NATIONAL BUREAU OF STANDARDS REPORT

6684

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

October through December 1959

For Official Distribution

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

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NBS PROJECT
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APPLIED MATHEMATICS DIVISION

October 1 through December 31, 1959

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**Part time

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*Only unclassified projects are included in this report.

Status of Projects

December 31, 1959

1. NUMERICAL ANALYSIS

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-12-11110/55-55

Origin: NBS Authorized 8/29/54
Manager: P. Davis
Full task description: July-Sept 1954 issue, p. 1

Status: CONTINUED. E. Haynsworth completed the paper on "Partitioned Matrices" and, in collaboration with Michael Drazin, RIAS, made additions to the paper on matrices with real roots. This paper has been retitled "Criteria for the Reality of Matrix Eigenvalues."

Dr. Haynsworth studied bounds for the condition numbers of matrices with positive roots, with the determinant of the matrix and one other elementary symmetric function of the roots (e.g., the trace of the matrix) known.

R. Silverman continued his investigation of the structure of metrized Cartesian product spaces. Characterizations were obtained for the distance matrix and its eigenvalues were determined. Dr. Silverman also obtained correspondences between various combinatorial configurations and subspaces of Euclidean n-space.

H. F. Weinberger derived the following results on approximation in L_i norm. It is well known that the problem of approximating a given function $f(x)$ by a linear combination

$$\sum_{1}^{n} a_i \phi_i(x)$$

of given functions ϕ_i in the sense of minimizing the L_2 deviation

$$\int_0^1 \left| f - \sum a_i \phi_i \right|^2 dx \tag{1}$$

leads to a set of n linear equations in the a_i. On the other hand, the problem of minimizing the L_1 deviation

$$\int_0^1 \left| f - \sum a_i \phi_i \right| dx \tag{2}$$

leads to the highly non-linear system

$$\int_0^1 \phi_j(x)\ \mathrm{sgn}(f - \Sigma a_i \phi_i)dx = 0. \tag{3}$$

By Schwarz's inequality

$$\left\{ \int_0^1 f - \Sigma a_i \phi_i \, dx \right\}^2 \leq \int_0^1 \frac{dx}{p(x)} \int_0^1 \left| f - \Sigma a_i \phi_i \right|^2 p(x)dx \tag{4}$$

for any positive function p(x). Equality is attained when

$$p = \left| f - \Sigma a_i \phi_i \right|^{-1}. \tag{5}$$

The problem of minimizing the second integral on the right of (5) again leads to linear equations for the a_i. This suggests an iteration process: The second integral on the right of (5) is minimized with respect to the a_i for fixed p. A new p is then chosen by means of (5), and the process is repeated. The convergence of this scheme is being investigated.

N. Bazley and P. Davis completed their theoretical and experimental study of the use of Monte Carlo methods for computing finite Markoff chains. A paper has been prepared for publication in a technical journal.

N. Bazley is investigating the calculation of eigenvalues for the spheroidal wave equation.

J. Rice studied a simple method of obtaining rational function approximations. An IBM 704 program was written to test it and to compare the results with known approximations.

Dr. Rice also studied an extrapolation procedure for sequences whose nth term is

$$\sum_{i=1}^{k} a_i b_i^n + c \quad \text{or} \quad \sum_{i=1}^{k} a_i b_i^n \cos(\phi_i + n\phi_i) + c.$$

Such sequences arise in many ways from operations involving matrices. The extrapolation formulas are based on the work in the paper "Tchebycheff approximations by ab^x+c and $ab^x\cos(\theta_o+\theta x) + c$." Both theoretical and numerical studies were made. In this connection Dr. Rice wrote a 704 program to evaluate two different algorithms for obtaining approximations by ab^x+c.

Dr. Rice also wrote two 704 programs to investigate the feasibility of an "optimized-split" integration method for simultaneous differential equations. Two particular differential equations were chosen. The programs will integrate them, each with different time steps which have been "optimized" so as to use a minimum of integration steps for a given interval.

Dr. Rice analyzed and P. J. Walsh programmed a problem to solve the partial differential equation

$$\frac{\partial}{\partial x}(h(x,y)\frac{\partial q}{\partial x}) + \frac{\partial}{\partial y}(h(x,y)\frac{\partial q}{\partial x}) = -\Lambda\frac{\partial(h(x,y)\sqrt{q})}{\partial x}$$

where h(x,y) is a given function and q(x,y) is to be determined. This equation is related to the theory of self-acting gas lubricated journal bearings. The program has three objectives: The first is to evaluate and study methods of accelerating the convergence of the numerical method to the solution. The second is to study the effect of using higher order difference approximations to the partial differential equation. It is suspected that large savings in computing may be made here. The third objective will be attempted only after the first two have been accomplished. If efficient means of integrating this equation are developed, then two sets of design curves would be computed for two particular bearing geometries.

Dr. Rice initiated work on another problem in this area concerned with numerical methods of satisfying an internal boundary condition involving a normal derivative. This problem comes from the theory of the infinite step bearing. He also made a theoretical study on the problem of approximation by convex polynomials. A tentative flow chart has been made of an IBM 704 code to obtain such approximations.

Dr. Rice made a long and unsuccessful attempt to prove the convergence of an algorithm for Tchebycheff approximations proposed by P. C. Curtis. Effort will continue on this problem.

In addition, he spent considerable effort on the analysis of a probability problem arising from an operations research analysis. The model under consideration may be described as follows: One has M machines and S space machines in which failures occur with an exponential distribution (i.e., at random). When a failure occurs the machine is sent to a repair station which will repair it in a fixed length of time. The repair station has unlimited capacity. The problem is to find various characteristics of the process so defined. The main effort so far has been to obtain a reasonable set of equations governing the process. This was accomplished only recently and in fact two different viewpoints are under development. This work is being done with some collaboration with W. E. Pruitt of Stanford University.

Publications:

(1) Basic theorems in matrix theory. M. Marcus. NBS Applied Mathematics Series 57. Available from U. S. Government Printing Office, Washington 25, D. C., 15 cents.
(2) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.
(3) Tchebycheff approximations by ab^x+c and $ab^x\cos(\theta_o+\theta x)+c$. J. Rice. Submitted to a technical journal.
(4) Split Runge-Kutta for simultaneous equations. J. Rice. In manuscript.
(5) A new representation of Gegenbauer's functions. J. Rice. Submitted to a technical journal.
(6) Split integration methods for simultaneous equations. J. Rice. In manuscript.

(7) Tchebycheff approximations by functions unisolvent of variable degree. J. Rice. Submitted to a technical journal.
(8) Applications of a theorem on partitioned matrices. E. Haynsworth. J. Research NBS, 63B, 73-78 (1959).
(9) Reduction formulas for partitioned matrices. E. Haynsworth. Submitted to a technical journal.
(10) Bounds for determinants with positive diagonals. E. Haynsworth. Submitted to a technical journal.
(11) Regions containing the characteristic roots of a matrix. E. Haynsworth. Submitted to a technical journal.
(12) Criteria for the reality of matrix eigenvalues. E. Haynsworth and M. Drazin. In manuscript.
(13) Partitioned matrices. E. Haynsworth. In manuscript.
(14) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam Milton Abramowitz.) P. J. Davis. American Math. Monthly, 66, 849-869 (1959).
(15) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. J. Research NBS 63B, No. 2, 131-169 (1959).
(16) Linear differential equations of the second order with large parameter. F. W. J. Olver. J. Soc. Indust. Appl. Math. 7, 306-310 (1959).

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-12-11411/55-56

Origin: NBS Authorized 8/13/54
Sponsor: Office of Naval Research
Manager: M. Newman
Full task description: July-Sept 1954 issue, p. 5

Status: CONTINUED. M. Newman prepared a manuscript in which it is shown that the members of a certain family of power series depending on a real parameter α are rational functions if and only if α is a rational number. One example is the power series $\sum_{n=0}^{\infty} x^{[n\alpha]}$. A set of notes on programming and coding from the mathematician's viewpoint was prepared by M. Newman. This was done in connection with the NBS-NSF Training Program in Numerical Analysis. These notes will appear as a chapter in a volume edited by Professor John Todd of C.I.T.

K. Goldberg continued his survey of recent work on incidence matrices. He also continued his investigation into the properties of the coefficients in the formal power series for $\log e^x e^y$.

K. Goldberg and E. C. Dade have completed a manuscript on the abstract properties of incidence algebras. A paper on the applications to v, k, λ designs, through group-generated incidence spaces, is now being prepared.

Dr. Goldberg studied the various forms of remainders in number theoretic sums and found relations among them. He also defined and studied

"pseudo multiplicative" Dirichlet series and showed that they are exactly the analytic functions of the logarithm of the Riemann zeta function.

Publications:

(1) Modular forms whose coefficients possess multiplicative properties. M. Newman. Ann. Math. 70, 478-489 (1959).
(2) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.
(3) Irrational power series. M. Newman. In manuscript.
(4) Generating functions for formal power series in non-commuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.
(5) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.
(6) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.
(7) The minimum of a certain linear form. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.
(8) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.
(9) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. Mich. J. Math. 6, 247-250 (1959).
(10) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors, V. (Usual Rayleigh quotient for non-Hermitian matrices and linear elementary divisors), Arch. Rat. Mech. Anal. 3, 472-481 (1959). Usual Rayleigh quotient for nonlinear elementary divisors, VI. Arch. Rat. Mech. Anal. 4, 153-165 (1959). A. M. Ostrowski.
(11) Weighted restricted partitions. M. Newman. To appear in Acta Mathematica.
(12) Über genaherte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. Monat. Math. u. Phys. 64, 51-63 (1960).
(13) Kantorovich's inequality. M. Newman. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

INFORMATION SELECTION SYSTEMS
Task 1101-12-11412/60-470

Origin: NBS Authorized 9/25/59
Sponsor: National Science Foundation
Managers: K. Goldberg, A. J. Goldman

Status: CONTINUED. K. Goldberg continued his investigations of the problem of classification and of the expected lengths of chains in an infinite flow of data through a computer with a finite memory.

