NATIONAL BUREAU OF STANDARDS REPORT

6847

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

January through March 1960

For Official Distribution

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT	NBS REPORT
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January 1 through March 31, 1960

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*On leave of absence **Part time

Contents

*Only unclassified projects are included in this report.

Status of Projects

March 31, 1960

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth continued work on partitioned matrices and on bounds for the P-condition number of matrices. The paper "A reduction formula for partitioned matrices" was revised and submitted to the NBS Journal of Research. Another paper "Special types of partitioned matrices" was completed, ready for submittal to a technical journal. Two additional papers, "Bounds for the P-condition number of matrices with positive roots" and "Criteria for the reality of matrix eigenvalues" (with M. Drazin) are in manuscript form.

H. F. Weinberger derived the following results on an error bound in the Rayleigh-Ritz approximation of eigenvectors. Let A be a linear hermitian operator with eigenvalues $\lambda_1 \leq \lambda_2 \leq \ldots$ and corresponding normalized eigenvectors $u_1, u_2, \ldots$. Upper bounds $k_1 \leq k_2 \leq \ldots \leq k_M$ for the first M eigenvalues are determined by the Rayleigh-Ritz method. Let $w_1, \ldots, w_M$ be the corresponding vectors, so that

$$(w_i, w_j) = \delta_{ij},$$

$$(Aw_i, w_j) = k_i \delta_{ij}.$$

Let lower bounds $\bar{\lambda}_1 \leq \bar{\lambda}_2 \leq \ldots \leq \bar{\lambda}_N$ for the first $N \geq M$ eigenvalues λ_i be known. A bound for the error in norm $(u_p - w_p, u_p - w_p)$ in the approximation of w_p to the pth eigenvector u_p is found in terms of the $\bar{\lambda}_i$ and k_i alone. This bound is sharp in the sense that there is an operator A for which equality holds.

N. Bazley completed the investigation of Monte Carlo methods for computing Markov chains by a study of the relationship of the nature of the chain to its eigenvalues with magnitude one. He also worked on the calculation of upper and lower bounds to the eigenvalues of the associated Legendre equation

$$[(x^2-1)y']' + \frac{m^2}{1-x^2} y - \lambda y = 0, \quad y \text{ and } y' \text{ finite at } \pm 1,$$

for non-integral vales of m. It is of interest to compare for integral m the results of the approximate methods with the known solutions.

J. R. Rice pursued the following tasks during this quarter:

(a) The manuscript, "Sequence transformations based on Tchebycheff approximations", was completed. This paper deals with convergence acceleration procedures for sequences whose nth term is

$$\sum_{i=1}^{k} a_i b_i^n + c \quad \text{or} \quad \sum_{i=1}^{k} a_i b_i^n \cos(\theta_i + n\phi_i) + c.$$

It is also shown that the transformations derived are effective for certain other types of sequences.

(b) The IBM 704 program to test a simple method of obtaining rational function approximations was checked out and examples are being run.

(c) Investigations were continued on the feasibility of an "optimized-split" integration method for simultaneous differential equations. The two IBM 704 codes were checked out, which will integrate two particular differential equations, each with different time steps that have been "optimized" so as to use a minimum of integration steps for a given interval. A series of tests were made and the following conclusions were reached: (i) The method is useful in certain situations; (ii) The usefulness depends critically on the nature of the particular equations being solved; (iii) The optimization procedure should be modified for each particular set of equations if maximum benefit is to be obtained.

(d) Work was continued on the numerical solution of the self-acting gas lubricated journal bearing:

$$\frac{\partial}{\partial x}\left[h(x,y)\frac{\partial q}{\partial x}\right] + \frac{\partial}{\partial y}\left[h(x,y)\frac{\partial q}{\partial y}\right] = -\frac{\partial[h(x,y)\sqrt{q}]}{\partial x} .$$

h(x,y) is a given function and q(x,y) is to be determined. At present the machine solutions are divergent. The reason for this is undetermined although the analysis and programming have been thoroughly checked.

P. J. Walsh (11.2) wrote a code to evaluate methods of numerically satisfying an internal boundary condition involving a normal derivative. This problem comes from the theory of the infinite step bearing. The code is now being checked out.

(e) A long analysis of the problem of approximation by convex and monotonic polynomials was almost completed. Many results were obtained for the general problem of approximating a function in a monotonic norm by a linear combination of functions whose coefficients satisfy a "convex constraint." Algorithms for obtaining both least squares and Tchebycheff approximations by convex and monotonic polynomials are being developed.

(f) It is well known that the best approximating (in a monotonic norm) nth degree polynomials to a function f(x) on [0,1] will interpolate the values of f(x) in at least n+1 points. This interpolation phenomenon is being studied for a broad class of approximation problems. Theorems of the following types have been established: Theorem 1. Let F(A,x) be a regular unisolvent function of variable degree and let f(x) be a given continuous function. If F(A*,x) is the best approximation (in the monotonic norm δ) to f(x) and F(A*,x)-f(x) has a finite number of zeros, then F(A*,x)-f(x) has at least k simple zeros where k is the degree of unisolvence of F at A*. Theorem 2. Let F(A,x),f(x) be as given above.

Assume that the monotonic norm δ satisfies "condition B" (a new concept in this study). If $F(A^*,x)$ is the best δ-approximation to $f(x)$, then $F(A^*,x)-f(x)$ has at least k strong sign changes where k is the degree of F at A^*.

Regularity and condition B are concepts developed in this study. An effort is now being made to establish the necessity of regularity and condition B for Theorems 1 and 2. An example is given of a non-regular unisolvent function of three parameters such that the best least squares approximation to $(x-\frac{1}{2})$ is identically zero.

(g) The paper "Tchebycheff approximations by ab^x+c and $ab^x\cos(\theta_o+\theta x)+c$ has been separated into two papers upon the advice of the referee. One of them, entitled "Tchebycheff approximation by ab^x+c" has been accepted by the Journal of the Society for Industrial and Applied Mathematics. The second paper has not yet been submitted.

Publications:

(1) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(2) Tchebycheff approximations by ab^x+c. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.
(3) Split Runge-Kutta for simultaneous equations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(4) A new representation of Gegenbauer's functions. J. R. Rice. Submitted to a technical journal.
(5) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.
(6) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Submitted to a technical journal.
(7) A reduction formula for partitioned matrices. E. Haynsworth. To appear in the Journal of Research, NBS, Sec. B.
(8) Special types of partitioned matrices. E. Haynsworth. In manuscript.
(9) Bounds for determinants with positive diagonals. E. Haynsworth. To appear in the Proceedings of the American Mathematical Society.
(10) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.
(11) Criteria for the reality of matrix eigenvalues. E. Haynsworth and M. Drazin (RIAS). In manuscript.
(12) Bounds for the P-condition number of matrices with positive roots. E. Haynsworth. In manuscript.
(13) A metrization for power sets with applications to combinatorial analysis. R. Silverman. Canadian J. Math. 12, 158-176 (1960).

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman has completed a study of periodicity of sequences of integers modulo a non-zero integer, and has obtained many theorems concerned with divisibility properties of partitions. An example is the fact that the unrestricted partition function fills all residue classes infinitely often modulo 2, 5, 13.

Dr. Newman also began work on certain diophantine problems. It was shown, for example, that $1 + \binom{n}{1}$, $1 + \binom{n}{1} + \binom{n}{2}$ are simultaneously squares if and only if $n = 15$.

K. Goldberg and E. C. Dade have extended their manuscript on the abstract properties of incidence algebras. A paper on the applications to v, k, λ designs, through group-generated incidence spaces, is in manuscript.

K. Goldberg has shown that if $\{p_n(t)\}$ is a set of Faber polynomials of a function $f(z)$ with Laurent expansion at $z = 1$ and if there exist constants a, b such that $\sum_{n=1}^{\infty}(p_n(a)-b)$ converges, then b is an integer. If b is positive it is the multiplicity of the root $z = 1$ of $f(z)-f(1)$, and $a = 1/f(1)$. If b is negative then $-b$ is the multiplicity of the pole $z = 1$ of $f(z)$, and $a = 0$. For an example see the task on Information Selection Systems (p. 5), in which $f(z) = ze^{-z}$, $a = e$, $b = 2$.

Publications:

(1) The minimum of a certain linear form. K. Goldberg. J. Research NBS, 64B, 49-50 (1960).
(2) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
(3) Weighted restricted partitions. M. Newman. Acta Arith. 5, 371-380 (1959).
(4) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
(5) Irrational power series. M. Newman. In manuscript.
(6) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
(7) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
(8) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.
(9) Kantorovich's inequality. M. Newman. J. Research NBS, 64B, 33-34 (1960).

