32.

1102-40-11645/60-461 DATA SYSTEMS LANGUAGES

Origin and Sponsor: NBS, Section 11.2
Manager: J. H. Wegstein
Full task description: July-Sept 1959 issue, p. 33
Status: Completed. Further editing of the proposal for a common business oriented language (COBOL) was completed by the Short-Range Data Systems Language Committee. With the completion of this task Mr. Wegstein resigned as chairman of the committee.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Continued. The computation was extended to include auxiliary evaluations requested by the sponsor.

1102-40-11645/60-464 CORRELATION MATRIX FOR PHS DATA

Origin and Sponsor: Public Health Service
Manager: J. M. Beiman
Full task description: July-Sept 1959 issue, p. 34

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS

Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1959 issue, p. 26
Status: Continued. Some production runs were made using the Central, A-Matrix and SCF programs which were written under task 3911-61-39952/57-236, Self-Consistent Fields (see p. 19).

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. Some experimental cases have been run to compare results with previous calculations and to gain experience in data preparation and set-ups required by the programs.
1102-40-11645/60-467  TRANSISTOR SIMULATION
Origin and Sponsor:  NBS, Section 12.1
Manager:  G. W. Reitwiesner
Full task description:  Oct-Dec 1959 issue, p. 27
Status:  Continued. A preliminary program is in the checkout stage, and changes in formulation are being accommodated.

1102-40-11645/60-471  INTERLABORATORY STANDARDIZATION OF TESTING PROCEDURES
Origin and Sponsor:  NBS, Section 7.3
Manager:  A. E. Beam
Full task description:  Oct-Dec 1959 issue, p. 28
Status:  Continued. The code was written and checked out, and some production runs were made under the direction of the sponsor.

1102-40-11645/60-475  IONOSPHERIC SOUNDINGS
Origin and Sponsor:  NBS, Section 82.40
Manager:  M. L. Paulsen
Full task description:  Oct-Dec 1959 issue, p. 29
Status:  Continued. Approximately 40,000 virtual height (h') data cards were processed using this program for its first production run. It was found that the transcription process from card to tape introduced illegal characters. Investigation revealed that the cards were put on tape using an 80x80 board instead of an 80x84 board which fills in or completes the 14th word (this meant the information in columns 79 and 80 was being ignored, because a partial word is treated as an E.O.R.). S. Peavy and R. Varner examined the input routine in the Bell system and made a correction enabling the routine to read in the partial word, thus eliminating the necessity of putting the 40,000 cards on tape again. The requested results were sent to N.B.S. Boulder on magnetic tape during the first week in March. (The amount of output is a little more than the input.) Another 20,000 cards were received to be put on tape.

1102-40-11645/60-476  GAS TUBE CHARACTERISTICS, II
Origin and Sponsor:  Diamond Ordnance Fuze Laboratories, Department of the Army
Manager:  H. Oser
Full task description:  Oct-Dec 1959 issue, p. 30
Status:  Continued. A program has been written to integrate simultaneously the two differential equations for field strength and negative ion density. The speed of convergence has been increased considerably as compared with that of earlier programs. Between 6 and 10 iterations are now required to establish a solution with 4 decimal place accuracy, and computing time has been reduced by 75 percent. The code has been checked out and will be turned over to the sponsor, who will conduct production runs directly. A study is now under way to obtain gas tube characteristics for very high total current densities.
1102-12-11122/60-479 PROCESSING OF DIAGRAMS
Origin and Sponsor: NBS, Section 11.0
Managers: F. L. Alt, S. T. Peavy, R. J. Herbold
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Two programs have been written for the 704, one to process diagrams made on another computer, and the other to analyze the diagrams. The codes are in the process of being checked.

1102-40-11645/60-480 LARGE SIGNAL CALCULATIONS FOR A VOLTAGE TUNEABLE MAGNETRON
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. E. Beam
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. The code was written and checked out, and production runs were started.