A. J. Goldman, with B. K. Bender (11.2) and R. B. Thomas (12.5) completed a paper on "Computer simplification of Boolean functions". Work continues on proving (or disproving) the invariance of the results of Phase III of the computer program under permutations of the individual steps. B. K. Bender and A. J. Goldman found a counterexample to a suggestion by W. V. Quine (Amer. Math. Monthly, "On cores and prime implicants of truth functions") that an absolutely superfluous prime implicant must imply the core. A manuscript is in preparation on the general techniques developed in searching for this example.

Publications:

(1) A note on algebras. A. J. Goldman. Amer. Math. Month. (Math. Notes) 66, 795-796 (1959).
(2) A continuous poker game. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Duke Mathematical Journal.
(3) Computer simplification of Boolean functions. B. K. Bender (11.2), A. J. Goldman, and R. B. Thomas (12.5). Submitted to a technical journal.
(4) Some results on Boolean functions. A. J. Goldman and B. K. Bender. In manuscript.
(5) A symmetric poker game. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

ORTHOGONAL FUNCTIONS IN THE THEORY OF PARTIAL DIFFERENTIAL EQUATIONS
Task 1101-12-11413/60-469

Origin: NBS Authorized 9/25/59
Sponsor: Atomic Energy Commission
Managers: P. Davis, P. Rabinowitz

Status: CONTINUED. P. Rabinowitz solved numerically Dirichlet problems for the sphere and the cube by orthonormalizing the first 25 independent harmonic polynomials in three variables.

P. Rabinowitz wrote a double precision floating routine to generate an arbitrary number of particular solutions of the Dirichlet problem for multiply-connected domains. These solutions consist of $1, R(z-z_1), I(z-z_1), \ldots, R(z-z_1)^{n_1}, I(z-z_1)^{n_1}, \log|z-z_2|, R(z-z_2)^{-1}, I(z-z_2)^{-1}, \ldots, R(z-z_2)^{-n_2}, I(z-z_2)^{-n_2}, \ldots, \log|z-z_n|, R(z-z_n)^{-1}, I(z-z_n)^{-1}, \ldots, R(z-z_n)^{-n_m}, I(z-z_m)^{-n_m}$. They are then orthonormalized and fitted to the boundary conditions by a previously written routine.

P. Rabinowitz wrote routines to compute the coefficients of the interpolating polynomial passing through a given set of points using both Newton's method with divided differences and Lagrange's method.

P. Davis and P. Rabinowitz have investigated numerically the stability of polynomial coefficient determinations. A variety of distributions of abscissas both real and complex were assumed. A paper describing the results is in process.

P. Davis and P. Rabinowitz have investigated theoretically and numerically certain questions relating to the asymptotic behavior of the abscissas and weights of integration formulas of Gaussian type.

STUDY OF DIFFERENTIAL EQUATIONS FOR NERVE EXCITATION
Task 1101-12-11414/56-148

Origin and Sponsor: National Institutes of Health Authorized 9/30/55
Manager: P. Rabinowitz
Full task description: July-Sept 1955 issue, p. 7

Status: CONTINUED. A few runs on the 704 were made during this quarter as requested by the sponsor, continuing for the slightly modified form of the one-dimensional case.

2. MATHEMATICAL TABLES AND PROGRAMMING RESEARCH

TABLES OF COULOMB WAVE FUNCTIONS
Task 1102-40-11112/47-2

Origin: NBS Authorized 7/1/47
Manager: I. A. Stegun
Full task description: Apr-June 1949 issue, p. 45

Status: INACTIVE.

TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
Task 1102-40-11112/51-8

Origin: Section 11.3, NBS Authorized 3/26/51
Manager: S. Peavy
Full task description: Apr-June 1951 issue, p. 49

Status: INACTIVE.

REVISION OF MATHEMATICAL TABLES
Task 1102-40-11112/52-7

Origin: NBS Authorized 8/10/51
Manager: I. A. Stegun
Full task description: July-Sept 1951 issue, p. 41

Status: INACTIVE.

SPHEROIDAL WAVE FUNCTIONS
Task 1102-40-11112/52-37

Origin: NBS Authorized 11/28/51
Manager: D. Liepman
Full task description: Oct-Dec 1951 issue, p. 38

Status: INACTIVE.

SIEVERT'S INTEGRAL
Task 1102-40-11112/52-57

Origin: NBS Authorized 2/12/52
Managers: M. Paulsen, P. O'Hara
Full task description: Jan-Mar 1952 issue, p. 46

Status: INACTIVE.

HANDBOOK OF MATHEMATICAL FUNCTIONS
Task 1102-40-11421/57-216

Origin and Sponsor: National Science Foundation Authorized 12/27/56
Manager: I. A. Stegun
Full task description: Oct-Dec 1956 issue, p. 10

Status: CONTINUED. The textual material for Chapter 15, Hypergeometric Functions, for Chapter 16, Elliptic Functions, and for Chapter 21, Spheroidal Wave Functions, has been prepared and distributed for comments. At the present time, it is planned not to include any tables for hypergeometric functions. Preliminary tables have been set up for elliptic functions and spheroidal functions; if interpolation requirements are not met, these functions will be modified or the existing ranges extended.

Revisions are being made in Chapter 8, Legendre Functions, in Chapter 18, Weierstrass Elliptic Functions, and in Chapter 20, Mathieu Functions.

Experiments with the final format for the Handbook were made using Chapter 6, Gamma Functions. A preliminary page proof was obtained with split-column arrangement for the textual material, and with the tabular material reduced to appropriate size. Various difficulties encountered are being corrected in the preparation of subsequent chapters.

AUTOMATIC CODING
Task 1102-12-11120/55-65

Origin: NBS Authorized 9/29/54
Manager: J. Wegstein
Full task description: July-Sept 1954 issue, p. 11

Status: CONTINUED. The 704 Tablemaker program was expanded with the addition of mean, standard deviation, and sum computation, data plotting, and an expanded curve fitting program. Several improvements were made in the Bell monitoring system. The system now reloads with each job, and the programmer can now change the system if necessary during the running of his problem.

MATHEMATICAL SUBROUTINES
Task 3911-61-39952/56-160

Origin: NBS Authorized 9/30/55
Managers: Staff
Full task description: July-Sept 1955 issue, p. 13

Status: CONTINUED. The polynomial curve fitting code described in the July-Sept 1959 issue (p. 11) has been written by P. Walsh and is being checked out. Given a set of points and their corresponding functional values, the program finds the best fitting polynomial (in the least squares sense) of each degree less than or equal to a maximum degree specified by the user. For each degree, the coefficients, deviations and standard deviations are printed. The covariance matrix is computed after the highest degree polynomial has been determined. Weights are optional, and the value of the polynomial for each degree is also computed for a given set of points supplied as input to the program.

The routine which computes the eigenvalues and eigenvectors of a real symmetric matrix has been merged by P. Walsh with another program, by R. Zucker, which sorts the roots algebraically and rearranges the vectors with their corresponding roots. The program then prints the roots and vectors in the rearranged compact form.

A Fortran program was written by P. Walsh to evaluate

$$F(a) = \int_a^b f(\alpha,x)dx$$

using Simpson's Rule for integration. The user must write his own Fortran subroutine to evaluate $f(\alpha,x)$. The routine evaluates n integrals

$$\int_{L_i}^{L_{i+1}} f(\alpha,x)dx$$

and stops whenever the difference between two successive approximations is less than a prescribed tolerance ϵ_i or when P_i points have been used to approximate the integral

$$F(\alpha) = \int_a^b f(\alpha,x)dx$$

$$= \int_{a=L_1}^{L_2} f(\alpha,x)dx + \int_{L_2}^{L_3} f(\alpha,x)dx + \ldots + \int_{L_n}^{L_{n+1}=b} f(\alpha,x)dx$$

Thus the programmer supplies n values ϵ_i, n values P_i, and (n+1) values L_i. Up to 10 values of the parameter α may be supplied at one time.

3. PROBABILITY AND MATHEMATICAL STATISTICS

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-12-11131/51-2

Origin: NBS Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

Status: CONTINUED. J. R. Rosenblatt worked out, for a special case, some properties of an experimental design procedure suggested by W. J. Youden, in which the treatment to be applied in the ith trial depends on the outcome of all previous trials. The procedure was proposed by Dr. Youden in connection with the preparation of a talk on "Problems of Experimental Design," which was given during the annual meeting of the Institute of Mathematical Statistics, December 30, at a session on the "Final Report of the Advisory Committee on Weather Control."

Publications:

(1) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. Joel. Ann. Math. Stat. 30, 885-895 (1959).
(2) Graphical computation of bivariate normal probabilities. M. Zelen and N. C. Severo. Submitted to a technical journal.
(3) Selected bibliography of statistical literature, 1930-1957. I. Correlation and Regression Theory. II. Time Series. Lola S. Deming. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-12-11131/53-1

Origin: NBS Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: INACTIVE.

Publications:

(1) Randomization and experimentation. W. J. Youden. To appear in Annals of Mathematical Statistics.
(2) Analysis of fractionally replicated 2^m3^n designs. R. C. Bose and W. S. Connor. To appear in the Proceedings of the International Statistical Institute, Brussels, 1958.

(3) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to Probability and Statistics," in press, Stanford University Press (1960).

STUDY OF NON-PARAMETRIC STATISTICAL TECHNIQUES
Task 1103-12-11131/56-170

Origin: NBS Authorized 12/15/55
Manager: J. R. Rosenblatt
Full task description: Oct-Dec 1955 issue, p. 14

Status: INACTIVE.