INFORMATION SELECTION SYSTEMS
Task 1101-12-11412/60-470

Origin: NBS Authorized 9/25/59
Sponsor: National Science Foundation
Managers: K. Goldberg, A. J. Goldman

Status: CONTINUED. K. Goldberg continued his investigations of the problem of classification and of the expected lengths of chains in an infinite flow of data through a computer with a finite memory. He has shown that the expected length of the nth increasing sequence in a random infinite sequence of numbers is $p_n(e)$, a value very close to 2, where $\{p_n(t)\}$ is the set of Faber polynomials of ze^{-z}.

A. J. Goldman began work on coding the boolean simplification program for the IBM 704. Investigation continued on the effects of permuting the individual steps in Phase III of the program. An analysis was started of the maximum number of basic k-cells in an n-variable Boolean function having no cell of dimension k+1.

Publications:

(1) A symmetric continuous poker model. A. J. Goldman and J. J. Stone (Stanford University). J. Research NBS, 64B, 35-40 (1960).
(2) A continuous poker game. A. J. Goldman and J. J. Stone (Stanford University). Duke Math. J. 27, 41-54 (1960).
(3) Computer simplification of boolean functions. B. K. Bender (11.2), A. J. Goldman, and R. B. Thomas (12.5). Submitted to a technical journal.
(4) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.
(5) Some results on boolean functions. B. K. Bender (11.2) and A. J. Goldman. In manuscript.
(6) Optimization models for distribution networks. B. K. Bender (11.2) and A. J. Goldman. In manuscript.

ORTHOGONAL FUNCTIONS IN THE THEORY OF PARTIAL DIFFERENTIAL EQUATIONS
Task 1101-12-11413/60-469

Origin: NBS Authorized 9/25/59
Sponsor: Atomic Energy Commission
Managers: P. Davis, P. Rabinowitz

Status: CONTINUED. The results of studies during this quarter pursued by P. Rabinowitz include the following:

(a) The Dirichlet problem was solved for various regions including two concentric ellipses, a circle with two holes punched out, and a square with corners removed. The dependence on the boundary functions of the convergence of the method of orthogonal functions for solving the Dirichlet problem was investigated on an ellipse and on a bean-shaped region.

(b) A routine was written for the solution of boundary value problems of the second and third kind for simply-connected planar regions.

(c) A routine was written for the solution of the biharmonic equation $\triangle\triangle u = 0$ for simply connected planar regions in which the function and its normal derivative are specified on the boundary. This routine was then used to solve such a problem on an ellipse.

(d) The transfinite diameter of two and three collinear segments was studied numerically by means of the orthonormalization routine. It was found that even double precision arithmetic was not good when the powers of x were orthonormalized. On the other hand, the 3-term recursion formula for orthogonal polynomials gave good results, even in single precision.

P. Davis and P. Rabinowitz have completed a manuscript entitled, "Some geometrical theorems for abscissas and weights of Gauss type." They also have in progress a manuscript entitled, "Advances in orthonormalizing computation," which, among other things, summarizes recent numerical experience.

Publications:

(1) Some geometrical theorems for abscissas and weights of Gauss type. P. Davis and P. Rabinowitz. In manuscript.
(2) Advances in orthonormalizing computation. P. Davis and P. Rabinowitz. In manuscript.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin:and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: P. Rabinowitz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. Additional runs on the 704 were made during this quarter as requested by the sponsor, still continuing for the slightly modified form of the one-dimensional case.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-11112/47-2

Origin: NBS — Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-11112/51-8

Origin: Section 11.3, NBS — Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-11112/52-7

Origin: NBS — Authorized 8/10/51
Manager: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-11112/52-37

Origin: NBS — Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-11112/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. Graphs and tabular material are being prepared for Chapter 8: Legendre Functions, Chapter 18: Weierstrass Elliptic Functions, and Chapter 20: Mathieu Functions. Textual material for these chapters is being revised. All chapters are undergoing review for consistency of notation and format.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. In January, J. H. Wegstein attended an international conference in Paris for the purpose of re-writing the international algorithmic language (ALGOL) proposal. The proposal prepared by this conference is called ALGOL 60. If this proposed language gains acceptance, it can be used for communicating scientific computer programs between people on an international level as well as for communicating directly with computers.

A group consisting of A. Beam, G. Galler, S. Peavy, P. Rabinowitz, G. Reitwiesner, G. Urban, J. Wegstein, and W. W. Youden began part-time work on a long-range project aimed at designing a generator for the purpose of generating data processing type computer programs. The group has begun by examining the features which are common to compilers, language translators, special data processing codes, and the intended generator itself. The characteristics sought for the generator are: an efficient object code, independence of any particular computer, notation convenient for humans (hopefully based on the ALGOL language), and capability of bootstrapping itself into existence. The group seeks to define a number of code "packages" which would be coded by hand in the language of any chosen computer (object code). These packages when strung together would constitute the generator

desired. In addition the generator should be capable of generating an even better version of itself as well as of "bit-chasing" and data processing type codes.

Publication: Report on the Algorithmic Language ALGOL 60. J. Backus, F. Bauer, J. Green, C. Katz, J. McCarthy, P. Naur, A. Perlis, H. Rutishauser, K. Samelson, B. Vanquois, J. Wegstein, A. van Wijngaarden, M. Woodger. To appear in Numerische Mathematik, and the Communications of the Association for Computing Machinery (May issue).

MATHEMATICAL SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. A General Purpose Orthonormalization Code (BSORTH) along with a complete description for its use was completed by P. J. Walsh and Emilie Haynsworth. This program was contributed to the SHARE organization for distribution to its membership. It can execute any of the following operations:

(a) orthonormalizing a set of vectors with respect to a general inner product;

(b) least squares approximation to a given function or functions by linear combinations of powers (polynomial approximations) or any linear combination of powers, rational functions, transcendental functions and special functions, such as those defined numerically by a set of values;

(c) surface fitting of empirical data in two or more dimensions;

(d) finding the best solution in the least squares sense to a system of m linear equations in n unknowns ($n \leq m$);

(e) matrix inversion and solution of linear systems of equations;

(f) expansion of a given function or functions in a series of orthogonal functions, such as a series of Legendre or Chebyshev polynomials.

The polynomial curve fitting code, prepared by P. J. Walsh and described in the July-Sept and Oct-Dec 1959 issues, pp. 11 and 10 respectively, has been checked out and applied to several functions. Successful approximations have been obtained for degrees as high as 20 using 50 and 75 points. Slight modifications will be introduced to prepare the routine as a Fortran function subroutine to be submitted to SHARE. The printout of intermediate calculations (starting at a specified minimum degree) will be made optional. Additional information will be available, such as the Fourier coefficients, residuals, and the sum of squares of residuals.

The introduction of a 32K core memory has made the formation of a matrix base code feasible. A code has been built by P. J. Walsh and J. D. Waggoner, which contains the more frequently used matrix operations listed below. Some of these subroutines were obtained from the SHARE library; others were written by members of the staff. For short matrix problems or for problems involving small matrices (which frequently arise), only a small

SAP code needs to be assembled and added to the binary base code.

1. $A(nxm) \pm B(nxm) = C(nxm)$

2. $A(nxm) \cdot B(mxr) = C(nxr)$

3. $A(nxm) \cdot B^T(mxr) = C(nxr)$
4. $B^T(nxm) \cdot A(mxr) = C(nxr)$

} Performs matrix multiplication without transposing B.

5. $A(nxn) \cdot B(nxr) = C(nxr)$
6. $B(mxn) \cdot A(nxn) = C(mxn)$

} Where A is a diagonal matrix stored as a vector with components a_{ii}, i=1,2,...,n.

7. $A^T(mxn) = B(nxm)$

8. $A^{-1}(nxn) = B(nxn)$

9. Replace the (i,j)-element of A(mxn) by the contents of the accumulator.

10. Replace the diagonal elements of A(nxn) by the components of vector b, with option of replacing all off-diagonal elements of A by zero or leaving them unaltered.
11. Select the diagonal elements of A(nxn) to form vector b.

Some experiments are being performed in curve fitting using the orthonormalization subroutine. Two problems have been presented which read as follows:

1. Given X_i and the corresponding functional values Y_i, i=0,1,2,...,n, find the best fitting polynomial in the least squares sense subject to the following constraints: (a) that the polynomial pass through (X_o,Y_o); and (b) that the derivative at X_o is K.

2. The second problem is the same as 1. but subject to the further restriction that Y(0) = 0.

In both problems, polynomials of degrees 2 through 5 have been requested. For case 1, the function (Y-Z) will be fitted by linear combinations of $(X-X_o)^2$, $(X-X_o)^3$,..., where $Z=a+bX_o$, the straight line passing through X_n and possessing slope K. For case 2, the function (Y-Z) will be fitted by linear combinations of $X(X-X_o)^2, X(X-X_o)^3$,..., where $Z=aX_o+bX_o^2$, the parabola passing through X_o and having derivative K.