1102-40-11645/60-484 POLYMER CRYSTALLIZATION
Origin and Sponsor: NBS, Section 7.6
Manager: H. Oser
Objective: To integrate a fourth order nonlinear differential equation of the following type:
\[ \lambda^{(4)} + Q(\lambda)\lambda'' + R(\lambda)(\lambda'')^2 + S(\lambda)\lambda''(\lambda')^2 + T(\lambda)(\lambda')^4 = U(\lambda). \]

Background: The above equation arose in the course of original research being conducted by F. Gornick (7.6), concerned with an extension of Mandelkern's research in the kinetics of crystallization of polymer systems (see Society of Plastics Engineers Journal, vol. 15, No. 1, Jan. 1959). If one assumes a law \( U(\lambda) \) describing the rate of nucleation in the liquid phase and allows for impingement of the growing nuclei, one arrives at a differential equation of this type. \( \lambda \) is the ratio of the weights of crystallized polymer at time \( t \) and time \( \infty \); \( Q, R, S, T \) are rather complicated functions of \( \lambda \) that involve \( t \) only implicitly.
Status: New. A Fortran program has been written for the integration and has been checked out completely. The results are extremely stable with respect to the choice of \( h \), the step length of the integration method. Some production runs have been made.

1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: R. Zucker
Objective: To compute the Franck-Condon factors
\[ q_{v',v''} = \left| \int_0^\infty \psi_{v'} \psi_{v''} \, dr \right|^2 \]
where \( \psi_{v'} \) and \( \psi_{v''} \) are the Morse wave functions for two different electronic states at various quantum numbers. The vibrational wave function for the
lowest level \((v = 0)\) is
\[
|\psi_0(r)| = \sqrt{\frac{a}{r(K-1)}} \exp\left\{- \frac{K}{2} \exp[-a(r-r_e)] \right\} \left\{ K \exp[-a(r-r_e)] \right\}^{K-1} 2
\]

where \(r\) denotes the internuclear separation, and \(a\) and \(K\) are functions of the velocity of light and the dissociation energy of the molecule.

Wave functions of higher order involve Associated Laguerre functions \(L_{K-2v-1}^{2v}(z), z = K \exp[-a(r-r_e)]\). Since there is an increasing loss of significant figures in obtaining these functions by recurrence relations, double precision arithmetic is to be used. The wave functions are to be normalized such that
\[
\int_0^\infty |\psi_v|^2 \, dr = 1.
\]
The following check is to be applied:
\[
\sum_v q_v'q_v'' = \sum_v q_v'' = 1.
\]

**Background:** This is part of a study of the vibrational transition probability arrays for molecular band systems of specific astrophysical interest. The bands usually include the vertex of the Condon parabola of the system concerned. For the theory, see (1) W. R. Jarmain and R. W. Nicholls, Vibrational Transition Probabilities to High Quantum Numbers for the Nitrogen First and Second Positive Band Systems, Can. J. Phys. 32, 201-204, 1954; and (2) R. W. Nicholls, Ann. Geophysique 14, 208-224, 1958.

The problem was submitted by R. W. Nicholls (3.0).

**Status:** New. The code has been written and checked out. Very good agreement was obtained with previous computations for the vibrational transition probabilities for the first positive \((B^3\Sigma_g^+ \rightarrow A^3\Sigma_u^+)\) band system of nitrogen.

1102-40-11645/60-489 INVERSION OF LINE PROBE DATA

**Origin and Sponsor:** NBS, Section 3.1

**Manager:** R. Herbold

**Objective:** To evaluate \(f(r)\) in the expression
\[
f(r) = \frac{1}{\pi} \int_0^\infty \frac{Q'(x) dx}{(x^2 - r^2)^{1/2}}
\]
for given discrete values of \(Q(x)\).

**Background:** This integral arises in calculations connected with radial distributions of physical properties of axially symmetric inhomogenous plasmas. Using an electric arc through a hot gas (the plasma) to create a cylindrical source, measurements \((Q(x))\) are made of the intensity of the light as a function of the vertical distance. The measurements are then used to obtain the radial distribution of the light intensity within the cylindrical source. The above integral, involving \(Q'(x)\), is the solution of Abel's integral equation expressing the geometry of the setup (see Courant and Hilbert, Vol. I, First English ed. 1953, p. 158).

**Status:** New. A working code was written and checked out. To improve and generalize the procedure, however, the code is being revised.
1102-40-11645/60-500  DENTAL RESEARCH COLORIMETRY

Origin and Sponsor:  NBS, Section 7.8

Manager:  J. P. Menard

Objective:  To compute the CIE tristimulus values and chromaticity coordinates of materials submerged in water.