Publications:
(1) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to Probability and Statistics," in press, Stanford University Press (1960).
(2) Exact and approximate distributions for the Wilcoxon statistic with ties. Shirley Young. Submitted to a technical journal.

MEASUREMENT OF RELIABILITY
Task 1103-12-11130/56-182

Origin: NBS Authorized 3/23/56
Manager: M. Zelen, J. R. Rosenblatt
Full task description: Jan-Mar 1956 issue, p. 13

Status: CONTINUED. J. R. Rosenblatt and M. Zelen participated in a Conference on Mathematics and Statistics for Reliability Problems held under ASA-ASQC sponsorship at New York University, October 16-18. Dr. Rosenblatt presented a paper on "Models for the analysis of component aging experiments". Dr. Zelen spoke on "The analysis of life tests at combined environmental conditions".

Dr. Zelen and Mary C. Dannemiller prepared a paper entitled "Are life testing procedures robust?", for presentation at the Sixth National Symposium on Reliability and Quality Control. Miss Dannemiller completed an additional investigation of these robustness questions and prepared a paper on "The robustness of certain life testing procedures derived from the exponential distribution". This work shows that certain life testing procedures should not be used when the underlying assumptions do not hold.

Publications:
(1) Analysis of two-factor classifications with respect to life tests. M. Zelen. To appear in "Contributions to Probability and Statistics," in press, Stanford University Press (1960).

(2) Are life testing procedures robust? M. Zelen and M. C. Dannemiller. To appear in the Proceedings of the Sixth National Symposium on Reliability and Quality Control (1960).

(3) The robustness of certain life testing procedures derived from the exponential distribution. M. C. Dannemiller. Submitted to a technical journal.

4. MATHEMATICAL PHYSICS

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-12-11141/55-57

Origin: NBS Authorized 9/1/54
Manager: W. H. Pell
Full task description: July-Sept 1954 issue, p. 27

Status: CONTINUED. A. Ghaffari has completed the manuscript of his paper on Rayleigh's self-excited quasi-harmonic equation. The location and the nature of the critical points at infinity are determined. The global qualitative description of the totality of the paths of Rayleigh's equation in the phase plane and in the projective plane near the line at infinity are investigated at length.

In the St. Venant flexure problem for an elastic isotropic beam one seeks the point in the plane end of the beam which has the property that any load (in the plane of the end) applied through this point produces torsionless bending. This load point is called the center of flexure. A method has been derived by L. E. Payne for obtaining improvable upper and lower bounds for the coordinates of the center of flexure. A paper on this subject has been written.

Publications:

(1) Turbulent motion. G. B. Schubauer (NBS Fluid Mechanics Section) and C. M. Tchen. Appears as Section B in Volume V of the Princeton Series, High Speed Aerodynamics and Jet Propulsion, "Turbulent Flows and Heat Transfer," pp. 75-190 (Princeton University Press, 1959).
(2) The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. Submitted to a technical journal.
(3) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. To appear in the Journal of Fluid Dynamics.
(4) The Stokes' flow problem for a class of axially symmetric bodies, II. The flow about a spindle. L. E. Payne and W. H. Pell. To appear in the Quarterly of Applied Mathematics.
(5) Upper and lower bounds for the center of flexure. L. E. Payne. In manuscript.

PLASMA RESEARCH
Task 1104-12-11140/59-422

Origin: NBS Authorized 6/30/59
Manager: C. M. Tchen
Full task description: Apr-June 1959 issue, p. 15

Status: CONTINUED. C. M. Tchen has continued his work on dynamics of plasma. Emphasis was given to the structure of the correlation function for a plasma. The theory was based on the hierarchy equations for the distribution functions, and on the degeneration of correlations which enables the closure of the differential system. The basis was similar to the derivation of the kinetic equation, made by Tchen (see Phys. Rev. 114, 394-411, 1959). Attention was given to the shielding mechanism provided by the plasma cloud. The results indicated that there does not exist a Debye sphere of shielding, as has been usually assumed, but that the geometry of shielding depends on the particle velocity. When the particle velocity exceeds its thermal velocity, the shielding is restricted within a trailing wake, comparable to the Cerenkov effect. This arises because the shielding cloud does not have the time to develop and catch up with the particle. A manuscript is in preparation.

Since September, Tchen has been conducting his research at the U. K. Atomic Energy Research Establishment at Harwell, England, as a Guggenheim Fellow.

RESEARCH ON SATELLITE ORBITS
Task 1104-12-11440/59-420

Origin: NBS Authorized 12/19/58
Sponsor: Office of Scientific Research, ARDC, USAF
Manager: J. P. Vinti
Full task description: Oct-Dec 1958 issue, p. 15

Status: CONTINUED. J. P. Vinti has now expanded the fundamental frequencies n_1, n_2 , and n_3 into power series through the second power of the oblateness parameter k. There are no singularities for any values of the orbital inclination or eccentricity. These results lead to expressions for the secular motions of the line of apsides relative to the line of nodes and of the line of nodes relative to the vernal equinox.

In finding these expressions it is necessary to use the theorem that in a separable Hamilton-Jacobi system the mean frequency f_k of a separation coordinate q_k is equal to the corresponding fundamental frequency n_k. This theorem seems to be familiar to all investigators in celestial mechanics, but a search of the literature and a number of inquiries failed to turn up any general proof. It therefore appeared worthwhile to devise a simple proof and this has been done.

A paper on satellite frequencies with a new gravitational potential is in preparation.

Publication:

(1) A new method of solution for unretarded satellite orbits. J. P. Vinti. J. Research NBS 63B, 105-116 (1959).

FOURIER TRANSFORMS OF PROBABILITY DISTRIBUTION FUNCTIONS
Task 1104-12-11626/56-154

Origin: NBS Authorized 9/30/55
Sponsor: Office of Naval Research
Manager: F. Oberhettinger
Full task description: July-Sept 1955 issue, p. 20

Status: CONTINUED. Preparation of the manuscript in a form suitable for publication continued.

5. MATHEMATICAL AND COMPUTATIONAL SERVICES

3911-61-39952/54-30 SPECTRUM ANALYSIS
Origin: NBS, Division 4
Managers: C. D. Coleman, W. Bozman (4.1)
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Wavenumbers were computed for about 6,000 ytterbium lines. The line list for 12,000 thorium lines has been put on tape preparatory to printing for publication. More than 12,000 lines of praseodymium have been measured and their wavelengths and wavenumbers computed, leaving about 4,000 more to be measured.

About 30,000 cards have been punched for the intensity tables of 70 elements. The code has been written and is now being used to calculate observed patterns from interferometry of a circular atomic beam. The code is in preparation for computation of a rectangular beam.

3911-61-39952/54-38 EQUATION OF STATE OF REAL GASES
(formerly COMPRESSIBILITY FACTORS OF DRY AIR)
Origin: NBS, Section 3.2
Manager: M. Paulsen
Full task description: Jan-Mar 1954 issue, p. 48
Status: Inactive.

3911-61-39952/55-68 CRYSTAL STRUCTURE CALCULATIONS
Origin: NBS, Division 9
Managers: P. O'Hara, S. Block (9.7)
Full task description: Jan-Mar 1955 issue, p. 18
Status: Continued. Routine structure determination calculations were carried out for the several crystals currently under investigation.

3911-61-39952/55-82 THERMOMETER CALIBRATIONS
Origin: NBS, Section 3.1
Manager: S. Prusch
Full task description: Jan-Mar 1955 issue, p. 20
Status: Continued. ITS constants and tables were computed for 65 thermometers under test. LTS constants were calculated for 18 thermometers.

1102-40-11645/56-186 MECHANICAL MEASUREMENTS OF GAGE BLOCKS
Origin and Sponsor: NBS, Section 2.5
Manager: S. Prusch
Full task description: July-Sept 1956 issue, p. 33
Status: Continued. Computations were performed for checking 10 laboratory sets of gage blocks.

1102-40-11645/57-219 THERMAL PROPERTIES
Origin and Sponsor: NBS, Section 3.2
Manager: R. Varner
Full task description: Oct-Dec 1956 issue, p. 30
Status: Continued. Production runs were continued under the direction of the sponsor.

1102-40-11645/57-221 BESSEL FUNCTIONS FOR COMPLEX ARGUMENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Oct-Dec 1956 issue, p. 31
Status: Inactive.

3911-61-39952/57-223 SELF-CONSISTENT FIELDS
Origin: NBS, Section 3.2
Manager: E. V. Haynsworth (11.1)
Full task description: Apr-June 1957 issue, p. 28
Status: Reactivated. The sponsor requested some modifications to the input of the A-matrix subprogram. These required minor changes in the program and revisions in the A-matrix write-up.

3911-61-39952/57-229 APPLICATION OF ELECTRONIC DATA PROCESSING MACHINERY TO PAYROLL OPERATIONS
Origin: NBS, Section 40.0
Managers: M. Paulsen, P. Ruttenberg
Full task description: Jan-Mar 1957 issue, p. 36
Status: Inactive.

1102-40-11645/57-236 SELF CONSISTENT FIELDS--EIGENVALUES
Origin and Sponsor: NBS, Section 3.6
Manager: E. Haynsworth (11.1)
Full task description: Apr-June 1957 issue, p. 30
Status: Inactive.

1102-40-11645/58-263 GAS TUBE CHARACTERISTIC
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: I. A. Stegun
Full task description: July-Sept 1957 issue, p. 35
Status: Terminated. For further related work, see task 1102-40-11645/60-476, p. 30, Gas Tube Characteristic II.

3911-61-39952/58-266 DEPOLYMERIZATION, II
Origin: NBS, Section 7.6
Manager: L. S. Joel
Full task description: July-Sept 1957 issue, p. 36
Status: Inactive.