A Fortran program was written by G. W. Reitwiesner to mechanize a part of the preparation of the Computation Laboratory monthly personnel time reports.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. M. Zelen and N. C. Severo have extended their work on normal approximations to the chi-square distribution to include approximations for small degrees of freedom.

C. Eisenhart and K. N. Dietmeier studied the extent to which the level of confidence would be lowered if, when σ is known, one always chose the narrower of the normal and the studentized confidence intervals for the population mean.

Publications:

(1) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.
(2) Selected bibliography of statistical literature, 1930-57. I. Correlation and regression theory. Lola S. Deming. NBS J. Research 64B, 55-68 (1960).
(3) Selected bibliography of statistical literature, 1930-57. II. Time series. Lola S. Deming. NBS J. Research 64B, 69-76 (1960).
(4) Selected bibliography of statistical literature, 1930-57. III. Limit theorems. Lola S. Deming. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(5) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(6) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52
Manager: J. M. Cameron
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. The catalog of fractional factorial designs for the 2^m3^n series prepared under Bureau of Ships sponsorship (see July-Sept 1959 issue, p. 14) is now being revised for publication in the light of comments received from a number of specialists in the theory and practice of experiment design.

Publications:

(1) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(2) Analysis of fractionally replicated 2^m3^n designs. R. C. Bose and W. S. Connor. To appear in "Proceedings of the 31st Session of the International Statistical Institute," Brussels, 1958.

(3) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to Probability and Statistics," Stanford University Press (1960).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

Publications:

(1) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to Probability and Statistics," Stanford University Press (1960).

(2) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. Submitted to a technical journal.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt completed a paper on "Statistical Models for Component Aging Experiments." A general framework is proposed for probabilistic models representing the results of experiments in which (say) electronic components are exposed to controlled "aging" conditions and measured periodically until "failure".

M. Zelen has found the exact distribution of the sum of

$$Y = \sum_{1}^{n} x_i$$

of random variables from a Weibull population having the cumulative distribution

$$F(x) = 1 - e^{-x^p/\theta},$$

namely,

$$\Pr(Y \leq y) = \left\{ \frac{py^p \Gamma(p)}{\theta} \right\}^M \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{y^p}{\theta} \right)^k \frac{E[(\Sigma W_i)^k]}{\Gamma(pk+pn+1)}$$

where the (W_i) are independent random variables each equal to the pth power of a gamma variable with parameter p.

M. Zelen and M. C. Dannemiller are preparing a paper summarizing their work on the robustness of statistical life testing procedures. They have found that many of the current life testing procedures are very sensitive to the assumption that failure times follow the exponential distribution. Incident to their studies on robustness, they have developed excellent approximations (i) to the distribution of a sum of Weibull random variables, and (ii) to the O.C. curve and average sample number when sequential tests are made on sums of Weibull random variables. These methods are based on a general life probability density function

$$p(x) = \frac{e^{-x} x^a}{\Gamma(a+1)} \sum_{s=0}^{k} a_s L_s^a(x)$$

where $L_s^a(x)$ are Laguerre polynomials.

Publications:

(1) Analysis of two-factor classifications with respect to life tests. M. Zelen. To appear in "Contributions to Probability and Statistics," in press, Stanford University Press (1960).

(2) Are life testing procedures robust? M. Zelen and M. C. Dannemiller. Appeared in "Proceedings of the Sixth National Symposium on Reliability and Quality Control" (1960), pp. 185-189.

(3) Statistical models for component aging experiments. Joan R. Rosenblatt. To appear in the "Institute of Radio Engineers National Convention Record."

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: E. W. Cannon
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. The most general solution of one-dimensional Brownian motion, which is governed by Chapman-Kolmogoroff functional equation

$$(1) \qquad f(x,s;y,t) = \int_V f(x,s;z,u)f(z,u;y,t)dz, \quad s < u < t,$$

was given by A. Ghaffari (Bull. Amer. Math. Soc. 57, 1951) in the form

$$(2) \qquad f(x,s;y,t) = \sum_{n=0}^{\infty} \theta^n(s,t)\varphi_n(x)\varphi_n(y).$$

V denotes the interval $(-\infty,\infty)$, and the function f, being Lebesgue-measurable in V, is the transition probability that the Brownian particle (assumed spherical) passes from state x at the instant s to any one of the states of a set V at the instant t. The infinite sequence of parabolic cylinder functions

$$\varphi_n(x) = \frac{e^{-x^2/2}H_n(x)}{\sqrt{2^n n!\sqrt{\pi}}}$$

form a complete orthonormal set over $(-\infty,\infty)$, and

$$\theta(s,t) = \frac{a(s)}{a(t)}$$

where a(s) is a positive increasing function $\neq 0$, and $H_n(x)$ is the nth Hermite polynomial. It was proved that, for s and t fixed such that $s < t$ and x,y varying arbitrarily over $(-\infty,\infty)$, the series solution (2) is absolutely and uniformly convergent. It was shown also that the series solution (2) is doubly square integrable with respect to Lebesgue measure. A. Kolmogoroff obtained (Math. Ann. 104, 1931) a class of particular solutions of equation (1) and showed that his solutions verify two linear partial differential equations of the second order and parabolic type.

Now A. Gnaffari has shown that the series solution (2), which is different from those given by A. Kolmogoroff, satisfies, for y and t fixed,

the parabolic linear partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + 2\,\frac{a(s)}{a'(s)}\,\frac{\partial f}{\partial s} + (1-x^2)f = 0$$

and, for x and s fixed, its formal adjoint. A paper is in preparation.

L. E. Payne has been working on a method for obtaining pointwise bounds in the Cauchy problem for the biharmonic equation. Such estimates are needed in the treatment of certain elastic plate problems.

Publications:

(1) The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. To appear in the Journal of Research, NBS, Section C. Engineering and Instrumentation.
(2) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. To appear in the Journal of Fluid Dynamics.
(3) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.
(4) Upper and lower bounds for the center of flexure. L. E. Payne. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
(5) On Stokes flow about a torus. W. H. Pell and L. E. Payne. Submitted to a technical journal.

PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen investigated the mechanism of shielding in the three-body plasma statistics. The system of B-B-G-K-Y equations (Bogoliubov, Born, Green, Kirkwood, and Yvon) was made closed by degenerating the triple correlation function, and reduced to a system of two equations relating the singlet distribution functions and the double correlation functions. The shielding, which modifies the static potential, was represented by such degenerated correlation functions (products of singlet distribution functions with double correlation functions). They caused some mathematical difficulties, as they entered in the form of an integral equation with different indices. Thus, in fact, one had to solve a system of integral equations.

The solution of the system was attempted by the following three methods: (a) Use of singular integral equations, (b) use of the dielectric property of the screening cloud, (c) use of immobilized screening cloud.

Method (a) was very much simplified when one assumed that the time scale of the correlation function was very small as compared with that

of the distribution function. Method (b) considered the screening cloud as a macroscopic medium having some dielectric property, determined by the Poisson equation. Method (c) considered the screening cloud as a medium with an average immobilized geometry. Method (c) had also served as a basis for the derivation of the kinetic equation by Tchen (see Phys. Rev. 114, 394-411, 1959).

Between September 1959 and February 1960, Dr. Tchen conducted his research at the Theoretical Physics Division of the U. K. Atomic Energy Research Establishment at Harwell, England, as a Guggenheim Fellow. His emphasis was on plasma dynamics and applications to thermonuclear fusion. Since March 1960, he has continued his research at the Max-Planck Institute for Physics and Astrophysics, at Munich, Germany. His emphasis has been on fundamental plasma problems and applications of magnetohydrodynamics to astrophysics.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti continued work on the problem of solving the kinetic equations of motion for a satellite orbit, expressed in spheroidal coordinates. The kinetic equations are the equations (63) given in his paper, J. Research NBS 63B, 105-116 (1959), expressing the spheroidal coordinates ρ, η, ϕ implicitly as functions of time. The development of the problem has lead to a solution, in which both secular terms and periodic terms are correct through order k^2. There remain many algebraic calculations to be done, however, in order to obtain the final formulas necessary to calculate an orbit. The polar coordinates will then follow directly from the spheroidal coordinates. It is of some interest to note that the secular terms calculated in Vinti's manner from the kinetic equations agree with the secular terms as calculated from the action variables J_i, with use of the theorem $\nu_i = \partial \alpha_1 / \partial J_i$ (Bull. Amer. Phys. Soc. 5, 8, 1960).

Dr. Vinti presented a paper on "Satellite Frequencies with a New Gravitational Potential," at the meeting of the American Physical Society, New York, January 27-30.