Background:  Existing codes for computing CIE tristimulus values and chromaticity coordinates do not compensate for measurements of samples submerged in water.  It seems to be more practical to write a special code for this purpose than to adapt one of the existing codes (see tasks 1102-40-11645/60-449, Spectral Line Colorimetry, and 3711-60-0009/57-250, Automatic Reduction of Spectrophotometric Data).

The problem was communicated by P. Oglesby (7.8).

Status:  New.

1102-40-11645/60-504  ELECTROSTATIC-FOCUSING PROBLEM

Origin and Sponsor:  Diamond Ordnance Fuze Laboratories, Department of the Army

Manager:  A. Beam

Objective:  To solve the differential equation

$$2VY'' + V'Y' + V''Y = A\sqrt{V}$$

where

$$V = 1 - B \sum_{N=1,3,5,7,...}^{\infty} \left[ (-1)^{(N-1)/2} \frac{\sin CN}{\cosh DN} \cos 2\pi NL \right].$$

The initial conditions given for L = 0 are:

$$Y(L) = 1, \ Y'(1) = \alpha.$$  

A, B, C, D are functions of input parameters.

Background:  It is of interest to study the motion of an electron sheet beam traveling between a set of electrostatic focusing plates where two different potentials are applied to alternating pairs of plates.  This motion will be studied on the IBM 704 computer by solving the paraxial-ray equation [Eq. (1)].  The potential function [Eq. (2)] to be used in the paraxial-ray equation was obtained from an approximate solution of the Laplace equation for the focusing structure.  It will be the purpose of this study to determine the amplitude and wavelength of the ripples present in the beam as a function of beam current and of the dimensions and voltages of the electrostatic focusing structure.  The paraxial-ray equation for this problem is of the form of an inhomogeneous Hill's equation; for certain values of the parameters the complimentary solution will become unstable.  An important part of the study will be a search for the critical values of the parameters for which instability occurs.

The problem was submitted by B. J. Udelson (DOFL).

Status:  New.  Coding of the problem was started.
6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS
RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Precision and accuracy of a standard cylinder. A study of the precision and accuracy of volume and density measurements on a standard cylinder was carried out by J. M. Cameron for C. T. Collett of the Capacity, Density and Fluid Meters Section.

(2) Analysis of data on flame spread on paint coated assemblies. A statistical analysis of the flame spread index for a number of paint coatings used on different base materials was carried out by H. H. Ku for D. Gross of the Fire Protection Section.

(3) Analysis of data on viscosity of water. Upon request of R. Marvin and R. C. Hardy of the Rheology Section the results of seven series of experiments on the viscosity of water were analysed by H. H. Ku. Several different parametric representations of the data were evaluated.

(4) Study of kaolin. An orthogonal design for estimating the effect of time type particle size and measuring equipment on the properties of kaolins was worked out for W. C. Ormsby of the Engineering Ceramics Section by W. J. Youden.

(5) Accuracy of cement tests. Dr. Youden participated in the planning phases and devised a scoring scheme for a long range program to determine the limits of accuracy of cement tests carried out at the state cement testing laboratories. This work was done for the Concreting Materials Section.

(6) Computations were carried out on the Bureau's electronic computer for M. J. Kerper (9.02), F. M. Reinhart (8.04), E. D. Tidwell (4.02), A. G. Strang (2.05), E. K. Plyler (4.02).

Publications:


(3) Variability of color-mixture data. I. Nimeroff (NBS Photometry and Colorimetry Section), J. Rosenblatt and M. C. Dannemiller. In manuscript.

Status of Projects


STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1
Origin and Sponsor: Ship Structure Committee, NRC Authorised 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93
Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. The complete set of tables for the Appendix, and some of the text has been turned over to the OOR contractor for preparation for final publication.

Publications:

STATISTICAL SERVICES
Task 1103-40-11625/58-346
Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) U. S. Geological Survey: Investigations were continued by J. R. Rosenblatt on several mathematical models associated with measurement processes in hydrology.