1102-40-11645/58-269 MOLECULAR STRUCTURE, IV
Origin and Sponsor: Naval Research Laboratory, USN
Manager: P. J. O'Hara
Full task description: July-Sept 1957 issue, p. 38
Status: Continued. Phases were computed for about 400 reflections of the cellobiose crystal, and these results will be used to calculate the three-dimensional Fourier map. Rational dependence corrections and a few trial phases were calculated for the arginine crystal.

1102-40-11645/58-270 MATHEMATICAL PROBLEMS RELATED TO POSTAL OPERATIONS
Origin: NBS
Sponsor: Post Office Department, Office of Research and Engineering
Managers: B. K. Bender, A. J. Goldman
Full task description: Oct-Dec 1958 issue, p. 22
Status: Continued. The optimal locations of mail-sorting installations in a large square area were determined under the following assumptions: (1) area is a uniform sink and source of mail; (2) area is divided into N^2 equal square regions, each with a single sorting center; (3) all transport routes are parallel to the sides of the square; (4) intra- and inter-region transportation costs per unit of mail per mile remain constant. Related mathematical models are being investigated.
Publications: (1) Analytic comparison of suggested configurations for automatic mail sorting equipment. B. K. Bender and A. J. Goldman. J. Research NBS 63 B, 83-104 (1959). (2) Capacity requirement of a mail sorting device, II. A. J. Goldman. J. Research NBS 63 B, 79-82 (1959).

1102-40-11645/58-272 THERMODYNAMIC PROPERTIES OF REAL GASES
Origin and Sponsor: NBS, Section 3.2
Manager: J. P. Menard
Full task description: Oct-Dec 1957 issue, p. 32
Status: Continued. In connection with the retabulation of existing tables of thermodynamic functions, the inverse interpolation and sub-tabulation code has been checked out completely and is ready for production runs. Investigations are being carried out with respect to differentiation subroutines. A code has been written which fits a second degree polynomial to 3, 5, or 7 points in the least squares sense, and evaluates the derivative of this second degree polynomial at the inner point of the interval, i.e., the second in the case of 3 points, the third in the case of 5 points, and the fourth in the case of 7 points. The end points of the interval are taken care of separately. Further investigations will be made in this direction.

1102-40-11645/58-274 CALCULATIONS FOR d-SPACINGS, II
Origin and Sponsor: NBS, Division 9
Manager: R. Zucker
Full task description: July-Sept 1957 issue, p. 38
Status: Terminated. The calculations will be continued as production runs under the direction of the sponsor. The machine time will be reported in the section of this Report entitled, "Application of Automatic Computer."

1102-40-11645/58-304 TRANSPORT PROPERTIES OF AIR AT ELEVATED TEMPERATURES
Origin and Sponsor: NBS, Section 3.2
Manager: P. J. Walsh, J. D. Waggoner
Full task description: Oct-Dec 1957 issue, p. 40
Status: Continued. A program was written to arrange in table form the output from the code which computes n (viscosity), K_o (thermal conductivity due to the transport of translational energies), C_{ij} (diffusion coefficients), etc. Further code checking on the potential parameter code is pending receipt of input data from the sponsor. Production runs were made using the force constants code.

1102-40-11645/58-307 STUDY OF SURFACE TENSION
Origin and Sponsor: NBS, Section 9.2
Manager: R. Arms
Full task description: Oct-Dec 1957 issue, p. 43
Status: Inactive.

1102-40-11645/58-316 INTERSECTION CAPACITY STUDY
Origin and Sponsor: Bureau of Public Roads
Managers: S. Peavy, J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 33
Status: Inactive.

1102-40-11645/58-339 COMPUTATION OF VISCOELASTICITY PROPERTIES OF MATERIALS
Origin and Sponsor: NBS, Section 3.4
Manager: H. Oser
Full task description: Jan-Mar 1958 issue, p. 38
Status: Continued. Stress-relaxation as well as creep functions have been computed over a wide range of over fifteen decades of time. The long time range expressions reach down to about 10^{-8} sec and can be obtained in different ways either by evaluating the inverse Laplace transform by way of the residue technique or by expanding the solution of the original boundary value problem in a series of eigenfunctions. Convergence has been found to be fairly good. A Fortran program has been written for computing these series. For $t<1$ sec, convergence is quite slow and summation has been terminated after 2500 terms. Below that and down to about 10^{-8} sec, the asymptotic expansion of these long time range expressions has been used.

For times shorter than 10^{-8} sec, the interference of a constant c_1 enters in, whereas it had been neglected in the expression mentioned above. Due to the fact that t is already so small, a power series expansion has been used to close the gap down to t=0.

Codes for the different true ranges are available and will be used soon to check the theory with experiments on polymers other than polyisobutylene.

1102-40-11645/59-348 RUSSIAN-TO-ENGLISH MACHINE TRANSLATION
Origin: NBS
Sponsor: Office of Ordnance Research, U. S. Army
Manager: I. Rhodes
Full task description: Oct-Dec 1958 issue, p. 26
Status: Continued. An NBS Report "A New Approach to the Mechanical Syntactic Analysis of Russian" has been completed and distributed. It details the status of the project as of the fall of 1959. Progress has been made principally in determining how to select among several possible grammatical interpretations of a Russian word. The present project is characterized, in comparison with other projects concerned with machine translation, by its emphasis on syntax in the conventional sense, and by a system of "predictions". A Russian word occurring in a sentence "predicts" certain other grammatical forms, e.g., a transitive verb predicts an accusative. The codes generating such predictions have been planned for a number of cases; so have codes for comparing actual occurrences with predictions.

1102-40-11645/58-358 REDUCED CROSS-SECTIONS
Origin and Sponsor: NBS, Section 3.2
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 30
Status: Inactive.

1102-40-11645/58-361 CALCULATIONS FOR SPECTRUM OF DIPOLE RADIATION
Origin and Sponsor: Naval Research Laboratory
Manager: R. J. Arms
Full task description: Apr-June 1958 issue, p. 33
Status: Continued. Production runs have been transmitted to the sponsor.

1102-40-11645/58-366 RADIATION PATTERNS OF ANTENNAS
Origin and Sponsor: U. S. Information Agency, Department of State
Managers: P. J. Walsh
Full task description: Apr-June 1958 issue, p. 35
Status: Inactive.

1102-40-11645/58-368 INTENSITY FUNCTIONS AND CROSS SECTIONS OF LIGHT SCATTERED BY SPHERICAL PARTICLES

Origin and Sponsor: U. S. Army Signal Research and Development Laboratories, Atmospheric Physics Branch, Belmar, N. J.

Manager: H. Oser

Full task description: July-Sept 1958 issue, p. 32

Status: Continued. The program to compute the scattering functions of monochromatic light incident to a sphere with a coating of different electromagnetic properties has been streamlined and made ready for production. About 130 scattering diagrams have been computed and submitted to the sponsor.

1102-40-11645/59-377 LOGICAL DIAGRAM REDUCTION

Origin and Sponsor: NBS, Section 12.3

Managers: W. Hall, C. Coleman (4.01)

Full task description: Apr-June 1959 issue, p. 25

Status: Continued. Production runs continued as the development changes in the logical design of the PILOT computer were made.

1102-40-11645/59-388 HEAT PUMP CALCULATIONS

Origin and Sponsor: NBS, Section 10.3

Manager: R. Zucker

Full task description: Jan-Mar 1959 issue, p. 26

Status: Continued. Additional production runs with heating and cooling data were made.

1102-40-11645/59-389 FREQUENCY ALLOCATION

Origin and Sponsor: Civil Aeronautics Administration

Manager: L. S. Joel

Full task description: Oct-Dec 1958 issue, p. 29

Status: Inactive. A production run with the "one interchange" program will be made as soon as amended data are received from the sponsor.

1102-40-11645/59-394 VARIATIONAL CALCULATION OF SLOW ELECTRON SCATTERING BY HYDROGEN ATOMS, II

Origin and Sponsor: NBS, Section 4.6

Manager: A. Beam

Full task description: Oct-Dec 1958 issue, p. 30

Status: Continued. Most of the production runs have been completed under the direction of the sponsor.

1102-40-11645/59-403 COMPUTATION OF COLOR FADINGS
Origin and Sponsor: NBS, Section 2.1
Manager: J. P. Menard
Full task description: Oct-Dec 1958 issue, p. 30
Status: Terminated. The program will continue in production under the sponsor's direction. Production time on the 704 will be reported hereafter in the section of this Report, "Application of Automatic Computer".

1102-40-11645/59-407 FOURIER COEFFICIENTS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 27
Status: Continued. Production runs continued on the 704.

1102-40-11645/59-412 DYNAMICS OF PNEUMATIC PRESSURE REDUCERS
Origin and Sponsor: NBS, Section 3.2
Manager: A. Beam
Full task description: Oct-Dec 1958 issue, p. 31
Status: Completed. Results have been transmitted to the sponsor.

1102-40-11645/59-414 INFINITE SYSTEMS
Origin and Sponsor: NBS, Division 3
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 28
Status: Inactive.

1102-40-11645/59-415 COMPLEX LEGENDRE FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: Jan-Mar 1959 issue, p. 29
Status: Continued. Additional production runs were made on the 704.

1102-40-11645/59-418 P-WAVE EQUATION
Origin and Sponsor: NBS, Section 4.8
Managers: S. Peavy, R. Varner
Full task description: Jan-Mar 1959 issue, p. 29
Status: Terminated. Production runs will be continued under the direction of the sponsor. Machine time on the 704 will appear in the section of this Report, "Application of Automatic Computer."