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: INACTIVE.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. The line list for 12,000 thorium lines has been prepared for publication. About 15,000 lines of praseodymium have now been measured, and their wave lengths and wave numbers calculated. The main table of intensities of 70 elements has been punched on cards, ready for listing and sorting into tables arranged by element and by wave length.

The interferometry codes have been completed and have been used for computation of observed patterns.

A code has been written to add every spectrum line of a list to every energy level of a second list, and then to select the sums that repeat, within a tolerance, a given number of times.

3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES
Origin: NBS, Section 3.2
Manager: M. L. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. Although no additional codes have been written, two codes written earlier, one for interpolation-editing and one that permits 3-, 4-, and 5-point interpolation in a table of many arguments (see Jan-Mar 1959 issue, p. 16) have been used by various members of the Thermodynamics Section (3.2) staff in other aspects of the project.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. J. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Least squares refinements were carried out for several crystals during the past quarter. Only in the case of the potassium cobalti tungstate crystal was the approximate structure sufficiently well determined to allow the least squares program to produce an improved structure.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: B. S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for 42 thermometers.

1102-40-11645/56-166 SCF-LCAO SOLUTION OF SOME HYDRIDES
Origin and Sponsor: NBS, Section 5.9
Managers: E. V. Haynsworth (11.1), P. J. Walsh
Full task description: Jan-Mar 1956 issue, p. 27
Status: Reactivated. Programs are being prepared to compute integrals and to sort them in a specific arrangement required by the A-matrix subroutine. Another program is planned which will generate a matrix (similar in form to the A-matrix) and select certain eigenvectors, computed by the SCF subroutine, for pre- and post-multiplication on sections of the matrix generated.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: B. S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed for 25 laboratory sets of gage blocks.

1102-40-11645/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: R. N. Varner
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. Production runs were made to determine the thermodynamic properties of a number of elements and their compounds. The results are included in an NBS Report prepared in the Thermodynamics Section (3.2), "Preliminary Report on the Thermodynamic Properties of Selected Light Element Compounds."

1102-40-11645/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Completed. Computations have been carried out as requested, and the results have been transmitted to the sponsor.

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. L. Paulsen, P. L. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-11645/57-236 SELF CONSISTENT FIELDS--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. V. Haynsworth (11.1)
Full task description: Apr-June 1957 issue, p. 30
Status: Continued. Calculations involving linear symmetric N_3 molecule were performed.

3911-61-39952/58-266 DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Continued. Work was resumed on the code for the solution of the differential equations with terminal initiation for large sized ($\approx$1000) polymer chains. Code checking is almost completed.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. The atomic coordinates and isotropic temperature factors of the N-benzyl dihydronics-tinamide crystal were refined using the least squares program of Dr. W. Busing (Oak Ridge National Laboratory). After detection and correction of a few errors in the observed data, it was possible to obtain a satisfactory molecular structure for the crystal.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman (11.1)
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. Investigation of mathematical models of distribution networks continued. The optimal value of the "decentralization parameter" N has been determined for the model described in the last report (Oct-Dec 1959, p. 19), assuming a simple formula for the cost of the routing-sorting operations. In particular, a highly centralized system proves optimal if efficiencies of scale can reduce operating cost by as much as 50 percent. A more general model was examined in which the service area is partitioned into (possibly unequal) regions by lines parallel to the sides; the optimal locations of the sorting installations for any such partition have been derived, and some progress has been made toward determining the optimal partition (for fixed N).

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Inactive. For status to date, see Oct-Dec 1959 issue, p. 19.

1102-40-11645/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1957 issue, p. 40
Status: Terminated. Some code checking was carried out; then the code was given to the sponsor who will directly supervise the production runs. Production time on the 704 will be reported hereafter in the section of this Report, "Current Applications of Automatic Computer."

1102-40-11645/58-307 STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. J. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Completed.

1102-40-11645/58-316 INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Managers: S. T. Peavy, J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 33
Status: Terminated, at the request of the sponsor.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. The theoretical investigations have been completed during this period. Expansions have been found to cover the whole t-axis for stress relaxation and creep. A Fortran program has been written for a small region from 10^{-3} to 1 sec which had not been covered before. We distinguish now four time regions (t in sec):

$$0 \leq t \leq 10^{-8},$$

$$10^{-8} \leq t \leq 10^{-4},$$

$$10^{-4} \leq t \leq 10^{2},$$

$$10^{2} \leq t \leq \infty.$$

For each of these regions expansions have been found which are numerically practical and allow one to compute stress relaxation and creep to any desired accuracy. A paper on the subject is being prepared by H. Oser together with R. S. Marvin (3.4).

1102-40-11645/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes (11.0)
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. Programming for the 704 computer is proceeding principally on that portion of the translation scheme which compares each incoming source word with the "predictions" arising from earlier words in the same sentence or from the rules of grammar. The program will consist of about 16 subroutines--one for each kind of prediction--and a main routine which accomplishes the necessary switching and bookkeeping. Each subroutine requires numerous branches corresponding to the different ways in which predictions can be satisfied. To date the main routine has been written and tested; subroutines are being coded one branch at a time, as required by sample texts. An outline has been made for the routines needed to set up predictions. Preliminary studies are under way for the treatment of clauses and phrases.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 30
Status: Inactive. For status to date, see July-Sept 1959 issue, p. 24.

3711-60-0009/58-360 DIFFUSION COEFFICIENTS
Origin: NBS, Section 5.2
Manager: J. P. Menard
Full task description: Apr-June 1958 issue, p. 32
Status: Reactivated. A code is now being written for the IBM 704, which will compute

$$D_t(c) = \frac{-1}{2t\left(\frac{\partial c}{\partial x}\right)_c} \int_{c_1}^{c} (x-\bar{x})dc$$

where

$$\bar{x} = \frac{1}{c_m - c_1} \int_{c_1}^{c_m} x(c)dc.$$

The fringe numbers x_i and the concentrations c_i, i=1,2,...,m, are input data.

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. Production runs were continued, and results were forwarded to the sponsor.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Manager: P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Continued. Calculations on Table (d), (see Apr-June 1958 issue, p. 35), were carried out for parameter $L/\lambda = 6.6(.1)8.0$. The code is being set up to calculate Table (d) for $4\lambda = 8.1(.1)10.0$. The antenna code was used to produce results for three different antennae, and results have been submitted to the sponsor.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES
Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.
Manager: H. Oser
Full task description: July-Sept 1958 issue, p. 32
Status: Continued. Production runs for scattering diagrams were made during the whole period, and the results were turned over to the sponsor.

1102-40-11645/59-377 LOGICAL DIAGRAM REDUCTION
Origin and Sponsor: NBS, Section 12.3
Managers: W. G. Hall, C. Coleman (4.1)
Full task description: Apr-June 1959 issue, p. 25
Status: Continued. Minor revisions were made in the code to make running automatic.

1102-40-11645/59-388 HEAT PUMP CALCULATIONS
Origin and Sponsor: NBS, Section 10.3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 26
Status: Continued. Production runs were continued using the heating and cooling data.

1102-40-11645/59-389 FREQUENCY ALLOCATION
Origin and Sponsor: Civil Aeronautics Administration
Manager: L. S. Joel
Full task description: Oct-Dec 1958 issue, p. 29
Status: Continued. A demonstration run of the "one interchange" code was

made for the sponsor. The code will be used in production as soon as a complete input data file is available. The program is to be extended to compute chains of single interchanges.

Study of the mathematical analysis of the station network was continued.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II

Origin and Sponsor: NBS, Section 4.6
Manager: A. E. Beam
Full task description: Oct-Dec 1958 issue, p. 30
Status: Continued. Production runs were continued under the direction of the sponsor. A new code is being written to compute the photodetachment cross section for H^-, making use of the symmetric p-wave results.

1102-40-11645/59-407 FOURIER COEFFICIENTS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 27
Status: Inactive.

1102-40-11645/59-414 INFINITE SYSTEMS

Origin and Sponsor: NBS, Division 3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 28
Status: Inactive. For status to date, see July-Sept 1959 issue, p. 26.

1102-40-11645/59-415 COMPLEX LEGENDRE FUNCTIONS

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 29
Status: Terminated. Additional production runs were made on the 704. Future production runs will be made under the direction of the sponsor, and machine time will be reported in the section of this report entitled, "Current Applications of Automatic Computer."

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS

Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. Exploratory experiments in the evaluation of electrocardiographic readings are being conducted. Various computational procedures are being applied to medical data to see if the results can be

used to reveal a meaningful medical interpretation. One experiment using Fourier analysis and the least squares method was carried out, but the results were unsuccessful. A revised procedure is being devised. An averaging routine has been checked out.