(2) Veterans' Administration Hospital, Perry Point, Md.: A statistical analysis of data from NP research studies was carried out on the Bureau's electronic computer by M. C. Dannemiller.
The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

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## Current Applications of Automatic Computer

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<tr>
<td>/60-476 DOFL</td>
<td>Gas tube characteristic II</td>
<td>9</td>
<td>335</td>
<td>106</td>
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<td>/60-480 DOFL</td>
<td>Large signal calculations for a voltage-tuneable magnetron</td>
<td>27</td>
<td>33</td>
<td>561</td>
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<td>/60-482 UMd</td>
<td>Fluid dynamics**</td>
<td>108</td>
<td>14</td>
<td>59</td>
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<td>/60-485 DOFL</td>
<td>Constant pressure data**</td>
<td>65</td>
<td>13</td>
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<tr>
<td>/60-488 NRL</td>
<td>Spectrum analysis**</td>
<td>4</td>
<td>7</td>
<td>3</td>
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<tr>
<td>/60-492 IMF</td>
<td>Monetary research reports**</td>
<td>88</td>
<td>33</td>
<td>7</td>
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<tr>
<td>/60-496 BPA</td>
<td>Short circuit program**</td>
<td>372</td>
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<td>/60-501 CEng</td>
<td>Kansas River Reservoir System**</td>
<td>41</td>
<td>7</td>
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<tr>
<td>/60-502 USA</td>
<td>Quartermaster Mathematics programming**</td>
<td></td>
<td></td>
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<td>Totals (Outside)</td>
<td></td>
<td>3,125</td>
<td>4,679</td>
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</table>

Total time for the quarter (MINUTES)..... 5,579 8,038 69,423

Total time for the quarter (HOURS) ..... 93 134 1,158

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.
** Problem programmed by sponsor and run under his direction.
° Classified task.
Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Colloquium

SAUER, R. (Mathematisches Institut der Technischen Hochschule, Munich)

Mathematical Statistics Seminar

SCALORA, Frank S. (International Business Machines Corp., New York City)

Statistical Engineering Laboratory Mathematical Seminars

BOSE, R. C. (Case Institute of Technology, Cleveland)
On powers of matrices with non-negative elements. March 29.

Statistical Engineering Laboratory Reliability Seminars

ROSENBLATT, J. R. Statistical models for component aging experiments. March 16.
WEISS, G. (Institute of Fluid Dynamics, University of Maryland)
Semi-Markov processes and applications to reliability. March 30.

Papers and Invited Talks
Presented by Members of the Staff
at Meetings of Outside Organizations

CANNON, E. W. The reflection of Logistics in electronic computer design.
Lectures and Technical Meetings

EISENHART, C. Some statistical aspects of measurement with a linear scale. Presented at a Graduate Colloquium, Statistics Department, Virginia Polytechnic Institute, Blacksburg, Va., February 26.


NEWMAN, M. Number theory and computers. Presented at the U. S. Naval Academy, Annapolis, Md., January 8.


(7) Interlaboratory comparison problems. Presented at a Seminar of the American Society for Quality Control, New York City, February 27.
(8) The sample, the procedure, and the laboratory. Presented before the Analytical Chemistry Group of the American Chemical Society, and Spectroscopy Society, Pittsburgh, Penn., March 1.

ZELLEN, M. (1) Are life testing procedures robust? Presented before the Committee on Statistics, University of Wisconsin, Madison, Wis., January 7; and at the Sixth National Symposium on Reliability and Quality Control in Electronics, Washington, D.C., January 12.
(2) Factorial experiments in life testing. Presented at a Seminar of the Department of Mathematics, University of Miami, Coral Gables, Fla., January 14.
Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Technical Notes, Manuals, and Bibliographies


1.3 Technical Papers

The following papers appeared in J. Research NBS 64B, Jan-Mar 1960 (Mathematics and Mathematical Physics):


* * * * *

The following paper appeared in J. Research NBS 64C, Jan-Mar 1960 (Engineering and Instrumentation):


* * * * *


2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Technical Notes, Manuals, and Bibliographies


2.3 Technical Papers


(16) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.


(23) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.


(29) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Submitted to a technical journal.


(34) How to evaluate accuracy. W. J. Youden. Submitted to a technical journal.


(38) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. Submitted to a technical journal.


THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.


• Office of Basic Instrumentation.

• Office of Weights and Measures.

BOULDER, COLORADO