1102-40-11645/59-434 PETROLOGICAL COMPUTATION
Origin and Sponsor: Geophysical Laboratories, Carnegie Institution of Washington
Manager: R. Varner
Full task description: Apr-June 1959 issue, p. 28
Status: Terminated. Additional runs on the 704 will be made under the direction of the sponsor. Machine time on the 704 will be reported in the section of this Report, "Application of Automatic Computer."

1102-40-11645/59-435 ELECTROCARDIOGRAPHIC ANALYSIS
Origin: NBS, Division 12.5
Sponsor: Veterans Administration
Manager: R. J. Arms
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. A sampling code has been checked out. Presently the code is being applied to a small experiment desired by the sponsor.

1102-40-11645/59-444 HOSPITAL SUBSISTENCE ITEMS SUPPLY
Origin and Sponsor: Veterans Administration
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 29
Status: Continued. A preliminary report has been submitted to the sponsor, which describes feasibility limitation in terms of problem size.

1102-40-11645/59-445 OIL SUPPLY
Origin and Sponsor: Military Petrolem Supply Agency, Department of the Navy
Manager: L. S. Joel
Full task description: Apr-June 1959 issue, p. 30
Status: Continued. Two allocation problems were solved using the Share 704 Transportation code. A new computer program by Massachusetts Institute of Technology Lincoln Laboratory with improved input editing will be investigated for future use in this project.

1102-40-11645/60-449 SPECTRAL LINE COLORIMETRY
Origin and Sponsor: NBS, Section 2.1
Manager: J. P. Menard
Full task description: July-Sept 1959 issue, p. 28
Status: Terminated. The program will continue in production under the sponsor's direction. Production time will be reported hereafter in the section, "Application of Automatic Computer."
Publications: (1) IBM 650 Computer program for CIE color specifications of objects illuminated by sources having continuous plus line spectra. John C. Schleter and John P. Menard, an NBS Report. (2) IBM 650 Computer program for CIE color specifications of objects illuminated by sources having continuous plus line spectra. John C. Schleter and John P. Menard, an NBS Report.

1102-40-11645/60-452 CONSULTING SERVICES FOR PACIFIC MISSILE RANGE
Origin and Sponsor: NBS, Section 12.3
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 29
Status: Continued. For orientation and background, existing and planned computer methods and equipment were observed during a visit to the Atlantic Missile Range at Cape Canaveral during December, 1959.

1102-40-11645/60-453 DATA CONVERSION
Origin and Sponsor: Army Map Service
Manager: J. Beiman
Full task description: July-Sept 1959 issue, p. 30
Status: Continued.

1102-40-11645/60-456 FIRING CIRCUIT EQUATIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 30
Status: Inactive.

1102-40-11645/60-458 DOMESTIC AIRLINE TRAFFIC SURVEY
Origin and Sponsor: Civil Aeronautics Board
Managers: J. Beiman, W. Hall
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. A code for the assembling of domestic airline passenger data was written and is being checked out.

1102-40-11645/60-459 TRANSCENDENTAL EQUATIONS
Origin and Sponsor: NBS, Section 9.01
Manager: R. Zucker
Full task description: July-Sept 1959 issue, p. 31
Status: Continued. The code was written and checked out. Sample calculations for u and γ were carried out.

1102-40-11645/60-460 DESIGN EQUATIONS FOR MAGNETRON INJECTION ELECTRON GUNS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: A. Beam
Full task description: July-Sept 1959 issue, p. 32
Status: Continued. The code was completed and checked out, and several production runs were made.

1102-40-11645/60-461 DATA SYSTEMS LANGUAGES
Origin and Sponsor: NBS, Section 11.2
Manager: J. H. Wegstein
Full task description: July-Sept 1959 issue, p. 33
Status: Continued. The Short-Range Data Systems Language Committee completed a proposal for a common business-oriented language called COBOL. This proposal in the form of a technical reference manual has been submitted, in manuscript form, for publication by the U. S. Government Printing Office.

1102-40-11645/60-462 CORRELATION OF FUNCTIONS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: G. W. Reitwiesner
Full task description: July-Sept 1959 issue, p. 33
Status: Reactivated. The computation has been generalized to determine the correlation between the functions

$$F_i = \frac{1 + a \cos (A_i + P_i \alpha)}{1 + a^2 + 2a \cos (A_i + P_i \alpha)} , \qquad i = 1,2,$$

for $P_1 = 1 < P_2 \leq 2$; $A_2 = 0 \leq A_1 < 2\pi$; $0 \leq \alpha < 2\pi$; a = 0.5, 0.9, 0.98, and has been expanded to include the determination of the mean and root mean square values of the functions:

$$\text{mean:} \quad \min (F_1, F_2)$$

$$\text{root mean square:} \quad \min (F_1, F_2) - 1/(1+a)...$$

1102-40-11645/60-464 CORRELATION MATRIX FOR PHS DATA
Origin and Sponsor: Public Health Service
Manager: J. Beiman
Full task description: July-Sept 1959 issue, p. 34
Status: Continued. Original objective has been obtained and results were submitted to sponsor. The program was then broadened to compute the Chi square and Phi coefficient of correlation. This has been code checked and sponsor has received his desired results.

1102-40-11645/60-465 CALCULATIONS IN MOLECULAR QUANTUM MECHANICS
Origin and Sponsor: NBS, Section 3.2
Managers: P. J. Walsh, J. D. Waggoner
Objective: To determine the separability of internuclear parameters from electronic coordinates in polyatomic systems.

Background: The standard LCAO-SCF procedure has been programmed on the 704 at NBS. This procedure is being used to calculate the interaction energies of atoms and molecules, as a function of internuclear distance for atoms and molecules. This project is making some of these calculations for selected systems in order to investigate the validity of the adiabatic approximation as well as the usual assumptions of the Born-Oppenheimer approximation.

The problem was submitted by V. Griffing, Consultant (3.2).

Status: New.

1102-40-11645/60-466 ELECTRONIC PROPERTIES OF SIMPLE MOLECULAR SYSTEMS

Origin and Sponsor: NBS, Section 3.2

Manager: P. J. Walsh

Objective: Computation of total energies, ionization potential and dissociation energies for the ground state and certain excited states of selected diatomic molecules; the calculation of potential curves for these molecules, to be used in calculation of thermodynamic and transport properties, is considered the ultimate objective.

Background: Information on the lowest excited levels is needed for thermodynamic studies of air mixtures and hydrides, oxides and hydrides of the light metals. Only fragmentary information is available at this time from the usual experimental source, and recourse is being made to *a priori* quantum mechanical calculation of the desired properties. Computer programs designed for this purpose have been constructed by R. K. Nesbet (Boston University), portions of which were coded by members of the NBS Computation Laboratory (see task 3911-61-39952/57-223, p. 18). The complete program, only recently compiled, will be utilized for this project. Its components consist of an extremely efficient "Diatomic Integral Program," and "A-matrix Program", an "SCF Program," and a "Control Program."

The problem was submitted by B. J. Ransil (3.2).

Status: New.

1102-40-11645/60-467 TRANSISTOR SIMULATION

Origin and Sponsor: NBS, Section 12.1

Manager: G. W. Reitwiesner

Objective: To simulate, on a digital computer, the behavior of a transistor in a digital circuit, where the transistor and circuit characteristics are expressed as parameters which may vary over representative ranges; and to study the optimization of circuit behavior through the adjustment of parameter assignments.

Background: A large signal equivalent circuit has been developed by NBS, Section 12.1, to characterize transistor behavior and has been successfully tested, using an analog computer. To exhaust the numerous practical combinations of values of parameters which describe the circuit, analysis by digital computer techniques is desired.

The problem was submitted by J. A. Cunningham (12.1).

Status: New. Preliminary programming for the central portion of the code has been prepared.

1102-40-11645/60-471 INTERLABORATORY STANDARDIZATION OF TESTING PROCEDURES
Origin and Sponsor: NBS, Section 7.3
Manager: A. Beam
Objective: To prepare a program for the rapid handling and analysis of the large amount of data required (1) for the proper evaluation of a testing procedure, and (2) for establishing the need for standard samples. The program is to provide for input data in several forms: (1) original observations, with the possibility of some missing observations; (2) averages and standard deviations; (3) original observations, including observations on standard samples.
Background: The evaluation and standardization of testing procedures must ultimately include an interlaboratory or "round robin" study. Because the conventional design and analysis of an interlaboratory study have generally yielded disappointing results, Mandel and Lashof (ASTM Bulletin No. 239, p. 53 (July 1959)) have proposed a new procedure based on a fundamental approach to the measuring process (Mandel, Technometrics 1, no. 3, p. 251 (August 1959)). The statistical analysis of the data, in accordance with the new procedure, will be greatly facilitated by the proposed 704 program.
The problem was submitted by T. W. Lashof (7.3).
Status: New.

1102-40-11645/60-472 PROBABILISTIC ENCOUNTER ANALYSIS
Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army
Manager: J. Beiman
Objective: To select sets of points at random, on the face of the earth, using degrees and minutes longitude and latitude and compensating for the convergence of the longitude lines at the poles. Also, to assign to each of the points of the second set a random number from 1 to 12 representing months of a year.
Background: Climactic environments and terrain features affect to varying degrees the utilization as well as the performance reliability of most military weapons systems. Randomly selected probable storage or tactical sites will be catalogued as to their predominant features and the data will be used to statistically analyze the reliability of a weapon on an objective basis.
The problem was communicated by P. Oakley (DOFL).
Status: Completed (New). The problem has been coded and checked out, and the results submitted to the sponsor.

1102-40-11645/60-474 GAGE BLOCK STABILITY
Origin and Sponsor: NBS, Section 2.05
Manager: J. Beiman
Objective: To prepare a 704 program for the analysis of electrical intercomparator results on gage blocks used in a study of the temporal stability of gage blocks of differing metallurgical background. The analysis provides estimates of differences between blocks and the appropriate statistical measures of precision necessary for the study of temporal effects and of the instrumentation involved.