1102-40-11645/59-444 HOSPITAL SUBSISTENCE ITEMS SUPPLY
Origin and Sponsor: Veterans Administration
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. A paper by G. Dantzig and P. Wolfe of the RAND Corporation, "Decomposition Principles for Linear Programming," describes a method that may be used for solution of multi-stage transportation problems. Preliminary study indicates that it may have application to the supplier-depot-consumer structure of VA's subsistence system.

1102-40-11645/59-445 OIL SUPPLY
Origin and Sponsor: Military Petrolem Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Continued. The Bureau has obtained a binary deck of the MIT transportation program and plans to test it in solving a 270x250 petroleum allocation schedule. Since the MIT program is not at present in SHARE, the significant operating differences between it and NYTR1 (the SHARE identification for the program customarily used to solve transportation problems) are noted:

(1) In data preparation, NYTR1 requires all c_{ij} to be listed explicitly, whereas the MIT program permits the use of lexicographic indices for listing only the nonzero elements of sparse matrices. The program generates the computational c_{ij} matrix. (DTMB has written a set of data preparation programs for use with NYTR1, but they apparently contain residual errors after debugging. These codes were produced ad hoc for a specific problem and were "discarded" after use; so they were not edited for submission to SHARE.)

(2) Intermediate feasible solutions may be saved and printed out with NYTR1, but not with the MIT code. Thus with NYTR1 it is sometimes possible to make changes in the c_{ij} and to start with a "good" feasible solution obtained from a previous computation. This can save many iterations.

(3) To solve disparate problems consecutively with NYTR1, the program must be read into the computer for each run. The MIT program permits the use of a consolidated data deck to solve several problems consecutively.

(4) NYTR1 can print alternate optima if they exist; the MIT code cannot.

(5) Both programs use fixed point arithmetic. To use the MIT code, a_i, b_j, c_{ij} must be less than 131,071 ($2^{17}-1$); for NYTR1 they may be as large as $10^{10}-1$.

Further evaluation of these two programs is planned: a set of problems will be run with both programs to test under what conditions NYTR1 is faster than the MIT code. Whenever a symbolic listing of the MIT program can be obtained, it will be studied to determine if it can be modified easily to obtain the output advantages of NYTR1 while retaining the convenience of the MIT data input.

In addition, a new method for solving the transportation problem, not based on the simplex method, that has been proposed by E. M. Beale, is being studied for computer application.

1102-40-11645/60-452 CONSULTING SERVICES FOR PACIFIC MISSILE RANGE
Origin and Sponsor: NBS, Section 12.3
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 29
Status. Continued. For further background, a visit was made by D. Mittleman and G. Reitwiesner to the Ballistic Research Laboratories at Aberdeen Proving Ground, Md.

1102-40-11645/60-453 DATA CONVERSION
Origin and Sponsor: Army Map Service
Manager: J. M. Beiman
Full task description: July-Sept 1959 issue, p. 30
Status: Continued. Production runs were continued as requested by the sponsor.

1102-40-11645/60-456 FIRING CIRCUIT EQUATIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 30
Status: Completed. The results were transmitted to the sponsor (see July-Sept 1959 issue, p. 30).

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. M. Beiman, W. G. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. Production runs are being made to check and process submitted data. A program is being written to edit the data processed by the above runs.

1102-40-11645/60-459 TRANSCENDENTAL EQUATIONS
Origin and Sponsor: NBS, Section 9.1
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. Production runs were made for 29 values of P for values of u=0(.05).5, and the results were submitted to the sponsor.

1102-40-11645/60-460 DESIGN EQUATIONS FOR MAGNETRON INJECTION ELECTRON GUNS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. E. Beam
Full task description: July-Sept 1959 issue, p. 32
Status: Inactive. For status to date, see Oct-Dec 1959 issue, p. 32.

1102-40-11645/60-461 DATA SYSTEMS LANGUAGES
Origin and Sponsor: NBS, Section 11.2
Manager: J. H. Wegstein
Full task description: July-Sept 1959 issue, p. 33
Status: Completed. Further editing of the proposal for a common business oriented language (COBOL) was completed by the Short-Range Data Systems Language Committee. With the completion of this task Mr. Wegstein resigned as chairman of the committee.
Publication: COBOL, Preliminary Specifications for a Common Business Oriented Language. In press, U. S. Government Printing Office.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Continued. The computation was extended to include auxiliary evaluations requested by the sponsor.

1102-40-11645/60-464 CORRELATION MATRIX FOR PHS DATA
Origin and Sponsor: Public Health Service
Manager: J. M. Beiman
Full task description: July-Sept 1959 issue, p. 34
Status: Inactive. For status to date, see Oct-Dec 1959 issue, p. 26.

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1959 issue, p. 26
Status: Continued. Some production runs were made using the Central, A-Matrix and SCF programs which were written under task 3911-61-39952/57-236, Self-Consistent Fields (see p. 19).

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. Some experimental cases have been run to compare results with previous calculations and to gain experience in data preparation and set-ups required by the programs.

1102-40-11645/60-467 TRANSISTOR SIMULATION
Origin and Sponsor: NBS, Section 12.1
Manager: G. W. Reitwiesner
Full task description: Oct-Dec 1959 issue, p. 27
Status: Continued. A preliminary program is in the checkout stage, and changes in formulation are being accommodated.

1102-40-11645/60-471 INTERLABORATORY STANDARDIZATION OF TESTING PROCEDURES
Origin and Sponsor: NBS, Section 7.3
Manager: A. E. Beam
Full task description: Oct-Dec 1959 issue, p. 28
Status: Continued. The code was written and checked out, and some production runs were made under the direction of the sponsor.

1102-40-11645/60-475 IONOSPHERIC SOUNDINGS
Origin and Sponsor: NBS, Section 82.40
Manager: M. L. Paulsen
Full task description: Oct-Dec 1959 issue, p. 29
Status: Continued. Approximately 40,000 virtual height (h'f) data cards were processed using this program for its first production run. It was found that the transcription process from card to tape introduced illegal characters. Investigation revealed that the cards were put on tape using an 80x80 board instead of an 80x84 board which fills in or completes the 14th word (this meant the information in columns 79 and 80 was being ignored, because a partial word is treated as an E.O.R.). S. Peavy and R. Varner examined the input routine in the Bell system and made a correction enabling the routine to read in the partial word, thus eliminating the necessity of putting the 40,000 cards on tape again. The requested results were sent to N.B.S. Boulder on magnetic tape during the first week in March. (The amount of output is a little more than the input.) Another 20,000 cards were received to be put on tape.

1102-40-11645/60-476 GAS TUBE CHARACTERISTICS, II
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: H. Oser
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. A program has been written to integrate simultaneously the two differential equations for field strength and negative ion density. The speed of convergence has been increased considerably as compared with that of earlier programs. Between 6 and 10 iterations are now required to establish a solution with 4 decimal place accuracy, and computing time has been reduced by 75 percent. The code has been checked out and will be turned over to the sponsor, who will conduct production runs directly.

A study is now under way to obtain gas tube characteristics for very high total current densities.

1102-12-11122/60-479 PROCESSING OF DIAGRAMS
Origin and Sponsor: NBS, Section 11.0
Managers: F. L. Alt, S. T. Peavy, R. J. Herbold
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. Two programs have been written for the 704, one to process diagrams made on another computer, and the other to analyze the diagrams. The codes are in the process of being checked.

1102-40-11645/60-480 LARGE SIGNAL CALCULATIONS FOR A VOLTAGE TUNEABLE MAGNETRON
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. E. Beam
Full task description: Oct-Dec 1959 issue, p. 30
Status: Continued. The code was written and checked out, and production runs were started.

1102-40-11645/60-484 POLYMER CRYSTALLIZATION
Origin and Sponsor: NBS, Section 7.6
Manager: H. Oser
Objective: To integrate a fourth order nonlinear differential equation of the following type:

$$\lambda^{(4)} + Q(\lambda)\lambda''\lambda' + R(\lambda)(\lambda'')^2 + S(\lambda)\lambda''(\lambda')^2 + T(\lambda)(\lambda')^4 = U(\lambda).$$

Background: The above equation arose in the course of original research being conducted by F. Gornick (7.6), concerned with an extension of Mandelkern's research in the kinetics of crystallization of polymer systems (see Society of Plastics Engineers Journal, vol. 15, No. 1, Jan. 1959). If one assumes a law $U(\lambda)$ describing the rate of nucleation in the liquid phase and allows for impingement of the growing nuclei, one arrives at a differential equation of this type. λ is the ratio of the weights of crystallized polymer at time t and time ∞; Q, R, S, T are rather complicated functions of λ that involve t only implicitly.
Status: New. A Fortran program has been written for the integration and has been checked out completely. The results are extremely stable with respect to the choice of h, the step length of the integration method. Some production runs have been made.