Background: This program, carried on in conjunction with the Metallurgy Division, has as its objective the development of gage blocks dimensionally stable to within 2 x 10^{-7}in/in. The objective has two aspects: (1) the development of suitable material and treatment to produce a gage block with the desired dimensional stability, surface quality, and other physical properties, and (2) the development of instrumentations and techniques for the precise measurement of temporal changes of length, so that stability of test samples may be determined to the required precision. The use of electronic computation for the analysis and evaluation of the large groups of measurement data resulting from this study represents a saving in both time and cost.

The problem was submitted by W. R. Ney (2.05).

Status: Terminated (New). The code has been written and a number of sets of data were processed. As the study progresses, additional sets will be analyzed under the direction of the sponsor. In the future, machine time will be reported in the section of this Report, "Application of Automatic Computer."

1102-40-11645/60-475 IONOSPHERIC SOUNDINGS

Origin and Sponsor: NBS, Section 82.40

Manager: M. Paulsen

Objective: (1) To compute hourly electron density profiles from virtual height data, in large quantity. (2) To express results in the form of tabulations of electron density at 10 km. levels of height between the bottom of the ionospheric E layer (daytime) or F_2 layer (nighttime) and the level h_{max} F_2. (3) To determine additional parameters including: h_{max} F_2, by fitting a parabola to the upper portion of the profile; Sh_{max}, by integrating the distribution between the limits h_{min} and h_{max}.

Background: This program has previously been developed, in substantially the present form, over the past two years at the Boulder Laboratories of NBS, for a 650 computer. The 704 permits a vastly greater quantity of data processing at greatly reduced cost. The results become basic research data for the ionospheric programs of the Boulder Laboratories, and afford opportunities for research into electron production, loss and movement processes. The data are also applied to numerous aspects of the U. S. Space program, including the tracking of launching vehicles, the determination of precise satellite orbits, and for comparison with satellite and rocket determinations of ionospheric characteristics.

The problem was submitted by J. W. Wright (82.40 Boulder).

Status: New. The program as requested has been written in Fortran. The code consists of approximately 250 Fortran statements, which assembled into a SAP code having more than 1500 orders. The code has been checked out, and a parallel run is underway to check the results against those obtained by using the 650 code.

1102-40-11645/60-476 GAS TUBE CHARACTERISTIC, II

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: H. Oser

Objective: To compute voltage current densities of low pressure cold-cathode gas discharges, attempting to obtain results over a fairly wide range of total current densities from 10^{-12} amp/cm^2 up to about 10^{-2} amp/cm^2.

Background: Because earlier computations could not be carried out for current densities higher than about 10^{-3} amp/cm^2 and also because they proved to be very slow, an attempt is being made to extend the range of the computations as far as possible and to speed up the convergence.

The computations are based on Townsend's basic ionization equations for cold-cathode gas discharges between parallel plates, which have been modified by Poisson's equation to account for space charge effects (see A. L. Ward, Phys. Rev. 112, 1852-1857, 1958). Townsend's equations are amended by a term proportional to the square of the electron current density. A new formulation of the positive-ion mobility is being used, and various approximations for the total current density are anticipated.

The problem was communicated by A. L. Ward, DOFL. It is an extension of task 1102-40-11645/58-263.

Status: New.

1102-12-11122/60-479 PROCESSING OF DIAGRAMS

Origin and Sponsor: NBS, Section 11.0

Managers: F. L. Alt, S. Peavy, R. Herbold

Objective: To investigate whether shapes of black and white diagrams, such as printed letters, can be described by their moments or related mathematical characteristics with a precision and simplicity adequate for their practical identification by digital computers.

Background: Problems of this kind arise, for example, in attempts to build machines for automatic print reading.

Status: New.

1102-40-11645/60-480 LARGE SIGNAL CALCULATIONS FOR A VOLTAGE TUNEABLE MAGNETRON

Origin and Sponsor: Diamond Ordnance Fuze Laboratories, Department of the Army

Manager: A. Beam

Objective: To solve the following set of equations simultaneously:

$$(1)\quad \ddot{R} - R\dot{\theta}\left[\,\dot{\theta} + A\,\right] = B\left[\frac{1}{R} - C\sin\left\{\frac{N}{2}(\theta - T)\right\}\right]$$

$$(2)\quad \ddot{\theta} + \frac{\dot{R}}{R}\left[\,2\dot{\theta} - A\,\right] = -D\cos\left\{\frac{N}{2}(\theta - T)\right\}$$

The initial conditions given for T=0 are:

$$R(T)=1;\ \dot{R}(T)=0;\ \theta(T)=i\frac{4\pi}{PN},\ i=0,1,\ldots,P-1;\ \dot{\theta}(T)=\alpha$$

A,B,C,D are functions of R and input parameters.
N = 8,12,or 16.

For each set of input parameters, the equations are to be solved P times, and the efficiency is to be computed.

Background: The purpose of this problem is to calculate the efficiency, as well as other parameters, of a so-called voltage tuneable magnetron, neglecting the space charge of the electron beam. Calculations will be made primarily for conditions in which the strength of the RF electric fields are not small compared with the DC electric fields; under this set of conditions one may not use perturbation techniques, but is faced with the problem of solving simultaneously a pair of differential equations for the motion of a given electron. Time is the independent variable; the equations are both nonlinear, and possess time-varying coefficients. It is necessary to solve the equations several times, say 16, for a given set of physical parameters, in order to determine the behavior of an "average" electron; each of the electrons of the set of 16 will have slightly different initial conditions.

The problem is to be run for a number of choices for each of the parameters. As examples of these parameters, one has the relative strengths of DC and RF electric fields, number of segments in the RF anode structure, the shape of the unperturbed orbits, the relative values of the cathode radius, anode radius, and unperturbed beam radius, if the unperturbed beam is circular, or the initial beam radius if the unperturbed beam is not circular. These parameters are to be varied in a systematic way so as to permit a search for maximum efficiency conditions, and the numerical values of these efficiencies. Other parameters of the motion will also be computed, in the hope of obtaining clues to still better tube design.

The problem was submitted by R. T. Young and W. E. Waters (DOFL).

Status: New.

6. STATISTICAL ENGINEERING SERVICES

COLLABORATION ON STATISTICAL ASPECTS OF NBS RESEARCH AND TESTING
Task 3911-61-39951/51-1

Origin: NBS Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

Status: CONTINUED. During this quarter members of the Section provided statistical assistance and advice to a number of Bureau personnel.

(1) Precision of measurements relating to new determination of the faraday. Statistical analyses of measurements on electrolytic deposits of silver and of mass-spectrometer measurements of the isotopic composition of silver were carried out by J. M. Cameron in connection with the work of D. N. Craig (Electrochemistry Section) and W. R. Shields (Mass Spectroscopy Section) on re-determination of the faraday and the atomic weight of silver.

(2) Polymer crystallization. A method of determining the probabilities arising in one form of model of nucleation of polymer crystals was worked out by J. R. Rosenblatt for F. Gornick (Polymer Structure Section).

(3) Life tests of cement. A ranking procedure for summarization of data from factorial life-testing experiments involving the freezing and thawing of cement with various kinds of embedded aggregates, developed by H. T. Arni of the Concreting Materials Section, was reviewed by J. R. Rosenblatt.

(4) Computations were carried out on the Bureau's eIectronic computer for J. S. Beers (Length Section), M. J. Kerper (Glass Section), F. M. Reinhart (Corrosion Section), E. D. Tidwell (Radiometry), A. G. Strang (Engineering Metrology), F. P. Knudsen (Engineering Ceramics).

Publications:

(1) Some canons of sound experimentation. C. Eisenhart. To appear in the "Proceedings of the 31st Session of the International Statistical Institute," Brussels, 1958.
(2) Statistical aspects of the cement testing program. W. J. Youden. To appear in the Proceedings of the American Society for Testing Materials.
(3) Statistics--engineering viewpoint. W. J. Youden. To appear in the Journal of Engineering Education.
(4) Variability of color-mixture data. I. Nimeroff (NBS Photometry and Colorimetry Section), J. Rosenblatt and M. C. Dannemiller. In manuscript.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-11430/52-1

Origin and Sponsor: Ship Structure Committee, NRC Authorized 12/1/51
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

Status: INACTIVE.

MANUAL ON EXPERIMENTAL STATISTICS FOR ORDNANCE ENGINEERS
Task 1103-40-11433/55-93

Origin and Sponsor: Office of Ordnance Research Authorized 12/29/54
Manager: C. Eisenhart
Full task description: Oct-Dec 1954 issue, p. 28

Status: REACTIVATED. Requisite mathematical tables and illustrative numerical examples are being prepared for inclusion in the manual. Cooperation with the John I. Thompson Company, contractor to OOR, on details of preparation of the manuscript for final publication is under way.

STATISTICAL SERVICES
Task 1103-40-11625/58-346

Origin and Sponsors: Various Agencies Authorized 3/31/58
Manager: J. M. Cameron
Full task description: Jan-Mar 1958 issue, p. 45

Status: CONTINUED. Work was done during the quarter for the following agencies:

(1) Veterans' Administration Hospital, Perry Point, Md.: A statistical analysis of data from NP research studies was carried out on the Bureau's electronic computer by M. C. Dannemiller under the direction of J. M. Cameron.

(2) U. S. Geological Survey: The investigations were continued on several mathematical models associated with measurement processes in hydrology.

(3) Bureau of Public Roads: See task 1102-40-11645/58-316 (p. 20) for write-up of this task done jointly with the Computation Laboratory.