1102-40-11645/60-486 MORSE WAVE FUNCTIONS AND FRANCK-CONDON FACTORS
Origin and Sponsor: NBS, Section 3.0
Manager: R. Zucker
Objective: To compute the Franck-Condon factors

$$q_{v'v''} = \left| \int_0^\infty \psi_{v'} \psi_{v''} \, dr \right|^2$$

where $\psi_{v'}$ and $\psi_{v''}$ are the Morse wave functions for two different electronic states at various quantum numbers. The vibrational wave function for the

lowest level (v = 0) is

$$|\psi_o(r)| = \sqrt{\frac{a}{\Gamma(K-1)}} \exp\left\{-\frac{K}{2}\exp[-a(r-r_e)]\right\}\left\{K\exp[-a(r-r_e)]\right\}^{\frac{K-1}{2}}$$

where r denotes the internuclear separation, and a and K are functions of the velocity of light and the dissociation energy of the molecule.

Wave functions of higher order involve Associated Laguerre functions $L^{K-2v-1}_{K-v-1}(z)$, $z = K\exp[-a(r-r_e)]$. Since there is an increasing loss of significant figures in obtaining these functions by recurrence relations, double precision arithmetic is to be used. The wave functions are to be normalized such that

$$\int_o^\infty |\psi_v|^2 \, dr = 1.$$

The following check is to be applied:

$$\Sigma_{v'} q_{v'v''} = \Sigma_{v''} q_{v'v''} = 1.$$

Background: This is part of a study of the vibrational transition probability arrays for molecular band systems of specific astrophysical interest. The bands usually include the vertex of the Condon parabola of the system concerned. For the theory, see (1) W. R. Jarmain and R. W. Nicholls, Vibrational Transition Probabilities to High Quantum Numbers for the Nitrogen First and Second Positive Band Systems, Can. J. Phys. 32, 201-204, 1954; and (2) R. W. Nicholls, Ann. Geophysique 14, 208-224, 1958.

The problem was submitted by R. W. Nicholls (3.0).

Status: New. The code has been written and checked out. Very good agreement was obtained with previous computations for the vibrational transition probabilities for the first positive ($B^3\pi_g \rightarrow A^3\Sigma u^+$) band system of nitrogen.

1102-40-11645/60-489 INVERSION OF LINE PROBE DATA

Origin and Sponsor: NBS, Section 3.1

Manager: R. Herbold

Objective: To evaluate f(r) in the expression

$$f(r) = -\frac{1}{\pi}\int_r^\infty \frac{Q'(x)dx}{(x^2-r^2)^{\frac{1}{2}}}$$

for given discrete values of Q(x).

Background: This integral arises in calculations connected with radial distributions of physical properties of axially symmetric inhomogenous plasmas. Using an electric arc through a hot gas (the plasma) to create a cylindrical source, measurements (Q(x)) are made of the intensity of the light as a function of the vertical distance. The measurements are then used to obtain the radial distribution of the light intensity within the cylindrical source. The above integral, involving Q'(x), is the solution of Abel's integral equation expressing the geometry of the setup (see Courant and Hilbert, Vol. I, First English ed., 1953, p. 158).

Status: New. A working code was written and checked out. To improve and generalize the procedure, however, the code is being revised.

1102-40-11645/60-500 DENTAL RESEARCH COLORIMETRY
Origin and Sponsor: NBS, Section 7.8
Manager: J. P. Menard
Objective: To compute the CIE tristimulus values and chromaticity coordinates of materials submerged in water.
Background: Existing codes for computing CIE tristimulus values and chromaticity coordinates do not compensate for measurements of samples submerged in water. It seems to be more practical to write a special code for this purpose than to adapt one of the existing codes (see tasks 1102-40-11645/60-449, Spectral Line Colorimetry, and 3711-60-0009/57-250, Automatic Reduction of Spectrophotometric Data).

The problem was communicated by P. Oglesby (7.8).

Status: New.

1102-40-11645/60-504 ELECTROSTATIC-FOCUSING PROBLEM
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. Beam
Objective: To solve the differential equation

$$2VY'' + V'Y' + V''Y = A\sqrt{V} \tag{1}$$

where

$$V = 1 - B \sum_{N=1,3,5,7,\ldots}^{\infty} \left[(-1)^{(N-1)/2} \frac{\sin CN}{\cosh DN} \cos 2\pi NL \right]. \tag{2}$$

The initial conditions given for L = 0 are:

$$Y(L) = 1, \quad Y'(1) = \alpha.$$

A, B, C, D are functions of input parameters.

Background: It is of interest to study the motion of an electron sheet beam traveling between a set of electrostatic focusing plates where two different potentials are applied to alternating pairs of plates. This motion will be studied on the IBM 704 computer by solving the paraxial-ray equation [Eq. (1)]. The potential function [Eq. (2)] to be used in the paraxial-ray equation was obtained from an approximate solution of the Laplace equation for the focusing structure. It will be the purpose of this study to determine the amplitude and wavelength of the ripples present in the beam as a function of beam current and of the dimensions and voltages of the electrostatic focusing structure. The paraxial-ray equation for this problem is of the form of an inhomogeneous Hill's equation; for certain values of the parameters the complimentary solution will become unstable. An important part of the study will be a search for the critical values of the parameters for which instability occurs.

The problem was submitted by B. J. Udelson (DOFL).

Status: New. Coding of the problem was started.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel. The following are representative examples:

(1) Precision and accuracy of a standard cylinder. A study of the precision and accuracy of volume and density measurements on a standard cylinder was carried out by J. M. Cameron for C. T. Collett of the Capacity, Density and Fluid Meters Section.

(2) Analysis of data on flame spread on paint coated assemblies. A statistical analysis of the flame spread index for a number of paint coatings used on different base materials was carried out by H. H. Ku for D. Gross of the Fire Protection Section.

(3) Analysis of data on viscosity of water. Upon request of R. Marvin and R. C. Hardy of the Rheology Section the results of seven series of experiments on the viscosity of water were analysed by H. H. Ku. Several different parametric representations of the data were evaluated.

(4) Study of kaolin. An orthogonal design for estimating the effect of time type particle size and measuring equipment on the properties of kaolins was worked out for W. C. Ormsby of the Engineering Ceramics Section by W. J. Youden.

(5) Accuracy of cement tests. Dr. Youden participated in the planning phases and devised a scoring scheme for a long range program to determine the limits of accuracy of cement tests carried out at the state cement testing laboratories. This work was done for the Concreting Materials Section.

(6) Computations were carried out on the Bureau's electronic computer for M. J. Kerper (9.02), F. M. Reinhart (8.04), E. D. Tidwell (4.02), A. G. Strang (2.05), E. K. Plyler (4.02).

Publications:

(1) Some canons of sound experimentation. C. Eisenhart. To appear in the "Proceedings of the 31st Session of the International Statistical Institute," Brussels, 1958.
(2) Statistical aspects of the cement testing program. W. J. Youden. To appear in the Proceedings of the American Society for Testing Materials.
(3) Variability of color-mixture data. I. Nimeroff (NBS Photometry and Colorimetry Section), J. Rosenblatt and M. C. Dannemiller. In manuscript.
(4) Graphical evaluation of analytical results. W. J. Youden. Appeared in "Proceedings of the Conference on Chemical Control Problems," National Plant Food Institute, Washington 6, D.C., 1959, pp. 1-15.

(5) How to evaluate accuracy. W. J. Youden. Submitted to a technical journal.
(6) Multivariable experimentation. W. J. Youden. To appear in the Transactions of the Society of Automotive Engineers.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: CONTINUED. The complete set of tables for the Appendix, and some of the text has been turned over to the OOR contractor for preparation for final publication.

Publications:
(1) The relation between confidence intervals and tests of significance--a teaching aid. M. G. Natrella. Amer. Stat. 14, 20-22 (Feb. 1960).

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) U. S. Geological Survey: Investigations were continued by J. R. Rosenblatt on several mathematical models associated with measurement processes in hydrology.

(2) Veterans' Administration Hospital, Perry Point, Md.: A statistical analysis of data from NP research studies was carried out on the Bureau's electronic computer by M. C. Dannemiller.