APPLICATION OF AUTOMATIC COMPUTER

The record of the use of the IBM 704 for the period October 1 through December 31 is as follows:

Task No.		Title	Assembly	Checking	Production
			(M I N U T E S)		
NBS:					
11110/55-55	11.1	Research in numerical analysis	18	40	325
11411/55-56	11.1	Research in mathematical topics applicable to numerical analysis	34	169	454
11413/60-469	11.1	Orthogonal functions in the theory of partial differential equations		82	
11120/55-65	11.2	Automatic coding	51	350	358
39951/56-160	11.2	Mathematical subroutines	38	201	7
39951/51-1	11.3	Statistical engineering	18	2	318
39952/54-30	4.1	Spectrum analysis	5	159	431
39952/55-68	9.7	Crystal structure calculations	132	150	1268
39952/55-82	3.1	Thermometer calibrations	1		223
11645/55-97	3.8	High temperature properties for air°	66	50	
39952/56-131	2.2	Calculations in optics*		5	
11645/56-171	3.2	Collision integrals used in transport theory**	52	84	575
11645/57-219	3.2	Thermal properties*		8	183
39952/57-223	3.2	Self-consistent fields			43
11645/57-246	4.8	Radiation diffusion**		24	33
39952/57-250	2.3	Automatic reduction in spectrophotometric data*			106
11645/57-252	4.11	Detecting efficiency in a neutral meson experiment**		101	
39952/58-254	2.3	Reproduction of color- and spectral-energy distribution of daylight*	2		16
11645/58-255	4.8	Chi functions**		5	54
/58-256	10.6	Composite walls**	25	13	436
/58-270	12.5	Mathematical problems related to postal operations			11

Task No.		Title	Code Assembly	Code Checking	Code Production
			(M I N U T E S)		
11645/58-272	3.2	Thermodynamic properties of real gases	7	61	20
/58-274	9.7	Calculations for d-spacings II		10	128
/58-275	7.8	Crystallography**	13	8	121
/58-304	3.2	Transport properties of air	45	44	238
/58-314	3.2	Approximations for gas mixtures*	4	11	178
/58-339	3.4	Viscoelasticity properties of materials	50	42	92
/58-357	3.3	Eigenvalues		40	7
/58-358	3.2	Reduced cross-sections	6	14	
/58-360	5.2	Diffusion coefficients*			6
/59-377	12.3	Logical diagram reduction	15	13	484
/59-387	30.4	Nuclear reactor design**			118
/59-388	10.3	Heat pump calculations			12
/59-394	4.6	Slow electron scattering by hydrogen atoms		179	1368
/59-395	7.7	Adsorption study**	4	61	8
/59-403	2.1	Computation of color fadings*	8	24	20
/59-409	12.5	Bank Board**	30	280	916
/59-417	2.4	Spectrum analysis of ruthenium**			74
/59-418	4.8	P-Wave equation*	36	240	89
/59-421	12.5	Traffic assimilation**		28	17
/59-433	2.3	Color of signals*			44
/59-440	82.10	Mapping	114	285	490
/59-446	85.10	Ionospheric data**		176	14
/60-449	2.1	Spectral line colorimetry*			6
/60-457	12.5	Public Housing problem**	64	266	626
/60-459	9.1	Transcendental equations	4	58	42
/60-471	7.3	Interlaboratory standardization of testing procedures		104	5
/60-474	2.5	Gage block stability*			39
/60-475	82.40	Ionospheric soundings	12	167	1
11122/60-479	11.0	Processing of diagrams		5	
		Miscellaneous	3	9	364
		Totals (NBS Services)...	857	3,568	10,368
OUTSIDE:					
11414/56-148	NIH	Differential equations for nerve excitation			45
11625/58-346	VA	Covariance analysis		75	
11645/53-45	SC	Air defense tactics°	27	61	258
/58-269	NRL	Molecular structure, IV	18	30	1334
/58-276	NOL	General kinetics, I**			5522
/58-278	NOL	Polaris**		2	146

Task No.		Title	Assembly	Code Checking	Production
				(M I N U T E S)	
11645/58-319	HPBA	Auto tag°		38	73
/58-335	DOFL	Roots of Bessel functions**		21	206
/58-340	DOFL	M5-17 Fuze Data°		27	50
/58-347	BURR	Computations for war games**		260	1512
/58-348	OOR	Russian-to-English machine translation	8		35
/58-361	NRL	Spectrum of dipole radiation	7	7	177
/58-368	SC	Intensity functions of light scattered by spherical particles	22	28	537
/58-370	NRL	Neutron diffusion study°			165
/59-371	NRL	ASWAP°	37	33	440
/59-407	DOFL	Fourier coefficients			89
/59-408	NASA	NASA**	884	688	28945
/59-411	HEW	Fitting of exponential curves**			341
/59-415	DOFL	Complex Legendre functions		74	439
/59-416	DOFL	Analysis of power supply experiments**		51	72
/59-419	DOFL	Neutrons°	1		2904
/59-423	WB	Weather Bureau**	25	417	24989
/59-424	NOL	Systems engineering**	29		66
/59-435	VA	Electrocardiographic analysis	9	63	77
/59-437	GE	GE Highway studies**	5		721
/59-441	GK	Systems engineering**	59	931	314
/59-445	NPSA	Oil supply			473
/59-447	BPRO	Public Roads study**			2324
/60-450	ACC	Chemical warfare		5	
/60-453	AMS	Data conversion			174
/60-454	GE	G.E.**			535
/60-458	CAB	Domestic airline traffic survey		56	6
/60-460	DOFL	Design equation for electron guns	19	17	1326
/60-462	DOFL	Correlation of functions			184
/60-464	PHS	Correlation matrix for PHS data			288
/60-465	CU	Calculations in molecular quantum mechanics			30
/60-468	NASA	NASA-Langley			9636
/60-472	DOFL	Probabilistic encounter analysis		86	13
/60-473	NSA	Operations research**			99

Task No.		Title	Code Assembly	Code Checking	Code Production
			(M I N U T E S)		
11645/60-476	DOFL	Gas tube characteristic II		229	192
/60-480	DOFL	Large signal calculations for a voltage-tuneable magnetron			220
		Totals (Outside)	1,150	3,199	84,957
		Total time for the quarter (MINUTES).....	2,007	6,767	95,325
		Total time for the quarter (HOURS)	33.9	112.8	1,588.8

* Problem programmed in the Computation Laboratory; production runs continued under direction of sponsor.
** Problem programmed by sponsor and run under his direction.
° Classified task.

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

NBS Scientific Staff Meeting

BOSE, R. C. (Case Institute of Technology) On some connections between design of experiments and information theory. November 6.

Applied Mathematics Division Colloquium

SHISHA, O. (Harvard University) Infrapolynomials with prescribed polynomials. November 5.

Mathematical Statistics Seminar

JENKINS, G. M. (Stanford University) Robustness of tests for dispersion based on the mean deviation. November 4.

Papers and Invited Talks Presented by Members of the Staff at Meetings of Outside Organizations

ALT, F. L. Problem analysis for digital computers. Presented at a Seminar on Computer Techniques of Interest and Usefulness in Medicine, held at the University of Pennsylvania, October 14.

BAZLEY, N. Lower bounds for eigenvalues. Presented at the Brookhaven National Laboratory, Upton, N. Y., November 13.

EISENHART, C. Statistical engineering. Presented as one of the Frontiers of Science Lectures sponsored by the Joint Board of Science Education for the Greater Washington Area, held at the Carnegie Institution of Washington, D. C., October 10.

GOLDMAN, A. J. Elementary combinatorial problems related to a mail sorting device. Presented at a meeting of the Mathematical Association of America, Washington, D. C., December 5.

RHODES, I. The wonderful world of computers. Presented before the Washington Chapter of the Association for Computing Machinery, Washington, D. C., December 17.

RICE, John R. The analog of Haar's problem for the existence and characterization of best Tchebycheff approximations. Presented at a meeting of the American Mathematical Society, Winston-Salem, N. C., November 20-21.

ROSENBLATT, J. R. (1) Models for the analysis of component aging experiments. Presented at a Conference on Statistics and Mathematics in Reliability Problems, sponsored by the American Statistical Association and the American Society for Quality Control, New York, N. Y., October 16. (2) Probability and Statistics. Presented at a Scientific Staff Meeting of the Diamond Ordnance Fuze Laboratories, Washington, D. C., November 24.

TCHEN, C. M. (1) Kinetic equation for a plasma. Presented before the Mathematics Department of the Queen Mary College, University of London, November 11. (2) The Fokker-Planck equation for a plasma. Presented at the National Physical Laboratory, Teddington, England, December 16.

WEGSTEIN, J. H. Small problems on a large computer. Presented at the Diamond Ordnance Fuze Laboratories, Washington, D. C., October 27.

YOUDEN, W. J. (1) Graphical evaluation of analytical results. Presented at a Conference on Chemical Control Problems, sponsored by the National Plant Food Institute, Washington, D. C., October 15. (2) Problems of experimental design. Presented at a meeting of the Institute of Mathematical Statistics, December 30, Washington, D. C.

ZELEN, M. (1) The analysis of life tests at combined environmental conditions. Presented before a meeting of the American Society for Quality Control and the American Statistical Association, New York, N. Y., October 17. (2) Life testing. Presented at a Course in Operations Research, General Electric Co., New York, N. Y., October 23. (3) Participation in panel discussion on statistical reliability, sponsored by American Society for Quality Control at Princeton University, Princeton, N. J., December 4. (4) Factorial experiments in life testing. Presented before the Department of Industrial Engineering, Cornell University, Ithaca, N. Y., December 8. (5) Factorial experiments in life testing. Presented before the Mathematics Department of Case Institute of Technology, Cleveland, Ohio, December 14.