Current Applications of Automatic Computer

The record of the use of the IBM 704 for the period January 1 through March 31 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
NBS:					
11110/55-55	11.1	Research in numerical analysis	134	154	362
11411/55-56	11.1	Research in mathematical topics applicable to numerical analysis	27	79	395
11413/60-469	11.1	Orthogonal functions in the theory of partial differential equations	25	46	475
11120/55-65	11.2	Automatic coding	383	117	605
39951/56-160	11.2	Mathematical subroutines	27	43	16
39951/51-1	11.3	Statistical engineering	21	2	432
39952/54-30	4.1	Spectrum analysis	122	148	746
39952/55-68	9.7	Crystal structure calculations	82	270	1424
39952/55-82	3.1	Thermometer calibrations			44
11645/55-97	3.8	High temperature properties for air°			14
39952/56-131	2.2	Calculations in optics*	44	30	71
11645/56-166	5.9	SCF-LCAO solution of some hydrides*	71	58	7
11645/56-171	3.2	Collision integrals used in transport theory**	90	142	139
11645/57-219	3.2	Thermal properties*			240
39952/57-236	3.2	Self-consistent fields-eigenvalues			73
11645/57-246	4.8	Radiation diffusion**	3	64	265
39952/57-247	6.1	Mechanical impedance*			17
11645/57-249	9.4	Color differences*			95
39952/57-250	2.3	Automatic reduction in spectrophotometric data*		3	26
11645/57-251	1.6	Current noise and fixed resistors	5		15
11645/57-252	4.11	Detecting efficiency in a neutral meson experiment**	204	295	550
39952/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*	3	4	9

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
11645/58-255	4.8	Chi functions**	95	7	40
/58-256	10.6	Composite walls**	175	121	333
/58-260	12.5	Prototype accounting**	8	94	34
/58-270	12.5	Mathematical problems related to postal operations	45	79	25
/58-271	6.3	Simultaneous equations for potential flow**	6		5
/58-274	9.7	Calculations for d-spacings II*	31	21	91
/58-275	7.8	Crystallography**			128
/58-304	3.2	Transport properties of air*	9	56	28
/58-308	3.4	Oscillating sphere*			10
/58-314	3.2	Approximations for gas mixtures*	25	9	86
/58-333	9.0	Calcium hydroxide*	3		295
/58-339	3.4	Viscoelasticity properties of materials	19	17	2
/58-357	3.3	Eigenvalues**	4	37	68
/58-360	5.2	Diffusion coefficients*	37	6	4
/59-377	12.3	Logical diagram reduction	15		27
/59-387	30.4	Nuclear reactor design**			34
/59-388	10.3	Heat pump calculations			7
/59-394	4.6	Slow electron scattering by hydrogen atoms	141	183	416
/59-395	7.7	Adsorption study**	26		9
/59-403	2.1	Computation of color fadings*			4
/59-409	12.5	Bank Board**	33	192	584
/59-417	2.4	Spectrum analysis of ruthenium**			133
/59-418	4.8	P-Wave equation*	61	43	132
/59-421	12.5	Traffic assimilation**		26	89
/59-428	12.5	Radio intensities**	80	106	73
/59-440	82.10	Mapping**	66	335	911
/59-446	85.10	Ionospheric data**	6	32	409
/60-449	2.1	Spectral line colorimetry*			12
/60-457	12.5	Public Housing problem**	1	22	393
/60-459	9.1	Transcendental equations		10	22
/60-466	3.2	Electronic properties of simple molecular systems		86	
/60-467	12.1	Transistor simulation	32	27	48
/60-471	7.3	Interlaboratory standardization of testing procedures	10	8	29
/60-474	2.5	Gage block stability*			20
/60-475	82.40	Ionospheric soundings		101	541
11122/60-479	11.0	Processing of diagrams	11		24
11645/60-484	7.6	Polymer crystallization	31	29	27
/60-486	3.6	Morse wave functions	19	95	2
/60-487	5.9	Parabolic curve fitting	2	5	33

Task No.		Title	Assembly	Checking	Production
				(M I N U T E S)	
11645/60-489	3.1	Inversion of line probe data	28	15	12
/60-493	3.7	Poisson distribution function**	130	75	41
/60-494	82.0	Atmospheric transmission**	22	54	4
/60-495	6.4	Engineering mechanics**	37	13	5
/60-500	7.8	Dental research colorimetry	5		1
		Totals (NBS Services)...	2,454	3,359	11,211
OUTSIDE:					
11414/56-148	NIH	Differential equations for nerve excitation			315
11645/53-45	SC	Air defense tactics°			59
/58-269	NRL	Molecular structure, IV	31	178	1,127
/58-276	NOL	General kinetics, I**			7,613
/58-278	NOL	Polaris**	24	96	150
/58-316	BPR	Intersection capacity study		3	
/58-319	HPBA	Auto tag°	3	75	46
/58-325	VA	Covariance analysis	5	20	177
/58-335	DOFL	Roots of Bessel functions**	6	11	26
/58-340	DOFL	M5-17 Fuze Data°	30	55	148
/58-347	BURR	Computations for war games**		6	438
/58-348	OOR	Russian-to-English machine translation	17	38	153
/58-361	NRL	Spectrum of dipole radiation	42	58	1,203
/58-366	USIA	Radiation patterns of antennas		5	39
/58-368	SC	Intensity functions of light scattered by spherical particles			333
/59-371	NRL	ASWAP°	167	254	64
/59-389	CAA	Frequency allocation	38	76	84
/59-407	DOFL	Fourier coefficients	37		70
/59-408	NASA	NASA**	1,574	870	12,834
/59-411	HEW	Fitting of exponential curves**	44		1,615
/59-415	DOFL	Complex Legendre functions*	37	9	92
/59-416	DOFL	Analysis of power supply experiments**	14		20
/59-419	DOFL	Neutrons°	86	3	231
/59-423	WB	Weather Bureau**		62	12,309
/59-425	CU	Molecular orbitals			8
/59-434	CIW	Petrological computations*	28	37	21
/59-435	VA	Electrocardiographic analysis	15	55	117
/59-437	GE	GE Highway studies**			430
/59-441	GK	Systems engineering**	114	1,850	309

Task No.	Title	Assembly	Checking	Production
		(M I N U T E S)		
11645/59-447 BPRO	Public Roads study**			5,194
/60-450 ACC	Chemical warfare°	11	4	
/60-453 AMS	Data conversion			608
/60-454 GE	G.E.**			1,563
/60-458 CAB	Domestic airline traffic survey	65	44	205
/60-462 DOFL	Correlation of functions	23	10	105
/60-465 CU	Calculations in molecular quantum mechanics			108
/60-468 NASA	NASA-Langley**		418	9,379
/60-476 DOFL	Gas tube characteristic II	9	335	106
/60-480 DOFL	Large signal calculations for a voltage-tuneable magnetron	27	33	561
/60-482 UMd	Fluid dynamics**	108	14	59
/60-485 DOFL	Constant pressure data**	65	13	78
/60-488 NRL	Spectrum analysis**	4	7	3
/60-492 IMF	Monetary research reports**	88	33	7
/60-496 BPA	Short circuit program**	372		199
/60-501 CEng	Kansas River Reservoir System**	41	7	
/60-502 USA	Quartermaster Mathematics programming**			6
	Totals (Outside)	3,125	4,679	58,212
	Total time for the quarter (MINUTES).....	5,579	8,038	69,423
	Total time for the quarter (HOURS)	93	134	1,158

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.

** Problem programmed by sponsor and run under his direction.

° Classified task.

Lectures and Technical Meetings

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

Applied Mathematics Division Colloquium

SAUER, R. (Mathematisches Institut der Technischen Hochschule, Munich)
Error analysis for finite difference methods in characteristic grids for hyperbolic differential equations. March 21.

Mathematical Statistics Seminar

SCALORA, Frank S. (International Business Machines Corp., New York City)
Abstract Martingale convergence theorems. January 15.

Statistical Engineering Laboratory Mathematical Seminars

BOSE, R. C. (Case Institute of Technology, Cleveland)
Construction of error correcting codes (A series of five lectures). January 25-29.
On powers of matrices with non-negative elements. March 29.

Statistical Engineering Laboratory Reliability Seminars

ZELEN, M. Are life testing procedures robust? January 6.

ROSENBLATT, J. R. Statistical models for component aging experiments. March 16.

WEISS, G. (Institute of Fluid Dynamics, University of Maryland)
Semi-Markov processes and applications to reliability. March 30.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

CANNON, E. W. The reflection of Logistics in electronic computer design. Presented at the Decennial Logistics Research Conference, Washington, D. C., January 20.

EISENHART, C. Some statistical aspects of measurement with a linear scale. Presented at a Graduate Colloquium, Statistics Department, Virginia Polytechnic Institute, Blacksburg, Va., February 26.

HAYNSWORTH, E. V. Bounds for determinants with positive diagonals. Presented at a meeting of the American Mathematical Society, Chicago, Ill., January 27-29.

NEWMAN, M. Number theory and computers. Presented at the U. S. Naval Academy, Annapolis, Md., January 8.

RABINOWITZ, P. (1) Some SEAC computations of subsonic flow by Bergman's method of integral operators. Presented before the Department of Aeronautics, University of California, Berkeley, Calif., January 12. (2) The use of orthogonal functions in numerical analysis. Presented at a Numerical Analysis Seminar, Stanford University, Stanford, Calif., January 13; before the Department of Mathematics, University of California, Berkeley, Calif., January 14; and at the Brookhaven National Laboratory, Upton, N. Y., January 18.

RHODES, I. (1) How the use of computers will affect the future careers of children. Presented at the Western High School, Baltimore, Md., January 20. (2) Current research in the field of machine translation. Presented at a National Symposium on Machine Translation held at the University of California, Los Angeles, Calif., February 2-5.

RICE, J. R. Tchebycheff approximations. Presented at the meeting of the American Mathematical Society, Chicago, Ill., January 27-29. Also presented by title at the same meeting: (1) A new representation of Gegenbauer's functions, (2) Split Runge-Kutta for simultaneous equations.

ROSENBLATT, J. Statistical models for component aging experiments. Presented at the Institute of Radio Engineers National Convention, New York City, March 23.

VINTI, J. Satellite frequencies with a new gravitational potential. Presented at a meeting of the American Physical Society, New York, N. Y., January 27.

YOUDEN, W. J. (1) Statistical ideas useful in experimentation. Presented at a meeting of the Society of Applied Spectroscopy, New York City, January 5. (2) Experiments involving several variables. Presented at Air Reduction Corporation Research Laboratory, Murray Hill, N. J., January 5. (3) Multivariable experimentation. Presented at a meeting of the Society of Automotive Engineers, Detroit, Mich., January 11. (4) Experiments involving several variables. Presented at the NBS Boulder Laboratories, January 20. (5) What is scientific evidence? Presented to the American Statistical Association, Colorado-Wyoming Chapter, Denver, Colo., January 21; also before the Cleveland Park Club, Washington, D. C., February 9.

(6) Errors in measurements. Presented at the Geophysical Laboratory of the Carnegie Institution of Washington, February 25. (7) Interlaboratory comparison problems. Presented at a Seminar of the American Society for Quality Control, New York City, February 27. (8) The sample, the procedure, and the laboratory. Presented before the Analytical Chemistry Group of the American Chemical Society, and Spectroscopy Society, Pittsburgh, Penn., March 1. (9) Applications of statistics in industry. Presented at the Shell Chemical Co., Union, N. J., March 25.

ZELEN, M. (1) Are life testing procedures robust? Presented before the Committee on Statistics, University of Wisconsin, Madison, Wis., January 7; and at the Sixth National Symposium on Reliability and Quality Control in Electronics, Washington, D. C., January 12. (2) Factorial experiments in life testing. Presented at a Seminar of the Department of Mathematics, University of Miami, Coral Gables, Fla., January 14. (3) The robustness of statistical life testing procedures. Presented at a meeting of the Department of Statistics, The George Washington University, Washington, D. C., March 30.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Technical Notes, Manuals, and Bibliographies

(1) Selected bibliography of statistical literature, 1930-1957. Lola S. Deming. I. Correlation and regression theory. J. Research NBS 64B, 55-68 (1960). II. Time series. J. Research NBS 64B, 69-76 (1960).

1.3 Technical Papers

The following papers appeared in J. Research NBS 64B, Jan-Mar 1960 (Mathematics and Mathematical Physics):

(1) The minimum of a certain linear form. K. Goldberg. Pp. 49-50.

(2) A symmetric continuous poker model. A. J. Goldman and J. J. Stone (Stanford University). Pp. 35-40.

(3) Space of k-commutative matrices. M. Marcus and N. A. Khan (University of British Columbia). Pp. 51-54.

(4) Kantorovich's inequality. M. Newman. Pp. 33-34.

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The following paper appeared in J. Research NBS 64C, Jan-Mar 1960 (Engineering and Instrumentation):

(1) A statistical chain ratio method for estimating relative volumes of mail to given destinations. N. C. Severo and A. E. Newman (12.3). Pp. 37-47.

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(1) A continuous poker game. A. J. Goldman and J. J. Stone (Stanford) University). Duke Math. J. 27, 41-54 (1960).

(2) The relation between confidence intervals and tests of significance --a teaching aid. M. G. Natrella. Amer. Stat. 14, 20-22 (Feb. 1960).

(3) Weighted restricted partitions. M. Newman. Acta Math. 5, 371-380 (1959).

(4) Graphical evaluation of analytical results. W. J. Youden. Appeared in "Proceedings of the Conference on Chemical Control Problems," National Plant Food Institute, Washington, D. C., 1959, pp. 1-15.

(5) Are life testing procedures robust? M. Zelen and M. C. Dannemiller. Appeared in "Proceedings of the Sixth National Symposium on Reliability and Quality Control in Electronics," held in Washington, D. C., January 1960, pp. 185-189.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION

2.2 Technical Notes, Manuals, and Bibliographies

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Selected bibliography of statistical literature, 1930-1957. III. Limit theorems. L. S. Deming. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(3) Index to the distributions of mathematical statistics. Frank A. Haight. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

2.3 Technical Papers

(1) The outlook for machine translation. F. L. Alt. To appear in the Proceedings of the Western Joint Computer Conference to be held in Los Angeles, May 1960.

(2) Computer simplification of Boolean functions. B. K. Bender, A. J. Goldman, and R. B. Thomas (Data Processing Systems). Submitted to a technical journal.

(3) Analysis of fractionally replicated 2^m3^n designs. R. C. Bose and W. S. Connor. To appear in "Proceedings of the 31st Session of the International Statistical Institute", Brussels, 1958.

(4) The reflection of logistics in electronic computer development. E. W. Cannon. To appear in the Proceedings of the Logistics Research Conference, held at the George Washington University, Washington, D. C., 1960.

(5) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to Probability and Statistics," in press, Stanford University Press (1960).

(6) Some SEAC computations of subsonic fluid flows by Bergman's method of integral operators. P. Davis and P. Rabinowitz. To appear as an Appendix in the book, "Bergman's Operator Method," by M. S. v. Krzywoblocki.

(7) Some canons of sound experimentation. C. Eisenhart. To appear in the "Proceedings of the 31st Session of the International Statistical Institute," Brussels, 1958.

(8) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(9) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(10) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.

(11) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.

(12) The range of a fleet of aircraft. A. J. Goldman. Submitted to a technical journal.

(13) On non-self-adjoint boundary value problems in ordinary differential equations. W. Greub and W. Rheinboldt. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(14) A reduction formula for partitioned matrices. E. V. Haynsworth. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(15) Bounds for determinants with positive diagonals. E. V. Haynsworth. To appear in the Proceedings of the American Mathematical Society.

(16) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.

(17) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.

(18) Upper and lower bounds for the center of flexure. L. E. Payne. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(19) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. To appear in the Journal of Fluid Dynamics.

(20) The Stokes flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.

(21) On Stokes flow about a torus. W. H. Pell and L. E. Payne. Submitted to a technical journal.

(22) Mechanized conversion of colorimetric data to Munsell renotations. W. C. Rheinboldt and J. P. Menard. To appear in the Journal of the Optical Society of America.

(23) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. To appear in Mechanical Translation.

(24) A new representation of Gegenbauer's functions. J. R. Rice. Submitted to a technical journal.

(25) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(26) Split integration methods for simultaneous equations. J. R. Rice. Submitted to a technical journal.

(27) Split Runge-Kutta for simultaneous equations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(28) Tchebycheff approximations by $ab^{x}+c$. J. R. Rice. To appear in the Journal of the Society for Industrial and Applied Mathematics.

(29) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Submitted to a technical journal.

(30) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to Probability and Statistics", in press, Stanford University Press (1960).

(31) Statistical models for component aging experiments. J. R. Rosenblatt. To appear in the "IRE National Convention Record."

(32) Normal approximation to the chi-square and non-central F probability functions. N. C. Severo and M. Zelen. Submitted to a technical journal.

(33) The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. To appear in the Journal of Research, NBS, Sec. C. Engineering and Instrumentation.

(34) How to evaluate accuracy. W. J. Youden. Submitted to a technical journal.

(35) Multivariable experimentation. W. J. Youden. To appear in the Transactions of the Society of Automotive Engineers.

(36) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.

(37) Statistical aspects of the cement testing program. W. J. Youden. To appear in the Proceedings of the American Society for Testing Materials.

(38) Exact and approximate distributions for the Wilcoxon statistic with ties. S. Young. Submitted to a technical journal.

(39) Analysis of two-factor classifications with respect to life tests. M. Zelen. To appear in "Contributions to Probability and Statistics", in press, Stanford University Press (1960).

(40) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. To appear in Annals of Mathematical Statistics.

USCOMM-NBS-DC-53215

U.S. DEPARTMENT OF COMMERCE
Frederick H. Mueller, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

• Office of Basic Instrumentation.

• Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Research. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation Obstacles Engineering. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High-Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. High Frequency and Very High Frequency Research. Ultra High Frequency and Super High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.


NBS