Publication Activities

1. PUBLICATIONS THAT APPEARED DURING THE QUARTER

1.2 Technical Notes, Manuals, and Bibliographies

(1) Basic theorems in matrix theory. M. Marcus. NBS Applied Mathematics Series 57. Available from U. S. Government Printing Office, Washington 25, D. C., 15 cents.

(2) Distribution of mail by destination at the San Francisco, Los Angeles, and Baltimore Post Offices. N. C. Severo and A. E. Newman. NBS Technical Note 27. Issued December 1959; price ¢ 1.50. Available from U. S. Department of Commerce, Office of Technical Services, Washington 25, D. C.

1.3 Technical Papers

The following papers appeared in J. Research NBS 63B, October-December 1959:

(1) Applications of a theorem on partitioned matrices. E. V. Haynsworth. Pp. 73-78.

(2) Capacity requirement of a mail sorting device: II. A. J. Goldman. Pp. 79-82.

(3) Analytic comparison of suggested configurations for automatic mail sorting equipment. B. K. Bender and A. J. Goldman. Pp. 83-104.

(4) New method of solution for unretarded satellite orbits. J. P. Vinti. Pp. 105-116.

(5) Effect of sudden water release on the reservoir free outflow hydrograph. V. M. Yevdjevich. Pp. 117-129.

(6) Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. F. W. J. Olver. Pp. 131-169.

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(7) The construction of Hadamard matrices. E. C. Dade and K. Goldberg. Mich. J. Math. 6, 247-250 (1959).

(8) Leonhard Euler's integral: A historical profile of the Gamma function. (In memoriam: Milton Abramowitz). P. J. Davis. Amer. Math. Month. 66, 849-869 (1959).

(9) Bending and stretching of corrugated diaphragms. R. F. Dressler. ASME Trans. 81D (J. Basic Eng.), 651-659 (1959).

(10) Modular forms whose coefficients possess multiplicative properties. M. Newman. Ann. Math. 70, 478-489 (1959).

(11) Linear differential equations of the second order with a large parameter. F. W. J. Olver. J. Soc. Indust. Appl. Math. 7, 306-310 (1959).

(12) On the convergence of Gauss' alternating procedure in the method of least squares, I. A. Ostrowski. Ann. Mat. Pura Appl. (IV), 48, 229-236 (1959).

(13) On the convergence of the Rayleigh quotient iteration for the computation of characteristic roots and vectors: V. Usual Rayleigh quotient for non-Hermitian matrices and linear elementary divisors; VI. Usual Rayleigh quotient for nonlinear elementary divisors. Arch. Rat. Mech. Anal. 3, 472-481 (1959); 4, 153-165 (1959). A. Ostrowski.

(14) Uber genaherte Determinanten und bedingte Tragheitsindizes. A. M. Ostrowski. Monat. Math. u. Phys. 64, 51-63 (1960).

(15) Evaluation of chemical analyses on two rocks. W. J. Youden. Technometrics 1, 409-417 (1959).

(16) The weighted compounding of two probabilities from independent significance tests. M. Zelen and L. S. Joel. Ann. Math. Stat. 30, 885-895 (1959).

1.4 Reviews and Notes

(1) A note on algebras. A. J. Goldman. Amer. Math. Month. (Math. Notes) 66, 795-796 (1959).

(2) Review of "Cours de Geometrie Differentielle Locale" by J. Favard. A. J. Goldman. Scripta Math. 24, 234-235 (1959).

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION DECEMBER 31, 1959

2.2 Technical Notes, Manuals, and Bibliographies

(1) Handbook of Mathematical Functions. To appear in the NBS Applied Mathematics Series.

(2) Selected bibliography of statistical literature, 1930-1957. I. Correlation and regression theory. II. Time series. L. S. Deming. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

2.3 Technical Papers

(1) Computer simplification of Boolean functions. B. K. Bender, A. J. Goldman, and R. B. Thomas (Data Processing Systems). Submitted to a technical journal.

(2) Analysis of fractionally replicated 2^m3^n designs. R. C. Bose and W. S. Connor. To appear in "Proceedings of the 31st Session of the International Statistical Institute", Brussels, 1958.

(3) Construction of fractional factorial designs of the mixed 2^m3^n series. W. S. Connor. To appear in "Contributions to probability and statistics, " in press, Stanford University Press (1960).

(4) The robustness of certain life testing procedures derived from the exponential distribution. M. C. Dannemiller. Submitted to a technical journal.

(5) Some SEAC computations of subsonic fluid flows by Bergman's method of integral operators. P. Davis and P. Rabinowitz. To appear as an Appendix in the book, "Bergman's Operator Method," by M. S. v. Krzywoblocki.

(6) Some canons of sound experimentation. C. Eisenhart. To appear in the "Proceedings of the 31st Session of the International Statistical Institute," Brussels, 1958.

(7) Generating functions for formal power series in noncommuting variables. K. Goldberg. To appear in the Proceedings of the American Mathematical Society.

(8) Note on a paper by S. Mukhoda and S. Sawaki. K. Goldberg. Submitted to a technical journal.

(9) The incidence equation $AA^T = aA$. K. Goldberg. To appear in the American Mathematical Monthly.

(10) The minima of cyclic sums. K. Goldberg. To appear in the Journal of the London Mathematical Society.

(11) The minimum of a certain linear form. K. Goldberg. To appear in the Journal of Research, NBS, Section B. Mathematics and Mathematical Physics.

(12) A continuous poker game. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Duke Mathematical Journal.

(13) A symmetric poker game. A. J. Goldman and J. J. Stone (Stanford University). To appear in the Journal of Research of the NBS. Section B. Mathematics and Mathematical Physics.

(14) On non-self-adjoint boundary value problems in ordinary differential equations. W. Greub and W. Rheinboldt. To appear in the Journal of Research of the NBS, Section B. Mathematics and Mathematical Physics.

(15) Bounds for determinants with positive diagonals. E. V. Haynsworth. Submitted to a technical journal.

(16) Reduction formulas for partitioned matrices. E. V. Haynsworth. Submitted to a technical journal.

(17) Regions containing the characteristic roots of a matrix. E. V. Haynsworth. Submitted to a technical journal.

(18) The relation between confidence intervals and tests of significance --a teaching aid. M. G. Natrella. To appear in the American Statistician.

(19) Kantorovich's inequality. M. Newman. To appear in the Journal of Research of the NBS, Section B. Mathematics and Mathematical Physics.

(20) Subgroups of the modular group and sums of squares. M. Newman. To appear in the American Journal of Mathematics.

(21) Weighted restricted partitions. M. Newman. To appear in Acta Arithmetica.

(22) Stokes flow problem for a class of axially symmetric bodies. L. E. Payne and W. H. Pell. To appear in the Journal of Fluid Dynamics.

(23) The Stokes flow problem for a class of axially symmetric bodies, II. The flow about a spindle. L. E. Payne (University of Maryland) and W. H. Pell. To appear in the Quarterly of Applied Mathematics.

(24) A new approach to the mechanical syntactic analysis of Russian. I. Rhodes. Submitted to a technical journal.

(25) A new representation of Gengenbauer's functions. J. R. Rice. Submitted to a technical journal.

(26) Criteria for the existence and equioscillation of best Tchebycheff approximations. J. R. Rice. To appear in the Journal of Research, NBS, Sec. B. Mathematics and Mathematical Physics.

(27) Tchebycheff approximations by $ab^{x}+c$ and $ab^{x}\cos(\theta_{o}+\theta x) + c$. J. R. Rice. Submitted to a technical journal.

(28) Tchebycheff approximations by functions unisolvent of variable degree. J. R. Rice. Submitted to a technical journal.

(29) On the power of some rank order two-sample tests. J. R. Rosenblatt. To appear in "Contributions to probability and statistics", in press, Stanford University Press (1960).

(30) A statistical chain ratio method for estimating relative volumes of mail to given destinations. N. C. Severo and A. E. Newman (12.3). To appear in the Journal of Research of the NBS, Section C.

(31) The functional synthesis of linear plots. J. P. Vinti and R. F. Dressler. Submitted to a technical journal.

(32) Graphical evaluation of analytical results. W. J. Youden. To appear in the "Proceedings of a Conference on Chemical Control Problems," sponsored by the National Plant Food Institute, Washington, D. C., October 1959.

(33) Multivariable experimentation. W. J. Youden. To appear in the Transactions of the Society of Automotive Engineers.

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U.S. DEPARTMENT OF COMMERCE
Frederick H. Mueller, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its major laboratories in Washington, D.C., and Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant publications, appears on the inside of the front cover.

WASHINGTON, D.C.

Electricity and Electronics. Resistance and Reactance. Electron Devices. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Photographic Technology. Length. Engineering Metrology.

Heat. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology. Molecular Kinetics. Free Radicals Research.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Neutron Physics. Radiation Theory. Radioactivity. X-rays. High Energy Radiation. Nucleonic Instrumentation. Radiological Equipment.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Molecular Structure and Properties of Gases. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Air Conditioning, Heating, and Refrigeration. Floor, Roof, and Wall Coverings. Codes and Safety Standards. Heat Transfer. Concreting Materials.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analog Systems. Application Engineering.

- Office of Basic Instrumentation.
- Office of Weights and Measures.

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships. VHF Research. Radio Warning Services. Airglow and Aurora. Radio Astronomy and Arctic Propagation.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Research. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Propagation Obstacles Engineering. Radio-Meteorology. Lower Atmosphere Physics.

Radio Standards. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Electronic Calibration Center. Microwave Physics. Microwave Circuit Standards.

Radio Communication and Systems. Low Frequency and Very Low Frequency Research. High Frequency and Very High Frequency Research. Ultra High Frequency and Super High Frequency Research. Modulation Research. Antenna Research. Navigation Systems. Systems Analysis. Field Operations.


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